The 2016 Horse Mackerel Assessment Model

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Furman (2014) developed and described the horse mackerel assessment model for his MSc thesis. This document extracts pertinent sections from his thesis which describe the population assessment model, and provides a description of assumptions made for projecting the resource into the future. The data, as updated in 2016, are also provided.

1.1 Population Model

An age-structured production model (ASPM) is used as the underlying assessment model for the Horse Mackerel. This model is able to fit to CPUE indices as well as catch-at-length data to allow for past recruitment fluctuations to be estimated. The assessment process involves developing a model of the resource dynamics and conditioning its output to the available data by minimising a log-likelihood function. A single-stock model is used which is based largely on the assessment model by Johnston and Butterworth (2007), except that a midwater CPUE series, time-varying selectivity, catch-at-length data and recruitment fluctuations are incorporated. Important features of the model are described below, with full specifications given in the Appendix.

1.1.1 Dynamics

The ASPM reflects the dynamics of the resource over the period 1949-2015. The resource is managed as a single stock (since 2001). It is assumed that the population was in equilibrium at its carrying capacity in 1949. In reality, horse mackerel catches have been taken as bycatch in other fisheries since the 1900s but these catches were recorded only from 1949 shortly after substantial development of the pelagic fishery commenced. Nevertheless, the cumulative catch before 1949 is unlikely to have been high.

Pope's approximation to the Baranov equations is used to determine fishing mortality (Pope 1972). It assumes that all catches are taken as a pulse in the middle of the fishing season, instead of continuously throughout. The number of recruits at the start of a new year is related to the biomass of the mature component of the population (i.e. spawning biomass) of the previous year by a stock-recruitment relationship. A Beverton-Holt form is assumed. Additionally, stock-recruitment residuals that reflect natural fluctuations about expected recruitment are estimated for the years 1986-2015. Despite this variability in recruitment, the model assumes that at the start of the fishery in 1949, the population is stable at its unexploited equilibrium. Selectivity functions for each fleet are estimated during the fitting procedure, and are assumed to have a Gaussian dependence on length. Demersal fishing selectivity is additionally assumed to vary over time, because the corresponding catch-at-length data show distinct patterns over the years. However, the confounding between time-varying selectivity and catchability introduces difficulties which are addressed by normalising under the assumption that the demersal survey catchability remains constant over time. The Appendix provides further details.

1.1.2 Likelihoods

The assessment model is conditioned on survey abundance and catch-at-length data, and on commercial CPUE and catch-at-length data. Additional contributions to the negative of the (penalised) log-likelihood come from the stock-recruitment residuals and various penalty functions which are discussed below.

The midwater CPUE and demersal survey time-series are both considered to be relative indices of abundance, each proportional to the biomass available to their respective fleets at midyear. However, without any estimates of biomass in absolute terms, the model is unable to estimate catchability coefficients for these indices reliable. The autumn survey is therefore treated as an absolute index by fixing its catchability to one of two values considered to be reasonable. The value of this catchability parameter is a key uncertainly of the model. Likelihoods are calculated by assuming that observed indices are log-normally distributed about their expected values. Although estimates of sampling variability are given for each demersal survey, the model estimates additional variance because there are likely to be other sources of variability; otherwise unrealistically high precision, and hence weight in the fitting procedure, would be accorded to these indices.

Because the assessment model is age-structured, catch-at-age estimates must be transformed into catch-at-length estimates before they can be compared to the observed catch-at-length data. This is done via an age-length matrix that is based on an input von Bertalanffy growth curve. The likelihood contributions are then calculated by comparing the model-predicted length distribution of horse mackerel catches with empirical data. Errors are assumed to be log-normally distributed.

The stock-recruitment residuals are also assumed to be log-normally distributed with no auto-correlation. Unfortunately, their variability cannot be estimated within the maximum likelihood framework used in this assessment, because the penalised likelihood function will always yield a minimum in the limit of the extent of this variability approaching zero. This issue is somewhat problematic, because recruitment fluctuations are of particular importance to the testing of pelagic MPs. While it could be dealt with by adopting a fully Bayesian methodology, it is simpler and adequate for present purposes to input the standard deviation for those residuals as a fixed value.

Finally, there are contributions to the negative log-likelihood from penalty functions. These do not correspond to any particular observed data or prior knowledge, but are instead included to discourage the optimisation from moving into unrealistic regions of parameter space, such as those resulting in negative population counts or fishing mortality. The models presented thus far for horse mackerel achieve convergence without triggering those penalty functions.

1.1.3 Parameters

Estimable parameters

A complete list of the 43 parameters estimated by the model fitting procedure is given below.

