Mathematical Literacy of Students Entering First Year at a South African University.

Vera Frith, Lynn Bowie, Kate Gray and Robert

Numeracy Centre, U.C.T.

Abstract:

The results of a mathematical Literacy questionnaire administered to school-leavers entering the Humanities, Health Sciences and Science Faculties at a South African university in 2002 are reported. The motivation for interest in the Mathematical Literacy of these students and the rationale behind the questionnaire are discussed. Some of the mathematics and contexts that caused the most difficulty are highlighted.

Introduction:

Many matriculants enter universities or the workplace without the necessary mathematical literacy, language competence or computer literacy to enable them to succeed in their chosen course of study or career. The traditional approach to various literacies at schools (and Universities) in South Africa is to teach them in a very compartmentalised way. It is generally assumed that a learner who has studied mathematics to a sufficiently high level in school will automatically be "mathematically literate" as well, which is not necessarily true.

The nature of Mathematical Literacy:

There is an ongoing debate about the meaning of the term "numeracy" (or mathematical or quantitative literacy) and its relationship to "literacy" (and to "mathematics"). We adopt the proposed working definition of "numerate behaviour" from the Adult Literacy and Lifeskills Survey as our definition of Mathematical Literacy:

> "Numerate behaviour is observed when people manage a situation or solve a problem in a real context, and involves responding to mathematical information that may be represented in multiple ways; it requires the activation of a range of enabling knowledge, behaviours and processes."

Hughes-Hallett (2001) summarises the difference between mathematical/quantitative literacy and mathematics as follows:

> "…mathematics focuses on climbing the ladder of abstraction, while quantitative literacy clings to context. Mathematics asks students to rise above context, while quantitative literacy asks students to stay in context. Mathematics is about general principles that can be applied in a range of contexts; quantitative literacy is about seeing every context through a quantitative lens."

This quotation sums up the idea, often mentioned in the literature, that an important component of mathematical literacy is the ability to operate in a context; yet the dominant pedagogical practice (particularly in South Africa) of teaching mathematical literacy in the restricted context of the formal mathematics classroom is at odds with this idea. Usiskin (2001) warns against the use of contrived "real-life" examples masquerading as "reality" in the mathematics classroom. Teaching mathematical literacy requires the use of real contexts, which need to be understood as clearly as the mathematical "skills" that are applied within the context. This is why students often experience a context-based mathematical literacy course as rather challenging, even if the mathematics skills required are quite elementary (Archer, Frith, Prince, 2002). For a learner to be mathematically literate, they would have to be able to see the contexts that they will encounter in the workplace or in their programme of study "through the quantitative lens"

Numerate behaviour can be thought of as a practice involving the exercise of several related competencies, not just the exercise of skills. Chapman and Lee (1990) argue that it is not possible to draw an artificial separation between the notions of numeracy and literacy, but rather that numeracy should be situated within a larger notion of literacy that involves many competencies:

> "The problem of recognising the need for and applying appropriate mathematics in non-mathematical contexts arises precisely because the skills of reading, writing and mathematics are inextricably interrelated in the ways in which they are used in communication and hence in learning."

Viewing mathematical literacy as a set of identifiable arithmetic skills, construes it as a set of techniques that can be taught and learnt without reference to the social contexts where they might be applied, and which are therefore universal across time and space. Baker, Clay and Fox (1996) use the term "numeracy" to mean "the collection of numeracy practices that people engage in – that is the contexts, power relations and activities – when they are doing mathematics".

Kemp (1995) stresses that the tertiary curriculum makes great demands on students' mathematical literacy, and that students often avoid or skim over the quantitative aspects they encounter in the contexts they are studying. She argues that mathematical literacy includes the ability to communicate clearly and fluently and to think critically and logically. In dealing with quantitative or mathematical ideas in context, students should be able to interpret ideas or messages presented either verbally, graphically, in tabular or symbolic form, and be able to make transformations between any of these forms.

The Numeracy Competency Questionnaire:

The Numeracy Competency Test Project has been running since 1999, aimed at testing the extent of the mathematical literacy of school-leavers who are registering for their first year of tertiary study. All students in specific faculties, or on particular programmes, fill in a "Numeracy Competency Questionnaire" during their first weeks at university (often during Orientation Week), which is administered under examination conditions. The questionnaire is intended to measure their knowledge of some of the mathematical literacy content that is often assumed by university first-year course curriculum designers. More significantly, though, the questionnaire is intended to measure the student's ability to interpret context-based information presented either verbally, graphically, in tabular or in symbolic form.

