MCOM DISSERTATION

An Investigation into Higher and Partial Moment Portfolio Selection Frameworks

By

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Abstract:

This dissertation highlights the importance of considering higher moments and partial moments of the distribution when conducting portfolio optimisation and selection. This is due partly to the weaknesses of mean-variance optimisation, as discussed throughout the dissertation, and the appropriateness of considering higher moments to better meet the investors utility functions. This dissertation investigates the usage of two bi-objective optimisation frameworks, a Skewness/Semivariance framework previously suggested by Brito et al (2016), and a proposed upside and downside semivariance framework (referred to as Semivariance/Semivariance), developed from Cumova and Nawrocki's (2014) general upper partial and lower partial moment framework. It solves the endogeneity issue present in the co-semivariance matrices, through the usage of a direct multi-search algorithm. The two frameworks were tested across multiple datasets, including one of pure stocks and one of asset classes, to test the ability to both allocate assets and select stocks. The performance was measured through nominal returns, statistical tests, Sharpe ratios, Sortino ratios, and Skewness/Semivariance ratios. The results reveal the Semivariance/Semivariance optimisation process to outperform the Skewness/Semivariance optimisation in the majority of the cases investigated. This suggests it may be a superior selection optimisation process. Furthermore, the Semivariance/Semivariance portfolios remain competitive with the benchmark portfolios selected in this dissertation, often outperforming them on an absolute return and ratio basis; however, this outperformance has not consistently proven to be statistically significant.

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Chapter 1

1.1 Introduction

Portfolio optimisation theory is, and will continue to be, an area of interest and debate in modern finance. Investors will continue to seek to improve on how their money is invested, to receive optimal risk adjusted returns. Therefore, this dissertation looks at the most taught portfolio optimisation theory: Markowitz mean-variance optimisation (1952). It investigates literature that addresses the weaknesses/limitations of Markowitz's method; specifically, the issues of normality in stock returns, the usage of quadratic utility functions, and the symmetrical basis of variance. This dissertation will analyse literature that proposes theories which suggest alternative constraints for portfolio optimisation. Sortino et al. (1999) suggested the use of upper partial moments and lower partial moments when selecting one's portfolio, to better align one's portfolio with Traversky and Kahneman's (1991) Prospect Theory. Following this, the dissertation investigates two main alternative portfolio selection processes: Skewness/Semivariance, and upside semivariance and downside semivariance (referred to as Semivariance/Semivariance), comparing the portfolios created through these methods with a set of benchmark portfolios.

The author acknowledges that the one method we investigate heavily has been conducted before by Brito, Sebastiao, and Godinho (2016); albeit not in a developing market such as South Africa nor one with such liquidity issues. This is the efficient Skewness/Semivariance portfolio selection method, which looks at the following equations to create an efficient frontier:

Maximising the Skewness:

$$\max_{w \in R} N \, \kappa(w) = E\big(R(w) - \mu(w)\big)^3, \quad (1)$$

Minimising the semivariance:

$$\min_{w \in R} N \;\; \textstyle\sum_B(w) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j CS_{ij} \; , \quad (2)$$

S.t. $w_i \geq 0, \; i \; .... \; N$,

where CS = $\frac{1}{T-1} \sum_{k \in U} (r_{k,i}$ - B)($r_{k,j}$ – B),

where, B = target rate, w = weight, CS = co-semivariance matrix, and r = return

Harvey et al (2010) found it benefits portfolios to take into account higher moments and not just mean and variance. Therefore, the suggestion by Brito et al. (2016) to use skewness as a constraint has credibility from other sources. Thus, this dissertation investigates the framework set out by Brito et al. (2016) throughout multiple market cycles and recessions/market crises, as well as in a developing market: South Africa.

The reason South Africa was chosen over other developing markets was due to ease of data accessibility, owing to the fact that the Johannesburg Stock Exchange (JSE) shifted all share price data to become electronic in 1996. This means that all the information needed to conduct the investigation is available from Bloomberg or the JSE itself. The time period was chosen to include two financial crises: the dotcom bubble in the early 2000s, and the global financial crisis of 2008, to compare the performance of the new methodology portfolios at times where the volatility experiences massive spikes.

Furthermore, this dissertation investigates the feasibility of a portfolio constrained by both upside and downside target semivariance. The logic behind this optimisation would be minimising the downside semivariance whilst maximising the upside semivariance. Thus, maximising potential upswings in returns while limiting gap downs, creating a 2-Dimensional efficient frontier, in the semivariance space. Each portfolio along the line representing an upside and downside semivariance optimal portfolio. The dissertation will then use different techniques to select portfolios along the line which are deemed more efficient. The techniques used are the following methods: the Max Sharpe ratio, the Max Sortino ratio and the Max Skewness/Semivariance ratio. To the author's best knowledge, pure semivariance portfolios have not be empirically tested before, and thus this dissertation aims to test the viability of these portfolios.

1.2 Research Question

How does the efficient Skewness/Semivariance optimisation proposed by Brito, Sebastiao, and Godinho (2016) perform in a developing market and throughout multiple market cycles, including recessions?

Furthermore, is it possible to construct portfolio's using upside semivariance and downside semivariance as the constraints that are competitive against traditional portfolio selection techniques?

1.3 Aims and Objectives

Therefore, as discussed above, the aims of this dissertation are to test the efficient Skewness/Semivariance portfolios in a developing market during periods of lower liquidity; aiming to find out if it can remain competitive with the mean-variance optimisation process; to test the efficient Skewness/Semivariance portfolios in multiple market cycles; to find out how the markets breaking down during the crisis would affect the Skewness/Semivariance selection process and the upside and downside semivariance portfolios.

The main aim of this dissertation is to expand on the suggestion by Sortino (1999), and Cumova and Nawrocki (2014) to use upper partial moments and lower partial moments in portfolio selection and propose the upside and downside semivariance portfolio and test its competitiveness against mean-variance and Skewness/Semivariance portfolios, throughout multiple market cycles, pre-, during-, and post- crisis.

1.4 Structure of the Dissertation

The remainder of this dissertation is structured into four chapters. Chapter 2: Literature Review, examines the importance of the issue, and literature surrounding the problem. Chapter 3: Data and Assumptions, describes the data used in this dissertation and the underlying assumptions made. Chapter 4: Methodology, will describe how the data is used, what the benchmark portfolios are and how they are formulated, the optimisation techniques used to generate the Skewness/Semivariance and Semivariance/Semivariance portfolios, and the subsequent measurement techniques used to compare them. Chapter 5: Results, discusses the empirical findings of the dissertation. Chapter 6: Conclusion and Future work, reiterates the key findings of this dissertation's results and suggests future expansions of this dissertation. It also highlights the key limitations of this dissertation.

Chapter 2: Literature Review

This literature review begins with an overview of the mean-variance optimisation model, examining the limitations and weaknesses of the model. It addresses the issues of using quadratic utility functions in portfolio selection, as well as seeking more optimal utility functions. Following this is an investigation into the issue of non-normality in stock returns by investigating literature that conducts empirical studies on the matter. This is followed by the dissertation addressing what happens to stock markets during times of financial crises, and how the distribution of returns is affected. It then focuses on issues with variance and alternative risk measures opposed to variance, specifically semivariance. It examines how to solve the issues surrounding the use of semivariance in portfolio selection. It investigates the preferences of individuals towards portfolios that have positive skewness. Furthermore, it investigates how the higher moments of the distribution could be included in portfolio selection to more efficiently select portfolios, specifically how skewness could be used. Finally, investigating some of the frameworks that make use of higher moments and/or lower partial moments proposed by the literature.

2.1 Mean-Variance

Markowitz (1952) put forward the mean-variance optimisation technique. The intention behind this type of optimisation is to minimise risk of the portfolio for a given level of expected return. Markowitz posited that through varying the expected return, the optimal portfolio at every given return level could be found, thus creating an efficient frontier. Each point along the frontier represents a mean-variance efficient portfolio. There are many ways to then select the portfolio that one uses, depending on their risk appetite. For example, one may select the Max Sharpe ratio, or simply the Minimum Variance portfolio, once the efficient frontier has been found. However, Markowitz's method has its drawbacks and limitations, which are discussed below. These limitations lead us to examine other alternative optimisation methods, which make use of higher moments as opposed to the first two moments of the distribution.

Mean-variance optimisation models have been found to be adequate with compact distributions of returns, and when the portfolios are rebalanced frequently, this forces the risk parameter to become sufficiently small (Samuelson, 1967). However, the mean variance optimisation doesn't consider the effects of transaction fees, and this means the shorter the period of rebalancing the higher the overall cost of the portfolio (Samuelson, 1967). The mean variance optimisation can be constructed in a way to consider transaction costs, but in general it does not.

Mean-Variance: Estimation Error

Fisher and Statman (1997) state mean-variance optimisation using historical means, variances and covariances will result in estimation errors and portfolios that are comprised of extreme weightings in a small set of stocks. This has led to many, including Frost and Savarino (1988), to attempt to improve ex-ante estimates of returns. Frost and Savarino attempted to use the capital asset pricing model instead of historical returns. However, this method resulted in their portfolios being comprised of a small number of stocks within their 200-stock dataset. Frost and Savarino concluded that this was due to biases in the estimation process, even though the method that was used was meant to eliminate such biases. This led them to constrain stock allocations to being greater than zero. i.e. disallowing short-sales (Fisher and Statman, 2004).

Michaud (1989) argued that mean-variance optimisation shifted majority of the portfolio's weightings to assets that exhibited high returns, low variances and low correlations in the past, but might not reexhibit these characteristics going forward. He argued that historical parameters as estimates of expected parameters exhibited too many estimation errors.

Merton (1980) argued that estimates of variances and covariances are more accurate than respective estimates of means. Merton found, however, that estimators based on the assumption of constant variance produced substantially variant estimates when compared with the properly fitted least-squares estimators, even when the time series was as long as fifty years (Merton, 1980). The paper he proposed went as far as to say trying to estimate expected returns was a "fool's errand" (Merton, 1980:5).

Best and Grauer (1991) investigated the mean-variance portfolio's sensitivity to asset return mean changes. It was found that small changes, such as an increase of 11.6% to any one asset mean in the portfolio of 100 assets, resulted in the equally weighted mean-variance portfolio to drop half of the stocks it was holding, and hold extreme positions in the stock that had a higher mean. However, for the overall portfolio, this had a small effect of roughly increasing expected returns and standard deviation of 2%. This shows the sensitivity to return estimates and leads this dissertation to use historical returns as a sample of the overall distribution, as one cannot estimate accurately returns going forward for each sample.

Fisher and Statman (1997), stated that if historical parameters are the inherent problem, better estimation is the solution. Fisher and Statman examined Green and Hollifield's (1992) work, who investigated mean-variance portfolios and how to improve their estimations and efficiency. Green and Hollifield (1992) found if you improved the estimates and constructed the mean-variance portfolio, the extreme weightings that made portfolios less appealing to investors were still present. This led them to recommend people accept that mean-variance optimised portfolios would have extreme weightings on occasions, and people should abandon their intuition on what a desirable portfolio might look like. This rationale crosses into all other types of optimisation, including our Skewness/Semivariance and Semivariance/Semivariance portfolios, as the more constraints you place on the optimisation, theoretically the less efficient it might become vs. an unconstrained optimisation, as Green and Hollifield showed with the mean-variance portfolio. Furthermore, Best and Grauers (1991) provided evidence that real-world portfolio problems will be guaranteed by binding constraints. Therefore, this dissertation will only employ the constraints of no short-selling and no leverage, implying the portfolio has weights that sum to one.

Mean-Variance: Issue of Quadratic Utility Functions

One key error to highlight is that Markowitz's optimisation requires the portfolio manager to know an investor's utility function in order to find the optimum level on the efficient frontier using Von Neumann's and Morgenstern's (1944) theory of expected utility maximisation. This raises a practical issue, as it is difficult to quantify each investor's own utility function or the utility functions of their clients. This leads the majority of practitioners and academics to assume quadratic utility functions.

The quadratic function is the only utility function that's expected utility only depends on the first two moments of the distribution (mean and variance); therefore, it is assumed for mean-variance optimisation (Pratt, 1964). A study conducted by Hanoch and Levy (1970) highlighted numerous drawbacks which arise from using quadratic utility functions. Tsiang (1972) backs up this claim, where he investigated the appropriateness of using quadratic utility functions in practice. Hanoch and Levy (1970), found that investor's utility functions can assume highly complex or irregular forms, differing from the simple quadratic form, often leading to the maximisation of the incorrect utility function in practice.

One major drawback is that a quadratic utility function implies an increasing absolute risk-aversion, which Arrow and Hicks "denounced as absurd." (Tsiang, 1972:355). This point is echoed in the literature as empirical and theoretical studies would lead one to assume decreasing absolute risk-aversion (Hanoch and Levy, 1970). Another key error with quadratic utility functions is that the positive marginal utility outcomes are required to be bounded within a certain range, as once the range is breached it leads to irrational behaviour (Quirk and Saposnik, 1962).

Hanoch and Levy (1970) put forward the usage of cubic utility functions and found the cubic form to be preferable to quadratic functions. The reason it may be desirable is because the expected utility now depends on skewness along with the traditional mean and variance. Adding the third moment allows for one to better approximate a general utility function (Hanoch and Levy, 1970). It also, most importantly, exhibits decreasing degrees of risk-aversion in certain intervals, which aligns with what Pratt (1964) finds about what the majority of investors risk-aversion preferences would be. Hogan and Warren (1974) found the same preference for decreasing degrees of risk-aversion, as well as a preference for positive skewness.

Mean-Variance: Issue of Non-Normality in Stock Returns

The second assumption required to satisfy the criteria for mean-variance optimisation is that the returns are normally distributed. This has been heavily scrutinised by financiers, as there have been multiple studies suggesting returns are not normally distributed. This was found by Beedles (1979) and Campbell and Hentschel (1992). The return distributions exhibit "fat tails" versus a typical normal distribution (Officer, 1972), which would imply that they have a higher degree of kurtosis than defined by the normal distribution.

During 1926-1965, Lawrence and Fisher (1970) found that for all periods, the wealth ratios tested exhibited positive skewness, suggesting the distribution to not share the characteristics of a normal distribution that has, 0.3 skewness. A second finding from Lawrence and Fisher (1970) is that for all periods except 1929, the kurtosis was greater than 3, which means that a greater amount of the observations fell extremely close to the mean than if it were a normal distribution, as a normal distribution has a kurtosis of 3.

A more recent study conducted by Chunhachinda (1997) investigated 14 key stock markets globally using the Wilks-Shapiro test to test for normality and found that none of them exhibited the characteristics of a normal distribution during the time period of 1988-1993. Macheado-Santos and Fernandes (2000) have provided empirical evidence of skewness when investigating data of the Portuguese stock markets.

It is important to examine the Johannesburg Stock Exchange (JSE) and the assumption of normality in stock returns, as this is where this dissertation will conduct its research. Page (1993) revealed evidence that the JSE's stock return distributions were of non-normal behaviour. Mangani (2007) found empirical evidence that the stock return distributions on the JSE were exhibiting leptokurtosis (excess kurtosis) and negative skewness. Therefore, these studies reveal that over the extended periods examined, distributions of stock returns do not necessarily exhibit the characteristics of a normal distribution, even though they may over certain periods. This implies that the higher moments of distributions should be considered when conducting the construction of one's portfolios.

Furthermore, this dissertation has periods where financial crises occur, and these crises affect stock markets in multiple ways. It can affect the market through financial contagion – this is when assets that were previously uncorrelated become correlated, therefore leading to different industries and markets tending towards similar stock return movements. It creates massive negative variance, i.e. great negative volatility. These high levels of negative volatility could last for extended periods of time, leading stock return distribution's to be altered, as they would have a lower mean with a higher probability of receiving lower returns or even greater losses. i.e. negative skewness (Sakthivel, 2014). This highlights an issue with using variance as a measure for risk, as the positive variance would slightly lessen the magnitude of the situation. This is owing to the fact that during these time periods, the amount of positive variance would be extremely small, but would bring the overall variance down. This implies the usage of semivariance as a better measure of risk than variance.

2.2 Semivariance

Nawrocki (1999) and Roy (1952), suggest if returns are non-normal, it would be more accurate to use semivariance as a measure for risk, as opposed to variance. Roy (1952), was the first to use semivariance in portfolio selection. This was as a “safety first” rule, where portfolios were compared on the probability of their returns falling below the threshold level expected/desired.

Markowitz (1952) originally suggested variance was the best proxy for risk. After investigating Roy (1952), Markowitz (1959) stated that semivariance using either below-mean or target semivariance would be a more accurate and plausible proxy of risk.

Following Markowitz’s (1959) statement, much research was conducted on variance versus semivariance, and into the most optimal way of measuring semivariance. Quirk and Saposnik (1962) found that semivariance was in fact theoretically superior to variance. Hogan and Warren (1974), stated that semivariance is more intuitively perceived as a failure to earn a target return, rather than simply being the half-variance (below mean semivariance). Hogan and Warren also found that if returns were symmetrical and a target semivariance was used, it provided alternative information to that of the variance, as the target return could be different to the mean. This contrasts the half-variance, as if the distribution is symmetric, half-variance (below mean semivariance) yields the same information as variance. Furthermore, Ang and Chua (1979) found target semivariance to be the lone measure of risk to satisfy the Von Neumann Morgenstern axioms of choice. Ang and Chua also presented the advantage of target semivariance when compared to half-variance (below mean semivariance) as mentioned by Hogan and Warren. Ang and Chua (1979) put forward the following methodology for measuring semivariance:

$$\sum B(w) = E\{[\min(R(w) - B, 0]^2\}, \quad (3)$$

where, B = target rate of return, and R(w) is the return, from an investment in w. Ang and Chua stated that the target rate of return should be independent of the probability distribution that one is ranking. This would ensure the information gained from it would be different to the information gathered from the variance or half-variance.

Another reason we suggest the use of semivariance is, as found by Tversky and Kahneman (1979), people generally value losses more severely than the benefit derived from gains. This is reiterated by Ballestero (2004). This should make intuitive sense, as investors gain disutility from the negative semivariance, although they do not receive disutility from the upside semivariance. Therefore, one would rationally want to look mainly at the downside semivariance as a constraint whilst trying to maximise the return gained. This separation of the undesirable downside and more desirable upside led many investors to pay attention to these new mean-semivariance models (Huang, 2007).

The reason a lot of practitioners avoid semivariance is due to the fact that it creates an issue of endogeneity in the co-semivariance matrix, and thus a closed form solution does not exist (Huang, 2007). Endogeneity in a matrix is when one or more rows in the matrix are linearly dependent on other rows in the matrix, which means one cannot invert the matrix or find a closed form solution, as one will end up with a range of variances. There have been numerous ways around the endogeneity issue, as numerous academics have addressed the mean-semivariance portfolio selection process. These include heuristic based algorithms, or the transformation of the endogenous co-semivariance matrix to an exogenous co-semivariance matrix (Cumova, 2007).

The issue of endogeneity was addressed by Brito et al. (2016) where he suggested the approach of a multi-objective optimisation derivative-free algorithm, which makes use of direct multi-search (DMS). The algorithm was first investigated for its usage in multi-objective optimisation by Custódio et al. (2011). This algorithm was first used in portfolio selection by Brito et al. in their 2014 paper: “Efficient cardinality/ Mean-Variance Portfolios”. This method is discussed further in Chapter 3: Methodology; or, if the reader desires an in-detail explanation, see Custódio et al. (2011).

2.3 Mean-Semivariance

An alternative to the direct multi-search solution is instead of using the endogenous co-semivariance matrix as it requires heavy computational power, one may use the exogenous asymmetric co-semivariance matrix proposed by Francis and Archer (1979). Albeit, this function has similar issues, as it does not always solve to a positive semi-definite matrix which has a closed form solution. Thus, Cumova and Nawrocki (2011), devised a solution whereby converting the exogenous matrix mentioned above into a symmetric matrix. This transformation allows for one to solve the semivariance matrix and thus allowing for a mean-semivariance framework to be solved using the critical line algorithm discussed in Markowtiz (1959). The benefit from using Cumova and Nawrocki's symmetric matrix is that it preserves the intercorrelation between securities, thus allowing one to more accurately observe the diversification benefit.

Cumova and Nawrocki (2011) highlight through empirical findings that using a mean-semivariance framework versus a mean-variance framework, one is able to increase one's portfolio's skewness, suggesting that skewness is something that investors crave. Although we need to note issues from the method suggested, Cumova and Nawrocki (2011) found that the method was only robust until the investable universe passed 50 stocks, at which the algorithm began to break down. The mean-semivariance frontier continued to add value until a point of 150 stocks was reached. This is not such an issue in a small market place such as South Africa, where the All Share Index is mainly comprised of companies within the Top 40, yet one cannot disregard the clear fault.

An issue using Cumova and Nawrocki's (2011) approach is that the portfolio semivariance computed from the co-semivariance matrix does not always agree with the portfolio semivariance computed from the portfolio's returns (Cumova and Nawrocki, 2011). This is most likely due to the multiple transformations of the co-semivarince matrix. The only way a solution that satisfies both the portfolio return semivariance and the co-semivariance matrices semivariance exists, is via the usage of heuristic-based highly iterative algorithms (Estrada, 2008). This is due to the trial and error nature of these techniques. This solidifies that the direct multi-search method is a superior technique for solving the co-semivariance matrix, as it preserves the original endogenous state.

Vasant et al. (2014), concluded that semivariance optimisation offers significant benefit in terms of risk adjusted returns on the JSE, even though other methods such as mean-variance could offer higher absolute returns. Therefore, this is promising in terms of outperforming mean-variance in an expected utility framework for investors whose utility functions are concave. As concavity represents risk aversion, thus outperformance increases with an increase in concavity of the utility function.

2.4 Mean-Semivariance vs Mean-Variance Portfolios

Levy and Markowitz (1979) defended the mean-variance portfolio selection from an angle of expected utility maximisation. They investigated whether an investor choosing their portfolio from mean and variance would approximately maximise their expected utility. Their findings were that, an investor who chose from mean-variance would approximately maximise their utility function, acknowledging that through other direct maximisation approaches, a more precise maximisation is possible, yet the mean-variance would be almost as accurate. Levy and Markowitz (1979) found that mean-variance was a superior approximation than if you used a mean-variance-skewness function. They did this through finding the correlation between the expected utility maximisation (EUM) outcome and a mean-variance and mean-variance-skewness portfolio. They found that in some instances, if you considered skewness and or kurtosis, it would worsen the approximation to expected utility. This was echoed by Briec et al. (2013), where they found that most mean-variance-skewness models solve the function by privileging one of the objectives at the cost of others, thus leading to a loss in optimisation power.

Estrada (2008) following this finding, conducted similar research using the same methodology, except using the mean-semivariance framework. He found theoretically that if the distribution was symmetrical, the methods should yield the same levels of expected utility, but if the distributions were either positively or negatively skewed, mean-semivariance would yield different levels of expected utility. Empirically he found that mean-semivariance was slightly more correlated with the EUM outcome, with an average correlation of 0.979, while mean-variance had an average correlation of 0.973; therefore, one may argue, a negligible difference.

Semivariance combines the information from variance and skewness, thus allowing for one to estimate required returns from a one factor model (Estrada, 2008). This information, coupled with Brito's Skewness/Semivariance portfolios discussed later, make the feasibility of a positive semivariance and negative semivariance portfolio selection process sound plausible.

2.5 Skewness and Mean-Variance

One cannot simply use mean-variance in every situation hoping it will align with the exact preference of the investor (Ryoo, 2007). This was previously highlighted with the massive extreme values unconstrained mean variance optimisation may provide, leading to unintuitive portfolios and investors placing binding individual weight limitations on securities or asset classes, to provide a more appealing portfolio.

Although Kane (1982) found that considering the third-moment will lead to an improvement on the mean variance optimisation, only if the underlying returns distribution is not excessively positively skewed, and the portfolio is not constrained to have individual stock weighting limitations, i.e. forced diversification. Kane (1982) concluded that there was a negative impact from diversification on the third-moment of returns, and thus the argument for a non-diversified portfolio is created. This provides a base for the extreme weightings one might find when using the Skewness/Semivariance optimisation, or any other type of optimisation that considers the third-moment of the distribution.

Skewness and appropriate utility functions:

Studies such as Arditti (1975), and Harvey and Siddique (2000) found generally an investor should have a theoretical preference for positive skewness. This is due to the fact that if an investor is assumed to be rational, he/she should prefer higher probabilities of achieving extreme profits whilst having a lower chance of receiving massive losses. This is essentially what positive skewness brings about; despite the fact that it lowers the mean (expected value), investors should forgo this to receive the increased frequency of extreme profits. This finding suggests that utility functions are not normally quadratic.

Alderfer and Bierman (1970) found through empirical studies that an investor does indeed have a preference for skewness. They found this as skewness helped explain the decisions people were making in their experiment. Alderfer and Bierman also found that the first three moments weren't enough to fully explain the decisions that investors would make. Coombs and Pruitt (1970) conducted experiments and found that an investor would prefer higher variance if his preference for skewness was met. This implies that if there is positive skewness in the distribution, the investor would prefer higher variability due to a higher probability of receiving returns, based on the shape of the distribution.

Kraus and Liztenberger (1976) discovered investors have an aversion for variance whilst maintaining a bias towards positive skewness. It revealed that incorporating skewness into the capital asset pricing model would lead to more accurate results, thus showing with empirical evidence that three-moment valuations are consistently superior to the original CAPM model. This implies that one should take into account these higher moments.

Bamberg and Dorfleitner (2013) argued that the most appropriate framework to use is a constant relative risk-aversion utility function if the investor wishes to use higher moments-based frameworks as expected. Utility relies on initial wealth while the higher moment functions do not; this is due to the fact that higher moment objective functions do not allow one to examine the reliance of portfolio weights on the initial level of wealth. Zakamouline and Koekebakker (2009) found that constant relative risk aversion investors tend to crave positive skewness and obtain disutility from negative skewness. Cont (2001) found empirical evidence that asset returns are generally below-mean returns, thus is representative of negative skewness. This is the opposite of what investors desire in their portfolios, and thus highlights the importance of being able to optimise your portfolio to be consistent with the properties desired, in the case of a typical rational investor as highlighted above, who desires positive skewness and not negative skewness. Mean variance optimisation by itself fails to take into account the skewness of a distribution.

Furthermore, higher moments cannot be ignored unless the distributions are symmetrical, which as stated earlier, was found to not be the case. Using expected utility reasoning, Zakamouline and Koekebakker (2009) suggested the use of skewness and semivariance for portfolio selection. This is due to the fact that they found that users expected utility functions are concave functions, with decreasing absolute risk aversion. This indicates that an investor’s expected utility function is an increasing function of skewness, this is in line with the findings of Arrow (1971).

Furthermore, Zakamouline and Koekebakker (2009) found that an investor's expected utility function is a decreasing function of semivariance. These characteristics make it plausible that skewness and semivariance could be used for portfolio selection. Zakamouline and Koekebakker's (2009) proofs for skewness and semivariance preference can be found below. Brito et al. (2016) provided a similar mathematical solution, showing the utility functions for skewness and semivariance.

The rationale for expected utility based on skewness:

Let R be a random variable representing the return on the investment in w.

$$u\big(R(w)\big) = u\big(\mu(w)\big) + u'\big(\mu(w)\big)\big(R(w) - \mu(w)\big)$$

$$+ \frac{1}{2!}u''\big(\mu(w)\big)\big(R(w) - \mu(w)\big)^2 + \frac{1}{3!}u'''\big(\mu(w)\big)\big(R(w) - \mu(w)\big)^3$$

$$+ \sum_{j=4}^{\infty} \frac{1}{j!}u^{j\prime}\big(\mu(w)\big)\big(R(w) - \mu(w)\big)^j,$$

Now applying the expectation operator:

$$E[u\big(R(w)\big)] = u\big(\mu(w)\big) + \frac{1}{2!}u''\big(\mu(w)\big)E[\big(R(w) - \mu(w)\big)^2] + \frac{1}{3!}u'''\big(\mu(w)\big)E[\big(R(w) - \mu(w)\big)]^3$$
$$+ \sum_{j=3}^{\infty} \frac{1}{j!}u^{j\prime}\big(\mu(w)\big)E[\big(R(w) - \mu(w)\big)]^j,$$

where each corresponding expansion represents the next central moment of R(w).

i.e. $E[\big(R(w) - \mu(w)\big)^2] = variance\ E[\big(R(w) - \mu(w)\big)^3] = skewness.$

Joro and Na (2006) created an approximation formula for the third-order, which looks like the following:

$$E[u\big(R(w)\big)] \approx u\big(\mu(w)\big) + \frac{1}{2}u''\big(\mu(w)\big)E[\big(R(w) - \mu(w)\big)^2]$$

$$+ \frac{1}{6}u'''\big(\mu(w)\big)E[\big(R(w) - \mu(w)\big)]^3, \qquad (4)$$

Arditti (1967) provided a similar expansion as the one provided above, explaining the rationale behind only focusing on the second- and third-moments of R(w)'s distribution. This was because the higher moments of R(w) i.e. kurtosis as the fourth and other moments, add very little extra information about the underlying distribution. Arditti states that u''(w) < 0, this is under the assumption that the investor is rational and risk-averse. This implies that one's marginal utility decreases with increasing wealth.

Furthermore, he proposes the following, using Pratt's 1964 approach to define the risk premium (γ) and prove positive skewness is preferred:

$$\gamma \approx \left[-\frac{\sigma^2}{2}\frac{u''(w)}{u'(w)}\right],$$

$$\frac{d}{dw}\left[-\frac{\sigma^2}{2}\frac{u''(w)}{u'(w)}\right] = \frac{\sigma^2}{2}\frac{-u(w)u'''(w) + [u''(w)]^2}{[u'(w)]^2} < 0,$$

where u(w) > 0, for the above function to hold. Therefore, this implies that u(w)'''>0. This means that the coefficient of the third moment, skewness is positive. This feeds into what was stated previously by other literature that investors prefer more positive skewness, given the same level of variance. These are in-line with Arrow's (1971) desirable investor utility function properties. These properties are the following u(w) > 0, u''(w) <0 and u'''(w) > 0.

The rational for expected utility based on semivariance:

$$u(R(w)) = \begin{cases} u_+(R(w)) if\ R(w) \geq B \\ u_-(R(w)) if\ R(w) < B \end{cases},$$

where the utility function is broken up into a piecewise function, with one for gains i.e. u_+ and one for losses, u_-.

This equation can be examined in a similar way to the skewness equations provided above. Investigating the second-order Taylor expansion for u(R(w)) around B results in the following:

$$u(R(w)) \approx \begin{cases} u_+(B) + u'_+(B)(R(w) - B) + \frac{1}{2}u''_+(B)(R(w) - B)^2\ if\ R(w) \geq B \\ u_-(B) + u'_-(B)(R(w) - B) + \frac{1}{2}u''_-(B)(R(w) - B)^2\ if\ R(w) < B \end{cases},$$

Similar to above, applying the expectation operator results in:

$$E[u(R(w))] \approx \begin{cases} u_+(B) + u'_+(B)E[(R(w) - B)] + \frac{1}{2}u''_+(B)E[(R(w) - B)^2]\ if\ R(w) \geq B \\ u_-(B) + u'_-(B)E[(R(w) - B)] + \frac{1}{2}u''_-(B)E[(R(w) - B)^2]\ if\ R(w) < B \end{cases},$$

Under the assumption that the Investor is rational, and as such risk-averse, specifically with regards to u_- meaning it is a concave function for losses, u''_- < 0. This implies the E[u(R(w))] is a decreasing function of semivariance. Therefore, Brito (2016) provided a mathematical proof that you would want to limit your exposure to downside semivariance, given a set return. This is, again, in line with Arrow's (1971) aforementioned properties for investors utility functions.

Downside semivariance can also be looked at through the same logic one applies to wanting to minimise variance. If we investigate the following portfolio variance equation:

$$\sigma^2 = \sum_i^n \sum_i^n w_i w \sigma_{ij}, \qquad (5)$$

where, w = the weight of the portfolio held in a security and σ_{ij} = the covariance of the securities when i ≠ j and the variance when i = j. Therefore, one can see from looking at the equation, the investor benefits the most when stocks have a negative or very small positive co-variance with each other, as this decreases the portfolio's over-all variance in general (Arrditi, 1967). This logic is confirmed by looking at the following equation:

$$\sigma_{ij} = r_{ij}\sigma_i\sigma_j, \qquad (6)$$

where, r_{ij} = correlation coefficient between the returns of the i and j securities and σ_i = the standard deviation of the returns of the i security.

Arrditi (1967), proves using the above two formulas that investors tend to prefer negatively correlated stocks in their investable universe, as it reduces the overall risk of the portfolio and thus the investor requires a lower rate of return for these securities. This logic can be applied to semivariance, as semivariance follows the same equation as equation (5), except the covariance (σ_{ij}) becomes co-semivariance, similarly if you are dealing with matrices, which this dissertation does, the covariance matrix becomes a co-semivariance matrix. Therefore, this supports our previous statements that investors would prefer to minimise their levels of semivariance given a specific return, as this would minimise the overall risk of the portfolio leading to higher risk-adjusted returns.

