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THE DESIGN AND EVALUATION OF

"SHORT-SIGHTED" STOCHASTIC OPTIMAL CONTROLLERS

by

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ABSTRACT

The quality of control provided by what is termed "short-sighted" stochastic optimal controllers controlling a linear heating system subjected to a random disturbance is evaluated by computer simulation and by experimentation.

The control laws evaluated respectively minimize the cost functions

$$I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2$$

(Control law 3)

and

$$I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2$$

(Control law 2)

and are called "short-sighted" since they do not take into account the effect that the present control action u_t will have on future outputs at lead times greater than the process time delay k.

Since control law 2 does not allow the mean value of control input u to drift, it was expected that it would be unable to control the system subjected to a non-stationary disturbance. Since control law 3 allows the mean value of control input u to drift, it was expected that it would be more appropriate to use in this case.

It was found that control law 3 provides a better quality of control than control law 2 when the disturbance is non-stationary and modelled as such <u>provided</u> that the process and disturbance models are very accurate. A stability analysis, simulation and experiment show that model inaccuracies result in a serious degradation of control quality. It was also found that control law 2 is inappropriate to use since the inability of the mean value of control input u to drift results in large mean errors.

When the disturbance acting on the system is stationary and modelled as such, minimizing I_2 results in a marginally better quality of control than obtained by minimizing I_3. However, a process model inaccuracy may result in an output offset error since a controller based on the minimization of I_2 does not have integral action. Using a controller based on the minimization of I_3 is advantageous in this case since the integral action of this controller cancels the offset.

For the situation where the disturbance acting on the system at a given time is stationary but may become non-stationary after a period of time, using I_3 and a stationary disturbance model results in a controller which provides the best overall control quality for the system evaluated. Sensitivity to model inaccuracies is less of a problem in this case.

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CHAPTER 1

INTRODUCTION

Stochastic control theory finds application in the design of controllers of systems subjected to large random disturbances. This dissertation evaluates by computer simulation as well as by experimentation what has been termed [2] "short-sighted" stochastic optimal control laws controlling a linear system subjected to respectively a stationary and a non-stationary disturbance.

Chapter 2 provides a brief background to the subject and describes the application of stochastic control in industry and various stochastic control strategies, including the "short-sighted" control laws. The scope and objectives of this dissertation are defined in more detail in this chapter.

Chapter 3 contains the theoretical analyses used to derive the respective control laws and to obtain an insight into the sensitivity and stability aspects. The process and disturbance parameters are defined and the process and disturbance models are derived from first principles.

Chapter 4 describes the design of the controller including hardware and software aspects.

The results obtained by simulation are presented in Chapter 5. Extensive use is made of diagrams and tables to portray the results obtained. A summary of the results obtained is given at the end of the chapter.

The results obtained by experimentation are presented in Chapter 6. Here also extensive use is made of diagrams and tables to portray the results obtained and a summary of the results obtained is given at the end of the chapter.

In Chapter 7 the results obtained by experimentation are critically compared to the results obtained by simulation with reference to the theoretical analyses. This is followed by a presentation of the final conclusions. Finally, suggestions are made for further work.

CHAPTER 2

BACKGROUND

2.0 WHY STOCHASTIC CONTROL?

When the magnitude of a random disturbance acting on a system is such that the system output becomes unacceptable i.t.o quality or performance, due consideration has to be taken of the disturbance in the design of the controller.

Using deterministic control theory to design a controller which effectively counters the effect of such a disturbance acting on a system poses a problem since it requires that the disturbance be postulated as an analytic function which is known a priori. This is usually not possible.

Stochastic control theory provides the necessary tools for the design of a controller of a system subjected to a random disturbance. Firstly, the disturbance is modelled as a stochastic process, i.e. as a sequence of random variables. Secondly, the controller is designed to consist of two parts, namely:

a. an optimal predictor which predicts the effect of the disturbance on the system output minimizing the prediction error, and

b. a controller which computes the control signal to make the predicted output equal to the desired value minimizing a given cost function.

The ability of a stochastic controller to predict the probable future value of the system output based on historical system input and output data and to drive the system in such a way as to minimize the probable future output error minimizing a given cost function, makes it a strong candidate for solving this kind of problem.

In addition, realising the prediction and control algorythm by means of a recursive calculation, makes real-time prediction and control by means of a digital computer a practical proposition.

2.1 APPLICATION IN INDUSTRY

The conventional PID controller has proved itself useful in feedback control applications. However, using it to control a process subjected to a large random disturbance may require a long parameter tuning time with subsequent production losses. Also, the PID controller is not able to adapt to a change in the statistical characteristics of the disturbance as is the case when the disturbance is non-stationary.

In such cases, the design and implementation of a stochastic controller may be profitable. The fact is, however, that stochastic control theory has found little application in the solution of process control problems in industry thus far. The following may be possible reasons for this:

a. Control engineers prefer to use the proven 3-term controller rather than to experiment with a novel and sophisticated control algorythm.

b. A stochastic control law must be implemented on a freely programmable process control computer. Previous generation process control systems were configured with dedicated hardware function blocks and were not freely programmable.

Åstrom [4] describes a stochastic control system which was designed by the IBM Nordic Laboratory in Stockholm and installed and commissioned in the Billerud Kraft Paper Mill at Grüvon, Sweden, in 1965. The system controls paper basis weight which is an important quality variable of kraft paper. The system is subjected to stationary disturbances.

Before the stochastic controller replaced the previous controller, the basis weight fluctuations had a standard deviation of 1,3 g/m^2. After installation of the stochastic controller, consistent standard deviations of 0,5g/m^2 wet basis weight and 0,3g/m^2 dry basis weight were achieved, thus a significant improvement in control quality. The controller was designed to minimize the variance of the output.

The acceptance criterion of kraft paper is based on the requirement that the quality variable of a representative sample is within the test limits with a specified probability. In order to compensate for fluctuations in quality during production, the set point for the basis weight controllers was previously set well above the lower test limit. However, in significantly reducing the variance of the output, it was possible to move the set point closer to the acceptance limit without changing the probability of acceptance. In doing so, the raw material was used more economically and the production was increased resulting in a capital gain.

A more recent documented application is the viscosity control of an industrial polymerisation process as described by MacGregor et al [2] (1976). Viscosity control of polymers is of importance in the manufacture of condensation polymers.

An identification study performed identified the process model as non-minimum phase and the disturbance model as non-stationary. The presence of a non-stationary disturbance necessitated minimizing the following cost function which results in a sophistication of the standard minimum variance control law used in the application discussed previously:

$$I = E\{y^2_{t+k} + \lambda(u_t - u_{t-1})^2\} \qquad (2.1.1)$$

where E is the expectation operand,

y_t is the system output at sampling moment t,

u_t is the system input at sampling moment t,

k is the system time delay, and

λ is a control weighting.

The objective in using this cost function is explained in paragraph 2.2.

The implementation of the stochastic control law resulted in a substantial improvement in viscosity control.

In conclusion then can be said that stochastic controllers have been successfully implemented in industry although on a small scale. However, with the trend of distributed microprocessor-based process control systems becoming more common on process control equipment markets, the ability to implement stochastic control laws more easily as a result of the free programmability of these controllers, may result in a wider application of stochastic control theory.

2.2 VARIOUS STOCHASTIC CONTROL STRATEGIES

The standard stochastic control law is the so-called minimum variance control law which minimizes the cost function.

$$I = E\{y_t^2\} \qquad (2.2.1)$$

This control law minimizes the variance of the system output and allows the system to be operated closer to the set point as explained in paragraph 2.1 with reference to the design by the IBM Nordic Laboratory.

However, as Clarke et al [1] point out, the minimum variance design has the following practical difficulties:

a. As such it cannot stably control a nonminimum - phase system (however, by modifying it to become a suboptimal control law, the problem may be avoided).

b. Since no constraint is placed on the variance of the control input, saturation of the control transducer may occur.

c. The controller does not have a tunable parameter which can be tuned if the controlled process has an unsatisfactory performance.

Clarke et al [1] proposed in 1971 a design to avoid the above difficulties. They proposed minimizing the cost function

$$I = E\{y^2_{t+k} + \lambda u_t^2\} \qquad (2.2.2)$$

where λ is an adjustable weighting factor which places a cost on the variance of control input u. They derived control laws for actual systems minimizing what they thought was the above cost function and obtained the following results:

a. No stability problems occurred in controlling nonminimum-phase systems.

b. By placing a weight on the variance of the control input, a significant reduction in the variance of the control input with only a small increase in the variance of the system output could be obtained.

c. The design had the flexibility to deal with cases where the closed - loop performance became unsatisfactory because of parameter drift or other causes.

However, in 1977 MacGregor et al [2] showed that Clarke et al erroneously dropped the expectation operand in their manipulations and in fact minimized the related but different cost function

$$I = \hat{y}^2_{t+k/t} + \lambda u_t^2 \qquad (2.2.3)$$

where $\hat{y}_{t+k/t}$ is the optimal k-step ahead predicted value of the system output y as predicted at time t.

MacGregor et al stated that minimizing the above cost function would appear to be sensible in that it selects the control action u_t which, at each instant of time t, attempts to drive the k-step-ahead forecast of the output deviation to zero, subject to a constraint on the magnitude of the present control action. They called it a "short-sighted" or "instantaneous action" stochastic optimal control action since it does not take into account the effect that the present control action u_t will have on future outputs at lead times greater than the process time delay k.

The cost function proposed by Clarke et al can be minimized using either of the following two methods [2]:

a. The discrete Wiener-Hopf equation which can be solved by a method of factorisation of discrete convariance generating functions. (This procedure is straightforward for single-input/single-output systems but becomes much more difficult for multivariable systems).

b. Transforming the discrete system transfer-function model into a state-variable form and minimizing the cost function by solving a Riccati equation.

MacGregor et al showed that minimizing the cost function proposed by Clarke et al does indeed provide a more optimal control than obtained by minimizing the cost function in equation (2.2.3). In the special case when the cost factor is zero, the two algorythms coincide.

In addition, MacGregor et al recognized that stochastic controllers are of greatest importance for controlling systems subjected to drifting or non-stationary disturbances. However, in order to deal with non-stationary disturbances, the control input must be allowed to drift i.e have "infinite" variance (limited of course between saturation boundaries). Controllers which respectively minimize the cost functions of equations (2.2.2) and (2.2.3) can therefore not stabilise the output of a system subjected to a non-stationary disturbance.

For this reason MacGregor et al proposed minimizing more appropriate cost functions such as

$$I = E\{y^2_{t+k} + \lambda(u_t - u_{t-1})^2\} \tag{2.2.4}$$

and

$$I = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \tag{2.2.5}$$

which allows the control input to drift.

The controllers have integral action as a result of a pole on the unit circle in the disturbance model and therefore also have the additional feature of being able to cancel offset due to set point changes or load step changes.

Cost function equation (2.2.4) is again minimized by the method of factorisation of a discrete Wiener-Hopf equation with slight modification or by solving an appropriate Riccati equation. This cost function was used by MacGregor et al to solve the industrial control problem described in paragraph 2.1.

Cost function equation (2.2.5) is easily used by applying the method of Clarke et al.

2.3 SCOPE OF THIS DISSERTATION

This dissertation evaluates by simulation and experimentation the so-called "short-sighted" or instantaneous action stochastic optimal control laws as described in paragraph 2.2. Three objectives were set as described in the following paragraphs.

The first objective was to evaluate the performance of a controller of a minimum-phase system subjected to a non-stationary disturbance which minimizes the cost function

$$I = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \qquad (2.3.1)$$

and to compare it to the performance of a controller which minimizes the cost function

$$I = \hat{y}^2_{t+k/t} + \lambda u_t^2 \qquad (2.3.2)$$

in order to determine whether using cost function equation (2.3.1) does provide a better quality of control than does using cost function equation (2.3.2) for non-stationary disturbances as stated by MacGregor et al.

The second objective was to evaluate which of the above two controllers provides the best quality of control if the disturbance is stationary and modelled as such.

The third objective was to evaluate for the situation in which it is not known a priori whether the disturbance is stationary or non-stationary, which combination of control law and disturbance model provides the best quality of control when the actual disturbance is stationary while the disturbance model is non-stationary and vice versa.

It was not within the scope of this dissertation to evaluate controllers based on the minimization of cost functions

$$I = E\{y^2_{t+k} + \lambda u_t^2\} \qquad (2.3.3)$$

and

$$I = E\{y^2_{t+k} + \lambda(u_t - u_{t-1})^2\} \qquad (2.3.4)$$

CHAPTER 3

THEORY

3.0 SYSTEM DEFINITION

A functional block diagram of the system is shown in Figure 3.0.1.

The sampled data temperature controller controls the power supply to the electric heating element of the hot water cylinder and attempts to maintain the measured temperature equal to the reference temperature.

The temperature is disturbed in a random manner by the disturbance generator which controls the outflow of warm water in a random manner. A level controller maintains a constant mass inventory in the cylinder by controlling the inflow of cold water.

The disturbance flow rate at any specific time is defined as the difference between the actual flow rate at the time and the nominal flow rate.

The values of the system parameters are specified in Table 3.0.1.

The mathematical models of the process and the disturbances are derived from first principles in paragraph 3.1.

In paragraph 3.2 the "instantaneous action" optimal control laws based on the minimization of the cost functions as described in paragraph 2.3 are derived. The minimum variance control law is derived as well but is shown to be a special case of one of the other two control laws. The description of the general procedure for the derivation of the control laws in paragraph 3.2.2 are followed by the derivation of the general forms of the control laws in paragraph 3.2.3 and the derivation of the specific forms of the control laws incorporating the process and disturbance models in paragraphs 3.2.4 and 3.2.5.

Table 3.0.1 Specifications of System Parameters

PARAMETER	SYMBOL	VALUE/RANGE
Specific heat capacity of water	H	4190 J/kg. K
Mass of water inventory	M	10 kg
Heater element power	Q	$0W < Q < 2700W$
Thermal resistance of cylinder wall	R	0,5 K/W
Measured temperature	T	$0°C < T < 100$ °C
Ambient temperature	T_e	18 °C
Cold Water temperature	T_i	16 °C
Reference temperature	T_{ref}	50 °C
Nominal disturbance flow rate	V	0,011 kg/s
Maximum disturbance flow rate	V_{max}	0,021 kg/s

In paragraph 3.3 the stability of the system subjected to respectively a stationary and a non-stationary disturbance using the respective control laws is analised mathematically.

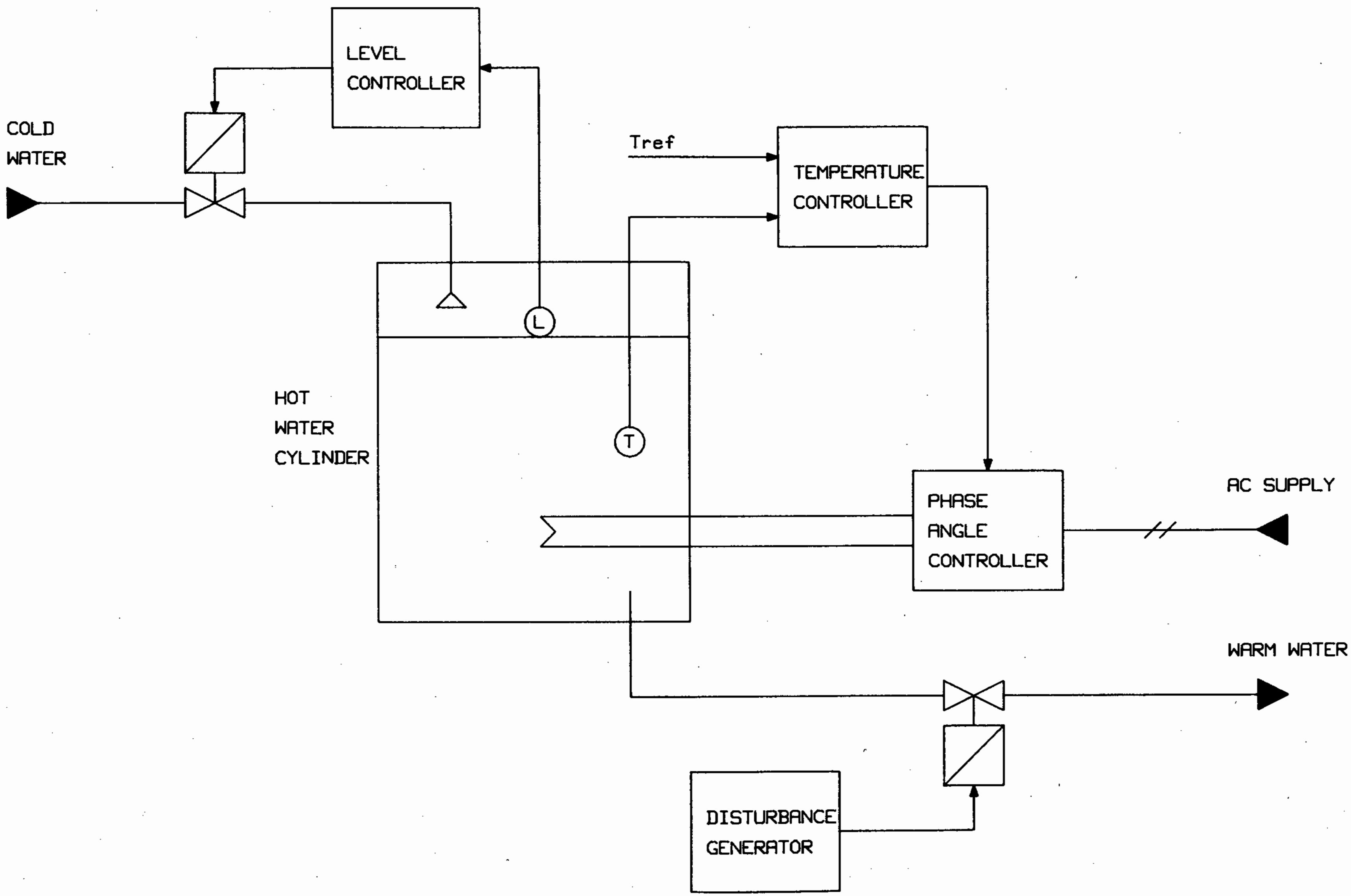


Figure 3.0.1 System Functional Block Diagram

3.1 PROCESS AND DISTURBANCE MODELS

The process and disturbance models are derived from first principles.

The derivation is based on the principle of conservation of energy:

$$Q = Q_c + Q_o - Q_i + Q_a \tag{3.1.1}$$

where:

Q = input heat from the heating element,

Q_c = heat stored in water mass,

Q_o = outflowing water heat loss,

Q_i = inflowing water heat input and

Q_a = heat loss to the ambient atmosphere

Writing equation (3.1.1) in differential form and using the symbols as defined in Table 3.0.1 gives:

$$Q = MH\frac{dT}{dt} + VHT - VHT_i + (T - T_e)/R \tag{3.1.2}$$

Writing equation (3.1.2) in an incremental form gives:

$$\Delta Q = MH\frac{d\Delta T}{dt} + (VH + 1/R)\Delta T + H(T - T_i)\Delta V - VH\Delta T_i - \Delta T_e/R \tag{3.1.3}$$

Taking the Laplace - transform of the equation and equating $\Delta T(o_+)$ to zero gives:

$$\Delta T(s) = \frac{1}{MHs + VH + 1/R} \cdot \left[\Delta Q(s) - H(T - T_i)\Delta V(s) + VH\Delta T_i(s) + \Delta T_e(s)/R\right] \tag{3.1.4}$$

with:

control input $\Delta Q(s)$,

output $\Delta T(s)$ and

disturbance inputs $\Delta V(s)$, $\Delta T_i(s)$ and $\Delta T_e(s)$.

For $\Delta V(s)$ a large forcing disturbance and $\Delta T_i(s)$ and $\Delta T_e(s)$ negligible, equation (3.1.4) approximates to:

$$\Delta T(s) = \frac{1}{MHS + VH + 1/R} \cdot \left[\Delta Q(s) - H(T - T_i) \Delta V(s) \right] \qquad (3.1.5)$$

Allowing for a dead time delay of kT_s seconds where:

k is a natural number and

T_s is the sampling period in seconds

gives:

$$\Delta T(s) = \frac{1}{MHS + VH + 1/R} \cdot \exp(-skT_s) \cdot \Delta Q(s) - \frac{H(T - T_i)}{MHS + VH + 1/R} \cdot \Delta V(s) \qquad (3.1.6)$$

Inserting a zero-order sample and hold circuit with sampling period T_s and transfer function

$[1-\exp(-sT_s)]/s$

at the input gives:

$$\Delta T(s) = \frac{1}{MHs + VH + 1/R} \cdot \frac{1-\exp(-sT_s)}{s} \cdot \exp(-skT_s) \cdot \Delta Q(s) - \frac{H(T-T_i)}{MHs + VH + 1/R} \cdot \Delta V(s) \quad (3.1.7)$$

Define in order to simplify the expression:

constant $K_1 = \dfrac{1}{VH + 1/R}$, (3.1.8)

constant $K_2 = \dfrac{T - T_i}{M}$, (3.1.9)

process time constant $T_p = \dfrac{MH}{VH + 1/R}$, (3.1.10)

disturbance time constant $T_n = \dfrac{MH}{VH + 1/R}$, (3.1.11)

input $u(t) = \Delta Q(t)$ (3.1.12)

where $\Delta Q(t)$ is the difference between the actual heat supplied by the heating element and the nominal heat required to maintain the temperature error equal to zero with the disturbance flow rate equal to the nominal value V,

output $y(t) = \Delta T(t)$ (3.1.13)

where $\Delta T(t)$ is the temperature error and

$$\xi(t) = K_2 \Delta V(t) \tag{3.1.14}$$

where:

$\xi(t)$ is a continuous stationary random variable with zero mean and
$\Delta V(t)$ is the difference between the actual disturbance flow rate at time t and the nominal disturbance flow rate V.

The relationship of the foregoing terms is shown in Figure 3.1.1 for the system subjected to a continuous stationary uncorrelated random disturbance with zero mean.

A non-stationary disturbance is generated by integrating the stationary random disturbance with zero mean with time. A proof is presented in Annexure 1 to Appendix A.

Figure 3.1.2 shows the continuous time presentation of the system subjected to a non-stationary disturbance.

The discrete time presentation of the system subjected to a stationary random disturbance with zero mean is:

$$y_t = \frac{K_1 [1 - \exp(-T_s/T_p)] z^{-(k+1)}}{1 - \exp(-T_s/T_p) z^{-1}} \cdot u_t + \frac{1}{1 - \exp(-T_s/T_n) z^{-1}} \cdot \xi_t \tag{3.1.15}$$

(The derivation of the z-transforms is shown in Annexure 2 to Appendix A).

with the symbols as defined before and:

discrete control input sequence $u_t = \Delta Q_t$,

discrete output sequence $y_t = \Delta T_t$ and

discrete stationary uncorrelated random sequence with zero mean

$$\xi_t = K_2 T_s \Delta V_t \tag{3.1.16}$$

The stationary disturbance sequence e_t which adds to the output is the filtered output of the disturbance model transfer function driven by the stationary ξ_t.

Figure 3.1.3 shows the discrete time presentation of the system subjected to a stationary disturbance.

The discrete time presentation of the system subjected to a non-stationary disturbance is:

$$y_t = \frac{K_1[1 - \exp(-T_s/T_p)] z^{-(k+1)}}{1 - \exp(-T_s/T_p) z^{-1}} \cdot u_t + \frac{1}{1 - z^{-1}} \cdot \frac{1}{1 - \exp(-T_s/T_n) z^{-1}} \cdot \xi_t \tag{3.1.17}$$

with the symbols as defined before.

The non-stationary disturbance sequence e_t which adds to the output is the filtered output of the disturbance model transfer function driven by the non-stationary

$$\frac{1}{1 - z^{-1}} \cdot \xi_t$$

Figure 3.1.4 shows the discrete time presentation of the system subjected to a non-stationary disturbance.

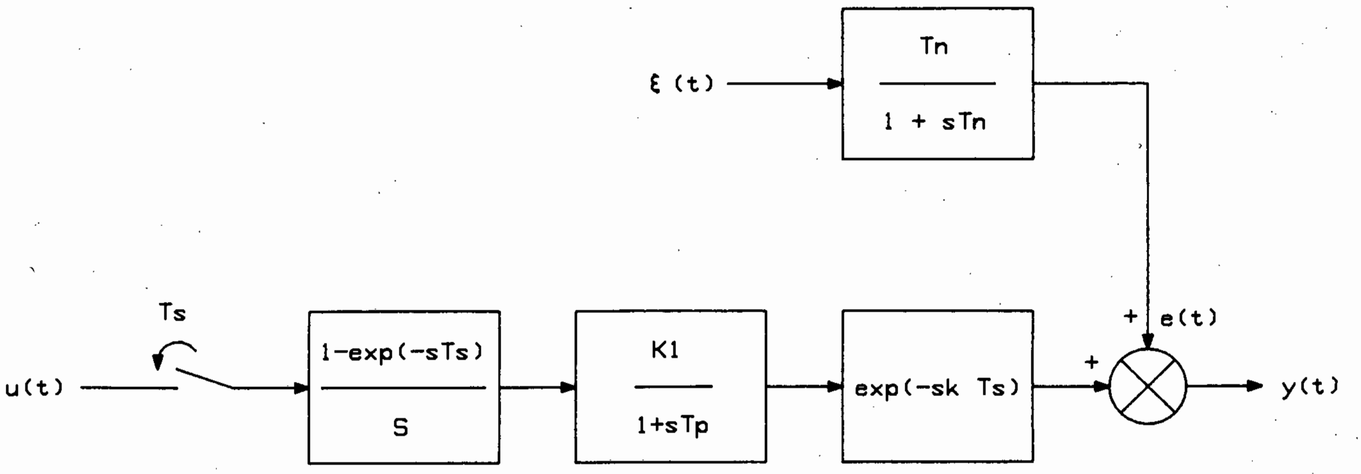


Figure 3.1.1 Continuous-time Presentation of the System Subjected to a Stationary Disturbance

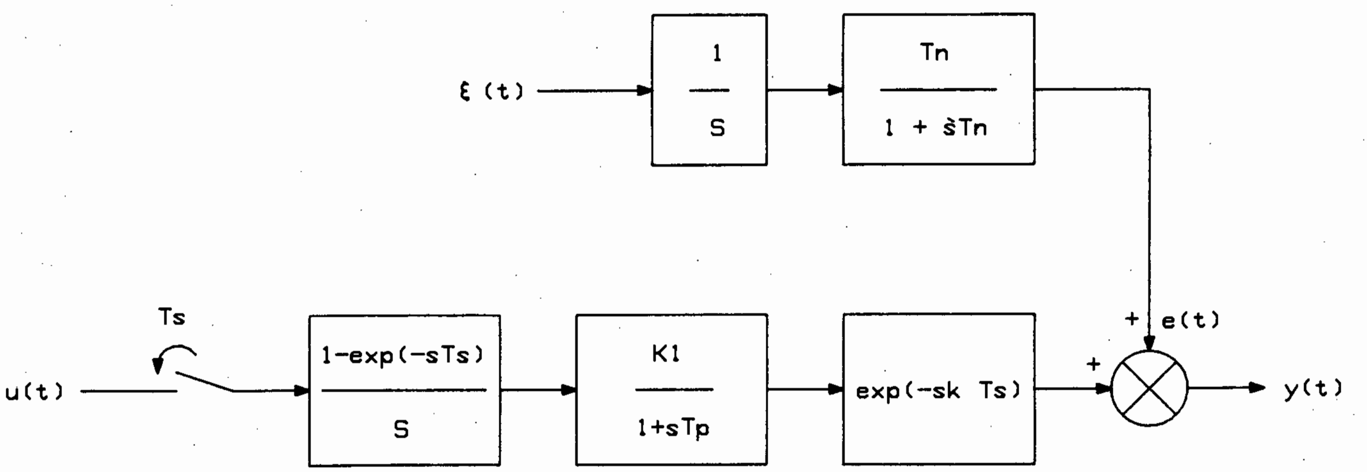


Figure 3.1.2 Continuous-time Presentation of the System Subjected to a Non-stationary Disturbance

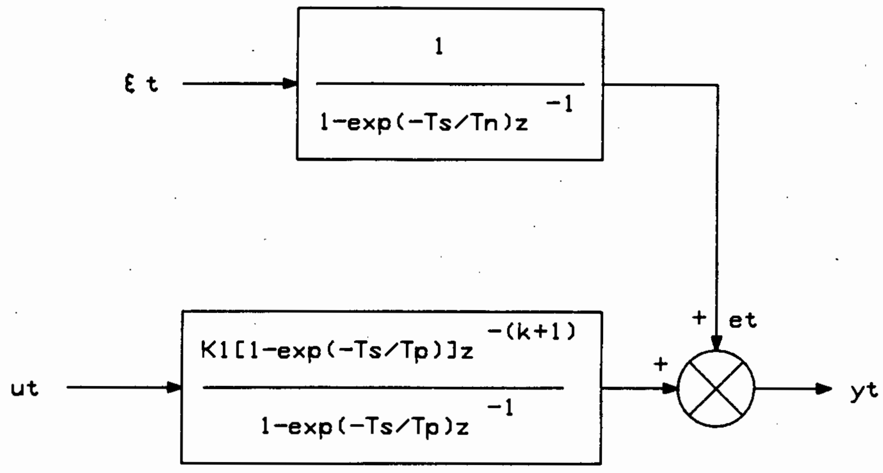


Figure 3.1.3 Discrete-time Presentation of the System Subjected to a Stationary Disturbance

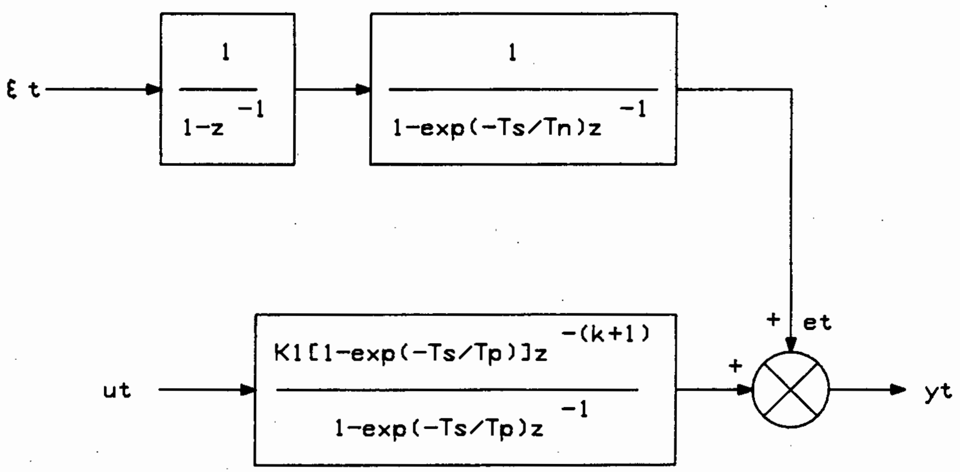


Figure 3.1.4 Discrete-time Presentation of the System Subjected to a Non-stationary Disturbance

3.2 DERIVATION OF CONTROL LAWS

3.2.1 Control Strategies

The three control strategies respectively implemented in the simulations and experiments are the following:

a. the minimization of the cost function

$$I_1 = E\{y_t^2\} \tag{3.2.1}$$

which is the standard minimum variance control strategy,

b. the minimization of the cost function

$$I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2 \tag{3.2.2}$$

as implemented by Clarke et al [1], [2],

where $\hat{y}_{t+k/t}$ is the optimal k-step ahead predictor of y at moment t and

c. the minimization of the cost function

$$I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \tag{3.2.3}$$

as suggested by MacGregor et al[2].

For both the system subjected to a stationary disturbance (equation (3.1.15)) and the system subjected to a non-stationary disturbance (equation (3.1.17)), three control laws which respectively minimize the above three cost functions are derived.

3.2.2 General Procedure for the Derivation of the Control Laws

The following general form of the model equation is used [1]:

$$y_t(z^{-1}) = z^{-k}G(z^{-1})u_t(z^{-1}) + N(z^{-1})\,\xi_t(z^{-1}) \qquad (3.2.4)$$

where:

G is the system polynomial in z^{-1} with a leading constant and

N is the disturbance polynomial in z^{-1} with a unity leading constant.

Rearranging equation (3.2.4) to obtain output y in the same form as that of the k-step ahead predictor of y at sampling moment t, $\hat{y}_{t+k/t}$, gives:

$$y_{t+k} = Gu_t + z^k N\,\xi_t \qquad (3.2.5)$$

Define: $$N = N_k + N_k^* \qquad (3.2.6)$$

where $$N_k^* = 1 + n_1 z^{-1} + \;....\; + n_{k-1} z^{-k+1} \qquad (3.2.7)$$

and $$N_k = n_k z^{-k} + n_{k+1} z^{-k-1} + \;.... \qquad (3.2.8)$$

Substituting into equation (3.2.5):

$$y_{t+k} = Gu_t + z^k N_k^*\,\xi_t + z^k N_k\,\xi_t \qquad (3.2.9)$$

For $\hat{y}_{t+k/t}$, the optimal k-step ahead predictor of y at sampling moment t, to be realizable, it must be based on present and past measurements of the output:

y_t, y_{t-1}, y_{t-2}, ...

only.

The predictor $\hat{y}$ is defined to be optimal if it minimizes the variance of the prediction error

$$E\{y_{t+k} - \hat{y}\}^2 .$$

Subtracting $\hat{y}$ on both sides of equation (3.2.9) gives:

$$y_{t+k} - \hat{y} = Gu_t + z^k N_k \xi_t \quad - \hat{y} + z^k N_k^* \xi_t$$

The variance of the prediction error is then given by:

$$E\{y_{t+k} - \hat{y}\}^2 = E\{Gu_t + z^k N_k \, _t - \hat{y}\}^2 + E\{z^k N_k^* \, _t\}^2 \tag{3.2.10}$$

The crossproduct terms drop away because of the independance of $z_k N_k^* \xi_t$ with all values of the output up to time t.

The variance of the prediction error is minimized to

$E\{z^k N_k^* \xi_t\}^2$ by equating

$$\hat{y} = \hat{y}_{t+k/t} = z^k N_k \xi_t + Gu_t \tag{3.2.11}$$

The control law which minimizes a given cost function is found by differentiating the respective cost function and equating it to zero.

To reduce the number of parameters in the control law derived, the following equivalent general form of the model equation is used:

$$y_t(z^{-1}) = \frac{z^{-k} B(z^{-1})}{1 + A(z^{-1})} \cdot u_t(z^{-1}) + \frac{1 + D(z^{-1})}{1 + C(z^{-1})} \cdot \xi_t \tag{3.2.12}$$

In Annexure 3 to Appendix A it is shown that the transfer function

$$H(z) = \frac{y(z)}{u(z)}$$

is realizable with:

A, C and D finite polynomials in z^{-1} with no leading constants and B a finite polynomial in z^{-1} with or without a leading constant.

Using, as Clarke at al [1] does, the identity

$$1+D = N_k^*(1+C) + z^{-k}D^1 \qquad (3.2.13)$$

and obtaining N_k^* and D^1 by equating coefficients, the control law is obtained in the finite-parameter form.

3.2.3 General Forms of Derived Control Laws

The general derivation of the three control laws which respectively minimize the three cost functions is shown in Annexure 4 to Appendix A.

The general form of the control law which minimizes $I_1=E\{y_t^2\}$ is:

$$u_t = -\frac{(1+A)D'}{N_k^* B(1+C)} \cdot y_t \qquad (3.2.14)$$

The general form of the control law which minimizes $I_2=\hat{y}^2_{t+k/t}+\lambda u^2_t$ is:

$$u_t = -\frac{(1+A)D'}{N_k^* B(1+C) + \lambda/b_0 \ (1+A)(1+D)} \cdot y_t \qquad (3.2.15)$$

Note: Equation (3.2.14) is a special form of equation (3.2.15) with the cost factor λ equal to zero.

The general form of the control law which minimizes $I_3=\hat{y}^2_{t+k/t}+\lambda(u_t-u_{t-1})^2$ is:

$$u_t - u_{t-1} = -\frac{(1+A)(1-z^{-1})D'}{N_k^* B(1+C) + \lambda/b_0 \ (1+A)(1-z^{-1})(1+D)} \cdot y_t \qquad (3.2.16)$$

3.2.4 Specific Control Laws for System Subjected to a Stationary Disturbance

Define:

$$a = \exp(-T_s/T_p) \qquad (3.2.17)$$

and

$$c = \exp(-T_s/T_n) \qquad (3.2.18)$$

For the system time delay k equal to zero, the discrete time presentation of the system subjected to a stationary disturbance with zero mean (equation (3.1.15)) becomes:

$$y_t = \frac{K_1(1-a)z^{-1}}{1-az^{-1}} \cdot u_t + \frac{1}{1-cz^{-1}} \cdot \xi_t \qquad (3.2.19)$$

The specific derivation of the three control laws which respectively minimize the three cost functions for the system subjected to a stationary disturbance is shown in Annexure 5 to Appendix A.

The control law which minimizes $I_1 = E\{y_t^2\}$ is:

$$u_t = \frac{1}{K_1(1-a)} \cdot \left\{ K_1(1-a)cu_{t-1} - cy_t + acy_{t-1} \right\} \tag{3.2.20}$$

The control law which minimizes $I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2$ is:

$$u_t = \frac{1}{K_1(1-a) + \frac{\lambda}{K_1(1-a)}} \cdot \left\{ \left[K_1(1-a) + \frac{\lambda a}{K_1(1-a)} \right] . u_{t-1} - cy_t + acy_{t-1} \right\} \tag{3.2.21}$$

The control law which minimizes $I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2$ is:

$$u_t = \frac{1}{K_1(1-a) + \frac{\lambda}{K_1(1-a)}} \cdot \left\{ \left[K_1(1-a)c + \frac{\lambda(a+1)}{K_1(1-a)} + K_1(1-a) + \frac{\lambda}{K_1(1-a)} \right] . u_{t-1} \right.$$

$$- \left[\frac{\lambda a}{K_1(1-a)} + K_1(1-a)c + \frac{\lambda(a+1)}{K_1(1-a)} \right] u_{t-2}$$

$$\left. + \frac{\lambda a}{K_1(1-a)} . u_{t-3} - cy_t + (a+1)cy_t - acy_{t-2} \right\} \tag{3.2.22}$$

3.2.5 Specific Control Laws for System Subjected to a Non-stationary Disturbance.

Using the parameters defined previously, the discrete time presentation of the system subjected to a non-stationary disturbance (equation (3.1.17)) is written in the form:

$$y_t = \frac{K_1(1-a)z^{-1}}{1-az^{-1}} \cdot u_t + \frac{1}{1-z^{-1}} \cdot \frac{1}{1-cz^{-1}} \cdot \xi_t$$

(3.2.23)

The specific derivation of the three control laws which respectively minimize the three cost functions for the system subjected to a non-stationary disturbance is shown in Annexure 6 to Appendix A.

The control law which minimizes $I_1 = E\{y_t^2\}$ is:

$$u_t = \frac{1}{K_1(1-a)} \{ K_1(1-a)(1+c)\, u_{t-1} - K_1(1-a)\, c\, u_{t-2} - y_t + a y_{t-1} \}$$ (3.2.24)

The control law which minimizes $I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2$ is:

$$u_t = \frac{1}{K_1(1-a) + \dfrac{\lambda}{K_1(1-a)}} \cdot \{ [K_1(1-a)(1+c) + \frac{\lambda a}{K_1(1-a)}]\, u_{t-1}$$

$$- K_1(1-a)\, c\, u_{t-2} - y_t + a y_{t-1} \}$$ (3.2.25)

The control law which minimizes $I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2$ is:

$$u_t = \frac{1}{K_1(1-a) + \dfrac{\lambda}{K_1(1-a)}} \cdot \{ [K_1(1-a)(1+c) + \frac{\lambda(a+1)}{K_1(1-a)} + K_1(1-a) + \frac{\lambda}{K_1(1-a)}]\, u_{t-1}$$

$$- [K_1(1-a)c + \frac{\lambda a}{K_1(1-a)} + K_1(1-a)(1+c) + \frac{\lambda(a+1)}{K_1(1-a)}]\, u_{t-2}$$

$$+ [K_1(1-a)c + \frac{\lambda a}{K_1(1-a)}]\, u_{t-3} - y_t + (a+1)\, y_{t-1} - a y_{t-2} \}$$

(3.2.26)

3.3 SENSITIVITY AND STABILITY

Suppose the actual process is described by

$$y_t = \frac{z^{-k} B^{\circ}}{1 + A^{\circ}} \cdot u_t + \frac{1 + D^{\circ}}{1 + C^{\circ}} \cdot \xi_t \tag{3.3.1}$$

while the derived control laws are based on the model parameters A,B,C and D which are not exactly equal to the actual process parameters.

The questions which arise are the following:

a. Does the control law which minimize I_2 (being a generalization of the control law which minimizes I_1) ensure stability respectively for the system with a stationary disturbance model and the system with a non-stationary disturbance model?

b. Does the control law which minimize I_3 ensure stability respectively for the system with a stationary disturbance model and the system with a non-stationary disturbance model?

The stability in each case is determined by the eigenvalues of the characteristic equation P(z) where

$$y_t = \frac{Q(z)}{P(z)} \cdot \xi_t \tag{3.3.2}$$

is obtained by:

a. Substituting the applicable control law into the system equation.

b. Using, as before, the identity

$$z^{-k} D^{1} = 1+D - N_k^{*}(1+C) \tag{3.3.3}$$

c. Equating the actual process and model parameters and substituting the parameters for the specific system (stationary or non-stationary disturbance model) into the equation derived.

For small deviations of the model parameters, the modes associated with the respective characteristic equation derived may be excited. This is of importance if some of the modes are unstable or marginally stable, i.e if eigenvalues of the characteristic equation lie on or outside the unit circle.

The detailed derivation in each case is shown in Annexure 7 to Appendix A.

3.3.1 Stability using I_2 on System with Stationary Disturbance Model

The characteristic equation P(z) as derived in Annexure 7 to Appendix A is:

$$P(z) = (z-c)(z-a)\left(z - \frac{\lambda a}{\lambda + [K_1(1-a)]^2}\right) \qquad (3.3.4)$$

where, as defined before:

$$a = \exp(-T_s/T_p) \quad , \qquad (3.3.5)$$

$$c = \exp(-T_s/T_n) \quad , \qquad (3.3.6)$$

$$K_1 = \frac{1}{VH + 1/R} \quad , \qquad (3.3.7)$$

and $$T_p = T_n = \frac{MH}{VH + 1/R} \qquad (3.3.8)$$

Defining the sampling period T_s equal to 60 seconds and using the parameter specifications as in Table 3.0.1, the following values are obtained:

$$T_p = T_n = 871 \text{ s} \qquad (3.3.9)$$

$$K_1 = 0{,}0208 \text{ K/W} \qquad (3.3.10)$$

$$a = c = 0{,}933 \qquad (3.3.11)$$

Substituting these values into equation (3.3.4) gives:

$$P(z) = (z - 0{,}933)(z - 0{,}933)\left(z - \frac{0{,}933\,\lambda}{\lambda + 1{,}917e-6}\right) \qquad (3.3.12)$$

The zeros of this equation are:

$$z_1 = z_2 = 0{,}933 \qquad (3.3.13)$$

$$0 < z_3 < 0{,}933 \text{ for } \lambda > 0 \qquad (3.3.14)$$

Since all the poles are within the unit circle for all values of the cost function λ, the system should be stable even if small model deviations occur.

3.3.2 Stability using I_2 on System with Non-stationary Disturbance Model

The characteristic equation P(z) as derived in Annexure 7 to Appendix A is:

$$P(z) = (z^2 - (1+c)z + c)(z - a)\left(z - \frac{\lambda a}{\lambda + [K_1(1-a)]^2}\right) \qquad (3.3.15)$$

Substituting the values of the constants as defined in paragraph 3.3.1 into equation (3.3.15) gives:

$$P(z) = (z^2 - 1{,}933z + 0{,}933)(z - 0{,}933)\left(z - \frac{0{,}933\,\lambda}{\lambda + 1{,}917e-6}\right) \qquad (3.3.16)$$

The zeros of this equation are:

$$z_1 = 1, \; z_2 = 0{,}933 \qquad (3.3.17)$$

$$z_3 = 0{,}933 \qquad (3.3.18)$$

$$0 < z_4 < 0{,}933 \text{ for } \lambda > 0 \qquad (3.3.19)$$

Since z_1 lies on the unit circle, the system is marginally stable. If model deviations cause z_1 to move outside the unit circle, an unstable mode may be excited.

3.3.3 Stability Using I_3 on System with Stationary Disturbance Model.

The characteristic equation P(z) as derived in Annexure 7 to Appendix A is:

$$P(z) = (z-a)(z-c)\left(z^2 - \frac{(a+1)\lambda}{\frac{\lambda}{K_1(1-a)} + K_1(1-a)} \cdot z + \frac{\frac{\lambda a}{K_1(1-a)} - K_1(1-a)c}{\frac{\lambda a}{K_1(1-a)} - K_1(1-a)}\right)$$

(3.3.20)

Substituting the values of the constants as defined in paragraph 3.3.1 into equation (3.3.20) gives:

$$P(z) = (z - 0{,}933)(z - 0{,}933)\left(z^2 - \frac{1{,}933\,\lambda}{718\,\lambda + 1{,}394e\text{-}3} \cdot z + \frac{670\,\lambda - 1{,}300\,e\text{-}3}{670\,\lambda - 1{,}394\,e\text{-}3}\right)$$

(3.3.21)

The zeros of this equation are:

$$z_1 = z_2 = 0,933 \tag{3.3.22}$$

$$0,933 < z_3 < 0,966 \quad \text{for } \lambda > 0 \tag{3.3.23}$$

$$0,966 < z_4 \leqslant 1 \quad \text{for } \lambda > 0 \tag{3.3.24}$$

Since z_4 lies on the unit circle when λ becomes infinitely large, the system becomes marginally stable in this case. However, since λ infinitely large corresponds to a situation of no control, it is not of any practical interest. Therefore, for finite values of λ, the system should be stable even if small model deviations occur.

3.3.4 Stability Using I_3 on System with Non-stationary Disturbance Model

The characteristic equation P(z) as derived in Annexure 7 to Appendix A is:

$$P(z) = \left(z^2 - (1+c)z + c\right)(z-1)(z-a)\left(z - \frac{\dfrac{\lambda a}{K_1(1-a)} - K_1(1-a)}{\dfrac{\lambda}{K_1(1-a)} + K_1(1-a)}\right)$$

(3.3.25)

Substituting the values of the constants as defined in paragraph 3.3.1 into equation (3.3.25) gives:

$$P(z) = \left(z^2 - 1,933z + 0,933\right)(z-1)(z-0,933)\left(z - \frac{670\lambda - 1,394e\text{-}3}{718\lambda + 1,394e\text{-}3}\right)$$

(3.3.26)

The zeros of this equation are:

$$z_1 = 1, \quad z_2 = 0{,}933 \tag{3.3.27}$$

$$z_3 = 1 \tag{3.3.28}$$

$$z_4 = 1 \tag{3.3.29}$$

$$-1 \leqslant z_5 < 0{,}933 \quad \text{for} \quad \lambda \geqslant 0 \tag{3.3.30}$$

Since z_1, z_3, z_4 and $\dot{z}_5$ for $\lambda = 0$ lie on the unit circle, the system is marginally stable. If model deviations cause any of the poles to move outside the unit circle, unstable modes may be excited.

3.4 SUMMARY OF THEORETICAL DERIVATIONS

In paragraph 3.1 the process and disturbance models were derived from first principles respectively for the disturbance stationary (Equation (3.1.15)) and for the disturbance non-stationary (Equation (3.1.17)).

In paragraph 3.2.2 the general procedure for the derivation of the control laws based on the minimization of the respective cost functions was described. The optimal predictor used minimized the variance of the prediction error.

Applying the general procedure minimizing the respective cost functions, the general forms of the respective control laws were derived in paragraph 3.2.3. It was shown that the control law obtained by minimizing

$$I_1 = E\{y_t^2\} \tag{3.4.1}$$

was a special case of the control law obtained by minimizing

$$I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2 \qquad (3.4.2)$$

by equating λ to zero.

