Voltage Calculation on Low Voltage Feeders with Distributed Generation

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DECLARATION

I know the meaning of plagiarism and declare that all the work in the document, save for that which is properly acknowledged and the normal guidance of my supervisor, is my own.

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2013 has been a good year. Thanks for the memories.

ABSTRACT

The increasing levels of greenhouse gas emission and the continued depletion of fossil fuels have been the driving factors for power utilities to utilize renewable energy sources for power generation. In South Africa, a target was set in 2008 to achieve 10000 GWh of renewable generation by 2013, which includes DG on LV feeders. This has seen the increase in small scale generators, close to load centres in low voltage distribution networks such as solar PV panels in residential houses, to supplement the energy needs of consumers. This has sparked much debate over the impacts, as well as benefits, of increasing the amount of generation on these low voltage (LV) feeders. However, the power utility holds the statutory role to preserve and maintain the quality of supply of electricity and must therefore assess any impact of increasing generation on LV distribution systems. This created the need for a planning tool to assess the impact of increasing DG on LV distribution networks. There has been a lot of work carried out by researchers to assess the impact of DG on the power system, using various indicators like frequency, power losses, current, voltage etc. Keeping the voltage of a DG-integrated feeder system within the pre-defined standards has been a major challenge for power utilities today. In this report, the voltage impact of DG in LV distribution systems is examined and analysed for increasing DG penetration, particularly solar PV panels in residential households.

In South Africa, the recommended method for voltage calculation in feeders is the Herman-Beta algorithm, which is used in the design of passive LV feeders. In 2011, Gaunt experimented with modelling DG as negative loads in the HB algorithm to extend the voltage calculation to include the presence of DG on LV feeders.

This work identifies and develops a tool(s) to enable power utility planners to analyse the voltage impact of DG on LV feeders. The work in this study adds onto the DG modelling approach, introduced by Gaunt in 2011, to produce an algorithm for voltage calculation in active LV feeders with DG. This involves three major steps. First step involves the thorough testing of the HB algorithm, written in Matlab, for passive LV feeders and validating it against voltage calculation through Monte Carlo Simulation (MCS). The second step involves ammending and extending the HB algorithm for voltage calculation in active LV feeders with DG, testing and validation against voltage calculation through Monte Carlo Simulation (MCS). With the HB algorithm fully tested and validated, the third step involves using the algorithm for voltage analysis of active feeders with increasing DG penetration. The third and final step, analysing the voltage rise constraints of active LV feeders, involves running the HB algorithm, analytical method, in a MCS to create various scenarios on the feeder.

Simulations have been performed to assess the voltage impact of increasing DG penetration on LV feeders for various test cases to mimic practical LV feeder conditions.

The outcome of this study presented an application tool for the design of active LV feeders, whose output/results are summarized into implications for voltage rise mitigation and providing useful information on the DG hosting capacity of LV feeders. The recommended DG penetration limit for LV feeders in this study has been DG capacity of 30 % of the actual ADMD, used to design the passive feeder. It has been shown that after this limit, the feeder should be reinforced to avoid incidents of voltage violations. In addition, the work done in this project has set a foundation upon which a variety of similar studies can be done with active LV feeders such as the effect of solar water heating and the penetration of other DG technologies such as wind.

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ACRONYMS AND ABBREVIATIONS

Below are some of the abbreviations that have been relevant in this report:

HB – Herman-Beta

MCS – Monte Carlo Simulation

DG – Distributed Generation

EG – Embedded Generation

PV – Photo Voltaic

GoF – Goodness of Fit

ADMD – After Diversity Maximum Demand

$\mathbf{HB_{p\text{-}MATLAB}}$ – HB Voltage Calculation for passive LV feeders in Matlab

$\mathbf{HB_{a\text{-}MATLAB}}$ – HB Voltage Calculation for active LV feeders with DG in Matlab

$\mathbf{HB_{p\text{-}EXCEL}}$ – HB Voltage Calculation for passive LV feeders in a MS-Excel spreadsheet

$\mathbf{HB_{a\text{-}EXCEL}}$ – HB Voltage Calculation for active LV feeders with DG in a MS-Excel spreadsheet

$\mathbf{VD_{p\text{-}MCS}}$ – Voltage Calculation using a MCS for passive LV feeders

$\mathbf{VD_{a\text{-}MCS}}$ – Voltage Calculation using a MCS for active LV feeders with DG

$\mathbf{I_{p\text{-}HB}}$ – HB Feeder Current Calculation for passive LV feeders

$\mathbf{I_{a\text{-}HB}}$ – HB Feeder Current Calculation for active LV feeders with DG

$\mathbf{I_{p\text{-}MCS}}$ – Feeder Current Calculation using a MCS for passive LV feeders

$\mathbf{I_{a\text{-}MCS}}$ – Feeder Current Calculation using a MCS for active LV feeders with DG

Chapter 1

1. INTRODUCTION

This report investigates a planning tool for Low Voltage (LV) distribution systems with Distributed Generation (DG), also known as Embedded Generation (EG), that complies with the voltage standards used in LV distribution network design in South Africa.

In this report, a DG model approach used in the Herman-Beta (HB) method for voltage calculation in active LV feeders with DG is examined. A Monte Carlo Simulation (MCS) method is used to validate the modelling method. After validation, the modified HB algorithm is used to analyse the voltage impact of DG on LV feeder/distribution systems.

1.1 Background

An electric power system can be described as a network of electrical components that generate, transmit, distribute and use electricity. Traditional power systems, as discussed by Jenkins, et al. [2000], are constructed in such a way that:

- Electricity generation sources, such as hydro-electric power dams, coal fired power plants etc. produce electricity and are normally located in distant areas from consumer areas.
- The electricity produced is supplied to the consumers by a large transmission and passive distribution network system, comprising high voltage (HV), medium voltage (MV) and Low Voltage (LV) networks.
- Power passes through all the above three network stages (generation, transmission and distribution) before it gets to the final consumer. The consumer utilises the power as required with the level of consumer demand varying among different consumers and areas.

The traditional concept of power systems considers one direction of power flow i.e. from the sources of generation through the large and extensive transmission and distribution system, to consumers (load) in a radial distribution network. This uni-directional power flow, shown in figure 1-1, is from higher voltage levels to lower voltage levels, used by consumers located along the radial feeders [Sarabia, 2011].

The demand for electricity has increased with increase in world population and development in technology, thereby reviving the interest in connecting more generation to the power system, particularly the distribution network, closer to the customer/consumer.

In addition, the environmental impact of the central power plants such as coal power plants has led to the call for renewable generation e.g. solar PV panels, wind turbines etc. embedded in the power systems to provide clean electricity [Jenkins, et al., 2000].

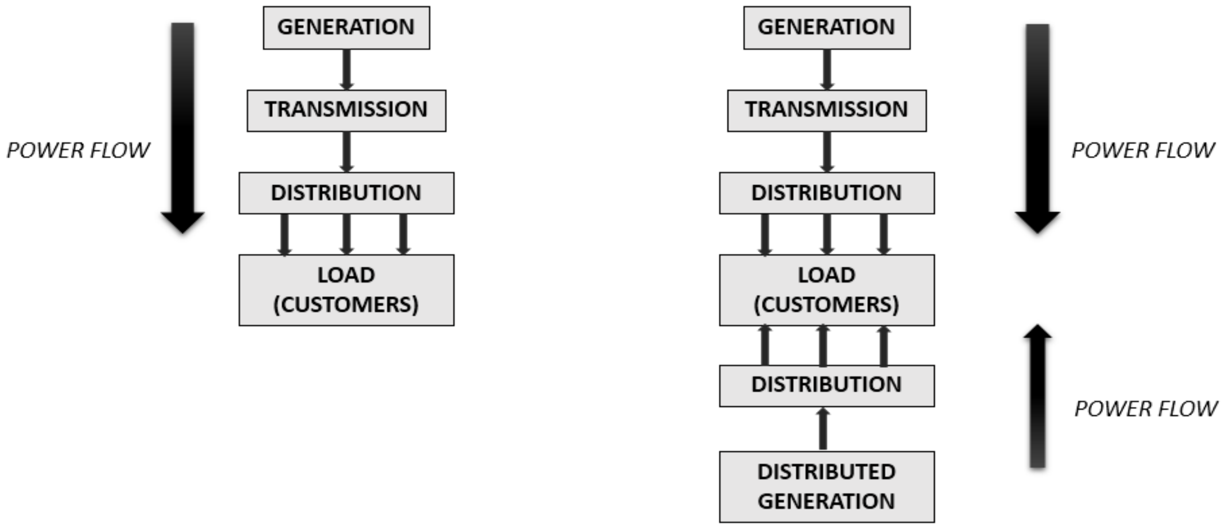


Figure 1-1: Concepts of the electric power system:

a) Traditional concept of a power system (left) b) New concept of a power system (right) [Sarabia, 2011]

Power systems have evolved in that there has been increased addition and integration of generation sources in the distribution network of the power system. These generation sources, also known as distributed generation (DG), present various challenges in the power system. This is because the power system, particularly the distribution system, is changed from a passive network, built and designed purely to deliver electricity to consumers, to an active network with power flows and voltage determined by generation and loads. The introduction of DG in the power system presents a new set of conditions on the network for operators regarding the direction of real power and non-active power flows as well as the amount of power that has to be transported in the power system, shown in figure 1-1 [Sarabia, 2011].

1.1.1 Distributed Generation

DG, also known as embedded/dispersed generation, has many descriptions and definitions based on the DG related issues like the purpose, location, rating, technology etc. DG, in this report, is considered as an electrical power source, connected to the power system close to or at the consumer site and is small in comparison to the centralized power generating plants [Sarabia, 2011].

Many DG technologies, such as solar PV, wind technologies etc. have become a common feature in power systems, particularly LV distribution systems in South Africa, as DG hardware and technology become much more reliable and economically feasible. In 2008, a target to achieve 10000 GWh of renewable generation in South Africa by 2013 was set to amount to 4 % of the projected demand in 2013 [Bello, 2008].

The introduction of DG in power systems offers a number of opportunities, which have been the driving reasons for their development and implementation. These benefits, discussed by Viral and Khatod [2012], include:

- DG facilitates cleaner production of power. The environmental impact mainly in terms of GHG emissions has become a major factor in the consideration of any power scheme leading to increased use of renewable DG.
- DG fulfils the national power requirement. DG provides the much needed power to meet the excess demand. This also provides increased security for critical loads i.e. allowing the system to withstand high loading conditions.
- DG improves the reliability of power supply to consumers by the use of local sources and consequently reduces the losses in the transmission and distribution system.

However, the introduction of DG affects the power network in certain aspects. These drawbacks, discussed by Sarabia [2011] include:

- *Voltage levels* – connection of DG to the power system can lead to the change in the voltage profile i.e. voltage fluctuations, voltage rise and unbalance. This occurs if coordination with the power utility and the DG installed is not achieved. Inappropriate allocation of DG in power systems causes a change in magnitude of power flows thereby leading to over voltage conditions.
- *Line loading* – DG may lead to increase of the load on a transmission line depending on the size and location
- *Fault levels* – The introduction of DG into the power system causes a change in short circuit levels consequently leading to power equipment damage if protection relay settings are not adjusted accordingly.
- *Harmonics and stability levels* – DG, connected to the power system, via inverters inject harmonics into the network.

The MV or LV distribution connected DG has significant impact on the voltage regulation because of the low capacities of distribution MV and LV networks. This is a major problem still faced with DG connected to distribution systems today.

1.1.2 Voltage Drop Calculation

Voltage regulation policies for LV distributors differ from country to country and every power utility is obliged to supply its customers at a voltage within these specified standard limits. These requirements are important in determining the design and cost of distribution systems and as a result, techniques have been developed over the years to ensure maximum use of distribution networks to supply consumers within required voltages [Jenkins, et al., 2000].

In South Africa, the nominal voltage is 230V with a maximum voltage deviation of ± 10 % at consumer's point of supply. The design of LV distribution systems is based on these technical constraints as indicated in the NRS 034 standard [2007].

The supply from substations to service distribution points (SDPs) is normally done with radial feeder networks. Voltage decreases along the length of the feeder due to the presence of loads connected to the feeder. As mentioned above, the presence of DG alters the voltage profile as shown in figure 1-2. The voltage is increased at the connection point of the DG on the LV feeder and may exceed the maximum allowed voltage limits when the DG power is higher than the load demand [Viawan, 2006].

The voltage rise caused by DG in LV networks is the biggest limitation towards its rapid development on these networks. This creates the need to plan and design power networks taking voltage variation caused by the introduction of DG in existing LV networks into consideration.

The calculation of the voltage drop is an essential procedure in the design and planning of distribution networks because any power system design should comply with the voltage requirements at consumer end points. From this voltage constraint, the sizing of most LV distribution feeders is carried out. Several methods of LV feeder voltage drop calculations, used in South Africa, were tested and compared by Sellick and Gaunt [1995], including the Monte Carlo Simulation, British method, DT VDrop method, Loss of Diversity method, Unbalance Voltage method and the Beta distribution method [Sellick & Gaunt, 1995].

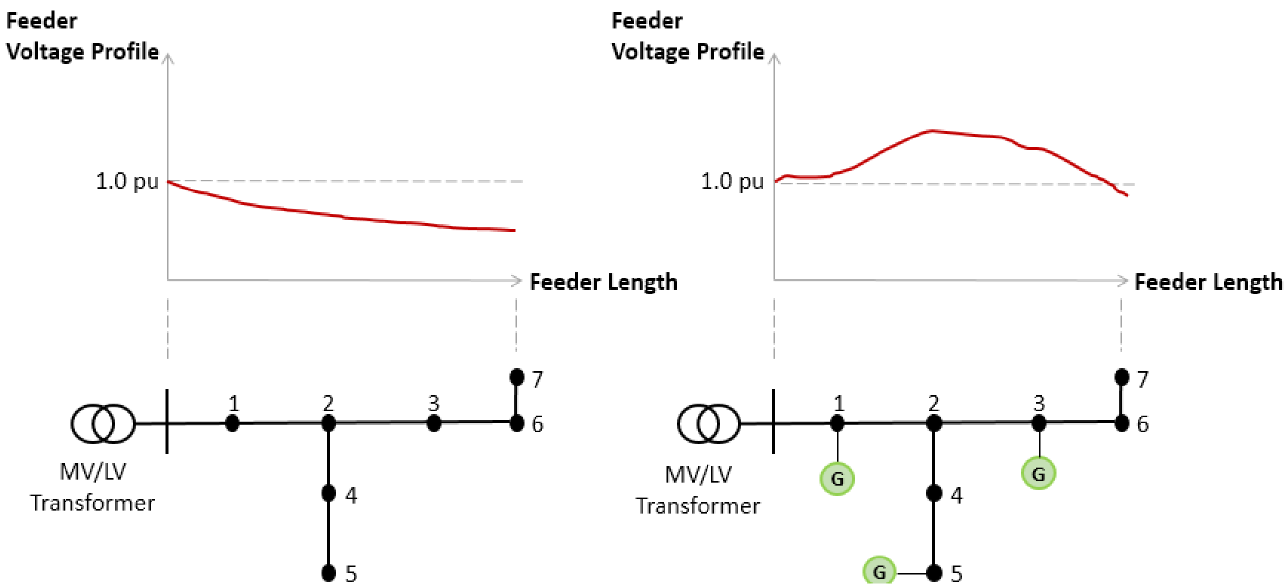


Figure 1-2: Feeder Voltage Profile for a LV feeder supplying 7 consumers:

a) LV feeder with no DG (left) *b) LV feeder with DG (right)*

McLaren [2012] summarised these methods and classified them into two groups namely empirical and statistical calculation methodologies, as discussed below:

1. Empirical Calculation methods

   This is a simplified approach to LV feeder voltage drop calculation, with two correction factors applied to the result voltage value to correct the statistically worst condition with some degree of confidence. The factors applied are the Unbalanced Voltage Correction Factor, UCF (N) and the Loss of Diversity Correction Factor, DCF (N). These methodologies assume equal consumer loading equivalent to the After Diversity Maximum Demand (ADMD) and balanced loading along distributor.

2. Statistical Calculation methods

   a) Herman-Beta (HB) method

      The Herman-Beta method is an analytical statistical method of voltage drop calculation that assumes the beta distribution as an acceptable descriptor of domestic load behaviour. It predicts voltage drop at the instant of maximum demand. The parameters used in this method include: alpha (α), beta (β) and maximum (c), normally taken as the circuit breaker size.

      Another major assumption was taken into account in the Herman-Beta before modification by Gaunt et al. [2011]. It was assumed with most loads that there were no generators or current sources in the networks modelled. This was based on the fact that the consumers on the LV feeder distribution system were taken as residential, as opposed to commercial or industrial [Sellick & Gaunt, 1995].

b) Monte Carlo Simulation (MCS)

This method uses measurements of consumers representing a future network under consideration. It uses a beta distribution for its load model and is often simply used as a base case against which to compare other algorithms being tested, as the calculations involved are time consuming.

In the comparative tests, done by Sellick and Gaunt [1995], it was found that the Herman-Beta method was the most consistently reliable method. In addition, it was established that all other methods exhibited large errors under various conditions, particularly single-phase networks. Eskom has adopted the Herman-Beta method as the preferred method for all LV domestic voltage drop calculations in South Africa and the MCS as a final check for design has been completed using the Herman-Beta method [McLaren, 2012].

1.2 Motivation for research

Given the rise of DG technology in power systems, distribution network planners need to plan power systems to include DG and operate within the statutory limits and requirements. According to Rycroft [2013], low initial penetration of DG in the power systems enables the grid to handle the problems of variability (in the case of renewable DG) as well as voltage. As penetration of DG increased, the grid capacity became strained until the voltage and variability problems have now become significant [Rycroft, 2013].

A lot of work and research has been carried out in the planning and design of electric power systems with the presence of DG to include factors such as best technology used, best location etc. whilst evaluating the impact of DG on the normal system operation such as voltage profile, stability and reliability [Viral & Khatod, 2012].

The work done, as discussed by Viral & Khatod [2012], has been used to develop planning tools to pave the way for the maximization of DG on the power system network. This was done through determining the location and optimal amount/size of DG that can be placed on the power network within the specified technical constraints and whether the power network can sustain one more DG unit at a specified location and whether it will present an adverse impact on the potential for future DG placement.

These planning tools would prove useful to the South African power utility, Eskom in its DG planning activities and are based on various power network performance indicators such as current, voltage, frequency etc.

In South Africa, the design of distribution networks has been carried out using Herman-Beta method, which uses voltage as a design parameter, ensuring the feeder voltage is within ± 10 % of the nominal voltage (230 V). The HB algorithm was modified and extended by Gaunt et al. [2011] to allow for voltage drop calculations of LV feeders with DG. The DG was represented and modelled in the algorithm as fixed negative loads at generator nodes. The output was a voltage profile of LV feeder under investigation [Gaunt, et al., 2011]. However, this DG modelling approach and the method for voltage calculation in active LV feeders with DG was not fully corrected and tested.

The HB method is the recommended method of voltage drop calculations for passive LV feeders. This new approach to include DG means that the HB method must be correctly modified and go through the thorough and rigorous testing before it is used as an active LV distribution feeder planning tool.

1.2.1 Contribution of the Research

Looking into the introduction of small scale DG in power distribution systems with the focus on the feeder voltage provides an understanding of the voltage impact of increasing DG penetration.

This dissertation outlines the research efforts in analysing the voltage impact of the increasing presence of small scale DG in distribution network systems using the Herman-Beta method. The contributions of this research are therefore:

- An analysis of the HB method for the design of passive LV feeders, as provided in the NRS 034 standard [2007], is performed. The method is thoroughly checked and validated against a Monte Carlo Simulation (MCS) for voltage drop calculation in passive LV feeders.
- The HB method, extended and developed into a tool, to enable planners and designers of power distribution systems calculate the voltage in active LV feeders (i.e. LV feeders with DG). This incorporates the DG model used by Gaunt et al. [2011] and is thoroughly tested and validated against a MCS for voltage calculation in LV feeders with DG penetration.
- Another tool developed for the voltage analysis of LV feeders with increasing DG penetration. This makes use of the HB method in a MCS to provide a much better understanding of the voltage rise constraints of an active LV feeder.

1.3 Hypothesis

The HB method is the chosen feeder voltage calculation in this research/study. To ensure the HB method is correctly implemented into a commercial design/planning tool, it should be rigorously checked and tested.

The following steps represent the part of the problem/topic that is tackled in this research project:

- A Monte Carlo Simulation (MCS) against which the validity of the HB method (for the passive LV feeders and then the active LV feeders) is thoroughly tested and validated
- HB algorithm is applied to enable the analysis of the voltage impact of DG on LV feeder systems using a variety of test scenarios to produce results suitable for LV feeder voltage rise mitigation, DG hosting capacity determination and other related tasks

The hypothesis this report addresses is therefore:

Using the negative load approach for DG models introduced by Gaunt et al. [2011], the Herman-Beta algorithm can be modified for voltage calculation in active LV feeders, validated using a MCS and applied as a tool to analyse the voltage rise constraints of active LV feeders.

1.4 Research Questions

In order to find the appropriate approach and identify the aim and purpose of this research, the following research questions have to be asked:

1. What impact does the presence of DG have on LV distribution feeder systems?
2. How successful have been attempts to model DG on LV feeders?
3. What approaches have been used to calculate voltage in LV feeder systems?
4. What assumptions underlie the Herman-Beta method and are they still valid in the presence of DG on LV feeders?
5. What changes must be made to the HB algorithm to allow for voltage calculation in LV feeders with DG and do they agree with the MCS for voltage calculation?
6. How can the HB algorithm be used as a tool for voltage analysis of LV feeders with DG?

1.5 Limitations

This research focuses on the voltage impact of increasing DG penetration in LV distribution systems. The HB method will be the centre of the work done to achieve the aims of this research. The HB method for passive LV feeders is based on the beta Probability Distribution Function (PDF) as a model for loads. The beta PDF is considered the best fit for load data obtained from the NRS Load Research Project. The load data is based on residential consumers in South Africa over several years. However, this study requires generation data in addition to the load data. This generation data is not readily available as the load data. Therefore, the beta PDF is used to model the generation at worst case scenario (i.e. DG producing maximum output at that interval of maximum generation – minimum load).

1.6 Outline of Dissertation

Chapter 1 states the background and motivation behind this research: the increased pressure from government and customers to increase the connection of DG onto distribution systems in order to satisfy their electricity demands while ensuring the voltage limit is not violated. This means that the utility should be able to plan for the increased penetration of DG in LV feeders in order to ensure the voltage in distribution systems are operating in accordance to the prescribed standards. The objective of the project was thus to develop software to enable voltage calculation in LV feeders with DG and further use this as a basis for which a planning tool can be created.

Chapter 2 does a review of literature into DG connected to distribution systems, particularly solar PV panels (PV-EG), and some of the problems that have arisen. It looks at some of the other planning tools for active LV distribution systems developed based on the various performance parameters and several approaches used to model PV-EG.

Chapter 3 introduces the HB method and gives a full description of the algorithm. A detailed analysis of the assumptions underlying the HB algorithm for passive LV feeders is carried out with the aim of assessing how it can be correctly extended to allow for voltage calculation in LV feeders with PV-EG (active feeders).

Chapter 4 thoroughly and rigorously tests the HB algorithm for passive feeder and validates the voltage calculation, based on the HB method, by Monte Carlo Simulation (MCS). The HB algorithm is then modified and extended for voltage drop calculation in active LV feeders. The HB algorithm for active LV feeders is validated using MCS. This chapter ensures that the HB algorithm is fully tested and correctly configured for both the passive and active LV feeder systems.

Chapter 5 describes in detail the development of a planning tool for active LV feeders. It applies the concept of continuously evaluating an analytical model using a MCS. The tool used for evaluation of active LV feeders, comprises the HB algorithm, used as the voltage calculating engine in a MCS to create random scenarios. All procedures, models of the load, feeder and DG are described in this chapter.

Chapter 6 describes the scenarios used to establish the effect of increasing PV-EG on the LV feeder. The results and discussion for each scenario is presented whilst referring to the base case scenario. The implications of the results are discussed with regards to the power distribution system.

Chapter 7 contains the conclusion of this study - the extent to which the work done in this dissertation answers and validates the hypothesis, recommendations, as well as final thoughts before the list of references used in this work.

Appendices contain all relevant additional material and results from the work done in this research.

Chapter 2

2. LITERATURE REVIEW

This chapter covers the relevant literature and answers some of the research questions. It provides a brief understanding of the concept of Distributed Generation, its impact on the distribution feeder systems and work done around analysis of this impact.

2.1 Distributed Generation

Distributed Generation (DG), also known as dispersed generation or embedded generation, are power generating technologies characterized by the source of energy, such as solar PV systems with the sun as the major source of energy. DG considered in this report is connected to the power system close to or at the consumer, that is small scale DG. The concept of DG has been described and grouped under various classifications based on various parameters such as principle of operation, construction and technology, size/rating etc. [Jenkins, et al., 2000].

It is relevant in this report to clarify and define DG according to size/rating of a generation unit. Katiraei & Aguero [2011] used the DG size as a classification parameter as shown in table 2-1.

Table 2-1: DG Size/Rating Classification

CLASSIFICATION TYPE	SIZE/RATING
Small Scale DG	1 W – 10 kW
Medium Scale DG	10 kW – 1 MW
Utility Scale DG	1 MW – 10 MW

There are various DG technologies that are available in the world today and their development has been driven by the increased need for electricity and development in technology. Some of the commonly used DG technologies are shown below as discussed by Sarabia [2011]:

1. **Solar Photovoltaic** – Solar Photovoltaic (PV) systems use solar cells to convert sunlight directly into electricity. The solar cells are constructed in a PV panel array that is either fixed or moving to track the sun movement for maximum power generation. The common applications for this technology include installations on residential rooftops, solar pumps, utility scale systems etc.

2. **Wind Power** – Wind power systems convert energy from wind speed to electricity. Wind, this source of energy, is highly stochastic and cannot be stored and hence turbines are used to harness its power. Wind turbines convert kinetic energy from the wind to mechanical energy (torque), which is converted to electricity by an attached generation system. Wind applications are mainly in wind farms on a utility and medium scale of power production.
3. **Fuel Cells** – Fuel Cells are devices that convert hydrogen and oxygen into electricity, heat and water in a mechanism that differs from conventional storage batteries that produce power from stored chemicals. Many of the fuel cells are named after their electrolytes such as direct methanol fuel cells (DMFCs). The main advantage of fuel cells is the lack of moving parts therefore low maintenance costs. The most common applications of fuel cells are as portable power devices in hospitals and utility power plants.
4. **Micro turbines** – Micro turbines (MTs) convert thermal energy in the form of hot gas into mechanical energy, which is then converted to electrical energy by the generation mechanism. MTs provide clean operation with no gas emissions as well as good efficiency. They have become a common feature in Combined Heat and Power (CHP) applications.

Table 2-2 shows how well the available resources and corresponding technologies in South Africa perform as well as the capacities employed.

Table 2-2 : Size and efficiency of the common DG in South Africa

DG	Solar PV	Wind turbines	Fuel Cells	Micro turbines
SIZE [kW]	1 – 1000	10 – 3000	100 – 3000	25 – 1000
EFFICIENCY RANGE[%]	6 – 19	25	36 - 50	25 – 30

Solar PV is one of the fastest growing renewable energy and DG technologies in the world today with high expectations for global electricity generation. Until a few years ago, solar PV applications were mainly off-grid such as water pumps as well as rural electrification. Today, the newly installed PV has been used directly in the distribution grid system. In this report, PV-EG will be the main focus and representation of DG on LV feeders. PV-EG, like the classification in Table 2-1, has been classified as a function of installed capacity. Gudimetla, et al. [2012] classified PV-EG as below:

1. Small-scale PV comprises few kVA single phase units e.g. 5 - 10 kW. An example is PV panels installed on rooftops of residential households. These units are usually connected to secondary distribution lines (230 V).
2. Medium-scale PV comprises larger kVA three phase units e.g. 100 – 500 kW. An example is PV panels installed on small commercial buildings.

These units may either be connected to secondary lines serving existing installations or to primary lines via independent interconnection transformers, depending on the PV-EG capacity.

3. Large-scale PV comprises MVA three phase units e.g. 500 kW - 10 MW. An example is PV of nominal capacity manageable by distribution feeders and substations. These units are connected to primary lines (11 kV) or distribution substations via dedicated feeders.

2.1.1 Impact of DG in power systems

Modern distribution systems are passive systems in which power flows (both real and non-active power) in one direction from higher voltages to lower levels. This means the behaviour of the network is well understood by system operators.

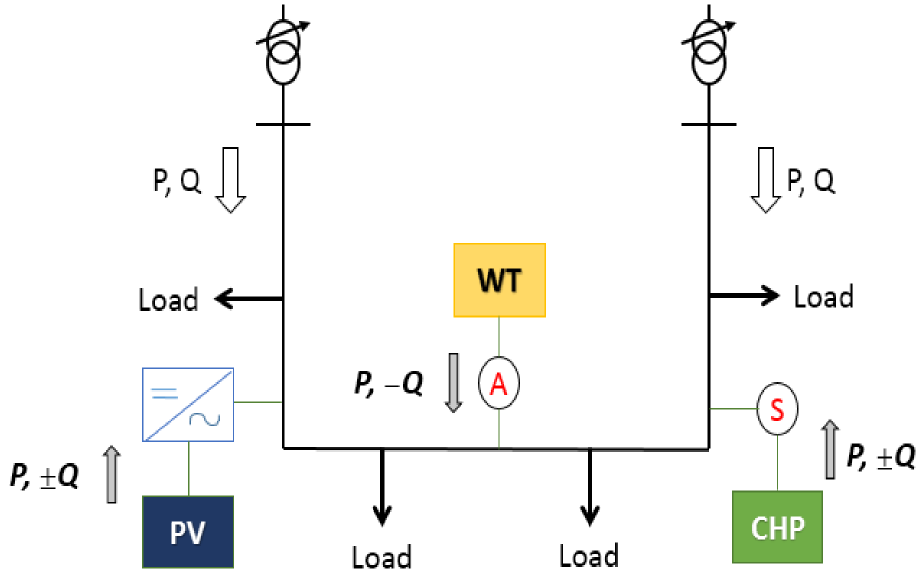


Figure 2-1: Schematic of Distribution System with DG (Jenkins, et al., 2000)

The introduction of DG into the power system, as shown in figure 2-1, causes reversed power flow. As a result, the distribution system is no longer a passive system simply supplying loads but an active system with power flows determined by generation as well as load conditions. This is shown in figure 2-1 and discussed below [Jenkins, et al.,2000]:

- The Combined Heat and Power (CHP) scheme is connected to the system through a synchronous generator. The scheme will export real power (P) when the electrical load it supplies falls below the generator output but may absorb/export non-active power (Q) depending on the excitation setting of the generator.
- The Wind Turbine (WT) is connected to the system through an asynchronous/induction generator. This generator requires non-active power (Q) to operate thereby absorbing non-active power from the system. The wind turbine also exports real power to the system.

- The solar Photovoltaic (PV) exports real power at a set power factor but may introduce harmonics.

The change in power flows, caused by the integration of DG into the power distribution system, also leads to other problems in various areas of distribution feeder systems such as feeder voltage profile and properties, feeder loading, power quality, feeder protection, feeder power losses and reliability and operation. Each of these areas is discussed further in this chapter.

The severity of the above problems on distribution feeder systems is dependent on the penetration level of DG, location of DG and electrical characteristics of the DG systems [Katiraei & Aguero, 2011]. The above factors, that influence the impact of DG on feeders, are as described by Freris & Infield [2008] below:

1. **DG penetration level** can be described as the ratio of amount of DG energy injected into the network to the feeder energy capacity. The higher levels of DG penetration bring more uncertainty in the power flows in the feeder and can lead to severe consequences.
2. **Location of DG** is the position on the feeder the DG is installed with respect to either the substation or the loads. Also known as siting, it is an important criterion in the consideration of DG impacts. Different locations on the feeder system affect the DG impacts like voltage profile, power losses etc. It is therefore important to find the optimum location for a higher level of reliability of the power system.
3. **Electrical characteristics of DG** can be described as the type of DG installed on the feeder with regards to the power output characteristics. Features of DG such as type of generator (synchronous and induction machines), availability of energy storage, as well as interconnection design can affect the power flows in the feeder and thereby leads to higher or lower severity of DG impact.

i. Feeder Voltage Regulation

One of the major obligations power utilities have is to supply their customers with high quality power at a voltage within specified limits. Voltage and voltage regulation is an important criterion in the quality of electrical supply point of view and has attracted a lot of attention in recent years. In order to regulate voltage in radial distribution systems, traditional measures are used such as Load Tap Changing (LTC) transformers, Line Drop Compensators (LDCs), line regulators on the feeders, as well as shunt capacitors. This voltage regulation is based on the one way power flow that was associated with passive distribution systems and assumes power flows from substation to loads [Sarabia, 2011].

The change in direction of power flow in feeders, caused by introduction of DG, leads to change in the voltage profile and other voltage related characteristics of a feeder. The stochastic variation of power output of DG like PV has also contributed negatively to the voltage on the LV feeder. The DG impact on voltage regulation include over-voltages, under-voltages, as well as positive impacts in LV feeders [Gonzalez, et al., 2012].

Some general rules of thumb stated by Baghzouz [2006] with regard to the DG impact on voltage regulation on an LV feeder include the following:

1. LV Feeder with a fixed substation voltage
   - DG placed on a LV feeder provides voltage support, much like a capacitor installation. However, large DG penetration may result in an over-voltage situation in some parts of the feeder, with the highest at the point of DG interconnection.
   - The closer the DG installed is to the substation, the higher the possibility of over-voltage occurence.
2. LV Feeder with a variable substation voltage
   - DG installed towards the end of the feeder will raise the feeder voltage thereby providing voltage support.
   - High DG penetration close to the substation causes under-voltage at feeder end. This is because the DG confuses the LTC causing it to set a voltage lower than what is required to meet the demand within the statutory limits.

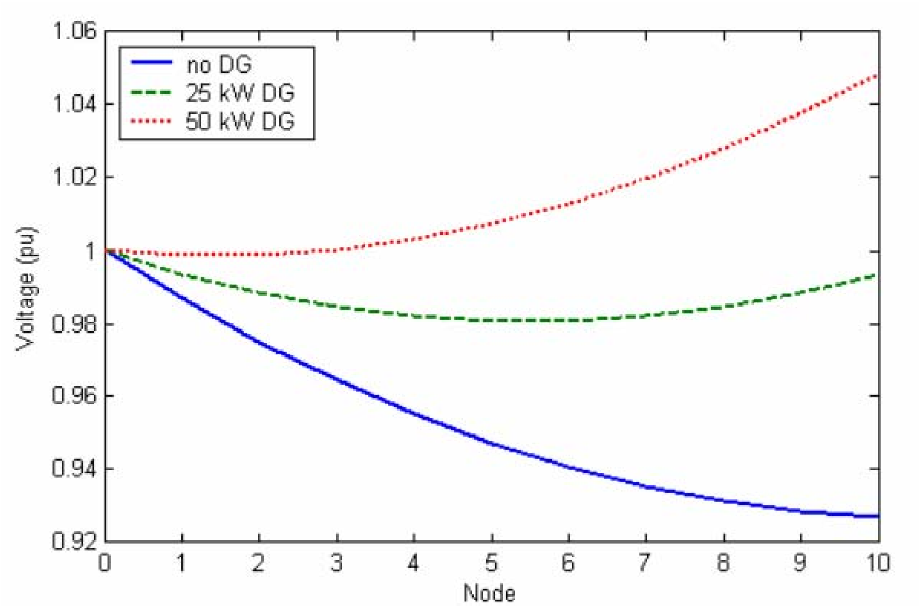


Figure 2-2: Voltage Profile along a feeder showing the effect of increasing DG penetrations [Viawan, 2006]

These DG conditions remain valid to date and have been proven correct by various researchers in the same field of study.

Work done by Mahmud, et al. [2011] and Viawan [2006] analysed the voltage rise caused by addition of DG to the distribution network, particularly focusing on the DG penetration levels. It was found that there were no serious voltage rise problems for conventional distribution networks or low penetration DG networks. Increase in DG penetration led to voltage rise problems on the network. These observations are shown in figure 2-2 above.

ii. ***Feeder Power Losses***

DG has major impact on the losses in the LV feeder, which heavily depends on the location of the DG on the feeder, feeder and load parameters. The location of DG is an important aspect that has to be analysed to achieve a reliable system with reduced losses. It is assumed that feeder losses are decreased when DG is located near loads, since most DG are consumer owned. However, increased power flow on the load side in LV cables may have undesired consequences and losses due to thermal rating violations [Sarabia, 2011].

Since DG has a positive impact on the power losses on the feeder due to its proximity to load centres, there is need to locate DG in places where they provide a higher reduction in power losses. The impact of DG on feeder losses can be analysed and studied in the same fashion as studies on capacitor placement in distribution system for loss reduction on feeders. Optimal capacitor size and location minimizes the power loss by reducing the flow of non-active power whereas the location and size of DG reduces power losses by reducing the flow of active power along the feeder [Baghzouz, 2006].

Barker & de Mello [2000] highlighted the effect of DG size on feeder losses, with consideration of the feeder capacity limits. The feeder capacity limits such as overhead line and cable thermal limits, should be considered when siting large DG units on feeders. In some cases, the DG can be allowed to inject power exceeding line thermal limits and it is more frequently the case that the voltage violations are the first limiting effect. A study carried out by Navigant Consulting Ltd. [2011] on the analysis of DG incorporation into distribution systems indicates that the feeder lines are typically designed to withstand the high levels of current for a short time interval, required for protective devices to electrically isolate and interrupt the fault before it damages equipment. Therefore, currents due to increased DG penetration can be allowed to exceed the thermal limits but only for short time intervals.

A study was done by Quezada, et al., [2006] on the impact of DG on losses, considering penetration and concentration levels. It was found that the annual power losses due to DG on a feeder took a U-shape trajectory, in that losses started to decrease with low DG penetration and increase after a minimum value with higher DG penetration levels. Also the DG concentration had impact on the power losses in that the more dispersed the DG units on the feeder, the greater the impact on power losses. Wind turbines were found to have a less positive impact on power losses given their intermittent power, which does not match well with feeder load patterns and the solar PV production was able to follow better the daily load patterns. Figure 2-3 shows the observations made on effect of DG on feeder losses.

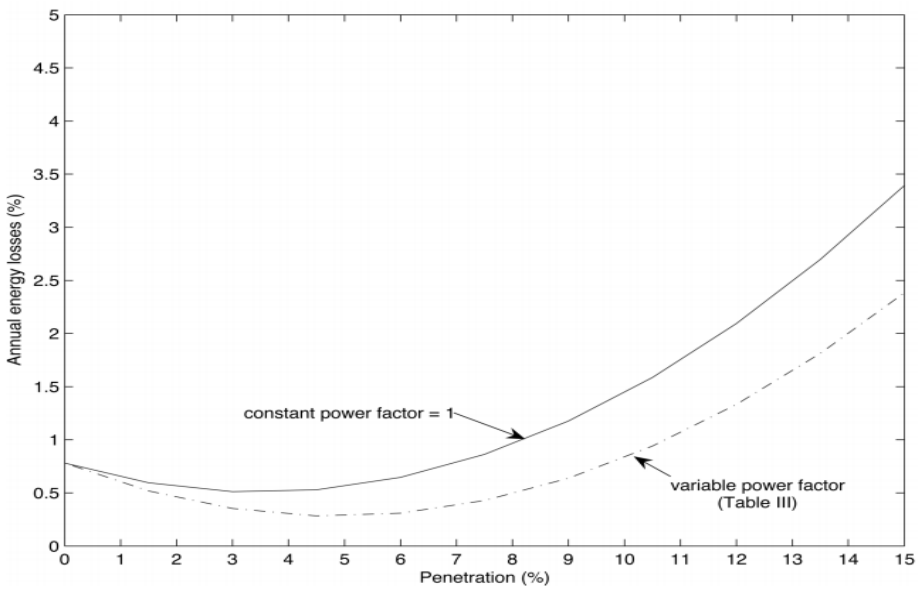


Figure 2-3: Effect of DG on feeder losses with regards to DG penetration [Quezada, et al., 2006]

iii. ***Feeder Protection***

In conventional radial LV feeder systems, protection schemes are designed as follows - overcurrent protection devices were usually graded bottom-up in opposition to the uni-directional power flow, short circuit currents were easily calculated in advance in compliance with grading intervals between relays. The addition of DG in LV feeders will alter power flows (thereby altering current flows) in various parts of the feeder during fault situations. In other words, the DG contributes to the fault current. For example, DG with generators based on rotating machines can have a continued fault current contribution of 3-5 times their rated current with an additional offset current contribution at the start of the fault. DG with inverter based generators contributes less than twice the inverter's rated output during a fault [Baghzouz, 2006].

Work done by Cheung, et al. [2007] and Hussain, et al. [2010] provides full and comprehensive studies on the effect of DG on feeder protection. The main concerns/issues of installing DG, with regards to protection in LV feeder systems are increase in short circuit currents, temporary faults, reduction of station breaker reach, sympathetic tripping, fuse-saving disruption, overcurrent protection and reverse power flow.

Some of these common protection issues are as described by Cheung, et al. [2007] and Hussain, et al. [2010] below:

- Sympathetic tripping

  This occurs when protective devices operates unnecessarily for faults in other protective zones. This is caused by the unexpected fault current contribution from the DG installed, which is a consequence of the changed power flows.

- Device discrimination

  Traditional power systems have a generation source at one end of the feeder network. This means that the fault current decreases with the feeder length due to the increase in line impedance. This property is used for discrimination of fault current magnitude based devices. However, in the case of Inverter-Interfaced DG, the fault current at all locations on the feeder is constant as the maximum fault current is limited. Consequently, the classical strategies and methods employed in current discrimination are no longer effective.

- Islanding

  The integration of DG into the power network can lead to islanding of that area of the network during the event of Loss of Mains (LOM) or Loss of Grid (LOG) i.e. when the supply from the utility is cut or interrupted. In the event of LOM, the utility has no control over the voltage or frequency. As a result, the DG continues to supply power during the disconnection, thereby supplying the fault allowing the fault to persist. This leads to unexpected voltage levels and frequency instability in case of islanded operation.

- Fuse-saving

  This is the practice of coordination of the feeder breaker/recloser to operate quickly relative to lateral fuses. These schemes are common on line reclosers installed at urban/rural boundaries. The presence of DG on feeder systems contribute to fault current during breaker/recloser operation rendering fuse-saving impossible.

- Reduction of station breaker reach

  Addition of DG leads to desensitization of the feeder over current protection devices, also defined as reduction of reach of these devices. Reach is the LV feeder distance downstream of the protective device at which the device is able to detect a fault. Utility protection engineers coordinate the protective devices by setting up the pick up current considering the expected fault current. The presence of DG reduces the sensitivity of the feeder protection as it raises the voltage profile reducing the current seen by the device.

Some general rules of thumb stated by Baghzouz [2006] with regard to the impact of DG on the protection operation of feeders are discussed below:

- The practice of fuse saving on LV feeders is not possible in the presence of DG due to the fault current contribution of the DG
- When a fault occurs downstream section of the installed DG on the feeder, the reach of the protective device upstream of DG will be reduced.
- DG of sufficient size/rating may result in sympathetic tripping of bi-directional relays when a fault occurs on adjacent feeders.

