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THE UNIVERSITY OF CAPE TOWN

Ambiguity, Ambiguity Aversion and the Coverage of Uncertain Risks: The Case of the Insurer.

By

GRIEVE CHELWA

(CHLGRI001)

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Supervised by

Dr Miquel Pellicer-Gallardo

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Abstract: Ambiguity refers to situations where the probability of an event cannot be objectively determined. Reasons for ambiguity include a lack of useful historical data or a general disagreement among “experts” on what the precise probability of an event is. Ambiguity aversion is defined as an aversion to any mean-preserving spread in the probability space. Using the Smooth Ambiguity Model proposed by Klibanoff, Marinacci and Mukerji (2005), we show that ambiguity aversion results in a reduction in the proportion of insurance coverage offered by an insurer. This is because an ambiguity averse insurer calculates expected utilities by using a “distorted” probability that raises the marginal disutility of wealth in the loss state. We also show that, in general, an ambiguity averse insurer will not offer more coverage to wealthier agents. Wealthier agents enjoy more coverage when the subjective average probability of loss is significantly high. Our results go a long way in reconciling theoretical models of insurance under ambiguity with the empirical finding that insurers are sensitive to ambiguity.

1. INTRODUCTION

Frank Knight (1921) pointed out the existence of a difference between risk and ambiguity¹. According to Knight, risk refers to situations where the probability of an event is known, or if not known, can be calculated from historical data. Or a risky event is one in which “experts” do not disagree on the probability of occurrence. On the other hand, ambiguity refers to situations where the probability of an event cannot be objectively determined. Reasons leading to ambiguity might include a lack of historical data or a general disagreement among “experts” on what the precise probability of an event is. Ambiguity aversion is defined by Klibanoff, Marinacci and Mukerji (2005) as an aversion to any mean-preserving spread in the probability space. That is an ambiguity averse agent prefers the lottery that yields a gain of 100 with probability $\frac{1}{2}$ to another lottery in which the probability of earning 100 is uncertain but has a subjective mean of $\frac{1}{2}$, i.e. the agent believes the probability of earning a 100 is either $\frac{3}{4}$ or $\frac{1}{4}$ with equal probability. We show, using the smooth ambiguity model proposed by Klibanoff, Marinacci and Mukerji (2005), that ambiguity averse insurers offer less insurance coverage than if they were not ambiguity averse. This result is consistent with empirical research that finds that insurers are sensitive to ambiguity (Hogarth and Kunreuther, 1985; Einhorn and Hogarth, 1986; Hogarth and Kunreuther, 1989; Kunreuther et al, 1993; Kunreuther et al, 1995; Cabantous, 2007). In addition, we show that ambiguity averse insurers do not necessarily increase insurance coverage as the policyholder’s wealth increases. Increase in insurance coverage for wealthier individuals is only assured when the subjective average loss probability is very large. That is, when insurers believe that there is a very high chance, on average, that the loss event will occur.

Despite numerous empirical studies showing that ambiguity affects insurers’ behaviour, there has been little or no theoretical research incorporating this empirical regularity. Most theoretical models begin from the premise that the insurer, while having access to historical records, knows the precise loss probability while the insured (policyholder) faces ambiguity. We are of the opinion that such an approach does not capture much of the experience of selling insurance in developing countries or providing insurance for new risks. Consider a subsistence farmer in a remote part of Africa trying to insure his plough, or consider a family in a high density township trying to insure their charcoal stove against theft or damage. In most developing countries, little or no data is collected regarding the number of ploughs stolen or damaged over a time period. The lack of such data places insurers in ambiguous environments forcing them to either offer less than full coverage or in the extreme situation decline to offer any coverage at all. New risks also have a similar impact on insurers. It is at present difficult to ascertain what the precise risks are from climate change (Treich, 2010). Similarly, insurers have a hard time grappling with the risks associated with new diseases such as the swine flu epidemic of 2009. The fact that ambiguity is a real problem for insurers has been identified by policymakers as an obstacle to providing insurance to the poor (Clarke and Dercon, 2009).

The traditional approach to fighting poverty has emphasised the importance of market imperfections (such as requirements for collateral) in preventing the poor from accumulating assets. There is now a realisation that poverty also has a vulnerability element attached to it. The poor may fail to accumulate assets, even in the absence of market imperfections, if they are vulnerable to risks (ibid.). Such vulnerability forces the poor to diversify into low-risk low-return projects or to

¹ In Knight’s original formulation, ambiguity was referred to as unmeasurable uncertainty.

accumulate savings in good times and only to deplete them when times are hard. Dercon (2004) shows that vulnerability to risk reduces the poor's welfare in a cross-section of countries. Our interest in modelling insurance market-failure in the presence of ambiguity is intended as an addition to the literature on how to better supply insurance to the poor.