2.6 Upper and Lower Partial Moment Portfolios

Cumova and Nawrocki (2014) put forward a generalised upper partial and lower partial moment portfolio. This portfolio was based on a reversed S shaped utility function, which exhibits the properties described in Traversky and Kahneman (1991), that states that individuals are risk-averse to losses while wanting to maximise their upside potential. Markowitz (1952) argued in favour of this, using an example that individuals are willing to purchase both insurance and lottery tickets. This implies a rational investor has a concave utility function for losses, but a convex utility function for gains. The UPM/LPM generalised model put forward by Cumova and Nawrocki (2014) takes this into account by attempting to maximise the upper partial moments, above some target rate, whilst minimising the lower partial moments, below some target rate. The framework he suggested for a general UPM/LPM portfolio looks as follows:

Maximise:

$$\max_{w \in R} N \ E(UPM_{ij}) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j E(UPM_{ij}), \qquad (6)$$

where:

$$E(UPM_{ij}) = \frac{1}{T} \sum_{i=1}^{T} [Max\{0, (r_{ij} - \mathrm{X})\}]^{c-1} (r_{ij} - \mathrm{X}),$$

Minimize:

$$\min_{w \in R} N \ E(LPM_{ij}) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j E(LPM_{ij}), \qquad (7)$$

where:

$$E(LPM_{ij}) = \frac{1}{T} \sum_{i=1}^{T} [Max\{0, (\mathrm{X} - r_{ij})\}]^{a-1} (\mathrm{X} - r_{ij}),$$

$$s.t \sum_{i=1}^{n} w_i = 1,$$

where, w = weights, X = mean (or target rate), r = return, T = number of observations, c = upside potential seeking variable, a = risk-aversion variable.

Therefore, this dissertation bases its proposed upside semivariance and downside semivariance portfolios on Cumova and Nawrocki's (2014) findings, and the proofs therein; that taking into account upper partial moments as well as lower partial moments may benefit the overall risk adjusted returns investors are looking for, as it takes into account the reverse S shaped utility function.

2.7 Skewness/Semivariance Portfolios

Brito (2016) examined how portfolio selection using an efficient frontier constrained by Skewness/Semivariance, would compete with the generic mean-variance efficient frontier's Max Sharpe ratio portfolio, 1/N Portfolio (equally weighted), and minimum variance portfolio. Brito selected three main techniques to select efficient portfolios from the new Skewness/Semivariance frontier, namely: the max skewness-to-semivariance ratio, the Max Sharpe ratio, and the Max Sortino ratio portfolios. Through his study using these techniques, he found that the portfolios selected from the Skewness/Semivariance frontier were consistently competitive with those selected from the mean-variance frontier, where one of the Skewness/Semivariance frontier portfolios would always outperform the 1/N portfolio (Brito, 2016). This study is extremely recent and calls for further research into the efficient Skewness/Semivariance frontier, where the portfolios selected from the frontier are selected using other target rates, such as a target rate of 0, or the risk-free rate.

Brito (2016) did not specifically examine the portfolios performance in a developing market which lacks liquidity and a vast investable universe. The new methodology has also not been tested in stressed market conditions such as the global financial crisis, and how contagion and volatility drying up would affect the model he proposed. Therefore, this dissertation seeks to address those facts whilst also proposing a new type of frontier using upside and downside semivariance and examining how it will compete with both the historical mean-variance frontier and the newly proposed Skewness/Semivariance frontiers portfolios.

2.8 Final Remarks from Literature

This literature review examined research on the limitations of the mean-variance optimisation function, showing how the quadratic utility function and normality of stock return distribution assumptions do not always hold in practice. It examined alternative portfolio selection techniques that may be more efficient than mean-variance. It examined how mean-variance portfolios performed in an expected utility ranking compared to mean-semivariance portfolios, finding that in this realm, they are almost identical. Furthermore, it was highlighted by Harvey et al (2010) that taking into account higher moments of the distribution may benefit the portfolio selection process. This led to the investigation of Brito's 2016 proposed framework, the efficient Skewness/Semivariance frontier and mean-semivariance-skewness portfolios. It was found here that under Brito's Skewness/Semivariance framework, certain portfolios along the frontier were competitive with those generated via the mean-variance frontier. However, these findings were in normal market conditions and not under stressed conditions, nor in a market deemed weaker than those of developed countries, thus leaving a gap in the literature, of how do Skewness/Semivariance portfolios compare in such conditions? Literature examined concluded that semivariance embodies skewness and variance in one factor, thus leading this dissertation to want to explore an upper and lower partial moments optimisation technique based on semivariance alone. It seeks to empirically study the general upper and lower partial framework suggested by Cumova and Nawrocki (2014), by providing a upside and downside semivariance framework where the endogenous issue of the semivariance matrix is solved through the usage of the DMS algorithm, and to test whether this framework truly returns superior risk adjusted returns.

Chapter 3: Data and Assumptions

3.1 Data

In this dissertation we compare the constructed Skewness/Semivariance portfolios and the Semivariance/Semivariance portfolios with each other and the benchmark portfolios using a set of stocks only and then using a set of asset class indices. The two datasets used in this dissertation are denoted such that dataset 1 is single stocks from the All Share index (ALSI), and dataset 2 is asset classes that are available to the typical South African investor. Dataset 1 is comprised of fifty stocks of the ALSI and dataset 2 is comprised of five asset classes. A breakdown of the contents of each dataset is attached in Appendix A: Figure1. These datasets were acquired from Bloomberg and are for the periods 1991/01/31-2018/01/31, and 2004/06/30-2018/07/31 respectively. The share price times series data being utilised is in a monthly format and it is denominated in South African Rands (ZAR). The data used is the end-of-month closing price. This price data is converted to monthly return data using the following:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}},$$

where, R_t = discrete return at time t, P_t = price at time t, and P_{t-1} = price for previous monthly period. This results in discrete returns for each period. These returns are transformed to continuous returns (r_t) as follows:

$$r_t = \ln(1 + R_t),$$

The risk-free rate used throughout this dissertation to calculate the respective efficient frontiers, and wherever mentioned, is representative of the South African 10-year treasury bond for that given period, converted to a monthly rate.

3.2 Assumptions

The author acknowledges that a data mining bias may be present in the data, as the outcomes may vary over time, and in different markets. However, this dissertation tries to limit this bias through the usage of rolling sample periods in order to increase the robustness of the findings. Furthermore, the dissertation attempts to eliminate selection bias in dataset 1 through randomly selecting fifty companies for inclusion in this dataset. The reason the entire ALSI dataset was not used was to avoid thin trading bias; thus, after adjusting for the companies who exhibited this, the dataset was substantially smaller than the ALSI. The limitation to fifty companies was also due to computational limitations. This dissertation aims to prove the concepts and optimisation processes work and remain competitive; a full study may be conducted that attempts to take into account all liquid companies.

This dissertation further acknowledges that survivorship bias may be present in analysis associated with dataset 1. As none of the randomly selected companies failed over the sample period, no adjustments are made for survivorship bias in this dissertation. The rebalancing periods in the quarterly rebalancing sample are relatively short, that if companies were liquidated or were simply unable to be traded going forward, they could easily be deleted and replaced in the portfolios. We attempt to improve the robustness of the findings is the inclusion of dataset 2 in this analysis.

Dataset 2 does not exhibit survivorship bias, as the asset class data taken would account for any companies that were liquidated over the period. This is due to the ALSI, SAPPY, and MSCI world indices being used. These indices' returns would be inclusive of company liquidations. This means for dataset 2, if a security is added or deleted from the investable universe, it would be treated equally by the benchmark portfolios and by the Skewness/Semivariance and Semivariance/Semivariance portfolios. This is due to the different optimisation techniques being compared to each other.

Chapter 4: Methodology

This chapter is structured the following way, we will initially discuss in brief the rolling window periods of the data. Secondly, what the benchmark portfolios are and how they are created. Followed by a description of the direct multi-search framework developed by Custódio et al. (2011). Then we will discuss how this dissertation employs it in a similar method to that of Brito et al. (2016). This will be followed by an explanation of the portfolio selection techniques used to generate the twenty-four portfolios, that are compared to the three benchmark portfolios and amongst themselves. Lastly, this chapter explains the performance measurement techniques used to compare the out-of-sample portfolios.

4.1 Rolling Sample Period

The optimisation for dataset 1 will be conducted on a 120-month rolling-window basis, where the portfolio is analysed and rebalanced quarterly. This means that the initial period 1991-2001 is used as the first estimation period which takes up 120 months of our 325-month sample. The portfolio is then observed for four out-of-sample months, following which it will drop the first four months of the current rolling-window and replace them with the next four months of the dataset. An example: months 1-120 are used to find the optimal weights of the portfolios, then months 121-124 are evaluated as out-of-sample data. Following this the second period uses months 5-124 to optimise the weights of the portfolio, followed by an examination of months 125-128. This process is repeated until all the data periods have been assessed. This gives us 51 optimised weights for the quarterly rebalancing, to be tested across 205 out-of-sample months.

Dataset 2 is a smaller dataset, but will still attribute great insight, as it allows us to observe the power of the optimisation when applied to overall asset classes. This dataset will also be subjected to the rolling window; albeit with only 169 months of data, we chose a smaller rolling window period of sixty months or five years, this means the initial period is 2004-2009. This will be subjected to the same quarterly rebalancing as mentioned above until the rolling window ends in 2018. This results in 109 out-of-sample months.

4.2 Benchmark Portfolios

This dissertation will compare the efficient Skewness/Semivariance and Semivariance/Semivariance portfolios to three main benchmarks. These benchmarks are the following: An Equally Weighted portfolio, the mean-variance Maximum Sharpe ratio portfolio, and the Minimum Variance portfolio. All portfolios will be created under the assumption that the companies in dataset 1 represent the complete investable universe as a simplifying assumption (i.e. the AltX and other JSE companies will be assumed to not exist). Furthermore, it assumes the asset classes in dataset 2, are the only asset classes available to South African investor.

4.2.1 Equally Weighted Portfolio (1/N)

The Equally Weighted portfolio: all constituent companies have the equal weightings of 100% divided by the number of variables included in the dataset:

$$w_i = \frac{1}{N}, i = 1, \dots, N.$$

This means that for the stock selection dataset, where we have 50 variables, each variable has a weighting of 0.02, while in the asset class dataset, each variable has the weighting of 0.2. The reason we include an equally weighted portfolio, is due to literature continuously finding the equally weighted portfolio to outperform other models in certain situations. DeMiguel et al. (2009) compared fourteen models and found that the equally weighted portfolio was not consistently outperformed by any of the fourteen models examined. Academics justify this outcome as a result from optimal diversification. Lawrence and Fisher (1970) found that a thirty-two-stock portfolio would lead to a total diversification of ninety-five percent of the entire New York Stock exchange. Adding more stocks from this point adds negligible diversification and leads to an asymptotic outcome approaching one hundred percent diversification. The equally weighted portfolio, as the dataset increases in-size, will begin to cause viability issues, as many stocks cannot be held in the same sizing to their counterparts, as their free-float is lower or simply due to liquidity constraints. Under these findings, we can justify using the equally weighted portfolio as our biggest dataset will only include fifty stocks.

4.2.2 Minimum Variance Portfolio

The minimum variance portfolio is easily solved for. One can solve for it through quadratic programming or through the usage of matrix algebra and a Lagrangian function. This dissertation makes use of the traditional approach put forward by most literature, specifically referencing work done by Kempf and Memmel (2006). The equation used is the following:

$$\min_{w \in R^N} = w^T M_2 w,$$
$$s.t.\ e^T w = 1,$$
$$w \geq 0, i, \ldots., N,$$

where e^T, is a column vector whose entries are 1s, of appropriate dimensions, M_2 is the covariance matrix, and w is a vector of portfolio weights. The minimum variance portfolio weights are given by the following equation:

$$w = \frac{M_2 e^T}{e M_2 e^T}.$$

This type of optimisation is done using only the co-variance matrix and removes the bias from estimating expected returns. The lower variance bound can only be calculated efficiently if the co-variance matrix of stock returns is known, albeit in reality this is not the case and one needs to estimate the co-variance matrix in real markets (Kempf and Memmel, 2006). Therefore, the minimum variance portfolio is found using historical distributions of variances and co-variances. This was found to be more effective, as the estimates of returns exhibit too much noise (Best and Grauer, 1991).

4.2.3 Max Sharpe Portfolio

The mean-variance Max Sharpe ratio portfolio is formulated using Markowtiz's (1959) proposed mean-variance optimisation technique. As mentioned previously, the mean-variance optimisation seeks to minimise the variance of the portfolio for a given expected return. According to Brito (2016), the mean-variance model can be altered to represent a bi-objective function, which can simultaneously maximise the expected return of the portfolio while minimising the variance of the portfolio, shown beneath in equation 9. Therefore, we can find a set of efficient portfolios; these portfolios either minimise variance given a certain target return, or maximise return constrained to a certain variance. Following this process allows us to create an efficient frontier. In this frontier, there is one particular portfolio that maximises the reward-to-risk ratio. This is called the Max Sharpe ratio portfolio, explained in greater detail in section 3.2. We will create a benchmark portfolio following this methodology for both datasets. This benchmark portfolio was created through quadratic programming to find a solution to equation 9, checked against the in-built MATLAB mean-variance Max Sharpe function.

$$\min_{w \in R^N} \sigma(w), \quad (8)$$

$$\text{s.t. } e^T w = 1,$$

$$w_i \geq 0. i = 1, 2, 3, \dots, N,$$

$$\mu(w) \geq r,$$

where $\sigma = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}$,

$$\min_{w \in R^N} \sigma(w) \qquad \max_{w \in R^N} \mu(w), \qquad (9)$$

Under the constraints:

$$\text{s.t. } e^T w = 1,$$

$$w_i \geq 0. i = 1, 2, 3, \dots, N,$$

where, w represents the weights, σ = variance, μ = mean

4.3 Optimisation Techniques

The following section is broken down into four subsections, to explain how we create the twenty-four constructed portfolios. These portfolios are made up of the twelve Semivariance/Semivariance portfolios and the twelve Skewness/Semivariance portfolios. It describes how we select each of these portfolios.

4.3.1 Direct Multi-Search

The usage of the direct multi-search algorithm, as mentioned in Chapter 1: is for the ability to solve the endogeneity issue in the semivariance matrix, without complex transformations, and to simultaneously be able to minimise the portfolios semivariance while maximising the portfolio's skewness or semivariance, depending on which optimisation is being run, in order to obtain the pareto dominant frontiers for the selected datasets and periods. Pareto dominance is defined to be the point of which there are no other portfolios that would result in the two conflicting objectives having more optimal outcomes, unless one of the two objective functions had a sub-optimal outcome. This means it may be impossible to find another portfolio which would simultaneously improve both objective functions values.

The direct multi-search algorithm uses methods of directional type for single- and multi-objective optimisation. It works in two steps: a search step, and a poll step. The poll step is responsible for the convergence properties of the algorithm. The search step is not necessarily needed but is included to increase the speed and fluidity of the algorithm. The algorithm tries to approximate the entire pareto frontier from the polling procedure. The initial search step finds a set of points which meet the criteria of the objective function; the poll step then chooses one of the non-dominated points stored from previous iterations or from the initial search step. Once it has selected a point, it uses this as a poll centre and performs a local search around it, removing any dominated points and only storing non-dominated points, effectively using Pareto dominance to maintain an array of non-dominated points i.e. efficient points, thus creating an efficient frontier. This frontier is then extracted, and the process runs a new iteration until the last extracted array dominates all previous arrays, giving us the optimal efficient frontier. This dissertation considers over 100 000 potential frontiers before ending the search for the most optimal frontier. This could theoretically leave room for minor improvements in the portfolios. This was tested, as some portfolios were recreated using 500 000 potential frontiers, and the results were not statistically significant when using bootstrapped p-values. Furthermore, they exhibited extremely high correlations to each other, and thus this dissertation chooses to use 100 000 frontiers as the limit when performing the optimisation.

This dissertation has left out the complex mathematical equations deriving the formula and the equations revolving around the barrier functions which limit infeasible points. If the reader would like an in-depth explanation of the direct multi-search algorithm, he/she is referred to Custódio et al. (2011).

4.3.2 Skewness/Semivariance

The function requiring solving, as mentioned previously, is maximising skewness while simultaneously minimising semivariance. The equation looks as follows:

Maximising the Skewness:

$$\max_{w \in R} N\ \kappa(w) = E\big(R(w) - \mu(w)\big)^3, \qquad (10)$$

Minimising the semivariance:

$$\min_{w \in R} N\ \textstyle\sum_B(w) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j CS_{ij}\ , \quad (11)$$

S.t. $w_i \geq 0,\ i\ \ldots .\ N$,

where CS = $\frac{1}{T-1}\sum_{k \in U}(r_{k,i}$ - B)($r_{k,j}$ – B),

where, B = target rate, w = weight, CS = co-semivariance matrix, and r = return

The method employed is the direct multi-search function method, mentioned above. The semi-variance matrix is found beforehand, through a function coded in MATLAB that follows Estrada's (2008) methodology, which looks as follows:

Figure 1: Semivariance MATLAB Function example

```
function[M2]=semivariance(R,T,underperforms,B)
n=size(R,2);
Semi=[];
if (size(underperforms,2)==0)
   M2=zeros(n,n);
else
   for i=1:size(R,2)
      for j=1:size(R,2)
         tempv=0;
         for t=1:size(underperforms,2)
            positions=underperforms(t);
            tempv=tempv+((R(positions,i)-B)*(R(positions,j)-B));
         end
         Semi(i,j)=tempv/(T-1);
      end
   end
M2=Semi;
end
```

where, R is the return matrix imported, T is the number of observations, underperforms is matrix coordinates that underperform the target rate, and B is the target rate.

A function called underperform first finds any points above the target rate B and will set those points equal to 0, which generates the variable: underperforms. Once this has occurred, the function semivariance will begin the looping process where it creates the semivariance matrix. This matrix can be used in the solving of equation (2) as the CS variable, where CS = $\frac{1}{T-1}\sum_{k\in U}(r_{k,i}$ - B)($r_{k,j}$ – B). The weights for equation (2) are generated via the direct-multisearch function as it searches for the pareto dominant frontier. Thus, we can solve the second part of the above equation minimising semivariance. The co-semivariance matrix is endogenous here, due to the fact that the portfolio's weights will affect the periods in which the portfolio underperforms the benchmark, and as such, the elements within the co-semivariance matrix will be affected (Estrada, 2008).

The first part of the above equation represents the skewness. The skewness of a distribution has multiple ways of being measured in literature, as shown in the literature review. Dealing with higher moments such as skewness can become increasingly algebraically complex, as a random vector by the nth dimension can have its moments seen as tensors. This implies the second tensor is co-variance, a n x n matrix and the third is skewness, a n x n x n cube in three-dimensional space. Filling this cube with values will result in identical values, similar to if you completed the full co-variance matrix. The actual number of skewness' one can calculate is not n^3 , but rather is equal to the combinations with repetition of three elements out of n, $\binom{n}{3}$ (Athayde and Flores, 2004).

We have taken the most practical measurement of skewness: equation (12) using Athayde and Flores (2004) methodology, where one transforms the skewness tensor into a nxn^2 matrix. This is done through the process of slicing each n x n layer and pasting them in the same order sideways, generating a singular matrix, meaning one can use matrix differential calculus and derivation in the expressions. Therefore, the calculation to find skewness given a portfolio's weights looks as follows:

$$\kappa(w) = E[(R(w) - \mu(w))^3] = w^T M_3 (w \otimes w), \text{ (12)}$$

Where w^T= weights transposed, M_3 = co-skewness matrix, and $(w \otimes w)$ = weights multiplied using the Kronecker product.

The Kronecker product is used, when solving or optimising a function where the unknown or answer is a matrix. The following is a brief example:

$$(A \otimes B) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{12}b_{22} & a_{12}b_{22} & a_{12}b_{21} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix}.$$

Therefore, if A is a m x n and B is a p x q matrix, $A \otimes B$ will be a mp x nq. This is useful when multiplying matrices of different dimensions, as it will always allow for a solution. This allows us to use direct multi-search to solve the bi-objective function.

An individual entry in the coskewness matrix may be found via the following:

$$\frac{1}{T-1} \sum_{i,j,k=1}^{N} \sum_{t=1}^{T} (r_{t,i} - \mu_i)(r_{t,j} - \mu_j)(r_{t,k} - \mu_k),$$

where, r is the period return, T represents all observations, and μ is the mean return.

Further using Athadye and Flores' (2004) notation for moments as $M,\ M_2$ and $M_3$to represent the mean, co-variance, and skewness of the distribution. The two functions optimised for skewness and semivariance are:

Maximising the skewness: $w^T M_3(w \otimes w)$,

Minimising the semivariance: $w^T M_2(w)^{-} w$.

The above process is repeated with four alternative target returns, namely: B = 0, B = Equally Weighted (1/N) benchmark portfolio's return, B = Max Sharpe benchmark portfolio's return, and B = Minimum Variance benchmark portfolio's return, resulting in optimised in-sample function values for both the skewness and semivariance equations. This allows for us to plot the pareto efficient frontier, as shown in Chapter 5.1, and to conduct out-of-sample analysis on portfolios selected from these frontiers, as done in Chapter 5.2.

4.3.3 Semivariance/Semivariance

This dissertation applies the Direct mutli-search to a second function. This function is a non-smooth, non-linear function. This is an adaption of Cumova and Nawrcoki's (2014) upper partial moment and lower partial moment general framework shown in Chapter 1. This is the first time, to the authors best knowledge, that an upside semivariance and a downside semivariance portfolio will be constructed and tested empirically. We design code that highlights the positions in the dataset that underperform and overperform the target rate. This means for downside semivariance it will identify all the positions that underperform the target rate B in the equation below, and only use those returns in equating the downside semivariance of the sample, setting other returns to be equal to 0. The same process is applied to find the upside semivariance, except we identify all the points that outperform instead of those that underperform. Look at the code presented in figure 1 for downside semivariance. A similar adaption is done for upside semivariance, except B>0. The equations being used for semivariance are similar to those provided by Estrada (2008) and Cumova and Nawrocki (2014).

Maximise upside semivariance

$$\max_{w \in R} N \ \ \textstyle\sum_{B+}(w) = \ \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j CS_{ij} \ , \qquad (13)$$

Minimising downside semivariance:

$$\min_{w \in R} N \ \ \textstyle\sum_{B-}(w) = \ \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j CS_{ij} \ , \qquad (14)$$

$$\text{S.t. } w_i \geq 0, \; i \; \dots . \; N,$$

$$\text{where CS} = \frac{1}{T-1} \sum_{k \in U} (r_{k,i} - B)(r_{k,j} - B),$$

where, B = target rate, w = weight, CS = co-semivariance matrix, and r = return

The Semivariance frontiers are found through this process. This process is repeated with the same alternative target returns, B, as the Skewness/Semivariance optimisation process. This leaves us with a frontier for each alternative target return. This allows an investigation into how much the target rate may affect the outcome of the frontier and subsequently the portfolios selected, as we know if the target rate is the mean, the downside semivariance is equal to using the half-variance method if the distribution is symmetrical. Therefore, an entire study to finding the optimal target rate could be conducted.

Once the frontiers are found, portfolio selection takes place, which is described below.

4.3.4 Portfolio Selection

Three portfolios are selected from each of the frontiers generated by the direct multi-search algorithm, resulting in twenty-four constructed portfolios. These portfolios are:

- The portfolio that maximises the Sharpe Ratio,

$$SR = \frac{(\mu * w) - rf}{\sqrt{\sigma^2}},$$

  where $\mu * w$ = expected return of portfolio (historical mean return). rf = risk free rate, and σ^2 = variance of portfolio.

- The portfolio that maximises the Sortino ratio,

$$SOR = \frac{(\mu * w) - B}{\sqrt{\sum B}},$$

  where $\mu * w$ = expected return of portfolio (historical mean return). B = target return, and $\sum B$ = semivariance of portfolio.

- The portfolio that maximises the Skewness/Semivariance,

$$SSR = \frac{\kappa_B}{\sum B},$$

where, κ_B = skewness of portfolio, and $\sum B$ = semivariance of portfolio.

These formulas are explained in more detail in Chapter 4.4. The main differences are 4.4 uses the out-of-sample return data of the portfolios to perform analysis, and the above is for when we are selecting the optimal portfolios; thus, we are optimising off the in-sample data, looking for the portfolio that maximises the above functions. The Sortino ratio's B in the formula above, represents the respective target rate returns, while in 4.4 we assume that B is always equal to 0. Therefore, the above formulas will result in three portfolios being selected from each of the eight unique efficient frontiers. Thus, we have twelve Semivariance/Semivariance portfolios, and twelve Skewness/Semivariance portfolios. The portfolio that maximises the Skewness/Semivariance ratio is also referred to as Max SkewSemi throughout the results chapter, to avoid confusion with the Skewness/Semivariance frontier.

The selected portfolios will be tested in the out-of-sample style described in Chapter 4.1: Rolling Sample Period, where the portfolios are optimised on a set rolling window, where rebalancing will take place every four mouths to reweight the securities to the new optimised weights, and are then tested in the consecutive four-month period where the out-of-sample returns are observed as discrete returns. These returns are converted into continuous returns using the following:

$$R_{t,i} = \ln{(1 + r_{t,i})},$$

where, $r_{t,i}$, is the discrete returns for the given periods.

This allows for proper time aggregation to take place. The portfolios are then compared to each other using the methodology presented below in Chapter 4.4

4.4 Out-of-sample portfolio analysis:

4.4.1 Descriptive statistics and Statistical tests

This subsection will compare how an initial portfolio value of 1 would fare over the investment period, under the assumption of no transaction or brokerage fees. The descriptive statistics of the portfolios will be presented in a table format, where the mean and variance will be tested against the other portfolios to test for significant differences. This entails the usage of a T-test, Mann-Whitney test, and an F-test. All tests are conducted under a significance level of 5%, unless otherwise stated.

F-test:

The F-Test is employed to test whether the variances of the portfolios are equal or differ significantly. It is used for the investigation of whether the newer methodologies provide a significantly different variance when compared to the benchmark styles of portfolio selection or compared to each other. The F-test can be used even when the data exhibits non-normality if the data being investigated is not extremely skewed (Tiku, 1971).

The formula used to obtain the F-statistic is as follows:

$$F = \frac{s_1^2}{s_2^2},$$

This value is compared to its critical value based on $F_{n1-1,n2-2}$ degrees of freedom and at the 5% significance level. If the F-statistic falls in the rejection region, we can reject the null hypothesis that the variances of the two portfolios are equal. This allows one to determine significant outperformance, as if the portfolio outperforms and has a significantly lower variance, it should be preferred by a risk averse investor.

T-Test:

A T-test is conducted to test the equality of the means between portfolios and check for significant differences in the expected return of each, even though observations are greater than 30, and the data exhibits non-normality in situations. The decision to use a t-test irrespective of these assumptions being violated, is validated by multiple sources of literature. Rhiel and Wilkie (1996) concluded the T-test to be superior than the Z-test in most cases. It found that the T-test is robust when n>30, the T-test to be more robust than the Z-test when the data exhibits non-normality, and finally, that the T-test is robust when dealing with slightly skewed distributions, but the Z-test is not. Therefore, we employ the usage of a T-test rather than a Z-test. If extreme skewness is present in the data, Rhiel and Wilkie recommended the usage of alternative tests.

This leads us to include the non-parametric test, the Mann-Whitney test, as a measure to add robustness to the findings of this dissertation. The investor would prefer a portfolio with a significantly higher mean if the variances of the two portfolios are equal; therefore, we investigate for this significance.

The hypothesis being tested will be: H_0:$\mu_x - \mu_y = 0$,

H_1: $\mu_x - \mu_y \neq 0$.

The formula for obtaining the t-statistic is provided below:

$$t = \frac{(x - y) - (\mu_x - \mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}},$$

Where x and y are representative of the portfolio's sample means, $\mu_x - \mu_y$ is the hypothesized difference between means, and s_x^2 s_y^2 represent the sample variances of the portfolios, and n represents the sample size.

The t-statistic is compared to the t-critical values, calculated from the degrees of freedom and the significance level. If the t-statistic falls within the rejection region, we reject the null hypothesis that the means are equal.

*Mann-Whitney Test***:**

The Mann-Whitney or Wilcoxon Ranked sum test is the non-parametric equivalent of the t-test. It tests whether the median of one sample is greater than or less than the median of the other sample used, or if the medians are equal on average. This test is employed to increase the robustness of the findings of the previous t-test used. It will explain whether the two portfolios are significantly different from each other in terms of their median values. This can be used to determine if one portfolio on average would deliver higher returns. If it did outperform and had lower or equal variance, it would be a preferred portfolio by a risk-averse investor.

The hypothesis' being tested are as follows:

H_0: medians are equal,

H_1: medians are not equal.

4.4.2 Sharpe Ratio

The Sharpe ratio is used to measure the risk-adjusted returns of a portfolio; thus, a higher Sharpe ratio is preferred. The Sharpe ratio is calculated for each portfolio, using the out-of-sample time series returns. This dissertation defines the Sharpe ratio as the sample mean (μ) of the portfolio's out-of-sample returns in excess of the risk-free rate (rf), where the rf is defined as the average 10-year treasury rate transformed into an average monthly rate, w = weights of the portfolio, σ^2 is the sample variance of the portfolio's out-of-sample returns.

$$SR = \frac{(\mu * w) - rf}{\sqrt{\sigma^2}},$$

Once the Sharpe ratios are calculated, the p-values are calculated using Ledoit and Wolf's (2008) bootstrap inference method to test for any statistically significant differences between the constructed efficient portfolios and the benchmark portfolios. Lahri (2003) finds that for time series data, bootstrap inference demonstrates improved inference accuracy when compared to standard inference based on asymptotic normality. This bootstrapping method calculates the inverted boostrapped confidence interval to allow the resampling from the observed data rather than the probability distribution where the Sharpe ratios are equal. The reader is referenced to Ledoit and Wolf's 2008 paper, Section 3.2, for an in-depth explanation into the bootstrapping methodology and the mathematical equations that are used to develop the p-values. This bootstrapping inference methodology is used not only for the Sharpe ratios p-values, but for the Sortino and Skewness/Semivariance ratios p-values as well.

4.4.3 Sortino Ratio

The Sharpe ratio analysis provides in-depth explanations into portfolio performance. The Sortino ratio, much like the Sharpe ratio is a metric used to analyse the risk-adjusted returns of a portfolio against a target rate, although the risk metric used is semivariance opposed to variance. The constructed Semivariance/Semivariance and Skewness/Semivariance portfolios are all optimised while having semivariance in the denominator, therefore it makes logical sense to compare them using a Sortino ratio, where semivariance is directly considered. Furthermore, as indicated by Markowitz (1959), semivariance is a better proxy for risk, thus we consider the Sortino ratio. The Sortino ratio is calculated for each portfolio, using the out-of-sample time series returns. This dissertation defines the Sortino ratio as the sample mean (μ) of the portfolio in excess of the target rate, B, divided by the square root of the downside semivariance ($\sum B$) of the portfolio. This dissertation will use 0 as the target rate, when conducting the performance measurement for the out-of-sample Sortino ratios. If this target rate is changed, it would affect the individual levels of the Sortino ratios.

$$SOR = \frac{(\mu * w) - B}{\sqrt{\sum B}},$$

If the numerator is negative, we make use of the Israelson (2005) methodology to obtain refined Sortino ratios, in order to obtain the correct ranking.

The p-values are calculated to account for statistical differences in the Sortino ratios. This is calculated through a modified version of Ledoit and Wolf's (2008) methodology for Sharpe ratio differences. The modification is replacing the variance variables for semivariance variables. The semivariance variable is calculated through the usage of Estrada's (2008) definition, refer to equation (3). Therefore, we are using a bootstrap inference method to calculate the p-values of the Sortino ratios.

4.4.4 Skewness Semivariance Ratio

The Skewness/Semivariance ratio is calculated in the same method that Brito (2016) used, It is calculated by taking the sample skewness κ_B and dividing it by the sample semivariance, $\sum B$. The sample skewness is obtained via equation (14)

$$SSR = \frac{\kappa_B}{\sum B},$$

The same adjustment to obtain a refined Skewness/Semivariance ratio is made in order to obtain the correct portfolio rankings when in the presence of a negative skewness. This is done in accordance with the Israelsen (2005) methodology. The formula is presented below:

$$SSR = \frac{\kappa_B}{\sum B^{\kappa_B/abs(\kappa_B)}},$$

where abs() represents an absolute function i.e. making the skewness within the absolute function positive. The bootstrapped p-values are calculated, as done with the previous ratios in order to compare for significant differences between the portfolios.

Due to the Skewness/Semivariance ratio, this dissertation places more weight on the outcomes of the more traditional measures, namely the Sharpe ratio and Sortino ratio. These are used by all practitioners when disclosing their performances and is used commonly throughout literature. The inclusion of the Skewness/Semivariance portfolio is to investigate whether the Skewness/Semivariance portfolio can maximise skewness in an out-of-sample period, when compared to the benchmark and the Semivariance/Semviariance optimisation style.

Chapter 5: Results

The results of this dissertation are presented in two ways: through the use of in-sample frontiers, proving that the direct multi-search algorithm can solve the endogeneity issue in semivariance and lead to efficient frontiers. Secondly, the results are presented and analysed in an out-of-sample basis, using the techniques described in Chapter 3.2.

5.1 In-Sample

The in-sample frontiers published below and in Appendix A, are created from the usage of the overall dataset to which each corresponds to save space and eliminate the repetitive nature of the frontiers. The frontiers are created using four different target returns in the calculation, namely: the equally weighted portfolio's return, the Max Sharpe portfolio's return, the minimum variance portfolio's return, and a target return of 0. The direct multi-search algorithm was able to find frontiers that are pareto dominant, thus confirming Brito's (2016) initial finding that DMS can be applied to find the efficient Skewness/Semivariance frontiers and analyse it, as well as this dissertation's finding that the DMS algorithm can be applied to find the Semivariance/Semivariance frontiers, which thus creates a way for one to investigate the efficient trade-off concerning upside semivariance and downside semivariance.

5.1.1 Skewness/Semivariance

To eliminate the repetitive nature of the frontiers, only two from each dataset will be presented below, with the remaining in-sample frontiers being published in the appendix. The curves are of a similar nature, as only the target rate for the semivariance is being adjusted.

