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A MULTIVARIATE ANALYSIS OF SHARES LISTED ON THE JOHANNESBURG STOCK EXCHANGE

by

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P R E F A C E

This thesis examines the usefulness of multivariate statistical techniques to portfolio theory by applying two different multivariate techniques to two separate classificatory problems concerning shares listed on the Johannesburg Stock Exchange.

In Chapter 1 the two techniques and two classificatory problems are introduced and their context within the general structure of portfolio theory is explained.

Chapter 2 gives a theoretical overview of the first technique used, namely Factor Analysis. Chapters 3 and 4 discuss the application of factor analytic techniques to shares listed on the Johannesburg Stock Exchange.

Chapter 5 gives a theoretical overview of Multiple Discriminant Analysis, the second multivariate technique used. Chapter 6 represents a survey of previous applications of Multiple Discriminant Analysis in the field of Finance, while Chapters 7 and 8 discuss the application of this technique to shares listed on the Johannesburg Stock Exchange.

Finally, Chapter 9 gives a brief summary of the main conclusions in this thesis.

Francesca Visser

C O N T E N T S

CHAPTER 1

INTRODUCTION

Harry Markowitz (1959), generally considered as the pioneer in the field of modern portfolio theory, once stated that

> *"a portfolio analysis starts with information concerning individual securities. It ends with conclusions concerning portfolios as a whole. The purpose of the analysis is to find portfolios which best meet the objectives of the investor."*

He thus divided portfolio theory into three phases, viz.,

(1) Security Analysis which concerns predictions about the future prospects of individual securities,

(2) Portfolio Analysis which involves the formation of portfolios and predictions concerning these portfolios,

(3) Portfolio Selection which involves choosing the portfolio best suited to the investor's requirements.

Although the specific requirements would depend on the individual investors, Markowitz identified two objectives common to all investors, namely:

(1) They want return to be high.

(2) They want this return to be dependable, stable, not subject to uncertainty.

From these two common objectives it follows that, apart from being concerned about the expected returns on securities, investors also have to take into consideration the risk associated with these returns. Another important concept is the interrelationships between securities.

It is generally assumed that all investors are risk averse. Hence for a given level of risk they aim to maximize return and for a given level of return they want to minimize risk. Risk is usually measured in terms of the variability in returns and since the variance of a combination of securities is a function of both the variances of the individual securities and the inter-relationships between securities, as described by their covariances or correlation coefficients, a combination of negatively correlated securities into a portfolio can serve to substantially reduce risk. In this way diversification forms the basis of portfolio theory. The utility of portfolio theory, however, depends on the diversification being efficient and this point is the main concern of the first part of this thesis.

King (1966) described the stock market as being "*subject to a steady inflow of information, much of which will have an effect on the set of anticipations that determine the price of security j.*" He further suggested that "*a single piece of information can affect more than one security price change, perhaps even the whole market, at a given time period,*" and hence concluded that "*if two variables share one or more common elements in their statistical makeup, they will exhibit correlated behaviour to some degree.*" It is usually assumed that the resulting groups of comoving shares correspond to the industry classifications to which shares belong. Since this belief often forms the basis for the diversification policies of investors, proof of its statistical soundness is of crucial importance. The first part of this thesis employs the multivariate statistical technique of factor analysis to determine the underlying structure of share returns on the Johannesburg Stock Exchange and its correspondence to existing industry classifications.

Returning to the first phase of portfolio theory, there are generally two approaches to the prediction of future share prices, viz., Technical Analysis and Fundamental Analysis. The technicians believe that future stock prices depend on past prices and hence use trends in the historical movement of prices to predict future trends. The fundamentalists emphasize the relationship between the stock price and the financial characteristics of firms as described in various financial statements such as balance sheets, income statements, and so forth. The utility of both these types of analysis has, however, been greatly reduced, if not ruled out, by the weak and semi-strong forms of the Efficient Market Hypothesis which respectively state that

(i) successive price changes are independent and hence no superior returns can be obtained through the analysis of past trends, and

(ii) no superior returns can be obtained through the analyses of publically available information, since this information is already reflected in the market price.

King gave a good summary of how the inflow of information bring about changes in security prices and subsequently differentiated between information affecting all shares, information relevant to individual shares only. Ball and Brown (1968) examined the relationship between information contained in accounting numbers and share performance and with respect to the annual income number concluded that "*most of the information contained in reported income is anticipated by the market before the annual report is released.*"

The second part of this thesis examines the relationship between information contained in the annual financial statements of firms and their relative performance on the stock market. More specifically, an attempt is made, through the employment of multiple discriminant analysis, to combine several measures of the financial characteristics of firms into meaningful models for the classification of these firms according to their relative performance on the stockmarket.

CHAPTER 2

THEORY OF FACTOR ANALYSIS

2.1 Introduction

Given a standardized variable $z_j = x_j - \bar{x}$, factor analysis aims to represent each variable x_j as a linear combination of a number of hypothetical factors, where these factors are chosen so as to either

(i) extract the maximum variance, or

(ii) best reproduce the observed correlations.

The well-known method aimed at extracting the maximum variance is principal component analysis, which describes a set of n variables z_j $(j = 1,\ldots,n)$ in terms of n uncorrelated components F_j. These components are determined in such a way that they successively maximally contribute to the total variance of the n variables. The underlying model is

$$z_j = a_{j1}F_1 + a_{j2}F_2 + \ldots + a_{jn}F_n \quad (j = 1,2,\ldots,n) \tag{2.1.1}$$

The classical factor analysis model was developed to primarily satisfy the second criterion stated above. However, most of the modern factor analytic techniques also aim at extracting maximum variance. The classical factor analysis model is

$$z_j = a_{j1}F_1 + a_{j2}F_2 + \dots + a_{jm}F_m + u_jV_j$$
$$(j = 1,2,\dots,n) \qquad (2.1.2)$$

where it is assumed that

(i) $m \ll n$, thus making factor analysis a very useful dimension reducing technique,

(ii) $E(F_p) = 0$, $Var(F_p) = 1 \quad \forall_p = 1,\dots,m$,

(iii) $E(Y_j) = 0$, $Var(Y_j) = 1 \quad \forall_j = 1,\dots,n$,

(iv) $E(Y_jY_k) = 0 \quad \forall_j \neq k$,

(v) $E(Y_jF_p) = 0 \quad \forall(j,p)$.

In matrix notation:

$$Z = Af + Uy,$$

where

- Z is the vector of observed variables,
- A is the matrix of factor loadings,
- f is the vector of hypothetical factors,
- U is the matrix of uniqueness, and
- y is the vector of unique factors.

Thus each z_j is described as a linear function of m common factors and a unique factor.

The basic problem of factor analysis is the estimation of the nm loadings of the common factors in equation (2.1.2). This is achieved through the analysis of the variance and covariance/correlation structure of the variables.

Returning to the first assumption stated above, it can be shown that a set of n variables can be expressed as linear functions of not less than m factors, where m is the rank of the correlation matrix. When the observed correlation matrix has unities in the diagonal, its rank is usually n, and thus the variables cannot be described by fewer than n factors. Replacing the unities with communalities usually reduces the rank of the correlation matrix to $m<<n$. This new correlation matrix is called the "reduced" correlation matrix. Furthermore, m is the smallest number of factors necessary to provide a base for the original variables and is thus the dimension of the "common-factor space".

The subsequent sections in this chapter will discuss several initial factorization procedures and subsequent rotation procedures. The aim is to provide a general overview of the subject, giving some insight into the historical development, and even more so into the practical procedures of factor analysis. The discussions are mainly based on Harman's (1960) excellent book on factor analysis, supplemented by information drawn from the more prominent papers by Guttman (1953), Harris (1962), and Kaiser (1970) in particular.

2.2 Composition of Variance

The sample variance for a variable X_j is

$$s_j^2 = \sum_{i=1}^{N} (X_{ji} - \bar{X}_j)^2 / N, \qquad (2.2.1)$$

and for a standardized variable z_j with zero mean,

$$s_j^2 = \sum_{i=1}^{N} z_{ji}^2 / N \ . \qquad (2.2.2)$$

Substituting equation (2.1.2) gives

$$s_j^2 = \sum_{p=1}^{m} a_{jp}^2\left(\sum_{i=1}^{N} F_{pi}^2/N\right) + u_j^2\sum_{i=1}^{N} Y_{ji}^2/N \tag{2.2.3}$$

$$+ 2\sum_{p<q=1}^{m} a_{jp}a_{jq}\left(\sum_{i=1}^{N} F_{pi}F_{qi}/N\right)$$

$$+ 2u_j^2 \sum_{p=1}^{m} a_{jp}\left(\sum_{i=1}^{N} F_{pi}Y_{ji}/N\right).$$

Then using the assumptions stated in section 2.1, equation (2.2,3) simplifies to

$$s_j^2 = 1 = a_{j1}^2 + a_{j2}^2 + \ldots + a_{jm}^2 + u_j^2$$

$$= \sum_{p=1}^{m} a_{jp}^2 + u_j^2$$

$$= h_j^2 + u_j^2 \tag{2.2.4}$$

Thus the variance of a variable can be separated into two parts, the communality (h_j^2) and the uniqueness (u_j^2).

Furthermore from equation (2.2.4) it follows that

a_{jp}^2 = contribution of factor F_p to the variance of z_j,

$V_p = \sum_{j=1}^{n} a_{jp}^2$ = total contribution of factor F_p to the variance of all the variables, and

$V = \sum_{p=1}^{m} V_p$ = total contribution of all the common factors to the total variance of all the variables.

Writing the classical factor analysis model in expanded form gives the first of two important matrices generated by factor analysis, viz.,

$$
\begin{aligned}
z_1 &= a_{11}F_1 + a_{12}F_1 + \ldots + a_{1m}F_m + u_1y_1 \\
z_2 &= a_{21}F_1 + a_{22}F_2 + \ldots + a_{2m}F_m \qquad + u_2y_2 \\
&\ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \\
z_n &= a_{n1}F_1 + a_{n2}F_2 + \ldots + a_{nm}F_m \qquad \ldots + u_ny_n
\end{aligned}
\tag{2.2.5}
$$

This matrix is called the "factor pattern". The other important matrix is the matrix of correlations between the variables and the factors. Typical elements of this matrix are

$$
r_{z_jF_p} = a_{j1}r_{F_pF_1} + a_{j2}r_{F_pF_2} + \ldots + a_{jp} + \ldots + a_{jm}r_{F_pF_m}
\tag{2.2.6}
$$

and

$$
r_{z_jy_j} = u_j \text{ .}
$$

This matrix is called the "factor structure". Clearly, in the case of uncorrelated common factors, the factor structure and factor pattern is identical.

The above derivation can be summarized in matrix notation as follows:

Let $Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_{22} & \cdots & z_{2N} \\ \ldots & \ldots & & \ldots \\ z_{n1} & z_{n2} & & z_{nN} \end{bmatrix}$,

$$f = \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ F_m \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \ldots & F_{1N} \\ \cdot\cdot & \cdot\cdot & \ldots & \cdot\cdot \\ & & & \\ F_{m1} & F_{m2} & \ldots & F_{mN} \end{bmatrix},$$

$$y = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{11} & Y_{12} & \ldots & Y_{1N} \\ \cdot\cdot & \cdot\cdot & \ldots & \cdot\cdot \\ & & & \\ Y_{n1} & Y_{n2} & \ldots & Y_{nN} \end{bmatrix}$$

Also, let M be the matrix of factor pattern coefficients, i.e.

$$M = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1m} & u_1 & 0 & \ldots & 0 \\ a_{21} & a_{22} & \ldots & a_{2m} & 0 & u_2 & \ldots & 0 \\ \cdot\cdot & \cdot\cdot & \ldots & \cdot\cdot & \cdot\cdot & \cdot\cdot & \ldots & \cdot\cdot \\ a_{n1} & a_{n2} & \ldots & a_{nm} & 0 & 0 & \ldots & u_n \end{bmatrix}$$

$$= (A|U)$$

so that the factor pattern may be written in matrix form as

$$Z = (A|U)\{f|y\} = Af + Uy = c + e \qquad (2.2.7)$$

If $s_{jp} = r_{z_j F_p}$, i.e. equal to the correlation between variable z_j and factor F_p, for $j = 1,\ldots,n$, and $p = 1,\ldots,m$, the factor structure can be written as

$$S = \begin{bmatrix} s_{11} & s_{12} & \ldots & s_{1m} \\ s_{21} & s_{22} & \ldots & s_{2m} \\ \cdot\cdot & \cdot\cdot & \ldots & \cdot\cdot \\ s_{n1} & s_{n2} & \ldots & s_{nm} \end{bmatrix}$$

It can be shown that

$$S = A\,\phi\ , \tag{2.2.8}$$

where

$$\phi = FF' = \begin{bmatrix} 1 & r_{F_1F_2} & \cdots & r_{F_1F_m} \\ r_{F_2F_1} & 1 & & r_{F_2F_m} \\ \cdot\cdot & \cdot\cdot & \cdots & \cdot\cdot \\ r_{F_mF_1} & r_{F_mF_2} & \cdots & 1 \end{bmatrix},$$

i.e. the structure matric is equal to the pattern matrix postmultiplied by the matrix of correlations among the factors. Equivalently,

$$A = S\phi^{-1}\ .$$

Clearly, in the case of uncorrelated common factors $\phi = I$, giving $S = A$ as was stated above.

Furthermore, the matrix of observed correlations are

$$R = ZZ'\ ,$$

where

$$Z = [z^*_{ji}] = [z_{ji}/\sqrt{N}]\ ,$$

from which the matrix of reproduced correlations follows, viz.

$$\hat{R} = AFF'A = A\phi A'\ . \tag{2.2.9}$$

Substituting equation (2.2.8) into equation (2.2.9) gives

$$\hat{R} = SA' = AS'\ , \tag{2.2.10}$$

and for uncorrelated factors,

$$\hat{R} = AA'\ , \tag{2.2.11}$$

which is known as the FUNDAMENTAL FACTOR THEOREM.

Thus $\hat{R}$ can be expressed in terms of the common factor coefficients alone, which means that in order for $\hat{R}$ to be a good approximation to R, the diagonal elements of R must also be functions of the common variance of the variables, i.e. R must have communalities in the diagonal. In this case the factor solution will involve both common and unique factors. Unities in the diagonal imply the principal components model with its associated factor solution involving only common factors. In this way the elements in the diagonal of the observed correlation matrix determine what proportions of the unit variances are factored into common factors.

2.3 The Problem of Communality

In the previous section it was shown that the diagonal elements of the correlation matrix affect its rank and determine the portions of the variances to be factored. Substituting communalities in the diagonal both reduces the rank of the correlation matrix and the portion of the variances to be factored. Any factor solution requires either the rank of the correlation matrix, or the values of its diagonal elements. However, no a priori knowledge of the values of the communalities is available and the theoretical estimation of them under known or assumed rank is so mathematically involved and requires so many computations, that it becomes impractical.

Although very efficient methods that do not require the estimation of communalities have become feasible by the use of computers, the estimation of communalities is still required for many important and popular methods of factor analysis. Various practical procedures for estimating communalities have thus been developed, some of which will be described in brief below.

(1) The HIGHEST CORRELATION of a given variable z_j from among its correlations with all the other variables in a given set. This procedure is useful for large correlation matrices.

(2) Employ a TRIAD, i.e.,

$$h_j^2 = r_{jk} r_{j\ell} / r_{k\ell} ,$$

where k and ℓ are the two variables which correlate highest with the given variable.

(3) The AVERAGE CORRELATION of a given variable with each of the remaining ones, viz.,

$$h_j^2 = \sum_{k=1}^{n} r_{jk}/(n-1) \quad , \qquad k \neq j \quad .$$

(4) Assume an approximate rank of the correlation matrix, based on a priori knowledge of the grouping of the variables. Divide the correlation matrix into sub-groups of p variables, each of approximately unit rank. Then determine the UNIT RANK ESTIMATES of the communalities as follows:

$$h_j^2 = \frac{1}{v} \sum_{k<\ell=1}^{p} r_{jk} r_{j\ell} / r_{k\ell}$$

where

$$v = \binom{p-1}{2}$$

= the total number of different triads obtainable for a given variable z_j out of a particular subset of p variables.

(5) The FIRST CENTROID FACTORS can be used as an estimate of the communalities, calculated as follows: Insert the highest correlation for each variable in the principal diagonal of the correlation matrix. Then estimate each communality by taking the ratio of the square of the column sum to the total sum of all the correlations in the matrix, i.e.,

$$h_j^2 = \left(\sum_{k=1}^{n} r_{jk}\right)^2 / \sum_{k=1}^{n} \sum_{\ell=1}^{n} r_{k\ell} ,$$

where

r_{jj} is the highest correlation of the given variable with all other variables. This method tends to under-estimate the communalities.

(6) If instead of inserting the highest correlations in the principal diagonal of the correlation matrix, the average correlations are inserted, the communalities are approximated by

$$h_j^2 = [n/n-1]\left[\left(\sum_{k=1}^{n} r_{jk}\right)^2 / \sum_{k=1}^{n} \sum_{\ell=1}^{n} r_{k\ell}\right], \quad (k \neq j,\ k \neq \ell).$$

In this case the diagonal values are actually ignored.

(7) A factor analysis may be preceded by a principal component analysis of the variables from which the dimension of the common factor space can be estimated as being equal to the number of principal components greater than one. The communalities are then taken to be equal to the variance contributed by the reduced number of principal components to each variable.

(8) ITERATION BY REFACTORING:
Start with unities or zero or any other values in the principal diagonal. Decide a priori on the dimension of the common factor space. Calculate a principal-factor solution. Determine the sum of squares of factor coefficients as the new estimates of the communalities. Calculate another factor solution. Repeat this process as many times as necessary until the recomputed diagonal values do not change from the preceding set. Wrigley (1956) showed that the squared multiple correlation of each variable with the remaining ones is the best starting point.

(9) The SQUARED MULTIPLE CORRELATION (SMC) of each variable with the remaining variables is a very popular estimate of the communality of a variable. It is given by

$$SMC_j = R^2_{j.12\ldots)j(\ldots n} = 1 - \frac{1}{r_{jj}} ,$$

where

r_{jj} is the diagonal element in R^{-1} corresponding to variable z_j.

The SMC's measure the predictable common variance among the observed correlations, and form the lower bounds for the communalities, i.e.,

$R_j^2(n-1) \leq h_j$.

The SMC's are generally considered as the "best possible" estimates of the communalities, since for many correlation matrices for which minimum rank m is attained, $R_j^2(n-1)$ actually equals h_j^2. Also the SMC's tend to approach the communalities if the ratio $m/n \to 0$ as $n \to \infty$. Furthermore, it helps to know the direction of the error in approximation of the communalities, as given above.

2.4 Properties of a Gramian Matrix

The computational procedures of factor analysis depends on the correlation matrix being "Gramian". Properties of a Gramian matrix include: (i) symmetry, and (ii) positive semidefiniteness. A matrix A is symmetric if and only if $A = A'$, i.e., $A = (a_{jk})$ is symmetric $\Leftrightarrow a_{jk} = a_{kj}$ $(j,k = 1,\ldots,n)$. A matrix is said to be positive semidefinite if all the principal minors are greater than or equal to zero.

All correlation matrices with unities in the diagonal are Gramian matrices and communality estimates are acceptable only if they preserve the Gramian properties of the matrix. By inserting squared multiple correlations in the principal diagonal of the correlation matrix, the Gramian properties of this matrix are usually destroyed. They can, however, be restored by adjusting the off-diagonal values. An important property of a Gramian matrix is that all its eigenvalues are greater than or equal to zero.

2.5 Principal Component Analysis

Since many of the factor analytic procedures are based on principal component analysis, a brief discussion of principal components will provide a useful background. Recall from section (2.1) that the basic model for principal component analysis is

$$z_j = a_{j1}F_1 + a_{j2}F_2 + \dots + a_{jn}F_n \quad (j = 1,2,\dots,n),$$

or in matrix notation,

$$z_j = a_j^T f \ , \tag{2.5.1}$$

where

$a_j^T = [a_{j1},\dots,a_{jn}]$ is a vector of constants. To ensure that the overall transformation is orthogonal, the following condition is imposed:

$$a_j^T a_j = \textstyle\sum_{k=1}^n a_{kj}^2 = 1 \ . \tag{2.5.2}$$

The problem is then to choose a_1 so as to maximise the variance of $a_1^T f$ subject to the constraint $a_1^T a_1 = 1$. Now the variance of $a_1^T f$ is given by $a_1^T \Sigma a_1$,

where

$$\Sigma = E[(X-\mu)(X-\mu)^T]$$

so that finally the problem can be formulated as

maximise $V_1' = a_1^T \Sigma a_1$

subject to

$$a_1^T a_1 = 1 \ .$$

It can be shown that V_1' is equal to the largest root of the characteristic equation

$$|\Sigma-\lambda I| = 0 \ . \qquad (2.5.3)$$

The coefficients of the second factor, a_2, must be selected so as to maximise the contribution of that factor to the total variance, subject to that factor being orthogonal to the first. Hence the problem can be stated as

maximise $V_2' = a_2^T\Sigma a_2$

subject to

$$a_2^T a_2 = 1$$

and $\quad \mathrm{cov}(f_2,f_1) = a_2^T\Sigma a_1 = 0$.

Since $\Sigma a_1 = \lambda_1 a_1$, the second constraint can be simplified to

$$a_2^T a_1 = 0 \ . \qquad (2.5.4)$$

It can be shown that the required maximum is in fact the second largest eigenvalue of Σ. In a similar way it can be shown that the j^{th} principal component equals the eigenvector associated with the j^{th} largest eigenvalue.

The above derivation has assumed the z_j not to be standardized variables. However, since they are in fact standardized variables with unit variances, the above procedure actually becomes a principal component analysis of the correlation matrix R, rather than the covariance matrix Σ. By substituting R for Σ in the above equations, the components can be shown to be equal to the eigenvectors of the correlation matrix.

2.6 The Principal Factor Model

The principal factor method is essentially a principal component analysis on the reduced correlation matrix and thus requires estimates of the values of the communalities. The method can be described in broad terms as follows.

Select the first-factor coefficients a_{ji} $(j = 1,\ldots,n)$ so as to make the sum of the contributions of that factor to the total communality a maximum, i.e.

maximise $V_1 = a_{11}^2 + a_{21}^2 + \ldots + a_{n1}^2$

subject to

$$r_{jk} = \sum_{p=1}^{m} a_{jp}a_{jk} \qquad (j,k = 1,2,\ldots,n),$$

where

$$r_{jk} = r_{kj} \quad \text{and} \quad r_{jj} = h_j^2 \quad .$$

It can be shown that V_1 is equal to the largest root of the characteristic equation

$$|\hat{R}-\lambda I| = 0,$$

where

$\hat{R}$ is the reduced correlation matrix.

By substituting the largest root λ_1 into

$$(\hat{R}-\lambda I)A = 0$$

an arbitrary solution $\alpha_{11},\alpha_{21},\ldots,\alpha_{n1}$ is obtained. The desired coefficients of F_1 in the factor pattern is then determined as

$$a_{j1} = \alpha_{j1}\sqrt{\lambda_1}/\sqrt{(\alpha_{11}^2+\alpha_{21}^2+\ldots+\alpha_{n1}^2)} \qquad j = 1,\ldots,n .$$

The coefficients of the second factor must be selected so as to maximise the contribution of that factor to the total residual communality, i.e. the communality after removal of the first factor. Thus the problem is to

maximise $V_2 = a_{12}^2 + a_{22}^2 + \ldots + a_{n2}^2$

subject to

$$ {}_1r_{jk} = r_{jk} - a_{j1}a_{k1} = a_{j2}a_{k2} + a_{j3}a_{k3} + \ldots + a_{jm}a_{km}. $$

It can be shown that the required maximum is in fact the second largest root of the original correlation matrix, $\hat{R}$.

Similarly, the successive largest roots and their associated vectors are obtained directly from the original reduced correlation matrix $\hat{R}$, until m factors have been extracted, where m is the rank of the reduced correlation matrix.

2.7 The Minres Method

Whereas the principal component and principal factor methods aim at extracting maximum variance, the main objective of the Minres method is to maximally reproduce the off-diagonal elements of the correlation matrix, and as a by-product, to obtain communalities consistent with this criterion. It therefore, also in contrast to the principal factor method, does not require a priori estimates of the values of the communalities, but do require an estimate of the number of common factors.

The aim of any factor analysis method is to obtain $\hat{R}$, the reproduced correlation matrix, as a "best" estimate of R, the observed correlation matrix. This can be done by ordinary least squares, either by fitting

$$R \text{ by } (\hat{R}^2+U^2), \tag{2.7.1}$$

or by fitting

$$(R-I) \text{ by } (\hat{R}-H^2), \tag{2.7.2}$$

where

$$H^2 = I-U^2 = \mathrm{diag}(AA'), \tag{2.7.3}$$

i.e., the diagonal matrix of communalities determined from the solution, A.

Equation (2.7.1) implies minimisation of the residuals of the total matrix which is achieved by principal component analysis, while equation (2.7.2) requires minimisation of the off-diagonal residuals and thus forms the basis of the minres model.

From equation (2.7.2) the objective can be derived as

$$\|[R-I] - [AA' - \mathrm{diag}(AA')]\|$$

which is equivalent to

$$f(A) = \sum_{k=j+1}^{n} \sum_{j=1}^{n-1} (r_{jk} - \sum_{p=1}^{m} a_{jp}a_{kp})^2 . \tag{2.7.4}$$

This function is minimised subject to the constraint

$$h_j^2 = \sum_{p=1}^{m} a_{jp}^2 \leq 1 , \quad j = 1,\ldots,n , \tag{2.7.5}$$

which is meant to restrict the communalities to numbers between zero and one.

It can be shown that a principal factor analysis of a correlation matrix with minres communalities produces a minres factor solution.

Although the majority of factor analytic methods do not make use of statistical estimation theory, it is clear that factor analysis solutions are subject to sampling errors. Thus, especially for judgment concerning the statistical significance of the number of common factors to be valid, it should take account of the actual sampling variations of the reproduced correlation matrix. Various attempts have been made to provide factor analysis with a sound statistical basis. The most important breakthrough was achieved by Lawley in 1940 through his "method of maximum likelihood", which will be discussed in the next section.

However, as far as the minres method is concerned, the statistical significance of the factorization can probably best be determined by use of a statistic developed by Rippe (1953). By assuming that the original variables have a multivariate normal distribution, from which it follows that the correlations have a Wishart distribution and that the sample values are maximum-likelihood estimates of the population correlations, he developed a statistic for testing the completeness of factorization, which is applicable to large samples and independent of the particular type of factor solution. For testing the significance of m factors, the statistic is as follows:

$$U_m = N\{\ell n|AA'+U^2| - \ell n|R| + tr[R(AA'+U^2)^{-1}] - n\},$$

which is χ^2_v, where

$$v = \tfrac{1}{2}[(n-m)^2 + n-m] .$$

If $U_m > \chi^2_v$ at significance level α, the hypothesis of m common factors is rejected.

It should be noted, however, that there is a distinction between the statistical significance of the number of common factors and the practical significance. It may often happen that statistically significant factors may not be interpretable in a practical sense. A practical approach which is therefore frequently used to determine the number of significant factors, is to consider the proportions of the total variance (or total communality) accounted for by each factor.

2.8 The Method of Maximum Likelihood

In contrast to the previous methods which were all based on pure mathematical theory, Lawley (1940) applied the statistical method of maximum likelihood estimation to obtain sample estimates of the universe factor loadings. His method is based on a a priori assumption of the number of common factors. Any variable may be expressed linearly as

$$x_j = a_{j1}F_1 + a_{j2}F_2 + \ldots + a_{jm}F_m + u_jY_j ,$$

where it is assumed that

(i) the test scores (F_i) have zero means;

(ii) all the factors (F's and Y's) are independent, normally distributed variables with zero means and unit variances, which means that the x's have a multivariate normal distribution.

To describe the derivation of the maximum likelihood method, Harman (1960) introduced the following notation:

Matrix			Order	Definition
Population	Estimator	Sample		
$\Sigma = (\sigma_{jk})$	$\hat{\Sigma}$	$S = (s_{jk})$	n×n	Covariance matrix
$\rho = (\rho_{jk})$	$\hat{\rho}$	$R = (r_{jk})$	n×n	Correlation matrix
$A = (a_{jp})$	$\hat{A}$	-	n×m	Matrix of common-factor coefficients
$U^2 = (u_j^2)$	$\hat{U}^2$	-	n×n	Diagonal matrix of uniqueness

From assumption (ii) above it follows that the elements of the covariance matrix follow a Wishart distribution, viz.,

$$df = k|\Sigma|^{-\frac{1}{2}(N-1)}|S|^{\frac{1}{2}(N-n-2)}\exp - \frac{N-1}{2}\sum_{j,k=1}^{n}\sigma^{jk}$$

$$s_{jk}\prod_{j<k=1}^{n} ds_{jk} ,$$

which, when considered as a function of the σ's, is the likelihood function L of the sample. Thus the problem is to find estimates $\hat{A}$ and $\hat{U}^2$ satisfying

$$\Sigma = AA' + U^2$$

which maximise L.

The results can be put in matrix form as follows:

$$\hat{\rho} = \hat{A}\hat{A}' + \hat{U}^2$$

$$\hat{A} = \hat{\rho}R^{-1}\hat{A}$$

$$\hat{U}^2 = I - \text{diag}\,\hat{A}\hat{A}'$$

$$\hat{A}'R^{-1}\hat{A} \quad \text{is diagonal} .$$

The procedure can be simplified by assuming

$$\hat{\rho} = R ,$$

so that

$$\hat{A}\hat{A}' + \hat{U}^2 = R .$$

The associated test statistic for the number of common factors depends on a theorem which states that *"-2 times the logarithm of the likelihood ratio is approximately distributed as* χ^2 *when* N *is large."* Thus the statistic is

$$U_m = -2 \ \ell n \ \lambda = N \ \ell n \frac{|\hat{\rho}|}{|R|} ,$$

which is χ_v^2 ,

where

$$v = \tfrac{1}{2}[(n-m)^2 - n-m].$$

The null hypothesis of m common factors is rejected when U_m exceeds the tabulated value at a given level of significance. The expression can be simplified to

$$U_m = N \sum_{j<k=1}^{n} \bar{r}_{jk}^2 / \hat{u}_j^2 u_k^2$$

where

$$\bar{r}_{jk}^2 = r_{jk} - \hat{r}_{jk} .$$

Note: N must be large.

2.9 Canonical Factor Analysis

Canonical correlation, as defined by Hotelling (1936) involves *"the weighting of variables in each of two sets so as to attain the maximum correlation between the two composites."* Rao (1955) suggested an alternative objective to the usual maximum variance objective of factor analysis, based on the idea of canonical correlation. Thus in his method of canonical factor analysis, the common factors are determined such that they are successively maximally related to the observed data, with the additional constraint of being orthogonal to each other.

Assuming n observed variables and m hypothetical factors generates the following matrix of correlations,

$$\begin{bmatrix} R & A \\ A' & I \end{bmatrix},$$

where

R = matrix of observed correlations,

A = matrix of correlations between the variables and the factors,

= factor pattern coefficients (because of orthogonality),

and I denotes the matrix of correlations among the common factors.

The squared canonical correlations are the roots, V_p, of the determinantal equation

$$|AA' - VR| = 0 \qquad (2.9.1)$$

Substituting $R - U^2$ as an approximation for AA', gives

$$[(R-U^2) - VR]b = 0 \qquad (2.9.2)$$

as the maximal relationship between the z's and the F's. Here b is a column vector of weights for the linear composite of the z's. Equation (2.9.2) can further be simplified to

$$[U^{-1}(R-U^2)U^{-1}-\lambda I]q = 0 \qquad (2.9.3)$$

or

$$[U^{-1}RU^{-1}-(\lambda+1)I]q = 0 \tag{2.9.4}$$

where

$$\lambda = \frac{v}{1-v}$$

and $q = Ub$.

Thus q is an eigenvector corresponding to the largest eigenvalue (λ_1+1) of the matrix $U^{-1}RU^{-1}$ and the factor pattern may be expressed as

$$A = UQ\Lambda^{\frac{1}{2}} \tag{2.9.5}$$

where

Q is the matrix of unit-length eigenvectors associated with the m largest eigenvalues of equation (2.9.3), and

Λ is the diagonal matrix of these eigenvalues.

Three observations can be made at this point:

(i) The number of real, nonzero canonical correlations, and hence the number of common factors, is equal to the number of roots of $U^{-1}RU^{-1}$ that are greater than one, or equivalently, is equal to the number of positive roots of $R-U^2$.

(ii) It can be shown that the elements of the matrix of factor loadings, A, are unaffected by arbitrary rescaling the observed variables.

(iii) Finally, it is clear that canonical analysis rescales the observed correlations in the metric of the unique parts, $U^{-1}RU^{-1}$.

2.10 Image Theory

While common factor theory employs a partial-correlation approach to define "commonness" among a set of n variables, breaking each variable into a common and unique part, viz.,

$$x_{ji} = c_{ji} + u_{ji} \quad , \tag{2.10.1}$$

Guttman (1953) developed his Image Theory based on a multiple-correlation approach where the squared multiple correlations, r_{ji}, describe commonness among the variables and in which each variable is partitioned as follows:

$$x_{ji} = \rho_{ji}^{(n)} + e_{ji}^{(n)} , \tag{2.10.2}$$

where

$\rho_{ji}^{(n)}$ is the predicted value of x_{ji} from the remaining $n-1$ variables in the sample, and

$e_{ji}^{(n)}$ is the related error of prediction.

The $\rho_{ji}^{(n)}$ are defined as

$$\rho_{ji}^{(n)} = \textstyle\sum_{k=1}^{n} w_{jk}^{(n)} w_{ki}^{(n)} \tag{2.10.3}$$

where

$w_{jk}^{(n)}$ denote the weight of x_k in the multiple regression for predicting x_j.

Furthermore,

(i) the e_j correlate zero with the ρ_j, i.e.,

$E_i \; e_{ji}^{(n)} \rho_{ji}^{(n)} = 0$, and

(ii) the e_j correlate zero with x_k, $\forall k \neq j$, hence

$E_i \; e_{ji}^{(n)} x_{ki} = 0$, $(j \neq k)$.

Guttman assumed the x_{ji} to be elements from a sample of n variables belonging to a "*universe of content of indefinitely many quantitative variables.*" He further assumed that the universe could be arbitrarily arranged such that the particular sample will be the first n variables.

The predicted value of x_{ji} from the remaining $n-1$ variables, $\rho_{ji}^{(n)}$, is called the "partial image" of x_{ji} and the related error of prediction, $e_{ji}^{(n)}$, is known as the "partial anti-image". Taking the limit of these values as $n \to \infty$ gives

$$\rho_{ji}^{(\infty)} = \lim_{n\to\infty} \rho_{ji}^{(n)} \qquad \text{and}$$

$$e_{ji}^{(\infty)} = \lim_{n\to\infty} e_{ji}^{(n)} ,$$

known respectively as the "total image" and "total anti-image" for x_{ji}.

To explain the correlation coefficients r_{jk}, Guttman makes use of the identity

$$r_{jk} = g_{jk}^{(n)} - \gamma_{jk}^{(n)} , \qquad (j \neq k) , \tag{2.10.4}$$

where

$g_{jk}^{(n)} = E_i \, \rho_{ji}^{(n)} \rho_{ki}^{(n)}$, the covariance of the partial images, and

$\gamma_{jk}^{(n)} = E_i \, e_{ji}^{(n)} e_{ki}^{(n)}$, the covariance of the partial anti-images.

It can be shown that

$$\gamma_{jk}^{(n)} = - \pi_{jk}^{(n)} \sigma_{jn} \sigma_{kn} , \tag{2.10.5}$$

where

$-\Pi_{jk}^{(n)} = \rho_{e_j e_k}^{(n)}$, the correlation between partial anti-images $e_j^{(n)}$ and $e_k^{(n)}$,

Thus equation (2.10.4) can be written as

$$r_{jk} = g_{jk}^{(n)} + \Pi_{jk}^{(n)} \sigma_{jn}\sigma_{kn} \qquad (2.10.6)$$

and any observed total correlation can be seen to be the sum of two parts, viz.,

(i) the covariance between the common parts of the two variables, and

(ii) "*a special pairwise linkage that may remain between the two variables after the remaining* $n-z$ *variables are partialed out.*"

Common-factor theory is in fact just a special case of image theory where zero pairwise linkages are assumed, thus giving

$$r_{jk} = E_i \; c_{ij}c_{ik} \; , \qquad (j \neq k) \qquad (2.10.7)$$

Common-factor theory does not in general result in unique solutions, a indeterminacy that can be overcome by introducing the concept of a "determinate" connom-factor space. In such a common-factor space there is a perfect regression for each common factor F_p on the observed x_j, thus implying zero error factors. Guttman states that it can be shown that if "*a common-factor space of rank* m *is determinate for an indefinitely large universe of content, then there is no other determinate common-factor space possible for the same universe - whether of rank* m *or any other rank. The communalities are uniquely determined and are equal to* *the corresponding squares of the total images. The common-factor scores are the total image scores, and the unique factor scores are the total anit-image scores."* Thus as $n \to \infty$, $c_{ji} = \rho^{\infty}_{ji}$ and there are no nonzero linkages so that image theory and common-factor theory are identical.

To summarize the above development in matrix notation, let

Γ_n be the Gramian matrix of the anti-image covariances, $\gamma^{(n)}_{jk}$,

G_n be the Gramian matrix of image covariances, $g^{(n)}_{jk}$, and

S^2_n be the diagonal matrix of elements $\sigma^2_{jn} = E_i(e^{(n)}_{ji})^2$.

Then

$$R_n = G_n - \Gamma_n + 2S^2_n \qquad (2.10.8)$$

As $n \to \infty$, $S^2_n \to U^2$ and

$$G \to R - S^2,$$

where $S^2 = \text{diag}(1/r_{jj})$.

Thus the traditional factor analysis procedure of the reduced correlation matrix $R\text{-}U^2$, can be approximated by factor analyzing the reduced correlation matrix $R\text{-}S^2$ with squared multiple correlations (SMC_j) in the diagonal. Since

$$SMC_j = 1 - \frac{1}{r_{jj}} \quad ,$$

the diagonal values of $R\text{-}S^2$ are the SMC's, and by taking

$$S^{-1} = \text{diag}(\sqrt{r_{jj}})$$

as a scale factor, we have the rescaled correlation matrix

$$R^* = S^{-1}RS^{-1} \;. \qquad (2.10.9)$$

2.11 Some Rao-Guttman Relationships

In 1962 C.W. Harris (1962) once again tackled the problem of determining the rank of R, and at the same time illustrated some very important relationships among various symmetric matrices, thereby high-lighting a very useful property of invariance among certain factor analytic methods. He approached the problem of determining common factors from the same direction as Rao (1955) did in the development of canonical factor analysis.

Observing that the number of real, nonzero canonical correlations equals the number of roots of $U^{-1}RU^{-1}$ that are greater than one, and also equals the number of positive roots of $R-U^2$, Harris deduced that the specific elements of U^2 determines both the sum of the roots of $U^{-1}RU$ and the number of real, nonzero canonical correlations. He then remarked that Guttman proved in the development of his image analysis that, by using the squared multiple correlations as estimates of communality, the number of nonnegative roots of $R-U^2$ forms a "best" lower bound to m, the rank of the observed correlation matrix.

Harris then transcribed Guttman's image theory as follows. He defined the scores on the image variables as

$$M = (I-U^2R^{-1})Z \;, \qquad (2.11.1)$$

where Z is such that $ZZ' = R$, and the scores on the anti-image variables as

$$A = U^2R^{-1}Z \ . \tag{2.11.2}$$

He then generated the following symmetric matrices:

$MM' = R + U^2R^{-1}U^2 - 2U^2 = G$,

Guttman's image covariance matrix;

$AA' = U^2R^{-1}U^2 = T$,

Guttman's anti-image covariance matrix;

$ZZ' = R$,

the observed correlation matrix;

$MA' = AM' = U^2 - U^2R^{-1}U^2$;

$MZ' = ZM' = R-U^2$; and

$AZ' = ZA' = U^2$.

Factorizing these matrices resulted in the following representations:

R	factors into	$UQ[\ b_i\ ]Q'U$,
U^2	factors into	$UQ[\ 1\ ]Q'U$,
$R-U^2$	factors into	$UQ[\ b_i-1\ ]Q'U$,
G	factors into	$UQ\left[\frac{(b_i-1)}{b_i}\right]Q'U$,
Γ	factors into	$UQ\left[\frac{1}{b_i}\right]Q'U$,
$AM' = MA'$	factors into	$UQ\left[\frac{(b_i-1)}{b_i}\right]Q'U$,

where

Q = complete set of characteristic vectors of $U^{-1}RU^{-1}$, and the internal matrix is a diagonal matrix of the roots

b_i, or of functions of them.

Hereby, Harris proved that all the matrices, except for $R-U = MZ'$ and ZA', could be expressed in the form

$$UQS^{\frac{1}{2}} \quad , \tag{2.11.3}$$

where $S^{\frac{1}{2}}$ differed for R, U^2, G and Γ, but was always a diagonal matrix of the square roots of the elements of the appropriate internal matrix. He thus showed that "*using Rao's principal to derive factors for each member of the system of matrices results in factors for* R, U^2, $R-U^2$, G, Γ *and the matrix* AM' *that are simply related to each other, by a change in scale of the columns ..... Thus the variables* Z, A, *and* M *may be expressed in terms of their (differing) factors but the same factor scores.*"

2.12 Indeterminancy of Factor Solutions and the Canonical Form

By factor analyzing a given correlation matrix one may come up with an indeterminate number of different factor solutions. This is because, although a factor solution determines the m-dimensional common-factor space uniquely, it does not determine the exact positions of these factors and hence the factor loadings a_{jp} are not unique. This problem of lack of uniqueness occurs when determining the initial factor solution, as well as at the stage of rotation to a more interpretable solution.

The indeterminancy at the initial stage can be overcome by rotation to canonical form, which is just a well-defined mathematical form in which equivalent solutions are identical, although they might have appeared quite different before the rotation. The procedure is as follows:

Let

A = arbitrary form of factor matrix $(n \times m)$,

B = canonical form of factor matrix $(n \times m)$,

T = orthogonal transformation matrix $(m \times m)$,

then

$$B = AT$$

where T can be shown to be the matrix of corresponding eigenvectors of the matrix $A'A$.

An exception to the general rule is the principal factor method, which produces a unique initial solution. The indeterminancy at the rotation stage, however, still remains.

2.13 Motivation Behind the Rotation of the Initial Factor Pattern

The general factor analysis model describes an observed variable linearly in terms of a number of hypothetical common factors and a unique component. The number of common factors involved in the description of a variable is called its "complexity". To ease the interpretation of factor analysis solutions, the complexity of each variable should be low. The ideal solution would be a uni-factor solution, i.e. one in which each variable would be of complexity one. This is hardly ever possible in practical situations but it is with this objective in mind that Thurstone (1947) proposed the "simple structure principles". They can be summarized as follows:

(i) Each row of the matrix should have at least one zero.

(ii) If there are m common factors, each column of the factor matrix should have at least m zeroes.

(iii) For every pair of columns of the factor matrix there should be several variables whose entries vanish in one column but not in the other.

(iv) For every pair of columns of the factor matrix, a large proportion of the variables should have vanishing entries in both columns when there are four or more factors.

(v) For every pair of columns of the factor matrix there should be only a small number of variables with non-vanishing entries in both columns.

Such a desired final solution is obtained through transformations of the initial factor solution, where the transformation usually involves an orthogonal or oblique rotation from the initial set of reference axes to a different set. The derived solution is known as a multiple factor solution and if it satisfies the five simple structure criteria, the graphical plot will have the following characteristics:

(i) many points near the two final factor axes;

(ii) a large number of points near the origin; and

(iii) only a small number of points removed from the origin and between the two axes.

The simple structure principles are just another way of stating the principle of parsimony. The rotational problem is mainly concerned with assigning a precise mathematical meaning to the measure of parsimony.

2.14 Objective Orthogonal Multiple-Factor Solutions

In this section the following notation will be employed. Let

$A = (a_{jp})$, the initial factor matrix,

$B = (b_{jp})$, the final factor matrix,

$T = (t_{qp})$, the orthogonal transformation matrix,

so that

$$B = AT \ . \tag{2.14.1}$$

Applying an orthogonal transformation to the initial factor matrix does not affect the communalities of the variables and hence

$$\sum_{p=1}^{m} b_{jp}^2 = \sum_{p=1}^{m} a_{jp}^2 = h_j^2 \ , \qquad j = 1,\ldots,n \ . \tag{2.14.2}$$

The squared communality of any variable also remains constant, viz.,

$$\left(\sum_{p=1}^{m} b_{jp}^2\right)^2 = \sum_{p=1}^{m} b_{jp}^4 + 2\sum_{p<q=1}^{m} b_{jp}^2 b_{jq}^2 = \text{constant} \tag{2.14.3}$$

Summing equation (2.14.3) over the n variables, gives

$$\sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^4 + 2\sum_{j=1}^{n}\sum_{p<q=1}^{m} b_{jp}^2 b_{jq}^2 = \text{constant.} \tag{2.14.4}$$

Thus maximisation of one of the terms is equivalent to minimization of the other term, and either term, or some function of these terms, could serve as a precise measure of parsimony.

Ferguson (1954) proposed the sum of squares of products of the coordinates as a measure of parsimony. This measure for the case of n variables and m orthogonal factors, is

$$\sum_{j=1}^{n}\sum_{p<q=1}^{m}(a_{jp}a_{jq})^2 \quad , \tag{2.14.5}$$

involving $m(m-1)/2$ sums of n pairs of coordinates. This implies minimisation of the second term of equation (2.14.4), or equivalently, the maximisation of

$$Q = \sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^4 \tag{2.14.6}$$

Neuhaus and Wrigley (1954) proposed that the variance in the distribution of squared factor loadings should be made a maximum, i.e. maximise

$$M = \frac{1}{mn}\sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^4 - (\bar{b}_2)^2 \quad , \tag{2.14.7}$$

where

$$\bar{b}_2 = \frac{1}{mn}\sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^2$$

which remains constant under orthogonal transformation.

Carroll (1953) proposed the minimisation of some sort of inner-product of the columns of the final factor-structure matrix, viz.,

$$N = \sum_{j=1}^{n}\sum_{p<q=1}^{m} b_{jp}^2 b_{jq}^2 \tag{2.14.8}$$

Saunders (1953) proposed as a criterion for a simple structure solution that the kurtosis of the doubled frequency distribution of rotated factor loadings be a maximum, viz.,

$$K = \sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^4 \Big/ \left(\sum_{j=1}^{n}\sum_{p=1}^{m} b_{jp}^2\right)^2 \tag{2.14.9}$$

If a uni-factor solution was practically possible, the variance of each variable would result from only one factor loading. With this objective in mind, a final factor solution with maximum inequality in the distribution of the variance among the several factors for each variable in the factor pattern, is usually aimed for. This objective is explicitly stated in the criterion developed by Neuhaus and Wrigley, but since from equation (2.14.4) it follows that all four criteria (Q, M, N and K) lead to identical results for an orthogonal solution, the objective can equivalently be described by any one of the four criteria. It is known as the QUARTIMAX criterion. The theoretical development of a computational procedure for the quartimax criterion will not be discussed here.

While the quartimax criterion emphasizes the simplification of the description of each row, or variable, of the factor matrix, Kaiser (1958) developed a criterion in which he put the emphasis on the simplification of the columns, or factors. The simplicity of a factor p is defined as the variance of its squared loadings, i.e.,

$$S_p^2 = \frac{1}{n}\sum_{j=1}^{n}(b_{jp}^2)^2 - \frac{1}{n^2}\left(\sum_{j=1}^{n} b_{jp}^2\right)^2 \quad , \qquad p = 1,\ldots,m \ .$$

Maximising the variance of a factor will improve its interpretability since it will cause its components (the b's) towards unity and zero. Maximum simplicity for a complete factor matrix is thus obtained by maximising the sum of the individual simplicities, viz.,

$$S^2 = \sum_{p=1}^{m} S_p^2 = \frac{1}{n}\sum_{p=1}^{m}\sum_{j=1}^{n} b_{jp}^4 - \frac{1}{n^2}\sum_{p=1}^{m}\left(\sum_{j=1}^{n} b_{jp}^2\right)^2 \quad (2.14.10)$$

This is known as the "RAW" VARIMAX CRITERION.

Both the quartimax and "raw" variance criterions are biased towards the more prominant factors in that these factors have larger values in both the large and small factor loadings than their counterparts in the less prominant factors. Kaiser related this bias directly to the size of the communality of each variable, since each variable contributes to S^2 as the square of its communality. He thus corrected for this bias by weighting the variables equally and so derived his (NORMAL) VARIMAX CRITERION as the maximisation of

$$V = n\sum_{p=1}^{m}\sum_{j=1}^{n}(b_{jp}/h_j)^4 - \left(\sum_{p=1}^{m} \sum_{j=1}^{n} b_{jp}^2 / h_j^2\right)^2 \quad (2.14.11)$$

Kaiser also proved an additional property of the varimax method for a special case, namely that "*the varimax solution is invariant under changes in the composition of the test battery.*" This principle of factorial invariance was stated by Thurstone (1947) as "*a fundamental requirement of a successful factorial method.*" Kaiser suggests that this criterion may even be a possible improvement to the simple structure criterion.

A general class of orthogonal criteria can be constructed from a weighted composite of the quartimax and varimax criteria, namely,

$\alpha Q + \beta V = \text{maximum},$

where

Q is from equation (2.14.6), and

V is the raw form of the varimax criterion, multiplied by n.

This is known as the ORTHOMAX criteria and may be explicitly written as

$$\sum_{p=1}^{m}\left(\sum_{j=1}^{n} b_{jp}^{4} - \frac{\gamma}{n}\left(\sum_{j=1}^{n} b_{jp}^{2}\right)^{2}\right) = \text{maximum}\ , \qquad (2.14.12)$$

where

$$\gamma = \beta/(\alpha+\beta)\ .$$

When $\gamma = 0$, equation (2.14.12) is equivalent to the quartimax criterion, $\gamma = 1$ gives the varimax criterion, and $\gamma = m/2$ is known as the equamax criterion.

2.15 Objective Oblique Multiple-Factor Solutions

In this section the restriction of orthogonality is removed, with the result that the four criteria derived in the previous section (Q, K, M, N) are no longer equivalent, and each method is not immediately generalizable to the oblique case. The following notation will be employed. Let

$A = (a_{jp})$, the initial factor matrix,

$V = (v_{jp})$, the final factor matrix,

$\Lambda = (\lambda_{qp})$, the oblique transformation matrix,

so that

$$V = A\Lambda\ . \qquad (2.15.1)$$

The oblique transformation matrix Λ which will carry A into V such that V satisfy the criterion K = maximum, proposed by Saunders (1953), is known as an OBLIMAX solution, and K may be expressed in terms of such an oblique solution as

$$K = \sum_{j=1}^{n}\sum_{p=1}^{m} v_{jp}^4 / \left(\sum_{j=1}^{n}\sum_{p=1}^{m} v_{jp}^2\right)^2 \tag{2.15.2}$$

Note that K is now not equivalent to Q.

By removing the restriction of orthogonality from the criterion N, suggested by Carroll, a QUARTIMIN solution is derived by minimising

$$N = \sum_{j=1}^{n}\sum_{p<q=1}^{m} v_{jp}^2 v_{jq}^2 \tag{2.15.3}$$

By considering his quartimin criterion and Kaiser's oblique version of the varimax criterion, Carroll (1960) derived a general class of methods involving oblique factors and a minimising criterion which he called OBLIMIN methods. Relaxing the restriction of orthogonality on the raw varimax criterion yields the minimization of

$$C^* = \sum_{p<q=1}^{m}\left(n\sum_{j=1}^{n} v_{jp}^2 v_{jq}^2 - \sum_{j=1}^{n} v_{jp}^2 \sum_{j=1} v_{jq}^2\right) , \tag{2.15.4}$$

which is in fact the minimization of the covariances of squared elements of the factor structure V. The corresponding normal oblique varimax criterion is

$$C = \sum_{p<q=1}^{m}\left[n\sum_{j=1}^{n}(v_{jp}^2/h_j^2)(v_{jq}^2/h_j^2) - \sum_{j=1}^{n} v_{jp}^2/h_j^2 \sum_{j=1}^{n} v_{jq}^2/h_j^2\right] \tag{2.15.5}$$

Since covarimin tends to be "too orthogonal" and quartimin "too oblique", Carroll (1957) proposed the following BIQUARTIMIN criterion as a compromise:

$$B^* = N + C^*/n = \text{minimum} . \tag{2.15.6}$$

Permitting varying weights of the quartimin and covarimin components yields

$$B^* = \alpha N + \beta C^*/n = \text{minimum} \qquad (2.15.7)$$

The general OBLIMIN criterion is given by

$$B^* = \sum_{p<q=1}^{m}\left(n\sum_{j=1}^{n} v_{jp}^2 v_{jq}^2 - \gamma\sum_{j=1}^{n} v_{jp}^2 \sum_{j=1} v_{jq}^2\right) \qquad (2.15.8)$$

where

$$\alpha = \beta\gamma/(\alpha+\beta) \ .$$

For normalized loadings,

$$B = \sum_{p<q=1}^{m}\left[n\sum_{j=1}^{n}(v_{jp}^2/h_j^2)(v_{jq}^2/h_j^2) - \gamma\sum_{j=1}^{n} v_{jp}^2/h_j^2\sum_{j=1}^{n} v_{jq}^2/h_j^2\right] \qquad (2.15.9)$$

When $\gamma = 1$ $B^* = C^*$, $\gamma = 0$ gives $B^* = K$, and $\gamma = \frac{1}{2}$ results in the biquartimin criterion.

Since the primary factor pattern, P, and the final multiple factor solution, V, are simply related by

$$P = VD^{-1} \quad \text{or} \quad V = PD,$$

it follows that simplifying P is equivalent to simplifying V. Jennrich and Sampson (1966) approached the problem of deriving a simple structure solution by developing a criterion involving the primary factor-pattern coefficients. By replacing the v_{jp} in equation (2.15.8) by these primary-factor-pattern coefficients, he arrived at the DIRECT OBLIMIN criterion, viz.,

$$F(P) = \sum_{p<q=1}^{m}\left(\sum_{j=1}^{n} b_{jp}^2 b_{jq}^2 - \frac{\delta}{n}\sum_{j=1}^{n} b_{jp}^2 \sum_{j=1}^{n} b_{jq}^2\right). \qquad (2.15.10)$$

2.16 Orthoblique Method of Rotation

Harris and Kaiser (1964) developed a procedure for arriving at an oblique solution by making use of orthogonal transformation matrices and positive-definite diagonal matrices. From fundamental factor theory it follows that the matrix of reproduced correlations, $\hat{R}$, can be expressed in terms of a direct orthogonal factor solution, A, as

$$\hat{R} = AA' \ .$$

The principal factor method gives

$$A = Q\Lambda^{\frac{1}{2}} \ ,$$

where Λ is the diagonal matrix of the m nonzero eigenvalues of $\hat{R}$, and

Q is the vector of corresponding normalized eigenvectors.

Thus

$$\hat{R} = Q\Lambda Q' \quad .$$

Using orthogonal matrices T and positive-definite matrices D, both types of order m, Harris and Kaiser showed that

$$\hat{R} = (Q\Lambda^{\frac{1}{2}}T_2D_2T_1D_1)(D_1^{-1}T_1'D_2^{-1}T_2'\Lambda^{-\frac{1}{2}}\Lambda\Lambda^{-\frac{1}{2}}T_2D_2^{-1}T_1D_1^{-1})$$
$$\times\ (D_1T_1'D_2T_2'\Lambda^{\frac{1}{2}}Q')$$

Clearly, the expression in the last set of parentheses is the transpose of the expression in the first set, so that

$$\hat{R} = P\phi P' \ ,$$

where

$$P = Q\Lambda^{\frac{1}{2}}T_2D_2T_1D_1 \text{ , the oblique pattern;}$$

$$\phi = D_1^{-1}T_1'D_2^{-1}T_2'\Lambda^{-\frac{1}{2}}\Lambda\Lambda^{-\frac{1}{2}}T_2D_2^{-1}T_1D_1^{-1}$$

$$= D_1^{-1}T_1'D_2^{-1}T_1D_1^{-1} \text{ , the matrix of factor correlations;}$$

and $S = P\phi = Q\Lambda^{\frac{1}{2}}T_2D_2^{-1}T_1D_1$, the oblique structure,

represents the entire class of orthoblique solutions. Different choices of T_1, T_2 and D_2 lead to different solutions. D_1 is merely a scaling matrix to ensure that ϕ has unities in the diagonal and is thus a proper correlation matrix.

2.17 A Second Generation Little Jiffy

In 1964 at a meeting of a "Working Group in Factor Analysis", Henry Kaiser's reply to a question about what factor analysts actually do in practice was, "*principal components with associated eigenvalues greater than one followed by normal varimax rotation.*" Because of its stark contrast to the highly involved theoretical issues of modern factor analysis with which they were concerned at the time, they called this simple procedure "Little Jiffy". Six years later in September 1970, in a presidential address at the annual meeting of the Psychometric Society, Kaiser (1970) presented his "Second Generation Little Jiffy". This was essentially a polished version of the original Little Jiffy in which he suggested the most practical solutions to the basic problems of factor analysis.

In the development of this new procedure, Kaiser was the whole time governed by two principles, the first of which implied the avoidance of crucial decisions and the second requiring the adherence to simple straightforward mathematics. His solutions to the four basic problems in factor analysis were as follows.

Problem 1. To determine the sampling adequacy of factor analytic data matrices.

Kaiser tried to develop a quantitative expression for Guttman's measure of sampling adequacy, namely that R^{-1} should be near-diagonal. Using the anti-image intercorrelation matrix,

$$Q = SR^{-1}S ,$$

he arrived at

$$MSA = 1 - \sum_{j \neq k}\sum q^2_{jk} / \sum_{j \neq k}\sum r^2_{jk} ,$$

or for each variable separately,

$$MSA(J) = 1 - \sum_{\substack{k \\ k \neq j}} q^2_{jk} / \sum_{\substack{k \\ k \neq j}} r^2_{jk} .$$

Clearly, $-\infty < MSA < 1$, and it turned out that good factor-analytic data is suggested by a MSA in the .80's, while for really excellent data MSA should be in the .90's. MSA was proved to be a function of four main effects. Holding the others constant, it can be shown that

(i) MSA improves as the number of variables, p, increases,

(ii) MSA improves as the (effective) number of factors,

q, decreases,

(iii) MSA improves as the number of subjects, n, increases, and

(iv) MSA improves as the general level of correlation, $\bar{r}$, increases.

Problem 2. Initial Factoring.

Of the three exploratory factor analysis models,

(i) component analysis,

(ii) common-factor analysis, and

(iii) image analysis,

Kaiser opted for image analysis because it seemed to him to be an ideal compromise between component and common-factor analysis. Like component analysis it satisfies the second principle in that it is mathematically simple, and furthermore, as the number of variables approaches infinity, image analysis and common-factor analysis become equivalent. In addition, by choosing image analysis, Kaiser was also proceeding in accordance with his first principle since Harris (1962) showed that all three methods have the same factors (though different loadings) and thus "Harris factors" are model-free.

Problem 3. The number of Harris factors.

Harris (1962) suggested Guttman's lower bound, namely the number of eigenvalues of $R-S^2$ greater than zero, as the "natural" number of factors to retain. It has been shown that this criterion leads to too many factors, and although too many factors can do no harm when using either image or common-factor analysis followed by orthogonal rotation, as soon as the restriction of orthogonality is removed, interpretation problems arise.

Kaiser arrived at another rule for the original Little Jiffy by using the expected value of the sum-of-squares of the projections of the test vectors, representing the original variables, on a unit-length factor vector, with common origin, as this factor sweeps over the entire space. For Little Jiffy this sum-of-squares turned out to be equal to one, which suggested that a factor should be retained "*if the sum-of-squares of the projections of the test vectors on it is greater than one.*" Applying the same procedure to component and common-factor analysis, but not, however, to image analysis, it turns out that only those factors greater than the mean eigenvalue of $S^{-1}RS^{-1}$ should be retained.

Although Kaiser was not able to prove it, practical examples suggested that this new rule always results in fewer or just as many factors as the original rule. There also exists the slight possibility that in rare cases this rule may prove to be too conservative.

Problem 4. The Transformation Procedure.
A special case of the orthoblique method of rotation which applies an orthonormal transformation T to the unit-length eigenvectors E, apparently always transforms the axes in the right directions to obtain a best simple

structure. It also adheres closely to the two principles stated above in that the unit-length eigenvectors are the same under all three mathematical models, and the T are generated by raw quartimax procedures, which is the simplest of the orthomax criteria and actually implies at the same time obtaining a varimax or any other orthomax solution.