The questionnaire is designed as a diagnostic test, to establish the level of Mathematical Literacy of participants and focuses on some of the basic competencies which comprise mathematical literacy (as discussed above). It consists of three sections which differ in complexity:

Section A consists of easy multiple choice questions requiring basic numerical ability, (mostly involving fractions, decimals and percentages) and a low level of interpretative skill.

Section B consists of a theme question involving the use of diagrams, tables and graphs. The questions test literacy and interpretative skills more than in the first section, and are organized around a particular context, for example HIV/AIDS or substance abuse. This is the part of the test where the mathematical literacy is tested most coherently. The ability to interpret and reason logically about information presented verbally, graphically and in tabular form is emphasized in this section.

Section C has an example of the application of 'everyday' mathematical literacy, and also tests the understanding, use and construction of formulae in 'everyday' situations.

Each question scores 1, 2 or 3, depending on the level of difficulty of the question. This difficulty level is a reflection of the literacy, interpretative and computational skills required to answer the question correctly. The questionnaire has not been designed to provide a standardised score which would indicate whether a participant is mathematically literate or not, but rather to survey the competence of particular cohorts of students in a comparative way. This is because there are currently no assessment standards for Mathematical Literacy at this level.

Performance of Humanities (All), Medical (MbChB) and Science (Earth and Biological Sciences) students on the Numeracy Questionnaire:

Results for the questionnaire as a whole:

The distribution of scores on the whole questionnaire for the all Humanities, Medical (MBChB) and Science (Earth and Biological) students are shown in Figure 1, Figure 2 and Figure 3 respectively.

As one might expect, the Medical students (median 72%) and the Science students (median 69%) performed considerably better than the Humanities students (median 55%), although the distribution for Medical students is only very slightly displaced towards the right when compared to that of the Biological and Earth Sciences students.

