AN INVESTIGATION INTO INDIVIDUAL METHOD AS APPLIED TO THE TEACHING OF MATHEMATICS, WITH SPECIAL REFERENCE TO TENACITY, INCENTIVE AND SOCIO-ECONOMIC CIRCUMSTANCES.

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THESIS

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ERRATUM: Page 200 should read page 190 and consecutively thereafter.

M.Ed. THESIS: SUMMARY. A. J. van Zyl.

AN INVESTIGATION INTO INDIVIDUAL METHOD AS APPLIED TO THE TEACHING OF MATHEMATICS, WITH SPECIAL REFERENCE TO TENACITY, INCENTIVE AND SOCIO-ECONOMIC CIRCUMSTANCES.

The aim of the investigation is to discover to what extent relative performance on the individual method is affected by factors such as tenacity, incentive socio-economic status and intelligence. In other words, will the pupil with a powerful incentive and a large amount of tenacity, for example, do relatively better on the individual method than the pupil with no incentive or no tenacity?

The Std. VII mathematics class of a mixed country school was taught on individual method lines (that is, by means of assignments) for six months and the test marks of this period were compared with those of the previous six months when the ordinary teaching method had been used. By means of this comparison the relative improvement (positive or negative) of each pupil was obtained. This relative improvement was compared with factors such as tenacity and incentive.

The results show that incentive and tenacity are two important factors in individual work. On the other hand, pupils with high I.Q.'s or good socio-economic circumstances will not necessarily be at a relatively greater advantage when the individual method is employed.

With respect to tenacity and I.Q. the results as obtained in the mixed country school were verified with larger numbers in two urban schools.

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Chapter I is of an introductory nature giving a review of a few aspects of individual method having a bearing on this investigation.

PREFACE.

The twentieth century has been called the age of experimental pedagogy - experimental from two points of view: firstly, in so far as the principles of scientific research have been applied in the field of education, and secondly, as regards the trying out of new methods. The educator of today has a whole gamut of methods from which to choose. Each method has its exponents and enthusiasts, but on the whole there are many common elements. South Africans have in general fought shy of the radically new, although our education cannot be stigmatised as being out-of-date. It is however our duty to take the best out of these methods and to test it out.

This investigation is an effort to study some aspects of the individual method. Its scientific value may be negligible, but the writer hopes that it will prove suggestive for further investigation in this field.

The introductory chapter gives a brief review of the development and present position as regards individual method. Most attention is given to the Dalton Plan as this investigation employs its fundamental feature, the assignment. The other chapters describe the investigation in a logical way. The first part of the investigation was carried out last year in the country school where the investigator was a teacher; the second part was carried out this year.

Assignments, tests, case studies and formulae are arranged in appendices so as to make the body of the thesis read as a whole.

Perhaps the most important part of the investigation was the drawing up of the assignments. The investigator was inexperienced and the reader will no doubt see many ways of improving on them.

The many limitations of technique and material make it impossible to attach much significance to the statistical results of the investigation. It points however to one important conclusion: the individual method can be applied in our schools and there are many advantages attached to it.

The investigator has to express his indebtedness to all those who so willingly assisted him in the collection of data: the principals and teachers of the Aberdeen High, Rustenburg Girls' High and Observatory Girls' High Schools; to Mr. Taylor of the Psychology Department for assistance with the statistical part of the investigation; to Professor Reyburn for suggesting the technique of the second part of the investigation; and to Professor Grant on whose suggestion this investigation was commenced and whose advice, aid and stimulation remained an indispensable factor throughout.

A. J. v. Z.

November, 1935.
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C O N T E N T S.

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COUNTRY SCHOOL.

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CHAPTER I.

INTRODUCTION.

There is a continual evolutionary change of society. In some places and at certain times the change is more rapid or less rapid than the average, like a fire spreading in a bush and sending out occasional tongues of flame in directions where the undergrowth is drier and thicker, or being kept back where it meets green wood. In relation to this general change there are subsidiary changes in the different phases of life, changes which often oscillate between extremes before the point of balance is reached.

In educational method the teacher has in the past occupied a prominent position as the imparter of knowledge. During recent years his dominant class-room personality seems to have faded away to some extent, the individual child becoming the centre of attention. The child was no longer looked upon as a miniature adult but its own peculiar needs were studied and provision made for it in its education. Towards the end of the nineteenth century and the beginning of the twentieth, the method of objective scientific research was applied in this field, so that the child-study movement has become one of the most important branches of educational study.

In gear with this general tendency, but standing for a more radical change in school procedure, are the so-called new methods, like the Montessori Method and the Dalton Plan, in which the teacher has become more of an adviser and organiser than a teacher who teaches. Like most new things, these methods are very attractive; hence from the outset one has to guard against being carried away with enthusiasm, accentuating only their advantages and forgetting their drawbacks. Americans made this mistake with the result that the pendulum is already swinging back; the authorities have become alarmed at the state of affairs to which the extreme individualism in education has led. We can only express the hope that the point of equilibrium will soon be reached, that is, normal progress according to the general social evolution, and be more careful in applying these methods to our own system of education.

Schools are usually thought of as consisting of different classes each with its own teacher, while the fact is lost sight of that education is fundamentally an individual matter, being the response of each individual pupil to what is said or done in the classroom. No matter what degree of uniformity is aimed at, innate individual differences will always be the cause of different responses.

HISTORICAL SURVEY.

The earliest forms of teaching were individual, there were no classes, and even during the eighteenth century the individual method of instruction was used in the vernacular schools. The imparting of information to a class as a whole was almost unknown. It was mainly a question of disciplining, handing out lessons and hearing each individual recite them. This method proved very wasteful of time and effort; some children might attend school for years and hardly get a start in reading and writing.

During the nineteenth century the systematic division of pupils into classes made considerable progress, especially in Germany. For a time the class system gave way to the mutual or monitorial system of instruction, associated with the names of Bell and Lancaster, but when the deficiency of teachers had been rectified by instituting sufficient normal colleges, it again came into force - only to be attacked once more in the twentieth century by Montessori, Parkhurst and other protagonists of the individual method.

Under the class system teachers were real "teachers" and considered it their duty to teach the facts into the children's brains. Typical of these were the German schoolmasters who were usually reserve officers. In the beginning of his "Great Didactic" comenius had written: "Let the main object of this, our Didactic, be as follows: To seek and to find a method of instruction by which teachers may teach less and learners may learn more; by which schools may be the scene of less noise, aversion, and useless labour, but of more leisure, enjoyment and solid progress." But these words, which might as well have been written by a Helen Parkhurst, were ignored or rather never read.

Before the eighteenth century the individual had been completely dominated by Church or State authorities and, if at all, was educated as such. But with the liberation of individuality new educational aims arose, especially as enunciated by the French political philosophers of the eighteenth century, which in most cases implied the perfection of the individual as a member of the democratic State. Thus was the <u>philosophic foundation</u> of individual education laid almost two centuries ago; and modern educational philosophy, as chiefly represented by Dewey and summed up in the words "individual development and social service balanced proportionally", is still much the same thing. Indeed, these few words almost exhaust the subject-matter of educational theory. In America, as has been noted, individual development as based on the findings of modern psychology has been overemphasised while an extremely nationalistic people like the Germans before the Great War, laid all the stress on social service, social duties and social sacrifice, "social" meaning "national" in their case. The wise ruler will however preserve the golden mean, or rather, aim at the maximum development of each without sacrificing the other.

The psychological justification of individual method was pronounced still earlier, by Locke in the seventeenth century, namely, that there can be no true education which does not adapt itself to the nature of the learner. For this reason he objected against the wholesale methods of class-teaching, such primitive forms of it as were in vogue, and preferred the tutorial system.

But whereas Locke hit on the truth almost by accident, and after him Rousseau, Pestalozzi and Froebel reiterated it, it was only the end of the nineteenth century and the beginning of the twentieth century that saw a very real dissatisfaction with the prevailing class methods of teaching and the vast amount of mechanical learning that was going on. The teachers found it hard to preserve discipline in the large classes which resulted as a response to industrial needs and from the desire to educate the masses, and usually resorted to methods of repressive discipline.

The need for a new and better education was felt very strongly, and was voiced in books like "What Is and What Might Be" (by Edmond Holmes, chief inspector of the Board of Education, England). Holmes gives a very clear exposition of the existing state of affairs. The first part of his book he devoted to "What is, or the path of mechanical obedience" and painted in most vivid style the schools of England, where the child read, but did it mechanically, and learned, but did it mechanically; where "the normal channel of his expansive energies was blocked by years of repression. He considered the function of education to be the fostering of growth, and on that ground condemned the prevailing system. "Whatever else the current system of education may do to the child, there is one thing which it cannot fail to do to him - to blight his mental growth." The second part of his book he titled "What might be, or the Path of Self-Realisation" and described a village school, which, although it actually existed, he called Utopia. The striking features of this school were the ceaseless activity of the children and the bright and happy look on every face. There the child was not considered a lump of clay or a "tabula rasa" but a living soul which grew to its own maturity.[1] There also, what he called the communicative instinct, the dramatic instinct, the artistic instinct, the musical instinct, the inquisitive instinct and the constructive instinct found adequate expression.

Books like these heralded the coming of a new education, the main aim of which was to be free from the mere lecture and recitation method which led to mechanical learning and a repressive discipline, and on the other hand to allow for freedom, self-activity, self-expression and self-discipline.

It was realised more than ever that rigid class-teaching could never be successful as it does not

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1. This statement may be criticised as being rather on the extreme side. It sounds very much like the unfolding potentiality theory of Pestalozzi and others; it implies the same fallacy, namely, that the individual, like the acorn, has potentialities which will unfold; and it forgets the very vital factor of environmental influence.

allow for individual differences. Locke had said this in the seventeenth century and after him many other great educationists had felt and said it, but never had it been so forcibly brought out as intelligence tests and scientific measurement of achievement in school now did. The very large differences between individuals were plainly indicated and it became clear to most thinking educationists that provision had to be made for these.

As a result many new methods developed and the common aim of all was to allow the child greater self-activity and greater freedom. To some extent America was a pioneer in this field: her great educational philosopher, Dewey, had laid the foundation on which practical teachers could build. England, although more conservative, followed America. In Germany after 1918 there was perhaps more "wild" experimentation in the practical educational field than anywhere else; it was in the same direction of greater freedom and activity for the individual.

NEW METHODS.

There are several methods which classify under the new education; some of them are group methods like the Decroly Method, Purposeful Activity or Project Method, and the part of Winnetka Technique which attains its aim though creative group activities. The outstanding feature of these group methods is that they are curriculum reformers, seeking to organise subject-matter round centres of interest; they deplore the artificial division of the curriculum into separate subjects like geography, history, economics. Starting with some point of interest, they build up other material round it so that in the end it forms a complete whole of knowledge.

But more important from our point of view are the individual methods which the new education has given to the world: the Montessori Method, the Dalton Plan, the Consinet Method, and the part of the Winnetka Technique which aims at mastering the knowledges and skills by means of individual work. Fundamentally these methods are the same as the group methods, being imbued with the same spirit of the new education, but there are differences the most important of which is their attitude towards the curriculum. The individual methods are not curriculum reformers but accept the existing curriculum with its subject divisions, and make it their aim instead to inculcate freedom, individual iniative and active learning through special devices.

The Montessori Method and the Dalton Plan are the most widely known and perhaps the most important among the individual methods.

THE MONTESSORI METHOD.-

Dr. Maria Montessori was a pioneer in the field of individual method, but her method is chiefly for young children, hence we shall be content with only a few words about it. The two main ideas of her system are the need for freedom and spontaneity on the part of the developing child, and the importance of training the muscles and senses in the first stages of education. Her conception of freedom is that the child should be allowed free physical and mental activity; there should be no fixed benches or forced learning. Each pupil does his own work and progresses in his own time. Such activity is "refreshing to the personality" and will cause no fatigue. The teacher's task mainly consists in preparing the correct environment. A notable fact is that her method originated in her work with subnormal children, the same as the Decroly Method.

THE DALTON PLAN:-

The Dalton Plan, discovered by Helen Parkhurst, is the most important of the individual methods which may be employed through the whole school, although it is of necessity not successful with young children who still find it difficult to read without much concentration. As this thesis makes use of its most outstanding feature, the assignment, it is of great importance for our discussion.

An assignment consists of written, typed or printed instructions concerning the work the pupil must do, other material he must use, and the questions which have to be answered and handed in. The drawing up of an assignment is the most important task under the Dalton Plan. It means that the whole syllabus of work for the school year (about nine months) has to be divided into nine equal portions, each of these divided into four weekly tasks and, if necessary, each of these again into daily tasks. Preparing assignments and correcting the work handed in constitute the teacher's most important task.

Advocates of the Dalton Plan claim that it has proved more successful than ordinary class-teaching under all circumstances, in all standards except the very lowest, in all subjects and with all types of children. Expressions like the following are spread throughout Dalton Plan literature:- "The Dalton Plan has taught me that there is no such thing as a dull boy"; "There is no limit to the application of this Plan". Even responsible people make themselves guilty of such sweeping statements. Others, who are not so enthusiastic say that it only works with clever pupils, which appears to be a logical conclusion but should nevertheless be subjected to scientific experiment before accepted as true.

The inception of the Plan dates out of the second decade of the twentieth century, and since then it has spread over the world, mostly in modified forms adapted to local circumstances. As examples may be quoted Lynch's work in England and the application of the method in Indian rural schools where children attend very irregularly and enter school at any time of the year.

The Dalton Plan does not set out to revolutionise the curriculum but simply offers a new method to teach the old curriculum. It does not even do away with the division of the school into grades or classes. Its essential feature is the monthly or weekly assignment of work given to the pupils in written form. Helen Parkhurst may indeed answer old Comenius: My method of instruction is through the assignment; teachers not only teach less but very seldom teach at all and learners not only learn more but learn as much as they can.[1] Another feature of the Plan is the division of the school into subject-rooms instead of class-rooms, each subject-room containing the requisite material for the subject, such as reference books, dictionaries, apparatus.

In England the Dalton Plan became very popular, especially as championed by Lynch and Rose. Mr. Lynch advocates a more radical break from the class system: he gives each individual pupil a new assignment as soon as he has completed the previous one, hence there are no classes. Miss Rose still keeps the class together, hands out assignments to all at the same time and expects them done in a fixed time. Mr. Lynch has an adjustment room for the weaker pupils who need so much help that they prove an obstacle for the smooth functioning of the Plan. In this room special attention is given to them and more teaching is done.

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1. Cf. to quotation from Comenius on page 3.

The Sub-Dalton Plan for young children developed as a link between the Montessori Method and the Dalton Plan. Instead of having different subject-rooms and subject-teachers, a big room is divided into a number of so-called "corners" for reading, writing, arithmetic, drawing, etc., respectively. Usually a few big tables are used. Here, as in the higher classes, the group method is of course retained for gymnastics, story-telling, singing and such subjects.

It is unnecessary to describe at greater length how the Dalton Plan operates in practice. It has been done often enough and any of the books by Parkhurst, Lynch, Kimmins and Rennie or Evelyn Dewey may be consulted on the matter.[1.]

Like all other methods the Dalton Plan has its shortcomings: it accentuates the division of learning into subjects; it encourages intellectualistic learning to a great extent; the setting of tasks and their correction involve a great amount of work; and finally, there is the all too often lack of suitable text-books and other books to which the assignments can refer.

CONCERNING THE FIELD OF THE PRESENT INVESTIGATION.

We have so far traced in very brief outline the evolution of individual method as a phase of the so-called new education, and must now narrow down the discussion to the actual field of our investigation. This consists mainly in the application of the assignment method to the teaching of mathematics in a mixed country school in South Africa; that is, instead of teaching to the class when its daily period of mathematics came round, each pupil was given written

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1. See Bibliography.

instructions and explanations of the work required of him, issued in the form of weekly assignments.
We are therefore interested in the Dalton Plan more particularly in so far as assignments are concerned and not in the general organisation of the whole school according to this method. The important question to answer thus becomes: What advantages has the assignment method over the ordinary teaching method, all other conditions being the same, and do these advantages hold good for all types of pupils under all circumstances?

ADVANTAGES OF THE ASSIGNMENT METHOD:

The most obvious and perhaps most important advantage of the assignment method is that each pupil can proceed at his own speed: the clever are not retarded by the stupid or the stupid dragged forward by the clever. Intelligence tests have shown that individual differences are as marked on the mental as on the physical side. The assignment method makes provision for these individual differences. The pupil has his work assigned to him in writing and is then left to himself to complete it. The clever pupil can progress as fast as he can and attain his maximum degree of development. On the other hand, the slow worker is not confused by making him go faster than he is capable of doing.

There is no immediate external stimulus like the teacher's voice (or his rod!) which forces the brain to react at once, perhaps unwillingly. The pupil is never forced to follow a line of argument at a speed perhaps just too fast for his brain. Before he has had the opportunity of facing the problem and considering possible solutions, the teacher might have obtained the correct answer from someone else in the class and continues to the next point. This is very dangerous: it often means that the solution of the problem is arrived at before the poor child has realised what the problem itself was, apart from the fact that he misses that inciting stimulus of coming face to face with an unsolved problem. A subject like Geometry will afterwards appear to him to consist of an unnecessary string of arguments about unnecessary things. It is not to be inferred that such is the case with all class-teaching. It should not be so. But the chances are that it happens daily in every class-room with at least a few pupils, for the teacher has to keep count with the average pupil in his class and not with the weakest.

But what about that type of lazy unambitious child who must have the teacher shouting about his ears to make him think at all? Dalton Plan enthusiasts declare that they disappear remarkably quickly where this method is applied, quite often becoming the most diligent and ambitious in the class. This is however a matter of doubt which can only be settled by objective experiment. It appears obvious that if there is such a person as a "natural sluggard", he will always do better under the direct stimulation of the teacher's voice and personality.

There is also that type of child who finds it difficult to follow written explanations; his aural intake is more efficient than his visual intake. After all, the spoken word comes earlier in the child's life and is learned in a more unconscious way than the written word. In addition, inflection, accent, tone and even facial expressions and gestures make the spoken word much more than a mere string of words as the written word must of necessity be.

On the other hand, in the individual method children have greater mental freedom. There is not that continual stimulation of the teacher's voice, commanding, directing, informing, warning, coercing, which is so harmful to normal mental growth. The child is given the opportunity of experiencing for himself and hence to develop a sense of responsibility and self-control. "The old school stood for information; the new school stands for experience." Daily we read that the chief function of the school is not the imparting of facts but the formation of the right attitude of mind, the development of character and "die verkryging van 'n gesonde en sterk lewensrigting."[1] It is the outspoken opinion of many that the assignment method is much more conducive to these aims than the ordinary teaching method.

The pupil learns to tackle a problem unaided, he is self-active, which means that he learns to think for himself. What Dewey meant when he talked about "learning by doing" is admirably illustrated by this method. The child can work in his own way using his own time, which increases his interest. At the same time, he has a definite contract to complete in a stipulated time, which gives him a new motive for work. He is still able to get all the help he wants from the teacher, and now he gets real help, the teacher tells him things which he really wants to know and he need not listen to things which he knows himself or can think out for himself.

Also physically and morally the child enjoys greater freedom: physical freedom of movement in the class and moral freedom from the excessively authoritative personal element provided by the teacher.

From the teacher's point of view the assignment method also has its advantages. He is forced to prepare

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1. Article in "Die Huisgenoot", 23rd November, 1934, by Dr. C.F. Visser: "Twee Skole vir elke Kind."

his assignments in good time and to do it well. How often does it not happen in our schools that teachers enter a class unprepared, not because of negligence, but because they thought that they knew enough about the lesson having had a vague idea what it should be about. Assignments, however, involve a much more tangible form of preparation. With class-teaching there is always the possibility of exacting abstract thinking from children at too early an age. As adults, many teachers revel in generalisations which are meaningless to children. This can be avoided by carefully planning the assignments.

The teacher is further benefited by the fact that no extra coaching is involved if pupils were absent from school. When they return they simply continue with their assignments from the point where they last left off. This is as beneficial to the child for now there can be no gaps in his knowledge. This has been well brought out in the Indian rural schools where attendance is very irregular.

Dalton Plan enthusiasts claim that discipline becomes comparatively easy under this method. The teacher very seldom need worry about it, for the pupils discipline themselves. There is an atmosphere of study and work which makes discipline of secondary importance. Under the old method the teacher, his voice and personality, remain in focus most of the time. Children are often good judges and their teachers full of weaknesses. How can they help noticing these weaknesses and criticisin them? In addition, clever pupils more often than not find teaching very dull for they know most of what is going to be said. How can they help thinking out practical jokes and becoming a general nuisance to the teacher instead of being idle all the time? Compare with this the new method, where the pupils are active and busy with their assignments while the teacher becomes a helping friend. He is more in the background and is less liable to incur the disrespect of his class. Under the old method much depended on this chance factor, the teacher's personality; under the individual method a poor personality is less likely to lead to disastrous results for the child's education. In some ways, however, it is a pity that those teachers who are capable of giving a really inspiring class-lesson very seldom gets the opportunity for doing so. But the exception cannot be allowed to determine the choice of method!

To sum up these advantages, we may say that the self-activity and greater freedom of the pupil result in self-discipline; more and better work is done; the teacher with his bold speeches and other aggressive methods which tended to impress his personality unduly on the child, makes way for the advising, helping friend. Indeed, Rousseau, Pestalozzi and Froebel would all have rubbed their hands in glee if they heard about this.

SOCIAL IMPLICATIONS:

The greater freedom and self-discipline of the individual method will have far-reaching social repercussions. Ithas been said that the present generation is being brought up too lax, that there is no discipline, and that the new tendency in education is to be blamed for it. It is true, the freedom which the new method allows might have been disastrous under the old method. Think, however, what it means if there enters into the world this new generation of people who have been self-disciplined and accustomed to freedom, who think for themselves and need no external coercion to make them peaceful, law-abiding citizens of their country. Contrast with this the old system where there were turned out into the world young people who had been accustomed to act only on teacher's commands and who now felt free to act as their whims and caprices directed, for teacher was no longer there to punish. No wonder the law had to make provision for a strong police force to take in teacher's place! We state this problem rather extremely but it is necessary to bring out its great importance.

The psychological influence of the new education is still greater than the social. The individual method is bound to produce much happier individuals. In the past the child has often been called the "victim of unhappiness". Modern psychology has time and again pointed out the evil results of repression and the complexes which it causes. One need only watch one's friends and acquaintances to realise how few people are unhampered by complexes which so often make life a burden. The aim of the individual method is self-development and self-realisation, to give the child the opportunity of tasting the joy of achievement.

There is another aspect of this matter: by means of the teacher's valiant efforts the backward child is pushed to work above his capacity. E.G. Malherbe says of the South African backward child: "Most of them are hard pushed to do school work that is beyond their mental level with the result that they develop anti-social tendencies in self-defence".[1] Similarly, average pupils are pushed to do work above their capacities; ambitious parents may even have them pushed right into the university where they may become perfectly unhappy because they find the work too much for them. Logical reasoning seems to indicate that under the individual method each child will find his own level

1. "Education and the Poor White", page 186.

to a much greater extent, which will lead to much greater all-round satisfaction. On the other hand, a brilliant pupil may develop habits of idleness under class-teaching which might ultimately mean the loss to the country of a great leader or scientist or administrator.

The habit that so many teachers develop of commanding, explaining, directing to the smallest detail - do this, don't do that, take care, draw a line, stop, continue, etc. etc. - is so often transferred into homes, especially by young mothers who had been teachers or had grown up in such an atmosphere. They think that it is their duty to direct their children in everything and prescribe in the greatest detail what they should do and what not, how and how not, why and why not. The phenomenon of the child who can never move without mother's advice or admonishment is not very uncommon. The poor child never gets the opportunity of experiencing for himself or using his own brains. The world at large does not consist of anxious mothers and teaching teachers, who will continually inform the grown-up boy or girl what is right and what is wrong, what should be done or what not. They will have to find out for themselves and bump their heads in the process, if necessary. Why not then let them bump their heads while they are still young and growing, and may easily grow out of the unpleasant consequences? The more one thinks upon the matter, the clearer it becomes that "teaching" in itself is an artificial method of education. Give the child his work, help him if required to do so, but otherwise leave him alone to continue on his own and to overcome his difficulties in his own way.

Having discussed in short some aspects of the individual method of education, we shall now proceed with the description of the present investigation noting how far the aforementioned claims are justified in practice and what new light is thrown on the subject in general.

CHAPTER II.

OBJECT AND NATURE OF THE INVESTIGATION.

Any investigation which claims scientific value must be carried out under controlled conditions, and must produce results which can be measured objectivel Subjective judgment, while giving general direction and purpose to the experiment, must be eliminated as far as possible where the actual results are concerned. If a different experimenter carries out the same experiment he must get the same results, whether he does it in China or the North Pole, in the year 2000 B.C. or in the year 2000 A.D. and even if he employs a different technique. The fourth decade of the Twentieth Century will be satisfied with nothing less - at least, in so far as it is dominated by the demands of physical scientists.

By them the method of the physical sciences which has developed since the time of Galileo, has been regarded as the only method which can produce such objectively true results, and consequently as its influence spread during the Nineteenth Century, it was applied to almost all phases of learning - history, psychology, philology and so on. In education it led to that craze for measurement which characterises American pedagogics.

Recently, however, there has been a tendency in the fields of the so-called "Geisteswissen~~wissen~~-schaften" to turn against the indiscriminate application of the methods of research suitable to the physical sciences, and they are trying to evolve methods of their own which are more particularly adapted to their nature. Those who take up this standpoint assert that human factors are not always measurable in the same way as the properties of things; they protest also against the mechanistic and materialistic interpretation of life to which the domination of the physical sciences has led.

In the present investigation human material, namely school children, and the appraisal of human factors such as tenacity and incentive are involved. The investigation deals with the application of individual method to the teaching of mathematics. A group of children was put on the individual method for a definite period of time and by means of careful observation, accurate measurement, a critical interpretation of data, and verification of results under different conditions, certain conclusions were arrived at concerning the applicability of individual method in our schools. The reader will note that throughout the investigation an attempt was made to use objective methods; a glance at the table of contents will show that the whole approach to the subject was on the lines of the method of the physical sciences;-- but where factors such as incentive, perseverance and industry were concerned, the personal judgment of the investigator had of necessity to play an important role. The investigator is at present unable to see how such an investigation can be carried out without this personal element entering into the field. In fact, such an investigation would be devoid of much that is essential if this personal factor is eliminated. The methods of research which have proved successful with the physical sciences seem to fall short where essentially human factors are concerned.

Initially, the object of the investigation was twofold: (1) to discover whether a pupil will do better when he works individually than when he receives instruction as a member of a class of twenty or thirty pupils: the circumstances under which the investigation was carried out did not admit of any definite results being reached as regards this point. It therefore falls away; a different technique is necessary for it.

(2) to find whether all pupils will benefit equally by the individual method, and if not, what type or types of pupil will benefit most: the present investigation is chiefly concerned with this question.

Its object may thus be stated as being the comparison of the ordinary class-teaching method as we have it in our schools to-day, to the individual method as represented by the "assignment" method, with reference to special pupil factors such as tenacity, incentive and socio-economic background; broadly speaking, will the individual method suit a certain type of pupil living under certain conditions better than a different type of pupil with a different background.

Obviously, the problem is not as simple as that of trying out overhead valves and side valves with different types of engines under different sets of conditions. One can compare the test scores obtained when the individual method is employed with those when the class-teaching method is used, and on that ground decide which method is more suitable for a particular pupil under certain conditions. But the pupil is not a machine where effects are directly measurable. For all we know the individual method might produce a much better type of person in the long run and yet produce worse examination results than the class-teaching method.

Success in life, character, personality, feeling of responsibility and other mental, moral and social qualities are not immediately measurable and it is doubtful whether they are measurable over a long period. How will such qualities be affected by individual or class-teaching method? It will be very difficult to evolve objective means of measurement to discover this, if indeed it is at all possible.

In making this statement, the writer is aware of the attempts of modern psychology to evaluate all kinds of human factors by isolating their essential constituent elements and then measuring certain specific external responses associated with them. Valuable as much of this work has proved, intelligence testing, for example, many results for which scientific validity is claimed, can only be viewed with scepticism.

For these reasons the present investigation does not set out to study the relative merits of individual and class methods with the object of discovering which is more suitable for different types of pupils in any absolute sense, that is as regards development of character and personality and so on, but only in so far as differences are revealed by modern testing technique. If this technique is good enough the pupils' scores ought to indicate which method gives them a better understanding of the subject - in this case mathematics. In this narrower sense no one can object to the scientific value of such an investigation. Even so, the validity of the results will be questionable as far as such technique has its failings and shortcomings; these will be dealt with after briefly explaining the method of investigation in the following section.

To be quite clear, it may be mentioned at this juncture that the first and main portion of the investigation was carried out with the mathematics section of a Std. VII class in a South African mixed country school: it is described in this and the following two chapters. The second part of the investigation is in the nature of a verification of the results obtained in the first, and it was carried out in two urban girls' schools: it is described in Chapter V.

METHOD OF INVESTIGATION.

As was mentioned above, the mathematics section - consisting of 19 pupils, 4 girls and 15 boys - of a Std. VII class in a South African mixed country school, was used for the purpose of the investigation. There were about 650 children in the school, 135 of whom were in the secondary standards - 55 in Std. VII. A feature of the school is the large number of poor children who attend it. The investigator was a full-time teacher on the staff of the school and taught methematics to all the secondary standards.

The experiment lasted for a full year, during which the usual Std. VII mathematics syllabus was covered.

SIX MONTHS' ORDINARY TEACHING:

During the first six months of the year the class was taught in the ordinary way, tests were given and a full record of results kept.

SIX MONTHS' INDIVIDUAL METHOD:

During the second six months of the year the class was put on the assignment method, that is, weekly assignments of work were handed out in typed form to each member of the class and he had to continue in his own way. The investigator was present all the time and gave all the necessary help.

Throughout the year the class had six periods of forty minutes each per week for mathematics. This meant that in school the pupils had four hours in which to complete the week's assignment. Those who could not complete it in school had to do so at home.

There were separate assignments for algebra and geometry. They were handed out every Monday morning at the beginning of the mathematics period. Usually it was necessary to point out one or two mistakes which resulted from faulty typing, and on some occasions to give them a few tips, but the aim was to have all the necessary instructions in the assignments.

As soon as a pupil had completed the week's assignment, he brought his work to the teacher who corrected it and pointed out mistakes while he looked on. This was possible in most cases seeing that there were only nineteen in the class. Quite often two pupils brought their work at the same time. The correcting did not take too long as the investigator had each assignment fully worked out in accordance with what was required. On some occasions, usually after a Friday, the work was corrected at home and handed back on the following school day, so that the mistakes could be pointed out before the next week's assignment was tackled; this was very important seeing that the subject was mathmatics. No marks were given, only mistakes indicated. In order to keep a check on the work done by each pupil, a record sheet was kept by the investigator showing the date of completion of each week's assignment and whether progress was good, satisfactory or unsatisfactory.

The second half of the year was wholly devoted to assignment work except for one and a half periods which were used for teaching: one full period on the introduction of the theorem and half of another period on explanations concerning theorems. On a few other occasions the attention of the whole class was directed to certain stumbling blocks in the course of the work.

During the first two weeks of individual work no algebra assignments were handed out as they were not yet ready. This accounts for the fact that the period covered by the geometry assignments exceeds the period covered by the algebra assignments by two weeks. This need not be considered a disturbing factor as no teaching was done in the algebra periods of those two weeks. The class was doing miscellaneous examples and test papers (see Durell's Algebra, pp. 59 - 61) and at the beginning of each of the two weeks the investigator informed them how many they had to complete during that week, which was much the same thing as handing out written assignments.

Tests of fifteen to thirty minutes each were given every month although more for the sake of helping the pupils than to discover progress. Their marks were not used for any purposes of comparison.

For the latter purpose the quarterly examination marks were used. Special care was taken with the setting and marking of these papers. The standard of marking was kept constant as far as possible throughout the year. The examinations were conducted at the end of each school term under proper examination conditions.

COMPARISON OF CLASS-TEACHING AND INDIVIDUAL METHOD TEST SCORES:

The marks of the first and second quarterly tests and also the marks of the third and fourth quarterly tests were added separately for each of the nineteen pupils. Some of them obtained a higher total for the last two terms, showing improvement under the individual method, while others obtained a smaller total, showing that the individual method did not suit them as well as the class-teaching method.

ADDITIONAL DATA:

Other data were collected for each of the nineteen pupils: intelligence quotients, tenacity, incentive, socio-economic status, health and school-record. These were then compared with the improvement as shown by the comparison of class-teaching and individual method test scores, with the object of discovering which personal qualities and environmental conditions are necessary for success in individual work. The technique which was applied in each case will be described in the following chapter.

LIMITATIONS:

The disadvantages of carrying out the investigation in this way were, firstly, the small number of pupils in the class; for reliable correlation coefficients there should have been at least thirty pupils.

The nineteen pupils were not selected according to the statistical requirements of accuracy; they represented the whole Std. VII Mathematics Class which is usually a select group. Altogether there were 55 pupils in Std. VII. Fortunately, the mathematics section proved to be a fairly good statistical group, as their general class marks were distributed fairly regularly among those of the non-mathematics section.

The investigation was carried out with a Std. VII class, hence mathematics was a new subject to them at the beginning of the year. It would have been better to have had a class which was already familiar with the subject. Would the same results have been obtained if the class had been put on the individual method for the first six months and on the class-teaching method for the second six months instead of the other way round?

The whole class was Afrikaans-speaking. Mathmatics is taught through the medium of English according to a decision of the school committee. During the six months of class-teaching Afrikaans was used quite often in explanations, while the assignments were written entirely in English.

Mathematics was the only subject taught by the individual method during the second six months of the year. The other school subjects were taught in the ordinary way and if the pupils were hard pressed they might have neglected their assignments - not in the sense of not doing them but by copying down the work from others.

ADVANTAGES:

On the other hand, the investigation was carried out over the period of a year during which time the investigator had full control of the class. This period was long enough to show definite results and it is claimed that much greater significance can be attached to these results than would have been the case if the investigator had been a stranger at the school where the experiment was performed, visiting it only two or three times per week over a period of three or four months.

The investigator knew each of the nineteen pupils well and was interested in each. It may be objected that this has no bearing on the objectivity of the experiment. It will be seen, however, that it would have been fatal to the investigation if the investigator had been a stranger to the situation.

The investigation did not disturb the ordinary school procedure, the usual Std. VII syllabus was completed (see assignments or text-books used), but at the same time an attempt was made to control such procedure so that the results would be scientifically reliable.

ALTERNATIVE METHOD OF INVESTIGATION:

It may be pointed out here that with the inception of the idea to investigate the application of individual method to the teaching of mathematics, it appeared that the best plan would be to run two parallel classes of the same average intelligence - other factors such as age, health, sex, socio-economic conditions being as far as possible the same in the two groups - the one class working on the individual method while the other was taught in the ordinary way. If they completed the same syllabus during the same time and wrote the same tests, the class which obtained the higher average mark would have indicated which method was the better. If then the same experiment was repeated with an older age group, say Std. IX, it could have been shown whether individual method works better with older pupils. Such an investigation, performed with at least thirty pupils in each class over a period of a year, ought to produce very valuable results. The teacher should of course not favour one group more than the other but should bestow the same amount of energy on the class-teaching as give individual attention to the other group. This technique would have given a completely different colour to the whole investigation.

Under the circumstances, however, it was impracticable. The Std. VII mathematics class which was available consisted of nineteen pupils. If it were divided into two groups there would have been too few in each group, while in addition it would have been very difficult to teach to the one section and at the same time give individual attention to the other.

All of which illustrates another important difference between investigations in the educational and in the physical science realms. The physical scientist can define his aim, choose his method, prepare his material and apparatus and control the experiment as he desires. The investigator in the pedagogical field very often gets no further than defining his object: for the rest he has to make use of what is available. Of course, in many cases it is possible to arrange and control everything as in the physical science experiment, but the practical difficulties are usually very great. It would be interesting to know how many educational investigations have been carried out during the past decade in which the investigators had all the necessary material and resources fully at their disposal. Surely, not a very big percentage! After all, human beings and their social institutions will not tolerate continual subjection to experimentation, and this leaves a very limited fund of resources at the experimenter's disposal.

ASSIGNMENTS.

(The full assignments are given in Appendix A).

In this investigation the main problem from the investigator's point of view was the drafting of assignments. It was quite clear to him that its whole success or failure would depend on the assignments.

As was mentioned in the previous chapter, the drawing up of assignments is no easy task. It requires more careful planning and more time than one would expect. The assignment is in a sense the substitute for the teacher's teaching, but it would be inadvisable to explain everything in detail by means of the written as by means of the spoken word - the assignment would become too bulky - hence it refers the pupil instead to suitable text-books or other sources. Contrary, however, to the view held by the authors of certain assignments,[1] the present writer holds the view that an assignment should be something more than mere references to text-books, even in the subject of mathematics. It should be alive; it should be of a much more personal nature.

Much will however depend on the text-books. In the present case the text-books used in the school were: "A New Algebra for Schools", Parts I and II by Durell (London, 1930); and "A New South African School Geometry" by Ward and Dick (Juta, 1933). The former has been found excellent for the purpose, with the result that the algebra assignments contained much less explanatory matter than the geometry assignments. In this book the concentric method is employed to a large extent, so that the pupil is never required to work a large number of similar examples in a mechanical way. Extra drill exercises are given which the teacher may require individual pupils to do if it appears from their assignment work that they are weak in certain parts. The variety of practical examples given in the book is commendable; unfortunately a large number of them were of no practical use as they refer to situations in a large city with which the present pupils were of course not familiar.

It may be mentioned here that for other subject assignments should contain special lists of reference books. For mathematics however, it is better to have one text-book and to prepare the assignments on the basis of that.

1. See, for example, "Dalton Plan Assignments" - Volume II, Mathematics and Science - Compiled by the Staff of the Streatham County Secondary School.

Separate assignments were drawn up for algebra and geometry: they are given in Appendix A. Each assignment had to be completed in one week. It was decided to have weekly instead of monthly assignments, for not only does this give the teacher a better control over the pupils in that he can see that they do not lag behind too far, but it is better for the pupils themselves. Especially in mathematics where one step is dependent upon the previous steps it is desirable that they should not be allowed to continue too long without the teacher seeing their work. As it was, it happened in quite a few instances that a pupil made a mistake in the beginning of the week and all his work that week suffered from it. If monthly assignments are given it will mean that such mistakes can only be pointed out at the end of the month when the teacher corrects the work, and by that time it might have become very difficult for the pupil to unlearn them.

For these reasons weekly assignments were given. The pupils were even encouraged to show their work more frequently to the teacher and to consult with him whenever they were uncertain about anything.

By a week's assignment of work is meant that amount of work which the average pupil can complete in one week. This means that the brilliant pupils will complete it in less and the dull pupils in more than a week. If new assignments are handed out as soon as those of the previous week have been completed, some pupils will complete the term's work in about half the time while others will not be able to complete it at all. Under the circumstances this was inadmissable as the class had to be kept together more or less. On the other hand, the ideal to hand each pupil a separate assignment suited to his personal requirements was impracticable. Hence the procedure adopted was to put in extra work for the better pupils while the weaker ones were encouraged to put in extra time at home on their assignments. In studying the assignments in Appendix A the reader will note these "extra questions" at the end of each assignment, except in the cases of a few where the pupil is advised to continue immediately with the next week's assignment at the end of which he will find extra questions. A few pupils did these extra questions most conscientiously, although it meant very hard work in some cases.

The third and fourth terms during which the class was on the assignment method consisted of eleven and nine and a half weeks respectively. In the third term assignments were handed out for nine weeks, one week was taken up by the quarterly examination and the last week was spent partly in revising test papers but mainly to allow those pupils who had lagged behind to get level with the rest of the class. The fact has already been mentioned that the first algebra assignment was only handed out in the third week thus giving only seven for the third term. This apparent irregularity could have no appreciable effect on the results, for the reasons already mentioned.[1] During the fourth term assignments were handed out for seven weeks. The final examination was written in the eighth week.

It should be realised that these assignments were of an experimental nature. On the whole they were found to be too full and too long - at least under the circumstances: the average I.Q. of the class was 106. For a class with a higher average I.Q. they may be of the right length. As was mentioned above, the last week of the third term had to be skipped in the handing out of new assignments in order to give the weaker pupils the opportunity of drawing level.

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1. Vide page 25.

According to the investigator's opinion, the best way to get an assignment of the right length is for the teacher to do it himself in the same way as the pupil is expected to do it and then to compare it with the work of an assignment which has already been found to be of the correct length. But even then it will be necessary to modify it in the light of new experience.

To obtain the best assignments is thus a matter of experiment. On correcting the work which is handed in weekly, note which mistakes occur most often; on such points the assignments should be improved. Also note down questions which are asked most often by the more clever pupils, for if they do not understand, it usually indicates a weak spot in the assignment. Of course, one has to guard against making them too bulky. All unnecessary side-issues and trivial distinctions should be avoided. Of necessity will the teacher be called upon to help the weaker pupils more often: but an attempt to augment the explanations in the assignments to suit them will only result in careless reading by the better pupils for they will consider it unnecessary to follow written explanations about things which they can explain for themselves.

Several alterations are necessary in the assignments given in Appendix A, as appeared from questions asked by the pupils, the mistakes they made and the time they required to complete each. The following are examples of points with respect to which such alterations are necessary:-

(i) Fourteenth week geometry: too long.
Tenth week geometry: too long.
Seventh week algebra: too short.

(ii) Twelfth week geometry: put similar questions together - numbers 3, 4, 6, 12.

(iii) Second week algebra: state units of scale in temperature graph. Take scale along vertical axis only between 60° F and 80° F.

(iv) Sixth week geometry: insert explanations of the word "diagonal". Never say $\Delta ABC = 2$ rt. Ls., but say: the Ls. of $\Delta ABC = 2$ rt. Ls. Insert question 10, p.61, under compulsory questions. In connection with question 11, draw attention to the fact that construction lines should not be erased.

In drawing up each of these assignments the following points were kept in view:

(a) It should provide a linking up with the work of the previous week, perhaps by means of a short summary of it, or in some way arouse the pupil's interest.

(b) It should state the goal of the week's work or give a short review of it, briefly indicating its nature.

(c) It should contain definite instructions about the work, reference to text books and necessary explanations.

(d) It should state which questions have to be answered.

It was considered important that, as far as possible, the form of the assignment should be of a standard type, so that on receiving it the pupil would know exactly where to look for everything. Also in referring to previous assignments he would have no difficulty in finding what he was looking for. Such a systematic form would save much aimless reading. While the aim was to maintain a standard form throughout, the nature and division of the work did not always permit it. The writer realises that the assignments can be improved considerably from this point of view.

They were however drawn up as clearly and definitely as possible. The utmost care was exercised in the choice of words and methods of explanation. Fundamentals were stressed so as to stand out among points of secondary importance. An attempt was made to maintain the interest of the pupils by means of practical problems and illustrations out of everyday-life. In short, the idea was to incorporate in every assignment all the good points of a good class-lesson as well as those of a clearly written explanation.

It was necessary to make the language expecially clear and simple, as the pupils were all Afrikaans-speaking and some of them were not completely at home in reading English.

It takes some time before pupils get used to work on the individual method. Some of them acquire wrong habits; they may even try to answer questions without having read the explanations or the text-book. It was necessary to instruct them on the following lines: "Do not be satisfied after having read the explanations once. Read them again and again until you thoroughly grasp what is said. Everything is said with a purpose. Hence do not continue until you completely understand an explanation. Parts which are stressed are all the more important and should be kept in mind continuously."