The other cost function minimized was

$$I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \qquad (3.4.3)$$

Using the general forms of the respective control laws and the derived process and disturbance models, the specific forms of the respective control laws were obtained in paragraph 3.2.4. These two derived control laws will be evaluated by simulation in Chapter 5 and by experimentation in Chapter 6 for the disturbance respectively stationary and non-stationary.

In paragraph 3.3 a stability analysis was conducted to determine the stability of the system using the respective control laws. This was done by determining the poles of the system transfer function for the respective cases.
It was found that the system is:

a. Asymptotically stable using I_2 and a stationary disturbance model.

b. Marginally stable with one pole on the unit circle using I_2 and a non-stationary disturbance model.

c. Asymptotically stable using I_3 and a stationary disturbance model.

d. Marginally stable with three to four poles on the unit circle depending on the value of λ using I_3 and a non-stationary disturbance model. From this may be concluded that this system may be sensitive to model inaccuracies which may excite unstable modes.

The stability analyses will be used to interpret the results obtained by simulation and the results obtained by experimentation in Chapters 5, 6 and 7.

CHAPTER 4

SYSTEM CONFIGURATION AND DESIGN

4.0 SYSTEM OBJECTIVE

The objective of the designed system is to serve both as an experimental controller and as a computer simulator which allows flexible implementation and evaluation of stochastic control laws.

4.1 PHYSICAL ENVIRONMENT OF THE SYSTEM

In order to obtain a clear definition of the boundaries of the controller, the structure of the environment it interfaces to is defined in this paragraph.

Figure 4.1.1 shows the functional interfaces of the controller to its interface. The controller interfaces to two environments, namely:

a. The operator environment by means of a man-machine interface.

b. The controlled system environment by means of a data acquisition and control interface.

The operator operates the controller and defines the control parameters. The controller controls the output of the controlled system according to the installed control law and randomly disturbs the output of the controlled system according to the installed disturbance generation function.

Figure 4.1.2 shows the functional configuration of the controlled system. The functional elements of the controlled system are specified in terms of the input and output specifications of the functional elements of the controller in paragraph 4.3.

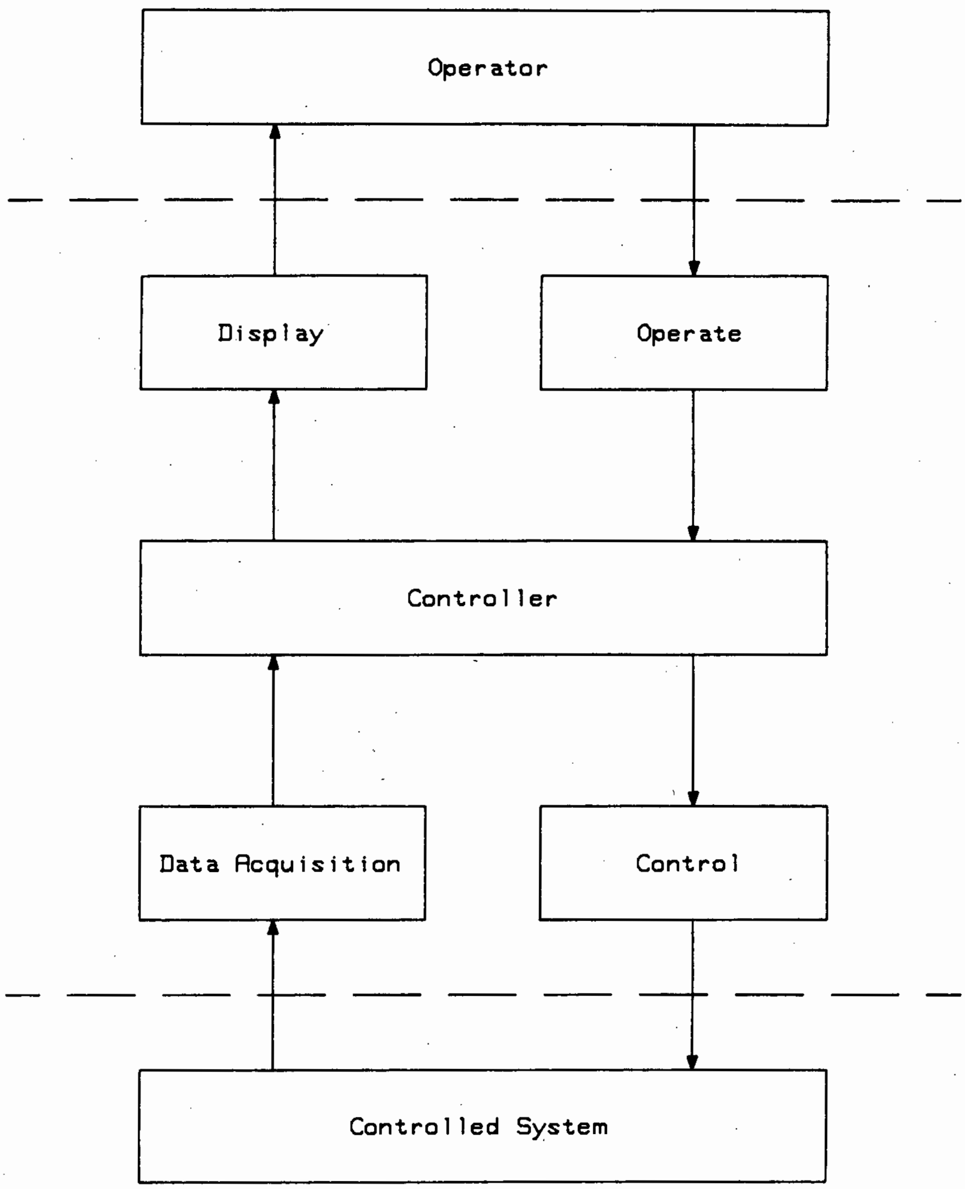


Figure 4.1.1. Functional Interfaces Between Controller and Environment

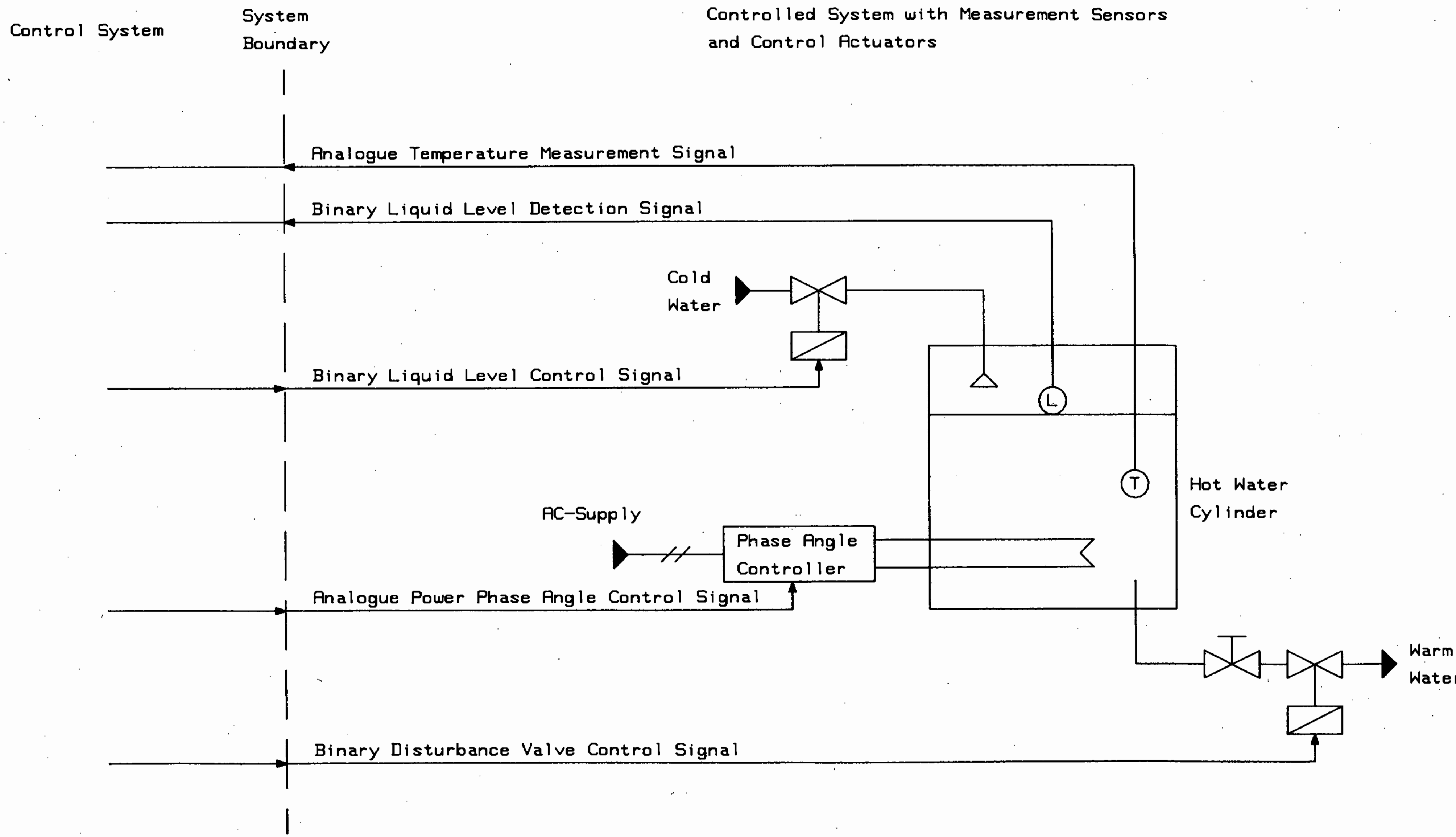


Figure 4.1.2. Functional Configuration of Controlled System

4.2 LIST OF SYSTEM FUNCTIONS

The following is a list of the system functions:

a. Function 1 : Temperature measurement and A/D-conversion

b. Function 2 : Liquid level control

c. Function 3 : Disturbance valve actuation

d. Function 4 : Power phase angle control

e. Function 5 : Computer parallel I/O-interfacing

f. Function 6 : Power supplies and earthing

g. Function 7 : Control parameters initialisation

h. Function 8 : Disturbance function computation

i. Function 9 : Control algorythm computation

j. Function 10 : Interrupt servicing

k. Function 11 : Mass storage

l. Function 12 : Man-machine interfacing

m. Function 13 : Data processing

4.3 FUNCTIONAL DESCRIPTION

4.3.1 Function 1 : Temperature Measurement and A/D-Conversion

Description:

The temperature of the water in the hot water cylinder is sensed by a precision temperature sensor immersed in the water. The analogue temperature signal is converted to a digital form.

Activities:

Figure 4.3.1 shows the circuit diagram of this functional element.

Operating as a 2-terminal zener, the LM335 precision temperature sensor has a breakdown voltage directly proportional to absolute temperature at 10mV/K. The breakdown voltage is calibrated by RV1 as 2,730 V at the melting point of ice (273K).

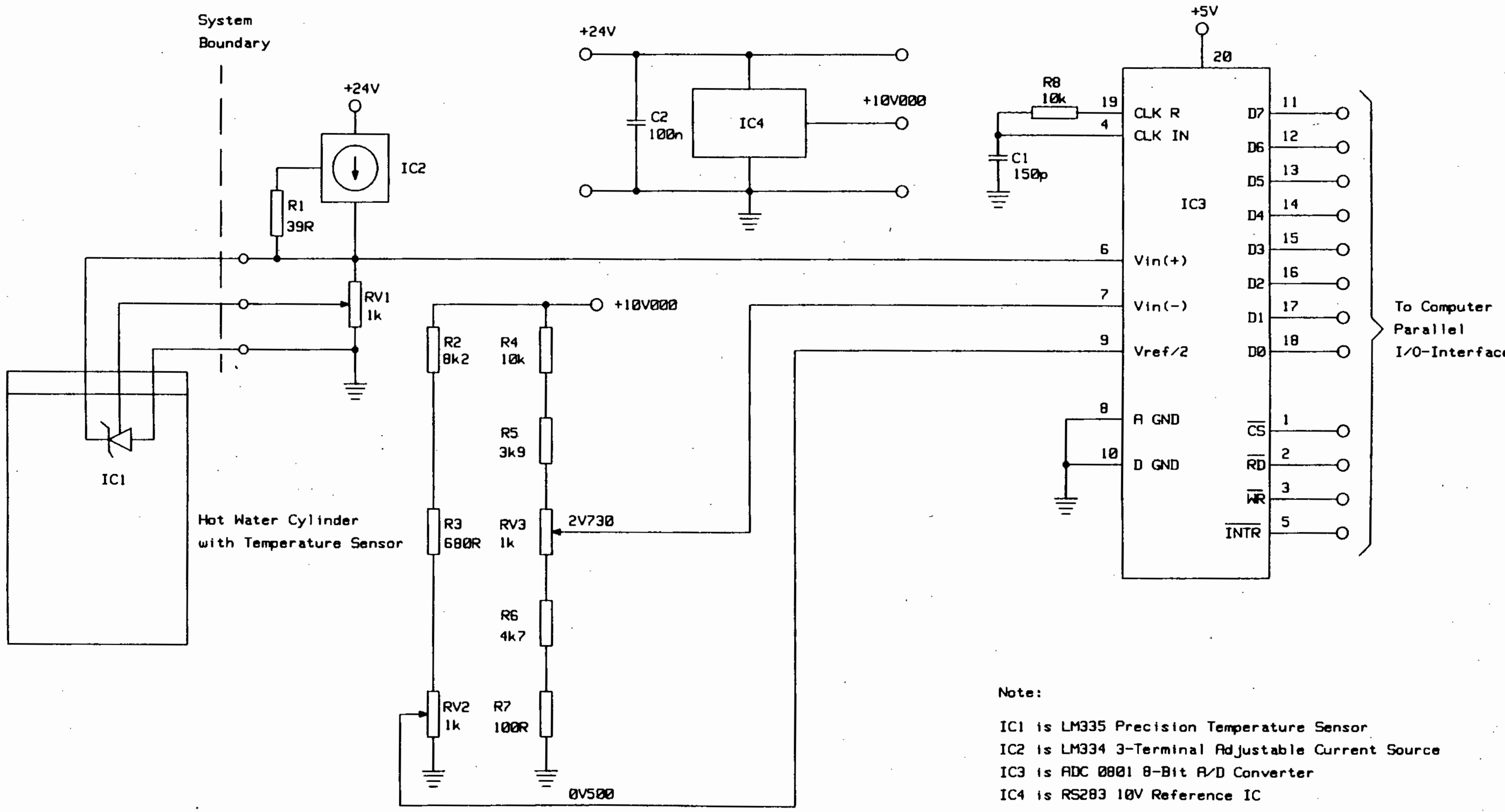
To eliminate self heating errors, the LM335 is supplied with a temperature compensated stable current by the LM334 3-terminal adjustable current source. The current is programmed at 1,7 mA by R1.

The analogue temperature voltage signal is applied to the positive differential input of the ADC 0801 8-bit A/D-converter. A reference voltage of 2,730V corresponding to 0 °C is applied to the negative differential input of the ADC 0801.

A reference voltage of 0,500V corresponding to an analog input voltage range of 1,000 V is applied to the $V_{REF}/2$ - input of the ADC 0801. The analogue input voltage range is therefore 2,730V to 3,730V. This corresponds to an input temperature range of 0 °C to 100 °C.

The accurate and stable reference voltages are obtained by means of resistor networks containing multiturn trim pots RV2 and RV3 supplied by a stable precision voltage of 10,000V. This voltage is supplied by the RS283 10,000V 3-terminal voltage reference IC.

Figure 4.3.2 shows the analogue to digital conversion timing diagrams. The digital output has 8-bit resolution over the analogue input voltage range.



Note:

IC1 is LM335 Precision Temperature Sensor
IC2 is LM334 3-Terminal Adjustable Current Source
IC3 is ADC 0801 8-Bit A/D Converter
IC4 is RS283 10V Reference IC

Figure 4.3.1. Precision Temperature Sensor and Analogue to Digital Converter

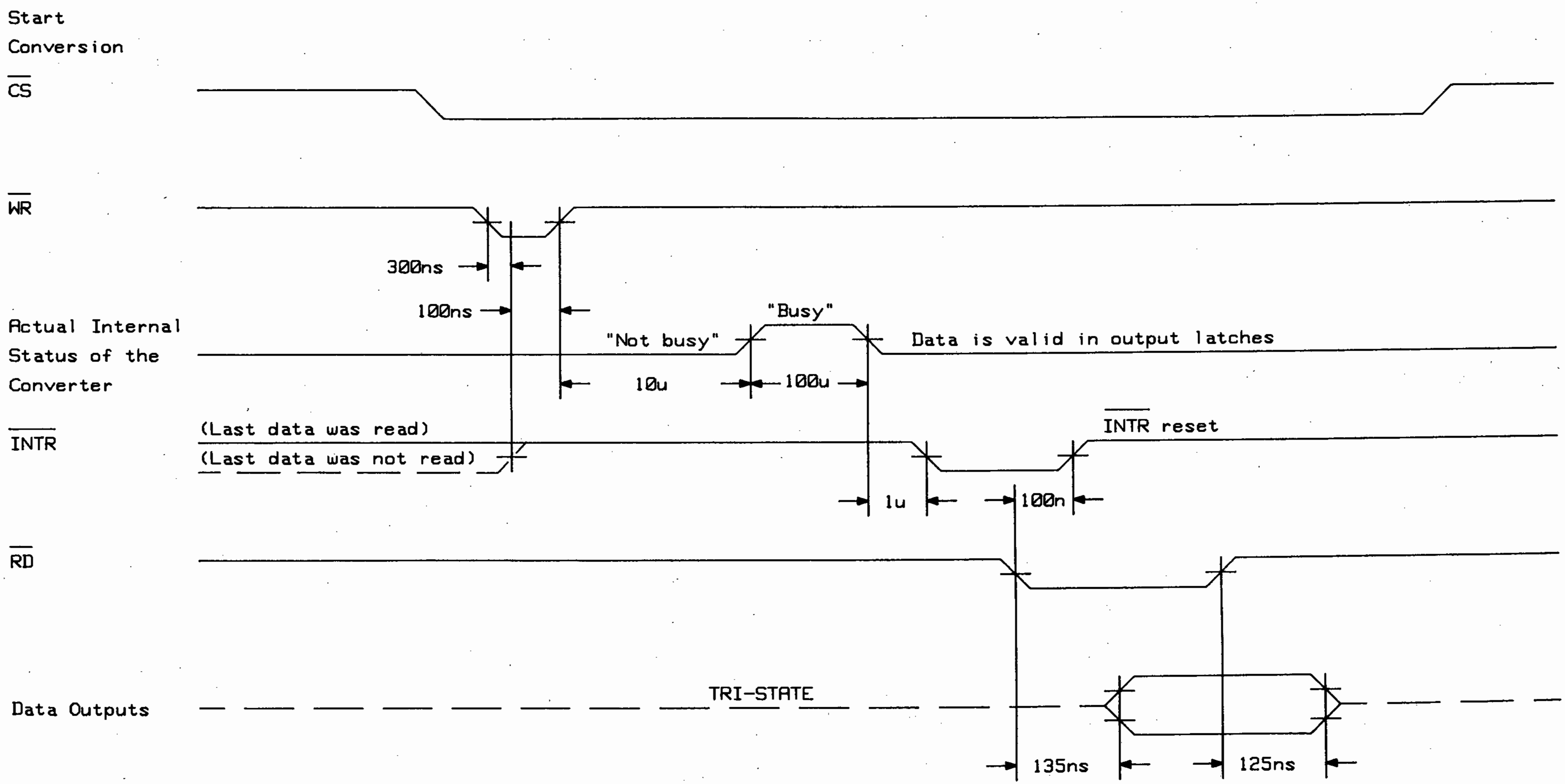


Figure 4.3.2. Analogue to Digital Conversion Timing Diagrams

Input Specifications:

Temperature measurement input range	: 0°C to 100°C
LM 335 proportional constant	: 10mV/K
LM 335 accuracy	: ± 1°C

Output Specifications:

Digital resolution	: 8 bits
Total conversion error	: ± 1/4 LSB
Conversion time	: 100 micro-seconds

4.3.2 Function 2 : Liquid Level Control

Description:

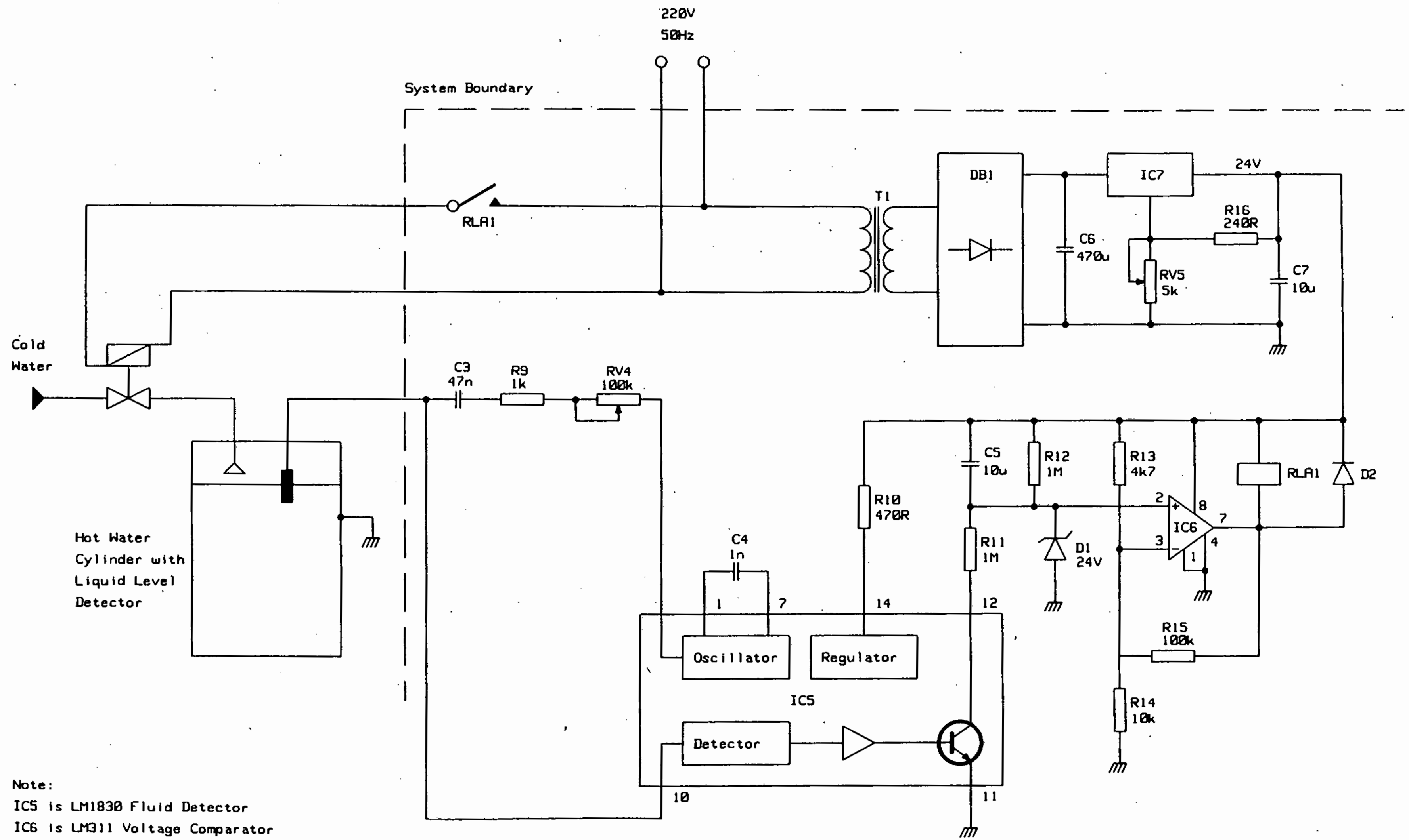
The level of water in the water cylinder is sensed by a fluid detector. A solenoid valve is actuated when the water level drops below a specific value allowing cold water to flow into the cylinder thus maintaining the required level.

Activities:

Figure 4.3.3 shows the circuit diagram of this functional element.

A phone plug installed at a height corresponding to a 10ℓ volume of water in the cylinder is used as a probe. An ac-signal is applied between the probe and the metallic cylinder wall (signal earth) by the LM1830 fluid detector.

When the resistance between these nodes drops below the calibrated value of RV4, the fluid detector's output transistor is switched hard on. The value of RV4 is determined by the conductivity of the water in the cylinder which has a typical range of 20 to 60 K-ohm.



Note:
IC5 is LM1830 Fluid Detector
IC6 is LM311 Voltage Comparator
IC7 is 3-Terminal Adjustable Regulator

Figure 4.3.3. Liquid Level Controller

The collector of the output transistor is connected to a RC-network with a 10 second time constant. Approximately 6 seconds after the transistor switches on, the output of the LM311 voltage comparator goes low, energizing relay RLA1.

With the relay energized, the relay contact closes and 220V 50Hz is applied to the solenoid valve causing it to open and water to flow into the cylinder.

When the probe becomes immersed in the water once again, the level detector's output transistor switches off. Approximately 4 seconds after the transistor switches off, the output of the voltage comparator goes high, de-energizing the relay causing the selenoid valve to shut.

The long RC-time constant and built-in hysteresis prevents jittering of the relay.

The electronics is supplied with a transformer isolated 24V dc-supply. This signal earth is isolated from the signal earth of the rest of the electronics for a reason explained in paragraph 4.3.6.

Input Specifications:

Resistance between probe and signal earth with:

a. probe above water level : larger than 100k-ohm

b. probe immersed in water : 20 to 40 k-ohm

Output Specifications:

Regulated volume of water in cylinder : 10ℓ

4.3.3 Function 3 : Disturbance Valve Actuation

Description:

The solenoid outlet valve is actuated by the computer generated disturbance function causing hot water to flow out of the cylinder according to the disturbance function. Inflowing cold water controlled by the liquid level controller replaces the outflowing hot water and causes the desired temperature disturbance characteristic.

Activities:

Figure 4.3.4 shows the circuit diagram of this functional element.

The output of the DM7406 inverter buffer is controlled by an output bit from the computer parallel input/output interface. With the input to the buffer high, relay RLA2 is energized and solenoid valve V2 is open. With the input to the buffer low, relay RLA2 is de-energized and solenoid valve V2 is shut.

Valve V1 is calibrated by hand before commencing an experiment to ensure an output flow-rate equal to V_{max} with valve V2 open.

Input Specifications:

Disturbance valve on command : Input logical 1

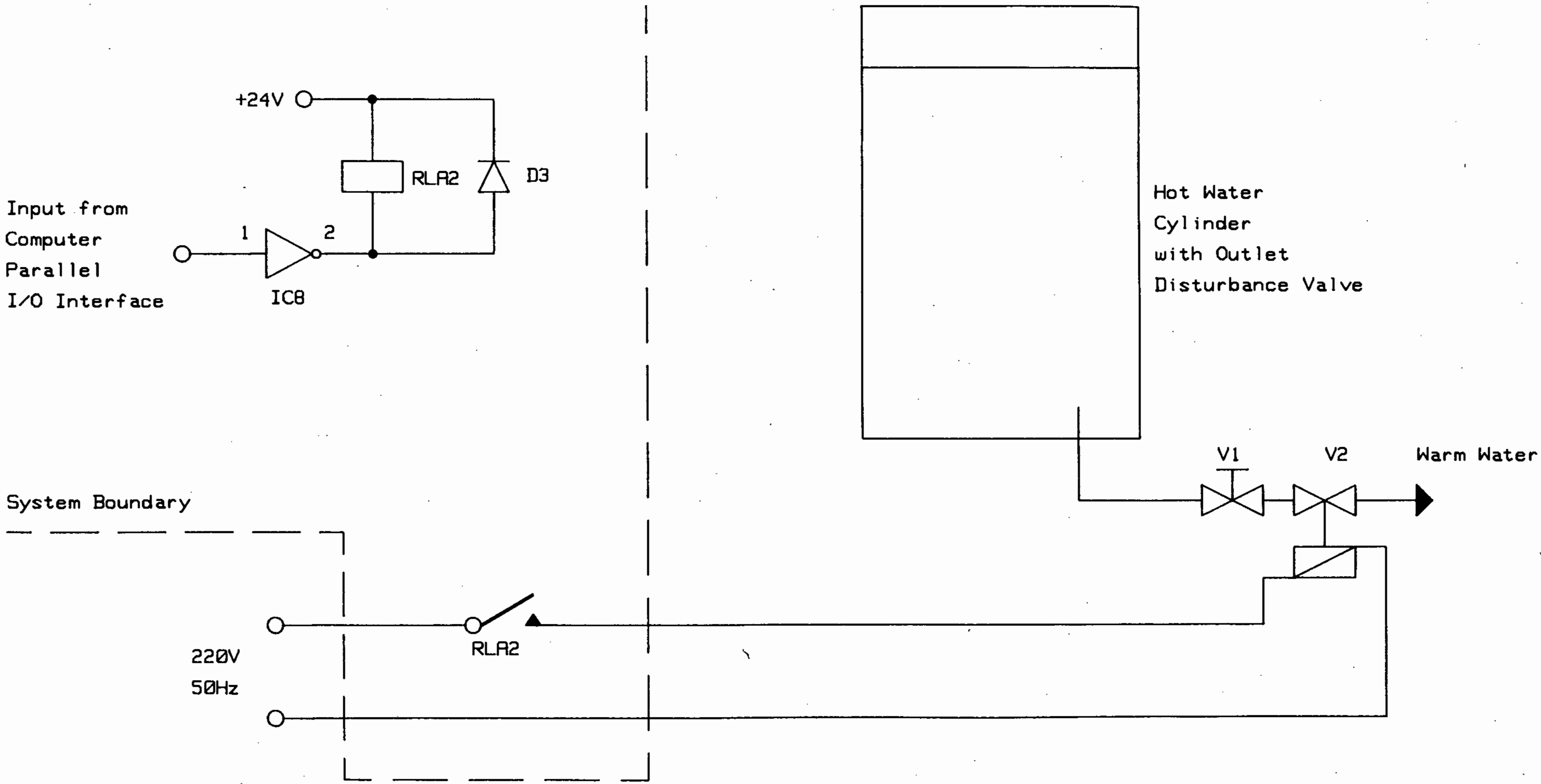
Disturbance valve off command : Input logical 0

Output Specifications:

Supply to outlet solenoid valve with :

a. Valve in shut state : 0V

b. Valve in open state : 220V, 50Hz



Note:
IC8 is DM7406 Hex Inverter Buffers with
Open-Collector High-Voltage Outputs

Figure 4.3.4. Disturbance Valve Actuator

4.3.4 Function 4 : Power Phase Angle Control

Description:

The phase angle 8-bit digital command signal between 0° and 175° at which each half wave of the sinusoidal ac-supply to the hot water cylinder heating element is to be triggered, is computed and supplied by the control computer as a function of the installed control law.

The phase angle controller converts and conditions the signal and switches the ac-supply to the heating element as a linear function of the input. The non-linear transformation from required average heating power to phase angle is described in paragraph 4.3.9.

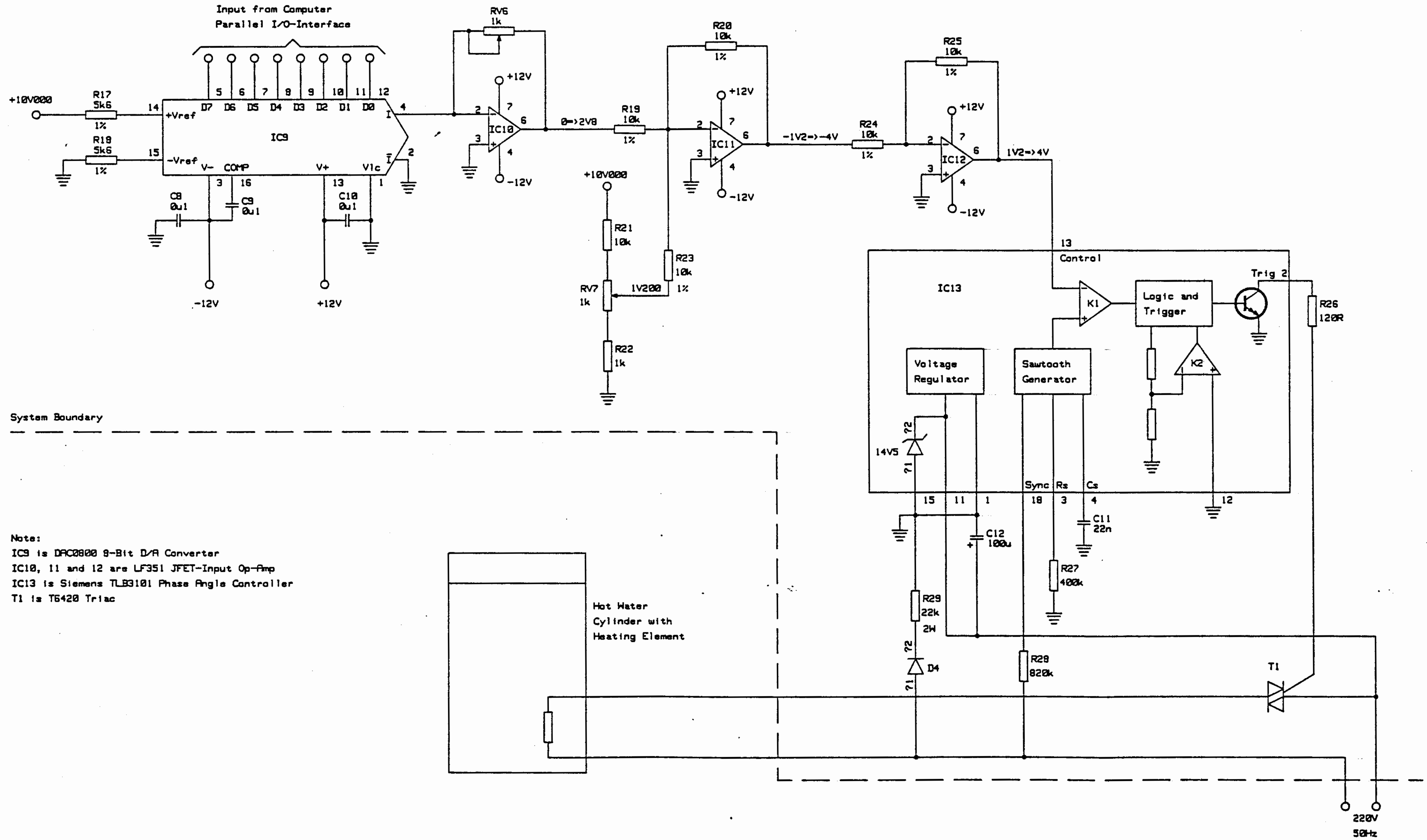
Activities:

Figure 4.3.5 shows the circuit diagram of this functional element.

The 8-bit phase angle command signal from the controller is applied at the data inputs of the DAC 0800 8-bit D/A-converter. The precision 10,000V reference voltage provides a stable 1,786 mA full scale sink current to the I-terminal of the converter.

Multiturn trim pot RV6 calibrates the output of the inverter stage to be 0,00V with digital input 00H and 2,80V with digital input 0FFH. This signal is added to a 1,20V reference voltage calibrated by trim pot RV7 by the summing stage which then has an output voltage span of -1,20V to -4,00V corresponding to a digital input range of 00H to 0FFH.

The resultant signal is inverted by the third Op-Amp stage to provide an analogue signal with a voltage span of 1,20V to 4,00V corresponding to a digital input range of 00H to 0FFH. This signal serves as control input to the TLB3101 phase angle controller integrated circuit.



Note:
IC9 is DAC0800 8-Bit D/A Converter
IC10, 11 and 12 are LF351 JFET-Input Op-Amp
IC13 is Siemens TLB3101 Phase Angle Controller
T1 is T6420 Triac

Figure 4.3.5. Power Phase Angle Controller

The phase angle controller controls the power supplied to the heating element by controlling the phase angle of each half wave of the sinusoidal ac-supply at which triac T1 is triggered. There is a linear relationship between the phase angle control voltage and the triggered phase angle as shown in Figure 4.3.6.

The phase angle controller's supply voltage of 10 to 20V is obtained directly from the 220V 50Hz power network by means of dropping resistor R29.

Input Specifications:

Digital input corresponding to minimum power	:	0FFH
Digital input corresponding to maximum power	:	00H
Full scale conversion error	:	$\pm$ 1 LSB

Output Specifications:

Triggered phase angle corresponding to minimum power	:	0 degrees
Triggered phase angle corresponding to maximum power	:	175 degrees

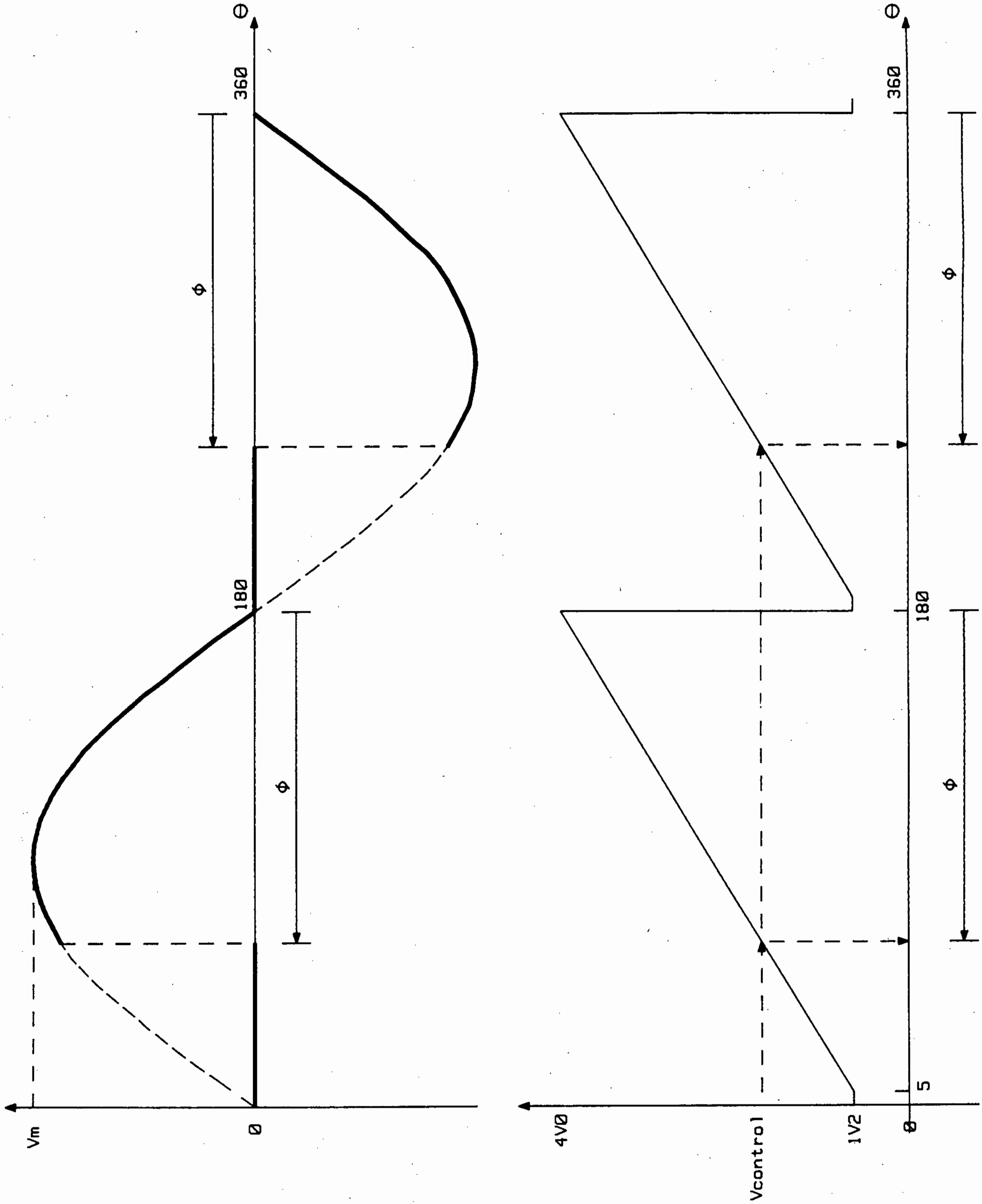
4.3.5 Function 5: Computer Parallel I/O - Interfacing

Description:

The exchange of data and control information between the computer controller and the data acquisition and control actuator circuitry is accomplished by means of the parallel input/output interface.

Activities:

Figure 4.3.7 shows the circuit diagram of this functional element.



50Hz Mains Supply

Phase Angle Control Voltage

Figure 4.3.6. Relationship between Phase Angle Control Voltage and Triggered Phase Angle

The computer control and address buses are buffered by 74LS244 octal buffers. The address bus is buffered by 74LS245 octal bus transceivers.

Two Z80 - PIOs are used as parallel I/O - devices. The I/O - space is addressed only when both A7 and Not-IORQ are logical zero. The OR-gate isolates the computer and I/O data buses when this condition is not true.

IC19 is chip-enabled by the 74LS138 3 to 8 decoder when the following conditions are true:
A7 logical zero, Not-IORQ logical zero, A4 logical zero, A5 logical zero, A6 logical zero.

IC20 is chip-enabled by the 74LS138 when the following conditions are true:
A7 logical zero, Not-IORQ logical zero, A4 logical one, A5 logical zero, A6 logical zero.

IC19 port A is programmed for control mode with all eight bits programmed as outputs. This port is used to output the data byte to the power phase angle controller D/A - converter.

IC19 port A is programmed by the following instructions:
Instruction 1 : OUT 0000 1011, 11xx 1111
Instruction 2 : OUT 0000 1011, 0000 0000

The address byte of both instructions addresses the control register (bit 3) of port A (bit 2) of IC19 (bits 4,5 and 6). Bits 6 and 7 of the data byte of instruction 1 selects the control mode for port A. The data byte of instruction 2 selects all the bits of port A as output bits.

A data byte is output from IC19 port A by the following instruction:
OUT 0000 0011, byte

In this case bit 3 of the address byte addresses the data register of port A of IC19.

IC19 port B is programmed for control mode with all eight bits programmed as inputs. This port is used to input the data byte from the temperature sensor A/D - converter.

IC19 port B is programmed by the following instructions:

Instruction 1 : OUT 0000 1111, 11xx 1111

Instruction 2 : OUT 0000 1111, 1111 1111

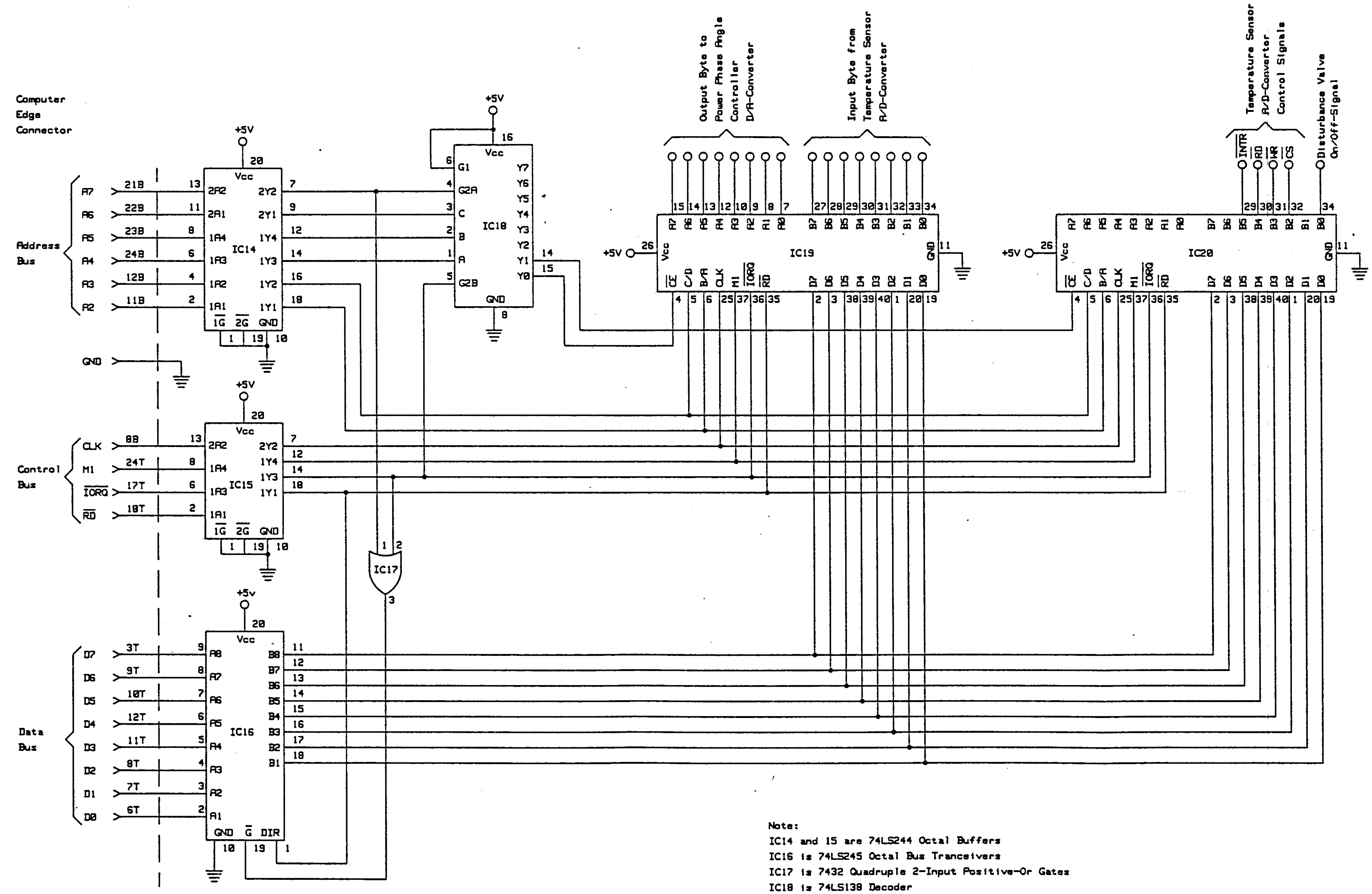
The address byte of both instructions addresses the control register (bit 3) of port B (bit 2) of IC19 (bits 4,5 and 6). Bits 6 and 7 of the data byte of instruction 1 selects the control mode for port B. The data byte of instruction 2 selects all the bits of port B as input bits.

A data byte is input to IC19 port B by the following instruction:

IN 0000 0111

In this case bit 3 of the address byte addresses the data register of port B of IC19.

IC20 port B is programmed for control mode. Bits 0,2,3 and 4 are programmed as output bits. Bit 5 is programmed as an input bit. Four of these bits are used to control the A/D - converter. The remaining bit is used to control the disturbance valve.



Note:
IC14 and 15 are 74LS244 Octal Buffers
IC16 is 74LS245 Octal Bus Tranceivers
IC17 is 7432 Quadruple 2-Input Positive-Or Gates
IC18 is 74LS138 Decoder
IC19 and 20 are Z80-PIO MK3881

Figure 4.3.7. Computer Parallel Input/Output-Interface

IC20 port B is programmed by the following instructions:

Instruction 1 : OUT 0001 1111, 11xx 1111

Instruction 2 : OUT 0001 1111, 0010 0000

The address byte of both instructions addresses the control register (bit 3) of port B (bit 2) of IC20 (bits 4,5 and 6). Bits 6 and 7 of the data byte of instruction 1 selects the control mode for port B. The data byte of instruction 2 selects bit 5 as an input bit and all the others as output bits.

A data byte is output from IC20 port B by the following instruction:

OUT 0001 0111, byte

In this case bit 3 of the address byte addresses the data register of port B of IC20. Bit 5 of the input port remains in a high impedance state.

A data byte is input to IC20 port B by the following instruction:

IN 0001 0111

In this case only bit 5 has significance as programmed.

4.3.6 Function 6 : Power Supplies and Earthing

Description:

The dc-supplies for the electronic circuitry are provided by transformer-isolated linear voltage regulators. The following supplies are provided : +24V(2x), +5V, +12V, -12V.

