An Analysis of the Low-Volatility Anomaly on the Johannesburg Stock Exchange

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Abstract:

The low-volatility anomaly can be described as the unexpected outperformance of low-volatility stocks compared to high-volatility stocks over the long-term. This dissertation investigates the low-volatility anomaly and its presence on the Johannesburg Stock Exchange (JSE). Possible reasons behind why low-volatility stocks consistently outperform their high volatility counterparts, as well as their own expected return, over the long-term are discussed. This includes analysing how financial risk is measured and whether this plays a role in obscuring the expected risk-return relationship, in addition to other fundamental factors impacting expected returns. It is found that the low-volatility anomaly is present on the JSE and that using a number of different risk metrics does not significantly change where a stock is ranked on the risk spectrum. Additionally, including an interest rate exposure factor, a value factor and a momentum factor lowers the unexpected portion (Alpha) of the returns of low volatility stocks, at the same time as narrowing the gap between the unexpected performance of the lowest and highest volatility stocks.

Keywords: Volatility, Alpha, Expected Returns, JSE, Beta, CAPM, GARCH, VaR

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1. Introduction

The low-volatility effect is an anomaly that has been found to occur in Equity markets the world over. Going against the long-believed theory that investing in a higher risk asset should compensate an investor with a higher return, empirical evidence has found the contrary to be true (Ang, Hodrick, Xing and Zhang, 2006). That is, lower risk stocks have been found to outperform higher risk stocks over the long run. This phenomenon is a particularly peculiar one as it goes against the traditional theories that are still taught academically in finance, consisting of Sharpe (1964), Lintner (1965) and Mossin's (1966) Capital Asset Pricing Model (CAPM), the Markowitz (1952) theory of the Efficient Frontier and the more recent Fama-French (1993) multifactor asset pricing models, all of which prescribe higher risk equating to higher returns. As a result, it is evidently an important topic that cannot be pushed aside if investors understanding of financial market predictions and pricing models are to continue to become more efficient.

The reason that the effect has earned the name of "low-volatility" as opposed to "low-risk" is due to the fact that volatility, as measured by the standard deviation of returns, is the most common metric in the financial world used to measure risk. Augmentations of studies in this area have also referred to it as the "low-beta" effect, as beta from the CAPM can be seen as a stock's exposure to systematic risk (Blitz and Van Vliet, 2007), which has also been found to incorrectly prescribe the risk-return relationship. Given the prior research into the topic, there is not as much of a need to prove the effects existence as there is to discover and explain its causes, as the effect has continuously been documented across both developed and emerging markets by respected academics such as Ang et al. (2008) and Blitz, Pang and van Vliet (2013) covering both areas.

One of the key areas of examination outlined in this paper is whether volatility, the metric commonly used to measure risk, is itself flawed in capturing how investors commonly perceive risk in equity markets. There has been extensive research into different statistical methods of calculating volatility by accounting for its relationship and variation with time, and how this might more accurately account for the variation of stock returns. These models are commonly known as Generalised Autoregressive Conditional Heteroskedasticity or GARCH volatility models. Additionally, other financial measures of risk that have recently become more prominent, including Value-at-Risk (VaR), Expected-Tail-Loss (ETL) and Downside Deviation are considered in terms of their relationship with returns. This paper looks at whether accounting for volatility or risk using these different derivations alters the results of the low-volatility anomaly and whether low-risk stocks continue to outperform on a relative basis.

Not only will the question of how to best measure risk and its impact on the low-volatility anomaly be examined, but in addition, a more fundamental analysis on what other factors might be impacting returns is looked at. It is entirely possible and in fact likely that the return generating process of equity assets and the fundamental characteristics that determine the risk behind these assets are not yet fully understood by financial academics and practitioners. This implies that only considering the variation in a stock's returns may be an overly simplistic way of looking at risk. Rather, by looking at other characteristics of the stock, such as its valuation, its exposure to macroeconomic variables, its ability to return cash to investors and how predictable the performance of the firm behind the stock is, alongside volatility, may paint a better picture of how best to measure expected return.

One of the nuances of the low-volatility anomaly is not only the outperformance of low-volatility stocks compared to their high-volatility counterparts, but the fact that this outperformance is unexpected based on current expected return models. Centered on this, the performance of each stock or portfolio of stocks needs to be assessed from an alpha (unexpected performance) return perspective as opposed to an absolute return one. The assumption here is that the alpha of low-volatility stocks or portfolios consisting of them is positive according to expected return models such as the CAPM or Fama-French three factor model. While it may not be feasible to expect to construct a model that perfectly predicts stock returns and thus results in a zero alpha, it may be possible to minimize the unexpected overperformance of low-volatility stocks or the unexpected underperformance of high-volatility stocks.

The South African equity market, the Johannesburg Stock Exchange (JSE), is the market landscape within which this research is considered. While still a developing market, the JSE has appeared to become more efficient over the last two decades as it has evolved into a more complex and liquid financial market for equity investors. Noakes and Rajaratnam (2016) find that although evidence of efficiency on the exchange is mixed, mid and large cap stocks appear to be more efficient than small cap stocks between the sample period used of 2005 to 2009. This is the main reason why the time frame of only data from 2003 and later is considered in this research, since an analysis of risk and return requires that the market within which this takes places is efficiently pricing assets in the first place. On the other hand, it must be acknowledged that only, approximately, the top seventy company stocks on the market could be considered to be liquid and efficiently priced assets. This is based on a combination of thirty-six large cap and forty-four mid cap stocks for a total of eighty stocks used by in prior research (Noakes and Rajartnam, 2016). Seventy stocks were used as a more conservative amount since this study covers a longer time-span. Thus, this study is restricted to using this smaller sample of stocks compared to what could be looked at in a more developed stock universe, such as that of the U.S. Nonetheless, a robust investigation is still able to be conducted and can still shed light on the low-volatility anomaly in an emerging market context.

This paper will discuss relevant previous literature and research on the low-volatility anomaly from a developed as well as emerging market perspective in Section 2. This is followed by the description of the data acquisition and cleaning required for this study in Section 3. Section 4 provides details on the methodology used behind the low-volatility exploration, followed by a preliminary analysis of the data in section 5. Sections 6 and 7 provide the results and findings of this research respectively. Finally, Section 8 concludes the paper with its implications, limitations and further research that should be considered on the topic.

2. Review of Prior Literature

The natural convention that has applied to financial markets and investments since the beginning of its academic analysis involves the idea that when investing in a riskier asset, the investor should be compensated for having taken on this higher risk with a commensurate higher return (Baker and Haugen, 2012). If this would not be the case, there would be no reason for the investor to invest in the riskier asset over the safer asset, which has become the tenant of finance teachings through any academically studied finance or financial economics course. This notion has more recently been challenged however, with empirical evidence across markets showing an opposite result, this being that a lower volatility or lower risk asset has actually accumulated higher returns than the equivalent higher volatility or higher risk asset over time.

One of the key assumptions to take note of when analysing the low-volatility anomaly is whether it is occurring in markets that are efficiently pricing assets. The Efficient Market Hypothesis (EMH) as described by Fama (1991) in its strongest form indicates that security prices fully reflect all available information. While trading costs and information costs make it unlikely for markets to follow this extreme level of efficiency, a more plausible version of the hypothesis is one where prices reflect information to the point where the marginal costs of acting on information exceed the marginal benefits (Jensen, 1978).

Congruent to the EMH is that security price changes follow a random walk, reacting to new information as it becomes readily available, while already accounting for all prior information (Malkiel, 2003). If the low-volatility anomaly is apparent on equity markets, this would suggest that either, equity markets are not efficient and do not follow a random walk process as described above, or, that current equilibrium pricing models against which market efficiency is being tested are inaccurate due to the joint-hypothesis problem, which points out that market efficiency can only be tested when compared to an accurate equilibrium price prediction model, such as the CAPM (Fama, 1991).

2.1. Initial Evidence of Low-volatility Effects

Clarke, de Silva and Thorley (2006) introduced one of the first comprehensive studies on what is more commonly being referred to as the “low-volatility” effect in which they analysed minimum variance portfolios in the U.S. Equity market between 1968 and 2005. They found that, going against Markowitz’s original hypothesis of the Equilibrium Portfolio Theory where the minimum variance portfolio returns should lie below that of the optimal risky portfolio returns, the returns from their constructed low or minimum variance portfolio actually clearly outperformed that of the optimal risky portfolio which can also be considered to be the market portfolio. In other words, one could “reduce portfolio volatility without sacrificing returns” (Clarke, de Silva and Thorley, 2006), something that would clearly be to the benefit of any rational investor.

In a further investigation, Ang, Hodrick, Xing and Zhang (2006) began to break down the anomaly into idiosyncratic volatility and systematic volatility while analysing the cross section of volatility and expected returns. This study started to look at possible factors in the cross section of returns that could explain the low-volatility effect, however the results showed that common factors originally used by Fama and French (1993) in their three-factor model analysis of multifactor portfolio models, such as size and book-to-market factors, as well as factors related to momentum and liquidity were unable to account for the low average returns earned by stocks with either high exposure to systematic volatility or high idiosyncratic volatility. Following these studies, research over the last decade has attempted to find explanations for the cause of the low-volatility effect.

Critics of the results that found the long believed high-risk high-return axiom to be false have questioned the validity of these studies. One of the most common issues arising is the way that risk and/or volatility is being measured, a problem that has long confronted practitioners in the finance world and is still being investigated in a number of different finance topics. Blitz and Van Vliet (2007) explained that a common metric that has been used to measure exposure to systematic risk is the beta of a stock as derived from the Capital Asset Pricing Model (CAPM). They empirically observed that portfolios consisting of stocks with low-volatility exhibit a low beta, while those consisting of stocks with high volatility exhibit a high beta, making beta a useful tool in the investigation of the differences in returns to low and high-risk stocks. Following this distinction, Blitz and Van Vliet (2007) expand the evidence of the low-volatility effect by analysing and finding it’s occurrence in not only the U.S. market, but also the European and Japanese markets in isolation, as well as concluding that size, value and momentum factors fail to subsume the volatility effect, meaning that their results indicate that low returns for high volatility stocks is in fact its own separate effect.

In a subsequent inquiry, Ang, Hodrick, Xing and Zhang (2008) following their original paper, further evidence is brought to the forefront. In their paper, they discovered significantly that across 23 developed markets, the difference in average returns between the opposite ends of quintiles of portfolios sorted on idiosyncratic volatility was -1.31% per month. This was also found after controlling for world market, size, and value factors. The negative difference points to the fact of the lower volatile portfolios outperforming the higher volatile portfolios. In an attempt to rule out other factors influencing their results, they also expel explanations for their findings specific to the U.S. based on trading frictions and information dissemination, both factors that could also lead to the misestimation of volatility itself. The particularly promising aspect of these results is that the negative relationship between high idiosyncratic volatility and average returns was found to be strongly statistically significant across all of the G7 countries (Canada, France, Germany, Italy, Japan, the U.S. and the U.K) and also observed in the larger sample of 23 developed markets. More importantly, the negative spread in returns between high and low idiosyncratic volatility stocks across the international markets exhibited strong co-movement, with this commonality indicating that broad and non-diversifiable factors lay behind the effect (Ang et al., 2008).

Following in a South African context, van Rensburg and Robertson (2003) conduct an analysis of returns on the Johannesburg Stock Exchange (JSE) in order to determine what factors explain the difference between the traditional CAPM model expected returns and actual returns found in the market. After specifically analysing the effects of size and price-to-earnings on quintile portfolios ranging from 1990 to 2000, their findings indicate that, in addition to smaller size and lower price-to-earnings ratio stocks earning higher returns, these same stocks also have lower betas. In other words, they find evidence of an inverse relationship between beta and returns for the first time on the JSE. Not only does this show the evidence of a low-beta anomaly occurring, but it also suggests that both small size and low price-to-earnings may be linked with this, although including these factors does not entirely explain away the phenomenon.

In an analysis of the low-volatility effect in emerging markets, Blitz, Pang and Van Vliet (2013) provide evidence that there is in fact a flat or even negative relationship between risk and return in emerging equity markets, much like that found in developed markets in prior research. In addition, they find that this volatility effect may be growing over time in emerging markets, perhaps due to the increased delegated portfolio management in these markets as they evolve. Contrasting the findings of Ang, Hodrick, Xing and Zhang (2008) however, the authors of this more recent paper find that while the low-volatility anomaly exists in emerging markets, it is only weakly related to the same effect in developed markets, which argues against the common-factor explanation previously thought to be prevalent. Investigating why there is the same effect in both types of markets despite the fact that it does not co-move between them may provide an inkling for future research into isolating the factors that are driving the effect overall.

2.2. Possible Explanations for Low-volatility Effects:

2.2.1. Behavioural Impacts

While explanations of the low-volatility effect using Fama and French (1993) factors such as size and book-to-market factors have not yet been exhaustive, other influences that may be causing the anomaly have been introduced by academics such as Baker, M., Bradley and Wurgler (2011). Describing the long-term success of low-volatility and low beta stock portfolios as being “among the many candidates for the greatest anomaly in finance", these authors apply the principles of behavioural finance to investigate the drivers of this anomaly and to assess the likelihood of it persisting. Using the behavioural finance principle of irrational investors making poor decisions, the hypothesis is that a preference for lotteries and the well-established human and investor biases of representativeness and over confidence may (incorrectly) lead to a demand for high volatility stocks that is not warranted by the true fundamentals of the stock, leading them to be over priced (Baker, Bradley and Wurgler 2011).

Adding on to the behavioural finance theory, a further explanation is one based on the concept of benchmarks as a limit to arbitrage. While the irrational investor theory makes sense, it doesn't explain why the educated institutional investors that should know better than to chase stocks on factors other than fundamentals would also follow the same suit. One would expect the demand by these investors to counteract the price impact of irrational investor's demand, however, one way in which these investors are limited in doing this is through the constraint of benchmarking, as many of the institutional investors with the ability to offset the irrational demand have fixed benchmark mandates which, by their nature, discourage investments in low-volatility stocks (Baker, Bradley and Wurgler, 2011). The question is why the institutional investors do not short the high volatility low performing stocks, however the issue is that a majority of these stocks are small cap stocks, which are expensive to trade in large quantities as well as far more illiquid, also making borrowing the stock more difficult for shorting (Baker, Bradley and Wurgler, 2011).

Despite a number of plausible explanations coming to light from the behavioural finance perspective on the low-volatility anomaly, there is yet to be concrete evidence proving that these theories are without a doubt accurate. The perspective of how market participants themselves impact securities however has been delved into further, such as by Baker and Haugen (2012). After confirming through their own research that there is clearly evidence of the anomaly's existence across markets, they analyse the possible explanations related to the nature of manager compensation and agency issues between professionals within an organisation and between these professionals and their clients.