K^{sp} is the pre-exploitation spawning biomass of horse mackerel;

q_{spr} is the catchability coefficient for the spring demersal survey abundance index;

ς_y is the fluctuation about the expected recruitment for year *y*, which is estimated for years 1986-2015;

μ^m is the centre of the Gaussian selectivity-at-length curve for the midwater fleet;

λ^m controls the width of the Gaussian selectivity-at-length curve for the midwater fleet;

μ^d_{y1-y2} is the centre of the Gaussian selectivity curve for the demersal fleet for years *y1-y2*, and is estimated for periods 1949-1993, 1994-1997, 2004-2006 and 2007+;

λ^d_{y1-y2} controls the width of the Gaussian selectivity-at-length curve for the demersal fleet for years *y1-y2*, and is estimated for the periods 1949-1993, 1994-1997, 2004-2006 and 2007+; and

σ^s_{add} is the square root of the additional variance for the survey abundance index *s* (*s* is either *aut* for the autumn survey of *spr* for the spring survey), and reflects variability not included in the corresponding survey CVs.

In order to fit the very low recent CPUE values, model variants are examined which allow for either a separate (and lower) catchability coefficient to be estimated for the 2014 and 2015 seasons, or a once off extra mortality which applies to the start of the 2014 season only – a later section on Model Variants describe these models in more detail.

Input parameters

Some parameters cannot be estimated by the model, or are adequately specified by other studies and need not be estimated. They are therefore input with fixed values. The following is a list of these parameters:

q_{aut} is the catchability coefficient for the autumn demersal survey abundance index, and is assumed to be 0.75;

h is the "steepness" of the Beverton-Holt stock-recruitment function, and is assumed to be 0.75;

M is the natural mortality rate of horse mackerel, and is fixed at 0.3 yr^{-1}; although this choice is somewhat arbitrary (Johnston and Butterworth 2007), Horsten (1999a) found key ASPM results to be fairly robust to alternative assumptions regarding this value.

a_m is the age-at-maturity for South African horse mackerel, and is described by a knife-edge function of age with 100% of the population being sexually mature at 3 years (Butterworth and

Clark 1996; Hecht 1990);

l_a is the expected length of a fish at age *a* in centimetres, and is based on the von Bertalanffy growth function given by Equation 1 and the growth parameters reported in Table 1;

w_a is the weight in metric tonnes of a fish at age *a*, and is based on the length-at-age relationship described above, in combination with the mass-at-length function given by Equation 2 and the growth parameters reported in Table 1;

$S^{p}_{a,y1-y2}$ is the fishing selectivity for the pelagic fleet for a fish at age *a* for years *y1-y2*, and is listed in Table 2 for the periods 1949-1962, 1963-1967 and 1968+;

σ_R is the standard deviation of the stock-recruitment log-residuals, and is assumed to be equal to 0.5, which is roughly typical for a species like horse mackerel;

γ is the CV of the length distribution of horse mackerel at any given age, and is assumed to be equal to 0.09 because this value provides good fits to catch-at-length data and lies within the expected range for a species like horse mackerel; and

w_{cal} is the weighting of the catch-at-length likelihood contributions, and is fixed at 0.35 (a weighting of 1 is equivalent of being “unweighted”).

Growth

The Cape horse mackerel has a maximum reported (fork) length of 60cm and may live to more than ten years of age (Bianchi *et al*. 1999). The length-at-age relationship used in the work presented in this thesis is taken from Kerstan (pers. commn) as quoted in Horsten (1999b). This relationship takes the form of a von Bertalanffy growth curve:

$$l_a = l_\infty(1 - e^{-\kappa(a-t_0)}) \qquad (1)$$

where

l_a is the expected total length of a fish of age a in years in centimetres;

l_∞ is the asymptotic total length in centimetres;

κ , the Brody growth coefficient, is a growth rate parameter; and

t_0 is the theoretical age at which length would be zero.

The mass-at-length relationship used for Cape horse mackerel is from Naish et al. (1991). It is provided by the power model:

$$w = \alpha(l)^\beta \qquad (2)$$

where

w is the expected weight in grams of fish;

l is the total length of the fish in centimetres; and

α and β are growth parameters.

Estimates for the parameters of these growth equations are reported in Table 1. Hecht (1990) found no difference between the mean length-at-age of males and females. This provides further support for a sex-aggregated model.

1.2 Model variants

Given the limited data available at present, the assessment model in unable to reliably estimate the parameters q_{aut} (autumn survey catchability) and *h* (stock-recruitment steepness). Hence they must be set externally. Note that q_{aut} can be thought of as a measure of the bias in the survey absolute biomass estimates. For example, a value of 0.5 means that actual biomass is twice as large as the swept-area estimate from the surveys, whereas a value of 1 would mean that these surveys provide unbiased results. *h* determines the productivity of the resource, with a larger *h* corresponding to greater productivity. Johnston and Butterworth (2007) and Furman (2014) identified four combinations of q_{aut} and *h* as covering a realistic range. These were:

- Model 1: q_{aut} = 1.0; *h* = 0.6 (most pessimistic)
- Model 2: q_{aut} = 0.5; *h* = 0.6
- Model 3: q_{aut} = 1.0; *h* = 0.9
- Model 4: q_{aut} = 0.5; *h* = 0.9 (most optimistic).

These four variants were considered to be equally plausible and formed the Reference Set of OMs. **Note however that the Base Case model, which is used in assessments and MP testing since 2014 assumes q_{aut} = 0.75; *h* = 0.75.** These values were selected as they fall in the middle of the bounds defined by the Reference Set. It is assumed that the Base Case model's results are reasonably representative of those of the original Reference Set and are assumed to apply for the 2016 updated assessments.

