

# TO PREY OR NOT TO PREY? WELFARE AND INDIVIDUAL LOSSES IN A CONFLICT MODEL<sup>1</sup>

MARTIN WITTENBERG\*

## *Abstract*

We analyse a generalised form of the Hirshleifer-Skaperdas predation model. In such a model agents have a choice between productive work and appropriation. We suggest that such a model can usefully be thought of as a continuous form of the Prisoners' Dilemma. We present closed form solutions for the interior equilibria and comparative statics for all Cournot equilibria and analyse the social welfare losses arising from predation. We show that predation is minimised under two quite different regimes, one in which claiming is very ineffective and another in which one of the players becomes marginalised. The worst outcomes seem to arise when claiming is effective, but inequality in power is significant but not extreme. This, arguably, is the situation in a number of transition societies.

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## 1. INTRODUCTION

Adam Smith is celebrated by modern economists as the inventor of the "invisible hand" argument for the virtue of the free market. Agents acting independently of each other and coordinating with each other only through the market mechanism manage to improve the welfare of everyone. This at least is how the argument is generally understood. Mittermaier (1999) has argued that the invisible hand argument is making a simpler point aimed at a different problem: Smith was concerned to show that parasitic behaviour was detrimental to general welfare. Parasitic behaviour involves any activities where effort is devoted to appropriating the fruits of other people's labour. If everyone were only to concentrate on producing the maximum that they were capable of, then, of necessity, the aggregate product would be as large as it could possibly be. The problem, of course, is that parasitic behaviour is frequently more attractive to the individual than honest toil. It is evident in the problems of crime and corruption which plague many societies. Protectionism and lobbying government for transfer payments or indeed any other kinds of rent-seeking behaviour would be other examples. In Smith's day it was the granting of monopoly rights which was at issue.

From the point of view of societal production, parasitism involves two costs: firstly there is the production foregone as a result of the energy devoted to it; secondly there is the cost incurred by producers in protecting themselves against it. Indeed, parasitism may be viewed as a negative sum game: the total product is lower when one agent is parasitic than when everyone cooperates.

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\* School of Economics and SALDRU University of Cape Town.

A simple model that has some of the flavour of this type of interaction is the Prisoners' Dilemma. Both players would be better off if they were to cooperate, but each player is tempted to "sucker" the opponent and so ends up with the "defect" strategy. It is easy to see why such Nash equilibria are suboptimal from a societal point of view. Of course, the Prisoners' Dilemma is too simple a game to allow one to examine how changes in the structure of the interaction might make the players more or less cooperative. In this paper we will discuss a generalised version of a predation model introduced by Hirshleifer (1988, 1991a, 1995) and Skaperdas (1991, 1992, 1996). In this model the choices available to the players are not discrete but continuous. One might think of it as a continuous version of the Prisoners' Dilemma (see also Killingback, Doebeli and Knowlton, 1999), in which the players can pick a level of cooperation anywhere between 0% (totally parasitic) to 100% (totally cooperative). We will analyse the nature of the equilibria in this game. Ultimately we are concerned with the size and determinants of the welfare loss incurred by society through predatory behaviour. We have four key parameters in our model: the level of inequality, the relative level of productive efficiency, the relative level of appropriation strength and the decisiveness of the appropriation effort. We will show that sufficiently large changes in these parameters will induce one of the players to become a complete parasite. Full parasitism by one of the players, however, does not necessarily translate into big welfare losses. Indeed we will show that there are two circumstances under which welfare losses are minimised: a regime in which institutions do not reward claiming behaviour, and a regime in which claiming is relatively successful but one of the players becomes marginalised. These results may provide some insights into why societies in transition from highly authoritarian regimes seem especially prone to high levels of crime and corruption.

The plan of the discussion is as follows. In the next section we briefly review some of the relevant literature. We show how our model relates to a number of other models of predation. We present the model formally in section 3. The first step in the analysis is to discuss the incentives of the players and we do this by deriving the nature of the reaction functions in section 4. We obtain expressions for the Cournot equilibria in section 5. In the case of interior equilibria these are closed form solutions while the corner equilibria can only be characterized implicitly. Nevertheless we can accurately sign the comparative static derivatives. We do this in section 6. We use these results to characterise the determinants of the welfare losses in section 7 and discuss these further in the conclusion.

## 2. PRODUCTION AND APPROPRIATION

The argument that appropriation is an interesting and in some ways fundamental economic issue has been made forcefully by Hirshleifer (1978, 1991b), and Garfinkel and Skaperdas (1996). Hirshleifer suggests that the "political economy" of trade and barter rests on a "natural economy" based on appropriation. The emergence of laws and cooperation (Axelrod, 1984) is what enables this transition. Nevertheless these institutions are inevitably imperfect.

"For that matter a perfectly law-abiding individual (if there is any such) could not have such confidence in third-party enforcement as to entirely forego personal vigilance and self-defense." (Hirshleifer, 1978, p. 238)

Garfinkel and Skaperdas draw attention to how the literature on rent-seeking reintroduces the “political” element to political economy. In these contexts while there may be property rights, they are only imperfectly defined and appropriation and conflict re-emerge as legitimate subjects of economic investigation. Similarly Baumol (1990) has suggested that the function of entrepreneurship is misunderstood, if it is viewed only in a narrow trading sense – pirates and monopolists are successful entrepreneurs too. What is important from the perspective of maximising the long-run well-being of society is whether it is appropriative or productive entrepreneurship that is being promoted.

A number of different approaches to modelling forcible appropriation have been presented in the literature. On the one hand, there are models of particular types of appropriation or conflict. Shleifer and Vishny (1993) model corruption; Murphy, Shleifer and Vishny (1993) look at rent-seeking; Garfinkel (1990) investigates arms races; Grossman (1991, 1994) looks at rural insurrections and land reform; while Usher (1989) looks at despotism and its breakdown.

Besides being focused on particular types of appropriation, some of these models posit an *ex ante* heterogeneity in agents: in Grossman’s insurrection model, for example, peasants have a choice about whether they produce or become bandits, but they cannot become rulers. In Grossman and Kim (1996), there is a potential predator and a potential prey, while in their later model (2000) there are moral and amoral agents. In some models this heterogeneity does not exist between agents, but applies to the particular roles that these agents can choose to play. Murphy, Shleifer and Vishny (1991) and Acemoglu (1995), for example, present models in which agents can engage in rent-seeking or in production but these roles are disjoint: a producer cannot, as it were, exercise any form of power in order to ward off the exactions of the predator. The choice is between paying up or joining them. Indeed as a result both papers have to treat the size of the transfer as exogenously fixed. If producers do not fight back, the size of the transfer cannot be endogenously determined.

The most general conflictual model of appropriation (and variants thereof) has been presented by Hirshleifer (1988, 1991a, 1995) and Skaperdas (1991, 1992, 1996). The key elements of this model are:

(i) *A resource partition function*

This indicates how the basic resource available to each player is divided between production and appropriation. It is assumed that these are the two types of activities open to each individual. Some authors, notably Grossman and Kim (1995, 1996, 2000), distinguish between expenditures on offence and defence. This introduces potential asymmetries between players which are interesting, but complicate the analysis.

(ii) *The aggregate production function*

In Hirshleifer (1988, 1991a) and Skaperdas (1992) total appropriable output is a function of the productive investments made by each player, while in Hirshleifer (1995) essentially private production functions for each player replace the aggregate production function.

(iii) *The contest success function*

This determines the probability of winning the contest<sup>2</sup> as a function of the strengths of the two players, although it can also be thought of as the “sharing rule” between the

<sup>2</sup> In most models the contest is about the output. In Hirshleifer (1995) it determines how the overall resource base is split between the players.

players, under the threat of conflict (Skaperdas, 1992, p.723). A number of possible functional forms have been investigated (see Hirshleifer (1989, 1991b) and Skaperdas (1996); also Dixit (1987)) but the most popular form is the ratio form

$$p_i = \frac{b_i s_i^m}{b_i s_i^m + b_j s_j^m}$$

where  $s_i$  and  $s_j$  are the relative fighting strengths of player  $i$  and  $j$  respectively and  $m$  is a “decisiveness” parameter.<sup>3</sup> This has the desirable property that scaling the contests up does not affect the outcome. Since the units in which these strengths are measured are arbitrary this is a positive feature.<sup>4</sup>

*(iv) The income distribution equation*

The final allocation, interpreted either as the expected payoff (if there is war) or actual payoff (if the contest success function is interpreted as a sharing rule), to each player is given by an equation of the sort

$$y_i = p_i y$$

where  $y$  is aggregate output.

The solution concept which is most usually applied to the analysis of this interaction is that of the Cournot equilibrium. Hirshleifer (1988) investigates the Stackelberg equilibrium as well although to his surprise the Stackelberg solution does not differ from the Cournot solution.<sup>5</sup>

One of the most striking results to have emerged from the analysis of this model is what Hirshleifer has dubbed the “paradox of power” (Hirshleifer, 1991a): that conflict can be an equalising force. An agent that initially has smaller resources may be motivated to fight much harder when facing a more powerful opponent. As a result we may see a transfer from the richer player to the poorer one in equilibrium. Hirshleifer notes that this result is contingent on the weaker player not being driven to a corner. If the weaker player is devoting all resources to fighting, then an increase in the strength of the more powerful player can no longer be matched by the weaker one, so that the larger resources eventually predominate. Furthermore the decisiveness parameter  $m$  plays an important part in determining how large the disequilibrium in resources can be, before the power of the richer player asserts itself.

Whereas Hirshleifer resorted to various simulations, we have explicit analytical solutions for all of the results. We are therefore not restricted to presenting analyses based on particular parameter values. Instead we can characterise the outcomes in a general way.

<sup>3</sup> We will discuss the interpretation of  $m$  in more detail below.

<sup>4</sup> The other functional form considered in the literature, the logit form does not have this feature, hence its relative lack of use, despite some attractive properties (Hirshleifer, 1989).