Figure 2: Stock Data, Skewness/Semivariance frontier, using Max Sharpe as the target rate for semivariance

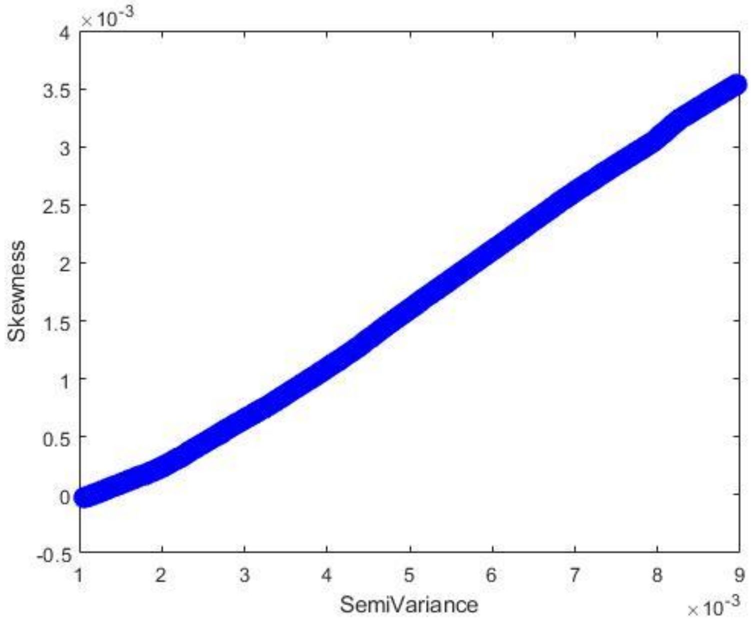


Figure 3: Stock Data, Skewness/Semivariance frontier, using Min Variance as the target rate for semivariance

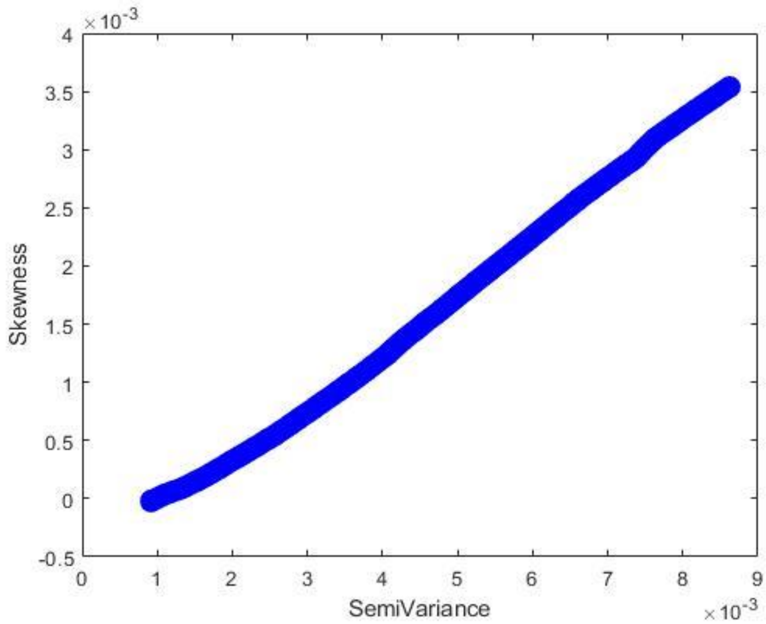


Figure 4: Asset Class Data, Skewness/Semivariance frontier, using 0 as the target rate for semivariance.

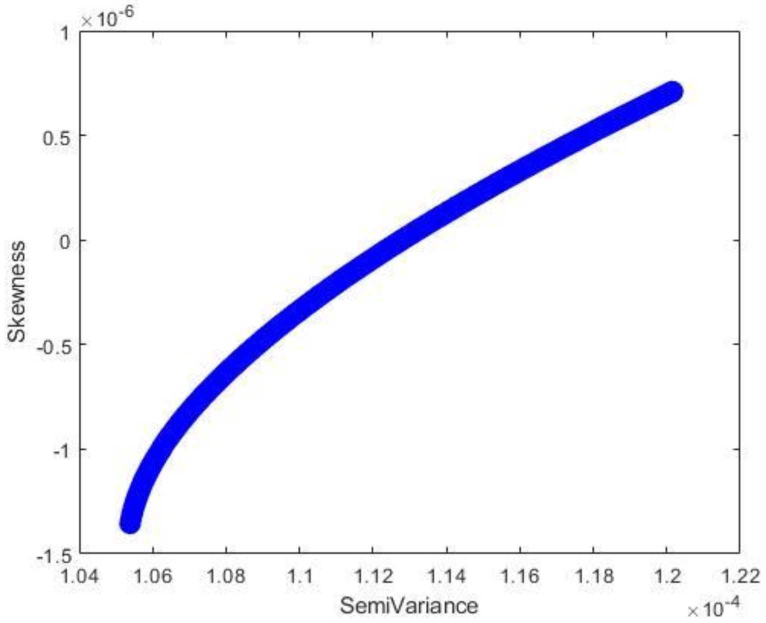
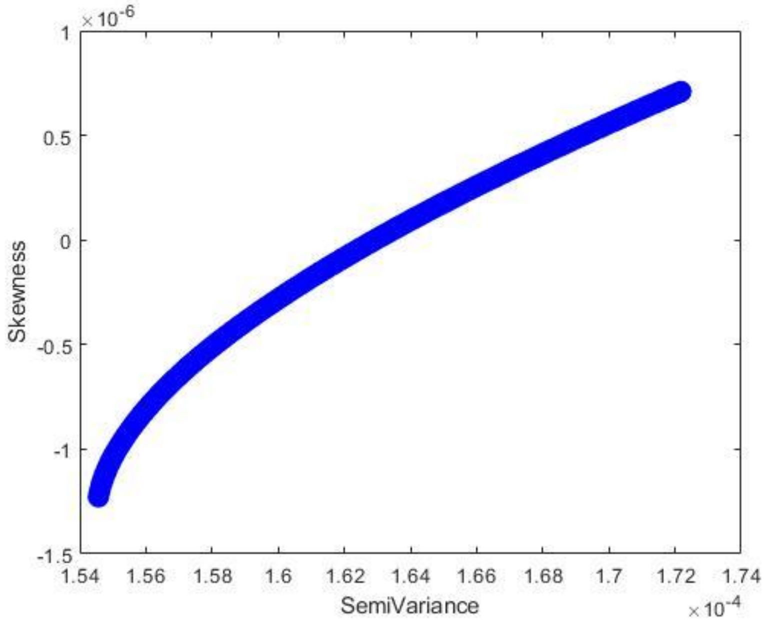


Figure 5: Asset Class Skewness/Semivariance frontier, using the equally weighted benchmark portfolio's return as its target rate for semivariance.



5.1.2 Semivariance/Semivariance

Figure 6: Stock Data Semivariance/Semivariance frontier, using Max Sharpe as its target rate for semivariance.

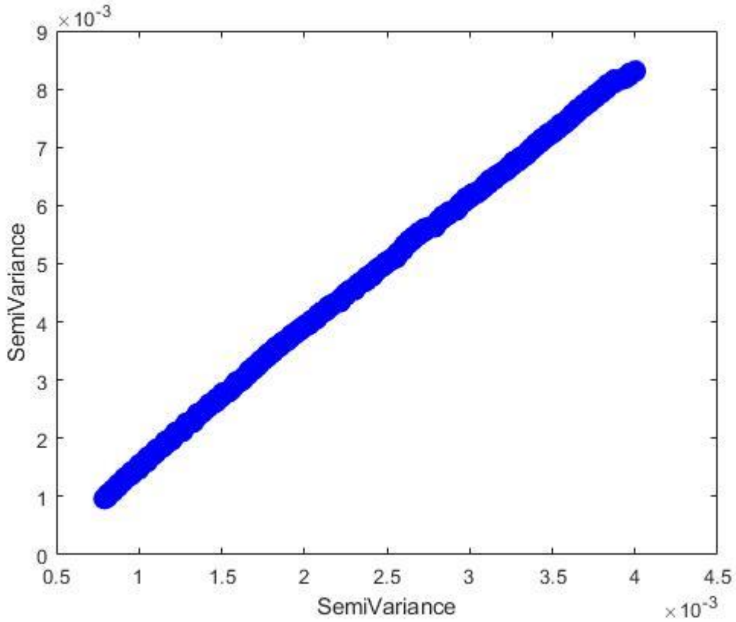


Figure 7: Stock Data Semivariance/Semivariance frontier, using Min Variance as its target rate for semivariance.

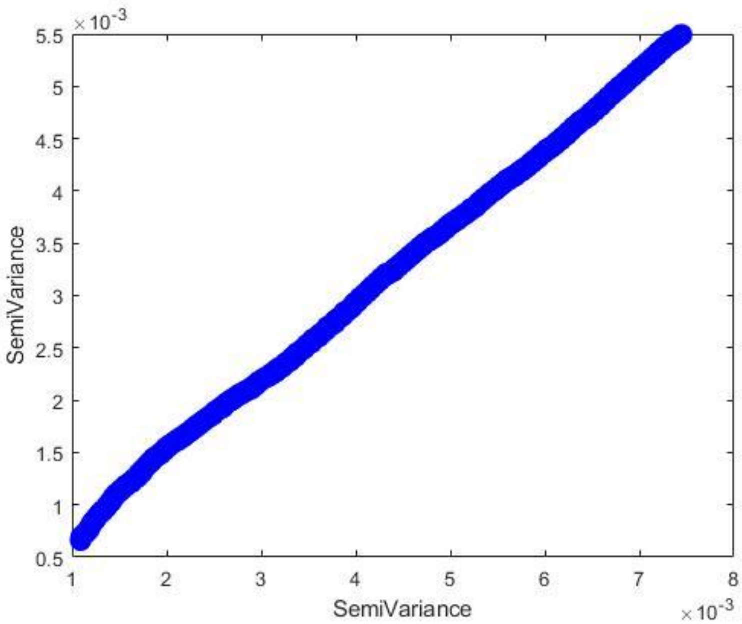


Figure 8: Asset Class Semivariance/Semivariance frontier, using the equally weighted benchmark portfolio's return as its target rate for semivariance.

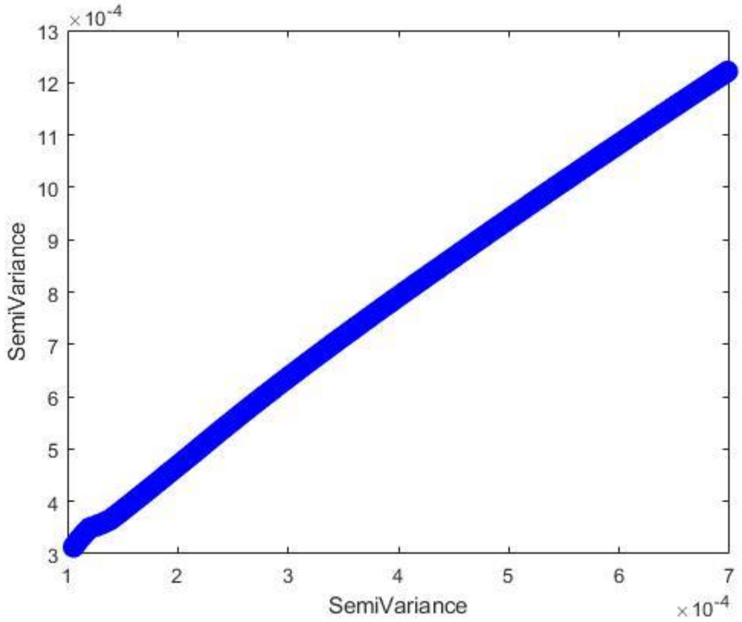
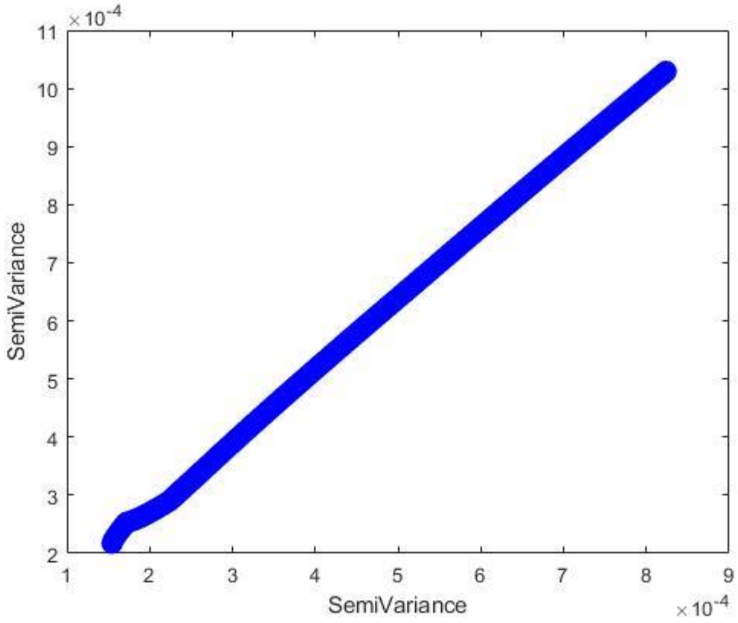


Figure 9: Asset Class Semivariance/Semivariance frontier, using 0 as its target return for semivariance.



Once frontiers are found for a respective in-sample period, this dissertation uses the portfolio selection techniques discussed in Chapter 4 to obtain the efficient portfolio weights. These optimised portfolio weights are then tested out-of-sample. This process is repeated until the entire rolling window period has been completed, resulting in 205 and 109 months of out-of-sample results to investigate for dataset 1 and 2, respectively.

5.2 Out-of-sample Analysis:

The results for the extensive out-of-sample analysis will be presented below, broken up into two sub-sections, one for each dataset. The Skewness/Semivariance and Semivariance/Semivariance portfolios will be compared to the benchmark portfolios, as well as each other, to investigate if any method might outperform the other over the periods and datasets selected. The out-of-sample analysis Chapter will follow the same layout as the performance measurement Chapter, initially starting with the descriptive statistics and statistical tests followed by the Sharpe, Sortino, and Skewness/Semivariance ratios and subsequent analysis.

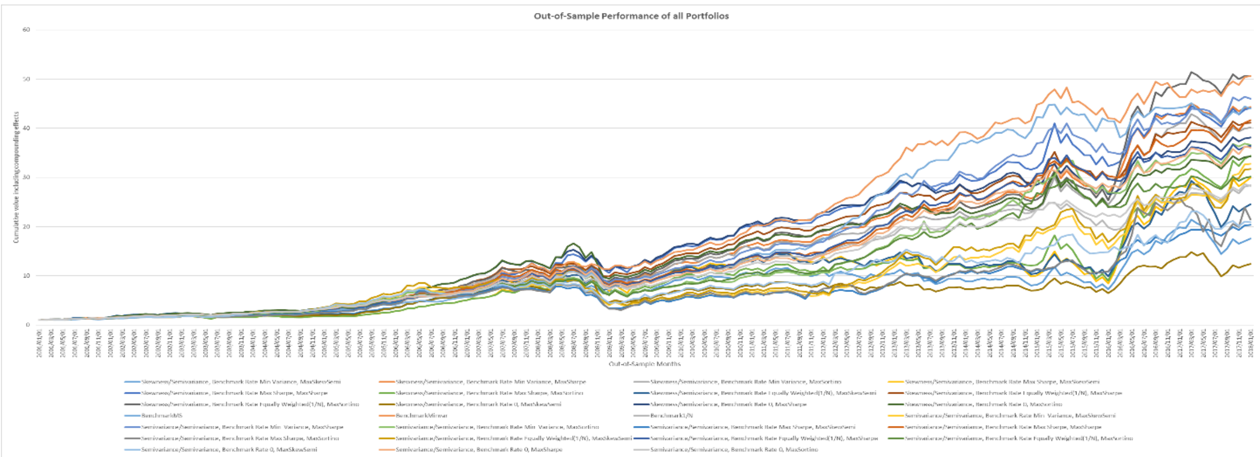
5.2.1 Dataset 1: Stocks

Descriptive statistics

All portfolios invested in are presented below in Figure 10. This includes all 27 portfolios created for the stock universe. It assumes a portfolio of value 1, at the initial date of portfolio inception: 2000/12/29 and that one reinvests all dividends and profits back into the portfolio until the final out-of-sample month, 2018/01/31. The figure reveals the Skewness/Semivariance Max Sortino portfolio using target rate of the equally weighted portfolio’s return, to have outperformed all other portfolios on a nominal basis, only edging out the benchmark minimum variance portfolio by 4%. Third and fourth best portfolios are the benchmark Maximum Sharpe and Semivariance/Semivariance Max Sharpe portfolio, using a target rate of the minimum variance portfolio. The three portfolios that returned the lowest cumulative value are all based off the Skewness/Semivariance framework, and all used the maximum Skewness/Semivariace ratio as their portfolio selection style. Six out of the bottom seven portfolios used this portfolio selection technique, representing that in terms of pure nominal return for this dataset and period, it may not be an optimal ratio to be maximising on either frontier. The range between the highest performing and the lowest performing portfolios is over four thousand percent and illustrates the importance in using the appropriate optimisation and portfolio selection techniques. Regardless of Skewness/Semivariance or Semivariance/Semivariance being used or what target rate was used, the portfolio that returned the highest nominal return was the portfolio selected via the Max Sharpe ratio technique.

One can deduce from the graph below (Figure 10) that the portfolios exhibit relatively high correlations with each other. All portfolios exhibited downturns over similar periods, albeit these periods were during market crises or recessions where the overall market performed poorly. The reader is referred to Figure 10, 2008, where one can see the bubble bursting. The correlations of the portfolios are found in Appendix B, Table 2B. It is noted from this table that combinations of portfolios from the same optimisation frontiers have higher correlations when comparing two portfolios from different optimisation frontiers; and furthermore, those selected with the same portfolio selection technique exhibited higher levels of correlation. Interestingly, the lower-end of portfolio correlations is 0.41. This is between the Skewness/Semivariance, Target Rate = Min Variance, MaxSkewSemi and the benchmark Max Sharpe portfolio. Furthermore, this Skewness/Semivariance portfolio exhibits a correlation of 0.47 when compared to the same portfolio generated via the Semivariance/Semivariance optimisation technique, which shows how each technique can result in different return profiles. This means that the portfolios, albeit moving in the same direction, will be less or more volatile than each other, given similar market conditions.

Figure 10: Stock Portfolios: How 1 initial wealth invested performs



Observing the descriptive statistics, in Appendix B, Table 1B, the minimum return in this table is representative of the biggest one period drawdown. The Skewness/Semivariance Max SkewSemi, Target Rate B=0 portfolio has the highest one period drawdown of negative thirty-two percent in one month. One notices that the benchmark Minimum Variance portfolio has the lowest variance, but of the constructed portfolios, Semivariance/ Semivariance Max Sharpe, Target Rate = Max Sharpe, and Semivariance/ Semivariance, Max Sharpe, Target Rate = Min Variance have the second and third lowest variance, respectively, giving up nominal returns to receive lower variations in your returns over time.

This dissertation conducted multiple statistical tests to investigate whether the differences in variance or expected returns were significantly different.

The first test computed was the F-test, to test the equality of variances. It's p-values are found in Appdenix B, Table 3B. This test found numerous variances to be significantly different. This is promising, as it means the portfolios selected are of a significantly different base. It is whether these portfolios have a significantly lower variance, while maintaining an expected return equal or greater than the comparable portfolio, that matters to the investor. The F-test reveals sixty-six percent of the portfolios have significantly different variances at the 5% level when compared to each other. This was expected due to the high dispersion in nominal returns of the portfolios. The benchmark Minimum Variance portfolio is significantly different from all portfolios selected via the Skewness/Semivariance frontier, and significantly different from seven of the twelve Semivariance/Semivariance frontiers. Investigating the results of the T-test and Mann-Whitney U test, Appendix B Tables 4B and 5B, respectively, result in none of the portfolios having significantly different means or medians at the 5% level, even though in the descriptive statistics there are differences in the means, they simply aren't statistically significant. Therefore, the main drivers in differences of portfolio returns are the underlying variances, semivariances and skewnesses. This leads to the investigation of the Sharpe, Sortino and Skewness/Semivariance ratios.

Sharpe ratio analysis

The table below represents the out-of-sample Sharpe ratios for the 205 months. All the Sharpe ratios are positive, resulting in positive risk adjusted returns for every portfolio. The top three portfolios, in order, are: Benchmark Minimum Variance, Semivariance/Semvariance Max Sharpe, Target rate = Minimum Variance, and Semivariance/Semivariance Max Sharpe, Target rate = Max Sharpe. These portfolios had Sharpe ratios of 0.367, 0.325, and 0.313, respectively. Interestingly, the highest nominal returning portfolio, the Skewness/Semivariance Max Sortino, Target rate = Equally Weighted, ranked eighth in terms of risk-adjusted returns. The Semivariance/Semivariance and Skewness/Semivariance portfolios selected using the Max Sharpe as its selection technique always outperform the equally weighted benchmark portfolio in terms of the Sharpe ratio. This is hard to do, as shown in DeMiguel et al. (2009). Interestingly, eight out of the twelve Semivariance/Semivariance portfolios outperform the Skewness/Semivariance portfolio selected using the same selection technique, suggesting it may be a better frontier selection technique.

Another observation from Table 1 below is that the portfolio selection technique is instrumental in the outcome, as in each instance for both Semivariance/Semivariance and Skewness/Semivariance the portfolio chosen using Max Sharpe outperforms the other techniques, namely Max Skewness/Semivariance ratio and Max Sortino ratio. This may be limited to our dataset. Furthermore, the Max Sortino ratio outperforms the Max Skewness/Semivariance ratio for each target rate and frontier selection style, except for Semivariance/Semivariance Max Sortino, Target rate = Max Sharpe. The target rate used does appear to affect the ratio, albeit to a lessor degree than the portfolio selection technique. This is seen through the negligible differences between the Sharpe ratios when comparing the same portfolio technique across different target rates.

The p-values, represented in Appendix B, Table 6B, reveal that thirty-one percent of portfolios are significantly different from each other. Therefore, one hundred and nine combinations of portfolios out of three hundred and fifty are statistically significant at the 5% level. The Minimum Variance portfolio is significantly different from seventeen of the twenty-six portfolios. This reiterates its outperformance to a significant level.

However, it is not significantly different from the second highest Sharpe ratio portfolio, the Semivariance/Semvariance Max Sharpe, Target rate = Minimum Variance portfolio. This portfolio is significantly different from fifteen of the twenty-six comparable portfolios, this is slightly less than the minimum variance portfolio but still boasts great outperformance. Albeit it is not significantly different from the any of the benchmark portfolios, it is significantly different compared to other portfolios selected from the same frontier using different target rates and techniques, as well as being significantly different from most Skewness/Semivariance portfolios. Therefore, one may make the argument that Semivariance/Semivariance framework is a better optimisation framework when compared to the Skewness/Semivariance.

Table 1: Stock Portfolios Sharpe Ratios

Stock Portfolios Sharpe Ratios			
Target Rate, B = Min Variance Return		**Target Rate, B = Min Variance Return**	
Semivariance/Semivariance Max SkewSemi	0.182	Skewness/Semivariance Max SkewSemi	0.122
Semivariance/Semivariance Max Sharpe	0.325	Skewness/Semivariance Max Sharpe	0.288
Semivariance/Semivariance Max Sortino	0.238	Skewness/Semivariance Max Sortino	0.244
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	0.158	Skewness/Semivariance Max SkewSemi	0.147
Semivariance/Semivariance Max Sharpe	0.294	Skewness/Semivariance Max Sharpe	0.289
Semivariance/Semivariance Max Sortino	0.261	Skewness/Semivariance Max Sortino	0.158
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	0.179	Skewness/Semivariance Max SkewSemi	0.142
Semivariance/Semivariance Max Sharpe	0.291	Skewness/Semivariance Max Sharpe	0.297
Semivariance/Semivariance Max Sortino	0.243	Skewness/Semivariance Max Sortino	0.285
Target Rate, B = Max Sharpe		**Target Rate, B = Max Sharpe**	
Semivariance/Semivariance Max SkewSemi	0.154	Skewness/Semivariance Max SkewSemi	0.097
Semivariance/Semivariance Max Sharpe	0.313	Skewness/Semivariance Max Sharpe	0.274
Semivariance/Semivariance Max Sortino	0.142	Skewness/Semivariance Max Sortino	0.238
Benchmark Min Variance	0.367	**Benchmark Max Sharpe**	0.305
Benchmark Equally Weighted (1/N)	0.262		

Sortino ratio analysis

The Sortino ratios are found in Table 2 below. The table reiterates the observations made by the Sharpe ratio analysis: the benchmark Minimum Variance portfolio has the highest Sortino ratio, followed by the Semivariance/Semivariance Max Sharpe, Target Rate B = Min Variance. The first difference is Skewness/Semivariance Max Sharpe, Target Rate B = Equally Weighted(1/N) return is now ranked third. The three aforementioned portfolios have the respective Sortino ratios of 1.281, 1.034, and 0.991. The Semivariance/Semivariance Max Sharpe, Target Rate B = Max Sharpe is now ranked fourth versus the Sharpe ratios where it was ranked third. Two of the Semivariance/Semivariance portfolios outperformed the benchmark Max Sharpe portfolio, and four of the Semivariance/Semivariance portfolios outperformed the Equally weighted benchmark portfolio. The Skewness/Semivariance portfolios had two that outperformed the benchmark Max Sharpe portfolio, and five that outperformed the Equally weighted benchmark portfolio. Comparing the portfolios selected from the same technique of the two frontier selection styles, seven of the twelve Semivariance/Semivariance portfolios outperformed the equivalent Skewness/Semivariance portfolios. An observation from the Sortino ratios is that the Max Sharpe portfolio selection technique always results in a higher Sortino ratio, regardless of frontier style and target rate used. It also results in the portfolios that outperform the benchmarks.

Investigating the p-values for differences in Sortino ratios, Appendix B Table 7B, sixty-one percent of the portfolios are significantly different from each other with regards to the Sortino ratios at the 5% level. This could be due to the Sortino ratio taking semivariance into account, whereas the Sharpe ratio does not. This means that majority of the portfolios are unique when compared to each other, and that the different target rate, portfolio selection technique, and frontier style all matter. Notably, the Minimum Variance benchmark portfolio is significantly different to all other twenty-six portfolios.

Eleven of the twenty-four constructed portfolios are significantly different to the equally weighted benchmark portfolio. The only portfolio that is significant for outperformance, however, is the Semivariance/ Semivariance, Target Rate Min Variance, MaxSharpe. The remainder are the portfolios that underperformed the benchmark by a statistically significant amount. The Skewness/ Semivariance, Target Rate Equally Weighted (1/N), MaxSharpe portfolio was extremely close to being significant at the 5% level with a p-value of 0.057, after adjusting for standard error in the p-value, it is significant. The portfolios that outperformed by smaller margins such as the Skewness/ Semivariance, Target Rate Min Variance, MaxSharpe, did so on an insignificant basis, as the difference was only 0.063, resulting in a p-value of 0.4. These findings are statistically inconclusive in ascertaining one dominant target rate and one optimal frontier selection technique. Once again, the use of the Max Sharpe as the selection technique always results in the highest ratios. While the Max Skewness/Semivariance ratio portfolios seem to produce the lowest ratios again, underperforming on a risk-adjusted basis.

Table 2: Stock Portfolios Sortino Ratios

Stock Portfolios Sortino Ratios			
Target Rate, B = Min Variance Return		**Target Rate, B = Min Variance Return**	
Semivariance/Semivariance Max SkewSemi	0.5019	Skewness/Semivariance Max SkewSemi	0.3864
Semivariance/Semivariance Max Sharpe	1.0345	Skewness/Semivariance Max Sharpe	0.9155
Semivariance/Semivariance Max Sortino	0.7014	Skewness/Semivariance Max Sortino	0.7890
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	0.4742	Skewness/Semivariance Max SkewSemi	0.2884
Semivariance/Semivariance Max Sharpe	0.8843	Skewness/Semivariance Max Sharpe	0.9003
Semivariance/Semivariance Max Sortino	0.8146	Skewness/Semivariance Max Sortino	0.6822
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	0.4921	Skewness/Semivariance Max SkewSemi	0.4361
Semivariance/Semivariance Max Sharpe	0.8874	Skewness/Semivariance Max Sharpe	0.9910
Semivariance/Semivariance Max Sortino	0.7136	Skewness/Semivariance Max Sortino	0.9230
Target Rate, B = Max Sharpe		**Target Rate, B = Max Sharpe**	
Semivariance/Semivariance Max SkewSemi	0.4431	Skewness/Semivariance Max SkewSemi	0.4335
Semivariance/Semivariance Max Sharpe	0.9835	Skewness/Semivariance Max Sharpe	0.9420
Semivariance/Semivariance Max Sortino	0.3959	Skewness/Semivariance Max Sortino	0.4594
Benchmark Min Variance	1.2815	Benchmark Max Sharpe	0.9380
Benchmark Equally Weighted (1/N)	0.8522		

Skewness/Semivariance ratio analysis

The out-of-sample skewness is defined as the same as the optimisation process [see equation (14)]. The Skewness/Semivariance ratios found below illustrate a different picture to the above Sharpe and Sortino ratios. It highlights that the Skewness/Semivariance optimisation has the highest six portfolios in terms of Skewness/Semivariance. This makes logical sense, as it is the only optimisation process that is set up to find and maximise positive skewness. This is something that investors are deemed to crave, according to multiple sources, see Kraus and Liztenberger (1976), Harvey (2010), and Arrow (1971). Therefore, in terms of this formula, ten of the twelve Skewness/Semivariance optimised portfolios outperform all the benchmark portfolios. Two of the Semivariance/Semivariance portfolios outperform all benchmark portfolios with an additional three that outperform the equally weighted portfolio only. Two of the bottom three portfolios using this ratio are using the Skewness/Semivariance optimisation framework and the Target Rate, B = 0.

Therefore, we can note the robustness of the direct multi-search algorithm in solving the presented optimisation problems, from the Skewness/Semivariance ratios.

Furthermore, if we investigate Appendix B, Table 8B the Skewness/Semivariance p-values, we find that none of the ratios exhibit statistical significance at the 5% level. Therefore, the outperformance of the Skewness/Semivariance optimisation style is noted due to its nominal magnitude but is given less importance when compared to the Sharpe or Sortino ratios that exhibit statistical significance. This implies that all of the portfolios compared have a similar Skewness/Semivariance ratio, and this makes logical sense as the highest high-low range is: 0.2.

Table 3: Stock Skewness/Semivariance ratios

Stock Portfolios Skewness/Semivariance Ratios			
Target Rate, B = Min Variance Return		**Target Rate, B = Min Variance Return**	
Semivariance/Semivariance Max SkewSemi	8.27E-03	Skewness/Semivariance Max SkewSe	1.03E-01
Semivariance/Semivariance Max Sharpe	-4.31E-09	Skewness/Semivariance Max Sharpe	1.59E-02
Semivariance/Semivariance Max Sortino	-9.43E-09	Skewness/Semivariance Max Sortino	1.10E-01
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	2.29E-02	Skewness/Semivariance Max SkewSe	-4.50E-07
Semivariance/Semivariance Max Sharpe	-1.47E-08	Skewness/Semivariance Max Sharpe	2.95E-02
Semivariance/Semivariance Max Sortino	-1.35E-08	Skewness/Semivariance Max Sortino	-3.63E-08
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	-2.22E-08	Skewness/Semivariance Max SkewSe	1.40E-01
Semivariance/Semivariance Max Sharpe	-1.23E-08	Skewness/Semivariance Max Sharpe	2.13E-02
Semivariance/Semivariance Max Sortino	-1.97E-08	Skewness/Semivariance Max Sortino	5.13E-02
Target Rate, B = Max Sharpe		**Target Rate, B = Max Sharpe**	
Semivariance/Semivariance Max SkewSemi	-4.89E-08	Skewness/Semivariance Max SkewSe	2.05E-01
Semivariance/Semivariance Max Sharpe	-6.13E-09	Skewness/Semivariance Max Sharpe	2.68E-02
Semivariance/Semivariance Max Sortino	3.88E-02	Skewness/Semivariance Max Sortino	1.95E-01
Benchmark Min Variance	-2.37E-10	Benchmark Max Sharpe	-3.47E-09
Benchmark Equally Weighted (1/N)	-6.98E-09		

5.2.2 Dataset 2: Asset Classes

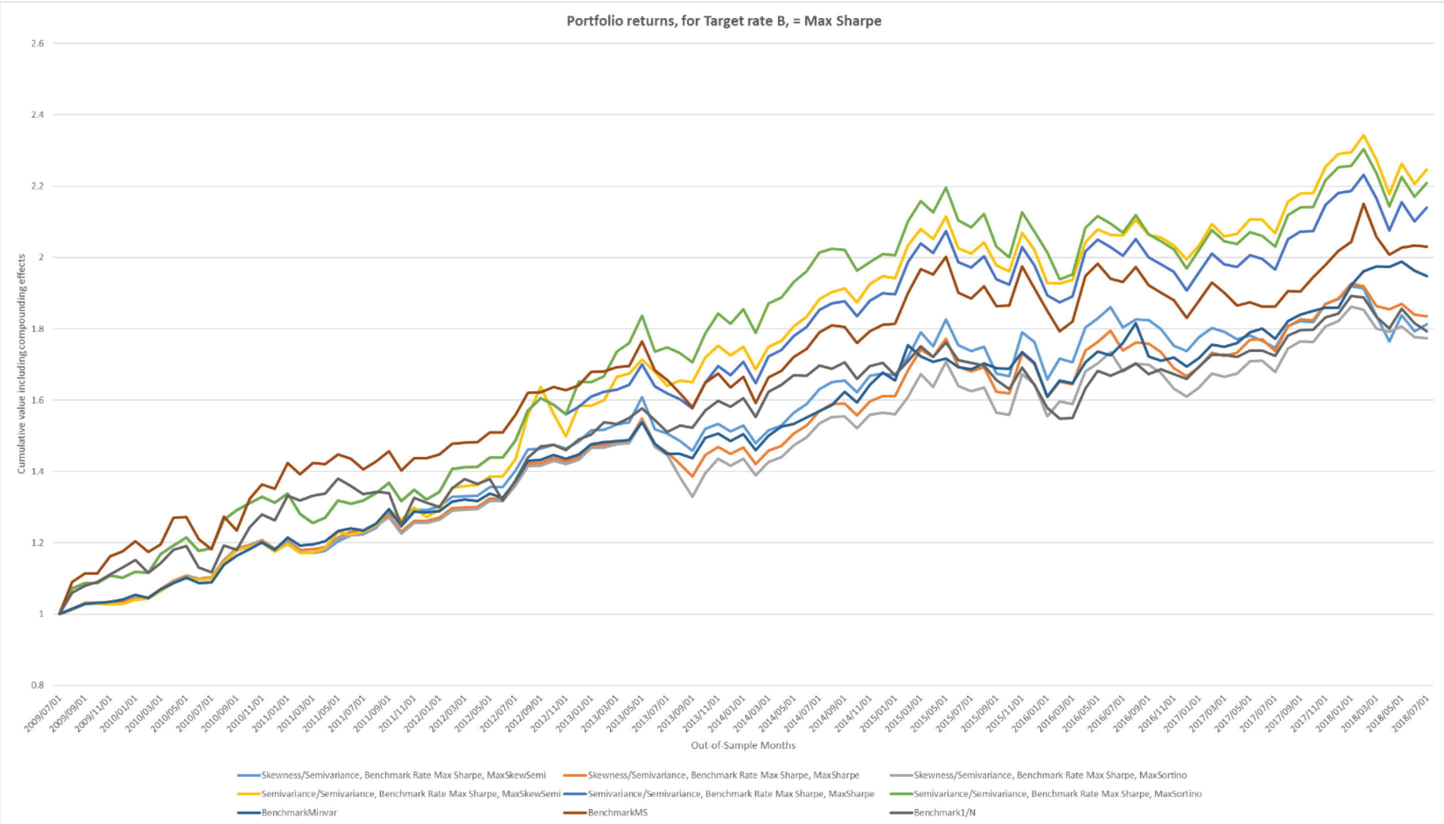
The inclusion of this second dataset, as stated earlier, is to test the robustness of the findings in dataset 1, and to test the ability of the different optimisation techniques when applied to Asset classes versus. Stock selection.