The rest of the paper is structured as follows: Section 2 reviews the literature on ambiguity and ambiguity aversion. Section 3 presents the main results of our paper and section 4 concludes.

2. LITERATURE REVIEW

In this section we review the main contributions to the theoretical and empirical literature on ambiguity aversion and uncertainty in general.

2.1. Theoretical Research

Von Neumann and Morgenstern (1944) are credited with having advanced consumer theory from the realm of certainty to one of risk. Before the contribution of von Neumann and Morgenstern, choice problems were framed in such a way that the consumer knew that any feasible consumption bundle could be obtained with certainty. But reality is fraught with many uncertainties. When buying a car, a consumer must consider the future price of petrol, expenditure on repairs and the resale value of the car several years later – none of which are known at the time of making the decision (Jehle and Reny, 2001). Further, the consumer must also grapple with the fact that the brand new car may not arrive as “advertised”. By defining preferences over gambles (or outcomes) von Neumann and Morgenstern put forward a set of axioms that guaranteed the existence of an expected utility function, essentially a utility function where the utility of each gamble is weighted by its probability. Thus a consumer chooses gamble A over B if the expected utility of A exceeds that of B.

In the von Neumann and Morgenstern (1944) formulation, the probabilities assigned to gambles are objective probabilities in the sense that there is no uncertainty regarding what the true probability is. But seldom is information sufficient for economic agents to discern objective probabilities. As pointed out in the introduction, a scarcity of historical data or a disagreement among “experts” might make it difficult for a decision maker to estimate probabilities. The main contribution of Savage (1954) was to extend the expected utility paradigm to situations of subjective probabilities – situations where objective probabilities were not known. Savage's approach considers choices between two or more uncertain acts. Choices between such gambles reflect both the desirability of the outcomes and the probability of the event in question. Consider the following example adapted from Gilboa et al (2007): “If horse A wins the race, you get a trip to Paris (otherwise you get nothing)” or “if horse B wins you get a trip to London (otherwise you get nothing)”. If you choose gamble A, an outside observer might suspect that you thought it more likely that horse A would win the race, or that you preferred Paris to London, or that some combination of your beliefs about the likely winner and your preferences between the two cities led to the observed choice. The Savage axioms were sufficient to guarantee the existence of a subjective expected utility function that represented the agent's preferences and beliefs. Thus the full armoury of von Neumann and

Morgenstern's formulation could be applied to situations of uncertainty where probabilities were subjective.

Savage essentially put forward four axioms. The first one is adapted from standard consumer theory, whereby agents are presumed to hold complete and transitive preferences over gambles. The next two axioms are technical axioms that separate tastes from beliefs. The fourth axiom, called the 'Sure Thing Principle', was crucial for Savage and has also been the source of much criticism. Consider the four gambles G1, G2, G3 and G4. Gamble G1 says "if horse A wins you get a trip to London, otherwise you win a trip to Philadelphia". G2 says "if horse A wins, you get a trip to Paris otherwise you get a trip to Philadelphia". G3 promises a trip to "London if horse A wins otherwise you get a trip to Montreal" while G4 says you win a trip to "Paris if horse A wins otherwise you get a trip to Montreal". G1 and G2 only differ if horse A wins. G3 and G4 only differ similarly. The sure thing principle says that if you prefer G1 to G2 then you must prefer G3 to G4. Thus the consolation prize is irrelevant as long as it is common to both gambles.

Ellsberg (1961) was the first to subject Savage's formulation to an empirical test. Ellsberg showed that agents violated the sure thing principle in situations where cognitive and informational constraints left them uncertain about what odds to apply to events (Klibanoff, et al, 2005). Ellsberg suggested the following thought experiment. A subject is shown two urns A and B each containing 100 balls that are either red or black. Urn A contains 50 black balls and 50 red balls while there is no additional information about the proportions of red and black balls in urn B. One ball is drawn at random from each urn. Bet 1 is "the ball drawn from urn A is red" and is denoted AR, with AB, BR and BB defined likewise. A successful bet carries a prize of \$100. Ellsberg was able to observe the following preferences over bets: AR AB BR BB. That is agents were indifferent between betting on a red ball or a black ball in urn A but preferred betting on urn A than betting on urn B even though the same agent was indifferent between red and black in urn B. The results showed that there was no probability measure supporting these preferences through subjective expected utility maximization. The subjects violated the sure thing principle due to the presence of ambiguity. The Ellsberg experiments gave rise to the term ambiguity aversion to describe instances where agents preferred bets on probabilized outcomes to non probabilized outcomes.

