A COMPUTER PROGRAM FOR THE ANALYSIS AND PLOTTING OF OCEAN WAVE REFRACTION DIAGRAMS

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by

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ABSTRACT.

The writer introduces the subject with a brief history of wave refraction analyses.

An alogrithm is developed for the computation of wave orthogonal progression, and hence a computer program is developed.

Various examples are investigated to verify the solutions obtained by the program.

The writer's opinions on the applications and interpretations of refraction diagrams in practical cases are presented.

The writer evaluates the program, and gives a method for modifying the program to incorporate a study area of almost limitless size.

A comprehensive User's Manual, documented according to the A.S.C.E. Engineering Computer Program Documentation Standard, is appended.

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NOMENCLATURE.

UPPER CASE CHARACTERS

H	wave height
K_s	shoaling coefficient of wave height
K_r	refraction coefficient of wave height
L	wave length
N	total number of nodes in a study area
P	energy flow per unit length of wave front
$R(t)$	instantaneous radius of curvature of orthogonal
T	wave period

LOWER CASE CHARACTERS

b	lane width between orthogonals
c	wave celerity
c_g	wave group velocity
g	acceleration of gravity
h	still water depth
$h(x,y)$	general depth in the grid system
$h(i,j)$	general nodal depth in the grid system
h_c	critical water depth
l_x	length of x-axis of study area
l_y	length of y-axis of study area
n	wave number
$n(t)$	instanteneous axis in direction of wave front
n_x	number of nodes along the x-axis
n_y	number of nodes along the y-axis
s	grid mesh spacing
$s(t)$	instantaneous axis in direction of orthogonal progression
t	time of orthogonal travel

GREEK CHARACTERS

θ	orthogonal direction
Δz	increment size
λ_x	proportion of s of which n value of point (n,y) is greater than a value of point (i,j)
λy	proportion of s of which y value of point (x,y) is greater than y value of point (i,j)

ϕ	compass bearing of x-axis
β	compass bearing of orthogonal direction
ψ	tidal shift

SECTION 1
INTRODUCTION

1.1 WAVE REFRACTION PHENOMENA

When an undisturbed wave train is travelling in deep water, it consists of a series of parallel wave crests, or wave fronts, moving at constant speed.

A wave orthogonal, or wave ray, is the path that a theoretical point, on a wave crest, would follow as the wave progressed, given that the point moves perpendicular to the wave fronts.

As a wave enters shallowing water the wave celerity (wave speed) decreases.

Consider the hypothetical case of a wave train approaching a beach, with parallel and straight depth contours, with the initial wave fronts parallel to the contours. It is evident that all points on a specific wave front shall experience the same depth effects as the wave approaches the beach, thus the wave would retard uniformly along the wave front. The wave fronts would remain parallel but would become more closely spaced. A set of wave orthogonals would thus also remain parallel.

Let us now consider the case of a wave train approaching the same beach at an oblique angle. It is apparent that if one considers an arbitrary section of a wave front in shallow water the inshore part shall be travelling slower than the offshore part, by virtue of the shallower depth, consequently the wave front shall tend to rotate and become parallel to the contours, similarly the orthogonals shall tend to become perpendicular to the shoreline.

As a general rule it can be said that for any wave travelling through shoaling water, if depth is reducing, the wave front shall approach parallelism, and the orthogonals shall approach perpendicularity with the contours.

1.2 REFRACTION DIAGRAMS

A refraction diagram is a diagram which shows the path followed by a set of wave orthogonals over a scaled plan of a specific area of water.

The orthogonals generally have selected starting positions on a wave of set period and deep water wave direction.

A refinement of the above mentioned refraction diagram is a diagram which has the cumulative time of travel, from the starting point, indicated on the orthogonals.

Refraction diagrams often have arbitrary wave fronts shown on the diagram.

1.3 THE USE OF REFRACTION DIAGRAMS

Generally refraction diagrams are used to determine wave conditions at a site of interest resulting from prevailing deep water wave conditions in the area.

The requisite data for a single refraction analysis (in a study area of known bottom topography) is the deep water wave direction, deep water wave height, and period of an incident wave.

The output of the analysis is the resultant wave height, or heights, and wave direction, or directions, at an arbitrary point in the study area.

It should be noted that owing to refraction phenomenon over various topographical features of the sea bed, (such as reefs), a single incident deep water wave train can result in a superposition of two or more wave trains, of different heights and directions.

Refraction diagrams which indicate cumulative time, as described in Section 1.2, can be used to determine time taken for a wave to travel between two points in the study area.

1.4 DETERMINATION OF REFRACTION DIAGRAMS

Two methods are commonly used for obtaining refraction diagrams, these are; (a) manual graphical methods, and (b) computer numerical methods.

(a) Manual graphical methods

The general method currently employed is as follows:

(i) Discrete contours, ignoring minor bottom irregularities, for the study area are established.
(ii) Mid-contour lines are drawn in on the plan.
(iii) The sea bottom is then assumed to be a set of horizontal steps with vertical rises occurring at the mid-contour lines.
(iv) Snell's law is applied directly to the orthogonals, with a discrete angle change occurring at each mid-contour line.

The application of this method is discussed in detail in the U.S. Army C.E.R.C. Shore Protection Manual (29).

(b) Computer numerical methods

The general method employed in computer analysis is to numerically calculate the progression of an orthogonal by incremental progression, with orthogonal curvature being determined for each increment.

To determine the curvature of an orthogonal at a point in the study area, it is necessary to ascertain, directly or indirectly, the rate of change of wave velocity with respect to distance travelled along the wave front (see Section 2.3). This derivative is most easily determined via the bed slopes, in two vertical perpendicular planes, at the point being considered.

The normal method of determining the bed slopes at a point in the study area, is to set up a rectangular grid system with the depth values given at each grid node. The bed slopes can thus be determined by fitting a surface through the relevant grid points.

The path of an orthogonal can thus be ascertained by analytical geometry, and can be represented on a scaled plan by use of an incremental plotter.

1.5 THE RELATIVE MERITS OF COMPUTER REFRACTION DIAGRAMS AND MANUAL REFRACTION DIAGRAMS

Generally a refraction sutdy involves the ascertainment of several individual refraction diagrams for a single study area.

The features of (i) the manual method and (ii) the computer method are as sketched below.

(i) Manual methods

(a) Preparation work

There is little preparation work needed for this method, namely a suitable contour map must be established.

(b) Construction

The actual construction of the refraction diagrams is a tedious and time consuming operation.

(c) Accuracy

The method lacks the accuracy that can be provided by a computer analysis.

(d) Complications

Complications occur when an orthogonal approaches parallelism with the contours.

(ii) Computer method

(a) Preparation work

There is considerable preparation work needed in establishing a grid, ascertaining nodal depth values, and physically punching the depth data. (For a practical example the number of grid nodes could be well in excess of 50 000).

(b) Construction

Once the preparation is done, the establishment of a refraction diagram is very simple and quick, the only requisite data being the incident wave characteristics, and the orthogonal starting positions.

(c) Accuracy

Computer methods can be considerably more accurate than the manual methods. It can be said that the accuracy of a refraction diagram, that can be conveniently obtained using a reasonable refraction program, exceeds the accuracy that the general wave theories model the true life situations.

(d) Complications

An efficient computer program which determines ray progression using orthogonal curvatures, as discussed in Section 1.3, is not subject to complications with various bottom topographies. Such as in (i) above.

It can be concluded that manual refraction diagrams are of considerable use when one or two refraction diagrams of an area are desired, especially if no great accuracy is required, however if a large study of the area is required, computer solutions have an undoubted advantage.

1.6 THE HISTORY OF WAVE REFRACTION ANALYSIS

The decrease in wave celerity resulting from decreasing water depth can be considered to be analogous to the decrease in the speed of light resulting from an increase in optical density of the transmitting medium through which the light is travelling.

In 1942 it was first suggested by Professor M.P. O'Brien (21) that this analogy may be used for the solution of water wave refraction caused by changing water depth.

Though O'Brien only published this in 1942, he had had graduate students, who were working under him, constructing refraction diagrams for complicated hydrography as far back as 1937.

In 1948, Johnson, O'Brien and Isaacs (15) developed graphical methods for application to the complex ocean bottoms of nature. The solutions are based upon the condition that Snell's law correctly predicts the refraction of water gravity waves.

The assumption that Snell's law can be applied was verified mathematically, for linear wave theory in shallow water,* by Lowell (18) in 1949.

Experimental verification of Snell's law in shallow water was established by Chien (5) in 1954.

Chien conducted his experiment in a 4 ft wide, 20 ft long, and 5 inch deep ripple tank. A plunger-type wave generator was located at the one end of the tank. The apparatus is shown in Figure 1.1.

A beach was made of a ¼ inch transparent lucite plate. The wave and beach characteristics are shown in Table 1.1.

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* Shallow water is considered as the region in which the wave celerity is given by $c = \sqrt{gh}$, where g is the acceleration of gravity, and h is the water depth. Generally this case is assumed when h/L is less than 0,05, where L is the wave length.

(Scale ratio of the model beach: length scale, 1/500; time scale, 1/22.4)

Run number	Beach			Waves				
	Still water depth	Orientation	Profile	Description	Wave period		Wave length in deep water	
					Model	Prototpye	Model	Prototype
	ft				sec	sec	sec	sec
1-A	0.068	50	Slightly concave upward	Uniform waves	0.83	18.5	3.50	1750
1-B					0.60	13.4	1.84	920
1-C					0.40	9.8	0.80	400
2-A	0.069	34	Essentially at 1/40 slope	Uniform waves	0.81	18.1	3.36	1680
2-B					0.60	13.5	1.86	930
2-C					0.39	8.7	0.77	385
2-D				Non-uniform waves	0.30	6.6	0.45	223
					0.53	11.7	1.42	710
3-A	0.069	15	Essentially at 1/40 slope	Uniform waves	0.77	17.2	3.03	1515
3-B					0.59	13.1	1.76	880
3-C					0.39	8.8	0.79	395

Table 1.1: Characteristics of waves and beaches, model tests by Chien (5)

Runs A represent long period uniform waves, Runs B represent intermediate period uniform waves, and Runs C represent short period uniform waves.

The non uniform waves, Run D, were generated by continuously and quickly increasing and reducing the speed of the wave generator motor.

The experiments were photographed at a speed of 28 frames per second, with a time clock being photographed for time reference.

The photographs were analysed to determine scale factors and wave periods.

For the non uniform waves, the time intervals at either side of the wave, T_1 and T_2, were measured, and the period T, of the relevant wave was determined by linear interpolation as follows;

$$T = \frac{T_1 T_2}{\frac{1}{2}(T_1 + T_2)} \quad .$$

The refraction angles, both measured and calculated were plotted. The results of the experiments are shown in Figure 1.2.

Chien concluded that "within the limits of accuracy of the experiment, Snell's law was verified, except that possibly the refraction angle was slightly less than that indicated by Snell's law for large wave periods".

In 1956 the investigations of Chien were extended by Ralls (25).

Ralls used the same apparatus that was used by Chien (see Figure 1.1) but used different beach slopes, namely 1:20; 1:40; and 1:60, and extended the investigation to include intermediate water depth zone.*

Ralls concluded that the study essentially verified Snell's law on refraction of uniform, long crested waves in the region $0{,}10 \leq d/L \leq 0{,}70$.

Ralls' graphical results for intermediate depths are shown in Figure 1.3.

In 1957 Wiegel and Arnold (31) performed model studies of wave refraction of uniform period waves travelling over various submerged shoals. The shoals had constant slopes with straight parallel contours. Slopes investigated were: vertical; 1:2,47; 1:5; and 1:10,62. Incident angles from 10° to 70° were investigated.

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* The intermediate depth zone is the zone between the deep water - where a change of depth does not affect the wave celerity - and shallow water, as described before. Generally the intermediate zone is assumed when h/L is between 0,5 and 0,05.

The wave tank used was 150 ft long by 64 ft wide by 2,5 ft deep, which is considerably larger than that used by Chien and Ralls.

It was found that Snell's law was valid over a large range of wave periods even for the case of refraction over a vertical step.

Wiegel and Arnold's graphical results are shown in Figure 1.4.

In the meanwhile work had progressed in the establishment of manual methods for constructing refraction diagrams.

Early refraction diagrams were drawn by sketching successive wave fronts. Each projection distance was based on the wave speed at the corresponding crest position, projection being perpendicular to the wave front direction.

In 1948 Johnson, O'Brien and Isaacs (15) greatly improved the construction of refraction diagrams, by devising a method for the direct construction of orthogonals, hence eliminating the need for prior sketching of the wave fronts.

This method was based on an approximate formula, which was derived from refraction geometry. The equation that was derived for determining the changes of angles experienced by an orthogonal travelling between two discrete contours was;

$$\Delta\alpha = (\Delta L/L_{ave}).\tan \alpha,$$

where α is the angle between the wave crest and the depth contour, and L is the wave length.

In 1952 Arthur, Munk and Isaacs (2) presented a modified procedure. They approached the problem by considering the curvature of an orthogonal by solving the governing differential equations.

The final equation derived was;

$$\Delta\alpha = \sin^{-1}\left[(1 + \Delta L/L)\sin\alpha\right] - \alpha .$$

This method yielded more accurate results.

The real significance of this approach however, is that it provided a stepping stone to the development of algorithms suited to computer usage.

Essentially all the significant computer programs developed have calculated refraction diagrams by solving the governing differential equations to establish the curvature of an orthogonal increment.

In November 1952, Munk and Arthur (20) further investigated the governing differential equations to establish wave intensity along a refracted ray. They developed a theory that predicts the relative wave height (neglecting shoaling) along an orthogonal, knowing the orthogonal path and the wave celerities both on and near the orthogonals.

Some programs have been written to include the calculation of these refraction coefficients.*

The first refraction program was developed by Griswold in 1963.

Griswold's algorithm had the following features:

(i) The study area is split up into a square $x - y$ grid system.

(ii) The increment size used was half a grid square length. Though Griswold used a constant distance increment, he pointed out the advantage of using a constant time step.

* The writer's view is that these coefficients should be carefully considered, especially when used in areas with complex bottom topography. The writer's opinions on interpretation of refraction diagrams is given in Section 7.

The advantage of this method is that increment step lengths decrease in shallow water, where curvatures tend to get greater. A greater accuracy is thus given in the regions where it is needed.

Griswold also pointed out that an advantage of having a constant time step is that wave front positions could be determined by joining points on adjacent orthogonals which had had equal times of travel. The writer, however, would like to point out that the orthogonal position at any arbitrary selected time of travel can be determined by interpolation, regardless of whether a constant time or constant distance increment is used. For a method of performing this operation the reader is referred to Section 5.8.

(iii) The curvature at any point was calculated by converting depth values at grid points to wave celerities and solving the governing differential equations by modelling the surface of wave celerity in the x and y directions.

The pertinent curvature equation is thus;

$$\frac{d\alpha}{ds} = \frac{1}{c}\left[\frac{dc}{dx}(\sin\alpha) - \frac{dc}{dy}(\cos\alpha)\right]^{**}.$$

(iv) Refraction coefficients were calculated along the orthogonal according to theory developed by Munk and Arthur (20).

Calculation of this coefficient essentially requires the calculation of the first and second derivative of wave celerity with respect to x and y. This is ascertained from the surface of wave celerities in the grid system.

** See Equation 2.22.

In 1964 Lepetit developed a computer program for refraction which used a constant time step increment. The major difference between Lepetit's algorithm and Griswold's is that; where Griswold had used a two dimensional array to store values of wave celerity at node points of the grid, Lepetit set up two dimensional arrays of celerity and the derivatives of wave celerity, with respect to x and y, in order to solve the governing differential equations.

Probably the best known, and most monumental, refraction program is that due to W.S. Wilson (32).*

Initially Harrison and Wilson (13) developed a computer program to calculate wave refraction in 1964. This work was extended by Wilson, and in 1966 the program was published by C.E.R.C. Locally, this program is currently used by the National Research Institute of Oceanology (branch of the C.S.I.R.), and the Fisheries Development Corporation of South Africa. Work performed by these institutes, especially the N.R.I.O., has shown that results obtained from the program indicate a very close correlation with observed ocean phenomena.

This program was the first one developed that not only calculated the orthogonal paths, but also traced them by use of an incremental plotter.

The other major development was that Wilson developed a method of calculating the curvature of the orthogonals by determining the derivatives of the water depths with respect to x and y, as opposed to determining the derivatives of the wave celerities.

Subsequent to Wilson various programs were written. The reader is referred to the works of Dobson (1967) (8), Jen (1969) (14),

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* This program has been used as a standard of comparison for the evaluation of the program developed by the writer.

Coudert and Raichlen (1970) (7), Keulegan and Harrison (1970) (16), Chao (1970) (4), Orr and Herbick (1970) (22), Collins (1972) (6), and Skovgaard, Jonsson and Bertelsen (1975) (28). The only significant development was that of the inclusion of the effects of bottom friction, this factor was included initially by Collins and subsequently by Skovgaard et al.

Generally speaking the algorithms employed by the various programs are similar. The accuracy of a specific program is generally dependent on two factors, namely (i) the accuracy with which the program models the topographical surface, and (ii) the accuracy with which the computer calculates the curvature, given that the surface is correctly modelled.

The general evaluation performed on programs is to run an analysis over a straight beach. The suitability of the example is that there is an exact solution given by Snell's Law (see Section 6.2).

The straight beach example is a good test of the second factor, mentioned above, but does not test the adaptability of the program to complicated bed topographies.

The ideal test on a program is to run an analysis on an example which has got both complex topographical features, and an exact solution. Every aspect of the program could thus be evaluated. Such examples are given by Arthur (1).

One such example is a point island on which any orthogonal shall follow the path of a logarithmic spiral (see Section 6.3).

The writer's opinion is that most published programs do not include enough verification by use of examples with exact solutions.

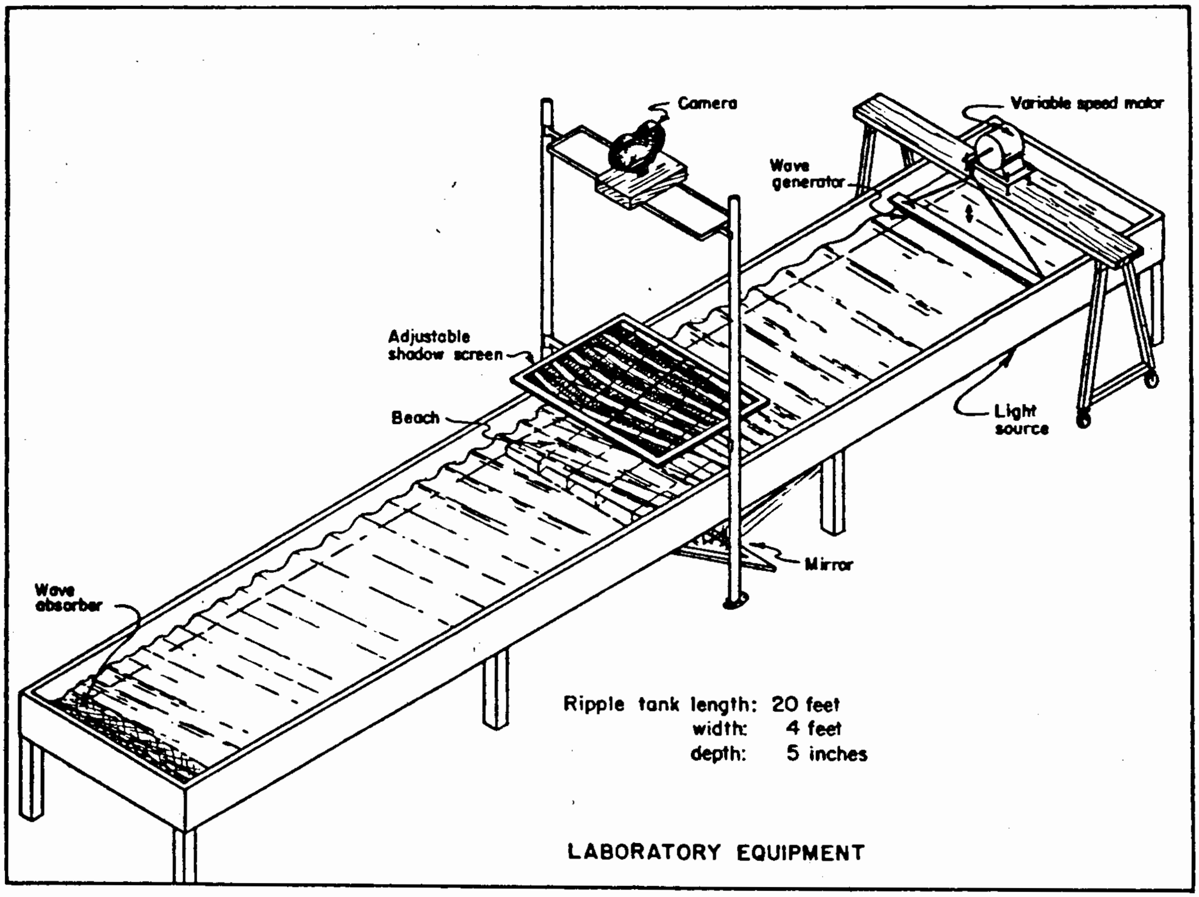
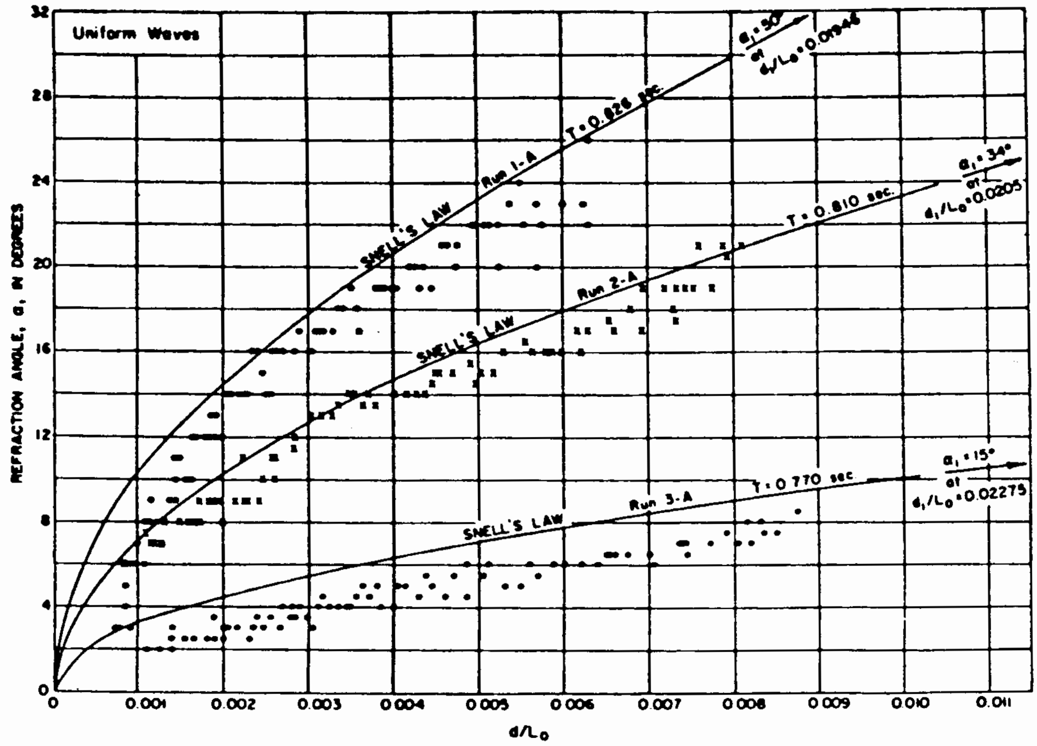
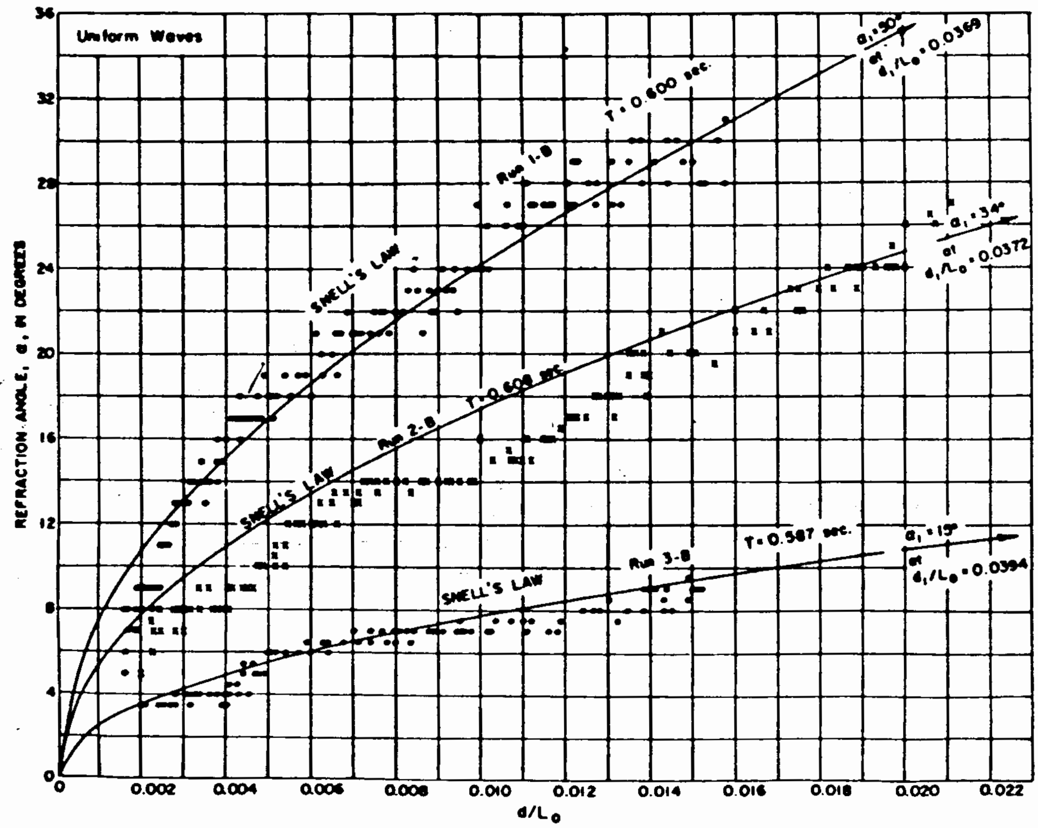


Fig. 1.1. Wave tank for model studies performed by Chien (5) and Ralls (25).

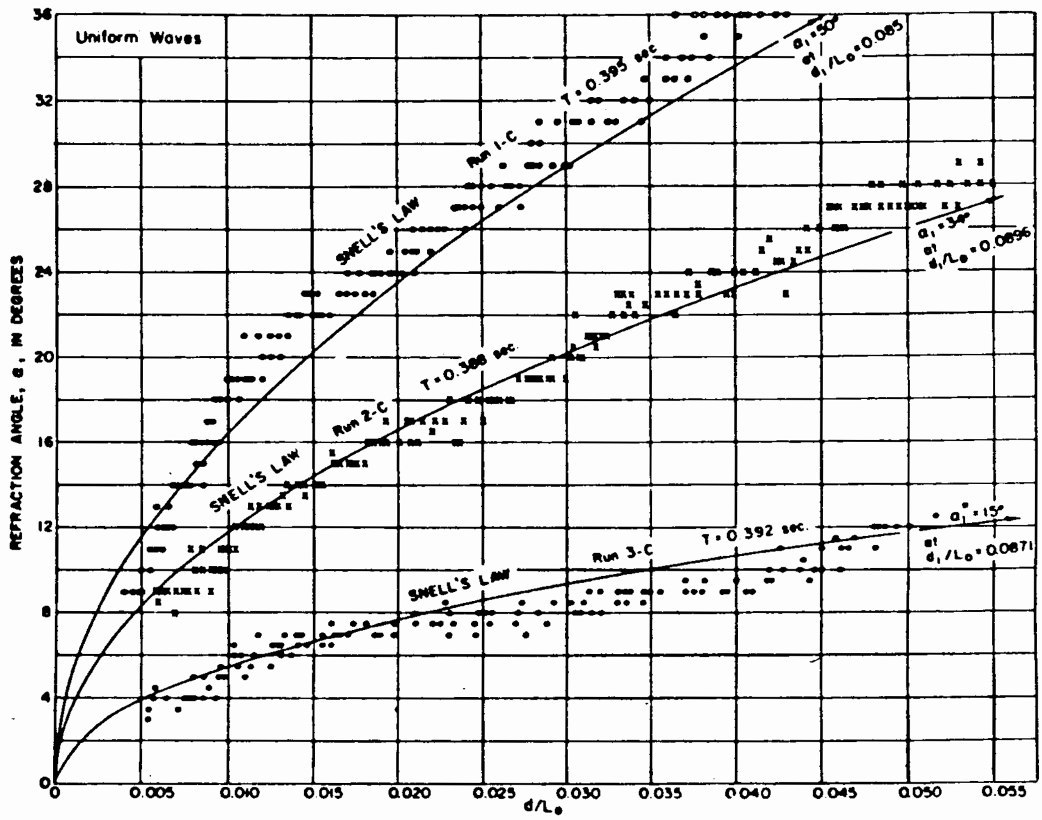


Refraction of long-period uniform waves

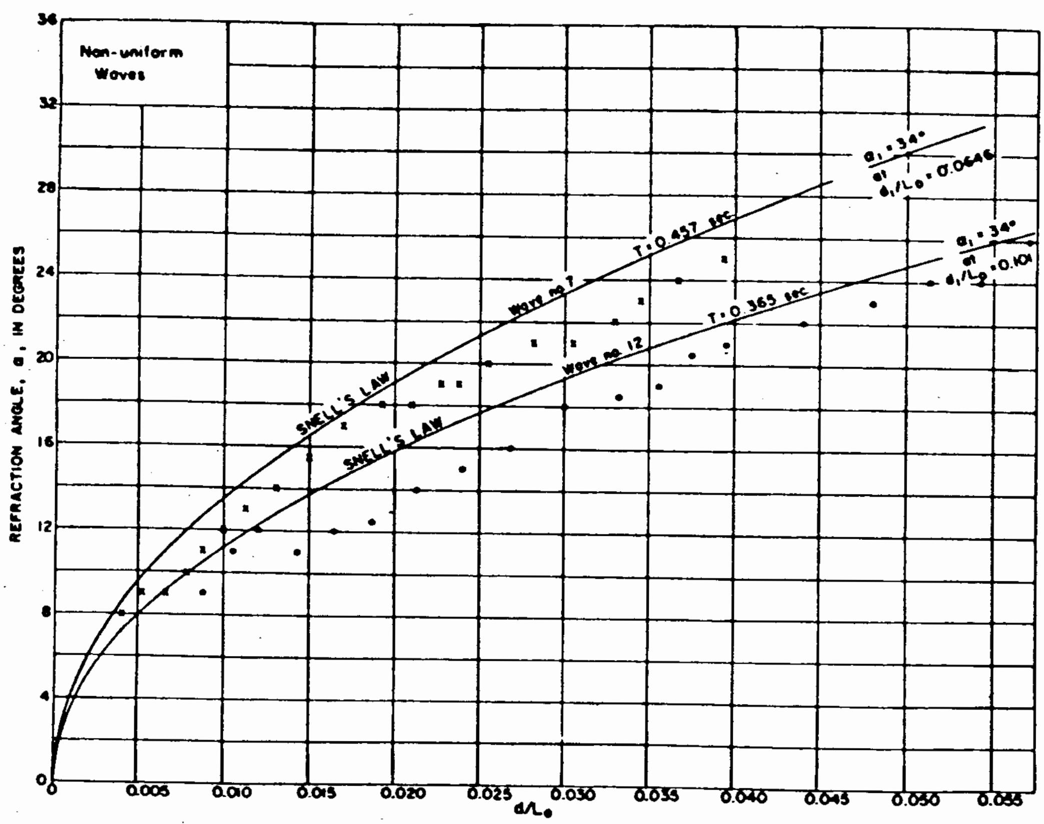


Refraction of intermediate-period uniform waves

Fig. 1.2(a): Wave refraction in a model basin. Chien (5) 1954.



Refraction of short-period uniform waves



Refraction of non-uniform waves

Fig. 1.2. Experimental results obtained by Chien (5).

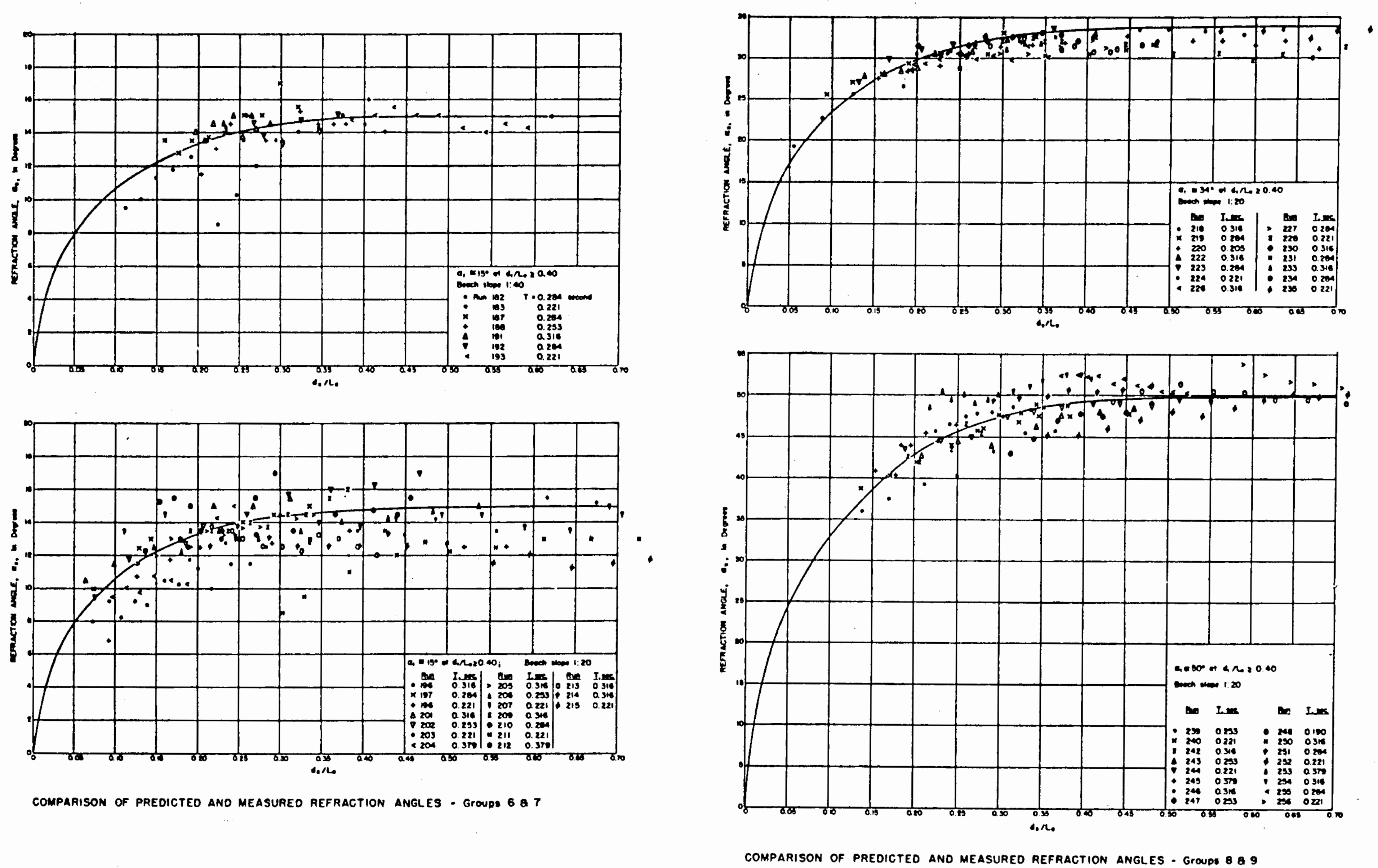


Fig. 1.3. Experimental results obtained by Ralls (25).

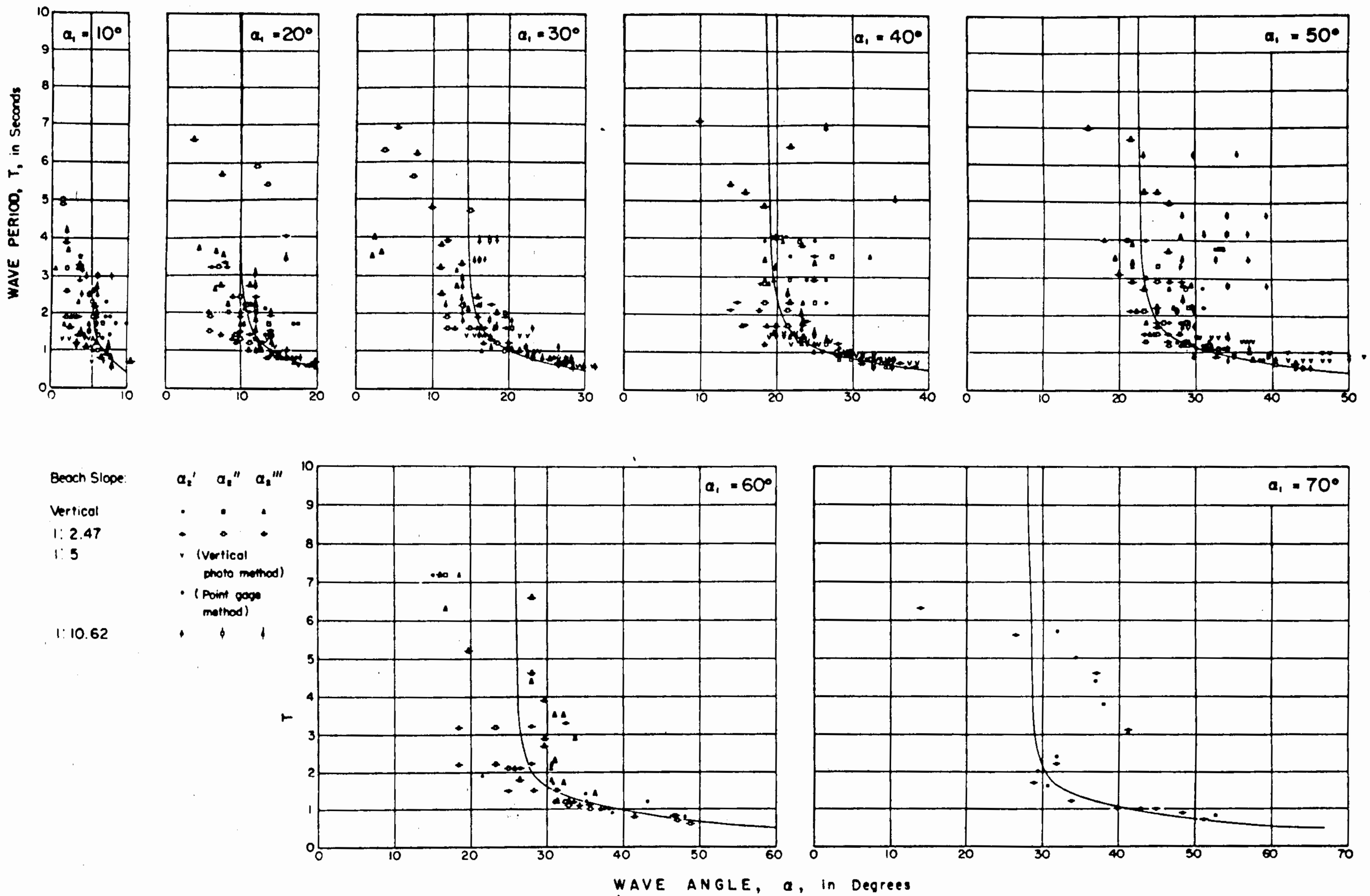


Fig. 1.4. Experimental results obtained by Wiegel and Arnold (31).

1.7 GOALS AND OBJECTIVES OF THIS THESIS*

The primary goal of this thesis is to develop a computer program to analyse and plot refraction diagrams for practical applications. The basic objectives of the program are listed below:

(i) Accuracy

The results achieved from a practical application of program must be sufficiently accurate to give a realistic representation of the natural conditions being investigated.

(ii) Output

The computer output must be in a suitable format such that the user can with ease determine the various parameters he requires.

(iii) Data

The requisite data for operating the program must be of a format such that the user can operate with maximum ease, and such that probability of erroneous data insertion is limited to a minimum.

(iv) Users Manual

A comprehensive users manual must be compiled, such that a user can, with ease, know what data is to be inserted, what the available options are, and in what format the data is to be inserted. For a user who is not accustomed to computer operating, ease of understanding the instructions is of primary importance.

(v) Internal Documentation of the Program

The program should be sufficiently internally documented to facilitate the determination of the program logic, such that a user can understand and locate the program sequences, in case he might want to make additions or alterations to the program

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* For assessment of achievement of objectives see Section 8.1.

for his own applications. This can be achieved by inserting 'comment' cards in the program.

(vi) Program Speed

It is advantageous that the program be relatively quick in operation, such that large computer expenses are avoided when running examples.

As a brief summary the ultimate goal of the thesis is to produce a well documented computer program, which can satisfactorily be used in practical applications for determining wave refraction diagrams for arbitrary prevailing deep sea wave conditions, in an area where the bottom topography is known.

SECTION 2

DEVELOPMENT OF THEORY

2.1 BASIC WAVE THEORY

A wave is a variation in water surface elevation which travels across a mass of water with a general deformed water surface as shown in Figure 2.1 below.

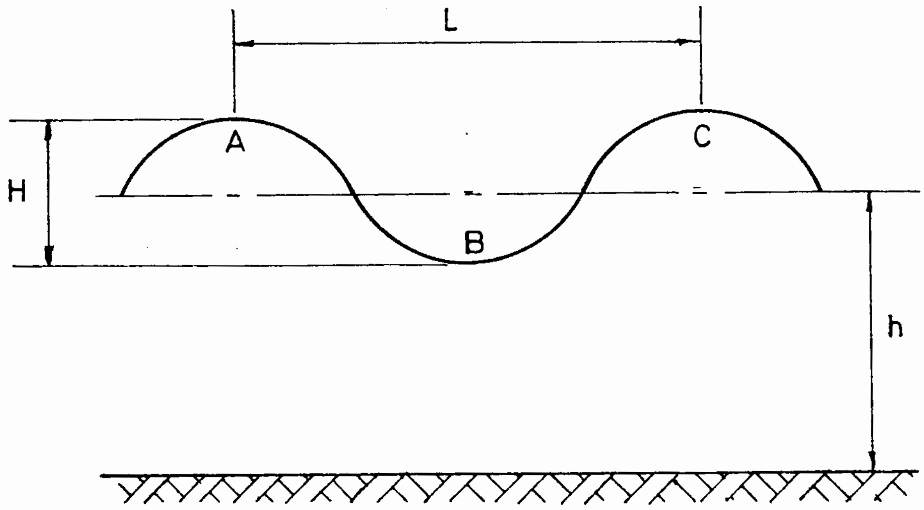


Figure 2.1: A typical wave

Points of maximum surface elevation are crests (A and C), and points of minimum surface elevation are troughs (B). The speed of travel of the wave is the wave celerity, c.

Essentially we do not deal with individual waves, but with wave trains, consisting of a series of similar waves. In a wave train the distance between adjacent crests (or troughs) is the wave length, L, and the time taken for a wave to travel a distance of one wave lenth is the period, T. It can thus be stated that;

$$L = c\,T. \tag{2.1}$$

The wave height, H, is the difference in elevation between the crest and the trough, and the water depth, h, is the undisturbed or still water depth, as shown in Figure 2.1.

The general wave used to simulate ocean waves is the Airy (first order) wave. According to the Airy wave theory, the wave celerity is given by;

$$c = \left[\frac{gL}{2\pi} \tanh \frac{2\pi h}{L}\right]^{\frac{1}{2}} . \qquad (2.2)$$

substitution of equation (2.1) in equation (2.2) yields;

$$c = \left[\frac{gT}{2\pi} \tanh \frac{2\pi h}{cT}\right] . \qquad (2.3)$$

As the period remains constant for a wave train, it can thus be seen that the wave celerity decreases with decreasing water depth. However, the tanh (x) function approaches unity asymptotically with increasing x, and it is thus generally considered that a wave is in deep water when the water depth is greater than half the wave length. The deep water wave length L_o is given by;

$$L_o = \frac{gT^2}{2\pi} . \qquad (2.4)$$

When h is equal to $L_o/2$, tanh $(2\pi h/L)$ is equal to 0,996. Hence we can assume the wave train shall be unaffected by the bottom topography in deep water. We can thus say that deep water occurs when,

$$h > \frac{gT^2}{4\pi} . \qquad (2.5)$$

2.2 THE THEORY OF REFRACTION DIAGRAMS

The main use of a refraction diagram is for determing wave conditions at a site of interest for prevailing deep water wave conditions in that area.

If we have a deep water wave of set period and wave direction, we can plot the path of a selected set of orthogonals with starting points on an arbitrary wave front in deep water. These orthogonals naturally represent the direction in which the wave is travelling, hence the direction of the wave at a specific point can be determined.

It can be assumed that there is no energy flow sideways along a wave front, and that all energy flows in the direction the wave is travelling - in the same direction as the orthogonals - thus we can assume the amount of energy flow between two adjacent wave orthogonals shall remain constant. Thus as adjacent orthogonals converge or diverge the energy concentration shall increase or decrease.

Quantitatively the energy flow per unit length of wave front, P, is given by;

$$P = wH^2 c_g/8, \tag{2.6}$$

where w is the specific weight of sea water, and c_g is the group velocity of the wave, as defined below. In the S.I. unit system P shall be in watts per metre.

The group velocity is the speed at which the wave energy progresses, and is generally less than the actual wave celerity. The group velocity is given by the expression,

$$c_g = nc, \tag{2.7}$$

where n is given by the expression,

$$n = 0{,}5 + \frac{2\pi h/L}{\sinh\ (4\pi h/L)} \tag{2.8}$$

In can thus be seen that the group velocity is equal to half the wave celerity in deep water, and is theoretically equal to the wave celerity in zero water depth.

Considering a specific lane between adjacent orthogonals, we specify initial wave height, H_o, and initial spacing between orthogonals, as length measured along the wave front, b_o. At a certain subsequent point between the orthogonals we specify a corresponding wave height and spacing of H, and b, respectively. As the total energy flow between orthogonals remains constant we can thus say that;

$$P_o \cdot b_o = P_1 . b_1 \quad . \qquad (2.9)$$

By combining equations (2.6) and (2.9) we can derive that;

$$\frac{(wH_o^2 \ cg_o/8)}{(wH_1^2 \ cg_1/8)} = \frac{b_1}{b_o} \quad , \qquad (2.10)$$

and hence;

$$H_1 = \sqrt{\frac{cg_o}{cg_1}} \cdot \sqrt{\frac{b_o}{b_1}} \cdot H_o \quad . \qquad (2.11)$$

Alternatively,

$$H_1 = K_s \cdot K_r \cdot H_o \quad , \qquad (2.12)$$

where K_s is a shoaling coefficient which can be determined from the shoaling wave charts in Appendix B, or by other means, and K_r is the refraction coefficient which can be determined from the refraction diagram.

2.3 THE DETERMINATION OF ORTHOGONAL CURVATURE

Consider a point on a wave crest at any time (t) in the x - y plane as shown in Figure 2.2.

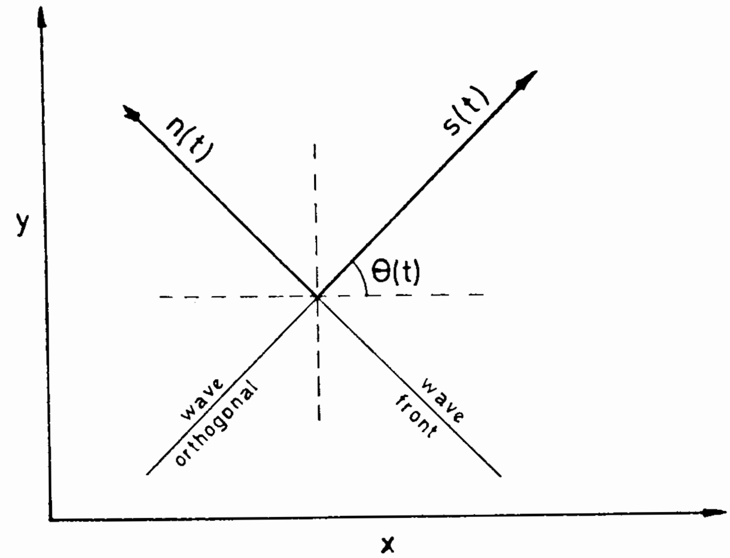


Figure 2.2 : Instantaneous orthogonal axis-system

We set up an instanteous orthogonal axis-system n(t) and s(t), where s(t) is the direction of the wave orthogonal in the direction the wave is travelling and n(t) is the direction of the wave crest at ninety degrees anti-clockwise from the s(t) direction. Let $\theta(t)$ be the angle between the x axis and the s(t) axis, measured anti-clockwise from the x axis. Hence;

$$\frac{dx}{ds(t)} = \cos\theta(t), \tag{2.13}$$

$$\frac{dx}{dn(t)} = -\sin\theta(t), \tag{2.14}$$

$$\frac{dy}{ds(t)} = \sin\theta(t), \tag{2.15}$$

and $$\frac{dy}{dn(t)} = \cos\theta(t). \tag{2.16}$$

If the wave celerity increases in the n(t) direction, (i.e. depth increases in the n(t) direction), the wave will swing round in a clockwise direction and will cause $\theta(t)$ to decrease.

Consider a time increment, dt, and an increase in wave celerity with respect to distance along the wave front in the n(t) direction, dc/dn(t), as shown in Figure 2.3, where point A moves to point B.

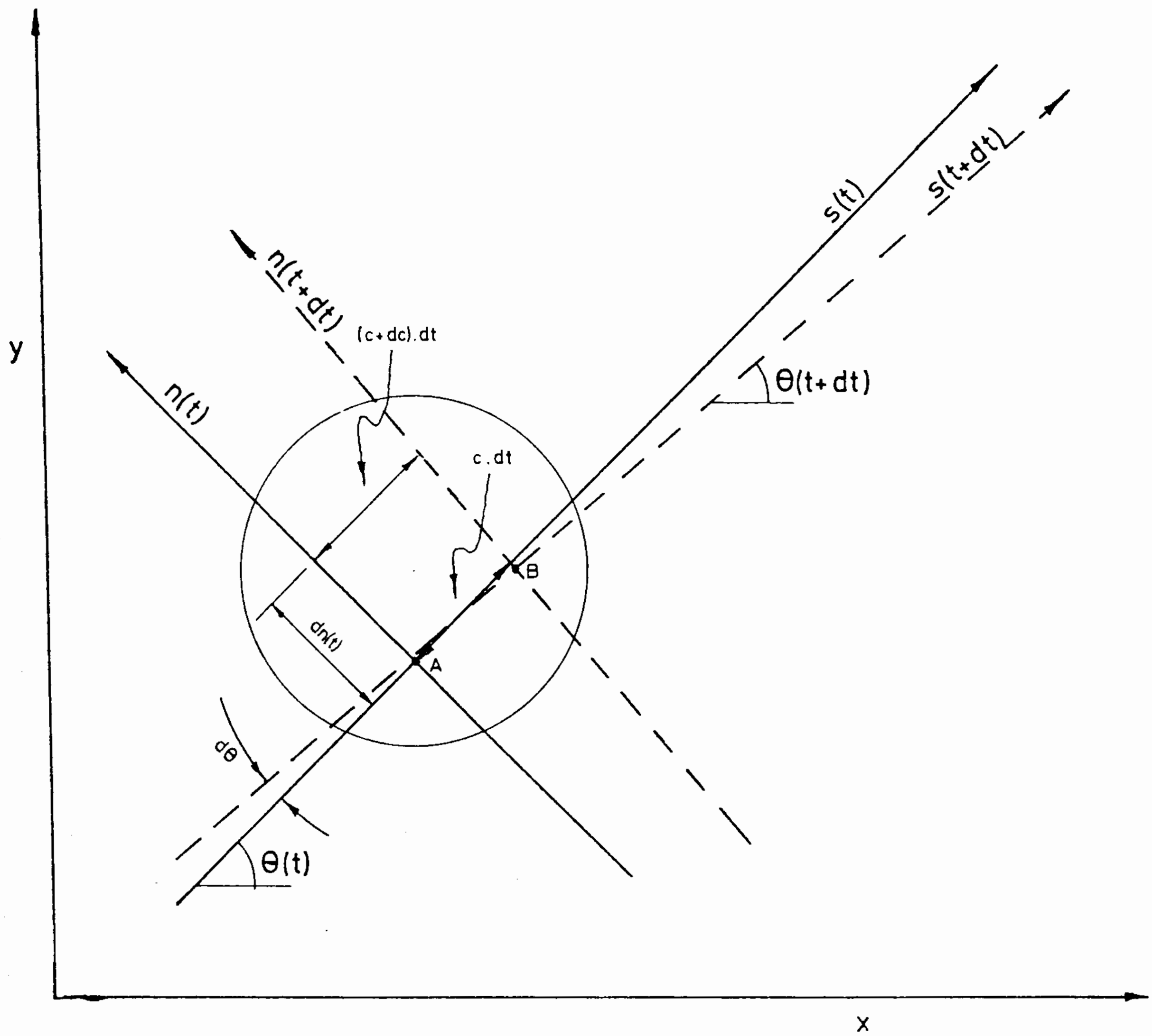


Figure 2.3: Increment of wave progression corresponding to time increment dt.

