DEFINITION OF APPARENT POWER IN 3-PHASE 4-WIRE NON-SINUSOIDAL POWER SYSTEMS

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THESIS PRESENTED FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN THE DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF CAPE TOWN

AUGUST 2008

Acknowledgement

The thesis is the result of associated work undertaken over the last 20 years in the field of 3-phase power systems. It began with a research of single phase to three phase converters, with passive elements and active converters. The work and interest diverted to three-phase voltage control inverters and their use for the use for quality of supply and renewable energy applications. The work then was redirected to the application of load balancers and voltage balancers and several commercial products resulted. Now, as a result of the above interest in active filters, instantaneous power theory and quality of supply measurement, this document which is concerned in particular with the definition and measurement of power factor and apparent power has been completed.

I am particularly thankful for all those who have encouraged me to publish and complete the work despite the many gaps, interruptions and other interests along the way. For this I am particularly thankful to Professor Trevor Gaunt for his continuous encouragement and rapid feedback.

Finally I am especially thankful for the patience of my wife Laurette, and my children Jean-Claude, Mireille and Gerard who had to bear my preoccupations with my thesis and encouraged me to finally complete it.

Michel Malengret
Cape Town
July 2008

Abstract

Concepts of apparent, active and reactive power, developed early in the 20th century, are widely applied in electrical power engineering. However, the distortions caused by large non-linear loads are increasingly significant. Conventional power theories are sometimes inadequate, or even incorrect, when dealing with unbalanced or non-sinusoidal systems. New power theories have been proposed and standard definitions revised, but inconsistencies are still reported, having technical and financial implications in power systems design, metering, reliability and quality of supply.

The thesis starts by collating and comparing most power theories in the instantaneous and average power domain. The instantaneous theories are reformulated and classified into three groups, which can be linked with one equation. Introducing a new instantaneous theory based on vector space allows the fundamental properties of the groups to be examined and provides the means to extend instantaneous power into the rms domain and calculate the compensating currents required. The approach results in power components that can be attributed a real physical interpretation. The new approach to three phase power theory has significant potential for education, the practical design of active compensators, and the revision of international standard definitions of power.

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Chapter 1

Apparent power and power components

Power theories developed in the first part of the 20th century have been of great benefit when dealing with symmetrical and sinusoidal systems in electrical engineering. Commonly used concepts such as those of apparent power (AP), active power and reactive power have been generally well received and widely used. However, with the recent increase of large non linear loads, the current and voltage distortions have become more significant and the conventional power theories sometimes become inadequate, or even incorrect when dealing with unbalanced or non-sinusoidal systems. For these reasons many new power theories have been proposed. Standards and definitions have been adopted and revised from time to time [IEC 1979, IEEE 2000], yet inconsistencies under non sinusoidal 3-phase supply voltage and load currents are still reported regarding what constitutes apparent power and reactive power. This can have major technical and financial implications in power systems design, metering, reliability, quality of supply and clarity, and consistency for educational purposes. For these reasons, the topic of power analysis has been much debated in the last 40 years. The formulation and universal acceptance of a general definition of at least apparent power has become necessary and overdue. To illustrate this point, the following are some of the many comments made recently by various prominent experts in the field:

- Ferrero et al [1993] report:

"Some attempts have been made to extend the definitions given in single-phase to three-phase systems. However a great difficulty is found in extending these definitions to unsymmetrical and unbalanced systems. A further difficulty arises when trying to adopt the same definitions for three-phase and four-wire systems. Even Czarnecki's approach, which is one of the most rigorous and formally correct ones does not consider unsymmetrical supply voltages and does not state clearly whether the given definitions are valid for both three-wire and four-wire systems or not. Moreover, the extension of the power given for single-phase systems to three-phase ones leads to non-unequivocal definitions of AP (and consequently of the power factor)."

- Filipski [1993] points out:

"The three phase AP is not an invariant quantity of the system. Its value depends on how the voltage and currents are defined or measured. The source phase voltage can have different rms values and can even have different harmonic contents! Not all authors realize this ambiguity and use reference points (even describing the same theory), sometimes from the centre of the voltage source star, sometimes from the centre of the load and sometimes from an artificial neutral. What is then the physical interpretation of AP? If AP has no definite physical meaning then it cannot be split into power components that all have a definite physical meaning...Thus all non-sinusoidal power theories are bound to fail in assigning practical significance…"

- Tolbert and Habetler [2000] state:

"Closely related to the definitions of active and non-active power are the definitions of apparent power and power factor. Several proposals have been made for the definition of both, but no consensus has been reached for these two conventional quantities. "

- Czarnecki [2004]:

"Power theory provides definitions of various powers in electrical circuits along with relations between them. Unfortunately, there is a major ambiguity with respect to one of the most commonly used power namely the apparent power, S, in three-phase systems. This ambiguity exists even at sinusoidal voltages and currents. Namely, according to the IEEE Standard Dictionary of Electrical and Electronics Terms, the apparent power is defined…There exists a third definition…but not referred by the IEEE Standard Dictionary…"

From the above comments it is clear that a consensus and clarity would be useful. Other power components such as reactive and non-active components can only be agreed if apparent power is uniquely defined for all cases normally encountered in power systems.

Similarly the definition of power factor is normally understood to be the ratio of real average power to AP. Therefore power factor can only be agreed if AP has been uniquely defined.

1.1 Definition of apparent power (AP) in single-phase systems

In single-phase systems the concept of AP is a direct result of a definition of the root mean square (rms) or effective value of a voltage or a current. The calculation of the rms value of any periodic waveform with a period T, is calculated as follows:

1) Squaring the values of the waveform at every point
2) Taking a mean value of the above over the interval T.
3) Taking the square root of the mean.

The result is a scalar which has proved to be effective when comparing AC with DC power. An AC current or voltage applied to a resistor, with the same rms value as a DC current or voltage will produce the same energy dissipation in a resistor. The rms definition is so widely accepted that if an AC voltage or current value is given, it is considered implicitly to be an rms value.

Apparent power S in a single-phase circuit is generally defined as the product of rms voltage V and rms current I where:

$S = V I$

$V = ((1/T) \int_t v(t)^2 \, dt)^{1/2}$ over interval T (from t to t +T)

$I = ((1/T) \int_t i(t)^2 \, dt)^{1/2}$ over interval T

In the case of sinusoidal voltage and current, S = VI in volt-amps (VA) is generally accepted as a useful figure which is directly associated with the size and cost of an electrical power device. A single-phase transformer is a good example. The size of a transformer operating under sinusoidal conditions is approximately a direct function of the amount of conductive material required and the maximum flux needed. These are determined by the rms value of current and voltage respectively. The product of the rms voltage and current in this case is a useful figure representing the size of the electrical equipment required.

A second figure of merit that is generally agreed to in the case of sinusoidal single-phase system is reactive power Q, which is obtained as the geometric difference between S and P the average power, and its magnitude is calculated as follows:

$Q = (S^2 - P^2)^{1/2}$ note that Q can be assigned a positive or negative value by convention

where the average power P is calculated as:

$P = 1/T \int_T u(t)\, i(t)\, dt$

and where u(t) is the voltage during a period T between the two wires and i(t) the current. Reactive power Q is also known as imaginary power or "useless" power as it does not contribute to the average power consumed by a load.

In the case of non-sinusoidal voltage and current waveform S and Q become less significant with increasing distortion. Nevertheless AP in single phase systems has a unique and invariant value for any particular current and voltage waveform and therefore the definition is generally accepted.

1.2 Present definitions of apparent power in three-phase systems

The 36th Annual Convention of the American Institute of Electrical Engineers conference in 1920 met to define the power factor λ and thereby apparent power S in 3-phase circuits. After lively debates two definitions were proposed [Emanuel 1993]:

Definition 1

$AP = S_1 + S_2 + S_3$ Eq. 1.1

where S_1 , S_2 , S_3 is the apparent power of each phase as defined in Section 1.1 The voltage reference is not specified, and is assumed to be some common point such as the neutral.

As no consensus was reached a second definition was also proposed.

Definition 2

$AP = |P_1 + P_2 + P_3 + j\,(Q_1 + Q_2 + Q_3)\,|$ Eq. 1.2

where Q_1 , Q_2 ,Q_3 is the reactive power of each phase as defined in Section 1.1 for a single-phase system.

Both definitions lead to identical values of power factor λ = P/S in balanced three-phase systems. For unbalanced systems however, the results may be quite different as demonstrated by a hypothetical three-phase circuit shown in Fig. 1.1 [Emanuel 1993]. Simple calculations using Eq. 1.1 and 1.2 for the above example give λ as 0.2 and 1.0 respectively. This unrealistic but vivid example illustrates the inadequacy of both definitions.

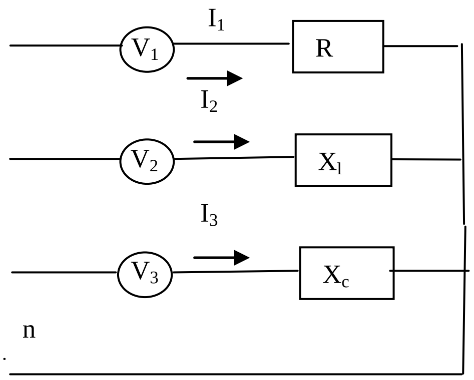

Fig. 1.1 Hypothetical unbalanced network where
$X_l = X_c = R/2$; I = V/R ; V=1 R=1 Ω

Emanuel [1993] points out that engineers such as Buchholtz [1929], Lurie [1951] and Manea [1960]), concerned with the imperfections of Eq. 1.1 and 1.2, considered a third definition which would have a closer physical interpretation to that of single-phase supply. It was proposed by them, that AP in three-phase systems should have the following meaning:

Definition 3

$S = 3I_eV_e$ Eq. 1.3

where

$V_e = ((V_1^2 + V_2^2 + V_3^2)/3)^{1/2}$ Eq. 1.4

$I_e = ((I_1^2 + I_2^2 + I_3^2)/3)^{1/2}$ Eq. 1.5

1.2.1 Implication of definition 3

The interpretation of Eq. 1.3 can be appreciated if one considers the line losses ΔP of a three-wire system.

If all three wires have an equal resistance r then, the line losses are:

$\Delta P = r\,(I_1^2 + I_2^2 + I_3^2)$ Eq. 1.6

From Eq. 1.5

$\Delta P = r\, 3\, I_e^2$

Consider an equivalent balanced resistive load consisting of three equal resistors R replacing the original load so that the line currents dissipate the same line losses ΔP. Then the power P delivered to the load would be:

$$P = R\,(I_1^2 + I_2^2 + I_3^2) \qquad \text{Eq. 1.7}$$

From Eq. 1.4 and Eq. 1.5, the power delivered would be:

$$\begin{aligned} P &= 3\,R\,I_e^2 = 3\,(R\,I_e)\,I_e \\ &= 3\,R\,((I_1^2 + I_2^2 + I_3^2)/3)^{1/2}\; I_e \\ &= 3\,((V_1^2 + V_2^2 + V_3^2)/3)^{1/2}\; I_e \end{aligned}$$

$$S = 3\,V_e\,I_e$$

and from Eq. 1.4 and 1.5 S as per Eq. 1.3 becomes:

$$S = (V_1^2 + V_2^2 + V_3^2)^{1/2}\,(I_1^2 + I_2^2 + I_3^2)^{1/2}$$

This can be expressed in linear algebra as the product of the norm of two vectors

$$S = \|\mathbf{V}\|\,\|\mathbf{I}\| \qquad \text{Eq 1. 8}$$

and where **V** and **I** are vectors

$\mathbf{V} = \{V_1, V_2, V_3\}$ and $\mathbf{I} = \{I_1, I_2, I_3\}$

Where:

$\|\mathbf{V}\|^2 = \mathbf{V} \bullet \mathbf{V}$ and $\|\mathbf{I}\|^2 = \mathbf{I} \bullet \mathbf{I}$

The definition as per Eq. 1.8 is therefore equivalent to that of Eq. 1.3. Eq. 1.8 was recommended and used by Czarnecki [1987, 1988, 1989, 1991, 1992, 1993a, 1995, 2004], Filipski [1993], and many others for non-sinusoidal situations.

Definition of AP according to Eq. 1.8 (or Eq. 1.3) is appealing as it represents the power dissipated by a balanced resistive load that would present the same line losses. This definition of AP in three-phase circuits appears to have a similar physical meaning to that of the definition of rms current and voltage in a single-phase system.

This may be useful, but the implication of Eq. 1.8 has a further physical meaning, which has been mentioned by only few authors such as Buchholtz [1950] and Emanuel [1993]

Emanuel [1993] points out that the main reason that definition 3 was proposed by Buchholtz and others was because definition 3 was meant to be equivalent to the following statement:

AP should be the theoretical maximum power which can be delivered by a voltage source during a specific time interval, whilst the total line losses are maintained constant.

In this thesis it is argued that Emanuel's statement is the most plausible one that should be more widely adopted so as to obtain a general consensus on the general definition of apparent power.

It is shown in the following section that, in order for Eq. 1.8 to be equivalent to Emanuel's statement, further considerations must be stated.

1.2.2 Definition of power factor and active current component in three wire systems

If S as defined by Eq. 1.8 is equivalent to Emanuel's statement then one could state that a unity factor is achieved when the power consumed by a load is equal to the theoretical maximum power under specific conditions that can be transmitted for the same line losses, or :

$\lambda = P/S = 1$

If λ is not unity, the power factor according to definition 3 can also be restated as the ratio of the average power of the load to the theoretical absolute maximum power that could be transmitted for the same line losses.

If the power transmitted was to remain at P after rearranging the currents with a compensator so as to obtain minimal losses. Then the reduced theoretical line currents current I_{a1} , I_{a2} , I_{a3} could be reduced to fraction

$$\lambda = P/S, \qquad \text{Eq. 1.9}$$

of the original line currents, whilst transmitting the same power P .

Since the line losses are proportional to the square of the rms current value, it can be deduced that the new line losses ΔP' would be reduced to:

$\Delta P' = (P/S)^2 \, \Delta P$

or from eq 1.9

$\Delta P' = \lambda^2 \, \Delta P$

therefore

$\lambda^2 = \Delta P' / \Delta P$ Eq. 1.10

If for a 3-wire system the line optimum rms current vector (set) after adjusting the load parameters or using a compensator using no average power is:

$\mathbf{I}_a = \{I_{a1}, I_{a2}, I_{a3}\}$ then the line losses ΔP', would be reduced to:

$\Delta P' = r\,(I_{a1}^2 + I_{a2}^2 + I_{a3}^2) = r\,\|\mathbf{I}_a\|^2$ Eq. 1.11

From Eq. 1.6, Eq. 1.10 and Eq. 1.11

$\lambda^2 = \|\mathbf{I}_a\|^2 / \|\mathbf{I}\|^2$

or $\lambda = \|\mathbf{I}_a\| / \|\mathbf{I}\|$ Eq. 1.12

Therefore from Eq. 1.9

$P = \lambda S = (\|\mathbf{I}_a\| / \|\mathbf{I}\|)\,(\|\mathbf{V}\|\,\|\mathbf{I}\|)$

$P = \|\mathbf{V}\|\,\|\mathbf{I}_a\|$ Eq. 1.13

It can therefore be said that if the load was adjusted or compensated so that the line current became $\mathbf{I}_a$, then the power P dissipated in the load would remain the same and the power factor corrected to:

$\lambda_a = P/P = 1$

Referring back to the example of Fig. 1 and using definition 3 (or Eq. 1.8) the power factor λ would be

$\lambda = P/S = 1 / \sqrt{27} = 0.1924$

This is below 0.2 and would effectively mean that if the load was adjusted (or compensated) one could carry more than 5 times the same real power for the same line losses (ignoring the neutral wire losses).

In order to calculate the AP correctly one has to find the optimum current vector that would transmit the same power P with minimum losses. This current vector is appropriately referred to as the active current vector component and the difference is referred to as non-active. Several power specialists, including Depenbrock [1962], and Czarnecki [1983], have defined active current in three phase systems as a current vector $\mathbf{I}_a$ where :

$$\mathbf{I}_a = G\,\mathbf{V} \quad \text{where} \quad G = P/\,\|\,\mathbf{V}\,\|^2 \qquad \text{Eq. 1.14}$$

This equation essentially means that the active line current components are proportional to the line voltages. Why this should be an optimum solution is not clearly explained.

If this equation was to be the solution consistent with Emanuel's statement then one would also have to make sure that the voltage "reference point" be chosen correctly so that Eq. 1.14 gives minimal active current norm, which implies the minimum theoretical line losses. It will be seen that the so called neutral or common base used as a voltage reference by many authors and existing standards [IEEE 1459, 2000] is not necessarily the correct one.

1.3 Examples of the effect of voltage reference on AP

If Eq. 1.14 was to be applicable to any kind of voltage waveforms, it should be valid for any voltage waveforms including those containing DC voltage components or zero sequence components. The following example applies to a 3-wire DC system [Malengret & Gaunt, 2005].

Example 1.3.1: DC voltages

Assume a resistive DC system as shown in Fig. 1.2. The bottom wire is used as the reference. It is assumed that all three wires have equal resistances equal to r.

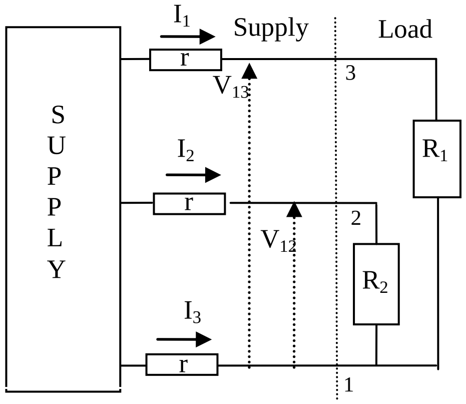

Fig. 1.2 3-wire system with DC voltages.

Let $V_{13} = 2$ VDC and $V_{12} = 1$ VDC

If one could adjust the load (or use a compensator), what value should I_1 or I_2 be in order to dissipate minimum line losses for an equivalent load drawing 100 Watts?

According to Eq. 1.14

$I_a = \{ I_{a1}, I_{a2}, I_{a3} \}$ where

$I_{a1} = (100/5)\ 2 = 40$ Amps

$I_{a2} = (100/5)\ 1 = 20$ Amps

$I_{a3} = - I_1 - I_2 = -60$ Amps

$P = 40 \times 2 + 20 \times 1 = 100$ Watts

$\Delta P' = r (40^2 + 20^2 + (-60)^2) = r (5600)$

$\Delta P'$ is not the optimum as a better solution exists:

If R_2 is open circuited and R_1 is adjusted to dissipate 100 Watts, then $I_1 = 100/2 = 50A$, $I_2 = 0$, and $I_3 = -50A$

$\Delta P' = r (2x\ 50^2) = r (5000)$

Eq. 1.14 would have given the above solution had one chosen line 2 as a reference.

According to Eq. 1.11 the power factor would be:

$\lambda = (5000/5600)^{1/2} = 0.945$ which is not optimal.

Therefore Eq 1.14 is not sufficient to derive the optimal solution.

Note that the concept of apparent power and power factor as given in Eq. 1.8 and 1.9 can therefore also be extended to DC circuits. The (DC) power factor in this case has nothing to do with phase angle between voltage waveforms and current waveforms but is simply a figure of merit reflecting the effective use of a particular voltage supply, with the intention to minimize transmission losses.

Example 1.3.2: Square waves

If the DC voltage supply waveform of Example 1.3.1 is replaced with two square waveforms, in phase but with different amplitudes as shown in Fig 1.3, then identical results to the DC example would be obtained. The power factor would be 0.945 and line 2 would be the correct reference.

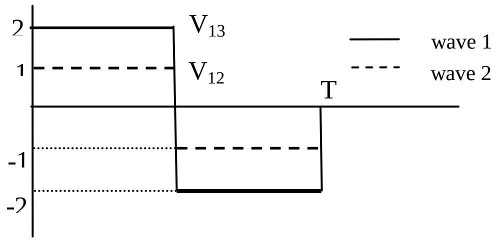

Fig. 1.3 Example of two square voltage .

It is obvious that the optimum reference may not necessary be the same throughout a period T. For example a waveform as shown in Fig. 1.4 would need a “dynamic virtual” reference.

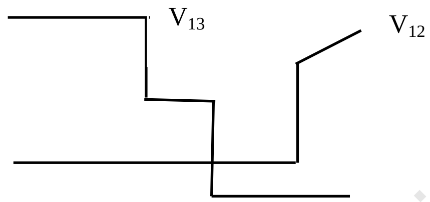

Fig. 1.4 Arbitrary voltage waveform example.

Note that a changing reference can be used when calculating $\mathbf{I_a}$ as per Eq. 1.11 as long as the law of energy conservation and circuit laws such as Kirchhoff’s current laws are respected.

1.4 A necessary additional condition in the definition of AP

As discussed following Eq. 1.18 and illustrated in example 1.3.2, AP can only be uniquely defined if the voltage reference is such that when S is calculated as S= $||\mathbf{V}||\ ||\mathbf{I}||$ it represents the maximum power that can be transmitted for the same losses.

Hypothesis 1: If S = ||V|| || I || is to represent the theoretical maximum power that can be transmitted for the same line losses. The voltage reference must be such that when ||V|| is calculated it has the maximum possible value.

The formulation of a definition of AP would then be equal to S and in accordance with Emanuel's statement would then give a meaningful and useful definition to AP, power factor and active power under all non-sinusoidal conditions, unbalanced and sinusoidal conditions.

The correct calculation of the ideal reference has to be clearly stated and justified. The thesis sets out to achieve this.

1.5 Decomposition of non-active power

The definition of non-active power and decomposition into sub-components can only be attempted once apparent power has been unequivocally defined. Only then would the definition/concept lead to a power component which does not contribute to the transfer of power and can therefore be correctly called **non-active power**. This power component could be decomposed in different ways into smaller portions (e.g. orthogonal components) that will depend on the most appropriate application or goal to be attained.

1.5.1 Application to 3-phase 4-wire systems

Most authors ignore the capacity of the fourth wire (neutral). It will be of interest to investigate the implication of considering the neutral current losses. In order for the hypothesis 1 to be valid it will be important to consider the result of including and excluding the effect of the neutral wire on both the losses and power transmission capacity, when defining AP to be the maximum power that can be transmitted for the same line losses.

Hypothesis 2

When defining Apparent Power (AP) S in a 3-phase 4-wire system it is also necessary to state how the wires are considered in both the losses and their influence on the power delivery capacity in accordance with hypothesis 1.

1.5.2 Instantaneous and average power theories

Emanuel's statement can be applied to any specific time interval. This can be for example an infinitely small time interval (instantaneous value at a specific time t) or over the period T of a periodic voltage and current waveform.

So called instantaneous collective power theory was first introduced by Buchholtz [1950] and later taken up by Depenbrock [1962]. Both theories were published initially in German.

A so called instantaneous power theory was popularised by Akagi et al [1983] and spurred a lot of publication and debates [Akagi et al, 1984, 1999; Peng & Lai, 1996; Willems, 1992, 1993, 2006 and Willems & Ghijselen 2004]. In these, some authors reformulated Akagi's theory and offered various "new" instantaneous theories; several were also extended from the instantaneous power theories into the average power domain. Other power specialists, such as Czarnecki, based their theory on average power and the frequency domain. Czarnecki mostly ignored the instantaneous power theory and still states that there is no plausible link between instantaneous power theories and the average power domain [Czarnecki, 2006] and, therefore according to him, it serves little purpose. This thesis reviews some of those relationships and their merit.

1.6 Structure and contribution of this thesis

In Chapter 2 the definition and analyses of apparent power for single-phase systems are reviewed. This is done in both the instantaneous and rms domain.

Chapter 3 reviews, compares and discusses valuable contributions made in the field of instantaneous power theory in multi-phase systems and in particular 3-phase. It is found by the author that all the instantaneous power theories reviewed for 3-phase systems falls into three distinct groups. The fact that each of the groups can be optimal (give better results than others) for a particular neutral wire resistance is, perhaps, an explanation of the lack of consensus between the various instantaneous power theoreticians.

In Chapter 4 the author demonstrates mathematically that the three instantaneous power groups can be reformulated into one general formula, where each of the groups uses a particular voltage reference and is respectively optimal depending to the value attributed to the neutral wire resistance.

Chapter 5 gives a mathematical explanation as to why each group can be optimal in its particular case. It then becomes clear and unambiguous as to the correct interpretation of instantaneous apparent power and therefore instantaneous non-active power. Then the analysis provided is practical and only then can be used as a basis for the decomposition of non-active power into various components.

Chapter 6 presents a new approach to instantaneous power based on vector space. This approach is specific to 3-phase 4-wire systems where all the conductors are considered to have equal resistance. This instantaneous power approach offers a unique and unprecedented way of comparing the non-active currents.

Chapter 7 reviews several rms or average domain theories for 3-phase systems. Several approaches can be seen. Most are based on extensions of the definition of single-phase theories. The apparent power in many cases is split into various power components to which a physical meaning is attributed. In a very few cases the instantaneous power theory is extended to the rms domain, but little consensus exists and some authors such as Czarnecki do not agree with the benefit or logic behind such attempts.

Chapter 8 demonstrates that instantaneous power theory based on the vector space approach, presented in chapter 6 can be naturally extended from the instantaneous domain into the average power domain, by simply redefining the inner products of the vectors. A coherent decomposition of currents and power components results. Moreover, the decomposition of currents and power components into four non-active powers is based on real physical properties and found to have merits for the design of active filters with and without energy storage devices.

Chapter 9 demonstrates the application of the average power theory described through practical results and the implication for the measurement of apparent power and power factor in particular examples in the rms domain.

Chapter 10 describes how to implement the theory into a practical instrument based on a digital signal processor (DSP) or a computer. The purpose of which would be to measure apparent power and power factor as well as generating the necessary control signals required by an active compensator. Experimental laboratory measurements of AP are taken on various load with an advance modern power meter and compared with the value calculated with the presented method of calculating AP.

Chapter 11 reviews the validity of the key hypotheses and summarises the proposed definitions of AP, power factor and non-active power and enumerates the key contributions of this thesis.

Chapter 2

Single phase power theory for non-sinusoidal systems

This chapter reviews theories related to non-sinusoidal systems in single-phase systems. The focus is on contributions concerned with defining and compensating unwanted power or current components related to transmission with minimum line losses.

The notations of the authors are partly used, but are often changed to a more uniform notation. Additional mathematical proofs above those given by the original authors have been added so as to familiarise the reader with the mathematical development that is to follow.

This chapter deals exclusively with single-phase systems.

2.1 General definition of power for single-phase systems

Active power P supplied by a periodical voltage source to a single-phase load is the average power supplied to a load over an observation time T, the duration of one cycle or an integer number of cycles. Where P is:

$P = 1/T \int_T u(t)\, i(t)\, dt$ Eq. 2.1

or

$P = 1/T \int_T p(t)dt$

where

$p(t) = u(t)\, i(t)$ Eq. 2.2

p(t) can be regarded as the time rate of energy transfer or consumption.

When the voltage and current are sinusoidal, it can be shown that the average power is the product of the rms value of the voltage and current respectively, multiplied by the cosine of the angle between the voltage and the current, which is also known as the power factor (p.f.).

$P = UI \cos \varphi$

When the waveforms are not sinusoidal and periodic, they can be expressed as a Fourier series and it can be shown from Fourier analysis that the average power P is the sum of each individual harmonic power and is:

$$P = \sum_n U_n I_n \cos \varphi_n$$

U_n, I_n are the rms voltage and current value of the respective harmonics of order n and φ_n is the phase angle between the sinusoidal harmonic voltage $u_n(t)$ and the current $i_n(t)$. There is no controversy about the definition of average power P and apparent power S; however apparent power and reactive powers are not based on a single well defined physical phenomenon. For sinusoidal voltage and currents, reactive power is defined as

$$Q = UI \sin \varphi$$

and apparent power as

$$S = UI$$

The following Pythagorean relationship then becomes evident

$$S^2 = P^2 + Q^2$$

In the case of non-sinusoidal single phase waveforms, the definition for apparent power S is also S = UI where U and I are the rms values of u(t) and i(t) respectively. S will also be equal to:

$$S = \left(\sum_n U_n^2 \sum_n I_n^2 \right)^{1/2}$$

In the case of non sinusoidal signals, non-active power has been defined by many. Some of those considered as more important for this thesis, will be covered in the following sections.

2.2 Budeanu

One of the early proposals of a definition for reactive power Q in non sinusoidal systems was given by Budeanu in the 1920's [1927, quoted in Czarnecki, 1987a].

$$Q = \sum_n U_n I_n \sin \varphi_n$$

The Pythagorean relationship between S, P and Q was no longer true, so a new quantity D was proposed by Budeanu, where

$$D^2 = S^2 - P^2 - Q^2$$

2.2.1 Relevance to the present hypothesis

- Reactive power as defined above does not necessarily lead to a Pythagorean relationship between S, P and Q.

- If Q is reduced to zero, $S^2 = P^2 + D^2$ and therefore $P \leq S$ and therefore the ratio of P/S generally known as power factor (p.f.) is never larger than 1.

- Q does not have a clear physical meaning and is not considered to provide useful information for any practical application [Czarnecki 1987] and in particular for an active or passive compensator that would minimise the transmission losses.

2.3 Fryze

Another of the early ideas of reactive power Q in non-sinusoidal single-phase systems was introduced by Fryze [1932] and recommended in 1980 by the International Electrotechnical Commission [IEC, 1979].

According to Fryze, if u(t) is the voltage of a single-phase system, then the source current i(t) can be decomposed in the time domain into component $i_a(t)$ and $i_b(t)$ defined as follows:

$$i_a(t) = (P/U^2)\ u(t) \qquad \text{Eq. 2.3}$$

where P is the average power supplied to a load during an interval T, and U is the rms value of the voltage applied to the load and the remaining current:

$$i_b = i(t) - i_a(t) \qquad \text{Eq. 2.4}$$

The power attributed to i_a over an interval T is:

$$P_a = 1/T \int_T u(t)\ i_a(t)\ dt$$

Substituting Eq. 2.2

$$P_a = (P/U^2)\ 1/T \int_T u(t)^2 = P \qquad \text{Eq. 2.5}$$

The average power delivered by a current i_a is the same as the average power delivered to the load by i. Hence **the average power** P_b contribution of i_b to the transmission of the average power must be zero.

S is defined as it is commonly known in a single-phase system as:

$S = UI$ Eq. 2.6

$S^2 = U^2 I^2 = U^2 \; 1/T \int_T i(t)^2 \, dt$

$= U^2/T \int_T i_a(t)^2 + U^2/T \int_T i_b(t)^2 + U^2/T \int_T 2i_a(t)\, i_b(t)$

$= P^2 + U^2 \; I_b^2 + U^2/T \int_T 2i_a(t)\, i_b(t)$

using

$Q = U\, I_b$ by definition, then

$S^2 = P^2 + Q^2 + U^2 \; 1/T \; 2\, P/\|V\|^2 \int_T 2v(t)\, i_b(t)$

but $1/T \int_T v(t)\, i_b(t) = 0$, since $P_b = 0$, hence

$S^2 = P^2 + Q^2$

dividing by U^2 gives:

$I^2 = P^2/V^2 + I_b^2$

from Eq. 2.3

$I_a^{\;2} = (P^2/V^4)\; 1/T \int_T v(t)^2 \, dt = P^2/V^2$

hence

$I^2 = I_a^2 + I_b^2$ Eq. 2.7

$i_a(t)$ and $i_b(t)$ are orthogonal under the inner product of two vectors defined in this case as

$< i_a(t) , i_b(t)> = 1/T \int_T i_a(t)\, i_b(t)\, dt = 0$

Relevant points.

- i_a (t) is proportional and in phase with respect to the voltage and is the current that would be drawn by a conductance G connected across the supply and of value $G = P/U^2$ so that it would dissipate the same average power P over an interval T.
- i_a (t) is optimum in the sense that it can be shown mathematically that no other current can be found that would result in lower line losses and deliver the same power P and therefore is in accordance with hypothesis 1 and 2.
- Q is defined without the use of Fourier series. It can be regarded as representing a measure of the under utilisation of a single-phase system. However Q does not have any physical interpretation as in the case of a sinusoidal single phase system and, in particular, does not provide any information relevant to the possibility of the p.f. improvement by means of a passive (static) filter consisting of capacitors and inductances.

- The Pythagorean equation $S^2 = P^2 + Q^2$ is always true, irrespective of the degree of voltage and current distortion.
- $i_b(t)$ is the current that a shunt connected filter (compensator) placed close to the load would require to inject, so as to minimize the supply losses.
- The active filter would require an energy storage element to store the difference between the instantaneous power $p(t)= v(t)i(t)$ and the average power P.
- It is assumed that the voltage remains constant after compensation and therefore the effect of the supply impedance is ignored.
- Fryze did not extend the theory to three-phase systems.

2.4 Kusters, Moore and Page

Fryze's definition is extended by a further split of the non-active current $i_b(t)$ into two orthogonal currents [Kusters & Moore, 1980]. The one component is calculated from the derivative and the integral functions of the voltage, and their respective rms values, the second non-active current is the residual. This method identifies the value of a single passive element, a capacitor or an inductance that will minimize the non-active current. Page extends it further to two passive elements; an inductance and a capacitor [Page, 1980].