The issue that comes into play here is that many investment managers compensation structure involves being paid a base salary, with additional bonus compensation for any outperformance over and above the benchmark of their portfolio. The benchmark return could be seen as the expected return of all stocks in the index and thus, assuming normality, the mean of the normal distribution of returns. Managers will then receive bonus compensation when their portfolio return exceeds this expected return by a certain amount and as a result, these managers naturally attempt to hold the more highly volatile stocks in their portfolio, as given the very definition of volatility as the standard deviation of returns, this gives the manager a higher chance of surpassing the expected return or in other words outperforming and receiving a higher level of compensation (Baker and Haugen, 2012). Exacerbating this problem is the fact that these managers are investing their client's money as opposed to their own, meaning that they face less personal risk if their risky bets do not pay off as planned.

2.2.2. Accuracy of Risk Measure Used

An issue that has been raised with the premise that a low-volatility anomaly is also a low-risk anomaly is whether or not volatility (also known as the standard deviation) is a truly accurate measure of what investors define as risk. As Sortino and van der Meer (1991) discuss, both the standard deviation and Beta's ability to accurately portray what risk means to investors have increasingly been questioned. The core of the problem is that these measures incorporate both downside and upside risk, while in reality, most investors are only concerned about the risk of possible losses on their investments and are only too happy when high variability brings about greater than expected gains (Sortino and van der Meer, 1991). If this is the case, one could argue that only the lower than expected portion of the variability of returns should be considered when analysing risk, with the higher than expected portion being ignored entirely.

The Sortino ratio (Sortino and van der Meer, 1991) is a risk adjusted performance measure similar to the Sharpe Ratio in that the measure of return is taken as the excess return on the portfolio, however, rather than dividing this return by the standard deviation, risk is measured as the downside deviations from the mean only. This lends itself to analysing risk adjusted returns on a more intuitive basis than the Sharpe Ratio and could change the results of any low-volatility stock analysis.

Another alternative to volatility that has increasingly received attention as its use has become more popular is Value at Risk (VaR). VaR is defined as the loss in portfolio value over a specific time horizon that can be expected to occur with a certain level of probability (Duffie and Pan, 1997). For example, if a portfolio has a 1% 10-day VaR of -5%, this implies that there is a 1% probability of the portfolio losing 5% or more of its value over that ten-day period, sometimes also stated as there being a 99% chance that, at a maximum, the portfolio will lose less than 5% of its value over that ten-day period. The benefit of using VaR as opposed to volatility as a risk measure is that it focuses on what most investors would consider as “risk”, namely extreme losses that could possibly occur with a certain level of confidence.

With regards to how the use of VaR instead of volatility may impact the evidence of any low-risk anomaly, a paper by Alexander and Baptista (2001) looks at the difference between creating mean-variance versus mean-VaR efficient portfolios. Their study finds that when comparing two mean-variance efficient portfolios, the higher variance portfolio may in fact have a lower VaR. This discovery does have the limitation that an efficient portfolio that globally minimizes VaR may not exist, but that it is possible for risk-averse investors to select portfolios with lower standard deviations that have a higher VaR, implying that a low-volatility anomaly may not actually translate into being a low-risk anomaly if VaR is used as the risk measure of choice. Instead, the higher VaR (or higher risk), lower standard deviation portfolio accrues the higher returns as would traditionally be expected.

An even more in-depth analysis of VaR as a risk measure might introduce Conditional VaR or CVaR. VaR provides the level of loss that can be reasonably assumed will not be exceeded with certain probability, while CVaR goes one step further than VaR in that it provides an estimate of the size of the loss an investor could expect given that this loss has exceeded VaR itself. In a later paper, Alexander and Baptista (2004) compare mean-variance portfolios imposing VaR as a constraint against imposing CVaR as a constraint and find that, under certain conditions, imposing the CVaR constraint may actually result in a portfolio that has both lower CVaR and lower standard deviation. The results change under different conditions however, also finding portfolios that have a lower CVaR but a higher standard deviation. This shows that although not yet fully determined, it is possible that using the correct risk measure may cause the low-risk anomaly to disappear.

More recently, Xiong, Idzorek and Ibbotson (2014) conducted a study specifically aimed at determining whether either volatility or tail risk are compensated in equity returns. In their analysis, they too found that there was no risk premium or compensation to investors for taking on higher levels of volatility in equity portfolios, however, tail risk was found to be economically significant in it's compensation of higher return. Tail risk is defined as CVaR and Excess CVaR, where Excess CVaR is the portfolio CVaR which is in excess of that implied by a normal distribution using the same mean and standard deviation. The measure was calculated using actual historical portfolio returns and finding the average of the returns that were less than VaR. Additionally, tail risk premium remained significant when controlling for size, value, portfolio Beta and portfolio momentum, as well as in both U.S. and non-U.S. equity mutual funds. These findings indicate that volatility may not be the correct measure of risk being used in the market, and that tail estimation is the metric with a positive risk premium.

Another angle through which to look at the low-volatility anomaly is not whether volatility is the incorrect risk metric to use, but rather the possibility that the calculation of volatility does not take into account the time-varying nature of financial markets. There is a large body of evidence on volatility clustering in financial markets, whereby large shocks to the market result in volatility changes and a higher probability of further large shocks taking place (Alexander, 2008). Risk metrics should take into account the ability of volatility to change over time rather than remain constant, and how time impacts what level of volatility could be expected to persist given its current movements. Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models of volatility that were introduced by Engle (1982) and Bollerslev (1986) were specifically designed to capture the volatility clustering of returns when calculating and predicting volatility (Alexander, 2008). It is not impossible that accounting for volatility in this way may alter the prevalence of the low-volatility anomaly in stock markets.

2.2.3. Fundamental Impacts

Continuing on a different path, there has also been research into factors explaining the low-volatility effect that is independent from the existence of market anomalies or inefficiencies of market participants that others have used to analyse the effect. In one such study, Dutt and Humphery-Jenner (2012) focuses on operating returns as a key driver of stock returns, specifically analysing whether low-volatility stock companies have higher operating returns, and thus whether the higher operating returns are driving the outperformance of such stocks as opposed to the low-volatility itself. They focus their research outside of the U.S., specifically in emerging markets, where they find this low-volatility effect to be apparent. They also conclude that low-volatility stocks do tend to exhibit strong operating performance and that strong operating returns would increase expected stock returns.

While operating returns are found to be a robust explainer of low-volatility anomalies, the authors do accept that this is simply an additional factor that may explain the effect and does not wholly explain the phenomenon on its own. Controlling for operating performance significantly influences the relationship between stock returns and volatility which leads to the possibility that it is one of the key drivers of such relationship (Dutt and Humphery-Jenner, 2012).

Given the mounting evidence of a low-volatility effect's existence across markets, and the new findings of Blitz (2016) as well as those mentioned previously that the value effect and other factors such as size, momentum and liquidity fail to account for this low-volatility anomaly, Fama and French (2016) have come out with new research including an augmented version of their three-factor model which includes an additional two factors, resulting in a five-factor model that specifically attempts to address the low-volatility question. The two factors being added to the model are those related to investment, and, along similar lines of the operating "return" or in other words "profit" results found by Dutt and Humphery-Jenner (2012), a factor for profit. In the results of this paper, exposure to stocks based on these two factors, with exposure being defined as the stock having high profitability for the profit factor and conservative investment for the investment factor, appear to account for the high average returns of low beta (and/or volatility) stocks by raising the predictions of their average returns, and conversely a lack of exposure to these factors accounts for the low average return of high beta (and/or volatility) stocks by lowering the predictions of their average returns (Fama and French, 2015).

The results that profitability and operating performance are to be the defining factors that may capture the low-volatility effect as discussed and finally answer the long-asked question about why a low-volatility effect exists in equity markets make logical and economic sense in addition to their statistical significance. When a firm is consistently profitable and has good recurring results, it makes sense that its stock price and thus stock returns are less volatile than one that exhibits volatile or low profit performance and hence has other more volatile factors driving the up-movements in the stock's price. It also makes sense that these same stocks with consistent profitability and good results outperform those with volatile or low profit performance over the long-run, thus linking the "low-volatility" stocks with higher returns. On the other hand, critics of these results still question whether there is in fact enough evidence to say that the low-volatility conundrum has been outright solved.

Blitz and Vidojevic (2017) conduct an investigation in a recent paper, in which they argue that although it seems that the low-volatility anomaly has been explained by the new Fama-French (2015) five-factor model, this conclusion is premature given the lack of empirical evidence for a positive relationship between risk and return. Their results find that "exposure to market beta in the cross-section is not rewarded with a positive premium, regardless of whether we control for the new factors in the five-factor model" as well as observing that there is a stronger mispricing of volatility than of beta, suggesting that the volatility related phenomenon is the dominant one. Despite these findings, the conclusion of this paper explains that it is just one attempt at finding a positive risk-return relation after controlling for factors that others have found might be significant in explaining why such a relation has failed to be found empirically. The fact that their attempt is unsuccessful does not entirely rule out that portfolios constructed in a different manner might find the positive risk-return relationship found by Fama and French in their models (Blitz and Vidojevic, 2017).

In what is probably the most recent and relevant paper to have been published on this topic in September of 2017, Driessen, Kuiper and Beilo examine whether the interest rate exposure of stocks explains the low-volatility anomaly in the U.S. market. They show that the outperformance of low-volatility stocks can in fact be explained by a premium for interest rate exposure, with low-volatility stock portfolios having negative exposure to interest rates and high volatility stock portfolios having positive exposure to interest rates. While there are several explanations as to why this might be the case, the most logical one is that low-volatility stocks also tend to be the ones that come from defensive sectors, such as utilities, consumer staples, healthcare or real estate. These companies are, in general, large, profitable, have generally little growth opportunities and often pay out dividends, all characteristics that make the cash flows of these firms more predictable and result in lower valuation uncertainty, making these stocks a lot more similar to bonds, and thus, just like bonds, having a negative exposure to interest rate movements (Driessen, Kuiper and Beilo, 2017). As a result, their paper finds that (depending on the assumptions used about the interest rate premium) between 20% and 80% of the excess return on low-volatility stocks can be explained by including an interest rate exposure factor.

Frazzini and Pederson (2014) produced one of the key papers with regards to taking advantage of the low risk and low beta effect that is evident in the context of the above research. As opposed to attempting to explain the reasons or causes of the effect, this paper outlines how to take advantage of its existence in equity markets. The idea is that because investors with constraints (such as the lack of ability to short stocks for mutual fund investors) bid up high beta assets, high beta is associated with low alpha or outperformance. As a result, introducing a new “Betting against beta” (BAB) factor, which consists of long leveraged positions in low-beta assets using the funds from shorting high-beta assets, produces significant positive risk adjusted returns. This finding is however dependent on the availability of funding, and finds that as funding constraints tighten, the return of the BAB factor is lower. It also depends on the assumption that more constrained investors hold riskier assets to find outperformance, which is empirically found to be true in the underlying research (Frazzini and Pedersen, 2014). This shows a significant measure for investors to outperform on by using leverage to go long in the higher-average-return low beta stocks and enhance this position by shorting the lower-average-return high beta stocks, which was found to rival so called outperformance from other factors such as value, momentum and size.

2.3. South African Market Context

In terms of conducting research on equity markets in South Africa, it is important to acknowledge the unique attributes that are inherent in this specific market. In the context of this paper, the most important aspect that must be discussed is the decision on the best applicable asset pricing model to use relevant to the JSE. In research conducted by Van Rensburg and Slaney in 1997 and then adjusted for the reclassification of industries on the JSE and updated by Van Rensburg in 2002, it was found that using a two factor Arbitrage Pricing Theory (APT) model better predicted expected asset returns than the traditional Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964), Lintner (1965) and Mossin (1966). These results showed that the All Share Index (an index representing the entirety of the JSE) is not mean-variance efficient due to the ability of off-shore investment, which suggests why the CAPM performed poorly. Rather, the dichotomy in the return generating process of mining and industrial firms on the JSE suggests that the Financial-Industrial (FINDI) and Resources (RESI) indices are better observable proxies obtained from the covariance matrix of JSE returns and should thus be used in a two factor APT model. On the other hand, this research was conducted in 2002, when the JSE looked very different to today in terms of its constituents. Resources stocks dominated the market back then, but now take up a much smaller share of the current overall stock listing's market capitilisation. This could mean that the two factor APT model is or is no longer as valid as in the past in explaining JSE returns.

In light of the prior research into the low-volatility effect, it is evident that while extensive research has been conducted over the last decade, there has yet to be a concrete and sound explanation for why the anomaly exists. In order to discover whether the anomaly is occurring due to some factors that may explain the effect, such as the factors of behavioural finance, market participants or of fundamental profitability and investment as discussed, or whether the effect is simply its own phenomenon that occurs independently of such factors, further investigation is required to improve on research conducted thus far and contribute to the eventual conclusion in order to answer the questions surrounding anomaly. The most recent and promising results rely on the effect of interest rate exposure in at least partially explaining the anomaly, and thus useful research may look into this factor's relevance in emerging market examples of the anomaly as well as including a combination of other fundamental factors to see if such factors together can more wholly explain the low-volatility effect.

3. Data

3.1. Data Acquisition

The data required for this research comprised solely of financial market data. The majority of the data used was therefore collated and acquired using Bloomberg Terminal, while any remaining data that was either not available or missing on Bloomberg Terminal was obtained from Thomson Reuters Datastream. All data was extracted over a 15-year period ranging from the 2nd of January 2003 to the 29th of December 2017. The time frame was chosen to be as recent and as long as possible, in order to keep the research current and relevant while also maximizing the number of observations being used to conduct statistical and economic analysis. Prior to 2003, the South African stock market was largely characterized as being highly inefficient due to low liquidity, a small number of listed companies including a number that were dual-listed foreign firms, and extreme levels of concentration, and hence only analysis from 2003 onwards was considered for this research as previous inefficient market anomalies may have skewed or interfered with results. For all categories, both daily and monthly data over the period were used for different aspects of this analysis.

3.2. Share and Index Returns Data

For all individual share returns as well as index returns, the Total Return Index was used as the relevant return measure in order to account for the effect of dividend payments over time when calculating absolute returns. To ensure that only shares with a high enough level of liquidity as possible over the period were used, the share returns of the top sixty companies listed on the JSE ranked by market capitalisation at the time of data acquisition (October 2018) were obtained. This is especially due to the nature of volatility calculations which would be heavily impacted by high levels of illiquidity, as well as the calculation of the returns of each share. This was later narrowed down to the top fifty-four shares due to some shares either not having been publicly listed over the full period duration, missing data, or an evident lack of liquidity in the earlier years of the period under analysis. A list of the final stocks included can be found in Appendix 1. In addition to individual share returns, the returns for the JSE All Share Index (ALSI), Top 40 Index (Top40), Resources Index (RESI), Finance-Industrial Index (FINDI) and All Bond Index (ALBI) were acquired for their use in asset pricing models with specific reference to the CAPM model using the ALSI as well as the two factor APT model using RESI and FINDI.

