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An empirical investigation of the value of High and Low price data to Modern Portfolio Theory

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1 Introduction

It is common practice to use the return series from closing prices in order to estimate the values of variables to be used in Modern Portfolio Theory (MPT). In fact the closing price series is generally what is referred to when price data or a financial time series is mentioned. We know this series to be made up of discrete points recorded as the last traded price on a specific day. But we also know this gives no indication of where the price has moved during the day. It is also widely believed that the price breaking through a certain level can be an indication of future movements. The highs and the lows, regardless of what one may believe they represent exactly, do, together with the closing prices, give a more complete view of the behaviour of a moving price. Yet they are for the most part left unused.

It is known that the series of highs and lows are published and are widely available, at least as available as closing prices. The question that needs to be answered is: are the highs and lows valuable enough to warrant their use in MPT, and if so, what would this entail?

It will be attempted, through an empirical study, to determine whether or not it is worthwhile to incorporate the highs and lows into the existing framework of MPT, and how this might be accomplished. It must be discovered more clearly whether there is extra information in the highs and lows that is not in the closings. The high and low series must also be used in core procedures like the calculation of the efficient frontier or the determination of actual portfolios in order to see if there is an appreciable difference to just using closings. This should give an indication of how one might use the highs and lows in MPT if they are indeed deemed valuable.

2 Literature Review and Discussion

The uses of high and low price data as relevant to Modern Portfolio Theory (MPT) seem not to be covered extensively in the literature. In a simplistic sense, it does seem natural when preoccupied with the calculation of expected return and standard deviation to simply use closing prices, so it might be easy to overlook the highs and lows. For this reason, and if it adds value, it may be prudent to try to find a way to easily incorporate the simple high and low series into the theory. It is obvious that if there is any valuable information in the highs and lows, not using them might be a mistake.

Cheung (2006) contends that "[the high and low] correspond to the prices at which the excess demand is changing its direction — the information that is not reflected by data on closing prices." And Parkinson (1980), which has been modified by Beekers (1983), Garman (1980) and others, showed that the price range is a more efficient estimator of volatility than the variance estimator that uses close to close data (under assumptions of course). This meaning that to completely overlook the high and low data might indeed be a mistake, specifically since it may be very useful for estimating variance.

Going further, Lin (1994) presents a model that highlights the relations between stock return variance and the high-low price range. While Beekers (1983) "shows on the basis of empirical tests that the daily price range does contain important new information concerning the stock price variability."

Therefore, we can be fairly certain that highs and lows do contain valuable information and cannot be overlooked in MPT. Based in this understanding the incorporation of high-low data into the the construction of portfolios can be attempted.

3 The Data

3.1 Specific Shares and Indices

The share data universe consists of 10 shares which will be used in the construction of portfolios, and which are listed in Table 1. The shares are also used as dependent variables in the Sharpe and Troskie-Hossain models, where the independent variables are the 6 indices listed in Table 1. All data were procured from McGregor BFA, which is available in the UCT main library.

All data are weekly (downloaded as such) and the High, Low and Closing price series for each share are used. The closing price is considered to be the last price of a given week, where the high and low are the supremum and infimum of the stock price in that week respectively. The data period is from 01 March 1996 to 08 January 2010 and comprises 730 observations.

Log returns of these prices will always be employed for analysis and dividends are not considered. The log return is defined as: $rt = \ln\left(\frac{St}{St-1}\right)$, where rt is the return for the period s to t ($s < t$), $t \in \mathcal{N}$ is a time index, \ln is the natural logarithm and St is the stock price at time t . The specific shares and indices were chosen for the following reasons.

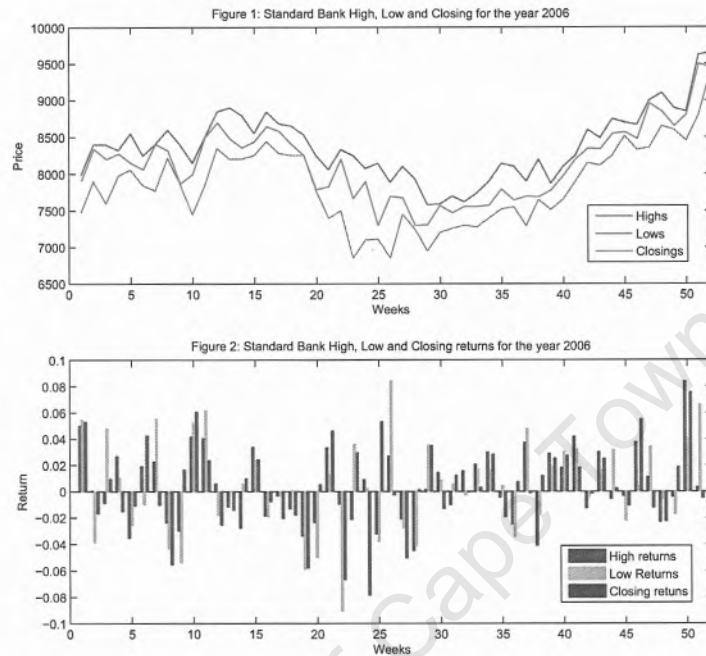
The shares are of companies that are all large, well established institutions. Therefore their shares are liquid and there is no problem of thin trading. This is an issue since prices are only recorded when a transaction takes place, if a share is thinly traded, the price on a given day could have been recorded relatively far in the past. But most importantly, the shares are diverse and represent different sectors of the economy which will allow us to effectively diversify our portfolios. The shares were also chosen in conjunction with the indices in light of their macroeconomic relationships.

Table 1: List of Shares and Indexes used

Shares:	ABSA	Anglo American	Barloworld	Bidvest
	Impala Plat	Lonmin	Nedbank	Pick 'n Pay
	Group 5	Illovo	Sappi	Santam
	Sasol	Shoprite	Standard Bank	
Indices:	All Share	TOP40	Fin. / Ind. 30	Gold mining
	Industrials	Oil and Gas		

The High, Low and Closing price series for Standard Bank are plotted in Figure 1 to illustrate the High/Low/Closing price relationship and the log returns

of the data used in Figure 1 are plotted in Figure 2.



In Figure 1 it is seen that the High, Low and Closing price drift together over time, though their distance from one another varies. This indicates changes in the volatility of the price. Figure 2 plots log returns from High, Low and Closing prices side by side for each week. This is illustrative of the sometimes large differences in the returns of the different series.

3.2 Descriptive statistics

As mentioned previously, High, Low and Closing (H, L C) data will be used in the analyses. Descriptive statistics of the log returns of the entire dataset (1996 - 2010) are now given in Table 2.