2.4.1 Relevant points.

- The advantage of this method is that the balance of the non-active current can be compensated by means of capacitors or inductances (passive elements), not needing an active filter, while the residual non-active current itself requires more (a passive filter in some cases – a combination of capacitors and inductors – or an active filter in other cases).
- The theory depends on rms or average values of voltages and its rms integral and derivative and, therefore, at least half a cycle of voltage and current samples are required before it is possible to determine the necessary compensating current.

2.5 Czarnecki single-phase theory.

For linear non-sinusoidal single-phase system the theory presented by Czarnecki [1987a] begins by extracting Fryze's time domain active current $i_a(t)$. He then proceeds to analyze the remaining current using the frequency domain. The justification given is that the frequency domain provides a more effective means of power factor improvement using passive elements and also offers a physical interpretation of power components and physical characteristic of the load.

The instantaneous voltage of a periodical voltage of ω_1 frequency is expressed by Czarnecki as a complex Fourier series:

$u(t) = \sqrt{2}\ \mathrm{Re} \sum_n U_n\, e^{jn\omega_1 t}$ Eq. 2.8

$i(t) = \sqrt{2}\ \mathrm{Re} \sum_n I_n\, e^{jn\omega_1 t}$ Eq. 2.9

the n^{th} harmonic admittance is $Y_n = I_n / U_n = G_n + j\, B_n$ Eq. 2.10

The current i(t) can be expressed as:

$i(t) = \sqrt{2}\ \mathrm{Re} \sum_n (G_n + j\, B_n) U_n\, e^{jn\omega_1 t}$ Eq. 2.11

Fryze's current $i_a(t)$ which supplies all the average power P is then deducted from i(t):

$i(t) - i_a(t) = \sqrt{2}\ \mathrm{Re} \sum_n (G_e - G_n + j\, B_n) U_n\, e^{jn\omega_1 t}$ Eq. 2.12

This current is then divided into two components:

$i_s(t) = \sqrt{2}\ \mathrm{Re} \sum_n (G_e - G_n) U_n\, e^{jn\omega_1 t}$ Eq. 2.13

which he calls the scattered current, which can be attributed to the load conductance varying with frequency (e.g. series connected reactance with resistor) and

$i_r(t) = \sqrt{2}\ \mathrm{Re} \sum_n j\, B U_n\, e^{jn\omega_1 t}$ Eq. 2.14

The three currents can be shown to be orthogonal under the inner product defined as in Section 2.3:

$< i(t) , i(t)> = < i_a(t) , i_a(t)> + < i_s(t) , i_s(t)> + < i_r(t) , i_r(t)>$

and therefore:

$I^2 = I_a^2 + I_s^2 + I_r^2$ Eq. 2.15

hence

$I^2 = P^2 / U^2 + \sum_n ((G_e - G_n)U_n^2 + \sum_n B_n U_n^2$ Eq. 2.16

multiplying by U^2 gives:

$S^2 = P^2 + D_s^2 + Q_r^2$ Eq. 2.17

The so called reactive power of a single-phase system with M harmonics can be compensated using a one-port multiple pole filter consisting of less than M(2M+1) elements inductances and capacitor passive elements [Emanuel, 1974]. The scattered power D_s cannot be compensated in a single-phase system by passive shunt compensation or filter.

In the case of non linear load [Czarnecki, 1993b] a further current component i_g is obtained and is referred to as a generated current harmonic component. These current harmonics exist due to the non-linear characteristic of the load and do not have corresponding voltage harmonics of the same order. The following rms equations result:

$I^2 = I_a^2 + I_s^2 + I_r^2 + I_g^2$ Eq. 2.18

and

$S^2 = P^2 + D_s^2 + Q_r^2 + D_g^2$ Eq. 2.19

2.5.1 Relevant points.

- One assumes that the voltage applied to the load remains constant and hence that the source impedance is negligible. Czarnecki's single-phase theory [1987a] provides a useful means to design a shunt connected passive filter that will eliminate the reactive power Q_r.
- The physical interpretation of the scattered current is a plausible one and can be explained as the result of change of conductance of a load circuit with frequency. D_s can only be eliminated with an active compensator.
- The above is true for linear loads. However, for non linear loads it would be difficult, if not impossible, to separate the effect of frequency on admittance of the load and that caused by non-linear elements. The physical meaning would be lost depending on the degree of non-linearity. Nevertheless, the theory still has practical merit in the design of one-port filters using capacitors and inductances. However the complexity of such passive filter may not find many useful applications.

2.6 Enslin and van Wyk

Enslin and van Wyk [1988] generalized Fryze's time-domain principle to single-phase non-periodic waveforms by using correlation techniques. The active current is the same as that of Fryze's if a time interval T is considered. The non-active current is decomposed into two orthogonal components. The one is called reactive and the other deactive.

However since the theory has not yet been extended to three-phase systems no attempt is made to link it to the present hypothesis.

2.7 Conclusion

The definition of apparent power as the product of rms voltage and current is clear and presents no ambiguity in the case of single-phase systems. The active current component as defined by Fryze (Eq. 2.3) represents the minimum current required to transmit the average power P in all possible cases.

Apparent power S defined as the product of rms voltage and current is the maximum power that can be transmitted for the same losses and is consistent with Emanuel's hypothesis. The rms value of the non-active current which consists of the difference between the actual line current and the active current is orthogonal to the active current (with respect to the inner product as defined in 2.3.1). The non-active current can be decomposed into various orthogonal components that in some cases can have a useful application in the design of active and passive filters and give physical interpretation to the various non-active components.

Chapter 3

Review of 3-phase instantaneous power theory

Single-phase systems have the advantage of simplicity, as they consist of two conductors only. However, above a certain electrical power rating three-phase systems become more suitable, for some of the following reasons:

- A balanced three phase systems does not necessarily require a return wire (neutral) as the sum of the collective instantaneous currents is zero for a three phase balanced system.
- The instantaneous power drawn from the supply of a balanced system is constant and continuous.
- Single-phase sinusoidal generation has a double frequency power oscillation component, which can cause undesirable vibrations in generators and motors. Single phase generators often need to be provided with very heavy and well constructed field dampers to store oscillating instantaneous power, otherwise severe mechanical vibration at twice the supply frequency will be experienced.
- Generation is more economical with a three-phase alternator than single phase as it utilises the available space and material of an electrical machine better.

The analysis of three phase systems has been generally approached from an average power point of view. However, due to the increased concern of voltage and current distortion caused by modern devices such as computers and power electronic converters, so called instantaneous power theories have assumed an important role, and have been intensely debated in the last 50 years. A thorough understanding of instantaneous power theory and its definition is necessary before analyzing the rms or average power domain.

Instantaneous power theory generally provides a means of decomposing currents into those contributing “useful” power and those “useless” components that can be provided from the source or by a local compensator. Suppressing these “useless” components with compensators, (also known as active filters) reduces the requirement on the supply and hence reduces the transmission losses.

Therefore this chapter reviews some of the more popular instantaneous power theories applied to 3-phase systems with 3 and 4 wires.

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3.1 Buchholtz

Buchholtz extended Fryze's single phase definition of non-active power to a poly-phase system [Buchholtz, 1950]. Buchholtz instantaneous power theory was not well known, apart from Depenbrock who included it in 1962 in what he called the FBD power theory. Buchholtz's and Depenbrock's work was published in German initially and was only appreciated and recognised more widely when it appeared in English publications from 1993 [Depenbrock 1993, Depenbrock et al 1994, 2003, Willems 2004].

Buchholtz described the **instantaneous collective** values of current and voltages in a system with M phases. He then defined the active instantaneous current i_{vp} of the v^{th} wire[1] in a m wire system as:

$$i_{vp} = g_p\, u_v \qquad \text{Eq 3.1}$$

for v = 1 to M (M is the number of conductors)

and the collective instantaneous collective voltage of the v th wire as:

$$u_v = 1/m \sum_{u=1}^{M} u_{vu} \qquad \text{Eq 3.2}$$

where:

$$g_p = p_\Sigma / u_\Sigma^{\,2}$$

$$p_\Sigma = \sum_{u=1}^{M} u_u\, i_u = \sum_{u=1}^{M} p_u$$

$$i_\Sigma^{\,2} = \sum_{u=1}^{M} i_u^{\,2}$$

$$u_\Sigma^{\,2} = \sum_{u=1}^{m} u_u^{\,2}$$

Fig 3.1 shows a 4-wire system with 3 line voltages measured from the neutral wire.

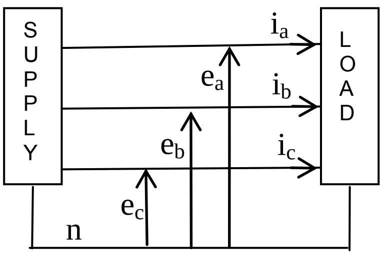

Fig.3.1 3-phase supply and load with 4-wire connection, with voltages measured from the neutral wire, n.

[1] The original author notations are in most instances retained when describing their theories, these will be changed later to a more general one when comparing with others.

The instantaneous voltages and currents in a non-sinusoidal system, at any moment, could be any arbitrary chosen positive or negative real number. The following instantaneous values for line voltages[2] and currents are arbitrarily chosen by the author for an example (not from Buchholtz) that will be used to illustrate several approaches in this thesis:

$e_a = -3$ V $i_a = -9$ A

$e_b = 9$ V $i_b = 2$ A

$e_c = -2$ V $i_c = -5$ A

Note that in this example $e_a + e_b + e_c = 4 \neq 0$

Let us consider two cases for a three phase system, where M can be 3 or 4 depending on the number of wires considered.

<u>Example 3.1 M = 3</u>

$p_\Sigma = (-3)(-9) + (9)(2) + (-2)(-5) = 55$ W

From Eq 3.2

$u_1 = ⅓ (u_{aa} + u_{ab} + u_{ac}) = ⅓ (0 + (-3 - 9) + (-3 - (-2)) = -4{,}333$

$u_2 = ⅓ (u_{ba} + u_{bb} + u_{bc}) = ⅓ ((9 - (-3)) + 0 + (9-(-2)) = 7{,}667$

$u_3 = ⅓ (u_{ca} + u_{cb} + u_{cc}) = ⅓ ((-2 - (-3)) + (-2 - 9) + 0 = -3.333$

$u_\Sigma(t)^2 = 88.67$

$g_p(t) = p_\Sigma(t) / u_\Sigma(t)^2 = 55/88.67$

From Eq 3.1

$i_{1p}(t) = -2.69$, $i_{2p}(t) = 4.76$ $i_{3p}(t) = -2.07$

giving:

$\| \mathbf{I}_p \|^2 = 34.12$

<u>Example 3.2 M = 4</u>

From Eq 3.2

$u_1 = ¼(u_{nn} + u_{na} + u_{nb} + u_{nc}) = ¼ (0 - (-3) - 9 - (-2)) = -1$

$u_2 = ¼(u_{an} + u_{aa} + u_{ab} + u_{ac}) = ¼ (-3 + 0 + (-3-9) + (-3 - (-2)) = -4$

[2] Symbol e for line voltages measured from the neutral is used thereafter for the sake of consistency when comparing with other instantaneous theories.

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$u_3 = \frac{1}{4} (u_{bn} + u_{ba} + u_{bb} + u_{bc}) = \frac{1}{4} (-2 - (-3)) + (-2 - 9) + 0 = 8$

$u_3 = \frac{1}{4} (u_{nc} + u_{ca} + u_{cb} + u_{cc}) = \frac{1}{4} (-2 - (-3)) + (-2 - 9) + 0 = -3$

$u_{\sum}^{2} = 90 \qquad g_p = p_{\sum} / u_{\sum}^{2} = 55/90 = 0.611$

From Eq 3.1 :

$i_{1p} = -0.611, \; i_{2p} = -2.444, \; i_{3p} = 4.889, \; i_{4p} = -1.833$

The line currents square sum is:

$\|\mathbf{I_{vp}}\|^2 = p^2 / u_{\sum}^{2} = 55^2 / 90 = 33.61$

3.1.1 Relevance to the present hypothesis

- Fryze's concept for active instantaneous currents in single phase was extended by Buchholtz for the first time to polyphase systems. The instantaneous active current of the v^{th} wire is proportional to g_p and a voltage u_v, which can be calculated at any instance, from the instantaneous values of voltages and currents.
- u_v is calculated from the voltage differences between the wires and is therefore independent of the voltage reference chosen.
- The theory applies to any number of wires.
- It is not clear that it is an optimum solution in the sense that the transmission losses will be minimum after compensation, as no mathematical proof is given.

3.2 Akagi's instantaneous p-q theory

Apparently independently unaware of Buchholtz's theory, Akagi et al [1984, 1989] introduced a time domain theory called the p-q instantaneous power theory for three-phase systems that is also based on instantaneous values of currents or voltages, instead of averages or rms values. A new electrical quantity q called instantaneous reactive power[3] was introduced. The p-q theory gained much interest as the concept of instantaneous reactive power was normally only attributed to sinusoidal waveforms. The concept was appealing as it enabled calculation in real time of compensating currents for active filters where no energy storage was necessary. The theory was applied to 3-phase, 3-wire and 4-wire

[3] Due to the controversy that the term instantaneous reactive power has generated, it is now generally referred in publications as non-active current or "useless current".

systems. The calculation did not require any average current or voltage values. Akagi began by defining a voltage $\mathbf{E}_{abc}$ and current $\mathbf{I}_{abc}$ vector:

$$\mathbf{E}_{abc} = \{e_a, e_b, e_c\}^T \qquad \text{Eq 3.3}$$

$$\mathbf{I}_{abc} = \{i_a, i_b, i_c\}^T \qquad \text{Eq 3.4}$$

Akagi then transformed the instantaneous line voltages and current vectors to the 0-α-β reference frame as follows:

$$\mathbf{E}_{0\alpha\beta} = \{e_0, e_\alpha, e_\beta\}^T = A\,\mathbf{E}_{abc} \qquad \text{Eq 3.5}$$

$$\mathbf{I}_{0\alpha\beta} = \{i_0, i_\alpha, i_\beta\}^T = A\,\mathbf{I}_{abc} \qquad \text{Eq 3.6}$$

where:

$$A = \sqrt{(2/3)}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{(3)}/2 & -\sqrt{3}/2 \end{bmatrix}$$

The instantaneous power $p = e_a i_a + e_b i_b + e_c i_c$.

The original theory defines two instantaneous real power components p_0 and $p_{\alpha\beta}$ and one instantaneous imaginary power $q_{\alpha\beta}$, which are calculated as follows:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 1 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \qquad \text{Eq 3.7}$$

This leads to the decomposition of the currents into three components in each phase. From the above the compensating currents in the 0-α-β are found. Then:

$$\mathbf{I}_{c0\alpha\beta} = B\,\mathbf{I}_{0\alpha\beta} \qquad \text{Eq 3.8}$$

where:

$$B = 1/e^2_{\alpha\beta}\begin{bmatrix} e^2_{\alpha\beta} & 0 & 0 \\ -e_0 e_\alpha & e_\beta^2 & -e_\alpha e_\beta \\ -e_0 e_\beta & -e_\alpha e_\beta & e_\alpha^2 \end{bmatrix}$$

and $e^2_{\alpha\beta} = e_\alpha^2 + e_\beta^2$

Eq 3.9

Compensating currents i_{ca}, i_{cb}, i_{cc} in the abc reference frame can then be obtained:

$$\mathbf{I}_{cabc} = A^{-1}\,\mathbf{I}_{c0\alpha\beta} \qquad \text{Eq 3.10}$$

where: $\{i_{ca}, i_{cb}, i_{cc}\} = \mathbf{I}_{cabc}^T$

`

and the inverse matrix of A is:

$$A^{-1} = \sqrt{(2/3)} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1//\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

The line currents after compensation are then :

$$\mathbf{I}_{sabc} = \mathbf{I}_{abc} - \mathbf{I}_{cabc} \qquad \text{Eq 3.11}$$

An active filter that requires no instantaneous real power, as depicted in Fig. 3.2, can supply the compensating currents.

The compensated supply current I_s still supplies the same real power, and the non-active (described as "useless") current components of I_c are supplied by the active filter, hence reducing the line losses.

Akagi's original method is said to also be valid in cases where the sum of the voltages are not zero (where, $e_a + e_b + e_c \neq 0$). These are depicted as containing instantaneous zero sequence voltage components.

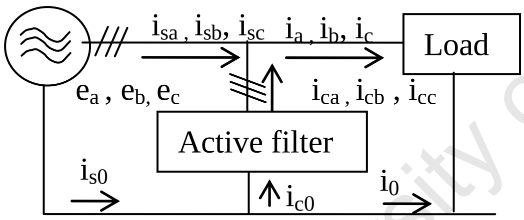

Fig. 3.2. Active filter for 4-wire system

For any four wire system, three voltages samples measured from the neutral e_a, e_b, e_c and three load current samples i_a, i_b , i_c are taken (See Fig. 3.1) This set of values can be taken at any instant in time and can be treated as any arbitrary real positive, negative numbers or zero.

<u>Example 3.3</u>

Choose the same arbitrary values as in the examples given for Buchholtz (Ex 3.1 and 3.2)

p = (-3)(-9) + (9)(2) + (-2)(-5) = 55 watts

$\{e_0 , e_\alpha , e_\beta \}$ = { 2.31 , -5.31, 7.78 } (from Eq 3.3)

$\{ i_0 , i_\alpha , i_\beta \}$ = {-6.93, -6.12, 4.95 } (from Eq 3.4)

$\{ i_{c0} , i_{c\alpha} , i_{c\beta} \}$ = {-6.93, -2.83, 0.12 } (from Eq 3.5)

$\{ i_{ca} , i_{cb} , i_{cc} \}$ = {-6.31, -2.76, -2.93 } (from Eq 3.6)

`

giving the supply current after compensation as:

$\{ i_{sa}, i_{cb}, i_{cc} \} = \{-2.69, 4.76, -2.07\}$ (from Eq 3.7)

The neutral current i_{s0} after compensation is

$i_{s0} = -2.69 + 4.76 - 2.07 = 0$

The power transmitted after compensation is:

$p = (-3)(-2.69) + (9)(4.76) + (-2)(-2.07) = 55$ W

p remains at 55 W but the sum of the square of the line currents is reduced from 254 to 34.12.

3.2.1 Relevance to present hypothesis

- The power and current components are calculated from instantaneous value and not average or rms values.
- The active filter required for compensation does not need to store energy as the instantaneous power needed is zero.
- The line losses are reduced if the instantaneous non-active current is supplied from a local compensator.
- The supply current losses have been reduced and the resultant neutral current in the case of a four wire system is zero after compensation, even if zero sequence voltage exists (i.e. $e_a + e_b + e_c \neq 0$).
- It is not clear that it is an optimum solution, as no mathematical proof or justification is given.
- Akagi does not consider the case of a 3-phase 4-wire system, where a zero neutral current in the fourth wire after compensation is not a constraint and therefore, whether the line losses could be reduced further, with a different method for calculating the compensating currents.
- Akagi's results are identical to those obtained using Buchholtz's equation with m=3.
- Buchholtz results with m=4 result in a neutral current and give lower total line losses in this example if all the wires are considered to have equal resistance.

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3.3 Willems

Willems [1992, 1993] proposes a ‘‘new’’ and more direct formula than Akagi’s to calculate instantaneous current components. His claim is that it is not necessary to define real and imaginary instantaneous power components, but that it suffices to define the instantaneous current components and voltages as vectors and through a simple process of vector projections where the inner product ($< \mathbf{v}, \mathbf{i} >$) is defined as the dot ($\mathbf{v} \bullet \mathbf{i}$) product of the current and voltage vector, it is possible to obtain the same results as those of Akagi. Moreover the theory can be applied to single phase as well as poly-phase systems. In 1993 he called it a new interpretation of the p-q theory.

Let the number of phases be denoted by m. The instantaneous currents and voltages in the m phases of the line are represented by the M-dimensional vectors $\mathbf{i}(t)$ and $\mathbf{v}(t)$. The instantaneous power transmitted to the load is the dot product of these two vectors:

$$p(t) = \mathbf{v}(t) \bullet \mathbf{i}(t) \qquad \text{Eq 3.12}$$

Let $\mathbf{i}_p(t)$ be the orthogonal current vector projection of the current vector $\mathbf{i}(t)$ onto the voltage vector $\mathbf{v}(t)$.

$\mathbf{i}_p(t)$ is proportional to $\mathbf{v}(t)$ and in the direction of $\mathbf{v}(t)$, and is calculated as follows:

$$\mathbf{i}_p(t) = (\mathbf{v}(t) \bullet \mathbf{i}(t)/\|\mathbf{v}(t)\|^2)\ \mathbf{v}(t) \qquad \text{Eq 3.13}$$

or

$$\mathbf{i}_p(t) = (p(t)/\|\mathbf{v}(t)\|^2)\ \mathbf{v}(t) = k(t)\ \mathbf{v}(t) \qquad \text{Eq 3.14}$$

$\|\mathbf{v}(t)\|$ denotes the length of a vector, i.e., $\|\mathbf{v}(t)\|^2 = \mathbf{v}(t) \cdot \mathbf{v}(t)$

The current $\mathbf{i}_q(t) = \mathbf{i}(t) - \mathbf{i}_p(t)$ is orthogonal to v(t) and therefore to $\mathbf{i}_p(t)$, such that

$$\mathbf{i}_p(t) \cdot \mathbf{i}_q(t) = 0 \qquad \text{Eq 3.15}$$

Summarizing, the instantaneous current vector can be decomposed into two components:

$$\mathbf{i}(t) = \mathbf{i}_p(t) + \mathbf{i}_q(t) \qquad \text{Eq 3.16}$$

- the *instantaneous active current* $\mathbf{i}_p(t)$ proportional to the voltage $\mathbf{v}(t)$ corresponds to the instantaneous power, p(t);
- the *instantaneous non-active* current $\mathbf{i}_q(t)$, which corresponds to a fictitious but useful quantity called instantaneous power q(t) with the dimension of power (product of voltage and current). It must be emphasised that the current $\mathbf{i}_q(t)$ contributes zero power at all time t.

`

The instantaneous power p(t) is also equal to

$p(t) = \mathbf{v}(t) \cdot \mathbf{i}(t) = \mathbf{v}(t) \cdot \mathbf{i}_p (t) + \mathbf{v}(t) \cdot \mathbf{i}_q (t) = \mathbf{v}(t) \cdot \mathbf{i}_p (t)$

The instantaneous imaginary power can be associated with

$q(t) = ||\mathbf{v}(t)||\ ||\mathbf{i}_q(t)||$ Eq 3.17

Since $\mathbf{i}_q(t)$ is orthogonal to $\mathbf{i}_p(t)$ then

$||\ \mathbf{i}(t)\ ||^2 = ||\ \mathbf{i}_p(t)\ ||^2 + ||\ \mathbf{i}_q(t)\ ||^2 = i_a(t)^2 + i_b(t)^2 + i_c(t)^2$ Eq 3.18

Moreover

$||\ \mathbf{i}_p(t)\ ||^2 = \mathbf{i}_p(t) \cdot \mathbf{i}_p (t)$ from Eq 3.14

$||\ \mathbf{i}_p(t)\ ||^2 = p(t)/||\mathbf{v}(t)||^2\ \mathbf{v}(t) \cdot p(t)^2/||\mathbf{v}(t)||^2\ \mathbf{v}(t)$

$||\ \mathbf{i}_p(t)\ ||^2 = p(t)^2 /||\mathbf{v}(t)||^2$ Eq 3.19

and

$||\ \mathbf{i}_q(t)\ ||^2 = q(t)^2 /||\mathbf{v}(t)||^2$ Eq 3.20

Therefore

$i_a(t)^2 + i_b(t)^2 + i_c(t)^2 = ||\ \mathbf{i}_p(t)||^2 + ||\ \mathbf{i}_q(t)\ ||^2$ Eq 3.21

$i_a(t)^2 + i_b(t)^2 + i_c(t)^2 = (p(t)^2 + q(t)^2) / ||\ \mathbf{v}(t)\ ||^2$ Eq 3.22

If q(t) is removed from the supply by local compensation, the supply line losses would be minimum.

<u>Example 3.4</u>

Choosing the same arbitrary currents and voltages as in examples 3.1 to 3.3, then:

Using Willems' method to calculate supply current I_s after compensation,

$\mathbf{v} = \{-3, 9, -2\}$ $||\mathbf{v}||^2 = 9 + 81 + 4 = 94$

From Eq 3.14

$\mathbf{i}_p = \mathbf{i}\ (p^2/||\mathbf{v}||^2)$

$\mathbf{i}_p = \{-1.76, 5.27, -1.17\}$

The neutral current i_{s0} is : (-1.76 + 5.27 -1.17) = -2.34.

The 4 wires current can be represented by a 4 dimension current vector where:

$\mathbf{i}_p = \{-2.24, -1.76, 5.27, -1.17\}$

$||\mathbf{i}_p|| = 37.65$

`

It is also evident from Eq 3.14 that Willems' original theory [1993][4] will result in some cases in a neutral current i_{s0} after compensation, when the sum of the three line voltages are not zero (as opposed to that of Akagi's). If the lines (including the neutral) have the same resistance then the line losses in this example are higher with Willems' solution. This is found to be the case for any other arbitrarily chosen, values of voltage and current.

The following questions arises:

1) Can it be shown that Akagi's methods always give a better result if the neutral current is constrained to be zero?

2) Can Willems' method give better results in some cases where the neutral current is not constrained to zero after compensation?

3) Can the line losses be reduced further than with any of the three above methods ?

3.3.1 Relevance to present hypothesis

- Willems has reformulated Akagi's equations in a much more simple form. However, in the case where zero sequence voltage components are present, the neutral wire would carry a current after compensation and Willems' equation would not be equivalent to Akagi's equations. Since the neutral wire is the sum of the line currents after compensation

$i_n(t) = - \sum_k \; p(t) / \|\mathbf{v}(t)\|^2 \, v_k$ for $k = 1 , 2, \ldots$

this would be zero only in the case were $\sum_k \; v_k = 0$

- In the case where the sum of the voltages is not zero and the neutral wire loss is neglected (i.e neutral wire resistance is zero), Willems' equation gives lower losses than Akagi and consistent with Buchholtz with m=3.
- In the case where the sum of the voltages is not zero and the neutral wire resistance is considered to be the same as the other wires, Willems original equation gives more losses than Akagi, and Buchholtz with m=3 or m=4.

[4] In 1993 it appears that Willems theory was only meant to be applied to three phase, 3 wire systems. Hence this may be an unfair comparison if the neutral wire resistance is not considered to be zero.

`

3.4 Nabae, et al

Nabae, et al [1995], formulated a “modified” theory of instantaneous power theory.

The modified theory in Nabae et al [1994, 1995] , and Akagi et al [1999] defines an instantaneous real power p and three instantaneous imaginary powers, q_0 , q_α , q_β , as follows:

$$\begin{bmatrix} p \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix} = \begin{bmatrix} e_0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \\ e_\beta & 0 & -e_0 \\ -e_\alpha & e_0 & 0 \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \qquad \text{Eq 3.23}$$

The following equations q_0 , q_α and q_β can be derived from above as follows

$$e_0 q_0 = -e_0 e_\beta i_\alpha + e_0 e_\alpha i_\beta$$

$$e_\alpha q_\alpha = e_\alpha e_\beta i_0 - e_0 e_\alpha i_\beta$$

$$e_\beta q_\beta = -e_\alpha e_\beta i_0 + e_0 e_\beta i_\alpha \qquad \text{Eq 3.24}$$

The sum of all the terms on the right hand side of the previous equations is always zero, thus leading to

$$e_0 q_0 + e_\alpha q_\alpha + e_\beta q_\beta = 0 \qquad \text{Eq 3.25}$$

The rank of the mapping matrix in the previous matrix representation is three so that the number of independent variables among q_0 , q_α and q_β is not three but two. As a result this leads to a three-dimensional power space on the modified theory, like that in the Akagi’s original theory. However, the two mapping matrices defined by Eq 3.7 and Eq 3.24 are different in formulation. The inverse transformation is performed as follows:

The instantaneous current in each phase can be obtained from the inverse transformation above and

$$i_0 = 1/ e^2_{0\alpha\beta} (e_{0p}) + (1/ e^2_0) (e_\beta q_\alpha - e_\alpha q_\beta)$$

where

$$e^2_{0\alpha\beta} = e^2_0 + e^2_\alpha + e^2_\beta$$

$$= i_{0p} + i_{0q}$$

where

i_{0p} : zero sequence instantaneous active current

`

i_{0q} : zero sequence instantaneous reactive current

i_α = $1/e^2_{0\alpha\beta}$ ($e_{\alpha p}$) + $1/e^2_{0\alpha\beta}$ ($e_0 q_\beta - e_\beta q_0$)

$i_{\alpha p}$ + $i_{\alpha q}$

i_β = $1/e^2_{0\alpha\beta}$ ($e_{\beta p}$) + $1/e^2_{0\alpha\beta}$ ($e_\alpha q_0 - e_0 q_\alpha$) Eq 3.26

= $i_{\beta p}$ + $i_{\beta q}$

The active current as given as :

$\mathbf{I}_{0\alpha\beta}$= { i_{0p} , $i_{\alpha p}$, $i_{\beta p}$ }^{T} and therefore from Eq 3.26

$\mathbf{I}_{0\alpha\beta}$= { $(p/e^2_{0\alpha\beta})\, e_0$, $(p/e^2_{\alpha\beta})\, e_\alpha$, $(p/e^2_{\alpha\beta})\, e_\beta$}^{T} Eq. 3.27

The active current in the abc normal frame is found by:

$\mathbf{I}_{abc} = A^{-1}\, \mathbf{I}_{0\alpha\beta}$ Eq. 3.28

Where $\mathbf{I}_{0\alpha\beta}$ = { i_{0p} , $i_{\alpha p}$, $i_{\beta p}$ }

Example 3.5

The equation illustrating the zero-sequence current i_0 implies that it can be divide into two instantaneous currents; $i_{\alpha p}$ and i_{0q} . Thus the following relations are obtained

$e_0 i_{0p} + e_\alpha i_{\alpha p} l + e_\beta i_{\beta p}$
$= (e_0/e_0^2)\, e_{0p} + (e_0/e_0^2)\, e_{\alpha p} + (e_0/e_0^2)\, e_\beta e_{\beta p}$
$= p_{0p} + p_{\alpha p} + p_{\beta p} = p$

$e_0 i_{0q} + e_\alpha i_{\alpha q} + e_\beta i_{\beta q}$
$= (e_0/e_0^2)(e_\beta q_\alpha - e_\alpha q_\beta) + (1/e_0^2)(e_0 q_\beta - e_\beta q_0) + e_\beta(1/e_0^2)(e_\alpha q_0 - e_0 q_\alpha)$
$= p_{0q} + p_{\alpha\theta} + p_{\beta\theta} = 0$

Again using the same values as example 3.1, the following results are obtained:

p = (-3)(-9) + (9)(2) + (-2)(-5) = 55 watts

{e_0 , e_α , e_β } = { 2.31 , -5.31, 7.78 } (from Eq. 3.3)

{ i_0 , i_α , i_β } = {-6.93, -6.12, 4.95 } (from Eq. 3.4)

{ i_{0p} , $i_{\alpha p}$, $i_{\beta p}$ } = { 1.35, -3.10, 4.55 } (from Eq. 3.27)

{ i_a , i_b , i_c } = {-1.76, 5.27 , -1.17 } (from Eq. 3.28)

The neutral current i_{s0} = -1.76 + 5.27 -1.17 = -2.34

It can be seen that Nabae's equation gives the same results as those of Willems's original equation.

`

3.4.1 Relevance to present hypothesis

- Nabae's equations first attempt to define various power components extends Akagi's 3-dimension matrix to a 4-dimension one. This results in a neutral current when zero sequence voltages exists (measured from neutral wire).
- The results are identical to those obtained by Willems' more simple equation, using the neutral wire as a voltage reference.
- No mathematical proof is offered to substantiate that the active current necessarily results in the least transmission losses or generation requirement to meet the original load power.

3.5 Peng and Lai

Peng and Lai [1996] announced a new "generalised theory of instantaneous reactive power" as the cross product of voltage and current of a three phase system. The instantaneous reactive power is given as a vector orthogonal to v(t) and i(t).