To simplify and speed up the weekly correction of the pupils' work, the investigator found it essential to have each assignment fully worked out in accordance with what was expected. In Appendix A (page 146) the reader will find the tenth week geometry worked out like this, Of course, it means extra work for the teacher but that is compensated for by the time saved in the correction.

The teacher who first attempts to draw up assignments will object to the great amount of work that it involves. The typing and cyclostyling involve still extra labour as well as expense - for the efficient functioning of the individual method the school will need a typist. But once the machinery has been set going the teacher will have much less work, although, as has been said, the assignments will have to be continually altered from year to year in the light of new experience.

To draw up satisfactory assignments is a study in itself. It is an art which many will never master thoroughly, but on the other hand one may truly say of it that it consists of 99% perspiration and 1% inspiration. There is however no reason why our teacher-training institutions should not give intending teachers a course in the drawing up of assignments whether or not it is required by the schools. It is hoped that this investigation will show of what invaluable assistance the assignment may be under certain circumstances and that the teacher who has received some general guidance in its drawing up will have a great advantage over others.[1]

DISCIPLINE AND ATMOSPHERE IN CLASS-ROOMS. GENERAL CONDITIONS.

During the six months of class-teaching the discipline was fairly rigid. A definite amount of homework was given two times per week and any neglect had to be compensated for by work in the detention class.

During the six months of individual work the discipline was not so rigid. Each pupil continued with his assignment in his own way. They were allowed to consult their friends if they did not disturb the others and could move about in the room at will. On very few occasions did it prove necessary to tell them to make less noise. The atmosphere was usually one of work and the few occasions on which they were reprimanded resulted from arguments concerning their work.

It was felt that it would have been a good idea to remove the benches out of their straight rows.

1. In Appendix E a few algebra assignments are given which were used with the Std. IX mathematics section of the school where the investigation was carried out.

but this was impossible as it would have interfered with the normal routine of the school. Hence the conditions under which the class worked were the same as during the first six months. The class-room, the class, the teacher, the periods, the text-books - everything remained unchanged. Instead of listening to the teacher's explanations for a considerable time every day, the pupils were busy with their assignments. It meant no formalities, no commands - but the full amount of time was utilised for work. The fact that a definite task was set for each week made all the differenc - it made each one in the class work.

On two occasions only was it necessary to require two pupils[1.] to come and do some work after school as they had been neglecting their mathematics.

Some of these children came out of very poor homes;[2.] in one or two cases the broomstick played a not unimportant role in the household. One might have thought that with such measures being applied at home, the freedom in school would be abused - but it was not the case. In addition, the other teachers of the school maintained a fairly rigid discipline, resorted very often to detention classes and other forms of punishment. One could have expected that this would have caused the pupils to neglect their mathematics assignments. But so eager were they to complete the task set for each week that they seldom caused trouble, except for the two cases mentioned. An important factor in such work is the satisfaction which it affords a child to hand in a concrete amount of work to be corrected each week.

A practical advantage which was experienced in having the class on the individual method concerned

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1. Pupils Q and R: see Appendix C.

2. See appendix C.

absences from school. Pupils F and G were absent for about a month as the result of sore eyes. When they returned to school they were told to continue with the assignments where they had last stopped. They were also told to delete some of the examples. They put in some extra work at home and caught up with the rest of the class without any additional trouble to the teacher. What is more, Table 2 (p.48) shows that they did relatively well on the individual method.

Pupil Q was also absent for six weeks as a result of sore eyes. After he had come back he did not seem to make any progress. He lagged behind the others till the end of the year and were it not for the fact that he was required to go to the detention class for quite a few times he would have lost more on the others instead of gaining. A scrutiny of his class percentages (Table 3, p.52) will however show that he deteriorated all round. Hence his poor performance cannot be ascribed entirely to the assignment method. It appeared at first that he had lost courage, but eventually it seemed as if it was due more to laziness than to anything else.

INDIVIDUAL PERFORMANCES ON THE INDIVIDUAL METHOD.

Before turning to the statistical part of the investigation in the following chapter, the reader is given below a general idea of the performance of each of the nineteen pupils on the individual method. For more general information about each pupil he should consult Appendix C. The pupils are named by the letters of the alphabet in the order of their relative improvement [1].

1. See Chapter III, in particular Table 2.

on the individual method. In a few cases certain factors such as health are named which might have influenced the pupils' performances on the individual method.

Pupils A, D and E: Three girls. Handed in excellent work every week. Very neat. Pupil A slower than the other two. Pupils D and E tried the extra questions very often. The questions asked by them showed that they had a good grasp of the work. Pupil E especially did very thorough work. There is little doubt that the individual method suited them.

Pupil B: Boy. Did good work; the facts that he is specially talented in number and that he was rather old for his class may have had something to do with it.

Pupil C: Quite satisfactory.

Pupils F and G: Their cases have been discussed above (sore eyes). Work quite satisfactory.

Pupil H: Satisfactory. At times tended to walk about too much in the class, discussing with others.

Pupil I: Very weak. Seemed to have been completely lost with assignment work. Average score throughout year ± 25%. His relative improvement of +5 must be due to chance, in so far that the individual method tests contained by chance a few more questions which he could do than the "class-teaching" tests.

Pupil J: Excellent. Best in class. Always first to complete the week's assignment. Did most of extra questions. The fact that he suffers from asthma and that it is usually troublesome with the coming of summer, may be the cause that he showed a smaller relative improvement than the above-named pupils. According to his father it definitely made him irritable and listless during these months. Judging from the standard of the work he handed in weekly, the individual method definitely suited him, perhaps best in the whole class.

Pupil K: Satisfactory.

Pupil L: Not satisfactory. His poor handwriting and unsystematic habits were definite handicaps in the individual work. Very untidy.

Pupil M: Girl. Not very satisfactory. Very slow but neat. Maths. not her strong point. Always seemed to work very hard. Extremely poor home circumstances - depressed, underfed appearance - might have affected performance on individual method.

Pupil N: Subnormal intelligence. Performance so poor that a change of method could hardly have any effect on it. Scraped together a few chance marks in the tests. Similar case to Pupil I - only much worse.

Pupil O: Quite satisfactory. Frequently consulted the teacher when he could not understand anything. Very hard worker. Poor health might have affected individual method performance.

Pupil P: Very unsatisfactory. Untidy in his work. It seemed as if he always copied down some of the work from others. No interest in work. Individual method does not seem to suit him.

Pupil Q: Absent from school with sore eyes for six weeks. Could make no progress after his return. (See above). Individual method will definitely not suit him.

Pupil R: Very unsatisfactory. Untidy. (Similar to Pupil P). Could not be trusted to work on his own. Teacher should see his work daily.

Pupil S: Unsatisfactory. Extremely slow. Although encouraged to hand in work earlier he always lagged behind. And yet he seemed to try his best. He seldom asked for an explanation of anything. It is very difficult to find a reason for the very big drop in his marks. Whenever anything was explained to him he said "yes" in a very understanding manner. His day-dreaming disposition as well as his inability to follow written explanations with ease might have influenced his performance. On the whole he was a problem to the investigator.

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CHAPTER III.

DATA: SOURCES AND COLLECTION.

Having outlined in brief the object and nature of the investigation, its limitations and possibilities, and the conditions under which it was carried out, we next come to the actual data and their collection. These will be dealt with under the following headings:-

I. Mathematics Scores: class-teaching scores and individual method scores.

II. Class percentages.

III. Intelligence Quotients (I.Q.).

IV. Tenacity Coefficients.

V. Incentive.

VI. Socio-economic Status (S.E.S.).

In addition, further information was collected about the nineteen pupils with regard to the following: Health, school-record and their personal opinions about the individual method.

I. MATHEMATICS SCORES.

During the year that the experiment lasted, the Std. VII class used for the purpose wrote the ordinary school examinations at the end of each term, and the marks so obtained were used for this investigation The mathematics marks of the first and second terms will be called class-teaching scores and those of the third and fourth terms individual method scores.

The fact that these examinations were part of the regular school routine and that the marks so obtained were put on the pupils' reports, was sufficient incentive to make them do their best. They were unaware of the fact that the marks were going to be used for the purposes of an investigation.

On the other hand, the investigator took special precautions in setting and marking the papers, in order to secure marks which would be as far as possible a true reflection of the pupils' progress in mathematics.

Each of these quarterly tests consisted of two papers - one for geometry and one for algebra - each of $1\frac{1}{2}$ hours, except at the end of the first term when one combined paper of $1\frac{1}{2}$ hours was given for algebra and geometry. Full details as regards these tests and the assignment of marks are given in Appendix B.

In setting the papers no consideration was given to whether the individual or class-teaching method had been used in the preceding term. The aim in each case was to test the pupils' progress in the work done up to the time of the test. The whole field was covered as far as possible while similar questions were not given in the same test.

In marking the utmost care was taken to keep the same standard throughout:

(i) The investigator first answered each test paper himself, then allotted marks to each step and fixed how many marks were to be deducted for each separate type of mistake, e.g. two marks for an arithmetical error like wrong addition or multiplication.

(ii) One, or in some cases two questions at a time were marked throughout the nineteen papers, then the next question and so on. This precaution is not usually as necessary with mathematics as with subjects like history or language, but it was taken in order to ensure complete fairness throughout the marking. In connection with this, attention may be drawn to the fact that it was a distinct advantage to have mathematics as the subject of investigation, as its test scores are much more reliable than those of languages where the examiner's "personal equation" assumes considerable proportions.

Except in the questions on graphs and accurate constructions, there is very small possibility that the marks of two examiners will differ appreciably once the scheme of marking has been decided upon, because usually it is a question of right or wrong. Hence the marks obtained by the pupils in the four tests may be taken to be completely reliable, at any rate as far as the marking itself is concerned.

The same type of test was given throughout the year. Of course, they became more difficult towards the end of the year when the work became more advanced. This is partly the reason why the average score of the last six months is lower than that of the first six months.

It is unnecessary to discuss these tests at further length. Suffice it to say that they were drawn up and marked in accordance with present day technique and requirements in this field.

The tests were written in the class-room under the supervision of a teacher. Pupils occupied single desks. The test papers were cyclostyled and each pupil handed a copy.

TABLE I:

Table I gives the mathematics scores obtained at each of the quarterly tests. Full marks in each case is one hundred. Column 3 gives the sum of the marks of the first two terms: the class-teaching scores; column 6 gives the sum of the marks of the third and fourth terms: the individual method scores. Full marks for columns 3 and 6 are therefore 200. The pupils are named by the letters of the alphabet.

TABLE I: MATHEMATICS SCORES obtained in quarterly TESTS.

Pupils	CLASS-TEACHING			INDIVIDUAL METHOD		
	1st Term	2nd Term	Total: 1st & 2nd Terms	3rd Term	4th Term	Total: 3rd & 4thTerms
	1	2	3	4	5	6
A	30	40	70	40.5	45	85.5
B	50	51	101	52.5	56	108.5
C	35	43	78	44.5	44	88.5
D	53	58	111	54	58	112
E	55	59	114	58.5	49	107.5
F	43	51	94	40.5	46	86.5
G	37	46	83	39	38	77
H	50	47	97	47.5	36	83.5
I	19	26	45	22.5	18	40.5
J	79	72	151	70	56	126
K	56	55	111	48.5	42	90.5
L	44	42	86	38.5	29	67.5
M	18	47	65	30	19	49
N	10	22	32	15.5	5	20.5
O	47	63	110	39	33	72
P	14	43	57	20.5	7	27.5
Q	43	46	89	32	20	52
R	30	59	89	29.5	16	45.5
S	71	64	135	34	22	56
Total			1718			1396
Average			90.4			73.5

The class-teaching average is 90.4 while the individual method average is 73.5. From this one might have concluded that the individual method is not as good as the class-teaching method - at least as far as examinations results are concerned. It has been pointed out that it would not mean that the individual method is the less desirable method of the two because its immediate results are not as good: it might prove more beneficial in the long run and might even give the pupil a better understanding of the subject. How many people do not pass examinations with flying colours, and yet after a few years they have forgotten almost everything about the subject.

But even as regards examination results it would be wrong to conclude from a comparison of columns 3 and 6 of Table 1 that the individual method is not as efficient as the class-teaching method. A scrutiny of the class percentages given in table 3 shows that they also decrease towards the end of the year, although to a smaller extent. This may be ascribed to the fact that in Std. VII several new subjects are started, the pupils enter a new school and hence have new teachers. In the beginning the examinations are of necessity very elementary, but towards the end of the year when more advanced subject matter covering a wider field is examined, the test scores decrease. This fact will appear when comparing the quarterly examination marks of almost any Std. VII class in almost any school, and especially is it the case with subjects like mathematics and science. It may be due to other reasons than these given above, but, important from an individual method point of view, it proves that the decrease in mathematics scores in the second six months does not necessarily indicate that the individual method is inefficient. It would even be doubtful whether such a conclusion would have been admissable if the standard of difficulty of the four tests had been exactly proportional to the degree of advancement which the class had reached in the subject at the time of each test. Such a proportional standard of difficulty is not claimed for these tests; and such standardised tests were not available.

They were however, not necessary for the purposes of this investigation, as its aim is not to compare the average marks obtained by individual and class-teaching methods. As was pointed out in the previous chapter, a different technique will have to be used for such a comparison, namely two parallel classes on individual and class-teaching methods respectively.

WEIGHTING OF INDIVIDUAL METHOD SCORES:-

The aim of this investigation is to compare the relative progress of pupils on the individual method with other factors such as tenacity, incentive, S.E.S. and I.Q. To facilitate such a comparison the following procedure was adopted:

The individual method marks were weighted so that their averages became the same as that of the class-teaching marks, that is, 90.4 instead of 73.5

The weighting was done by the following simple proportion method: (Pupil A's mark is used in the example)

If average mark is 73.5, then pupil A scores 85.5
" " " " 90.4, " " " "

$$85.5 \times \frac{90.4}{73.5}$$

$$= 105.$$

TABLE 2: RELATIVE IMPROVEMENT OF PUPILS ON THE INDIVIDUAL METHOD.

Pupils	Class-teaching scores	Individual Method scores (Weighted)	Relative Improvemt	Rank (Rel.Impr.
	1	2	3	4
A	70	105	+35	1
B	101	134	+33	2
C	78	109	+31	3
D	111	138	+27	4
E	114	132	+18	5
F	94	106	+12	6
G	83	95	+12	7
H	97	103	+,6	8
I	45	50	+5	9
J	151	155	+4	10
K	111	111	0	11
L	86	83	-3	12
M	65	60	-5	13
N	32	25	-7	14
O	110	89	-21	15
P	57	34	-23	16
Q	89	64	-25	17
R	89	56	-33	18
S	135	69	-66	19
Total	1718	1718	0	
Average	90.4	90.4	0	

Note: Pupils F and G are ranked as 6 and 7 respectively, instead of ranking both as 6.5, because their relative improvements, calculated correct to one decimal place, are 12.4 and 11.7 respectively, which show a difference of 0.7 and this is nearer to one than to zero.

This means that each of the scores in column 6, table 1, was multiplied by $\frac{90.4}{73.5}$ or 1.23.

TABLE 2: shows the resulting scores, given correct to the nearest whole mark. Column 3 shows the difference between class-teaching and weighted individual method scores and thus represents the relative improvement of pupils on the individual method on a basis of equal averages for individual and class-teaching scores.

Pupils A to J show positive improvement on this basis, while pupils L to S show negative improvement, that is, their performances are relatively worse on the individu method than those of pupils A to J. The total positive improvement is equal to the total negative improvement because the average marks of columns 1 and 2 are the same as a result of the weighting.

Column 4 shows the rank of improvement: pupil A improved relatively most, pupil B second most and pupil S the least. The reader should note that the pupils have been named by the letters of the alphabet according to their ranks of improvement. Thus whenever pupil A is mentioned, we know that it is the pupil who made relatively the most progress on the individual method, while pupil O, for example, made relatively smaller improvement than all those who are named by letters preceding O in the alphabet.

REASON FOR WEIGHTING SCORES:

The procedure of weighting the scores has eliminated the effects of unequal averages for the first and second six months of the year, which might have been due to other causes than merely a change of method - causes such as the newness of the subject in the beginning of the year, the more advanced work and wider field which have to be tested towards the end of the year.

Since standardised tests were not used and the scores were weighted, we are not justified in talking of absolute improvement: column 3 shows relative improvement that is, the improvement of each pupil in the class in relation to the improvement of the other pupils in the class. Thus whenever "improvement" is mentioned it will mean "relative improvement".

For the correlations of Chapter IV, the method of rank-differences is used for reasons stated there, hence the rank of improvement (column 4, table 2) is used instead of the improvement scores themselves (column 3). The rank of improvement has been affected by the weighting but not to a very great extent, as for example the improvement scores have been. Hence the process of weighting will not influence the results considerably. The rank of improvement as obtained by this method approximates more nearly to that obtained from the percentage improvement scores, than the rank of improvement as obtained from a mere subtraction of the class-teaching scores from the unweighted individual method scores. The reader can verify this from the marks in table 1.

II CLASS PERCENTAGES.

TABLE 3 gives the average class percentage of each pupil for the four quarterly examinations. Each class percentage is obtained by adding the separate marks (total 100 in each case) of the six subjects of the curriculum - Afrikaans, English, History, Science, Mathematics and German or Bookkeeping and Commercial Arithmetic - and then dividing the total by six. These class percentages are used to get the class position of each of the pupils as is shown in column 6.

The irregular distribution of the marks should not be taken as a disturbing factor, for, as has been pointed out, these 19 pupils constituted the mathematics section of the Std. VII class which consisted of 55 pupils.

III INTELLIGENCE QUOTIENTS.

The intelligence quotients of the pupils were obtained by means of the South African Group Test of Intelligence. Forms I and II were used, fifteen minutes rest was allowed between parts 1 and 2 of the test and all the other necessary precautions were taken.

The test was given at the end of the year (10th December). Pupils C, P and Q were absent. Pupils P and Q were given individual tests, but as pupil C had left the school the week before, his I.Q. was not obtained. He had to be eliminated in all correlations where I.Q. was concerned.

A few of the others were also given individual tests in order to verify the results of the group test. As table 4 indicates, these results show extraordinary similarity. The individual test used was the "Kaap-provinsiale Indiwiduele Intelligensieskaal vir Afrikaans-sprekende Kinders," published in 1929 by the Department of Education of the Cape Province.

Table 4 also shows the sex and age of each of the pupils.

It may be noted here that the average intelligence of the group is 106.2, which is rather low for such a group; compare, for example, the average intelligence quotients of 123.4 and 121.7 of the Std. VII mathematics sections[1] of the two urban schools where the second part of the investigation was carried out.

According to the investigator's judgment the intelligence of pupil B is higher than 99, although this judgment might have been influenced by the fact that this pupil is specially talented as regards mathematics.

(continued on page 54).

1. Vide pp. 124 and 127

TABLE 3: CLASS PERCENTAGES
(Average percentages for all subjects of curriculum).

Pupils	First Term	Second Term	Third Term	Fourth Term	Total	Rank
	1	2	3	4	5	6
A	49.5	59.3	59.6	55.5	223.9	4
B	51.8	58.3	56.5	49.8	216.4	6
C	40.0	54.5	48.5	42.2	185.2	14
D	40.2	50.5	51.7	56.5	198.9	9
E	54.8	64.2	57.8	54.8	231.6	2
F	51.8	52.3	61.3	54.2	219.6	5
G	42.5	58.7	45.0	48.2	194.4	11
H	43.2	45.7	53.3	42.0	184.2	15
I	35.0	34.7	34.0	32.7	136.4	18
J	64.3	64.7	64.0	56.2	249.2	1
K	53.3	53.7	51.5	47.2	205.7	7
L	51.8	60.0	59.8	54.0	225.6	3
M	55.3	56.3	46.2	47.2	205.0	8
N	21.7	21.5	22.5	21.5	87.2	19
O	39.3	57.2	51.3	47.5	195.3	10
P	32.8	44.8	39.2	33.7	150.5	17
Q	54.3	49.0	39.8	24.8	167.9	16
R	51.5	51.0	45.5	46.0	194.0	12
S	41.7	56.3	46.0	42.2	186.2	13
Total	874.8	992.7	933.5	856.2	3657.2	
Average	46.0	52.2	49.1	45.1	192.5	

TABLE 4: INTELLIGENCE QUOTIENTS.

Pupils	Age	Sex. B boy G girl	INTELLIGENCE QUOTIENTS Group Test	Individual Test	Rank
	1	2	3	4	5
A	16.1	G	104	-	13
B	16.9	B	99	-	16
C	14.8	B	-	-	-
D	14.5	G	121	-	1
E	15.10	G	109	-	7.5
F	16.4	B	110	112	6
G	16.2	B	108	-	10
H	15.11	B	114	-	3
I	15.3	B	100	-	14.5
J	14.10	B	112	-	5
K	15.3	B	108	-	10
L	14.4	B	116	115	2
M	15.0	G	109	-	7.5
N	17.2	B	80	-	18
O	18.11	B	95	-	17
P	15.1	B	-	108	10
Q	15.0	B	-	106	12
R	16.3	B	100	100	14.5
S	15.2	B	113	-	4
Total	298.5	4 girls	1912		
Average	15.8	15 boys	106.2		

Note: 1. Column 1 gives ages in years and months: 16.1 means 16 years 1 month.

2. For pupils P and Q the individual test scores are taken; for the others the group test scores are taken.

Unfortunately the opportunity was not available to give him an individual test. The fact that he was fairly old, 17 years 9 months, might have affected the result. The same applies to pupil O. On the whole the class was too old for the intelligence tests to be very reliable.

IV TENACITY COEFFICIENTS.

Achievement in school and intelligence do not always correlate very highly.[1] Some very intelligent children may show no outstanding performance, while others with much lower I.Q.'s may do very well. This apparent discrepancy may be due to faulty measurements either of intelligence or of achievement, but more probably - indeed, we may be fairly certain of it - it is the natural result of other factors, chief among which are certain traits or types of behaviour which are popularly designated as "industry", "perseverance", "ambition", "disposition towards work" and so on. We are more concerned with the first two: industry (or diligence) and perseverance.

One often hears of a boy who is not very intelligent but is very successful because of his persevering and industrious disposition; or, on the other hand, of a very intelligent boy "who has no back-bone". It is true that in the popular conception which is so often based on inferences from individual observations, such isolated cases may become the general rule. According to certain investigations[2] it is probably nearer the truth to say that the most intelligent pupils are in the majority of cases also the most persevering.

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1. Almost any correlation between school marks and I.Q.'s will confirm this statement. See also correlations between I.Q. and maths. scores, and between I.Q. and class-marks on pp 101, Nos. 8, 9, 14.

2. See, for example, "Gifted Children" by L.S. Hollingworth

It may however be possible that factors such as perseverance and diligence will play a relatively more important role in successful individual work than in successful ~~sax~~ class work. Some pupils find it harder than others to concentrate on their work and to persevere with what they do if there is not the continual stimulation of the teacher's voice and personality. From a purely logical consideration one could say that such a type would not do well under the individual method where he is thrown on to his own resources.

To verify such a conclusion objectively would require some technique of arriving at an evaluation of the amount of perseverance or diligence possessed by each individual.

The investigator at first attempted to devise tests by means of which one could find a coefficient of tenacity for each pupil on the same lines as intelligence tests, without the intention however of standardising such tests as it would have been impossible under the d rcumstances. But the idea was eventually given up, as all possible tests seem to test something else as well as tenacity.

Hence a less objective procedure was followed: the teachers of the school were each requested to estimate the perseverance of the separate pupils by giving them marks according to the following scale:-

<u>Perseverance Scale</u>.

5 very persevering.
4 persevering.
3 average.
2 little perseverance.
1 very little perseverance.

Similarly they were requested to estimate the diligence of each pupil according to the following scale:-

Diligence Scale.

5 very diligent (very hard worker).
4 diligent (hard worker).
3 average.
2 little diligence (lazy).
1 very little diligence (very lazy).

For the sake of uniformity and to help the teacher, the terms "perseverance" and "diligence" were defined as follows:

Perseverance: the quality of tackling hard work or a difficult problem and keeping at it till success is met with.

Diligence: industry; steady application to work; the quality of the good hard worker.

Actually the estimates of perseverance corresponded very closely with the estimates of diligence except in a few cases. Hence the two estimates were compounded to form one estimate of tenacity. By tenacity therefore, is meant readiness to work and the ability of continuous application.

The average estimate of the tenacity of each pupil was divided by ten and the fraction so obtained was called the coefficient of tenacity. This means that the amount of tenacity varies from zero to one. The procedure will become clear on consulting tables 5, 6 and 7.

TABLES 5, 6 and 7: Columns 1 to 6 in tables 5 and 6 give the teachers' estimates of the pupils' perseverance and diligence respectively. Column 8 gives the average estimate.

Table 7 shows the tenacity coefficients: column 3 is obtained by adding columns 1 and 2, and hence its maximum mark would be ten. Column 4 is obtained by dividing column 3 by ten, so that the maximum mark for tenacity becomes one.

The estimates of some teachers correspond much more closely than those of others; for example, column 4 shows greater disagreement from the average than any of the others. One concludes that the "personal equation" of that teacher is further removed from the normal than that of the other teachers, or that in the teaching of science the teacher gets a different impression of the pupils' perseverance and diligence, or that the attitude between this teacher and his class is different from that of the other teachers. Perhaps the children really worked harder for him than for the other teachers, because of a greater interest in his subject or because they liked him better.

The coefficients of tenacity as given in column 4, table 7, are the quantitative evaluations of the pupils' perseverance and diligence. They will be compared with the relative improvement given in column 4, table 2.

V INCENTIVE.

In the previous section it was suggested that the reason why intelligence and achievement in school do not usually show a very high correlation might be the coming into play of factors such as industry and perseverance, and that they may be relatively more effective in individual than in class-teaching method. In this section we shall describe the collection of data in conection with a third factor, namely, incentive, with a view to studying its effect on performance in individual work.

GENERAL DISCUSSION OF INCENTIVES:

What motives of study has the vast majority of children? Before trying to measure or to make an estimate of such motives, one has to give a tentative answer to this question. There is little doubt that many children will not be able to say why they do their

(continued on p. 61).

TABLE 5: Teachers' Estimates of Pupils' PERSEVERANCE.

Pupils	Perseverance Estimates made by Class Teachers						Total	Average
	Maths	German	Eng. & Hist.	Science	Bookk. & Comm. Ar.	Afrik.		
	1	2	3	4	5	6	7	8
A	4	3	4	3	3	4	21	3.5
B	4	2	3	3	4	2	18	3.0
C	3	3	3	3	4	2	18	3.0
D	4	4	4	3	-	4	19	3.8
E	4	4	4	4	-	4	20	4.0
F	3	2	3	3	4	3	18	3.0
G	4	3	5	3	3	3	21	3.5
H	3	2	3	3	2	2	15	2.5
I	3	3	1	2	3	1	13	2.2
J	4	4	4	3	4	4	23	3.8
K	3	3	4	3	-	3	16	3.2
L	2	3	3	3	3	3	17	2.8
M	2.5	3	1	3	-	1	10.5	2.1
N	0	2	0	4	-	0	6	1.2
O	4	1	3	4	3	4	19	3.2
P	2	1	2	3	2	1	11	1.8
Q	1	1	1	2	-	1	6	1.2
R	1	2	1	3	3	2	12	2.0
S	3.5	3	4	4	-	2	16.5	3.3
Total	55	49	53	59	38	46	300	53.1
Average	2.9	2.6	2.8	3.1	3.2	2.4		2.8

Note: The teacher of Bookkeeping andCommerciak Arithmetic (Column 5) had only 12 of the 19 pupils in his class and hence could not make the estimates in the other cases.

TABLE 6: Teachers' Estimates of pupils' DILIGENCE.

Pupils	Diligence Estimates made by Class Teachers						Total	Average
	Maths	German	Eng.& Hist.	Science	Bookk.& Comm.Ar.	Afrik,		
	1	2	3	4	5	6	7	8
A	4	4	4	3	3	4	22	3.7
B	3	2	4	3	4	3	19	3.2
C	3	3	3	4	5	-	18	3.6
D	4	4	5	4	-	4	21	4.2
E	4	5	5	4	-	4	22	4.4
F	3	2	4	3	4	2	18	3.0
G	4	3	5	3	2	4	21	3.5
H	3	2	3	3	1	2	14	2.3
I	3	3	2	3	2	1	14	2.3
J	4	5	4	3	3	4	23	3.8
K	3	4	4	4	-	3	18	3.6
L	3	3	4	3	3	3	19	3.2
M	3	3	3	4	-	4	17	3.4
N	0	2	0	3	-	0	5	1.0
O	4	1	4	4	3	4	20	3.3
P	1	1	2	4	3	0	11	1.8
Q	1	1	2	2	-	2	8	1.6
R	1	1	2	3	2	2	11	1.8
S	4	3	3	4	-	3	17	3.4
Total	55	52	63	64	35	49	318	57.1
Average	2.9	2.7	3.3	3.4	2.9	2.7		3.0

Note: The Afrikaans teacher did not make an estimate in the case of Pupil C, as he did not know him well enough to make a just estimate.

TABLE 7: TENACITY COEFFICIENTS.

Pupils	Average Teachers' Estimates of		Total	Tenacity	Rank
	Perseverance	Diligence	Max.Mark 10	10 Coefficients	
	1	2	3	4	5
A	3.5	3.7	7.2	.72	4
B	3.0	3.2	6.2	.62	10
C	3.0	3.6	6.6	.66	8
D	3.8	4.2	8.0	.80	2
E	4.0	4.4	8.4	.84	1
F	3.0	3.0	6.0	.60	11.5
G	3.5	3.5	7.0	.70	5
H	2.5	2.3	4.8	.48	14
I	2.2	2.3	4.5	.45	15
J	3.8	3.8	7.6	.76	3
K	3.2	3.6	6.8	.68	6
L	2.8	3.2	6.0	.60	11.5
M	2.1	3.4	5.5	.55	13
N	1.2	1.0	2.2	.22	19
O	3.2	3.3	6.5	.65	9
P	1.8	1.8	3.6	.36	17
Q	1.2	1.6	2.8	.28	18
R	2.0	1.8	3.8	.38	16
S	3.3	3.4	6.7	.67	7
Total	53.1	57.1	110.2	11.02	
Average	2.8	3.0	5.8	.58	

daily schoolwork, or if they give a reason it will probably be one that strikes them at the moment of reply. The fact that they do not always consciously think about such motives does not however mean that there are none. Every child has motives such as fear or love of teachers or parents, the desire to please someone dear to them by doing well at school, interested in the work, rewards, punishments, social approval or disapproval, competition with class-mates, a sort of pride or even vanity - a form of self-assertion; an ambition which spurs them on to do well at school so that they might enter some vocation - perhaps traditional to the family or because father wants it, or that they may get employment which will make them independent - earning their own money, or so that they may be able to enter the university for further study.

Some incentives are of an immediate and direct character - their effects are only temporary: a child may work hard because he is for the moment interested in what he does, or because he wants to win a prize or because teacher has promised him a caning. Such effects as they have will change with circumstances, hence they fall outside the scope of this investigation. They have been studied by other investigators.[1] The general conclusions are that such incentives play a very important role in successful schoolwork. Two investigators[2] found that extrinsic motivation such as offering prizes, stimulating competition and informing the learner of his progress is most effective, and that "the prime function of motivation is to make drill or practice more palatable"

Other incentives are of a more permanent nature: they are usually concerned with the pupil's future career,

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1. Vide bibliography.

2. Symonds and Chase - Vide bibliography.

home circumstances, and sensitivity to social approval. These are of greater importance for our purpose.

With some children such incentives are very strong: they have definite ideals and plans for the future and they consciously strive to realise them - the urge to get on in life is very strong with them. Others are of a much more easy-going type, they attend school as a matter of course and do not seem to have a motive at all. But all children have to a greater or smaller extent some form of incentive, although the general consensus of opinion is that the average pupil usually works below his maximum capacity because he lacks adequate incentive.

It is to be expected that the presence of a greater or smaller degree of motivation will appreciably effect individual work. With any method, one could say, the pupil with a powerful incentive will do better. But with the individual method strong or weak incentives could be expected to have a greater differentiating effect: the pupil with a strong incentive will shoot ahead while the pupil with a weak incentive will lag behind to a greater extent than with the class-teaching method where the teacher's voice and explanations always constitute an immediate incentive - We have mentioned before that the spoken word is much more alive than the written word.

METHOD:

After considering the question of incentives from this angle, the investigator was faced with the problem of devising some means of arriving at an evaluation of the amount of motivation in respect of each of the 19 pupils with whom the experiment was being carried out. On the face of it this seemed to be fairly easy: people so often in a very cocksure way designate one boy as ambitious and another as not ambitious. But when one honestly tries to classify 19 pupils according to strength of motivation, one is apt to lose all confidence in one's ability to do it. For this reason, partly, the investigator decided not to obtain the teachers' estimates as was done in the case of "tenacity". Such estimates ~~no~~ would also overlap too much with those of "diligence."

At the same time it was clear that, the available criteria not being very specific, it would be wellnigh impossible to distinguish between small variations in degree of motivation, whatever technique was adopted. Hence instead of ranking the 19 pupils in descending order as regards degree of motivation, it was decided to classify them into three groups:-

The A-group	consisting	of	pupils	with	definite	motivation
The B-group	"	"	"	"	average	"
The C-group	"	"	"	"	least	"

Such a threefold grouping presents a smaller field for error in a case like this where the criteria on which the classification has to be based, are not very definite. While some of the pupils may still be classified in wrong groups, the chances are that the majority will be in the right groups.

The data on which the classification into these three groups was based, were obtained by

(a) questioning the pupils individually,

(b) controlling such data by securing confirmatory information from the principal of the school and one of the Std. VI teachers, both of whom were well acquainted with the pupils, their homes and local conditions,

(c) visiting the pupils' homes and conversing with the parents and other with whom the pupils daily associated.

(a) <u>Questioning the Pupils.</u>

The questions had to be directed so as to supply data on which classification into one of the above groups could be made. One of the most important incentives of a more permanent nature, to the investigator's mind, is that which is concerned with the pupils' future vocational plans. The questions had to reveal, if possible, the strength of such motivation. Secondly, they had to show any incentive in connection with home circumstances, such as interest of parents in pupils' school progress, parents' intellectual development, their occupations and concomitant interests. Thirdly, the questions had to gauge whether the pupil was sensitive to the interest that members of the social circle in which he moved took in him and whether he tried to live up to it.

The following questions were put verbally by the investigator to each of the pupils, and their answers afterwards noted down:-

(1) Do you like school?

(2) Do you like Mathematics?

(3) Have you any special reason for continuing school after Std. VI ?

(4) What are your plans for the future? What are you going to do on leaving school?

(5) What is the occupation of (a) your father, (b) your mother (before marriage or now), (c) your brothers and sisters, (d) other near relatives?

(6) Did any of your parents have other than primary education? (N.B. Till recently a very large percentage of children in that locality left school at Std. VI).

(7) Do your parents help you with your schoolwork? Do they often enquire how you are getting on at school? Do they take an interest in your school progress?

(8) Is there any other person who has had a great influence on your lives?

(9) Does it worry you if your schoolwork is not up to date?

The method of oral questioning was adopted in preference to a written questionnaire because it gave opportunity (i) of explaining what was required if the pupil did not understand the question as given above: (ii) of following up unsatisfactory answers by further questioning: (iii) of judging whether the answer was given sincerely and spontaneously or whether it was invented and (iv) of allowing the pupil to talk about himself.

It must be explained here that these questions were given together with other questions concerning the pupils' socio-economic circumstances, health and school records, which will be dealt with in the following section. It took from twenty to thirty minutes to get all the necessary information from each pupil.

The following were the answers of four sample pupils:-

Pupil D: I.Q. 121. Girl. A-group.

(1) Yes.

(2) Yes.

(3) I would like to know more about the different subjects.

(4) I want to get Matric. and then become a nurse.

(5) (a) Farmer, (b) On farm, (c) Farmers, (d) Most of them farmers.

(6) Yes, father matriculated.

(7) No one helps me with my work (parents are on farm). My father is anxious that I should do well at school.

(8) No.

(9) Yes.

Pupil O: I.Q. 95. Boy. A-group.

(1) Yes.

(2) Yes.

(3 & 4) I want to become a farmer. When I am twenty-one I must take over the farm from my mother. I want to get my schoolhigher first and then go on to an agricultural school for two years. If possible I want to continue private study while farming.

(5) (a) Farmer (died), (b) On farm, (c) - (only a sister), (d) Mostly farming, one or two in offices.

(6) Yes, my mother takes an interest in my school-work. Does not help me.

(7) No, only my mother.

(8) Yes, very much.

Pupil H: I.Q. 114. Boy, B-group.

(1) Yes.

(2) Yes.

(3) One must study further. One will need the things one learns later in life.

(4) Advocate if possible. Otherwise I do not know.

(5) All are farmers except one brother who is a policeman.

(6) Father matriculated.

(7) Parents on farm. No. Take an interest.

(8) No.

(9) Yes, I feel uneasy about it.

Pupil Q: I.Q. 106. Boy. C-group.

(1) Yes.

(2) Yes.

(3) No.

(4) I do not know as yet. I shall decide later.

(5) Parents and relatives are all farmers.

(6) No.

(7) No.

(8) No. (9) Yes.

It is unnecessary to give here the full answers obtained from the other pupils as innumerable inconsistencies made the investigator give up his attempt to classify them statistically.

In a sense the answers to these questions did not prove very satisfactory. Questions 1, 2 and 9 were definitely of no use.

Questions 1 and 2:- Eighteen out of the nineteen pupils answered "yes". Only pupil N (I.Q. 80) stated that he did not like school or mathematics. According to the pupils themselves, therefore, they all had a definite incentive as regards direct interest in schoolwork and mathematics. Other factors however showed that some of them had no such incentive.

Apart from this, however, it was clear to the investigator that positive answers were given because they thought that was the fit thing to do: the investigator would be pleased to hear that they liked mathematics because that is his special subject.

Hence the answers to these questions were left out of account in the classification.

Questions 3 and 4:- These two questions are considered together because many of the pupils gave an answer concerning their future plans to question 3. It was generally necessary to follow up the answers given to question 4 by further questions in order to discover, if possible, whether the answer had been genuine or whether it had been invented on the spur of the moment.

The answers to question 3 of pupils D, E, H, J, L, were to the effect that they would like to increase their knowledge or that they would require it in later life. Pupils B, F, I, K, P stated that it was their parents' desire that they should study further. The other either answered "no" or gave an answer in connection with their future plans.

As regards question 4 in which pupils were requested to express their ideas of career, pupils A, B, C, D, E, H, J, L, M, O, R, S had more or less definite plans for the future. In column 1, table 8, they were all classified in the A-group, except pupils C, M and S who were classified in the B-group for the following reasons: pupil C was rather vague about going into an office like his brother - it appeared that his future plans could not be considered a very definite incentive as regards work in school; pupil M showed the same vagueness. Pupil S stated that the wanted to become an engine driver; as he had never been on a train and had seen it only a few times, it looked more like a day-dream than a definite incentive. The other pupils - F, G, I, K, N, P, Q-who stated that they had no plans for the future, were classified in the C-group.

Questions 5, 6 and 7:

The tendency was to give positive answers to question 6, and hence much significance could not be attached to them. The answers to question 7 are given in column 6, table 10. Most information concerning these questions were obtained from the other sources: principal of the school and visiting the homes.

Questions 8 and 9:

The answers to question 8 were negative throughout, while those to question 9 were positive throughout. Hence they proved to be of no value.

In drawing up a list of questions like the above it seems advisable to test them out on a few preliminary pupils before fixing the final list. The latter would thus contain no unnecessary or useless questions and the other questions could be altered to suit the purposes better. The investigator was, however, inexperienced as a result of which much unnecessary labour was involved and even then the questions chosen were not all of the most suitable.

(b) Confirmatory Information.

The data collected by questioning the pupils were controlled by consultation with the principal of the school, who had been in the school for eight years, and secondly, with one of the Std. VI teachers who had been there for fifteen years. Both of them associate freely with the parents and have perhaps a more comprehensive view of local circumstances than most other inhabitants of the village.

According to the latter, (the class had passed through his hands the year before), pupils P, Q and R have no ambition at all. Pupil M has a little but it has been damped by extreme poverty. Pupils A, D and E are very ambitious - they definitely set the standard in a class. Pupil F tries to get on ("vooruitstrewend") and does well considering the poor circumstances of his home. Pupil B is old for his class and it augurs well for him that he still tries to improve his position - his brother left school at an early age. Pupil Q has no ambition at all - just like his father. He once ran away from boarding school. He wants to "loaf" around in the streets the whole day long.

(c) Visiting the Pupils' Homes.

These visits were made in an informal way and the investigator tried to bring the conversation to bear on the child in a tactful way. This proved an easy task as most parents are eager to talk about their children. In most cases they manifested a strong interest in their children, but direct questions were not put so as not to give them any suspicion that such interest was being gauged.

TABLE 8: DATA SUMMED UP:

The data as obtained by the above three methods were used to classify the pupils into the A, B or C groups after a consideration of the following factors:-

1) Strength of incentive in connection with pupil's future vocational plans: column 1 of table 8 shows this classification according to data as discussed on page 68 and confirmed by other evidence.

2) Home circumstances, intellectual development of parents, interest of parents in pupil's progress, social prestige: whether these constitued an encouraging factor in the pupil's studies. Column 2 summarises this classification.

3) Other factors such as the pupil's sensitivity to the interest that his immediate social circle manifests in his achievement at school, and his ambitions in general, as judged by the investigator. This classification is given in column 3.

It may be pointed out that the classification in column 1 is a fairly objective representation of the pupils' statements of career; the classification in column 2 is partly objective in so far as the parents' intellectual status was gauged by the amount of schooling[1] that they had, but otherwise the investigator had to rely on his own judgment because of the unreliability of pupils' and parents' statements; the classification in column 3 is purely subjective - the investigator had the opinion of the Std. VI teacher and further he used his own judgment.

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1. Vide column 6, table 10.

TABLE 8: Classification of pupils into A, B or C groups according to strength of INCENTIVE.

A-group: pupils with definite incentive.
B-group: " " average "
C-group: " " least "

Pupils	Career	Home	Other Factors	Final Classification
	1	2	3	4
A	A	C-	A++	A
B	A	A	A	A
C	B	B	B	B
D	A	A	A+	A
E	A	A+	A+	A
F	C	C-	C	C
G	C	B	A	B
H	A	B	C	B
I	C	B	C	C
J	A	A++	A+	A
K	C	B	C+	C
L	A	A++	C-	B
M	B	C- -	C+	C
N	C	C- -	C-	C
O	A	A	A	A
P	C	C-	C	C
Q	C	C-	C-,-	C
R	A	B	C+	B
S	B	C-	C-	C
Summary	9 A's 3 B's 7 C's	6 A's 6 B's 7 C's	7 A's 1 B 11 C's	6 A's 5B's 8 C's

Note: The + or - signs are used in columns 2 and 3 to indicate stronger or weaker motivation in the particular group.

To understand table 8 better, the reader should refer to the general information about each of the 19 pupils given in appendix C.

Column 4 gives the final classification. There was no doubt as to the categories in which pupils B, C, D, E, F, J, N, O, P, Q should be placed: the data of coumns 1, 2 and 3 definitely pointed to one of the three categories. As regards pupils G, H, I, K, M, R S, the data are more conflicting, but the final classification is the average of the other three columns. The final classification of pupils A and L proved more difficult: in their cases the investigator ultimately allowed his own judgment as represented in column 3 to override the slightly more objective classifications in columns 1 and 2, by putting pupil A in category A and pupil L in category B, although, strictly statistically, they should be put in the same category. The general evidence was clear enough to allow such a discrimination.