Enlarging circled area we get Figure 2.4.

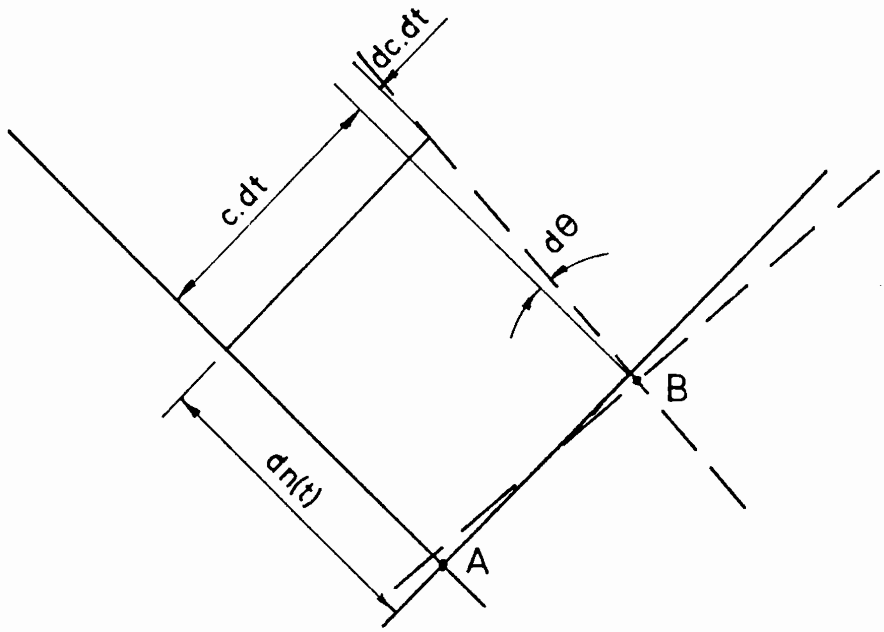


Figure 2.4: Enlargement of Figure 2.3

As Θ is decreasing, dΘ is negative, hence from Figure 2.4 we get;

$$- d\Theta = \frac{dc \cdot dt}{dn(t)} \quad . \qquad (2.17)$$

Hence the curvature, dΘ/ds, is given by;

$$\frac{d\Theta}{ds(t)} = - \frac{dc \cdot dt}{dn(t)\ ds(t)} \quad . \qquad (2.18)$$

But,

$$ds(t) = c\ dt. \qquad (2.19)$$

Substituting eqtn. (2.19) in eqtn. (2.18) we get;

$$\frac{d\Theta}{ds(t)} = \frac{1}{c} \frac{dc}{dn(t)} \quad . \qquad (2.20)$$

The Chain rule states that;

$$\frac{d}{dn(t)} = \frac{dx}{dn(t)} \cdot \frac{\partial}{\partial n} + \frac{dy}{dn(t)} \cdot \frac{\partial}{\partial y} .$$

From equations (2.14) and (2.16) we can thus see that;

$$\frac{d}{dn(t)} = - \sin\theta \frac{\partial}{\partial x} + \cos\theta \frac{\partial}{\partial y} . \qquad (2.21)$$

Combining equations (2.21) and (2.20) we get that;

$$\frac{d\theta}{ds(t)} = \frac{1}{c} \quad \sin\theta \cdot \frac{\partial c}{\partial x} - \cos\theta \cdot \frac{\partial c}{\partial y} . \qquad (2.22)$$

Hence we can state that the curvature of the wave orthogonal, $d\theta/ds$, depends on the wave celerity, c, and the gradient of the wave velocity in the x and y directions, $\partial c/\partial x$ and $\partial c/\partial y$ respectively.

Restating equation (2.3) the wave celerity is given by;

$$c = \frac{gT}{2\pi} \tanh \frac{2\pi h}{cT} . \qquad (2.23)$$

Differentiation of this equation with respect to water depth, h, yields;

$$\frac{dc}{dh} = \frac{\frac{g}{c} \cdot \text{sech}^2 \left(\frac{2\pi h}{cT}\right)}{1 + \frac{gh}{c^2} \text{sech}^2 \left(\frac{2\pi h}{cT}\right)} . \qquad (2.24)^*$$

As c and $\frac{dc}{dh}$ are functions of h alone, (g and T are constants), and h, c, $\frac{\partial h}{\partial x}$ and $\frac{dc}{dh}$ are all continuous, we can state that;

$$\frac{\partial c}{\partial x} = \frac{\partial h}{\partial x} \cdot \frac{dc}{dh} . \qquad (2.24)$$

* For derivation of Eqtn. (2.24), see Appendix D.

That is the rate of change of wave celerity with respect to x, (keeping y constant), equals the rate of change of depth with respect to x, (keeping y constant), times the rate of change of wave celerity with respect to depth.

Similarly,

$$\frac{\partial c}{\partial y} = \frac{\partial h}{\partial y} \cdot \frac{dc}{dh} \quad .$$

We can thus rewrite equation (2.22) as,

$$\frac{d\theta}{ds(t)} = \frac{1}{c}\left[\sin\theta \, \frac{\partial h}{\partial x} \cdot \frac{dc}{dh} - \cos\theta \, \frac{\partial h}{\partial y} \cdot \frac{dc}{dh}\right] , \quad (2.25)$$

and hence,

$$\frac{d\theta}{ds(t)} = \frac{1}{c}\,\frac{dc}{dh}\left[\sin\theta \, \frac{\partial h}{\partial x} - \cos\theta \, \frac{\partial h}{\partial y}\right] . \quad (2.26)$$

Substituting eqtn. (2.24) into eqtn. (2.26) gives our equation for curvature at time t as;

$$\frac{d\theta}{ds(t)} = \frac{\frac{g}{c^2}\,\mathrm{sech}^2\left(\frac{2\pi h}{cT}\right)}{1 + \frac{gh}{c^2}\,\mathrm{sech}^2\left(\frac{2\pi h}{cT}\right)} \cdot \left[\sin\theta \, \frac{\partial h}{\partial x} - \cos\theta \frac{\partial h}{\partial y}\right] . \quad (2.27)$$

As c is a function of g, h and T, we can say that the curvature of a wave orthogonal in the x - y plane is dependent on:

1. The acceleration of gravity, g.
2. The water depth at that point, h.
3. The wave period, T.
4. The wave direction at that point,
5. The rate of change of depth with respect to distance travelled in the x and y directions, $\partial h/\partial x$ and $\partial h/\partial y$ respectively. (These two values are in fact the negative bed slopes in the x and y directions respectively).

2.4 THE GEOMETRY OF INCREMENTAL ORTHOGONAL PROGRESSION

Assume we have a wave of known period travelling in the x - y plane, for which we know the water depth and negative bed slopes in the x and y directions at any point.

Consider a small increment in wave orthogonal progression distance of length Δz, corresponding to a time increment of Δt, and an orthogonal rotation of $\Delta\theta$. Let the increment have starting conditions t_o, x_o, y_o and θ_o; and final conditions t_2, x_2, y_2 and θ_2; with central point x_1, y_1 and θ_1. As the increment is small we can assume it has constant curvature, $d\theta/ds(t_o)$.

As shown in equation (2.27) the curvature at a point can be determined from our known information. Hence the curvature of the increment can be approximated by the value of $d\theta/ds$ at the central point (x_1, y_1).

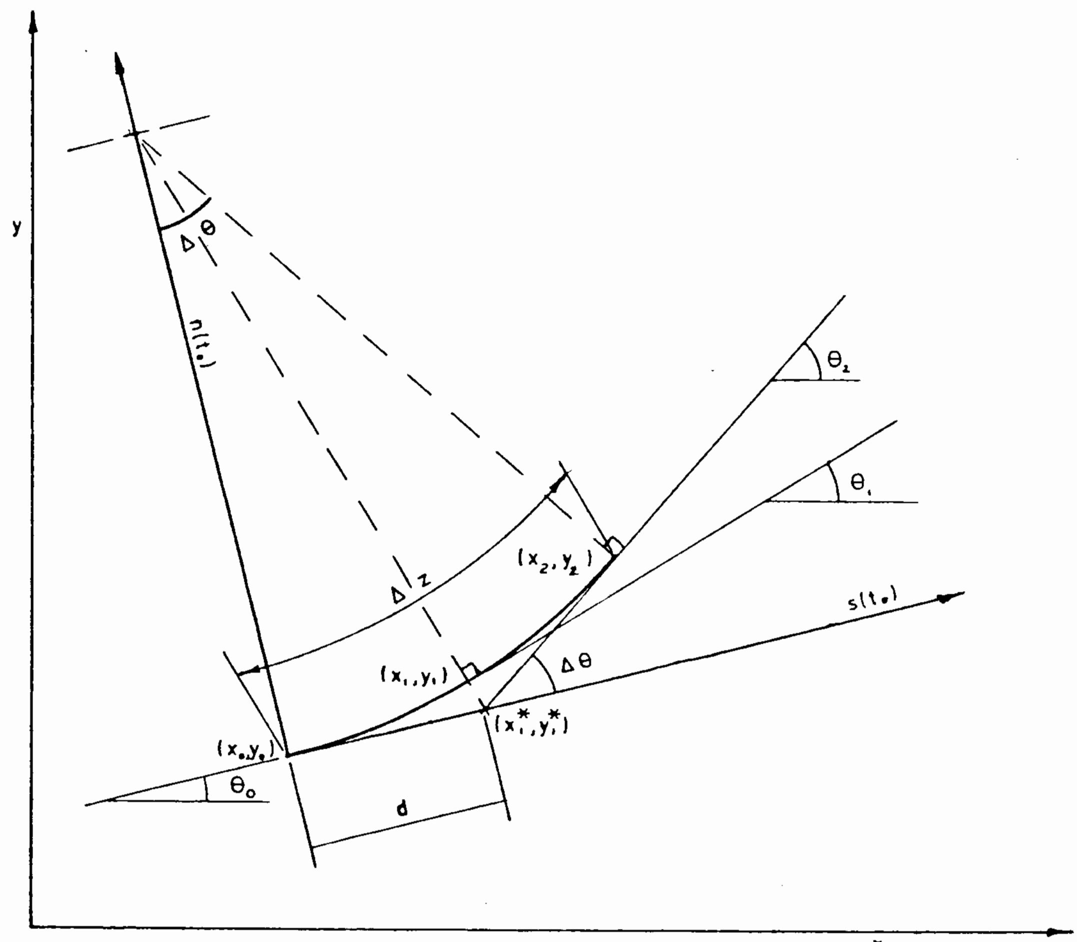


Figure 2.5: A typical increment

Considering Figure 2.5, as $\Delta\theta$ is very small we may make the following assumptions for the purpose of calculating the curvature of the increment:

$$\theta_1 = \theta_o, \tag{2.28}$$

$$x_1 = x_1^*, \tag{2.29a}$$

$$y_1 = y_1^*, \tag{2.29b}$$

$$\text{and} \quad d = \Delta z/2. \tag{2.30}$$

Hence,

$$x_1 = x_o + \tfrac{1}{2}\,\Delta z \cos\theta_o, \tag{2.31}$$

and

$$y_1 = y_o + \tfrac{1}{2}\,\Delta z \sin\theta_o. \tag{2.32}$$

Hence the curvature of our increment, $d\theta/ds(t_o)$, can be determined by using the appropriate values for point (x_1, y_1), as given in eqtns (2.31) and (2.32), in equation (2.27).

We can thus determine the radius of the circular increment according to;

$$R(t_o) = \frac{1}{d\theta/ds(t_o)}. \tag{2.33}$$

The change of direction of the orthogonal is given by;

$$\Delta\theta = \frac{d\theta}{ds(t_o)} \cdot \Delta z, \tag{2.34}$$

and hence;

$$\theta_2 = \theta_o + \frac{d\theta}{ds(t_o)} \cdot \Delta z. \tag{2.35}$$

The time taken for the orthogonal to travel the distance of the increment shall be;

$$\Delta t = \frac{\Delta z}{c_1} , \qquad (2.36)$$

where c_1 is the speed of the wave for a depth corresponding to the depth at point (x_1, y_1), as given in equations (2.31) and (2.32). Hence;

$$t_2 = t_o + \frac{\Delta z}{c_1} . \qquad (2.37)$$

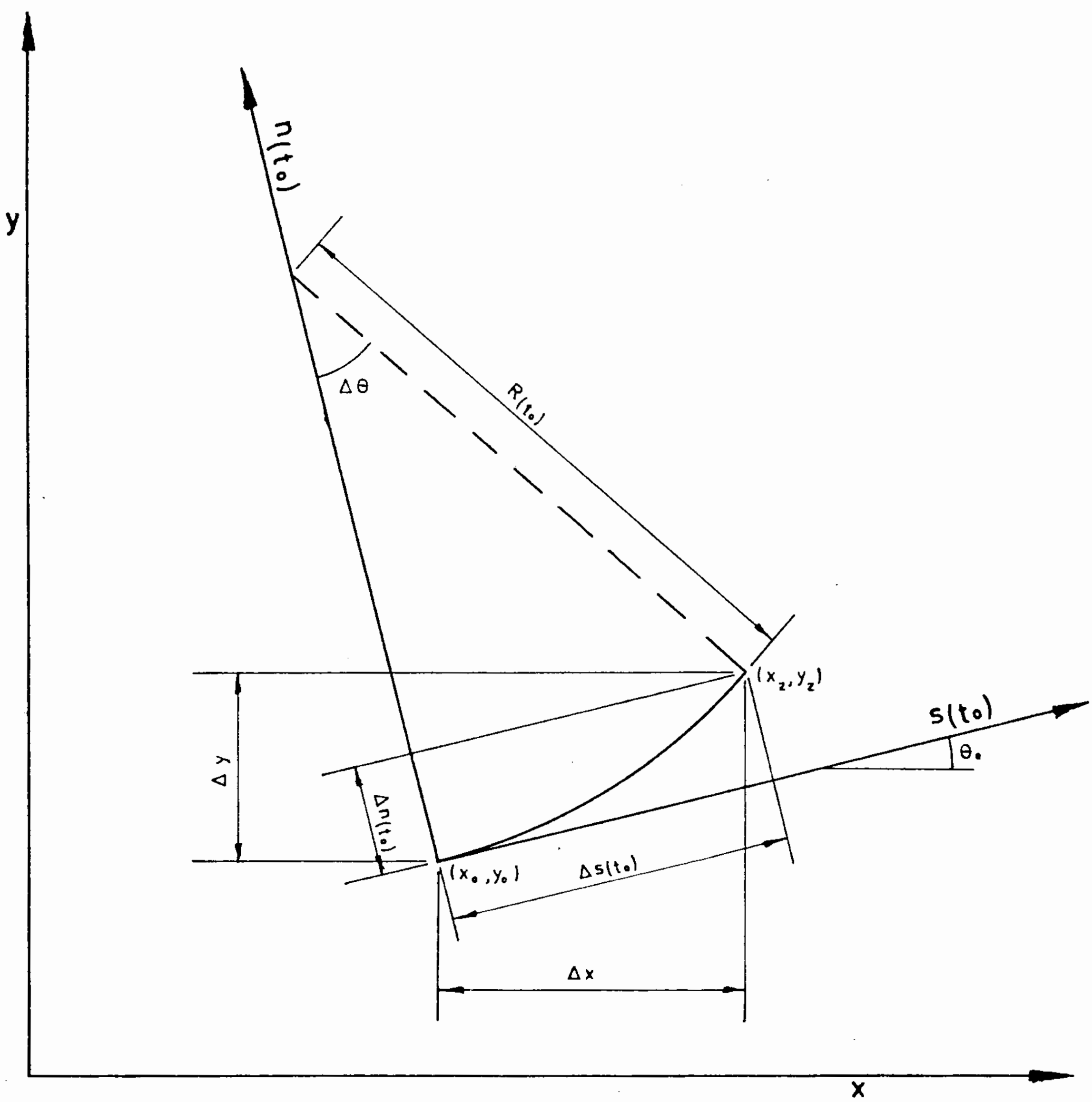


Figure 2.6: A typical increment

Considering Figure 2.6 we see that;

$$\Delta s_{(t_o)} = R_{(t_o)} \sin \Delta\theta \ , \tag{2.38}$$

and $$\Delta n_{(t_o)} = R_{(t_o)} \cdot (1 - \cos \Delta\theta) \ . \tag{2.39}$$

Alternatively if $\Delta\theta$ is very small we may assume that;

$$\Delta s_{(t_o)} = \Delta z \ , \tag{2.40}$$

and $$\Delta n_{(t_o)} = \frac{\Delta z \cdot \Delta\theta}{z} . \tag{2.41}$$

Equation (2.39) is prone to computational errors for small $\Delta\theta$, hence we can use equations (2.38) and (2.39) for $\Delta\theta$ greater than 10^{-2} radians (~$0{,}5^o$), and equations (2.40) and (2.41) for $\Delta\theta$ smaller than this value. (For $\Delta\theta$ equal to 10^{-2} radians we get approximately 2×10^{-3}% error in $\Delta s_{(t_o)}$ and 5×10^{-6}% error in $\Delta n_{(t_o)}$ when using the approximations in equations (2.40) and (2.41).

We can now obtain x and y from;

$$\Delta x = \Delta s_{(t_o)} \cos \theta_o - \Delta n_{(t_o)} \sin \theta_o \ , \tag{2.42}$$

and $$\Delta y = \Delta s_{(t_o)} \sin \theta_o + \Delta n_{(t_o)} \cos \theta_o . \tag{2.43}$$

We now know t_2, x_2, y_2 and θ_2 and could now proceed to the next point.

2.5 SUMMARY

It has been shown that if we have an area represented on an x - y plane, such that at all points the water depth and negative bed slopes in the x and y directions can be determined, then for a wave of set period, we can compute the path traced by an orthogonal of arbitrary starting point (usually in deep water) and initial direction.

Assuming the determination of depth and bed slopes is exact, the only factor to affect the accuracy shall be the chosen increment size.

One problem that arises is the fact that the wave celerity needs to be calculated for determination of the curvature, as given in equation (2.27). We see that the equation for determining wave celerity, equation (2.23), is implicit, hence some form of iterative process shall be needed to determine this parameter.

SECTION 3
DETERMINATION OF DEPTHS AND BED SLOPES

3.1 GRID DEFINITION

In order to determine depths and bed slopes, the area through which the orthogonals travel is divided into a rectangular grid system consisting of a set of equally sized squares.

The rectangular grid has an x dimension of ℓ_x, and a y dimension of ℓ_y. The rectangular area is divided into n_x equally spaced lines parallel to the y-axis, and n_y equally spaced lines parallel to the x-axis. The spacing between adjacent lines, in both directions, is s, corresponding to the side lengths of the individual grid squares.

It follows that the x-axis shall be divided into $(n_x - 1)$ equal divisions, and the y-axis shall be divided into $(n_y - 1)$ equal divisions.

Thus; $$\ell_x = (n_x - 1)\cdot s, \qquad (3.1)$$

and $$\ell_y = (n_y - 1)\cdot s \; . \qquad (3.2)$$

From equations (3.1) and (3.2) it is seen that;

$$\ell_y = \frac{(n_y - 1)}{(n_x - 1)} \cdot \ell_x \; . \qquad (3.3)$$

There are a series of grid points set up which are generally referred to as points (i, j), where i refers to the x value, and j refers to the y value. Thus i can be any integer value ranging between 1 and n inclusive, and j can be any integer between 1 and n_y inclusive.

Special note must be given to the fact that the origin of the grid is point (1,1), and not (0,0) as in the normal cartesian system.

It is convenient to refer to the grid points as nodes. The total number of nodes, N, is given by;

$$N = n_x \cdot n_y . \qquad (3.4)$$

Consider for example an arbitrary case where $n_x = 10$, $n_y = 7$ and s = 100 m., as shown in Figure 3.1.

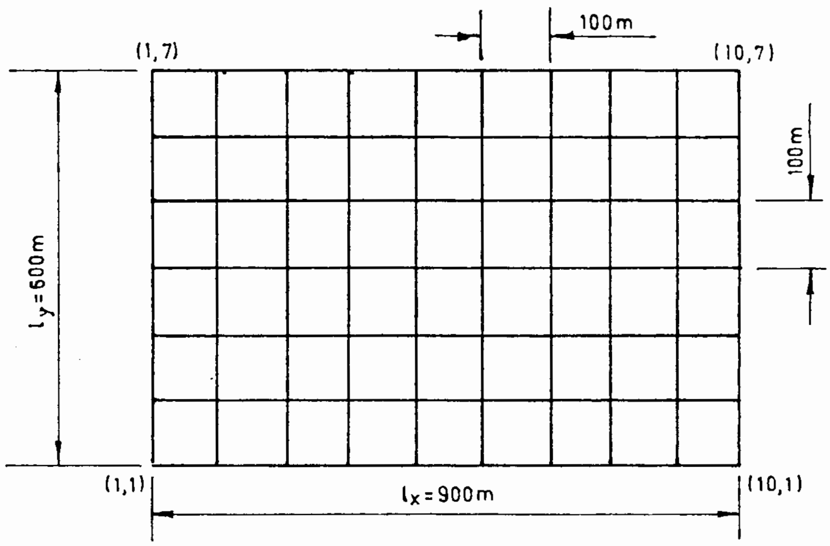


Figure 3.1: Example of a typical grid

For depth and bed slope determination the water depth at each node is needed. The general nodal depth is referred to as h (i, j), which corresponds to a point (i, j) as discussed above. There shall thus be N depth values.

In a practical example one could superimpose a rectangular grid over a contour map of the area through which the orthogonals shall travel, and read off the nodal depth values.

3.2 DETERMINATION OF DEPTHS IN THE GRID SYSTEM

As discussed in Section 2.3 we need to be able to determine the depth at any point in our grid system.

We now define our general depth as h(x,y), where x ranges between 1 and n_x, and y ranges between 1 and n_y. Note that x and y have the same range as i and j respectively, but that x and y are real values, whereas i and j are integers only.

The problem is thus to determine the generalised depth, h (x,y), from the known set of nodal depths, h(i,j).

Let us assume a specific value of (x, y), and a corresponding value of (i, j) such that $0 \leq (x - i) < 1$, and $0 \leq (y - j) < 1$. Thus i is the integer part of x, and j is the integer part of y.

We now define λx and λy as;

$$\lambda x = x - i, \tag{3.5}$$

and $$\lambda y = y - j. \tag{3.6}$$

Diagramatically point (x, y) and the four closest nodal points are shown in Figure 3.2.

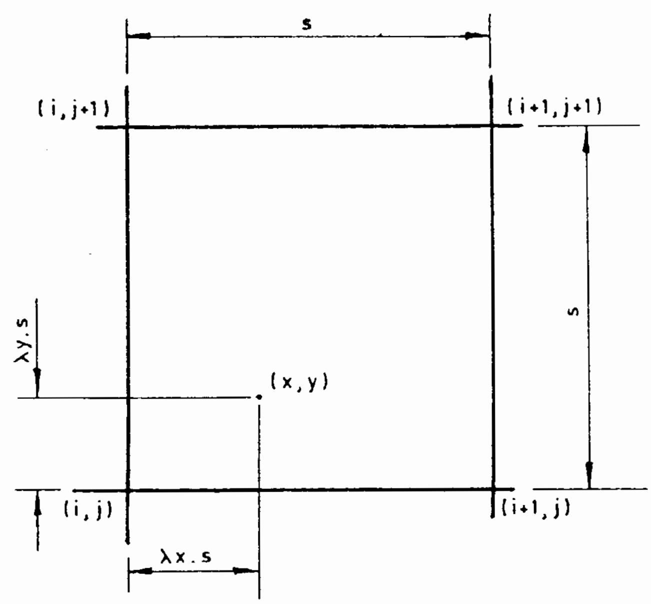


Figure 3.2: A typical grid square

A surface is fitted through the four closest nodal points such that there is continuity of depth over the boundaries. The surface that satisfies the above is one which has linear displacement variation in both the x and y direction for a general fixed y or x value. The equation for depth shall thus take the form;

$$h(x, y) = a + b \cdot \lambda x + c \cdot \lambda y + d \cdot \lambda x \cdot \lambda y, \qquad (3.7)$$

where a, b, c and d are constants which are determined by substitution of the nodal depths with the corresponding values of λx and λy.

Solution of equation (3.7)*yields;

$$h(x, y) = K_1 \, h(i, j) + K_2 \cdot h(i, j + 1) + K_3 \cdot h(i + 1, j)$$

$$+ K_4 \cdot h(i + 1, j + 1), \qquad (3.8)$$

where;

$$K_1 = 1 - \lambda x - \lambda y + \lambda x \cdot \lambda y, \qquad (3.9)$$

$$K_2 = \lambda y - \lambda x \cdot \lambda y, \qquad (3.10)$$

$$K_3 = \lambda x - \lambda x \cdot \lambda y, \text{ and} \qquad (3.11)$$

$$K_4 = \lambda x \cdot \lambda y. \qquad (3.12)$$

Thus the depth can be determined at any point in our typical grid square.

3.3 DETERMINATION OF BED SLOPES

Bed slopes are generally determined by partial differentiation in the x and y direction of the equation of a surface which has been fitted through a set of points.

---

* For solution of equation (3.7) see Appendix C.

Wilson (32) used a least squares method to fit a plane through the four closest grid points, whereas Keulegan and Harrison (16) used a modified second degree surface fitted through ten of the twelve closest points.

The writer decided not to determine the bed slopes by fitting a whole surface, but by using the twelve closest points as described below.

Assuming we are determining $\partial h/\partial x$ and $\partial h/\partial y$ for the same point (x, y) as discussed in Section 3.2.

Considering Figure 3.3,

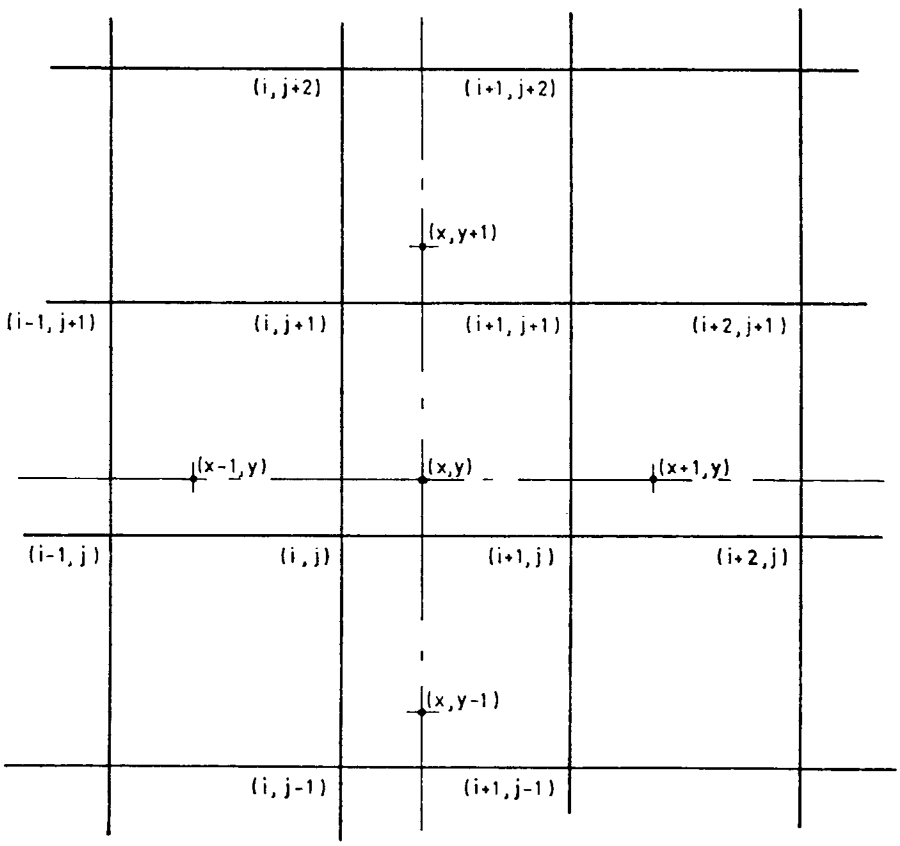


Figure 3.3: Localised grid system

we see, from equation (3.8), that,

$$h(x-1,y) = K_1 \cdot h(i-1,j) + K_2 \cdot h(i-1,j+1) + K_3 \cdot h(i,j) + K_4 \cdot h(i,j+1), \quad (3.13)$$

$$h(x,y-1) = K_1 \cdot h(i,j-1) + K_2 \cdot h(i,j) + K_3 \cdot h(i+1,j-1) + K_4 \cdot h(i+1,j), \quad (3.14)$$

$$h(x,y+1) = K_1 \cdot h(i,j+1) + K_2 \cdot h(i,j+2) + K_3 \cdot h(i+1,j+1) + K_4 \cdot h(i+1,J+2), \quad (3.15)$$

$$h(x+1,y) = K_1 \cdot h(i+1,j) + K_2 \cdot h(i+1,j+1) + K_3 \cdot h(i+2,j) + K_4 \cdot h(i+2,j+1). \quad (3.16)$$

where K_1, K_2, K_3 and K_4 have the values stated in equations (3.9) - (3.12).

To determine $\partial h/\partial x$ at point (x, y) a parabola is fitted through points (x - 1, y), (x, y) and (x + 1, y). The slope of the parabola at point (x, y) is given by;

$$\partial h/\partial x \ (x, y) = \frac{\{h(x + 1, y) - h(x - 1, y)\}}{2s} . \quad (3.17)$$

Similarly, by fitting a parabola through points (x, y - 1), (x, y) and x, y +1), we see that;

$$\partial h/\partial y \ (x, y) = \frac{\{h(x, y + 1) - h(x, y - 1)\}}{2s} . \quad (3.18)$$

As the depths are continuous over boundaries it is clear from equations (3.17) and (3.18) that the bed slopes shall also be continuous over the boundaries.

It should be noted that though one model has been used for fitting a surface to the nodal depths, as described in Section 2.2, the modification of the model used for bed slope determination shall have an effect similar to smoothing of the ocean bottom.

In conclusion it should also be noted that the bed slopes cannot be calculated, by this method, for a point closer than one grid square length, s, to any of the boundaries.

SECTION 4

PRACTICAL GRID AND NODAL DEPTH SELECTION

4.1 INITIAL REQUIREMENTS

Let the coastal area of interest be specified, and the set of relevant periods and initial wave directions $\{T_i, \gamma_i\}$ be given. A bathymetric chart of the area is needed so that nodal depth values can be determined.

4.2 SELECTION OF GRID BOUNDARIES

A rectangular x, y co-ordinate grid boundary is imposed on the chart of the region.

The boundaries represent the lines; $x = 1$, $x = n_x$, $y = 1$, and $y = n_y$, as described in Section 3.1. The grid may be oriented at any angle on the chart.

It is preferable that the x dimension of the grid be the larger dimension because, for the purpose of plotting, the y axis length is restricted to the width of the plotting paper, whereas the x axis length is unrestricted.

The position of the boundaries generally depends on three factors. The factors are; firstly the maximum water depth at which refraction occurs for the given set of periods $\{T_i\}$, secondly the given set of deep water wave directions $\{\gamma_i\}$, and finally the bathymetry of the coastal area.

(i) The sub critical-depth region

It is convenient to initially establish a depth region, in which refraction would occur, around our area of interest. This region shall be referred to as the sub critical-depth region.

The critical-depth, h_c, where refraction starts is given by

$$h_c = \frac{gT^2}{4\pi} , \qquad (4.1)$$

as stated in equation (2.5). In the S.I. system we have

$$h_c = 0{,}780\ T^2 \text{ metres.} \qquad (4.2)$$

The largest value of $\{T_i\}$ shall be taken to determine h_c.

The seaward extent of the sub critical-depth region can thus be determined by drawing the h_c contour on the chart. If a reef or island lies seaward of this contour, the sub critical-depth region must be extended to incorporate all the surrounding water of a lesser depth than h_c. For example see Figure 4.1.

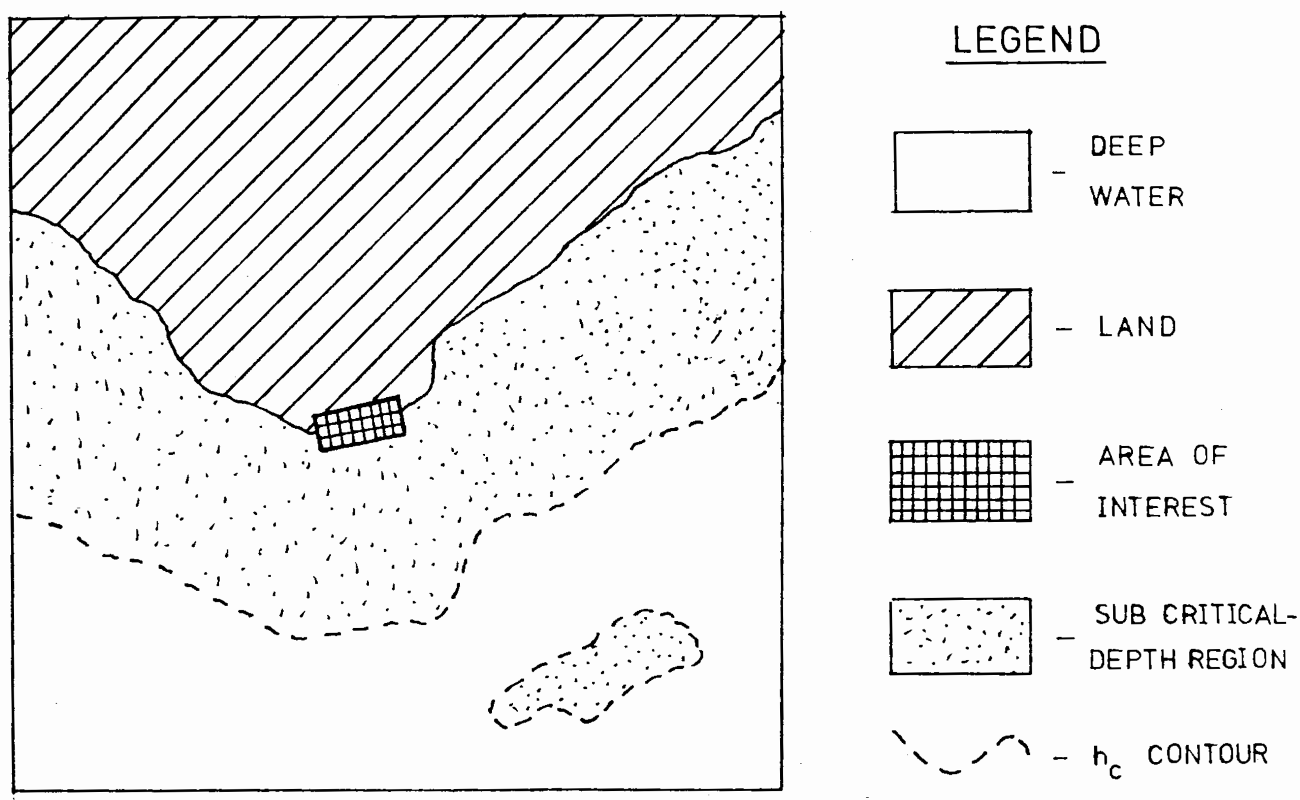


Figure 4.1: Example of the sub critical-depth region

The lateral extent of the sub critical-depth region that now must be considered is dependent on both the set of deep water wave directions $\{\gamma_i\}$ and also the bathymetry. Two regions are set up independently, they shall be termed the angular region and the intra-perpendicular region.

(ii) The angular region

Let the outer limits of the set $\{\gamma_i\}$ be γ_1 and γ_2. The two directions γ_1 and γ_2 are projected from the outside limits of the area of interest, so as to give the widest lateral spread. The angular region is the portion of the sub critical-depth region which is bounded by the h_c contour, γ_1 and γ_2, as shown in Figure 4.2.

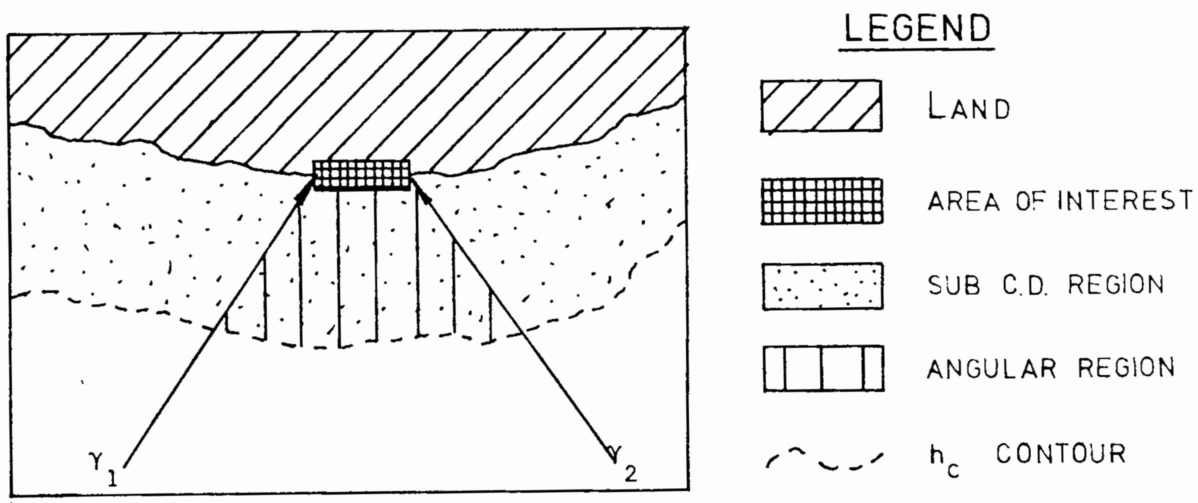


Figure 4.2: Example of the angular region

(iii) The intra-perpendicular region

From each extremity of the area of interest a line is drawn such that it remains perpendicular to the contours till the h_c contour is reached. These lines shall be called L_1 and L_2. The intra-perpendicular region is the region bounded by the h_c contour, L_1 and L_2, as shown in Figure 4.3.

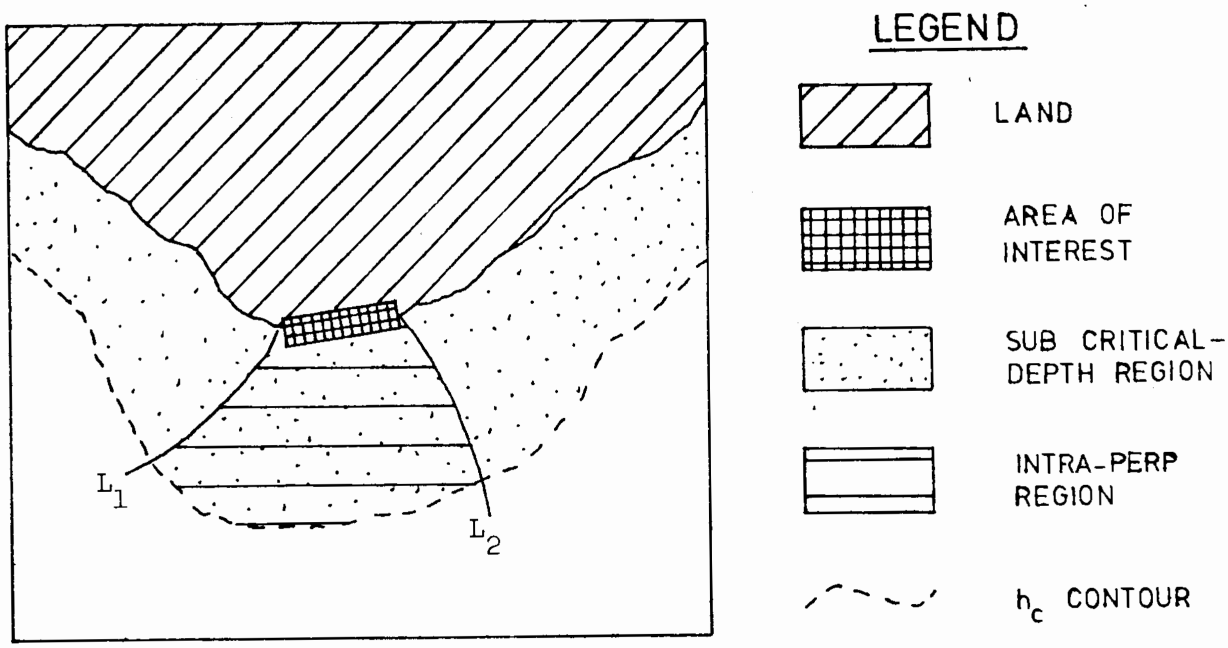


Figure 4.3: Example of the intra-perpendicular region

(iv) The pertinent depth region

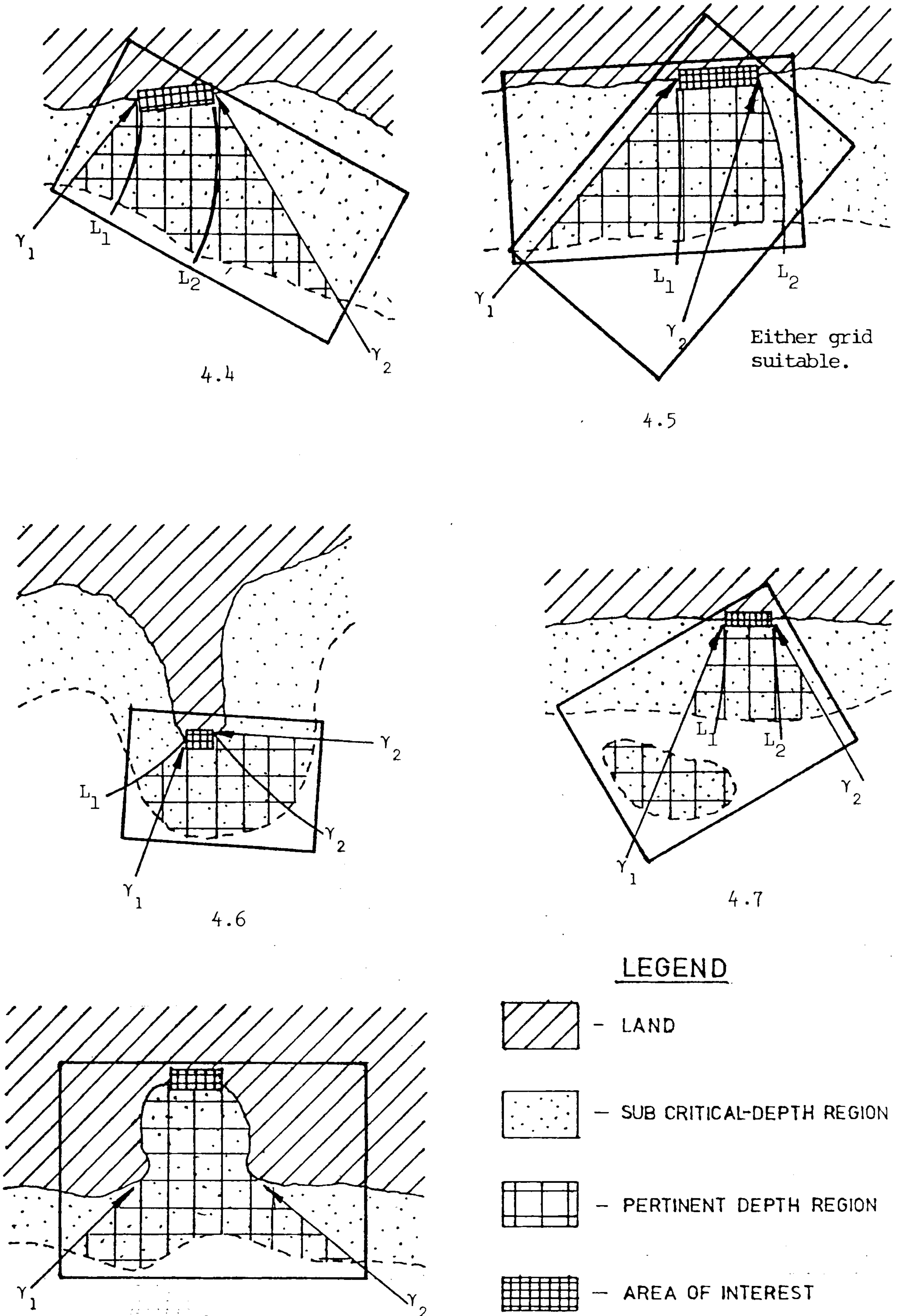
The pertinent depth region includes all areas in which nodal depths need to be given. This region can generally be said to be the union of the angular region and the intra-perpendicular region.

It should however be stressed that this is a general rule, and in cases of unusual bathymetry the users discretion must be used. The two most common cases are when offshore reefs or islands occur, and when the area of interest lies in a bay.

In the case of the former it should be noted that a reef generally acts as a converging lens, and thus should be included in the pertinent depth region. For example see Figure 4.7.

In the case of the bay, γ_1 and γ_2 should be drawn from the headlands, as shown in Figure 4.8.

The grid boundaries are set up such that the pertinent depth region is all included. It is however best to set up the grid



Figs 4.4-4.8. Examples of grid selection.

with the smallest area, as this shall correspondingly have the fewest nodal points for a set grid square size.

4.3 SELECTION OF GRID INTERVALS

Generally speaking the smaller the grid interval, s, the more accurate the determination of bed slopes shall be.

For practical examples, s should be chosen such that the bottom topography would be reasonably represented by fitting a parabola through three consecutive nodal depths of fixed x or y value.

The storage space is of course limited, depending on the computer. In general the maximum number of grid points should be limited to about ten thousand less than the available core space for computer being used.

Once the grid interval has been chosen, and the origin selected, the grid can be superimposed on the chart.

4.4 SELECTION OF DEPTH VALUES

From the bathymetric chart the depth values at each nodal point, which does not occur on land, can be determined.

As refraction does not occur in water depth greater than h_c, given by equation (4.1) all depths in our grid area with actual values greater than this may be equated to h_c.

Generally the nodal points that lie on the land can be given a value of zero. For the purpose of bed slope determination in the shallow water, however, the nodal points on land but adjacent to the shoreline should be given appropriate values as discussed below.

It can be seen from Section 3.2 that for slope determination near the shore, nodal depths shall be needed for all grid points lying two grid units or less from the shoreline.

Normally these values shall be negative, however in the case of a cliff shoreline they could be positive. In general the bottom should be extrapolated so as to give a true representation of the bed slope till the shoreline.

A method suggested by Wilson (32) recommends drawing 'reflected depth contours' on the land. In effect this method gives the depth at a point on land as the negative value of the depth at a point an equal distance from the shore, on a line drawn from the inland point, perpendicular to the shoreline. For example in Figure 4.9, $h_1 = -h_2$.

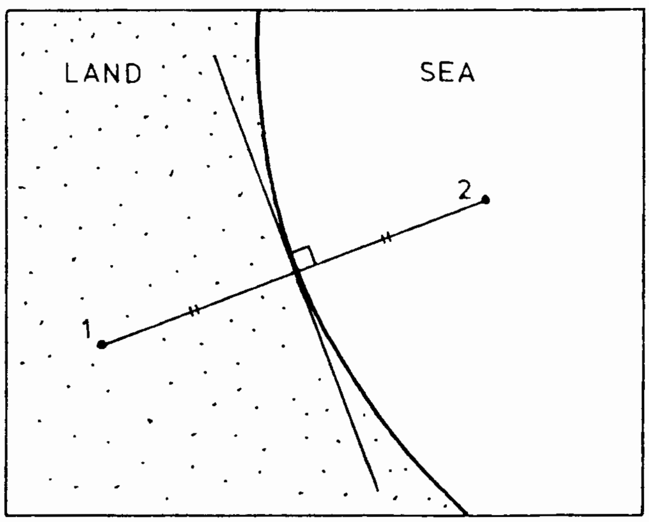


Figure 4.9: Example of a reflected depth

This method is reasonable as long as the shoreline is not clifflike. (i.e. the bed slope is not discontinuous over the shoreline).

Alternatively, if the chart has land contours on it, the appropriate value could be read off.

SECTION 5

COMPUTER OPERATIONS

5.1 OPTIONAL USE OF COMPASS BEARINGS

As yet only cartesian angles have been discussed, where positive angles are measured anti-clockwise from the direction of increasing x. It is however often convenient to refer to directions as compass bearings, where angles are measured clockwise from the direction of North.

All calculations are performed using cartesian angles, thus if compass bearings are being used they need to be converted to cartesian angles after data input, and similarly the cartesian angles must be converted to compass bearings before output.

Given that compass bearings are being used, we specify ϕ as the compass bearing of the increasing x direction. Assume an arbitrary direction with angle θ in the cartesian system, and a corresponding compass bearing of β as shown in Figure 5.1.

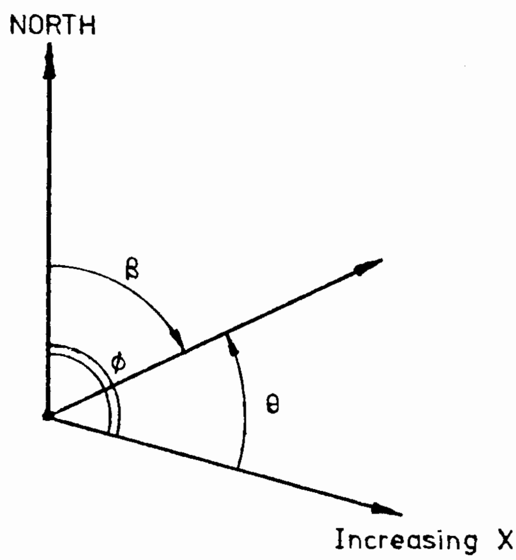


Figure 5.1: A typical wave direction

It is clear that;

$$\phi = \theta + \beta . \tag{5.1}$$

Thus if a general compass bearing, β is read, the corresponding cartesian angle, θ, is given by;

$$\theta = \phi - \beta . \tag{5.2}$$

Similarly if the direction with cartesian angle needs to be converted to compass bearing for output, we get the compass bearing is given by;

$$\beta = \phi - \theta . \tag{5.3}$$

5.2 SELECTION OF RAY ORIGINS

To determine the refraction diagram for a given wave, a set of ray origins are required. Generally speaking these origins should be in deep water.

Normally a segment of a single wave crest is drawn on the chart. Origin points for the rays are selected along this crest, and the co-ordinate values are recorded.

To determine what section of the segment should be considered for a particular area of interest the reverse tracing procedure can be used, as described in Section 5.11.

5.3 OPTIONAL RAY ORIGIN GENERATION

Normally a refraction diagram has its ray origins equally spaced on a section of a straight wave front in deep water. If this is the case, the set of ray origins can be computed, given one ray origin, the wave direction, and the spacing between orthogonals in deep water, b_o.

We specify the position (x_o, y_o) as the ray origin on the extreme left of the section when facing the direction in which the wave is travelling. This orthogonal shall be called the leading orthogonal. Let there be m orthogonals.