3.3. Fundamental Variables Data

For each share under consideration, a number of fundamental variables about the company were obtained for use in determining whether any of these factors played a role in the unexpected outperformance of low-volatility ranked stocks and unexpected underperformance of high-volatility ranked stocks. The variables chosen were decided based on prior research using the same or similar factors as discussed in Section 2, the availability of factors on the relevant data bases and finally, hypotheses about intuitive factors that might have the highest potential of explaining the underlying anomaly. These variables (in no specific order) include: Industry Sector, Return on Assets, Operating Margin, Net Margin, Current Market Capitalisation, Market Capitalisation to Book Value, Dividend Pay-out Ratio, Price-to-Earnings Ratio, Price-to-Cash-Flow Ratio, Relative Share Price Momentum, Revenue Growth, Total Liabilities, Total Assets, Total Asset Turnover and Total Return Index. A description of each of these variables and how they are calculated can be found in Appendix 2.

3.4. Macroeconomic Variables Data

The macroeconomic data used in this research includes the three-month South African Government Treasury Bill Yield. This is used as a proxy for the South African risk-free interest rate in the calculation of excess returns on individual shares and indices. The forward-looking rate at each month was obtained including changes in this yield over time in order to account for the most accurate risk-free rate at each specific point in time. These yields were obtained directly from the South African Reserve Bank (SARB) website, since they could not be found for the entire period under analysis on the other relevant databases that were used.

3.5. Data Cleaning

All data was cleaned to ensure that there were as few missing data points as possible, either by searching for such data points in alternative places or extrapolating using the other data available. Weekends were already excluded from the extracted dataset, however, national public holidays as well as other days on which financial markets were closed were included and thus had to be removed and adjusted for. Some of the variables were also standardised for use in an equal weighted portfolio to ensure a more accurate scale of results in regression analysis. In the context of share and index returns, all returns were calculated as log-returns using the Total Return Index variable. Any future reference to “returns” or “excess returns” may be assumed as being log-returns and excess log-returns. This is due to the additive nature of log-returns over time. Excess returns for each share and index is calculated by subtracting the risk-free rate for the relevant time period from the share or index’s log-return.

4. Methodology

A total of seven measures of volatility and/or risk were calculated to determine whether there was a possibility that the occurrence of the low-volatility anomaly was in fact a result of either calculating volatility in relation to stocks incorrectly, or that volatility was the wrong risk measure to use when analysing the risk-return relationship of stocks altogether. Once calculated, the risk-return relationship of each risk measure could be compared by creating equally weighted portfolios of stocks ranked by the specific risk metric and comparing these portfolios performance across each type of risk.

These risk metrics can be broken down into four alternative methods of calculating volatility and three alternative risk measures that differ to traditional volatility. The four alternative methods of calculating stock volatility are all slightly different variations of the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) volatility model, which takes into account the time-varying nature of volatility and its non-constant relationship with time. These include the symmetric normal GARCH model, the asymmetric-GARCH (A-GARCH) model, the asymmetric Glosten-Jagannathan-Runkle-GARCH (GJR-GARCH) model and the asymmetric Exponential-GARCH (E-GARCH) model. The three risk measures used as alternates to volatility include the Downside Deviation (DD), Value-at-Risk (VaR) and Expected Tail Loss (ETL).

4.1. GARCH Volatility Models

There is a large body of evidence on volatility clustering in financial markets, whereby large shocks to the market result in volatility changes and a higher probability of further large shocks subsequently taking place (Alexander, 2008). The GARCH model and its derivatives were produced to account for the volatility clustering of returns, which by their nature indicate the ability of the level of stock return volatility to vary with time.

GARCH volatility distinguishes between unconditional and conditional volatility. Unconditional volatility is simply the volatility of the unconditional returns distribution, that is to say, the volatility of the returns distribution calculated in a manner that does not take the ordering of returns into account but simply estimates volatility as an average of return deviations from the mean. This is essentially how traditional volatility is calculated and assumed to be constant. Conditional volatility, on the contrary, does account for the ordering of returns and changes at every point in time by being calculated based on the previous history of returns up until that time period. GARCH volatility methods are therefore employed in this context to determine whether accounting for the relationship between the variation of stock returns, their history and time could be a more accurate measure of the risk faced by a deviation in stock returns, and thus better explain the positive relationship between risk and return that the low-volatility anomaly seems to refute.

Daily return data was used in the estimation of the GARCH models, as volatility clustering effects in financial asset returns tend to fade when returns are measured over longer time intervals (Alexander, 2008).

4.1.1. Normal Symmetric GARCH Volatility

The symmetric normal GARCH volatility assumes that the conditional variance (σ^2) at time (t) is given by the following equation, and that conditional volatility is taken as the square root of the conditional variance:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 , \quad (1.1)$$

where (ω) is the GARCH constant parameter, ε_{t-1} denotes the market shock or unexpected return (deviation from mean) in the previous period and (σ_{t-1}^2) is the conditional variance in the previous period.

The parameters (ω), (α) and (β) of the model are calculated by the method of maximizing the value of the log likelihood function using a computational iterative procedure that finds the maximum value possible of the log likelihood equation of the given dataset. The log likelihood function is given as:

$$\ln L(\theta) = -\frac{1}{2}\sum_{t=1}^{T} [\ln(\sigma_t^2) + (\frac{\varepsilon_t}{\sigma_t})^2] , \quad (1.2)$$

where (θ) represents the parameters (ω), (α) and (β). There are parameter constraints which are considered in the log likelihood maximisation procedure which ensure that the long term or unconditional volatility that is calculated using the GARCH conditional volatility is finite and positive. These are that (ω) is greater than zero, and that (α) and (β) when added together are less than one. This is also to ensure a positive conditional GARCH volatility value.

By using the excess share returns of each stock in the sample, the conditional normal symmetric GARCH volatility of each stock on each day was calculated using this procedure.

4.1.2. A-GARCH Volatility

The variations to the standard normal GARCH model modify the conditional variance equation to include additional features that might be relevant to the asset under consideration. It has long been understood that equity market returns often do not follow a perfectly normal distribution, and that most equity market return distributions are asymmetrically skewed. This has been empirically shown by the fact that increases in volatility of equity markets are larger following a large negative return than for a same size positive return. This is explained by the leverage effect, which is when a decrease in the firm's stock price causes an increase in the debt-equity ratio, resulting in the firm's future becoming less certain and thus a higher stock price volatility (Alexander, 2008). The A-GARCH volatility model accounts for this asymmetry in volatility to provide a more accurate estimate of equity market volatility.

The A-GARCH volatility model assumes that the conditional variance (σ^2) at time (t) is given by the following adjusted equation, which adds another parameter to the conditional GARCH volatility equation to capture the asymmetric volatility response. Conditional volatility is taken as the square root of the conditional variance:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}^2 - \lambda)^2 + \beta\sigma_{t-1}^2, \qquad (2.1)$$

where (λ) represents the capturing of the leverage effect. The parameters (ω), (α), (λ) and (β) of the model are calculated as they were for GARCH volatility, by the method of maximizing the value of the same log likelihood function (equation 1.2) using a computational iterative procedure, but now taking into account that (α)'s estimation is also dependent on (λ). The constraints in this model are the same as for the GARCH model, and there is no constraint on (λ), which when positive indicates that there is a larger increase in volatility when market shocks are negative than when they are positive, showing the presence of the leverage effect.

By using the excess share returns of each stock in the sample, the conditional A-GARCH volatility of each stock on each day was calculated using this procedure.

4.1.3. GJR-GARCH Volatility

An alternate GARCH volatility model to the A-GARCH model that also attempts to capture the asymmetric nature of equity returns and volatility calculations used is the GJR-GARCH model. This model also includes an additional parameter that captures the leverage effect as discussed before, however, the asymmetric response factor is adapted to specifically augment the volatility response from only decreases in share prices, while ignoring increases.

The GJR-GARCH volatility model assumes that the conditional variance (σ^2) at time (t) is given by the following adjusted equation and that conditional volatility is taken as the square root of the conditional variance:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \lambda 1_{\{\varepsilon_{t-1}<0\}}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \qquad (3.1)$$

where (λ) again represents the asymmetric response factor, and the indicator function represented by (1) is equal to 1 if $\varepsilon_t < 0$ and otherwise equal to 0. This is very similar to the A-GARCH volatility model and often yields very similar results, but was tested in this research to ensure completeness of analysis on a number of GARCH models. Parameter estimation for (ω), (α), (λ) and (β) are calculated using the same maximisation of the log likelihood function (equation 1.2) as before, and the constraints also remain the same.

By using the excess share returns of each stock in the sample, the conditional GJR-GARCH volatility of each stock on each day was calculated using this procedure.

4.1.4. E-GARCH Volatility

The final GARCH method that was employed is the Exponential or E-GARCH volatility model. In terms of calculating the volatility of stock returns in an asymmetric GARCH context, this model often yields the most accurate forecasting of future volatility when compared with the other GARCH methods (Alexander, 2008). Rather than imposing constraints on the parameters to ensure that the variance is positive as in the previous models, the E-GARCH volatility is calculated by formulating the conditional variance equation in terms of the log variance. This ensures that while the log variance may indeed be negative, the actual variance will always be positive when transformed out of its log equivalent form.

To calculate the E-GARCH conditional variance and volatility, first the asymmetric response function must be defined using an independent and identically distributed (i.i.d.) normal variable Z_t. This initial function can be written in the form:

$$g(z_t) = \theta z_t + \gamma(|z_t| - \sqrt{2/\pi}). \qquad (4.1)$$

This captures deviations of the realisation of Z_t from its expected value. Due to the nature of this function, a range of asymmetric responses to market shocks can be captured by E-GARCH Volatility. The simplest specification of the E-GARCH conditional variance, which assumes that Z_t is normally distributed with a population mean of 0 and standard deviation of 1, can be written in its log form as:

$$\ln(\sigma_t^2) = \omega + g(z_{t-1}) + \beta \ln(\sigma_{t-1}^2), \qquad (4.2)$$

where g(Z_{t-1}) is the previous period asymmetric response function defined in equation 4.1. Just as with the previous GARCH volatility models, the parameters can be estimated using the same log likelihood maximisation function as before, however, now the parameters that are being estimated to give the maximum log likelihood include those in the asymmetric response function: (θ), (Y), (ω) and (β). There are no constraints on these parameters due to the calculation and use of log conditional variance. The conditional variance is then simply transformed from its log equivalent form to its standard form.

By using the excess share returns of each stock in the sample, the conditional E-GARCH volatility of each stock on each day was calculated using this procedure.

4.2. Alternative Risk Measures

In addition to calculating different derivations of volatility, the most common risk metric used in finance, alternative risk measures were also considered to determine if the low-volatility anomaly was not also a low-risk anomaly. The DD, VaR and ETL are all risk measures that have been receiving growing attention in finance, and by using daily excess share returns, each was calculated to be used in comparison with the GARCH volatility model results when constructing portfolios of stocks ranked by each specific measure of risk.

4.2.1. Downside Deviation

Similar to the asymmetric GARCH models discussed above, DD is a method of calculating the standard deviation of returns, but by only considering negative returns or returns on the downside. This measure is widely recognised in its use in the Sortino ratio, which is similar to the Sharpe ratio but uses DD instead of standard deviation in its denominator. It has the potential to better represent what most investors consider to be risk, which is the risk of the portfolio losing value, while not considering the risk of extraordinary gains, what most investors would consider to be reward, as the straight forward standard deviation does.

To calculate the DD, first a Minimum Acceptable Return (MAR) is determined, which is used as the target below which returns are included as deviations in return and above which returns are considered to have zero deviation. In this case, the MAR was chosen as 0% due to the fact that excess returns were used, and therefore a reasonable return would be considered to be one greater than the risk-free rate, or greater than 0% in terms of excess returns. The next step is to calculate the sum of the squared deviation of those daily excess returns (*ExcessReturn*) that are less than the MAR and divide this by the total number of days minus one, as is seen in the normal formula to calculate sample variance. Finally, the square root is taken to arrive at the DD. The DD can be written by formula as:

$$DD = \sqrt{\frac{\sum_{t=1}^{n}[\min(ExcessReturn - MAR, 0)]^2}{N-1}} \, . \qquad (5.1)$$

By using the excess share returns for each stock in the sample, the DD of each stock on each day was calculated.

4.2.2. Value-at-Risk

VaR is probably the most significantly different risk measure to the others that have been employed and discussed thus far. While the GARCH volatility and DD methods centre around how large returns deviate from the mean or an MAR value, VaR centres around the distribution of returns and focuses on the extreme loss amount that may not be exceeded with certain probability or level of confidence. Many argue that this is a more accurate measure of risk as it is based on what most investors intuitively view as risk which is the probability of extreme losses in stock or portfolio value.

The simplest method for calculating daily VaR values first relies on the assumption that returns are normally distributed. Out of the sample of fifty-four stocks under analysis, each of the log excess returns were checked for normality by constructing histograms based on the daily returns of the full fifteen-year period. Every stock exhibited approximately normal distribution characteristics, although not matching the perfectly normal distribution with slight negative skewness and kurtosis being prevalent in the distributions of some stocks. It was determined, however, that the stocks were close enough to being normally distributed to employ this VaR method as their distributions were not drastically or distinctively different to the normal distribution.

Calculating VaR for individual shares is also a useful way of bringing systematic risk into the consideration of total risk, as it focuses on the co-movement of market risk parameters, the sensitivity of the stock to each risk factor and the volatility of the risk factors. The normal linear VaR formula incorporates these aspects by multiplying the inverse of the standard normal distribution at the given level of significance $(\phi^{-1}(\alpha))$, the stock Beta to the index at time t (β_t) and the index standard deviation (σ_t). This can then be adjusted for the desired time frame by being multiplied by the square root of time $(\sqrt{T})$, which in this case is the number of days. The standard normal linear VaR formula can be written in equation form as:

$$VaR_{(T,\alpha,t)} = \sqrt{T} \times \phi^{-1}(\alpha) \times \beta_t \times \sigma_t \ . \qquad (6.1)$$

In this research, VaR was calculated using a rolling 250 day time frame, and 1% as the significance level to signify extreme losses. The choice of 250 days was due to the nature of investing in equities which is more commonly done over longer time periods such as one year rather than a few weeks. The 1% level of significance was chosen to calculate extreme levels of risk in opposition to the standard volatility measure. The

JSE All Share Index was the relevant benchmark index for each share and was used to calculate a rolling 250-day index standard deviation each day, as well as the stock's Beta to the index, calculated as the Covariance between the stock and the index divided by the index variance.

The interpretation of the VaR number calculated on each day is important to underscore for a robust understanding of the risk measure. For example, if the 1% 250-day VaR calculated on one day is 20%, this means that there is a 1% chance that the stock will lose 20% or more of its value over that 250-day period, or in other words, there is a 99% level of confidence that the stock will not lose 20% or more of its value over that 250-day period. The 1% 250-day VaR was calculated on each day in the sample for all fifty-four stocks in order to be used as another risk metric by which to rank stocks from low to high risk and assess whether this would influence the risk-return relationship being perceived in the sample.

4.2.3. Expected Tail Loss

ETL, also known as Expected Shortfall and Conditional VaR, is a by-product of VaR itself. While VaR gives investors an estimate of what loss should not be exceeded with a given probability, ETL gives an estimate of what size loss could be expected in the case that the VaR loss amount is in fact exceeded. Again, assuming a normal distribution of returns, this is essentially calculated as an average of the negative returns in the extreme left tail of the distribution. In this case, since a 1% VaR is being employed, this means an average of the negative returns in the far-left tail of the distribution with significance levels or probabilities of less than 1%.