$\mathbf{q}(t) = \mathbf{v}(t) \times \mathbf{i}(t)$

$\| \mathbf{q}(t)\| = \| \mathbf{v}(t) \times \mathbf{i}(t) \|$

The instantaneous current vector, $\mathbf{i}_p(t)$, the instantaneous reactive current, $\mathbf{i}_q(t)$, the instantaneous apparent power s(t), and the instantaneous power factor are defined as

$\mathbf{i}_p(t) = p(t)/\| \mathbf{v}(t)\|^2 \ \mathbf{v}(t)$ Eq. 3.27

$\mathbf{i}_q(t) = \mathbf{q}(t) \times \mathbf{v}(t) / \| \mathbf{v}(t)\|^2$ Eq. 3.28

$s(t) = \| \mathbf{v}(t)\| \, \| \mathbf{i}(t) \|$ Eq. 3.29

$\lambda(t) = p(t) / s(t)$ Eq. 3.30

The sum of $\mathbf{i}_p(t)$ and $\mathbf{i}_q(t)$ is always equal to $\mathbf{i}(t)$ since:

$$
\begin{aligned}
\mathbf{i}_p + \mathbf{i}_q &= \mathbf{v}\ p/\|\mathbf{v}\|^2 + \mathbf{q} \times \mathbf{v} / \|\mathbf{v}\|^2 \\
&= ((\mathbf{v} \times i) \times \mathbf{v} + (\mathbf{v} \cdot \mathbf{v})\, i)/ \|\mathbf{v}\|^2 \\
&= ((\mathbf{v} \cdot \mathbf{i})\, \mathbf{v} + (\mathbf{i} \cdot \mathbf{v})\mathbf{v} + (\mathbf{v} \cdot \mathbf{v})\, i)/\|\mathbf{v}\|^2 \\
&= (\mathbf{v} \cdot \mathbf{v})\, i / \|\mathbf{v}\|^2 \\
&= i
\end{aligned}
$$

therefore

$i_q = i - i_p$ Eq. 3.31

`

3.5.1 Relevance to present hypothesis

- The more complicated cross product approach of Peng & Lai and the definition of power components for each phase is interesting for a mathematical power definition for 3-phase systems. However the method gives exactly the same active and non-active currents expressed in essentially an equivalent equation by Willems.

3.6 Ferrero and Superti-Furga

Ferrero and Superti-Furga's approach [1991] employs the Park transformation and they show how the method fits with other proposed methods such as Akagi's.
They state:
" This method leads to the definition of quantities that are intrinsically apparent powers… drawbacks can be overcome if the Park transformation is employed in describing three-phase systems: in fact, this mathematical approach represents a powerful, synthetic, and universal way to represent the behaviour of three-phase systems in any possible working condition (unsymmetrical, unbalanced, non-sinusoidal, etc.)… It was proven that the application of this method leads to the definition of two quantities, the real and the imaginary power, that are measured in a quite simpler way than those proposed by other theories proposed by other authors."

3.6.1 Instantaneous values

The approach takes three original quantities $y_a(t)$, $y_b(t)$, $y_c(t)$ (that represent three voltages measured from an arbitrary voltage reference or the three line currents), and changes them to the complex Park vector:

$$y(t) = \sqrt{(2/3)}\ (\ y_a(t) + a\ y_b(t) + a^2\ y_c\ (t)), \qquad \text{Eq. 3.32}$$

where a is the complex operator $e^{j\,2\pi/3}$

After separating the real and imaginary part

$$y = y_d(t) + jy_q(t) \qquad \text{Eq. 3.33}$$

the zero sequence component is given as

`

$y_0(t) = (1/\sqrt{3})\ (y_a(t) + y_b(t) + y_c(t))$ Eq. 3.34

This is expressed in matrix form as:

$[y_d(t), y_q(t)\ , y_0(t)\ ]^T = [\ T\ ]\ [y_a(t), y_b(t)\ , y_c(t)\ ]^T$ Eq. 3.35

$$[\ T\ ] = \begin{bmatrix} \sqrt{(2/3)} & -\sqrt{(1/6)} & -\sqrt{(1/6)} \\ 0 & \sqrt{(1/2)} & -\sqrt{(1/2)} \\ \sqrt{(1/3)} & \sqrt{(1/3)} & \sqrt{(1/3)} \end{bmatrix}$$

[T] is orthogonal hence $[T]^{-1} = [T]^T$

$[y_a(t), y_b(t)\ , y_c(t)\ ]^T = [T]^{-1}\ [y_d(t), y_q(t)\ , y_0(t)\ ]^T$ Eq. 3.36

$[y_a(t), y_b(t)\ , y_c(t)\ ]^T = [T]^T\ [y_d(t), y_q(t)\ , y_0(t)\ ]^T$ Eq. 3.37

If the zero-sequence component is not present only the two d-q quantities are needed.
The Park vector is invariant with respect to additive terms [Ferrero & Superti-Furga, 1991, Ferrero et al 1993] ; it represents the "pure" three-phase component of the system.

3.6.2 Power Definition

They define the Park instantaneous complex power as:

$a_p(t) = v(t) \cdot i^*(t)$ Eq. 3.38

which results in the Park real power $p_p(t)$ and the Park imaginary power $q_p(t)$, where:

$a_p(t) = p_p\ (t) + j\ q_p(t)$ Eq. 3.39

The zero sequence power is introduced as $p_0\ (t) = v_0(t)\ i_0(t)$

It follows that the real instantaneous power is:

$p(t) = p_p(t) + p_0(t)\ = p_d(t) + p_q(t) + p_0(t)$ Eq. 3.40

Example 3.6

Using the same arbitrarily chosen numbers as in Example 3.1 but adding a further arbitrarily chosen number to all 3 values, for example -2 to the voltage, then

$\mathbf{v}(t) = \sqrt{(2/3)}\ (\ -3\text{-}2 + a\ (9\text{-}2)\text{+}\ a^2\ (\text{-}2\ \text{-}2)$ (from Eq 3.29)

$\mathbf{v}(t) = \sqrt{(2/3)}\ ((\ -3 + a\ (9)\text{+}\ a^2\ (-2) + -2\ (\ 1 + a + a^2\ )$

Note : it can be seen how the added voltage cancels out since $1 + a + a^2$

$\mathbf{v}(t) = \sqrt{(2/3)}\ (\ -3 + a\ (9) + a^2\ (-2))$

$\mathbf{v}(t) = \sqrt{(2/3)}\ (\ -1 + a\ (11) = \sqrt{(2/3)}\ ((\ -1 + -11/2 + j\ 11\sqrt{(3/2)})$

`

$\mathbf{v}(t) = v_d(t) + j\, v(t) = -5.31 + j\, 7.78$

$\mathbf{i}(t) = \sqrt{(2/3)}\,(-9 + a\,(2) + a^2\,(-5))$ (from Eq 3.29)

$\mathbf{i}(t) = i_d(t) + j\, i_q(t) = -6.12 + j\, 4.95$

$p_d = v_d(t)\, i_d(t) = 32.5$ W $\quad p_q = v_q(t)\, i_q(t) = 38.5 \quad p_0 = v_0(t)\, i_0(t) = -16$

$p = p_d + p_q + p_0 = 32.5+38.5 -16 = 55$ W

It can be seen that the Ferrero and Superti-Furga's approach gives identical power components $i_d(t) = i_\alpha(t)$ of Akagi and $i_q(t) = i_\beta(t)$ of Akagi.

Moreover a change in initial voltage reference in both cases does not alter the results.

The active current is identical to that of Akagi.

3.6.3 Relevance to present hypothesis

The following points are particularly relevant:

- Ferrero & Superti-Furga [1991] use the Park transform and deal with the zero sequence values separately. The Park transform is said to be in agreement with Akagi's approach and can be said to be equivalent and of a more general method.
- The concept of "pure" seems to be brought about by the fact that the Park transform is invariant with respect to an additive term and, therefore, zero sequence components. Similarly to Buchholtz (m=3), this formulation is independent of the voltage reference chosen.
- The resultant neutral current in a four wire system is zero after compensation. The theory does not consider the case where the neutral wire current is not constrained to zero after compensation.

3.7 Rossetto and Tenti.

Rossetto and Tenti [1994], propose yet an alternative way to calculate, the instantaneous active current vector $\mathbf{i}_p(t)$, as follows:

$$\mathbf{i}_p(t) = k(t)\, \mathbf{u}(t) \qquad \text{Eq. 3.41}$$

where $k(t) = (p(t) / \| \mathbf{u}(t) \|^2)$

u(t) is assumed to be such that the reference is chosen so that the sum of the voltages is zero at any one time. The reason given is said to be based on Lagrange multipliers method of finding a minima for the line current losses.

`

The instantaneous balance of the current is called reactive current:

$\mathbf{i}_q(t) = \mathbf{i}(t) - \mathbf{i}_p(t)$

Example 3.7

p = (-3)(-9) + (9)(2) + (-2)(-5) = 55 W

from Eq 3.2

Using the same values as the previous examples and shifting the reference so that the sum of the voltages to the three wires is zero, and found by subtracting $e_z/3$ where

$e_z = e_a + e_b + e_c = 4$

The new voltages measured from the shifted reference by $e_z/3$ are now refered as:

$e_{3a} = -3- 4/3 = - 4.33$ $e_{3b} = 9-4/3 = 7.67$ $e_{3c} = -2-4/3 = -3.33$

$\mathbf{e}_3(t) = \{ -4.33 , 7.67, -3.33 \}$

$\| \mathbf{e}_3(t) \|^2 = 88.67$

k(t) = 55/88.67 and from Eq 3.38

$\mathbf{i}_p(t) = \{-2.69, 4.76, -2.07\}$ giving: $\| \mathbf{i_p} \|^2 = 34.12$

3.7.1 Relevance to present hypothesis

- The virtual star point voltage reference concept is new and highly relevant as it is such that the sum of voltages is always zero. This effectively ensures that there are no zero sequence instantaneous voltage components.
- A mathematical justification based on Lagrange multipliers is given for the first time.
- The case where the neutral wire current is not constrained after compensation is not considered.
- The approach gives identical results to Buchholtz-Depenbrock (m=3) , Akagi is the most simple and direct from a computational point of view.

3.8 Comparison of results from different approaches.

It can be seen from Table 1 that when the neutral current is constrained to zero after compensation, only group 2 approaches will be valid and the other approaches which do not incorporate this constraint will give different results.

In the case where the neutral wire current is not restricted and the neutral wire resistance is the same as the other wires then group 1 results are lower than group 2 and 3. Group 3 results with the highest transmission losses.

`

Table 3.1

Comparison of instantaneous current before and after compensation

Assuming that all wires have equal resistances.

	i_{so}	i_{sa}	i_{sb}	i_{sc}	$\| \mathbf{I_s} \|^2$
Before compensation	12	-9	2	-5	**254**
Group 1					
Buchholz m=4	-0.611	-2.444	4.889	1.833	**33.61**
Group 2					
Buchholz m=3	0.00	-2.69	4.76	-2.08	**34.12**
Akagi	0.00	-2.69	4.76	-2.08	**34.12**
Rossetti Tenti	0.00	-2.69	4.76	-2.08	**34.12**
Ferrero Superti-Furga	0.00	-2.69	4.76	-2.08	**34.12**
Group 3					
Nabae et al	-2.34	-1. 75	5.26	1.17	**37.65**
Willems	-2.34	-1. 75	5.26	1.17	**37.65**
Peng	-2.34	-1. 75	5.26	1.17	**37.65**

If we now neglect the neutral wire current losses, one could look at the sum of the squares of the line currents only. Group 2 method is then not relevant.

The results of Group 1 and Group 3 methods, with:

$\| \mathbf{I_s} \|^2 = i_{sa}^2 + i_{sb}^2 + i_{sc}^2$ are presented in Table 2

Table 3.2

Comparison of instantaneous current before and after compensation

Assuming that 3 wires have equal resistances and the neutral wire resistance is zero.

	i_{so}	i_{sa}	i_{sb}	i_{sc}	$\| \mathbf{I_s} \|^2$
Before compensation	12	-9	2	-5	**110**
Group 1					
Buchholtz m=4	-0.61	-2.44	4.89	1.84	**33.23**
Group 3					
Nabae	-2.34	-1. 75	5.26	1.17	**32.17**
Willems	-2.34	-1. 75	5.26	1.17	**32.17**
Peng	-2.34	-1. 75	5.26	1.17	**32.17**

`

It can now be seen that in this case group 3 gives the lowest results for $\| \mathbf{I}_s \|^2$, when the neutral wire current is not restricted to zero and the neutral wire resistance is considered to be zero.

The approaches can be tested with any set of arbitrary chosen numbers, but the same group characteristics are evident.

3.9 Conclusion

Instantaneous power theory can be defined as power theory calculated at any instant from instantaneous values of voltage and currents, without prior information of the voltages and currents.

It was shown in terms of examples using arbitrarily chosen voltage and current values that all the instantaneous power reviewed in this chapter can be grouped by observation into 3 groups that give identical results:

1) Buchholtz's with m=4 (Group 1) gives best results for an active current when a neutral current can exist and is not restricted to zero after compensation, and the neutral wire is considered to have the same resistance as the other wires. The supply losses are lower than if there is no neutral wire ($r_n = \infty$).

2) Akagi, Buchholtz with m=3, Rossetto and Tenti, Ferrero and Superti-Furga, all give identical results (Group 2). They represent the condition that has no neutral current ($r_n=\infty$). The line losses are minimal using these methods only when the neutral wire active current is constrained to zero.

3) Willems, Nabae, and Peng's approaches (Group 3) result in neutral wire current in the active current. The supply losses are lowest if the neutral wire resistance is considered to be zero. However, the supply losses are the highest if the neutral wire is considered to have the same resistance as the other wires.

The above observations were tested with many other arbitrarily chosen values and found to be true. These observations do not seem to have been previously described and categorised into groups as this chapter has done. Moreover no mathematical reasons, justification or comprehensive proofs of the above observations regarding minimum losses have been found in published material. This will be addressed in the next two chapters.

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Chapter 4

Reformulation of instantaneous power theories

In the previous chapter it was seen that instantaneous power theory has become an important part of electrical engineering theories associated with line losses. The appropriate and unequivocal definition of instantaneous active power, power factor and apparent power could became possible if it can be shown that minimum line losses can be achieved under various constraints.

Various authors still emphasise their own theories in preference to others and there still exists a lack of clarity in even recent publications as to which approach is optimal and, in particular in 4-wire 3-phase systems, where a zero sequence voltage exists.

It was also seen in the previous chapter in terms of examples that all the various instantaneous power theories reviewed could be grouped into three groups, within which results are identical and, in turn, are superior under different criteria of constraints relating to the neutral current.

In this chapter an approach is developed in which all three groups of the instantaneous power theories can be expressed as only one general formula and each group is characterised by the particular voltage reference adopted.

4.1 Group 1 Reformulated

In group 1, the neutral current is not constrained to zero and the resistances of all wires including the neutral, are equal.

4.1.1 Buchholtz (M = 4) reformulated.

From Section 3.1 it was seen how Buchholtz defined the active current in an M-wire supply according to Eq 3.1 where:

$\mathbf{i}_{vp} = g_p \mathbf{u}_v$

u_v is the v^{th} wire voltage measured from a voltage reference defined as per Eq 3.2.

If individual wires of a system with M wires are referred as the m^{th} wire instead of the v^{th} and a new voltage reference is chosen so that the m^{th} voltage becomes v_{0m} where :

$v_{0m} = u_m - e_z/M \quad$ where $e_z = \sum_1^M u_n$

then $\sum_1^M v_{0m} = \sum_1^M u_n - \sum_1^M e_z/m = \sum_1^M u_n - M\, e_z/M = e_z - e_z = 0$

Therefore the <u>sum</u> of all the wires voltages measured from this reference is <u>zero</u>.

This voltage reference v_{0m} will now be referred to as the 'null' voltage reference for the m^{th} wire.

If the null reference is chosen then u_v , the voltage of the v^{th} wire as defined by Buchholtz becomes:

$$u_v = 1/M \sum_{u=1}^{M} u_{vu} = 1/M \sum_{\mu=1}^{M} (u_{0\mu} - u_{0v}) = 1/M \sum_{\mu=1}^{M} u_{0v} = u_{0v} = v_{0m}$$

Therefore Buchholtz u_v is equivalent to v_{0m} the voltage measured from the null point.

Buchholtz' Eq 3.1 can therefore be reformulated in vector form as follows:

$$\mathbf{i}_{am} = k_M \mathbf{v}_{0m} \qquad \text{Eq. 4.1}$$

where: $k_M = p/ \| \mathbf{v}_{0M} \|^2$,

$\mathbf{v}_{0M} = \{ v_{01}, v_{02}, \ldots, v_{0M} \} = \mathbf{v} - \mathbf{e}_{zM}$,

Eq 4-2

$\mathbf{e}_{zM} = \{ e_z/M, e_z/M, \ldots e_z/M \}$,

and

$\| \mathbf{v}_{0m} \|^2 = \mathbf{v}_{0m} \bullet \mathbf{v}_{0m}$

$$p = \sum_{v=1}^{M} p_m = \sum_{m=1}^{M} \mathbf{v} \bullet \mathrm{i}$$

Eq 4.1 can be stated simply as: the reassignment of the supply line currents to be proportional to the line voltages measured from a "null" reference where the sum of the voltages are zero, so the instantaneous power transferred remains the same.

If now M = 4 (e.g. 3-phase with neutral), then from Eq 4.1

$$\mathbf{i}_{a4} = k_4 \mathbf{v}_{04} \qquad \text{Eq. 4.3}$$

where $k_4 = p/ \| \mathbf{v}_{04} \|^2$

$\mathbf{v}_{04} = \{ v_{01}, v_{02}, v_{03}, v_{04} \} = \mathbf{v} - \mathbf{e}_{z4}$

$\mathbf{e}_{z4}$ = { $e_z/4$, $e_z/4$, $e_z/4$, $e_z/4$ } this is illustrated in Fig 4.1.

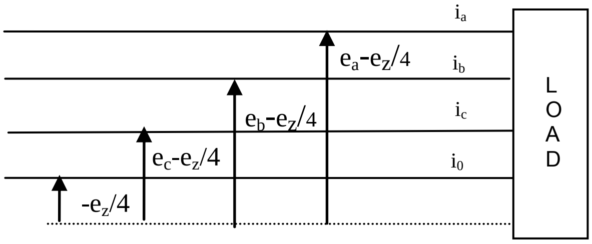

Fig.4.1. Virtual voltage reference shifted by -$e_z/4$ from neutral reference

The compensating current is:

$\mathbf{i}_{c4} = \mathbf{i} - \mathbf{i}_{a4}$ Eq. 4.4

4.2 Group 2 reformulated

In Group 2 the neutral current is constrained to zero.

4.2.1 Buchholtz (M = 3) reformulated

If then M = 3 from eq 4.1

$\mathbf{i}_{a3} = k\,\mathbf{v}_{03}$ Eq. 4.5

where $k_3 = p/\,\|\,\mathbf{v}_{03}\,\|^2$

$\mathbf{v}_{03}$ = { 0, v_{01} , v_{02} , v_{03} } = $\mathbf{v} - \mathbf{e}_{z3}$

$\mathbf{e}_{z3}$ = { 0, $e_z/3$, $e_z/3$, $e_z/3$ } this is illustrated in Fig 4.2.

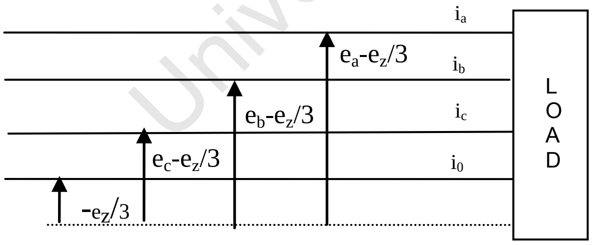

Fig.4.2. Virtual voltage reference shifted by -$e_z/3$ from neutral reference

The non active current is defined as:

$\mathbf{i}_{c3} = \mathbf{i} - \mathbf{i}_{a3}$ Eq 4.6

4.2.2 Akagi reformulated

Akagi's formulae can also be similarly reformulated (Malengret & Gaunt 2008).

If we decompose B as seen in Eq 3.9

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} -1/e^2_{\alpha\beta} \begin{bmatrix} 0 & 0 & 0 \\ e_0e_\alpha & e_\alpha^2 & e_\alpha e_\beta \\ e_0e_\beta & e_\alpha e_\beta & e^2_\beta \end{bmatrix}$$

Then from Eq 3.3 to 3.6

$\mathbf{I}_{c0\alpha\beta} = \mathbf{I}_{0\alpha\beta} - 1/e_{\alpha\beta}^2\{0, p\, e_\alpha , p\, e_\beta\}^T$

$p = e_0 i_0 + e_\alpha i_\alpha + e_\beta\, i_\beta$

$\mathbf{I}_{c0\alpha\beta} = \mathbf{I}_{\,0\alpha\beta} - p/e_{\alpha\beta}^2\{0,\ e_\alpha ,\ e_\beta\ \}^T$

$\mathbf{I}_{c0\alpha\beta} = \mathbf{I}_{\,0\alpha\beta} - p/e_{\alpha\beta}^2\,(\{\ e_0 ,\ e_\alpha ,\ e_\beta\ \} - \{\ e_0 ,\ 0\ ,\ 0\ \})^T$

$\mathbf{I}_{cabc} = A^{-1}\,\mathbf{I}_{c0\alpha\beta}$ from Eq 3.6

$e_{\alpha\beta}^2 = e_\alpha^2 + e_\beta^2 = e_\alpha^2 + e_\beta^2 + e_0^2 - e_0^2 = \|\mathbf{v}\|^2 - e_0^2$

Where $\|\mathbf{v}\|^2 = e_a^2 + e_b^2 + e_c^2$ but $e_0^2 = e_z^2/3$

Hence $e_{\alpha\beta}^2 = \|\mathbf{v}\|^2 - e_z^2/3 = \|\mathbf{v}_{03}\|^2$

where $\mathbf{v}_{03} = \mathbf{v} - \{\ e_z/3 , e_z/3 , e_z/3\ \}$

and $e_{z3} = \{\ e_1 + e_2 + e_3\ \}$

$\mathbf{i}_{cabc} = \mathbf{i} - p\,(\,\mathbf{v} - \{\ e_0/\sqrt{3} , e_0/\sqrt{3} , e_0/\sqrt{3}\ \}/\ \|\mathbf{v}_{03}\|^2$

but $e_0 = (e_a + e_b + e_c)/\sqrt{3} = e_z/\sqrt{3}$ hence:

$\mathbf{i}_{cabc} = \mathbf{i} - (p/\|\mathbf{i}_{a3}\|^2)\,\mathbf{i}_{a3} = \mathbf{i} - k_3\,\mathbf{i}_{a3}$

therefore from Eq 4.6

$\mathbf{i}_{cabc} = \mathbf{i}_{c3}$

Therefore the active instantaneous current as defined by Akagi is equivalent to:

$$\mathbf{i}_{a3} = k_3\,\mathbf{v}_{03} \qquad \text{Eq. 4.7}$$

where $k_3 = p/\|\mathbf{v}_{03}\|^2$

$\mathbf{v}_{03} = \{0, e_a - e_z/3, e_b - e_z/3, e_c - e_z/3\}$

$e_z = e_a + e_b + e_c$ and $\|\mathbf{v}_{03}\|^2 = \mathbf{v}_{03} \bullet \mathbf{v}_{03}$

Eq 4.7 is illustrated in Fig 4.2, It can be stated simply as the reassignment of the supply line currents to be proportional to the line voltages measured from a “ null ” reference where the sum of the three line voltages are zero. However the neutral current is zero.

The null point voltage can be obtained simply by subtracting $e_z/3$ at every instant from the three line voltages measured from any reference (including the neutral wire if it exists).

$\mathbf{v}_{03} = \mathbf{v} - \{ 0, e_z/3, e_z/3, e_z/3\}$, where $e_z = e_a + e_b + e_b$

4.2.3 Rosseti & Tenti reformulated

The formulae given in Eq 3.38 is already identical to that Eq 4.4 (Buchholtz (m=3)) and Eq 4.6 (Akagi reformulated).

4.3 Group 3 reformulated

In group 3, the neutral resistance in zero and the neutral current is not constrained.

4.3.1 Willems

Willems' equation was given in the previous chapter as:

$\mathbf{i}_a(t) = k(t)\, \mathbf{v}(t)$ Eq. 4.8

where $k(t) = p(t)/\|\mathbf{v}(t)\|^2$

4.3.2 Peng

Peng's active current is already in the same format as the equation Eq 4.7 provided that the neutral wire is taken as the reference and is therefore equivalent to Willems using the neutral wire as a voltage reference.

4.3.3 Nabae reformulated

The active current as seen in Eq 3.27 was given by Nabae as:

$\mathbf{I}_{0\alpha\beta} = \{ (p/e^2_{0\alpha\beta})\, e_0 ,\ (p/e^2_{\alpha\beta})\, e_\alpha ,\ (p/e^2_{\alpha\beta})\, e_\beta\}^T$

Where $e^2_{0\alpha\beta} = e^2_0 + e_\alpha{}^2 + e_\beta{}^2$

$\mathbf{I}_{0\alpha\beta} = (p/e^2_{0\alpha\beta})\, \{ e_0 ,\ e_\alpha , e_\beta\}^T$

The active current in the abc coordinate is:

$\mathbf{I}_{abc} = A^{-1}\, \mathbf{I}_{0\alpha\beta} = (p/e^2_{\alpha\beta})\, A^{-1}\, \{ e_0 ,\ e_\alpha , e_\beta\}^T$

$\mathbf{I}_{abc} = (p/e^2_{0\alpha\beta})\{ e_a ,\ e_b , e_c\}^T$

This the same as

$\mathbf{i_a} = k\,\mathbf{v}$ where $k = p/\|\mathbf{v}\|^2$ Eq. 4.9

where $k = p/\|\mathbf{v}\|^2$ and $\mathbf{v} = \{e_a, e_b, e_c\}$ is a 3 dimension voltage vector measured from the neutral wire.

4.4 Mathematical Comparison of the three groups.

4.4.1 All wires have equal resistance (Group 1 gives the best results).

Let $\|\mathbf{i}_a\|^2$ and $\|\mathbf{i}_{a3}\|^2$ be the sum of the squares of the 4 line currents obtained by group 3 (Eq 4.7) and group 2 (Eq 4.2) respectively. Then:

$\|\mathbf{i}_a\|^2 > \|\mathbf{i}_{a3}\|^2$ if $e_z \neq o$

$\|\mathbf{i}_a\|^2 = \|\mathbf{i}_{a3}\|^2$ if $e_z = o$

Proof:

$\|\mathbf{i}_a\|^2 = i_{sa}^2 + i_{sb}^2 + i_{sc}^2 + i_{s0}^2$ (including neutral losses)

From Eq 4.7

$\|\mathbf{i}_a\|^2 = p^2\, \mathbf{v} \bullet \mathbf{v} / (\|\mathbf{v}\|^2 \|\mathbf{v}\|^2) + i_{s0}^2 = p^2 / \|\mathbf{v}\|^2 + i_{s0}^2$

Now by Kirchhoff $i_{s0} = -(i_{sa} + i_{sb} + i_{sc})$ hence:

$i_{s0}^2 = p^2 e_z^2 / \|\mathbf{v}\|^4$

$\|\mathbf{i}_a\|^2 = p^2 (\|\mathbf{v}\|^2 + e_z^2) / \|\mathbf{v}\|^4$ This has to be compared with :

$\|\mathbf{i}_{a3}\|^2 = p^2 / \|\mathbf{v}_{a3}\|^2$

$\|\mathbf{v}_{a3}\|^2 = \|\mathbf{v}\|^2 - e_z^2/3$

Let us compare $\|\mathbf{i}_a\|^2$ with $\|\mathbf{i}_{a3}\|^2$

In order to have a common denominator, we rather compare their inverse, hence if we subtract the one from the other; then:

$1/\|\mathbf{i}_a\|^2 - 1/\|\mathbf{i}_{a3}\|^2$

$= (\|\mathbf{v}\|^4 - (\|\mathbf{v}\|^2 - e_z^2/3)(\|\mathbf{v}\|^2 + e_z^2) / (\|\mathbf{v}\|^2 + e_z^2)$

$= -1/3\, e_z^2 (2\|\mathbf{v}\|^2 - 1/3\, e_z^2) / (\|\mathbf{v}\|^2 + e_z^2)$

since the square of a sum is less or equal the sum of the square (Cauchy-Schwarz inequality theorem).

But: $e_z^2 \leq e_a^2 + e_b^2 + e_c^2 = \|\mathbf{v}\|^2$

Hence:

$1/\|\mathbf{i}_a\|^2 - 1/\|\mathbf{i}_{a3}\|^2 \leq -(5/9)\, e_z^4 / (\|\mathbf{v}\|^2 + e_z^2)$

The right hand side of the above is always negative or zero, hence:

$1/\|\mathbf{i}_a\|^2 \leq 1/\|\mathbf{i}_{a3}\|^2$ or $\|\mathbf{i}_a\|^2 \geq \|\mathbf{i}_{a3}\|^2$

$\|\mathbf{i}_a\|^2 > \|\mathbf{i}_{a3}\|^2$ if $e_z \neq o$

$\|\mathbf{i}_a\|^2 = \|\mathbf{i}_{a3}\|^2$ if $e_z = o$

Hence group 2 gives better results that group 3 when the all wires have equal resistance. Moreover group 2 neutral wire active current is zero. This is in agreement with the conclusion of Akagi et al [1999] and Depenbrock et al [2004].

A comparison of group1 and 2 is now made where all wires are considered to have equal resistances.

From Eq 4.1and 4.2

$\|\mathbf{i}_{a4}\|^2 = p^2 / \|\mathbf{v}_{a3}\|^2$ and $\|\mathbf{i}_{a3}\|^2 = p^2 / \|\mathbf{v}_{a3}\|^2$

$\|\mathbf{v}_{a4}\|^2 = \|\mathbf{v}\|^2 - e_z^2/4$ and $\|\mathbf{v}_{a3}\|^2 = \|\mathbf{v}\|^2 - e_z^2/3$

hence

$\|\mathbf{v}_{a4}\|^2 > \|\mathbf{v}_{a3}\|^2$

Therefore

$\|\mathbf{i}_{a4}\|^2 < \|\mathbf{i}_{a3}\|^2$

This proves that group 1 gives better results than group 2 and group 2 gives better results than group 3, when all the wires have equal resistances.

4.4.2 Neutral wire resistance is infinite (Group 2).

An infinite resistance implies that no neutral current can flow and therefore is equivalent to a 3 wire system. Therefore the only solution possible is group 2. Groups 1 and 3 would be invalid.

4.4.3 Neutral wire resistance is zero (Group 3)

$\|\mathbf{i}_a\|^2 = i_{sa}^2 + i_{sb}^2 + i_{sc}^2$ (no neutral wire losses)

$\|\mathbf{i}_a\|^2 = p^2\, \mathbf{v} \bullet \mathbf{v} / (\|\mathbf{v}\|^2 \|\mathbf{v}\|^2)$

$\|\mathbf{i}_a\|^2 = p^2 / \|\mathbf{v}\|^2$

$\|\mathbf{i}_{a4}\|^2 = p^2 / \|\mathbf{v}_{a4}\|^2$

$\| \mathbf{i}_{a3}\|^2 = p^2 / \|\mathbf{v}_{a3}\|^2$

$\|\mathbf{v}_{a4}\|^2 = \|\mathbf{v}\|^2 - e_z^2 /4$

$\|\mathbf{v}_{a3}\|^2 = \|\mathbf{v}\|^2 - e_z^2 /3$

Therefore $\|\mathbf{v}\|^2 > \|\mathbf{v}_{a4}\|^2 > \|\mathbf{v}_{a3}\|^2$

Hence:

$\|\mathbf{i}_a\|^2 < \| \mathbf{i}_{a4}\|^2 < \| \mathbf{i}_{a3}\|^2$

Therefore group 3 gives better results than group 1 and group 2 if the neutral wire resistance is zero (i.e neutral wire losses neglected).

4.4 Conclusion

When defining active instantaneous currents in a three-phase 4 wire system the three groups of instantaneous active currents can be calculated with one general formulae Eq 4.1, with the appropriate voltage reference used.

Group 1 (Buchholtz with M = 4)

Can be expressed as Eq 4.2 with M = 4 and the voltage reference to be the virtual null point of all four wires.

Group 2 (Buchholtz, Akagi, Ferrero & Superti-Furga , Rossetto & Tinto)

Can be expressed as Eq 4.6 with M = 3 and the voltage reference chosen to be the virtual point of three wires only (excluding the neutral even if it exists).

Group 3 (Willems original method, Peng, Nabae)

Can also be expressed by Eq 4.7 where the voltage reference chosen is the neutral wire.

All three groups gave identical results if the sum of the voltages measured from the neutral is zero.

In the case where the sum of the voltages measured from the neutral wire is not zero then the following was proved mathematically:

1) Group 1 gave the least losses of the 3 groups if the neutral wire was not constrained to zero and the neutral wire resistance was considered to be equal to the other three wires.
2) Group 2 gave the least losses of the 3 groups if the neutral wire current was constrained to zero (i.e. the neutral wire resistance is infinite).
3) Group 3 gave the least losses of the 3 groups if the neutral wire current was not constrained to zero and its resistance considered to be zero.

Thus the grouping of the methods on the bases of the examples developed in chapter 3 is supported by generalised mathematical approach.

However the above proofs do not necessarily ascertain that no better solution exists for the three cases considered (i.e neutral wire resistance is equal the other wires, infinite, or zero.)

This is the subject of the next chapter.

Chapter 5

Optimal instantaneous power solution

It was seen in the previous chapter that the reviewed instantaneous power theories can be reformulated as one general formula with the appropriate voltage reference, corresponding to a particular resistance value attributed to the neutral wire. Mathematical proofs were given to show under what criteria each of the groups would give the best result. However it was not yet proven that these were optimal.

In this chapter a mathematical proof based on Lagrange multipliers is given that applies to any number of wires with any resistance for each of the individual wires. In the particular case of a 3-phase 4-wire system it is shown that the optimal solution depends on how the neutral wire resistance is accounted. The appropriate voltage reference required then becomes a necessary condition.