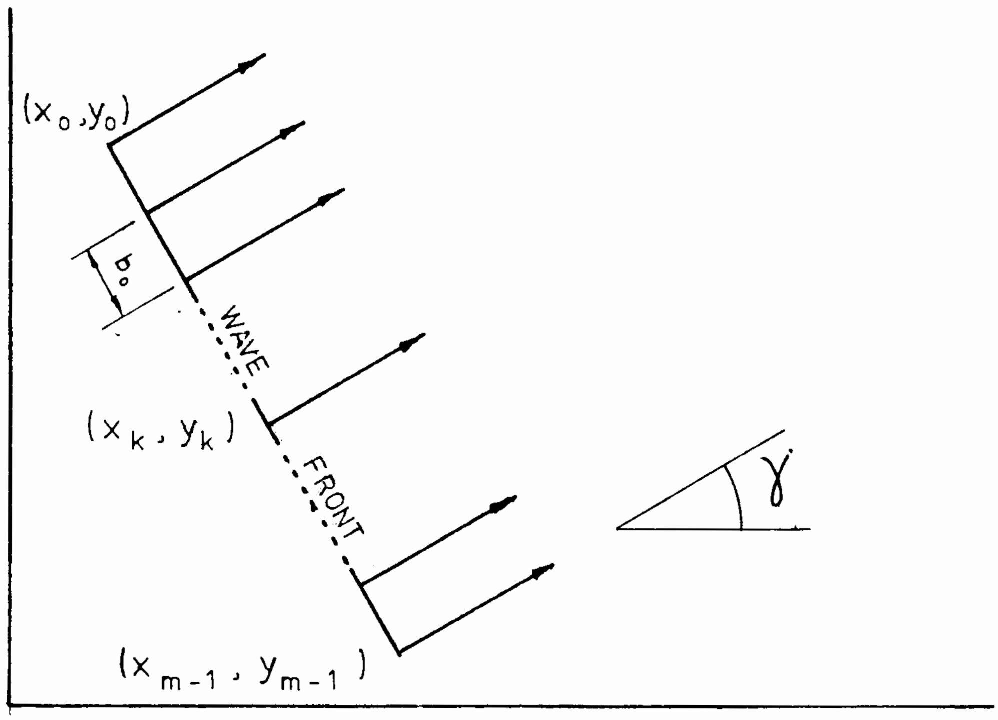


Figure 5.2 : Generated orthogonal starting positions

From Figure 5.2 we see that the general ray origin, (x_k, y_k), is given by;

$$x_k = x_o + k \cdot b_o \cdot \cos(\gamma - 90^o) , \qquad (5.4a)$$

and $$y_k = y_o + k \cdot b_o \cdot \sin(\gamma - 90^o) , \qquad (5.4b)$$

where k ranges between zero and (m - 1).

5.4 TIDAL SHIFT MODIFICATION

The values read for the general depths, h(i,j), pertain to a set tidal condition , usually mean sea level. Thus if the refraction diagram is desired for a different tidal condition, the tidal shift, ψ , can be added to each of the general depths.

Our modified general depths, $h^*(i,j)$, are thus given by;

$$h^*(i,j) = h(i,j) + \psi . \qquad (5.5)$$

ψ shall be positive for a rise in tide (i.e. an increased water depth).

5.5 COMPUTATION OF WAVE CELERITY

The equation for wave celerity, as stated in equation 2.3, is;

$$c = \frac{gT}{2\pi} \tanh \frac{2\pi h}{cT} \qquad (5.6)$$

As there is an implicit relationship between wave celerity and water depth, for a particular value of water depth, h, the wave celerity, c, must be computed by some iterative procedure.

As the wave celerity needs to be computed a great number of times in a typical refraction problem, the writer has decided to initially compute wave celerities corresponding to set water depths between zero and deep water. Subsequently wave celerities are determined by interpolation between these set values.

The writer has chosen to use a bisection method to determine the wave celerities for the set water depths.

The celerities are computed at half metre intervals in depth from zero to ten metres depth, and at one metre intervals from ten metres depth to deep water. This is done because the rate of change of wave celerity with respect to depth is greater in the shallow water, hence more accuracy is required.

The wave celerity for a general water depth is determined by performing a third degree interpolation between the wave celerities for the four closest depth values, as shown below.

(a) <u>Computation of wave celerity for water depth greater than ten metres</u>

Assume the wave celerity is needed for a water depth h, greater than ten metres, with the corresponding integer value k satisfying

$$1 > h - k \geq o \ , \qquad (5.7)$$

(i.e. k is the highest integer less than h).

$$\text{Let} \quad \Delta h = h - k \, . \qquad (5.8)$$

The wave celerities for the four closest set water depths of (k - 1), (k), (k + 1) and (k + 2) metres are c_{k-1}, c_k, c_{k+1} and c_{k+2} respectively.

Assuming that the relationship between water depth and wave celerity can be approximated by a third degree polynomial in that range, by substituting the known values of c for the four closest integer depths it can be shown that the wave celerity at depth h, $c_{(h)}$, is given by;*

$$c(\Delta h) = a_1 + a_2 \cdot \Delta h + a_3 (\Delta h)^2 + a_4 (\Delta h)^3 \ , \qquad (5.9)$$

where a_1, a_2, a_3 and a_4 have the following values;

$$a_1 = c_k \ , \qquad (5.10)$$

$$a_2 = \frac{\{- 2 \cdot c_{k-1} - 3 \cdot c_k + 6 \cdot c_{k+1} - c_{k+2}\}}{6} \ , \qquad (5.11)$$

$$a_3 = \frac{\{c_{k-1} - 2c_k + c_{k+1}\}}{2} \ , \qquad (5.12)$$

---

* For solution of equation (5.9) see Appendix E.

$$a_4 = \frac{\{- c_{k-1} + 3c_k - 3c_{k+1} + c_{k+2}\}}{6} \quad . \qquad (5.13)$$

(b) <u>Computation of wave celerity for water depth less than ten metres</u>

Assume the wave celerity is needed for a water depth h, less than ten metres such that the integer value k satisfies,

$$1 > 2h - k \geq o \quad . \qquad (5.14)$$

Let $\Delta h = 2h - k$. (5.15)

For example, if h is 7,8 metres, k shall be 15, and Δh shall be 0,6.

The wave celerities for the four closest set half metre depth values are once again c_{k-1}, c_k, c_{k+1} and c_{k+2}, however these speeds correspond to water depths of (k-1)/2, (k)/2, (k+1)/2 and (k+2)/2 metres respectively.

Thus in the previous example where h was 7,8m, c_{k-1} is the celerity at depth 7 metres, c_k is the celerity at depth 7,5m, c_{k+1} is the celerity at 8m and c_{k+2} is the celerity at 8,5m.

Equations (5.9) to (5.13) shall still hold provided the appropriate value of Δh, as given by equation (5.15), is used.

5.6 SELECTION OF INCREMENT SIZE

Clearly the smaller the increment size the more accurate the results obtained.

Since the computation of orthogonal progression is more error prone in shallower water a smaller increment is desired in this region.

This is obtained by using a constant time increment rather than a constant increment in Δz. Because the wave speed reduces in shallowing water a constant time step will cause correspondingly smaller increments in Δz.

It is however necessary to choose an initial increment size, Δz_o, which corresponds to the increment size in deep water. The desirable increment size is dependent on the specific example, and though the smaller the increment size is, the more accurate the results shall be, the increased computation time should also be considered.

Griswold (11) suggests using an increment size equivalent to one half of a grid side length, the writer however found one quarter of a grid side length as being more appropriate for this program. Various examples are considered in Section 5.

5.7 COMPUTATION OF A TYPICAL ORTHOGONAL

Given the starting parameters of a typical orthogonal as position, (x_o, y_o) and direction, θ_o, at time $t_o = 0$. After n increments in Δz the parameters have values (x_n, y_n), θ_n and t_n. To calculate the parameters after the (n + 1)th increment we proceed as follows.

The increment size Δz is given by;

$$\Delta z = \frac{\Delta z_o \cdot c_{(previous)}}{c_{(max)}} \tag{5.16}$$

where $c_{(previous)}$ is taken as the computed wave celerity for the previous increment, and $c_{(max)}$ is the deep water wave celerity.

The central point of the increment, (x^*, y^*), is approximated by equations (2.31) and (2.32).

The water depth, h, and bed slopes, $\partial h/\partial x$ and $\partial h/\partial y$, are given by equations (3.7), (3.17) and (3.18) respectively.

The wave celerity is now determined by interpolation according to equations (5.9) to (5.15).

The curvature of the element, $d\theta/ds$, the radius of curvature, R, and the angular change across the element, $\Delta\theta$, are given by equations (2.27), (2.33) and (2.34) respectively.

Δs and Δn are now given by equations (2.38) and (2.39) or equations (2.40) and (2.41) depending on the value of $\Delta\theta$.

The final parameters (x_n, y_n), θ_n and t_n are now given by equations (2.42) and (2.43), (2.35) and (2.36) respectively.

Successive points are computed until the central point, (x^*,y^*), is in less than one metre water depth, or falls within one grid length of any of the boundaries.

5.8 PRINTED OUTPUT

As data the user can specify the time interval, t^*, in seconds between successive output time values of the orthogonal.

We specify the general output values as $(x_i^\dagger, y_i^\dagger)$, $\theta_i^\dagger$ and $t_i^\dagger$, representing the various orthogonal parameters corresponding to a time value of $i \cdot t^*$ from t_o, (i being any integer).

In order to determine the parameters assume that t^* lies between t_n and t_{n+1} as discussed in Section 5.7. A linear interpolation is performed between t_n and t_{n+1} to determine the relative position of t^*.

We define p as;

$$p = \frac{t^* - t_n}{t_{n+1} - t_n} \qquad (5.17)$$

Assuming a linear relationship between time and (a) the x_{value}, (b) the y_{value} and, (c) the θ_{value}.

$$x_i^{\dagger} = x_n \cdot p(x_{n+1} - x_n) \; , \tag{5.18}$$

$$y_i^{\dagger} = y_n \cdot p(y_{n+1} - y_n) \; , \tag{5.19}$$

and $$\theta_i^{\dagger} = \theta_n \cdot p(\theta_{n+1} - \theta_n) \; . \tag{5.20}$$

There is an error in the above assumption, but if Δz is small this error shall be very small. Also the error is not cumulative.

5.9 PLOTTING OPERATIONS

This is an optional operation, and a plot is only given if specified in the data.

The y-axis length in inches is specified in the data and the x-axis length is calculated according to equation (3.3). The shoreline is entered in the data as a set of consecutive points along the shoreline which are to be joined by straight lines, the accuracy of the plotted shoreline is thus dependent on the users selection of representative points. It should also be noted that the shoreline will keep the same position regardless of the tidal shift value, ψ.

The orthogonal is plotted by joining successive points $(x_i^{\dagger}, y_i^{\dagger})$, as described in Section 5.8, by straight lines.

It is also optional to have the wave front directions - represented by a dash - drawn on the orthogonals at set time values. If this is desired the crest marks shall be drawn at a chosen integer, n, multiple value of t*. That is for a selected n, specified in the data, we shall have a crest mark drawn at every n_{th} $(x_i^{\dagger}, y_i^{\dagger})$ position. It is also optional to have the time value printed.

For example if we want to have crest marks at every ten minute interval, and desire printed values of the orthogonal at every twenty seconds, (this may be desired to obtain a smooth plot), we would specify t* as 20 seconds and n as 30.

Each orthogonal is numbered at its termination point for easy identification.

5.10 REVERSE TRACING PROCEDURE

This method as described by Dorrestien (9), is very useful as an initial plot to determine the ray origins (see Section 5.2) for a set period, if only interested in a point of interest. It can also be used to determine the range of wave directions which can be expected at the point of interest, and the deep water wave directions that cause them.

Dorrestien suggested that the value of the refraction coefficient, k_r, could be determined for a general deep water wave direction from this diagram alone (drawn for a set period). The writer investigated this but did not get answers in agreement with the standard theory.

In effect this method sends out a set of rays from the point of interest at varying angles, and traces the path backwards to deep water. Examples are given in Section 6, and Appendix A.

SECTION 6

EXAMPLES

6.1 INTRODUCTION

In this section the writer's program, REFRAC, is verified and compared with the program written by Wilson. It should be noted that whereas the writer has a changeable deep water increment size z_o, which is read as data, Wilson sets his z_o as s/2, where s is the side length of an individual grid square.

Four examples are covered. These are; a plane beach with straight parallel depth contours, a point island, a conical island and a real bay.

6.2 EXAMPLE 1 - A STRAIGHT PARALLEL CONTOURED BEACH

The advantage of this example is that exact theoretical values of orthogonal direction can be determined for any general depth. This can be determined because according to Snell's law, which states that c/sin θ must remain constant, we get that the wave direction is given by

$$\theta = \arc \sin \left(c \cdot \frac{\sin \Theta_o}{c_o}\right) . \qquad (6.1)$$

The beach chosen has a slope of 1:25 and the grid has the following properties, as defined in Section 3.1:

$$\ell_x = \ell_y = 4\ 200\text{m} ,$$

$$n_x = n_y = 43 ,$$

$$s = 100\text{m} .$$

The shoreline is the line y = 42, and the generalised depth is thus;

$$h(x, y) = (42 - y) \cdot 4 . \qquad (6.2)$$

Hence for a given period and deep water wave direction the true angle of the orthogonal can be determined. The grid is shown in Figure 6.1.

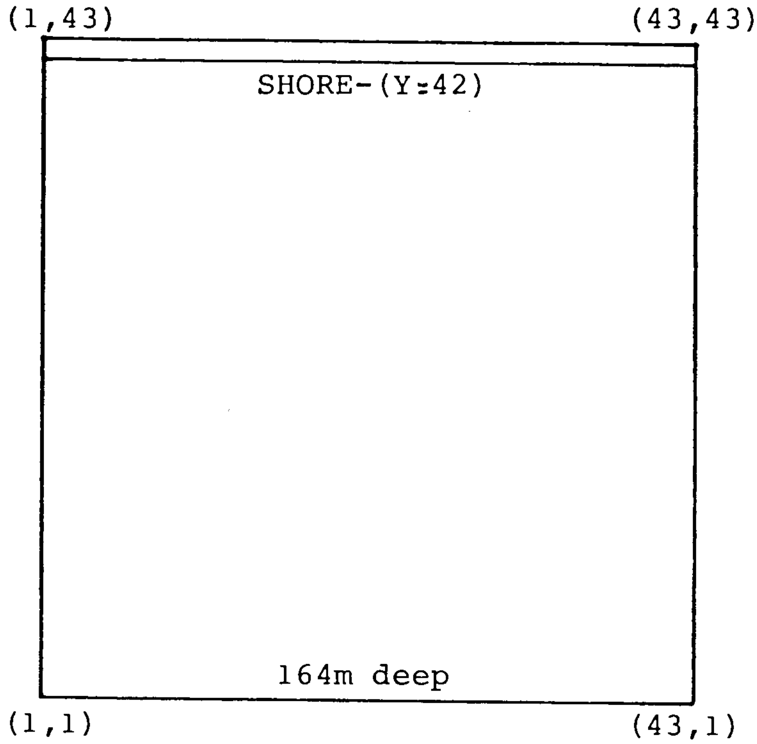


Figure 6.1: Example 1, a straight beach. Grid properties.

It should be noted that the computation of bed slopes shall be exact in this example, it shall thus only test the other aspects of the program.

The writer has arbitrarily chosen a wave of twelve second period and deep water wave direction of fourty five degrees for this example.

The results of three orthogonals computed by the writer's program, with different deep water step lengths are shown in Table 6.1; where the theta 1 values are the computed values of orthogonal direction, and the theta 2 values are exact values obtained from equations (6.1) and (6.2). Consecutive results are at ten second intervals.

The three orthogonals, all with starting point (2,2), have initial step length, z_o, of 1m, 10m and 50m respectively. The computer plot of the orthogonals is shown in Figure 6.2, and it is seen that the difference is too small for visual separation of the orthogonals.

The writer has combined his program with the Wilson program to plot on the same axes, with the Wilson solution being a solid line, and the writer's solution being a dashed line. The initial step size is 50m (orthogonal three in Table 6.1) and the plot is shown in Figure 6.3. It can be seen that the two orthogonals differ.

Figure 6.4 shows a graph of error in degrees versus depth in metres for solutions obtained by the Wilson program (line 1), the writer's program with z_o being 1 metre (line 2), and the writer's program with z_o being 50m (line 3).

The initial error of - 0,02 degrees exists because at 160 metres water depth a change in direction of 0,02 degrees would already have occurred.

The error in line 2 is so small that it can reasonably be assumed that the writer's solution when using a one metre step length is correct.

The Wilson program, represented by line 1 in Figure 6.4, assumes that no refraction occurs in water of greater depth than h_c (112 metres for a 12 second wave), and thus accumulates a relatively large initial error of - 0,2 degrees. The error is thus carried through till the wave ray is stopped, and hence, as would be expected, the ray lies on the clockwise side of the correct solution.

In the shallow water the writer's solution with z_o being fifty metres, represented by line 3 in Figure 6.4, develops a larger angular error than the Wilson program. This is because Wilson's

ORTHOGONAL NO. 1, STARTING PT.:(2.00, 2.00) z_o=1m.

X	Y	THETA1	THETA2	DEPTH
2.0000	2.0000	45.000		
3.3194	3.3195	45.005	45.019	154.72
4.6387	4.6392	45.012	45.025	149.44
5.9578	5.9590	45.021	45.035	144.16
7.2766	7.2790	45.033	45.047	138.88
8.5951	8.5994	45.049	45.063	133.60
9.9132	9.9202	45.070	45.085	128.32
11.231	11.242	45.099	45.115	123.03
12.548	12.564	45.138	45.154	117.74
13.863	13.887	45.191	45.206	112.45
15.177	15.211	45.260	45.276	107.16
16.487	16.535	45.352	45.367	101.86
17.792	17.859	45.473	45.489	96.565
19.090	19.182	45.632	45.648	91.270
20.381	20.506	45.840	45.856	85.976
21.660	21.829	46.110	46.125	80.683
22.925	23.152	46.457	46.472	75.393
24.172	24.474	46.900	46.915	70.106
25.396	25.794	47.459	47.474	64.825
26.590	27.111	48.158	48.173	59.554
27.750	28.425	49.023	49.037	54.300
28.865	29.732	50.078	50.091	49.070
29.928	31.030	51.350	51.363	43.879
30.927	32.314	52.863	52.875	38.745
31.854	33.577	54.640	54.651	33.694
32.698	34.810	56.699	56.709	28.760
33.447	36.003	59.053	59.062	23.990
34.096	37.140	61.709	61.716	19.438
34.636	38.207	64.664	64.669	15.173
35.065	39.182	67.909	67.913	11.272
35.386	40.045	71.424	71.425	7.8202
35.606	40.774	75.179	75.178	4.9036
35.738	41.348	79.135	79.131	2.6065
35.801	41.750	83.244	83.238	1.0013

ORTHOGONAL NO. 2, STARTING PT.:(2.00, 2.00) z_o=10m.

X	Y	THETA1	THETA2	DEPTH
2.0000	2.0000	45.000		
3.3194	3.3195	45.005	45.019	154.72
4.6387	4.6392	45.012	45.025	149.44
5.9578	5.9590	45.021	45.035	144.16
7.2766	7.2790	45.033	45.047	138.88
8.5951	8.5994	45.049	45.063	133.60
9.9132	9.9202	45.071	45.085	128.32
11.231	11.242	45.100	45.115	123.03
12.547	12.564	45.139	45.154	117.74
13.863	13.887	45.191	45.206	112.45
15.177	15.211	45.260	45.276	107.16
16.487	16.535	45.352	45.367	101.86
17.792	17.859	45.474	45.489	96.565
19.090	19.183	45.633	45.648	91.270
20.381	20.506	45.841	45.856	85.976
21.660	21.829	46.111	46.125	80.683
22.925	23.152	46.458	46.472	75.392
24.172	24.474	46.900	46.915	70.105
25.395	25.794	47.460	47.474	64.824
26.590	27.112	48.160	48.173	59.554
27.749	28.425	49.024	49.037	54.299
28.865	29.733	50.080	50.091	49.069
29.927	31.030	51.353	51.363	43.878
30.927	32.314	52.868	52.876	38.744
31.854	33.577	54.646	54.652	33.692
32.697	34.810	56.708	56.710	28.758
33.447	36.003	59.065	59.063	23.987
34.094	37.141	61.725	61.718	19.435
34.634	38.208	64.686	64.672	15.170
35.063	39.183	67.938	67.916	11.265
35.384	40.046	71.463	71.429	7.8163
35.603	40.775	75.230	75.183	4.9000
35.734	41.349	79.200	79.136	2.6032

ORTHOGONAL NO. 3, STARTING PT.:(2.00, 2.00) z_o=50m.

X	Y	THETA1	THETA2	DEPTH
2.0000	2.0000	45.000		
3.3194	3.3195	45.005	45.019	154.72
4.6387	4.6392	45.012	45.025	149.44
5.9578	5.9590	45.021	45.035	144.16
7.2766	7.2790	45.033	45.047	138.88
8.5951	8.5994	45.049	45.063	133.60
9.9132	9.9203	45.071	45.085	128.32
11.231	11.242	45.100	45.115	123.03
12.547	12.564	45.139	45.154	117.74
13.863	13.887	45.191	45.206	112.45
15.177	15.211	45.261	45.276	107.16
16.487	16.535	45.352	45.367	101.86
17.792	17.859	45.474	45.489	96.565
19.090	19.183	45.633	45.648	91.270
20.380	20.506	45.841	45.856	85.976
21.660	21.829	46.111	46.125	80.683
22.925	23.152	46.458	46.472	75.392
24.171	24.474	46.902	46.915	70.105
25.395	25.794	47.462	47.474	64.824
26.590	27.112	48.162	48.173	59.553
27.749	28.426	49.029	49.037	54.298
28.864	29.733	50.088	50.092	49.068
29.926	31.031	51.363	51.364	43.876
30.926	32.315	52.883	52.877	38.740
31.852	33.578	54.670	54.653	33.688
32.694	34.812	56.742	56.713	28.753
33.444	36.005	59.112	59.068	23.979
34.090	37.143	61.794	61.723	19.426
34.620	38.210	64.781	64.630	15.159
35.055	39.187	68.067	67.930	11.254
35.373	40.049	71.637	71.444	7.8039
35.590	40.780	75.458	75.210	4.8819
35.718	41.352	79.503	79.160	2.5927
35.778	41.752	83.725	83.272	.99106

Table 6.1. Example 1, a straight Beach.
Comparison of varying initial step length.

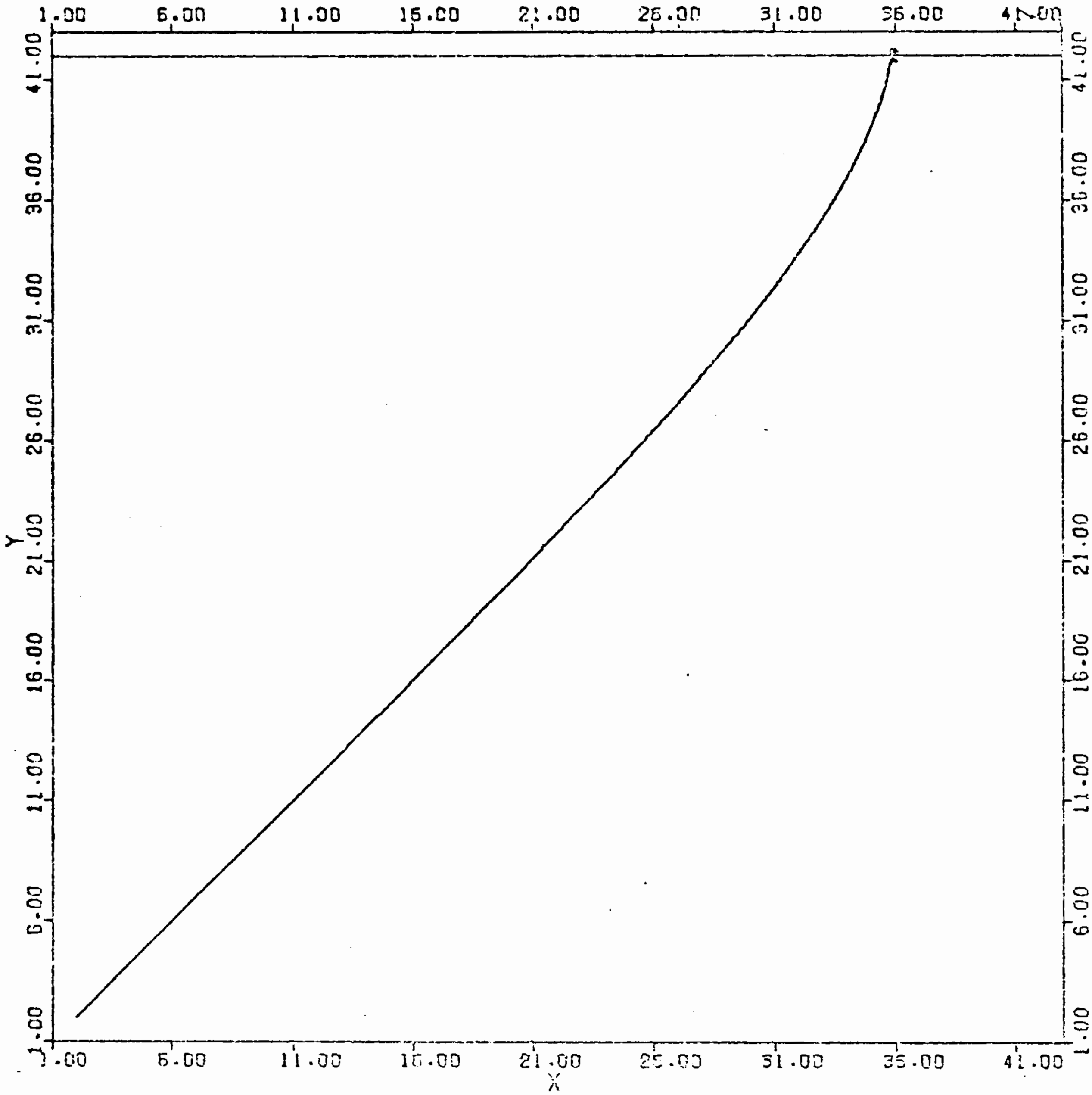


Figure 6.2. Example 1, a Straight Beach.
Comparison of varying initial step length.

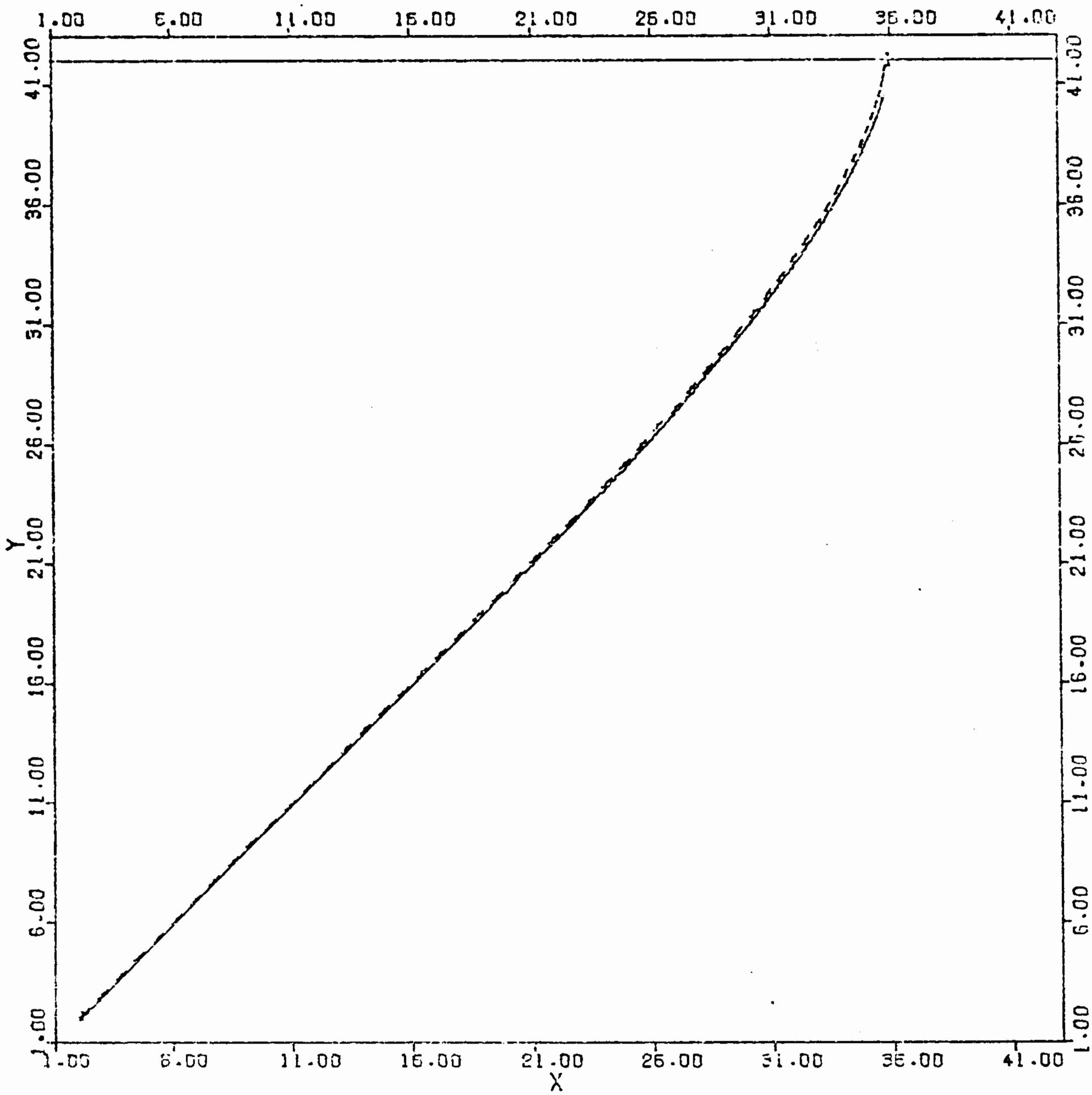


Figure 6.3. Example 1, a Straight Beach.
Comparison of results obtained by writer's program and Wilson's program. (Wilson: continuous line, writer: dotted line.)

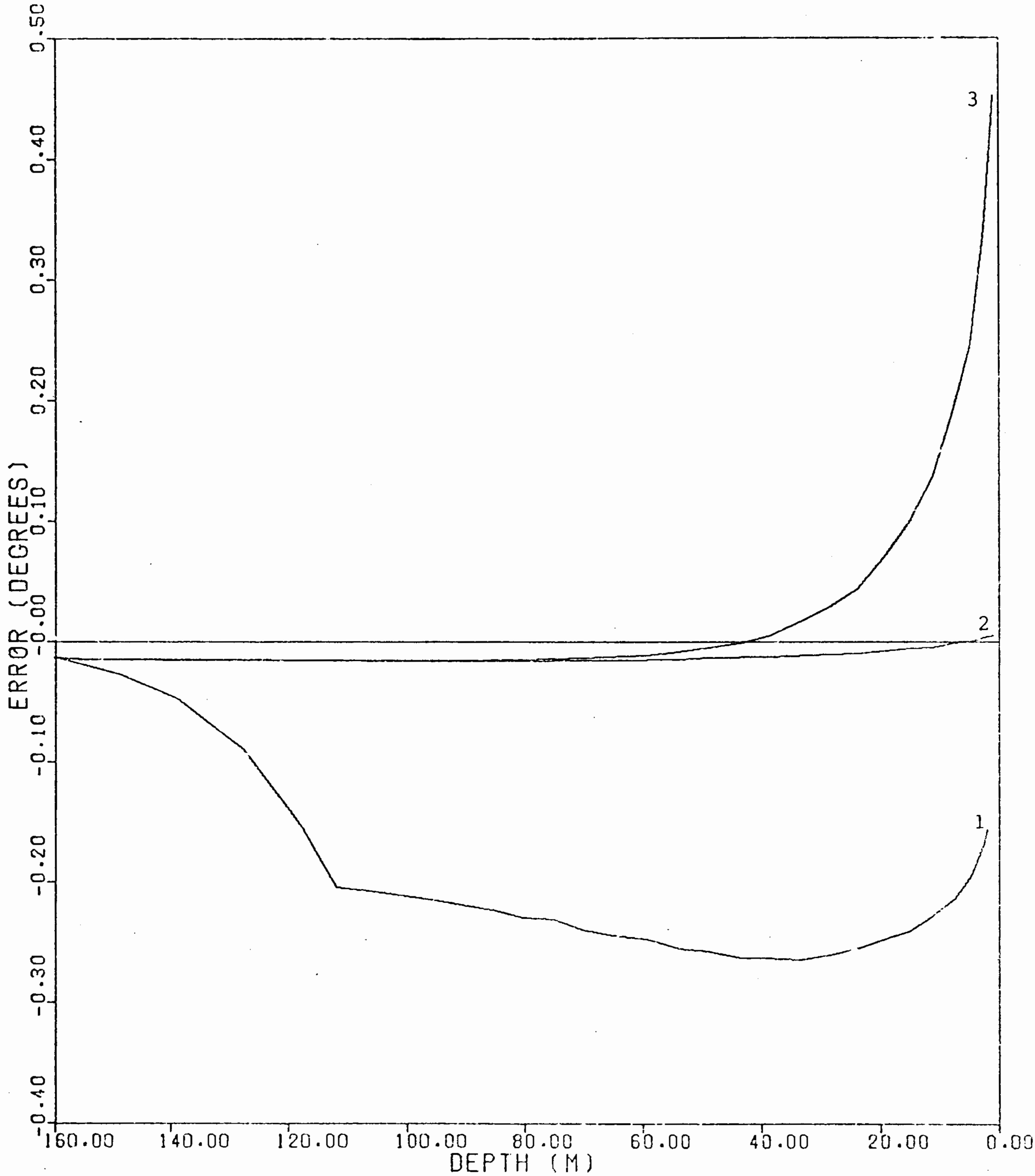


Figure 6.4. Example 1, a Straight Beach.

Angular error versus depth for various methods. (Line 1: Wilson, Line 2: writer with z_o = 1m., Line 3: writer with z_o = 50m.)

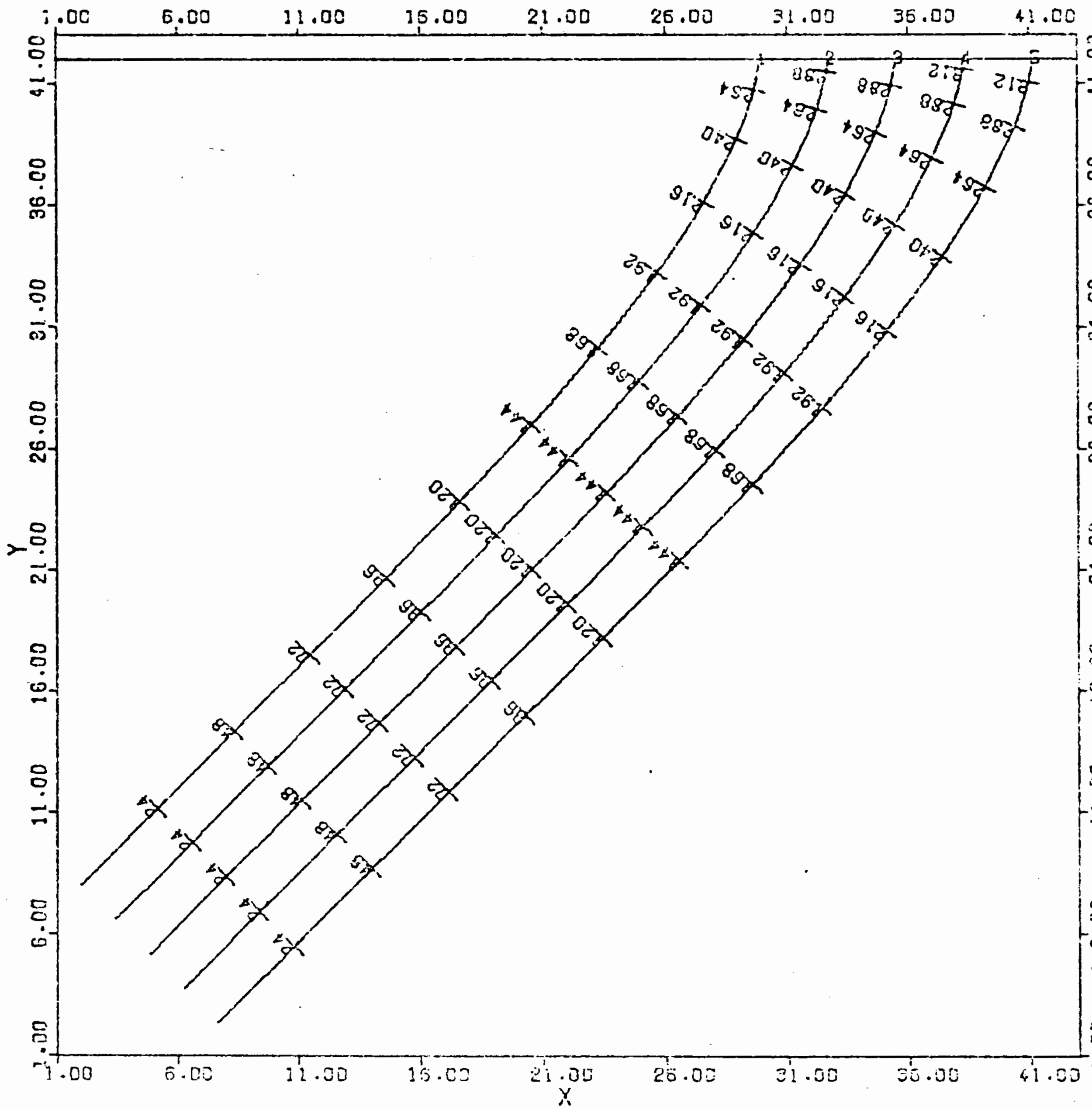


Figure 6.5. Example 1, a Straight Beach.

Typical refraction diagram drawn by writer's program.

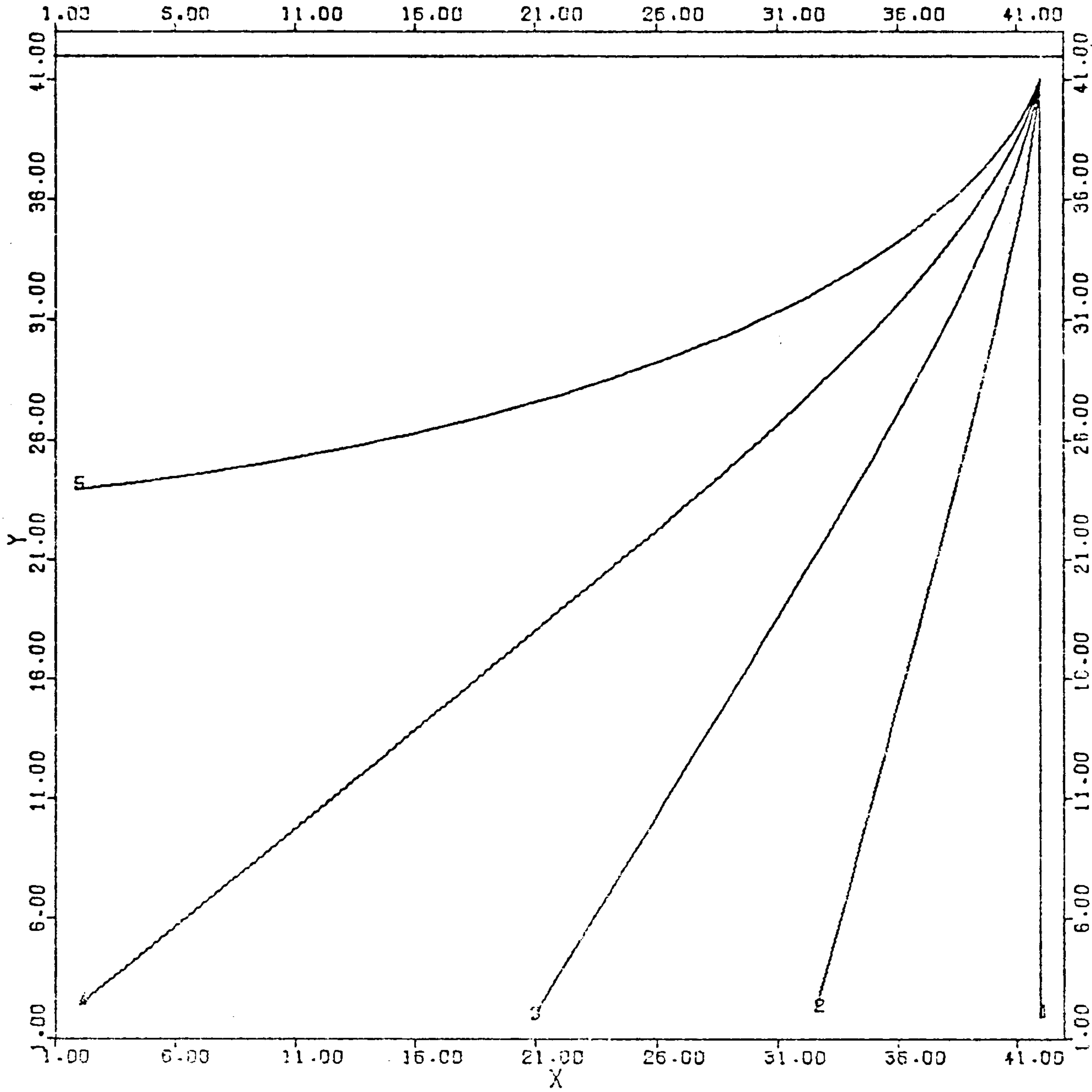


Figure 6.6. Example 1, a Straight Beach.
Reverse tracing method.

step lengths are more greatly reduced in the shallow water than the writers. (The writer uses a constant time increment, as described in Section 5.7, whereas Wilson uses a reducing time increment.)

This error however only occurs at the end of ray progression, and as errors are cumulative it is far less important than errors incurred towards the beginning of ray progression. This point is confirmed by Figure 6.2 in which, the difference between the orthogonals with z_o being 1 metre, 10 metres and 50 metres cannot be seen.

A full plot of five orthogonals for a wave with the following parameters is shown in Figure 6.5.

T = 12 secs, $\theta_o = 45^o$, b_o = 200 metres (2s), leading orthogonal starting position (2,8), and successive crest marks at 24 second intervals (every alternate crest).

An example of the reverse tracing method, as discussed in Section 5.10, is shown in Figure 6.6 This example is for a twelve second wave, with initial angles of 270, 265, 260, 255 and 250 degrees. The ray origin is point (42,41).

6.3 EXAMPLE 2 - A POINT ISLAND

In 1964 Arthur (1) investigated the refraction of waves by islands and shoals with circular bottom-contours from an analytical standpoint, to obtain a precise determination of resulting wave patterns.

Arthur applied Fermat's principle to obtain the exact solution for five examples. One such example was a point island.

The island has the following features:

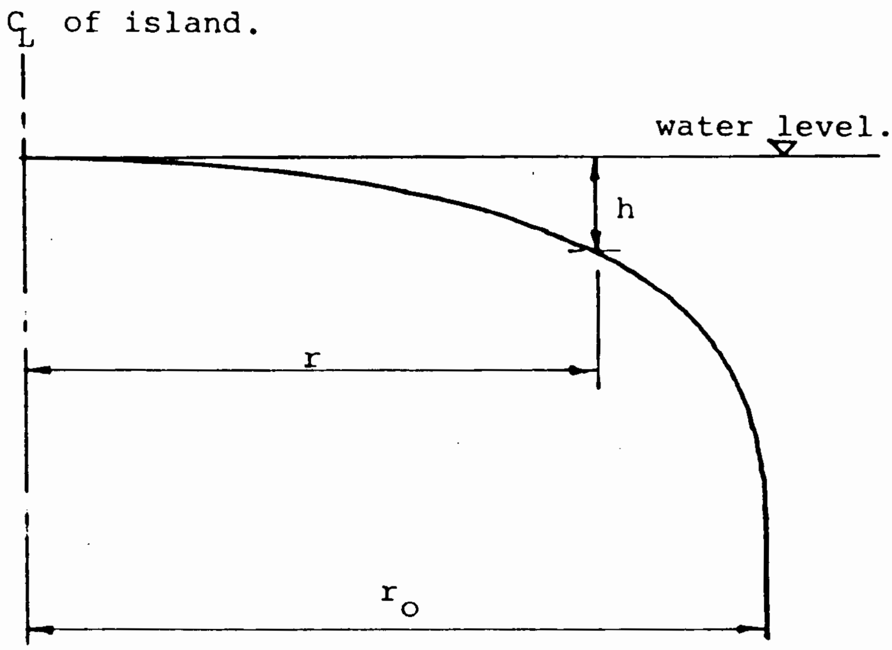


Figure 6.7: Example 2, a point island. Radial section.

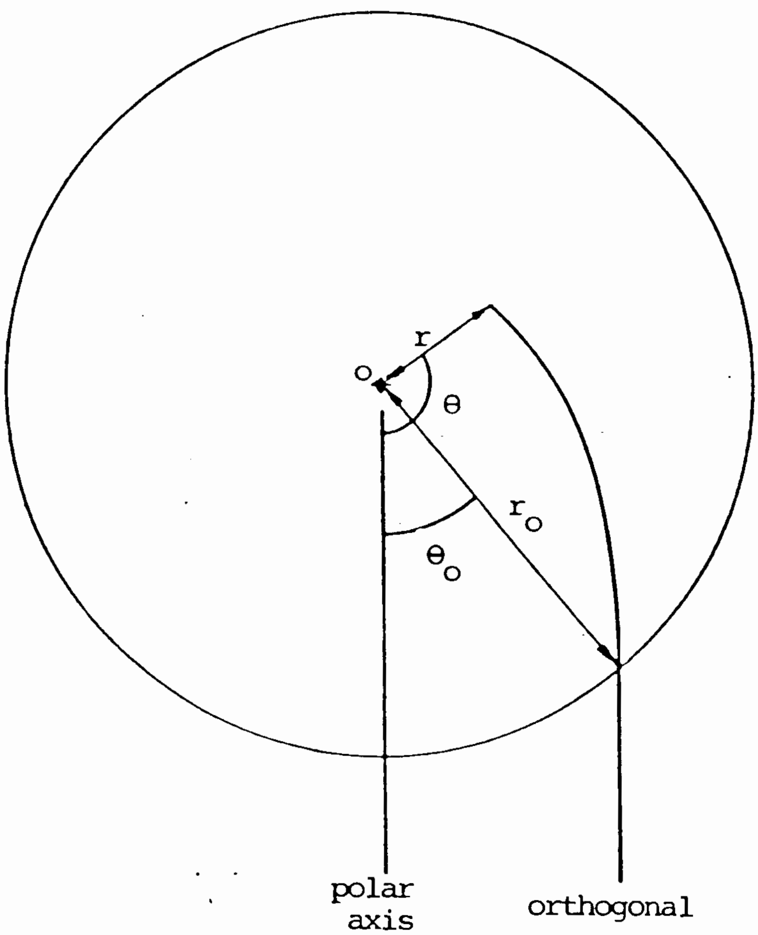


Figure 6.8: Example 2, a point island. Polar co-ordinate system.

i. The island has an outside radius of r_o.

ii The water depth at any value of r, greater than r_o, is infinite.

iii The water depth, h, at r, given that r is less than r_o, is such that the wave celerity at this point c is given by;

$$c = \frac{c_o \cdot r}{r_o} . \qquad (6.3)$$

where : c_o is the deep water wave celerity. (The geometry of the island is thus dependent on the wave period and the outside radius.)

The generalised polar co-ordinate of an orthogonal, travelling over the island is shown in Figure 6.8.

Arthur determined the solution to be;

$$r = \frac{r_o}{\exp\{(\theta - \theta)_o \cot\theta_o\}} . \qquad (6.4)$$

Equation 6.4 describes a logarithmic spiral. All orthogonals thus converge towards the centre.

The writer selected a point island, with outside radius of 200m, corresponding to a twelve second wave.

Wave celerity is given by;

$$c = \frac{gT}{2\pi} . \tanh \frac{2\pi h}{cT} . \qquad (6.5)$$

Thus deep water wave celerity is given by;

$$c_o = \frac{gt}{2\pi} . \qquad (6.6)$$

Deep water wave celerity for a twelve second wave is thus 18,736 m/s.

The writer established a study area with the following parameters;

$\ell x = \ell y = 1400m$,

$n_x = n_y = 141$,

$s = 3m$.

The centre of the island was centrally placed in the study area, i.e. co-ordinates (71,71). The radius of any co-ordinate (x,y) in the study area is thus given by;

$$r = 3\sqrt{(x - 71)^2 + (y - 71)^2} \quad . \qquad (6.7)$$

According to equation (6.3) the depth, h, at any point, (x,y) in the study area, provided that $r < r_o$, must be such that the wave celerity for a twelve second wave in h metres of water is given by;

$$c = \frac{18,736 \quad r}{200} \quad . \qquad (6.8)$$

Combining equations (6.8) and (6.6) yields;

$$\frac{12\ g}{2\pi} \tanh \frac{2\pi h}{\frac{18,736r}{200}} = \frac{18,736\ r}{200} \quad . \qquad (6.9)$$

Solution of equation (6.9) would yield the corresponding depth. This equation has however an implicit relationship between r and h, and must be solved by an iterative method.

The writer developed a program* to calculate the depths at each grid node using a bisection method, and to write the depths into a data file, which would be compatible as input for his program.

For any value $r \geq r_o$, the depth was set at 112.5m (the standard approximation of deep water).

The writer considered five orthogonals with the following values of θ_o: 0^o, 10^o, 45^o, 60^o and 75^o. Orthogonal progression was terminated when a water depth of one metre was reached.

The exact solution of r as given by equation 6.4 for various values of θ is given in Table 6.2.

---

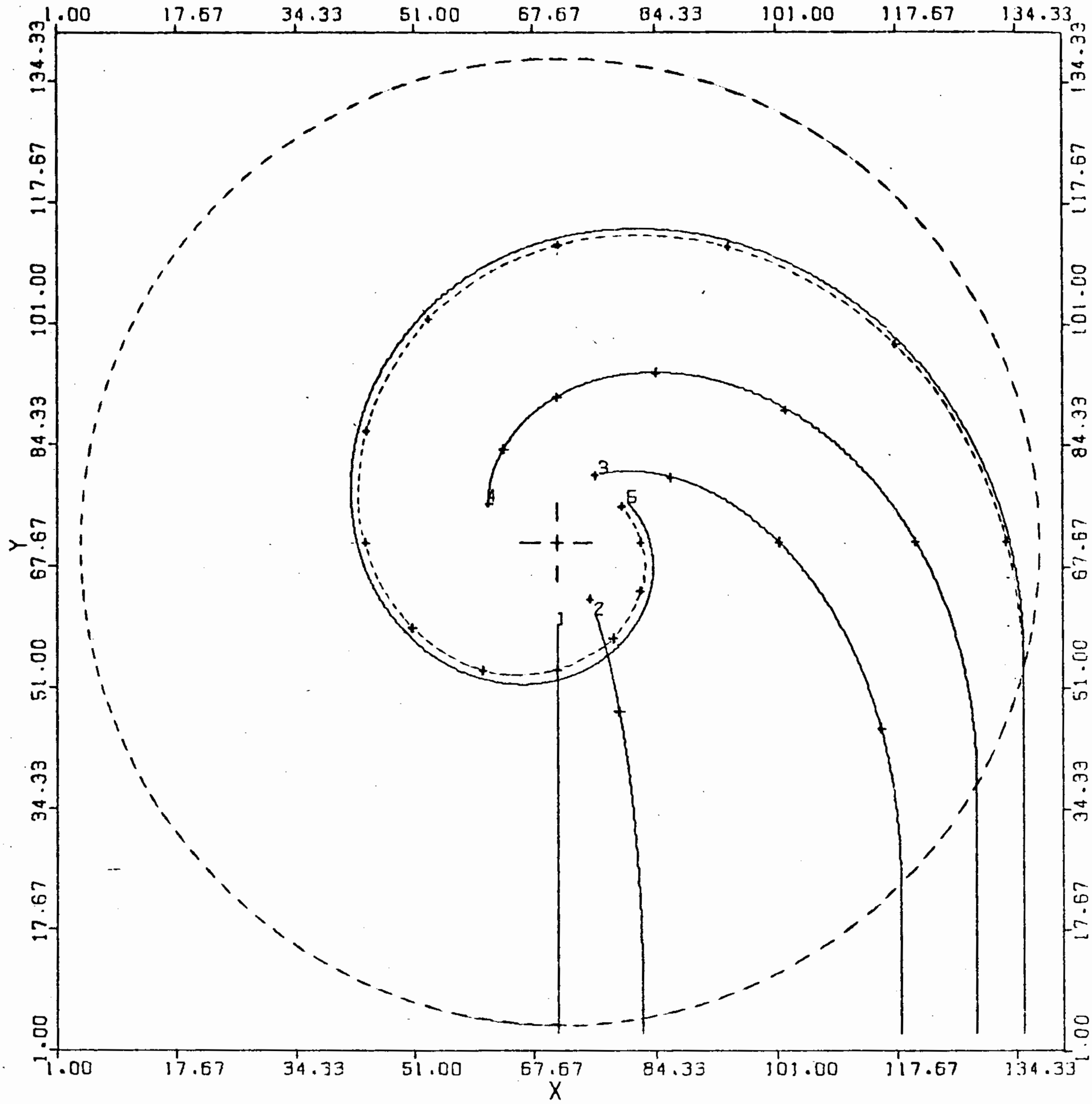
* The program is given in Appendix G.