The Ellsberg paradox prompted a search for decision models that incorporated ambiguity. The first model that gained influence was the Choquet Expected Utility (CEU) model proposed by Schmeidler (1989). Schmeidler weakened Savage's sure thing principle and obtained a representation where beliefs are characterized not by a subjective probability but by a capacity, that is, a non-necessary additive, increasing set function. The capacity is defined as follows: Let Ω be an outcome space, and let \mathcal{A} be an algebra on Ω . A capacity is a set function $\nu: \mathcal{A} \rightarrow [0, 1]$ that satisfies $\nu(\emptyset) = 0$ and $\nu(\Omega) = 1$. The capacity is non-additive in the sense that for any two disjoint events A and B , $\nu(A \cup B) > \nu(A) + \nu(B)$. The last condition would hold with equality if ν were a probability distribution function (additive capacity). The representation of preferences derived by Schmeidler is, letting the random variable X take only two values on Ω ,

(1)

where the last step uses μ and v is a utility function. The capacity reflects the relative (subjective) likelihoods of the various outcomes, as well as the attitudes towards ambiguity. The definition in (1) is equivalent to putting the “left-over” probability mass on the smallest value of v over the outcome space. Stated differently, the decision maker underweights bad outcomes relative to good outcomes and in this way the theory capture the notion of pessimism. Note that, if the capacity is additive, we get back to subjective expected utility as per Savage (1954).

A setback in Schmeidler’s (1989) model was the difficulty encountered in giving a realistic interpretation to the capacity function μ and in justifying why such a function represented ambiguity. The “Multi Prior” model (or “Maxmin expected utility” model) by Gilboa and Schmeidler (1989) was an improvement over the former. With the multi prior model, ambiguity is not represented by a unique probability estimate generated by a capacity but by a range of probabilities where each probability in the range has a distribution function. Instead of an ambiguous decision maker putting the chance of rainfall at 60 percent, the multi prior model says the decision maker is better off estimating a range of probabilities, say between 55 and 65 percent. The decision maker can then calculate his expected utility for each of the probability distributions and make a decision based on the set of expected values obtained. In making a decision, the agent proceeds as follows: Assign to each alternative probability estimate the minimum expected value over the set of priors and choose the alternative that yields the highest minimum, hence the term “maxmin”. Interestingly, the CEU model of Schmeidler (1989) and the multi prior model of Gilboa and Schmeidler (1989) are related. Under the assumption of uncertainty aversion (a key axiom in the formulation of the multi prior model), the CEU model is a special case of the multi prior model. In that case, the set of priors over which the decision maker takes the minimum has the property of being the core of the capacity ². The core of the capacity μ is the set of all probability measures relevant to the decision maker; hence the core coincides with the range of probabilities under the multi prior model.

Both the CEU model and the multi prior model have been influential in shaping the way economists think about decision making under ambiguity. The two models have been used as points of departure in later models of ambiguity aversion. Both the CEU and multi prior models confound the concepts of ambiguity and ambiguity aversion. The model proposed by Klibanoff, Marinacci and Mukerji (2005) achieves a separation of the two where ambiguity is represented by second order probabilities over first order probabilities (probability over probability) and ambiguity aversion is captured by a concave function that transforms expected utility. This model forms the basis of our results and is discussed in detail in section 3.

In as far as insurance is concerned, the CEU and multi prior models have been used to model the insurance purchasing decisions of ambiguity averse agents. Andersson (1999) considers a simple insurance model where the distribution of accident probabilities in the population is known, but where the actual probability of each policyholder is unknown to the policyholder and the insurer. In such a setting, with ambiguity averse policyholders, Andersson shows that deductibles are distorted downwards. Jeleva (2000) deals with the demand for insurance with a background risk (uninsurable

² Given a capacity μ , we can define the set of all probability functions on Ω which dominate μ (the core of μ) to be:

Core $C(\mu)$ is additive, μ is additive, μ is additive, μ is additive. See Mukerji and Tallon (2001).

risk) in an ambiguous framework. Jeleva uses the CEU model and finds that when the insurable and background risk are comonotone (have positive covariance), the impact of the background risk on insurance demand is related to the attitude towards wealth. When the two risks are anticomonotone, the attitude towards ambiguity is determinant. Vergote (2009) models insurance contracts with one-sided ambiguity: the policyholder is ambiguous about the likelihood of relevant events while the insurer does not face ambiguity but is aware that the policyholder does. Vergote's contribution is unique in that he views insurance contracts as possible information transmitters from the informed insurer to the uninformed policyholder. Thus in designing a contract, the insurer is aware of this fact and the contract is designed so as not to lose this information advantage. Finally, Alary, Gollier and Treich (2010) operationalize the smooth ambiguity model of Klibanoff et al (2005). Like other studies, Alary et al assume ambiguity on the part of the policyholder and conclude that ambiguity aversion always raises the demand for self-insurance but may well increase the demand for self-protection. Self-insurance is defined as a reduction in the amount of loss whereas the self-protection refers to a reduction in the loss probability. Alary et al also show that ambiguity aversion raises the optimal insurance coverage.