The paper by Hussain, et al. [2010] goes on to list the possible solutions to the above mentioned protection-related issues and can be used as a guideline for protection design in DG LV feeders.

iv. ***Feeder Power Quality***

Power quality in distribution systems is characterised by the parameters that express the level of harmonic pollution, non-active power as well as voltage unbalance and flicker. In other words, the power quality shows how close the system resembles the ideal system, i.e. delivering power at constant magnitude and frequency sinusoidal voltage waveform. It can be demonstrated by how well and efficiently loads connected to the distribution system run. The most common power quality aspects considered with regards to DG include harmonic distortion, voltage variations [Jenkins, et al., 2000].

1. **Transient Voltage Variations**

   Addition of DG to the distribution system can cause noticeable voltage flicker, seen in two ways as far as analysis is concerned – simple case or complex case. In the simple cases, it can be as a result of starting DG induction generators or voltage step changes in DG outputs which result in significant voltage changes in the feeder. Another cause of voltage flicker can be the stochastic variation in the DG power output such as PV-DG [Barker & de Mello, 2000].

2. **Harmonics**

   DG introduces harmonics into the distribution systems and lead to unacceptable voltage distortions. In addition, directly connected DG can lower the harmonic impedance of the system thereby reducing the network harmonic voltage at the expense of increased harmonic currents [Jenkins, et al., 2000].

   The type and severity of harmonics introduced into the distribution system by DG depends on the DG power converter technology as well as interconnection configuration. The effect of large penetration levels of PV-DG on harmonics is important given the current trend and development of additional power electronics-interfaced equipment to be connected to the distribution system such as electric vehicles, distributed energy storage etc [Begovic, et al., 2012].

3. **Voltage Unbalance**

   PV-DG connections are single phase in most cases and are connected to low voltage networks through a 'fit and inform' policy by customers. This leads to voltage unbalance in LV feeders which are normally three phase due to their varying generation characteristics throughout the day. Voltage unbalance in LV feeders lead to deterioration of performance of distribution transformers and three phase induction machines [Huat Chua, et al., 2012].

   A study by Begovic, et al. [2012] indicated that single-phase PV, usually on residential rooftops offset currents leading to voltage unbalance for example phase A of the three phase LV feeder may experience reverse power flow due to PV-DG while phase B and C remain unaffected. Such situations increase the voltage unbalance factor and this increases with penetration level.

As discussed in this section, PV-DG may have very adverse effects on LV distribution systems (feeders) and affect the quality of power delivered to the customers. There is need for utility planners and designers to plan, assess and quantify the impact of increasing PV-DG penetration on LV feeders in order to attain a clear understanding of the technical requirements placed on both the customer and system operator sides to achieve a reliable and high quality LV distribution system. This study is focused on the voltage impact of DG on LV distribution systems and therefore it is important to accurately calculate the voltage in LV feeders.

2.2 Analysing the impact of DG penetration on LV feeder

The analysis of the impact of DG on LV feeders results in the determination of the DG hosting capacity of LV feeder system. The DG hosting capacity of an LV feeder is the maximum DG penetration on the LV feeder for which the feeder operates satisfactorily. It can also be described as the amount of DG for which the LV feeder performance becomes unacceptable. DG hosting capacity of an LV feeder is determined by comparing a performance index/parameter, which is calculated as a function of DG penetration level, with its limit (i.e. the satisfactory level of operation of the LV feeder). The determination of DG hosting capacity of an LV feeder using performance indices (as a function of DG penetration) such as voltage, protection etc and this can be illustrated using figure 2-4 [Yang & Bollen, 2008].

Figure 2-4 shows the maximum amount of DG penetration for which the limit of improvement is exceeded and can be used to determine the DG hosting capacity of the LV feeder. There are various performance parameters used for the determination of DG hosting capacity of LV feeders. Performance indices can be defined such that a lower value corresponds to better performance or a higher value corresponds to better performance [Bollen & Hassan, 2011].

Vega [2012] discussed some of the performance indices as shown in table 2-3. The DG hosting capacity approach can be applied to these performance indices that can be measured in the network.

Table 2-3: Common Performance Indices/Parameters for DG Hosting Capacity

PHENOMENON	PERFORMANCE INDEX
Voltage	Over-voltage, Under-voltage, Voltage Distortion, Imbalance, Flicker
Power	Overload equipment, Losses
Current	Cable Ampacity, Harmonic Distortion
Frequency	Over-frequency, Under-frequency
Reliability	Number of outages, Number of voltage sags, Number of failures in equipment

A lot of work has been done on determining the DG hosting capacity of distribution systems based on the above mentioned performance indices. Čuvičin, et al. [2008] considered the effect of DG on frequency control during normal and emergency operating conditions and based the methodology of their work on the hosting capacity of transmission and distribution systems to determine the frequency control efficiency. The hosting capacity was obtained as the penetration level for which the performance index exceeds its frequency limit.

Bertini, et al. [2011] developed and described an approach to evaluate the hosting capacity of LV distribution networks in Italy according to three technical constraints/performance indices - transformer and line thermal limits, steady-state voltage variations and rapid voltage changes limits. Etherden & Bollen [2011] applied the hosting capacity method to a realistic distribution system to assess the degree of DG penetration acceptable to the system without endangering reliability or power quality.

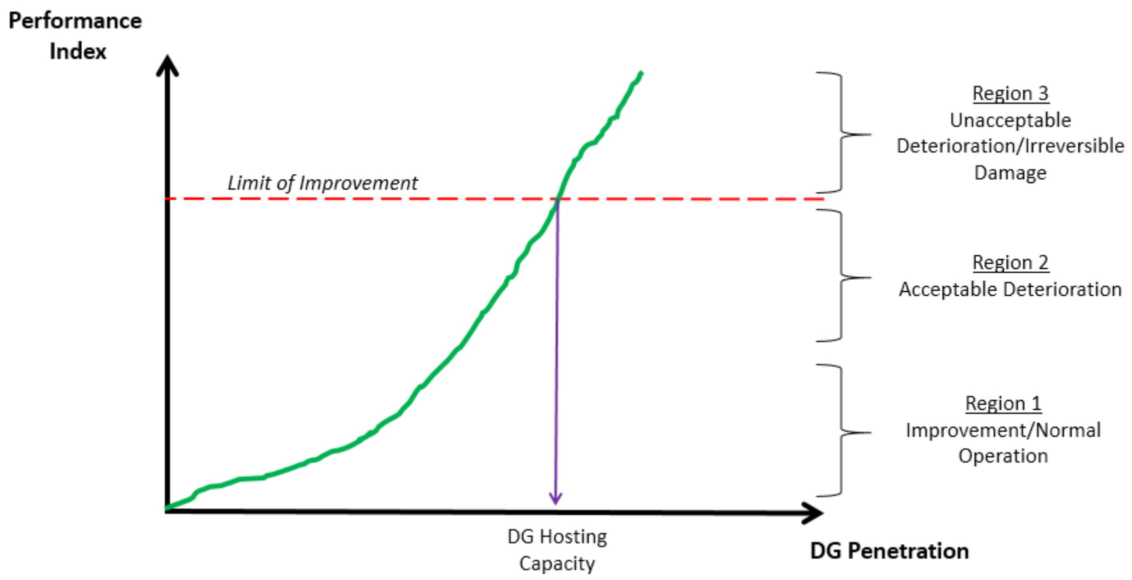


Figure 2-4: Hosting Capacity Approach in which the performance index Low in the ideal case [Bollen & Hassan, 2011]

In this study, the area of interest is the voltage impact of DG on the LV feeder, which therefore makes use of over-voltage as the limit of improvement. The calculation of DG hosting capacity of LV feeders requires appropriate voltage calculation techniques, using strong and reliable load and DG models in order to accurately assess the voltage impact of increasing DG penetration on LV feeders.

2.2.1 Voltage calculation techniques in LV distribution networks

The analysis of voltage impact of PV-EG on LV distribution systems has been carried out in various studies using appropriate voltage calculation methods. There have been various methods put forward for LV feeder voltage calculation and can be divided into two main categories – statistical and deterministic techniques

i. Statistical Techniques

These techniques are strongly believed to be far more accurate and appropriate than traditional deterministic and empirical methods since they model load and DG models with diversity much more accurately. Such methods include Monte Carlo Simulation (MCS), method of Cumulants and the Herman-Beta method.

The MCS technique for voltage calculation is the most commonly used method comprising repeated random sampling from a large data base of discrete load or PV values or simply a PDF describing the load or PV. Its popularity can be attributed to the improvement in technology allowing for higher computer speeds. Demailly, et al. [2005] used the MCS approach in the calculation of the voltage profile for a whole year in which the year was divided into periods in which statistical parameters of the loads and DG are constant. The drawback in using the MCS is the large number of iterations required, which becomes time consuming, as mentioned by Viawan [2006]. Soroudi, et al. [2012] implement the MCS as a method of verifying and checking other probabilistic/statistical methods. This approach has been taken up in this study to verify and check the selected voltage calculation technique.

Ruiz-Rodriguez, et al. [2012] introduced a method of Cumulants and moments, which replaces the convolution of random variables with the sum of their Cumulants to reduce computational burden. It makes use of the Pearson familiy of PDFs, which includes the beta PDF to model load.

In South Africa, an analytical probabilistic method of voltage drop calculation was developed by Dr. Ron Herman of Stellenbosch University called the Herman-Beta (HB) method and is the recommended method of voltage calculation in passive LV feeders in South Africa. It makes use of the beta PDF, as an acceptable load model taking into account the stochastic load behaviour, to calculate the voltage drop in LV feeders. Like the method of Cumulants in some aspects, it involves the calculation of moments from the load model to obtain the consumer voltage. The advantages of the HB method are that it is simple to use and is not time consuming like the MCS [Herman & Gaunt, 2008].

ii. Deterministic Techniques

These are the most common methods used in various studies and rely on the use of average inputs for loads and DG, as in traditional power flow methods. Some of the common methods include the Newton Raphson iterative method, and the backward-forward sweep technique, a technique first used by Cheng & Shirmohammadi [1995] and later used by Tant, et al. [2013] in a load flow method to determine the supply terminal voltage of a household. These methods do not take into account the variation and stochastic properties of loads and DG.

Modified deterministic approaches have been developed to cater for the variation of loads and DG with time. McLaren, et al. [2012] discussed the use of empirical methods which assumed equal loading of each consumer equal to the ADMD and used correction factors to correct for the statistically worst condition to take into consideration the variation of the load. Masoum, et al. [2012] made use of recorded load and PV data in a time step process to account for variation of load and PV throughout the day but did not account for the variation of the load in a set of consumers at specified time steps.

The use of statistical techniques have become popular in order to fully account for the stochastic behaviour of load and DG profiles.

2.2.2 DG Modelling in LV feeders

In this report, DG on LV feeders is represented by solar PV. In order to assess the impacts of PV-DG on LV feeders, a suitable model must be used to describe and mimic the behaviour of the solar PV power output. In most work done, solar PV modelling involves the modelling of solar irradiance at a site and converting this data to power output by the solar array. This is because the sun is the source of energy for solar PV panels and is only available during the day. In addition to this time limitation, there are many factors that affect the amount of sunlight that reaches the solar panels such as clouds, dirt, shadows from trees and buildings, etc. The amount of power drawn from sunlight depends on the PV technology and its efficiency. The power output of a solar PV panel usually depends on the area of the solar panel (A), panel efficiency (η) and solar irradiance (I_t), which is stochastic in nature, shown below in equation (2.1):

$$P_{PV} = A\eta I_t \text{........(2.1)}$$

Studies in papers by Conti et al. [2007] and Ruiz-Rodriguez, et al. [2012] account for the difference between values of solar radiation outside the atmosphere and on the earth surface by use of an hourly clearness index, k_t. This index is the ratio of actual irradiance to that above the atmosphere and from it, it is possible to determine solar irradiance on a solar panel considering its inclination from the horizontal earth surface. The output power from the PV system was obtained from the solar irradiance using equation (2.1).

In load flow studies, PV-DG has been commonly modelled as a 'negative load'. Work done by Harrison & Wallace [2005] successfully used a negative load approach to model fixed power factor steady state DG to evaluate the capacity of distribution networks for connection of DG. The use of negative loads as a modelling approach for DG is also discussed by Brown & Freeman, [2001] in a paper on reliability assessment of DG.

It indicates that from a utility perspective, modelling DG as a negative load is reasonable since utilities will require that DG units be disconnected like other loads during faults or disconnection of utility supply. A number of PDFs have been used for solar parameters (such as clearness index, solar irradiance) in order to represent and predict the PV-DG power output. A normal (Gaussian) distribution has been used by Di Piazza et al. [2008] to represent daily solar irradiance and the energy capability of the PV plant, in the study performed, was obtained by describing the atmospheric variation trends using continuous functions, defined by two parameters. A beta PDF has been used to represent how the random behaviour of solar irradiation, from solar PV power can be obtained, by Karaki et al. [1999], Atwa et al. [2010] and Soroudi et al. [2012]. The beta PDF is the most commonly used PDF and usually gives the best fit for solar data. Work done by Barton, [2007] and Herman & Gaunt [2008] highlight the advantages of the beta PDF above other distributions – its useful property that allows all possible values (solar irradiance, clearness index etc.) to lie within a finite range.

2.3 Concluding Remarks

In conclusion, the literature review has highlighted the following:

- The presence of DG can have an adverse impact on the power quality on LV feeder distribution systems with its impact on the feeder voltage being the most discussed and examined issue. Consequently, the performance index examined in this study to assess the impact of DG on LV distribution systems will be voltage, with an interest in the over-voltage (voltage rise) caused by presence of DG.
- Given the choice of performance index, i.e. voltage, combined with the associated network measurement of voltage rise, the voltage impact analysis approach used will be such that the value of performance index is low for better performance (ideal case). The voltage rise constraints of LV feeders with DG is one of the objectives of this study.
- The Herman-Beta (HB) method can be used to evaluate the voltage on LV feeders with DG and can therefore be used as a tool in the process of assessing the voltage impact of DG on LV feeders using the voltage as the performance index.
- The ‘negative load’ DG modelling approach has been used and found acceptable in many recent research material published and therefore is acceptable in this study. This study will use probabilistic negative load to model the DG performance with the beta PDF as the probability distribution of choice.

Chapter 3

3. THE HERMAN-BETA (HB) METHOD

The chapter provides a brief description of the HB method i.e. its assumptions and methodology in voltage calculation in passive LV feeders. In addition, the effect of introduction of DG on the understanding of the workings of the HB method is analysed and recommendations made. Any additional information relevant to this chapter can be found in Appendix A.

3.1 Introduction

In order to ensure an acceptable quality of voltage at the terminals of customers, there is need for LV feeder designers to accurately determine or calculate the maximum voltage drop at worst case conditions. In South Africa, the Herman Beta method is a recommended and nationally accepted method of voltage drop calculation in LV feeders as in the NRS 034 and SANS 507 standards [2007]. The Herman-Beta algorithm is an analytical probabilistic voltage drop calculation tool developed to transform currents, which are assumed to be beta-distributed to beta-distributed voltages/voltage drops. It is important to note that because the algorithm is a transformation based on analytical statistics, it therefore automatically includes the effects of diversity. It is designed to calculate voltage drop for the three network topologies – three-phase, bi-phase and single phase network systems [Herman & Gaunt, 2008].

The basic assumptions that underlie the Herman-Beta method for LV feeder design purposes, as stated by Gaunt, et al. [2011], include the following:

1. The maximum voltage drop occurs at the interval of maximum demand for the anticipated group of consumers
2. The loads are represented as currents at unity power factor – as maximum demand residential loads tend to unity power factor
3. At any specified interval, the collective loads may be represented as a statistic whose description fits the Beta PDF.
4. At this time interval the load currents are assumed to be independently distributed – this is valid if the statistic is considered at a single time interval
5. LV feeder impedance is regarded as resistive at a specified temperature – these feeders typically have small reactances (phase spacing about 200mm for bare conductors and less than 20mm for aerial bundled conductors)

Based on the assumptions mentioned, the following are the basic steps used in the HB method as described by Herman and Gaunt [2008]:

- Set up network topology, including the beta distribution of the load on the feeder
- Calculate the possible maximum and minimum consumer voltages
- Obtain the first and second statistical moments of the consumer voltage, $E(V_c)$ and $E(V_c^2)$
- Using the statistical moments, derive the parameters, α_v and β_v of the consumer voltage, $V_{a'n'}$
- The design feeder end (consumer) voltage is the quantile value calculated according to chosen risk, which is simply an indication of the extent to which the designer is willing to allow for uncertainty.

It is also important to note that all steps in the HB algorithm are linear. Figure 3-1 shows the general procedure used in the HB algorithm. The full HB algorithm can be found in Appendix A.

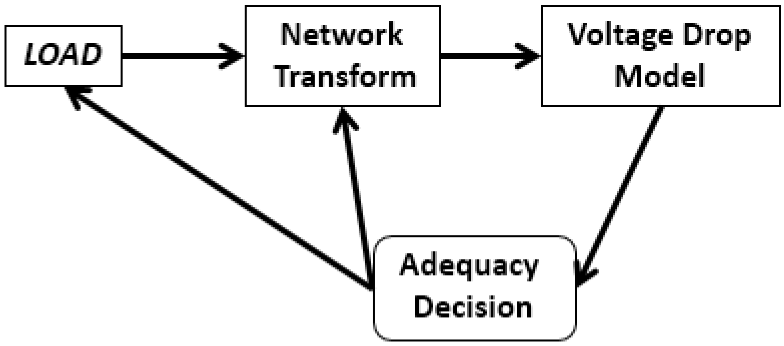


Figure 3-1: Voltage Drop Calculation in passive LV feeders using the HB algorithm [Herman & Gaunt, 2008]

3.2 Incorporating Distributed Generation in the HB method

There is need to analyse and cross check the validity of the above mentioned assumptions in the presence of DG in power systems. Some of the important and relevant assumptions and the effect of the introduction of DG on their validity are discussed below.

3.2.1 Variation of DG and load with respect to time

In passive LV distribution systems, the voltage decreases along the LV feeder length away from the substation. In calculations of voltage drop along LV distributed feeders, for passive feeder design, the maximum voltage drop was found to occur in intervals of peak (maximum) load demand. This condition has been used by various LV feeder designers and is the design assumption used in the Herman-Beta method [Herman & Gaunt, 2008].

However, the introduction of DG in power systems creates the need to consider the whole DG and load profile with respect to time. The amount of DG that can be connected (i.e. DG penetration limit) is usually calculated using a deterministic approach which is also referred to as "worst-case approach". In this approach, the DG penetration limit is determined by the lowest amount of consumption (load) and the highest amount of production (DG). In reality, DG power output can vary stochastically, particularly renewable-based DG such as solar PV, wind etc. and the load demand cannot be known with certainty, also given its varying behaviour.

The disadvantage of using a worst case scenario (minimum load - maximum DG) is that it leads to under design i.e. under estimation of DG penetration limit on LV feeders. This happens because minimum load and maximum DG may not occur at the same time [Viawan, 2006].

The varying nature of solar PV power output has significant impact on the occurrence of over-voltages on LV feeders with residential loads. This is because the typical residential load peaks during the evening/night time, when there is little/no PV generation. It is also important to note that the load demand is relatively low during daytime when PV generation peaks causing over-voltages due to reversed power flows. This load – PV generation relationship is illustrated in figure 3-2 below. It is also of significance to note the good correlation of typical industrial and commercial load profiles with solar PV profile (i.e. peaks during day and minimum during evening/night times). This tends to reduce the likelihood of over-voltages [Tonkoski, et al., 2012].

Bollen & Hassan [2011] suggest the use of probabilistic design methods to consider the stochastic behaviour of both the load demand and DG. The introduction of DG on feeder systems implies that original feeder assumptions no longer hold. The correlation between the production and load typical profiles is a suitable design parameter [Bollen & Hassan, 2011]

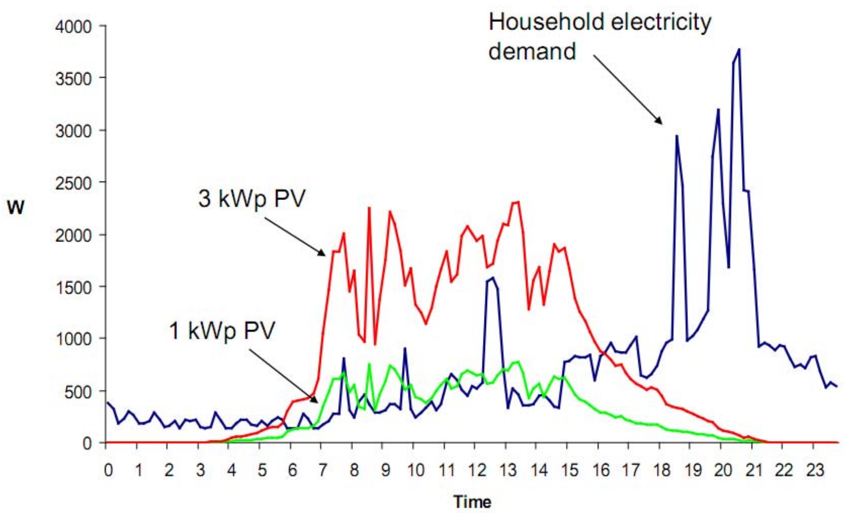


Figure 3-2: On-site PV generation in distribution grids [Widén, 2012]

Paraskevadaki, et al. [2009] indicated that there are three typical load profiles – residential, light industrial and commercial as shown in figure 3-3. Although it may be useful to note that both the commercial and industrial load profiles are similar to the solar PV profiles as shown in figure 3-3, this report is focused on the residential consumer loads.

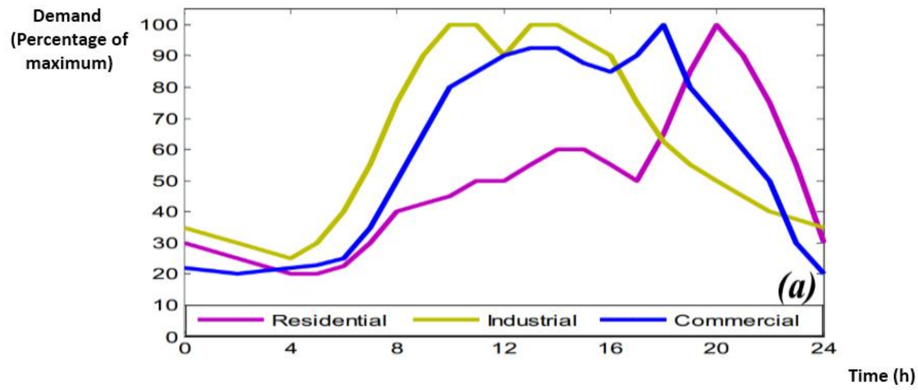


Figure 3-3: Daily load profiles for the different load types [Paraskevadaki, et al., 2009]

Furthermore, Reinecke et al. [2013] conducted a study to identify opportunities and hurdles towards maximising the rooftop solar PV within mid-sized municipalities and referenced Eskom's proposed 'Embedded Generation on Low Voltage networks' connection criteria, drafted by Carter-Brown [2012], as a guideline in their study. This Eskom draft has been approved by the working group and is at the South African Bureau of Standards (SABS) for language editing before publication into a standard/code.

Carter-Brown [2012] provided restrictions on the installed EG generation size, irrespective of whether more roofspace was available. These restrictions provide a guideline on the maximum size of PV installation that may connect to the distribution grid without the need for further network studies. The maximum sizes were defined within three main distinctions of the main distribution network:

- **Shared LV feeder** (typical in normal residential areas)
  The maximum generation should be less than 25% of the Notified Maximum Demand (NMD), which is defined as the ADMD in this report.
- **Dedicated LV feeders** (typical in light commercial, industrial and agricultural areas)
  The maximum generation should be less than 75% of the Notified Maximum Demand (NMD)
- **Medium Voltage (MV) feeder** (typical in larger industrial and commercial areas)
  The maximum generation should be less than 75% of the Notified Maximum Demand (NMD) for customers connected to the MV feeder and should not exceed 15% of the MV feeder peak load.

It is very important to know that in most tests carried out for DG hosting capacity, these scenarios are often based on a worst case methodology where high load and low load conditions are taken into account. These conditions are treated separately so as to observe the extreme impacts on the LV systems [Conti & Raiti, 2007].

Povlsen [2002] describes the significance of the high-load and low-load conditions in the analysis of PV penetration. In power systems, PV penetration is more suitable when its generation coincides with heavy load situations as it describes conditions typical in areas with increased use of air conditioners during summer. PV penetration is considered less suitable for areas where its generation coincides with low load conditions, where use of air conditioners is limited and the low load conditions are due to limited use of home appliances (such as lighting) during the day (i.e. low load demands from households and industry due to holiday seasons).

3.2.2 Power factor

In the Herman-Beta method, the voltage drop is calculated under the assumption that the load is at unity power factor (upf). This is because the majority of consumers' high load appliances are resistive, and this assumption was validated through measurements by supply utilities at the time of system peak loading [Herman & Gaunt, 2008].

The effect of DG on power system LV feeders depends on the generator and technology implemented, as well as operating conditions. The various DG power system interfaces include synchronous machines (deliver power and absorb non-active power), asynchronous machines (deliver power but likely to absorb non-active power to operate) and power electronic inverter (deliver power at unity pf but likely to introduce harmonics). The DG considered in this report is solar PV and has a power electronic inverter as an interface [Zobaa & Bansal, 2011].

A lot of work/research involving DG on LV feeders has assumed operation at unity power factor [Begovic, et al., 2001; Mahat, 2010; Olivieri, et al., 2012].

It was suggested and found that most DG units in the world, particularly PV, are designed to operate at unity power factor, which means they only provide active power to the feeder system. An allowance is however given for some DG systems to operate at non-unity power factor. The operation of DG units at unity power factor complies with the CEI 11-20 and IEEE 1547 standards.

Work was done by Paraskevadaki, et al. [2009] to investigate the DG power factor regulation capability and its benefits on LV feeders. It was suggested that the capability of DG, normally operated at unity power factor, to regulate their reactive power output through pf regulation was unexploited.

It was found that although pf regulation capabilities create benefits of voltage regulation, energy losses reduction and resulting higher total power factor, these benefits are not spectacular. The assumption of operation at unity power factor holds, given that the DG in question is solar PV.

Begovic, et al. [2001] discusses the use of PV-DG systems by residential consumers. PV systems are designed for operation at unity power factor (i.e. providing only active power) for the benefit of the residential customers. This is because the utility charges customers only based on the active power they draw from the grid. However, it should be noted that PV systems may also be operated at non-unity power factor for situations that may permit the generation of limited amounts of non-active power by PV systems. Utility regulations on operations of grid-tied PV-DG, as in the IEEE 929-2000 Standard, indicate PV systems should in fact operate at power factor greater than 0.85 (leading or lagging) when output is greater than 10 % of its rating. Any systems that provide non-active power compensation must be operated with approval of the utility.

3.2.3 Modelling DG performance

The HB method is based on the assumption that the beta distribution can be used as a suitable representation of load currents. The negative load model approach, used to model DG on the feeder, means that like loads, DG may be represented as resistance, current or power in an electrical sense.

Therefore the same reasoning, used by Herman & Gaunt [2008] to support the representation of loads as current sinks, can be used in this study to support the representation of DG performance as current sources. These reasons include:

- It is the best representation of the mixed DG units typical of residential consumers. For instance, PV panels installed in residential households differ in various ways that may affect their output such as orientation, actual size/rating etc.
- The measurement of DG performance as currents can be carried out accurately and inexpensively. This is similar to the work done in the Load Research Project in South Africa.
- The magnitude of current-modelled DG, like load, is independent of the voltage drop or rise that may occur in the feeder during operation.
- In traditional methods of voltage calculation in LV feeders, loads and generation units specified as power are usually converted to equivalent current at rated voltage. The use of currents as a representation of DG performance has already been successfully carried out in various related work.

The conclusions made in Chapter 2 highlighted the usefulness of the beta PDF as a good model for the performance of DG, particularly PV-DG. The advantages of the beta PDF as a suitable model fit have already been discussed by Herman & Gaunt [2008] and these can be applied to DG performance:

- It is constrained to a finite base in the same way that the DG currents are usually confined from 0 to the maximum rated output current.
- It can be negatively and positively skewed enabling it to model low and high DG output
- Its parameters alpha(α) and beta (β) can easily be calculated and obtained from the data
- It is conducive to convenient statistical analysis since the moments are gamma functions
- It can be readily incorporated into voltage calculations on feeders, eliminating the need to use diversity correction curves.

Like the load in the HB method, the DG performance can be appropriately represented by a beta PDF. Figure 3-4 shows a simple illustration of how the beta PDF is used to represent DG performance in this study using the arbitrary DG unit parameter records in table 3-1.

Table 3-1: Data for selected DG units rated at 230 V

DG unit	DG unit power	DG unit current		Beta PDF parameters	
		Minimum	Maximum	α	β
A	13.8	0	60	1.5	4.0
B	11.5	0	50	4	2
C	18.4	20	80	4	4

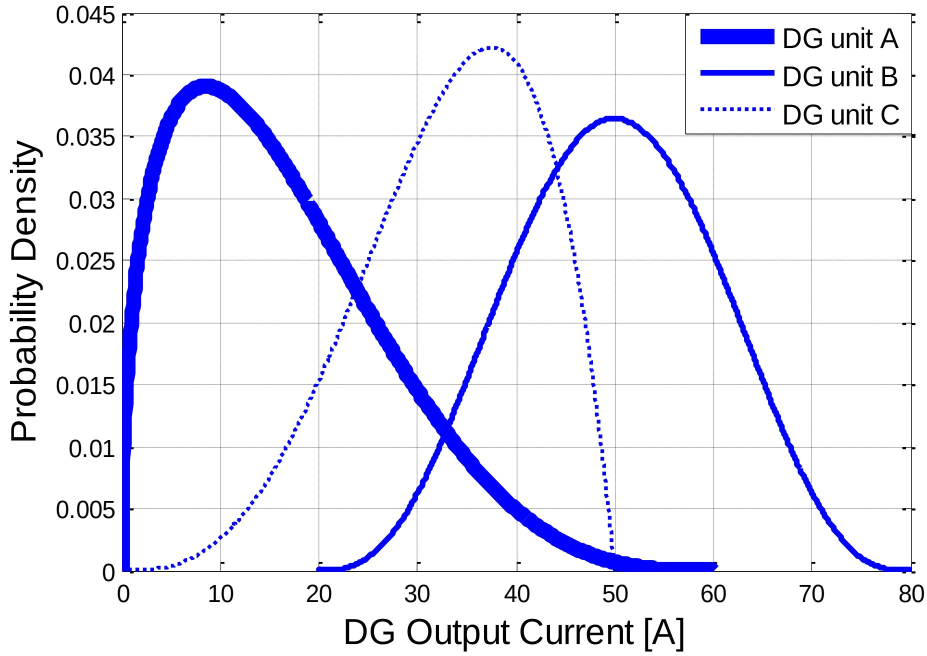


Figure 3-4: Illustration of beta PDF representation of performance for 3 selected DG units

From figure 3-4, the following observations can be derived from the beta PDF representing the DG units A, B and C:

- The beta PDFs for the DG units have a finite base from minimum DG current to the maximum rated current of the unit e.g. the currents recorded for DG unit C range from 20 A to 80 A. This allows the beta PDF to be scaled for a DG unit of any size/rating.
- The difference in skewness of the beta PDFs for the different DG units can be noted. The beta PDF for A is positively skewed to indicate that most of the current values recorded from the DG unit A are below the mean current and the beta PDF for C is negatively skewed to indicate that most of the current values recorded from DG unit C are above the mean current. The beta PDF for DG unit B takes the shape of a normal distribution (i.e. beta-normal distribution) and is neither positively or negatively skewed to indicate that there are as many currents recorded that are above the mean as there are below the mean.

One of the limitations as mentioned in Chapter 1 of this report was the lack of readily available high resolution PV-DG generation data (recorded current values) for use in this study. A viable solution to this limitation was to use the beta PDF to model the maximum output of a DG unit, as this is the interval of interest in this work i.e. the period of maximum generation presents a problem on the voltage of the feeder. One way to represent this interval is by using the beta PDF to represent a generation output that has very little deviation from selected average current output i.e. a beta PDF to model the generation deterministically.

3.2.4 The concept of the design risk used in the HB method

The design risk is an indicator to the designer as to what extent he/she is to allow for uncertainty. The concept of risk can be described as the state of uncertainty where some possible outcomes/events have an undesired effect or significant loss. It is the complement of confidence level. In statistics, a confidence interval is defined as an estimated range of values which is likely to include an unknown population parameter, which can be estimated at a specified quantile or level of confidence [Ip Cho, 2013].

In the HB method of voltage calculation, the beta PDF of resultant voltages provides the confidence interval i.e. an estimated range of voltage values in which the required design voltage lies. The desired voltage extracted is associated with particular levels of confidence (conversely levels of risk), as described by Herman [2001].

In the HB algorithm for passive feeders, the chosen design risk is usually 10 %, which is also equivalent to 90 % confidence level. However, with the introduction of DG to passive LV feeders, the design risk chosen needs to change to accommodate the use of the HB method in active LV feeders. This is explained below.

i. Case study – Validation of the change in risk level used in the HB algorithm

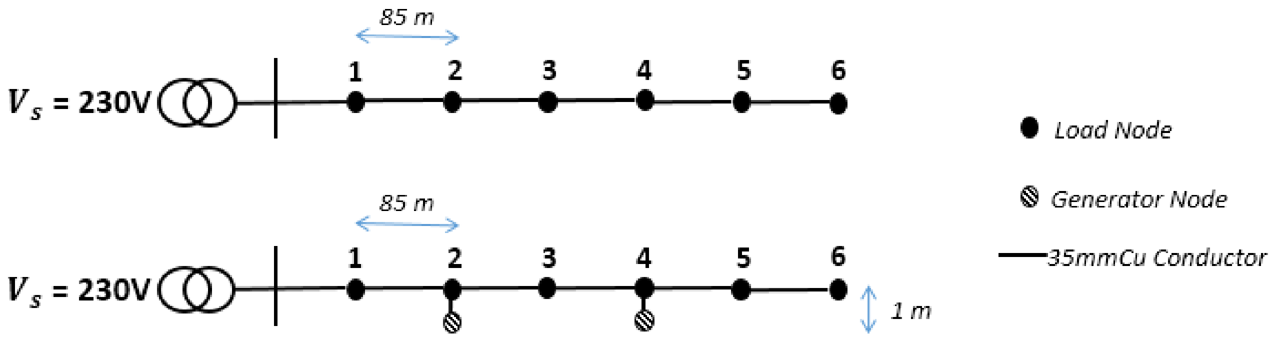


Figure 3-5: Passive LV feeder (above) and Active LV feeder (below)

The LV feeders in figure 3-5 are the test feeders used to illustrate and validate the need to change the risk level used in the HB algorithm for active feeders. The LV feeders have 6 nodes (3 customers on each node) and inter-node spacing using 35 mm^2 Copper cable conductor of length 85 metres for load nodes and 1 metre for distance between load and generation nodes (i.e. nodes with PV-DG). The passive feeder has no PV-DG while the active LV feeder has one three phase solar PV panel at nodes 2 and 4. Using the HB algorithm, the resulting beta distribution of voltages is examined for each feeder to explain the significance of design risk levels (10 % and 90 %).

ii. Voltage Drop in a Passive LV Distribution Feeder (Loads only - no DG)

Figure 3-6 shows the beta distribution of feeder voltages from which **voltage drop** is calculated and the low risk level at which a decision is made on the resultant voltage chosen. From figure, it can be observed that the **voltage drop**, caused by the loads on the feeder, results in a beta distribution towards the left from the supply voltage.

The HB algorithm usually uses a 10 % risk level to determine the voltage at the end of the feeder. This 10 % risk, equivalent to 90 % confidence level, shows an indication of the extent to which the designer/planner is willing to allow for uncertainty in the value selected.

Loads on a passive feeder cause the voltage to drop from the supply voltage, 230 V, progressively to the end of the feeder away from the supply substation. This means the selected design risk level should be associated with a low probability of occurrence of feeder end voltages, less than the selected design feeder end voltage. The beta PDF of resultant voltages of the passive LV feeder is shown in figure 3-7.

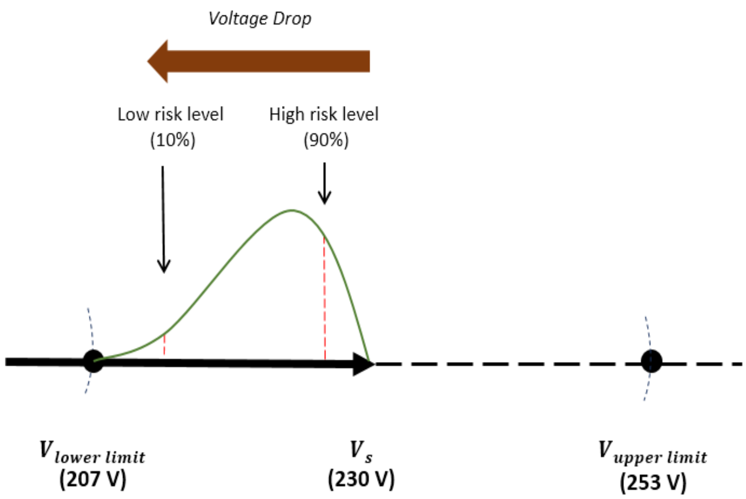


Figure 3-6: Design Risk for Passive LV feeder (Loads only)

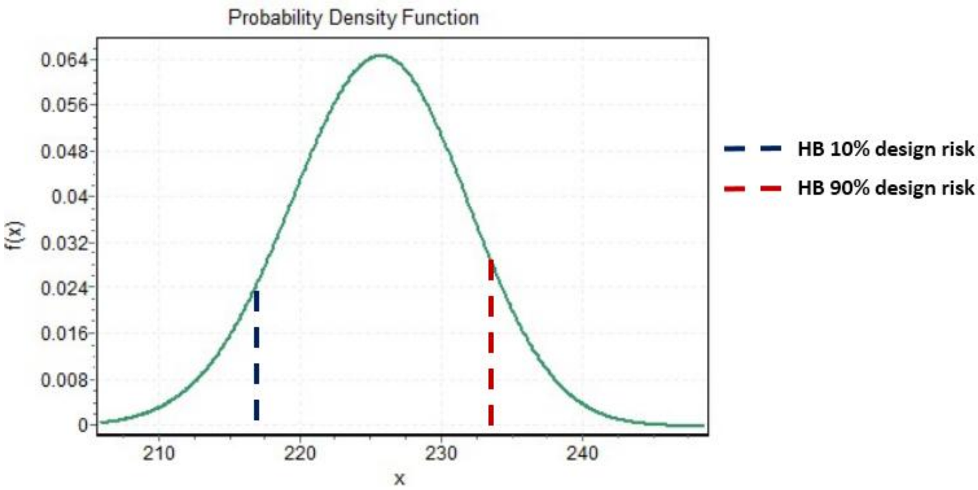


Figure 3-7: Beta Distribution of resultant end voltage for passive LV feeder

The information in table 3-2 is obtained from the beta PDF shown in figure 3-7. From table 3-2, it is observed that the probability of occurrence of voltages less (or voltage drops higher) than the chosen end voltage (or voltage drop) is lower than with a higher design risk. The basis of the design of passive feeders is the voltage drop and therefore a design risk of 10 % is used to obtain the design voltage at the 10th percentile of the range of resultant voltages. There is a low probability of occurrence of voltages below this value.

Table 3-2: Significance of design risk levels

DESIGN RISK [%]	FEEDER END VOLTAGE (V_{con}) [Volts]	P (x < V_{con})	VOLTAGE DROP [%]
10	217.48	0.1006	5.85
50	225.54	0.5001	2.36
90	233.32	0.89997	-1.00

iii. Voltage Rise in an Active LV Distribution Feeder (Loads with DG)

The addition of PV-DG in an LV feeder can lead to reverse power flow in the feeder thereby resulting in a voltage rise in the feeder. From figure 3-8, it can be observed, that the **voltage rise**, caused by the PV-DG on the feeder, results in the beta distribution towards the right from the supply voltage. This means that the level of low risk used in the HB algorithm, as shown in figure 3-8, is changed to 90 % design risk for active LV feeders.

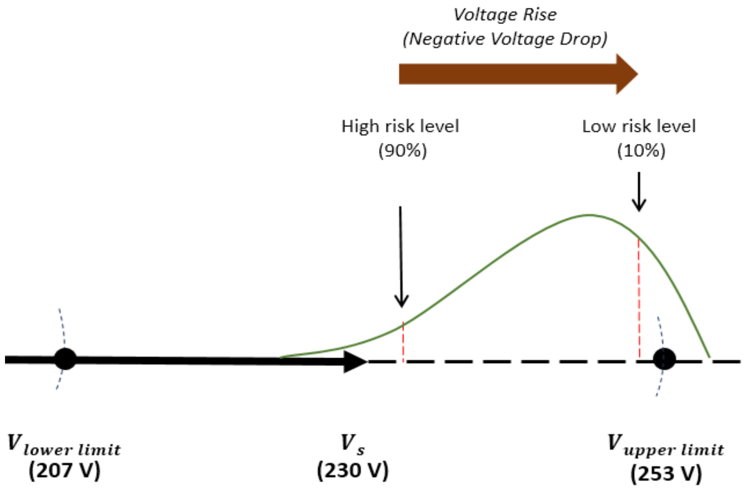


Figure 3-8: Design Risk for Active LV feeder (Loads with DG)

Addition of PV-DG on a passive LV feeder makes the feeder an active feeder. The introduction of PV-DG on the feeder causes reversed power flow causing voltage rise on the feeder at points/nodes where the PV-DG is located.

This means the selected design risk level should be associated with a low probability of occurrence of feeder end voltages higher than the selected design feeder end voltage. In the Herman-Beta algorithm, the design risk level of 90 % is used for voltage rise assessment in active feeders. The beta PDF of resultant voltages of the passive LV feeder is shown in figure 3-9.

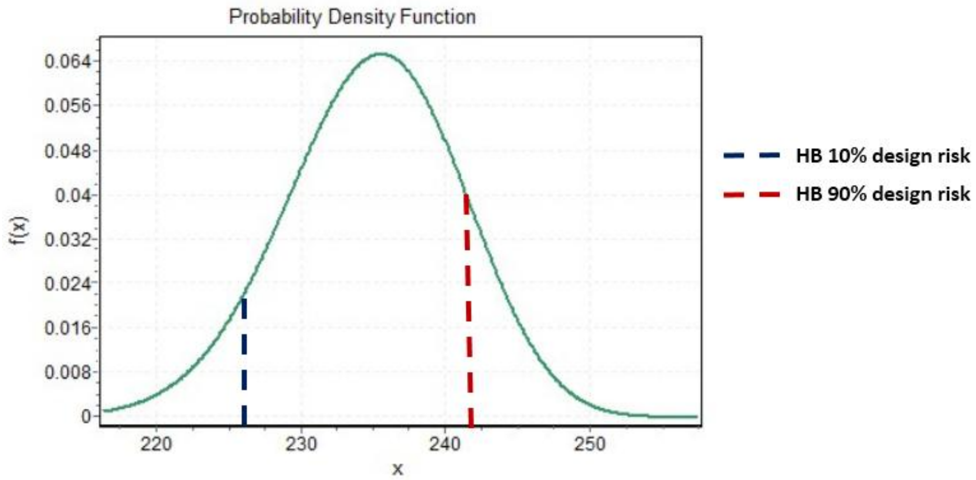


Figure 3-9: Beta Distribution of resultant end voltage for active LV feeder

The information in table 3-3 is obtained from the beta PDF shown in figure 3-9. Table 3-3 shows the probability of occurrence of voltages higher (or voltage rise higher) than the chosen end voltage (or voltage rise) is lower than with a higher design risk. This means that, with a lower design risk than 90 %, it is increasingly likely that there will be larger voltage rises (than the selected voltage rise) on the feeder. The basis of the design of feeders with DG is the voltage rise and therefore a design risk of 90 % is used to obtain the design voltage at the 90th percentile of the range of resultant voltages. There is a low probability of occurrence of voltages above this value.