The 2016 assessment model is further extended to allow for better fits to the midwater CPUE data for 2014 and 2015 (which are particularly low). The model either assumes these low CPUE values are due to reduced selectivity, or that extra mortality of fish occurred at the start of 2014. Thus

Variant I) $q = q_1$ for years up to and including 2013

$q = q_2$ for years 2014 and 2015

q = either a) q_1 for 2016+ (i.e. reverts to normal for 2016+) or

b) q_2 for 2016+ (i.e. remains at the lower estimated q_2 value into the future.

Variant II) Extra mortality occurs at the start of 2014 (numbers-at-age in 2014 reduced by an estimated additional proportion M^{extra}). This extra mortality is a once-off event.

1.4 Projections

Projections are simulations of the future state of a fishery given present understanding of the resource dynamics as represented by an assessment model. By providing a basis to calculate fishery performance statistics, they give means of testing candidate MPs and enable stake-holders to make informed decisions about trade-offs. In this section we look at projections under the assumption of constant future catches for both the future pelagic catches and future midwater catches. The horse mackerel resource is projected 30 years into the future. Because there are stochastic elements in the model dynamics, 1000 projections, each using different random numbers, are simulated for each future catch scenario as explained below. This allows for realistic estimates of performance statistics. Additionally, the random number generator is seeded with the same value at the start of each set of 1000 projections in order to eliminate the variability that would results from u sing different seeds; this allows for readier comparisons between scenarios.

To simplify projections, the time-varying fishing selectivities for the pelagic and demersal fleets are assumed to remain in the future at their 2012 values. Future stock-assessment residuals are drawn randomly from a normal distribution with a stand deviation of σ_R. Additionally, they are assumed to be serially correlated, with a Pearson correlation coefficient of 0.47. This *r* value is taken from the serial correlation of the mode-estimated residuals.

Future "observed" midwater CPUE and autumn demersal and pelagic survey biomass estimates are generated during projections, because these indices of abundance are potentially useful as inputs to many MPs. Realistic observation errors are added to the expected values of these abundance indices by drawing them at random from the same log-normal distribution assumed in the assessment model (Equation A.24, A.26. A.28 and A.29). The variance of the error distribution for the CPUE and pelagic survey indices are estimated in the assessment (Equation A.30), while the variance for the autumn demersal survey abundance estimate is a combination of the estimated σ_{add}^{aut} (additional variance) and a CV (Equation A.27). Future CVs are drawn randomly with replacement from historic autumn survey CVs.

1.3 Data

The various data are reported in Tables 1-5 and Figure 1.

References

Bianchi, G., Carpenter, K.E., Roux, J.P., Molloy, F.J., Boyer, D. and Boyer, H.J. 1999. FAO species identification field guide for fishery purposes. *The living marine resources of Namibia*. Rome: FAO. 265 pp.

Butterworth, D.S. and Clark, E.D. 1996. Calculation of the proportion of a recent biomass estimate that should be taken to provide a precautionary maximum catch limit for horse mackerel. DAFF report, WG/10/6/D:HM:28.

Furman, L.B 2014. Using management strategy evaluation to address problems arising as a result of competing users of the South African horse mackerel resource. MSc thesis, University of Cape Town.

Hecht, T. 1990. On the life history of Cape horse mackerel *Trachurus trachurus capensis* off the south-east coast of South Africa. *South African Journal of Marine Science* 9.1:317-326.

Horsten, M.B. 1999a. Horse mackerel modelling – progress report. DAFF report, WG/09/99/D:HM:29, 11pp.

Horsten, M.B 1999b. Update on horse mackerel research. DAFF document, WG/08/09/D:HM:16, 4pp.

Johnston, S.J. and Butterworth, D.S. 2007. The South African horse mackerel assessment for 2007 using an age-structured production model, with future biomass projections. DAFF report, 2007:WG-DEM:HM:10, 33pp.

Naish, K.A., Hecht, T. and Payne, A.I.L. 1991. Growth of Cape horse *mackerel Trachurus trachurus capensis* off South Africa. *South African Journal of Marine Science* 10.1:29-35.

Pope, J. G. 1972. An investigation of the accuracy of virtual population analysis using cohort analysis. *Research Bulletin International Commission of the Northwest Atlantic Fisheries* 9:65-74.

Table 1: Parameter values for the von Bertalanffy growth curve (Equation 1) and mass-at-length relationship (Equation 2) for Cape horse mackerel. Values reported are taken from Kerstan *pers. commn*) as quoted in Horsten (1999b) and from Naish *et al.* (1991) respectively.

Parameter	Value
l_∞ (cm)	54.56
κ (yr^{-1})	0.183
t_0 (yr)	-0.654
α (g/cm^β)	0.0078
β	3.011

Table 2: Selectivity-at-age vectors assumed for the pelagic fleet over three different periods (Johnston and Butterworth 2007).