LIMITATIONS:

The subjective element as represented by the investigator's judgment entered strongly into the above grouping, in spite of the fact that the available criteria were treated as objectively as possible.

On the other hand, were it not for this subjective element, the classification would have had to be in accordance with degree of motivation as taken in a very restricted sense, for example, as represented by column 1 of table 8. Incentive would have meant whatever the few objective data claimed to measure.

The attempt to secure data which were amenable to statistical treatment proved a failure because of the unreliability of the human being where the verbal expression of his inner drives and motives are concerned. Very few people are willing to confess openly why they act in the ways that they do, and try instead to create the impression that they are more ambitious and progressive than they really are, but have become the prey of circumstances. The investigator observed time and again that pupils and parents alike tried to create such impressions which are in accordance with their conceptions of how things should be.

The only way to overcome this difficulty is to employ means which would show indirectly what the real incentives are, as was partly done. An example of this is the amount of schooling that the parents had: their interests would to a large extent be conditioned by it; in the case of parents of superior standard the intellectual status of the home and the intellectual values set before the pupil would determine his incentive for the reason that the pupil necessarily attempts to attain the parental standard. While such means were used where possible, it was found inadvisable to base the whole classification on it, partly because of the restricted meaning which that would ascribe to "incentive", but mainly because of the impractibility of such a method under the circumstances.

The limitations of the measuring devices at our disposal at present are distinctly revealed when it comes to measuring the inner drives and motives of the human being. They can only be evaluated by measuring responses which are made to environmental stimuli, but these responses become more variable and difficult to isolate the more deep-seated and subtle the motives are, so that increasingly larger numbers of cases are required in order to attain a satisfactory degree of reliability. The result is that the exasperated scientist often utters a fervent prayer that all the powers of the world should be directed in his cause so that he might complete his researches

- only to realise afterwards that this is impossible as living itself is more important than the reason for living.

With these limitations of method and means the investigator had to make the best of the available material by supplementing it with his own critical common-sense judgment. Hence, whatever the results may indicate, no great importance can be attached to this part of the investigation from the strictly objective scientific point of view. The investigator however offers it as being suggestive of further investigation in this field.

VI <u>SOCIO-ECONOMIC STATUS (S.E.S.)</u>

In order to study the effect of socio-economic status on performance in assignment work, an evaluation of it had to be obtained. The data according to which this evaluation was made was collected in more or less the same way as outlined in the previous section:-

(a) by questioning pupils

(b) by consulting the principal of the school who was personally acquainted with the circumstances of each of the pupils.

(c) by visiting the homes

(d) by obtaining the Divisional Council evaluations in the cases where the parents owned properties (where the other sources did not prove sufficiently reliable).

The method of questioning the pupils was not really ~~successful~~ necessary, but it was the easiest way of obtaining the bulk of the information, reserving the other sources for controlling such information and for securing additional information. As was pointed out before, this questioning was done in conjunction with that concerning incentive.

As socio-economic status is a much more tangible factor than incentive, it was decided to rank the 19 pupils separately instead of classifying them into three different groups. This was done in two ways:-

A. The investigator, in consultation with the principal of the school, ranked them according to his judgment of their socio-economic status.

B. A number of factors which condition S.E.S. were rated separately: the sum of these separate ratings indicated the S.E.S. of each pupil. From a scientific point of view this is the better method of the two but it has its shortcomings as will be pointed out later.

These two methods will be dealt with separately.

A. <u>Ranking by Investigator in consultation with Principal.</u>

The first step was to classify each of the 19 pupils into one of the following groups:-

1. Very good socio-economic circumstances.
2. Good " "
3. Average " "
4. Poor " "
5. Very poor " "

The locality where the experiment was performed has many poor people. The general socio-economic level may be gauged from the fact that more than seventy-five percent of the secondary school children receive all or some of their books free. Hence a local standard of classification had to be set up.

After a general survey which produced a tentative standard of comparison, each pupil was discussed separately and classified in one of the above groups on the following considerations:-

a) Economic status of parents: whether they were well-off, or just lived comfortably, or whether they could just afford the bare necessities of life or whether there was at times a shortage of food in the house. It may be mentioned here that in the homes of pupils M and N, there was usually a shortage of food, while the homes of pupils A, F and P were not much better.

b) Social status of family: whether they were the leading people of the district, whether they belonged to the average respectable class or whether they were of the "bywoner"-type.

The second step consisted of ranking the pupils of each group separately. Where there was any doubt which of two or more pupils should be assigned a particular rank, they were rather given the same rank.

It will not leave the reader much the wiser if the cases of the separate pupils are discussed here, as such a ranking primarily depends on a first-hand acquaintance with the circumstances of each of the families and of local conditions. He is, however, referred to appendix C for information about the separate pupils.

<u>Table 9</u>, Column 1 gives the grouping according to very good (V.G.), good (G), average (A), poor (P) or very poor (V.P.) socio-economic conditions.

Column 2 gives the separate ranks.

It should be explained that this method was adopted in preference to the method of obtaining the average of a number of separate estimates by people who are familiar with local conditions and families, firstly, because such people are difficult to find and secondly, because few of them are able to rise above their prejudices even for the sake of an objective investigation.

TABLE 9: Ranking (A) of pupils according to SOCIO-ECONOMIC STATUS (investigator's judgment).

Pupils	Preliminary Grouping	Final Ranking
	1	2
A	V.P.	16
B	A	9
C	P	12
D	G	6
E	G	3
F	V.P.	17
G	A	10
H	A	8
I	G	5
J	V.G.	1
K	A	7
L	G	3
M	V.P.	19
N	V.P.	18
O	G	3
P	V.P.	15
Q	P	13
R	P	11
S	P	14
Summary	1 V.G. 5 G. 4 A. 4 P. 5 V.P.	

B. <u>Rating of Specific Factors</u>:

This method consists of assigning marks to a number of specific factors which determine the pupil's socio-economic status: for example, one mark is given if the father possesses a motor-car, another mark if he subscribes to one daily newspaper and so on. If a sufficiently large number of representative factors are taken, in order to allow for chance factors such as individual eccentricities and idiosyncracies, a fairly objective quantative measurement of S.E.S. can be obtained. Attempts have been made in other countries to draw up and standardise scales for this purpose.[1.]

The best that the investigator could do under the circumstances was to decide upon a number of factors which would determine S.E.S. and assign marks to each according to its relative importance as judged by him.

The following information was obtained for this purpose and marks assigned as shown in brackets:-

1) Does family possess a motor-car? (1 mark - ½ a mark if motor-car is very old. Also 1 or ½ mark if it possesses a cart and horses or some other vehicles.)

2) Does family possess a piano, radio, harmonium or gramophone or something similar? (1 mark for each of first two, ½ mark for each of last two). Maximum of 2 marks).

3) Is or was father a member of schoolboard, school committee, divisional council or municipality? (1 mark for each. Maximum of 2 marks.)

4) How many periodicals and newspapers in house? (1 mark for each daily newspaper, ½ a mark for each periodical. Maximum of 2 marks).

5) How many books in house? (1, ½ or 0 marks)

6) Did father or mother have education other than primary? (1 mark for each. Maximum of 2 marks).

7) Does pupil take private lessons like music? Does he play tennis? (1 mark for either. Maximum of 1 mark).

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1. Vide Journal of Educational Psychology 1925: "The Quantitive Measurement of Certain Aspects of Socio-economic Status" by Chapman and Sims.

8) Does pupil get free books in school? (1 or ½ mark if he pays for all/for some of his books respectively)

9) Value of parents' properties or their salaries? (4, 3, 2, 1 or 0 marks).

Factors such as furniture, kitchen utensils, number of rooms in house, occupation of parents and so on were omitted because of the difficulty of assigning marks objectively in their cases.

TABLE 10:

In table 10, columns 1 to 9, the marks as assigned to the above factors are given. Column 10 gives the total mark for each pupil and column 11 the rank.

LIMITATIONS OF RANKING B AND COMPARISON WITH RANKING A:-

The number of factors were not sufficiently large or comprehensive; they do not distinguish between pupils A, F, M and N. Now that the investigator has completed the experiment, he sees several ways of improving on the above.

Socio-economic status as measured in this way will mean whatever the above factors measure, and the reader, will agree that that meaning is not as comprehensive as the meaning attached to it in the first method of ranking. In addition, full marks were assigned to each of the factors in accordance with what the investigator judged to be its importante as no standardising technique could be applied.

Where the investigator knew exactly what was required as well as the circumstances of each pupil, and where all the pupils were inhabitants of one and the same district, the ranking by the first method could claim to be a truer representation of the situation than a scale like the above which has not been standardised and which is not sufficiently comprehensive to allow for chance factors.

(Continued on page 81).

TABLE 10: RANKING (B) of pupils according to their SOCIO-ECONOMIC STATUS (rating of specific factors).

Pupils	Con-veyance	music. instr.	Public body	News-papers	Books	Intell. status	Private lessons	Free books	Property	TOTAL	RANK
	1	2	3	4	5	6	7	8	9	10	11
A	-	-	-	-	-	-	-	-	-	0	17.5
B	½	-	1	1	1	-	-	½	2½	6½	9
C	-	1	-	1	½	-	1	-	1	4½	12
D	½	1	1	1½	½	1	1	½	2½	9½	3
E	1	1	-	1½	½	1	1	½	2	8½	7
F	-	-	-	-	-	-	-	-	-	0	17.5
G	½	-	-	2	½	-	-	-	2	5	11
H	½	½	-	2	1	1	-	½	2½	8	8
I	1	1	-	1½	½	-	1	1	4	10	2
J	1	1½	2	2	1	1	1	1	4	14½	1
K	½	½	-	1½	½	1	-	-	3	9	5
L	½	½	-	2	1	1	-	1	3	9	5
M	-	-	-	-	-	-	-	-	-	0	17.5
N	-	-	-	-	-	-	-	-	-	0	17.5
O	1	1	-	1½	½	-	-	1	4	9	5
P	-	½	-	-	-	-	1	-	-	1½	13
Q	-	-	-	-	-	-	-	-	1	1	14
R	½	½	-	1½	1	-	1	½	1	6	10
S	-	-	-	-	-	-	-	-	½	½	15
Total	7½	9	4	19	8½	6	7	6½	33	102½	

In addition, there was the difficulty that five of the pupils were in indigent boarding-houses (pupils F, G, K, M, Q) and four others boarded privately in the village as their parents farmed in the district (pupils B, D, E, H). Their immediate environment in the village was a factor which had to be considered.

TABLE 11:

In table 11, the two different rankings are correlated by Spearman's method of rank-differences. They show a very high correlation, .995: this may be ascribed to the fact that many of the factors used in method B necessarily affected the investigator's judgment in method A, although the first ranking was completed some time before the second was made.

TABLE 11: CORRELATING the RANKS as determined by Methods A and B.

Pupils	Ranking A	Ranking B	d	d^2
	1	2	3	4
A	16	17.5	1.5	2.3
B	9	9	0	0
C	12	12	0	0
D	6	3	3	9
E	3	7	4	16
F	17	17.5	0.5	.3
G	10	11	1	1
H	8	8	0	0
I	5	2	3	9
J	1	1	0	0
K	7	5	2	4
L	3	5	2	4
M	19	17.5	1.5	2.3
N	18	17.5	0.5	.3
O	3	5	2	4
P	15	13	2	4
Q	13	14	1	1
R	11	10	1	1
S	14	15	1	1
			Sum	59

$$\rho = 1 - \frac{6 \times 59}{19(19^2 - 1)}$$

$$= 1 - .0052$$

$$= .9948$$

VII ADDITIONAL DATA.

The following additional information about each pupil was collected: a) health, b) extra-mural activities, c) school record, d) general behaviour in class, e) pupils' opinions of individual method.

Much of this is embodied in the general description of each pupil given in Appendix C. The purpose was to discover any abnormalities which might affect individual study, or any other facts which might throw light on the general achievement of the pupils.

a) Health: Data was obtained

1) by questioning pupils: Have you ever been ill? For how long? When? Any after effects? Any other ailments? Eyes or ears? Doyou easily feel fatigued when doing schoolwork?

2) from school medical doctor's Reports where available.

3) from parents

4) by personal observation: strong or delicate, healthy or sickly appearance; active and lively or passive and dull disposition.

The only facts worth while mentioning here are the following: the asthma of pupil J; sore eyes of pupils F, G and Q which kept them out of school for 4, 5 and 6 weeks respectively in the second six months of the year; the underfed appearance of pupils N and S.

b) Extra-Mural Activities:-

The active participation of pupils in sport, debates, concerts and so on may throw some light on their general dispositions. We may mention here: the all-round sporting capabilities of pupils C and J; the artistic efforts of pupil D (handword and drawing);

the talent for reciting of pupil M; musical talents of pupils P and R. Pupils G, M, N, Q, R and S took no part in any form of sport.

c) School Record:-

Information concerning the following points was obtained from the Std. VI teachers as well as from the pupils themselves:

1) Average symbol (pupils are awarded symbols as the result of quarterly examinations in this school, and not class positions); quality of schoolwork - very superior, superior, average, inferior or very inferior; any special abilities.

2) Any standards failed (accelerated or retarded).

3) Age of school entrance (normal or late).

4) Whether absent from school for any long period.

5) How many different schools attended.

The following facts may be given here:- Pupils D, G and I attended farm schools till Std. VI; pupil B entered school late, at age of nine, hence he was very old for Std. VII (16¾ years); pupil O entered the village school at age of eight, then was out of school for 1½ years before attending a farm school till Std. 4, failed Std. 6 - hence retarded, age 18.11; pupil Q was chosen in 1933 as the best indigent boarding-house pupil in Std. 6, hence he could stay on for Std. 7, receiving free books, but since then his progress has been hopeless; pupil N, whose intelligence was definitely below normal, should never have been in Std. VII - he understood very little of mathematics, or of any of the other work done in school. He failed Std. VII and left school after that. Pupils I and P also failed Std. VII; pupils B, H, Q and R were reported by Std. VI teacher to be good at arithmetic, although only pupil B did well at mathematics; pupils A, D and E were commended for their good behaviour and in general setting the standard of the class - they were also the neatest in the class, especially pupil A.

For further information the reader is referred to Appendix C where each of the pupils are discussed separately.

d) General Behaviour in Class:-

The investigator made observations in the class with regard to the following:-

i) Which pupils are attentive in class and lively in answering questions (during 6 months of class-teaching)

ii) Which pupils are systematic and neat in their work or careless.

iii) Types of questions asked: intelligent or not?

iv) Any other peculiarities: in writing, unnecessary use of erasers, etc.

Pupil L was definitely handicapped by his untidy and unsystematic habits of work as well as by a poor handwriting. Pupil I who was one of the weakest in mathematics, very seldom followed any explanations during the class-teaching period - he seemed to follow the written explanations in the assignments better, although it made no appreciable difference to his performance.

e) Pupils' Opinions about Individual Method:-

The pupils were asked whether they liked the individual method better than the class-teaching method, and why.

Pupils A, B, C, D, E, J, K, O replied in the affirmative and the rest in the negative.

The following were some of the answers obtained

Pupil A: Yes (emphatically). I can follow the written explanations better. I cannot always follow the teacher explaining.

Pupil H: I do not like it because you have much more work to do.

Pupil O: Yes, because I can work on my mathematics whenever I have little other homework to do.

CHAPTER IV.

RESULTS.

The different groups of data which had been collected as described in the previous chapter, were compared in order to discover which factors were instrumental in bringing about the changes in individual performances when the class was put on the individual method.

CALCULATION[1].

RAW CORRELATIONS: There were two ways of comparing the data:-

1) The differences between class-teaching and individual method scores, in other words the relative improvement, were correlated with tenacity, S.E.S., I.Q., class percentages and mathematics scores respectively.

2) The class-teaching scores and individual method scores were correlated separately with each of the other factors and in each case the class-teaching correlation and individual method correlation were compared in order to discover whether the difference between them was significant or not.

This may be illustrated by taking the example of I.Q.: firstly, the relative improvement was correlated with I.Q. and showed a negligible correlation; secondly, the class-teaching and individual method scores were separately correlated with I.Q., and the two correlation coefficients which were obtained differed insignificantly, showing that I.Q. does not favour any

1. All the formulae used in the calculations are given in Appendix D.

of the two methods and thus confirming the first-mentioned result.

For this part of the investigation, Spearman's method of rank-differences is used to calculate correlation coefficients: the relative improvement scores consist of positive and negative quantities which make it impossible to use the product-moment method. The symbol "ρ" (rho) is employed to denote a correlation coefficient as obtained by this method, reserving the symbol "r" for the product-moment method which is the best method.

In appendix D, the formulae are given which were used to calculate "ρ" and "r", and their probable errors. For a few correlations, like I.Q. and mathematics scores, the product-moment method was possible, but it was thought advisable to retain the other method throughout for the sake of uniformity. In any case it would not make an appreciable difference.

It was considered unnecessary to give the actual tables of calculation of all the correlations that were made.

TABLES 12 to 16 show the correlations of relative improvement with total mathematics scores for the year, with class percentages, with I.Q., with tenacity and with S.E.S. respectively.

TABLES 17 and 18 show the correlations of tenacity with class-teaching and with individual method scores respectively.

These tables are self-explanatory.

From the data and tables already given, the reader can verify any of the correlation coefficients given below with very little extra trouble.

(Continued on page 99).

TABLE 12: CORRELATION between RELATIVE IMPROVEMENT and TOTAL MATHEMATICS SCORES for the year.

(The total maths. scores are obtained by adding columns 3 and 6 of table 1).

Pupils	Relative Improvement (rank)	Total Maths. Scores		d (rank-difference)	d^2
		score	rank		
A	1	155.5	12	11	121
B	2	209.5	4	2	4
C	3	166.5	10	7	49
D	4	223	2	2	4
E	5	221.5	3	2	4
F	6	180.5	8.5	2.5	6.25
G	7	160	11	4	16
H	8	180.5	8.5	0.5	0.25
I	9	85.5	17	8	64
J	10	277	1	9	81
K	11	201.5	5	6	36
L	12	153.5	13	1	1
M	13	114	16	3	9
N	14	52.5	19	5	25
O	15	182	7	8	64
P	16	84.5	18	2	4
Q	17	141	14	3	9
R	18	134.5	15	3	9
S	19	191	6	13	169
n = 19					675.5

Note: For formulae used below see appendix D.

$$\rho = 1 - \frac{6 \times 675.5}{19(19^2 - 1)} = .4074$$

$$P.E. = .706 \times \frac{1 - .4074^2}{\sqrt{19}} = .1348$$

$$\therefore \rho = .41 \pm .14$$

TABLE 13: CORRELATION between RELATIVE IMPROVEMENT and AVERAGE CLASS POSITION during the year.

Pupils	Relative Improvement (rank)	Average Class Position (Col. 6, Table 3).	d	d^2
A	1	4	3	9
B	2	6	4	16
C	3	14	11	121
D	4	9	5	25
E	5	2	3	9
F	6	5	1	1
G	7	11	4	16
H	8	15	7	49
I	9	18	9	81
J	10	1	9	81
K	11	7	4	16
L	12	3	9	81
M	13	8	5	25
N	14	19	5	25
O	15	10	5	25
P	16	17	1	1
Q	17	16	1	1
R	18	12	6	36
S	19	13	6	36
n = 19				654

$$\rho = 1 - \frac{6 \times 654}{19(19^2 - 1)} = .4264$$

$$P.E. = .706 \times \frac{1 - .4264^2}{\sqrt{19}} = .1320$$

$$\therefore \rho = .43 \pm .13$$

TABLE 14: CORRELATION between RELATIVE IMPROVEMENT and I.Q. (I.Q. of Pupil C was not obtained. See p. 51)

Pupils	Relative Improvement (rank)	I.Q. (Col.5,tab.3)	d	d^2
A	1	13	12	144
B	2	16	14	196
C	-	-	-	-
D	3	1	2	4
E	4	7.5	3.5	12.25
F	5	6	1	1
G	6	10	4	16
H	7	3	4	16
I	8	14.5	6.5	42.25
J	9	5	4	16
K	10	10	0	0
L	11	2	9	81
M	12	7.5	4.5	20.25
N	13	18	5	25
O	14	17	3	9
P	15	10	5	25
Q	16	12	4	16
R	17	14.5	2.5	6.25
S	18	4	14	196
n = 18				826

$$\rho = 1 - \frac{6 \times 826}{18(18^2 - 1)}$$

$$= .1477$$

$$P.E. = .706 \times \frac{1 - .1477^2}{\sqrt{18}}$$

$$= .1627$$

$$\therefore \rho = .15 \pm .16$$

TABLE 15: CORRELATION between RELATIVE IMPROVEMENT and TENACITY.
(See also tables 17 and 18, where tenacity is correlated with class-teaching and individual method scores respectively).

Pupils	Relative Improvement (rank)	Tenacity (Col.5, Tab. 7)	d	d^2
A	1	4	3	9
B	2	10	8	64
C	3	8	5	25
D	4	2	2	4
E	5	1	4	16
F	6	11.5	5.5	30.25
G	7	5	2	4
H	8	14	6	36
I	9	15	6	36
J	10	3	7	49
K	11	6	5	25
L	12	11.5	0.5	0.25
M	13	13	0	0
N	14	19	5	25
O	15	9	6	36
P	16	17	1	1
Q	17	18	1	1
R	18	16	2	4
S	19	7	12	144
n = 19				509.5

$$\rho = 1 - \frac{6 \times 509.5}{19(19^2 - 1)} = .5531$$

$$P.E. = .706 \times \frac{1 - .5531^2}{\sqrt{19}} = .1124$$

$$\therefore \rho = .55 \pm .11$$

TABLE 16: CORRELATION between RELATIVE IMPROVEMENT and SOCIO-ECONOMIC STATUS.
(Note: The ranking according to S.E.S. as given in column 2, table 9, is used in this correlation. If the ranks in column 11, table 10, are taken, then $\rho = .12$).

Pupils	Relative Improvement (rank)	S.E.S. (Col.2,tab.9)	d	d^2
A	1	16	15	225
B	2	9	7	49
C	3	12	9	81
D	4	6	2	4
E	5	3	2	4
F	6	17	11	121
G	7	10	3	9
H	8	8	0	0
I	9	5	4	16
J	10	1	9	81
K	11	7	4	16
L	12	3	9	81
M	13	19	6	36
N	14	18	4	16
O	15	3	12	144
P	16	15	1	1
Q	17	13	4	16
R	18	11	7	49
S	19	14	5	25
n = 19				974

$$\rho = 1 - \frac{6 \times 974}{19(19^2 - 1)}$$

$$= .1457$$

$$\text{P.E.} = .706 \times \frac{1 - .1457^2}{\sqrt{19}}$$

$$= .1583$$

$$\rho = .15 \pm .16$$

TABLE 17: CORRELATION between TENACITY and CLASS-TEACHING SCORES.

Pupils	Class-Teaching scores (col.3, tab.1)		Tenacity (Table 7)		d	d^2
	Scores	Rank	Coefficients	Rank		
A	70	15	.72	4	11	121
B	101	7	.62	10	3	9
C	78	14	.66	8	6	36
D	111	4.5	.80	2	2.5	6.25
E	114	3	.84	1	2	4
F	94	9	.60	11.5	2.5	6.25
G	83	13	.70	5	8	64
H	97	8	.48	14	6	36
I	45	18	.45	15	3	9
J	151	1	.76	3	2	4
K	111	4.5	.68	6	1.5	2.25
L	86	12	.60	11.5	0.5	.25
M	65	16	.55	13	3	9
N	32	19	.22	19	0	0
O	110	6	.65	9	3	9
P	57	17	.36	17	0	0
Q	89	10.5	.28	18	7.5	56.25
R	89	10.5	.38	16	5.5	30.25
S	135	2	.67	7	5	25
n = 19.						427.5

$$\rho = 1 - \frac{6 \times 427.5}{19(19^2 - 1)} = .6249$$

$$P.E. = .706 \times \frac{1 - .6249^2}{\sqrt{19}} = .0997.$$

$$\therefore \rho = .62 \pm .10$$

TABLE 18: CORRELATION between TENACITY and INDIVIDUAL METHOD SCORES.

Pupils	Individual Method Scores (Col.6, tab.1)		Tenacity (Table 7)		d	d^2
	Scores	Rank	Coefficients	Rank		
A	85.5	8	.72	4	4	16
B	108.5	3	.62	10	7	49
C	88.5	6	.66	8	2	4
D	112	2	.80	2	0	0
E	107.5	4	.84	1	3	9
F	86.5	7	.60	11.5	4.5	20.25
G	77	10	.70	5	5	25
H	83.5	9	.48	14	5	25
I	40.5	17	.45	15	2	4
J	126	1	.76	3	2	4
K	90.5	5	.68	6	1	1
L	67.5	12	.60	11.5	0.5	0.25
M	49	15	.55	13	2	4
N	20.5	19	.22	19	0	0
O	72	11	.65	9	2	4
P	27.5	18	.36	17	1	1
Q	52	14	.28	18	4	16
R	45.5	16	.38	16	0	0
S	56	13	.67	7	6	36
n = 19						218.5

$$\rho = 1 - \frac{6 \times 218.5}{19(19^2 - 1)}$$

$$= .8084$$

$$P.E. = .706 \times \frac{1 - .8084^2}{\sqrt{19}}$$

$$= .0557$$

$$\therefore \rho = .81 \pm .06$$

TABLE 19: RELATIVE IMPROVEMENT and INCENTIVE.

Pupils	Incentive Grouping (Col. 4, tab. 8)	Relative Improvement for pupils falling in		
		A-Group	B-Group	C-Group
A	A	+35		
B	A	+33		
C	B		+31	
D	A	+27		
E	A	+18		
F	C			+12
G	B		+12	
H	B		+6	
I	C			+5
J	A	+4		
K	C			0
L	B		-3	
M	C			-5
N	C			-7
O	A	-21		
P	C			-23
Q	C			-25
R	B		-33	
S	C			-66
Total		+96	+13	-109
Average	~~16x0~~	+16.0	+2.6	-13.6
	σ	19.53	21.0	23.1
	E^2	28.9	40.1	30.3

Note: 1) See appendix D for the different formulae used in the calculations on this page.

2) σ = standard deviation
E = P.E. of average.
$E_{diff AB}$ = P.E. of the difference between the averages of the A- and B-groups.
$C.R._{AB}$ = Critical ratio for the A- and B-group averages

$E_{diff AB} = \sqrt{28.9 + 40.1} = 8.31$

$E_{diff BC} = \sqrt{40.1 + 30.3} = 8.39$

$E_{diff AC} = \sqrt{28.9 + 30.3} = 7.69$

$C.R._{AB} = \frac{13.4}{8.31} = 1.6$

$C.R._{BC} = \frac{16.2}{8.39} = 1.9$

$C.R._{AC} = \frac{29.6}{7.69} = 3.9$

TABLE 20: COEFFICIENTS of VARIABILITY of CLASS-TEACHING and INDIVIDUAL METHOD SCORES.

Pupils	CLASS-TEACHING			INDIVIDUAL METHOD		
	Scores	Deviation from Av. D	D^2	Scores (Weighted)	Deviation from Av. D_1	D_1^2
A	70	20.4	416	105	14.6	213
B	101	10.6	112	134	43.6	1901
C	78	12.4	154	109	18.6	346
D	111	20.6	424	138	47.6	2266
E	114	23.6	557	.32	41.6	1731
F	94	3.6	13	106	15.6	243
G	83	7.4	55	95	4.6	21
H	97	6.6	44	103	12.6	159
I	45	45.4	2061	50	40.4	1632
J	151	60.6	3672	155	64.6	4173
K	111	20.6	424	111	20.6	424
L	86	4.4	19	83	7.4	55
M	65	25.4	645	60	30.4	924
N	32	58.4	3411	25	65.4	4277
O	110	19.6	384	89	1.4	2
P	57	33.4	1116	34	56.4	3181
Q	89	1.4	2	64	26.4	697
R	89	1.4	2	56	34.4	1183
S	135	44.6	1989	69	21.4	458
Total	1718	420.4	15500	1718	567.6	23886
Average	90.4	22.1		90.4	29.9	

Class-teaching Sobres:

Standard dev. $= \sqrt{\frac{15500}{19}}$

$= 28.6 \pm 3.13$

Coefficient of variability $= \frac{28.6}{90.4}$

$= .32$

Individual Method Scores:

Standard dev. $= \sqrt{\frac{23886}{19}}$

$= 35.5 \pm 3.89$

Coeff. of variability $= \frac{35.5}{90.4}$

$= .39$

P.E. of diff. between 28.6 and 35.5
$= 4.99$
Critical ratio $= 1.4$
Probbability coefficient $= .345$

CALCULATIONS.

MULTIPLE CORRELATION COEFFICIENTS:-

(a) Relative Improvement with I.Q. + tenacity.

1 - Relative Improvement.
2 - I.Q.
3 - Tenacity.

$r_{12} = .15$ $r_{13} = .55$ $r_{23} = .40$

$$r_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}\ r_{13}\ r_{23}}{1 - r_{23}^2}}$$

$$= \sqrt{\frac{.15^2 + .55^2 - 2 \times .15 \times .55 \times .40}{1 - .40^2}}$$

$$= \underline{.5552}$$

(b) Relative Improvement (1) with Maths scores (2) plus class scores (3).

$r_{12} = .41$ $r_{13} = .43$ $r_{23} = .62$

$\underline{r_{1.23} = .4673}$ (calculated as above).

PARTIAL CORRELATION COEFFICIENTS:-

(a) S.E.S.(1) and Class-teaching scores(2) with I.Q.(3) and Tenacity(4) partialed out.

Tenacity was first partialed out thus giving the following partial correlation coefficients:-

S.E.S. and Class-teaching scores: $r_{12.4} = .38$
S.E.S. and I.Q. : $r_{13.4} = -.05$
Class-teaching scores and I.Q. : $r_{23.4} = .27$

These three coefficients were then used to partial out I.Q. :-

$$r_{12.34} = \frac{r_{12.4} - r_{13.4}\ r_{23.4}}{(1 - r_{13.4}^2)^{\frac{1}{2}}(1 - r_{23.4}^2)^{\frac{1}{2}}}$$

$$= \frac{.38 + .05 \times .27}{(1 - .05^2)^{\frac{1}{2}}\ (1 - .27^2)^{\frac{1}{2}}}$$

$$= .4043$$

(b) S.E.S. and Individual Method Scores with I.Q. and Tenacity partialed out.

Calculated as above, $r_{12.34} = \underline{.2234}$.

MULTIPLE CORRELATIONS: On p. 98 it is shown how the coefficients of multiple correlation were calculated in the cases of

(a) Relative improvement with I.Q. + tenacity.
(b) Relative improvement with maths. scores + class scores.

By means of the formula used the correlation of relative improvement with the combined score of I.Q. and tenacity, for example, was calculated without actually combining the scores. This formula gives the maximum correlation with relative improvement that can be obtained by combining the I.Q. and tenacity scores.[1]

PARTIAL CORRELATIONS: Two variables may show a high correlation without there being any causal connection between them; a third variable may be operating to produce the same changes in both, hence before assuming that it is a real correlation, that is, that the one variable is the cause of the other, such other variables must be partialed out.

The formula for partialing out such variables is given in appendix D. Only one variable can be partialed out at a time. On page 99 it is shown how a second variable can be partialed out after the first one has been partialed out.

It was not considered necessary to partial out all the possible variables for all the correlation coefficients obtained in this part of the investigation, as it was felt that on the whole the raw correlations were more reliable: the reliability of the coefficient of partial correlation will suffer from the cumulative effect of the limitations of all the methods employed in the collecting of the different groups of data, while the reliability of the raw correlation is only affected

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1. For further explanation see, for example, Otis: "Statistical Method in Educational Measurement", chapter nineteen.

by the limitations of two methods. Hence it will be advisable to attach most significance to partial correlations where I.Q. has been partialed out, because the latter is the most objective portion of our data.

THE INCENTIVE DATA: TABLE 19: (p.96)

It was not possible to correlate the incentive data with the relative improvement, as the pupils were classified into three groups with definite, average and least incentive respectively instead of ranking them. Table 19 shows the relative improvement of these three groups in three different columns. In the A-group (definite incentive), there are six pupils with a total "relative improvement" of +96, hence the average "relative improvement" is +16.0. Similarly, the average improvements of the B and C groups are +2.6 and -13.6 respectively.

On page 96 also are shown the calculations for finding whether the differences between these average improvements are significant.

COEFFICIENTS OF VARIABILITY OF CLASS-TEACHING AND INDIVIDUAL METHOD SCORES: TABLE 20: (p.97)

From Table 20 (i) the average deviations (22.1 and 29.9 respectively), (ii) the standard deviations (28.6 ± 3.13 and 35.5 ± 3.89 respectively), (iii) the coefficients of variability (.32 and .39 respectively) of class-teaching and individual method scores are obtained.

The weighted individual method scores were used in order to make the two standard deviations comparable, which is exactly the same as calculating the coefficients of variability of the unweighted scores. The standard deviation of the unweighted individual method scores is 28.8 (calculation not shown); if this is divided by 73.5 (the average of the unweighted individual method scores - see column 6, table 1), a coefficient of variability of .39 is obtained which is the same as obtained by the method on page 97 .

SUMMARY OF RESULTS.

Before discussing their meanings separately we give a summary of the results below:-

A. RAW CORRELATIONS:- (rank difference method).

(see tables 12 to 18)

1.	Relative Improvement	with Average Maths. Scores	.41 ± .14
2.	" "	" " Class Position	.43 ± .13
3.	" "	" I.Q.	.15 ± .16
4.	" "	" Tenacity	.55 ± .11
5.	" "	" S.E.S.	.15 ± .16
6.	Tenacity	" Class-teaching Scores	.62 ± .10
7.	Tenacity	" Individual Method " (Calculations not shown)	.81 ± .06
8.	I.Q.	with Class-teach. Scores	.43 ± .13
9.	I.Q.	with Individual Method "	.41 ± .14
10.	S.E.S.	with Class-teach. Scores	.55 ± .11
11.	S.E.S.	with Individual Method "	.47 ± .13
12.	Relative Improvement	with Class-teach. Scores	.04 ± .16
13.	" "	" Individual Method "	.69 ± .08
14.	I.Q.	with Av. Class Position	.37 ± .14
15.	I.Q.	with Tenacity	.40 ± .14
16.	I.Q.	with S.E.S.	.14 ± .16
17.	S.E.S.	with Tenacity	.45 ± .13
18.	Average Class Position	with Av. Maths. Scores	.62 ± .10
19.	Class-teaching Scores	with Indiv. Method Scores	.70 ± .08
20.	Tenacity	with Average Class Position	.69 ± .08

B. MULTIPLE CORRELATIONS:-

(see page 98)

1.	Relative Improvement	with I.Q. + Tenacity	.56
2.	" "	" Av. Maths. + Av. Cl. Scs.	.47
3.	" "	" Tenacity + Av. Cl. Pos.	.55

C. PARTIAL CORRELATIONS:

1.	Tenacity with Class-Teach. Scs.	(I.Q. constant)	.54
2.	Tenacity with Indv. Method Scs.	(" ")	.77
3.	S.E.S. with Class-teach. Scs.	(" ")	.55
4.	S.E.S. with Indiv. Method Scs.	(" ")	.45
5.	Relative Imp. with Tenacity	(" ")	.54
6.	Relative Imp. with S.E.S.	(" ")	.13
7.	I.Q. with Class-Teach. Scs.	(Tenacity" ")	.25
8.	I.Q. with Indiv. Method Scs.	(" ")	.21
9.	S.E.S. with Class-Teach. Scs.	(" ")	.38
10.	S.E.S. with Individ. Meth. Scs.	(" ")	.21
11.	I.Q. with Class-Teach. Scores	(S.E.S. ")	.42
12.	I.Q. with Indiv. Method Scs.	(" ")	.39
13.	Relative Imp. with I.Q.	(" ")	.13
14.	Tenacity with Av. Class Pos. (see page 98)	(I.Q. ")	.64
15.	S.E.S. with Class-Teach. Scs.	(I.Q. and Tenacity constant)	.40
16.	S.E.S. with Indiv. Method Scs	(do)	.22

D. RELATIVE IMPROVEMENT AND INCENTIVE:

(see table 19)

Average Improvement of pupils with DEFINITE INCENTIVE +16.0

" " " " " AVERAGE " +2.6

" " " " " LEAST " -13.6

E. STANDARD DEVIATIONS AND COEFFICIENTS OF VARIABILITY OF CLASS-TEACHING AND INDIVIDUAL METHOD SCORES.

(see table 20)

$\sigma_c = 28.6 \pm 3.14$ (class-teaching scores)

$\sigma_i = 35.5 \pm 3.89$ (indiv. method scores)

P.E. of difference = 4.99

Critical ratio = 1.4

Probability of the difference between σ_c and σ_i being due to chance is 34.5%

Coefficient of variability of class-teach. scs. = .32 or 32%

" " " " indiv. meth. scs. = .39 or 39%

CRITICAL INTERPRETATION OF RESULTS.

In discussing these results we must bear in mind: 1) that they cannot lay claim to any great scientific value, for they have been obtained from too small a number of cases. Whatever conclusions are based on them must merely be taken as suggestive of further investigation in this field.

2) that a correlation coefficient which is smaller than three times its probable error must be looked upon as statistically insignificant. If it is three times its P.E. there is a probability of 95.7% of its being right. To be quite significant it should be four or probably five times its P.E.

3) that it is not known how to calculate the P.E. of a partial correlation coefficient, that is, the extent to which it may vary by chance sampling, hence it is impossible to discover whether the difference between two partial correlations is significant or not.

4) that the difference between two averages or between two raw correlations is significant only if it is more than three times its P.E.

The results are discussed in groups; the reader may refer to the summary given above for any of the results not included in these groups.

I (a) RELATIVE IMPROVEMENT and I.Q.: $\rho = .15 \pm .16$

(b) I.Q. and CLASSTEACHING SCORES: $\rho = .43 \pm .13$

(c) I.Q. and INDIVIDUAL METHOD SCORES: $\rho = .41 \pm .14$

See also partial correlations 7 and 8, 11 and 12, and 13 in summary.

Correlation (a) may be discarded as being of no significance, the P.E. being greater than the coefficient. This implies that intelligence as measured by modern testing technique does not discriminate betweem individual and class methods.

Correlations (b) and (c) point to the same conclusion, there being no significant difference between them. Neither do the partial correlations 7 and 9 (see summary), or 11 and 12 where tenacity and S.E.S. have been partialed out respectively show significant differences.

According to this investigation, therefore, pupils with high intelligence will not necessarily do relatively better on the individual method than pupils with low intelligence.

II (a) <u>RELATIVE IMPROVEMENT AND TENACITY</u>: $\rho = .55 \pm .11$

(b) <u>RELATIVE IMPROVEMENT AND TENACITY</u> (I.Q. constant) $\rho = .54$

(c) <u>TENACITY AND CLASS-TEACHING SCORES</u> (" ") $\rho = .54$

(d) <u>TENACITY AND INDIVIDUAL METH. SCS.</u> (" ") $\rho = .77$

(e) <u>TENACITY AND AVERAGE CLASS POS.</u> (" ") $\rho = .64$

See also raw correlations 6 and 7 in summary.

Correlation (a) is highly significant, being five times its P.E.; that is, the teachers' estimates of the pupils' readiness to work and their ability to apply themselves continuously to what they do correlate highly with the relative improvement. Heboe the factors perseverance and industry, for which we use the combined term tenacity, may be said to be more important with individual than with class-teaching method.

The partial correlation (b) does not differ appreciably from (a) and hence does not warrant any conclusion with regard to the effect of I.Q.

The partial correlations (c) and (d) show that tenacity is a more important factor in individual than in class-teaching method. This is the same conclusion as the above - only the data have been treated differently as was pointed out in the beginning of this chapter.

If the raw correlations 6 and 7 (see summary) are compared, they show a difference of .19 with a P.E. of .12, and hence a critical ration of $\frac{.19}{.12}$ or 1.6. The probability of the difference being due to chance is therefore .28[1]. Although this method of treating the data shows a less significant result than correlation (a) as regards the greater scope that the individual method allows for tenacity, the fact that it points in the same direction confirms the conclusion based on correlation (a).

Correlation (e) shows that either the teachers' estimates of tenacity have been ~~have been~~ strongly influenced by the pupils' achievement in school, or that the most persevering and diligent pupils usually do best at school. It will be difficult to discover which of these two is the true interpretation. If it is the former, then correlation (a) given above will mean the same as the correlation of .43 between relative improvement and average class position (see summary: raw correlation No. 2)

III (a) <u>RELATIVE IMPROVEMENT AND S.E.S.</u>: $\rho = .15 \pm .16$

(b) <u>RELATIVE IMPROVEMENT AND S.E.S. (I.Q. constant)</u>. $\rho = .13$

(c) <u>S.E.S. AND CLASS-TEACHING SCORES</u>(" ") $\rho = .55$

(d) <u>S.E.S. AND INDIV. METHOD SCORES</u> (" ") $\rho = .45$

See also raw correlations 10 and 11, and partial correlations 9 and 10, 14 and 15.

Correlation (a) is negligible, hence the socio-economic status of a pupil may be said to have no differentiating effect on his performances, in individual teaching and class-teaching methods; a pupil in favourable social and economic circumstances, for example, will not necessarily do better or worse on the individual

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1. Read off from probability tables.

method than a pupil in less favourable circumstances.

If I.Q. is partialed out (correlation b), the correlation is also insignificant, hence we conclude that there is no causal connection between S.E.S. and relative improvement.

Treating the data in a different manner by correlating S.E.S. with class-teaching and individual method scores respectively, raw correlations of .55 and .47 are obtained (see summary): the P.E. of their difference is .17 and the critical ratio .05, hence the difference may be due to chance and not be a real difference.

If I.Q. is partialed out, as shown in (c) and (d) above, partial coefficients of .55 and .45 are obtained. If tenacity is partialed out, partial coefficients of .38 and .21 are obtained (see summary: 9 and 10). If tenacity and I.Q. are both partialed out, partial coefficients of .40 and .22 are obtained (see summary: 14 and 15). The differences are larger in the last two cases, but the correlations themselves being too small to be significant, no significance can be attached to the differences between them.

The raw correlations .55 and .47 (summary: 10 and 11) themselves are however quite significant, hence while these correlations confirm the conclusions arrived at by other investigators that pupils coming out of good homes do well at school (because "intelligent parents" usually have "good homes"), they do not point to any connection between home conditions and the type of teaching-method employed at school. In other words, a pupil with a good home and intelligent parents will most probably do well at school irrespective of the method of teaching used.

IV (a) RELATIVE IMPROVEMENT AND AV. CLASS POS.: $\rho = .43 \pm .13$

(b) RELATIVE IMPROVEMENT AND AV. MATHS SCS.: $\rho = .41 \pm .14$

(c) RELATIVE IMPROVEMENT AND CLASS-TEACH. SCS.: $\rho = .04 \pm .16$

(d) RELATIVE IMPROVEMENT AND INDIV. METH. SCS.: $\rho = .69 \pm .08$

(e) RELATIVE IMPROVEMENT & AV.MATHS AV. CLASS SCS.: $\rho = .47$

(f) STANDARD DEVIATION OF CLASS-TEACH. SCORES: $\sigma_c = 28.6$

(g) STANDARD DEVIATION OF INDIV. METHOD SCORES: $\sigma_i = 35.5$

Correlation (a) is more than three times its P.E. and is therefore almost hundred percent significant, although of course data on larger groups are necessary to establish it with certainty. This result indicates that there is a tendency for pupils who ranked highest in class position (for all subjects of curriculum) throughout the year to make relatively the greatest improvement on the individual method. In other words, those pupils who are all-round good in their schoolwork will very probably benefit most by the individual method.