Table 3-3: Significance of design risk levels

DESIGN RISK [%]	FEEDER END VOLTAGE (V_{con}) [Volts]	P (x > V_{con})	VOLTAGE RISE [%]
90	242.62	0.09991	5.03
50	235.11	0.50024	1.78
10	226.96	0.89994	-1.75

3.3 Concluding Remarks

In conclusion, this section on the HB algorithm has highlighted the following:

- The chosen risk for calculating voltage rise in the HB algorithm for active LV feeders is 90 % to account for the effect of the introduction of DG in LV feeders. The power factor is kept at unity, which is common for studies involving DG analysis on feeder systems
- In order to solve the task of determining the LV feeder DG hosting capacity the high-load and low-load conditions are tested to observe both extremes of the active LV feeders
- The HB algorithm has to be re-written in a suitable program for both active and passive LV feeders and tested against the MCS method of calculating voltage drop before being applied to DG hosting capacity problem

Chapter 4

4. HB METHOD FOR ACTIVE LV FEEDERS WITH DISTRIBUTED GENERATION

This chapter concerns the general procedure used in this study aimed at successfully extending the passive feeder HB algorithm to enable voltage calculation in active LV feeders. Before this is accomplished, a series of tests must be carried out to ensure the correctness of the HB algorithm in passive LV feeders. Any additional information and full results relevant to this chapter can be found in Appendix B.

4.1 General Methodology

The general procedure used in this chapter is described below:

1. Write passive feeder HB algorithm in suitable software (e.g. Matlab, C) - $HB_{p\text{-SOFTWARE}}$
2. Compare the passive feeder HB algorithm, written in the selected software, with the passive feeder Excel Spreadsheet ($HB_{p\text{-EXCEL}}$) for consistency. In addition, carry out relevant tests to check the correctness of the voltage drop calculation implemented in the passive feeder HB algorithm
3. Set up MCS for voltage drop calculation in passive feeders in Matlab ($VD_{p\text{-MCS}}$). Check the performance of the $VD_{p\text{-MCS}}$ and edit to eliminate errors
4. Test the passive feeder HB algorithm against $VD_{p\text{-MCS}}$ to validate the HB algorithm.
5. Using appropriate statistical and electrical theory, extend the passive feeder algorithm to enable voltage calculation in active feeders. Repeat steps 3-4 for the active feeder HB algorithm ($HB_{a\text{-SOFTWARE}}$), whilst comparing it with the active feeder Excel Spreadsheet ($HB_{a\text{-EXCEL}}$) to ensure both platforms agree. In addition, the testing of the $HB_{a\text{-SOFTWARE}}$ against the $VD_{a\text{-MCS}}$ also validates the algorithm for active feeder voltage calculation

   Figure 4-1 shows the general procedure used to validate the HB algorithm.

4.2 Program Software used

Matlab is selected because it is best suited to handle Monte Carlo Simulations (MCSs) for probabilistic methods. Its ease of use, predefined functions and extensive documentation are just some of the reasons Matlab has been used as a platform for the development and testing of locally derived methods.

Barton [2007] highlighted Matlab's suitability for quick manipulation of matrices and vectors in his probabilistic study into modelling energy storage in hybrid solar/wind systems. This property is very useful in implementing the HB algorithm, which involves voltage calculation node by node. The statistical toolbox installed in Matlab is very useful in carrying out the complex calculations comprising the HB algorithm. The HB algorithm for passive LV feeders is written in Matlab and is denoted by $HB_{p\text{-}MATLAB}$ in the rest of the report.

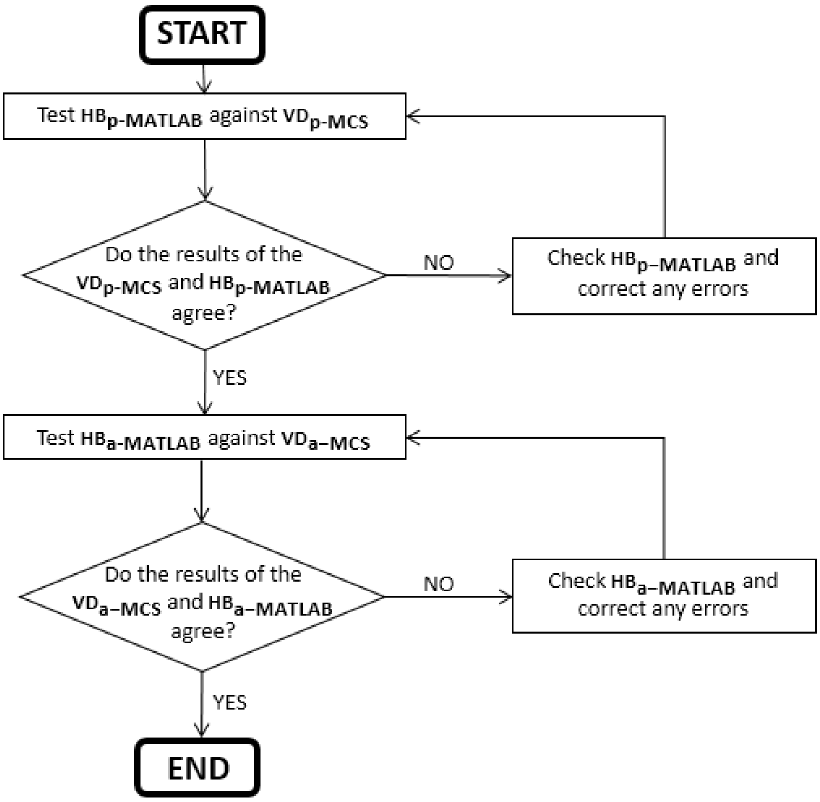


Figure 4-1: Process used to validate the HB algorithm

4.3 Monte Carlo Simulations

Monte Carlo Simulation (MCS) is a powerful method used in complex mathematical calculations, stochastic process simulations, engineering system analysis and reliability. It is a statistical simulation method that can be used for testing probabilistic calculation methods by using sequences of generated random samples to perform the simulation. The main concept behind the MCS is the creation or generation of data to be simulated while taking into account as much uncertainty as possible [Viawan, 2006].

4.3.1 MCS Procedure

Since the MCS comprises many repetitions/iterations carried out for each calculation, it is considered impractical for use in normal design work/projects. MCS is mainly applied in the verification of other methods such as statistical analytical methods [Herman & Heunis, 2001].

Herman & Heunis [2001] also identified two sources of error in the Monte Carlo method, described below:

1. **Random number generation**

   A reliable random number generator is vital in this method. The error in random generation arises when some random generators used in certain software packages start each round of generation with the same seed value, leading to same random pattern to be repeated.

   This led to the need to validate random number generators prior to their use in the MCS. This source of error can be minimised by using a uniform probability distribution as the resultant distribution of random numbers generated, allowing equal probability for each value.

2. **Number of iterations**

   The MCS is based on n repeated calculations and the resultant error is inversely proportional to n^2. This source of error can be minimised by the use of a large number of iterations to achieve acceptable accuracy.

   However, simply increasing the number of iterations does not definitely lead to a smaller error. Ip Cho [2013] used the convergence approach in the MCS by checking the resultant error with a set error limit to minimise the source of error. In order to accomplish this, the co-efficient of variation was used as a convergence criterion in the MCS.

The HB algorithm is a transform that converts a beta distribution of load currents to a beta distribution of voltages from which the resultant voltage drop is calculated. It is an analytical method that gives a unique result for the same input parameters. The MCS, usually categorized as a sampling method, randomly generates inputs from probability distribution to simulate the core process, the HB algorithm.

Below are the steps taken to perform probabilistic voltage drop calculations using the MCS (VD_{MCS}) as described by Herman & Heunis [2001]:

1. Use a random number generator to draw load currents from a PDF describing the load currents. Random selection is done with replacement.
2. Calculate a single deterministic voltage drop for each current drawn using appropriate circuit theory, which is the HB algorithm in this case.

3. Repeat steps 1 and 2 several thousand times to produce a large set of voltage drop values.
4. The resultant voltage drop is determined using the required risk level (in the case of active feeders: 90 %)

In addition, the resultant set of voltage values are fitted to a suitable probability distribution and compared to the beta distribution of resultant voltages produced by the HB algorithm. The beta distribution is chosen as the distribution suitable to perform this because the HB algorithm also produces a beta distribution, as well as its good performance in 'Goodness of Fit' (GoF) tests.

In this study, increasing the number of iterations is suitable enough to minimise the error for the MCS set up for each case without implementing the convergence approach. The results of the MCS are described using a probability distribution and therefore a large sample of resultant values is required. The number of iterations recommended in VD_{MCS} is 10 000 iterations and not time consuming. The current samples are taken from a distribution that describes the load.

4.4 HB method for voltage calculation in passive LV feeders

The HB algorithm, as found in the NRS 034 standard [2007] and in literature published by Herman & Gaunt [2008], used for passive LV feeder design (i.e. feeders with loads only) is written in Matlab and checked thoroughly for consistency before being tested against the $VD_{p\text{-}MCS}$. The following tests are carried out to test the $HB_{p\text{-}MATLAB}$. All detailed results of the testing can be found in Appendix B.

4.4.1 Feeder configuration

The default test feeder configuration, used for testing the HB algorithm for passive LV feeders in Matlab, is described in table 4-1 with an illustration shown in figure 4-2. Any other feeder configuration may be used to test the algorithm.

Table 4-1: Test Feeder Configuration

FEEDER PARAMETER	Bi-phase/Three-phase		
Number of nodes	2		
Inter-node length [m]	100		
Load (abc parameters)	α = 1.5	β = 4.0	c = 60
Temperature [^oC]	T_1=20; T_2=40		
Feeder Conductor	35 mm^2 Copper conductor (35mmCu)		

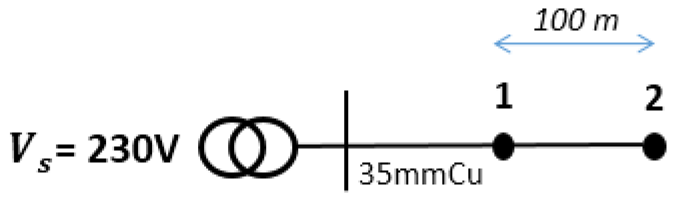


Figure 4-2: Illustration of Test Feeder configuration

It is important to know the convention of phase assignment used for each test. Table 4-2 shows the connection convention used in the HB method for overhead line topologies e.g. Bal111 in a three phase network to mean 1 consumer at each phase of a node (total of 3 consumers per node).
Other common connection patterns to represent 3 consumers per node/kiosk include Cyc210, Cos210, Cyc300, Cos300 etc.

Table 4-2: Connection Patterns [Gaunt, et al., 2011]

Connection Pattern	Bi phase	Three phase
Cyclic (Cyc)	R,B,R,B,…	R,W,B,R,W,B,…
Cosine (Cos)	R,B,B,R,B…	R,W,B,B,W,R,R…
Balanced (Bal)	Same Number on all phases	Same Number on all phases

4.4.2 Testing the HB algorithm in Matlab

i. **Test 1 – $HB_{p\text{-}MATLAB}$ and $HB_{p\text{-}EXCEL}$ comparison**

The first test to check the correctness of the $HB_{p\text{-}MATLAB}$ is to simply compare the resultant voltages with the results obtained using $HB_{p\text{-}EXCEL}$ for both the bi-phase and three phase feeders.

From table 4-3, it can be seen that results from the $HB_{p\text{-}MATLAB}$ are in exact agreement with the $HB_{p\text{-}EXCEL}$, which indicates that the HB algorithm has been correctly written in Matlab.

Table 4-3: Selected Test 1 Results

FEEDER NETWORK TOPOLOGY	NODE [CONNECTION]	VOLTAGE DROP [%]		ERROR
		$HB_{p\text{-}EXCEL}$	$HB_{p\text{-}MATLAB}$	
Three phase system	2 nodes [Bal 111]	2.80	2.80	0.00
Bi phase system	2 nodes [Bal 11]	2.89	2.89	0.00

ii. **Test 2 – Reverse Feeder Calculation**

This test is carried out to ensure the algorithm is correctly implemented for voltage drop calculation. A default feeder is set up and the voltage drop is calculated. Changing the direction of voltage drop calculation and using the feeder end voltage as the source voltage with negative loads, the voltage drop is calculated. The feeder end voltage for the reverse feeder is calculated ($V_{s\text{-}calc}$) compared to the original supply voltage (V_s). Figure 4-3 shows an illustration of the approach used in test 2.

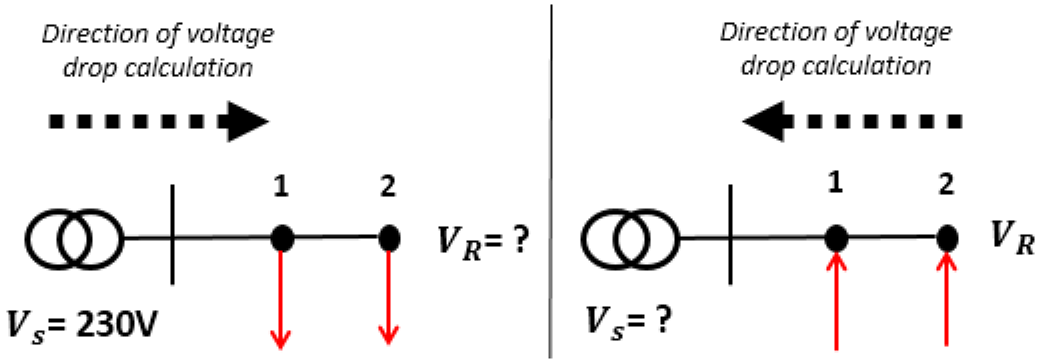


Figure 4-3: Feeder set up for Reverse Feeder Calculation

a) Normal Feeder (left) *b) Reversed Feeder (right)*

Table 4-4: Selected Test 2 results

FEEDER NETWORK TOPOLOGY	NODE [CONNECTION]	VOLTAGE		ERROR	%ERROR
		V_s	$V_{s\text{-}calc}$		
Three phase system	2 nodes [Bal 111]	230.00	229.86	0.14	0.06
Bi phase system	2 nodes [Bal 11]	230.00	229.85	0.15	0.07

From table 4-4, it can be seen that the difference in values, $V_{s\text{-}calc}$ and V_s, is very small. This is due to the successive use of the value taken at the HB design risk level. The results from this test indicate that correct calculation of voltage drop by the algorithm is in accordance with electrical circuit theory.

iii. **Test 3 – Single Phase Feeder Tests**

After the above mentioned tests, it is also necessary to check the voltage calculation formula is implemented correctly for each of the phases.

The default feeder configuration is set up and a single feeder is loaded for the bi-phase system, as well as similar loading for a single feeder on a three-phase system.

Table 4-5: Selected Test 3 Results (Loaded phase)

FEEDER NETWORK TOPOLOGY	NODE [CONNECTION]	VOLTAGE DROP [%]		ERROR
		$HB_{p\text{-}EXCEL}$	$HB_{p\text{-}MATLAB}$	
Three phase system	2 nodes [Bal 100]	3.99	3.99	0.00
Bi phase system	2 nodes [Bal 10]	3.99	3.99	0.00

Table 4-6: Selected Test 3 Results (Unloaded phase)

FEEDER NETWORK TOPOLOGY	NODE [CONNECTION]	VOLTAGE DROP [%]		ERROR
		$HB_{p\text{-}EXCEL}$	$HB_{p\text{-}MATLAB}$	
Three phase system	2 nodes [Bal 100]	-0.28	-0.28	0.00
Bi phase system	2 nodes [Bal 10]	-0.50	-0.50	0.00

The results in tables 4-5 and 4-6 (i.e. no difference in results for the loaded phase in the single feeder in both three phase and bi phase systems) indicate that the voltage drop calculation is correctly programmed and implemented in the $HB_{p\text{-}MATLAB}$. It is important to note that the difference in voltage drop in the unloaded phase(s) in the bi phase and three phase system is due to the difference in number and relationship between the phases in each network topology. In the bi phase system, the two phases are 180 degrees out of phase while in the three phase system, the three phases are 120 degrees out of phase.

iv. **Test 4 – Benchmark Tests**

The NRS 034 standard [2007] prescribes a total of 30 benchmark tests, which cover a variety of network and load combinations, to examine the accuracy of the $HB_{p\text{-}EXCEL}$. In test 4, the same benchmark tests were carried out for $HB_{p\text{-}MATLAB}$ to make sure that it was in agreement with the results of the benchmark tests in the NRS 034 standard. It is important to note that there are a few corrections to the feeder configuration, prescribed in the NRS 034 standard due to errors in the values given, particularly the design temperature, T_2, choice of cable conductor and cable length. The rest of the parameters mentioned for the benchmark tests were correctly given in the NRS 034 Standard. Table 4-7 shows the parameters that were amended for the feeder configuration in the NRS 034 for the first 24 benchmark tests.

Table 4-7: Feeder parameter amendments for the first 24 benchmark tests

PARAMETER	BI PHASE SYSTEM		THREE PHASE SYSTEM	
T_1 [°C]	20		20	
T_2 [°C]	22	64	22	64
Design risk [%]	10	20	10	20
Cable Conductor	70 mm^2 Copper [70mmCu]		70 mm^2 Copper [70mmCu]	
Cable Length [m]	100		100	

The results from this test, indicate the $HB_{p\text{-}MATLAB}$ and $HB_{p\text{-}EXCEL}$ are in agreement as the results for all of the benchmark tests were as published in the NRS 034 standard. This was the final test and produced the expected results for testing passive LV feeders.

The above results indicate that the $HB_{p\text{-}MATLAB}$ and the $HB_{p\text{-}EXCEL}$ agree with each other and are one and the same in formulation. The two can be used inter-changeably for the rest of the report.

4.4.3 Voltage Drop Calculation using Monte Carlo Simulation ($VD_{p\text{-}MCS}$)

In this section of testing the HB algorithm, the MCS is used to iteratively calculate the voltage drop for the given test feeder using relevant electrical circuit theory. The procedure $VD_{p\text{-}MCS}$, as given in section 4.3.1 is used to produce a beta PDF of resultant voltages, which is compared to the beta PDF produced by $HB_{p\text{-}MATLAB}$/$HB_{p\text{-}EXCEL}$. A comparison of the two beta PDFs should give an indication of the validity of the $HB_{p\text{-}MATLAB}$. In addition, the first and second statistical moments, E (V_{con}) and E (V_{con}^2), can be obtained through calculation and analysis of the raw set of resultant voltages from the $VD_{p\text{-}MCS}$. These can be compared to first and second statistical moments of the feeder end voltage from the $HB_{p\text{-}MATLAB}$.

i. Test Feeder Configuration

The test feeder configuration used in this section is similar to the feeder described in section 4.4.1 above as shown in figure 4-4. However, the customer allocations are changed as shown in table 4-8.

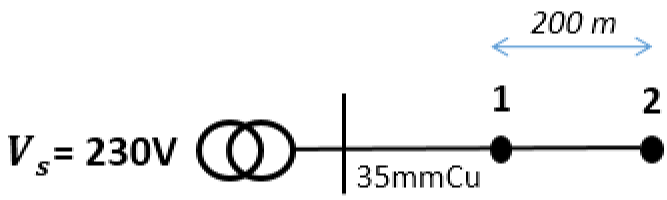


Figure 4-4: $VD_{p\text{-}MCS}$ *Test Feeder*

Table 4-8: $VD_{p\text{-}MCS}$ *Test Feeder Configuration*

FEEDER PARAMETER	Bi phase/Three phase Network Topology		
Number of nodes	2 [3 customers per node]		
Inter-node length [m]	200		
Load (abc parameters)	α = 1.5	β = 4.0	c = 60
Temperature [^{o}C]	T_1=20; T_2=40		
Feeder Conductor	35 mm^2 Copper conductor (35mmCu)		

The feeder configuration can be changed for various scenarios and tested against the $VD_{p\text{-}MCS}$ in order to ensure thorough testing. The feeder configuration can be altered to test against the $VD_{p\text{-}MCS}$ for a single phase feeder, as well as unbalanced customer allocation.

ii. Software used

Matlab is the software used to carry out the $VD_{p\text{-}MCS}$ because of its good random number generator as well as its quick manipulation of formulae and vectors, enabling the use of a large number of iterations (10 000), as prescribed to reduce the resultant error.

The $VD_{p\text{-}MCS}$ results are analysed by fitting a beta PDF to the resultant voltages. This is accomplished using *EasyFit Professional®* software developed by MathWave Technologies. It is the program in which StatAssist is embedded. Further comparisons using the beta parameters are done using Matlab.

iii. $VD_{p\text{-}MCS}$ Procedure

Validation of the $VD_{p\text{-}MCS}$

The $VD_{p\text{-}MCS}$ formulation and code is verified/validated by using a single phase feeder set up for both the bi phase and three phase network topologies. Using the test feeder described in figure 4-4, the customer allocation is set up such that there are 3 customers in the red phase for the first and second node and no customers on the blue phase. The voltage calculation is carried out using the $VD_{p\text{-}MCS}$ for both bi phase and three phase network topologies and the results are compared.

Random Current generator

Matlab has its own random number generator embedded in its software to aid with this task. The Matlab random generator allows for sampling from various probability distributions, the beta PDF in this case.

The *betarnd* function in Matlab enables sampling from a beta distribution describing the load currents on the feeder, inputting α, β and *n,* the total number of customers on a particular phase. The snippet of the Matlab code used for generating the random numbers for the test bi phase feeder example, using the configuration shown in figure 4-5a, is in figure 4-5b.

	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	α	β	[A]	[m]	Code
1	1	2	1.500	4.000	60	200	35mmCu
2	3	4	1.500	4.000	60	200	35mmCu

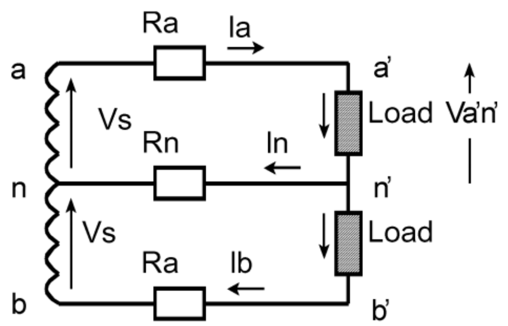
```
%No. of customers (red phase)              %No. of customers (blue phase)
ma1 = 1;    ma2 = 3;                       mb1 = 2;    mb2 = 4;
ma = ma1+ma2;                              mb = mb1+mb2;

%Randomly sample from the beta distribution described by the alpha, beta and c
parameters
Y1 = 60*betarnd (1.500, 4.000, ma,1);      Y2 = 60*betarnd (1.500, 4.000, mb,1);

Ia1 = sum (Y1 (1:ma1)); %Total current in node 1 - red phase
Ia2 = sum (Y2 (1:ma2)); %Total current in node 2 - red phase
Ib1 = sum (Y1 (((ma1+1):(ma1+mb1))));      %Total current in node 1 - blue phase
Ib2 = sum (Y2 (((ma2+1):(ma2+mb2))));      %Total current in node 2 - blue phase
```

Figure 4-5: a) Bi phase feeder configuration example from $HB_{p\text{-}EXCEL}$ *(above) b) MATLAB Code snippet (below)*

A similar approach is used for a 2-node three phase test feeder system, involving 6 currents for each load on each of the three phases.

Voltage Drop Calculation

a) Bi phase Network Topology




```
Vd1 = Ia1*(Rp1+Rn1)+Ia2*(Rp1+Rn1)-Rn1*(Ib1+Ib2); %Voltage drop due to node 1 loads
Vd2 = Ia2*(Rp2+Rn2)-Rn2*Ib2;                      %Voltage drop due to node 2 loads
Vcon = Vs - Vd1 - Vd2;                            %Consumer voltage
```

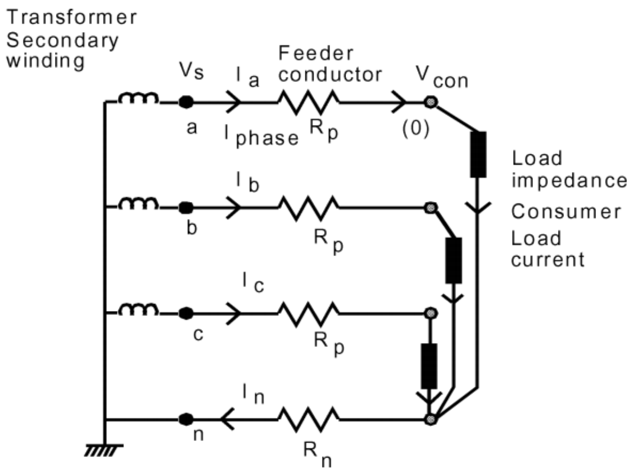
Figure 4-6: a) Single section of bi phase feeder [Herman & Gaunt, 2008] (above) b) MATLAB Code snippet (below)

Figure 4-6 shows the simple electrical circuit diagram for single section of a bi phase feeder. For the test feeder, with 2 nodes, the voltage calculation is such that the total voltage drop is due to the voltage drop from the loads in node 1 and the voltage drop from the loads in node 2.

The snippet of the Matlab code used for calculating the voltage drop in feeder example, shown in figure 4-6a, is in figure 4-6b. The variables used are a continuation from the code written in figure 4-5b above.

b) Three phase Network Topology

Figure 4-7 shows the simple electrical circuit diagram for single section of a three phase feeder. The snippet of the Matlab code used for calculating the voltage drop in feeder example, shown in figure 4-7a, is in figure 4-7b. The variables used are a continuation from the code written in figure 4-8b.




```
%Real and Imaginary component of voltage drop
Vra = I4*(Rp1+Rp2+Rn1+Rn2)+I1*(Rp2+Rn2)-0.5*(Rn1+Rn2)*(I5+I6)-0.5*Rn2*(I2+I3);
Via = sqrt(3)*0.5*((Rn1+Rn2)*(I5-I6)+Rn2*(I2-I3));
Vcona = sqrt((Vs - Vra)^2+(Via)^2);                     %Consumer voltage
```

Figure 4-7: a) *Single section of three phase feeder [Herman & Gaunt, 2008] (above) b) MATLAB Code snippet (below)*

The same concept used in the bi phase to calculate the voltage drop is applied. However, because the phases are 120 degrees out of phase, it is important to account for the real and imaginary parts of the voltage drop.

Like in the bi phase network, the snippet of the Matlab code used for generating the random numbers for the test three phase feeder example, using the configuration in figure 4-8a, is shown in figure 4-8b. It is important to note that the customers are arbitrarily allocated so as to easily demonstrate the MATLAB code.

	No Consumers on phases			Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1	1	2	3	1.500	4.000	60	200	35mmCu
2	4	5	6	1.500	4.000	60	200	35mmCu

```
ma1 = 1;     ma2 = 4;     ma = ma1 + ma2;    %No. of customers (red phase)
mb1 = 2;     mb2 = 5;     mb = mb1 + mb2;    %No. of customers (white phase)
mc1 = 3;     mc2 = 6;     mc = ma1 + ma2;    %No. of customers (blue phase)
%Randomly sample from the beta distribution described by the alpha, beta and c
parameters and find the currents in each section of each phase
Y = 60*betarnd (1.500, 4.000, ma,1);       Y2 = 60*betarnd (1.500, 4.000, mb,1);
% -------------------------1st section-------------------------------------
    I1 = sum(Y(1:ma1));
    I2 = sum(Y(((ma1+1):(ma1+mb1))));
    I3 = sum(Y((ma1+mb1+1):(ma1+mb1+mc1)));
% -------------------------2nd section-------------------------------------
    I4 = sum(Y1(1:ma2));
    I5 = sum(Y1(((ma2+1):(ma2+mb2))));
    I6 = sum(Y1((ma2+mb2+1):(ma2+mb2+mc2)));
```

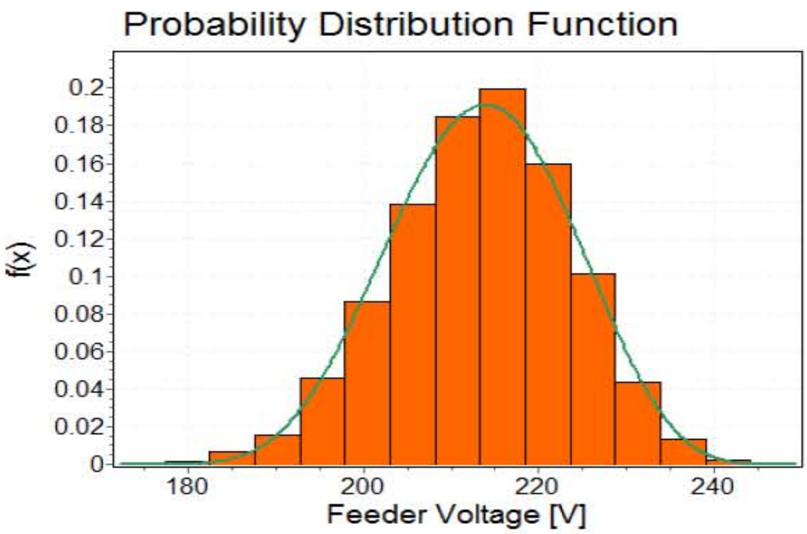
Figure 4-8: a) Three phase feeder configuration from $HB_{p\text{-}EXCEL}$ *9 (above) b) MATLAB Code snippet (below)*

Distribution Fitting

The resultant voltages from the $VD_{p\text{-}MCS}$ are compiled and a beta PDF is fitted to it. The reason for the choice of probability distribution is because the HB algorithm provides a beta PDF of resultant voltages.

The fitting function in the *EasyFit Professional®* software enables us to easily obtain the beta PDF parameters for the resultant voltage data. For the test feeder, the software is tested in order to assess how well the beta PDF fits the data. The results of the GoF test as indicated in figure 4-9b indicate that the *EasyFit Professional®* software gives a rank 1 model for the beta PDF, which translates to 'the very best model obtainable', in comparison to the commonly used PDFs – Normal, Weibull and Lognormal. The results in figure 4-9c give a much detailed view of the GoF test of the beta PDF. It lists the critical values of the GoF statistics calculated for various significance levels (alpha), as well as the acceptance of the null hypothesis for each of the level values. From figure 4-9c, the large number of acceptances is enough to draw the deduction that the beta PDF is a good fit for the data and that the software is suitable for the task of fitting a beta PDF on the voltage data. In addition, the resultant set of voltages from the $VD_{p\text{-}MCS}$ can be analysed and through known formulae, the first and second statistical moments can be calculated.

Since the $HB_{p\text{-}MATLAB}$ also calculates these two moments before they are used to obtain the beta PDF of resultant voltages, the two sets of statistical moments can be compared as an extra step to checking that the $HB_{p\text{-}MATLAB}$ and the $VD_{p\text{-}MCS}$ are in agreement.



#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.02777	1	3.1183	3	8.66	1
2	Lognormal	0.03917	3	2.5904	2	15.182	3
3	Normal	0.03534	2	1.9357	1	12.861	2
4	Weibull	0.04679	4	5.4948	4	24.306	4

Beta [#1]					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	1000 0.02777 0.41585 1				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.03393	0.03867	0.04294	0.048	0.05151
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic Rank	1000 3.1183 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	No	No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	9 8.66 0.46923 1				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	12.242	14.684	16.919	19.679	21.666
Reject?	No	No	No	No	No

Figure 4-9: a) beta PDF fit of resultant voltages from the $VD_{p\text{-}MCS}$ (above) b) GoF Test Summary (middle) c) Detailed information about the GoF of the top ranked beta PDF (below)

Results

Before the $VD_{p\text{-}MCS}$ was used to validate the HB algorithm, a single phase feeder test was carried out to ensure the MCS voltage calculation was correctly written. The results of this test are shown in table 4-9 and figure 4-10.

Table 4-9: Result data Comparison for single phase feeder $VD_{p\text{-}MCS}$ *tests*

Topology	**α**	**β**	V_{min}	V_{max}	$E(V_{con})$	σ^2
Bi phase	31.036	11.042	95.61	232.72	196.74	84.467
Three phase	17.975	7.8964	124.92	228.32	196.76	84.358

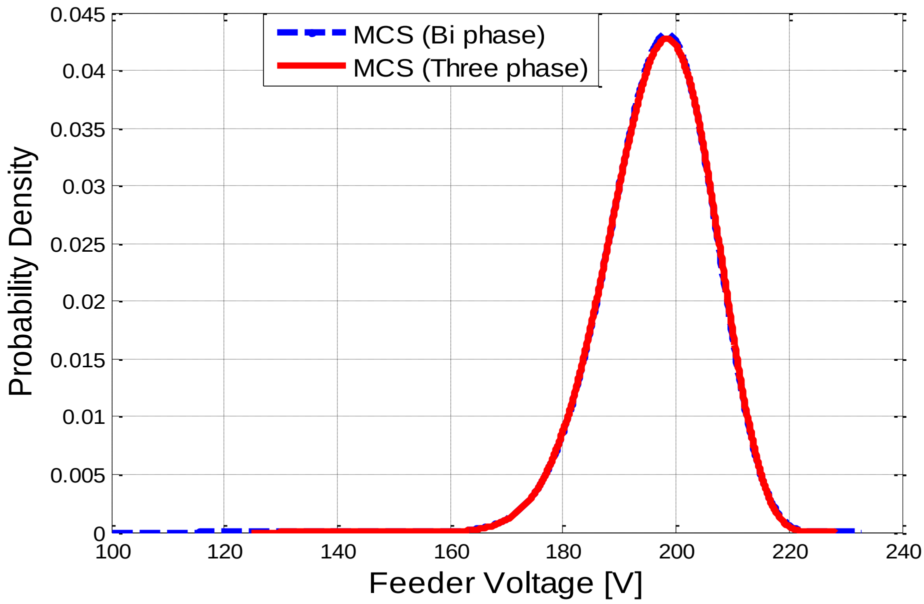


Figure 4-10: Comparison of beta PDFs from $VD_{p\text{-}MCS}$ *single phase feeder tests for bi phase and three phase networks*

From table 4-9 and figure 4-10, it can be observed that the $VD_{p\text{-}MCS}$ gives the same results for both the bi phase and three phase networks because of the overlapping beta PDFs. This validates the $VD_{p\text{-}MCS}$ used because it is expected that a single phase feeder, configured in the same way in either bi phase or three phase network, should give the same result despite the difference in calculations employed in each network.

Table 4-10 shows the comparison of the statistical moments from the $HB_{p\text{-}MATLAB}$ and $VD_{p\text{-}MCS}$ while figure 4-11 and figure 4-12 compare the resultant beta PDFs from $HB_{p\text{-}MATLAB}$ and $VD_{p\text{-}MCS}$ in the bi phase and three phase network topologies. The results of the other feeder configurations used to further test the $HB_{p\text{-}MATLAB}$ against the $VD_{p\text{-}MCS}$ can be found in Appendix B.

The overlapping of the beta PDFs indicates that the $HB_{p\text{-}MATLAB}$ correctly calculates the voltage drop for the test feeder. The $HB_{p\text{-}MATLAB}$ is further validated by looking at the values of the first and second statistical moments in table 4-10, which have a small difference (i.e. less than 5 % error).

Table 4-10: Result data Comparison for $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

Topology	**Voltage Calculation**	**α**	**β**	V_{min}	V_{max}	$E(V_{con})$	σ^2
Bi phase	$HB_{p\text{-}MATLAB}$	44.22	32.58	108.14	290.93	213.38	102.45
	$VD_{p\text{-}MCS}$	7.08	6.16	172.17	249.26	213.39	103.83
Three phase	$HB_{p\text{-}MATLAB}$	48.05	35.35	108.14	290.93	213.45	96.675
	$VD_{p\text{-}MCS}$	6.76	5.46	174.53	245.08	213.55	93.059

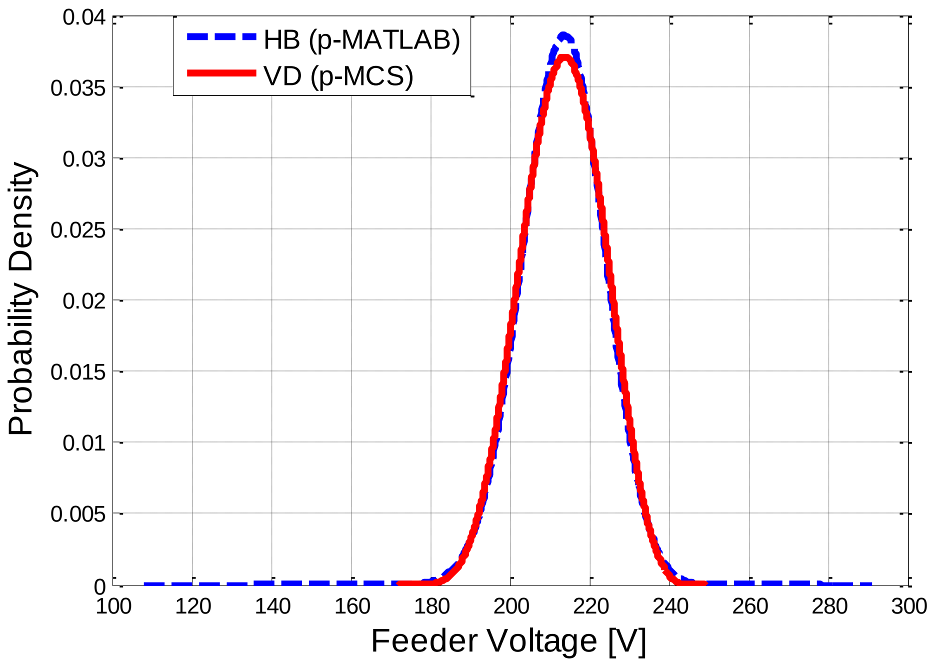


Figure 4-11: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

It is also important to note in figures from these tests, comparing the beta distribution for the HB and MCS, that the beta PDF from the $HB_{p\text{-}MATLAB}$ spans a wider range than the $VD_{p\text{-}MCS}$ because the former calculates the extreme (worst case) values V_{max} and V_{min}, for a particular configuration, which may only be achieved in the latter through 'infinite' sampling. This can be further demonstrated by specifying the quantile range from 5 % to 95 % for the beta PDF from the $HB_{p\text{-}MATLAB}$ and $VD_{p\text{-}MCS}$. Using the beta PDFs for the bi phase network in table 4-10, the (5 – 95 %) quantile range for the $HB_{p\text{-}MATLAB}$ is 33.72 while that of the $VD_{p\text{-}MCS}$ is 33.61.

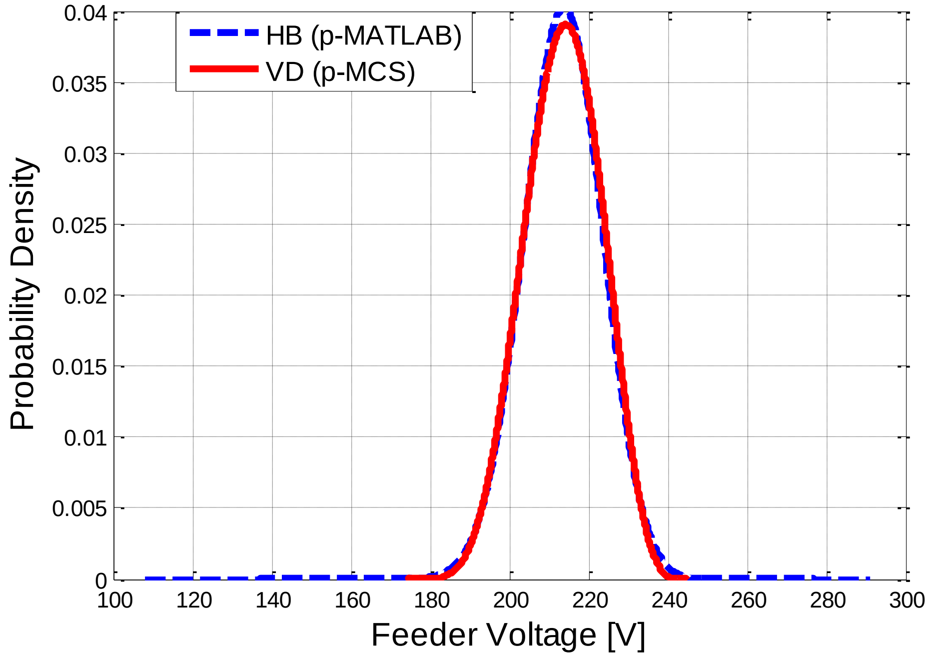


Figure 4-12: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for three phase network*

The results of the three phase network feeder are also shown in figure 4-12 and the same observations as in the bi phase case can be made, therefore validating the $HB_{p\text{-}MATLAB}$ for both bi phase and three phase networks.

4.4.4 Concluding Remarks

Based on the thorough and rigorous testing carried out in this section for both bi phase and three phase network topologies, the following conclusions were drawn:

- The HB algorithm is a valid voltage drop calculation method/tool for passive LV feeders. This was based on the thorough testing against the $VD_{p\text{-}MCS}$
- The $HB_{p\text{-}MATLAB}$ was formulated correctly and agrees with the $HB_{p\text{-}EXCEL}$ as per the testing and inspection
- The reverse feeder test supports the property already reported by other authors to the effect that DG can be modelled as negative loads
- The $HB_{p\text{-}EXCEL}$/$HB_{p\text{-}MATLAB}$ can therefore be extended to allow for voltage calculation in active LV feeders, whilst modelling DG as negative load.

4.5 HB method for voltage calculation in active LV feeders with DG

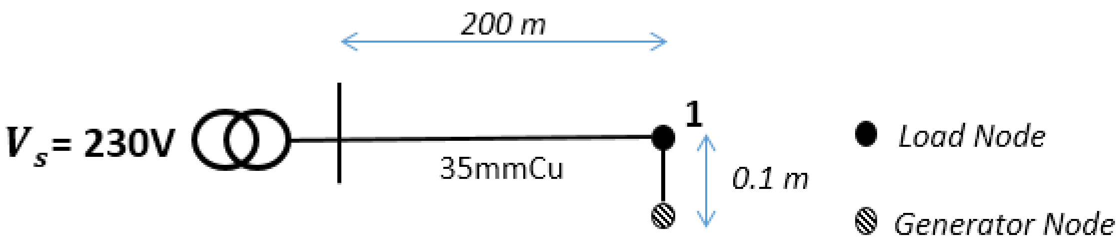
The next step, after the validation of the HB algorithm for passive LV feeders, is to extend the algorithm to calculate voltage in feeders with DG (i.e. active LV feeders). It has been accepted that DG can be modelled as negative load and this has been supported by various research authors.