	Period		
Age (yr)	**1949-1962**	**1963-1967**	**1968+**
0	0.00	0.14	0.28
1	0.00	0.50	1.00
2	0.30	0.40	0.50
3	1.00	0.50	0.00
4	0.50	0.25	0.00
5	0.50	0.25	0.00
6	0.25	0.13	0.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	0.00	0.00
10+	0.00	0.00	0.00

Table 3: Horse Mackerel catch data for the three different fleets (values in kilo tonnes KT).

Year	Pelagic catch	Demersal catch	Midwater catch
1949	3360	0.00001	0.00001
1950	49900	445	0.00001
1951	98900	1105	0.00001
1952	102600	1226	0.00001
1953	85200	1456	0.00001
1954	118100	2550	0.00001
1955	78800	1926	0.00001
1956	45800	1334	0.00001
1957	84600	959	0.00001
1958	56400	2073	0.00001
1959	17700	2075	0.00001
1960	62900	3712	0.00001
1961	38900	3627	0.00001
1962	66700	3079	0.00001
1963	23300	1401	0.00001
1964	24400	9522	0.00001
1965	55000	7017	0.00001
1966	26300	7596	0.00001
1967	8800	6189	0.00001
1968	1400	9116	0.00001
1969	26800	12252	0.00001
1970	7900	17872	0.00001
1971	2200	33329	0.00001
1972	1300	20560	0.00001
1973	1600	33900	0.00001
1974	2500	38391	0.00001
1975	1600	55459	0.00001
1976	400	50981	0.00001
1977	1900	116400	0.00001
1978	3600	37290	0.00001
1979	4300	53584.5	0.00001
1980	400	39187.5	0.00001
1981	6100	41215	0.00001
1982	1100	32176	0.00001
1983	2100	38332	0.00001
1984	2800	37969	0.00001
1985	700	27278	0.00001
1986	500	31378	0.00001
1987	2834	38571	0.00001
1988	6403	41482	0.00001
1989	25872	58206	0.00001

1990	7645	56721	0.00001
1991	582	39759	0.00001
1992	2057	37208	0.00001
1993	11651	35998	0.00001
1994	8207	20030	0.00001
1995	1986	10790	0.00001
1996	18920	31846	0.00001
1997	12654	34671	0.00001
1998	26680	36279	15770
1999	2057	21580	2161
2000	4503	9259	15376
2001	915	8824	19220
2002	8148	4863	11098
2003	1012	3578	25291
2004	2048	4932	27154
2005	5627	5272	29005
2006	4824	4122	18068
2007	1903	4799	25041
2008	2280	4333	23888
2009	2087	3737	29410
2010	4385	5594	23479
2011	10990	5036	29241
2012	2199	4940	22581
2013	596	2657	21444
2014	2760	3085	10055
2015	2040	4467	7968

Table 4: GLM standardised CPUE (for the *Desert Diamond*) and survey abundance data for South African horse mackerel for the period 1986-2015. Data were provided by Coetzee, Fairweather and Singh (DAFF, *pers. commn*).

Year	CPUE	Autumn demersal survey		Spring demersal survey	
		Biomass (KT)	CV	Biomass (KT)	CV
1986				97.36	0.13
1987				332.97	0.14
1988		159.07	0.29		
1989					
1990					
1991		352.19	0.23		
1992		422.21	0.23		
1993		435.28	0.20		
1994		340.72	0.26		
1995		195.13	0.24		
1996		261.77	0.23		
1997		241.02	0.23		
1998					
1999		330.63	0.24		
2000					
2001				316.72	0.18
2002					
2003	0.721	146.72	0.24	231.36*	0.20*
2004	0.637	195.73*	0.32*	366.50*	0.19*
2005	0.896	175.04*	0.21*		
2006	0.945	386.57	0.20	350.28	0.19
2007	1.482	243.58*	0.40*	473.22*	0.19*
2008	1.020	279.86*	0.27*	300.00*	0.17*
2009	1.072	337.16*	0.24*		
2010	1.276	271.79	0.37		
2011	1.472	213.09*	0.22*		
2012	0.633				
2013	1.456	522.69	0.28		
2014	0.390	180.08	0.17		
2015	RC=0.181 Sensitivity=0.259				

*These values correspond to surveys that used the new trawl net, which was introduced in September 2003.

Table 5a: Spring demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers) as used in the assessment model. Provided by Fairweather (DAFF, *pers commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
1986	0.000	0.000	0.002	0.090	0.238	0.164	0.169	0.231	0.105
1987	0.000	0.000	0.116	0.223	0.160	0.206	0.124	0.129	0.043
2001	0.002	0.015	0.375	0.255	0.124	0.136	0.075	0.015	0.004
2003	0.000	0.050	0.068	0.376	0.367	0.091	0.040	0.008	0.001
2004	0.001	0.238	0.256	0.161	0.226	0.074	0.035	0.008	0.001
2006	0.008	0.267	0.243	0.288	0.144	0.041	0.008	0.001	0.000
2007	0.000	0.223	0.634	0.095	0.044	0.003	0.001	0.000	0.000
2008	0.001	0.027	0.458	0.429	0.068	0.010	0.005	0.002	0.000