Descriptive statistics

The best constructed portfolios and the benchmark portfolios invested in over the period given each optimisation style will be presented, showing how each would have fared if you invested 1 initial wealth into them. To eliminate redundancy, only one target return graph is provided below, showing nine portfolios. This is due to the different target rates performing closely in line with each other. This is shown in Appendix C, Table 2C. Their return distributions are extremely highly correlated with each other, with the average correlation being around 0.8 for most portfolios. This means that they move extremely closely with each other in terms of returns, which could be due to only five asset classes being investigated.

Figure 11 reveals that there is a dispersion between strategies and optimisation styles over the period. The Semivariance/Semivariance portfolio using the Max Sharpe return as its target return and Max Skewness/Semivariance ratio as its portfolio selection style outperforms the Skewness/Semivariance portfolio that is created using the same target return and portfolio selection style, returning 125.63% versus 79.8% over the same out-of-sample period. This is economically significant to the investor, as it is a nominal outperformance of 45.83% over the one hundred and nine out-of-sample months examined. The same Semivariance/Semivariance portfolio outshines all benchmark portfolios in terms of total return for the given period.

Figure 11: Asset Class: How 1 initial wealth invested performs, using a Target return, B = Max Sharpe portfolio return.



The table of the descriptive statistics is provided in Appendix C: Table 1C, these statistics are used to conduct the statistical tests mentioned in Chapter 4. The F-Test finds that the majority of the portfolios have significantly different variances, shown in Table 3C (the highlighted red cells represent statistical significance). This leads us to use the two-sample T-test for different variances and the Mann-Whitney U-test. The Mann-Whitney U-test rejected the null hypothesis when comparing every single portfolio. The p-values from the Mann-Whitney U-test are found in Appendix C: Table 5C. This implies that the medians of each portfolio's return distribution are not significantly different from each other. The two sample T-test confirms this finding, as none of the means compared exhibit statistical significance, illustrated in Appendix C: Table 4C. This means the differences in the portfolio returns is generated from the differences in their variances, semivariances, and skewness'. Therefore, the investigation of the Sharpe, Sortino and Skewness/Semivariance ratios is necessary to determine if any portfolio truly outperforms on a statistically significant basis.

Sharpe ratio analysis

The table below presents the out-of-sample Sharpe ratios, for the 109 month out-of-sample periods. The Sharpe ratios are poor, which could be due to the period examined. This is illustrated via the negative Sharpe ratios. The equally weighted benchmark portfolio exhibits a negative Sharpe ratio. The Skewness/Semivariance portfolios all exhibit negative Sharpe ratios, whereas seven out of the twelve variations of the Skewness/Semivariance portfolio have a less negative Sharpe ratio compared to the equally weighted Benchmark portfolio's negative Sharpe of -0.0506. The Semivariance/Semivariance portfolio optimisation technique presented in this dissertation results in multiple portfolios that outperform all the benchmarks on a Sharpe ratio basis. Nine out of the twelve Semivariance/Semvariance portfolios selected using the selection techniques described in Chapter 4 outperform all the benchmark portfolios and Skewness/Semivariance portfolios, while two of the remaining three outperform both the minimum variance and equally weighted benchmark portfolios on a Sharpe ratio basis. The last Semivariance/Semivariance portfolio selected via Max Sharpe, with a Target rate B = 0, underperforms all of the benchmark portfolios, as well as all of the Skewness/Semivariance portfolios on a Sharpe ratio basis.

Examining the Skewness/Semivariance portfolios leads to the conclusion that for Dataset 2, this frontier selection method underperforms the Semivariance/Semivariance portfolio selection method on a Sharpe ratio basis, but seven out of the twelve portfolios outperform the equally weighted benchmark portfolio. One of the twelve outperforms the minimum variance portfolio slightly, the Skewness/Semivariance portfolio selected using the Max Sortino ratio, and a target return of the equally weighted portfolio. Zero of the Skewness/Semivariance portfolios outperformed the Max Sharpe benchmark portfolio in this dataset. Unlike dataset 1, there is no portfolio selection technique that constantly outperforms the others.

Investigating the bootstrapped p-values, refer to Appendix C: Table 8C, results in none of the portfolios' Sharpe ratios being significantly different compared to the benchmark portfolios, whereas multiple Semivariance/Semivariance portfolios are significantly different compared to each other using different target rates and significantly different to multiple of the Skewness/Semivariance portfolios selected. This finding highlights the importance in the selection of the target rate once again, as it can alter the trajectory of the portfolio immensely. Furthermore, it highlights that Semivariance/Semivariance portfolio optimisation may be superior to Skewness/Semivariance, as the portfolios outperform in terms of Sharpe ratios, with some bearing statistical significance. Twenty-two out of three hundred and fifty combinations of the portfolios are statistically different at the 5% level after considering standard error, while at the 10% level, this number increases to around sixty-eight out of three hundred and fifty, yet still no portfolios are significantly different from the benchmark portfolios.

Table 4: Asset Class Portfolio Sharpe Ratios.

Asset Class Portfolios Sharpe Ratios			
Target Rate, B = Min Variance		**Target Rate, B = Min Variance**	
Semivariance/Semivariance Max SkewSemi	0.0271	Skewness/Semivariance Max SkewSemi	-0.05265
Semivariance/Semivariance Max Sharpe	-0.0144	Skewness/Semivariance Max Sharpe	-0.03852
Semivariance/Semivariance Max Sortino	0.0161	Skewness/Semivariance Max Sortino	-0.04813
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	0.0074	Skewness/Semivariance Max SkewSemi	-0.0250
Semivariance/Semivariance Max Sharpe	-0.0603	Skewness/Semivariance Max Sharpe	-0.0474
Semivariance/Semivariance Max Sortino	-0.0201	Skewness/Semivariance Max Sortino	-0.05762
Target Rate, B = Max Sharpe Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	0.0338	Skewness/Semivariance Max SkewSemi	-0.05696
Semivariance/Semivariance Max Sharpe	0.0158	Skewness/Semivariance Max Sharpe	-0.04778
Semivariance/Semivariance Max Sortino	0.0274	Skewness/Semivariance Max Sortino	-0.01277
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Max Sharpe Return**	
Semivariance/Semivariance Max SkewSemi	0.0162	Skewness/Semivariance Max SkewSemi	-0.05692
Semivariance/Semivariance Max Sharpe	0.0235	Skewness/Semivariance Max Sharpe	-0.04344
Semivariance/Semivariance Max Sortino	0.0158	Skewness/Semivariance Max Sortino	-0.05449
Benchmark Min Variance	-0.026	**Benchmark Equally Weighted (1/N)**	-0.0506
Benchmark Max Sharpe	0.006		

Sortino ratio analysis

The Sortino ratios represent a brighter picture. When compared to the Sharpe ratios, none of the ratios are negative. The highest performing portfolio based on the Sortino ratio is the Semivariance/Semivariance Max Sortino, Target rate B = Equally Weighted(1/N). This portfolio outperforms all of the benchmark and other constructed portfolios. The second highest ratio is the Skewness/Semivariance Max Sortino, Target rate B = 0. Eight out of the twelve Semivariance/Semivariance portfolios outperform the equivalent Skewness/Semivariance portfolios. This suggests it may be the better frontier style. Four of the Semivariance/Semivariance portfolios outperform all benchmark portfolios, with an additional five outperforming the Equally Weighted and Max Sharpe benchmark portfolios. The Skewness/Semivariance portfolios, on the other hand, only has two portfolios that outperform all benchmarks, albeit slightly. All twelve Skewness/Semivariance portfolios outperform the Equally Weighted benchmark portfolio. This signifies that both frontier selection processes can outperform the benchmarks, with the edge tipping toward Semivariance/Semivariance portfolios in terms of magnitude.

Investigating the Sortino p-values in Appendix C: Table 7C, fifty-five of the three hundred and fifty portfolio combinations are significantly different from each other. In terms of outperformance, none of the constructed portfolios significantly outperform the benchmark portfolios at the 5% level. The Semivariance/Semivariance portfolios constructed using the target rate of the equally weighted portfolio, and portfolio selection techniques of Max Sortino and Max Skewness/Semivariance, both exhibit statistically significant outperformance when compared to ten of the twelve Skewness/Semivariance portfolios. This reiterates that the Semivariance/Semivariance optimisation style may be superior to the Skewness/Semivariance optimisation style.

Table 5: Asset Class Portfolio Sortino Ratios.

Asset Class Portfolios Sortino Ratios			
Target Rate, B = Min Variance		**Target Rate, B = Min Variance**	
Semivariance/Semivariance Max SkewSemi	0.5392	Skewness/Semivariance Max SkewSemi	0.4573
Semivariance/Semivariance Max Sharpe	0.4359	Skewness/Semivariance Max Sharpe	0.4569
Semivariance/Semivariance Max Sortino	0.5441	Skewness/Semivariance Max Sortino	0.4492
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	0.5185	Skewness/Semivariance Max SkewSemi	0.4135
Semivariance/Semivariance Max Sharpe	0.3392	Skewness/Semivariance Max Sharpe	0.4528
Semivariance/Semivariance Max Sortino	0.6116	Skewness/Semivariance Max Sortino	0.6215
Target Rate, B = Max Sharpe Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	0.5824	Skewness/Semivariance Max SkewSemi	0.4189
Semivariance/Semivariance Max Sharpe	0.5307	Skewness/Semivariance Max Sharpe	0.4545
Semivariance/Semivariance Max Sortino	0.5187	Skewness/Semivariance Max Sortino	0.5785
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Max Sharpe Return**	
Semivariance/Semivariance Max SkewSemi	0.6040	Skewness/Semivariance Max SkewSemi	0.4105
Semivariance/Semivariance Max Sharpe	0.5347	Skewness/Semivariance Max Sharpe	0.4653
Semivariance/Semivariance Max Sortino	0.6406	Skewness/Semivariance Max Sortino	0.4140
Benchmark Min Variance	0.5740	**Benchmark Equally Weighted (1/N)**	0.4004
Benchmark Max Sharpe	0.4637		

Skewness/Semivariance ratio analysis

Investigating the Skewness/Semivariance ratios for the asset classes results in an interesting outcome, where most of the out-of-sample Semivariance/Semvariance portfolios have positive skewness, while the out-of-sample Skewness/Semivariance portfolios have negative skewness. This could be due to the semivariance being a proxy variable for skewness, but the difference is also extremely negligible, with the outcomes of both portfolios being extremely close to zero, with the Skewness/Semivariance portfolios being approximately equal to zero. This negligible difference is confirmed through the investigation of the p-values provided in Appendix C Table 8C, where none of the p-values exhibit statistical significance. Therefore, in terms of a Skewness/Semivariance ratio, the argument could be made that all of the portfolios performed in line with each other.

Therefore, in proving whether one portfolio optimisation technique is superior, an argument could be made for Semivariance/Semivariance on a nominal basis, but the margin is too small to have any true significant finding.

Table 6: Asset Class Skewness/Semivariance ratios

Asset Class Portfolios Skewness/Semivariance Ratios			
Target Rate, B = Min Variance		**Target Rate, B = Min Variance**	
Semivariance/Semivariance Max SkewSemi	2.89E-02	Skewness/Semivariance Max SkewSemi	-2.40E-10
Semivariance/Semivariance Max Sharpe	7.52E-03	Skewness/Semivariance Max Sharpe	-2.91E-10
Semivariance/Semivariance Max Sortino	-1.69E-11	Skewness/Semivariance Max Sortino	-3.79E-10
Target Rate, B = 0		**Target Rate, B = 0**	
Semivariance/Semivariance Max SkewSemi	-4.82E-10	Skewness/Semivariance Max SkewSemi	-3.74E-10
Semivariance/Semivariance Max Sharpe	5.12E-03	Skewness/Semivariance Max Sharpe	-3.32E-10
Semivariance/Semivariance Max Sortino	-1.21E-10	Skewness/Semivariance Max Sortino	-3.73E-10
Target Rate, B = Max Sharpe Return		**Target Rate, B = Equally Weighted(1/N) Return**	
Semivariance/Semivariance Max SkewSemi	8.28E-03	Skewness/Semivariance Max SkewSemi	-1.98E-10
Semivariance/Semivariance Max Sharpe	1.15E-02	Skewness/Semivariance Max Sharpe	-3.54E-10
Semivariance/Semivariance Max Sortino	7.83E-03	Skewness/Semivariance Max Sortino	-4.21E-10
Target Rate, B = Equally Weighted(1/N) Return		**Target Rate, B = Max Sharpe Return**	
Semivariance/Semivariance Max SkewSemi	-1.56E-10	Skewness/Semivariance Max SkewSemi	-2.92E-10
Semivariance/Semivariance Max Sharpe	2.66E-02	Skewness/Semivariance Max Sharpe	-3.56E-10
Semivariance/Semivariance Max Sortino	-8.85E-11	Skewness/Semivariance Max Sortino	-3.54E-10
Benchmark Min Variance	-3.38E-10	**Benchmark Equally Weighted (1/N)**	2.50E-03
Benchmark Max Sharpe	3.18E-02		

Chapter 6: Conclusion and future work

This dissertation tested the robustness of the Skewness/Semivariance framework put forward by Brito et al. (2016), via the usage of this methodology in a developing market such as South Africa and over an extremely long sample period. Furthermore, this paper proposed an upper and lower partial moment optimisation framework, derived from Cumova and Nawrocki’s (2014) general framework. This dissertation found that it is possible to create such a framework via maximising upside semivariance and minimising downside semivariance in a 2-dimensional plane, through the use of the direct multi-search algorithm. Therefore, showing the robustness of this algorithm in overcoming the endogeneity issue in the co-semivariance matrix. The extensive out-of-sample analysis resulted in findings that are inconclusive for the time periods and datasets used, as the Sharpe ratios and Sortino ratios attained from both Semivariance/Semivariance and Skewness/Semvariance optimisations outperformed the benchmarks on numerous occasions on a nominal basis, but this outperformance was rarely statistically significant at the 5% level using bootstrapped p-values. The Semivariance/Semvariance framework has been shown to outperform the Skewness/Semivariance optimisation in most aspects. Therefore, for the datasets and time period used, it may be deemed a superior optimisation technique. This dissertation also found that in most of the cases investigated, the Max Sharpe ratio portfolio selection technique outperforms its corresponding Max Sortino and Max Skewness/Semivariance portfolio selection techniques. This dissertation had some limitations, which may be addressed by future work, through the expansion of the dataset to include all the companies on the ALSI, while adjusting for thin trading bias. A study conducted into optimising the target return for semivariance could be conducted to find the most optimal return to be used in optimising the semivariance/semivarince frontier. The sample used was also investigated on a rolling-sample window, which could be conducted on set sample periods without rebalancing. Furthermore, this dissertation ignored the impact of transaction costs, and this could be considered to more accurately compare optimisation techniques.

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Appendix A: Datasets and In-Sample graphs:

All tickers in the table below, are representative of Bloomberg tickers.

Table 1A: List of dataset securities

Dataset 1: Ticker:	Name:	Dataset2: Ticker	Name:
CFR SJ Equity	Cie Financiere Richemont	JALSH Index	FTSE/JSE Africa All Share Index
FSR SJ Equity	Firstrand	SAPY Index	FTSE/JSE South Africa Listed Property Index
SBK SJ Equity	Standard Bank	BCOM Index	Bloomberg Commodity Index
SOL SJ Equity	Sasol	MXWD Index	MSCI All Country World Index
BGA SJ Equity	ABSA Bank Group	ALBI Index	JSE All Bond Index
SHP SJ Equity	Shoprite		
NED SJ Equity	Nedbank Group		
INL SJ Equity	Investec Limited		
AMS SJ Equity	Anglo American Platinum		
TBS SJ Equity	Tiger Brands		
BVT SJ Equity	Bidvest		
IPL SJ Equity	Imperial Logistics		
TFG SJ Equity	The Foschini Group		
SAP SJ Equity	Sappi		
AVI SJ Equity	AVI Ltd		
BAW SJ Equity	Barloworld Ltd		
PIK SJ Equity	Pick 'n Pay		
SNT SJ Equity	Santam Ltd		
DST SJ Equity	Distell Group		
NHM SJ Equity	Northam Platinum		
ITE SJ Equity	Italtile		
RCL SJ Equity	Rhodes Food Ltd		
TON SJ Equity	Tongaat		
RLO SJ Equity	Reunert Ltd		
AFE SJ Equity	AECI Ltd		
OCE SJ Equity	Oceana Group		
NPK SJ Equity	Nampak Ltd		
HAR SJ Equity	Harmony Gold Mining		
GND SJ Equity	Grindrod Ltd		
OMN SJ Equity	Omnia Holdings Ltd		
AFX SJ Equity	African Oxygen Ltd		
TRE SJ Equity	Trencor Ltd		
SUI SJ Equity	Sun International Ltd		
HDC SJ Equity	Hudaco Industries Ltd		
AEL SJ Equity	Allied Electronics Ltd		
MUR SJ Equity	Murray & Roberts Holdings		
CAT SJ Equity	Caxton and CTP Publishers		
MTA SJ Equity	Metair Investments		
LON SJ Equity	Lonmin		
ACT SJ Equity	Afrocentric Investment Corp		
APN SJ Equity	Aspen Pharmacare Holdings		
GRT SJ Equity	Growthpoint Properties Ltd		
MRP SJ Equity	Mr Price Group Ltd		
ASR SJ Equity	Assore Ltd		
GFI SJ Equity	Goldfields Ltd		
ARI SJ Equity	African Rainbow Minerals		
IMP SJ Equity	Impala Platinum Holdings		
CSB SJ Equity	Cashbuild Ltd		
IVT SJ Equity	Invicta Holdings Ltd		
OCT SJ Equity	Octodec Investments Ltd		

In-sample Frontiers: Skewness/Semivariance

Figure 1A: Asset Class Skewness/Semivariance frontier, using Max Sharpe as its target return for semivariance.

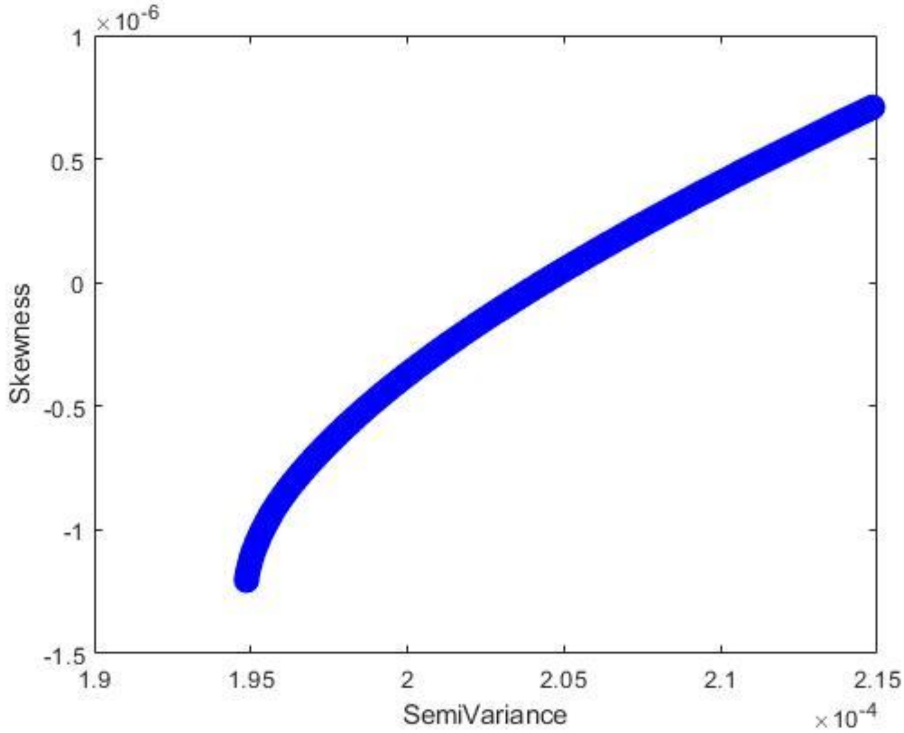


Figure 2A: Asset Class Skewness/Semivariance frontier, using Min Variance as its target return for semivariance.

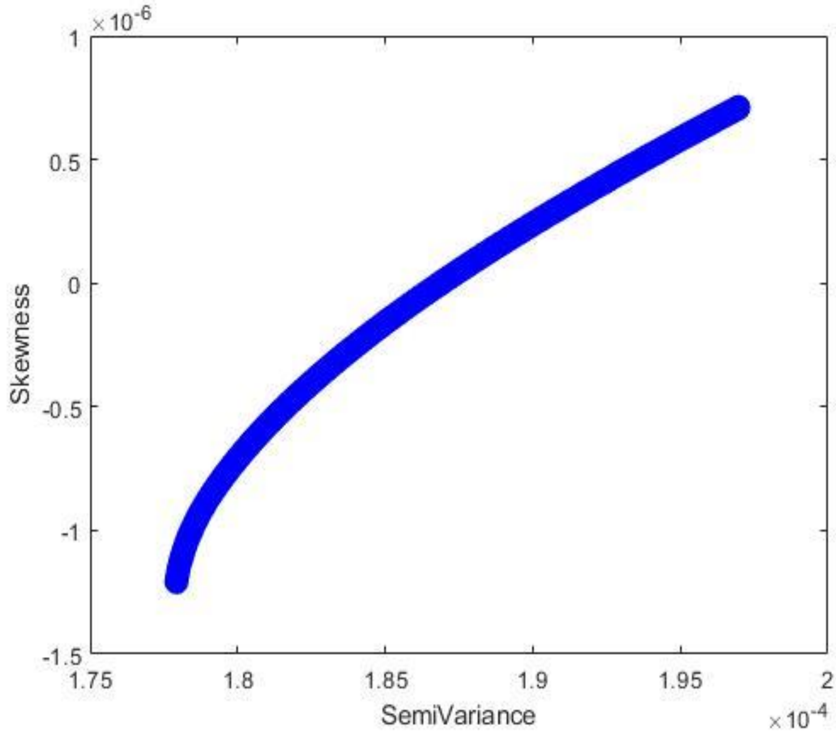


Figure 3A: Stock Data Skewness/Semivariance frontier, using Equally weighted benchmark portfolio as its target return for semivariance.

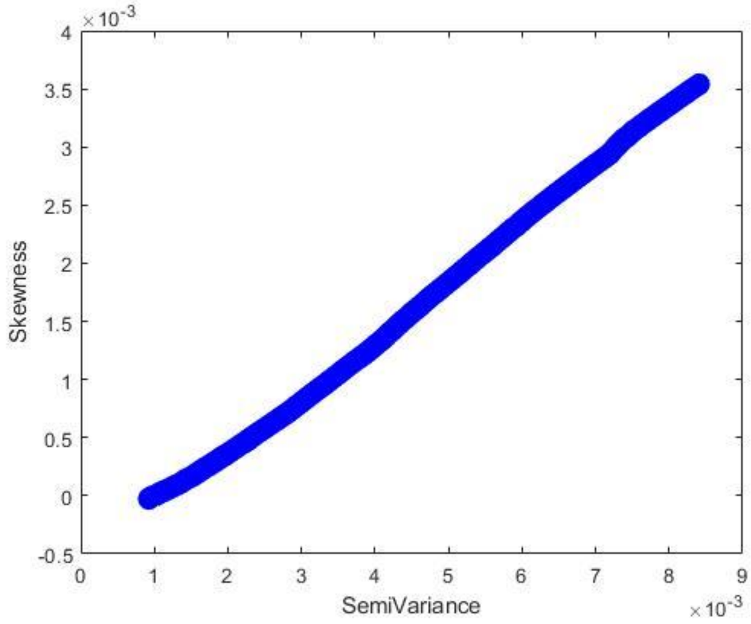


Figure 4A: Stock Data Skewness/Semivariance frontier, using 0 as its target return for semivariance.

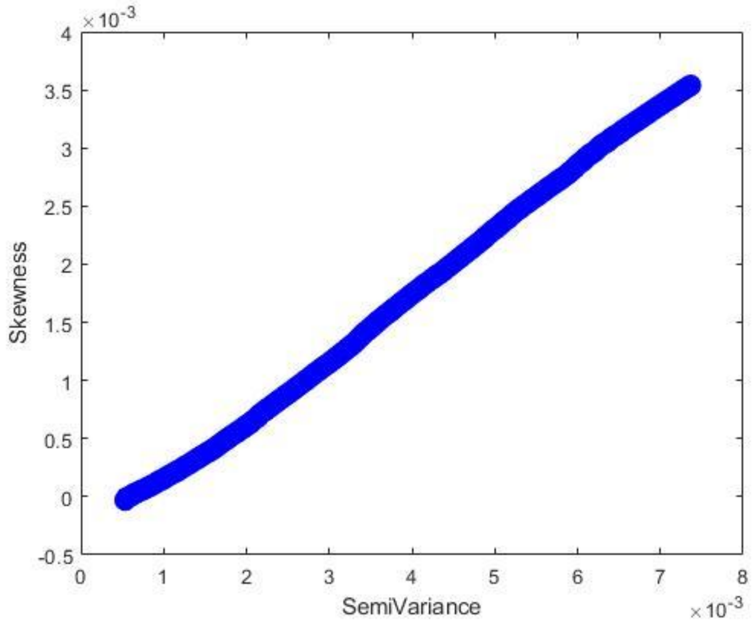


Figure 5A: Asset Class Semivariance /Semivariance frontier, using Min Variance as its target return for semivariance.

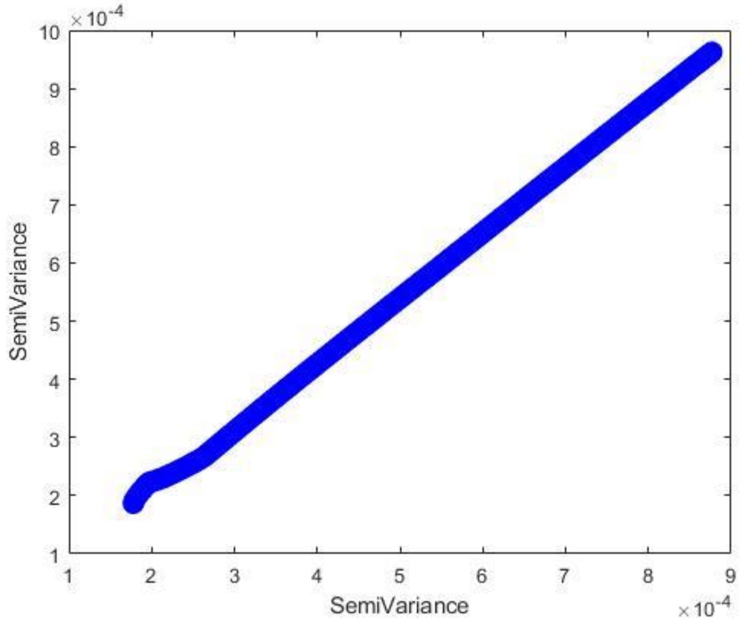


Figure 6A: Asset Class Semivariance /Semivariance frontier, using Max Sharpe as its target return for semivariance.

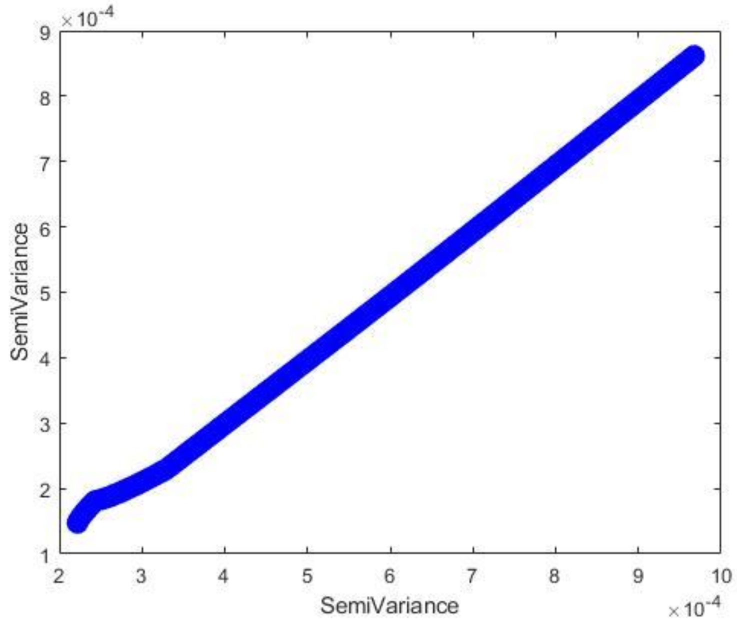


Figure 7A: Stock Data Semivariance /Semivariance frontier, 0 as its target return for semivariance.

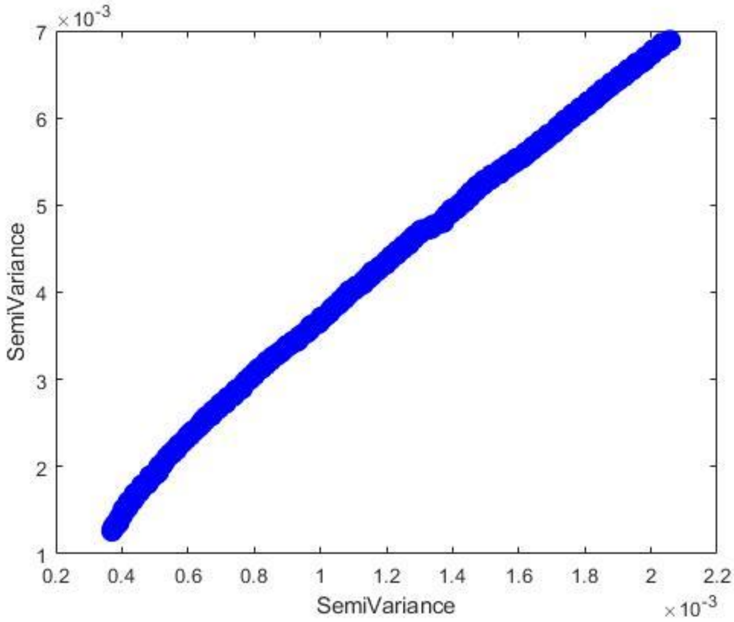
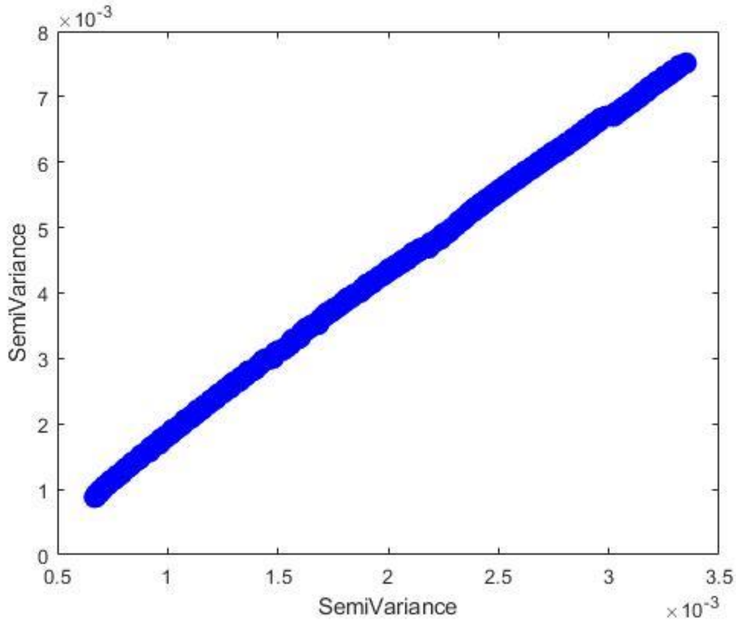


Figure 8A: Stock Data Semivariance /Semivariance frontier, using Equally weighted benchmark portfolio as its target return for semivariance.