	H	SE H	L	SE L	C	SE C
ABSA	0.0437	0.0016	0.0509	0.0019	0.052	0.0019
BIDVEST	0.0336	0.0012	0.0405	0.0015	0.04	0.0015
GROUP 5	0.0644	0.0024	0.0664	0.0025	0.0673	0.0025
ILLOVO	0.0444	0.0016	0.0473	0.0018	0.05	0.0019
NEDBANK	0.0419	0.0016	0.0458	0.0017	0.0471	0.0017
PNP	0.0543	0.002	0.0533	0.002	0.0568	0.0021
SAPPI	0.0562	0.0021	0.0646	0.0024	0.0663	0.0025
SANTAM	0.0435	0.0016	0.0427	0.0016	0.0459	0.0017
SHOPRITE	0.042	0.0016	0.0675	0.0025	0.0463	0.0017
STANDARD	0.0434	0.0016	0.0484	0.0018	0.0491	0.0018

Table 2 shows that for the majority of the shares, the standard deviation calculated from the highs is the lowest, with that from the closings higher and that of the lows being the highest.

The calculations of the means reveal that they are all close to zero (mean of log returns) and therefore very similar. As is evident from Figure 1, highs, lows and closings do not drift apart over time, the average return must therefore be equal or very similar.

Pearson correlation coefficients between H, L and C for each share are presented in Table 3.

	C&H	C&L	H&L
ABSA	0.602	0.511	0.495
BIDVEST	0.5759	0.5209	0.5193
GROUP 5	0.6438	0.5721	0.4797
ILLOVO	0.6604	0.5084	0.49
NEDBANK	0.5533	0.5356	0.5667
PNP	0.7689	0.6974	0.7266
SAPPI	0.6158	0.5786	0.5471
SANTAM	0.6726	0.656	0.5264
SHOPRITE	0.6404	0.3786	0.3532
STANDARD	0.5644	0.5569	0.5641

Table 3 is a compact way of expressing correlation matrices of the form

$$\begin{pmatrix} & H & L & C \\ H & 1 & 0.9273 & 0.9439 \\ L & 0.9273 & 1 & 0.9245 \\ C & 0.9439 & 0.9245 & 1 \end{pmatrix}, \text{ which would be the example for ABSA.}$$

It can be seen from Table 3 that for the majority of shares except for Sappi and Shoprite, the correlation between the closings (C) and the highs (H) are larger than those between C & L, and larger than those between H & L. Also, correlations between C & L are larger than between H & L, except for ABSA. In summary it can be seen in general that correlations can be ordered as: C&H > C&L > H&L.

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4 The Markowitz, Sharpe and Troskie-Hossain Models

4.1 The Markowitz Model

Markowitz's seminal 1952 paper on "Portfolio Selection" describes the manner in which a rational investor, who desires more return for less risk, might proceed. It explains how one might consider choosing a portfolio as a constrained maximisation/minimisation problem in which one tries to achieve a portfolio with minimum variance for a given level of return, or with maximum return for a given variance, since variance is considered essentially as risk. The model is briefly outlined below.

Suppose a portfolio consists of p shares, with returns, expected returns and covariance matrix respectively:

$$\underline{r} = \begin{pmatrix} r_1 \\ \cdot \\ \cdot \\ \cdot \\ r_p \end{pmatrix} \quad E[\underline{r}] = \underline{\mu} = \begin{pmatrix} \mu_1 \\ \cdot \\ \cdot \\ \cdot \\ \mu_p \end{pmatrix} \quad \underline{\Sigma} = E[(\underline{r} - \underline{\mu})(\underline{r} - \underline{\mu})']$$

The holding in each stock is then defined as: $\underline{w}' = [w_1 \quad \cdot \quad \cdot \quad \cdot \quad w_p]$. Where each member of \underline{w}' is a proportion of the overall amount of cash available for investment such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^p w_i = 1$. Therefore, the portfolio can be represented by $P = \underline{w}'\underline{r}$. The portfolio has expected return: $E(P) = \underline{w}'E(\underline{r}) = \underline{w}'\underline{\mu}$ and variance: $var(P) = \underline{w}'\underline{\Sigma}\underline{w}$.

Markowitz also described an efficient portfolio, going hand in hand with the concept of an efficient frontier.

A portfolio is efficient if:

- For a given variance, expected return is maximised, or:
- For a given return, expected variance is minimised.
- The portfolio may not have negative weights (no short selling).

One would want to maximise the expected return while minimising the portfolio variance. This problem is summarised by:

$$\max \{\underline{w}'\underline{\mu}\} \quad \text{and} \quad \min \{\underline{w}'\Sigma\underline{w}\} \quad \text{s.t.} \quad \sum_{i=1}^p w_i = 1$$

$$0 \leq w_i \leq 1 \quad i = 1, \dots, p$$

Minimising $\underline{w}'\Sigma\underline{w}$ subject to $\underline{w}'\underline{\mu} = \mu_p$, $0 \leq w_i \leq 1$ and $i = 1, \dots, p$ is a quadratic programming problem. By fixing μ_p and solving for every possible value, the efficient frontier is developed. This is the set of portfolios that offer maximal return for minimum variance. Sharpe (1970) offers another approach which follows.

Draw a straight line with equation $\mu_p = B\sigma_p + A$ through the efficient frontier in mean - standard deviation space. Let the gradient be $B = \frac{1}{\phi}$, then for a line parallel to the σ_p axis, $\phi = \infty$ and for a line parallel to the μ_p axis, $\phi = 0$. Now consider the line $\frac{A}{B} = \frac{1}{B}\mu_p - \sigma_p$ and maximise A while keeping B fixed, it will give a point on the efficient frontier. Varying the gradient will generate the efficient frontier.

4.2 The Sharpe Multiple Index Model

The Sharpe Multiple Index Model models asset returns as a function of market indices and can be represented as:

$$R_{tj} = \alpha_j + \beta_{j1}I_{t1} + \beta_{j2}I_{t2} + \dots + \beta_{jM}I_{tM} + e_{tj}$$

$$j = 1, 2, \dots, p \quad t = 1, 2, \dots, N$$

where R_{tj} is return, j represent shares and t indexes time. The following assumptions are made:

$$E(e_{tj}^2) = \sigma_j^2 \tag{1}$$

$$E(e_{tj}e_{sj}) = 0 \quad s \neq t = 1, \dots, N \tag{2}$$

$$E(e_{tj}I_{tk}) = 0 \quad k = 1, 2, \dots, M \quad (3)$$

$$E(I_{tk}I_{tl}) = c_{kl} \quad k = l = 1, 2, \dots, M \quad (4)$$

$$E(e_{ti}e_{tj}) = 0 \quad i \neq j = 1, \dots, p \quad (5)$$

These assumptions state that in the model, with regard to the error terms, their mean is zero, there is homoskedasticity, nonautocorrelation and the errors from fitting different dependent variables are not correlated with the indexes or with each other. The indexes may be correlated.