The effect of the introduction of DG (negative load) in the HB algorithm leads to the following observations, which form the basis of the HB algorithm for active LV feeders ($HB_{a\text{-}EXCEL}$/$HB_{a\text{-}MATLAB}$):

- DG can be modelled as a negative customer current. DG current can be modelled using a beta PDF, just like the load
- DG and loads cannot be mixed at the same node in order to maintain algebraic identity. Two nodes are created (i.e. generator node and load node), with the generator node adjacent to the load. The DG is then assigned to the phases as 'negative number of customers'
- The formulae should largely remain unchanged with a few terms in the algorithm changed to include the effect of DG on the voltage of a feeder.

4.5.1 Extending the HB algorithm to active LV feeder voltage calculation

$HB_{a\text{-}EXCEL}$ is created by changing the terms in the $HB_{p\text{-}EXCEL}$ that need to be changed to account for the introduction of DG on passive LV feeders. In order to perform the investigation and identification of these terms, a 2-node feeder system is set up as shown in figure 4-13. The loads and an equal number of DG units were allocated to each phase.



	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb			[A]	[m]	Code
1L	6	6	1.500	4.000	60	200	35mmCu
1G	- 6	- 6	1.500	4.000	60	0.1	35mmCu

				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	6	6	6	1.500	4.000	60	200	35mmCu
1G	- 6	- 6	- 6	1.500	4.000	60	0.1	35mmCu

Figure 4-13: Test Feeder for HB algorithm extension:

a) General feeder layout (above) b) bi phase system (middle) c) three phase system (below)

From electrical theory, it is important to note that *n* loads cause a voltage drop of equal magnitude to voltage rise caused by *n* DG on a feeder. This property is important in allowing the $HB_{p\text{-}EXCEL}$ to be extended to allow for voltage calculation in active LV feeders. Based on this property, the terms that need to be altered to enable correct calculation in active LV feeders are:

- The E (V_{con}^2) term
- Beta distribution parameters – alpha (α_v) and beta (β_v)
- Maximum (V_{max}) and minimum (V_{min}) consumer voltage

i. The E (V_{con}^2) term

Statistical moments are statistical properties used as a way to describe probability distributions – the first moment (mean); second moment (variance); third moment (skewness). In the HB algorithm, the first moment of the voltage, E (V_{con}) is the mean or expected value of the consumer voltage being calculated. The second moment of the voltage, variance, gives a measure of the spread of the voltages from the mean voltage value in the beta PDF of resultant voltages. The variance is given by the formula: E (V_{con}^2) - (E (V_{con}))^2. The term E (V_{con}^2) is a parameter within the variance (i.e. a term from which the variance can be calculated).

Since variance is a non-negative number, it should be modified to ensure this holds true for all combinations of feeder configurations. This means the term E (V_{con}^2) should always be positive and greater than the square of the mean, (E (V_{con}))^2. Figures 4-14 and 4-15 show the modified section of $HB_{p\text{-}EXCEL}$ necessary to have a correct value for E (V_{con}^2), that holds true for all cases.

$$ri = ci^2 \times Ri^2\ [|mbi| \times ki^2 + |mai|(1+ki)^2]$$

$$si = ci^2 \times Ri^2\ [|mbi|(|mbi|-1)ki^2 - 2|mai| \times |mbi|(ki+1)ki + |mai|(|mai|-1)(1+ki)^2]$$

Figure 4-14: New formula for ri and si terms in $HB_{a\text{-}EXCEL}$ *for bi phase systems*

$$F1i = |mai|(|mai|-1) - |mai|(|mbi|+|mci|) + 0.25(|mbi|+|mci|-1)(|mbi|+|mci|)$$

$$F2i = |mai|(2|mai|-|mbi|-|mci|-2)$$

$$F3i = |mai|(|mai|-1)$$

$$C2i = ki^2[(|mai| + 0.25|mbi| + 0.25|mci|) + |mai|(2ki+1)]$$

$$C4i = \frac{3ki^2}{4}(|mbi|+|mci|)$$

$$C5i = \frac{3ki^2}{4}[(|mbi|-|mci|)^2 - (|mbi|+|mci|)]$$

Figure 4-15: New formula for constants F1i, F2i, F3i, C2i, C4i and C5i in $HB_{a\text{-}EXCEL}$ *for three phase systems*

ii. Beta distribution parameters of consumer voltage – alpha, beta

The beta probability distribution is only defined for α, β > 0. In order to avoid any errors in this regard, the absolute values of alpha and beta are used to calculate the percentile value at the selected risk, p %.

Figure 4-16 shows the correction made in the $HB_{a\text{-}EXCEL}$ for bi phase and three phase network topologies to avoid any errors in this regard. In figure 4-16, av is alpha and bv is beta, which are the beta parameters of the resultant beta PDF of voltages.

$$v\% = betainv[\left(\frac{p}{100}\right), |av|, |bv|]$$

Figure 4-16: The use of absolute values of the alpha and beta parameters in the $HB_{a\text{-}EXCEL}$

After all these modifications were made, the HB algorithm for active feeders in Excel ($HB_{a\text{-}EXCEL}$) is re-written in Matlab ($HB_{a\text{-}MATLAB}$) and tested against the MCS ($VD_{p\text{-}MCS}$) to validate the algorithm, as done with the $HB_{p\text{-}MATLAB}$

iii. Minimum and maximum consumer voltage – V_{min} and V_{max}

In the $HB_{a\text{-}MATLAB}$, the use of separate nodes for loads and DG requires the change in formulation of the V_{min} and V_{max} terms. The extreme limits of the consumer voltage for the load nodes are calculated based on the condition that:

- Minimum voltage in a particular phase is obtained if the phase(s) is loaded and the other phase is unloaded
- Maximum voltage in a particular phase is obtained if the phase is unloaded and the other phase is loaded

In active LV feeders, with the addition of DG, the conditions for the formulation of the V_{min} and V_{max} terms changes for DG (i.e. inverse to the conditions for load nodes). The extreme limits of the consumer voltage for the generation nodes are calculated based on the condition that:

- Minimum voltage in a particular phase is obtained if the phase has no generator and the other phase has a generator
- Maximum voltage in a particular phase is obtained if the phase has a generator and the other phase has no generator.

a) Bi phase Network Topology

Table 4-11 shows the calculation of the extreme V_{con} limits for the load and generator nodes for the bi phase network

Table 4-11: Extreme consumer voltage limit calculation for phase a in bi phase network

NODE	Minimum voltage (V_{min})	Maximum voltage(V_{max})
Load node	$V_s - I_{a(LOAD)}.(1+k).R$	$V_s + I_{b(LOAD)}.k.R$
Generator node	$V_s + I_{b(DG)}.k.R$	$V_s - I_{a(DG)}.(1+k).R$

b) Three phase Network Topology

Tables 4-12and 4-13 show the calculation of the extreme V_{con} limits for the load and generator nodes for the three phase network

Table 4-12: Minimum consumer voltage real and imaginary components for phase a in three phase network

NODE	Real component ($\Delta V_{re, min}$)	Imaginary component ($\Delta V_{im, min}$)
Load node	$I_{a(LOAD)}.R.(1+k)$	0
Generator node	$-((I_{b(DG)} + I_{c(DG)}).R.k).\frac{1}{2}$	$((I_{b(DG)} - I_{c(DG)}).R.k).\frac{\sqrt{3}}{2}$

Table 4-13: Maximum consumer voltage real and imaginary components for phase a in three phase network

NODE	Real component ($\Delta V_{re, max}$)	Imaginary component ($\Delta V_{im, max}$)
Load node	$-((I_{b(LOAD)} + I_{c(LOAD)}).R.k).\frac{1}{2}$	$((I_{b(LOAD)} - I_{c(LOAD)}).R.k).\frac{\sqrt{3}}{2}$
Generator node	$I_{a(DG)}.R.(1+k)$	0

For calculation of the extreme V_{con} limits in the three phase network, using the above real and imaginary voltage drop components appropriately, the following equations are used:

$$V_{con\,(min)} = \sqrt{(V_s - \Delta V_{re,\,min})^2 + (\Delta V_{im,min})^2} \quad ......(4.1)$$

$$V_{con\,(max)} = \sqrt{(V_s - \Delta V_{re,\,max})^2 + (\Delta V_{im,max})^2} \quad ......(4.2)$$

4.5.2 Voltage Drop Calculation using Monte Carlo Simulation ($VD_{a\text{-}MCS}$)

The $VD_{a\text{-}MCS}$ is carried out using the same procedure as the $VD_{p\text{-}MCS}$ in section 4.4.3 above, with the difference being the negative load model for DG. This means that the current samples from the beta PDF are negative.

i. **Test Feeder Configuration**

The feeder is a 2-node feeder with the generator node placed very near the load as residential PV units are usually placed on the rooftops of households.

The test feeder used in this testing is shown in figure 4-17.

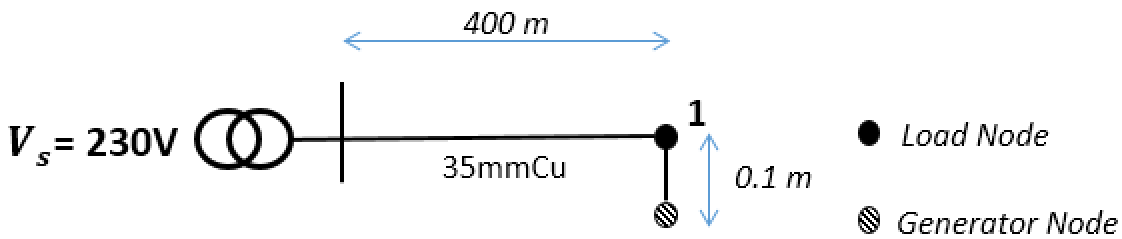


Figure 4-17: $VD_{a\text{-}MCS}$ *Test Feeder*

ii. **$VD_{a\text{-}MCS}$ Procedure**

The basic procedure is used in this testing of the $HB_{a\text{-}MATLAB}$ as with $HB_{p\text{-}MATLAB}$. The tests carried out are described below.

1. **Test 1 – DG Only Feeder**

   The first test carried out is a feeder with only DG for bi phase and three phase networks. With no loads on the load node, the feeder has only generation, 6 DG on each phase on the generator node. In addition, the beta PDF of the equivalent 'Load Only' feeder (i.e. 6 loads on each phase on the load node and no DG on the generator node) can be compared to see the relationship between the two cases. The purpose of this test is to ensure the $HB_{a\text{-}MATLAB}$ is correctly amended and formulated for DG.

2. **Test 2 – Constant Load with varying DG**

   The purpose of this test is to validate the $HB_{a\text{-}MATLAB}$ while monitoring the movement of the beta PDF of resultant voltages with uniformly varying DG. This test starts with a passive feeder (6 loads on the load node and no DG at the generator node) and the $HB_{a\text{-}MATLAB}$ and $VD_{a\text{-}MCS}$ comparison is carried out.

   Then DG is added one at a time to the generator node, while keeping the load constant (6 loads) and then carrying out the $HB_{a\text{-}MATLAB}$ and $VD_{a\text{-}MCS}$ comparison for each case. The beta PDFs are obtained for each scenario up to when the DG is just exceeding the load connected.

Results

1. Test 1 – DG Only Feeder

The results of this test can be seen in table 4-14, and figures 4-18 and 4-19, for bi phase and three phase network topologies.

Table 4-14: Result data Comparison for $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests*

Topology	Voltage Calculation	α	β	V_{min}	V_{max}	$E(V_{con})$	σ^2
Bi phase	$HB_{p\text{-}MATLAB}$	36.25	49.20	148.74	392.52	252.16	167.91
	$VD_{p\text{-}MCS}$	32.542	71.848	163.54	448.10	252.25	164.85
Three phase	$HB_{p\text{-}MATLAB}$	41.74	56.54	148.74	392.52	252.27	146.26
	$VD_{p\text{-}MCS}$	5.5279	7.1526	212.47	303.67	252.23	149.50

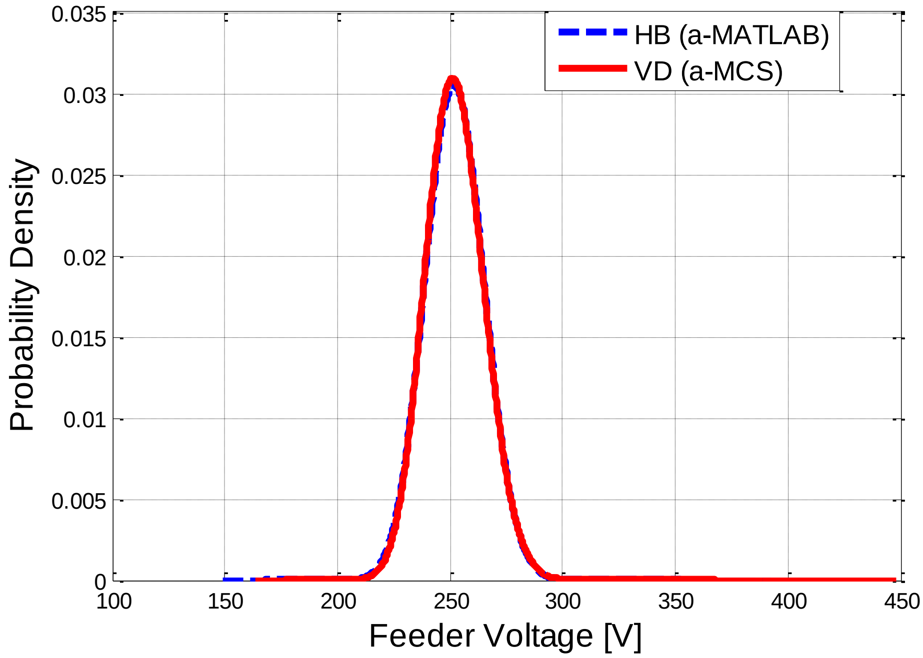


Figure 4-18: Comparison of beta PDFs from $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for bi phase network*

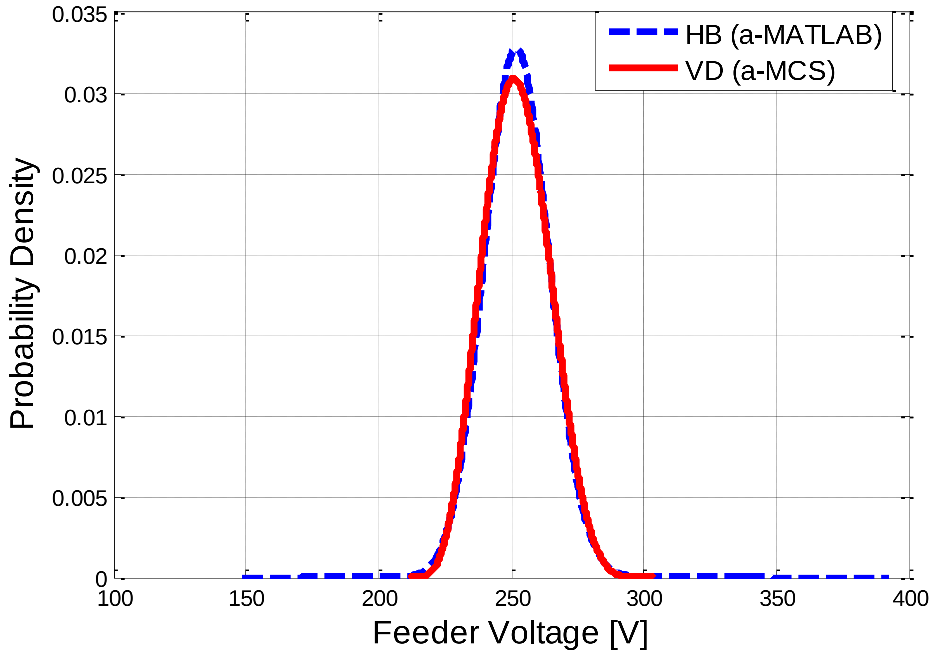


Figure 4-19: Comparison of beta PDFs from $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for three phase network*

The identical beta PDFs, as well as values of the first and second statistical moments, indicates that the $HB_{a\text{-}MATLAB}$ is correctly formulated and extended from the $HB_{p\text{-}MATLAB}$. The next and last step would be to check the validity of the $HB_{a\text{-}MATLAB}$ with load and DG on the feeder.

2. Test 2 – Constant Load with varying DG

The result for the test feeder for 12 loads and 12 generators (for bi phase) and 18 loads and 18 generators can be seen in table 4-15 and figures 4-20 and 4-21.

Table 4-15: Result data Comparison for $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for equal loading and generation*

Topology	Voltage Calculation	α	β	V_{min}	V_{max}	$E(V_{con})$	σ^2
Bi phase	$HB_{p\text{-}MATLAB}$	87.98	87.98	-13.73	473.75	230.01	335.72
	$VD_{p\text{-}MCS}$	9.7299	7.8995	142.42	301.40	230.16	335.52
Three phase	$HB_{p\text{-}MATLAB}$	81.66	91.86	13.73	473.75	230.22	302.10
	$VD_{p\text{-}MCS}$	8.6519	8.4569	155.30	303.10	230.04	301.54

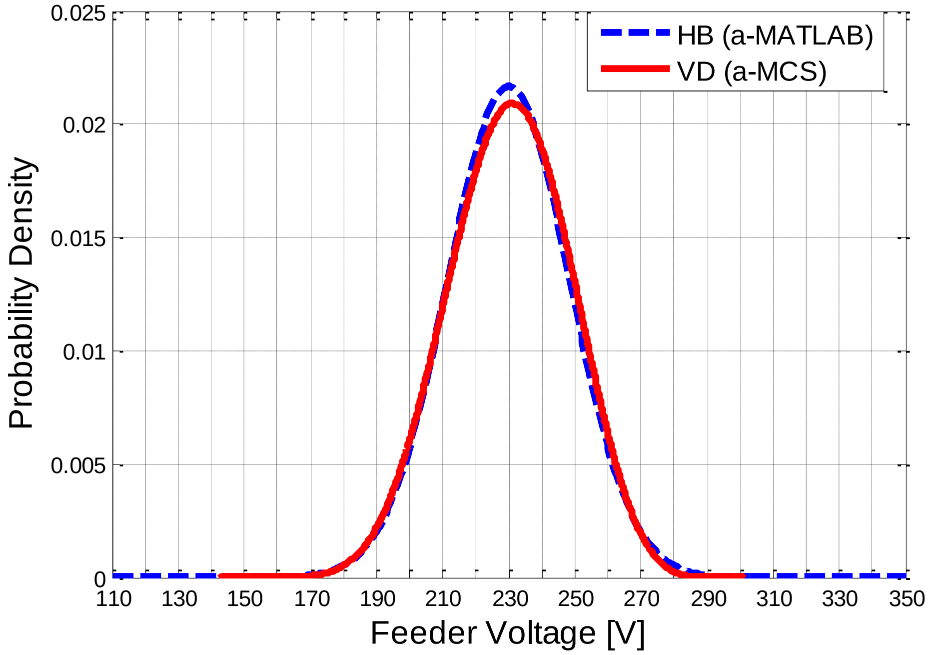


Figure 4-20: Comparison of beta PDFs from $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for bi phase network*

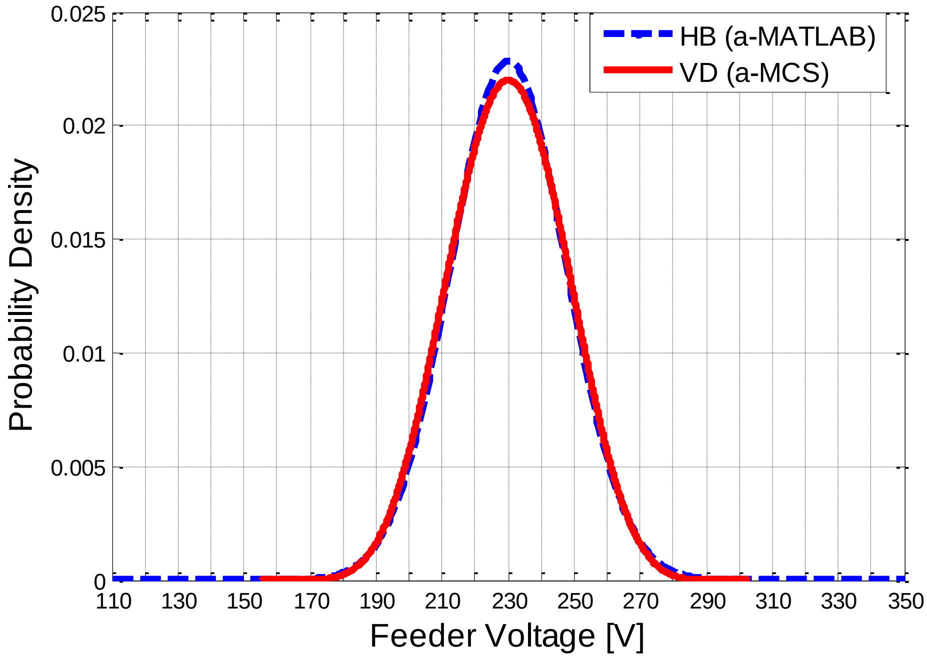


Figure 4-21: Comparison of beta PDFs from $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for three phase network*

The result of the feeders with equal loading and generators for $HB_{a\text{-}MATLAB}$ and $VD_{a\text{-}MCS}$ produce the expected beta PDF – which is symmetrical about the nominal supply voltage (230 V), the expected mean voltage on this feeder.

The rest of the results can be found in the Appendix B.

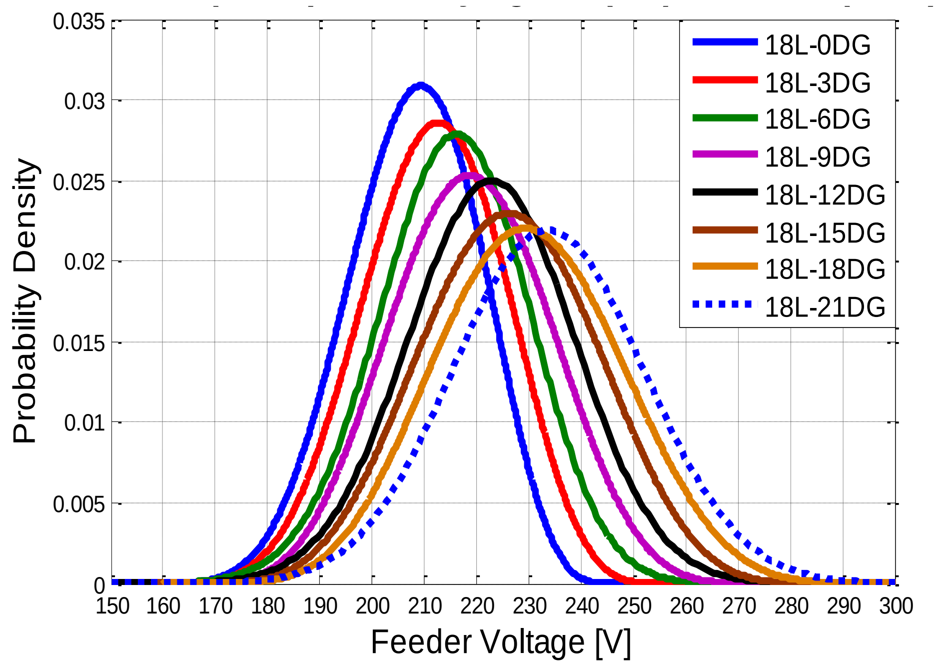


Figure 4-22: The transition of the beta PDF with increasing generation while load is constant for $VD_{a\text{-}MCS}$

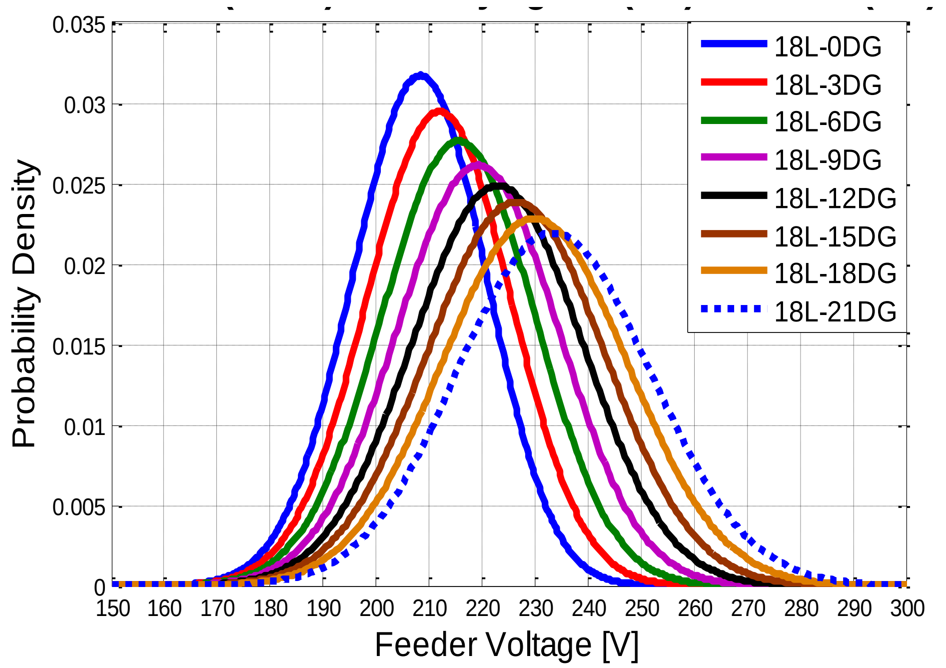


Figure 4-23: The transition of the beta PDF with increasing generation while load is constant for $HB_{a\text{-}MATLAB}$

The results in figure 4-22 and figure 4-23 show the shift of the beta PDF caused by increasing DG penetration on an originally passive feeder. Given that the beta PDFs for the $HB_{a\text{-}MATLAB}$ and $VD_{a\text{-}MCS}$ both agree in all cases, it can be said that the HB algorithm for active feeders is valid.

4.5.3 Concluding Remarks

Based on the testing carried out in this section for both bi phase and three phase network topologies, the following conclusions were drawn:

- The HB algorithm for passive LV feeders ($HB_{p\text{-}MATLAB}$) has been successfully extended to allow for voltage calculation in active LV feeders ($HB_{a\text{-}MATLAB}$). This has been validated through the use of $VD_{a\text{-}MCS}$
- The concept of modelling DG as negative load in $HB_{a\text{-}MATLAB}$ is a welcome and valid approach
- The $HB_{a\text{-}MATLAB}$ can now be successfully used in the voltage analysis of active LV feeders.

Chapter 5

5. VOLTAGE ANALYSIS ON LV FEEDERS WITH DISTRIBUTED GENERATION

This chapter describes the application of the HB algorithm as a tool for voltage analysis of feeders with DG present. The HB algorithm has been thoroughly and rigorously tested, as well as validated for its application in this chapter. The detailed construction of the voltage analysis model is provided in this chapter, which comprises passive LV feeder design before the voltage analysis model of the active LV feeder with increasing DG penetration

5.1 Construction of voltage analysis model

5.1.1 Passive LV Feeder design

The application of the HB method in passive feeder design is shown in figure 5-1 as indicated by Herman [2001]. In this study, the two most important aspects in the design of the passive LV feeder will be to ensure a maximum of 10 % voltage drop on the feeder and ensure that the thermal rating of the feeder conductor is not exceeded (i.e. overloaded feeder).

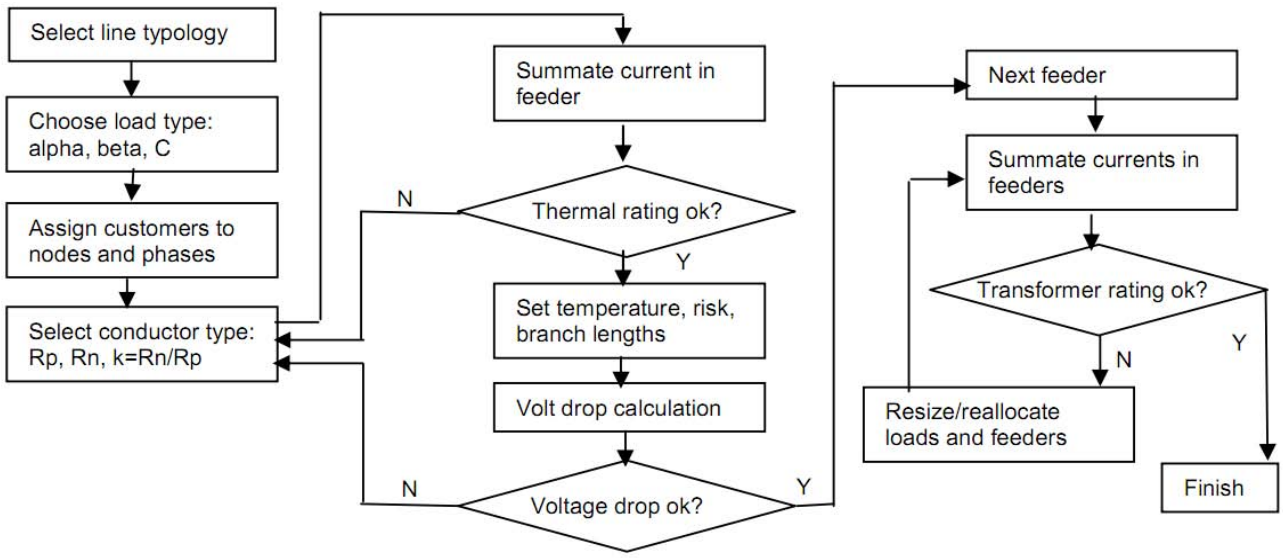


Figure 5-1: Reticulation design Procedure

The feeder network used in this design is a 6-node three phase network configuration, with the nominal LV source voltage of 230 V/phase.

Using this feeder topology, the feeder is adjusted to design it for passive loads for various test scenarios required. It should be noted that various feeder configurations and topologies can be used for the analysis.

i. Characteristic Load

As discussed earlier in the report, the beta distribution is used to model the load demand, for use in the HB algorithm. The load demand differs from customer to customer based on various factors. The most relevant factors in this study include:

- Time of day and season

  Figure 5-2, shows the variation between the load demand for the heaviest day in winter and the typical day in summer. It can be seen that the winter load demand is greater than the summer load because of increased heating required.

- Class of customer (Income group)

  Considering this study looks at the residential solar PV, it is relevant that the load demand for a high end customer is used for this study. This is because they are most likely to afford to purchase solar PV panels for their homes. It may also be relevant to look at low end customers who may dwell in rural areas, which are a big target for solar home projects.

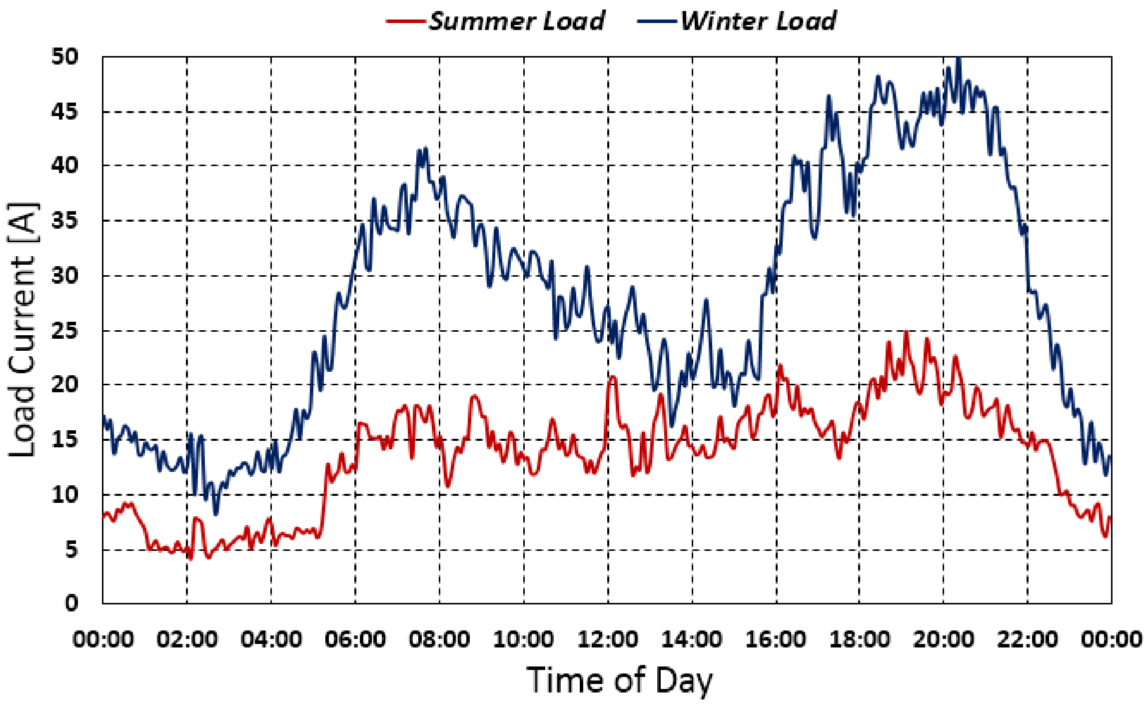


Figure 5-2: 5-min daily summer and winter load profiles for Claremont site [NRS 034 Domestic Load Research Project, 1998]

The load data used for the two sites chosen for the study was obtained from the NRS 034 Domestic Load Research Project [1998]. The load data obtained is measured over 5-minute intervals. Table 5-1 shows the load beta parameters for the sites.

The Claremont site is chosen as its customers belong to the high end classification of customers while customers in Orient Hills are of a lower classification.

Table 5-1: Load parameters for the chosen sites

SITE	YEAR	SEASON	α	β	C [A]	Power [kVA]	LSM
Claremont	1998	Winter	0.8625	1.8953	50.1270	3.61	7
		Summer	0.4366	1.2877	24.8680	1.45	
Orient Hills	2000	Winter	0.6865	1.8022	26.0160	1.65	5
		Summer	0.3572	1.1672	13.6270	0.74	

Based on the NRS 034 standard [2007], classification of customers based on their income is showed using the Living Standard Measure (LSM). Putting the chosen sites in perspective, table 5-2 shows the characteristics of the chosen sites.

Table 5-2: Classification of customers [NRS 034 Standard, 2007]

SITE	YEAR	SEASON	LSM	CONSUMER CLASS	INCOME RANGE [gross R/month]
Claremont	1998	Winter	7	Urban Residential II	5 500 – 8 000
Orient Hills	2000	Winter	5	Township area	1 500 – 3 000

ii. Characteristic PV Generation

The NRS 034 Domestic Load Research Project [1994] carried out extensive load measurements for various sites in South Africa, which were used to develop statistical models for the load demand. However, with PV, there are no such extensive measurements and therefore making the statistical models relatively unknown.

Therefore, in order to analyse the impact of PV generation on feeders, the following characteristics have to be considered:

- Daily generation profile

  Figure 5-3 shows the typical daily generation profile of a solar PV panel, which follows the average irradiance profile of the sun throughout the day. As seen in figure 5-3, the time period of interest in this study is the time during which the PV generation exceeds the load demand, particularly the interval of maximum difference between the generation and load, as indicated in figure 5-3.

- Power output (rating)

  The PV panels of interest in this study include a single phase PV panel (rated 1-kW or 3.5-kW) and a three phase lumped PV panel (rated 10-kW).

Using a negative load approach model for DG in the HB method implied that they can be modelled using the beta PDF. This can be done using measured solar data. The lack of such data prompts the use of beta PDF models to represent equivalent deterministic values. It is expected that the solar PV panel will produce its maximum possible power at the time of highest solar irradiation.

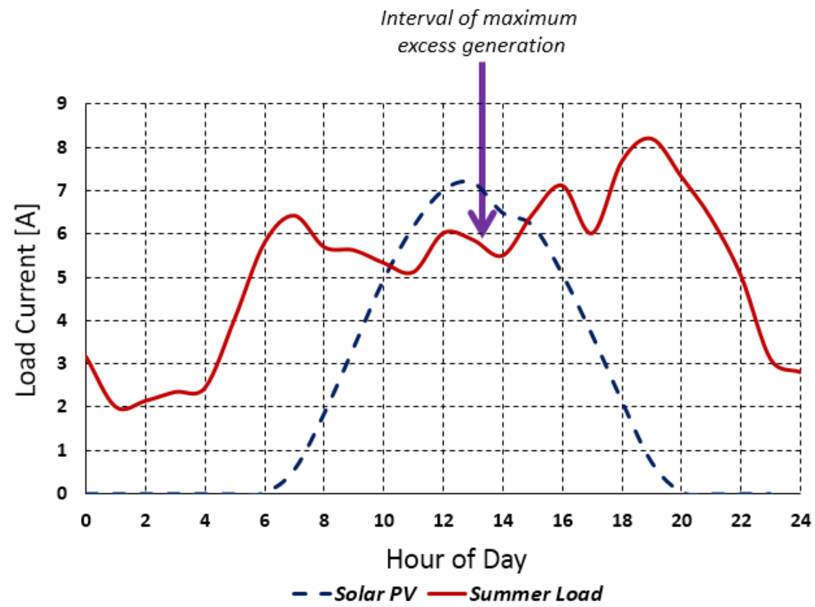


Figure 5-3: Correlation between the daily summer load profile and PV generation profile

The beta distribution can be used to model the deterministic PV by:

- using high values of alpha and beta as well as twice the maximum current for the value of c.
- using the mean and maximum current output expected to generate the alpha and beta parameters. A value for alpha is chosen and beta calculated using the formula $\beta = \frac{(\alpha - \alpha \times \frac{\mu}{I_{max}})}{\frac{\mu}{I_{max}}}$

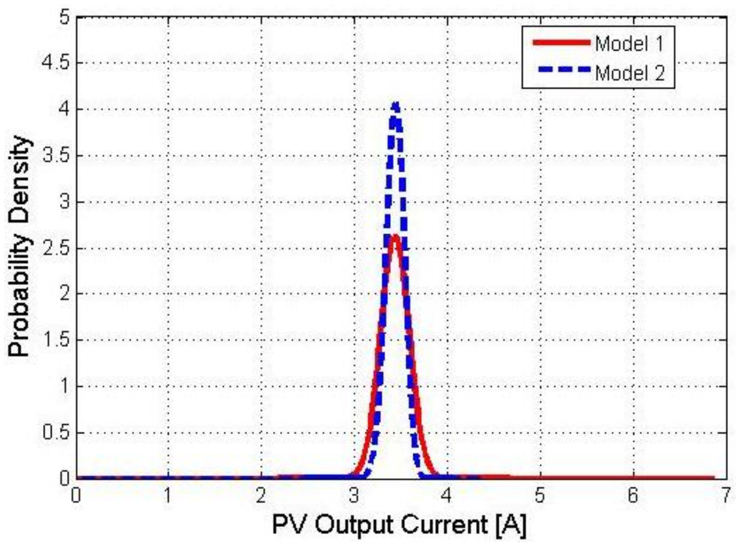


Figure 5-4: Modelling PV-EG using the beta PDF

From figure 5-4, it is observed that the maximum current output is 4.3478 A, which is the rated output current of the 1-kW solar PV panel. The two models are similar as they have the same means and the current values sampled from both distributions are very close to the mean.

Table 5-3 shows the beta PDF parameters used to model the PV-EG.

Table 5-3: Using the beta PDF to model the deterministic PV rated 1-kW

PV-EG MODEL	α	β	c	μ	σ
Model 1	255.50	255.50	6.8696	3.4348	0.1520
Model 2	255.00	67.78	4.3478	3.4348	0.0984

iii. Feeder Conductor

The designer should take the current carrying capacity of the feeder conductor used into consideration. The feeder conductor used was the 35 mm^2 4-core Copper conductor, denoted as 35mmCu. Using the current calculation in the HB method, the design feeder current of the conductor is obtained to ensure that the conductor thermal rating is not exceeded, which would mean overloading the conductor. From table in figure B-2 in Appendix B, the thermal rating of this conductor is 143 A.

The results from the current calculation in the HB method are shown in table 5-4. The beta PDF parameters of the resultant current distribution are α = 6.7387; β = 14.8081; c = 300.7620.

Table 5-4: Results from current calculation in HB method

CURRENT	FEEDER PHASES		
	R	W	B
90^{th} percentile	133.28 A	133.28 A	133.28 A
Mean	94.06	94.06	94.06

From the resultant beta PDF of resultant currents, it can found that the probability of occurrence of currents exceeding the thermal rating of the feeder conductor (143 A) causing the conductor to become overloaded is 0.05771. This is a very small probability and therefore justifies the use of the 35 mm^2 4-core Copper conductor in designing the passive LV feeder as the current estimated for the feeder is within the thermal limit of the cable, 143 A.

The passive LV feeder configuration to ensure a maximum voltage drop of 10 % on the feeder, whilst not overloading the feeder, is shown in figure 5-5. This feeder configuration is as seen in the HB Excel spreadsheet. The beta PDF load model used was the winter load demand for Claremont in 1998 and the beta PDF DG model used was model 1 as shown in table 5-3.

				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

Figure 5-5: Passive LV feeder

5.1.2 Active LV Feeder Voltage Performance Evaluation

i. Expected Output

The proposed illustration of output from this study is shown in figure 5-6, illustrating the voltage rise constraints of a LV feeder with increasing PV-EG. The reasoning behind the choice of the horizontal axis parameter %EG/ADMD, which is an independent parameter, was that the results can be scaled to feeders of any capacity (ADMD).

This is because the ratio used depends on two values:

- EG – the rated capacity of the connected PV-EG units
- ADMD – After Diversity Maximum Demand is the average load, usually the winter load demand used to design the passive LV feeder

The effect of increasing penetration of a particular capacity/rating of PV-EG on a feeder of known ADMD can be obtained. In order to analyse the effect of the same PV-EG on another feeder of different ADMD, the first result can be scaled accordingly.

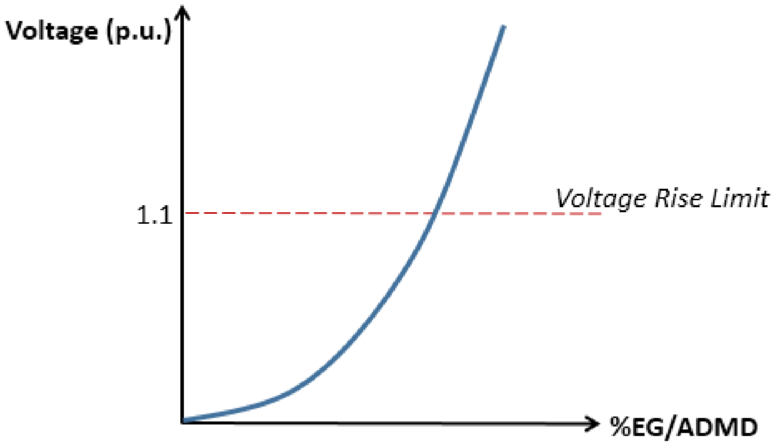


Figure 5-6: Voltage Rise constraints for active LV feeders

For each penetration level (%EG/ADMD), there are many possibilities/combinations of PV allocation throughout the feeder. Therefore in order to produce reliable results and estimates of the feeder conditions, a MCS is required to randomly allocate the PV-EG units to create a possible combination for a particular penetration level.

ii. Assumptions and Considerations

In order to increase the accuracy of the results as well as produce reliable results that can be extended as close to practical LV distribution systems as possible, the following assumptions and considerations were made:

1. The maximum rating of PV units a customer can install on his/her household may not exceed 50 % of the nominal connection capacity of the household, which is taken as the circuit breaker size of that household. The existing guideline is that the maximum rating of PV-EG may not exceed 25 % of the nominal connection capacity. For the purpose of this study, this limit was related to 50 %. Therefore, there is a PV unit limit – maximum number of PV units that a household can install, denoted by PV_unit_max as in the flow chart in figure 5-7.
2. Only phases with customers can be allocated PV units
3. All PV-units are considered to be facing the sun in an optimal position to allow for maximum power output
4. At the interval of maximum excess generation, the PV units provide 80 % of the rated output power. 80 % is the rule of thumb and is used to make up for the inefficiency of conversion of DC to AC in the inverter.