<sup>5</sup> Indeed in Hirshleifer’s “anarchy” model (1995) the Stackelberg leader performs relatively less well than the follower. The reason for this difference is that in this model the contest is over the underlying resource and not over the output of production.

### 3. THE MODEL

We assume that there are two players who we will label  $A$  and  $B$  (and some times  $1$  and  $2$ ), which follows the template of the appropriation model outlined above.

*(i) Wealth partition function*

Each player has an initial endowment of wealth  $W_A$  and  $W_B$  respectively, and the choice confronting them is how to allocate these resources to productive  $E_i$  and claiming activities  $F_i$  respectively. Since we will be interested in characterising how “cooperative” or “predatory” a player is, we will focus mainly on the proportions of resources,  $\alpha$  and  $\beta$ , devoted to production by each player. We have

$$E_A = \alpha W_A \text{ and } E_B = \beta W_B \tag{1}$$

$$F_A = (1 - \alpha) W_A, F_B = (1 - \beta) W_B \tag{2}$$

*(ii) Aggregate production function*

We assume that the aggregate production function  $Y_{AB}$  is such that  $\partial Y_{AB} / \partial E_i > 0$  and  $Y_{AB}(0, 0) = 0$ . Some of the results can be derived for this arbitrary specification, but we will generally make the more restrictive assumption that the production function is separable in the products of  $A$  and  $B$ , *i.e.*  $Y_{AB} = Y_A + Y_B$ , where  $Y_A$  and  $Y_B$  are the outputs of  $A$  and  $B$  respectively.<sup>6</sup> With this specification it is possible to unambiguously identify the contributions of  $A$  and  $B$ , which helps to identify whether the final allocation exhibits parasitism or not. In the discussions below we have chosen particular functional forms given by:

$$Y_A = c_A E_A^b, Y_B = c_B E_B^b \tag{3}$$

where  $c_A$  and  $c_B$  are productivity parameters and  $b$  is a return to scale parameter. We will restrict our attention here to the constant returns case  $b = 1$ .

*(iii) The appropriation function*

Since we are interested in situations of long-run predation, we interpret the contest success function as a “sharing rule” in the sense of Skaperdas (1992). The function therefore determines how much each player is able to appropriate, rather than each player’s probability of winning the entire amount. We assume that the functions are given by the ratio form

$$g_A = \frac{s_1^m}{s_1^m + s_2^m}, g_B = \frac{s_2^m}{s_1^m + s_2^m} \tag{4}$$

where  $s_1(\cdot)$  and  $s_2(\cdot)$  are functions that depend on  $F_A$  and  $F_B$  only. In particular we assume

$$s_1 = f_A F_A, s_2 = f_B F_B \tag{5}$$

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<sup>6</sup> Many of the simulations presented by Hirshleifer (1988, 1991a) fit into this model, so the results can be compared to the analytical results presented below.

where the  $f_i$  are “appropriation efficiency” parameters. They indicate how effective each player is in turning her fighting resources into claiming strength. We allow these to be different, because different agents may not have access to the same fighting or claiming technology. In the case of feudal societies knowledge about how to produce certain types of weapons would have been closely guarded secrets of the nobility. In more recent times, apartheid South Africa imposed legal restrictions on the ability of black South Africans to get access to guns. There were also restrictions on the kinds of legal claims that blacks could make on property. All of these would have severely impaired the efficiency with which claims could be established.

The choice of the ratio form for the appropriation function is motivated not only by its popularity in other contexts, such as its use in the rent-seeking literature (Rosen, 1986, Gradstein and Konrad, 1999). It is also motivated by the fact that it is homogeneous of degree zero in the arguments  $F_A$  and  $F_B$ . This means that a balanced increase in the resources available to each player and devoted to claiming, will not affect the overall shares. Furthermore, as Hirshleifer has argued (1991b, p.104), the ratio form is probably most appropriate for contexts in which there is a lot of mutual information and there is no place to hide.

One implication of the ratio form is that a player that does not manage to stake any claim, will receive nothing, even if that player has contributed the largest share of the total output. Claiming activities should therefore not be thought of as intrinsically illegitimate. People who do not engage in efforts to establish their rights to particular resources or then to defend those rights are likely to be taken advantage of. In this sense claiming is not only an alternative to production, but also a necessary complement (if the other player is likely to engage in claiming, that is).

This view of human nature is not that far fetched. Entire professions have grown up around the establishment and enforcement of claims. Litigation, the registration of title deeds or the registration of patents are all examples of claiming activities in this sense. Lobbying government for welfare payments or for a reduction of taxes would be others. Furthermore the outcomes of these contestations need not be related to the intrinsic merits of the cases, but may often just reflect the relative skills of the lawyers or politicians involved.

The parameter  $m$  in the appropriation function deserves a more detailed comment. It is a decisiveness parameter – it records how sensitive the final division of aggregate output is to claiming behaviour. With a low  $m$  claiming activities are relatively ineffective and the final output is more or less equally divided. With high  $m$  claims become highly effective and the final shares come to reflect the respective energy that was put into making claims on the output. With an extremely large  $m$ , the person with the largest muscle gets to keep everything.

It should be noted that  $m$  is a reflection of the social values and technologies available within a society. We might list some of them as follows:

- cultural factors: A society’s attitude towards wealth and inequality would definitely affect  $m$ . A great belief in equality would tend to reduce  $m$ , while a high tolerance for inequality would drive up  $m$ ;
- military technology: The more sophisticated the tools of destruction, the more leverage the owners of those implements would tend to have on the division of the product, *i.e.* this would drive up  $m$ ;

- storage technology: Limits on the ability to store and transport wealth (e.g. the presence or absence of grain silos) would tend to reduce  $m$ .

(iv) *The income distribution equation*

The final payoffs to each player are given by

$$Y_1 = g_A [c_A E_A^b + c_B E_B^b], Y_2 = g_B [c_A E_A^b + c_B E_B^b]$$

Our key concern is to analyse the effects of the strategic interactions around appropriation and production. It is evident that if both players claim only and do not produce, then there will be no product to split. This, however, cannot be an equilibrium: if the other player is determined to be an absolute parasite, it would be in my interest to produce something, because even a small share of a positive output would be preferable to absolute no return at all. The balance between appropriation and production that we will see ought to depend on the productiveness of the players, their effectiveness in establishing claims, their respective wealth and the degree to which claiming is an effective activity.

We will in general be more concerned with analysing the effects of relative changes in wealth and productivity. We therefore reparameterise our model, letting

$$k = W_A/W_B, p = c_A/c_B, f = f_A/f_B \tag{6}$$

We interpret  $k$  as our index of inequality,  $p$  as an index of productivity differentials and  $f$  as an index of  $A$ 's relative claiming strength. Without loss of generality we will assume throughout that  $B$  is the less productive individual, i.e.  $p \geq 1$ . With this reparameterisation,  $c_B$  and  $W_B$ , now function as scale parameters. Increases in  $c_B$  and  $W_B$ , (for fixed values of  $p$  and  $k$ ) lead to increases in the productivity or wealth of both players. To signal this change, we drop the subscript.<sup>7</sup>

By substituting equation (1) into the production function; equation (2) into the "strength equation" (5) and that in turn into the appropriation success function (4) and then reparameterising as above, we can write the payoffs as

$$Y_1 = \frac{(1 - \alpha)^m f^m k^m}{(1 - \alpha)^m f^m k^m + (1 - \beta)^m} c (p\alpha k + \beta) W \tag{7}$$

$$Y_2 = \frac{(1 - \beta)^m}{(1 - \alpha)^m f^m k^m + (1 - \beta)^m} c (p\alpha k + \beta) W \tag{8}$$

We will make the Cournot assumption that players treat their opponent's choice of cooperativeness as fixed. This means that the one-period equilibrium will be at the intersection of the respective reaction functions, where these give the optimal values of  $\alpha$  (or  $\beta$ ) given the opponents choice of  $\beta$  (or  $\alpha$ ). For interior solutions the reaction functions will be given by the loci of the solutions to

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<sup>7</sup> A similar point applies to  $f$  of course. The "baseline" appropriation efficiency  $f_B$  does not feature in the payoff function, however, so there is no need to concern ourselves with this issue.



$$\partial Y_1 / \partial \alpha = 0, \quad \partial Y_2 / \partial \beta = 0$$

Hirshleifer’s results, however, should alert us to the fact that we are not guaranteed to get interior solutions. There will be combinations of the parameters for which one of the players becomes completely parasitic. Indeed the possibility of complete parasitism and its effects on the interactions with the other player turn out to be absolutely crucial for the behaviour of the model.

#### 4. REACTION FUNCTIONS

We begin by analyzing the reaction functions. By definition a reaction function plots the optimal choice for a particular player, given a variety of choices by the opponent. In this particular case this optimum has to balance two countervailing forces: on the one hand raising production will increase the pie for everyone; on the other this will reduce the effectiveness of one’s claim. In some cases (particularly when the opponent is contributing very little) increasing the size of the pie might benefit the player more than squabbling more vigorously over non-existent crumbs. These sort of trade-offs underlie the reaction functions that we have graphed in Fig. 1. This diagram shows some reaction functions and the locus of Cournot equilibria for a particular set of parameters. It is evident that beyond a critical level of inequality ( $k = 5$  in the diagram), Player  $B$  becomes completely parasitic. Indeed for higher levels of inequality  $B$ ’s reaction function moves very sharply down to zero and then stays at zero until  $A$  shows excessively high degrees of cooperation.