All the circuits with the exception of the liquid level controller have a common signal earth. From Figure 4.3.5 can be seen that the signal earth voltage of the TLB3101 phase angle controller has a constant value equal to the supply voltage to the TLB3101 (approximately 20V) lower than the 220V 50Hz mains supply voltage w.r.t safety earth at all times. This also applies to the signal earth of the other electronics with the exception of the liquid level controller.

From Figure 4.3.3 can be seen that the signal earth voltage of the liquid level controller is the same as that of that of the hot water cylinder wall. Since the cylinder wall is connected to safety earth, the signal earth of the liquid level controller is transformer-isolated from the other signal earth to prevent earth leakage.

4.3.7 Function 7 : Control Parameters Inisialisation

Description:

The program "LOADER" inputs process and program control parameters from the keyboard, calculates coefficients used by the control algorythms and stores the data in the form of arrays on magnetic tape. To run the main program for simulation and experimentation, the process and program control parameters are loaded from magnetic tape into memory. The objective of this is to minimize the demands placed by the main program on the limited (48 Kbyte) read-write memory.

Activities:

The program "LOADER" calls three routines. The major portion of the task is done by subroutine "SR/DATADEF". Subroutine "SR/DECBIN" converts a decimal number to a 16-bit binary number. This routine is used to convert parameters which will be used by the interrupt service machine code routine to a binary form. Subroutine "SR/TITLDEF" inputs a string-array which can be used for the later identification of the simulation or experiment.

The listings of these programs appear in Annexure 1 to Appendix B.

4.3.8 Function 8 : Disturbance Function Computation

Description:

In the case of an experiment, the disturbance valve on/off - ratio is calculated by formulae respectively for stationary and non-stationary disturbances. In the case of a simulation, the actual temperature disturbance as a result of the mass flow corresponding to the calculated valve on/off - ratio is calculated as well.

Activities:

Figure 4.3.8 is a schematic presentation of the disturbance valve on-off cycle.

To approximate a continuous disturbance, the sampling period T_s (60 seconds) is broken down into two valve cycles with period T_{vv} (30 seconds) each. The valve on-time T_{von} per unit valve cycle is the actual random variable.

For a stationary disturbance, T_{von} has a pseudo-random probability density function symmetrical around $T_{vv}/2$. The difference between T_{von} and $T_{vv}/2$, ΔT_{von}, therefore has a probability density function which is symmetrical around zero as shown in Figure 4.3.8.

Valve V1 in Figure 4.3.4 is calibrated to allow a maximum flow rate of 0,021 kg/s. This corresponds to a power loss equal to the heating element rating for the system parameters with specifications as in Table 3.0.1. The nominal flow rate is therefore 0,011 kg/s. The width of the pseudo-random probability density function was chosen to be 0,0135 kg/s for all experiments and simulations thus allowing a large variance of the control input.

With the pseudo-random variable generated by the BASIC program's RND function, ΔT_{von} and T_{von} are calculated for each valve cycle. Using equation (3.1.16) and equating ΔV to $(\Delta T_{von} \times V_{max})/T_{vv}$, ξ_t is obtained. Using the disturbance component of equation (3.1.15), the stationary temperature disturbance e_t is calculated.

For a non-stationary disturbance, ΔT_{von} is obtained by adding a pseudo-random variable with a probability density function with a width of 3 seconds and symmetrical around zero to the previous value of ΔT_{von}. This is equivalent to the integration of ξ_t in equation (3.1.17) allowing ΔT_{von} to drift slowly. ΔT_{von} is not allowed to drift outside the basic valve period boundaries. Using the disturbance component of equation (3.1.17), the non-stationary temperature disturbance e_t is calculated.

The program listing of the subroutine "DISTROUTS" which calculates these disturbances appears in Annexure 2 to Appendix B.

4.3.9 Function 9 : Control Algorythm Computation

Description:

The main program computes the power output to the heating element at each sampling moment with present and past values of measured temperature and control input as inputs using the selected control algorythm.

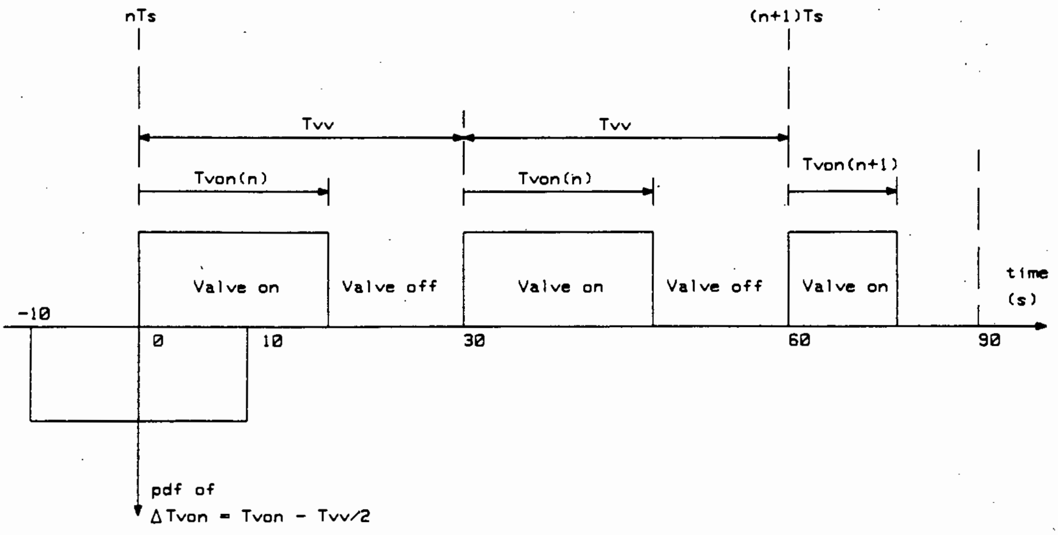


Figure 4.3.8. Schematic Presentation of Disturbance Valve On-Off Cycle with pdf of ΔTvon Superimposed

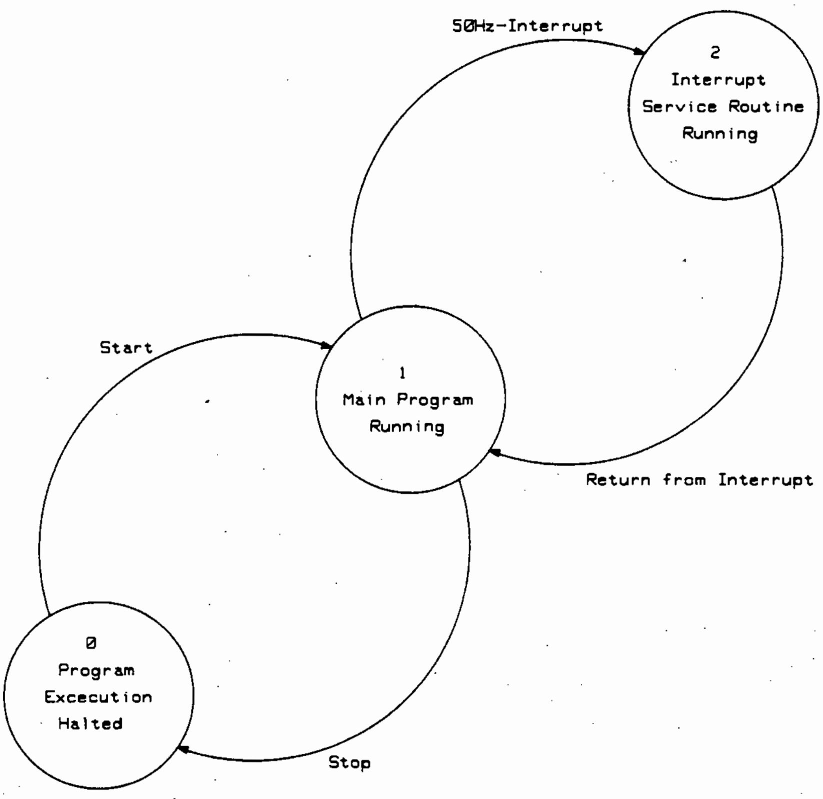


Figure 4.3.9. State Diagram of Program Execution

Activities:

Figure 4.3.9 shows a state diagram of program execution. The control algorythms are computed by the BASIC main program which is interrupted every 20 ms by the machine code interrupt service routine when conducting an experiment. The interrupt service routine is described in paragraph 4.3.10.

Figure 4.3.10 shows the flow diagram of the main program written in BASIC. A ZX-Spectrum home computer is used as control computer. A listing of "MAINPROG" appears in Annexure 3 to Appendix B.

The program allows the selection of the following options by the operator:

a. simulation/experiment

b. disturbance stationary/non-stationary

c. control laws 1 and 2 or 3

d. process model accurate/perturbed

The main program communicates with the interrupt driven machine code program which controls the temperature measurement and disturbance valve on/off-switching in the case of an experiment by means of parameter-passing to and from defined memory locations.

The computed required average power as output from the selected control algorythm is a non-linear function of the required power phase angle voltage which is the final output from the program when conducting an experiment.

The relationship between average heating power P_{ave} and phase angle Φ (radians) is the following:

$$P_{ave} = \frac{2 \cdot P_{rating}}{\pi} \left[\frac{\phi}{2} + \frac{\sin 2(\pi - \phi)}{4} \right] \quad (4.3.1)$$

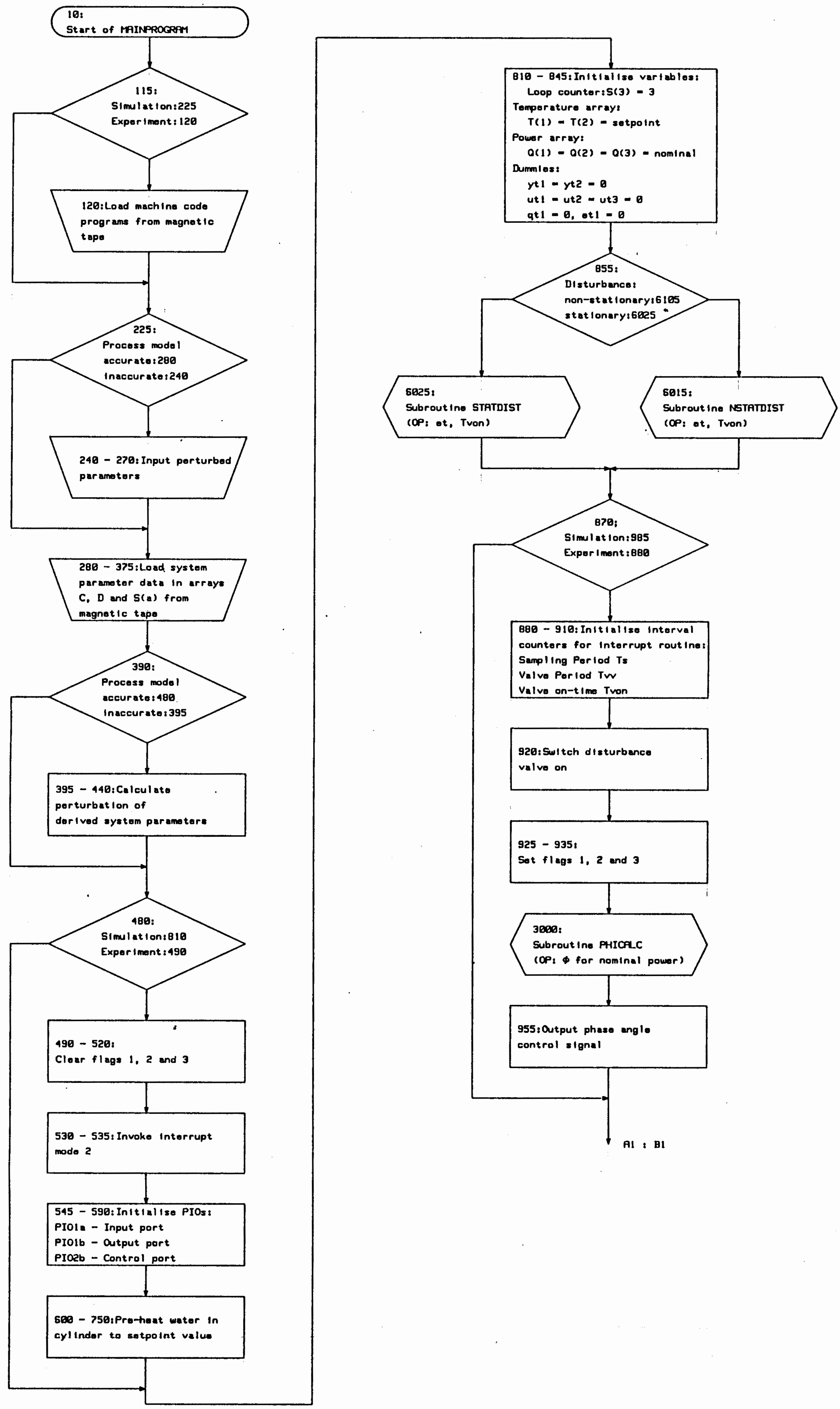


Figure 4.3.10. (page 1 of 2) Flow Diagram of Main Program

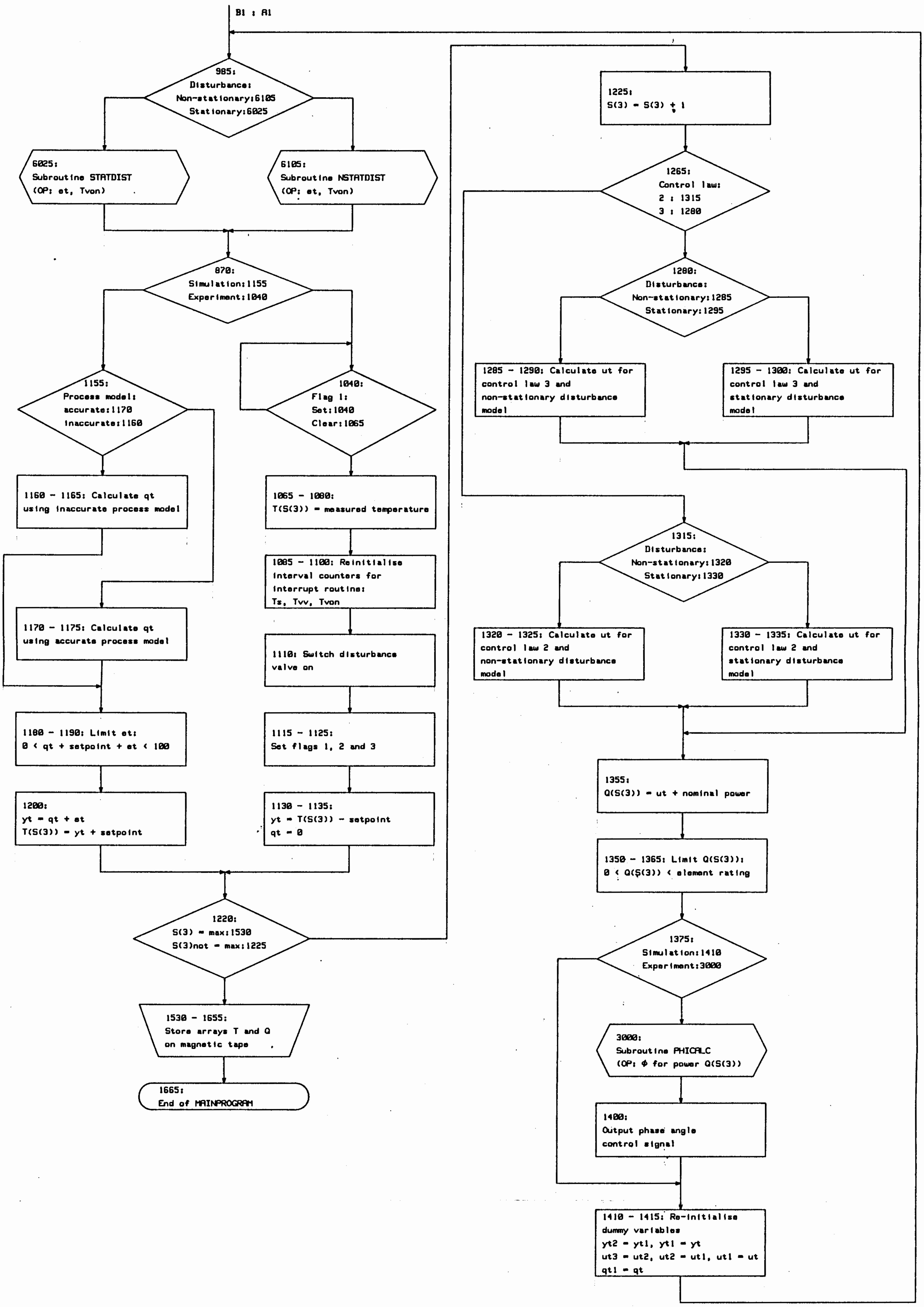


Figure 4.3.10. (page 2 of 2) Flow Diagram of Main Program

To solve for Φ, the Newton-Raphson method is used. This is executed by subroutine "SR/PHICALC" a listing of which appears in Annexure 4 to Appendix B. The calculation is iterated until the accuracy of Φ corresponds to a power accuracy of 1W.

Finally, the output voltage to the phase angle controller is obtained using the linear relationship between the control voltage and phase angle as shown in Figure 4.3.6.

The measured temperature and output power at all sampling points during the experiment or simulation are stored in arrays which are saved on magnetic tape after the completion of a simulation or experiment for data processing at a later stage.

4.3.10 Function 10 : Interrupt Servicing

Description:

The machine code interrupt service routine serves as real time counter for the timing of sampling periods, disturbance valve switching and as control routine for temperature measurement analogue to digital conversion during experiments. In addition it calls the keyboard read subroutine.

Activities:

Figure 4.3.11 shows a flow diagram of the interrupt service routine which interrupts the main program every 20 ms.

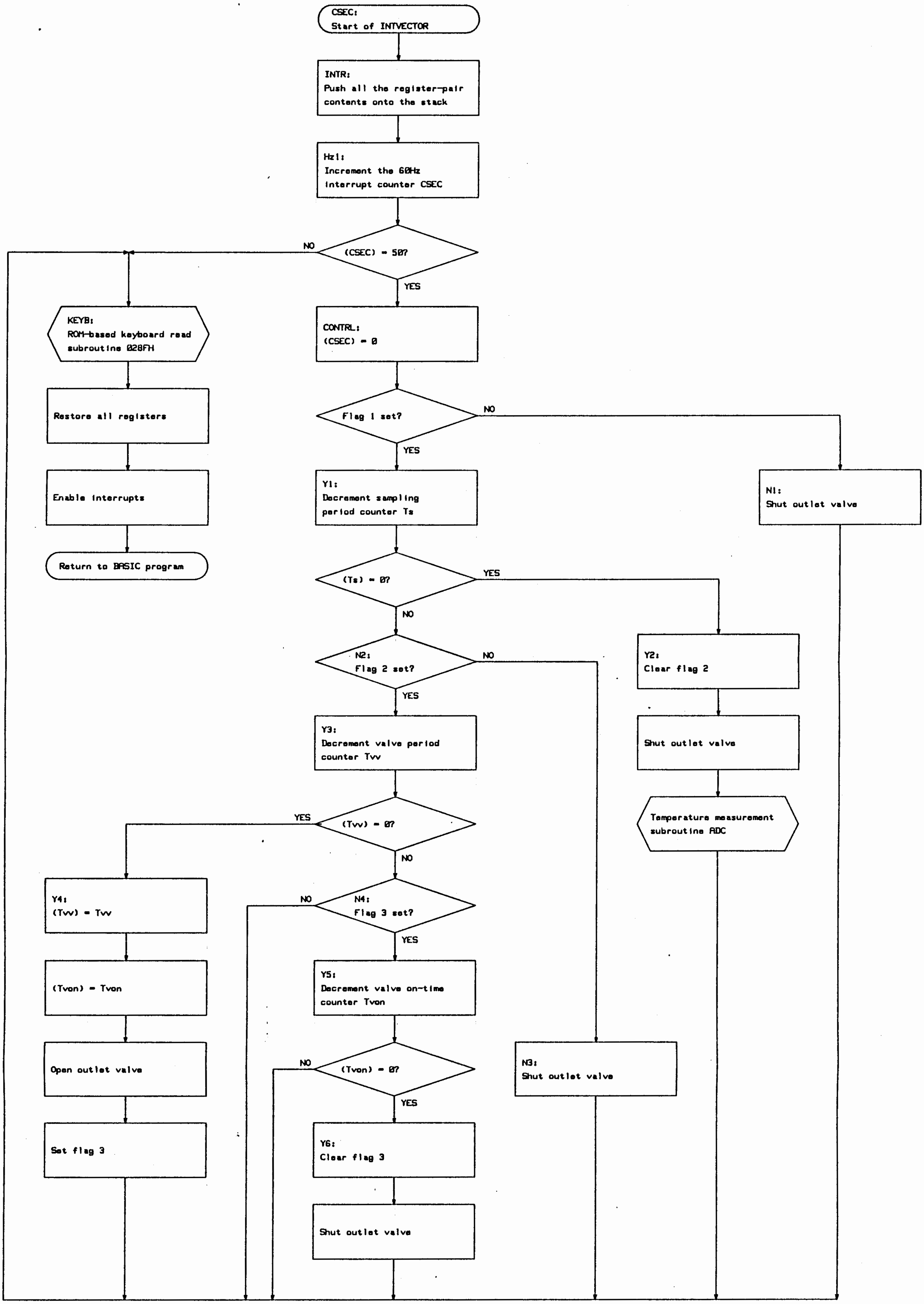


Figure 4.3.11. Flow Diagram of Interrupt Service Routine

The computer has a Z80 8-bit CPU. The interrupt service routine requires a 16-bit address vector since it is located at address E128H. Therefore the Z80 is programmed for interrupt mode 2 by calling routine "INTMODES" located at address 57601 using the "USR" function. The top of the address space for the BASIC programs is set to 57600 using the "CLEAR" statement before programs are loaded from magnetic tape into memory.

"INTMODES" programs the contents of the I-register of the CPU to be 31H. A 50Hz interrupt signal generated from line voltage is applied to the CPU. When an interrupt occurs, a 16 bit address of 31FFH is formed using the contents of the I-register as upper byte and the contents of the data bus (FFH, tri-state) as lower byte.

Using this address, a 16-bit interrupt vector is formed by using as lower byte the contents of memory 31FFH and as upper byte the contents of memory 3200H. These two memory locations fall in the ROM address space with the contents of 31FFH 28H and 3200H E1H respectively. The program therefore vectors to the interrupt service routine at address E128H.

The interrupt service routine updates the real time counters associated with the switching of the disturbance valve and the sampling period, controls the state of the disturbance valve, calls the temperature measurement subroutine "ADC" on sampling moments, sets/clears flags for communication with the main program and calls a ROM-based keyboard read routine.

"ADC" controls the temperature measurement A/D-conversion of 128 samples in quick succession and obtains the average value to minimize measurement noise. This occurs in approximately 25 ms. Figure 4.3.12 shows a flow diagram of this subroutine.

On completion of the interrupt service routine, the registers are restored, interrupts enabled and control returned to the BASIC main program.

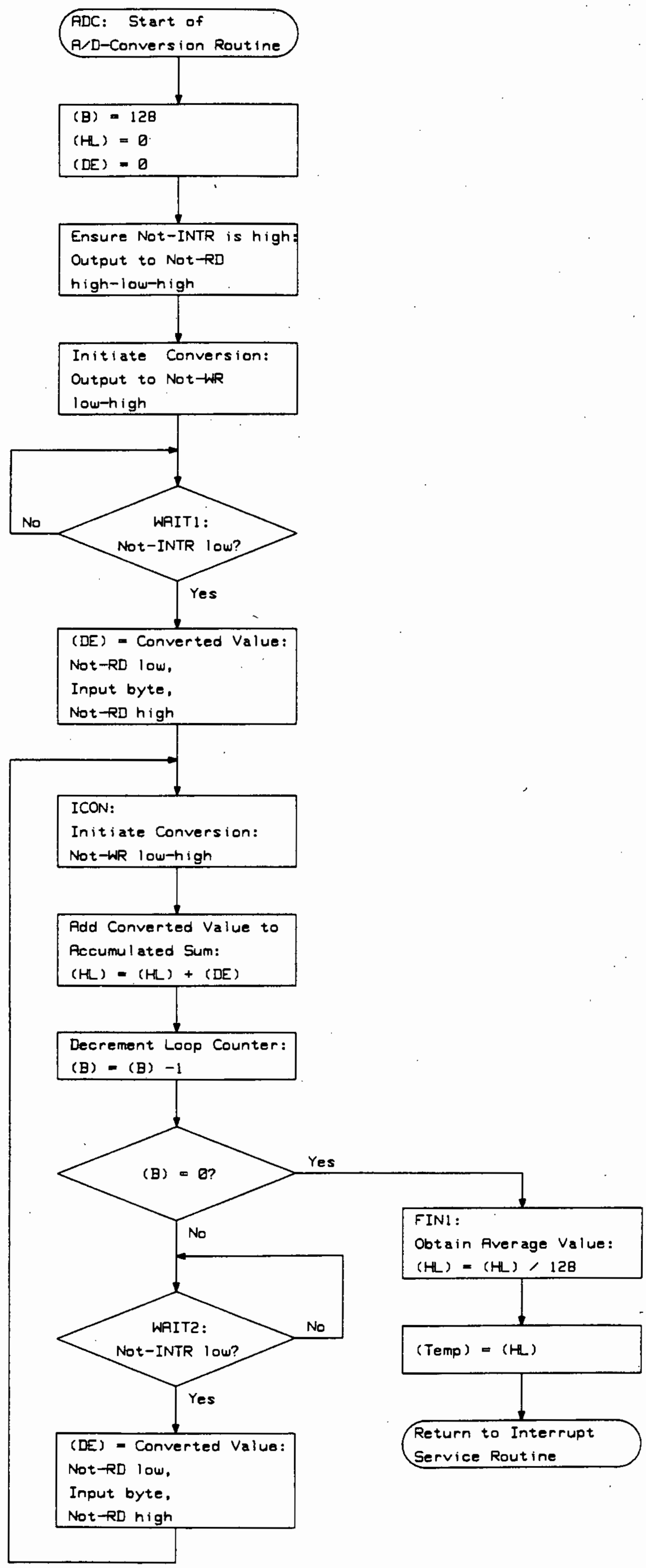


Figure 4.3.12. Flow Diagram of Machine Code A/D-Conversion Routine

Listings of "INTMODES", "INTVECTOR" and "ADC" appear in Annexure 5 to Appendix B.

4.3.11 Function 11 : Mass Storage

Description:

A magnetic tape recorder is used as mass storage medium for the storage of programs and data as described in the previous paragraphs.

4.3.12 Function 12 : Man-Machine Interfacing

Description:

The operator communicates with the controller by means of the computer's keyboard and the TV-display connected to the computer.

4.3.13 Function 13 : Data Processing

Description:

Following the completion of a simulation or an experiment, the measured temperature and control input data arrays stored on magnetic tape are reloaded into memory and processed by data processing programs to obtain the data in a compressed form which can be easily interpreted and analised.

Activities:

Program "PLOTPACK" draws the input and output time responses on a graphic printer.

Program "STATPACK" computes and prints the mean and variances of the input and output.

Program "STATPACK2" computes and prints the autocorrelated values of the input and output.

Listings of these programs appear in Annexure 6 to Appendix B.

4.4 CONSTRUCTION

The physical construction of the system is briefly described in this paragraph.

The following system components are "off the shelf"- items:

a. Computer
b. Monochrome television receiver
c. Graphic printer
d. Tape recorder
e. Heating cylinder
f. Solenoid valves and piping

The computer parallel interface was built on Vero-board using wire-wrap interconnections. It was packaged in a plastic enclosure and plugged directly into the computer's edge connector.

The computer parallel interface connected by means of a flat ribbon cable to a DIL-socket inside a steel enclosure which housed the following circuitry:

a. The A/D and D/A conversion plus signal conditioning circuitry.
b. The precision voltage reference.
c. The digital output buffer which actuates the disturbance valve relay.
d. The liquid level control circuitry.
e. The relays which control the supply to the solenoid valves.
f. The dc-power supplies.

The circuits were built on Vero-board using soldered wire inter-connections and interfaced by means of shielded twisted pairs to the respective sensors and actuators.

The phase angle controller was packaged in a seperate steel enclosure to minimize RF-interference. The circuit was built on Vero-board. The triac was mounted on a heat sink inside the enclosure.

Figure 4.4.1 shows a photograph of the electronic equipment. The computer parallel interface is to the left of the computer. The flat ribbon cable is connected to the enclosure to the top of the computer. The phase angle controller is to the right of the television receiver.

Figure 4.4.2 shows a photograph of the heating cylinder with associated cabling and piping.

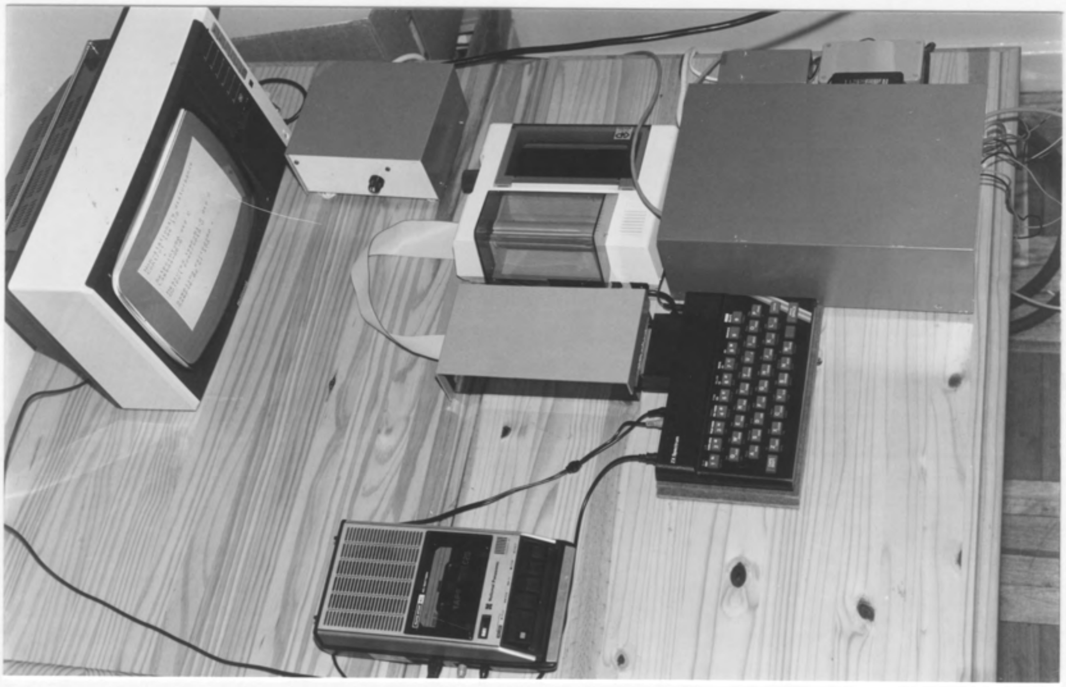


Figure 4.4.1 Photograph of the System Electronic Equipment

Figure 4.4.2 Photograph of the Heating Cylinder

CHAPTER 5

RESULTS OBTAINED BY SIMULATION

5.0 SCOPE

Having derived theoretically the control laws which respectively minimize cost functions 1, 2 and 3 (henceforth referred to as control laws 1, 2 and 3) in Chapter 3, the performance of the respective control laws was evaluated by computer simulation using the computer programs documented in Chapter 4.

The following aspects were evaluated :

a. The quality of control provided respectively by control law 2 (being a general form of control law 1) and control law 3, given accurate process and disturbance models. Stationary as well as non-stationary disturbances were simulated. The results appear in paragraph 5.1.

b. The quality of control provided respectively by control laws 2 and 3, given an accurate process model but an inaccurate disturbance model. The disturbance model in each case was inaccurate in the sense that when the simulated disturbance model was stationary, the actual simulated disturbance was non-stationary and vice versa. The results appear in paragraph 5.2.

c. The quality of control provided respectively by control laws 2 and 3 given an accurate disturbance model but an inaccurate process model. The process model inaccuracy was realized by incorporating an arbitrary process model parameter error. The results appear in paragraph 5.3.

A summary of the results obtained by simulation is given in paragraph 5.4.

Each simulation was conducted over 2000 sampling points simulating a sampling period of 60 seconds. In total 60 simulation runs, each requiring approximately 30 minutes of computer processing time, were made.

The system parameters specified in Tabel 3.0.1 were used as model parameters. The width of the probability density function of the discrete stationary uncorrelated random sequence ξ_t was 0,0135 Kg/s for all the simulations.

5.1 RESULTS OBTAINED BY SIMULATION USING ACCURATE PROCESS AND DISTURBANCE MODELS

5.1.1 Time Responses Obtained

Figure 5.1.1 shows the input and output time responses obtained by simulation using cost function 2, a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.21) and cost factor λ equal to 2×10^{-6}. The variance of input u is 87290 W^2. Output y has a variance of 0,803 $°C^2$ and a mean of -0,009 °C.

Regarding the interpretation of the sample autocorrelation function of output y shown in Figure 5.1.2, there are three types of sample autocorrelation functions, [6] namely :

a. A function which dies down in a damped non-oscillatory exponential fashion.

b. A function which dies down in a damped oscillatory exponential fashion.

c. A function which dies down in a damped sine-wave fashion.

If the sample autocorrelation function of a time series dies away relatively quickly, the time series may be assumed to be stationary, implying that it has a constant mean. If the sample autocorrelation function of a time series dies away slowly, the time series may be assumed to be non-stationary, implying that it has a drifting mean.

The sample autocorrelation function shown in Figure 5.1.2 has a quick sinusoidal decay and the corresponding output time response shown in Figure 5.1.1 is therefore stationary.

Figure 5.1.3 shows the input and output time responses obtained by simulation using cost function 2, a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.25)) and cost factor λ equal to 2 x 10^{-6}. The variance of input u is 380550 W^2. Output y has a variance of 0,841 °C^2 and a mean of 0,427 °C.

The sample autocorrelation function shown in Figure 5.1.4 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 5.1.3 is therefore non-stationary.

Figure 5.1.5 shows the input and output time responses obtained by simulation using cost function 3, a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.22)) and cost factor λ equal to 2 x 10^{-6}. The variance of input u is 169601 W^2. Output y has a variance of 0,869 °C^2 and a mean of -0,017 °C.

The sample autocorrelation function shown in Figure 5.1.6 has a quick sinusoidal decay and the corresponding output time response shown in Figure 5.1.5 is therefore stationary.

Figure 5.1.7 shows the input and output time responses obtained by simulation using cost function 3, a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.26)) and cost factor λ equal to 2 x 10^{-6}.

The variance of input u is 392784 W^2. Output y has a variance of 0,225 $°C^2$ and a mean of -0,001 °C.

The sample autocorrelation function shown in Figure 5.1.8 has a slow sinusoidal decay and the corresponding output time response shown in Figure 5.1.7 is therefore non-stationary.

Table 5.1.1 tabulates the variance of input u and the variance and mean of output y obtained by simulation for λ equal to 2×10^{-6} using accurate process and disturbance models.

Table 5.1.2 tabulates the nature of the sample autocorrelation function and the output time response for the above cases.

The following observations can be made:

a. When the disturbance acting on the system was non-stationary and modelled as such, control law 3 provided a better quality of control than did control law 2. Control law 2 provided an output y which had a relatively large mean error.

b. When the disturbance acting on the system was stationary and modelled as such, control law 2 provided a marginally better quality of control than did control law 3.

c. In all cases, the nature of the output time response obtained was the same as that of the disturbance and disturbance model.

Table 5.1.1 Variance of Input u and Variance and Mean of Output y Obtained by Simulation for λ Equal to 2 x 10^{-6} Using Accurate Process and Disturbance Models

Control Law	Nature of Disturbance and Disturbance Model	Variance of Input u (W^2)	Variance of Output y (°C^2)	Mean of Output y (°C)
2	Stationary	87290	0,803	-0,009
2	Non-Stationary	380550	0,841	0,427
3	Stationary	169601	0,869	-0,017
3	Non-Stationary	392784	0,255	-0,001

Table 5.1.2 Nature of the Sample Autocorrelation Function and the Output Time Series Obtained by Simulation for λ Equal to 2 x 10^{-6} Using Accurate Process and Disturbance Models

Control Law	Nature of Disturbance and Disturbance Model	Nature of Sample Autocorrelation Function	Nature of Output Time Response
2	Stationary	Quick sinusoidal decay	Stationary
2	Non-Stationary	Slow non-oscillatory exponential decay	Non-Stationary
3	Stationary	Quick sinusoidal decay	Stationary
3	Non-Stationary	Slow sinusoidal decay	Non-Stationary

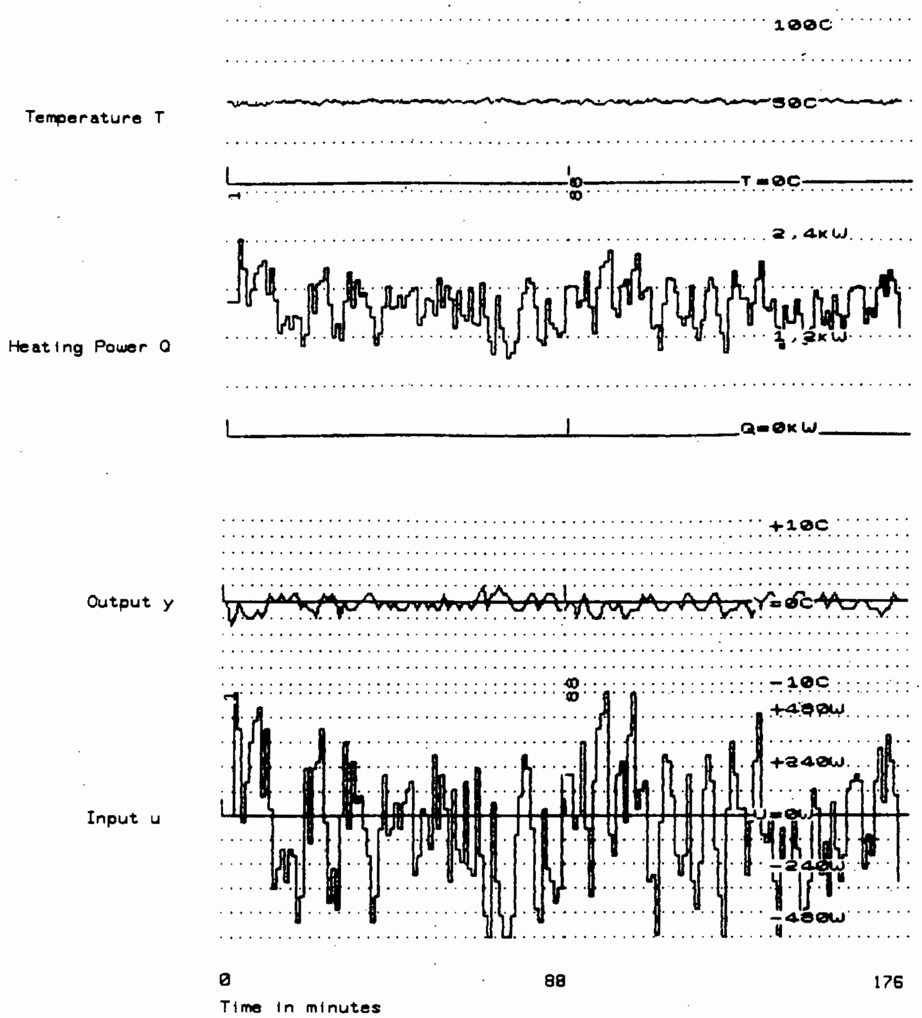


Figure 5.1.1. Input and Output Time Responses by Simulation using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Stationary

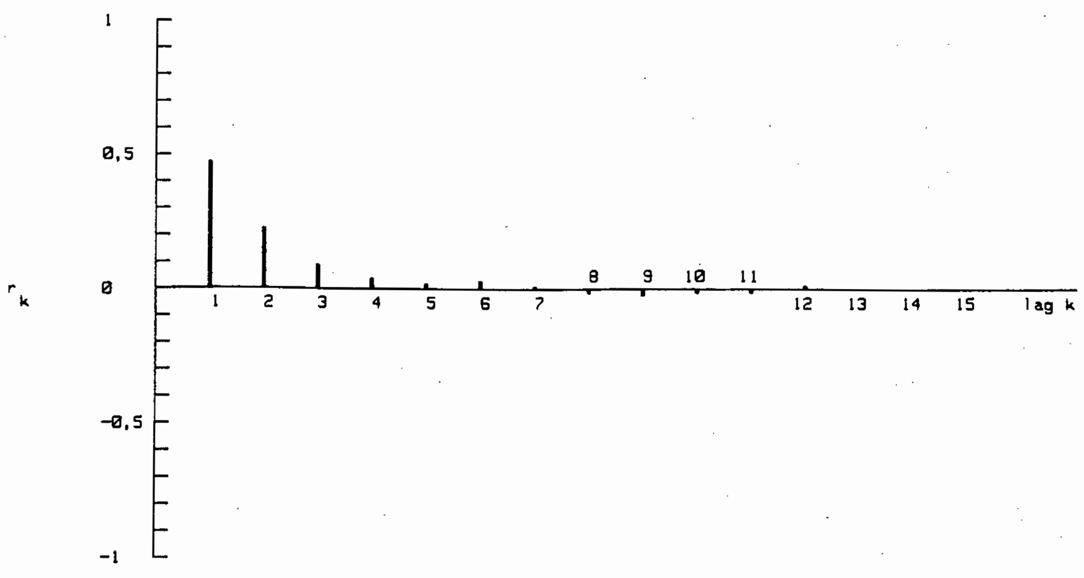


Figure 5.1.2. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

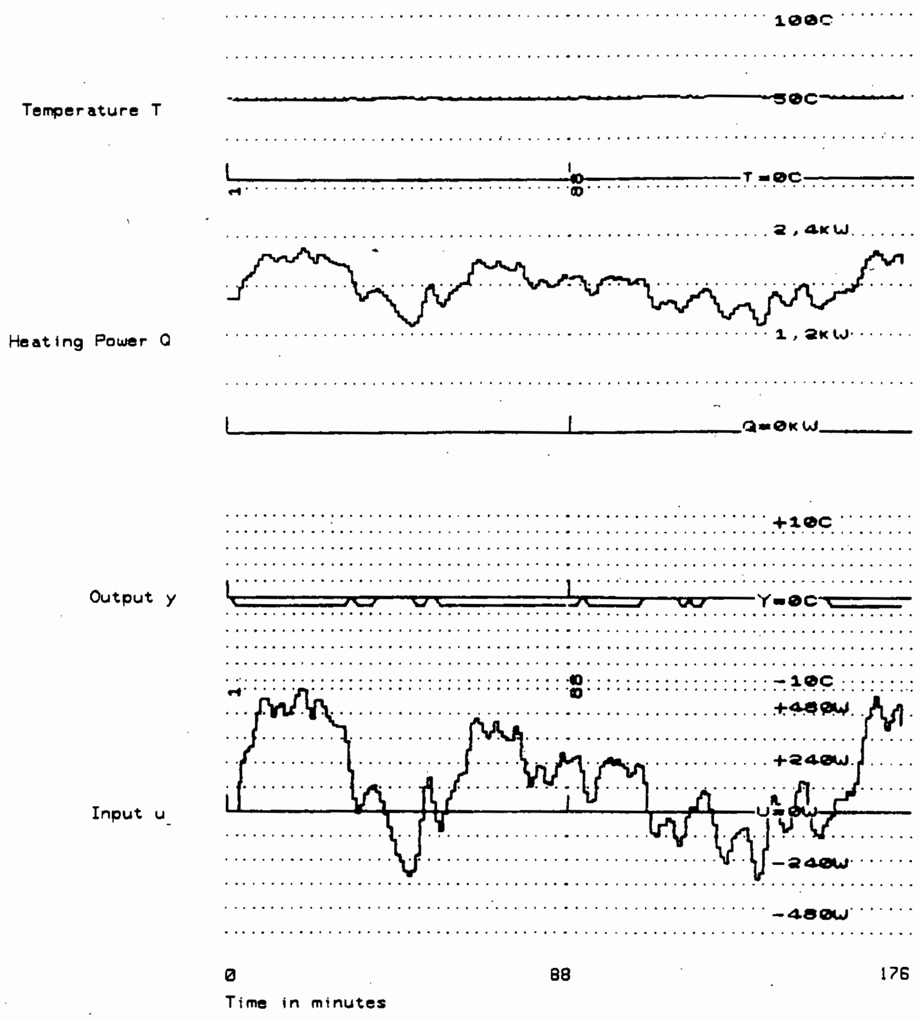


Figure 5.1.3. Input and Output Time Responses by Simulation using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Non-stationary

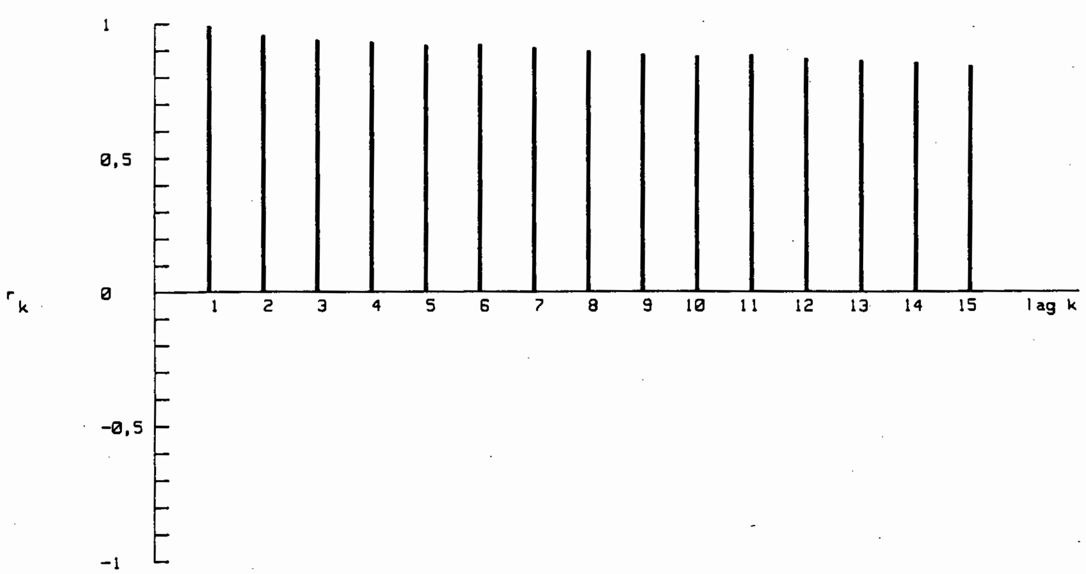


Figure 5.1.4. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

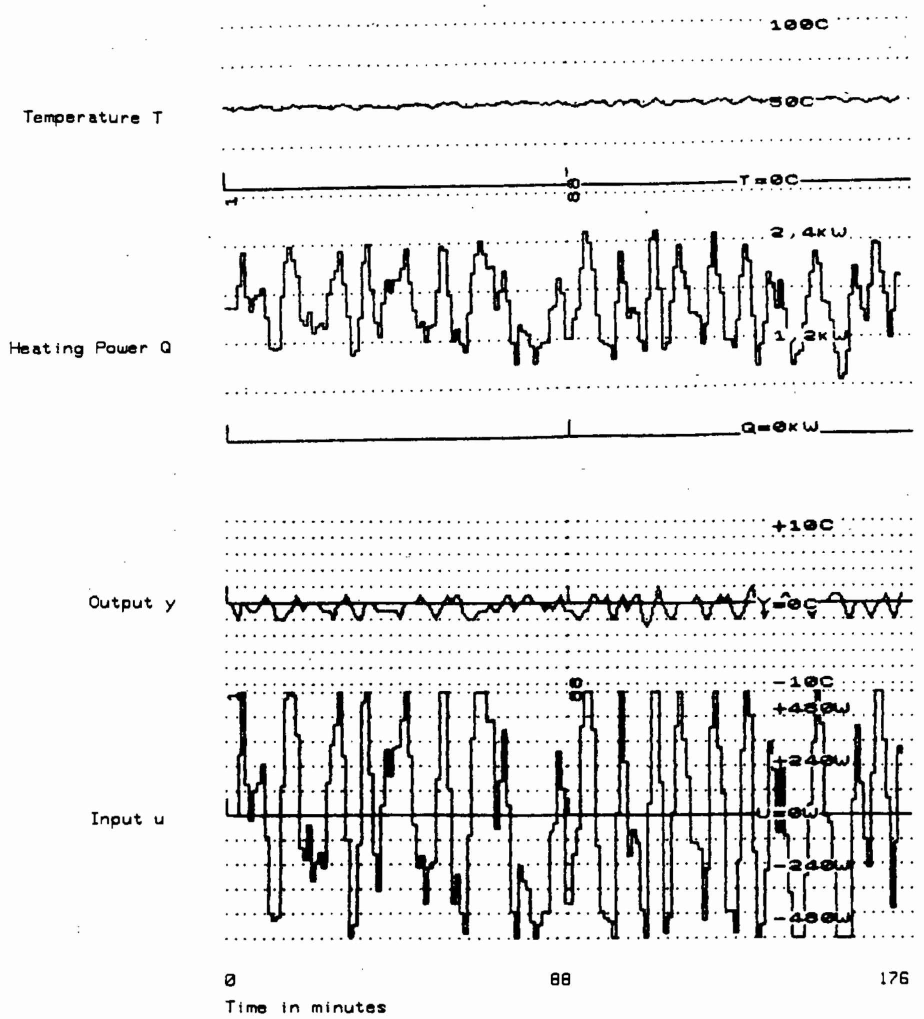


Figure 5.1.5. Input and Output Time Responses by Simulation using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Stationary