In matrix form:

$$R_t = \alpha + \beta I_t + e_t$$

where

$$\alpha = \begin{pmatrix} \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_p \end{pmatrix}, \beta = \begin{pmatrix} \beta_{11} & \cdot & \cdot & \cdot & \beta_{1M} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{p1} & \cdot & \cdot & \cdot & \beta_{pM} \end{pmatrix}, e_t = \begin{pmatrix} e_{t1} \\ \cdot \\ \cdot \\ \cdot \\ e_{tp} \end{pmatrix}$$

$$R_t = \begin{pmatrix} R_{t1} \\ \cdot \\ \cdot \\ \cdot \\ R_{tp} \end{pmatrix}, I_t = \begin{pmatrix} I_{t1} \\ \cdot \\ \cdot \\ \cdot \\ I_{tM} \end{pmatrix}$$

$$\text{Thus } E(R_t) = \alpha + \beta \mu_I \text{ where } \mu_I = E(I_t) = \begin{pmatrix} \mu_1 \\ \cdot \\ \cdot \\ \cdot \\ \mu_M \end{pmatrix}$$

and

$$\text{cov}(e) = \begin{pmatrix} \sigma_{e1}^2 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \sigma_{e2}^2 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \sigma_{ep}^2 \end{pmatrix}$$

Now consider the covariance matrix of R_t :

$$\begin{aligned} \text{cov}(R_t) &= E[R_t - E(R_t)][R_t - E(R_t)]' \\ &= E[\alpha + \beta I_t + e_t - \alpha - \beta \mu_I][\alpha + \beta I_t + e_t - \alpha - \beta \mu_I]' \\ &= E[\beta(I_t - \mu_I) + e_t][\beta(I_t - \mu_I) + e_t]' \\ &= \beta E(I_t - \mu_I)(I_t - \mu_I)' \beta' + E(e_t e_t') \quad \text{since : } E(I_t e_t') = 0 \\ &= \beta C \beta' + \Omega \\ &= \Phi \end{aligned}$$

where $\Omega = \text{cov}(e)$.

A covariance matrix is thus obtained for R_t , this fits into the previously described optimisation problem and together with the vector of expected returns is able to produce the efficient frontier in the Markowitz setting.

4.3 The Troskie-Hossain Improved Multiple Index Model

This model is similar to the Sharpe Multiple Index Model, but includes an important addition. Specifically, the Troskie-Hossain Model does not assume that the residuals of different shares are uncorrelated, where the Sharpe Multiple Index Model does. The definition of the Sharpe model is retained here, including the assumptions, however, the following adjustments are made:

In the Sharpe model, it is assumed that

$$E(e_{ti} e_{tj}) = 0 \quad i \neq j = 1, \dots, p$$

The Troskie-Hossain model instead assumes

$$E(e_{ti} e_{tj}) = \sigma_{ij} \quad i \neq j = 1, \dots, p$$

now consider the covariance matrix of R_t :

$$\begin{aligned}
cov(R_t) &= E[R_t - E(R_t)][R_t - E(R_t)]' \\
&= E[\alpha + \beta I_t + e_t - \alpha - \beta \mu_I][\alpha + \beta I_t + e_t - \alpha - \beta \mu_I]' \\
&= E[\beta(I_t - \mu_I) + e_t][\beta(I_t - \mu_I) + e_t]' \\
&= \beta E(I_t - \mu_I)(I_t - \mu_I)' \beta' + E(e_t e_t') \quad \text{since : } E(I_t e_t') = 0 \\
&= \beta C \beta' + \Omega_{Troskie} \\
&= \Phi
\end{aligned}$$

where

$$cov(e) = \begin{pmatrix} \sigma_{e1}^2 & \sigma_{12} & \cdot & \cdot & \cdot & \sigma_{1p} \\ \sigma_{21} & \sigma_{e2}^2 & \cdot & \cdot & \cdot & \sigma_{2p} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{p1} & \cdot & \cdot & \cdot & \cdot & \sigma_{ep}^2 \end{pmatrix}$$

The matrix $\Omega_{Troskie}$ would be a diagonal matrix in the Sharpe Multiple Index Model but this is obviously not the case here. In order to estimate $\Omega_{Troskie}$ from a dataset of N points and a model with $M + 1$ parameters, Troskie (2005) suggests using the following:

$$\hat{\Omega}_{Troskie} = \frac{1}{N - M - 1} \hat{E} \hat{E}'$$

$$\text{where } \hat{E} = \begin{pmatrix} \hat{e}_{11} & \cdot & \cdot & \cdot & \hat{e}_{N1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hat{e}_{1p} & \cdot & \cdot & \cdot & \hat{e}_{Np} \end{pmatrix}$$

where \hat{e}_{tj} are the estimates for the individual error terms.

4.4 Dynamic Time Series Models

The Index models are easily extended to include autocorrelation and heteroskedasticity in estimated residuals and squared residuals respectively. Autocorrelation and heteroskedasticity are tested for and remedied through use of the following models.

Consider first the AR(k) model:

$$R_{tj} = \alpha_j + \beta_{j1}I_1 + \beta_{j2}I_2 + \dots + \beta_{jM}I_M + e_{tj}$$

$$e_{jt} = \phi_{j1}e_{jt-1} + \phi_{j2}e_{jt-2} + \dots + \phi_{jk}e_{jt-k} + v_{jt}$$

$$j = 1, \dots, p \quad t = 1, \dots, n$$

where v_{jt} is normally distributed with mean 0 and variance σ_{iv}^2 . Specifically the errors are modelled as AR(k) processes.

Now consider the GARCH(1,1) volatility process.

$$R_{tj} = \alpha_j + \beta_{j1}I_1 + \beta_{j2}I_2 + \dots + \beta_{jM}I_M + e_{tj} \quad e_{jt} = \sigma_{jt}\varepsilon_{jt}$$

$$\sigma_{it}^2 = \theta_j + \alpha_j e_{jt-1}^2 + \gamma_j \sigma_{jt-1}^2$$

where ε_{jt} is normally distributed with mean 0 and variance 1. Where errors are modelled as GARCH(1,1).