	θ_o			
θ	10°	45°	60°	75°
10°	200,0			
20°	74,3			
30°	27,6			
60°		153,9	200,0	
90°		91,2	147,8	186,5
120°		54,0	109,3	162,0
150°		32,0	80,8	140,8
180°			59,7	122,4
210°			44,1	106,4
240°			32,6	92,5
270°				80,4
300°				69,8
330°				60.7
360°				52,8
390°				45,8
420°				39,8
450°				34,6
480°				30,1

Table 6.2 : Example 2, a point island.
Radius in meters for orthogonals, for various values of θ.

The comparison of the solution given by the writer's program and the exact solution is shown graphically in Figure 6.9.

It is reasonable to say that this example tests every aspect of a refraction program, as it necessitates the correct modelling of a complex surface, to determine depths and rates of change of depth in the x and y directions, over and above the factors tested by the straight beach example.



<u>Fig 6.9.</u> Example 2, a point island.
Graphical comparison of solution obtained by writer's program, and exact solution.
(Solid lines are writer's solution, dotted lines and crosses represent exact solution, as given by Equation 6.4.)

6.4 EXAMPLE 3 - A CONICAL ISLAND

This example has been used with two grids with the same boundaries but with different grid square sizes to test the sensitivity of the program to grid coarseness.

The geometric features of the island are shown in Figure 6.10.

The two grids are grid 1 and grid 2. They have the parameters listed in Table 6.3.

	Grid 1	Grid 2
S	22,5m	90m
n_x	141	36
n_y	101	26
island centre	(61,51)	(16,13,5)
general pt (x, y)*	$\frac{x}{22,5} + \frac{y}{22,5} + 1$	$\frac{x}{90} + 1,\ \frac{y}{90} + 1$

Table 6.3 : Example 3, a conical island. Grid system characteristics.

* The general point (x,y) represents a point which is x metres from the y axis and y metres from the x axis.

For the purpose of this example eleven orthogonals shall be considered, each with starting angle of zero degrees and with starting position listed in Table 6.4.

A plot was drawn using the writer's program with grid 1, with deep water step length, z_o, set at one metre. The solution obtained should be very accurate in view of the small step length, as discussed in Section 6.2, and the fineness of the grid.

A subsequent plot was drawn using the writer's program with z_o set at 11,25 metres on the same grid. There was no visible difference between the two solutions.

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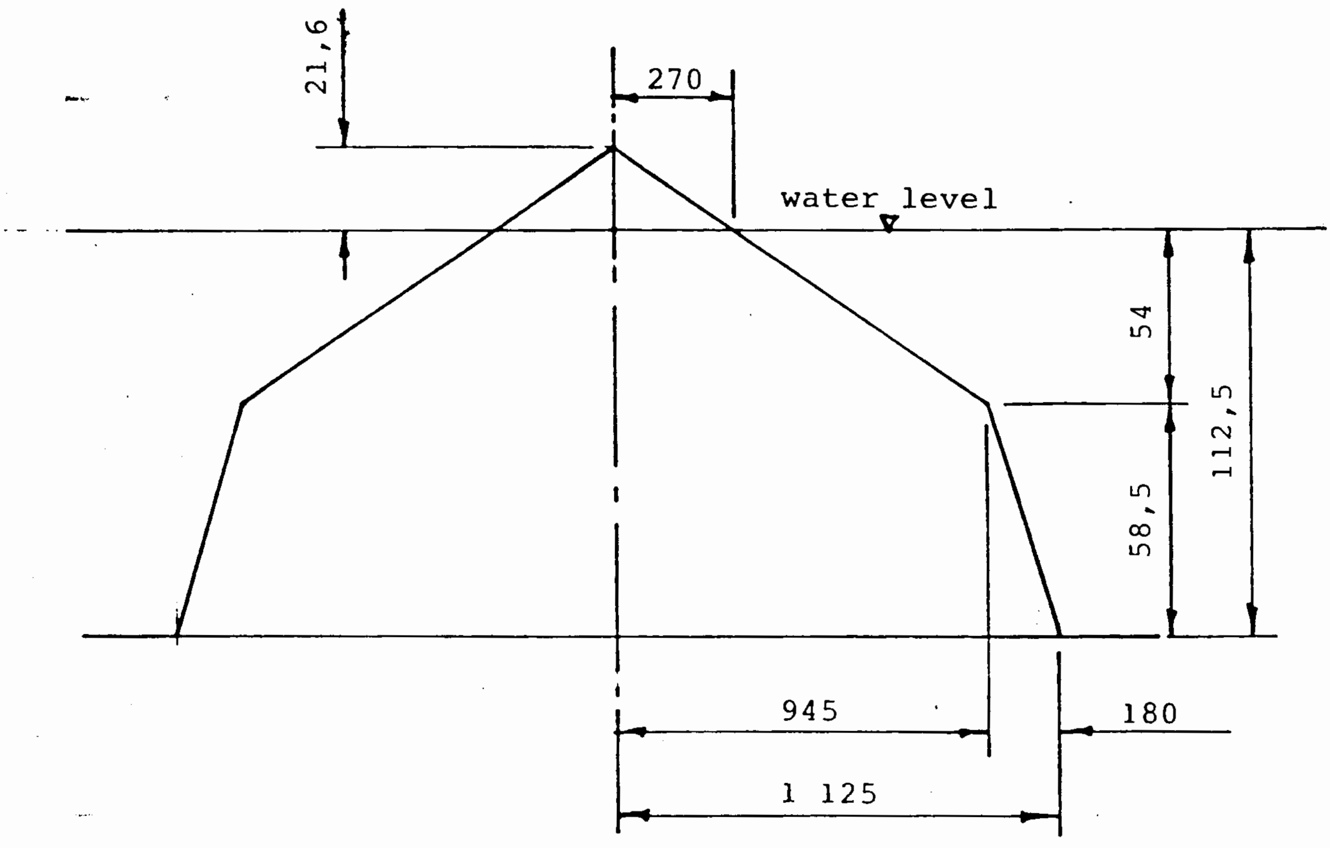


SCALES: HORIZ: 1:20 000 VERT: 1:2 000

DIMENSIONS IN METRES

Fig. 6.10. Example 3, a conical island. Geometric features.

Orthogonal No.	General Starting Position	Grid 1	Grid 2
1	(90, 225)	(5, 11)	(2, 3,5)
2	(90, 405)	(5, 19)	(2, 5,5)
3	(90, 585)	(5, 27)	(2, 7,5)
4	(90, 765)	(5, 35)	(2, 9,5)
5	(90, 945)	(5, 43)	(2, 11,5)
6	(90, 1125)	(5, 51)	(2, 13,5)
7	(90, 1305)	(5, 59)	(2, 15,5)
8	(90, 1485)	(5, 67)	(2, 17,5)
9	(90, 1665)	(5, 78)	(2, 19,5)
10	(90, 1845)	(5, 83)	(2, 21,5)
11	(90, 2025)	(5, 91)	(2, 23,5)

Table 6.4 : Example 3, a conical island. Orthogonal characteristics in the two grid systems.

A plot was drawn, showing the refraction diagrams obtained by both the writer's program, and Wilson's program, each using grid 1, with an initial step-length of 11,25 metres. The solutions are presented in Figure 6.11 where the Wilson solution is a full line, and the writer's solution is a dotted line.

It can be seen that visible differences occur only between the two orthogonal pairs, 1 and 11, and 2 and 10. The differences are relatively small so it would be reasonable to assume the true solution must lie close to these solutions.

A plot was then obtained using the writer's program with the coarser grid and with z_o set at one metre. This example would thus test the effect of a coarser grid on determination of bed slopes and depths.

It was found that a deviation occurred in the orthogonal pair 1 and 11. This difference can be seen in Figure 6.12, where the solid line is the solution obtained using grid 2, and the dotted line is the solution obtained using grid 1. Both grids were used with z_o being one metre.

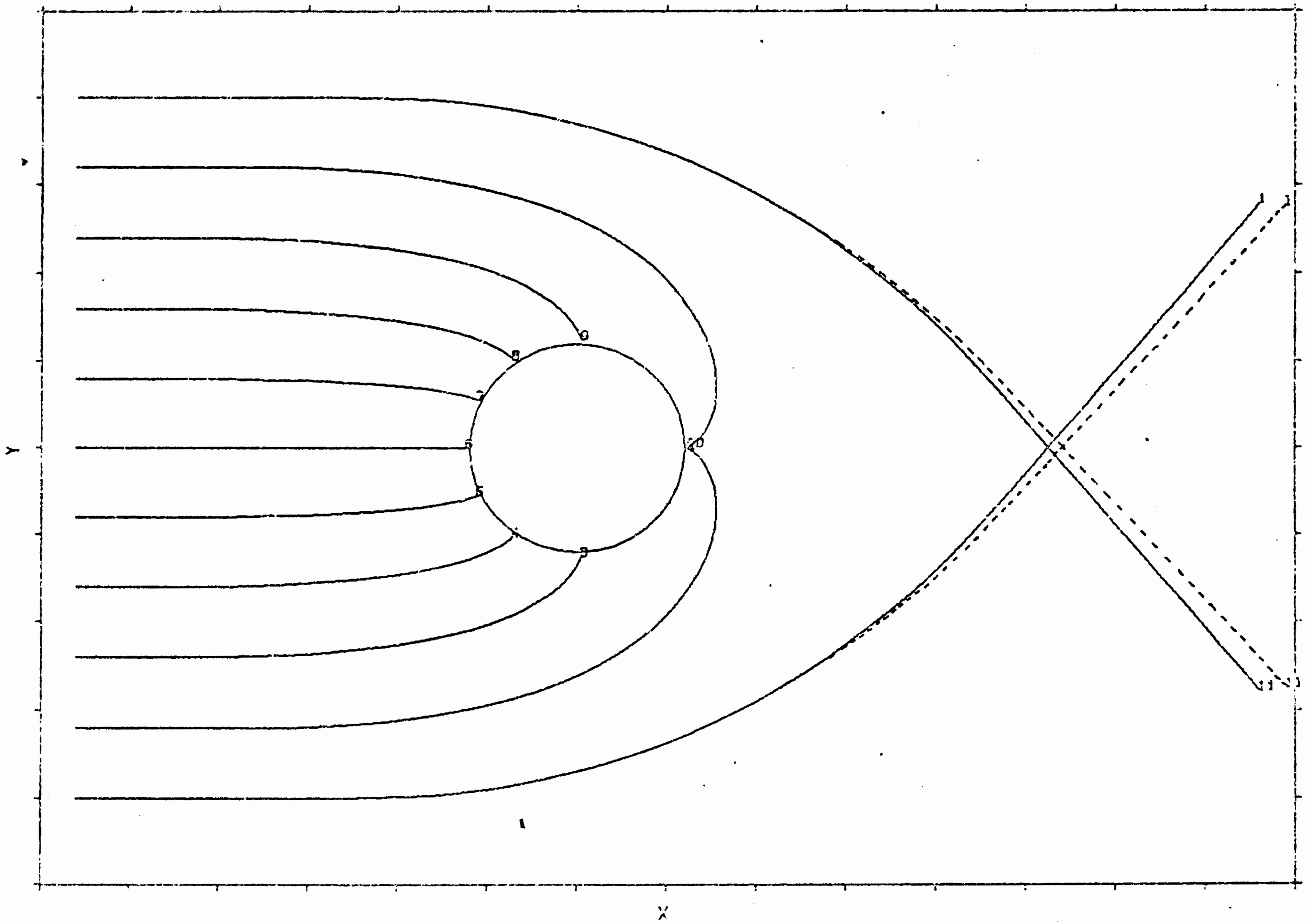
A plot was then obtained using the writer's program on grid 2 with z_o set at 45m. This solution was compared with the solution obtained using the same grid with z_o being 1 metre. It was found that the only difference occurred in the orthogonal pair 2 and 10. This difference was however very small, with divergence only occurring in the very shallow water.

Superposition of Figure 6.11 on Figure 6.13 yields Figure 6.14. This figure represents the complete set of results using both programs on both grids. Writer's solutions are dotted lines, Wilson's solutions are full lines.

The most critical differences occur on the orthogonal pair 1 and 11. It can be seen that the Wilson solutions for the two grids, represented by the solid lines, differ far more than the writer's solutions.

As the writer's two solutions and the Wilson solution using the finer grid are all in close agreement, it is reasonable to assume these solutions are satisfactory. It could thus be concluded that the writer's program seems to give better solutions than Wilson's when using a coarser grid.

In conclusion a full refraction diagram for this example is shown in Figure 6.15. Crest marks are given every twelve seconds, these would thus represent the positions of successive wave fronts.



<u>Fig. 6.12.</u> Conical Island. Comparison of writer's solutions on grid 1 & grid 2, with z_o of 1m.

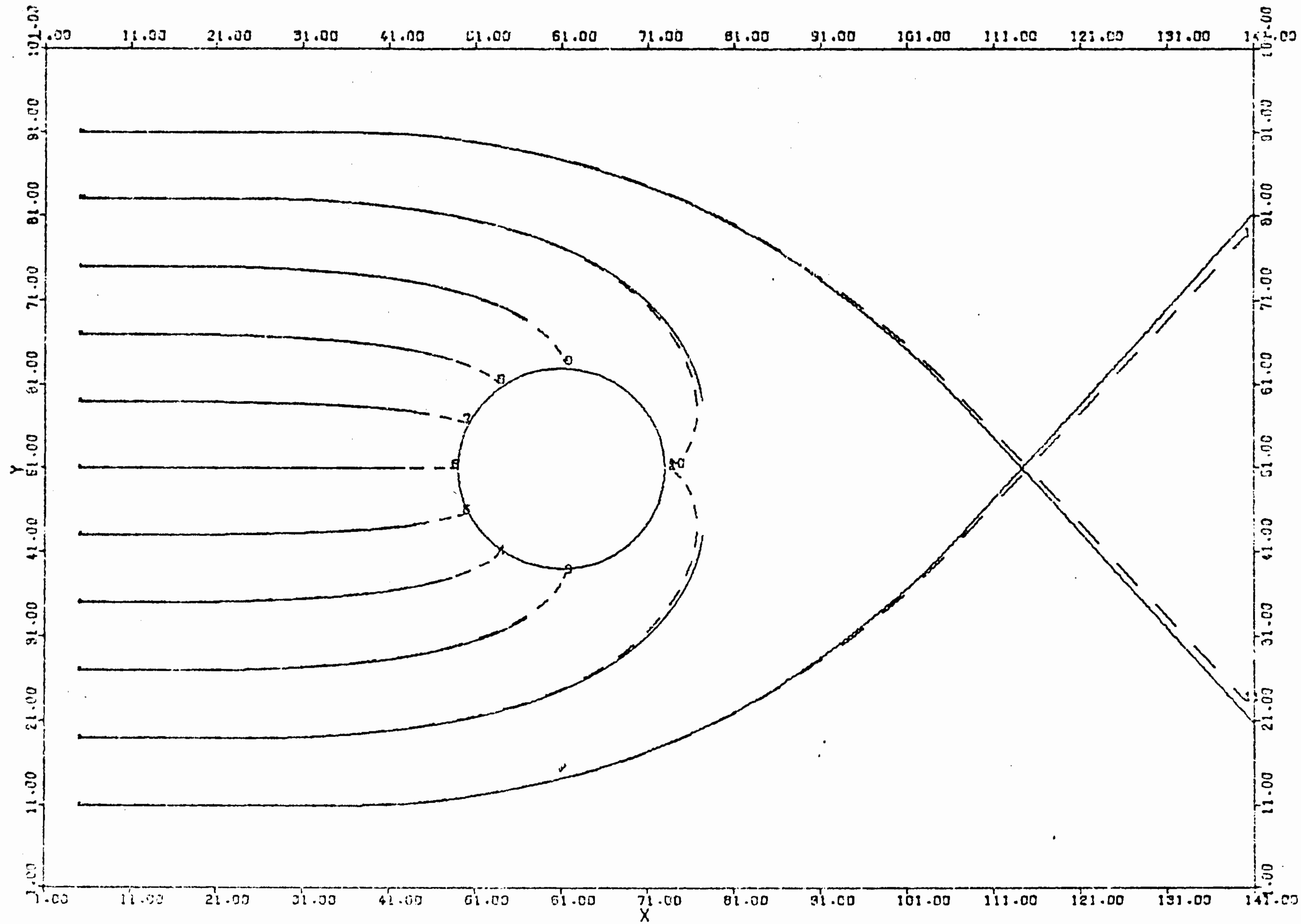


Fig. 6.11. Conical Island. Comparison of Wilson's & writer's solutions, with grid 1 & z_o of 11.25m.

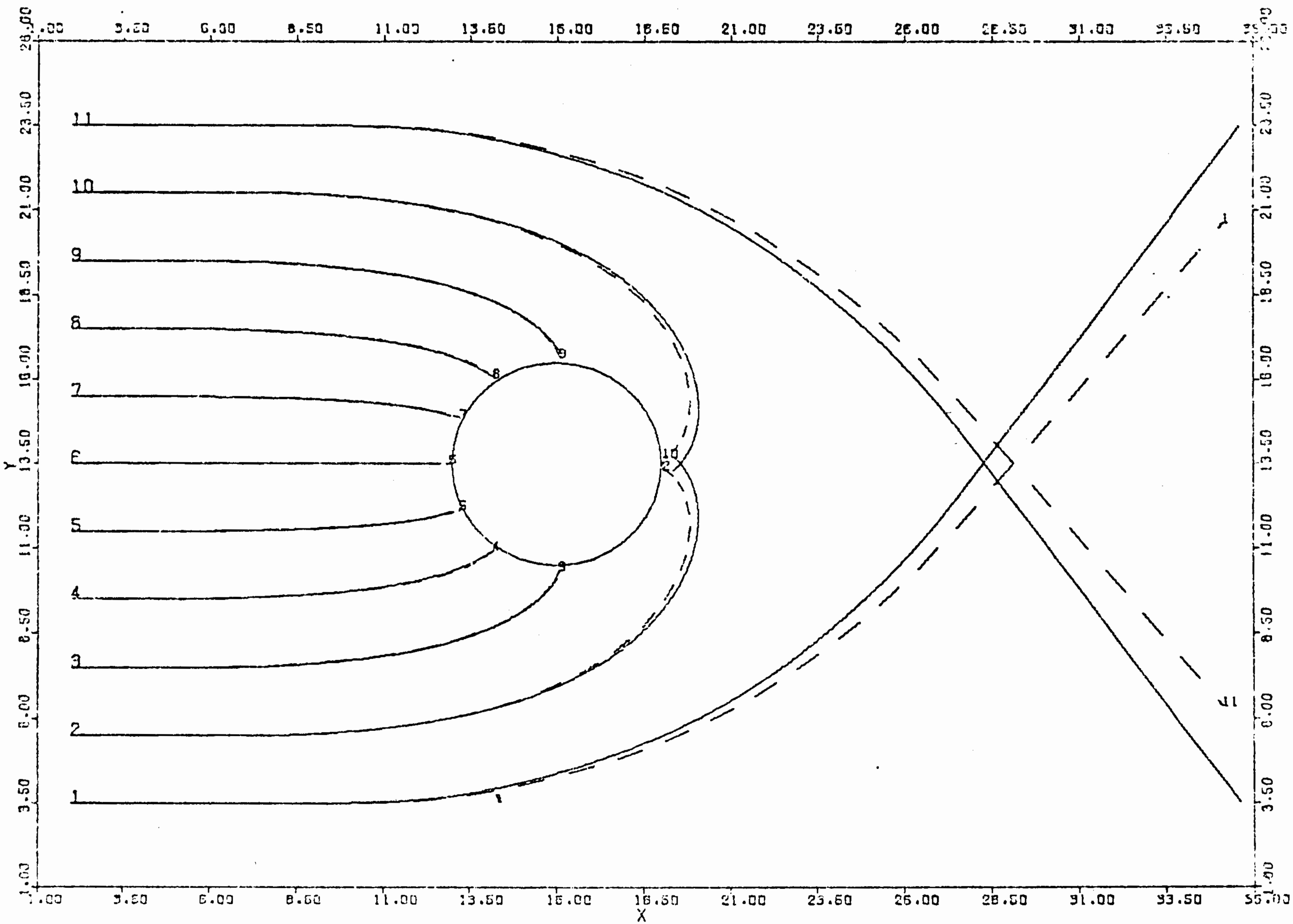


Fig. 6.13. Conical Island. Comparison of Wilson's & writer's solutions with grid 2 & z_o of 45m.

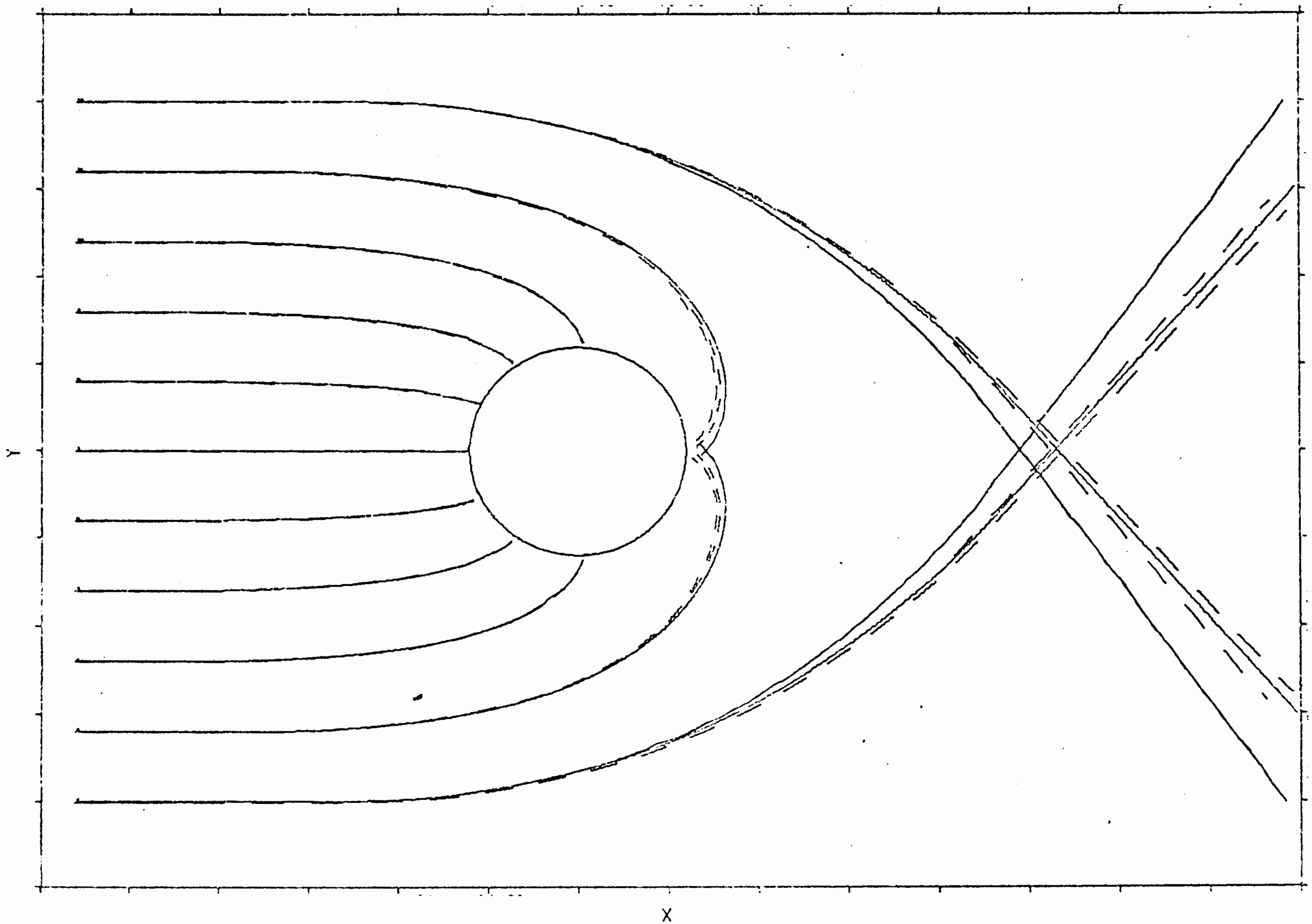


Fig. 6.14. Conical Island. Wilson's solutions & writer's solutions for both grids.

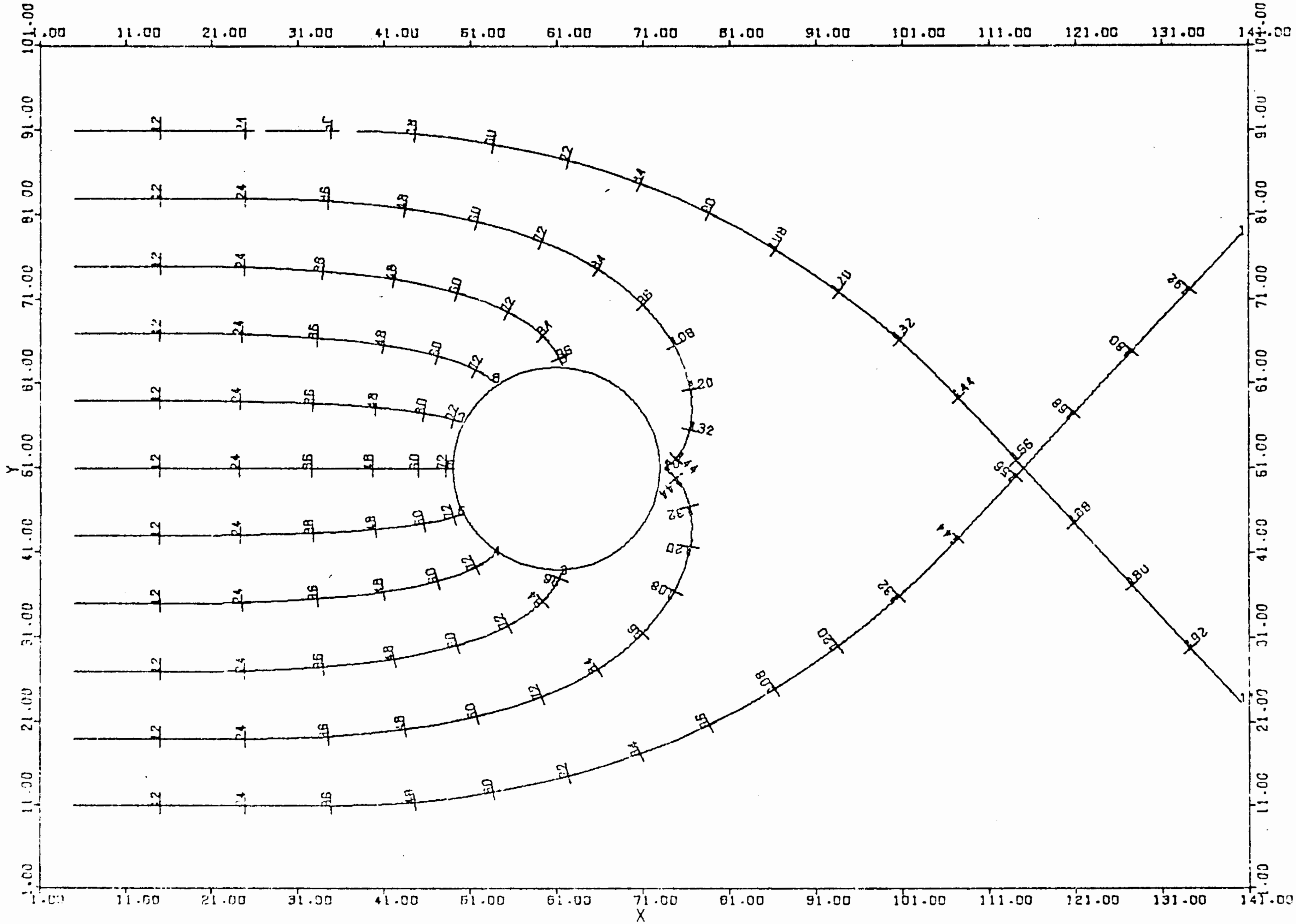


Fig. 6.15. Conical Island. Full refraction diagram, by writer's program, showing consecutive fronts.

6.5 EXAMPLE 4 - A REAL BAY

The bay considered in this example is False Bay, which is situated at the Cape of Good Hope.

A contour map of the area and the selected study area are shown in Figure 6.16.

A detailed contour plan, showing ten metre contours inside the bay is shown in Figure 6.17.

The study area as shown in Figure 6.16 has the following properties.

$$\ell_x = 60\text{km} ,$$

$$\ell_y = 66{,}75\text{km} ,$$

$$n_x = 81 ,$$

$$n_y = 90 ,$$

$$s = 750\text{m} .$$

The actual depth data used in running this example was supplied by the Fisheries Development Corporation of South Africa. This data had previously been used by them in doing a refraction study on the area using Wilson's program.

A typical refraction diagram for False Bay is shown in Figure 6.18.

The diagram pertains to a twelve second wave, with a deep water wave direction of 45^{o}, which approximately simulates the general prevailing conditions.

It is interesting to note that the eastern shore of False Bay is subject to sporadic freak waves, of exceptional wave heights.

This phenomenon is explained by the focusing effect of Rocky Bank (see Figure 6.17) and is evident in the refraction diagram given in Figure 6.18.

Further examples of False Bay, illustrating the program features, are given in Appendix A, section A 2.9.

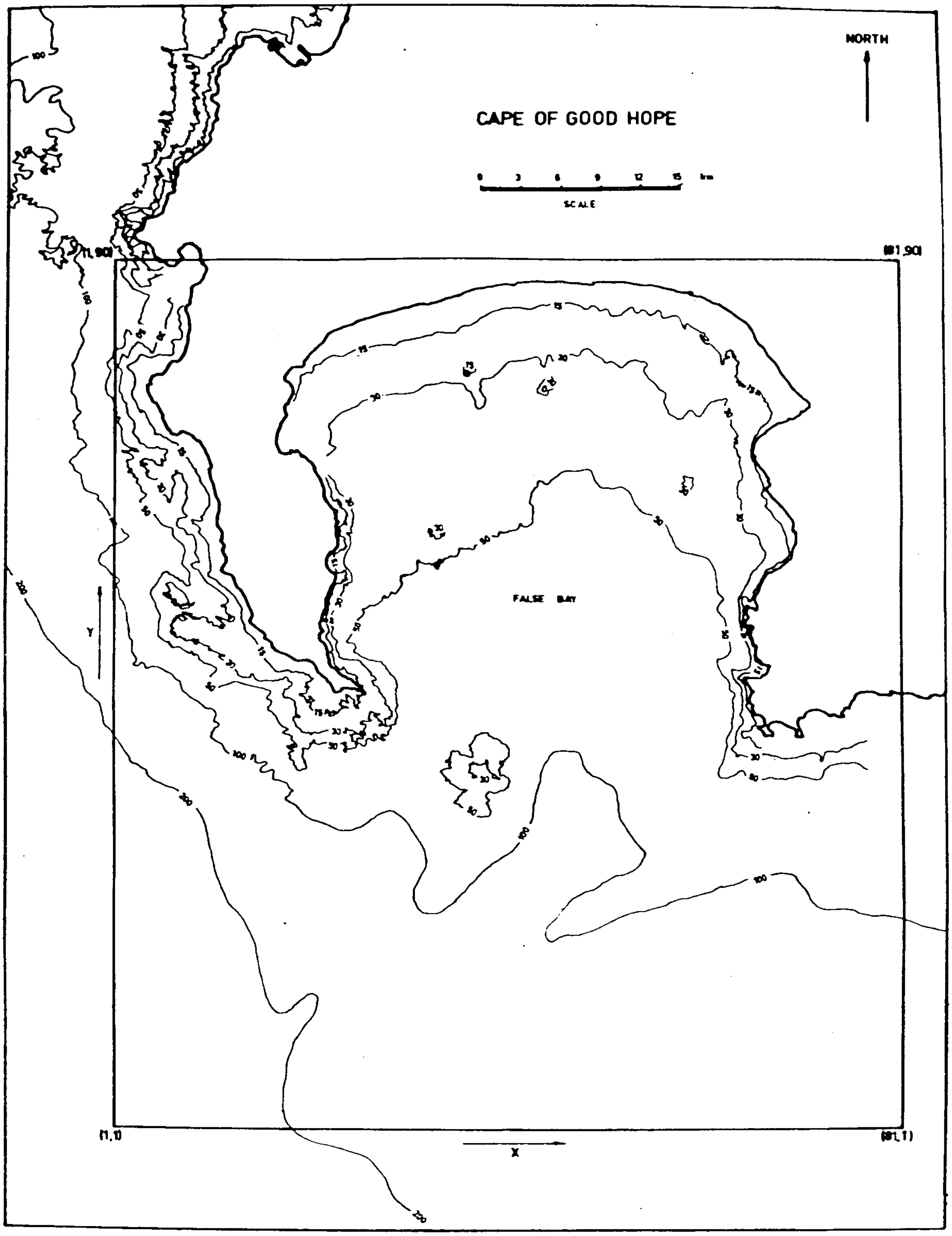


Fig. 6.16. Example 4, False Bay. A contour map of False Bay, showing the selected study area.

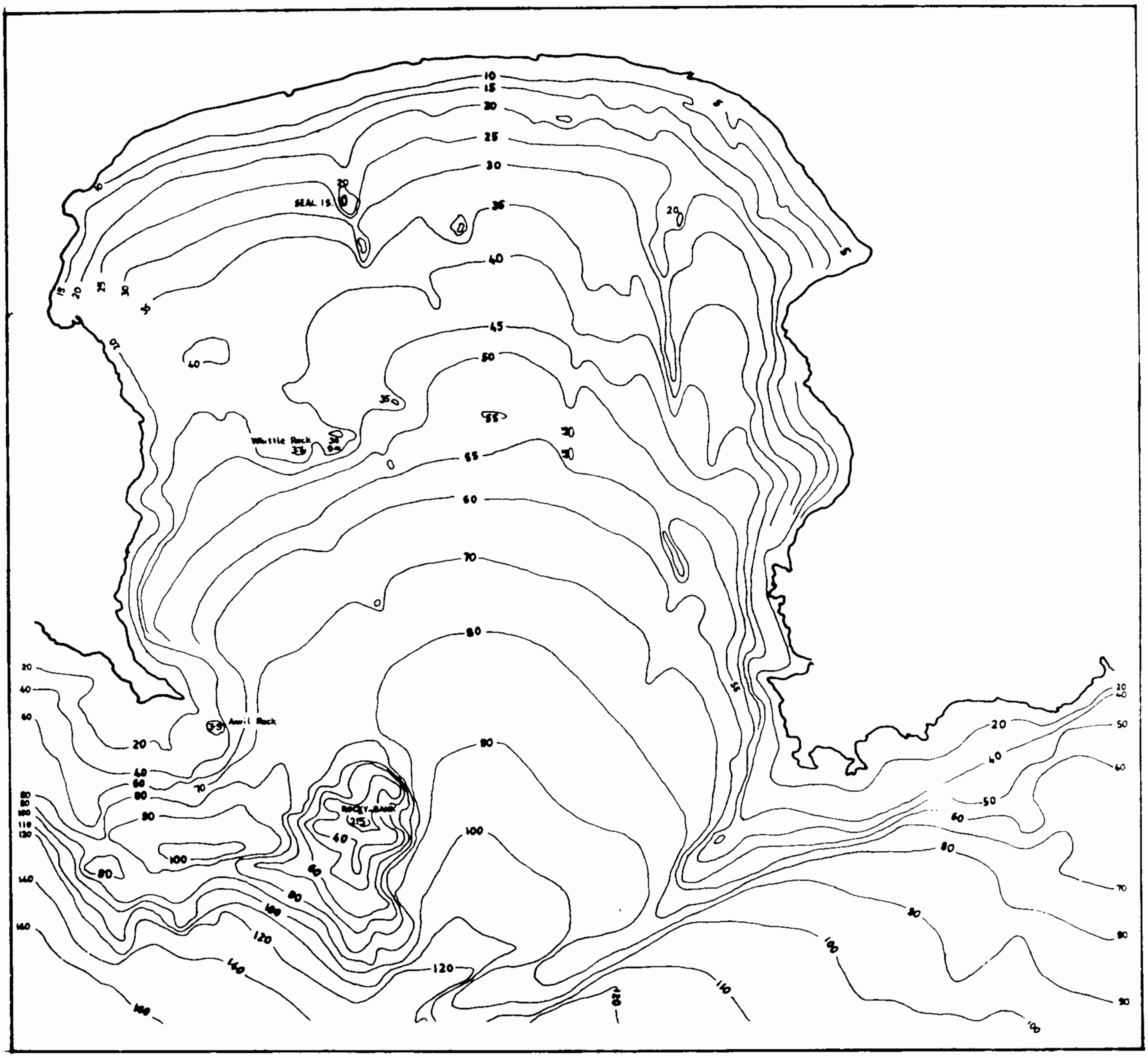


Fig. 6.17. False Bay. Contour map showing ten and five metre contours.

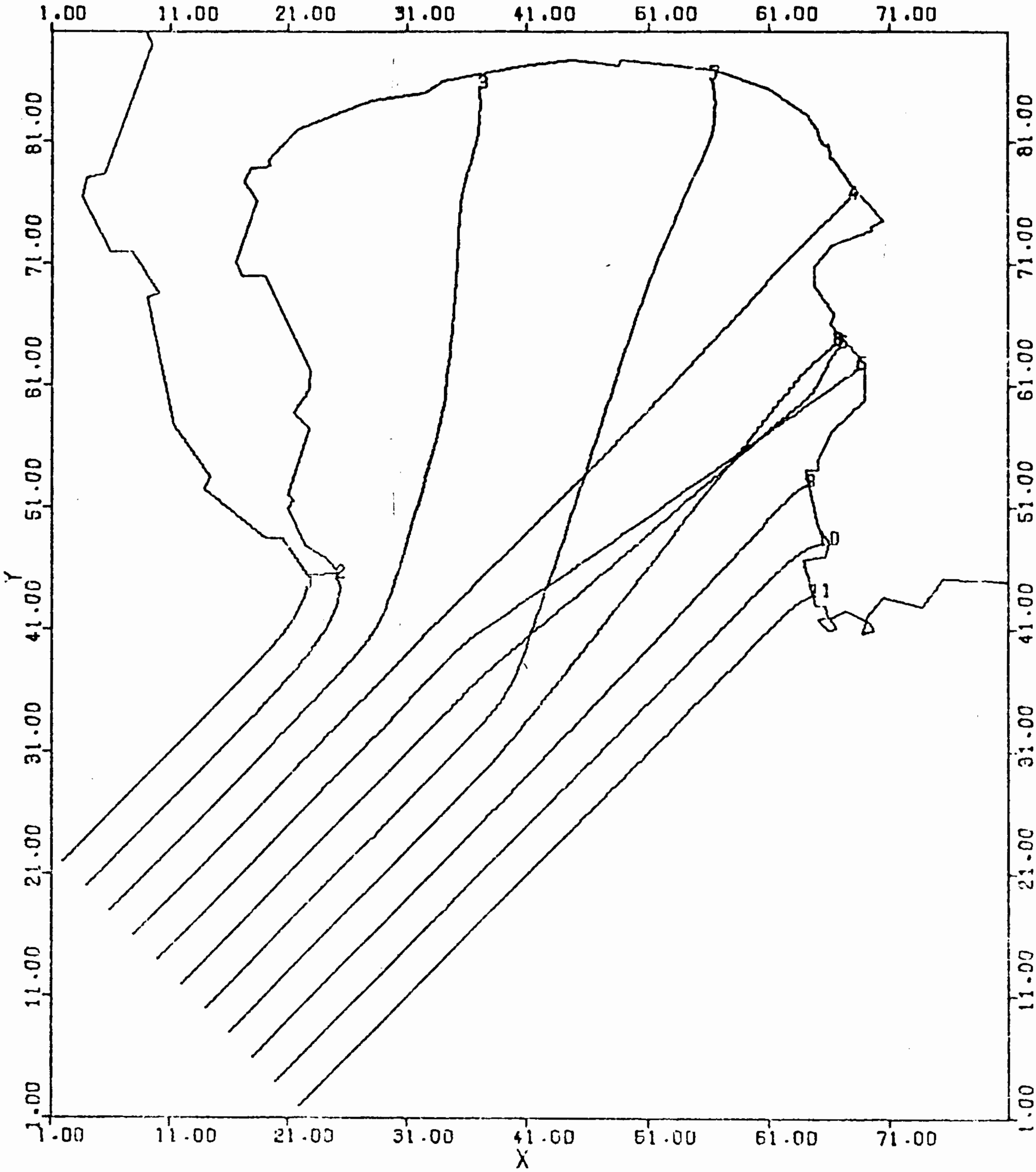


Fig. 6.18. False Bay. A typical refraction diagram.

SECTION 7

THE APPLICATION OF REFRACTION DIAGRAMS

7.1 GENERAL

As previously mentioned a refraction diagram will give the wave direction and wave heights at a point of interest in the study area, resulting from a selected prevailing deep water wave direction, wave period, and wave height.

In reality two main problems arise when considering a refraction analysis. These problems are; firstly, when a wave train travels over a submerged shoal or round an island, orthogonals often cross each other and a caustic condition arises; and secondly, the prevailing deep sea condition is generally not a monochromatic wave train of set height, but is a spectrum of waves with various heights, periods and deep sea directions.

7.2 THE INTERPRETATION OF REFRACTION DIAGRAMS WITHOUT CAUSTICS

According to the standard theory, the refraction coefficient at a point of interest can be obtained from the relative breadths of the relevant orthogonal lane in deep water and at the point of interest.

The refraction coefficient, as described in Section 2.2, is given by;

$$K_r = \frac{b_o}{b_1} \quad . \tag{7.1}$$

Considering Figure 7.1, according to the theory, the wave height at point A shall be considerably bigger than the wave height at point B.

The theory in fact implies that wave heights along a particular wave front shall be discontinuous across orthogonals, unless the breadths of the adjacent lanes are the same width. In

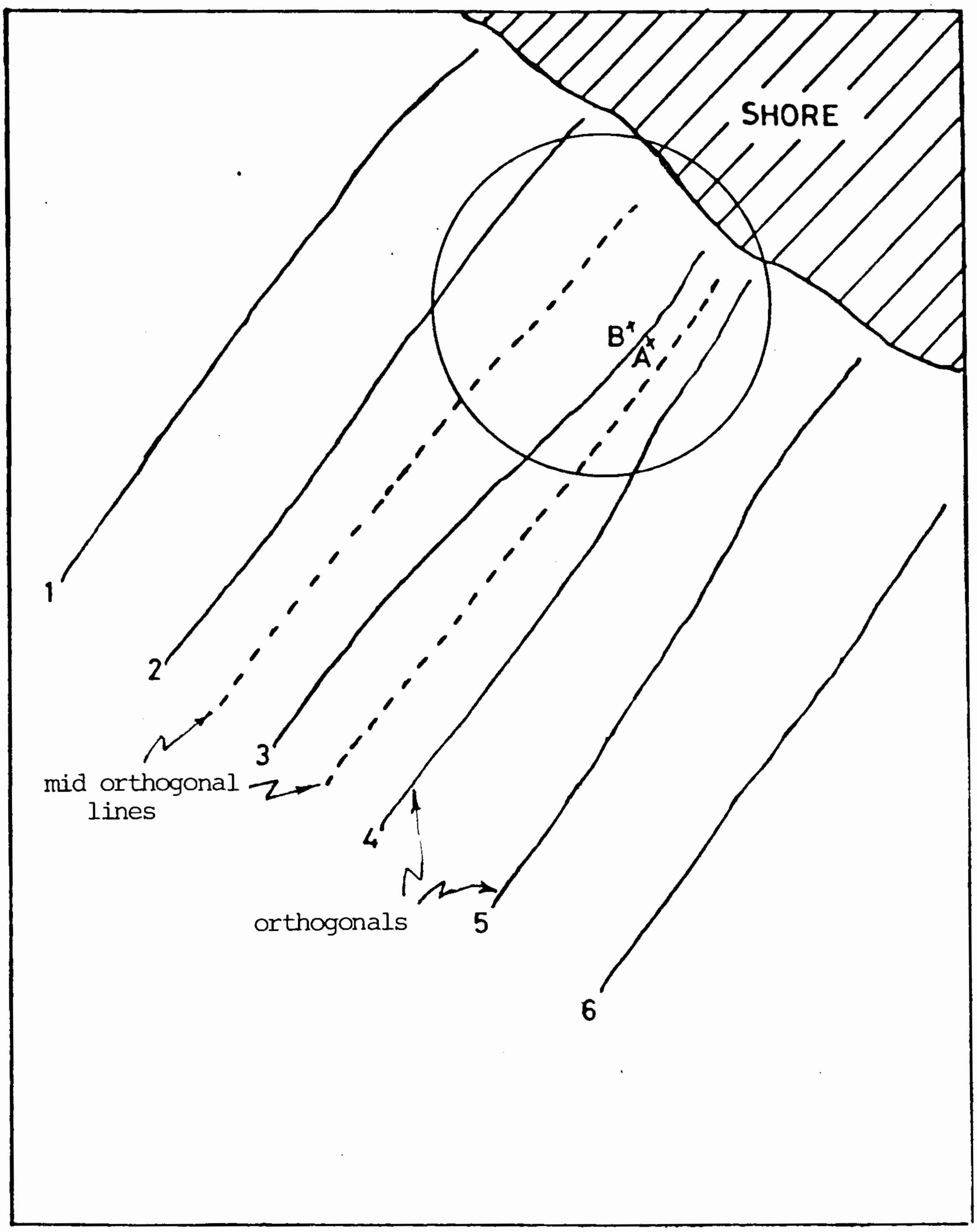


Fig. 7.1. A hypothetical refraction diagram showing mid orthogonal lines.

reality this does not occur. Two suitable methods for overcoming this factor are as follows.

(a) Mid orthogonal method

The two mid orthogonal lines shown in Figure 7.1 represent the lines that shall bisect the wave fronts between the corresponding lanes.

Figure 7.2 shows the wave front that passes through point A. The distance, along the wave front, between A and the two adjacent mid orthogonal lines is d_a and d_b, as shown in Figure 7.2.

A linear interpolation shall give the relevant lane width, b_l, for use in equation (7.1) as;

$$d_l = \frac{d_b}{d_a+d_b} \cdot b_{la} + \frac{d_a}{d_a+d_b} \cdot b_{lb} \quad (7.2)$$

If $d_a = 0$ (that is, point A lies centrally between orthogonals 3 and 4), equation (7.2) gives d_l as b_{la}, as one would expect.

Also, if $d_a = d_b$, (that is point A lies centrally between the two mid orthogonal lines), equation (7.2) gives d_l as being the average of b_{la} and b_{lb} (i.e. $\frac{1}{2}\{b_{la} + b_{lb}\}$) as one would also expect.

This method gives wave height continuity across the orthogonals.

The deep water lane width, b_o, for use in equation (7.1) is given by;

$$b_o = \frac{1}{2}\{b_{ob} + b_{oa}\}, \quad (7.3)$$

where b_{ob} and b_{oa} are the deep water lane widths between orthogonal pairs 2 and 3, and 3 and 4 respectively.

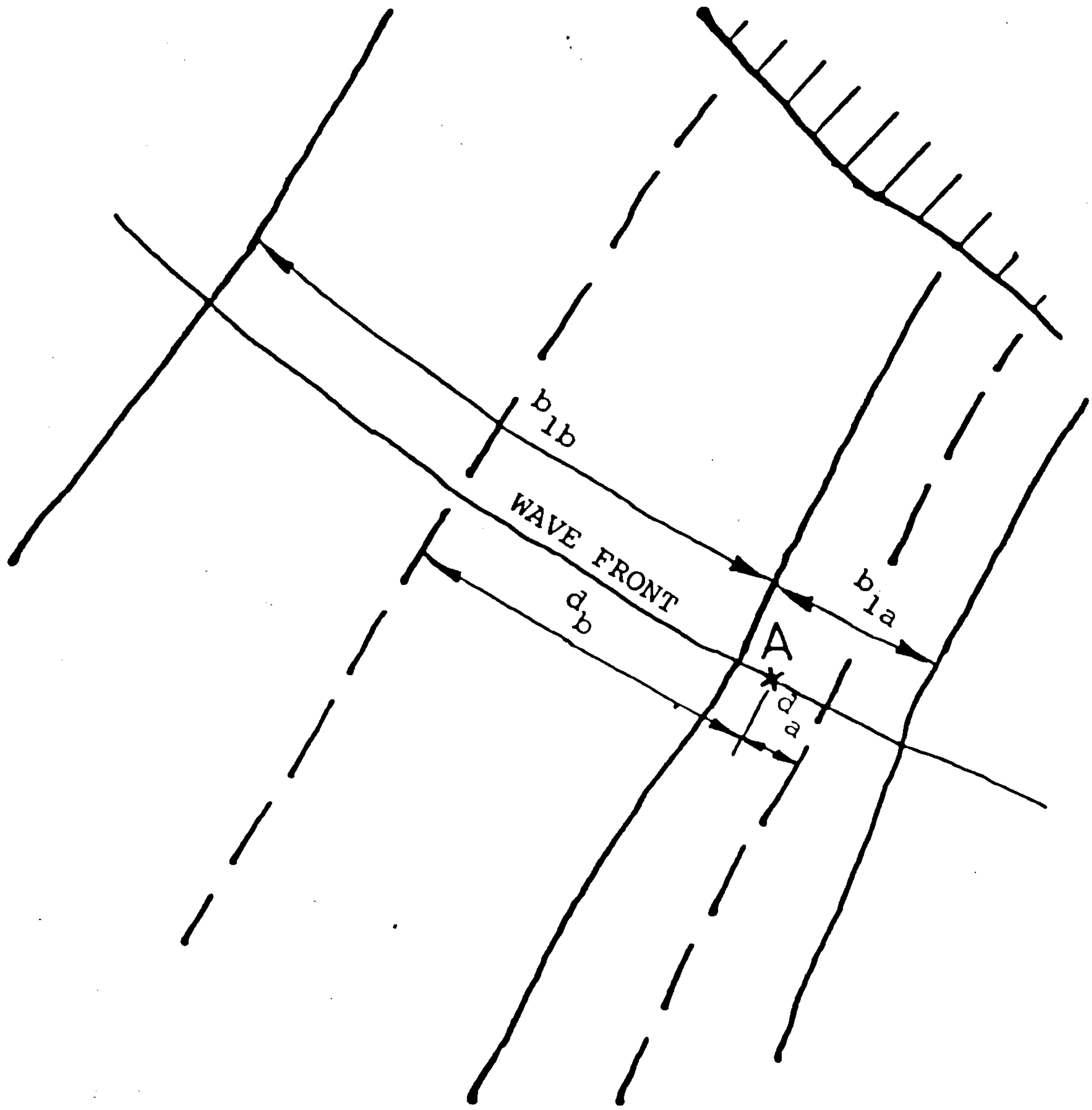


Fig 7.2. Enlargement of circled area in Fig. 7.1.

Generally, however, the values of b_o shall be constant for all lanes.

In a practical example, the mid orthogonal lines need only be determined in the localised region of the point of interest.

(b) Averaging method

This method only applies to a refraction study where all deep water lane widths are the same.

The solution obtained at a point of interest by equation (7.1) is generally dependent on the actual starting positions of the orthogonals in deep water.

Considering Figure 7.3. The solid lines represent Run (a), a typical refraction diagram with equal deep water lane widths. The dotted lines represent Run (b), another refraction diagram with the same deep water lane width as Run (a), but with the orthogonal starting positions off-set by half a lane width.

Clearly the refraction coefficient obtained from Run (a) shall be far greater than that obtained for Run (b). measured where?

The writer's suggestion for obtaining a representative solution for the refraction coefficient at the point of interest, is to perform five individual analyses, each time shifting the starting positions of the orthogonals by one fifth of a lane width, in a direction along the initial wave front.

The refraction coefficient is then given as the mean of the five results obtained from the individual studies.