These assumptions are made, in addition to the assumptions that underlie the HB algorithm for active and passive LV feeders as stated in Chapter 3. However, these assumptions underlie the MCS used in the voltage analysis of active LV feeders with DG.

iii. Procedure

The approach used to carry out the voltage analysis for LV feeders with increasing PV-EG employs a MCS to iteratively evaluate the HB algorithm, which is an analytical model. The general procedure used is discussed below, with the flow chart in figure 5-7:

Step 1 Design the passive LV feeder, using the winter load ADMD beta parameters (α, β and c), for maximum voltage drop (10 %) for the required test scenario.

Step 2 Locate the interval of maximum excess generation, as shown in figure 5-3, from the summer load and PV data. Obtain the beta parameters for this interval – summer load beta parameters. Replace the winter load beta parameters with the summer beta parameters on the feeder.

Step 3 Calculate the maximum number of PV units that a household can install (*PV_unit_max*) through the formula – $\frac{0.5 \times \text{Circuit breaker size}}{\text{rated PV output current}}$. The total number of PV units (*PV_feeder_max*) is calculated as the product of the total number of customers and *PV_unit_max.*

Step 4 For each penetration level:

- Randomly allocate the associated number of PV units on a randomly selected node and phase, while checking that the selected phase has a customer allocate it and that the *PV_unit_max* has not been exceeded.
- Use the HB algorithm to calculate the feeder voltage profile for all the 3 phases and store the maximum voltage seen on all phases on all nodes.
- Calculate the total ADMD value of the feeder using, ADMD = $\frac{\alpha \times \text{number of customers} \times V_S \times c}{(\alpha+\beta)}$ and calculate the %EG/ADMD = $\frac{\text{Power rating of connected PV} \times 100}{\text{ADMD}}$

Step 5 Carry out *Step 4* 1000 times for each penetration level till the feeder is 'full' (i.e. when the total number of PV units equals to *PV_feeder_max.*

Step 6 Plot the results as a scatter plot on Voltage (p.u.) vs. %EG/ADMD graph.

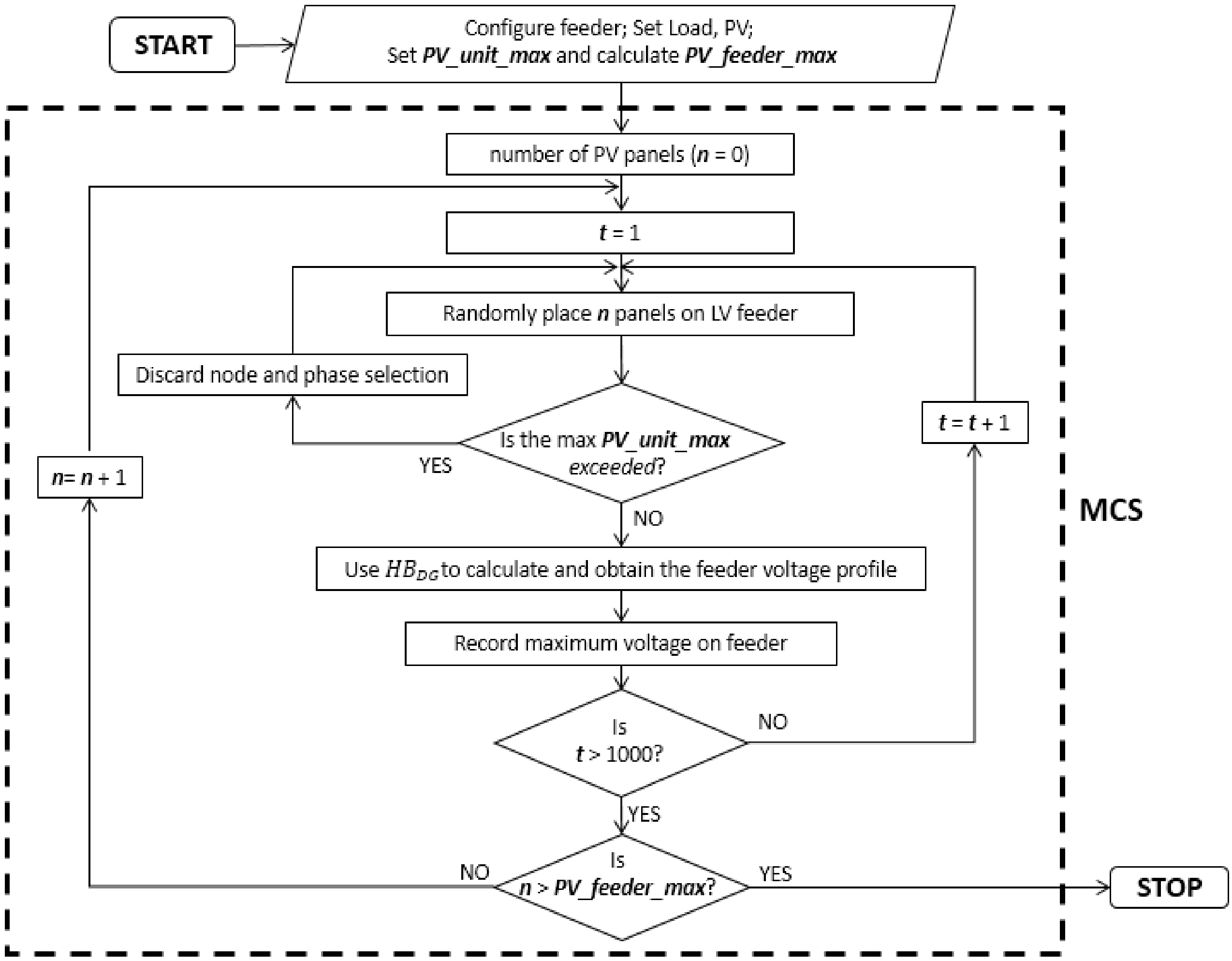


Figure 5-7: Procedure for voltage analysis on LV feeders with DG

Using the procedure stated above, the following test scenarios are investigated in chapter 6:

1. Case 1 – Base Case
    - Load – High LSM load (Claremont); Single phase profiles; 18 customers (3 per node)
    - PV – Single phase, rated 1-kW
2. Case 2 – As in base case but with lumped single phase PV units, larger in capacity, rated 3.5-kW
3. Case 3 – As in base case but with lumped PV units in the form of 10-kW three phase PV
4. Case 4 – As in base case but with a lateral branch of 2 nodes located at node 2. The location of the branch may be moved farther down the feeder away from the voltage source
5. Case 5 – As in base case but with unbalanced/uneven allocation of customers on the feeder
6. Case 6 – As in base case but with the variation of source voltage included (1.03 p.u.) to model the variation on MV networks such as times of light loading on the MV network
7. Case 7 – As in base case but with customers of a lower LSM (Orient Hills)

Chapter 6

6. SIMULATIONS AND RESULTS

This chapter describes the various test scenarios that were used to evaluate the performance of LV feeders under conditions of increasing PV penetration. The results of each test are discussed in detail in this chapter. Any surplus testing, extra information and results relevant to this chapter can be found in Appendix C.

6.1 Test Scenarios

6.1.1 Case 1: Base Case

For the base case, a simple 6-node radial feeder, shown in figure 6-1, with 18 customers is designed using the winter load (ADMD). The feeder conductor used is a 35 mm^2 4-core Copper conductor, with the thermal limits considered to design the passive LV feeder as shown in figure 6-1.

In order to carry out the voltage analysis of the active feeder with DG, the feeder load used is the summer load, particularly at the interval of maximum generation and minimum load. The PV-EG units allocated randomly on the feeder are 1-kW single phase PV units.

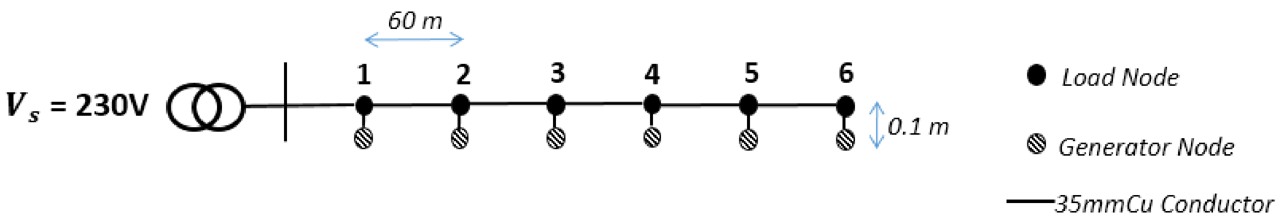


Figure 6-1: Base Case Passive LV feeder

In summary, the feeder parameters used for the base case are shown in table 6-1.

Table 6-1: Base Case LV Feeder parameters

FEEDER PARAMETER	VALUE		
Winter load (ADMD) beta parameters	α = 0.8625	β = 1.8953	c = 50.1270
Summer load beta parameters	α = 0.4366	β = 1.2877	c = 24.8680
1-kW 1-phase PV unit beta parameters	α = 255.50	β = 255.50	c = 6.8696
Maximum number of PV units (per customer)	7		

Result

Total design feeder load (ADMD) = $\mu_{winter\ load}$ × no. of customers = (15.677 × 0.23) × 18 = 64.90 kW

Total load (active feeder) = $\mu_{summer\ load}$ × no. of customers = (6.297 × 0.23) × 18 = 26.07 kW

The scatter plot as a result of the MCS for active feeder voltage analysis is shown in figure 6-2.

The response to increasing PV penetration on an originally passive LV feeder is an almost proportional increase in feeder voltage. The active feeder load is exceeded at PV capacity of 26.07 kW (approximately 40 % EG/ADMD) and at this point reverse power flow occurs in the feeder. Initially, before 40 % EG/ADMD, the increase in feeder voltage profile is due to the dispersion in the load demand (mean of 6.2967 A and standard deviation of 6.5516 A) as well as the unbalance in the phase voltages and voltages along length of the feeder due to the random allocation of PV units on the feeder (randomly selected phase and node. Beyond 40 % EG/ADMD, the feeder voltage increase is because current fed back into the feeder as a result of the excess generation causing reverse power flow. It is also important to note that the effect of dispersion in the load diminishes as stated in the central limit theorem.

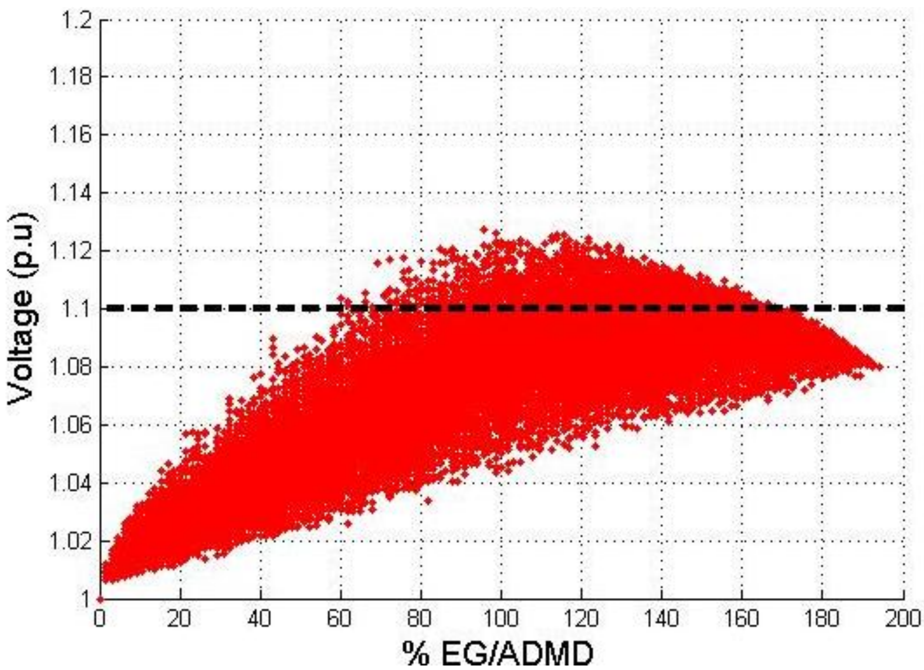


Figure 6-2: Feeder over-voltage profile for penetration of 1-kW single phase PV units (Base Case)

There is also an increase in the scatter caused by the random allocations of the PV units to different nodes and phases of the feeder (i.e. for each level of PV penetration, there is a range of possible feeder voltages arising from the various combinations of PV unit allocation in the feeder). The scatter plot converges to a single point because of the PV unit constraint (capacity of 50 % of the nominal household capacity), as well as the diminishing effect of the random PV allocations on the feeder (i.e. increased likelihood towards the generation being more balanced across the phases and length of the feeder). At the end point of the scatter plot in figure 6-2, the feeder is considered 'full' i.e. each customer has installed the legal maximum PV capacity – 50 % of the nominal household capacity. All voltages across the phases and along the length of the feeder are balanced.

The voltage analysis algorithm can be tested for consistency to ensure that it is correctly implemented and formulated as described in Appendix C. The test indicated that the voltage analysis algorithm is correctly formulated and has been implemented correctly. Further tests can be carried out using the base case to obtain more information and understanding of the results shown on figure 6-2.

i. Effect of dispersion in load demand

The effect of the dispersion in the load demand on the over-voltage profile of the active feeder can be illustrated by using a probabilistic model of a deterministic load demand (i.e. limited dispersion) with the same mean as the load model used in the base case. The load model parameters used are shown in table 6-2. Looking at table 6-2, two different beta PDFs are used to create load models – summer load model used in the base case and a load model with the same mean as the summer load model used in the base case but with limited dispersion. The beta PDFs representing these load models can be found in the Appendix C.

Table 6-2: Probabilistic model of varying load demand used and the probabilistic model of deterministic load demand

LOAD MODEL	α	β	c	μ	σ
With dispersion	0.4366	1.2877	24.8680	6.2967	6.5516
With limited dispersion	255.50	255.50	12.5926	6.2963	0.2783

Result

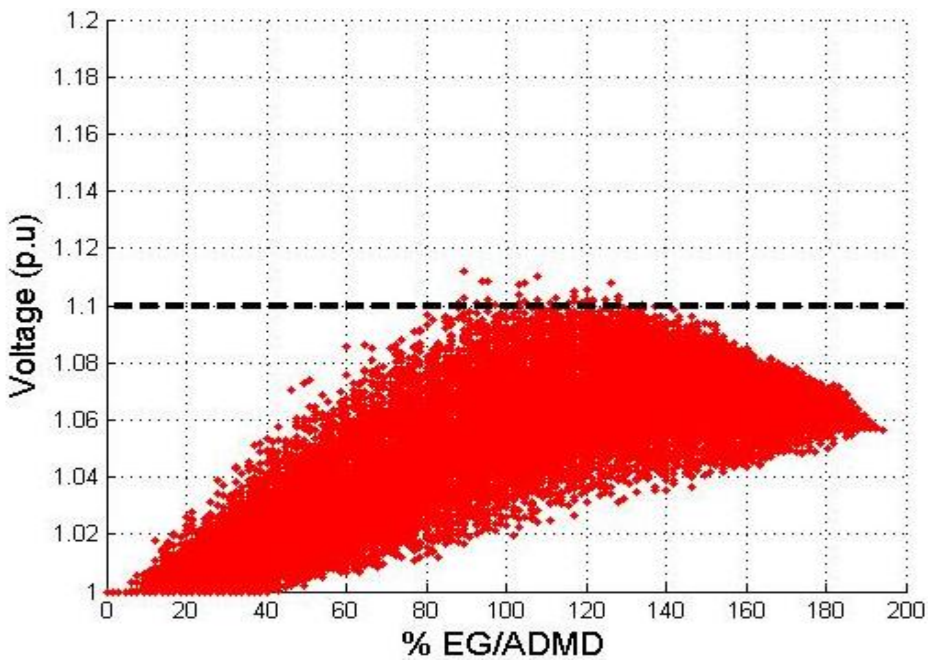


Figure 6-3: Base Case using a load model with limited dispersion

The result of this test can be seen in figure 6-3. The voltage rise in the base case result in figure 6-2 is higher than in this test. From the result shown in figure 6-3, it can be observed that the increase in feeder voltage due to the dispersion in the load demand in the active feeder has been diminished compared to the result in the base case in figure 6-2. This is because limited dispersion in the load means that the load currents on the feeder are close to the mean. Therefore, for the same voltage increase margin, as in the base case result figure 6-2, a larger DG penetration is required in this test. The increase in feeder voltage before reverse power flow (at 40 % EG/ADMD) is mainly due to unbalance in voltages across the phases and along the length of the feeder, caused by the random allocation of PV units on the feeder.

ii. **Analysing the unbalance of voltage across the phases and along the feeder length**

This effect of unbalance in voltages across the phase voltages and along the length of the feeder can be demonstrated by testing the scatter plot obtained from the base case by deterministically allocating/placing PV units on the feeder to cause maximum and minimum unbalance in the phase voltages. The procedure used to achieve this is described in detail in section C.1.3 in Appendix C. The deterministic envelope obtained provides the worst case (extreme conditions) of the balanced and unbalanced PV unit allocation. This is illustrated in figure 6-4.

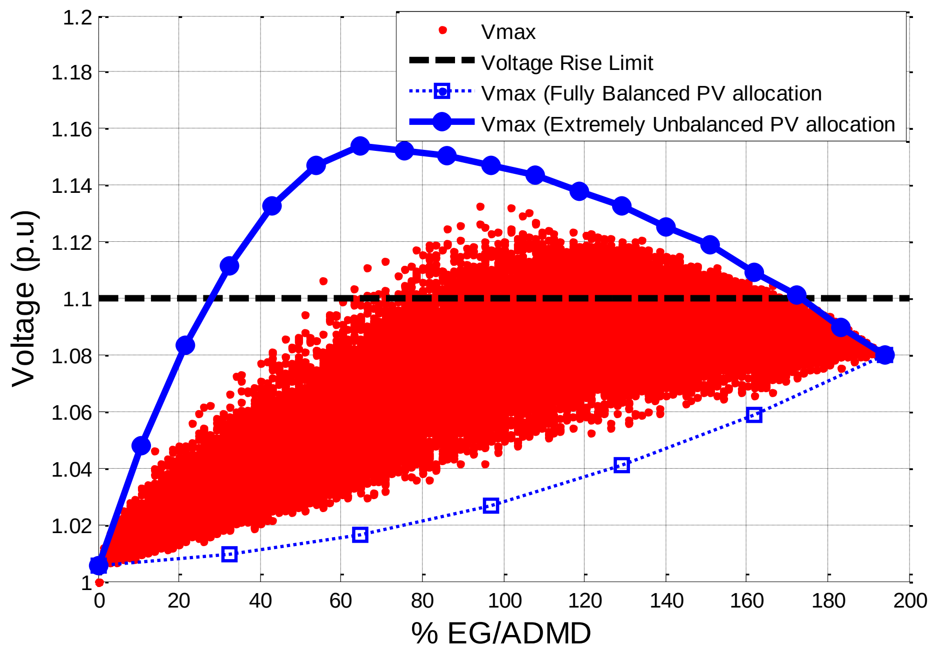


Figure 6-4: Overlay of the deterministically determined envelope on the probabilistic scatter plot

From figure 6-4, the envelope produced is as expected and encompasses the probabilistic scatter plot. With iterations tending to infinity per penetration level, it is expected that the scatter plot will cover all the space within the envelope.

The very low probabilities of obtaining PV allocation scenarios that produce close to maximum and minimum unbalance means that a large density of the maximum voltage plots will be in the probabilistic scatter plot shown in figure 6-4.

iii. Interpreting the scatter plot

The base case scatter plot also provides useful distributions that can be used by planning engineers in the voltage analysis of active LV feeders. There are two distributions that can be obtained from the scatter plot that can be very useful to planners, as shown in figure C-4 found in section C.1.4 in Appendix C. These distributions are at:

1. Specified penetration level (%EG/ADMD)
2. Specified feeder voltage level (p.u.)

Analysing the distribution of maximum voltages at a specified penetration level (%EG/ADMD)

At any particular level of PV penetration (%EG/ADMD), a vertical distribution of maximum voltages can be obtained. This distribution can be useful in that a planner may use it to obtain a design voltage at given/desired risk levels, as shown in figure C-4. This is useful in that it provides the planner with the highest voltage (depending on the selected risk level) he/she can expect for a particular DG penetration level.

The understanding of the impact of the risk level selected is important and is explained in table 6-3. Suppose a planner or utility decides to allow a maximum DG penetration level of 85 % EG/ADMD on the LV feeder. The maximum voltages at 85 % EG/ADMD are obtained and a beta PDF is fit onto the data. The parameters of the beta PDF obtained are alpha = 2.4997, beta = 2.989, minimum = 1.041 and maximum = 1.1177. Using the distribution at that penetration level, a planner can observe that the highest voltage expected on the feeder would be 1.1013 p.u. at a 5 % risk level.

A planner may decide to opt to determine the highest voltage expected at a risk level other than 5 % for various reasons such as investment capital available to reinforce the feeder in case of voltage violation, willingness of consumers in a certain area to install PV units etc. Table 6-3 shows the significance of the risk level used in determining the design voltage. The use of a design voltage at a risk level lower than 5 % would mean the feeder would be exposed to a higher likelihood of occurrence of voltages above the design voltage and vice versa.

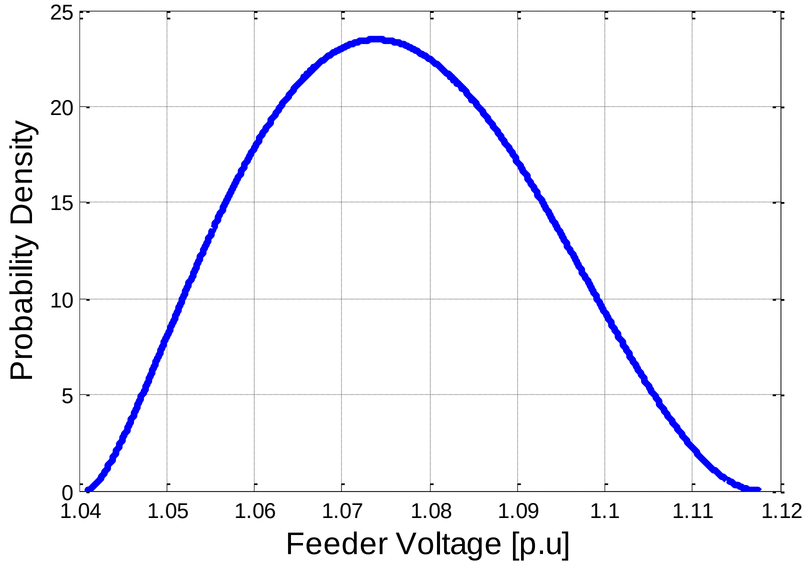


Figure 6-5: Beta PDF representing the maximum voltages at penetration level 85 % EG/ADMD

Table 6-3: Maximum voltages at selected risk levels at penetration level - 85 % EG/ADMD

RISK LEVEL [%]	VOLTAGE [p.u.]	P (X > V)
5	1.1013	0.05
50	1.0755	0.50
95	1.0521	0.96

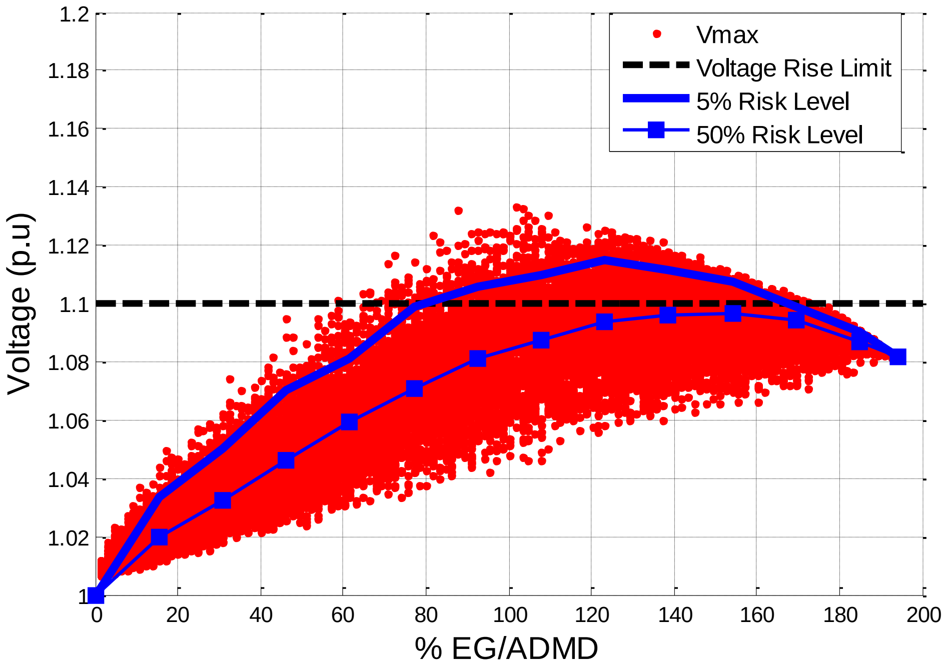


Figure 6-6: 5 % and 50 % risk voltage contours for the scatter plot (below)

Using the above mentioned approach for each level of DG penetration, one can extend the results of the scatter plot by adding a plot of contours of design voltages at any desired risk level, as shown in figure 6-6. This new result can be useful in determining the hosting capacity of the LV feeder. Using the 5 % risk level, as shown in figure 6-6, the hosting capacity of the LV feeder is a total DG capacity 80 % of the ADMD used to design the LV feeder. This approach used is not limited to the risk levels shown in figure 6-6, but can be adapted to the planner's requirements.

Analysing the distribution of maximum voltages at a specified voltage level (p.u.)

Utility/distribution planners may opt to set a voltage limit, lower than the statutory 1.1 p.u., to use as a design parameter for active LV feeders with DG. At any particular voltage level selected, a horizontal distribution of maximum voltages can be obtained, as shown in figure C-4. This distribution can be useful in that a planner may use it to obtain a design penetration level at a desired probability of exceeding the selected voltage level. In this case, for each penetration level, the number of occurrences of voltages exceeding the selected voltage limit is recorded and the probability obtained. A CDF is plotted based on the probability of occurrence of voltages above the selected voltage limit. Suppose a planner selected a voltage limit of 1.05 p.u. for which to design the active LV feeder with DG. A CDF is plotted as shown in figure 6-7.

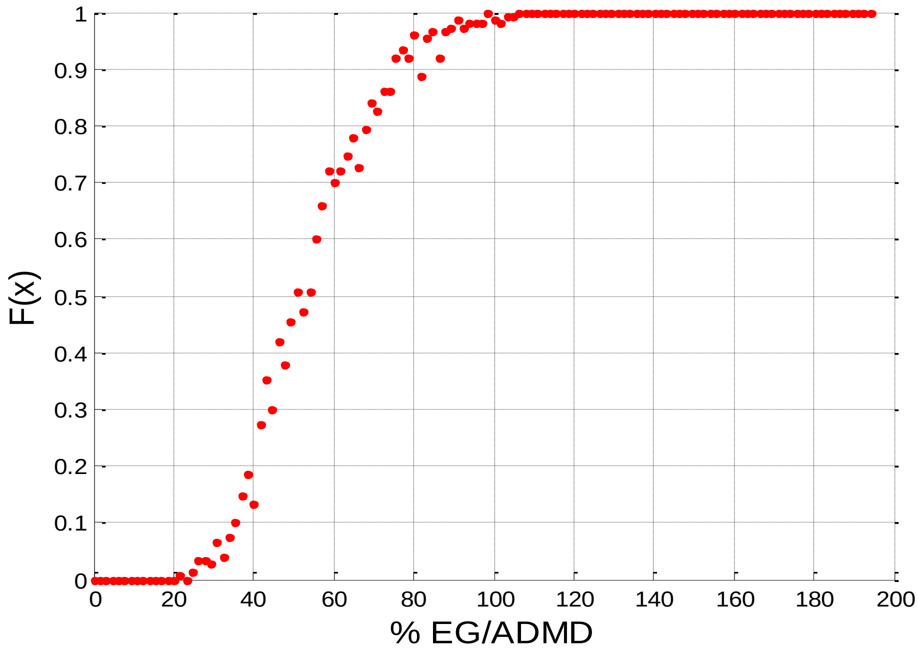


Figure 6-7: CDF plot representing occurrence of voltages above 1.05 p.u. for each level of penetration

From figure 6-7, the planner can choose the probability acceptable to him/her for occurrence of voltages above the 1.05 p.u. limit. For a probability less than 0.1 (10 occurrences for every 100 cases of feeder operation), the planner can choose to limit the DG on the feeder to a capacity 35 % of the ADMD used to design the feeder, as observed from figure 6-7.

The scatter plot result can be utilised as presented above for various plans and implementations as desired by the planner.

6.1.2 Cases 2-3: PV-EG Characteristics

The sensitivity of the results from the base case is tested by extending the base case set up to carry out tests whilst varying the characteristics of the PV units.

i. Case 2: PV Maximum Power Output rating

In this test, the maximum power output of the PV units is increased from 1-kW to 3.5-kW single phase PV units randomly allocated on the feeder. The effect of increasing the current output from a discrete PV unit is tested. The changed feeder parameters are shown in table 6-4.

Table 6-4: LV feeder parameters changed for this test

FEEDER PARAMETER	VALUE		
3.5-kW 1-phase PV unit beta parameters	α = 255.50	β = 255.50	c = 24.0435
Maximum number of PV units (per customer)	2		

Result

Figure 6-8 shows the scatter plot of the over voltage profile for penetration of 3.5-kW PV units. Compared to the base case scatter plot (figure 6-2), the scatter plot of maximum possible voltages in figure 6-8 has increased in both value and spread/range. The voltage rise in this case is higher than in the base case. The increased current output from a discrete PV unit means that for a particular penetration level, the feeder will experience higher voltages than in the base case.

In addition, the spread/range of possible maximum voltages for a particular penetration level is increased in this case compared to the base case because the increased current output from a discrete PV unit enhances the effect of unbalance in voltage across the phases and along the length of the feeder, as a consequence of random allocation of PV units on the feeder. In this scenario (figure 6-8), although there are a fewer number of PV units installed on the feeder, the risk of voltage violation is observed just after 30 % EG/ADMD while in the base case(in figure 6-2), the risk of voltage violation is observed just after 60 % EG/ADMD.

Although the PV units used in this scenario have a larger power rating than in the base case (therefore a larger current output), the scatter plot converges at 1.08 p.u., like in the base case scenario (figure 6-2). This is because of the PV unit limit per household (50 % of the nominal connection capacity of the household) which allows only maximum of two 3.5-kW PV units compared to the maximum of seven 1-kW PV units.

Nevertheless, in both cases, the feeder is 'full' when each household has the maximum PV unit installed equating to the same current output leading to convergence to the same point for this case and the base case.

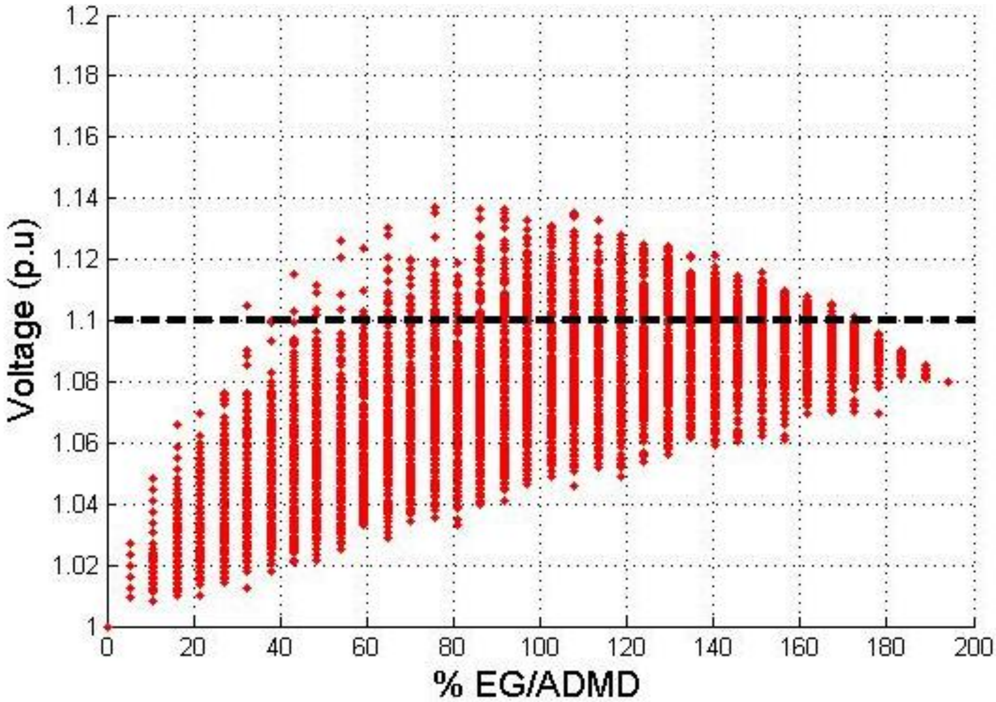


Figure 6-8: Feeder over-voltage profile for penetration of 3.5-kW single phase PV units

ii. Case 3: PV Construction Type

The effect of type of PV unit construction used in the feeder on the sensitivity of the results in the base case, is tested in this scenario. In the base case, 1-kW single phase PV units are allocated on the feeder. While keeping the feeder topology the same as the base case, the PV units placed on the feeder are changed from 1-kW single phase to 10-kW three phase PV units. The changed feeder parameters are shown on table 6-5.

Table 6-5: LV feeder parameters changed for this test

FEEDER PARAMETER	VALUE		
10-kW 3-phase PV unit beta parameters	α = 255.50	β = 255.50	c = 22.8053
Maximum number of PV units (per customer)	2		

Result

Figure 6-9 shows the scatter plot of the over voltage profile for penetration of 10-kW three phase PV units. The voltage rise in this scenario is lower than in the base (figure 6-2). Although the current output in the PV units used in this scenario is greater than in the base case, the three phase construction of these PV units means that the voltages are always balanced across the phases in the same node.

As a result, the unbalance in voltages is mainly along the length of the feeder causing the scatter plot to have a smaller spread/range than the base case (figure 6-2) and the case 2 (figure 6-8). In the base case and case 2, the unbalance of voltages across the phases (same node) and along the feeder causes the scatter plot results to have a larger spread than in this case.

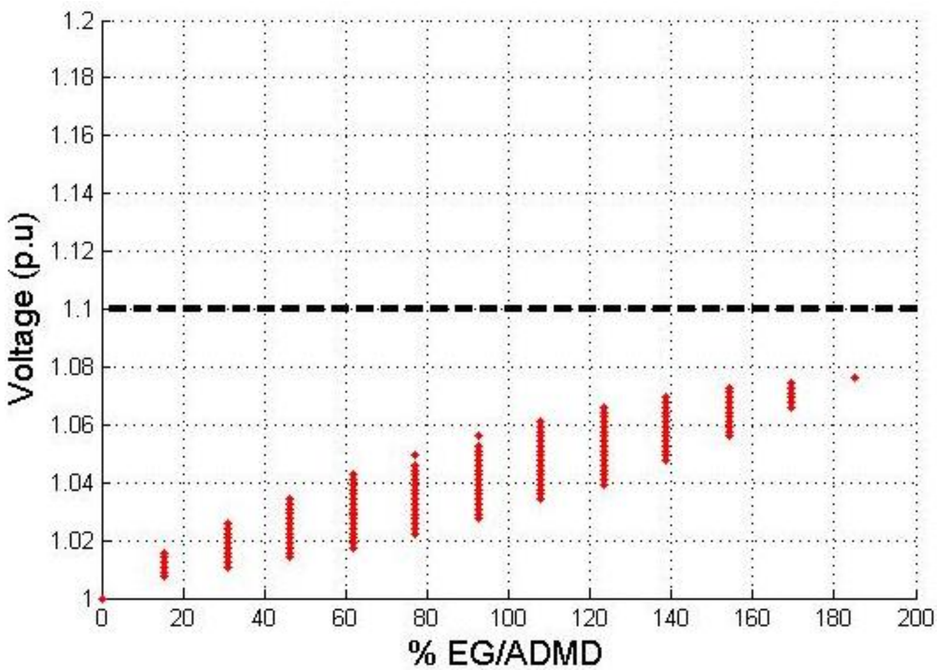


Figure 6-9: Feeder over-voltage profile for penetration of 10-kW three phase PV units

6.1.3 Cases 4-7: Feeder Characteristics

In practice, not all feeders are as simple and balanced as in the base case. In this section, the feeder is modified and its characteristics varied to analyse their effects.

i. Case 4: Lateral Branch

In this scenario, the base case feeder is changed to a feeder with a short lateral branch at node 2 as shown in figure 6-10. The passive feeder is designed for maximum voltage drop of 10 %. The branch is located near the start of the feeder.

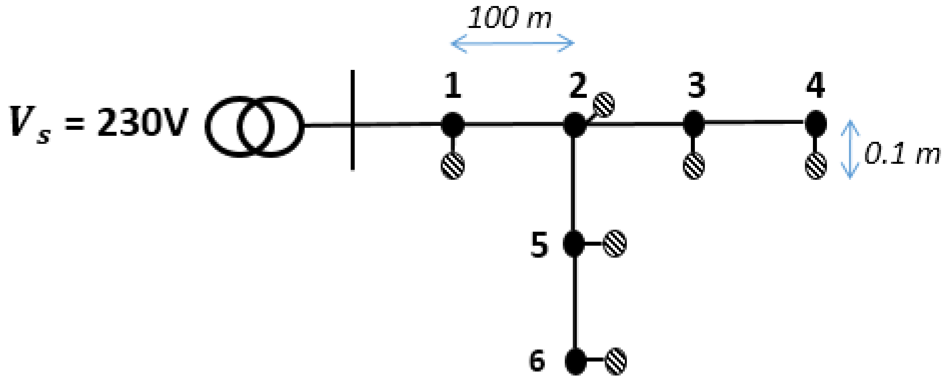


Figure 6-10: Feeder with lateral branch at node 2

Result

The passive feeder is designed for maximum voltage drop of 10 % at the farthest node. In a simple equally loaded LV feeder with no branches, the feeder load, like the feeder voltage, decreases proportionally along the feeder length away from voltage source. In branched feeders, the feeder load is not proportionally distributed along the length of the feeder but most of the feeder load is concentrated on the node(s) with branches. Let us consider nodes 1-2-3-4 as the main feeder line and nodes 2-5-6 as the lateral branch on the main feeder, as shown in figure 6-10.

The lateral branch, located at node 2, causes a larger proportion of the overall voltage drop on the feeder (nodes 1-2-3-4) to occur before the node 2. This is because:

- Before node 2:

  The total feeder load decreases proportionally along the feeder length away from the voltage source, resulting in an equally proportional decrease in voltage from the voltage source.

- After node 2:

  The feeder load decreases sharply after node 2 along the feeder (nodes 1-2-3-4) length away from the voltage source because a larger portion of the total feeder load is concentrated on node 2 as a result of the lateral branch at node 2. The voltage gradient along the feeder (node 1-2-3-4) length after node 2 is less than the voltage gradient along the feeder length before node 2.

Figure 6-11 shows the scatter plot for the over voltage profile for penetration for 1-kW single phase PV units on a branched feeder. The voltage rise in this scenario is higher than in the base case (figure 6-2). This is because the presence of a branch means that there is increased concentration of PV-EG on the location of the branch node on the feeder thereby increasing the margin of voltage rise. This also means the likelihood of exceeding the voltage limit at a much lower penetration level is increased.

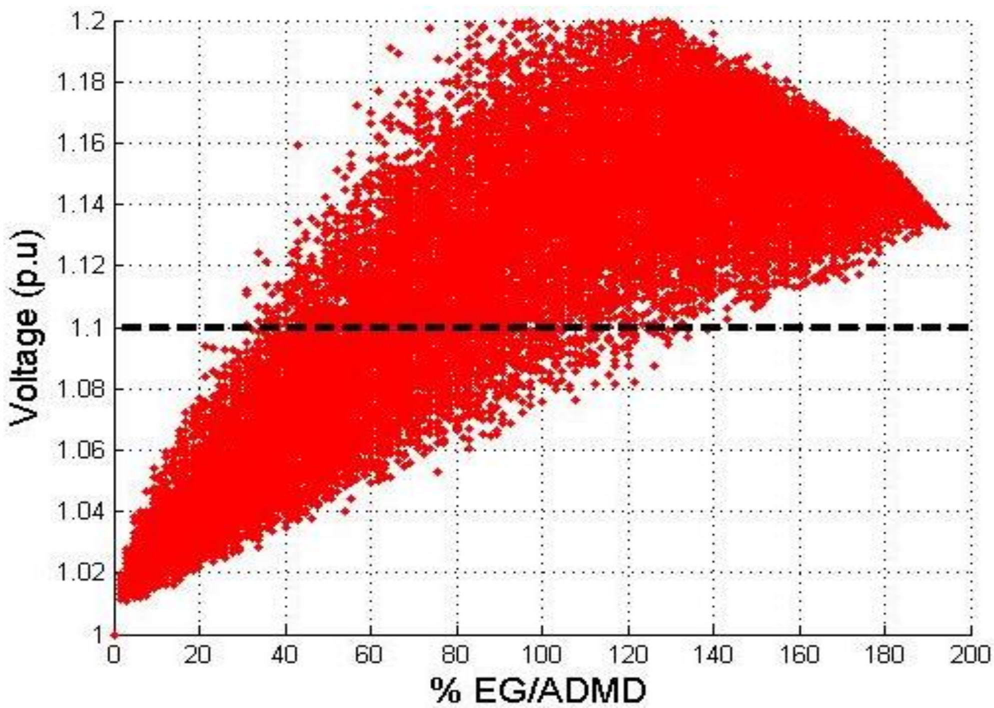


Figure 6-11: Feeder over-voltage profile for penetration of 1-kW single phase PV units on a branched feeder

The effect of changing the position of the lateral branch was also investigated by having the lateral branch farther away from the source voltage (start of the feeder). The results can be found in Appendix C. It was found that the position of the branch on the feeder also affected the scatter plot in that:

- The effect of the adding a branch to the feeder compared to the base case produced similar results – significant increase in voltage rise shown in the scatter plot
- The voltage rise, in the feeder with the lateral branch farther away from the voltage source, was lower than that in the feeder, with the lateral branch nearer the voltage source. This is because having the branch nearer the source voltage means that the feeder load density after the branch is lower and therefore allowing for increased voltage rise with increased PV penetration

ii. **Case 5: Unbalanced customer allocation**

This test was carried out to test the effect of introducing more unbalance in the feeder by having unbalanced customer allocation in the passive LV feeder. The feeder topology is designed as the base case feeder, however the customer allocation used was [Cos300], keeping the same total number of customers as the base case. The Cos300 consumer pattern means that there are 3 consumers on the node/kiosk with all 3 consumers placed on only one phase per node and the phase of allocation alternates from node to node according to the Cosine pattern. The PV feeder allocation, as described in chapter 5, is only limited to phases with customers and maximum number of allowable PV units is a characteristic of the number of customers in each phase.