Appendix B: Dataset 1, Stock Portfolios’ Tables and Figures

Table 1B: Stock Portfolios’ descriptive statistics

Benchmark Max Sharpe	*Benchmark Max Sharpe*	*Benchmark Min Variance*	*Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi*	*Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe*	*Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino*	*Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi*	*Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe*	*Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino*	*Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi*	*Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe*	*Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino*	*Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi*	*Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe*	*Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino*
Mean	0.02010	0.02013	0.01987	0.01984	0.01924	0.01730	0.01935	0.01853	0.01974	0.01872	0.01800	0.01723	0.01866	0.01747
Standard Error	0.00300	0.00250	0.00495	0.00277	0.00359	0.00469	0.00276	0.00570	0.00497	0.00282	0.00317	0.00453	0.00278	0.00281
Median	0.02081	0.02169	0.01852	0.01891	0.02055	0.01877	0.02030	0.01856	0.01702	0.02082	0.02178	0.01367	0.02162	0.01931
Standard Deviation	0.04307	0.03589	0.07112	0.03969	0.05151	0.06732	0.03966	0.08180	0.07129	0.04041	0.04545	0.06497	0.03985	0.04033
Sample Variance	0.00185	0.00129	0.00506	0.00158	0.00265	0.00453	0.00157	0.00669	0.00508	0.00163	0.00207	0.00422	0.00159	0.00163
Kurtosis	2.41435	0.13520	1.28583	1.84885	1.07475	2.20541	2.00398	1.82322	1.17624	2.42974	2.20862	1.02982	1.89322	1.55712
Skewness	-0.10481	-0.03153	0.03949	-0.19760	-0.09919	-0.10252	-0.26501	0.15205	-0.03437	-0.43341	-0.34167	0.11663	-0.53614	-0.45841
Range	0.37078	0.21346	0.52070	0.31994	0.34464	0.51215	0.32159	0.57103	0.52282	0.32988	0.33464	0.44405	0.29995	0.29837
Minimum	-0.16922	-0.08305	-0.24836	-0.15618	-0.17080	-0.26352	-0.16033	-0.27671	-0.25068	-0.17643	-0.18280	-0.16392	-0.16245	-0.16245
Maximum	0.20156	0.13042	0.27234	0.16376	0.17384	0.24863	0.16126	0.29432	0.27214	0.15345	0.15184	0.28014	0.13750	0.13592

	Benchmark Equally Weighted (1/N)	*Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi*	*Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe*	*Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino*	*Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi*	*Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe*	*Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino*	*Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi*	*Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe*	*Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino*	*Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi*	*Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe*	*Skewness/ Semivariance, Benchmark Rate 0, MaxSortino*
Mean	0.01747	0.01597	0.01986	0.01971	0.01915	0.01986	0.01976	0.01761	0.01936	0.02067	0.01462	0.01907	0.01884
Standard Error	0.00279	0.00517	0.00312	0.00365	0.00578	0.00311	0.00563	0.00523	0.00291	0.00335	0.00553	0.00308	0.00348
Median	0.01915	0.01299	0.01851	0.01516	0.01845	0.01527	0.01601	0.01363	0.01829	0.01674	0.01782	0.01623	0.01673
Standard Deviation	0.04010	0.07416	0.04483	0.05232	0.08291	0.04463	0.08079	0.07507	0.04181	0.04815	0.07936	0.04428	0.04994
Sample Variance	0.00161	0.00550	0.00201	0.00274	0.00687	0.00199	0.00653	0.00564	0.00175	0.00232	0.00630	0.00196	0.00249
Kurtosis	0.07241	1.00413	1.21711	2.73038	1.94652	0.61706	2.33339	1.45109	0.91666	1.13050	3.68381	1.31603	3.36336
Skewness	-0.26830	0.40204	0.07587	0.47815	0.68358	0.12730	0.66586	0.51489	0.10342	0.22503	-0.38274	0.14464	-0.39471
Range	0.23099	0.42529	0.33160	0.42139	0.50481	0.29109	0.52315	0.48478	0.29903	0.33384	0.66912	0.33739	0.41261
Minimum	-0.11017	-0.15731	-0.14097	-0.15899	-0.16591	-0.11970	-0.18196	-0.17058	-0.12683	-0.15060	-0.32993	-0.13934	-0.21456
Maximum	0.12081	0.26798	0.19063	0.26240	0.33890	0.17139	0.34119	0.31421	0.17219	0.18324	0.33920	0.19805	0.19805

Table 2B: Stock Portfolios' Correlation Matrix

	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	1																										
Benchmark Min Variance	0.838260614	1																									
Benchmark Equally Weighted (1/N)	0.760191131	0.820054905	1																								
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.553300296	0.496708406	0.641820125	1																							
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.88174674	0.846339337	0.827881083	0.660018932	1																						
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.730068953	0.634623738	0.707439786	0.851316498	0.836713128	1																					
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.520724274	0.47479257	0.57586925	0.747525679	0.616015817	0.681191399	1																				
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.857439521	0.835920881	0.842818817	0.638710728	0.952633045	0.811633953	0.665093759	1																			
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.514413434	0.442349548	0.553894746	0.663422606	0.594054129	0.611347768	0.891424689	0.630846663	1																		
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.548269335	0.503818362	0.637808433	0.97241893	0.643887314	0.838891564	0.758374261	0.639511251	0.670132014	1																	
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.899210191	0.859531021	0.829063959	0.65055024	0.976403193	0.824155576	0.605285349	0.950732449	0.582991674	0.645933105	1																
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.806045324	0.714822879	0.767624172	0.821860533	0.874472043	0.935572391	0.652659064	0.8576346	0.596662827	0.823082372	0.892574226	1															
Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.534062458	0.488920484	0.546130123	0.807898263	0.583151593	0.723881449	0.716292715	0.58895491	0.643685103	0.84207044	0.587926289	0.705715509	1														
Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	0.894506445	0.877684766	0.822411167	0.661926338	0.945566579	0.813601008	0.620929844	0.928685431	0.596382753	0.664079678	0.958938014	0.876296706	0.651457349	1													
Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	0.883630456	0.855519176	0.81612569	0.699920228	0.940145527	0.843633071	0.629315256	0.918083572	0.598707042	0.693424291	0.948203205	0.887500792	0.699612714	0.98403807	1												
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.418964747	0.493767193	0.662624402	0.478616255	0.528060136	0.43235279	0.468116026	0.545354626	0.44940435	0.451790048	0.527886674	0.487059174	0.349561372	0.538290599	0.520314903	1											
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.723868007	0.770336676	0.852816937	0.579016407	0.810639515	0.66626574	0.570577348	0.818860786	0.560886616	0.555062421	0.820399461	0.731298957	0.49441844	0.814123839	0.805748046	0.808078394	1										
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.663448775	0.677652909	0.770645095	0.522679135	0.747721632	0.612543714	0.53310802	0.763320124	0.530162024	0.497114986	0.755197939	0.67862161	0.445890626	0.753200524	0.735498606	0.85546566	0.943602519	1									
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.448310976	0.507742767	0.632237061	0.545629447	0.552707261	0.482487277	0.512649193	0.550585643	0.499382862	0.507927149	0.570568864	0.544931079	0.391809486	0.567773997	0.552110816	0.866238138	0.805226138	0.788271282	1								
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.761869167	0.81198387	0.852710662	0.601771624	0.836434524	0.685624405	0.566797683	0.840827798	0.544259006	0.578550311	0.843579044	0.757344753	0.512939764	0.831661088	0.82008043	0.720741166	0.928147643	0.866796244	0.798935018	1							
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.460506331	0.52001852	0.641494197	0.565565393	0.566931919	0.51238724	0.50751831	0.562334671	0.491692307	0.529781644	0.587011575	0.575576007	0.410541179	0.584980324	0.568295495	0.850285376	0.810069001	0.788752141	0.979771483	0.802099233	1						
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.425973418	0.497906842	0.647510955	0.476944078	0.537361301	0.426635514	0.489865746	0.553760589	0.474806299	0.453509233	0.534464144	0.487232134	0.355564031	0.537848612	0.514499866	0.977437776	0.788797734	0.847196758	0.861053869	0.716981987	0.836684291	1					
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.726574829	0.800771687	0.86628894	0.564167926	0.820456185	0.647469216	0.580864032	0.834989451	0.557178686	0.551900796	0.819111869	0.712580759	0.508768046	0.81044948	0.795929979	0.772487198	0.92464845	0.888890159	0.713614146	0.908636608	0.709116101	0.796444213	1				
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.638023081	0.700817317	0.796231321	0.553716888	0.769811432	0.619431251	0.582628625	0.781611918	0.57475079	0.524583423	0.766890998	0.672245628	0.456904031	0.747284105	0.736098979	0.831529358	0.929860562	0.903523974	0.803513368	0.859025115	0.801097849	0.854565901	0.935462688	1			
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.549048682	0.532094394	0.674919141	0.49580607	0.610533856	0.501415551	0.501768501	0.609527714	0.483135728	0.465456088	0.627111468	0.566907368	0.395919947	0.620725219	0.601979616	0.884674156	0.80787554	0.842667939	0.833263153	0.750706922	0.814040646	0.891901611	0.774885164	0.819301974	1		
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.739672232	0.783475312	0.859869056	0.552029734	0.818694908	0.659306548	0.574254382	0.816664684	0.551068721	0.531470712	0.824210563	0.718167208	0.486081146	0.806370226	0.796026495	0.78616658	0.925084524	0.913267001	0.734529092	0.898357049	0.729164304	0.797745862	0.937961616	0.905642237	0.848242301	1	
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.741843539	0.746425252	0.816878963	0.53239478	0.805843568	0.642957582	0.553132064	0.802759847	0.528841782	0.507401044	0.821510209	0.707210436	0.471890651	0.803863252	0.791359445	0.767802149	0.926166723	0.912074055	0.74215393	0.899216088	0.73981562	0.768779105	0.913169558	0.892513505	0.878084605	0.971534032	1

Table 3B: Stock Portfolios' F-test P-values, for equal variances.

F-test	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0.000	0.009	0.307	0.000	0.243	0.011	0.000	0.239	0.000	0.000	0.362	0.441	0.000	0.266	0.348	0.000	0.568	0.006	0.000	0.611	0.000	0.000	0.671	0.111	0.000	0.693	0.035
Benchmark Min Variance	0.009	0.000	0.113	0.000	0.150	0.000	0.000	0.153	0.000	0.000	0.090	0.001	0.000	0.135	0.095	0.000	0.002	0.000	0.000	0.002	0.000	0.000	0.029	0.000	0.000	0.003	0.000
Benchmark Equally Weighted (1/N)	0.307	0.113	0.000	0.000	0.884	0.000	0.000	0.877	0.000	0.000	0.912	0.073	0.000	0.929	0.933	0.000	0.111	0.000	0.000	0.126	0.000	0.000	0.550	0.009	0.000	0.156	0.002
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.000	0.000	0.000	0.000	0.000	0.000	0.433	0.000	0.046	0.973	0.000	0.000	0.196	0.000	0.000	0.549	0.000	0.000	0.029	0.000	0.069	0.439	0.000	0.000	0.117	0.000	0.000
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.243	0.150	0.884	0.000	0.000	0.000	0.000	0.993	0.000	0.000	0.798	0.053	0.000	0.955	0.818	0.000	0.082	0.000	0.000	0.094	0.000	0.000	0.457	0.006	0.000	0.118	0.001
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.074	0.001	0.000	0.001	0.000	0.047	0.823	0.000	0.041	0.000	0.000	0.003	0.335	0.000	0.031	0.658
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.000	0.000	0.000	0.433	0.000	0.000	0.000	0.000	0.005	0.413	0.000	0.000	0.610	0.000	0.000	0.167	0.000	0.000	0.003	0.000	0.009	0.120	0.000	0.000	0.019	0.000	0.000
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.239	0.153	0.877	0.000	0.993	0.000	0.000	0.000	0.000	0.000	0.791	0.052	0.000	0.948	0.811	0.000	0.081	0.000	0.000	0.092	0.000	0.000	0.452	0.006	0.000	0.116	0.001
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.000	0.000	0.000	0.046	0.000	0.000	0.005	0.000	0.000	0.050	0.000	0.000	0.001	0.000	0.000	0.161	0.000	0.000	0.847	0.000	0.859	0.220	0.000	0.000	0.665	0.000	0.000
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.000	0.000	0.000	0.973	0.000	0.000	0.413	0.000	0.050	0.000	0.000	0.000	0.185	0.000	0.000	0.572	0.000	0.000	0.031	0.000	0.074	0.460	0.000	0.000	0.125	0.000	0.000
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.362	0.090	0.912	0.000	0.798	0.001	0.000	0.791	0.000	0.000	0.000	0.093	0.000	0.842	0.979	0.000	0.138	0.000	0.000	0.155	0.000	0.000	0.626	0.012	0.000	0.191	0.003
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.441	0.001	0.073	0.000	0.053	0.074	0.000	0.052	0.000	0.000	0.093	0.000	0.000	0.060	0.088	0.000	0.842	0.045	0.000	0.794	0.000	0.000	0.232	0.410	0.000	0.707	0.179
Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.000	0.000	0.000	0.196	0.000	0.001	0.610	0.000	0.001	0.185	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.002	0.001	0.000	0.002	0.039	0.000	0.000	0.004	0.000	0.000
Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	0.266	0.135	0.929	0.000	0.955	0.000	0.000	0.948	0.000	0.000	0.842	0.060	0.000	0.000	0.862	0.000	0.093	0.000	0.000	0.105	0.000	0.000	0.492	0.007	0.000	0.132	0.001
Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	0.348	0.095	0.933	0.000	0.818	0.001	0.000	0.811	0.000	0.000	0.979	0.088	0.000	0.862	0.000	0.000	0.131	0.000	0.000	0.148	0.000	0.000	0.608	0.012	0.000	0.182	0.002
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.000	0.000	0.000	0.549	0.000	0.000	0.167	0.000	0.161	0.572	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.221	0.862	0.000	0.000	0.333	0.000	0.000
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.568	0.002	0.111	0.000	0.082	0.047	0.000	0.081	0.000	0.000	0.138	0.842	0.000	0.093	0.131	0.000	0.000	0.027	0.000	0.950	0.000	0.000	0.319	0.306	0.000	0.860	0.123
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.006	0.000	0.000	0.000	0.000	0.823	0.000	0.000	0.000	0.000	0.000	0.045	0.002	0.000	0.000	0.000	0.027	0.000	0.000	0.023	0.000	0.000	0.001	0.235	0.000	0.017	0.506
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.000	0.000	0.000	0.029	0.000	0.000	0.003	0.000	0.847	0.031	0.000	0.000	0.001	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.711	0.156	0.000	0.000	0.532	0.000	0.000
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.611	0.002	0.126	0.000	0.094	0.041	0.000	0.092	0.000	0.000	0.155	0.794	0.000	0.105	0.148	0.000	0.950	0.023	0.000	0.000	0.000	0.000	0.350	0.278	0.000	0.909	0.108
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.000	0.000	0.000	0.069	0.000	0.000	0.009	0.000	0.859	0.074	0.000	0.000	0.002	0.000	0.000	0.221	0.000	0.000	0.711	0.000	0.000	0.294	0.000	0.000	0.799	0.000	0.000
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.000	0.000	0.000	0.439	0.000	0.000	0.120	0.000	0.220	0.460	0.000	0.000	0.039	0.000	0.000	0.862	0.000	0.000	0.156	0.000	0.294	0.000	0.000	0.000	0.427	0.000	0.000
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.671	0.029	0.550	0.000	0.457	0.003	0.000	0.452	0.000	0.000	0.626	0.232	0.000	0.492	0.608	0.000	0.319	0.001	0.000	0.350	0.000	0.000	0.000	0.044	0.000	0.412	0.011
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.111	0.000	0.009	0.000	0.006	0.335	0.000	0.006	0.000	0.000	0.012	0.410	0.000	0.007	0.012	0.000	0.306	0.235	0.000	0.278	0.000	0.000	0.044	0.000	0.000	0.231	0.602
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.000	0.000	0.000	0.117	0.000	0.000	0.019	0.000	0.665	0.125	0.000	0.000	0.004	0.000	0.000	0.333	0.000	0.000	0.532	0.000	0.799	0.427	0.000	0.000	0.000	0.000	0.000
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.693	0.003	0.156	0.000	0.118	0.031	0.000	0.116	0.000	0.000	0.191	0.707	0.000	0.132	0.182	0.000	0.860	0.017	0.000	0.909	0.000	0.000	0.412	0.231	0.000	0.000	0.086
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.035	0.000	0.002	0.000	0.001	0.658	0.000	0.001	0.000	0.000	0.003	0.179	0.000	0.001	0.002	0.000	0.123	0.506	0.000	0.108	0.000	0.000	0.011	0.602	0.000	0.086	0.000

Red highlighting represents significance at the 5% level

Table 4B: Stock Portfolios' T-test P-Values.

T-test	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Se mivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0	0.994190139	0.521141796	0.968669134	0.948375342	0.853405408	0.615563061	0.85435946	0.807505978	0.949975492	0.736803526	0.630204091	0.597130106	0.724160057	0.522385563	0.49011643	0.955939823	0.934512907	0.884207709	0.954995369	0.957775407	0.680304016	0.860030505	0.899626291	0.384673626	0.811198717	0.783391681
Benchmark Min Variance	0.994190139	0	0.478228067	0.963218273	0.937430498	0.838263753	0.59519071	0.834791022	0.797410342	0.943732521	0.707793687	0.597798121	0.575137336	0.693769065	0.479776313	0.469550964	0.946645341	0.924953926	0.876729366	0.945589552	0.952597626	0.664688594	0.841864977	0.897447184	0.365080203	0.790201208	0.763097492
Benchmark Equally Weighted (1/N)	0.521141796	0.478228067	0	0.67271715	0.547167651	0.697711988	0.975774248	0.631988745	0.867225212	0.690846181	0.753013286	0.900062087	0.964009876	0.7628885	0.999869259	0.799183893	0.568218569	0.625318608	0.793171399	0.568023917	0.715238445	0.980396624	0.638977276	0.463887752	0.646524798	0.700141503	0.759190819
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.968669134	0.963218273	0.67271715	0	0.994846826	0.91709929	0.706513148	0.926805625	0.858947774	0.984471038	0.839361028	0.750119172	0.693676685	0.830607354	0.673238804	0.586146592	0.998400996	0.979196686	0.924559231	0.99777416	0.988271955	0.754006239	0.929320107	0.894276139	0.480001455	0.890923674	0.864200721
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.948375342	0.937430498	0.547167651	0.994846826	0	0.894610092	0.641929805	0.901345377	0.836767862	0.985981597	0.776796705	0.662282588	0.623191304	0.76359277	0.548471183	0.510177862	0.995234493	0.978422175	0.914964139	0.996101342	0.990641315	0.707428014	0.906204563	0.848250783	0.399844659	0.853715792	0.822173998
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.853405408	0.838263753	0.697711988	0.91709929	0.894610092	0	0.743534416	0.979644588	0.916609317	0.934931306	0.909483634	0.796180142	0.728319552	0.898566212	0.698457077	0.604264865	0.895462733	0.925792209	0.990168354	0.896030862	0.937127377	0.798326712	0.978087126	0.770661556	0.484643318	0.972409684	0.936419259
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.615563061	0.59519071	0.975774248	0.706513148	0.641929805	0.743534416	0	0.706838977	0.867956402	0.721776752	0.796090673	0.902111584	0.990914405	0.803836948	0.975716974	0.848979096	0.650002551	0.685140893	0.803802255	0.650152259	0.73717433	0.964630379	0.70921291	0.559704055	0.712082787	0.752748886	0.792859708
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.85435946	0.834791022	0.631988745	0.926805625	0.901345377	0.979644588	0.706838977	0	0.896911588	0.946042781	0.872292855	0.747746692	0.68909492	0.859386824	0.633121656	0.564590362	0.902755161	0.93712314	0.975166934	0.903390629	0.947746012	0.769158403	0.997737333	0.761980657	0.44502075	0.946245088	0.907855647
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.807505978	0.797410342	0.867225212	0.858947774	0.836767862	0.916609317	0.867956402	0.896911588	0	0.873299244	0.976575284	0.935096232	0.85808773	0.984036152	0.867455911	0.73970468	0.837831342	0.861352943	0.938953556	0.83822224	0.877776308	0.905773267	0.89656754	0.746598251	0.62306873	0.933408498	0.963388512
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.949975492	0.943732521	0.690846181	0.984471038	0.985981597	0.934931306	0.721776752	0.946042781	0.873299244	0	0.858383168	0.768171288	0.709116767	0.849668336	0.691345594	0.599747711	0.983029483	0.996910316	0.938884233	0.983632236	0.997202357	0.768697826	0.948308639	0.876460449	0.491960821	0.909596034	0.88215281
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.736803526	0.707793687	0.753013286	0.839361028	0.776796705	0.909483634	0.796090673	0.872292855	0.976575284	0.858383168	0	0.865499062	0.780148373	0.987930065	0.75383644	0.641246771	0.785631879	0.828969693	0.946069464	0.785968378	0.868127905	0.852799667	0.873391205	0.656065148	0.51000597	0.932278501	0.978637158
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.630204091	0.597798121	0.900062087	0.750119172	0.662282588	0.796180142	0.902111584	0.747746692	0.935096232	0.768171288	0.865499062	0	0.889099939	0.875863427	0.900440116	0.738323541	0.675614382	0.722838162	0.861145044	0.675711446	0.784925575	0.949849536	0.751383822	0.563071257	0.596764049	0.808306374	0.858704445
Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.597130106	0.575137336	0.964009876	0.693676685	0.623191304	0.728319552	0.990914405	0.68909492	0.85808773	0.709116767	0.780148373	0.889099939	0	0.787975428	0.963971284	0.855077219	0.632318489	0.669226195	0.793317517	0.632434503	0.725813821	0.955524872	0.691872051	0.54173697	0.715783625	0.736559775	0.778229832
Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	0.724160057	0.693769065	0.7628885	0.830607354	0.76359277	0.898566212	0.803836948	0.859386824	0.984036152	0.849668336	0.987930065	0.875863427	0.787975428	0	0.763696318	0.647500396	0.773352104	0.817940651	0.938501718	0.773656163	0.860276025	0.860274999	0.860854415	0.644342217	0.515019987	0.920402075	0.967804567
Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	0.522385563	0.479776313	0.999869259	0.673238804	0.548471183	0.698457077	0.975716974	0.633121656	0.867455911	0.691345594	0.75383644	0.900440116	0.963971284	0.763696318	0	0.799360844	0.569337756	0.626173306	0.79347528	0.569148607	0.715632878	0.9805095	0.640039544	0.465057587	0.646844555	0.701015116	0.759840949
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.49011643	0.469550964	0.799183893	0.586146592	0.510177862	0.604264865	0.848979096	0.564590362	0.73970468	0.599747711	0.641246771	0.738323541	0.855077219	0.647500396	0.799360844	0	0.519948011	0.554574987	0.681842633	0.519955609	0.620117538	0.823446823	0.568000999	0.446382547	0.858700414	0.606853093	0.645923461
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.955939823	0.946645341	0.568218569	0.998400996	0.995234493	0.895462733	0.650002551	0.902755161	0.837831342	0.983029483	0.785631879	0.675614382	0.632318489	0.773352104	0.569337756	0.519948011	0	0.975292202	0.914046056	0.999170507	0.987794487	0.712416064	0.907141876	0.860240535	0.410153661	0.857402093	0.826669233
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.934512907	0.924953926	0.625318608	0.979196686	0.978422175	0.925792209	0.685140893	0.93712314	0.861352943	0.996910316	0.828969693	0.722838162	0.669226195	0.817940651	0.626173306	0.554574987	0.975292202	0	0.934636341	0.976009281	0.99403023	0.742199804	0.940296759	0.847029651	0.442812719	0.893368884	0.862123988
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.884207709	0.876729366	0.793171399	0.924559231	0.914964139	0.990168354	0.803802255	0.975166934	0.938953556	0.938884233	0.946069464	0.861145044	0.793317517	0.938501718	0.79347528	0.681842633	0.914046056	0.934636341	0	0.914515866	0.939669444	0.843630193	0.974015598	0.820503617	0.571550419	0.990270769	0.96275607
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.954995369	0.945589552	0.568023917	0.99777416	0.996101342	0.896030862	0.650152259	0.903390629	0.83822224	0.983632236	0.785968378	0.675711446	0.632434503	0.773656163	0.569148607	0.519955609	0.999170507	0.976009281	0.914515866	0	0.988350343	0.712663433	0.907778462	0.859173955	0.410072742	0.857912005	0.827102448
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.957775407	0.952597626	0.715238445	0.988271955	0.990641315	0.937127377	0.73717433	0.947746012	0.877776308	0.997202357	0.868127905	0.784925575	0.725813821	0.860276025	0.715632878	0.620117538	0.987794487	0.99403023	0.939669444	0.988350343	0	0.779873304	0.949728304	0.890105102	0.515249256	0.914371636	0.888801652
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.680304016	0.664688594	0.980396624	0.754006239	0.707428014	0.798326712	0.964630379	0.769158403	0.905773267	0.768697826	0.852799667	0.949849536	0.955524872	0.860274999	0.9805095	0.823446823	0.712416064	0.742199804	0.843630193	0.712663433	0.779873304	0	0.770366554	0.623280609	0.694675968	0.810375041	0.845754071
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.860030505	0.841864977	0.638977276	0.929320107	0.906204563	0.978087126	0.70921291	0.997737333	0.89656754	0.948308639	0.873391205	0.751383822	0.691872051	0.860854415	0.640039544	0.568000999	0.907141876	0.940296759	0.974015598	0.907778462	0.949728304	0.770366554	0	0.768948618	0.448889052	0.945386888	0.907773149
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.899626291	0.897447184	0.463887752	0.894276139	0.848250783	0.770661556	0.559704055	0.761980657	0.746598251	0.876460449	0.656065148	0.563071257	0.54173697	0.644342217	0.465057587	0.446382547	0.860240535	0.847029651	0.820503617	0.859173955	0.890105102	0.623280609	0.768948618	0	0.35064919	0.726247932	0.704856086
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.384673626	0.365080203	0.646524798	0.480001455	0.399844659	0.484643318	0.712082787	0.44502075	0.62306873	0.491960821	0.51000597	0.596764049	0.715783625	0.515019987	0.646844555	0.858700414	0.410153661	0.442812719	0.571550419	0.410072742	0.515249256	0.694675968	0.448889052	0.35064919	0	0.482891988	0.519466922
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.811198717	0.790201208	0.700141503	0.890923674	0.853715792	0.972409684	0.752748886	0.946245088	0.933408498	0.909596034	0.932278501	0.808306374	0.736559775	0.920402075	0.701015116	0.606853093	0.857402093	0.893368884	0.990270769	0.857912005	0.914371636	0.810375041	0.945386888	0.726247932	0.482891988	0	0.959684398
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.783391681	0.763097492	0.759190819	0.864200721	0.822173998	0.936419259	0.792859708	0.907855647	0.963388512	0.88215281	0.978637158	0.858704445	0.778229832	0.967804567	0.759840949	0.645923461	0.826669233	0.862123988	0.96275607	0.827102448	0.888801652	0.845754071	0.907773149	0.704856086	0.519466922	0.959684398	0

Red highlighting represents significance at the 5% level

Table 5B: Stock Portfolios' Mann-Whitney U(ranksum) test, P-Values

Mann-Whitney	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0	0.966008977	0.594399	0.822246821	0.908103482	0.787032132	0.543339177	0.872791262	0.482099696	0.854575998	0.770530867	0.670908081	0.508255587	0.840964687	0.607064624	0.275997578	0.80043063	0.571685656	0.56271732	0.769265726	0.5643937	0.321337092	0.666089042	0.719815758	0.421944414	0.635107891	0.711166784
Benchmark Min Variance	0.966008977	0	0.601292569	0.820959526	0.95941401	0.809395965	0.545536902	0.866928515	0.461713665	0.857173673	0.758537133	0.663684666	0.48779046	0.868230727	0.611119726	0.246850195	0.779403783	0.507195249	0.537318251	0.756648603	0.532960552	0.292365665	0.652908074	0.676950948	0.414806535	0.594399	0.670908081
Benchmark Equally Weighted (1/N)	0.594399	0.601292569	0	0.856524108	0.67271873	0.790216699	0.873443116	0.709934215	0.749736496	0.823534594	0.791491511	0.871487813	0.823534594	0.732856427	0.985145916	0.471336489	0.77306298	0.927147256	0.808754738	0.838377308	0.846792535	0.549945541	0.89761843	0.873443116	0.676950948	0.977225516	0.907447678
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.822246821	0.820959526	0.856524108	0	0.84031768	0.937014584	0.741593519	0.89761843	0.682407381	0.983165648	0.96864764	0.931092929	0.737844895	0.926489815	0.876703665	0.464235298	0.883230991	0.952162772	0.696426653	0.886497657	0.770530867	0.545536902	0.942281215	0.878661003	0.585833214	0.989766876	0.94755037
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.908103482	0.95941401	0.67271873	0.84031768	0	0.831916558	0.574502135	0.93898926	0.47593299	0.872139494	0.812604009	0.768001199	0.529162285	0.908103482	0.65888631	0.268052449	0.851331123	0.552709826	0.56551262	0.844201326	0.580721397	0.32988913	0.729744339	0.754761525	0.423380583	0.654699228	0.734725793
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.787032132	0.809395965	0.790216699	0.937014584	0.831916558	0	0.684837822	0.921232134	0.609959901	0.953480902	0.987126276	0.962711175	0.636878669	0.919261379	0.850682448	0.357716648	0.981845527	0.750991767	0.647546639	0.985145916	0.66729252	0.453696058	0.911383435	0.951503755	0.526457694	0.842259024	0.921889161
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.543339177	0.545536902	0.873443116	0.741593519	0.574502135	0.684837822	0	0.62218643	0.926489815	0.720434932	0.694592035	0.833207814	0.995048384	0.670304956	0.833853612	0.643982457	0.612280528	0.785123097	0.898928123	0.62863294	0.982505582	0.754132823	0.694592035	0.608801054	0.788305535	0.766105566	0.713634186
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.872791262	0.866928515	0.709934215	0.89761843	0.93898926	0.921232134	0.62218643	0	0.540050875	0.953480902	0.905480645	0.80043063	0.580721397	0.987786414	0.727879217	0.305842372	0.934382288	0.621601808	0.607643189	0.862373639	0.624527299	0.341939112	0.765473999	0.802989528	0.460707157	0.708702405	0.785123097
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.482099696	0.461713665	0.749736496	0.682407381	0.47593299	0.609959901	0.926489815	0.540050875	0	0.652311467	0.609380355	0.762318509	0.94755037	0.56160107	0.724774263	0.737220727	0.564953027	0.706241071	0.976565573	0.581288344	0.916634478	0.823534594	0.600716728	0.535137165	0.868881967	0.675135881	0.636288178
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.854575998	0.857173673	0.823534594	0.983165648	0.872139494	0.953480902	0.720434932	0.953480902	0.652311467	0	0.993067781	0.891074221	0.698875574	0.981845527	0.829980524	0.460707157	0.927147256	0.91860457	0.686054269	0.923203374	0.773696386	0.550497853	0.977225516	0.908759349	0.579022111	0.964030222	0.997689234
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.770530867	0.758537133	0.791491511	0.96864764	0.812604009	0.987126276	0.694592035	0.905480645	0.609380355	0.993067781	0	0.916634478	0.641018543	0.883230991	0.816457812	0.357284294	0.971286646	0.732856427	0.670908081	0.991747402	0.697038588	0.425780544	0.842259024	0.896963688	0.525917624	0.814530349	0.898928123
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.670908081	0.663684666	0.871487813	0.931092929	0.768001199	0.962711175	0.833207814	0.80043063	0.762318509	0.891074221	0.916634478	0	0.710550405	0.822246821	0.94755037	0.453696058	0.936356444	0.806191121	0.728500746	0.934382288	0.79723502	0.521069774	0.865626664	0.960732796	0.612861294	0.904825096	0.991747402
Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.508255587	0.48779046	0.823534594	0.737844895	0.529162285	0.636878669	0.995048384	0.580721397	0.94755037	0.698875574	0.641018543	0.710550405	0	0.629220402	0.807472671	0.661883653	0.551603295	0.742844429	0.936356444	0.62218643	0.955458335	0.767369166	0.6298081	0.549945541	0.811962146	0.685445943	0.658287497
Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	0.840964687	0.868230727	0.732856427	0.926489815	0.908103482	0.919261379	0.670304956	0.987786414	0.56160107	0.981845527	0.883230991	0.822246821	0.629220402	0	0.743470139	0.324984363	0.914008487	0.661883653	0.6298081	0.942281215	0.656492372	0.402604695	0.764842587	0.823534594	0.505606922	0.747227944	0.817743405
Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	0.607064624	0.611119726	0.985145916	0.876703665	0.65888631	0.850682448	0.833853612	0.727879217	0.724774263	0.829980524	0.816457812	0.94755037	0.807472671	0.743470139	0	0.449220894	0.797873872	0.905480645	0.79531928	0.801070155	0.850682448	0.544987058	0.948209181	0.898273242	0.648736502	0.975905646	0.89761843
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.275997578	0.246850195	0.471336489	0.464235298	0.268052449	0.357716648	0.643982457	0.305842372	0.737220727	0.460707157	0.357284294	0.453696058	0.661883653	0.324984363	0.449220894	0	0.317316044	0.429636952	0.740343291	0.330711228	0.625699157	0.879966308	0.33858719	0.274904793	0.877356028	0.395203992	0.371725511
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.80043063	0.779403783	0.77306298	0.883230991	0.851331123	0.981845527	0.612280528	0.934382288	0.564953027	0.927147256	0.971286646	0.936356444	0.551603295	0.914008487	0.797873872	0.317316044	0	0.69765072	0.600716728	0.956776776	0.62863294	0.387431517	0.874747081	0.922546241	0.464740543	0.825467144	0.898273242
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.571685656	0.507195249	0.927147256	0.952162772	0.552709826	0.750991767	0.785123097	0.621601808	0.706241071	0.91860457	0.732856427	0.806191121	0.742844429	0.661883653	0.905480645	0.429636952	0.69765072	0	0.717340903	0.738469236	0.825467144	0.516779972	0.794042804	0.735972912	0.581288344	0.885844168	0.831271099
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.56271732	0.537318251	0.808754738	0.696426653	0.56551262	0.647546639	0.898928123	0.607643189	0.976565573	0.686054269	0.670908081	0.728500746	0.936356444	0.6298081	0.79531928	0.740343291	0.600716728	0.717340903	0	0.636288178	0.917947816	0.874747081	0.646952044	0.545536902	0.878661003	0.706856118	0.666089042
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.769265726	0.756648603	0.838377308	0.886497657	0.844201326	0.985145916	0.62863294	0.862373639	0.581288344	0.923203374	0.991747402	0.934382288	0.62218643	0.942281215	0.801070155	0.330711228	0.956776776	0.738469236	0.636288178	0	0.69765072	0.403535589	0.940964312	0.987126276	0.467778427	0.85522527	0.914008487
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.5643937	0.532960552	0.846792535	0.770530867	0.580721397	0.66729252	0.982505582	0.624527299	0.916634478	0.773696386	0.697038588	0.79723502	0.955458335	0.656492372	0.850682448	0.625699157	0.62863294	0.825467144	0.917947816	0.69765072	0	0.754132823	0.705011554	0.6180991	0.774329944	0.785759299	0.736596733
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.321337092	0.292365665	0.549945541	0.545536902	0.32988913	0.453696058	0.754132823	0.341939112	0.823534594	0.550497853	0.425780544	0.521069774	0.767369166	0.402604695	0.544987058	0.879966308	0.387431517	0.516779972	0.874747081	0.403535589	0.754132823	0	0.420511106	0.36336719	0.975905646	0.491951856	0.45519335
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.666089042	0.652908074	0.89761843	0.942281215	0.729744339	0.911383435	0.694592035	0.765473999	0.600716728	0.977225516	0.842259024	0.865626664	0.6298081	0.764842587	0.948209181	0.33858719	0.874747081	0.794042804	0.646952044	0.940964312	0.705011554	0.420511106	0	0.996368805	0.531873928	0.923860559	0.990427045
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.719815758	0.676950948	0.873443116	0.878661003	0.754761525	0.951503755	0.608801054	0.802989528	0.535137165	0.908759349	0.896963688	0.960732796	0.549945541	0.823534594	0.898273242	0.274904793	0.922546241	0.735972912	0.545536902	0.987126276	0.6180991	0.36336719	0.996368805	0	0.445264078	0.869533296	0.935040296
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.421944414	0.414806535	0.676950948	0.585833214	0.423380583	0.526457694	0.788305535	0.460707157	0.868881967	0.579022111	0.525917624	0.612861294	0.811962146	0.505606922	0.648736502	0.877356028	0.464740543	0.581288344	0.878661003	0.467778427	0.774329944	0.975905646	0.531873928	0.445264078	0	0.571123149	0.534048299
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.635107891	0.594399	0.977225516	0.989766876	0.654699228	0.842259024	0.766105566	0.708702405	0.675135881	0.964030222	0.814530349	0.904825096	0.685445943	0.747227944	0.975905646	0.395203992	0.825467144	0.885844168	0.706856118	0.85522527	0.785759299	0.491951856	0.923860559	0.869533296	0.571123149	0	0.936356412
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.711166784	0.670908081	0.907447678	0.94755037	0.734725793	0.921889161	0.713634186	0.785123097	0.636288178	0.997689234	0.898928123	0.991747402	0.658287497	0.817743405	0.89761843	0.371725511	0.898273242	0.831271099	0.666089042	0.914008487	0.736596733	0.45519335	0.990427045	0.935040296	0.534048299	0.936356412	0