In trying to solve Thurstone's (1947) famous box problem, Kaiser abandoned his first principle completely and after a lot of trouble involving complicated winscrizing procedures, arrived at a solution which correlates 90% with the original orthoblique method.

Finally, to find the intercorrelation matrix of the factors, the first principle had to be abondoned once more since the elements of this matrix are functions of the eigenvalues, which are not the same for the three different methods. Similarly, the factor score matrices for the three models are also different.

Thus Kaiser's Second Generation Little Jiffy represents an excellent agglomeration of factor analytic techniques and provides a very general and practical procedure.

CHAPTER 3

A FACTOR ANALYSIS OF SHARES LISTED ON THE JSE

3.1 Introduction

It is generally known that prices of stocks move together and hence there is considerable speculation about the movement of the market as a whole. However, averages, indexes and separate analysis of industrial, gold, mining or financial sectors indicate that, apart from being influenced by the general movement of the market, stocks tend to group together according to the similarity of their performance. What is more, it is usually assumed that these groups of comoving shares correspond to the industry classifications to which the shares belong.

Investors usually try to minimize the risk involved in obtaining their desired returns by diversifying their portfolios. Since effective diversification is only achieved by investing in stocks that do not fluctuate in a similar way, it is important to test whether the assumption of the grouping of shares according to their industry classification is statistically sound.

3.2 King's Study of the NYSE

King (1966) used factor analytic techniques to determine firstly whether security price changes could be sufficiently accounted for by three effects, namely a market effect, an industry effect, and an individual firm effect, and secondly if this proved to be the case, to determine "*to what extent do the industry-like clustering within this ensemble of sicty-three securities corresponded to the six two-digit SEC classifications represented therein.*"

As his basic data King used the monthly rate of return. This was derived from the actual prices as follows. If P_{jk} = price of security j at end of month k, then $P_{jk}-P_{jk-1}$ is the actual monthly price change. In order to make this independent of the magnitude of the price, the change in the logarithm of the price is usually used, i.e.

$$\log P_{jk} - \log P_{jk-1}$$

and thus the random variable describing monthly returns is given by

$$Z_{jk} = \log(P_{jk}/P_{jk-1}) .$$

His data comprised of monthly closing prices of sixty-three securities listed on the New York Stock Exchange continuously for the time period May, 1927 through December, 1960. This resulted in four hundred and three changes in log price (or equivalently, rates of return) for each security. The securities represented six different SEC industry classifications, viz.,

(1) Tobacco

(2) Petroleum products

(3) Metals (ferrous and non-ferrous)

(4) Railroads

(5) Utilities, and

(6) Retail Stores

To investigate the stability of both his data and the factor analysis results over time, he divided his total period of observations into four subperiods of about one hundred and one months each, and performed the analysis for the total period, as well as for each of the subperiods.

King used the squared multiple correlation coefficients, R_j^2, resulting from the regression of each variable on the remaining sixty-two as the initial estimates of communality. The R_j^2 could then be considered as estimators of the degree of affiliation to the market, or equivalently, of the variance due to communality. Analysis of the R_j^2 for each security and over all four subperiods showed a general weakening of comovement at time progressed. The mean R_j^2 for the total period was 0.722.

To extract a market component from his data, King used two different methods of factor analysis. The first was the centroid method. This method provided a computational compromise for the principal factor method before the general availability of computers and, as the name indicates, is closely related to the mechanical concept of a centroid or centre of gravity. This involved a regression of each stock on the standardized cross-sectional mean.

King called his second method the "Guttman-Harris" technique, referring to Harris's (1962) refinement of previous work done by Guttman (1953) and Rao (1955). His main reason for using this latter method, was that it rescales the observed correlation matrix in the metric of the variable set's estimated uniqueness, thus preventing "*variables with large components of unique variance from playing an inordinate role in the composition of the first component.*"

Both methods resulted in very similar first-factor loadings. For the overall period the first-factor accounted on the average for 52% of the total variance, implying that about half of a typical stock's price variations was due to changes in the over-all market. The difference between the percentage of variance explained by the market, indicated an upper bound of approximately 20% for the possible effect of industry groupings on the total variance.

By removing the market effect from the data, King now formed a new matrix,

$$G_1 = G - a_1 a_1' ,$$

where G was the original covariance matrix, and a_1 denotes the column vector of first factor loadings. This was called the "*matrix of residual covariances after removal of the market factor*" and was subsequently analyzed using three different techniques.

King called the first technique the "*quick and dirty*" method of factor analysis. This basically involved transforming the residual covariance matrix into a correlation matrix and then performing an average linkage clustering of the variables in terms of these residual correlations. He did not use any stopping rule but allowed the clustering procedure to be repeated until all the variables were grouped into one super-cluster. At the end of fifty-six iterations the groupings, or clusters, corresponded exactly, to the SEC two-digit classifications, thus providing King with a very significant proof of industry comovement.

The second method employed by King in the analysis of the residual covariance matrix, was the multiple-factor method. This method involves the factoring of a covariance or correlation matrix into several multiple factors simulteneously and results in factors which are usually oblique to one another. It depends on an a priori grouping of the data (the SEC-industry classifications in King's case). It is now mainly of historical interest and is only used in cases where a specific hypothesis involving groups of variables is tested. The results obtained from this method once again verified the very significant grouping of stocks into their SEC-industry classifications.

The last method used by King, was the further implementation of the Guttman-Harris technique followed by Kaiser's Varimax method of orthogonal rotation. In contrast to the multiple-factor method, this method takes no account of any a priori knowledge of structure. The results showed considerable variation in the component contributions over time. For the overall period, the first seven components accounted for 91,5% of the common variance. However, especially for the latter subperiods, it seemed as if a greater number of factors was required in order to explain the comovement. Once again King was able to identify the SEC-industry classifications in the solution for the over-all period.

This lead King (1966) to conclude perhaps a trifle over-confidently, that "*grouping according to two-digit industrial classifications is so strong that almost any method of factor analysis would bring it out.*"

3.3 Resemblance between the present study and that of King

With the same objective as that of King in mind, namely the testing of the statistical significance of grouping shares by their industry classifications, King's analysis was repeated for a selection of shares listed on the Johannesburg Stock Exchange (JSE). There were, however, a few differences between King's study and this study.

Firstly, a smaller number of shares (49) was used. Secondly, the time period was shorter, namely, March 1973 to June 1981. The reason for this shorter time period was due to restricted availability of data. As a result of this, weekly instead of monthly rates of return had to be used to ensure that, when the data set was divided into subperiods, the number of observations (weekly returns) still exceeded the number of variables (shares). The data set did, however, correspond to King's in that it too satisfied the objective of getting a representative number of shares in each of seven industry classifications.

The other main difference was the specific factor analytic techniques used. King employed the centroid method to extract the market component and later forced a multiple-factor solution on the residual covariance matrix. Both these methods are now of historical interest and have been replaced by the principal-factor method.

The multiple-factor analysis of the residual covariance matrix provided King with the most significant back-up for his conclusions. In his criticism of certain aspects of King's study, Meyers (1973) identified the use of the multiple-factor method as the "*primary basis for exaggeration of the strength of industry factors.*" In the light of the above discussion, and also because computer programs for the implementation of the centroid and multiple-factor methods were not readily available, these two methods were replaced by the principal-factor method of factor analysis.

The Guttman-Harris technique employed by King also forms the basis of the method of initial factor-extraction used by Kaiser (1970) in his Second Generation Little Jiffy. King employed the varimax method of orthogonal rotation on his Guttman-Harris solution but remarked that it would be interesting to see the effect of oblique rotations on the factor pattern. In this study the third method of analyzing the residual covariance matrix thus comprised of an initial factor extraction using Little Jiffy, followed by a varimax rotation, as well as the Kaiser-Harris orthoblique rotation, which is also the method of rotation usually employed by Little Jiffy. The BMDP (1977) statistical package was used for the cluster analysis (P1M) and for two of the methods of initial factor extraction (P4M). Since only the totation of the second and subsequent factors was of interest, the STATJOB-ROTATE 1 (1974) package had to be used for determining the derived solutions, since BMDP does not allow for a choice of which factors to rotate.

3.4 The Data

Weekly rates of return were used for the period March 1973 to June 1981. Since the data available comprised weekly prices, the weekly returns were calculated as

$$Z_{it} = \log(P_{it}) - \log(P_{i(t-1)}) \ ,$$

where P_{it} = price of share i at end of week t.

The data comprised of forty-nine shares from seven different sectors of the JSE, with seven shares per sector. These are listed in Appendix A.

To test the stability of the factor analysis over time the over-all time period was divided into two subperiods, viz.,

(i) February 1973 to March 1977, and
(ii) April 1977 to June 1981.

The analysis was repeated for the total period, as well as for each of the two subperiods. Because of the very dominant effects of the gold and coal shares, the analysis was also repeated with the gold and coal shares excluded.

Note that all tables and figures referred to in the subsequent sections of this chapter are to be found in Appendix B.

3.5 Estimation of Communalities and Determination of Market Effect

For both methods of factor analysis the squared multiple correlations, obtained from regressing each variable on the remaining 48 variables, were used as initial estimates of communality. Table 1 shows the R_j^2 for each variable for the total period, as well as for each of the two subperiods. It also shows the average R_j^2 for each industry classification and the overall average.

One may interpret R_j^2 as an indicator of the degree of affiliation with the rest of the market. From Table 1 it can be seen that in general the gold shares have the highest R_j^2's. Analyzing the R_j^2's over time seems to indicate a downward trend in the strength of affiliation with the market. The average value of R_j^2 for the total period is .35801, indicating that on average about 36% of a share's variance is due to its comovement with other shares. The discrepancy between the mean for the total period and the mean of the two subperiods (.43515) is due to the fact that R_j^2 calculated using N observations is not a linear function of $R_j^2(1)$, calculated using n_1 observations and $R_j^2(2)$ calculated using n_2 observations, where $n_1 = n_2 = N$.

It is generally assumed that the major portion of a share's variation in price is due to the over-all effect of the market. Since the first factor resulting from any factor analytic method is always determined so as to either extract the maximum variance from the observed data, or to exhibit the highest correlation with the observed data, the first factor was considered to denote the general market effect.

Table 2 shows the percentage of variance explained by the first factor which was calculated as

$$\frac{a_{j1}}{\sigma_j^2} \qquad (j = 1,\ldots,49) ,$$

where a_{j1} = loading of share j for first factor.

The results generated by both the principal-factor method and Little Jiffy for the total period, as well as for the two sub-periods are given. In addition, Table 2 also shows average percentage contributions for each industry classification and for the whole market.

In all cases the gold shares seem to be closest to the market. On the average, it can be seen that about 40% of the total variance of the gold shares is accounted for by the first factor. The results for the first subperiod as generated by Little Jiffy show this figure to be as high as 54%, compared to relatively small percentages for the other industry groups. Thus in this case the first factor seems to be more of a gold factor than an overall market factor. This is probably also the reason for the lower overall average percentage for this period as given by Little Jiffy.

From the overall averages for the total period it can be concluded that the market accounts on the average for about 16% of the total variation in weekly returns of shares. The difference between the percentage of variance due to communality, as given by $\bar{R}^2_j$, and the percentage of variance due to the market, can be determined as being equal to 36% - 16% = 20%, If industry effects can thus be proved to be present, this figure can be regarded as an upper bound of their possible contribution to the total variance.

3.6 Direct Analysis of the Residual Covariance Matrix

By removing the market effect, the original covariance matrix can be transformed to a new matrix,

$$G_1 = G - a_1'a_1 \ ,$$

whose elements can be referred to as residual covariances. This new matrix was subsequently analyzed to determine the comovement structure of the shares.

If the hypothesis that shares move together according to their industry classifications is true, then by looking at the elements of the residual covariance matrix, one should observe high within-group covariances in the sub-matrices along the diagonal, compared to low between-group covariances in the off-diagonal sub-matrices. Figure 1 shows the residual covariance matrix. All elements greater than .000 750 in absolute value are encircled. It can be observed that 75 out of a possible 1225 entries are more than .000 750 in absolute value. In particular, all but two of the within-group covariances for the gold shares are encircled. For the coal and motor shares 12 and 8 entries respectively are greater than .000 750. For the other industry classifications, only the self-covariances are encircled (with the exception of the covariance between the two chemical firms Lanchem and Triomf). Only 2 covariances from the off-diagonal sub-matrices are encircled.

Hence it appears that the gold shares form a distinct group on their own with possible groupings of the coal shares and motor shares also being suggested. There does not, however, seem to be any strong factor of comovement among shares from any of the other industry classifications.

3.7 A Cluster Analysis of the Residuals

An average linkage clustering was subsequently performed on the residual covariance matrix. The procedure can be briefly summarised as follows.

1. Convert the residual covariance matrix into a correlation matrix.

2. Combine the two variables/clusters with the highest pair-wise correlation to form one group or cluster.

3. Recompute the correlation matrix to make allowance for the correlation of the newly formed variable/cluster with the other variables/clusters. These new correlations are defined as being equal to

$$\Sigma\Sigma \; r_{ij}/(I \; J) \; ,$$

where variable i is contained in the first cluster and variable j in the second cluster, and I and J are the number of variables in the two clusters. The summation is over all possible pairings of the variables between the two clusters.

4. Repeat steps 2 and 3 until all the variables have merged to form one super-cluster.

The cluster analysis results are shown in the dendrogram in Figure 2. The values at which the clusters were formed have been scaled so as to fall between 0 and 100, according to the following table.

Value Above	=	Correlation Above	Value Above	=	Correlation Above
0		-1.000	55		.100
5		-.900	60		.200
10		-.800	65		.300
15		-.700	70		.400
20		-.600	75		.500
25		-.500	80		.600
30		-.400	85		.700
35		-.300	90		.800
40		-.200	95		.900
45		-.100	100		1.000
50		.000			

Figure 2 shows how the gold shares form a very distinct group on their own, having merged at correlations between .300 and .500. The coal shares also form a group on their own with correlations between .100 and .500. Looking at the groupings formed at values above 56, a group containing 5 motor shares apart from one odd building share can be identified. The bank and building shares seem to cluster together in one big group. After this point the clusters become "diluted" and no distinct groups emerge.

3.8 Principal-Factor Analysis of the Residual Covariances

The principal-factor method extracted four factors in the total period and five factors in each of the two subperiods.

The unrotated and varimax rotated factor patterns for the total period and the two subperiods are given in Tables 3 and 5, 7 and 9, and 11 and 13, respectively. The factor loadings have also been sub-totaled for the different industry groups. These sub-totals are indicative of the impact of the respective factors on the specific industry groups but the significance of their meaning is overriden by the magnitudes of the individual loadings. It should also be noted that only the second and subsequent factors were rotated and therefore the varimax rotated factor patterns as given in Tables 5, 7 and 9 do not include the first factor loadings.

The first factor representing the influence of the over-all market has already been discussed. In addition to a market factor, the other factors can be identified as representing a gold, a coal and a motor factor, while a fourth factor is mainly due to variation in returns of the two chemical firms, Lanchem and Triomf.

In the two subperiods the very large correlation of the gold shares with the market is illustrated by the high first factor loadings for these shares. It can also be seen that in the varimax rotated patterns, the gold shares have relatively high negative loadings for all the factors. All the factors can thus be regarded as made up of a small gold component in addition to any other identifiable components. In fact, in the second subperiod, instead of emerging as a distinct factor the effect of the gold shares is divided among the second to fifth factors in the varimax rotated pattern. The above remarks serve to illustrate the very significant influence of gold on the market.

Tables 4 and 6, 8 and 10, and 12 and 14 give the percentage contributions of the factors extracted in the total period and two subperiods respectively, to the communality and total variance. On the average, a gold factor plus a much smaller motor component contribute about 9% towards the total variance. A coal factor contributes a further 4,5% while the two chemical firms, Lanchem and Triomf, contribute about another 2%.

The weakening of comovement over time is illustrated by the decline in the proportion of the total variance explained by the common factors - from 35,5% in the first subperiod to 31,4% in the second subperiod. For the total period this figure is 30%, which is slightly less than was predicted by the mean R_j^2. The two subperiods respectively show the "industry factors" to account for more or less 18,4% and 16% of the total variance, while for the total period this value is about 14%. If the less interpretable factors during the subperiods are ignored these figures become more compatible.

3.9 <u>Analyzing the residual covariance matrix using Little Jiffy</u>

Little Jiffy extracted thirteen factors in the total period. The unrotated, varimax rotated and obliquely rotated factor patterns are given in Tables 15, 17 and 19, respectively. The same remarks concerning the representation as for the principal factor analysis, apply to these tables as well. Tables

16, 18 and 20 give the corresponding percentage contributions of the factors to the communality and total variance.

Apart from a market effect, contributing 18% to the total variance, the other factors that can be identified are a gold factor, a coal factor and a motor factor. The sixth factor is mainly due to variation in returns of the two chemical firms, Lanchem and Triomf, while no further interpretation of the other factors is possible. Their contribution to the total variance is in any case so small that they can be considered as being insignificant.

Because of the orthogonality constraint inherent in the varimax rotation of the factors, the varimax rotated factor pattern gives the clearest indication of the relative importance of the various group effects that emerged. From Table 18 it can be seen that gold accounts for about 5% of the total variance. A coal factor contributes 3,4%, a motor factor 2,7% and a factor due to Lanchem and Triomf a further 0,6%. The other less interpretable factors together account for a further 4,8%.

With the orthogonality constraint removed, the results for the obliquely rotated factor pattern show the above contributions to be somewhat smaller since the effects of the groups are now divided among more than one of the factors. This is particularly the case for the gold shares. The third factor can now be regarded as a small gold factor contributing 1,1% towards the total variance, while the residual effect of gold is now distributed among the other factors, as can be seen from the relatively large negative loadings for the gold shares with all the other factors. The positive correlations among all factors, except for the correlations of the third factor with the others, as shown in Table 21, confirm the influence of gold on all the factors.

In the first subperiod 12 factors were extracted. The results are presented in Tables 22 to 28, in a similar fashion as before. In addition to the market effect, the results once again show the emergency of a gold, a coal and a motor factor. The very high first factor loadings for the gold shares and the relatively high loadings for the other shares with both the first and second factors, seem to indicate the presence of two gold factors affecting the rest of the market, rather than a single market effect. Nonetheless, the first factor was again considered as representing a market effect and only the second and subsequent factors were rotated.

From the results of the varimax rotation it can be seen that gold contributes 6% towards the total variance while a coal factor contributes 4% and a motor factor 2,7%. The other less interpretable factors together account for a further 6,2% of the total variance. These figures all include the effect of a small gold components whose presence is illustrated by the consistent regative loadings of the gold shares with the other factors.

When the orthogonality constraint is removed no single gold factor emerge. The effect of gold is now spread among all the factors, as shown by the relatively large negative loadings of the gold shares with all the factors, in addition, of course, to the original impact of gold on the first factor. The positive correlations among the second and subsequent factors, as shown in Table 28, once again confirm the above remarks.

The results for the second subperiod, as given in Tables 29 to 34, also show 12 factors. Once again the gold shares have fairly high first factor loadings and the first and second factors both influence all the shares quite significantly but these effects are not as strong as is the case of the first subperiod. Apart from a market effect, a gold factor, a coal factor and a motor factor can be identified. A further factor can once again be attributed to the variation in returns of Lanchem and Triomf.

In this subperiod, rotation to an orthogonal or oblique frame of reference did not change the picture significantly. It can be concluded that on the average gold contributes about 10% to the total variance, while a coal factor accounts for a further 2,5 to 3%, a motor factor for about another 2%, and Lanchem and Triomf for about 1%. The other factors together contribute a further 4 to 6%.

To summarize the results generated by Little Jiffy, it can be concluded that for the total period the common factors contribute 31% towards the total variance. This is again slightly less than was predicted by the mean R_j^2 but is in are given in Tables 35 to 37. These tables also show the percentage contributions of the factors to the communality and total variance.

From these percentages it can be seen that the only significant factor is in fact the market factor, accounting for 85% of the communality and 19% of the total variance in the total period. The contributions of the other factor(s) is insignificantly small and it seems to be mainly due to variation in returns of a few specific firms, especially Lanchem and Triomf. The smaller contribution of the first factor in the second subperiod and the emergence of one extra factor is again an indication of the weakening of comovement with time.

Little Jiffy extracted 11 factors in the total period and the two subperiods. Tables 38 to 40 show the unrotated factor patterns and the percentage contributions of the factors to the communality and total variance.

The first factor, representing the market as a whole, once again seems to be the only really significant factor. In the total period it accounts for 74% of the communality and 17% of the total variance. The decline in the corresponding figures for the two subperiods, from 66,8% and 19% in the first subperiod, to 56,5% and 16,7% in the second subperiod, again illustrates the weakening in comovement as time progressed. This also confirms the suspicion that the apparent stronger comovement reported by Little Jiffy for the second subperiod in the analysis of all the variables, was due to the effect of the gold shares.

The contributions of the second and subsequent factors are again insignificantly small. Apart from being able to discern a possible grouping of motor shares, as shown by factor 2 in the total period and first subperiod, and factor 3 in the second subperiod, these factors are once again due to the variation in returns of a few specific firms.

Because of the insignificant contributions of the second and subsequent factors compared to that of the first factor, it was decided that rotation of the factor patterns would not improve the results significantly.

3.11 To Summarize

From the various tables generated by the two different analytic techniques the following conclusions can be drawn. An overall market effect accounts for approximately 16% of the total variance in weekly returns for shares listed on the JSE. A further 14% of the variance can be attributed to common factors. The most significant common factor is the effect of gold on the market, while the coal shares, and to a certain extent the motor shares, also form distinct groups on their own. Apart from a linkage between the two chemical firms, Lanchem and Triomf, no further significant groupings into industry classifications are apparent.

The effect of gold on the JSE is further illustrated by the very high first factor loadings of the gold shares, indicating that the movement of gold determines the movement of the market as a whole. The influence of gold extends to most of the other factors as well, as can be seen from the consistently relatively high negative loadings of the gold shares with all the factors. Thus it seems that gold can possibly be regarded as the single effect on the market.

The results generated after removal of the gold and coal shares further confirm the absence of any additional significant groupings. One can thus conclude that the variance of shares listed on the JSE, with the possible exception of the coal and motor shares, is a function of only two components, viz., a gold-dominated market effect and a unique effect. This is in accordance with the assumptions inherent in Sharpe's diagonal model (1963) and seems to indicate that, for shares listed on the JSE, the division of the total risk associated with a share's return into a systematic component and an unsystematic component is justifiable.

CHAPTER 4

A FACTOR ANALYSIS OF GOLD SHARES LISTED ON THE JSE

4.1 Introduction

Because of the very significant influence of gold on the JSE and the large gold mining sector, gold shares form a major part of most South African investors' portfolios. It was thus decided to analyse the gold shares on their own using similar techniques to those used in the analysis of the whole market. A list of the gold shares that was used in the analysis appears in Appendix C. It was thought that apart from a market effect, some of the characteristics of the gold mines, such as their location, life, ore grade, mining costs, profits, etc., would be identified as having an effect on the comovement of the gold shares.

The information on the location of the mines, their group membership, life and in some case their ore grade were taken from a Quarterly Gold Review done by Simpson, Frankel and Kruger Inc. (1982). The information on the mining costs and profits came from various gold and uranium quarterly reports as published in the Financial Mail. It should be noted that the information for all the mines was not always available. Furthermore, especially for the later periods analysed, Elsburg and Western Areas, Welkom and Western Holdings, and

Zandpan and Hartebeesfontein were often linked to each other, since the only significant assets for the first-mentioned company in each of the three cases were its shares in the company mentioned with it.

The symbols which were used to describe the life of the mines can be translated into years as follows:

B = Break-up = 1 → 5 years,
S = Short = 6 → 10 years,
M = Medium = 11 → 16 years, and
L = Long = 17 → 20+ years.

The analysis was repeated for two time periods, viz.,

(i) April 1968 to December 1971, and
(ii) February 1973 to July 1981.

The first period corresponds to a time when the modified gold standard, as designed by the Bretton Woods (Morgan and Morgan, 1976) agreement, which was aimed at stabilizing exchange rates and assuring a continual flow of new gold into official reserves, was operational. By the early 1970's, however, the Bretton Woods Agreement had achieved quite the opposite and the USA's decision to renounce its obligation to sell gold at the official price of US $35 an ounce marked the beginning of the demonitisation process of gold (which was completed in 1974-75). The second period of analysis thus corresponds to a period without any effective monetary control over gold and it was hoped that a comparison of the results of the two periods would throw light on any effect the abolishment of an international gold standard had on the comovement of the gold shares.

All tables and figures referred to in the subsequent sections of this chapter are to be found in Appendix D.

4.2 Analysis of the Gold Shares for the period April 1968 to December 1971

Figure 1 shows the dendogram resulting from the average-linkage cluster analysis of the gold shares for the period April 1968 to December 1971. The values at which the clusters were formed have been scaled so as to fall between 0 and 100, according to the following table.

Value Above	=	Correlation Above	Value Above	=	Correlation Above
0		-1.000	55		0.100
5		-0.900	60		0.200
10		-0.800	65		0.300
15		-0.700	70		0.400
20		-0.600	75		0.500
25		-o.500	80		0.600
30		-0.400	85		0.700
35		-0.300	90		0.800
40		-0.200	95		0.900
45		-0.100	100		1.000
50		0.000			

The clustering process was thus carried out in a range of correlations between 0.100 and 0.800.

Table 1 shows the resulting groupings and indicates within what range of correlations these groupings were formed. It also gives the most important information for each gold mine as reported in the gold quarterly report of December 1969 (Financial Mail (1970)). Analysing the information given in Table 1, it is clear that none of the resulting clusters of gold shares showed common location, life, grade or any other characteristics for the individual members.

The weekly returns of the gold shares for the above period were then factor analysed using Little Jiffy followed by both a varimax and orthoblique rotation of the second and subsequent factors. Table 2 shows the unrotated factor pattern. Although nine factors were extracted, from the percentage contributions of each factor to the communality and total variance it is clear that only the first factor is really significant. It accounted for 75% of the communality and 33,4% of the total variance, compared to the very small contributions made by the other factors.

Nonetheless, the shares with relatively high loadings for the second and subsequent factors were identified and analysed to see whether any factor could be regarded as corresponding to a specific characteristic of the gold mines. The characteristic information for the gold mines was again taken from the gold and uranium quarterly report of December 1969. This report is shown in Appendix E. Gold shares with relatively high loadings for a specific factor did not, however, seem to have any characteristics in common. Analysis of the rotated patterns in Tables 3 and 4 shows that rotation to an orthogonal or oblique frame of reference did not change the results at all. It can thus be concluded that for the period April 1968 to December 1971, any comovement of the gold shares was only due to their common affiliation to a market factor.

4.3 Analysis of the Gold Shares for the period February 1973 to July 1981 ...

Figure 2 shows the dendogram resulting from an average-linkage clustering of the gold shares for the period February 1973 to July 1981. The values at which the clusters were formed have been scaled in a similar manner as before and the clusters were formed at correlations ranging from 0.242 to 0.878. Table 5 shows the resulting groupings and indicates within what range of correlations these groupings were formed. It also gives information concerning the more important characteristics of each mine. The information on the location, group, life and grade characteristics of each mine was obtained from Simpson et al (1982) while the cost and profit columns came from the gold quarterly report of September 1980 (Financial Mail (1980)). In all the tables the figures in brackets refer to statistics from one quarter earlier for profit and cost columns, and one year earlier for the grade column.

It is immediately clear from Table 5 that shares clustered together according to the location of the mines. Other characteristics shared by all the mines in some of the clusters can be seen to have been group membership and grade but since these do not appear consistent as common group characteristics, their apparent influence on some clusters was probably due to their linkage with location. At the bottom of Table 5 is a list of the shares that showed no strong clustering with any other shares. It can be seen that all of these, with the exception of WSTNDP and AFRLEASE, are Rand mines. It can thus be said that although the Rand mines did not cluster together to form a group on their own, they did not show any strong links with any of the other groups either.

The weekly returns of the gold shares for the period February 1973 to July 1981 were then factor analysed using the initial factor extraction method of Little Jiffy followed by a varimax and orthoblique rotation of the second and subsequent factors. Table 6 shows the unrotated factor pattern. Four factors were extracted with the first factor markedly dominating the others in importance. In fact, the large percentage contributions of this factor to the communality and total variance, 92% and 56% respectively, as compared to the very small contributions made by the other factors, suggests that only the first factor can be considered as having had a significant influence on the weekly returns of the gold shares.

Nonetheless, the shares with relatively high loadings for the second and subsequent factors were again identified and analysed to see whether any factor could be regarded as corresponding to a specific characteristic of the gold mines. The characteristic information was obtained from the same sources as mentioned in the cluster analysis of the mines. In Table 7 the shares with relatively high loadings for each factor have been listed together with their characteristic information.

It is clear that mines which showed high loadings for a specific factor share, on the whole, a common location, or two locations in the case of both positive and negative loadings. This is especially true for factors 2 and 4. Factor 2 further seems to contrast low-profit mines to high-profit mines, although it could be a coincidental characteristic of the Free State Mines to have shown high profits over the period. It is clear that the Free State Mines in fact formed a very homogeneous group with respect to all the characteristics. Although there is evidence of grouping according to location for the mines listed under factor 3, the distinction between high-grade and low-grade mines appear to be more consistent.

In the varimax rotated pattern (Tables 8 and 9), the grouping of mines according to location is apparent in the results for all the factors, Factors 2 and 4 again seem to contrast low-grade mines to high-grade mines while all the shares with high loadings for Factor 3 are low grade mines. Distinctions between low-cost and high-cost, as well as low-profit versus high-profit mines are also apparent, especially for Factors 2 and 4.

The factors also seem to suggest a distinction between longer life mines as against mines of medium or shorter lives.

Apart from the OFS mines which are all AAC mines, no grouping according to "Mining House" is apparent. The orthoblique solution presented in Tables 10 and 11 again confirms that grouping took place according to the location of the mines. A distinction between low and hihg-profit mines is apparent in all the factors, while only Factor 4 now suggests the possible effect of the grade and cost of the mines on the clustering of the shares. Factor Pãtterns with the shares ordered or grouped according to their different characteristics emphasize the above results. Tables giving these ordered factor patterns for an unrotated first factor and varimax rotated second to fourth factors are given in Appendix F. In addition these tables also show average loadings for sub-grouped shares.

The period February 1973 to July 1981 was then divided into two subperiods, viz.,

(1) February 1973 to April 1977,

(2) May 1977 to July 1981,

and the weekly returns for the two subperiods factor analysed to test the stability of the results over time. Table 12 shows the unrotated factor pattern for the first subperiod. Four factors were extracted with the first factor once again dominating the others by far in importance.

Notwithstanding their insignificantly small contributions to the communality and total variance, the second and subsequent factors were again analysed in a similar manner as for the total period. The characteristic information for each share as given in Tables 13, 15 and 17, was obtained from Simpson et al (1982) and the gold and uranium quarterly report of September 1973 (Financial Mail (1973)).

From Table 13 the grouping of shares according to their location is once again visible in the results for all the factors. The Free State Mines again seem to have emerged as a very homogeneous group. Varimax rotation of the results emphasized the grouping according to location as can be seen from Table 15. Neither the unrotated or varimax rotated factor patterns, however, show any possible effect the other characteristics of the mines could have had on the clustering process. Apart from the influence of the location of the mines, a further distinction between low-grade and high-grade mines can be seen for the shares listed under Factors 3 and 4 of the orthoblique rotated factor pattern (Tables 16 and 17). Factor 4 shows a further possible distinction between low-cost and high-cost mines.

Table 18 shows the unrotated factor pattern for the second subperiod. It can be seen that seven factors were extracted, compared to the four in the first subperiod. The very high contributions of the first factor to the communality and total variance, viz., 84% and 51% respectively, as compared to the insignificantly small contributions from the other factors, again suggests that the first factor was the only really significant influence on the weekly returns of the gold shares for the period May 1977 to July 1981. The values are, however, smaller than the corresponding first factor contributions for the first subperiod which were respectively 91,6% and 61%. This decline in contributions, as well as the increase in the number of factors extracted indicate a weakening of comovement over time.

Tables 19, 21 and 23 show a similar analysis of the second and subsequent factors to that presented before. The information was obtained from Simpson et al (1982) and the gold and uranium quarterly report of September 1980 (1980). Table 19 shows the groupings corresponding to the unrotated factor pattern in Table 18. The location of the mines again seem to have had an effect on the formation of the groups, though the influence appeared to be not as strong as before. In fact, Factor 2 shows a much more emphasized distinction between low-grade versus high-grade, high-cost versus low-cost and low-profit versus high-profit mines, than a grouping according to location. In Factor 3 the Free State mines again emerged as a strong homogeneous group.

In the varimax solution the grouping according to location is even less apparent than in the unrotated factor pattern and the only really significant grouping seems to have been that of the Free State mines. Similar remarks apply to the orthoblique solution presented in Tables 22 and 23, with the possible additional emergence of a distinction between low-grade and high-grade, and low-profit and high-profit mines shown for Factor 2.

4.4 To Summarize

From the analysis of the comovement of gold shares for the two periods April 1968 to December 1971 and February 1973 to July 1981, the following can be concluded.

The first period (which was before the collapse of the Gold Standard) showed no clustering of the gold shares into homogeneous groups with respect to any characteristics of the mines. The only significant factor, accounting for 33,4% of the total variance was due to the general influence of the market as a whole.

In the second period, which was after the collapse of the Gold Standard and which also corresponded to the period for which the whole market was analysed previously, some clustering of the gold shares into homogeneous groups with respect to their mine characteristics became apparent. The most significant characteristic seemed to be the location of the mines, although characteristics like grade, mining costs and profits also had some effect. These effects were, however, so small in comparison to the effect of the first factor that they could virtually be ignored as insignificant and they would certainly cause no threat to the risk/return relationships postulated by Sharpe (1963). Thus once again comovement of the gold shares was only due to a market factor which accounted for 56% of the total variance.

The results for the last period seemed to be fairly consistent over time, though a weakening of comovement was indicated by the decline in the contributions of the common factors to the total variance for the two subperiods, from 66% to 60%, and by an increase in the number of factors extracted. Comparing the contributions of the common factors to the total variance for the two periods April 1968 to December 1971 and February 1973 to July 1981, it can be seen that not only did the contributions of all the common factors to the total variance increase from 42,8% in the first period to 60,6% in the second period, but the contributions of the general market factor alone also showed a considerable increase from 33,4% to 56%. Thus even though the subperiod analysis for the last period showed a slight weakening of comovement, on the whole the gold shares seem to form a much more cohesive sector now than they did before the demonitisation of gold.

CHAPTER 5

THEORY OF DISCRIMINANT ANALYSIS

5.1 Introduction

Discriminant analysis is a statistical technique for

(i) the analysis and description of groups of populations relative to one another, and

(ii) the formulation of classification schemes to be used for the prediction of group membership of previously unclassified observations from a random sample.

The general appeal of discriminant analysis lies in its multivariate nature, in that both the inferential tests and classification functions are based on a vector of measurements for each observation.

5.2 Assumptions

The necessary assumptions required to hold for the successful application of a discriminant analysis are as follows:

(i) The different groups or populations have to be distinct and identifiable.

(ii) Each observation in each group must be characterized by a set of measurements on m variables.

(iii) These m variables are assumed to have a multivariate normal distribution in each population.

(iv) The m variables have equal dispersion matrices in the different groups or populations.

The first two assumptions form the basis of the general development of discriminant analysis, whereas the last two assumptions are required to hold for the development of the standard discriminant procedures. Deviations from the last two assumptions and remedial procedures will be discussed in later sections. Since deviation from any one of these assumptions affects the discriminant analysis procedures and results in one way or another, it is necessary to precede any discriminant analysis with hypothesis tests of the validity of these assumptions.

A test of the discriminatory power of the set of measurements involves testing whether there is significant evidence of differences in the group means based on the set of m characteristics, and is thus closely related to the first two assumptions. Since the test statistic derived for testing this hypothesis is also based on the assumptions of multivariate normality and equal group dispersions, tests for the latter two assumptions will be discussed first.

Although some tests have been derived for testing multivariate normality, they have very seldom been implemented in practice. Most empirical studies have made use of univariate tests. Univariate normality is a necessary, though not sufficient condition for multivariate normality to hold, and it is generally believed that a set of univariate normally distributed variables have a good probability of conforming to the assumption of multivariate normality.

To test the hypothesis of equal group dispersions, viz., $H_1 : \Delta_k = \Delta,\ k = 1,\dots,g$, Box (1949) developed the following test statistic:

$$M = (N-g)\ell n|D_w| - \textstyle\sum_{k=1}^{g}(N_k-1)\ell n|D_k|,$$

where

D_k = variance/covariance estimate for the kth sample,

and D_w = pooled-groups estimate based on W, the within groups dispersion matrix.

Defining

$$A_1 = \left(\sum_{k=1}^{g} \frac{1}{N_{k-1}} - \frac{1}{N-g}\right)\frac{2p^2+3p-1}{6(g-1)(p+1)}, \quad \text{and}$$

$$A_2 = \left(\sum_{k=1}^{g} \frac{1}{(N_k-1)^2} - \frac{1}{(N-g)^2}\right)\frac{(p-1)(p+2)}{6(g-1)},$$

then if $A_2 - A_1^2$ is positive,

$$n_1 = \frac{(g-1)p(p+1)}{2},$$

$$n_2 = \frac{n_1+2}{A_2-A_1^2},$$

$$b = \frac{n_1}{1-A_1-(n_1/n_2)},$$

$$F^{n_1}_{n_2} = \frac{M}{b}.$$

If $A_2 - A_1^2$ is negative,

$$n_1 = \frac{(g-1)p(p+1)}{2},$$

$$n_2 = \frac{n_1+2}{A_1^2-A_2},$$

$$b = \frac{n_2}{1-A_1+(2/n_2)} \quad , \quad \text{and}$$

$$F^{n_1}_{n_2} = \frac{n_2 M}{n_1(b-M)} \quad .$$

Note: g = the number of groups, and

p = the number of variables.

To test for the equality of group means, i.e.,

$$H_2 : \mu_k = \mu, \qquad k = 1,\ldots,g \; ;$$

Wilks (1932) derived the following statistic:

$$\Lambda = \frac{|W|}{|T|} \; ,$$

where

$$W = \text{within groups dispersion matrix}$$
$$= \textstyle\sum_{k=1}^{g}\sum_{i=1}^{N_k}(X_{ki}-\bar{X}_k)(X_{ki}-\bar{X})'$$

$$T = \text{total dispersion matrix}$$
$$= \textstyle\sum_{k=1}^{g}\sum_{i=1}^{N_k}(X_{ti}-\bar{X})(X_{ki}-\bar{X})'$$

Note: N_k = the number of observations in group k,

$\bar{X}_k$ = kth group mean

$\bar{X}$ = overall mean.

Transformations of this statistic are available that approximate χ^2 and F distributions. Rao's F approximation is generally considered to be superior and is based on the design parameters:

p = number of variables,

g = number of groups, and

N = total number of observations in all groups.

Using these parameters, the following functions are computed:

$$s = \sqrt{\frac{p^2(g-1)^2-4}{p^2+(g-1)^2-5}}$$

$$n_1 = p(g-1)$$

$$n_2 = s\left[(N-1) - \frac{p+(g-1)+1}{2}\right] - \frac{p(g-1)-2}{2}$$

from which

$$y = \Lambda^{1/s}$$

and

$$F^{n_1}_{n_2} = \left(\frac{1-y}{y}\right)\left(\frac{n_2}{n_1}\right) \quad .$$

The subsequent development of discriminant analysis, as well as the successful application of any discriminant analysis in practice, assumes that

$$H_1 : \Delta_k = \Delta, \qquad k = 1,\ldots,g \quad \text{is accepted,}$$

and

$$H_2 : \mu_k = \mu, \qquad k = 1,\ldots,g \quad \text{is rejected.}$$

5.3 Derivation of a Discriminant Function

As mentioned before, a discriminant function can be developed with two goals in mind. An attempt can be made to develop a function that will optimally characterize the degree of separation between the groups, or a classification function can be derived that will assign a random observation to one of the groups with the minimum probability of error. When the assumptions of normality and equal group dispersion matrices hold, it can be shown that striving to satisfy either of these two goals leads to the same discriminant function.

The following developments will all be for the two-group case. Extension to the multi-group case is straightforward and will be discussed briefly in a later section. The algebraic derivations presented here follow Lachenbruch (1975) closely.

The first major contribution to the development of discriminant analysis was made by Fisher (1936). With the purpose of deriving a function based upon the m variables which will optimally characterize the degree of separation between the groups, he proposed a linear combination of the observation vectors

$$y = \lambda' X \,, \tag{5.3.1}$$

where λ is such that the ratio of the between-groups variance of y to the within-groups variance of y is a maximum.

Defining

π_1 = population (or group) 1, and

π_2 = population (or group) 2,

the mean of y in π_1 is $\lambda'\underline{\mu}_1$ and in π_2 is $\lambda'\underline{\mu}_2$. If Σ denotes the common variance matrix in the two populations, then the variance of y in both populations is $\lambda'\Sigma\lambda$.

The optimum linear combination is then found by maximizing

$$\phi = \frac{(\lambda'\underline{\mu}_1 - \lambda'\underline{\mu}_2)^2}{\lambda'\Sigma\lambda} \tag{5.3.2}$$

with respect to λ.

Differentiation gives

$$\underline{\mu}_1 - \underline{\mu}_2 = \Sigma\lambda \frac{\lambda'\underline{\mu}_1 - \lambda'\underline{\mu}_2}{\lambda'\Sigma\lambda}$$

and since λ is only used to separate the groups, it may be multiplied by any derived constant. Thus λ is proportional to

$$\Sigma^{-1}(\underline{\mu}_1 - \underline{\mu}_2) \; , \tag{5.3.3}$$

or, in the case of unknown parameters,

$$S^{-1}(\bar{x}_1 - \bar{x}_2) \; . \tag{5.3.4}$$

The classification rule based on this function is then to assign an observation to π_1 if

$$y = (\bar{x}_1 - \bar{x}_2)'S^{-1}x \tag{5.3.5}$$

is closer to $\bar{y}_1 = (\bar{x}_1 - \bar{x}_2)'S^{-1}\bar{x}_1$ than to $\bar{y}_2$, and to π_2 otherwise.

There exist several variations on the general procedure of deriving a function that will minimize the probability of misclassification, all of which lead to a classification rule involving the ratio of the two population density functions. Welch (1939) suggested minimizing the total probability of misclassification. Let

$f_1(x)$ denote the density function of x in π_1, and

$f_2(x)$ denote the density function of x in π_2,

q_1 be the prior probability that an observation comes from π_1, and

q_2 be the prior probability that an observation comes from π_2.

Then, if an observation is viewed as a point in p-dimensional space which is divided into two mutually exclusive regions such that if the observation falls in R_1 it is classified as belonging to π_1, and if falls in R_2 it is classified as belonging to π_2, the probability of misclassification within each group is given by

$$P_1 = \int_{R_2} f_1(x)\ dx \qquad (5.3.6)$$

and

$$P_2 = \int_{R_1} f_2(x)\ dx\ . \qquad (5.3.7)$$

The total probability of misclassification is then

$$\begin{aligned} T(R,f) &= q_1\int_{R_2} f_1(x)\ dx + q_2\int_{R_1} f_2(x)\ dx \\ &= q_1[1-\int_{R_1} f_1(x)\ dx] + q_2\int_{R_1} f_2(x)\ dx \\ &= q_1 + \int_{R_1}[q_2 f_2(x) - q_1 f_1(x)]\ dx \qquad (5.3.8) \end{aligned}$$

Clearly, $T(R,f)$ is minimized if R_1 is chosen such that $q_2 f_2(x) - q_1 f_2(x) < 0$ for all points in R. This leads to the classification rule:

Assign x to π_1 if

$$\frac{f_1(x)}{f_2(x)} > \frac{q_2}{q_1} \qquad (5.3.9)$$

and to π_2 otherwise.

Sometimes, misclassifying observations from one group can have more serious consequences than misclassifying observations from another group. In such a case the costs of misclassification should also be taken into account. If

C_1 = the cost of misclassifying a member of π_1, and

C_2 = the cost of misclassifying a member of π_2, then the function to be minimized is

$$T = C_1q_1\int_{R_2} f_1(x)dx + C_2q_2\int_{R_1} f_2(x)dx$$

$$= C_1q_1 + \int_{R_1} [C_2q_2f_2(x) - C_1q_1f_1(x)]dx \qquad (5.3.10)$$

This leads to the classification rule:

Assign x to π_1 if

$$\frac{f_1(x)}{f_2(x)} > \frac{C_2q_2}{C_1q_1} \qquad (5.3.11)$$

and to π_2 otherwise.

Rule (5.3.9) could also have been derived by following a Bayesian approach. If the conditional probability of coming from population π_1, given an observation x, is

$$\frac{q_1f_1(x)}{q_1f_1(x)+q_2f_2(x)} ,$$

then, for a given observation x, the probability of misclassification is minimized by assigning x to that population that has the higher conditional probability. Thus if

$$\frac{q_1f_1(x)}{q_1f_1(x)+q_2f_2(x)} \geq \frac{q_2f_2(x)}{q_1f_1(x)+q_2f_2(x)}$$

assign x to π_1, otherwise to π_2.

Since the probability of misclassification is minimized at each point, it is minimized over the whole space. Clearly (5.3.9) and (5.3.12) are equivalent.

Another approach is to aim at minimizing the maximum probability of misclassification. This entails setting up regions R_1 and R_2 which minimizes

$$P_1 = \int_{R_2} f_1(x)dx = \int_{R_1} f_2(x)dx = P_2 .$$

Since $\int_{R_2} f_1(x)dx = 1 - \int_{R_1} f_1(x)dx = \int_{R_1} f_2(x)dx$,

$$_{R_1}[f_1(x)dx + f_2(x)dx] = 1 \quad .$$

Using La Grange multipliers, the problem is thus to minimize

$$\int_{R_1} f_2(x) - \alpha[f_1(x) + f_2(x)]dx + \alpha$$

$$\alpha \int_{R_1} [\beta f_2(x) - f_1(x)]dx + C \quad . \tag{5.3.13}$$

This is minimized if $\beta f_2(x) - f_1(x)$ is negative throughout R_1, which leads to the minimax rule:

Assign to π_1 if

$$\frac{f_1(x)}{f_2(x)} > \beta \tag{5.3.14}$$

and to π_2 otherwise.

The value of β depends on the distribution involved. For the normal case, with equal dispersion matrices, $\beta = 1$.

5.4 Classification into One of Two Multivariate Normal Populations

Assuming two multivariate normal populations with equal dispersions, i.e. $N_1(\underline{\mu}_1,\Sigma)$ and $N_2(\underline{\mu}_2,\Sigma)$, the ith density is given by

$$f_i(x) = \frac{1}{(2\pi)^{\frac{1}{2}P}|\Sigma|^{\frac{1}{2}}} \exp[-\tfrac{1}{2}(x-\underline{\mu}_i)'\Sigma^{-1}(x-\underline{\mu}_i)] \quad . \tag{5.4.1}$$

The ratio of the densities is then

$$\frac{f_1(x)}{f_2(x)} = \frac{\exp[-\frac{1}{2}(x-\underline{\mu}_1)'\Sigma^{-1}(x-\underline{\mu}_1)]}{\exp[-\frac{1}{2}(x-\underline{\mu}_2)'\Sigma^{-1}(x-\underline{\mu}_2)]}$$

$$= \exp[x'\Sigma^{-1}(\underline{\mu}_1-\underline{\mu}_2)-\tfrac{1}{2}(\underline{\mu}_1+\underline{\mu}_2)'\Sigma^{-1}(\underline{\mu}_1-\underline{\mu}_2)] . \tag{5.4.2}$$

Taking logarithms on both sides of (5.3.9) gives the classification rule as:

Assign x to π_1 if

$$D_T(x) = [x-\tfrac{1}{2}(\underline{\mu}_1+\underline{\mu}_2)]'\Sigma^{-1}(\underline{\mu}_1-\underline{\mu}_2) > \ell n \frac{q_2}{q_1} . \qquad (5.4.3)$$

$D_T(x)$ is called the true discriminant function.

If the population parameters are not known their sample estimates can be used. These are given by

$$\hat{\underline{\mu}}_1 = \bar{x}_1 = \textstyle\sum_{j=1}^{n_1} x_{1(j)}/n_1 ,$$

$$\hat{\underline{\mu}}_2 = \bar{x}_2 \quad \textstyle\sum_{j=1}^{n_2} x_{2(j)}/n_2 , \quad \text{and}$$

$$S = \left[\textstyle\sum_{j=1}^{n_1}(x_{1(j)}-\bar{x}_1)(x_{1(j)}-\bar{x}_1)'+\sum_{j=1}^{n_2}(x_{2(j)}-\bar{x}_2)(x_{2(j)}-\bar{x}_2)'\right]$$
$$\div (n \ +n_2-2) .$$

Substituting these estimates for the population parameters gives

$$Ds(x) = [x-\tfrac{1}{2}(\bar{x}_1+\bar{x}_2)]'S^{-1}(\bar{x}_1+\bar{x}_2) . \qquad (5.4.4)$$

Comparison of the coefficients of x from this function and Fisher's linear discriminant function shows them to be equivalent.

5.5 Regression Analogy

It can be shown that two-group discriminant analysis is proportional to the regression of a dichotomous variable on p explanatory variables. Define a dichotomous variable y_i as

$$y_i = \frac{n_2}{n_1+n_2} \quad \text{if} \quad x_i \quad \text{is a member of} \quad \pi_1, \quad \text{and}$$

$$y_i = -\frac{n_1}{n_1+n_2} \quad \text{if} \quad x_i \quad \text{is a member of} \quad \pi_2.$$

It follows that $\bar{y} = 0$.

The problem is to find parameters λ which best fit the model

$$E(y_i) = \lambda'(x_i-\bar{x}),$$

where

$$\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2} \quad .$$

Now

$$\begin{aligned}\sum_{i=1}^{n_1+n_2} y_i(x_i-\bar{x}) &= \frac{n_2}{n_1+n_2}\, n_1(\bar{x}_1-\bar{x}) - \frac{n_1}{n_1+n_2}\, n_2(\bar{x}_2-\bar{x}) \\ &= \frac{n_1 n_2}{n_1+n_2}(\bar{x}_1-\bar{x}_2)\end{aligned}$$

and

$$\begin{aligned}\sum_{i=1}^{n_1+n_2}(x_i-\bar{x})(x_i-\bar{x})' &= \sum_{i=1}^{n_1}(x_i-\bar{x})(x_i-\bar{x})' \\ &\quad + \sum_{i=n\ \ +1}^{n_2}(x_i-\bar{x})(x_i-\bar{x})'\end{aligned}$$

$$= \sum_{i=1}^{n_1}(x_i-\bar{x}_1)(x_i-\bar{x}_1)' + n_1(\bar{x}_1-\bar{x})(\bar{x}_1-\bar{x})'$$

$$+ \sum_{i=n_1+1}^{n_2}(x_i-\bar{x}_2)(x_i-\bar{x}_2)' + n_2(\bar{x}_2-\bar{x})(\bar{x}_2-\bar{x})'$$

$$= (n_1+n_2-2)S + \frac{n_1 n_2}{n_1+n_2}(\bar{x}_1-\bar{x}_2)(\bar{x}_1-\bar{x}_2)' \quad .$$

Thus the normal equations for the regression are

$$[(n_1+n_2-2)S + \frac{n_1 n_2}{n_1+n_2}(\bar{x}_1-\bar{x}_2)(\bar{x}_1-\bar{x}_2)']\lambda = \frac{n_1 . n_2}{n_1+n_2}(\bar{x}_1-\bar{x}_2).$$

Letting $A = (\bar{x}_1-\bar{x}_2)'\lambda$,

$$(n_1+n_2-2)S\lambda = \frac{n_1 . n_2}{n_1+n_2}(\bar{x}_1-\bar{x}_2)(1-A) \quad .$$

Thus λ is proportional to $S^{-1}(\bar{x}_1-\bar{x}_2)$, the discriminant function coefficients obtained in the previous section.

5.6 Multiple-Group Problems

The procedures described for two-group discriminant analysis can easily be adapted to handle multi-group problems. In the case of g groups, and assuming multivariate normality, the optimal assignment rule is as follows:

Assign to π_i if

$$q_i \frac{1}{(2\pi)^{k/2}|\Sigma_i|^{\frac{1}{2}}} \exp[-\tfrac{1}{2}(x-\underline{\mu}_i)'\Sigma_i^{-1}(x-\underline{\mu}_i)]$$

$$= \max_j q_j \frac{1}{(2\pi)^{k/2}|\Sigma_j|^{\frac{1}{2}}} \exp[-\tfrac{1}{2}(x-\underline{\mu}_j)'\Sigma_j^{-1}(x-\underline{\mu}_j)] \qquad (5.6.1)$$

or, when taking logarithms,

assign to π_i if

$$\ell n\, q_i - \tfrac{1}{2}\,\ell n|\Sigma_i| - \tfrac{1}{2}(x-\underline{\mu}_i)'\Sigma_i^{-1}(x-\underline{\mu}_i)$$

$$= \max_j\{\ell n\, q_j - \tfrac{1}{2}\,\ell n|\Sigma_j| - \tfrac{1}{2}(x-\underline{\mu}_j)'\Sigma_j^{-1}(x-\underline{\mu}_j)\} \qquad (5.6.2)$$

If $\Sigma_i = \Sigma$, for $i = 1,\ldots,q$, the rule becomes:

Assign to π_i if

$$\ell n\, q_i + (x-\underline{\mu}_i/2)'\Sigma^{-1}\underline{\mu}_i = \max_j[\ell n\, q_j + (x-\underline{\mu}_j/2)'\Sigma^{-1}\underline{\mu}_j] \quad (5.6.3)$$

Clearly the multiple-group case involves, among other things, more types of error and more complex sampling situations. In practice, questions about the robustness of the above rule to violations of the normality and equal covariances assumptions remain unanswered.

5.7 Evaluating a Discriminant Function

In the evaluation of a discriminant function there are three basic questions that have to be answered. The first one involves testing the between-group differences. This has already been dealt with in section (5.2) where Wilks' lambda was proposed as an appropriate statistic for testing the discriminatory power of the included variables. Another statistic can be derived based on the relationship between Fisher's linear discriminant function and Mahalanobis's D^2 statistic. It is easily shown that

$$\begin{aligned}\bar{y}_1-\bar{y}_2 &= (\bar{x}_1-\bar{x}_2)'S^{-1}\bar{x}_1-(\bar{x}_1-\bar{x}_2)'S^{-1}\bar{x}_2\\ &= (\bar{x}_1-\bar{x}_2)'S^{-1}(\bar{x}_1-\bar{x}_2)\\ &= D^2\end{aligned}$$

and

$$\begin{aligned}\mathrm{var}(y) &= \lambda'S\lambda\\ &= (\bar{x}_1-\bar{x}_2)'S^{-1}SS^{-1}(\bar{x}_1-\bar{x}_2)\\ &= D^2\end{aligned}$$

The statistic

$$\frac{n_1n_2(n_1+n_2-k-1)}{(n_1+n_2)(n_1+n_2-2)k}\,D^2 \qquad (5.7.1)$$

then has an F-distribution with k and n_1+n_2-k-1 degrees of freedom where k = the number of variables included in the model, and can be used to test for significant differences between the groups.

Alternatively, if the regression approach is used the Anova table can be constructed as

SOURCE	SUM OF SQUARES	D.O.F
Due to Regression	$\frac{n_1 n_2}{n_1+n_2} \lambda'(\bar{x}_1+\bar{x}_2)$	k
About Regression	$\frac{n_1 n_2}{n_1+n_2}[1-\lambda'(\bar{x}_1-\bar{x}_2)]$	n_1+n_2-k-1
Total	$\frac{n_1 n_2}{n_1+n_2}$	n_1+n_2-1

and the usual F-test of homogeneity may be applied.

The second question concerns the adequacy of the subset of variables for discrimination. This entails testing the relative discriminatory power of the individual variables. Since the discriminant function coefficients are not unique, the relative importance of individual variables cannot be determined by straightforward tests of the significance of the particular coefficients. Several other procedures have been proposed, some of which will be described below.

(i) Rank variables on the basis of their univariate F-statistics. This entails comparing the univariate Wilks' lambdas, which for the ith variable is

$$\Lambda_i = \frac{(W)_{ii}}{(T)_{ii}} , \tag{5.7.2}$$

where

$$\frac{(1-\Lambda_i)(N-k)}{\Lambda_i(k-1)} \sim F^{k-1}_{N-K} .$$

The lower the value of Λ_i, the greater is the discriminatory power of the variable.

(ii) Calculate scaled discriminant function coefficients by weighting the original coefficients with the square roots of the corresponding diagonal elements of the pooled within-groups deviation sums-of-squares matrix, W.

Both the above two methods are, however, univariate in nature and thus do not take into consideration the correlation among the variables. This can be a rather serious limitation since it is often found that although a variable may on its own be quite a poor discriminator between groups, it may prove quite valuable when combined with other variables.

(iii) Several stepwise procedures have been developed which do take into account the correlations among the variables. The stepwise forward and stepwise backward methods measure the relative contribution of a given variable to the multivariate F-statistic against an increasing or decreasing number of variables.

(iv) The conditional deletion method involves removing each variable in turn and calculating the residual Wilks' lambda with all the other variables included. The variables are then ranked according to the corresponding Wilks' lambda values, the best discriminator being the variable yielding the highest Wilks' lambda when deleted. The conditional deletion method is generally quite popular, mainly because it measures the relative discriminatory power of each variable conditional on all the other variables being included.

(v) Mosteller and Wallace (1963) proposed another measure for the significance of the individual variables, viz.,

$$b_j(\bar{x}_{j1}-\bar{x}_{j2})/\{\Sigma_i\ b_i(\bar{x}_{i1}-\bar{x}_{i2})\} \tag{5.7.3}$$

which represents the contribution of the jth variable to the Mahalanobis' distance between group means divided by the total Mahalanobis' distance. This method is often rejected because the discriminant function coefficients, b_j, (i) are signed, (ii) can be greater than one, and (iii) do not sum to one, all of which makes it difficult to interpret them. Furthermore, the method cannot be easily adapted to handle the multi-group case.

Apart from testing the significance of individual variables, it also makes sense to test the discriminatory power of a subset of variables. The conditional deletion method can be adapted so as to test the significance of a subset containing $p-i$ variables, $i = 1,\ldots,p-1$. The "best" subset of size $p-i$ will be that set which has the minimum Wilks' lambda. Rao (1970) also derived a statistic to test whether a subset of variables $x_1,\ldots,x_{k_1}$, say, are sufficient as discriminators. This is given by

$$F = \frac{n_1+n_2-k-1}{k-k_1} \cdot \frac{C(D_k^2-D_{k_1}^2)}{1+CD_{k_1}^2} \qquad (5.7.9)$$

where

D_k^2 and $D_{k_1}^2$ are the Mahalanobis' D^2 statistics on the full set and subset, respectively and

$$C = \frac{n_1 n_2}{(n_1+n_2)(n_1+n_2-2)}$$

This statistic has an F-distribution with $k-k_1$ and n_1+n_2-k-1 d.o.f.

However, since testing the significance of all possible subsets that can be generated by p observed variables will require 2^p-1 tests, all with different degrees of freedom, this procedure becomes infeasible and stepwise procedures of variable selection are preferred.

The third question to be answered with regard to the evaluation of a discriminant function, concerns its success in classifying observations into their correct groups. Recall that the initial aim in the derivation of a successful discriminant function was the minimization of the probability of misclassification, It thus seems logical that the evaluation of a discriminant function should involve the determination of this probability. Using this point of view as a basis, a number of error rates can be defined.

Following Lachenbruch (1975), let $T(R,f)$ define the error rates, with R referring to the classification regions and f to the presumed distribution of the observations that will be classified. Then

$$T(R,f) = q_1 \int_{R_2} f_1(x)dx + q_2 \int_{R_1} f_2(x)dx \qquad (5.7.5)$$

is the optimum error rate. When the parameters are known this equals the total probability of misclassification, i.e.,

$$T(R,f) = q_1 P_1 + q_2 P_2 \ .$$

When $f_i(x)$ is multivariate normal with mean μ_i and covariance Σ, P_1 and P_2 can be easily calculated. It can be shown that

$$D_T(x) \sim N(\tfrac{1}{2}\delta^2, \delta^2) \quad \text{in} \quad \pi_1, \quad \text{and}$$

$$D_T(x) \sim N(-\tfrac{1}{2}\delta^2, \delta^2) \quad \text{in} \quad \pi_2 .$$

where

$$\delta^2 = (\underline{\mu}_1 - \underline{\mu}_2)' \Sigma^{-1} (\underline{\mu}_1 - \underline{\mu}_2)$$

= Mahalanobis' distance for known parameters.