Correlation (b) shows the same result for the average mathematics scores of the whole year. It is however contradicted by correlations (c) and (d): the former shows that improvement does not correlate significantly with class-teaching scores, in other words, those pupils who do best in mathematics when the class-teaching method is used will not necessarily do best when the individual method is used. It must however be borne in mind that the whole year's average gives more reliable results than the average of six months.

Correlation (d) is high because in a way it represents the correlation of improvement with itself, but in conjunction with it should be taken the fact that the marks of the second six months of the year is a better indication of a pupil's "mathematics ability" than those of the first six months. Hence the above correlation coefficient of .04 is probably too low if it is to be taken as an indication of the connection between improvement and performance in mathematics. From this investigation it is however not possible to discover to what extent the shuffling of the marks in the second six months of the year was due to the change of method and to what extent it was due to more advanced work. As was pointed out before[1] this is one of the shortcomings of the investigation as a whole.

A comparison of the distributions of class-teaching and individual method scores shows a larger standard deviation for the latter. The individual method marks deviate more from the average mark than the class-teaching marks. The coefficients of variability are 32% and 39% respectively. As is pointed out in table 20, the probability of this difference being due to chance is 34.5%, hence the chances are fairly big that the difference is a real difference.

It may be concluded, therefore, that the individual method has a greater discriminating power between good and weak pupils. Its high marks tend to be higher while its low marks tend to be lower than those on the class-teaching method. This result seems to confirm the old complaint that in our present system teachers pay too much attention to the weaker pupils and leave the brighter pupils to their own devices. The better pupils do not work at their full capacity while the weaker are pushed to work above their capacity, hence the former get lower marks while the latter get higher marks than would have been the case if they had all worked according to their true capacities, and consequently the marks cluster round the average. In the individual method each pupil works at his own pace - the weaker pupils are not forced to work above their capacity while the clever

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1. Vide page 26-7

pupils need waste no time in listening to repeated explanations. Hence there is a bigger spread of marks.

V **RELATIVE IMPROVEMENT AND INCENTIVE:**

A. Average improvement of pupils with definite incentive + 16.0

B. Average improvement of pupils with ~~definite~~ average incentive + 2.6

C. Average improvement of pupils with ~~definite~~ least incentive −13.6

The difference between the average improvements of the A and B groups, as well as that of the B and C groups, is not quite significant (the probability of their being due to chance is .28 and .20 respectively) But the difference between the average improvements of the A and C groups is nearly hundred percent significant because it is nearly four times as large as its probable error (critical ratio = 3.9: see table 19).

This means that those pupils with most motivation show much greater relative improvement than those with least motivation. The individual method affords greater opportunity for the pupil with a powerful incentive[1] than the class-teaching method.

TENTATIVE HYPOTHESIS.

Summing up these results, it appears that tenacity and incentive are causally connected with relative improvement while I.Q. and S.E.S. are not.

In other words, there is a tendency for those pupils who have been judged to be most persevering and diligent, and most powerfully motivated to make relativel[y] more progress on the individual method than pupils who have been judged to be less persevering and diligent and to be only very slightly motivated.

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1. Vide Chapter III with regard to type of incentives judged.

With respect to intelligence as measured by modern technique, and socio-economic status, it appears that pupils favourably placed will most probably show a good performance irrespective of the method of teaching used.

Finally, there is also an indication that those pupils who show an all-round good performance in school, though not necessarily in mathematics itself, will find relatively more scope in the individual method than pupils with an all-round poor performance. The explanation which the investigator offers in connection with this result is that pupils who are all-round good in their schoolwork are on the whole better equipped with the instrumentalities of learning - reading, writing and number - and this will be a definite aid in individual work - more so than with ordinary teaching. Together with this appears the result that the individual method scores deviate further away from their average than the class-teaching scores. It would appear as if there is greater opportunity for the brilliant pupil to shine, and that the weak pupil is more likely to go under.

The investigator wishes to set up the above conclusions as a tentative hypothesis in this field. It is realised that statistically very little significance can be attached to it because of the small number of pupils used in the experiment. The strongest argument in favour of it is that it agrees with critical common-sense experience. It is hoped, however, that future investigators will find it suggestive as a starting-point for further investigation in this field.

In the second part of this investigation an attempt is made to verify a part of the above hypothesis.

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CHAPTER V.

VERIFICATION OF RESULTS IN TWO URBAN GIRLS' SCHOOLS.

The application of individual method to the teaching of mathematics in the mixed country school produced very interesting and suggestive results, especially in connection with tenacity and incentive. These results were, however, not very reliable because of the small number of pupils with whom the experiment was performed.[1] Hence it was desirable to repeat the experiment with a larger group.

In addition, there was the fact to be considered that these results were representative of a special set of circumstances and a special section of the South African population, namely, that of the country. The same experiment performed with town children might produce different results.

There was however no time or opportunity to repeat the experiment fully in a town school with a larger group. The investigator would have had to take complete control of a mathematics class for a year, or six months at least, doing three months ordinary class-teaching and three months "assignment" work. This was impossible in the circumstances, hence a different technique was employed.

The Std. VII mathematics sections of two different schools were selected for the purpose of the investigation: in the one section the discipline was fairly rigid and the ordinary class method of instruction was being employed by the mathematics teacher of the school; in the other section the discipline was much less rigid

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1. The P.E. of a correlation coefficient becomes very large if the number of cases is smalll. See formulae in appendix D.

and a fair amount of individual work was being done.

For each of these sections the following information was obtained:

1) Mathematics scores.
2) Intelligence quotients.
3) Tenacity coefficients.

Groups of the first type (rigid control, ordinary class-teaching) could be found in almost any of the urban schools, but the problem was to find a group of the second type (non-rigid control, individual method) The investigator inquired at several boys' schools but the unanimous reply of teachers was that they do not believe in such a method, with the result that it was necessary to resort to girls' schools.

The I-group:

Eventually the Std. VII mathematics section of one of the big urban girls' schools of the Cape Peninsula was selected: according to the principal and the mathematics mistress of the school, much individual work[1]. was being done by the class, the discipline was fairly loose, clever pupils were given extra work to do - in short, in spite of the fact that regular weekly assignments of work were not handed out, the method approximated inmany of its aspects to that employed by the investigator in the country school.

The Std. VII mathematics section alone was too small, hence the Std. VIII section, with which the same method was being employed, had to be included. This gave a group of 41 girls: 23 in Std. VII and 18 in Std. VIII. Results were however calculated for only 34 of them, as full data could not be obtained for the other seven: three were absent when the intelligence tests

1. "Individual work" is used to denote "work on the individual method" as the reader will have noticed. Strictly speaking, of course, all school work is individual, as was pointed out in Chapter I.

were given and four had not written the June test. Of the 34 that remained there were 21 in Std. VII and 13 in Std. VIII. For the sake of clearness we shall henceforward refer to them as the I-Group (individual method, non-rigid control).

The collection of the required data was a considerably easy matter:-

1) The mathematics scores of the June quarterly examination were obtained from the principal of the school. (A record book is kept in which all the quarterly test scores are written).

2) The intelligence quotients of the pupils were also obtained from the principal, as they had been tested on entrance into the school.

3) The teachers of the school very kindly consented to rate the pupils' perseverance and diligence. This was done in the same way as described in Chapter III, and tenacity coefficients were calculated as before. See tables 21, 22 and 23: the pupils are numbered from 1 to 34 according to their ranks in mathematics scores; pupil I obtained the highest mark and pupil 34 the lowest in the June test.

The R-Group:

As mentioned above, it was easy to find a rigidly controlled group. The Std. VII mathematics section of another big urban girls' school of the Cape Peninsula was selected: the principal expressed the belief that a firm control over girls is necessary to get the best out of them. Hence this class could be taken as representative of rigid control method.

There were 32 girls in the class; the Std. VIII methematics section was not included as in the case of the I-group. The final correlations were made for only 28 of these girls, as all the data could not be obtained for the other four: three were absent when the intelligence test was given and one did not write the June test. Henceforward we shall refer to them as the R-Group (rigid control, ordinary class-teaching).

The same data were collected for this group as for the I-Group:-

1) The mathematics scores of the June quarterly examination.

2) The intelligence quotients: the investigator had to give the intelligence tests himself, as the pupils had not been tested before. The class teacher was present to assist him, forms I and II of the South African Group Test of Intelligence were used for alternative rows and further all the necessary precautions were taken in giving the tests. The I.Q.'s are given in correlation table 31.

3) Tenacity coefficients: Obtained as before. See tables 24, 25 and 26. The separate estimates of perseverance and diligence correspond very closely. According to the teachers' judgments, therefore, these factors are usually present more or less to the same extent.

Retest of I.Q.'s of I-Group:

The correlation between mathematics scores and I.Q. for the I-Group was found to be exceptionally low (.25). Hence the investigator decided to retest this group, in order to check the I.Q.'s obtained from the school record book. This would at the same time bring greater uniformity in the method of investigation as the intelligence tests would then have been given by the same person to the I and R-Groups.

The retest was carried out with the usual precautions. The investigator tried to get hold of the answer booklets of the previous test in order to be able to give each pupil a different form for this test - if a pupil had form I for the previous test she would get form II for the retest and vice versa - but he was not successful as they had been destroyed.

The retest gave an average I.Q. of 123.4 which was 3.0 units higher than the first test. The correlation between the I.Q.'s of the first test and those of the retest is .87. For the investigation the I.Q.'s as obtained by the retest are used.

GENERAL SOCIO-ECONOMIC STATUS OF THE PUPILS OF THE TWO GROUPS:

The schools out of which the two groups were selected, are both large urban girls' schools in the Cape Peninsula, located a few miles from each other. The school of the rigid control group is fee-paying while the other is not, and on the whole the girls of this school come out of "better homes". One could expect that their health would be better: they get what they require, parents would not think twice before calling the doctor if it seemed necessary, they would get sufficient nutritious food and would not suffer from fatigue and other disturbances as the result of an unbalanced diet. As the principal of the school ran her finger down the list of names which constitute the R-Group, she pointed out that they all come out of homes where there was seldom a lack of anything.

On the other hand, these girls are subjected to many more social distractions than those of the non-feepaying school: parties, drives, theatres and so on, which leave less time for study. Hence, as regards schoolwork, good socio-economic circumstances may be a handicap as well as an advantage. The less favourable home conditions of girls of the I-Group may in some ways prove a handicap in their schoolwork, but it is an advantage in so far as it produces fewer social distractions and hence more time for study. These two factors will thus balance each other, so that the difference between the girls of the R and I-Groups as regards S.E.S. need not be considered a significant disturbing factor in the investigation.

The average I.Q. of the R-Group is 121.7, while that of the I-Group is 123.4 according to the retest (120.4 according to the first test). Hence in spite of evidence that well-to-do homes usually produce more intelligent children, the I.Q.'s of these two groups were more or less the same. As far as intelligence quotients are concerned, these two groups serve the purpose of the investigation very well.

WEIGHTING THE MATHEMATICS SCORES OF THE STD. VIII SECTION OF THE I-GROUP: TABLE 27:

As was mentioned, the I-Group consisted of 21 girls of Std. VII and 13 of Std. VIII. The average score of the former in the June mathematics test was 57.48% (column 2, table 27) while that of the latter was 45.9% (obtained by dividing 137.8 by 3 - column 4, table 27). The Std. VII papers were marked out of 100 while the Std. VIII papers were marked out of 300.

Hence the Std. VIII marks were weighted by multiplying each by $\frac{57.48}{137.8}$ or .4171. The weighted marks are shown in column 5, table 27. Thus the average score of the Std. VIII section was made equal to that of the Std. VII section in order to compensate for the fact that they did not write the same test papers.

This procedure would have been perfectly justifiable if we had been dealing with two large homogeneous groups - their average I.Q.'s as well as their average tenacity coefficients would have been the same - for then the difference between their average test scores could have been ascribed wholly to the fact that they had written different test papers. Examining these two sections constituting the I-Group, the following is noticed:-

Average I.Q. of Std. VII section:	122.2
" " " " VIII " :	125.5
Average tenacity coeff. of Std. VII section:	53.7
" " " " " VIII " :	60.8

(continued on page 131)

TABLE 21: Teachers' Estimates of Pupils' PERSEVERANCE (I-Group).

I-Group Pupils	Teachers' Estimates of Perseverance				Total	Average
	Biology	History	English	Maths.		
	1	2	3	4	5	6
1	4	4	5	5	18	4.5
2	3	4	5	5	18	4.25
3	2	3	4	4	13	3.25
4	2	4	4	4	14	3.5
5	3	3	3	3	12	3.0
6	2	3	3	5	13	3.25
7	2	4	3	4	13	3.25
8	3	3	4	4	14	3.5
9	3	4	5	4	14	4.0
10	1.5	4	4	3	12.5	3.13
11	3	3	4	5	15	3.75
12	2	2	2	3	9	2.25
13	3	3	3	4	13	3.25
14	2	3	3	3	11	2.75
15	2	4	2	3	11	2.75
16	2	2	3	3	10	2.5
17	2	2	2	3	9	2.25
18	3	4	4	4	15	3.75
19	2	2	2	4	10	2.5
20	2	4	3	3	12	3.0
21	2.5	2	2	3	9.5	2.38
22	2	3	3	4	12	3.0
23	2	2	2	1	7	1.75
24	2	2	1	2	7	1.75
25	1	2	1	1	5	1.25
26	2.5	4	2	2	10.5	2.63
27	1	1	1	1	4	1.0
28	2	4	2	5	13	3.25
29	1	2	3	1	7	1.75
30	2.5	3	2	3	10.5	2.63
31	2	-	2	2	6	2.0
32	2	2	2	2	8	2.0
33	2	3	2	2	9	2.25
34	1	1	2	1	5	1.25
Total	74	96	95	106	371	93.27
Average	2.2	2.9	2.8	3.1		2.7

TABLE 22: Teachers' Estimates of Pupils' DILIGENCE. (I-Group)

I-Group Pupils	Teachers' Estimates of Diligence				Total	Average
	Biology	History	English	Maths.		
	1	2	3	4	5	6
1	4	5	5	4	18	4.5
2	3.5	4	5	5	17.5	4.38
3	3	3	3	3	12	3.0
4	2	4	4	4	14	3.5
5	3	3	3	3	12	3.0
6	2	3	3	5	13	3.25
7	3	3	4	5	15	3.75
8	3	3	4	3	13	3.25
9	3.5	4	5	4	16.5	4.13
10	2.5	4	4	4	14.5	3.63
11	3.5	3	3	5	14.5	3.63
12	2	2	2	3	9	2.25
13	3	3	3	4	13	3.25
14	2	3	3	3	11	2.75
15	2	5	2	3	12	3.0
16	2	2	2	3	9	2.25
17	3	2	3	3	11	2.75
18	3	5	4	4	16	4.0
19	2	3	3	3	11	2.75
20	1.5	4	4	3	12.5	3.13
21	3	2	2	3	10	2.5
22	2	3	3	3	11	2.75
23	1	2	2	2	7	1.75
24	1.5	2	2	2	7.5	1.88
25	1.5	2	1	1	5.5	1.38
26	2	3	2	2	9	2.25
27	2	1	1	1	5	1.25
28	2	4	3	4	13	3.25
29	2	3	3	2	10	2.5
30	2.5	3	2	2	9.5	2.38
31	2.5	-	3	2	7.5	2.5
32	3	2	2	3	10	2.5
33	2	3	2	3	10	2.5
34	2	1	2	1	6	1.5
Total	82.5	99	99	105	385.5	97.04
Average	2.4	3.0	2.9	3.1		2.9

TABLE 23: TENACITY COEFFICIENTS (I-Group).

I-Group Pupils	Average Teachers'Estimates		Total Max. 10	Tenacity Coefficients
	Perseverance	Diligence		
	1	2	3	4
1	4.5	4.5	9.0	.90
2	4.25	4.38	8.63	.86
3	3.25	3.0	6.25	.63
4	3.5	3.5	7.0	.70
5	3.0	3.0	6.0	.60
6	3.25	3.25	6.5	.65
7	3.25	3.75	7.0	.70
8	3.5	3.25	6.75	.68
9	4.0	4.13	8.13	.81
10	3.13	3.63	6.76	.68
11	3.75	3.63	7.38	.74
12	2.25	2.25	4.5	.45
13	3.25	3.25	6.5	.65
14	2.75	2.75	5.5	.55
15	2.75	3.0	5.75	.58
16	2.5	2.25	4.75	.48
17	2.25	2.75	5.0	.50
18	3.75	4.0	7.75	.78
19	2.5	2.75	5.25	.53
20	3.0	3.13	6.13	.61
21	2.38	2.5	4.88	.49
22	3.0	2.75	5.75	.58
23	1.75	1.75	3.5	.35
24	1.75	1.88	3.63	.36
25	1.25	1.38	2.63	.26
26	2.63	2.25	4.88	.49
27	1.0	1.25	2.25	.23
28	3.25	3.25	6.5	.65
29	1.75	2.5	4.25	.43
30	2.63	2.38	5.01	.50
31	2.0	2.5	4.5	.45
32	2.0	2.5	4.5	.45
33	2.25	2.5	4.75	.48
34	1.25	1.5	2.75	.28
Total	93.27	97.04	190.31	19.08
Average	2.7	2.9	5.6	5.6

Note: In colum 4 the tenacity coefficients are given correct to two decimal places. This explains the total of 19.08 instead of 19.03

TABLE 24: Teachers' Estimates of Pupils' PERSEVERANCE. (R-Group)

R-Group Pupils	Teachers' Estimates of Perseverance					Total	Average
	History	Biology	Maths	Afr.	Eng. Lat.		
	1	2	3	4	5	6	7
1	4	4	4.5	5	5	22.5	4.5
2	3	3	4	4	4	18	3.6
3	5	2	4	4	5	20	4.0
4	5	3	4	4	5	21	4.2
5	4	4	4	4	4	20	4.0
6	3	3	4	3	3	16	3.2
7	3	3	4	2	4	16	3.2
8	2	2.5	3	2	2	11.5	2.3
9	4.5	4	4	4	4	20.5	4.1
10	3	2.5	2	4	3	14.5	2.9
11	4	3	3	3	4	17	3.4
12	3	3	4	4	3	17	3.4
13	2	2	2	-	2	8	2.0
14	3	3	4	3	3	16	3.2
15	5	4	3.5	3	4	19.5	3.9
16	5	3	4	3	4	19	3.8
17	2	2	3	4	2	13	2.6
18	2	1.5	2	2	3	10.5	2.1
19	3	4	3.5	2.5	2	15	3.0
20	3	3	3	3	3	15	3.0
21	3	2.5	1.5	3	3.5	13.5	2.7
22	2	3	2	3	2	13	2.4
23	2.5	2.5	3	4	3	15	3.0
24	2	3	2	2	1	10	2.0
25	3	3	2	2.5	3	13.5	2.7
26	2.5	2	1	3	3	11.5	2.3
27	1.5	2	2	2.5	1	9	1.8
28	3	2	1	2	3	11	2.2
Total	88	79.5	84	85.5	88.5	425.5	85.5
Average	3.1	2.8	3.0	3.2	3.2		3.1

TABLE 25: Teachers' Estimates of Pupils' DILIGENCE. (R-Group).

R-Group Pupils	Teachers' Estimates of Diligence					Total	Average
	History	Biology	Maths	Afr.	Eng. Lat.		
	1	2	3	4	5	6	7
1	5	4	4.5	5	5	23.5	4.7
2	3	2.5	4	4	3	16.5	3.3
3	3	3	4	4	5	19	3.8
4	4	3	4	4	5	20	4.0
5	4	4	4	4	4	20	4.0
6	3	3	4	3	3	16	3.2
7	3	3	4	2.5	4	16.5	3.3
8	2	2.5	3	2	2	11.5	2.3
9	4	4	4	4	4	20	4.0
10	3.5	3	2	4	3	15.5	3.1
11	4	3	3	3	5	18	3.6
12	3	3	4	4	3	17	3.4
13	2	2.5	2	-	2	8.5	2.1
14	3	3	4	3	3	16	3.2
15	4	3.5	3.5	3	4	18	3.6
16	4	3	4	3	4	18	3.6
17	3	3	3	4	3	16	3.2
18	2	1.5	2	2	2	9.5	1.9
19	3	4	3.5	2.5	2	15	3.0
20	2.5	2.5	3	3	3	14	2.8
21	3	2.5	1.5	2.5	3.5	13	2.6
22	2.5	2.5	2	3	2	12	2.4
23	2.5	2	3	4	3	14.5	2.9
24	2.5	3	2	2	1	10.5	2.1
25	3	3	2	2.5	3	13.5	2.7
26	2.5	2.5	1	2.5	3	11.5	2.3
27	2.5	3	2	2.5	1	11	2.2
28	2.5	2.5	1	2	2	10	2.0
Total	86	82	84	85	87.5	424.5	85.3
Average	3.1	2.9	3.0	3.1	3.1		3.0

TABLE: TENACITY COEFFICIENTS (R-Group).

R-Group Pupils	Average Teachers' Estimates			Tenacity Coefficients
	Perseverance	Diligence	Total Max. 10	
	1	2	3	4
1	4.5	4.7	9.2	.92
2	3.6	3.3	6.9	.69
3	4.0	3.8	7.8	.78
4	4.2	4.0	8.2	.82
5	4.0	4.0	8.0	.80
6	3.2	3.2	6.4	.64
7	3.2	3.3	6.5	.65
8	2.3	2.3	4.6	.46
9	4.1	4.0	8.1	.81
10	2.9	3.1	6.0	.60
11	3.4	3.6	7.0	.70
12	3.4	3.4	6.8	.68
13	2.0	2.1	4.1	.41
14	3.2	3.2	6.4	.64
15	3.9	3.6	7.5	.75
16	3.8	3.6	7.4	.74
17	2.6	3.2	5.8	.58
18	2.1	1.9	4.0	.40
19	3.0	3.0	6.0	.60
20	3.0	2.8	5.8	.58
21	2.7	2.6	5.3	.53
22	2.4	2.4	4.8	.48
23	3.0	2.9	5.9	.59
24	2.0	2.1	4.1	.41
25	2.7	2.7	5.4	.54
26	2.3	2.3	4.6	.46
27	1.8	2.2	4.0	.40
28	2.2	2.0	4.2	.42
Total	85.5	85.3	170.8	17.08
Average	3.1	3.0	6.1	.61

TABLE 27: MATHS. SCORES of the Std. VIII section of the I-Group WEIGHTED.

Std. VII		Std. VIII		
Pupils	Scores Max. 100	Pupils	Scores Max. 300	Scores Weighted Max. 100.
1	2	3	4	5
2	87.5	1	213	88.8
4	82	3	193	82.6
5	78			
6	76.5			
7	76.6	8	180	75.1
9	75			
10	71			
11	68.5			
12	68	13	162	67.6
14	67	15	159	66.3
16	66			
17	63.5	18	151	63.0
20	59	19	150	62.6
		21	140	58.4
23	48.5	22	132	55.1
24	45.5			
25	37.5	26	89	37.1
27	36	28	79	33.0
29	32.5	30	74	30.9
32	27	31	65	27.1
33	24.5			
34	17			
Total	1207		1792	747.6
Average	57.48		137.8	57.5

Note: 1) The last column was obtained by multiplying each of the marks in the second last column by $\frac{57.48}{137.8}$, that is, by .4171

2) The pupils have been numbered according to their percentage scores as shown in columns 1 and 3: pupil 1 obtained the highest score, pupil 2 the second highest and so on. The pupils in column 3 have been so spaced so that they fall alongside those of column 1 to whom they are nearest in rank

TABLE 28: CORRELATION between MATHEMATICS SCORES and I.Q. (I-Group).

I-Group Pupils	Maths Marks	x	x^2	I.Q. (Retest)	y	y^2	xy
1	88.8	31.3	979.7	140	17	289	532.1
2	87.5	30.0	900.0	135	12	144	360
3	82.6	25.1	630.0	115	- 8	64	- 200.8
4	82	24.5	600.3	126	3	9	73.5
5	78	20.5	420.3	117	- 6	36	- 123.0
6	76.5	19.0	361	115	- 8	64	- 152.0
7	76.5	19.0	361	125	2	4	38
8	75.1	17.6	309.8	129	6	36	105.6
9	75	17.5	306.3	128	5	25	87.5
10	71	13.5	182.3	109	- 14	196	- 189
11	68.5	11.0	121	135	12	144	132
12	68	10.5	110.3	131	8	64	84
13	67.6	10.1	102.0	128	5	25	50.5
14	67	9.5	90.3	129	6	36	57
15	66.3	8.8	77.4	121	- 2	4	- 17.6
16	66	8.5	72.3	115	- 8	64	- 68
17	63.5	6.0	36	144	21	441	126
18	63	5.5	30.2	134	11	121	60.5
19	62.6	5.1	26.0	127	4	16	20.4
20	59	1.5	2.2	113	- 10	100	- 15
21	58.4	0.9	.8	130	7	49	6.3
22	55.1	- 2.4	5.8	122	- 1	1	2.4
23	48.5	- 9.0	81.0	127	4	16	- 36
24	45.5	-12.0	144.0	130	7	49	- 84
25	37.5	-20.0	400.0	126	3	9	- 60
26	37.1	-20.4	416.2	121	- 2	4	40.8
27	36	-21.5	462.2	109	-14	196	301
28	33	-24.5	600.2	125	2	4	- 49
29	32.5	-25.0	625.0	97	-26	676	650
30	30.9	-26.6	707.6	119	- 4	16	106.4
31	27.1	-30.4	924.2	119	- 4	16	121.6
32	27	-30.5	930.2	127	4	16	-122
33	24.5	-33.0	1089.0	114	- 9	81	297
34	17	-40.5	1640.0	114	- 9	81	364.5
Total	1954.6		13744.6	4196		3096	3617.1
Average	57.5			123.4			-1116.4
				$e_2 = -0.4$			2500.7

$$r = \frac{\Sigma(xy) - n\, e_1 e_2}{\{(\Sigma x^2 - n e_1^2)(\Sigma y^2 - n e_2^2)\}^{\frac{1}{2}}}$$

$$= \frac{2500.7 - 34 \times 0 \times (-0.4)}{\{13744.6(3096 - 34 \times (-0.4)^2)\}^{\frac{1}{2}}}$$

$$= .3837$$

$$\text{P.E.} = .6745 \times \frac{1 - .3837^2}{\sqrt{34}}$$

$$= .0997$$

$$\therefore r_{12} = .38 \pm .10$$

TABLE 29: CORRELATION between MATHEMATICS SCORES and TENACITY. (I-Group).
Note: The decimal point is omitted for the Tenacity Coefficients.

I-Group Pupils	Maths Marks	x	x^2	Tenacity Coeffics	z	z^2	xz
1	88.8	31.3	979.7	90	34	1156	1064.2
2	87.5	30.0	900.0	86	30	900	900
3	82.6	25.1	630.0	63	7	49	175.7
4	82	24.5	600.3	70	14	196	343
5	78	20.5	420.3	60	4	16	82
6	76.5	19.0	361	65	9	81	171
7	76.5	19.0	361	70	14	196	266
8	75.1	17.6	309.8	68	12	144	211.2
9	75	17.5	306.3	81	25	625	437.5
10	71	13.5	182.3	68	12	144	162
11	68.5	11.0	121	74	18	324	198
12	68	10.5	110.3	45	- 11	121	- 115.5
13	67.6	10.1	102.0	65	9	81	90.9
14	67	9.5	90.3	55	- 1	1	- 9.5
15	66.3	8.8	77.4	58	2	4	17.6
16	66	8.5	72.3	48	- 8	64	- 68
17	63.5	6.0	36	50	- 6	36	- 36
18	63	5.5	30.2	78	22	484	121
19	62.6	5.1	26.0	53	- 3	9	- 15.3
20	59	1.5	2.2	61	5	25	7.5
21	58.4	0.9	.8	49	- 7	49	- 6.3
22	55.1	- 2.4	5.8	58	2	4	- 4.8
23	48.5	- 9.0	81.0	35	- 21	441	189
24	45.5	-12.0	144.0	36	- 20	400	240
25	37.5	-20.0	400.0	26	- 30	900	600
26	37.1	-20.4	416.2	49	- 7	49	142.8
27	36	-21.5	462.2	23	-33	1089	709.5
28	33	-24.5	600.2	65	9	81	- 220.5
29	32.5	-25.0	625.0	43	- 13	169	325
30	30.9	-26.6	707.6	50	- 6	36	159.6
31	27.1	-30.4	924.2	45	- 11	121	334.4
32	27	-30.5	930.2	45	- 11	121	335.5
33	24.5	-33.0	1089.0	48	- 8	64	264
34	17	-40.5	1640.0	28	- 28	784	1134
Total	1954.6		13744.6	1908		8964	8681.4
Average	57.5			56.1			-475.9
				$e_z = -0.1$			8205.5

$$r = \frac{8205.5}{\{13744.6\,[8964 - 34 \times (-0.1)^2]\}^{1/2}}$$

$$= .7392$$

$$P.E. = .6745 \times \frac{1 - .7392^2}{\sqrt{34}}$$

$$= .0523$$

$$\therefore r_3 = .74 \pm .05$$

TABLE 30: CORRELATION between I.Q. and TENACITY (I-Group).

I-Group Pupils	I.Q.	y	y^2	Tenacity	z	z^2	yz
1	140	17	289	90	34	1156	578
2	135	12	144	86	30	900	360
3	115	- 8	64	63	7	49	- 56
4	126	3	9	70	14	196	42
5	117	- 6	36	60	4	16	- 24
6	115	- 8	64	65	9	81	- 72
7	125	2	4	70	14	196	28
8	129	6	36	68	12	144	72
9	128	5	25	81	25	625	125
10	109	-14	196	68	12	144	- 168
11	135	12	144	74	18	324	216
12	131	8	64	45	- 11	121	- 88
13	128	5	25	65	9	81	45
14	129	6	36	55	- 1	1	- 6
15	121	- 2	4	58	2	4	- 4
16	115	- 8	64	48	- 8	64	64
17	144	21	441	50	- 6	36	- 126
18	134	11	121	78	22	484	242
19	127	4	16	53	- 3	9	- 12
20	113	-10	100	61	5	25	- 50
21	130	7	49	49	- 7	49	- 49
22	122	- 1	1	58	2	4	- 2
23	127	4	16	35	- 21	441	- 84
24	130	7	49	36	- 20	400	- 140
25	126	3	9	26	- 30	900	- 90
26	121	- 2	4	49	- 7	49	14
27	109	- 14	196	23	- 33	1089	462
28	125	2	4	65	9	81	18
29	97	- 26	676	43	- 13	169	338
30	119	- 4	16	50	- 6	36	24
31	119	- 4	16	45	- 11	121	44
32	127	4	16	45	- 11	121	- 44
33	114	- 9	81	48	- 8	64	72
34	114	- 9	81	28	- 28	784	252
Total	4196		3096	1908		8964	2996
Average	123.4			56.1			-1015
	$e_2 = -0.4$			$e_3 = -0.1$			1981

$$r = \frac{1981 - 34 \times (-.4) \times (-.1)}{[3096 - 34(-.4)^2]^{\frac{1}{2}}[8964 - 34(-.1)^2]^{\frac{1}{2}}}$$

$$= .3761$$

$$P.E. = .6745 \times \frac{1 - .3761^2}{\sqrt{34}}$$

$$= .0993$$

$$r_{23} = .38 \pm .10$$

TABLE 31: CORRELATION between MATHEMATICS SCORES and I.Q. (R-Group).

R-Group Pupils	Maths Marks	x	x^2	I.Q.	y	y^2	xy
1	87	26	676	129	7	49	182
2	87	26	676	121	- 1	1	- 26
3	87	26	676	138	16	256	416
4	86	25	625	135	13	169	325
5	86	25	625	123	1	1	25
6	85	24	576	144	22	484	528
7	76	15	225	120	- 2	4	- 30
8	68	7	49	120	- 2	4	- 14
9	68	7	49	124	2	4	14
10	67	6	36	114	- 8	64	- 48
11	65	4	16	128	6	36	24
12	63	2	4	131	9	81	18
13	63	2	4	114	- 8	64	- 16
14	62	1	1	128	6	36	6
15	61	0	0	118	- 4	16	0
16	61	0	0	129	7	49	0
17	58	- 3	9	125	3	9	- 9
18	58	- 3	9	115	- 7	49	21
19	57	- 4	16	114	- 8	64	32
20	54	- 7	49	115	- 7	49	49
21	52	- 9	81	123	1	1	- 9
22	49	- 12	144	112	- 10	100	120
23	40	- 21	441	115	- 7	49	147
24	40	- 21	441	116	- 6	36	126
25	39	- 22	484	116	- 6	36	132
26	37	- 24	576	118	- 4	16	96
27	31	- 30	900	111	- 11	121	330
28	27	- 34	1156	111	- 11	121	374
Total	1714		8544	3407		1969	2965
Average	61.2			121.7			- 152
	$e_1 = -0.2$			$e_2 = +0.3$			2813

$$r = \frac{2813 - 28 \times (-0.2) \times (0.3)}{[8544 - 28(-0.2)^2]^{\frac{1}{2}}\ [1969 - 28(0.3)^2]^{\frac{1}{2}}}$$

$$= .6869$$

$$P.E. = .6745 \times \frac{1 - .6869^2}{\sqrt{28}}$$

$$= .0673$$

$$\therefore r_{12} = .69 \pm .07$$

TABLE 32: CORRELATION between MATHEMATICS SCORES and TENACITY (R-Group).

R-Group Pupils	Maths Marks	x	x^2	Tenacity Coeffics.	z	z^2	xz
1	87	26	676	92	31	961	806
2	87	26	676	69	8	64	208
3	87	26	676	78	17	289	442
4	86	25	625	82	21	441	525
5	86	25	625	80	19	361	475
6	85	24	576	64	3	9	72
7	76	15	225	65	4	16	60
8	68	7	49	46	-15	225	-105
9	68	7	49	81	20	400	140
10	67	6	36	60	- 1	1	- 6
11	65	4	16	70	9	81	36
12	63	2	4	68	7	49	14
13	63	2	4	41	- 20	400	- 40
14	62	1	1	64	3	9	3
15	61	0	0	75	14	196	0
16	61	0	0	74	13	169	0
17	58	- 3	9	58	- 3	9	9
18	58	- 3	9	40	- 21	441	63
19	57	- 4	16	60	- 1	1	4
20	54	- 7	49	58	- 3	9	21
21	52	- 9	81	53	- 8	64	72
22	49	-12	144	48	- 13	169	156
23	40	-21	441	59	- 2	4	42
24	40	-21	441	41	- 20	400	420
25	39	-22	484	54	- 7	49	154
26	37	-24	576	46	- 15	225	360
27	31	-30	900	40	- 21	441	630
28	27	-34	1156	42	- 19	361	646
Total	1714		8544	1708		5844	5358
Average	61.2			~~6.10~~ 61.0			-151
	$e_1 = -0.2$			$e_3 = 0$			5207

$$r = \frac{5207 - 28(-0.2) \times 0}{[8544 - 28 \times (-0.2)^2]^{\frac{1}{2}} \, [5844]^{\frac{1}{2}}}$$

$$= .7369$$

$$P.E. = .6745 \times \frac{1 - .7369^2}{\sqrt{28}}$$

$$= .0583$$

$$\therefore r_{13} = .74 \pm .06$$

TABLE 33: CORRELATION between I.Q. and TENACITY (R-Group)

R-Group Pupils	I.Q.	y	y^2	Tenacity Coeffics.	z	z^2	yz
1	129	7	49	92	31	961	217
2	121	- 1	1	69	8	64	- 8
3	138	16	256	78	17	289	272
4	135	13	169	82	21	441	274
5	123	1	1	80	19	361	19
6	144	22	484	64	3	9	66
7	120	- 2	4	65	4	16	- 8
8	120	- 2	4	46	- 15	225	30
9	124	2	4	81	20	400	40
10	114	- 8	64	60	- 1	1	8
11	128	6	36	70	9	81	54
12	131	9	81	68	7	49	63
13	114	- 8	64	41	- 20	400	160
14	128	6	36	64	3	9	18
15	118	- 4	16	75	14	196	- 56
16	129	7	49	74	13	169	91
17	125	3	9	58	- 3	9	- 9
18	115	- 7	49	40	- 21	441	147
19	114	- 8	64	60	- 1	1	8
20	115	- 7	49	58	- 3	9	21
21	123	1	1	53	- 8	64	- 8
22	112	- 10	100	48	- 13	169	130
23	115	- 7	49	59	- 2	4	14
24	116	- 6	36	41	- 20	400	120
25	116	- 6	36	54	- 7	49	42
26	118	- 4	16	46	- 15	225	60
27	111	- 11	121	40	- 21	441	231
28	111	- 11	121	42	- 19	361	209
Total	3407		1969	1708		5844	2294
Average	121.7			61.0			- 89
	$e_2 = 0.3$			$e_3 = 0$			2205

$$r = \frac{2205}{[5844]^{\frac{1}{2}}[1969 - 28(0.3)^2]^{\frac{1}{2}}}$$

$$= .6504$$

$$P.E. = .6745 \times \frac{1 - .6504^2}{\sqrt{28}}$$

$$= .0736$$

$r_{23} = .65 \pm .07$

PARTIAL CORRELATIONS.

I-Group: 1 - Maths. $r_{12} = .38 \pm .10$ (Table 28).

2 - I.Q. $r_{13} = .74 \pm .05$ (Table 29).

3 - Tenacity $r_{23} = .38 \pm .10$ (Table 30).

$$\therefore\ r_{12.3} = \frac{r_{12} - r_{13}\ r_{23}}{(1 - r_{13}^2)^{\frac{1}{2}}\ (1 - r_{23}^2)^{\frac{1}{2}}}$$

$$= \frac{.38 - .74 \times .38}{(1 - .74^2)^{\frac{1}{2}}\ (1 - .38^2)^{\frac{1}{2}}}$$

$$= .1375$$

Similarly $r_{13.2} = .7013$

$r_{23.1} = .1375$

R-Group: 1 - Maths. $r_{12} = .69 \pm .07$ (Table 31).

2 - I.Q. $r_{13} = .74 \pm .06$ (Table 32).

3 - Tenacity $r_{23} = .65 \pm .07$ (Table 33).

$r_{12.3} = .4108$ (calculated above).

$r_{13.2} = .5272$

$r_{23.1} = .2876$

(Note: As shown in column 3, table 27, pupils 1, 3, 8, 13, 15, 18, 19, 21, 22, 26, 28, 30 and 31 were in Std. VIII

These averages show a higher average I.Q. as well as a higher average tenacity coefficient for Std. VIII although the differences are not significant. The two sections could thus not be considered completely homogeneous as regards I.Q. and tenacity, although the difference is not appreciable enough to affect the results. While the process of weighting necessarily decreased the reliability of the results it was unavoidable.

CORRELATIONS:

For each of the I- and R-Groups the mathematics scores, intelligence quotients and tenacity coefficients were intercorrelated. Spearman's product-moment method was used.

Tables 28, 29 and 30 show the correlations of the I-Group.

Tables 31, 32 and 33 show the correlations of the R-Group.

These tables are self-explanatory. The x, y and z columns show the deviations from their averages of mathematics scores, intelligence quotients and tenacity coefficients respectively. To avoid undue arithmetical labour the deviations were calculated from the whole number nearest to the average: for example, in table 28 the deviations of intelligence quotients were calculated from 123 instead from 123.4, and allowance was made in the calculation of the correlation coefficient for the error, $e_2 = -0.4$, thus incurred.[1]

Mathematics scores, I.Q.'s and tenacity coefficients are designated by the numbers 1, 2 and 3 respectively; r_{12} therefore means the correlation between

1. By using formula (5), appendix D.

mathematics scores and I.Q.

On page 130 the partial correlations are calculated. For this part of the investigation the partial correlations are very important. We are here dealing with two different groups with different mathematics scores, I.Q.'s and tenacity coefficients. Hence to discover the relation between any two of these variables, the third variable has to be partialed out, because it is different for the two groups. In order to compare, for example, the correlation between mathematics and tenacity for the I-Group with the correlation between mathematics and tenacity for the R-Group, it is necessary to partial out I.Q. because the I.Q.'s of the two groups are not the same.

In the following table the partial correlation coefficients are arranged so as to make a comparison of them easy:

TABLE 34: PARTIAL CORRELATION COEFFICIENTS.

PARTIAL CORRELATIONS between	I-Group	R-Group
Maths. scores ($r_{12.3}$) and I.Q. (tenacity constant)	.14	.41
Maths. scores ($r_{13.2}$) and tenacity (I.Q. constant)	.70	.53

CRITICAL EXAMINATION OF RESULTS:

The above show that the correlation between mathematics scores and I.Q. is much lower for the I-Group than for the R-Group. On the other hand the correlation between mathematics scores and tenacity is much higher for the I-Group than for the R-Group. Hence, we deduce that intelligence is of relatively smaller importance while tenacity is of relatively greater importance in a group where there is non-rigid control and some individual method work.

This agrees with the result obtained in the first part of the investigation, that there is a significant correlation (.54) between relative improvement and tenacity, or, taking it the other way, that the correlations between tenacity and class-teaching scores is .54 while that between tenacity and individual method scores is .77.

As regards I.Q., however, this part of the investigation produced a much bigger differance between the correlation of I.Q. with I-Group and R-Group mathematics scores respectively (that is, .14 and .41) than the first part of the investigation (.21 and .25). Indeed, as regards the latter the conclusion was drawn that I.Q. has no differentiating effect on the type of teaching method used, because the difference is clearly negligible. To make a comparison easy, the results are represented in tabular form, before stating the final conclusion:-

TABLE 35: COUNTRY SCHOOL RESULTS compared with TOWN SCHOOL RESULTS.

PARTIAL CORRELATIONS between	Country School		Town Schools	
	Indiv. Method	Class-Teach. Method	I-Group	R-Group
Maths. scores and I.Q. (tenacity constant)	.21	.25	.14	.41
Maths. scores and Tenacity (I.Q. constant)	.77	.54	.70	.53

From the table it appears that the results as regards tenacity agree very closely in the two parts of the investigation, but as regards I.Q. the negligible difference (.04) shown by the first part of the investigation has become fairly appreciable (.27) in the second part, although it is in the same direction in each case. From a common-sense point of view, however, it is difficult to see why I.Q. should be of less importance in individual method work than in class-teaching work, as the second part of the investigation indicates; and, as it does not agree with the first part of the investigation, the tentative hypothesis as set up by the latter may be taken to stand, namely, that I.Q. has no differentiating effect as regards the type of teaching method used. But we may add, that if it has any effect, it seems likely that the class-teaching method is more suitable for pupils with a high I.Q.

As far as tenacity is concerned, however, the hypothesis suggested by the first experiment may be said to have been verified under an entirely new set of circumstances by means of a different technique. Unfortunately the opportunity was not available to test out the hypotheses in connection with incentive and socio-economic status in the same way.

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CHAPTER VI.

CONCLUSIONS AND THEIR IMPLICATIONS FOR EDUCATION.

From the results which were obtained in the country mixed school by applying the individual method to the teaching of mathematics, and which were partly verified in two urban girls' schools by means of a different technique, the following conclusions are drawn:-

A. The individual method will suit those pupils best

1) who are persevering and hardworking
2) who have powerful incentives
3) who do well at school in general, ~~that is, who have a high fund of "g", in Spearman's terminology.~~

B. The individual method will not necessarily prove more beneficial to those pupils

1) whose socio-economic circumstances are favourable
2) who have high intelligence quotients - indeed, there is an indication that I.Q. is of relatively less importance under the individual method.