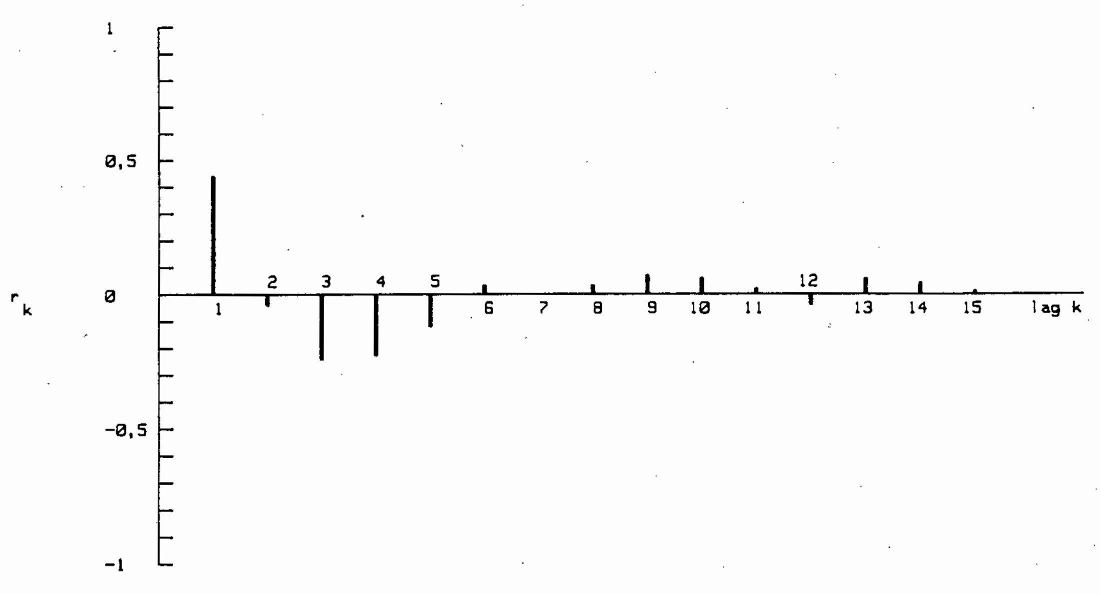


Figure 5.1.6. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

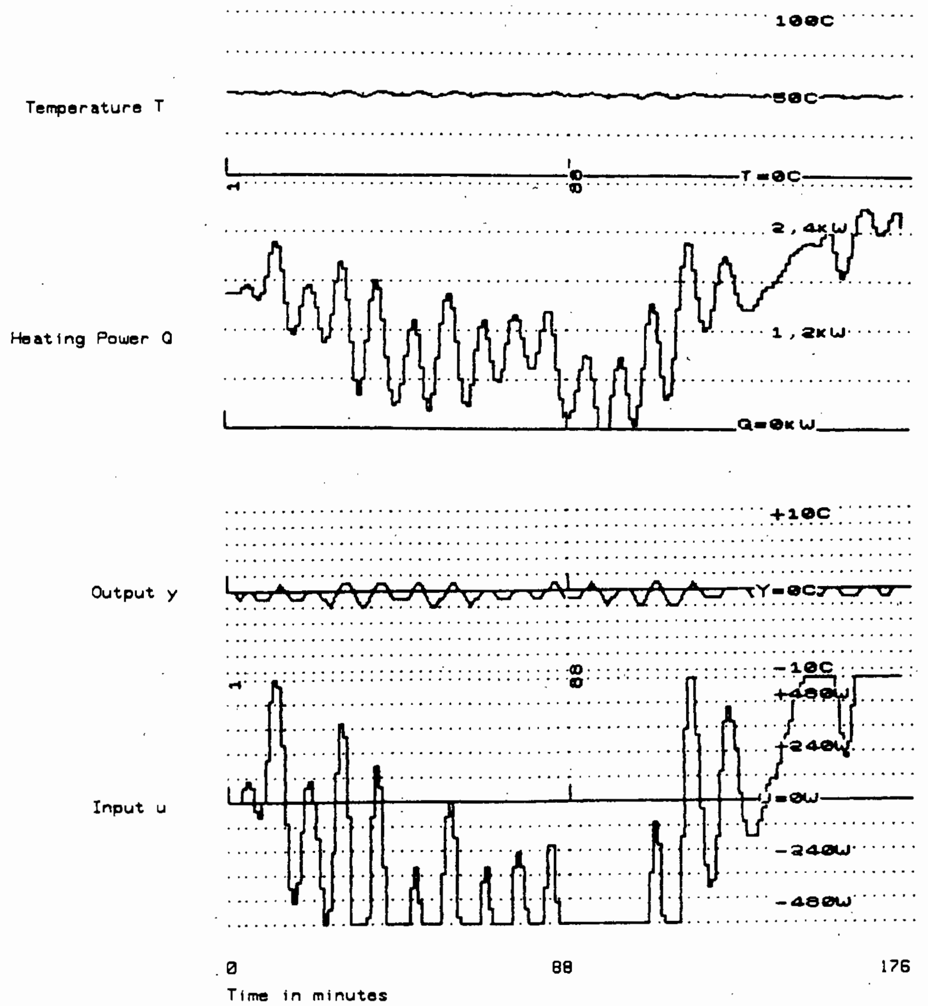


Figure 5.1.7. Input and Output Time Responses by Simulation using Cost Function 3, a Non-Stationary Disturbance Model and with the actual Disturbance Non-stationary

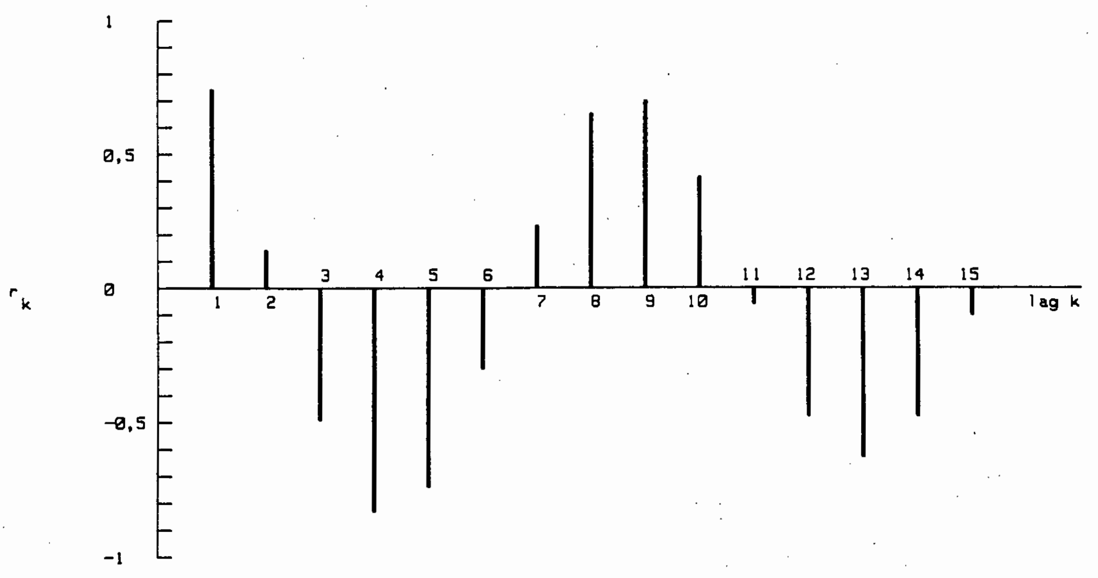


Figure 5.1.8. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

5.1.2 Variance of Input u and Output y as a Function of Cost Factor λ

Figure 5.1.9 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 2 and a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.21)). This shows that the variance of control input u can be significantly reduced from the variance of input u obtained by using control law 1 (λ equal to zero) with only a small increase in the variance of output y by using a value for λ larger then zero. This agrees with the results obtained by Clarke at al [1].

Figure 5.1.10 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 2 and a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.25)). This shows that the variance of output y increases sharply with increasing values of λ larger then zero. This provides additional support to the claim that a controller designed to mimimize control law 2 has difficulty to stabilise the output of a system subjected to a non-stationary disturbance. However, control law 1 (λ equal to zero) provides a small variance of output y.

Figure 5.1.11 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 3 and a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.22)). This shows that it is not possible to significantly reduce the variance of control input u without significantly increasing the variance of output y by using a value larger than zero for λ .

Figure 5.1.12 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 3 and a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.26)). Using a value for λ larger than zero in this case, does not reduce the variance of control input u. This is because control input u obtained by minimizing control law 3 has the inherent nature to drift. However, the rate of change of control input u is reduced with increasing values of λ. This may be of

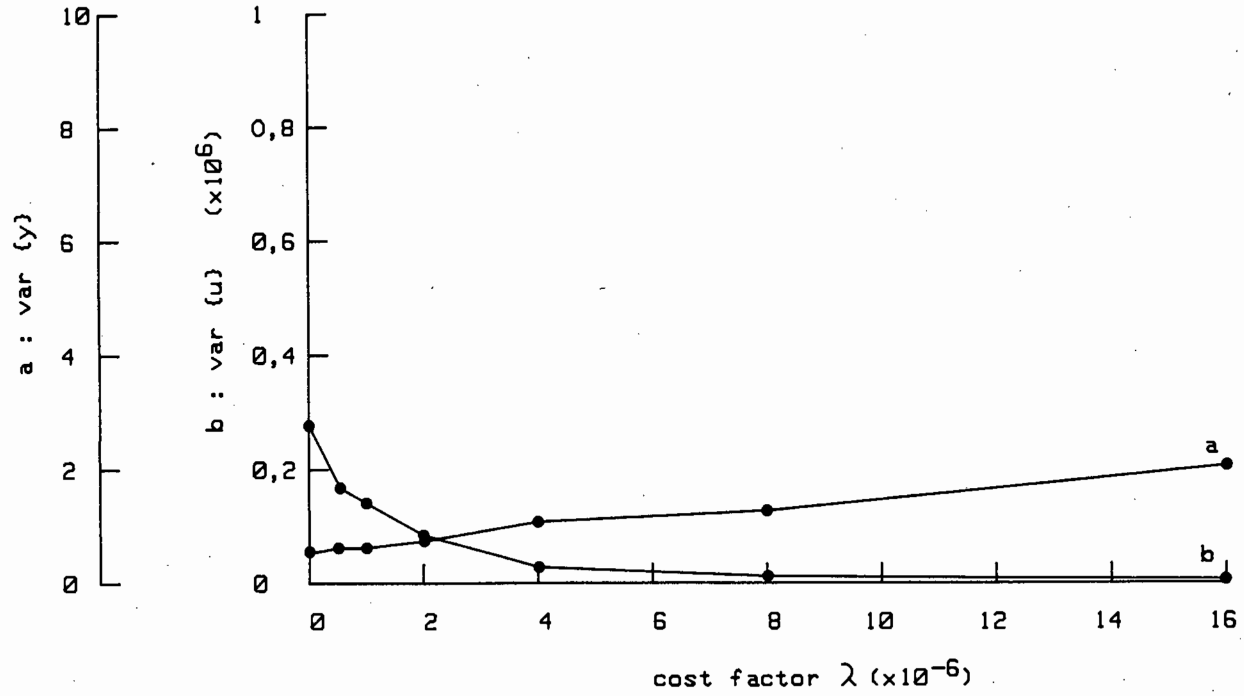


Figure 5.1.9. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Stationary

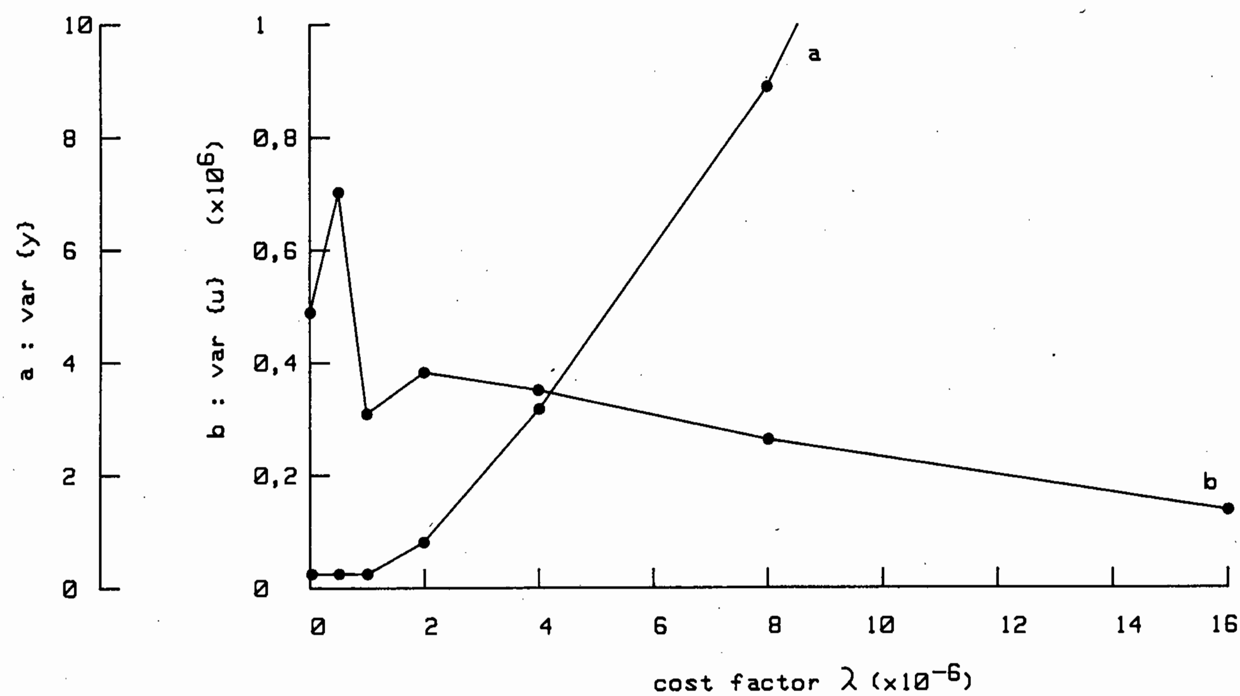


Figure 5.1.10. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Non-stationary

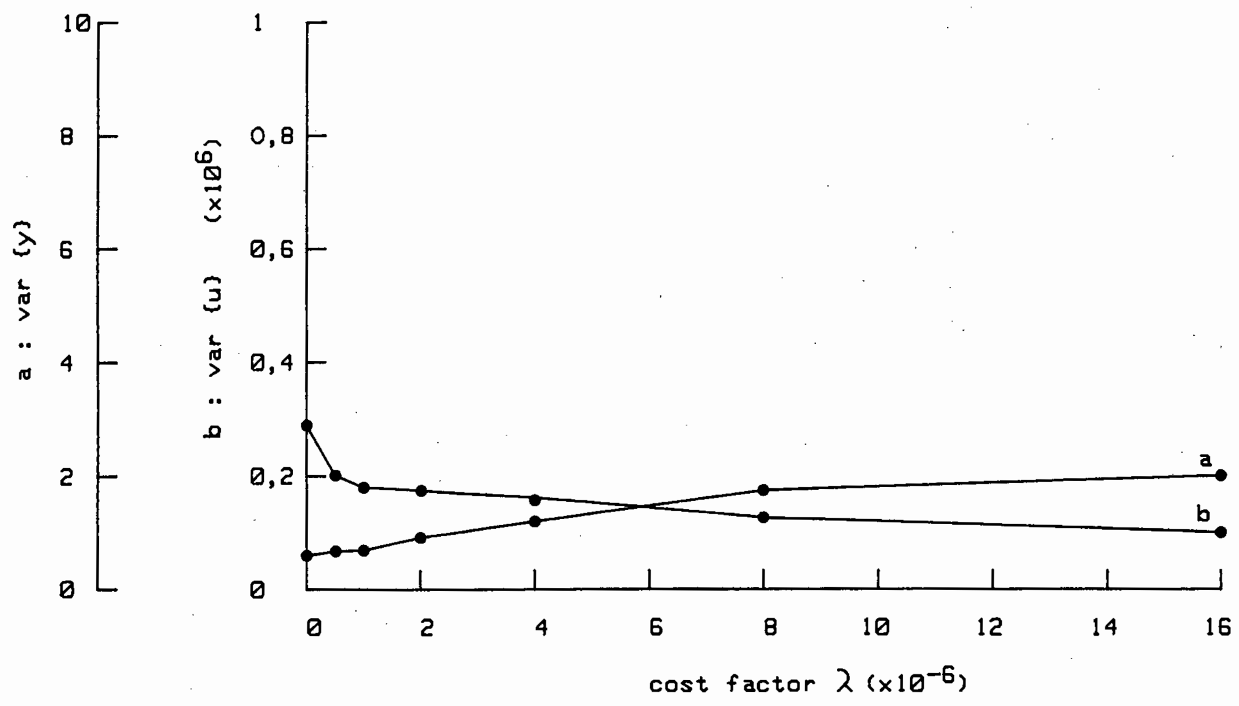


Figure 5.1.11. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Stationary

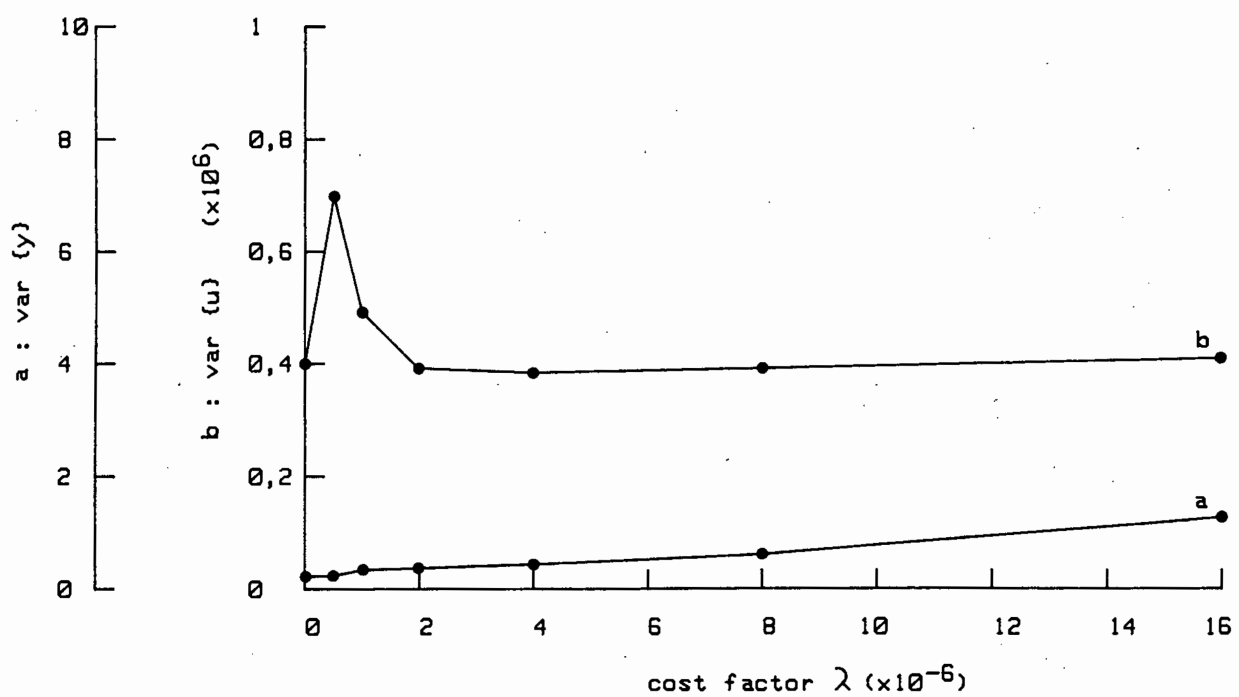


Figure 5.1.12. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 3, a Non-stationary Disturbance Model and with the actual Disturbance Non-stationary

importance when saturation of the control transducer with the corresponding introduction of non-linearities is likely to occur. A larger value for λ may reduce the chances of this happening.

5.2 RESULTS OBTAINED BY SIMULATION USING AN ACCURATE PROCESS MODEL BUT AN INACCURATE DISTURBANCE MODEL

5.2.1 Time Responses Obtained

As stated in paragraph 5.0, the disturbance model in each case was inaccurate in the sense that when the simulated disturbance model was stationary, the actual simulated disturbance was non-stationary and vice versa.

Figure 5.2.1 shows the input and output time responses obtained by simulation using cost function 2, a stationary disturbance model (Equation (3.2.21)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was non-stationary (Figure 3.1.4). The variance of input u is 523989 W^2. Output y has a variance of 5,242 $°C^2$ and a mean of -0,385 °C.

The sample autocorrelation function shown in Figure 5.2.2 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 5.2.1 is therefore non-stationary.

Figure 5.2.3 shows the input and output time responses obtained by simulation using cost function 2, a non-stationary disturbance model (Equation (3.2.25)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was stationary (Figure 3.1.3). The variance of input u is 159861 W^2. Output y has a variance of 0,788 $°C^2$ and a mean of 0,005 °C.

The sample autocorrelation function shown in Figure 5.2.4 has a quick sinusoidal decay and the corresponding output time response shown in Figure 5.2.3 in therefore stationary.

Figure 5.2.5 shows the input and output time responses obtained by simulation using cost function 3, a stationary disturbance model (Equation (3.2.22)) and cost factor λ equal to 2 x 10^{-6} while the actual disturbance was non-stationary (Figure 3.1.4). The variance of input u is 477002 W^2. Output y has a variance of 1,126 $°C^2$ and a mean of 0,499 °C.

The sample autocorrelation function shown in Figure 5.2.6 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 5.2.5 is therefore non-stationary.

Figure 5.2.7 shows the input and output time responses obtained by simulation using cost function 3, a non-stationary disturbance model (Equation (3.2.26)) and cost factor λ equal to 2 x 10^{-6} while the actual disturbance was stationary (Figure 3.1.3). The variance of input u is 782254 W^2. Output y has a variance of 5,411 $°C^2$ and a mean of -0,001 °C.

The sample autocorrelation function shown in Figure 5.2.8 has a slow sinusoidal decay and the corresponding output time response shown in Figure 5.2.7 in therefore non-stationary.

Table 5.2.1 tabulates the variance of input u and the variance and mean of output y obtained by simulation for λ equal to 2 x 10^{-6} using an accurate process model but inaccurate disturbance models.

Table 5.2.2 tabulates the nature of the sample autocorrelation function and the output time responses for the above cases.

The following observations can be made :

a. When the disturbance acting on the system was non-stationary but modelled as stationary, control law 3 provided a better quality of control than did control law 2. Control law 2 provided a very poor quality of control in this case.

b. When the disturbance acting on the system was stationary but modelled as non-stationary, control law 2 provided a better quality of control than did control law 3. Control law 3 provided a very poor quality of control in this case.

c. Referring to Tables 5.1.2 and 5.2.2, the only case where the nature of the output time responses was not the same as that of the actual disturbance, was for the case when control law 3 and a non-stationary disturbance model was used while the actual disturbance was stationary. This may be interpreted as an inability on the part of this controller to stably control the system.

Table 5.2.1 Variance of Input u and Variance and Mean of Output y Obtained by Simulation for λ Equal to 2×10^{-6} Using an Accurate Process Model but with the Disturbance Model Inaccurate as Defined

Control Law	Nature of Disturbance Model	Nature of Actual Disturbance	Variance of Input u (W^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)
2	Stationary	Non-stationary	523989	5,242	-0,385
2	Non-Stationary	Stationary	159861	0,788	0,005
3	Stationary	Non-Stationary	477002	1,126	0,499
3	Non-Stationary	Stationary	782254	5,411	-0,001

Table 5.2.2 Nature of the Sample Autocorrelation Function and the Output Time Series Obtained by Simulation for λ Equal to 2×10^{-6} Using an Accurate Process Model but with the Disturbance Model Inaccurate as Defined

Control Law	Nature of Disturbance Model	Nature of Actual Disturbance	Nature of Sample Autocorrelation Function	Nature of Output Time Response
2	Stationary	Non-Stationary	Slow non-oscillatory exponential decay	Non-Stationary
2	Non-Stationary	Stationary	Quick sinusoidal decay	Stationary
3	Stationary	Non-Stationary	Slow non-oscillatory exponential decay	Non-Stationary
3	Non-Stationary	Stationary	Slow sinusoidal decay	Non-Stationary

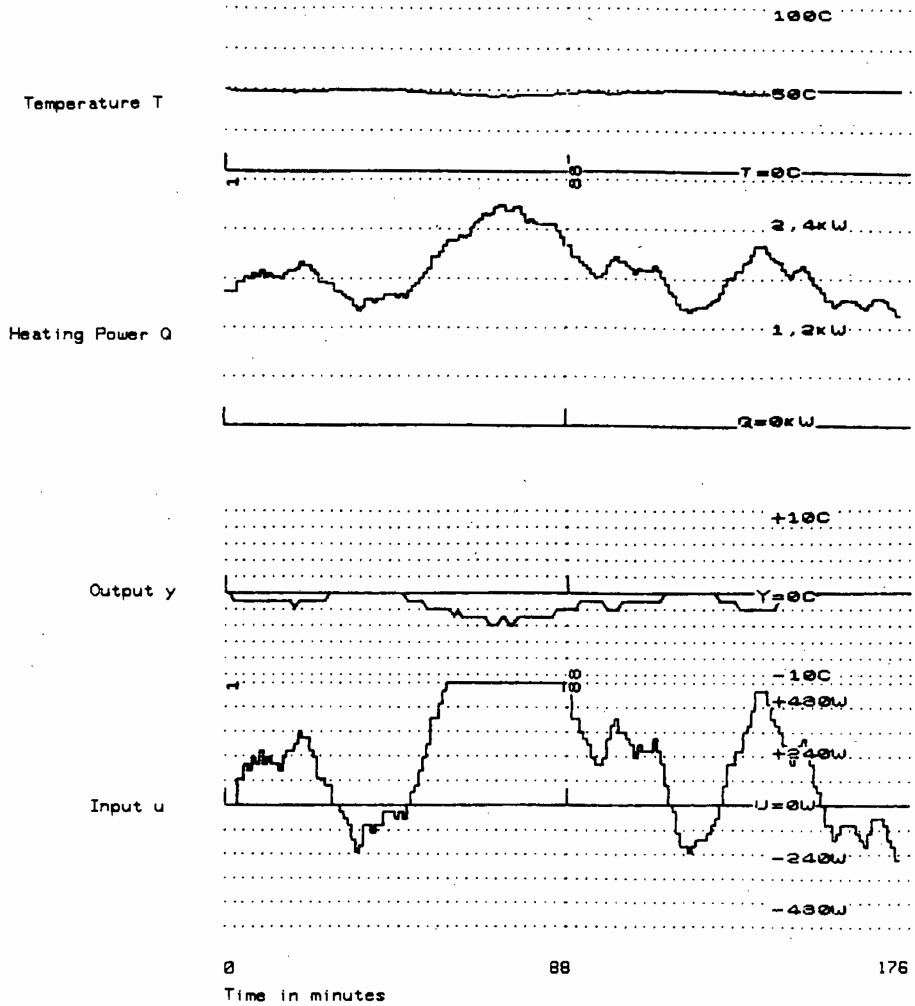


Figure 5.2.1. Input and Output Time Responses by Simulation using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

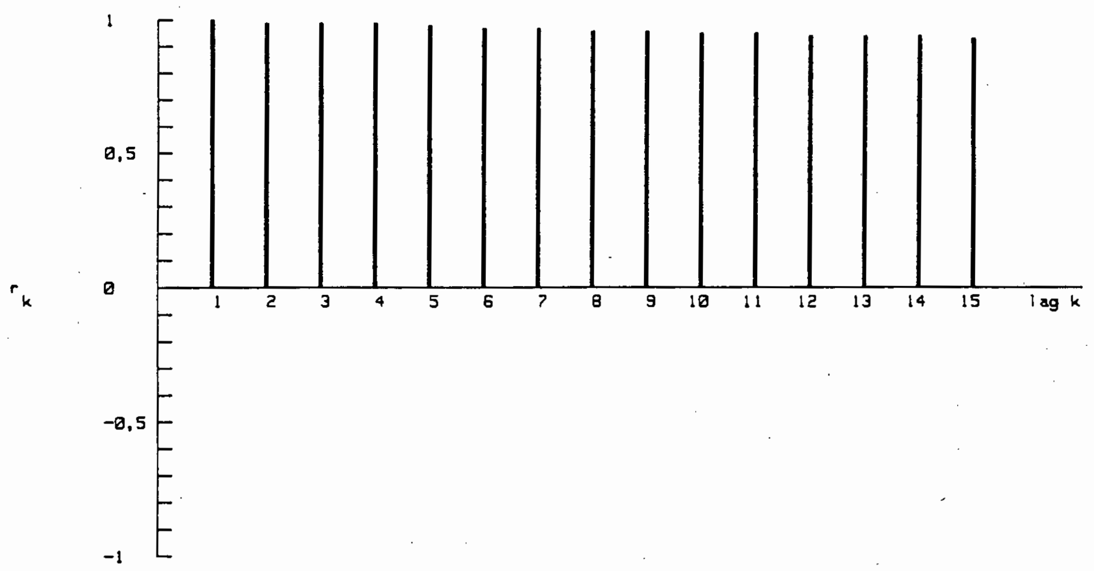


Figure 5.2.2. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

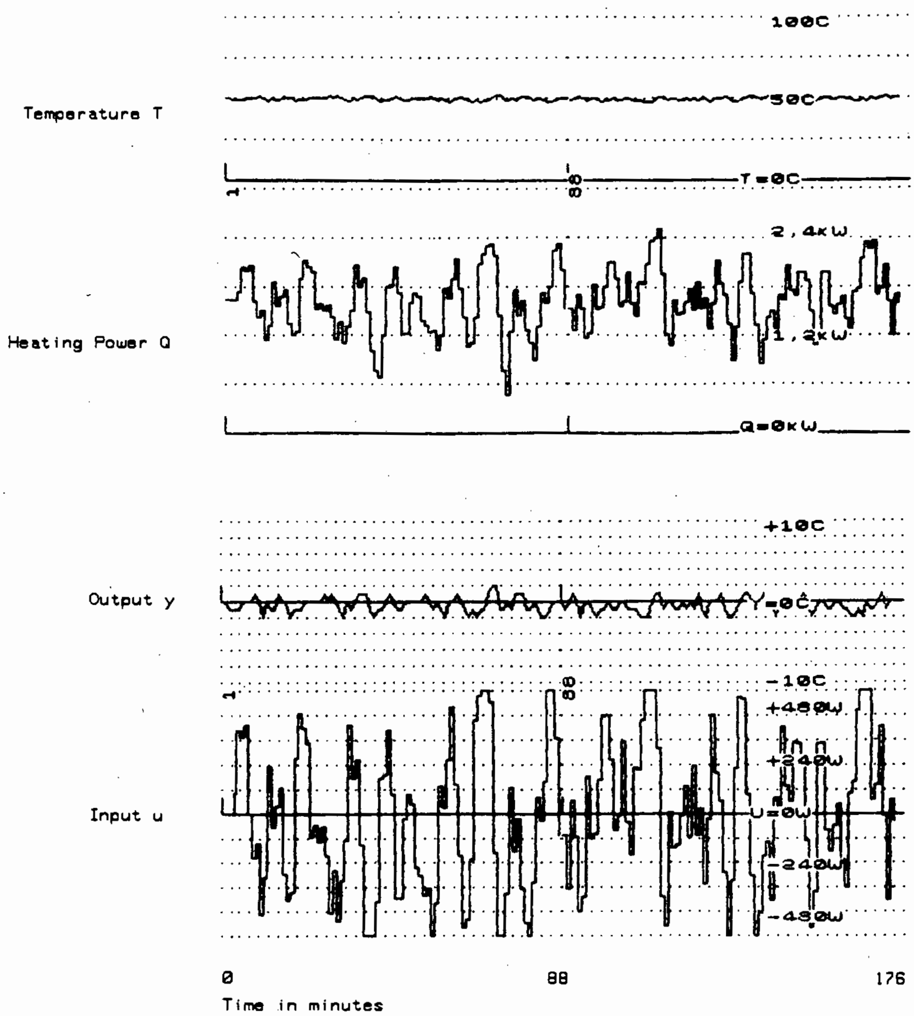


Figure 5.2.3. Input and Output Time Responses by Simulation using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

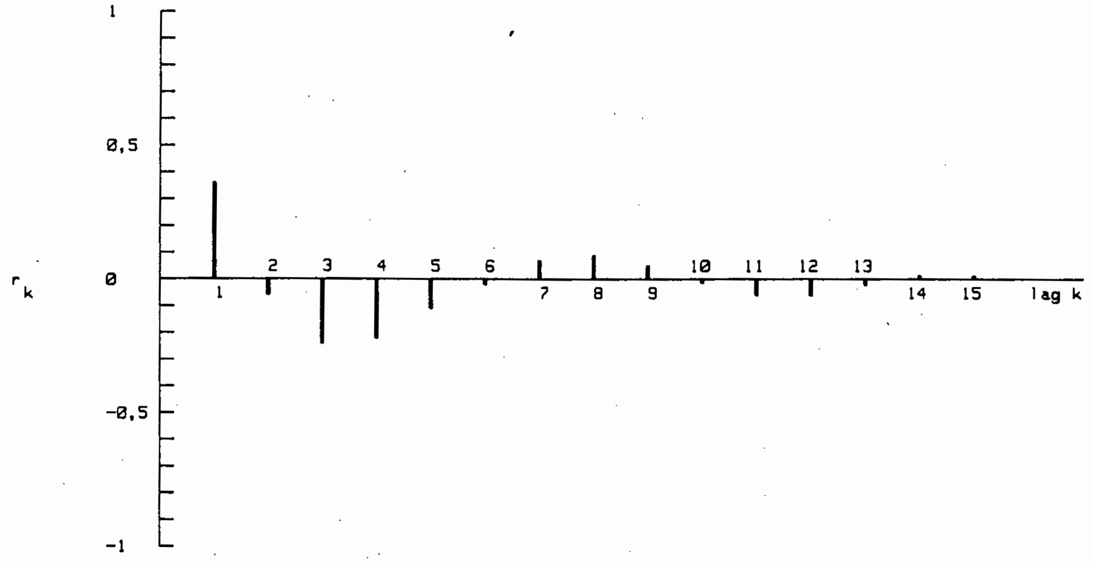


Figure 5.2.4. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

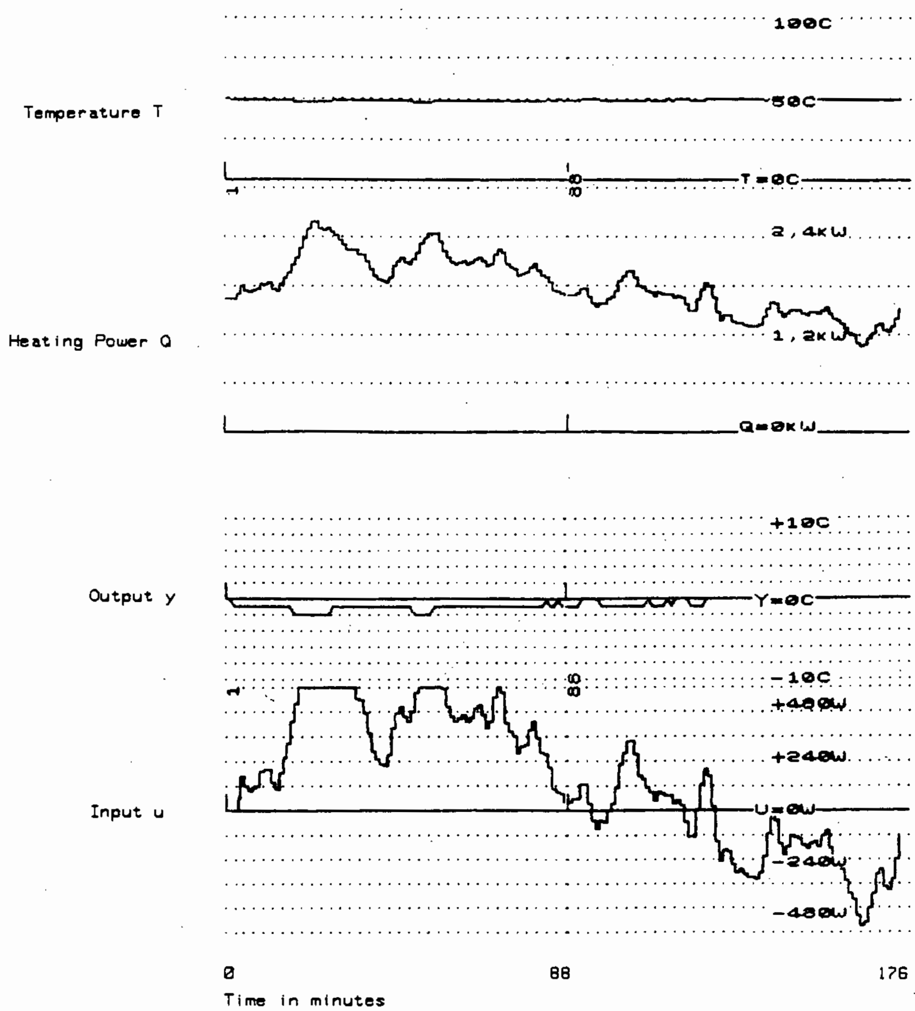


Figure 5.2.5. Input and Output Time Responses by Simulation using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

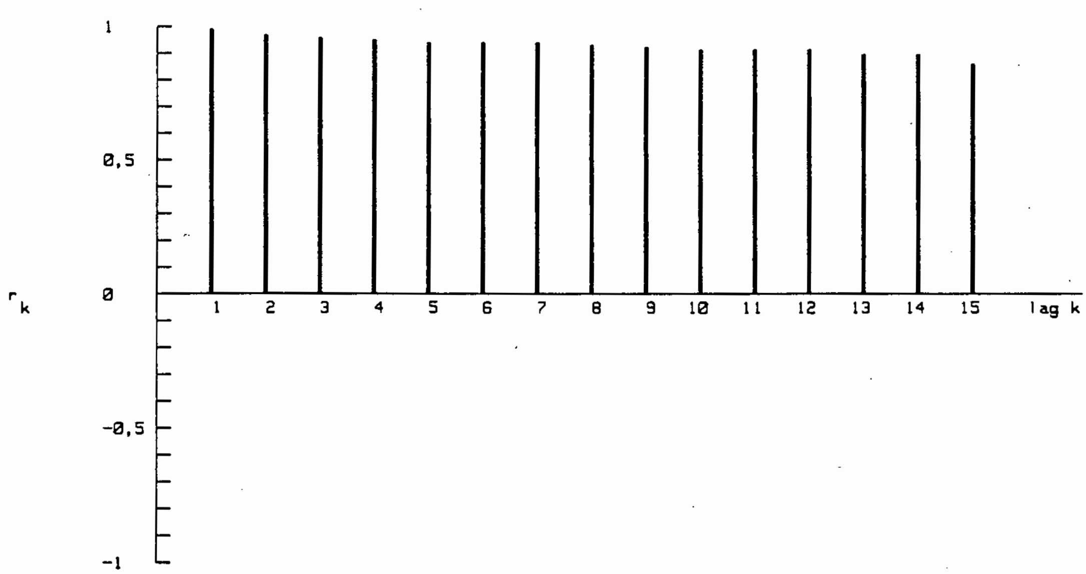


Figure 5.2.6. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

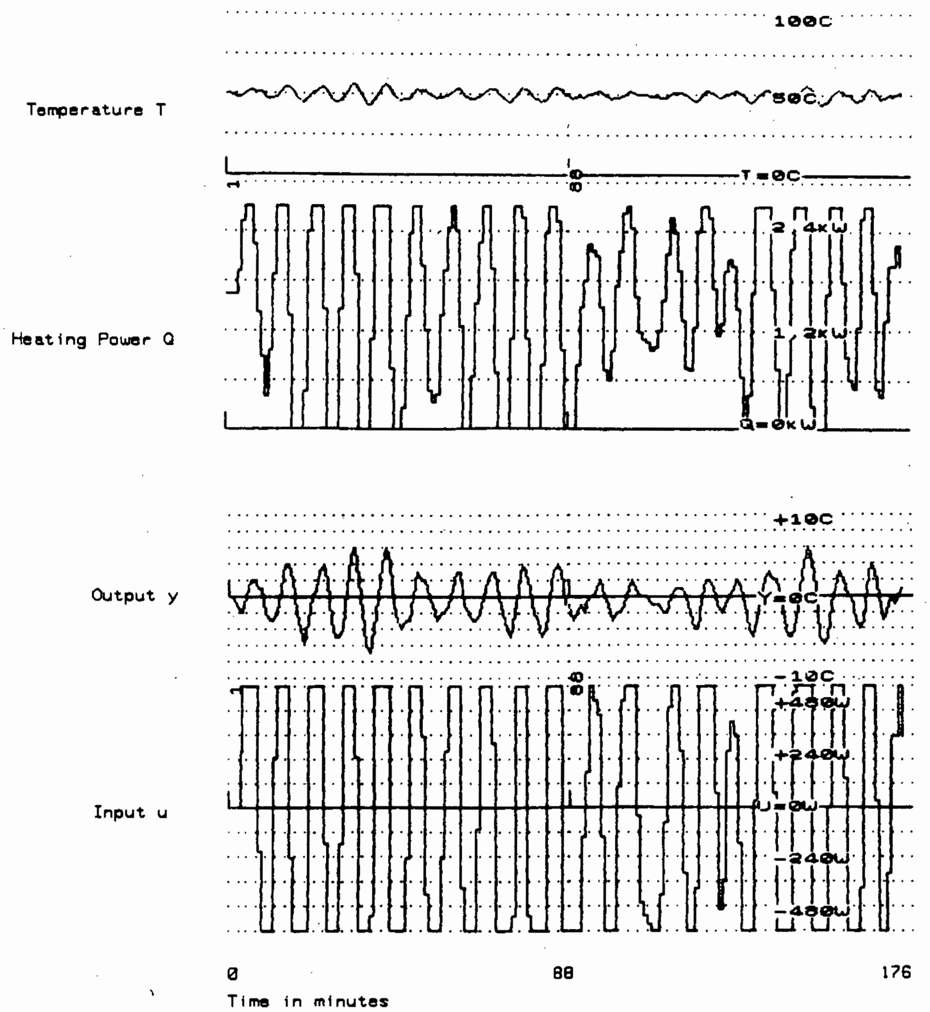


Figure 5.2.7. Input and Output Time Responses by Simulation using Cost Function 3, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

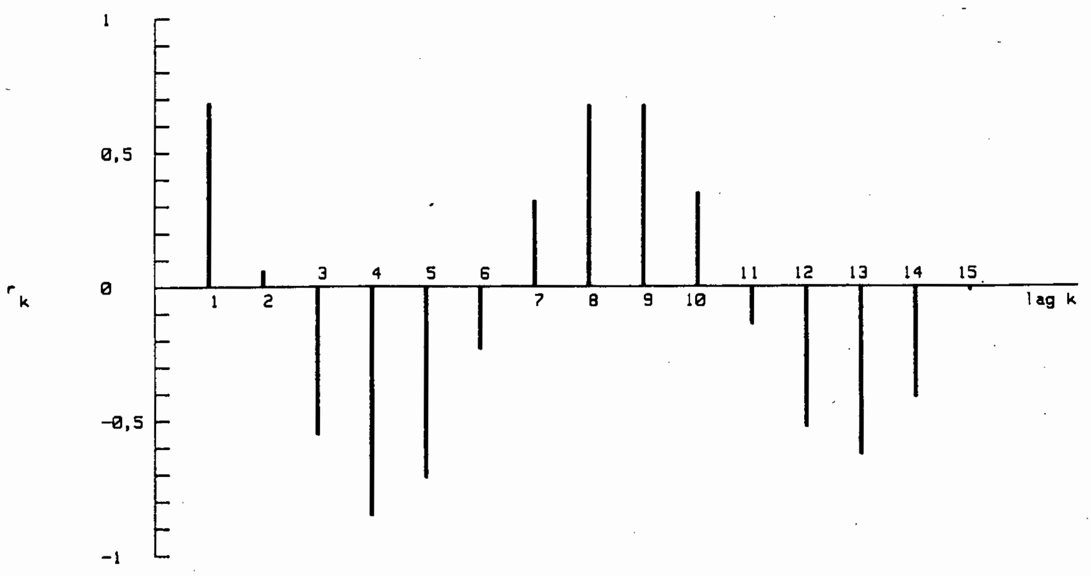


Figure 5.2.8. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

5.2.2 Variance of Input u and Output y as a Function of Cost Factor λ

Figure 5.2.9 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 2 and a stationary disturbance model (Equation (3.2.21)) while the disturbance was actually non-stationary (Figure 3.1.4). This shows that the variance of output y increases sharply with increasing values of λ . This is similar to the characteristics obtained using cost function 2 and a non-stationary disturbance and disturbance model shown in Figure 5.1.10.

Figure 5.2.10 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 2 and a non-stationary disturbance model (Equation (3.2.25)) while the disturbance was actually stationary (Figure 3.1.3). The poor control quality provided by control law 1 (λequal to zero), may be the result of non-linearities introduced by control element saturation.

Figure 5.2.11 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 3 and a stationary disturbance model (Equation (3.2.22)) while the disturbance was actually non-stationary (Figure 3.1.4). This shows that the variance of the output y is relatively small and varies little over the range of values for λ shown.

Figure 5.2.12 shows the variance of input u and output y as a function of cost factor λ obtained by simulation using cost function 3 and a non-stationary disturbance model (Equation (3.2.26)) while the disturbance was actually stationary (Figure 3.1.3). This shows that the quality of control is extremely poor for all values of λ shown.