Result

The passive LV feeder in this case is designed for maximum voltage drop of 10 % but is different from the base case in that the phase voltages are unbalanced. Figure 6-12 shows the scatter plot for the over voltage profile for penetration for 1-kW PV units on a feeder with intrinsic unbalance in customer allocation. The voltage rise in this scenario is higher than in the base case (figure 6-2). This is because of the increased unbalance in voltages across the phases and along the length of the feeder. Consequently, the voltage rise is higher even at low penetration levels and increases the likelihood of voltage violations due to PV-EG. Unbalanced allocation of customers on LV feeders, like in practical feeders used in power systems today, increases the likelihood of the voltage violations at penetration levels lower than in the base case.

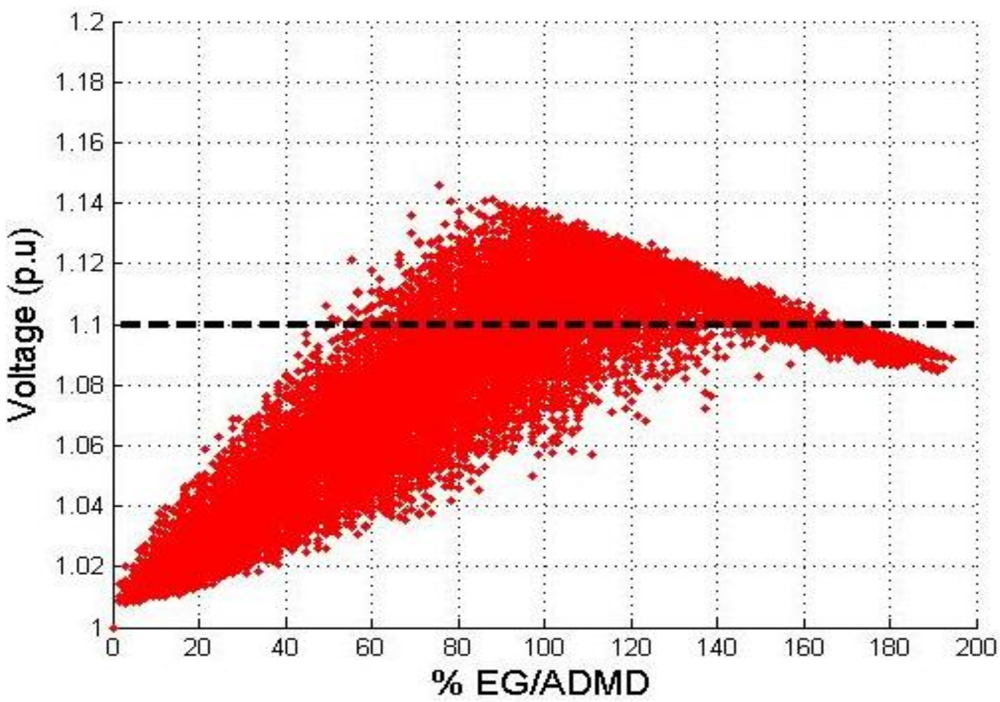


Figure 6-12: Feeder over-voltage profile for penetration of 1-kW single phase PV units on a feeder with unbalanced customer allocation

iii. Case 6: Variation in source voltage

From the results of the base case (figure 6-2), there is likelihood of voltage violation when the PV-EG penetration level exceeds 60 % EG/ADMD (i.e. 0.01 p.u. voltage rise per PV-EG penetration level of 6 % EG/ADMD).

A typical new distribution (MV/LV) transformer has a rated secondary voltage of 420 V, which is 242.49 V single phase – 1.05 p.u. of the nominal voltage (230 V). The purpose of this specification was to provide voltage support on heavily loaded feeders, on the basis that the MV feeder voltage might fall to 0.97 p.u. of the nominal voltage during these heavy loading conditions. However, during lightly loaded times of the day, if the feeder voltage rises to about 1.03 p.u., the voltage boost would take the LV source voltage to 1.08 p.u. of the nominal voltage, 230 V.

Result

Figure 6-13 shows the scatter plot of the over voltage profile for penetration of 1-kW single phase PV units on a feeder during light loading conditions of the MV network. These conditions cause an increase in source voltage to 1.03 p.u. leaving very little scope for injection of PV-EG into the feeder. In the base case, it is observed that the likelihood of voltage violations occurs for penetration level exceeding 60 % EG/ADMD.

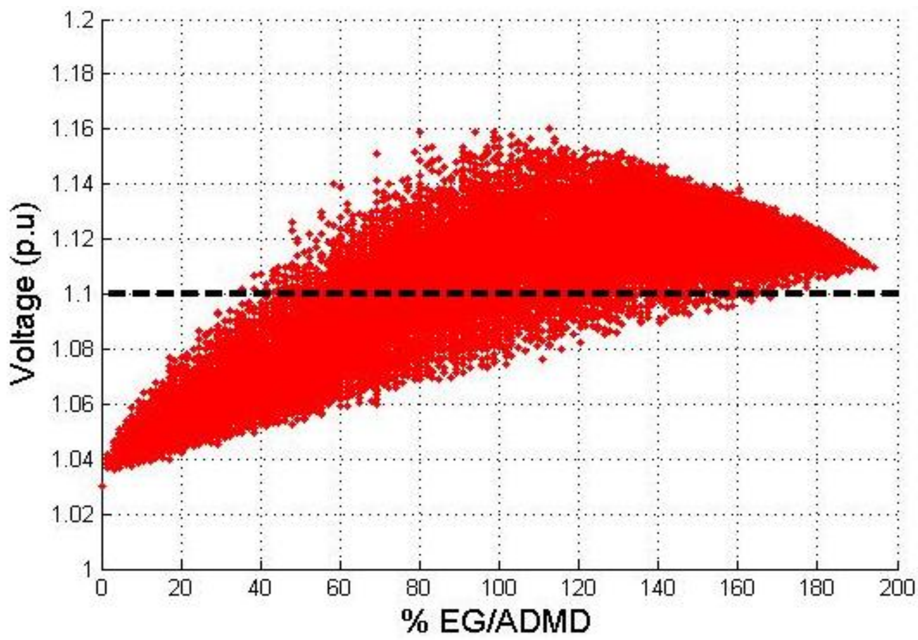


Figure 6-13: Over-voltage profile for feeder with source voltage 1.03 p.u.

However, during times when the MV network is lightly loaded during the day, causing the source voltage to rise to 1.03 p.u., the likelihood of voltage violations occurs for penetration levels of as low as 38 % EG/ADMD.

iv. Case 7: Customer Classification

The use of the %EG/ADMD parameter for the horizontal axis of the voltage analysis output graph allows the results to be scaled to feeders of any capacity since it is an independent parameter. A feeder is designed for an area of lower Living Standard Measure (LSM) than the base case site.

The details of this site, as obtained from the load data for Orient Hills [NRS 034 Domestic Load Research Project, 2000], is displayed in table 6-6. The LV feeder topology is shown in figure 6-14.

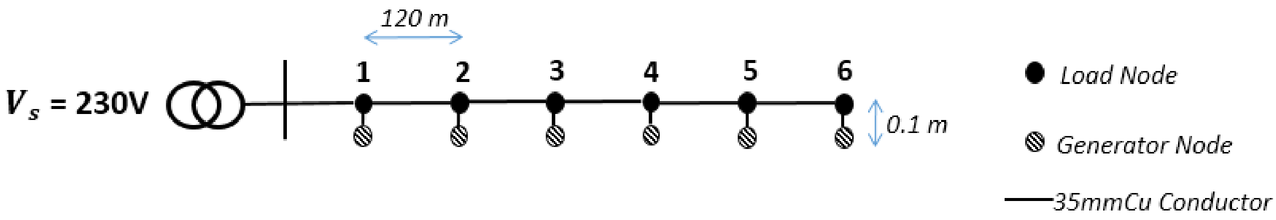


Figure 6-14: Passive LV feeder

Table 6-6: LV feeder parameters for feeder designed for Orient Hills

FEEDER PARAMETER	VALUE		
Winter load (ADMD) beta parameters	α = 0.6865	β = 1.8022	c = 26.0160
Summer load beta parameters	α = 0.3572	β = 1.1627	c = 13.6270
1-kW 1-phase PV unit beta parameters	α = 255.50	β = 255.50	c = 6.8696
Maximum number of PV units (per customer)	7		

Result

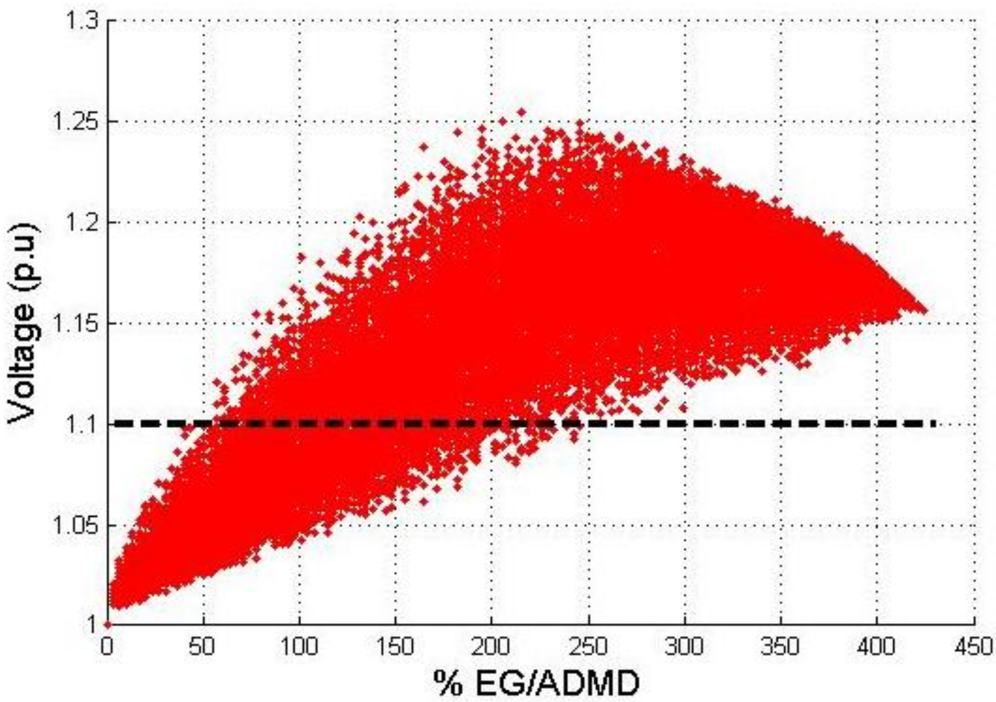


Figure 6-15: Over-voltage profile for feeder with lower LSM customers

Figure 6-15 shows the scatter plot for the over voltage profile for penetration for 1-kW single phase PV units on a feeder with customers of a lower LSM. The ADMD used to design the passive feeder for this test is 29.71 kW while the base case ADMD is 64.90 kW (i.e. the base case is 2.18 times the ADMD used to design the passive feeder for this test).

It is observed that the scatter plot of voltage rise in this scenario is greater than the base case scatter plot. This is because the load on the active feeder in this scenario is lower than the active feeder load in the base case, thereby causing larger excess generation for a particular penetration level leading to higher maximum voltages.

It is also important to note that the use of the parameter %EG/ADMD allows us to scale the base case scatter plot to give a feeder of another capacity, which in this case has a capacity $\frac{1}{2.18}$(0.4587) of the base case feeder capacity. This is illustrated in figure 6-16, which compares the voltage contours of the scatter plots from case 7 and the base case at a risk level of 95 %.

From figure 6-16, it can be observed that the voltage plot result from case 7 is roughly scaled to be twice the voltage plot result of the base case since the ADMD used to design the feeder in case 7 is approximately half that used in the base case feeder. This result indicates the usefulness of the voltage analysis tool in that it can be applied to feeders of any ADMD and the result of the base case (figure 6-2) scaled accordingly.

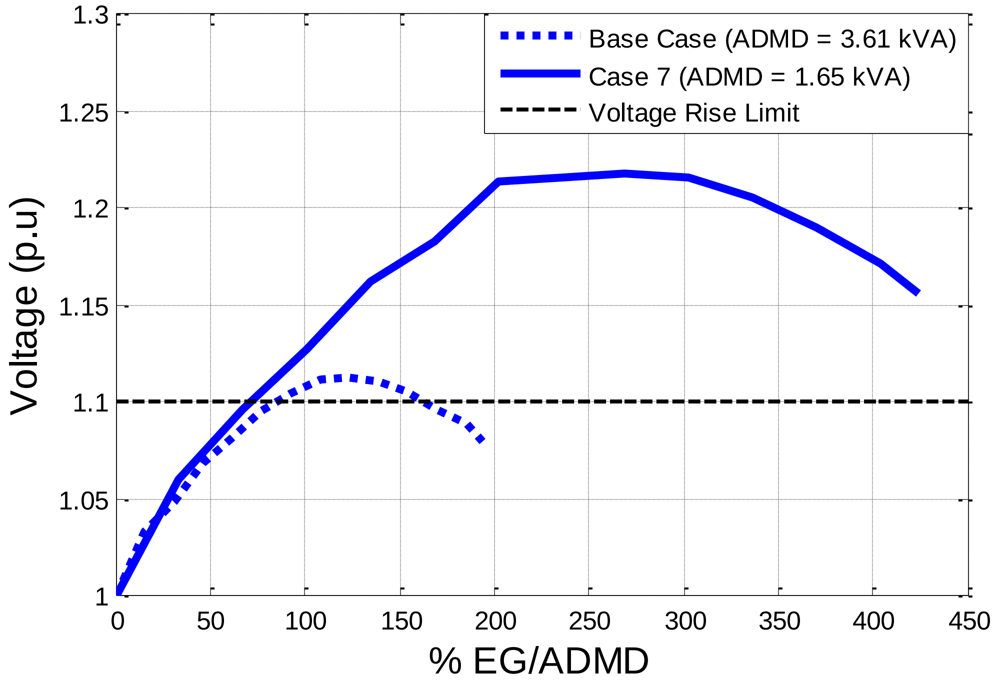


Figure 6-16:Comparing the voltage rise plots for the Base Case and Case 7 at 95 % risk level

6.2 Implications of results

The testing of several scenarios done in this chapter indicates that a general application, i.e. 'active feeder voltage analysis', can be carried out by distribution planners for any practical LV feeders used today. There are many factors that play an important role in the planning of LV feeders with DG such as the PV-DG characteristics, feeder characteristics, which include the topology and configuration, consumer classification, feeder parameters like source voltage etc. Consequently, it is important for a utility to have a comprehensive and concise parameter (i.e. the rated connected DG capacity, expressed as a percentage/ratio of the design ADMD) with which to set the statutory limits for DG penetration on distribution LV feeders. In South Africa, feeders are designed using an ADMD, known to the utility, and therefore a limit can be set based on the ADMD irrespective of the above mentioned factors that are different for the various LV feeders set up in the country.

The result of the voltage analysis of active LV feeders using MCS indicated that it can be recommended that the utility can allow connected PV-DG capacity of up to 30 % of the actual ADMD used to design the passive radial LV feeder. Once the penetration is limited to 30 %, the maximum voltage expected on the feeder will not exceed 242.03 V (1.0523 p.u. of the source voltage), with 5 % risk. This choice of this penetration level is to allow for an additional voltage rise due to the above mentioned factors such as variation of source voltage during conditions of lightly loaded MV networks, feeders with lateral branches, feeders with unbalanced consumer allocation along the feeder etc. In addition, the utility may not be able to control the rate and pattern pertaining to the installation of solar PV panels on households on a particular feeder, as well as the legality of those connections. Having a prescribed DG penetration limit of 30 % EG/ADMD will ensure there is little to no likelihood of voltage violations. The results from this study agree with the restrictions put forward by Carter-Brown [2012] for shared LV feeders, common in residential areas.

Feeders of lower capacity, usually found in rural areas, are much more sensitive to increasing PV-DG penetration than feeders of higher capacity, found in urban areas. As a result, the utility should pay much more attention to feeders in rural areas given the fast growing drive to install solar PV panels to supplement their electricity.

Although the tests in this chapter were carried out in three phase systems, it should be noted that the voltage analysis tool can be used for bi phase and single phase LV feeder technologies. This provides an application that caters for all LV feeder technologies used in South Africa.

Chapter 7

7. CONCLUSIONS

This study was motivated by the desire of the utility to plan for increased penetration of Distributed Generation (DG) in low voltage distribution systems in South Africa. The purpose of this study was therefore to investigate the need for a planning tool for LV feeders with DG and develop such a tool in order to enable voltage calculation in LV feeders with DG. The tool would then be used to analyse the voltage rise constraints of active LV feeders with DG. The tool was developed based on the Herman-Beta algorithm, initially developed for passive LV feeder design. This chapter discusses the outcomes of the study.

7.1 Summary of Answers to the Research Questions

This research/study has been aimed at investigating the validity of the hypothesis stated in Chapter 1. The hypothesis put forward was as follows:

"*The Herman-Beta algorithm, using the negative load approach for DG models introduced by Gaunt et al. [2011], can be modified for active LV feeders, validated using a MCS and applied as a tool to analyse the voltage rise constraints of active LV feeders*"

The results and findings of this study have been used to answer the research questions, which were derived from the hypothesis. The summarized answers to the research questions formulated are as follows:

- ***What impact does the presence of DG have on LV distribution feeder systems?***

  DG is defined as an electrical source, which is small compared to the centralized power generation plants, connected to the power network system at a point close to or at the consumer site (household). Generally, DG has been installed to provide back-up generation and on-site power supply to meet the ever increasing demand for electricity and improve the power quality of the distribution systems. DG is very beneficial in distribution systems from reducing distribution losses, voltage support and reliability points of view. However, many broad issues arise from increased penetration of DG on distribution systems today. The various problems caused by increasing penetration could include: reverse power flow, voltage rise, voltage fluctuations, voltage and current unbalance, increase in power losses, increase in total harmonic distortion (THD), and protection mal-protection.

It has become imperative that utilities plan for the increasing presence of DG on distribution feeder systems. This can be achieved by developing appropriate models for DG as well as a suitable tool for analysing the effect of increasing DG on LV feeders.

- ***How successful have been attempts to model DG on LV feeders?***

DG models used in various studies vary depending on the aspect of the DG relevant to the investigation such as the generator and technology implemented, operating conditions, DG performance etc. The important issues relevant in this study are the operating conditions and DG performance. Like the load, renewable DG varies with time and season and therefore this variation has to be considered. Modelling of DG performance based on actual generation data has been difficult over the years given the lack of high resolution raw data available. As a result, DG performance has been modelled statistically. There have been many statistical models based on various probability distributions such as the Gaussian PDF for solar PV, the Weibull PDF for wind models etc. There have been many issues regarding the operation of DG in the power system such as whether the role of DG in voltage regulation is steady state or dynamic, whether PV-EG can be considered a negative load or grid asset etc. The idea of using ‘negative loads’ to model the effect of DG on the grid has been successfully used by various authors thereby prompting the use of this approach in this study. In this study, the DG used was a negative load with its performance modelled using a beta PDF to have a negative probabilistic load. The usefulness of the beta PDF to model DG arises from its finite base, its ability to be negatively or positively skewed depending on the DG performance, and the ease of calculating the beta parameters (α and β) from data.

- ***What approaches have been used to calculate voltage in LV feeder systems?***

Several voltage calculation techniques have been developed and used by many researchers. Deterministic techniques (e.g. Newton-Raphson and forward-backward-sweep methods) make use of average inputs of load and DG, similar to traditional power flow methods, to calculate voltage in feeders. These techniques do not consider the time variation in loads and DG installed on the feeder, which is very important in the optimal design of both active and passive LV feeders. Monte Carlo Simulation (MCS) techniques, which have become popular with increased computer speed over the years, make use of random sampling from a large data base of discrete values or a PDF describing the data before performing successive network voltage calculations.

According to Zhang & T.Lee [2004], despite its ability to provide accurate results, the MCS is time consuming and not suitable to handle practical systems but is a reliable tool for comparison purposes.

Another technique, which combined the concept of Cumulants and Gram-Charlier expansion theory, was used to compute a probabilistic load flow in extensive power systems. In addition, this method significantly reduced the computational time, compared to the MCS, while maintaining a high degree of accuracy.

However, the Herman-Beta (HB) method, based on the beta PDF model of load currents, provides a faster approach (less computational time) used for calculating voltage, compared to the MCS and the method of Cumulants. The HB method is a statistical method and includes the effects of diversity and variation in the load demand. The HB method is the recommended method of voltage calculation in LV feeders (passive LV feeder design) in South Africa. This study extends the use of the HB algorithm to calculate voltage in active LV feeders.

- ***What assumptions underlie the Herman-Beta algorithm and are they still valid in the presence of DG on LV feeders?***

The HB method has been used for the design of passive LV feeders in South Africa. This study looks into two sections i.e. the passive LV feeder design and voltage analysis of the active LV feeder with DG. Some of the assumptions taken into account in the HB algorithm in the design for passive LV feeders and the effect they may have on the results of this study are as follows:

- Maximum volt-drop occurs at interval of maximum demand for the anticipated group of consumers

  This assumption remains valid in this study as it remains the parameter used to design passive LV feeders (i.e. LV feeders are designed for a maximum voltage drop of 10 %). In this study, a prerequisite for the voltage analysis of active LV feeders with DG is a passive LV feeder designed for maximum voltage drop, as is done for practical LV feeders. This assumption is used in the design of passive LV feeders. The introduction of DG in LV feeders requires a look at the correlation of the load and DG profiles, varying with time. This is because the load profile may not necessarily match the DG profile. In the analysis of feeders with DG, the period that is of interest is the interval the minimum load and maximum generation coincide. In active LV feeders with DG, the maximum voltage rise occurs at the interval of minimum load-maximum generation.

- <u>Loads represented as currents at unity power factor (pf =1)</u>

  This assumption remains valid in this study as it remains the parameter used to design passive LV feeders. In the voltage analysis of active LV feeders with DG, time interval of interest in the load demand profile differs from that in passive LV feeder design.

  However, it remains appropriate to consider loads as currents at unity power factor as the residential load still remains largely resistive because of its large heating component. In addition, the use of unity power factor, as applied to DG has been investigated in this study. Review into literature indicated that with regards to DG, the power factor used depends on the technology implemented, generator as well as the operating conditions of the DG. Many researchers have gone on to successfully use unity power factor in voltage calculation, thereby allowing this assumption to remain unchanged for DG.

- <u>At any specified interval the collective loads may be represented as a statistic that fits the Beta PDF</u>

  The suitability of the beta PDF to represent the load and DG at any specified interval as discussed in Chapter 3 remains the basis of the Herman Beta method and is valid. This application of the beta PDF has been backed up in various research studies and the beta PDF has been a very useful PDF to represent the load at any interval to include its variation. This assumption remains valid in this study as it remains the parameter used to design passive LV feeders. It remains valid in the voltage analysis of active LV feeders with DG as a group of PV-EG units at any particular interval may be represented using the beta PDF. Beta PDFs representing the load and DG at that interval are used in the HB method for voltage calculation in active LV feeders.

- ***What changes must be made to the HB algorithm to allow for voltage calculation in LV feeders with DG and did it agree with the MCS for voltage calculation?***

The HB algorithm can be seen as a transform of a beta distribution of load currents to a beta distribution of voltages, from which a voltage can be selected at the desired risk level. It uses the first and second statistical moments, calculated from the beta PDF of load currents, to calculate the first and second statistical moments of the consumer voltage, which in turn can be used to calculate the alpha and beta parameters of the beta PDF of consumer voltages. With the knowledge that loads cause voltage drop and generation causes voltage rise in feeders, the parameters of the beta PDF of resultant voltages (first and second statistical moments, minimum and maximum voltage) must be modified to model DG effect on feeders.

These were successfully modified to allow for voltage calculation (voltage rise) in active feeders, as a result of thorough and rigorous testing, as well as validation against the MCS for voltage calculation. The HB method for both passive and active LV feeders, in both bi phase and three phase network topologies, has been tested against the MCS for voltage calculation for a variety of scenarios and produced results that agree with the MCS. This means that the voltage calculation using the HB method is valid for both passive LV and active LV feeders.

- ***How can the HB algorithm be used as a tool for voltage analysis of LV feeders with DG?***

In work done related to evaluating the impact of DG penetration in LV feeders, several performance parameters (such as voltage, frequency, power losses, current etc.) have been used to assess the effect of increasing DG penetration. In this study, the parameter of interest is voltage, which can be obtained using the HB method. A tool was developed using MCS techniques to create various combinations of PV-EG allocation on the feeder and apply the HB method for voltage calculation in order to fully analyse the voltage impact of increasing DG on LV feeders. The output of the voltage analysis using MCS was a graphical solution (voltage vs. penetration level), with a limit of improvement (10 % voltage rise). The penetration level in this study was defined as the rated capacity of the DG connected as a percentage of the actual ADMD used to design the passive feeder.

7.2 Validity of the Hypothesis

The hypothesis, "*The Herman-Beta algorithm, using the negative load approach for DG models introduced by Gaunt et al. [2011], can be modified for active LV feeders, validated using a MCS and applied as a tool to analyse the voltage rise constraints of active LV feeders*" has been addressed in this study and is demonstrated to be valid. The study has demonstrated that the HB algorithm is still a powerful tool for voltage calculation and has been successfully modified to allow voltage calculation in active LV feeders, in addition to passive LV feeders.

It can also be used as an engine for various studies and tests concerning the voltage in passive and active LV feeders. In this study, it formed part of another important section of this project – voltage analysis of LV feeders with increasing DG penetration.

7.3 Recommendations

This study has brought to light very significant results pertaining to active LV distribution systems.

1. **National Standard/Code**

   The NRS-034 standard [2007] contains guidelines are provided for the planning and design of power distribution systems. The work carried out in this project unearthed some errors in the standard with regards to the HB algorithm, which have been successfully corrected. In addition, the work done in this study (complete study and formulation of the HB algorithm for both passive and active LV feeders) is very relevant material that should be included in the standard, which only covers passive LV feeders.

   The NRS097-2-3 draft by Carter-Brown [2012], which is close to publication, provides guidelines on the maximum amount of DG penetration on LV networks to ensure continued compliance to voltage range standards. This study provides a useful application tool which can be used as a supplement to the work done in the draft.

2. **DG Hosting Capacity of LV feeder distribution systems**

   This study has shown the application of the HB algorithm in analysing voltage impact of DG on LV feeders, from which the DG hosting capacity of a feeder can be determined thereby answering the question – "*How much PV-DG capacity on a LV feeder is legally acceptable to the utility?*"

   Based on the results, the recommended capacity should be limited to PV capacity 30 % of the actual ADMD. With this information, the utility can plan and make strategies for what penetration level feeder reinforcement can start to be considered.

3. **Further work and projects**

   The work done in this study has led to the development of very useful tools in the design of distribution systems and analysing the effect of increasing DG penetration. However, a lot more can be done to build onto this work and application.

   Such extensions to this work include:

   - *Collection of solar PV data*

     The voltage analysis carried out in this study makes use of beta PDF deterministic PV models because of the lack of actual PV data. The NRS 034 Domestic Load Research Project started in South Africa was successfully carried out and has provided a better understanding of the variation of load data. It is a worthwhile recommendation to carry out the same project in order to collect high resolution generation data for various sites in South Africa in order to better understand the behaviour of various DG generation patterns.

From this venture, much stronger models for DG can be used to represent the long term and short term variation in the DG generation. For instance, in this study using PV technology, the availability of high resolution PV data would enable the effect of clouds, as well as ageing of PV units be added to the model.

- *Effect of solar water heaters*

  The adoption of rooftop solar PV panels is at times associated with the adoption of solar water heating. There is a need to investigate the correlation between the two. If it is significant, it is worthwhile because of its likely effect on the daily and seasonal load profiles as well as the interval of maximum excess demand.

- *Analysis of the effect of DG on a dedicated LV feeder and MV feeder*

  Carter-Brown [2012] provides restrictions on the installed DG on three proposed distinctions of the distribution network based – shared LV feeders, dedicated LV feeders and MV feeders. This research has been focused purely on residential loads on a shared LV feeder but the same approach can be modified to allow for similar studies on industrial, agricultural and commercial loads on dedicated and MV feeders. As highlighted in the literature, particularly in figure 3-3, the correlation between the PV generation profile and the agricultural, industrial or commercial load profiles is an important factor to take into consideration.

4. **The need for feeder voltage monitoring systems**

   From the voltage analysis using MCS tool, it was observed that the maximum voltage on the feeder is not necessarily restricted to the feeder ends, but can occur anywhere near the feeder. This increases the need for feeder voltage monitoring systems/devices to alert the system operator about any voltage violations on the feeder to maintain a high quality of power supplied to consumers.

7.4 Dissertation Conclusion and Final Thoughts

Arising from this study are two very useful easy-to-use tools, which have been extensively developed and tested. These tools include:

1. *Modified HB algorithm*

   The HB algorithm has been written in both Microsoft Excel and Matlab software. The algorithm allows for voltage calculation in passive and active LV feeders for both the bi phase and three phase network topologies.

2. *Voltage analysis of LV feeders with DG using MCS*

   The MCS program, written in Matlab, can be used to investigate the voltage rise constraints of a feeder, of any type or capacity. Graphical outputs show the effect of increasing PV-DG penetration on the LV feeder. The results obtained from this tool enable designers to recommend the maximum DG allowable on a feeder as a percentage of the ADMD for which a passive feeder has been designed.

The approach of modelling PV-DG using 'negative load', described by a beta PDF, has been applied successfully in this study. This approach can be extended to other types of DG technologies such as wind turbines, provided the statistical models of the generation output in terms of the beta PDF are obtained.

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APPENDIX A

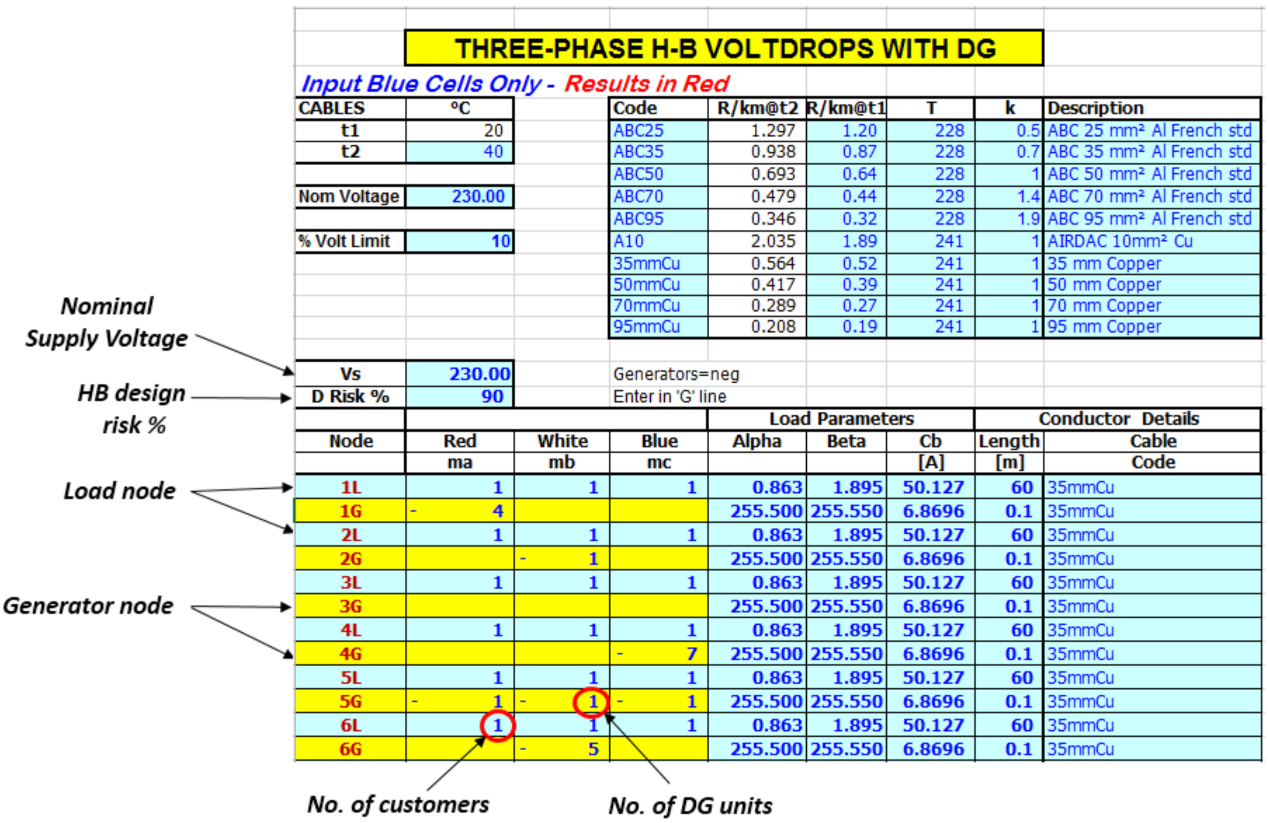
Appendix A – Herman-Beta Algorithm

This appendix contains the full description and equations of the correct and up-to-date version of Herman-Beta algorithm for passive LV feeders and active LV feeders with DG. The original algorithm can be found in the NRS-034 standard but required correction of two errors i.e. calculation of Hi and $E(Vd^2)$.

The modification of the algorithm for active LV feeders is discussed in chapter 4.

A.1 Illustration

Figures A-1 and A-2 show a section of the HB spreadsheet for voltage calculation in LV feeders with DG.



THREE-PHASE H-B VOLTDROPS WITH DG

Input Blue Cells Only - Results in Red

CABLES	°C
t1	20
t2	40
Nom Voltage	230.00
% Volt Limit	10

Code	R/km@t2	R/km@t1	T	k	Description
ABC25	1.297	1.20	228	0.5	ABC 25 mm² Al French std
ABC35	0.938	0.87	228	0.7	ABC 35 mm² Al French std
ABC50	0.693	0.64	228	1	ABC 50 mm² Al French std
ABC70	0.479	0.44	228	1.4	ABC 70 mm² Al French std
ABC95	0.346	0.32	228	1.9	ABC 95 mm² Al French std
A10	2.035	1.89	241	1	AIRDAC 10mm² Cu
35mmCu	0.564	0.52	241	1	35 mm Copper
50mmCu	0.417	0.39	241	1	50 mm Copper
70mmCu	0.289	0.27	241	1	70 mm Copper
95mmCu	0.208	0.19	241	1	95 mm Copper

Vs	230.00
D Risk %	90

Generators=neg
Enter in 'G' line

				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G	- 4			255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G		- 1		255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G			- 7	255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G	- 1	- 1	- 1	255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G		- 5		255.500	255.550	6.8696	0.1	35mmCu

Figure A-1: The section of HB spreadsheet in which the feeder parameters are input

Results	Red	White	Blue	Unit
%-tile Vcon	227.56	235.67	234.53	V
%Volt drop	1.06	-2.47	-1.97	%V
%-tile Isum	39.83	32.90	29.43	A
Mean Isum	76.89	70.02	66.59	A
Stdev Isum	29.37	29.37	29.37	A

Figure A-2: The voltage and current results of the HB method

Figure A-1 shows the data input required for the feeder and this is replicated in the Matlab program of the HB method. From figure A-1, we can observe that the load and generator nodes are separate from each other i.e. each node has a load and generator node on which the number of consumers and DG units are placed respectively. The design risk used for the LV feeder with DG is 90 % as discussed in chapter 3.

Figure A-2 provides an illustration of the output of the HB calculations as seen in the HB spreadsheet. The voltage results include the voltage at the end of the feeder and the resultant voltage drop. The resultant voltage is taken at the 90^{th} percentile (selected design risk). The current results include the phase current at the 90th percentile, the mean/average phase current and the measure of spread the resultant currents are from the mean current. The current calculations in the HB method are in Appendix D.

A.2 Voltage Calculation Algorithm in the Herman-Beta method

A.2.1 List of symbols

The symbols used in the formulations were chosen to avoid Greek or other non-alphabetic symbols and, where possible, to avoid subscripts and superscripts. For example, ai is used rather than a_i

Vs	is the nominal supply voltage, in volts
ai	is the Beta probability density function parameter, alpha at node i
bi	is the Beta probability density function parameter, beta at node i
ci	is the scaling factor in amperes (usually the circuit-breaker size) at node i
mai	is the number of consumers connected to the a-phase at node I
mbi	is the number of consumers connected to the b-phase at node i
mci	is the number of consumers connected to the c-phase at node i
N	is the total number of nodes in the radial feeder section
Rp	is the temperature-corrected resistance of the phase conductor per span
Rn	is the temperature-corrected resistance of the neutral conductor per span
p	is the percentage risk in the probabilistic calculation
G	is the first statistical moment
H	is the second statistical moment
ki	is a resistance ratio
Ri	is a phase resistance index
Vmax	is the maximum consumer voltage
Vmin	is the minimum consumer voltage
Vd	is a voltage drop
L, K	are node counters
Vc	is the consumer voltage
vc	is the normalized consumer voltage
av	is the alpha parameter of scaled consumer voltage
bv	is the beta parameter of scaled consumer voltage
betainv	is the Beta inverse function
DVrmaxi	is the real component of maximum volt drop at node i
DVjmaxi	is the imaginary component of maximum volt drop at node i
DVrmini	is the real component of minimum volt drop at node i
DVjmini	is the imaginary component of minimum volt drop at node i
E()	is the expected value of ()

C1i, C2i, C3i, C4i, C5i, C6i, pi, qi and F1i, F2i ,F3i are constants.

A.2.2 Algorithm for Passive LV feeders

i. **Step-wise procedure for calculating bi-phase system voltage drops**

Step 1 – Select the network parameters

1. Supply voltage, ***Vs***.
2. Load description in Beta pdf form: ***ai, bi, ci.*** Where ***ci*** is the scaling factor – usually the circuit-breaker size.
3. Specify the number of consumer connections at each node, i: ***mai*** and ***mbi***
4. Specify total number of nodes in the radial section, ***N***.
5. Specify the phase and neutral conductor resistances for each section: ***Rp*** and ***Rn***, allowing for temperature rise.
6. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$Gi = \frac{ai}{(ai+bi)} \qquad Hi = \frac{ai(ai+1)}{(ai+bi)(ai+bi+1)}$$

Step 3 – Calculate Ri, Rp and ki

$$Ri = \sum_{j=1}^{i} Rn(j) \qquad Rp = \sum_{j=1}^{i} Rp(j) \qquad ki = \frac{Ri}{Rp}$$

Where

Rn (j) is the neutral conductor resistance for section *(i–1)* to *(i)*;

Rp (j) is the phase conductor resistance for section *(i–1)* to *(i)*.

Step 4 – Calculate constants qi and pi, and maximum and minimum voltages, Vmax and Vmin

$$pi = (1+ki) \times ci \times mai \times Ri \qquad qi = ki \times ci \times mbi \times Ri$$

$$Vimax = qi \qquad Vimin = pi$$

$$Vmax = Vs + \sum_{i=1}^{N} Vimax \qquad Vmin = Vs - \sum_{i=1}^{N} Vimin$$

Step 5 – Calculate the expected values: E (Vi) and E (Vd)

$$E(Vi) = Gi(qi-pi) \qquad E(Vd) = \sum_{i=1}^{N} E(Vi)$$

Step 6 – Calculate ri and si and the expected values: E (Vi^2) and E (Vd^2)

$$ri = ci^2 \times Ri^2 \left[|mbi| \times ki^2 + |mai|\,(1+ki)^2\right]$$

$$si = ci^2 \times Ri^2 \left[|mbi|(|mbi|-1)ki^2 - 2|mai| \times |mbi|(ki+1)ki + |mai|(|mai|-1)(1+ki)^2\right]$$

$$E\left(Vi^2\right) = ri \times Hi + si \times Gi^2$$

$$E\left(Vd^2\right) = \sum_{i=1}^{N} E\left(Vi^2\right) + \sum_{K=1}^{N} \sum_{\substack{K=1 \\ L \neq K}}^{N} E\left(Vd_K\right) \times E\left(Vd_L\right)$$

Step 7 – Calculate expected values E (Vc) and E (Vc²)

$$E(Vc) = Vs + E(Vd)$$

$$E\left(Vc^2\right) = Vs^2 + 2Vs \times E(Vd) + E(Vd^2)$$

Step 8 – Calculate the scaled values of E (vc) and E (vc²)

$$E(vc) = \frac{E(Vc) - Vmin}{Vmax - Vmin}$$

$$E(vc^2) = \frac{E\left(Vc^2\right) - 2Vmin \times E(Vc) + Vmin^2}{(Vmax - Vmin)^2}$$

Step 9 – Calculate the Beta parameters of vc: av and bv

$$av = \frac{E\left(vc^2\right) - E(vc)}{E(vc) - \frac{E\left(vc^2\right)}{E(vc)}} \qquad bv = \frac{av}{E(vc)} - av$$

Step 10 – Select a risk percentage p and calculate percentile value v %

Use the Beta inverse function:

$$v\,\% = betainv[\left(\frac{p}{100}\right), |av|, |bv|]$$

A percentile value of 10 % (less commonly 5 %) is used for calculating voltage drop in passive feeders.