7.3 THE INTERPRETATION OF CROSSED ORTHOGONALS

The fundamental theory of wave refraction analyses assumes that energy flows perpendicular to the wave fronts, and hence the total energy flow between an adjacent set of orthogonals remains constant. It is thus apparent that if two adjacent

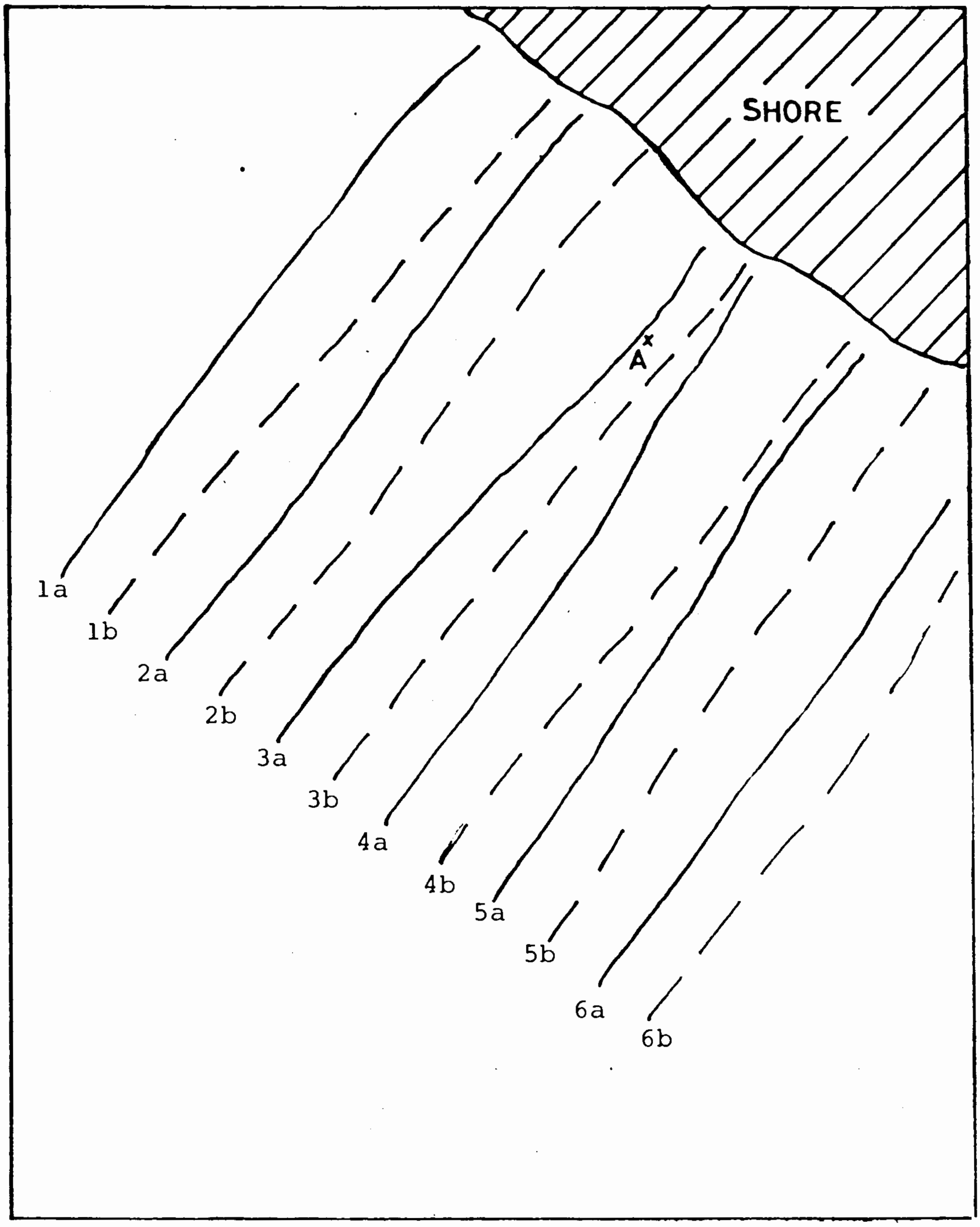


Fig 7.3. A hypothetical refraction study, showing two refraction diagrams with off-set starting positions.

sets of orthogonals cross, at the point of crossing there would be infinite energy flow concentration and thus infinite wave height.

In reality however, the wave front between adjacent orthogonals discontinues when convergence of the two orthogonals begins, this phenomena can be seen in Figure A8,between orthgonals 6 and 7 after travelling over Rocky Bank.

The energy is thus not focused on a single point, but forms what is called a caustic envelope. Furthermore, the convergence of wave orthogonals is accompanied by the generation of a discontinuous wave crest which has a phase lag w.r.t. the original wave front. This phenomena is shown in Figures 7.4 and 7.5 from Pierson (23).

Pierson has discussed the subject in detail, in a memorandum of the Beach Erosion Board (23). After the orthogonals have crossed, and divergence takes place, the distance between adjacent orthogonals is still a reasonable measure of the wave heights.

When orthogonals have passed over a topographical feature that causes caustics, a superposition of wave trains occur.

The writer's opinion on determining the wave parameters at a point of interest, in a caustic condition, for a particular incident wave, is as follows.

(i) Determine the set of resultant wave directions at the point of interest.

This can be done by using a reverse tracing diagram. A set of rays are sent out from the point of interest, with a general angle, θ , and the corresponding deep water wave direction, θ_o, is ascertained for each θ. (When using the computer program the actual angles used will differ by 180^o, as the wave is being reversed.)

Fig.7.4. Shadowgraph for waves passing over a clock glass, from Pierson (23).

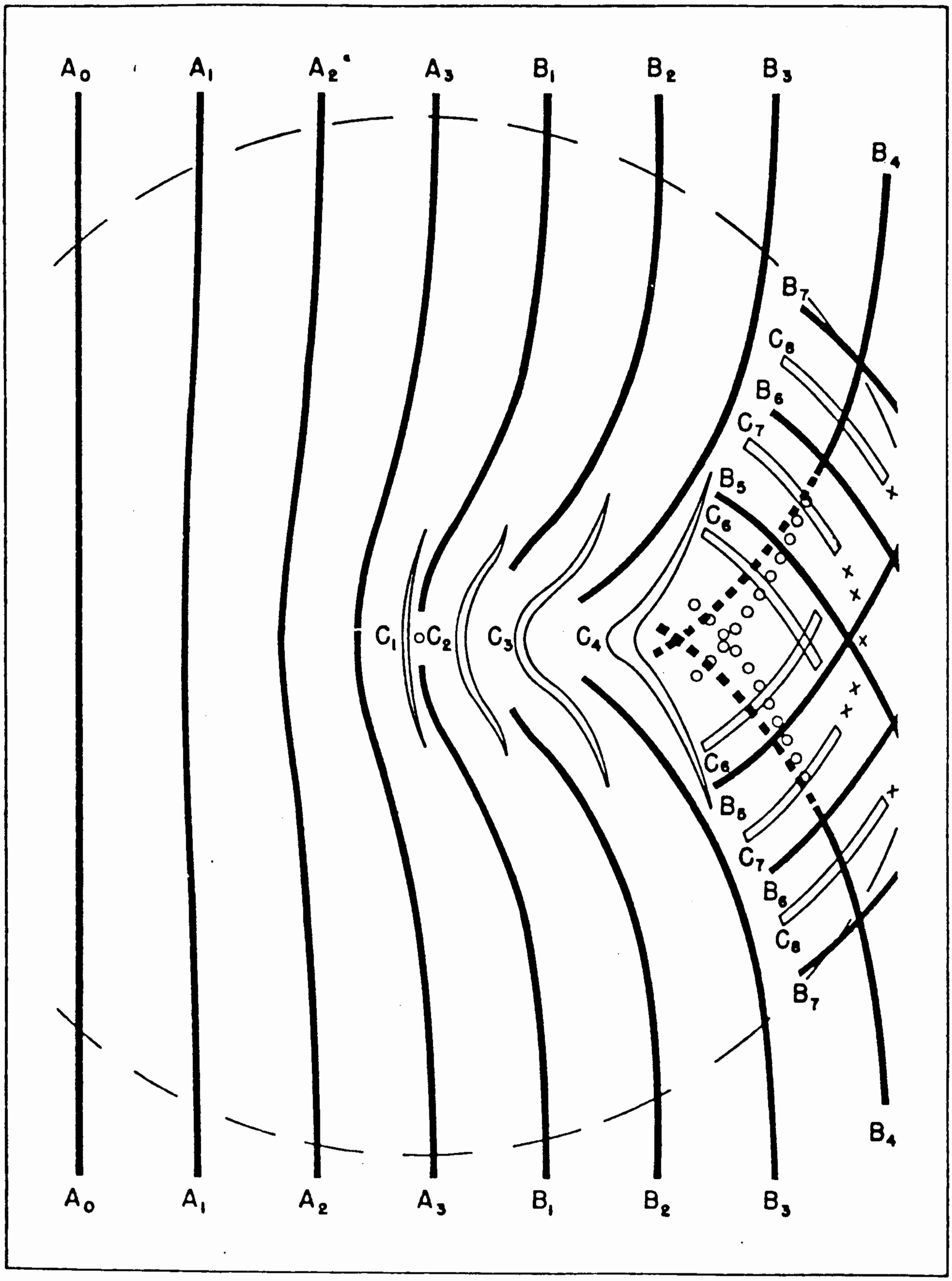


Fig.7.5. Schematic wave crest pattern for waves passing over a clock glass, from Pierson (23).

The values of θ versus θ_o are graphed, and hence the resultant wave directions, at the point of interest, for any incident wave direction can be ascertained.

For example, in Figure 7.6 an incident deep water wave with a deep water direction, θ_o, of 60^o shall have three resultant wave sets, at the point of interest, with directions 73^o, 95^o and 111^o.

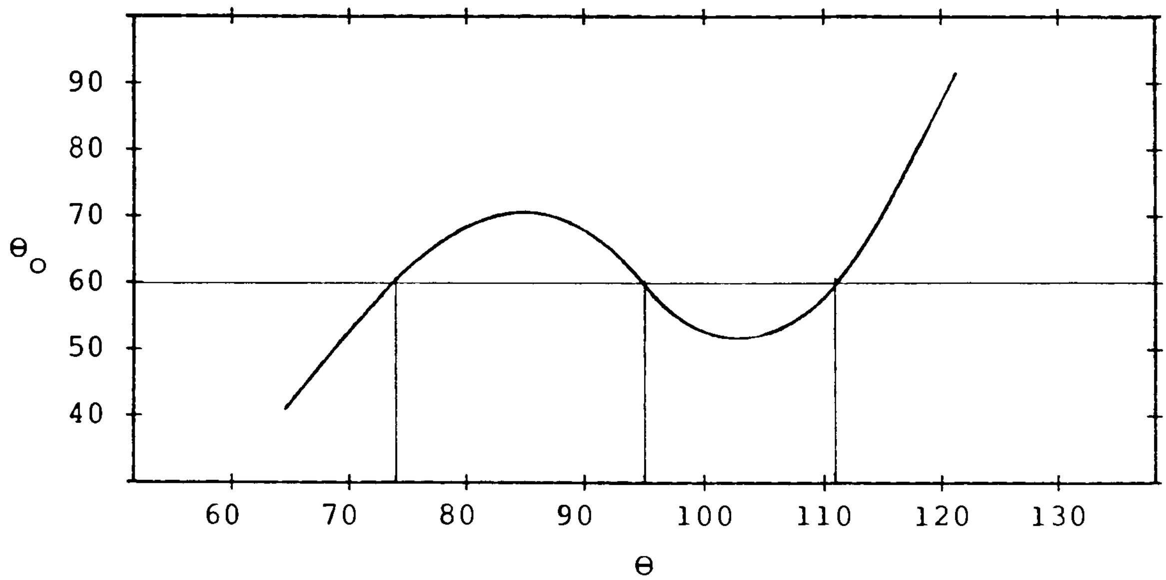


Figure 7.6 : A typical example of θ versus θ_o, for a point of interest, as obtained from a reverse tracing diagram.

(ii) Determine the refraction diagram for an arbitrary set of orthogonal starting positions, with equal deep water lane widths.

(iii) Ascertain between which sets of adjacent orthogonals the point of interest lies, regardless of whether the adjacent orthogonals have crossed or not.

For example, in Figure 7.7, point A lies between orthogonal pairs 2 and 3, 3 and 4, and 4 and 5.

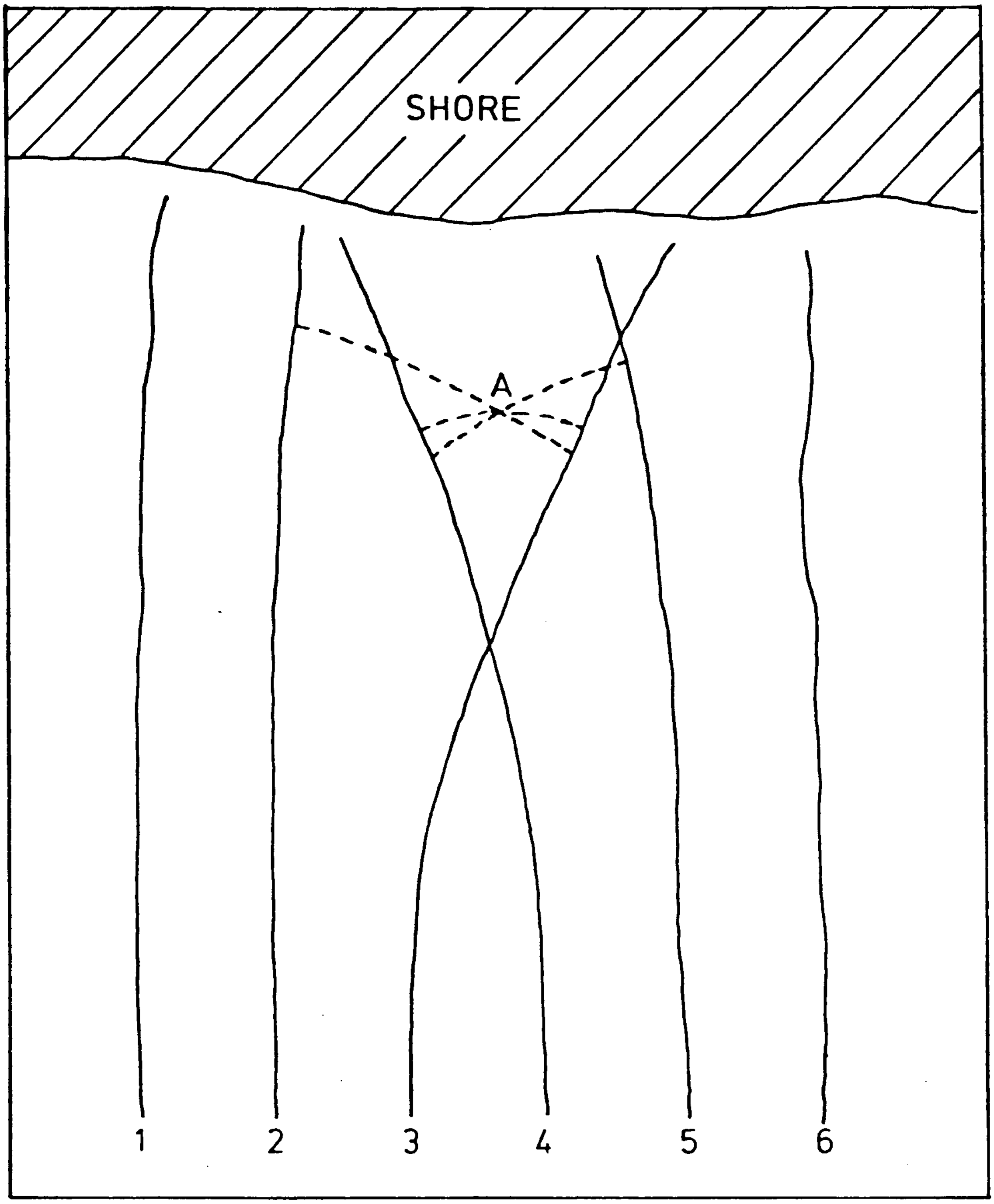


Fig. 7.7. A refraction diagram showing a point of interest in a caustic zone.

These lanes will generally represent the wave sets ascertained in 1.

(iv) Ascertain the refraction coefficient and directions for each set, using equation (7.1), given that if the ratio b_o/b_1 is greater than four, it shall be assumed to be four.

(v) Repeat the analysis five times as described in the averaging method in Section 7.2.

A typical example of the results obtained for a point of interest with three resultant wave sets, for a specific incident wave, is shown in Table 7.1.

ANALYSIS	SET 1		SET 2		SET 3	
	K_r	θ	K_r	θ	K_r	θ
1	1,5	98°	0,3	73°	0,6	112°
2	1,7	96°	0,3	78°	0,9	108°
3	*	*	*	*	1,2	115°
4	0,9	91°	0,1	69°	0,8	109°
5	1,2	93°	*	*	*	*
MEAN	1,3	95°	0,1	73°	0,7	111°

* Indicates that for that particular analysis, the point of interest did not fall inside an adjacent set of orthogonals representing the corresponding wave set. For K_r determination assume the value to be zero, for angular determination neglect when calculating the mean.

Table 7.1 : A typical example of wave parameters, at a point of interest, obtained from a single refraction analysis.

The mean angular values obtained should correspond with those ascertained in part (i).

The wave condition at the point of interest shall be a superposition of the wave sets ascertained.

7.4 THE APPLICATION OF MONOCHROMATIC WAVE REFRACTION DIAGRAMS TO WAVE SPECTRA

The prevailing deep water wave parameters are usually given as continuous wave spectra of heights, periods and directions.

These spectra can be approximated by a finite sum of discrete long crested waves. These waves can be each separately refracted and then recombined to approximate the resultant refracted spectra at the point of interest.

An explanation of the method is given by Pierson, Neumann and James (24).

SECTION 8

EVALUATION AND CONCLUSIONS

8.1 EVALUATION OF OBJECTIVES

The objectives of the program were given in Section 1.7. A brief discussion of the attainment of these objectives is given below.

i. Accuracy: A set of examples were given in Section 6, which tested all aspects of the programs accuracy. The results show that the program is sufficient to give an accurate representation of the refraction phenomena implied by the basic wave theory.

ii. Output: The user can select any time interval of orthogonal travel, such that the orthogonal parameters are printed at each multiple of that interval. The parameters are: the x and y co-ordinates; the wave direction in degrees as a cartesian angle, and if desired, as a compass bearing; and the time of travel in seconds.

    The plotted output can take one of three forms: (a) a plot of the orthogonals alone, (b) a plot of the orthogonals with selected wave front directions, and positions being shown (at multiples of a selected time value), or (c) a plot of the orthogonals showing the wave front directions with the time of travel being printed adjacent.

    Examples of printed and plotted output are given in Appendix A, Section A2.9.

iii. Data: All data is entered in free format, as this is the simplest method of data insertion. A comprehensive documentation of the data input is given in Appendix A, Section A2.5.

iv. Users Manual: A comprehensive users manual is given in Appendix A. The documentation of the program was done according to the American Society of Civil Engineers (ASCE) Engineering Computer Program Documentation Standard. This standard is included in Appendix F.

v. Internal documentation of the program: The source program listing, which is given in Appendix A, Section A3.3, includes a set of comment cards.

At the start of each subroutine, the function of the subroutine is given. All major program operations are preceded by a short explanation of the operation being performed.

vi. Program speed: The speed of execution is discussed in Appendix A, Section 4.5. This program operates accurately with sufficient speed to be economically viable for practical use.

The program itself has been used successfully for a refraction study by Gara (10), for a B.Sc Civil Engineering Thesis at the University of Cape Town in 1976.

The program was subsequently used by Rosenthal (26) for teaching purposes in a postgraduate M.Sc course on Coastal Engineering Practice in 1977.

8.2 SCOPE FOR MODIFICATIONS

The basic limiting factor for a computer refraction study, is the size of the depth matrix, which corresponds to the total number of nodes in the study area.

The writer's program has been written with a databank core space of 27K, of which 22½K is for the depth matrix.

When specific depths are wanted, the corresponding values in the depth matrix are used. This is the most efficient way of running the program, but the permissible size of the study area, in combination with the fineness of the grid, is limited by the core space of the computer.

An alternative, though inefficient, method exists whereby a study area, with a depth matrix of unlimited size can be considered.

The method, in its simplest form, is as follows:
The depth data is written into a data file in which each line represents a nodal depth. The sequence of inserting the data is the same as that described in Appendix A, Section A2.4.

If the program wants to use a depth h(i,j), this depth is represented by line N in the data file, where N is given by;

$$N = (i - 1)n_y + j \quad , \qquad (8.1)$$

where n_y is the number of y co-ordinates in the study area.

The program can then be made to read a variable, h, N times, starting at the beginning of the data file.

When this has been performed, the value of h stored by the computer shall be the correct value of h(i,j).

Each increment requires the reading of twelve depth values, hence the inefficiency of the method can be easily appreciated.

More complicated methods, which are in turn more efficient, exist for performing this operation.

One such method is to write every set of depth values, for a specific x value, into a data element of the program file. There shall be thus n_x data elements.

The jth line of the ith data element shall thus correspond to the depth h(i,j).

When using this method, the appropriate data element is inspected, hence less data needs to be read to get to the correct value. The operation shall thus be quicker than that mentioned previously.

The most efficient method is to use random access files, or direct access files.

These methods are however very hardware dependent, for example a method developed for a Univac computer will not be compatible with an IBM computer.

For the Univac the direct access files form part of the ERTRAN system. (Exclusive to the Univac.).

The method employed is to set up a file with a specified number of records, each record having a specific number of words.

Logically the specified number of records would correspond to the number of x co-ordinates, and the number of words per record would correspond to the number of y co-ordinates.

When an increment is projected, with the central point of the increment being (x,y), the twelve depths needed for calculation of the curvature fall in four sets as shown below:

Set (i)	h(i-1,j); h(i-1, j+1)
Set (ii)	h(i,j-1); h(i,j); h(i,j+1); h(i,j+2)
Set (iii)	h(i+1,j-1); h(i+1,j); h(i+1,j+1); h(i+1,j+2)
Set (iv)	h(i+2,j); h(i+2,j+1)

(See Figure 3.3).

Each set has a specific integer x value. For computation the computer shall in turn access each specific record into core and select the desired depth values.

For example, when record (i+1) - set (iii) - is accessed from the direct access file, words no (j-1), (j), (j+1) and (j+2) are selected and recorded as the appropriate depths.

It can thus be seen that at any time the maximum number of depth values using core space is n_y.

This will enable a study area of almost unlimited magnitude to be investigated.

These methods need only be employed when a study area of great size and grid fineness is to be investigated, as the method used by the writer's program will suffice, and be superior, in most practical applications.

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A P P E N D I C E S

ERRATUM

Page No. A25, of Appendix A does not exist.
Script reads from A24 directly to A26.

APPENDIX A

USERS MANUAL

FOREWORD

The structure of the program documentation herein has been based on the proposed ASCE Standard for Engineering Computer Program Documentation, as described by the Sub-Committee on Program Documentation of the Committee on Computer Applications of the Soil Mechanics and Foundations Division.

A copy of the standard is given in Appendix F.

Figure A1 shows an outline of the standard.

AMERICAN SOCIETY OF CIVIL ENGINEERS (ASCE) Engineering Computer Program Documentation Standard			
1.0	PROGRAM IDENTIFICATION		
1.1	Program Title	1.6	Updates, Version
1.2	Program Code Name	1.7	Source Language
1.3	Writer	1.8	Availability
1.4	Organisation	1.9	Abstract
1.5	Date		
2.0	ENGINEERING DOCUMENTATION		
2.1	Narrative Description	2.6	Printed Output
2.2	Method of Solution	2.7	Other Output
2.3	Program Capabilities	2.8	Flowchart
2.4	Program Options	2.9	Sample Run
2.5	Data Input		
3.0	SYSTEM DOCUMENTATION		
3.1	Computer Equipment	3.5	Data Structures
3.2	Peripheral Equipment	3.6	Storage Requirements
3.3	Source Program	3.7	Maintenance and Updates
3.4	Variables and Subroutines		
4.0	OPERATING DOCUMENTATION		
4.1	Operator Instructions	4.4	Error Recovery
4.2	Operating Messages	4.5	Run Time
4.3	Control Cards		

A1 PROGRAM IDENTIFICATION

A1.1 Program Title: Calculation and plotting of ocean wave refraction.

A1.2 Program Code Name: REFRAC

A1.3 Writer: A. Wale

A1.4 Organisation: University of Cape Town, Department of Civil Engineering.

A1.5 Date: May 1977

A1.6 Updates: None Version No.: 0

A1.7 Source Language: Univac 1106 Fortran V.

A1.9 Abstract: The Program computes and plots orthogonal paths for selected orthogonals pertaining to a wave of set period and deep water direction travelling over known bottom topography.

Orthogonal starting positions and directions may be individually read, but if starting positions are to be equally spaced along a straight wave front (all orthogonals having same starting direction), then starting positions may be computed.

The rectangular study area is subdivided into a square grid system, with water depths being given as data at each grid node.

Orthogonal directions may either be cartesian, (measured anti - clockwise from the horizontal x-axis), or compass bearings, (measured clockwise from north).

A tidal shift can be entered in order to investigate the effect of tide on refraction.

Orthogonal computation is based on incremental progression. For each increment the water depth is estimated by fitting a first degree surface (hyperbolic paraboloid) to the relevant grid square. Bed slopes are estimated by performing a quadratic interpolation between the adjacent grid square surfaces. Wave celerities and orthogonal curvatures for the increment are hence computed according to Airy's classic wave theory.

The numerical results may be plotted automatically. Selected integer time multiple values of wave position and direction are printed and corresponding wave front orientations on each orthogonal may be plotted on the refraction diagram.

A2 - ENGINEERING DOCUMENTATION

A2.1 Narrative Description: For a description of the wave refraction problem the reader is referred to Section 1.

For the application of the program to the solution of the wave refraction problems, the reader is referred to Sections 5 and 7.

A2.2 Method of Solution: For the method of solution of the wave refraction problem the reader is referred to Sections 2, 3, 4 and 5.

A2.3 Program Capabilities:

(i) Range of wave periods

The program has been developed to solve practical cases of ocean wave refraction. As a typical refraction analysis involves numerous calculations of the wave celerity (each time an increment is progressed) the writer has decided not to solve the implicit wave celerity equation (Eqtn. 2.3) for each increment, but rather to initially compute wave celerities for set depths, and subsequently, during computation of orthoganal projections, to determine celerities by interpolation (see Section 5.5).

The advantage of the method is that because interpolation is far quicker than solving an implicit relationship by an iterative method, the total computer time used to calculate wave celerities shall be less in a normal application, where thousands of increments are computed.

In the program the set depths considered are at half metre intervals from zero to 10 metres, and at one metre intervals from 10 metres to deep water.

It is thus apparent that unless adapted, the program is not suitable for obtaining refraction diagrams for model studies, (unless the model is dimensionally scaled up to represent true ocean parameters).

As the program stands the dimension of the array of depths is three hundred, which corresponds to a wave with a critical depth of about two hundred and eighty metres. This represents a nineteen second wave.

The program would thus not be suitable for studying Tsunamis waves, which have a period of about 15 minutes, but could easily be modified by adapting the program to interpolate between larger depth intervals (e.g. instead of one metre intervals, take five metre intervals).

If wave periods of up to a minute are to be investigated the dimension of the array of wave celerities can be increased to three thousand. A one minute period wave has a critical depth of two thousand eight hundred metres (see Eqtn. 4.2).

As it stands however, the program is capable of analysing any wave with a period in the normal range.

(ii) Size of grid

The program reads, as data, the set of depth values at each grid node - as discussed in Section 3.1. This data is stored in a two dimensional matrix.

The matrix has a general element h(x,y). The maximum permissible size of the matrix, as shown in the program listing dimensions, is 150 x 150 elements. This constitutes a core space of 22½K.

These limiting dimensions can be changed by editing the dimension statements in the main program and subroutines. This may be necessary for a particular study.

There is no need for this matrix to be dimensioned as square. For example; if the study area is rectangular the dimensions could be altered to 200 x 100.

As a general rule the limiting dimensions should be such that the total number of elements does not exceed 10K less than the available core space for the computer being used.

(iii) Number of orthogonals

The starting parameters, namely x,y, and direction values of the orthogonals, are read, and then stored in three vectors - one for x, one for y, and one for direction. Each vector has limiting dimension set at 100. This corresponds to a maximum number of orthogonals at 100.

The figure was arbitrarily chosen as the maximum number of orthogonals needed for a single refraction diagram. If a particular study requires the analysis of more orthogonals, the relevant dimension statements may be altered accordingly.

(iv) Maximum length of plot

The maximum length of time for which the actual plot may be run is set at 40 minutes. (See card No. 47 of subroutine REFRAC.STPLOT If longer plots are required, this card may be changed accordingly. If the card is omitted, the computer shall assume a maximum plot of 20 minutes.

(v) Position of orthogonal starting points

For refraction calculations the bed slopes are computed from the adjacent grid squares' nodal depths. The orthogonal starting positions must thus have a minimum margin, from any boundary, of one grid square length. It should be noted that the grid origin for this program is not (0,0), but (1,1).

For example; take a study area with the x axis boundary being two hundred metres long, and with a grid spacing of two metres. The permissible range of x values would thus be from x = 1 to x = 101. The corresponding range of permissible x values for orthogonal starting position is thus from x = 2, to x = 100.

(vi) Termination of orthogonal progression

The progression of orthogonals shall terminate either when the orthogonal reaches one metre of water, or when it reaches one grid spacing of any of the boundaries.

(vii) Limitations of method of analysis

The method of analysis used for calcualting refraction used by this program makes the following assumptions.

(a) The direction of wave advance is perpendicular to the wave crest, that is, in the direction of the orthogonals.
(b) The speed of a wave of given period at a particular location depends only on the depth at that location.
(c) Changes in bottom topography are gradual enough

to be modelled by fitting a surface to the grid square nodes.

(d) Waves are long crested, constant period, small amplitude and monochromatic.

(e) Effects of current, winds and reflections are considered negligible.

(f) Effects of earth curvature are negligible.

(viii) General

The writer has tested this program on a comprehensive set of examples, (see Section 6). Provided the limitations previously mentioned in this section are adhered to, there should be no malfunctions.

A2.4 Program Options: Essentially the user has six options. These are; (i) orthogonal starting point generation; (ii) the use of compass bearings for wave directions; (iii) the use of a tidal shift for depth values; (iv) the option of having a plot drawn; (v) the sketching of wave front directions (drawn as a dash) at selected time positions along the orthogonal, given that option (iv) is selected; and (vi) the inclusion of printed time values where wave front directions are shown, given that option (v) is selected.

(i) Orthogonal starting point generation (see Section 5.3)

If the orthogonal starting points are equally spaced along a section of a straight wave front, the set of orthogonal starting points can be computed. The requisite data would be:

(a) the starting point of one orthogonal;
(b) the wave orthogonal starting direction;
(c) the number of orthogonals; and
(d) the initial spacing between the orthogonals.

The specific orthogonal whose starting point is selected will be the one on the extreme left when looking in the direction of wave progression.

(ii) <u>The use of compass bearings for wave directions (see Section 5.1)</u>

Generally the program uses cartesian angles for refraction calculations. The user may however, use compass bearings for input, which the computer then converts to cartesian angles - calculates - and then reconverts the calculated cartesian angles to compass bearings for printed output.

If the user intends using compass bearings, the necessary additional input data is the compass bearing of the x-axis.

Cartesian angles are specified as being angles measured counter clockwise from the x-axis. Compass bearings are specified as being angles measured clockwise from the direction of north.

(iii) <u>Tidal shift</u> (see Section 5.4)

The set of depths read as data shall be relative to a certain arbitrary datum, usually G.M.S.L. If a different tidal condition is to be considered, the tidal shift (in metres) taken as positive for increasing depth can be read. This shift shall be added to all depth values.

(iv) <u>Plot option</u>

The user can specify whether or not a plot is to be drawn. If not only the numerical results shall be given.

(v) <u>Sketching of wave front positions</u>

Refraction diagrams are drawn by joining specific consecutive positions of each orthogonal, by straight lines. These positions correspond with the printed positions of orthogonal progression, and are obtained by interpolation between the basic incremental progression values. (For explanation see Sections 5.7 and 5.8).

Any integer multiple of the plotted positions can be accompanied by the sketch of a wave front direction indication, given as a dash with centre on the orthogonal.

(vi) The inclusion of printed time values on the orthogonal

The wave front position indications described above may be accompanied by a print of the relevant time taken for the wave to travel from orthogonal starting point to that position.

A2.5 Data Input: The requisite data can be categorised into 12 sections. Each section must start on a new data card. The consecutive sections (in the order in which the data is to be inserted) are:

i) orthogonal starting position type;
ii) direction type;
iii)* compass correction;
iv) refraction parameters;
v) plot option;
vi)* starting parameters of leading orthogonal;
vii)* starting parameters of each orthogonal;
viii) depth data;
ix)* plotting controls;
x)* plot size;
xi)* number of shore co-ordinates to be joined; and
xii)* co-ordinates of consecutive shore points.

NOTE: * Indicates sections which might be inserted in data depending on selected options. All other sections must be inserted regardless of options.

The explanation of the individual sections is:

i) Orthogonal starting position

Computer symbol START. START is an alphanumeric word which represents option no. (i) as described in Section A2.4. START may assume one of two values, namely 'NORM' or 'CALC'. When data is inserted the first letter must start in the first column of the data card. If START reads 'NORM', the starting positions of the wave orthogonals shall be read individually. If START reads 'CALC', the starting position of the orthogonals shall be calculated as discussed in Section A2.4, option no. (i). START occupies one data card.

(ii) Direction type

Computer symbol ORIEN. ORIEN is an alphanumeric word which represents option no. (ii) in Section A2.4. ORIEN may assume one of two values namely 'GRID' or 'COMPAS'. When data is inserted the first letter must start in the first column of the data card. If ORIEN reads 'GRID', all directions are given as cartesian angles. If ORIEN reads 'COMPAS', then input and output angles are compass bearings. ORIEN occupies one data card.

(iii) Compass correction

Computer symbol GAMMA. GAMMA is real and represents compass bearing of the x-axis of the study area. GAMMA is measured in degrees clockwise from north. GAMMA occupies one data card, and is only inserted if the direction type, ORIEN, reads 'COMPAS'. If ORIEN reads 'GRID' this card must be omitted. GAMMA is read in open format.*

(iv) Refraction parameters

The refraction parameters are read on one card. This section consists of eight parameters, which have symbols; T, N, NX, NY, DEL, DIST, TIM and TIDE respectively.

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* When open format is used for reading data, the individual data may be expressed in F format or E format.

Consecutive elements of data, on a card, may be separated by a comma, a set of blanks, or both. The last element of data on a card should not be followed by a comma, as a comma is assumed at the end of the card.

(a) T is real, and represents the wave period in seconds.

(b) N is an integer which represents the number of wave orthogonals to be calculated.

(c) NX is an integer which represents the number of X co-ordinates in the study area grid (as specified in Section 3.1).

(d) NY is an integer which represents the number of Y co-ordinates in the study area grid.

(e) DEL is real, and represents the length in metres of one grid square side. That is the grid spacing (see symbol 's' in Section 3.1).

(f) DIST is real, and represents the increment size in metres for orthogonal progression in deep water. The basic increment is a constant time step, which is calculated by the computer from DIST and the deep water wave celerity. For practical examples DIST should be given a value of between one tenth and one half of the grid spacing, DEL. The time increment related to DIST is given by: (DIST/1.6T).

(g) TIM is real, and represents the time increment in seconds between successive printed and plotted points along the orthogonal. Values of position and direction for the orthogonal are determined at each integer multiple of TIM, by linear interpolation between the previous and successive values obtained from the basic increments of DIST (see also Section 5.8). It is thus necessary that the time increment TIM is greater than the corresponding time increment related to DIST. The writer suggests that TIM is selected to be at least twice the corresponding time increment related to DIST. That is TIM > DIST/0.8T. If TIM is too large an orthogonal shall be plotted as a set of visible straight lines, rather than a curve.

(h) TIDE is real, and represents the tidal shift in metres, which is to be added to each of the input depth values (see option no. (iii) of Section A2.4).

A positive value of TIDE represents an increased depth, i.e. a state of tide higher than the datum taken for measurement of water depths as given in the basic depth matrix. If no tidal shift is required this symbol should read 0.0. This data card is read in open format.

(v) Plot options

Computer symbol DES. DES is an alphanumeric word which represents option no. (iv) in Section A2.4. DES may assume one of two values, namely 'NP' or 'PLOT'. When data is inserted the first letter must start in the first column of the data card. If DES reads 'PLOT', the refraction diagram will be plotted. If DES reads 'NP', the numerical results alone shall be given.

(vi) Starting parameters of the leading orthogonal

This section of the data is read only if the starting positions of the orthogonals are to be calculated, as specified in section (i). That is, the first data card START reads 'CALC'. If START reads 'NORM' section (vii) below shall apply, and this data shall be omitted. The leading orthogonal is defined as the orthogonal on the extreme left when looking in the direction of wave progression. This section includes four symbols, namely X11, Y11, WDN11 and DIST1. All symbols are read on the same data card.

(a) X11 is real, and represents the X-value of the starting position of the leading orthogonal, measured relative to the grid system.

(b) Y11 is real, and represents the Y-value of the starting position of the leading orthogonal measured relative to the grid system.

(c) WDN11 is real and represents the starting direction of all the orthogonals. This parameter will be measured in degrees counter clockwise from the x-axis (cartesian angle) if

ORIEN is equal to 'GRID' (see section (ii)). Alternatively it will be measured in degrees measured clockwise from north if ORIEN is equal to 'COMPAS'.

(d) DIST1 is real, and represents the spacing between the orthogonals expressed as a number of grid spacings. That is, DIST1 is the spacing in metres divided by DEL (specified in section (vi)). This data card is read in open format.

(vii) Starting parameters of each orthogonal

This section of the data is read only if the starting positions of the orthogonals are to be individually read, as specified in section (i). That is, the first data card, START, reads 'NORM'. If START reads 'CALC' this data is to be omitted.

This section consists of a set of N data cards, corresponding to the starting parameters for each of the N orthogonals.

Each data card shall consist of three symbols, namely XI (I), YI (I), and WDN (I); which represent the starting parameters of the Ith orthogonal.

(a) XI (I) is real, and represents the X-value of the starting position of the Ith orthogonal, measured relative to the grid system.

(b) YI (I) is real, and represents the Y-value of the starting position of the Ith orthogonal, measured relative to the grid system.

(c) WDN (I) is real, and represents the starting direction of the Ith orthogonal. This symbol may be the cartesian angle or the compass bearing, as discussed in (vi) (c) above. These data cards are read in open format.

(viii) Depth data

This section consists of NX times NY depth values which have a general symbol H (I,J).

H(I,J) is real, and represents the depth at the point where X is equal to I, and Y is equal to J. The method of reading is to keep X constant, while reading the corresponding set of NY Y values, for that specific X, and when completed to increase X by one and read the next corresponding set of Y values. This is repeated till NX sets of Y values have been read.

There shall thus be NX sets of data, each with NY values therein. For example the fifteenth value in the sixth set, shall correspond to the depth at X = 6 and Y = 15. That is H(6,15). Each set of data will generally spread over a number of data cards. Each set must however, start on a new card, and not continue on the last card of the previous set. This data is read in open format.

(ix) Plotting controls

This section is read only if a plot is required. That is DES reads PLOT in section (v). This section consists of two symbols, namely NUM and INT.

(a) NUM is an integer which represents what is to be shown on the orthogonal (see options (v) and (vi) in Section A2.4). NUM may be assigned a value of 1, 2 or 3. If NUM equals 1, then the orthogonals alone are drawn. If NUM equals 2, then wave front direction are sketched at selected positions along orthogonal. If NUM equals 3, then the wave front direction sketches, described above, are accompanied by a print of the corresponding time of travel from the orthogonal starting position.

(b) INT is an integer which represents the integer multiple of TIM values, (see section (iv) (g), at which the wave front directions are sketched on the orthogonal. If NUM equals 1, i.e. no wave front sketches are to be drawn, this symbol should be given a value of 1. (The program shall in fact operate in this case if INT is given any arbitrary integer value.) This data is read in open format on one data card.

(x) Plot size

This section is read only if a plot is required. That is DES reads 'PLOT' in section (v). This section consists of two symbols, namely A and FACT. Both symbols are read in open format on one data card.

(a) A is real, and represents the length of the Y-axis in inches, to be plotted assuming no scale factor is applied. (See part (b) below). When the scale factor has been applied, the Y-axis length must not exceed the width of the paper (30 inches).

(b) FACT is real, and represents the scale factor to be applied to the plot. For example; if FACT is equal to 0.5, the Y-axis shall be half that length specified by A above. If no scaling is required FACT must read 1.0.

(xi) Number of shore co-ordinates to be joined

The shoreline is drawn by joining a set of points by straight lines. The number of consecutive points to be joined by straight lines is represented by the integer IA. If no shoreline is to be drawn this symbol must read 2. This data is read on one card and is only read if DES reads 'PLOT' in section (v).

(xii) Co-ordinates of consecutive shore points

This section is read only if DES reads 'PLOT' in section (v). This section consists of IA data cards with general symbols SHX (I), and SHY (I), corresponding to the Ith data card in the section.

(a) SHX (I) is real, and represents the X-value of the Ith shoreline point, relative to the grid system.

(b) SHY (I) is real, and represents the Y-value of the Ith shoreline point relative to the grid system.

If no shoreline is to be drawn two data cards are to be inserted. Both cards shall give the SHX (I), and SHY (I) values as being 1.0. All data is read in open format.

A summarised representation of the DATA input is shown in Figure A2.

A2.6 Printed Output: The parameters of each orthogonal are printed at each integer multiple of TIM, as described in Section A2.5 (iv). If cartesian angles are used as described in Section A2.5 (i) the parameters printed are: X8, Y8, THED8, and TIME8 respectively, where;

(i) X8 is the interpolated X co-ordinate value,

(ii) Y8 is the interpolated Y co-ordinate value,

(iii) THED8 is the interpolated cartesian angle of the wave direction, and

(iv) TIME8 is the time taken for the orthogonal to travel to that position from the starting point.

If compass bearings are used, the parameters printed are; X8, Y8, THED8, THEDC8, and TIME8 respectively, where THEDC8 is the interpolated compass bearing of the wave direction. Examples are shown in Section A2.9.

A2.7 Other Outputs: The refraction diagram may be plotted by the computer, that is DES reads 'PLOT' as described in Section A2.5 (v).

(i) Orthogonal Starting Position Type — (START)

'NORM'	Starting positions of orthogonals read individually
'CALC'	Starting positions of orthogonals generated by program from one starting point and direction.

(ii) Direction Type — (ORIEN)

'GRID'	Directions are counter-clockwise from X-axis (Cartesian)
'COMPAS'	Directions are clockwise compass bearing from north.

(iii) Compass Correction

GAMMA (real)
Compass bearing of X-axis
* DO NOT USE IF 'GRID' WAS READ IN (ii).

(iv) Refraction Parameters

T (real)	N (integer)	NX (integer)	NY (integer)	DEL (real)	DIST (real)	TIM (real)	TIDE (real)
Wave period in seconds	No of Orthogonals	No of X-coords in Grid	No of Y-coords in Grid	Grid spacing in metres	Orthogonal increment in metres	Increment of plotted time in seconds	Correction added to depth vals. in metres

(v) Plot Option — (DES)

'NP'	No Plot
'PLOT'	Plot required

(vi) Starting parameters of leading orthogonal**

X11 (real) Y11 (real)	WDN11 (real)	DIST1 (real)
X & Y Coords of starting position of orthogonal on left when looking in direction of wave progression.	Wave direction	Spacing between orthogonals measured along wave front in Grid Mesh Units.

** DO NOT USE IF 'NORM' WAS READ IN (i).

(vii) Starting parameters of each orthogonal* — N cards (see iv)

XI(I) (real) YI(I) (real) — X & Y of starting position of Ith orthogonal

WDN (I) (real) — Wave direction of Ith orthogonal

* DO NOT USE IF 'CALC' WAS READ IN (i)

(viii) Depth Data — NX sets (see iv)

H(I,1) H(I,2) H(I,3) ....(all real)

NY values (see iv) ⟶

Depth values of corresponding grid points in metres.

(ix) Plotting Controls **

NUM (integer)

1 orthogonal only drawn
2 wave front directions are drawn
3 as for 2 but results listed

INT (integer) — Integer multiple of TIM (see iv) for wave front direction sketch on orthogonals.

(x) Plot Size **

A (real) — Length of Y axis in inches

FACT (real) — Scale factor

(xi) No of shore coords to be joined **

IA (integer)

(xii) Co-ords of consecutive shore points ** — IA cards (see xi)

SHX (I) (real) — X- co ordinate of the Ith shore point.

SHY (I) (real) — Y-co ordinate of the Ith shore point.

** DO NOT USE IF 'NP' WAS READ IN (v)

Fig. A2. Coding Format for data input for writer's program.

The program uses the following standard CALCOMP subroutines for plotting; PLOTS, FACTOR, PLOT, AXIS, NUMBER and SYMBOL. The CALCOMP manual is included in Appendix H. The size of the plot is controlled by the stipulation of the length of the Y-axis in combination with a scaling factor. See Section A2.5 (x).

The advantage of having the scale factor is that firstly, when initially running an example, an idea of the complexity of the diagram can be obtained, without necessitating a long plot (the time of plot is approximately proportional to the scale factor). Secondly, a small plot may be required, e.g. for reproduction on an A4 size sheet. If the size of the plot is just stipulated by specifying a small Y-axis length, the symbols will become disproportionately large. If is preferable to specify a large Y-axis length (about 24 inches) and to specify the relevant scale factor. For special operater messages see Section A4.2.

A2.8 Flow Charts: Macro-Flow-Charts of the MAIN program and the subroutines are presented in Figures A3 - A7.

A2.9 Sample Runs: Various sample runs are included to illustrate the various features of the program. The study area used is False Bay, which is the same as that used in Section 6.

The depth data has been written onto a data file, namely VISDAT. . The depth data was obtained from the Fisheries Development Corporation of South Africa.

The shoreline data has been written into a program element, namely REFRAC.VISSHO. The shoreline data was obtained from a scaled plan of the area. Eighty seven points were determined.