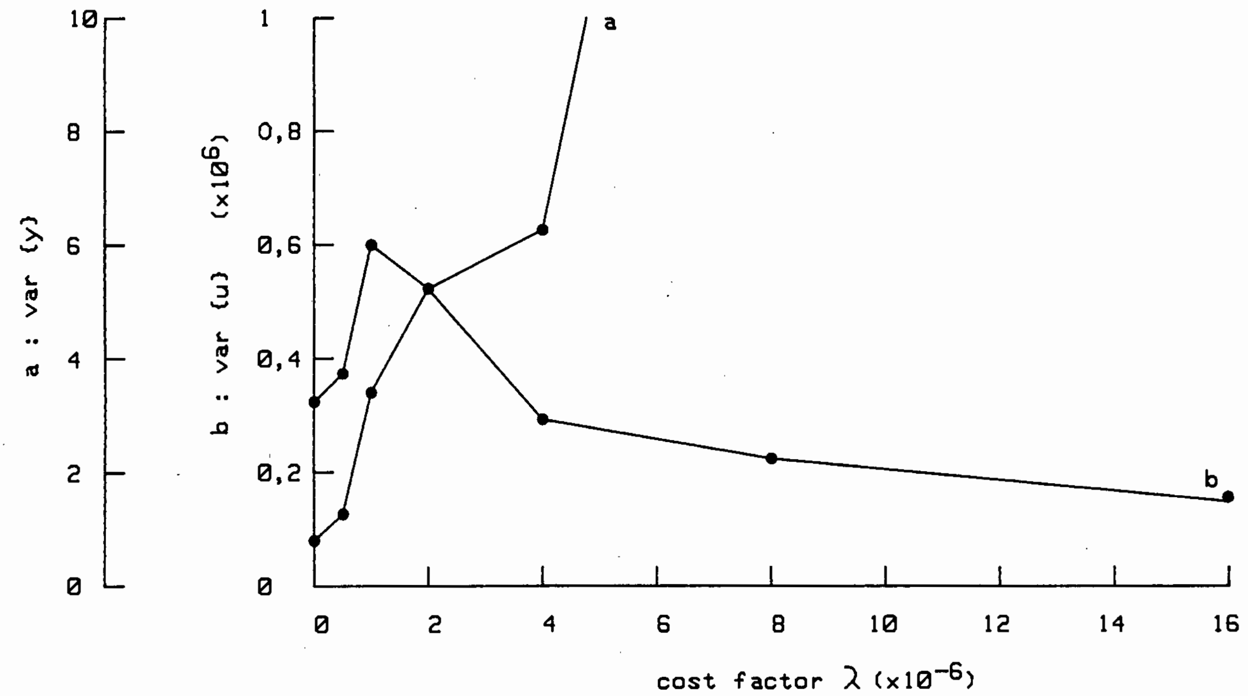


Figure 5.2.9. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

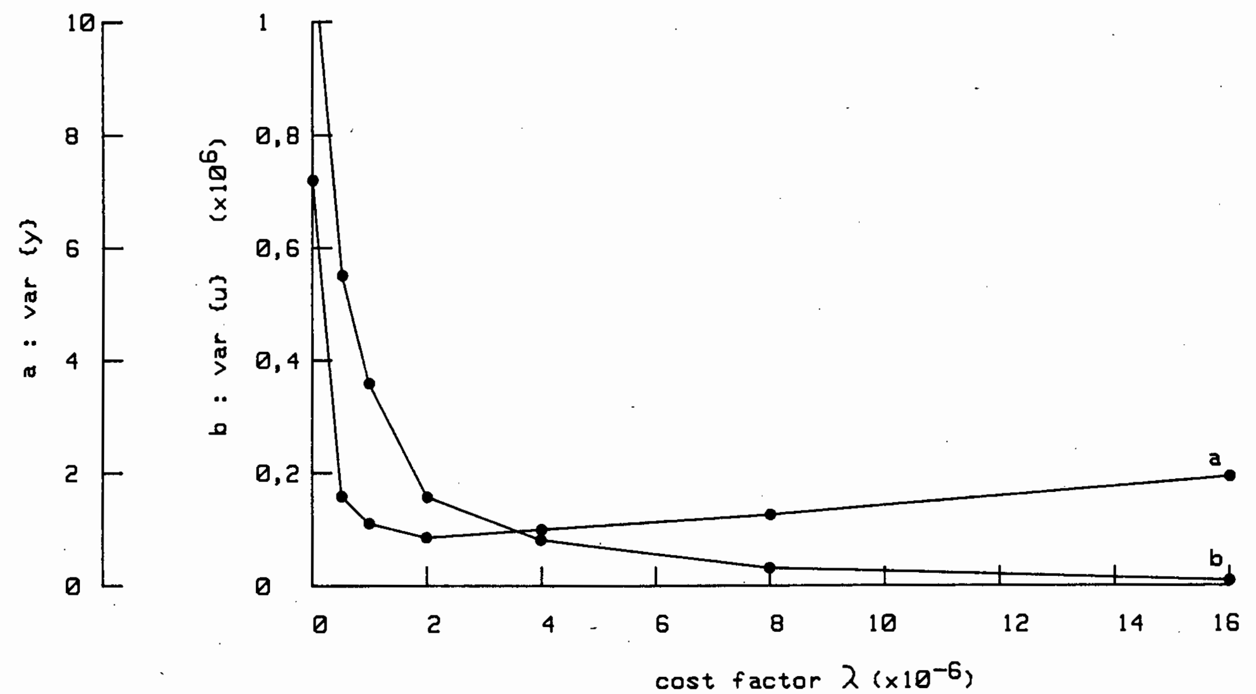


Figure 5.2.10. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

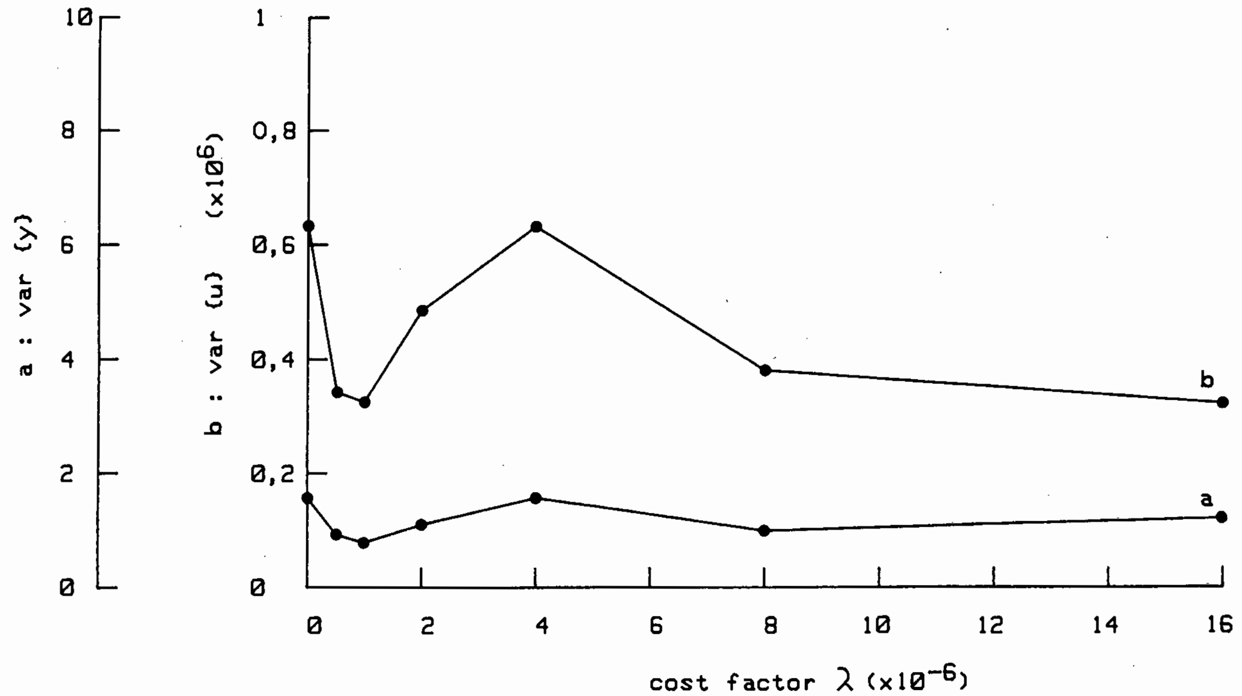


Figure 5.2.11. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

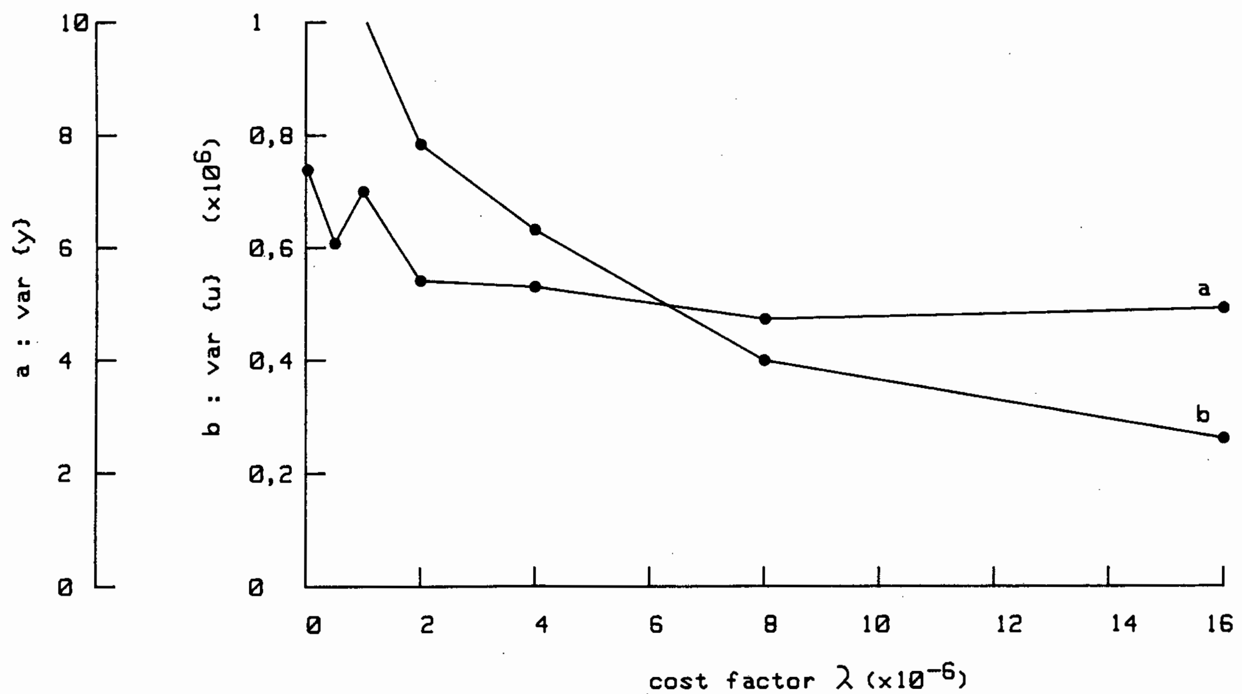


Figure 5.2.12. Control u and Output y Variances as a Function of the Cost Factor λ by Simulation using Cost Function 3, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

5.3 RESULTS OBTAINED BY SIMULATION USING AN ACCURATE DISTURBANCE MODEL BUT AN INACCURATE PROCESS MODEL

The aim of doing these simulations was to evaluate the effect of an arbitrary process model inaccuracy in the quality of control provided by the respective controllers. The actual mass of water in the cylinder was simulated to be 13 kg instead of the model value of 10 kg.

Using control law 2 and a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.21)), a variance of 100470 W^2 for input u and a variance of 0,924 $°C^2$ and mean of -0,009 °C for output y were obtained.

Using control law 2 and a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.25)), a variance of 614980 W^2 for input u and a variance of 1,507 $°C^2$ and mean of -0,480 °C for output y were obtained.

Using control law 3 and a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.22)), a variance of 219827 W^2 for input u and a variance of 1,009 $°C^2$ and mean of 0,008 °C for output y were obtained.

Using control law 3 and a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.26)), a variance of 586766 W^2 for input u and a variance of 0,524 $°C^2$ and mean of 0,000 °C for output y were obtained.

Table 5.3.1 tabulates the percentage increase in the variance of input u and output y relative to the values obtained using accurate process and disturbance models (Table 5.1.1) as a result of the specified process model inaccuracy.

Table 5.3.1 Percentage Increase in the Variance of Input u and Output y Obtained by Simulation for λ Equal to 2×10^{-6} as a Result of the Specified Process Model Inaccuracy (M_{model} = 10 Kg, $M_{process}$ = 13 Kg)

Control Law	Nature of Disturbance and Disturbance model	Percentage Increase in the Variance of Input u	Percentage Increase in the Variance of Output y
2	Stationary	15	15
2	Non-Stationary	62	79
3	Stationary	30	16
3	Non-Stationary	49	106

The variance of output y obtained using control law 2 and a stationary disturbance model has a small sensitivity to the process model inaccuracy. This can be explained by the stability analysis of this controller in paragraph 3.3.1. Since the corresponding characteristic equation has no eigenvalues on the unit circle, model perturbations can be expected to have a smaller effect on the variance of output y than it would have had if there were poles on the unit circle.

The variance of output y obtained using control law 2 and a non-stationary disturbance model has a relatively high sensitivity to the process model inaccuracy. This can be explained by the stability analysis of this controller in paragraph 3.3.2. Since the corresponding characteristic equation has an eigenvalue on the unit circle, the variance of output y can be expected to be sensitive to model perturbations.

The variance of output y obtained using control law 3 and a stationary disturbance model has a small sensitivity to the process model inaccuracy. This can be explained by the stability analysis of this controller in paragraph 3.3.3 which shows that the corresponding characteristic equation has no eigenvalues on the unit circle.

The variance of output y obtained using control law 3 and a non-stationary disturbance model has a high sensitivity to the process model inaccuracy. This can be explained by the stability analysis of the controller in paragraph 3.3.4 which shows that the corresponding characteristic equation has three eigenvalues on the unit circle.

5.4 SUMMARY OF RESULTS OBTAINED BY SIMULATION

The results obtained by simulation are summarized in the following paragraphs.

The following is a summary of the results obtained with the disturbance non-stationary and modelled as such:

a. Table 5.1.1 shows that control law 3 provided a substantially better quality of control than did control law 2. Also compare Figure 5.1.10 and Figure 5.1.12.

b. The relatively large output mean error obtained using control law 2 is attributed to the inability of the mean value of control input u to drift when control law 2 is used.

c. Table 5.3.1 shows that the variance of output y obtained using control law 3 was very sensitive to the simulated process model inaccuracy.

The following is a summary of the results obtained with the disturbance stationary and modelled as such:

a. Table 5.1.1 shows that control law 2 provided a marginally better quality of control than did control law 3. Also compare Figure 5.1.9 and Figure 5.1.11.

b. Table 5.3.1 shows that the variance of output y in both cases was relatively insensitive to the simulated process model inaccuracy.

The following is a summary of the results obtained with the disturbance non-stationary but modelled as stationary:

a. Table 5.2.1 shows that control law 3 provided a substantially better quality of control than did control law 2 which provided a very poor quality of control. Also compare Figure 5.2.9 and Figure 5.2.11.

b. In both cases the mean error was relatively large.

The following is a summary of the results obtained with the disturbance stationary but modelled as non-stationary:

a. Table 5.2.1 shows that control law 2 provided a substantially better quality of control than did control law 3 which provided a very poor quality of control. Also compare Figure 5.2.10 and Figure 5.2.12.

b. In both cases the mean error was relatively small.

c. Table 5.2.2 shows that output y using control law 3 was non-stationary although the disturbance was stationary. The fact that the output drifted while the disturbance was stationary may be interpreted as a sign of instability.

Comparing Figures 5.1.9 and 5.1.11 as well as Figures 5.1.10 and 5.1.12 shows that the variances of input u and output y using control law 3 are less dependant on cost factor λ than using control law 2. This is attributed to the ability of control input u using control law 3 to drift.

CHAPTER 6

RESULTS OBTAINED BY EXPERIMENTATION

6.0 SCOPE

In order to verify the results obtained by simulation and documented in Chapter 5, a number of experiments was conducted on the actual system.

The following aspects were evaluated :

a. The quality of control provided respectively by control law 2 (being a general form of control law 1) and control law 3, given accurate process and disturbance models. Stationary as well as non-stationary disturbances were simulated. The results appear in paragraph 6.1.

b. The quality of control provided respectively by control laws 2 and 3, given an accurate process model but an inaccurate disturbance model. As before, the disturbance model in each case was inaccurate in the sense that when the disturbance model was stationary, the actual disturbance was non-stationary and vice versa. The results appear in paragraph 6.2.

A summary of the results obtained by experimentation is given in paragraph 6.3

Each experiment was conducted over 173 sampling points with a sampling period of 60 seconds for a duration of 2 hours and 53 minutes per experiment. Eight experiments were conducted.

The system parameters specified in Table 3.0.1 were used as model parameters. The width of the probability density function of the discrete stationary uncorrelated random sequence ξ_t was 0,0135 kg/s for each experiment.

6.1 RESULTS OBTAINED BY EXPERIMENT USING ACCURATE PROCESS AND DISTURBANCE MODELS

Figure 6.1.1 shows the input and output time responses obtained by experiment using cost function 2, a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.21)) and cost factor λ equal to 2×10^{-6}. The variance of input u is 83986 W^2. Output y has a variance of 0,767 $°C^2$ and a mean of 0,852 °C.

The sample autocorrelation function shown in Figure 6.1.2 has a quick sinusoidal decay and the corresponding output time response shown in Figure 6.1.1 is therefore stationary.

Figure 6.1.3 shows the input and output time responses obtained by experiment using cost function 2, a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.25)) and cost factor λ equal to 2×10^{-6}. The variance of input u is 304181 W^2. Output y has a variance of 0,762 $°C^2$ and a mean of -0,457 °C.

The sample autocorrelation function shown in Figure 6.1.4 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 6.1.3 is therefore non-stationary.

Figure 6.1.5 shows the input and output time responses obtained by experiment using cost function 3, a stationary disturbance (Figure 3.1.3) and disturbance model (Equation (3.2.22)) and cost factor λ equal to 2×10^{-6}. The variance of input u is 196860 W^2. Output y has a variance of 0,856 $°C^2$ and a mean of 0,183 °C.

The sample autocorrelation function shown in Figure 6.1.6 has a relatively quick sinusoidal decay and the corresponding output time response shown in Figure 6.1.5 is therefore stationary.

Figure 6.1.7 shows the input and output time responses obtained by experiment using cost function 3, a non-stationary disturbance (Figure 3.1.4) and disturbance model (Equation (3.2.26)) and cost factor λ equal to 2×10^{-6}. The variance of input u is 814185 W^2. Output y has a variance of 5,159 $°C^2$ and a mean of -0,775 °C.

The sample autocorrelation function shown in Figure 6.1.8 has a slow sinusoidal decay and the corresponding output time response shown in Figure 6.1.7 is therefore non-stationary.

Table 6.1.1 tabulates the variance of input u and the variance and mean of output y obtained by experiment for λ equal to 2×10^{-6} using accurate process and disturbance models.

Table 6.1.2 tabulates the nature of the sample autocorrelation function and the output time response for the above cases.

The following observations can be made:

a. When the disturbance acting on the system was non-stationary and modelled as such, control law 3 provided a very poor quality of control. Control law 2 provided an output y which had a relatively large mean error.

b. When the disturbance acting on the system was stationary and modelled as such, control law 2 provided an output y that had a marginally smaller variance but a considerable larger mean error than provided by control law 3.

c. In all the cases, the nature of the output time response obtained was the same as that of the disturbance and disturbance model.

Table 6.1.1 Variance of Input u and Variance and Mean of Output y Obtained by Experiment for λ Equal to 2 x 10^{-6} using Accurate Process and Disturbance Models

Control Law	Nature of Disturbance and Disturbance Model	Variance of Input u (W^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)
2	Stationary	83986	0,767	0,852
2	Non-Stationary	304181	0,762	-0,457
3	Stationary	196860	0,856	0,183
3	Non-Stationary	814185	5,159	-0,775

Table 6.1.2 Nature of the Sample Autocorrelation Function and the Output Time Series Obtained by Experiment for λ Equal to 2 x 10^{-6} Using Accurate Process and Disturbance Models

Control Law	Nature of Disturbance and Disturbance Model	Nature of Sample Autocorrelation Function	Nature of Output Time Response
2	Stationary	Quick sinusoidal decay	Stationary
2	Non-Stationary	Slow non-oscillatory exponential decay	Non-Stationary
3	Stationary	Quick sinusoidal decay	Stationary
3	Non-Stationary	Slow sinusoidal decay	Non-Stationary

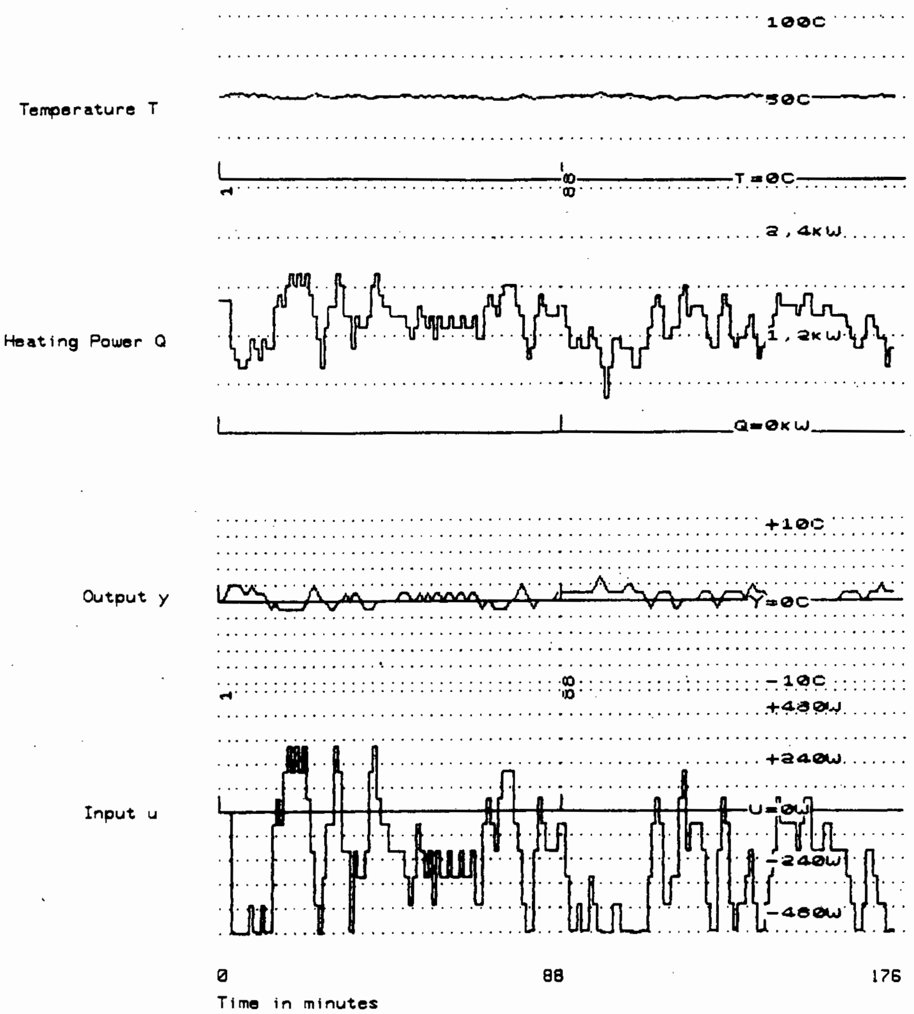


Figure 6.1.1. Input and Output Time Responses by Experiment using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Stationary

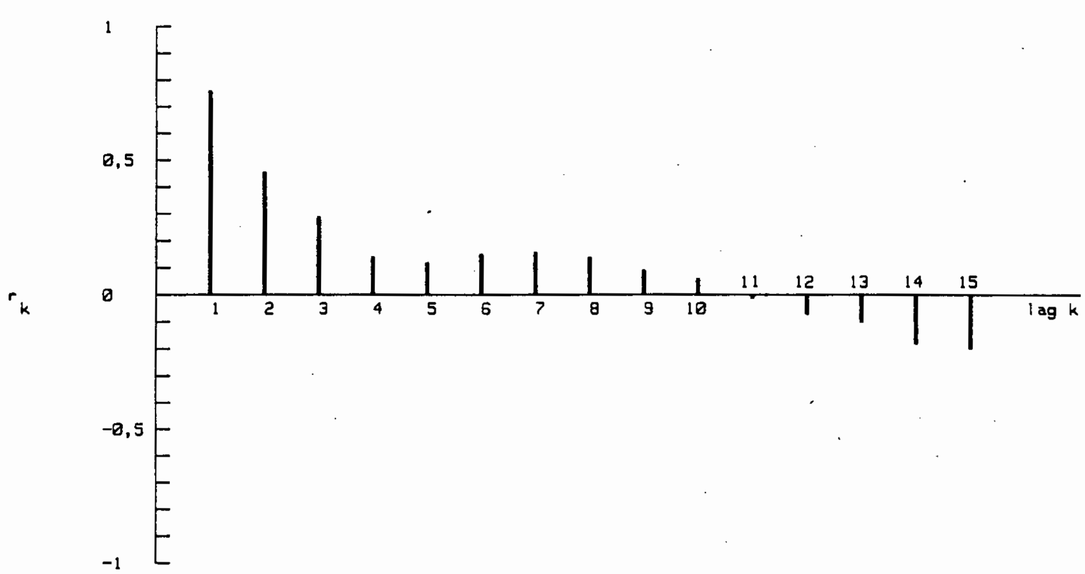


Figure 6.1.2. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

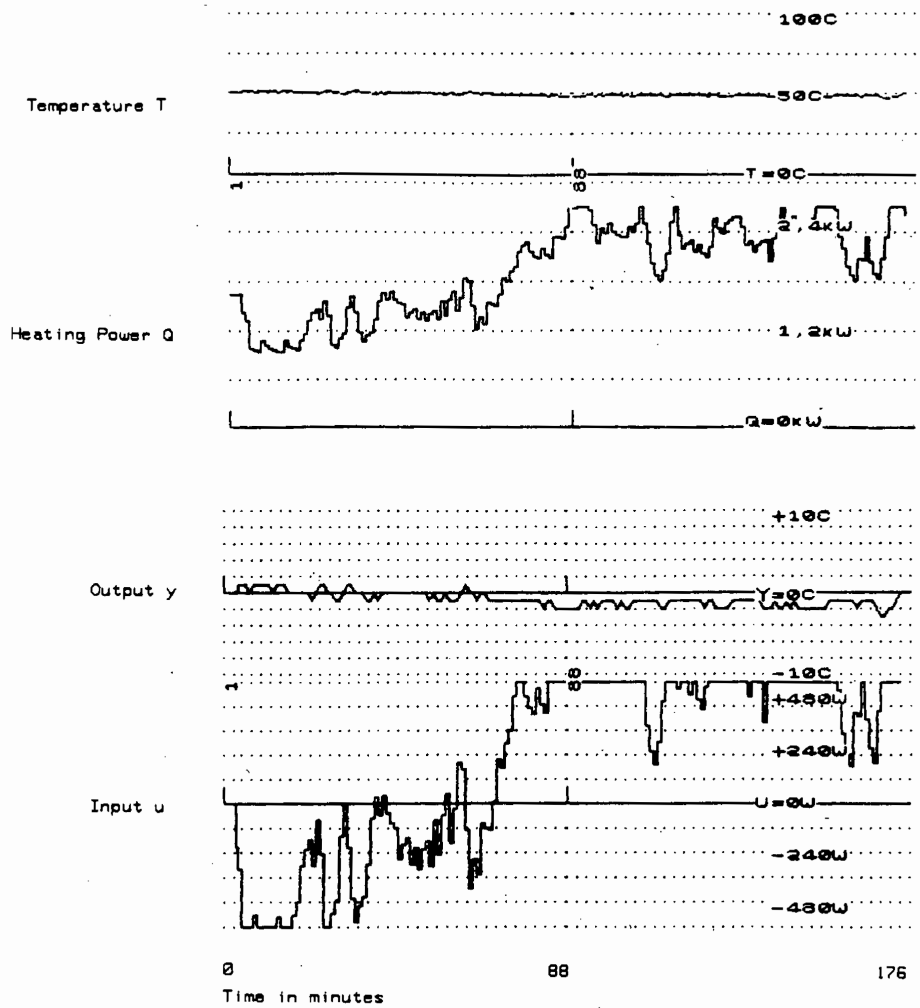


Figure 6.1.3. Input and Output Time Responses by Experiment using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Non-stationary

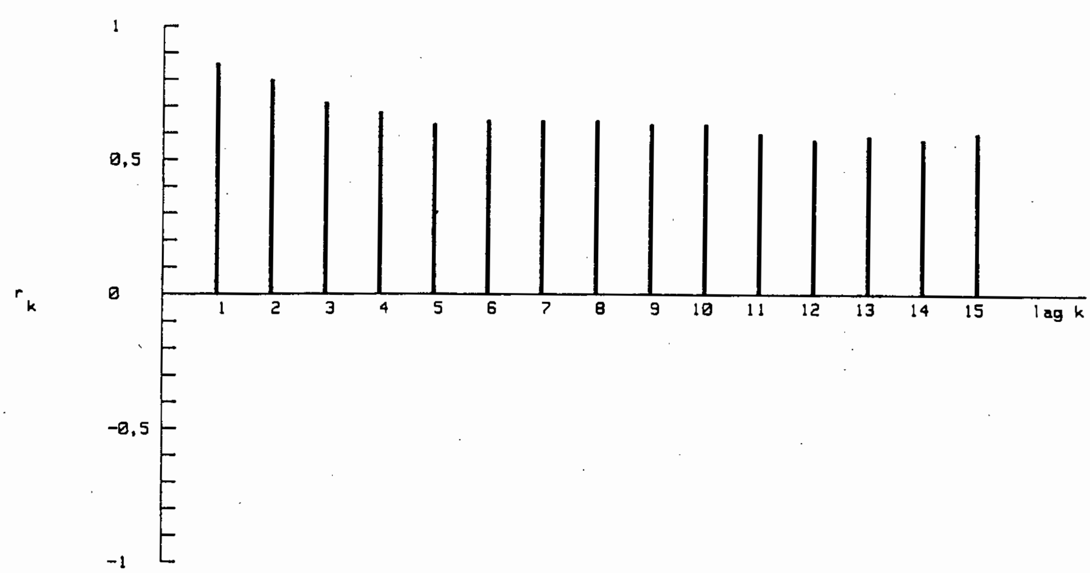


Figure 6.1.4. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

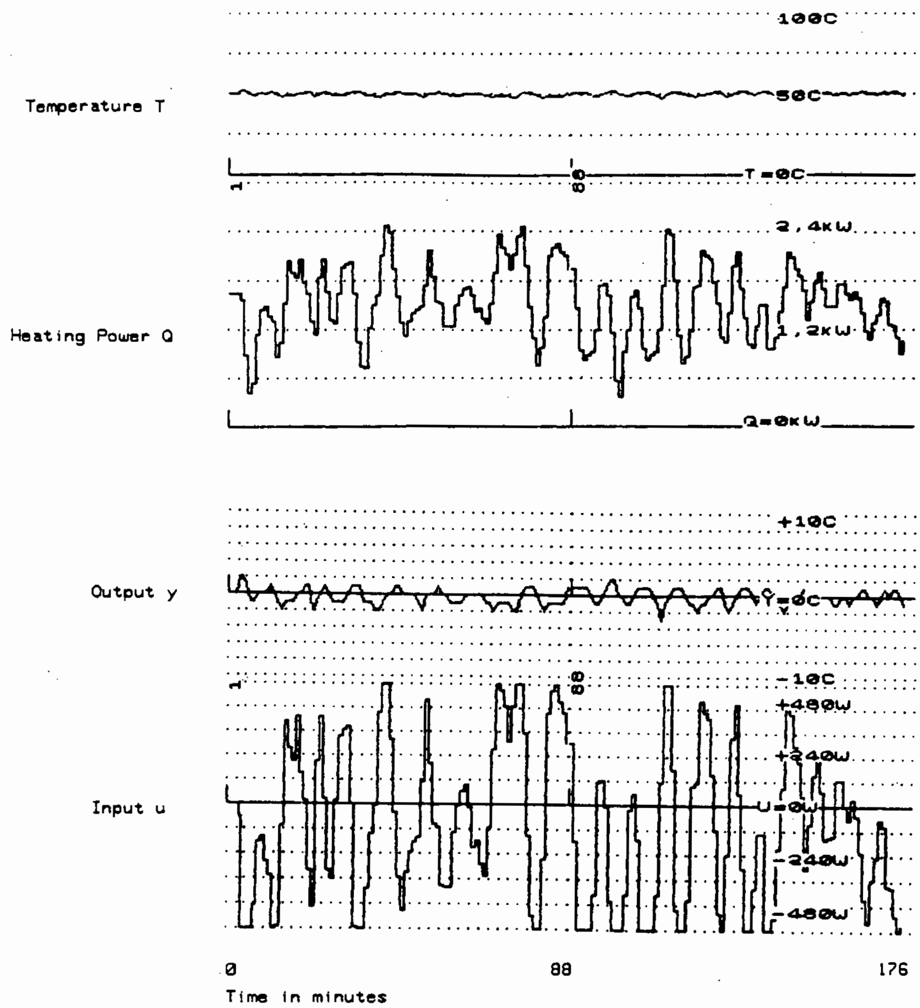


Figure 6.1.5. Input and Output Time Responses by Experiment using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Stationary

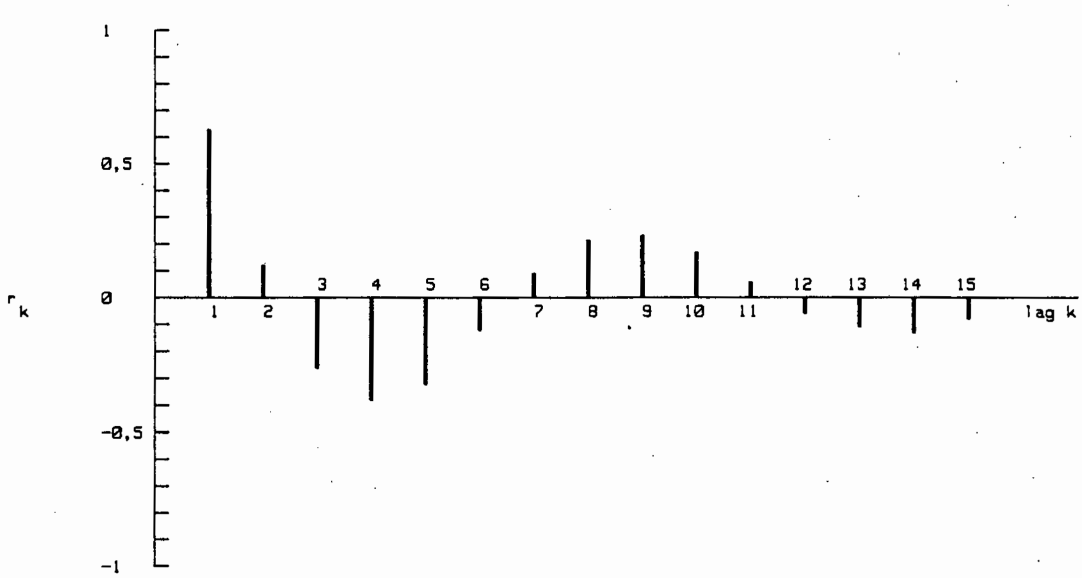


Figure 6.1.6. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

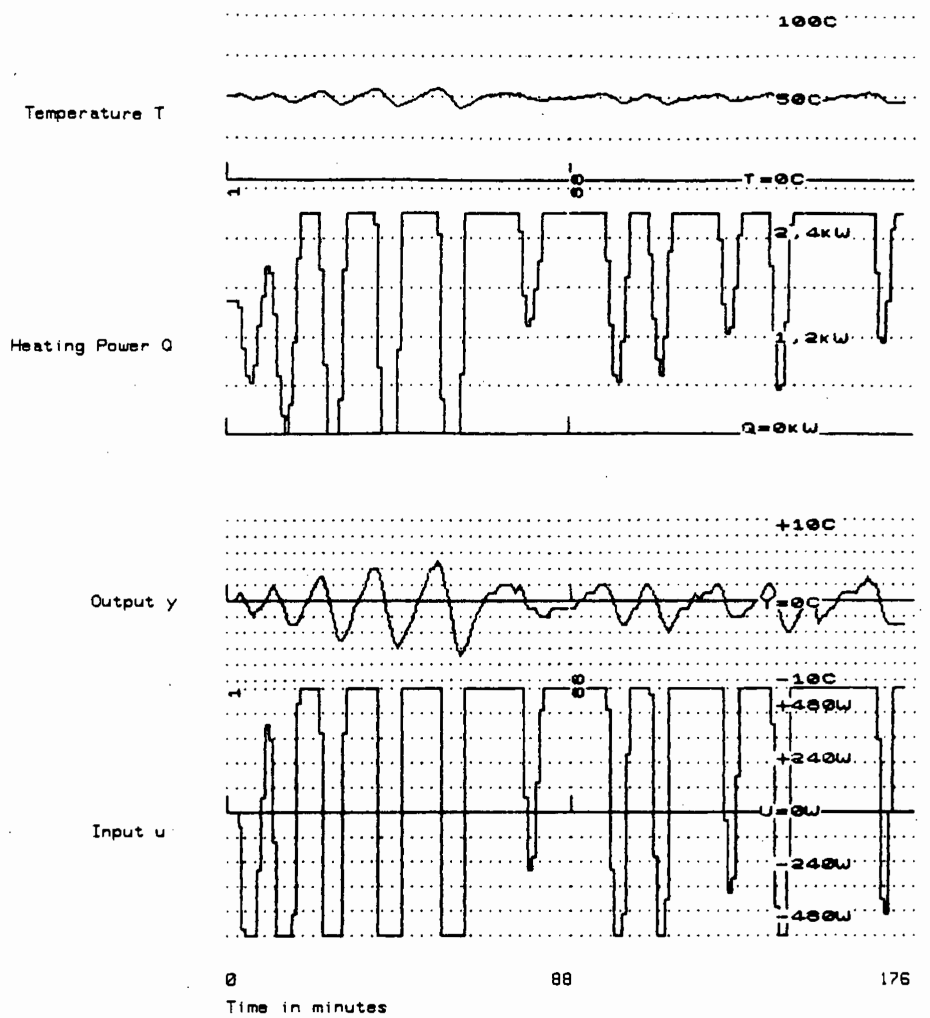


Figure 6.1.7. Input and Output Time Responses by Experiment using Cost Function 3, a Non-stationary Disturbance Model and with the actual Disturbance Non-stationary

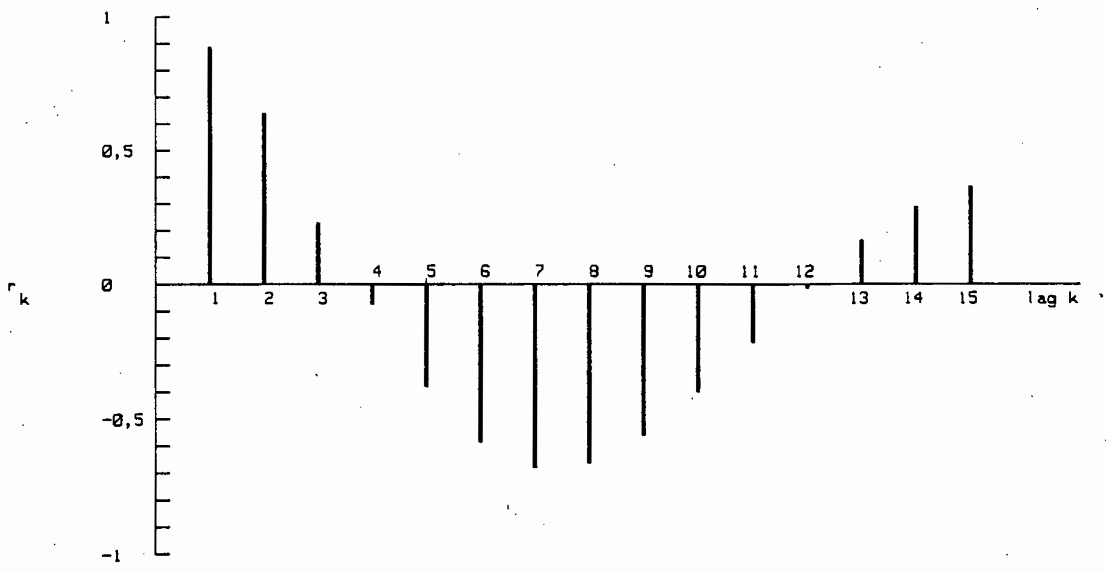


Figure 6.1.8. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

6.2 RESULTS OBTAINED BY EXPERIMENT USING AN ACCURATE PROCESS MODEL BUT AN INACCURATE DISTURBANCE MODEL

As before, the disturbance model in each case was inaccurate in the sense that when the disturbance model was stationary, the actual disturbance was non-stationary and vice versa.

Figure 6.2.1 shows the input and output time responses obtained by experiment using cost function 2, a stationary disturbance model (Equation (3.2.21)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was non-stationary (Figure 3.1.4). The variance of input u is 278358 W^2. Output y has a variance of 3,062 $°C^2$ and a mean of -0,402°C.

The sample autocorrelation function shown in Figure 6.2.2 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 6.2.1 is therefore non-stationary.

Figure 6.2.3 shows the input and output time responses obtained by experiment using cost function 2, a non-stationary disturbance model (Equation (3.2.25)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was stationary (Figure 3.1.3). The variance of input u is 214392 W^2. Output y has a variance of 0,782 $°C^2$ and a mean of 0,180 °C.

The sample autocorrelation function shown in Figure 6.2.4 has a quick sinusoidal decay and the corresponding output time response shown in Figure 6.2.3 is therefore stationary.

Figure 6.2.5 shows the input and output time responses obtained by simulation using cost function 3, a stationary disturbance model (Equation (3.2.22)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was non-stationary (Figure 3.1.4). The variance of input u is 117696 W^2. Output y has a variance of 2,380 $°C^2$ and a mean of -2,315 °C.

The sample autocorrelation function shown in Figure 6.2.6 has a slow non-oscillatory exponential decay and the corresponding output time response shown in Figure 6.2.5 is therefore non-stationary.

Figure 6.2.7 shows the input and output time responses obtained by experiment using cost function 3, a non-stationary disturbance model (Equation (3.2.26)) and cost factor λ equal to 2×10^{-6} while the actual disturbance was stationary (Figure 3.1.3). The variance of input u is 852640 W^2. Output y has a variance of 4,202 $°C^2$ and a mean of -0,025 °C.

The sample autocorrelation function shown in Figure 6.2.8 has a slow sinusoidal decay and the corresponding output time response shown in Figure 6.2.7 is therefore non-stationary.

Table 6.2.1 tabulates the variance of input u and the variance and mean of output y obtained by experiment for λ equal to 2×10^{-6} using an accurate process model but inaccurate disturbance models.

Table 6.2.2 tabulates the nature of the sample autocorrelation function and the output time response for the above cases.

The following observations can be made:

a. When the disturbance acting on the system was non-stationary but modelled as stationary, control law 3 provided an output y with a smaller variance but with a larger mean error than provided by control law 2. However, from Figure 6.2.5 can be seen that the control element was saturated for most of the experiment with control law 3 without managing to cancel the effect of the disturbance. From this must be concluded that the disturbance flow rate was incorrectly calibrated before the experiment commenced and provided a load which exceeded the capacity of the control transducer. The data obtained from this experiment is therefore not conclusive.

b. When the disturbance acting on the system was stationary but modelled as non-stationary, control law 2 provided a better quality of control than did control law 3. Control law 3 provided a very poor quality of control in this case.

c. Referring to Tables 6.1.2 and 6.2.2, the only case where the nature of the output time response was not the same as that of the actual disturbance, was for the case when control law 3 and a non-stationary disturbance model was used while the actual disturbance was stationary. This may be interpreted as an inability on the part of this controller to stably control the system.