These conclusions are based on results which have been obtained by methods not entirely in accord with objective scientific requirements. From the strictest statistical point of view all the results are invalid because they have been calculated from too small a number of cases - except those concerning tenacity and I.Q. which have been verified under different conditions with a larger number of cases. The latter should thus receive serious consideration while the rest should merely be looked upon as tentative hypotheses for further advance in this field

DISCUSSION:

It would appear that a pupil with a high I.Q. and a high S.E.S. will most probably do work of a high order on the individual method but not necessarily of a higher order than his work on the class method. A pupil with a low I.Q. and poor S.E.S. will most probably do work or a low order on both methods. In short, I.Q. and S.E.S. have no discriminating effect as regards the type of method used - if there is any effect in the case of I.Q., the indication is that the pupil with a high I.Q. will do relatively better on the class-teaching method.

The investigation shows however that some pupils do relatively better than others on the individual method, and points strongly to factors such as "tenacity" and "incentive" as being instrumental in causing these differences in performance.

The social implications of these conclusions are very significant. Which people are generally speaking most successful in life? Intelligence tests are of too recent origin to have shown definite results, but it seems as if the most intelligent people are not always the most successful. Many people of relatively poor intelligence get on well in life. What factors then besides intelligence will make for success? Common-sense experience tells us that tenacity and incentive are not among the least important of them. And so the next step in the argument becomes clear: performance on the individual method agrees more closely with tenacity and incentive evaluations than performance on class method. Hence the individual method may be said to give a truer indication as to which pupils are most likely to make a success of their future careers.

In other words, an employer who employs the top boy or girl of the individual method class is more likely to get a good employee than the employer who employs the top boy or girl of the ordinary class.

These results also contain important conclusions with regard to intelligence tests. The investigator has been forced to the conclusion that the dominant factor in these tests is the stimulus - response situation, and that, together with Behaviorism in general, they commit the error of taking this to be the main fact in our existence. This is indeed no argument against the application of intelligence tests; according to behaviourists this will be an argument in favour of their objectivity. There is little doubt that they measure what they set out to measure, but the argument is that what they measure should not be taken as the only important criterion of the mental make-up of the human being. And let it be said here that the exponents of the new type of examination comprising numerous short questions of the Ballard type, lay themselves open to the same criticism. Experimental educationists to-day in their desire to eliminate unscientific opinion tend to emphasise the measurement of such factors as are susceptible of objective treatment, and to the writer it would appear that this tendency has as its complement a further tendency to disregard other factors which are not so susceptible to quantitative treatment. Such factors, for example, as tenacity and incentive are admittedly elusive; the investigator's attempts to assign values to them have been fraught with difficulty, his methods and results can be subjected to criticism; yet it is felt that these are the very factors which determine the relative success not only with the methods with which this thesis deals, but with others throughout the field of education. This has forced the investigator to the conclusion that difficult to evaluate as these factors are, they cannot be disregarded. It would appear unsound to utilise only the results of standardised intelligence tests.

On the other hand, an unscientific terminology is to a large extent meaningless. In educational literature such phrases as "inner drives", "instinctive motivating power", "mainspring of existence" and "unconscious propelling force" are too often employed without any scientific basis, although most probably there is more in them than the mere string of reactions which so many of our modern scientists try to measure in their quest for objective data.

Which modern tests that can lay claim to scientific objectivity set out to evaluate that ability to organise and arrange one's observations and thoughts systematically, to pick out the essentials from a mass of detail, to think clearly when faced with complex situations instead of relapsing into a state of mental asphyxiation, and to persevere until success is attained? Advocates of the short type of question argue that this is what they wish to test - only, it is on a much "smaller scale". And here lies the danger!

In his comparison of the short type and essay type of questions, Ballard states that the former "deals with a smaller unit but not with a unit of different quality". These tests "bring into play the same complex trains of thought as are evoked by the broader questions of the old examination".[1] This may be true, but experience tells us that it is a completely different matter to think

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1. P.B. Ballard: "The New Examiner" (p. 123).

clearly when faced with a short direct question than to think clearly when faced with a complex situation involving a large number of details. The mere fact that most children enjoy doing intelligence and other standardised tests is significant in that it show that little effort is involved, not in the sense that enjoyment and effort never accompany each other, but that the effort is of a very short duration in each case. It involves no long and continued application in order to achieve success, for each question can be answered in a very short time. As regards this it might be mentioned here that the answering of some of the letter and figure tests of the South African Group Test of Intelligence requires more sustained effort than is usually the case with the short type of question.

It would appear that "perseverance" and "industry" count a great deal towards success in life. Some people have certain qualities in their make-up - physical or psychic - which enable them to apply themselve continuously to whatever they attempt until succes is attained. A person may not have a very high I.Q. but his persevering industrious disposition will bring him success. On the other hand the career of a person with a fairly high I.Q. may be very disappointing; as long as he is stimulated by questions and problems such as intelligence tests provide, he reacts very successfuly, but he may be found lacking in some of the important qualities which ensure successful adjustment in later life This reminds the investigator of the instance of a debater who desired to be heckled as much as possible otherwise he would have nothing to say for himself.

Once more should it be emphasised that this is no argument against the application of intelligence tests. These tests have proved so successful that the modern educationist could hardly do without them. Neither should it be inferred that this is a criticism of intelligence tests for claiming to measure that which they do not really measure. The aim of this discussion is merely to draw attention to the tendency among certain groups of educationists who, in their attempt to employ a scientific technique, attach undue importance to some factors while they disregard others. They rarely fathom the stimulating power of an incentive, or the effect which the presence or absence of a factor such as tenacity may have as regards success or failure.

This investigation shows that tenacity and incentive are very important in individual method work; and this seems to be in accordance with critical common-sense experience.

CONCLUSIONS BASED ON OBSERVATION.

In addition to the above conclusions based on statistical evidence the following conclusions based on observation may be mentioned.

The individual method has the practical advantage that pupils who were absent from school can make up for the lost time without involving extra coaching. They need have no gaps in their knowledge for they continue with the assignments from the point where they last stopped. Two pupils (F, G) who were absent, actually showed improvement in comparison with the general performance of the class on the individual method, in spite of the shorter time they had in which to complete their assignments. It is true, the third pupil (Q), who had been absent for a considerable time, completely lost courage and crumpled up, but then it was not only in mathematics. His work in all the other subjects deteriorated, hence it cannot be ascribed only to a weakness of the assignment method.

As far as amount of work and discipline are concerned, the investigation has shown the individual method to be very successful. It is not to be inferred that it will be a success under all conditions and with all classes of pupils. This investigation has vindicated the claim of individual method enthusiasts that the problem of discipline becomes relatively insignificant. Disciplinary troubles so often result from want of occupation; if everyone in the class is busy there can be no such troubles. For this statement the investigator offers no statistical confirmation - it is the conclusion to which he has come after working with the class and watching them work on the individual method for six months.

In spite of the fact however that the six months of individual method produced a better discipline than the six months of class-teaching, the fact that two pupils had to be disciplined because of neglect of work suggests that enthusiasts tend to over-emphasise the virtues of individual method with regard to freedom, self-discipline and self-expression. The individual method provides opportunity for a much freer atmosphere and a greater degree of self-discipline, but the element of restraint cannot be completely absent. Civilisation involves a number of inhibitions; a child cannot be allowed to express himself just as his caprices and feelings dictate. In every class there are usually a few pupils who seem incapable of controlling themselves, even under favourable conditions, and who need direct external restraint; on the whole children show less all-round sensitivity than adults to the approval or disapproval of their immediate circle of contacts - a sensitivity which will lead to self-discipline. The behaviour of such pupils might prove infectious, hence disciplinary measures are necessary in the individual as well as in the class method. Needless to say, the teacher's personality here once more enters the field, and as with the class method much will depend on his tact and understanding.

Much more work was done during the second six months of the year than during the first six months, and, according to the investigator's judgment, much more than would have been done if the ordinary teaching method had been retained throughout the year. Especially is this true for the better pupils in the class. The time that is usually wasted in listening to unnecessary explanations could be fruitfully utilised in working a larger variety of examples.

In the beginning it appeared that pupils read their assignments very carelessly. It is the common experience of many people that it is a completely different thing to follow a written explanation than to listen to a verbal exposition. The teacher may use the same words as the text-book and yet many pupils will assimilate the teacher's explanation much better. As was mentioned before, the fact that the voice can be inflected, that it is alive while the written word is dead, has a great deal to do with it. But apart from this, some people react much better to an auditory than to a visual stimulus. Perhaps it is largely a matter of training - such is the investigator's personal experience. There is of course no general faculty of concentration. The pupil who has been used to have everything explained to him by the teacher will find it very difficult in later life to acquire the habit of easily assimilating written explanations. Especially are pupils handicapped on entering the university where lectures become of less importance and study out of books take in first place. The Assignment method will teach pupils in school already to follow written instructions and explanations, and even how to use books. Pupil S may be mentioned here as a case in point: he seemed to have developed the art of listening to a nicety; wh alone with his assignments he was completely lost.

SOME CONSTRUCTIVE SUGGESTIONS.

With the general introduction of the individual method in the school attention will have to be paid to the careful reading of assignments. This seems to be essential for the efficient functioning of the method. Especially will it be necessary with the all-round weaker pupils. A few periods spent in the beginning by carefully reading one or two assignments with them will save much unnecessary work later on.

Schools should consider it their duty to give pupils some practice in assignment work. The investigato strongly believes that such pupils will be better equipped to continue self-education. As regards the university it speaks for itself that such pupils will be more capable of adaptation to the type of work they have to do there.

In big schools one or two of the sections of each standard could be put on the individual method. If possible, it should be arranged that those pupils whose class percentages are the highest will get into these sections. At a staff-meeting where the division of pupil into sections is discussed "tenacity" and "incentive" should act as guiding factors. The writer realises of course that practical difficulties in connection with school organisation may make such an arrangement impossibl

But wherever the assignment is introduced there should still be some class-teaching. The assignment could provide the foundation of school method, but provision should be made for frequent teaching. Class-teaching should thus supplement assignment work. The investigator now realises that if he had arranged for two or three teaching lessons in every month of individual work, some pupils would have done much better.

To those who are very sceptical as regards the merits of the assignment the following suggestion might prove useful: Decide which part of the syllabus will be covered each month, how many lessons it will be necessary to teach and which examples the class will be required to do; then draw up an assignment consisting of groups of exercises which will follow on each lesson. After having taught the first lesson the teacher need only instruct the class to continue with the exercises in Group I, also informing them when these have to be completed. There may be extra exercises for fast pupils. Similarly, after the second lesson has been taught, the pupils will be referred to Group II. This will be of great help to teacher and pupils, especially in mathematics. This is thus the reverse of the above suggestion: here the assignment supplements the ordinary class-teaching.

If the whole school is to be organised on individual method lines, it is essential that teachers should come to an agreement among themselves as to some general principles to be kept in view in the drawing up of assignments. There should for example be some uniformity in the assignments for different subjects as regards the arrangement of contents and general form. It will not be advisable to introduce the individual method simultaneously for all the subjects. The first year or two will be an experiment for teachers and pupils alike and the newness may cause too great a loss of efficiency for the time being. It will be better if only half of the teachers employ it for the first year, or its introduction may even be spread over a period of two or three years. There will of course not be the overwhelming enthusiasm (which may die down) which an immediate change of method seems to involve, but principal and teachers will gradually feel their way, allothe test of time to decide in some cases, and eventually that modified form of the method will have been brought into practice which best suits the local conditions. Needless to say, during this introductory period the school will still have its ordinary time-table, indicating the daily periods for the different subjects.

After the full introduction there should still be times set aside for teaching, in some subjects more than in others. From his very short experience with the standard IX class,[1] the investigator would judge the individual method to be more suitable for older children. Hence, there could be less teaching in the advanced classes and more in the lower classes. Especially in Std. VII with the introduction of new subjects it might be adviable to have more teaching. It appears to be a very sound idea to have less and less teaching every year so that in the last school year assignment work will occupy most of the time. On leaving school pupils will not miss their teachers voice and explanations to such a great extent, and if they have any inclination for further study they will not find it such a formidable task as at present.

Finally, it may be suggested that the individual method, like all other methods, will have to be modified to suit local conditions. No hard and fast rules can be laid down as to its general application. It has been successfully applied by many educators. The investigator's personal view is that it will not produce as good immediate results as the class-teaching method if it is not adequately supplemented by teaching, but it will in most cases produce young men and women better equipped to hold their own in future occupations. It is worth while giving it a trial even if it is decided to discontinue it afterwards.

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1. See appendix E.

APPENDICES.

APPENDIX A.

ASSIGNMENTS USED IN COUNTRY MIXED SCHOOL.

ALGEBRA ASSIGNMENTS.

(For the second six months of the Std. VII year).

BOOK USED: "New Algebra for Schools " Parts I and II - by Durell.

GENERAL INSTRUCTIONS: The explanations given in these assignments and in the text-book should be read and re-read until you thoroughly understand them. If you still cannot understand them, consult with your friends or come to me.

The questions which you are required to do every week may be done in pencil if you work neatly. Otherwise they must be done in ink. Bring them to me as soon as you have completed the week's work.

In mathematics each new step is built upon a previous step. Hence, you will only confuse yourself if you continue with new work before you thoroughly understand the part you have done.

FIRST WEEK.

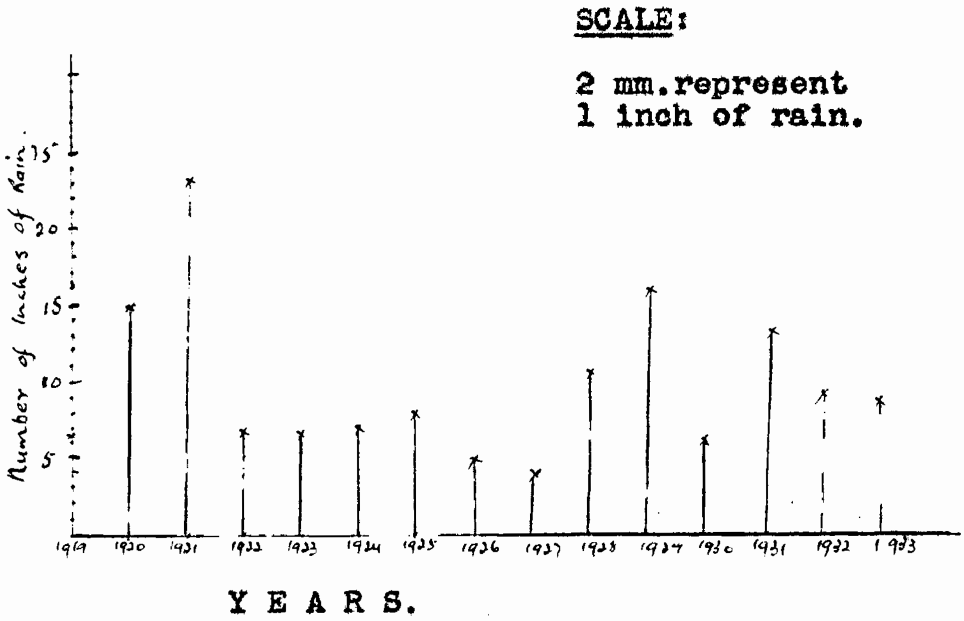
Imagine you are a stranger to Aberdeen and you want to know how much it rains here every year with the object of buying a farm in the district. On enquiring at the Magistrate's Office they will supply you with the following information: (They have a raingauge there.)

YEAR.	RAINFALL IN INCHES.	YEAR.	RAINFALL IN INCHES.
1919	$6\frac{1}{2}$	1926	5
1920	15	1927	4
1921	23.5	1928	11
1922	7	1929	16
1923	$6\frac{1}{2}$	1930	6
1924	7	1931	$13\frac{1}{2}$
1925	8	1932	10
		1933	9

If you are an intelligent person, you will make certain calculations from these data. (data = facts given). C a l c u l a t e the AVERAGE, ANNUAL RAINFALL during the above period of years.

You will note there are some dry years and also some wet years. Do the wet years occur after equal intervals?

The above information may be represented in the following way:-



When it is so represented the facts stand out clearly. Study the diagram and use it to answer the following questions:-

1. In which year did it rain least?
2. In which year did it rain most?
3. Which successive years were the driest?
4. What is the average annual rainfall of Aberdeen? (Look opposite which inch-mark on the upright on the left of the figure you see the most crosses).
5. Do the wet years occur at regular intervals?
6. What would you guess the rainfall to be in 1935? This year's rainfall so far is 18 inches.

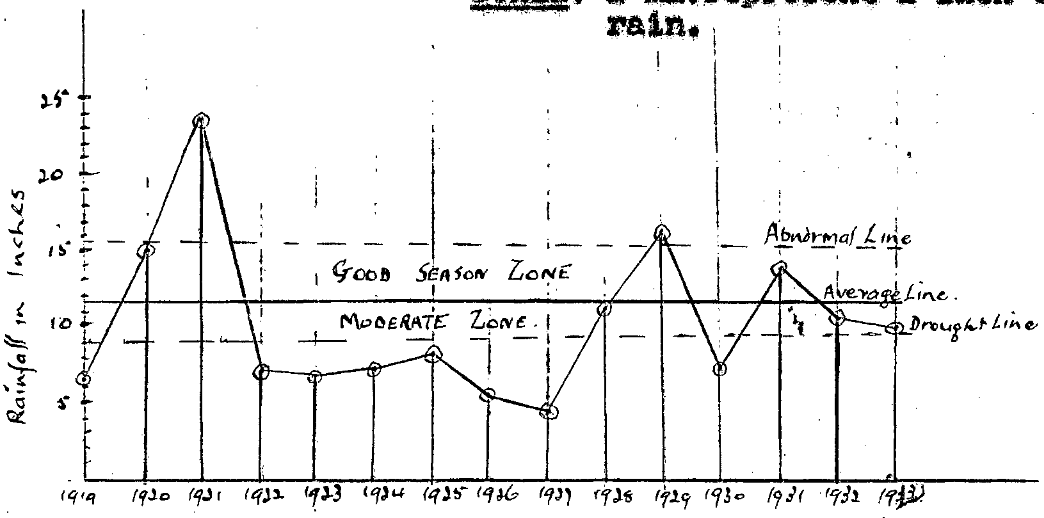
The above is a GRAPHICAL REPRESENTATION, or a GRAPH, of the rainfall of Aberdeen during the years 1920 - 1932. We have drawn a picture of the amounts of rain we get every year. Every upright line stands for the number of inches during that year. If the upright is 2 mm. high it represents 1 inch of rain: if it is 4 mm. high it represents 2 inches, and so on.

Is it easier to draw conclusions from the table of facts as the Magistrate's Office gave it or from our diagram, or graph as it is called?

To make our graph more interesting we can join the crosses to show better how the rainfall goes up or down every year: and also draw horizontal dotted lines showing the average rainfall etc.

RAINFALL OF ABERDEEN 1919 - 1933.

SCALE: 2 mm. represent 1 inch of rain.



YEARS.

Draw this graph on a piece of good paper or cardboard so that you can keep it somewhere in your house and add on to it the rainfall of future years. You might discover very interesting facts from such a graph about Aberdeen's annual rainfall. You might even become rain prophets!! If anyone can bring a suitable piece of cardboard (fairly big) to school, we can make such a graph for our class room.

Remember the following points when you draw your graph:-

1. Its name: RAINFALL OF ABERDEEN 1919 - 1933.
2. Write along the vertical upright or axis on the left: I n c h e s of R a i n and measure off equal distances to represent these inches of rain, say 1 inch for every 5 inches of rain (it depends on how big your graph is).
3. Along the horizontal axis mark off the successive years.

READ:

Example 1, Chapter V in Durell and answer the questions (Ex. V.a.) That is another interesting example of the graphical representation of facts.

QUESTIONS:

Draw graphs to represent the figures tabulated below. Do it just as above in the first graph: do not join the crosses. Give each graph a name and write along each axis what that axis represents.

1. Draw a graph showing the European population of the villages lying near Aberdeen. The following data is to be found on pg. 92 of the Official Year Book of the Union of S.A. (No. 14. 1931 - 32 issue).

Name of Village	Graaff-Reinet	Beaufort West	Aberdeen	Willowmore	Murraysburg	Jansenville	Steytlerville
European population.	4447	3337	1737	912	733	679	614

2. (a) Those of you who are wool farmer's sons or are interested in wool-farming will draw a graph to show the amount of wool exported during the years 1918 - 1932. You will find the necessary statistics on pg. 380 of the Union Year Book (No.14)

OR

(b) Those interested in any other products of our country, may draw the graph of the export of such products instead of drawing the graph for the export of wool. Find the necessary information in the Union Year Book.

3. Ex. V.b. No. 4 (pg. 66 Durell). Can you read off from your graph what the weight of boys of age 12½ years will be more or less?

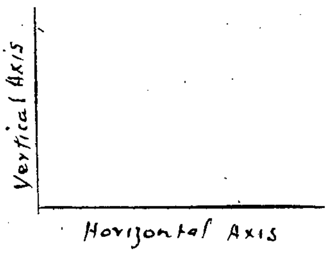
GRAPHS ON SQUARED PAPER.

As you will have noticed, these graphs take some time to draw in spite of being so useful. Every upright has to be measured and you are never sure that it is quite accurate.

For these reasons squared paper is used. Instead of measuring each upright, we just count off a number of divisions. This makes the drawing of graphs a much easier and accurate job.

READ:

Example 2, pg. 67 and answer the questions on it. (Note: A piece of squared paper is taken, the AXES are drawn on it and a suitable SCALE is chosen. This means, it is decided what number of marks will be represented by one division on the squared paper along the vertical axis, and what number of divisions will represent a week along the horizontal axis. In this case 1 div. along the vert. axis represents 2 marks: and along the horizontal axis 2½ divisions represent 1 week.)



QUESTIONS:

On the piece of graph paper supplied, draw a graph to represent the following:-

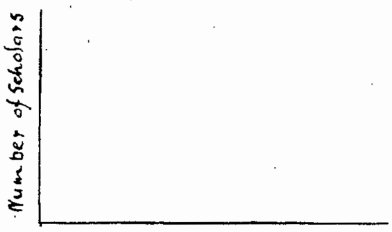
Number of scholars who passed the Senior Certificate Examination in the Aberdeen High School:-

YEAR.	1925	1926	1927	1928	1929	1930	1931	1932	1933
Number of Scholars	9	6	6	12	5	10	13	14	21

NOTE:

1. Write its name as shown in figure.
2. Write along vert. axis: Number of Scholars.
3. Write along horizontal axis: Years.

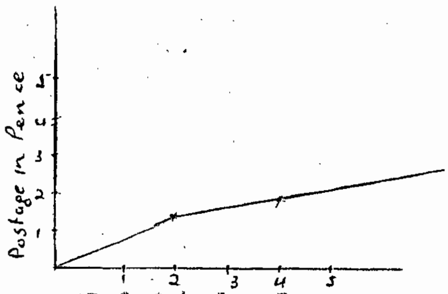
Sen. Cert. passes in Aberdeen High School 1925 - 1933.



YEARS.

A graph records how one quantity varies in size when another quantity, on which it depends, varies. Let us take an example:-

The postage on parcels of various weights. The postage varies as the weight of the parcel changes. The postage is thus dependent on the weight of the parcel. Such quantities as the postage which depend on other quantities we call DEPENDENT VARIABLES. The quantities on which they depend (in this case the parcel's weights) are the INDEPENDENT VARIABLES.



Weight in Ounces.

The INDEPENDENT VARIABLE is always measured along the horizontal axis. The DEPENDENT VARIABLE is always measured along the vertical axis.

QUESTIONS:

1. Which are the independent and the dependent variables in the following:-

   (a) Average height of boys of different ages.
   (b) Our gold export during the last 10 years.
   (c) Expenditure of an hotel and its number of boarders.

2. Pg. 69 No's 1 to 15 (Read pg. 68 to understand this better.

Come and show your graphs to me as soon as you have done them.

If there is anything you cannot understand, discuss it with your friends. If they are unable to help you, come to me. Do not proceed with new work until you are quite sure about the work you have done.

SECOND WEEK.

INTERPOLATION: (A big word, the meaning of which you will know quite soon). Before continuing we want to point out a very important difference between some of these graphs. Compare the following two graphs which you drew:-

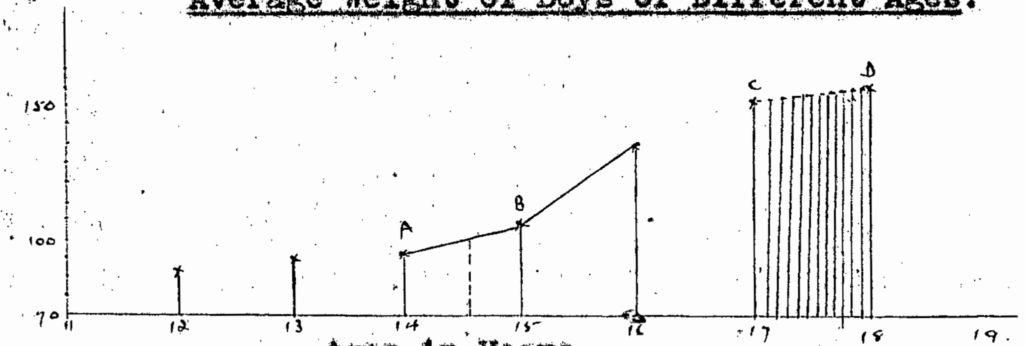
(a) Annual rainfall of Aberdeen.
(b) Average weights of boys of different ages.

and answer the following questions for yourself:-

1. Is it possible in (a) to insert an upright between 1931 and 1932 and attach any importance to it?

2. In (b) is it possible to give any meaning to intermediate uprights? What is, for example, the average weight of boys of $13\frac{1}{2}$ yrs., or of $14\frac{1}{2}$ yrs., or of $13\frac{1}{4}$ yrs., or of $12\frac{3}{4}$ yrs. etc.? In other words, can you put uprights midway or quarter-way between two ages and thus find the average weight of boys of these ages?

Average Weight of Boys of Different Ages.



Ages in Years.

In the figure we have joined the crosses between 14 and 16 yrs. Make an extra upright (dotted lines) between 14 and 15 yrs. and see where it cuts AB. That will give the average weight for boys of age $14\frac{1}{2}$ yrs. From your graph find the average weights of boys of ages $13\frac{1}{4}$, $16\frac{3}{4}$, $17\frac{1}{2}$ yrs. respectively. Deducing these values from our graph, we call "interpolation".

Can you interpolate on your rainfall graph? We come to the following conclusions:-

1. In the rainfall graph we can have no uprights in between the others. That is, we cannot interpolate, we cannot fill up the gaps if some readings are missing. We cannot get any extra readings from the graph. It is therefore not quite right to join the crosses. Why we did it last week in one of our graphs, was to show better how the rainfall went up and down in successive years.

2. In the "Average weight of boys" graph we can have intermediate uprights as we have shown i.e., we can interpolate, i.e., we can take readings from our graph which have not been actually measured. To draw the graph the average weights of boys of ages 11, 12, 13, 14, 15, 16, 17 and 18 were found. Now from the graph we can deduce the average weights for ages $13\frac{1}{2}$, $17\frac{1}{4}$, etc. etc. In this case it is quite right to join the tops of the uprights. The line so found is really the approximate path of the tops of an infinite number of intermediate uprights. (See graph: CD is the path between ages 17 and 18.)

In future whenever you draw a graph, note whether interpolation is possible or not. This is rather a long explanation of the very easy but important idea of interpolation. If you still cannot grasp what is meant by it, discuss it with one of your friends or come to me.

This week we must get some practice in drawing graphs on squared paper. As was said last week, we spare ourselves all the trouble of drawing and measuring an upright for each separate reading, by making use of the lines on squared paper. We need only mark the points on the squared paper and never draw the uprights (See Example 3. pg. 71 and also graph on pg. 73).

Before proceeding, however, just a word of explanation about choosing a <u>SCALE</u>:-

As it was pointed out to you in your drawings to scale last term, you must always choose as <u>large</u> a scale as the paper allows you to do. In drawing graphs on squared paper we also have to choose a CONVENIENT scale e.g. 1 inch represents 1, 10, 100 or 5, 50 etc. Never let one inch represent 3 or 7 or 9 etc. as it makes it very difficult to find (or to <u>plot</u>) your points on the squared paper. (READ on pg. 68 what they say about SCALES).

In order that your graph may be useful and easy to understand, always remember the following:-

1. Write above the graph its TITLE or NAME.
2. Write along each axis what that axis represents (independent variable along horizontal axis and dependent variable along vertical axis.)
3. Graduate each axis so as to show clearly the scale for that axis. (See also instructions on pg. 69).

<u>QUESTIONS</u>:

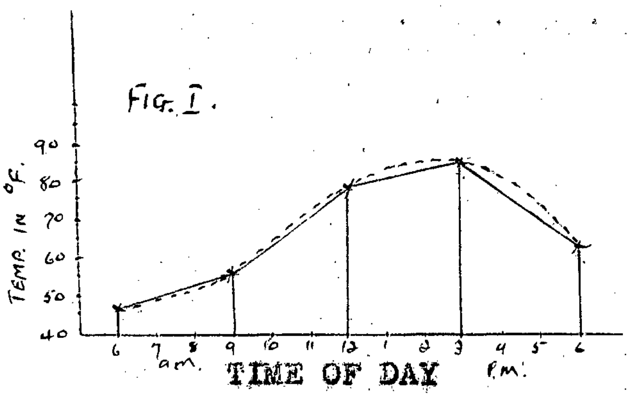
1. Pg. 72 No. 3. Follow the instructions given at the end of the question very carefully. Use graph paper. Can you have interpolation?

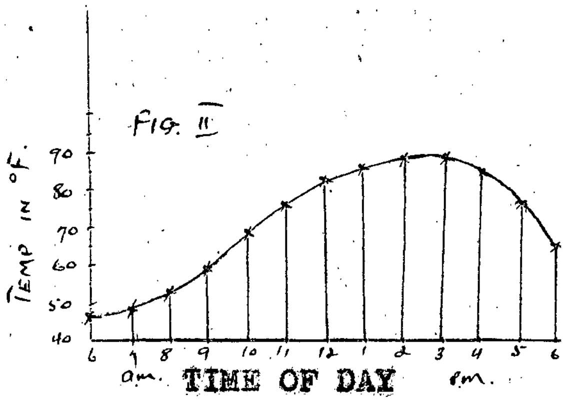
2. Pg. 73 No. 5 (Use squared paper).
N.B. To find the points on the squared paper after the axes have been drawn, and graduated, we call <u>PLOTTING</u> the points. This expression "to plot a point" you will use very often in connection with graphs.

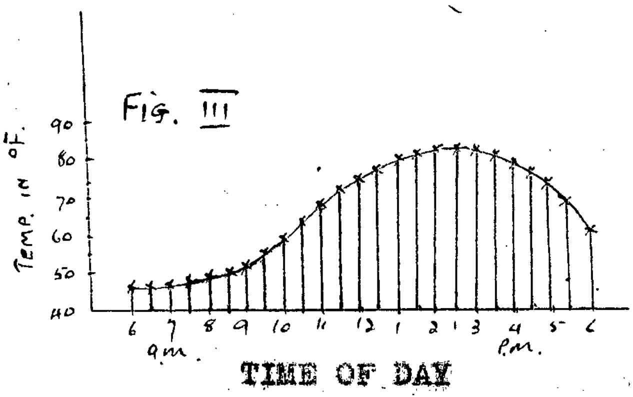
L O C U S   G R A P H S

Look at the following graphs:-

TEMP. CHART for Aug. 23rd, 1934. 6am. - 6Pm.







These three graphs represent exactly the same thing:- the readings on a Fahrenheit thermometer at different times between 6 a.m. and 6 p.m. on the 23rd Aug., 1934. BUT, for fig. 1 readings were taken every 3 hrs., for fig. 2, every hour and for fig. 3, every half-an-hour.

Interpolation is possible on these graphs i.e. we can read off from our graphs the temperatures for times when no readings were taken.

Which of these graphs represents the temp. changes on Aug. 23rd the best? We can get a still better graph by taking readings every 5 minutes. We can even take readings every minute although it will be a very tiring job, but such a graph will be an excellent representation of how the temperature rose and fell on that day. And if we then join the points which we have plotted, we shall get a fairly SMOOTH CURVE. Even fig. 3 gives a fair curve.

Fig. 1 is not a good graph, especially because the pts. are joined with straight lines. If we had drawn a curve (as shown in dotted lines) it would have been better.

Fig. 2 gives quite a good idea of the temp. changes. In such a case the best is to join the plotted points in a smooth curve. Then it will be possible to interpolate with fair accuracy.

Such a curve represents fairly accurately the LOCUS of the top-points of all the intermediate uprights (locus = path traced out under certain conditions). We therefore call it a LOCUS-GRAPH.

READ:

Examples 3 and 4 on pg. 73, 74, 75 which also explain locus-graphs.

ANSWER:

The oral questions on them for yourselves or get one of your friends to ask you these questions and to listen to whether you answer them correctly. They are very important and it is essential that you must be able to answer them.

IMPORTANT CONCLUSION:

We can only join the plotted points (i.e. the tops of the uprights) when interpolation is possible and then we join them so as to get a smooth curve.

In other graphs such as our rainfall graph, where no interpolation is possible and intermediate uprights have no meaning, we cannot really join the points. If we join them we just do it to show better the ups and downs on the graph, but the joined lines really have no meaning.

QUESTIONS:

(A graph must look neat: All lines are drawn in pencil. All writing in ink - neatly printed).

3. Take hourly readings (as the church-clock strikes the hours) of the thermometer which hangs in your classroom on Thursday, 30th Aug. from 9 a.m. to 3.30 p.m. when the school closes. Plot these readings on squared paper and draw IN PENCIL a smooth curve through them.

   Read off from your graph the temperatures at 9.30 a.m. 11.45 a.m., 2.30 p.m.

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THIRD WEEK.

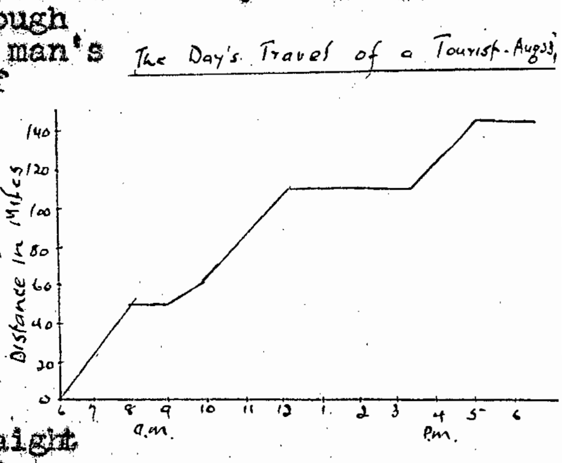
We are all convinced by now that a graph is a much better way to record a number of readings or observations than the numerical table. When you see such a large number of figures, your brain becomes confused but when you see them neatly represented in a picture - for that is really what a graph is - you take it all in with one glance and you remember it much better.

In addition, from your graph you can deduce readings which you have not actually taken - interpolation, we call it.

Furthermore, a graph shows so beautifully the relation between two groups of quantities e.g. the realtion between the weights of parcels and the postage on them. How does the one vary as the other varies? It shows very clearly the tendencies of these variations - keep this in mind when doing question 1.

QUESTIONS:

1. (a) Describe as well as you can what you deduce from the following rough graph as regards the man's tour, speed, times of meals etc.

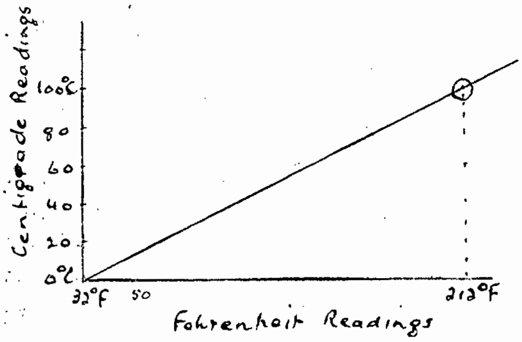


   (b) Questions 5 - 8 pg. 76 - 77.

2. Questions 2 pg. 76.

3. (a) Questions 9, 10, 11, pg. 77 - 78.
   Note: If we get a graph which is a straight line as in question 9, then the two quantities are in proportion.

   (b) Draw a graph to convert Fahrenheit readings to Centigrade readings.
   We know that the readings 32°F and 0°C: 212°F and 100°C correspond. This is also a straight-lined graph, hence plot these two points, join them in a straight line and convert the following into Centigrade readings: 40°F, 160°F, 200°F, 88°F.



4. Questions 15, 16, 17, pg. 80.

The sheet of graph paper is for questions 2 and 3(b) - one side of the sheet for each question.

EXTRA QUESTIONS:

If you find this work interesting and time permits you may also do questions 12, 13, 18, 19, 20. Get graph paper from me.

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FOURTH WEEK

We are now going to leave graphs for some time, but shall return to them later on. In the meanwhile be on the lookout for graphs published in newspapers or other periodicals or books:- e.g. rainfall graphs of certain countries (annual <u>or</u> seasonal rainfall graphs): graphs showing our exports and imports during the different months of the year or during successive years: population graphs: graphs showing the earnings of the railways etc. etc. If any of you see such interesting graphs the rest of the calss will be very pleased to see them if you can bring them to school. You will also find a few interesting graphs in the Union Year Book and in geography books. If you notice anything peculiar about a graph we shall be very glad to discuss it with you if you bring it to school.

Before continuing with other work, think back about all the graphs you have drawn, and then do the following:-

<u>QUESTIONS</u>:-

1. Write a short essay on why you think a graph is such a very useful way to represent data which are related. (About 5 - 10 lines, briefly enumerating your reasons.)

<u>B R A C K E T S</u>. (Ch. VI Durell).

Brackets are used to <u>g r o u p</u> those quantities together which have to be treated in the same way (i.e. on which the same operation have to be performed).

<u>Example 1</u>:

Subtract 6 ~~from~~ and 4 from 19. We may write this as follows 19 - (6 4). The two numbers 6 and 4 are grouped together because both have to be subtracted from 19. We thus have:-

$$19 - (6+4) = 19 - 10 = 9.$$

<u>Example 2</u>:

I have 25/-. I spend 2/- on fruit, 11/- on books and 3/- for going to a concert. How much have I left.

<u>Answer</u>:

$$25 - (2+11+3) = 25 - 16 = 9 \text{ shillings.}$$

<u>Example 3</u>:

John has a/-. He buys tops for b/- and sweets for c/-. How much has he left?

<u>Answer</u>:

$$a - (b+c) \text{ shilling.}$$

<u>Take Example 1 and look at it from a different point of view</u>:-

Subtract 6 from 19, and then subtract 4 from the remainder.

Answer:-

$19 - 6 - 4 = 13 - 4 = 9$ which is the same answer as above.

HENCE $19 - (6+4)$ is the same as $19 - 6 - 4$.
Similarly $25 - (2+11+3) = 25 - 2 - 11 - 3$.
$a - (b+c) = a - b - c$.

ON THE OTHER HAND:

$21+(5+9)$ is exactly the same as $21+5+9$ both being equal to 35.
Similarly $a+(b+c) = a+b+c$. The brackets do not change the signs.

Perhaps it will simplify matters if we make two rules about the removal of brackets:-

1. If a bracket has a - sign in front of it, any connecting terms in the brackets are changed when the bracket is removed.

   e.g. $a - (p - s) = a - p + s$.

2. If a bracket has a sign in front of it, any sign connecting terms in the brackets remains the same when the bracket is removed.

   e.g. $a + (p - s) = a + p - s$.

QUESTIONS:-

1. Pg. 82 Nos. 1 - 8. Do them mentally. They need not be handed in.
2. Nos. 9 - 25.
3. All the even numbers from 26 - 50. Remember to add like terms after you have removed the brackets.
4. Nos. 51 - 56. First remove the brackets and then solve in the usual way.
5. Nos. 57 - 60.

The above must be done by everyone. If you have completed them you may spend any time that is left in doing extra examples of the third week.

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FIFTH WEEK.

It is of the utmost importance that you must not continue with new work until you have completed and thoroughly understand the work you have done.

This week we shall continue doing examples in which we use brackets to group terms together which have to be treated alike. Not only when adding and subtracting are brackets of great use, as we saw last week, but also in MULTIPLICATION and DIVISION.

If we want to MULTIPLY each of a, b, and c by the same number, say 5, then we write the sum of the results as follows: $5(a+b+c)$.

$\therefore$ $5(a+b+c)$ is the same as $5a+5b+5c$.

Similarly $p(3x+y) = 3xp+yp$. And $\frac{1}{4}(4a - 9b) = a - \frac{9}{4}b$.

READ:

Example 1, pg. 83.

RULE:

When an expression in a bracket is multiplied by a number, each term in the bracket must be multiplied by that number when the bracket is removed.
Note: $2ab(b+c)$ may also be written $2 \times a \times b \times (b+c)$.
In words: 2 times a times b times (b+c).

QUESTIONS:

1. pg. 85, Nos. 1 - 18.

2. Say the following in words:- $2a(b+c)$; $x^2(2y - 4)$.

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In doing the examples that follow now, we also make use of what was learned about brackets last week.

EXAMPLE:

Subtract 3 times $(a+b)$ from c. That is, $c - 3(a+b)$. We multiply first and then remove the bracket.

$$c - 3(a+b) = c - (3a+3b)$$
$$= c - 3a - 3b. \quad \text{(Signs change).}$$

QUESTIONS:

3) Pg. 85, Nos. 19 - 28.

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In No. 13, pg. 85 you had to divide $a^2 - ab$ by a. The easiest way is to divide each term by a and then your quotient is $a - b$. Now $(a^2 - ab)$ divide by a or $\frac{a^2-ab}{a}$ or $(a^2 - ab)\frac{1}{a}$ are different ways of writing the same thing, but the second way, $\frac{a^2-ab}{a}$ we shall use most often. It means that the whole of $a^2 - ab$ must be divided by a.

Query: Why are $(a^2 - ab) \div a$ and $(a^2 - ab)\frac{1}{a}$ the same?

READ:

Example 4, pg. 84. It is very important and you must understand it well before you continue. Note that if we write A for $2x - 1$ and B for $3x+1$, the expression becomes $\frac{A}{4} - \frac{B}{7}$. Always look upon the $2x - 1$ (also $3x+1$)

as one quantity which has to be treated as a whole.

<u>READ</u>:

Example 5, pg. 84. Note that the L.C.M. of 4a and 6b is 12ab.

<u>QUESTIONS</u>:

4) Pg. 85, Nos. 29 - 42, omitting Nos. 36, 38, 39, 40.

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We shall now apply our knowledge of brackets to do other types of examples where they occur.

<u>READ</u>:

Example 6, pg. 85. Remove the brackets and then solve.

<u>QUESTIONS</u>:

5) pg. 86, Nos. 43, 45, 47, (multiply each term of both sides by 7) 48, 49, 51, 53, 55, 56, 57 to 64.

No extra examples are given this week. Continue with the seventh week as soon as you have completed the above. At end of 6th week you will find a number of extra examples.

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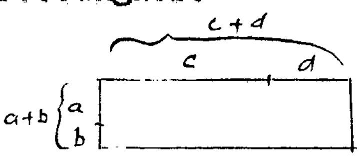
<u>SIXTH WEEK</u>.

This week we shall do further examples on brackets. The product of two quantities such as <u>a</u> and $b+c$ is written $a(b+c)$. The <u>factors</u> of this product are <u>a</u> and $b+c$, just as 2 and 3 are the factors of 6.

Similarly the product of $(a+b)$ and $(c+d)$ is written $(a+b)(c+d)$ or $(a+b)\times(c+d)$. The factors are $(a+b)$ and $(c+d)$. <u>Each of these factors</u> consists of <u>two</u> terms, therefore we call them <u>binomial</u> factors. (bi = two).