ETL is used here in conjunction with VaR, and individual stocks were ranked by ETL and compared to the VaR stock rankings to see if any stocks may have had a lower VaR but higher ETL than another stock, in which case ETL may better explain the risk-return relationship being sought out than VaR itself. To calculate ETL in the normal linear model, which also relies on some of the factors involved in the normal linear VaR formula, the following formula is used:

$$ETL_{(h,\alpha)}(X) = \alpha^{-1}\varphi\left(\phi^{-1}(\alpha)\right)\sigma_t - \mu_h \,. \qquad (7.1)$$

The factors also used in the VaR calculation (equation 6.1) include the inverse normal distribution (ϕ^{-1}) with significance level (α), however in this case the stock or portfolio standard deviation (σ) is used which is derived using the index standard deviation and Beta. Here, the mean level (μ_h) is allocated a zero-value given the use of excess returns and definition of risk as negative excess returns. In addition, to ensure consistency with the VaR calculations, the ETL was calculated over a 250-day period by multiplying by the square root of 250. The 250-day ETL was calculated on each day for each of the fifty-four stocks in the sample.

4.3. Stock Ranking and Portfolio Construction

Upon completion of calculating each of the seven measures of volatility and risk for every stock, each of the fifty-four stocks in the sample were ranked in order from lowest to highest risk for each specific risk metric. This resulted in seven sets of rankings. For the four GARCH models, VaR and ETL, stocks were ranked based on the average of the daily metric calculated over the entire period. For the DD, the measure returned was a calculation for the total period and did not require being averaged. Once ranked, the analysis of returns comparison between low and high-risk stocks could be conducted. VaR and ETL had the exact same stock ranking result.

The returns to low and high-risk stocks were calculated on an absolute, average and risk adjusted basis to examine the evidence found in prior studies that low risk stocks had higher returns than high risk stocks across a number of return measures. This involved calculating the total excess return of each stock over the total period between 2 January 2003 and 29 December 2017 as well as calculating the average daily excess returns for each stock. In terms of risk adjusted measures, the Sharpe ratio was calculated for GARCH volatility rankings (by simply replacing the standard deviation in the original Sharpe ratio with the GARCH standard deviation) the Sortino ratio in the case of DD rankings and for VaR and ETL, a risk adjusted return was calculated by dividing the excess return by the VaR or ETL for every ranked stock. The results regarding these rankings and their specific returns are discussed in more detail in the preliminary analysis.

Once the stocks had been ranked and measures of return had been calculated for each type of risk, stocks were allocated to portfolios based on their ranking. The stocks were split into a total of six portfolios with nine stocks in each portfolio to make up the total of fifty-four stocks in the sample. The decision on how best to allocate stocks was made based on the following criteria: ensuring there was an equal number of stocks in each portfolio, ensuring there were enough stocks in each portfolio to rule out the effect of outliers, and ensuring there were enough portfolios to reasonably measure the differences between them across various levels of perceived risk. In total, based on the seven risk metrics covered, there were forty-two portfolios created - six portfolios ranked from lowest to highest risk for each risk metric.

Each portfolio was constructed as an equally weighted portfolio. For the purpose of portfolio regressions and calculations, monthly returns data was used. The portfolio excess return for each month was therefore calculated as an average of the excess returns of the nine stocks for each portfolio. In addition to excess returns, each of the fundamental variables being tested that related to each stock in the portfolio, such as Market Capitalisation to Book Value ratios, were also averaged in order to compute a portfolio level variable i.e. a portfolio Market Capitalisation to Book Value ratio.

4.4. Portfolio Regressions

In addition to analysing whether or not the low-volatility anomaly was also a low-risk anomaly by measuring risk in a number of different ways as discussed, fundamental factors that could possibly explain why the anomaly is occurring were also tested. In order to test these factors however, there was a need to calculate what the expected excess return was for each portfolio, examine the difference between expected and actual returns seen, and finally, add factors that might better explain the actual returns or cause a smaller difference between expected and actual returns. This procedure involves running expected return regressions before and after adding factors that may explain the outperformance of low-volatility stocks and assessing how the alpha or unexpected return generated by the regression model changes with the addition of such factors.

In other words, if the low-volatility portfolio is outperforming, it could be determined whether this was because low-volatility portfolios outperform in general, or whether there was some other factor that could explain the outperformance that happens to also be a characteristic of low-volatility stocks. For example, a profitability factor could explain the low-volatility anomaly if low-volatility stocks also tend to be more profitable and thus it is more profitable firm stocks that outperform as opposed to the fact that the stock is lower-risk being the reason behind outperformance. By adding the profitability factor to an expected returns formula, the unexpected outperformance of low-volatility stocks, or high alpha, would become an expected performance, or lower alpha, instead.

Initially, the expected excess return formula that was chosen to be used was that of the two factor APT model by Van Rensburg and Slaney in 1997 and updated by Van Rensburg in 2002. They found that using two market factors on the JSE, which were the Resources and Financial-Industrial Indices (RESI and FINDI respectively), better explained returns than the one factor CAPM model. However, applying this model by running regressions for each portfolio, with the excess return as the dependent variable and excess RESI and FINDI returns as independent variables, found in most cases that either one or both of the variable coefficients were statistically insignificant in explaining the portfolio returns. The same can be said for the Top 40 Index which was tested in the same manner with the same result.

On the other hand, the CAPM regression was also applied using the JSE All Share Index (ALSI) excess returns as the proxy for the market (independent variable), and the ALSI coefficient was found to be extremely statistically significant in almost every portfolio regression. This does not disprove the two factor APT model on the JSE, however in the context of this research using equally weighted portfolios, the aim is not to discover the most accurate asset pricing model but to use a model that can show any alpha, or unexpected performance, with statistically significant relevance to this data and methodology. Therefore, the CAPM was used as the chosen expected return formula against which regressions with additional factors could be compared to. This was the excess return version of the CAPM, which can be written in equation form as:

$$R_i - R_f = \alpha + \beta(R_{ALSI} - R_f) + \epsilon \,, \qquad (8.1)$$

where R_i represents the return on portfolio i, R_f represents the risk-free rate, α represents the unexpected return on the portfolio, β is the coefficient of the portfolio to the excess market return, R_{ALSI} is the Return on the All Share Index or market and ε captures the error term.

Once the CAPM was chosen as the relevant asset pricing model, regressions were run by adding fundamental variables or factors to the model. As seen above, the constant returned from the regression output can be interpreted as alpha rather than the risk-free rate as seen in the traditional CAPM formula, since the excess returns are being applied in the independent and dependent variables. As discussed in the Data section, the fundamental variables that were applied to this regression model and assessed for their impact on the explanation of portfolio excess returns included: Industry Sector, Return on Assets, Operating Margin, Net Margin, Current Market

Capitalisation, Market Capitalisation to Book Value, Dividend Pay-out Ratio, Price-to-Earnings Ratio, Price-to-Cash-Flow Ratio, Relative Share Price Momentum, Total Assets and Total Asset Turnover.

In addition to these fundamental factors, the All Bond Index (ALBI) excess return was applied as a measure of average interest rate exposure in light of the research by Driessen, Kuiper and Beilo (2017) recognised in the review of prior literature on the anomaly. By adding and ALBI factor, it could be determined whether low-volatility stocks were also those stocks that have bond like characteristics i.e. constant and predictable cash flows in the form of dividends and continuous operating performance on which these cash flows rely. The assumption is that if low-volatility stocks do exhibit these characteristics, they should be negatively exposed to increases in interest rates just as bond prices are. If this is the case, the returns of these stocks should be positively correlated with the ALBI as a proxy for the bond market in South Africa.

To arrive at the final model, only those variables that were found to be economically significant in that their relationship with the explanatory variable followed previously researched economic theory and hypothesis, and variables that were found to have a direct impact on the alpha (or intercept) of the portfolio regression were chosen to be included for analysis. This is in line with Fama and French (1993), where it is discussed that "*in such regressions, a well-specified asset-pricing model produces intercepts that are indistinguishable from zero".* The aim is therefore to find factors that reduce the intercept (or alpha) as close to zero as possible. Furthermore, in terms of strict statistical significance, Fama and French (2016) explain that "*Asset pricing models, [in this context], are simplified propositions about expected returns that are rejected in* *tests with power. We are less interested in whether competing models are rejected than in their relative performance [at reducing regression intercepts]."* Although this will be discussed in more detail in the Results section, the variables that had a notable influence on the expected excess returns and alphas of the regression and were thus included in the final model other than the ALSI excess return were the ALBI excess return, Portfolio Market Capitalisation to Book Value (MB), and Portfolio Relative Share Price Momentum (MOM). The final model employed is therefore written in equation form as:

$$R_i - R_f = \alpha + \beta_1(ALSI) + \beta_2(ALBI) + \beta_3(MB) + \beta_4(MOM) + \epsilon \ . \quad (8.2)$$

The outcomes of the above methodology components are presented and discussed in the Preliminary Analysis and Results sections.

4.5. Portfolio VaR and ETL

As a final check of robustness for the methods of ranking stocks by risk and allocating them to portfolios correctly, the one-month 1% VaR and ETL was calculated of the actual portfolios themselves once they had been constructed. This is separate to the VaR and ETL calculations of individual stocks that was used for ranking as discussed above. The same normal linear VaR and ETL formulas (equations 6.1 and 7.1) as for the individual stock rankings were used, with the only difference being that the Betas involved in the calculation were portfolio Betas and that this was calculated using monthly returns data. As a result, the Beta of each portfolio in each month was calculated as the sum of the covariances of each stock in the portfolio with the ALSI index, multiplied by its portfolio weight, divided by the variance of the ALSI index. Each monthly VaR and ETL was annualised by multiplying by the square root of twelve, after-which the average of all of the monthly values was calculated to work out the average portfolio VaR and ETL for each portfolio. As expected, it was found that the more risky the portfolio of stocks is, the larger its VaR and ETL is.

5. Preliminary Analysis

An analysis of the returns of stocks ranked by risk metric for the seven risk metrics measured in this research was conducted to determine firstly whether the low-volatility anomaly is present in the sample of fifty-four stocks on the JSE and secondly whether the different risk metrics resulted in either significantly different ranking of stocks or a different relationship between risk and return than dictated by the low-volatility hypothesis.

5.1. Stock Rankings for Each Risk Metric

Appendix 3 provides a table of the ranking of each of the fifty-four stocks in the sample by each risk metric covered and outlined by portfolio. As can be seen, amongst the GARCH volatility models, many of the stocks are ranked in similar positions. In particular, the lowest and highest risk portfolio groups of stocks are virtually unchanged, with only one or two stocks changing between adjacent groups amongst the different rankings. The stocks in the middle four portfolios that fall between the lowest and highest risk portfolios are much more variable and change positions and groups more frequently. This may indicate that the key difference in returns between low and high-risk stocks may mostly apply to a comparison between those ranked on either end of the spectrum, and that the middle risk stocks are not as heavily influenced by the low-volatility anomaly.

For the DD ranked stocks, the same effect can be seen when compared with the GARCH volatility ranked stocks. Again, the lowest and highest risk portfolio groups of stock rankings may slightly change within the groups, but most of the stocks that are members of these groups for the other ranking methods remain in these groups for the DD rankings. The middle risk stocks again are the ones that vary more and jump between portfolios in comparison to the other rankings. Finally, the VaR and ETL ranked stocks came out to be identically ranked. This is unsurprising since ETL is a derivation of VaR. The lowest and highest risk portfolios in these rankings do also have a number of the same stocks as in the other rankings, with these stocks exhibiting a clearly consistent risk profile across the measures of risk, however there is much more variation in rankings with most stocks changing positions and moving between portfolios far more than before. One such example is the PSG stock, which falls in either portfolio 5 or portfolio 6 for the other risk metric rankings but jumps all the way up to the second stock in portfolio 2 for the VaR and ETL rankings.

5.2. Evidence of a Low-Risk Anomaly on the JSE

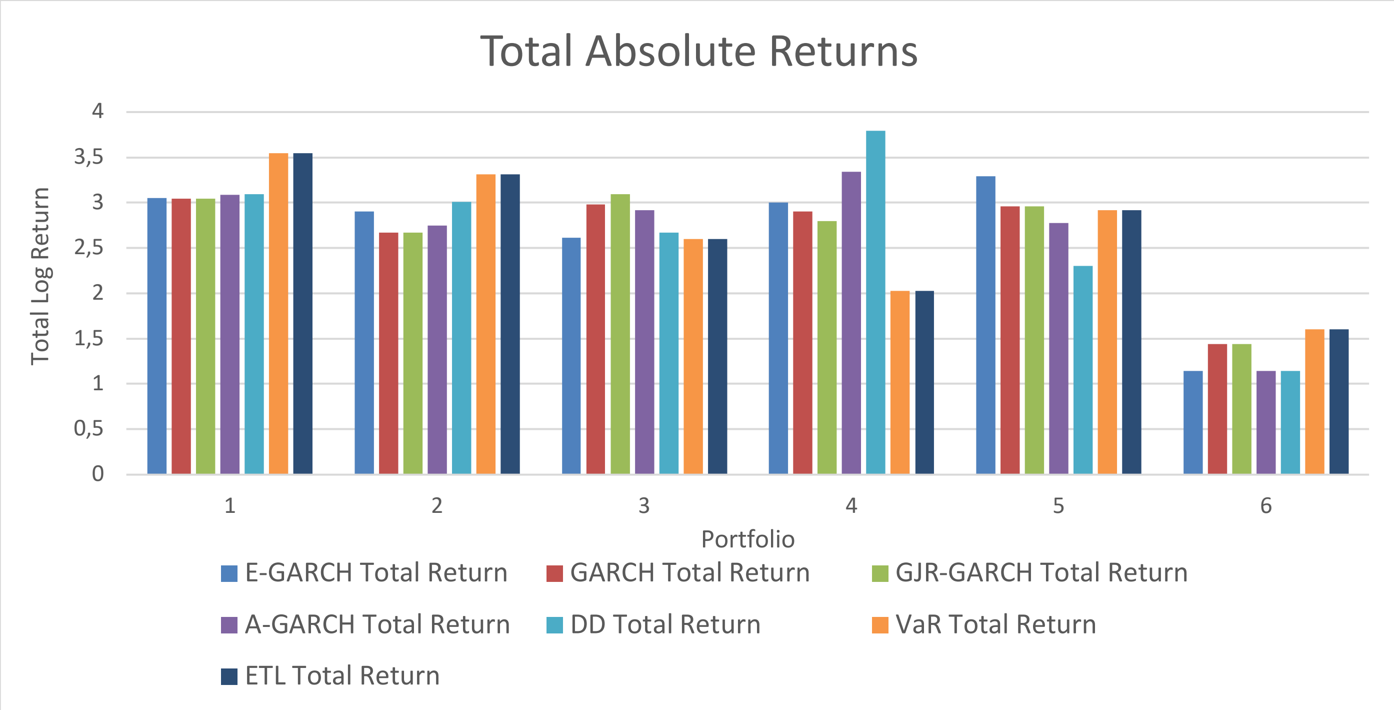

Figure 1: Total Absolute Returns Ranked from Lowest to Highest Risk Portfolio

The low-volatility anomaly does in fact appear to be a low-risk anomaly on initial examination. As can be seen in Figure 1, for each of the portfolios ranked from 1 to 6 in order of lowest risk to highest risk, no matter the risk metric used in ranking and portfolio construction, the high-risk portfolio consistently and significantly underperforms, while the low-risk portfolio continues to offer high levels of performance. This is evident on a total absolute return basis measured as the return over the entire 15-year period. One noticeable characteristic of these returns is that their relationship with risk is not perfectly linear. For example, for most of the risk metrics, there is actually a slight jump in returns in portfolios 4 and 5, particularly noticeable with the DD performance in portfolio 4. This may however simply be due to one or two outlier stocks in these medium to high risk portfolios that performed particularly well over the period. It is still clear that the difference between the lowest risk and highest risk portfolio at the extremities of the spectrum is extremely large, with portfolio 6 returns being less than half (or around 43% on average) of portfolio 1 returns in every case.