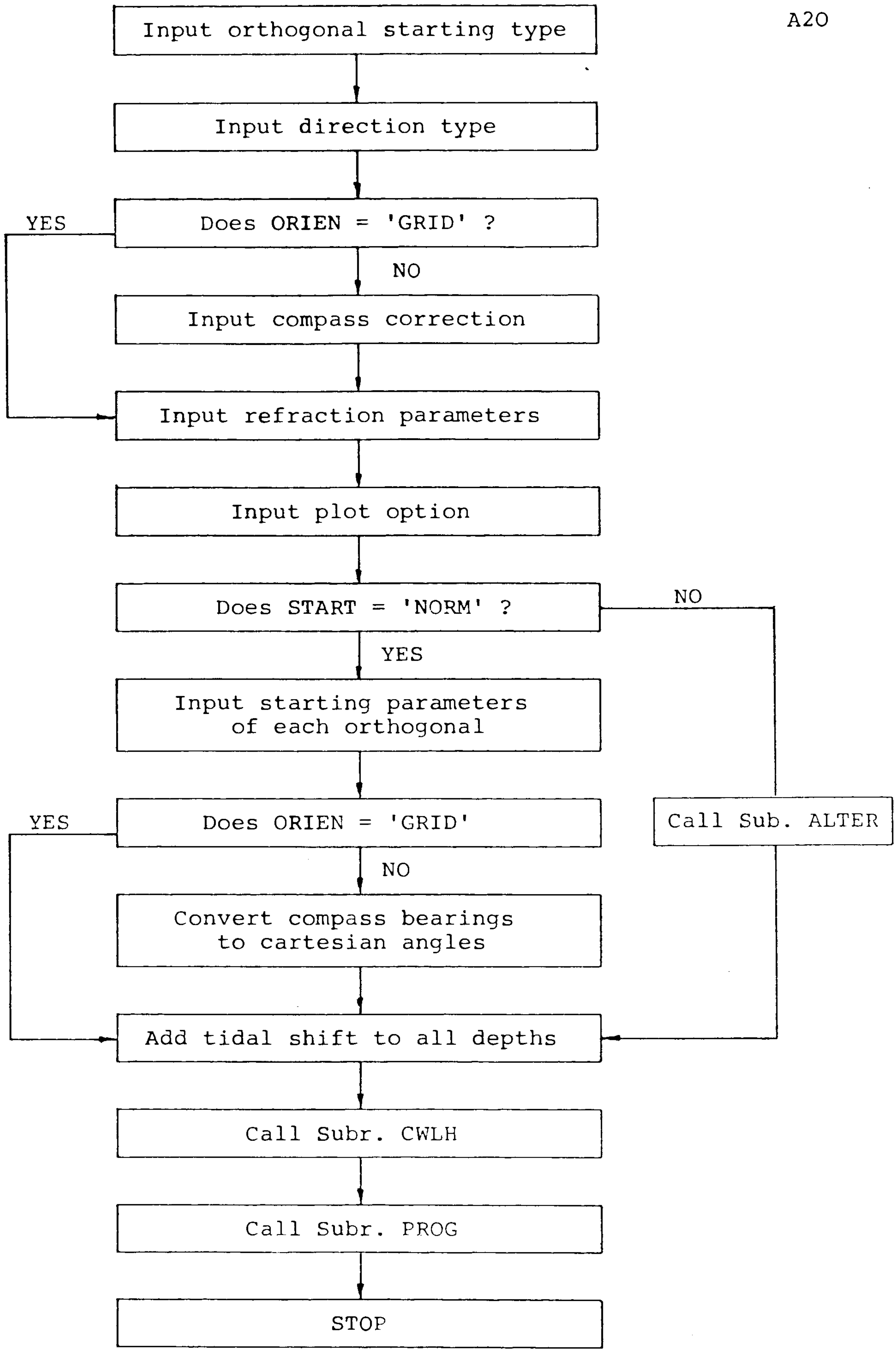


Figure A3   Macro-Flow-Chart for MAIN Program

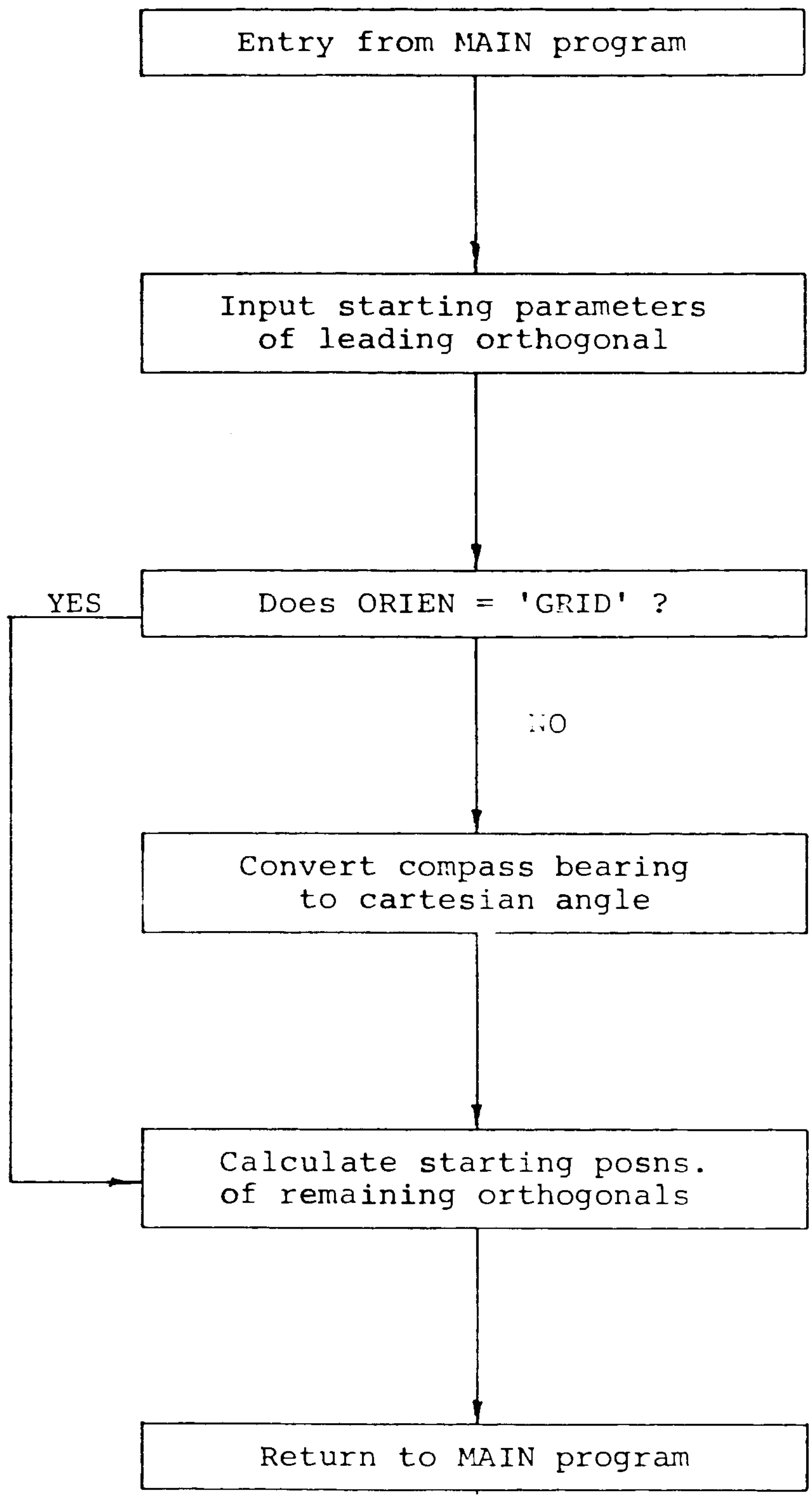
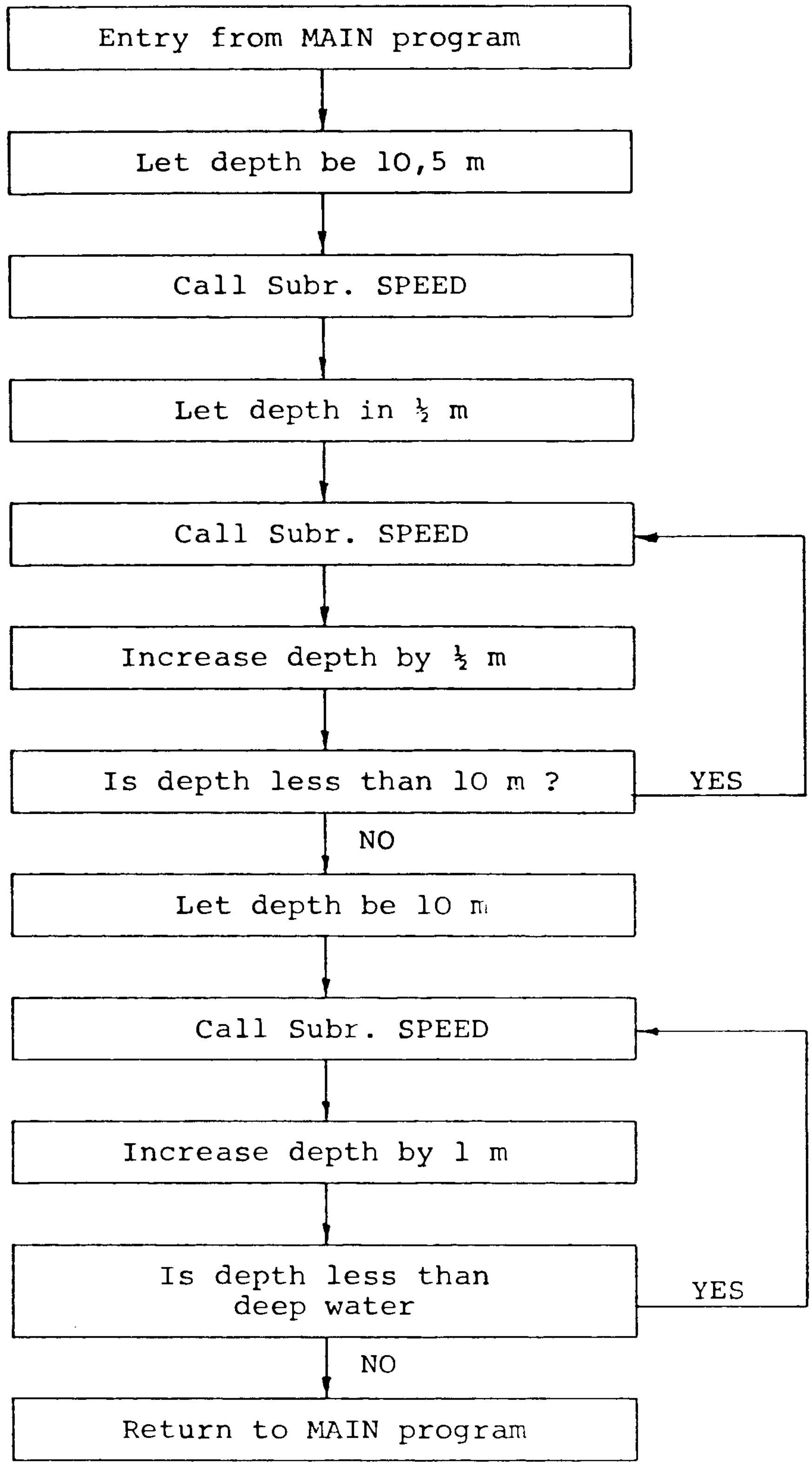
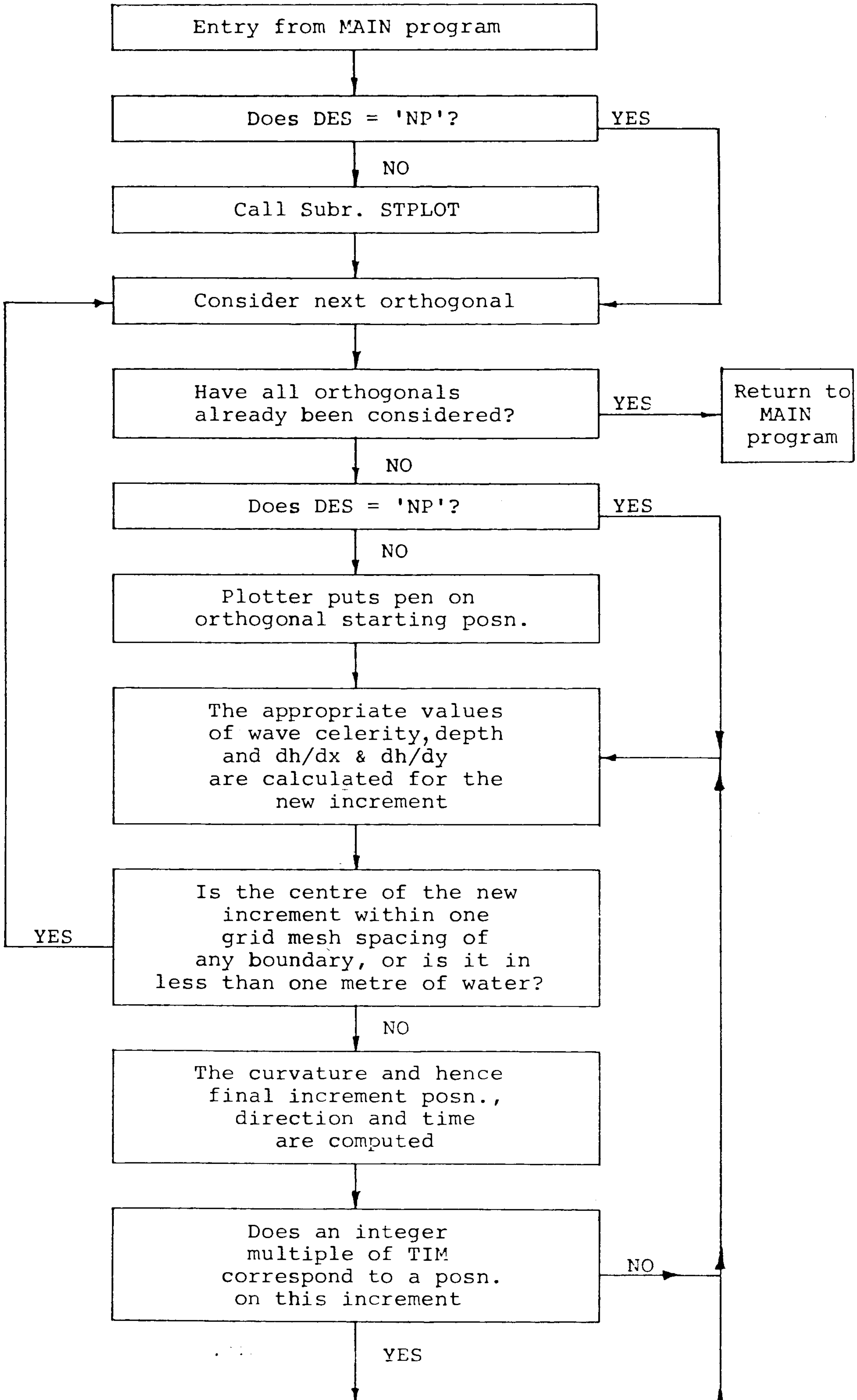


Figure A 4 Macro-Flow-Chart for subroutine ALTER



Subroutine SPEED is an internal subroutine which performs a bisection method to solve the implicit wave celerity equation for a specific depth. Termination occurs when the range is less than 10^{-5} m/sec (That is, max error is 0,5 x 10^{-5} m/sec)

Figure A5 Macro-Flow-Chart for subroutine CWLH


Entry from MAIN program
Does DES = 'NP'?
YES
NO
Call Subr. STPLOT
Consider next orthogonal
Have all orthogonals already been considered?
YES
Return to MAIN program
NO
Does DES = 'NP'?
YES
NO
Plotter puts pen on orthogonal starting posn.
The appropriate values of wave celerity, depth and dh/dx & dh/dy are calculated for the new increment
Is the centre of the new increment within one grid mesh spacing of any boundary, or is it in less than one metre of water?
YES
NO
The curvature and hence final increment posn., direction and time are computed
Does an integer multiple of TIM correspond to a posn. on this increment
NO
YES

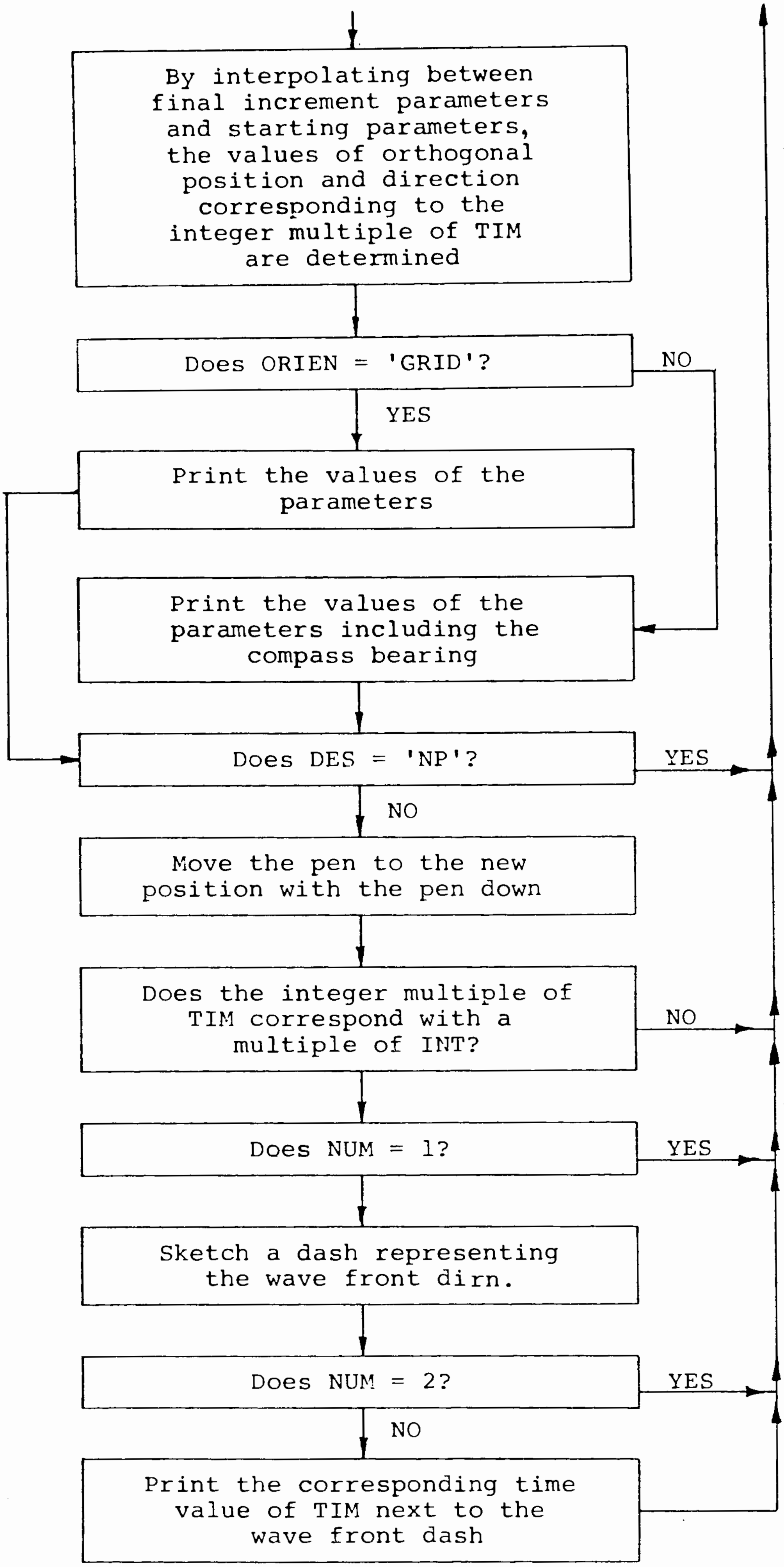


Figure A6 Macro-Flow-Chart for subroutine PROG

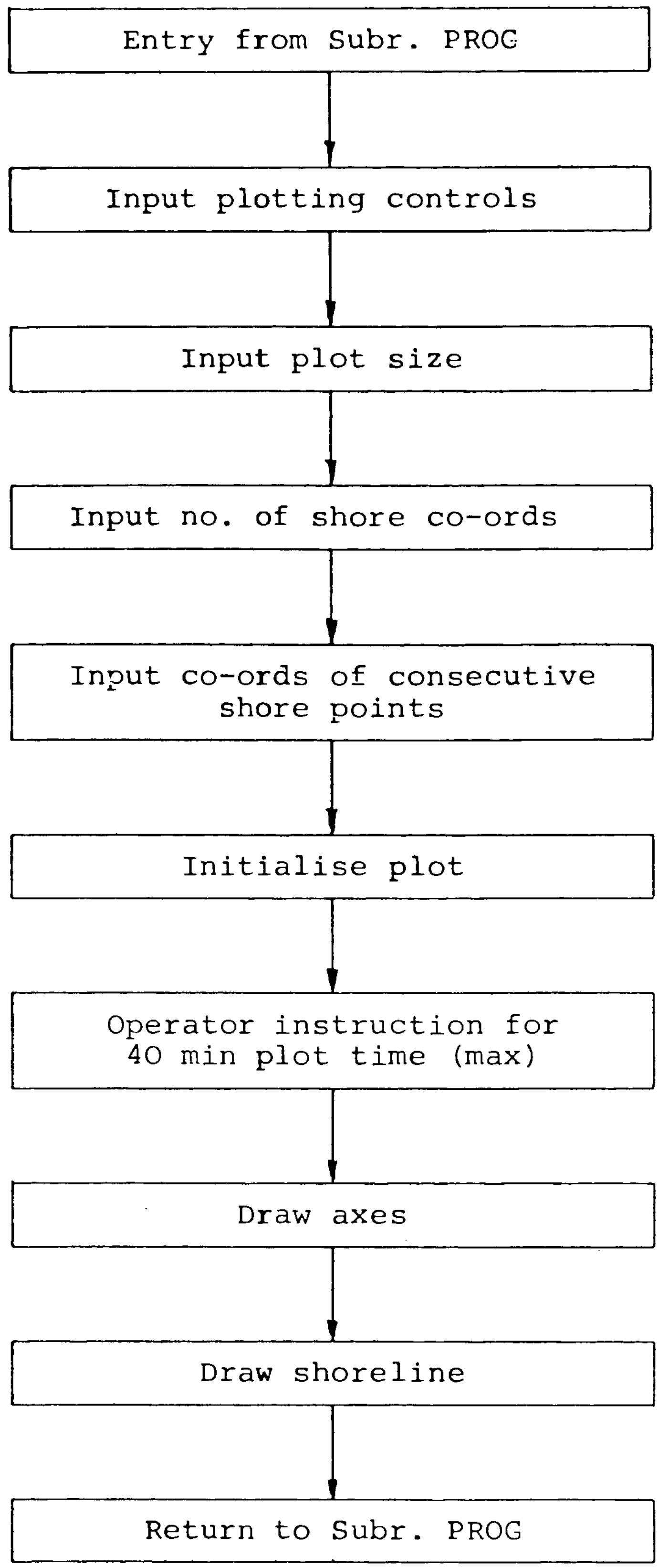


Figure A7 Macro-Flow-Chart for subroutine STPLOT

The overall study area has an X-axis length of 60 kilometres and a Y-axis length of 66,75 kilometres.

The grid mesh spacing is 750 metres, giving a corresponding number of X co-ordinates of 81, and number of Y co-ordinates of 90. The deep water step length for all examples was taken as three hundred and seventy five metres, and the time increment between printed and plotted values was 36 seconds. The wave period was 12 seconds. No tidal shift was applied for any of the examples. North is the direction of the Y-axis.

EXAMPLE 1

Example 1 was run using cartesian angles, with eleven orthogonals at 45^{o}, each being individually entered. A plot was drawn with wave front directions being shown every 144 seconds (4 x 36). These would thus represent every twelfth wave front.

The plot size was such that the Y-axis was 8,9 inches long, with no scale factor being applied.

The data used for this example is shown below, and the resultant plot is shown in Figure A8. Printed output for orthogonal No. 1 is shown in Figure A9.

```
@XQT REFRAC.ABS
NORM
GRID
12.,11,81,9Ø,75Ø.,375.,36.,Ø.
PLOT
2.,22.,45.
4.,2Ø.,45.
6.,18.,45.
8.,16.,45.
1Ø.,14.,45.
12.,12.,45.
14.,1Ø.,45.
16.,8.,45.
18.,6.,45.
2Ø.,4.,45.
22.,2.,45.
@ADD VISDAT.
2,4
8.9,1.
87
@ADD REFRAC.VISSHO
```

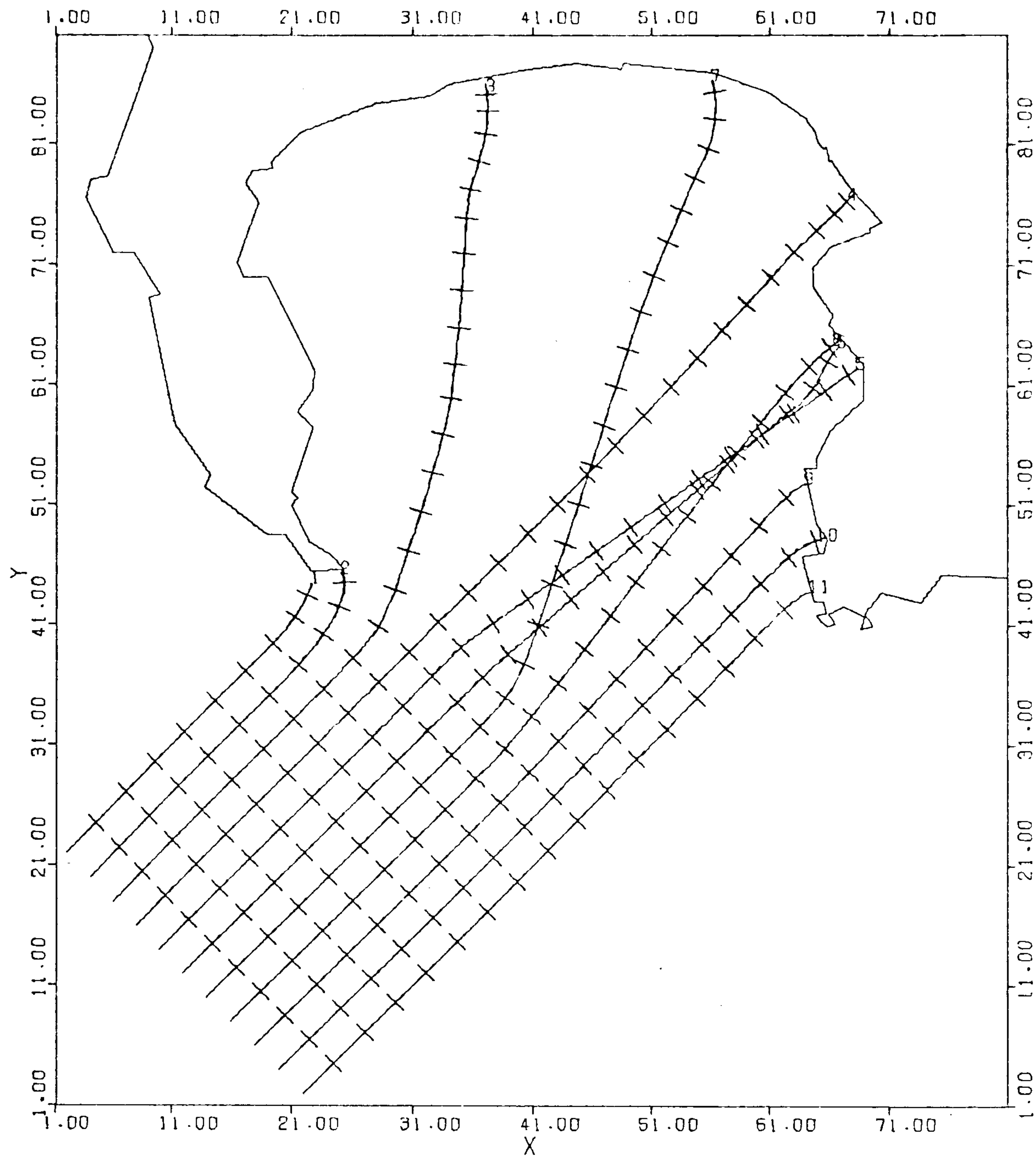


Fig. A8. Example 1. Plot showing wave front directions.

ORTHOGONAL NO. 1, STARTING PT.:(2.00, 22.00)

X	Y	THETA	TIME
-	-	-----	----
2.0000	22.000	45.000	0.00000
2.6333	22.633	45.000	36.000
3.2667	23.267	45.000	72.000
3.9000	23.900	45.000	108.00
4.5334	24.533	45.000	144.00
5.1667	25.167	45.000	180.00
5.8000	25.800	45.000	216.00
6.4334	26.433	45.000	252.00
7.0667	27.067	45.000	288.00
7.7001	27.700	45.000	324.00
8.3334	28.333	45.000	360.00
8.9667	28.967	45.000	396.00
9.6001	29.600	45.000	432.00
10.233	30.233	45.000	468.00
10.867	30.867	45.000	504.00
11.500	31.500	44.999	540.00
12.133	32.133	44.999	576.00
12.767	32.767	44.999	612.00
13.400	33.400	45.000	648.00
14.033	34.033	45.003	684.00
14.667	34.667	45.018	720.00
15.300	35.301	45.066	756.00
15.931	35.934	45.133	792.00
16.557	36.564	45.191	828.00
17.175	37.186	45.074	864.00
17.779	37.788	44.798	900.00
18.368	38.370	44.583	936.00
18.947	38.944	45.093	972.00
19.504	39.515	46.407	1008.0
20.027	40.082	48.259	1044.0
20.501	40.636	50.588	1080.0
20.923	41.173	53.211	1116.0
21.289	41.686	55.918	1152.0
21.600	42.169	58.539	1188.0
21.872	42.638	61.112	1224.0
22.111	43.094	63.719	1260.0
22.310	43.519	66.377	1296.0
22.459	43.892	69.962	1332.0
22.561	44.205	74.411	1368.0
22.617	44.433	78.090	1404.0

Fig. A9. Example 1. Printed output for typical orthogonal, using cartesian angles.

EXAMPLE 2

This example is the same as example 1, except that compass bearings were used instead of cartesian angles. No plot was drawn. The data used for this example is shown below. Printed output for orthogonal No. 1 is shown in Figure A10.

```
@XQT REFRAC.ABS
NORM
COMPAS
9Ø.
12.,11,81,9Ø,75Ø.,375.,36.,Ø.
NP
2.,22.,45.
4.,2Ø.,45.
6.,18.,45.
8.,16.,45.
1Ø.,14.,45.
12.,12.,45.
14.,1Ø.,45.
16.,8.,45.
18.,6.,45.
2Ø.,4.,45.
22.,2.,45.
@ADD VISDAT.
```

EXAMPLE 3

This example is the same as example 1, except that the starting positions of the orthogonals are generated. No plot was drawn.

```
@XQT REFRAC.ABS
CALC
GRID
12.,11,81,9Ø,75Ø.,375.,36.,Ø.
NP
2.,22.,45.,2.82843
@ADD VISDAT
```

ORTHOGONAL NO. 1, STARTING PT.:(2.00, 22.00)

X	Y	THETA	BEARING	TIME
2.0000	22.000	45.000	45.000	0.00000
2.6333	22.633	45.000	45.000	36.000
3.2667	23.267	45.000	45.000	72.000
3.9000	23.900	45.000	45.000	108.00
4.5334	24.533	45.000	45.000	144.00
5.1667	25.167	45.000	45.000	180.00
5.8000	25.800	45.000	45.000	216.00
6.4334	26.433	45.000	45.000	252.00
7.0667	27.067	45.000	45.000	288.00
7.7001	27.700	45.000	45.000	324.00
8.3334	28.333	45.000	45.000	360.00
8.9667	28.967	45.000	45.000	396.00
9.6001	29.600	45.000	45.000	432.00
10.233	30.233	45.000	45.000	468.00
10.867	30.867	45.000	45.000	504.00
11.500	31.500	44.999	45.001	540.00
12.133	32.133	44.999	45.001	576.00
12.767	32.767	44.999	45.001	612.00
13.400	33.400	45.000	45.000	648.00
14.033	34.033	45.003	44.997	684.00
14.667	34.667	45.018	44.982	720.00
15.300	35.301	45.066	44.934	756.00
15.931	35.934	45.133	44.867	792.00
16.557	36.564	45.191	44.809	828.00
17.175	37.186	45.074	44.926	864.00
17.779	37.788	44.798	45.202	900.00
18.368	38.370	44.583	45.417	936.00
18.947	38.944	45.093	44.907	972.00
19.504	39.515	46.407	43.593	1008.0
20.027	40.082	48.259	41.741	1044.0
20.501	40.636	50.588	39.412	1080.0
20.923	41.173	53.211	36.789	1116.0
21.289	41.686	55.918	34.082	1152.0
21.600	42.169	58.539	31.461	1188.0
21.872	42.638	61.112	28.888	1224.0
22.111	43.094	63.719	26.281	1260.0
22.310	43.519	66.377	23.623	1296.0
22.459	43.892	69.962	20.038	1332.0
22.561	44.205	74.411	15.589	1368.0
22.617	44.433	78.090	11.910	1404.0

Fig. A10. Example 2. Printed output for typical orthogonal, using compass bearings.

EXAMPLE 4

This example is the same as example 1, except that no wave front positions are shown in the plot. The data for this example is shown below and the plot is shown in Figure All.

```
@XQT REFRAC.ABS
NORM
GRID
12.,11,81,9Ø,75Ø.,375.,36.,Ø.
PLOT
2.,22.,45.
4.,2Ø.,45.
6.,18.,45.
8.,16.,45.
1Ø.,14.,45.
12.,12.,45.
14.,1Ø.,45.
16.,8.,45.
18.,6.,45.
2Ø.,4.,45.
22.,2.,45.
@ADD VISDAT.
1,4
8.9,1.
87
@ADD REFRAC.VISSHO
```

EXAMPLE 5

This example considers only one orthogonal, orthogonal No. 1 in example 1. For this orthogonal the time values are given with the wave front directions.

The plot size was such that the Y-axis length is 17.8 inches, but is scaled by a factor of one half, to bring the plot size to the same as that in example 1.

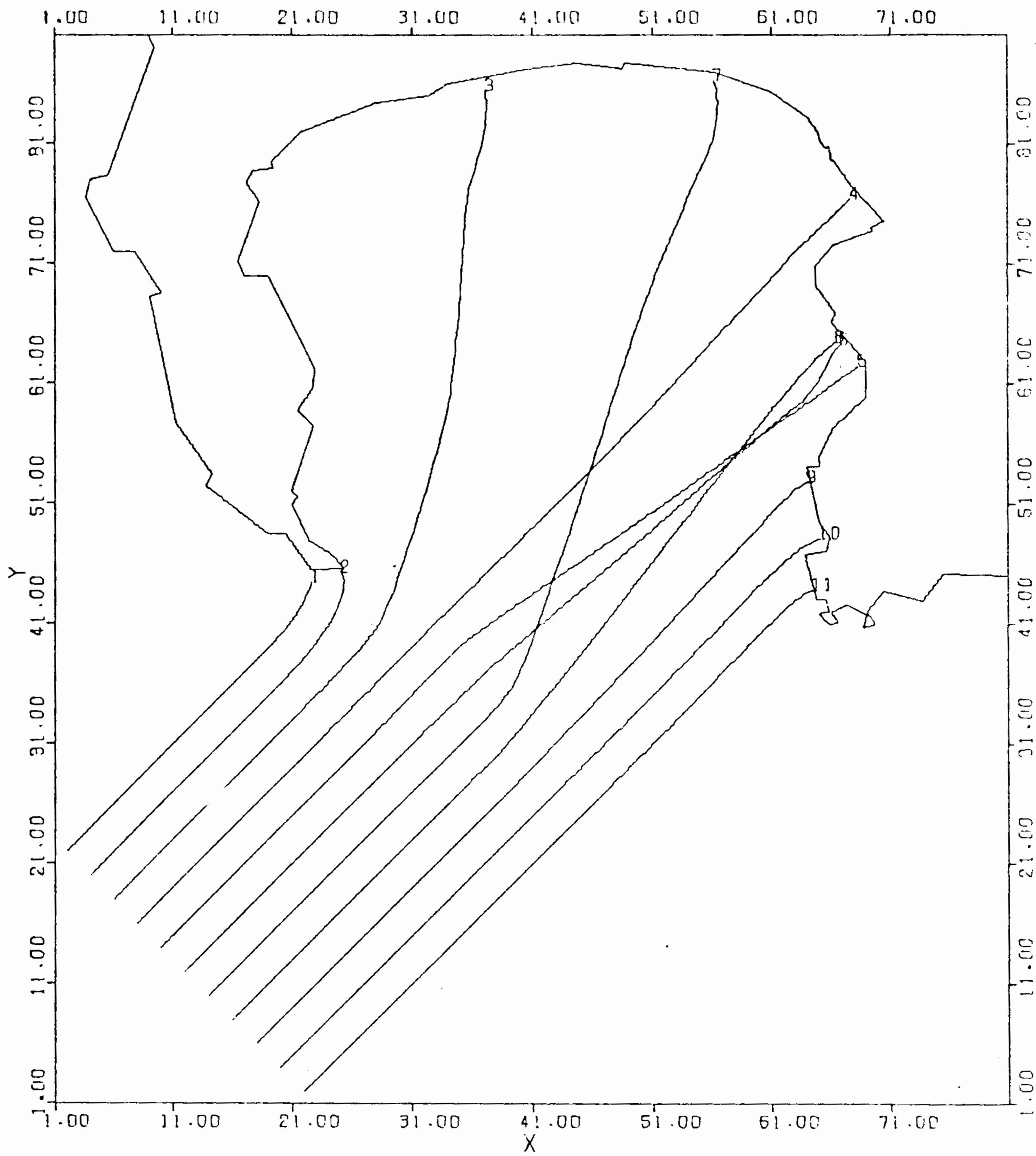


Fig. All. Example 4. Plot without wave front directions.

Same as page 58?

The plot is shown in Figure A12, and the data is shown below.

```
@XQT REFRAC.ABS
NORM
GRID
12.,1,81,9Ø,75Ø.,375.,36.,Ø.
PLOT
2.,22.,45.
@ADD VISDAT.
3,4
17.8,Ø.5
87
@ADD REFRAC.VISSHO
```

<u>EXAMPLE 6</u>

This example is an example of a reverse tracing procedure. The origin of the rays is at point (61,56) and eight rays are projected at ten degree angles from 200° till 270°. The plot is shown in Figure A13, and the data is shown below. The printed output for orthogonal No. 6 is shown in Figure A14.

```
@XQT REFRAC.ABS
NORM
GRID
12.,8,81,9Ø,75Ø.,375.,36.,Ø.
PLOT
61.,56.,2ØØ.
61.,56.,21Ø.
61.,56.,22Ø.
61.,56.,23Ø.
61.,56.,24Ø.
61.,56.,25Ø.
61.,56.,26Ø.
61.,56.,27Ø.
@ADD VISDAT.
1,4
8.9,1.
87
@ADD REFRAC.VISSHO
```

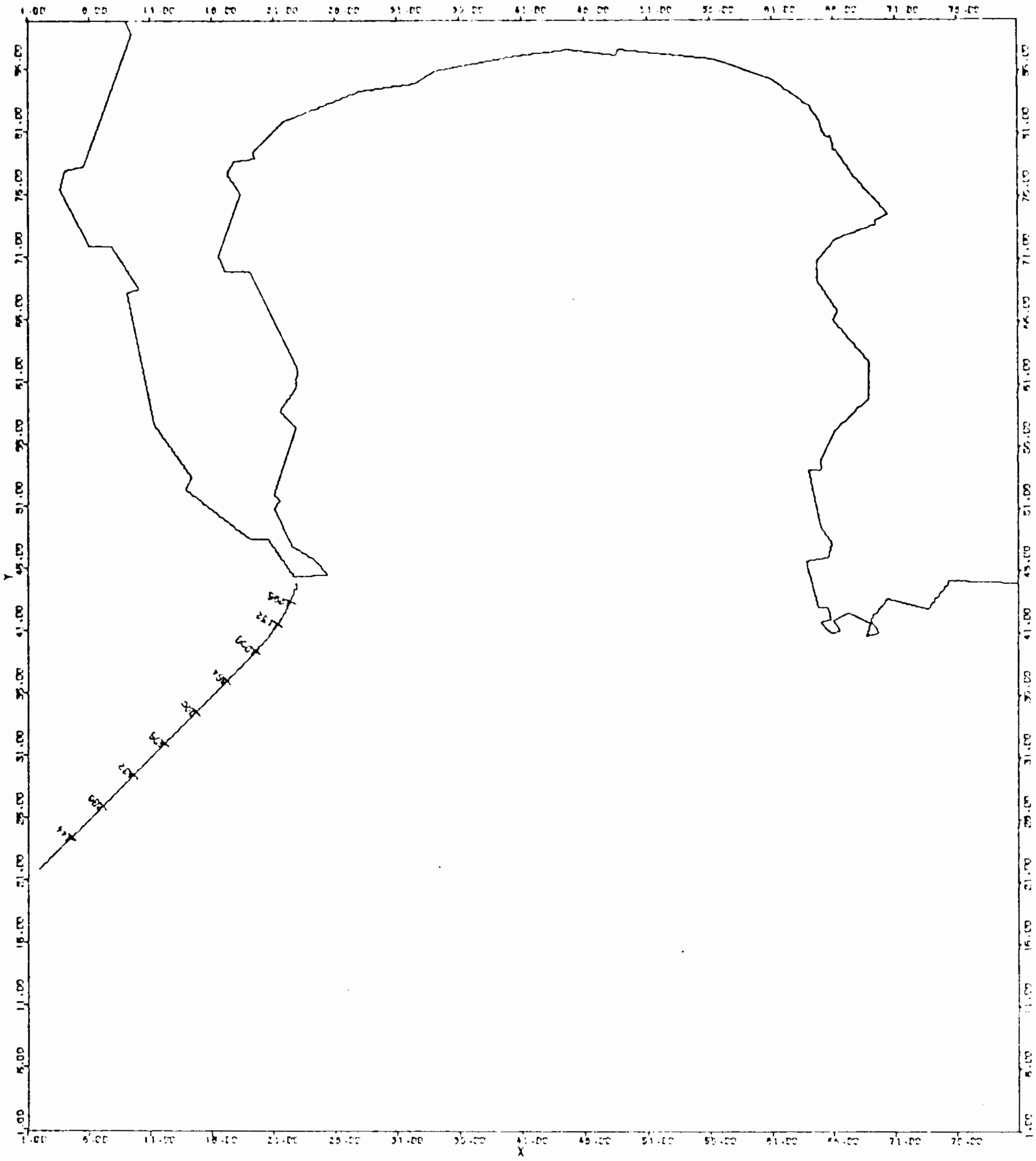


Fig. A12. Example 5. Plot for a typical orthogonal showing wave front directions, and time values, with a scale factor applied.

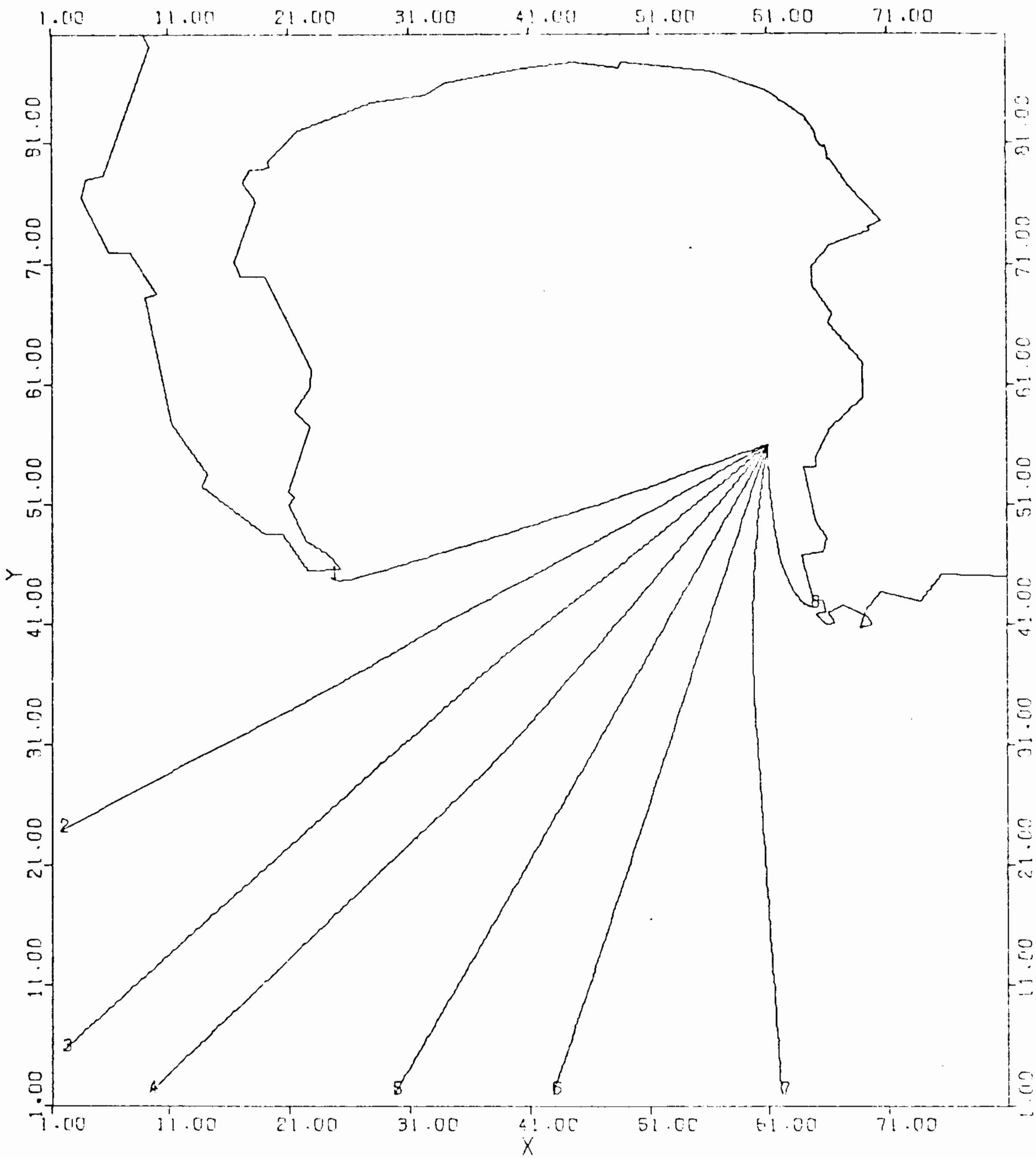


Fig. A13. Example 6. Plot of reverse tracing procedure.

ORTHOGONAL NO. 6, STARTING PT.:(61.00, 56.00)

X	Y	THETA	TIME
61.000	56.000	250.00	0.00000
60.724	55.230	250.52	36.000
60.452	54.452	250.95	72.000
60.184	53.665	251.34	108.00
59.916	52.867	251.52	144.00
59.646	52.064	251.58	180.00
59.380	51.260	251.57	216.00
59.112	50.456	251.52	252.00
58.842	49.651	251.44	288.00
58.570	48.842	251.39	324.00
58.297	48.031	251.33	360.00
58.021	47.217	251.25	396.00
57.743	46.401	251.16	432.00
57.464	45.583	251.15	468.00
57.184	44.763	251.16	504.00
56.903	43.938	251.18	540.00
56.621	43.110	251.20	576.00
56.339	42.280	251.25	612.00
56.056	41.448	251.31	648.00
55.774	40.613	251.36	684.00
55.493	39.776	251.41	720.00
55.211	38.938	251.46	756.00
54.930	38.098	251.50	792.00
54.648	37.257	251.50	828.00
54.367	36.415	251.50	864.00
54.084	35.572	251.46	900.00
53.802	34.728	251.47	936.00
53.519	33.884	251.46	972.00
53.235	33.039	251.46	1008.0
52.951	32.193	251.46	1044.0
52.668	31.346	251.46	1080.0
52.383	30.499	251.45	1116.0
52.099	29.652	251.44	1152.0
51.814	28.804	251.43	1188.0
51.529	27.956	251.42	1224.0
51.244	27.108	251.41	1260.0
50.959	26.259	251.49	1296.0
50.675	25.409	251.59	1332.0
50.393	24.559	251.61	1368.0
50.110	23.709	251.61	1404.0
49.828	22.859	251.60	1440.0
49.545	22.009	251.60	1476.0
49.262	21.160	251.59	1512.0
48.979	20.310	251.59	1548.0
48.696	19.460	251.59	1584.0
48.414	18.610	251.59	1620.0
48.131	17.760	251.59	1656.0
47.848	16.910	251.60	1692.0
47.565	16.060	251.60	1728.0
47.282	15.211	251.60	1764.0
47.000	14.361	251.60	1800.0
46.717	13.511	251.60	1836.0
46.434	12.661	251.60	1872.0
46.152	11.811	251.60	1908.0
45.869	10.961	251.60	1944.0
45.586	10.111	251.60	1980.0
45.304	9.2613	251.60	2016.0
45.021	8.4114	251.60	2052.0
44.738	7.5615	251.61	2088.0
44.456	6.7116	251.61	2124.0
44.173	5.8616	251.61	2160.0
43.890	5.0117	251.61	2196.0
43.606	4.1618	251.61	2232.0
43.325	3.3119	251.61	2268.0
43.043	2.4620	251.61	2304.0

Fig. A14. Example 6. Printed output for typical orthogonal, reverse tracing procedure.

A3 - SYSTEM DOCUMENTATION

A3.1 Computer Equipment: The program REFRAC was developed on a UNIVAC 1106 computer. This computer has a 36 bit word processor, and a code access speed of 1.5 μ seconds.

A3.2 Peripheral Equipment: The following peripheral equipment was used in developing the program.

(i) 2 line printers, namely the Univac 0768 drum printer, (900 lines/min), and the Univac 9300 bar printer, (600 lines/min).

(ii) A Univac 9300 card reader, (600 cards/min).

(iii) An Olivetti TE 318 tele-type interactive terminal.

(iv) A Calcomp 760/953 off-line plotter, which has a 30 inch drum, and uses plot-increments of 1/100 of an inch. Plot speed is 300 increments/sec.

Any similar set of peripheral equipment compatible with Fortran V, including the CALCOMP standard subroutines, may be used to operate this program.

A3.3 Source Program: The source listings of the MAIN program and the subroutines are shown in Figure A15.

A3.4 Variables and Subroutines: The variables used in the main program and the subroutines are defined in Figure A16. The program consists of the following parts.

(i) REFRAC.MAIN·

The main program reads all data, except plotting instructions, and calls subroutines ALTER, CWLH and PROG.

(ii) REFRAC.ALTER

This subroutine is called by the main program if starting positions of orthogonals, in deep water, are to be calculated.

(iii) REFRAC.CWLH

This subroutine calculates and stores wave celerities for specific water depths from one metre to deep water, for interpolation in subroutine PROG.

As the real relationship between depth and wave celerity is implicit, a bisection method is used. The bisection method is incorporated in an internal subroutine, subroutine SPEED.

(iv) REFRAC.PROG

This subroutine calculates the progression of each orthogonal by progressing from the given starting point in a set of circular increments.

For each increment the curvature and travel time are computed, hence the final co-ordinates, direction, and time values are computed.

The orthogonal calculation is discontinued in the event of the water depth being less than one metre, or the orthogonal position being within one grid length of any of the borders.

The orthogonal parameters are printed at multiples of a selected time value, and, if desired, are plotted on an off-line drum plotter.

This subroutine calls subroutine STPLOT.

(v) REFRAC.STPLOT

This subroutine is called by subroutine PROG if a plot is required. It initialises the plot, and draws the axes. This subroutine reads the plotting instruction data and shoreline data.

The Calcomp library subroutines used are: PLOTS, FACTOR, PLTIME, PLOT, AXIS, NUMBER, and SYMBOL.

Other library subroutines which are provided by the system are: COS, SIN, TANH, and COSH.

A3.5 Data Structures: Data may be all on cards, or user may use data files, data elements, and data cards in any combination compatible with the computer being used.

A3.6 Storage Requirements: The requisite core space for running the program on the Univac 1106 system is 33 363 words of which 27 270 words are for data bank. These core requirements apply for non segmental mapping.

A3.7 Maintenance and Updates: Besides the development of the program no updates have taken place.

SECTION A4 - OPERATING DOCUMENTATION

A4.1 Operator Instructions: The operation of the program shall be dependent on the specific system being used by the user.

A4.2 Operating Messages: A special operating message is produced by the program if a plot is to be drawn. This message is on card number 47 of subroutine STPLOT and is a request for a maximum of 40 minutes plotting time (normal maximum time is 20 minutes).

A4.3 Control Cards: The program is compiled, mapped and executed by using standard Univac Exec 8 control cards.

A4.4 Error Recovery: The program must be restarted on error.

A4.5 Run Time: The actual run time is dependent on the example considered, and is hard to predict without having previously done a similar example. As a basic indication the run time taken for example 1 in Section A2.9 was 38 seconds, of which 21 seconds was CPU time. The cost of the run was R4,85.