Lastly, the AR(k)-GARCH(1,1) models:

$$R_{tj} = \alpha_j + \beta_{j1}I_1 + \beta_{j2}I_2 + \dots + \beta_{jM}I_M + e_{tj}$$

$$e_{jt} = \phi_{j1}e_{jt-1} + \phi_{j2}e_{jt-2} + \dots + \phi_{jk}e_{jt-k} + v_{jt} \quad v_{jt} = \sigma_{jt}\varepsilon_{jt}$$

$$\sigma_{it}^2 = \theta_j + \alpha_j e_{jt-1}^2 + \gamma_j \sigma_{jt-1}^2$$

Both the Sharpe and Troskie - Hossain models assume homoskedasticity and no autocorrelation. This is however rarely true in practise. It is also known from the Gauss - Markov theorem that the least squares estimates are the best linear unbiased estimators (BLUE), which will not be the case in the presence

of heteroskedasticity and autocorrelation. It is for these reasons that these dynamic time series models are used. They account for heteroskedasticity and autocorrelation, thereby insuring that BLUE is arrived at.

4.5 Empirical examples

Efficient frontiers for the models that have been discussed are now calculated using Closing data, the entire data period is used. Examples of optimal portfolios will then be given. In these plots of efficient frontiers and for all to follow, values for mean and standard deviation are annual.

4.5.1 Efficient Frontiers

Efficient frontiers from all models will be plotted together. Efficient frontiers for individual models using least squares, AR and GARCH models will also be plotted. The Markowitz model will be included in all figures for reference.

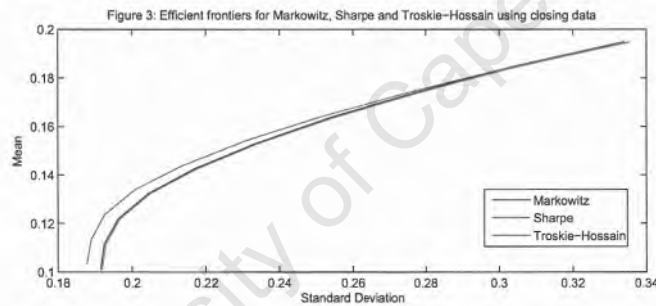


Figure 3 shows that the Troskie-Hossain model tracks the Markowitz and that the Sharpe shows lower risk for decreasing standard deviation. This is expected since the Troskie-Hossain takes covariances between shares into account whereas the Sharpe does not. Generally negative covariances will decrease risk and positive covariances will increase risk. The Sharpe model takes neither into account and is seen to underestimate the risk in Figure 3.

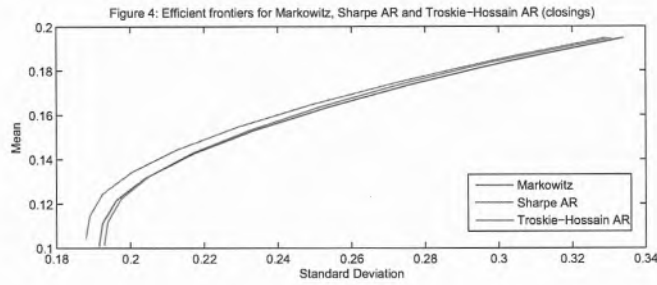


Figure 4 shows the effect of the introduction of the AR models. The effect on the Troskie-Hossain model is evident, with the Sharpe reacting in much the same way.

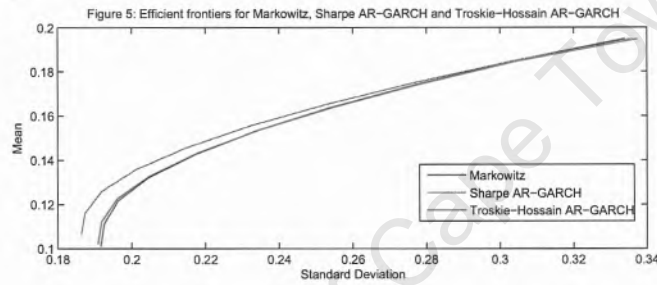


Figure 5 shows the effects of AR and GARCH modelling. When compared to the standard Sharpe and Troskie-Hossain models, the AR-GARCH modelling adjusts risk downward at low standard deviations. Figures 3 to 5 summarise the performance of the separate models. For investigations in later sections the Troskie-Hossain model is favoured due to its use of more information than the Sharpe.

Figures 6 and 7 show better how the efficient frontier is moved away from the Markowitz frontier through use of the Sharpe, Troskie-Hossain and AR-GARCH modelling.

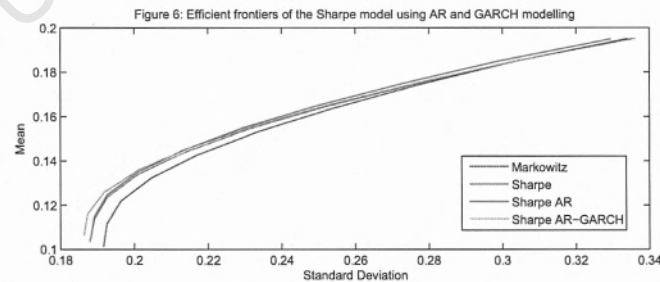


Figure 6 shows the progressing of the Sharpe efficient frontier through the use of different modelling. The Sharpe model underestimates risk and AR-GARCH modelling reduces it further resulting in much lower apparent risk. This might be attractive to an investor who seeks out lower risk portfolios, but the model may not be accurate.

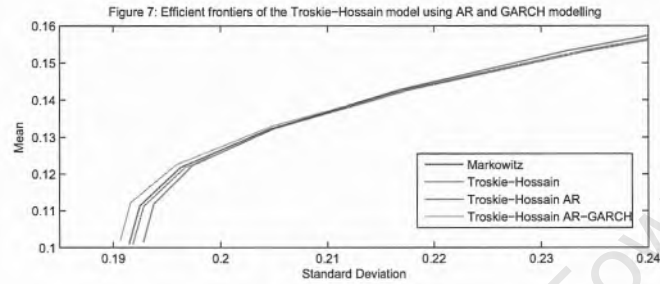


Figure 7 also shows the shifting of the efficient frontier toward lower risk. The Troskie-Hossain model is most likely a better estimate of the true risk over the efficient frontier. The identified low risk area might well be very useful to an investor. Figure 7's x axis (standard deviation) has been shortened to give a better view of the movement of the efficient frontiers, it initially ran to 0.33.