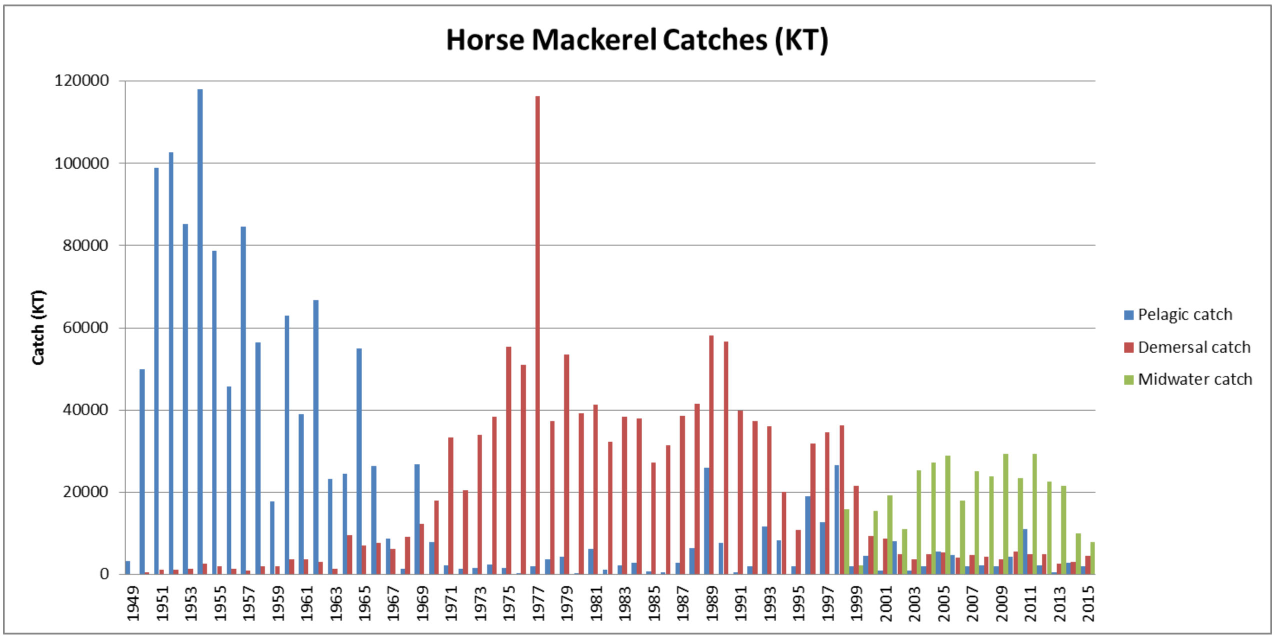
Table 5b: Autumn demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers) as used in the assessment model. Provided by Fairweather (DAFF, *pers commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
1988	0.000	0.015	0.051	0.014	0.156	0.166	0.180	0.291	0.127
1992	0.000	0.072	0.046	0.105	0.374	0.273	0.056	0.043	0.030
1993	0.000	0.092	0.353	0.075	0.198	0.118	0.076	0.065	0.023
1994	0.000	0.027	0.157	0.220	0.298	0.254	0.029	0.010	0.004
1995	0.000	0.000	0.023	0.109	0.460	0.271	0.092	0.033	0.011
1996	0.000	0.000	0.001	0.023	0.542	0.308	0.111	0.013	0.002
1997	0.000	0.003	0.024	0.005	0.468	0.401	0.079	0.016	0.005
1999	0.000	0.010	0.169	0.063	0.082	0.522	0.114	0.033	0.006
2003	0.000	0.001	0.393	0.329	0.120	0.060	0.082	0.015	0.001
2004	0.022	0.142	0.432	0.055	0.186	0.100	0.053	0.008	0.001
2005	0.000	0.354	0.198	0.148	0.186	0.057	0.050	0.007	0.000
2006	0.001	0.033	0.239	0.345	0.282	0.063	0.030	0.006	0.000
2007	0.108	0.463	0.319	0.088	0.016	0.004	0.002	0.001	0.000
2008	0.001	0.071	0.382	0.384	0.150	0.009	0.001	0.002	0.000
2009	0.000	0.068	0.155	0.525	0.220	0.028	0.002	0.001	0.000
2010	0.000	0.056	0.068	0.527	0.294	0.044	0.003	0.006	0.001
2011	0.141	0.770	0.032	0.033	0.022	0.001	0.000	0.000	0.000

Table 5c: Commercial midwater catch-at-length for South African horse mackerel (shown as proportions of numbers) as used in the assessment model. Provided by Singh (DAFF, *pers commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
2003	0.000	0.000	0.000	0.001	0.135	0.256	0.505	0.102	0.001
2004	0.000	0.000	0.000	0.012	0.241	0.382	0.328	0.036	0.001
2005	0.000	0.000	0.004	0.079	0.288	0.388	0.190	0.035	0.016
2006	0.000	0.000	0.006	0.113	0.339	0.403	0.126	0.010	0.003
2007	0.000	0.000	0.003	0.090	0.293	0.359	0.187	0.054	0.014
2008	0.000	0.001	0.043	0.256	0.328	0.246	0.111	0.014	0.001
2009	0.000	0.000	0.001	0.088	0.386	0.318	0.170	0.034	0.002
2010	0.000	0.000	0.018	0.220	0.378	0.255	0.100	0.026	0.003
2011	0.000	0.000	0.001	0.146	0.490	0.236	0.104	0.022	0.001
2012	0.000	0.000	0.076	0.147	0.266	0.342	0.120	0.045	0.004
2013	0.000	0.000	0.220	0.474	0.076	0.164	0.058	0.007	0.002
2014	0.000	0.000	0.000	0.019	0.071	0.492	0.383	0.032	0.003

Figure 1: Plot of the Horse Mackerel catch series split into the three fleets.