From this it follows that

$$P_1 = P\left[D_T(x) < \ln \frac{1-q_1}{q_1}\right]$$

$$= P\left[\frac{D_T(x) - \frac{1}{2}\delta^2}{\delta} < \frac{\ln \frac{1-q_1}{q_1} - \delta^2/2}{\delta}\right]$$

$$= \phi\left(\frac{\ln \frac{1-q_1}{q_1} - \delta^2/2}{\delta}\right) \qquad (5.7.6)$$

and, similarly,

$$P_2 = \phi\left(-\frac{\ln \frac{1-q_1}{q_1} + \delta^2/2}{\delta}\right) \quad . \qquad (5.7.7)$$

When $q_1 = q_2 = 0.5$, then $\ln \frac{1-q_1}{q_1} = 0$, and $P_1 = P_2 = \phi(-\delta/2)$.

The actual error rate involves the probability of misclassifying observations from future samples, and is given by

$$T(\hat{R}, f) = q_1 \int_{\hat{R}_2} f_1(x)dx + q_2 \int_{\hat{R}_1} f_2(x)dx \quad . \qquad (5.7.8)$$

This can be shown to equal

$$\tfrac{1}{2}\phi\left[\frac{-D_S(\underline{\mu}_1)}{\sqrt{V_D}}\right] + \tfrac{1}{2}\phi\left[\frac{D_S(\underline{\mu}_2)}{\sqrt{V_D}}\right] ,$$

where

$$D_S(\underline{\mu}_i) = [\underline{\mu}_i - \tfrac{1}{2}(\bar{x}_1 + \bar{x}_2)\ 'S^{-1}(\bar{x}_1 - \bar{x}_2)]$$

and $V_D = (\bar{x}_1 - \bar{x}_2)' S^{-1} \Sigma S^{-1} (\bar{x}_1 - \bar{x}_2)$.

When the parameters of the functions are not known, their sample estimates have to be used. This leads to the plug-in estimate of the error rate, viz.,

$$T(\hat{R},\hat{f}) = q_1 \int_{\hat{R}_2} \hat{f}\ (x)dx + q_2 \int_{\hat{R}} \hat{f}_2(x)dx \quad . \qquad (5.7.9)$$

It can be shown that

$$D_S(\hat{\underline{\mu}}_1) = \tfrac{1}{2}D^2 \ ,$$
$$D_S(\hat{\underline{\mu}}_2) = -\tfrac{1}{2}D^2 \ , \quad \text{and}$$
$$\hat{V}_D = D^2 \ ,$$

hence

$$T(\hat{R},\hat{f}) = \phi(-D/2) \ . \qquad (5.7.10)$$

The expectation of the plug-in error rate is given by

$$E(T(\hat{R},\hat{f})) = E(q_1 \int_{\hat{R}_2} \hat{f}_1(x)dx + q_2 \int_{\hat{R}_1} \hat{f}_2(x)dx)$$

and can be found by determining the expected value of $T(\hat{R},\hat{f})$ over all possible samples of size N_1 and N_2. It can be shown that

$$E(T(\hat{R},\hat{f})) < T(R,f) < T(\hat{R},f),$$

i.e., the actual error rate is greater than the optimum error rate, which in turn is greater than the expectation of the plug-in estimation of the error rate.

The calculation of the above error rates are, however, all based upon the assumption of normality. When this assumption does not hold, alternative procedures have to be used. These usually involve using samples to evaluate classification efficiency. Using the discriminant function to classify observations into one of two groups may result in two kinds of possible errors. These can be illustrated by an "accuracy-matrix" as follows:

	PREDICTED GROUP MEMBERSHIP		
ACTUAL GROUP MEMBERSHIP	π_1	π_2	
π_1	n_{11}	n_{12}	$n_{1.}$
π_2	n_{21}	n_{22}	$n_{2.}$
	n_1	n_2	$n_{..}$

where n_{12} represents the number of observations actually from π_1 classified as belonging to π_2, while n_{21} represents the number of observations actually from π_2 classified as belonging to π_1. If the null hypothesis is stated as

H_0 : The observation comes from π_1,

then the n_{12} observations represent Type I errors and the n_{21} observations represent Type II errors.

From the accuracy-matrix, three general measures of discriminatory efficiency can be calculated. They are:

(i) total efficiency, measured by $\dfrac{(n_{11}+n_{22})}{n_{..}}$,
   and which equals

$$1 - (n_{12}+n_{21})/n_{..}$$
$$= 1 - \text{apparent error rate};$$

(ii) the probability of correctly identifying an observation given its group membership, calculated as $n_{11}/n_{1.}$ and $n_{22}/n_{2.}$, respectively;

(iii) the probability that the actual membership corresponds to the assigned membership, given by $n_{11}/n_{.1}$ and $n_{22}/n_{.2}$, respectively.

The statistical significance of these measures can be tested by comparing them to results generated by chance models. Various such models have been developed of which the most appropriate one is considered to be the proportional chance

(i) the overall fraction correctly classified by chance will be

$$(n_1./n..)^2 + (n_2./n..)^2 ,$$

(ii) the chance probability of correctly classifying an observation given in its group membership are $n_1./n..$ and $n_2./n..$, respectively, and

(iii) the chance probabilities of correspondence between actual and assigned membership are given by $n_{11}/n._1$ and $n_{22}/n._2$, respectively.

The classifactory process can easily be recognised as a binomial experiment from which it follows that

$$X \sim B(n;\ p).$$

where

X = the number of correct classifications,

n = the number of trials (classifications), and

p = probability of correct classification generated by chance model,

or using the normal approximation of the binomial distribution,

$$X \mathrel{\dot\sim} N(np,np(1-p)) .$$

An important consideration when using these measures is the choice of the appropriate sample of observations to be used for testing the classificatory power of the discriminant function. The proportion of observations correctly classified by the discriminant function is in general determined by three factors, viz.,

(i) true differences between the groups,
(ii) sampling errors in the original sample, and
(iii) intensive search for the variables that work best for the sample.

Since these last two factors lead to an upward bias in the number of observations correctly classified in the original sample, this procedure is not usually recommended, and is in fact often criticised as providing no meaningful information. Thus the need for a "holdout" sample arises. The most common procedure is to randomly divide all observations available for analysis into two samples. Observations from one of the samples are then used to derive the discriminant function, which is then used to classify the observations from the second sample.

Joy and Tollefson (1975) critisized researchers for using the classifactory results arising from this procedure as a means of testing the predictive power of the discriminant function. They regard the split-sample procedure as merely a cross-validation, verifying the importance of the independent variables in the discriminant function. Altman and Eisenbeis (1978) suggested that all ex post classification actually does, is to provide *"an index of the overlaps among the variable distributions in the groups for the sample period."* They further argued that, just as the classificatory results cannot be extrapolated to a future time period, the importance of the variables will also only be valid for the sample period.

There is thus general consensus about the need for classifying a sample of observations from a future time period in order to determine the predictive ability of a particular discriminant function. It is further asserted that comparison of classification results from within the sample period with those outside the same period may serve as a crude test for stationarity. If stationarity holds, then the extrapolation of classification results to future time periods is of course valid.

Lachenbruch considered the split-sample approach as *"wasteful of data"* and often infeasible because large enough samples might not be readily available. He suggested the *"leaving-one-out"* method, often also referred to as the jackknife procedure, which involves the estimation of the discriminant function with one observation omitted and the classification of that observation using the estimated function. This is done for all observations and the number of misclassifications is counted. It can be shown that this method results in an almost unbiased estimate of the expected actual error rate, $E(T(\hat{R},f))$.

5.8 The Application of Discriminant Analysis and Associated Problems

In section (5.2) the necessary assumptions required to hold for the successful application of the standard discriminant analysis procedures were listed. However, in practice it is very often found that one or more of these assumptions are violated. Remedial procedures, robustness tests and common errors in application will be discussed in this section, which is based mainly on two papers by Joy and Tollefson (1975) and Eisenbeis (1977).

5.8.1 Sample Design

The first assumption in section (5.2) required the different groups or populations under investigation to be discrete and identifiable. This assumption is often violated when groups are formed based on the segmentation of a continuous variable. According to Eisenbeis, such segmentation of a continuous variable "*effectively discards information about the relationships between the independent or explanatory variables and the grouping criterion variable.*" He further argues that "*the only time it really makes sense to form groups based upon the distribution of a particular variable is if natural breaks or discontinuities appear.*" Otherwise regression is a more appropriate technique.

Eisenbeis emphasized the importance of using similar populations for obtaining the analysis and validation or prediction samples. He also warned against the use of arbitrarily defined groups such as bond classes, since these original assignments may already have been in error, thereby introducing an additional source of error into the assessment of any classification results.

With respect to sample sizes, Joy and Tollefson pointed out that there are actually no real reasons for any a priori proportions, and hence for equal sample proportions. They argued that the determinants of sample proportions should rather be the cost of sampling, minimum sample size considerations, (for example a sufficiently large number of observations should be obtained from the smaller group), and data handling facilities. However, consideration should be given to the fact that equal sample sizes reduces the effect of unequal covariances on the significance tests for group differences.

5.8.2 <u>The Choice of the Appropriate a priori Probabilities</u>

The standard discriminant analysis classification rules incorporate a priori probabilities denoting the relative occurrence of observations in different populations. Assuming equal prior probabilities when the groups are in fact not euqally likely, may cause the estimated error rates to be quite different from what might be expected in the population. When the population priors are not known, sample proportions are often used. This is acceptable if the data represents a random sample from the population.

The estimation of priors when using discriminant analysis in a time series context raise new problems. For example, when observations from a single time period are used to form a classification rule to be used on observations from a future time period, a problem in the estimation of priors arises because the relative expected occurrences of the groups in the population may vary from period to period. It would probably be reasonable in this case to use an average of relative frequencies over several time periods to estimate the priors. A second problem arises when data on the groups are obtained by pooling observations from different time periods.

5.8.3 The Distribution of the Variables

The development of discriminant analysis as described in previous sections is based on the assumption that the variables have multivariate normal distributions. If this is not the case then the tests of significance and the estimated error rates will be biased. Because of the scarcity of tests for multivariate normality and the virtual impossibility to derive appropriate alternative joint density functions, most researchers generally prefer to assume the results produced by standard discriminant procedures as being reasonable approximations.

An alternative procedure that has been suggested if the specific case of nonnormality, caused by the inclusion of discrete or dichotomous variables in addition to continuous variables, hold, is to split the samples using the discrete or dichotomous variables and then to employ standard discriminant procedures on the subdivided samples. Discrete discriminant analysis procedures and nonparametric classification rules have also been developed. These will, however, not be discussed here and the reader is referred to Lachenbruch (1975) and Goldstein and Dillon (1978).

Investigations into the robustness of standard linear procedures when normality does not hold, have indicated that the standard procedures are quite sensitive to the violation of the normality assumption. Bounding the distributions usually helps to decrease the sensitivity. Estimated overall classification errors are usually not as much affected as individual group error rates. In many cases quadratic rules have been shown to perform even worse than linear rules in the case of nonnormality.

Transformation of the variables, by using logarithms for example, prior to estimating a discriminant function in an attempt to improve normality has been recommended. Disadvantages of this procedure, however, are that transformations may affect the inter-relationships among the variables, as well as the relative positions of the observations in the groups. Furthermore, since negative values cannot be transformed by taking logarithms, this leads to the exclusion of some cases.

5.8.4 Equal Versus Unequal Dispersions

Classical linear discriminant analysis assumes that the group dispersion matrices are equal across all groups. If this does not hold, the significance tests for the differences in group means, the usefulness of "reduced-space transformations" and the appropriate form of the classification rules are affected.

The effect on the tests of equality of group means seems to be related to the number of variables and relative sample sizes in the groups. Large differences in sample sizes cause the actual significance level to be greater than the hypothesized level which leads to the null hypothesis being wrongly rejected an increased number of times. An increase in the number of variables, increases the significance level and thus at the same time the sensitivity to unequal sample sizes.

Reduced-space discriminant analysis reduces the original m-dimensional variable test space to an r-dimensional problem, where r equals the minimum of m and one minus the number of groups. The linear transformation by which the reduction is achieved preserves relative linear Euclidean distances among observations and leaves the significance and classification results unaffected - but ONLY IF THE GROUP DISPERSION MATRICES ARE EQUAL.

Unequal dispersion matrices implies that a quadratic classification rule should be used, viz., Assign to π_1 if

$$Q(x) = \ell n \frac{f_1(x)}{f_2(x)} > \ell n \frac{1-q_1}{q_1}$$

$$= \tfrac{1}{2} \ell n \frac{|\Sigma_2|}{|\Sigma_1|} - \tfrac{1}{2}(x-\underline{\mu}_1)'\Sigma_1^{-1}(x-\underline{\mu}_1)+\tfrac{1}{2}(x-\underline{\mu}_2)'\Sigma_2^{-1}(x-\underline{\mu}_2)$$

$$= C_0 - \tfrac{1}{2}[x'(\Sigma_1^{-1}-\Sigma_2^{-1})x-2x'(\Sigma_1^{-1}\underline{\mu}_1-\Sigma_2^{-1}\underline{\mu}_2)] \ .$$

Gilbert (1969) compared the classificatory power of the linear function, $L(x)$, and the quadratic function, $Q(x)$ in the presence of unequal dispersion matrices. It was assumed that the parameters were known and attention was restricted to the two-group case. The results indicated that differences in classification results generated by the two rules are related to (i) the differences in dispersions, (ii) the number of variables, and (iii) the separation among groups. It was found that the quadratic form can utilize discrepancies in variance to decrease the probability of misclassification. This proved to be especially the case when the separation among the groups are small and the number of variables is large. Gilbert concluded that the classification results generated by the two rules could be considered as satisfactorily equivalent only for moderate differences in dispersions and adequate separation among the groups. Furthermore, agreement between the two rules worsened as the number of variables increased.

CHAPTER 6

INVESTIGATIONS INTO THE USEFULNESS OF FINANCIAL RATIO ANALYSIS

6.1 Introduction

Business enterprises compile various financial statements, like balance sheets, income statements and application of funds statements, to keep record of their financial position at all times. The analysis of these financial statements entails an evaluation of the current and past operations and financial profile of an enterprise, with the hope of formulating good estimates of future performance. A very popular tool of financial analysis is ratio analysis. This entails the calculation and comparison of various mathematical relationships between one quantity and another to describe certain characteristics of a business enterprise, like for example liquidity, leverage, profitability, turnover and so forth.

It is also clear that, apart from being highly dependent on internal management for successful performance, the financial position of all business enterprises is in addition influenced by the general state of the economy. Various statistics like the number of building plans passed, the number of residential houses completed, interest rates and money supply, to name but a few, are generally used as indicators of the current economic position.

Many studies have been conducted in the past to determine the usefulness of ratio analysis. Most of these involved attempts to formulate models, based on financial ratios and economic indicators, for the prediction of bankruptcy or failure of different kinds of business enterprises. The next section will attempt to give a general overview of some of the more important studies.

6.2 Previous Research in the USA and UK

Beaver (1966) is generally recognized as having done the pioneering work in the determination of the usefulness of financial ratios for the prediction of firm failure. He computed thirty ratios from financial statements for each of the five years prior to failure for a sample containing seventy-nine failed and seventy-nine non-failed firms for the period 1954 to 1964. The ratios were selected on the basis of (i) their popularity, (ii) their performance in previous studies, and (iii) their definability in terms of a "cash-flow" concept, and represented six "common element" groups, viz.,

(i) Cash-flow ratios,
(ii) Net-income ratios,
(iii) Debt to total-asset ratios,
(iv) Liquid-asset to total-asset ratios,
(v) Liquid-asset to current debt ratios, and
(vi) Turnover ratios.

Beaver used profile analysis to describe the general relationships between failed and nonfailed firms and thereby confirmed the existence of a difference in the ratios of failed and nonfailed firms. He also showed that this difference increased as the year of failure approached. Using a dichotomous classification procedure, based on a ranking of the observations for each ratio, it was found that not all variables had the same predictive ability and that the ratios did not predict failed and nonfailed firms with the same degree of success.

The last technique employed by Beaver was essentially a Bayesian approach in which the usefulness of the ratios was evaluated in terms of the degree to which likelihood ratios changed the prior probabilities of failure or nonfailure when converting them into posterior probabilities conditional upon the value of the ratios. The results showed that the ratios conveyed useful information in determining solvency for at least five years prior to failure.

Beaver's analysis was univariate in nature and thus ignored the fact that, due to relations among the variables, a combination of ratios could prove to be superior to individual ratios in predicting failure. Beaver acknowledged this but argued that at the time of his research, no developed multi-ratio models had been proved to be consistently superior to the best single ratio. He furthermore pointed out the serious imperfections that could arise from applying these multi-ratio models assuming multivariate normality, when in fact the assumption does not hold.

Subsequently, however, many very successful models were developed and were proved to be superior to single ratios in the prediction of failure. Most of these models were derived using multiple discriminant analysis, which was considered an appropriate technique because of its ability to simulteneouslt take into account various characteristics, as described by the financial ratios, of firms, as well as the interactions of these properties.

Altman (1968) employed multiple discriminant analysis to investigate the usefulness of financial and economic ratios in the prediction of corporate bankruptcy for the period 1946 to 1965. Together with Haldeman and Narayanan, Altman (1977) revised this model, taking into account new developments with respect to business failures, accounting practices and the practical application of discriminant analysis. Edmister (1972) used a multiple discriminant analysis approach to determine the usefulness of financial ratio analysis for predicting small business failure, where as Pinches and Mingo (1973) tested the usefulness of thirty-five different financial and economic variables for the prediction of industrial bond ratings. In 1982 Taffler (1982) incorporated all the most recent refinements in the application of multiple discriminant analysis to derive a model for identifying British companies at risk of failure.

Most of the reserachers followed Beaver in using a paired-sample design for the selection of their non-failed or non-bankrupt firms to ensure that their two samples came from the same population. The non-failed firms were usually matched to firms from the failed group by industry, asset size and financial year. In doing this, however, they ran the risk of selecting a sample of non-failed firms that might not be representative of the general population of non-failed firms. In view of this possible defect, Taffler abondoned the paired-sample approach.

The variables employed in most of the studies were financial ratios calculated from profit and loss accounts and balance sheets of the firms for one to, in some cases, five years prior to failure or bankruptcy. These ratios were usually selected on the basis of their (i) popularity in the literature, (ii) proven usefulness or relevance in previous studies, and (iii) availability, and measured various characteristics of the business enterprises, of which the most common were

(i) profitability,
(ii) leverage,
(iii) liquidity,
(iv) capitalization,
(v) earnings variability, and
(vi) size.

In addition, economic variables were sometimes included, for example in the studies done by Altman and Pinches and Mingo. Edmister also examined the usefulness of industry-relative ratios, trend measures and trend-level combinations. Transformation and winsorizing procedures were employed in most of the studies in an attempt to improve normality and lessen the negative effects arising from nonnormality.

Right from the very early studies an attempt was made to avoid including variables in the final classification function that measured the same characteristics of the firms and were as such collinear. Beaver, for example, divided the ratios into six "common element" groups and selected only one ratio from each group for inclusion on the basis of the lowest percentage error for their group over the five-year period. There is in fact much dispute over the possible adverse effects of multicollinearity on discriminant analysis. Some researchers have claimed that these are the same as that for regression, in that it affects the stability of the parameter estimates and hence the success of forecasting. For this reason the discriminant analysis procedure is often preceded by a factor analysis of the data, as was the case for the study carried out by Pinches and Mingo. Alternatively, a stepwise variable selection procedure can be used.

Altman and Eisenbeis (1978), however, argued that multicollinearity only becomes a concern when it reaches the degree where dispersion matrices become singular. They were subsequently critized by Taffler for ignoring the probability of sample bias being introduced by the presence of multicollinearity and hence Taffler also factor analysed his data, not only to limit multicollinearity, but also to identify the underlying dimensionality of the data and to aid the interpretation of the derived models. Apart from reducing multicollinearity, these techniques also reduces the dimension of the problem considerably, thereby simplifying any potential classification functions. Care should, however, be taken that dimension reduction is not carried out at the expense of classification efficiency, as discussed by Eisenbeis (1977).

All the studies mentioned above employed linear discriminant analysis procedures. While this is strictly correct, only when the assumption of equal group dispersions hold, most of the studies made no mention of this assumption. Taffler did test for the validity of this assumption, but although it turned out that it was not quite satisfied, he decided in the light of (i) departure from the assumption of multivariate normality, and (ii) small data samples relative to the number of variables, that the use of linear procedures would nonetheless be more correct than employing quadratic formulations. Altman et al actually derived both a quadratic and linear classification model and compared their accuracy in several ways. The results showed the linear function to perform at least as well or better than the quadratic function in its classificatory ability.

In addition, the possibility that the quadratic parameters could be highly sensitive to individual sample observations, gave further support for the general preference of the linear rule.

In most studies Rao's (1970) F-transformation of the Wilks' lambda statistic was used to determine the overall discriminatory power of the models. While the earlier studies mostly employed ranking procedures univariate based on F-statistics and scaled discriminant coefficients to determine the relative importance of the individual variables included in the discriminant function, the limitations inherent in these methods due to their univariate nature were soon recognised. Since no single measure for the relative importance of the variables, however, exists, subsequent studies compared the rankings based on various different procedures, some univariate and others multivariate in nature. Altman et al compared the rankings of the variables included in their zeta-model using the five tests described in section (5.7) and observed consistent results for all the procedures. In contrast to this however, Altman and Eisenbeis (1978) had to conclude that for the variables included in the Altman-1968-study, the rankings were very sensitive to the criterion employed.

To test the classificatory power of the derived models, most studies constructed accuracy matrices resulting from classifying observations from various samples. All of these studies recognised the potential bias involved in reclassifying observations from the original sample, and hence tested the models on different holdout samples. Many, however, failed to test the predictive ability of the models by classifying observations from a future period, and erroneously reported the results generated by using data from two to five years prior to bankruptcy or failure as appropriate measures of the predictive accuracy.

The only two studies of those mentioned above that did employ future samples were those done by Pinches and Mingo and by Taffler. In general the overall efficiency of the models on the holdout samples were quite high, ranging from 64.58% for the Pinches and Mingo study to about 90% for the Altman studies. The percentage of correct classifications for the future sample in the Pinches and Mingo study was somewhat lower, viz. 56%, while the Type I and Type II errors associated with the classification of observations from Taffler's future sample were 12.1% and 0% respectively.

Apart from determining a cutoff point that would lead to optimal overall efficiency, Edmister also provided alternative z-scores which improved the predictive accuracy of one condition at the expense of the other. From these a z-score could be selected *"so as to equate the probability of Type I and Type II errors with the ratio of the explicit cost of accepting a failure to the opportunity cost of rejecting a success."* Alternatively, he suggested the use of the *"black-gray-white"* method which essentially divide the z-scores into three intervals, one corresponding to scores depicting definite loss loans, another corresponding to scores depicting definite good loans, and a *"gray area"* in the centre containing the borderline cases. A similar approach was adopted by Altman (1968).

Most studies seem to have avoided the inclusion of unequal prior probabilities and costs of misclassification, probably due to the difficulty of obtaining good estimators of these parameters. Altman et al, however, explicitly admitted the potential bias involved in assuming equal prior probabilities and equal costs of errors. Although they had no precise estimates of bankruptcy priors, they gave a good example of the adjustments involved in the incorporation of unequal prior probabilities and costs of misclassification. They proved their zeta model to be considerably more efficient than any chance models for various reasonable combinations of prior probabilities and costs of misclassification.

6.3 Research done in South Africa

Some of the studies described in the previous section have been repeated using data in respect of South African companies. Daya (1977) conducted a similar study to that of Beaver, while Amiras et al (1978) attempted to develop an Altman-type model for South African businesses. Le Roux (1980) used discriminant analysis to determine the usefulness of financial ratios for predicting failure in respect of industrial companies listed on the Johannesburg Stock Exchange. Since his study was essentially done on a data set very similar to that used for the present study, it will be discussed in some detail and will later be used for comparison purposes.

Le Roux selected fifty-four industrial companies that had failed between 1975 and 1979. A matching sample of fifty-four non-failed companies, in the same line of business as the failed companies and of a similar asset size, were selected. Financial variables were chosen on the basis of their acceptable performance in previous studies and the availability of information.

With the purpose in mind of dealing with the controvercy surrounding the effect of multicollinearity on discriminant analysis, two sets of discriminant analysis were performed. The first discriminant analysis was performed without specifically providing for collinearity. The second discriminant analysis was preceded by a factor analysis of the data. The variables with the highest correlations with the various factors were then selected for inclusion in the second discriminant run.

Discriminant functions were calculated for each of the five years before failure. The first set of discriminant analysis resulted in some functions having high degrees of collinearity among the variables. The low degrees of collinearity in the second set of functions served as evidence of the success of factor analysis in the reduction of multicollinearity. Those functions generated by the first set of discriminant runs which did not carry a high degree of collinearity, together with the discriminant functions from the post-factor analysis discriminant functions, were considered to find the best predictor.

The five best functions and their accuracy in classifying observations from a holdout sample in the first year of failure were as follows:

(i) 0.20731 (return on assets) - 0.02670 (total debt + share capital) + 0.39500
(84%)

(ii) -0.07558 (interest bearing debt plus preference share capital) + 1.69452
(67%)

(iii) 0.23649 (return on assets) - 1.22412
(87%)

(iv) 0.07075 (interest bearing debt ratio) + 1.53854
(66%)

(v) -0.05927 (current ratio) + 0.30506 (debtors ratio) - 0.04543 (interest bearing debt) + 0.01923 (roe) + 0.01479 (cash flow/current liabilities) + 0.12589
(75%)

The accuracy levels dropped significantly in the fourth and fifth years before failure indicating that at these early stages the financial characteristics of potential failures were not much different from that of their successful counterparts.

Le Roux did not mention whether the necessary assumptions concerning multivariate normality and equal group dispersions held or not. He did not present any measures of the relative importance of the individual variables. He also failed to incorporate a test sample from a future time period and hence provided no measure of the predictive accuracy of the model.

6.4 Aim of Present Study

Sparked on by the proven usefulness of financial ratio analysis in the formulation of discriminatory models for various kinds of business enterprises, as discussed in section (6.2), and the relatively little research done in this context in South Africa, it was decided to once more employ discriminant analysis procedures to determine the usefulness of financial and economic variables in a classificatory problem with respect of South African business enterprises. However, the objective was not to derive a model for bankruptcy prediction, as was the case for most of the studies discussed in the previous sections, but rather to derive a model which would group industrial shares listed on the Johannesburg Stock Exchange into two groups, one containing bad performers, the other good performers, on the basis of their financial characteristics, where performance would be measured in terms of yearly returns.

Thus in contrast to the probability of ruin models which only served to prevent investors from incurring large losses by investing in shares of firms which are likely to go bankrupt, this study followed a more positive approach in that the identification of both good and bad performers will provide investors with guidelines with respect to both opportunities for large gains, as well as warnings against possible losses. Furthermore, the primary aim was not the development of prediction models, since any chance of deriving models that will prove to be successful in a predictory sense was ruled out by assuming the Johannesburg Stock Exchange to be an efficient market and by taking cogniscance of the conclusions in the Ball and Brown study mentioned in the introduction. The emphasis was thus on the classificatory power of the functions and the degree of correspondence between the classificatory results of the derived functions and the actual performance on the stock market will serve as a measure of the validity of the semi-strong form of the Efficient Market Hypothesis with respect to shares listed on the Johannesburg Stock Exchange, as well as of the usefulness of engaging in extensive fundamental analysis.

Three different samples were chosen for analysis. The first sample contained more or less equal proportions of good and bad performers for each of the seven years from 1973 to 1979. In contrast, the other two samples were each restricted to one specific year. One comprised of firms that had returns substantially lower or higher than the average return of industrial shares in 1973, which represented a bear market, while the other sample contained good and bad performers of 1979, which was a bullish year for industrial shares on the Johannesburg Stock Exchange. It was hoped that using these three samples models would be derived that could be used to classify industrial shares on the J.S.E. during any time period, and also more specifically during bull and bear markets. The analysis of these three samples will be discussed separately in the next chapters.

CHAPTER 7

A DISCRIMINANT ANALYSIS OF SHARES LISTED ON THE JSE FOR THE PERIOD 1973 TO 1979

7.1 Sample Design

By selecting more or less equal proportions of "good" and "bad" performers for each of the years 1973 to 1979, where performance was measured in terms of yearly returns relative to the return on the Composite Index for industrial shares, a sample containing one hundred and thirty-five industrial firms was formed, the composition of which is shown in Table 7.1. The specific firms included in the analysis together with their yearly returns are given in Appendix G. This initial sample was then randomly divided into

(i) an analysis sample containing eighty-seven shares, viz., forty-six good performers and forty-one bad performers; and

(ii) a holdout sample containing forty-eight shares, viz., twenty-three good performers and twenty-five bad performers.

Although the criterions for determining good and bad performers for each year were quite uniform when viewed relative to the return on the composite industrial index, they seemed to differ substantially when viewed on their own, as can be seen from Table 7.1. For example, whereas firms were required to have shown a return in excess of 100% to be classified as "good" performers in 1979, in 1973 they needed only to have had positive returns. Thus, although relative performance intuitively seemed to be the correct criterion for classification purposes, it was nonetheless feared that such considerable variation as described above could perhaps lead to erroneous initial classifications. Hence it was decided to

TABLE 7.1

COMPOSITION OF 1973-1979 SAMPLE

YEAR	GOOD PERFORMERS ‡ > CUTOFF POINT	BAD PERFORMERS ‡ < CUTOFF POINT	COMPOSITE INDUSTRIAL INDEX RETURN
1973	10 > 20.00	10 < -45.00	-20.48
1974	9 > 0.00	15 < -60.00	-15.82
1975	13 > 40.00	9 < -20.00	10.54
1976	9 > 10.00	7 < -50.00	-13.17
1977	8 > 40.00	8 < -50.00	18.75
1978	10 > 45.00	9 < -10.00	26.90
1979	10 > 100.00	8 < 0.00	59.25
TOTAL	69	66	

impose an additional constraint that in all years "good" performers were required to have had a return in excess of 30%, while "bad" performers were those shares with returns less than -30%. Clearly this lead to the exclusion of some observations in some of the years and a smaller sample was thus formed containing ninety-nine shares - forty-eight good performers and fifty-one bad performers. This smaller sample was again randomly divided into

(i) an analysis sample containing sixty shares, viz., twenty-nine good performers and thirty-one bad performers, and

(ii) a holdout sample containing thirty-nine shares, viz., nineteen good performers and twenty bad performers.

Both the larger sample of one hundred and thirty-five shares and the smaller sample containing ninety-nine shares were analyzed and the results compared.

7.2 Selection of Variables

Twenty-two financial ratios were selected on the basis of

(i) their inclusion in Le Roux's (1980) study, and

(ii) the availability of data.

The data was obtained from the University of Stellenbosch GSB-Ratio Analyses of Selected Companies (1981) and included liquidity, capital structure, leverage, return on investment, and cash flow ratios. By dividing the ratios by the sector averages, twenty-two relative ratios were calculated and included in the analysis. In addition a second set of discriminant analyses was performed with seven economic indicators added to the set of financial ratios. Thus a set of fifty-one variables, as listed in Table 7.2, was formed to describe the characteristics of each observation and on the basis of which classification functions could be derived.

Logarithmic transformations were used to improve normality. This lead to the exclusion of a few cases due to negative values for a few ratios. The variables that were transformed by taking logarithms were as follows:

X_1 = log(current ratio + 1),
X_2 = log(quick ratio + 1),
X_{11} = log(interest cover + 1),
X_{17} = log(ebit on selected liabilities +1),
X_{18} = log(return on book capital +1),
X_{23} = log(current ratio/sector average +1),
X_{24} = log(quick ratio/sector average + 1),
X_{33} = log(interest cover/sector average +1),
X_{43} = log(cash flow to debt/sector average +1),
X_{44} = log(cash flow to current liab./sector average +1).

Although these transformations did improve the normality of the specific variables, distributions of some other variables, in particular the average tax rate ratio and the corresponding relative ratio, still had quite serious violations of the normality assumption. Since Eisenbeis (1977) reported that bounding the distributions helped to decrease the sensitivity of the standard discriminant procedures to violation of the normality assumption, a winsorizing procedure was employed which replaced the upper and lower 5% tails of the distributions with the corresponding cutoff points.

Another problem arose with respect to the distributions of the economic indicators. There were only six distinct values

TABLE 7.2

FINANCIAL AND ECONOMIC VARIABLES USED IN THE ANALYSIS

VARIABLE NO.	VARIABLE
X_1	Current ratio
X_2	Quick ratio
X_3	Debtors ratio
X_4	Stock ratio
X_5	Asset composition
X_6	Total debt ratio
X_7	Long term debt ratio
X_8	Long term + short term debt ratio
X_9	Interest bearing debt
X_{10}	Interest bearing debt + prefs
X_{11}	Interest cover
X_{12}	Fixed cost cover
X_{13}	Return on assets
X_{14}	Return + deftax on assets
X_{15}	Return on equity
X_{16}	Ebit on total assets
X_{17}	Ebit on selected liabilities
X_{18}	Return on book capital
X_{19}	Average tax rate
X_{20}	Cash flow to assets
X_{21}	Cash flow to debt
X_{22}	Cash flow to current liabilities
For $I = 23,44$, $X_I = X_{I-22}$/sector average	
X_{45}	Construction building plans passed
X_{46}	Residential building plans passed
X_{47}	Total buildings completed
X_{48}	Residential buildings completed
X_{49}	Money supply
X_{50}	Interest Rates
X_{51}	Index of coinciding indicators

for each of these variables corresponding to the six different years and they were thus as such discrete variables. This meant that a split-sample approach or discrete discriminant analysis procedures should actually have been used. However, it was decided to employ standard procedures, just keeping in mind the possible adverse effects of biased tests of significance and biased estimates of error rates.

The BMDP07M (1981) computer package was used in the analysis. This is a stepwise discriminant analysis procedure in which variables are selected for inclusion depending on their contribution to the separation between groups as measured by the F-statistic:

$$F = \frac{n-q-p}{q-1} \cdot \frac{1-\Lambda(u.x)}{\Lambda(u.x)}$$

where

$\Lambda(u.x) = \dfrac{\Lambda((x,y))}{\Lambda(x)}$, i.e. the multiplicative increment in $\Lambda(x)$ resulting from adding a variable u to the set $x = (x_1,\ldots,x_p)$, and

$$\Lambda(x) = \frac{|W(x)|}{|T(x)|} .$$

The entry and removal of variables is further guided by F-to-enter and F-to-remove threshold values which can be varied depending on the degree of multicollinearity considered acceptable in the function. Due to this control on the multicollinearity on the data set, it was not considered necessary to precede the discriminant analysis with a factor analysis of the data.

After the variables for inclusion have been selected, a linear discriminant function of the form

$$d = a + b_1x_1 + \ldots + b_px_p$$

is derived, where d is such that the ratio of the total to the within group sums of squares $\frac{T(d)}{W(d)}$ is a maximum. By varying the combinations of samples, variable sets and F-threshold values, eight different discriminant functions were derived. The eight parameter combinations were as follows:

(i) Larger sample, financial ratios, F-to-enter = 4.00, F-to-remove = 3.996;

(ii) Larger sample, financial ratios, F-to-enter = 2.00, F-to-remove = 1.996;

(iii) Smaller sample, financial ratios, F-to.enter = 4.00, F-to-remove = 3.996;

(iv) Smaller sample, financial ratios, F-to-enter = 2.00, F-to-remove = 1.996;

and four parameter combinations similar to the above four, the only difference being that in addition to financial ratios, seven economic indicators were also included. These functions will be discussed in the following sections.

7.3 Discriminant Analysis on Larger Sample Using Financial Ratios and with F-to-Enter Threshold = 4.00, F-to-Remove Threshold = 3.996

With the above stated threshold values only one variable, namely X_{15} = return on equity, was selected for inclusion in the discriminant function. The final discriminant function was

$$d_1 = -1.63509 + 0.0992\ X_{15}\ .$$

The overall discriminatory power of the model was measured by comparing the F-transformation of the Wilks' lambda statistic with the corresponding tabulated value at a 5% significance level. For d_1 this turned out to be

$$\hat{F} = 26.564 > F^{0.05}_{1,85} \doteqdot 3.96\ ,$$

thus demonstrating a very significant difference between the groups on the basis of the return on equity ratio.

The classificatory power of the function was determined by using the function to classify observations from

(i) the original sample,
(ii) a Lachenbruch jackknife sample, and
(iii) a randomly selected holdout sample.

Because of the bias involved in using the derived model to classify observations from the original sample and the relative uselessness of the associated error rates as discussed in Chapter 5, section 5.7, these results will not be reported here. Instead, results for the Jackknife and holdout samples, which were both designed so as to eliminate the causes of potential bias in the original sample, will be given. The classificatory efficiency of the derived functions were tested against the performance of appropriate chance models, as discussed in Chapter 5, section 5.7. The calculated z values will be given in brackets after each classificatory measure. These can be compared against tabulated values of, for example, 1.96 at a 5% significance level and 2.58 at a 1% significance level.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	31	15	46
	BAD	8	33	41
	TOTAL	39	48	87

Thus

(1) overall efficiency $= \frac{31+33}{87} = 73.56\%$ (4.3649),

(2) proportion of original good performers correctly classified $= \frac{31}{46} = 67.39\%$ (1.973) ,

(3) proportion of original bad performers correctly classified $= \frac{33}{41} = 80.49\%$ (4.2789)

(ii) The holdout sample

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	11	12	23
	BAD	4	21	25
	TOTAL	15	33	48

Thus

(1) overall efficiency $= \frac{11+21}{48} = 66.67\%$ (2.2969),

(2) proportion of original good performers correctly classified $= \frac{11}{23} = 47.83\%$ (-0.0090),

(3) proportion of original bad performers correctly classified $= \frac{21}{25} = 84.00\%$ (3.1948) .

7.4 Discriminant Analysis on Larger Sample Using Financial Ratios and with F-to-Enter Threshold = 2.00, F-to.Remove = 1.996

With the above stated threshold values the following three variables were selected for inclusion in the discriminant function:

X_2 = quick ratio,

X_{12} = fixed cost cover ratio,

X_{15} = return on equity.

The relative importance of the individual variables were determined by ranking them according to three criteria, viz., (i) their univariate F-statistics, (ii) the associated scaled discriminant coefficients, and (iii) their relative contributions to the multivariate F-statistic as measured by the stepwise procedure. The rankings for the above three variables were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_2	3	2	2
X_{12}	2	3	3
X_{15}	1	1	1

Thus from all three rankings, the return on equity ratio appeared to be the best single discriminator. It should be noted that the stepwise procedure is the only ranking procedure employed that takes into account the relations among the variables. In the presence of any inconsistencies, more weight will thus always be given to the ranking of the variables in terms of this criterion.

The final discriminant function was

$$d_2 = -0.88422 - 5.15037X_2 + 0.98684X_{12} + 0.09568X_{15} .$$

The overall discriminatory power of the model was given as

$$\hat{F} = 11.538 > F^{0.05}_{3,83} \doteq 2.72 ,$$

demonstrating a significant difference between the groups on the basis of the three included variables.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	31	15	46
	BAD	11	30	41
	TOTAL	42	45	87

Thus

(1) overall efficiency $= \frac{31+30}{87} = 70.11\%$ (3.7207),

(2) proportion of original good performers correctly classified $= \frac{31}{46} = 67.39\%$ (1.9730),

(3) proportion of original bad performers correctly classified $= \frac{30}{41} = 73.17\%$ (3.3403)

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	11	12	23
	BAD	6	19	25
	TOTAL	17	31	48

Thus

(1) overall efficiency $= \frac{11+19}{48} = 62.50\%$ (1.7196),

(2) proportion of original good performers correctly classified $= \frac{11}{23} = 47.83\%$ (-0.0090),

(3) proportion of original bad performers correctly classified $= \frac{19}{25} = 76.00\%$ (2.3941) .

7.5 Discriminant Analysis on Smaller Sample Using Financial Ratios and with F-to-Enter Threshold = 4.00, F-to-Remove Threshold = 3.996

With the above stated threshold values the following three variables were selected for inclusion in the discriminant function:

X_1 = current ratio,

X_3 = debtors ratio,

X_{15} = return on equity ratio.

The relative importance of the individual variables as given by their rankings were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_1	2	2	2
X_3	3	3	3
X_{15}	1	1	1

Once again the return on equity ratio were shown to be the best single discriminator.

The final discriminant function was

$$d_3 = 2.56554 - 16.83407X_1 + 2.55416X_3 + 0.16384X_{15}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 12.851 > F^{0.05}_{3,56} \doteqdot 2.77 ,$$

demonstrating a significant difference between the groups on the basis of the three included variables.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	22	7	29
	BAD	6	25	31
	TOTAL	28	32	60

Thus

(1) overall efficiency $= \dfrac{22+25}{60} = 78.33\%$ (4.3801),

(2) proportion of original good performers correctly classified $= \dfrac{22}{29} = 75.86\%$ (2.9669),

(3) proportion of original bad performers correctly classified $= \frac{25}{31} = 80.65\%$ (3.2283)

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	11	8	19
	BAD	7	13	20
	TOTAL	18	21	39

Thus

(1) overall efficiency $= \frac{11+13}{39} = 61.54\%$ (1.8323),

(2) proportion of original good performers correctly classified $= \frac{11}{19} = 57.89\%$ (0.8001),

(3) proportion of original bad performers correctly classified $= \frac{13}{20} = 65.00\%$ (1.2276)

7.6 Discriminant Analysis on Smaller Sample Using Financial Ratios and with F-to-Enter Threshold = 2.00, F-to-Remove Threshold = 1.996

With the above stated threshold values the following variables were selected for inclusion in the discriminant function:

X_1 = current ratio,

X_2 = quick ratio,

X_3 = debtors ratio,

X_{14} = return + deftax on assets,

X_{19} = average tax rate,

X_{42} = X_{19}/sector average.

The relative importance of the individual variables as given by their rankings were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_1	2	6	1
X_2	3	5	4
X_3	5	4	2
X_{14}	1	2	3
X_{19}	6	1	5
X_{42}	4	3	6

The inclusion of both the average tax rate ratio and its associated relative ratio, as well as the inclusion of two liquidity ratios, viz., X_1 and X_2, indicated the possible presence of a fairly high degree of multicollinearity among the included variables, due to the relaxation of the threshold values. This could then also have been the reason for the inconsistency in the rankings of the variables for the three different procedures.

The final discriminant function was

$$d_4 = 3.84870 - 13.79635X_1 - 21.84591X_2 + 6.37900X_3 + 0.65057X_{14} - 0.19230X_{19} + 4.63065X_{42} .$$

The overall discriminatory power of the model was given as

$$\hat{F} = 11.084 > F^{0.05}_{6,53} \doteq 2.27,$$

demonstrating a significant difference between the groups on the basis of the included variables.

The accuracy matrices and associated measures of classification efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	25	4	29
	BAD	4	27	31
	TOTAL	29	31	60

Thus

(1) overall efficiency $= \frac{25+27}{60} = 86.67\%$ (5.6711),

(2) proportion of original good performers correctly classified $= \frac{25}{29} = 86.21\%$ (4.0817),

(3) proportion of original bad performers correctly classified $= \frac{27}{31} = 87.10\%$ (3.9472).

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	11	8	19
	BAD	6	14	20
	TOTAL	17	22	39

Thus

(1) overall efficiency $= \frac{11+14}{39} = 64.10\%$ (1.7577),

(2) proportion of original good performers correctly classified $= \frac{11}{19} = 57.89\%$ (0.8001),

(3) proportion of original bad performers correctly classified $= \frac{14}{20} = 70.00\%$ (1.6749).

7.7 Comparison of Previous Four Functions and Determination of Their Predictive Power

The results for all four discriminant functions derived above showed remarkable differences in the accuracy of the derived functions when classifying observations from the "good" and "bad" samples. At first it was thought that it could have been due to initial incorrect classifications and for this reason the smaller sample was analyzed. Although this improved the results for the Jackknife samples considerably, the improvement for the holdout samples were not that impressive. In fact in the latter case it seemed as if the improvement in the results for the "good" samples had been obtained at the expense of the satisfactory performance of the functions in classifying observations from the "bad" samples. A decision as to which situation would be more desirable probably depend on the different opportunity costs. This point will again be referred to at a later stage.

The drop in the accuracy of the derived functions when applied to the hold out samples in contrast to when applied to the jackknife samples can be explained by the fact that the variables included in the functions were selected on the basis of their discriminatory ability between the good and bad performers of the original, and hence also Jackknife, samples without taking any cognition of the observations in the holdout samples. Thus, though in most cases these variables demonstrated significant power to discriminate between good and bad performers in the holdout samples, this significance was much lower than that corresponding to the original samples.

The above mentioned facts did not leave much hope for the successful application of the derived functions on observations from future samples. It was nonetheless decided to ignore the Efficient Market Hypothesis for the moment and to test the predictive power of the two "best" functions on a sample of observations from 1980 using data from the preceding financial year. The two functions chosen for application to observations from this sample were d_1 and d_4.

The accuracy matrices and associated measures of classificatory efficiency for the two functions when applied to the future sample were as follows:

(i) Function d_1:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	10	3	13
	BAD	3	6	9
	TOTAL	13	9	22

Thus

(1) overall efficiency $= \frac{10+6}{22} = 72.73\%$ (1.9783),

(2) proportion of original good performers correctly classified $= \frac{10}{13} = 76.92\%$ (1.3078),

(3) proportion of original bad performers correctly classified $= \frac{6}{9} = 66.67\%$ (1.5716).

When using the discriminant function for predictive purposes, the original membership is actually unknown and the conditional probabilities should actually be calculated in the reverse order. Hence

(4) proportion of "good" predictions proven to be correct

$$= \frac{10}{13} = 76.92\% \ (1.3078),$$

(5) proportion of "bad" predictions proven to be correct

$$= \frac{6}{9} = 66.67\% \ (1.5716).$$

(ii) Function d_4:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	9	4	13
	BAD	3	6	9
	TOTAL	12	10	22

Thus

(1) overall efficiency $= \frac{9+6}{22} = 68.18\% \ (1.5517)$,

(2) proportion of original good performers correctly classified $= \frac{9}{13} = 69.23\% \ (1.0770)$,

(3) proportion of original bad performers correctly classified $= \frac{6}{9} = 66.67\% \ (1.5716)$,

(4) proportion of "good" predictions proven to be correct

$$= \frac{9}{12} = 75.00\% \ (1.4227),$$

(5) proportion of "bad" predictions proven to be correct

$$= \frac{6}{10} = 60.00\% \ (0.9241).$$

Although both functions performed with higher accuracy on the future sample than on the holdout sample with respect to the classification of original good performers, the overall accuracy of classification of observations from the future sample was lower than for observations from the holdout sample. The poor results can be ascribed to the influence of three factors:

(i) The variables included in the function were not selected on the basis of their discriminatory ability between the specific observations contained in the 1980 sample.
(ii) The functions were not derived for the specific time period.
(iii) The functions were derived so as to maximally discriminate between groups using data from the financial years corresponding to the calendar years for which relative performance was measured, and not data from the preceding financial years.

The usefulness of the discriminant functions thus seemed to be restricted to the specific sample of observations used, and even more so to the specific time period in which they were derived. Above all, the poor results obtained from applying the functions in a predictive role further validated the relevance of the semi-strong form of the Efficient Market Hypothesis to the Johannesburg Stock Exchange.

Share prices are not only subject to information specific to the individual firms but also to information concerning the economy as a whole. Subsequently seven economic indicators, which took cognisance of the changes in conditions over time, were included to examine the degree to which the information contained in these variables was already reflected in the market prices. Discriminant analyses were performed on both the larger and smaller samples, using both financial ratios and economic variables, and again varying the F-threshold values. For the larger sample, however, no economic variables were selected for inclusion and exact similar functions to d_1 and d_2 were derived. Some economic variables were, however, selected for inclusion into the functions derived for the smaller sample. These functions will be discussed in the following sections.

7.8 Discriminant Analysis on Smaller Sample Using Financial and Economic Variables with F-to-Enter = 4.00, F-to-Remove = 3.996

With the above stated threshold values the following three variables were selected for inclusion in the discriminant function:

X_{15} = return on equity ratio,

X_{45} = construction building plans passed,

X_{48} = residential buildings completed.

The relative importance of the individual variables as given by their rankings were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_{15}	1	1	1
X_{45}	3	2	2
X_{48}	2	3	3

The final discriminant function was

$$d_5 = 22.39847 + 0.12511X_{15} - 0.15559X_{45} - 0.24552X_{48}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 14.198 > F^{0.05}_{3,56} \doteq 2.77 ,$$

demonstrating a significant difference between the groups on the basis of the three included variables.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	25	4	29
	BAD	6	25	31
	TOTAL	31	29	60

Thus

(1) overall efficiency $= \frac{25+25}{60} = 83.33\%$ (5.1547),

(2) proportion of original good performers correctly classified $= \frac{25}{29} = 86.21\%$ (4.0817),

(3) proportion of original bad performers correctly classified $= \frac{25}{31} = 80.65\%$ (3.2283).

(ii) The hold out sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	13	6	19
	BAD	4	16	20
	TOTAL	17	22	39

Thus

(1) overall efficiency $= \frac{13+16}{39} = 74.36$ (3.0387)

(2) proportion of original good performers correctly classified $= \frac{13}{19} = 68.42\%$ (1.7181),

(3) proportion of original bad performers correctly classified $= \frac{16}{20} = 80.00\%$ (2.5696).

7.9 Discriminant Analysis on Smaller Sample Using Financial and Economic Variables with F-to-Enter = 2.00, F-to-Remove = 1.996

With the above stated threshold values the following variables were selected for inclusion in the discriminant function:

X_2 = quick ratio,

X_{14} = return + deftax on assets,

X_{18} = return on book capital,

X_{20} = cash flow to assets,

X_{23} = current ratio/sector average,

X_{25} = debtors ratio/sector average,

X_{45} = construction building plans passed,

X_{48} = residential buildings completed.

The relative importance of the individual variables as given by their rankings were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_2	6	4	5
X_{14}	1	1	6
X_{18}	3	8	8
X_{20}	2	2	7
X_{23}	7	6	3
X_{25}	8	3	4
X_{45}	5	5	1
X_{48}	4	7	2

The two univariate procedures were consistent in ranking X_{14} = return + deftax on assets and X_{20} = cash flow to assets, respectively, as the best and second-best single discriminators. The rankings according to these univariate criteria, however, differed remarkedly from the ranking given by the stepwise procedure. The procedure ranked the two economic indicators included in the function as the two best individual discriminators.

The final discriminant function was

$$d_6 = 34.69050 - 17.60278X_2 + 0.76194X_{14} + 3.85349X_{18}$$
$$- 0.39075X_{20} - 20.02391X_{23} + 3.44415X_{25}$$
$$- 0.20497X_{45} - 0.37759X_{48}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 9.915 > F^{0.05}_{8,51} \doteq 2.13 ,$$

demonstrating a significant difference between the groups on the basis of the three included variables.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	27	2	29
	BAD	6	25	31
	TOTAL	33	27	60

Thus

(1) overall efficiency $= \frac{27+25}{60} = 86.67\%$ (5.6711)

(2) proportion of original good performers correctly classified $= \frac{27}{29} = 93.10\%$ (4.8249),

(3) proportion of original bad performers correctly classified $= \frac{25}{31} = 80.65\%$ (3.2283) .

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	15	4	19
	BAD	5	15	20
	TOTAL	20	19	39

Thus

(1) overall efficiency $= \frac{15+15}{39} = 76.92\%$ (3.3589),

(2) proportion of original good performers correctly classified $= \frac{15}{19} = 78.95\%$ (2.6360),

(3) proportion of original bad performers correctly classified $= \frac{15}{20} = 75.00\%$ (2.1223).

Comparing the results for these last two discriminant functions with the four discriminant functions derived before the inclusion of the economic indicators, showed a significant improvement in the classificatory power of the functions. The improvement in overall efficiency was mainly due to the increased accuracy with which original good performers were classified. However, these last two functions performed very unsatisfactorily when applied to predict the performance of industrial firms in 1980 based on data from the June 1979 to June 1980 financial year. The results were as follows:

(i) Function d_5:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	10	3	13
	BAD	6	3	9
	TOTAL	16	6	22

Thus

(1) overall efficiency $= \frac{10+3}{22} = 59.09\%$ (0.6984),

(2) proportion of good performers correctly classified $= \frac{10}{13} = 76.92\%$ (1.3078),

(3) proportion of bad performers correctly classified $= \frac{3}{9} = 33.33\%$ (-0.4623),

(4) proportion of "good" predictions proven to be correct

$$= \frac{10}{16} = 62.50\% \ (-0.9188),$$

(5) proportion of "bad" predictions proven to be correct

$$= \frac{3}{6} = 50.00\% \ (1.2502).$$

(ii) Function d_6:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	9	4	13
	BAD	4	5	9
	TOTAL	13	9	22

Thus

(1) overall efficiency $= \frac{9+5}{22} = 63.64\% \ (1.250)$,

(2) proportion of good performers correctly classified

$$= \frac{9}{13} = 69.23\% \ (0.7437),$$

(3) proportion of bad performers correctly classified

$$= \frac{5}{9} = 55.56\% \ (0.8936),$$

(4) proportion of "good" predictions proven to be correct

$$= \frac{9}{13} = 69.23\% \ (0.7437),$$

(5) proportion of "bad" predictions proven to be correct

$$= \frac{5}{9} = 55.56\% \ (0.8936).$$

Thus, while the results for the Jackknife and holdout samples indicated that share performance was directly related, not only to financial characteristics specific to the firms, but also to the general economic conditions, the inclusion of economic indicators seemed to render no beneficial contributions to the development of predictive models. This illustrated that, not only did the market reflect all information contained in annual financial statements prior to the release of these statements, but it also anticipated any changes in economic conditions over time.

7.12 Further Refinements with Respect to Discriminant Analysis Procedures Employed

The violation of the normality assumption have already been discussed or referred to at different stages during the analysis. It seemed not to have affected the results too much. Since a few of the financial ratios also appeared to be discrete in nature, especially due to a large number of zero values in some cases, more consideration should perhaps have been given to the employment of discrete discriminant analysis procedures.

The linear discriminant analysis procedures employed were of course all based on the assumption of equal group dispersions. Box's (1949) test statistic for the equality of group dispersions, as discussed in Chapter 5, section 5.2, confirmed that this assumption was satisfied for the variables included in most of the functions. In the one or two cases where it was not satisfied, the differences were small, and in view of the sensitivity of quadratic procedures to non-normality, it was decided that the linear procedures were more appropriate.

Furthermore, equal prior probabilities and costs of misclassification were assumed throughout the analysis. This was done in the absence of any estimates of these parameters and in any event the assumption of equal prior probabilities seemed intuitively to be correct, since on the average about half of the shares would be expected to have performed better than average and about half to have performed worse. The different degrees of success of the derived discriminant functions in the classification of observations from the "good" and "bad" samples complicated the decision about choosing a single best function. As was mentioned in Chapter 7, section 7.7, this decision would probably depend on the different costs of misclassification for the "good" and "bad" samples. Inclusion of such differential costs prior to estimating the functions could probably simplify the decision about a preferred function. Estimates of these costs are, however, very difficult to obtain and would depend on the preferences of the individual investors.

Finally, although the poor predictory performances of the derived functions could mainly be ascribed to the futility of attempting to predict share performance based on information contained in financial statements in the light of the semi-strong form of the Efficient Market Hypothesis, a more successful attempt at the derivation of predictive functions could probably have been made by using data from financial years prior to the calendar years for which predictions were required at the derivation stage of the functions.

7.13 Summary of Results

Comparing the classificatory efficiency of the six derived functions, two functions were chosen as being the most useful in specific circumstances. The best function based on the inclusion of financial ratios alone appeared to be

$$\begin{aligned} d_4 = {} & 3.84870 - 13.79633 \text{ (current ratio)} \\ & - 21.84591 \text{ (quick ratio)} + 6.37900 \text{ (debtors ratio)} \\ & + 0.65057 \text{ (return + def.tax on assets)} \\ & - 0.19230 \text{ (average tax rate)} \\ & + 4.63065 \text{ (cash flow to current liab./sector average)}. \end{aligned}$$

Its overall classificatory ability on the holdout sample was 64.10% and on the Jackknife sample 86.67%.

The function based on the inclusion of financial and economic variables that performed best on the holdout and Jackknife samples was

$$\begin{aligned} d = {} & 34.69050 - 17.60278 \text{ (quick ratio)} \\ & + 0.76194 \text{ (return + def.tax on assets)} \\ & + 3.85349 \text{ (return on book capital)} \\ & - 0.39075 \text{ (cash flow to assets)} \\ & - 20.02391 \text{ (current ratio/sector average)} \\ & + 3.4415 \text{ (debtors ratio/sector average)} \\ & - 0.20497 \text{ (construction building plans passed)} \\ & - 0.37759 \text{ (residential buildings completed)}. \end{aligned}$$

Its overall discriminatory ability on the hold out sample was 76.92% and on the Jackknife sample 86.67%. The major difference in the performance of functions d_4 and d_6 was in the proportion of good performers correctly classified which improved from 57.89% for d_4 on the holdout sample to 78.95% for d_6.

Although not included in any one of the two "best" functions, the most important single discriminator was X_{15} = return on equity, which from a logical point of view would have been expected to be the variable most closely related to share performance on the stock market. A different measure of return on investment, viz., X_{14} = return + def. tax on assets, were instead included. The other characteristics of an industrial company that seemed to be significantly related to its performance on the stock market, was its liquidity position. This was shown by the fact that all three best functions included three measures of liquidity, viz., the current quick and debtors ratios, or the corresponding sector-relative ratios.

Comparison of the above two functions with Le Roux's five functions given in Chapter 6, section 6.3, showed some agreement on the individual variables selected for inclusion in the final discriminant functions. When comparing the classificatory efficiency of the models with that of Le Roux's functions, the results for the Jackknife samples should be used since Le Roux's results were for the performance of his functions on the Jackknife samples. An examination of these accuracy measures showed the three functions given above to have performed at least as well or better than any of Le Roux's functions.

It should be remembered that the two sets of functions were not derived with the same objective in mind. Le Roux attempted to derive models for predicting bankruptcy and hence the performance of his functions on the Jackknife samples illustrates the relationship between financial statement data and firm failure, whereas the aim of this study has been to derive models to classify shares in terms of their relative performance on the stock market, and as such determine the relationship between financial statement data and share prices. Since this latter objective intuitively seems to be more difficult to achieve than accurate bankruptcy prediction, the superiority of the two models given above is further confirmed. Furthermore, assuming the semi-strong form of the Efficient Market Hypothesis to hold for shares listed on the Johannesburg Stock Exchange, the poor results obtained from using the functions in a predictory sense, do not affect the above superiority.

CHAPTER 8

DISCRIMINANT ANALYSES OF SHARES LISTED ON THE JSE FOR THE YEARS 1973 AND 1979

8.1 Introduction

In view of the instability of economic conditions from year to year it was decided to calculate separate discriminant functions using data from two specific years. The two years chosen were

(i) 1973, during which the return on the industrial composite index was -20.48% and which thus represented a bear market, and

(ii) 1979, during which the overall return on the market was 59.25% and which thus represented a bull market.

It was hoped that through the analysis of these two samples two functions could be derived that would not only successfully discriminate between good and bad performers in the specific years, but that could also be applied with reasonable degrees of accuracy to classify shares during future bull and bear markets.

8.2 Sample Design

By defining "good" performers as those shares with returns greater than 70% for 1979, and "bad" performers as those shares with returns less than 20% for 1979, thirty-one "good" performers and thirty "bad" performers were selected to form an initial sample containing sixty-one industrial shares. This sample was randomly divided into

(i) an analysis sample containing forty-two shares, viz., twenty good performers and twenty-two bad performers, and

(ii) a holdout sample containing nineteen shares, viz., eleven good performers and eight bad performers.

Similarly, be defining "good" performers as those shares with returns greater than zero, and "bad" performers as those shares with returns less than -40%, thirty-two "good" performers and thirty "bad" performers for 1973 were selected to form an initial sample containing sixty-two industrial shares. This sample was randomly divided into

(i) an analysis sample containing forty-two shares, viz., twenty-two good performers and twenty bad performers, and

(ii) a holdout sample containing twenty shares, viz., eleven good performers and nine bad performers.

The specific shares included in the two samples together with their yearly returns are given in Appendix H.