Table 6B: Stock Portfolios' Sharpe Ratio Differences, P-Values

	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0	0.2666	0.4466	0.0406	0.5912	0.1726	0.0336	0.8622	0.027	0.0374	0.6962	0.1394	0.015	0.7322	0.214	0.077	0.7574	0.335	0.1038	0.7496	0.1156	0.1188	0.8834	0.7442	0.008	0.5942	0.2328
Benchmark Min Variance	0.2666	0	0.0086	0.004	0.404	0.0192	0.0078	0.2976	0.0038	0.0042	0.1926	0.026	0.0014	0.1996	0.0478	0.0004	0.0882	0.0228	0.0038	0.0744	0.0114	0.0036	0.1006	0.1348	0.0008	0.0402	0.0474
Benchmark Equally Weighted (1/N)	0.4466	0.0086	0	0.1472	0.1596	0.6152	0.0982	0.2366	0.0748	0.1478	0.5604	0.6632	0.1034	0.549	0.9754	0.0438	0.524	0.7016	0.129	0.4844	0.1564	0.095	0.3078	0.6218	0.022	0.7672	0.6612
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.0406	0.004	0.1472	0	0.004	0.0494	0.4982	0.0112	0.3894	0.9038	0.0298	0.0888	0.5732	0.0366	0.1098	0.409	0.0568	0.2394	0.5942	0.0426	0.6962	0.6002	0.0498	0.0766	0.1794	0.121	0.3432
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.5912	0.404	0.1596	0.004	0	0.026	0.004	0.579	0.0052	0.0044	0.024	0.0056	0.004	0.1824	0.0066	0.023	0.4342	0.1276	0.047	0.3972	0.0496	0.0482	0.5306	0.3976	0.0016	0.2716	0.0386
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.1726	0.0192	0.6152	0.0494	0.026	0	0.0966	0.0646	0.0826	0.0754	0.1926	0.847	0.0592	0.1742	0.5298	0.1378	0.3356	0.9174	0.2364	0.2774	0.2548	0.2346	0.2594	0.4054	0.0294	0.4894	0.9938
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.0336	0.0078	0.0982	0.4982	0.004	0.0966	0	0.0036	0.6782	0.5342	0.018	0.1056	0.9266	0.0194	0.0662	0.6624	0.0228	0.1196	0.9184	0.0218	0.9392	0.869	0.021	0.0196	0.327	0.038	0.1374
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.8622	0.2976	0.2366	0.0112	0.579	0.0646	0.0036	0	0.0114	0.0084	0.3384	0.0256	0.006	0.4936	0.0604	0.035	0.6134	0.1912	0.0628	0.5522	0.0668	0.062	0.7144	0.561	0.0032	0.3852	0.0838
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.027	0.0038	0.0748	0.3894	0.0052	0.0826	0.6782	0.0114	0	0.4374	0.0224	0.0922	0.7484	0.0254	0.0696	0.7634	0.0156	0.07	0.928	0.017	0.7726	0.9942	0.0212	0.0164	0.4328	0.0286	0.1294
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.0374	0.0042	0.1478	0.9038	0.0044	0.0754	0.5342	0.0084	0.4374	0	0.0348	0.0858	0.5586	0.0328	0.1014	0.45	0.0688	0.2734	0.6464	0.0554	0.7474	0.6294	0.0664	0.1034	0.2138	0.1272	0.343
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.6962	0.1926	0.5604	0.0298	0.024	0.1926	0.018	0.3384	0.0224	0.0348	0	0.0592	0.0238	0.8996	0.1484	0.0594	0.9518	0.3552	0.1064	0.9604	0.12	0.096	0.9016	0.9044	0.0034	0.6984	0.1762
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.1394	0.026	0.6632	0.0888	0.0056	0.847	0.1056	0.0256	0.0922	0.0858	0.0592	0	0.0898	0.079	0.5084	0.1558	0.3538	0.988	0.257	0.287	0.28	0.2584	0.3106	0.4506	0.0296	0.5248	0.9136
Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.015	0.0014	0.1034	0.5732	0.004	0.0592	0.9266	0.006	0.7484	0.5586	0.0238	0.0898	0	0.0178	0.051	0.6284	0.036	0.1386	0.874	0.0248	0.9958	0.8302	0.0204	0.0466	0.3868	0.062	0.228
Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	0.7322	0.1996	0.549	0.0366	0.1824	0.1742	0.0194	0.4936	0.0254	0.0328	0.8996	0.079	0.0178	0	0.057	0.0518	0.9056	0.3276	0.1112	0.913	0.1148	0.0876	0.952	0.8558	0.0054	0.6698	0.1648
Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	0.214	0.0478	0.9754	0.1098	0.0066	0.5298	0.0662	0.0604	0.0696	0.1014	0.1484	0.5084	0.051	0.057	0	0.0938	0.5312	0.7316	0.2018	0.4788	0.213	0.1548	0.4292	0.6406	0.0122	0.7718	0.5608
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.077	0.0004	0.0438	0.409	0.023	0.1378	0.6624	0.035	0.7634	0.45	0.0594	0.1558	0.6284	0.0518	0.0938	0	0.004	0.005	0.432	0.0118	0.2928	0.1156	0.0046	0.001	0.6292	0.017	0.168
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.7574	0.0882	0.524	0.0568	0.4342	0.3356	0.0228	0.6134	0.0156	0.0688	0.9518	0.3538	0.036	0.9056	0.5312	0.004	0	0.0408	0.006	0.9626	0.0054	0.0116	0.7206	0.8792	0.0002	0.5674	0.181
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.335	0.0228	0.7016	0.2394	0.1276	0.9174	0.1196	0.1912	0.07	0.2734	0.3552	0.988	0.1386	0.3276	0.7316	0.005	0.0408	0	0.0194	0.1194	0.0304	0.0156	0.0776	0.0942	0.0008	0.2956	0.8768
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.1038	0.0038	0.129	0.5942	0.047	0.2364	0.9184	0.0628	0.928	0.6464	0.1064	0.257	0.874	0.1112	0.2018	0.432	0.006	0.0194	0	0.0156	0.5622	0.871	0.0092	0.0026	0.242	0.023	0.1804
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.7496	0.0744	0.4844	0.0426	0.3972	0.2774	0.0218	0.5522	0.017	0.0554	0.9604	0.287	0.0248	0.913	0.4788	0.0118	0.9626	0.1194	0.0156	0	0.018	0.0248	0.7662	0.8868	0.0004	0.5626	0.162
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.1156	0.0114	0.1564	0.6962	0.0496	0.2548	0.9392	0.0668	0.7726	0.7474	0.12	0.28	0.9958	0.1148	0.213	0.2928	0.0054	0.0304	0.5622	0.018	0	0.681	0.0124	0.0068	0.1818	0.0406	0.2236
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.1188	0.0036	0.095	0.6002	0.0482	0.2346	0.869	0.062	0.9942	0.6294	0.096	0.2584	0.8302	0.0876	0.1548	0.1156	0.0116	0.0156	0.871	0.0248	0.681	0	0.0128	0.0042	0.3598	0.0326	0.234
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.8834	0.1006	0.3078	0.0498	0.5306	0.2594	0.021	0.7144	0.0212	0.0664	0.9016	0.3106	0.0204	0.952	0.4292	0.0046	0.7206	0.0776	0.0092	0.7662	0.0124	0.0128	0	0.648	0.0006	0.3322	0.1392
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.7442	0.1348	0.6218	0.0766	0.3976	0.4054	0.0196	0.561	0.0164	0.1034	0.9044	0.4506	0.0466	0.8558	0.6406	0.001	0.8792	0.0942	0.0026	0.8868	0.0068	0.0042	0.648	0	0.0002	0.6938	0.2808
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.008	0.0008	0.022	0.1794	0.0016	0.0294	0.327	0.0032	0.4328	0.2138	0.0034	0.0296	0.3868	0.0054	0.0122	0.6292	0.0002	0.0008	0.242	0.0004	0.1818	0.3598	0.0006	0.0002	0	0.0008	0.0024
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.5942	0.0402	0.7672	0.121	0.2716	0.4894	0.038	0.3852	0.0286	0.1272	0.6984	0.5248	0.062	0.6698	0.7718	0.017	0.5674	0.2956	0.023	0.5626	0.0406	0.0326	0.3322	0.6938	0.0008	0	0.313
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.2328	0.0474	0.6612	0.3432	0.0386	0.9938	0.1374	0.0838	0.1294	0.343	0.1762	0.9136	0.228	0.1648	0.5608	0.168	0.181	0.8768	0.1804	0.162	0.2236	0.234	0.1392	0.2808	0.0024	0.313	0

Red highlighting represents significance at the 5% level, green highlighting is significance at the 10% level

Table 7B: Stock Portfolios' Sortino Ratio Differences, P-values

	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino
Benchmark Max Sharpe	0.000	0.003	0.437	0.001	0.141	0.019	0.001	0.539	0.001
Benchmark Min Variance	0.001	0.000	0.001	0.001	0.016	0.001	0.001	0.007	0.001
Benchmark Equally Weighted (1/N)	0.450	0.001	0.000	0.001	0.055	0.095	0.001	0.121	0.002
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.001	0.001	0.001	0.000	0.001	0.002	0.468	0.001	0.244
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.152	0.019	0.047	0.001	0.000	0.001	0.001	0.247	0.001
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.009	0.001	0.078	0.001	0.001	0.000	0.010	0.002	0.005
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.001	0.001	0.001	0.422	0.001	0.007	0.000	0.001	0.366
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.590	0.008	0.137	0.001	0.237	0.001	0.001	0.000	0.001
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.002	0.001	0.001	0.224	0.001	0.004	0.348	0.001	0.000
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.002	0.001	0.001	0.748	0.001	0.001	0.532	0.001	0.281
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.457	0.009	0.709	0.001	0.002	0.010	0.001	0.037	0.001
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.011	0.001	0.122	0.004	0.001	0.778	0.008	0.001	0.002
Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.001	0.001	0.004	0.697	0.001	0.006	0.730	0.001	0.438
Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	0.385	0.001	0.736	0.001	0.002	0.012	0.001	0.089	0.002
Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	0.062	0.001	0.688	0.001	0.001	0.100	0.003	0.004	0.001
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.004	0.001	0.001	0.413	0.001	0.024	0.674	0.001	0.929
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.853	0.002	0.461	0.001	0.238	0.035	0.001	0.474	0.001
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.257	0.001	0.504	0.016	0.031	0.425	0.007	0.068	0.003
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.009	0.001	0.012	0.590	0.004	0.063	0.923	0.001	0.755
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.979	0.002	0.291	0.001	0.295	0.017	0.001	0.631	0.001
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.006	0.001	0.007	0.737	0.002	0.095	0.896	0.005	0.616
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.014	0.001	0.006	0.671	0.002	0.064	0.949	0.002	0.742
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.686	0.006	0.057	0.001	0.641	0.011	0.002	0.916	0.001
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.910	0.009	0.494	0.003	0.320	0.074	0.001	0.546	0.001
Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.001	0.001	0.001	0.029	0.001	0.001	0.090	0.001	0.265
Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	0.755	0.001	0.522	0.002	0.144	0.052	0.001	0.343	0.001
Skewness/ Semivariance, Benchmark Rate 0, MaxSortino	0.020	0.001	0.117	0.113	0.001	0.849	0.016	0.002	0.013

	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino
Benchmark Max Sharpe	0.001	0.426	0.007	0.001	0.405	0.071	0.006	0.859	0.270
Benchmark Min Variance	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.002	0.001
Benchmark Equally Weighted (1/N)	0.002	0.717	0.110	0.001	0.745	0.687	0.001	0.467	0.541
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.762	0.001	0.003	0.696	0.001	0.004	0.398	0.001	0.023
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.001	0.001	0.001	0.001	0.005	0.001	0.001	0.232	0.024
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.002	0.019	0.750	0.002	0.016	0.100	0.033	0.033	0.433
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.533	0.001	0.007	0.718	0.001	0.003	0.686	0.001	0.006
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.001	0.036	0.001	0.001	0.063	0.003	0.001	0.502	0.050
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.264	0.001	0.009	0.442	0.001	0.002	0.931	0.001	0.001
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.000	0.001	0.003	0.798	0.001	0.001	0.466	0.002	0.016
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.002	0.000	0.003	0.001	0.931	0.066	0.002	0.766	0.318
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.002	0.006	0.000	0.007	0.005	0.066	0.036	0.060	0.489
Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.778	0.002	0.013	0.000	0.001	0.001	0.526	0.001	0.013
Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	0.001	0.935	0.008	0.001	0.000	0.022	0.003	0.757	0.341
Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	0.005	0.057	0.044	0.001	0.017	0.000	0.006	0.267	0.816
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.428	0.002	0.033	0.523	0.002	0.007	0.000	0.001	0.001
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.002	0.766	0.061	0.001	0.768	0.294	0.001	0.000	0.005
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.022	0.321	0.466	0.017	0.343	0.797	0.001	0.008	0.000
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.648	0.012	0.085	0.775	0.015	0.029	0.441	0.001	0.001
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.002	0.517	0.015	0.001	0.488	0.113	0.001	0.649	0.019
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.800	0.009	0.105	0.935	0.014	0.031	0.297	0.001	0.002
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.702	0.004	0.080	0.777	0.003	0.012	0.050	0.001	0.001
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.001	0.259	0.014	0.001	0.273	0.067	0.001	0.207	0.003
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.008	0.783	0.077	0.002	0.723	0.333	0.001	0.894	0.020
Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.062	0.001	0.001	0.123	0.001	0.001	0.251	0.001	0.001
Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	0.001	0.890	0.068	0.003	0.860	0.357	0.001	0.789	0.059
Skewness/ Semivariance, Benchmark Rate 0, MaxSortino	0.098	0.011	0.713	0.097	0.013	0.066	0.026	0.003	0.143

	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0.010	0.966	0.013	0.012	0.652	0.925	0.001	0.773	0.018
Benchmark Min Variance	0.001	0.001	0.001	0.001	0.008	0.005	0.001	0.001	0.001
Benchmark Equally Weighted (1/N)	0.012	0.262	0.006	0.002	0.061	0.464	0.001	0.530	0.096
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.607	0.001	0.699	0.636	0.001	0.003	0.036	0.002	0.098
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.001	0.299	0.002	0.001	0.640	0.306	0.001	0.162	0.001
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.062	0.010	0.068	0.071	0.015	0.082	0.002	0.083	0.850
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.945	0.001	0.889	0.959	0.001	0.001	0.102	0.002	0.021
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.002	0.623	0.006	0.001	0.932	0.559	0.001	0.375	0.002
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.752	0.001	0.586	0.743	0.001	0.001	0.270	0.001	0.014
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.676	0.001	0.791	0.686	0.001	0.004	0.065	0.003	0.107
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.011	0.525	0.015	0.004	0.243	0.741	0.001	0.883	0.007
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.080	0.014	0.104	0.098	0.015	0.096	0.002	0.087	0.733
Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.770	0.002	0.911	0.789	0.001	0.003	0.112	0.002	0.091
Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	0.015	0.502	0.010	0.009	0.290	0.711	0.001	0.835	0.012
Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	0.029	0.133	0.020	0.016	0.056	0.352	0.001	0.333	0.087
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.462	0.001	0.277	0.038	0.001	0.001	0.256	0.001	0.034
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.001	0.639	0.002	0.001	0.207	0.869	0.001	0.777	0.004
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.002	0.031	0.001	0.002	0.004	0.030	0.001	0.078	0.156
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.000	0.001	0.450	0.977	0.001	0.001	0.077	0.001	0.045
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.001	0.000	0.001	0.001	0.449	0.802	0.001	0.512	0.001
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.425	0.001	0.000	0.757	0.001	0.001	0.061	0.001	0.078
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.968	0.001	0.765	0.000	0.001	0.001	0.100	0.001	0.058
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.001	0.432	0.001	0.001	0.000	0.295	0.001	0.066	0.002
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.001	0.805	0.001	0.001	0.309	0.000	0.001	0.761	0.029
Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.086	0.001	0.041	0.099	0.001	0.001	0.000	0.001	0.001
Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	0.001	0.473	0.001	0.001	0.065	0.723	0.001	0.000	0.031
Skewness/ Semivariance, Benchmark Rate 0, MaxSortino	0.038	0.002	0.082	0.066	0.001	0.020	0.001	0.029	0.000

Red Highlights represent the 5% significance level

Table 8B: Stock Portfolios' Skewness/Semivariance Ratio Differences, P-values

Skewness/Semivariance P-Values	Benchmark Max Sharpe	Benchmark Min Variance	Benchmark Equally Weighted (1/N)	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino
Benchmark Max Sharpe	0.0000	0.8887	0.6155	0.8024	0.6301	0.9184	0.7524	0.4602	0.6881	0.9400	0.3459	0.4702	0.9004	0.3482	0.3279	0.5162	0.5392	0.2666	0.2889	0.4882	0.3266	0.3479	0.5488	0.4085	0.2216	0.4898	0.2619
Benchmark Min Variance	0.8964	0.0000	0.2016	0.9134	0.7451	0.7744	0.7441	0.6518	0.8367	0.9507	0.5412	0.4125	0.9920	0.4145	0.3442	0.4822	0.8234	0.3852	0.3072	0.6614	0.3299	0.3082	0.8291	0.5388	0.5571	0.7941	0.6088
Benchmark Equally Weighted (1/N)	0.6161	0.1979	0.0000	0.3945	0.6581	0.4029	0.7747	0.7764	0.4372	0.4765	0.9284	0.9800	0.4608	0.7884	0.7551	0.1899	0.3416	0.2582	0.1686	0.1759	0.1513	0.1496	0.2732	0.2069	0.8237	0.3682	0.9777
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.7911	0.9044	0.3812	0.0000	0.5861	0.5315	0.5222	0.4489	0.8540	0.2373	0.3066	0.2396	0.8441	0.2236	0.2236	0.6774	0.9530	0.5641	0.3602	0.8617	0.3759	0.5072	0.9730	0.6001	0.4195	0.9337	0.4139
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.6178	0.7491	0.6691	0.5735	0.0000	0.8680	0.8620	0.3419	0.4898	0.7451	0.1783	0.4245	0.7677	0.1999	0.2006	0.4089	0.3072	0.2549	0.1919	0.3026	0.2193	0.2736	0.3562	0.2176	0.3822	0.3762	0.3246
Semivariance/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.9137	0.7774	0.4022	0.5268	0.8590	0.0000	0.7614	0.6421	0.5821	0.7438	0.4239	0.3356	0.7188	0.2136	0.1943	0.4052	0.5961	0.3492	0.2066	0.4102	0.2396	0.2906	0.5378	0.2876	0.4975	0.5981	0.5462
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.7727	0.7414	0.7861	0.5325	0.8690	0.7624	0.0000	0.9097	0.3789	0.6734	0.5885	0.6561	0.7294	0.4435	0.4598	0.4379	0.4992	0.3519	0.2323	0.4245	0.2696	0.3179	0.4905	0.2373	0.5595	0.5238	0.6571
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.4588	0.6524	0.7991	0.4352	0.3372	0.6488	0.9194	0.0000	0.3645	0.5881	0.1653	0.5571	0.6631	0.2106	0.2006	0.3665	0.2343	0.2239	0.1699	0.2279	0.1739	0.2196	0.2396	0.1666	0.4845	0.2972	0.5358
Semivariance/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.6761	0.8364	0.4375	0.8620	0.4642	0.5838	0.3822	0.3915	0.0000	0.7468	0.2979	0.3382	0.8454	0.2979	0.2842	0.7511	0.8824	0.5785	0.3249	0.9750	0.3422	0.5488	0.8690	0.6588	0.2982	0.9094	0.3406
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.9437	0.9533	0.4738	0.2353	0.7358	0.7481	0.6731	0.5981	0.7134	0.0000	0.3892	0.3286	0.8727	0.2976	0.2806	0.5925	0.8087	0.4885	0.3346	0.6934	0.3599	0.4199	0.8071	0.4865	0.4725	0.8017	0.5058
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.3259	0.5278	0.9337	0.3062	0.1986	0.4162	0.5725	0.1729	0.3036	0.3972	0.0000	0.7641	0.5042	0.5462	0.5048	0.2786	0.1783	0.1999	0.1450	0.1583	0.1669	0.1879	0.1706	0.1476	0.6778	0.2133	0.9437
Semivariance/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.4555	0.4069	0.9837	0.2466	0.4089	0.3189	0.6841	0.5651	0.3362	0.3376	0.7631	0.0000	0.4295	0.3839	0.3366	0.2359	0.2469	0.2389	0.1296	0.1546	0.1326	0.1879	0.2046	0.1619	0.6898	0.2566	0.9217
Semivariance/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.9020	0.9913	0.4602	0.8374	0.7767	0.7248	0.7224	0.6598	0.8361	0.8644	0.4975	0.4229	0.0000	0.4095	0.3882	0.6215	0.8840	0.5118	0.3472	0.7684	0.3446	0.4122	0.8910	0.6091	0.5288	0.8704	0.5998
Semivariance/ Semivariance, Benchmark Rate 0, MaxSharpe	0.3416	0.4095	0.7897	0.2449	0.2089	0.2209	0.4595	0.2146	0.2999	0.2876	0.5358	0.3952	0.3885	0.0000	0.8234	0.2056	0.1686	0.1726	0.1426	0.1296	0.1616	0.1436	0.1443	0.1396	0.8157	0.1853	0.7531
Semivariance/ Semivariance, Benchmark Rate 0, MaxSortino	0.3522	0.3372	0.7587	0.2183	0.2059	0.2056	0.4588	0.1859	0.2842	0.2782	0.4815	0.3376	0.3805	0.8117	0.0000	0.2063	0.1563	0.1583	0.1183	0.1066	0.1306	0.1456	0.1260	0.1336	0.8294	0.1649	0.7298
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.5365	0.4972	0.1833	0.6908	0.4205	0.4009	0.4399	0.3559	0.7777	0.5955	0.2916	0.2466	0.5991	0.2063	0.1996	0.0000	0.5871	0.6761	0.1536	0.6534	0.2023	0.2146	0.5392	0.9703	0.3775	0.5895	0.3755
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.5478	0.8307	0.3222	0.9567	0.3202	0.5815	0.5088	0.2419	0.8764	0.8137	0.1736	0.2333	0.8767	0.1663	0.1526	0.5951	0.0000	0.3216	0.2919	0.7031	0.3092	0.3106	0.9387	0.3629	0.3339	0.9250	0.2396
Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	0.2749	0.3799	0.2636	0.5698	0.2566	0.3515	0.3605	0.2266	0.5921	0.4938	0.1926	0.2209	0.5092	0.1833	0.1676	0.6778	0.3276	0.0000	0.5868	0.4469	0.6298	0.8940	0.3196	0.6904	0.1430	0.2493	0.1916
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.2886	0.3169	0.1513	0.3549	0.1929	0.2143	0.2366	0.1430	0.3146	0.3406	0.1606	0.1243	0.3412	0.1350	0.1236	0.1483	0.2906	0.5961	0.0000	0.3689	0.9387	0.3839	0.3069	0.2652	0.2443	0.3299	0.2622
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.4918	0.6541	0.1653	0.8640	0.3022	0.4299	0.4239	0.2193	0.9757	0.7014	0.1416	0.1536	0.7937	0.1266	0.1183	0.6628	0.7001	0.4422	0.3659	0.0000	0.4049	0.4179	0.6688	0.5955	0.2423	0.8444	0.2659
Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.3279	0.3286	0.1636	0.3902	0.2236	0.2313	0.2539	0.1906	0.3439	0.3595	0.1703	0.1513	0.3855	0.1683	0.1393	0.1846	0.3202	0.6355	0.9377	0.4009	0.0000	0.4632	0.3306	0.3019	0.2512	0.3825	0.2816
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.3386	0.2972	0.1573	0.5065	0.2702	0.2769	0.3119	0.2293	0.5288	0.4345	0.1969	0.1936	0.4345	0.1390	0.1293	0.2013	0.3316	0.9030	0.3735	0.4075	0.4722	0.0000	0.2869	0.5998	0.2746	0.2932	0.2379
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.5712	0.8111	0.2634	0.9685	0.3583	0.5597	0.4873	0.2459	0.8701	0.7991	0.1744	0.2129	0.8966	0.1404	0.1369	0.5347	0.9275	0.3278	0.2924	0.6587	0.3483	0.3018	0.0000	0.4658	0.2829	0.8656	0.2584
Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.3998	0.5327	0.2084	0.6067	0.2284	0.2864	0.2539	0.1694	0.6422	0.4733	0.1544	0.1659	0.6127	0.1329	0.1344	0.9760	0.3708	0.6987	0.2809	0.5777	0.3228	0.5932	0.4413	0.0000	0.3193	0.5162	0.2064
Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	0.2144	0.5727	0.8436	0.4223	0.4208	0.4978	0.5452	0.4908	0.3208	0.4883	0.6847	0.6927	0.5502	0.8101	0.8451	0.3778	0.3313	0.1314	0.2239	0.2439	0.2734	0.2889	0.2924	0.3358	0.0000	0.4643	0.4258
Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	0.5047	0.8026	0.3763	0.9325	0.3973	0.5757	0.5257	0.3003	0.9020	0.7966	0.2184	0.2619	0.8776	0.1824	0.1844	0.5827	0.9180	0.2619	0.3488	0.8306	0.3633	0.2964	0.8666	0.4943	0.4433	0.0000	0.2069
Skewness/ Semivariance, Benchmark Rate 0, MaxSortino	0.2534	0.6047	0.9705	0.4178	0.3253	0.5462	0.6807	0.5512	0.3498	0.5057	0.9385	0.9220	0.5812	0.7376	0.7481	0.3738	0.2514	0.1914	0.2464	0.2529	0.2604	0.2489	0.2599	0.2109	0.4258	0.2009	0.0000

Red Represents significance at the 5% level

Appendix C: Dataset 2, Asset Class Portfolios' Tables and Figures
Table 1C: Asset Class Descriptive Statistics of Portfolios

	Semivariance/ Semivariance Benchmark Rate MinVar MaxSkewSemi	*Semivariance/ Semivariance Benchmark Rate MinVar MaxSharpe*	*Semivariance/ Semivariance Benchmark Rate MinVar Max Sortino*	*Semivariance/ Semivariance Benchmark Rate 0 MaxSkewSemi*	*Semivariance/ Semivariance Benchmark Rate 0 MaxSharpe*	*Semivariance/ Semivariance Benchmark Rate 0 Max Sortino*	*Semivariance/ Semivariance Benchmark Rate 1/N MaxSkewSemi*	*Semivariance/ Semivariance Benchmark Rate 1/N MaxSharpe*	*Semivariance/ Semivariance Benchmark Rate 1/N Max Sortino*	*Semivariance/ Semivariance Benchmark Rate 1/N MaxSkewSemi*	*Semivariance/ Semivariance Benchmark Rate MaxSharpe MaxSharpe*	*Semivariance/ Semivariance Benchmark Rate MaxSharpe Max Sortino*	*Benchmark Minimum Variance*	*Benchmark Max Sharpe*
Mean	0.0078	0.0066	0.0074	0.0071	0.0054	0.0066	0.0073	0.0076	0.0073	0.0078	0.0074	0.0077	0.0065	0.0071
Standard Error	0.0026	0.0025	0.0023	0.0023	0.0025	0.0019	0.0021	0.0025	0.0020	0.0024	0.0024	0.0026	0.0018	0.0027
Median	0.0077	0.0106	0.0103	0.0063	0.0077	0.0072	0.0066	0.0081	0.0081	0.0088	0.0100	0.0099	0.0060	0.0072
Standard Deviation	0.0271	0.0264	0.0244	0.0239	0.0262	0.0194	0.0219	0.0265	0.0209	0.0246	0.0254	0.0270	0.0193	0.0282
Sample Variance	0.0007	0.0007	0.0006	0.0006	0.0007	0.0004	0.0005	0.0007	0.0004	0.0006	0.0006	0.0007	0.0004	0.0008
Kurtosis	0.3024	-0.3112	-0.3320	0.1348	-0.1393	0.4228	0.0162	0.2007	-0.1983	0.2802	-0.1980	-0.2759	1.0224	0.2383
Skewness	0.3063	0.0947	-0.0067	-0.1951	0.0718	-0.1514	-0.1052	0.2926	-0.0777	0.1011	0.1360	0.0891	-0.3878	0.3412
Range	0.1397	0.1179	0.1131	0.1190	0.1193	0.1028	0.1094	0.1250	0.0981	0.1321	0.1152	0.1269	0.1153	0.1401
Minimum	-0.0517	-0.0456	-0.0456	-0.0559	-0.0471	-0.0496	-0.0495	-0.0442	-0.0428	-0.0462	-0.0430	-0.0547	-0.0555	-0.0503
Maximum	0.0879	0.0722	0.0675	0.0632	0.0722	0.0533	0.0599	0.0807	0.0553	0.0860	0.0722	0.0722	0.0599	0.0898