5.1 Obtaining a general optimal solution for M-wire system

In order to find a general solution that will give minimal line losses, it is assumed that there are M-wires and that they have the resistances $r_1, r_2, r_3, \dots, r_M$

The condition for minimal line losses has to be found under the following systems constraints:

1) $p = \sum_{n=1}^{M} p_n$ for n = 1 to M the number of wires Eq. 5.1

2) $\sum_{n=1}^{M} i_n = 0$ Kirchhoffs current law Eq. 5.2

The minimum total line losses are found through the method of Lagrangian multipliers [Sokolnikoff, 1941; Smith, 1953]. The method calls for the introduction of a new expression, where the line currents can be different; under the constraints that power p delivered to the load remains constant and that the sum of all the wires currents is zero at all time t.

If $\mathbf{i}_n$ is the modified current vector, delivering the same power p, and the line loss in any individual line is referred to as p_{ln}, then the total line losses p_l can be expressed as:

$$p_l = \sum_{n=1}^{M} p_{ln} + \lambda_L \left(p - \sum_{u=1}^{M} p_n \right) \qquad \text{Eq. 5.3}$$

Note: the last term equal to zero from Eq. 5.1 r_n is the respective line resistance, p_{ln} are the individual line losses and λ_L a variable introduced by the Lagrangian method.

Minimum total line losses occur where λ_L is such that:

$\delta p_l / \delta p_n = 0$ for all n.

Minimum total power loss p_l for all the wires supplying a load p is obtained by partial differentiation of Eq. 5.3 with respect to p_n .

$\delta p_l / \delta p_n = \delta p_{ln} / \delta p_n - \lambda = 0$

$$\delta p_{ln} / \delta p_n = \lambda_L \qquad \text{Eq. 5. 4}$$

Since partial differentiation indicates that we are considering the effect of only changing p_n , it follows that a change in p_n can only affect p_{ln}. Therefore:

$\delta p_{ln} / \delta p_n = dp_{ln} / d p_n$

which is recognised as the incremental line loss, which is from Eq. 5.4 for λ_L:

$\lambda_L = dp_{ln} / dp_n$ or

$\lambda_L = d (i_n^2 r_n) / d (i_n v_n)$

since the line voltage v_n is considered constant.

$\lambda_L = 2 i_n r_n / v_n$ or

$$i_n = v_n \lambda_L / 2r_n \qquad \text{Eq. 5.5}$$

Now introducing the second constraint Eq 5.2 (Kirchhoff).

$$\sum_{n=1}^{M} i_n = 1/2 \lambda_L \sum_{n=1}^{M} (v_n / r_n) = 0$$

It can be concluded that minimal line losses will occur <u>if and only if</u> the voltage reference is such that

$$\sum_{n=1}^{M} (v_n / r_n) = 0 \qquad \text{Eq. 5.6}$$

Now since

$$p = \sum_{n=1}^{M} p_n = \sum_{n=1}^{M} v_n i_n$$

from Eq. 5.5

$$p = \lambda_L /2 \sum_{n=1}^{M} v_n^2 / r_n$$

$$\lambda_L = 2\,p \;/ \sum_{n=1}^{M} v_n^2 / r_n$$

therefore substituting λ_L in Eq. 5.5 the optimal line currents are:

$$\mathbf{i}_a = (p / \| \mathbf{v} \|^2)\, \mathbf{v}_n \qquad \text{Eq. 5.7}$$

where $\| \mathbf{v} \|^2 = \sum_{n=1}^{M} v_n^2$

Note: Eq. 5.7 is only valid if the voltage reference is in accordance with Eq. 5.6

5.2 Optimal solution for 3-phase 4-wire systems

The following conclusion for a 4-wire system in the neutral wire resistance can assume one of three values:

1) If all four of the wires resistances are considered to be equal so that $r_1 = r_2 = r_3 = r_4$ then the voltage reference should be chosen so that the sum of four wires voltages adds to zero, which was the case with Buchholtz reformulated with M=4 (Group 1).
2) If the neutral wire resistance is considered to be such that $r_N = \infty$, then $v_N / r_N = v_N / \infty = 0$. Then according to Eq. 5.6, $v_1 + v_2 + v_3$ must be zero. This was the case with Group 2 (Akagi reformulated, etc).
3) In the case of a three-phase system with the neutral wire resistance considered to be zero, this could be viewed as three line wires plus a very large number M_∞ of wires in parallel forming the neutral wire, all having the same resistance r.
   Then from Eq 5.6 it can be said:
   $v_1 + v_2 + v_3 + M_\infty\, v_N = 0$

This would only be true if the neutral wire is used as a reference. v_N would tend to zero even if the sum of the line voltages is not zero (since M_∞ tends to infinity).
This explains why Willems, Peng and Nabae group gave optimal results if the neutral wire losses are considered as negligible.

5.3 Conclusion

The necessary and sufficient condition that a 4-wire system with four non-equal resistances delivers power with minimal wire losses is that the currents in the wires after compensation be calculated as per Eq. 5.7 and that the instantaneous voltage reference used be such that Eq. 5.6 is true.

In particular, this implies that:

1) In the case of a 4-wire system with all wires considered to have equal resistances then the voltage reference must be chosen so that the sum of the instantaneous voltages be zero at all times. This corresponds to Group 1 as identified in the previous chapter.

2) In the case of a 4-wire system with three equal resistances and the neutral wire current being restricted to zero then this is equivalent to considering the neutral wire to have an infinite resistance or to having three wires only. In such cases the instantaneous voltage reference must be such that the sum of the voltages of the three wires is zero at all time. This corresponds to Group 2.

3) In the case of a 4-wire system with three equal resistances and a neutral with zero resistance (or where the neutral wires losses are ignored) then the neutral wire must be chosen to be the reference. This corresponds to Group 3.

This approach clarifies instantaneous power theory in 4-wire systems, removes all the confusion and ambiguities that permeate many of the publications on instantaneous power theories and is extendable to any m-wire system.

The next chapter offers a different approach that is restricted to the case where all wires have equal resistance but also offers a means of comparing compensating currents. It introduces a link for the instantaneous power theory to be extended to the rms domain.

Chapter 6

Space vector approach to instantaneous power in 3- and 4-wire systems

In this chapter a new and different approach is presented. It is based on the properties of space vectors where the load currents are separated into mutually orthogonal components with the property that only one contributes to the transfer of real power. The process relies on the Grahm-Schmidt process of deriving a set of appropriate orthogonal vectors representing the constraints of the system (Kirchhoff's current law and the conservation of energy). These can be represented in the form of a vector space (see classical linear algebra vector space [Anton 2000]). Following this and defining the inner product ("dot" in this case) of two vectors, a set of orthogonal current components are obtained by projecting the load current onto the orthogonal vectors. This approach has the advantage of being able to compare the active and non-active components. The comparison of non-active current components allows the comparison of previously described methods such as, for example, Group 1 and Group 2.

6.1 Vector Space approach for 3- phase 4-wire systems

One of the basic problems as seen in the previous chapter was to find an optimum vector representing the optimum line current after compensation that will deliver the same power p under a set of conditions. The voltages in the case of 3-phase could be any set of three arbitrarily chosen numbers representing the voltage measured from the neutral wire. The three line to neutral voltages could be any real positive or negative numbers or zero.

For any 4-wire system, three arbitrarily chosen voltage measured from the neutral e_a, e_b, e_c and three load arbitrarily chosen current values i_a, i_b , i_c are taken (See Fig. 3.1) This set of values can be taken at any instant in time and can be treated as any arbitrary real numbers.

Let us consider the four dimensional current vector **i** and voltage vector **v** in R^4

i = $\{ i_z , i_a , i_b , i_c \}$ where $i_z = -(i_a + i_b + i_c)$ and Eq. 6.1

v = $\{0 , e_a , e_b , e_c\}$ where $e_z = e_a + e_b + e_c$ Eq. 6.2

According to the laws of linear algebra, a four dimensional vector can be decomposed into four orthogonal vectors. There are an infinite number of such orthogonal sets. However the electrical system must conform to Kirchhoff's current law and the law of conservation of energy. These are expressed as two independent equations.

$i_z + i_a + i_b + i_c = 0$

$e_a i_a + e_b i_b + e_c i_c = p$

p is the instantaneous power delivered to the load.

The equation can be expressed in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & e_a & e_b & e_c \end{bmatrix} \begin{bmatrix} i_z \\ i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0 \\ p \end{bmatrix}$$

There is still an infinite number of solutions because there are four unknowns and only two equations. If we include the special case where the neutral current has to be zero, then a third equation is obtained:

$i_a + i_b + i_c = 0$,

Three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are obtained by considering the left side of the equations (homogeneous part), where:

$\mathbf{u}_1 = \{1 , 1 , 1 , 1 \}$ (Kirchhoff current law)

$\mathbf{u}_2 = \{0 , e_a , e_b , e_c \}$ (Energy conservation)

$\mathbf{u}_3 = \{0 , 1 , 1 , 1 \}$ (Zero neutral current case, Group 2).

The above three equations of vectors can be said to span a three dimensional subspace S^3 of R^4 [Anton 2000]. If we include **i** then it can be said that S^3 is a subspace of S^4 in R^4 determined by the 4 vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and **i** ,

where **i** = $\{ i_z + i_a + i_b + i_c \}$

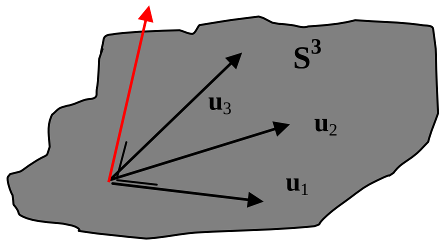

Fig.6.1. Projection of I in R^4 spanned by $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{U}_3$

6.2 Deriving the appropriate orthogonal basis

The three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are linearly independent but not orthogonal to each other. According to linear algebra [Anton 2000] they can define a unique vector space S^3 which is in R^3 . If the vector **i** is also considered then a vector space S^4 which is in R^4 can be defined. S^3 is a subspace of a 4 dimension space in R^4. The same vector space S^3 can also be defined by three orthogonal vectors. By Grahm-Schmidt [Anton 2000] we can change the basis to an orthogonal basis representing the same vector space S^3. Every vector space in R^3 has at least one orthogonal set of vectors (provided they are not in the same direction i.e. multiples). The above non orthogonal vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ can be transformed to an appropriate basis of orthogonal vectors: $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

6.2.1 Grahm-Schmidt method

The Grahm-Schmidt method [Anton 2000] is used to derive three appropriate orthogonal vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ that define the same space vector space S^3.

Step 1:

Let $\mathbf{v}_1 = \mathbf{u}_1 = \{1, 1 , 1 , 1\}$

Step 2:

The Grahm-Schmidt method requires to subtract from $\mathbf{v}_1$ the projection of $\mathbf{u}_2$ onto $\mathbf{v}_1$.

$\mathbf{v}_2 = \mathbf{u}_2$ - Proj $\mathbf{u}_2$ on $\mathbf{v}_1 = \mathbf{u}_2 - (\mathbf{u}_2 \bullet \mathbf{v}_1)\mathbf{v}_1/\|\mathbf{v}_1\|^2$

$(\mathbf{u}_2 \bullet \mathbf{v}_1) = (0 + e_a + e_b + e_c) = e_{z4}$ (see Buchholtz reformulated Eq. 4.2 with M=4)

$\|\mathbf{v}_1\|^2 = 4$

hence

$\mathbf{v}_2$ = {0 , e_a, e_b, e_c} - {$-e_{z4}/4$, $-e_{z4}/4$, $-e_{z4}/4$, $-e_{z4}/4$}

$\mathbf{v}_2 = \mathbf{u}_2 - \mathbf{e}_{z4}$ where $\mathbf{e}_{z4}$ - {$-e_{z4}/4$, $-e_{z4}/4$, $-e_{z4}/4$, $-e_{z4}/4$}

$\mathbf{v}_2$ = {$-e_z/4$, $e_a - e_z/4$, $e_b - e_z/4$, $e_c - e_z/4$}

Note: this is the same as $\mathbf{v}_{0m}$ in chapter 4 (Eq. 4.2 with M=4)

Step 3:

Similarly

$\mathbf{v}_3' = \mathbf{u}_2$ - Proj $\mathbf{u}_2$ on $\mathbf{u}_3$

$\mathbf{v}_3'$ ={0 , $e_a - e_z/3$, $e_b - e_z/3$, $e_c - e_z/3$ } (Same as Akagi reformulated as in Eq. 4.5, $\mathbf{v}_3' = \mathbf{v}_{03}$)

continuing with the Grahm-Schmidt method and substituting $\mathbf{v}_3'$ for $\mathbf{v}_{03}$ a third orthogonal vector $\mathbf{v}_3$ is obtained.

$\mathbf{v}_3 = \mathbf{v}_{03}$ - Proj $\mathbf{v}_{03}$ on $\mathbf{v}_2$

then: $\mathbf{v}_3 = \mathbf{v}_{03} - k_3\mathbf{v}_2$

where k_3 is a scalar $= (\mathbf{v}_{03} \bullet \mathbf{v}_2) / \|\mathbf{v}_2\|^2$

but $(\mathbf{v}_{03} \bullet \mathbf{v}_2) = \|\mathbf{v}_{03}\|^2$

hence $k_3 = \|\mathbf{v}_{03}\|^2 / \|\mathbf{v}_2\|^2$

Therefore the vector space S^3 can be defined by the following three orthogonal vector bases:

$$\mathbf{v}_1 = \{ 1 , 1 , 1 , 1 \} \qquad \text{Eq. 6.3}$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \mathbf{e}_{z4} \qquad \text{Eq. 6.4}$$

$$\mathbf{v}_3 = \mathbf{v}_{03} - k_3 \mathbf{v}_2 \qquad \text{Eq. 6.5}$$

where $k_3 = \|\mathbf{v}_{03}\|^2 / \|\mathbf{v}_2\|^2$.

These vectors form an orthogonal basis for S^3, hence:

$\mathbf{v}_1 \bullet \mathbf{v}_2 = \mathbf{v}_1 \bullet \mathbf{v}_3 = \mathbf{v}_2 \bullet \mathbf{v}_3 = 0$

Note that $\mathbf{v}_1$ is a result of Kirchhoff's current law, $\mathbf{v}_2$ is a 4 dimensional voltage vector where the sum of the voltages (including the neutral wire) is always zero (see Fig. 6.4). $\mathbf{v}_3$ is then the third vector that results from the Grahm-Schmidt process. Vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ span exactly the same vector space S^3 in R^3 as the previous non-orthogonal vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$. The current vector $\mathbf{i}$ which is in R^4 can then be projected onto the subspace S^3 which is in R^3 and R^4. This is illustrated in Fig. 6.2.

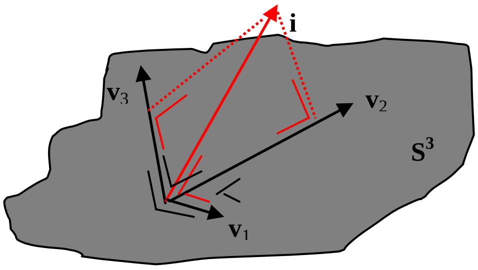

Fig. 6.2 Projection of I which is in R^4 onto an orthogonal set of vectors coordinates representing subspace S^3.

Example 6.1: $\mathbf{v} = \{0, -2, 9, -3\}$ $\quad \mathbf{e_z = e_a + e_b + e_c = 4}$

$\mathbf{v}_{03} = \{0, -3.33, 7.67, -4.33\}$ $\quad \|\mathbf{v}_{03}\|^2 = 88.67$

$k_3 = \|\mathbf{v}_{03}\|^2 / \|\mathbf{v}_2\|^2 = 0.985$ then:

$\mathbf{v}_1 = \{1, 1, 1, 1\}$ $\quad \|\mathbf{v}_1\|^2 = 4$

$\mathbf{v}_2 = \{-1, -3, 8, -4\}$ $\quad \|\mathbf{v}_2\|^2 = 90$

$\mathbf{v}_3 = \{0.99, -0.38, -0.21, -0.39\}$ $\quad \|\mathbf{v}_3\|^2 = 1.31$

6.2.2 Decomposing the current vector into four orthogonal vectors

Having obtained an appropriate orthogonal voltage basis one is now able to split the load current vector **i** into four mutually orthogonal vector components. Three of these current vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are in R^3 and are obtained by projecting **i** (which is in R^4) on $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ respectively (see Fig. 6.2).

The fourth orthogonal vector $\mathbf{i}_4$ in R^4 but not in S^3 is obtained by subtracting the projection of **i** onto S^3 from **i**.

The following current four current components in R^4 are obtained:

$$\mathbf{i}_1 = (\mathbf{i} \bullet \mathbf{v}_1)\, \mathbf{v}_1 / \|\mathbf{v}_1\|^2 = 0 \quad (\text{ since } \mathbf{i} \text{ and } \mathbf{v}_1 \text{ are orthogonal }) \qquad \text{Eq. 6.6}$$

$$\mathbf{i}_2 = (\mathbf{i} \bullet \mathbf{v}_2)\, \mathbf{v}_2 / \|\mathbf{v}_2\|^2 = (p/\|\mathbf{v}_2\|^2)\, \mathbf{v}_2 = k_2\, \mathbf{v}_2 \qquad \text{Eq. 6.7}$$

$$\mathbf{i}_3 = (\mathbf{i} \bullet \mathbf{v}_3)\, \mathbf{v}_3 / \|\mathbf{v}_3\|^2 \qquad \text{Eq. 6.8}$$

$$\mathbf{i}_4 = \mathbf{i} - \mathbf{i}_1 - \mathbf{i}_2 - \mathbf{i}_3 \qquad \text{Eq. 6.9}$$

and since these current vectors are projections onto orthogonal voltage vector it can therefore be concluded that:

$$\| \mathbf{i} \|^2 = \| \mathbf{i}_2 \|^2 + \| \mathbf{i}_3 \|^2 + \| \mathbf{i}_4 \|^2 \qquad \text{Eq. 6.10}$$

i can be decomposed into three orthogonal vectors in R^4 (note $i_1 = 0$, i_2 and i_3 in S^3, i_4 is in R^4 but not in S^3); see Fig. 6.3.

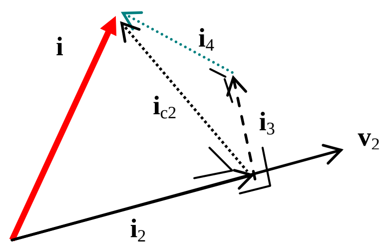

Fig.6.3. Orthogonality of current components

The instantaneous power transmitted by each of these current vector is p_2, p_3 and p_4 and they are calculated using voltage reference represented by $\mathbf{v}_2$ (power is invariant from the reference chosen). Then:

$p_2 = p$ $\quad$ ($p_2 = \mathbf{v}_2 \bullet \mathbf{i}_2 = (\mathbf{v}_2 \bullet \mathbf{v}_2)\, p / \|\mathbf{v}_2\|^2 = p$)

$p_3 = 0$ $\quad$ ($p_3 = \mathbf{v}_2 \bullet \mathbf{i}_3$ where $\mathbf{i}_3$ is orthogonal to $\mathbf{v}_2$)

$p_4 = 0$ since $p_4 = \mathbf{v}_2 \bullet \mathbf{i}_4$ where $\mathbf{i}_4$ is orthogonal to $\mathbf{v}_2$

$\mathbf{i}_2$ is in the direction $\mathbf{v}_2$ and since $\mathbf{i}_2$ is found to be the only component of **i** which delivers power p. Therefore $\mathbf{i}_2$ can be concluded to be the instantaneous active current.

Note that Eq. 6.7 is identical to that of Eq. 4.1 (Buchholtz and Depenbrock M=4 reformulated or Group 1) .

6.2.3 Compensating current required

In order to minimize instantaneous transmission losses using a compensator, it is necessary to calculate the required optimum compensating current vector $\mathbf{i}_{c4}$, that gives the minimum active current vector.

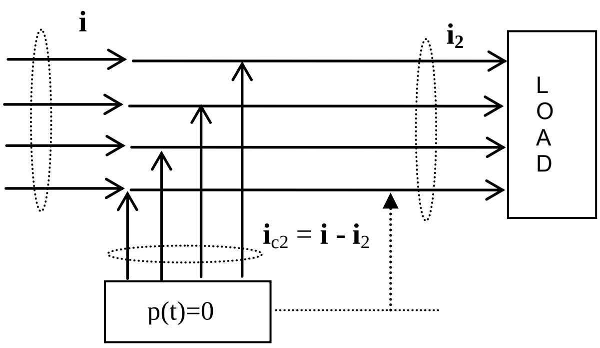

Fig. 6.4. Optimum load current compensator without energy storage

The instantaneous non-active power component is (see Fig 6.4) :

$$\mathbf{i}_{c2}(t) = \mathbf{i}(t) - \mathbf{i}_2(t) \qquad \text{Eq. 6.11}$$

The compensating current vector norm determines the size and the losses of the compensator. These in conjunction with the transmission losses avoided would help decide which method to use in an individual case.

6.3 Comparison of compensating current required by Akagi and Buchholtz/Depenbrock methods.

The compensating current for Akagi as calculated in Eq. 4.6 was found to be:

$\mathbf{i}_{c3} = \mathbf{i} - \mathbf{i}_{a3}$ where $\mathbf{i}_{a3} = k_3\,\mathbf{v}_{03}$ (note $\mathbf{v}_{03} = \mathbf{v}_3 + k_3\,\mathbf{v}_2$ see Eq. 6.5)

and for Buchholtz in Eq. 4.4

$\mathbf{i}_{c2} = \mathbf{i}_{c4} = \mathbf{i} - \mathbf{i}_4$ where $\mathbf{i}_{a4} = k_4\,\mathbf{v}_{04}$ (note $\mathbf{v}_{04}$ is the same as v_2)

The above equations are now mapped in $S^{3.}$ This is illustrated in Fig. 6.5

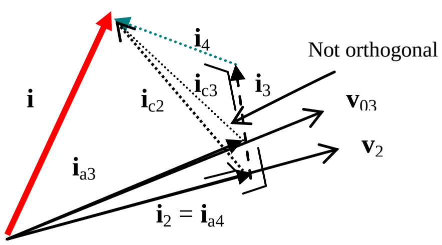

Fig.6.5 Comparison of compensating currents for group 2 (Akagi) and group 1 (Depenbrock), where group 2 $\mathbf{i}_{c3}$ has a component in the direction of $\mathbf{i}_3$

It can be seen that $\mathbf{i}_{a3}$ has a current component in the direction of $\mathbf{i}_2$ and $\mathbf{i}_3$ and therefore is larger than $\mathbf{i}_{a4}$, which has only $\mathbf{i}_2$ as a component.

It is important to note that $\mathbf{i}_{a3}$ has a component in the direction of $\mathbf{i}_3$ that can be in the positive or negative direction (See Fig. 6.6). This explains what was observed when comparing the compensating currents in the results of Example 3.3, that where $\| \mathbf{i}_{c3}\| < \| \mathbf{i}_{c4} \|$ was not necessarily true for all arbitrarily chosen voltage values. For example if v_b was -9V instead of 9V,then it is found that $\| \mathbf{i}_{c3}\| > \| \mathbf{i}_{c2} \|$.

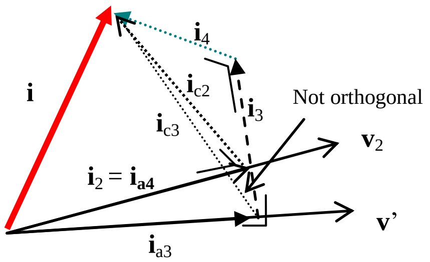

Fig. 6.6. Comparison of compensating currents for Akagi and Depenbrock where Akagi i` has a negative component in the direction of $\mathbf{i}_3$

Therefore the compensating currents required in the case of Group 2 (Akagi) are not necessarily less than Group 1 (Depenbrock), and depend on the individual set of voltages chosen.

Note that $\mathbf{i}_{c3}$ is not orthogonal to $\mathbf{i}_3$; $\mathbf{i}_{c3}$ contains the same non-active vector $\mathbf{i}_4$ as $\mathbf{i}_{c2}$ but contains a non active component in the direction of $\mathbf{i}_3$ of different length from that of $\mathbf{i}_{c4}$.

6.4 Conclusion

It has been shown through linear algebra and in particular the properties of space vectors that:

- From the property of vector space it can be concluded that Eq. 6.5 is indeed optimum as far as minimizing line losses and there exists no better solution that will minimize line losses when all four wires have equal resistances and the neutral wire current after compensation is not constrained
- However the norm of the compensating current required for four equal wires is not necessarily less than for a three wire system. This depends on the individual case of load current. Therefore the cost and losses incurred in the compensator need to be also assessed before drawing any conclusion as to which method to use.

- Such comparisons of compensating current have not been found in the literature reviewed.

The approach of this chapter will be extended to the rms domain in Chapter 8, after first reviewing the existing rms theories in Chapter 7.

Chapter 7

Review of average power theory for 3-phase systems

This chapter reviews various rms or average power domain theories. The approach developed in Chapter 6 will be continued in Chapter 8.

7.1 Depenbrock

Following on Buchholtz theory, Depenbrock extended the theory of Bucholtz to the rms domain and called it the FBD (i.e. Fryze, Buchholtz, and Depenbrock) method [Depenbrock 1962] which is as follows.

The collective admittance G is first defined as:

$$G = P_{\Sigma} / U_{\Sigma}^{2} \qquad \text{Eq. 7.1}$$

P_{Σ} is the average of $p_{\Sigma}(t)$ and U_{Σ}^{2} is the average of $u_{\Sigma}(t)^2$ over time interval T.

Then Depenbrock defines the active currents of the v^{th} line as i_{va} in the **rms domain** as:

$$i_{va}(t) = G\, u_v(t) \qquad \text{Eq. 7.2}$$

He then subtracts the above active currents from the original line currents $i_v(t)$ and obtains a non-active component in the rms domain called $i_{vn}(t)$. Hence:

$$i_v(t) = i_{va}(t) + i_{vn}(t) \qquad \text{Eq. 7.3}$$

The original current is made of two components. One active and the other non-active.

He then splits the non active current $i_{vn}(t)$ into two components $i_{vz}(t)$ and $i_{vv}(t)$.

These are obtained as follows:

$$i_{vv}(t) = i_{va}(t) - i_{vp}(t) \quad \text{where} \qquad \text{Eq. 7. 4}$$

$i_{vp}(t)$ is the instantaneous active current as defined by Buchholtz (see Eq. 3.1),

$i_{vp}(t) = g_p(t)\, u_v(t)$

and the remaining non active power component.

$$i_{vz}(t) = i_{vn}(t) - i_{vv}(t) \qquad \text{Eq. 7.5}$$

Hence the original line current $i_v(t)$ can be separated into three components $i_{va}(t)$ which is active and $i_{vz}(t)$ and $i_{vv}(t)$ which are non active. Such that:

$$i_v(t) = i_{va}(t) + i_{vz}(t) + i_{vv}(t) \qquad \text{Eq. 7.6}$$

The first is active in the rms domain whilst the other two are non active.

7.1.1 Relevant points to the present hypothesis

- Depenbrock's current $i_{vn}(t)$ is the total current that ***collectively*** does not contribute to any **average** power.
- Depenbrock's zero current $i_{vz}(t)$ is a current that ***collectively*** does not contribute to any instantaneous power, as originally formulated by Buchholtz.
- Depenbrock's current $i_{vv}(t)$ is the deviation between the two above currents; $i_{vn}(t)$ and i_{vz}.
- The concept of orthogonal vectors under any form of inner product is not used by Depenbrock.

7.2 Czarnecki

Czarnecki [1987b, 1988] extended his single-phase frequency domain decomposition method as seen in Chapter 2, to 3-phase, 3-wire systems with symmetrical voltages. He decomposed the currents into five orthogonal 3-dimension current vector components to which he attributes physical meanings. The first four are based on his single-phase theory with only a change in their mathematical characterization. An additional orthogonal component appears on the source current, when the load is not symmetrical.

In all his publications on non-sinusoidal three-phase systems Czarnecki [1989, 1991, 1992, 1993b, 1995] considers voltage supplies restricted to symmetrical three-phase voltages only, and with a zero instantaneous mean of the currents and voltages. The theory is therefore restricted to a supply that contains only positive sequence fundamental components voltages and voltage harmonics that are either individually of positive or negative sequence characterisation. The calculations of the current components are all based on 3-dimensional vector, and in particular the inner product of two such vectors $\mathbf{X}(t)$, $\mathbf{Y}(t)$ over a period T , defined as:

$$< \mathbf{X}(t) , \mathbf{Y}(t) > = 1/T \int_T \mathbf{X}(t) \cdot \mathbf{Y}(t) dt$$

Czarnecki refers to this expression as a scalar product but it is referred to here, for the sake of mathematical consistency, as a specific inner product definition in conformity with the rules that apply to the generalised inner product. It must be noted that the inner product can be defined in many ways [Anton, 2000]. This definition of the inner product is more specifically appropriate to the situation where all the wires have equal resistances and when the losses are considered over a complete cycle.

Czarnecki first defines the following 3 dimensional vector:

$\mathbf{U}(t) = \{u_R(t), u_S(t), u_T(t)\}$

where $u_S(t) = u_R(t - T/3)$ and $u_T(t) = u_R(t + T/3)$

Note the above implies symmetrical voltages only, hence not unbalanced voltages.

Expressing it in a complex Fourier series, with $n \neq 3k$ k= 1, 2, 3… (i.e. also no zero sequence voltage components)

$u_X(t) = \sqrt{2}\,\mathrm{Re}\sum_n U_{Xn}\, e^{jn\omega_1 t}$ x denotes one of the phases R, S, T

U_{Xn} is a complex number (not a vector) .

If the load is symmetrical, each current is expressed as a series of Fourier sinusoidal components and expressed a current vector $\mathbf{i}(t) = \{i_R(t), i_S(t), i_T(t)\}$ where:

$$\mathbf{i}(t) = \{\sqrt{2}\,\mathrm{Re}\sum I_{Rn}\, e^{jn\omega_1 t}, \sqrt{2}\,\mathrm{Re}\sum_n I_{Sn}\, e^{jn\omega_1 t}, \sqrt{2}\,\mathrm{Re}\sum_n I_{Tn}\, e^{jn\omega_1 t}\} \quad \text{Eq. 7.7}$$

The norm of a vector $\mathbf{i}(t)$ is depicted as $\| \mathbf{I} \|^2 = < \mathbf{i}(t), \mathbf{i}(t) >$ which is obtained by taking its inner product with itself. The inner product (in this case) is the integral of the dot product divided by T (T is the period of the periodic waveform), namely:

$$< \mathbf{i}(t), \mathbf{i}(t) > = 1/T \int_T \mathbf{i}(t) \cdot \mathbf{i}(t)\, dt$$

It follows that since the Fourier components are orthogonal to each other:

$$\| \mathbf{I}_n \|^2 = I_{Rn}^2 + I_{Sn}^2 + I_{Tn}^2 \quad \text{Eq. 7.8}$$

Which, by Fourier, implies that:

$$\| \mathbf{I} \|^2 = I_R^2 + I_S^2 + I_T^2 \quad \text{Eq. 7.9}$$

Also, if I_{Xn} and U_{Xn} is the complex harmonic (of order n) rms current and voltage,

$$I_{Xn} = I_{Xn}\, e^{j\alpha} = \sqrt{2}/T \int_T i_x(t)\, e^{-jn\omega_1 t} \quad \text{Eq. 7.10}$$

$$U_{Xn} = U_{Xn}\, e^{j\alpha} = \sqrt{2}/T \int_T u_x(t)\, e^{-jn\omega_1 t} \quad \text{Eq. 7.11}$$

The complex rms values of the line harmonic voltage and current are arranged as vectors:

$\mathbf{U}_n = \{ U_{Rn}, U_{Sn}, U_{Tn} \}$ $\mathbf{I}_n = \{ I_{Rn}, I_{Sn}, I_{Tn} \}$

The average power $P = < \mathbf{u}, \mathbf{i} > = 1/T \int_T (u_R(t)\, i_R(t) + u_S(t)\, i_S(t) + u_T(t)\, i_T(t))\, dt$ Eq. 7.12

The complex apparent power of the n^{th} order harmonic is defined as:

$S_n = \mathbf{U}_n \cdot \mathbf{I}_n^* = \boldsymbol{U}_{Rn} \boldsymbol{I}_{Rn}^* + \boldsymbol{U}_{Sn} \boldsymbol{I}_{Sn}^* + \boldsymbol{U}_{Tn} \boldsymbol{I}_{Tn}^*$ Eq. 7.13

$S_n = P_n + j\, Q_n$ Eq. 7.14

From the conjugate apparent complex power S_n, the voltage and voltage harmonic rms values $\| \mathbf{U} \|$ and $\| \mathbf{U}_n \|$, the following parameters of the symmetrical load are found:

$G_e = P / \| \mathbf{U} \|^2$ Eq. 7.15

$Y_n = G_n + j\, B_n = S^*_n / \| \mathbf{U}_n \|^2$ Eq. 7.16

The conductance G_e stands for the phase conductance dissipating the total average power P with symmetrical voltage vector **u** applied to three star connected admittances of value G_e each . Y_n stands for the phase admittance with harmonic total apparent power S_n with symmetrical voltage vector $\mathbf{U}_n$ applied to three equal star connected admittances Y_n . Such a load would draw a symmetrical current:

$\mathbf{I}_n(t) = G_n \sqrt{2}\mathrm{Re}\,(\mathbf{U}_n\, e^{-jn\,\omega_1 t}) + B_n \sqrt{2}\mathrm{Re}\,(\,j\, \mathbf{U}_n\, e^{-jn\,\omega_1 t}\,)$ Eq. 7.17

Czarnecki proceeds by defining the active current component vector as did Depenbrock in Eq. 7.7.