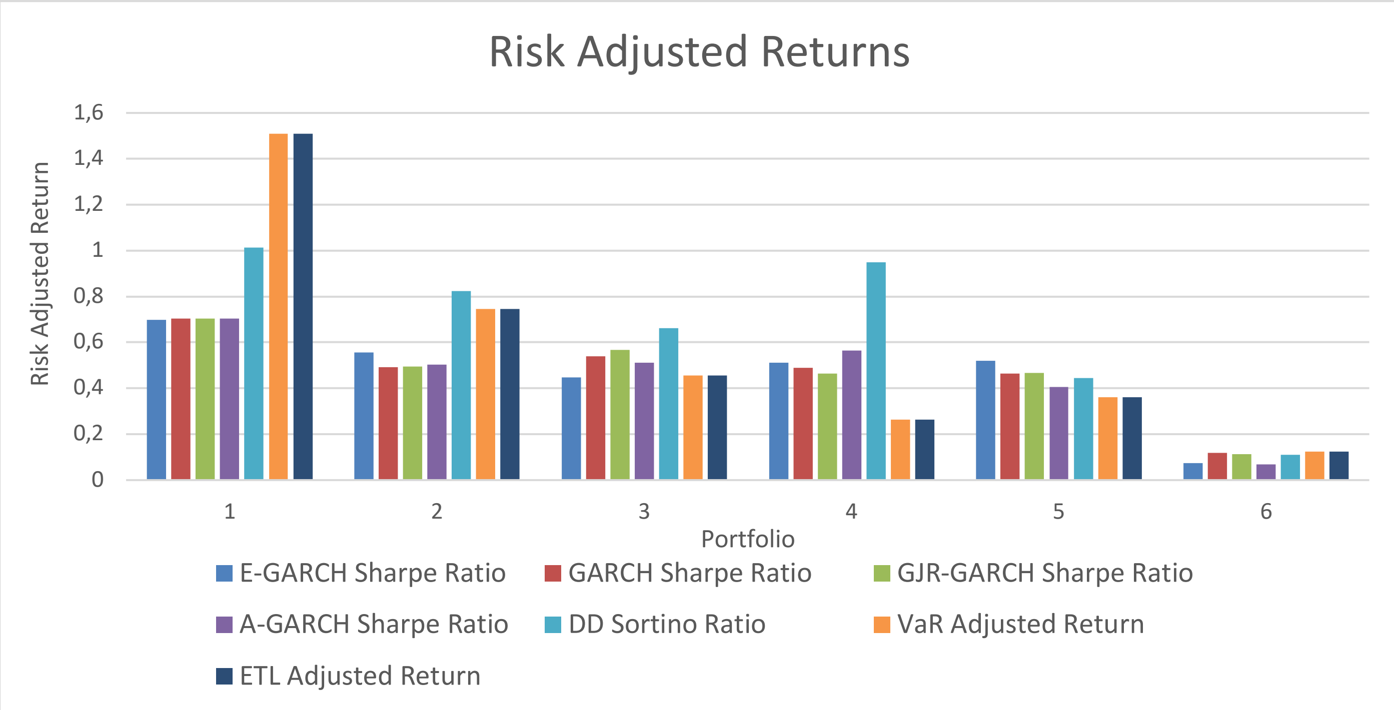

Figure 2: Risk Adjusted Returns Ranked from Lowest to Highest Risk Portfolio

Analysing the risk-return relationship on a risk adjusted basis, as is seen in Figure 2, provides even more evidence of the presence of a low-risk anomaly. When using risk adjusted returns, the relationship between returns and risk ranked portfolios does become slightly more linear although still not an exact linear relationship. The biggest exception is seen in the DD portfolio 4 returns once again. These results show that not only are investors not being rewarded for taking on higher levels of risk, but they are actually being penalized for it on a per-unit-of-risk basis. When comparing the lowest and highest risk portfolios, portfolio 6 risk adjusted returns make up only 11.5% of portfolio 1 risk adjusted returns on average.

A surprising result of both the total and risk adjusted returns is that of the VaR and ETL ranked portfolios. It may be expected that the GARCH volatility portfolios provide similar results to each other, with stock rankings definitely changing between them although not very dramatically. VaR and ETL rankings however resulted in the largest change in rankings among stocks, with some stocks moving more than ten ranks compared to GARCH volatility ranks, yet on a risk adjusted basis, VaR and ETL see the highest evidence of low-risk stocks heavily outperforming their high-risk counterparts. This suggests that the low-volatility anomaly may not be as a result of how volatility is calculated, or of using volatility to measure risk, since the relationship persists when using VaR, ETL and DD. As a result, the anomaly may instead be occurring due to fundamental factor differences between low and high-risk stocks themselves.

5.3. Total Risk vs Systematic Risk

Thus far the discussion has been centered around total risk, which includes both systematic and unsystematic risk. To break down the relationship further, the portfolios can be compared on a systematic risk basis. One of the best measures of systematic risk is the portfolio’s Beta to the market, which according to the original CAPM theory should be positively related to portfolio returns. Table 1 shows a different story altogether. The average monthly Beta computed for portfolios 1 to 6 on each risk metric ranking is positively related to the rankings of total risk as expected, i.e. the low-risk portfolios have lower Betas and high-risk portfolios have higher Betas. However as previously shown, this means that Betas are in fact negatively related to returns, confirming what has been found in prior literature on the matter.

Portfolio	1	2	3	4	5	6
E-GARCH Beta	0.384	0.43	0.494	0.468	0.636	1.206
GARCH Beta	0.363	0.435	0.51	0.462	0.721	1.122
GJR-GARCH Beta	0.363	0.435	0.502	0.473	0.721	1.122
A-GARCH Beta	0.384	0.414	0.502	0.489	0626	1.206
DD Beta	0.316	0.451	0.64	0.272	0.746	1.206
VaR Beta	0.255	0.373	0.402	0.488	0.724	1.386
ETL Beta	0.255	0.373	0.402	0.488	0.724	1.386

Table 1: Average Betas Ranked from Lowest to Highest Risk Portfolio

Taking a closer look at the Betas also helps explain the non-linear relationship seen in graph's 1 and 2 on a total risk basis. When looking at the DD ranking Betas, the Beta for portfolio 4 sees a sudden large decrease, while simultaneously, the total return and risk adjusted return sees a large increase. This provides further evidence of the negative relationship taking place between Beta and returns. In fact, the same can be said for the GARCH volatility models, as the portfolio 4 decrease in Beta also causes an increase in returns, but on a smaller scale.

5.4. Bond Market Correlation

Before the regression analysis which will deliver the main results of this research, the correlation between the risk-ranked portfolios and the ALBI were examined. As was hypothesized, it is evident that the low-volatility portfolio exhibits substantially higher correlations with the bond market, and that as the portfolio becomes riskier the correlation decreases. While the correlations for the lowest risk portfolio are not that high in absolute terms, ranging from 29.6% to 35.1%, they are still more than double that of even the second lowest-risk portfolio. In addition, the high-risk portfolios appear to have almost no correlation with the bond market at all, with correlation even entering negative values for the VaR and ETL high-risk ranked portfolios. This provides partial evidence that low-volatility stocks are more closely linked and positively related to bond returns and might provide an explanation of their outperformance compared to high-volatility stocks.

Portfolio	1	2	3	4	5	6
E-GARCH Bond Correlation	0.317	0.154	0.099	0.184	0.04	0.027
GARCH Bond Correlation	0.351	0.120	0.110	0.154	0.094	0.011
GJR-GARCH Bond Correlation	0.351	0.120	0.122	0.143	0.094	0.011
A-GARCH Bond Correlation	0.317	0.141	0.123	0.174	0.041	0.027
DD Bond Correlation	0.293	0.155	0.114	0.176	0.039	0.027
VaR + ETL Bond Correlation	0.296	0.079	0.143	0.083	0.210	-0.007

Table 2: Correlation of Each Ranked Portfolio with the ALBI Bond Market

A further examination of the link between the bond market, interest rate exposure and low- and high-risk stocks can be seen in Figures 3a and 3b. This shows the ALBI excess return alongside the excess return of the lowest risk portfolio in Figure 3a, and the highest risk portfolio in figure 3b, plotted over the 2003-2018 date range under analysis. The GARCH-volatility ranked portfolios are used here as a representative sample of the broader risk ranked portfolios. As suspected, the lowest risk portfolio excess returns appear to more or less move in the same direction as that of the ALBI excess return, as can be seen for example in the two spikes that occur between February 2008 and 2009, further highlighting their co-movement. On the other hand, the highest risk portfolio excess returns show almost no pattern or link with that of the ALBI, even moving in the opposite direction in some instances, such as February 2008 and 2009, and August 2017. This underscores the idea that low-risk stocks exhibit bond-like characteristics and are thus significantly exposed to movements in interest rates, while high-risk stocks do not have the same level of exposure to interest rates.

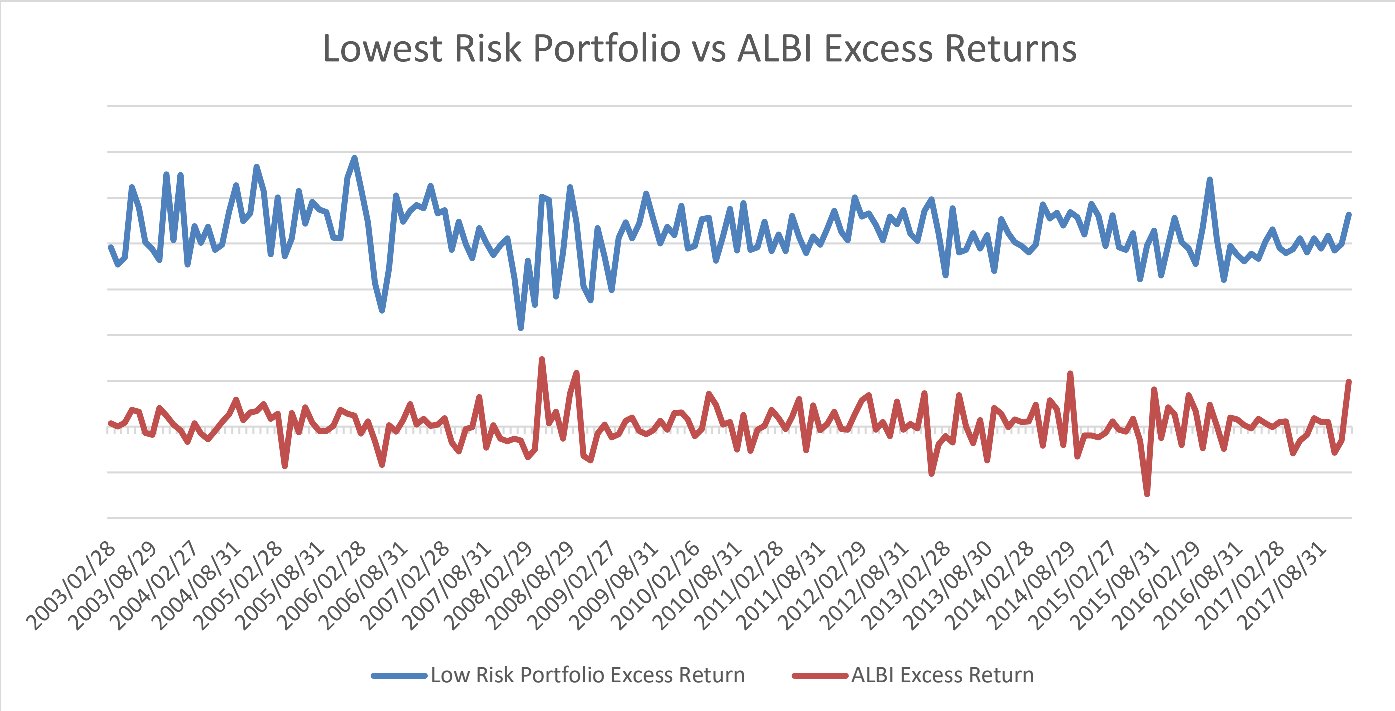

Figure 3a: Lowest Risk Excess Return vs ALBI Excess Return

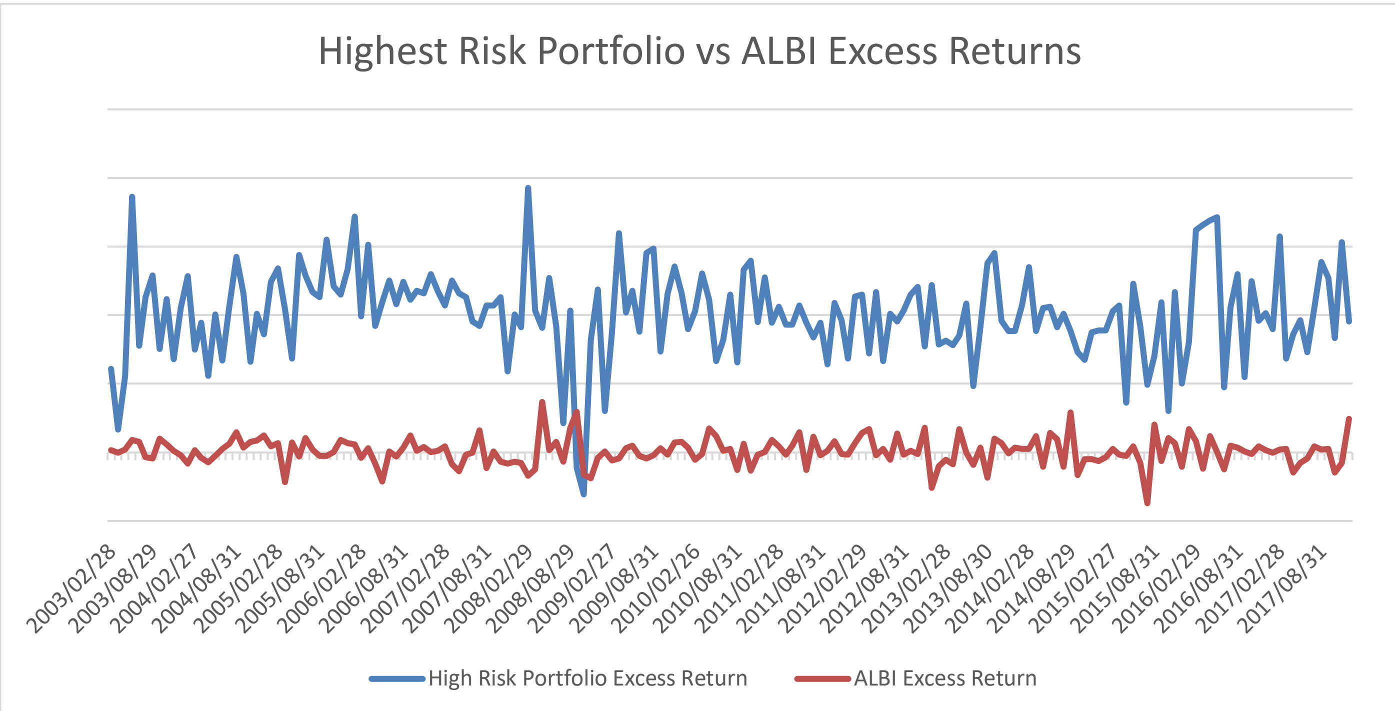

Figure 3b: Highest Risk Excess Return vs ALBI Excess Return

6. Results

The results can be broken down into portfolio regressions for each risk metric that stocks were ranked by. Detailed tables of the regression output for each portfolio in one of the risk metrics used can be found in the Appendix 4. There were six portfolios for each of the six risk metrics analysed, given that the VaR and ETL rankings were the same and thus have the same regression output. This means, with four regression models run for each portfolio, a total of 144 regression outputs, the relevant results of which are summarised here. Regression outputs for the risk metrics not displayed in Appendix 4 are available upon request.