8.3 Selection of Variables

The twenty-two financial ratios and their corresponding sector relative ratios used in the analysis of the 1973-1979 pooled sample and listed in Table 7.2, were used in the analysis of the 1979 and 1973 samples. Logarithmic transformations were once again employed in an attempt to improve normality. The variables that were transformed for the two specific samples are given in Appendix I. These transformations lead to the exclusion of a few cases in the 1979 sample due to negative values for some of the ratios. Even after these transformations some violations of the normality assumption remained, mainly caused by a large number of zero values and a few extreme values for some variables. A winsorizing procedure similar to that described in Chapter 7, section 7.2, was employed to limit the possible adverse effects caused by the presence of nonnormality.

The data was again taken from the University of Stellenbosch GSB-Ratio Analyses of Selected Companies. Ratios calculated from financial year-end statements between June 1979 and June 1980 were used in the 1979 sample, and between June 1973 and June 1974 in the 1973 sample. In addition data was also collected for the two preceding financial years in both cases. The results obtained from substituting these past values of the ratios into the derived functions would give an indication of how long in advance the financial characteristics of a firm resembled its characteristics as a good or bad performer in 1979 and 1973, respectively.

The stepwise linear discriminant analysis computer program, BMDP07M (1981), was again used to analyze the date and different analyses corresponding to different F-threshold values were performed. These will be discussed in the following sections.

8.4 Discriminant Analysis of 1979 Sample with F-to-Enter = 4.00, F-to-Remove = 3.996

With the above stated threshold values the following variables were selected for inclusion in the discriminant function:

X_{15} = return on equity ratio,

X_{34} = fixed cost cover/sector average,

X_{44} = cash flow to current liab./sector average.

The relative importance of the individual variables were determined by ranking them according to the same three different criteria as was discussed in Chapter 7, section 7.4. The rankings for the above three variables were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_{15}	1	2	1
X_{34}	2	1	2
X_{44}	3	3	3

From these rankings the return on equity ratio once again appeared to be the best single discriminator. Furthermore, the inclusion of two relative ratios seemed to suggest the usefulness of dividing ratios by their sector averages.

The final discriminant function was

$$d_{79(1)} = -1.03551 + 0.11922X_{15} - 8.72697X_{34} + 0.85204X_{44}.$$

The overall discriminatory power of the function was given as

$$\hat{F} = 5.894 > F^{0.05}_{3,34} \doteq 2.88 ,$$

demonstrating a significant difference between the groups on the basis of the three included variables. Note, however, that this $\hat{F}$-value is much lower than any of those obtained for the functions derived in Chapter 7.

The classificatory power of the function was tested on observations from

(i) the original sample,
(ii) a Lachenbruch jackknife sample,
(iii) a randomly selected holdout sample, and
(iv) a sample of future observations from 1980.

Once again only the results for the last three samples will be reported. The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	16	4	20
	BAD	6	12	18
	TOTAL	22	16	38

Thus

(1) overall efficiency $= \frac{16+12}{38} = 73.68\%$ (2.9027),

(2) proportion of original good performers correctly classified $= \frac{16}{20} = 80.00\%$ (2.4514),

(3) proportion of original bad performers correctly classified $= \frac{12}{18} = 66.7\%$ (1.6396).

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP		
		GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	7	4	11
	BAD	4	8	12
	TOTAL	11	12	23

Thus

(1) overall efficiency $= \frac{7+8}{23} = 65.22\%$ (1.4510),

(2) proportion of original good performers correctly classified $= \frac{7}{11} = 63.64\%$ (1.0495),

(3) proportion of original bad performers correctly classified $= \frac{8}{12} = 66.67\%$ (1.0053).

(iii) The 1980 sample:

		ASSIGNED MEMBERSHIP		
		GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	8	4	12
	BAD	7	2	9
	TOTAL	15	6	21

Thus

(1) overall efficiency $= \frac{8+2}{21} = 47.62\%$ (-0.4110),

(2) proportion of good performers correctly classified $= \frac{8}{12} = 66.67\%$ (0.6669),

(3) proportion of bad performers correctly classified $= \frac{2}{9} = 22.22\%$ (-1.2511).

When substituting the 1978 and 1977 values of the ratios into the above function, the following measures of classificatory efficiency were obtained. For 1978,

(1) overall efficiency = 57.14% (0.5345)

(2) proportion of good performers correctly classified
= 42.86% (-0.3780),

(3) proportion of bad performers correctly classified
= 71.43% (1.1339).

When using 1978 data to classify observations according to their expected performance in 1979, their original membership is actually unknown and the conditional probabilities should actually be calculated in the reverse order. Hence

(4) proportion of "good" predictions proven to be correct
= 60.00% (2.2671),

(5) proportion of "bad" predictions proven to be correct
= 55.56% (-1.0939).

For 1977,

(1) overall efficiency = 56.14% (0.9242),

(2) proportion of good performers correctly classified
= 46.43% (-0.2799),

(3) proportion of bad performers correctly classified
= 65.52% (1.5767),

(4) proportion of "good" predictions proven to be correct
= 56.52% (1.5809),

(5) proportion of "bad" predictions proven to be correct
= 55.88% (-0.4478).

8.5 Discriminant Analysis of 1979 Sample with F-to-Enter = 2.00, F-to-Remove = 1.996

With the above stated threshold values the following variables were selected for inclusion in the discriminant function:

X_3 = debtors ratio,

X_4 = stock ratio,

X_{10} = interest bearing debt + prefs,

X_{12} = fixed cost cover,

X_{14} = return + def. tax on assets,

X_{18} = return on book capital,

X_{19} = average tax rate,

X_{27} = asset composition/sector average,

X_{30} = long term and short term debt ratio/sector average,

X_{41} = average tax rate/sector average,

X_{42} = cash flow to assets/sector average,

X_{44} = cash flow to current liabilities/sector average.

The relative importance of the individual variables as given by their rankings were as follows:

$$d_{79(2)} = -16.58774 + 18.36406X_3 - 4.39396X_4 - 0.35833X_{10}$$
$$-12.11301X_{12} + 3.34028X_{14} - 0.71143X_{18} - 0.83658X_{19}$$
$$-12.99237X_{27} + 34.09726X_{30} + 18.73383X_{41}$$
$$-16.76053X_{42} + 6.87727X_{44}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 10.896 > F^{0.05}_{12,25} \doteqdot 2.16 ,$$

demonstrating increased significance in the difference between the groups on the basis of the variables included in this function compared to the results for function $d_{79(1)}$.

The accuracy matrices and associated measures of classification efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	19	1	20
	BAD	1	17	18
	TOTAL	20	18	38

Thus

(1) overall efficiency $= \frac{19+17}{38} = 94.74\%$ (5.4983),

(2) proportion of good performers correctly classified $= \frac{19}{20} = 95.00\%$ (3.7949),

(3) proportion of bad performers correctly classified $= \frac{17}{18} = 94.44\%$ (3.9999).

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	6	5	11
	BAD	4	8	12
	TOTAL	10	13	23

Thus

(1) overall efficiency = $\frac{6+8}{23}$ = 60.87% (1.0339),

(2) proportion of good performers correctly classified

$= \frac{6}{11} = 54.55\%$ (0.4459),

(3) proportion of bad performers correctly classified

$= \frac{8}{12} = 66.67\%$ (1.0053).

(iii) The 1980 sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	6	6	12
	BAD	8	1	9
	TOTAL	14	7	21

Thus

(1) overall efficiency = $\frac{6+1}{21}$ = 40.00% (-1.6213),

(2) proportion of good performers correctly classified

$= \frac{6}{12} = 50.00\%$ (-0.4998),

(3) proportion of bad performers correctly classified

$= \frac{1}{9} = 0.11\%$ (-1.9247).

When substituting the 1978 and 1977 values of the ratios into the above function, the following measures of classificatory efficiency were obtained. For 1978,

(1) overall efficiency = 82.14% (2.4054),

(2) proportion of good performers correctly classified
= 75.00% (1.3229),

(3) proportion of bad performers correctly classified
= 89.29% (2.0788),

(4) proportion of "good" predictions proven to be correct
= 87.50% (4.4191),

(5) proportion of "bad" predictions proven to be correct
= 78.13% (2.3988).

For 1977,

(1) overall efficiency = 71.93% (3.3083),

(2) proportion of good performers correctly classified
= 53.57% (0.4712),

(3) proportion of bad performers correctly classified
= 89.60% (4.1768),

(4) proportion of "good" predictions proven to be correct
= 83.33% (4.7236),

(5) proportion of "bad" predictions proven to be correct
= 66.67% (-0.2356).

8.6 Evaluation of Discriminant Functions Derived for 1979

The drop in the accuracy of the derived functions when applied to the holdout sample in contrast to when applied to the Jackknife sample can once again be explained by the fact that the variables included in the functions were selected on the basis of their discriminatory ability between the good and bad performers of the original, and hence also Jackknife, samples without taking any cognition of the observations in the holdout sample. The same remark applies to the 1980 sample. The poor results for the 1980 sample can further be explained by the instability in the financial and economic conditions over time. Although the average return for industrial shares in 1980 was also positive, it was much lower than in 1979.

The classificatory results obtained by substituting 1978 and 1977 values of the ratios into the derived functions gave an indication of the resemblance in the financial characteristics of the firms for the three different years. Thus according to function $d_{79(1)}$ about 57% of the firms had similar financial characteristics in 1978 as in 1979, while about 82% of the firms had similar financial profiles in 1978 as in 1979 according to function $d_{79(2)}$. For 1977 these figures were somewhat lower.

However, the above-quoted figures could be considered as underestimating the real resemblance in the financial profiles of firms for the different years, since they were obtained for functions that did not perform with 100% accuracy on the 1979 data form which thev were derived. The discrepancy in the classificatory results of the two functions when using 1978 and 1977 ratios could then also be ascribed to the different degrees of success of the two functions in discriminating between good and bad performers of the Jackknife sample. Function $d_{79(2)}$ outperformed function $d_{79(1)}$ by far on the Jackknife sample and hence the 1978 and 1977 results for function $d_{79(2)}$ were accordingly also much better than for function $d_{79(1)}$. In fact, since function $d_{79(2)}$ performed with 95% accuracy on the Jackknife sample, the results obtained from substituting 1978 and 1977 values of the ratios into this function could be considered as reasonably accurate estimates of the real resemblance in the financial profiles of firms for the three years.

As was stated in section 8.2, the values of the financial ratios used for deriving the discriminant functions were obtained from financial year-end statements between June 1979 and June 1980. This was done because it was thought that the information contained in these financial statements would most accurately refer to the financial profiles of the firms in 1979. The data used for the 1980 sample was thus correspondingly obtained from the subsequent financial year. This, however, implied that none of the results derived for the Jackknife, holdout, or 1980 samples could be viewed as measures of the predictive ability of the derived functions, since at the time when these predictions would have been required, namely at the beginning of the calendar year, the financial statements necessary for the calculation of the ratios would not as yet have been available. Hence the above classificatory results strictly only provided measures of the correspondence between the financial characteristics of firms, as described by fundamental financial analysis, and the relative performance of these shares on the stock market for the corresponding year, and as such satisfied the primary aim of this study.

The very good results obtained for the Jackknife samples illustrated the degree to which accounting information was reflected in the share prices and thus confirmed the validity of the semi-strong form of the Efficient Market Hypothesis with respect to shares listed on the J.S.E. This, together with the conclusions of the Ball and Brown study mentioned in Chapter 1, implied that there would be no purpose in attempting to derive predictive functions based on financial statement information.

Nonetheless, to determine the predictive powers, if any of the derived functions, firms contained in the 1980 sample were reclassified by substituting the values of the ratios obtained from the June 1979 to June 1980 financial statements into function $d_{79(2)}$. The results were as follows:

(1) overall efficiency = 41.18%,

(2) proportion of good performers correctly classified = 55.56%,

(3) proportion of bad performers correctly classified = 25,00%,

(4) proportion of "good" predictions proven to be correct = 45.45%,

(5) proportion of "bad" predictions proven to be correct = 33.33%.

Although these results were better than those obtained when 1980 data was used, they were very poor when compared to the results obtained when 1978 and 1977 data were used to classify firms according to their relative performance in 1979. This could be due to the fact that more similar average returns were earned by industrial shares in 1977, 1978 and 1979, than in 1980 and it would be interesting to see the results obtained from applying function $d_{\bar{79}(2)}$ to a future year with average returns for industrial shares more similar to that of 1979.

8.7 Discriminant Analysis of 1973 Sample with F-to-Enter = 4,00, F-to.Remove = 3.996

With the above threshold values only one variable, viz., X_{41} = average tax rate/sector average was selected for inclusion in the discriminant function. The final discriminant function was

$$d_{73(1)} = -2.48732 + 2.90896X_{41}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 10.083 > F^{0.05}_{1,36} = 4.11 ,$$

demonstrating a significant difference between the groups on the basis of the one included variable.

The accuracy matrices and associated measures of classification efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	18	3	21
	BAD	6	11	17
	TOTAL	24	14	38

Thus

(1) overall efficiency $= \frac{18+11}{38} = 76.32\%$ (3.1768),

(2) proportion of good performers correctly classified

$= \frac{18}{21} = 85.71\%$ (2.8068),

(3) proportion of bad performers correctly classified

$= \frac{11}{17} = 64.71\%$ (1.6556).

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	10	1	11
	BAD	7	6	13
	TOTAL	17	7	24

Thus

(1) overall efficiency $= \frac{10+6}{24} = 66.67\%$ (1.5987),

(2) proportion of good performers correctly classified

$= \frac{10}{11} = 90.91\%$ (3.0007),

(3) proportion of bad performers correctly classified

$= \frac{6}{13} = 46.15\%$ (-0.5801).

(iii) The 1974 sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	10	0	10
	BAD	4	6	10
	TOTAL	14	6	20

Thus

(1) overall efficiency = $\frac{10+6}{20}$ = 80.00% (2.6833),

(2) proportion of good performers correctly classified

= $\frac{10}{10}$ = 100.00% (3.1623),

(3) proportion of bad performers correctly classified

= $\frac{6}{10}$ = 60.00% (0.6325).

When substituting the 1972 and 1971 values of the ratios into the above function, the following measures of classificatory efficiency were obtained.

For 1972,

(1) overall efficiency = 54.84% (0.8707),

(2) proportion of good performers correctly classified

= 71.88% (2.2939),

(3) proportion of bad performers correctly classified

= 36.67% (-1.2849),

(4) proportion of "good" predictions proven to be correct

= 54.76% (-1.7992),

(5) proportion of "bad" predictions proven to be correct

= 55.00% (2.1755).

For 1971,

(1) overall efficiency = 70.97% (3.4110),

(2) proportion of good performers correctly classified = 93.75% (4.7701),

(3) proportion of bad performers correctly classified = 46.67% (-0.1889),

(4) proportion of "good" predictions proven to be correct = 65.22% (-1.3907),

(5) proportion of "bad" predictions proven to be correct = 87.50% (5.6391).

8.8 Discriminant Analysis of 1973 Sample with F-to-Enter = 2.00, F.to.Remove = 1.996

With the above stated threshold values the following variables were selected for inclusion in the discriminant function:

X_{14} = return + def. tax on assets,

X_{37} = return on equity/sector average,

X_{41} = average tax rate/sector average.

The relative importance of the individual variables as given by their rankings were as follows:

VARIABLE	RANKING ACCORDING TO		
	(i) F-STATISTICS	(ii) SCALED COEFFICIENTS	(iii) STEPWISE PROCEDURE
X_{14}	2	1	2
X_{37}	3	3	3
X_{41}	1	2	1

The final discriminant function was

$$d_{73(2)} = -3.16918 + 0.19490X_{14} - 1.45835X_{37} + 3.32264X_{41}.$$

The overall discriminatory power of the model was given as

$$\hat{F} = 6.287 > F^{0.05}_{3,34} \doteq 2.88,$$

demonstrating a significant difference between the groups on the basis of the variables included.

The accuracy matrices and associated measures of classificatory efficiency were as follows:

(i) The Jackknife sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	16	5	21
	BAD	6	11	17
	TOTAL	22	16	38

Thus

(1) overall efficiency $= \frac{16+11}{38} = 71.05\%$ (2.5279),

(2) proportion of good performers correctly classified $= \frac{16}{21} = 76.19\%$ (1.9290),

(3) proportion of bad performers correctly classified $= \frac{11}{17} = 64.71\%$ (1.6556).

(ii) The holdout sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	9	2	11
	BAD	5	8	13
	TOTAL	14	10	24

Thus

(1) overall efficiency $= \frac{9+8}{24} = 70.83\%$ (2.0070),

(2) proportion of good performers correctly classified

$= \frac{9}{11} = 81.82\%$ (2.3955),

(3) proportion of bad performers correctly classified

$= \frac{8}{13} = 61.54\%$ (0.5332).

(ii) The 1974 sample:

		ASSIGNED MEMBERSHIP GOOD	BAD	TOTAL
ORIGINAL MEMBERSHIP	GOOD	8	2	10
	BAD	4	6	10
	TOTAL	12	8	20

Thus

(1) overall efficiency $= \frac{8+6}{20} = 70.00\%$ (1.7889),

(2) proportion of good performers correctly classified

$= \frac{8}{10} = 80.00\%$ (1.8974),

(3) proportion of bad performers correctly classified

$= \frac{6}{10} = 60.00\%$ (0.6325).

When substituting the 1972 and 1971 values of the ratios into the above function, the following measures of classificatory efficiency were obtained.

For 1972,

(1) overall efficiency = 64.52% (2.3949),

(2) proportion of good performers correctly classified

= 71.88% (2.2931)

(3) proportion of bad performers correctly classified

= 56.67% (0.9071),

(4) proportion of "good" predictions proven to be correct

= 63.89% (0.7081),

(5) proportion of "bad" predictions proven to be correct

= 65.38% (2.2381)

For 1971,

(1) overall efficiency = 69.35% (3.1570),

(2) proportion of good performers correctly classified

= 71.88% (2.2931),

(3) proportion of bad performers correctly classified

= 66.67% (2.0031),

(4) proportion of "good" predictions proven to be correct

= 69.70% (1.8959),

(5) proportion of "bad" predictions proven to be correct

= 68,97% (2.3955).

8.9 Evaluation of Discriminant Functions Derived for 1973

In contrast to the results for the functions derived for the 1979 sample, the two functions derived for 1973 both performed better on the random sample of observations from 1974 than on the Jackknife or holdout samples. Furthermore, the results obtained for the 1974 sample were also superior to those obtained after substitution of 1972 and 1971 values of the ratios into the derived functions. This was probably due to the greater degree of similarity in the economic conditions influencing the performance of industrial firms in 1973 and in 1974, compared to the correspondence in the economic conditions of 1972 and 1971 to those in 1973.

The resemblance in the financial profiles of firms in 1972 and 1973 as given by the results obtained after substitution of the 1972 values into the derived functions were 54.84% according to function $d_{73(1)}$ and 64.52% according to function $d_{73(2)}$. The corresponding figures for the 1971 data were somewhat higher, thus suggesting that conditions in 1971 and 1973 were more similar than conditions in 1972 and 1973. These figures were of course again underestimates of the real relations for the same reasons as was given in section 8.6. Since function $d_{73(1)}$ was the more accurate of the two in the classification of observations from the Jackknife samples, the estimates according to this function would probably be the more accurate estimates of the resemblance in the financial profiles of the firms during the three years.

Although the results obtained for the Jackknife samples were not quite as good as the corresponding results obtained for the 1979 functions, they once again indicated that the semi-strong form of the Efficient Market Hypothesis was valid and that a significant amount of information contained in financail statements was anticipated by the market. However, the predictive power of the derived functions were once again tested by reclassifying some of the firms contained in the 1974 sample by substituting the values of the ratios obtained from the June 1973 to June 1974 financial statements into both functions $d_{73(1)}$ and $d_{73(2)}$. The results were as follows:

For function $d_{73(1)}$,

(1) overall efficiency = 83.33% (1.9362),

(2) proportion of good performers correctly classified = 100.00% (2.8286),

(3) proportion of bad performers correctly classified = 75.00% (0.4998),

(4) proportion of "good" predictions proven to be correct = 66.67% (0.4082),

(5) proportion of "bad" predictions proven to be correct = 100.00% (1.2247).

For function $d_{73(2)}$,

(1) overall efficiency = 75.00% (1.3552),

(2) proportion of good performers correctly classified = 100.00% (2.8286),

(3) proportion of bad performers correctly classified = 62.50% (0.2502),

(4) proportion of "good" predictions proven to be correct = 57.14% (0.0002),

(5) proportion of "bad" predictions proven to be correct = 100.00% (1.1180).

It should be noted that these results were obtained for a very small number of randomly selected shares and hence the low significance levels in spite of apparently high percentages of accuracy. Nonetheless, the results implied that both functions derived for the 1973 sample could be used with some expectancy of success to predict the relative performance of industrial shares in years with similar economic profiles to that of 1973, and as such represented possible indications of violations of the semi-strong form of the Efficient Market Hypothesis.

8.10 Summary of Results

There was not much difference between the two functions derived for each of the two years under consideration. The better function for 1979 could probably be taken as

$$
\begin{aligned}
d_{79(2)} = &- 16.58774 + 18.36406 \text{ (debtors ratio)} \\
&- 4.39396 \text{ (stock ratio)} \\
&- 0.35833 \text{ (interest bearing debt + prefs)} \\
&- 12.11301 \text{ (fixed cost cover)} \\
&+ 3.34028 \text{ (return + def. tax on assets)} \\
&- 0.71143 \text{ (return on book capital)} \\
&- 0.83658 \text{ (average tax rate)} \\
&- 12.99237 \text{ (asset composition/sector average)} \\
&+ 34.09726 \text{ (long term \& short term debt/sector average)} \\
&+ 18.73383 \text{ (average tax rate/sector average)} \\
&- 16.76053 \text{ (cash flow to assets/sector average)} \\
&+ 6.87727 \text{ (cash flow to current liab./sector average).}
\end{aligned}
$$

The overall classificatory ability of this function on the

Jackknife sample was 94.74% and on the holdout sample was 60.87%. The function, however, performed poorly when applied to observations from a future time period.

For 1973 the better function was

$$d_{73(1)} = -2.48732 + 2.90896 \text{ (average tax rate/sector average).}$$

Not only did this function perform reasonably well on the Jackknife and holdout samples, classifying respectively 76.32% and 66.67% of the observations correctly, but it also demonstrated some predictive ability

The poor results obtained from applying function $d_{79(2)}$ to 1980 data, compared to the results obtained from substituting 1977 and 1978 values of the ratios into the function, were due to the more similar economic conditions in 1977, 1978 and 1979, than in 1980. For the 1973 analysis, the reverse situation was true in that 1974 turned out to be more similar to 1973 than either of 1971 and 1972. The superior predictive results obtained by the functions derived for 1973 could then also be ascribed to the fact that the predictive powers of these functions were tested on time periods very similar in financial characteristics to 1973 in which the functions were derived. This served to illustrate the high dependence of the predictive utility of the derived functions on the similarity in economic conditions for the specific time periods. It should be noted that more useful predictive models could possibly have been derived by using data from the financial years preceding the calendar years under consideration.

However, it should be remembered that the aim of this study was not to derive predictive models, since on the basis of the assumption of market efficiency and the conclusions drawn by Ball and Brown, it was believed that no useful predictive procedures could be obtained from the analysis of financial statements. The excellent classificatory results obtained from the application of the derived functions to the Jackknife samples confirmed the validity of the semi-strong form of the Efficient Market Hypothesis with respect to shares listed on the Johannesburg Stock Exchange and, as was the case in the Ball and Brown study, it could be concluded that most of the information contained in financial statements is anticipated by the market.

Once again an agreement in the variables selected for inclusion into the functions derived for 1973 and 1979 and the functions derived in Chapter 7 or by Le Roux was apparent. The return on investment and capital structure characteristics of firms seemed to be quite important. Furthermore, the average tax rate ratio and/or the corresponding relative ratio appeared in three of the four functions. The inclusion of sector relative ratios in all the functions suggested the usefulness of dividing ratios by their sector averages.

The equality of group dispersions assumption was satisfied for most of the functions. In the odd case where it was not satisfied differences were small and in view of the violation of the normality assumption, linear procedures were considered to be more appropriate than quadratic procedures. Similar remarks with respect to the violation of the normality assumptions and the inclusion of different prior probabilities and costs of misclassification as was made in Chapter 7, section 7.12, apply to the procedures employed in this chapter.

CHAPTER 9

SUMMARY AND CONCLUSIONS

Taking into account the multivariate nature of stockmarket data, the aim of this thesis was to examine the usefulness of multivariate statistical techniques to portfolio theory. More specifically, two different multivariate techniques were used in two separate classificatory problems concerning shares listed on the Johannesburg Stock Exchange (JSE).

Firstly, factor analytic techniques were used to determine the statistical significance of grouping shares by their industry classifications. Chapter 2 provided a theoretical background of factor analysis, while the application of these techniques on shares listed on the JSE were discussed in Chapters 3 and 4. The specific factor analytic techniques used were the principal factor method and Kaiser's Second Generation Little Jiffy for initial factor extraction, both followed by orthogonal varimax and orthoblique rotations.

In Chapter 3 the weekly returns of shares listed on the JSE for the period March 1973 to June 1981 were analyzed. It was found that about 16% of the total variation in weekly returns of shares was due to the influence of a general market factor. The high impact of the gold shares on the market was indicated by the fact that about 40% of the total variance of the gold shares was accounted for by the market factor.

An average-linkage clustering of the residual covariances after removal of the market factor, revealed very distinct groupings of gold and coal shares, and to a lesser extent of motor shares. The significant factors extracted by the two methods of factor analysis could then also be identified as being gold, coal and motor factors. These factors together accounted for about 14 to 18% of the total variance. Sub-period analysis showed a general weakening in comovement over time. It also served to emphasize the impact of gold on the market, since in addition to high first factor loadings for gold shares, the effect of the gold shares was in some cases divided among all the factors.

Because of the very dominant effect of the gold shares, and to a lesser extent the coal shares, on the market, the analysis was repeated with the gold and coal shares excluded. Apart from a market factor no significant factors were extracted, thereby confirming the absence of any additional significant groupings. It was thus concluded that the total risk associated with a share's return could be divided into a systematic component due to a gold-dominated market effect and an unsystematic component due to effects unique to the individual shares.

Furthermore it could be concluded that apart from some evidence of comovement among the motor shares and a strong correlation in the returns of the two chemical firms Lanchem and Triomf, the industry classifications do not provide any guidelines for efficient diversification opportunities.

Due to the importance of gold shares on the JSE, Chapter 4 was devoted towards examining the underlying dimensions in the variations in returns of gold shares. It was thought that these dimensions would be related to some mining characteristics like location, life, ore grade, costs, profits and so forth. The data comprised of weekly returns of gold shares for two periods, one before the demonitisation of gold and one thereafter, viz.,

(i) April 1968 to December 1971, and
(ii) February 1973 to July 1981.

Both periods were once again divided into subperiods to determine the stability of the factor analysis results.

For both periods the market effect turned out to be the only significant factor, accounting for 33,4% of the total variance in the first period and 56% in the second period. Although the subperiod analysis for the second period showed a weakening in the comovement of gold shares over time, the above figures illustrated that gold shares formed a much more cohesive sector after the collapse of the Gold Standard than before.

While no groupings of gold shares according to the characteristics of the mines were apparent for the first period, some correspondence in the clustering of gold shares and these characteristics was detected for the second period. The most prominent of these seemed to be a clustering of gold shares according to the location of the mines, while groupings according to similar ore grades, mining costs and profits were also discernable. The effects of these groupings were, however, insignificant when compared to the general market effect and proved to be useless for diversification purposes.

Whereas the first part of this thesis was concerned with the comovement of shares and hence with the composition of portfolios, the second part was confined to the analysis of individual securities. The specific classificatory problem was that of classifying firms into groups according to their relative performance on the stock market. The problem was multivariate in nature in that the classificatory process involved the derivation of models based on various financial characteristics of the firms, as well as in some economic characteristics of the specific time periods. The statistical technique considered appropriate for use in this problem was Multiple Discriminant Analysis, of which a theoretical overview was given in Chapter 5. Chapter 6 gave an overview of the previous successful applications of Multiple Discriminant Analysis in determining the usefulness of financial ratio analysis. In Chapters 7 and 8 a stepwise linear discrimininant analysis procedure was employed to derive classificatory functions based on data from three different samples.

In Chapter 7 a pooled sample was constructed containing shares that had performed well or poorly relative to the average performance of industrial shares for each of the years 1973 to 1979. A smaller, more restrictive sample, where a "no-man's-land" range of 60% was required between the returns of "good" and "bad" performers, was also constructed. By including only financial ratios, or financial and economic variables, and by varying the parameters of the variable selection procedure, several functions were derived for the two samples. Two functions were selected as being the most efficient in the classification of firms, one based on the inclusion of financial ratios alone, the other including both financial and economic variables. In Chapter 8 models were derived for the classification of shares in bull and bear markets based on data for 1979 and 1973, respectively.

The derived functions performed very well in discriminating between observations from the samples used in the derivation process. Less successful results were obtained when these functions were applied to observations from the same time period but not included in the original samples. The accuracy of classification dropped even further when the functions were applied to future time periods, except when these future time periods corresponded very closely in their financial and economic characteristics to those for which the functions were initially derived. Thus it could be concluded that the classificatory powers of the functions depended to some extent on the specific observations used in the derivation of these functions, and very much on the specific time periods under consideration.

An examination of the variables included in the most successful functions indicated that the characteristics of firms most closely related to their performance on the stock market were those describing their returns on investment, liquidity position and capital structure. Analysis of these characteristics could thus possibly provide some indications as to expected share performance. Furthermore, the superior performance of the function derived on the basis of both financial and economic variables indicated the usefulness of including economic indicators in the analysis.

The data used for the derivation of the discriminant functions were obtained from financial year-end statements corresponding to the second six months of the calendar years in which classifications were to be made, or to the first six months of the following calendar years. This was done because it was thought that the information contained in these financial statements would most accurately reflect the financial profiles of firms in the specific calendar years under consideration, and hence the extent to which this information was reflected in share prices could be determined. The excellent classificatory results obtained for the functions on the Jackknife samples indicated that a significant amount of this information was in fact anticipated by the market prior to the release of the financial statements. This was in accordance with the results of the Ball and Brown study mentioned in Chapter 1 and did not leave much hope for the successful application of the derived models in the prediction of future share performance. The predictory results obtained for the functions were then also poor.

However, the possibility of deriving predictive models was already ruled out before the start of the analysis by assuming the semi-strong form of the Efficient Market Hypothesis to hold for shares listed on the Johannesburg Stock Exchange. Thus the procedure adopted for the derivation of the discriminant functions was not aimed at the prediction of share performance, but rather at the determination of the relationship between information concerning financial characteristics of firms and their relative performance on the stock market. Superior predictive models could possibly have been obtained by using data from earlier financial years in the derivation of the functions but only if the Efficient Market Hypothesis is invalid. Furthermore, the inclusion of lagged values of the variables most closely related to share performance could possibly have proved to be useful in a predictive approach. However, due to "creative accounting" techniques, the values of the ratios would possibly have turned out to be fairly stable over time, thus causing additional multicollinearity problems.

The definition of the initial groups based on an inherently continuous variable like share returns lead to a problem with respect to the performance of the discriminant functions in the classification of intermediate shares. In previous studies the use of a "black-gray-white" method was suggested, as discussed in Chapter 6, section 6.2 With reference to this study such an approach would have involved dividing the z-scores into three intervals, one corresponding to scores depicting only bad performers, another corresponding to scores depicting only good performers, and a "gray area" in the centre for the intermediate and borderline cases. An analysis of the z-scores for the individual observations as generated by the various derived functions, however, showed a tremendous overlap in the z-scores of good and bad performers and any attempts to define black-gray-white zones proved useless. The problem of the classification of intermediate shares would, however, mainly arise in predictive problems and since prediction was not the primary aim of this study, the above deficiency was not considered to be too serious.

Various shortcomings of the specific discriminant analysis techniques employed have been discussed in previous chapters and possible refinements with respect to the inclusion of unequal prior probabilities of membership and costs of misclassification were suggested, as well as the use of discrete discriminant analysis procedures instead of standard linear procedures. The practical employment of these refinements and the determination of their usefulness in the derivation of possible superior discriminatory models, as well as the adoption of an approach more directly geared towards the derivation of possible predictive models, provide opportunities for further research.

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APPENDIX A

Names of Shares used in this study

COAL	:	1.	Amcoal	
		2.	Apex Mines Ltd	(APEX)
		3.	Clydesdale	(CLYSDL)
		4.	Tavistock	(TAVISTK)
		5.	Trans-Natal	(TRNSNTL)
		6.	Vierfontein	(VIERFNT)
		7.	Welgedacht	(WELGDCT)
GOLD	:	8.	Blyvooruitzicht	(BLYVOOR)
		9.	Doornfontein	(DOORNS)
		10.	Driefontein Consolidated	(DRIECON)
		11.	Kloof	
		12.	Western Areas	(WAREAS)
		13.	Wes Drie	
		14.	Western Deep Levels	(WSTNDP)
BANKS	:	15.	Bankorp	
		16.	Boland	
		17.	ICLEF	
		18.	Nedbank	
		19.	Stanbic	
		20.	T & T	
		21.	Volkskas	
BUILDING	:	22.	Alpha	
		23.	Boumat	
		24.	Everite Ltd	(EVRITE)
		25.	Grinaker Holdings Ltd	(GRNAKR)
		26.	LTA	
		27.	Murray and Roberts	(M & R)
		28.	Pretoria Portland Cement	(PPCEM)

CHEMICAL	:	29.	AECI	
		30.	Chemical Holdings	(CHEMHD)
		31.	De Beers Industrial Corporation	(DEBERL)
		32.	Lanchem	
		33.	Sentrachem	(SENCHEM)
		34.	Trek	
		35.	Triomf	
FOOD	:	36.	Cadbury Schwepps	(CADSWP)
		37.	Fedfood	
		38.	ICS	
		39.	I & J	
		40.	Kanhym	
		41.	Premier Group.	
		42.	Tiger Oats	(TIGOATS)
MOTOR	:	43.	Asseng	
		44.	Dunlop	
		45.	Gentra	
		46.	McCarthy	
		47.	Saficon	
		48.	Toyota	
		49.	WMHunt	

APPENDIX B

TABLES AND FIGURES GENERATED BY A FACTOR ANALYSIS OF SHARES LISTED ON THE JSE

TABLE 1

SQUARED MULTIPLE CORRELATIONS

	Shares	Total Period	1st Subperiod	2nd Subperiod
1	AMCOAL	.44844	.52201	.53882
2	APEXMIN	.38662	.46573	.47235
3	CLYSDL	.43384	.54497	.46969
4	TAVISTK	.43186	.46958	.53383
5	TRNSNTL	.51253	.59782	.54041
6	VIERFNT	.22213	.32201	.29416
7	WELGDCT	.41032	.47730	.47976
	COAL	.40653	.48563	.47557
8	BLYVOOR	.63159	.72542	.62938
9	DOORNS	.56867	.60890	.63663
10	DRIECON	.72500	.80671	.72341
11	KLOOF	.66618	.73944	.65983
12	WARGAS	.66840	.77481	.65554
13	WESDRIE	.63690	.70120	.68658
14	WSTNDP	.38408	.73469	.33389
	GOLD	.61155	.72731	.61789
15	BANKORP	.21850	.20224	.37539
16	BOLAND	.27693	.36565	.37494
17	ICLEF	.24637	.40671	.22956
18	NEDBANK	.41794	.47685	.50150
19	STANBIC	.31525	.30540	.46126
20	T & T	.17943	.28233	.24389
21	VOLKSKAS	.44022	.55223	.51005
	BANKS	.29923	.37020	.38523
22	ALPHA	.26489	.31314	.41746
23	BOUMAT	.38629	.53143	.35457
24	EVRITE	.21125	.29337	.26750
25	GRNAKR	.31738	.40330	.43072
26	LTA	.38002	.49560	.45525
27	M & R	.40073	.46413	.45598
28	PPCEM	.32347	.38729	.40391
	BUILDINGS	.32629	.41261	.39791
29	AECI	.43576	.50510	.45606
30	CHEMHD	.22247	.27891	.28648
31	DEBERL	.28134	.37594	.29975
32	LANCHEM	.19244	.30986	.30955
33	SENCHM	.32184	.41619	.40946
34	TREK	.24762	.34115	.33258
35	TRIOMF	.24330	.27576	.38402
	CHEMICALS	.27782	.35756	.35399
36	CADSWP	.20806	.36684	.33630
37	FEDFOOD	.24043	.36646	.29610
38	ICS	.39394	.43431	.49352
39	I & J	.19397	.24384	.34325
40	KANHYM	.22404	.28048	.31891
41	PREMGRP	.34893	.50370	.33452
42	TIGOATS	.49976	.65635	.40081
	FOOD	.30130	.40743	.36049
43	ASSENG	.18432	.32904	.24503
44	DUNLOP	.27485	.34634	.39096
45	GENTRA	.28570	.38830	.29923
46	MCCARTHY	.39674	.53980	.49197
47	SAFICON	.29149	.41122	.37578
48	TOYOTA	.30695	.42365	.31697
49	WMHUNT	.24337	.36379	.25988
	MOTOR	.28335	.40031	.33997
	AVERAGE	.35801	.45158	.41872

(0.43515)

TABLE 2

PROPORTION OF VARIANCE EXPLAINED BY FIRST FACTOR

(A) Principal Factor Analysis.

	Share	Total Period	1st Subperiod	2nd Subperiod
1	AMCOAL	.292 918	.341 424	.232 883
2	APEXMIN	.100 069	.093 785	.112 148
3	CLYSDL	.159 493	.210 298	.096 043
4	TAVISTK	.133 469	.158 249	.108 455
5	TRNSNTL	.153 280	.158 751	.145 040
6	VIERFNT	.064 202	.069 828	.060 746
7	WELGDCT	.123 734	.148 661	.099 172
	COAL	.146 735	.168 714	.122 070
8	BLYVOOR	.391 407	.412 798	.375 414
9	DOORNS	.391 449	.359 338	.453 412
10	DRIECON	.454 541	.463 782	.448 854
11	KLOOF	.460 650	.440 941	.481 592
12	WAREAS	.476 445	.442 837	.542 844
13	WESDRIE	.441 019	.408 001	.485 123
14	WSTNDP	.190 543	.374 967	.118 213
	GOLD	.400 865	.414 666	.415 065
15	BANKORP	.061 820	.030 736	.092 058
16	BOLAND	.084 571	.086 737	.068 483
17	ICLEF	.030 731	.049 404	.006 627
18	NEDBANK	.221 045	.217 492	.247 158
19	STANBIC	.118 837	.113 704	.107 259
20	T & T	.067 781	.090 306	.034 311
21	VOLKSKAS	.210 467	.180 614	.243 631
	BANKS	.113 607	.109 856	.114 218
22	ALPHA	.117 839	.077 761	.165 196
23	BOUMAT	.142 150	.157 143	.127 156
24	EVRITE	.085 219	.093 810	.065 481
25	GRNAKR	.158 831	.150 209	.159 855
26	LTA	.110 191	.083 978	.168 832
27	M & R	.186 633	.213 954	.137 873
28	PPCEM	.137 098	.122 792	.159 098
	BUILDING	.113 994	.128 521	.140 499
29	AECI	.240 216	.254 779	.205 954
30	CHEMHD	.067 095	.056 746	.066 710
31	DEBERL	.114 310	.126 195	.089 479
32	LANCHEM	.038 281	.079 678	.014 650
33	SENCHM	.191 187	.182 838	.201 348
34	TREK	.061 705	.064 145	.057 418
35	TRIOMF	.066 257	.056 586	.074 462
	CHEMICALS	.111 293	.117 281	.101 432
36	CADSWP	.072 090	.083 834	.041 896
37	FEDFOOD	.124 109	.136 886	.096 098
38	ICS	.204 172	.203 788	.199 597
39	I & J	.047 517	.045 680	.046 383
40	KANHYM	.034 704	.059 784	.069 524
41	PREMGRP	.189 771	.226 971	.142 550
42	TIGOATS	.267 333	.356 195	.154 041
	FOOD	.138 528	.159 020	.107 156
43	ASSENG	.010 050	.053 263	.000 069
44	DUNLOP	.082 862	.067 537	.099 723
45	GENTRA	.036 064	.048 430	.017 402
46	MCCARTHY	.191 084	.209 225	.172 335
47	SAFICON	.093 331	.095 645	.086 590
48	TOYOTA	.118 509	.127 781	.105 441
49	WMHUNT	.085 688	.110 249	.051 689
	MOTOR	.088 227	.101 733	.076 178
	AVERAGE	.161 893	.171 399	.153 803

TABLE 2

(CONTINUED)

(B) Little Jiffy.

	Share	Total Period	1st Subperiod	2nd Subperiod
1	AMCOAL	.280 689	.281 954	.263 356
2	APEXMIN	.089 813	.055 494	.130 682
3	CLYSDL	.159 473	.154 708	.128 674
4	TAVISTK	.130 519	.099 541	.145 618
5	TRNSNTL	.156 524	.113 566	.188 859
6	VIERFNT	.061 860	.053 818	.064 677
7	WELGDCT	.118 272	.110 259	.124 348
	COAL	.142 450	.124 191	.149 459
8	BLYVOOR	.356 724	.516 716	.275 551
9	DOORNS	.362 343	.445 114	.334 730
10	DRIECON	.420 075	.606 827	.332 587
11	KLOOF	.423 062	.565 191	.365 863
12	WAREAS	.429 992	.579 913	.398 556
13	WESDRIE	.415 751	.520 893	.422 763
14	WSTNDP	.165 377	.519 823	.060 973
	GOLD	.367 618	.536 354	.313 003
15	BANKORP	.059 670	.019 157	.116 333
16	BOLAND	.085 999	.047 809	.088 735
17	ICLEF	.028 079	.026 521	.007 730
18	NEDBANK	.225 401	.137 489	.283 005
19	STANBIC	.122 865	.074 233	.157 971
20	T & T	.066 054	.057 024	.031 585
21	VOLKSKAS	.213 058	.092 817	.296 885
	BANKS	.114 447	.004 803	.140 321
22	ALPHA	.121 112	.050 176	.221 728
23	BOUMAT	.140 502	.092 984	.380 756
24	EVRITE	.079 633	.048 854	.078 099
25	GRNAKR	.146 130	.083 137	.167 203
26	LTA	.099 675	.029 816	.215 650
27	M & R	.193 810	.140 909	.200 354
28	PPCEM	.137 098	.065 058	.194 682
	BUILDING	.131 137	.072 991	.208 353
29	AECI	.250 069	.194 774	.262 357
30	CHEMHD	.055 450	.036 507	.065 267
31	DEBERL	.114 310	.086 936	.097 996
32	LANCHEM	.031 895	.047 963	.011 937
33	SENCHEM	.187 075	.121 276	.215 391
34	TREK	.058 324	.031 721	.077 302
35	TRIOMF	.048 483	.031 504	.051 085
	CHEMICAL	.106 599	.078 669	.111 619
36	CADSWP	.071 026	.062 364	.066 046
37	FEDFOOD	.117 704	.108 926	.107 637
38	ICS	.212 178	.137 959	.258 362
39	I & J	.042 135	.023 149	.047 173
40	KANHYM	.060 813	.034 610	.068 380
41	PREMGRP	.192 196	.160 756	.179 293
42	TIGOATS	.285 555	.263 602	.207 157
	FOOD	.140 230	.113 052	.133 435
43	ASSENG	.007 865	.020 379	.000 044
44	DUNLOP	.073 788	.026 817	.115 783
45	GENTRA	.035 121	.019 974	.022 572
46	MCCARTHY	.178 419	.001 152	.001 894
47	SAFICON	.075 674	.041 834	.084 687
48	TOYOTA	.106 191	.062 975	.106 467
49	WMHUNT	.080 634	.064 682	.070 664
	MOTOR	.079 670	.033 973	.057 444
	AVERAGE	.154 593	.137 719	.159 091

(B-4)

FIGURE 1: RESIDUAL COVARIANCE MATRIX

		AMCOAL 1	APEXMIN 2	CLYSOL 3	TAVISTK 4	TRNSNTL 5	VIERFNT 6	WELGDCT 7	BLYVOOR 8	DOORNS 9	DRIECON 10	KLOOF 11	WAREAS 12	WESDRIE 13	WSTNDP 14
AMCOAL	1	.001 350													
APEXMIN	2	.000 470	.002 080												
CLYSOL	3	.000 412	.000 504	.001 990											
TAVISTK	4	.000 488	.000 788	.000 601	.002 110										
TRNSNTL	5	.000 529	.000 548	.001 010	.000 654	.002 010									
VIERFNT	6	.000 295	.000 293	.000 387	.000 370	.000 570	.003 880								
WELGDCT	7	.000 558	.001 040	.000 750	.000 980	.000 866	.000 700	.003 580							
BLYVOOR	8	-.000 288	-.000 304	-.000 244	-.000 561	-.000 275	-.000 310	-.000 378	.002 270						
DOORNS	9	-.000 316	-.000 444	-.000 339	-.000 462	-.000 548	-.000 416	-.000 321	.001 150	.003 690					
DRIECON	10	-.000 302	-.000 381	-.000 382	-.000 486	-.000 484	-.000 315	-.000 411	.001 010	.001 080	.001 900				
KLOOF	11	-.000 319	-.000 359	-.000 213	-.000 474	-.000 204	-.000 286	-.000 531	.001 050	.001 050	.001 040	.002 290			
WAREAS	12	-.000 254	-.000 325	-.000 238	-.000 341	-.000 248	-.000 163	-.000 260	.000 905	.001 230	.001 150	.001 060	.002 680		
WESDRIE	13	-.000 161	-.000 131	-.000 228	-.000 309	-.000 150	-.000 147	-.000 255	.000 775	.000 697	.000 831	.000 774	.000 756	.001 530	
WSTNDP	14	-.000 202	-.000 282	-.000 315	-.000 574	-.000 269	-.000 443	-.000 466	.001 040	.001 060	.001 020	.000 976	.001 290	.000 494	.005 350
BANKORP	15	.000 136	.000 051	.000 044	.000 075	.000 114	.000 220	.000 198	-.000 486	-.000 335	-.000 360	-.000 237	-.000 306	-.000 188	-.000 288
BOLAND	16	.000 096	-.000 008	.000 069	.000 033	.000 115	-.000 031	-.000 066	-.000 332	-.000 381	-.000 373	-.000 243	-.000 293	-.000 252	-.000 195
ICLEF	17	-.000 096	.000 126	-.000 093	-.000 045	.000 055	.000 113	-.000 090	-.000 150	-.000 264	-.000 149	-.000 224	-.000 160	-.000 089	-.000 102
NEDBANK	18	.000 076	.000 012	.000 136	.000 114	.000 144	.000 109	-.000 164	-.000 355	-.000 333	-.000 350	-.000 391	-.000 528	-.000 344	-.000 490
STANBIC	19	.000 179	-.000 010	.000 112	.000 019	.000 132	.000 134	.000 130	-.000 221	-.000 319	-.000 274	-.000 304	-.000 358	-.000 287	-.000 337
T & T	20	.000 046	-.000 006	-.000 165	.000 084	-.000 295	-.000 191	.000 029	-.000 273	-.000 343	-.000 325	-.000 133	-.000 440	-.000 171	-.000 258
VOLKSKAS	21	.000 100	.000 077	.000 166	.000 201	.000 211	.000 065	.000 080	-.000 454	-.000 385	-.000 505	-.000 455	-.000 466	-.000 287	-.000 500
ALPHA	22	.000 070	.000 012	.000 070	.000 189	.000 011	-.000 174	.000 075	-.000 230	-.000 118	-.000 261	-.000 313	-.000 218	-.000 264	-.000 577
BOUMAT	23	.000 099	-.000 113	.000 061	.000 104	-.000 140	.000 186	-.000 038	-.000 435	-.000 189	-.000 338	-.000 417	-.000 568	-.000 347	-.000 438
EVRITE	24	.000 005	.000 128	.000 040	.000 218	-.000 012	.000 041	.000 200	-.000 277	-.000 155	-.000 242	-.000 269	-.000 181	-.000 143	-.000 256
GRNAKR	25	.000 017	.000 125	-.000 130	-.000 088	-.000 040	-.000 017	.000 071	-.000 292	-.000 365	-.000 413	-.000 393	-.000 501	-.000 320	-.000 088
LTA	26	-.000 062	.000 095	-.000 041	.000 078	-.000 091	-.000 170	-.000 109	-.000 690	-.000 507	-.000 450	-.000 662	-.000 666	-.000 338	-.000 592
M & R	27	.000 056	.000 058	.000 044	.000 089	.000 048	-.000 057	-.000 074	-.000 500	-.000 491	-.000 444	-.000 535	-.000 306	-.000 299	-.000 446
PPCEM	28	.000 127	.000 097	.000 124	.000 134	.000 069	-.000 073	.000 054	-.000 247	-.000 279	-.000 359	-.000 344	-.000 440	-.000 312	-.000 330
AECI	29	.000 037	-.000 066	.000 060	-.000 054	.000 196	.000 100	.000 008	-.000 240	-.000 348	-.000 287	-.000 286	-.000 365	-.000 198	-.000 315
CHEMHD	30	.000 059	.000 091	-.000 039	.000 042	-.000 112	.000 043	.000 157	-.000 208	-.000 196	-.000 230	.000 301	-.000 228	-.000 147	-.000 092
DEBERL	31	.000 073	-.000 057	-.000 001	.000 056	.000 029	.000 072	.000 205	-.000 126	-.000 062	-.000 152	-.000 181	-.000 164	-.000 148	-.000 152
LANCHEM	32	-.000 044	.000 206	-.000 006	.000 137	.000 120	.000 532	.000 015	-.000 210	-.000 520	-.000 319	-.000 198	-.000 337	-.000 278	-.000 263
SENCHM	33	-.000 008	.000 015	.000 038	-.000 028	-.000 008	.000 054	.000 161	-.000 394	-.000 374	-.000 252	-.000 323	-.000 317	-.000 200	-.000 265
TREK	34	.000 046	.000 064	-.000 104	-.000 002	.000 136	.000 344	.000 191	-.000 311	-.000 480	-.000 454	-.000 460	-.000 399	-.000 342	-.000 387
TRIOMF	35	.000 116	.000 228	.000 022	.000 171	.000 029	.000 528	.000 197	-.000 445	-.000 571	-.000 506	-.000 246	-.000 336	-.000 384	.000 134
CAOSWP	36	.000 100	.000 105	.000 039	.000 110	.000 053	.000 093	.000 156	-.000 142	-.000 178	-.000 256	-.000 338	-.000 332	-.000 160	-.000 311
FEDFOOD	37	-.000 009	-.000 056	-.000 052	.000 083	-.000 142	-.000 159	.000 106	-.000 146	-.000 077	-.000 222	-.000 220	-.000 246	-.000 207	-.000 215
ICS	38	-.000 030	-.000 051	.000 035	.000 144	.000 078	.000 179	-.000 015	-.000 357	-.000 596	-.000 390	-.000 356	-.000 482	-.000 289	-.000 498
I & J	39	.000 197	.000 095	.000 104	.000 189	.000 039	.000 258	-.000 033	-.000 474	-.000 332	-.000 348	-.000 302	-.000 370	-.000 452	-.000 178
KANHYM	40	.000 198	.000 137	.000 115	.000 256	.000 255	.000 229	.000 202	-.000 360	-.000 521	-.000 388	-.000 270	-.000 448	-.000 269	-.000 241
PREMGRP	41	-.000 002	-.000 096	-.000 049	.000 207	.000 023	-.000 011	.000 053	-.000 328	-.000 136	-.000 227	-.000 250	-.000 274	-.000 254	-.000 315
TIGOATS	42	.000 049	-.000 078	.000 029	.000 114	.000 047	.000 178	-.000 069	-.000 335	-.000 357	-.000 205	-.000 226	-.000 364	-.000 245	-.000 404
ASSENG	43	.000 041	-.000 076	.000 086	.000 159	-.000 103	.000 373	.000 062	-.000 395	-.000 315	-.000 377	-.000 431	-.000 376	-.000 262	-.000 286
DUNLOP	44	-.000 073	-.000 002	-.000 092	-.000 112	-.000 094	-.000 086	.000 071	-.000 445	-.000 356	-.000 504	-.000 432	-.000 472	-.000 267	-.000 355
GENTRA	45	-.000 067	-.000 051	.000 052	.000 069	-.000 034	.000 009	.000 005	-.000 309	-.000 194	-.000 280	-.000 193	-.000 347	-.000 315	-.000 336
MCCARTHY	46	.000 058	-.000 022	-.000 170	.000 109	-.000 196	-.000 298	-.000 246	-.000 598	-.000 454	-.000 468	-.000 528	-.000 518	-.000 302	-.000 875
SAFICON	47	-.000 064	.000 068	-.000 035	.000 041	-.000 142	-.000 027	-.000 173	-.000 421	-.000 500	-.000 594	-.000 550	-.000 584	-.000 511	-.000 662
TOYOTA	48	-.000 032	.000 115	.000 060	.000 102	-.000 107	-.000 235	-.000 125	-.000 514	-.000 525	-.000 558	-.000 368	-.000 444	-.000 210	-.000 340
WMHUNT	49	.000 115	-.000 072	.000 139	.000 324	.000 225	.000 231	.000 286	-.000 440	-.000 495	-.000 407	-.000 351	-.000 633	-.000 337	-.000 589

FIGURE I (CONTINUED):

		BANKORP 15	BOLAND 16	ICLEF 17	NEDBANK 18	STANBK 19	T&T 20	VOLKSKAS 21	ALPHA 22	BOUMAT 23	EVRITG 24	GRNAKR 25	LTA 26	M&R 27	PPCEM 28
BANKORP	15	.001 970													
BOLAND	16	.000 224	001 530												
ICLEF	17	.000 008	.000 174	.001 460											
NEDBANK	18	.000 259	.000 171	.000 026	.001 460										
STANBIC	19	.000 217	.000 156	-.000 014	.000 196	.001 040									
T&T	20	.000 067	.000 236	-.000 043	.000 222	.000 122	.003 340								
VOLKSKAS	21	.000 198	.000 206	.000 046	.000 310	.000 141	.000 227	.000 989							
ALPHA	22	.000 208	.000 040	-.000 012	.000 279	.000 132	.000 189	.000 197	001 570						
BOUMAT	23	.000 103	.000 321	.000 186	.000 096	.000 244	.000 133	.000 149	.000 113	001 780					
EVRITE	24	.000 090	.000 005	.000 069	.000 115	.000 079	.000 210	.000 097	.000 150	.000 098	000 866				
GRNAKR	25	-.000 029	.000 201	.000 051	.000 087	.000 270	.000 201	.000 165	.000 089	.000 224	.000 083	.002 030			
LTA	26	.000 249	.000 275	.000 315	.000 233	-.000 003	.000 252	.000 373	.000 087	.000 107	.000 100	.000 523	002 740		
M&R	27	.000 103	.000 232	.000 163	.000 338	.000 154	.000 261	.000 309	.000 235	.000 251	.000 107	.000 369	.000 557	001 920	
PPCEM	28	.000 051	.000 280	.000 082	.000 152	.000 181	.000 231	.000 192	.000 198	.000 227	.000 064	.000 195	.000 153	.000 183	.001 110
AECI	29	.000 137	.000 189	.000 141	.000 301	.000 301	.000 149	.000 221	.000 199	.000 107	-.000 016	.000 152	.000 164	.000 283	.000 191
CHEMHD	30	.000 101	.000 113	.000 043	-.000 079	.000 116	.000 200	-.000 069	.000 056	.000 162	.000 157	.000 082	.000 234	.000 024	.000 003
DEBERL	31	.000 069	.000 061	-.000 094	.000 086	.000 171	.000 120	.000 063	.000 031	.000 062	.000 042	.000 088	.000 004	.000 216	.000 030
LANCHEM	32	-.000 124	-.000 005	.000 104	.000 483	.000 072	.000 234	.000 217	-.000 036	-.000 380	-.000 012	.000 321	.000 246	.000 054	.000 152
SENCHM	33	.000 251	.000 162	.000 067	.000 216	.000 075	.000 043	.000 103	.000 151	.000 175	.000 075	.000 134	.000 378	.000 203	.000 094
TREK	34	.000 230	-.000 079	.000 232	.000 083	.000 132	-.000 358	.000 190	.000 072	.000 191	.000 053	.000 180	.000 446	.000 094	.000 202
TRIOMF	35	.000 271	-.000 105	.000 123	.000 087	.000 118	.000 094	.000 077	.000 065	.000 162	-.000 056	.000 139	.000 048	-.000 102	.000 024
CADSWP	36	.000 063	.000 154	-.000 163	.000 037	.000 184	.000 247	.000 113	.000 196	.000 342	.000 157	.000 150	.000 103	.000 067	.000 130
FEDFOOD	37	-.000 075	.000 020	.000 087	.000 040	.000 024	.000 141	.000 134	.000 088	.000 165	.000 040	.000 141	.000 347	-.000 014	.000 191
ICS	38	.000 197	.000 149	-.000 006	.000 361	.000 188	.000 217	.000 348	.000 209	.000 244	.000 063	.000 269	.000 312	.000 393	.000 192
I&J	39	.000 043	.000 224	.000 107	.000 172	.000 130	.000 516	.000 176	.000 107	.000 185	.000 018	-.000 126	.000 270	.000 232	-.000 020
KANHYM	40	.000 062	.000 070	.000 257	.000 212	.000 269	.000 090	.000 237	.000 049	.000 171	.000 076	.000 141	.000 047	.000 004	.000 021
PREMGRP	41	.000 012	.000 182	.000 071	.000 156	.000 083	.000 161	.000 230	.000 129	.000 276	.000 083	.000 088	.000 104	.000 202	.000 065
TIGOATS	42	.000 089	.000 060	.000 047	.000 293	.000 125	.000 176	.000 230	.000 048	.000 227	.000 022	.000 175	.000 195	.000 326	.000 142
ASSENG	43	.000 225	.000 104	.000 149	.000 114	.000 199	.000 159	.000 079	.000 085	.000 375	.000 164	.000 178	.000 305	.000 267	.000 146
DUNLOP	44	.000 109	.000 056	.000 076	.000 169	.000 239	.000 135	.000 179	.000 075	.000 319	.000 165	.000 158	.000 383	.000 310	.000 297
GENTRA	45	.000 076	.000 053	.000 399	.000 084	-.000 055	.000 028	.000 193	.000 105	.000 406	.000 106	.000 035	.000 338	.000 305	.000 085
MCCARTHY	46	.000 287	.000 282	.000 156	.000 368	.000 112	.000 380	.000 304	.000 275	.000 422	.000 231	.000 265	.000 567	.000 300	.000 281
SAFICON	47	.000 371	.000 159	.000 165	.000 151	.000 114	.000 277	.000 184	-.000 009	.000 505	.000 073	.000 337	.000 986	.000 309	.000 042
TOYOTA	48	-.000 039	.000 161	-.000 068	.000 210	.000 047	.000 241	.000 198	.000 231	.000 076	.000 194	.000 359	.000 655	.000 314	.000 218
WMHUNT	49	.000 236	.000 199	.000 201	.000 107	.000 126	-.000 069	.000 167	.000 149	.000 344	.000 127	.000 093	.000 245	.000 113	.000 250

FIGURE 1 (CONTINUED):