Appendix A: Mathematical details of the ASPM

A.1 Dynamics

The population dynamics are described by the following equations:

$$N_{y+1,0} = R_{y+1} \tag{A.1}$$

$$N_{y+1,a+1} = (N_{y,a}e^{-M/2} - C_{y,a})e^{-M/2} \qquad 0 \le a \le m-2 \tag{A.2}$$

$$N_{y+1,m} = \left(N_{y,m}e^{-\frac{M}{2}} - C_{y,m}\right)e^{-\frac{M}{2}} + (N_{y,m-1}e^{-\frac{M}{2}} - C_{y,m-1})e^{-M/2} \tag{A.3}$$

where

$N_{y,a}$ is the number of horse mackerel of age *a* at the start of year *y*;

$C_{y,a}$ is the total number of horse mackerel of age *a* taken in year *y* by the pelagic, midwater and demersal fleets combined;

R_y is the number of recruits (0-year olds) at the start of year *y*;

M is the natural mortality rate for horse mackerel; and

m is the minimum age within the plus-group and is set here to ten years old.

The approximation of the fishery as a pulse catch in the middle of the season is considered of sufficient accuracy for present purposes. Note that the model also assumes that recruitment to the population occurs at the start of a new year (Equation A.1), even though in reality there are two pawning peaks roughly two months apart.

The total number of horse mackerel of age a caught each year is given by:

$$C_{y,a} = \sum_f C_{y,a}^f \tag{A.4}$$

where *f* indicates the fishery concerned and in this case is either *p* for pelagic, *d* for demersal or *m* for midwater.

The annual catch by mass for fleet *f* is given by:

$$C_y^f = \sum_{a=0}^{m} w_{y,a}^f C_{y,a}^f$$

$$= \sum_{a=0}^{m} w_{y,a}^f S_{y,a}^f F_y^f N_{y,a}e^{-M/2} \tag{A.5}$$

where

$S_{y,a}^f$ is the fishing selectivity-at-age for fleet *f* for fish of age *a* in year *y*;

F_y^f is the fleet-specific fishing mortality for a fully selected age class in year *y*; and

$w_{y,a}^f$ is the effective weight of a horse mackerel of age *a* for fleet *f* in year *y*.

Fishing selectivity for the pelagic fleet is described by a selectivity-at-age function; therefore that fleet's effective weight-at-age (in gm) is simply given by a combination of the length-at-age (in cm) and weight-at-length relationships discussed in 1.3:

$$l_a = 54.56\left[1 - e^{-0.183(a+0.654)}\right] \quad \text{(A.6)}$$

$$w_a^p = 0.0078 l_{a+1/2}^{3.011} \text{x} 10^{-6} \quad \text{(A.7)}$$

Because the fishing selectivities of the midwater and demersal fleets are modelled by selectivity-at-length functions, their effective weights-at-age must be calculated differently:

$$w_{y,a}^f = \frac{\sum_l w_l S_{y,l}^f A_{l,a}}{\sum_l S_{y,l}^f A_{l,a}} \quad \text{(A.8)}$$

where

w_l is the weight of a horse mackerel of length l (Equation 2);

$S_{y,l}^f$ is the fishing selectivity for fleet f for fish of length l in year y; and

$A_{l,a}$ is the age-length key, which gives the proportion of fish of age *a* that are of length *l* (detailed later in Equation A.16).

Note that fishing selectivity for the midwater fleet is assumed to be time-invariant; therefore the *y* subscript may be dropped when determining the effective weight-at-age for that fleet.

The fleet-specific exploitable component of abundance is taken to be given by exploitable biomass at midyear:

$$B_y^f = \sum_{a=0}^m w_{y,a}^f S_{y,a}^f N_{y,a}\, e^{-M/2} \quad \text{(A.9)}$$

or in terms of numbers of individuals:

$$N_y^f = \sum_{a=0}^m S_{y,a}^f N_{y,a}\, e^{-M/2} \quad \text{(A.10)}$$

The proportion of the resource harvested each year by fleet *y* is therefore given by:

$$F_y^f = \frac{C_y^f}{B_y^f} \quad \text{(A.11)}$$

and

$$C_{y,a}^f = S_{y,a}^f F_y^f N_{y,a} e^{-M/2} \quad \text{(A.12)}$$

Note that in terms of Equations A.11 and A.12 the model assumes the same fishing selectivity for the commercial demersal fleet and both demersal surveys. This simplifying assumption has been made because there are no catch-at-length data available to estimate selectivity functions for the commercial fleet.