All of the following results discussed were found to be similar and consistent among all methods of stock ranking. This is particularly due to the similarity of the specific stocks that make up the lowest and highest risk portfolios among almost every ranking method as previously shown. Therefore, only the details of the GARCH ranked portfolios are displayed here as a sample of the overall six portfolio ranking method results.

6.1. Unexpected Performance (α)

An explanation for part of the low-volatility anomaly is given by adding additional explanatory factors, namely a Bond market factor (ALBI), Market to Book Value factor (MB) and Relative Share Price Momentum factor (MOM), to the regression model. The largest impact of these variables on the expected returns actually occurs in the lowest volatility portfolio itself. This is evidence of a part explanation of the anomaly because it reduces what is considered to be “unexpected” returns to this portfolio (the definition of the anomaly) by providing factors that result in the high returns to low-volatility portfolios having a more logical explanation. In other words, a reduction in the alpha of the low-volatility portfolio does not mean that it does not still outperform the higher volatility portfolios, but rather that this outperformance can be expected based on its exposure to fundamental factors that are impacting returns.

The initial CAPM regression of the ranked portfolios results in alphas that are consistent with what could be expected from the low-volatility anomaly. As can be seen in Table 3, the low-risk portfolio has the highest unexpected positive performance or alpha of 0.8414%, while the high-risk portfolio has the lowest unexpected performance alpha of -0.4859%. This trend is consistent among the other stock rankings. Portfolios 2 to 5 also exhibit alpha's that have a negative relationship with risk level, generally decreasing in each level of risk with slight exception to the middle portfolios, which can be seen as having similar levels of risk and, as a result, similar alphas. The difference between the portfolio 1 and 6 alpha is shown throughout to illustrate how adding the explanatory factors changes the distinction in unexpected performance on either end of the risk spectrum and to see whether there is any narrowing of this differentiation in alpha.

Portfolio	CAPM α	Adding Bond Factor α	Adding MB Factor α	Adding Momentum α
1	0.8414***	0.7575***	0.2747	0.1309
2	0.7864***	0.7530***	0.7565***	0.4007
3	0.7159***	0.6845***	0.6961***	0.3649
4	0.7230***	0.6798***	0.6112**	0.0611
5	0.5744**	0.5461**	0.5561**	-0.0241
6	-0.4859	-0.4867	-0.4717	-0.5839
1-6	1.3273	1.2442	0.7464	0.7147

Table 3: Comparison of GARCH Ranked Portfolio a's Before and After Adding Explanation Factors
***** Significant at 1% level*
*** Significant at 5% level*
** Significant at 10% level*

Adding the ALBI factor to the regression causes a small decrease in portfolio alphas. On average, portfolio alphas decrease around 0.04% in each portfolio. In saying this, the ALBI factor causes the largest decrease in alpha for portfolio 1, this being decrease of 0.084%, and the smallest decrease in alpha for portfolio 6, being a decrease of just 0.0008%. The difference between portfolio 1 and 6 alphas therefore also slightly decreases by 0.083%, providing explanation for a small portion of the distinction in unexpected returns between the two.

Subsequently, adding the MB factor causes a large decrease in portfolio 1 alpha of 0.483%. At the same time, the alpha of portfolio 6 actually sees a slight increase of 0.015%, causing a small convergence of the two portfolio alphas from both directions. This leads to a large reduction in the difference in alpha between the lowest and highest risk portfolio which is actually the biggest jump that occurs from adding such fundamental factors to the expected return formula, seeing the difference moving from 1.2442% to 0.7464% between them. This is consistent among the other stock ranked portfolios. Finally, by adding the MOM factor to the regression model, there is a fairly large reduction in the alphas of the portfolios that lie between portfolios 1 and 6, and a smaller decrease in the alphas of these portfolios themselves.

While the difference in unexpected performance between the initial CAPM model and the final model that includes all factors does not completely explain the low-volatility anomaly in its entirety, the important results here are that the lowest-risk portfolio sees the biggest decrease in alpha or increase in explanation of returns, while the highest-risk portfolio alpha only slightly changes overall. The decrease in the gap between the two portfolios has occurred due to the low-risk portfolio outperformance being better explained, while there are still questions surrounding the explanation of the underperformance of high-risk stocks.

6.2. Significance of Interest Rate Exposure

Despite the ALBI factor not making an extremely large difference to the distinction between low and high-risk portfolio alphas, it's inclusion in expected return regressions still provides insight into the cause of the low-volatility phenomenon. Table 4 displays the Bond Beta (the beta of the ALBI factor in expected return equation) as well as p-value or statistical significance for each of the ranked portfolios. As can be seen, there is a systematic decrease in both the Bond Beta and its statistical significance when moving from lowest to highest risk portfolio, with exception to portfolio 4. In the regression output of the alternative risk metric ranked portfolios, the coefficient can be seen to be even less significant in each of the 5th highest risk portfolios. In portfolio 1, the Bond Beta is 0.526, meaning that an increase of 1%-point in the ALBI excess return results in a 0.526%-point increase in the excess return of the portfolio. In portfolio 6, the Bond Beat is -0.026, meaning that an increase of 1%-point in the ALBI excess return results in a -0.026%-point decrease in the portfolio excess return, however, with a p-value of 0.889 this coefficient is extremely unreliable.

Portfolio	Bond B	P-Value
1	0.526	0.000
2	0.249	0.036
3	0.225	0.065
4	0.337	0.008
5	0.197	0.080
6	-0.026	0.889

Table 4: Bond Betas and their Significance in GARCH Ranked Portfolios

These results indicate that there is in fact a link between low-volatility stocks and the bond market. Rather than detract from the reliability of these results, the outcome that high-volatility stocks are not significantly impacted by the bond market further substantiates the evidence that this may be one of the key differentiators between the returns accumulated by low and high-risk stocks. Since the low-volatility portfolio is significantly positively related to the bond market, and the bond market is negatively exposed to interest rates, this shows that low-volatility stocks are also negatively exposed to interest rate movements.

6.3. High-Volatility Portfolios are High-Risk Portfolios

In addition to ranking stocks by VaR and ETL as already described, the VaR and ETL of each portfolio itself was computed to determine whether the low-volatility portfolio might still exhibit a higher VaR or ETL than the high-volatility portfolio. This was found not to be the case across all six risk metric ranked portfolios.

As can be seen in Table 5, the VaR calculated for each portfolio increased with the level of risk which that portfolio is associated with. Portfolio 1 sees an average 1-month 1% annualised VaR of 12.69%. This indicates that, in annualised terms and on average, there is a 1% probability that the portfolio will lose 12.69% or more in value over one month. The associated ETL of 14.53% suggests that given the portfolio does lose 12.69% or more of its value, it will lose 14.53% of its value on average. Meanwhile, the VaR for Portfolio 6 is far greater than any of the other portfolios, along with its ETL, with these values being calculated at 40.11% and 45.96% respectively. This suggests that the ranking of stocks by the different risk measures under consideration has correctly assigned each portfolio with the level of risk being aimed at, with the high and low-volatility portfolios also being the high and low-risk portfolios based on these alternative measures of risk. The VaR and ETL results for the alternative risk measure ranked portfolios provide similar results to that of the GARCH ranked portfolio.

Portfolio	**Annualised 1% 1-month VaR**	**Annualised ETL corresponding to VaR**
1	0.1269	0.1453
2	0.1612	0.1847
3	0.1675	0.1919
4	0.1714	0.1964
5	0.2437	0.2916
6	0.4011	0.4596

Table 5: Annualised 1% 1-Month VaR and ETL of Each GARCH Ranked Portfolio

7. Findings

7.1. Sector and Industry Based Risk

One of the most interesting and substantial findings of this paper has to do with which sector the respective risk ranked stocks in each portfolio is a part of based on generally recognised macro-industries. Appendix 1 gives a full break down of the fifty-four stocks in the sample as well as which industry they belong to, while Appendix 3 shows the ranking of these stocks under each risk metric.

In terms of the low-risk portfolios, portfolio 1 includes almost every single Real Estate Investment Trust (REIT) company listed in the sample in every risk metric ranking. Out of the five REIT companies in the sample, four of them make it into portfolio 1 on each and every occasion. At the same time, REIT companies are those companies that can be seen to have high, predictable and constant levels of cash flow in the form of rental income received from properties under their ownership, while paying out these cash flows consistently in the form of high dividends as this is one of the regulations required in being listed as a REIT.

This brings the story back to the bond like characteristics of low-risk stocks. It makes sense that these stocks are consistently ranked as low risk since their operating performance and returns to investors can be determined and forecast with a reasonable level of certainty. The outperformance of these stocks can be attributed to the fact that they are non-cyclical and are less exposed to fluctuations in economic activity as the economy works its way through business cycles.

On the other hand, in terms of the high-risk portfolios, portfolios 5 and 6 are constantly and almost exclusively made up of companies from the resources/mining sector. Out of the eleven resource/mining industry companies in the sample, eight to nine make it into the highest risk portfolio for every risk metric other than the VaR and ETL ranked portfolio, which still contains six of these companies in its portfolio 6. Resource/mining companies' operating performance is invariably uncertain thanks to the fact that these companies' revenue relies heavily on the prices of the commodities which they mine and sell. The prices of most commodities fluctuate significantly over economic cycles and over time, especially those that make up the products of these companies such as gold, platinum and iron ore. Not only do the prices of the commodities vary significantly, but they are also mostly priced in U.S. Dollars. This lends itself to a double knock on effect for such South African firms, as they are thus also exposed to exchange rate fluctuations and the South African Rand over the past two decades has been one of the more volatile currencies when compared to most other countries' currency. This explains the extreme levels of volatility and risk that resource/mining company stocks exhibit on the JSE as found in this research.

In addition to such high levels of risk, resource/mining companies used to make up an extremely large proportion of both the JSE total market capitalisation and South Africa's Gross Domestic Product (GDP). Since the beginning of the 21st century however, this dominance in market size and contribution to GDP has fallen sharply, which may explain the extremely poor returns to investors seen in these stocks. In addition, as a highly cyclical industry, there is a lot more uncertainty regarding the operating performance of these firms.

Even further evidence of the sector-based risk phenomenon is the fact that amongst the other industry firms in the sample, their risk levels and rankings were more much more flexible, with specific industries not falling into one particular grouping but rather being more spread out. This points to the fact that, at least in a South African context, the low-volatility anomaly can also be viewed as a sector-based risk and return relationship between the lowest and highest risk sectors.

7.2. Value vs Low Risk

Another important finding of this research is the impact of including a value factor in the expected returns formula on the low-volatility anomaly. A number of factors that can be seen as a proxy for the value of a stock were tested, for example Return on Assets or Price-to-Earnings. Of all factors tested, the most significant and relevant value proxy was the Market Capitalisation to Book Value, which measures the difference between the market price of a share and the price at which it was originally issued or is measured in the company's financial statements. This is indicative of the value of a stock as it shows how cheaply or expensively it is currently priced at in the market relative to its original worth.

While previous studies as mentioned in section 2 have had mixed results surrounding the impact of a value factor, with some having failed to find that including value better explains the returns to low-volatility stocks, in this case it is found to cause the largest decrease in the alpha of the low-volatility portfolio out of all of the factors that landed up in the final regression model. Standardising the value variable and calculating the portfolio Market to Book Value by averaging that of each stock in the portfolio allows an equal comparison of the variable between all portfolios.

It is found that the lower risk portfolios exhibit portfolio Market to Book values that, on a monthly basis over the total period, exceed their mean value (or 1 in standardised terms) less times than the higher risk portfolios. This indicates that the cheaper the stock is, the better its overall performance will be, and that low-volatility stocks also tend to be cheaper stocks. This is not the first time that this fact has been discovered, as many previous studies on the outperformance of cheap stocks as measured by Price-to-Earnings, another proxy of value, have also found this to be the case. This makes intuitive sense as investing in cheaper stocks allows investors to receive more ‘value for money’, with the investment being under-priced by the market and having a lot more room to grow than stocks that are already considered to be expensive or over-priced. The key difference with the results found in this research however is the finding of a link between this ‘value anomaly’ and the low-risk anomaly.

7.3. Low Risk Stocks and Interest Rate Exposure

As mentioned in section 6.2 within the results section, the inclusion of the ALBI factor partially confirmed what had been discussed in more recent research on the low-volatility anomaly. While the inclusion of a Bond Beta only narrows the gap between the alpha of the lowest and highest risk portfolio by 0.083%, it is notable that there is a far larger reduction in the alpha of the lowest risk portfolio (-0.084%) than the highest risk portfolio (-0.0008%), pointing to the likelihood that lower risk stocks are more heavily exposed to the bond market and, therefore, to interest rates.

It is evident that interest rate exposure does not fully explain the low-volatility anomaly, however it can certainly be said that it does play a role in the difference in returns to more and less risky stocks. Of note, interest rates and more specifically the repurchase

(Repo) rate as determined by the South African Reserve Bank (SARB) in South Africa have gradually declined on average since early 2003, falling from 13.5% to 6.75% by the end of 2017. Although these rates have naturally fluctuated up and down within this period, the general trend has seen them decreasing, particularly in the wake of the 2008 global financial crisis.