Figure1: Distribution of scores for Humanities students on the NQ2002

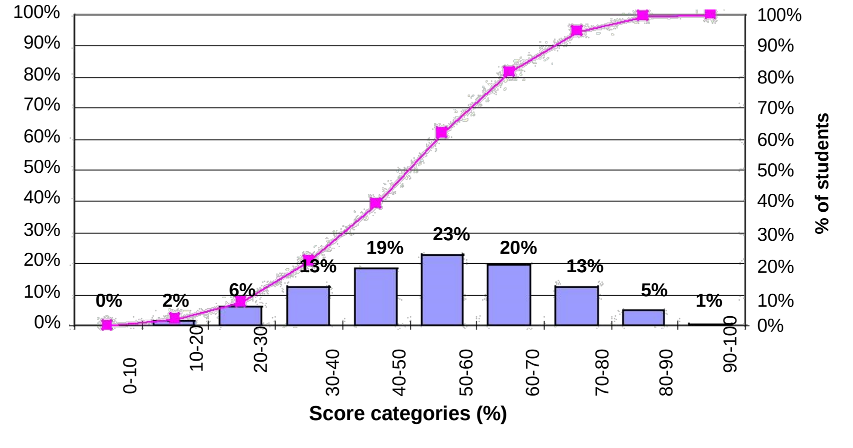


Figure 2: Distribution of scores for Medical students on the NQ2002

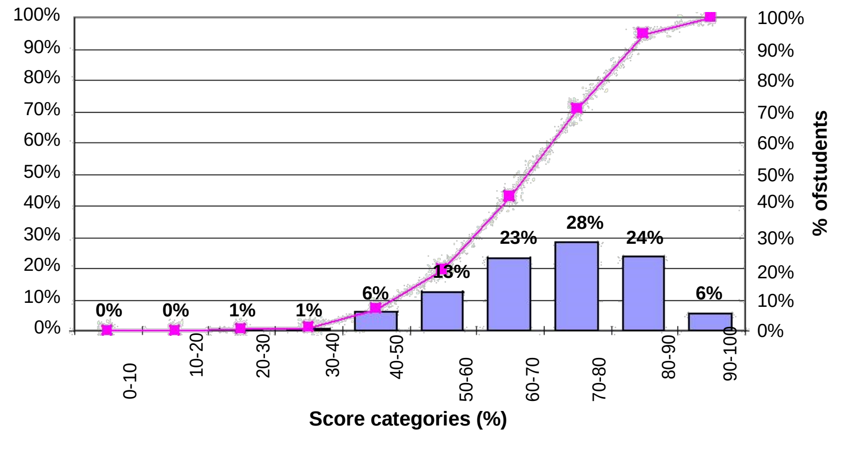
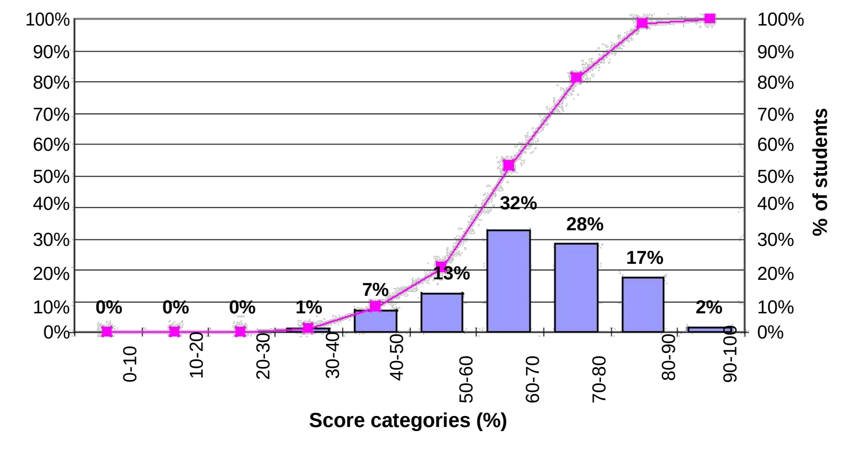


Figure 3: Distribution of scores for Science students on the NQ2002



Results for the different sections of the questionnaire:

The descriptive statistics that summarise the distributions of scores for Sections A, B and C and for the questionnaire as a whole are tabulated in Table 1 (Humanities), Table 2 (Medical) and Table 3 (Science).

The relative performance of the three different student cohorts on Section A is similar to their relative performance on the test as a whole (although the medians are all higher for Section A, since this is the easier section). Section B is the part of the questionnaire that really explores the students Mathematical Literacy (as defined above) and it is interesting that in this section the Science students (median 69%) did slightly better than the Medical students (median 67%). Also, the difference between the performance of the three groups is not as marked as for the test as a whole. As expected, in Section C (where the use of simple variables is required) the Humanities students performed much worse than the others (median 48%), and once again the Medical students (median 69%) performed somewhat better than the Science students (median 64%).

Table 1:
Results by Section and Overall for Humanities students on NCQ 2002

	Mean	Median	Minimum	Maximum	1st Quartile	3rd Quartile	Std.Dev.
Section A	60%	62%	3%	100%	44%	77%	21%
Section B	54%	54%	6%	94%	44%	65%	16%
Section C	48%	48%	0%	97%	35%	62%	20%
Overall	54%	55%	10%	97%	43%	66%	16%

Table 2:
Results by Section and Overall for MBChB students on NCQ 2002

	Mean	Median	Minimum	Maximum	1st Quartile	3rd Quartile	Std.Dev.
Section A	81%	85%	21%	100%	73%	92%	16%
Section B	66%	67%	38%	96%	56%	77%	14%
Section C	68%	69%	17%	100%	57%	83%	17%
Overall	71%	72%	28%	95%	62%	82%	13%

Table 3:
Results by Section and Overall for Science students on NCQ 2002

	Mean	Median	Minimum	Maximum	1st Quartile	3rd Quartile	Std.Dev.
Section A	76%	77%	31%	100%	67%	87%	15%
Section B	66%	69%	29%	94%	58%	75%	13%
Section C	63%	64%	10%	97%	52%	76%	15%
Overall	69%	69%	35%	95%	61%	78%	12%

Results for some individual questions:

The percentages reported in Table 4 highlight relevant areas of weakness for the three different categories of student. The percentages in the first 3 columns show the proportion of students in each cohort who could not answer the specific question correctly. Concepts that presented roughly the same high level of difficulty to all students were: calculation of percentage increase (70-73% incorrect), the understanding that percentage change is not the same as change in absolute magnitude (73-88% incorrect), graphical representation of growth rate (29-31% incorrect) and basic statistics (around 50% incorrect) . These figures, more than any others, highlight the rather alarming lack of basic mathematical Literacy in this fairly representative cross-section of entry-level students.

In the questions classified as “numerical”, the areas of particular weakness for all groups were decimals (30-40% incorrect) and working with ratios (between about 30 and 60% incorrect). It is particularly noteworthy that 20% of the science students could not recognize the correct scientific notation representation of a large number and that 32% of medical students could not work with a ratio expressed as “5 to 1”.