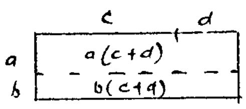
Look at the following rectangles:-

(1) Area = length $\times$ breadth.
$= (a+b)(c+d)$.



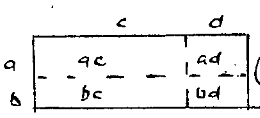
(1)

(2) Area of big rectangle
= areas of two small rectangles.
$= a(c+d) + b(c+d)$



(2)

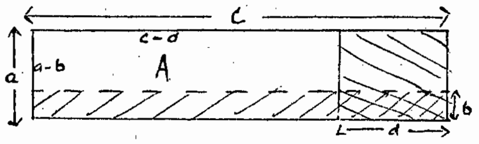
(3) Area of big rectangle
= areas of 4 small rectangles
$= ac + ad + bc + bd$



(3)

But these three rectangles are the same.
Hence $(a+b)(c+d) = a(c+d)+b(c+d) = ac+ad+bc+bd.$
Similarly $(a-b)(c-d) = a(c-d) - b(c-d)$
$= ac - ad - (bc - bd)$
$= ac - ad - bc + bd.$

Check this result from the following figure:-



Area of rectangle $A = (a-b)(c-d)$ and it is equal in area to whole rectangle minus other parts etc.

READ:

On pg. 86, 87 everything that is said about Binomial Products.

QUESTIONS:

1. Pg. 87 Nos 1, 3, 5, 7, 9, 10, 11, 12, 14, 15, 16, 19, 17, 18, 22, 24, 25, 26, 27, 28, 30.

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BRACKETS INSIDE BRACKETS:

We sometimes get brackets inside other brackets as the foll. example shows:-

Subtract $4a - (3b+c)$ from $8a - (2b - c)$.

This may be written: $\{8a - (2b - c)\} - \{4a - (3b \quad c)\}$

We use a different shape of bracket for the outside ones, otherwise we may become confused. To simplify this, we first remove the inner brackets and collect like terms before removing outer brackets.

Thus $\{8a - (2b - c)\} - \{4a - (3b \quad c)\}$
$= \{8a - 2b + c\} - \{4a - 3b - c\}$
$= 8a - 2b + c - 4a + 3b + c$
$= 4a + b + 2c.$

There are different shapes of brackets such as $a - [b+c]$: $a - \{b+c\}$: $a - \overline{b+c}$. The "line" bracket or vinculum as it is called, is exactly the same as the other brackets, it groups the terms under it together.

READ:

On pg. 88 what they say about systems of brackets. Note especially example 8.

QUESTIONS:

2) Pg. 89 Nos. 1 - 16.

EXTRA QUESTIONS:

Pg. 89 Nos. 17 - 24.

SEVENTH WEEK.

The quarterly examinations being written on Thursday and Friday of this week, we shall do some revision examples.

SUMMARY:

1. Addition and subtraction: Only like terms can be added or subtracted.

2. Equations: Whatever you do to the left side of the equation, also do it to the right side.

3. Problems: Let the unknown quantity be X, i.e., the quantity which it is your aim to find.

4. Multiplication and division: To multiply add the indices: to divide, subtract the indices.

5. Graphs: Take independent variable along horizontal axis and dependent variable along vertical axis.

REVISION EXAMPLES:

1. Pg. 257, Revision Ex. R.1. Do every third example from 1 - 39, and any others which you want to do.

2. Pg. 259, Revision Ex. R.2. Do every fourth example from 1 - 55, and any others.

3. Take a look through your assignments and note important points.

Show these revision examples which you have done to me as soon as possible so that I can help you with mistakes you might have made. Come to me with any other difficulties you have.

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EIGHTH WEEK.

This plan to group quantities together by means of brackets, comes in very useful when working out practical problems such as you will do this week. Some general examples are also given to test whether you thoroughly understand how to remove brackets, to insert them etc. This week will conclude for the time being our work on brackets, but you will use them quite often in later examples.

You may make use of the foll. formulae in solving the practical problems set for this week:-

1. The circumference of a circle of radius "r" $= 2\pi r$. in.

2. The area " .... "r" $= \pi r^2$ sq.in.

3. The volume of a cylinder whose cross-section is A sq. in. and whose length is L in. = AL cu. in.

EXAMPLE:

(Same as Example 10, pg. 90).

Find the area of the square plate, as shown in book, from which four equal semicircles and one complete circle have been cut away.

Length of one side of plate is 3a in.

Area of plate as in figure = (Area of sq. plate)
− (Area of 3 whole circles cut out)
$= (3a)^2 - 3\pi(\frac{a}{2})^2$

Now READ example 10 as it is explained in the book. Come to me if you cannot understand it.

QUESTIONS:

(1) Pg. 91: Nos. 1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, (In No. 8, note that the area of a $\Delta = \frac{1}{2}$ base $\times$ height. In No. 17, note that a quadrant is a quarter of a circle.)

(2) Test paper A.20, pg. 96. (Get graph paper from me for question 2.)

EXTRA QUESTIONS:

1. Pg. 91: Nos. 2, 3.
2. Pg. 93: Nos. 19 - 25.
3. Any one of the test Papers on pg. 94.

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NINTH WEEK.

As you have seen by now, in algebra we employ the same rules of addition, subtraction, multiplication and division as in arithmetic, only we use letters as well as numbers: algebra may be called generalised arithmetic. Most of the ideas and devices we use are after all not so very new and strange to us. For example, to group those quantities together that have to be treated alike by means of brackets, is quite a common-sense plan. Think clearly and algebra will be your easiest subject.

NEGATIVE NUMBERS:

Look at the foll. wool-market report:-

Class of wool.	Price in pence.		Rise or fall.
	January	July.	
First class	13	12	-1
Second class	11	11	0
Third class	9	$9\frac{1}{2}$	$+\frac{1}{2}$

Clearly, the plus sign in the last column is used to indicate a rise in price, and the minus sign a fall.

The plus and minus signs always have opposing meanings: if the plus stands for profit, the minus stands for loss, e.g. A man buys two horses for £30 and £20 respectively and sells them for £28 and £24 resp. His loss on first horse is £2, i.e., his gain is £(-2). His gain on second horse is £4, i.e.£(+4).

Another example:- John has 10/-, Paul has no money and so has Henry. But Henry also owes 5/-. The shortest way of writing down how much each has is as follows:-

John	Paul	Henry.
10/-	0/-	-5/-

Henry has 5/- less than nothing, therefore we say he has -5/-. John has 10/- more than 0/-, therefore we say he has +10/- or just 10/-.

Up to now we have only worked with numbers greater than zero (i.e. nought), hence we never took the trouble to put the sign in front. We are also not going to do it in future, but we shall keep in mind that all numbers with no sign are greater than zero.

Such numbers greater than zero we call POSITIVE numbers.

Numbers less than zero are NEGATIVE numbers and we put -signs in front of them.

If we count backwards from 5 we shall get
5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6 etc.

Positive numbers (5, 4, 3, 2, 1) — Negative numbers (-1 ... -6).

As you see, this is really nothing new to you. You might want to know: Why should we say a man has -5/- if we can just as well say that he owes 5/-? The answer is: This is a shorter way and mathematics always chooses the shorter way.

Before we continue discussing negative numbers, answer the foll. questions for yourselves. If you feel doubtful about any of them, consult a friend or come to me:

1. John owes £5. Express this mathematically. Peter has £10. How much has Peter more than John?

2. I owe a man 100 apples. I give him 80, which is all that I had. How many apples do I than possess?

3. John has £(-5). Peter has £6. How much do the two together possess?

4. I owe £20 in the bank. How much money must I get to pay this and still have £10 left?

5. What is the meaning of: I agree to pay you £(-10) for helping you?

Positive and negative numbers are discussed in Durell Ch. VII: DIRECTED NUMBERS. But why directed numbers? Because pos. and neg. correspond to forward and backward or up and down. A direction can either be pos. or neg. We decide for ourselves which direction is pos. and then the direct opposite is negative. Up is usually taken as pos. and down as neg.

To the right is usually pos. and to the left neg.

"John walked -3 miles South", means that he walked 3 miles North. What do I mean by saying "The mountains are -10 miles East of Aberdeen?"

← Negative direction. Positive direction. →

READ:

Pgs. 99 and 100 and then do the following:-

QUESTIONS:

1. Pg. 100 Nos. 1 - 15, omitting Nos. 4, 9 and 12.

READ:

On pg. 102, 103 what is said about the NUMBER - scale and about addition and subtraction. The same rules apply here as for removal of brackets.

QUESTIONS:

2. Pg. 103, Nos. 1 - 30. (In Nos. 8 and 9 write the answers and do not first copy the questions out of the book as we usually do.)

You will find extra examples to do at the end of the tenth week.

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TENTH WEEK.

The tenth and eleventh weeks will also be devoted to directed numbers.

READ:

Pgs. 105 and 106. There you are given the rules for multiplication and division of pos. and neg. numbers. This is extremely important.

We shall give the rules here:-

In multiplication like signs give a positive product and unlike signs give a negative product. (The same rule holds for division).

Like Signs	Unlike Signs
Thus $4 \times 5 = 20$	$4 \times (-5) = -20$
$(-4) \times (-5) = 20$	$(-4) \times 5 = -20$
$a \times b = ab$	$(-a) \times b = -ab$
$(-a) \times (-b) = ab$	$a \times (-b) = -ab$

QUESTIONS:

1. pg. 106: Nos. 1 - 9.
2. pg. 108: Nos. 1 - 42. (Write down answers only).
3. Pg. 108: All odd numbers from 43 to 66.

BRACKETS:

READ on pgs. 108, 109 what they say about brackets when used in connection with directed numbers.

QUESTIONS:

4. Pg. 109: Nos. 1 - 28 (Answers only).
5. Pg. 109: all odd numbers from 29 - 65.

EXTRA QUESTIONS:

All odd numbers from 30 - 64, pg. 109.

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ELEVENTH WEEK.

In the previous examples, brackets have been used for performing addition, subtraction, multiplication and division. The working may, however, be arranged as in arithmetic. READ examples 6, 7, 8 on pg. 11 to see how this is done.

In addition and subtraction like terms are put below each other. You will find the addition very easy. Also subtraction is easy, but you need moreepractice to be able to subtract easily, quickly and correctly. Hence you are given a large number of subtraction sums to do. For subtraction, note that the signs of each of the terms of the lower line are changed (mentally) and then the like terms are added.

RULE:

Change signs and add.

QUESTIONS:

1. Pg. 112: All odd numbers from 1 - 14.
2. Pg. 112: Nos. 15 - 52.

The large number of exercises you have just completed were given so as to make you experts at the game of adding, subtracting etc. in algebra. We have now practised enough and will use our skill thus attained to solve other problems.

READ:

Examples 9, 10 on pg. 113.

QUESTIONS:

3. Pg. 114: Nos. 1 - 15.

EXTRA QUESTIONS:

Pg. 115: Nos. 16 - 18.

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TWELFTH WEEK.

During the past few weeks we have learned to make use of brackets and to work with negative numbers. A short summary of what we did will be very helpful:-

BRACKETS:

When you remove brackets, change signs when there is a negative sign in front, but leave them unchanged if there is a positive sign.

SUBTRACTION:

Change signs of lower line and add.

MULTIPLICATION & DIVISION:

Like signs give plus, unlike signs minus.

We learn algebra to enable us to solve problems which me may come across in daily life. All these practice examples in brackets, add., subtract, mult. etc. we did simply to make it easy for us when we want to solve problems.

Most problems are solved by means of formulae or equations, hence we shall now first see whether we can apply our skill in adding, subtr., mult., etc. to solve a few equations.

m Later in this week and also in the following two weeks we shall do some problems to sum up the work we have done in algebra during this year.

READ:

Example 1, pg. 116. Note that the steps enclosed in square brackets can be omitted as soon as the process is understood.

In solving the following equations simplify first i.e. remove brackets or multiply both sides by the same number to do away with the fractions or add like terms. Remember only that whatever you do to one side of the equation, must also be done to the other side. If you multiply, see that you multiply every term of both sides by the same number.

QUESTIONS:

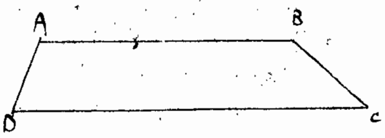
1) Pg. 116: Nos. 1 - 22, 29 - 35, 38, 41, 44.

FORMULAE.

Example 2 on page 117 shows how you can use your skill in performing all these operations to find the value of one of the unknowns in a formula if the values of the others are given.

You may read it if you understand how a camera lens works. Otherwise read the foll. example:-

Example: The area of a trapezium is given by the formula $A\ \ \frac{1}{2}h(a\ \ b)$, where a and b are the parallel sides, and h is the perp. distance between them. (A trapezium is a quadrilateral having only one pair of opposite sides parallel. As in figure where only AB//CD.



Find b if $A=5.4$, $a=1.7$, $h=3.6$

ANSWER:

Substitute the given values for A, a and h in the above formula, then

$$5.4=\frac{1}{2}\times 3.6\ (1.7+b)$$
$$\therefore 5.4=1.8\ (1.7+b)$$
$$\therefore 5.4=3.06+1.8b$$
$$\therefore 5.4-3.06=1.8b$$
$$\therefore 2.34=1.8b$$
$$\therefore b=\frac{2.34}{1.8}$$
$$=1.3$$

QUESTIONS:

2) Pg. 118: Nos. 1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15.

EXTRA QUESTIONS:

Pg. 118, Nos. 8, 11, 16.

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THIRTEENTH WEEK

PROBLEMS:

Read very carefully the instructions given at foot of pg. 119 for solving problems: then read example 3 pg. 120. It is very interesting to solve these problems. They make one think, but it is very gratifying to be able to solve them.

QUESTIONS:

1) Pg. 121: Nos. 1 - 20.
2) Test Paper A. 26, Pg. 127.

EXTRA QUESTIONS:

Pg. 124 Nos 1 - 10. (First read what is said about the transformation of formulae.

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FOURTEENTH WEEK.

This week concludes the work in algebra which we shall do this year. For revision for the examination you may work any of the extra practice exercises on pgs. 132 - 144, especially the simple equations on pg. 144. Also try a few more problems on pg. 123, Nos. 21 - 30.

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GEOMETRY ASSIGNMENTS.

(For the second six months of the Std. VII year).

BOOK USED: "Geometry": Ward and Dick.

FIRST WEEK.

During the first six months of the year we have learned the meanings of certain words which we are going to use in Geometry: "triangle", "quadrilateral", "angle" etc. We were not satisfied with just knowing what these words mean in the ordinary way, but found that each has a very definite meaning attached to it. A point is not a mark on a paper but is only a position. Hence we know now exactly what we mean when we talk about:-

points, lines (parallel and perp. lines), surfaces, angles, (right-angles, str. Ls., acute Ls., obtuse Ls., reflex Ls., adj. Ls., suppl. Ls., compl. Ls., vert. opp. Ls., altern. Ls., corresp. Ls., ext. Ls., int. Ls.), triangles (think about all the different kinds of triangles you know), quadrilaterals, circles (arcs, chords, radii).

We also measured these lines, angles, triangles with our instruments. We learned how to bisect lines and angles, how to draw perpendiculars and parallel lines. (A part of the job of a good mathematician is to be an expert with his instruments).

Then we went further: we compared some of our measurements and in that way discovered certain properties of geometrical figures:

e.g. The int. Ls. of any triangle = 2 rt. Ls. Also, if two lines are parallel and a third line cuts them, the corresp. and altern. Ls. are equal.

By constructing triangles we learned under which circumstances two triangles are equal in all respects. (See whether you still know the three instances). We found that these facts or truths are very useful in solving certain problems by drawing figures to scale e.g. to find the height of our church spire.

For the next 6 months we shall deal with these facts and many more from a theoretical standpoint. We call it theoretical geometry.

THEORETICAL GEOMETRY.

Those of you who are practical boys and girls and have practical fathers and mothers may not like this term "theoretical" at first sight. They say: theoretical people can talk a lot but can do nothing! We shall see!

The facts which we discovered by measurement (and also some additional ones) we shall now try to prove by r e a s o n i n g. For example, by measuring the int. angles of a large number of triangles we found that their sum always came to 2 rt. Ls.: we shall prove this same fact by arguments based on pure reasoning.

The arguments or discussion about any such property or fact we call a THEOREM. We shall have a large number of theorems, the one based upon another.

On page 48 you will find theorem 1. It states: If a str. line stands on another str. line the sum of the adj. angles so formed is equal to 2 rt. Ls. This is a clear s t a t e m e n t of what the theorem is about. We call it the ENUNCIATION of the theorem.

The portion underlined gives us certain information about the figure (we call it our hypothesis or given).

The portion which is not underlined gives the property of the figure which has to be proved.

READ the enunciation of theorems 1, 2, and 3. You are already familiar with them. Learn them by heart.

If once we have proved a theorem we may always use it in later theorems. The later theorems in the book therefore depend upon the earlier theorems. In theorem 3, as we shall see, we are going to make use of theorem 1.

Remember we must give a reason for every statement we make. Nothing is taken for granted, except of course certain self-evident facts. For example, the halves of two equal lines are themselves equal. Such truths we call AXIOMS. On pg. 46 a few important axioms are given. See that you understand them.

READ the proof of theorem 3. Note the steps:

1. figure.
2. given (hypothesis).
3. required to prove.
4. proof.

You are already familiar with the first 3 steps (we used them last term to describe our constructions).

Note the neat way the steps follow one another in the proof. No unnecessary words are used. Neatness counts a lot in geometry.

Without looking at your book write out completely the proof of Theorem 3.

If there is anything at this stage not quite clear to you, come and discuss it with me before you continue.

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In connection with the above note:-

1. We are not only going to prove certain facts (theorems) but we are also going to argue about the way in which certain constructions have to be made. The latter we call "p r o b l e m s". There is a general name for problems and theorems:- p r o p o s i t i o n s. Do not however trouble yourself about this at present. Only be sure that you know what is meant by a theorem.

2. Theorem 2 is the <u>CONVERSE</u> of theorem 1. This means the <u>given</u> of Th. 1 is the <u>required to prove</u> of Th. 2, and the <u>required to prove</u> of Th. 1 is the <u>given</u> of Th. 2. You may think it unnecessary to give a proof of the converse of a theorem, but think about the following statement: "A horse is an animal". Is the opposite true

3. No formal proofs of theorems 1 and 2 are given. They are <u>a x i o m a t i c</u>, that is, we can see that they are true from the figures. (Think about the definitions of an angle, rt. L., str. L.)

4. The first three theorems are all concerned with <u>angles round a point</u>. If you read pp. 45, 46 in your text book you may understand the above better.

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<u>Summary of work you have to know:-</u>

1. Do you grasp the meanings of the following completely: enunciation, theorem, axiom, converse of a theorem?

2. Learn the abbreviations on pg. 47.

3. Learn the enunciation of theorems 1, 2 and 3. (These theorems are the bricks of our geometrical building).

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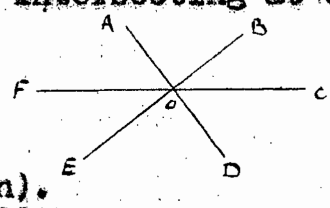
<u>SECOND WEEK</u>.

<u>QUESTIONS</u>:

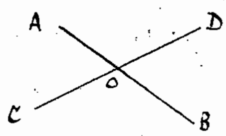
In doing the following exercises you may make use of the facts stated in theorem 1, 2 and 3.

1. Complete the following:- (Str. lines intersecting at O).

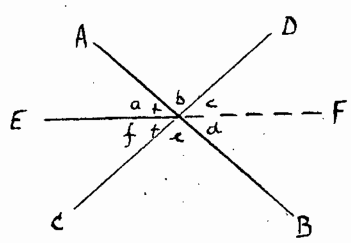


L.AOB=.......
L.AOC=.......
L.COE=.......
L.BOF+.......= 2 rt. Ls.
L.AOF + L.AOB + L.BOC = ... (Give reason).
L.AOB + L.BOC + L.COD + L.DOE + L.BOF + L.FOA = ...

2. In the figure L.AOD = 33°.
Calculate calues of Ls.AOC, COB, BOD.



3. Supply the missing parts in the following proof:-



<u>Given</u>: Two str. lines AB, CD intersecting at O.
EO bisects L.AOC, (Angles marked equal), and is produced to F.

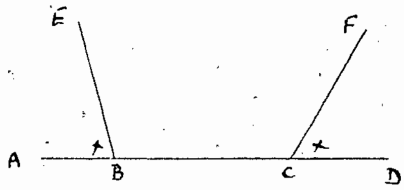
<u>Required to prove</u>: that OF bisects L.BOD.

<u>Proof</u>: a + b + c = ...
also d + e + f = ...
∴ a + b + c = d + e + f.

of these a f .... (Given)
b ...... (vert. opp. Ls.)
c d
and FO bisects L.DOB. Q.E.D.

4. Do questions 1, 2, 3, 4 on page 49.

5. ABE, DCF are equal angles (marked equal in figure) on the same side of AD.
Prove L.EBC = L.FCB.
(Remember the four steps in doing this:- 1. Figure. 2. given. 3. required. 4. proof. Follow the method in which Th. 3 is proved.)



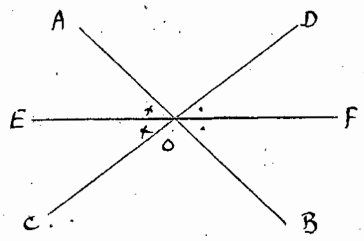
The above exercises must be done by __everyone.__ Do them __very neatly__ in your scriblers. All lines must be drawn with rulers. Come and show them to me as soon as you have finished them.

__EXTRA QUESTIONS:__

Those who have found the above easy may attempt the foll:-

1. Question 5, pg. 49.

2. AB, CD intersect at O. EO, FO bisect Ls. AOC, BOD resp. (Equal points are marked in the figure). Prove that EO and OF are in the same str. line.



3. PQR and PQS are two adj. suppl. Ls. QM, QN, bisect Ls. PQR, PQS resp. Prove that MQ is perp. to QN.

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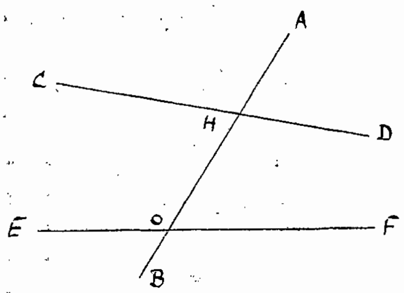
In the first three theorems we proved certain facts about the angles formed when lines meet at a point.

In theorems 4, 5 and 6 we have to work with __parallel lines.__

Let us first revise a few points about it:-

1. What do we call the line AB?

2. Complete:-



Ls. AHC, HOE are .... Ls.
Ls. DHO, HOE are .... Ls.
Ls. BOF, OHD are .... Ls.
Ls. FOH, OHC are .... Ls.
Ls. AHD, HOF are .... Ls.
Ls. DHO, HOF are two ....Ls. on the same side of AB.
Ls. AHD, AHC, BOE, BOF are .... Ls.

3. What are parallel lines?
Mention some parallel lines in the classroom.
Do you think all horizontal lines are parallel?
Do you think all vertical lines are parallel?

4. What did we discover last term experimentally (i.e. by measuring the angles in a large number of cases) about corresp. and altern. Ls. if the lines CD, EF are parallel (in above figure).
Conversely: What did we discover about the lines CD, EF if the corresp. and altern. Ls. are equal?

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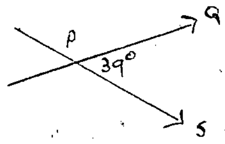
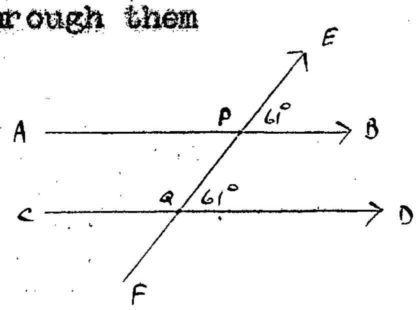
THIRD WEEK.

You still remember what we learned about DIRECTION last term.
Two lines PQ, PS differ in direction by 39° if the angle between them is equal to 39°. (Read again pg. 19, 20 as it is important in connection with Th. 4.)



Draw any line EF. Take any two points P and Q in EF and through them draw lines AB, CD so that each of their directions differs from the direction of FE by 61°.



Thus the direction of AB differs from the direction of FE by the same amount as the direction of CD differs from it.

Hence AB, CD point in the same direction, and therefore AB is parallel to CD (See definition of parallel lines on pg. 20).
(Note: Ls. EPB, PQD are corresp. Ls.)

Let us now state this idea as a theorem:
When a straight line cuts two other straight lines and a pair of corresponding angles are equal, the two straight lines are parallel.

Note: 1. The portion with double lines is the hypothesis or given, the other part is the conclusion or that which has to be proved.

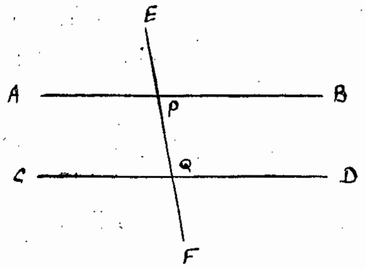
2. This theorem is also axiomatic, just as theorems 1 and 2. Its truth is self-evident from the figure: hence no proof is given in the book.

At once the following question enters the interested mind: Will the lines also be parallel if two alternate angles are equal? The answer is: Yes, and it is stated in Th. 4 (ii) pg. 51. There we also find a third condition (Th. 4(iii)) when lines are parallel, namely, when two angles on the same side of the transversal are equal to two rt. Ls.

READ: the proof of theorem 4(ii) and (iii): it is quite easy to understand. Note the neat systematic way in which these theorems are proved. ("By hypothesis" means "it is given").

SUMMARY:-

The conditions under which any two lines AB, CD will be parallel when cut by the transversal EF, are given below:-



1. The corresp. Ls. must be equal e.g. L.EPB = L.PQD.

2. The altern. Ls. must be equal e.g. L.APQ = L.PQD.

3. Two interior Ls. on the same side of EF must be supplementary e.g. L.BPQ + L.PQD = 2 rt. Ls.

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Is the converse of Th. 4 true? (State it). We shall have to prove it - our animal may not be a horse this time!

Theorem 5 states: If a straight line cuts two parallel straight lines, then

(i) the corresp. Ls. are equal. (pg. 50).
(ii) the altern. Ls. are equal. (pg. 52).
and (iii) the two interior Ls. on the same side of the transversal are supplementary. (pg. 52).

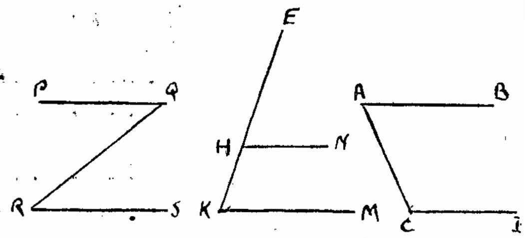
Note:(i) is axiomatic (think about it in terms of direction.)

(ii) and (iii) are proved on pg. 52. READ the proof.

Before answering the following questions you must know the enunciations of theorems 4 and 5 thoroughly and must be able to prove Theorem 4 (ii) and (iii) and 5 (ii) and (iii). Close your books and write out the proofs - this is the only way to test whether you know them. (Instead of using the Greek letters $\alpha, \beta, \gamma, \delta$ for the angles use the other way of angle notation e.g. L.DEF, L.APE etc.)

Come to me if there is anything you do not understand in these proofs.

QUESTIONS:



1. Under which conditions will
   (a) PQ be parallel to RS. (fig. 1) Why?
   (b) HN be parallel to KM. (fig. 2) Why?
   (c) AB be parallel to CD. (fig. 3) Why?

2. Do questions 2, 3 on pg. 53.

3. CD, EF are drawn perpendicular to a straight line AB. Give two reasons why CD is parallel to EF.

4. Do questions 5, 6, 7, 8 on pg. 53.

5. If you stand on the ground looking up at a man standing on the roof of a high building, your angle of elevation is equal to his angle of depression. Why is this?

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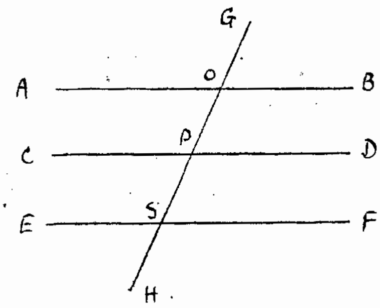
FOURTH WEEK.

This week we shall do a few exercises on parallel lines. Before we start with them, however, read over THEOREM 6 on pg. 53. The proof of this theorem is based on theorem 5. It was not quite necessary to give this proof as this theorem is also axiomatic. Of theorems 4, 5 and 6 which all go about parallel lines, this theorem is the least important.

QUESTIONS:

1. In the figure AB, CD and EF are parallel lines and L.GOB = 44°. Write down the values of the other angles. Give reasons.



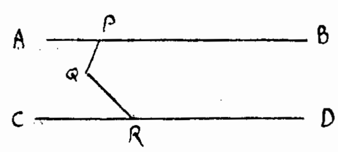
2. Question 4 pg. 54.

3. Question 12 pg. 55. (Give the steps 1. figure, 2. given, 3. required, 4. proof.)

4. Parallel lines CD, EF are cut by AB at M and N resp. Prove that the bisector of L.AMD is parallel to the bisector of L.ENB. (Given, required, proof).

5. Using only your ruler and compasses, draw two lines parallel to each other, the perpendicular distance between them being 1.5 ins. Then draw two other parallel lines cutting the first two at angles of 45°, the distance between them being 2 inches. Measure the angles of the quadrilateral so obtained. (Note: 1. For this accurate construction use only ruler and compasses. You may not use your protractor except for measuring the angles after the figure has been constructed. 2. Such a quadrilateral with its opposite sides parallel is called a PARALLELOGRAM.)

6. Fill in the missing parts of the following proof:-



Given: Two parallel lines AB, CD. Q is any point between them and QP, QR are drawn to AB, CD resp.

Reqd. to prove: L.PQR = L.APQ + L.QRC.

Construction: Through Q draw QS parallel to AB or CD.

Proof: AB // QS

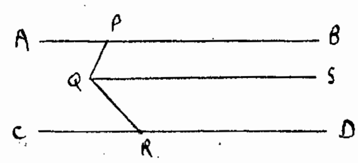
∴ L.APQ = altern. L. .....

QS // CD

∴ L.QRC = altern. L. .....

∴ L.APQ + L.QRC = ... + ...

= L.PQR



QED.

(Note: We had to draw a line QS parallel to AB or CD in order to prove the required fact. So far we have been able to give our proofs without making additional constructions to our figures, but sometimes as in the above we have to put in this step so as to make it possible to prove the desired fact.

The above Questions must be done by everyone. Come and show them to me as soon as you have finished them.

EXTRA QUESTIONS:

Those who have found the above easy may try the following:-

1. Question 14, pg. 55. (Here you will have to draw lines parallel to AB or CD through Q and through R in order to prove what is reqd.)

2. Question 15, pg. 55. (Construction: Through O draw a line // AB or CD.)

3. Question 9, pg. 54. (Construction: Produce DC, FE to meet each other.)

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FIFTH WEEK.

So far we have done six theorems:-

A. Theorems 1, 2, 3 dealing with ANGLES ROUND A POINT.

B. Theorems 4, 5, 6 dealing with PARALLEL LINES.

We can almost say we have laid six bricks of our geometrical building. This week we are going to build on these bricks by doing two theorems. These two theorems deal with the ANGLES OF RECTILINEAL FIGURES (Theorems 7 and 8). (What are rectilineal figures? What are three-sided, four-sided, five-sided, - - - many-sided rectilineal figures called, respectively?)

Theorem 7: The sum of the angles of a triangle is equal to two rt. Ls. (We discovered this experimentally last term by measuring the angles of different triangles and finding the sum of the angles of each triangle.)

You will find the proof of theorem 7 (pg. 56) very interesting. A construction is necessary to prove it, i.e., lines are added to the given figure -- in this case one side of the triangle is produced and a line is drawn parallel to one of the sides of the triangle.

QUESTIONS:

(To be done only after you understand proof of Th. 7).

1. Which of the first 6 theorems are used to prove Th. 7?

2. Prove Th. 7 using the other way of angle notation (e.g. L. ACB, L.ABC etc.)
(Do this question without looking at your books.)

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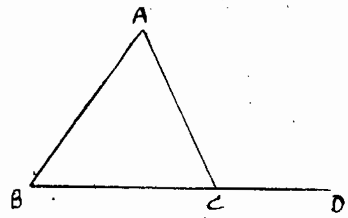
Other properties of a triangle which follow directly on what has been proved in Th. 7 can now be stated:



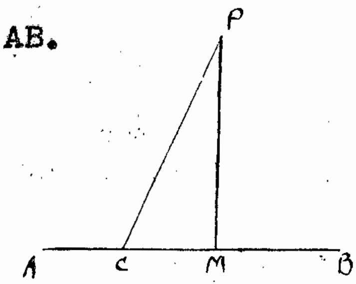
a) In the figure L.ACD = L.ABC + L.BAC. (State this in words). Also L.ACD > L.ABC or L.BAC.
b) See pg. 56.
c) See pg. 56.

Such important properties which follow as a consequence of a theorem just proved, are stated as COROLLARIES to that theorem. On pg. 56 you will find the three corollaries on Th. 7. Especially corollary 1. is very important. These corollaries should be remembered in connection with their respective theorems as we may want to use them in proving later theroems.

QUESTIONS:

3) Pg. 57, No. 1, 2.

4) Supply the missing parts of the following proof: (No. 3., pg. 57).



Given: Any pt. P. outside a str. line AB.

Reqd. to rpove: that from P. only one perp. can be drawn to AB.

Construction: Draw PM perp. to AB. Draw any other line PC to AB.

Proof: L.PMC = ... (because PM $\perp^r$ AB)
∴ L.PCM < ...... (because a triangle can have only one rt. L. Corollary 3, Th. 7.)
∴ PC is not perp. to AB.
Similarly any other line which we draw from P to AB can be proved not perp. to AB.
∴ PM is the only perp. that can be drawn from P to AB.

5) Pg. 57, Nos. 4, 5, 6, 7, 8, 9.

The above must be done by everyone. If you cannot understand a question or statement, discuss it with your friends. If they cannot help you come to me.

EXTRA QUESTIONS:

Those who have completed the above may attempt Nos. 10, 11, 12, 14, on pg. 58.

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SIXTH WEEK.

The fact that the sum of the three angles of any triangle is 180° is very interesting. But what about the interior angles of other rectilineal figures: quadrilaterals, pentagons, hexagons etc.? Our first impulse may be to draw these figures and measure their angles - this was our way of doing things last term. After all, we may argue, what is the protractor for?

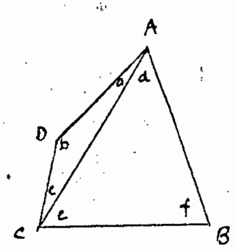
But then we have to remember that we are busy with theoretical geometry this term. We reason things out instead of laboriously measuring them. We have become reasoning, calculating beings instead of measurers.

It is however quite often a good plan to draw the figure and measure its angles just to get a clue to what the conclusion will be. You will find it interesting to draw and measure the angles of a quadrilateral, a pentagon etc. Do not waste much time about it as you did it last term. We then found that the angles of a quadrilateral are equal to 4 rt. Ls.

Now let us reason out this same fact making use of Th. 7.

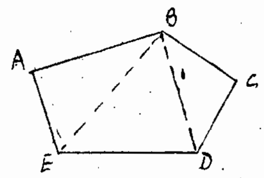
QUADRILATERAL:

By joining AC we have two triangles.
In △ADC, $a + b + c = 2$ rt. Ls.
In △ACB, $d + e + f = 2$ rt. Ls.
∴ $a + b + d + c + e + f = 4$ rt. Ls.
The interior angles of ABCD
= 4 rt. Ls.



PENTAGON:

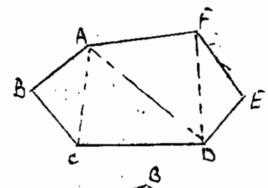
Complete for yourself:
The interior angles of a Pentagon=....



Note: We have divided the pentagon into three triangles.

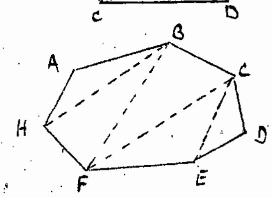
HEXAGON:

The interior angles of a hexagon=....



HEPTAGON:

Interior angles of a heptagon=......



DECAGON:

(deca = ten) Interior angles of a decagon = ...
(Draw your own figure and divide it into triangles).

So we can continue with any other polygon. All these facts are included in Th. 8 which states:

<u>In a polygon of "n" sides the sum of the interior angles is (2n - 4) rt. angles.</u>

Theorem 8 thus gives us a formula for finding the sum of the interior angles of any rectilineal figure no matter how many sides it has - from a triangle to a hundred-sided figure. We can state Th. 8 as follows

The sum of the interior angles of any polygon is equal to twice as many rt. Ls. as the polygon has sides minus 4 rt. Ls.

A quadrilateral has 4 sides.

∴ Sum of its interior Ls. = $(2 \times 4 - 4)$ rt. Ls.
= $(8 - 4)$ rt. Ls.
= 4 rt. Ls.

A pentagon has 5 sides.

∴ Sum of its int. Ls. = $(2 \times 5 - 4)$ rt. Ls.
= $(10 - 4)$ rt. Ls.
= 6 rt. Ls.

A decagon has 10 sides.

∴ Sum of its int. Ls. = $(2 \times 10 - 4)$ rt. Ls.
= $(20 - 4)$ rt. Ls.
= 16 rt. Ls.

Similarly by using this formula $(2n - 4)$, we can calculate the sum of the interior angles of any rectilineal figure.

<u>READ</u>:

The proof of Th. 8 on pg. 59. If you do not understand it, ask one of your friends to explain it to you or come to me. Make sure of this proof. On which <u>two</u> previous theorems is the proof of this theorem based?

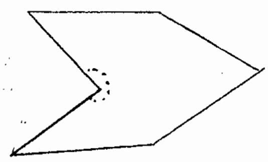
<u>READ</u>: also

1) <u>definition</u> of <u>regular polygon</u> on pg. 59.

2) Corollary of Th. 8 on pg. 60. This is another of the facts we discovered by measurement last term. The proof of this corollary is important and must be known. Read over the alternative proof given, but do not waste too much time about it if you cannot understand it, as it is quite sufficient if you can understand one way of proving it.

<u>Note</u>:

A <u>convex</u> figure is one having no reflex angles as interior angles. The accompanying figure is not convex as one of its interior angles is a reflex angle.



<u>QUESTIONS</u>:

(These questions must be done by <u>everyone</u> and haded in to me as soon as they have been completed. You may do them in pencil but very <u>neatly</u>.)

1. What is another name for a regular triangle, al so for a regular quadrilateral?

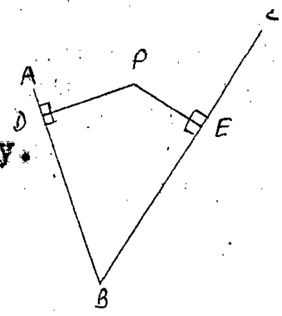
Calculate the number of degrees in one angle of (a) a regular pentagon. (b) a regular hexagon. (c) a regular octagon. (d) a regular decagon. (Note: a regular ~~has~~ hexagon has 6 <u>equal</u> Ls. The sum of these

6 angles can be found by means of Th. 8. Hence the size of one of its angles).

2. Pg. Nos. 1, 2, 3, 4, 5, 6, 11. In No. 5, draw a figure. Do No. 6 by means of an equation. Let one of the unknown equal angles be X°.

3. TYPICAL EXAMPLE:

P is any point between the arms of L.ABC PD, PE are drawn perp. to AB, BC resp. Prove that L.DPE and L.ABC are supplementary.



Try your best to give a very neat and systematic proof, as this is the type of thing you are often required to prove in this part of the work.

EXTRA QUESTIONS:

The following questions may be attempted if time permits and handed in with the others: Pg. 61, Nos. 8, 9, 10, 12.

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SEVENTH WEEK.

Our geometrical building is progressing very well. Let us look back and see what we have done:-

A. Theorems 1, 2, 3 about ANGLES ROUND A POINT.
B. Theorems 4, 5, 6 about PARALLEL LINES.
C. Theorems 7, 8 about the ANGLES OF RECTILINEAL FIGURES, and now in the following two weeks we are going to add:-

D. Theorems 9, 10, 11, 12, 13, 14 about CONGRUENCY OF TRIANGLES.

You are all quite familiar with this word "congruent". By drawing triangles last term we discovered that from certain data only one triangle can be constructed. If twenty of you each construct a triangle with sides 4, 5, 6, inches you will have twenty triangles exactly equal in shape and size: In other words, you will have twenty congruent triangles. But if twenty of you each construct a triangle with angles 30°, 70°, and 80°, you will get twenty different triangles. Their shapes will be the same but not their sizes.

Last term we discovered three such sets of data from which one and only one triangle can be drawn. These sets of data are:-

(1) the lengths of two sides and the size of the included angle (Th. 9).
(2) the length of one particular side and the sizes of two angles. (Th. 10).
(3) the lengths of three sides. (Th. 13.)

Any of these three sets of data are sufficient to determine the exact shape and size of a triangle, and hence if two triangles agree in any one set of them, the triangles must also agree in the unspecified pats, i.e., the triangles must be congruent (equal in all respects).

For example, if two triangles have three sides of the one equal to three sides of the other, each to each, then also the three angles of the one are equal to the three angles of the other, each to each, their areas are equal etc. i.e. they are congruent.

In theorems 9, 10 and 13 these three conditions of congruency are stated. (Th. 9: two sides and included ∠.: Th. 10: two ∠s. and corresp. side: Th. 13: three sides).

LEARN:

The enunciations of theorems 9, 10, 13.

READ:

The proofs of theorems 9, 10, 13, but if you cannot understand them rather leave them alone. These proofs are not really essential in our scheme, as it is quite clear that from a given set of data one and only one triangle can be constructed.

(Note: 1. △ ABC ≡ △ DEF means △s ABC, DEF are congruent.

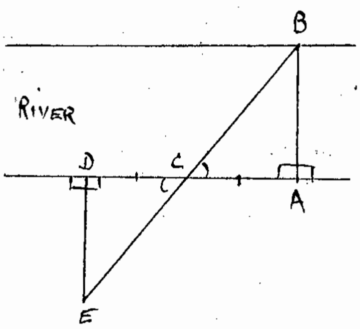
2. Theorems 9,10, 13 are proved by applying one triangle to the other i.e. fitting one triangle on the other so that equal pats fall on to one another or coincide).

WHAT IS THE USE OF KNOWING WHETHER TRIANGLES ARE CONGRUENT?

To give an answer to this question we shall consider the following practical example:

The Kraairiver is in flood. You stand on one of its banks and you would like to know the width of the river. The following is a very easy way of getting the width:-

Look for a tree (B) or any other object on the opposite bank and stand directly opposite it at (A). From A walk along the bank a distance of, say 10 paces, to C. At C put a pole in the ground and then walk another 10 paces to D so that AC = DC. From D walk straight back from the river bank (i.e. perp. to bank) until you see C and B in a straight line.



The distance DE which you have walked away from the bank, gives the width of the river.

What right have we to say this? In other words, what right have we to say that DE = AB?