422 Cards

TRIG*REFRAC(1).MAIN

```
C* THE FUNCTION OF THE MAIN PROGRAM IS TO READ THE DATA,
C* AND TO CALL THE SUBROUTINES.
      COMMON H(150,150),C(300),XI(100),YI(100)
     1,IBUF(2000),WDN(100)
C**********
C* THE FIRST DATA CARD STIPULATES WHETHER STARTING POSITIONS
C* OF THE ORTHOGONALS ARE TO BE READ INDIVIDUALLY, OR ARE
C* GOING TO BE COMPUTED ACCORDING TO THE WAVE DIRECTION AND
C* ONE STARTING POINT.
C* IF THE STARTING POSITIONS ARE TO BE INDIVIDUALLY ENTERED
C* THIS CARD SHOULD READ 'NORM', IF NOT IT SHOULD READ 'CALC'.
      READ 160,START
C**********
C* THE SECOND CARD STIPULATES WHETHER THE DIRECTIONS READ ARE
C* CARTESIAN, MEASURED ANTI-CLOCKWISE FROM THE HORIZONTAL
C* X-AXIS, OR COMPAS  BEARINGS, MEASURED CLOCKWISE FROM NORTH.
C* IF THEY ARE CARTESIAN THIS CARD SHOULD READ 'GRID',
C* IF NOT IT SHOULD READ 'COMPAS'.
      READ 160,ORIEN
  160 FORMAT(A4)
      IF(ORIEN.EQ.'GRID') GO TO 57
C**********
C* THE NEXT DATA CARD IS READ ONLY IF COMPASS BEARINGS ARE
C* USED. THIS CARD READS THE COMPASS BEARING OF THE X-AXIS.
C* IF ALL DIRECTIONS ARE CARTESIAN THIS CARD MUST NOT BE
C* INSERTED.
      READ 100,GAMMA
   57 CONTINUE
C**********
C* THE NEXT DATA CARD IS READ HERE. IT CONTAINS:
C* 1- T -THE WAVE PERIOD IN SECONDS - REAL.
C* 2- N -THE NUMBER OF WAVE ORTHOGONALS -INTEGER.
C* 3- NX -THE NUMBER OF X COORDINATES IN THE GRID - INTEGER.
C* 4- NY -THE NUMBER OF Y COORDINATES IN THE GRID - INTEGER.
C* 5- DEL -THE LENGTH OF EACH GRID SIDE IN METRES - REAL.
C* 6- DIST -THE LENGTH IN METRES OF EACH INCREMENT - REAL.
C* 7- TIM -THE TIME INCREMENT , IN SECONDS, BETWEEN CONSECUTIVE
C* PRINTED POSITION, DIRECTION & TIME VALUES - REAL.
C* 8- TIDE -THE TIDAL SHIFT, IN METRES, WHICH IS TO BE ADDED TO
C* ALL THE DEPTH VALUES - REAL.
      READ 100,T,N,NX,NY,DEL,DIST,TIM,TIDE
      DS=DIST/DEL
  100 FORMAT()
C**********
C* THE NEXT CARD STIPULATES WHETHER A PLOT IS TO BE DRAWN OR NOT.
C* IF NO PLOT IS DESIRED, THE CARD SHOULD READ 'NP'.
C* IF A PLOT IS DESIRED, THE CARD SHOULD READ 'PLOT'.
C* (IF PLOT IS DESIRED SEE ADDITIONAL DATA REQUIREMENTS
C* FOR PLOTTING STIPULATED IN SUBROUTINE STPLOT.)
      READ 111,DES
  111 FORMAT(A2)
      IF(START.EQ.'NORM') GO TO 11
      CALL ALTER(ORIEN,GAMMA,N)
      GO TO 21
   11 CONTINUE
C**********
C* THE NEXT SET OF DATA CARDS IS READ ONLY IF THE STARTING
C* POSITION OF EACH ORTHOGONAL IS READ INDIVIDUALLY ( SEE THE
C* FIRST DATA CARD ).
C* IF THE STARTING POSITIONS ARE COMPUTED WE WILL READ JUST
```

```
C* ONE DATA CARD HERE. FOR STIPULATIONS SEE SUBROUTINE ALTER.
C* WE NOW READ N CARDS - N BEING THE NO. OF WAVE ORTHOGONALS -
C* EACH DATA CARD WILL HAVE THE X & Y COORDINATE AND ORTHOGONAL
C* DIRECTION VALUES RESPECTIVELY AT THE STARTING POSITION OF
C* THE CORESPONDING ORTHOGONAL.
      DO 1 I=1,N
      READ 100,XI(I),YI(I),WDN(I)
      IF (ORIEN.EQ.'GRID') GO TO 1
      WDN(I)=GAMMA-WDN(I)
    1 CONTINUE
   21 CONTINUE
C**********
C* WE NOW READ NX DATA SETS, WHICH HAVE THE DEPTH COORDINATES
C* OF THE GRID POINTS.
C* EACH SET MUST HAVE NY VALUES.
C* HENCE, FOR EXAMPLE THE EIGTH VALUE IN THE FIFTH SET
C* WOULD BE THE DEPTH AT GRID POINT X=5, Y=8.
C* EACH DATA SET CAN STRETCH OVER ANY NUMBER OF CARDS,
C* BUT MUST START ON A NEW CARD.
C* WHERE A POINT APPEARS ON LAND IT CAN BE GIVEN A VALUE
C* OF ZERO (REAL), HOWEVER FOR SLOPE DETERMINATION CLOSE TO
C* THE SHORELINE IT IS MORE ACCURATE TO INSERT APPROPRIATE
C* NEGATIVE VALUES FOR GRID POINTS ADJACENT TO THE SHORELINE.
      DO 2 I=1,NX
    2 READ 100,(H(I,J),J=1,NY)
      IF(ABS(TIDE).LT.1.E-5) GO TO 44
      DO 43 I=1,NX
      DO 43 J=1,NY
   43 H(I,J)=H(I,J)+TIDE
   44 CONTINUE
      CALL CALH(T,CC)
      CALL PROG(N,T,NX,NY,DEL,DS,DES,TIM,CC,ORIEN,GAMMA)
      STOP
      END
```

TRIG*REFRAC(1).ALTER

```
      SUBROUTINE ALTER(ORIEN,GAMMA,N)
C* THIS SUBROUTINE IS CALLED BY THE MAIN PROGRAM IF  STARTING
C* POSITIONS AND DIRECTIONS OF ORTHOGONALS ARE NOT INDIVIDUALLY
C* READ. IT CAN BE USED FOR ORTHOGONALS THAT ARE STILL IN DEEP
C* WATER, I.E. AS REFRACTION HAS NOT YET OCCURRED THE WAVE FRONT
C* IS STRAIGHT AND ORTHOGONALS ARE THUS PARALLEL.
C* WE NEED THE STARTING POINT OF ONE ORTHOGONAL, THE ORTHOGONAL
C* IS THE ONE ON THE EXTREME LEFT OF THE WAVE FRONT WHEN
C* LOOKING IN THE DIRECTION OF THE WAVE PROGRESSION.
      COMMON H(150,150),C(300),XI(100),YI(100)
     1,IBUF(2000),WDN(100)
C**********
C* HERE WE READ THE INITIAL X & Y VALUES OF THE ORTHOGONAL,
C* THE WAVE DIRECTION AND THE DESIRED SPACING BETWEEN ADJACENT
C* ORTHOGONALS, MEASURED ALONG THE WAVE FRONT, EXPRESSED AS
C* NUMBER OF GRID LENGTHS.
      READ 100,XI1,YI1,WDNI1,DIST1
  100 FORMAT()
      IF(ORIEN.EQ.'GRID') GO TO 14
      WDNI1=GAMMA-WDNI1
   14 ANG=(WDNI1-90.)*1.745329E-2
      XI(1)=XI1
      YI(1)=YI1
      WDN(1)=WDNI1
      DX=DIST1*COS(ANG)
      DY=DIST1*SIN(ANG)
      DO 11 I=2,N
      XI(I)=XI(I-1)+DX
      YI(I)=YI(I-1)+DY
   11 WDN(I)=WDNI1
      RETURN
      END
```

TRIG*REFRAC(1).CWLH

```
      SUBROUTINE CWLH(T,CC)
C* THIS SUBROUTINE CALCULATES AND STORES WAVE CELERITIES FOR
C* SPECIFIC DEPTHS UP TO DEEP WATER, FOR INTERPOLATION LATER
C* IN THE PROGRAM. DEPTHS CHOSEN ARE AT HALF METRE INTERVALS
C* TILL TEN METRES WATER DEPTH, AND AT ONE METRE INTERVALS
C* FROM TEN METRES TO DEEP WATER.
C* AS THE RELATIONSHIP BETWEEN DEPTH AND WAVE CELERITY IS
C* IMPLICIT A BISECTION METHOD IS USED.
      COMMON H(150,150),C(300)
      CO=1.5608*T
      HC=0.780*T*T+14.
      IHC=HC
   31 D=10.5
      CALL SPEED(T,D,WC)
      CC=WC
   32 DO 1 I=1,19
      D=I
      D=D/2.
      CALL SPEED(T,D,WC)
    1 C(I)=WC
   33 DO 2 I=20,IHC
      D=I-10
      CALL SPEED(T,D,WC)
    2 C(I)=WC
      RETURN
      SUBROUTINE SPEED(T,D,WC)
      A=0.
      B=1.6*T
      CP=0.8*T
      FA=CO
   22 FC=CO*TANH(6.2832*D/CP/T)-CP
      IF(FA*FC.LT.0.) GO TO 17
      A=CP
      FA=FC
      GO TO 19
   17 B=CP
   19 CP=(A+B)/2.
      IF(ABS(B-A).LT.1.E-5) GO TO 21
      GO TO 22
   21 WC=CP
      RETURN
      END
```

TRIG*REFRAC(1).PROG

```
      SUBROUTINE PROG(N,T,NX,NY,DEL,DS,DES,TIM,CC,ORIEN,GAMMA)
C* THIS SUBROUTINE CALCULATES THE PROGRESSION OF EACH ORTHOGONAL,
C* BY PROGRESSING FROM THE GIVEN STARTING POINT IN A SET OF
C* CIRCULAR INCREMENTS.
C* FOR EACH INCREMENT THE CURVATURE  AND THE TRAVEL TIME ARE
C* COMPUTED. HENCE THE FINAL COORDINATES, DIRECTION AND TIME
C* VALUES ARE COMPUTED.
C* THE ORTHOGONAL CALCULATION IS DISCONTINUED IN THE EVENT OF
C* THE WATER DEPTH BEING LESS THAN ONE METRE, OR THE ORTHOGONAL
C* POSITION BEING WITHIN ONE GRIDLENGTH OF ANY OF THE BORDERS.
C* THESE ARRAYS ARE PRINTED, AND IF DESIRED ARE PLOTED. (SEE
C* SUBROUTINE PPLOT.)
      COMMON H(150,150),C(300),XI(100),YI(100)
     1,IBUF(2000),WDN(100)
      REAL K1,K2,K3,K4
      HC=0.780*T*T
      IHC=HC
  100 FORMAT()
      IF(DES.EQ.'NP') GO TO 1000
      CALL STPLOT(NX,NY,DELTAV,NUM,XAXLEN,INT)
 1000 CONTINUE
      DCS=DS
C-----------------------------------------------------------------
C* THE FIRST LOOP STARTS HERE. ON EACH SUCCESSIVE TRAVEL A NEW
C* WAVE ORTHOGONAL IS COMPUTED.
C------------------------------------------------------------------
      DO 98 II=1,N
C* THE ORTHOGONAL PROJECTION IS INITIATED BY SETTING THE
C* STARTING VALUES.
      WC=2.
      H3=2.
      TIME=0.
      X=XI(II)
      Y=YI(II)
      THE=WDN(II)*1.745329E-2
      THED=THE*57.295788
      MM=1
  400 FORMAT(1H1)
      PRINT 200,II,X,Y
  200 FORMAT(1H1,' ORTHOGONAL NO.',I3,',  STARTING PT.:(',F6.2,',',F6.2
     1,')')
  194 FORMAT(1H ,/,7X,'X',12X,'Y',10X,'THETA',8X,'TIME'
     1,/,7X,'-',12X,'-',10X,'-----',8X,'----')
  193 FORMAT(1H ,/,7X,'X',12X,'Y',10X,'THETA',7X,
     1'BEARING',7X,'TIME',/,7X,'-',12X,'-',10X,
     2'-----',7X,'-------',7X,'----')
      IF(ORIEN.EQ.'GRID') GO TO 1230
      THEDC=GAMMA-THED
      PRINT 193
      PRINT 100,X,Y,THED,THEDC,TIME
      GO TO 1231
 1230 PRINT 194
      PRINT 100,X,Y,THED,TIME
 1231 IF(DES.EQ.'NP') GO TO 1001
      MMM=0
      X8=X/DELTAV
      Y8=Y/DELTAV
      CALL PLOT(X8,Y8,3)
 1001 CONTINUE
      Z0=C(2)
```

```
C-----------------------------------------------------------------
C* THE SECOND LOOP STARTS HERE. ON EACH SUCCESSIVE TRAVEL THE
C* WAVE ORTHOGONAL IS PROJECTED BY A DISTANCE CORRESPONDING
C* TO A TIME STEP REQUIRED FOR THE WAVE TO TRAVEL A DISTANCE
C* DIST (SEE FOURTH DATA CARD IN MAIN PROGRAM) IN DEEP WATER.
C-----------------------------------------------------------------
    9 CONTINUE
C* THE APPROPRIATE VALUES OF DH/DX, DH/DY AND WAVE CELERITY
C* ARE COMPUTED.
      IIHC=IHC+10
      DS=DCS*WC/1.5608/T
      X1=X+0.5*DS*COS(THE)
      Y1=Y+0.5*DS*SIN(THE)
      IF(H3.LT.1.) GO TO 99
      IF(X1.LT.2.OR.X1.GT.(NX-1)) GO TO 99
      IF(Y1.LT.2.OR.Y1.GT.(NY-1)) GO TO 99
      I=X1
      J=Y1
      DX=X1-I
      DY=Y1-J
      K1=1-DX-DY+DX*DY
      K2=DY-DX*DY
      K3=DX-DX*DY
      K4=DX*DY
      H1=H(I-1,J)*K1+H(I-1,J+1)*K2+H(I,J)*K3+H(I,J+1)*K4
      H2=H(I,J-1)*K1+H(I,J)*K2+H(I+1,J-1)*K3+H(I+1,J)*K4
      H3=H(I,J)*K1+H(I,J+1)*K2+H(I+1,J)*K3+H(I+1,J+1)*K4
      H4=H(I,J+1)*K1+H(I,J+2)*K2+H(I+1,J+1)*K3+H(I+1,J+2)*K4
      H5=H(I+1,J)*K1+H(I+1,J+1)*K2+H(I+2,J)*K3+H(I+2,J+1)*K4
      IF(H3.LT.1.) GO TO 99
      IF(H3.LT.10.) GO TO 421
      K=H3+10
      IF(K.GT.(IIHC)) K=IIHC
      DDH=H3+10.-K
      IF(K.EQ.(IIHC)) DDH=0.
      GO TO 422
  421 RK=2.*H3
      K=RK
      DDH=2.*H3-K
  422 CONTINUE
      ZM=C(K-1)
      Z0=C(K)
      Z1=C(K+1)
      Z2=C(K+2)
      IF(K.EQ.20) ZM=C(19)
      IF(K.EQ.19) Z2=CC
      A1=Z0
      B1=(-2.*ZM-3.*Z0+6.*Z1-Z2)/6.
      C1=(ZM-2.*Z0+Z1)/2.
      D1=(-ZM+3.*Z0-3.*Z1+Z2)/6.
      WC=A1+B1*DDH+C1*DDH*DDH+D1*DDH*DDH*DDH
      DHDX=(H5-H1)/2./DEL
      DHDY=(H4-H2)/2./DEL
C* THE CURVATURE AND HENCE FINAL POSITION & DIRECTION FOR
C* THE INCREMENT ARE COMPUTED.
      AA1=1./(COSH(6.28319*H3/WC/T))**2
      AA2=9.80665/(WC*WC)*AA1
      CURV=AA2/(1.+H3*AA2)*(SIN(THE)*DHDX-COS(THE)*DHDY)
      R=1./CURV
      DTHETA=DS*CURV*DEL
      IF(ABS(DTHETA).GT.1.E-2) GO TO 11
      DN=DS*DTHETA/2.
      DS1=DS
      GO TO 12
   11 DS1=R*SIN(DTHETA)/DEL
      DN=R*(1.-COS(DTHETA))/DEL
```

```
   12 DEX=DSI*COS(THE)-DN*SIN(THE)
      DEY=DSI*SIN(THE)+DN*COS(THE)
      DT=DS/AC*DEL
C* ALL TIME VALUES CORRESPONDING TO INTEGER MULTIPLES OF
C* TIM (SEE FOURTH DATA CARD IN MAIN PROGRAM) ARE PRINTED
C* AND PLOTTED.
      X9=X
      Y9=Y
      THED9=THED
      TIME9=TIME
      X=X+DEX
      Y=Y+DEY
      THE=THE+DTHETA
      THED=THE*57.295788
      TIME=TIME+DT
      FT=TIME/TIM
      FT9=TIME9/TIM
      IFT=FT
      IFT9=FT9
      IF(IFT9.EQ.IFT) GO TO 9
      ZZ=MM
      MM=MM+1
      PRO=(TIM*ZZ-TIME9)/(TIME-TIME9)
      X8=X9+PRO*(X-X9)
      Y8=Y9+PRO*(Y-Y9)
      THED8=THED9+PRO*(THED-THED9)
      TIME8=TIM*ZZ
      IF(ORIEN.EQ.'GRID') GO TO 1240
      THEDC8=GAMMA-THED8
      PRINT 100,X8,Y8,THED8,THEDC8,TIME8
      GO TO 1241
 1240 PRINT 100,X8,Y8,THED8,TIME8
 1241 IF(DES.EQ.'NP') GO TO 1002
      MMM=MMM+1
      X8=X8/DELTAV
      Y8=Y8/DELTAV
      ANGLE=THED8+90.
      CALL PLOT(X8,Y8,2)
      IF(MMM.LT.INT) GO TO 1002
      MMM=0
      GO TO (1002,1102,1103),NUM
 1103 CALL NUMBER(X8,Y8,0.10,TIME8,ANGLE,-1)
 1102 CALL SYMBOL(X8,Y8,0.20,3,ANGLE,-1)
      CALL PLOT(X8,Y8,3)
 1002 CONTINUE
C-----------------------------
C* THE SECOND LOOP ENDS HERE.
C-----------------------------
      GO TO 9
C* THE LAST VALUE OF THE ORTHOGONAL POSITION IS PLOTTED.
   99 CONTINUE
      IF(DES.EQ.'NP') GO TO 98
      X8=X/DELTAV
      Y8=Y/DELTAV
      CALL PLOT(X8,Y8,2)
      RII=II+.1
      CALL NUMBER(X8,Y8,0.1,RII,0.0,-1)
C---------------------------
C* THE FIRST LOOP ENDS HERE.
C---------------------------
   98 CONTINUE
      PRINT 400
      IF(DES.EQ.'NP') GO TO 900
      DIST=XAXLEN+4.
      CALL PLOT(DIST,0.0,999)
  900 RETURN
      END
```

```
TRIG*REFRAC(1).STPLOT
          SUBROUTINE STPLOT(NX,NY,DELTAV,NUM,XAXLEN,INT)
C* IF A PLOT IS REQUIRED ( SEE MAIN PROGRAM ) THIS SUBROUTINE
C* INITIALISES THE PLOT, AND DRAWS THE AXES.
C* THE DATA READ IN THIS SUBROUTINE IS READ AFTER ALL THE OTHER
C* DATA, AND MAY BE OMITTED IF A PLOT IS NOT REQUIRED.
      COMMON H(150,150),C(300),XI(100),YI(100)
     1,IBUF(2000)
      DIMENSION SHX(300),SHY(300)
C**********
C* HERE WE READ WHETHER WE WANT TIME & ORTHOGONAL DIRN. VALUES
C* PRINTED, AND AT WHAT TIME SPACING THEY MUST BE.
C* THE FIRST VARIABLE CAN ASSUME A VALUE OF 1,2 OR 3.
C* IF IT IS 1, THE ORTHOGONAL ALONE IS DRAWN.
C* IF IT IS 2, WAVE FRONT DIRECTIONS ARE DRAWN AT THE APPROPRIATE
C* TIME INTERVALS.
C* IF IT IS 3, BOTH WAVE FRONT DIRECTIONS AND TIME VALUES ARE
C* DRAWN AND PRINTED
C* THE SECOND VARIABLE IS THE INTEGER MULTIPLE OF THE TIME
C* INCREMENT VALUE ( SEE TIM ON THE FOURTH DATA CARD IN
C* THE MAIN PROGRAM ) FOR WHICH DIRECTION AND TIME VALUES
C* ARE DRAWN AND PRINTED ON THE PLOT.
C* BOTH VARIABLES ARE ENTERED ON ONE DATA CARD AS INTEGERS.
      READ 100,NUM,INT
      RNX=NX
      RNY=NY
C**********
C* HERE WE READ, ON ONE DATA CARD, THE LENGTH OF THE Y-AXIS
C* IN INCHES, AND THE SCALE FACTOR. BOTH ARE REAL.
      READ 100,A,FACT
      DELTAV=(RNY-1.)/A
      AX=1./DELTAV
C**********
C* THE SHORELINE DATA IS READ HERE.
C* THE FIRST CARD REPRESENTS THE NO. OF SHORE COORDINATES,
C* SAY N, TO BE READ (AN INTEGER)- SUCCESSIVE POINTS TO BE
C* JOINED BY STRAIGHT LINES.
C* THE SUBSEQUENT N DATA CARDS EACH HAVE THE GRID COORDINATES
C* - X & Y RESPECTIVELY - OF THE CORRESPONDING POINT.
      READ 100,IA
      DO 5 I=1,IA
      READ 100,SHX(I),SHY(I)
      SHX(I)=SHX(I)/DELTAV
    5 SHY(I)=SHY(I)/DELTAV
  100 FORMAT()
      CALL PLOTS(IBUF,2000,6)
      CALL FACTOR(FACT)
      CALL PLTIME(40)
      CALL PLOT(.5,.5,-3)
      YAXLEN=A
      XAXLEN=A*(RNX-1.)/(RNY-1.)
      YAX=A+AX
      XAX=XAXLEN+AX
      CALL AXIS(AX,AX,1HY,1,YAXLEN,90.0,1.0,DELTAV)
      CALL AXIS(AX,YAX,1H ,1,XAXLEN,0.0,1.0,DELTAV)
      CALL AXIS(AX,AX,1HX,-1,XAXLEN,0.0,1.0,DELTAV)
      CALL AXIS(XAX,AX,1H ,-1,YAXLEN,90.0,1.0,DELTAV)
      CALL PLOT(SHX(1),SHY(1),3)
      DO 60 I=2,IA
   60 CALL PLOT(SHX(I),SHY(I),2)
      RETURN
      END
```

Fig. A15. Source program listing.

COMPUTER SYMBOL	SYMBOL IN TEXT	DESCRIPTION
CØ	c(max)	Deep water wave celerity
CURV	$d\theta/ds$	The instanteneous curvature of an orthogonal
DEL	s	The spacing of the grid mesh in metres
DES		An alphanumeric word specifying whether a plot is to be drawn (see Section A2.5)
DS	Δz	The increment size (in grid mesh spacing units)
DCS	Δz_o	The increment size in deep water (in grid mesh spacing units)
DIST	Δz_o	The increment size in deep water (in metres)
DELTAV		The factor by which a grid mesh spacing must be divided to give an inch unit for use on the plotter
DX	dx	The incremental change in x
DY	dy	The incremental change in y
DTHETA	$d\theta$	The incremental change in θ
DHDX	$\delta h/\delta x$	The instantaneous rate of change of depth with respect to x
DHDY	$\delta h/\delta y$	The instantaneous rate of change of depth with respect to y
FACT		The scale factor for the size of plot (see Section A2.5)
GAMMA	ϕ	The compass bearing of the X-axis (see Section A2.5)
HC	h_c	The critical depth in metres
H1	h(x-1,y)	See Section 3.3
H2	h(x,y-1)	See Section 3.3
H3	h(x,y)	See Section 3.3 All depths in metres
H4	h(x,y+1)	See Section 3.3
H5	h(x+1,y)	See Section 3.3
IA		The number of shore co-ordinates (see Section A2.5)
INT		Integer multiple of TIM at which wave fronts are to be drawn (see Section A2.5)
I	i	Integer representing first subscript of an array
J	j	Integer representing second subscript of an array
K1	K_1	See Section 3.3
K2	K_2	See Section 3.3
K3	K_3	See Section 3.3
K4	K_4	See Section 3.3
N	m	Number of orthogonals

COMPUTER SYMBOL	SYMBOL IN TEXT	DESCRIPTION
NX	n_x	Number of x co-ordinates in study area
NY	n_y	Number of y co-ordinates in study area
NUM		Specification of whether wave fronts are to be drawn (see Section A2.5)
ORIEN		An alphanumeric word specifying whether directions are compass bearing or cartesian angles (see Section A2.5)
R	R	Instantaneous radius of curvature of an orthogonal
START		An alphanumeric word specifying whether orthogonal starting positions are to be generated (see Section A2.5)
SHX		The x co-ordinate of a shoreline co-ordinate
SHY		The y co-ordinate of a shoreline co-ordinate
T	T	The wave period in seconds
TIDE	ψ	The tidal shift in metres
TIM		The time increment of orthogonal (travel) in seconds, between positions joined by straight lines in the plot, and positions at which printed parameters are given (in seconds)
THE	θ	The instantaneous wave direction as a cartesian angle in radians
THED*	θ	The instantaneous wave direction as a cartesian angle in degrees
THEDC*	β	The instantaneous wave direction as a compass bearing
TIME*	t	The instantaneous time of travel of an orthogonal, from the starting position (in seconds)
X,Y*	x,y	Instantaneous co-ordinates of orthogonal
X1,Y1		Starting co-ordinates of orthogonal
XAXLEN		Length in inches of the X-axis in the plot
YAXLEN		Length in inches of the Y-axis in the plot

* These symbols followed by 8, e.g. THED8, correspond to the interpolated values which are printed and plotted (see Sections A2.6 and A2.7).

Figure A16 : Major variables used in writer's program

APPENDIX B.

Shoaling Wave Charts.

The following charts have been compiled for determining wave heights, lengths and celerities for a wave of period T, and deep water wave height H_o, travelling through shoaling water with a general undisturbed depth of D. The charts are also applicable to waves which are subject to refraction; but when refraction occurs the relative height (H/H_o) must be modified by a refraction coefficient.

The range of wave periods covered is from six seconds to sixteen seconds, which covers most waves encountered in practice.

The values were obtained from the small amplitude wave theory and constant power transmission.

T(s) / D(m)	6	7	8	9	10	11	12	13	14	15	16	T(s) / D(m)
1	1,26	1,35	1,43	1,52	1,60	1,67	1,74	1,81	1,88	1,94	2,01	1
2	1,09	1,16	1,23	1,29	1,36	1,42	1,48	1,53	1,59	1,64	1,69	2
3	1,01	1,07	1,13	1,18	1,24	1,29	1,34	1,39	1,44	1,49	1,54	3
4	0,97	1,02	1,06	1,11	1,16	1,21	1,26	1,30	1,35	1,39	1,43	4
5	0,94	0,98	1,02	1,07	1,11	1,16	1,20	1,24	1,28	1,32	1,36	5
6	0,93	0,96	0,99	1,03	1,07	1,11	1,15	1,19	1,23	1,27	1,31	6
7	0,92	0,94	0,97	1,01	1,04	1,08	1,12	1,15	1,19	1,23	1,26	7
8	0,91	0,93	0,96	0,99	1,02	1,05	1,09	1,12	1,16	1,19	1,23	8
9	0,91	0,92	0,94	0,97	1,00	1,03	1,06	1,10	1,13	1,16	1,20	9
10	0,91	0,92	0,93	0,96	0,98	1,01	1,04	1,08	1,11	1,14	1,17	10
11	0,92	0,91	0,93	0,95	0,97	1,00	1,03	1,06	1,09	1,12	1,15	11
12	0,92	0,91	0,92	0,94	0,95	0,99	1,01	1,04	1,07	1,10	1,13	12
13	0,93	0,91	0,92	0,93	0,96	0,97	1,00	1,03	1,05	1,08	1,11	13
14	0,93	0,92	0,92	0,93	0,94	0,96	0,99	1,01	1,04	1,07	1,09	14
15	0,94	0,92	0,91	0,92	0,94	0,96	0,98	1,00	1,03	1,05	1,08	15
16	0,94	0,92	0,91	0,92	0,93	0,95	0,97	0,99	1,02	1,04	1,06	16
17	0,95	0,92	0,91	0,92	0,93	0,94	0,96	0,98	1,01	1,03	1,05	17
18	0,96	0,93	0,91	0,91	0,92	0,94	0,96	0,98	1,00	1,02	1,04	18
19	0,96	0,93	0,92	0,91	0,92	0,93	0,95	0,97	0,99	1,01	1,03	19
20	0,97	0,94	0,92	0,91	0,92	0,93	0,94	0,96	0,98	1,00	1,02	20
30	0,99	0,97	0,95	0,93	0,92	0,91	0,92	0,92	0,93	0,95	0,96	30
40	1,00	0,99	0,98	0,95	0,93	0,92	0,91	0,91	0,92	0,92	0,93	40
50	1,00	1,00	0,99	0,98	0,96	0,94	0,92	0,92	0,91	0,91	0,92	50
60	1,00	1,00	1,00	0,99	0,97	0,96	0,94	0,93	0,92	0,91	0,91	60
70	1,00	1,00	1,00	0,99	0,98	0,97	0,95	0,94	0,93	0,92	0,91	70
80	1,00	1,00	1,00	1,00	0,99	0,98	0,97	0,95	0,94	0,93	0,92	80
90	1,00	1,00	1,00	1,00	1,00	0,99	0,98	0,96	0,95	0,93	0,93	90
100	1,00	1,00	1,00	1,00	1,00	0,99	0,98	0,97	0,96	0,94	0,93	100
120	1,00	1,00	1,00	1,00	1,00	1,00	0,99	0,99	0,97	0,96	0,95	120
140	1,00	1,00	1,00	1,00	1,00	1,00	1,00	0,99	0,99	0,98	0,96	140
160	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	0,99	0,99	0,98	160
180	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	0,99	0,99	180
200	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	0,99	200
D(m) / T(s)	6	7	8	9	10	11	12	13	14	15	16	D(m) / T(s)

RELATIVE WAVE HEIGHT (H/Ho)

T(s) / D(m)	6	7	8	9	10	11	12	13	14	15	16	T(s) / D(m)
1	3,1	3,1	3,1	3,1	3,1	3,1	3,1	3,1	3,1	3,1	3,1	1
2	4,3	4,3	4,3	4,4	4,4	4,4	4,4	4,4	4,4	4,4	4,4	2
3	5,1	5,2	5,3	5,3	5,3	5,3	5,3	5,3	5,4	5,4	5,4	3
4	5,8	5,9	6,0	6,1	6,1	6,1	6,1	6,2	6,2	6,2	6,2	4
5	6,3	6,5	6,6	6,7	6,8	6,8	6,8	6,9	6,9	6,9	6,9	5
6	6,8	7,0	7,1	7,3	7,4	7,4	7,5	7,5	7,5	7,5	7,6	6
7	7,2	7,5	7,7	7,8	7,9	8,0	8,0	8,1	8,1	8,1	8,1	7
8	7,5	7,9	8,1	8,3	8,4	8,5	8,5	8,6	8,6	8,6	8,7	8
9	7,8	8,2	8,5	8,7	8,8	8,9	9,0	9,1	9,1	9,1	9,2	9
10	8,1	8,5	8,9	9,1	9,2	9,4	9,4	9,5	9,6	9,6	9,6	10
11	8,3	8,8	9,1	9,4	9,6	9,8	9,9	9,9	10,0	10,0	10,1	11
12	8,5	9,1	9,5	9,8	10,0	10,1	10,2	10,3	10,4	10,5	10,5	12
13	8,6	9,3	9,7	10,1	10,3	10,4	10,6	10,7	10,8	10,9	10,9	13
14	8,7	9,5	10,0	10,4	10,6	10,8	11,0	11,1	11,2	11,2	11,3	14
15	8,8	9,7	10,2	10,6	10,9	11,1	11,3	11,4	11,5	11,6	11,7	15
16	8,9	9,8	10,4	10,9	11,2	11,4	11,6	11,7	11,8	11,9	12,0	16
17	9,0	10,0	10.6	11,1	11,4	11,7	11,9	12,0	12,2	12,3	12,3	17
18	9,1	10,1	10,7	11,3	11,7	12,0	12,2	12,3	12,5	12,6	12,7	18
19	9,1	10,2	11,0	11,5	11,9	12,2	12,4	12,6	12,8	12,9	13,0	19
20	9,2	10,3	11,1	11,7	12,1	12,4	12,7	12,9	13,0	13,2	13,3	20
30	9,3	10,8	12,0	13,0	13,7	14,3	14,8	15,1	15,4	15,6	15,8	30
40	9,4	10,9	12,3	13,6	14,6	15,5	16,1	16,7	17,1	17,4	17,7	40
50	9,4	10,9	12,4	13,9	15,1	16,2	17,1	17,8	18,4	18,8	19,2	50
60	9,4	10,9	12,5	14,0	15,4	16,6	17,7	18,6	19,3	19,9	20,4	60
70	9,4	10,9	12,5	14,0	15,5	16,9	18,1	19,1	20,0	20,8	21,4	70
80	9,4	10,9	12,5	14,0	15,6	17,0	18,3	19,5	20,6	21,4	22,2	80
90	9,4	10,9	12,5	14,0	15,6	17,1	18,5	19,8	20,9	22,0	22,8	90
100	9,4	10,9	12,5	14,0	15,6	17,1	18,6	20,0	21,2	22,3	23,3	100
120	9,4	10,9	12,5	14,0	15,6	17,2	18,7	20,2	21,6	22,8	24,0	120
140	9,4	10,9	12,5	14,0	15,6	17,2	18,7	20,2	21,7	23,1	24,4	140
160	9,4	10,9	12,5	14,0	15,6	17,2	18,7	20,3	21,8	23,3	24,7	160
180	9,4	10,9	12,5	14,0	15,6	17,2	18,7	20,3	21,8	23,3	24,8	180
200	9,4	10,9	12,5	14,0	15,6	17,2	18,7	20,3	21,8	23,4	24,9	200
D(m) / T(s)	6	7	8	9	10	11	12	13	14	15	16	D(m) / T(s)

WAVE CELERITY (m/s)

D(m) \ T(s)	6	7	8	9	10	11	12	13	14	15	16	T(s) / D(m)
1	18	22	25	28	31	34	37	41	44	47	50	1
2	26	30	35	39	44	48	53	57	62	66	70	2
3	31	36	42	48	53	59	64	70	75	81	86	3
4	35	41	48	54	61	67	74	80	86	93	99	4
5	38	46	53	60	68	75	82	89	96	103	111	5
6	41	49	57	66	74	82	89	97	105	113	121	6
7	43	52	61	70	79	88	96	105	113	122	130	7
8	45	55	65	74	84	93	102	111	121	130	139	8
9	47	58	68	78	88	98	108	118	127	137	147	9
10	48	60	71	82	92	103	113	124	134	144	154	10
11	50	62	73	85	96	107	118	129	140	151	161	11
12	51	63	76	88	100	111	123	134	146	157	168	12
13	52	65	78	91	103	115	127	139	151	163	174	13
14	52	66	80	93	106	119	131	144	156	168	181	14
15	53	68	82	96	109	122	135	148	161	174	186	15
16	54	69	83	98	112	126	139	152	166	179	192	16
17	54	70	85	100	114	129	143	156	170	184	197	17
18	54	71	86	102	117	131	146	160	174	189	203	18
19	55	71	88	104	119	134	149	164	179	193	207	19
20	55	72	89	105	121	137	152	168	183	197	212	20
30	56	75	96	117	137	157	177	196	215	234	253	30
40	56	76	99	122	146	170	194	217	239	262	284	40
50	56	76	100	125	151	178	205	231	257	283	308	50
60	56	76	100	126	154	183	212	242	270	299	327	60
70	56	76	100	126	155	186	217	249	280	312	343	70
80	56	76	100	126	156	187	220	254	288	322	355	80
90	56	76	100	126	156	188	222	257	293	329	365	90
100	56	76	100	126	156	188	223	260	297	335	373	100
120	56	76	100	126	156	189	224	262	302	343	384	120
140	56	76	100	126	156	189	225	263	304	347	391	140
160	56	76	100	126	156	189	225	264	305	349	395	160
180	56	76	100	126	156	189	225	264	306	350	397	180
200	56	76	100	126	156	189	225	264	306	351	398	200
D(m) / T(s)	6	7	8	9	10	11	12	13	14	15	16	D(m) / T(s)

WAVE LENGTH (m)

APPENDIX C

Solution of equation (3.7) for surface fit through four closest nodal points.

$$h(x,y) = a + b.\lambda x + c.\lambda y + d.\lambda x.\lambda y. \qquad (C.1)$$

When $\lambda x = 0$ and $\lambda y = 0$, $h(x,y) = h(i,j)$. Thus equation (C.1) yields

$$a = h(i,j). \qquad (C.2)$$

Substitution of equation (C.2) in equation (C.1) yields

$$h(x,y) = h(i,j) + b.\lambda x + c.\lambda y + d.\lambda x.\lambda y. \qquad (C.3)$$

When $\lambda x = 1$ and $\lambda y = 0$, $h(x,y) = h(i+1,j)$. Thus equation (C.3) yields

$$b = h(i+1,j) - h(i,j). \qquad (C.4)$$

Substitution of equation (C.4) in equation (C.3) yields

$$h(x,y) = h(i,j) + \{h(i+1,j) - h(i,j)\}.\lambda x + c.\lambda y + d.\lambda x.\lambda y. \qquad (C.5)$$

When $\lambda y = 1$ and $\lambda x = 0$, $h(x,y) = h(i,j+1)$. Thus equation (C.5) yields

$$c = h(i,j+1) - h(i,j). \qquad (C.6)$$

Substitution of equation (C.6) in equation (C.5) yields

$$h(x,y) = h(i,j) + \{h(i+1,j) - h(i,j)\}.\lambda x + \{h(i,j+1) - h(i,j)\}.\lambda y + d.\lambda x.\lambda y. \qquad (C.7)$$

When $\lambda x = 1$ and $\lambda y = 1$, $h(x,y) = h(i+1,j+1)$. Thus equation (C.7) yields

$$h(i+1,j+1) = h(i,j) + \{h(i+1,j) - h(i,j)\} + \{h(i,j+1) - h(i,j)\} + d. \qquad (C.8)$$

Hence

$$d = h(i,j) - h(i+1,j) - h(i,j+1) + h(i+1,j+1) \qquad (C.9)$$

Substitution of equation (C.9) in equation (C.7) yields

$$h(x,y) = h(i,j) + \{h(i+1,j) - h(i,j)\}.\lambda x + \{h(i,j+1) - h(i,j)\}.\lambda y$$
$$+ \{h(i,j) - h(i+1,j) - h(i,j+1) + h(i+1,j+1)\}.\lambda x.\lambda y. \qquad (C.10)$$

Hence

$$h(x,y) = h(i,j).\{1 -\lambda x-\lambda y-\lambda x.\lambda y\} + h(i+1,j).\{\lambda x-\lambda x.\lambda y\}$$
$$+ h(i,j+1).\{\lambda y-\lambda x.\lambda y\} + h(i+1,j+1).\{\lambda x.\lambda y\}. \qquad (C.11)$$

APPENDIX D

The determination of the rate of change of wave celerity with respect to depth (See equation 2.24).

Equation (2.23) states:

$$c = \frac{gT}{2\pi} \tanh \left[\frac{2\pi h}{cT}\right] \qquad (D.1)$$

$$\therefore \frac{dc}{dh} = \frac{gT}{2\pi} \frac{d}{dh} \tanh \left[\frac{2\pi h}{cT}\right] . \qquad (D.2)$$

Now $$\frac{d}{dh} \tanh \left[\frac{2\pi h}{cT}\right] = \operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right] \frac{d}{dh} \frac{2\pi h}{cT} \qquad (D.3)$$

Now $$\frac{d}{dh} \left(\frac{2\pi h}{cT}\right) = \frac{2\pi}{cT} + \frac{2\pi h}{T} \left(- \frac{1}{c^2}\right) \frac{dc}{dh}$$

$$\therefore \frac{d}{dh} \left(\frac{2\pi h}{cT}\right) = \frac{2\pi}{cT} \left(1 - \frac{h}{c} \frac{dc}{dh}\right) \qquad (D.4)$$

Substitution of (D.4) in (D.3) yields

$$\frac{d}{dh} \tanh \left[\frac{2\pi h}{cT}\right] = \operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right] \frac{2\pi}{cT} \left(1 - \frac{h}{c} \frac{dc}{dh}\right) \qquad (D.5)$$

Substitution of (D.5) in (D.2) yields

$$\frac{dc}{dh} = \frac{gT}{2\pi} \left(\operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right]\right) \frac{2\pi}{cT} \left(1 - \frac{h}{c} \frac{dc}{dh}\right)$$

$$\therefore \frac{dc}{dh} = \frac{g}{c} \left(\operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right]\right) \left(1 - \frac{h}{c} \frac{dc}{dh}\right)$$

$$\therefore \frac{dc}{dh} = \frac{g}{c} \operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right] - \frac{gh}{c^2} \operatorname{sech}^2 \left[\frac{2\pi h}{cT}\right] \frac{dc}{dh}$$

$$\therefore \quad \frac{dc}{dh}\left\{1 + \frac{gh}{c^2}\,\mathrm{sech}^2\left[\frac{2\pi h}{cT}\right]\right\} = \frac{g}{c}\,\mathrm{sech}^2\left[\frac{2\pi h}{cT}\right]$$

$$\therefore \quad \frac{dc}{dh} = \frac{\frac{g}{c}\,\mathrm{sech}^2\left[\frac{2\pi h}{cT}\right]}{1 + \frac{gh}{c^2}\,\mathrm{sech}^2\left[\frac{2\pi h}{cT}\right]} \qquad \text{(D.6)}$$

APPENDIX E

Third degree interpolation of wave celerities, solution of equation 5.9

The wave celerity in the area considered is approximated by the function where c is to be calculated for the range of ranging between 0 and 1.

$$c(\Delta h) = a_1 + a_2\Delta h + a_3(\Delta h)^2 + a_4(\Delta h)^3 \ldots \qquad (E.1)$$

when $\Delta h = -1, \quad c(\Delta h) = c_{k-1},$ (E.2)

when $\Delta h = 0, \quad c(\Delta h) = c_k,$ (E.3)

when $\Delta h = +1, \quad c(\Delta h) = c_{k+1},$ (E.4)

when $\Delta h = +2, \quad c(\Delta h) = c_{k+2}.$ (E.5)

Considering equation (E.1) and condition (E.3);

$$c_k = a_1 + a_2(0) + a_3(0)^2 + a_4(0)^3.$$

$$\therefore \; a_1 = c_k. \qquad (E.6)$$

Thus equation (E.1) can be re-written as;

$$c(\Delta h) = c_k + a_2\Delta h + a_3(\Delta h)^2 + a_4(\Delta h)^3. \qquad (E.7)$$

Considering equation (E.7) and condition (E.2),

$$c_{k-1} = c_k + a_2(-1) + a_3(-1)^2 + a_4(-1)^3$$

$$\therefore \; c_{k-1} = c_k - a_2 + a_3 - a_4. \qquad (E.8)$$

Considering equation (E.7) and condition (E.4),

$$c_{k+1} = c_k + a_2(+1) + a_3(+1)^2 + a_4(+1)^3$$

$$\therefore \; c_{k+1} = c_k + a_2 + a_3 + a_4. \qquad (E.9)$$

Adding equations (E.8) and (E.9) yields;

$$c_{k-1} + c_{k+1} = 2c_k + 2a_3,$$

$$\therefore \quad a_3 = \frac{c_{k-1} - 2c_k + c_{k+1}}{2} \tag{E.10}$$

Substracting equation (E.8) from equation (E.9) yields;

$$-c_{k-1} + c_{k+1} = 2a_2 + 2a_4. \tag{E.11}$$

Combining equations (E.7) and (E.10) yields;

$$c(\Delta h) = c_k + a_2\Delta h + \frac{c_{k-1} - 2c_k + c_{k+1}}{2}\Delta h^2 + a_4\Delta h^3 \tag{E.12}$$

Considering equation (E.12) and condition (E.5),

$$c_{k+2} = c_k + a_2(2) + \frac{c_{k-1} - 2c_k + c_{k+1}}{2}(2)^2 + a_4(2)^3,$$

$$\therefore \quad -2c_{k-1} + 3c_k - 2c_{k+1} + c_{k+2} = 2a_2 + 8a_4. \tag{E.13}$$

Substracting (E.11) from (E.13) yields;

$$-c_{k-1} + 3c_k - 3c_{k+1} + c_{k+2} = 6a_4,$$

$$\therefore \quad a_4 = \frac{-c_{k-1} + 3c_k - 3c_{k+1} + c_{k+2}}{6}. \tag{E.14}$$

Combining equations (E.14) and (E.13) yields;

$$-2c_{k-1} + 3c_k - 2c_{k+1} + c_{k+2} = 2a_2$$

$$+ 8\frac{-c_{k-1} + 3c_k - 3c_{k+1} + c_{k+2}}{6}$$

$$\therefore \quad a_2 = \frac{-2c_{k-1} - 3c_k + 6c_{k+1} - c_{k+2}}{6}. \tag{E.15}$$

Hence a_1, a_2, a_3 and a_4, are given by equations (E.6), (E.15), (E.10), and (E.14) respectively.

ASCE Engineering Computer Program Documentation Standard.

Engineering Computer Program Documentation Standard

For documenting a computer program, the user of this standard should follow the sequence of topic items presented in the following. Only the items applicable to the particular program should be implemented. Some entries may need expansion or modification, and addition of special topics may be necessary for adequate documentation of certain computer programs. The numbering system is used to refer to the checklist of key items to be covered in program documentation, in Fig. 1, and may be omitted in the program documentation itself.

Section I: Program Identification

1.1 Program Title: A one-line title, describing the problem solved by the program. The title may include the problem that is solved, the method used for the solution, or a name for the procedure used in the program. The title should be less than 80 characters long (one punched card). Example: "Slope Stability Analysis—Bishop Method."

1.2 Program Code Name: A four- to eight-character long code name to identify the program. This can be a key word or a program number. Examples: SLOPE, EP33."

1.3 Writer: Name of the person who prepared the program, the documentation, or who is most familiar with the program. He is to deal with questions relating to the program.

1.4 Organization: Name (and address) of the company, agency, or organization where the program was written.

1.5 Date: Date of completion of first documentation.

1.6 Updates, Versions: Date of any updates or modifications. The original version is numbered zero; new versions are numbered consecutively.

1.7 Source Language: Programming language used in the source program. Example: ASA Fortran, Fortran II, Fortran IV, Basic, Algol, etc. Indicate if the language is associated with a manufacturer or a specific computer: e.g., IBM 1130 Fortran.

1.8 Availability: Organization where the program may be obtained. A complete listing of the source program is part of the Standard ASCE Documentation—indicate if the card deck is available separately. Usually the user is responsible for the results produced by the computer program. The documentation may include a disclaimer, pointing out the limits of the liability of the programmer and of the organization issuing the program.

AMERICAN SOCIETY OF CIVIL ENGINEERS (ASCE)

Engineering Computer Program Documentation Standard

1.0 PROGRAM IDENTIFICATION

1.1 Program Title	1.6 Updates, Version
1.2 Program Code Name	1.7 Source Language
1.3 Writer	1.8 Availability
1.4 Organization	1.9 Abstract
1.5 Date	

2.0 ENGINEERING DOCUMENTATION

2.1 Narrative Description	2.6 Printed Output
2.2 Method of Solution	2.7 Other Output
2.3 Program Capabilities	2.8 Flowchart
2.4 Data Inputs	2.9 Sample Run
2.5 Program Options	

3.0 SYSTEM DOCUMENTATION

3.1 Computer Equipment	3.5 Data Structures
3.2 Peripheral Equipment	3.6 Storage Requirements
3.3 Source Program	3.7 Maintenance and Updates
3.4 Variables & Subroutines	

4.0 OPERATING DOCUMENTATION

4.1 Operator Instructions	4.4 Error Recovery
4.2 Operating Messages	4.5 Run Time
4.3 Control Cards	

Note: The numbering system in this outline is used to refer to the text and is not necessary in the program documentation.

Fig. 1.—Outline of Standard Engineering Computer Program Documentation

1.9 Abstract: Concise description of the program for use in indexes, catalogs, etc. Limit to 10 lines (80 characters each, including spaces) to permit computer processing.

Section II: Engineering Documentation

2.1 Narrative Description: Briefly describe, using engineering terms, the problem solved by the program, its background, and its occurrence. Discuss the application of the program to the solution of the specific engineering problem. List the key references related to the problem and to the method of solution.

2.2 Method of Solution: Describe the basic mathematical expressions used in the solution of the problem. For commonly used formulas, reference to standard methods of analysis (e.g., Earth Pressure Calculation by the Rankine Method) is usually sufficient. Describe any computational procedures, such as iterations, approximations, numerical solutions, etc., unique to the computer solution.

Discuss the major variables manipulated in the program, identifying those expected as input and those produced as output. (Symbols representing the variables should be discussed in Section 3.4.)

Discuss any constant values built into the program and any engineering assumptions implicit in the analysis. Example: Some programs use a preset value of 62.4 pcf for the unit weight of water. Some use certain assumptions regarding the distribution of vertical stress in soil masses—such as the Boussinesq or the Westergaard solution. Identify such procedures, or values, or both, in the documentation.

2.3 Program Capabilities: Discuss the range of values that can be assigned to variables or arrays. State the external and internal limitations of the program to avoid unexplained program failures.

Discuss any limitations of the method of analysis. For example, most slope stability programs cannot handle slip circles whose center is below the uppermost soil layer. Some settlement analysis programs do not include the effects of elastic compression. Such engineering features limiting the use of the program should be clearly explained.

Some programs malfunction if some variables take on certain values, such as zero, or very large values. Describe limiting values for any variables. Example: certain slope stability programs cannot process vertical lines in the slope geometry.

Discuss the sensitivity of the mathematical procedure to a change in the input variables. Which variables produce large changes in the results and which ones are relatively unimportant? Describe the experiences of the programmer in solving various types of problems.

2.4 Data Inputs: Describe, in engineering terms, the data needed for the program—the units, the valid range, and commonly used values for the input data. Give sources for difficult-to-find data. Example: Commonly used subgrade reaction coefficients may be found in: Terzaghi "Evaluation of Coefficients of Subgrade Reaction," *Geotechnique* Vol. V, 1955, pages 297-376.

Data input forms for direct keypunching of data cards are helpful. Such forms specify the input sequence and the data format, and help save labor and prevent errors. A sample data form is shown in Fig. 2. Each field should be identified with a title and the units to be used. Data fields requiring additional explanation can be keyed to notes on the data form. If data input forms are not given, all input formats must be provided.

2.5 Program Options: Describe all the program options available to the user. For example, *Data input codes* are used to repeat some data values in subsequent runs without reentering them into the program. *Mode of operation codes* will branch to alternate program features for special conditions. *Continuation codes* will restart the program without having to reenter all the data.

A *debugging option* provides output for following the intermediate steps of the analysis, before printing the final results. This makes it easy to check the calculations and locate possible sources of trouble.

2.6 Printed Output: For flexibility, some programs include several *output options*. These provide varying amounts of information regarding the solution of the problem. Initially, users usually wish to see the results of the analysis in great detail. After developing confidence in the reliability and accuracy of the program, output of the final answer is usually sufficient. The input data used in the program is usually reprinted in formatted, labeled, and tabulated form to facilitate checking.

2.7 Other Outputs: If any punched cards are produced, their format should be described. Often, another computer program will use such cards for input, and the card formats must be compatible. Sometimes card output can be suppressed.

The engineering significance of all message outputs should be fully described. Describe special operator messages in section 4.2. If plotted output is produced, describe its engineering features. Operator instructions for plotter operation should be given in section 4.1.

If the program writes output on magnetic tape, describe the tapes, the data on each, and their identification. The operational features of magnetic tape output should be described in section 4.1.

2.8 Flow chart: Present the major steps of the program in a condensed flow chart (Macro-Flow-Chart), illustrating the program logic and the steps in the program. Do not include every detail, only the over-all flow c the analysis. Conventional symbols should be used for flow charts.

2.9 Sample Runs: At least one (but preferably several) sample runs should be included to illustrate the program's operation. They should include a brief description of the problem, the input data, and all the output produced by the program. The sample problems should illustrate as many options of the program as practical. If needed, notes should be used to clarify sample runs. Make the sample runs relatively simple to illustrate clearly the input and the output.

Section III: System Documentation

3.1 Computer Equipment: Describe the computer hardware (manufacturer and model number, word length, core access speed) used for running the program. Word length helps establish the accuracy of the calculations and is a clue to potential problems in running the program on computers with different word lengths. Core speed gives an indication of the relative running time on different computers.

3.2 Peripheral Equipment: Describe the necessary peripheral and communication equipment of the computer on which the program was developed, such as: line printers, card-readers, card-punches, paper-tape readers, paper-tape punches, disk drives, magnetic tape drives, plotters, etc. Include the manufacturer, model number, and key features of each item. For magnetic tape drives, the density and the number of tracks should be given.

3.3 Source Program: Provide a complete listing of the source program. Source program listings should contain enough comment cards to explain the major steps of the program. (On the average, one comment card for each 10 program statements is desirable.) Comment cards should include a meaningful description of the operation—not just a restatement of an instruction in the English language. Comment cards, at the beginning of the program, can also be used to describe briefly the nature of the program, its author and date, and the major input variables.

Fortran program cards should be sequentially numbered in columns 72 to 80. Input/output unit numbers should be identified to facilitate conversion to other computer systems.

3.4 Variables and Subroutines: List and describe all the variables used in the program. List and describe all library and systems subroutines and their calling sequence. Describe program sections that are linked (chained or overlaid) to save storage.

3.5 Data Structures: Describe the structure of any files that are created or read by the program, as well as the structure of the data being processed. For magnetic tape files, define file and record lengths and blocking factors. Define any sequential or random access disk files and the page sizes for direct access files. Describe the structure of any matrices used for processing, including the type of structure, tables, and pointers that are used.

3.6 Storage Requirements: State the core storage required for the main program. List core storage requirements of subroutines called by the program. If helpful, provide a core map to facilitate debugging.

3.7 Maintenance and Updates: Describe all program changes, including the reason for the changes and their effects on program output.

Section IV: Operating Documentation

4.1 Operator Instructions: Describe the hardware settings on the computer and the peripheral equipment for operating the program, such as: (a) Console switch settings; (b) magnetic tape unit numbers; (c) disk drives; (d) plotter; and (e) any other special requirements.

4.2 Operating Messages: List and explain any operator actions and check-point messages produced by the program.

4.3 Control Cards: Describe fully all control cards required for running the programs, including the deck setup, definitions of parameters, file descriptions, etc. The file descriptions should explain the format and content of each file, including lables, blocking, units, etc.

4.4 Error Recovery: Describe all required operator actions in case of program malfunction. These might include: (a) Program restart procedures; (b) file re-initializations; (c) core dumps; and (d) other special procedures.

4.5 Run Time: Provide a simple formula for estimating program execution time. The estimated execution time gives the user an idea of the cost of running the program and can help the operator recognize program malfunctions if the program runs substantially longer than anticipated.

APPENDIX G.

Computer Program for generating depth data for Point Island.

```
      DIMENSION H(150,150)
  100 FORMAT()
      DO 10 I=1,141
      DO 10 J=1,141
      X=I
      Y=J
      DIST=SQRT((X-71.)*(X-71.)+(Y-71.)*(Y-71.))*3.
      IF(DIST.GE.200.) H(I,J)=112.5
      IF(DIST.GE.200.) GO TO 10
      A=0.
      B=120.
      CP=60.
      R=DIST/200.
      C=18.736*R
      FA=C-18.736*TANH(0.5236*A/C)
   22 FCP=C-18.736*TANH(0.5236*CP/C)
      IF(FA*FCP.LT.0.) GO TO 17
      A=CP
      FA=FCP
      GO TO 19
   17 B=CP
   19 CP=(A+B)/2.
      IF(ABS(B-A).LT.1.E-3) GO TO 21
      GO TO 22
   21 H(I,J)=CP
   10 CONTINUE
      DO 2 I=1,141
    2 WRITE(11,100),(H(I,J),J=1,141)
      STOP
      END
```

APPENDIX H.

Extract from CALCOMP plotting manual describing CALCOMP subroutines used in writer's program.

PROGRAMMING CALCOMP PEN PLOTTERS

SEPTEMBER 1969

CALIFORNIA COMPUTER PRODUCTS, INC.

PLOT SUBROUTINE

Most graphic applications require the generation of X-Y graphs to show the relationship between two or more sets of data. Usually these graphs can be produced easily and quickly by a suitably programmed combination of the five supporting subroutines SCALE, AXIS, LINE, SYMBOL, and NUMBER. These subroutines do not directly produce plotter commands; they only compute appropriate arguments that define pen positions, and then call the PLOT subroutine, which generates the actual plotter commands.

When unique plotting requirements cannot be satisfied by using the supporting subroutines, the user can resort to the PLOT subroutine, which gives him direct control of pen movement (to any X, Y coordinates position), pen status (up or down), and generation of Search records. (See SEARCH RECORDS under OPERATING CONSIDERATIONS.)