The results in Table 4 under the heading “Percentage and percentage increase” give particular cause for concern, given the frequency with which these concepts are encountered in a range of contexts, including those in Social Sciences and in Medicine. For example, at the simplest level, 14% of medical students and 18% of Science students could not calculate 10% of a given number, while 33% of medical students and 36% of Science students did not recognize 5% as being the same as 0.05.

Only two contexts are highlighted in Table 4, but they both reveal a fairly high degree of mathematical illiteracy in the realm of “everyday mathematics”. Twenty-three percent (23%) of Medical and Science students and 41% of Humanities students could not correctly calculate the bank charge for an ATM transaction and between 25% and 40% of the students could not correctly calculate the charge for a Telkom call to a cell-phone (given the information provided in the telephone directory).

The questions in Section C of the questionnaire required the formulation and manipulation of very simple algebraic formulae. It is not surprising that Humanities students performed particularly badly on these (between 41% and 82% incorrect) but the degree of difficulty experienced by Medical and Science students gives serious cause for concern. For example 66% of Science students did not recognize that “32% larger than N” is represented by 1.32N, and 57% could not answer a question that required the construction of a simple algebraic formula.

Table 4: Relevant areas of weakness for Humanities, Medical and Science students

The percentages show the proportion of students in each cohort who could not answer the specific question correctly.

Humanities	Medical	Science	Description of difficulty
Numerical:			
28%	8%	11%	did not recognise 0.125 as 1/8 ;
25%	5%	9%	could not identify the largest of a list of 5 decimal numbers
44%	33%	29%	could not identify a decimal number closest to 8 from a list of 5 decimal numbers
37%	9%	13%	could not answer a question that required the conversion from mm and cm to m
29%	11%	9%	could not find 1/3 of 1/3
51%	13%	20%	could not recognise 9 600 000 000 as $9.6x10^9$
66%	32%	42%	could not work with a ratio given as "5 to 1"
55%	27%	34%	could not work with ratios given as 1:3:5
Percentage and percentage increase:			
32%	11%	19%	could not perform an easy simple interest calculation
66%	37%	50%	could not perform an easy compound interest calculation
32%	20%	25%	could not calculate a simple percentage reduction
55%	33%	36%	could not recognise 5% as 0.05
29%	14%	18%	could not calculate 10% of a number
88%	73%	76%	showed a lack of understanding that if the same percentage of people die in 2 different groups, this does not necessarily mean that the same number of people die in each group
73%	72%	70%	could not calculate percentage increase, and many choose the option that showed a complete lack of understanding of the concept
25%	25%	25%	wrongly identified the greatest percentage increase as resulting in the greatest increase in numbers, ignoring the original size of the groups
Charts:			
31%	29%	29%	could not identify the chart showing the highest growth rate
57%	28%	25%	could not identify the chart showing an increasing rate of increase
35%	12%	11%	could not read a stacked bar chart
Applications:			
41%	23%	23%	could not calculate the bank charges on an Automatic Teller Machine (ATM) transaction
40%	34%	25%	could not calculate the Telkom charge for a call to a cell-phone
Basic statistics:			
45%	21%	29%	could not identify that "1 in a 100 chance" is a probability of 0.01
65%	50%	62%	could not apply the understanding of the terms "mean, median and mode"
48%	48%	53%	could not identify the median from a ranked list of five numbers
Formulae:			
41%	16%	27%	could not express "M is 3 less than N" as M = N – 3
82%	47%	66%	could not recognise '32% larger than N' as 1.32N
58%	26%	32%	could not answer a question which required the manipulation of a formula
75%	47%	57%	could not answer the question which required the construction of a simple formula

Conclusion:

The results of the testing of the Mathematical Literacy of entry-level students at U.C.T reveal that there are many areas of weakness, the extent of which is generally surprisingly great, even for Medical and Science students. The implications for University teachers are that they need to examine more carefully their assumptions about what competencies school-leavers can be expected to have, and more attention must be given to ways of integrating development of the necessary competencies into the university curriculum.

As far as schools are concerned, it is clear that currently even adequate exposure to the Mathematics curriculum is not equipping students with sufficient mathematical Literacy, and the introduction of the new FET Mathematical Literacy curriculum is a very welcome development. In the light of the definition of Mathematical Literacy discussed at the beginning of this paper, a key concern in curriculum change will be to consider how to promote the development of Mathematically Literate behaviour, rather than to define outcomes in terms of content (or even contexts) studied.

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