More generally, interest rates around the globe have been steadily decreasing over an even longer period, such as the Federal Funds Rate in the U.S. which has trended downwards since its peak at 20% in 1981 to its lowest levels ever recorded in recent times at 0% to 0.5%. As referenced in the section 2 literature review, most studies on the low-volatility anomaly have been conducted with the substantial or even entire portion of the sample of data being tested taking place since 1981 within this global interest rate environment. It is possible that in the opposite environment, with a long-term upward trend in interest rates, that the low-volatility anomaly may no longer appear to exist.

This general interest rate trend in the local market may have contributed to the outperformance of low-risk stocks as examined over the sample period given the relationship that has been confirmed between low risk stocks and interest rate exposure. It would be interesting to see if the low-risk anomaly persists when there is instead an increasing trend in global and local interest rates and thus determine how much of the out or under performance of low-volatility stocks can be accounted for by interest rate movements.

7.4. Lowest Volatility Portfolio is the Most Impacted

Out of all of the portfolios, the lowest-volatility portfolio in each case was the one that was most impacted by the addition of each new factor in its expected return formula. This is true on an individual factor basis as well as the overall case between the first CAPM model and the final model including all three additional factors. The difference between the original CAPM model alpha and the final expected return formula alpha is 0.7105 percentage points for portfolio 1, the largest out of any portfolio. Interestingly, the highest-volatility portfolio actually saw the smallest impact on alpha from adding in the additional ALBI, MB and MOM factors with a change of 0.0977 percentage points.

The significance of this outcome is that the anomaly lives up to its name in that it is the expected return of the low-volatility stocks that is varying based on what factors are used to predict their return as opposed to that of the high-volatility stocks. What this shows is that using “risk” (as it is currently defined in the market) alone to assess the potential gains that an investor might expect to see in a “low-risk” stock over the long-term fails to account for the other factors that play a role in the stock’s ability to perform.

This leads to a change in the way that investors should look at stocks, to take on a far more multi-faceted approach by accounting for interest rate movements and a stock’s exposure to this, the stock’s valuation, its momentum and there are likely other factors that have yet to be uncovered as key metrics impacting expected returns. By doing so, the idea that a stock is riskier in the long-term based on how much its returns vary throughout time and that it should thus offer a higher return in compensation for taking on such risk can be disparaged as being an overly simplistic way of predicting just how much return can potentially be realized by a given stock in the equity market. This in essence argues that volatility and its many variations, some of which have been utilized in this research, is not the most accurate measure of long-term risk. Rather, a stock's exposure to the factors discussed and others, especially if it exhibits a low level of volatility, account for a large proportion of its downside risk, which most investors would define as the probability of making a loss or underperforming other comparable assets.

8. Conclusion

8.1. Implications

The low-volatility anomaly is still present on the JSE in South Africa. This research suggests that the low-volatility anomaly is in fact also a low-risk anomaly as has been previously questioned in prior research on the subject. This is based on the fact that using different statistical methods to calculate volatility and its relationship with time, including GARCH, A-GARCH, E-GARCH and GJR-GARCH volatilities, as well as different financial measures of risk such as Downside Deviation, VaR and ETL, all fail to explain why low-volatility stocks tend to outperform their high-volatility counterparts despite conventional finance theory prescribing the opposite. It is of course possible that there is an alternative volatility calculating method and/or risk metric that has not been assessed here, or which is yet to be uncovered by academics that are continually searching for the most useful way to analyse financial risk. However, those risk measures looked at in this paper provide a robust enough assessment of current methods of measuring variations in returns to conclude that this idea of risk does not accurately portray the true risk-return relationship that a rational investor would accept when choosing between assets, which would be high-risk equal to high-reward.

Instead, it is the acknowledgement of other factors that inherently effect a stock’s performance, namely its exposure to interest rates, level of valuation and its momentum, that need to be accounted for when trying to explain why one stock may outperform another and what constitutes “risk” in the true sense of the word. As it turns out, based on including these factors in an expected return formula to determine if outperformance of low-volatility stocks should be expected or unexpected, the unexpected portion of returns to low-volatility stocks is significantly reduced, while that of the high-volatility stocks is relatively unchanged in comparison. While low-volatility portfolios still perform better than expected and high-volatility portfolios perform worse than expected, the difference between the unexpected returns to the two, or difference between the portfolio alphas, is narrowed to allow a partial explanation of the phenomenon that is perplexing academics and practitioners alike in the finance world.

8.2 Research Limitations

There are certain limitations and research biases based on the research methodology used in this study that may impact specific aspects of the results and findings. The use of the Total Return Index, while taking dividend payments into account, has not been adjusted for complications that may arise related to shares that underwent corporate actions such as share repurchases, share splits, merger and acquisition transactions or divestitures among others. This may impact the variable’s use in analysing true stock returns. Additionally, the sample of stocks chosen based on being within the largest seventy stocks on the JSE by market capitilisation at a certain point in time adds the potential for survivorship bias in the study whereby only stocks that have maintained a certain relative size over the sample period are included, while stocks that may have performed poorly and dropped out of the sample threshold are not considered.

In terms of look-ahead bias, it should be noted that in the context of this research, the use of financial ratios is an attempt to explain stock performance retroactively, as opposed to predict future stock returns based on information that would not have been available at that time. The financial ratios are used to assess if stock's return over the same period were in line with and could be explained by their financial performance for that period. Lastly, future studies on the anomaly may want to consider a different approach of selecting a sample of shares that can be considered liquid such as by using daily or monthly turnover as a liquidity metric in order to assess if there is a difference in results relating to liquidity bias based on the current methodology.

8.3 Further Research

In light of the results of this paper, the recommended further research on explaining the low-volatility anomaly would involve delving deeper into the three factors discussed here, be it exposure to interest rates through a bond market beta, valuation based on the market capitilisation to book value ratio and finally, relative share price momentum, as well as the inclusion of other factors not mentioned under this analysis. In particular, going forward as the global and South African interest rate environment evolves, it would be especially relevant to look at whether low-volatility stocks still outperform on an expectation basis when interest rates are gradually increasing as opposed to decreasing over time in the manner that they have.

Another potential adjustment that could be made to research in this area going forward would be to look at the portfolio weightings of high and low-volatility stocks. This paper used a simple equal weighted method for each of the portfolios constructed, however, using either each stock's relative index weighting in the portfolio or adding more or less weight to the lower and higher volatility stocks within specific portfolios may alter the findings of this research and add more colour to the continual analysis of the anomaly. In addition to this, it would be interesting to look at the feasibility of trading strategies based on the knowledge that low-volatility stocks will outperform high-volatility stocks on a relative and absolute basis, such as using the ability to go short high-volatility stocks and long low-volatility stocks using the leverage garnered from the short position as discussed in the literature review.

Finally, while prior research has been conducted on both developed and emerging markets, an analysis using similar methodologies and ideas that have been employed in this paper could be extended to other emerging markets, especially those that have not seen as much attention as others in the past. It would be useful to compare the results between a number of emerging markets and look at both the similarities and differences with the South African market case. An interesting part of this comparison would be looking at the impact of the efficiency and liquidity in a specific market on the anomaly and seeing how this effects the risk-return relationship.

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Appendices

Appendix 1: List of Stocks in Sample Used

Company	**Ticker**	**Sector/Industry**
Resilient	RES	Real Estate Investment Trust
Growthpoint	GRT	Real Estate Investment Trust
Hyprop	HYP	Real Estate Investment Trust
Remgro	REM	Investment Holdings
Tiger Brands	TBS	Consumer Goods
Redefine	RDF	Real Estate Investment Trust
Santam	SNT	Financial Services
Liberty Holdings	LBH	Financial Services
Bidvest Group	BVT	Conglomerate
Pick n Pay	PIK	Retail
Intu Properties	ITU	Real Estate Investment Trust
Discovery Group	DSY	Financial Services
Clicks	CLS	Retail Pharmacy
Avi Limited	AVI	Consumer Goods
Sanlam	SLM	Financial Services
Netcare	NTC	Healthcare
MMI Holdings	MMI	Financial Services
Distell	DST	Consumer Beverages
Absa Group	ABG	Financial Services
Shoprite	SHP	Retail
Standard Bank Group	SBK	Financial Services
Nedbank Group	NED	Financial Services
Aspen Pharmacare Holdings	APN	Pharmaceutical
Woolworths	WHL	Retail
First Rand Limited	FSR	Financial Services
Coronation Fund Managers	CML	Financial Services
Investec PLC	INP	Financial Services
The Foschini Group	TFG	Clothing Retail
Massmart Holdings	MSM	Retail
RMB Holdings	RMH	Financial Services
Tsogo Sun	TSH	Entertainment
Compagnie Financiere Richemont South Africa	CFR	Consumer Luxury Goods
Investec Limited	INL	Financial Services
Mr Price	MRP	Clothing Retail
Imperial Logistics Holdings	IPL	Logistics
Barloworld Limited	BAW	Conglomorate
SASOL	SOL	Chemicals
Naspers	NPN	Media/Entertainment

Truworths Limited	TRU	Clothing Retail
Capitec Bank Holdings	CPI	Financial Services
Old Mutual	OML	Financial Services
MTN Group	MTN	Telecommunications
BHP Billiton	BHP	Resources/Mining
Exxaroo	EXX	Resources/Mining
AngloGold Ashanti	ANG	Resources/Mining
PSG Group	PSG	Financial Services
Sappi Limited	SAP	Resources/Timber
Assore Limited	ASR	Resources/Mining
African Rainbow Minerals	ARI	Resources/Mining
Impala Platinum Holdings	IMP	Resources/Mining
Anglo American	AGL	Resources/Mining
Goldfields Limited	GFI	Resources/Mining
Anglo American Platinum	AMS	Resources/Mining
Northam Platinum	NHM	Resources/Mining

Appendix 2: Fundamental Variables

Variable	Description	Calculation Method
Industry Sector	Shows which Industry/Sector the specific stock belongs to as dictated by the Exchange (JSE)	-
Return on Assets	Shows what return or profit a company has made as a percentage of Total Assets	Net Income divided by Average Total Assets
Operating Margin	Shows what Operating return or profit a company has made as a percentage of Revenue	Operating Income divided by Revenue
Net Margin	Shows what Net return or profit a company has made as a percentage of Revenue	Net Income divided by Revenue
Current Market Capitalisation	Shows the size of the company or Market Capitalisation as of the date on which the data was extracted	Number of Shares Outstanding Multiplied by Current Share Price
Market Capitalisation to Book Value Ratio	Compares the historical cost or accounting value of a company's shares to its current Market Value	Current Market Capitalisation divided by Accounting or Book of the firm
Dividend Pay-out Ratio	Shows the size of the dividend paid to shareholders as a percentage of total earnings that year.	Dividend divided by Total Net Income

Price-to-Earnings Ratio	A ratio comparing the current share price of the stock to its forecasted Earnings per Share	Current Share Price divided by Forecasted Earnings Per Share
Price-to-Cash-Flow Ratio	A ratio comparing the current share price of the stock to its forecasted Net Cash-Flow per Share	Current Share Price divided by Forecasted Net Cash-Flow per Share
Relative Share Price Momentum	Momentum is the rate of change or acceleration of movements in a stock's share price either up or down.	Closing Share Price minus Three Month's prior Closing Share Price
Revenue Growth	Percentage change in Revenue Period over Period	(Revenue minus Previous Period Revenue) divided by Previous Period Revenue
Total Liabilities	Total Liabilities in a Company's Statement of Financial Position during the Period	-
Total Assets	Total Assets in a Company's Statement of Financial Position during the Period	-
Total Asset Turnover	Shows the efficiency of a Company's assets by indicating the amount of Revenue per Assets that has been generated	Total Revenue/Sales divided by Average Total Assets
Total Return Index	Tracks the total return on a stock or index by including bot the capital appreciation as well as distributions/dividends paid out.	Add Dividends/Distributions to the capital appreciation of the share price over time.

Appendix 3: List of Stocks Ranked by Each Risk Metric

Portfolio	Rank	E-GARCH	GARCH	A-GARCH	GJR-GARCH	DD	VaR/ETL
1	1	RES	RES	RES	RES	RES	DST
	2	GRT	GRT	GRT	GRT	GRT	RES
	3	HYP	HYP	HYP	HYP	HYP	HYP
	4	REM	REM	RDF	REM	REM	SNT
	5	RDF	TBS	REM	TBS	TBS	TSH
	6	TBS	RDF	TBS	RDF	RDF	RDF
	7	AVI	SNT	SNT	SNT	SNT	GRT
	8	SNT	LBH	PIK	LBH	PIK	CPI
	9	LBH	BVT	BVT	BVT	AVI	AVI
2	10	BVT	PIK	LBH	PIK	BVT	CML
	11	PIK	ITU	AVI	ITU	DSY	PSG
	12	DSY	DSY	DSY	DSY	LBH	CLS
	13	CLS	CLS	CLS	AVI	CLS	LBH
	14	NTC	AVI	ITU	CLS	SHP	PIK
	15	SLM	SLM	MMI	SLM	NTC	DSY
	16	DST	NTC	NTC	NTC	MMI	NTC
	17	ABG	MMI	SLM	MMI	SLM	TBS
	18	SBK	DST	SHP	DST	DST	APN
3	19	MMI	ABG	DST	ABG	CML	SHP
	20	SHP	SHP	NED	SHP	ITU	MSM
	21	NED	SBK	WHL	SBK	APN	MMI
	22	ITU	NED	ABG	NED	WHL	ASR
	23	APN	APN	APN	APN	SBK	MRP
	24	WHL	WHL	SBK	WHL	ABG	TRU
	25	INP	FSR	CML	CFR	CFR	ANG
	26	TFG	CML	MSM	CML	NED	TFG
	27	MSM	INP	CFR	FSR	TSH	ITU
4	28	FSR	TFG	FSR	INP	MSM	WHL
	29	CML	MSM	TFG	TFG	TFG	GFI
	30	MRP	RMH	MRP	MSM	FSR	REM
	31	OML	TSH	RMH	RMH	RMH	BVT
	32	IPL	CFR	TSH	INL	MRP	BAW
	33	TSH	INL	IPL	TSH	NPN	ABG
	34	BAW	MRP	NPN	MRP	TRU	NED
	35	RMH	IPL	TRU	IPL	IPL	IPL
	36	CFR	BAW	BAW	BAW	CPI	SAP
	37	SOL	SOL	SOL	SOL	OML	SLM
	38	NPN	NPN	OML	NPN	INP	NHM
	39	INL	TRU	INP	TRU	BAW	SBK