Statistics for the AR - GARCH modelling are quoted in Table 4.

	Adjusted \hat{R}^2	S. E. of Regression	F - Stat.	Durbin - Watson stat.
ABSA	0.522141	0.035963	66.56015	1.912641
BIDVEST	0.453664	0.029613	51.09934	2.094026
GROUP 5	0.098151	0.063587	7.502741	2.036931
ILLOVO	0.083432	0.047601	5.655385	2.002254
NEDBANK	0.477175	0.034036	74.72466	2.082861
PNP	0.042514	0.055724	3.66039	1.522224
SAPPI	0.219989	0.058681	15.50455	2.283961
SANTAM	0.180658	0.041606	10.86702	1.931048
SHOPRITE	0.193702	0.041798	10.54274	1.966188
STANDARD	0.551174	0.033025	64.0684	1.943884

4.5.2 Optimal portfolios

With the introduction of the riskless asset, it is possible to calculate the Capital Market Line (CML) and tangency portfolio. If risk free borrowing and lending

can be engaged in, the line from the risk free asset in mean - standard deviation space that is tangent to the efficient frontier is the CML. Any efficient point on the CML is expressed as a combination of the risk free asset and the tangency portfolio. This theory is due to Tobin (1958). Every point on the CML is more efficient than any on the efficient frontier. Therefore the tangency or optimal portfolio is the most efficient portfolio an investor can invest in without holding or borrowing cash.

The optimal portfolios for the Markowitz, Sharpe and Troskie-Hossain Models using AR-GARCH modelling are calculated. The risk free rate is taken as the the current 91 day Treasury Bill rate of 7.10%, which converts to an annual rate of 7.29%. The weights of the portfolios for each model are shown in Table 5.

	Markowitz	Sharpe	Troskie-Hossain
ABSA	0.0055	0.053	0.002
BIDVEST	0	0	0.0002
GROUP 5	0	0	0
ILLOVO	0.0113	0.0385	0.0061
NEDBANK	0	0	0
PNP	0	0	0
SAPPI	0	0	0
SANTAM	0	0.0082	0.001
SHOPRITE	0.8615	0.7903	0.8321
STANDARD	0.1217	0.11	0.1586

Table 5 shows that Shoprite and Standard bank are clearly the dominating assets in these portfolios. The portfolios differ appreciably for different models.

5 Efficient Frontiers using High, Low and Closing Prices

The differences between high, low and closing data are apparent. Bearing in mind that the literature found value in high and low prices, it is desirable to observe the differences these data will cause in core structures of Modern Portfolio theory. Namely; the efficient frontier and its associated optimal portfolio. The first and simplest usage of H, L and C data considered is the creation of separate efficient frontiers and optimal portfolios for the 10 shares in the portfolio. These can then be compared and differences noted.

5.1 Efficient Frontiers

The 3 separate efficient frontiers for H, L and C data will be plotted together on the same figure for the entire data period such that relevant differences might become apparent. Note that the shape and location of the efficient frontier in mean - standard deviation space depends on the expected return and standard deviation of individual shares as well as the covariance structure between shares. The shape and location in the following figures are indicative of the differences in the data used. As was shown in section 2.

The models developed in the previous section will be useful when plotting the efficient frontiers. The Markowitz Model's efficient frontier will be plotted first in Figure 8 using H, L and C data such that the initial differences in shape and location can be observed. The Sharpe model will not be employed due to the Troskie-Hossain model representing more information. The Troskie Hossain model including AR and GARCH models for autocorrelation and heteroskedasticity will be plotted for H, L and C. This model gives the most complete description of the positions of the H, L and C efficient frontiers relative to one another and is shown in Figure 9.

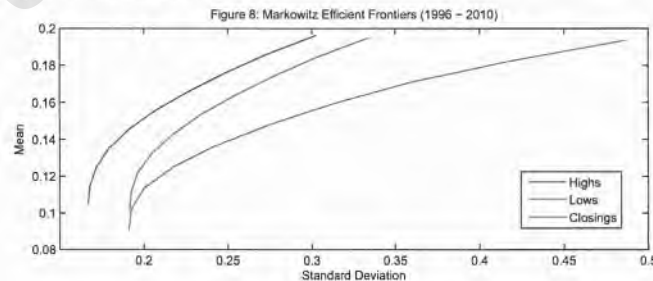


Figure 9 shows some of the differences in the data that were shown in section 2. The lower volatility of the highs and closings compared to the lows along with the similarities in expected return are evident. The difference in the shape of the efficient frontier of the lows is noticeable, caused by the higher volatilities exhibited by the lows as well as the covariance structure. Table 6 shows the standard correlation matrix of the lows. The correlation matrix for the highs is also given for comparison.

Table 6: Correlation Matrix (Pearson coeffs.) for 10 Shares (Low data)									
1	0.4508	0.1568	0.1964	0.587	0.2811	0.3075	0.3415	0.1522	0.6507
0.4508	1	0.1739	0.2043	0.4755	0.2158	0.3023	0.2676	0.2252	0.5069
0.1568	0.1739	1	0.0572	0.1542	0.1224	0.0859	0.1117	0.0499	0.1512
0.1964	0.2043	0.0572	1	0.1772	0.087	0.1614	0.106	0.1467	0.1253
0.587	0.4755	0.1542	0.1772	1	0.2514	0.3013	0.2975	0.2556	0.6315
0.2811	0.2158	0.1224	0.087	0.2514	1	0.1914	0.139	0.1565	0.2827
0.3075	0.3023	0.0859	0.1614	0.3013	0.1914	1	0.1331	0.1439	0.2951
0.3415	0.2676	0.1117	0.106	0.2975	0.139	0.1331	1	0.1167	0.3136
0.1522	0.2252	0.0499	0.1467	0.2556	0.1565	0.1439	0.1167	1	0.1953
0.6507	0.5069	0.1512	0.1253	0.6315	0.2827	0.2951	0.3136	0.1953	1
Table 7: Correlation Matrix (Pearson coeffs.) for 10 Shares (High data)									
1	0.3631	0.238	0.0873	0.5942	0.275	0.1296	0.2181	0.33	0.6178
0.3631	1	0.2338	0.1298	0.4311	0.2203	0.1759	0.1938	0.2384	0.3623
0.238	0.2338	1	0.0838	0.2359	0.1806	0.2101	0.0766	0.0922	0.1608
0.0873	0.1298	0.0838	1	0.1028	0.055	0.1218	0.1105	0.1356	0.0863
0.5942	0.4311	0.2359	0.1028	1	0.2828	0.1503	0.1885	0.3448	0.6009
0.275	0.2203	0.1806	0.055	0.2828	1	0.1443	0.1092	0.16	0.2857
0.1296	0.1759	0.2101	0.1218	0.1503	0.1443	1	0.093	0.1357	0.0958
0.2181	0.1938	0.0766	0.1105	0.1885	0.1092	0.093	1	0.2354	0.2259
0.33	0.2384	0.0922	0.1356	0.3448	0.16	0.1357	0.2354	1	0.2789
0.6178	0.3623	0.1608	0.0863	0.6009	0.2857	0.0958	0.2259	0.2789	1