Table 6.2.1 Variance of Input u and Variance and Mean of Output y Obtained by Experiment for λ Equal to 2×10^{-6} Using an Accurate Process Model but with the Disturbance Model Inaccurate as Defined

Control Law	Nature of of Disturbance Model	Nature of Actual Disturbance	Variance of Input u (W^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)
2	Stationary	Non-Stationary	278358	3,062	-0,402
2	Non-Stationary	Stationary	214392	0,782	0,180
3	Stationary	Non-Stationary	117696	2,380	-2,315
3	Non-Stationary	Stationary	852640	4,202	-0,025

Table 6.2.2 Nature of the Sample Autocorrelation Function and the Output Time Series Obtained by Experiment for λ Equal to 2×10^{-6} Using an Accurate Process Model but with the Disturbance Model Inaccurate as Defined

Control Law	Nature of Disturbance Model	Nature of Actual Disturbance	Nature of Sample Autocorrelation Function	Nature of output Time Response
2	Stationary	Non-Stationary	Slow non-oscillatory exponential decay	Non-stationary
2	Non-Stationary	Stationary	Quick sinusoidal decay	Stationary
3	Stationary	Non-Stationary	Slow non-oscillatory exponential decay	Non-stationary
3	Non-Stationary	Stationary	Slow sinusoidal decay	Non-stationary

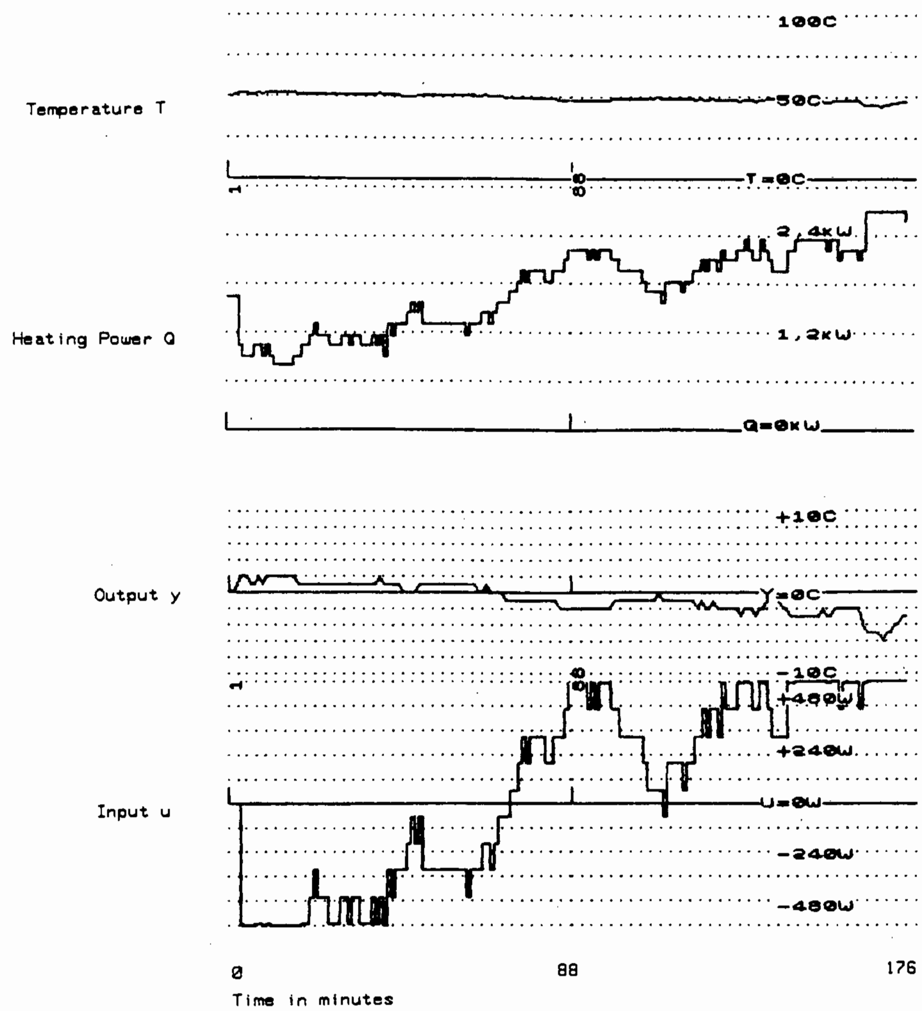


Figure 6.2.1. Input and Output Time Responses by Experiment using Cost Function 2, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

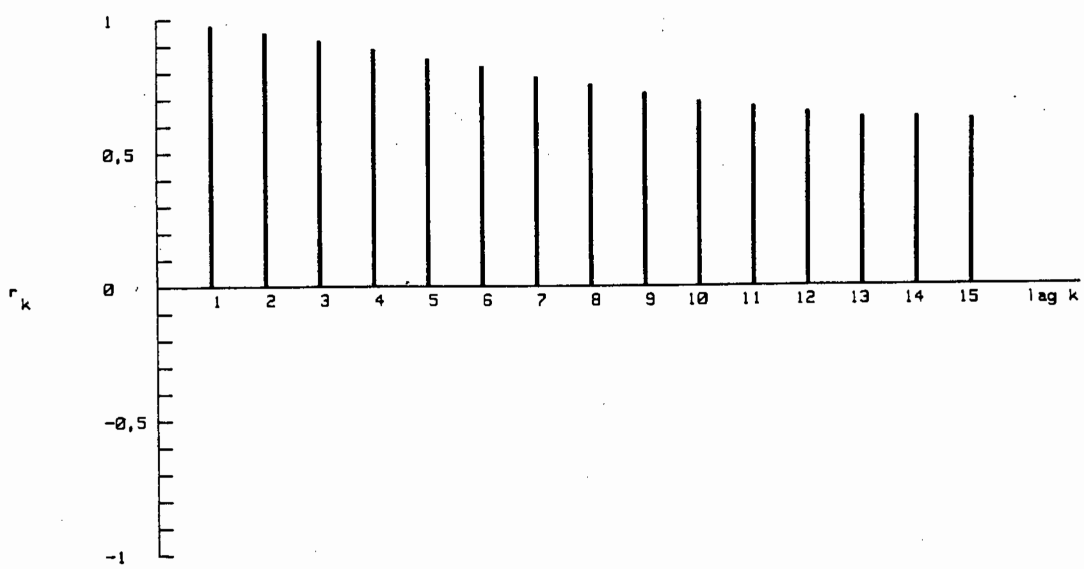


Figure 6.2.2. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

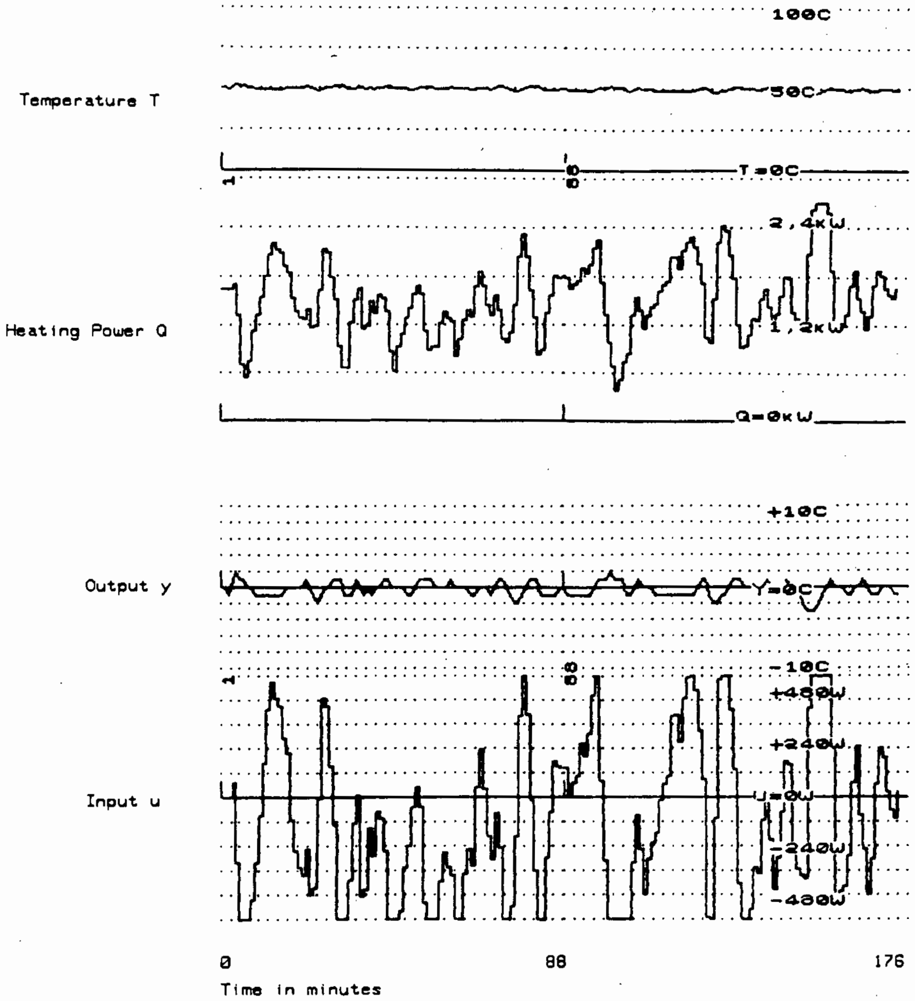


Figure 6.2.3. Input and Output Time Responses by Experiment using Cost Function 2, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

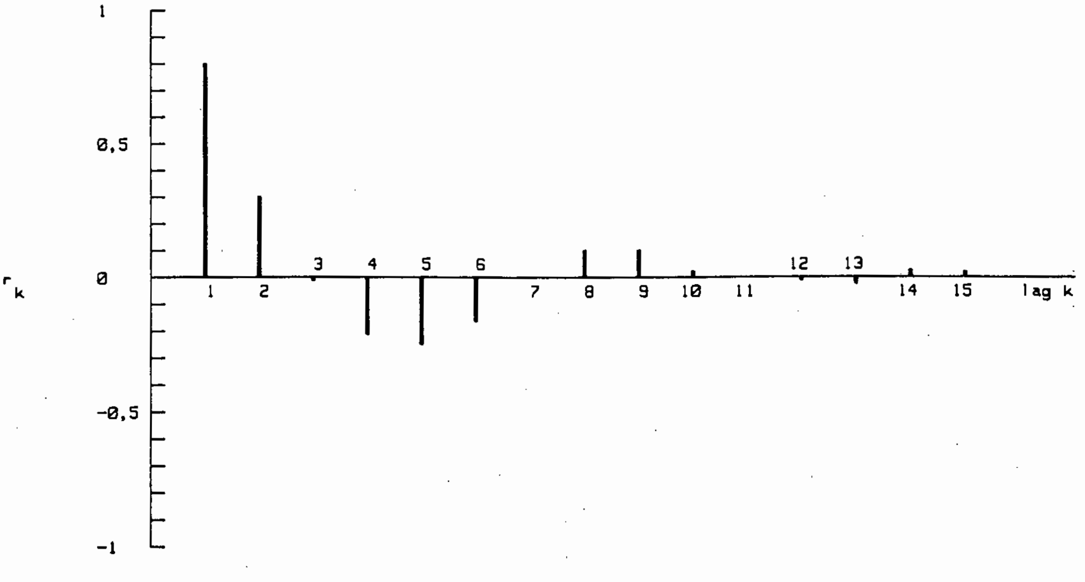


Figure 6.2.4. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

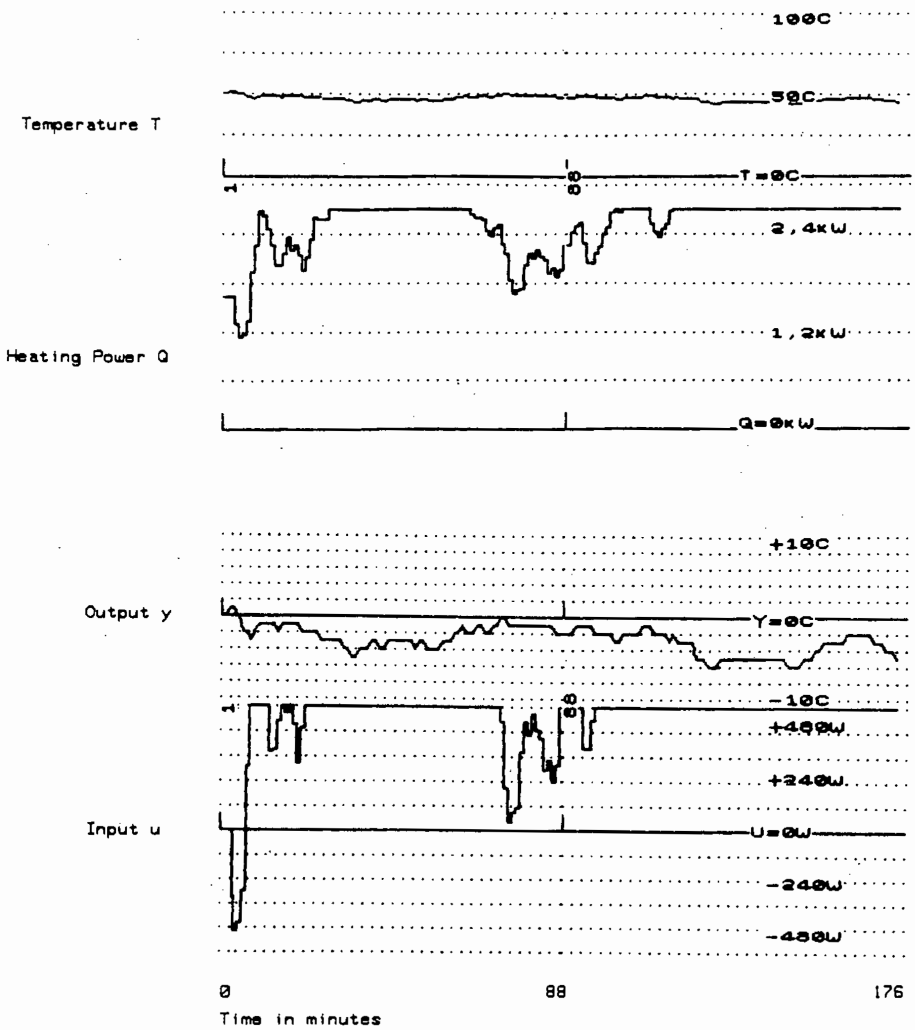


Figure 6.2.5. Input and Output Time Responses by Experiment using Cost Function 3, a Stationary Disturbance Model and with the actual Disturbance Non-stationary

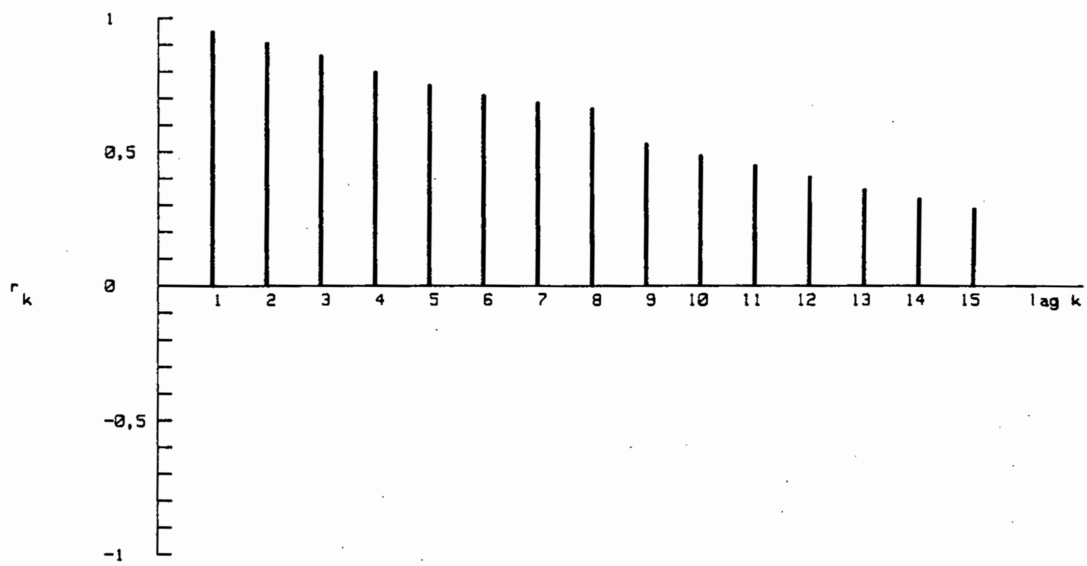


Figure 6.2.6. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

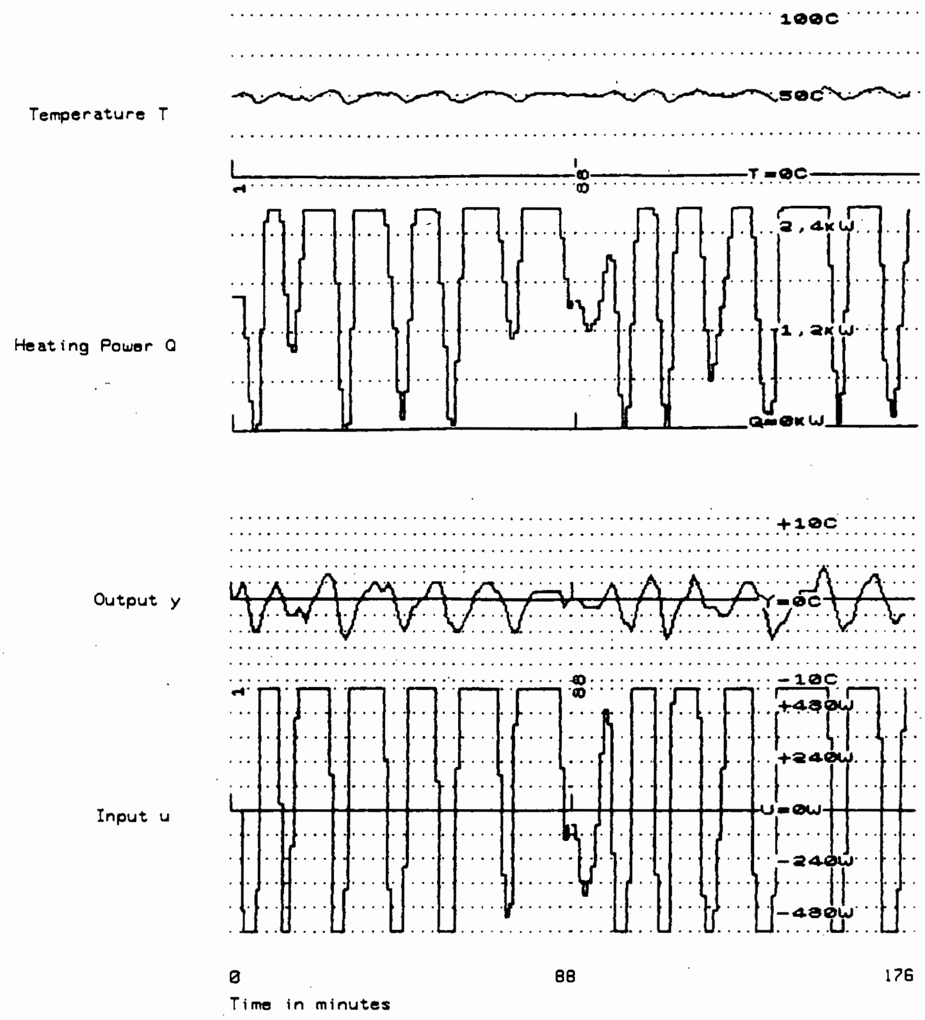


Figure 6.2.7. Input and Output Time Responses by Experiment using Cost Function 3, a Non-stationary Disturbance Model and with the actual Disturbance Stationary

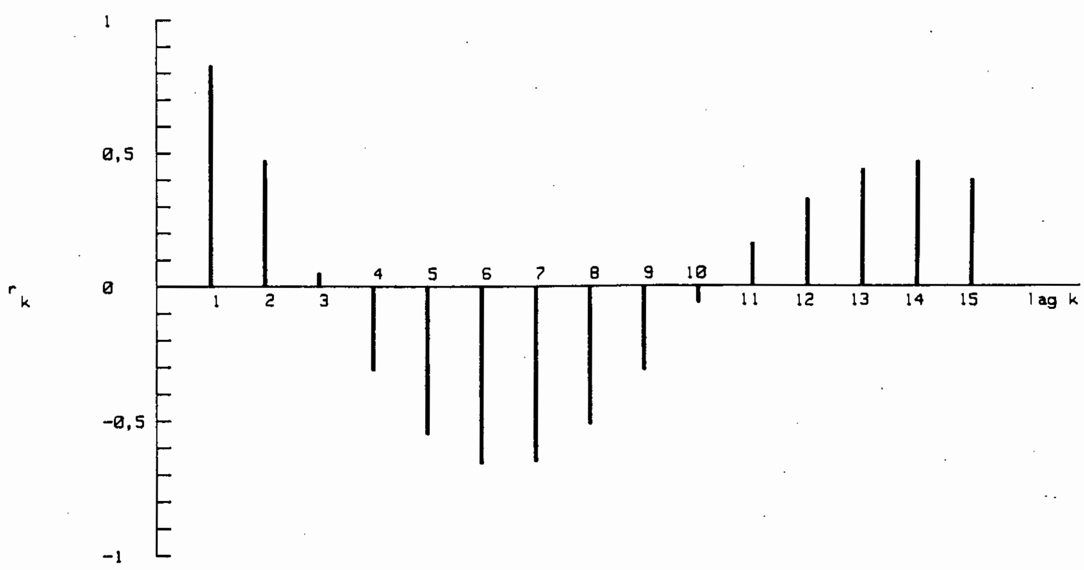


Figure 6.2.8. Sample Autocorrelation Function of the Output Time Response shown in the Figure above

6.3 SUMMARY OF RESULTS OBTAINED BY EXPERIMENTATION

In the following paragraphs the results obtained by experimentation are summarized.

The following is a summary of the results obtained with the disturbance non-stationary and modelled as such:

a. Table 6.1.1 shows that control law 2 provided a substantially better quality of control than did control law 3 which provided a very poor quality of control.

b. In both cases the output mean error was relatively large.

The following is a summary of the results obtained with the disturbance stationary and modelled as such:

a. Table 6.1.1 shows that control law 2 provided an output y with a variance marginally smaller than provided by control law 3.

b. The mean error of output y obtained using control law 3 was substantially smaller than obtained using control law 2 which provided a relatively large mean error.

The following is a summary of the results obtained with the disturbance non-stationary but modelled as stationary:

a. Table 6.2.1 shows that the variance of output y obtained using control law 3 was marginally smaller than obtained using control law 2.

b. From Figure 6.2.5 is concluded that the large mean error of output y obtained using control law 3 as shown in Table 6.2.1 is because of a flow-rate calibration error.

The following is a summary of the results obtained with the disturbance stationary but modelled as non-stationary:

a. Table 6.2.1 shows that control law 2 provided a substantially better quality of control than did control law 3 which provided a very poor quality of control.

b. In both cases the mean error was relatively small.

c. Table 6.2.2 shows that output y obtained using control law 3 was non-stationary although the disturbance was stationary. The fact that the output drifted while the disturbance was stationary may be interpreted as a sign of instability.

In Chapter 7 the results obtained by experimentation are compared to the results obtained by simulation and interpreted before the final conclusions are made.

CHAPTER 7

CONCLUSION

7.0 COMPARISON BETWEEN RESULTS OBTAINED BY SIMULATION AND RESULTS OBTAINED BY EXPERIMENTATION

In the following paragraphs the results obtained by simulation are compared to the results obtained by experimentation. Where the results differ, the reasons for the differences as deducted from the data documented in Chapters 5 and 6 are presented.

7.0.1 Comparison of Results with the Disturbance and Disturbance Model Non-Stationary

Table 7.0.1 compares the results obtained by simulation with the results obtained by experimentation when the disturbance was non-stationary and modelled as such.

From Table 7.0.1 can be seen that there is good agreement between the results obtained by simulation and the results obtained by experimentation using control law 2.

Table 7.0.1 shows that there is a very large difference between the results obtained by simulation and the results obtained by experimentation using control law 3. The very poor quality of control obtained in the experiment with control law 3 is attributed to the high sensitivity of the control quality provided by control law 3 to process model inaccuracies as shown by simulation in Table 5.3.1 and supported by the stability analysis in paragraph 3.3.4. It is conceivable that the process model is slightly inaccurate since the complex dynamics of heat convection in the heating cylinder was not modelled.

Table 7.0.1 Comparison between Results Obtained by Simulation and Results Obtained by Experimentation with the Disturbance Non-Stationary and Modelled as such

Control Law	Experiment/ Simulation	Variance of Input u (W^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)
2	Simulation	380550	0,841	0,427
2	Experiment	304181	0,762	-0,457
3	Simulation	392784	0,255	-0,001
3	Experiment	814185	5,159	-0,775

7.0.2 Comparison of Results with the Disturbance and Disturbance Model Stationary

Table 7.0.2 compares the results obtained by simulation with the results obtained by experimentation when the disturbance was stationary and modelled as such.

Table 7.0.2 shows that the mean of output y obtained using control law 2 is considerable larger for the experiment then for the simulation. Since the mean value of control input u using control law 2 is unable to drift, it had difficulty to compensate for process model inaccuracies which presumably existed.

From Table 7.0.2 can be seen that there is good agreement between the results obtained by simulation and the results obtained by experimentation using control law 3.

Table 7.0.2 Comparison between Results Obtained by Simulation and Results Obtained by Experimentation with the Disturbance Stationary and Modelled as such

Control Law	Experiment/ Simulation	Variance of Input u (W^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)
2	Simulation	87290	0,803	-0,009
2	Experiment	83986	0,767	0,852
3	Simulation	169601	0,869	-0,017
3	Experiment	196860	0,856	0,183

7.0.3 Comparison of Results with the Disturbance Non-Stationary but with the Disturbance Model Stationary

Table 7.0.3 compares the results obtained by simulation with the results obtained by experimentation when the disturbance was non-stationary but modelled as stationary.

From Table 7.0.3 can be seen that using control law 2, the results obtained by simulation agrees reasonably well with the results obtained by experimentation.

Table 7.0.3 shows that using control law 2 there is a large difference between the mean of output y obtained by simulation and the mean of output y obtained by experimentation. The high value of the mean of output y obtained with the experiment is attributed to a flow-rate calibration error (paragraph 6.3 refers).

Table 7.0.3 Comparison between Results Obtained by Simulation and Results Obtained by Experimentation with the Disturbance Non-Stationary but Modelled as Stationary

Control Law	Experiment/ Simulation	Variance of Input u (W^2)	Variance of Output y (°C^2)	Mean of Output y (°C)
2	Simulation	523989	5,242	-0,385
2	Experiment	278358	3,062	-0,402
3	Simulation	477002	1,126	0,499
3	Experiment	117696	2,380	-2,315

7.0.4 Comparison of Results with the Disturbance Stationary but with the Disturbance Model Non-Stationary

Table 7.0.4 compares the results obtained by simulation with the results obtained by experimentation when the disturbance was stationary but modelled as non-stationary.

From Table 7.0.4 can be seen that for both control law 2 and control law 3 there is good agreement between the results obtained by simulation and the results obtained by experimentation.

Table 7.0.4 Comparison between Results Obtained by Simulation and Results Obtained by Experimentation with the Disturbance Stationary but Modelled as Non-Stationary

Control Law	Experiment/ Simulation	Variance of Input u (W^2)	Variance of Output y (°C^2)	Mean of Output y (°C)
2	Simulation	159861	0,788	0,005
2	Experiment	214392	0,782	0,180
3	Simulation	782254	5,411	-0,001
3	Experiment	852640	4,202	-0,025

7.1 CONSOLIDATION OF RESULTS

In order to be able to make clear conclusions, the primary results obtained are consolidated in the following paragraphs. The information is ordered in such a way as to make it possible to answer the following questions:

a. Given that the system is subjected to a non-stationary disturbance, using which one of the cost functions

$$I = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \qquad (7.1.1)$$

(Control law 3)

and

$$I = \hat{y}^2_{t+k/t} + \lambda u_t^2 \qquad (7.1.2)$$

(Control law 2)

provides the best quality of control?

b. Given that the system is subjected to a stationary disturbance, using which one of the cost functions in equations (7.1.1) and (7.1.2) provides the best quality of control?

c. Given that it is not known a priori whether the disturbance is stationary or non-stationary, the use of which one of the cost functions in equations (7.1.1) and (7.1.2) in combination with which disturbance model (stationary/non-stationary) is the best overall choice?

7.1.1 Primary Results Obtained for the Case where the Disturbance is Known A Priori to be Non-Stationary

The primary results are:

a. Simulation shows that using control law 3 and a non-stationary disturbance model provides the best quality of control.

b. Stability analysis, simulation as well as experimentation indicate that model inaccuracies can drastically deteriorate the quality of control obtained when using control law 3 and a non-stationary disturbance model.

c. Using control law 2 causes a relatively large mean error.

7.1.2 Primary Results Obtained for the Case where the Disturbance is Known A Priori to be Stationary

The primary results are:

a. Simulation shows that using control law 2 and a stationary disturbance model provides the best quality of control.

b. Experimentation indicates that process model inaccuracies may result in a relatively large output mean error when control law 2 is used.

7.1.3 Primary results Obtained for the Case where it is Not Known A Priori whether the Disturbance is Stationary or Non-Stationary

The primary results are:

a. Using control law 3 and a non-stationary disturbance model while the actual disturbance is stationary results in an extremely poor quality of control.

b. Using control law 2 and a stationary disturbance model while the actual disturbance is non-stationary results in an extremely poor quality of control.

c. Table 7.1.1 consolidates the performances of respectively

i. control law 2 and a non-stationary disturbance model

and

ii. control law 3 and a stationary disturbance model

using the results obtained by simulation.

. Taking into account the behaviour of the variances of the input and output as a function of cost factor λ , control law 3 and a stationary disturbance model is the best overall combination.

Table 7.1.1 Comparison between Simulational Results Obtained Respectively by Control Law 2 and a Non-Stationary Disturbance Model and Control Law 3 and a Stationary Disturbance Model

Control Law	Actual Disturbance	Variance of Input u (w^2)	Variance of Output y ($°C^2$)	Mean of Output y (°C)	Variances of Input and Output as function of λ
2	Stationary	159861	0,788	0,005	Figure 5.2.10
3	Stationary	169601	0,869	-0,017	Figure 5.1.11
2	Non-Stationary	380550	0,841	0,427	Figure 5.1.10
3	Non-Stationary	477002	1,126	0,499	Figure 5.2.11

7.2 CONCLUSIONS

The final conclusions arrived at are the following:

a. When the disturbance acting on the system is non-stationary and modelled as such, minimizing cost function

$$I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2$$

provides a better quality of control than obtained by minimizing cost function

$$I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2$$

<u>provided</u> that the process and disturbance models are very accurate.

Model inaccuracies result in a serious degredation of control quality. Due consideration must therefore be taken of sensitivity aspects in the design of a controller based on the minimization of I_3 which must control a system subjected to a non-stationary disturbance.

It is not appropriate to use cost function I_2 in this case since the inability of the mean value of the control input to drift may result in large mean output errors.

b. When the disturbance acting on the system is stationary and modelled as such, minimizing I_2 results in a marginally better quality of control than obtained minimizing I_3. However, a process model inaccuracy may result in an output offset error since a controller based on the minimization of I_2 does not have integral action. Using a controller based on the minimization of I_3 is advantageous in this case since the integral action of the controller will cancel the error.

c. If the situation exists where the disturbance acting on the system at a given time is stationary but may become non-stationary after a period of time, using I_3 and a stationary disturbance model results in a controller which provides the best overall control quality for the system evaluated. Sensitivity to model inaccuracies is less of a problem in this case.

7.3 USE OF THE METHOD AND SUGGESTIONS FOR FURTHER WORK

As stated in paragraph 2.1, the facility of free programmability on modern microprocessor-based process control systems encourages, where justified by the problem, the implementation of more sophisticated control algorythms such as those evaluated by this thesis.

Implementing a recursive stochastic control algorythm on a digital computer is a relatively simple task as such. However, this must be accompanied by a relatively large amount of theoretical analysis, eg. identifying the process and disturbance dynamics, deciding on a control strategy, deriving the specific control law, conducting sensitivity analyses, evaluation by computer simulation, and so forth. The cost of doing this must be less than the production costs saved thanks to the implementation of the method in order to economically justify its use.

However, where large random disturbances are a serious problem and steps need to be taken to improve the control quality, the issue becomes one of defining the options available to the designer.

The approach taken as described by this dissertation is certainly one way of approaching the problem. A logical extension to the work described is minimizing the cost functions:

$$I = E\{y^2_{t+k} + \lambda u_t^2\} \tag{7.3.1}$$

or

$$I = E\{y^2_{t+k} + \lambda(u_t - u_{t-1})^2\} \tag{7.3.2}$$

which should provide more optimal control than obtained by the two cost functions evaluated.

An interesting adaptation of the method described could be to use an adaptive predictor rather than the optimal predictor used. Trigg and Leach [11] have developed a procedure for the adaptive control of a single exponential smoothing constant. The smoothing constant automatically becomes larger when the prediction error increases, reducing the memory of the predictor. After having settled down, the smoothing constant's value decreases again.

This predictor reacts much more quickly to the step function than does conventional, or nonadaptive, prediction. The performance of the predictor before and after a step change is the same as that of conventional prediction. This predictor may be particularly useful if the disturbance is non-stationary.

Chow [12] has developed an alternative adaptive prediction scheme. Van der Bij and Schuppen [9] have used adaptive Kalman filtering to predict railway power demand which is a stochastic variable. However, if these predictors contain non-linearities, difficulty may be encountered in using linear stochastic control theory to design and analise the control algorythm.

Linear stochastic control theory can also be applied to multivariable systems. Work has been done by Dugard et al [10] in this regard.

Fuller [5] follows a different approach in the design of stochastic controllers. He shows that for the case when the disturbance is a highly smoothed white noise (which seems relevant in many practical circumstances), by applying Wiener filtering theory and adapting simplified models, closed-form expressions for the optimal controller which approximates a conventional type such as proportional plus integral can sometimes be obtained.

It may be possible therefore to use a conventional type of controller and to use the theory to tune the controller to obtain the desired stability margin and control quality. However, in order to obtain the closed-form expressions, relatively complex algebraic manipulations and sensible approximations of transfer functions are required.

Finally, a literature survey on the subject of stochastic control theory indicates that comparatively little work has been done in this area, particularly w.r.t practical implementation, and there is therefore much scope for further work.

APPENDIX A : MATHEMATICAL PROOFS AND DERIVATIONS

Annexure 1 to

Appendix A

PROOF THAT INTEGRATING A STATIONARY UNCORRELATED RANDOM SEQUENCE WITH ZERO MEAN PRODUCES A NON-STATIONARY RANDOM SEQUENCE

Consider a pseudo-random sequence with zero mean and finite variance σ_x^2 to be the input to a filter with transfer function H(s).

The spectral density of the output y is

$$S_y(\omega) = \sigma_x^2 \cdot \left|H(\omega)\right|^2 \tag{A1.1}$$

where

$$y_n = ay_{n-1} + bx_n \tag{A1.2}$$

Taking the discrete Fourier-transforms gives:

$$Y(\omega) = ae^{-j\omega T_s} \cdot Y(\omega) + bX(\omega) \tag{A1.3}$$

where T_s is the sampling period.

It follows[8] then that

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{b}{1 - a \cdot \exp(-j\omega T_s)} \tag{A1.4}$$

and

$$S_y(\omega) = \frac{b^2 \sigma_x^2}{(1+a^2) - 2a\cos\omega T_s} \tag{A1.5}$$

A1-2

With the auto-correlation function defined as

$$R_y(k) = \frac{T_s}{2\pi} \int_{-\pi/T_s}^{\pi/T_s} S_y(\omega)\, e^{j\omega k T_s}\, d\omega \tag{A1.6}$$

it follows[8] that

$$R_y(k) = \frac{\sigma_x^2 b^2}{1 - a^2} \cdot a^{-|k|} \tag{A1.7}$$

The variance of the output signal is then given by:

$$\sigma_y^2 = R_y(0) = \frac{T_s}{\pi} \int_0^{\pi/T_s} S_y(\omega)\, d\omega = \frac{\sigma_x^2 b^2}{1 - a^2} \tag{A1.8}$$

The filter being considered is an integrator with s-domain transfer function

$$H(s) = \frac{1}{s} \tag{A1.9}$$

and z-domain transfer function

$$H(z) = \frac{1}{1 - z^{-1}} \tag{A1.10}$$

which gives

$$y_n = y_{n-1} + x_n \tag{A1.11}$$

and therefore a = 1 and b = 1

Using equation (A1.8), the variance of the output signal is found to be

$$\sigma_y^2 = R_y(0) = \frac{\sigma_x^2}{1-1} \tag{A1.12}$$

which is infinite for a finite input variance and the output signal is therefore a non-stationary random sequence.

Annexure 2 to

Appendix A

DERIVATION OF Z-TRANSFORMS OF PROCESS AND DISTURBANCE MODELS

Define for both systems the forward transfer function

$$G(s) = [1 - \exp(-sT_s)].\exp(-skT_s).\frac{K_1}{s(1+sT_p)} \quad \text{(A2.1)}$$

$$= G_1(s).G_2(s) \quad \text{(A2.2)}$$

where

$$G_1(s) = [1-\exp(-sT_s)].\exp(-skT_s) \quad \text{(A2.3)}$$

and

$$G_2(s) = \frac{K_1}{s(1+sT_p)} \quad \text{(A2.4)}$$

With $z = \exp(sT_s)$,

$$G_1(z) = z^{-k}(1-z^{-1}) \quad \text{(A2.5)}$$

Expressing $G_2(s)$ in partial fractions gives:

$$G_2(s) = \frac{K_1}{s} - \frac{K_1T_p}{1+sT_p} \quad \text{(A2.6)}$$

A2-2

Using transform tables gives:

$$G_2(z) = \frac{K_1 z}{z-1} - \frac{K_1 z}{z - \exp(-T_s/T_p)} \qquad \text{(A2.7)}$$

$$= \frac{K_1 z(1 - \exp(-T_s/T_p))}{(z-1)(z - \exp(-T_s/T_p))} \qquad \text{(A2.8)}$$

$G(z) = G_1(z).G_2(z)$ is found to be

$$G(z) = \frac{K_1 [1 - \exp(-T_s/T_p)].z^{-(K+1)}}{1 - \exp(-T_s/T_p)\, z^{-1}} \qquad \text{(A2.9)}$$

The disturbance transfer function is given by

$$N(s) = \frac{T_n}{1 + sT_n} \qquad \text{(A2.10)}$$

and using transform tables gives

$$N(z) = \frac{1}{1 - \exp(-T_s/T_n)\, z^{-1}} \qquad \text{(A2.11)}$$

Using the derived z-domain transfer functions for the respective blocks, equations (3.1.16) and (3.1.17) are obtained.

Annexure 3 to

Appendix A

REQUIREMENTS FOR TRANSFER FUNCTION H(z) TO BE REALIZABLE

Consider a system with transfer function

$$H(z) = \frac{Y(z)}{U(z)} \tag{A3.1}$$

$$= \frac{a_{m+1} z^{m} + a_{m} z^{m-1} + \dots + a_{1}}{b_{n+1} z^{n} + b_{n} z^{n-1} + \dots + b_{1}} \tag{A3.2}$$

where Y(z) is the output from and U(z) is the input to the system.

In developing H(z) as a power series in z^{-1}, the coefficients of the series represent the values of the weighted sequence of the digital system. The coefficient of the z^{-k} - term corresponds to the value of the weighted sequence at $t = kT_s$ after the input appeared at t=0.

For H(z) to be physically realizable, the power series of H(z) may not have any positive power in z. A positive power in z implies prediction of the input or the appearance of the output before the input has been applied and is therefore not physically realizable.

Therefore, for H(z) to be a physically realizable transfer function, the highest power of U(z) must be equal to or greater than the highest power of Y(z), i.e. $n \geq m$.

Factorize H(z) :

$$H(z) = \frac{\text{const}\ (z + c_1)(z + c_2) \dots (z + c_m)}{(z + d_1)(z + d_2) \dots (z + d_n)} \tag{A3.3}$$

with $n \geq m$ and m finite zeros and n finite poles.

Develop power series:

$$H(z) = \frac{\text{const}\,[z^{m} + e_1 z^{m-1} + \ldots + e_m]}{z^{n} + f_1 z^{n-1} + \ldots + f_n} \tag{A3.4}$$

$$= \text{const.}\, z^{m-n} \cdot \frac{1 + e_1 z^{-1} + \ldots + e_m z^{-m}}{1 + f_1 z^{-1} + \ldots + f_n z^{-n}} \tag{A3.5}$$

$$= z^{m-n} \cdot \frac{g_0 + g_1 z^{-1} + \ldots + g_m z^{-m}}{1 + f_1 z^{-1} + \ldots + f_n z^{-n}} \tag{A3.6}$$

If m = n then:

$$H(z) = \frac{g_0 + g_1 z^{-1} + \ldots + g_n z^{-n}}{1 + f_1 z^{-1} + \ldots + f_n z^{-n}} \tag{A3.7}$$

The system of equation (3.2.12)

$$y_t = \frac{z^{-k} B(z^{-1})}{1 + A(z^{-1})} \cdot u_t(z^{-1}) + \frac{1 + D(z^{-1})}{1 + C(z^{-1})} \cdot \xi_t(z^{-1}) \tag{A3.8}$$

with

$$\begin{aligned} A(z^{-1}) &= a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n} \\ B(z^{-1}) &= b_o + b_1 z^{-1} + \ldots + b_n z^{-n} \\ C(z^{-1}) &= c_1 z^{-1} + c_2 z^{-2} + \ldots + c_n z^{-n} \\ D(z^{-1}) &= d_1 z^{-1} + d_2 z^{-2} + \ldots + d_n z^{-n} \end{aligned} \tag{A3.9}$$

is therefore physically realizable since both

$$\frac{B(z^{-1})}{1 + A(z^{-1})} \quad \text{and} \quad \frac{1 + D(z^{-1})}{1 + C(z^{-1})}$$

have the same form as equation (A3.7).

Annexure 4 to

Appendix A

GENERAL DERIVATION OF THE CONTROL LAWS WHICH RESPECTIVELY MINIMIZE COST FUNCTIONS I_1, I_2 and I_3

The control laws which respectively minimize cost functions I_1, I_2 and I_3 are derived using the general procedure described in paragraph 3.2.2.

It will be shown that the control law which minimizes I_1 is a special case of the control law which minimizes I_2 with cost factor λ equal to zero. The control law which minimizes I_2 is therefore derived first.

Derivation of Control Law which Minimizes Cost Function I_2

$$I_2 = \hat{y}^2_{t+k/t} + \lambda u_t^2 \qquad \text{(A4.1)}$$

where

$$\hat{y}_{t+k/t} = z^k . N_k . \xi_t + G u_t \qquad \text{(A4.2)}$$

as derived in paragraph 3.2.2.

Differentiating I_2 wrt u_t and equating it to zero gives:

$$\frac{\partial I_2}{\partial u_t} = 0 = 2g_0 (z^k N_k \xi_t + G u_t) + 2 \lambda u_t \qquad \text{(A4.3)}$$

The control law which minimizes I_2 is therefore:

$$u_t = - \frac{z^k N_k \xi_t}{G + \lambda / g_0} \qquad \text{(A4.4)}$$

Substituting

$$\xi_t = \frac{y_t - z^{-k} G u_t}{N} \tag{A4.5}$$

and

$$N = N_k^* + N_k \tag{A4.6}$$

into equation A4.4 gives:

$$u_t = -\frac{z^k N_k / N}{G N_k^* / N + \lambda / g_0} \cdot y_t \tag{A4.7}$$

Using the identities

$$G = \frac{B}{1 + A} \quad , \tag{A4.8}$$

$$N = \frac{1 + D}{1 + C} \quad \text{and} \tag{A4.9}$$

$$1 + D = N_k^* (1 + C) + z^{-k} D' \tag{A4.10}$$

the finite-parameter form of the control law is found to be:

$$u_t = -\frac{D'(1 + A)}{B N_k^* (1 + C) + \lambda / b_0 \, (1 + A)(1 + D)} \cdot y_t \tag{A4.11}$$

A4-3

Derivation of Control Law which Minimizes Cost Function I_1

$$I_1 = E\{y^2_{/t}\} \tag{A4.12}$$

$$\equiv E\{\hat{y}^2_{t+k/t}\} \tag{A4.13}$$

as shown in paragraph 3.2.2 where

$$\hat{y}_{t+k/t} = z^k N_k \xi_t + G u_t \quad \text{as before.}$$

Differentiating I_1 wrt u_t and equating it to zero gives:

$$\frac{\partial I_1}{\partial u_t} = 0 = 2E\{z^k N_k \xi_t + G u_t\} \cdot g_0 \tag{A4.14}$$

The control law which minimizes I_1 is therefore:

$$u_t = -\frac{z^k N_k \xi_t}{G} \tag{A4.15}$$

Since equation (A4.15) is a special case of equation (A4.4) with cost factor λ equal to zero, the infinite-parameter and finite-parameter forms of the control law can be obtained from equations (A4.7) and (A4.11) directly.

$$u_t = -\frac{z^k N_k / N}{G N_k^* / N} \cdot y_{/t} \tag{A4.16}$$

$$= -\frac{D'(1+A)}{B N_k^* (1+C)} \cdot y_{/t} \tag{A4.17}$$

A4-4

Derivation of Control Law which Minimizes Cost Function I_3

$$I_3 = \hat{y}^2_{t+k/t} + \lambda(u_t - u_{t-1})^2 \qquad \text{(A4.18)}$$

where

$$\hat{y}_{t+k/t} = z^k N_k \xi_t + G u_t$$ as before.

Differentiating I_3 wrt $(u_t - u_{t-1})$ and equating it to zero gives:

$$\frac{\partial I_3}{\partial(u_t - u_{t-1})} = 0 = 2g_0 \left[z^k N_k \xi_t + \frac{G}{1-z^{-1}} \cdot (u_t - u_{t-1}) \right] + 2\lambda(u_t - u_{t-1}) \qquad \text{(A4.19)}$$

The control law which minimizes I_3 is therefore:

$$u_t - u_{t-1} = - \frac{z^k N_k \xi_t}{\frac{G}{1-z^{-1}} + \lambda/g_0} \qquad \text{(A4.20)}$$

Substituting

$$\xi_t = \frac{y_t - z^{-k} G u_t}{N}$$

and

$$N = N_k^* + N_k$$

into equation (A4.20) gives:

$$u_t - u_{t-1} = - \frac{z^k N_k / N}{\frac{G}{1-z^{-1}} \cdot N_k^*/N + \lambda/g_0} \cdot y_t \qquad \text{(A4.21)}$$

A4-5

Using the identities

$$G = \frac{B}{1+A} \quad ,$$

$$N = \frac{1+D}{1+C} \qquad \text{and}$$

$$1 + D = N_k^{*}(1+C) + z^{-k}D'$$

the finite- parameter form of the control law is found to be:

$$u_t - u_{t-1} = - \frac{(1+A)(1-z^{-1})D'}{N_k^{*}B(1+C) + \lambda/b_o\,(1+A)(1-z^{-1})(1+D)} \cdot y_t$$

(A4.22)

Annexure 5 to

Appendix A

SPECIFIC DERIVATION OF THE CONTROL LAWS WHICH RESPECTIVELY MINIMIZE COST FUNCTIONS I_1, I_2 and I_3 FOR THE DISTURBANCE STATIONARY

Repeating the discrete time presentation of the system subjected to a stationary disturbance with zero mean of equation (3.2.19).

$$y_t = \frac{K_1(1-a)z^{-1}}{1-az^{-1}} \cdot u_t + \frac{1}{1-cz^{-1}} \cdot \xi_t \qquad (A5.1)$$

Using the general form of the equation (equation 3.2.12)

$$y_t = \frac{z^{-k}B}{1+A} \cdot u_t + \frac{1+D}{1+C} \xi_t \qquad (A5.2)$$

define:

$k = 1$ (A5.3)

$A = -az^{-1}$ (A5.4)

$B = K_1(1-a)$ (A5.5)

$C = -cz^{-1}$ (A5.6)

$D = 0$ (A5.7)

Using the identity

$$1 + D = N_k^*(1+C) + z^{-k}D' \quad , \qquad (A5.8)$$

obtain

$$N_k^* = 1 + n_1 z^{-1} + \dots + n_{k-1} z^{-k+1} \quad \text{(A5.9)}$$

$$= 1 \cdot \text{ for } k = 1 \quad \text{(A5.10)}$$

and

$$1 + 0 = 1(1 - cz^{-1}) + z^{-1}(d_0' + d_1' z^{-1} + \dots) \quad \text{(A5.11)}$$

Comparing coefficients:

$$z^0 : 1 = 1$$
$$z^{-1} : 0 = -c + d_0'$$
$$z^{-2} : 0 = d_1'$$

gives:

$$D' = d_0' = c \quad \text{(A5.12)}$$

The general form of the control law which minimizes I_2 (equation (3.2.15)) is:

$$u_t = - \frac{(1+A)D'}{N_k^* B(1+C) + \lambda/b_0 (1+A)(1+D)} \cdot y_t \quad \text{(A5.13)}$$

Substituting the predefined polynomials into equation (A5.13), the following specific form of the control law which minimizes I_2 for the disturbance stationary is obtained:

$$u_t = \frac{1}{K_1(1-a) + \frac{\lambda}{K_1(1-a)}} \cdot \left\{ \left[K_1(1-a) + \frac{\lambda a}{K_1(1-a)} \right] u_{t-1} - c y_t + ac y_{t-1} \right\}$$

(A5.14)

Equating cost factor λ in equation (A5.14) to zero, the specific form of the control law which minimizes I_1 for the disturbance stationary is found to be:

$$u_t = \frac{1}{K_1(1-a)} \cdot \{ K_1(1-a)c\,u_{t-1} - c\,y_t + ac\,y_{t-1} \} \tag{A5.15}$$

The general form of the control law which minimizes I_3 (equation 3.2.16) is:

$$u_t - u_{t-1} = -\frac{(1+A)(1-z^{-1})\,D'}{N_k^* B(1+C) + \lambda/b_0\,(1+A)(1-z^{-1})(1+D)} \cdot y_t \tag{A5.16}$$

Substituting the predefined polynomials into equation (A5.16), the following specific form of the control law which minimizes I_3 for the disturbance stationary is obtained:

$$u_t = \frac{1}{K_1(1-a) + \frac{\lambda}{K_1(1-a)}} \cdot \Big\{ \Big[K_1(1-a)c + \frac{\lambda(a+1)}{K_1(1-a)} + K_1(1-a) + \frac{\lambda}{K_1(1-a)} \Big] \cdot u_{t-1}$$

$$- \Big[\frac{\lambda a}{K_1(1-a)} + K_1(1-a)c + \frac{\lambda(a+1)}{K_1(1-a)} \Big] \cdot u_{t-2}$$

$$+ \frac{\lambda a}{K_1(1-a)} \cdot u_{t-3} - c\,y_t + (a+1)c\,y_{t-1} - ac\,y_{t-2} \Big\}$$

(A5.17)

Annexure 6 to

Appendix A

SPECIFIC DERIVATION OF THE CONTROL LAWS WHICH RESPECTIVELY MINIMIZE COST FUNCTIONS I_1, I_2 AND I_3 FOR THE DISTURBANCE NON-STATIONARY

Repeating the discrete time presentation of the system subjected to a non-stationary disturbance of equation (3.2.23):

$$y_t = \frac{K_1(1-a)z^{-1}}{1-az^{-1}} \cdot u_t + \frac{1}{1-z^{-1}} \cdot \frac{1}{1-cz^{-1}} \cdot \xi_t \qquad \text{(A6.1)}$$

Using the general form of the equation (equation 3.2.12)

$$y_t = \frac{z^{-k}B}{1+A} \cdot u_t + \frac{1+D}{1+C} \cdot \xi_t \qquad \text{(A6.2)}$$

define:

$k = 1$ (A6.3)

$A = -az^{-1}$ (A6.4)

$B = K_1(1-a)$ (A6.5)

$C = -(1+c)z^{-1} + cz^{-2}$ (A6.6)

$D = 0$ (A6.7)

Using the identity

$$1 + D = N_k^*(1+C) + z^{-k}D' \quad , \qquad \text{(A6.8)}$$

obtain

$$N_k^* = 1 + n_1 z^{-1} + \ \ldots \ + n_{k-1} z^{-k+1} \qquad \text{(A6.9)}$$

$$= 1 \quad \text{for} \quad k = 1 \qquad \text{(A6.10)}$$

and

$$1 + 0 = 1\left[1 - (1+c)z^{-1} + cz^{-2}\right] + z^{-1}\left(d_0' + d_1' z^{-1} + d_2' z^{-2} + \ldots\right) \qquad \text{(A6.11)}$$

Comparing coefficients:

$$z^0 : \quad 1 = 1$$

$$z^{-1} : \quad 0 = -(1+c) + d_0'$$

$$z^{-2} : \quad 0 = c + d_1'$$

$$z^{-3} : \quad 0 = d_2'$$

gives:

$$D' = d_0' + d_1' = (1+c) - c = 1 \qquad \text{(A6.12)}$$

The general form of the control law which minimizes I_2 (equation (3.2.15)) is:

$$u_t = - \frac{(1+A)D'}{N_k^* B(1+C) + \lambda/b_0 \ (1+A)(1+D)} \cdot y_t \qquad \text{(A6.13)}$$

Substituting the predefined polynomials into equation (A6.13), the following specific form of the control law which minimizes I_2 for the disturbance non-stationary is obtained:

$$u_t = \frac{1}{K_1(1-a) + \dfrac{\lambda}{K_1(1-a)}} \cdot \left\{ \left[K_1(1-a)(1+c) + \frac{\lambda a}{K_1(1-a)} \right] u_{t-1} - K_1(1-a)c\,u_{t-2} - y_t + a y_{t-1} \right\} \qquad \text{(A6.14)}$$

Equating cost factor λ in equation (A6.14) to zero, the specific form of the control law which minimizes I_1 for the disturbance non-stationary is found to be:

$$u_t = \frac{1}{K_1(1-a)} \cdot \left\{ K_1(1-a)(1+c)\,u_{t-1} - K_1(1-a)c\,u_{t-2} - y_t + a y_{t-1} \right\} \qquad \text{(A6.15)}$$

The general form of the control law which minimizes I_3 (equation (3.2.16)) is:

$$u_t - u_{t-1} = - \frac{(1+A)(1-z^{-1})D'}{N_k^* B(1+C) + \lambda/b_0\,(1+A)(1-z^{-1})(1+D)}\, y_t \qquad \text{(A6.16)}$$

Substituting the predefined polynomials into equation (A6.16), the following specific form of the control law which minimizes I_3 for the disturbance non-stationary is obtained:

$$u_t = \frac{1}{K_1(1-a) + \dfrac{\lambda}{K_1(1-a)}} \cdot \Big\{ \Big[K_1(1-a)c + \frac{\lambda(a+1)}{K_1(1-a)} + K_1(1-a) + \frac{\lambda}{K_1(1-a)} \Big] u_{t-1}$$

$$- \Big[\frac{\lambda a}{K_1(1-a)} + K_1(1-a).c + \frac{\lambda(a+1)}{K_1(1-a)} \Big] u_{t-2}$$

$$+ \frac{\lambda a}{K_1(1-a)} \cdot u_{t-3} - cy_t + (a+1)cy_{t-1} - acy_{t-2} \Big\}$$

(A6.17)

Annexure 7 to

Appendix A

DERIVATION OF CHARACTERISTIC EQUATIONS RESPECTIVELY FOR SYSTEMS WITH STATIONARY AND NON-STATIONARY DISTURBANCE MODELS AND CONTROL LAWS WHICH RESPECTIVELY MINIMIZE COST FUNCTIONS I_2 AND I_3

Given the actual system description

$$y_t = \frac{z^{-k} B^o}{1 + A^o} \cdot u_t + \frac{1 + D^o}{1 + C^o} \cdot \xi_t \qquad \text{(A7.1)}$$

while the derived control laws are based on the model parameters A,B,C and D which are not exactly equal to the actual process parameters.