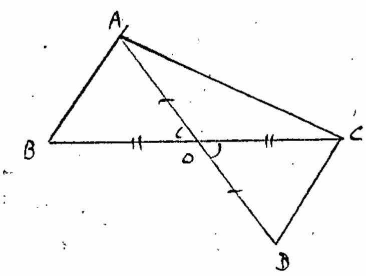
The answer is: Because △s BAC, EDC are congruent (AC = DC, L.BAC = L.EDC - - - rt. Ls., L.BCA = L.DCE - - - vert. opp. Ls. i.e. two Ls. and a corresp. side - Th. 10).

Try this method of finding the width of a river or of any distance where you cannot come. It may come in very useful to you some day.

This then is how we make use of the congruency of triangles. <u>If we want to know whether two lines are equal, we look for two congruent triangles which contain those lines.</u> Similarly if we want to know whether two angles are equal, we look for two triangles which contain them and compare the triangles to find whether they are congruent.

<u>TYPICAL EXAMPLE:</u>

O is the mid-point of the base BC of △ ABC. AO is joined and produced its own length to D. Prove that AB = CD.



<u>HINTS how to prove that AB CD:-</u>

Look for two triangles of which AB and CD are sides. Compare those triangles to see whether they are congruent. Mark all the angles and sides which are equal as shown in figure. The best way to write down this proof is as follows:

<u>PROOF:</u>

In triangles ABO, CDO
BO = OC .... given.
AO = OD .... given.
and incl. L.AOB incl. L.DOC .... vert. opp. Ls.

∴ △ AOB ≡ △ CDO
∴ AB = CD Q.E.D.

This is a very neat way to show that triangles are congruent. Always use this method of writing it down. <u>Note: a reason is given why each of the two pairs of sides are equal and why the included angles are equal.</u> It is <u>given</u> that BO = OC and that AO = OD. But L.AOB = L.DOC because they are vert. opp. Ls. (Th. 3).

<u>QUESTIONS:</u>

(In answering these questions draw neat and large figures, <u>and mark the angles and sides which are given equal.</u> Thus you get a clear picture of all your data in front of you and it is much easier to argue about it.)

(Do not take special cases to prove general facts e.g. if you are given <u>any</u> triangle about which you must prove something, do not take a rt. Ld. or isosceles or equilateral triangle as these are special types of triangle take a scalene triangle.)

1. Draw the following figures: mark equal angles and sides:
(a) AOB and COD <u>intersect</u> at O. Join AC, BD. Are the two triangles so formed congruent? Give a reason for your answer.

(b) AOB and COD bisect each other at O. Join AC, BD. Are the two triangles so formed congruent? Give reason. What about AC, BD? Are they equal? Are they parallel? Give reasons.

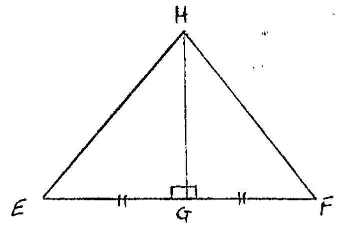
(c) AOB bisects COD. Join AC, BD. Are the triangles congruent?

(d) AOB is bisected by COD.

2. Fill in the missing parts in the following proof (No 4, pg. 64

Given: G is the mid-point of EF. At G, GH is drawn perp. to EF. HE, HF are joined.

Reqd. to prove: that HE = HF.



Proof: In $\triangle$s HGE, HGF
EG = ... (...)
HG = ...
and incl. L.HGE = ... (...)
$\therefore$ $\triangle$ HGE $\equiv$ $\triangle$HGF
$\therefore$ HE = HF

Q.E.D.

(Note: We can take any point, say S, on HG and prove that SE = SF. We can say that any point on HG, the perp. bisector of EF, is equidistant from E and F.)

3. ABCDE is a regular pentagon. Join AC and AD.
(i) Prove that AC = AD (Compare triangles ABC and AED)
(ii) Hoin EB and prove that AC = EB.
(iii) How many different diagonals such as AC, EB, etc. may be drawn in a pentagon?
(iv) Hence, what property about the diagonals of a regular pentagon can be deduced? Is this true of any pentagon?

4. Pg. 64, Nos. 3, 5, 6, 7, 8, 14.

The above questions must be done by everyone and handed in to me as soon as they have been completed.

EXTRA QUESTIONS:

The following may be done if, after having completed the above satisfactorily, you find the time to do so:

Pg. 64, Nos. 9, 10, 11, 12, 13.

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EIGHTH WEEK.

THEOREM 11 states:

If two sides of a triangle are equal, the angles opposite those sides are equal.

Note: The underlined portion is our given or hypothesis, the rest is what has to be proved. What is a triangle with two equal sides called? Hence what can we say instead of the portion underlined? Hence, another way of stating Theorem 11 is as follows:- The angles at the base of an isosceles triangle are equal.

How to prove theorem 11:- We want to prove two angles equal to one another, hence we look for two triangles which contain those angles. But, to our dismay, we see that there is only one triangle, namely, the given triangle. The only thing left to do is to make two triangles by means of a construction. We bisect L.BAC by drawing AD (see figure in book, pg. 65) and thus we get the two triangles ABD and ACD, which we can show to be congruent.

THEOREM 12 states:

If two angles of a triangle are equal the sides opposite those angles are equal.

Can we write this enunciation in a different way? Can we say: If two angles of a triangle are equal, then the triangle is isosceles?

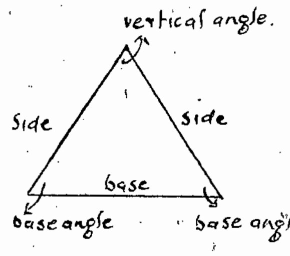
LEARN:

The enunciations of Ths. 11 and 12 and their corollaries. READ over their proofs and make sure that you understand them. It is essential that you should be able to prove these two theorems.

QUESTIONS:

1. Define an isosceles triangle.
2. What do we prove in Th. 11 about an isosceles triangle?
3. On which previous theorem is the proof of Th. 11 based?
4. On which previous theorem is the proof of Th. 12 based?
5. Pg. 67, Nos. 1, 2, 4, 5, 6, 7, 9, 10.

Note: In an isosceles triangle we talk of two sides and a base: also of the two base angles and of the vertical angle.



EXTRA QUESTIONS:

The above must be done by everyone

Those who could do them all, may also try the following: Pg. 67, Nos. 8, 11, 12, 13, 14.

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NINTH WEEK.

The following is a summary of the work you have to know for the coming quarterly examination:-

1. The enunciations of Theorems 1 - 13.
2. The proofs of Theorems 7, 8, 11, 12. The rest of the theorems are axiomatic, hence no proofs required.
3. Turn back through assignments and note important points which have been stressed.

The following are a few typical exercises and problems based on these theorems. Pg. 49 No. 5: Pg. 54 No. 7, 8: Pg. 57 Nos 1 (VIII) and (X), 2, 5: Pg. 61 No. 2, 3, 4: Pg. 64 No. 1, 4: Pg. 67 No. 4, 7: You have done most of these, but it will be a good thing to see whether you can do all of them.

This is an outline of how you should revise the work done so far. Those who have always done their assignments thoroughly, will barely find any revision necessary. You need not stick to the above for revision. Everyone knows his own weak points and should now make use of this opportunity to polish up the rusted parts.

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TENTH WEEK.

This week we shall complete the group of theorems on the congruency of triangles. (i.e. Ths. 9, 10, 13, 14.)

THEOREM 14: If two triangles have two sides of the one equal to two sides of the other, each to each, and the angles opposite to one pair of equal sides are right angles, the triangles are congruent.

This theorem is almost the same as Th. 9. In Th. 9 we had two sides and an included angle. In Th. 14 we have two sides and a right angle.

READ: over the proof of Th. 14 on pg. 68 and try to understand it

In doing the questions use the following way of comparing triangles. The proof of Th. 12, pg. 66, is taken as an example:-

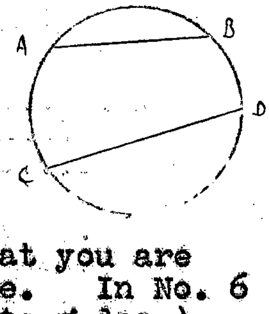
Proof: In the triangles ADB, ADC

because { L.ABD = L.ACD .... Given.
L.BAD = L.CAD .... having been bisected
AD = AD

∴ △ ADB ≡ △ ADC
∴ AB = AC Q.E.D.

The bracket groups ~~are~~ the three equal parts together which are the cause that the triangles are congruent. Instead of writing "because" use the abbreviation

QUESTIONS:



1. Pg. 69, No. 1. (Note: In figure AB, CD are chords of the circle. What is another name for the longest chord?)

2. Pg. 69, No. 2, 5, 6, 7, 8, 10, 12. (Note: In No. 2 look very carefully what you are given and what you are required to prove. In No. 6 note that there are two pairs of opposite sides.)

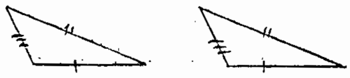
EXTRA QUESTIONS:

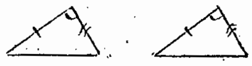
Those who found the above easy may also attempt the following:-

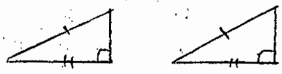
Pg. 69, Nos. 4, 11, 13, 14, 15.

SUMMARY OF CONGRUENCY THEOREMS.

Two triangles are congruent if the following parts of the one are respectively equal to the corresponding parts of the other:-

(1) 3 sides (Th. 13). 

(2) 2 sides and an included angle (Th. 9). 

(3) 2 sides and a right angle (Th. 14). 

(4) 1 side and 2 angles (Th. 10). 

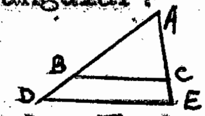
In all these cases three par ts of the one are respectively equal to the three corresponding part of the other. Now we may think:-

(A) Are equiangular triangles congruent? (i.e. triangles having 3 angles of the one respectively equal to 3 angles of the other). Note that in case (4) above we are really given 3 angles, because if we are given 2 Ls. of a triangle we can always calculate what the third one wi ll be.

(B) Why are triangles not congruent when two sides and <u>any</u> angle of the one are respectively equal to two sides and a corresponding angle of the other?

We shall now investigate these two cases.

(A) <u>THREE ANGLES</u>:- (a) Draw two triangles (i) with sides 3", $4\frac{1}{2}$", 5" and (ii) with sides $1\frac{1}{2}$", $2\frac{1}{4}$", $2\frac{1}{2}$". Measure their angles. Are they equiangular? Are they congruent?



(b) In the figure BC // DE. Are the triangles ABC, ADE equiangular? Why? Are they congruent?

<u>CONCLUSION</u>: Triangles which are equiangular have similar shapes but their sizes differ and hence they are not congruent.

(B) <u>TWO SIDES AND A NON-INCLUDED ANGLE</u>:

Construct a triangle ABC having L.A = 40°, a = 1", b = 1.4".
<u>Construction</u>:- Draw AX. At A make L.CAX = 40°. Make CA = 1.4". With centre C and radius 1" describe an arc of a circle to cut AX at B and B_1. Now ACB has L.A = 40°, b = 1.4", a = 1". Also ACB_1 has L.A = 40°, b = 1.4", a = 1". <u>READ</u> further pg. 71 and make quite sure that you understand everything. If you are unable to follow the explanation come to me.

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<u>ELEVENTH WEEK</u>.

Think again for a few moments what this subject <u>geometry</u> is about. What did we do during the past ten weeks? Think back and take a general view of the work we have done. What was our object in playing about with these lines, angles, triangles etc.? Try to answer this for yourselves before reading further.

Our object was to discover and prove different properties of lines, angles, triangles etc. in a systematic way, taking nothing for granted.

The most important properties we stated as theorems and arranged them in a definite order so that they follow each other logically, each new theorem depending on those going before.

These theorems are of the utmost importance as we use them to prove further properties of these lines, angles, triangles and other geometrical figures, and also relations between them. Our whole ability to solve problems or to prove further properties depends upon a thorough understanding of these theorems. Before continuing therefore, we shall give a summary of the theorems we have had so far:-

A. Theorems 1, 2, 3 about ANGLES ROUND A POINT.
B. Theorems 4, 5, 6 about PARALLEL LINES.
C. Theorems 7, 8 about ANGLES OF RECTILINEAL FIGURES.
D. (a) Theorems 9, 10, 13, 14 about CONGRUENCY OF TRIANGLES.
(b) Theorems 11, 12 about equal base-angles and equal sides of triangles.

Now if you want to prove something about parallel lines, you at once think of what was proved in theorems 4, 5, 6: also think about the definition of parallel lines, i.e., lines which point in the same direction.

There are FOUR sets of data for which triangles are congruent. (Name them for yourselves.) Each of them consists of 3 pieces of data i.e. 3 sides: 2 sides and incl. L.: 2 angles and 1 side: rt. L. and 2 sides. We also considered two cases when triangles are not congruent (i) 3 angles (ii) 2 sides and a non-included L. (ambiguous case).

NEW THEOREMS: Almost all the theorems so far were about equal lines, equal angles, equal triangles etc. The next few theorems fall under the heading: INEQUALITIES.

THEOREM 15: If two sides of a triangle are unequal the greater side has the greater angle opposite it.

READ: its proof on pg. 74. It is important. Note that a construction is made: from the longer of the two unequal sides a piece is cut off equal to the shorter.

QUESTIONS:

1. In △ABC, AB = 5", BC = 3", L.B = rt. L.
What can we say about the relative sizes of the angles?

2. Pg. 74, Nos. 1, 2, 3.

The above must be done by everyone.

EXTRA QUESTION:

Pg. 74, No. 4.

THEOREM 16: See pg. 75. It is the converse of Th. 15. Its proof is of a new kind, called PROOF BY EXHAUSTION In the book it is also sometimes called by the process of elimination. All the possibilities are taken, those which are impossible are eliminated and the remaining possibility is then accepted as correct.

QUESTIONS:

3. Pg. 75, Nos. 1, 2, 3.

4. Pg. 75, No. 4. Is it really necessary to prove that any two sides of a triangle are together greater than the third side? Does this not follow automatically from the definition of a straight line?

Note: Distinguish very carefully between the signs $>$ and $<$. Make them very neatly.

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TWELFTH WEEK.

REVISION of Theorems 15 and 16. Are you quite sure that you can distinguish clearly between these two theorems? To prove theorem 16 it is impossible to make a construction as in Th. 15, for there you will not know which is the longer side so that you can cut off a piece equal to the shorter side.

THEOREM 17 is based on Th. 16. READ its proof and make sure that you understand it.

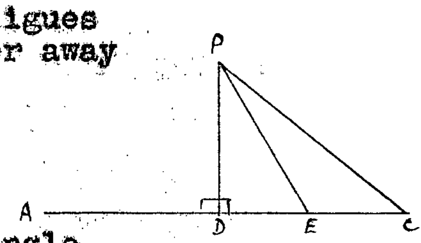
Note: 1. If we say a pt. is 2" away from a line, we mean that the perpendicular distance from the pt. to the line is 2".

2. What are obliques?

QUESTIONS: (Based on Theorems 15, 16, 17.)

1. Read over the following very carefully and supply the missing parts: (Important Exercise (1) at foot of pg. 76).

Given: $PD \perp AB$ and PE, PC are two obliques from P to AB, PC being further away from the perp. than PE.



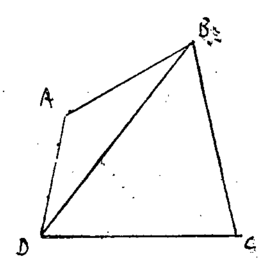
Reqd. to prove: $PE < PC$.

Proof: In $\triangle$PDE, L.PDE = ...
∴ L.PED is an .... angle.
∴ L.PEC is an ..... angle (L.PED + L.PED = 2 rt. Ls.)
∴ L.PCE is an ..... angle (a $\triangle$ can have only one ..... L.)
and ∴ L.PEC $>$ L.PCE
∴ $>$

Q.E.D.

2. Pg. 77: Nos. 1, 2.

3. Copy and complete: (No. 3, Pg. 77).
Given: Any quadrilateral ABCD.



Reqd. to Prove: that any 3 sides are together greater than the fourth side.

Construction: Join BD.

Proof: In$\triangle$ABD, AB + AD $>$ ..... (Any two sides of a are together greater than the third side.)
Also in$\triangle$BDC, BD + DC $>$ ....
Still more is ....$>$DC.
Similarly any other three sides may be proved greater than the fourth. Q.E.D.

4. Pg. 77: Nos. 4, 6, 12. In these questions you may make use of the fact that any two sides of a triangle are together greater than the third side.

5. Pg. 77: Nos. 8, 10.

EXTRA QUESTIONS:

Pg. 77: Nos., 5, 7, 9, 11, 13, 17. In No. 7 from the longer of the two sides cut off a piece equal to the shorter. That will be the difference between the two sides and you can then prove it less than the third side.

WARNING:

Take great care that your reasoning is sound from step to step. It never pays to make any statement or write down a step if you have no reason for it. The extra questions or riders, as they are called, are a little harder than the others. Attempt them if you find time. Some good hard thinking is a good thing. The more you have tried, the greater will be your satisfaction if you get them right.

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THIRTEENTH WEEK.

We have been so busy with the "theoretical geometry" that we might have forgotten completely how to work with our instruments. For this reason we shall spend the following two weeks in constructing triangles from given data.

QUESTIONS:

1. Make a table as below and supply the answers to the foll. questions:-

A. Is it possible to construct triangles from the following sets of data?

B. How many possible triangles will there be in each case? (In other words, will the whole class get congruent triangles? if each one constructs his triangle from the data, or will there be two or more shapes or sizes of triangles?) Give reasons.

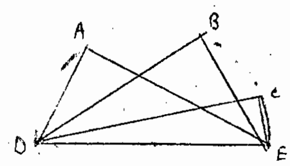
	Data for constructing ABC	A.	B.
1.	a = 2", b = 3", c = 4".		
2.	a = 2", b = 4".		
3.	a = 2", b = 4", L.C = 30°.		
4.	L.A = 30°, L.B = 40°.		
5.	L.A. = 45°, L.B = 40°, c = 3".		
6.	L.A = 90°, a = 2", b = 3".		
7.	L.B = 40°, c = 2", b = 1.9".		
8.	a = 5", b = 1", c = 3.1".		
9.	L.A = 50°, L.B = 70°, L.C = 70°.		
10.	L.C = 41°, b = 3", c = 3.1"		

It is clear by now that the same sets of data ~~xxxx~~ required for triangles to be congruent, are also required for constructing a triangle uniquely i.e. so that if the whole class construct the triangle, each one will get a triangle, which is exactly equal in shape and size to those of the others in the class. READ pg. 78.

PROBLEM A pg. 79 shows how to construct a rt. Ld. triangle having given the hypotenuse and one side.
Note: a proof is given that the construction gives the required triangle.

If you do not follow the construction or proof, ask one of your friends to explain it to you or come to me. But do not continue before it is quite clear to you.

THEOREM 40 pg. 80, will be easy to understand, for its proof is the same as that of problem A. You can verify this theorem experimentally by drawing any semi-circle as shown in figure, taking points A, B, C etc. on the circumference and then measuring the angles at A, B, C, etc. to find whether they are rt. angles.



This is an important theorem, as you see in the book, and you must understand it well before you continue.

READ the alternative construction for Problem A on pg. 80.

QUESTIONS:

2. Pg. 83, No. 1 (i), (ii), (iii), (iv), (vi).
Read very carefully the instructions given at top of pg. 83. Very neat and accurate constructions are required, and in each case give under the headings 1) construction and 2) by measurement a short description of how you constructed the triangle and the length of the line or size of angle measured, respectively.

PROBLEM B: Shows how to construct an equilateral triangle when the altitude is given. You may think that this is impossible as you are not given sufficient data: but the properties of an equilateral triangle make up for this.

In the proof which you are required to supply, you have to prove that the triangle which has been constructed, is an equilateral triangle with the altitude equal to the given altitude. Supply the proof and show it to me before you continue.

PROBLEM C: Supply proof in same way as for problem B.

QUESTION:

3. pg. 83, Nos. 1 (v), (vii), (ix).

EXTRA QUESTIONS:

Pg. 83, No 1, (x), (xi), (xii).

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FOURTEENTH WEEK.

PROBLEM D: Read the whole of pg. 82 and supply the proof. This problem is very important.

Note: The new heading "Analysis" is not to be given in any answer. It is simply how you think and argue about the construction before you start with it.

In the following questions state the steps of your construction clearly in each case. Use the short clear mathematical way of doing it: see the constructions of all the problems you have had so far, for examples. Accuracy can only be attained by using straight rulers and pencils with sharp points.

QUESTIONS:

Pg. 83, Nos. 2 - 11.

EXTRA QUESTIONS:

Pg. 84, Nos. 12, 13, 14.

READ: on pg. 84 and 85 the definitions that are given, you must know what each of the following is:- Quadrilateral, diagonal, parallelogram, rectangle, square, rhombus, trapezium, convex polygon.

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FIFTEENTH WEEK.

In theorems 18, 19, 20, 21 we prove certain properties of PARALLELOGRAMS.

THEOREM 18: The opposite angles of a parallelogram are equal.

THEOREM 19: The opposite sides of a parallelogram are equal, and each diagonal bisects the parallelogram.

THEOREM 20: The diagonals of a parallelogram bisect each other.

These theorems are important and you must understand their proofs thoroughly. For practice rewrite the proofs with closed books, using the "three-letter" way of angle notation.

READ: the converse of Th. 20 on pg. 88, and the corollaries at the foot of pp. 86, 87.

QUESTIONS:

1. On which previous theorems does each of Ths. 18, 19, and 20 depend?

2. Pg. 89: Nos. 1, 6, 11. (These are all proved by congruent triangles).

3. Pg. 89: Nos. 4, 10, 12.

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SIXTEENTH WEEK.

THEOREM 21: If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram. READ ~~the~~ and learn its proof.

Is this theorem the converse of any of the previous theorems?

This theorem concludes the group of theorems on parallelograms, viz. 18, 19, 20, 21.

QUESTIONS:

1. Pg. 89: Nos. 2, 3, 7, 8, 13, 14, 16, 17.

EXTRA QUESTIONS:

Pg. 89: No. 15.

The following are a few general questions based on the theorems we have done so far. Revise the theorems and then attempt them:

1. A hexagon has one angle of 110 and two others each of 135°. Of the remaining angles, one is 13 greater than each of the other two, which are equal. Find the size of each of the last three Ls.

2. ABC is a triangle. CA is produced to D and AB to E, so that AD = AE. If the line joining DE is perp. to CB produced, show that ABC is isosceles. (HINT: see whether the two rt. Ld. triangles in the figure are equiangular.)

3. ABC is a triangle with three unequal sides. CD, the perp. from C to the bisector of L.A, is produced to cut AB, produced if necessary at E. Show that EB is equal to the diff. in length of AB and AC.

4. In ABC, L.C 90 and D is any pt. in the side AC. Prove that BD is greater than BC but less than BA.

For revision you may also attempt any of the questions in your book, also those not given so far. Come and show them to me so that you can be sure whether they are correct.

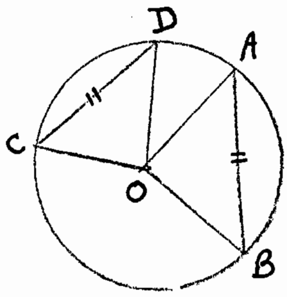
\- - - - - - - -

EXAMPLE OF AN ASSIGNMENT AS WORKED OUT IN ORDER TO SERVE AS A REFERENCE IN CORRECTING PUPILS' WORK.

(See p. 35)

GEOMETRY: TENTH WEEK.

Question 1 (pg. 69).



Given: AB, CD two equal chords of a circle with centre O.

Required to prove: L.AOB = L.COD.

Proof: In Δs AOB, COD

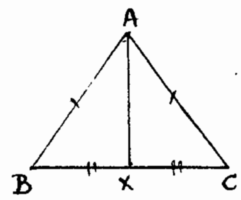
$\because$ { AB = CD .... given.
AO = DO .... Equal radii.
BO = CO .... equal radii. }

$\therefore \Delta AOB \equiv \Delta COD$

$\therefore$ L.AOB = L.DOC. Q.E.D.

Question 2 (pg. 69).

Given: An isosceles Δ ABC having AB = AC. Also X, the mid-pt of BC, joined to A.



Required to prove: (i) L.BAX = L.CAX.
(ii) AX $\perp$ BC.

Proof: In Δs ABX, ACX

$\because$ { AB = AC .... given.
BX = CX .... given.
AX = AX }

$\therefore \Delta ABX \equiv \Delta ACX$

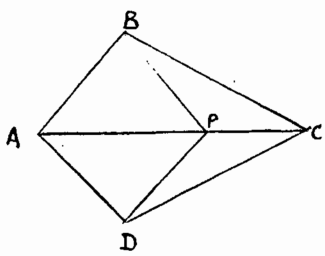
$\therefore$ L.BAX = L.CAX

and L.AXB = L.AXC

and being adjacent supplementary Ls., each is a rt. L.

$\therefore$ AX $\perp$ BC. Q.E.D.

Question 5 (pg. 69).



Given: A quadr. ABCD. having AB = AD and BC = CD.
P is any pt. on AC.

Required to prove: PB = PD

Proof: In $\triangle$s ABC, ADC

$\because$ {AB = AD .... given.
BC = CD .... given.
AC = AC

$\therefore$ $\triangle$ABC $\equiv$ $\triangle$ADC
$\therefore$ L.BAC = L.DAC.

In $\triangle$s BAP, DAP

$\because$ {AB = AD .... given.
AP = AP
Incl. L.BAP = incl. L.DAP .... proved above.

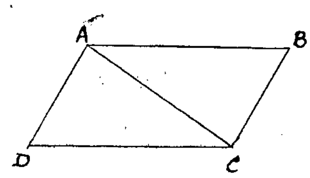
$\therefore$ $\triangle$BAP $\equiv$ $\triangle$DAP
$\therefore$ BP = PD.

Similarly any other pt. on AC can be proved equidistant from B and D.

Q.E.D.

Question 6, p. 69.

Given: Quadr. ABCD having AB = DC and AD = BC.



Required to prove: AB // DC and AD // BC.

Construction: Join AC.

Proof: In ADC, ABC

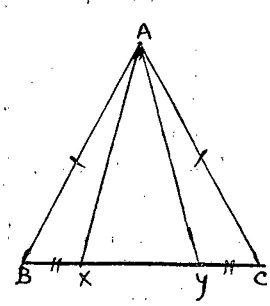
$\because$ {AD = BC .... given.
DC = AB .... given.
AC = AC

$\therefore$ $\triangle$ADC $\equiv$ $\triangle$ABC
$\therefore$ L.DAC = L.ACB and being alternate Ls.,
$\therefore$ AD // BC
also L.DCA = L.CAB and being alternate Ls.,
$\therefore$ AB // DC.

Q.E.D.

Question 7, p. 69.

Given: An isosceles $\triangle$ABC having AB = AC.
X, Y are pts in BC such that BX = CY.



Reqd. to prove: AX = AY.

Proof: AB = AC .... given.
$\therefore$ L.ABC = L.ACB

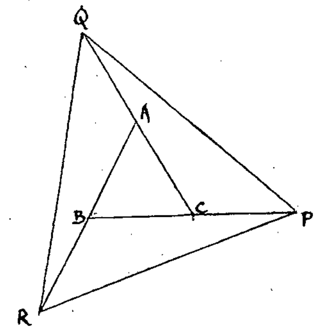
In $\triangle$s ABX, ACY

$\because$ {AB = AC .... given.
BX = CY .... given.
incl.L.ABX = incl.L.ACY ..... proved above.

$\therefore$ $\triangle$ABX $\equiv$ $\triangle$ACY
$\therefore$ AX = AY.

Q.E.D.

Question 8, p.69.



Given: An equilateral ΔABC. Its sides are produced to P, Q, R. resp., each its own length.

Reqd. to prove: ΔPQR is also equilateral.

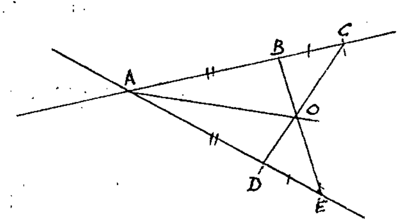
Proof: AB = AC .... given.
∴ L.ABC = L.ACB
∴ L.RBC = L.QCP ..... supplements of the equal Ls. ABC, ACB.

Now in Δs QCP, RBP
∵ { QC = BP ...... each is twice the side of the equil. Δ
CP = RB ...... each is equal to a side of the equil. Δ
incl. L. QCP = incl. L. RBP ...... proved above.
∴ $\Delta QCP \equiv \Delta RBP$
∴ QP = RP

Similarly by comparing Δs QRA, RBP we prove RP = QR.

∴ QP = RP = QP.
∴ Δ QRP is equilateral. Q.E.D.

Question 10(a), p. 69.



Given: Two intersecting lines ABC, ADE having AB = AD, AC = AE. Also BE, CD cut each other at O.

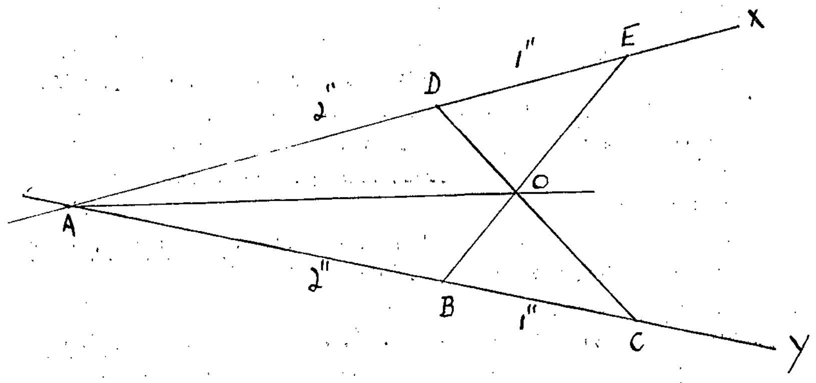
Reqd. to prove: AO bisects L.A.

Proof: In Δs ACD and AEB
∵ { AC = AE ..... each is equal to the sum of two equal lines.
AD = AB ...... given.
incl. L. CAD = incl. L. CAD.
∴ Δ ACD $\equiv \Delta$AEB.
∴ L.ACD = L.AED.

In Δs BOC, DOE
∵ { L.BOC = L.DOE ..... vert. opp. Ls.
L.BCO = L.DEO ..... proved above.
BC = DE ..... given.
∴ Δ BOC $\equiv \Delta$ DOE
∴ BO = DO.

In Δs ABO, ADO
∵ { AB = AD .... given.
BO = DO .... proved above.
AO = AO
∴ $\Delta ABO \equiv \Delta ADO$
∴ L. BAO = L. DAO
∴ ao bisects L.BAD. Q.E.D.

Question 10(b), p.69.



To show how any ∠A can be bisected using only a ruler graduated in inches.

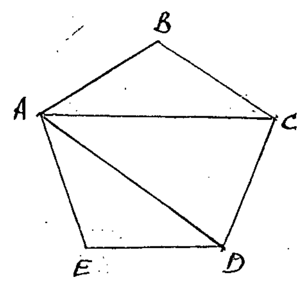
Construction: Along AX mark off AD = 2", DE = 1". also along AY mark off AB = 2", BC = 1". Join BE, CD and let them intersect at O. Join AO.

Proof: AO bisects ∠A (See proof of 10(a)).

Check: By measurement with protractor:

$$\angle DAO = 15.3^\circ$$
$$\angle BAO = 15.4^\circ$$

Question 12, p. 69.



Given: ABCDE is a regular pentagon.

Reqq. to prove: ACD is an isosceles △.

Proof: In △s AED, ABC

∵ AE = AB ..... given.
ED = BC ..... given.
incl. ∠AED = incl. ∠ABC ... given.
∴ △AED ≡ △ABC
∴ AD = AC
∴ ADC is an isosceles △.

Q.E.D.

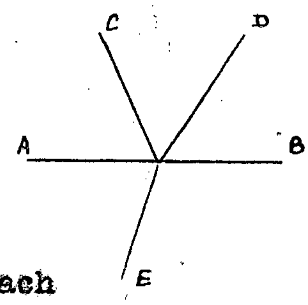
APPENDIX B.

QUARTERLY TESTS.

FIRST TERM.

March, 1934. Mathematics - Std. VII. Time: 1½ hrs.

1. In the figure AB is a straight line.



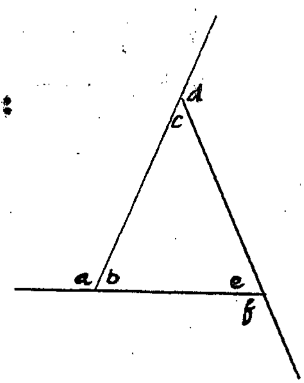
(a) Write one angle for:

(i) L.DOB + L.DOC =
(ii) L.DOE – L.BOE =
(iii) L.COB + L.COE =

(b) Equal to how many rt. Ls. are each of the following:

(i) L.DOB + L.DOC + L.COA =
(ii) L.EOA + L.EOB =
(iii) L.COB + L.COE + L.EOB =

2. Write one angle or the number of rt. angles for each of the following:



(i) b + c + e =
(ii) c + e =
(iii) c + d =
(iv) a + d + f =
(v) b + e =

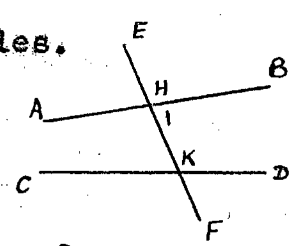
3. Complete the following:

(i) The corresponding angle of L.EHA is ....

(ii) Ls. AHK and HKD are ..... angles.

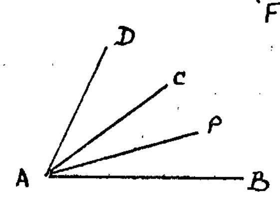
(iii) Ls. EHB and AHK are ...angles.

(iv) Ls. FKD and KHB are ...angles.



(v) Ls. BHK and HKD are two ...... angles on the same side of EF.

4. In the figure PA bisects L.CAB. L.DAB = x° and L.DAC = y°



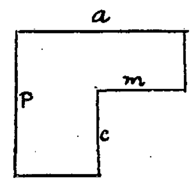
Express in degrees:

(i) L.BAC (ii) L.BAP (iii) L.PAD.

5. Remove the brackets in the following:

(i) $a(3a + b)$ (ii) $3x(y + z)$
(iii) $a + b(c + d)$ (iv) $5a(b - 2c)$
(v) ~~$5a(b - 2c)$~~ $3a^2(b^2 + a)$

6. Find the area and the perimeter of the given figure. The measurements are in feet.



7. Draw an angle equal to 64° with your protractor. Bisect it with your compasses and give a short description of how you did it.

8. The angle of elevation of the church spire, 200 yds. from its foot, is 42°. Find the height of the spire by making an accurate drawing to scale.

<u>Marks were assigned as follows:-</u>

1. Two marks to each of the six divisions.

TOTAL 12.

2. Two marks to each of the five divisions.

TOTAL 10.

3. Two marks to each of the five divisions.

TOTAL 10.

4. Four marks to each of the three divisions.

TOTAL 12.

5. Two marks to each of the five divisions.

TOTAL 10.

6. Five marks for perimeter and area each.

TOTAL 10.

7. Three marks for drawing angle correctly. Six marks for bisecting. Six marks for description.

TOTAL 15.

8. Two marks for scale. Two marks for stating answer clearly. Eight marks for correct drawing. Nine marks for accuracy.

TOTAL 21.

<u>GRAND TOTAL 100.</u>

SECOND TERM.

June, 1934. Algebra - Std. VII. Time: $1\frac{1}{2}$ hrs.

1. (a) Find the value of $\frac{5xy - 3yz}{2zx}$ when $x = 4$, $y = 3$, $z = 5$.

   (b) If $a = 1$, $b = \frac{1}{2}$, $c = 2$, find the value of:

   (i) $a^2 + b^2$ (ii) $(a + b + c)^2$
   (iii) $a(a + b) + ac$ (iv) $(a+b)^2 - (c^2 - a^2) + c^2$

   (v) $\sqrt{\frac{5(a^2+b^2)}{3abc}}$

2. Solve the following equations:

   (i) $\frac{n}{4} = \frac{3}{8}$ (ii) $\frac{2}{3}(x+3) = 6$

   (iii) $\frac{5}{n} = \frac{6}{7}$ (iv) $\frac{3}{5}y - \frac{y}{2} = \frac{1}{2}$

   (v) $1 + \frac{4}{7}a - \frac{2}{3}a = 4 + \frac{4}{3}a$

   (vi) $\frac{t+3}{5} = \frac{t-2}{3}$
   Verify equation (vi)

3. (a) Divide £98 between A and B so that nine times A's share may equal fine times B's share.

   (b) Find a number so that the difference between one-fifth and one-sixth of it equals nine.

   (c) A silver collection was composed of shillings and sixpenses only. There were twelve more sixpenses than shillings. If the collection amounted to £3. 9. 0., how many coins of each sort were there?

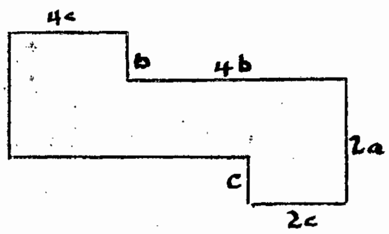
4. Simplify:

   (i) $\frac{3N+5N}{6}$ (ii) $3b+2+b$

   (iii) $2k+\frac{k}{2}$ (iv) $aba+bab+baa$

   (v) $c^2-1-c-1$ (vi) $5t+txs-4xt\div 2$.

5. Find the perimeter and area of the given figure. The angles are all rt. Ls. The measurements are given in feet.



Marks were assigned as follows:-

1. (a) 3.
   (b) (i) 1. (ii) 1. (iii) 2. (iv) 2. (v) 3

   TOTAL 12.

2. (i) 1. (ii) 1. (iii) 1. (iv) 2. (v) 2.
   (vi) 2. Verification 1.

   TOTAL 10.

3. (a) 5. (b) 5. (c) 5.

TOTAL 15.

4. One mark to each of the six sections.

TOTAL 6.

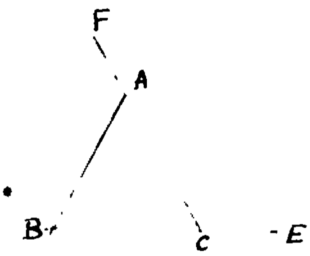
5. Three marks for perimeter. Four marks for area.

TOTAL 7.

GRAND TOTAL 50.

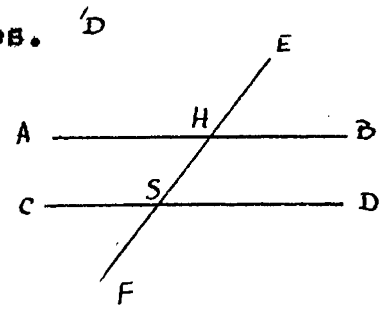
June, 1934. Geometry - Std. VII. Time: 1½ hrs.

1. Complete the following statements:



(a) L.ACE + ...= 2 rt. Ls.
(b) L.ACE = L.ABC + ...
(c) L.ABC + L.BAC + ... = 2 rt. Ls.
(d) L.FAB + L.DBC + L.ACE = ...

2. AB, CD are two parallel straight lines. Answer or complete the following:



(a) What is the line EF called?
(b) L.EHA = ... (vert. opp.L.)
(c) L.HSC = ... (corresp. L.)
(d) L.AHS = ... (altern. L.)
(e) By what common name can we call Ls. AHE, EHB, SFD, CSF ?

3. What do we mean by saying that triangles are congruent? Name three sets of conditions for which triangles are congruent.

4. Draw any angle and show how to bisect it using your compasses and ruler only. Explain how you have done it under the headings: given, required and construction.

5. On the bank of a river there is a tree 35 ft. high. From a point on the other bank and directly opposite the tree the angle of elevation of the top of the tree is 22°. Find the width of the river by making an accurate drawing to scale.

6. A village B is situated 70 miles due East from another village A. A third village C is 60 miles from B and in the direction 20° W. of S. How long will a person take to travel directly from C to A at 20 miles per hour?

Marks were assigned as follows:

1. One mark to each of the four divisions.

   TOTAL 4.

2. One mark to each of the five divisions.

   TOTAL 5.

3. Two marks for definition. Two marks to each of the 3 sets.

   TOTAL 8.

4. Correct figure and construction lines: 4 marks. Description: 6 marks.

   TOTAL 10.

5. Scale: 1 mark, correct statement of an swer: 1 mark, correct figure: 3 marks, accuracy and answer: 5 marks.

   TOTAL 10.

6. Scale: 1 mark, figure: 5marks, answer: 5 marks, calculation: 2 marks.

   TOTAL 13.

GRAND TOTAL 50.

THIRD TERM.

September, 1934. Algebra - Std. VII. Time: 1½ hrs.

1. Solve for x: $\frac{9-x}{10} + \frac{7-x}{15} = \frac{1}{30}$

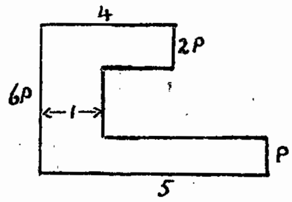
2. Simplify:

(a) $\frac{(3xy)^2}{3xy^2}$ (b) $\frac{b}{c} - \frac{c}{d} + \frac{d}{b}$

(c) $\frac{2ab}{15c^2} \times \frac{5bc}{4a^2}$

3. I buy 20 pencils for 3/-: some cost 1½d each, the rest 2½d each. How many of the cheaper kind did I buy?

4. Find the area and the perimeter of the given figure. The measurements are in inches. The corners are all rt. Ls.



5. From the ages 5 to 14 years Joan Brown's height was measured (in inches) on each birthday, and the results were as follows:

AGE	5	6	7	8	9	10	11	12	13	14
HEIGHT (in inches)	41	42	46	49	52	53	55	57	59	61

Draw a graph to illustrate this. From your graph answer the following questions:-

(a) State clearly which three years show the greatest increase in growth.

(b) What was her height when she was 6½ years of age?

6. If you have to draw the following graphs, which quantity would you measure along the horizontal axis:-

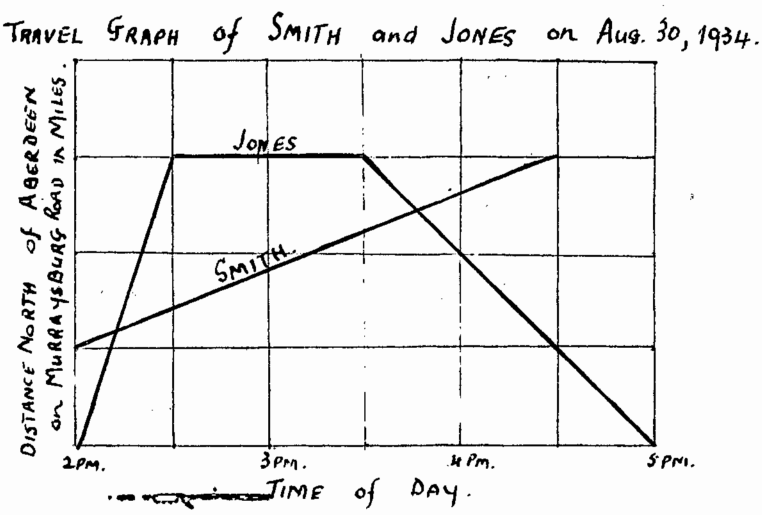
(i) A graph to convert degrees Fahrenheit to degrees Centigrade.

(ii) The amount of stretch of an india-rubber cord and the loads suspended from it.

(iii) The weight of a motor-car and the tax on it.

(iv) Record times for races of various lengths.

7. Interpret the following travel graphs of Smith and Jones in detail (i.e. explain what the graphs tell us about their travels on the Aberdeen)Murraysburg road on the 30th Aug., 1934). When and where do Smith and Jones pass one another?