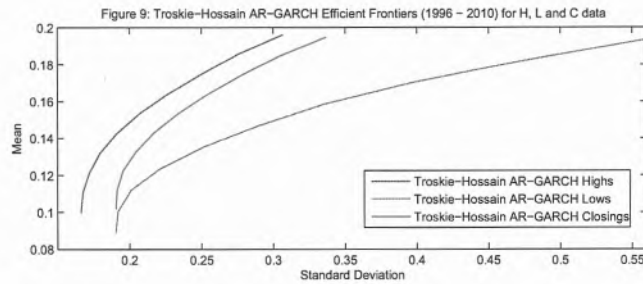


Table 6 is very similar to Table 7 in the relative positions of the efficient frontiers. In further tests that compare H, L and C data the Troskie-Hossain model will be used for simplicity, the above Figures suggest that relative information is preserved and that AR and GARCH modelling is not absolutely necessary.

6 Backtesting of Optimal Portfolios based on High, Low and Closing Data

From the previous section it is evident that optimal portfolios can be constructed using High, Low or Closing data. In order to compare these 3 portfolios in a practical manner, a backtest is performed. The portfolios will be constructed using historical data together with the Troskie - Hossain Improved Multiple Index Model. The Troskie - Hossain model includes more information than the Sharpe model and can be incorporated into a backtest in a straightforward manner. AR and GARCH modelling is not included for any dataset since relative performance is of interest and so as not to overcomplicate the backtests. The performance (with respect to portfolio value) over differing periods will be measured and will give an indication as to whether either H, L or C data is preferable for the construction of optimal portfolios for this dataset.

The nature of the tests that will be carried out is as follows.

There will be an initial optimisation period of set length over which optimal portfolios for H, L and C data will be created. For the year immediately following this period, an amount of 1 unit will be invested in each of the 3 portfolios. The return on each share for this year is measured using the closing prices at the beginning and end of the year respectively, that is

$$r_{1yr} = \frac{S_1}{S_0} - 1$$

where S_0 and S_1 indicate the closing price at the beginning and end of a year. These returns are then used to determine the portfolio return.

At the end of this first year, the optimisation will again be performed, but this time over the length of the optimisation period immediately preceding the current date. This amounts to rolling the initial optimisation forward by one year. The portfolio is then rebalanced according to the new efficient portfolio. When this is done, no cash is injected into the portfolio or withdrawn from it. It is self financing.

This procedure is repeated year after year until the end of the data period. The optimisation period is variable, but only initially. These periods are not altered once the rolling over of optimisations has begun.

This is the core test and Diagram 1 gives a visual representation. Table 8 details the lengths of different quantities for all tests performed.

After the results of these tests have been given the investment period of 1 year will be lengthened to 2 years for reasons to follow. The optimisation periods will remain variable for these tests and will now in effect be rolled over by 2 years for each rebalancing of the portfolio.

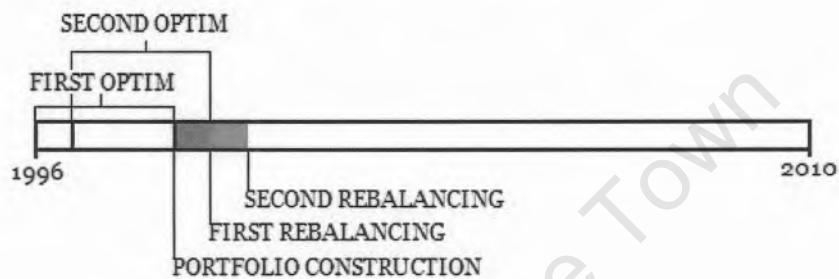
	Roll Optim. Period length	No. of rebalancings
1 year rebalancing	7	5
	6	6
	5	7
	4	8
	3	9
2 year rebalancing	5	3
	4	3
	3	4

6.1 Annual portfolio rebalancing

The tests will run over the entire data period (1996 - 2010). This means that portfolios with shorter optimisation periods will be invested for longer. For instance, a portfolio with an initial optimisation of 5 years will be invested for

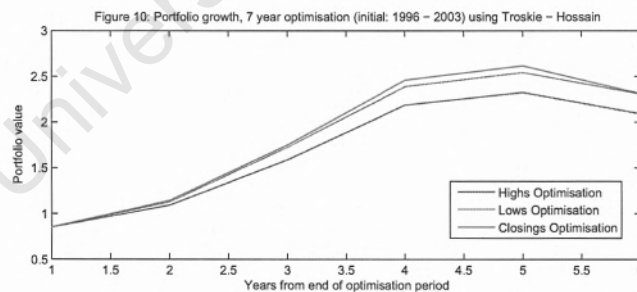
9 years (until 2010), a portfolio optimising over 4 years will invest in the first portfolio 1 year earlier and be invested for 10 years (until 2010). As stated above, the portfolio is self financing and therefore cumulative. It is expected to grow over its investment period and this growth is shown in the plots to follow. Immediately below, the test structure is shown in Diagram 1.