The system characteristic equation P(z) is obtained by writing the system equation in the form

$$y_t = \frac{Q(z)}{P(z)} \cdot \xi_t \qquad \text{(A7.2)}$$

System with Stationary Disturbance Model and Control Law which Minimizes Cost function I_2

Substituting the control law which minimizes cost function I_2

$$u_t = - \frac{(1+A) D'}{N_k^* B (1+C) + \lambda/b_o \; (1+A)(1+D)} \cdot y_t \qquad \text{(A7.3)}$$

and the identity

$$D' = z^k (1 + D) - z^k N_k^* (1 + C) \qquad \text{(A7.4)}$$

into equation (A7.1) gives:

$$P(z) = (1+C^{o})\{(1+C)N_k^{*}[B(1+A^{o}) - B^{o}(1+A)] + (1+A)(1+D)[\lambda/b_o(1+A^{o}) + B^{o}]\}$$

(A7.5)

Equating A = A°, B = B°, C = C° and D = D° gives:

$$P(z) = (1+C^{o})(1+A^{o})(1+D^{o})[\lambda/b_o(1+A^{o}) + B^{o}] \quad (A7.6)$$

As shown in Annexure 5 to Appendix A, the parameters for the disturbance model stationary are:

$k = 1$ (A7.7)

$A = -az^{-1}$ (A7.8)

$B = K_1(1-a)$ (A7.9)

$C = -cz^{-1}$ (A7.10)

$D = 0$ (A7.11)

$D^1 = c$ (A7.12)

Substituting these parameters into equation (A7.6) gives:

$$P(z) = (z-c)(z-a)\left(z - \frac{\lambda a}{\lambda + [K_1(1-a)]^2}\right) \quad (A7.13)$$

System with Non-stationary Disturbance Model and Control Law which Minimizes Cost function I_2

As shown in Annexure 6 to Appendix A, the parameters for the disturbance model non-stationary are:

$$k = 1 \tag{A7.14}$$

$$A = -az^{-1} \tag{A7.15}$$

$$B = K_1(1-a) \tag{A7.16}$$

$$C = -(1+c)z^{-1} + cz^{-2} \tag{A7.17}$$

$$D = 0 \tag{A7.18}$$

$$D^1 = 1 \tag{A7.19}$$

Substituting these parameters into equation (A7.6) gives:

$$P(z) = (z^2 - (1+c)z + c)(z-a)\left(z - \frac{\lambda a}{\lambda + [k_1(1-a)]^2}\right) \tag{A7.20}$$

System with Stationary Disturbance Model and Control Law which Minimizes Cost Function I_3

Substituting the control law which minimizes cost function I_3

$$u_t = -\frac{(1+A)D'}{N_k^* B(1+C) + \lambda/b_0\,(1+A)(1-z^{-1})(1+D)} \cdot y_t$$

and the identity

$$D' = z^k(1+D) - z^k N_k^*(1+C) \tag{A7.21}$$

into equation (A7.1) gives:

$$P(z) = (1+C^\circ)\{(1+C)N_k^*[B(1+A^\circ) - B^\circ(1+A)(1-z^{-1})]$$

$$+ (1+A)(1-z^{-1})(1+D)[\lambda/b_0\,(1+A^\circ) + B^\circ]\} \tag{A7.22}$$

Equating A = A°, B = B°, C = C° and D = D° gives:

$$P(z) = (1+C^{\circ})\{(1+C^{\circ})N_k^{*}z^{-1}B^{\circ}(1+A^{\circ})$$

$$+ (1+A^{\circ})(1-z^{-1})(1+D^{\circ})[\lambda/b_0 (1+A^{\circ}) + B^{\circ}]\} \quad (A7.23)$$

Substituting the parameters for the disturbance model stationary as in equations (A7.7) to (A7.12) into equation (A7.23) gives:

$$P(z) = (z-a)(z-c)\left(z^2 - \frac{\lambda(a+1)}{\frac{\lambda}{K_1(1-a)} + K_1(1-a)} z + \frac{\frac{\lambda a}{K_1(1-a)} - K_1(1-a)c}{\frac{\lambda a}{K_1(1-a)} - K_1(1-a)}\right) \quad (A7.24)$$

System with Non-stationary Disturbance Model and Control Law which Minimizes Cost Function I_3

Substituting the parameters for the disturbance model non-stationary as in equations (A7.14) to (A7.19) into equation (A7.23) gives:

$$P(z) = (z^2 - (1+c)z + c)(z-1)(z-a)\left(z - \frac{\frac{\lambda a}{K_1(1-a)} - K_1(1-a)}{\frac{\lambda}{K_1(1-a)} + K_1(1-a)}\right) \quad (A7.25)$$

APPENDIX B : PROGRAM LISTINGS

Annexure 1 : Listings of "LOADER", "SR/DATADEF", "SR/DECBIN" and "SR/TITLDEF"

2 : Listing of "DISTROUTS"

3 : Listing of "MAINPROG"

4 : Listing of "SR/PHICALC"

5 : Listing of "INTMODES", "INTVECTOR" and "ADC"

6 : Listing of "PLOTPACK", "STATPACK" and "STATPACK2"

Annexure 1 to

Appendix B

LISTINGS OF "LOADER", "SR/DATADEF", "SR/DECBIN" and "SR/TITLDEF"

```
10 REM **LOADER**
15 REM *Tape 1.3b-000/007*
20 REM *Purpose-
Conduct SR/DATADEF & save (.)
T$,C,D & S(a)
25 REM *OP-(.) T$,C,D & S(a)
30 REM *Subroutines called-
35 REM  SR/DATADEF(3900)
40 REM  SR/DECBIN(3300)
45 REM  SR/TITLDEF(3600)
50 REM
55 REM *Def SR-addresses
60 LET DATADEF=3900
65 LET DECBIN=3300
70 LET TITLDEF=3600
75 REM
80 REM *Go SR/DATADEF
85 GO SUB DATADEF
90 REM
95 REM *Save arrays
100 CLS
105 PRINT AT 9,1;
"Remove program cassette":
    PRINT AT 10,1;
"(Press CONT when done)"
110 IF INKEY$="c" OR INKEY$="C"
THEN GO TO 120
115 GO TO 110
120 CLS
125 INPUT "Storage code=?";Z$
130 PRINT AT  9,1;
"Save (.) T$,C,D & S(a)"
135 PRINT AT 11,1;
"1) Remove earphone socket"
140 PRINT AT 12,1;
"2) Position data tape & press
    RECORD"
145 PRINT AT 14,1;
"3) Press KB PB as prompted"
150 LET Y$="T$"+Z$:
    SAVE Y$ DATA T$()
155 LET Y$="C"+Z$:
    SAVE Y$ DATA C()
160 LET Y$="D"+Z$:
    SAVE Y$ DATA D()
165 LET Y$="S(a)"+Z$:
    SAVE Y$ DATA S()
170 REM
175 REM *Verify (.)T$,C,D&S(a)
180 CLS
185 PRINT AT 9,1;
"Verify (.) T$,C,D & S(a)"
190 PRINT AT 11,1;
"1) Insert earphone socket"
195 PRINT AT 12,1;
"2) Rewind to start of series"
200 LET Y$="T$"+Z$:
    VERIFY Y$ DATA T$()
205 LET Y$="C"+Z$:
    VERIFY Y$ DATA C()
210 LET Y$="D"+Z$:
    VERIFY Y$ DATA D()
215 LET Y$="S(a)"+Z$:
    VERIFY Y$ DATA S()
220 REM
225 CLS : PRINT AT 9,1;
"Remove data tape"
230 PRINT AT 11,1;
"Loading completed"
235 REM
240 REM *Done
245 STOP
```

```
3300 REM **SR/DECBIN**
3305 REM *Tape 1.3a-015/019*
3310 REM *Purpose-
         Convert a decimal
         number DEC to a 16-bit
         binary number with 2
         bytes,hibyte & lobyte*
3315 REM *IP-DEC (0<=DEC<=65025)
3320 REM *OP-hibyte,lobyte*
3325 REM *Var-,b$,c$,
              A$(.),A(.),i*
3330 REM
3335 REM *Convert DEC to string
          number c$-
3340 LET c$=""
3345 LET delta=INT (DEC/2)
3350 LET romeo=DEC-2*delta
3355 IF romeo=0 THEN LET b$="0"
3360 IF romeo=1 THEN LET b$="1"
3365 LET c$=b$+c$
3370 IF delta=0 THEN GO TO 3390
3375 LET DEC=delta
3380 GO TO 3345
3385 REM *Calc numeric value
          A(.) of string number-
3390 DIM A$(16): DIM A(16)
3395 REM *Initialise A(.)-
3400 FOR i=1 TO 16
3405 LET A(i)=0
3410 NEXT i
3415 LET bravo=LEN c$: LET A$=c$
3420 FOR i=1 TO bravo
3425 LET A(i)=VAL A$(bravo+1-i)
3430 NEXT i
3435 REM *Calc lower and higher
          bytes of A(.)-
3440 LET lobyte=0: LET hibyte=0
3445 FOR i=1 TO 8
3450 LET lobyte=lobyte+2↑(i-1)*
     A(i)
3455 LET hibyte=hibyte+2↑(i-1)*
     A(i+8)
3460 NEXT i
3465 RETURN
```

```
3600 REM **SR/TITLDEF**
3605 REM *Tape 1.3a-030/034*
3610 REM *Purpose-Inputs string
          description of the
          control function U(t)
          from the user*
3615 REM *IP (from operator)-
          String data for
          T$(.)*
3620 REM *OP-T$(.)*
3625 REM *Var-i,j,s$
3630 REM
3635 REM *Dimension T$(.)-
3640 CLS : PRINT AT 9,2;
"Input no. of lines (21 max)
 in title of U(t):":
 INPUT NSTRING:
 DIM T$(NSTRING,30): CLS
3645 REM *Load T$(.)-
3650 FOR i=1 TO NSTRING
3655 PRINT AT 21,5;
"Input title line ";i: INPUT s$:
 LET T$(i)=s$: CLS
3660 FOR j=1 TO i
3665 PRINT AT j,1;T$(j)
3670 NEXT j
3675 NEXT i
3680 RETURN
```

```
3900 REM **SR/DATADEF**
3904 REM *Tape 1.3a-090/105*
3908 REM *Purpose-
a)IP data for fixed constants
  array D(.)
b)IP+calc data for the system
  data constants array C(.)
c)IP data for the program status
  control array S(.)
3912 REM *Var-i,j
3916 REM *SR called-
         TITLDEF & DECBIN
3920 REM
3924 REM *Input string descrip=
         tion of U(t)-
3928 GO SUB TITLDEF
3932 REM **Define system con=
stants which remain unchanged
throughout the experiment &
store as D(.)-
3936 REM *Dim D(.)-
3940 DIM D(7)
3944 REM *Specific heat capacity
         H(J/kg.K) for water
3948 LET H=4190: LET D(1)=H
3952 REM *Mass M(kg) of water in
         cylinder-
3956 LET M=10: LET D(2)=M
3960 REM *Thermal resistance R
(K/W) between water and ambient-
3964 LET R=.5: LET D(3)=R
3968 REM *Ambient temp Te(C)-
3972 CLS : PRINT AT 9,1;
"Input ambient temp Te (deg C):"
: INPUT Te: CLS : LET D(4)=Te
3976 REM *Cold inlet water temp
         Ti(C)-
3980 PRINT AT 9,2;
"Input cold inlet water temp
 Ti (deg C):": INPUT Ti: CLS :
LET D(5)=Ti
3984 REM *OP flow-rate Vmax
(kg/s) with the OP v/v open-
3988 PRINT AT 9,2;
"Input output flow-rate Vmax
 (kg/s) with the output valve
 open:": INPUT Vmax: CLS : LET
D(6)=Vmax
3992 REM *Power rating(W) of
heater element-
3996 LET Prating=2700: LET D(7)=
Prating
4000 REM **Define system con=
stants which may change during
the experiment and load array
C(.) with these as well as the
constants allready defined-
4004 REM *Define the number of
different value BPS (called data
sets these constants will assume
during the experiment-
4008 PRINT AT 9,1;
"Input the number of different
 values BPS (called data-sets)
 the system constants will
 assume during the experiment:":
     INPUT BPS: CLS
4012 REM ***Load C(.)-
4016 REM *Dim C(.)-
4020 DIM C(BPS,40)
4024 REM ***********************
4028 FOR j=1 TO BPS
4032 REM
4036 REM **Input sampling data-
4040 REM *Input no. of samples
NsplBp in data-set j-
4044 PRINT AT 9,2;
"Input the number of samples
 NsplBp using data-set ";j;" of
";BPS: INPUT NsplBp: CLS :
LET C(j,1)=INT (NsplBp)
4048 REM *IP the sampling period
Ts(s) for data-set j-
4052 PRINT AT 9,1;
"Input the sampling period Ts(s)
 for data-set ";j;" of ";BPS:
     PRINT AT 12,4;
"(0<=Ts<=65535)": INPUT Ts: CLS
:    IF Ts<0 THEN LET Ts=0
4056 IF Ts>65535 THEN LET
     Ts=65535
4060 LET C(j,2)=INT (Ts)
```

```
4064 REM *Convert Ts to 16-bit
          number-
4068 LET DEC=Ts: GO SUB DECBIN
4072 LET TsLB=lobyte:
     LET TsHB=hibyte:
     LET C(j,3)=TsLB:
     LET C(j,4)=TsHB
4076 REM
4080 REM **IP disturbance con=
          trol data-
4084 REM *Initilize-
Calc Tvv for each C(j,5)
4088 REM *30s<=Tvv<=45s,Ts/Tvv
          is an integer
4092 LET i=1
4096 IF C(j,2)/i<=45 AND
C(j,2)/i>=30 THEN LET
Tvv=INT (C(j,2)/i):
LET C(j,5)=Tvv: GO TO 4104
4100 LET i=i+1: GO TO 4096
4104 REM
4108 REM *Convert Tvv to 16-bit
          number-
4112 LET DEC=Tvv: GO SUB DECBIN
4116 LET TvvLB=lobyte:
     LET TvvHB=hibyte:
     LET C(j,6)=TvvLB:
     LET C(j,7)=TvvHB
4120 REM *IP the nominal flow-
rate V(kg/s) for data-set j-
4124 PRINT AT 9,1;
"Input the nominal flow-rate V
 (kg/s) for data-set ";j;" of ";
 BPS: PRINT AT 13,4;
 "(0<=V<=Vmax)";: PRINT AT 14,4;
 "Vmax= ";Vmax;" kg/s": INPUT V:
 IF V>Vmax THEN LET V=Vmax
4128 IF V<0 THEN LET V=0
4132 LET C(j,9)=V: CLS
4136 REM *Input the width DELV
of the white noise flow-rate
pdf-
4140 REM *Calc max allowable
value DELVmax for DELV-
4144 IF 2*(Vmax-V)<=2*V THEN LET
     DELVmax=2*(Vmax-V)
4148 IF 2*V<=2*(Vmax-V) THEN LET
     DELVmax=2*V
4152 PRINT AT 9,1;
"Input the width DELV(kg/s) of
 the white noise flow-rate pdf
 which is symmetrical around V
 for data-set ";j;" of ";BPS:
     PRINT AT 14,7;
"DELV<=";DELVmax;" kg/s":
     INPUT DELV
4156 IF DELV>DELVmax THEN
     LET DELV=DELVmax
4160 LET C(j,10)=DELV: CLS
4164 REM
4168 REM **IP model constants-
4172 REM *IP the setpoint temp
Tt(C) for data-set j-
4176 PRINT AT 9,1;
"Input the setpoint temperature
 Tt(deg C) for data-set ";j;" of
 ";BPS: PRINT AT 13,5;  "(0<=Tt
<=100)": INPUT Tt
4180 IF Tt>100 THEN LET Tt=100
4184 IF Tt<0 THEN LET Tt=0
4188 LET C(j,11)=Tt: CLS
4192 REM *IP control cost-func=
tion factor LAMBDA-
4196 PRINT AT 9,2;
"Input control cost-function
 factor LAMBDA for data-set
 ";j;" of ";BPS: INPUT LAMBDA:
     LET C(j,16)=LAMBDA: CLS
4200 REM *IP power Qh to heater-
4204 LET Qh=V*H*(Tt-Ti)+
(Tt-Te)/R:   LET C(j,17)=Qh
4208 REM *Calc system constant
          K1(K/W)-
4212 LET K1=1/(V*H+1/R):
     LET C(j,18)=K1
4216 REM *Calc system constant
          K2(K/kg)-
4220 LET K2=(Tt-Ti)/M:
     LET C(j,19)=K2
4224 REM *Calc system constant
          Tn(s)-
4228 LET Tn=M*H/(V*H+1/R):
     LET C(j,20)=Tn
4232 REM *Calc system constant
          Tp(s)-
4236 LET Tp=M*H/(V*H+1/R):
     LET C(j,21)=Tp
4240 REM *Calc system constant
          ALPHA-
4244 LET ALPHA=1/EXP (Ts/Tp):
     LET C(j,22)=ALPHA
4248 REM *Calc system constant
          GAMMA-
4252 LET GAMMA=1/EXP (Ts/Tn):
     LET C(j,23)=GAMMA
4256 REM
4260 REM **Calculate control
          function constants-
4264 REM
4268 REM *c16=C(j,37) (dum var)
4272 LET c16=K1*(1-ALPHA):
     LET C(j,37)=c16
4276 REM *c1=C(j,24)-
4280 LET C(j,24)=1/(c16+LAMBDA/
c16)
4284 REM *c2=C(j,25)-
4288 LET C(j,25)=c16*GAMMA+
LAMBDA*ALPHA/c16
4292 REM *c3=-C(j,23) as defined
4296 REM *c4=C(j,26)-
4300 LET C(j,26)=ALPHA*GAMMA
4304 REM *c5=C(j,27)-
4308 LET C(j,27)=c16*(GAMMA+1)+
LAMBDA*(ALPHA+2)/c16
4312 REM *c6=C(j,28)-
4316 LET C(j,28)=-(LAMBDA*
(2*ALPHA+1)/c16+c16*GAMMA)
4320 REM *c7=C(j,29)-
4324 LET C(j,29)=LAMBDA*ALPHA/
c16
4328 REM *c8=C(j,30)-
4332 LET C(j,30)=(ALPHA+1)*
GAMMA
4336 REM *c9=-C(j,26) as defined
4340 REM *c10=C(j,31)-
4344 LET C(j,31)=c16*(1+GAMMA)+
LAMBDA*ALPHA/c16
4348 REM *c11=C(j,32)-
4352 LET C(j,32)=-c16*GAMMA
4356 REM *c12=C(j,33)-
4360 LET C(j,33)=c16*(2+GAMMA)+
LAMBDA*(ALPHA+2)/c16
4364 REM *c13=C(j,34)-
4368 LET C(j,34)=-(c16*(2*GAMMA+
1)+LAMBDA*(2*ALPHA+1)/c16)
4372 REM *c14=C(j,35)-
4376 LET C(j,35)=c16*GAMMA+
LAMBDA*ALPHA/c16
4380 REM *c15=C(j,36)-
4384 LET C(j,36)=ALPHA+1
4388 NEXT j
4392 REM
4396 REM **********************
4400 REM **Define program status
          control variables-
4404 REM *Dimension S(.)-
4408 DIM S(4)
4412 REM *Calculate the total
number of samples for the expe=
riment NSAMPLE as the sum of all
BPS values of NsplBp-
4416 LET NSAMPLE=0
4420 FOR i=1 TO BPS
4424 LET NSAMPLE=NSAMPLE+C(i,1)
4428 NEXT i
4432 LET S(1)=NSAMPLE
4436 REM *Save the number of
data-sets used in the experiment
4440 LET S(2)=BPS
4444 REM '*Initialize the number
of samples completed NCOMPL-
4448 LET NCOMPL=0: LET S(3)=NCOM
PL
4452 REM *Save the number of
lines NSTRING in the title of
U(t)-
4456 LET S(4)=NSTRING
4460 REM *Finish
4464 RETURN
```

Annexure 2 to

Appendix B

LISTING OF "DISTROUTS"

```
6000 REM **DISTROUTS**
6005 REM *Tape 1.3a-040/046*
6010 REM *Consists of-
b)SR/STATDIST (line 6025)
b)SR/NSTATDIST (line 6105)
6015 REM *Purpose-Generates
random variables for both stat &
non-stat dist & both experiment
(Tvon) & simulation(et)
6020 REM
6025 REM **SR/STATDIST**
6030 REM *Purpose-Generates stat
          rand dist var for both
          experiment(Tvon) &
          simulation (et)
6035 REM *IP-Data-set j,S(.),
          C(.),Vmax
6040 REM *OP-Tvon(v/v on-time)&
          et(simulated temp dist
6045 REM
6050 LET rnd=RND
6055 REM *Calc delTvon-
6060 LET delTvon=(rnd-.5)*
C(j,5)*C(j,10)/Vmax
6065 REM *Calc Tvon-
6070 LET Tvon=C(j,9)*C(j,5)/
Vmax+delTvon
6075 REM Calc zetat-
6080 LET zetat=C(j,19)*C(j,2)*
Vmax*delTvon/C(j,5)
6085 LET et=C(j,23)*et1+zetat
6090 LET et1=et
6095 RETURN
6100 REM
6105 REM **SR/NSTATDIST**
6110 REM *Purpose-As SR/STATDIST
          but for non-stat dist
6115 REM *IP&OP-As SR/STATDIST
6120 REM
6125 LET rnd=RND
6130 REM *Calc delTvonact
6135 LET delTvonact=delTvonact1
+(rnd-.5)*3
6140 IF delTvonact>C(j,5)-
C(j,9)*C(j,5)/Vmax THEN LET
delTvonact=.9*(C(j,5)-C(j,9)*
C(j,5)/Vmax)
6145 IF delTvonact<-C(j,9)*
C(j,5)/Vmax THEN LET delTvonact
=.9*(-C(j,9)*C(j,5)/Vmax)
6150 LET delTvonact1=delTvonact
6155 REM *Calc Tvon-
6160 LET Tvon=C(j,9)*C(j,5)/
Vmax+delTvonact
6165 REM *Calc zetat
6170 LET zetat=C(j,19)*C(j,2)*
Vmax*delTvonact/C(j,5)
6175 REM *Calc et
6180 LET et=C(j,23)*et1+zetat
6185 LET et1=et
6190 RETURN
```

Annexure 3 to

Appendix B

LISTING OF "MAINPROG"

```
  10 REM **MAINPROG**
  15 REM *Tape 1.3b-035/054*
  20 REM *Purpose-
Conduct stochastic control simu=
lation/experiment of 1st order
linear system subject to statio=
ary/non-stationary disturbance
using control law 1&2/3.
  25 REM *IP-(.)C,D & S(a)
  30 REM *OP-(.)Q,T & S(b)
  35 REM *Subroutines called-
  40 REM  SR/DECBIN (3300)
  45 REM  SR/PHICALC (3000)
  50 REM  SR/STATDIST (6025) or
          SR/NSTATDIST (6105)
  55 REM *Machine code routines
  called for experiment-
  60 REM  INTVECTORC & IM1/IM2
  65 REM
  70 REM **Select options-
  75 REM
  80 REM *Sim or exp?
  85 CLS : PRINT AT 9,1;
  "Simulation? (y/n)"
  90 IF INKEY$="y" OR INKEY$="Y"
  THEN LET SIM=1: GO TO 125
  95 IF INKEY$="n" OR INKEY$="N"
 THEN LET SIM=0: GO TO 125
 100 GO TO 90
 105 REM
 110 REM *Load machine code pro=
         grams if exp-
 115 IF SIM=1 THEN GO TO 125
 120 CLS : PRINT AT 9,1;
"Load machine code": PRINT AT
10,1;"(↑ CONT when loaded)":
STOP
 125 REM *Stat or non-stat dist
         model?
 130 PAUSE 100: CLS :
     PRINT AT 9,1;
"Stat(y)/Non-stat(n) dist
 model ?"
 135 IF INKEY$="y" OR INKEY$="Y"
THEN LET DIST=1: GO TO 155
 140 IF INKEY$="n" OR INKEY$="N"
THEN LET DIST=0: GO TO 155
 145 GO TO 135
 150 REM
 155 REM *Real dist stat or
         non-stat?
 160 PAUSE 100: CLS :
     PRINT AT 9,1;
"Real dist Stat(y)/Non-stat(n)?
"
 165 IF INKEY$="y" OR INKEY$="Y"
THEN LET DISTR=1: GO TO 185
 170 IF INKEY$="n" OR INKEY$="N"
THEN LET DISTR=0: GO TO 185
 175 GO TO 165
 180 REM
 185 REM *Control law 1&2/3?
 190 PAUSE 100: CLS :
     PRINT AT 9,1;
"Control law 1&2(y)/3(n)?"
 195 IF INKEY$="y" OR INKEY$="Y"
THEN LET CNTLAW=2: GO TO 215
 200 IF INKEY$="n" OR INKEY$="N"
THEN LET CNTLAW=3: GO TO 215
 205 GO TO 195
 210 REM
 215 REM *Perturbation required
         for sensitivity anal?
 220 CLS : PRINT AT 9,1;
"Parameters perturbed(p)/
 correct(c) ?"
 225 IF INKEY$="p" OR INKEY$="P"
THEN LET PERT=1: GO TO 240
 230 IF INKEY$="c" OR INKEY$="C"
THEN LET PERT=0: GO TO 280
 235 GO TO 225
 240 REM *Input correct values
         of perturbed params
 245 REM
 250 DIM P(9)
 255 CLS : INPUT
"Correct m(kg)=?";P(1): CLS
 260 INPUT "Correct V(kg/s)=?";
     P(2): CLS
 265 INPUT "Correct R(K/W)=?";
     P(3): CLS
 270 INPUT "Correct Ti(deg C)=?"
;    P(4): CLS
 275 REM
```

```
280 REM *Def SR-addresses
285 LET DECBIN=3300
290 LET PHICALC=3000
295 LET STATDIST=6025
300 LET NSTATDIST=6105
305 REM
310 REM *Load (.)C,D & S(a)-
315 CLS : PRINT AT 9,1;
"Remove program tape"
320 PRINT AT 10,1;
"(Press CONT when done)"
325 IF INKEY$="c" OR INKEY$="C"
    THEN GO TO 335
330 GO TO 325
335 CLS :
    INPUT "Storage code=?";Z$
340 CLS : PRINT AT 9,1;
"Load (.) C,D & S(a)"
345 PRINT AT 11,1;
"1) Insert & rewind data tape
    to start of series"
350 PRINT AT 13,1;
"2) Press PLAY"
355 LET Y$="C"+Z$:
    LOAD Y$ DATA C()
360 LET Y$="D"+Z$:
    LOAD Y$ DATA D()
365 LET Y$="S(a)"+Z$:
    LOAD Y$ DATA S()
370 CLS : PRINT AT 9,1;
"Stop & leave data tape in"
375 PAUSE 150
380 REM
385 REM *If params perturbed
        calc correct values
390 IF PERT=0 THEN GO TO 450
395 REM *Calc correct K2
400 LET P(5)=(C(1,11)-P(4))/
    P(1)
405 REM *Calc correct K1
410 LET P(6)=1/(P(2)*D(1)+1/
    P(3))
415 REM *Calc correct Tp & Tn
420 LET P(7)=P(1)*D(1)/
    (P(2)*D(1)+1/P(3))
425 REM *Calc correct ALPHA &
        GAMMA
430 LET P(8)=1/EXP (C(1,2)/
  P(7))
435 REM *Calc correct
        K1.(1-ALPHA)
440 LET P(9)=P(6)*(1-P(8))
445 REM
450 REM *Allocate D(.) names-
455 LET H=D(1): LET M=D(2):
    LET R=D(3): LET Te=D(4):
    LET Ti=D(5): LET Vmax=D(6):
    LET Prating=D(7)
460 CLS
465 REM
470 REM **Init condition if exp
475 REM
480 IF SIM=1 THEN GO TO 765
485 REM
490 REM *Zero flags-
495 LET Flag1=0
500 POKE 60001,Flag1
505 LET Flag2=0
510 POKE 60002,Flag2
515 LET Flag3=0
520 POKE 60003,Flag3
525 REM
530 REM *Invoke IM2-
535 LET usr=USR 57601
540 REM
545 REM *Program PIO's-
550 REM
555 REM *PIO1a OP-port-
560 OUT 11,207: OUT 11,0:
    LET PIO1a=3
565 REM *OP zero power-
570 OUT PIO1a,255
575 REM *PIO1b IP-port-
580 OUT 15,207: OUT 15,255
585 REM *PIO2b Control-port-
590 OUT 31,207: OUT 31,32:
    LET PIO2b=23
595 REM
```

```
600 REM **Pre-heating-
605 REM
610 CLS : PRINT AT 9,1;
"Pre-heating required? (y/n)"
615 IF INKEY$="n" OR INKEY$="N"
THEN GO TO 750
620 IF INKEY$="y" OR INKEY$="Y"
THEN GO TO 635
625 GO TO 615
630 REM
635 REM *Pre-heating required
640 CLS : INPUT
"Pre-heating power(W) ?";Pave:
CLS
645 GO SUB PHICALC
650 OUT PIO1a,Pwrbyte
655 REM
660 REM *Display temp each 5s-
665 REM *Init timer CTsHb&Lb-
670 LET CTsHb=0: LET CTsLb=5:
    POKE 60004,CTsHb:
    POKE 60005,CTsLb
675 REM *Activate timer-
680 LET Flag1=1:
    POKE 60001,Flag1
685 REM *Re-init PIO1a
690 OUT 11,207: OUT 11,0:
    OUT PIO1a,Pwrbyte
695 REM *Display temp & power-
700 PRINT AT 18,1;
    "(↑ s to stop)"
705 IF INKEY$="s" OR INKEY$="S"
    THEN GO TO 610
710 IF PEEK 60001=0 THEN LET
TEMP=PEEK 60014*1/2.55: CLS :
PRINT AT 9,1;"Temperature=";
INT TEMP;" deg C": PRINT AT
10,1; "Power=";Pave;" W":
GO TO 730
715 GO TO 685
720 REM *Limit power if req-
725 IF TEMP>C(1,11)+3 THEN
GO TO 750
730 REM *Safety limit
735 IF TEMP=C(1,11)+5 THEN
    GO TO 750
740 GO TO 665
745 REM
750 REM *End pre-heating,0W out
755 OUT PIO1a,255: CLS
760 REM
765 REM *Sample-class j calc
         init-
770 DIM L(S(2)): DIM H(S(2))
775 LET l=1
780 FOR i=1 TO S(2)
785 LET h=C(i,1)+l-1
790 LET L(i)=l: LET H(i)=h
795 LET l=h+1
800 NEXT i
805 REM
810 REM **Initialization-
815 REM
820 REM *Init j & S(3)-
825 LET j=1: LET S(3)=3
830 REM *Dimension (.) Q & T-
835 DIM Q(S(1)): DIM T(S(1))
840 REM *Init variables-
845 LET T(1)=C(1,11): LET yt2=0
    LET T(2)=C(1,11): LET yt1=0
    LET Q(1)=C(1,17): LET ut3=0
    LET Q(2)=C(1,17): LET ut2=0
    LET Q(3)=C(1,17): LET ut1=0
    LET qt1=0: LET et1=0:
    LET delTvonact1=0
850 REM *Activate real
    disturbance
855 IF DISTR=1 THEN GO SUB
    STATDIST
860 IF DISTR=0 THEN GO SUB
    NSTATDIST
865 REM **Init counters & flags
          and OP-power if exp-
870 IF SIM=1 THEN GO TO 960
875 REM *Convert Tvon to hex-
880 LET DEC=INT Tvon
885 GO SUB DECBIN
890 LET TvoHb=hibyte:
    LET TvoLb=lobyte
895 REM *Load counters-
900 LET CTsHb=C(1,4):
    LET CTsLb=C(1,3):
    LET CTvvHb=C(1,7):
    LET CTvvLb=C(1,6):
    LET CTvoHb=TvoHb:
    LET CTvoLb=TvoLb:
    LET TvvHb=C(1,7):
    LET TvvLb=C(1,6)
905 POKE 60004,CTsHb:
    POKE 60005,CTsLb:
    POKE 60006,CTvvHb:
    POKE 60007,CTvvLb:
    POKE 60008,CTvoHb:
    POKE 60009,CTvoLb
910 POKE 60010,TvvHb:
    POKE 60011,TvvLb:
    POKE 60012,TvoHb:
    POKE 60013,TvoLb
915 REM *Switch dist v/v on-
920 OUT PIO2b,BIN 11111101
925 REM *Set flags-
930 LET Flag3=1: LET Flag2=1:
    LET Flag1=1
935 POKE 60003,Flag3:
    POKE 60002,Flag2:
    POKE 60001,Flag1
940 REM *OP nominal power-
945 LET Pave=C(1,17)
950 GO SUB PHICALC
955 OUT PIO1a,Pwrbyte
960 REM *End of initialization
965 REM
970 REM **Start of main loop
975 REM
980 REM *Activate real
        disturbance
985 IF DISTR=1 THEN GO SUB
    STATDIST
990 IF DISTR=0 THEN GO SUB
    NSTATDIST
995 REM
1000 REM **Measure/calc yt &
          T(S(3)) for exp/sim-
1005 IF SIM=1 THEN GO TO 1145
1010 REM *Exp-
1015 REM *Convert Tvon to hex-
1020 LET DEC=Tvon
1025 GO SUB DECBIN
1030 LET TvoHb=hibyte:
     LET TvoLb=lobyte
1035 REM *Check if Flag1=0
1040 LET Flag1=PEEK 60001
1045 REM *Re-init PIOs
1050 OUT 11,207: OUT 11,0:
     OUT 15,207: OUT 15,255:
     OUT 31,207: OUT 31,32:
     OUT PIO1a,Pwrbyte
1055 IF flag1<>0 THEN GO TO 1040
1060 CLS
1065 REM *Scale  TEMP-
1070 LET TEMP=PEEK 60014*1/2.55
1075 REM *Round-off to 1 dec &
          save TEMP-
1080 LET TEMP=INT (TEMP*10+.5)/
     10: LET T(S(3))=TEMP
1085 REM *Load counters-
1090 LET CTsHb=C(1,4):
     LET CTsLb=C(1,3):
     LET CTvvHb=C(1,7):
     LET CTvvLb=C(1,6):
     LET CTvoHb=TvoHb:
     LET CTvoLb=TvoLb:
     LET TvvHb=C(1,7):
     LET TvvLb=C(1,6)
1095 POKE 60004,CTsHb:
     POKE 60005,CTsLb:
     POKE 60006,CTvvHb:
     POKE 60007,CTvvLb:
     POKE 60008,CTvoHb:
     POKE 60009,CTvoLb
1100 POKE 60010,TvvHb:
     POKE 60011,TvvLb:
     POKE 60012,TvoHb:
     POKE 60013,TvoLb
```

```
1105 REM *Switch dist v/v on-
1110 OUT PIO2b,BIN 11111101
1115 REM *Set flags-
1120 LET Flag3=1: LET Flag2=1:
     LET Flag1=1
1125 POKE 60003,Flag3:
     POKE 60002,Flag2:
     POKE 60001,Flag1
1130 REM *Calc  yt-
1135 LET yt=T(S(3))-C(j,11):
     LET qt=0
1140 GO TO 1205
1145 REM *Sim-
1150 REM *Calc qt-
1155 IF PERT=0 THEN GO TO 1175
1160 REM *For perturbed params-
1165 LET qt=P(8)*qt1+P(9)*ut1:
     GO TO 1180
1170 REM *Params not perturbed-
1175 LET qt=C(j,22)*qt1+C(j,37)*
     ut1
1180 REM *Limit et if necessary-
1185 IF qt+C(j,11)+et>100 THEN
     LET et=100-qt-C(j,11)
1190 IF qt+C(j,11)+et<0 THEN LET
     et=-qt-C(j,11)
1195 REM *Calc yt,round-off to
          one dec & calc T(S(3))
1200 LET yt=qt+et:
     LET yt=INT (yt*10+.5)/10:
     LET T(S(3))=yt+C(j,11)
1205 REM *End of yt & T(S(3))
          calc/measurement-
1210 REM
1215 REM **Program j-count con=
          trol-
1220 IF S(3)=S(1) THEN GO TO
     1530
1225 LET S(3)=S(3)+1
1230 IF S(3)<=H(j) AND S(3)>=
     L(j) THEN GO TO 1245
1235 IF j=S(2) THEN LET j=0
1240 LET j=j+1: GO TO 1230
1245 REM *End of j-count control
1250 REM
1255 REM **Calc control ut-
1260 REM
1265 IF CNTLAW=2 THEN GO TO 1310
1270 REM
1275 REM *Control law 3-
1280 IF DIST=1 THEN GO TO 1295
1285 REM *Non-stationary dist-
1290 LET ut=C(j,24)*(C(j,33)*
     ut1+C(j,34)*ut2+C(j,35)*
     ut3-yt+C(j,36)*yt1-C(j,22)*
     yt2): GO TO 1340
1295 REM *Stationary dist-
1300 LET ut=C(j,24)*(C(j,27)*
     ut1+C(j,28)*ut2+C(j,29)*
     ut3-C(j,23)*yt+C(j,30)*yt1-
     C(j,26)*yt2): GO TO 1340
1305 REM
1310 REM *Control law 1&2-
1315 IF DIST=1 THEN GO TO 1330
1320 REM *Non-stationary dist-
1325 LET ut=C(j,24)*(C(j,31)*
     ut1+C(j,32)*ut2-yt+c(j,22)*
     yt1): GO TO 1340
1330 REM *Stationary dist-
1335 LET ut=C(j,24)*(C(j,25)*
     ut1-C(j,23)*yt+C(j,26)*yt1)
1340 REM *End of ut-calc
1345 REM
1350 REM *Limit Q(S(3)) if
          necessary-
1355 LET Q(S(3))=
     INT (ut+C(j,17))
1360 IF ut+C(j,17)>Prating THEN
     LET Q(S(3))=Prating
1365 IF ut+C(j,17)<0 THEN LET
     Q(S(3))=0
1370 REM *Output power-
1375 IF SIM=1 THEN GO TO 1405
1380 LET Pave=Q(S(3))
1385 GO SUB PHICALC
1390 REM *Re-init PIOs
1395 OUT 11,207: OUT 11,0:
     OUT 15,207: OUT 15,255:
     OUT 31,207: OUT 31,32
1400 OUT PIO1a,Pwrbyte
1405 REM
1410 REM *Re-init dummy-var
1415 LET yt2=yt1: LET yt1=yt:
     LET ut3=ut2: LET ut2=ut1:
     LET ut1=ut: LET qt1=qt
1420 REM
1425 REM **Display data-
1430 REM
1435 REM *Display after every
          sample for exp and
          after every 10 samples
          for sim-
1440 IF NOT (SIM=0 OR (SIM=1
     AND S(3)/10=INT (S(3)/10)))
     THEN GO TO 970
1445 CLS
1450 IF DIST=1 THEN PRINT AT 1,1
;"Stationary disturbance"
1455 IF DIST=0 THEN PRINT AT 1,1
;"Non-stationary disturbance"
1460 IF CNTLAW=2 THEN PRINT AT
 2,1;"Control law 1/2"
1465 IF CNTLAW=3 THEN PRINT AT
 2,1;"Control law 3"
1470 PRINT AT 5,1;
 "Setpoint=";C(j,11);" deg C"
1475 PRINT AT 6,1;
 "Lambda=";C(j,16)
1480 PRINT AT 9,1;
 "Measured temp=";T(S(3)-1);
 " deg C"
1485 PRINT AT 10,1;
 "Output power=";Q(S(3));" W"
1490 PRINT AT 13,1;
 "Sampling period=";C(j,2);" s"
1495 PRINT AT 14,1;
 "Sample ";S(3);" of ";S(1)
1500 REM *End of display
1505 REM
1510 REM *Go to start of main
          loop-
1515 GO TO 970
1520 REM
1525 REM
1530 REM **End of run-
1535 REM
1540 REM *OP zero power-
1545 IF SIM=1 THEN GO TO 1560
1550 OUT PIO1a,255
1555 REM
1560 REM *Save (.) S(b),Q & T-
1565 CLS : PRINT AT 9,1;
"Save (.)S(b),Q & T"
1570 PRINT AT 11,1;
"1) Remove earphone socket"
1575 PRINT AT 12,1;
"2) Note count & press RECORD"
1580 PRINT AT 13,1;
"Press KB PB as prompted"
1585 LET Y$="S(b)"+Z$:
     SAVE Y$ DATA S()
1590 LET Y$="Q"+Z$:
     SAVE Y$ DATA Q()
1595 LET Y$="T"+Z$:
     SAVE Y$ DATA T()
1600 REM
1605 REM *Verify (.)S(b),Q & T-
1610 CLS
1615 PRINT AT 9,1;
"Verify (.)S(b),Q & T"
1620 PRINT AT 11,1;
"1) Insert earphone socket"
1625 PRINT AT 12,1;
"2) Rewind to previous count"
1630 PRINT AT 13,1;
"3) Press PLAY"
1635 LET Y$="S(b)"+Z$:
     VERIFY Y$ DATA S()
1640 LET Y$="Q"+Z$:
     VERIFY Y$ DATA Q()
1645 LET Y$="T"+Z$:
     VERIFY Y$ DATA T()
1650 REM
1655 CLS : PRINT AT 9,1;
"Remove & store data tape"
1660 PRINT AT 11,1;
"End of run"
1665 STOP
```

Annexure 4 to

Appendix B

LISTING OF "SR/PHICALC"

```
3000 REM **SR/PHICALC**
3005 REM *Tape 1.3a-000/007*
3010 REM *Purpose-
     1)Calc and OP byte to DAC
        for phase control for
        required Pave
     2)Enable interrupt
3015 REM *IP-Prating,Pave
3020 REM *OP-Pwrbyte (to port 3
           (OP-port for dac))*
3025 REM *SR called-SR/ENABLE
3030 REM *Var-PHI,Psol,Vadc
3035 REM
3040 REM *Limit Pave if req-
3045 IF Pave>Prating THEN
     LET Pave=Prating
3050 IF Pave<0 THEN LET Pave=0
3055 REM **Newton-Raphson-
3060 REM *Initialise PHI-
3065 LET PHI=PI/2
3070 REM *Calc new PHI-
3075 LET PHI=PHI+
     (SIN (2*(PI-PHI))+2*PHI-
     4*PI/6000*Pave)/
     (2*COS (2*(PI-PHI))-2)
3080 REM *Calc Psol for new PHI-
3085 LET Psol=6000/PI*
     (.5*PHI+.25*
        SIN (2*(PI-PHI)))
3090 REM *Psol close to Pave?
3095 IF ABS (Pave-Psol)>1
     THEN GO TO 3075
3100 REM *175<=PHI<=180 ?
           Yes,let Vadc=1.2
3105 IF PHI<=PI AND
     PHI>175/180*PI THEN
     LET Vadc=1.2
3110 IF PHI<=PI AND
     PHI>175/180*PI THEN
     GO TO 3130
3115 REM *Calc Vadc-
3120 LET Vadc=4-2.8/175*PHI/PI*
           180
3125 REM *Calc Pwrbyte-
3130 LET Pwrbyte=INT (255/2.8*
     Vadc-1.2*255/2.8+.5)
3145 RETURN
```