	Benchmark 1/N	*Skewness/ Semivariance Benchmark Rate MinVar MaxSkewSemi*	*Skewness/ Semivariance Benchmark Rate MinVar MaxSharpe*	*Skewness/ Semivariance Benchmark Rate MinVar Max Sortino*	*Skewness/ Semivariance Benchmark Rate 0 MaxSkewSemi*	*Skewness/ Semivariance Benchmark Rate 0 MaxSharpe*	*Skewness/ Semivariance Benchmark Rate 0 Max Sortino*	*Skewness/ Semivariance Benchmark Rate 1/N MaxSkewSemi*	*Skewness/ Semivariance Benchmark Rate 1/N MaxSharpe*	*Skewness/ Semivariance Benchmark Rate 1/N Max Sortino*	*Skewness/ Semivariance Benchmark Rate 1/N MaxSkewSemi*	*Skewness/ Semivariance Benchmark Rate MaxSharpe MaxSharpe*	*Skewness/ Semivariance Benchmark Rate MaxSharpe Max Sortino*	
Mean	0.0057	0.0058	0.0059	0.0061	0.0056	0.0059	0.0065	0.0057	0.0059	0.0067	0.0057	0.0060	0.0057	
Standard Error	0.0024	0.0021	0.0021	0.0022	0.0022	0.0021	0.0018	0.0022	0.0021	0.0019	0.0022	0.0021	0.0022	
Median	0.0059	0.0054	0.0070	0.0066	0.0057	0.0072	0.0076	0.0057	0.0070	0.0072	0.0067	0.0072	0.0068	
Standard Deviation	0.0246	0.0217	0.0218	0.0227	0.0228	0.0219	0.0185	0.0226	0.0218	0.0203	0.0229	0.0218	0.0228	
Sample Variance	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0003	0.0005	0.0005	0.0004	0.0005	0.0005	0.0005	
Kurtosis	0.3145	0.9033	0.6302	0.6051	0.6630	0.6071	0.7796	0.7151	0.6301	0.7296	0.7255	0.6527	0.5428	
Skewness	0.0345	-0.1487	-0.1792	-0.2003	-0.1777	-0.1733	-0.3526	-0.1608	-0.1907	-0.3065	-0.1869	-0.1872	-0.1847	
Range	0.1346	0.1326	0.1257	0.1286	0.1313	0.1272	0.1072	0.1327	0.1259	0.1191	0.1328	0.1254	0.1282	
Minimum	-0.0674	-0.0610	-0.0553	-0.0553	-0.0592	-0.0552	-0.0552	-0.0594	-0.0555	-0.0555	-0.0596	-0.0550	-0.0550	
Maximum	0.0672	0.0715	0.0704	0.0732	0.0722	0.0720	0.0520	0.0733	0.0704	0.0636	0.0733	0.0704	0.0732	

Table 2C: Asset Class Portfolios' Correlation Matrix

	Semivariance/ Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/ Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/ Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/ Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/ Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/ Semivariance Benchmark Rate 0, MaxSortino	Semivariance/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/ Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/ Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/ Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted (1/N)	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance Benchmark Rate 0, MaxSortino	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	1																										
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.865982178	1																									
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.869554595	0.933011025	1																								
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.926428197	0.848660907	0.846406065	1																							
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.827185078	0.980176733	0.923776641	0.845092728	1																						
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.843209834	0.845006652	0.872769669	0.877080549	0.852338826	1																					
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.904032746	0.868904913	0.872525631	0.961902855	0.86842617	0.870254609	1																				
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.761439005	0.912594646	0.862592613	0.794733835	0.920590272	0.781220179	0.825687698	1																			
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.845030063	0.916780827	0.957699434	0.871708106	0.926525661	0.905714981	0.907532365	0.86469623	1																		
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.954994299	0.860469061	0.843233427	0.936656628	0.81922579	0.840217794	0.907738162	0.76255012	0.837318714	1																	
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.855158546	0.974357466	0.929825086	0.861775851	0.962283454	0.857746121	0.873096652	0.920958961	0.923582316	0.874946732	1																
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.858544176	0.952342081	0.912675937	0.872336563	0.937014687	0.813324798	0.895149446	0.896146389	0.90636917	0.88338339	0.961501751	1															
Benchmark Min Variance	0.70673023	0.726367534	0.759237572	0.732820814	0.730705105	0.91768885	0.750828374	0.702856397	0.802567621	0.703943676	0.75312458	0.69094764	1														
Benchmark Max Sharpe	0.66385279	0.816710386	0.817385903	0.674259971	0.82910484	0.726879439	0.713547799	0.880114543	0.816971771	0.656208648	0.826537354	0.803278773	0.681015532	1													
Benchmark Equally Weighted (1/N)	0.714375373	0.808822221	0.7660568	0.700609955	0.802863303	0.737940116	0.696468888	0.810053148	0.771768144	0.704590994	0.804163938	0.775702282	0.724603824	0.823041704	1												
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0.799072481	0.795500226	0.836546593	0.877064119	0.813910247	0.869580061	0.877070255	0.763375475	0.880459389	0.792954436	0.832365837	0.791447013	0.740327271	0.70037839	0.677348954	1											
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.78651522	0.811214112	0.866849227	0.859471582	0.831677108	0.893087651	0.865465776	0.780870514	0.900275012	0.779814871	0.844369352	0.800261865	0.76948637	0.724965287	0.693645944	0.967334004	1										
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	0.775968312	0.795868457	0.878121277	0.841821667	0.816475871	0.873341072	0.854930417	0.765831292	0.886382337	0.760311422	0.825904817	0.783340189	0.746005121	0.716300811	0.657150869	0.953810136	0.986991947	1									
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	0.812043671	0.808019034	0.820574559	0.904646084	0.830764245	0.843023924	0.898790014	0.776889677	0.861274305	0.811820848	0.845840383	0.808067651	0.707291066	0.689917354	0.674369482	0.973178357	0.942375416	0.931188562	1								
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	0.800712597	0.825906849	0.868621625	0.877505094	0.847638937	0.892955785	0.885061417	0.796098374	0.901729287	0.79565892	0.858737834	0.814273211	0.76640595	0.726765259	0.699906968	0.968156513	0.99663908	0.983771797	0.95819241	1							
Skewness/Semivariance Benchmark Rate 0, MaxSortino	0.790812404	0.793147537	0.830088925	0.85932843	0.815270927	0.955103925	0.863225097	0.757024972	0.883776076	0.784480336	0.82403106	0.765825359	0.907062762	0.68403959	0.703763682	0.879480427	0.885299493	0.868209944	0.857485743	0.888327998	1						
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.808785715	0.805549941	0.818772923	0.900778374	0.828529955	0.842866582	0.89609908	0.774608421	0.861529237	0.808941541	0.843708583	0.804055206	0.70858781	0.689710723	0.674603506	0.97496238	0.944522333	0.932810453	0.999705943	0.959665388	0.857243563	1					
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.784808932	0.809349739	0.866088205	0.858352746	0.829956997	0.892482357	0.863661983	0.77883135	0.899686677	0.778681296	0.842755097	0.798648491	0.768957905	0.724331119	0.693323041	0.965843717	0.999865147	0.987003781	0.941112306	0.996077644	0.884457509	0.943340794	1				
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.801445007	0.809735402	0.864150216	0.88621772	0.829870824	0.9248264	0.886852376	0.77160558	0.923702949	0.79860774	0.837177297	0.791862268	0.82549068	0.725034421	0.706639933	0.937929286	0.95383525	0.94010337	0.915047209	0.951531205	0.933904637	0.917284531	0.954801097	1			
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.807295953	0.806476952	0.822767206	0.901890026	0.829734768	0.841768845	0.898222531	0.776368912	0.860114213	0.805435765	0.843353361	0.807634955	0.706926116	0.689988925	0.670695737	0.97311064	0.945088654	0.937198402	0.999353046	0.960529248	0.856890175	0.999210066	0.943886542	0.915176801	1		
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.788188087	0.809963335	0.867821913	0.857202991	0.831285591	0.889342279	0.867721681	0.77944122	0.901381906	0.778399036	0.843168924	0.799414319	0.770812769	0.727254758	0.694666385	0.964551149	0.998574247	0.986770533	0.94418674	0.996345669	0.887132823	0.946366365	0.998494269	0.953089893	0.947020941	1	
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.769768361	0.799652321	0.868417109	0.841678321	0.82001208	0.861266108	0.847878838	0.770433644	0.876008713	0.75801892	0.831974814	0.79009295	0.734879271	0.713880959	0.677619915	0.947905609	0.987037649	0.991502382	0.931668628	0.985079711	0.854011773	0.933196428	0.987272179	0.930658972	0.93759938	0.98678682	1

Table 3C: Asset Class F-Test outcomes, P-values

F-test	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance Benchmark Rate 0, MaxSortino	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted (1/N)	Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance Benchmark Rate 0, MaxSharpe	Skewness/Semivariance Benchmark Rate 0, MaxSortino	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0	0.780820883	0.276227961	0.194690594	0.745114564	0.000568101	0.028959939	0.81174007	0.008085478	0.318383379	0.518362443	0.971923446	0.00051853	0.684685657	0.320488479	0.023544811	0.024396495	0.065667664	0.073042791	0.030084799	0.000105238	0.064854049	0.026867219	0.002921553	0.082238136	0.024112671	0.0732321
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.780820883	0	0.41727855	0.307999307	0.962634162	0.001498004	0.056332976	0.96803431	0.017585128	0.471525527	0.713040623	0.807959726	0.0013754	0.493881895	0.474199203	0.046689122	0.048220164	0.117750826	0.129504496	0.058311013	0.0003077	0.116444475	0.052630787	0.006877142	0.143953265	0.047710571	0.129804188
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.276227961	0.41727855	0	0.834700359	0.444646327	0.017411365	0.270795133	0.394679711	0.116404832	0.927273193	0.657271791	0.292006773	0.016267808	0.135336998	0.923824817	0.237063611	0.242568488	0.450039988	0.479617888	0.277456714	0.004857873	0.446679597	0.25810217	0.057282838	0.514490244	0.240742949	0.480357253
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.194690594	0.307999307	0.834700359	0	0.330708198	0.029878149	0.371840334	0.289416443	0.173072188	0.764222989	0.514259242	0.207033521	0.02804163	0.088973736	0.760914661	0.329938608	0.336825735	0.584467269	0.618194608	0.380033808	0.009024015	0.58061364	0.356155612	0.090261708	0.657530884	0.334543996	0.619033355
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.745114564	0.962634162	0.444646327	0.330708198	0	0.001752346	0.062605045	0.930733783	0.019912855	0.50081783	0.748255797	0.771893076	0.001610536	0.464814779	0.503580234	0.052055066	0.053732615	0.129091843	0.141710342	0.064764286	0.000366299	0.127687604	0.058559585	0.007891968	0.157185188	0.053174375	0.142031722
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.000568101	0.001498004	0.017411365	0.029878149	0.001752346	0	0.198998532	0.001308024	0.415153461	0.013588884	0.004880944	0.000644535	0.979832647	0.000122869	0.013427258	0.228848315	0.223567896	0.103252252	0.093450151	0.193746217	0.656151606	0.104439843	0.20955853	0.6299734	0.083234917	0.225299745	0.093219265
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.028959939	0.056332976	0.270795133	0.371840334	0.062605045	0.198998532	0	0.051393265	0.637860737	0.233170256	0.122788725	0.03161734	0.190334438	0.00974215	0.231478623	0.935147651	0.946136825	0.728927517	0.692822056	0.987852605	0.084102753	0.733154278	0.976273947	0.421514291	0.652624124	0.942511082	0.691943946
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.81174007	0.96803431	0.394679711	0.289416443	0.930733783	0.001308024	0.051393265	0	0.015787708	0.447241383	0.683394556	0.839143028	0.001199943	0.519500944	0.449836902	0.04247755	0.043891081	0.108686312	0.119728494	0.053225316	0.000264691	0.107460359	0.047967293	0.006104215	0.133330397	0.043420517	0.120010317
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.008085478	0.017585128	0.116404832	0.173072188	0.019912855	0.415153461	0.637860737	0.015787708	0	0.096759468	0.044378422	0.008953065	0.40085408	0.002310811	0.095895979	0.697012474	0.686828432	0.413965234	0.386848213	0.627031715	0.208018181	0.417180308	0.659237843	0.738834389	0.357384769	0.690181184	0.38619642
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.318383379	0.471525527	0.927273193	0.764222989	0.50081783	0.013588884	0.233170256	0.447241383	0.096759468	0	0.724508322	0.335714266	0.012671403	0.160751518	0.996536473	0.202905789	0.207829496	0.397279872	0.42482752	0.239172984	0.003662213	0.394157894	0.221755873	0.046412048	0.457460142	0.206196006	0.425517686
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.518362443	0.713040623	0.657271791	0.514259242	0.748255797	0.004880944	0.122788725	0.683394556	0.044378422	0.724508322	0	0.541392984	0.004516942	0.293115998	0.727764847	0.104360325	0.107322664	0.23087741	0.250240587	0.126504801	0.00115106	0.228703853	0.115777358	0.019268754	0.27359197	0.106338303	0.250729916
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.971923446	0.807959726	0.292006773	0.207033521	0.771893076	0.000644535	0.03161734	0.839143028	0.008953065	0.335714266	0.541392984	0	0.000588732	0.659025591	0.337893273	0.025768097	0.026689146	0.070955969	0.078811688	0.032830442	0.00012096	0.070088522	0.029358745	0.00326753	0.088588799	0.026382253	0.079013164
Benchmark Min Variance	0.00051853	0.0013754	0.016267808	0.02804163	0.001610536	0.979832647	0.190334438	0.001199943	0.40085408	0.012671403	0.004516942	0.000588732	0	0.000111197	0.012519516	0.219250657	0.214131145	0.098044526	0.088645855	0.18525262	0.674511073	0.099183993	0.200557408	0.612134688	0.078863566	0.215810025	0.088424609
Benchmark Max Sharpe	0.684685657	0.493881895	0.135336998	0.088973736	0.464814779	0.000122869	0.00974215	0.519500944	0.002310811	0.160751518	0.293115998	0.659025591	0.000111197	0	0.162044828	0.007700289	0.008017112	0.02490291	0.028166251	0.010174077	1.96E-05	0.024546761	0.008945502	0.000744406	0.032318892	0.007911343	0.028250825
Benchmark Equally Weighted (1/N)	0.320488479	0.474199203	0.923824817	0.760914661	0.503580234	0.013427258	0.231478623	0.449836902	0.095895979	0.996536473	0.727764847	0.337893273	0.012519516	0.162044828	0	0.201376049	0.206272692	0.394867738	0.422316693	0.237450464	0.003612697	0.391757312	0.220124067	0.04594791	0.454839712	0.204648148	0.42300446
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0.023544811	0.046689122	0.237063611	0.329938608	0.052055066	0.228848315	0.935147651	0.04247755	0.697012474	0.202905789	0.104360325	0.025768097	0.219250657	0.007700289	0.201376049	0	0.988980439	0.668727852	0.633815943	0.92304812	0.099655623	0.672823897	0.958823691	0.469964798	0.595110692	0.992614833	0.632968586
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.024396495	0.048220164	0.242568488	0.336825735	0.053732615	0.223567896	0.946136825	0.043891081	0.686828432	0.207829496	0.107322664	0.026689146	0.214131145	0.008017112	0.206272692	0.988980439	0	0.678809021	0.64368091	0.934023507	0.096864223	0.682928892	0.96983245	0.461529461	0.604708042	0.996365373	0.642828011
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	0.065667664	0.117750826	0.450039988	0.584467269	0.129091843	0.103252252	0.728927517	0.108686312	0.413965234	0.397279872	0.23087741	0.070955969	0.098044526	0.02490291	0.394867738	0.668727852	0.678809021	0	0.961324858	0.740394107	0.038353365	0.995514789	0.70670569	0.250305393	0.91748308	0.675477506	0.960376072
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	0.073042791	0.129504496	0.479617888	0.618194608	0.141710342	0.093450151	0.692822056	0.119728494	0.386848213	0.42482752	0.250240587	0.078811688	0.088645855	0.028166251	0.422316693	0.633815943	0.64368091	0.961324858	0	0.70408762	0.034085427	0.956845555	0.671013656	0.230938626	0.956047472	0.640420111	0.999050065
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	0.030084799	0.058311013	0.277456714	0.380033808	0.064764286	0.193746217	0.987852605	0.053225316	0.627031715	0.239172984	0.126504801	0.032830442	0.18525262	0.010174077	0.237450464	0.92304812	0.934023507	0.740394107	0.70408762	0	0.081424326	0.74464265	0.964134709	0.412787129	0.663633901	0.930402057	0.703204285
Skewness/Semivariance Benchmark Rate 0, MaxSortino	0.000105238	0.0003077	0.004857873	0.009024015	0.000366299	0.656151606	0.084102753	0.000264691	0.208018181	0.003662213	0.00115106	0.00012096	0.674511073	1.96E-05	0.003612697	0.099655623	0.096864223	0.038353365	0.034085427	0.081424326	0	0.038876468	0.089541329	0.354167264	0.029735932	0.097777869	0.033985987
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.064854049	0.116444475	0.446679597	0.58061364	0.127687604	0.104439843	0.733154278	0.107460359	0.417180308	0.394157894	0.228703853	0.070088522	0.099183993	0.024546761	0.391757312	0.672823897	0.682928892	0.995514789	0.956845555	0.74464265	0.038876468	0	0.710887418	0.252622134	0.913023293	0.679589591	0.955897047
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.026867219	0.052630787	0.25810217	0.356155612	0.058559585	0.20955853	0.976273947	0.047967293	0.659237843	0.221755873	0.115777358	0.029358745	0.200557408	0.008945502	0.220124067	0.958823691	0.96983245	0.70670569	0.671013656	0.964134709	0.089541329	0.710887418	0	0.438872141	0.63133733	0.966200747	0.670146246
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.002921553	0.006877142	0.057282838	0.090261708	0.007891968	0.6299734	0.421514291	0.006104215	0.738834389	0.046412048	0.019268754	0.00326753	0.612134688	0.000744406	0.04594791	0.469964798	0.461529461	0.250305393	0.230938626	0.412787129	0.354167264	0.252622134	0.438872141	0	0.210253986	0.464302215	0.23047697
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.082238136	0.143953265	0.514490244	0.657530884	0.157185188	0.083234917	0.652624124	0.133330397	0.357384769	0.457460142	0.27359197	0.088588799	0.078863566	0.032318892	0.454839712	0.595110692	0.604708042	0.91748308	0.956047472	0.663633901	0.029735932	0.913023293	0.63133733	0.210253986	0	0.601534892	0.95699599
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.024112671	0.047710571	0.240742949	0.334543996	0.053174375	0.225299745	0.942511082	0.043420517	0.690181184	0.206196006	0.106338303	0.026382253	0.215810025	0.007911343	0.204648148	0.992614833	0.996365373	0.675477506	0.640420111	0.930402057	0.097777869	0.679589591	0.966200747	0.464302215	0.601534892	0	0.639569026
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.0732321	0.129804188	0.480357253	0.619033355	0.142031722	0.093219265	0.691943946	0.120010317	0.38619642	0.425517686	0.250729916	0.079013164	0.088424609	0.028250825	0.42300446	0.632968586	0.642828011	0.960376072	0.999050065	0.703204285	0.033985987	0.955897047	0.670146246	0.23047697	0.95699599	0.639569026	0

Red Highlights represent significance at the 5% level

Table 4C: Asset Class T-Test outcomes, P-values

	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance Benchmark Rate 0, MaxSortino	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted(1/N)	Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance Benchmark Rate 0, MaxSharpe	Skewness/Semivariance Benchmark Rate 0, MaxSortino	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0	0.744039957	0.906316953	0.856334184	0.509525426	0.708595564	0.893110263	0.960224931	0.885463621	0.993702271	0.910319619	0.985946113	0.681908233	0.8653517	0.559230754	0.569521028	0.58959423	0.631815447	0.543536596	0.59230491	0.700024776	0.54695603	0.590941001	0.755848336	0.546151979	0.610915085	0.557531659
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.744039957	0	0.82255061	0.870444272	0.735984591	0.997455111	0.823233112	0.77962907	0.825853426	0.726140787	0.823854212	0.757144656	0.968362802	0.88220166	0.80219259	0.828102636	0.85128337	0.895084195	0.792564863	0.853540697	0.992341464	0.797175823	0.852441307	0.956397051	0.795064221	0.875846308	0.809024262
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.906316953	0.82255061	0	0.94743877	0.565422439	0.793326618	0.990422398	0.946825159	0.984018184	0.894850792	0.997673169	0.920829068	0.763756679	0.949975393	0.621867786	0.635822426	0.658208594	0.704231488	0.60592238	0.661063756	0.784889602	0.609838382	0.659634501	0.844163572	0.608744759	0.681973581	0.621543713
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.856334184	0.870444272	0.94743877	0	0.605304004	0.847687859	0.954335671	0.896237439	0.959679689	0.842183896	0.94625187	0.870749575	0.817280328	0.998128191	0.665356306	0.68236606	0.705610151	0.752461155	0.650331238	0.708413399	0.839872387	0.654513638	0.707017321	0.898849494	0.653167515	0.73027486	0.666596785
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.509525426	0.735984591	0.565422439	0.605304004	0	0.702964633	0.554791645	0.53740003	0.552545991	0.484219445	0.57133469	0.520310867	0.730027054	0.635330882	0.922274791	0.88053953	0.857292266	0.818471602	0.922459168	0.855965971	0.703512026	0.917062282	0.856558251	0.665091061	0.920531015	0.832719518	0.905533392
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.708595564	0.997455111	0.793326618	0.847687859	0.702964633	0	0.790671296	0.747548263	0.792253033	0.68420664	0.796171667	0.723098628	0.965235674	0.864788331	0.775513294	0.801607516	0.828659856	0.880526578	0.76189521	0.831488147	0.99389663	0.767033935	0.830100397	0.945322561	0.764955988	0.8573339	0.780881761
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.893110263	0.823233112	0.990422398	0.954335671	0.554791645	0.790671296	0	0.935216136	0.993412654	0.88007885	0.988209569	0.908225091	0.759216224	0.956575255	0.612536577	0.625352601	0.648902905	0.697809724	0.594693038	0.652011762	0.78139873	0.598710786	0.650451215	0.844948204	0.597729433	0.673909444	0.61105056
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.960224931	0.77962907	0.946825159	0.896237439	0.53740003	0.747548263	0.935216136	0	0.928150041	0.951948598	0.950110303	0.974359393	0.71996302	0.902451752	0.589817316	0.601935048	0.622794536	0.666121849	0.574385197	0.625519629	0.739190381	0.5780023	0.624152381	0.795711614	0.577049585	0.644944094	0.588949293
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.885463621	0.825853426	0.984018184	0.959679689	0.552545991	0.792253033	0.993412654	0.928150041	0	0.871621071	0.981926821	0.90080646	0.759981651	0.961530339	0.610949986	0.623403866	0.647465477	0.697588962	0.592359143	0.650677398	0.782661294	0.596427768	0.649063756	0.847957659	0.5954844	0.673016761	0.609042235
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.993702271	0.726140787	0.894850792	0.842183896	0.484219445	0.68420664	0.88007885	0.951948598	0.871621071	0	0.899390125	0.978976828	0.656177021	0.85330421	0.533630266	0.541539164	0.562307654	0.606851344	0.515777997	0.56527657	0.674536861	0.519177263	0.563776128	0.73455406	0.518584528	0.584376955	0.530175647
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.910319619	0.823854212	0.997673169	0.94625187	0.57133469	0.796171667	0.988209569	0.950110303	0.981926821	0.899390125	0	0.924566996	0.76735561	0.948748479	0.627291192	0.641749913	0.663670311	0.708520367	0.612138228	0.66642122	0.788085547	0.616015384	0.665046011	0.845603326	0.614873171	0.686938344	0.627465127
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.985946113	0.757144656	0.920829068	0.870749575	0.520310867	0.723098628	0.908225091	0.974359393	0.90080646	0.978976828	0.924566996	0	0.696162175	0.878731036	0.570951654	0.58196014	0.602271219	0.644772499	0.555435839	0.60497625	0.714642952	0.558923168	0.603616776	0.770535423	0.558059951	0.623842144	0.56960734
Benchmark Min Variance	0.681908233	0.968362802	0.763756679	0.817280328	0.730027054	0.965235674	0.759216224	0.71996302	0.759981651	0.656177021	0.76735561	0.696162175	0	0.837329486	0.804644556	0.833226377	0.860522222	0.912067898	0.792246557	0.863233698	0.970602213	0.797540365	0.86191087	0.911443419	0.795251157	0.889444975	0.811456871
Benchmark Max Sharpe	0.8653517	0.88220166	0.949975393	0.998128191	0.635330882	0.864788331	0.956575255	0.902451752	0.961530339	0.85330421	0.948748479	0.878731036	0.837329486	0	0.69331285	0.712024696	0.733409734	0.775704636	0.68127907	0.735820131	0.858211121	0.685291035	0.734627161	0.910597613	0.683778284	0.756092369	0.696362647
Benchmark Equally Weighted (1/N)	0.559230754	0.80219259	0.621867786	0.665356306	0.922274791	0.775513294	0.612536577	0.589817316	0.610949986	0.533630266	0.627291192	0.570951654	0.804644556	0.69331285	0	0.960831012	0.936375441	0.894192296	0.996957727	0.934719945	0.777095745	0.997606345	0.935484555	0.733631559	0.999157045	0.910509623	0.985392539
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0.569521028	0.828102636	0.635822426	0.68236606	0.88053953	0.801607516	0.625352601	0.601935048	0.623403866	0.541539164	0.641749913	0.58196014	0.833226377	0.712024696	0.960831012	0	0.973815562	0.928046399	0.955918003	0.971910315	0.803523125	0.961587223	0.972801006	0.75604512	0.95837004	0.946114179	0.974694039
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.58959423	0.85128337	0.658208594	0.705610151	0.857292266	0.828659856	0.648902905	0.622794536	0.647465477	0.562307654	0.663670311	0.602271219	0.860522222	0.733409734	0.936375441	0.973815562	0	0.953664148	0.930430903	0.99799007	0.831099746	0.936014849	0.998937706	0.782146661	0.932946594	0.972291868	0.949160548
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	0.631815447	0.895084195	0.704231488	0.752461155	0.818471602	0.880526578	0.697809724	0.666121849	0.697588962	0.606851344	0.708520367	0.644772499	0.912067898	0.775704636	0.894192296	0.928046399	0.953664148	0	0.886714395	0.955808442	0.883940166	0.892045658	0.95478481	0.833254909	0.889279225	0.980784609	0.904979826
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	0.543536596	0.792564863	0.60592238	0.650331238	0.922459168	0.76189521	0.594693038	0.574385197	0.592359143	0.515777997	0.612138228	0.555435839	0.792246557	0.68127907	0.996957727	0.955918003	0.930430903	0.886714395	0	0.928738847	0.763126953	0.994332267	0.929517513	0.718824486	0.997718669	0.903477102	0.981626967
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	0.59230491	0.853540697	0.661063756	0.708413399	0.855965971	0.831488147	0.652011762	0.625519629	0.650677398	0.56527657	0.66642122	0.60497625	0.863233698	0.735820131	0.934719945	0.971910315	0.99799007	0.955808442	0.928738847	0	0.833985392	0.93429647	0.999051768	0.785058507	0.931248439	0.974411954	0.947393875
Skewness/Semivariance Benchmark Rate 0, MaxSortino	0.700024776	0.992341464	0.784889602	0.839872387	0.703512026	0.99389663	0.78139873	0.739190381	0.782661294	0.674536861	0.788085547	0.714642952	0.970602213	0.858211121	0.777095745	0.803523125	0.831099746	0.883940166	0.763126953	0.833985392	0	0.768350958	0.832569486	0.938281423	0.766252157	0.860329113	0.782460214
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.54695603	0.797175823	0.609838382	0.654513638	0.917062282	0.767033935	0.598710786	0.5780023	0.596427768	0.519177263	0.616015384	0.558923168	0.797540365	0.685291035	0.997606345	0.961587223	0.936014849	0.892045658	0.994332267	0.93429647	0.768350958	0	0.935088975	0.723655791	0.99663464	0.908969611	0.987245452
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.590941001	0.852441307	0.659634501	0.707017321	0.856558251	0.830100397	0.650451215	0.624152381	0.649063756	0.563776128	0.665046011	0.603616776	0.86191087	0.734627161	0.935484555	0.972801006	0.998937706	0.95478481	0.929517513	0.999051768	0.832569486	0.935088975	0	0.783619832	0.932030784	0.973404357	0.948212873
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.755848336	0.956397051	0.844163572	0.898849494	0.665091061	0.945322561	0.844948204	0.795711614	0.847957659	0.73455406	0.845603326	0.770535423	0.911443419	0.910597613	0.733631559	0.75604512	0.782146661	0.833254909	0.718824486	0.785058507	0.938281423	0.723655791	0.783619832	0	0.721881769	0.809825585	0.737097809
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.546151979	0.795064221	0.608744759	0.653167515	0.920531015	0.764955988	0.597729433	0.577049585	0.5954844	0.518584528	0.614873171	0.558059951	0.795251157	0.683778284	0.999157045	0.95837004	0.932946594	0.889279225	0.997718669	0.931248439	0.766252157	0.99663464	0.932030784	0.721881769	0	0.90605939	0.983956521
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.610915085	0.875846308	0.681973581	0.73027486	0.832719518	0.8573339	0.673909444	0.644944094	0.673016761	0.584376955	0.686938344	0.623842144	0.889444975	0.756092369	0.910509623	0.946114179	0.972291868	0.980784609	0.903477102	0.974411954	0.860329113	0.908969611	0.973404357	0.809825585	0.90605939	0	0.922153116
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.557531659	0.809024262	0.621543713	0.666596785	0.905533392	0.780881761	0.61105056	0.588949293	0.609042235	0.530175647	0.627465127	0.56960734	0.811456871	0.696362647	0.985392539	0.974694039	0.949160548	0.904979826	0.981626967	0.947393875	0.782460214	0.987245452	0.948212873	0.737097809	0.983956521	0.922153116	0

Table 5C: Asset Class Portfolios' Mann-Whitney U(ranksum) test, P-Values.