The main theme running through all the models that apply the concept of ambiguity to insurance is that the policyholder is ignorant of the loss probability whereas the insurer is not. We take a different approach in this paper, justified by the experience of practitioners and empirical evidence (see section 2.2), that ambiguity and the aversion to it, in developing countries at least, falls on the part of the insurer.

2.2 Empirical Research

In the insurance literature, most empirical studies begin from the assumption that ambiguity and ambiguity aversion are prominent among insurers than the insured. These studies then proceed to investigate how ambiguity affects insurance purchase decisions. Einhorn and Hogarth (1986) pioneered the testing of the ambiguity aversion hypothesis in insurance markets. Einhorn and Hogarth begin by assuming ambiguity on the part of both the insurer and the policyholder. They assume that only the insurer is averse to ambiguity. The rationale for this is based on the greater cost to sellers of insurance from underestimating loss probabilities. Their experimental results confirm the hypothesis that ambiguity averse insurers tend to charge higher premiums, than otherwise, over the entire range of loss probabilities. Kunreuther, Hogarth and Meszaros (1993) conduct a series of experiments investigating the decision processes of actuaries, underwriters and reinsurers in setting premiums for ambiguous and uncertain risks. Their survey data on prices suggest that all three types of these insurance decision makers set higher premiums when facing ambiguity. The follow-up paper by Kunreuther, Meszaros, Hogarth and Spranca (1995) investigates whether ambiguity aversion is also present among underwriters who are essentially line managers that determine what prices the insurer actually charges in the market place³. They conduct a field study of primary-insurance underwriters in a random sample of commercial property-and-casualty insurance companies and find that premiums are significantly higher for risks when there is ambiguity regarding the probability of a particular event occurring. Cabantous (2007) conducts a survey of 78 professional actuaries to test the ambiguity aversion hypothesis. His findings reveal that

³ In an insurance company, the analysis of loss probabilities conducted by an actuary feeds into the pricing decisions of an underwriter, who is essentially a line manager.

premiums are significantly higher for risks when there is ambiguity regarding the loss probability. Secondly, insurers are sensitive to sources of ambiguity whereby insurers charge a higher premium when ambiguity comes from conflict and disagreement regarding the probability of loss than when ambiguity comes from imprecision (imprecise forecast about the probability of loss).

Ambiguity aversion has also been used in finance models to shed light on previous anomalies such as the equity premium puzzle, i.e. the finding that the excess return of risky assets is too high to be explained by traditional measures of risk and plausible risk aversion parameters (Mehra and Prescott, 1985). Studies that incorporate ambiguity aversion in asset pricing show that, because financial assets entail ambiguity they are priced pessimistically. Thus such models exhibit an ambiguity related premium, which is not captured in risk-based models (Chen and Epstein, 2002; Epstein and Schneider 2008; Gollier 2009). Antoniou, Galariotis and Read (2010) investigate the source of the size-effect (Banz, 1981) whereby smaller companies tend to outperform larger ones without necessarily having larger betas. By using analyst earnings forecasts, they find that the size-effect arises because small companies are relatively more ambiguous, thus ambiguity averse investors respond pessimistically to information about them.

Ambiguity aversion has been used to investigate a wide range of interesting phenomena. Anagol et al (2010) find that ambiguity aversion affects the costume choices of children during Halloween. Hayden, Heilbronner and Platt (2010) demonstrate that the preference for unambiguous options by human beings is also shared by rhesus macaques – monkeys. These findings are also replicated in Rosati and Hare (2011).

3. THE MODEL AND RESULTS

In this section we state and prove the paper's main results. We, however, begin by briefly reviewing the smooth ambiguity model proposed by Klibanoff, Marinacci and Mukerji (2005) hereafter referred to as KMM. The representation of ambiguity and ambiguity aversion in KMM forms the basis of our results.