$\mathbf{I}_a(t) = G_e\, \mathbf{U}(t)$ Eq. 7.18

He then also proceeds by subtracting $\mathbf{i}_a(t)$ from $\mathbf{i}(t)$, and obtains:

$\mathbf{I}(t) - \mathbf{I}_a(t) = \sum_n (G_e - G_n)\, \sqrt{2}\, \mathrm{Re}\,(\mathbf{U}_n\, e^{-jn\,\omega_1 t}) + \sum_n \sqrt{2}\, B_n\, \mathrm{Re}\,(\,j\, \mathbf{U}_n\, e^{-jn\,\omega_1 t}\,)$ Eq. 7.19

and decomposes this current into two more components:

$\mathbf{I}_s(t) = \sum_n \; j\, \sqrt{2}\, \mathrm{Re}\,(G_e - G_n)\mathbf{U}_n e^{-jn\,\omega_1 t}$ (which he calls the scattered current) Eq. 7.20

$\mathbf{I}_r(t) = \sum_n \sqrt{2}\, \mathrm{Re}\; j\, B_n \mathbf{U}_n\, e^{-jn\,\omega_1 t}$ (the reactive current vector) Eq. 7.21

$\mathbf{I}(t) = \mathbf{I}_a(t) + \mathbf{I}_s(t) + \mathbf{I}_r(t)$ Eq. 7.22

The three current vector components are shown to be orthogonal under the inner product as defined above, hence:

$|| \mathbf{I} ||^2 = || \mathbf{I}_a ||^2 + || \mathbf{I}_s ||^2 + || \mathbf{I}_r ||^2$ Eq. 7.23

multiplying by $|| \mathbf{U} ||^2$ results the power component equation :

$S^2 = P^2 + D_s^2 + Q_s^2$ Eq. 7.24

Where $D_s^2 = || \mathbf{I}_s ||^2 \, || \mathbf{U} ||^2$

7.2.1 Czarnecki definition for three-phase with asymmetrical load

The above theory assumed both the supply voltage and the load to be symmetrical. In the case of asymmetrical load (but with symmetrical voltage), Czarnecki [1991] develops his theory further as follows.

An equivalent admittance G_e is defined as above but stands for the phase conductance of a resistive symmetrical load which is equivalent to the original load with the active power P and the voltage **U**. Similarly the admittance Y_{en} stands for the phase admittance of a balanced load which is equivalent to the original load with the complex apparent power S_n at voltage $\mathbf{U}_n$. The active, scattered, and reactive currents are defined as for the symmetrical load case. An additional current component vector results and is equal to:

$\mathbf{I}_{un}(t) = \mathbf{I}_n(t) - \mathbf{I}_{en}(t) = \sqrt{2}\, \mathrm{Re}\, [\, \mathbf{I}_n - Y_{en} \mathbf{U}_n \,]\, e^{-jn\omega_1 t} = \sqrt{2}\, \mathrm{Re}\, [\, \mathbf{A}_n \,]\, e^{-jn\omega_1 t}$ Eq. 7.25

$\mathbf{I}_{un}(t)$ is referred to as the unbalance current vector component.

Vector $\mathbf{I}_{un}(t)$ is shown to have an opposite sequence to that of $\mathbf{U}_n$

The supply is assumed to have either positive or negative sequence. $\mathbf{U}_n$ has a positive sequence component only when n = 3k + 1, and a negative sequence when n= 3k -1,

$\mathbf{U}_n$ does not exist when n = 3k (so as to have no zero sequence)

$I_n(t)$ for linear unbalanced load can therefore be decomposed into four components:

$\mathbf{I}_n(t) = \mathbf{I}_{en}(t) + \mathbf{I}_{un}(t) = \mathbf{I}_{an}(t) + \mathbf{I}_{sn}(t) + \mathbf{I}_{bn}(t) + \mathbf{I}_{un}(t)$ Eq. 7.26

In the case of generated harmonic current a further fifth component $\mathbf{I}_{hn}(t)$ is proposed [Czarnecki 1992]. These are harmonic currents that exist with no corresponding voltage harmonics.

These components are orthogonal under the scalar product and therefore the sum of the squares of all the currents harmonic rms values leads to the relationship:

$$\| \mathbf{i} \|^2 = \| \mathbf{i}_a \|^2 + \| \mathbf{i}_s \|^2 + \| \mathbf{i}_r \|^2 + \| \mathbf{i}_u \|^2 + \| \mathbf{i}_h \|^2 \qquad \text{Eq. 7.27}$$

Multiplying by $\| \mathbf{U} \|^2$ results into:

$$S^2 = P^2 + D_s^2 + Q_s^2 + D_u^2 + D_h^2 \qquad \text{Eq. 7.28}$$

7.2.2 Relevance to present hypothesis

i. Three-phase symmetrical supply with symmetrical load

- Czarnecki [1987] extends the single-phase theory to three-phase system with symmetrical supply voltage and symmetrical linear load. His active current component in the rms domain is identical to that of Depenbrock.
- The three non-active components are identified and shown to be orthogonal under this specific inner product definition.
- The reactive power component can be compensated using three passive one port networks or an active filter; the scattered power component D_s can only be compensated with an active filter.

ii. Three-phase symmetrical supply with asymmetrical load

- Czarnecki [1988] extends his theory to include unbalanced loads with symmetrical voltage supply. The Czarnecki unbalanced theory is particularly useful in restoring current fundamental balance.
- He shows how the reactive Q_s and unbalanced D_u power component can be compensated completely using three one port networks [1995] . However the complex passive filters are unlikely to be practical if total compensation of these two components is required. The fundamental and a limited number of harmonics may possibly be considered in some cases.
- Unbalanced harmonic current compensation is based on the existence of single sequence harmonic (e.g. positive or negative voltages). He therefore assumes the three-phase voltage supply to be symmetrical.
- No zero sequence voltages are considered.

- It is not clear (mentioned) if the theory is applicable to 4-wire systems as well.
- Czarnecki's active current is proportional to the supply instantaneous voltages but the proportionality constant G_e (Eq. 7.15) requires integration over an interval T and in that sense cannot be calculated instantaneously.
- Czarnecki's active current (Eq. 7.18) is equivalent to that of Depenbrock (Eq. 7.2).
- In the case of unbalanced loads, energy storage elements are required if the supply current is to be compensated and reduced to active currents only

iii. Three-phase asymmetrical supply

- Czarnecki's theory does not extend to unsymmetrical voltages[1]. In the case of non-linear load there would be no way of differentiating between generated sequence components and the sequence components due to asymmetrical loads. The same would be true if there were positive and negative sequence voltages of the same harmonic, which would be the case in asymmetrical voltage supplies with asymmetrical linear loads. This is probably the reason that Czarnecki refrains from considering the more general case of asymmetrical voltage supply.
- Czarnecki's approach is entirely based on the frequency domain.
- He ignores the instantaneous power theory and he does not support its merit or recognise any plausible link between instantaneous power and the rms domain [Czarnecki 2006].

7.3 Ferrero and Superti-Furga

Ferrero and Superti-Furga's approach [Ferrero et al, 1993] employs the Park transformation and they show how the method fits with other proposed methods such as Akagi and Czarnecki. They specifically deal with three-phase 3-wire systems as follows:

7.3.1 RMS values

Ferero et al [1993] take three original quantities $y_a(t)$, $y_b(t)$, $y_c(t)$ (that represent three voltages measured from an arbitrary reference or the three line currents) that are changed to the Park vector:

[1] Symmetrical voltage containing only positive sequence fundamental and harmonics voltages whilst unsymmetrical voltage also contains negative sequence and/or zero sequence voltage components.

$\mathbf{y}(t) = \sqrt{3}/2\ (y_a(t) + a\ y_b(t)\ a^2\ y_c(t)\ )$

In periodical conditions (with period T) they use three-phase rms value given as:

$$Y = \sqrt{1/T \cdot \int_T \mathbf{y}(t) \cdot \mathbf{y}^*(t) \cdot dt}$$

Eq. 7.29

$y^*(t)$ is the conjugate of y(t)

If the zero-sequence component is nil, it follows:

$$Y = \sqrt{V_a^{\ 2} + V_b^{\ 2} + V_c^{\ 2}}$$

Eq. 7.30

The rms value of the Park vector is defined as the "pure" three-phase rms value.

7.3.2 Power Definition

They define the Park instantaneous complex power as:

$a_p(t) = v(t) \cdot i^*(t)$ Eq. 7.31

which results with the Park real power $p_p(t)$ and the Park imaginary power $q_p(t)$, where:

$a_p(t) = p_p\ (t) + j\ q_p(t)$ Eq. 7.32

The zero sequence power is introduced as $p_0\ (t) = v_0(t)\ i_0(t)$

It follows that the real instantaneous power is:

$p(t) = p_p(t) + p_0(t)$ Eq. 7.33

The average value of $a_p(t)$ can be evaluated as:

$A_p = P_p + jQ_p = 1/T \int_T v(t) \cdot i^*(t) \cdot dt$ Eq. 7.34

P_p is the active power if $p_0\ (t) = 0$

7.3.3 Apparent power and power factor

Starting from the given definitions of three-phase rms values of voltage and currents, the following "pure" three-phase apparent power can be defined:

$S = V \cdot I$ Eq. 7.35

The three-phase power factor can be defined as

$\lambda = P_p / S$ Eq. 7.36

Taking the amplitude of the average complex power A_p defined as Eq. 3.44

$$A_p = \sqrt{P_p^{\,2} + Q_p^{\,2}}$$

Eq. 7.37

it can be proved that if $S \geq A_p$

the quantity $D_p^2 = S^2 - A_p^2$ Eq. 7.38

can be defined so that the apparent power can be rewritten as:

$S^2 = P_p + Q_p^2 + D_p^2$ Eq. 7.39

7.3.4 Power compensation

Ferrero stipulates, that [1999] as far as power compensation is concerned, total compensation is achieved if $\lambda = 1$ is obtained. To obtain this goal he says that, the total compensation of the instantaneous value of $q_p(t)$ is necessary, **although not sufficient**. Moreover, the compensation of $q_p(t)$ leads to the maximum compensation without energy storage elements.

However, in the optimal compensation, the instantaneous power $p_p(t)$ must be considered. An indication on how to operate is given in the following paragraph this is obtained by the extension of Fryze's time domain decomposition in terms of Park vectors.

7.3.5 Time domain decomposition using Park vectors

The current Park vector of a three-phase system with null zero sequence components can be decomposed into an active component (that is associated with active power) and a residual one. The active current is defined by:

$\mathbf{i}_a(t) = k\,\mathbf{v}(t)$ where $k = P_p/V^2$ P_p is the average value of the real power and V the rms value of the voltage Park vector.

The residual current is obtained by the difference:

$\mathbf{i}_x(t) = \mathbf{i}(t) - \mathbf{i}_a(t)$ Eq. 7.40

The functions $\mathbf{i}_a(t)$ and $\mathbf{i}_x(t)$ are orthogonal, their average inner product is define as:

$$1/T \int_T \mathbf{i}_a(t) \cdot \mathbf{i}_x(t) \cdot dt = 0 \qquad \text{Eq. 7.41}$$

This leads to the following equation for the rms values:

$$I^2 = I_a^2 + I_x^2 \qquad \text{Eq. 7.42}$$

and consequently to

$$P_p = 1/T \int_T v(t) \cdot i_a(t)\, dt = V I_a \qquad \text{Eq. 7.43}$$

This means that if i_x is completely compensated , so that $i = i_a$, then the power factor λ becomes 1 . It follows that the current component i_a in a three-phase system is the minimum current that determines the active power P_p for a specified voltage.

7.3.6 Frequency-domain decomposition

Ferrero et al [1993] demonstrates that the frequency domain decomposition of the current Park vector leads to a generalization equivalent to that introduced by Czarnecki namely:

$$i = i_a + i_s + i_r + i_f \qquad \text{Eq. 7.44}$$

where

i_a : active, i_s scattered, i_r reactive, i_f caused by current sequence harmonics that exist without corresponding voltage sequence harmonics component,

He disregards a further decomposition of i_f in order to reveal explicitly the effect of the circuit asymmetry on the source current. The reason given is that Czarnecki's unbalance component decomposition is only valid if the supply voltage is symmetrical.

7.3.7 Relevance to present hypothesis

The following points are particularly relevant:

- Ferrero et al [1993] uses the Park transform and deals with the zero sequence values separately. The Park transform in the instantaneous domain is said to be in agreement with Akagi's approach and be of a more general method.

- The term "pure" is repeated throughout and the message conveyed seems to be that the Park vector, is the "key" that extract the "pure" three-phase component of the system. The concept of "pure" seems to be brought about by the fact that the Park transform is invariant with respect to an additive term and **therefore zero sequence components** (equivalent to selecting a "null voltage reference point").
- The Park transform approach is also extended to the rms and frequency domain.
- The definition of apparent power as given can be decomposed (without resorting to the frequency domain) into three power terms namely; the real Park average power P_p, the imaginary average power Q_p , and a term D_p which represent the geometric balance (the "rest").
- In his Park approach Ferrero deals with both the instantaneous, frequency, and rms domain.
- The frequency domain decomposition includes asymmetrical voltages. The current components corresponding to non existent sequence voltages are not decomposed further.
- The application refers to three-phase without specifying if it is applicable to 4-wire systems as well. The application to four wires is however implied as it takes into account zero sequence currents. However the presence and therefore the consequent effect of a neutral current is not mentioned.

7.4 Rossetto and Tenti

Rossetto and Tenti [1992] propose an alternative way to calculate instantaneous active current vector $\mathbf{i}_p(t)$, using Lagrange multipliers as follows:

$\mathbf{i}_p(t) = \mathbf{u}\ (p/ \| \mathbf{u} \|^2)$

where **u** is assumed to be such that the reference is chosen so that the sum of the voltages is zero at any one time.

The instantaneous balance of the current is called reactive current:

$\mathbf{i}_q(t) = \mathbf{i}(t) - \mathbf{i}_p(t)$

the active current is calculated as:

$i_a(t)= (P/\|U\|^2)\ u(t)$

The difference between the instantaneous active current $\mathbf{i}_p(t)$ and the active current $\mathbf{i}_a(t)$ is then called the fluctuating current $\mathbf{i}_f(t)$:

$\mathbf{i}_f(t) = \mathbf{i}_p(t) - \mathbf{i}_a(t) = \mathbf{u}(t)\ [\ p/\|\ u\ \|^2\ - p/\|U\|^2\ ]$

This current does not affect the average power transfer, but its compensator requires some energy storage.

Rossetto and Tenti gives results that correspond to the generalised theory of Buchholtz (i.e. extension of Fryze) and to the instantaneous power theory such as that of Akagi. Similarly to Willems and Peng the results are obtained without the need of a three-phase to quadrature transformation such as the Park transformation.

7.4.1 Relevance to present hypothesis

- The instantaneous active current is identical to Willems' and Peng.
- The active current is identical to Buchholtz and therefore Czarnecki.
- The non-active current (The difference between the load current and the active current) is identical to the FBD theory
- The voltage reference is assumed to be zero at any one time, but no justification is given.
- The equations are similar to those obtained by the Depenbrock FBD method. There is essentially no difference between the results obtained. No reference to the FBD method is given.
- The mathematical derivation is more elegant, than by linear vector algebra.

7.5 Depenbrock , Marshall and van Wyk

More recently Depenbrock restates his theory in English [Depenbrock 1993] and receives support from authors such as Marshall and van Wyk [Depenbrock et al 1994]. They also point out that the FBD method gives identical results to those of Rossetto and Tenti and also a more general solution than other authors such as Akagi. They also state that the definition of collective apparent power as originally introduced by Buchholtz fulfils the condition that in no case the active power can exceed the apparent power, but no justification for this statement is given.

7.5.1 Relevance to present hypothesis

- The voltage reference is specific and sum of the voltages of all m wires is always zero.
- A very important point is raised as that this method fulfils the condition that **in no case the active power can exceed the apparent power**. The reason for this is not explained.

7.6 Willems

More recently Willems [2004, 2006] offers a more generalized power theory in the rms domain that includes the case that losses in the return conductor are negligibly small and shows that the virtual star point is not an obvious reference point and not always the point to be used. He states that an objection to the analysis in the previous section is that the two situations considered (identity of the resistances of the neutral conductor and of the other conductors on the one hand, and no resistance in the neutral conductor on the other hand) are extreme cases; the real situation is often somewhere in between. He presents a more general analysis where all conductor resistances may be different.

7.6.1 Apparent power in the general case

In that case the maximum value of the active power is required with respect to the currents in the terminals for given voltages and given conductor losses, subject to the conditions that the sum of the currents vanishes. If we do not assume the conductors to be identical, the conductor losses are given by an expression of the following form (see Willems [2004, 2006]) :

$$\sum_{k=1}^{m+1} R_k \; \| I_k \|^2 = A \qquad \text{Eq. 7.45}$$

where R_k denotes the resistance of conductor k, I_k the rms current of the k^{th} wire.

Using the Lagrange multiplier technique this is equivalent to finding the maximum value of the function:

$$\sum_{k=1}^{m+1} R_e(V_k I_k^*) - \lambda/2 \left(\sum_{k=1}^{m+1} R_k I_k^{\,2} - A \right) - R_e\left(\mu \sum_{k=1}^{m+1} I_k \right) \qquad \text{Eq. 7.46}$$

with respect to the currents and the Lagrange multipliers λ (real) and μ (complex). This leads to the equations.

$$V_k = \lambda R_k I_k + \mu \qquad (k = 1, \ldots, m+1) \qquad \text{Eq. 7.47}$$

$$\sum_{k=1}^{m+1} R_k \; | I_k |^2 = A \qquad \text{Eq. 7.48}$$

$$\sum_{k=1}^{m+1} I_k = 0 \qquad \text{Eq. 7.49}$$

The solution of the set of the three equations is

$$\mu = \left(\sum_{k=1}^{m+1} V_k / R_k \right) / \left(\sum_{k=1}^{m+1} 1 / R_k \right) = V_{ref} \qquad \text{Eq. 7.50}$$

$$\lambda = \left(\left(\sum_{k=1}^{m+1} |V_k - V_{ref}|^2 / R_k \right) \left(\sum_{k=1}^{m+1} R_k |I_k|^2 \right) \right)^{1/2} \qquad \text{Eq. 7.51}$$

$$I_{kg} = (V_k - V_{ref}) / \lambda R_k \quad (k = 1, \dots, m+1) \qquad \text{Eq. 7.52}$$

Hence the currents yielding the maximal power satisfy

The solution according to Willems [2006] of the set of equations is

$$I_{kg} = \left(\left(\sum_{k=1}^{m+1} R_k |I_k|^2 \right) / \left(\sum_{k=1}^{m+1} |V_k - V_{ref}|^2 / R_k \right) \right)^{1/2} \quad k = 1, \dots, m+1) \qquad \text{Eq. 7.53}$$

The apparent power, the maximal average active power that can be delivered with the given voltage and the given magnitude of the current characterized by the conductor losses or a proportional quantity is according to Willems:

$$S_g = \left(\sum_{k=1}^{m+1} (V_k - V_{ref})^2 / R_k \right)^{1/2} \left(\sum_{k=1}^{m+1} R_k I_k^2 \right)^{1/2} \qquad \text{Eq. 7. 54}$$

This expression of the apparent power contains a product of a voltage-dependent factor and a current dependent factor. It is clear from the voltage-dependent factor that the voltages of the terminals should be referred to the reference value obtained in the above analysis.

7.6.2 Norms of current and voltage vectors

From a mathematical point of view expression Eq. 7.54 leads to the fundamental question whether the two factors in S_g can still be seen as the norms of the current and voltage vectors respectively. In other words can the expression of power and the two factors in S_g be considered as derived from a single definition of the inner product? More explicitly the question is whether or not it is possible to define the inner product in the vector space (denoted by $x \bullet y$) such that

$$P = V \bullet I \qquad \text{Eq. 7.55}$$

$$V \bullet V = \| V \|^2 = \sum_{k=1}^{m+1} |V_k - V_{ref}|^2 / R_k \qquad \text{Eq. 7.56}$$

$$I \bullet I = \| I \|^2 = \sum_{k=1}^{m+1} R_k |I_k|^2 \qquad \text{Eq. 7.57}$$

The answer according to Willems is yes; it is achieved by considering the voltage and current vector components.

$(V_k – V_{ref}) / \sqrt{R_k}$

for the voltage and

$I_k \sqrt{R_k}$

for the current, and using the inner product.

$$x \bullet y = R_e \left(\sum_{k=1}^{m+1} x_k y_k^* \right) \qquad \text{Eq. 7.58}$$

Of course some normalizing proportionality factor can be introduced, such that the norms of the voltage and the current have dimensions of voltage and current respectively; the restriction is that the product of the two proportionality factors should be equal to 1. Hence Willems' expression for apparent power is

$$S_g = \| V \|_g \| I \|_g \qquad \text{Eq. 7.59}$$

7.6.3 Relevance to the hypothesis

- Willems' recent theory is of a general nature in which the wires can have any resistance value.
- The theory is specifically applied to the rms domain.
- The theory is based on obtaining optimal active current that will transmit the maximum power and therefore in agreement with Emanuel's hypothesis.
- It is interesting to note that despite this generalized formulation he did not apply this kind of approach to explain the differences obtained between the three groups covered. Some interesting remarks were made [Willems 2006] recently: "The question is what one gains by using this higher-dimensional quantity, or explicitly what additional information the reactive power tensor may provide compared to the reactive vector". This remark is in contrast with the conclusion made in Chapter 5 with respect of the neutral wire resistance.

7.7 Conclusion

The rms domain decomposition of currents has been reviewed and in some cases has been extended from the instantaneous domain to the rms domain by various authors for three wires only. Having shown some of the issues that instantaneous power theories can still raise and having shown that in the case where all four wires have equal resistances the reference voltage has to be first chosen so that the sum of all four wires are zero, then the load current four dimension vector can be separated into four orthogonal current vectors. It is now shown that the extension to the rms domain is a natural and logical extension of the theory developed in Chapter 6.

Chapter 8

4-wire systems with equal wire resistances.

In this chapter it is shown how the instantaneous theory developed in Chapter 6 for 4-wire systems with equal resistances, can be extended to the rms or average power domain. Four orthogonal current vector components in the rms domain are identified.

8.1 Optimal compensation over a cycle without energy storage

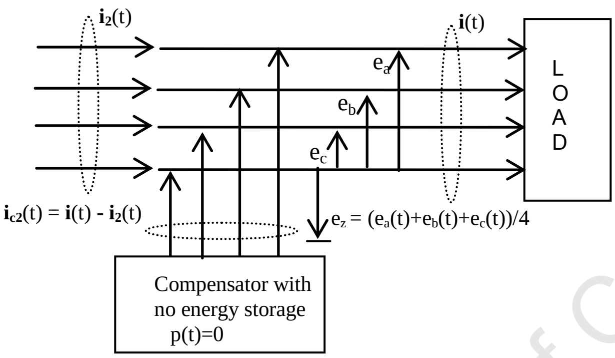

Fig 8.1 Load current compensator on 3-phase, 4-wire supply without energy storage

It was shown in Chapter 6, that the instantaneous active current obtained through Eq. 6.7

$\mathbf{i}_2(t) = k_2(t)\, \mathbf{v}_2(t)$ where $k_2(t) = p(t)/\|\mathbf{v}(t)\|^2$

offers theoretical minimum transmission losses if no energy storage elements are used in the compensator. It was also shown in Chapter 5 that the minimum instantaneous active norm could be calculated using Eq. 5.9

$\| \mathbf{i}_2 \|^2 = p^2 / \|\mathbf{v}_2\|^2$

The difference between i(t) and $i_2(t)$ was referred to as $i_{c2}(t)$ the instantaneous non-active current.

When the voltages and the currents are periodic and vary over a time interval then the rms values are more relevant. The rms value of the line current $\|\mathbf{I}\|$ can be calculated as follows:

$\|\mathbf{I}\|^2 = 1/T \int \mathbf{i}(t) \bullet \mathbf{i}(t)\, dt$

Therefore it is appropriate to redefine the inner products of two vectors **f**(t), **g**(t) as follows:

$$< \mathbf{f}(t), \mathbf{g}(t) > = (1/T) \int (\mathbf{f}(t) \bullet \mathbf{g}(t))\, dt \qquad \text{Eq. 8.1}$$

then from Eq. 8.1 and Eq. 6.11 (note: i_{c4} is now referred to i_{c2} for consistency with i_2)

$||\mathbf{I}||^2 = 1/T \int \mathbf{i}(t) \bullet (\mathbf{i}_2(t) + \mathbf{i}_{c2})$

$||\mathbf{I}||^2 = 1/T \int \mathbf{i}(t) \bullet \mathbf{i}_2(t)\, dt + 1/T \int \mathbf{i}(t) \bullet \mathbf{i}_{c2}(t)\, dt$

$||\mathbf{I}||^2 = 1/T \int \mathbf{i}_2(t) \bullet \mathbf{i}_2(t)\, dt + 1/T \int \mathbf{i}_{c2}(t) \bullet \mathbf{i}_{c2}(t)\, dt$

$$||\mathbf{I}||^2 = ||\mathbf{I}_2||^2 + ||\mathbf{I}_{c2}||^2 \qquad \text{Eq. 8.2}$$

The rms current components now defined in the rms domain are orthogonal and therefore form a Pythagorean triangle. This is demonstrated in Example 8.1 and illustrated in Fig. 8.2.

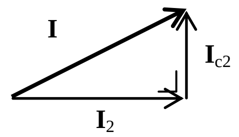

Fig 8.2 Orthogonal property of RMS values of current components.

Example 8.1

This example considers a simple 4-wire case where the voltages and currents are constant for the first half of an interval T and then change to another arbitrary constant value for the second half. Let

$\mathbf{v}(t_1)$={0, -3, 9, -2} and $\mathbf{i}(t_1)$ ={12, -9, 2, -5} for the first half cycle

and

$\mathbf{v}(t_2)$ ={0, -6, 18, -4}and $\mathbf{i}(t_2)$ ={7, -3, 4 , -8} for the second half cycle

then

$||\mathbf{v}(t_1)||^2 = 0 + 9 + 81+ 4 = 94$

and

$||\mathbf{v}(t_2)||^2 = 0 + 36 + 324 + 16 = 376$

$||\mathbf{V}||^2$ is the average of $||\mathbf{v}(t)||^2$ during period T

hence

$||\mathbf{V}||^2$ is in this example the average of $||\mathbf{v}(t_1)||^2$ and $||\mathbf{v}(t_2)||^2 = (376 + 94)/2 = 235$

$p(t_1)$ and $p(t_2)$ are the power used by the load for the first and second half of the cycle:

$p(t_1) = (-3)(-9) + (9)(2) + (-2)(-5) = 55$ watts

$p(t_2) = (-6)(-3) + (18)(4) + (-4)(-8) = 122$ watts

The average power is

$P = (1/T) \int p(t)\, dt = (55 + 122)/2 = 88.5$ watts

The average line losses would be:

$\| \mathbf{I} \|^2 = (1/T) \int \|\mathbf{i}(t)\|^2 dt = (\|\mathbf{i}(t_1)\|^2 + \|\mathbf{i}(t_2)\|^2)/2 = (254+138)/2=196$

$\| \mathbf{I} \| = 196^{1/2} = 14$

$\mathbf{v}_2(t_1)= \{-1,-4,8,-3\}$ $\|\mathbf{v}_2(t_1)\|^2 = 1+16+64+9=90$ (see Example 6.1)

Similarly,

$\mathbf{v}_2 (t_2) = \{-2, -8, 16, -6 \}$ $\|\mathbf{v}_2 (t_2)\|^2 = 4 + 64 + 256 + 36 = 360$

$\|\mathbf{v}_2\|^2 = (\|\mathbf{v}_2(t_1)\|^2+\| \mathbf{v}_2 (t_2) \|^2)/2 = (90 + 360)/2 = 225$

From Eq. 6.7 (The instantaneous active current is $i_2(t) = k_2(t)\, v_2(t)$ where $k_2(t) = p(t)/\|\mathbf{v(t)}\|^2$).

$k_2(t_1) = p(t_1)/ \|\mathbf{v}_2(t_1)\|^2 = 55/90 = 0.6111$

$\mathbf{i}_2(t_1) = 0.6111\{-1, -4, 8, -3 \}$ which gives

$\|\mathbf{i}_2(t_1)\|^2 = p(t_1)^2/ \|\mathbf{v}_2(t_1)\|^2 = 55^2 /90 = 33.61$

Similarly

$k_2 (t_2) = p(t_2)/ \|\mathbf{v}_2(t_2)\|^2 = 122/360 = 0.3389$

$\mathbf{i}_2 (t_2) = 0.3389\{-2, -8, 16, -6 \} = \{-0.6778, -2.711, 5.42, -2.03\}$

The non-active instantaneous current is given by Eq. 6.11 ($\mathbf{i}_{c2}(t) = \mathbf{i}(t) - \mathbf{i}_2(t)$)

$\mathbf{i}_{c2}(t_1) = \mathbf{i}(t_1) - \mathbf{i}_2(t_1) =\{12, -9, 2, -5\} - \{-0.61, -2.44, 4.88, -1.83\}$

$\mathbf{i}_{c2}(t_1) = \{12.61, -6.56, -2.88, -3.17\}$ which gives

$\|\mathbf{i}_{c2}(t_1)\|^2 = 220.39$ $\|\mathbf{i}_{c2}(t_1)\| = 14.845$

$\|\mathbf{i}_2(t_2)\|^2 = 41.344$

Note the same results can be obtained using Eq. 3.19 $\|\mathbf{i}_2(t)\|^2 = p(t)^2 / \|\mathbf{v}_2 (t)\|^2$

$p(t_2)^2/ \|\mathbf{v}_2 (t_2) \|^2 = 1222 /360 = 41.344$

Similarly

$\mathbf{i}_{c2} (t_2) = \mathbf{i}(t_2) - \mathbf{i}_2(t_2) =\{7, -3, 4 , -8\} - \{0.68 , -2.71, 5.42, -2.03\}$

$\mathbf{i}_{c2}(t_2) = \{ 7.68, -0,29, -1,42, -5.96\}$ $\|\mathbf{i}_{c2} (t_2) \|^2 = 96.65$

Taking the average of $\|\mathbf{i}_2(t)\|^2$

$\|\mathbf{I}_2\|^2 = (33.611+41.344)/2 = 37.5$ which gives $\|\mathbf{I}_2\| = 6.12$

$\|\mathbf{i}_{c2}(t_1)\|^2 = 254 - 33.61 = 220.39$

$\|\mathbf{i}_{c2} (t_2)\|^2 = 138 - 41.34 = 96.65$

$\|\mathbf{I}_{c2}\|^2 = (220.39 + 96.65)/2 = 317.04/2= 158.52$

$\|\mathbf{I}_2\|^2 + \|\mathbf{I}_{c2}\|^2 = 37.5 +158.5 =196 = \| \mathbf{I} \|^2$ which agrees with Eq. 8.2

So far the approach only deals with compensation without energy storage. It achieves theoretical optimum compensation under such conditions, but it will now be seen how further energy transmission losses can be reduced if energy can be stored and released during a cycle. Therefore not all non-active current (yet to be defined) can be compensated without using energy storage.

8.2 Compensation over a cycle with energy storage

The orthogonal voltage vector basis $\mathbf{v}_1(t)$, $\mathbf{v}_2(t)$, $\mathbf{v}_3(t)$ as in Chapter 6 (see Eq. 6.3, 6.4 and 6.5) is used. Let $\mathbf{i}^{\#}(t)$ be the projection of vector $\mathbf{i}(t)$ onto $\mathbf{v}_2(t)$ over the interval T, with the inner product of two vectors defined as in Eq. 8.1, then the projection of the current vectors onto the voltage vector $\mathbf{v}_2(t)$ according to linear algebra [Anton 2000] is found as follows:

$\mathbf{i}^{\#}(t) = \langle \mathbf{i}(t)\,,\, \mathbf{v}_2\,(t) \rangle\, \mathbf{v}_2(t) \,/\, \|\mathbf{V}_2\|^2$

where $\|\mathbf{V}_2\,\|^2 = \langle \mathbf{v}_2(t), \mathbf{v}_2(t\,) \rangle = (1/T) \int \mathbf{v}_2(t) \bullet \mathbf{v}_2(t)\, dt$

and since $\langle \mathbf{i}(t)\,,\, \mathbf{v}_2\,(t) \rangle = (1/T) \int \mathbf{v}_2\,(t) \bullet i(t)\, dt = (1/\,T) \int p(t)\, dt = P$

hence:

$\mathbf{i}^{\#}(t)=k^{\#}\mathbf{v}_2(t)$ Eq. 8.3

where $k^{\#} = P \,/\, \|\mathbf{V}_2\|^2$

$\mathbf{i}^{\#}(t)$ is also a 4-dimension time varying current vector in the same direction as $\mathbf{v}_2(t)$, the instantaneous voltage vector measured from the time dependent "null" point.