	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance Benchmark Rate 0, MaxSortino	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted(1/N)	Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance Benchmark Rate 0, MaxSharpe	Skewness/Semivariance Benchmark Rate 0, MaxSortino	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0	0.946920769	0.886438818	0.92215997	0.645821538	0.878811816	0.93410554	0.988862653	0.929837409	0.904275128	0.933251735	0.894925705	0.989719331	0.804933331	0.66287197	0.71665349	0.747353001	0.806595796	0.677739231	0.776812125	0.856843504	0.670680891	0.757134151	0.938375676	0.691149464	0.773522119	0.734374742
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.946920769	0	0.841697784	0.888135186	0.652000345	0.796634257	0.911934649	0.927277038	0.927277416	0.835823762	0.880505025	0.830795686	0.921307372	0.996573026	0.762039462	0.788358346	0.843378342	0.895775163	0.71665349	0.848422636	0.92642442	0.721471916	0.841698327	0.974302062	0.731141831	0.868659938	0.841698327
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.886438818	0.841697784	0	0.96231717	0.541215177	0.702266423	0.916193435	0.905976277	0.907678185	0.998286506	0.952906219	0.985436104	0.801611053	0.763676704	0.543350014	0.616809118	0.630481068	0.695905893	0.570747057	0.648908298	0.742477529	0.556245684	0.63813203	0.806595796	0.567832426	0.667552993	0.625909367
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.92215997	0.888135186	0.96231717	0	0.626669627	0.82410293	0.964884723	0.978583901	0.953761622	0.940938508	0.989719319	0.92386548	0.899173813	0.757134151	0.549063012	0.622869523	0.710246457	0.785054556	0.621352003	0.735992968	0.798292358	0.619836092	0.724690366	0.890680791	0.630481068	0.760403285	0.723080529
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.645821538	0.652000345	0.541215177	0.626669627	0	0.892378114	0.623628784	0.613033824	0.604017164	0.5341296	0.592087797	0.55696635	0.770236176	0.699083518	0.976014723	0.964028858	0.853473145	0.819926383	0.976014723	0.836662634	0.765315003	0.986292737	0.860216475	0.705454558	0.993146116	0.813254327	0.861903922
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.878811816	0.796634257	0.702266423	0.82410293	0.892378114	0	0.758768179	0.770236046	0.71504984	0.70625216	0.72630127	0.674598533	0.902574314	0.994859559	0.702266423	0.806595796	0.955472341	0.98457953	0.776812125	0.986292737	0.948630774	0.785054556	0.962317191	0.872041577	0.803271748	0.988006006	0.952051116
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.93410554	0.911934649	0.916193435	0.964884723	0.623628784	0.758768179	0	0.960605653	0.978583864	0.941792823	0.984579521	0.929837409	0.836662634	0.790011674	0.532013523	0.606266157	0.700674313	0.750608955	0.592088011	0.724690366	0.775166617	0.586903557	0.70385984	0.855157994	0.616809118	0.740854654	0.694319076
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.988862653	0.927277038	0.905976277	0.978583901	0.613033824	0.770236046	0.960605653	0	0.970876967	0.886438818	0.974301943	0.931544434	0.836662634	0.911083253	0.624388646	0.692733597	0.781754633	0.829958808	0.65200091	0.796634372	0.861903922	0.658204076	0.790011674	0.942647473	0.667552993	0.818257127	0.763676704
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.929837409	0.927277416	0.907678185	0.953761622	0.604017164	0.71504984	0.978583864	0.970876967	0	0.945211314	0.998286508	0.931544394	0.804933331	0.798292358	0.544775485	0.60027803	0.700674313	0.766954352	0.586164486	0.727913689	0.778458635	0.585426064	0.708647873	0.852630928	0.597294032	0.757134151	0.695905893
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.904275128	0.835823762	0.998286506	0.940938508	0.5341296	0.70625216	0.941792823	0.886438818	0.945211314	0	0.926424165	0.990575982	0.778458635	0.707050571	0.498738253	0.534836376	0.616809118	0.68167325	0.539084772	0.642741245	0.70385984	0.52639055	0.621352003	0.778458635	0.549063012	0.656651056	0.606266157
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.933251735	0.880505025	0.952906219	0.989719319	0.592087797	0.72630127	0.984579521	0.974301943	0.998286508	0.926424165	0	0.955471981	0.833309206	0.880505781	0.639666893	0.673814507	0.702266423	0.763676704	0.644280723	0.703859676	0.826611499	0.645821729	0.699083518	0.899173813	0.648908298	0.734374742	0.699083518
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.894925705	0.830795686	0.985436104	0.92386548	0.55696635	0.674598533	0.929837409	0.931544434	0.931544394	0.990575982	0.955471981	0	0.781754633	0.84505907	0.585426064	0.633536761	0.665991203	0.710246457	0.603268752	0.669898109	0.766954352	0.604766622	0.667552993	0.831633625	0.615298066	0.697494042	0.655099518
Benchmark Min Variance	0.989719331	0.921307372	0.801611053	0.899173813	0.770236176	0.902574314	0.836662634	0.836662634	0.804933331	0.778458635	0.833309206	0.781754633	0	0.91619358	0.647364256	0.71504984	0.872041577	0.924718388	0.727913689	0.882200293	0.946920953	0.731141831	0.868659938	0.958894388	0.74572671	0.873733278	0.848422636
Benchmark Max Sharpe	0.804933331	0.996573026	0.763676704	0.757134151	0.699083518	0.994859559	0.790011674	0.911083253	0.798292358	0.707050571	0.880505781	0.84505907	0.91619358	0	0.763676704	0.902574314	0.933251851	0.989719331	0.866970014	0.96745268	0.941792991	0.860216475	0.93666744	0.863591999	0.872041577	0.989719331	0.92642442
Benchmark Equally Weighted (1/N)	0.66287197	0.762039462	0.543350014	0.549063012	0.976014723	0.702266423	0.532013523	0.624388646	0.544775485	0.498738253	0.639666893	0.585426064	0.647364256	0.763676704	0	0.840019032	0.798292358	0.721471916	0.872041577	0.801611053	0.656651056	0.866970014	0.798292358	0.591346365	0.866970014	0.748980419	0.801611053
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0.71665349	0.788358346	0.616809118	0.622869523	0.964028858	0.806595796	0.606266157	0.692733597	0.60027803	0.534836376	0.673814507	0.633536761	0.71504984	0.902574314	0.840019032	0	0.905976548	0.831633528	0.976871062	0.87457931	0.799121636	0.981153262	0.900023681	0.730334122	0.958894316	0.858529667	0.912786247
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.747353001	0.843378342	0.630481068	0.710246457	0.853473145	0.955472341	0.700674313	0.781754633	0.700674313	0.616809118	0.702266423	0.665991203	0.872041577	0.933251851	0.798292358	0.905976548	0	0.92557048	0.909380618	0.946920309	0.894925342	0.900023681	0.986292562	0.807425881	0.92386548	0.952906056	0.983722798
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	0.806595796	0.895775163	0.695905893	0.785054556	0.819926383	0.98457953	0.750608955	0.829958808	0.766954352	0.68167325	0.763676704	0.710246457	0.924718388	0.989719331	0.721471916	0.831633528	0.92557048	0	0.856843422	0.966596385	0.979440262	0.846740329	0.923012036	0.911082689	0.867814863	0.964028608	0.898323184
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	0.677739231	0.71665349	0.570747057	0.621352003	0.976014723	0.776812125	0.592088011	0.65200091	0.586164486	0.539084772	0.644280723	0.603268752	0.727913689	0.866970014	0.872041577	0.976871062	0.909380618	0.856843422	0	0.876271875	0.804102373	0.985436037	0.90172399	0.753053772	0.964028691	0.869504937	0.920454773
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	0.776812125	0.848422636	0.648908298	0.735992968	0.836662634	0.986292737	0.724690366	0.796634372	0.727913689	0.642741245	0.703859676	0.669898109	0.882200293	0.96745268	0.801611053	0.87457931	0.946920309	0.966596385	0.876271875	0	0.942647175	0.872041577	0.954616496	0.856842438	0.880505712	0.995716269	0.930690501
Skewness/Semivariance Benchmark Rate 0, MaxSortino	0.856843504	0.92642442	0.742477529	0.798292358	0.765315003	0.948630774	0.775166617	0.861903922	0.778458635	0.70385984	0.826611499	0.766954352	0.946920953	0.941792991	0.656651056	0.799121636	0.894925342	0.979440262	0.804102373	0.942647175	0	0.785054191	0.894925403	0.917045284	0.804933221	0.926424165	0.893227026
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.670680891	0.721471916	0.556245684	0.619836092	0.986292737	0.785054556	0.586903557	0.658204076	0.585426064	0.52639055	0.645821729	0.604766622	0.731141831	0.860216475	0.866970014	0.981153262	0.900023681	0.846740329	0.985436037	0.872041577	0.785054191	0	0.893227334	0.736802301	0.988862601	0.858529667	0.917045572
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.757134151	0.841698327	0.63813203	0.724690366	0.860216475	0.962317191	0.70385984	0.790011674	0.708647873	0.621352003	0.699083518	0.667552993	0.868659938	0.93666744	0.798292358	0.900023681	0.986292562	0.923012036	0.90172399	0.954616496	0.894925403	0.893227334	0	0.80992183	0.912786247	0.955472058	0.974301809
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.938375676	0.974302062	0.806595796	0.890680791	0.705454558	0.872041577	0.855157994	0.942647473	0.852630928	0.778458635	0.899173813	0.831633625	0.958894388	0.863591999	0.591346365	0.730334122	0.807425881	0.911082689	0.753053772	0.856842438	0.917045284	0.736802301	0.80992183	0	0.752238464	0.865280235	0.811587445
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.691149464	0.731141831	0.567832426	0.630481068	0.993146116	0.803271748	0.616809118	0.667552993	0.597294032	0.549063012	0.648908298	0.615298066	0.74572671	0.872041577	0.866970014	0.958894316	0.92386548	0.867814863	0.964028691	0.880505712	0.804933221	0.988862601	0.912786247	0.752238464	0	0.874579166	0.931544315
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.773522119	0.868659938	0.667552993	0.760403285	0.813254327	0.988006006	0.740854654	0.818257127	0.757134151	0.656651056	0.734374742	0.697494042	0.873733278	0.989719331	0.748980419	0.858529667	0.952906056	0.964028608	0.869504937	0.995716269	0.926424165	0.858529667	0.955472058	0.865280235	0.874579166	0	0.947775456
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.734374742	0.841698327	0.625909367	0.723080529	0.861903922	0.952051116	0.694319076	0.763676704	0.695905893	0.606266157	0.699083518	0.655099518	0.848422636	0.92642442	0.801611053	0.912786247	0.983722798	0.898323184	0.920454773	0.930690501	0.893227026	0.917045572	0.974301809	0.811587445	0.931544315	0.947775456	0

Table 6C: Asset Class Differences of Sharpe Ratios P-Values

	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance, Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance, Benchmark Rate 0, MaxSortino	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted (1/N)	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance, Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance, Benchmark Rate 0, MaxSortino	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance, Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0	0.3504	0.7748	0.4432	0.0944	0.3094	0.801	0.912	0.817	0.79	0.7402	0.9576	0.4126	0.7088	0.2528	0.0962	0.121	0.2078	0.0654	0.1296	0.2854	0.0682	0.1286	0.3972	0.0726	0.1464	0.1224
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.3504	0	0.368	0.6178	0.0354	0.9586	0.4306	0.2664	0.3546	0.2938	0.2026	0.1982	0.8886	0.7212	0.5896	0.4332	0.4402	0.641	0.36	0.4436	0.8802	0.3726	0.4542	0.912	0.373	0.5102	0.3916
Semivariance/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.7748	0.359	0	0.8424	0.0412	0.437	0.974	0.8816	0.9644	0.7212	0.985	0.8258	0.5144	0.8434	0.3642	0.1352	0.0842	0.1136	0.1164	0.0928	0.3788	0.1124	0.0902	0.514	0.0954	0.1086	0.0526
Semivariance/Semivariance, Benchmark Rate 0,MaxSkewSemi	0.4432	0.6132	0.8384	0	0.028	0.542	0.7074	0.728	0.7878	0.344	0.8468	0.6304	0.6144	0.9562	0.392	0.0956	0.193	0.3056	0.0606	0.1742	0.387	0.063	0.1936	0.585	0.0648	0.2364	0.1744
Semivariance/Semivariance, Benchmark Rate 0,MaxSharpe	0.0944	0.0368	0.0426	0.028	0	0.0102	0.0466	0.0138	0.0216	0.078	0.0158	0.0202	0.5214	0.2038	0.869	0.8066	0.708	0.5614	0.9166	0.69	0.3464	0.8986	0.6896	0.1944	0.893	0.6004	0.843
Semivariance/Semivariance, Benchmark Rate 0,MaxSortino	0.3094	0.958	0.4556	0.5362	0.0102	0	0.3718	0.366	0.2998	0.2686	0.4304	0.3628	0.8972	0.7058	0.6318	0.4646	0.4732	0.6626	0.4292	0.4884	0.8688	0.4306	0.4754	0.849	0.421	0.5466	0.4324
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.801	0.4322	0.9712	0.6978	0.043	0.379	0	0.8908	0.998	0.7268	0.9622	0.8004	0.477	0.8202	0.3326	0.0566	0.0658	0.1454	0.0378	0.0592	0.2624	0.0432	0.0646	0.3328	0.0414	0.0884	0.0792
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.912	0.277	0.8752	0.7446	0.0132	0.381	0.8862	0	0.8864	0.836	0.824	0.9154	0.4022	0.663	0.2392	0.095	0.0716	0.1806	0.0792	0.0802	0.2946	0.082	0.0738	0.4218	0.0756	0.105	0.071
Semivariance/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.817	0.3552	0.9684	0.7894	0.0168	0.302	0.997	0.888	0	0.7454	0.9518	0.8148	0.4468	0.8022	0.2986	0.0584	0.0454	0.1496	0.0392	0.0476	0.2214	0.0458	0.0508	0.3018	0.0456	0.0648	0.0802
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.79	0.298	0.7294	0.3486	0.078	0.272	0.7388	0.843	0.7412	0	0.6384	0.8588	0.396	0.672	0.2294	0.076	0.1212	0.2218	0.046	0.1266	0.2548	0.0562	0.1312	0.3368	0.053	0.161	0.131
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.7402	0.2006	0.982	0.847	0.0156	0.4458	0.9628	0.8278	0.9546	0.6452	0	0.647	0.5104	0.8276	0.3192	0.0936	0.086	0.2298	0.075	0.092	0.3486	0.0772	0.0946	0.5082	0.0696	0.1288	0.0968
Semivariance/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.9576	0.1972	0.8234	0.636	0.0206	0.3644	0.7898	0.9182	0.8142	0.8552	0.6552	0	0.4118	0.681	0.2098	0.0812	0.0844	0.208	0.0522	0.0912	0.3164	0.0612	0.0836	0.3922	0.0592	0.1266	0.0918
Benchmark Min Variance	0.4126	0.8794	0.4964	0.6074	0.5238	0.891	0.4778	0.4038	0.4456	0.4002	0.4908	0.4084	0	0.6348	0.6762	0.6824	0.6938	0.8216	0.6112	0.7086	0.9784	0.6236	0.6926	0.8218	0.6168	0.7768	0.6298
Benchmark Max Sharpe	0.7088	0.7088	0.8332	0.956	0.2164	0.71	0.8208	0.666	0.809	0.6682	0.8382	0.6834	0.6488	0	0.2728	0.3544	0.3596	0.4698	0.2916	0.3566	0.674	0.2982	0.3612	0.7922	0.2906	0.405	0.3214
Benchmark Equally Weighted (1/N)	0.2528	0.6054	0.36	0.3744	0.8698	0.6208	0.3216	0.2362	0.3114	0.2226	0.313	0.21	0.6796	0.2758	0	0.9944	0.9562	0.8706	0.934	0.9482	0.6856	0.9424	0.9478	0.564	0.9376	0.9006	0.971
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSkewSemi	0.0962	0.4302	0.1356	0.0956	0.8152	0.473	0.0512	0.0974	0.0626	0.0848	0.096	0.0832	0.6736	0.3542	0.9932	0	0.9004	0.7818	0.6132	0.8842	0.4862	0.671	0.8992	0.299	0.6664	0.826	0.96
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSharpe	0.121	0.437	0.092	0.1824	0.6972	0.4772	0.0618	0.0772	0.0482	0.1304	0.0894	0.0848	0.714	0.3546	0.952	0.8982	0	0.7076	0.7394	0.914	0.536	0.7566	0.8396	0.2122	0.754	0.355	0.7152
Skewness/Semivariance, Benchmark Rate Min Variance, MaxSortino	0.2078	0.6326	0.1032	0.3144	0.5734	0.6514	0.1484	0.1768	0.1534	0.2234	0.2312	0.2054	0.83	0.497	0.8658	0.7858	0.7076	0	0.5894	0.6996	0.7342	0.5992	0.7216	0.445	0.5834	0.8482	0.3956
Skewness/Semivariance, Benchmark Rate 0, MaxSkewSemi	0.0654	0.37	0.1096	0.058	0.9092	0.4142	0.045	0.0716	0.0464	0.052	0.0716	0.0594	0.5994	0.293	0.9298	0.6194	0.73	0.603	0	0.713	0.4302	0.7184	0.7328	0.1696	0.8708	0.6388	0.9336
Skewness/Semivariance, Benchmark Rate 0, MaxSharpe	0.1296	0.4488	0.0878	0.1766	0.686	0.4914	0.0522	0.072	0.051	0.1306	0.0952	0.0812	0.7178	0.3654	0.9492	0.891	0.9104	0.7058	0.7124	0	0.5504	0.7318	0.9648	0.1918	0.7158	0.4344	0.6668
Skewness/Semivariance, Benchmark Rate 0, MaxSortino	0.2854	0.8876	0.3728	0.3786	0.3634	0.8642	0.2624	0.3068	0.2164	0.2664	0.3474	0.3138	0.979	0.6654	0.6716	0.4934	0.5324	0.732	0.4218	0.5578	0	0.4266	0.552	0.7	0.4306	0.6318	0.484
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.0682	0.378	0.1084	0.065	0.903	0.4222	0.039	0.0788	0.0494	0.055	0.0794	0.0616	0.6248	0.2936	0.9392	0.6566	0.74	0.6044	0.7156	0.7366	0.4398	0	0.7574	0.1898	0.9828	0.6522	0.9472
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.1286	0.4506	0.0884	0.1846	0.6994	0.4714	0.0616	0.077	0.048	0.129	0.082	0.0888	0.696	0.3752	0.9506	0.8928	0.8356	0.722	0.7374	0.971	0.5398	0.751	0	0.2066	0.7354	0.3324	0.7076
Skewness/Semivariance, Benchmark Rate Equally Weighted(1/N), MaxSortino	0.3972	0.9094	0.4988	0.583	0.2046	0.8402	0.3368	0.4292	0.292	0.356	0.5154	0.3958	0.821	0.7904	0.5682	0.2802	0.2114	0.4332	0.1724	0.204	0.7024	0.1728	0.21	0	0.1776	0.269	0.2502
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSkewSemi	0.0726	0.3756	0.1062	0.0678	0.8948	0.4302	0.0406	0.0818	0.0414	0.0602	0.0782	0.055	0.6118	0.2982	0.9402	0.6682	0.7432	0.5874	0.8656	0.7266	0.449	0.9846	0.7516	0.181	0	0.653	0.9428
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSharpe	0.1464	0.5234	0.112	0.2306	0.6112	0.5346	0.081	0.1004	0.0634	0.1608	0.1212	0.106	0.762	0.4204	0.9022	0.8314	0.366	0.8602	0.641	0.4298	0.6226	0.6554	0.3316	0.2678	0.6442	0	0.5386
Skewness/Semivariance, Benchmark Rate Max Sharpe, MaxSortino	0.1224	0.3782	0.0476	0.176	0.85	0.437	0.072	0.0706	0.0762	0.1358	0.0934	0.0924	0.6262	0.322	0.9664	0.9644	0.7038	0.3886	0.936	0.668	0.486	0.9514	0.6892	0.253	0.9424	0.5438	0

Red highlights represent 5% significance and green represents 10% significance, after taking into account standard error

Table 7C: Asset Class Differences of Sortino Ratios P-Values

	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance Benchmark Rate 0, MaxSortino	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted (1/N)	Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/Semivariance Benchmark Rate 0, MaxSharpe	Skewness/Semivariance Benchmark Rate 0, MaxSortino	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0	0.293235588	0.95868044	0.754081973	0.077640786	0.556481173	0.498167278	0.967344219	0.330223259	0.357214262	0.922359214	0.833055648	0.83772076	0.590803066	0.392869044	0.42652492	0.440519827	0.428857048	0.209263579	0.374875042	0.527490836	0.202932356	0.416861046	0.728090636	0.185271576	0.47050983	0.290236588
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.279906698	0	0.182272576	0.390536488	0.021326225	0.12262791	0.081639454	0.195268244	0.026324558	0.206931023	0.105631456	0.227257581	0.318893702	0.787070976	0.785738087	0.82292536	0.794068644	0.880706431	0.803732089	0.837387537	0.12857014	0.848383872	0.825058314	0.17994002	0.762745751	0.733422193	0.799733422
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.961012996	0.183272243	0	0.807064312	0.02065978	0.570143286	0.529156948	0.920693102	0.29056981	0.764745085	0.895701433	0.821059647	0.838387204	0.50583139	0.37320893	0.382872376	0.27724092	0.165611463	0.2005998	0.262245918	0.51316279	0.224925025	0.244918361	0.718760413	0.19326891	0.331222926	0.089636788
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.77440853	0.386204598	0.807397534	0	0.061646118	0.37020993	0.11362879	0.880373209	0.132955681	0.389870043	0.88637121	0.999000333	0.698433855	0.659113629	0.418527158	0.381539487	0.458847051	0.485171609	0.08863712	0.401532822	0.282572476	0.107964012	0.434521826	0.411862712	0.083305565	0.522492502	0.304565145
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.075974675	0.025658114	0.023325558	0.059980007	0	0.0279067	0.00733089	0.013995335	0.001999334	0.059980007	0.009330223	0.018327224	0.121292902	0.217594135	0.623125625	0.188937021	0.136287904	0.2255947	0.341886038	0.133288904	0.028323892	0.329223592	0.133288904	0.029990003	0.355548151	0.106964345	0.395534822
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.530823059	0.128623792	0.558147284	0.392869044	0.024658447	0	0.939353549	0.424858381	0.666111296	0.752415861	0.378540487	0.347550816	0.665111629	0.205931356	0.109296901	0.091969344	0.043985338	0.053315561	0.052315895	0.04665116	0.849383539	0.056981006	0.035321559	0.604131956	0.053648784	0.052649117	0.029990003
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.419526824	0.040319893	0.429856714	0.056981006	0.000666445	0.92335888	0	0.399866711	0.524491836	0.810063312	0.374541819	0.202932356	0.791069643	0.226591136	0.130956348	0.01661113	0.019660113	0.037320893	0.03665445	0.010996335	0.81062979	0.002999	0.019660113	0.645784738	0.004331889	0.030656448	0.015661446
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.966677774	0.14285238	0.913695435	0.856381206	0.005331556	0.437854049	0.415194935	0	0.144618461	0.665444852	0.950683106	0.812395868	0.727757414	0.393202266	0.247584139	0.373875375	0.305564812	0.340219927	0.14617794	0.292902366	0.343552149	0.181272909	0.2979007	0.628123959	0.143618794	0.378540487	0.1666113
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.219593469	0.009996668	0.138953682	0.08263912	0.000666445	0.664111963	0.514161946	0.137954015	0	0.515161613	0.116294568	0.098967011	0.503498834	0.08863712	0.065978007	0.007997334	0.002999	0.021992669	0.001332889	0.003332223	0.761412862	0.001666111	0.002332556	0.23158947	0.003665445	0.008997001	0.004331889
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.252915695	0.117627458	0.709096968	0.227257581	0.019326891	0.763745418	0.813395535	0.665444852	0.527157614	0	0.570476508	0.410863046	0.953682106	0.365211596	0.175941353	0.141952682	0.168943685	0.21826058	0.04931684	0.161279573	0.695101633	0.05631462	0.193935355	0.965011663	0.046984339	0.223258914	0.113962013
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.915361546	0.05898034	0.852715761	0.885038321	0.001666111	0.385204932	0.375541486	0.956014662	0.115294902	0.550483172	0	0.790069977	0.712095968	0.507830723	0.277907364	0.334221926	0.299566811	0.354215262	0.108297234	0.268577141	0.307230923	0.14951683	0.29356881	0.543485505	0.106631123	0.372875708	0.161612796
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.793068977	0.140619793	0.76841053	0.998333889	0.006331223	0.337220926	0.204598467	0.809063645	0.093968677	0.427190936	0.772409197	0	0.641119627	0.594801733	0.294235255	0.459513496	0.423858714	0.464845052	0.161612796	0.405198267	0.29056981	0.199933356	0.416527824	0.45718094	0.170276574	0.513828724	0.248250583
Benchmark Min Variance	0.802732423	0.205598134	0.792735755	0.625791403	0.033322226	0.677107631	0.808730423	0.739420193	0.51182939	0.955348217	0.713095635	0.642119294	0	0.35688104	0.157947351	0.29690103	0.251582806	0.267577474	0.175608131	0.245584805	0.659446851	0.179273575	0.244585138	0.968010663	0.158947018	0.289903366	0.143952016
Benchmark Max Sharpe	0.531822726	0.78173942	0.46751083	0.623125625	0.176274575	0.213262246	0.22257914	0.370876375	0.089636788	0.371542819	0.516494502	0.594135288	0.353548817	0	0.51649502	0.945018327	0.945018327	0.893702099	0.622125958	0.907364212	0.194935022	0.688437188	0.931689437	0.314895035	0.608797068	0.987337554	0.651449517
Benchmark Equally Weighted (1/N)	0.304231923	0.760079973	0.289236921	0.347217594	0.575474842	0.109296901	0.136621126	0.247584139	0.077640786	0.178273909	0.292569144	0.290236588	0.14895035	0.4955015	0	0.656781073	0.654781739	0.724091969	0.916361213	0.690103299	0.107964012	0.88937021	0.651449517	0.180939687	0.939020327	0.592469177	0.919360213
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate 0, MaxSortino	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0

Red highlights significance at the 5% level

Table 8C: Asset Class Differences of Skewness/Semivariance Ratios P-Values

	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	Semivariance/Semivariance Benchmark Rate 0, MaxSortino	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	Benchmark Min Variance	Benchmark Max Sharpe	Benchmark Equally Weighted (1/N)	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSharpe	Skewness/ Semivariance Benchmark Rate Min Variance, MaxSortino	Skewness/ Semivariance Benchmark Rate 0, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate 0, MaxSharpe	Skewness/ Semivariance Benchmark Rate 0, MaxSortino	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	Skewness/ Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSharpe	Skewness/ Semivariance Benchmark Rate Max Sharpe, MaxSortino
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0	0.51974013	0.215892054	0.103448276	0.557221389	0.202398801	0.301349325	0.938530735	0.224387806	0.086456772	0.452773613	0.444277861	0.165417291	0.779610195	0.446776612	0.212393803	0.176411794	0.237381309	0.197901049	0.211394303	0.165917041	0.208895552	0.194402799	0.166916542	0.2003998	0.202898551	0.224887556
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.522738631	0	0.512243878	0.254372814	0.937031484	0.382808596	0.369815092	0.347326337	0.408795602	0.906546727	0.812093953	0.904047976	0.237381309	0.2013993	0.783608196	0.381309345	0.302848576	0.301349325	0.286356822	0.328335832	0.226886557	0.327336332	0.294852574	0.243878061	0.287856072	0.296851574	0.329835082
Semivariance/Semivariance Benchmark Rate Min Variance, MaxSortino	0.232383808	0.526236882	0	0.370314843	0.655172414	0.619690155	0.686656672	0.24037981	0.769615192	0.662168916	0.410294853	0.553723138	0.342328836	0.190404798	0.871564218	0.581209395	0.453773113	0.386806597	0.473263368	0.496751624	0.355822089	0.525737131	0.434782609	0.341329335	0.450274863	0.44077961	0.412293853
Semivariance/Semivariance Benchmark Rate 0, MaxSkewSemi	0.107446277	0.254872564	0.371814093	0	0.339830085	0.557221389	0.459270365	0.127436282	0.467766117	0.215892054	0.171914043	0.206896552	0.898550725	0.133433283	0.52173913	0.793103448	0.882558721	0.992003998	0.989505247	0.871564218	0.937031484	0.910044978	0.931534233	0.984507746	0.955022489	0.898550725	0.993503248
Semivariance/Semivariance Benchmark Rate 0, MaxSharpe	0.541229385	0.933033483	0.665167416	0.343328336	0	0.503248376	0.532233883	0.408795602	0.540729635	0.944527736	0.850074963	0.964017991	0.385807096	0.218390805	0.810594703	0.409295352	0.350324838	0.393303348	0.373313343	0.372313843	0.372313843	0.429285357	0.332833583	0.389305347	0.406796602	0.341829085	0.351824088
Semivariance/Semivariance Benchmark Rate 0, MaxSortino	0.19890055	0.387806097	0.631184408	0.546226887	0.506746627	0	0.968015992	0.141429285	0.712143928	0.494252874	0.257371314	0.383308346	0.362818591	0.168415792	0.692653673	0.835082459	0.712643678	0.660669665	0.693653173	0.755622189	0.203398301	0.75812094	0.715642179	0.477261369	0.672163918	0.71964018	0.696651674
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.306346827	0.378810595	0.71814093	0.44077961	0.530234883	0.95802099	0	0.148925537	0.804597701	0.620689655	0.266366817	0.351324338	0.473263368	0.147926037	0.698150925	0.783108446	0.670664668	0.615692154	0.607196402	0.685157421	0.510744628	0.672163918	0.636181909	0.4007996	0.586206897	0.656671664	0.632183908
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.940529735	0.367316342	0.257371314	0.12043978	0.402798601	0.162918541	0.15992004	0	0.174912544	0.485257371	0.410794603	0.330334833	0.122438781	0.47876062	0.31834083	0.170914543	0.16091954	0.166916542	0.147426287	0.15992004	0.099950025	0.261369315	0.145927036	0.109945027	0.145427286	0.156921539	0.177411294
Semivariance/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.224887556	0.417791104	0.754622689	0.464267866	0.55872064	0.732633683	0.794102949	0.168915542	0	0.586206897	0.227886057	0.35882059	0.313843078	0.15992004	0.747626187	0.669665167	0.632683658	0.505247376	0.51974013	0.602698651	0.294352824	0.595202399	0.6001999	0.237381309	0.52173913	0.572213893	0.554222889
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.106946527	0.893053473	0.672663668	0.228885557	0.944027986	0.490754623	0.596701649	0.511744128	0.587206397	0	0.828085957	0.944527736	0.340829585	0.355322339	0.895552224	0.541229385	0.493253373	0.459270365	0.456771614	0.482258871	0.35982009	0.473263368	0.475262369	0.381309345	0.466266867	0.475762119	0.496751624
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.460269865	0.832083958	0.393803098	0.164417791	0.83958021	0.258870565	0.248875562	0.415292354	0.24137931	0.834082959	0	0.754122939	0.179910045	0.24087956	0.708645677	0.304347826	0.243378311	0.252373813	0.24877561	0.257371314	0.170414793	0.270364818	0.232383808	0.167416292	0.257371314	0.258870565	0.289855072
Semivariance/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.443278361	0.926036982	0.568715642	0.193403298	0.974512744	0.383808096	0.367816092	0.326836582	0.36031984	0.950524738	0.765617191	0	0.285857071	0.203898051	0.829585207	0.378310845	0.306346827	0.256371814	0.317841079	0.328335832	0.272363818	0.404797601	0.294852574	0.225887056	0.31984008	0.306346827	0.429285357
Benchmark Min Variance	0.168415792	0.235382309	0.35982009	0.911544228	0.387806097	0.384307846	0.467766117	0.116441779	0.331834083	0.331834083	0.174912544	0.28035982	0	0.135932034	0.455772114	0.755122439	0.810094953	0.91854073	0.906546727	0.820589705	0.697151424	0.868065967	0.846076962	0.885557221	0.948525737	0.843078461	0.928535732
Benchmark Max Sharpe	0.768115942	0.210394803	0.196901549	0.12093953	0.212393803	0.16091954	0.156921539	0.463268366	0.143428286	0.377811094	0.223888056	0.211394303	0.126936532	0	0.191404298	0.15992004	0.142428786	0.134432784	0.140429785	0.141929035	0.12143928	0.170414793	0.140929535	0.111944028	0.152423788	0.143928036	0.166916542
Benchmark Equally Weighted (1/N)	0.452773613	0.783608196	0.873063468	0.542728636	0.831584208	0.699650175	0.723138431	0.297851074	0.75862069	0.903548226	0.705147426	0.820589705	0.452773613	0.192903548	0	0.646176912	0.576211894	0.578710645	0.566716642	0.589205397	0.517241379	0.609695152	0.556221889	0.497251374	0.566716642	0.573713143	0.577211394
Skewness/Semivariance Benchmark Rate Min Variance, MaxSkewSemi	0.192903548	0.372313843	0.576211894	0.817091454	0.38805597	0.825587206	0.768615692	0.172913543	0.693153423	0.51924038	0.309345327	0.385307346	0.772613693	0.163918041	0.632683658	0	0.8005997	0.707646177	0.463768116	0.826086957	0.842578711	0.625687156	0.728135932	0.646176912	0.422288856	0.788605697	0.689655172
Skewness/Semivariance Benchmark Rate Min Variance, MaxSharpe	0.2003998	0.295352324	0.455272364	0.88055972	0.370314843	0.741629185	0.675162419	0.151924038	0.651174413	0.513743128	0.249375312	0.31884058	0.7996002	0.141929035	0.575212394	0.808595702	0	0.740129935	0.803598201	0.926036982	0.941529235	0.939530235	0.487756122	0.740129935	0.724637681	0.710644678	0.767616192
Skewness/Semivariance Benchmark Rate Min Variance, MaxSortino	0.245877061	0.312843578	0.374312844	0.986506747	0.382808596	0.651674163	0.617691154	0.175412294	0.505747126	0.464267866	0.256871564	0.23838081	0.930034983	0.146926537	0.572713643	0.688155922	0.757621189	0	0.967516242	0.750624688	0.943528236	0.869065467	0.835082459	0.998001	0.963018491	0.783608196	0.994502749
Skewness/Semivariance Benchmark Rate 0, MaxSkewSemi	0.194402799	0.290854573	0.458770615	0.993003498	0.375812094	0.700649675	0.611694153	0.141429285	0.552723638	0.469765117	0.247876062	0.31934033	0.909045477	0.150424788	0.578710645	0.451774113	0.7996002	0.971014493	0	0.771614193	0.970514743	0.330834583	0.899050475	0.965017491	0.522738631	0.84107946	0.972513743
Skewness/Semivariance Benchmark Rate 0, MaxSharpe	0.204897551	0.326836582	0.510244878	0.886556722	0.348325837	0.776611694	0.687656172	0.163418291	0.607696152	0.475762119	0.275362319	0.323338331	0.820089955	0.154922539	0.576711644	0.813593203	0.931534233	0.746626687	0.777111444	0	0.953023488	0.909545227	0.564717641	0.750124938	0.703648176	0.795602199	0.752123938
Skewness/Semivariance Benchmark Rate 0, MaxSortino	0.183408296	0.231884058	0.366816592	0.930034983	0.383308346	0.196901549	0.514742629	0.099950025	0.312343828	0.369315342	0.183908046	0.275862069	0.711144428	0.11844078	0.515742129	0.864067966	0.942028986	0.941029485	0.967516242	0.954522739	0	0.972513743	0.989005497	0.906546727	0.925037481	0.941529235	0.936531734
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSkewSemi	0.207896052	0.31834083	0.51924038	0.92153923	0.457771114	0.756621689	0.68015992	0.242378811	0.594202899	0.492253873	0.277361319	0.386306847	0.868565717	0.149925037	0.607696152	0.592203898	0.936031984	0.860569715	0.341329335	0.917541229	0.966016992	0	0.976511744	0.902548726	0.396801599	0.970014993	0.871064468
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSharpe	0.2003998	0.28085957	0.442778611	0.928035982	0.328835582	0.697651174	0.633683158	0.149425287	0.582208896	0.487756122	0.24137931	0.28855572	0.83858071	0.143428286	0.552723638	0.737131434	0.486756622	0.837081459	0.886556722	0.56171914	0.986506747	0.976011994	0	0.83908046	0.7986007	0.544727636	0.845577211
Skewness/Semivariance Benchmark Rate Equally Weighted(1/N), MaxSortino	0.156921539	0.247876062	0.335832084	0.982008996	0.367816092	0.469765117	0.412793603	0.116441779	0.24137931	0.372313843	0.171414293	0.218890555	0.891554223	0.12193903	0.512743628	0.669165417	0.75862069	0.994002999	0.968515742	0.756621689	0.890554723	0.888555722	0.833083458	0	0.96151924	0.770614693	0.99950025
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSkewSemi	0.203898051	0.297851074	0.467266367	0.970014993	0.388305847	0.668165917	0.587706147	0.143428286	0.534732634	0.473763118	0.261369315	0.310344828	0.945527236	0.142428786	0.550724638	0.419790105	0.713143428	0.969515242	0.533233383	0.711644178	0.911044478	0.378310845	0.809095452	0.956021989	0	0.749125437	0.96001999
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSharpe	0.211894053	0.272363818	0.429285357	0.899050475	0.351824088	0.700649675	0.657171414	0.141429285	0.56021989	0.463768116	0.233883058	0.295852074	0.829085457	0.138930535	0.554722639	0.775612194	0.689155422	0.785607196	0.830584708	0.812093953	0.95852074	0.969015492	0.541729135	0.772113943	0.753623188	0	0.781109445
Skewness/Semivariance Benchmark Rate Max Sharpe, MaxSortino	0.229385307	0.314842579	0.431284358	0.989505247	0.331334333	0.694152924	0.619190405	0.168915542	0.568715642	0.460269865	0.303848076	0.445777111	0.922538731	0.166916542	0.572713643	0.697151424	0.753623188	0.997001499	0.981009495	0.751624188	0.944527736	0.88055972	0.83958021	0.981	0.96001999	0.783108446	0

Red highlights significance at the 5% level