		AECI 29	CHEMHD 30	DEBERL 31	LANCHEM 32	SENCHM 33	TREK 34	TRIOMF 35	CADSWP 36	FEDFOOD 37	ICS 38	I&J 39	KANHYM 40	PREMGRP 41	TIGOATS 42
AECI	29	.001 090													
CHEMHD	30	-.000 044	.001 360												
DEBERL	31	.000 044	.000 082	.000 725											
LANCHEM	32	.000 170	-.000 338	.000 132	.007 450										
SENCHM	33	.000 289	.000 120	.000 048	.000 057	.001 440									
TREK	34	.000 376	.000 310	.000 058	-.000 062	.000 143	.003 150								
TRIOMF	35	.000 065	.000 206	.000 141	.001 330	.000 174	.000 116	.003 890							
CADSWP	36	.000 005	.000 060	.000 212	-.000 077	.000 043	.000 264	-.000 141	.002 340						
FEDFOOD	37	.000 086	.000 042	.000 083	-.000 093	.000 058	.000 161	.000 095	.000 086	.001 650					
ICS	38	.000 203	-.000 009	.000 053	.000 201	.000 178	.000 262	.000 051	.000 255	.000 053	.001 650				
I&J	39	.000 202	.000 047	-.000 068	.000 299	.000 130	.000 020	.000 016	-.000 099	.000 104	.000 217	.002 880			
KANHYM	40	.000 071	.000 037	.000 010	.000 504	.000 079	.000 299	.000 431	.000 101	.000 026	.000 224	.000 378	.002 480		
PREMGRP	41	.000 154	.000 055	.000 087	.000 052	.000 093	.000 049	-.000 008	.000 018	.000 104	.000 275	.000 080	.000 072	.001 050	
TIGOATS	42	.000 269	-.000 024	.000 057	.000 074	.000 179	.000 231	.000 003	.000 063	.000 208	.000 380	.000 188	.000 205	.000 233	.000 870
ASSENG	43	-.000 033	.000 203	.000 022	-.000 194	.000 001	.000 076	.000 228	.000 156	-.000 050	.000 155	-.000 025	-.000 321	.000 088	.000 116
DUNLOP	44	.000 182	.000 144	.000 104	.000 172	.000 044	.000 482	-.000 028	.000 287	.000 113	.000 208	.000 087	.000 099	.000 126	.000 140
GENTRA	45	.000 099	.000 092	.000 099	-.000 207	.000 119	-.000 084	.000 078	-.000 036	.000 112	.000 205	.000 170	.000 071	.000 130	.000 099
MCCARTHY	46	.000 098	.000 351	-.000 053	.000 059	.000 280	.000 495	.000 231	.000 114	.000 136	.000 276	.000 262	.000 130	.000 122	.000 091
SAFICON	47	.000 024	.000 322	-.000 122	.000 153	.000 119	.000 330	.000 520	.000 099	.000 297	.000 174	.000 522	.000 094	.000 115	-.000 004
TOYOTA	48	.000 008	.000 295	-.000 049	.000 005	.000 230	.000 196	-.000 019	-.000 033	.000 158	.000 240	.000 305	.000 244	.000 226	.000 078
WMHUNT	49	.000 170	.000 129	.000 108	.000 211	.000 274	.000 240	.000 128	.000 367	.000 185	.000 135	.000 185	.000 073	.000 122	.000 095

FIGURE 1 (CONTINUED)

		ASSENG 43	DUNLOP 44	GENTRA 45	MCCARTHY 46	SAFICON 47	TOYOTA 48	WM HUNT 49
ASSENG	43	.002 660						
DUNLOP	44	.000 296	.002 230					
GENTRA	45	.000 179	.000 207	.001 540				
MCCARTHY	46	.000 490	.000 435	.000 213	.003 030			
SAFICON	47	.000 522	.000 490	.000 535	.000 855	.004 760		
TOYOTA	48	.000 342	.000 633	.000 119	.000 644	.000 577	.003 160	
WMHUNT	49	.000 241	.000 330	.000 342	.000 629	.000 529	.000 281	.002 970

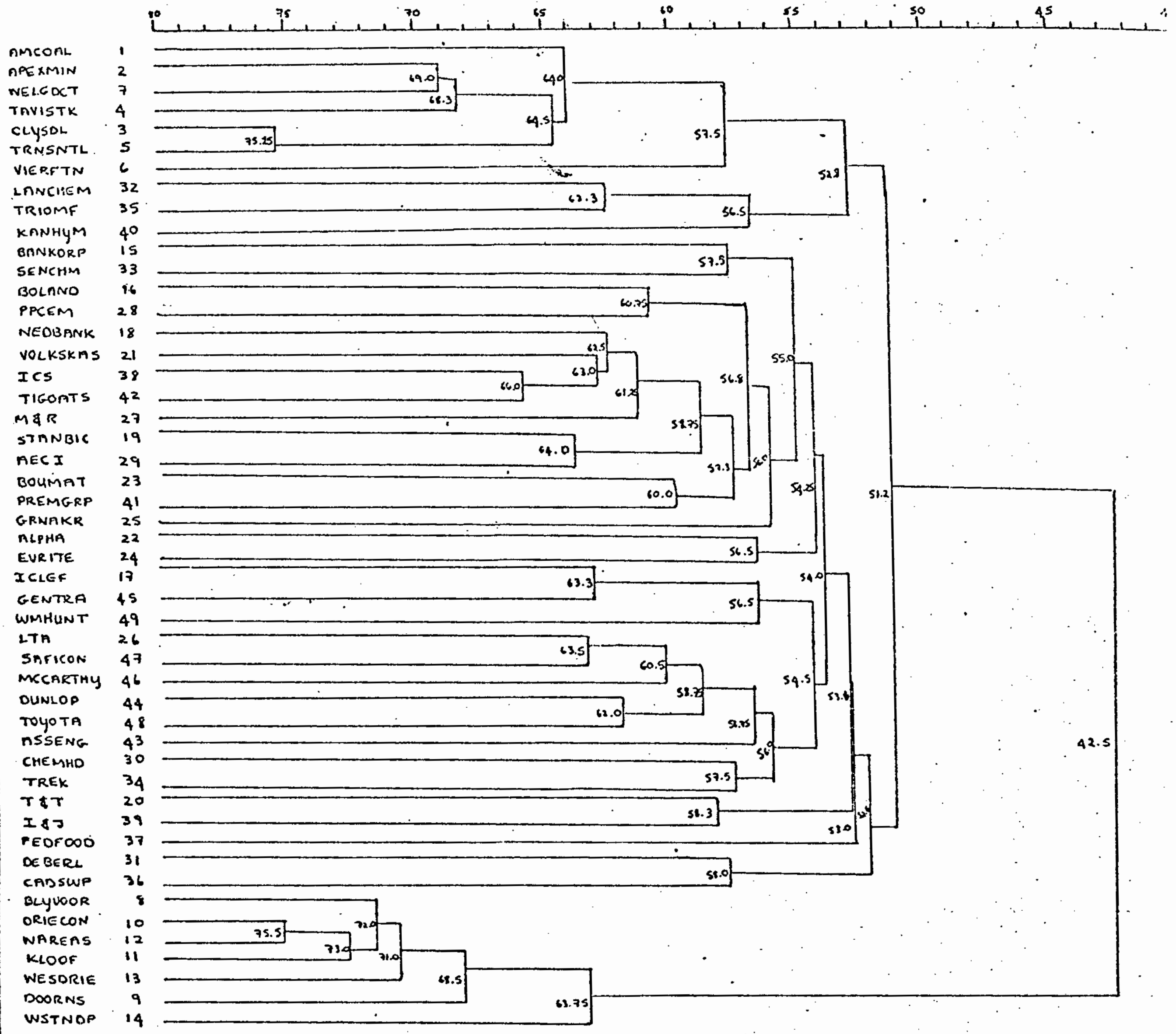

Figure 2: Dendrogram resulting from cluster analysis of residual covariance matrix

TABLE 3

UNROTATED FACTOR PATTERN, PRINCIPAL FACTOR ANALYSIS, TOTAL PERIOD

Share		Factor 1	Factor 2	Factor 3	Factor 4
AMCOAL	1	.0237	-.0089	.0145	-.0048
APEXMIN	2	.0152	-.0106	.0206	-.0030
CLYSDL	3	.0194	-.0093	.0214	-.0075
TAVISTK	4	.0180	-.0152	.0207	-.0071
TRANSNTL	5	.0190	-.0103	.0264	-.0042
VIERFNT	6	.0163	-.0102	.0188	.0073
WELGDCT	7	.0224	-.0127	.0302	-.0099
COAL		.1340	-.0772	.1526	-.0293
BLYVOOR	8	.0375	.0299	-.0015	.0005
DOORNS	9	.0475	.0314	-.0057	-.0069
DRIECON	10	.0388	.0307	-.0039	-.0010
KLOOF	11	.0432	.0296	-.0016	.0035
WAREAS	12	.0480	.0319	.0013	-.0009
WESDRIE	13	.0344	.0214	-.0002	-.0029
WSTNDP	14	.0351	.0313	-.0011	.0077
GOLD		.2845	.2062	-.0127	.0000
BANKORP	15	.0114	-.0111	-.0001	-.0014
BOLAND	16	.0119	-.0104	-.0039	-.0027
ICLEF	17	.0068	-.0065	-.0031	.0031
NEDBANK	18	.0204	-.0139	-.0026	.0058
STANBIC	19	.0119	-.0101	.0006	-.0003
T & T	20	.0156	-.0102	-.0076	.0016
VOLKSKAS	21	.0163	-.0150	-.0008	.0005
BANKS		.0943	-.0772	-.0175	-.0066
ALPHA	22	.0145	-.0096	-.0022	-.0047
BOUMAT	23	.0172	-.0143	-.0076	-.0081
EVRITE	24	.0090	-.0075	-.0003	-.0045
GRNAKR	25	.0196	-.0125	-.0063	.0040
LTA	26	.0184	-.0210	-.0145	.0001
M & R	27	.0210	-.0157	-.0074	-.0030
PPCEM	28	.0133	-.0113	-.0021	-.0023
BUILDING		.1130	-.0919	-.0404	-.0185
AECI	29	.0186	-.0103	-.0013	.0025
CHEMHD	30	.0099	-.0075	-.0041	-.0056
DEBERL	31	.0096	-.0046	.0017	.0002
LANCHEM	32	.0172	-.0113	.0067	.0349
SENCHEM	33	.0185	-.0108	-.0028	.0005
TREK	34	.0144	-.0146	-.0030	-.0011
TRIOMF	35	.0166	-.0122	.0058	.0257
CHEMICALS		.1048	-.0713	.0031	.0571
CADSWP	36	.0135	-.0088	-.0003	-.0072
FEDFOOD	37	.0153	-.0070	-.0051	-.0023
ICS	38	.0206	-.0152	-.0038	.0017
I & J	39	.0120	-.0122	-.0013	.0047
KANHYM	40	.0131	-.0121	.0057	.0110
PREMGRP	41	.0157	-.0089	-.0031	.0020
TIGOATS	42	.0179	-.0107	-.0024	.0010
FOOD		.1081	-.0749	-.0103	.0069
ASSENG	43	.0052	-.0125	-.0059	-.0060
DUNLOP	44	.0142	-.0150	-.0099	-.0024
GENTRA	45	.0076	-.0103	-.0065	-.0045
MCCARTHY	46	.0267	-.0209	-.0165	-.0034
SAFICON	47	.0221	-.0217	-.0159	.0017
TOYOTA	48	.0206	-.0164	-.0119	-.0030
WMHUNT	49	.0167	-.0176	-.0014	-.0037
MOTOR		.1131	-.1144	-.0680	-.0213

TABLE 4

PRINCIPAL-FACTOR ANALYSIS, NO ROTATION, TOTAL PERIOD

(A) Percentage of Communality Explained

(B) Percentage of Total Variance Explained

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	52.83	52.83	16.19	16.19
2	Gold, (-) Motor	29.45	82.28	8.76	24.95
3	Coal	11.47	93.74	3.64	28.59
4	Lanchem & Triomf	6.26	100.00	1.43	30.02

TABLE 5

VARIMAX ROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS

TOTAL PERIOD

Share		Factor 2	Factor 3	Factor 4
AMCOAL	1	.003	.017	.001
APEXMIN	2	.001	.023	.004
CLYSDL	3	.001	.024	-.001
TAVISTK	4	.007	.026	.002
TRANSNTL	5	-.001	.028	.003
VIERFNT	6	-.001	.019	.013
WELGDCT	7	.001	.034	-.001
COAL		.011	.171	.021
BLYVOOR	8	-.025	-.012	-.011
DOORNS	9	-.022	-.014	-.019
DRIECON	10	-.024	-.014	-.013
KLOOF	11	-.026	-.013	-.008
WAREAS	12	-.028	-.010	-.013
WESDRIE	13	-.017	-.007	-.011
WSTNDP	14	-.029	-.014	-.005
GOLD		-.171	-.084	-.080
BANKORP	15	.010	.004	.003
BOLAND	16	.011	.001	.001
ICLEF	17	.006	-.001	.005
NEDBANK	18	.011	.001	.010
STANBIC	19	.008	.004	.004
T & T	20	.011	-.004	.004
VOLKSKAS	21	.013	.004	.006
BANKS		.070	.009	.033
ALPHA	22	.011	.003	-.001
BOUMAT	23	.018	.000	-.003
EVRITE	24	.008	.004	-.001
GRNAKR	25	.012	-.002	.008
LTA	26	.024	-.006	.006
M & R	27	.017	-.000	.002
PPCEM	28	.011	.003	.002
BUILDINGS		.101	.002	.013
AECI	29	.009	.002	.006
CHEMHD	30	.010	.000	-.003
DEBERL	31	.003	.003	.002
LANCHEM	32	-.004	.001	.037
SENCHEM	33	.010	.001	.004
TREK	34	.014	.003	.004
TRIOMF	35	.000	.003	.029
CHEMICALS		.042	.013	.060
CADSWP	36	.010	.005	-.003
FEDFOOD	37	.009	-.002	-.000
ICS	38	.014	.001	.007
I & J	39	.010	.002	.009
KANHYM	40	.005	.006	.015
PREMGRP	41	.009	.001	.001
TIGOATS	42	.010	.001	.005
FOOD		.067	.014	.034
ASSENG	43	.015	.001	-.001
DUNLOP	44	.018	-.003	.002
GENTRA	45	.013	-.001	-.001
MCCARTHY	46	.026	-.007	.003
SAFICON	47	.025	-.007	.008
TOYOTA	48	.020	-.004	.002
WMHUNT	49	.017	.006	.003
MOTOR		.134	-.015	.016

TABLE 6

PRINCIPAL-FACTOR ANALYSIS, VARIMAX ROTATION, TOTAL PERIOD

(A) Percentage of Communality Explained

(B) Percentage of Total Variance Explained

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	52.30	52.30	16.19	16.19
2	Gold, Motor	25.00	77.30	7.40	23.59
3	Coal	13.60	90.90	4.10	27.69
4	Lanchem & Triomf	9.10	100.00	2.20	29.89

TABLE 7

UNROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS, FIRST SUBPERIOD

Share		Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
AMCOAL	1	.0263	-.0037	.0161	.0004	-.0052
APEXMIN	2	.0156	-.0065	.0260	.0087	.0082
CLYSDL	3	.0253	-.0065	.0258	.0058	-.0001
TAVISTK	4	.0232	-.0011	.0275	.0061	-.0026
TRNSNTL	5	.0214	-.0075	.0318	.0050	.0077
VIERFNT	6	.0172	-.0053	.0244	-.0070	.0031
WELGDCT	7	.0281	-.0050	.0385	.0145	-.0005
COAL		.1571	-.0356	.1901	.0335	.0106
BLYVOOR	8	.0404	.0318	-.0066	-.0034	.0035
DOORNS	9	.0478	.0345	-.0109	.0094	-.0069
DRIECON	10	.0424	.0342	-.0088	.0003	-.0027
KLOOF	11	.0454	.0356	-.0038	-.0055	.0035
WAREAS	12	.0478	.0393	-.0010	.0038	.0033
WESDRIE	13	.0331	.0257	-.0026	.0013	.0017
WSTNDP	14	.0372	.0352	-.0058	-.0022	.0022
GOLD		.2941	.2363	-.0395	.0037	.0046
BANKORP	15	.0077	-.0069	.0012	.0021	-.0068
BOLAND	16	.0132	-.0124	.0019	-.0043	-.0046
ICLEF	17	.0101	-.0107	-.0077	.0044	.0054
NEDBANK	18	.0244	-.0188	.0003	-.0174	.0006
STANBIC	19	.0125	-.0088	.0035	-.0069	-.0043
T & T	20	.0224	-.0158	-.0056	-.0113	-.0059
VOLKSKAS	21	.0166	-.0177	.0008	-.0024	-.0039
BANKS		.1069	-.0911	-.0056	-.0358	-.0195
ALPHA	22	.0122	-.0074	-.0026	-.0075	-.0077
BOUMAT	23	.0221	-.0195	-.0072	.0013	-.0253
EVRITE	24	.0097	-.0080	.0023	.0029	-.0032
GRNAKR	25	.0207	-.0176	-.0050	-.0036	.0057
LTA	26	.0193	-.0283	-.0187	.0191	.0129
M & R	27	.0260	-.0183	-.0059	-.0054	-.0021
PPCEM	28	.0147	-.0143	.0006	-.0026	-.0046
BUILDING		.1247	-.1134	-.0365	.0042	-.0243
AECI	29	.0191	-.0097	-.0038	-.0087	-.0003
CHEMHD	30	.0096	-.0052	-.0029	.0093	-.0045
DEBERL	31	.0100	-.0048	.0044	-.0016	-.0069
LANCHEM	32	.0232	-.0130	.0016	-.0197	.0280
SENCHM	33	.0194	-.0135	-.0026	-.0011	-.0054
TREK	34	.0155	-.0155	-.0081	.0039	.0084
TRIOMF	35	.0130	-.0107	.0010	-.0071	.0066
CHEMICALS		.1098	-.0724	-.0104	-.0250	.0259
CADSWP	36	.0160	-.0052	.0031	.0005	-.0140
FEDFOOD	37	.0176	-.0057	-.0078	.0069	-.0024
ICS	38	.0237	-.0017	-.0023	-.0117	-.0067
I & J	39	.0118	-.0012	-.0043	-.0054	.0031
KANHYM	40	.0138	-.0013	.0043	-.0106	.0040
PREMGRP	41	.0202	-.0011	-.0028	-.0041	-.0070
TIGOATS	42	.0229	-.0012	-.0019	-.0106	-.0068
FOOD		.1260	-.0174	-.0117	-.0350	-.0298
ASSENG	43	.0097	-.0146	-.0036	.0038	-.0084
DUNLOP	44	.0146	-.0185	-.0067	.0057	.0010
GENTRA	45	.0109	-.0149	-.0087	.0120	-.0082
MCCARTHY	46	.0283	-.0252	-.0135	-.0007	.0059
SAFICON	47	.0251	-.0257	-.0228	.0214	.0116
TOYOTA	48	.0198	-.0208	-.0058	.0025	.0089
WMHUNT	49	.0188	-.0153	-.0014	.0074	-.0022
MOTOR		.1272	-.1350	-.0625	.0521	.0086

TABLE 8

PRINCIPAL-FACTOR ANALYSIS, NO ROTATION, FIRST SUBPERIOD

(A) Percentage of Communality Explained

(B) Percentage of Total Variance Explained

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	47.40	47.40	17.14	17.14
2	Gold, (−) Motor	28.46	75.86	9.91	27.05
3	Coal	12.88	88.74	4.36	31.41
4		5.85	94.59	2.09	33.50
5		5.41	100.00	2.00	35.50

TABLE 9

VARIMAX ROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS FIRST SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
AMCOAL	1	-.001	.002	.016	.006
APEXMIN	2	.009	-.003	.027	-.005
CLYSDL	3	.005	-.001	.027	.002
TAVISTK	4	.007	.002	.029	006
TRNSNTL	5	.007	.000	.033	-.005
VIERFNT	6	-.003	.008	.024	-.004
WELGDCT	7	.008	-.009	.040	.004
COAL		.032	-.001	.196	.004
BLYVOOR	8	-.022	-.019	-.010	-.011
DOORNS	9	-.019	-.030	-.015	.001
DRIECON	10	-.023	-.023	-.013	-.005
KLOOF	11	.027	-.020	-.008	-.012
WAREAS	12	-.024	-.030	-.005	-.011
WESDRIE	13	-.016	-.018	-.006	-.007
WSTNDP	14	-.024	-.022	-.010	-.010
GOLD		-.155	-.162	-.067	-.055
BANKORP	15	.004	.003	.002	.008
BOLAND	16	.004	.012	.003	.006
ICLEF	17	.013	.004	-.006	-.002
NEDBANK	18	.002	.025	.002	-.001
STANBIC	19	-.000	.011	.004	.004
T & T	20	.003	.019	-.004	.006
VOLKSKAS	21	+.009	.014	.003	.007
BANKS		.035	.088	.004	.028
ALPHA	22	-.002	.011	-.002	.007
BOUMAT	23	.007	.013	-.005	.029
EVRITE	24	.006	.003	.003	.005
GRNAKR	25	.012	.015	-.003	-.002
LTA	26	.038	.006	-.014	-.001
M & R	27	.009	.017	-.004	.005
PPCEM	28	.007	.012	.002	.007
BUILDING		.077	.077	-.023	.050
AECI	29	.002	.013	.003	.000
CHEMHD	30	.008	-.003	-.002	.008
DEBERL	31	-.000	.004	.005	.007
LANCHEM	32	.005	.023	.003	-.028
SENCHM	33	.007	.010	-.001	.008
TREK	34	.017	.008	-.006	-.004
TRIOMF	35	.005	.012	.002	-.006
CHEMICALS		.044	.067	-.022	-.015

TABLE 9 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS FIRST SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
CADSWP	36	-.001	.003	.003	.014
FEDFOOD	37	.009	-.001	-.007	.005
ICS	38	.003	.021	-.001	.007
I & J	39	.007	.012	-.003	-.001
KANHYM	40	.003	.016	.005	-.004
PREMGRP	41	.003	.011	-.002	.008
TIGOATS	42	-.000	.016	-.001	.007
FOOD		.024	.078	-.006	.036
ASSENG	43	.010	.007	-.002	.012
DUNLOP	44	.018	.009	-.004	.004
GENTRA	45	.016	.002	-.006	.014
MCCARTHY	46	.021	.018	.010	-.000
SAFICON	47	.038	.002	.018	-.000
TOYOTA	48	.020	.012	-.003	-.003
WMHUNT	49	.015	.005	.001	.007
MOTOR		.138	.055	-.042	.034

TABLE 10

PRINCIPAL-FACTOR ANALYSIS, VARIMAX ROTATION, FIRST SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	47.40	47.40	17.14	17.14
2	Gold, Motor	17.50	64.90	5.30	22.44
3	Gold	15.80	80.70	6.00	28.44
4	Coal	14.00	94.70	4.50	32.94
5		5.30	100.00	2.60	35.54

TABLE 11

UNROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS SECOND SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
AMCOAL	1	.0205	-.0151	.0030	-.0076	-.0088
APEXMIN	2	.0151	-.0155	.0060	-.0052	-.0100
CLYSDL	3	.0127	-.0111	.0014	-.0169	-.0099
TAVISTK	4	.0126	-.0185	.0028	-.0041	-.0072
TRNSNTL	5	.0163	-.0123	.0047	-.0223	-.0061
VIERFNT	6	.0157	-.0160	.0111	-.0049	-.0065
WELGDCT	7	.0167	-.0206	.0031	-.0140	-.0126
COAL		.1096	-.1091	.0321	-.0750	-.0611
BLYVOOR	8	0349	.0277	-.0018	-.0064	-.0017
DOORNS	9	.0483	.0272	-.0040	.0000	-.0021
DRIECON10	10	.0352	.0265	-.0011	-.0035	.0029
KLOOF	11	.0405	.0228	-.0008	-.0002	.0040
WAREAS	12	.0496	.0228	.0019	.0026	-.0009
WESDRIE	13	.0361	.0156	-.0063	-.0079	.0068
WSTNDP	14	.0330	.0286	.0168	.0125	-.0172
GOLD		.2776	.1712	.0047	-.0029	-.0082
BANKORP	15	.0145	-.0167	-.0009	.0023	-.0006
BOLAND	16	.0094	-.0058	-.0085	.0036	.0020
ICLEF	17	.0025	-.0015	.0026	-.0027	-.0021
NEDBANK	18	.0157	-.0072	-.0021	-.0007	.0064
STANBIC	19	0103	-.0108	-.0034	-.0009	.0015
T & T	20	.0074	-.0023	-.0021	.0062	.0040
VOLKSKAS	21	0154	-.0118	.0010	-.0030	.0050
BANKS		.0752	-.0561	-.0134	.0108	.0162
ALPHA	22	.0164	.0132	-.0074	-.0065	.0051
BOUMAT	23	.0115	.0077	-.0044	.0030	-.0069
EVRITE	24	.0076	.0061	-.0052	.0015	.0007
GRNAKR	25	.0176	.0057	-.0012	.0023	.0050
LTA	26	.0169	.0135	-.0037	.0038	.0103
M & R	27	.0146	.0118	-.0091	-.0023	.0093
PPCEM	28	.0113	.0072	-.0035	-.0013	.0038
BUILDING		.0959	.0652	-.0345	.0005	.0273
AECI	29	.0171	-.0117	-.0019	-.0092	.0050
CHEMHD	30	.0092	.0099	-.0033	.0078	-.0055
DEBERL	31	.0086	-.0033	.0022	-.0021	.0067
LANCHEM	32	.0113	-.0117	.0442	.0038	.0249
SENCHM	33	.0175	-.0082	-.0015	-.0024	.0059
TREK	34	.0131	-.0156	-.0035	-.0074	-.0041
TRIOMF	35	.0198	-.0168	.0362	.0152	-.0038
CHEMICALS		.0966	-.0772	.0724	.0057	.0291

TABLE 11 (CONTINUED)

UNROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS SECOND SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
CADSWP	36	.0090	-.0115	-.0091	-.0041	.0064
FEDFOOD	37	.0120	-.0087	-.0036	.0032	.0017
ICS	38	.0167	-.0123	-.0024	-.0010	.0122
I & J	39	.0118	-.0135	.0041	.0045	-.0095
KANHYM	40	.0121	-.0121	.0122	-.0009	-.0025
PREMGRP	41	.0107	-.0061	-.0041	-.0010	.0044
TIGOATS	42	.0119	-.0081	-.0038	-.0037	.0057
FOOD		.0824	-.0723	-.0067	-.0030	.0184
ASSENG	43	-.0005	-.0093	-.0062	.0129	-.0093
DUNLOP	44	.0129	-.0098	-.0072	.0075	.0041
GENTRA	45	.0036	-.0049	-.0026	.0006	-.0012
MCCARTHY	46	.0248	-.0168	-.0193	.0170	.0029
SAFICON	47	.0181	-.0196	-.0045	.0176	-.0127
TOYOTA	48	.0206	-.0104	-.0139	.0154	.0023
WMHUNT	49	.0130	-.0203	-.0062	.0126	.0006
MOTOR		.0925	-.0911	-.0599	.0836	-.0133

TABLE 12

PRINCIPAL-FACTOR ANALYSIS, NO ROTATION, SECOND SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	47.58	47.58	15.38	15.38
2	Gold, Coal	25.59	73.17	8.54	23.92
3	Lanchem & Triomf	12.59	85.76	2.56	26.48
4	(-) Coal, Motor	7.57	93.33	2.58	29.06
5		6.67	100.00	2.31	31.37

TABLE 13

VARIMAX UNROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS FIRST SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
AMCOAL	1	.019	.003	.000	.003
APEXMIN	2	.020	.004	.002	.000
CLYSDL	3	.020	-.005	-.006	.006
TAVISTK	4	.019	.007	.003	004
TRNSNTL	5	.022	-.009	-.003	.011
VIERFNT	6	.019	.001	.008	-.000
WELGDCT	7	.027	.002	-.002	.006
COAL		.146	.003	.002	.030
BLYVOOR	8	-.015	-.019	-.012	-.008
DOORNS	9	-.018	-.014	-.012	-.011
DRIECON	10	-.018	-.017	-.008	-.006
KLOOF	11	-.017	-.013	-.005	-.006
WAREAS	12	-.015	-.012	-.005	-.012
WESDRIE	13	-.012	-.012	-.009	.005
WSTNDP	14	-.010	-.015	-.000	-.035
GOLD		-.105	-.102	-.051	-.073
BANKORP	15	.010	.012	.005	.005
BOLAND	16	-.001	.010	-.003	.005
ICLEF	17	.004	-.002	.000	.000
NEDBANK	18	.001	.004	.003	.008
STANBIC	19	.006	.007	.001	.007
T & T	20	-.004	.006	.003	.001
VOLKSKAS	21	-.006	.004	.006	.009
BANKS		.022	.041	.015	.035
ALPHA	22	.006	.006	-.001	.015
BOUMAT	23	.006	.009	-.004	-.001
EVRITE	24	.001	.007	-.001	.004
GRNAKR	25	.000	.005	.004	.005
LTA	26	.000	.011	.007	.012
M & R	27	.001	.009	.000	.015
PPCEM	28	.002	.005	.001	.007
BUILDING		.016	.052	.006	.057
AECI	29	.008	.001	.001	.013
CHEMHD	30	.005	.013	.000	-.003
DEBERL	31	.000	-.001	.005	.006
LANCHEM	32	.005	-.013	.050	.003
SENCHM	33	.003	.003	.003	.009
TREK	34	.015	.006	-.002	.008
TRIOMF	35	.017	.004	.035	-.017
CHEMICALS		.053	.013	.092	.019

TABLE 13 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, PRINCIPAL-FACTOR ANALYSIS SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
CADSWP	36	.003	.008	-.002	.014
FEDFOOD	37	.002	.009	.002	.004
ICS	38	.001	.007	.007	.014
I & J	39	.013	.010	.004	-.005
KANHYM	40	.013	.001	.011	-.001
PREMGRP	41	.001	.004	.001	.007
TIGOATS	42	-.003	.004	.001	.010
FOOD		.036	.043	.024	.043
ASSENG	43	.004	.018	-.002	-.007
DUNLOP	44	-.001	.014	.002	.005
GENTRA	45	.003	.005	-.001	.002
MCCARTHY	46	-.004	.030	-.003	.007
SAFICON	47	.011	.026	.002	-.008
TOYOTA	48	-.005	.023	-.001	.003
WMHUNT	49	.010	.015	.002	.010
MOTOR		.018	.131	-.001	.012

TABLE 14

PRINCIPAL-FACTOR ANALYSIS, VARIMAX ROTATION, SECOND SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	47.58	47.58	15.38	15.38
2	Coal, Gold	14.40	61.98	5.30	20.68
3	Motor, Gold	16.42	78.40	4.90	25.58
4	Lanchem & Triomf	12.00	90.40	2.00	27.58
5		9.60	100.00	3.80	31.38

TABLE 15

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
AMCOAL	1	.0232	-.0086	.0130	.0012	-.0024	.0008
APEXMIN	2	.0144	-.0092	.0188	.0076	.0033	.0030
CLYSDL	3	.0194	-.0093	.0198	-.0011	.0014	-.0043
TAVISTK	4	.0178	-.0142	.0184	.0047	-.0002	-.0041
TRNSNTL	5	.0192	-.0104	.0230	-.0047	.0035	-.0015
VIERFNT	6	.0160	-.0088	.0138	-.0025	.0014	.0025
WELGDCT	7	.0219	-.0120	.0266	.0079	-.0047	.0009
COAL		.1319	-.0725	.1334	.0131	.0023	-.0027
BLYVOOR	8	.0358	.0290	-.0008	-.0008	-.0018	.0016
DOORNS	9	.0457	.0302	-.0045	.0040	-.0050	-.0044
DRIECON	10	.0373	.0298	-.0036	-.0002	.0001	-.0013
KLOOF	11	.0414	.0291	-.0009	-.0020	.0022	-.0000
WAREAS	12	.0456	.0310	.0017	.0024	.0016	-.0007
WESDRIE	13	.0334	.0216	.0005	0016	.0015	.0008
WSTNDP	14	0327	.0299	-.0005	.0021	.0025	.0058
GOLD		.2719	.2006	-.0081	.0071	.0111	.0018
BANKORP	15	.0112	-.0104	-.0005	.0005	-.0005	.0023
BOLAND	16	0120	-.0104	-.0033	-.0010	-.0005	-.0019
ICLEF	17	.0065	-.0056	-.0036	.0025	.0086	-.0031
NEDBANK	18	.0206	-.0134	-.0034	-.0076	.0010	.0013
STANBIC	19	.0121	-.0098	-.0037	-.0024	-.0066	.0032
T & T	20	.0154	-.0095	-.0072	.0019	-.0052	.0033
VOLKSKAS	21	.0164	-.0145	-.0013	-.0041	.0014	- 0010
BANKS		.0942	-.0736	-.0230	-.0102	-.0018	.0041
ALPHA	22	.0147	-.0092	-.0021	-.0008	-.0036	-.0013
BOUMAT	23	.0171	-.0136	-.0073	.0036	-.0050	-.0066
EVRITE	24	.0087	-.0069	-.0002	.0046	-.0024	-.0006
GRNAKR	25	.0188	-.0118	-.0061	.0012	-.0015	.0063
LTA	26	.0175	-.0181	-.0112	.0072	.0096	.0019
M & R	27	.0214	-.0154	-.0069	-.0029	.0011	-.0028
PPCEM	28	.0133	-.0111	-.0020	-.0006	-.0031	.0002
BUILDING		.1115	-.0861	-.0358	.0123	-.0049	-.0029
AECI	29	.0190	-.0102	-.0030	-.0079	.0020	.0008
CHEMHD	30	.0090	-.0062	-.0025	.0099	-.0025	.0026
DEBERL	31	.0097	-.0047	-.0000	-.0012	-.0060	.0011
LANCHEM	32	.0157	-.0089	.0023	-.0088	.0094	.0179
SENCHM	33	.0183	-.0100	-.0038	-.0007	.0022	.0015
TREK	34	.0140	-.0132	-.0036	.0016	.0007	.0077
TRIOMF	35	.0142	-.0101	.0028	.0033	.0048	0124
CHEMICALS		.0999	-.0633	.0078	-.0038	.0106	.0440

TABLE 15 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
CADSWP	36	.0134	-.0083	-.0005	.0022	-.0108	.0011
FEDFOOD	37	.0149	-.0066	-.0046	.0019	-.0006	-.0006
ICS	38	.0210	-.0144	-.0053	-.0065	-.0003	-.0002
I & J	39	.0113	-.0114	-.0012	-.0012	.0070	-.0016
KANHYM	40	.0127	-.0112	.0034	-.0033	.0047	.0050
PREMGRP	41	.0158	-.0090	-.0038	-.0026	-.0016	-.0039
TIGOATS	42	.0185	-.0104	-.0042	-.0073	.0002	-.0012
FOOD		.1076	-.0713	-.0162	-.0168	-.0014	-.0014
ASSENG	43	.0046	-.0113	-.0047	.0081	-.0035	-.0025
DUNLOP	44	.0134	-.0136	-.0085	.0048	-.0027	.0028
GENTRA	45	.0075	-.0096	-.0057	.0037	.0046	-.0091
MCCARTHY	46	.0258	-.0174	-.0124	.0107	.0019	.0017
SAFICON	47	.0199	-.0174	-.0116	.0158	.0087	.0008
TOYOTA	48	.0195	-.0143	-.0080	.0091	.0036	.0035
WMHUNT	49	.0162	-.0154	-.0013	.0058	-.0007	-.0030
MOTOR		.1069	-.0990	-.0522	.0580	.0119	-.0058

TABLE 15 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 13
AMCOAL	1	.0004	-.0006	.0018	.0015	.0001	.0002	-.0030
APEXMIN	2	-.0039	.0012	-.0033	.0010	.0004	-.0009	-.0014
CLYSDL	3	-.0008	-.0041	.0022	.0023	.0001	.0017	.0022
TAVISTK	4	-.0038	.0063	.0000	-.0004	-.0019	.0000	-.0008
TRNSNTL	5	.0009	-.0057	.0006	.0004	.0003	.0013	.0016
VIERFNT	6	.0119	.0033	.0011	-.0053	.0016	.0054	.0020
WELGDCT	7	.0012	.0027	-.0061	-.0035	.0015	-.0025	-.0001
COAL		.0059	.0031	-.0037	-.0041	.0021	-.0052	.0005
BLYVOOR	8	.0011	-.0031	-.0016	.0026	-.0027	-.0004	.0027
DOORNS	9	-.0006	.0018	-.0002	.0012	-.0010	-.0033	.0020
DRIECON	10	.0005	.0014	-.0004	-.0021	.0000	.0001	-.0011
KLOOF	11	.0007	.0004	.0027	.0020	-.0015	.0014	.0005
WAREAS	12	.0008	.0014	.0010	-.0014	.0034	-.0002	-.0006
WESDRIE	13	.0024	-.0020	-.0002	- 0031	-.0003	.0014	.0002
WSTNDP	14	.0050	-.0003	-.0017	.0082	.0057	.0005	-.0036
GOLD		.0035	-.0004	-.0004	.0095	.0036	-.0005	.0001
BANKORP	15	.0025	-.0013	.0067	-.0055	.0036	-.0021	-.0008
BOLAND	16	0005	-.0045	.0030	.0051	.0027	-.0017	-.0045
ICLEF	17	.0059	-.0014	-.0034	.0006	-.0000	-.0027	-.0005
NEDBANK	18	-.0030	.0011	.0044	-.0009	-.0001	-.0014	.0019
STANBIC	19	.0031	-.0019	.0018	.0015	.0010	.0008	-.0004
T & T	20	-.0059	.0051	.0013	.0039	.0002	-.0005	-.0017
VOLKSKAS	21	-.0031	.0006	.0003	.0006	-.0006	- 0004	.0013
BANKS		.0000	-.0023	.0141	.0053	.0068	-.0080	-.0047
ALPHA	22	-.0045	.0006	.0029	-.0019	-.0014	-.0051	.0009
BOUMAT	23	.0079	.0005	.0008	.0025	-.0019	.0019	-.0018
EVRITE	24	-.0023	.0022	.0001	-.0017	-.0001	-.0009	.0005
GRNAKR	25	-.0010	-.0014	-.0052	.0050	.0034	.0015	-.0001
LTA	26	-.0052	-.0020	-.0044	-.0012	.0039	.0005	.0005
M & R	27	-.0039	-.0003	-.0025	.0004	.0075	.0011	.0012
PPCEM	28	-.0013	-.0039	-.0009	.0036	-.0027	-.0023	.0001
BUILDING		-.0103	-.0043	-.0092	.0069	.0087	-.0033	.0013
AECI	29	.0019	-.0038	-.0015	-.0021	-.0000	-.0027	-.0008
CHEMHD	30	.0022	-.0001	.0001	-.0017	.0013	.0004	-.0034
DEBERL	31	.0018	.0019	-.0019	.0001	.0031	-.0015	.0019
LANCHEM	32	.0012	.0098	.0022	.0063	-.0014	-.0040	.0095
SENCHM	33	.0009	-.0008	.0004	-.0040	.0024	-.0034	-.0028
TREK	34	.0049	-.0054	-.0060	-.0078	-.0053	.0018	-.0014
TRIOMF	35	.0107	.0091	.0044	.0029	0012	-.0031	.0032
CHEMICALS		.0236	.0107	-.0023	-.0063	.0013	-.0125	.0062

TABLE 15 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 13
CADSWP	36	.0006	-.0017	-.0017	-.0006	-.0022	.0011	.0018
FEDFOOD	37	-.0002	.0019	-.0054	.0018	-.0041	-.0023	-.0005
ICS	38	-.0016	.0022	-.0001	-.0013	-.0005	.0040	.0010
I & J	39	.0017	.0034	.0042	.0026	-.0017	.0014	-.0048
KANHYM	40	.0038	.0047	.0000	.0024	-.0057	.0009	-.0030
PREMGRP	41	-.0006	.0040	-.0009	.0004	-.0003	.0006	-.0015
TIGOATS	42	.0006	.0025	-.0023	-.0016	-.0009	.0027	-.0015
FOOD		.0043	.0170	-.0062	.0037	-.0154	.0084	-.0085
ASSENG	43	.0026	-.0008	.0040	-.0012	.0042	.0054	.0034
DUNLOP	44	-.0003	-.0040	-.0025	-.0004	-.0024	.0027	.0039
GENTRA	45	.0045	.0019	-.0015	.0012	.0012	-.0019	.0036
MCCARTHY	46	-.0031	-.0019	.0071	-.0025	-.0042	-.0010	-.0006
SAFICON	47	.0037	-.0006	.0044	.0028	-.0007	.0034	.0042
TOYOTA	48	-.0097	-.0015	.0013	.0021	-.0018	.0046	-.0043
WMHUNT	49	.0053	-.0031	.0017	-.0024	-.0042	-.0030	.0032
MOTOR		.0030	-.0100	-.0145	-.0004	-.0079	.0102	.0134

TABLE 16

LITTLE JIFFY, NO ROTATION, TOTAL PERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	50.15	50.15	15.46	15.46
2	Gold, Motor	26.47	76.62	7.98	23.44
3	Coal	8.59	85.21	2.82	26.26
4	Motor	3.12	88.33	1.09	27.35
5		2.07	90.40	0.78	28.13
6	Lanchem & Triomf	2.22	92.62	0.59	28.72
7		1.82	94.44	0.58	29.30
8		1.25	95.69	0.40	29.70
9		1.03	96.72	0.35	30.05
10		1.04	97.66	0.32	30.37
11		0.81	98.57	0.28	30.65
12		0.64	99.21	0.23	30.88
13		0.79	100.00	0.23	31.11

TABLE 17

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.001	.015	.000	.000	.003	-.001
APEXMIN	2	-.002	.022	.004	.002	-.001	-.002
CLYSDL	3	.002	.022	-.002	-.002	-.001	.001
TAVISTK	4	.004	.023	.003	.000	.000	.001
TRNSNTL	5	.003	.025	-.005	.000	-.001	.000
VIERFNT	6	.003	.015	.002	.008	.004	.003
WELGDCT	7	-.004	.029	.001	.000	.007	.001
COAL		.007	.151	.003	.008	.011	.003
BLYVOOR	8	-.022	-.013	-.013	-.001	-.004	-.002
DOORNS	9	-.024	-.016	-.010	-.005	-.004	.002
DRIECON	10	-.021	-.016	-.012	-.003	-.008	-.001
KLOOF	11	-.020	-.013	-.013	.000	-.009	-.002
WAREAS	12	-.024	-.011	-.012	-.002	-.010	-.002
WESDRIE	13	-.017	-.008	-.007	- 003	-.007	-.004
WSTNDP	14	-.025	-.013	-.011	.004	-.006	-.001
GOLD		-.153	-.090	.078	-.010	-.048	-.010
BANKORP	15	.007	.003	.006	.002	.003	-.001
BOLAND	16	.009	.002	.005	-.002	.003	.002
ICLEF	17	.004	.000	.005	.003	-.004	.009
NEDBANK	18	.016	.002	.002	.003	.001	-.002
STANBIC	19	.008	.000	.003	.001	.010	-.001
T & T	20	.008	.002	.007	.001	.006	-.004
VOLKSKAS	21	.014	.005	.004	.002	-.001	001
BANKS		.066	.014	.032	.010	.018	.004
ALPHA	22	.008	.002	.003	-.002	.003	-.001
BOUMAT	23	.009	-.001	.009	-.002	.008	.009
EVRITE	24	.003	.003	.006	-.001	.003	.000
GRNAKR	25	.009	.000	.008	.003	.007	-.002
LTA	26	.015	-.001	.019	.000	-.003	.001
M & R	27	.016	.001	.006	-.003	.003	.002
PPCEM	28	.008	.003	.004	-.001	.006	.001
BUILDING		.068	.007	.055	-.006	.027	.010
AECI	29	.013	.001	-.001	.001	.002	.001
CHEMHD	30	-.001	.001	.011	.000	.005	.000
DEBERL	31	.003	.001	-.000	.001	.008	.001
LANCHEM	32	.010	.004	-.002	.025	-.001	-.006
SENCHM	33	.009	.001	.005	.001	.001	.001
TREK	34	.009	.002	.009	.002	.006	-.002
TRIOMF	35	.003	.006	.007	.020	.004	001
CHEMICALS		.046	.016	.029	.050	.025	-.004

TABLE 17 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.003	.003	.004	-.003	.012	-.001
FEDFOOD	37	.005	-.001	.004	.000	.003	.003
ICS	38	.016	.001	.003	.001	.003	-.001
I & J	39	.011	.004	.005	.004	-.005	.002
KANHYM	40	.009	.007	.002	.009	-.001	-.000
PREMGRP	41	.010	.000	.002	-.001	.002	.003
TIGOATS	42	.016	.002	.001	.001	.003	.001
FOOD		.070	.016	.021	.0118	.017	.007
ASSENG	43	.005	.001	.013	-.002	.007	.003
DUNLOP	44	.009	-.001	.013	-.001	.008	.000
GENTRA	45	.008	.000	.008	-.000	-.001	.013
MCCARTHY	46	.011	-.003	.022	.000	.001	-.001
SAFICON	47	.008	-.001	.026	.006	-.001	.006
TOYOTA	48	.010	.001	.018	-.002	-.002	-.007
WMHUNT	49	.008	.006	.011	.000	.005	-.007
MOTOR		.059	.003	.111	.001	.017	.021

TABLE 17 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 13
AMCOAL	1	.000	-.001	-.002	.000	.003	.002
APEXMIN	2	.000	.003	.004	.001	-.001	.003
CLYSDL	3	.000	-.002	-.001	-.001	.002	-.006
TAVISTK	4	-.005	.002	-.003	.003	-.003	.003
TRNSNTL	5	.004	-.003	.000	-.001	.002	-.005
VIERFNT	6	.006	-.008	-.005	-.006	-.004	.001
WELGDCT	7	.002	.001	.002	.002	-.005	.005
COAL		.007	-.008	-.005	-.002	-.006	.003
BLYVOOR	8	.001	.002	.001	-.001	.000	-.005
DOORNS	9	-.005	.000	.000	.003	-.002	-.001
DRIECON	10	.000	-.002	.000	-.001	-.003	.001
KLOOF	11	-.001	-.002	-.002	.002	.000	-.003
WAREAS	12	-.002	-.004	.002	-.001	-.002	.001
WESDRIE	13	.002	-.001	.001	- 001	-.003	-.001
WSTNDP	14	-.001	.000	.005	-.008	.006	.002
GOLD		-.006	-.007	.007	-.011	-.004	.006
BANKORP	15	.003	-.008	-.001	.003	.002	.002
BOLAND	16	-.001	-.001	.001	.000	.009	.001
ICLEF	17	.005	.001	.002	-.001	.001	.001
NEDBANK	18	-.001	-.002	.000	.004	.001	-.002
STANBIC	19	.001	-.001	-.001	-.001	.004	-.000
T & T	20	-.008	.004	.001	.002	.001	.004
VOLKSKAS	21	-.001	.001	.001	.001	.000	- 002
BANKS		-.002	-.006	.002	.008	.018	.004
ALPHA	22	-.002	-.000	.000	.008	.000	.000
BOUMAT	23	-.001	.001	-.007	-.002	.003	.002
EVRITE	24	-.002	.001	.000	.003	-.002	.002
GRNAKR	25	.000	.005	.006	-.004	.002	.001
LTA	26	.001	.003	.008	-.002	-.001	.002
M & R	27	-.003	-.001	.007	-.002	.000	.001
PPCEM	28	.001	.004	.000	.003	.004	-.002
BUILDING		-.006	.013	.014	.004	.006	.006
AECI	29	.006	.000	.002	.001	.002	.000
CHEMHD	30	.001	.000	-.001	.000	.001	.005
DEBERL	31	-.001	-.001	.000	.000	-.001	.002
LANCHEM	32	-.001	.003	.005	.002	-.002	-.004
SENCHM	33	.004	-.002	.002	.002	.001	.004
TREK	34	.014	.004	-.001	-.001	-.003	.002
TRIOMF	35	.001	-.003	-.001	.000	.001	003
CHEMICALS		.024	.001	.008	.004	-.001	.012

TABLE 17 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 12
CADSWP	36	.000	.002	-.002	.001	-.001	-.001
FEDFOOD	37	.000	.007	.000	.002	-.001	.002
ICS	38	-.001	.000	-.002	-.002	-.002	-.001
I & J	39	-.001	.000	-.005	-.002	.004	.003
KANHYM	40	.002	.004	-.005	-.001	.001	.002
PREMGRP	41	-.003	.001	-.002	.000	-.001	.003
TIGOATS	42	.001	.001	-.002	-.002	-.002	.002
FOOD		-.002	.015	-.018	-.004	-.002	.010
ASSENG	43	-.003	-.006	-.002	-.003	-.001	-.001
DUNLOP	44	.003	.004	.000	-.001	-.001	-.003
GENTRA	45	-.002	.000	.000	-.000	-.001	-.001
MCCARTHY	46	.000	.001	-.004	.006	.002	.000
SAFICON	47	-.001	.000	-.001	-.002	.001	-.003
TOYOTA	48	-.003	.006	.001	-.001	.003	.002
WMHUNT	49	.004	.000	-.004	.005	.000	-.002
MOTOR		-.002	.005	-.010	.004	.003	-.008

TABLE 18

LITTLE JIFFY, VARIMAX ROTATION, TOTAL PERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	50.15	50.15	15.46	15.46
2	Gold	16.00	66.15	5.10	20.56
3	Coal	11.40	77.55	3.40	23.96
4	Motor	9.10	86.65	2.70	26.66
5	Lanchem & Triomf	2.30	88.95	0.60	27.20
6		2.30	91.20	1.20	28.40
7		2.30	93.50	0.60	29.00
8		2.30	95.80	0.40	29.40
9		1.00	96.80	0.40	29.80
10		0.80	97.60	0.40	31.10
11		0.80	98.40	0.30	31.40
12		0.80	99.20	0.30	31.70
13		0.80	100.00	0.30	32.00

TABLE 19

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.011	.000	-.001	-.003	.003	.004
APEXMIN	2	.022	.001	.005	.000	.001	-.001
CLYSDL	3	.014	-.004	.001	-.003	.000	-.001
TAVISTK	4	.021	-.008	.002	-.001	.005	-.001
TRNSNTL	5	.016	-.003	-.002	-.001	.000	-.000
VIERFNT	6	.009	.000	-.001	.000	.002	.002
WELGDCT	7	.031	.000	-.001	-.001	-.002	.002
COAL		.124	-.014	.003	-.009	.009	.005
BLYVOOR	8	-.007	.008	-.004	-.010	-.008	-.007
DOORNS	9	-.006	.006	-.003	-.009	-.009	-.005
DRIECON	10	-.007	.007	-.005	-.005	-.005	-.005
KLOOF	11	-.009	.007	-.003	-.008	-.004	-.006
WAREAS	12	-.002	.008	-.003	-.004	-.008	-.004
WESDRIE	13	-.003	.003	-.001	- 004	-.007	-.005
WSTNDP	14	-.005	.022	-.002	-.003	-.003	-.002
GOLD		-.039	.061	-.021	-.043	-.044	-.034
BANKORP	15	.000	-.004	.001	.002	-.002	.012
BOLAND	16	-.003	.003	.001	.002	.005	.007
ICLEF	17	.001	.005	.005	.003	.004	.001
NEDBANK	18	-.004	-.010	-.002	.004	.002	.004
STANBIC	19	-.005	-.000	-.002	.001	.001	.005
T & T	20	-.002	-.003	.002	.003	.004	.002
VOLKSKAS	21	-.001	-.007	.001	.005	.004	-.000
BANKS		-.012	-.016	.006	.020	.018	.031
ALPHA	22	.000	-.009	-.001	-.001	-.001	.005
BOUMAT	23	-.005	.000	.005	-.001	.007	.003
EVRITE	24	.004	-.004	.003	.001	-.001	.002
GRNAKR	25	-.001	.005	.003	.008	.001	.000
LTA	26	.000	-.001	.013	.015	.001	.001
M & R	27	-.002	-.003	.001	.015	.000	.002
PPCEM	28	-.001	-.002	.001	-.001	.002	.002
BUILDING		-.005	-.014	.025	.036	.009	.015
AECI	29	-.003	-.002	-.005	.004	.002	.004
CHEMHD	30	.003	.003	.007	.000	.001	.006
DEBERL	31	.002	.000	-.004	.003	-.003	.003
LANCHEM	32	.002	-.001	.000	.002	.001	-.001
SENCHM	33	.000	-.001	-.001	.005	.002	.008
TREK	34	.000	-.001	.002	.001	.002	.002
TRIOMF	35	.005	.006	.006	-.002	.003	008
CHEMICALS		.009	.004	.005	.013	.008	.030

TABLE 19 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.001	-.004	-.000	-.003	-.003	.001
FEDFOOD	37	.001	-.000	.002	.000	.004	-.002
ICS	38	-.004	-.008	-.001	.007	.005	-.002
I & J	39	-.001	-.001	.004	.001	.013	.002
KANHYM	40	.003	-.000	.000	-.003	.012	-.001
PREMGRP	41	-.001	-.004	-.002	.004	.006	-.000
TIGOATS	42	-.003	-.005	-.004	.007	.008	-.001
FOOD		-.004	-.022	-.001	.013	.045	-.003
ASSENG	43	-.002	-.003	.011	.004	-.003	.003
DUNLOP	44	-.004	-.004	.009	.002	-.002	-.001
GENTRA	45	.000	-.001	.008	.004	.002	.000
MCCARTHY	46	-.006	-.009	.015	-.002	.002	.008
SAFICON	47	-.004	.000	.025	.002	.002	.002
TOYOTA	48	-.001	-.004	.014	.005	.006	-.001
WMHUNT	49	.002	-.005	.008	-.004	.000	.006
MOTOR		-.015	-.026	.090	.011	.007	.017

TABLE 19 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 13
AMCOAL	1	-.001	-.001	.004	.005	-.002	.000
APEXMIN	2	.000	.001	.001	.002	-.003	-.003
CLYSDL	3	-.004	-.001	.000	.013	.002	.001
TAVISTK	4	-.005	-.001	.002	.000	.000	.001
TRNSNTL	5	.000	.001	-.002	.015	.001	.001
VIERFNT	6	.005	.004	-.002	.005	-.001	.014
WELGDCT	7	.003	-.002	.004	-.001	.001	.002
COAL		-.002	.001	.007	.039	-.002	.016
BLYVOOR	8	-.001	.000	-.002	.000	-.001	-.002
DOORNS	9	-.007	-.004	-.002	-.009	.002	-.001
DRIECON	10	-.004	-.003	-.006	-.006	-.004	.001
KLOOF	11	-.006	.000	-.005	-.001	-.004	.001
WAREAS	12	-.008	-.003	-.007	-.004	-.006	.001
WESDRIE	13	-.001	-.003	-.005	- 002	-.006	-.001
WSTNDP	14	-.006	.001	-.001	.000	-.004	-.001
GOLD		-.033	-.012	-.028	-.022	-.023	-.002
BANKORP	15	.001	.001	-.001	.001	-.003	.003
BOLAND	16	-.002	-.002	.003	.006	.003	-.004
ICLEF	17	.003	.000	-.008	.000	.007	-.001
NEDBANK	18	-.001	.006	.000	.003	.000	-.001
STANBIC	19	.003	.001	.008	.003	.001	.002
T & T	20	-.003	.002	.010	-.006	-.001	-.003
VOLKSKAS	21	.000	.004	-.001	.003	.002	- 002
BANKS		.001	.012	.011	.010	.009	-.006
ALPHA	22	-.001	.001	.002	-.002	.002	-.004
BOUMAT	23	.002	-.005	.006	-.001	.009	.005
EVRITE	24	.000	-.001	.003	-.004	.000	.000
GRNAKR	25	.005	.003	.009	.001	.001	-.003
LTA	26	.004	.000	-.003	-.002	.000	-.005
M & R	27	-.001	.000	.003	.002	.003	.000
PPCEM	28	.003	.001	.005	.003	.005	-.005
BUILDING		.012	-.001	.025	-.003	.020	-.012
AECI	29	.006	.002	-.003	.004	.003	-.002
CHEMHD	30	.003	-.004	.004	-.004	-.002	.001
DEBERL	31	.001	.002	.005	-.002	.003	.002
LANCHEM	32	.001	.028	.000	.000	-.002	-.002
SENCHM	33	.003	.000	-.003	-.001	.000	-.001
TREK	34	.019	-.001	-.001	.000	-.003	.001
TRIOMF	35	.001	.016	.001	-.003	.000	006
CHEMICALS		.034	.043	.003	-.006	-.001	.005

TABLE 19 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12	FACTOR 13
CADSWP	36	.005	-.002	.010	-.001	.002	.002
FEDFOOD	37	.004	.000	.002	-.005	.005	-.004
ICS	38	.003	.002	.003	.001	.000	.003
I & J	39	-.003	.001	-.003	.002	.000	.001
KANHYM	40	.004	.007	.000	.001	-.001	.001
PREMGRP	41	-.001	.001	.002	-.003	.003	.001
TIGOATS	42	.004	.001	.002	.000	.002	.003
FOOD		.016	.008	.015	-.005	.011	.007
ASSENG	43	-.001	-.003	.006	.001	.001	.008
DUNLOP	44	.008	.000	.006	.001	.002	-.001
GENTRA	45	-.002	-.001	-.004	-.002	.012	.002
MCCARTHY	46	.003	-.001	.001	-.003	-.002	-.003
SAFICON	47	.001	.002	-.000	.000	.003	.002
TOYOTA	48	.001	-.003	.005	-.000	-.007	-.006
WMHUNT	49	.006	-.001	-.001	.001	.008	.001
MOTOR		.016	-.007	.013	-.002	.017	.003

TABLE 20

LITTLE JIFFY, ORTHOBLIQUE ROTATION, TOTAL PERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	57.00	57.00	15.46	15.46
2	Coal	8.20	65.20	2.20	17.66
3		5.30	70.50	1.10	18.76
4	Motor	5.20	75.70	1.30	20.06
5		2.70	78.40	1.00	21.06
6		2.70	81.10	0.90	21.96
7		2.70	83.80	0.70	22.66
8		2.70	86.50	0.70	23.36
9	Lanchem & Triomf	2.70	89.20	0.60	23.96
10		2.70	91.90	0.90	24.86
11		2.70	94.60	0.70	25.56
12		2.70	97.30	0.70	26.26
13		2.70	100.00	0.40	26.66

TABLE 21

LITTLE JIFFY AND ORTHOBLIQUE ROTATION,
TOTAL PERIOD CORRELATION MATRIX OF FACTORS

	2	3	4	5	6	7	8	9	10	11	12	13
2	1.000											
3	-.177	1.000										
4	.017	-.211	1.000									
5	-.020	-.391	.300	1.000								
6	.155	-.403	.273	.406	1.000							
7	.126	-.377	.353	.326	.358	1.000						
8	.069	-.293	.253	.315	.286	.342	1.000					
9	.079	-.049	.026	.139	.233	.125	.130	1.000				
10	.105	-.306	.185	.237	.213	.305	.248	.024	1.000			
11	.358	-.219	-.057	.120	.218	.143	.113	.092	.028	1.000		
12	.022	-.293	.169	.253	.275	.252	.210	-.036	.117	.107	1.000	
13	.165	.037	-.057	-.118	.017	.115	.041	.054	-.036	.099	.103	1.000

TABLE 22

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
AMCOAL	1	.0239	.0105	.0145	.0003	.0042
APEXMIN	2	.0120	.0100	.0244	.0078	-.0049
CLYSDL	3	.0217	.0138	.0254	.0005	-.0017
TAVISTK	4	.0184	.0180	.0243	.0007	-.0014
TRNSNTL	5	.0181	.0138	.0297	.0006	-.0077
VIERFNT	6	.0151	.0098	.0186	-.0044	.0019
WELGDCT	7	.0242	.0129	.0345	.0096	.0050
COAL		.1334	.0888	.1714	.0151	-.0046
BLYVOOR	8	0452	-.0214	-.0026	.0006	.0031
DOORNS	9	.0532	-.0212	-.0067	.0038	.0039
DRIECON10	10	.0485	-.0229	-.0067	.0003	.0019
KLOOF	11	.0514	-.0231	-.0004	-.0035	-.0039
WAREAS	12	.0547	-.0262	.0020	.0001	-.0053
WESDRIE	13	0374	-.0174	-.0002	.0048	.0009
WSTNDP	14	.0438	-.0246	-.0027	.0001	-.0019
GOLD		.3342	-.1568	-.0173	.0062	-.0013
BANKORP	15	.0060	.0083	-.0001	.0009	.0029
BOLAND	16	.0098	.0151	.0012	.0004	.0024
ICLEF	17	.0074	.0113	-.0083	.0059	-.0096
NEDBANK	18	.0194	.0243	-.0018	-.0114	-.0031
STANBIC	19	.0101	.0116	-.0020	-.0020	.0071
T & T	20	.0178	.0195	-.0060	-.0019	.0082
VOLKSKAS	21	.0119	.0214	-.0005	-.0044	-.0040
BANKS		.0824	.1115	-.0135	-.0125	.0039
ALPHA	22	.0098	.0103	-.0036	-.0032	.0029
BOUMAT	23	.0170	.0243	-.0095	.0021	.0158
EVRITE	24	.0070	.0095	.0013	.0040	.0024
GRNAKR	25	.0154	.0020	-.0055	.0046	-.0013
LTA	26	.0115	.0028	-.0015	.0160	-.0158
M & R	27	.0211	.0024	-.0062	-.0036	-.0053
PPCEM	28	.0107	.0017	-.0003	.0014	.0030
BUILDING		.0925	.0530	-.0253	.0213	.0017
AECI	29	.0167	.0143	-.0054	-.0077	-.0038
CHEMHD	30	.0077	.0060	-.0019	.0104	.0055
DEBERL	31	.0083	.0075	.0029	-.0016	.0069
LANCHEM	32	.0180	.0150	.0011	-.0021	-.0101
SENCHM	33	.0158	.0172	-.0048	.0011	.0020
TREK	34	.0109	.0166	-.0088	.0068	-.0044
TRIOMF	35	.0097	.0120	-.0011	.0006	.0025
CHEMICALS		.0871	.0886	-.0180	.0075	-.0014