Step 11 – Rescale the consumer voltage, Vc %

$$Vc\,\% = v\,\%(Vmax - Vmin) + Vmin$$

ii. Step-wise procedure for calculating three-phase system voltage drops

Step 1 – Select the network parameters

1. Supply voltage, ***Vs***.
2. Load description in Beta pdf form: ***ai***, ***bi***, ***ci***. Where ***ci*** is the scaling factor – usually the circuit-breaker size.
3. Specify the number of consumer connections at each load node, i: ***mai***, ***mbi*** and ***mci***
4. Specify total number of nodes in the radial section, ***N***.
5. Specify the phase and neutral conductor resistances for each section: ***Rp*** and ***Rn***, allowing for temperature rise.
6. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$Gi = \frac{ai}{(ai+bi)} \qquad Hi = \frac{ai(ai+1)}{(ai+bi)(ai+bi+1)}$$

Step 3 – Calculate Ri, Rp and ki

$$Ri = \sum_{j=1}^{i} Rn(j) \qquad Rp = \sum_{j=1}^{i} Rp(j) \qquad ki = \frac{Ri}{Rp}$$

Where

Rn (j) is the neutral conductor resistance for section *(i–1)* to *(i)*;

Rp (j) is the phase conductor resistance for section *(i–1)* to *(i)*;

Step 4 – Calculate maximum and minimum voltages, Vmax and Vmin

Real parts are indexed with *r* and imaginary parts with *j*. The symbol D is used to indicate voltage drop and *i* index indicates the *i*-th node.

$$DVrmaxi = -0.5ki \times Ri \times ci(mbi + mci) \qquad DVjmaxi = \frac{\sqrt{3}}{2} ki \times Ri \times ci(mbi - mci)$$

$$DVrmini = (1 + ki) \times Ri \times ci \times mai \qquad DVjmini = 0$$

$$Vmax = \sqrt{\left(Vs - \sum_{i=1}^{N} DVrmaxi\right)^2 + \left(\sum_{i=1}^{N} DVjmaxi\right)^2}$$

$$Vmin = \sqrt{\left(Vs - \sum_{i=1}^{N} DVrmini\right)^2 + \left(\sum_{i=1}^{N} DVjmini\right)^2}$$

Step 5 – Calculate the constants: C1i, C2i, C3i, C4i, C5i and C6i and F1i, F2i and F3i

$$F1i = |mai|(|mai| - 1) - |mai|(|mbi| + |mci|) + 0.25(|mbi| + |mci| - 1)(|mbi| + |mci|)$$

$$F2i = |mai|(2|mai| - |mbi| - |mci| - 2)$$

$$F3i = |mai|(|mai| - 1)$$

$$C1i = (1+ki)mai - 0.5ki(mbi + mci)$$

$$C2i = ki^2[(|mai| + 0.25|mbi| + 0.25|mci|)+|mai|(2ki + 1)]$$

$$C3i = F1i \times ki^2 + F2i \times ki + F3i$$

$$C4i = \frac{3ki^2}{4}(|mbi| + |mci|)$$

$$C5i = \frac{3ki^2}{4}[(|mbi| - |mci|)^2 - (|mbi| + |mci|)]$$

$$C6i = \frac{\sqrt{3}}{2}ki(mbi - mci)$$

Step 6 – Calculate the expected values: E (Dvri), E (DVr), E (DV²ri) and E (DV²r)

$$E(DVri) = C1i \times Ri \times ci \times Gi$$

$$E(DVr)=\sum_{i=1}^{N} E(DVri)$$

$$E(DV^2ri) = Ri^2 \times ci^2[C2i \times Hi + C3i \times Gi^2]$$

$$E(DV^2r) = \sum_{i=1}^{N} E(DV^2ri) + \sum_{K=1}^{N}\sum_{\substack{K=1\\L\neq K}}^{N} E(DVr_K) \times E(DVr_L)$$

Step 7 – Calculate expected values: E (DVji), E (DVj), E (DV²ji) and E (DV²j)

$$E(DVji) = C6i \times Ri \times ci \times Gi$$

$$E(DVj)=\sum_{i=1}^{N} E(DVji)$$

$$E(DV^2ji) = Ri^2 \times ci^2[C4i \times Hi + C5i \times Gi^2]$$

$$E(V^2j) = \sum_{i=1}^{N} E(DV^2ji) + \sum_{K=1}^{N}\sum_{\substack{K=1\\L\neq K}}^{N} E(DVj_K) \times E(DVj_L)$$

Step 8 – Calculate E (Vc) and E (V²c)

$$E(Vc) = Vs[1 - \frac{E(DVr)}{Vs} + 0.5\frac{E(DV^2j)}{Vs^2}]$$

$$E(Vc^2) = Vs^2 - 2Vs \times E(DVr) + E(DV^2r) + E(DV^2j)$$

Step 9 – Calculate the scaled values of E (vc) and E (vc²)

$$E(vc) = \frac{E(Vc) - Vmin}{Vmax - Vmin}$$

$$E(vc^2) = \frac{E(Vc^2) - 2Vmin \times E(Vc) + Vmin^2}{(Vmax - Vmin)^2}$$

Step 10 – Calculate the Beta parameters of vc: av and bv

$$av = \frac{E(vc^2) - E(vc)}{E(vc) - \frac{E(vc^2)}{E(vc)}} \qquad bv = \frac{av}{E(vc)} - av$$

Step 11 – Select a risk percentage p and calculate percentile value v %

Use the Beta inverse function:

$$v\,\% = betainv[\left(\frac{p}{100}\right), |av|, |bv|]$$

A percentile value of 10 % (less commonly 5 %) is used for calculating voltage drop in passive feeders.

Step 12 – Rescale the consumer voltage, Vc %

$$Vc\,\% = v\,\%(Vmax - Vmin) + Vmin$$

A.2.3 Algorithm for active LV feeders with DG

Note: It should be noted that for all formulae apply to all nodes (load and generator nodes) except symbols denoted with *'symbol'*$_{LOAD}$ for load nodes and *'symbol'*$_{DG}$ for generator nodes

i. **Step-wise procedure for calculating bi-phase system voltage drops**

Step 1 – Select the network parameters

1. Supply voltage, ***Vs***.
2. Load description in Beta pdf form: ***ai, bi, ci.*** Where ***ci*** is the scaling factor – usually the circuit-breaker size.
3. Specify the number of consumer connections at each node, i: ***mai*** and ***mbi***, (loads in load nodes and embedded generators in generator nodes).
4. A positive number represents a load and a negative number is an embedded generator.
5. Specify total number of nodes in the radial section, ***N***.
6. Specify the phase and neutral conductor resistances for each section: ***Rp*** and ***Rn***, allowing for temperature rise.
7. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$Gi = \frac{ai}{(ai+bi)} \qquad Hi = \frac{ai(ai+1)}{(ai+bi)(ai+bi+1)}$$

Step 3 – Calculate Ri, Rp and ki

$$Ri = \sum_{j=1}^{i} Rn(j) \qquad Rp = \sum_{j=1}^{i} Rp(j) \qquad ki = \frac{Ri}{Rp}$$

Where

Rn (j) is the neutral conductor resistance for section *(i–1)* to *(i)*;

Rp (j) is the phase conductor resistance for section *(i–1)* to *(i)*.

Step 4 – Calculate constants qi and pi, and maximum and minimum voltages, Vmax and Vmin

$$pi_{LOAD} = (1+ki) \times ci \times mai \times Ri \qquad qi_{LOAD} = ki \times ci \times mbi \times Ri$$

$$pi_{DG} = (1+ki) \times ci \times mai \times Ri \qquad qi_{DG} = ki \times ci \times mbi \times Ri$$

$$Vimax_{LOAD} = qi_{LOAD} \qquad Vimax_{DG} = -\,pi_{DG}$$

$$Vimin_{LOAD} = pi_{LOAD} \qquad Vimin_{DG} = -\,qi_{LOAD}$$

$$Vmax = Vs + \sum_{i=1}^{N} Vimax \qquad Vmin = Vs - \sum_{i=1}^{N} Vimin$$

Step 5 – Calculate the expected values: E (Vi) and E (Vd)

$$E(Vi) = Gi(qi\text{-}pi) \qquad E(Vd) = \sum_{i=1}^{N} E(Vi)$$

Step 6 – Calculate ri and si and the expected values: E (Vi²) and E (Vd²)

$$ri = ci^2 \times Ri^2\ [|mbi| \times ki^2 + |mai|\ (1+ki)^2]$$

$$si = ci^2 \times Ri^2\ [|mbi|(|mbi|-1)ki^2\ - 2|mai| \times |mbi|(ki+1)ki + |mai|(|mai|-1)(1+ki)^2]$$

$$E\left(Vi^2\right) = ri \times Hi + si \times Gi^2$$

$$E\left(Vd^2\right) = \sum_{i=1}^{N} E\left(Vi^2\right) + \sum_{K=1}^{N} \sum_{\substack{K=1 \\ L \neq K}}^{N} E(Vd_K) \times E(Vd_L)$$

Step 7 – Calculate expected values E (Vc) and E (Vc²)

$$E(Vc) = Vs + E(Vd)$$

$$E\left(Vc^2\right) = Vs^2 + 2Vs \times E(Vd) + E(Vd^2)$$

Step 8 – Calculate the scaled values of E (vc) and E (vc²)

$$E(vc) = \frac{E(Vc)\ \text{-}\ Vmin}{Vmax\ \text{-}\ Vmin}$$

$$E(vc^2) = \frac{E\left(Vc^2\right)\ \text{-}\ 2Vmin \times E(Vc) + Vmin^2}{(Vmax\ \text{-}\ Vmin)^2}$$

Step 9 – Calculate the Beta parameters of vc: av and bv

$$av = \frac{E\,(vc^2)\text{-}E(vc)}{E(vc)\text{-}\dfrac{E(vc^2)}{E(vc)}} \qquad bv = \frac{av}{E(vc)} - av$$

Step 10 – Select a risk percentage p and calculate percentile value v %

Use the Beta inverse function:

$$v\ \% = betainv[\ \left(\frac{p}{100}\right), |av|,\ |bv|]$$

A percentile value of 90 % (or 95 %) is used for calculating voltage rise in active feeders with embedded generation.

Step 11 – Rescale the consumer voltage, Vc % according to whether the net current is into or out of the feeder

$$Vc\ \% = v\ \%\ (Vmax\ \text{-}\ Vmin) + Vmin$$

ii. Step-wise procedure for calculating three-phase system voltage drops

Step 1 – Select the network parameters

1. Supply voltage, ***Vs***.
2. Load description in Beta pdf form: ***ai***, ***bi***, ***ci***. Where ***ci*** is the scaling factor – usually the circuit-breaker size.
3. Specify the number of consumer connections at each load node, i: ***mai***, ***mbi*** and ***mci*** (loads in load nodes and embedded generators in generator nodes).
   A positive number represents a load and a negative number is an embedded generator.
4. Specify total number of nodes in the radial section, ***N***.
5. Specify the phase and neutral conductor resistances for each section: ***Rp*** and ***Rn***, allowing for temperature rise.
6. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$Gi = \frac{ai}{(ai+bi)} \qquad Hi = \frac{ai(ai+1)}{(ai+bi)(ai+bi+1)}$$

Step 3 – Calculate Ri, Rp and ki

$$Ri = \sum_{j=1}^{i} Rn(j) \qquad Rp = \sum_{j=1}^{i} Rp(j) \qquad ki = \frac{Ri}{Rp}$$

Where

Rn (j) is the neutral conductor resistance for section *(i–1)* to *(i)*;
Rp (j) is the phase conductor resistance for section *(i–1)* to *(i)*.

Step 4 – Calculate maximum and minimum voltages, Vmax and Vmin

Real parts are indexed with *r* and imaginary parts with *j*. The symbol D is used to indicate voltage drop and *i* index indicates the *i*-th node.

$$DVrmaxi_{LOAD} = -0.5ki \times Ri \times ci(mbi + mci) \qquad DVrmaxi_{DG} = (1 + ki) \times Ri \times ci \times mai$$

$$DVjmaxi_{LOAD} = \frac{\sqrt{3}}{2} ki \times Ri \times ci(mbi - mci) \qquad DVjmaxi_{DG} = 0$$

$$DVrmini_{LOAD} = (1 + ki) \times Ri \times ci \times mai \qquad DVrmini_{DG} = -0.5ki \times Ri \times ci(mbi + mci)$$

$$DVjmini_{LOAD} = 0 \qquad DVjmini_{DG} = \frac{\sqrt{3}}{2} ki \times Ri \times ci(mbi - mci)$$

$$Vmax = \sqrt{\left(Vs - \sum_{i=1}^{N} DVrmaxi\right)^2 + \left(\sum_{i=1}^{N} DVjmaxi\right)^2}$$

$$Vmin = \sqrt{\left(Vs - \sum_{i=1}^{N} DVrmini\right)^2 + \left(\sum_{i=1}^{N} DVjmini\right)^2}$$

Step 5 – Calculate the constants: C1i, C2i, C3i, C4i, C5i and C6i and F1i, F2i and F3i

$$F1i = |mai|(|mai| - 1) - |mai|(|mbi| + |mci|) + 0.25(|mbi| + |mci| - 1)(|mbi| + |mci|)$$

$$F2i = |mai|(2|mai| - |mbi| - |mci| - 2)$$

$$F3i = |mai|(|mai| - 1)$$

$$C1i = (1+ki)mai - 0.5ki(mbi + mci)$$

$$C2i = ki^2[(|mai| + 0.25|mbi| + 0.25|mci|)+|mai|(2ki + 1)]$$

$$C3i = F1i \times ki^2 + F2i \times ki + F3i$$

$$C4i = \frac{3ki^2}{4}(|mbi| + |mci|)$$

$$C5i = \frac{3ki^2}{4}[(|mbi| - |mci|)^2 - (|mbi| + |mci|)]$$

$$C6i = \frac{\sqrt{3}}{2}ki(mbi - mci)$$

Step 6 – Calculate the expected values: E (Dvri), E (DVr), E (DV²ri) and E (DV²r)

$$E(DVri) = C1i \times Ri \times ci \times Gi$$

$$E(DVr) = \sum_{i=1}^{N} E(DVri)$$

$$E(DV^2ri) = Ri^2 \times ci^2[C2i \times Hi + C3i \times Gi^2]$$

$$E(DV^2r) = \sum_{i=1}^{N} E(DV^2ri) + \sum_{K=1}^{N}\sum_{\substack{K=1\\ L\neq K}}^{N} E(DVr_K) \times E(DVr_L)$$

Step 7 – Calculate expected values: E (DVji), E (DVj), E (DV²ji) and E (DV²j)

$$E(DVji) = C6i \times Ri \times ci \times Gi$$

$$E(DVj) = \sum_{i=1}^{N} E(DVji)$$

$$E(DV^2ji) = Ri^2 \times ci^2[C4i \times Hi + C5i \times Gi^2]$$

$$E(V^2j) = \sum_{i=1}^{N} E(DV^2ji) + \sum_{K=1}^{N}\sum_{\substack{K=1\\ L\neq K}}^{N} E(DVj_K) \times E(DVj_L)$$

Step 8 – Calculate E (Vc) and E (V²c)

$$E(Vc) = Vs[1 - \frac{E(DVr)}{Vs} + 0.5\frac{E(DV^2j)}{Vs^2}]$$

$$E\left(Vc^2\right) = Vs^2 - 2Vs \times E(DVr) + E\left(DV^2r\right) + E\left(DV^2j\right)$$

Step 9 – Calculate the scaled values of E (vc) and E (vc²)

$$E(vc) = \frac{E(Vc)\ \text{-}\ Vmin}{Vmax\ \text{-}\ Vmin}$$

$$E(vc^2) = \frac{E\left(Vc^2\right) - 2Vmin \times E(Vc) + Vmin^2}{(Vmax - Vmin)^2}$$

Step 10 – Calculate the Beta parameters of vc: av and bv

$$av = \frac{E(vc^2)\text{-}E(vc)}{E(vc)\text{-}\frac{E(vc^2)}{E(vc)}} \qquad bv = \frac{av}{E(vc)} - av$$

Step 11 – Select a risk percentage p and calculate percentile value v %

Use the Beta inverse function:

$$v\ \% = betainv[\left(\frac{p}{100}\right), |av|, |bv|]$$

A percentile value of 90 % (or 95 %) is used for calculating voltage rise in active feeders with embedded generation.

Step 12 – Rescale the consumer voltage, Vc % according to whether the net current is into or out of the feeder

$$Vc\ \% = v\ \%(Vmax - Vmin) + Vmin$$

APPENDIX B

Appendix B – Testing the HB Algorithm

This appendix contains all results and graphs from simulations from the testing phase of the HB algorithm. HB_{MATLAB} *is denoted by* HB_M *and* HB_{EXCEL} *is denoted by* HB_{ES} *for presentation of the results in tables.*

B.1 Feeder cable conductor parameters

Figure B-1 shows a table of cable conductors used in the HB algorithm, with t1 = 20°C and t2 = 40°C.

Code	R/km@t2	R/km@t1	T	k	Description
ABC25	1.297	1.20	228	0.5	ABC 25 mm² Al French std
ABC35	0.938	0.87	228	0.7	ABC 35 mm² Al French std
ABC50	0.693	0.64	228	1	ABC 50 mm² Al French std
ABC70	0.479	0.44	228	1.4	ABC 70 mm² Al French std
ABC95	0.346	0.32	228	1.9	ABC 95 mm² Al French std
A10	2.035	1.89	241	1	AIRDAC 10mm² Cu
35mmCu	0.564	0.52	241	1	35 mm Copper
50mmCu	0.417	0.39	241	1	50 mm Copper
70mmCu	0.289	0.27	241	1	70 mm Copper
95mmCu	0.208	0.19	241	1	95 mm Copper

Figure B-1: Feeder cable conductor characteristics

Figure B-2 shows a table with the electrical and physical Properties of 3 and 4 core PV C Insulated PVC bedded SWA PVC sheathed 600/1000 V cables manufactured to SANS 1507-3 [Aberdare Cables, 2007].

COPPER CONDUCTORS

Cable Size	Electrical Properties						Physical Properties							
	Current Rating			Impe-dance	3ϕ Volt drop	1ϕ Volt drop	Nominal Diameters						Approx. Mass	
	Ground	Ducts	Air				D1		d		D2			
							3c	4c	3c	4c	3c	4c	3c	4c
(mm²)	(A)	(A)	(A)	(Ω/km)	(mV/A/m)	(mV/A/m)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(kg/km)	(kg/km)
1,5	24	20	19	14,48	25,080	28,956	8,51	9,33	1,25	1,25	14,13	14,95	448	501
2,5	32	26	26	8,87	15,363	17,734	9,61	10,56	1,25	1,25	15,23	16,18	522	597
4	42	34	35	5,52	9,561	11,034	11,40	12,57	1,25	1,25	17,02	18,39	667	762
6	53	43	45	3,69	6,391	7,374	12,58	13,90	1,25	1,25	18,40	19,72	790	910
10	70	58	62	2,19	3,793	4,384	14,59	16,14	1,25	1,25	20,41	21,96	996	1169
16	91	75	83	1,38	2,390	2,759	16,55	19,18	1,25	1,60	22,37	25,92	1295	1768
25	119	96	110	0,8749	1,515	1,749	19,46	21,34	1,60	1,60	26,46	28,34	1838	2196
35	143	116	135	0,6335	1,097	1,267	20,89	23,97	1,60	1,60	27,89	31,17	2215	2732
50	169	138	163	0,4718	0,817	0,944	24,26	28,14	1,60	2,00	31,46	36,54	2871	3893
70	210	171	207	0,3325	0,576	0,665	27,07	31,29	2,00	2,00	35,47	40,09	3617	4837
95	251	205	251	0,2460	0,427	0,492	31,19	35,82	2,00	2,00	39,99	44,62	4901	6115
120	285	234	290	0,2012	0,348	0,402	33,38	38,10	2,00	2,00	42,18	47,40	5720	7269
150	320	263	332	0,1698	0,294	0,339	36,68	42,05	2,00	2,50	45,98	52,65	6908	9250
185	361	298	378	0,1445	0,250	0,289	40,82	46,75	2,50	2,50	51,12	57,45	8690	11039
240	416	344	445	0,1220	0,211	0,244	46,43	53,06	2,50	2,50	57,13	64,16	10767	13726
300	465	385	510	0,1090	0,189	0,218	51,10	58,53	2,50	2,50	62,20	70,13	12950	16544

Figure B-2: Electrical and Physical Properties of 3 and 4 core Copper cables [Aberdare Cables, 2007]

B.2 Testing the HB algorithm in Matlab

The details of the tests in this section B.2.1 can be found in section 4.4.2.

B.2.1 Test 1 – Comparison with the HB Excel Spreadsheet

Table B-1: Passive Feeder Test 1 – Bi-phase systems

NODES (CONNECTION PATTERN)	MAXIMUM VOLTAGE DROP [%]					
	RED PHASE			BLUE PHASE		
	HB_M	HB_{ES}	Error	HB_M	HB_{ES}	Error
1 node (Bal11)	1.17	1.17	0.00	1.17	1.17	0.00
2 nodes (Cyc20)	1.33	1.33	0.00	4.40	4.40	0.00
6 nodes (Cos21)	20.65	20.65	0.00	21.99	21.99	0.00

Table B-2: Passive Feeder Test 1 – Three-phase systems

NODES (CONNECTION PATTERN)	MAXIMUM VOLTAGE DROP [%]								
	RED PHASE			WHITE PHASE			BLUE PHASE		
	HB_M	HB_{ES}	Error	HB_M	HB_{ES}	Error	HB_M	HB_{ES}	Error
1 node (Bal111)	1.12	1.12	0.00	1.12	1.12	0.00	1.12	1.12	0.00
3 nodes (Cyc200)	0.85	0.85	0.00	3.65	3.65	0.00	6.56	6.56	0.00
6 nodes (Cos211)	19.26	19.26	0.00	19.05	19.05	0.00	18.95	18.95	0.00

B.2.2 Test 2 – Reverse Feeder Calculation

Table B-3: Passive Feeder Test 2 – Bi-phase systems

NODES (CONNECTION PATTERN)	VOLTAGE [V]					
	RED PHASE			BLUE PHASE		
	V_s	$V_{s\text{-calc}}$	Error	V_s	$V_{s\text{-calc}}$	Error
1 node (Bal11)	230.00	229.88	0.12	230.00	229.88	0.12
2 nodes (Cyc20)	230.00	229.91	0.09	230.00	229.80	0.20
6 nodes (Cos21)	230.00	229.81	0.19	230.00	229.81	0.19

Table B-4: Passive Feeder Test 2 – Three-phase systems

NODES (CONNECTION PATTERN)	VOLTAGE [V]								
	RED PHASE			WHITE PHASE			BLUE PHASE		
	V_s	$V_{s\text{-calc}}$	Error	V_s	$V_{s\text{-calc}}$	Error	V_s	$V_{s\text{-calc}}$	Error
1 node (Bal111)	230.00	229.90	0.10	230.00	229.90	0.10	230.00	229.90	0.10
3 nodes (Cyc200)	230.00	229.97	0.03	230.00	229.87	0.13	230.00	229.87	0.13
6 nodes (Cos211)	230.00	229.08	0.92	230.00	229.05	0.95	230.00	229.03	0.97

B.2.3 Test 3 – Single-Phase Testing Of Feeders

Table B-5: Passive Feeder Test 3 – Bi-phase systems

FEEDER PHASES (Bal10)	MAXIMUM VOLTAGE DROP [%]					
	1 NODE			6 NODES		
	HB_M	HB_{ES}	Error	HB_M	HB_{ES}	Error
R (load)	1.54	1.54	0.00	23.38	23.38	0.00
B	-0.10	-0.10	0.00	-5.37	-5.37	0.00
R	-0.10	-0.10	0.00	-5.37	-5.37	0.00
B (load)	1.54	1.54	0.00	23.38	23.38	0.00

Table B-6: Passive Feeder Test 3 – Three-phase systems

FEEDER PHASES (Bal100)	MAXIMUM VOLTAGE DROP [%]					
	1 NODE			6 NODES		
	HB_M	HB_{ES}	Error	HB_M	HB_{ES}	Error
R (load)	1.54	1.54	0.00	23.38	23.38	0.00
W	-0.05	-0.05	0.00	-3.26	-3.26	0.00
B	-0.05	-0.05	0.00	-3.26	-3.26	0.00
R	-0.00	-0.00	0.00	-0.55	-0.55	0.00
W (load)	1.54	1.54	0.00	23.38	23.38	0.00
B	-0.00	-0.00	0.00	-0.55	-0.55	0.00
R	-0.05	-0.05	0.00	-3.26	-3.26	0.00
W	-0.05	-0.05	0.00	-3.26	-3.26	0.00
B (load)	1.54	1.54	0.00	23.38	23.38	0.00

B.2.4 Validation of HB Algorithm for passive LV feeders using $VD_{p\text{-}MCS}$

i. Single Phase Test Feeder

Bi phase network topology

1. **Feeder Configuration**

	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	α	β	[A]	[m]	Code
1	3		1.500	4.000	60	200	35mmCu
2	3		1.500	4.000	60	200	35mmCu

Figure B-3: Single Phase Bi phase Passive test LV feeder

2. **Results**

Table B-7: Result data Comparison for $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^{2})$
$HB_{p\text{-}MATLAB}$	24.80	9.30	108.14	230.00	196.77	38801
$VD_{p\text{-}MCS}$	31.036	11.042	95.61	232.72	196.74	38791

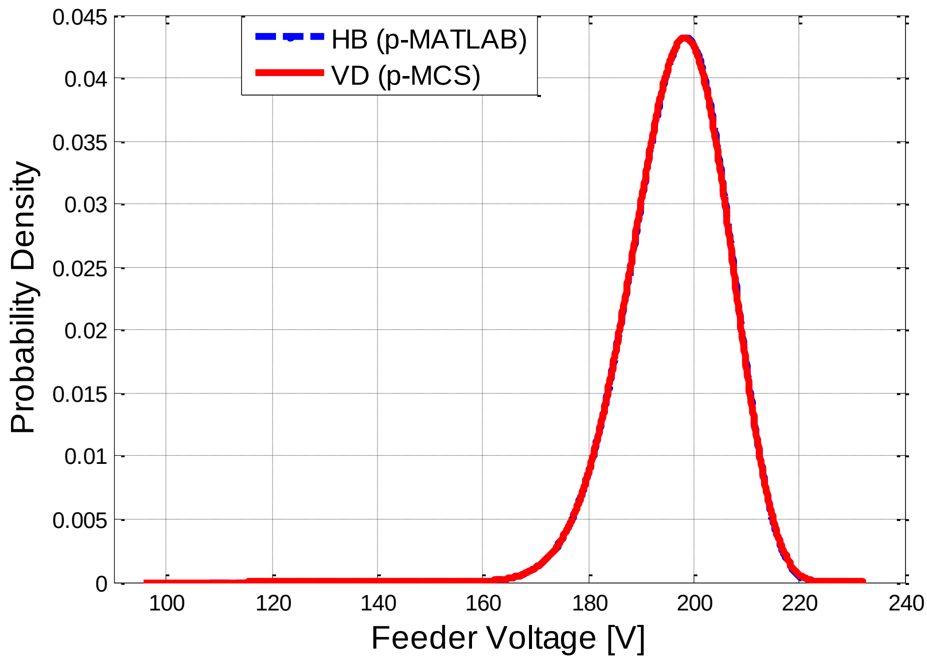


Figure B-4: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

Three phase network topology

1. Feeder Configuration

	No Consumers on phases			Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1	3			1.500	4.000	60	200	35mmCu
2	3			1.500	4.000	60	200	35mmCu

Figure B-5: Single Phase Three phase Passive test LV feeder

2. Results

Table B-8: Result data Comparison for $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for three phase network*

	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^2)$
$HB_{p\text{-}MATLAB}$	24.80	9.30	108.14	230.00	196.77	38801
$VD_{p\text{-}MCS}$	17.975	7.8964	124.92	228.32	196.76	38798

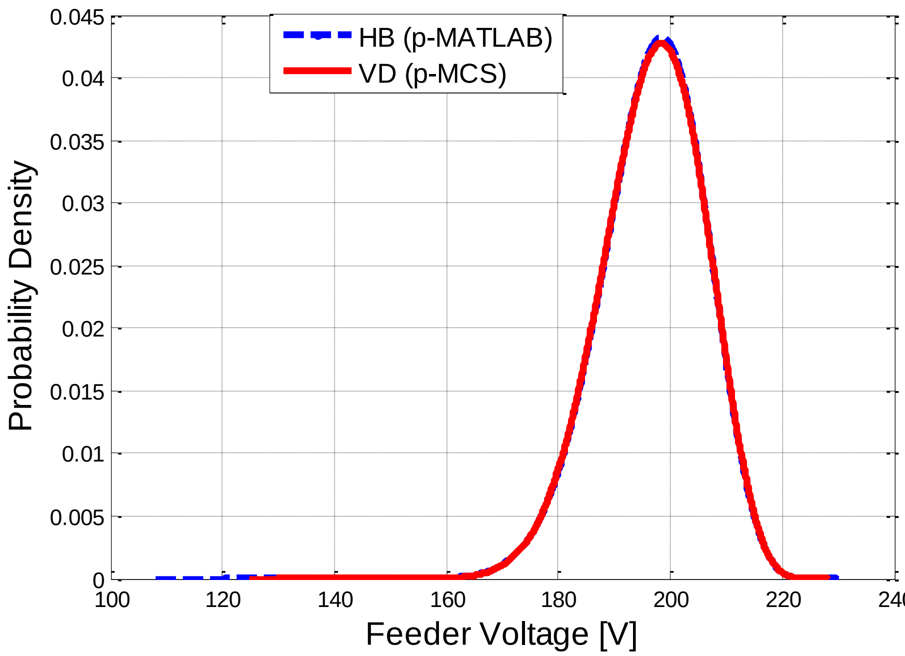


Figure B-6: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for three phase network*

ii. Test Feeder with unbalanced loading

Bi phase network topology

1. **Feeder Configuration**

	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	α	β	[A]	[m]	Code
1	3		1.500	4.000	60	200	35mmCu
2		3	1.500	4.000	60	200	35mmCu

Figure B-7: Bi phase Test Feeder with unbalanced loading

2. **Results**

Table B-9: Result data Comparison for $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^2)$
$HB_{p\text{-}MATLAB}$	24.08	24.08	189.38	270.62	230.00	52934
$VD_{p\text{-}MCS}$	6.996	6.3127	206.94	250.86	230.03	52946

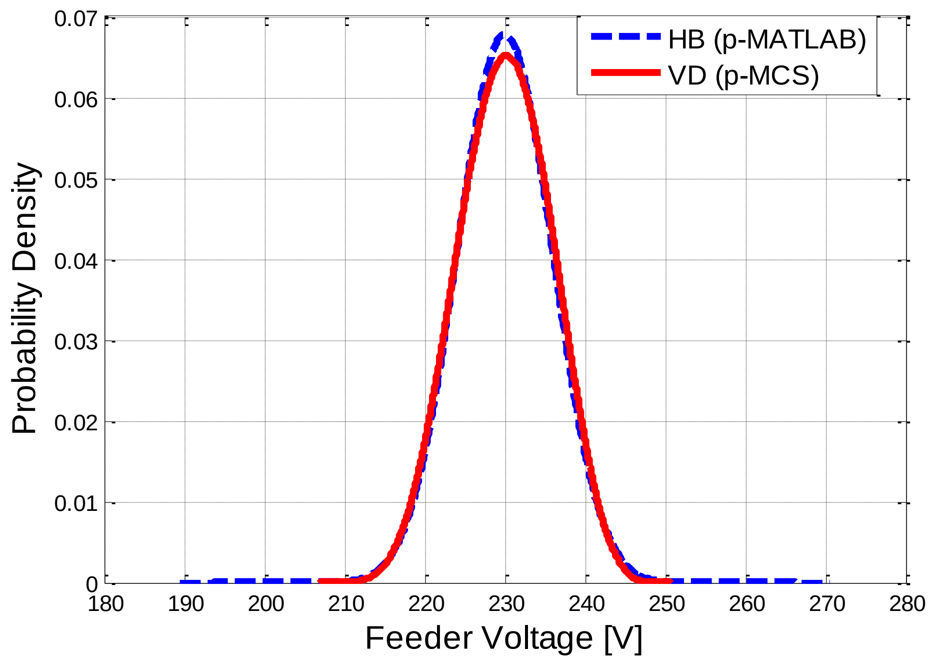


Figure B-8: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for bi phase network*

Three phase network topology

1. Feeder Configuration

	No Consumers on phases			Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1	1	2	3	1.500	4.000	60	200	35mmCu
2	4	5	6	1.500	4.000	60	200	35mmCu

Figure B-9: Three phase Test Feeder with unbalanced loading

2. Results

Table B-10: Result data Comparison for $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for three phase network*

	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^2)$
$HB_{p\text{-}MATLAB}$	52.13	45.86	108.14	321.87	221.85	49334
$VD_{p\text{-}MCS}$	9.1399	6.1713	169.95	257.0	221.91	49357

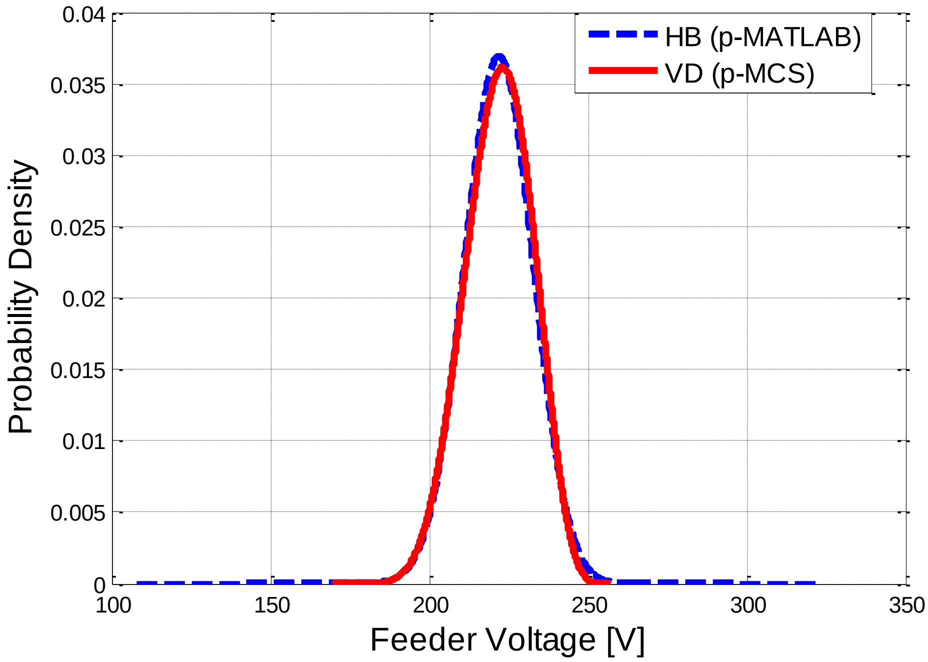


Figure B-10: Comparison of beta PDFs from $HB_{p\text{-}MATLAB}$ *and* $VD_{p\text{-}MCS}$ *tests for three phase network*

B.2.5 Validation of HB Algorithm for active LV feeders using $VD_{a\text{-}MCS}$

i. **Test 2 – Constant Load with varying DG**

Bi phase Network Topology

1. Feeder Configuration

	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb			[A]	[m]	Code
1L	6	6	1.500	4.000	60	400	35mmCu
1G	- 1	- 1	1.500	4.000	60	0.1	35mmCu

Figure B-11: Example - Bi phase LV feeder for 12 loads and 2 generators

2. Results

a) 12 Loads and 2 Generators (DG)

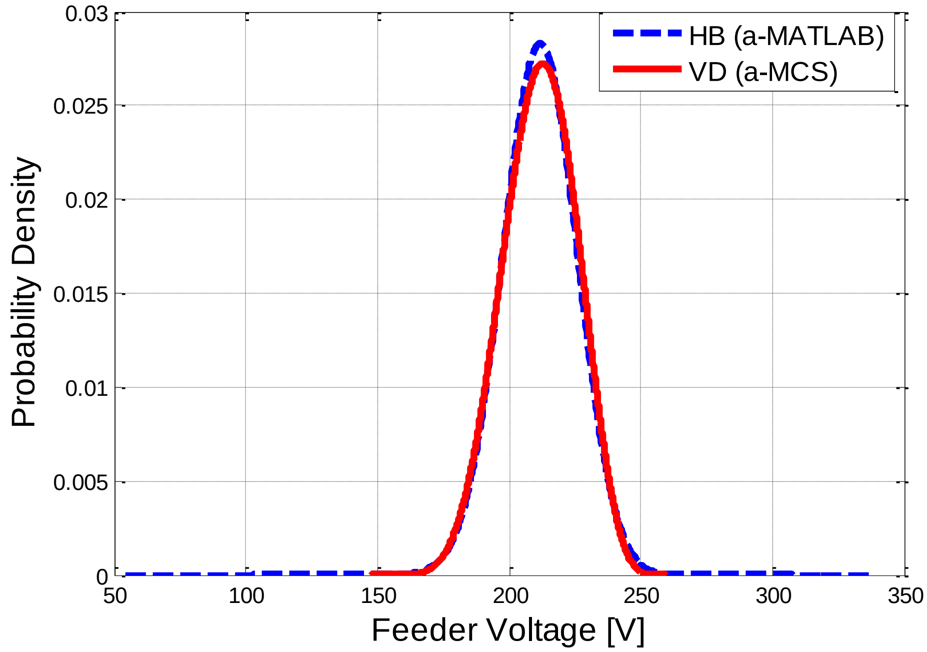


Figure B-12: Bi phase LV feeder - 12 loads and 2 generators for $VD_{a\text{-}MCS}$ *and* $HB_{a\text{-}MATLAB}$

b) 12 Loads and 6 Generators (DG)

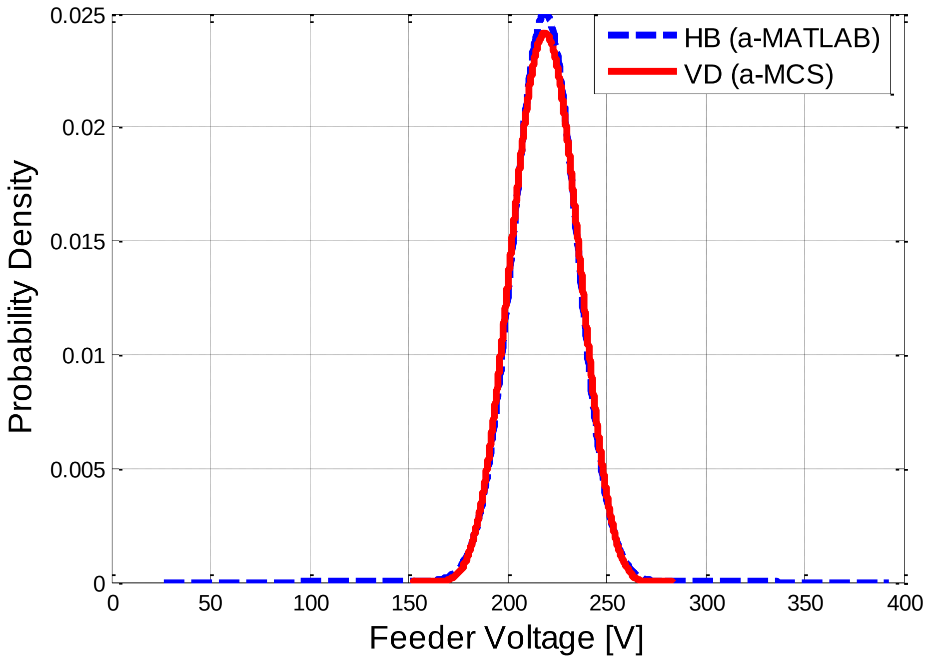


Figure B-13: Bi phase LV feeder - 12 loads and 6 generators for $VD_{a\text{-}MCS}$ *and* $HB_{a\text{-}MATLAB}$

Table B-11: Results for Comparison for $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for Constant Load with varying DG in bi phase*

Loads	DG	Voltage Calculation	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^2)$
12	0	$HB_{a\text{-}MATLAB}$	49.20	36.25	67.52	311.24	207.84	43367
		$VD_{a\text{-}MCS}$	6.9587	5.5294	154.28	240.47	207.88	43383
12	2	$HB_{a\text{-}MATLAB}$	55.98	45.04	53.98	338.32	211.54	44944
		$VD_{a\text{-}MCS}$	8.4529	6.3341	147.50	259.66	211.62	44976
12	4	$HB_{a\text{-}MATLAB}$	62.55	53.74	40.44	365.41	215.23	46548
		$VD_{a\text{-}MCS}$	8.55	7.71	149.54	274.35	215.19	46527
12	6	$HB_{a\text{-}MATLAB}$	69.01	62.38	26.89	392.50	218.92	48180
		$VD_{a\text{-}MCS}$	8.2927	8.0894	152.21	284.07	218.82	48133
12	8	$HB_{a\text{-}MATLAB}$	75.38	70.95	13.35	419.58	222.62	49.839
		$VD_{a\text{-}MCS}$	9.0398	8.9838	150.18	294.68	222.65	49849
12	10	$HB_{a\text{-}MATLAB}$	81.70	79.48	-0.19	446.67	226.31	51525
		$VD_{a\text{-}MCS}$	7.4112	7.7586	157.37	298.62	226.38	51555
12	12	$HB_{a\text{-}MATLAB}$	87.98	87.98	-13.73	473.75	230.01	53238
		$VD_{a\text{-}MCS}$	9.7299	7.8995	142.42	301.40	230.16	53311
12	14	$HB_{a\text{-}MATLAB}$	94.23	96.45	-27.28	500.84	233.70	54979
		$VD_{a\text{-}MCS}$	116.65	131.43	-49.516	553.32	233.95	55096

Three Phase Network Topology

1. Feeder Configuration

Node	Red	White	Blue	Load Parameters			Conductor Details	
				Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L				1.500	4.000	60	400	35mmCu
1G	- 6	- 6	- 6	1.500	4.000	60	0.1	35mmCu

Figure B-14: DG only three phase LV feeder

2. Results

a) 18 Loads and 6 Generators (DG)

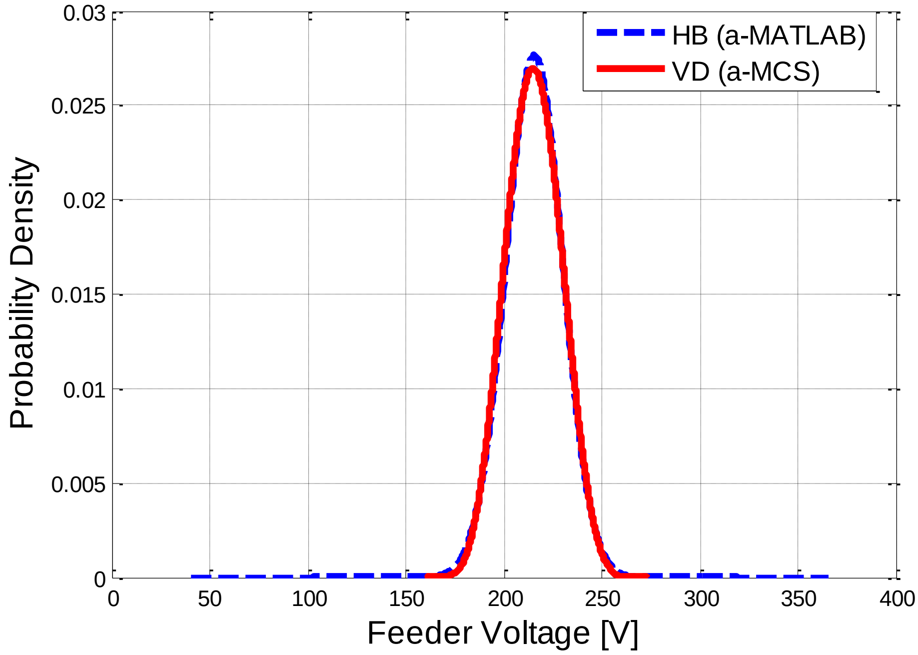


Figure B-15: Three phase LV feeder - 18 loads and 6 generators for $VD_{a\text{-}MCS}$ *and* $HB_{a\text{-}MATLAB}$

b) 18 Loads and 15 generators (DG)

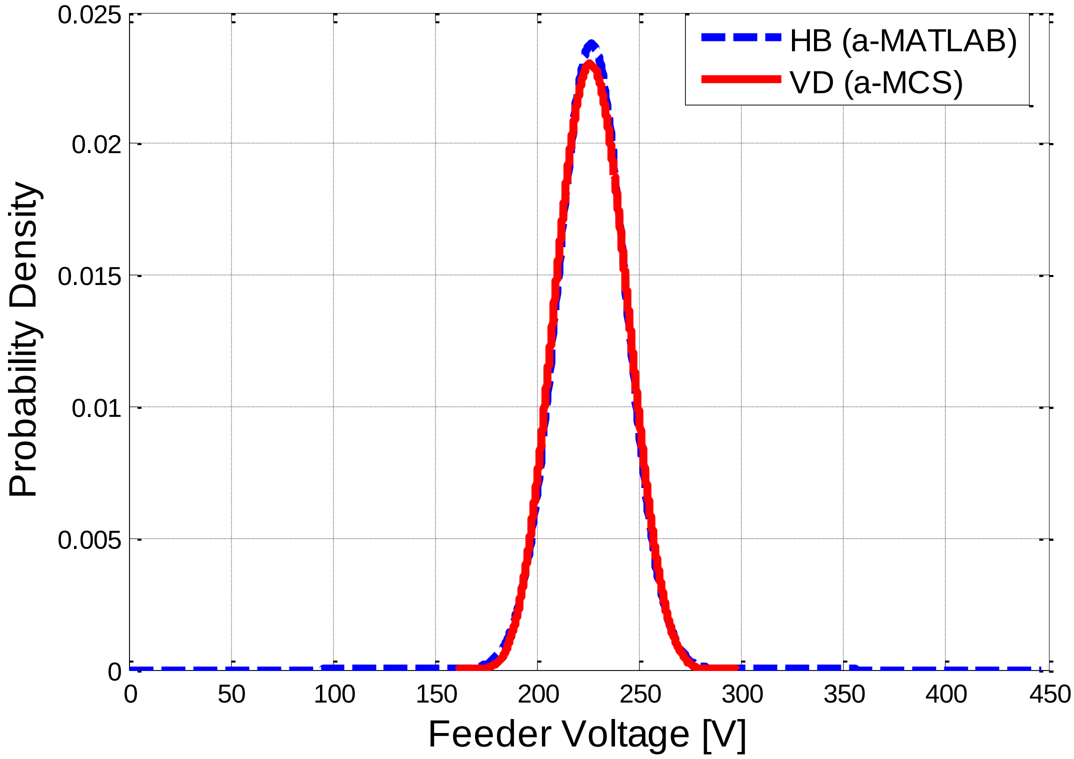


Figure B-16: Three phase LV feeder - 18 loads and 15 generators for $VD_{a\text{-}MCS}$ *and* $HB_{a\text{-}MATLAB}$

Table B-12: Results for Comparison for $HB_{a\text{-}MATLAB}$ *and* $VD_{a\text{-}MCS}$ *tests for Constant Load with varying DG in three phase*

Loads	DG	Voltage Calculation	α	β	V_{min}	V_{max}	$E(V_{con})$	$E(V_{con}^2)$
18	0	$HB_{a\text{-}MATLAB}$	53.04	39.01	67.52	311.24	207.95	43401
		$VD_{a\text{-}MCS}$	8.2814	6.3699	152.61	250.64	208.02	43423
18	3	$HB_{a\text{-}MATLAB}$	60.66	48.73	53.98	338.32	211.67	44983
		$VD_{a\text{-}MCS}$	7.7674	6.1053	154.25	256.63	211.57	44937
18	6	$HB_{a\text{-}MATLAB}$	68.15	58.45	40.44	365.41	215.38	46593
		$VD_{a\text{-}MCS}$	7.2033	7.6596	160.92	273.17	215.32	46562
18	9	$HB_{a\text{-}MATLAB}$	75.58	68.19	26.89	392.50	219.09	48230
		$VD_{a\text{-}MCS}$	9.3868	7.8407	149.38	277.49	219.18	48265
18	12	$HB_{a\text{-}MATLAB}$	83.00	77.98	13.35	419.58	222.80	49895
		$VD_{a\text{-}MCS}$	8.6511	7.6393	152.30	284.98	222.76	49876
18	15	$HB_{a\text{-}MATLAB}$	90.21	87.75	0.19	446.67	226.51	51586
		$VD_{a\text{-}MCS}$	7.5599	8.3638	161.90	298.15	226.59	51615
18	18	$HB_{a\text{-}MATLAB}$	81.66	91.86	13.73	473.75	230.22	53305
		$VD_{a\text{-}MCS}$	8.6519	8.4569	155.30	303.10	230.04	53221
18	21	$HB_{a\text{-}MATLAB}$	73.51	94.93	27.28	500.84	233.94	55052
		$VD_{a\text{-}MCS}$	7.2876	7.8441	163.92	309.37	233.97	55069

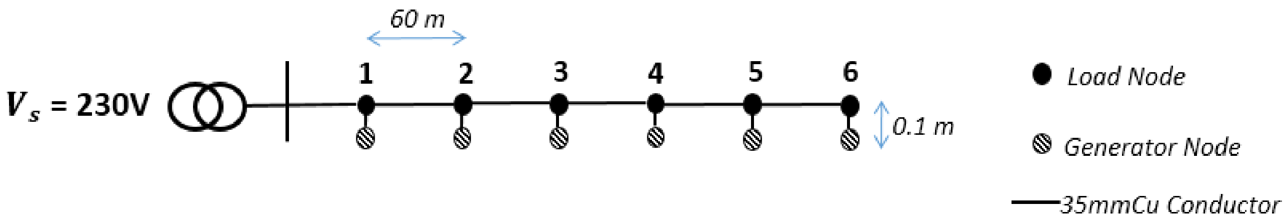
APPENDIX C

Appendix C – Voltage Analysis of Active LV Feeders with DG

C.1 Test Scenario Passive LV feeder configurations and designs

All passive LV feeders were designed for a maximum voltage drop of 10 % before active feeder voltage analysis carried out for increasing PV-EG penetration.