We observe a number of features. When  $k = 10$  we note that  $A$ ’s reaction function  $r_A$  increases monotonically while  $B$ ’s reaction function  $r_B$  very rapidly goes to zero, then stays at zero and finally increases again towards one. By contrast, when  $k = 0.5$  or 1 or 2 we see

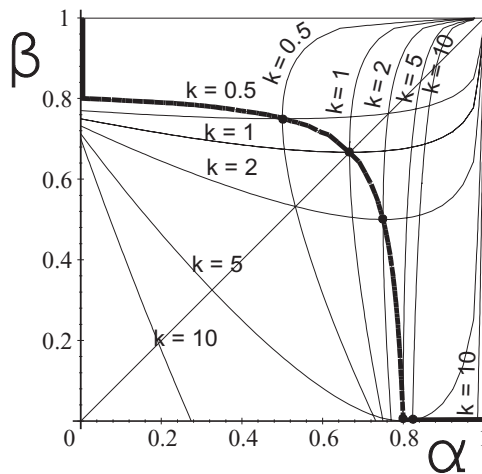


Figure 1. Reaction functions and Cournot equilibria for a various levels of inequality  $k$ . The reaction functions are indicated by the light lines, while the locus of equilibria is indicated by the heavy curve. Large dots indicate intersections of the reaction functions shown in the diagram. Parameters:  $f = 1, p = 1, m = 0.5$



that both reaction functions have a unique turning point in the interior of the unit square. Indeed, this is our first result:

*Proposition 1 There are three types of reaction functions:*

1. *Type one increases monotonically on the interval  $[0, 1]$  and is such that  $r_i \geq 1/(m + 1)$*
2. *Type two is differentiable with a unique turning point in  $(0, 1)$*
3. *Type three is differentiable except at two points  $\delta_1$  and  $\delta_2$ . It decreases on an interval  $[0, \delta_1]$ , is equal to zero on an interval  $[\delta_1, \delta_2]$  and increases on  $[\delta_2, 1)$ .*

*In all cases the reaction function is quasi-convex with  $r_i(0) \geq 1/(m + 1)$  and  $r_i(x) \rightarrow 1$  as  $x \rightarrow 1$ . Furthermore if any player has a reaction function of type one, then the opponent will have a reaction function of type three, while if any player has a reaction function of type two, then the opponent will also have a type two reaction function.*

The fact that all reaction functions start at a point higher than  $1/(m + 1)$  is evident in Figure 1. Indeed it is clear why this has to be the case. If  $A$  is completely parasitic then  $B$  is compelled to produce, if she wants a positive payoff. As  $\alpha$  increases, this compulsion gradually disappears. In our diagram  $B$ 's response is to immediately increase her levels of claiming (*i.e.*  $B$ 's reaction functions are of type 2 or 3). This is due to two factors. At higher levels of  $\alpha$   $A$  is producing positive output which adds to aggregate output. This increases the incentive for  $B$  to claim. On the other hand, as  $\alpha$  increases,  $A$ 's level of claiming activity goes down. This makes  $B$ 's claims relatively much more effective. The combination of increased incentive with increased effectiveness leads to the decrease in the reaction functions observed in the diagram. This decrease happens until either  $B$  becomes a complete parasite or until the Cournot equilibrium is reached. In the case of complete parasitism the reaction function eventually reappears and converges on  $\beta = 1$  at a high enough level of  $\alpha$ . Essentially at these levels of  $\alpha$   $A$  is doing such little claiming that  $B$  is left with almost the total output. In these situations it is in  $B$ 's interests to start producing to increase the total output. Although  $B$  becomes more cooperative, the reaction function stays far below the  $45^\circ$  line in this region, so  $B$  devotes relatively much larger resources to claiming than  $A$ .

$A$ 's optimal choice is obviously influenced by similar considerations. One additional point to note is that if  $A$  is substantially wealthier than  $B$  (*e.g.*  $k = 10$  in the diagram), then we have a type 1 reaction function: there is no longer any incentive for  $A$  to increase her claiming activities as  $B$  becomes more cooperative. The additional resources that she can obtain from her own production far outweigh the benefits she might gain by claiming from  $B$ .

## 5. COURNOT EQUILIBRIA

A Cournot equilibrium (or Nash equilibrium) occurs when both players have no incentive to change their strategy even when they know what the other player will do. In this case two forces will exactly counterbalance each other: the additional gains to either player from additional production will be exactly offset by the losses accrued from a decreasing share of the pie. In Fig. 1 we see that the intersection always seems to occur at the respective minima of the functions. When  $k = 10$ , the Cournot equilibrium occurs where  $B$  is completely parasitic (*i.e.*  $B$ 's reaction function is at its minimum). Furthermore  $A$ 's reaction function increases monotonically, so that in fact the value of  $\alpha$  at the

equilibrium is also at a minimum. A visual inspection of the interior Cournot equilibria also suggests that the reaction functions intersect at their respective turning points. This is our second result:

*Proposition 2 There are two types of Cournot equilibria*

1. *A type one equilibrium is at the intersection of a type one and a type three reaction function. In this case one player will be completely parasitic and the other player will fix the degree of cooperativeness at  $r_1(0)$*
2. *A type two equilibrium is at the intersection of two type two reaction functions. These curves intersect at right angles at their respective minima.*

In all cases, therefore, the Cournot equilibrium has the curious property that it represents the maximally uncooperative point on either player's reaction function. Equivalently, it is the point at which both players spend the most energy on claiming, given that this level must be rational on some hypothesis about the opponent's behaviour. It can be shown that this property depends only on the fact that the production function is separable in the inputs of the two players.

The exact equilibrium that is reached is dependent on the parameters of the model. We can, however, derive expressions for all equilibria. These are collected in the following theorem.

*Theorem 3 Let  $(\bar{\alpha}, \bar{\beta})$  be a Cournot solution. Then*

1. *If  $(\bar{\alpha}, \bar{\beta})$  is an interior Cournot equilibrium then*

$$\bar{\alpha} = 1 - \frac{m}{m+1} \frac{p^{\frac{m}{m+1}}}{\left(p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}\right)} \frac{pk+1}{pk} \quad (9)$$

$$\bar{\beta} = 1 - \frac{m}{m+1} \frac{f^{\frac{m}{m+1}}}{\left(p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}\right)} (pk+1) \quad (10)$$

2. *If  $(0, \bar{\beta})$  is a corner Cournot equilibrium where **A** is completely parasitic, then  $\bar{\beta}$  is implicitly defined by*

$$(1 - \bar{\beta})^{m+1} = f^m k^m [\bar{\beta}(m+1) - 1] \quad (11)$$

3. *If  $(\bar{\alpha}, 0)$  is a corner Cournot equilibrium where **B** is completely parasitic, then  $\bar{\alpha}$  is implicitly defined by*

$$(1 - \bar{\alpha})^{m+1} f^m k^m = \bar{\alpha}(m+1) - 1 \quad (12)$$

*Furthermore  $(\bar{\alpha}, \bar{\beta})$  will be an interior Cournot equilibrium if, and only if,*

$$p \frac{m}{\left[ 1 + (m+1) \left( \frac{f}{p} \right)^{\frac{m}{m+1}} \right]} \leq k \leq \frac{1 + (m+1) \left( \frac{p}{f} \right)^{\frac{m}{m+1}}}{mp} \tag{13}$$

*A will be completely parasitic if the first inequality is reversed and B will be completely parasitic if the last inequality is reversed.*

The theorem is useful not only because it provides expressions for the equilibrium levels of cooperativeness of each player, but because it also indicates some of the more subtle properties of the solution. In the first instance, none of the solutions depend on either  $c$  or  $W$ . This implies that the optimal level of production never depends on the baseline wealth or productiveness of society. Secondly, the equations for the optimal cooperativeness at a corner are independent of  $p$ . This implies that once one of the players has become completely parasitic, changes in the relative productivity of the players no longer has an effect (except in so far as it may shift a player right out of the corner). The player who is left as the sole producer will fix the optimal level of production only in terms of the relative wealth and relative ability to appropriate the product. We will return to this point below.

### 6. THE IMPACT OF THE PARAMETERS

The results in the theorem above can be used also to explore the comparative statics of the model. As diagram 1 suggests, increases in the relative wealth of  $A$  (i.e.  $k$ ) would make  $A$  more cooperative and  $B$  less so. It is less evident how the other parameters would affect the outcome, particularly at the corners. The comparative statics can be summarised in the following Theorem:

*Theorem 4 Let  $(\bar{\alpha}, \bar{\beta})$  be any Cournot equilibrium. Then*

$$\partial \bar{\alpha} / \partial k \geq 0, \partial \bar{\beta} / \partial k \leq 0$$

$$\partial \bar{\alpha} / \partial p \geq 0, \partial \bar{\beta} / \partial p \leq 0$$

$$\partial \bar{\alpha} / \partial f \geq 0, \partial \bar{\beta} / \partial f \leq 0$$

$$\partial \bar{\alpha} / \partial m \geq (\leq) 0 \text{ iff } g_A / g_B \geq (\leq) \zeta, \partial \bar{\beta} / \partial m \geq (\leq) 0 \text{ iff } g_B / g_A \geq (\leq) \zeta$$

where  $\zeta$  is the solution of the equation  $\ln \zeta = 1/\zeta + 1$ , i.e.  $\zeta \approx 3.591121$

These results, except for the last one, seem intuitively obvious and extend the findings of Hirshleifer's simulations (1988, 1991a). We will discuss them briefly one by one.

(1). *Inequality of wealth:* An increase in  $A$ 's wealth relative to  $B$  makes  $A$  more cooperative and  $B$  less so. As a player becomes more wealthy, a smaller proportion of resources devoted to claiming will have the same effect. Consequently the individual who becomes more

affluent can afford to devote more resources to production. The poorer individual, becomes more specialised in claiming. This is part of what drives Hirshleifer's "paradox of power".

(2). *Relative productivity*: If  $A$  becomes more productive relative to  $B$ , then  $A$  would tend to become more cooperative. The result is again intuitive: the costs of production foregone increase for the person who becomes more productive. For the less productive individual the gains from own production start looking less attractive relative to what can be gained by claiming from the other player.