5	40	CPI	CPI	INL	CPI	INL	CFR
	41	TRU	OML	CPI	OML	SOP	FSR
	42	MTN	MTN	MTN	MTN	MTN	NPN
	43	BIL	BIL	BIL	BIL	BIL	MTN
	44	PSG	EXX	PSG	EXX	PSG	RMH
	45	SAP	ANG	SAP	ANG	SAP	OML
6	46	ANG	PSG	EXX	SAP	EXX	INP
	47	EXX	SAP	ANG	ASR	AGL	INL
	48	ASR	ASR	AGL	ARI	ANG	ARI
	49	ARI	ARI	ARI	PSG	ARI	EXX
	50	AGL	IMP	AMS	IMP	ASR	SOL
	51	IMP	AGL	NHM	AGL	AMS	AMS
	52	GFI	GFI	IMP	GFI	NHM	IMP
	53	AMS	AMS	ASR	AMS	GFI	BIL
	54	NHM	NHM	GFI	NHM	IMP	AGL

Appendix 4: Regression Output of the GARCH-Ranked Portfolios

A: Portfolio 1

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		43,71
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0321	1	0,0321	R^2 =		0,1980
Residual	0,1298	177	0,0007	Adjusted R^2 =		0,1935
Total	0,1619	178	0,0009	Root Mean Square Error =		0,0271
Portfolio 1	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3034	0,0459	6,61	0,0000	0,2128	0,3939
Constant (α)	0,0084	0,0021	4,10	0,0000	0,0044	0,0125

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		40,74
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0512	2	0,0256	R^2 =		0,3165
Residual	0,1107	176	0,0006	Adjusted R^2 =		0,3087
Total	0,1619	178	0,0009	Root Mean Square Error =		0,0251
Portfolio 1	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3000	0,0425	7,06	0,0000	0,2162	0,3839
ALBI Excess Return	0,5169	0,0936	5,52	0,0000	0,3322	0,7017
Constant (α)	0,0076	0,0019	3,97	0,0000	0,0038	0,0113

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		30,17
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0552	3	0,0184	R^2 =		0,3422
Residual	0,1062	175	0,0006	Adjusted R^2 =		0,3308
Total	0,1615	178	0,0009	Root Mean Square Error =		0,0247
Portfolio 1	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,2814	0,0425	6,63	0,0000	0,1976	0,3652
ALBI Excess Return	0,5239	0,0938	5,59	0,0000	0,3389	0,7090
Market-Book	0,0005	0,0002	2,70	0,0080	0,0001	0,0009
Constant (α)	0,0027	0,0026	1,06	0,2920	-0,0024	0,0079

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		24,00
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0576	4	0,0144	R^2 =		0,3569
Residual	0,1038	174	0,0006	Adjusted R^2 =		0,3421
Total	0,1615	178	0,0009	Root Mean Square Error =		0,0245
Portfolio 1	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3085	0,0442	6,97	0,0000	0,2212	0,3958
ALBI Excess Return	0,5260	0,0930	5,66	0,0000	0,3425	0,7095
Market-Book	0,0005	0,0002	2,61	0,0100	0,0001	0,0009
MOM	0,0004	0,0002	1,99	0,0480	0,0000	0,0007
Constant (α)	0,0013	0,0027	0,49	0,6250	-0,0040	0,0066

B: Portfolio 2

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		49,64
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0517	1	0,0517	R^2 =		0,2190
Residual	0,1842	177	0,0010	Adjusted R^2 =		0,2146
Total	0,2359	178	0,0013	Root Mean Square Error =		0,0323
Portfolio 2	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3851	0,0547	7,05	0,0000	0,2773	0,4930
Constant (a)	0,0079	0,0024	3,22	0,0020	0,0030	0,0127

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		26,58
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0547	2	0,0274	R^2 =		0,2320
Residual	0,1812	176	0,0010	Adjusted R^2 =		0,2232
Total	0,2359	178	0,0013	Root Mean Square Error =		0,0321
Portfolio 2	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3839	0,0544	7,06	0,0000	0,2765	0,4911
ALBI Excess Return	0,2062	0,1198	1,72	0,0870	-0,0302	0,4425
Constant (a)	0,0075	0,0024	3,09	0,0020	0,0027	0,0123

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		17,78
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0551	3	0,0184	R^2 =		0,2336
Residual	0,1808	175	0,001	Adjusted R^2 =		0,2205
Total	0,2359	178	0,0013	Root Mean Square Error =		0,0321
Portfolio 2	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3781	0,0552	6,85	0,0000	0,2692	0,4871
ALBI Excess Return	0,2100	0,1201	1,75	0,0820	-0,0271	0,4472
Market-Book	0,0030	0,0048	0,62	0,5360	-0,0065	0,0124
Constant (a)	0,0076	0,0024	3,09	0,0020	0,0027	0,0124

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		16,71
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0655	4	0,0164	R^2 =		0,2776
Residual	0,1704	174	0,0010	Adjusted R^2 =		0,2610
Total	0,2359	178	0,0013	Root Mean Square Error =		0,0313
Portfolio 2	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,3986	0,0541	7,36	0,0000	0,2917	0,5054
ALBI Excess Return	0,2487	0,1176	2,12	0,0360	0,0166	0,4808
Market-Book	0,0063	0,0048	1,32	0,1880	-0,0031	0,0157
MOM	0,0007	0,0002	3,25	0,0010	0,0003	0,0012
Constant (a)	0,004	0,0026	1,53	0,1280	-0,0012	0,0092

C: Portfolio 3

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		46,68
Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F. Stat P-Value =		0,0000
Model	0,0521	1	0,0521	R^2 =		0,2087
Residual	0,1974	177	0,0011	Adjusted R^2 =		0,2042
Total	0,2494	178	0,0014	Root Mean Square Error =		0,0334
Portfolio 3	**Coefficient**	**Standard Error**	**t-Statistic**	**P-Value**	**95% Confidence Interval**	
JSE ALSI Excess Return	0,3866	0,0566	6,83	0,0000	0,2749	0,4982
Constant (a)	0,0072	0,0025	2,83	0,0050	0,0022	0,0122

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		24,74
Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F. Stat P-Value =		0,0000
Model	0,0547	2	0,0274	R^2 =		0,2194
Residual	0,1947	176	0,0011	Adjusted R^2 =		0,2106
Total	0,2494	178	0,0014	Root Mean Square Error =		0,0333
Portfolio 3	**Coefficient**	**Standard Error**	**t-Statistic**	**P-Value**	**95% Confidence Interval**	
JSE ALSI Excess Return	0,3853	0,0564	6,84	0,0000	0,2741	0,4966
ALBI Excess Return	0,1933	0,1242	1,56	0,1210	-0,0518	0,4383
Constant (a)	0,0068	0,0025	2,71	0,0070	0,0019	0,0118

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		17,21
Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F. Stat P-Value =		0,0000
Model	0,0568	3	0,0189	R^2 =		0,2278
Residual	0,1926	175	0,0011	Adjusted R^2 =		0,2145
Total	0,2494	178	0,0014	Root Mean Square Error =		0,0332
Portfolio 3	**Coefficient**	**Standard Error**	**t-Statistic**	**P-Value**	**95% Confidence Interval**	
JSE ALSI Excess Return	0,3693	0,0574	6,43	0,0000	0,2559	0,4826
ALBI Excess Return	0,1962	0,1239	1,58	0,1150	-0,0483	0,4407
Market-Book	0,0060	0,0044	1,37	0,1710	-0,0026	0,0147
Constant (a)	0,0070	0,0025	2,76	0,0060	0,0020	0,0119

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		15,97
Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F. Stat P-Value =		0,0000
Model	0,067	4	0,0167	R^2 =		0,2686
Residual	0,1824	174	0,0010	Adjusted R^2 =		0,2518
Total	0,2494	178	0,0014	Root Mean Square Error =		0,0324
Portfolio 3	**Coefficient**	**Standard Error**	**t-Statistic**	**P-Value**	**95% Confidence Interval**	
JSE ALSI Excess Return	0,3864	0,0563	6,86	0,0000	0,2753	0,4976
ALBI Excess Return	0,2249	0,1213	1,85	0,0650	-0,0144	0,4642
Market-Book	0,0082	0,0043	1,89	0,0600	-0,0004	0,0168
MOM	0,0006	0,0002	3,12	0,0020	0,0002	0,0009
Constant (a)	0,0036	0,0026	1,36	0,1760	-0,0016	0,0089

D: Portfolio 4

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		60,46
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0714	1	0,0714	R^2 =		0,2601
Residual	0,2031	177	0,0011	Adjusted R^2 =		0,2558
Total	0,2744	178	0,0015	Root Mean Square Error =		0,0344
Portfolio 4	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,4732	0,0609	7,78	0,0000	0,3531	0,5933
Constant (α)	0,0072	0,0027	2,73	0,0070	0,0020	0,0125

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		33,74
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0777	2	0,0388	R^2 =		0,2829
Residual	0,1968	176	0,0012	Adjusted R^2 =		0,2746
Total	0,2744	178	0,0016	Root Mean Square Error =		0,0339
Portfolio 4	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,4724	0,0600	7,86	0,0000	0,3537	0,5910
ALBI Excess Return	0,2967	0,1271	2,33	0,0210	0,0459	0,5476
Constant (α)	0,0068	0,0026	2,59	0,0100	0,0016	0,0120

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		19,72
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0703	3	0,0234	R^2 =		0,2712
Residual	0,1889	175	0,0012	Adjusted R^2 =		0,2575
Total	0,2592	178	0,0016	Root Mean Square Error =		0,0345
Portfolio 4	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,4487	0,0638	7,03	0,0000	0,3226	0,5748
ALBI Excess Return	0,3184	0,1313	2,42	0,0160	0,0591	0,5777
Market-Book	0,0027	0,0043	0,65	0,5190	-0,0056	0,0111
Constant (α)	0,0061	0,0028	2,22	0,0280	0,0007	0,0116

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		20,09
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,0874	4	0,0218	R^2 =		0,3371
Residual	0,1718	174	0,0011	Adjusted R^2 =		0,3204
Total	0,2592	178	0,0016	Root Mean Square Error =		0,0330
Portfolio 4	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,4343	0,0611	7,10	0,0000	0,3135	0,5552
ALBI Excess Return	0,3380	0,1257	2,69	0,0080	0,0897	0,5862
Market-Book	0,0039	0,0041	0,96	0,3380	-0,0041	0,0120
MOM	0,0012	0,0003	3,96	0,0000	0,0006	0,0018
Constant (α)	0,0006	0,0030	0,20	0,8380	-0,0053	0,0065

E: Portfolio 5

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		147,12
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,1396	1	0,1396	R^2 =		0,4539
Residual	0,168	177	0,0009	Adjusted R^2 =		0,4508
Total	0,3076	178	0,0017	Root Mean Square Error =		0,0308
Portfolio 5	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,6331	0,0522	12,13	0,0000	0,5301	0,7361
Constant (α)	0,0057	0,0023	2,46	0,0150	0,0011	0,0104

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		75,27
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,1418	2	0,0709	R^2 =		0,4610
Residual	0,1658	176	0,0009	Adjusted R^2 =		0,4549
Total	0,3076	178	0,0017	Root Mean Square Error =		0,0307
Portfolio 5	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,6320	0,0052	12,15	0,0000	0,5293	0,7346
ALBI Excess Return	0,1745	0,1146	1,52	0,1290	-0,0516	0,4007
Constant (α)	0,0055	0,0023	2,34	0,0200	0,0009	0,0101

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		50,78
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,1431	3	0,0477	R^2 =		0,4654
Residual	0,1644	175	0,0009	Adjusted R^2 =		0,4562
Total	0,3076	178	0,0017	Root Mean Square Error =		0,0307
Portfolio 5	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,6191	0,0531	11,67	0,0000	0,5144	0,7238
ALBI Excess Return	0,1767	0,1145	1,54	0,1240	-0,0492	0,4026
Market-Book	0,0052	0,0043	1,20	0,2340	-0,0034	0,0137
Constant (α)	0,0056	0,0023	2,38	0,0180	0,0010	0,0102

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		42,30
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,1516	4	0,0379	R^2 =		0,493
Residual	0,1559	174	0,0009	Adjusted R^2 =		0,4814
Total	0,3076	178	0,0017	Root Mean Square Error =		0,0299
Portfolio 5	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,6181	0,0518	11,93	0,0000	0,5159	0,7204
ALBI Excess Return	0,1975	0,1120	1,76	0,0080	-0,0236	0,4185
Market-Book	0,0065	0,0042	1,54	0,1260	-0,0019	0,0149
MOM	0,0011	0,0003	3,08	0,0020	0,0004	0,0017
Constant (α)	-0,0002	0,0030	-0,08	0,9350	-0,0061	0,0056

F: Portfolio 6

CAPM Regression Output				Number of Observations =		179
				F(1, 177) =		128,48
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,3264	1	0,3264	R^2 =		0,4206
Residual	0,4497	177	0,0025	Adjusted R^2 =		0,4173
Total	0,7761	178	0,0044	Root Mean Square Error =		0,0504
Portfolio 6	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,9680	0,0854	11,33	0,0000	0,7995	1,1365
Constant (a)	-0,0049	0,0038	-1,27	0,2050	-0,0124	0,0027

CAPM + ALBI Regression Output				Number of Observations =		179
				F(2, 177) =		63,88
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,3264	2	0,1632	R^2 =		0,4206
Residual	0,4497	176	0,0026	Adjusted R^2 =		0,4173
Total	0,7761	178	0,0044	Root Mean Square Error =		0,0505
Portfolio 6	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,9680	0,0857	11,30	0,0000	0,7989	1,1370
ALBI Excess Return	0,0047	0,1887	0,02	0,9800	-0,3677	0,3771
Constant (a)	-0,0049	0,0038	-1,27	0,2070	-0,0124	0,0027

CAPM + ALBI + Market-Book Regression Output				Number of Observations =		179
				F(3, 177) =		45,75
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,3411	3	0,1137	R^2 =		0,4396
Residual	0,4349	175	0,0025	Adjusted R^2 =		0,4300
Total	0,7761	178	0,0044	Root Mean Square Error =		0,0499
Portfolio 6	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,9514	0,0848	11,23	0,0000	0,7841	1,1187
ALBI Excess Return	0,0025	0,1861	0,01	0,9890	-0,3648	0,3698
Market-Book	0,0160	0,0066	2,43	0,0160	0,0030	0,0290
Constant (a)	-0,0047	0,0038	-1,24	0,2150	-0,0122	0,0028

CAPM + ALBI + Market-Book + MOM Regression Output				Number of Observations =		179
				F(4, 177) =		36,76
Source	Sum of Squares	Degrees of Freedom	Mean Square	F. Stat P-Value =		0,0000
Model	0,3565	4	0,0891	R^2 =		0,4594
Residual	0,4195	174	0,0024	Adjusted R^2 =		0,4469
Total	0,7760	178	0,0044	Root Mean Square Error =		0,0492
Portfolio 6	Coefficient	Standard Error	t-Statistic	P-Value	95% Confidence Interval	
JSE ALSI Excess Return	0,9582	0,0838	11,44	0,0000	0,7928	1,1235
ALBI Excess Return	-0,0260	0,1869	-0,14	0,8890	-0,3950	0,3429
Market-Book	0,0077	0,0073	1,05	0,2960	-0,0068	0,0222
MOM	0,0008	0,0003	2,51	0,0130	0,0002	0,0014
Constant (a)	-0,0058	0,0038	-1,55	0,1240	-0,0133	0,0016