3.1. Smooth Model of Decision Making Under Ambiguity

KMM propose a model that achieves a separation between ambiguity tastes and ambiguity. The former is captured by a preference relation while latter is shown to be consistent with having a second order probability distribution over a first order distribution, i.e. a probability over probabilities. Such a separation between ambiguity tastes and ambiguity was not possible in the pioneering models of Schmeidler (1989) and Gilboa and Schmeidler (1989). Such separation leads to smooth preferences which permit, among other things, comparative statics. In KMM, preferences are shown to be represented by a functional of the double expectational form,

(2)

where u is a real valued function defined on the state space Ω , i.e. u is an act⁴. u is a von Neumann-Morgenstern utility function. μ is a probability measure on Ω and π is a map from reals to reals. π represents the decision maker's subjective probability distribution over Ω , the set of all probabilities on Ω . Therefore, π measures the subjective relevance of a particular ω as the "right" probability. The relationship between μ and π gives rise to what is referred to as having a 'probability over probabilities' and thus introduces ambiguity into the setting. E_π is the expectation operator.

π , as usual, measures attitudes towards risk whereas the shape of μ determines whether an individual is ambiguity averse, ambiguity neutral or ambiguity loving. Analogous with the literature on risk, concavity, linearity and convexity of μ corresponds respectively with ambiguity aversion, ambiguity neutrality and ambiguity loving. Whereas risk aversion is an aversion to any mean-preserving spread in the payoff lottery, ambiguity aversion is defined by KMM as an aversion to any mean-preserving spread in the space of probabilities. Consider the example: Ambiguity averse agents prefer the lottery that yields a gain of 100 with probability $\frac{1}{2}$ to another lottery in which the probability of earning 100 is uncertain but has a subjective mean of $\frac{1}{2}$, i.e. the agent believes the probability of earning a 100 is either $\frac{3}{4}$ or $\frac{1}{4}$ with equal probability.

Within the KMM framework, the evaluation of an act proceeds as follows: "first, compute all possible expected utilities of u , each expected utility corresponding to a μ . Next, compute the expectation (with respect to the measure μ) of the expected utility transformed by the increasing function π " (Klibanoff et al, p.1857, 2005).

3.2. Main Results

We now state and prove the main results of our paper. As point of departure, we adopt the model in Alary, Gollier and Treich (2010). Their model is, to the best of our knowledge, the first attempt at operationalizing KMM.

Preliminaries and Assumptions

We assume that there are two types of agents: Insurers and policyholders. Insurers supply insurance and policyholders purchase insurance against the prospect of losing an amount x with probability μ . The probability μ is ambiguous in the sense that it depends upon an unknown parameter θ . The ambiguity takes the form of a probability distribution for θ where μ_θ denotes the subjective probability of θ . With respect to the KMM model above, μ_θ is identical in interpretation with μ – the first order probability distribution. μ_θ is identical to μ – the second order probability distribution over Ω (or θ).

Let α be the coverage rate offered by the insurer where $\alpha = 1$ implies full cover and $\alpha = 0$ implies no coverage offered. Total premiums collected by the insurer depend only the amount of coverage αx and are defined as,

$$(3)$$

⁴ An act maps outcomes from the sample space onto the real line.

where μ is the mean of the ambiguous probability distribution π . P is the “full premium” which is the premium amount that would be collected if the insurer were to offer full coverage (i.e. $P = \int \pi(\omega) d\omega$ for π). The representation in (3) captures the premium setting behaviour of an insurance company in a competitive market structure. In such a scenario, a profit maximizing firm will set the insurance premium per dollar covered equal to the loss probability (which is a precise number if the probability is known or an average of the probability distribution if it is unknown). Total premiums collected will then equal the premium per dollar covered “times” the amount of coverage sold. A numerical example: Suppose a farmer stands to lose \$100 worth of crop with average loss probability equal to 0.5. Suppose the farmer opts to cover \$80 of the loss by purchasing an insurance policy. A competitive insurer might then decide to set the premium at 50cents per dollar covered, thus equating the premium to the average loss probability. This pricing rule will lead to zero profits in equilibrium – which is essentially the equilibrium condition for a firm in a competitive market structure. The insurer then collects a total of \$40 (i.e. $\$80 \times 0.50 = \40). Using (3) we can obtain the \$40 by setting $\pi = 0.8$ (80% coverage), $L = 100$ (total loss) and $P = 40$. Thus the representation in (3) is consistent with price setting behaviour in a competitive insurance setting (see Hogarth et al, 1989). Note also that π is constant as L is always constant (it’s an average) and the loss amount L is fixed apriori. In as far as meeting his objectives is concerned, the insurer is only free to adjust the coverage rate π . The insurer’s value function is thus given by,

$$(4)$$

where E is the expectation operator, ϕ is an increasing transformation function from reals to reals whose shape (concave, linear or convex) determines the insurer’s attitude towards ambiguity. U represents the expected utility resulting from offering coverage π and is given by,

$$(5)$$

where U is a standard concave utility function. W_0 and W_1 represent the insurer’s wealth in the loss and no loss states respectively (assuming only two states of nature). W_0 and W_1 are defined as,

$$\text{and } W_1 = W_0 - \pi L \quad (6)$$

with W_0 representing initial insurer wealth and W_1 is the indemnity or the amount paid out in the event the policyholder suffers a loss⁵. Clearly $W_1 < W_0$.