(3). *Changes in claiming efficiency*: If  $A$  becomes more effective in establishing claims relative to  $B$ , then  $A$  becomes more cooperative. This result makes sense if one remembers that an increasing claiming effectiveness implies that  $A$  gets to keep a larger share of her output. It therefore becomes in her interest to enlarge the output. For the player who loses ground in the claiming stakes, it is in their interest to increase their claiming effort and so reduce the productive effort.

(4). *Changes in the decisiveness of claiming*: If claiming becomes more decisive, then we would expect both players to spend more time claiming and less time producing. In fact the result above is nicely ambivalent. If the players are relatively evenly matched, then an increase in the decisiveness parameter would unambiguously increase the amount of claiming activities. However, if there is a large imbalance in the power of the players, then the player who currently extracts the lion's share would actually become more productive, since this power now obviously translates into a much larger impact. This feature – greater decisiveness increases all claiming when players are evenly matched, but may reduce it when the contest is lopsided – seems to be an important insight that is not apparent from the Hirshleifer simulations.

The above results are true for marginal changes in the parameters around a given equilibrium. Larger changes in the parameters will potentially change the nature of the equilibrium, e.g. driving an interior solution to one or other of the corners. We can characterise the impact of these changes also:

*Theorem 5 The limiting behaviour of the Cournot equilibrium  $(\bar{\alpha}, \bar{\beta})$  consequent on changes in the parameters is as follows:*

1. *Impact of  $p$*

As  $p \rightarrow \infty$  player  $B$  becomes completely parasitic. The limiting value of  $\bar{\alpha}$  will be implicitly defined by equation 12.

As  $p \rightarrow 0$  player  $A$  becomes completely parasitic and the limiting value of  $\bar{\beta}$  will be defined by equation 11.

2. *Impact of  $f$*

$$\lim_{f \rightarrow \infty} \bar{\alpha} = 1 \text{ and } \lim_{f \rightarrow \infty} \bar{\beta} = (1 - mpk)/(m + 1) \text{ if } k \leq 1/mp$$

$$\lim_{f \rightarrow \infty} \bar{\alpha} = 1 \text{ and } \lim_{f \rightarrow \infty} \bar{\beta} = 0 \text{ if } k > 1/mp$$

$$\lim_{f \rightarrow 0} \bar{\alpha} = (pk - m)/[(m + 1)pk] \text{ and } \lim_{f \rightarrow 0} \bar{\beta} = 1 \text{ if } k \geq m/p$$

$\lim_{f \rightarrow 0} \bar{\alpha} = 0$  and  $\lim_{f \rightarrow 0} \bar{\beta} = 1$  if  $k < m/p$

3. Impact of  $k$

$\lim_{k \rightarrow \infty} \bar{\alpha} = 1$  and  $\lim_{k \rightarrow \infty} \bar{\beta} = 0$

$\lim_{k \rightarrow 0} \bar{\alpha} = 0$  and  $\lim_{k \rightarrow 0} \bar{\beta} = 1$

4. Impact of  $m$

$\lim_{m \rightarrow \infty} \bar{\alpha} = 1 - 1/fk$  and  $\lim_{m \rightarrow \infty} \bar{\beta} = 0$  if  $fk > 1$

$\lim_{m \rightarrow \infty} \bar{\alpha} = 0$  and  $\lim_{m \rightarrow \infty} \bar{\beta} = 0$  if  $fk = 1$

$\lim_{m \rightarrow \infty} \bar{\alpha} = 0$  and  $\lim_{m \rightarrow \infty} \bar{\beta} = 1 - fk$  if  $fk < 1$

$\lim_{m \rightarrow 0} \bar{\alpha} = 1$  and  $\lim_{m \rightarrow 0} \bar{\beta} = 1$

In most cases large enough changes in the parameters will shift the solution to where one of the players becomes completely parasitic. Often the player who is richest and most effective in claiming also becomes most focussed on production, while the weak and poor player ends up as the parasite. This is one of the dimensions of Hirshleifer’s “Paradox of Power”.

7. WELFARE ANALYSIS

We are now ready to analyse the societal impacts of predation and how these are affected by changes in the parameters. In order to do this we define as our index of the welfare loss:

$$I = (Y_{\max} - Y_{AB})/Y_{\max} \tag{14}$$

where  $Y_{\max} = c(pk + 1)W$ , i.e. it is the maximum output that could be produced if both players produced at full capacity. Obviously  $I$  is bounded between zero and one.

*Theorem 6* Let  $(\bar{\alpha}, \bar{\beta})$  be any Cournot equilibrium. Then

1. If  $(\bar{\alpha}, \bar{\beta})$  is an interior Cournot equilibrium then  $I = m/(m + 1)$  and hence

$$\partial I / \partial p = \partial I / \partial k = \partial I / \partial f = 0$$

2. If  $(\bar{\alpha}, 0)$  is a corner Cournot equilibrium where **B** is completely parasitic, then

$$\partial I / \partial p < 0, \partial I / \partial k < 0, \partial I / \partial f < 0$$

3. If  $(0, \bar{\beta})$  is a corner Cournot equilibrium where **A** is completely parasitic, then

$$\partial I / \partial p > 0, \partial I / \partial k > 0, \partial I / \partial f > 0$$

4. The behaviour of  $I$  in the limit is given by

$$\lim_{p \rightarrow \infty} I = 1 - a \text{ where } a \text{ is defined by equation 12}$$

$$\lim_{p \rightarrow 0} I = 1 - \beta \text{ where } \beta \text{ is defined by equation 11}$$

$$\lim_{k \rightarrow \infty} I = 0, \lim_{k \rightarrow 0} I = 0$$

$$\lim_{f \rightarrow \infty} I = 0 \text{ if } k > 1/mp, \lim_{f \rightarrow \infty} I = m/(m+1) \text{ if } k \leq 1/mp$$

$$\lim_{f \rightarrow 0} I = 0 \text{ if } k < m/p, \lim_{f \rightarrow 0} I = m/(m+1) \text{ if } k \geq m/p$$

$$\lim_{m \rightarrow \infty} I = (p+f)/(pfk+f) \text{ if } fk > 1$$

$$\lim_{m \rightarrow \infty} I = 1 \text{ if } fk = 1$$

$$\lim_{m \rightarrow \infty} I = (pk+fk)/(pk+1) \text{ if } fk < 1$$

$$\lim_{m \rightarrow 0} I = 0$$

The most startling implication of these findings is that the welfare losses tend to be biggest when the players are relatively evenly matched. It is only in the “corner” equilibria that the losses start to decline. The reason for these relatively big losses is due to the fact that evenly matched players can inflict a lot of damage on each other. They therefore have to invest a lot in claiming capacity. This result is not so surprising when one considers the level of military expenditure that relatively evenly matched countries deem necessary for their own survival. The reason why this finding strikes us as paradoxical is that in fairly equal societies the net parasitism rate should be fairly low. Indeed this is the case, even in our model. In Fig. 2 we contrast the evolution of social welfare losses attendant on increasing inequality ( $k$ ) with the evolution of the individual losses. The latter we define as the proportional difference between what the individual produces versus what they end up getting. In this case it is  $(Y_A - Y_1)/Y_A$ . At very low levels of inequality the welfare losses are high, but the individual losses are low. The reason for this divergence is that the “claiming” expenditures of the two players cancel each other out. This welfare loss is large, but since each player gets more or less what they produce, the players would not find the end result unfair. The net parasitism rate is low, but the welfare loss is high.

At high levels of inequality both the individual losses and the social welfare losses start to decline. The reason for this is that the weaker player simply lacks the resources to inflict meaningful damage on the stronger one. Again this is a result that has resonance in real developments: it was only when the poorer sections of the South African population gained access to weaponry, the money to engage lawyers or to corrupt the police that crime for the richer citizens became a notable problem. We observe (in Figure 2) that the individual losses can be substantially larger than the social losses. The reason for this is that the actual losses sustained by the richer individuals are transferred to the poorer

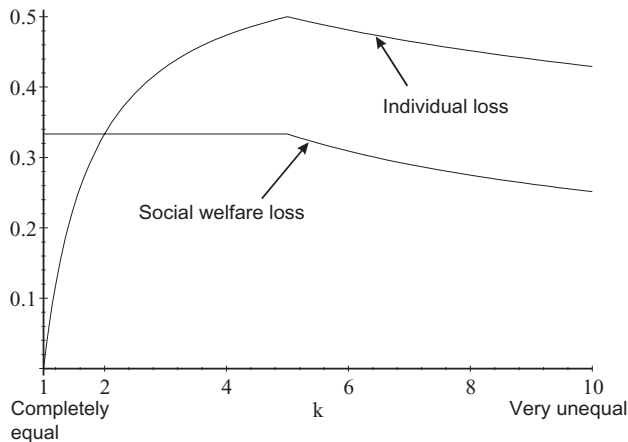


Figure 2. Social welfare losses and individual losses (to the richer player) with increases in inequality. At  $k = 1$  social welfare losses are high, but individual losses are zero. Parameter settings:  $f = 1$ ,  $p = 1$ ,  $m = 0.5$

player. The social losses, however, are the deadweight losses: the overall reduction in output irrespective of how this output is actually divided.

An efficient outcome, *i.e.* one in which  $I = 0$ , is reached under two scenarios: firstly if  $m = 0$ , *i.e.* if the allocation rule is completely insensitive to claiming activity. In this case both players focus entirely on production because there is simply no point in doing anything else. The second efficient outcome occurs if the strength of one of the players becomes completely overwhelming. This happens either when inequality  $k$  is driven to either extreme, or when  $f$  becomes extreme, with  $k$  sufficiently large (or small) also. In this case the outcome is reasonably efficient simply because the parasitic player is so insignificant, that the output lost is so small relative to the output produced that it hardly matters.

Interestingly enough, increasing the productivity, strength or wealth of the parasitic player will increase the welfare losses. There are several effects at work here. Small changes in these parameters will not induce the player to start becoming productive. They do, however, increase the damage that such claiming can inflict on the productive player, thus increasing defensive claiming activities. On top of this, some of these changes push out the production possibilities frontier, *i.e.*  $Y_{\max}$ , and hence increase the size of the potential loss.