TABLE 22 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
CADSWP	36	.0138	.0084	.0024	.0025	.0192
FEDFOOD	37	.0157	.0096	-.0069	.0003	-.0005
ICS	38	.0195	.0023	-.0045	-.0094	.0003
I & J	39	.0084	.0136	-.0045	-.0033	-.0068
KANHYM	40	.0105	.0152	.0021	-.0039	-.0043
PREMGRP	41	.0170	.0162	-.0047	-.0069	.0006
TIGOATS	42	.0197	.0181	-.0046	-.0104	-.0004
FOOD		.1046	.0834	-.0207	-.0311	.0081
ASSENG	43	.0060	.0157	-.0042	-.0048	.0061
DUNLOP	44	.0092	.0194	-.0062	.0091	.0043
GENTRA	45	.0070	.0163	-.0078	.0046	-.0022
MCCARTHY	46	.0021	.0270	-.0012	.0138	-.0027
SAFICON	47	.0166	.0247	-.0016	.0220	-.0068
TOYOTA	48	.0139	.0220	-.0043	.0085	-.0066
WMHUNT	49	.0144	.0173	-.0025	.0097	.0055
MOTOR		.0692	.1424	-.0278	.0725	-.0024

TABLE 22 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	.0003	-.0001	.0034	-.0013	.0035	-.0020	-.0022
APEXMIN	2	.0042	-.0055	.0060	.0000	.0035	-.0016	-.0017
CLYSDL	3	-.0035	-.0001	-.0095	.0018	-.0029	.0024	-.0015
TAVISTK	4	-.0070	-.0073	.0019	-.0010	.0033	.0059	.0054
TRNSNTL	5	-.0005	.0067	-.0057	.0023	-.0015	-.0018	.0005
VIERFNT	6	.0014	.0127	.0101	-.0021	-.0095	.0041	-.0012
WELGDCT	7	-.0035	-.0033	.0107	.0025	-.0004	.0009	.0019
COAL		-.0086	.0031	.0169	.0022	-.0040	.0079	.0012
BLYVOOR	8	.0095	.0009	-.0035	.0074	.0014	.0006	-.0041
DOORNS	9	-.0060	-.0104	-.0017	.0053	.0026	-.0007	.0006
DRIECON	10	-.0036	-.0001	.0048	-.0023	.0009	.0024	.0034
KLOOF	11	.0000	.0012	-.0068	.0008	.0001	.0031	.0000
WAREAS	12	-.0053	-.0023	.0026	-.0050	-.0010	-.0008	.0004
WESDRIE	13	.0018	.0019	-.0021	-.0024	-.0025	-.0018	.0003
WSTNDP	14	.0021	.0031	-.0002	-.0003	-.0007	-.0047	-.0012
GOLD		-.0015	-.0057	-.0069	.0035	.0008	-.0019	-.0006
BANKORP	15	-.0047	.0052	.0017	-.0051	-.0014	-.0038	-.0004
BOLAND	16	-.0005	.0048	-.0027	-.0031	.0076	-.0105	-.0015
ICLEF	17	-.0057	.0099	.0044	.0054	.0067	.0004	-.0011
NEDBANK	18	.0043	-.0011	-.0017	-.0030	.0004	.0018	-.0010
STANBIC	19	.0048	.0047	.0003	-.0004	-.0018	-.0005	-.0018
T & T	20	.0093	-.0013	-.0005	-.0076	.0057	.0016	-.0048
VOLKSKAS	21	-.0017	.0047	-.0035	-.0008	-.0016	-.0050	.0018
BANKS		.0058	.0175	-.0020	-.0130	.0156	-.0160	-.0088
ALPHA	22	.0007	-.0068	-.0040	-.0028	.0082	.0062	.0014
BOUMAT	23	-.0099	.0058	-.0028	.0009	.0020	.0006	.0010
EVRITE	24	-.0023	-.0027	.0021	-.0013	-.0001	.0014	.0007
GRNAKR	25	.0111	-.0015	.0067	-.0009	.0006	.0012	-.0057
LTA	26	-.0008	-.0069	.0070	.0016	-.0065	-.0052	-.0015
M & R	27	-.0017	-.0034	-.0002	-.0059	-.0003	-.0031	-.0075
PPCEM	28	.0043	-.0019	-.0020	.0029	.0091	-.0033	.0011
BUILDING		.0014	-.0174	.0068	-.0055	.0130	-.0022	-.0105
AECI	29	.0018	.0037	.0009	.0031	.0003	-.0023	.0006
CHEMHD	30	-.0005	.0000	.0035	-.0042	-.0015	-.0002	-.0021
DEBERL	31	.0000	-.0023	.0014	.0013	-.0012	.0002	-.0031
LANCHEM	32	.0198	.0059	-.0047	-.0001	.0041	.0092	.0019
SENCHM	33	-.0017	.0051	.0049	-.0045	-.0012	-.0002	-.0017
TREK	34	.0097	.0023	.0093	.0071	-.0024	-.0008	.0113
TRIOMF	35	.0072	.0049	.0002	.0021	-.0016	.0112	-.0072
CHEMICALS		.0363	.0196	.0155	.0048	-.0035	.0171	-.0003

TABLE 22 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
CADSWP	36	.0067	-.0028	.0000	.0055	-.0037	-.0057	.0037
FEDFOOD	37	-.0024	-.0067	.0051	.0123	.0014	.0135	-.0021
ICS	38	.0028	.0008	-.0012	-.0027	-.0033	-.0014	.0002
I & J	39	-.0027	.0033	.0022	-.0049	.0027	.0012	-.0014
KANHYM	40	.0030	.0068	.0021	.0007	.0022	.0035	.0050
PREMGRP	41	-.0057	-.0031	-.0004	-.0015	-.0019	.0005	.0036
TIGOATS	42	-.0009	-.0006	.0035	.0021	-.0021	.0014	.0005
FOOD		.0008	-.0023	.0113	.0115	-.0047	.0130	.0095
ASSENG	43	-.0021	.0020	-.0038	-.0051	-.0050	.0018	-.0008
DUNLOP	44	.0052	-.0014	-.0068	.0040	-.0055	-.0030	.0086
GENTRA	45	-.0146	.0008	-.0053	.0062	.0011	.0019	-.0064
MCCARTHY	46	.0044	.0042	-.0035	-.0100	.0072	.0039	.0043
SAFICON	47	-.0024	-.0041	-.0079	.0054	-.0084	.0026	-.0074
TOYOTA	48	.0070	-.0061	-.0064	-.0045	-.0057	.0023	.0023
WMHUNT	49	-.0037	.0103	-.0027	.0071	.0005	.0039	.0044
MOTOR		.0062	.0057	-.0364	.0031	-.0158	.0134	.0050

TABLE 23

LITTLE JIFFY, NO ROTATION, FIRST SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	45.91	45.91	14.63	14.63
2	Gold, Motor	23.75	69.66	9.09	23.72
3	Coal	9.69	79.35	3.36	27.08
4		4.04	83.39	1.48	28.56
5		3.41	86.80	1.33	29.89
6		3.09	89.89	1.02	30.91
7		2.18	92.07	0.85	31.76
8		1.95	94.02	0.68	32.44
9		1.71	95.73	0.68	33.12
10		1.43	97.16	0.58	33.70
11		1.58	98.74	0.66	34.36
12		1.26	100.00	0.44	34.80

TABLE 24

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.004	.017	.006	-.002	-.003	.002
APEXMIN	2	-.001	.028	-.000	.006	-.001	.006
CLYSDL	3	.004	.028	.003	-.002	.011	-.006
TAVISTK	4	.010	.029	.001	.001	.000	-.007
TRNSNTL	5	.004	.032	-.002	-.002	.010	-.002
VIERFNT	6	.004	.020	.005	-.006	-.002	.007
WELGDCT	7	-.003	.038	.009	.004	-.007	-.001
COAL		.022	.192	.022	-.001	.008	-.001
BLYVOOR	8	-.019	-.010	-.004	-.006	.001	.005
DOORNS	9	-.017	-.012	-.005	.002	-.007	-.009
DRIECON	10	-.016	-.013	-.008	-.003	-.009	-.003
KLOOF	11	-.016	-.008	-.013	-.008	.003	-.003
WAREAS	12	-.019	-.005	-.015	-.002	-.007	-.004
WESDRIE	13	-.017	-.006	-.004	- 002	.000	.000
WSTNDP	14	-.019	-.010	-.010	-.004	-.004	-.001
GOLD		-.123	-.064	-.059	-.023	-.023	-.013
BANKORP	15	.006	.002	.006	.001	-.002	-.002
BOLAND	16	.011	.005	.008	.000	.000	-.001
ICLEF	17	.010	-.003	-.002	.012	.000	.002
NEDBANK	18	.026	.005	.004	-.003	.005	.005
STANBIC	19	.007	.004	.011	-.004	.002	.005
T & T	20	.015	-.001	.014	-.003	.003	.010
VOLKSKAS	21	.021	.006	.004	.005	.004	-.003
BANKS		.096	.018	.045	.008	.012	.016
ALPHA	22	.010	.000	.003	-.003	.003	-.001
BOUMAT	23	.018	-.003	.023	.002	.001	-.009
EVRITE	24	.005	.005	.006	.005	.000	-.001
GRNAKR	25	.000	-.003	.002	.005	-.001	.014
LTA	26	.000	.003	-.006	.024	.000	.004
M & R	27	.006	-.005	-.004	.003	-.001	.002
PPCEM	28	.000	.000	.002	-.001	-.001	.001
BUILDING		.039	-.003	.026	.035	.001	.010
AECI	29	.017	-.001	.001	.000	.001	.003
CHEMHD	30	-.001	.001	.010	.008	-.001	.002
DEBERL	31	.004	.005	.008	-.001	-.002	.001
LANCHEM	32	.012	.005	-.003	-.003	.016	.017
SENCHM	33	.014	.001	.009	.005	-.001	.004
TREK	34	.013	-.002	.006	.012	.001	.009
TRIOMF	35	.007	.003	.008	.000	.008	011
CHEMICALS		.066	.012	.039	.021	.022	.047

TABLE 24 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.001	.003	.022	-.003	-.002	.000
FEDFOOD	37	.009	-.002	.002	.007	.000	.002
ICS	38	.007	-.005	.000	-.006	.001	.002
I & J	39	.015	.000	-.002	.003	.001	.002
KANHYM	40	.014	.006	.001	-.001	.003	.004
PREMGRP	41	.018	.000	.004	.001	.000	-.005
TIGOATS	42	.021	.001	.004	-.001	-.001	.002
FOOD		.085	.003	.031	.000	.002	.007
ASSENG	43	.010	.000	.013	.004	.007	-.002
DUNLOP	44	.012	-.001	.015	.009	.011	-.002
GENTRA	45	.013	-.001	.006	.012	.005	-.009
MCCARTHY	46	.015	.008	.010	.011	.012	.005
SAFICON	47	.010	.008	.012	.027	.018	.000
TOYOTA	48	.016	.003	.006	.013	.015	.005
WMHUNT	49	.008	.003	.014	.007	.007	-.004
MOTOR		.084	.020	.076	.083	.075	-.007

TABLE 24 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	.001	.001	.003	-.002	-.002
APEXMIN	2	-.003	.000	.005	-.001	.001
CLYSDL	3	.002	-.002	-.001	.000	-.003
TAVISTK	4	-.002	.006	.003	.006	.002
TRNSNTL	5	.005	-.002	-.002	-.007	.002
VIERFNT	6	.005	.004	-.015	-.003	.005
WELGDCT	7	.000	.003	-.001	.002	.004
COAL		.007	.010	-.008	-.005	.009
BLYVOOR	8	001	-.010	.002	.003	.000
DOORNS	9	-.004	-.006	.004	.007	-.003
DRIECON10	10	-.002	.003	-.003	.003	.002
KLOOF	11	.000	-.004	-.003	.002	-.002
WAREAS	12	-.005	.001	-.006	-.001	-.003
WESDRIE	13	-.003	-.001	-.003	-.004	.000
WSTNDP	14	-.001	-.005	-.003	-.005	-.001
GOLD		-.014	-.022	-.012	.005	-.007
BANKORP	15	.002	.005	-.003	-.007	-.001
BOLAND	16	.005	.001	.009	-.012	-.002
ICLEF	17	.016	.005	.003	-.003	.004
NEDBANK	18	-.001	.001	.002	.001	-.002
STANBIC	19	.002	.000	-.001	-.002	.001
T & T	20	-.002	.007	.009	.000	-.004
VOLKSKAS	21	-.002	.003	.003	-.002	-.001
BANKS		.020	.016	.022	-.025	-.005
ALPHA	22	-.002	.005	.010	.007	-.002
BOUMAT	23	.009	.008	.003	.001	.000
EVRITE	24	-.001	.004	.001	.002	.000
GRNAKR	25	-.002	.000	.003	.001	.000
LTA	26	-.005	-.002	-.002	-.003	.001
M & R	27	-.002	.001	.000	-.003	-.010
PPCEM	28	.001	-.002	.011	-.001	.001
BUILDING		-.002	.014	.026	.004	-.010
AECI	29	.004	-.003	.000	-.002	.003
CHEMHD	30	-.001	.007	.001	-.001	-.001
DEBERL	31	.000	-.001	.000	.003	-.003
LANCHEM	32	.002	.002	.007	.001	.008
SENCHM	33	.004	.007	-.002	-.003	.000
TREK	34	.000	-.001	.002	.000	.018
TRIOMF	35	.006	.004	-.002	.007	.001
CHEMICALS		.015	.015	.006	.005	.024

TABLE 24 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
CADSWP	36	-.004	-.007	.003	.001	.005
FEDFOOD	37	.006	-.001	.001	.019	.002
ICS	38	-.003	-.002	-.003	-.002	-.001
I & J	39	.004	.007	.000	-.003	-.002
KANHYM	40	.005	.004	.001	-.001	.008
PREMGRP	41	-.002	.002	-.001	.002	.000
TIGOATS	42	.002	-.001	-.002	.003	.001
FOOD		.008	.002	-.001	.019	.013
ASSENG	43	.000	.007	-.002	-.001	-.002
DUNLOP	44	-.003	-.002	.005	-.002	.010
GENTRA	45	.012	.002	.001	.004	-.007
MCCARTHY	46	.002	.017	.012	-.005	.005
SAFICON	47	.003	.001	.001	.002	-.004
TOYOTA	48	-.008	.004	.004	-.000	.002
WMHUNT	49	.012	.005	.000	.001	.009
MOTOR		.018	.034	.023	-.001	.013

TABLE 25

LITTLE JIFFY, VARIMAX ROTATION, FIRST SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	45.50	45.50	14.63	14.63
2	Gold	14.50	60.00	6.00	20.63
3	Coal	12.00	72.00	4.10	24.73
4		7.30	79.30	2.70	27.43
5	Motor	5.00	84.30	1.60	29.03
6	Motor	3.60	87.90	1.00	30.03
7		3.10	91.00	0.90	30.93
8		1.80	92.80	0.90	31.83
9		1.80	94.60	0.80	32.60
10		1.80	96.40	0.80	33.40
11		1.80	98.20	0.10	33.50
12		1.80	100.00	0.06	33.56

TABLE 26

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.017	-.001	.000	.005	.001	.002
APEXMIN	2	.028	.007	.004	.001	-.003	-.004
CLYSDL	3	.020	.008	.006	.001	.005	.002
TAVISTK	4	.029	.000	.004	.000	.006	-.003
TRNSNTL	5	.022	.006	.010	.005	.006	.000
VIERFNT	6	.014	-.003	.002	.000	.000	.001
WELGDCT	7	.039	.003	-.006	-.002	.002	.001
COAL		.169	.020	.020	.010	.017	-.002
BLYVOOR	8	-.009	-.002	.000	-.007	-.006	.002
DOORNS	9	-.005	-.002	-.014	-.006	-.003	-.002
DRIECON	10	-.007	-.010	-.009	-.010	.000	-.007
KLOOF	11	-.009	-.005	.000	-.008	-.001	-.007
WAREAS	12	-.002	-.006	-.009	-.007	-.004	-.012
WESDRIE	13	-.005	.000	-.005	- 009	-.001	-.002
WSTNDP	14	-.009	-.004	-.006	-.004	-.005	-.004
GOLD		-.046	-.029	-.043	-.051	-.020	-.032
BANKORP	15	.001	.000	-.004	.005	.005	.002
BOLAND	16	.003	.001	.002	.013	.007	.006
ICLEF	17	-.006	.004	.002	.011	.012	-.008
NEDBANK	18	.001	.000	.015	.011	-.002	.004
STANBIC	19	.002	.000	.003	.002	.001	.009
T & T	20	.000	.001	.009	.004	-.001	.006
VOLKSKAS	21	.004	.006	.006	.011	-.001	.007
BANKS		.005	.012	.033	.057	.021	.008
ALPHA	22	.002	-.003	.008	.000	.004	.000
BOUMAT	23	-.003	.001	-.006	.007	.016	.013
EVRITE	24	.006	.003	-.002	.000	.002	.001
GRNAKR	25	-.002	.005	.004	-.001	-.008	-.002
LTA	26	.003	.019	-.005	.002	-.007	-.009
M & R	27	-.005	.003	.000	.009	-.006	-.006
PPCEM	28	.002	-.002	.003	.003	.002	.004
BUILDING		.003	.026	.002	.020	.003	.001
AECI	29	-.004	-.001	.008	.011	.000	.004
CHEMHD	30	.002	.007	-.007	-.003	.003	.001
DEBERL	31	.005	.001	-.002	.003	-.002	.006
LANCHEM	32	-.001	.003	.029	-.004	.001	-.001
SENCHM	33	-.001	.003	.000	.006	.004	.002
TREK	34	-.001	.005	.008	-.001	.001	.009
TRIOMF	35	-.002	.005	.009	-.002	.001	000
CHEMICALS		-.002	.023	.045	.010	.008	.021

TABLE 26 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.006	.001	-.005	-.003	-.002	.023
FEDFOOD	37	.000	.003	.002	.001	-.001	-.003
ICS	38	-.006	-.004	.005	.003	-.005	.004
I & J	39	-.002	.000	.006	.009	.004	-.006
KANHYM	40	.003	-.003	.012	.003	.006	.001
PREMGRP	41	.000	-.001	.002	.006	.002	.004
TIGOATS	42	.000	-.003	.006	.010	-.002	.005
FOOD		.001	-.007	.028	.029	.002	.028
ASSENG	43	-.003	.008	-.001	-.001	.006	.006
DUNLOP	44	-.002	.013	.005	-.004	.005	.017
GENTRA	45	-.006	.011	-.006	.013	.011	-.003
MCCARTHY	46	.005	.010	.013	-.003	.016	-.002
SAFICON	47	.000	.034	.000	.002	.005	.000
TOYOTA	48	-.001	.018	.012	-.003	-.001	.003
WMHUNT	49	-.001	.006	.000	-.001	.018	.008
MOTOR		-.008	.100	.023	.003	.060	.039

TABLE 26 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	.005	-.003	-.001	-.001	.001
APEXMIN	2	.002	-.005	-.002	.003	-.002
CLYSDL	3	-.007	.001	-.001	-.014	.002
TAVISTK	4	-.002	.007	.006	-.003	-.003
TRNSNTL	5	-.011	-.003	-.008	-.007	.005
VIERFNT	6	-.002	.002	-.003	.001	.021
WELGDCT	7	.000	-.003	.001	.002	.005
COAL		-.015	.002	-.008	-.019	.029
BLYVOOR	8	-.002	-.015	.001	-.003	-.001
DOORNS	9	-.003	-.003	.006	-.002	-.011
DRIECON	10	.000	.001	.000	.003	-.001
KLOOF	11	-.007	-.004	-.002	-.007	-.002
WAREAS	12	-.004	.002	-.006	-.001	-.002
WESDRIE	13	-.001	-.003	-.007	-.001	.000
WSTNDP	14	-.004	-.007	-.008	.000	.000
GOLD		-.021	-.029	-.016	-.011	-.017
BANKORP	15	.004	.004	-.005	.001	.004
BOLAND	16	.006	-.003	-.008	.001	-.004
ICLEF	17	-.002	-.005	.002	.011	.002
NEDBANK	18	.004	.008	.004	-.002	.001
STANBIC	19	.005	-.001	-.001	-.001	.006
T & T	20	.018	.000	.002	-.001	.000
VOLKSKAS	21	-.002	.008	.001	.000	-.004
BANKS		.033	.011	-.005	.009	.005
ALPHA	22	.008	.003	.008	-.003	-.008
BOUMAT	23	.008	.004	.007	-.001	.003
EVRITE	24	.004	.004	.003	.002	.000
GRNAKR	25	.009	-.006	.001	.007	.002
LTA	26	-.004	.003	-.003	.013	-.003
M & R	27	.005	.004	-.002	-.002	-.001
PPCEM	28	.003	-.008	.000	.002	-.008
BUILDING		-.033	.004	.014	.018	-.015
AECI	29	-.003	.002	.002	.004	.002
CHEMHD	30	.009	.001	-.001	.003	.002
DEBERL	31	.004	.000	.004	-.003	.002
LANCHEM	32	.001	-.005	.001	.002	.002
SENCHM	33	.008	.005	.000	.005	.007
TREK	34	-.004	.002	.003	.021	.001
TRIOMF	35	.007	-.005	.009	-.003	.011
CHEMICALS		.022	.000	.018	.028	.027

TABLE 26 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
CADSWP	36	.004	-.003	.002	.001	.000
FEDFOOD	37	-.001	-.001	.023	.004	.001
ICS	38	.000	.005	-.002	-.002	.002
I & J	39	.003	.006	.000	.003	.002
KANHYM	40	-.002	.002	.001	.005	.004
PREMGRP	41	.000	.012	.005	.000	-.001
TIGOATS	42	.000	.007	.007	.002	.004
FOOD		.004	.028	.036	.013	.012
ASSENG	43	.007	.007	.000	-.003	.004
DUNLOP	44	-.002	.004	.000	.005	-.004
GENTRA	45	-.002	.002	.009	-.004	.000
MCCARTHY	46	.013	.004	-.004	.006	-.003
SAFICON	47	.000	.001	.005	-.001	.001
TOYOTA	48	.004	.010	.000	.002	-.003
WMHUNT	49	-.002	-.003	.004	.004	.005
MOTOR		.018	.025	.014	.009	.000

TABLE 27

LITTLE JIFFY, ORTHOBLIQUE ROTATION, FIRST SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	48.10	48.10	14.63	14.63
2	Coal	10.40	58.50	3.30	17.93
3	Motor	5.90	64.40	1.60	19.53
4	Lanchem	5.90	70.30	1.70	21.23
5	Banks	3.90	74.20	1.60	22.83
6	Motor	9.70	83.90	3.40	26.23
7		3.90	87.80	1.60	27.83
8		3.90	91.70	1.10	28.93
9		2.30	94.00	1.10	30.03
10		2.10	96.10	1.20	31.23
11		2.00	98.10	0.90	32.13
12		1.90	100.00	0.80	32.93

TABLE 28

LITTLE JIFFY AND ORTHOBLIQUE ROTATION, FIRST SUBPERIOD CORRELATION MATRIX OF FACTORS

	2	3	4	5	6	7	8	9	10	11	12
2	1.000										
3	.178	1.000									
4	.153	.210	1.000								
5	.103	.174	.279	1.000							
6	.156	.295	.160	.300	1.000						
7	.086	.137	.181	.188	.241	1.000					
8	-.038	.136	.179	.183	.183	.282	1.000				
9	.051	.176	.180	.376	.201	.158	.172	1.000			
10	-.037	.148	.106	.184	.167	.174	.140	.163	1.000		
11	-.097	.200	.109	.072	.079	.057	.124	.067	.097	1.000	
12	.199	.025	.163	.137	.137	.114	.042	.045	.007	.029	1.000

TABLE 29

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.0218	-.0119	.0117	.0000	.0017	-.0024
APEXMIN	2	.0163	-.0118	.0137	.0092	-.0056	-.0012
CLYSDL	3	.0147	-.0091	.0150	-.0064	-.0006	-.0057
TAVISTK	4	.0146	-.0150	.0101	.0081	-.0064	-.0010
TRNSNTL	5	.0186	-.0102	.0161	-.0106	-.0022	-.0013
VIERFNT	6	.0162	-.0100	.0101	.0020	.0076	.0038
WELGDCT	7	.0187	-.0175	.0147	.0031	-.0057	.0037
COAL		.1209	.0855	.0914	.0063	-.0112	-.0041
BLYVOOR	8	.0299	.0312	.0046	-.0000	-.0008	-.0022
DOORNS	9	.0415	.0327	-.0004	.0040	.0079	-.0015
DRIECON	10	.0303	.0304	.0013	-.0009	.0001	-.0018
KLOOF	11	.0353	.0285	-.0008	.0005	.0012	.0023
WAREAS	12	.0425	.0281	.0014	.0049	-.0004	.0052
WESDRIE	13	.0337	.0213	.0018	-.0005	-.0077	-.0025
WSTNDP	14	.0237	.0289	.0075	.0079	.0083	.0018
GOLD		.2369	.2011	.0154	.0153	.0086	.0013
BANKORP	15	.0163	-.0133	-.0024	-.0019	.0088	-.0026
BOLAND	16	.0107	-.0504	-.0075	-.0017	-.0004	-.0020
ICLEF	17	.0027	-.0009	.0035	.0012	-.0045	.0017
NEDBANK	18	.0168	-.0041	-.0027	-.0035	.0067	-.0059
STANBIC	19	.0125	-.0094	-.0041	-.0034	.0055	.0016
T & T	20	.0071	-.0016	-.0061	.0043	-.0037	.0037
VOLKSKAS	21	.0170	-.0091	.0000	-.0028	.0034	-.0007
BANKS		.0831	-.0888	-.0193	-.0078	.0158	-.0042
ALPHA	22	.0190	-.0104	-.0021	-.0038	.0005	-.0034
BOUMAT	23	.0119	-.0059	-.0021	.0037	.0022	.0005
EVRITE	24	.0083	-.0047	-.0022	.0043	-.0025	-.0022
GRNAKR	25	.0180	-.0050	-.0072	-.0032	-.0022	.0090
LTA	26	.0191	-.0097	-.0074	-.0008	-.0034	-.0004
M & R	27	.0176	-.0101	-.0086	-.0060	-.0069	.0022
PPCEM	28	.0125	-.0055	-.0031	-.0024	.0023	-.0010
BUILDING		.1064	-.0513	-.0327	-.0082	-.0100	.0047
AECI	29	.0193	-.0087	-.0005	-.0035	.0014	.0004
CHEMHD	30	.0091	-.0075	-.0027	.0087	-.0008	.0016
DEBERL	31	.0090	-.0022	-.0030	-.0036	-.0006	.0064
LANCHEM	32	.0102	-.0063	.0088	-.0026	.0118	.0243
SENCHM	33	.0181	-.0050	-.0020	-.0030	.0000	.0001
TREK	34	.0152	-.0013	.0010	-.0052	.0018	.0025
TRIOMF	35	.0164	-.0011	.0128	.0151	.0160	0176
CHEMICALS		.0973	.0321	.0144	.0009	.0296	.0529

TABLE 29 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.0113	-.0097	-.0053	-.0015	-.0067	.0005
FEDFOOD	37	.0127	-.0068	-.0039	.0036	-.0008	.0013
ICS	38	.0190	-.0087	-.0064	-.0020	-.0043	.0029
I & J	39	.0119	-.0111	.0051	.0074	.0086	-.0009
KANHYM	40	.0120	-.0092	.0075	.0033	.0059	.0074
PREMGRP	41	.0120	-.0047	-.0036	.0009	-.0039	.0013
TIGOATS	42	.0138	-.0055	-.0034	-.0043	-.0006	.0007
FOOD		.0927	.0557	-.0100	.0074	-.0018	.0132
ASSENG	43	-.0004	-.0083	-.0058	.0100	-.0005	-.0039
DUNLOP	44	.0139	-.0080	-.0109	.0022	-.0004	.0039
GENTRA	45	.0041	-.0036	-.0017	.0033	-.0041	.0023
MCCARTHY	46	.0026	-.0098	-.0144	.0148	.0029	-.0133
SAFICON	47	.0179	-.0133	-.0033	.0169	.0101	-.0032
TOYOTA	48	.0207	-.0066	-.0120	.0113	-.0028	-.0049
WMHUNT	49	.0152	-.0017	-.0020	.0045	-.0015	-.0034
MOTOR		.0740	-.0513	-.0501	.0630	.0037	-.0225

TABLE 29 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	-.0034	.0017	-.0053	-.0040	.0006	.0036
APEXMIN	2	.0006	.0034	.0046	.0000	-.0043	.0011
CLYSDL	3	-.0032	.0020	.0044	.0009	-.0021	-.0011
TAVISTK	4	.0011	.0011	-.0031	-.0045	.0021	.0007
TRNSNTL	5	-.0014	-.0000	-.0015	.0006	.0025	.0002
VIERFNT	6	-.0015	.0013	-.0011	.0058	.0151	-.0082
WELGDCT	7	-.0027	-.0063	-.0032	.0025	-.0055	-.0007
COAL		-.0105	.0032	-.0052	.0013	.0075	-.0044
BLYVOOR	8	-.0021	-.0021	.0011	-.0015	.0021	.0001
DOORNS	9	-.0047	-.0080	-.0044	-.0039	-.0002	-.0049
DRIECON	10	.0009	.0025	.0034	-.0015	.0000	-.0011
KLOOF	11	.0006	.0014	.0005	.0025	.0007	.0011
WAREAS	12	-.0033	-.0033	-.0011	.0027	-.0032	-.0020
WESDRIE	13	.0045	.0030	-.0039	.0016	.0021	.0021
WSTNDP	14	-.0108	.0049	.0030	.0003	-.0082	.0085
GOLD		-.0149	-.0016	-.0014	.0002	-.0067	.0038
BANKORP	15	.0021	.0061	-.0064	.0056	-.0027	-.0041
BOLAND	16	-.0035	.0047	.0065	-.0015	.0014	.0010
ICLEF	17	-.0011	.0040	.0035	.0027	.0033	-.0032
NEDBANK	18	.0036	.0020	.0006	-.0009	-.0020	-.0017
STANBIC	19	-.0054	-.0021	-.0007	-.0039	-.0007	-.0001
T & T	20	.0029	.0007	-.0063	.0002	.0001	.0015
VOLKSKAS	21	.0036	-.0024	.0015	-.0040	.0011	-.0004
BANKS		.0022	.0130	-.0013	-.0018	.0005	-.0070
ALPHA	22	-.0025	-.0042	.0009	.0000	-.0058	-.0034
BOUMAT	23	-.0084	.0014	.0014	.0002	.0034	.0002
EVRITE	24	.0009	-.0042	-.0031	-.0003	-.0028	-.0025
GRNAKR	25	-.0079	.0012	.0042	-.0041	-.0041	.0019
LTA	26	.0009	.0108	.0003	-.0032	-.0025	-.0013
M & R	27	-.0023	.0038	-.0005	-.0002	-.0015	-.0004
PPCEM	28	-.0014	-.0010	-.0034	.0001	.0025	.0031
BUILDING		-.0207	.0078	-.0002	-.0075	-.0108	-.0024
AECI	29	-.0004	-.0013	.0021	.0051	.0009	.0011
CHEMHD	30	-.0020	-.0008	.0004	.0053	-.0019	-.0003
DEBERL	31	-.0007	-.0011	-.0034	-.0012	-.0009	-.0038
LANCHEM	32	.0237	.0092	.0004	-.0102	-.0044	-.0039
SENCHM	33	.0046	-.0007	.0041	.0081	-.0065	.0014
TREK	34	-.0024	-.0016	-.0079	.0144	.0027	-.0003
TRIOMF	35	.0083	.0095	-.0032	.0013	-.0022	-.0053
CHEMICALS		.0311	.0132	-.0075	.0228	-.0123	-.0111

TABLE 29 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	-.0006	-.0060	.0039	-.0015	-.0019	-.0068
FEDFOOD	37	.0009	-.0034	-.0012	-.0012	.0002	.0053
ICS	38	.0071	.0000	.0016	-.0021	.0027	-.0007
I & J	39	.0020	-.0053	.0120	-.0029	.0066	.0030
KANHYM	40	.0056	-.0060	.0036	.0005	.0002	.0044
PREMGRP	41	.0008	-.0029	.0002	-.0035	.0025	-.0002
TIGOATS	42	.0035	.0002	-.0005	.0017	.0050	.0031
FOOD		.0193	-.0234	.0196	-.0090	.0153	.0081
ASSENG	43	-.0098	.0092	-.0078	-.0015	.0051	-.0081
DUNLOP	44	-.0032	-.0002	-.0023	.0024	.0022	.0034
GENTRA	45	.0003	-.0024	.0049	.0028	.0036	-.0041
MCCARTHY	46	.0065	-.0023	-.0025	-.0001	-.0003	-.0025
SAFICON	47	-.0050	.0054	.0070	.0044	.0015	.0011
TOYOTA	48	.0010	.0000	.0015	.0039	-.0052	.0114
WMHUNT	49	.0017	-.0060	.0003	.0018	.0019	-.0026
MOTOR		-.0085	.0037	.0011	.0137	.0088	-.0014

TABLE 30

LITTLE JIFFY, NO ROTATION, SECOND SUBPERIOD

(A) Percentage of Communality Explained.
(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	41.90	41.90	15.91	15.91
2	Gold	27.37	69.27	10.52	26.43
3	Coal	6.30	75.57	2.51	28.94
4	Motor	4.51	80.08	1.60	30.54
5	Lanchem & Triomf	3.28	83.36	1.11	31.65
6	Lanchem & Triomf	3.76	87.12	0.94	32.59
7		3.80	90.92	0.96	33.55
8		2.17	93.09	0.76	34.31
9		1.82	94.91	0.77	35.08
10		1.76	96.67	0.68	35.76
11		1.74	98.40	0.63	36.39
12		1.60	100.00	0.55	36.94

TABLE 30

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	-.004	.016	.000	-.004	.005	-.001
APEXMIN	2	-.002	.020	.002	.002	-.001	.006
CLYSDL	3	-.004	.014	-.004	-.010	.000	.004
TAVISTK	4	-.007	.019	.001	.004	.000	.000
TRNSNTL	5	-.006	.014	-.002	-.013	-.002	-.002
VIERFNT	6	-.003	.008	.007	-.001	.009	.003
WELGDCT	7	-.007	.023	.001	.001	.000	-.004
COAL		-.033	.114	.005	-.021	.011	.006
BLYVOOR	8	.027	-.012	-.005	-.007	-.006	-.002
DOORNS	9	.030	-.016	-.002	.001	.000	-.008
DRIECON	10	.024	-.015	-.002	-.007	-.007	.000
KLOOF	11	.023	-.015	.000	-.004	-.005	-.001
WAREAS	12	.026	-.011	.000	.000	-.004	-.006
WESDRIE	13	.016	-.008	-.004	- 003	-.012	-.002
WSTNDP	14	.034	-.004	-.003	-.009	.011	.001
GOLD		.180	-.081	-.016	-.029	-.023	-.018
BANKORP	15	-.011	.003	.004	.004	.005	.001
BOLAND	16	-.045	.018	.001	.006	.015	.009
ICLEF	17	.000	.003	.000	-.001	.000	.002
NEDBANK	18	-.005	-.002	.001	.000	.000	.003
STANBIC	19	-.009	.000	.001	-.001	.009	-.003
T & T	20	-.003	-.001	.002	.007	-.002	-.004
VOLKSKAS	21	-.009	.002	.003	-.001	.001	.001
BANKS		-.082	.023	.012	.014	.028	.009
ALPHA	22	-.010	.003	-.004	.002	.002	-.002
BOUMAT	23	-.003	.002	-.002	.002	.011	.002
EVRITE	24	-.003	.003	-.002	.008	-.001	-.002
GRNAKR	25	-.007	-.003	.002	-.003	.007	-.004
LTA	26	-.012	.001	.001	.002	.002	.000
M & R	27	-.014	-.001	-.003	.000	.001	-.004
PPCEM	28	-.006	.000	-.002	.000	.004	.000
BUILDING		-.055	.005	.000	.011	.012	-.010
AECI	29	-.010	.000	-.001	-.005	.001	.002
CHEMHD	30	-.003	.005	.001	.009	.004	.003
DEBERL	31	-.004	-.002	.003	.000	.001	-.008
LANCHEM	32	-.006	.004	.037	-.007	-.009	-.004
SENCHM	33	-.006	.000	.001	.001	-.005	.005
TREK	34	.000	.000	-.001	-.001	-.001	-.003
TRIOMF	35	.010	.011	.029	.005	.005	001
CHEMICALS		-.019	.018	.069	.002	-.002	-.004

TABLE 30 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	-.011	.001	-.003	.005	-.001	-.003
FEDFOOD	37	-.006	.002	.000	.005	.002	.001
ICS	38	-.012	-.002	.003	.003	-.004	.000
I & J	39	-.005	.007	.006	.001	.007	.014
KANHYM	40	-.004	.009	.010	-.001	.000	.005
PREMGRP	41	-.006	.000	-.001	.003	.000	-.002
TIGOATS	42	-.008	-.002	.000	-.001	-.001	.002
FOOD		-.052	.015	.015	.015	.003	.017
ASSENG	43	-.004	.004	-.003	.012	.013	-.004
DUNLOP	44	-.009	-.003	.000	.006	.007	-.001
GENTRA	45	-.003	.001	.000	.004	.000	.002
MCCARTHY	46	-.008	-.002	-.004	.023	.002	.008
SAFICON	47	-.004	.007	.004	.012	.015	.015
TOYOTA	48	-.005	-.000	-.005	.013	.001	.009
WMHUNT	49	-.001	.000	-.003	.007	-.001	.002
MOTOR		-.033	.007	-.011	.077	.037	.031

TABLE 30 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	-.003	.000	-.002	.005	-.002
APEXMIN	2	.002	-.003	.002	-.003	.002
CLYSDL	3	-.003	.002	.001	.002	.008
TAVISTK	4	-.003	-.003	.000	-.002	-.004
TRNSNTL	5	-.006	.006	-.002	.000	.003
VIERFNT	6	-.015	.011	-.001	-.005	.002
WELGDCT	7	.003	.004	-.006	-.004	.004
COAL		-.025	.017	-.008	-.007	.018
BLYVOOR	8	-.003	-.004	.000	-.002	-.001
DOORNS	9	-.005	-.004	-.005	.003	.002
DRIECON	10	-.002	-.005	.004	-.001	.000
KLOOF	11	.001	.000	.003	-.001	-.003
WAREAS	12	.003	.000	.000	-.004	.000
WESDRIE	13	-.002	.000	.006	-.001	-.007
WSTNDP	14	.011	-.004	.003	.002	-.001
GOLD		.003	-.017	.011	-.004	-.010
BANKORP	15	-.001	.009	.004	.010	.004
BOLAND	16	.003	.000	.001	-.001	.004
ICLEF	17	-.002	.002	.005	-.007	.001
NEDBANK	18	-.002	.000	.001	.008	.004
STANBIC	19	-.001	-.001	-.004	.003	.002
T & T	20	.003	.001	.001	-.001	-.007
VOLKSKAS	21	-.003	-.002	-.004	.002	.002
BANKS		-.001	.009	.004	.014	.010
ALPHA	22	.003	.000	-.003	.003	.008
BOUMAT	23	.000	.000	.001	-.003	-.001
EVRITE	24	.001	-.001	-.002	.001	.001
GRNAKR	25	.010	-.004	.001	-.006	.000
LTA	26	.003	-.004	.011	.002	-.001
M & R	27	.005	.000	.005	-.003	-.001
PPCEM	28	.000	.002	-.002	.003	-.003
BUILDING		.022	-.007	.011	-.003	.003
AECI	29	.001	.007	-.002	.000	.003
CHEMHD	30	.005	.003	.000	-.003	.000
DEBERL	31	.001	.001	.000	-.002	.001
LANCHEM	32	-.002	-.004	.000	.004	.000
SENCHM	33	.007	.006	-.001	.001	.005
TREK	34	.000	.018	-.001	.000	-.001
TRIOMF	35	-.002	.004	.003	.003	.000
CHEMICALS		.010	.035	-.001	.003	.008

TABLE 30 (CONTINUED)

VARIMAX ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
CADSWP	36	.001	-.003	-.002	-.007	.007
FEDFOOD	37	.003	-.001	-.005	-.000	-.005
ICS	38	-.001	-.002	.000	-.003	-.002
I & J	39	-.005	-.005	-.010	-.003	.002
KANHYM	40	-.001	.002	-.011	-.001	.000
PREMGRP	41	-.001	-.003	-.002	-.004	-.002
TIGOATS	42	-.002	.003	-.001	-.000	-.004
FOOD		-.004	-.009	-.031	-.018	-.004
ASSENG	43	-.006	.000	.012	.000	-.002
DUNLOP	44	.005	.003	-.000	-.002	-.006
GENTRA	45	-.001	.001	.000	-.009	.002
MCCARTHY	46	-.003	-.004	.000	.008	-.001
SAFICON	47	.002	.000	.003	.001	.001
TOYOTA	48	.012	-.002	.000	.003	-.008
WMHUNT	49	-.003	.000	-.004	-.002	.001
MOTOR		.006	-.002	.011	-.001	-.013

TABLE 31

LITTLE JIFFY, VARIMAX ROTATION, SECOND SUBPERIOD

(A) Percentage of Communality Explained.

(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	41.90	41.90	15.91	15.91
2	Gold	21.40	63.30	8.10	24.01
3	Coal, Gold	9.50	72.80	3.60	27.61
4	Lanchem & Triomf	7.10	79.90	1.10	28.71
5	Motor	4.50	84.40	1.70	30.41
6		4.30	88.70	1.60	32.01
7		2.20	90.90	1.00	33.01
8		1.80	92.70	0.80	33.81
9		1.80	94.50	0.80	34.61
10		1.70	96.20	0.80	35.41
11		2.00	98.20	1.00	36.41
12		1.80	100.00	0.70	37.11

TABLE 32

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AMCOAL	1	.005	.000	.011	-.004	.003	.004
APEXMIN	2	.007	.003	.018	.002	.004	.001
CLYSDL	3	.008	-.004	.009	-.010	.003	-.001
TAVISTK	4	.005	.002	.016	.001	-.002	.006
TRNSNTL	5	.007	-.003	.009	-.015	-.002	-.001
VIERFNT	6	.002	.003	-.001	-.010	-.005	.014
WELGDCT	7	.003	.000	.022	-.004	-.001	-.001
COAL		.037	.001	.084	-.040	.000	.022
BLYVOOR	8	-.022	-.004	-.005	-.005	.005	-.002
DOORNS	9	-.032	-.001	-.007	-.001	.004	.003
DRIECON	10	-.018	-.001	-.009	-.003	.005	-.002
KLOOF	11	-.018	.001	-.009	-.001	.005	-.003
WAREAS	12	-.023	.001	-.002	-.001	.007	-.002
WESDRIE	13	-.015	-.003	-.001	- 001	-.002	-.002
WSTNDP	14	-.016	-.002	-.002	-.000	.030	-.001
GOLD		-.144	-.009	-.035	-.012	.054	-.009
BANKORP	15	.006	.005	-.001	.003	-.002	.005
BOLAND	16	.044	-.001	.004	.005	-.003	.005
ICLEF	17	.004	.000	.002	-.004	.000	.004
NEDBANK	18	.003	.001	-.005	.002	-.002	.000
STANBIC	19	.008	.000	-.003	-.002	.002	.001
T & T	20	-.001	.003	.001	.006	-.003	.001
VOLKSKAS	21	.006	.002	-.001	-.001	-.004	-.001
BANKS		.070	.010	-.003	.009	-.012	.015
ALPHA	22	.006	-.003	.003	.000	-.002	-.002
BOUMAT	23	.007	-.003	-.002	.001	.005	.006
EVRITE	24	-.003	-.001	.006	.005	-.004	.001
GRNAKR	25	.013	.001	-.003	-.002	.008	-.002
LTA	26	.015	.003	-.002	.003	.001	.004
M & R	27	.014	-.002	-.002	-.001	-.001	.000
PPCEM	28	.004	-.002	-.003	.001	-.001	.000
BUILDING		.056	-.007	-.003	.007	.006	.007
AECI	29	.009	-.003	-.003	-.004	-.002	-.004
CHEMHD	30	.003	.000	.005	.008	.001	.002
DEBERL	31	.001	.003	-.001	-.004	-.002	.001
LANCHEM	32	.005	.038	-.001	-.007	-.005	-.006
SENCHM	33	.005	.001	.001	.004	-.001	-.009
TREK	34	-.004	-.004	-.001	-.003	-.002	.000
TRIOMF	35	-.007	.029	.005	.002	.007	008
CHEMICALS		.012	.064	.005	-.004	-.004	-.008

TABLE 32 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
CADSWP	36	.007	-.003	.003	.000	-.007	.000
FEDFOOD	37	.003	.000	.002	.006	-.001	-.002
ICS	38	.009	.003	-.002	.002	-.009	-.002
I & J	39	.008	.003	.000	.003	.000	-.001
KANHYM	40	.003	.008	.005	.001	-.001	-.007
PREMGRP	41	.003	-.001	.001	.001	-.004	.001
TIGOATS	42	.006	-.001	-.005	.000	-.005	-.002
FOOD		.039	.009	.004	.013	-.027	-.013
ASSENG	43	.004	-.002	.001	.004	.002	.021
DUNLOP	44	.008	-.001	-.004	.006	.000	.001
GENTRA	45	.003	-.001	.001	.000	-.004	.002
MCCARTHY	46	-.002	-.002	.000	.023	-.010	.007
SAFICON	47	.010	.003	-.001	.015	.009	.009
TOYOTA	48	.006	-.003	.002	.021	.004	-.006
WMHUNT	49	-.004	-.003	.002	.005	-.006	.001
MOTOR		.025	-.009	.001	.074	-.005	.035

TABLE 32 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
AMCOAL	1	.002	.001	.000	.000	-.010
APEXMIN	2	.004	-.006	-.003	.002	.002
CLYSDL	3	.004	-.002	-.001	.009	-.001
TAVISTK	4	.002	-.003	-.003	-.004	-.004
TRNSNTL	5	.002	-.002	.003	.003	-.005
VIERFNT	6	.012	-.002	.009	-.001	.001
WELGDCT	7	.001	.005	.005	-.002	.001
COAL		.027	-.009	.010	.007	-.016
BLYVOOR	8	.002	-.005	-.005	-.001	.001
DOORNS	9	.002	.007	-.005	.000	.001
DRIECON	10	-.001	-.007	-.006	.002	.003
KLOOF	11	-.001	-.006	.000	-.001	.003
WAREAS	12	-.002	.000	.001	-.003	.007
WESDRIE	13	-.005	-.012	-.002	-.002	-.001
WSTNDP	14	.001	.000	-.001	.000	.000
GOLD		-.004	-.023	-.018	-.005	.014
BANKORP	15	-.005	.003	.008	.010	-.003
BOLAND	16	.003	.006	.002	.001	-.001
ICLEF	17	.001	-.006	.001	-.001	.005
NEDBANK	18	.000	.002	-.001	.009	-.003
STANBIC	19	.000	.010	.000	.000	-.002
T & T	20	-.006	.000	.002	-.006	-.002
VOLKSKAS	21	.004	.004	-.002	.002	-.003
BANKS		-.003	.019	.010	.015	-.009
ALPHA	22	-.002	.008	.000	.006	.002
BOUMAT	23	.003	.002	.001	-.003	.001
EVRITE	24	-.002	.004	-.001	.000	.001
GRNAKR	25	-.005	.006	-.002	-.006	.005
LTA	26	-.009	-.003	-.004	.002	.001
M & R	27	-.009	.001	.000	-.002	.002
PPCEM	28	.000	.003	.003	-.002	-.005
BUILDING		-.024	.021	-.003	-.005	-.007
AECI	29	.001	.001	.006	.002	.000
CHEMHD	30	.001	.001	.005	-.002	.004
DEBERL	31	-.005	.005	.001	-.003	.002
LANCHEM	32	-.001	.000	-.003	.000	-.001
SENCHM	33	-.001	-.001	.006	.006	.005
TREK	34	-.003	.000	.017	.000	-.001
TRIOMF	35	.004	-.001	.007	.001	.001
CHEMICALS		-.004	.005	.039	.004	.010

TABLE 32 (CONTINUED)

ORTHOBLIQUE ROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

SHARE		FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11	FACTOR 12
CADSWP	36	-.001	.006	-.003	-.001	.009
FEDFOOD	37	.001	.002	.001	-.005	-.003
ICS	38	-.001	-.001	-.002	-.004	.001
I & J	39	.020	.001	-.003	-.001	.000
KANHYM	40	.010	.003	.004	-.003	-.001
PREMGRP	41	.000	.002	-.003	-.005	.000
TIGOATS	42	.001	-.002	.003	-.003	-.003
FOOD		.030	.011	-.003	-.022	.003
ASSENG	43	-.006	.001	-.001	-.001	.000
DUNLOP	44	-.002	.003	.005	-.007	.000
GENTRA	45	.004	-.001	.001	-.003	.008
MCCARTHY	46	.002	.002	-.003	.005	-.001
SAFICON	47	.010	-.001	.003	.003	.003
TOYOTA	48	-.001	-.003	.002	-.003	-.002
WMHUNT	49	.004	.002	.000	.000	.003
MOTOR		.011	.003	.007	-.006	.011

TABLE 33

LITTLE JIFFY, ORTHOBLIQUE ROTATION, SECOND SUBPERIOD

(A) Percentage of Communality Explained.
(B) Percentage of Total Variance Explained.

Factor	Dominant Shares	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
1	Market	35.30	35.30	15.91	15.91
2	Gold	19.60	54.90	9.60	25.51
3	Lanchem & Triomf	5.90	60.80	1.10	26.61
4	Coal	7.80	68.60	3.70	30.31
5	Motor	5.90	74.50	2.20	32.51
6		7.80	82.30	2.60	35.11
7		3.90	86.20	1.70	36.81
8		3.90	90.10	1.60	38.41
9		3.00	93.10	2.60	41.01
10		3.00	96.10	1.50	42.51
11		2.00	98.10	1.10	43.61
12		1.90	100.00	1.20	44.81

TABLE 34

LITTLE JIFFY AND ORTHOBLIQUE ROTATION
SECOND SUBPERIOD CORRELATION MATRIX OF FACTORS

	2	3	4	5	6	7	8	9	10	11	12
2	1.000										
3	.043	1.000									
4	.316	.072	1.000								
5	.150	-.065	-.037	1.000							
6	-.323	-.024	-.081	-.047	1.000						
7	.155	.014	.113	.158	.006	1.000					
8	.077	.114	.203	.007	.040	.103	1.000				
9	.305	-.013	.048	.111	-.122	.077	.035	1.000			
10	.190	.075	.121	-.013	-.065	.131	.076	.108	1.000		
11	.065	.010	.057	-.062	-.016	.032	.097	-.001	.052	1.000	
12	-.138	-.037	-.114	.055	.074	-.046	-.050	-.045	-.062	-.141	1.000

TABLE 35

UNROTATED FACTOR PATTERN, PRINCIPAL FACTOR ANALYSIS, TOTAL PERIOD
VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	
BANKORP	15	.0154	-.0018	
BOLAND	16	.0163	-.0030	
ICLEF	17	.0099	-.0003	
NEDBANK	18	.0250	.0084	
STANBIC	19	.0153	.0017	
T & T	20	.0198	.0014	
VOLKSKAS	21	.0219	.0033	
BANKS		.1236	.0097	
ALPHA	22	.0173	-.0002	
BOUMAT	23	.0231	-.0100	
EVRITE	24	.0113	-.0030	
GRNAKR	25	.0241	.0022	
LTA	26	.0296	-.0059	
M & R	27	.0271	-.0029	
PPCEM	28	.0176	-.0003	
BUILDING		.1230	-.0201	
AECI	29	.0211	.0048	
CHEMDH	30	.0126	-.0085	
DEBERL	31	.0101	.0033	
LANCHEM	32	.0199	.0351	
SENCHM	33	.0216	.0012	
TREK	34	.0205	-.0034	
TRIOMF	35	.0195	-.0217	
CHEMICAL		.1253	.0542	
CADSWP	36	.0156	-.0044	
FEDFOOD	37	.0170	-.0019	
ICS	38	.0262	.0028	
I & J	39	.0167	.0031	
KANHYM	40	.0167	.0132	
PREMGRP	41	.0180	.0000	
TIGOATS	42	.0209	.0029	
FOOD		.1311	.0157	
ASSENG	43	.0122	-.0113	
DUNLOP	44	.0211	-.0058	
GENTRA	45	.0132	-.0078	
MCCARTHY	46	.0361	-.0076	
SAFICON	47	.0329	-.0083	
TOYOTA	48	.0278	-.0070	
WMHUNT	49	.0234	-.0038	
MOTOR		.1667	-.0516	
				(TOTAL)
% AGE CONTRIBUTION TO COMMUNALITY		84,83	15,17	100,0%
% AGE CONTRIBUTION TO TOTAL VARIANCE		19,25	5,06	21,31%

TABLE 36

UNROTATED FACTOR PATTERN, PRINCIPAL FACTOR ANALYSIS, FIRST SUBPERIOD
VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	
BANKORP	15	.0099	-.0007	
BOLAND	16	.0174	-.0062	
ICLEF	17	.0156	.0088	
NEDBANK	18	.0307	-.0163	
STANBIC	19	.0146	-.0095	
T & T	20	.0276	-.0108	
VOLKSKAS	21	.0235	-.0051	
BANKS		.1393	-.0398	
ALPHA	22	.0144	-.0100	
BOUMAT	23	.0294	-.0039	
EVRITE	24	.0118	-.0010	
GRNAKR	25	.0274	-.0005	
LTA	26	.0351	.0268	
M & R	27	.0320	-.0047	
PPCEM	28	.0200	-.0063	
BUILDING		.1701	.00040	
AECI	29	.0251	-.0078	
CHEMDH	30	.0107	.0062	
DEBERL	31	.0098	-.0074	
LANCHEM	32	.0244	-.0074	
SENCHM	33	.0236	-.0026	
TREK	34	.0223	.0077	
TRIOMF	35	.0163	-.0032	
CHEMICAL		.1286	-.0145	
CADSWP	36	.0143	-.0070	
FEDFOOD	37	.0177	.0044	
ICS	38	.0296	-.0111	
I & J	39	.0172	-.0002	
KANHYM	40	.0175	-.0072	
PREMGRP	41	.0226	-.0069	
TIGOATS	42	.0257	-.0120	
FOOD		.1446	-.0400	
ASSENG	43	.0171	.0022	
DUNLOP	44	.0236	.0051	
GENTRA	45	.0185	.0109	
MCCARTHY	46	.0391	.0049	
SAFICON	47	.0379	.0278	
TOYOTA	48	.0286	.0060	
WMHUNT	49	.0233	.0051	
MOTOR		.1881	.0620	
				(TOTAL)
% AGE CONTRIBUTION TO COMMUNALITY		85,43	14,57	100,0%
% AGE CONTRIBUTION TO TOTAL VARIANCE		20,10	3,11	23,21%

TABLE 37

UNROTATED FACTOR PATTERN, PRINCIPAL FACTOR ANALYSIS, SECOND SUBPERIOD

VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	
BANKORP	15	.0220	.0008	.0030	
BOLAND	16	.0129	-.0078	-.0024	
ICLEF	17	.0017	.0020	-.0027	
NEDBANK	18	.0176	.0011	-.0022	
STANBIC	19	.0155	-.0018	-.0036	
T & T	20	.0083	-.0011	-.0009	
VOLKSKAS	21	.0194	.0037	-.0046	
BANKS		.0974	-.0031	-.0134	
ALPHA	22	.0209	-.0043	-.0081	
BOUMAT	23	.0136	-.0047	.0045	
EVRITE	24	.0099	-.0044	.0001	
GRNAKR	25	.0187	-.0008	-.0113	
LTA	26	.0229	-.0005	-.0053	
M & R	27	.0201	-.0068	-.0127	
PPCEM	28	.0137	-.0020	-.0050	
BUILDING		.1198	-.0235	-.0378	
AECI	29	.0203	.0041	-.0109	
CHEMDH	30	.0142	-.0034	.0070	
DEBERL	31	.0090	.0040	-.0084	
LANCHEM	32	.0145	.0470	-.0062	
SENCHM	33	.0193	.0008	-.0068	
TREK	34	.0188	-.0034	-.0033	
TRIOMF	35	.0231	.0347	.0150	
CHEMICAL		.1192	.0838	-.0136	
CADSWP	36	.0149	-.0068	-.0080	
FEDFOOD	37	.0154	-.0021	-.0004	
ICS	38	.0217	.0015	-.0086	
I & J	39	.0163	.0035	.0106	
KANHYM	40	.0156	.0137	.0035	
PREMGRP	41	.0125	-.0026	-.0051	
TIGOATS	42	.0147	-.0019	-.0067	
FOOD		.0894	.0053	-.0147	
ASSENG	43	.0055	-.0087	.0122	
DUNLOP	44	.0184	-.0058	-.0027	
GENTRA	45	.0059	-.0028	.0008	
MCCARTHY	46	.0329	-.0137	.0143	
SAFICON	47	.0274	-.0043	.0230	
TOYOTA	48	.0256	-.0116	.0054	
WMHUNT	49	.0225	-.0042	.0022	
MOTOR		.1382	-.0511	.0552	
					(TOTAL)
% AGE CONTRIBUTION TO COMMUNALITY		62,55	24,58	12,87	100,0%
% AGE CONTRIBUTION TO TOTAL VARIANCE		17,76	2,77	3,10	23,21%

TABLE 38

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD

VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
BANKORP	15	.0148	.0006	-.0018	.0024	.0029	-.0045
BOLAND	16	.0162	.0005	-.0011	-.0031	.0004	-.0026
ICLEF	17	.0091	.0048	.0078	-.0035	.0050	.0010
NEDBANK	18	.0247	-.0069	.0015	.0024	-.0014	-.0038
STANBIC	19	.0153	-.0027	-.0065	0010	.0027	-.0005
T & T	20	.0192	-.0001	-.0048	.0020	-.0041	-.0034
VOLKSKAS	21	.0214	-.0036	.0020	-.0001	-.0016	-.0013
BANKS		.1207	-.0074	-.0029	.0011	.0039	-.0151
ALPHA	22	.0173	-.0022	-.0026	.0006	.-0014	-.0027
BOUMAT	23	.0225	.0052	-.0035	-.0075	.0056	.0002
EVRITE	24	.0107	.0027	-.0030	.0004	-.0009	-.0015
GRNAKR	25	.0229	.0006	-.0020	.0025	-.0027	.0024
LTA	26	.0264	.0095	.0078	.0032	-.0065	.0019
M & R	27	.0271	-.0018	.0018	-.0041	-.0051	-.0021
PPCEM	28	.0172	-.0007	-.0027	-.0008	-.0004	.0018
BUILDING		.1170	.0133	-.0042	-.0057	.0114	.0000
AECI	29	.0216	-.0065	.0029	-.0007	.0016	.0025
CHEMDH	30	.0109	.0086	-.0043	.0021	.0015	.0009
DEBERL	31	.0103	-.0028	-.0049	-.0005	.0016	-.0007
LANCHEM	32	.0174	-.0110	.0075	.0186	.0053	-.0048
SENCHM	33	.0210	-.0009	.0019	.0013	.0012	-.0001
TREK	34	.0195	.0025	-.0007	.0049	.0030	.0114
TRIOMF	35	.0163	.0010	.0020	.0117	.0119	-.0040
CHEMICAL		.1170	-.0091	.0044	.0374	.0261	.0052
CADSWP	36	.0153	.0000	-.0110	-.0011	.0011	.0020
FEDFOOD	37	.0164	.0011	.0010	-.0005	.0003	.0049
ICS	38	.0261	-.0054	.0004	-.0011	-.0022	-.0043
I & J	39	.0155	.0003	.0072	.0008	.0014	-.0020
KANHYM	40	.0157	-.0039	.0040	.0053	.0066	.0013
PREMGRP	41	.0183	-.0026	.0001	-.0034	-.0005	-.0005
TIGOATS	42	.0214	-.0064	.0018	-.0023	-.0004	.0024
FOOD		.1287	-.0169	.0035	-.0028	.0063	.0038
ASSENG	43	.0108	.0097	-.0046	-.0024	-.0015	-.0035
DUNLOP	44	.0203	.0058	-.0037	.0011	-.0017	.0042
GENTRA	45	.0216	.0067	.0057	-.0075	.0028	-.0026
MCCARTHY	46	.0325	.0114	-.0007	.0047	-.0012	-.0018
SAFICON	47	.0276	.0193	.0056	.0043	.0011	-.0024
TOYOTA	48	.0250	.0091	.0004	.0059	-.0092	.0012
WMHUNT	49	.0215	.0069	-.0014	-.0007	.0053	.0002
MOTOR		.1503	.0689	.0013	.0054	-.0044	-.0047
% AGE CONTRIBUTION TO COMMUNALITY		73,98	7,52	3,55	4,53	2,94	1,29
% AGE CONTRIBUTION TO TOTAL VARIANCE		16,84	1,93	0,92	1,30	0,72	0,38