Fishing selectivities

Selectivity-at-age for the pelagic fleet is input and assumed to change with time. The same values are used as for the 2007 assessment model (Johnston and Butterworth 2007). Essentially there is one selectivity function for the pre-1993 period and another for the post-1967 period, while for the period between (1963-1967) the average of those two selectivity functions is used.

In contrast, selectivity-at-length is estimated for both the midwater and demersal fleets. These are assumed to have a Gaussian form with length:

$$S_{y,l}^{f} = \frac{e^{-(l-\mu_y^f)^2}}{2(\lambda_y^f)^2} \qquad \text{if } l_{min}^{f} \leq l \leq l_{max}^{f} \text{ or}$$

$$= 0 \qquad \text{otherwise} \qquad \text{(A.13)}$$

where

μ_y^f is an estimated selectivity parameter that determines the centre of the Gaussian for fleet *f* in year *y*;

$\lambda_y\, f$ is an estimated parameter that determines the width of the Gaussian for fleet *f* in year *y*;

l_{min}^{f} is a fixed selectivity parameter that determines the smallest length class with non-zero selectivity for fleet *f*, and is set equal to 10cm for both the demersal or the midwater fleets, and

l_{max}^{f} is a fixed selectivity parameter that determines the largest length class with non-zero selectivity for fleet *f*, and is set equal to 50cm or 60cm for the demersal or midwater fleets respectively.

Note again that the y subscript may be dropped when dealing with selectivity for the midwater fleet because it is time-invariant. Selectivity-at-length is then normalised according to:

$$S_{y,l}^{f} \rightarrow S_{y,l}^{*,f} = \frac{S_{y,l}^{f}}{\sum_{l'=l_1}^{l_2} \frac{S_{y,l'}^{f}}{l_2 - l_1 + 1}} \qquad \text{(A.14)}$$

In other words, the selectivity function is scaled by the inverse of its average value over a certain length range. l_1 and l_2 are the same for both midwater and the demersal fleets and are set equal to 10cm and 40cm respectively.

Because the model is age-structured, selectivity-at-length must be transformed into selectivity-at-age using an age-length relationship:

$$S_{y,a}^{f} = \sum_{l} A_{l,a} S_{y,l}^{f} \tag{A.15}$$

It is assumed that the length distribution for horse mackerel of age *a* is described by a normal distribution with mean which is given by the von Bertalanffy growth curve input, and with a standard deviation that is proportional to this mean. Consequently, with length classes of 1cm, $A_{l,a}$ is computed according to:

$$A_{l,a} = \frac{1}{2}\left[\text{erf}\left(\frac{l+0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})}\right) - \text{erf}\left(\frac{l-0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})}\right)\right] \tag{A.16}$$

where

erf is the error function;

$l_{a+0.5}$ is the expected midyear length for a horse mackerel of age a, which is calculated using the input von Bertalanffy growth curve given by Equation 1; and

γ is the CV of the length-at-age distribution, which is fixed at 0.9.

Stock-recruitment relationship

The spawning biomass in year y is given by:

$$B_y^{sp} = \sum_{a=a_m}^{m} w_a N_{y,a} \tag{A.17}$$

where

a_m is the age corresponding to 100% sexual maturity, which is assumed here to be described by a knife-edge function of age; and

w_a is the mass of a horse mackerel of age *a* at the start of the year.

The number of recruits at the start of fishing year y is related to the spawner stock size by a Beverton-Holt stock-recruitment relationship:

$$R\left(B_y^{sp}\right) = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}} e^{\varsigma_y} \tag{A.18}$$

where

α and β are stock-recruitment parameters; and

ς_y are stock-recruitment residuals reflecting fluctuations about expected recruitment in year *y*.

In order to work with estimable parameters that are more biologically meaningful than α and β, the stock-recruitment relationship is re-parametersied in terms of pre-exploitation equilibrium spawning biomass, K^{sp}, and the steepness of the stock-recruitment relationship, *h*, where steepness is the fraction of pristine recruitment, R_0, that results when spawning biomass drops to 20% of its pristine level:

$$hR_0 = R(0.2K^{sp}) \quad \text{(A.19)}$$

from which it follows that:

$$h = \frac{0.2(\beta + K^{sp})}{\beta + 0.2K^{sp}} \quad \text{(A.20)}$$

and hence:

$$\alpha = \frac{4hR_0}{5h-1} \quad \text{(A.21)}$$

and

$$\beta = \frac{K^{sp(1-h)}}{5h-1} \quad \text{(A.22)}$$

Given a value for the pre-exploitation spawning biomass K^{sp} of horse mackerel, together with the assumption of an initial equilibrium age-structure, pristine recruitment can be determined from:

$$R_0 = \frac{K_{sp}}{[\sum_{a=a_m}^{m-1} w_a e^{-aM} + w_m e^{-mM}/(1-e^{-M})]} \quad \text{(A.23)}$$

A.2 Likelihood functions

The model is fitted to three biomass indices and three sets of catch-at-length data. Stock recruitment residuals also contribute to the penalised negative log-likelihood that is minimised in the fitting process.