The implication of this analysis is that high inequality societies might paradoxically be more efficient than low inequality societies, when the effectiveness of claiming is large. Theft, malicious litigation and political lobbying all require resources. If the poor player is sufficiently marginalised she does not have sufficient resources to launch serious challenges. Furthermore she would not be capable of producing all that much either. On both counts the efficiency losses from the predation are consequently small.

## 8. CONCLUSION

In this paper we have set up and solved out a generalised version of the Hirshleifer-Skaperdas predation model. This has allowed us to investigate the determinants of



predation and the attendant welfare losses in more detail than has hitherto been possible. The most striking conclusion to arise from this analysis is that predation is minimised under two quite different regimes. If the institutional framework makes claiming very ineffective we would expect people to focus on productive work. This is the Smithian solution. Strong, clear property rights can minimise unproductive claiming. If, however, claiming is effective, then the effects of predation can be minimised only if one of the players becomes marginalised. The worst outcomes seem to arise when claiming is effective, but inequality in power is significant but not extreme. This, arguably, is the situation in a number of transition societies.

A third route through which the effect of predation might be minimised is through repeated interactions. It is, for instance, well known that cooperative outcomes can be sustained as subgame perfect equilibria in the repeated version of the Prisoners' Dilemma. Since our game can be thought of as a continuous version of the discrete choice Prisoners' Dilemma the same logic indicates that 100% cooperation should be sustainable in the repeated version of the predation game. A simple trigger strategy of the sort "cooperate at 100% until any player defects, then play the Nash equilibrium" should be sufficient. This suggests that cooperation may be sustainable even in contexts where claiming is effective, provided that the agents in that society expect repercussions from their actions in the future. The question of which institutions would promote such a long-term horizon is beyond the scope of this paper. Social norms, moral codes and religion may all have a role to play. Indeed this is a fascinating avenue for research. Nevertheless we leave a proper analysis of a dynamic version of this game for future work. The fact that we have closed form solutions of the static model should facilitate such analyses. Indeed it should also enable extensions to this predation framework and its articulation with other types of models. One possible direction in which such work could go would be to analyse the "evolutionary" logic of such a system given agents of limited memory. This line of enquiry pioneered by Axelrod (1984) has led to a burgeoning research programme (see for instance Nowak and Sigmund, 1992, Nowak and Sigmund, 1993).

#### APPENDIX – Proofs

We will generally present proofs for only one of the players. By a suitable retranslation of the parameters, we can deduce the results for the other player.

*Proposition A.1 For a given value of  $\beta$  we have that  $\partial Y_1/\partial a \geq (\leq) 0$  as  $Y_2 \leq (\geq) (1 - a)/m \cdot \partial Y_{AB}/\partial a$ . Similarly for a given value of  $a$  we have that  $\partial Y_2/\partial \beta \geq (\leq) 0$  as  $Y_1 \leq (\geq) (1 - \beta)/m \cdot \partial Y_{AB}/\partial \beta$*

*Proof.* This follows by straightforward differentiation of the payoff functions  $Y_1$  and  $Y_2$ .

We have  $Y_1 = g_A Y_{AB}$  where  $g_A = \frac{s_1^m}{s_1^m + s_2^m}$  so

$$\frac{\partial Y_1}{\partial \alpha} = \frac{m s_1^{m-1} \frac{\partial s_1}{\partial \alpha}}{s_1^m + s_2^m} \left[ \frac{s_2^m}{s_1^m + s_2^m} Y_{AB} + \frac{s_1}{m} \frac{\partial Y_{AB}}{\partial \alpha} \right]$$

since  $s_2$  does not depend on  $\alpha$ . The sign of  $\partial Y_1/\partial\alpha$  therefore depends only on the expression in square brackets. Furthermore we have  $\partial s_1/\partial\alpha < 0$  so the sign of  $\partial Y_1/\partial\alpha$  will be the opposite of the term in brackets. We can simplify the latter by noting that  $s_1 = f_A(1 - \alpha)W_A$ , so  $s_1 \left/ \frac{\partial s_1}{\partial\alpha} \right. = -(1 - \alpha)$ , i.e. the expression in square brackets can be written as  $g_B Y_{AB} - (1 - \alpha)/m \cdot \partial Y_{AB}/\partial\alpha$ . Since  $Y_2 = g_B Y_{AB}$  the result follows.

Note that this result depends only on the nature of the appropriation function, it does not depend on the nature of the aggregate production function.

*Proposition A.2 Given the choices of functional forms in section 3, the payoff function  $Y_1(\alpha|\beta, k, p, f, m, c, W)$  can be of two types:*

1. It can have its global maximum in the interior, i.e.  $\alpha \in (0, 1)$
2. It can have its global maximum at  $\alpha = 0$

*Proof.* The proof follows from proposition A.1. We note that  $(1 - \alpha)/m \cdot \partial Y_{AB}/\partial\alpha$  is monotonically decreasing in  $\alpha$  with  $(1 - \alpha)/m \cdot \partial Y_{AB}/\partial\alpha = 0$  when  $\alpha = 1$ , while at this point  $Y_2 > 0$ , hence  $\partial Y_1/\partial\alpha < 0$  near  $\alpha = 1$ . Furthermore  $Y_2$  is monotonically increasing in  $\alpha$ , hence we need consider only what happens at  $\alpha = 0$ . If  $Y_2 > 1/m \cdot \partial Y_{AB}/\partial\alpha$  at  $\alpha = 0$ , then  $\partial Y_1/\partial\alpha$  will be negative on the entire interval  $[0, 1)$ . If, however, the payoff function initially slopes upward, then it must reach a turning point from where it decreases. Hence it must have a global maximum in the interior. It follows from this proposition that  $A$ 's reaction function will be given by the solutions to the equation  $\partial Y_1/\partial\alpha = 0$  (where a turning point on the payoff function exists) and otherwise by  $\alpha = 0$ .

*Proposition A.3 The conditions  $\partial Y_1/\partial\alpha = 0$  and  $\partial Y_2/\partial\beta = 0$  can be written respectively as*

$$\frac{(1 - \beta)^m}{(1 - \alpha)^m f^m k^m + (1 - \beta)^m} (p\alpha k + \beta) = \frac{(1 - \alpha)pk}{m} \tag{15}$$

$$\frac{(1 - \alpha)^m f^m k^m}{(1 - \alpha)^m f^m k^m + (1 - \beta)^m} (p\alpha k + \beta) = \frac{(1 - \beta)}{m} \tag{16}$$

*Proof.* This follows from proposition A.1

*Proposition A.4 The slope of  $A$ 's reaction function  $\alpha = r_A(\beta)$  at any interior point will be given by*

$$\frac{\partial\alpha}{\partial\beta} = - \frac{-\frac{m}{(1 - \alpha)} \frac{\partial Y_2}{\partial\beta} + \frac{\partial^2 Y_{AB}}{\partial\beta\partial\alpha}}{-\frac{m}{(1 - \alpha)^2} Y_2 - \frac{m}{(1 - \alpha)} \frac{\partial Y_2}{\partial\alpha} + \frac{\partial^2 Y_{AB}}{\partial\alpha^2}} \tag{17}$$

*provided that the denominator is not zero.*

*Proof.* This follows from the implicit function theorem. An interior optimum will be at a point where  $\partial Y_1/\partial\alpha = 0$ . From proposition A.1 we note that the locus of solutions can be defined implicitly by  $-m/(1 - \alpha)Y_2 + \partial Y_{AB}/\partial\alpha \equiv 0$ . Letting  $F(\alpha, \beta) = -m/(1 - \alpha)Y_2 + \partial Y_{AB}/\partial\alpha$  we have that  $\partial\alpha/\partial\beta = -F_\beta/F_\alpha$ . The result follows.

Proposition A.5 *With our choice of functional forms, the slope of A's reaction function at any point  $(\alpha, \beta)$  with  $\alpha = r_A(\beta)$  will be such that  $\partial\alpha/\partial\beta \leq (\geq) 0$  if, and only if  $\partial Y_2(a, \beta)/\partial\beta \geq (\leq) 0$*

Proof. We note that with our choice of functional form we have  $\partial^2 Y_{AB}/\partial\beta\partial\alpha = \partial^2 Y_{AB}/\partial\alpha^2 = 0$ , i.e. equation 17 simplifies to

$$\frac{\partial\alpha}{\partial\beta} = -\frac{(1-\alpha)\frac{\partial Y_2}{\partial\beta}}{Y_2 + (1-\alpha)\frac{\partial Y_2}{\partial\alpha}}$$

and the denominator of this is guaranteed to be positive.

We are now ready to prove Proposition 1 of the main text.

Proof of Proposition 1. The proof is in three parts. First we show that near  $\beta = 0$  and  $\beta = 1$  A's optimal response is in the interior, and, indeed, A's optimal response will be large. In other words both if B is completely parasitic and if B is completely cooperative, it is in A's interest to produce a positive amount. Secondly we then consider the slope of the reaction function  $\partial\alpha/\partial\beta$ . We show that if this reaction function is ever positive at any  $\beta^*$ , it will be positive for all  $\beta$  to the right of  $\beta^*$ . From this we can deduce what shapes the reaction functions can take. In the third part, we show that the shape of A's reaction function has implications for the shape of B's reaction function and *vice versa*.

The proof of the first part is a straightforward application of proposition A.1. We show that if  $\beta = 0$  then if  $\alpha = 0$  we evidently have  $Y_2 = 0$ , while  $(1-\alpha)/m \cdot \partial Y_{AB}/\partial\alpha > 0$ . It follows that the payoff function has an interior maximum. This maximum satisfies equation 15. Substituting in  $\beta = 0$  we see that the optimal  $\alpha$  has to satisfy the equation

$$(1-\alpha)^{m+1} f^m k^m = \alpha(m+1) - 1$$

This will have a solution in the interval  $(1/(m+1), 1)$ . Hence it follows that all A's reaction functions start with  $\alpha > 1/(m+1)$ . Near  $\beta = 1$  we will have  $Y_2$  near zero and  $(1-\alpha)/m \cdot \partial Y_{AB}/\partial\alpha > 0$ , hence there will also be an interior solution. From this it follows that as  $\beta \rightarrow 1$  we must have  $\alpha \rightarrow 1$ , since otherwise equality in equation 15 could not obtain.