TABLE 38 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, TOTAL PERIOD
VARIABLES 15-49

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11
BANKORP	15	-.0062	-.0019	-.0031	-.0004	.0020
BOLAND	16	-.0026	-.0019	.0060	-.0038	-.0006
ICLEF	17	-.0003	-.0012	.0019	.0013	-.0008
NEDBANK	18	-.0020	.0009	-.0007	.0013	.0005
STANBIC	19	.0000	-.0009	.0008	-.0010	.0022
T & T	20	.0036	.0013	.0019	-.0019	-.0026
VOLKSKAS	21	.0002	.0013	.0007	.0011	.0003
BANKS		-.0073	-.0024	.0075	-.0034	.0010
ALPHA	22	-.0034	.0019	-.0007	.0021	-.0029
BOUMAT	23	.0014	.0019	.0012	-.0017	.0017
EVRITE	24	.0002	.0020	-.0012	.0012	-.0022
GRNAKR	25	.0042	-.0046	.0026	-.0020	.0006
LTA	26	.0009	-.0051	-.0006	-.0008	-.0003
M & R	27	.0019	-.0041	-.0015	.0007	.0007
PPCEM	28	-.0015	-.0036	.0050	.0020	-.0006
BUILDING		.0037	-.0116	.0048	.0015	-.0030
AECI	29	-.0038	-.0020	-.0000	.0004	-.0004
CHEMDH	30	-.0004	-.0001	-.0027	-.0035	-.0022
DEBERL	31	.0029	-.0028	-.0019	.0006	-.0018
LANCHEM	32	.0087	-.0028	.0060	.0053	.0002
SENCHM	33	-.0039	-.0022	-.0021	-.0021	-.0029
TREK	34	-.0041	.0000	-.0033	.0013	.0022
TRIOMF	35	.0050	-.0024	-.0012	.0001	-.0000
CHEMICAL		.0044	-.0123	-.0052	.0021	-.0049
CADSWP	36	.0012	.0009	.0000	.0017	-.0002
FEDFOOD	37	.0027	.0005	.0006	-.0002	-.0035
ICS	38	.0013	.0021	-.0016	-.0002	.0027
I & J	39	-.0007	.0046	.0006	-.0035	.0013
KANHYM	40	.0033	.0058	.0009	-.0018	-.0001
PREMGRP	41	.0019	.0022	-.0005	-.0016	-.0010
TIGOATS	42	.0012	.0008	-.0017	-.0011	.0011
FOOD		.0109	.0169	-.0017	-.0067	.0003
ASSENG	43	-.0001	-.0018	-.0009	.0015	.0051
DUNLOP	44	.0008	.0013	.0004	.0045	.0025
GENTRA	45	.0024	-.0001	-.0018	.0028	-.0013
MCCARTHY	46	-.0041	.0030	.0011	.0004	.0001
SAFICON	47	.0025	-.0004	-.0010	-.0013	.0044
TOYOTA	48	.0106	.0041	.0010	-.0010	-.0016
WMHUNT	49	-.0027	.0006	.0012	.0038	-.0019
MOTOR		.0094	.0067	.0000	.0107	.0073
% AGE CONTRIBUTION TO COMMUNALITY		2,36	1,25	0,94	0,86	0,76
% AGE CONTRIBUTION TO TOTAL VARIANCE		0,54	0,24	0,22	0,23	0,22

TABLE 39

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
BANKORP	15	.0101	.0004	.0043	-.0050	.0032	-.0028
BOLAND	16	.0173	-.0008	.0046	-.0018	.0077	-.0027
ICLEF	17	.0145	.0097	-.0080	-.0088	.0073	.0026
NEDBANK	18	.0309	-.0105	-.0048	.0036	.0010	-.0045
STANBIC	19	.0148	-.0044	.0069	0025	.0022	.0023
T & T	20	.0271	-.0053	.0061	.0090	-.0044	-.0059
VOLKSKAS	21	.0235	-.0034	.0032	-.0010	-.0036	-.0031
BANKS		.1382	-.0143	.0059	-.0015	.0134	-.0141
ALPHA	22	.0144	-.0036	.0014	.0020	.0007	-.0053
BOUMAT	23	.0302	.0018	.0013	-.0111	.0028	.0018
EVRITE	24	.0112	.0020	.0036	-.0000	-.0019	-.0013
GRNAKR	25	.0259	.0036	-.0013	.0084	-.0002	.0030
LTA	26	.0306	.0207	-.0127	.0018	-.0082	.0012
M & R	27	.0320	-.0022	-.0047	-.0011	-.0017	-.0085
PPCEM	28	.0195	-.0004	.0034	.0028	.0032	.0016
BUILDING		.1638	.0219	-.0090	.0028	-.0053	-.0075
AECI	29	.0222	-.0060	-.0058	-.0014	.0030	.0035
CHEMDH	30	.0095	.0079	.0077	.0011	-.0011	.0003
DEBERL	31	.0102	-.0046	.0057	.0000	-.0031	.0000
LANCHEM	32	.0225	-.0034	-.0086	.0187	.0113	.0010
SENCHM	33	.0236	.0008	.0013	-.0022	-.0045	.0006
TREK	34	.0208	.0085	-.0047	.0073	.0006	.0137
TRIOMF	35	.0155	-.0003	.0021	.0038	.0022	.0025
CHEMICAL		.1243	.0029	-.0023	.0273	.0174	.0216
CADSWP	36	.0142	-.0046	.0162	.0032	-.0059	.0067
FEDFOOD	37	.0179	.0005	-.0039	-.0041	-.0078	.0096
ICS	38	.0301	-.0076	-.0013	.0004	-.0012	-.0026
I & J	39	.0162	.0007	-.0076	-.0018	.0047	-.0025
KANHYM	40	.0173	-.0024	-.0046	.0016	.0051	.0019
PREMGRP	41	.0233	-.0057	-.0010	-.0028	-.0021	-.0018
TIGOATS	42	.0265	-.0088	-.0036	-.0023	-.0021	.0035
FOOD		.1455	-.0279	-.0058	-.0058	-.0093	.0148
ASSENG	43	.0164	.0049	.0078	-.0005	-.0005	-.0033
DUNLOP	44	.0218	.0076	.0069	.0056	-.0033	.0026
GENTRA	45	.0177	.0087	-.0014	-.0144	-.0033	-.0047
MCCARTHY	46	.0352	.0145	.0024	.0071	.0094	-.0025
SAFICON	47	.0314	-.0252	-.0020	-.0008	-.0092	-.0020
TOYOTA	48	.0259	.0090	-.0022	.0105	-.0056	-.0059
WMHUNT	49	.0219	.0079	.0069	-.0044	.0042	.0056
MOTOR		.1703	.0778	.0184	.0031	-.0083	-.0102
% AGE CONTRIBUTION TO COMMUNALITY		66,82	8,73	4,63	5,02	3,16	2,70
% AGE CONTRIBUTION TO TOTAL VARIANCE		19,08	2,01	1,41	1,13	0,79	0,73

TABLE 39 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, FIRST SUBPERIOD

VARIABLES 15-49

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11
BANKORP	15	-.0019	-.0041	.0125	.0003	.0056
BOLAND	16	.0045	-.0023	.0090	.0036	.0006
ICLEF	17	.0018	.0035	-.0011	.0010	-.0025
NEDBANK	18	.0009	.0019	-.0021	.0022	.0020
STANBIC	19	-.0017	-.0002	-.0012	.0029	.0012
T & T	20	-.0027	.0030	.0072	-.0023	-.0040
VOLKSKAS	21	.0051	-.0018	.0021	-.0014	.0012
BANKS		.0060	.0000	.0264	.0063	.0041
ALPHA	22	.0004	.0038	-.0046	-.0053	-.0048
BOUMAT	23	.0007	-.0019	-.0025	-.0024	-.0030
EVRITE	24	-.0001	.0015	-.0002	.0020	-.0016
GRNAKR	25	-.0068	.0014	.0031	-.0037	-.0007
LTA	26	-.0021	-.0047	.0064	.0017	.0034
M & R	27	-.0008	.0009	.0002	.0024	-.0013
PPCEM	28	.0050	.0021	.0053	-.0045	-.0026
BUILDING		-.0037	.0031	.0077	-.0098	-.0106
AECI	29	.0002	-.0010	-.0001	-.0003	.0013
CHEMDH	30	-.0063	-.0020	.0009	.0042	-.0042
DEBERL	31	-.0008	.0030	.0007	.0046	-.0006
LANCHEM	32	.0058	.0122	-.0016	.0047	.0077
SENCHM	33	-.0068	-.0021	.0019	.0006	.0034
TREK	34	.0028	-.0068	-.0041	.0007	-.0036
TRIOMF	35	-.0068	.0107	-.0024	-.0025	.0038
CHEMICAL		-.0119	.0140	-.0047	.0120	.0078
CADSWP	36	.0051	-.0020	.0013	-.0018	.0035
FEDFOOD	37	-.0007	.0052	.0047	-.0005	-.0024
ICS	38	-.0003	-.0050	-.0023	-.0026	-.0001
I & J	39	-.0031	-.0031	.0007	.0046	-.0023
KANHYM	40	.0026	-.0014	-.0054	.0054	-.0037
PREMGRP	41	-.0004	-.0014	-.0015	.0013	-.0015
TIGOATS	42	-.0022	-.0007	-.0013	-.0013	.0001
FOOD		.0010	-.0084	-.0038	.0051	-.0064
ASSENG	43	-.0030	-.0011	-.0037	-.0045	.0034
DUNLOP	44	.0106	-.0020	-.0057	.0020	.0013
GENTRA	45	.0027	.0054	-.0024	-.0004	.0000
MCCARTHY	46	-.0015	-.0006	-.0012	-.0038	-.0028
SAFICON	47	.0005	.0038	.0000	.0005	.0086
TOYOTA	48	.0014	-.0010	-.0036	.0019	-.0026
WMHUNT	49	.0039	.0046	-.0015	.0021	.0032
MOTOR		.0146	.0091	-.0181	-.0022	.0111
% AGE CONTRIBUTION TO COMMUNALITY		1,96	2,18	2,18	1,13	1,49
% AGE CONTRIBUTION TO TOTAL VARIANCE		0,58	0,52	0,69	0,40	0,36

TABLE 40

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

VARIABLES 15-49

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
BANKORP	15	.0205	.0055	.0013	.0102	-.0014	.0049
BOLAND	16	.0134	-.0049	.0006	.0003	.0011	-.0006
ICLEF	17	.0018	-.0008	.0005	-.0029	.0037	.0005
NEDBANK	18	.0170	.0040	-.0003	.0053	-.0042	.0018
STANBIC	19	.0161	.0007	-.0021	0039	.0040	-.0036
T & T	20	.0077	-.0034	.0022	-.0039	-.0010	.0071
VOLKSKAS	21	.0188	.0052	-.0017	-.0020	-.0019	-.0007
BANKS		.0953	-.0063	.0005	.0109	.0003	.0094
ALPHA	22	.0213	.0001	-.0024	.0028	-.0025	-.0040
BOUMAT	23	.0219	-.0027	.0041	.0028	.0066	-.0038
EVRITE	24	.0094	-.0018	.0032	-.0016	-.0037	.0006
GRNAKR	25	.0194	-.0055	-.0054	-.0011	.0082	-.0019
LTA	26	.0221	-.0053	-.0010	.0013	.0020	.0090
M & R	27	.0214	-.0077	-.0068	.0006	.0016	.0018
PPCEM	28	.0139	-.0002	-.0016	.0028	-.0009	-.0010
BUILDING		.1204	-.0231	-.0099	.0076	.0113	.0007
AECI	29	.0208	.0029	-.0061	.0019	-.0016	-.0044
CHEMDH	30	.0118	-.0011	.0079	-.0022	.0023	-.0002
DEBERL	31	.0093	-.0005	-.0057	-.0009	.0033	.0021
LANCHEM	32	.0110	.0273	-.0128	-.0088	.0090	.0187
SENCHM	33	.0184	.0032	-.0023	-.0002	-.0052	-.0002
TREK	34	.0186	.0030	-.0009	.0051	.0012	-.0026
TRIOMF	35	.0165	.0224	.0096	-.0020	.0143	.0118
CHEMICAL		.1064	.0572	-.0103	-.0071	.0233	.0252
CADSWP	36	.0152	-.0048	-.0017	-.0061	-.0006	-.0024
FEDFOOD	37	.0147	-.0005	.0024	-.0022	-.0004	-.0001
ICS	38	.0215	-.0003	-.0031	-.0053	-.0012	.0031
I & J	39	.0142	.0094	.0083	-.0060	.0002	-.0078
KANHYM	40	.0130	.0139	.0026	-.0063	.0034	-.0041
PREMGRP	41	.0130	-.0027	-.0008	-.0049	-.0003	-.0006
TIGOATS	42	.0152	.0009	-.0035	-.0007	-.0016	-.0009
FOOD		.1068	.0159	.0042	-.0315	-.0005	-.0128
ASSENG	43	.0038	-.0104	.0096	.0075	.0058	.0064
DUNLOP	44	.0179	-.0055	.0013	.0000	.0028	.0007
GENTRA	45	.0056	-.0024	.0021	-.0064	.0010	-.0017
MCCARTHY	46	.0297	-.0026	.0165	.0001	-.0096	.0041
SAFICON	47	.0220	.0033	.0195	.0061	.0075	-.0034
TOYOTA	48	.0230	-.0067	.0115	-.0025	-.0061	-.0019
WMHUNT	49	.0204	-.0047	.0055	-.0046	-.0045	-.0023
MOTOR		.1224	-.0290	.0660	.0002	-.0031	.0057
% AGE CONTRIBUTION TO COMMUNALITY		56,46	11,75	8,80	3,73	4,51	5,18
% AGE CONTRIBUTION TO TOTAL VARIANCE		16,65	1,67	1,57	1,02	0,92	0,83

TABLE 40 (CONTINUED)

UNROTATED FACTOR PATTERN, LITTLE JIFFY, SECOND SUBPERIOD

VARIABLES 15-49

SHARE		FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10	FACTOR 11
BANKORP	15	.0039	-.0036	.0011	-.0032	-.0009
BOLAND	16	-.0068	-.0040	-.0001	-.0016	-.0066
ICLEF	17	.0019	-.0031	-.0003	-.0037	-.0027
NEDBANK	18	-.0031	-.0002	.0011	-.0006	.0005
STANBIC	19	-.0007	.0037	.0039	-.0001	.0003
T & T	20	.0022	.0024	.0001	-.0009	-.0043
VOLKSKAS	21	-.0020	.0008	.0026	-.0016	.0004
BANKS		-.0046	-.0040	.0084	-.0117	-.0133
ALPHA	22	.0008	-.0030	.0046	.0033	.0015
BOUMAT	23	-.0008	.0003	-.0008	-.0007	-.0008
EVRITE	24	.0028	.0013	.0049	.0011	.0000
GRNAKR	25	-.0040	-.0008	.0010	- .0057	.0000
LTA	26	-.0037	-.0025	-.0008	.0003	.0004
M & R	27	.0004	-.0024	-.0016	-.0001	.0029
PPCEM	28	.0013	.0049	-.0019	-.0014	-.0009
BUILDING		-.0032	-.0022	.0054	.0082	.0031
AECI	29	.0016	-.0034	-.0025	-.0010	-.0005
CHEMDH	30	-.0056	-.0022	-.0003	.0031	-.0019
DEBERL	31	.0032	.0005	.0021	.0002	.0011
LANCHEM	32	-.0041	.0015	-.0003	.0050	.0059
SENCHM	33	.0019	-.0056	-.0042	.0044	.0004
TREK	34	.0143	.0001	-.0023	-.0010	-.0038
TRIOMF	35	.0040	-.0019	.0010	.0004	-.0013
CHEMICAL		.0265	-.0110	-.0065	.0111	-.0001
CADSWP	36	.0022	-.0016	.0071	.0017	.0000
FEDFOOD	37	-.0011	.0060	-.0030	.0030	-.0023
ICS	38	-.0015	.0011	-.0010	-.0020	.0005
I & J	39	-.0061	-.0001	-.0024	-.0056	.0058
KANHYM	40	-.0001	.0024	.0003	.0025	-.0037
PREMGRP	41	-.0008	.0017	.0018	-.0010	-.0015
TIGOATS	42	-.0002	.0011	-.0044	-.0017	-.0016
FOOD		-.0072	.0108	-.0016	-.0031	-.0028
ASSENG	43	.0021	.0012	.0040	-.0054	.0024
DUNLOP	44	.0028	.0055	-.0044	-.0005	.0048
GENTRA	45	.0021	-.0034	-.0008	-.0029	.0026
MCCARTHY	46	-.0008	.0015	.0037	-.0012	.0005
SAFICON	47	-.0044	-.0046	-.0029	.0081	.0034
TOYOTA	48	-.0018	.0033	-.0075	.0085	.0006
WMHUNT	49	-.0028	-.0029	.0010	-.0012	.0023
MOTOR		.0028	.0006	-.0069	-.0008	.0166
% AGE CONTRIBUTION TO COMMUNALITY		2,88	1,69	1,81	1,75	1,44
% AGE CONTRIBUTION TO TOTAL VARIANCE		0,70	0,58	0,55	0,41	0,38

APPENDIX C: LIST OF GOLD SHARES ANALYSED.

1. African Lease.
2. Blyvooruitzicht.
3. Bracken.
4. Buffelsfontein.
5. Durban Roodepoort Deep.
6. Doornfontein.
7. ERPM
8. E.T. Consolidated
9. Elsburg.
10. Free State Geduld
11. Grootvlei.
12. Harmony.
13. Hartebeesfontein.
14. Kinross.
15. Kloof.
16. Leslie.
17. Libanon.
18. Loraine.
19. Marievale.
20. President Brand.
21. President Steyn.
22. Randfontein.
23. S.A. Land.
24. Southvaal.
25. St. Helena.
26. Stilfontein.
27. Vaal Reefs.
28. Venterspost.
29. Village.
30. Vlakfontein.
31. W.R. Consolidated.
32. Welkom.
33. Western Areas.
34. Wes Driefontein.
35. Western Holdings.
36. Winkelhaak.
37. Western Deep Levels.
38. Zandpan.

APPENDIX D

TABLES AND FIGURES GENERATED BY A FACTOR ANALYSIS OF GOLD SHARES LISTED ON THE JSE.

(D-1)

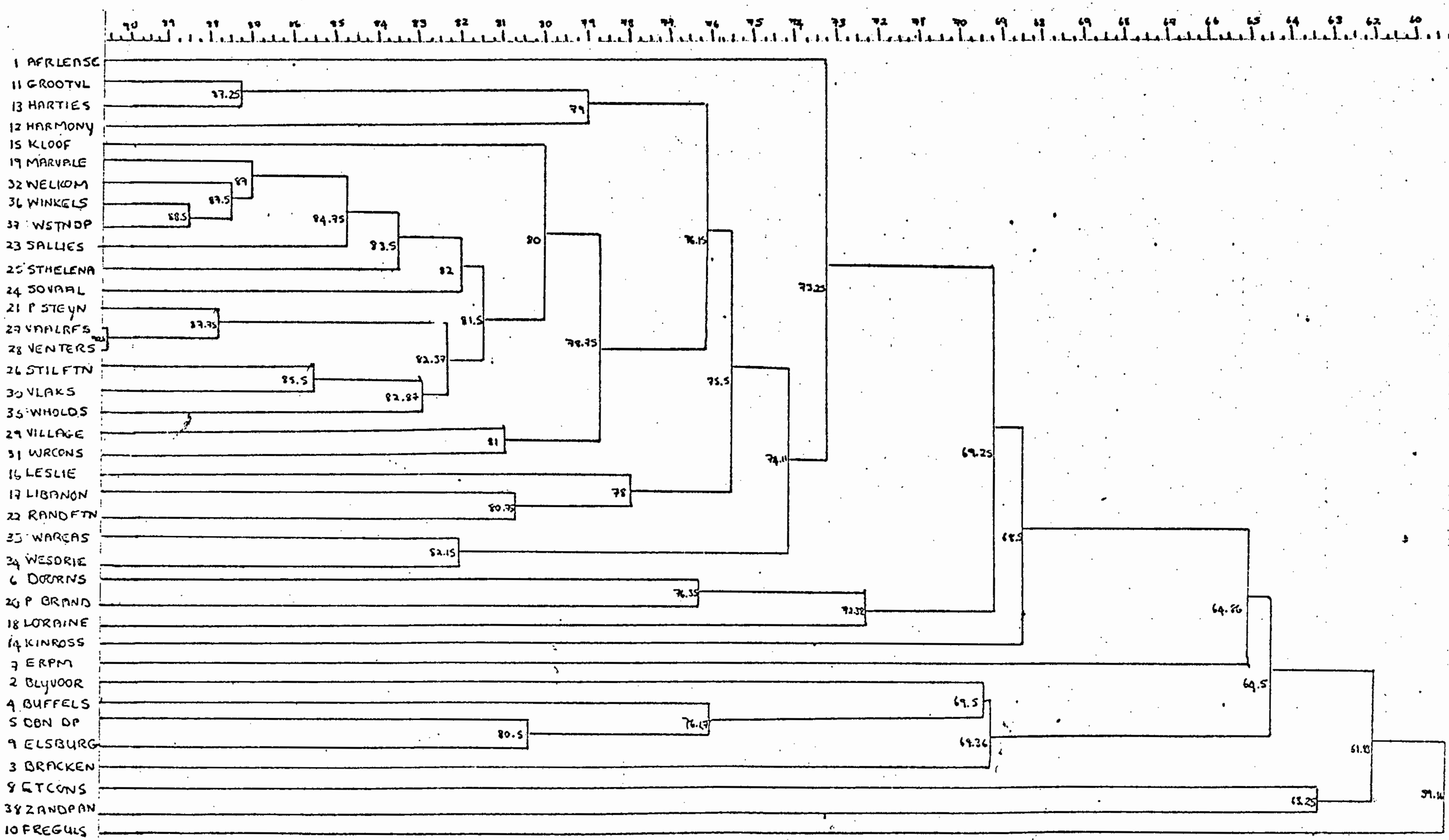

Figure 1 : Cluster Analysis of Gold Shares ... = 4/68 → 12/71

TABLE 1

GROUPS GENERATED BY APPLYING AVERAGE-LINKAGE CLUSTER ANALYSIS
GOLD SHARES FOR THE PERIOD 4/1968 TO 12/1971

Share	Location	Group	Life	Grade	Cost in R/t	Capex (R000)	Working Profit (Gold)	Gross Uran Profit
CORRELATION: .640 → .770								
WINKELS	EVANDER	UNION	L	5.91	5.75	161	871	-
WSTNDP	WEST WITS	AAC	L	11.92	6.97	2994	7521	-
WELKOM	OFS	AAC	M	6.45	6.60	117	1143	-
MARVALE	RAND	GEN.MINING	S	4.83	4.23	-	575	-
SALLIES	RAND	AAC	M	4.70	5.44	-	272	-
ST. HELENA	OFS	UNION	L	9.20	4.95	63	3983	-
SOVAAL	KLERKSDORP	AAC						
CORRELATION: .650 → .810								
VAALRFS	KLERKSDORP	AAC	L	9.07	7.69	4525	2626	687
VENTERS	WEST WITS	GOLDFIELDS	M	6.93	7.53	26	445	-
P. STEYN	OFS	AAC	L	6.70	6.68	895	1502	-
STILFTN	KLERKSDORP	GEN.MINING	M	7.28	9.05	42	287	-
VLAKS	RAND	G F S A	S	8.11	7.55	-	471	-
WHOLDS	OFS	AAC	M	12.91	6.00	269	7905	-
CORRELATION: .390 → .610								
DBN DP	RAND	BARLOW	M	2.98	4.72	-	-588	-
ELSBURG	WEST WITS	JCI	L	5.27	5.51	-	953	-
BUFFELS	KLERKSDORP	GEN.MINING	L	8.53	7.81	2157	2583	616
BLYVOOR	WEST WITS	BARLOW	M	11.80	7.51	484	3670	-
BRACKEN	EVANDER	UNION	M	8.40	5.61	3	1381	-

TABLE 1 (continued)

Share	Location	Group	Life	Grade	Cost in R/t	Capex (R000)	Working Profit Gold	Gross Uran Profit
CORRELATION: .450 → .530								
DOORNS	WEST WITS	GFSA	L	9.38	8.03	446	1453	-
P BRAND	OFS	AAC	M	12.57	6.62	792	6218	-
LORRAINE	OFS	ANGLOVAAL	S	6.35	8.45	86	119	-
CORRELATION: .580 → .740								
GROOTVL	RAND	UNION	B	3.90	3.80	-	540	-
HARTIES	KLERKSDORP	ANGLOVAAL	L	7.30	8.10	589	1089	589
HARMONY	OFS	BARLOW	M	6.15	6.41	408	1163	230
CORRELATION: .560 → .615								
LESLIE	EVANDER	UNION	L	5.60	4.62	162	1128	-
LIBANON	WEST WITS	GFSA	M	8.12	6.73	65	1370	-
RANDFTN	RAND	JCI						

TABLE 2

UNROTATED FACTOR PATTERN - GOLD SHARES - 4/68 to 12/71

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
AFRLEASE	1	.0194	-.0060	.0085	-.0040	0087
BLYVOOR	2	.0191	.0166	.0066	-.0106	.0113
BRACKEN	3	.0124	-.0023	.0175	.0041	.0094
BUFFELS	4	.0245	.0114	.0122	-.0039	.0012
DBN OP	5	.0183	.0032	.0146	.0012	.0009
DOORNS	6	.0235	.0041	.0089	.0077	.0053
ERPM	7	.0212	-.0045	.0047	-.0052	.0063
ETCONS	8	.0075	.0101	.0064	-.0023	-.0037
ELSBURG	9	.0210	.0099	.0109	-.0044	-.0048
FREGULS	10	.0093	-.0019	0111	.0008	0011
GROOTVL	11	.0259	-.0091	.0061	-.0086	-.0049
HARMONY	12	.0278	-.0044	.0031	.0038	-.0042
HARTIES	13	.0277	-.0075	.0096	-.0027	-0034
KINROSS	14	.0295	-.0005	.0056	.0033	-.0034
KLOOF	15	.0307	.0097	-.0067	-.0048	.0025
LESLIE	16	.0369	-.0039	.0039	.0028	-.0020
LIBANON	17	.0304	.0032	-.0050	.0056	.0035
LORAINE	18	.0251	-.0044	.0111	.0020	.0140
MARVALE	19	0311	-.0012	-.0043	.0047	.0034
P BRAND	20	.0223	-.0035	.0129	.0031	.0111
P STEYN	21	.0223	-.0095	-.0010	-.0053	-.0016
RANDFNT	22	.0190	-.0027	.0041	.0023	0042
SALLIES	23	.0500	-.0009	.0014	.0146	.0048
SOVAAL	24	.0284	-.0012	-.0081	-.0016	.0023
STHELENA	25	.0335	-.0061	-.0022	.0008	.0001
STILFTN	26	.0271	-.0024	-.0017	-.0009	-.0026
VAALRFS	27	.0351	-.0095	.0034	-.0078	-.0031
VENTERS	28	.0219	-.0123	-.0013	-.0028	.0003
VILLAGE	29	.0360	.0074	-.0010	-.0073	-.0011
VLAKS	30	.0266	.0053	.0095	-.0002	-.0062
WRCONS	31	.0401	.0006	-.0025	.0058	-.0054
WELKOM	32	.0367	.0023	-.0059	.0091	-.0027
WAREAS	33	.0316	.0119	-.0026	-.0045	-.0044
WESDRIE	34	.0285	.0129	.0082	-.0025	-.0028
WHOLDS	35	.0292	.0021	-.0068	-.0054	.0017
WINKELS	36	.0395	.0003	-.0043	.0099	-.0021
WSTNDP	37	.0280	.0031	-.0050	-.0057	0057
ZANDPAN	38	.0225	0049	.0073	.0033	-.0128
% Contribution to communality		75.27	4.68	6.05	2.97	3.05
% Contribution to total variance		33.43	2.05	2.65	1.09	0.99

TABLE 2 (CONTINUED)

UNROTATED FACTOR PATTERN - GOLD SHARES - 4/68 to 12/71

SHARE		FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	
AFRLEASE	1	-.0035	.0027	.0110	.0030	
BLYVOOR	2	.0064	-.0019	-.0048	-.0010	
BRACKEN	3	-.0004	-.0048	.0035	-.0043	
BUFFELS	4	-.0064	.0052	-.0046	-.0038	
DBN OP	5	-.0066	.0053	-.0020	-.0039	
DOORNS	6	-.0005	.0098	-.0032	-.0017	
ERPM	7	.0020	.0023	.0208	-.0015	
ETCONS	8	.0032	-.0025	.0163	-.0012	
ELSBURG	9	-.0044	.0033	-.0030	-.0017	
FREGULS	10	.0168	.0052	-.0016	-.0031	
GROOTVL	11	.0028	-.0068	.0035	.0008	
HARMONY	12	.0057	.0006	-.0031	-.0072	
HARTIES	13	-.0009	-.0051	-.0059	-.0035	
KINROSS	14	.0041	-.0059	-.0032	-.0007	
KLOOF	15	.0069	-.0046	-.0023	.0046	
LESLIE	16	-.0011	.0051	.0001	.0097	
LIBANON	17	.0041	-.0007	.0001	-.0003	
LORAINE	18	-.0030	.0000	-.0020	.0058	
MARVALE	19	.0037	.0046	.0041	-.0003	
P BRAND	20	.0010	-.0047	-.0024	.0007	
P STEYN	21	-.0019	.0005	-.0044	.0024	
RANDFNT	22	.0001	-.0053	-.0028	-.0023	
SALLIES	23	-.0099	-.0097	-.0022	.0064	
SOVAAL	24	-.0023	-.0007	-.0007	-.0014	
STHELENA	25	-.0048	-.0036	.0030	-.0020	
STILFTN	26	-.0002	-.0005	.0009	.0008	
VAALRFS	27	.0004	.0034	-.0023	.0030	
VENTERS	28	-.0003	.0049	-.0018	.0004	
VILLAGE	29	-.0033	.0010	.0015	-.0056	
VLAKS	30	.0045	-.0003	.0015	.0004	
WRCONS	31	-.0003	-.0043	-.0035	-.0053	
WELKOM	32	.0001	.0039	.0025	.0001	
WAREAS	33	-.0026	.0002	-.0047	.0025	
WESDRIE	34	-.0028	.0009	.0032	.0072	
WHOLDS	35	-.0005	.0011	-.0026	-.0061	
WINKELS	36	-.0010	-.0030	.0005	.0015	
WSTNDP	37	.0008	.0007	.0009	-.0003	
ZANDPAN	38	-.0020	.0078	.0059	-.0045	
% Contribution to communality		2.04	1.82	2.88	1.23	100%
% Contribution to total variance		0.82	0.69	0.63	0.45	42.80

TABLE 3

VARIMAX ROTATED FACTOR PATTERN - GOLD SHARES - 4/68 to 12/71

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9
AFRLEASE	1	-.002	.009	.012	-.003	.003	-.006	-.004	.006
BLYVOOR	2	.008	.008	.001	.020	.003	-.006	.000	-.007
BRACKEN	3	.006	.018	.008	.003	.005	.002	.003	.000
BUFFELS	4	.017	.008	-.004	-.000	-.002	-.002	-.003	-.002
DBN OP	5	.013	.008	-.000	-.004	.004	-.006	-.002	-.000
DOORNS	6	.009	.010	-.003	-.002	-.008	-.005	.002	.003
ERPM	7	-.003	.003	.022	-.003	.000	-.006	-.001	.001
ETCONS	8	.010	-.004	.018	.003	-.000	.002	.000	-.003
ELSBURG	9	.016	.000	-.002	.002	.005	-.004	-.002	-.000
FREGULS	10	.003	.006	.003	.003	-.001	-.007	.019	.001
GROOTVL	11	-.003	-.002	.006	-.002	.014	-.000	.007	.002
HARMONY	12	.000	-.000	-.002	-.003	.000	.001	.009	.002
HARTIES	13	.001	.003	.008	-.008	.010	.001	.004	-.000
KINROSS	14	.003	.002	-.001	-.000	.004	.006	.007	.000
KLOOF	15	-.002	-.004	-.001	.015	-.001	.002	-.000	-.002
LESLIE	16	.001	.000	-.001	-.002	-.000	-.002	.000	.012
LIBANON	17	-.003	.001	-.000	.004	-.008	.004	.002	-.002
LORAINE	18	-.001	.017	.001	.001	.003	-.002	-.002	.008
MARVALE	19	-.004	.000	.003	-.001	-.009	-.001	.002	.001
P BRAND	20	.001	.017	.002	.001	.005	.002	.003	.003
P STEYN	21	-.006	-.001	-.004	-.004	.007	-.004	-.000	.004
RANDFNT	22	-.001	.007	-.001	-.001	.003	.004	-.002	-.002
SALLIES	23	.000	.009	-.004	-.003	-.002	.016	-.007	.007
SOVAAL	24	-.006	-.002	-.002	.000	-.002	-.000	-.004	-.003
STHELENA	25	-.005	.000	.002	-.006	.002	.003	-.003	-.001
STILFTN	26	-.002	-.003	.000	-.002	.001	.000	.000	.001
VAALRFS	27	-.003	-.002	-.001	-.004	.009	-.008	.003	.005
VENTERS	28	-.015	-.006	-.005	-.005	-.002	-.005	-.001	.001
VILLAGE	29	.005	-.003	.002	.003	.002	-.004	-.005	-.008
VLAKS	30	.010	-.001	.004	.002	.003	.001	.007	.001
WRCONS	31	.001	-.002	-.005	-.004	-.001	.007	.003	-.005
WELKOM	32	.000	-.004	-.001	-.003	-.011	.003	-.000	.001
WAREAS	33	.007	-.007	-.001	.007	-.000	-.000	-.006	-.002
WESDRIE	34	.014	-.001	.004	.007	.001	.001	-.004	.005
WHOLDS	35	-.003	-.003	-.003	.002	-.001	-.004	-.003	-.008
WINKELS	36	-.001	-.001	-.002	-.003	-.006	.009	-.000	.002
WSTNDP	37	-.004	-.000	.001	.006	-.001	-.004	-.004	-.004
ZANDPAN	38	.015	-.007	.004	-.009	-.003	-.002	.003	-.000
% Contribution to communality		5.19	4.34	3.42	2.99	2.52	2.30	2.60	1.67
% Contribution to total variance		2.29	1.56	0.70	1.14	1.19	0.81	0.98	0.63

TABLE 4

ORTHOBLIQUE ROTATED FACTOR PATTERN - GOLD SHARES - 4/68 to 12/71

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9
AFRLEASE	1	.010	-.001	-.000	.012	-.005	-.004	-.001	.004
BLYVOOR	2	.003	.001	.025	-.001	-.001	.002	.000	-.000
BRACKEN	3	.019	.003	.002	.004	.001	.005	.002	-.003
BUFFELS	4	.006	.016	.005	-.004	.000	.001	-.004	.001
DBN OP	5	.009	.014	.001	-.001	-.006	-.000	-.000	.001
DOORNS	6	.007	.008	.001	-.001	-.003	.006	-.011	.002
ERPM	7	.004	-.002	-.000	.023	-.003	-.001	-.000	-.000
ETCONS	8	-.003	.006	.005	.016	.008	.003	.005	.001
ELSBURG	9	000	.013	.006	-.003	-.002	-.000	.004	.005
FREGULS	10	.002	-.003	.004	.001	-.006	.019	-.000	.000
GROOTVL	11	.002	-.005	-.002	.003	-.004	.002	.014	.001
HARMONY	12	-.000	-.001	-.005	-.003	-.001	.008	.002	.000
HARTIES	13	.007	.002	-.005	.005	-.002	.001	.010	-.002
KINROSS	14	.003	-.000	-.001	-.004	.004	.006	.006	-.001
KLOOF	15	-.007	-.007	.013	-.002	.005	.000	.001	.002
LESLIE	16	.001	-.001	-.006	-.001	-.002	.001	-.002	.011
LIBANON	17	-.001	-.004	.002	.000	.005	.003	-.006	-.002
LORAINE	18	.017	-.004	.003	-.001	-.003	-.001	-.004	.006
MARVALE	19	-.002	-.004	-.002	.005	.001	.003	-.008	-.001
P BRAND	20	.017	-.003	.004	-.001	.000	.003	.001	.000
P STEYN	21	.001	-.004	-.004	-.004	-.008	-.003	.004	.002
RANDFNT	22	.008	-.002	.001	-.003	.002	.001	.002	-.004
SALLIES	23	.013	-.001	-.006	-.006	.014	-.006	-.002	.003
SOVAAL	24	-.002	-.003	.000	-.001	-.000	-.005	-.002	-.004
STHELENA	25	.003	-.001	-.006	.002	.001	-.005	.002	-.004
STILFTN	26	-.002	-.001	-.003	.000	-.000	-.001	.002	.001
VAALRFS	27	.000	-.002	-.004	-.002	-.011	-.000	.005	.004
VENTERS	28	-.006	-.009	-.008	-.001	-.008	-.005	-.004	-.002
VILLAGE	29	-.004	.007	.006	.002	-.002	-.005	.002	-.004
VLAKS	30	-.001	.006	.002	.001	.002	.008	.006	.004
WRCONS	31	-.002	.003	-.004	-.005	.005	.002	.002	-.006
WELKOM	32	-.005	.001	-.006	.002	.005	.002	-.007	-.000
WAREAS	33	-.008	.006	.007	-.001	.003	-.004	.001	.003
WESDRIE	34	-.001	.008	.007	.002	.005	-.001	.002	.010
WHOLDS	35	-.004	.000	.005	-.001	-.004	-.004	-.002	-.007
WINKELS	36	-.001	-.001	-.006	-.002	.009	.001	.003	.000
WSTNDP	37	-.002	-.003	.007	.002	-.002	-.004	-.003	-.002
ZANDPAN	38	-.006	.016	-.007	.005	-.000	.005	.001	.001
% Contribution to communality		5.77	3.74	3.98	3.10	2.38	2.26	2.15	1.37
% Contribution to total variance		2.11	1.47	1.48	0.70	0.96	1.04	1.01	0.48

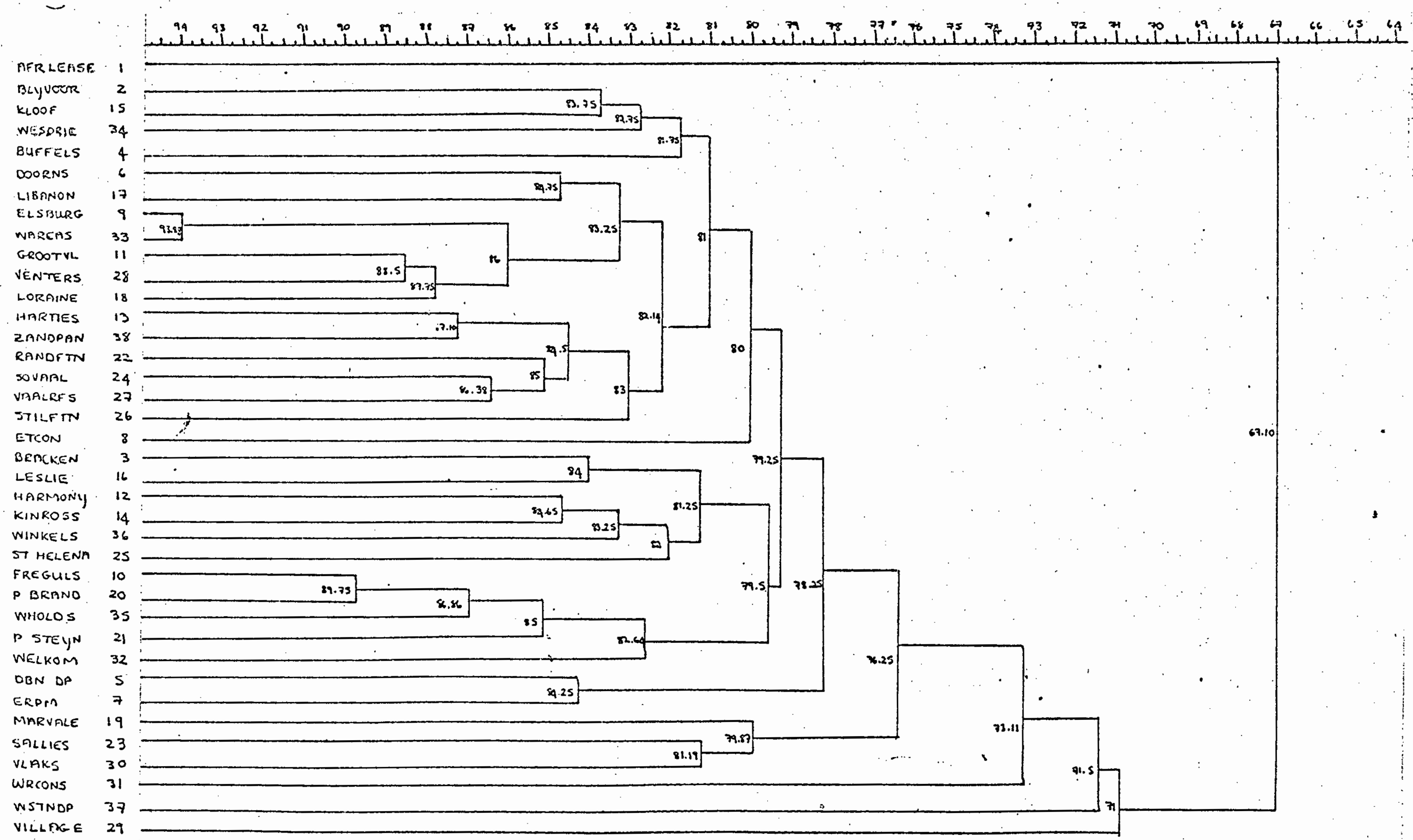

Figure 2: Cluster Analysis of Gold Shares - 2/73 → 7/81

TABLE 5

GROUPS GENERATED BY APPLYING AVERAGE-LINKING CLUSTERING

TO GOLD SHARES FOR THE PERIOD 2/73 to 7/81

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
CORRELATION .670 .878							
ELSBURG	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
WAREAS	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
GROOTVL	RAND	UNION	M	3.7	6 615	16 074	(244)
VENTERS	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
LORAINE	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
LIBANON	WEST WITS	GOLDFIELDS	M	6.0	5 627	25 244	1 046
DOORNS	WEST WITS	GOLDFIELDS	L	8.2	4 845	32 352	1 195
CORRELATION .660 .742							
HARTIES	KERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
ZANDPAN	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
RANDFTN	RAND	JCI	L	5.2	7 353	47 049	7 699
SOVAAL	KLERKSDORP	AAC	L	10.9			
VAALRFS	KLERKSDORP	AAC	L	8.6	4 090	223 098	14 476
STILFTN	KLERKSDORP	GEN.MINING	S	7.8	5 100	51 681	(3 191)
CORRELATION .635 .675							
BLYVOOR	WEST WITS	BARLOW	S	8.	5 022	52 347	1 963
KLOOF	WEST WITS	GOLDFIELDS	L	14.6	3 071	96 449	4 838
WESDRIE	WEST WITS	GOLDFIELDS			2 562	138 763	7 588
BUFFELS	KLERKSDORP	GEN.MINING	L	8.4	5 610	70 518	(1 657)
CORRELATION .625 .693							
BRACKEN	EVANDER	UNION	L	5.9	6 294	8 994	264
LESLIE	EVANDER	UNION	S	3.2	8 206	7 760	3
HARMONY	OFS	BARLOW	L	4.2		80 799	
KINROSS	EVANDER	UNION	L	5.9	4 780	25 515	218
WINKELS	EVANDER	UNION	L	6.5	3 317	47 523	1 214
ST HELENA	OFS	UNION	L	7.3	3 546	54 232	816
CORRELATION .653 .795							
FREGULS	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND	OFS	AAC	L	8.0	3 412	95 765	6 502
P STEYN	OFS	AAC	L	6.5	5 116	77 284	7 872
WELKOM	OFS	AAC	L	5.3	6 192	30 099	2 020
W HOLDS	OFS	AAC	L	5.3	3 507	91 940	8 798
NO CLUSTERING							
ETCONS	RAND	ANGLOVAAL	M	7.4	5 243	5 678	140
DBNDP	RAND	BARLOW	M	3.7	9 914	11 492	412
ERPM	RAND	BARLOW	M	4.5	9 801	18 740	641
MARVALE	RAND	GEN.MINING	B	(1.4)	7 583	2 888	24
SALLIES	RAND			(1.1)			
VLAKS	RAND	GOLDFIELDS	B	(1.2)	8 292	1 507	171
WRCONS	RAND	GEN.MINING	B		22 550	(4 553)	6 910
WSTNDP	WEST WITS	AAC	L	12.4	3 207	152 133	4 299
VILLAGE	RAND						
AFRLEASE	KLERKSDORP		M				

TABLE 6

UNROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 7/81

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	
AFRLEASE	1	.0414	-.0051	-.0030	.0012	
BLYVOOR	2	.0448	-.0011	.0117	-.0017	
BRACKEN	3	.0524	-.0025	-.0100	.0028	
BUFFELS	4	.0468	-.0073	.0100	-.0056	
DBN OP	5	.0709	.0134	.0133	.0055	
DOORNS	6	.0579	.0067	.0063	.0019	
ERPM	7	.0558	.0068	-.0030	-.0061	
ETCONS	8	.0626	.0127	.0008	.0075	
ELSBURG	9	.0599	.0003	-.0135	-.0144	
FREGULS	10	.0459	-.0209	-.0054	.0050	
GROOTVL	11	.0664	.0239	-.0013	.0084	
HARMONY	12	.0541	-.0076	.0006	-.0018	
HARTIES	13	.0497	-.0014	.0170	-.0021	
KINROSS	14	.0537	-.0047	-.0052	.0025	
KLOOF	15	.0505	-.0034	.0079	-.0009	
LESLIE	16	.0549	.0010	-.0099	.0039	
LIBANON	17	.0593	.0120	.0055	-.0006	
LORAINE	18	.0726	.0155	-.0128	.0002	
MARVALE	19	.0605	.0276	.0084	.0135	
P BRAND	20	.0487	-.0171	-.0036	.0068	
P STEYN	21	.0538	-.0150	.0010	.0069	
RANDFNT	22	.0508	.0008	.0059	-.0018	
SALLIES	23	.0682	.0185	-.0044	.0156	
SOVAAL	24	.0540	.0025	.0036	.0007	
STHELENA	25	.0420	-.0097	.0004	.0082	
STILFTN	26	.0491	.0022	.0058	-.0057	
VAALRFS	27	.0453	-.0044	.0089	-.0029	
VENTERS	28	.0737	.0214	-.0083	-.0029	
VILLAGE	29	.0599	.0183	-.0074	-.0014	
VLAKS	30	.0567	.0220	.0010	.0136	
WRCONS	31	.0556	.0115	-.0042	.0057	
WELKOM	32	.0519	-.0142	-.0043	.0046	
WAREAS	33	.0606	.0002	-.0090	-.0135	
WESDRIE	34	.0386	-.0037	.0119	-.0032	
WHOLDS	35	.0404	-.0181	-.0053	.0046	
WINKELS	36	.0432	-.0062	-.0023	.0012	
WSTNDP	37	.0451	-.0045	.0019	-.0060	
ZANDPAN	38	0500	.0016	.0125	-.0035	
% Contribution to communality		92.11	4.69	1.87	1.33	100%
% Contribution to total variance		55.97	2.57	1.29	0.76	60.59

TABLE 7.

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS WITH FACTORS IN THE UNROTATED FACTOR PATTERN.

Share		Location	Group	Life	Grade	Cost in R/Kg	Working Profit (Gold) R(000)	Gross Uran Profit
FACTOR 2 :								
GROOTVL	(11)	RAND	UNION	M	3.7	6 615	16 074	(244)
MARVALE	(19)	RAND	GEN. MIN.	B	(1,43)	7 583	2 888	24
VENTERS	(28)	WEST WITS	GOLD FIELDS	M	4.0	9 415	9 530	783
VLAKS	(30)	RAND	GOLD FIELDS	B	(1,2)	8 293	1 507	171
DBNDP	(5)	RAND	BARLOW	M	3.7	9 914	11 492	412
ETCONS	(8)	RAND	ANGLOVAAL	M	4.5	5 243	5 676	140
LIBANON	(17)	WEST WITS	GOLD FIELDS	M	6.0	5 627	25 244	1 046
LORAINE	(18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
SALLIES	(23)	RAND	AAC	S	(1,11)			
VILLAGE	(29)	RAND						
WRCONS	(31)	RAND	GEN. MIN.	M		22 550	(4 553)	6 910
FREGULS	(10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND	(20)	OFS	AAC	L	8.0	3 412	95 765	8 582
P STEYN	(21)	OFS	AAC	L	6.5	5 116	77 284	7 872
WELKOM	(32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS	(35)	OFS	AAC	L	5.3	3 507	91 940	8 798
FACTOR 3:								
BLYVOOR	(2)	WEST WITS	BARLOW	S	8.8	5 022	52 347	1 963
BUFFELS	(4)	KLERKSDORP	GEN. MIN.	L	8.4	6 294	70 518	223.5
MARVALE	(19)	RAND	GEN. MIN.	B	(1,43)	7 583	2 888	24
SALLIES	(23)	RAND	AAC	S	(1,11)			
VLAKS	(30)	RAND	GOLD FIELDS	B	(1,2)	8 292	1 507	171
HARTIES	(13)	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
ZANDPAN	(38)							

Table 7: Continued

Share		Location	Group	Life	Grade	Cost in R/Kg	Working Profit (Gold) R(000)	Gross Uran Profit R(000)
BRACKEN	(3)	EVANDER	UNION	B	3.5	6 294	8 994	264
ELSBURG	(9)	WEST WITS	JCI	L	4.1	8 456	5 747	314
WAREAS	(33)							
LESLIE	(16)	EVANDER	UNION	S	3.2	8 206	7 780	3
LORAINE	(18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
FACTOR 4:								
MARVALE	(19)	RAND	GEN. MIN.	B	(1,43)	7 583	2 888	24
SALLIES	(23)	RAND	AAC	S	(1,11)			
VLAKS	(30)	RAND	GOLD FIELDS	B	(1,2)	8 292	1 507	171
ELSBURG	(9)	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
WAREAS	(33)							

TABLE 8

VARIMAX ROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 7/81

SHARE		FACTOR 2	FACTOR 3	FACTOR 4
AFRLEASE	1	.005	-.003	.002
BLYVOOR	2	-.002	.001	-.012
BRACKEN	3	.005	-.001	.009
BUFFELS	4	.001	-.006	-.012
DBN OP	5	-.006	.009	.017
DOORNS	6	-.005	.007	-.004
ERPM	7	-.009	-.002	.004
ETCONS	8	-.006	.013	.003
ELSBURG	9	-.007	-.015	.011
FREGULS	10	.020	-.009	.001
GROOTVL	11	-.014	.019	.008
HARMONY	12	.005	-.005	-.003
HARTIES	13	-.002	.002	-.017
KINROSS	14	.006	-.002	.004
KLOOF	15	.001	-.001	-.009
LESLIE	16	.003	.001	.010
LIBANON	17	-.011	.007	-.003
LORAINE	18	-.011	.006	.016
MARVALE	19	-.015	0.28	-.000
P BRAND	20	.018	-.005	.000
P STEYN	21	.016	-.002	-.004
RANDFNT	22	-.002	.000	-.006
SALLIES	23	-.005	.022	.011
SOVAAL	24	-.002	.003	-.003
STHELENA	25	.013	.001	-.002
STILFTN	26	-.006	-.002	-.006
VAALRFS	27	.001	-.003	-.010
VENTERS	28	-.018	.007	.013
VILLAGE	29	-.015	.007	.011
VLAKS	30	-.010	.023	.006
WRCONS	31	.005	.010	.007
WELKOM	32	.015	-.005	.001
WAREAS	33	-.007	-.013	.007
WESDRIE	34	.000	-.002	-.013
WHOLDS	35	.018	-.007	.001
WINKELS	36	.006	-.003	.001
WSTNDP	37	-.000	-.007	-.004
ZANDPAN	38	-.005	.001	-.012
% Contribution to communality		2.90	2.83	2.22
% Contribution to total variance		1.88	1.37	1.41

TABLE 9

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS WITH FACTORS IN THE VARIMAX ROTATED FACTOR PATTERN

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
PBRAND (20)	OFS	AAC	L	8.0	3 412	95 765	8 582
P STEYN (20)	OFS	AAC	L	6.5	5 116	77 284	7 872
STHELENA (25)	OFS	UNION	L	7.3	3 546	54 232	816
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS (35)	OFS	AAC	L	5.3	3. 507	91 940	8 798
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
LIBANON (17)	WEST WITS	GOLDFIELDS	M	6.0	5 627	25 244	1 046
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
MARVALE (19)	RAND	GEN.MINING	M		7 583	2 888	24
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 451	9 530	783
VILLAGE (29)	RAND						
VLAKS (30)	RAND	GOLDFIELDS	B	(1.2)	8 292	1 507	171
FACTOR 3							
MARVALE (19)	RAND	GEN.MINING	B	(1.43)	7 583	2 888	24
SALLIES (23)	RAND	AAC	S	1.11)			
VLAKS (30)	RAND	GOLDFIELDS	B	(1.2)	8 292	1 507	171
ETCONS (8)	RAND	ANGLOVAAL	M	7.4	5 243	5 676	140
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
WRCONS (31)	RAND	GEN.MINING	M		22 550	(4 553)	6 910
ELSBURG (9))							
WAREAS (33))	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
FACTOR 4							
DURBAN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
ELSBURG (9)	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
LESLIE (16)	EVANDER	UNION	S	3.2	8 206	7 760	3
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
SALLIES (23)	RAND	AAC	S				
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.0	9 415	9 530	783
VILLAGE (29)	RAND						
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	8.4	5 610	70 518	(1 657)
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
VAALRFS (27)	KLERKSDORP	AAC	L	8.6	4 090	223 098	14 476
WESDRIE (34)	WEST WITS	GOLDFIELDS		(15.1)	2 562	138 763	7 988

TABLE 10

ORTHOBLIQUE ROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 7/81

SHARE		FACTOR 2	FACTOR 3	FACTOR 4
AFRLEASE	1	-.004	.005	.002
BLYVOOR	2	.000	-.001	-.012
BRACKEN	3	-.002	.004	.009
BUFFELS	4	-.007	.002	-.012
DBN OP	5	.011	-.007	.017
DOORNS	6	.008	-.004	-.004
ERPM	7	.001	-.009	.004
ETCONS	8	.015	-.006	.004
ELSBURG	9	-.011	-.008	.011
FREGULS	10	-.015	.020	.000
GROOTVL	11	.024	-.014	.009
HARMONY	12	-.007	.005	-.003
HARTIES	13	.001	-.001	-.017
KINROSS	14	-.003	.005	.004
KLOOF	15	-.002	.002	-.009
LESLIE	16	.001	.002	.010
LIBANON	17	.010	-.010	-.002
LORAINE	18	.010	-.012	.016
MARVALE	19	.031	-.014	.001
P BRAND	20	-.010	.018	-.000
P STEYN	21	-.008	.016	-.004
RANDFNT	22	.001	-.002	-.006
SALLIES	23	.023	-.005	.012
SOVAAL	24	.003	-.002	-.003
STHELENA	25	-.003	.013	-.002
STILFTN	26	-.000	-.005	-.006
VAALRFS	27	-.004	.001	-.010
VENTERS	28	.014	-.019	.013
VILLAGE	29	.012	-.015	.012
VLAKS	30	.026	-.010	.007
WRCONS	31	.012	-.006	.008
WELKOM	32	-.009	.014	.001
WAREAS	33	-.009	-.008	.007
WESDRIE	34	-.003	.001	-.013
WHOLDS	35	-.013	.017	.001
WINKELS	36	-.005	.006	.001
WSTNDP	37	-.007	-.000	-.004
ZANDPAN	38	.002	-.004	-.012
% Contribution to communality		3.96	2.82	2.27
% Contribution to total variance		2.01	1.83	1.44

TABLE 11

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE ORTHOBOLIQUE ROTATED FACTOR PATTERN

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	244
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
SALLIES (23)	RAND	AAC	S				
VLAKS (30)	RAND	GOLDFIELDS	B		8 292	1 507	171
DNB OP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
ETCONS (8)	RAND	ANGLOVAAL	M	7.4	5 243	5 676	140
LIBANON (17)	WEST WITS	GOLDFIELDS	M	4.0	5 627	25 244	1 046
LORAINE (17)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
VENTERS (28)	WEST WITS	GOLDFIELDS	M	9 415	9 415	9 530	783
VILLAGE (29)	RAND						
WRCONS (31)	RAND	GEN.MINING	M		22 550	4 553	6 910
ELSBURG (9)	WEST WITS	JCI					
FREGUL (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND (20)	OFS	AAC	L	8.0	3 412	95 765	6 582
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91 940	8 798
FACTOR 3:							
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND (20)	OFS	AAC	L	8.0	3 412	95 765	6 582
P STEYN (21)	OFS	AAC	L	6.5	5 116	77 284	7 872
STHELENA (25)	OFS	UNION	L	7.3	3 546	54 232	816
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91 940	8 798
GROOTVL (11)	RAND	UNION	L	5.2	6 615	16 074	(244)
LIBANON (17)	WEST WITS	GOLDFIELDS	M	6.0	5 627	25 244	1 046
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.0	9 415	9 530	783
VILLAGE (29)	RAND						
VLAKS (30)	RAND	GOLDFIELDS	B		8 292	1 507	171
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
ELSBURG (9)	WEST WITS	JCI	L		8 456	39 508	2 262
LESLIE (16)	EVANDER	UNION	S	3.2	8 206	7 760	3
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
SALLIES (23)	RAND		S				
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
VILLAGE (19)	RAND						
BLYVOOR (2)	WEST WITS	GOLDFIELDS	L	8.8	5 022	52 347	1 983
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	8.4	5 610	70 518	1 657
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
VAALRFS (27)	KLERKSDORP	AAC	L	8.6	4 090	223 098	14 476
WESDRIE (34)	WEST WITS	GOLDFIELDS			2 562	138 763	7 988
ZANDPAN (38)	KLERKSDORP	ANGLOVAAL					

TABLE 12

UNROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 4/77(1)

SHARE		FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	
AFRLEASE	1	.0430	-.0061	-.0127	-.0087	
BLYVOOR	2	.0487	-.0032	-.0104	-.0075	
BRACKEN	3	.0576	-.0006	.0096	.0139	
BUFFELS	4	.0515	-.0136	.0115	-.0017	
DBN OP	5	.0795	.0106	.0092	.0035	
DOORNS	6	.0593	-.0007	.0002	-.0118	
ERPM	7	.0629	.0083	-.0089	.0055	
ETCONS	8	.0737	.0159	.0123	-.0048	
ELSBURG	9	.0600	-.0022	-.0037	.0181	
FREGULS	10	.0501	-.0247	.0093	.0009	
GROOTVL	11	.0784	.0283	.0086	-.0003	
HARMONY	12	.0612	-.0074	-.0043	.0036	
HARTIES	13	.0587	-.0031	-.0094	-.0080	
KINROSS	14	.0596	-.0031	.0051	.0043	
KLOOF	15	.0555	-.0052	-.0049	-.0026	
LESLIE	16	.0616	.0013	.0105	.0100	
LIBANON	17	.0680	.0131	-.0025	-.0014	
LORAINE	18	.0833	.0191	.0055	-.0004	
MARVALE	19	.0753	.0273	.0075	-.0130	
P BRAND	20	.0541	-.0192	.0106	-.0034	
P STEYN	21	.0584	-.0170	.0064	-.0053	
RANDFNT	22	.0638	.0029	-.0064	-.0040	
SALLIES	23	.0781	.0227	.0165	-.0173	
SOVAAL	24	.0623	.0039	-.0015	-.0026	
STHELENA	25	.0464	-.0140	.0004	-.0089	
STILFTN	26	.0584	.0055	-.0121	.0005	
VAALRFS	27	.0536	-.0041	-.0099	-.0031	
VENTERS	28	.0809	.0234	.0033	.0057	
VILLAGE	29	.0637	.0259	.0043	.0014	
VLAKS	30	.0643	.0288	.0104	-.0073	
WRCONS	31	.0705	.0213	-.0035	-.0004	
WELKOM	32	.0581	-.0143	.0000	.0003	
WAREAS	33	.0634	-.0014	-.0055	.0131	
WESDRIE	34	.0396	-.0072	.0105	-.0058	
WHOLDS	35	.0449	-.0022	.0098	.0017	
WINKELS	36	.0466	-.0030	.0048	.0010	
WSTNDP	37	.0500	-.0038	-.0111	-.0026	
ZANDPAN	38	0605	-.0021	-.0099	-.0025	
% Contribution to communality		91.67	5.18	1.81	1.34	100%
% Contribution to total variance		60.81	3.08	1.38	0.89	66.16%