Annexure 5 to

Appendix B

LISTINGS OF "INTMODES", "INTVECTOR" AND "ADC"

```
00010 ;**INTVECTOR1**
00020 ;Tape 1.5a-036
00030 ;Another version of this text
00040 ;file but without comment
00050 ;statements is stored as
00060 ;INTVECTOR2 on Tape 1.5a
00070 ;The code is stored as
00080 ;INTVECTORC on Tape 1.5a
00090 ;
00100 ;Interrupt vector=57640=E128H
00110 ;PIO-ports defined in subroutine
00120 ;ADC
00130 ;
00140 ;Origen of code address 57640D
00150 ;or E128H
00160         ORG   57640D
00170 ;
00180 ;50-Hz counter CSEC address
00190 CSEC    EQU   60000D
00200 ;
00210 ;50-Hz loop
00220 ;Save registers on stack
00230 INTR    PUSH AF
00240         PUSH HL
00250         PUSH BC
00260         PUSH DE
00270 ;1-Hz clock (50-Hz counter)
00280 Hz1     LD    A,(CSEC)
00290         INC   A
00300         LD    (CSEC),A
00310         CP    50
00320         JP    Z,CONTRL
00330 ;Call keyboard read routine
00340 KEYB    CALL  02BFH
00350 ;Restore registers
00360         POP   DE
00370         POP   BC
00380         POP   HL
00390         POP   AF
00400 ;Enable interrupts
00410         EI
00420 ;Return
00430         RET
00440
00450
00460
00470
00480 ;Temperature measurement and
00490 ;disturbance v/v control routine
00500 ;Entry once/second
00510 ;
00520 ;Flags addresses
00530 flag1   EQU   CSEC+1
00540 flag2   EQU   flag1+1
00550 flag3   EQU   flag2+1
00560 ;
00570 ;Counters addresses
00580 CTsHb   EQU   flag3+1
00590 CTsLb   EQU   CTsHb+1
00600 CTvvHb  EQU   CTsLb+1
00610 CTvvLb  EQU   CTvvHb+1
00620 CTvoHb  EQU   CTvvLb+1
00630 CTvoLb  EQU   CTvoHb+1
00640 ;
00650 ;Next sampling period (Counters)
00660 ;addresses
00670 TvvHb   EQU   CTvoLb+1
00680 TvvLb   EQU   TvvHb+1
00690 TvoHb   EQU   TvvLb+1
00700 TvoLb   EQU   TvoHb+1
00710 ;
00720 ;PIOs' addresses
00730 PIO1B   EQU   7D
00740 PIO2B   EQU   23D
00750 ;
00760 ;
00770 ;Reset 50-Hz counter
00780 CONTRL  LD    A,0
00790         LD    (CSEC),A
00800 ;
```

```
0000            00010 ;**INTVECTOR2*
0000            00020 ;Tape 1.5a-050
0000            00030
0000            00040
0000            00060
0000            00140
0000            00160        ORG   57640D
E128            00170
E128            00180
EA60            00190 CSEC   EQU   60000D
E128            00200
E128            00210
E128 F5         00230 INTR   PUSH  AF
E129 E5         00240        PUSH  HL
E12A C5         00250        PUSH  BC
E12B D5         00260        PUSH  DE
E12C            00270
E12C 3A60EA     00280 Hz1    LD    A,(CSEC)
E12F 3C         00290        INC   A
E130 3260EA     00300        LD    (CSEC),A
E133 FE32       00310        CP    50
E135 CA41E1     00320        JP    Z,CONTRL
E138            00330
E138 CDBF02     00340 KEYB   CALL  02BFH
E13B            00350
E13B D1         00360        POP   DE
E13C C1         00370        POP   BC
E13D E1         00380        POP   HL
E13E F1         00390        POP   AF
E13F            00400
E13F FB         00410        EI
E140            00420
E140 C9         00430        RET
E141            00440
E141            00450
E141            00460
E141            00470
E141            00520
EA61            00530 flag1  EQU   CSEC+1
EA62            00540 flag2  EQU   flag1+1
EA63            00550 flag3  EQU   flag2+1
E141            00560
EA64            00580 CTsHb  EQU   flag3+1
EA65            00590 CTsLb  EQU   CTsHb+1
EA66            00600 CTvvHb EQU   CTsLb+1
EA67            00610 CTvvLb EQU   CTvvHb+1
EA68            00620 CTvoHb EQU   CTvvLb+1
EA69            00630 CTvoLb EQU   CTvoHb+1
E141            00640
EA6A            00670 TvvHb  EQU   CTvoLb+1
EA6B            00680 TvvLb  EQU   TvvHb+1
EA6C            00690 TvoHb  EQU   TvvLb+1
EA6D            00700 TvoLb  EQU   TvoHb+1
E141            00710
E141            00720
0007            00730 PIO1B  EQU   7D
0017            00740 PIO2B  EQU   23D
E141            00750
E141            00760
E141 3E00       00780 CONTRL LD    A,0
E143 3260EA     00790        LD    (CSEC),A
E146            00800
```

```
;Check if flag1 is set
        LD   A,(flag1)
        CP   1
        JP   NZ,N1
;
;Yes, flag1 is set
;Decrement Counter Ts
Y1      LD   A,(CTsHb)
        LD   B,A
        LD   A,(CTsLb)
        LD   C,A
        DEC  BC
        LD   A,B
        LD   (CTsHb),A
        LD   A,C
        LD   (CTsLb),A
;Check if decremented value of
;Counter Ts=0
        LD   A,C
        OR   B
        JP   Z,Y2
;
;No,Counter Ts not= 0
;Check if flag2 is set
N2      LD   A,(flag2)
        CP   1
        JP   NZ,N3
;
;Yes, flag2 is set
;Decrement Counter Tvv
Y3      LD   A,(CTvvHb)
        LD   B,A
        LD   A,(CTvvLb)
        LD   C,A
        DEC  BC
        LD   A,B
        LD   (CTvvHb),A
        LD   A,C
        LD   (CTvvLb),A
;Check if decremented value of
;Counter Tvv=0
        LD   A,C
        OR   B
        JP   Z,Y4
;
;No, Counter Tvv not= 0
;Check if flag3 is set
N4      LD   A,(flag3)
        CP   1
        JP   NZ,N5
;



;Yes, flag3 is set
;Decrement Counter Tvon
Y5      LD   A,(CTvoHb)
        LD   B,A
        LD   A,(CTvoLb)
        LD   C,A
        DEC  BC
        LD   A,B
        LD   (CTvoHb),A
        LD   A,C
        LD   (CTvoLb),A
;Check if decremented value of
;Counter Tvon=0
        LD   A,C
        OR   B
        JP   NZ,N6
;
;Yes, Counter Tvon=0
;Clear flag3
Y6      LD   A,0
        LD   (flag3),A
;Switch v/v off
        LD   A,11111100B
        OUT  (PIO2B),A
;Finish
        JP   FINISH
;
;No, flag1 is not set
;Switch v/v off
N1      LD   A,11111100B
        OUT  (PIO2B),A
;Finish
        JP   FINISH
;
```

```
E146
E146 3A61EA          LD   A,(flag1)
E149 FE01            CP   1
E14B C2ACE1          JP   NZ,N1
E14E
E14E
E14E 3A64EA     Y1   LD   A,(CTsHb)
E151 47              LD   B,A
E152 3A65EA          LD   A,(CTsLb)
E155 4F              LD   C,A
E156 0B              DEC  BC
E157 78              LD   A,B
E158 3264EA          LD   (CTsHb),A
E15B 79              LD   A,C
E15C 3265EA          LD   (CTsLb),A
E15F
E15F
E15F 79              LD   A,C
E160 B0              OR   B
E161 CAB3E1          JP   Z,Y2
E164
E164
E164 3A62EA     N2   LD   A,(flag2)
E167 FE01            CP   1
E169 C2C2E1          JP   NZ,N3
E16C
E16C
E16C 3A66EA     Y3   LD   A,(CTvvHb)
E16F 47              LD   B,A
E170 3A67EA          LD   A,(CTvvLb)
E173 4F              LD   C,A
E174 0B              DEC  BC
E175 78              LD   A,B
E176 3266EA          LD   (CTvvHb),A
E179 79              LD   A,C
E17A 3267EA          LD   (CTvvLb),A
E17D
E17D
E17D 79              LD   A,C
E17E B0              OR   B
E17F CAC9E1          JP   Z,Y4
E182
E182
E182 3A63EA     N4   LD   A,(flag3)
E185 FE01            CP   1
E187 C2EDE1          JP   NZ,N5
E18A
E18A
E18A
E18A
E18A 3A68EA     Y5   LD   A,(CTvoHb)
E18D 47              LD   B,A
E18E 3A69EA          LD   A,(CTvoLb)
E191 4F              LD   C,A
E192 0B              DEC  BC
E193 78              LD   A,B
E194 3268EA          LD   (CTvoHb),A
E197 79              LD   A,C
E198 3269EA          LD   (CTvoLb),A
E19B
E19B
E19B 79              LD   A,C
E19C B0              OR   B
E19D C2F0E1          JP   NZ,N6
E1A0
E1A0
E1A0 3E00       Y6   LD   A,0
E1A2 3263EA          LD   (flag3),A
E1A5
E1A5 3EFC            LD   A,11111100B
E1A7 D317            OUT  (PIO2B),A
E1A9
E1A9 C3F3E1          JP   FINISH
E1AC
E1AC
E1AC
E1AC 3EFC       N1   LD   A,11111100B
E1AE D317            OUT  (PIO2B),A
E1B0
E1B0 C3F3E1          JP   FINISH
E1B3
```

```
;Yes, Counter Ts=0
;Clear flag1
Y2      LD    A,0
        LD    (flag1),A
;Switch v/v off
        LD    A,11111100B
        OUT   (PIO2B),A
;Measure and store temperature
        CALL ADC
;Finish
        JP    FINISH
;
;No, flag2 is not set
;Switch v/v off
N3      LD    A,11111100B
        OUT   (PIO2B),A
;Finish
        JP    FINISH
;
;Yes, Counter Tvv=0
;Reload Counter Tvv with Tvv
Y4      LD    A,(TvvHb)
        LD    (CTvvHb),A
        LD    A,(TvvLb)
        LD    (CTvvLb),A
;Reload Counter Tvon with Tvon
        LD    A,(TvoHb)
        LD    (CTvoHb),A
        LD    A,(TvoLb)
        LD    (CTvoLb),A
;Switch v/v on
        LD    A,11111101B
        OUT   (PIO2B),A
;Set flag3
        LD    A,1
        LD    (flag3),A
;Finish
        JP    FINISH
;
;No, flag3 is not set
;Finish
N5      JP    FINISH
;
;No, Counter Tvon not= 0
;Finish
N6      JP    FINISH
;
;Return to 50-Hz loop
FINISH JP   KEYB




;********************************
;** ADC **
;TEMPERATURE MEASUREMENT SR
;Converts 128 analog values to
;8-bit bytes in succession and
;calculates the average
;
;Temperature storage byte
;"Temp" address
Temp    EQU   TvoLb+1
;
;PIO1 port B: 8-bit converted
;                temperature value
;PIO2 port B control bits:
; Outputs B0 & B1 low for v/v
; shut, B0 high and B1 low for
; v/v open
; Output B2: Not-CS
; Output B3: Not-WR
; Output B4: Not-RD
; Input  B5: Not-INTR
; Outputs B6,B7 not used,leave
; high
;
;Initialize counter reg B
ADC     LD    B,128
;Initialize sum reg pair HL
        LD    HL,0
;Initialize reg pair DE
        LD    DE,0
;
```

```
E1B3            01700
E1B3 3E00       01720 Y2      LD    A,0
E1B5 3261EA     01730         LD    (flag1),A
E1B8            01740
E1B8 3EFC       01750         LD    A,11111100
                                    B
E1BA D317       01760         OUT   (PIO2B),A
E1BC            01770
E1BC CDF6E1     01780         CALL ADC
E1BF            01790
E1BF C3F3E1     01800         JP    FINISH
E1C2            01810
E1C2            01820
E1C2            01830
E1C2 3EFC       01840 N3      LD    A,11111100
                                    B
E1C4 D317       01850         OUT   (PIO2B),A
E1C6            01860
E1C6 C3F3E1     01870         JP    FINISH
E1C9            01880
E1C9            01890
E1C9 3A6AEA     01910 Y4      LD    A,(TvvHb)
E1CC 3266EA     01920         LD    (CTvvHb),A
E1CF 3A6BEA     01930         LD    A,(TvvLb)
E1D2 3267EA     01940         LD    (CTvvLb),A
E1D5            01950
E1D5 3A6CEA     01960         LD    A,(TvoHb)
E1D8 3268EA     01970         LD    (CTvoHb),A
E1DB 3A6DEA     01980         LD    A,(TvoLb)
E1DE 3269EA     01990         LD    (CTvoLb),A
E1E1            02000
E1E1 3EFD       02010         LD    A,11111101
                                    B
E1E3 D317       02020         OUT   (PIO2B),A
E1E5            02030
E1E5 3E01       02040         LD    A,1
E1E7 3263EA     02050         LD    (flag3),A
E1EA            02060
E1EA C3F3E1     02070         JP    FINISH
E1ED            02080
E1ED            02090
E1ED            02100
E1ED C3F3E1     02110 N5      JP    FINISH
E1F0            02120
E1F0            02130
E1F0            02140
E1F0 C3F3E1     02150 N6      JP    FINISH
E1F3            02160
E1F3            02170
E1F3 C338E1     02180 FINISH JP    KEYB
E1F6            02190
E1F6            02200
E1F6            02220
E1F6            02250 ;** ADC **
E1F6            02260
E1F6            02270
EA6E            02330 Temp    EQU   TvoLb+1
E1F6            02340
E1F6            02350
E1F6 0680       02490 ADC     LD    B,128
E1F8            02500
E1F8 210000     02510         LD    HL,0
E1FB            02520
E1FB 110000     02530         LD    DE,0
E1FE            02540
```

```
02550 ;Let Not-CS stay low
02560 ;Let Not-RD go hi-lo-hi to
02570 ;ensure Not-INTR high
02580 ;Not-RD hi
02590         LD    A,11111000B
02600         OUT   (PIO2B),A
02610 ;Not-RD lo
02620         LD    A,11101000B
02630         OUT   (PIO2B),A
02640 ;Not-RD hi
02650         LD    A,11111000B
02660         OUT   (PIO2B),A
02670 ;
02680 ;Initiate conversion
02690 ;Not-WR lo
02700         LD    A,11110000B
02710         OUT   (PIO2B),A
02720 ;Not-WR hi
02730         LD    A,11111000B
02740         OUT   (PIO2B),A
02750 ;
02760 ;Wait for conversion to end
02770 WAIT1   IN    A,(PIO2B)
02780         BIT   5,A
02790         JR    NZ,WAIT1
02800 ;
02810 ;Latch and load converted value
02820 ;into reg pair 2 (DE)
02830 ;Latch
02840 ;Not-RD lo
02850         LD    A,11101000B
02860         OUT   (PIO2B),A
02870 ;Read value
02880         IN    A,(PIO1B)
02890         LD    E,A
02900 ;Not-RD hi
02910         LD    A,11111000B
02920         OUT   (PIO2B),A
02930 ;
02940
02950
02960
02970 ;Initiate conversion
02980 ;Not-WR lo
02990 ICON    LD    A,11110000B
03000         OUT   (PIO2B),A
03010 ;Not-WR hi
03020         LD    A,11111000B
03030         OUT   (PIO2B),A
03040 ;
03050 ;Sum reg pair HL and reg pair DE
03060         ADD   HL,DE
03070 ;Check if counter in reg B is 0
03080         DEC   B
03090         LD    A,0
03100         CP    B
03110 ;If (reg B)=0 jump to FIN1
03120         JR    Z,FIN1
03130 ;
03140 ;No, (reg B) not= 0
03150 ;Wait for conversion to finish
03160 WAIT2   IN    A,(PIO2B)
03170         BIT   5,A
03180         JR    NZ,WAIT2
03190 ;
```

```
E1FE            02550
E1FE 3EF8       02590         LD    A,11111000
                                    B
E200 D317       02600         OUT   (PIO2B),A
E202            02610
E202 3EE8       02620         LD    A,11101000
                                    B
E204 D317       02630         OUT   (PIO2B),A
E206            02640
E206 3EF8       02650         LD    A,11111000
                                    B
E208 D317       02660         OUT   (PIO2B),A
E20A            02670
E20A            02680
E20A            02690
E20A 3EF0       02700         LD    A,11110000
                                    B
E20C D317       02710         OUT   (PIO2B),A
E20E            02720
E20E 3EF8       02730         LD    A,11111000
                                    B
E210 D317       02740         OUT   (PIO2B),A
E212            02750
E212            02760
E212 DB17       02770 WAIT1   IN    A,(PIO2B)
E214 CB6F       02780         BIT   5,A
E216 20FA       02790         JR    NZ,WAIT1
E218            02800
E218            02810
E218 3EE8       02850         LD    A,11101000
                                    B
E21A D317       02860         OUT   (PIO2B),A
E21C            02870
E21C DB07       02880         IN    A,(PIO1B)
E21E 5F         02890         LD    E,A
E21F            02900
E21F 3EF8       02910         LD    A,11111000
                                    B
E221 D317       02920         OUT   (PIO2B),A
E223            02930
E223            02940
E223            02950
E223 3EF0       02990 ICON    LD    A,11110000
                                    B
E225 D317       03000         OUT   (PIO2B),A
E227            03010
E227 3EF8       03020         LD    A,11111000
                                    B
E229 D317       03030         OUT   (PIO2B),A
E22B            03040
E22B            03050
E22B 19         03060         ADD   HL,DE
E22C            03070
E22C 05         03080         DEC   B
E22D 3E00       03090         LD    A,0
E22F B8         03100         CP    B
E230            03110
E230 2813       03120         JR    Z,FIN1
E232            03130
E232            03140
E232            03150
E232 DB17       03160 WAIT2   IN    A,(PIO2B)
E234 CB6F       03170         BIT   5,A
E236 20FA       03180         JR    NZ,WAIT2
E238            03190
```

```
;Latch and load converted value
;into reg pair 2 (DE)
;Latch
;Not-RD lo
        LD   A,11101000B
        OUT  (PIO2B),A
;Read value
        IN   A,(PIO1B)
        LD   E,A
;Not-RD hi
        LD   A,11111000B
        OUT  (PIO2B),A
;
;Jump to ICON
        JR   ICON
;
;Yes, (reg B)=0
;Calculate average of sum in
;reg pair 1 (HL)
;Average=(HL)/128
;Devision by 128 equivalent to
;shifting (HL) 7 bits to the
;right
;
;Manipulate lower byte
FIN1    LD   A,L
;Clear lower 7 bits
        AND  10000000B
;Shift 7 bits to the right
        LD   B,7
SHIFT   SRL  A
        DJNZ SHIFT
;Store intermediate result in
;reg C
        LD   C,A
;Manipulate higher byte
        LD   A,H
;Clear MSB
        AND  01111111B
;Shift 1 bit to the left
        SLA  A
;Logical OR manipulated hi and
;lo bytes
        OR   C
;
;Save temperature
        LD   (Temp),A
;Return
        RET
;******************************
;
;End of source code
        END
```

```
E238          03200
E238          03210
E238 3EE8     03240         LD   A,11101000
                                 B
E23A D317     03250         OUT  (PIO2B),A
E23C          03260
E23C DB07     03270         IN   A,(PIO1B)
E23E 5F       03280         LD   E,A
E23F          03290
E23F 3EF8     03300         LD   A,11111000
                                 B
E241 D317     03310         OUT  (PIO2B),A
E243          03320
E243          03330
E243 18DE     03340         JR   ICON
E245          03350
E245          03360
E245 7D       03450 FIN1    LD   A,L
E246          03460
E246 E680     03470         AND  10000000B
E248          03480
E248 0607     03490         LD   B,7
E24A CB3F     03500 SHIFT   SRL  A
E24C 10FC     03510         DJNZ SHIFT
E24E          03520
E24E          03530
E24E 4F       03540         LD   C,A
E24F          03550
E24F 7C       03560         LD   A,H
E250          03570
E250 E67F     03580         AND  01111111B
E252          03590
E252 CB27     03600         SLA  A
E254          03610
E254          03620
E254 B1       03630         OR   C
E255          03640
E255          03650
E255 326EEA   03660         LD   (Temp),A
E258          03670
E258 C9       03680         RET
E259          03690
E259          03700
E259          03720         END

00000 TOTAL ERRORS

ADC            E1F6
CONTRL         E141
CSEC           EA60
CTSHB          EA64
CTSLB          EA65
CTVOHB         EA68
CTVOLB         EA69
CTVVHB         EA66
CTVVLB         EA67
FIN1           E245
FINISH         E1F3
FLAG1          EA61
FLAG2          EA62
FLAG3          EA63
HZ1            E12C
ICON           E223
INTR           E128
KEYB           E138
N1             E1AC
N2             E164
N3             E1C2
N4             E182
N5             E1ED
N6             E1F0
PIO1B          0007
PIO2B          0017
SHIFT          E24A
TEMP           EA6E
TVOHB          EA6C
TVOLB          EA6D
TVVHB          EA6A
TVVLB          EA6B
WAIT1          E212
WAIT2          E232
Y1             E14E
Y2             E183
Y3             E16C
Y4             E1C9
Y5             E18A
Y6             E1A0
```

```
00010  ;**INTMODES1**
00020  ;Tape 1.5a-065
00030  ;Another version of this text
00040  ;file but without comment
00050  ;statements is stored as
00060  ;INTMODES2 on Tape 1.5a
00070  ;The code is stored as
00080  ;INTMODESC on Tape 1.5a
00090  ;
00100  ;Purpose:
00110  ;Either set IM2 and load I-reg
00120  ;or set IM1
00130  ;To initialize IM2:
00140  ;"LET usr=USR 57601"
00150  ;To initialize IM1:
00160  ;"LET usr=USR 57612"
00170  ;
00180  ;Origen of code address 57601
00190  ;or 0E101H
00200          ORG   57601
00210  ;
00220  ;Initialize IM2
00230  ;(I)=31H; (databus)=0FFH
00240  ;(31FFH)=28H (ROM)
00250  ;(3200H)=0E1H (ROM)
00260  ;Interrupt service routine at
00270  ;0E128H=57640
00280  IM2     DI
00290          PUSH  AF
00300          LD    A,31H
00310          LD    I,A
00320          IM    2
00330          POP   AF
00340          EI
00350          RET
00360  ;
00370  ;Initialize IM1
00380  IM1     IM    1
00390          RET
00400  ;
00410          END
00420
```

```
0000            00010 ;**INTMODES2**
0000            00020 ;Tape 1.5a-072
0000            00090 ;
0000            00200           ORG   57601
E101            00210 ;
E101 F3         00280 IM2       DI
E102 F5         00290           PUSH  AF
E103 3E31       00300           LD    A,31H
E105 ED47       00310           LD    I,A
E107 ED5E       00320           IM    2
E109 F1         00330           POP   AF
E10A FB         00340           EI
E10B C9         00350           RET
E10C            00360 ;
E10C ED56       00380 IM1       IM    1
E10E C9         00390           RET
E10F            00400 ;
E10F            00410           END

00000 TOTAL ERRORS

IM1             E10C
IM2             E101
```

Annexure 6 to

Appendix B

LISTINGS OF "PLOTPACK", "STATPACK" AND "STATPACK 2"

```
5000>REM **PLOTPACK**
5002 REM *Tape 1.3a-055/077*
5004 REM *Purpose-
Plot Q(t)&T(t) &/or U(t)&Y(t) on
request (screen &/or hard-copy)
5006 REM *Input-(.)C-S-Q-T
5008 REM *Output-plots
5010 REM *Approx 13,5K code
5012 REM *Arrays used-L(.)&H(.)
5014 REM *Range-
a)0<=temperature<=100  (deg C)
b)0<=power<=3000  (W)
5016 REM
5018 REM
5020 REM *Data-poking of UDG for
          subscripts
5022 REM
5024 RESTORE 5194
5026 FOR i=0 TO 7
5028 READ row
5030 POKE USR "A"+i,row
5032 NEXT i
5034 FOR i=0 TO 7
5036 READ row
5038 POKE USR "B"+i,row
5040 NEXT i
5042 FOR i=0 TO 7
5044 READ row
5046 POKE USR "C"+i,row
5048 NEXT i
5050 FOR i=0 TO 7
5052 READ row
5054 POKE USR "D"+i,row
5056 NEXT i
5058 FOR i=0 TO 7
5060 READ row
5062 POKE USR "E"+i,row
5064 NEXT i
5066 FOR i=0 TO 7
5068 READ row
5070 POKE USR "F"+i,row
5072 NEXT i
5074 FOR i=0 TO 7
5076 READ row
5078 POKE USR "G"+i,row
5080 NEXT i
5082 FOR i=0 TO 7
5084 READ row
5086 POKE USR "H"+i,row
5088 NEXT i
5090 FOR i=0 TO 7
5092 READ row
5094 POKE USR "I"+i,row
5096 NEXT i
5098 FOR i=0 TO 7
5100 READ row
5102 POKE USR "J"+i,row
5104 NEXT i
5106 FOR i=0 TO 7
5108 READ row
5110 POKE USR "K"+i,row
5112 NEXT i
5114 FOR i=0 TO 7
5116 READ row
5118 POKE USR "L"+i,row
5120 NEXT i
5122 FOR i=0 TO 7
5124 READ row
5126 POKE USR "M"+i,row
5128 NEXT i
5130 FOR i=0 TO 7
5132 READ row
5134 POKE USR "N"+i,row
5136 NEXT i
5138 FOR i=0 TO 7
5140 READ row
5142 POKE USR "O"+i,row
5144 NEXT i
5146 FOR i=0 TO 7
5148 READ row
5150 POKE USR "P"+i,row
5152 NEXT i
5154 FOR i=0 TO 7
5156 READ row
5158 POKE USR "Q"+i,row
5160 NEXT i
5162 FOR i=0 TO 7
5164 READ row
5166 POKE USR "R"+i,row
5168 NEXT i
5170 FOR i=0 TO 7
5172 READ row
5174 POKE USR "S"+i,row
5176 NEXT i
5178 FOR i=0 TO 7
5180 READ row
5182 POKE USR "T"+i,row
5184 NEXT i
5186 FOR i=0 TO 7
5188 READ row
5190 POKE USR "U"+i,row
5192 NEXT i
5194 DATA 0,0,0,128,96,0,0,0
5196 DATA 0,0,68,66,126,64,64,0
5198 DATA 0,36,82,82,82,82,76,0
5200 DATA 0,36,66,66,74,74,52,0
5202 DATA 0,48,40,36,34,126,32,
          32
5204 DATA 0,46,74,74,74,74,50,0
5206 DATA 0,60,74,74,74,74,48,0
5208 DATA 0,2,66,34,18,10,6,0
5210 DATA 0,52,74,74,74,74,52,0
5212 DATA 0,12,82,82,82,82,60,0
5214 DATA 0,60,98,82,74,70,60,0
5216 DATA 0,60,66,66,66,66,36,0
5218 DATA 0,2,2,126,2,2,0,0
5220 DATA 0,62,64,64,64,64,62,0
5222 DATA 0,1,2,4,248,4,2,1
5224 DATA 0,124,16,40,68,0,0,0
5226 DATA 62,64,64,48,64,64,62,0
5228 DATA 0,60,66,66,82,98,124,
          128
5230 DATA 0,40,40,40,40,40,40,0
5232 DATA 0,16,16,16,254,16,16,
          16
5234 DATA 0,16,16,16,16,16,16,0
5236 REM
5238 REM ***Start main routine
5240 REM
5242 CLS
5244 PRINT AT 9,1;
"Is plot1 (Qh&Tm) required?
 (y/n)"
5246 IF INKEY$="n" OR INKEY$="N"
     THEN GO TO 5264
5248 IF INKEY$="y" OR INKEY$="Y"
     THEN GO TO 5256
5250 GO TO 5246
5252 REM
5254 REM *Go plotting-parameters
          input routine
5256 GO SUB 5298
5258 REM
5260 REM *Go Q&T plotting-
          control routine
5262 GO SUB 5356
5264 CLS
5266 PAUSE 150
5268 PRINT AT 9,1;
"Is plot2 (U&Y) required?
 (y/n)"
5270 IF INKEY$="n" OR INKEY$="N"
     THEN GO TO 5292
5272 IF INKEY$="y" OR INKEY$="Y"
     THEN GO TO 5280
5274 GO TO 5270
5276 REM
5278 REM *Go plotting-parameters
          input routine
5280 GO SUB 5298
5282 REM *Go U&Y plotting-
          control routine
5284 GO SUB 5420
5286 REM
5288 REM ***End main routine
5290 REM
5292 CLS : RETURN
5294 REM
5296 REM *Plotting-parameters
          input routine
5298 CLS
5300 PAUSE 150
5302 REM
5304 REM *Go Sample-class ini-
          tialization routine
5306 GO SUB 5926
5308 PRINT AT 9,1;
"Is a hard-copy required?
 (y/n)"
```

```
5310 IF INKEY$="n" OR INKEY$="N"
     THEN GO TO 5316
5312 IF INKEY$="y" OR INKEY$="Y"
     THEN GO TO 5316
5314 GO TO 5310
5316 LET SwHC=1: GO TO 5320
5318 LET SwHC=0
5320 CLS
5322 IF SwHC=1 THEN PRINT AT
14,1;"(260 samples=A4)"
5324 PRINT AT 9,1;
"Input 1st sampling point
 SP1stPlot to be plotted"
5326 PRINT AT 11,1;"(";1;
     "<=SP1stPlot<=";S(3)-1;")"
5328 INPUT SP1stPlot
5330 IF SP1stPlot<1 OR
     SP1stPlot>S(3)-1 THEN
    GO TO 5328
5332 CLS
5334 IF SwHC=1 THEN PRINT AT
14,1;"(260 samples=A4)"
5336 PRINT AT 9,1;
"Input last sampling point
 SPlastPlot to be plotted"
5338 PRINT AT 11,1;"(";SP1stPlot
;    "<SPlastPlot<=";S(3);")"
5340 INPUT SPlastPlot
5342 IF SPlastPlot<=SP1stPlot
OR SPlastPlot>S(3) THEN GO TO
5340
5344 REM
5346 REM *End plotting-params
          input routine
5348 RETURN
5350 REM
5352 REM *Start Q&T plotting
          control-routine
5354 REM
5356 LET Page=1
5358 LET Plot1=SP1stPlot
5360 LET Plotf=87+SP1stPlot
5362 CLS
5364 PRINT AT 9,1;
"Are graphs of setpoint Tt and
 nom input power required on
 plot1? (It takes longer)
 (y/n)"
5366 IF INKEY$="n" OR INKEY$="N"
     THEN LET SwSP=0: GO TO 5376
5368 IF INKEY$="y" OR INKEY$="Y"
     THEN LET SwSP=1: GO TO 5376
5370 GO TO 5366
5372 REM
5374 REM *Go Q&T page-plotting
          routine
5376 GO SUB 5474
5378 PRINT AT 0,8;"Sample ";
     Plot1
5380 PRINT AT 21,8;"Sample ";
     Plotf
5382 PRINT AT 10,8;"↑C to cont"
5384 IF Plotf<=SPlastPlot THEN
     PRINT AT 21,8;"Sample ";
     Plotf
5386 IF Plotf>SPlastPlot THEN
     PRINT AT 21,8;"Sample ";
     SPlastPlot
5388 IF INKEY$<>"c" AND INKEY$
     <>"C" THEN GO TO 5388
5390 IF SwHC=0 THEN GO TO 5400
5392 REM
5394 REM *Go Q&T page-plotting
          routine
5396 GO SUB 5474
5398 IF SwHC=1 THEN PRINT AT
     0,18;Plot1: COPY
5400 LET Page=Page+1
5402 LET Plot1=Plotf
5404 LET Plotf=87*(Page-1)+87+
     SP1stPlot
5406 IF i<SPlastPlot THEN GO TO
     5376
5408 REM
5410 REM *End Q&T plotting
          control routine
5412 RETURN
5414 REM
5416 REM *U&Y plotting control
          routine
5418 REM
5420 LET Page=1
5422 LET Plot1=SP1stPlot
5424 LET Plotf=87+SP1stPlot
5426 REM
5428 REM *Go U&Y page-plotting
          routine
5430 GO SUB 5672
5432 PRINT AT 0,8;"Sample ";
     Plot1
5434 PRINT AT 21,8;"Sample ";
     Plotf
5436 PRINT AT 10,9;"↑C to cont"
5438 IF Plotf<=SPlastPlot THEN
     PRINT AT 21,8;"Sample ";
     Plotf
5440 IF Plotf>SPlastPlot THEN
     PRINT AT 21,8;"Sample ";
     SPlastPlot
5442 IF INKEY$<>"c" AND INKEY$
     <>"C" THEN GO TO 5442
5444 IF SwHC=0 THEN GO TO 5454
5446 REM
5448 REM *Go U&Y page-plotting
          routine
5450 GO SUB 5672
5452 IF SwHC=1 THEN PRINT AT
     0,18;Plot1: COPY
5454 LET Page=Page+1
5456 LET Plot1=Plotf
5458 LET Plotf=87*(Page-1)+87+
     SP1stPlot
5460 IF i<SPlastPlot THEN GO TO
     5430
5462 CLS
5464 REM
5466 REM *End U&Y plotting
          control-routine
5468 RETURN
5470 REM
5472 REM **Q&T page-plotting
          routine
5474 CLS
5476 REM
5478 REM *Plot grid
5480 REM
5482 PLOT 0,175: DRAW 0,-175
5484 PLOT 0,175: DRAW 10,0
5486 PLOT 155,175: DRAW 0,-175
5488 PLOT 155,175: DRAW 10,0
5490 FOR k=30 TO 150 STEP 30
5492 FOR i=175 TO 0 STEP -5
5494 PLOT k,i
5496 NEXT i
5498 NEXT k
5500 FOR k=180 TO 255 STEP 25
5502 FOR i=175 TO 0 STEP -5
5504 PLOT k,i
5506 NEXT i
5508 NEXT k
5510 REM
5512 REM *Plot Q
5514 REM
5516 PLOT INT ((Q(Plot1)+10)/20)
     ,175
5518 FOR i=Plot1+1 TO plotf
5520 DRAW 0,-2
5522 DRAW INT ((Q(i)+10)/20)-
     INT ((Q(i-1)+10)/20),0
5524 IF i=SPlastplot THEN GO TO
     5534
5526 NEXT i
5528 REM
5530 REM *Plot T
5532 REM
5534 PLOT 155+INT T(Plot1),175
5536 FOR i=Plot1+1 TO Plotf
5538 DRAW INT T(i)-INT T(i-1),
     -2
5540 IF i=SPlastPlot THEN GO TO
     5544
5542 NEXT i
5544 IF SwSP=0 THEN GO TO 5606
5546 REM
5548 REM *Plot setpoint and nom
          Q if required
5550 REM
5552 LET yax=175
5554 LET DSno=1
5556 FOR i=Plot1 TO Plotf
5558 LET Sampleno=i
5560 REM
5562 REM *Go sample-class
          routine
5564 GO SUB 5952
5566 PLOT 155+INT C(DSno,11),yax
5568 IF INT C(DSno,11)=100 THEN
     DRAW -1,0
5570 DRAW 0,-1: DRAW 0,1
5572 DRAW 1,0
5574 DRAW 0,-1: DRAW 0,1
5576 DRAW -2,0
5578 DRAW 0,-1: DRAW 0,1
5580 PLOT INT ((C(DSno,17)+10)/
     20),yax
5582 IF INT ((C(DSno,17)+10)/20)
     =0 THEN DRAW 1,0
```

```
5584 DRAW 0,-1: DRAW 0,1
5586 DRAW 1,0
5588 DRAW 0,-1: DRAW 0,1
5590 DRAW -2,0
5592 DRAW 0,-1: DRAW 0,1
5594 LET yax=yax-2
5596 IF i=SPlastPlot THEN GO TO
     5606
5598 NEXT i
5600 REM
5602 REM *Print subsripts on all
         even pages
5604 REM
5606 IF INT (Page/2)<>Page/2
     THEN GO TO 5664
5608 PRINT AT 11,0;"Ø"
5610 PRINT AT 11,19;"┤"
5612 PRINT AT 12,0;"‖"
5614 PRINT AT 12,19;"‖"
5616 PRINT AT 13,0;"0"
5618 PRINT AT 13,7;"P"
5620 PRINT AT 13,15;"ŵ"
5622 PRINT AT 13,19;"0"
5624 PRINT AT 13,25;"ŵ"
5626 PRINT AT 13,31;"P"
5628 PRINT AT 14,0;"ᴺ"
5630 PRINT AT 14,7;"ˎ"
5632 PRINT AT 14,15;"ˎ"
5634 PRINT AT 14,19;"O"
5636 PRINT AT 14,25;"0"
5638 PRINT AT 14,31;"0"
5640 PRINT AT 15,0;"E"
5642 PRINT AT 15,7;"ŵ"
5644 PRINT AT 15,15;"P"
5646 PRINT AT 15,25;"O"
5648 PRINT AT 15,31;"0"
5650 PRINT AT 16,7;"ᴺ"
5652 PRINT AT 16,15;"ᴺ"
5654 PRINT AT 16,31;"O"
5656 PRINT AT 17,7;"E"
5658 PRINT AT 17,15;"E"
5660 REM
5662 REM **End Q&T page plotting
         routine
5664 RETURN
5666 REM
5668 REM **U&Y page-plotting
         routine
5670 REM
5672 CLS
5674 REM
5676 REM *Plot grid
5678 REM
5680 PLOT 75,175: DRAW 0,-175
5682 PLOT 75,175: DRAW 10,0
5684 PLOT 205,175: DRAW 0,-175
5686 PLOT 205,175: DRAW 10,0
5688 FOR k=0 TO 60 STEP 15
5690 FOR i=175 TO 0 STEP -5
5692 PLOT k,i
5694 NEXT i
5696 NEXT k
5698 FOR k=90 TO 150 STEP 15
5700 FOR i=175 TO 0 STEP -5
5702 PLOT k,i
5704 NEXT i
5706 NEXT k
5708 FOR k=155 TO 195 STEP 10
5710 FOR i=175 TO 0 STEP -5
5712 PLOT k,i
5714 NEXT i
5716 NEXT k
5718 FOR k=215 TO 255 STEP 10
5720 FOR i=175 TO 0 STEP -5
5722 PLOT k,i
5724 NEXT i
5726 NEXT k
5728 REM
5730 REM *Plot U&Y
5732 REM
5734 LET Sampleno=Plot1
5736 LET DSno=1
5738 REM
5740 REM *Go Sample-class
         routine
5742 GO SUB 5952
5744 LET Ordinate1=O(Plot1)-
     C(DSno,17)
5746 LET Ordinate1=
     INT ((Ordinate1+4)/8)
5748 IF Ordinate1>75 THEN LET
     Ordinate1=75
5750 IF Ordinate1<-75 THEN LET
     Ordinate1=-75
5752 PLOT 75+Ordinate1,175
5754 FOR i=Plot1+1 TO Plotf
5756 DRAW 0,-2
5758 LET Sampleno=i
5760 REM
5762 REM *Go Sample-class
         routine
5764 GO SUB 5952
5766 LET Ordinate2=O(i)-
     C(DSno,17)
5768 LET Ordinate2=
     INT ((Ordinate2+4)/8)
5770 IF Ordinate2>75 THEN LET
     Ordinate2=75
5772 IF Ordinate2<-75 THEN LET
     Ordinate2=-75
5774 DRAW Ordinate2-Ordinate1,0
5776 LET Ordinate1=Ordinate2
5778 IF i=SPlastPlot THEN GO TO
     5788
5780 NEXT i
5782 REM
5784 REM *Plot T
5786 REM
5788 LET Sampleno=Plot1
5790 REM
5792 REM *Go Sample-class
         routine
5794 GO SUB 5952
5796 LET Ordinate1=T(Plot1)-
     C(DSno,11)
5798 LET Ordinate1=5*
     INT Ordinate1
5800 IF Ordinate1>50 THEN LET
     Ordinate1=50
5802 IF Ordinate1<-50 THEN LET
     Ordinate1=-50
5804 PLOT 205+Ordinate1,175
5806 LET DSno=1
5808 FOR i=Plot1+1 TO Plotf
5810 LET Sampleno=i
5812 REM
5814 REM *Go Sample-class
         routine
5816 GO SUB 5952
5818 LET Ordinate2=T(i)-
     C(DSno,11)
5820 LET Ordinate2=5*
     INT Ordinate2
5822 IF Ordinate2>50 THEN LET
     Ordinate2=50
5824 IF Ordinate2<-50 THEN LET
     Ordinate2=-50
5826 DRAW Ordinate2-Ordinate1,-2
5828 LET Ordinate1=Ordinate2
5830 IF i=SPlastPlot THEN GO TO
     5840
5832 NEXT i
5834 REM
```

```
5836 REM *Print subscripts on
          even page numbers
5838 REM
5840 IF INT (Page/2)<>Page/2
     THEN GO TO 5918
5842 PRINT AT 12,9;"C"
5844 PRINT AT 12,25;"-<"
5846 PRINT AT 13,1;"|"
5848 PRINT AT 13,9;"‖"
5850 PRINT AT 13,13;"+"
5852 PRINT AT 13,17;"+"
5854 PRINT AT 13,19;"|"
5856 PRINT AT 13,25;"‖"
5858 PRINT AT 13,31;"+"
5860 PRINT AT 14,1;"Ꮭ"
5862 PRINT AT 14,5;"⑩"
5864 PRINT AT 14,9;"0"
5866 PRINT AT 14,13;"⑩"
5868 PRINT AT 14,17;"Ꮭ"
5870 PRINT AT 14,19;"ト"
5872 PRINT AT 14,25;"0"
5874 PRINT AT 14,31;"ト"
5876 PRINT AT 13,5;"|"
5878 PRINT AT 15,1;"⑩"
5880 PRINT AT 15,5;"Ꮭ"
5882 PRINT AT 15,9;"Ꮸ"
5884 PRINT AT 15,13;"Ꮭ"
5886 PRINT AT 15,17;"⑩"
5888 PRINT AT 15,19;"0"
5890 PRINT AT 15,25;"0"
5892 PRINT AT 15,31;"0"
5894 PRINT AT 16,1;"0"
5896 PRINT AT 16,5;"0"
5898 PRINT AT 16,13;"0"
5900 PRINT AT 16,17;"0"
5902 PRINT AT 16,19;"0"
5904 PRINT AT 16,31;"0"
5906 PRINT AT 17,1;"Ꮸ"
5908 PRINT AT 17,5;"Ꮸ"
5910 PRINT AT 17,13;"Ꮸ"
5912 PRINT AT 17,17;"Ꮸ"
5914 REM
5916 REM **End U&Y page-plotting
          routine
5918 RETURN
5920 REM
5922 REM *Sample-class initiali-
          sation routine
5924 REM
5926 DIM L(S(2)): DIM H(S(2))
5928 LET l=1
5930 FOR j=1 TO S(2)
5932 LET h=C(j,1)+l-1
5934 LET L(j)=l: LET H(j)=h
5936 LET l=h+1
5938 NEXT j
5940 REM
5942 REM *End Sample-class
          initialisation routine
5944 RETURN
5946 REM
5948 REM *Sample-class routine
5950 REM
5952 IF Sampleno<=H(DSno) AND
     Sampleno>=L(DSno) THEN
     RETURN
5954 IF DSno=S(2) THEN LET
     DSno=0
5956 LET DSno=DSno+1
5958 GO TO 5952
```

```
 10 REM **STATPACK**
 15 REM *Tape 1.3a-073
 20 REM *Purpose-Calc mean &
         variance of ut & yt
 25 REM *IP-(.)C,S(b),Q & T
 30 REM *OP-Printed mean & var
         of ut & yt
 35 REM
 40 REM *Load (.)C,S(b),Q & T-
 45 CLS
 50 INPUT "Storage code=?";Z$
 55 PRINT AT 9,1;
"Load (.)C,S(b),Q & T"
 60 PRINT AT 11,1;
"1) Insert & rewind data-tape
    to start of series"
 65 PRINT AT 13,1;
"2) Press PLAY"
 70 LET Y$="C"+Z$:
    LOAD Y$ DATA C()
 75 LET Y$="S(b)"+Z$:
    LOAD Y$ DATA S()
 80 LET Y$="Q"+Z$:
    LOAD Y$ DATA Q()
 85 LET Y$="T"+Z$:
    LOAD Y$ DATA T()
 90 CLS
 95 REM
100 REM *Calc mean of ut &
         mean of yt-
105 LET sumut=0: LET sumyt=0
110 FOR i=1 TO S(3)
115 LET sumut=sumut+Q(i)-
    C(1,17):
    LET sumyt=sumyt+T(i)-
    C(1,11)
120 NEXT i
125 LET meanut=sumut/S(3):
    LET meanyt=sumyt/S(3)
130 REM
135 REM *Calc var of ut &
         var of yt-
140 LET sumut=0: LET sumyt=0
145 FOR i=1 TO S(3)
150 LET wut=Q(i)-C(1,17)-meanut
155 LET wyt=T(i)-C(1,11)-meanyt
160 LET sumut=sumut+wut*wut:
    LET sumyt=sumyt+wyt*wyt
165 NEXT i
170 LET varut=sumut/(S(3)-1):
    LET varyt=sumyt/(S(3)-1)
175 REM
180 REM *Print results-
185 LPRINT Z$;":": LPRINT
190 LPRINT "ut:"
195 LPRINT "mean=";meanut:
    LPRINT "var=";varut
200 LPRINT
205 LPRINT "yt:"
210 LPRINT "mean=";meanyt:
    LPRINT "var=";varyt
215 LPRINT : LPRINT : LPRINT
220 CLS
225 RUN
```

```
10  REM **STATPACK2**
15  REM *Tape 1.3a-
20  REM *Purpose-Calc mean &
         variance of ut & yt
25  REM *IP-(.)C,S(b),Q & T
30  REM *OP-Printed mean & var
         of ut & yt
35  REM
40  REM *Load (.)C,S(b),Q & T-
45  CLS
50  INPUT "Storage code=?";Z$
55  PRINT AT 9,1;
"Load (.)C,S(b),Q & T"
60  PRINT AT 11,1;
"1) Insert & rewind data-tape
    to start of series"
65  PRINT AT 13,1;
"2) Press PLAY"
70  LET Y$="C"+Z$:
    LOAD Y$ DATA C()
75  LET Y$="S(b)"+Z$:
    LOAD Y$ DATA S()
80  LET Y$="Q"+Z$:
    LOAD Y$ DATA Q()
85  LET Y$="T"+Z$:
    LOAD Y$ DATA T()
90  CLS
95  REM
100 REM *Input data-
105 INPUT
 "No of ut autocor shifts=?";
 kut: LET kut=INT kut:
 DIM U(kut+1)
110 INPUT
 "No of yt autocor shifts=?";
 kyt: LET kyt=INT kyt:
 DIM Y(kyt+1)
115 REM
120 REM *Calc mean of ut &
         mean of yt-
125 LET sumut=0: LET sumyt=0
130 FOR i=1 TO S(3)
135 LET sumut=sumut+Q(i)-
    C(1,17):
    LET sumyt=sumyt+T(i)-
    C(1,11)
140 NEXT i
145 LET meanut=sumut/S(3):
    LET meanyt=sumyt/S(3)
150 REM
155 REM *Calc auto-correllation
         of ut-
160 FOR j=0 TO kut
162 CLS : PRINT AT 9,5;"ja=";j:
    PRINT AT 11,5;Z$
165 LET sum=0
170 FOR i=1 TO S(3)
175 LET wa=Q(i)-C(1,17)-meanut:
 LET wb=Q(i+j)-C(1,17)-meanut:
 LET sum=sum+wa*wb
180 IF i+j=S(3) THEN GO TO 195
190 NEXT i
195 LET U(j+1)=sum/(S(3)-1-j)
200 NEXT j
205 REM
210 REM *Calc auto-correllation
         of yt-
215 FOR j=0 TO kyt
217 CLS : PRINT AT 9,5;"jb=";j:
    PRINT AT 11,5;Z$
220 LET sum=0
225 FOR i=1 TO S(3)
230 LET wa=T(i)-C(1,11)-meanyt:
  LET wb=T(i+j)-C(1,11)-meanyt:
  LET sum=sum+wa*wb
235 IF i+j=S(3) THEN GO TO 250
245 NEXT i
250 LET Y(j+1)=sum/(S(3)-1-j)
255 NEXT j
260 CLS
265 REM
270 REM *Print results-
275 LPRINT Z$;":": LPRINT
280 LPRINT "mean of ut=";meanut
285 LPRINT "mean of yt=";meanyt
290 LPRINT
295 LPRINT "Auto-correllation:"
300 LPRINT "K";TAB 5;"ut";
    TAB 20;"yt"
305 LET kmax=0
310 IF kut>kmax THEN LET kmax=
    kut
315 IF kyt>kmax THEN LET kmax=
    kyt
320 FOR i=1 TO kmax+1
325 IF kut+1<i THEN LPRINT i-1;
    TAB 18;Y(i): GO TO 340
330 IF kyt+1<i THEN LPRINT i-1;
    TAB 3;U(i): GO TO 340
335 LPRINT i-1;TAB 3;U(i);
    TAB 18;Y(i)
340 NEXT i
345 LPRINT : LPRINT : LPRINT
350 CLS
355 RUN
```

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