To prove the second part, we consider a point  $(\alpha^*, \beta^*)$  on A's reaction curve, i.e.  $\alpha^* = r_A(\beta^*)$  where by assumption  $\partial\alpha/\partial\beta > 0$ . We have, by proposition A.5 that  $\partial\alpha/\partial\beta > 0$  if, and only if  $\partial Y_2(\alpha, \beta)/\partial\beta < 0$ . Now of course proposition A.1 implies that  $\partial Y_2(\alpha, \beta)/\partial\beta < 0$  if, and only if,  $Y_1 > (1-\beta)/m \cdot \partial Y_{AB}/\partial\beta$ . We show that along A's reaction curve we must have  $Y_1$  increasing. Totally differentiating, we have

$$dY_1/d\beta = \partial Y_1/\partial\beta + \partial Y_1/\partial\alpha \cdot d\alpha/d\beta$$

Now at every point along A's reaction function we have  $\partial Y_1/\partial\alpha = 0$  so we must have  $dY_1/d\beta > 0$ . So if at  $\beta^*$  we have  $Y_1 > (1-\beta)/m \cdot \partial Y_{AB}/\partial\beta$  then it must be true at every  $\beta > \beta^*$ , since the right hand side of this inequality is independent of  $\alpha$  and decreasing in  $\beta$ . It follows that if  $\partial\alpha/\partial\beta > 0$  at  $(\alpha^*, \beta^*)$  then it will stay positive for  $\beta > \beta^*$ . This means that the behaviour of the reaction function depends on its behaviour at  $\beta = 0$ . If it is

increasing at this point, then it will continue to increase, *i.e.* we will have a type one reaction function. If it decreases, then it must eventually reverse its direction, since we have seen that as  $\beta \rightarrow 1$  we must have  $\alpha \rightarrow 1$ . This reversal of direction can happen in one of two ways: the locus of solutions to the equation  $\partial Y_1/\partial \alpha = 0$  can have a turning point, *i.e.* at this point  $\partial \alpha/\partial \beta = 0$ , or it can become undefined, in which case the reaction function has reached the corner solution  $\alpha = 0$  at some value  $\beta_1$ . At a higher value  $\beta_2$ , however, the reaction function will take on positive values again and from this point on (since the slope is now positive), it will converge on one. The former corresponds to our type two reaction curves, while the latter to a type 3 curve.

To prove the third part, we note that if  $A$  has a type 1 reaction function then at  $\beta = 0$  we have  $\partial \alpha/\partial \beta > 0$ . By proposition A.5 we must have at  $A$ 's optimal response  $\alpha^* = r_A(0)$  that  $\partial Y_2/\partial \beta < 0$ . This means, in turn, by proposition A.2 that  $B$ 's payoff function when  $\alpha = \alpha^*$  is monotonically decreasing in  $\beta$ . There is therefore no interior solution to the equation  $\partial Y_2/\partial \beta = 0$  at this  $\alpha$ .  $B$ 's reaction function is therefore of type 3 and it is at a corner when  $\alpha = \alpha^*$ .

If  $A$  has a type two reaction function, then its turning point will be at a point  $(\bar{\alpha}, \bar{\beta})$  where  $\partial \alpha/\partial \beta = 0$ . By proposition A.5 we must have  $\partial Y_2/\partial \beta = 0$  at this point. This, however, implies that  $B$ 's payoff function reaches its maximum also at  $(\bar{\alpha}, \bar{\beta})$ , *i.e.* this is the point at which the reaction functions intersect. This is sufficient to show that  $B$  also has a type two reaction function.

If  $A$  has a type three reaction function, then there will be a range of values of  $\beta$  for which the best response is at  $\alpha = 0$ . For these values of  $\beta$  we will have  $\partial Y_1/\partial \alpha < 0$  at  $\alpha = 0$ . By proposition A.1 we must have  $Y_2 > 1/m \cdot \partial Y_{AB}/\partial \alpha$  in this range. But  $B$ 's optimal response  $\beta^*$  to  $A$ 's choice  $\alpha = 0$ , *i.e.*  $\beta^* = r_B(0)$  must fall into this range, since if for some  $\beta$  we have  $Y_2 > 1/m \cdot \partial Y_{AB}/\partial \alpha$ , then certainly  $\max \beta Y_2 > 1/m \cdot \partial Y_{AB}/\partial \alpha$ . Now at  $\beta^*$  we have  $\partial Y_1/\partial \alpha < 0$ , so by proposition A.5 (reinterpreted for  $B$ ) we must have the slope of  $r_B$  upwards sloping, *i.e.* it is a type one reaction function.

Proof of Proposition 2. We have proved proposition 2 in the process of proving proposition one.

Proposition A.6 *Assume that we have an interior Cournot equilibrium. At this equilibrium we will have*

$$\frac{g_A}{g_B} = \frac{(1-\beta) \frac{\partial Y_{AB}}{\partial \beta}}{(1-\alpha) \frac{\partial Y_{AB}}{\partial \alpha}} \tag{18}$$

$$Y_{AB} = \frac{(1-\alpha)}{m} \frac{\partial Y_{AB}}{\partial \alpha} + \frac{(1-\beta)}{m} \frac{\partial Y_{AB}}{\partial \beta} \tag{19}$$

Furthermore if  $Y_{AB}$  is homogeneous of degree one in  $\alpha$  and  $\beta$ , then

$$Y_{AB} = \frac{1}{m+1} \left( \frac{\partial Y_{AB}}{\partial \alpha} + \frac{\partial Y_{AB}}{\partial \beta} \right)$$

Proof. By proposition A.1 the reaction functions must satisfy (respectively)

$$g_B Y_{AB} = (1 - \alpha) / m \cdot \partial Y_{AB} / \partial \alpha$$

$$g_A Y_{AB} = (1 - \beta) / m \cdot \partial Y_{AB} / \partial \beta$$

Dividing the latter equation by the former produces the first equation. Adding the two equations produces the second. Now equation 19 implies that  $m Y_{AB} = \partial Y_{AB} / \partial \alpha + \partial Y_{AB} / \partial \beta - (\alpha \cdot \partial Y_{AB} / \partial \alpha + \beta \cdot \partial Y_{AB} / \partial \beta)$ . If  $Y_{AB}$  is homogeneous of degree one in  $\alpha$  and  $\beta$  then  $\alpha \cdot \partial Y_{AB} / \partial \alpha + \beta \cdot \partial Y_{AB} / \partial \beta = Y_{AB}$  (Euler's theorem). Rearranging produces the result.

Proposition A.7 *Let  $(\bar{\alpha}, \bar{\beta})$  be any interior Cournot equilibrium. Then with our functional choices*

$$\frac{(1 - \alpha)^{m+1} f^m k^{m+1}}{(1 - \beta)^{m+1}} = \frac{1}{p} \tag{20}$$

$$pk[a(m + 1) - 1] + \beta(m + 1) - 1 = 0 \tag{21}$$

Proof. Substitute into equations 18 and 19.

Proposition A.8 *At any interior Cournot equilibrium we will have*

$$g_A = \frac{f^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}}, g_B = \frac{p^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}}$$

$$\text{and } Y_1 = \frac{f^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}} \frac{c(pk + 1)W}{m + 1}, Y_2 = \frac{p^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}} \frac{c(pk + 1)W}{m + 1}$$

Proof. We can rewrite equation 20 as  $\frac{(1 - \alpha)^{m+1} f^{m+1} k^{m+1}}{(1 - \beta)^{m+1}} = \frac{f}{p}$ , hence

$$\frac{(1 - \alpha)^m f^m k^m}{(1 - \beta)^m} = \left(\frac{f}{p}\right)^{\frac{m}{m+1}}$$

The left hand side of this is just  $g_A/g_B$ . Then exploiting the fact that  $g_A + g_B = 1$ , we can solve for  $g_A$  and  $g_B$ . To get the expressions for  $Y_1$  and  $Y_2$  we note that  $Y_1 = g_A Y_{AB}$  and  $Y_2 = g_B Y_{AB}$ . By proposition A.6 we have that  $Y_{AB} = c(pk + 1)W/(m + 1)$ . The result follows.

Proof of Theorem 3. We will derive the conditions for the interior equilibrium first.

Equation 20 in proposition A.7 can be rewritten as  $\frac{(1 - \alpha)^{m+1} f^{m+1} k^{m+1}}{(1 - \beta)^{m+1}} = \frac{f}{p}$ , i.e.

$$\frac{(1 - \alpha)fk}{(1 - \beta)} = \left(\frac{f}{p}\right)^{\frac{1}{m+1}} \tag{22}$$

so  $(1 - \alpha)fk = (1 - \beta) \left(\frac{f}{p}\right)^{\frac{1}{m+1}}$ . From equation 21 we get  $k = [1 - \beta(m + 1)] /$

$[p(\alpha(m + 1) - 1)]$ . Substituting this in, and cross-multiplying (including the term  $f/p$  on the LHS) we get

$$(1 - \alpha)[1 - \beta(m + 1)] = \left(\frac{p}{f}\right)^{\frac{m}{m+1}} (1 - \beta)[\alpha(m + 1) - 1]$$

Collecting up terms in  $\alpha$  and simplifying we get

$$1 - \alpha = \frac{\left(\frac{p}{f}\right)^{\frac{m}{m+1}} (1 - \beta)m}{\left(\frac{p}{f}\right)^{\frac{m}{m+1}} (m + 1)(1 - \beta) - \beta(m + 1) + 1}$$

We substitute this expression back into the equation 22 and solve for  $\beta$ . This gives us equation 10 of the Theorem. With this expression we substitute back into the equation above, to obtain the equilibrium value of  $\alpha$ , given in equation 9 of the Theorem.