By calling the PLOT subroutine with a different entry name (PLOTS, FACTOR, WHERE, or NEWPEN), the user also has control of: plot output record length; output device opening and closing; enlarging or reducing a plot; locating the pen's current position; and selecting any pen in a Model 618 or 718 multi-pen system. Each entry is described separately below.

PLOT ENTRY

The PLOT entry to the PLOT subroutine is used primarily to move the pen in a straight line to a new position, with the pen either up or down during the movement. It converts the arguments to the appropriate sequence of plotter commands, and outputs these to the attached device (tape, disc, drum, or plotter controller). Note that the PLOTS entry must be called before any other entries are called.

The calling sequence has three arguments:

CALL PLOT (XPAGE, YPAGE, ±IPEN)

XPAGE, YPAGE — are the X, Y coordinates, in inches from the current reference point (origin), of the position to which the pen is to be moved. An origin (where both X, Y equal zero) may be established anywhere on (or off) the plotting surface, as explained below for negative IPEN values.

±IPEN — is a signed integer which controls pen up/down status, origin definition, and the generation of Search records.

If IPEN = 2, the pen is <u>down</u> during movement, thus drawing a visible line.

If IPEN = 3, the pen is <u>up</u> during movement.

If IPEN = −2 or −3, a new origin is defined at the terminal position after the movement is completed as if IPEN were positive. The logical X, Y coordinates of the new pen position are set equal to zero, so that that position is the reference point for succeeding pen movements. In addition, all of the plotter commands accumulated in the output buffer area are transmitted to the output device.

If the plotting system is offline, a Search record with the next sequential Search address is also produced. (See SEARCH RECORDS under OPERATING CONSIDERATIONS.)

If IPEN = 999, the effects are the same as if IPEN = −3, except that a Search record with Search address 999 is written, and the output device is closed. IPEN = 999 may be used only once in a given program, and must be the last plotting call in the program.

The examples in Figure 3 show the pen movements that result from a series of calls to the PLOT subroutine. The initial call to PLOTS and an appropriate DIMENSION statement for the PLOT buffer area, as well as a call to FACTOR, are included. Opposite each call is shown the Search-record address that would be produced for an offline plotting system.

PLOTS ENTRY

The PLOTS entry is used to initialize the PLOT subroutine. It must be called only once – before any other call to PLOT, SYMBOL, NUMBER, AXIS, or LINE is given. This entry sets up certain constants and the plot buffer area from which the plotter commands are written, and it opens the plot output device by performing standard file-opening procedures through the computer's operating system. If the output device is a tape unit, the first Search record, with Search address No. 001, is written out. Figure 3 includes an example of the use of PLOTS.

This entry's calling sequence also has three arguments:

CALL PLOTS (IBUF, NLOC, LDEV)

IBUF — is the name of a large area of storage assigned to accumulate the plotter commands produced by PLOT and to buffer the output. This area should be defined by a DIMENSION statement as an array.

NLOC — is the number of locations reserved for the buffer area IBUF. Consult your computer-oriented supplement for the particular manner of defining the size, which is specified in words for some computers and in characters or bytes for others. Typically, the size should be large enough to produce plot records of at least several hundred commands. This argument's value should correspond to the array size specified in the DIMENSION statement for IBUF.

For users of 600-series plotters driven by either a Model 750 or 760 tape unit, the sign of NLOC is used to set in the program the logical increment size to match the actual switch setting used on the plotter. +NLOC corresponds to the small increment size, and –NLOC corresponds to the large increment size.

LDEV — is the logical output-device number, which is assigned by the user. In some versions of PLOT this argument may not be applicable, depending on the characteristics of the operating system. Consult your computer-oriented supplement for details.

FACTOR ENTRY

The FACTOR entry to the PLOT subroutine enables the user to enlarge or reduce the size of the entire plot by changing the ratio of the desired plot size to the normal plot size. A sample FACTOR statement is shown in Figure 3.

CALL FACTOR (FACT)

FACT — is the ratio of the desired plot size to the normal plot size. For example, if FACT = 2.0, all subsequent pen movements will be twice their normal size. When FACT is reset to 1.0, all plotting returns to normal size. During the debugging of a plotting application program, computer and plotting time can be saved by reducing the size of the entire plot output. This is done by calling FACTOR with a value less than 1.0, after calling PLOTS. When debugging is completed, this call statement can be removed.

SYMBOL SUBROUTINE

The SYMBOL subroutine produces plot annotation at any angle and in practically any size. There are two SYMBOL call formats: 1) the "standard" call, which can be used to draw text such as titles, captions, and legends; and 2) the "special" call, which is used to draw special centered symbols such as a box, octagon, triangle, etc., for plotting data points.

The standard characters that are drawn by SYMBOL include the letters A-Z, digits 0-9, and certain special characters. See your computer-oriented supplement to this manual for other characters available in your particular SYMBOL subroutine.

Both forms of the SYMBOL calling sequence have six arguments. The "standard" call is:

CALL SYMBOL (XPAGE, YPAGE, HEIGHT, IBCD, ANGLE, +NCHAR)

XPAGE, YPAGE — are the coordinates, in inches, of the lower left-hand corner (before character rotation) of the first character to be produced. The pen is up while moving to this point.

Annotation may be continued from the position following that at which the last annotation ended. Continuation occurs when XPAGE and/or YPAGE equals 999.0, and may be applied to X or Y independently. (Calling WHERE to obtain the current pen position and using RXPAGE, RYPAGE in another call to SYMBOL would not give the same results as using 999.)

HEIGHT — is the height, in inches, of the character to be plotted. For best results, it should be a multiple of seven times the plotter increment size (e.g., .07, .14, .21), but other values are acceptable. The width of a character, including spacing, is normally the same as the height (e.g., a string of 10 characters 0.14 inch high is 1.4 inches wide).

IBCD — is the text, in internal computer representation (usually BCD or A-type format), to be used as annotation. The character(s) must be left-justified and contiguous in: a single variable, an array, or in a Hollerith literal (if the compiler permits). Blanks in the text do not cause any pen movement until the next non-blank character is started.

The text must be right-justified in IBCD if a single character is desired and NCHAR = 0.

ANGLE — is the angle, in degrees from the X-axis, at which the annotation is to be plotted. If ANGLE = 0, the character(s) will be plotted right side up and parallel to the X-AXIS.

+NCHAR — is the number of characters to be plotted from IBCD. If NCHAR > 0, the data must be left-justified in the first element of IBCD.

If NCHAR = 0, one alphameric character is produced, using a single character which is right-justified in the first element of IBCD.

Some examples of using the "standard" call to SYMBOL are shown in Figure 4.

The second form is the "special" call, which produces only a single symbol based on the index value of INTEQ—not on the BCD representation of a character.

CALL SYMBOL (XPAGE, YPAGE, HEIGHT, INTEQ, ANGLE, -ICODE)

XPAGE, YPAGE, and ANGLE	are the same as described for the "standard" call. If the symbol to be produced is one of the centered symbols (i.e., if INTEQ is less than 14), XPAGE, YPAGE represent the geometric center of the character produced.
HEIGHT	is the height (and width), in inches, of the centered symbol to be drawn. Preferably, it should be a multiple of four times the plotter's increment size.
INTEQ	is the integer equivalent of the desired symbol. Valid integers and their symbols are listed in the Symbol Table of the applicable computer supplement. If INTEQ is 0 through 13, a centered symbol is produced. (See Figure 5.)
-ICODE	is negative and determines whether the pen is up or down during the move to XPAGE, YPAGE.

When -ICODE is:

-1, the pen is up during the move, after which a single symbol is produced;

-2, or less, the pen is down during the move, after which a single symbol is produced.

NUMBER SUBROUTINE

NUMBER converts a floating-point number to the appropriate decimal equivalent so that the number may be plotted in the FORTRAN F-type format. The NUMBER calling sequence has six arguments.

CALL NUMBER (XPAGE, YPAGE, HEIGHT, FPN, ANGLE, ±NDEC)

XPAGE, YPAGE, HEIGHT, and ANGLE	are the same as those arguments described for Subroutine SYMBOL. The continuation feature, where XPAGE or YPAGE equals 999., may also be used.
FPN	is the floating-point number that is to be converted and plotted.
±NDEC	controls the precision of the conversion of the number FPN. If the value of NDEC > 0, it specifies the number of digits to the right of the decimal point that are to be converted and plotted, after proper rounding. For example, assume an internal value (perhaps in binary form) of -0.12345678×10^3. If NDEC were 2, the plotted number would be -123.46.

If NDEC = 0, only the number's integer portion and a decimal point are plotted, after rounding.

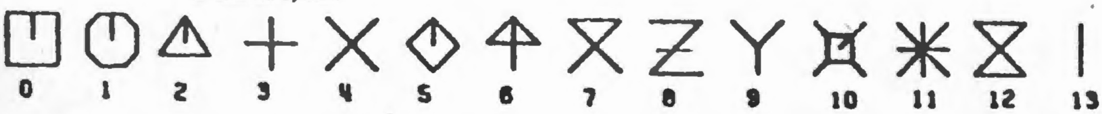
If NDEC = -1, only the number's integer portion is plotted, after rounding. (The above example would be plotted as -123 with no decimal point.)

If NDEC < -1, |NDEC| -1 digits are truncated from the integer portion, after rounding.

The magnitude of NDEC should not exceed 9.

Figure 5 illustrates various uses of SYMBOL and NUMBER.

A: Centred Symbols



B: Plotting Data Points

```
CALL SYMBOL (1.5,0.5,.14,3,0.,-1)
CALL SYMBOL (3.25,0.,.14,0,0.,-2)
CALL SYMBOL (4.5,0.0,.14,5,0.,-2)
```

Pen is up ------

Pen is down }
Pen is down } ——

C: Combining SYMBOL and NUMBER and Drawing a Superscript

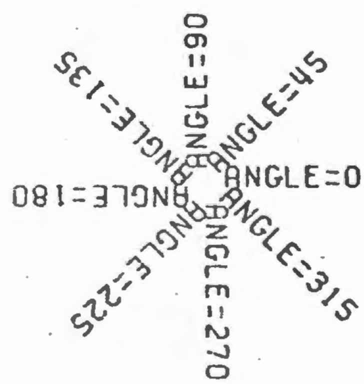
```
CALL SYMBOL (X,Y,.14,10HVALUE OF X,0.,10)
CALL SYMBOL (999.,Y+.1,.07,2H2  ,0.,2)
CALL SYMBOL (999.,Y,.14,2H=  ,0.,2)
CALL NUMBER (999.,999.,.14,VALUE,0.,3)
```

Superscript .

VALUE OF X^2 = 12.123

D: Drawing Text and Numbers at Various Angles

```
   DO 10 I=0,315,45
   ANGLE =I
   CALL SYMBOL (X,Y,.17H  ANGLE = ,ANGLE,7)
10 CALL NUMBEP (999.,999.,.1,ANGLE,ANGLE,-1)
```



Sample Uses of SYMBOL and NUMBER

AXIS SUBROUTINE

Most graphs require axis lines and scales to indicate the orientation and values of the plotted data points. The most common type of scaled axis is easily produced by the AXIS subroutine, which draws any length line at any angle, divides it into one-inch segments, annotates the divisions with appropriate scale values, and labels the axis with a centered title. When both the X and the Y axes are needed, AXIS is called separately for each one.

There are eight arguments in the calling sequence:

CALL AXIS (XPAGE, YPAGE, IBCD, ±NCHAR, AXLEN, ANGLE, FIRSTV, DELTAV)

XPAGE, YPAGE are the coordinates, in inches, of the axis line's starting point. The entire line and terminal ends should be at least one-half inch from any side to allow space for the scale annotation and axis title. Usually, both the X and the Y axes are joined at the origin of the graph, where XPAGE and YPAGE equal zero; but other starting points can be used if desired. When using the LINE subroutine to plot data on an axis, at least one of the coordinates must be 0; i.e., for an X-axis, XPAGE = 0, and for a Y-axis, YPAGE = 0.

IBCD is the title, which is centered and placed parallel to the axis line. This parameter may be an alphameric array, or it may be a Hollerith literal if the FORTRAN compiler being used permits it. The characters have a fixed height of 0.14 inch (about seven characters per inch).

±NCHAR specifies the number of characters in the axis title, and determines by its sign which side of the line the scale (tick) marks and labeling information shall be placed. Since the axis line may be drawn at any angle, the line itself is used as a reference.

If the sign is positive, all annotation appears on the positive (counterclockwise) side of the axis, which condition is normally desired for the Y-axis.

If the sign is negative, all annotation appears on the negative (clockwise) side of the axis, which condition is normally desired for the X-axis.

AXLEN is the length of the axis line, in inches.

ANGLE is the angle, in positive or negative degrees, at which the axis is to be drawn. Normally, this value is zero for the X-axis and 90.0 for the Y-axis.

FIRSTV is the starting value (either minimum or maximum) which will appear at the first tick mark on the axis. This value may be computed by the SCALE subroutine and stored at subscripted location ARRAY (NPTS*INC+1), or may be determined by the user and stored anywhere.

This number and each scale value along the axis is always drawn with two decimal places. Since the digit size is 0.105 inch (about 10 characters per inch), and since a scale value appears every inch, no more than six digits and a sign should appear to the left of the decimal point.

DELTAV represents the number of data units per inch of axis. This value (increment or decrement), which is added to FIRSTV for each succeeding one-inch division along the axis, may be computed by SCALE and stored beyond FIRSTV at ARRAY (NPTS*INC+INC+1), or may be determined by the user and stored anywhere.

In order to use a standard format of two decimal places, the size of DELTAV is adjusted to less than 100, but not less than 0.01. As a result, the decimal point may be shifted left or right in the scale values as drawn, and the axis title is then followed by "*10n," where n is the power-of-ten adjustment factor. (See X-axis example in Figure 6.)

Figure 6 illustrates typical X and Y axis elements controlled by the arguments of AXIS.

APPENDIX I. COURSE RECORD FOR COURSES DONE BY WRITER IN PARTIAL FULFILMENT OF M.Sc. DEGREE.

Course		Date Completed.	Credi Value.
CE 508	Skeletal Structures	1975	5
CE 504	Probabilities and Statistics for Engineers	1975	4
CE 525	Coastal Engineering	1976	5
CE 506	Properties of Concrete	1976	4
CE 531	Urban Transportation Planning and Modeling	1977	4
		Total	22

Total credit requirements for the M.Sc.(Eng) Degree: 40

Course Credits:	22
Half Thesis:	20
Total:	42

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY EXAMINATION: JUNE, 1975

COURSE CE 508 - SKELETAL STRUCTURES

Time allowed: 4 hours | Notes are allowed

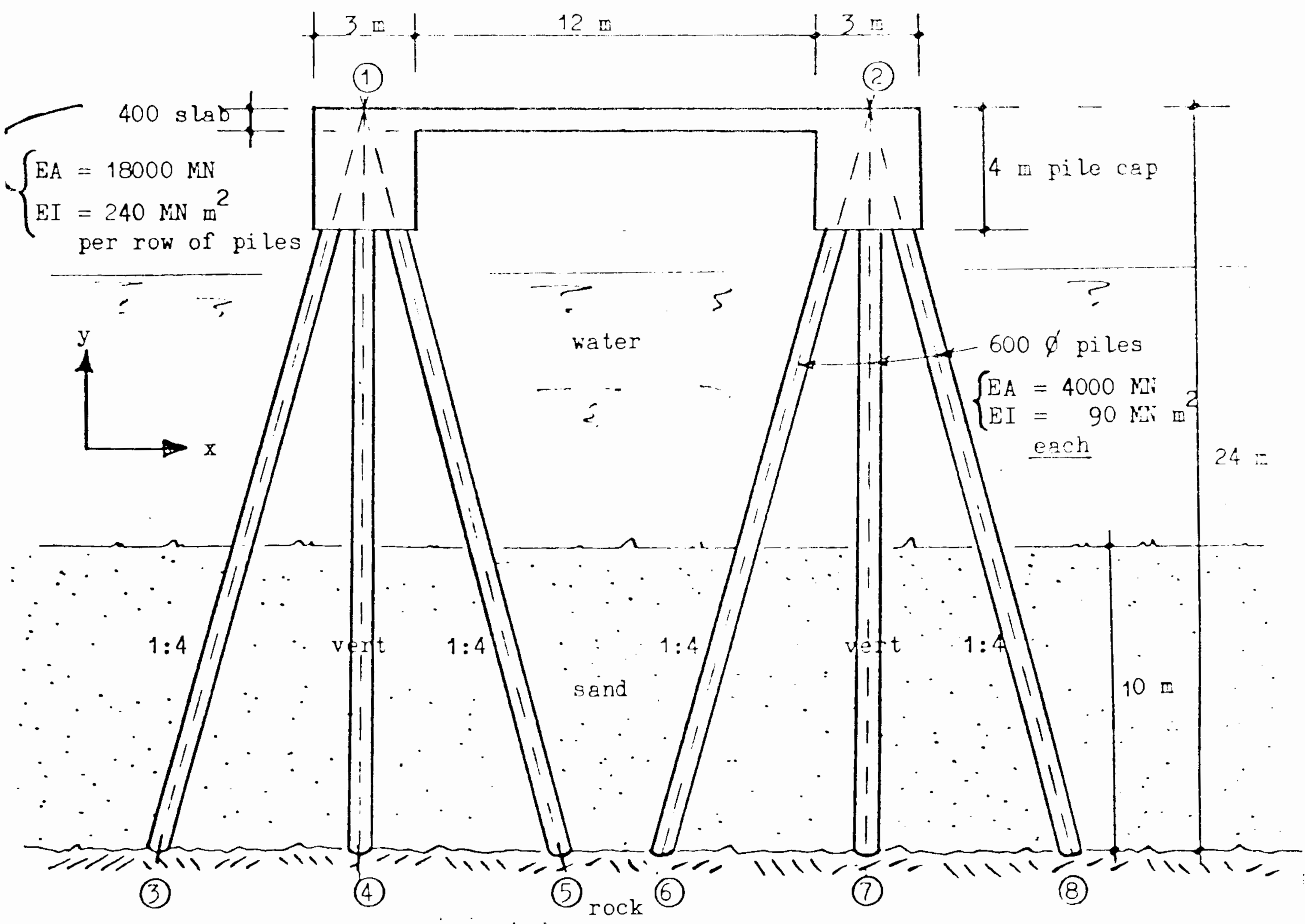
Part A: For each of the five structures shown below determine the degree of static and of effective kinematic indeterminateness, select the most suitable method of analysis, give the order of all the relevant matrices required for solution by the chosen method. State clearly what assumptions are made.

[40 marks]

Part B: Compile the matrices for any two of these structures; one analysed by the FORCE method and one analysed by the DISPLACEMENT method. Do not attempt to complete all the arithmetic processes, but give sufficient detail to show clearly the principles and operations involved.

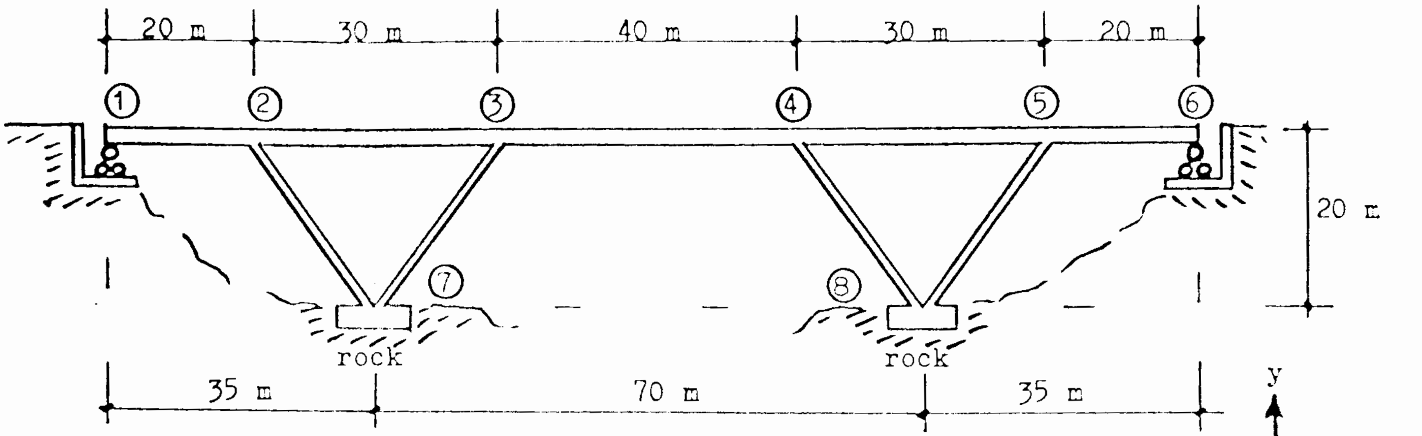
[60 marks]

1. Jetty with vertical and horizontal loads applied to the top surface in the xy plane. The pile rows are at 3 m spacing along the jetty.



CROSS-SECTION

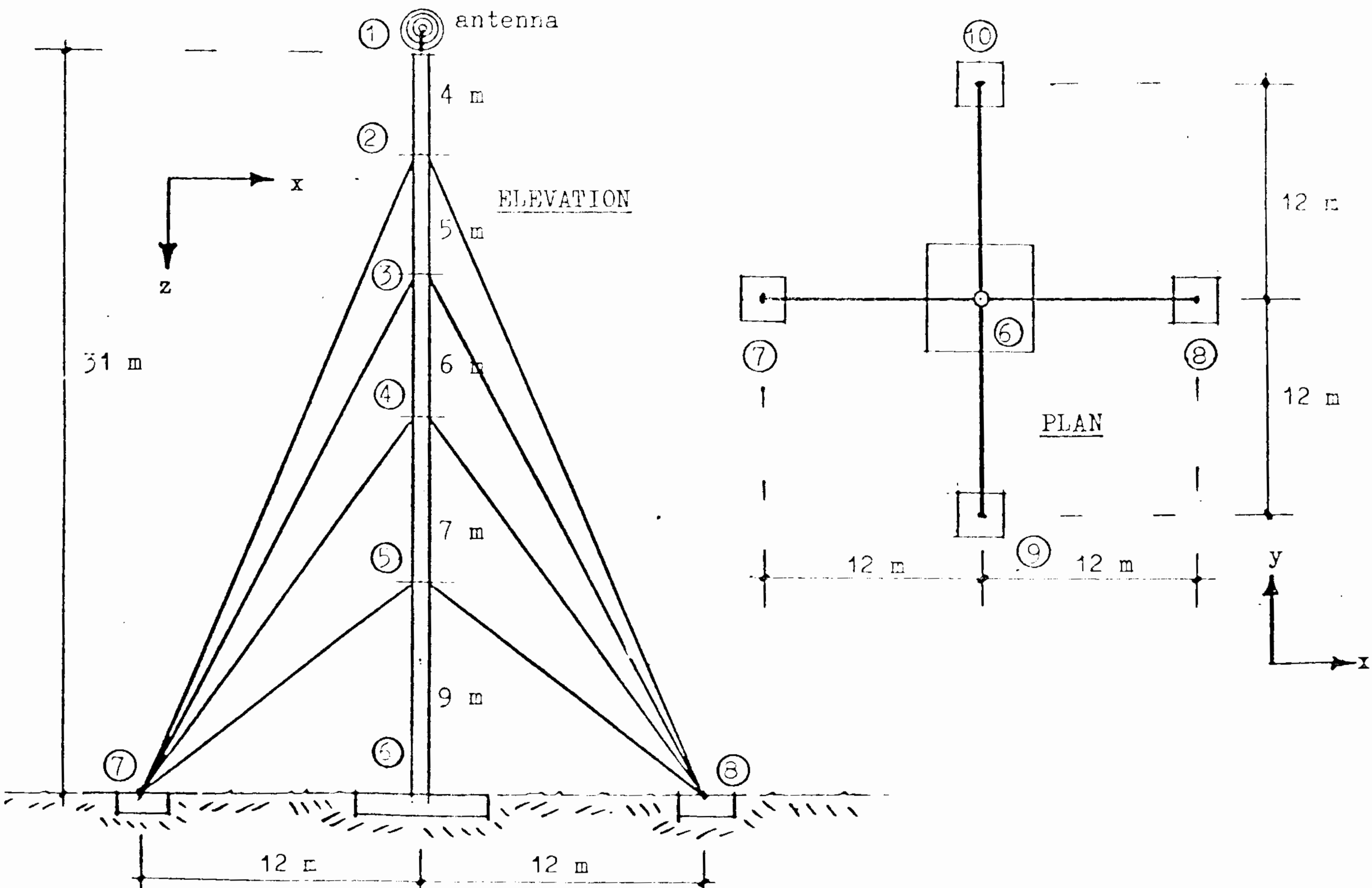
2. Bridge, monolithic concrete beam-slab deck and inclined columns, with vertical and horizontal loading applied to the deck.



deck : EA = 140 000 MN; EI = 40 000 MN m^2

columns: EA = 36 000 MN; EI = 3 000 MN m^2

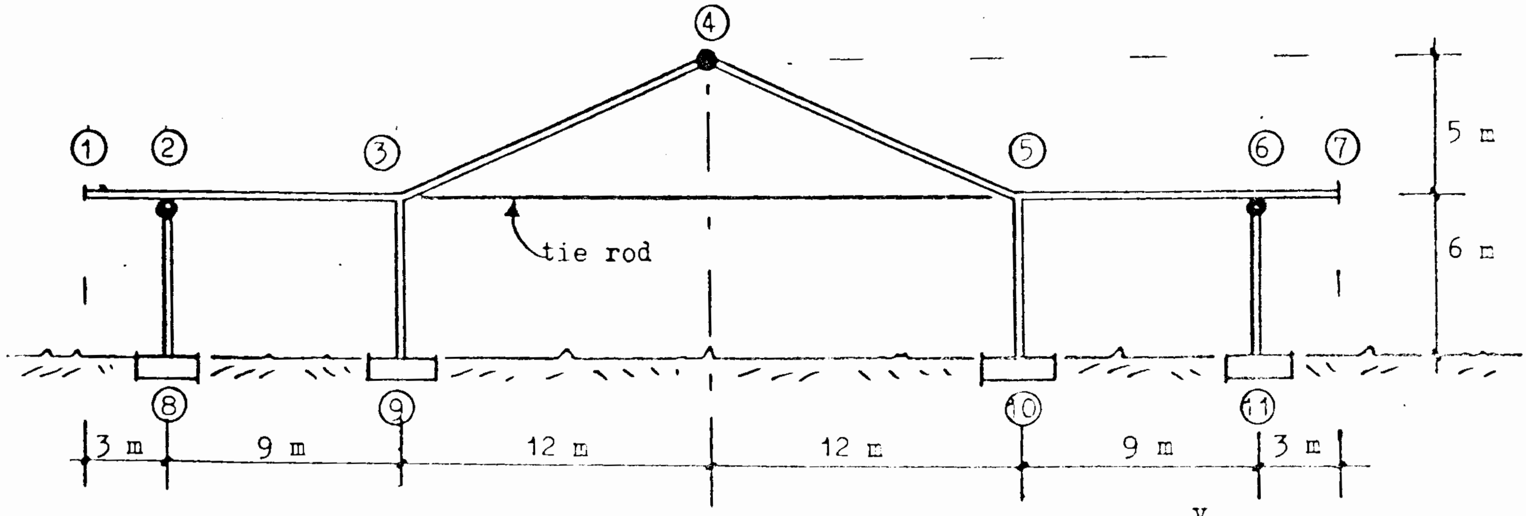
3. Tower, consisting of a single vertical tubular column fixed at the base and stayed at right angles on four levels with steel wire guy ropes, which are sufficiently pretensioned not to go slack. Loading is applied at the top only.



column : EA = 1300 MN; EI = 7 MN m^2 (200 ∅)

wire rope: EA = 36 MN each: (15 mm ∅.)

4. Building, tied steel portal frame with two side bays. Wind, dead and imposed roof loading.



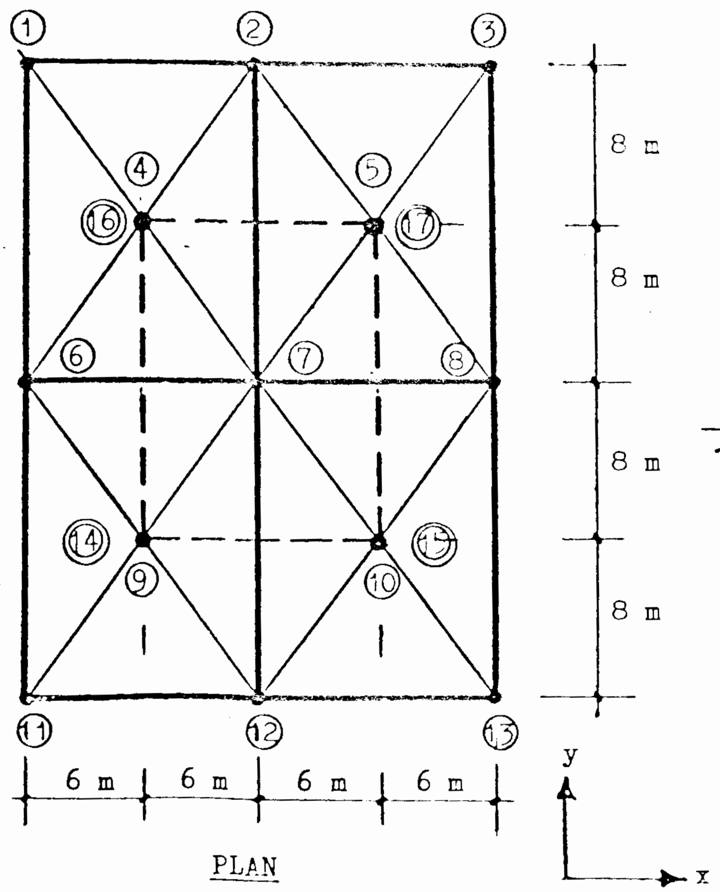
Rafters: EA = 1000 MN; EI = 20 MN m^2

Columns: EA = 1200 MN; EI = 9 MN m^2

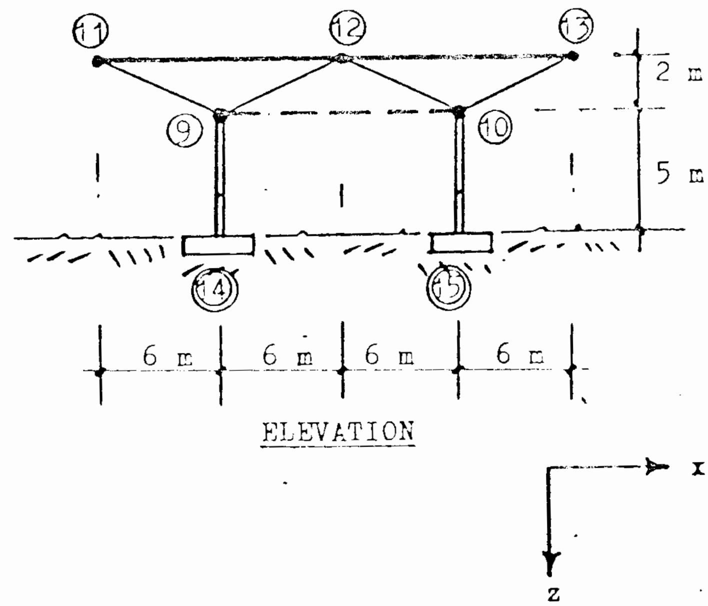
Tie rod: EA = 60 MN (20 ∅).

---

5. Roof, ball-jointed, double-layer, three-way grid on four columns fixed at their bases, all of tubular steel construction, with vertical and horizontal loading applied to the top joints.



PLAN



ELEVATION

▬▬ top layer	:	EA = 600 MN, (200 mm ∅)
—— inclined	:	EA = 900 MN, (300 mm ∅)
— — bottom layer	:	EA = 1200 MN, (400 mm ∅)
columns	:	EA = 3000 MN, (500 mm ∅); EI = 100 MN m^2

- - - - - - - - - - - - - -

UNIVERSITY OF CAPE TOWN

UNIVERSITY EXAMINATION OCTOBER 1975

PROBABILITY AND STATISTICS FOR ENGINEERS

EXTERNAL EXAMINER : PROFESSOR D.M. SCHULTZ
INTERNAL EXAMINER : MR. A.M. HURWITZ

TIME : 3 HOURS

ANSWER ANY SIX OF THE FOLLOWING NINE QUESTIONS.

---

1. (a) Briefly explain the meaning of each of the following terms (and give an example in each case):

"sample space"; "element of a sample space"; "event"; "null event".

(b) Prove: $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ where P denotes "Probability", and E_1, E_2 are arbitrary events defined on a sample space S.

(c) 8 short stories are to be arranged in a book. If the arrangement is to be done in a random fashion, what is the probability that (i) neither the longest nor the shortest story will be placed first, (ii) the longest is placed last but the shortest is not placed first?

2. (a) Write down the probability functions of the hypergeometric, binomial and Poisson distributions, and in each case briefly describe the sitration(s) in which they are used (i.e. the conditions under which they are valid and applicable).

(b) Suppose that 5% of the asprin tablets pressed by a certain type of machine are chipped. The tablets are boxed 12 per box. What percent of the boxes would you estimate:

 (i) to be free of chipped tablets
 (ii) to have exactly x chipped tablets.

(c) A bag of grass seeds is known to contain 1% weed seeds. A sample of 100 seeds is drawn randomly from the bag. Find the probabilities of 0,1,2,3 weed seeds being in the sample.

3. (a) What is an "operating characteristic curve" - how is it derived? What is the "Average Outgoing Quality"? Briefly explain its significance.

3. continued ....

(b) In a quality control scheme, suppose that it is desired to reject with certainty all lots with more than 7% defectives, and to accept all lots with exactly 7% or less defectives.

What would the sample scheme and operating characteristic curve be in this case?

(c) Under the sampling scheme $n = 2$ and $c = 1$ (c = allowable number of defectives), and given the probabilities:

$P(x = 0 | \theta = 0,1) = 0,81$
$P(x = 0 | \theta = 0,2) = 0,64$
$P(x = 0 | \theta = 0,3) = 0,49$
$P(x = 0 | \theta = 0,4) =$ 0,3[illegible]
$P(x = 0 | \theta = 0,5) =$ 0,2[illegible]
$P(x = 1 | \theta = 0,1) = 0,18$
$P(x = 1 | \theta = 0,2) = 0,32$
$P(x = 1 | \theta = 0,3) = 0,42$
$P(x = 1 | \theta = 0,4) = 0,48$
$P(x = 1 | \theta = 0,5) = 0,50$

sketch the operating characteristic curve with respect to the scheme.

4. (a) Find the mean and variance of the binomial distribution using the moment generating function.

(b) Suppose that a point, e, is picked at random inside the unit circle in the (x,y)-plane.

What is the sample space in this situation?

Let r be the distance from the point e to the centre of the circle. What is the cumulative distribution function of r? What is the probability density function and the sample space of r?

(c) Let x_1 be a number taken at random on the interval $(0,1)$ and x_2 a number taken at random on the interval $(x_1,1)$. Show that the distribution of x_2 has probability density function

$$f(x_2) = -\log(1-x_2) \qquad 0 < x_2 < 1$$
$$= 0 \qquad \text{elsewhere}$$

(Hint: Use conditional distributions.)

5. (a) A random variable is said to have the Normal Distribution $N(\mu,\sigma^2)$ if its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

Show that μ is the mean of the distribution.

(b) What is the "Standard Normal Distribution", and what is its use? How do we approximate a binomial distribution to a normal?

(c) It is known that the probability of dealing a bridge hand with at least one ace is approximately 0,7. If a person plays 100 hands of bridge, what is the approximate probability that the number of hands he will receive containing at least one ace will be between 60 and 80 inclusive?

6. (a) What is a "statistic"? a "parameter"? If we are considering a population whose distribution has a parameter θ, what do we mean by an "unbiased estimator" for θ?

(b) Show how to construct a $100(1-\alpha)\%$ confidence interval for the difference of means from two normal populations with equal but unknown variances.

(c) Suppose that random samples of 25 are taken from two large lots of bulbs, and the standard deviations of the bulb lives were found to be: $S_A = \sqrt{10}$

$$S_B = \sqrt{12}$$

Find 95% confidence limits for $\frac{\sigma_A^2}{\sigma_B^2}$, the ratio of the population variances.

(Note: $F(n_1,n_2;\, 1-\frac{\alpha}{2}) \doteq \frac{1}{F(n_1,n_2;\, \frac{\alpha}{2})}$)

7. (a) What do we mean by the "Power" of a statistical test?

(b) If, in a left-sided statistical test of $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1 < \mu_0$, we let

α = Probability of a type I error
β = probability of a type II error.

Show, with the aid of a sketch, what the power function value is at μ_0.

4/............

7. continued ....

(c) The standard deviation of muzzle velocities of a random sample of nine rounds of ammunition was found to be $s = 93,2$ ft. per second. If the "standard" value of σ for the muzzle velocity of this type of ammunition is 70 ft. per second is the value of s significantly large at the 5% level of significance?

8. (a) In "Goodness of Fit" tests the Chi-squared statistic

$$\chi^2 = \sum_{i=1}^{K} \frac{(f_i - n\theta_i)^2}{n\theta_i}$$

is used. 'xplain the theory underlying this statistical test.

(b) How is the χ^2 statistic of part (a) used to test independence of factors in a 2×2 "contingency table" when the probabilities of the two factors are unknown?

(c) Pieces of Vulcanite were examined according to porosity and dimensionai defects, and the results are shown in the following table:

	Porous	Non-Porous
With Defective Dimensions	142	331
Without Defective Dimensions	1233	5099

Test the hypothesis that the two criteria of classification are independent at the 5% level.

9. (a) Breifly explain, with the aid of a sketch, the concept of fitting a straight line to a set of data points, (x_i, y_i) $i = 1...n$, by the methoc of least quares. If we assume that the straight line has an equation o the form $E(y|x) = \alpha + \beta x$, write down the least squares formul e for estimating α and β.

(b) Write down the formula giving the sample correlation coefficient r.

If the population correlation coefficient is ρ, how would you test the hypothesis $H_0 : \rho = 0$ against the alternative hypothesis $H_1 : \rho \neq 0$?

(c) The data shown below was obtained in an experiment to study the relationship between the amount of beta-enthyroidine x (in milligrams) in an aqueous solution and colorimeter reading of turbidity y of the solution.

5/..........

9. (c) continued ....

y	x
89	40
175	50
272	60
335	70
390	80
415	90

(i) Determine the regression line of y on x.
(ii) Estimate the variance, σ^2, of y.

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

M.Sc IN CIVIL ENGINEERING

UNIVERSITY EXAMINATION : FEBRUARY 1976

CE 525 : Coastal Engineering

All Questions may be attempted | Time : 3 hrs

Constants

Sea water density = 1025 kg/m^3

Sea water weight = 10 kN/m^3

1. A swell of 10 second period with a deep water wave height of 3 m approaches a beach with the wave crests parallel to the shore. Trace the progress of this wave in shoaling water through to the breaker point including the following calculations :-
   - (a) the wave length and wave celerity in deep water
   - (b) the water depth at which the wave begins to be affected by the presence of the sea bed.
   - (c) the wave length and wave celerity for water depths at 10 m intervals between d=80 m and d=10 m, and at 1 m intervals between d=10 m and d=1 m.
   - (d) the depth of water in which the wave breaks, the type of breaker and the wave height at breaking. Ignore the effect of wave set up or down. *Bed slope 1 in 50*
   - (e) sketch the effect of wave set up and down including an estimate of depths.
   - (f) estimate the wave heights in the surf zone.
   - (g) calculate the energy flow in W/m in water depths of 10 m, 5 m, and 2 m.

2. A cylindrical pipe is laid on the sea bed across a harbour entrance in 10 m of water, the pipe diameter being 1 m and the axis of the pipe is parallel to the local wave crests. If the local wave length is 50 m, estimate the wave period, and find the peak magnitudes of the velocity and acceleration force components per metre length of pipe. Estimate the peak resultant force in the inshore direction, and the timing of this in relation to the passage of a wave crest

   $H=2m$ $C_D = 1{,}2$ $C_M = 2{,}5$

3.(a). A storm at sea generates waves with a period range of 6 to 12 seconds. The resulting swell travels towards a harbour 400 km away. Estimate the time required for the longest waves to cover the intervening distance, assuming deep water throughout. Also estimate how much later the shortest waves will begin to arrive.

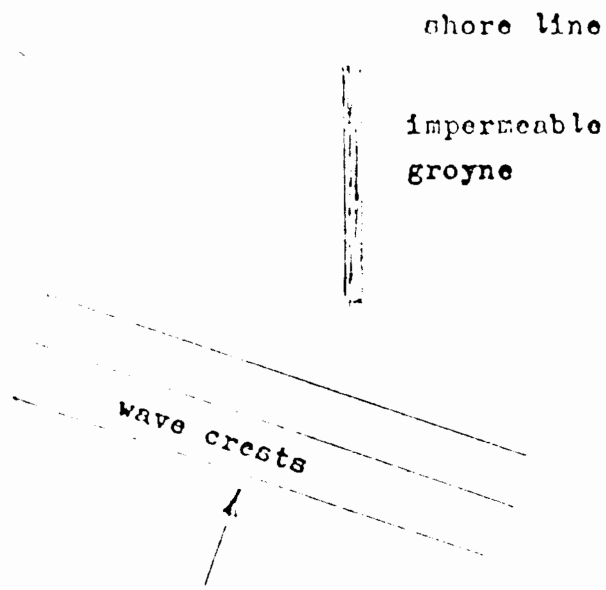
(b). A refraction diagram is constructed for a bay and the spacing between a particular pair of adjacent orthogonals doubles in travelling from deep water to the 10 m depth, the wave period being 7 seconds. Estimate the percentage change in wave height ocurring between these zones on the assumption that no breaking waves are present between the zones.

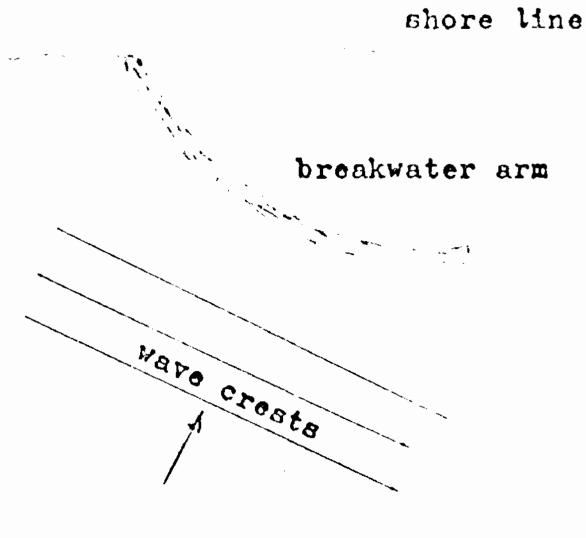
(c). Suggest some of the requirements you would incorporate into a specification for armour blocks.

4. The overleaf page shows the plan views of three separate coastal structures on which oblique waves impinge. In each case indicate areas where you consider deposition or erosion will occur, and also estimate the shape of the breaker line once stable conditions are established

5. There is a continuous dissipation of energy due to tidal movements of water over the earths surface, and in some instances useful power is abstracted from the sea in tidal power schemes. Suggest what effect this may have on the dynamics of the earth-moon system over very long periods of time.

Feb 1976


shore line
impermeable
groyne
wave crests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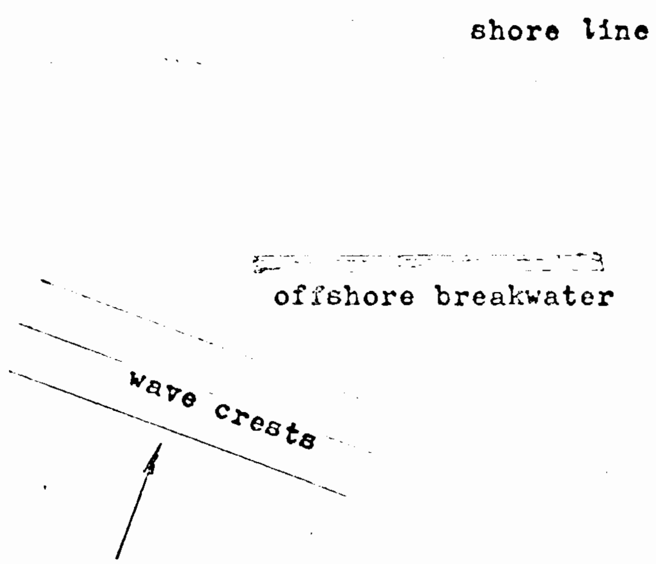
breakwater arm
wave crests


shore line
offshore breakwater
wave crests

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY EXAMINATION: JUNE 1976

COURSE CE 506 - PROPERTIES OF CONCRETE

Time allowed: 3 hours

5th June, 1976

Part A consists of fifteen multiple-choice questions. Each question is followed by five suggested answers; select the one which is best in each case and circle one of (a), (b), (c), (d) or (e) for each question. This portion of the examination paper must NOT be removed from the Examination Room and must be handed in for marking.
Part B consists of five questions. Answer all questions.

PART A - Multiple-Choice Section (All questions of equal value)

Question A1: In controlling the quality of concrete produced for a project, a test is needed which:

(a) gives the true strength of the material;

(b) gives, for variations in testing procedures, the least variation in results;

(c) gives the true strength of the specimen;

(d) gives a clearly defined stress pattern;

(e) is easy to carry out.

Question A2: In design of concrete mixes according to CP 110 Concrete Structures Code, the target strength chosen is directly related to:

(a) the design strength f_{cu};

(b) the design strength f_{cu} plus 1,65 times the standard deviation 'σ'.

(c) the design strength f_{cu} plus the standard deviation 'σ';

(d) the design strength f_{cu} plus the coefficient of variation 'v';

(e) the design strength f_{cu} plus 1,65 times the coefficient of variation 'v'.

Question A3: The most important aspect of sampling from a pre-mixed concrete truck is to:

(a) protect the sample from wind and sun;

(b) obtain a representative sample in order to carry out further tests;

(c) ensure that the concrete is properly mixed;

(d) check the workability and slump;

(e) obtain a sufficient quantity of concrete to carry out further tests.

/Question A4: ....

Question A4: For a water/cement ratio of 0,6 by weight the use of rounded river gravel in place of crushed aggregate of cubic shape and rough texture will:

(a) show little difference in compressive strength but increase flexural strength;

(b) increase compressive strength by about 10% and also increase flexural strength;

(c) decrease compressive strength by about 10% but increase flexural strength;

(d) increase compressive strength slightly but lower flexural strength;

(e) decrease slightly, both compressive and flexural strengths.

Question A5: The Unit Water Method of Mix Design, described in lectures, suggests that the grading of the combined aggregate be made finer than the recommended grading when:

(a) the maximum aggregate size is larger;

(b) the maximum aggregate size is smaller;

(c) the coarse aggregate is crushed material;

(d) the cement content is higher;

(e) the cement content is lower.

Question A6: An increase in the proportion of aggregate material in the sieve range 2,00 mm to 9,5 mm (No. 8 to 3/8") will tend to:

(a) make the concrete harsh and liable to honeycomb;

(b) make the finishability of the concrete better;

(c) improve the economy of the mix;

(d) increase the amount of water required;

(e) reduce the amount of water required.

Question A7: The addition of an air entraining agent to a concrete mix usually leads to:

(a) a more economical mix;

(b) a stronger concrete;

(c) a decrease in the required sand percentage;

(d) a decrease in cement content;

(e) a denser concrete because of improved workability.

/Question A8: .....

Question A8: In the Unit Water Method of Mix Design, described in lectures, the estimated water content for a particular slump is fixed by:

(a) the maximum size of the aggregate;

(b) the grading of the aggregate;

(c) the shape of the aggregate;

(d) (a) and (b) above;

(e) (a) and (c) above.

Question A9: Capillary water in hydrated cement paste is:

(a) water held in areas of restricted adsorption of the gel structure;

(b) water occupying space beyond the range of surface forces of the solid phase of the gel structure.

(c) water existing in cavities and channels up to 100 times greater than the size of gel pores;

(d) both (b) and (c) above;

(e) water chemically combined such that it is part of the solid matter in the hardened paste.

Question A10: Plastic shrinkage of concrete is caused by:

(a) removal of capillary and gel pore water;

(b) the absorbtion of mixing water by porous or dry aggregates;

(c) sedimentation and settling of solids in the concrete mix;

(d) bleeding of free water to the top surface of the concrete where it is often lost by evaporation or drainage;

(e) all of (b), (c) and (d) above.

Question A11: The secant elastic modulus of concrete is increased by:

(a) increased water:cement ratio and increased paste content;

(b) constant water:cement ratio and increased paste content;

(c) increased water:cement ratio and decreased water content;

(d) constant water:cement ratio and air entrainment;

(e) decreased water:cement ratio and decreased paste content;

Question A13: Decreasing the water/cement ratio influences the ultrasonic pulse velocity because:

(a) poor compaction leads to voids;

(b) a decrease in the density causes the pulse velocity to increase;

(c) an increase in strength (due to a lowering of the water cement ratio) causes the pulse velocity to increase;

(d) an increase in the density causes the pulse velocity to increase;

(e) an excess of paste causes the pulse velocity to decrease.

/Question A14: ....

Question A14: Rapid Hardening Portland cement can be manufactured by:

(a) more finely grinding the Portland cement;

(b) changing the ratio of $C_2S:C_3S$;

(c) intergrinding some high alumina cement with the Portland cement;

(d) both (a) and (b) above;

(e) all of (a), (b) and (c) above.

Question A15: Excessive bleeding of concrete can be corrected by:

(a) adding more cement;

(b) adding crusher dust or other fine material;

(c) by air entrainment;

(d) both of (a) and (b) above;

(e) all of (a), (b) and (c) above

[Total 20 marks]

PART B

Question B1:

(a) A laboratory trial mix of concrete with 30 kg of water, 50 kg of cement, 130 kg of sand and 180 kg of stone gave a 28-day strength which was too low, a slump of 110 mm and real mortar excess of 8%. It is decided that a reduction in water/cement ratio to 0,56 will probably correct the strength requirement. What mix would you suggest for a second trial to give a slump of 60 mm and a real mortar excess of 2% given that the densities of the water, cement, sand and stone are 1000, 3150, 2600 and 2750 kg/m^3 respectively.

(b) The compressive strength of the second trial mix after 28 days' storage at 18°C is 33 MPa. Using Plowman's method, determine how long it would take to reach the same strength at 25°C. What will be the compressive strength after 3 days at 25°C?

[20 marks]

Question B2:

Consider an average structural grade concrete made with 20 mm river gravel aggregate (irregular gravel), normal Portland cement, water/cement ratio (by weight) 0,60 aggregate/cement ratio 6,0, and slump of 75 mm.

(i) Calculate the effect on strength of adding water so as to increase the slump to 150 mm.

(ii) How does this strength change compare with that expected to result from changing from gravel to crushed coarse aggregate but maintaining the aggregate/cement ratio at 6,0 and slump at 75 mm?

(iii) If a graded river gravel with maximum size 80 mm was used in place of the 20 mm gravel, comment on the expected water demand, water/cement ratio and resulting compressive strength of the concrete.

Clearly state the assumptions made in each case.

[15 marks]

Question B3:

(i) Explain briefly how the progressive hydration of cement may lead to self-desiccation of concrete.

(ii) Calculate the gel/space ratio for a concrete with a water/cement ratio of 0,60 at an age of 14 days at which time 60 per cent of the cement had hydrated. Comment on the expected compressive strength corresponding to this gel/space ratio.

(iii) 100 g of cement and 20 g of water are placed in one sealed container and 100 g of cement and 60 g of water are placed in another sealed container. Calculate in both instances the maximum degree of hydration possible, the volume of gel formed, the weight of chemically combined water and the weight of free water in the capillary pores.

[20 marks]

/Question B4: ....

Question B4: "When concrete specimens are loaded axially in compression they always fail in tension". Briefly discuss this statement and go on to discuss the effect of specimen size and shape, and also the effectiveness of capping materials on the apparent ultimate compressive strength of concrete test specimens.

[10 marks]

Question B5: A considerable number of different types of test procedures have been devised to measure "workability" of concrete. Discuss the reasons for the multiplicity of methods used. List ways in which the workability of concrete can be increased without increasing the water content.

[~~20~~ 15 marks]

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

1977

COURSE CE 531 - URBAN TRANSPORTATION PLANNING AND MODELING

The course was evaluated by a set of assignments and a major project.

The project was a comprehensive report which was to constructively criticize a transportation study of Cape Town.