The proportionality constant $k^{\#}$ is not time dependant as was the case with $k_2(t)$ of Eq. 6.7 and is a constant throughout a cycle. $k^{\#}$ is proportional to the average power rather than the instantaneous power and inversely proportional to $\|\mathbf{V}_2\|^2$ which is the average of $\mathbf{v}_2(t) \bullet \mathbf{v}_2(t)$ over the interval T.

The power transmitted by $\mathbf{i}^{\#}(t)$ over an interval T and calculated from the voltage reference $\mathbf{v}_2(t)$ is:

$= (1/T) \int \mathbf{v}_2(t) \bullet \mathbf{i}^{\#}(t)\, dt$ From Eq 8.3

$= (1/T) \int \mathbf{v}_2(t) \bullet k^{\#}\, \mathbf{v}_2(t)\, dt$

$= (1/T)\, k^{\#} \int \|\mathbf{v}_2(t)\|^2\, dt = k^{\#}\|\mathbf{V}_2\|^2 = P$

Hence $\mathbf{i}^{\#}(t)$ delivers the same average power P over an interval T, as $\mathbf{i}_2(t)$ or the original current vector $\mathbf{i}(t)$. The average power contributed by the compensator which requires an energy storage is zero (see fig 8.3).

$\mathbf{i}_c^{\#}(t) = \mathbf{i}_2(t) - \mathbf{i}^{\#}(t)$ Eq. 8.4

over T must therefore be zero.

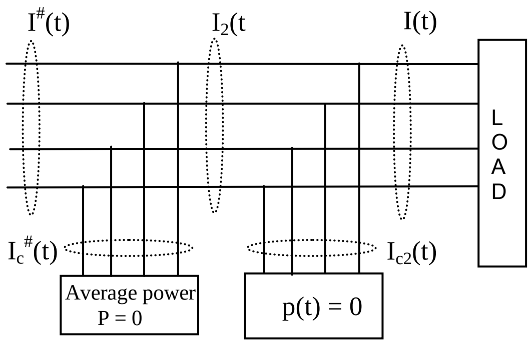

Fig. 8.3 Compensators with and without energy storage

It will now be shown that $\mathbf{i}_c^{\#}(t)$, $\mathbf{i}^{\#}(t)$ are orthogonal

$\|\mathbf{I}_c^{\#}\|^2 = < \mathbf{i}_c^{\#}(t) , \mathbf{i}_c^{\#}(t) > = < \mathbf{i}_2(t) - \mathbf{i}^{\#}(t) , \mathbf{i}_2(t) - \mathbf{i}^{\#}(t) >$

Then $\|\mathbf{I}_c^{\#}\|^2 = < \mathbf{i}_2(t), \mathbf{i}_2(t)> + < \mathbf{i}^{\#}(t), \mathbf{i}^{\#}(t)> -2 < \mathbf{i}_2(t), \mathbf{i}^{\#}(t >$

From Eq. 8.3, 8.4

$\|\mathbf{I}_c^{\#}\|^2 =\|\mathbf{I}_2\|^2+P^2/\|V_2\|^2 - (2/T) \int k_2(t)\mathbf{v}_2(t) \bullet k^{\#}\mathbf{v}_2(t)$

$\|\mathbf{I}_c^{\#}\|^2 =\|\mathbf{I}_2\|^2+P^2/\|V_2\|^2 - (2/T) k^{\#}\int \mathbf{i}_2(t) \bullet \mathbf{v}_2(t)$

$\|\mathbf{I}_c^{\#}\|^2 =\|\mathbf{I}_2\|^2+P^2/\|V_2\|^2 - (2/T) k^{\#}\int p(t)$

From Eq. 8.3

$\|\mathbf{I}_c^{\#}\|2 =\|\mathbf{I}_2\|^2+P^2/\|V_2\|^2 - 2 P^2/\|V_2\|^2$

$\|\mathbf{I}_c^{\#}\|^2 = \|\mathbf{I}_2\|^2 - P^2/\|V_2\|^2$ from Eq 8.3

$P = (1/T) \int \mathbf{i}^{\#}(t) \bullet \mathbf{v}_2(t)\, dt = <\mathbf{i}^{\#}(t), \mathbf{v}_2(t)>$

$P = (1/T) \int < \mathbf{i}^{\#}(t), \mathbf{i}^{\#}(t) > / k^{\#}$ hence

$P = \|\mathbf{I}^{\#}\|^2 /k^{\#} = \|\mathbf{I}^{\#}\|^2\|\mathbf{V}_2\|^2 / P$, and

$P_2= \|\mathbf{I}^{\#}\|^2 \|\mathbf{V}_2\|^2$ giving

$$P = \|\mathbf{I}^{\#}\| \|\mathbf{V}_2\| \qquad \text{Eq. 8.5}$$

Hence:

$\|\mathbf{I}_c^{\#}\|^2 = \|\mathbf{I}_2\|^2 - \|\mathbf{I}^{\#}\|^2$, or :

$$\|\mathbf{I}_2\|^2 = \|\mathbf{I}^{\#}\|^2 + \|\mathbf{I}_c^{\#}\|^2 \qquad \text{Eq. 8.6}$$

Which proves that $\mathbf{i}^{\#}(t)$ and $\mathbf{i}_c^{\#}(t)$ are orthogonal to each other under the definition of the inner product give by Eq. 8.1. This is illustrated in Example 8.2 and Fig. 8.3.

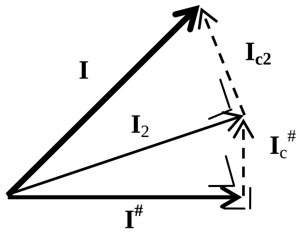

Fig 8.4 Orthogonality between current components in RMS domain

Example 8.2

Using the same conditions as in Example 8.1, from Eq. 8.3

$k^{\#} = P /\|V_2\|^2 = 88.5/ 225 = 0.3933$

The instantaneous active current vector is:

$\mathbf{i}^{\#}(t_1) = 0.3933 \{-1, -4, 8, -3 \}$

Similarly $\mathbf{i}^{\#}(t_2) = 0.393\{-2, -8, 16, -6 \}$

$P_{t1av} = < \mathbf{v}(t_1) , i^{\#}(t1) > = < \mathbf{v}(t) , i^{\#}(t) >$

$P_{t1av} = \{0,-3,9,-2 \} \bullet 0.393\{-1, -4, 8, -3 \} = 35.40$

Similarly

$P_{t2av} = 0.393\{0,-6,18,-4\} \bullet \{24, -8, 16, -6 \} = 141.60$

$P = (P_{t1av} + P_{t2av})/2 = (35.40 +141.60)/2 = 88.5$

$\|\mathbf{i}^{\#}_{t1}\|^2 = .393\ 2 \times 90 = 13.92$

$\|\mathbf{i}^{\#}_{t1}\|^2 = .393\ 2 \times 360 = 55.69$

The average is:

$\|\mathbf{I}^{\#}\|^2 = (13.92 + 55.69)/2 = 34.81$

or from Eq 12

$\|\mathbf{I}^{\#}\|^2 = P_2/\|V_2\|^2 = 88.52 / 225 = 34.81$

This illustrates the validity of Eq. 8.3

$\|\mathbf{I}^{\#}_c\|^2 = \|\mathbf{I}_2\|^2 - \|\mathbf{I}^{\#}\|^2 = 37.48-34.81= 2.67$

From Eq 8.5

$\|\mathbf{I}_2\|^2 = \|\mathbf{I}^{\#}\|^2 + \|\mathbf{I}_c^{\#}\|^2 = 34.81 + 2.67 = 37.48$

Which is in agreement with $\|\mathbf{I}_2\|^2$ obtained in Example 8.1

Hence from Eq. 8.6 it can be said that : $\|\mathbf{I}^{\#}\|^2 \leq \|\mathbf{I}_2\|^2$ which means that compensation with energy storage can reduce the average line losses further than was possible without energy storage. $\mathbf{i}_c^{\#}(t)$ is the portion of the compensating current that can only be compensated with energy storage devices.

If $\mathbf{i}_c(t) = \mathbf{i}(t) - \mathbf{i}^{\#}(t)$,

then

$$\mathbf{i}_c(t) = \mathbf{i}_{c2}(t) + \mathbf{i}_c^{\#}(t) \qquad \text{Eq. 8.7}$$

In Fig 6.4 The first component of the non-active current $i_{c2}(t)$ does not require energy storage whilst the second $\mathbf{i}_c^{\#}(t)$ does. These two components are also orthogonal to each other since:

$< \mathbf{i}_{c2}(t) , \mathbf{i}_c^{\#}(t) > \; = < \mathbf{i}(t) - \mathbf{i}_2(t), \; \mathbf{i}_2(t) - \mathbf{i}^{\#}(t) >$

$= < \mathbf{i}(t) , \mathbf{i}_2(t) > - < \mathbf{i}(t) , \mathbf{i}^{\#}(t) > - < \mathbf{i}_2(t) , \mathbf{i}_2(t) > + < \mathbf{i}_2(t) , \mathbf{i}^{\#}(t) >$

$= < \mathbf{i}_2(t) , \mathbf{i}_2(t) > - \; \mathbf{k}^{\#} < \mathbf{i}(t) , v_2(t) > - < \mathbf{i}_2(t) , \mathbf{i}_2(t) > + \mathbf{k}^{\#} < \mathbf{i}_2(t) , \mathbf{v}_2(t) >$

$= - \; \mathbf{k}^{\#} (P – P) = 0$

therefore

$$||\mathbf{I}_c||^2 = ||\mathbf{I}_{c2}||^2 + ||\mathbf{I}_c^{\#}||^2 \qquad \text{Eq. 8.8}$$

The load current $\mathbf{i}(t)$ consists therefore of three orthogonal components such that:

$i(t) = i^{\#}(t) + i_c^{\#}(t) + i_{c2}(t)$

and can from Eq. 8.3 and Eq. 8.6 be said to be orthogonal to each other, hence:

$$||\mathbf{I}||^2 = ||\mathbf{I}^{\#}||^2 + ||\mathbf{I}_c^{\#}||^2 + ||\mathbf{I}_{c2}||^2 \qquad \text{Eq. 8.9}$$

Example 8.3

$\mathbf{i}^{\#}_c (t_1) = (k_2 (t_1) - k\#) \, \mathbf{v}_2(t_1)$

$k_2 (t_1) = 0.611$ and $k\# = 0.393$ (from Example 8.1and 8.2 respectively)

$\mathbf{i}^{\#}_c(t_1) = 0.218\{-1, -4, 8, -3 \}$

the average power during the first half cycle is

$P_{t1av} = < \mathbf{v}_2(t_1) , i^{\#}(t1) > = 0.218\{-1, -4, 8, -3 \} \bullet \{-1, -4, 8, -3 \}$

$P_{ct1av} = 0.218 \text{ x } 90 = 19.62$

Similarly

$\mathbf{i}^{\#}_c(t_2) = (k_2(t_2) - k^{\#}) \, \mathbf{v}_2(t_2) = -0.0545\{-2, -8, 16, -6\}$

$P_{ct2av} = -0.0545 \text{ x } 360 = -19.62$

hence $P_{cav} = P_{ct1} + P_{ct2} = 19.62-19.63 = 0$

The average power required from the compensator is therefore zero

$||\mathbf{i}^{\#}_c (t_1)||^2 = ||0.218\{-1, -4, 8, -3 \}||^2 = (0.2182)^2 \, (90) = 4.28$

similarly

$||\mathbf{i}^{\#}{}_{c}(t_2)||^2$ = (-0.0545)2(360) =1.07

$||\mathbf{I}^{\#}{}_{c}||^2$ = (4.28 +1.07)/2 = 2.67 as above

Using $||\mathbf{I}^{\#}||^2$ from example 8.2

$||\mathbf{I}||^2 = ||\mathbf{I}_c||^2 + ||\mathbf{I}_c{}^{\#}||^2$ = 161.19 + 34.81 = 196

$||\mathbf{I}_c||^2 = ||\mathbf{I}_{c2}||^2 + ||\mathbf{I}^{\#}||^2$ = 158.52 + 2.67 = 161.19

Which is in agreement with $||\mathbf{I}||^2$ obtained in Example 8.1

The results of Example 8.1, 8.2, 8.3 are illustrated in Fig. 8.5

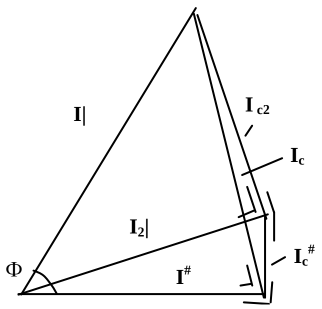

Fig .8.5 Orthonormal relationship of rms current components see examples 2 and 3.

Note: $||\mathbf{I}^{\#}||^2 + ||\mathbf{I}_c{}^{\#}||^2 + ||\mathbf{I}_{c2}||^2 = ||\mathbf{I}||^2$ = 34.81+2.67+158.52 =196 $||\mathbf{I}^{\#}||^2 + ||\mathbf{I}_c{}^{\#}||^2 = ||\mathbf{I}_2||^2$ = 34.81 + 2.67 = 37.48

$||\mathbf{I}_{c2}||^2 + ||\mathbf{I}_c{}^{\#}||^2 = ||\mathbf{I}_c||^2$ = 58.52+2.67=161.19 $||\mathbf{I}_2||^2 + ||\mathbf{I}_{c2}||^2$ = 37.48+158.52 =196

This approach to current decomposition successfully links the instantaneous current theory to the rms domain, by separating the non-active current vector into two orthogonal components. One component can be compensated without energy storage, and one requires temporary energy storage for the duration of a cycle. Furthermore with this approach, the neutral current losses are included, which is not the case with the methods such as Group 2 and Group 3 as seen in Chapter 3.

8.3 AP and non-active power

If the same average power P can be transmitted by a current vector $i^{\#}(t)$ then one would be able to transmit more power in proportion to the ratio of the norm of the current before compensation over the norm of the current after compensation (this is assuming that the line voltages do not change.)

AP as defined by Emanuel and explained in Chapter 1 is by definition the maximum power hence

$S = P \times \|\mathbf{I}\| / \|\mathbf{I}^{\#}\|$

From Eq 8.5

$P = \|\mathbf{I}^{\#}\| \, \|\mathbf{V}_2\|$ Eq. 8.10

Therefore

$S = \|\mathbf{I}\| \, \|\mathbf{V}_2\|$ Eq. 8.11

Note that $\|\mathbf{V}_2\|^2 = \|\mathbf{V}\|^2 + \|\mathbf{E}_{z4}\|^2$

Where: $\|\mathbf{E}_{z4}\|^2 = \langle \mathbf{e}_{z4}(t), \mathbf{e}_{z4}(t) \rangle$ (see definition in Chapter 4, Eq. 4.1)

Hence: $\|\mathbf{V}_2\|^2 \geq \|\mathbf{V}\|^2$

And therefore $S = \|\mathbf{I}\| \, \|\mathbf{V}\|$ is not the maximum power, and therefore not in agreement with Emanuel's definition of AP.

The power factor λ was defined as the ratio of the power transmitted over the power that can be transmitted if compensated optimally (For the same line losses)

$\lambda = P/S$ Eq. 8.12

Note: λ is not a function of time. From Eq. 8.5 and 8.10 hence it follows that the power factor is:

$\lambda = \|\mathbf{I}^{\#}\| / \|\mathbf{I}\|$ Eq. 8.13

This ratio indicates a goodness factor of how effectively a supply is used compared with its maximum capacity for the same losses,

If Q is defined as:

$Q^2 = S^2 - P^2$ Eq. 8.14

This is an indication of the capacity (size) of the compensator required to bring the power factor to unity

from Eq. 8.8 and 8-9

$Q = (\|\mathbf{I}\|^2 - \|\mathbf{I}^{\#}\|^2)^{1/2} \|\mathbf{V}_2\|^2$

$Q_{c2} = \|\mathbf{I}_{c2}\| \, \|\mathbf{V}_2\|$ Eq. 8.15

Q can be considered as the total non-active power.

Since $\mathbf{I}_{c2}$ can be split into two non active currents $\mathbf{I}_c^{\#}$ and $\mathbf{I}_{c2}$ (see Eq 8.8). One can say that Q the apparent power consists of two non-active components: one that can only be compensated with energy storage, whilst the second (the balance) can be compensated without energy storage.

$Q = Q^{\#} + Q_2$ Eq. 8.16

where

$Q^{\#} = \|\mathbf{I}_c^{\#}\| \, \|\mathbf{V}_2\|$ Eq. 8.17

and the balance

$Q_2 = \|\mathbf{I}_{c2}\| \, \|\mathbf{V}_2\|$ Eq. 8.18

Therefore:

$S^2 = P^2 + Q^{\#\,2} + Q_2^{\,2}$ Eq. 8.19

The orthogonality property of the three power components is demonstrated in Example 8.4 and illustrated in Fig. 8.6.

If λ_2 be the power factor that can be achieved without energy storage compensator then

$\lambda_2 = P / S_2$ Eq. 8.20

where

$S_2 = (S^2 - Q^{\#\,2})^{1/2}$ Eq. 8.21

From Eq. 8.10 and 8.17

$S_2 = \|\mathbf{I}_2\| \, \|\mathbf{V}_2\|$ Eq. 8.22

If λ^1 is the power factor after compensating and $S^{\#}$ the AP after compensation then

$\lambda^1 = P / S^{\#} = P / \|\mathbf{V}_2\| \left(\| \mathbf{I} - \mathbf{I}_{c2} - \mathbf{I}_c^{\#} \| \right) = P / \|\mathbf{V}_2\| \, \|I^{\#}\| = P / P = 1$ Eq. 8.23

Example 8.4

Using values from Examples 8.2 and 8.3

$S = \|I\| \, \|V_2\| = = (\|I\|^2 \, \|V_2\|^2)^{1/2} = (196 \times 225)^{1/2} = 210$ VA

$P = \|I^{\#}\| \, \|V_2\| = (34.8/225)^{1/2} = 88.5$W

$\lambda = P / S = 88.5/210 = 0.4225$ or

$\lambda = \|I^{\#}\| / \|I\| = (34.81/196)^{1/2} = 0.4225$

$\lambda_2 = P/S_2 = 88.5/91.80 = 0.964$

$Q_2 = \|\mathbf{I}_{c2}\| \, \|\mathbf{V}_2\| = (158.52 \times 225)^{1/2} = 188.85$

$S^{\#} = \|V_2\|\ \|\mathbf{I}^{\#}\| = P = 88.5$

$\lambda^1 = 88.5/88.5 = 1$

$Q^{\#} = \|\mathbf{I}_c^{\#}\|\ \|\mathbf{V}_2\| = (2.68 \text{ x } 225)^{1/2} = 24.56$

$Q^2 = (161.19 \text{ x } 225)^{1/2} = 190.44$

$Q^{\#} = (Q^2 - Q_2^{\ 2})^{1/2} = (190.44^2 - 188.85^2)^{1/2} = 24.56$

$S_2 = \|\mathbf{I}_2\|\ \|\mathbf{V}_2\| = (37.48 \text{ x } 225)^{1/2} = 91.80$

$(S_2^{\ 2} + Q_2^{\ 2})^{1/2} = (91.80^2 + 188.85^2)^{1/2} = 210 = S$

$S = (P^2 + Q^{\#2} + Q_2^{\ 2})^{1/2} = (88.5^2 + 24.56^2 + 188.26^2)^{1/2} = 210$

$S^{\#} = \|V_2\|\ \|I^{\#}\| = P = 88.5 \qquad \lambda^1 = 88.5/88.5 = 1$

Using values from Examples 8.1, 8.2 and 8.3:

$\mathbf{v}(t_1) = \{-3, 9, -2\}$ $\mathbf{v}(t_2) = \{-6, 18, -4\}$

$\|\mathbf{V}(t_1)\|^2 = v(t_1) \bullet v(t_1) = 9 + 81 + 4 = 94$

$\|\mathbf{V}(t_1)\|^2 = \mathbf{v}(t_1) \bullet \mathbf{v}(t_1) = 36 + 324 + 16 = 376$

$Q^{\#} = \|I_c^{\#}\|\ \|V_2\| = (2.68 \text{ x } 225)^{1/2} = 24.56$

$Q^{\#} = (Q^2 - Q_2^{\ 2})^{1/2} = (190^2 - 188.26^2)^{1/2} = 24.56$

$\lambda_2 = P/S_2 = 88.5/91.80 = 0.964$

$(S_2^{\ 2} + Q_2^{\ 2})^{1/2} = (91.80^2 + 188.85^2)^{1/2} = 210 = S$

$S = (P^2 + Q^{\#2} + Q_2^{\ 2})^{1/2} = (88.5^2 + 24.56^2 + 188.85^2)^{1/2} = 210$

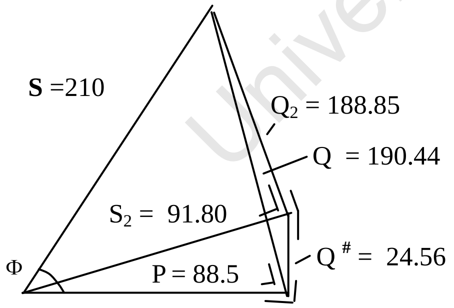

Fig. 8.6 Active and none-active power components
Note: $S = (88.5^2 + 25.65^2 + 188.85^2)^{1/2} = 210$

8.4 Further decomposition

In previous chapters it was shown how $\mathbf{i}_{c2}(t)$, the compensating current component to be compensated without energy storage, could be separated into two orthogonal instantaneous current components $\mathbf{i}_3(t)$ an $\mathbf{i}_4(t)$ (see Fig. 6.5). These were obtained projecting $\mathbf{i}(t)$ onto $\mathbf{v}_3(t)$ and $\mathbf{v}_4(t)$. The projection was obtained using the inner product as the dot product.

It was noticed that $\mathbf{i}_3(t)$ is the compensating current component required to change the neutral current to its optimum value without changing the instantaneous power.

If now $\mathbf{I}_3(t)$ and $\mathbf{I}_4(t)$ are the projection of $\mathbf{i}(t)$ onto $\mathbf{v}_3(t)$ and $\mathbf{v}_4(t)$ respectively with the inner product, not simply the dot product, but defined as in Eq. 8.1 then:

$\mathbf{I}_3(t) = < \mathbf{i}(t) \bullet \mathbf{v}_3 (t)> \mathbf{v}_3 (t)/ \| \mathbf{V}_3 \|^2$

$\mathbf{I}_4(t) = < \mathbf{i}(t) \bullet \mathbf{v}_4 (t) > \mathbf{v}_4 (t) / \| \mathbf{V}_4 \|^2$

where:

$\|\mathbf{V}_3\|^2 = < \mathbf{v}_3(t), \mathbf{v}_3(t) > = 1/T \int \mathbf{v}_3(t) \bullet \mathbf{v}_3(t)\, dt$

$\|V_4\|^2 = < \mathbf{v}_3(t), \mathbf{v}_3(t) > = 1/T \int \mathbf{v}_3(t) \bullet \mathbf{v}_3(t)\, dt$

$\|\mathbf{I}_{c2}\|^2 = < \mathbf{I}_{c2}(t), \mathbf{I}_{c2}(t) >$

$\|\mathbf{I}_{c2}\|^2 = < \mathbf{i}_3(t) + \mathbf{i}_4(t), \mathbf{i}_3(t) + \mathbf{i}_4(t) >$

$\|\mathbf{I}_{c2}\|^2 = < \mathbf{i}_3(t) + \mathbf{i}_3(t)> + < \mathbf{i}_4(t)+\mathbf{i}_4(t)> + 2< \mathbf{i}_3(t). \mathbf{i}_4(t)>$

and since $\mathbf{i}_3(t)$ and $\mathbf{i}_4(t)$ are orthogonal

$$\|\mathbf{I}_{c2}\|^2 = \|\mathbf{I}_3\|^2 + \|\mathbf{I}_4\|^2 \qquad \text{Eq. 8.24}$$

where

$\|\mathbf{I}_3\|^2 = 1/T \int \mathbf{i}_3(t) \bullet \mathbf{i}_3(t)\, dt$ and $\|\mathbf{I}_4\|^2 = 1/T \int \mathbf{i}_4(t) \bullet \mathbf{i}_4(t)\, dt$

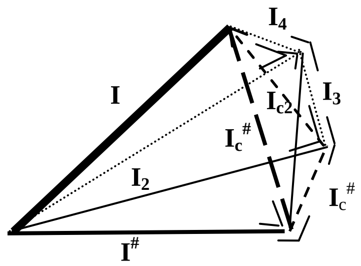

Fig. 8.7 Orthogonality of 4 rms domain current components

$\|\mathbf{I}\|^2 = \|\mathbf{I}^{\#}\|^2 + \|\mathbf{I}_c^{\#}\|^2 + \|\mathbf{I}_3\|^2 + \|\mathbf{I}_4\|^2$

$\|\mathbf{I}\|^2 = \|\mathbf{I}^{\#}\|^2 + (\|\mathbf{I}_c^{\#}\| + \|\mathbf{I}_3\| + \|\mathbf{I}_4\|)^2$

$\|\mathbf{I}\|^2 = (\|\mathbf{I}^{\#}\|^2 + \|\mathbf{I}_c^{\#}\|)^2 + (\|\mathbf{I}_3\| + \|\mathbf{I}_4\|)^2$

$\|\mathbf{I}\|^2 = (\|\mathbf{I}^{\#}\| + \|\mathbf{I}_c^{\#}\| + \|\mathbf{I}_3\|)^2 + \|\mathbf{I}_4\|^2$

Example 8.5

From Examples 8.1 and 8.2

$\mathbf{v}(t_1)=\{0, -3, 9, -2\}$ and $v(t_1) =\{12, -9, 2, -5\}$ for the first half $v(t_2)=\{0, -6, 18, -4\}$ and

$\mathbf{i}(t_2) =\{7, -3, 4, -8\}$ for the second half

$\mathbf{i}_{c2}(t_1)= \{12.6, -6.56, -2.88, -3.17\}$ $\quad \|\mathbf{i}_{c2}(t_1)\|^2 = 220.39$

$\mathbf{i}_{c2}(t_2)= \{7.68, -0{,}29, -1{,}42, -5.96\}$ $\quad \|\mathbf{i}_{c2}(t_2)\|^2 = 96.65$

$\|\mathbf{i}_{c2}\|^2 = (220.39 + 96.65)/2 = 317.04/2= 158.52$

Now:

$\mathbf{i}_{c3}(t_1) = (\mathbf{i}(t_1) \bullet \mathbf{v}_3(t))\, \mathbf{v}_3(t) / \|\mathbf{V}_3(t)\|^2$

$\mathbf{v}(t_1)' = \{0, e_a - e_z(t1)/3, e_b - e_z(t1)/3, e_c - e_z(t1)/3\} = \{0, -4.33, 7.67, -3.33\}$

$\|\mathbf{v}(t_1)'\|^2 = 88.67$ $\quad \|\mathbf{v}_2(t_1)\|^2 = 90$

$k_3(t_1) = \|\mathbf{v}(t_1)'\|^2 / \|\mathbf{V}_2(t_1)\|^2 = 88.67/90 = 0.985$

$\mathbf{v}_3(t_1) = \mathbf{v}'(t_1) - k_3(t_1)\, \mathbf{v}_2(t_1) = \{0, -4.33, 7.67, -3.33\} - 0.985\{-1, -4, 8, -3\}$

$\mathbf{v}_3(t_1) = \{0.985, -0.393, -0.215, -0.378\}$ $\quad \|\mathbf{v}_3(t_1)\|^2 = 1.31$

$\mathbf{i}_3(t_1) = (i\, t_1 \bullet v_3(t_1) \mathbf{v}_3(t_1) / \|\mathbf{v}_3(t_1)\|^2$

$\mathbf{i}_3(t_1) = \{12.6, -5.03, -2.75, -4.84\}$ $\quad \| i_3 t_1 \|2 = 215.24$

$\mathbf{i}_4(t_1) = i(t_1) - i_2(t_1) - i_3(t_1)$

$\mathbf{i}_4(t_1) = \{0, -1.53, -0.139, 1.67\}$ $\quad \| i_4 t_1 \|^2 = 5.15$

Similarly

$\| \mathbf{i}_3(t_2) \|^2 = 79.78$ $\quad \| \mathbf{i}_4(t_2) \|^2 = 16.67$

The average of the square over period T is:

$\| \mathbf{I}_3 \|^2 = (\| \mathbf{i}_3(t_1) \|^2 + \| \mathbf{i}_3(t_2) \|^2)/2 = (215.24+ 79.78)/2 = 147.51$

$\| \mathbf{I}_4 \|^2 = (\| \mathbf{i}_4(t_1) \|^2 + \| \mathbf{i}_4(t_2) \|^2)/2 = (5.15 + 16.68)/2 = 11$

$\| \mathbf{I}_2 \|^2 + \| \mathbf{I}_3 \|^2 + \| \mathbf{I}_4 \|^2 = 37.8 + 147.51 + 11 = 196 = \|I\|^2$

and

$\| \mathbf{I}_3 \|^2 + \| \mathbf{I}_4 \|^2 = 158.51 = \| \mathbf{I}_{c2} \|^2$ see Example 8.1

$\mathbf{I}_3(t)$ and $\mathbf{I}_4(t)$ are two components that can both be compensated without energy storage. The physical interpretation of $\mathbf{I}_3(t)$ appears to be related to that of the compensating current required to render the neutral current optimum without changing the average power P. This observation is not pursued any further and left for future work.

8.5 Conclusions

The load currents can be considered to be the sum of three orthogonal current components:

1) $\mathbf{i}^{\#}(t)$ which carries all the average real power
2) $\mathbf{i}_{c2}(t)$ which does not carry instantaneous power and can be compensated without energy storage. This power could therefore be considered as power that oscillates between the phases (wires), since it is not being stored at any stage.
3) $\mathbf{i}_c^{\#}(t)$ which does not contribute to the delivery of average power and can only be compensated with energy storage elements. This current component could be considered as power that oscillates between the supply and the load.

All three current components are orthogonal to each other. $\mathbf{i}^{\#}(t)$ cannot be reduced by any further compensation as $\mathbf{i}^{\#}(t)$ is the projection of $\mathbf{i}(t)$ on $\mathbf{v}_2(t)$ and is the only component that carries average power and has therefore the minimum possible norm and therefore minimum transmission losses if all wires are considered to have equal resistance.

The decomposition based on the nature of the energy storage requirement has a physical meaning associated with each of the current or power components. The decomposition based on energy storage is of particular interest for active compensator design and, in particular, if active and passive compensators are to be used in the most economical combination. This will become even more relevant as the cost of semiconductor compensators (without capacitors or inductances) become increasingly competitive.

Further decomposition is possible and it was shown that, for example, the component $i_{c2}(t)$ could be split into two more orthogonal components. One of these seems to be associated with the neutral wire current but the concept will not be pursued any further in this thesis.

The calculation of the compensation currents is quick, being limited to simple arithmetic processes, with obvious implications for real time practical implementation in the design and sizing of active.

The approach used here shows a linkage between instantaneous power theory and the rms domain. The potential therefore exists to demonstrate the physical representation of some of the components described by other theoreticians. For example, it would be interesting to investigate Czarnecki's unbalanced current components relative to $i_c^{\#}$ (t) in the case of symmetrical voltages. Further, the approach could allow the effect of unsymmetrical voltages (not considered by Czarnecki) to be distinguished from unsymmetrical loads. However such further investigation is left for further work.

Chapter 9

Practical examples and real time measurement of distorted supply.

The examples previously seen in the rms domain were used on simple arbitrary voltage and current values. In this chapter examples with sinusoidal voltages with some harmonic components are demonstrated. Four models of 3-phase, star-connected voltage supplies, consisting of various combinations of classical (Fortescue) fundamental sequence voltage components, illustrated in Fig. 9.1, are examined. One example includes a 5^{th} harmonic zero sequence component.

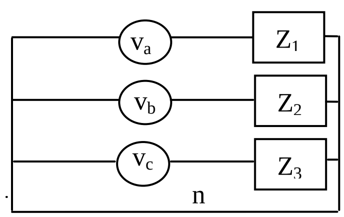

Fig. 9.1. 3-phase 4-wire supply with star load

9.1 Examples with linear non-time variant loads

Four types of linear time invariant loads are considered for each supply model. The four loads are:

1) Balanced resistive: $Z_1 = Z_2 = Z_3 = 1 + j0$
2) Unbalanced resistive: $Z_1 = 1 + j0, \quad Z_2 = Z_3 = \infty$
3) Balanced complex: $Z_1 = Z_2 = Z_3 = 0.8 + j0.6$
4) Unbalanced complex: $Z_1 = 0.8 + j0.6, \ Z_2 = Z_3 = \infty$

The supply voltages $v_a(t)$, $v_b(t)$, $v_c(t)$ are a linear combination of positive, negative and zero sequence components and fifth harmonic zero sequence component, measured from the reference of the neutral wire. The line voltages sequence components for line a, b, c are a combination of

1) Positive sequence line voltages:
   $v_p(t)$, $v_p(t+T/3)$, $v_p(t-T/3)$
   where: $v_p(t) = \sqrt{2}\, V_p \sin(\omega t)$ and $V_p = 100V$
2) Negative sequence line voltages:
   $v_n(t)$, $v_n(t-T/3)$, $v_n(t+T/3)$
   where: $v_n(t) = \sqrt{2}\, V_n \sin(\omega t)$ and $V_n = 10V$
3) Zero sequence line voltages:
   $v_{0-1}(t)$, $v_{0-1}(t)$, $v_{0-1}(t)$
   where: $v_{0-1}(t) = \sqrt{2}\, V_{0-1} \sin(\omega t)$ and $V_{0-1} = 10V$
4) 5^{th} harmonic zero sequence line voltages:
   $v_{0-5}(5\omega t)$, $v_{0-5}(5\omega t)$, $v_{0-5}(5\omega t)$
   where : $v_{0-5}(t) = \sqrt{2}\, V_{0-5} \sin(5\omega t)$ and $V_{0-5} = 10V$

The supply voltage components are combined in the following ways to create the supply models.