The expressions for the corner equilibria follow by substituting  $\beta = 0$  into  $A$ 's reaction function (equation 15) and  $\alpha = 0$  into  $B$ 's reaction function respectively.

The solutions given in equations 9 and 10 represent a legitimate equilibrium only if both formulae evaluate to non-negative quantities (they are guaranteed to produce values less than one). Imposing the conditions  $\bar{\alpha} \geq 0$  and  $\bar{\beta} \geq 0$  and simplifying the respective equations yields the condition in inequality 13 of the Theorem.

To get the reverse implication, we consider the corner equilibrium  $(\bar{\alpha}, \bar{\beta})$  with  $B$  parasitic and show that the inequality must be reversed. In this case the following two conditions must be satisfied at the point  $(\bar{\alpha}, 0)$ :

$$\frac{1}{(1 - \alpha)^m f^m k^m + 1} (p\alpha k) = \frac{(1 - \alpha)pk}{m} \tag{23}$$

$$\frac{(1 - \alpha)^m f^m k^m}{(1 - \alpha)^m f^m k^m + 1} (p\alpha k) \geq \frac{1}{m} \tag{24}$$

The first follows from equation 15 with  $\beta = 0$  and the second from the fact that  $\partial Y_2 / \partial \beta \leq 0$  at  $\alpha$  which implies by proposition A.1 that  $Y_1 \geq (1 - \beta)/m \cdot \partial Y_{AB} / \partial \beta$ . When we divide each side of inequality 24 by the corresponding side of equation 23 we get

$$(1 - \alpha)^m f^m k^m \geq 1 / [(1 - \alpha)pk]$$

*i.e.*  $(1 - \alpha)^{m+1} f^{m+1} k^{m+1} > fp$ , and thus

$$(1 - \alpha)^m f^m k^m \geq \left(\frac{f}{p}\right)^{\frac{m}{m+1}} \tag{25}$$

Equation 23 can be rearranged to show that  $(1 - \alpha)^m f^m k^m = [\alpha(m + 1) - 1] / (1 - \alpha)$ . Substituting this into the left-hand side of the inequality above and simplifying we get

$$\alpha \geq \frac{1 + \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{(m+1)\left(\frac{p}{f}\right)^{\frac{m}{m+1}} + 1}$$

From this it follows that

$$\frac{1}{1-\alpha} \geq \frac{(m+1)\left(\frac{p}{f}\right)^{\frac{m}{m+1}} + 1}{m\left(\frac{p}{f}\right)^{\frac{m}{m+1}}}$$

We can rewrite inequality 25 as

$$k \geq \frac{1}{(1-\alpha)f} \left(\frac{f}{p}\right)^{\frac{1}{m+1}}$$

Substituting in for  $1/(1-\alpha)$  and simplifying yields

$$k \geq \frac{(m+1)\left(\frac{p}{f}\right)^{\frac{m}{m+1}} + 1}{mp}$$

We have therefore shown that  $(\bar{\alpha}, \bar{\beta})$  is a corner Cournot equilibrium with  $B$  parasitic *only if* this last condition holds. The conditions for the other corner equilibrium follow similarly.

Since there is always guaranteed to be a Cournot equilibrium; since the three cases exhaust all possibilities; and because these equilibria are uniquely defined by  $k, p, f$  and  $m$ , the opposite implications follow.

Proof of Theorem 4. In order to prove Theorem 4 we need to consider separately the interior from the corner Cournot equilibria. Starting with the interior equilibria,  $\alpha$  will be given by equation 9. Differentiating this we get

$$\frac{\partial \bar{\alpha}}{\partial k} = \frac{m}{m+1} \frac{p^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}} \frac{1}{pk^2} > 0$$

$$\frac{\partial \bar{\alpha}}{\partial p} = \frac{m}{m+1} \frac{p^{\frac{m}{m+1}}}{p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}} \frac{\bar{\beta}}{p^2 k} \geq 0$$



$$\frac{\partial \bar{\alpha}}{\partial f} = \frac{m^2}{(m+1)^2} \frac{p^{\frac{m}{m+1}}}{f^{\frac{m}{m+1}} \left( p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}} \right)^2} \frac{pk+1}{pk} > 0$$

In order to calculate the comparative statics with respect to  $m$ , it is in fact easier to calculate them with respect to the variable  $\mu = m/(m + 1)$ . Since  $d\mu/dm > 0$  the sign of  $\partial \bar{\alpha} / \partial \mu$  will correspond to the sign of  $\partial \bar{\alpha} / \partial m$ . It is also more convenient to rewrite  $\bar{\alpha}$  as

$$\bar{\alpha} = 1 - \frac{\mu}{1 + \left(\frac{f}{p}\right)^\mu} \frac{pk+1}{pk}$$

Then

$$\frac{\partial \bar{\alpha}}{\partial \mu} = - \frac{1 + \left(\frac{f}{p}\right)^\mu - \mu \left(\frac{f}{p}\right)^\mu \ln\left(\frac{f}{p}\right)}{\left[1 + \left(\frac{f}{p}\right)^\mu\right]^2} \left(\frac{pk+1}{pk}\right)$$

The sign of this expression depends only on the sign of  $1 + \left(\frac{f}{p}\right)^\mu - \mu \left(\frac{f}{p}\right)^\mu \ln\left(\frac{f}{p}\right)$ . We

can rewrite this as  $-\left(\frac{f}{p}\right)^\mu \left[ \ln\left(\frac{f}{p}\right) - 1 - \frac{1}{(f/p)^\mu} \right]$ . The expression in square brackets is

of the form  $\ln y - 1 - 1/y$ . Consequently  $\partial \bar{\alpha} / \partial \mu \leq (\geq) 0$  as  $(fp)^\mu \leq (\geq) \zeta$  where  $\zeta$  is the root of the equation  $\ln y - 1 - 1/y = 0$ . The result follows, since proposition A.8 shows that at an interior equilibrium  $g_A/g_B$  is equal to the left hand side of the last inequality.

For the corner solution, we proceed by implicit differentiation. We assume that  $B$  is parasitic, so that the optimal  $\alpha$  is given by the solution to the equation

$$(1 - \alpha)^{m+1} f^m k^m = \alpha (m + 1) - 1$$

Let  $q(\alpha) = (1 - \alpha)^{m+1} f^m k^m - \alpha (m + 1) + 1$ . Then  $\bar{\alpha}$  is defined implicitly by  $q(\alpha) = 0$ . We can get the comparative statics on  $\bar{\alpha}$  from the implicit function theorem provided that  $q_\alpha \neq 0$ . We have  $q_\alpha = -(m + 1)(1 - \alpha)^m f^m k^m - (m + 1) < 0$ , so that the numerator will determine the sign.  $q_k = m(1 - \alpha)^{m+1} f^m k^{m-1} > 0$ , hence  $\partial \bar{\alpha} / \partial k > 0$ . Also  $q_f = m(1 - \alpha)^{m+1} f^{m-1} k^m > 0$ , i.e.  $\partial \bar{\alpha} / \partial f > 0$ .

In fact

$$\begin{aligned} \frac{\partial \bar{\alpha}}{\partial k} &= \frac{(1 - \alpha)[\alpha(m + 1) - 1]}{(m + 1)\alpha k} \\ \frac{\partial \bar{\alpha}}{\partial f} &= \frac{m(1 - \alpha)^{m+1} f^{m-1} k^m}{(m + 1)[(1 - \alpha)^m f^m k^m + 1]} \end{aligned} \tag{26}$$

In the case of  $m$  we have  $q_m = (1 - \alpha)^{m+1} f^m k^m \ln [(1 - \alpha) fk] - \alpha$ . It helps to rewrite this. Let  $y = (1 - \alpha)^m f^m k^m$ , then  $q_m = (1 - \alpha) y \ln y^{1/m} - \alpha$ , *i.e.*  $q_m = (1 - \alpha)/m [y \ln y - (\alpha m)/(1 - \alpha)]$ . This expression has to be evaluated at a solution to  $q(\alpha) = 0$ . At such a solution we have  $\alpha m = (1 - \alpha)^{m+1} f^m k^m + (1 - \alpha)$ . Substituting this into the expression for  $q_m$  we get  $q_m = (1 - \alpha)/m [y \ln y - y + 1]$ , *i.e.*  $q_m = (1 - \alpha)/m y [\ln y - 1 + 1/y]$ . So  $q_m \leq (\geq) 0$  as  $(1 - \alpha)^m f^m k^m \leq (\geq) \zeta$ . This final expression, however, is just  $g_A/g_B$  when  $\beta = 0$ .

Proof of Theorem 5. The proof relies on analysing the behaviour of the inequalities in condition 13 to examine which of the three types of equilibria we will get in the limit. Then we examine the limiting behaviour of the appropriate equation (*i.e.* 9 or 12; 10 or 11). Beginning with the parameter  $p$  we have

$$\lim_{p \rightarrow \infty} \frac{1 + (m + 1) \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{mp} = 0$$

$$\lim_{p \rightarrow 0} \frac{m}{p \left[ 1 + (m + 1) \left(\frac{f}{p}\right)^{\frac{m}{m+1}} \right]} = \infty$$

So it is evident that the equilibrium ends up in one or the other corner. The conditions in Theorem 3 then indicate what the equilibrium levels of  $\alpha$  and  $\beta$  will be, since these expressions do not involve  $p$ .

In the case of  $f$

$$\lim_{f \rightarrow \infty} \frac{1 + (m + 1) \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{mp} = \frac{1}{mp}$$

$$\lim_{f \rightarrow 0} \frac{m}{p \left[ 1 + (m + 1) \left(\frac{f}{p}\right)^{\frac{m}{m+1}} \right]} = \frac{m}{p}$$

The behaviour of the model therefore depends on the size of  $k$  relative to these quantities.