TABLE 13

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS WITH FACTORS IN THE UNROTATED FACTOR PATTERN - 2/73 to 4/77 (1)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 800	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	
VENTERS (28)	RAND	WEST WITS	M	6.90	11.28	2 043	
VILLAGE (29)	RAND						
VLAKS (30)	RAND	GOLDFIELDS	B	6.50	10.15	846	
WRCONS (31)	RAND	GEN.MINING	M				
LORAINE (18)	OFS	ANGLOVAAL	M	8.90	14 07	1 937	
LIBANON (17)	WEST WITS	GOLDFIELDS	M	10.80	10.21	6 094	
ETCONS (8)	RAND	ANGLOVAAL	M				
DNB OP (5)	RAND	BARLOW	M	4.29	8.51	1 122	
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	11.12	14.00	9 444	-379
FREGULS (10)	OFS	OFS	L	21.95	12.11	19 582	
P BRAND (20)	OFS	AAC	L	15.25	11.84	18 288	
P STEYN (21)	OFS	AAC	L	12.50	12.33	11 450	
STHELENA (25)	OFS	UNION	L	12.40	7.91	13 387	
WELKOM (32)	OFS	AAC	L	9.08	11.77	5 752	
FACTOR 3:							
ETCONS (8)	RAND	ANGLOVAAL	M				
LESLIE (16)	EVANDER	UNION	B	6.20	7.00	2 707	
P BRAND (20)	OFS	AAC	L	15.25	11.84	18 288	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	
STHELENA (25)	OFS	UNION	L	12.40	7.91	13 387	
VLAKS (30)	RAND	GOLDFIELDS	B	6.50	10.95	846	
AFRLEASE (1)	KLERKSDORP		M				
BLYVOOR (2)	WEST WITS	BARLOW	M	17.03	13.28	13 419	81
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	11.12	14.00	9 444	-379
STILFTN (26)	KLERKSDORP	GEN.MINING	S	9.91	13.61	5 100	
WESDRIE (34)	WEST WITS	GOLDFIELDS	M	28.00	14.00	36 171	59
WSTNDP (37)	WEST WITS	AAC	L	18.50	12.51	25 429	-4
FACTOR 4:							
BRACKEN (3)	EVANDER	UNION	B	9.30	7.88	3 751	
LESLIE (16)	EVANDER	UNION	S	6.20	7.00	2 707	
VILLAGE (29)	RAND						
ELSBURG (9)	WEST WITS	JCI	L	8.45	12.53	1 865	
WAREAS (33)	WEST WITS	JVI	L	6.67	10.50	3 963	
DOORNS (6)	WEST WITS	GOLDFIELDS	L	13.10	13.22	6 663	
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 803	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	

TABLE 14

VARIMAX ROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 4/77 (1)

SHARE		FACTOR 2	FACTOR 3	FACTOR 4
AFRLEASE	1	.000	-.017	.001
BLYVOOR	2	.002	-.013	.002
BRACKEN	3	-.004	.014	-.009
BUFFELS	4	-.006	-.015	-.007
DBN OP	5	.005	.013	.004
DOORNS	6	-.002	-.006	.010
ERPM	7	.012	-.001	-.005
ETCONS	8	.007	.014	.014
ELSBURG	9	.003	.005	-.018
FREGULS	10	-.026	-.002	-.006
GROOTVL	11	.020	.018	.013
HARMONY	12	-.004	-.005	-.007
HARTIES	13	.001	-.012	.003
KINROSS	14	-.005	.005	-.003
KLOOF	15	-.002	-.007	-.001
LESLIE	16	-.003	.014	-.005
LIBANON	17	.012	.003	.005
LORAINE	18	.013	.012	.009
MARVALE	19	.018	0.10	.024
P BRAND	20	-.022	-.001	-.000
P STEYN	21	-.018	-.004	.001
RANDFNT	22	.005	-.006	.002
SALLIES	23	.008	.014	.029
SOVAAL	24	.004	-.001	.003
STHELENA	25	-.020	.000	.007
STILFTN	26	.011	-.007	-.003
VAALRFS	27	.001	-.011	-.002
VENTERS	28	.019	.015	.004
VILLAGE	29	.021	.019	.001
VLAKS	30	.018	.016	.020
WRCONS	31	.020	.005	.007
WELKOM	32	.012	-.005	.005
WAREAS	33	-.003	.001	-.014
WESDRIE	34	-.002	-.014	-.001
WHOLDS	35	-.024	-.000	-.006
WINKELS	36	-.005	-.002	-.001
WSTNDP	37	.002	-.011	-.003
ZANDPAN	38	.003	-.010	-.002
% Contribution to communality		3.68	2.59	2.18
% Contribution to total variance		2.55	1.66	1.19

TABLE 15

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE VARIMAX FACTOR PATTERN - 2/73 to 4/77 (1)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
ERPM (7)	RAND	BARLOW	M	6.01	9.89	2 313	
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
LIBANON (17)	WEST WITS	GOLDFIELDS	M	10.80	10.21	6 094	
LORAINE (18)	OFS	ANGLOVAAL	M	8.90	14.07	1 937	
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 800	
STILFTN (26)	KLERKSDORP	GEN.MINIGN	S	9.91	13.61	5 100	
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.90	11.28	2 043	
VILLAGE (29)	RAND						
VLAKS (30)	RAND	AAC	B	6.50	10.15	846	
WRCONS (31)	RAND	GEN.MINING	M				
FREGULS (10)	OFS	AAC	L	21.95	12.11	19 582	
P BRAND (20)	OFS	AAC	L	15.25	11.84	18 288	
P STYEN (21)	OFS	AAC	L	12.50	12.33	11 450	
STHELENA (25)	OFS	UNION	L	12.40	7.91	13 387	
WELKOM (32)	OFS	AAC	L	9.08	11.77	5 752	
WHOLDS (35)	OFS	AAC	L	17.90	10.50	23 374	
FACTOR 3:							
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
DBN DP (5)	RAND	BARLOW	M	4.29	8.51	1 122	
ETCONS (8)	RAND	ANGLOVAAL	M				
LESLIE (16)	EVANDER	UNION	S	6.20	7.00	2 707	
LORAINE (18)	OFS	ANGLOVAAL	M	8.90	14.07	1 937	
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 800	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.90	11.28	2 043	
VILLAGE (29)	RAND						
VLAKS (30)	RAND	GOLDFIELDS	B	28.00	14.00	36 171	59
AFRLEASE (1)	KLERKSDORP		M				
BLYVOOR (2)	WEST WITS	BARLOW	S	17.03	13.28	13 419	81
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	12.90	14.00	9 444	-379
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	12.90	14.48	11 230	838
VAALRFS (27)	KLERKSDORP	AAC	L	12.52	13.17	23 003	371
WESDRIES (34)	WEST WITS			28.00	14.00	36 717	59
WSTNDP (37)	WEST WITS	AAC	L	18.50	12.51	25 429	-4
ZANDPAN (38)	KLERKSDORP	ANGLOVAAL					
FACTOR 4:							
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 800	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	
VLAKS (30)	RAND	GOLDFIELDS	B	28.00	14.00	36 171	59
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
ETCONS (8)	RAND	ANGLOVAAL	M				
DOORNS (6)	WEST WITS	GOLDFIELDS	L	13.10	13.22	6 663	
ELSBURG (9)	WEST WITS	JCF	L	8.45	12.53	1 855	
WAREAS (33)	WEST WITS	JCF	L	6.67	10.50	3 963	

TABLE 16

ORTHOBLIQUE ROTATED FACTOR PATTERN - GOLD SHARES - 2/73 to 4/77 (1)

SHARE		FACTOR 2	FACTOR 3	FACTOR 4
AFRLEASE	1	-.004	-.002	-.016
BLYVOOR	2	-.002	-.003	-.013
BRACKEN	3	-.005	.002	.017
BUFFELS	4	-.013	.001	-.010
DBN OP	5	.008	-.002	.010
DOORNS	6	.008	.005	-.009
ERPM	7	-.004	-.014	.000
ETCONS	8	.019	.000	.014
ELSBURG	9	-.016	-.008	.012
FREGULS	10	-.009	.023	.003
GROOTVL	11	.021	-.012	.009
HARMONY	12	-.009	.000	-.001
HARTIES	13	.001	-.002	-.013
KINROSS	14	-.002	.004	.006
KLOOF	15	-.004	.001	-.006
LESLIE	16	-.001	.003	.015
LIBANON	17	.007	-.010	-.001
LORAINE	18	.014	-.008	.006
MARVALE	19	.028	-.008	-.002
P BRAND	20	-.002	.021	.001
P STEYN	21	-.002	.018	-.003
RANDFNT	22	.001	-.005	-.007
SALLIES	23	.034	.003	-.001
SOVAAL	24	.003	-.003	-.002
STHELENA	25	.006	.022	-.001
STILFTN	26	-.004	-.012	-.006
VAALRFS	27	-.005	-.003	-.009
VENTERS	28	.011	-.015	.010
VILLAGE	29	.009	.018	.015
VLAKS	30	.027	.009	.004
WRCONS	31	.010	-.016	.000
WELKOM	32	-.008	.009	-.002
WAREAS	33	-.013	-.008	.007
WESDRIE	34	-.005	-.001	-.012
WHOLDS	35	-.008	.021	.004
WINKELS	36	.002	.006	.002
WSTNDP	37	.006	-.005	-.009
ZANDPAN	38	.005	-.005	-.008
% Contribution to communality		3.37	2.59	2.18
% Contribution to total variance		1.78	2.06	1.24

TABLE 17

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE OROTHOBLIQUE FACTOR PATTERN - 2/73 to 4/77 (1)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
ETCONS (8)	RAND	ANGLOVAAL	M				
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
LORAINE (18)	OFS	ANGLOVAAL	M	8.90	14.07	1 937	
MARVALE (19)	RAND	GEN.MINING	B	5.00	5.85	1 800	
SALLIES (23)	RAND	AAC	S	5.51	9.35	1 248	
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.90	11.28	2 043	
VLAKS (30)	RAND		B	6.50	10.15	846	
WRCONS (31)	RAND	GEN.MINING	M				
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	11.12	14.00	9 444	-379
ELSBURG (9)	WEST WITS	JCI	L	8.45	12.53	1 865	
WAREAS (33)	WEST WITS	JCI	L	6.67	10.50	3 963	
FACTOR 3:							
FREGULS (10)	OFS	AAC	L	21.95	12.11	19 582	
P BRAND (20)	OFS	AAC	L	15.52	11.84	18 288	
P STEYN (21)	OFS	AAC	L	12.50	12.33	11 450	
STHELENA (25)	OFS	UNION	L	12.40	7.91	13 387	
WHOLDS (35)	OFS	AAC	L	17.90	10.50	23 374	
ERPM (7)	RAND	BARLOW	M	6.01	9.89	2 313	
GROOTVL (11)	RAND	UNION	M	4.00	5.49	1 639	
LIBANON (17)	WEST WITS	GOLDFIELDS	M	10.80	10.1	6 094	
STILFNT (26)	KLERKSDORP	GEN.MINING	S	9.91	13.61	5 100	
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.90	11.28	2 043	
VILLAGE (29)	RAND						
WRCONS (31)	RAND	GEN.MINING	B				
FACTOR 4:							
BRACKEN (3)	EVANDER	UNION	B	9.30	7.88	3 751	
DBN DP (5)	RAND	BARLOW	M	4.29	8.57	1 122	
ELSBURG (9)	WEST WITS	JCI	L	8.45	12.53	1 865	
LESLIE (16)	EVANDER	UNION	S	6.20	7.00	2 707	
VENTERS (28)	WEST WITS	GOLDFIELDS	M	6.90	11.28	2 043	
VILLAGE (29)	RAND						
AFRLEASE (1)	KLERKSDORP		M				
BLYVOOR (2)	WEST WITS	BARLOW	S	17.03	13.28	13 419	81
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	11.12	14.00	9 444	-379
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	12.90	14.48	11 230	838
WESDRIE (34)	WEST WITS	GOLDFIELDS	M	28.00	14.00	36 171	59

TABLE 18

UNROTATED FACTOR PATTERN - GOLD SHARES - 5/77 to 7/81 (2)

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	
AFRLEASE	1	.0414	.0120	-.0045	-.0030	.0082	.0028	-.0034	
BLYVOOR	2	.0408	-.0143	.0036	-.0023	.0022	-.0063	.0053	
BRACKEN	3	.0467	.0007	-.0083	.0221	.0093	-.0047	-.0062	
BUFFELS	4	.0415	-.0101	.0008	-.0001	-.0026	-.0050	.0092	
DBN OP	5	.0607	.0183	.0144	.0004	.0165	.0193	-.0027	
DOORNS	6	.0568	-.0024	.0153	.0020	.0048	-.0041	.0015	
ERPM	7	0481	.0041	.0045	.0038	.0035	.0212	-.0004	
ETCONS	8	.0497	-.0025	.0113	-.0025	-.0005	.0021	.0056	
ELSBURG	9	.0615	.0020	.0028	.0024	-.0123	-.0011	.0000	
FREGULS	10	.0414	.0046	-.0182	-.0119	.0032	-.0017	-.0002	
GROOTVL	11	.0522	-.0007	.0207	-.0060	.0132	-.0033	.0003	
HARMONY	12	.0463	-.0058	-.0096	.0123	.0010	.0082	-.0005	
HARTIES	13	.0389	-.0199	.0029	-.0019	-.0054	.0064	-.0032	
KINROSS	14	.0473	-.0001	-.0074	.0098	.0041	-.0013	-.0064	
KLOOF	15	.0449	-.0113	-.0007	-.0026	-.0014	-.0060	-.0041	
LESLIE	16	.0468	.0029	-.0022	.0196	.0086	.0008	.0037	
LIBANON	17	.0493	-.0046	.0114	-.0047	-.0024	-.0060	-.0125	
LORAINE	18	.0608	.0259	.0096	-.0004	-.0001	-.0007	.0066	
MARVALE	19	.0422	-.0076	.0284	-.0092	.0132	-.0049	-.0027	
P BRAND	20	.0428	.0027	-.0162	-.0071	.0032	.0001	.0016	
P STEYN	21	.0488	-.0027	-.0130	-.0061	.0042	.0000	.0003	
RANDFNT	22	.0359	-.0041	-.0012	.0002	-.0070	.0026	.0004	
SALLIES	23	.0567	.0095	.0104	.0028	.0090	-.0058	.0095	
SOVAAL	24	.0442	-.0054	-.0005	.0017	.0010	-.0041	.0059	
STHELENA	25	.0372	-.0078	-.0069	.0148	.0054	-.0023	-.0013	
STILFTN	26	.0383	-.0052	-.0011	.0031	-.0069	.0004	.0097	
VAALRFS	27	.0355	-.0091	-.0053	.0011	-.0043	-.0018	.0062	
VENTERS	28	.0622	.0129	.0204	-.0011	.0040	-.0013	.0018	
VILLAGE	29	.0514	.0067	.0095	-.0001	.0008	.0048	-.0016	
VLAKS	30	.0483	-.0008	.0163	-.0092	.0116	-.0084	.0000	
WRCONS	31	.0376	.0066	-.0023	.0118	.0107	.0090	.0037	
WELKOM	32	.0448	.0075	-.0157	-.0120	.0088	-.0027	.0005	
WAREAS	33	.0584	.0138	.0255	-.0002	-.0120	-.0011	-.0029	
WESDRIE	34	.0383	-.0138	.0030	-.0061	-.0041	-.0018	-.0059	
WHOLDS	35	.0352	.0039	-.0138	-.0161	.0028	.0071	-.0037	
WINKELS	36	.0399	-.0014	-.0116	.0123	-.0016	-.0061	-.0077	
WSTNDP	37	.0394	.0024	-.0056	-.0033	-.0019	.0057	.0179	
ZANDPAN	38	.0374	-.0141	.0074	-.0038	-.0027	.0089	.0022	
% Contribution to communality		84.02	3.83	4.75	2.65	1.91	1.62	1.22	100%
% Contribution to total variance		50.86	2.39	2.70	1.77	1.01	0.88	0.71	60.32%

TABLE 19

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE UNROTATED FACTOR PATTERN - 5/77 to 7/81 (2)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
AFRLEASE (1)	KLERKSDORP		M				
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
WAREAS (33)	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
BLYVOOR (2)	WEST WITS	BARLOW	S	8.8	5 022	52 347	1 963
BUFFELS (4)	KLERKSDORP	ANGLOVAAL	L	10.9	5 610	70 518	(1 657)
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
KLOOF (15)	WEST WITS	GOLDFIELDS	L	14.6	3 071	96 449	4 838
WESDRIE (34)	WEST WITS		L		2 562	138 763	7 988
ZANDPAN (38)	KLERKSDORP	ANGLOVAAL	L				
FACTOR 3:							
DNB DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
DOORNS (6)	WEST WITS	GOLDFIELDS	L	8.2	4 845	32 352	1 195
ETCONS (8)	RAND	ANGLOVAAL	M	7.4	5 243	5 676	140
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
LIBANON (17)	WEST WITS	GOLDFIELDS	M	6.0	5 627	25 244	1 046
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
SALLIES (23)	RAND	AAC	S				
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
WAREAS (33)	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND (20)	OFS	AAC	L	8.0	3 412	95 765	9 502
P STEYN (21)	OFS	AAC	L	6.5	5 116	77 284	7 872
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91940	8 798
WINKELS (36)	EVANDER	UNION	L	6.5	3 317	47 523	1 214
FACTOR 4:							
BRACKEN (3)	EVANDER	UNION	B	3.5	6 294	8 994	264
HARMONY (12)	OFS	BARLOW	L	4.2		80 799	
LESLIE (16)	EVANDER	UNION	S	3.2	8 206	7 760	3
STHELENA (25)	OFS	UNION	L	7.3	3 546	54 232	816
WRCONS (31)	RAND	GEN.MINING	M		22 550	(4 553)	6 910
WINKELS (36)	EVANDER	UNION	L	6.5	3 317	47 525	1 214
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	3 020
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91 940	8 798
FACTOR 5:							
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
VLAKS(30)	RAND		B		8 292	1 507	171
WRCONS (31)	RAND	GEN.MINING	M		22 550	(4 553)	6 910
ELSBURG (9))	WEST WITS	JCI					
WAREAS (33))	WEST WITS	JCI	L	4.1	8 456	39 508	2 262

TABLE 20

VARIMAX ROTATED FACTOR PATTERN - GOLD SHARES - 5/77 to 7/81 (2)

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AFRLEASE	1	.011	.008	.002	.000	.007	-.006
BLYVOOR	2	-.006	-.004	-.012	-.003	-.008	.004
BRACKEN	3	.007	-.003	-.002	.025	-003	-.003
BUFFELS	4	-.004	-.003	-.006	-.002	-.008	.010
DBN DP	5	.016	-.007	-.001	-.004	.028	-.008
DOORNS	6	.003	-.015	-.006	-.004	-.001	-.002
ERPM	7	-.002	-.003	.001	.002	.022	.001
ETCONS	8	-.004	-.009	-.002	-.006	.003	-.007
ELSBURG	9	.009	-.003	.022	-.003	.002	.000
FREGULS	10	.004	.022	.000	-.002	-.003	-.001
GROOTVL	11	.008	-.014	-.013	-.012	.003	-.008
HARMONY	12	-.006	.003	-.002	.016	.005	.005
HARTIES	13	-.020	-.003	-.008	-.001	.000	.001
KINROSS	14	.001	.002	-.000	.014	-.001	-.004
KLOOF	15	-.008	.000	-.006	-.000	-.009	-.003
LESLIE	16	.009	-.006	-.002	.018	.004	.005
LIBANON	17	-.006	-.009	-.001	-.006	-.005	-.014
LORAINE	18	.021	-.006	.014	-.009	.007	.001
MARVALE	19	.003	-.020	-.018	-.017	.001	-.011
P BRAND	20	.003	.018	-.001	.001	-.001	.002
P STEYN	21	-.001	.014	-.005	.001	-.002	.000
RANDFNT	22	-.007	.000	.003	-.000	-.000	.003
SALLIES	23	.017	-.010	-.003	-.004	.000	.003
SOVAAL	24	-.000	-.001	-.005	.001	-.005	.006
STHELENA	25	-.001	-.001	-.006	.018	-.003	.002
STILFTN	26	-.005	-.002	.001	.000	-.003	.012
VAALRFS	27	-.007	.002	-.003	.002	-.006	.009
VENTERS	28	.013	-.016	.003	-.010	-.006	.009
VILLAGE	29	.004	-.007	.003	-.004	.006	-.004
VLAKS	30	.008	-.010	-.012	-.013	-.003	-.008
WRCONS	31	-.052	.004	.034	.005	.027	-.004
WELKOM	32	.009	.020	-.003	-.003	-.001	-.003
WAREAS	33	.003	-.002	.018	-.004	.000	-.002
WESDRIE	34	-.014	-.001	-.006	-.004	-.005	-.004
WHOLDS	35	-.001	.021	-.000	-.007	.006	-.004
WINKELS	36	-.001	.004	.004	.017	-.008	-.003
WSTHDP	37	.003	.006	.000	-.005	.004	.018
ZANDPAN	38	-.014	-.005	-.008	-.006	.005	.004
% Contribution to communality		5.41	3.57	3.28	3.04	2.68	1.47
% Contribution to total variance		3.08	2.36	1.84	1.94	1.38	0.90

TABLE 21

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE VARIMAX ROTATED FACTOR PATTERN - 5/77 to 7/81 (2)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold)	Gross Uran. Profit
						(R000)	(R000)
FACTOR 2:							
AFRLEASE (1)	KLERKSDORP		M				
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
SALLIES (23)	RAND	AAC	S				
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
WRCONS (31)	RAND	GEN.MINING	M		22 550	(4 553)	6 910
-							
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L		4 547	94 589	6 834
WESDRIES (34)	WEST WITS				2 562	138 763	7 988
ZANDPAN (38)	KLERKSDORP	ANGLOVAAL	L				
FACTOR 3:							
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P BRAND (20)	OFS	AAC	L	8.0	3 412	95 765	8 582
P STEYN (21)	OFS	AAC	L	6.5	5 116	77 284	7 872
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91 940	8 793
-							
DOORS (6)	WEST WITS	GOLDFIELDS	L	8.2	4 845	32 352	1 195
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
SALLIES (23)	RAND		S				
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
VLAKS (30)	RAND		B		8 292	1 507	171
FACTOR 4:							
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
WRCONS (31)	RADN	GEN.MINING	M		22 550	(4 553)	6 910
ELSBURG (9))	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
WAREAS (33))							
-							
BLYVOOR (2)	WEST WITS	BARLOW	S	8.8	5 022	52 347	1 963
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	244
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
VILLAGE (29)	RAND						
FACTOR 5:							
BRACKEN (3)	EVANDER	UNION	S	3.5	6 294	8 994	264
HARMONY (12)	OFS	BARLOW	L	4.2		80 799	
KINROSS (14)	EVANDER	UNION	L	5.9	4 780	25 515	218
LESLIE (16)	EVANDER	UNION	S	32.	8 206	7 760	3
STHELENA (25)	OFS	UNION	L	7.3	3 546	54 232	816
WINKELS (36)	EVANDER	UNION	L	8.4	3 317	47 523	1 214
-							
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
VENTERS (28)	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
VLAKS (30)	RAND		B		8 292	1 507	171

TABLE 22

ORTHOBLIQUE ROTATED FACTOR PATTERN - GOLD SHARES - 5/77 to 7/81 (2)

SHARE		FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
AFRLEASE	1	.013	.002	.001	.009	.003	-.003
BLYVOOR	2	-.014	.008	.002	.000	-.004	.006
BRACKEN	3	.006	-.001	.025	-.003	-.002	-.003
BUFFELS	4	-.010	.002	-.001	-.003	-.005	.009
DBN DP	5	.021	.012	-.000	.001	.022	-.001
DOORNS	6	-.002	.014	-.001	-.007	-.000	.001
ERPM	7	.006	-.004	-.000	-.003	.021	.002
ETCONS	8	-.004	.006	-.004	-.004	.005	-.006
ELSBURG	9	.018	-.008	-.010	-.010	-.006	-.004
FREGULS	10	.003	-.007	-.003	.019	-.004	.001
GROOTVL	11	-.001	.025	-.002	.000	-.003	-.000
HARMONY	12	-.002	-.010	.012	-.002	.008	.003
HARTIES	13	-.020	-.004	-.002	-.004	.009	-.002
KINROSS	14	-.002	-.003	.013	.001	-.000	-.005
KLOOF	15	-.012	.001	.001	.002	-.004	-.004
LESLIE	16	.008	.001	.019	-.006	.002	.006
LIBANON	17	-.007	.008	-.004	-.003	.002	-.014
LORAINE	18	.025	.006	-.009	-.006	-.004	.003
MARVALE	19	-.008	.031	-.004	-.002	.004	-.003
P BRAND	20	.002	-.007	.000	.015	-.002	.003
P STEYN	21	-.003	-.004	.002	.013	-.001	.002
RANDFNT	22	-.005	-.007	-.003	-.004	.001	.000
SALLIES	23	.011	.015	.002	-.003	-.005	.008
SOVAAL	24	-.005	.002	.003	-.001	-.004	.006
STHELENA	25	-.004	-.002	.018	-.002	-.000	.001
STILFTN	26	-.005	-.005	-.002	-.007	-.002	-.009
VAALRFS	27	-.009	-.005	.001	-.001	-.003	.007
VENTERS	28	.012	.016	-.006	-.009	.001	-.001
VILLAGE	29	.006	.004	-.004	-.005	.005	-.002
VLAKS	30	-.002	.023	-.003	.004	-.003	-.001
WRCONS	31	.069	-.002	.000	-.000	.001	.000
WELKOM	32	.007	-.001	-.001	0.21	-.004	.001
WAREAS	33	.011	-.007	-.010	-.008	-.005	-.006
WESDRIE	34	-.016	.001	-.004	.000	.001	-.006
WHOLDS	35	.002	-.007	-.008	.019	.005	-.002
WINKELS	36	.000	-.009	.014	-.001	-.006	-.007
WSTNDP	37	.003	-.005	-.006	.002	.001	.018
ZANDPAN	38	.014	.001	-.005	-.004	.011	.003
% Contribution to communality		8.30	3.80	2.33	2.16	1.54	1.17
% Contribution to total variance		4.61	1.91	1.56	1.61	0.88	0.73

TABLE 23

GOLD SHARES GROUPED ACCORDING TO THEIR RELATIVELY HIGH LOADINGS

WITH FACTORS IN THE ORTHOBLIQUE ROTATED FACTOR PATTERN - 5/77 to 7/81 (2)

Share	Location	Group	Life	Grade	Cost in R/kg	Working Profit (Gold) (R000)	Gross Uran. Profit (R000)
FACTOR 2:							
AFRLEASE (1)	KLERKSDORP		M				
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
ELSBURG (9))	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
WAREAS (35))							
LORAINE (18)	OFS	ANGLOVAAL	M	4.0	12 713	5 747	314
SALLIES (23)	RAND	AAC	S				
VENTERS	WEST WITS	GOLDFIELDS	M	4.0	9 415	9 530	783
WRCONS (31)	RAND	GEN.MINING	M		22 550	(4 553)	6 910
-							
BLYVOOR (2)	WEST WITS	BARLOW	S	8.8	5 022	52 347	1 963
BUFFELS (4)	KLERKSDORP	GEN.MINING	L	8.4	5 610	70 518	(1 657)
HARTIES (13)	KLERKSDORP	ANGLOVAAL	L	10.2	4 547	94 589	6 834
KLOOF (15)	WEST WITS	GOLDFIELDS	L	14.6	3 071	96 449	4 838
WESDRIE (34)	WEST WITS				2 562	138 763	7 988
ZANDPAN (38)	KLERKSDORP	ANGLOVAAL	L				
FACTOR 3:							
DBN DP (5)	RAND	BARLOW	M	3.7	9 914	11 492	412
DOORNS (6)	WEST WITS	GOLDFIELDS	L	8.2	4 845	32 352	1 195
GROOTVL (11)	RAND	UNION	M	3.7	6 615	16 074	(244)
MARVALE (19)	RAND	GEN.MINING	B		7 583	2 888	24
SALLIES (23)	RAND	AAC					
VENTERS (28)	WEST WITS	GOLDFIELDS		4.0	9 415	9 530	783
VLAKS (30)							
-							
HARMONY (12)	OFS	BARLOW	L	4.2		80 799	
FACTOR 4:							
BRACKEN (3)	EVANDER	UNION	S	3.5	6 294	8 994	264
HARMONY (12)	OFS	BARLOW	L	4.2		80 799	
KINROSS (14)	EVANDER	UNION	L	5.9	4 780	25 515	218
LESLIE (16)	EVANDER	UNION	S	3.2	8 206	7 760	3
STHELENA (25)	OFS	UNION	L	7.3	3 546	54 232	816
WINKELS (36)	EVANDER	UNION	L	8.4	3 317	47 523	1 214
-							
ELSBURG (9))							
WAREAS (33))	WEST WITS	JCI	L	4.1	8 456	39 508	2 262
FACTOR 5:							
FREGULS (10)	OFS	AAC	L	9.3	4 030	91 750	4 428
P. BRAND (20)	OFS	AAC	L	8.0	3 412	95 765	8 582
P STEYN (21)	OFS	AAC	L	6.5	5 116	77 284	7 872
WELKOM (32)	OFS	AAC	L	5.3	6 192	30 099	2 020
WHOLDS (35)	OFS	AAC	L	5.3	3 507	91 940	8 793

APPENDIX E

EXTRACT FROM GOLD AND URANIUM QUARTERLY REPORT FOR DECEMBER 1969

GOLD MINES	GRADE dwt ton	COST R/ton	WKG PROFIT GOLD (R000)	GROSS URANIUM PROFIT (R000)	LIFE
ANGLO AMERICAN					
FS GEDULD	18.60	7.71	8 482	-	M
PRES.BRAND	12.57	6.62	6 218	-	
PRES.STEYN	6.70	6.68	1 502	-	L
SA LANDS	4.70	5.44	272	-	M
VAAL REEFS	9.07	7.69	2 626	687	L
WELKOM	6.45	6.60	1 143	-	M
WSTN DP LEVELS	11.92	6.97	7 521	-	L
WSTN HOLDINGS	12.91	6.00	7 905	-	M
ANGLOVAAL					
HARTBEESFONTEIN	7.30	8.10	1 089	598	L
LORAINE	6.35	8.45	119	-	S
ZANDPAN	6.60	8.30	194	-	M
GENERAL MINING					
BUFFELSFONTEIN	8.53	7.81	2 583	616	L
STILFONTEIN	7.28	9.05	287	-	M
GOLDFIELDS					
DOORNFONTEIN	9.38	8.03	1 453	-	L
KLOOF	9.78	7.17	2 577	-	L
LIBANON	8.12	6.73	1 370	-	M
WEST DRIEFONTEIN	18.88	9.92	9 357	-	L
VENTERSPOST	6.93	7.53	445	-	M
VLAKFONTEIN	8.11	7.55	471	-	S
UNION CORPORATION					
BRACKEN	8.40	5.61	1 381	-	M
GROOTVLEI	3.90	3.80	540	-	B
LESLIE	5.60	4.62	1 128		L
MARIEVALE	4.83	4.23	575	-	S
ST HELENA	9.20	4.95	3 983	-	L
WINKELHAAK	5.91	5.75	871	-	L
JCI					
WESTERN AREAS	5.27	5.51	953	-	L
RAND MINES					
BLYVOORUITZICHT	11.80	7.51	3 670	-	M
DURBAN DEEP	2.98	4.72	-588	-	M
ERPM	4.13	5.79	-478	-	M
HARMONY	6.15	6.41	1 163	230	M

APPENDIX F: FACTOR PATTERNS FOR AN UNROTATED FIRST FACTOR AND VARIMAX ROTATED SECOND TO FOURTH FACTORS.

TABLE 24

FACTOR PATTERN ORDERED ACCORDING TO PROFIT - GOLD SHARES - 2/73 to 7/81

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
WESDRIE	.0386	.000	-.002	-.013
FREGULS	.0459	.020	-.009	.001
ZANDPAN	.0500	-.005	.001	-.012
VAALRFS	.0453	.001	-.003	-.010
WINKELS	.0432	.006	-.003	.001
R75 000		.004	-.003	-.007
P BRAND	.0487	.018	-.005	.000
RANDFNT	-.0508	-.002	.000	-.006
HARTIES	.0497	-.002	-.002	-.017
ST HELENA	.0420	.013	.001	-.002
BUFFELS	.0468	.001	-.006	-.012
SOVAAL	.0540	-.002	.003	-.003
WSTNDP	.0451	.000	-.007	-.004
HARMONGY	.0541	.005	-.005	-.003
R30 000		.004	-.002	-.006
P STEYN	.0538	.016	-.002	-.004
KLOOF	.0505	.001	-.001	-.009
KINROSS	.0537	.006	-.002	.004
WAREAS	.0606	-.007	-.013	.007
LINBANON	.0593	-.011	.007	-.003
R10 000		.005	-.002	-.001
WELKOM	.0519	.015	-.005	.001
BRACKEN	.0524	.005	-.001	.009
GROOTVL	.0664	-.014	.019	.008
DOORNS	.0579	-.005	.007	-.004
STILFNT	.0491	-.006	-.002	-.006
MARVALE	.0605	-.015	.028	.000
VLAKS	.-567	-.010	.023	.006
SALLIES	.0682	-.005	.022	.011
LESLIE	.0549	.003	.001	.010
BLYVOOR	.0448	-.002	.001	-.012
R000		-.003	.009	.002
WRCONS	.0356	-.005	.010	.007
LORAINE	.0726	-.011	.006	.016
VENTERS	.0737	-.018	.007	.013
ERPM	.0558	-.009	-.002	.004
R000		-.011	.005	.010
AFRLEASE	.0414	.005	-.003	.002
ETCONS	.0626	-.006	.013	.003
ELSBURG	.0599	-.007	-.015	.011
VILLAGE	.0599	-.015	.007	.011
ZANDPAN	.0500	-.005	.001	-.012
VENTERS	-.0737	.-018	.007	.013

TABLE 25

FACTOR PATTERN ORDERED ACCORDING TO COST - GOLD SHARES - 2/73 to 7/81

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
STILFNT	.0491	-.006	-.002	-.006
ERPM	.0558	-.009	-.002	.004
WSTNDP	.0451	.000	-.007	-.004
LORAINE	.0726	-.011	.006	.016
KLOOF	.0505	.001	-.001	-.009
VAALRFS	.0453	.001	-.003	-.010
SOVAAL	.0540	-.002	.003	-.003
DOORNS	.0579	-.005	.007	-.004
HARTIES	.0497	-.002	.002	-.017
ZANDPAN	.0500	-.005	.001	-.012
BLYVOOR	.0448	-.002	.001	-.012
WESDRIE	.0386	.000	-.002	-.013
BUFFESL	.0468	.001	-.006	-.012
P STEYN	.0538	.016	-.002	-.004
VENTERS	.0737	-.018	.007	.013
WAREAS	.0606	-.007	-.013	.007
ELSBURG	.0599	-.007	-.015	.011
FREGULS	.459	.020	-.009	.001
LIBANON	.0593	-.011	.007	-.003
WELKOM	.0519	.015	-.005	.001
R20 t		-.002	-.002	-.003
WINKELS	..0432	.006	-.003	.001
DNB DP	0709	-.006	.009	.017
ST HELENA	.0420	.013	.001	-.002
LESLIE	.0549	.003	.001	.010
HARMONY	.0541	.005	-.005	-.003
BRACKEN	.0524	.005	-.001	.009
KINROSS	.0537	.006	-.002	.004
WHOLDS	.0404	.-18	-.007	.001
GROOTVL	.0664	-.014	.019	.008
P BRAND	.0487	.018	-.005	.000
VLAKS	.0556	-.010	.023	.006
MARVALE	.0605	-.015	.028	.000
R20 t		-.002	-.002	-.003
AFRLEASE	.0414	.005	-.003	.002
ETCONS	.0626	-.006	.013	.003
RANDFONT	.0508	-.002	.000	-.006
SALLIES	.0682	-.005	.022	.011
VILLAGE	.0599	-.015	.007	.011
WRCONS	.0556	-.055	.010	.007

TABLE 26

FACTOR PATTERN ORDERED ACCORDING TO GRADE GOLD SHARES - 2/73 to 7/81

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
WESDRIE	.0386	.000	-.002	-.013
RANDFNT	.0508	-.002	+.000	-.006
WSTNDP	.0451	.000	-.007	-.004
FREGULS	.0459	.020	-.009	.001
BRACKEN	.0524	.005	-.001	.009
KLOOF	.0505	.001	-.001	-.009
WHOLDS	.0404	.018	-.007	.001
HARTIES	.0497	-.002	.002	-.017
ZANDPAN	.0500	-.005	.001	-.012
P BRAND	.0487	.018	-.005	.000
ST HELENA	.0420	.013	.001	-.002
10 g/t		.006	-.003	-.005
SOVAAL	.0540	-.002	.003	-.003
BUFFESL	.0468	.001	-.006	-.012
VAALRFS	.0453	.001	-.003	-.010
DOORSN	.0579	-.005	.007	-.004
P STEYN	.0538	.016	-.002	-.004
STILFNT	.0491	-.006	-.002	-.006
LIBANON	.0593	-.011	.007	-.003
WINKELS	.0432	.006	-.003	.001
KINROSS	.0537	.006	-.002	.004
BRACKEN	.0524	.005	-.001	.009
LORAINE	.0726	-.011	.006	.016
ERPM	.0558	-.009	-.002	.004
WELKOM	.0519	.015	-.005	.001
WAREAS	.0606	-.007	-.013	.007
ELSBURG	.0599	-.007	-.015	.011
6 g/t		-.001	-.002	.011
VENTERS	.0737	-.018	.007	.013
HARMONY	.0541	.005	-.005	-.003
LESLIE	.0549	.003	.001	.010
GROOTVL	.0664	-.014	.019	.008
DBN DP	.0709	-.006	.009	.017
MARVALE	.0605	-.015	.028	.000
VLAKS	.0567	-.010	.023	.006
WRCONS	.0556	-.005	.010	.007
SALLIES	.0682	-.005	.022	.011
6 g/t		-.007	.013	.008
VILLAGE	.0599	-.015	.007	.011
ETCONS	.0626	-.066	.013	.003
AFRLEASE	.0414	.005	-.033	.002

TABLE 27

FACTOR PATTERN ORDERED ACCORDING TO LOCATION GOLD SHARES - 2/73 to 7/81

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
DBN DP	.0709	-.006	.009	.017
ERPM	.0558	-.009	-.002	.004
ETCONS	.0626	-.006	.013	.003
GROOTVL	.0664	-.014	.019	.008
MARVALE	.0605	-.015	.028	.000
RANDFNT	.0508	-.002	.000	-.006
SALLIES	.0682	-.005	.022	.011
VILLAGE	.0599	-.015	.007	.011
VLAKS	.0567	-.010	.023	.006
WRCONS	.0556	-.005	.010	.007
RAND		-.009	-.013	.006
BLYVOOR	.0448	-.002	-.003	.002
DOORNS	.0579	-.005	.007	-.004
KLOOF	.0505	.001	-.001	-.009
LIBANON	.593	-.011	.007	-.003
VENTERS	.0737	-.018	.007	.013
WAREAS	.0606	-.007	-.013	.007
ELSBURG	.0599	-.007	-.015	.011
WESDRIE	.0386	.000	-.002	-.013
WSTNDP	.0451	.000	-.007	-.004
WESTVITS		-.005	-.002	.000
BRACKEN	.0524	.005	-.001	.009
KINROSS	.0537	.006	-.002	.004
LESLIE	.0549	.003	.001	.010
WINKELS	.0432	.006	-.003	.001
EVANDER		.005	.001	.006
BUFFELS	.0468	.001	-.006	-.012
HARTIES	.0497	-.002	.002	-.017
SOVAAL	.0540	-.002	.003	-.003
STILFNT	.0491	.006	-.002	-.006
VAALRFS	.0453	.001	-.003	-.010
AFRLEASE	.0414	.005	-.003	.002
ZANDPAN	.0500	-.005	.001	-.012
KLERKSDORP		-.001	-.001	-.008
FREGULS	0459	.020	-.009	.001
LORAINE	.0726	-.011	.006	.016
P BRAND	.0487	.018	-.005	.000
P STEYN	.0538	.016	-.002	-.004
ST HELENA	.0420	.013	.001	-.002
WHLDS	.0404	.018	-.007	.001
WELKOM	.0519	.015	-.005	.001
HARMONY	.0541	.005	-.005	-.003
OFS		.012	-.003	.001

TABLE 28

FACTOR PATTERN GROUPED ACCORDING TO LIFE OF MINES GOLD SHARES - 2/73 to 7/81

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
RANDFNT	.0508	-.002	.000	-.006
KINROSS	.0537	.006	-.002	.004
WINKELS	.0432	.006	-.003	.001
BUFFELS	.0468	.001	-.006	-.012
HARTIES	.0497	-.002	.002	-.017
SOVAAL	.0540	-.002	.003	-.003
VAALRFS	.0453	.001	-.003	-.010
FREGULS	.0459	.020	-.009	.001
P BRAND	.0487	.018	-.005	.000
P STEYN	.0538	.016	-.002	-.004
ST HELENA	.0420	.013	.001	-.002
WHOLDS	.0404	.018	-.007	.001
DOORNS	.0579	-.005	.007	-.004
KLOOF	.0505	.001	-.001	-.009
WAREAS	.060	-.007	-.013	.007
WSTN DP	0451	-.000	-.007	-.004
ELSBURG	.0599	-.077	-.015	.011
HARMONY	.0541	.005	-.005	-.003
ZANDPAN	.0500	-.005	.001	-.012
WELKOM	.0519	.015	-.005	.001
L		.005	-.003	.003
ETCONS	.0626	-.006	.013	.003
DBN DP	.0709	-.006	.009	.017
ERPM	.0558	-.009	-.002	.004
GROOTVL	.0664	-.014	.019	.008
LORAINE	.0726	-.011	.006	.016
LIBANON	.0593	-.011	.007	-.003
VENTERS	.0737	-.018	.007	.013
AFRLEASE	.0414	.005	-.003	.002
WRCONS	.0556	-.005	.010	.007
M		-.008	.007	.007
LESLIE	.0549	.003	.001	.010
STILFNT	.0491	-.006	-.002	-.006
BLYVOOR	.0448	-.002	.001	-.012
SALLIES	.0682	-.005	.022	.011
S		-.003	.006	.001
BRACKEN	.0524	.005	-.001	.009
VLAKS	.0367	-.010	.023	.006
MARVALE	.0605	-.015	.028	.000
B		-.007	.017	.005
VILLAGE	.0599	-.015	.007	.011
WESDRIE	.0386	.000	-.002	-.013

FACTOR PATTERN GROUPED ACCORDING TO MINING COMPANY GOLD SHARES - 2/73 to 7/81

(F-6)

SHARE	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
FREGULS	.0459	.020	-.009	.001
P BRAND	.0487	.018	-.005	.000
P STEYN	.0538	.016	-.002	-.004
VAALRFS	.0453	.015	-.005	.001
WELKOM	.0519	.015	-.005	.001
WSTNDP	.0451	.000	-.007	-.004
WHOLDS	.0404	.018	-.007	.001
AFRLEASE	.0414	.005	-.003	.002
SALLIES	.0682	-.005	.022	.011
SOVAAL	.0540	-.022	.003	-.003
ACC		.009	-.002	.001
ETCONS	.0626	-.006	.013	.003
HARTIES	.0497	-.002	.002	-.017
LORAINE	.0726	-.011	.006	.016
ZANDPAN	.0500	-.005	.001	-.012
VILLAGE	.0599	-.015	.007	.011
ANGLOVAAL		-.008	.006	.0002
GROOTVL	.0664	-.014	.019	.008
BRACKEN	.0524	.005	-.001	.009
KINROSS	.0537	.006	-.002	.004
LESLIE	.0549	.003	.001	.010
WINKELS	.0432	.006	-.003	.001
ST HELENA	.0420	.013	.001	-.002
UNION		.003	.003	.005
BUFFELS	.0468	.001	-.006	-.012
STILFNT	.0491	-.006	-.002	-.006
WRCONS	.0556	-.005	.010	.007
MARVALE	.0605	-.015	.028	.000
GEN MINING		-.006	.008	-.003
DOORNS	.0579	-.005	.007	-.004
KLOOF.	.0505	.001	-.001	-.009
LIBANON	.0593	-.011	.007	-.003
VENTERS	.0737	-.018	.007	.013
VLAKS	.0567	-.010	.023	.006
WESDRIE	.0386	.00	-.002	.013
GOLDFIELDS		-.007	.007	-.002
RANDFNT	.0508	-.002	.000	-.006
ELSBURG	.0399	-.007	-.015	.011
WAREAS	.0606	-.007	-.013	.007
JCI		-.005	-.009	.004
BLYVOOR	.0448	-.002	.001	-.012
DBN DP	.0709	-.006	.009	.017
ERPM	.0558	-.009	-.002	.004
HARMONY	.0541	.005	-.005	-.003
BARLOW RAND		-.003	.001	.002

APPENDIX G.

COMPANIES INCLUDED IN 1973 - 1979 SAMPLE.

COMPANY	RETURN %	YEAR FOR WHICH RETURN WAS MEASURED.	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Picardi Hotelle Bpk	33.65	1973	G
Picardi Hotelle Bpk	-63.76	1974	B
Suncrush Ltd.	2.99	1974	G
Uniewyn Bpk.	-76.21	1974	B
Goodhope Concrete Pipes Ltd.	112.21	1979	G
Gough Cooper Ltd.	-69.31	1974	B
Grinaker Holdings Ltd.	25.59	1973	G
Grinaker Holdings Ltd.	16.33	1974	G
Grinaker Holdings Ltd.	-24.74	1975	B
Gypsum Industries Ltd.	-47.00	1975	B
Group Five Engineering Ltd.	-39.30	1979	B
LTA Ltd.	-53.90	1973	B
LTA Ltd.	42.09	1975	G
Masonite Ltd.	-62.42	1974	B
Masonite Ltd.	74.92	1978	G
Murray & Roberts Holdings Ltd.	48.72	1977	G
National Veneer Holdings Ltd.	-48.55	1975	B
Premier Portland Cement Ltd.	-34.17	1975	B
Lanchem Ltd.	182.45	1979	G
Sentrachem Ltd.	25.82	1974	G
Sentrachem Ltd.	14.95	1976	G
Sentrachem Ltd.	45.95	1977	G
Triomf Fertilizer Investments Ltd	-99.45	1977	B
Triomf Fertilizer Investments Ltd	140.28	1979	G
Adonis Knitwear Holdings Ltd.	103.41	1979	G
African & Overseas Enterprises Ltd.	-48.06	1973	B
Berkshire International (S.A.) Ltd.	-69.31	1974	B
Berkshire International (S.A.) Ltd	71.29	1977	G
Bristol Industrial Corporation Ltd.	23.36	1973	G
Bristol Industrial Corporation Ltd.	- 5.72	1979	B

COMPANY	RETURN %	YEAR FOR WHICH RETURN WAS MEASURED.	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Delswa Ltd	-15.42	1978	B
Dugson Holdings Ltd.	-78.85	1974	B
Dugson Holdings Ltd.	51.08	1975	G
Dugson Holdings Ltd.	-53.90	1977	B
Ninian & Lester Holdings Ltd.	-68.12	1977	B
Seardel Investment Corporation Ltd.	85.50	1975	G
Svenmill Ltd.	100.68	1979	G
Towles, Edgar Jacobs Ltd.	-55.96	1977	B
Towles, Edgar Jacobs Ltd.	61.52	1978	G
Veka Ltd.	-53.90	1973	B
Veka Ltd.	-61.09	1974	B
T.W. Beckett & Company Ltd.	-22.31	1975	B
Cadbury Schweppes (S.A.) Ltd.	47.96	1977	G
Jabula Foods Ltd.	-15.17	1978	B
Monis & Fattis Industries Ltd.	24.85	1973	G
Monis & Fattis Industries Ltd.	-10.01	1979	B
Kaap Kunene Beleggings Bpk	-56.80	1976	B
Kaap Kunene Beleggings Bpk	-19.32	1978	B
Lamberts Bay Holdings Ltd.	40.55	1973	G
Lamberts Bay Holdings Ltd.	15.82	1976	G
Ovenstone Investments Ltd.	-18.72	1978	B
Sea Products (S.W.A.) Ltd.	-10.54	1978	B
Amalgamated Retail Ltd.	110.71	1979	G
Beares Ltd.	66.14	1978	G
Duros Ltd.	-69.31	1974	B
Television & Electrical Holdings Ltd.	1.06	1974	G
Television & Electrical Holdings Ltd.	70.36	1975	G
Television & Electrical Holdings Ltd.	-60.35	1976	B
Television & Electrical Holdings Ltd.	117.87	1979	G
Berzack Brothers (Holdings) Ltd.	82.20	1978	G

COMPANY	RETURN %	YEAR FOR WHICH RETURN WAS MEASURED.	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Chubb Holdings Ltd.	-26.72	1978	B
Claude Neon Lights (S.A.) Ltd.	-69.31	1974	B
Field Industries Ltd.	40.55	1975	G
Fintec Ltd.	28.77	1976	G
Fintec Ltd.	-30.75	1978	B
Globe Engineering Works Ltd.	-27.19	1975	B
Goldfields Industrial Corporation Ltd.	84.73	1977	G
Metair Investments Ltd.	-98.08	1976	B
Metair Investments Ltd.	113.94	1979	G
Metkor Investments Ltd.	-69.31	1977	B
Metkor Investments Ltd.	-10.54	1978	B
National Trading Co. Ltd.	37.16	1973	G
S.A. Selected Holdings Ltd.	136.83	1975	G
Steelmetals Ltd.	12.26	1974	G
African Cables Ltd.	-64.66	1979	B
ASEA Electric South Africa Ltd.	-23.48	1975	B
Scottish Cables (S.A.) Ltd.	25.45	1973	G
Currie Motors (1946) Ltd.	-53.49	1976	B
Dunlop South Africa Ltd.	-86.50	1973	B
Eriksen Consolidated Holdings Ltd.	-64.19	1974	B
Eriksen Consolidated Holdings Ltd.	40.55	1975	G
McCarthy Group Ltd.	-51.08	1976	B
Northern Free State Motors Ltd.	-50.21	1979	B
Quinton Hazell Superite Holdings	-99.85	1974	B
Schus Holdings Ltd.	-81.09	1976	B
Williams Hunt S.A. Ltd.	-69.31	1974	B
Welfit Oddy Holdings Ltd.	-69.31	1974	B
Canadian Overseas Packaging Ind.	-82.93	1979	B
Kohler Brothers Ltd.	53.90	1973	G
Kohler Brothers Ltd.	44.47	1977	G

COMPANY	RETURN %	YEAR FOR WHICH RETURN WAS MEASURED.	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Metal Closures Group S.A. Ltd.	45.49	1975	G
Premier Paper Ltd.	78.85	1978	G
Premier Paper Ltd.	113.58	1979	G
Adcock Ingram Ltd.	-63.56	1973	B
Alex Lipworth Ltd.	-69.31	1974	B
General Optical Co. Ltd.	15.47	1975	G
South African Druggists Ltd.	- 2.99	1979	B
Shulton Africa Ltd.	-65.92	1973	B
Shulton Africa Ltd.	90.21	1978	G
Argus Printing & Publishing Co. Ltd.	80.23	1978	G
Caxton Ltd.	19.51	1976	G
South African Associated Newspapers Ltd.	-31.51	1975	B
Vaderland Beleggings Bpk.	-60.61	1976	B
Cullinan Holdings Ltd.	75.91	1978	G
Dunswart Iron & Steel Works Ltd.	101.39	1979	G
The Union Steel Corporation of S.A.	10.27	1974	G
Mobile Industries Ltd.	-55.96	1973	B
Mobile Industries Ltd.	-69.31	1974	B
Mobile Industries Ltd.	40.55	1975	G
Mobile Industries Ltd.	49.64	1977	G
Putco Ltd.	10.35	1974	G
Putco Ltd.	75.26	1978	G
Edgars Stores Ltd.	3.02	1974	G
Foschini Ltd.	50.39	1975	G
Frasers Ltd.	22.31	1976	G
Grand Bazaars Ltd.	103.82	1978	G
Greatermans Stores Ltd.	-53.19	1977	B
Harrowe's Ltd.	-66.33	1977	B
Hepworths Ltd.	-87.55	1977	B
Hepworths	-22.31	1978	B
John Orr Holdings Ltd.	37.27	1973	G

COMPANY	RETURN %	YEAR FOR WHICH RETURN WAS MEASURED.	INITIAL CLASSIFICATION AS GOOD (G) BAD (B) PERFORMER.
Lewis Foschini Investment Co. Ltd	-53.06	1973	B
Lewis Foschini Investment Co. Ltd	43.08	1975	G
Pep Stores Ltd.	- 6.35	1979	B
Pick 'n Pay Stores Ltd.	44.09	1975	G
Metcash Ltd.	69.31	1973	G
Scotts Stores Ltd.	44.09	1975	G
M & S Spitz Footwear Holdings Ltd	43.08	1977	G
Hullet's Corporation Ltd.	1.31	1974	G
The Lion Match Co. Ltd.	11.78	1976	G
Rembrandt Beherende Beleggings Bpk.	-54.65	1973	B
Tegniese Beleggingskorporasie Bpk.	-63.97	1973	B
Tegniese Beleggingskorporasie Bpk	21.51	1976	G
Utico Holdings Ltd.	-23.64	1975	B

APPENDIX H: COMPANIES INCLUDED IN 1973 AND 1979 SAMPLES:

1973 SAMPLE:

COMPANY.	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Picardi Hotelle Bpk.	G
Uniewyn Bpk.	B
Grinaker Holdings Ltd.	G
National Veneer Holdings Ltd.	G
Premier Portland Cement Ltd.	G
Gough Cooper.	B
LTA Ltd.	B
Masonite Ltd.	B
Consolidated Textile Mills Investment Corporation Ltd.	G
Delswa Ltd.	G
Dugson Holdings Ltd.	G
African and Overseas Enterprises Ltd.	B
Berkshire International (S.A.) Ltd.	B
Seardel Investment Corporation Ltd.	B
Veka Ltd.	B
Irvin and Johnson Ltd.	G
Jabula Foods Ltd.	G
Kanhym Investments Ltd.	G
Monis and Fattis Industries Ltd.	G
Kaap Kunene Beleggings Bpk.	G
Lamberts Bay Holdings Ltd.	G
Sea Products (S.W.A.) Ltd.	G
South West Africa Fishing Industries Ltd.	G
Willem Barends Ltd.	G
Associated Furniture Companies Ltd.	G
Bradlow's Stores Ltd.	G
Duros Ltd.	G
World Furnishers Group Ltd.	G
National Trading Co. Ltd.	G
Berzack Illman Investment Corporation Ltd.	B

1973 SAMPLE (CONT):

COMPANY	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Field Industries Ltd.	B
African Cables Ltd.	G
L.H. Marthinusen Ltd.	G
Central African Cables Ltd.	G
Scottish Cables (S.A.) Ltd.	G
Evelyn Haddon & Co. Ltd.	G
Kohler Brothers Ltd.	G
Frasers Ltd.	G
John Orr Holdings Ltd.	G
O.K. Bazaars (1929) Ltd.	G
Pep Stores Ltd.	G
Metcash Ltd.	G
Foschini Ltd.	B
Greatermans Stores Ltd.	B
Lewis Foschini Investment Company Ltd.	B
Currie Motors Ltd.	B
Dunlop S.A. Ltd.	B
Eriksen Consolidated Holdings Ltd.	B
Quinton Hazell Superite Holdings Ltd.	B
Schus Holdings Ltd.	B
Toyota (S.A.) Ltd.	B
Wesco Investments Ltd.	B
Adcock Ingram Ltd.	B
South African Druggists Ltd.	B
Shulton Africa Ltd.	B
Die Afrikaanse Pers (1962) Bpk.	B
Vaderland Beleggings Bpk.	B
Mobile Industries Ltd.	B
Rembrandt Beherende Beleggings Bpk.	B
Rembrandt Group Ltd.	B
Tegniese Beleggingskorporasie.	B
Utico Holdings Ltd.	B

1979 SAMPLE.

COMPANY	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Picardi Hotelle Bpk.	B
The South African Breweries Ltd.	G
Plate Glass & Shatterprufe Industries Ltd.	G
Placor Holdings Ltd.	G
Goodhope Concrete Pipes Ltd.	G
Gypsum Industries Ltd.	G
Grinaker Holdings Ltd.	B
Group Five Engineering Ltd.	B
Lanchem Ltd.	G
Sentrachem Ltd.	G
Triomf Fertilizer Investments Ltd.	G
Adonis Knitwear Holdings Ltd.	G
Ninian & Lester Holdings Ltd.	G
Searles Holdings Ltd.	G
Svenmill Ltd.	G
Veka Ltd.	G
Ensign Clothing Ltd.	B
Natal Canvas Rubber Manufacturers Ltd.	B
Jabula Foods Ltd.	B
Monis and Fattis Industries Ltd.	B
Tiger Oats & National Milling Co. Ltd.	B
Kanhym Investments Ltd.	G
Kaap Kunene Beleggings Bpk.	G
Ovenstone Investments Ltd.	B
Amalgamated Retail Ltd.	G
Associated Furniture Companies Ltd.	G
Duros Ltd.	G
World Furnishers Group Ltd.	G
Piccan Ltd.	B
Globe Engineering Works Ltd.	G
Metair Investments Ltd.	G
Metkor Investments Ltd.	G

1979 SAMPLE (CONT):

COMPANY	INITIAL CLASSIFICATION AS GOOD (G) OR BAD (B) PERFORMER.
Central African Cables Ltd.	G
Scottish Cables (S.A.) Ltd.	G
African Cables Ltd.	B
L.H. Marthinusen Ltd.	B
Alderson & Flitton Holdings Ltd.	B
Associated Engineering S.A. Ltd.	B
Currie Motors (1946) Ltd.	B
Eriksen Consolidated Holdings Ltd.	B
Northern Free State Motors Ltd.	B
Quinton Hazell Superite Holdings Ltd.	B
Premier Paper Ltd.	G
Sappi Ltd.	G
Canadian Overseas Packaging Industries Ltd.	B
Argus Printing & Publishing Co. Ltd.	B
Adcock Ingram Ltd.	B
Amalgamated Industrial Investment Corporation Ltd.	B
South African Druggists Ltd.	B
The Union Cold Storage of South Africa Ltd.	B
Dunswart Iron & Steel Works Ltd.	G
The Union Steel Corporation of South Africa Ltd.	G
Greatermans Stores Ltd.	G
Katz & Louri Ltd.	B
Pep Stores Ltd.	B
Metcash Ltd.	B
Woolworths Truworths Ltd.	B
Lonhro Sugor Corporation Ltd.	B
Crookes Brothers Ltd.	G
Hulett's Corporation Ltd.	G
Putco Ltd.	B

APPENDIX I: VARIABLE TRANSFORMATIONS FOR 1973 AND 1979 SAMPLES.

For 1973:

X_{11} = log (interest cover + 1),
X_{12} = log (fixed cost cover +1),
X_{21} = log (cash flow to debt +1),
X_{22} = log (cash flow to current liabilities +1),
X_{33} = log (interest cover/sector average +1),
X_{34} = log (fixed cost cover/sector average +1),
X_{43} = log (cash flow to debt/sector average +1).

For 1979:

X_{1} = log (current ratio +1),
X_{2} = log (quick ratio + 1),
X_{11} = log (interest cover +1),
X_{12} = log (fixed cost cover +1),
X_{21} = log (cash flow to debt +1),
X_{23} = log (current ratio/sector average +1),
X_{24} = log (quick ratio/sector average +1),
X_{33} = log (interest cover/sector average +1),
X_{34} = log (fixed cost cover/sector average +),
X_{43} = log (cash flow to debt/sector average +1).