Model 1: Balanced supply

Applying the approach to the balanced supply:

$\mathbf{V}(t) = \{0, v_p(t), v_p(t + T/3), v_p(t - T/3)\}$

From Eq. 6.1

$e_z(t) = (v_p(t) + v_p(t + T/3) + v_p(t-T/3))/4 = 0$

$\mathbf{e_z}(t) = \{e_z(t)/4, e_z(t)/4, e_z(t)/4, e_z(t)/4\} = \mathbf{0}$

hence from Eq 6.4

$\mathbf{V}_2(t) = \mathbf{V}(t) - \mathbf{e}_z(t) = \mathbf{V}(t)$

$||\mathbf{V}_2||^2 = (1/T) \int \mathbf{V}_2(t) \bullet \mathbf{V}_2(t)$

$= (1/T) \int (v_p(t)^2 + v_p(t+T/3)^2 + v_p(t-T/3)^2)$

$= 3\, V_p^2 = 3 \times 100^2 = 30000$

Model 2: Unbalanced supply with positive and zero sequence fundamental voltages only

$v_a(t) = v_p(t) + v_{0-1}(t)$,

$v_b(t) = v_p(t+T/3) + v_{0-1}(t)$

$v_c(t) = v_p(t\text{-}T/3) + v_{0\text{-}1}(t)$

or in vector form $\mathbf{V}(t) = \mathbf{V}_p(t) + \mathbf{V}_{0\text{-}1}(t)$

$e_z(t) = v_a(t) + v_b(t) + v_c(t)$

$= (v_p(t) + v_{0\text{-}1}(t) + v_p(t+T/3) + v_{0\text{-}1}(t) + v_p(t\text{-}T/3) + v_{0\text{-}1}(t))/4$

$= ¾\, v_0(t)$

Then from Eq. 6.4

$\mathbf{V}_2(t) = \mathbf{V}_p(t) - \{¾\, v_{0\text{-}1}(t), ¾\, v_{0\text{-}1}(t), ¾\, v_{\text{-}1}(t), ¾\, v_{0\text{-}1}(t)\}$

$=\{-¾\, v_{0\text{-}1}(t), v_p(t) + ¼\, v_{0\text{-}1}(t), v_p(t+T/3) + ¼\, v_{0\text{-}1}(t), v_p(t\text{-}T/3) + ¼\, v_{0\text{-}1}(t)\}$

$\|\mathbf{V}_2(t)\|^2 = \mathbf{V}_2(t) \bullet \mathbf{V}_2(t) = 9/16 v_{0\text{-}1}{}^2(t) + 3V_p{}^2 + 3/16\, v_{0\text{-}1}{}^2(t)$

$= 3V_p{}^2 + 3/4\, v_{0\text{-}1}{}^2(t) = 3V_p{}^2 + 3/2 V_{0\text{-}1}{}^2 \sin^2(\omega t)$

$= (1/T) \int \| \mathbf{V_2}(t) \|^2 dt = 3V_p{}^2 + 3/4 V_{0\text{-}1}{}^2$

With $V_p = 100$ and $V_{0\text{-}1} = 10$

$\|\mathbf{V}_2(t)\|^2 = 30000 + (1/T) \int 150 \sin^2(\omega t)$

$\|\mathbf{V}_2\| = 30000 + 75 = 30075$

Model 3: Supply with positive and negative sequence voltages

Three line voltages measured from the neutral are:

$v_a(t) = v_p(t) + v_n(t)$,

$v_b(t) = v_p(t+T/3) + v_n(t\text{-}T/3)$,

$v_c(t) = v_p(t\text{-}T/3) + v_n(t+T/3)$ where

or in vector form: $\mathbf{V}(t) = \mathbf{V}_+(t) + \mathbf{V}_n(t)$

$\mathbf{V}(t) = \{0, v_a(t), v_b(t), v_c(t)\}$

Hence from Eq. 6.4

$\mathbf{V}_2(t) = \mathbf{V}(t) - \mathbf{e}_z(t)$ but $e_z(t) = 0$

$\|\mathbf{V}_2\|^2 = (1/T) \int \mathbf{V}(t) \bullet \mathbf{V}(t) = 3V_p{}^2 + 3V_0{}^2 = 30300$

Model 4: Supply with positive fundamental sequence voltages and 5th harmonic zero sequence voltage

Including a 5th zero sequence harmonic voltage with a fundamental positive sequence gives three line voltages measured from the neutral as:

$v_a(t) = v_p(t) + v_{0\text{-}5}(5\omega t)$, $v_b(t) = v_p(t+T/3) + v_{0\text{-}5}(5\omega t)$, $v_c(t) = v_p(t-T/3) + v_{0\text{-}5}(5\omega t)$

Letting $V_p = 100$ $V_{0\text{-}5} = 10$ then

$\|\mathbf{V}_2\|^2 = (1/T)\int \|\mathbf{V}_2(t)\|^2 = 3V_p + 3V_{0\text{-}5}$

$= 3x100^2 + 3x\ 10^2 = 30300$

Table 9.1: Balanced supply	Balanced resistive load	Balance complex load	Unbalanced resistive load	Unbalanced complex load
Balanced 3 phase supply $V_p = 100$ $V_n = 0$ $V_0 = 0$ $\|\mathbf{V}\|^2 = \|\mathbf{V}_2\|^2 = V_a^2 + V_b^2 + V_c^2 = 30000$	$Z_a = (1 + j0)$ $Z_a = (1 + j0)$ $Z_a = (1 + j0)$	$Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$	$Z_a = (1 + j0)$ $Z_a = (\infty + j\infty)$ $Z_a = (\infty + j\infty)$	$Z_a = (0.8 + j0.6)$ $Z_a = (\infty + j\infty)$ $Z_a = (\infty + j\infty)$
$\mathbf{I} = \{I_n, I_a, I_b, I_c\} = \{I_n, V_a/Z_a, V_b/Z_b, V_c/Z_c\}$	{0 , 100, 100, 100}	{0 , 100, 100, 100}	{100 , 100, 0, 0}	{100 , 100, 0, 0}
$\|I\|^2 = I_n^2 + I_a^2 + I_b^2 + I_c^2$	30000	30000	20000	20000
$\|I_2\|^2 = 1/T\int p^2(t)/\|\mathbf{V}_2(t)\|^2$ ……….. **Eq. 5.9**	30000	19200	5000	3800
$\|\mathbf{I}_{c2}\|^2 = \|\mathbf{I}\|^2 - \|\mathbf{I}_2\|^2$…………………..… **Eq. 8.2**	0	10800	15000	16200
$\|\mathbf{I}^{\#}\|^2 = P^2/\|V_2\|^2$ ………….………. **Eq. 8.5**	30000	19200	3333	2133
$\|\mathbf{I}_c^{\#}\|^2 = \|\mathbf{I}_2\|^2 - \|\mathbf{I}^{\#}\|^2$…………………. **Eq. 8.6**	0	0	1667	1667
$\|\mathbf{I}_c\|^2 = \|\mathbf{I}_{c2}\|^2 + \|\mathbf{I}_c^{\#}\|^2 = \|\mathbf{I}\|^2 - \|\mathbf{I}^{\#}\|^2$…. **Eq. 8.9**	0	10800	16667	17867
$\mathbf{S} = \|\mathbf{V}_2\|\,\|\mathbf{I}\|$…………….………… **Eq. 8.11**	30000	30000	24495	24995
$\mathbf{P} = 1/T\int p(t)dt = 1/T\int v_2(t)\, i(t)\, dt$ =	30000	24000	10000	8000
$\lambda = P/S$ …………………………….. **Eq. 8.12** **Power factor before compensation**	**1**	**0.8**	**0.408**	**0.327**
$\mathbf{Q} = (S^2 - P^2)^{1/2}$……………………… **Eq. 8.14**	0	19200	22361	23151
$S_2 = \|\mathbf{I}_2\|\,\|\mathbf{V}_2\|$ ……………………… **Eq. 8.21**	30000	24000	12247	10677
$\lambda_2 = P/S_2$ ..……………………………. **Eq. 8.20** **Power factor compensated without energy storage**	**1**	**1**	**0.816**	**0.749**
$\mathbf{Q}_2 = \|\mathbf{I}_{c2}\|\,\|\mathbf{V}_2\|$…………………….. .. **Eq. 8.18**	0	19200	21213	22045
$\mathbf{Q}^{\#} = \|\mathbf{I}_c^{\#}\|\,\|\mathbf{V}_2\|$ ……………………… **Eq. 8.17**	0	0	7071	7071
$\lambda^1 = P/\|V_2\|\,\|\mathbf{I}^{\#}\|$ ……………………. **Eq. 8.23** **Power factor fully compensated**	**1**	**1**	**1**	**1**

Table 9.2: Unbalanced supply with positive and zero sequence fundamental voltages only	Balanced resistive load	Balance complex load	Unbalanced resistive load	Unbalanced complex load
Unbalance 3 phase voltage $V_p = 100$ $V_n = 0$ $V_0 = 10$ $\|\|V''\|\|^2 = V_a^2 + V_b^2 + V_c^2 = 30075$	$Z_a = (1 + j0)$ $Z_a = (1 + j0)$ $Z_a = (1 + j0)$	$Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$	$Z_a = (1 + j0)$ $Z_a = (\infty + j\,\infty)$ $Z_a = (\infty + j\,\infty)$	$Z_a = (0.8 + j0.6)$ $Z_a = (\infty + j\,\infty)$ $Z_a = (\infty + j\,\infty)$
$I = \{I_n, I_a, I_b, I_c\} = \{I_n, V_a/Z_a, V_b/Z_b, V_c/Z_c\}$	{30,110,95.4,95.4}	{30,110,95.4,95.4}	{110 , 0, 0, 110}	{110 , 0, 0,110}
$\|\|I\|\|^2 = I_n^2 + I_a^2 + I_b^2 + I_c^2$	31200	31200	24200	24200
$\|\|I_2\|\|^2 = 1/T \int p^2(t) / \|\|V_2(t)\|\|^2$ ……….. **Eq. 5.9**	30527	19538	7290	5542
$\|\|I_{c2}\|\|^2 = \|\|I\|\|^2 - \|\|I_2\|\|^2$……………………..… **Eq. 8.2**	672	11662	16910	18658
$\|\|I^{\#}\|\|^2 = P^2 / \|\|V_2\|\|^2$ …………..………. **Eq. 8.5**	30527	19537	4868	3116
$\|\|I_c^{\#}\|\|^2 = \|\|I_2\|\|^2 - \|\|I^{\#}\|\|^2$…………………. **Eq. 8.6**	0.84	1.1	2422	2426
$\|\|I_c\|\|^2 = \|\|I_{c2}\|\|^2 + \|\|I_c^{\#}\|\|^2 = \|\|I\|\|^2 - \|\|I^{\#}\|\|^2$…. **Eq. 8.9**	673	11663	19332	21084
$S = \|\|V_2\|\| \, \|\|I\|\|$……………….………… **Eq. 8.11**	30632	30632	26978	26978
$P = 1/T \int p(t)dt = R_a I_a^2 + R_b I_b^2 + R_c I_c^2$	30300	24240	12100	9680
$\lambda = P/S$ …………………………….. **Eq. 8.12** **Power factor before compensation**	**0.989**	**0.791**	**0.449**	**0.359**
$Q = (S^2 - P^2)^{1/2}$……………………… **Eq. 8.14**	4500	18729	24112	25182
$S_2 = \|\|I_2\|\| \, \|\|V_2\|\|$ ……………………….. **Eq. 8.21**	30300	24241	14807	12910
$\lambda_2 = P/S_2$ ..…………………………. . **Eq. 8.20** **Power factor compensated without energy storage**	**0.9999**	**0.9997**	**0.817**	**0.750**
$Q_2 = \|\|I_{c2}\|\| \, \|\|V_2\|\|$…………………….. .. **Eq. 8.18**	4497	18729	22551	23688
$Q^{\#} = \|\|I_c^{\#}\|\| \, \|\|V_2\|\|$ ……………………… **Eq. 8.17**	158	189.8	8535	8542
$\lambda 1 = P / \|\|V_2\|\| \, \|\|I^{\#}\|\|$ ……………………. **Eq. 8.23** **Power factor fully compensated**	**1**	**1**	**1**	**1**

Table 9.3: Supply with positive and negative sequence voltages.	Balanced resistive load	Balance complex load	Unbalanced resistive load	Unbalanced complex load
Unbalance 3 phase voltage Positive and Negative seq voltages $V_p = 100$ $V_n = 10$ $V_0 = 0$ $\|\|V''\|\|^2 = V_a^2 + V_b^2 + V_c^2 = 30300$	$Z_a = (1 + j0)$ $Z_a = (1 + j0)$ $Z_a = (1 + j0)$	$Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$	$Z_a = (1 + j0)$ $Z_a = (\infty + j\infty)$ $Z_a = (\infty + j\infty)$	$Z_a = (0.8 + j0.6)$ $Z_a = (\infty + j\infty)$ $Z_a = (\infty + j\infty)$
$I = \{I_n, I_a, I_b, I_c\} = \{I_n, V_a/Z_a, V_b/Z_b, V_c/Z_c\}$	{0,110,95.5,95.4 }	{0,110,95.5, 95.4 }	{110,110, 0, 0 }	{110, 110, 0, 0 }
$\|\|I\|\|^2 = I_n^2 + I_a^2 + I_b^2 + I_c^2$	30300	30300	24200	24200
$\|\|I_2\|\|^2 = 1/T \int p^2(t) / \|\|V_2(t)\|\|^2$ ……….. **Eq. 5.9**	30300	19608	6433	4995
$\|\|I_{c2}\|\|^2 = \|\|I\|\|^2 - \|\|I_2\|\|^2$……………………. **Eq. 8.2**	0	10692	17767	19204
$\|\|I^{\#}\|\|^2 = P^2 / \|\|V_2\|\|^2$ …………..………. **Eq. 8.5**	30300	19392	4832	3092
$\|\|I_c^{\#}\|\|^2 = \|\|I_2\|\|^2 - \|\|I^{\#}\|\|^2$…………………. **Eq. 8.6**	0	216	1601	1903
$\|\|I_c\|\|^2 = \|\|I_{c2}\|\|^2 + \|\|I_c^{\#}\|\|^2 = \|\|I\|\|^2 - \|\|I^{\#}\|\|^2$…. **Eq. 8.9**	0	10908	19368	21108
$S = \|\|V_2\|\| \, \|\|I\|\|$…………………………. **Eq. 8.11**	30300	30300	27078	27078
$P = 1/T \int p(t)dt = 1/T \int v_2(t)\, i(t)\, dt$ =	30300	24240	12100	9690
$\lambda = P/S$ …………………………….. **Eq. 8.12** **Power factor before compensation**	**1**	**0.8**	**0.447**	**0.357**
$Q = (S^2 - P^2)^{1/2}$……………………… **Eq. 8.14**	0	18180	24225	25289
$S_2 = \|\|I_2\|\| \, \|\|V_2\|\|$ ………………………. **Eq. 8.21**	30300	24375	13961	12303
$\lambda_2 = P/S_2$ ..……………………………. **Eq. 8.20** **Power factor compensated without energy storage**	**1**	**0.995**	**0.867**	**0.787**
$Q_2 = \|\|I_{c2}\|\| \, \|\|V_2\|\|$…………………….. .. **Eq. 8.18**	0	1800	23202	24122
$Q^{\#} = \|\|I_c^{\#}\|\| \, \|\|V_2\|\|$ ……………………… **Eq. 8.17**	0	2558	6965	7594
$\lambda^1 = P / \|\|V_2\|\| \, \|\|I^{\#}\|\|$ ……………………. **Eq. 8.23** **Power factor fully compensated**	**1**	**1**	**1**	**1**

Table 9.4: Supply with positive fundamental and 5th harmonic zero sequence voltages.	Balanced resistive load	Balance complex load	Unbalanced resistive load	Unbalanced complex load
Unbalance 3 phase voltage Positive and Negative seq voltages $V_p = 100$ $V_n = 0$ $V_0 = 0$ $V_{50} = 10$ $\|\|V''\|\|^2 = V_a^2 + V_b^2 + V_c^2 = 30075$	$Z_a = (1 + j0)$ $Z_a = (1 + j0)$ $Z_a = (1 + j0)$	$Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$ $Z_a = (0.8 + j0.6)$	$Z_a = (1 + j0)$ $Z_a = (\infty + j\infty)$ $Z_a = (\infty + j\infty)$	
$I = \{I_n, I_a, I_b, I_c\} = \{I_n, V_a/Z_a, V_b/Z_b, V_c/Z_c\}$	{30,100.5,100.5,100.5}	{9.66,100.1, 100.1,100.1}	{100.5,100.50, 0, 0 }	{100.05, 100.05, 0, 0}
$\|\|I\|\|^2 = I_n^2 + I_a^2 + I_b^2 + I_c^2$	31200	10125	20200	20124
$\|\|I_2\|\|^2 = 1/T \int p^2(t) / \|\|V_2(t)\|\|^2$ ……….. **Eq 5.9**	30527	19192	5183	3837
$\|\|I_{c2}\|\|^2 = \|\|I\|\|^2 - \|\|I_2\|\|^2$……………………. **Eq 8.2**	672	10932	15012	16287
$\|\|I^{\#}\|\|^2 = P^2 / \|\|V_2\|\|^2$ ……………………. **Eq 8.5**	30527	19192	3392	2141
$\|\|I_c^{\#}\|\|^2 = \|\|I_2\|\|^2 - \|\|I^{\#}\|\|^2$…………………. **Eq 8.6**	0.84	0.17	1795	1696
$\|\|I_c\|\|^2 = \|\|I_{c2}\|\|^2 + \|\|I_c^{\#}\|\|^2 = \|\|I\|\|^2 - \|\|I^{\#}\|\|^2$…. **Eq 8.9**	673	10932.61	16808	17983
$S = \|\|V_2\|\| \, \|\|I\|\|$…………………………. **Eq 8.11**	30632	10100	24648	24501
$P = 1/T \int p(t)dt = 1/T \int v_2(t)\, i(t)\, dt$ =	30300	24025	10100	8025
$\lambda = P/S$ …………………………….. **Eq 8.12** **Power factor before compensation**	**0.989**	**0.798**	**0.410**	**0.326**
$Q = (S^2 - P^2)^{1/2}$……………………… **Eq 8.14**	4500	18133	22483	23256
$S_2 = \|\|I_2\|\| \, \|\|V_2\|\|$ ……………………… **Eq 8.21**	30300	24025	12490	10743
$\lambda_2 = P/S_2$ ……………………………. **Eq 8.20** **Power factor compensated without energy storage**	**0.9999**	**0.999**	**0.8087**	**0.747**
$Q_2 = \|\|I_{c2}\|\| \, \|\|V_2\|\|$…………………….. .. **Eq 8.18**	4497	18133	21249	22132
$Q^{\#} = \|\|I_c^{\#}\|\| \, \|\|V_2\|\|$ ……………………… **Eq 8.17**	158	70.51	7348	7142
$\lambda^1 = P / \|\|V_2\|\| \, \|\|I^{\#}\|\|$ …………………… **Eq 8.23** **Power factor fully compensated**	**1**	**1**	**1**	**1**

The values for each model have been calculated using the equations developed in the above analysis. The calculations require only simple arithmetic and were carried out in a spreadsheet. The following observations are evident and are all in agreement with Chapter 8.

- Unity power factor ($\lambda_2 = 1$) can be obtained without energy storage in the cases where the supply and the load are balanced. The load can be of a complex nature (reactive).
- The power factor can generally be improved substantially without energy storage.
- All current components and power components are indeed orthogonal as the sum of the squares is equal to the original current and apparent power respectively.

- Power factor measurement with voltages measured from a common point (the neutral in this case) can give results which are not in accord with Emanuel's definition of AP.
- Unity power factor can be obtained in all cases if energy storage elements are used.

The following chapter illustrates the approach required to design an instrument to measure apparent factor and power factor as described in the previous chapters. This instrument is referred as a true power factor meter (TPM).

Chapter 10

True power factor meter for 4-wire system.

This chapter describes how an instrument based on the theory presented can be implemented in practice using a computer or a digital signal processor (DSP) capable of processing in real-time measured instantaneous values of currents and voltages. The true power factor meter (TPM) displays power, apparent power and power factor, under the various conditions described in the previous chapters. The TPM is also capable of providing the necessary compensating signals required by an active compensator to compensate the line currents. Two sets of signal output are provided, one for a compensator without and one with energy storage.

A prototype TPM was assembled and built around the Texas Instrument DSP TMS320F240 and used to measure power factor on unbalanced and distorted 3-phase supplies, comparing results with conventional meters. Due to the difficulty of obtaining accurate results with such laboratory prototype a more reliable laboratory proof-of-concept demonstration was carried out by taking voltage and current samples with a conventional (commercial) 3-phase pf meter and carrying out the calculations on sampled signals using a computer which avoids the risk of error in construction and calibration of the physical TPM.

10.1 True power factor meter block diagram description

Fig. 10.1 describes in block diagram form the TPM. The meter input consists of up to four currents and voltages derived from the lines supplying the load (see Fig. 10.2).

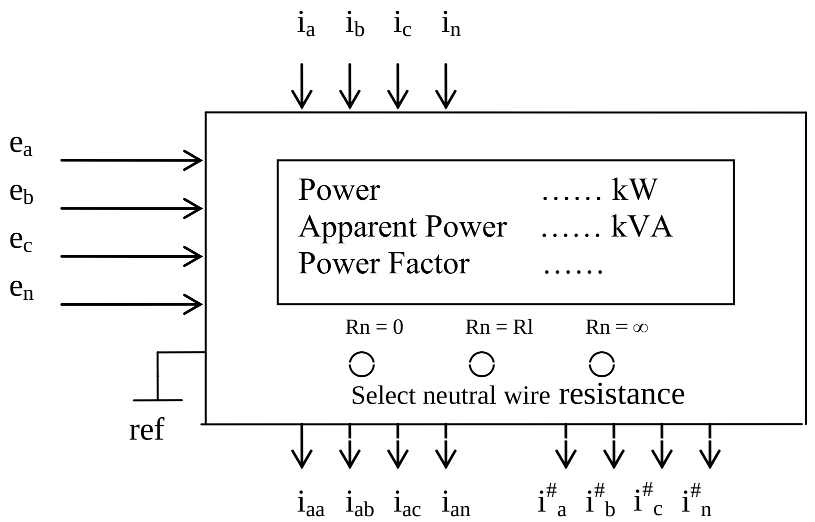

Fig. 10.1 The TPM with input and output signals, display and selection buttons.

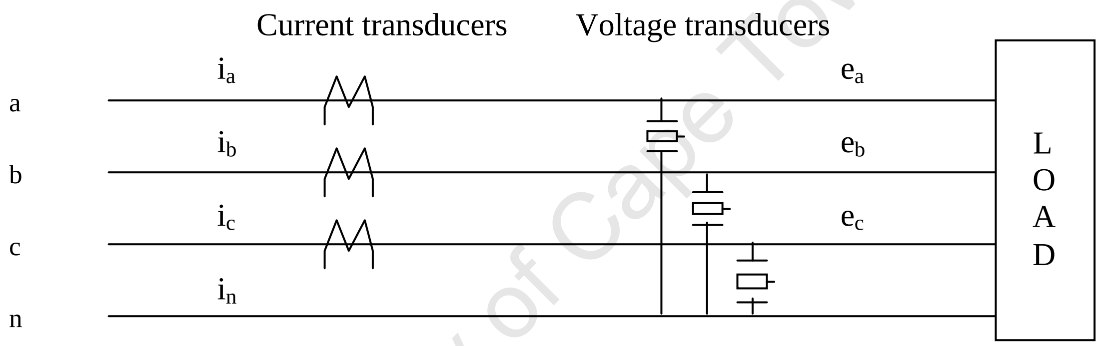

Fig. 10.2 Connection diagram for the TPM.

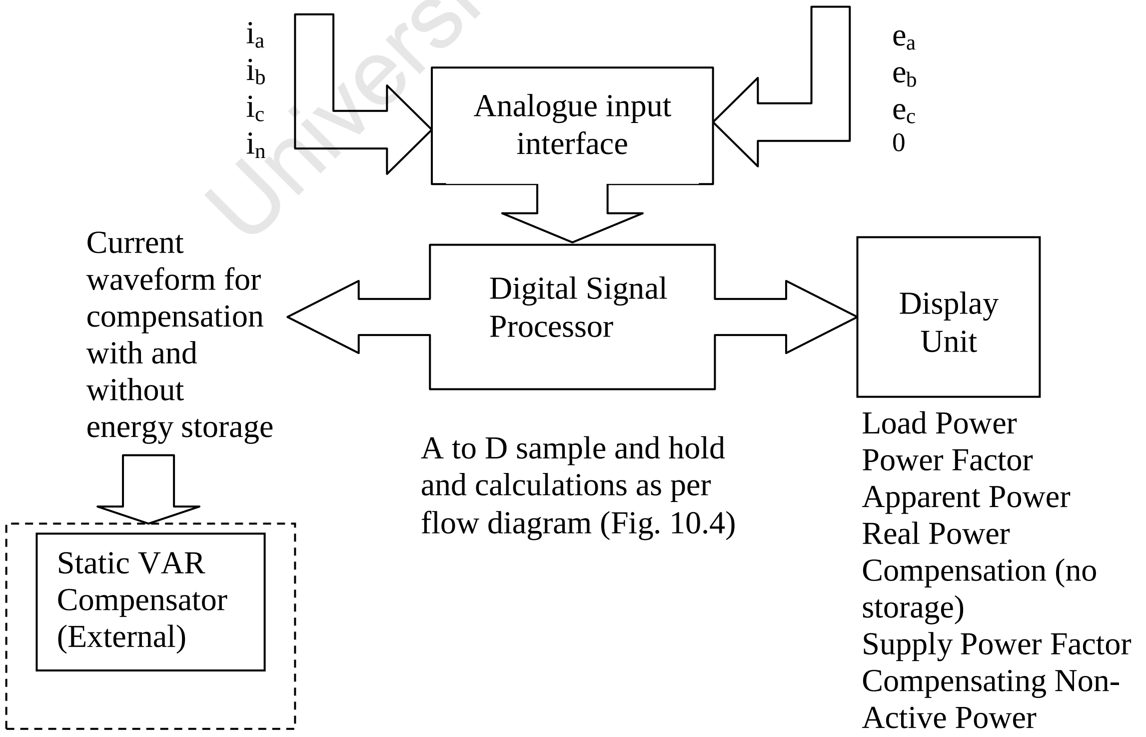

Fig. 10.3 Processor block diagram for the TPM.

Fig. 10.3 illustrates how the instantaneous values of line currents and voltages are processed by a DSP or computer. The user of the instrument can select three conditions depending on how the neutral wire is treated. The instrument output is calculated according to the value attributed to

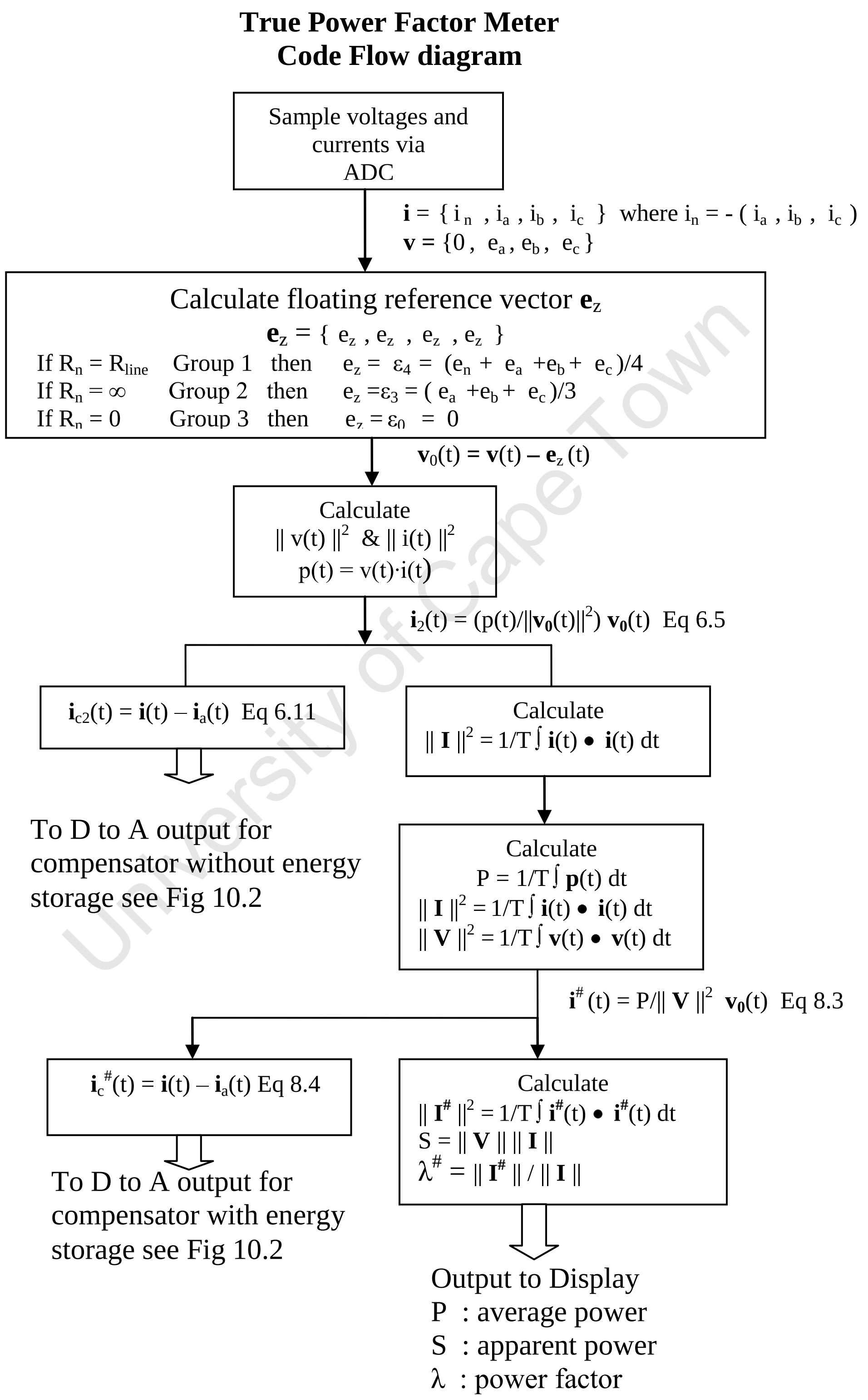

Fig. 10.4 Processor flow chart for the TPM.

the neutral wire. Fig. 4.4 illustrates the TPM calculation flow chart.

10.2 Comparison of experimental laboratory measurement and methods of calculation

Four different experiments were undertaken, namely:

1) Balanced supply[1] and balanced resistive[2] loads
2) Balanced supply and balanced reactive loads
3) Balanced supply and unbalanced resistive loads
4) Positive sequence supply with zero sequence voltage and balanced resistive load. Zero sequence was introduced by connecting a single phase voltage between the star point of the load and the neutral wire. A step down single phase isolation transformer was used for this purpose.

Readings were taken with a Yokogawa WT1600 Digital Power Meter connected as three-phase, four-wire system (3-P 4-W) with equal wire resistances as per Fig. 10.2. The Yokogawa readings[3] for each experiment can be seen in Tables 10.1 to 10.2.

For each of the experiments a set of 1002 instantaneous values of voltages and 1002 instantaneous values of current recorded by the Yokogawa power meter over four cycles were transferred to a PC. The instantaneous values of voltages and currents were then used and the AP and power factor were calculated using four different methods. The four calculation methods presented are compared with the readings from the meter. The results for each of the 4 experiments with equal 4-wire resistances are tabulated. Calculation with zero wire neutral resistances and ∞ were also included to illustrate the change in results if the voltage reference is changed accordingly, given the same measurement.

Method 1 (This method is given in definition 2 of Chapter 1)

$I_1 = (\text{sigma } i_1^2)^{1/2}$, $I_2 = (\text{ sigma } i_2^2)^{1/2}$, $I_3 = (\text{ sigma } i_3^2)^{1/2}$,

where sigma x is $\sum x_s /1002$ for s = 1 to 1002 sampled values considered.

[1] Approximate balance and sinusoidal supply from a three-phase variac connected to UCT machine lab supply
[2] 1 kW heating elements
[3] According to the Yokogawa manual, power is measured as per standard IEC76-1(1976). AP is the sum of the 3 separately measured products of rms voltages and currents.