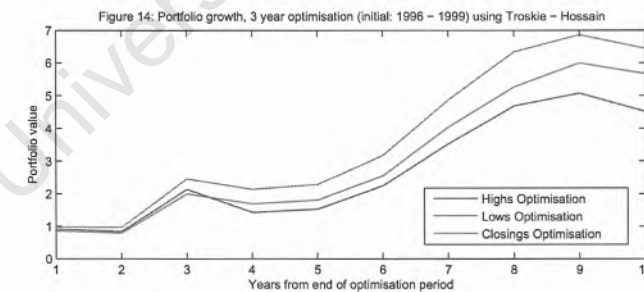
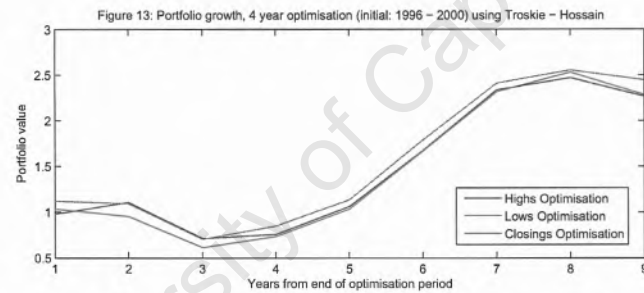
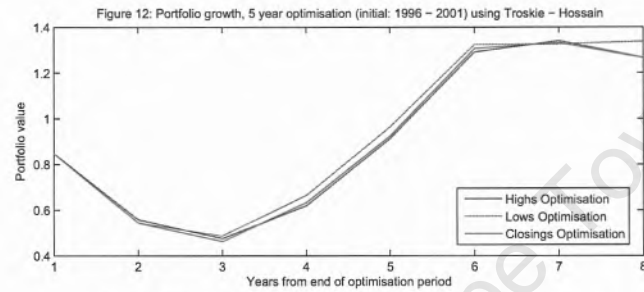
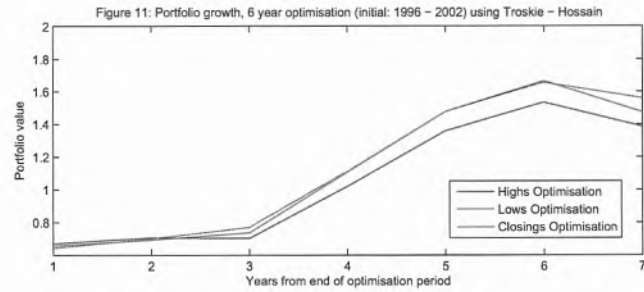
Diagram 1: Optimisation and rebalancing of backtest



In Diagram 1 the optimisation period is seen to roll forward. This period is variable for different tests, the one year growth period (shown in colour in Diagram 1) is not for these initial tests.

The 5 year optimisation period will be considered a benchmark as it is a satisfactory tradeoff between large enough sample size and period over which betas are stable, as detailed in Bradfield (2003). Optimisation periods from 3 to 7 years are used as they are realistic and will give concise and informative results which are shown in Figures 10 to 14.





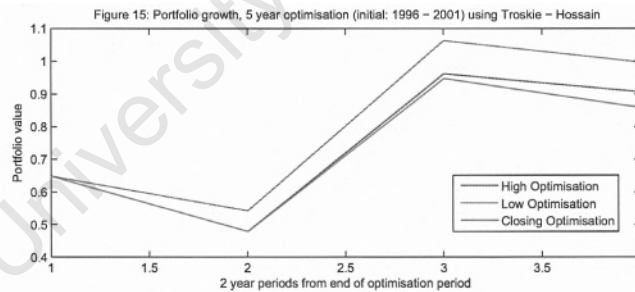
Figures 10 to 14 show the dominance of optimal portfolios created from Low and Closing data. The optimal portfolio from the Highs falls behind in value over the vast majority of both optimisation periods and life of the portfolios. Further, optimal portfolios from Low data are dominant over shorter optimisation periods

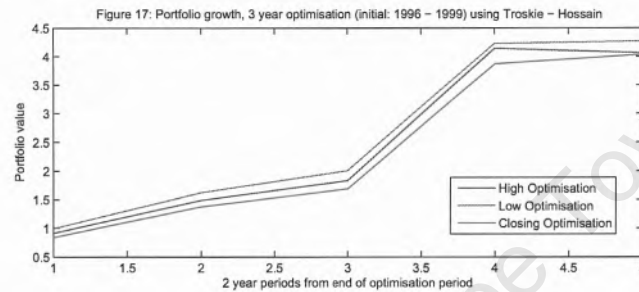
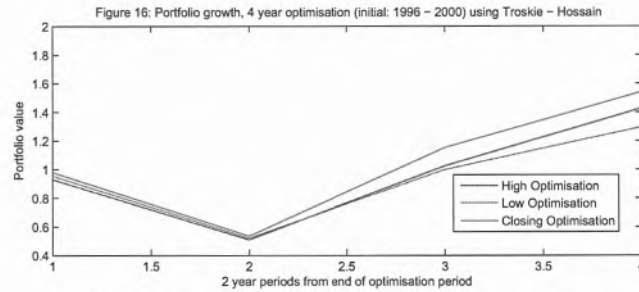
whereas those from Closing data seem to overtake the Low portfolios at longer optimisation periods. The relative performances of H, L and C portfolios are summarised in Table 9. A position of 1 is best. 3 is worst.

Optimisation Period	Position		
	High	Low	Closing
7	3	2	1
6	3	1	2
5	3	1	2
4	3	1	2
3	3	1	2

6.2 Biennial portfolio rebalancing

In the previous section, different optimisation periods were considered but the rebalancing/forecast period was fixed. The period between rebalancings was 1 year as it is logical to include the most recent information available into the structure of a portfolio. In the following figures, rebalancings are moved 2 years apart. This would not be practically ideal, but rebalancing a portfolio can be costly in terms of fees and tax. Therefore, the results of holding a portfolio for a longer period are shown in Figures 15 to 17. Optimisation periods remain variable.





Figures 15 to 17 only optimise over 3, 4 and 5 years. This is due to the data period, longer investment periods imply less data points in the results.

The benchmark optimisation period of 5 years again shows the dominance of the optimal portfolio created from Low data. It is now substantially more pronounced in its dominance than the benchmark portfolio using 1 year rebalancings. The 3 year optimisation shows the same structure, with the 4 year optimisation showing an inversion in the order. Relative positions of portfolios are summarised in Table 10. The similarity to Figures 10 to 14 is that for all 3 optimisation periods it would have been better to use either the Lows or Closings.

Optimisation Period	Position		
	High	Low	Closing
5	2	1	3
4	2	3	1
3	3	1	2

6.3 Results

From an empirical point of view, for this particular dataset, it can be concluded that on average over different optimisation and rebalancing periods it would

have been better to optimise a portfolio using Low data.

It is clear by looking at Table 9 that (for annual rebalancing) the portfolio from Low data is superior to both those of High and Closing Data. The only difference being a change of order between Low and Closing portfolios at an optimisation period of 7 years, a difference which is not influential since optimising over 7 years is not advisable and far from the benchmark. Figures 10 to 14 paint a clear picture and give an appreciation for the differences in the portfolios.

Table 10 alludes to the further dominance of the Low data portfolio, though the data period is not long enough to acquire a full appreciation for biennial portfolio rebalancing. It is however helpful and does not contradict what was previously said.