Note also that  $\frac{m}{p \left[ 1 + (m + 1) \left(\frac{f}{p}\right)^{\frac{m}{m+1}} \right]} \leq \frac{m}{p}$  and  $\frac{1}{mp} \leq \frac{1 + (m + 1) \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{mp}$ , so if

$m/p \leq k \leq 1/(mp)$  we will have an interior solution regardless of the size of  $f$ . At an interior equilibrium we obtain the limiting behaviour by examining equations 9 and 10. If  $k > 1/(mp)$  there will be some value  $f_0$  so that for  $f > f_0$  we will have

$k > \frac{1 + (m + 1) \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{mp}$ , so we end up at a corner with  $B$  completely parasitic. At this corner the value of  $\alpha$  will be defined implicitly by equation 12, which can also be written as

$$f^m k^m = [\alpha(m + 1) - 1] / (1 - \alpha)^{m+1}$$

As  $f \rightarrow \infty$  the left hand side of this expression becomes unbounded. Equality can obtain only if  $\alpha \rightarrow 1$ . Similarly we can show that if  $k < m/p$  we end up at the corner with  $A$  parasitic as  $f \rightarrow 0$  and that at this corner we must have  $\beta \rightarrow 1$ .

It is trivial to note that as  $k \rightarrow \infty$  or  $k \rightarrow 0$  we have to be at the corner where  $B$  or  $A$  are parasitic, respectively. The same logic used above then shows that  $\alpha \rightarrow 1$  or  $\beta \rightarrow 1$  at these corners.

It is also trivial to note that as  $m \rightarrow 0$  we must have an interior solution and that  $\lim_{m \rightarrow 0} \alpha = \lim_{m \rightarrow 0} \beta = 1$ . Now

$$\lim_{m \rightarrow \infty} \frac{m}{p \left[ 1 + (m + 1) \left(\frac{f}{p}\right)^{\frac{m}{m+1}} \right]} = \lim_{m \rightarrow \infty} \frac{1 + (m + 1) \left(\frac{p}{f}\right)^{\frac{m}{m+1}}}{mp} = \frac{1}{f}$$

It follows that if  $k > 1/f$  we must end up in the corner with  $B$  parasitic as  $m \rightarrow \infty$ , while we end up in the opposite corner if  $k < 1/f$ . Consider the former case first. At this corner the equilibrium value of  $\alpha$  is defined by equation 12 which we now write as

$$\frac{1}{fk} = \left( \frac{(1 - \alpha)^{m+1}}{\alpha(m + 1) - 1} \right)^{\frac{1}{m}}$$

Furthermore

$$\lim_{m \rightarrow \infty} \left( \frac{(1 - \alpha)^{m+1}}{\alpha(m + 1) - 1} \right)^{\frac{1}{m}} = 1 - \alpha$$

So the limiting value of  $\alpha$  as  $m \rightarrow \infty$  must be such that  $1 - \alpha = 1/(fk)$  and hence we obtain  $\alpha = 1 - 1/(fk)$ . By a similar logic we show that  $\beta \rightarrow 1 - fk$  as  $m \rightarrow \infty$  if  $k < 1/f$ .

It remains to consider the case where  $k = 1/f$ . In this case it is possible to construct cases where we have interior equilibria for all finite values of  $m$  (e.g. pick  $k = p = f = 1$ ) and where we have one or the other of the corner equilibria. Note, however, that in this case

$$\lim_{m \rightarrow \infty} = 1 - \frac{m}{m+1} \frac{p^{\frac{m}{m+1}}}{\left(p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}\right)} \frac{pk+1}{pk} = 1 - \frac{p}{p+f} \frac{f^{\frac{p}{f}+1}}{\frac{p}{f}} = 0$$

$$\lim_{m \rightarrow \infty} = 1 - \frac{m}{m+1} \frac{f^{\frac{m}{m+1}}}{\left(p^{\frac{m}{m+1}} + f^{\frac{m}{m+1}}\right)} (pk+1) = 1 - \frac{f}{p+f} \left(\frac{p}{f} + 1\right) = 0$$

and  $1 - 1/(fk) = 1 - fk = 0$  in this case also. It does not matter therefore what precise series of equilibria the model traverses as  $m \rightarrow \infty$ . In all cases  $\alpha \rightarrow 0$  and  $\beta \rightarrow 0$ .

Proof of Theorem 6. The proof of the first part follows from the proof of Proposition A.8, i.e.  $Y_{AB} = c(pk+1)W/(m+1)$ . In the case of a corner equilibrium where  $B$  is completely parasitic we have  $Y_{AB} = cp\alpha kW$ . Consequently

$$I = 1 - pk\alpha/(pk+1) \tag{27}$$

hence

$$\partial I/\partial p = -k\alpha/(pk+1)^2 - pk/(pk+1) \cdot \partial\alpha/\partial p < 0$$

$$\partial I/\partial k = -p\alpha/(pk+1)^2 - pk/(pk+1) \cdot \partial\alpha/\partial k < 0$$

$$\partial I/\partial f = -pk/(pk+1) \partial\alpha/\partial f \leq 0$$

where we have also taken cognisance of the signs of the derivatives in Theorem 4. The results for the other corner equilibrium follow analogously.

To establish what happens at the limit we use the results of Theorem 5. We first observe which type of equilibrium we will end in, and then investigate the limit of  $I$ . For instance we know that as  $k \rightarrow \infty$  we end up with a corner equilibrium in which  $B$  is completely parasitic. In this case  $I$  is given by equation 27 and  $\lim_{k \rightarrow \infty} [1 - pk\alpha/(pk+1)] = 0$ , since  $\lim_{k \rightarrow \infty} \alpha = 1$ . The other results follow analogously.

## REFERENCES

- ACEMOGLU, D. (1995). Reward structures and the allocation of talent. *European Economic Review*. 39, 17-33.
- AXELROD, R. M. (1984). *The evolution of cooperation*. New York. Basic Books.
- BAUMOL, W. J. (1990). Entrepreneurship: Productive, Unproductive and Destructive. *Journal of Political Economy*. 98 (5 pt.1), 893-921.
- DIXIT, A. (1987). Strategic Behavior in Contests. *American Economic Review*. 77 (5), 891-898.
- GARFINKEL, M. R. (1990). Arming as a strategic investment in a cooperative equilibrium. *American Economic Review*. 80 (1), 50-68.
- and SKAPERDAS, S. (1996). Introduction: Conflict and Appropriation as economic activities. In Garfinkel, M.R. and Skaperdas, S. (Eds.), pp. 1-14.
- and ——— (Eds.) (1996). *The political economy of conflict and appropriation*. Cambridge. Cambridge University Press.
- GRADSTEIN, M. and KONRAD, K. A. (1999). Orchestrating rent seeking contests. *Economic Journal*. 109, 536-545.
- GROSSMAN, H. I. (1991). A general equilibrium model of insurrections. *American Economic Review*. 81 (4), 912-921.
- (1994). Production, appropriation and land reform. *American Economic Review*. 84 (3), 705-712.

- and KIM, M. (1995). Swords or Plowshares? A Theory of the Security of Claims to Property. *Journal of Political Economy*. 103 (6), 1275-1288.
- and ——— (1996). Predation and production. In Garfinkel, M.R. and Skaperdas, S. (Eds.), pp. 57-71.
- and ——— (2000). Predators, moral decay, and moral revivals. *European Journal of Political Economy*. 16, 173-187.
- HIRSHLEIFER, J. (1978). Competition, Cooperation, and Conflict in Economics and Biology. *American Economic Review*. 68 (2), 238-243.
- (1988). The Analytics of Continuing Conflict. *Synthese*. 76, 201-233.
- (1989). Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice*. 63, 101-112.
- (1991a). The paradox of power. *Economics and Politics*. 3 (3), 177-200.
- (1991b). The Technology of Conflict as an Economic Activity. *American Economic Review*. 81 (2), 130-134.
- (1995). Anarchy and its breakdown. *Journal of Political Economy*. 103, 26-52.
- KILLINGBACK, T., DOEBELI, M. and KNOWLTON, N. (1999). Variable Investment, the Continuous Prisoner's Dilemma, and the Origin of Cooperation. *Proceedings of the Royal Society B: Biological Sciences*. 266 (1430), 1723-1728.
- MITTERMAIER, K. (1999). The delicate nature of markets. Ludwig Lachmann Memorial Lecture. Economics Department, University of the Witwatersrand.
- MURPHY, K. M., SHLEIFER, A. and VISHNY, R. W. (1991). The allocation of talent: implications for growth. *Quarterly Journal of Economics*. 106, 503-530.
- and ——— (1993). Why is Rent-Seeking so costly to growth? *American Economic Review*. 83 (2), 409-414.
- NOWAK, M. and SIGMUND, K. (1992). Tit for tat in heterogeneous populations. *Nature*. 355, 250-253.
- and ——— (1993). A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoners' Dilemma game. *Nature*. 364, 56-58.
- ROSEN, S. (1986). Prizes and incentives in elimination tournaments. *American Economic Review*. 76 (4), 701-715.
- SHLEIFER, A. and VISHNY, R. W. (1993). Corruption. *Quarterly Journal of Economics*. 108 (3), 599-617.
- SKAPERDAS, STERGIOS (1991). Conflict and attitudes toward risk. *American Economic Review*. 81 (2), 116-120.
- (1992). Cooperation, conflict, and power in the absence of property rights. *American Economic Review*. 82 (4), 720-739.
- (1996). Contest success functions. *Economic Theory*. 7, 283-290.
- USHER, D. (1989). The dynastic cycle and the stationary state. *American Economic Review*. 79 (5), 1031-1044.
- WITTENBERG, M. (1999). Parasites and producers: A model of crime, corruption and contending claims. Econometrics Research Southern Africa Working Paper 9. Johannesburg. University of the Witwatersrand.