$V_1 = (\text{sigma } v_1^2)^{1/2}$,

$P_1 = \text{sigma } (v_1 * i_1)$, $P_2 = \text{sigma } (v_2 * i_2)$, $P_3 = \text{sigma } (v_3 * i_3)$

$P = P_1 + P_2 + P_3$

$S_1 = I_1 * V_1$, $S_2 = I_2 * V_2$, $S_3 = I_3 * V_3$

$S = S_1 + S_2 + S_3$

$\lambda = P/S$

Method 2 (Corresponding to Group1 $r_n = r_1 = r_2 = r_3$).

$\| I^{\#} \| = \| I_4 \| = [(\text{sigma } i_1^2 + \text{sigma } i_2^2 + \text{sigma } i_3^2 + \text{sigma } i_n^2) / (1002)]^{1/2}$

where $i_n = - (i_1 + i_2 + i_3)$

$\|V_{\varepsilon 4}\| = [(\text{sigma } v_{1\varepsilon 4}^2 + \text{sigma } v_{2\varepsilon 4}^2 + \text{sigma } v_{3\varepsilon 4}^2 + \text{sigma } v_{n\varepsilon 4}^2) / (1002)]^{1/2}$

where $v_{1\varepsilon 4} = v_1 - e_z /4$, $v_{2\varepsilon 4} = v_2 - e_z /4$, ${}_{3\varepsilon 4} = v_3 - e_z /4$, $v_{n\varepsilon 4} = 0 - e_z /4$

$P = \text{sigma } (v_{1\varepsilon 4} * i_1) + \text{sigma } (v_{2\varepsilon 4} * i_3) + \text{sigma } (v_{3\varepsilon 4} * i_3) + \text{sigma } (v_{n\varepsilon 4} * i_n)$

$S = \| I_4 \| \ \|V_{\varepsilon 4}\|$

$\lambda^{\#} = \lambda_{\varepsilon 4} = P/S$

Method 3 (Group 3 $r_n = 0$)

$\| I \| = [(\text{sigma } i_1^2 + \text{sigma } i_2^2 + \text{sigma } i_3^2) / (1002)]^{1/2}$

$\|V\| = [(\text{sigma } v_1^2 + \text{sigma } v_2^2 + \text{sigma } v_3^2) / (1002)]^{1/2}$

$P = [\text{sigma } (v_1 * i_1) + \text{sigma } (v_2 * i_3) + \text{sigma } (v_3 * i_3)] / 1002$

$S = \| I \| \ \|V\|$

$\lambda = P/S$

Method 4 (Group 2 $r_n = \infty$).

$\| I_3 \| = [(\text{sigma } i_1^2 + \text{sigma } i_2^2 + \text{sigma } i_3^2) / (1002)]^{1/2}$

$\|V_3\| = [(\text{sigma } v_{1\varepsilon 3}^2 + \text{sigma } v_{2\varepsilon 3}^2 + \text{sigma } v_{3\varepsilon 3}^2 + \text{sigma } v_{n\varepsilon 3}^2) / (1002)]^{1/2}$

where $v_{1\varepsilon 3} = v_1 - e_z /3$, $v_{2\varepsilon 4} = v_2 - e_z /3$, $v_{3\varepsilon 3} = v_3 - e_z /3$, $v_{n\varepsilon 3} = 0 - e_z /3$

$P = [\text{sigma } (v_{1\varepsilon} * i_1) + \text{sigma } (v_{2\varepsilon 3} * i_3) + \text{sigma } (v_{3\varepsilon 3} * i_3) + \text{sigma } (v_{n\varepsilon 3} * i_n)] /1002$

$S = \| I_3 \| \ \|V_{\varepsilon 3}\|$

$\lambda_{\varepsilon 3} = P/S$

Experiment 1 Balance supply with balance resistive load.

Table 10.1 Experiment 1 Yokogawa WT 1600 power meter readings

	Urms	Irms	S	P	λ
Phase 1	230.12	4.15	955.73	955.63	0.99989
Phase 2	232.2	4.29	995.96	995.9	0.99994
Phase 3	230.99	4.1	948.02	947.98	0.99995
3-phase	231.1	4.18	2899.71	2899.51	0.999931

Table 10.2 Experiment 1 Calculated values as per method 1.

	U	I	S	P	λ
Phase 1	230.2538	4.159628	957.7701	957.4492	0.999665
Phase 2	232.3601	4.288493	996.4745	994.9218	0.998442
Phase 3	230.899	4.105977	948.066	947.8976	0.999822
3-phase			2902.311	2900.269	0.999296

Table 10.3 Experiment 1 Calculated values as per method 2.
(Group 1 $R_n = R_1 = R_2 = R_3 = R_3$)

	$\|\| V_{\varepsilon 4} \|\|$	$\|\| I_{\varepsilon 4} \|\|$	$S_{\varepsilon 4}$	$P_{\varepsilon 4}$	$\lambda^{\#}$
3-phase	400.0223	7.282925	2913.333	2900.269	0.995516

Table 10.4 Experiment 1 Calculated values as per method 3.
(Group 2 R_n= 0)

	$\|\| V \|\|$	$\|\| I \|\|$	S	P	λ
3-phase	400.4028	7.249325	2916.103	2900.269	0.99457

Table 10.5 Experiment 1 Calculated values as per method 4.
(Group 3 R_n= ∞)

	$\|\| V_{\varepsilon 3} \|\|$	$\|\| I_{\varepsilon 3} \|\|$	$S_{\varepsilon 3}$	$P_{\varepsilon 3}$	$\lambda_{\varepsilon 3}$
3-phase	399.9567	7.282925	2900.05	2894.128	0.997958

Experiment 2 Balance supply with balanced reactive load

Table 10.6 Experiment 2 Yokogawa WT 1600 power meter readings

	Urms	Irms	S	P	λ
Phase 1	230.08	2.8125	647.11	451.35	0.697486
Phase 2	232.05	2.9624	687.43	479.58	0.697642
Phase 3	230.93	2.7546	636.11	436.02	0.685447
3-phase	231.02	2.8432	1970.65	1366.95	0.693654

Table 10.7 Experiment 2 Calculated values as per method 1.

	U	I	S	P	λ
Phase 1	230.1299	2.81679	648.2276	451.9814	0.697257
Phase 2	232.2113	2.966278	688.8032	480.4289	0.697484
Phase 3	230.9794	2.758778	637.2208	437.05	0.685869
3-phase			1974.252	1369.46	0.69366

Table 10.8 Experiment 2 Calculated values as per method 2.
(Group 1 $R_n = R_1 = R_2 = R_3 = R_3$)

Item	$\|\| V_{\varepsilon 4} \|\|$	$\|\| I_{\varepsilon 4} \|\|$	$S_{\varepsilon 4}$	$P_{\varepsilon 4}$	$\lambda^{\#}$
3 phase	400.022	4.990495	1996.308	1369.46	0.685997

Table 10.9 Experiment 2 Calculated values as per method 3.
(Group 2 R_n= 0)

Item	$\|\| V \|\|$	$\|\| I \|\|$	S	P	λ
3-phase	400.2915	4.990495	1997.653	1369.46	0.685535

Table 10.10 Experiment 2 Calculated values as per method 4.
(Group 3 $R_n = \infty$)

Item	$V_{\varepsilon 3}$	$I_{\varepsilon 3}$	$S_{\varepsilon 3}$	$P_{\varepsilon 3}$	$\lambda_{\varepsilon 3}$
3-phase	399.9566	4.933961	1973.37	1363.72	0.691061

Experiment 3 Balance supply with unbalanced resistive load

Table 10.11 Experiment 3 Yokogawa WT 1600 power meter readings

	Urms	Irms	S	P	λ
Phase 1	227.26	2.2336	507.62	507.13	0.999035
Phase 2	228.19	4.2214	963.29	963.06	0.999761
Phase 3	228.12	4.0536	924.71	924.6	0.999881
3-phase	227.86	3.5029	2395.62	2394.8	0.999658

Table 10.12 Experiment 3 Calculated values as per method 1.

	U	I	S	P	λ
Phase 1	227.2957	2.23776	964.9142	963.7896	0.998835
Phase 2	228.3885	4.224881	925.4856	925.2382	0.999733
Phase 3	228.1496	4.056486	2399.033	2396.713	0.999033
3-phase	227.2957	2.23776	508.6334	507.6847	0.998135

Table 10.13 Experiment 3 Calculated values as per method 2.
(Group 1 $R_n = R_1 = R_2 = R_3 = R_3$)

	$\|\| V_{\varepsilon 4} \|\|$	$\|\| I_{\varepsilon 4} \|\|$	$S_{\varepsilon 4}$	$P_{\varepsilon 4}$	$\lambda^{\#}$
3-phase	394.5195	6.581822	2596.657	2396.713	0.922999

Table 10.14 (Group 2 R_n= 0)	Experiment 3 Calculated values as per method 3.				
	II V II	II I II	S	P	λ
3-phase	394.8125	6.581822	2598.586	2396.713	0.922314

Table 10.15 (Group 3 R_n= ∞)	Experiment 3 Calculated values as per method 4.				
Item	$V_{\varepsilon 3}$	$I_{\varepsilon 3}$	$S_{\varepsilon 3}$	$P_{\varepsilon 3}$	$\lambda_{\varepsilon 3}$
3-phase	394.4544	6.26995	2473.209	2391.853	0.967105

Experiment 4 Positive sequence supply with zero sequence voltage with balanced resistive load.

Table 10.16	Experiment 4	Yokogawa WT 1600 power meter readings			
	Urms	Irms	S	P	λ
Phase 1	208	3.85	800	800	1
Phase 2	209	3.97	830	830	1
Phase 3	289	5.16	1490	1490	1
3-phase	235	4.33	3120	3120	1

Table 10.17	Experiment 4 Calculated values as per method 1.				
	Urms	Irms	S	P	λ
Phase 1	207.5823	3.860129	830.8086	831.466	1.000791
Phase 2	209.0904	3.973442	1491.327	1491.131	0.999869
Phase 3	288.7379	5.164986	3123.43	3123.635	1.000066
3-phase	207.5823	3.860129	801.2944	801.0378	0.99968

Table 10.18 (Group 1 $R_n = R_1 = R_2 = R_3 = R_3$)	Experiment 4 Calculated values as per method 2.				
	II $V_{\varepsilon 4}$ II	II $I_{\varepsilon 4}$ II	$S_{\varepsilon 4}$	$P_{\varepsilon 4}$	$\lambda^{\#}$
3-phase	403.1661	8.176312	3296.412	3123.635	0.947587

Table 10.19 (Group 2 R_n= 0)	Experiment 4 Calculated values as per method 3.				
	II V II	II I II	S	P	λ
3-phase	412.5273	8.176312	3372.952	3123.635	0.926084

Table 10.20 (Group 3 R_n= ∞)	Experiment 4 Calculated values as per method 4.				
	$V_{\varepsilon 3}$	$I_{\varepsilon 3}$	$S_{\varepsilon 3}$	$P_{\varepsilon 3}$	$\lambda_{\varepsilon 3}$
3-phase	399.9635	7.574029	3029.335	2943.966	0.971819

10.3 Conclusion

Results of AP and pf obtained from various 3-phase 4-wire loads have been tabulated and compared with the methods described in this thesis which uses a null point in accordance with the neutral wire resistances. The following comparative observations from the results obtained can be made.

1) Method 1 results correspond closely to those obtained from the Yokogawa WT1666 power meter in all four experiments. The results of this method are identical to those using Definition 2 (Eq. 1.2 of Chapter1), in which phase components of power are added and the phase components of single-phase are added.
2) Definition 3 of Chapter 1 gives three sets of results depending on the null point chosen.
3) In this particular experiment all 4-wires had the same resistances and therefore method 2 is the only valid one and gives the true AP and power factor.
4) If different null points are used assuming different neutral wire resistances, different AP and pf values are obtained.
5) Had the currents and voltages remained constant for different neutral wire resistances, the power factor results calculated would decrease with the neutral wire resistance.
6) The maximum power that can be transmitted (i.e. AP) increases as the value of the resistance attributed to the neutral wire decreased from ∞ to 0. It is logical that the amount of power that can be transmitted for the same line losses increases as the neutral wire resistance decreases and hence increases its contribution to the transmission of power.

The above conclusion is in agreement with Emanuel's statement.

Chapter 11

Conclusion

11.1 Interpretation of the impact of the work

Inconsistencies with measuring AP under non-sinusoidal 3-phase supply voltage and load currents have been much debated. Even recently there has been disagreement over what constitutes apparent power and power factor.

Theories such as instantaneous power have generated much interest and have been shown to have much merit for the purpose of definition of power components, measurement, education and the design of active and passive filters in single-phase and polyphase systems. However, these theories have failed to achieve a wider practical application and implementation, due to their complex and sometimes inconsistent formulations. This thesis should give the reader a better appreciation of existing instantaneous power theories and a clearer understanding of them. The reformulation of the major existing instantaneous power theories and their classification by the author into three groups which can all be formulated in one simple general equation should simplify the understanding of instantaneous power and avoid the complex and ambiguous formulations that have been published in the past. It is shown that each of the three groups of reformulated methods is equivalent provided that the appropriate "null" point voltage reference is correctly selected. It is mathematically evident that the voltage reference necessary to obtain 'optimally compensated' solutions with minimum transmission losses is simply related to the value of the neutral wire resistance in each case (all other wires being equal). Thus it is possible to unequivocally define the instantaneous active and non-active current and power components.

This thesis also shows that instantaneous power theory can be logically extended to the rms domain and become a necessary and integral part of power theory in the average power domain. The separation of non-active power into components that can be compensated with and without energy storage clarifies presently published work. This thesis should encourage readers to adopt a definition of apparent power which is in accordance with Emanuel's definition.

11.2 Summary of chapters

Chapter 1 introduced the need for a universally acceptable general definition of apparent power. Comments by various experts in the field and some examples illustrating the point were given. Emanuel's definition of AP as the theoretical maximum power delivered by a voltage source, whilst the total line losses are maintained constant, is the most plausible. It provides a figure of merit related to the effective transmission of power with minimal losses. This thesis agrees that the concept and definition of apparent power for a periodic voltage waveforms is $S = \|\mathbf{V}\|\,\|\mathbf{I}\|$ (that is, the product of the norm of two vectors representing the voltage and current of a poly-phase system), provided that the voltage reference renders S to be a maximum. It is also shown by example that the correct voltage reference is a function of the resistances of the wires and how the neutral wire contributes to the power delivery capacity.

Chapter 2 reviews theories related to non-sinusoidal systems in single phase systems. It shows that the apparent power S in single-phase systems can be defined without ambiguity as the product of rms voltage and current. The maximum power that can be transmitted for the same losses is consistent with Emanuel's definition of AP.

Chapter 3 reviews the most well known instantaneous power theories such as those of Fryze, Buchholtz, Depenbrock, Akagi, Nabae, Willems, Peng, Ferrero and Superti-Furga. It shows in terms of examples using arbitrarily chosen voltage and current values that all the instantaneous power approaches reviewed form three groups, that within each group the various approaches give identical results and, further, that:

Group 1 (Fryze, Depenbrock, Buchholtz with M=4) gives best results for an active current, when a neutral current can exist and is not restricted to zero after compensation, and if the neutral wire is considered to have the same resistance as the other wires.

Group 2 (Akagi, Buchholtz with M=3, Rossetto and Tenti, Ferrero and Superti-Furga) results in minimum instantaneous active current when the neutral wire current is constrained to zero

Group 3 (Willems, Nabae and Peng) results in the lowest supply losses if the neutral wire resistance is considered to be zero.

Chapter 4 reformulates the formulae of the above three groups into one universal equation for the active current, calculated as:

$\mathbf{i_a} = k\,\mathbf{v}$ where $k = p/\|\mathbf{v}\|^2$

where the voltage reference for each of the three groups is as follows:

Group 1: (condition: $r_n = r_1 = r_2 = r_3$)

The voltage reference is the virtual null point of all four wires.

Group 2: (condition: $r_n = \infty$)

The voltage reference is the virtual null point of three wires only (excluding the neutral even if it exists).

Group 3: (condition: $r_n = 0$)

The voltage reference is the neutral wire.

Chapter 5 shows mathematically that the necessary and sufficient condition for the instantaneous power to be in accordance with chapter one hypothesis is,

$$\sum_{n=1}^{M} (v_n / r_n) = 0$$

From this equation it is shown that the specific voltage reference for each of the groups described in Chapter 4 is appropriate.

This approach is the key that clarifies instantaneous power theory in 3- and 4-wire systems and should help to remove the confusion and ambiguities that permeate many of the publications on instantaneous power theories.

Chapter 6 introduces a different new instantaneous method based on the properties of space vectors for the case of a 4-wire system with equal resistances (Group 1). It shows that the norm (size) of the compensating current required depends not only on the neutral wire, but on the individual case of supply current. Therefore the cost and losses incurred in the compensator need to be assessed before drawing any conclusion as to which method to use. Such comparisons of compensating current have not been found in the literature reviewed.

Chapter 7 reviews various rms or average power domain theories. The concept of maximum power transmission is fundamental to most of these theories. Not all authors recognise any link between instantaneous domains to the rms domain and ignore it altogether. Most theories considered all wires to have equal resistance whilst some ignore the effect of the neutral wire in the case of 3- phase 4- wire systems. The importance of the “null” point is evident. Willems recent method is of a general nature and in full agreement with the hypothesis 1 discussed in Chapter 1.

Chapter 8 shows how the vector space instantaneous power theory given in Chapter 6 is consistant with Emanuel's definition of AP of Chapter 1 and that it can be extended naturally and meaningfully into the rms domain. The active current obtained in the rms domain is in agreement with this definition and the non-active current can be decomposed into orthogonal components with physical meaning based on energy storage requirements for the compensator. The calculation of the compensation currents is quick, with obvious implications for real time practical implementation in the design and sizing of active filters.

Chapter 9 demonstrates theoretical examples with sinusoidal voltages with some harmonic components. Four models of 3-phase, star-connected voltage supplies, consisting of various combinations of classical (Fortescue) fundamental sequence voltage components are examined.

Chapter 10 describes how a true power factor meter (TPM) based on the theory presented can be implemented in practice using a computer or a digital signal processor (DSP) capable of real time processing of measured instantaneous values of currents and voltages. The TPM displays power, apparent power and power factor, under the various conditions described in the previous chapters. The TPM is also capable of providing compensating signals required by an active compensator to compensate the line currents. Two sets of signals output are provided, one for a compensator without energy storage and one with.

A laboratory experiment compares the new method with the results obtained with a good commercial power meter. It can be seen how the value attributed to the neutral wire can have a significant influence in determining the true value of apparent power and power factor.

11.3 Key contributions

In summary, this thesis makes the following contributions:

1) A comprehensive collation of most power theories in the instantaneous and average power domain.
2) Comparison of the theories and their application by means of numerical and experimental results.
3) Reformulation of instantaneous theories in a consistent mathematical approach.
4) New classification of major instantaneous power theories into three groups.
5) Linking the three groups by one general equation and showing that optimal compensation results can be obtained for each group, depending on the value of the resistance attributed to the neutral wire, including the mathematical proof of the compensation being optimal.
6) Introducing a new instantaneous theory based on vector space, which enables the extension of instantaneous power into the rms domain in a logical and meaningful way and results in power components with real physical interpretation.
7) Separation of compensating currents in the rms domain into two components, one of which requires energy storage and one which does not.
8) The description of an instrument to measure true power factor and provide the inputs for power compensation.
9) A new method of comparing instantaneous compensating currents that can find applications in practical design and for the purpose of sizing active compensators. This may encourage new and more extensive work in this area.

This approach could have an important educational impact. Some parts could be included in the curriculum of electrical power engineering students and be more logical than the way complex power theory is generally introduced. Concepts of apparent power, imaginary power and phase angle are difficult for students to relate to the real physical world. The approach presented in this thesis is based on fundamental physical and mathematical principles, including Kirchhoff's current law, conservation of energy and the properties of linear algebra. The results are easily visualised in terms of compensation with and without energy storage. Apparent power and power factor be introduced as a logical consequence of Emanuel's definition of apparent power rather than attempting to relate it to the cosine of the phase angle of a single-phase system.

It is hoped that this work will facilitate the wider use of the concept of the correct "null" point and encourage its wider use in practice. It should assist the better definition and standards for the definition of apparent power and power factor in balanced and unbalanced loads, with and without sinusoidal currents and voltages. It should contribute to the standardisation of consistent measurements in poly-phase power systems.

Finally, this thesis provides a reference source for familiarising readers with various power theories and foster appreciation of progress and the valuable contributions made by various specialists over the years.

11.4 Conclusions

Present definitions of AP have often been reported by many to be inadequate in three-wire and four-wire system. In the case where the voltage is considered to be invariant with respect to the current, one of the ways AP can be clearly defined is as follows:

AP is the theoretical maximum power which can be delivered by a voltage source whilst the total line losses remain constant.

Power factor is then the ratio of average power and apparent power, this can be interpreted as a figure of merit of how well the transmission system is used with respect of minimizing transmission losses.

This definition of AP can be applied to an infinitively small interval as well as a period T of a periodic voltage source. The first one is referred to as the instantaneous AP and the second as the rms (or average) AP. A clear and plausible link between the two has been demonstrated.

In both cases AP is calculated as:

$$S = \|\mathbf{V}\| \, \|\mathbf{I}\| \qquad \text{Eq 1.8}$$

However, the equation yields different results according to the choice of the reference for the voltage. The product of voltage and current represented by Eq 1.8, often thought of as the AP, is equal to the AP as defined above only when the voltage reference is chosen such that the AP has its maximum value**.**

The correct voltage reference depends on the resistances of the wires. In the case of 4-wire systems having three wires of equal resistances, three cases are considered, where the fourth wire has a resistance that is:

1) Equal to the other three wires.
2) Infinite.
3) Zero.

Then the voltage reference has to be respectively:

1) The 'null' point where the sum of the voltages of all four wires is zero.
2) The 'null' point where the sum of the voltage of the other three wires is zero.
3) The neutral wire.

In the case of 4-wire systems, a new space vector space technique enables the current to be decomposed into active current (the optimum in respect of minimizing transmission losses after compensation) and non-active current (the balance which can be compensated with an active filter). The non-active current can compensated into two stages with a compensator that:

1) does not require energy storage, and
2) one that requires energy storage.

This result of the space vector technique provides a new means of comparing the compensating current where the neutral current is restricted to zero with that where it is not restricted.

I believe that this approach to the definition of apparent power provides methods that may ultimately lead to the adoption of better standard definitions of AP, power factor, and active and non-active current. The approach also has implications in the design of compensating filters.

A similar vector space approach can be extended to the case of m wires with different resistances.

References

Akagi H, Kanazawa Y, Nabae A (1983), “Generalized theory of the instantaneous reactive power in three-phase circuits.” Proc. IEEJ International Power Electronics Conf. (IPEC-Tokyo), pp 1375-1386, 1983.

Akagi H. Kanazawa Y, and Nabae A (1984), “Instantaneous reactive power compensators comprising switching devices without energy storage components.” IEEE Trans. Ind. Appl., Vol. IA-20, No. 3, pp. 625-630, May/June 1984.

Akagi H, Ogasawara S and Kim H (1999), “The theory of instantaneous power in three-phase four-wire systems: a comprehensive approach.” IEEE-IAS conference record, Vol. 1, pp. 431-439, Oct 1999.

AIEE (1941), American Standard Definition of Electrical Terms. New York/USA: AIEE, 1941.

Anton, H (2000), “Elementary Linear Algebra.” 8th Edition John Wiley & Sons. ISBN 0-471-17055-0.

Budeanu C. I. (1927), **“**Reactive and Fictitious Powers”, Publ. No. 2 of the Rumanian National Institute of Energy, Bucharest, Romania, 1927.

Buchholtz, F (1950), “ Das Begriffsystem Rechtleistung, Wirkleistung, totale Blindleistung.”, Selbstverlag Munchen, 1950.

Czarnecki LS (1987), “Minimisation of Reactive Power under Non-sinusoidal Conditions” , IEEE Trans, Inst. Meas., Vol. IM-36, No.1, pp. 18-22, March 1987.

Czarnecki LS (1988), “Orthogonal decomposition of the currents in a 3-phase non-linear asymmetrical circuit with non-sinusoidal voltage source “,IEEE Trans. Inst. Meas., Vol IM-37, (1), pp.30-34 , 1987.

Czarnecki LS (1989), Comments "Reactive and unbalanced currents compensation in three-phase asymmetrical circuits under non-sinusoidal conditions", IEEE Trans. Inst. Meas., Vol 38. No. 3, pp. 839-841, June 1989.

Czarnecki LS (1991), "Scattered and reactive current, voltage and power in circuits with non-sinusoidal waveforms and their compensation", IEEE Trans. Inst. Meas., Vol. 40, No. 3, pp. 563-567, June 1991.

Czarnecki LS (1992), "Minimization of unbalanced and reactive currents in three-phase asymmetrical circuits with non-sinusoidal voltage ", IEE Proceedings–B, Vol. 139, No.4, pp. 347-354, July 1992.

Czarnecki LS (1993a), "Power factor improvement of three-phase unbalanced loads with non-sinusoidal supply voltage" ETEP Vol.3, pp. 67-74, No.1, Jan/Feb 1993.

Czarnecki LS (1993b), "Physical reasons of currents rms value increase in power systems with non-sinusoidal voltage", IEEE Trans. on Power Delivery, Vol. 8, No.1. pp. 437-447, Jan 1993.

Czarnecki LS (1995), "Adaptive balancing compensator", IEEE Trans. on Power Delivery, Vol. 10, No 3, pp. 1663-1669, July 1995.

Czarnecki LS (2004), "On some Misinterpretation of the instantaneous reactive power p-q theory. ", IEEE Trans on Power Electronics, Vol. 19. No.3, pp. 828-836, May 2004.

Depenbrock M (1962a), **"**Untersuchungen uber die Spannungsund Leistungsverhaltnisse bei Umrichtern ohne Energiespeicher.", Ph.D. thesis, TH. Hannover 1962, pp 15-25.

Depenbrock M (1962b), "Some remarks to active and fictitious power in poly-phase and single-phase systems," ETEP, Vol.3, pp. 15-19, Jan 1962.

Depenbrock M (1993), "The FBD-Methods a generally applicable tool for analyzing power relations " IEEE Trans. on Power Systems, Vol.8, pp 381-387 No.2. May 1993.

Depenbrock M, Marshall DA Van Wyk JD (1994), " Formulating requirements for a universally applicable power theory as control algorithm in power compensators" ETEP Vol.4, No.6, pp. 445-455, Nov / Dec 1994.

Depenbrock M, Staudt V, Wrede H **(2003)**, "A theoretical investigation of original and modified instantaneous power theory applied to four-wire systems " IEEE Trans. on Ind. App., Vol. 39, No. 4, pp. 1160-1167, July/Aug 2003.

Depenbrock M, Staudt V and Wrede H (2004a), "Concerning "Instantaneous Power Compensation in three-phase systems by using p-q-r Theory " IEEE Trans. on Power Electronics, Vol. 19, No. 4, pp. 1151-1152, July 2004.

Emanuel AE (1990), "Powers in non-sinusoidal situations. A review of definitions and physical meaning.", IEEE Trans on Power Delivery, Vol 5, pp 1377-1384, July 1990.

Emanuel AE (1993), "Apparent and Reactive Powers in Three-Phase Systems: In Search of a physical Meaning and a Better Resolution" ETEP Vol 3. , pp 7-14, No 1. Jan/Feb.

Enslin JHC., Van Wyk JD (1988), " Measurement and Compensation of fictitious power under non-sinusoidal voltages and current conditions.", IEEE Trans. on Inst. Meas., Vol. 37, No. 4, pp. 403-408, Sep1988.

Ferrero A, Morando AP, Ottoboni R, Superti-Furga (1993), " On the Meaning of the Park power components in three-phase systems under non-sinusoidal conditions.", ETEP Vol 3, pp. 33-43, No 1, Jan/Feb 1993.

Ferrero A, Superti-Furga G (1991), "A New Approach to the definition of Power Components in Three-phase Systems Under Non-sinusoidal Conditions.", IEEE Trans. Inst. Meas., Vol 40, No. 3, pp.568=577, June 1991.

Filipski PS (1980), "A new approach to reactive current and reactive power measurement in non-sinusoidal systems," , IEEE Trans. Inst. Meas., Vol. IM-29, pp. 423–426, Dec. 1980.

Filipski PS (1993), "AP – a misleading quantity in the non-sinusoidal power theories doomed to fail ?", ETEP Vol.3, No 1. pp. 21-26, Jan/Feb 1993.

Fryze S. (1932) , "Active, reactive and apparent powers in circuits with distorted voltages and currents," ETZ Elekttotech. Z., Vol.53, pp. 596-599., 1932.

Kusters NL, Moore W J M (1980), "On the definition of reactive power under non-sinusoidal conditions," IEEE Trans. Power Appl. Sys., Vol. PAS-99, pp. 1845–1854, Sept/Oct 1980.

IEC (1979) Electrotechnical Commission, Technical Committee No. 25, Working Group 7, Report: "Reactive power and distortion power", Dec. No. 25, pp. 13 December, 1979.

IEEE Standard 1459 – 2000 (2000), "Definitions for the measurement of electric quantities under sinusoidal, non-sinusoidal, balanced, or unbalanced condition. " IEEE Trans. Std 1449-2000", January 2000.

Le Roux W, van Wyk JD (1998), "Evaluation of residual network distortion during compensation according to the " instantaneous power theory.", ETEP Vol.8, pp. 337-344, No 5, Sep/Oct 1998.

Malengret M, Gaunt CT (2005), " Definition of apparent power in poly-phase systems.", South African University Power Engineering Conference (SAUPEC), CD Rom, Jan 2005.

Malengret M, Gaunt CT (2008), **"**Decomposition of currents in three- and four-wire Systems." IEEE Trans of Inst. and Meas. , Vol. 57, No 5, pp. 963-972 May 2008.

Nabae A, Nakano H and Togasawa S (1995), "An instantaneous distortion current compensator without any coordinate transformation." Proc. IEEJ International Power Electronics Conf. (IPEC-Yokohama), pp. 1651-16555, 1995.

.

Page CH (1980), "Reactive power in non-sinusoidal situations," IEEE Trans. Inst. Meas., Vol. IM-29, pp. 420–423, Dec. 1980.

Peng FZ , Lai JS (1996), "Generalized instantaneous reactive power theory for three-phase power systems." IEEE Trans. Inst. Meas., vol. 45, No.1, pp. 293-297, Feb. 1996.

Rossetto L, Tenti P (1994), "Evaluation of instantaneous power terms in multi-phase systems: Techniques and application to power-conditioning equipment." ETEP Vol. 4, pp. 469-475, No. 6, Nov/Dec 1994.

Salmerón P, Montano JC, Vazquez JR, Prieto J, Perez A (2004), "Compensation in non-sinusoidal unbalanced three-phase four-wire systems with active power-line conditioner", IEEE Trans. on Power Delivery, Vol. 19, No. 4, pp. 1968-1974 , Oct 2004.

Shepherd, Zakikhani P (1972), "Suggested definition of reactive power in non-sinusoidal systems.", Proc. Inst. Elec. Eng., Vol. 119, pp. 1361–1362, 1972.

Smith LP (1953), " Mathematical Methods for Scientists and Engineers.", pp. 39-42, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1953.

Sokolnikoff IS, (1941), " Higher Mathematics for Engineers and Physicists.", 2nd ed., pp. 163-167, McGraw-Hill Book company, Inc., New York, 1941.

Staudt V, Wrede H (2004), "Composition Strategies for four-conductor systems.", L'Energia Elettrica , " Richerche". Vol. 81, pp 182-189, 2004.

Tolbert LM, Habetler TG (2000), "Comparison of time-based non-active power definition for active filtering." 7th International Power Electronics Congress, Technical Proceedings, CIEP 2000 (Cat. No. 00TH8529). Piscataway, NJ, USA: IEEE, pp. 73-9., 2000.

White N, Ilié MD (1997), "Vector Space Decomposition of Reactive Power for Periodic Nonsinusoidal Signals.", IEEE Trans. On Circuits and Systems Vol. 44, No. 4, pp 338-346, April 1997.

Willems JL (1992), "A new interpretation of the Akagi-Nabae power components for non-sinusoidal three-phase situations.", IEEE Trans. Inst. Meas., vol. 41, No. 4, pp. 523-527, Aug 1992.

Willems JL (1993), "Current compensation in three-phase power systems" ETEP Vol.3. No. 1, pp. 61-66, Jan/Feb 1993.

Willems JL, Ghijselen JA (2004), "The relationship between the Generalized Apparent Power and Voltage Reference.", L'Energia Elettrica "Richerche", Vol. 81, pp 37-34, 2004.

Willems JL (2006), Discussion of: "Generalized Theory of Instantaneous Reactive Quantity for Multiphase Power Systems.", IEEE Trans on Power Delivery.Vol.21 No.1, pp. 964, January 2006.

Xu Y, Tolbert LM, Peng FZ, Chiasson JN, Chen J (2003), "Compensation-Based Non Active Power Definition.", IEEE Power Electronics Letters,Vol.1, No.2, June 2003.

Yokogawa (2001) Yokogawa WT 1600 meter Digital Power Meter user manual second edition, manual No. IM 760101-01E., August 2001.