Considering the standard use of closing price data it is reassuring to note that the portfolios from the Closings rival those of the Lows. But the interesting observation is the poor performance of the portfolios from the Highs.

The observation most clear in the above figures is that for differing periods in optimisation and investment, portfolios from different data sets differ in value relative to one another. And since we see an obvious dominance of the Low portfolio, it would appear that the selection of a preferable dataset is possible. That is to say that though the Low series seems to be preferable, it is not certain to be so.

This suggests that the 3 data sets all contain information that might be useful. Indeed, a combination of H, L and C series is certainly possible according to Parkinson (1980) and others, and might be preferable to using any single series.

7 Combining Highs, Lows and Closings

The two elements central to the current focus are the mean vector (μ) and the covariance matrix (Σ) of the log returns. These will have to be reconstructed in order to combine High, Low and Closing data. The mean vector for a combination can easily be constructed, initially by an average across H, L and C. The covariance matrix is more complicated, but an initial approximation is possible.

New estimators for variance that rely on the High, Low and Closing price are readily available in the literature, combining H, L and C into a more efficient

estimators as first detailed in Parkinson (1980). This type of estimator provides combined estimates of variance for individual share returns and can therefore be used to replace the main diagonal of the covariance matrix with the estimated variance for each share. The off-diagonal elements will require the combination of the covariance matrices from High, Low and Closing data. The off-diagonal elements from the Closing series matrix provide an approximation as they are well representative of the covariance structure, the covariances between shares for Highs, Lows and Closings are indeed similar.

An approximation to the mean vector and covariance matrix for the combination is constructed below in order to view the efficient frontier and to compare optimal portfolios of the combination to those of the individual series. A practical version of the Garman and Klass estimator (shown in the next section) for variance will be used to replace the main diagonal in the covariance matrix from the Closing series. There are other estimators available, but the Garman and Klass is appropriate for illustrative purposes due to its simplicity.

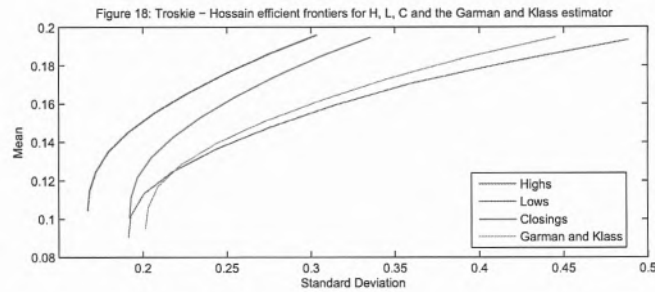
7.1 Efficient Frontier using the Garman and Klass estimator

As detailed in Beckers (1983), a practical version of the Garman and Klass estimator is:

$$\sigma^2 = \frac{1}{2}(H_t - L_t)^2 - 0.39(C_t - C_{t-1})^2$$

where σ^2 is variance, H_t , L_t and C_t are high, low and closing log returns.

Efficient frontiers are plotted in Figure 18 for High, Low and Closing data as well as for the new estimator using the Troskie-Hossain Improved Multiple Index Model. The covariance matrix including the new estimator is constructed as discussed above. The mean vector to be used with the new estimates is taken as the average vector of H, L and C mean vectors. Figure 18 shows the results.



The efficient frontier from the Garman and Klass estimator appears to be closest to the Lows. This is possibly a positive outcome considering the performance of the Lows in the backtesting.

7.2 The Optimal Portfolio

Optimal portfolios are constructed using these efficient frontiers for comparison.

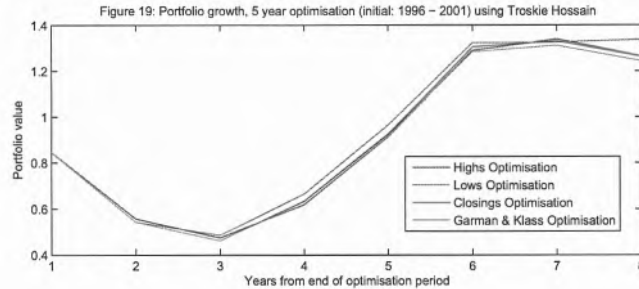
	Highs	Lows	Closings	Garman & Klass
ABSA	0.0022	0.1991	0.0063	0.0867
BIDVEST	0.077	0.0077	0	0.0576
GROUP 5	0	0	0	0
ILLOVO	0.0149	0	0.0107	0.0275
NEDBANK	0	0	0	0
PNP	0	0	0	0
SAPPI	0	0	0	0
SANTAM	0	0.0564	0	0.0306
SHOPRITE	0.7832	0.529	0.859	0.6136
STANDARD	0.1227	0.2078	0.124	0.184

Table 11 shows that the optimal portfolio generated from the Garman and Klass estimator is distinct from those of the H, L and C in the weightings it assigns to different shares. To further compare this new optimal portfolio a backtest similar to the previously performed ones is conducted.

7.3 Further backtesting

The optimisation period used for backtesting here will include periods from 3 to 7 years while the rebalancing period will be 1 year. Figure 19 shows the results

for the 5 year optimisation.



Figures of portfolio growth for more optimisation periods are not shown to avoid repetition. They do however show that the portfolio from the new estimator performs comparably to the other portfolios. It would remain better to simply use the Lows for this dataset, but there are no immediate problems with using the new estimator, and there is room for development. Table 12 summarises the relative performance of portfolios, where a 1 again indicates the best performing portfolio and 4 indicates the worst.

Optimisation Period	Position			
	High	Low	Closing	Garman & Klass
7	4	2	1	3
6	4	1	3	2
5	3	1	2	4
4	4	2	3	1
3	4	1	3	2

8 Concluding Remarks

It has been shown that the High, Low and Closing price series are certainly of interest to Modern Portfolio Theory. In terms of the use of the individual series, the positions of the efficient frontiers relative to one another in mean - standard deviation space are certainly different, which adds information to the establishment of risk versus return. It is indeed possible to optimise a portfolio using High or Low data in the place of Closing data, and the backtests suggest that at least for the dataset in use, optimising using the Low data may achieve better results than by using Closing data.

It would seem that obtaining an accurate combination of High, Low and Closing data in order to estimate variance is advisable and very likely required. This because though the 3 datasets are similar, their differences are valuable. The correct philosophy is perhaps to prioritise obtaining the best estimates of variance and covariance between shares possible, and to achieve this, High and Low data must be incorporated along with the Closing data.

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