We now state and prove the paper’s four propositions.

Proposition 1 *An ambiguity averse insurer offers less coverage than an ambiguity neutral insurer.*

Proof

An ambiguity averse insurer faces the following maximization problem

⁵ Continuing with our example on the farmer, the insurer would pay out \$80 as indemnity in the event of loss. Recall that the farmer’s insurance policy provides 80% coverage for a loss of \$100.

(7)

where u is concave as the insurer is ambiguity averse. v is also concave due to the concavity of the utility function u . The first order condition for the problem in (7) is

(8)

The first and second order conditions are satisfied due to the concavity of the value function v .⁶ The first order condition in (8) essentially gives the insurer's marginal change in expected utility adjusted for ambiguity. v' is the expected marginal utility of x and is given as

(9).

From (9) the expected marginal utility of coverage is the difference between marginal utility in the loss state, which is non-positive as u' ,⁷ and marginal utility in the no loss state which always positive. An ambiguity neutral insurer has a v that is linear and their first order condition is given as

(10)

Recall that an ambiguity neutral agent is indifferent to a mean preserving spread in the probability space (see 3.1 on the KMM model). Hence v for the ambiguity neutral agent takes on the following form

(11)

whereas before μ is the mean of the probability of loss. Let x^* be the optimal insurance coverage under ambiguity neutrality so that

(12)

Since the value function in (4) is concave, an ambiguity averse insurer will offer less coverage than x^* if and only if

(13)

Since v by the assumption of ambiguity aversion (i.e. concavity assumption), for (13) to hold we need to show that $v' < 0$ or which is the same thing

⁸ (14).

Similarly (12) can be rewritten as

(15).

⁶ The value function is concave because it is a concave transformation of a concave function.

⁷ Recall that u' and since u is concave, it must be that $u' < 0$.

⁸ Obtained by expanding and collecting 'like' terms.

Alary, Gollier and Treich (2010) show that an ambiguity averse agent computes expected utility by using a ‘distorted’ probability (see appendix for proof) where

$$(16)$$

Thus we can rewrite (14) as

$$(17).$$

Comparing (15) and (17) and noting that concludes the proof

In evaluating expected utility, the ambiguity averse insurer settles on the distorted probability as the “right” probability of loss. This has the effect of raising the marginal disutility of coverage offer in the loss state (which always non-positive as) relative to the marginal utility in the no loss state which always positive. On a net basis, therefore, the ambiguity averse insurer experiences expected marginal disutility in offering additional coverage and this generates the result.

Proposition 2 *For an ambiguity averse insurer, an increase in ambiguity results in a reduction in the insurance coverage offered.*

Proof

Let be differentiable so that . This assumption induces , i.e.

$$(18)$$

is less than zero because by assumption and we know that (wealth in the loss state is less than wealth in the no loss state) . (18) tells us that an increase in results in a decrease in the insurer’s expected utility. Therefore, increasing must be akin to increasing the amount of ambiguity (or vagueness) around the occurrence of loss. We can, therefore, prove proposition 2 by showing that

$$— (18).$$

The derivative in (18) can be signed by using the implicit function theorem. The implicit-function theorem states that: Provided an implicit function for is known to exist for the equation , then the partial derivative of with respect to is given by,

$$— (1, 2, \dots,) (19).$$

By setting equal to the first order condition (8) we have,

$$(20).$$

By the implicit function theorem, (18) is defined as

$$\text{---} \quad (21)$$

Since our objective function is concave and the choice set is convex, the Weierstrass extreme value theorem (see appendix for statement) tells us that an implicit function for does exist for the equation in (20). From (21) is defined (by using the product rule of differentiation) as

$$(22).$$

From (22) we need to determine the sign of . Taking the derivative of (9) with respect to yields,

$$(23)$$

=

(23) is less than zero because by assumption and we know that (recall that and since , it must be that), implying that the term in square brackets in the second line of (23) is negative.