Marks were assigned as follows:

1. Two marks for each correct step. (5 steps).

TOTAL 10.

2. (a) 5. (3 for removing bracket).
(b) 5.
(c) 5.

TOTAL 15.

3. Eight marks for correct equation. Six marks for solving. One mark for answer.

TOTAL 15.

4. Five marks for area and perimeter each.

TOTAL 10.

5. Name 2, scale 3, axes 3, plotting points 10, graph 4.
(a) 3 (b) 3.

TOTAL 28.

6. Two marks to each.

TOTAL 8.

7. Jones: 3 stages - time and distance for each: 6 marks.
Smith: time and distance: 2 marks.
Time and place of passing each other: 3 marks each.

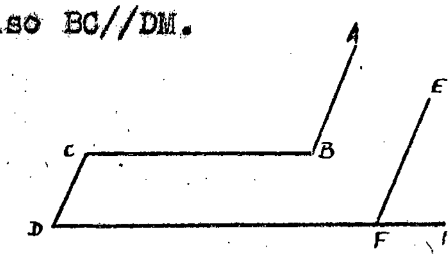
TOTAL 14.

GRAND TOTAL 100.

September, 1934. Geometry - Std. VII. Time: 1½ hrs.

1. (a) If a line cuts across two other parallel lines, what conclusions can you draw with regard to the angles which are so formed (Name 3 conclusions: they must be the result of the lines being parallel).

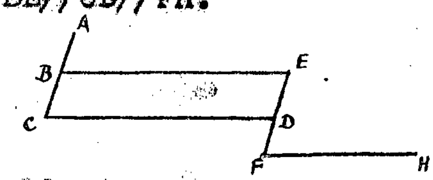
(b) In the figure AB//CD//EF. Also BC//DM.



Complete and give reasons:

L.ABC = ...
L.EFM = ...
L.CDF + ... = ...

(c) In the figure AC//EF and also BE//CD//FH.



Prove that (i) L.ABE = L.EFH
(ii) L.BCD = L.BED

2. State the converse of each of the following theorems:-

(a) If a straight line stands on another straight line, the sum of the adjacent angles so formed is equal to 2 rt. Ls.

(b) When a straight line cuts two other straight lines, those lines are parallel if

(i) a pair of corresponding angles are equal.
(ii) a pair of alternate angles are equal.
(iii) a pair of interior angles on the same side of the cutting line are supplementary.

(c) If two sides of a triangle are equal, the angles opposite those sides are equal.

3. (a) Prove that the sum of the angles of a triangle is equal to two right angles.

(b) Find the sizes of the angles of a triangle from the following information:

(i) L.B = 30°: rt. Ld. at C.
(ii) Ext. L. at A = 113°: L.B – L.C = 15°.

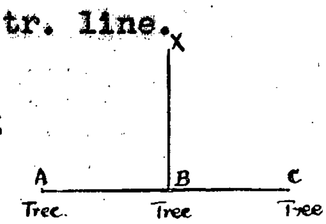
(c) In $\triangle$ PQR, L.Q = L.R and PS is perp. to QR. Prove that L.QPS = L.RPS.

4. (a) Name three sets of data which make triangles congruent.

(b) DEF is any triangle with H the mid-point of EF. DH is joined and produced to S so that DH = HS. SF is joined. Prove that L.DEH = L.SFH.

(c) As a result of your proof in (b), what can you say about the lines DE and SF? (Two properties).

(d) Three trees A, B and C stand in a str. line. I start walking from the middle tree B, and walk along BX perp. to AC. Prove that at any point along my path BX, I am equidistant from the two trees A and C.



<u>Marks were assigned as follows:</u>

1. (a) One mark for each of the three conclusions.

   (b) Two marks for each: one for completing it and one for the reason.

   (c) (i) Five marks: 2 and 3, or 1, 2 and 2 for the different steps according to method of proof used.

   (ii) Five marks: 2 and 3, or 2, 1, 1, 1,.

   TOTAL 19.

2. (a) 2 (b) 3 (c) 2.

   TOTAL 7.

3. (a) 2 for construction, 1 for each logical step. Maximum 9.

   ~~(b) 2 for construction, 1 for ea~~

   (b) (i) 2 (ii) 7.

   (c) 8 marks: 2 marks subtracted if reasons for equalities are not given.

   TOTAL 26.

4. (a) 3 for each set of data and maximum 9.

   (b) 15 marks: 2 marks for correct figure and statement, 13 for proof, 2 marks subtracted for omitting to give reasons.

   (c) 2 marks for each property.

   (d) 20 marks.

   TOTAL 48.

<u>GRAND TOTAL 100.</u>

<u>NOTE:</u> To get mathematics score for third term the algebra and geometry marks were added and then divided by two in order to get a percentage mark.

FOURTH TERM.

December, 1934. **Algebra - Std. VII.** **Time: 1½ hrs.**

1. Simplify: (a) $3(x+3)-(5-x)-2(\frac{x}{2}-6)$

   (b) $(3x+2)(5x-6)$

   (c) $x-6\left[x-4\{3x-(2x-\overline{1+x})\}\right]$

2. Solve: (a) $\frac{y}{3}-\frac{y}{4}=6$

   (b) $\frac{3}{2a}-\frac{2}{3a}=1.2$

   (c) $\frac{a+2}{5}-\frac{a-1}{3}=6$

   (d) $\frac{2}{3}(y+5)=y-\frac{1}{5}(y-10)$

   (e) $15=-4p$

3. John is four times as old as William. Twenty years hence he will be twice as old as William. Find their present ages.

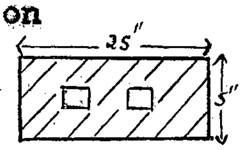
4. (a) Subtract $2a-b-2c$ from $-5b-9c$.

   (b) Divide $4p^3-6p^2y-2py^2$ by $-2p$.

   (c) Simplify: $\frac{cd}{ce}\div de$

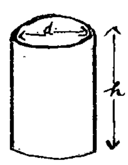
   (d) Simplify: $\frac{z}{3yz}+\frac{x}{6xy}$

5. (a) (i) Find the area of the striped portion of the figure. Two squares of side l inches are cut out.



   (ii) What is its area if $s=3$, $1=\frac{1}{2}$

   (b) (i) Find the area of the curved surface of a cylinder, with diameter d inches and height h inches.



   (ii) What is its area if $d=4$, $h=21$?

6. Draw a graph to show the connection between circumferences and corresponding diameters of different circles from the following data:

Diameter	2	5	7	8	10
Circumference	6.3	15.7	22	25.1	31.8

Find approximately from your graph:

(a) The circumference of a circle 9.5 inches in diameter.
(b) The diameter of a circle which has a circumference of 18.9 inches.

Marks were assigned as follows:

1. (a) 2 (b) 1 (c) 3. Give $\frac{1}{2}$ a mark for each step.

TOTAL 6.

2. (a) 2 (b) 2 (c) 3 (d) 2 (e) 1.

TOTAL 10.

3. 4 for equation, 3 for solution.

TOTAL 7.

4. (a) 2 (b) 2 (c) 1 (d) 2

TOTAL 7.

5. (a) (i) 2 (ii) 1 (b) (i) 2 (ii) 1.

TOTAL 6.

6. Graph 10. Readings 2 each.

TOTAL 14.

GRAND TOTAL 50.

December, 1934. Geometry - Std. VII. Time: $1\frac{1}{2}$ hrs.

1. (a) Prove that the sum of the angles of a triangle is equal to two right angles.

   (b) ABC is an isosceles triangle having AB = AC. BA is produced to D and L.DAC = 130°. Calculate the sizes of the interior angles of the △ ABC.

   (c) ACB is any straight line drawn through the vertex C of △CDE. Prove that

   L.ACD + L.BCE = L.CDE + L.CED.

2. (a) Will triangles BCD, EFH be congruent if

   (i) L.B = L.F, L.C = L.H, DC = EH.
   (ii) L.B = L.E, BC = EF, DB = EH.
   (iii) L.B = L.H, L.C = L.E, L.D = L.F.

   Give reasons in each case.

   (b) AB, DC are perpendicular to BC, making AC = BD. Prove that AB = DC.

   (c) ABC is a scalene triangle. From C, CD is drawn perpendicular to the line bisecting L.A. CD is produced to cut AB (AB produced if necessary) at E. Prove that EB is equal to the difference in length of AB and AC.

3. (a) Construct an isosceles △ ABC having its base BC = 2.3 inches and L.A = 71°. Give a short description of your construction. Measure AC.

(b) Construct a quadrilateral ABCD having AD = 1.9", BC = 1.6", L.A = 81°, L.C = 90° and the diagonal BD = 2.6". Measure AB and CD. Give a short description of your construction.

4. (a) If two angles of a triangle are unequal, the greater angle has the greater side opposd te to it.

(b) △ DEF has an obtuse angle at F. If P is any point in the base EF, show that DE > DP > DF.

Marks were assigned as follows:

1. (a) 4 (b) 1 (c) 6.

TOTAL 11.

2. (a) One mark for each case together with its reason. Max. 3.

(b) 4.

(c) 6.

TOTAL 13.

3. (a) 2 for construction, 1 for description, 1 for measurement. Maximum 4.

(b) 6 for construction, 1 for description, 1 for measurement. Maximum 8.

TOTAL 12.

4. (a) 5 (b) 9.

TOTAL 14.

GRAND TOTAL 50.

APPENDIX C.

CASE STUDIES OF THE NINETEEN PUPILS WITH WHOM THE FIRST PART OF THE INVESTIGATION WAS CARRIED OUT.

The following are summaries of the data collected in connection with the separate pupils from sources such as interviews, consultations with teachers and people familiar with local conditions, visiting their homes, observation in class, and school-records. They do not claim to be full case studies; only the main facts are touched on in the following order.

1. Family history and home circumstances.
2. Health, physique, temperament, participation in sport.
3. School record, quality of schoolwork, behaviour in class.
4. General: incentive, special abilities, peculiarities. Resumé.

Note: i) "Normal school-record" means normal age of school entrance, passed all standards without failing, attended only one school.

ii) "Good home circumstances" means good according to the local standard.

Pupil A: Girl. I.Q. 104. Age 14.10.

1. Her father had been a transport-driver, one brother a policeman, others still at school (6 children). Very poor, not always enough food in house. Neat house. Pupil A has several domestic duties.

2. Healthy but a little anaemic; head-ache at times: had enteric fever, no bad after-effects. Very neat in habits and appearance. Well-behaved, quiet type of girl: teachers think very highly of her. Plays basketball.

3. Normal school-record. Schoolwork very satisfactory: very hard worker, conscientious. Known as the neatest girl in her class. Achievement higher than intelligence. Assignment work very satisfactory, completed in time, no careless mistakes.

4. Wants to become a teacher (training college). No outstanding abilities. Very reliable. Gives her best in school - a credit to any school. Her success under individual method probably due to hard work, neatness, systematic habits and a strong desire to please others.

Pupil B: Boy. I.Q. 99. Age 16.9

1. Father is a farmer (landowner): uneducated (no post-primary education): very ambitious with regard to his three children and takes great interest in them. Home circumstances fair.

2. Excellent health, never been ill except with measles. Physically well-developed, tall, healthy appearance. Plays rugby.

3. Entered school late - at age of nine. Attended farm school till Std. I. Otherwise normal. Quality of schoolwork above class average. His best subjects are commercial arithmetic and mathematics. According to Std. VI teacher he was outstanding in arithmetic. Behaves well in school.

4. He wants to enter post-office, has already made enquiries as regards qualifications for entrance. He is old for his class, but seems to be fairly ambitious: his brother and sister left school at Std. VI, he wants to continue for a few years longer. His assignment work satisfactory, always handed in in time. His relative success on the individual method probably due to fact that he found it easy to follow the explanations in assignment and text-book, being good in mathematics.

Pupil C: Boy. Age 14.8. I.Q. probably between 105 and 110.

1. Father is a pensioned soldier from Anglo-Boer War - by profession a soldier: intellectually undeveloped. Mother backward as regards mental qualities. Financially poor, but standard of living not so very low.

2. Health excellent. Lively in appearance. Very good-natured. Very good rugby player. Popular.

3. Normal school-record. Schoolwork on the whole satisfactory (about 46%). Assignment work satisfactory, worked together with pupil J.

4. Wants to work in an office like his brother. Seems to take life very easily. Popular in his class and in school. It is difficult to say why the individual method suited him relatively better than some of the others who came below him.

Pupil D: Girl. I.Q. 121. Age 14.6

1. Father has shop in district, also owns land, poor type of personality but fairly developed. He manages to get the State to pay for the education of his children. Four children. Mother has a very good character: educates children very thoroughly according to her ideals at home: pupil D has to make her own clothes. Home conditions good. Pupil D boards in the village with private people.

2. Health excellent. Well-built, walks very erect. Very neat in appearance, clothes and habits. Responds quickly. Plays basketball for first team (school has six teams.)

3. Attended farm school (19 pupils) up to Std. VI: only two in her class. Very neat in her work. Quality of schoolwork above average - good all-rounder. Attentive in class - sets the standard.

4. She wants to become a nurse after having obtained her Senior Certificate. Artistic: draws well and does very pretty needlework. Takes music lessons. She is a leader type of girl. As regards character, manners, neatness, she is a typical product of her mother. In spite of numerous other activities, she does not neglect her schoolwork. It could have been expected that she would do well under the individual work.

<u>Pupil E</u>: Girl. I.Q. 109. Age 15.10.

1. Father is a farmer: tries to give the impression that he is poorer than he actually is. Financially not so well-off but standard of living fairly high (like that of pupil J). Her mother was a teacher in a farmschool. She has one brother who attended an industrial school and is now in special service battalion. Her identical twin sister also in Std. VII - does not take maths. Both are of the best in the class. Pupil E boards in village with well-off people - conditions for study ideal.

2. Excellent health: well-built, good-looking Good-natured. Plays tennis and basketball.

3. Normal school record. Quality of school work superior. Good all-rounder. Exemplary conduct. Neat and hardworker. Lively and attentive in class. Popular in school.

4. Wants to become a teacher (training college) Like pupils A and D she is a real credit to any class. Does pretty needlework. She appears to be very interested in mathmatics, asks intelligent questions. She gives the impression that she has a good grip of the work. The fact that she does her best and is good all round probably explains her relative success on the individual method.

<u>Pupil F</u>: Boy. I.Q. 110. Age 16.4

1. Extreme poverty in home. Father has very little brains, hence the poverty: very unstable irresolute character - poor-white type. Not always enough food in house. Two uncles in police force. Pupil F himself is in indigent boardinghouse.

2. Eyes weak. Had to stay out of school for four weeks while class was on individual method. Not a very healthy appearance. Depressed type: weighed down by hardships of poverty, scarcely ever smiles.

3. School record normal, except for having been in farm school till Std. I. Schoolwork above average. Bookkeeping and Commercial Arithmetic his best subject. Good in number. Although four weeks out of school because of sore eyes he caught up with the rest in the assignment work.

4. Would like to become a teacher (training college) but seems very undecided about it. Quiet, dull type, very irresolute like father. Still, he does well in school and tries to get on - this and his ability in number probably affected individual work to the good.

Pupil G: Boy. I.Q. 108. Age 16.2

1. Father is a farmer (landowner): intellectual level of home more or less the same as that of pupil B. Thirteen children: father is anxious that his children should do well at school, but finds it hard to get them all educated. Pupil G is in indigent boarding house.

2. Eyes weak. During period of individual work he stayed out of school for five weeks. Now wears glasses. Not very healthy appearance, physically undeveloped. No sport.

3. Farm school till Std. VI (20 pupils in school). Schoolwork average. Commercial arithmetic his best subject. Very neat: systematic hard worker. In spite of five weeks' absence he caught up with the others in assignment work. Very quiet dull type. Good conduct. He seems to be a hard worker by nature.

4. Has no idea as regards career. Seems to work because he thinks it is his duty. Seldom smiles or laughs: depressed by circumstances, perhaps because he is in indigent boardinghouse. The fact that he is a hard worker, that he is neat and systematic must have helped him considerably in individual work.

Pupil H: Boy. I.Q. 114. Age 15.11.

1. Financially not very well-off but live well. Farmer. Father matriculated, also mother - intellectual development of house fairly high. Pupil stays alone with his brother in father's town-house - take their meals with other people. Three brothers and two sisters.

2. Health good. Self-assertive type. Plays rugby.

3. Attended farm school till Std. I, otherwise normal. Schoolwork average. Argumentative type - tries to argue with teachers. Fond of "playing the fool" with class-mates. Tries to bluff teachers that he is working very hard, although his work does not always warrant it. In assignment work he tended to consult too often with others, walked about the room too much.

4. Wants to become an advocate: if not that he does not know. He has to be handled very carefully by teacher - easily obstinate. It seems as if the class method will suit him better than individual method. According to himself he prefers the former because then he need not work so hard.

Pupil I: Boy. I.Q. 100. Age 15.3

1. Home circumstances very good. Father is a landowner, very well-off. This boy bicycles in from farm to school every morning (7 miles) and back in the afternoon. He seems to be very strongly attached to his home.

2. Health excellent. Very childish in his ways. Appears not to have reached the puberty stage in spite of age of 15. Appearance and ways that of boy of 11 or 12.

3. Farm school (8 pupils) till Std. VI. Was alone in his standard till Std. VI. Well-behaved and quiet in class. Schoolwork inferior. Very seldom follows what is done in class. Failed Std. VII.

4. No idea of career. Likes engineering. He will find it hard to pass J.C. He would probably do better in an industrial or agricultural school.

<u>Pupil J</u>: Boy. I.Q. 112. Age 14.10.

1. Father a farmer, landowner, well-off. Also mother has property. Father educated in one of the large Western Province Boys' High Schools. Standard of living by far the best of the nineteen pupils. Father has a first-class town-house which is run for his three sons who are in school. Children get whatever is necessary.

2. Had a serious sickbed of fever when younger: as a result asthma which is very troublesome especially during change from winter to summer. Has taken a Rex Ferrus Nature cure course. According to his father who takes great interest in his children's school work, this asthma is a great handicap to him. Note: during six months of individual work asthma worse than before. Otherwise pupil has a very healthy appearance. Has a very attractive, somewhat reserved nature. Liked by teachers and pupils. A good tennis and rugby player.

3. Normal school record. As class percentages indicate he is first in class. Probably his I.Q. is higher than 112. He was always first to hand in his assignments. A systematic steady worker.

4. Wants to become a medical doctor, seems to be quite determined about it. There is little doubt that he will make a success of his studies. Very reliable type. Gives his best in whatever he does. Questions asked by him show that he desires to know everything well. His sympathetic, though serious, nature makes him a leading personality in school. The assignment method suited him very well in spite of the fact that he came only tenth in class with respect to relative improvement. Perhaps the asthma affected his maths scores towards end of year.

<u>Pupil K</u>: Boy. I.Q. 108. Age 15.3

1. Father a farmer (landowner). Home circumstances good. Relatives all farmers. Three brothers and three sisters. One brother a carpenter. Pupil K is in indigent boardinghouse. Father pays something.

2. Healthy. No striking features. Plays a little rugby.

3. In a neighbouring village school till Std. I, otherwise school record normal. Schoolwork satisfactory. Better in number than in language.

4. He "really does not know" what career he would like - does not know father's plans - perhaps a teacher. (Note: the fact that quite a number of pupils expressed this desire is probably explained by the fact that there is a training college 36 miles away). It is difficult to say which method would suit this pupil better

Pupil L: Boy. I.Q. 116. Age 14.4

1. Father is teacher on primary school staff. Home circumstances very good. His elder brother in Std. IX, does well, obtained an A for J.C. maths.

2. Health good. Complains that his eyes burn if he works too much. Finds difficulty in expressing himself well when addressing a teacher. Plays rugby.

3. Normal school record. Schoolwork superior. His best subjects are arithmetic and history. His writing deplorable - not neat at all - this was a handicap in assignment work.

4. Wants to become a medical doctor. Father is very interested in his children and would most probably encourage him in whatever he tries. This pupil seems to have inferiority feeling, speaks indistinctly, quite often he does not attend in class. Seems to find it very hard to keep up with assignment work.

Pupil M: Girl. I.Q. 109. Age 15.0

1. Father is a bywoner. One of most needy families in the district. Extremely poor - not always enough food in house. Three brothers and five sisters. One brother works in a shop, another is a ganger. Pupil M seems to be underfed, depressed, without spirit, as result of poverty of home - looks like bursting into tears at any moment. She is now in girls' indigent boardinghouse.

2. Healthy. Appearance underfed. No sport.

3. Normal school record. Better in languages. Schoolwork quite satisfactory. Fairly neat.

4. Wants to become teacher (training college). veryGood at reciting - with remarkable pathos. Often recites at debates and concerts. Slow in handing in her assignment work, but seemed to work intelligently.

Pupil N: Boy. I.Q. 80. Age 17.2

1. Father may be classified as one of the poorest and most backward poor-white type. Lack of food in house. Whole family low down on intellectual scale. A brother and a sister mentally deficient.

2. Health good. Underfed appearance. Good temperament. No sport.

3. Attended farm school till Std. VI: would otherwise never have reached Std. VII. Weakest pupil in Std. VII. Teachers gave him up as being unable to do secondary schoolwork.

4. Should have spent a year or two at an agricultural school after Std. VI. If left to himself will probably join the ranks of the poor-whites. Needless to say, he derived no benefit from the maths classes irrespective of method used.

Pupil O: Boy. I.Q. 95. Age 18.11.

1. Financially very well-off. Father committed suicide. Mother married foreman on farm presumably because of his industrious nature. Pupil O has to take over farm as soon as he is 21, he has his own motor-car. Business-like people, though intellectually not very highly developed.

2. Not very healthy: has been under doctor's treatment for more than a year for ulcer in the stomach. Whenever he eats he does not feel well. It disturbs him in his work. He is well-built. Plays rugby: one of school's best forwards.

3. Entered village school at age of eight. Then stayed out of school for 1½ years. Then attended farm school till Std. IV and after that village school. Failed Std. VI once. Irregular school attendance as result of bad health. Work satisfactory, tries his best. His best subject is history.

4. His plans for the future are to attend an agricultural school for two years after having passed J.C. After that he is going to farm and wants to continue studies in private. His conduct is very exemplary - polite and reserved. He does well in school considering his low I.Q. All the teachers are perfectly satisfied with his schoolwork. The investigator personally judged him to be progressing better on individual method than before, although his marks showed a considerable decrease.

Pupil P: Boy. I.Q. 108. Age 15.1

1. Father has small shop, makes a poor living: he does not like work, rather does hawking - had been better off but today can barely supply his house with food Pupil P seems to be of the same type as his father - amusing himself with his friends is much more in his line than work.

2. Very healthy appearance. Complains of head-ache. Left eye very weak, he needs glasses. Very friendly appearance.

3. Normal school record. Never failed except Std. VII. He should do much better at school. Achievement far below I.Q. Quality of schoolwork inferior. Very untidy and unsystematic.

4. Has no idea as regards career. He is fairly musical: taught himself to play piano, guitar and is very good at singing, repeating, for example, gramophone records. It seems as if he would spend all his time amusing himself or his friends. Looks upon schoolwork as a nuisance. Only works when forced to. Appeared as if he copied his assignment work from others. The individual method would definitely not suit him - except if it could bring about a complete transformance in him! It is doubtful whether he could be interested in this type of work, whatever method is used.

Pupil Q: Boy. I.Q. 106. Age 15.0

1. Home circumstances poor. Father a bywoner, has no "moral backbone", allows himself to be persuaded by anyone, easy-going unambitious type. Pupil Q seems to take after his father, he has no ambition. His mother more ambitious and hardworking. Intellectually undeveloped. Pupil Q is in indigent boarding house.

2. Eyes weak. Stayed out of school for six weeks because of sore eyes. Health otherwise good. No sport.

3. Normal school record. In Std. VI he did fairly well. He was the best of the indigent boarding house pupils and hence was selected to have free boarding during Std. VII year. Best in languages. Very poor performance in Std. VII - class percentage decreased from 54% in first term to 25% in fourth term. Perhaps his weak eyes affected it, it certainly did, but he made no effort to catch up with rest of class after returning to school. Assignment work very poor. Conduct in class very good - too good. He ran away from school once.

4. He does not know at all what he is going to do on leaving school. The general feeling among the teachers is that he is a "loafer". In connection with assignment work he had to attend the detention class for a few days, but showed little improvement afterwards.

Pupil R: Boy. I.Q. 100. Age 16.3

1. Father financially very weak, insolvent, but standard of living fairly high. Father had been a farmer, then had a shop, now an undertaker. Family either farmers or mechanics.

2. Health good. Has sciatica in legs (mild form). No sport.

3. Normal school record. Likes languages and history best - reads a lot. Schoolwork average. Very weak in mathematics: unneat: unsystematic. He is not to be trusted to work on his own. Would rather get work from others than do it himself. Copied in examination once and had his paper torn up.

4. Wants to enter magistrate's office. Takes singing lessons. His work under individual method of the worst in the class. Was required to attend detention class because he neglected his assignments.

Pupil S: Boy. I.Q. 113. Age 15.2

1. Lives comfortably as regards food and bare necessities of life. Father dead. Mother is assistant matron in girls' indigent boardinghouse, very undeveloped intellectually. One uncle in police force. Further he comes into contact with none of his family.

2. Health good, although his appearance is that of an underfed person. Quiet type. No friends. No sport.

3. Normal school record. Schoolwork average. One would never expect his I.Q. to 113, probably it is lower. Hard worker.

4. He wants to be come an engine-driver - probably it is merely a day-dream. Very lonely type of person, no friends, always all by himself. He seems to work continuously and never really gets on. Very dull and humble. Will probably be satisfied with a very humble position in life. Socially very backward. On the whole, he is quite a problem, one does not know what to make of him.

APPENDIX D.

FORMULAE USED IN CALCULATIONS.

I. RANK COEFFICIENT OF CORRELATION: (Spearman).

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} \quad \ldots\ldots (1)$$

where rho = coefft. of correlation (rho is used since it is only an approximation; "r" is used for the exact coefft. of correlation).

n = number in class.

d = difference between ranks in the two variables to be correlated.

If correlations are calculated from less than 30 cases the probable error due to chance factors is very big.

$$P.E. = .706 \times \frac{1 - \rho^2}{\sqrt{n}} \quad \ldots\ldots (2)$$

The P.E. of a correlation coefft. as obtained by formula (1) is larger than P.E. of coefficients obtained by other methods. "Probable error" is not a good name, for the correlation coefft. is not probably wrong by this amount. P.E. only represents the extent to which "r" may vary by chance sampling. If "r" is three times its P.E., the probability that it is due to chance is only 4.3%. If $n < 30$ it is wise to discard correlations when they do not exceed four times their P.E.

II PRODUCT-MOMENT COEFFICIENT OF CORRELATION:

$$r = \frac{\sum(xy)}{\sqrt{\sum x^2 \ \sum y^2}} \quad \ldots\ldots (3)$$

where x = deviation of each mark from its average for the one variable

y = deviation of each mark from its average for the other variable.

$$P.E. = .6745 \times \frac{1 - r^2}{\sqrt{n}} \quad \ldots\ldots (4)$$

Note: the smaller the number of cases the bigger is the P.E.

$$r = \frac{\sum(xy) - n\, e_1 e_2}{\sqrt{(\sum x^2 - n\, e_1^2)(\sum y^2 - n\, e_2^2)}} \quad \ldots\ldots (5$$

where e_1 = error involved by taking the nearest whole number instead of a fraction for the average in the case of the first variable.

e_2 = error in case of second variable

III. Formula for finding whether two unconnected correlation coefficients differ significantly from one another.

$$\text{P.E. of difference} = \sqrt{p_1^2 + p_2^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

where p_1 and p_2 are the probable errors of the two coefficients.

$$\text{Critical Ratio} = \frac{\text{difference}}{\text{P.E. of difference}} \quad \ldots\ldots\ldots \quad (7)$$

The difference cannot be looked upon as a real difference if the critical ratio is less than three.

IV. MULTIPLE CORRELATION:

$$r_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

where 1, 2 and 3 are the three variables

$r_{1.23}$ = correlation between 1 and the combined score of 2 and 3.

r_{12} = correlation between 1 and 2.

V. PARTIAL CORRELATION:

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{(1 - r_{13}^2)^{\frac{1}{2}}(1 - r_{23}^2)^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

where $r_{12.3}$ = correlation between 1 and 2 if 3 is constant.

VI. Formula for determining whether the difference between two averages is significant.

Use formulae (6) and (7) given above.

$$E_A = \frac{.6745\,\sigma}{\sqrt{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where E_A = P.E. of average
σ = standard deviation of scores
n = number of cases.

VII. STANDARD DEVIATION:

$$\sigma = \sqrt{\frac{\Sigma(A - x)^2}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where x is any value
A is average
σ = standard deviation.

$$E_\sigma = \frac{.6745\,\sigma}{\sqrt{2n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where E_σ = P.E. of σ

Note: "Standard deviation" is a more reliable measure of the deviation of scores from their average than "average deviation".

VIII. COEFFICIENT OF VARIABILITY:

$$\text{Coefft. of variability} = \frac{\sigma}{\text{Average}} \quad \ldots\ldots (\ldots$$

APPENDIX E.

INDIVIDUAL METHOD APPLIED TO THE TEACHING OF MATHEMATICS TO EIGHT PUPILS IN STD. IX FOR A PERIOD OF SIX WEEKS.

During the first term of this year the Std. IX mathematics section of the country school where the main investigation was carried out, was put on the individual method for six weeks. They were handed out weekly assignments and they wrote tests at the end of every two weeks, that is, three tests in all.

In the following table the results are given: the pupils are named by the letters of the alphabet according to their ranks in total maths. scores:

Columns 1, 2, 3 give sex, age, I.Q. respectively.

Column 4 gives the average maths. scores obtained by the pupils in the three quarterly tests written in their J.C. year.

Column 6 gives the average maths. score obtained in the three fortnightly tests written during the six weeks of individual work.

Column 5 gives their J.C. maths. symbols and the values of the symbols in percent.

Pupils	Sex B boy G girl	Age	I.Q.	MATHEMATICS SCORES: Class-method during J.C.year	J.C. symbol	Individual meth. in Std. IX.
	1	2	3	4	5	6
A	B	15.10	120	75	A 90	73
B	B	15.10	117	61	D 55	68
C	B	17.4	109	56	E 45	61
D	G	14.8	114	36	D 55	58
E	G	15.6	114	36	E 45	35
F	G	17.10	103	35	E 45	31
G	G	14.9	105	35	F 35	35
H	B	18.2	97	27	H 10	9
Total		129.11	879	361	380	370
Average		16.3	109.9	45.1	47.5	46.3

From these data no conclusions can be drawn but according to observation it appeared that on the whole this older age-group did better on the individual method than the Std. VII group described before.

The assignments drawn up for this class contain much less explanatory matter than those of Std. VII. Below are given the algebra assignments used during the six weeks; note the different procedure followed as regards extra questions for the better pupils:-

ALGEBRA ASSIGNMENTS - STD. IX.

Book Used: "Elementary Algebra": Hall & Knight.

General Instructions: The explanations given in these assignments and in the text-book should be read and re-read very carefully until you completely understand them.

All questions marked with an A must be done by everyone. In addition questions B or C must be done. Questions C are more difficult.

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FIRST WEEK

QUADRATIC EQUATIONS - Chapter XXV.

Read: articles 189, 190.

Are the following simple, pure quadratic or adfected equations?

$5x + 8 = 0$; $3x^2 + 7x - 4 = 0$; $5x^2 - 80 = 0$

What suitable names would you give to each of the following two equations?

$$x^3 + 4x^2 - 6x = 4$$

$$x^6 = 4x^2 + 6.$$

To solve pure quadratic equations is a very simple performance: our real task lies with adfected quadr. equations.

PURE QUADRATIC EQUATIONS:

Read: article 191.

Note: $x^2 = 36$
$x = +6$ or -6, because $(+6) \times (+6) = 36$ and also $(-6) \times (-6) = 36$.

A short way of writing this is $x = \pm 6$.

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Questions A: Solve the following pure quadratic equations. In each case find two values for the unknown.

1. $a^2 = 169$, $5x^2 = 125$, $4x^2 - 16 = 0$
2. $(x - 4)^2 = 9$ (Taking the square root of both sides you get $x - 4 = 3$ or $x - 4 = -3$ from each of which you can find a value for x).
3. $3(x + 6)^2 - 48 = 0$.

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ADFECTED QUADRATIC EQUATIONS:

There are three methods of solving adfected quadratic equations:

1. by factorising
2. by completing the square.
3. by drawing its graph.

We shall get busy with the first two methods for the next 4 weeks. The graphical method will be left till later.

SOLUTION BY FACTORS.

This is the shorter and easier method of the two, and whenever possible you will use this method to solve quadratic equations.

Example 1: Solve $3x^2 + 7x - 6 = 0$
factorising, $(3x - 2)(x + 3) = 0$
To make is easier to understand, let us write A for the factor $3x - 2$ and B for $x + 3$

then $A \times B = 0$

Now if $A = 0$, then $A \times B = 0$: that is, the equation is satisfied by the value of $A = 0$

Similarly if $B = 0$, then $A \times B = 0$
That is, either A can be 0 or B can be 0

$\therefore$ the solution is $A = 0$ or $B = 0$

$\therefore 3x - 2 = 0$, or $x + 3 = 0$

From which $x = \frac{2}{3}$ or $x = -3$.

Example 2:

Solve $4x^2 - 3x = 85$
i.e. $4x^2 - 3x - 85 = 0$
i.e. $(4x + 17)(x - 5) = 0$
$\therefore 4x + 17 = 0$ or $x - 5 = 0$

$\therefore x = -\frac{17}{4}$ or 5.

If you do not follow these two examples, read the explanations given in Hall & Knight, article 201.
If you still cannot understand consult a friend or come to me.

QUESTIONS A: Examples xxv. c., pg. 201, Nos. 13 - 20, 23, 25
B: Nos. 27, 28, 29. (See article 201, example 1)

OR

C: Nos. 27, 29 (See article 201, example 1)
Nos. 31, 32, 35, 36 (These are equations of the 4th degree and each will have 4 roots.
See art. 202, example 1)

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SECOND WEEK.

SOLUTAION BY THE METHOD OF "COMPLETING THE SQUARE".

We shall first show how to complete a square.

To make $x^2 + 6x$ a perfect square, we have to add 9, then $x^2 + 6x + 9 = (x + 3)^2$

What must be added to each of the following to make them a perfect squares:

$x^2 + 4x$, $x^2 + 18x$, $x^2 - 14x$, $x^2 - 10x$.

Note that each time you had to add the square of half the coefficient of x,

e.g. $x^2 + 18x$ becomes a perfect square if we add $(\frac{18}{2})^2$, that is 9^2 or 81
And

$x^2 - \frac{9}{2}x$ becomes a perfect square if we add $(\frac{9}{4})^2$ to it (because $(\frac{9}{4})$ is half of $\frac{9}{2}$)

Now complete the following squares using this rule:-

$x^2 + 20x$, $x^2 - 9x$, $x^2 - 5x$, $x^2 + \frac{4}{3}x$, $x^2 - \frac{15}{6}x$

(Their answers need not be handed in, but if you are not certain that they are correct, show them to me before you continue).

<u>We shall now show how this method is used to solve quadratic equations.</u>

Solve $x^2 - 6x - 16 = 0$

Transpose 16, $x^2 - 6x = 16$

Add 3^2 to each side, $x^2 - 6x + 3^2 = 16 + 9$

$\therefore (x - 3)^2 = 25$

Take the square root of each side

$\therefore x - 3 = +5$ or $x - 3 = -5$

$\therefore x = 8$ or $x = -2$

We made the lefthand side of the equation a perfect square by adding the square of half the coefficient of x to it, i.e., the square of $\frac{6}{2}$, i.e. 3^2 or 9 So we also had to add 9 to the right side.

If you find any difficulty in solving the following equations, refer to article 193 where this method is also explained with examples.

<u>QUESTIONS A</u>: 1) Examples XXV. a. pg. 194. All odd numbers from 3 - 19. Solve them by the method of completing the square.

<u>READ</u>: articles 194, 195. The steps in article 195 must be understood and remembered very thoroughly.

<u>QUESTIONS A</u>: 2) Examples XXV. b. pg. 197. Nos. 1, 4, 7, 10, 13, 15, 19.

B: Nos. 25, 27, 29 (Simplify first as is shown in Art. 196, ex. 1)

OR

C: Questions in B and also Nos. 36, 37.

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<u>THIRD WEEK</u>.

This week we shall study what at first sight will appear to be a different method of solving quadratic equations. But it is really only the method of xompleting the square in a shortened form.

After simplification, a quadratic equation can have 3 terms at most: the term in x^2, the term x and the constant term. Hence any quadratic equation can be ~~redux~~ reduced to the general form:

$$ax^2 + bx + c = 0$$

where a, b, c, may have any <u>numerical</u> values whatever. If you solve $ax^2 + bx + c = 0$ using the method of completing the square, you will find its roots to be $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ Or in short $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula is <u>very important</u> and must be committed to memory without delay.

<u>READ</u>: article 198 (It is very important).

<u>READ</u>: article 199: here it is shown how the above formula is used to find the roots of any quadratic equation by ~~the~~ substituting numerical values for a, b, c in the formula.

<u>QUESTIONS A</u>: 1) Pg. 201: Solve Nos. 13 - 17 using the formula (Compare your answers with those obtained by factorising).

<u>READ</u>: article 200 very carefully.

<u>QUESTIONS A</u>: 2) Pg. 201, Nos. 1 - 6, 10 - 12 (use formula).

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<u>FOURTH WEEK.</u>

<u>SUMMARY of first 3 weeks' work</u>:

PURE QUADRATIC EQUATIONS are solved by extracting the square roots of both sides e.g. $x^2 = 81 \therefore x = \pm 9$

ADFECTED quadratic equations may be solved by three different methods:-

1. <u>By factors</u>: This is the easiest and is used whenever possible.

2. <u>By completing the square</u>: When you cannot factorise, you use this method. In practice however you never actually complete the square but you use the <u>formula</u>.

3. <u>By drawing a graph</u>: This method will be treated later

<u>THIS WEEK</u> we shall be busy solving a few more difficult quadratic equations.

<u>READ</u>: article 202a.

<u>QUESTIONS A</u>: 1) pg. 201, b. Nos. 1 - 3.

The following is an example of an equation of the <u>third degree</u>.

<u>Example</u>: Solve:

$$x^3 + 2x = 3x^2$$

$$x^3 - 3x^2 + 2x = 0$$

$$x(x^2 - 3x + 2) = 0$$

$$x(x - 2)(x - 1) = 0$$

Equate each of the three factors equal to zero.

$$\therefore x = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\therefore x = 0 \quad \text{or} \quad 2 \quad \text{or} \quad 1.$$

<u>QUESTIONS A</u>: 2) Solve by factors:

$$x^3 - 4x^2 + 4x = 0$$

$$3x^3 + 8x = 3x$$

<u>READ</u>: article 202 b.

QUESTIONS A: 3) pg. 201 b, Nos. 8 - 11 (if $x = 5$ is a root, then $x - 5 = 0$ is a factor of $x^2 - 39x + 70 = 0$ Hence divide $x - 5$ into $x^3 - 39x + 70$ and then factorise the quotient in the ordinary way.

QUESTIONS A: 4) Pg. 201 b, Nos. 12, 15, 16.
B: Pg. 201 b, Nos. 18, 19.
C: Pg. 201 b, Nos. 18, 19, 20, 21.
Pg. 201 a, Nos. 39, 40 (See article 202, Ex. 2.)

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FIFTH WEEK.

For most of the time this week we shall be busy with a few problems leading to quadratic equations.

See Hall & Knight, Chapter XXVII. READ article 209, Example 1: X has two values:- 25 or -30. Since the train cannot travel at -30 miles per hour we neglect this value, the correct solution being 25 miles per hour. (Note: In article 209 they explain what the -30 m.p.h. means. You may read it if you are interested.)

See also examples 2, 3, 4 on page 210.

QUESTIONS A: pg. 211; Nos. 2, 3, 6, 7.
B: pg. 211: Nos. 12, 13.
C: pg. 211: Nos. 13, 16, 19, 23.

If you understand everything well so far, you will be able to solve any quadratic equation; and this is quite sufficient for our purposes. But it will more than repay the trouble to consider a few interesting points about the roots of quatratic equations.

On solving $x^2 - 10x + 16 = 0$ we find its roots to be 8 or 2 Now the sum of the roots i.e. $8 + 2 = 10$, which is the coefficient of x with the sign changed.
Also the product of the roots, i.e. $8 \times 2 = 16$ which is the constant term.

The above two conclusions are true only if the coefficient of x is unity. Hence the equation $4x^2 + 3x - 10 = 0$ must first be written in the form $x^2 + \frac{3}{4}x - \frac{10}{4} = 0$ (dividing both sides by 4) and then we can say that

the sum of its roots = $-\frac{3}{4}$
and the product of its roots = $-\frac{10}{4}$

READ: article 339, pg. 313. There the roots of the general quadratic equation $Ax^2 + Bx + C = 0$ are taken and then the sum and product are calculated.

These two results are very important for they may be used to check the solution of any quadratic equation in the form $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ by seeing whether the sum of its roots $= -\frac{b}{a}$ and the product $= \frac{c}{a}$

QUESTIONS A: 2) Solve the following equations and check your answers by finding the sum and product of the roots:

$x^2 + 12x + 35 = 0$

$2x^2 + 5x = 12$

$6x^2 = 5x + 6$

$x^2 = 9$

$x^2 - 6x = 0$

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SIXTH WEEK.

Before concluding this week with a few miscellaneous examples on quadratic equations, we shall discuss another interesting point.

Solve $3x^2 - 2x + 11 = 0$ using the formula

$$\therefore x = \frac{2 \pm \sqrt{4 - 132}}{6}$$

$$= \frac{2 \pm \sqrt{-128}}{6}$$

Now since it is impossible to find the square root of a negative quantity, the roots of this equation will have to be left in this form. We say the roots are IMAGINARY.

Thus in the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ the quantity, $b^2 - 4ac$, tells us whether the roots of the equation are

REAL OR IMAGINARY.

If $b^2 - 4ac$ is positive, the roots are real.
If $b^2 - 4ac$ is negative, the roots are imaginary

QUESTIONS A: 1) Say whether the roots of the following eqns. are real or imaginary.

$3x^2 - 8x + 10 = 0$, $5x^2 + 4x - 16 = 0$,

$7x^2 - 8x = 10$, $4 - 15x^2 = 3x$

$8x + 16x^2 = -3$

READ: article 337, 338 on page 312 for further interesting points about this quantity $b^2 - 4ac$.

REVISION EXAMPLES.

QUESTIONS A: 1) Solve by completing the square:

a) $ax^2 - bx + c = 0$

b) $\frac{12}{5}x = \frac{4}{5} - x^2$

2) Solve by factors if possible, otherwise using the formula (answers correct to 2 decimal places if $b^2 - 4ac$ is not a perfect square). (Check 2 of them).

a) $5(x^2 + 5) = 6x^2$
b) $12x^2 + 23Kx + 10K^2 = 0$

c) $\frac{4}{x - 1} - \frac{5}{x+2} = \frac{3}{x}$

d) $2x^2 - 3x - 1 = 0$

e) $3x^2 + 4x + 8 = 0$

QUESTIONS B:

1) Solve $x^3 - 1 = 0$ (factorise first and then use formula for the one factor).

2) $\frac{1}{x} + \frac{5}{3x - 1} = \frac{1}{x+5} + \frac{10}{3(2x-1)}$

QUESTIONS C:

1) The questions in B.

2) $3x^2 - 2ax - bx = 0$

3) $216 - x^3 = 0.$

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