Abundance indices

The assessment model is ordinarily fitted to three abundance indices: spring and autumn demersal biomass estimates, and a commercial midwater CPUE series. The associated likelihood contribution are calculated by assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^s = \widehat{I}_y^s e^{\epsilon_y^s} \text{ or } \epsilon_y^s = \ln(I_y^s) - \ln(\hat{I}_y^s) \quad \text{(A.24)}$$

where

s indicates the abundance index concerned and is either *aut* for the autumn survey, or *spr* for the

spring survey, *cpue* for CPUE or *pel* for the pelagic index;

I_y^s is the observed value of index *s* in year *y*;

$\hat{I}_y^s$ is the model predicted value of *s* in year *y*.

The negative of the log-likelihood function (after removal of the constant) is then given by:

$$-lnL = \sum_s \sum_y [ln\sigma_y^s + \left(\epsilon_y^s\right)^2 / 2(\sigma_y^s)^2] \quad \text{(A.25)}$$

The spring and autumn demersal survey biomass estimates are assumed to reflect demersal exploitable biomass:

$$\hat{I}_y^s = q_s B_y^d \quad \text{(A.26)}$$

where q_s is the catchability coefficient corresponding to index *s*. Note that the same demersal exploitable biomass B_y^d is used to fit both the autumn and spring demersal surveys even though they occur several months apart. Because a mid-year pulse catch assumption is made (Equation A.9), this exploitable biomass does not account for fishing mortality that may occur between the surveys. For these series, reliable estimates of sampling variability and additional variance are available; therefore the standard deviations are calculated according to the following formula:

$$\sigma_y^s = \sqrt{\ln\left[1 + (CV_y^s)^2\right] + (\sigma_{add}^s)^2} \quad \text{(A.27)}$$

where

CV_y^s is the CV for survey s in year y, which is given in Table 3, and

σ_{add}^s is the model estimated additional variance for survey abundance index s.

The midwater CPUE index is assumed to reflect the midwater exploitable biomass:

$$\hat{I}_y^{cpue} = q_{cpue} B_y^m \quad \text{(A.28)}$$

and the pelagic hydro-accoustic survey index from November of year *y* is assumed to reflect recruitment in year y+1:

$$\hat{I}_y^{pel} = q_{pel} R_{y+1} \quad \text{(A.29)}$$

Reliable estimates of CVs and catchability are unavailable for the CPUE and pelagic abundance indexes. Therefore, they are set to their maximum likelihood estimates:

$$\sigma^s = \sqrt{1/n \sum_y (\epsilon_y^s)^2} \quad \text{(A.30)}$$

$$lnq_s = 1/n \sum_y \epsilon_y^s \quad \text{(A.31)}$$

Catch-at-length

Model estimated catch-at-length proportions are fitted to spring and autumn demersal survey length-frequency data, and commercial midwater length frequency data.

Catch-at-age estimates (Equation A.12) are transformed into catch-at-length estimates using the age-length relationship $A_{l,a}$ (Equation A.16):

$$C_{y,l}^{f} = \sum_{a=0}^{m} A_{l,a} C_{y,a}^{f} \qquad \text{(A.32)}$$

where $C_{y,l}^{f}$ is the total number of horse mackerel of length *l* caught in year *y*.

The contribution of catch-at-length data to the negative log-likelihood function is then given by:

$$-lnL = w_{cal} \sum_{s} \sum_{y} \sum_{l} \left[ln\sigma_{cal}^{s} + \left(\sqrt{p_{y,l}^{s}} - \sqrt{\hat{p}_{y,l}^{s}}\right)^{2} / 2(\sigma_{cal}^{s})^{2}\right] \qquad \text{(A.33)}$$

where

w_{cal} is a weighting for this likelihood contribution, and is fixed at 0.35;

$p_{y,l}^{s}$ is the observed proportion of fish caught in year *y* that are of length *l* for dataset *s*;

$\hat{p}_{y,l}^{s}$ is equal to $C_{y,l}^{f} / \sum_{l} C_{y,l}^{f}$ and is the model predicted proportion of fish caught in year *y* that are of length *l* in dataset *s*, where *f* is the appropriate fleet; and

σ_{cal}^{s} is the standard deviation associated with catch-at-length dataset *s*, which is estimated in the fitting procedure by:

$$\sigma_{cal}^{s} = \sqrt{\sum_{y} \sum_{l} \left(\sqrt{p_{y,l}^{s}} - \sqrt{\hat{p}_{y,l}^{s}}\right)^{2} / \sum_{y} \sum_{l} 1} \qquad \text{(A.34)}$$

Note that allowance is made for a minus group (fish smaller than 10 cm) and a plus group (fish 46 cm and larger). Length classes are specified with intervals of 5 cm.

Stock-recruitment residuals

It is assumed that these residuals are log-normally distributed and are not serially correlated. Therefore, their contribution to the penalised negative log-likelihood is given by:

$$-lnL = \sum_{y} \frac{\varsigma_{y}^{2}}{2\sigma_{R}^{2}} \qquad \text{(A.35)}$$

where

ς_{y} is the estimated stock-recruitment residual for year y; and

σ_{R} is the input standard deviation of the log-residuals, which is assumed to be equal to 0.5.