Concavity implies that in (22), and are positive. If we assume that the cross-partial derivative is positive⁹ and recalling from (18) and (23) that and are both negative, we have the result that is negative. We now need to determine the sign of . By the product rule of differentiation we have

$$(24)$$

In (24), and are both less than zero by concavity. is positive by concavity. The implication is that the two terms on the RHS of (24) are both negative, hence .

To recap, since we have established that and are less than zero, from the implicit function theorem we must have

— . This concludes the proof

Proposition 3 *Given two ambiguity averse insurers with the same probability distribution functions and , with insurer A more ambiguity averse than insurer B (in the sense that A's transformation is more concave than B's transformation function), then insurer A will offer less coverage than insurer B.*

Proof

Since is more concave than , we can define as

⁹ The assumption that a cross-partial derivative is positive is often assumed in neoclassical economics.

(25)

where v is a strictly increasing and concave function. In other words, $v \circ \pi$ is a concave transformation of v . Let π^B be the optimal coverage offered by insurer B such that

(26).

To show that π^B is more coverage than would be offered by insurer A, it must be that

(27)

because A's objective function is concave. We can use (25) to rewrite (27) as

(28).

By Jensen's Inequality (see appendix for statement) we have

(29)

By (26), the LHS of (29) equals zero. Therefore, (29) becomes

And this concludes the proof

3.3. An Extension: Ambiguity Aversion and Wealth Inequality

Fischer (1973) proposes a life cycle model of insurance purchases under conditions of quantifiable risk. Fischer's main result is that insurance purchases follow the life cycle pattern of earnings. That is, individuals tend to purchase less coverage in earlier times when incomes are little and purchase more insurance in later life when incomes rise. Fischer's result also has a cross sectional interpretation: At any point in time, wealthier individuals tend to purchase more insurance than less wealthier individuals. Fischer's prediction has been confirmed by empirical evidence that suggests that insurance companies tend to over-sell insurance to wealthier agents for quantifiable risks (Schoen et al, 2000). In this section we try to establish whether Fischer's prediction is robust to the introduction of ambiguity. This leads us to the following proposition.

Proposition 4 *Provided that the subjective average loss probability π is very large, an ambiguity averse insurer will provide more insurance coverage to wealthier individuals.*

Proof

To prove the proposition, we need to adjust the model to incorporate differing wealth levels on the part of the policyholder. Assume that the loss suffered in the bad state is a function of policyholder wealth w , i.e. $L(w)$ ¹⁰. Further assume that $L(w)$ is differentiable with $L'(w) > 0$, i.e. losses increase in policyholder wealth. This assumption says that wealthier policyholders invest more of their wealth in risky projects. In other words, we are assuming that policyholders have absolute risk aversion. The insurer's problem is now given as

¹⁰ Note that w , the policyholder's wealth, should be distinguished from W , the insurer's wealth.

(30)

where,

(31)

with insurer wealth in loss and no loss states defined respectively as

and (32).

As before, the 'full premium' is given as and is the insurer's initial wealth. First order conditions for the problem in (30) are given as

(33)

To show that an increase in policyholder wealth increases coverage, we need to show that

— (34)

We again invoke the implicit function theorem (see proposition 2) to sign the derivative in (34). By the implicit function theorem

— (35)

where (36).

From (24) we know that . Thus the derivative in (35) is positive if and only if . By the product rule we have

(37).

Evaluating yields,

(38).

Where is the derivative of the utility function with respect to . We need to evaluate . Recall that from (9) is

. (39)

Applying the product rule to (39) yields as¹¹

¹¹ Note that must be distinguished from . The former (big "U") refers to the expected utility function and the latter (small "u") refers to the utility function.

$$\frac{\partial}{\partial \omega} \left(\frac{\partial}{\partial \omega} \right) + 1 - \frac{\partial}{\partial \omega} \left(\frac{\partial}{\partial \omega} \right) \quad (40)$$

The expression in (40) can be simplified by collecting like terms to obtain

$$\frac{\partial}{\partial \omega} \left(\frac{\partial}{\partial \omega} \right) + \frac{\partial}{\partial \omega} \left(\frac{\partial}{\partial \omega} \right) \quad (41)$$

Looking at (37) we know that $\frac{\partial}{\partial \omega}$ and $\frac{\partial}{\partial \omega}$ by concavity. The cross partial derivative by assumption. Thus (37) is positive if and only if $\frac{\partial}{\partial \omega}$ and $\frac{\partial}{\partial \omega}$. The two derivatives are simultaneously positive when the average loss probability is very large¹², i.e. when $\frac{\partial}{\partial \omega}$. This concludes the proof

Stated differently, the result above suggests that if the insurer does not consider the average probability of loss to be very high (i.e. $\frac{\partial}{\partial \omega}$), the effect of increased policyholder wealth on coverage offered is ambiguous. For a very small $\frac{\partial}{\partial \omega}$, the net result will depend on the type of utility function adopted by the insurer. Proposition 4 arises from the fact that a very high $\frac{\partial}{\partial \omega}$ significantly reduces the marginal disutility of coverage in the loss state relative to the marginal utility of coverage in the no loss state. In the extreme condition where $\frac{\partial}{\partial \omega}$, marginal disutility in the loss state is equal to zero. This result essentially stems from the fact that a high $\frac{\partial}{\partial \omega}$ increases insurer wealth in the loss state relative to the no loss state (see definitions in (32)).

4. CONCLUDING REMARKS

Using the smooth ambiguity model proposed by Klibanoff, Marinacci and Mukerji (2005), our paper has shown that ambiguity and the aversion to it on the part of the insurer leads to a decrease in the amount of coverage offered for uncertain risks. Specifically, we have shown that an ambiguity averse insurer offers less coverage than an ambiguity neutral insurer. Correspondingly, given two ambiguity averse insurers whereby one is more averse to ambiguity than the other, the more averse insurer offers less coverage. Further, our paper shows that the Fischer (1973) result whereby wealthier individuals are offered more coverage is not robust to the introduction of ambiguity. Specifically, ambiguity averse insurers will offer more coverage to wealthier agents when the subjective average loss probability is sufficiently high.

Our results are in line with most of the empirical research that finds that insurers are sensitive to ambiguity. This goes a long way in reconciling the theoretical modelling of ambiguity in insurance with empirical regularities. Most of the modelling of ambiguity in insurance begins from the premise that the insurer possesses sufficient knowledge to calculate objective loss probabilities. We are, however, of the opinion that this is not usually the case for insurers covering risks in developing countries or for insurers covering new risks. The lack of basic data capture on such variables as crime patterns in developing countries makes the calculation of loss probabilities problematic. Further, the

¹² By examining the second line in (38), we notice that a very large $\frac{\partial}{\partial \omega}$ makes the term in square bracket equal to zero thus making the derivative positive. Similarly examining (41) reveals that a large $\frac{\partial}{\partial \omega}$ makes the term in braces equal zero making the derivative positive.

onset of new diseases such as the swine flu epidemic of 2009 makes assessment of risks emanating from such epidemics difficult to discern. The position we take is supported by the experiences of insurance policymakers who identify insurer ambiguity as an obstacle in supplying insurance to the poor.

APPENDIX 1: Alary, Gollier and Treich (2010) result on Distorted Probabilities.

Alary, Gollier and Treich consider the problem of an ambiguity averse individual attempting to self-protect against uncertainty¹³. Let Δp be the infinitesimal reduction in the loss probability resulting from successful self-protection. And let ΔW be the willingness to pay for this reduction in risk. Assume that the agent suffers a loss of L in the loss state and assume that the agent has no insurance. The value for this type of agent is given as:

$$(A1.1)$$

$$(A1.2)$$

whereas before u is a standard concave utility function, \tilde{u} is concave, p is the loss probability and W is the agent's initial wealth. Straight forward computations lead to

$$\frac{\Delta W}{\Delta p} = \frac{L \cdot p \cdot u'(W - Lp)}{p - \tilde{p}} \quad (A1.3)$$

This can be rewritten as follows:

$$\frac{\Delta W}{\Delta p} = \frac{L \cdot p \cdot u'(W - Lp)}{p - \tilde{p}} \quad (A1.4)$$

In which \tilde{p} is the distorted probability of loss that is defined as

$$(A1.5)$$

To show that $\frac{\Delta W}{\Delta p} > 0$: Suppose without loss of generality that \tilde{p} is increasing in p . It implies from (A1.2) that \tilde{p} is decreasing in p . Under ambiguity aversion \tilde{p} is decreasing, which implies that \tilde{p} is increasing in p . By the covariance rule, it implies that the numerator in equation (A1.5) is larger than $L \cdot p \cdot u'(W - Lp)$. It implies that

APPENDIX 2: Weierstrass Existence of Extreme Values (Jehle and Reny 2001).

Let f be a continuous real-valued mapping from where S is a nonempty compact subset of \mathbb{R}^n . Then there exists a vector x^* and a vector y^* such that

for all

¹³ As pointed out in the introduction, the standard approach in this literature is to assume ambiguity aversion on the part of the prospective policyholder.

APPENDIX 3: Jensen's Inequality (Varian, 1992)

Let X be a nondegenerate random variable and f be a strictly concave function of this random variable. Then $E[f(X)] < f(E[X])$. The opposite is true if f is convex.

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