C.1.1 Case 1: Penetration of 1-kW single phase PV units (Base Case)



				Load Parameters			Conductor Details	
Node	Red ma	White mb	Blue mc	Alpha	Beta	Cb [A]	Length [m]	Cable Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

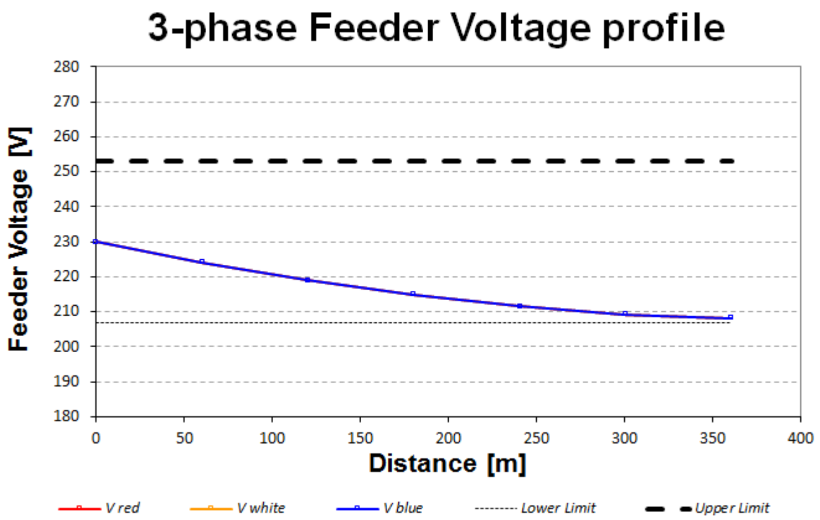
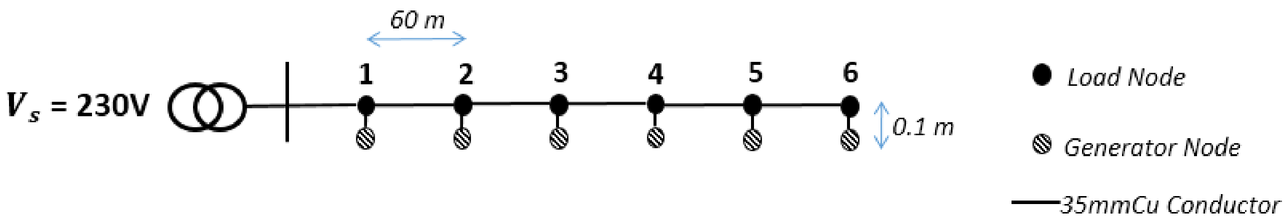


Figure C-1: Base Case LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.2 Case 2: Penetration of 3.5-kW single phase PV units



				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	24.044	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	24.044	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	24.044	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	24.044	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	24.044	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	24.044	0.1	35mmCu

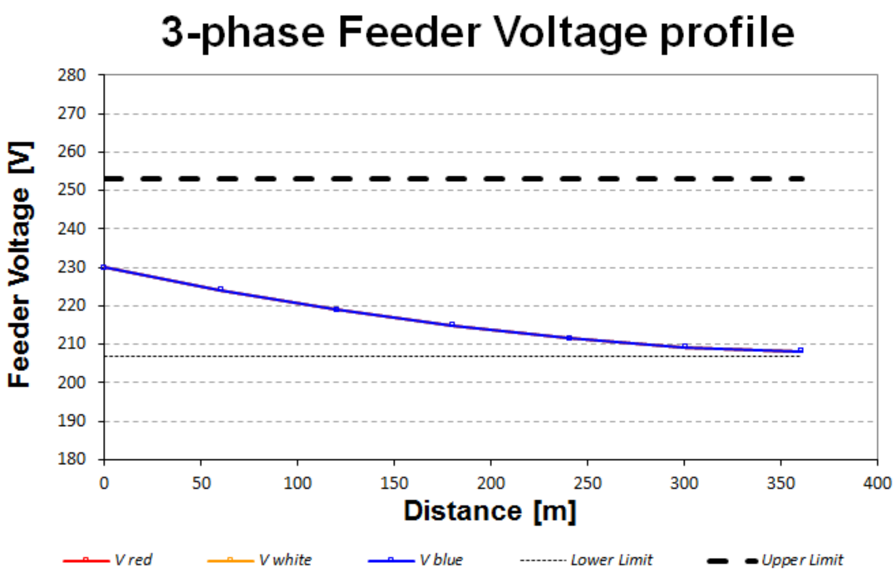
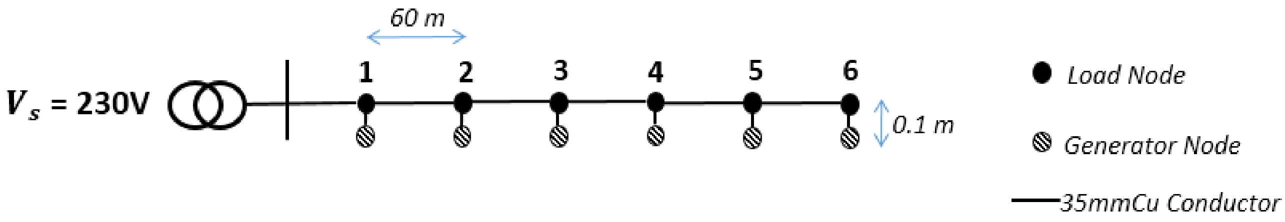


Figure C-2: Case 2 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.3 Case 3: Penetration of 10-kW three phase PV units



				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	22.805	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	22.805	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	22.805	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	22.805	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	22.805	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	22.805	0.1	35mmCu

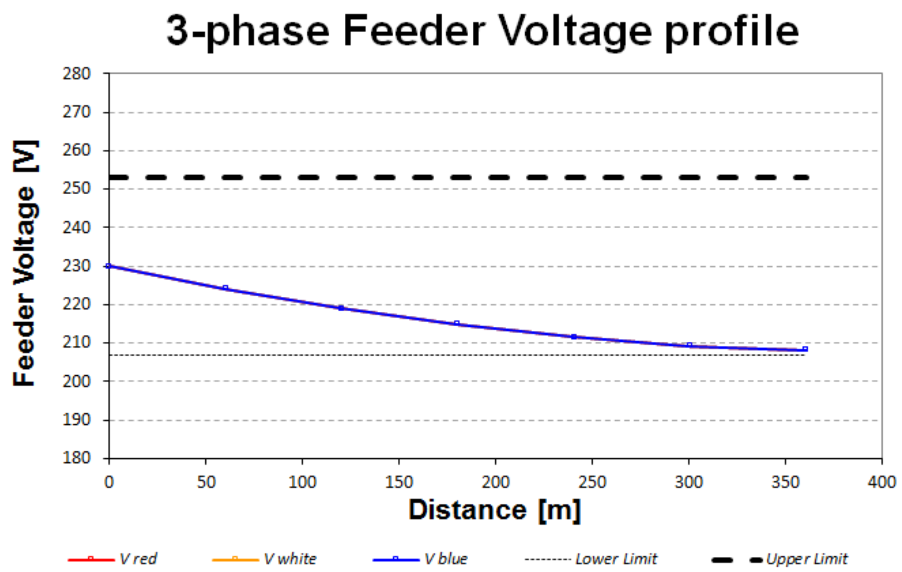
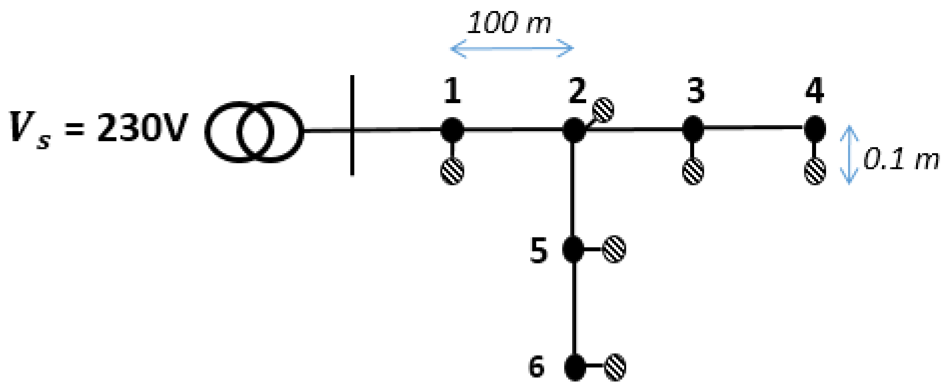


Figure C-3: Case 3 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.4 Case 4: Feeder with Lateral Branch



				Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	1	1	1	0.863	1.895	50.127	100	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	100	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	0.1	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	0.1	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	100	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	100	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

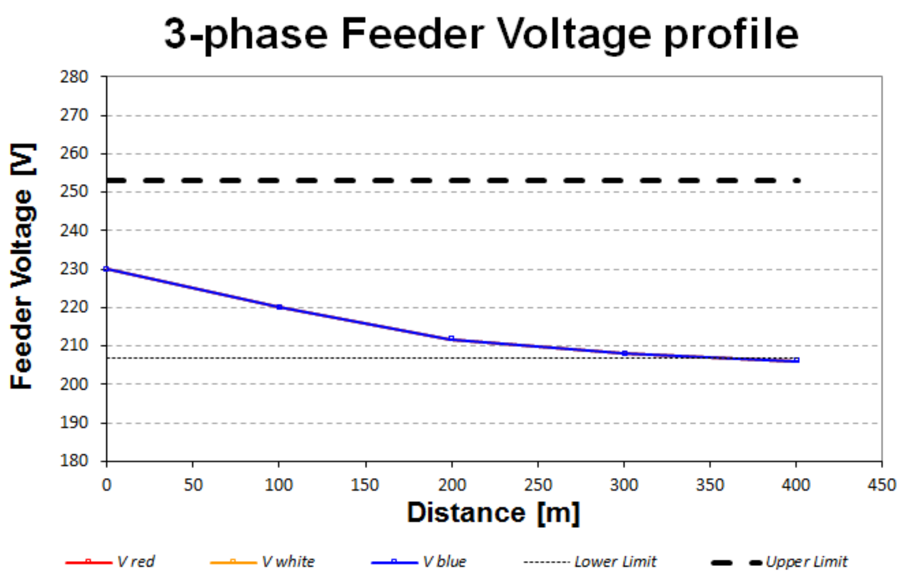
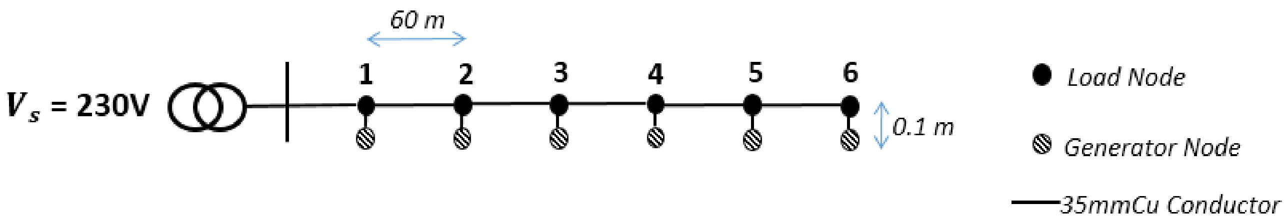


Figure C-4: Case 4 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.5 Case 5: Feeder with unbalanced consumer allocation



Node	Red	White	Blue	Load Parameters Alpha	Beta	Cb	Conductor Details Length	Cable
	ma	mb	mc			[A]	[m]	Code
1L	3	-	-	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	-	3	-	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	-	-	3	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	-	-	3	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	-	3	-	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	3	-	-	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

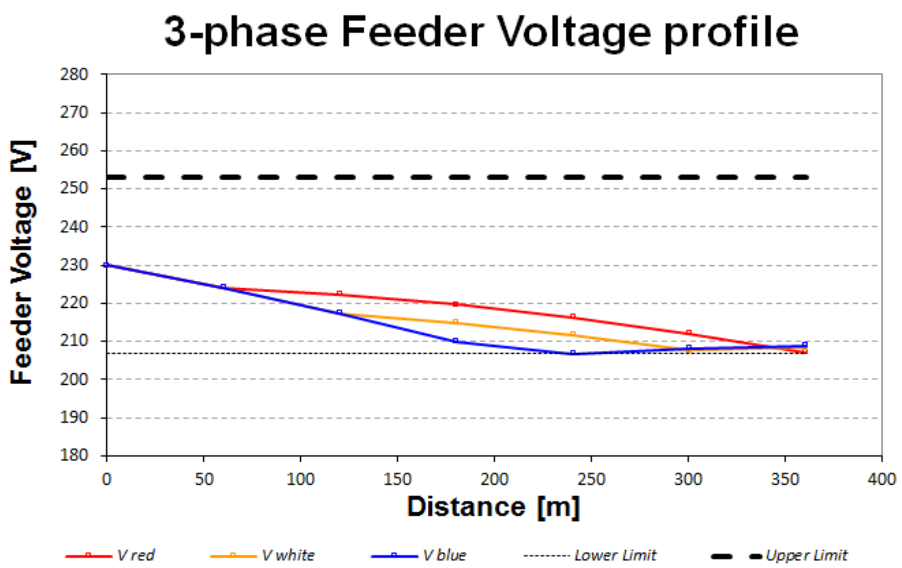
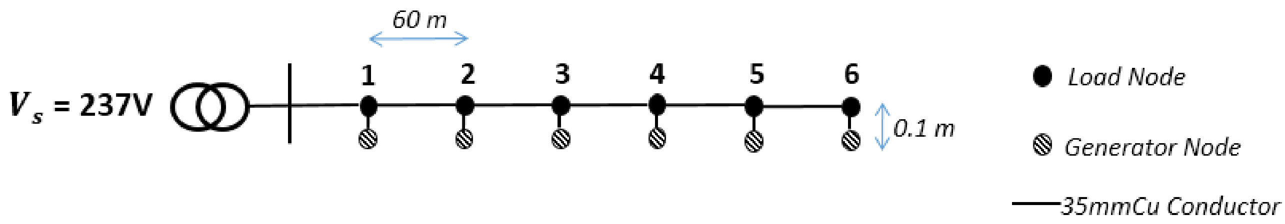


Figure C-5: Case 5 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.6 Case 6: Feeder with variation in source voltage



				Load Parameters			Conductor Details	
Node	Red ma	White mb	Blue mc	Alpha	Beta	Cb [A]	Length [m]	Cable Code
1L	1	1	1	0.863	1.895	50.127	60	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.863	1.895	50.127	60	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.863	1.895	50.127	60	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.863	1.895	50.127	60	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.863	1.895	50.127	60	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.863	1.895	50.127	60	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

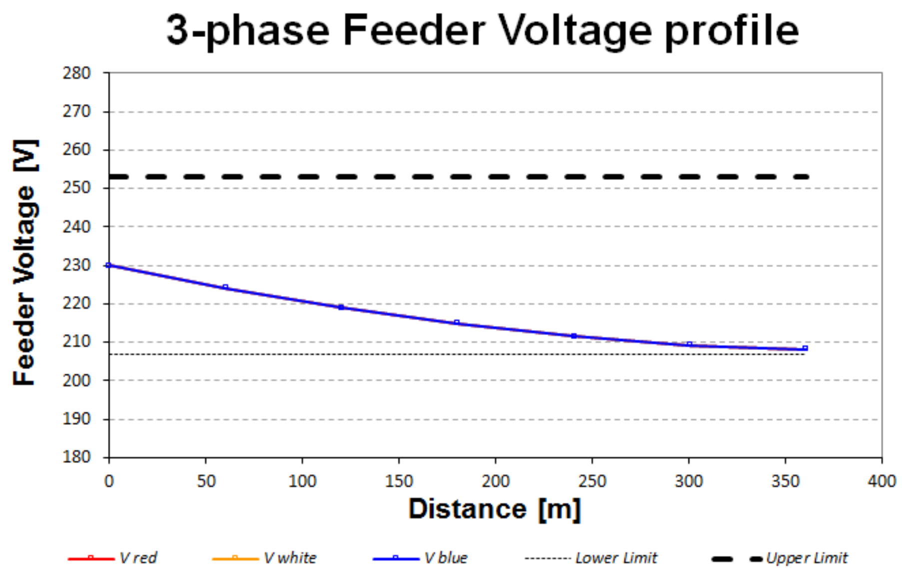
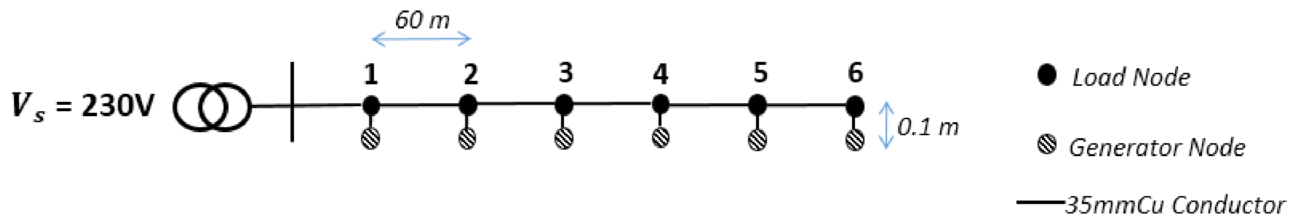


Figure C-6: Case 6 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.1.7 Case 7: Feeder with consumers of lower customer classification (LSM)



				Load Parameters			Conductor Details	
Node	Red ma	White mb	Blue mc	Alpha	Beta	Cb [A]	Length [m]	Cable Code
1L	1	1	1	0.687	1.802	26.016	120	35mmCu
1G				255.500	255.550	6.8696	0.1	35mmCu
2L	1	1	1	0.687	1.802	26.016	120	35mmCu
2G				255.500	255.550	6.8696	0.1	35mmCu
3L	1	1	1	0.687	1.802	26.016	120	35mmCu
3G				255.500	255.550	6.8696	0.1	35mmCu
4L	1	1	1	0.687	1.802	26.016	120	35mmCu
4G				255.500	255.550	6.8696	0.1	35mmCu
5L	1	1	1	0.687	1.802	26.016	120	35mmCu
5G				255.500	255.550	6.8696	0.1	35mmCu
6L	1	1	1	0.687	1.802	26.016	120	35mmCu
6G				255.500	255.550	6.8696	0.1	35mmCu

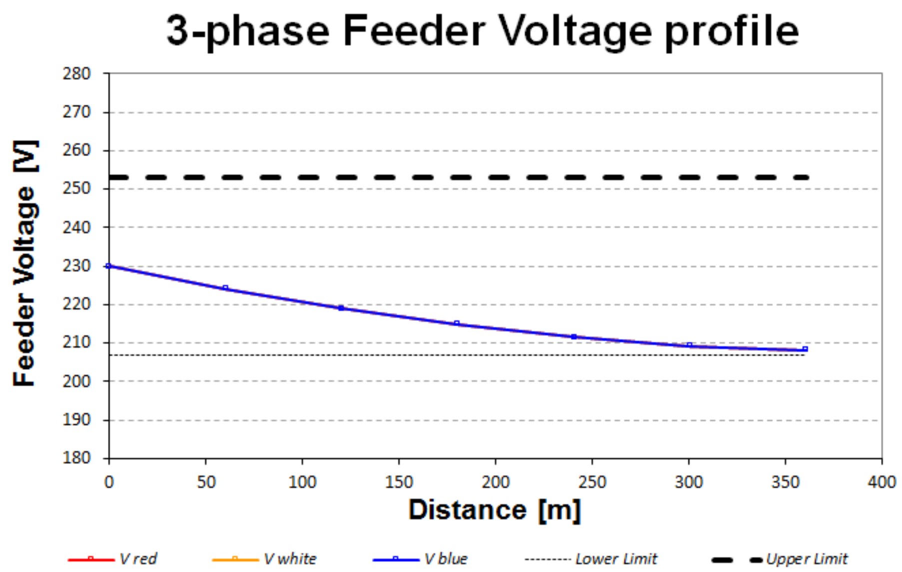


Figure C-7: Case 7 LV feeder configuration

a) LV feeder configuration (above) b) HB feeder configuration on Excel (middle) c) Feeder profile (below)

C.2 Additional Selected Scenario Tests

C.2.1 Case 1: Base Case

i. Testing the voltage analysis tool

This test is carried using the same configuration as in the base case but using another beta PDF model for the same DG output, as in the base case. The DG model used in the base case is used to represent a 1-kW PV unit that has a mean output of 3.43 A, as stated in the assumptions in Chapter 5. In the base case, a beta PDF model used provides a deterministic model of the DG performance as shown in figure C-8. Another beta PDF model for the same DG performance, can be modelled to have the same mean current 3.43 A and a maximum current output of 4.35 A, the rated maximum current output for a 1-kW PV unit. Using the mean, maximum current and an arbitrary value for the alpha parameter, the beta parameter can be calculated using the equation (C-1). A similar process can be done to obtain the alpha parameter using the mean, maximum current and an arbitrary value for the beta parameter. The two beta PDFs are statistically the same and should provide the same result as the base case.

$$\beta = \frac{(\alpha - \alpha \times \frac{\mu}{I_{max}})}{\frac{\mu}{I_{max}}} \; \ldots \ldots (C\text{-}1)$$

Table C-1: Probabilistic models of the deterministic PV used

PV-EG MODEL	α	β	c	μ	σ
Model 1 (Base Case)	255.50	255.50	6.8696	3.4348	0.1520
Model 2	255.00	67.78	4.3478	3.4348	0.0984

Result

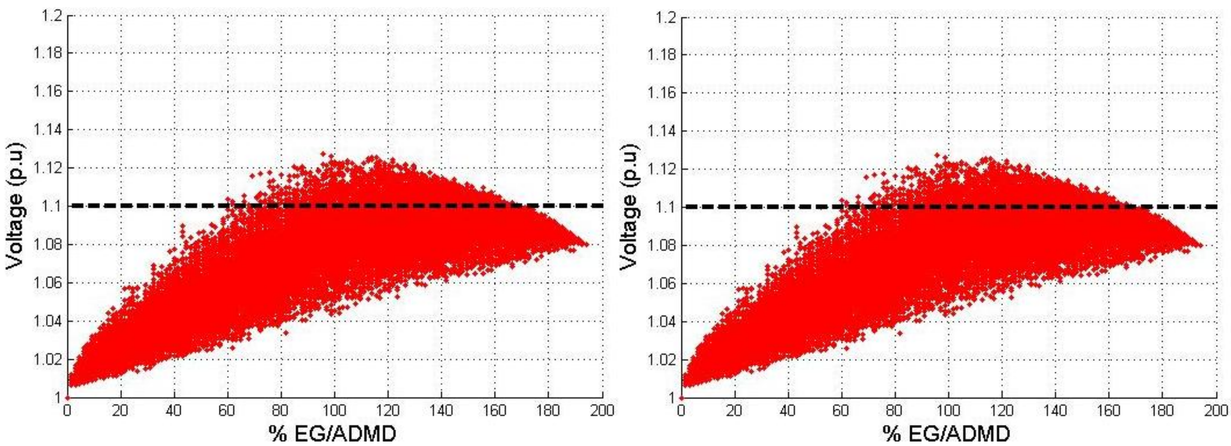


Figure C-8: Results of the base case test using DG model 1 (left) and DG model 2 (right)

As expected, it can be observed in figure C-8 that the base case scatter plots are the same which indicates that the MCS for the voltage analysis of active LV feeders is correctly implemented and formulated.

ii. Effect of dispersion in load demand

In statistics, standard deviation is a common measure of spread of a given data set or range and is used by statisticians to assess variability or diversity. It is used to explain how much variation or dispersion the current values are from the mean current [Navidi, 2010].

In this case, a high standard deviation indicates the current values are spread out in excess of a large range of values while a low standard deviation indicates that the current values are very close to the mean current. The two beta parameters used to represent different loads in this test are shown in table C-2 and the beta PDFs are shown in figure C-9.

Table C-2: Probabilistic model of varying load demand used and the probabilistic model of deterministic load demand

LOAD MODEL	α	β	c	μ	σ
With dispersion	0.4366	1.2877	24.8680	6.2967	6.5516
With limited dispersion	255.50	255.50	12.5926	6.2963	0.2783

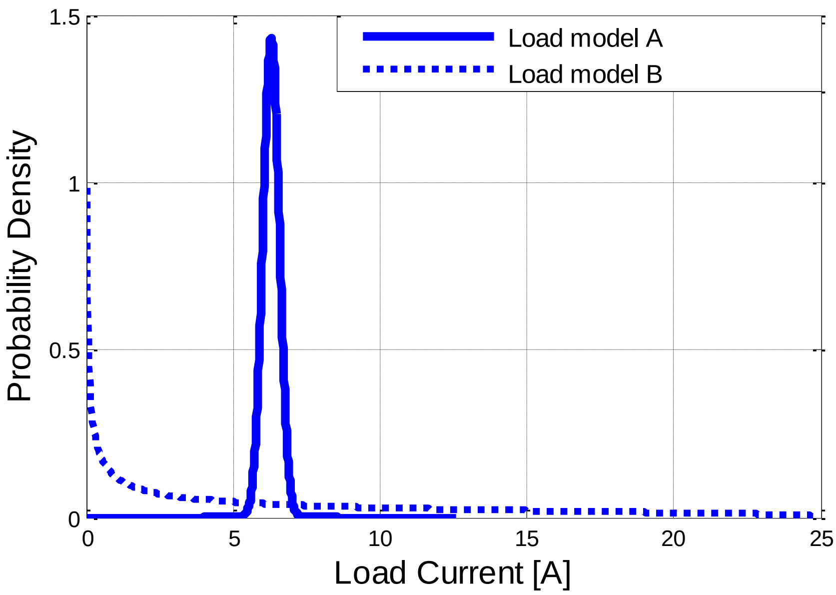


Figure C-9: Illustration of the beta PDFs for the load models

From figure C-9, it can be observed that the beta PDF load A has a lower spread/base than the beta PDF for load B. This indicates that the current values in load A model are very close to the mean current (low dispersion).

iii. Demonstrating the effect of unbalance in voltages in the feeder

In general, voltage unbalance in the feeder across the phases and along the length of a passive LV feeder can be increased as a result of the uncertainties in the random connection point of DG units to the network, their nominal capacity and time of operation time [Shahnia, 2011].

The uncertain allocation of PV-units is important and relevant in this study as the nominal capacity and time of operation are already set. The MCS used for voltage analysis of the active LV feeder with DG is based on the random allocation of PV units to consumers to produce a scatter plot of possible maximum voltages on the feeder as seen in the base case result in figure 6-2. Although the random allocation is done iteratively and with a large number of runs, it is unlikely to produce all possible PV unit allocation combinations as the successive probability of choosing a nodes and phases in a different and specific order is very small. Therefore, the effect of unbalance in voltage across the phases and the length of the feeder can be best demonstrated by deterministically allocating the PV units to create the extreme conditions of voltage unbalance and balance. Unbalance in voltage in the LV feeder due to uncertainty in allocation of PV units in the LV feeder can be seen caused by random allocation of PV units on:

- Different locations (nodes) along the length of feeder – Same phase
- Different locations (nodes) along the length of feeder – Different phases
- Same location (node) on the feeder – Different phases

This test is carried out as described below:

- The active LV feeder is configured as in the base case in the HB Excel spreadsheet.
- With the knowledge of the maximum number of units a consumer can install on his household, the PV units are placed 7 units at a time per phase. The allocation is done in such a way to establish an envelope to show the extreme limits of most unbalance allocations and most balanced allocation.
- For the most unbalanced allocations, the allocation was as follows:
  - 7 units are placed on the red phase on last node (node 6) of the feeder and the maximum voltage on the feeder profile is recorded. The red phase is filled progressively with 7 units each time from node 6 to node 1. For every 7-unit fill on the phase, the maximum voltage on the feeder profile is recorded.
  - Once the red phase is fully loaded with PV units, PV units are added to the white and blue phases progressively from the start of the feeder (node 1) to the end of the feeder (node 6).
  - When 7 PV units are assigned to every phase in the feeder, the feeder is considered full and the maximum voltage recordings plotted on the graph to form upper limit of the deterministic envelope.

This allocation is considered as the most unbalanced allocation because for each allocation of 7 units, it ensures there is a largest possible distance between the loaded and unloaded phases (for voltage unbalance along the feeder length) and ensures the largest difference in the number of PV units between the phases (for voltage unbalance across the phases).

- For the most balanced allocations, the allocation was as follows:
  - 7 units are placed on the every phase (red, white and blue phases) at the node 1 (start of the feeder) and the maximum voltage on feeder profile recorded. The nodes are loaded progressively from node 1 to node 6.
  - When 7 PV units are assigned to every phase in the feeder, the feeder is considered full and the maximum voltage recordings plotted on the graph to form lower limit of the deterministic envelope.

    This allocation is considered as the most balanced allocation because for each allocation of 7 units, the phases are balanced and the allocation is from the node 1 to node 6.

Result

Tables C-3 and C-4 show the results of this test and the resulting deterministic envelope is shown in figure C-10.

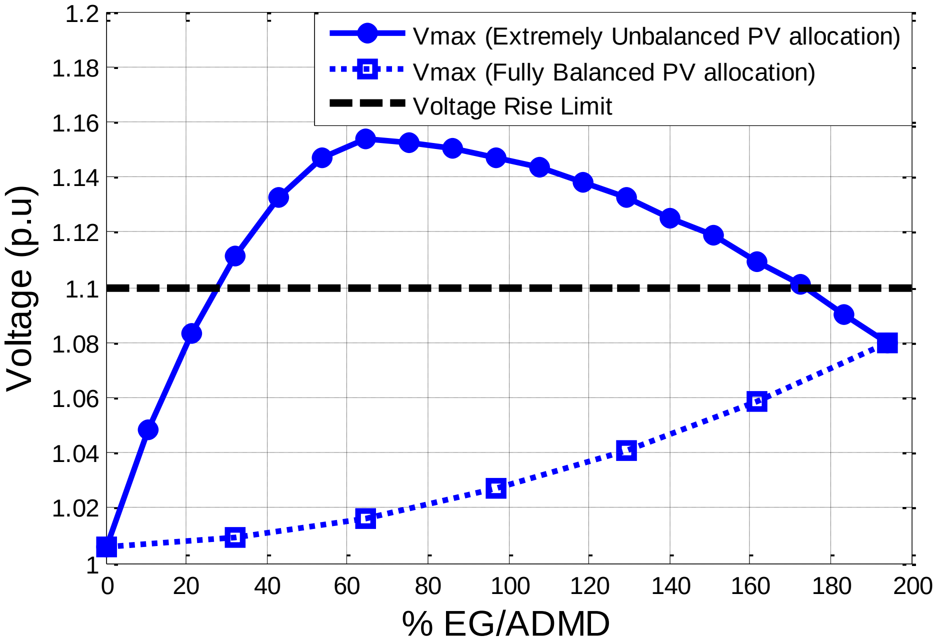


Figure C-10: Deterministic Envelope of maximum voltages due to extreme conditions of balanced and unbalanced PV allocation

Table C-3: Results from the most balanced allocation of PV units

Total no. of PV units on feeder	Penetration Level [%EG/ADMD]	Maximum voltage on feeder profile [V]	Maximum voltage on feeder profile [p.u]
0	0.00	231.31	1.0057
21	32.36	232.12	1.0092
42	64.71	233.75	1.0163
63	97.06	236.18	1.0269
84	129.42	239.43	1.0410
105	161.77	243.49	1.0587
126	194.13	248.37	1.0799

Table C-4: Results from the most unbalanced allocation of PV units

Total no. of PV units on feeder	Penetration Level [%EG/ADMD]	Maximum voltage on feeder profile [V]	Maximum voltage on feeder profile [p.u]
0	0.00	231.31	1.01
7	10.78	241.06	1.05
14	21.57	249.18	1.08
21	32.35	255.67	1.11
28	43.14	260.54	1.13
35	53.92	263.79	1.15
42	64.71	265.42	1.15
49	75.49	265.00	1.15
56	86.28	264.60	1.15
63	97.06	263.76	1.15
70	107.85	262.98	1.14
77	118.63	261.70	1.14
84	129.42	260.54	1.13
91	140.20	258.82	1.13
98	150.99	257.30	1.12
105	161.77	255.14	1.11
112	172.56	253.24	1.10
119	183.34	250.67	1.09
126	194.13	248.37	1.08

iv. Interpreting the scatter plot result

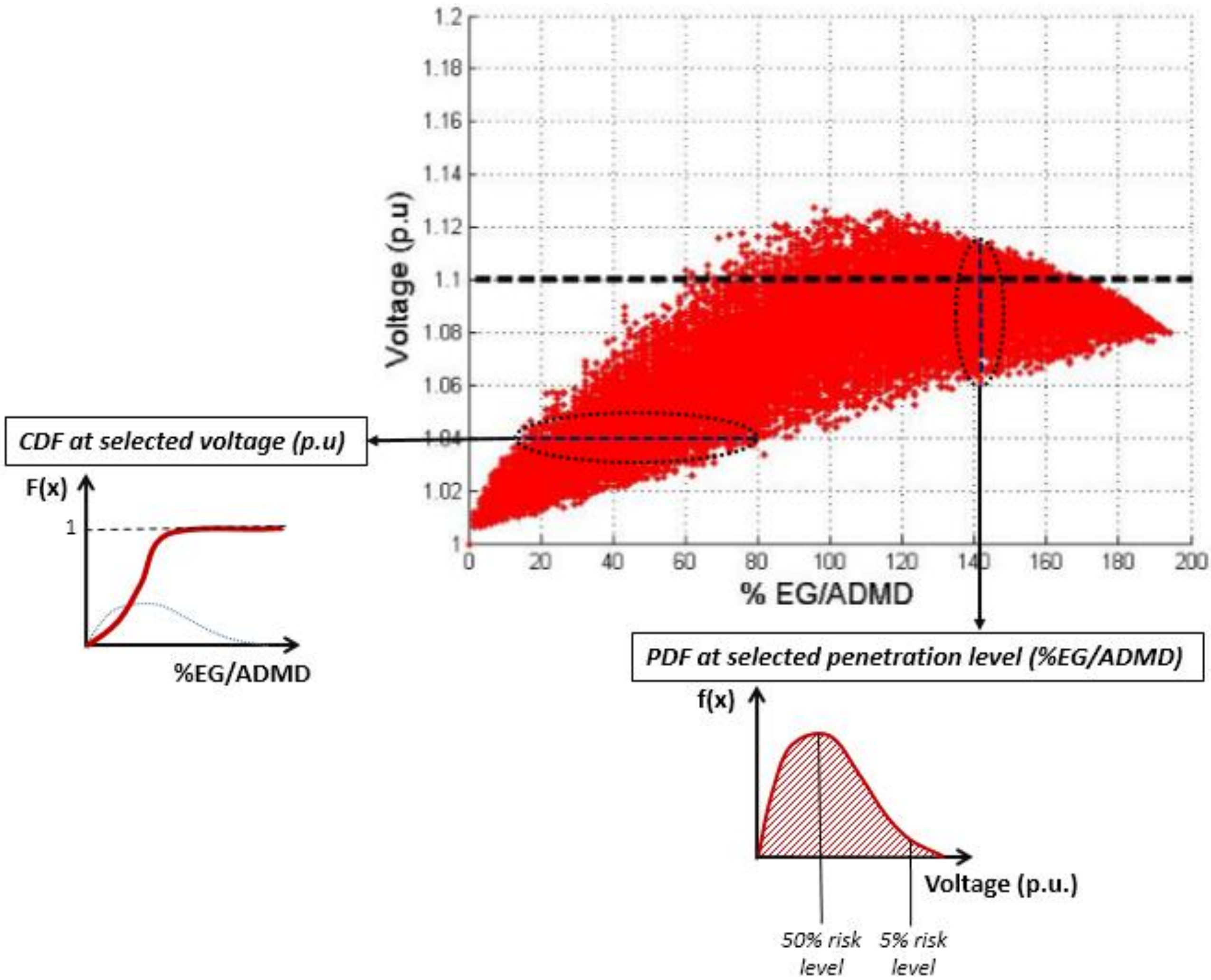


Figure C-11: Obtaining the distributions of voltages from the scatter plot result

C.3 Case 4: Branched feeder scenario

i. The effect of the position of the lateral branch

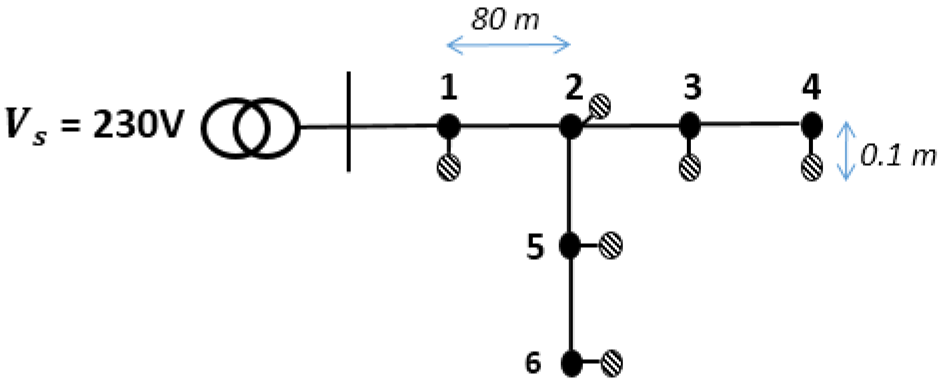


Figure C-12: Feeder with lateral branch at node 3

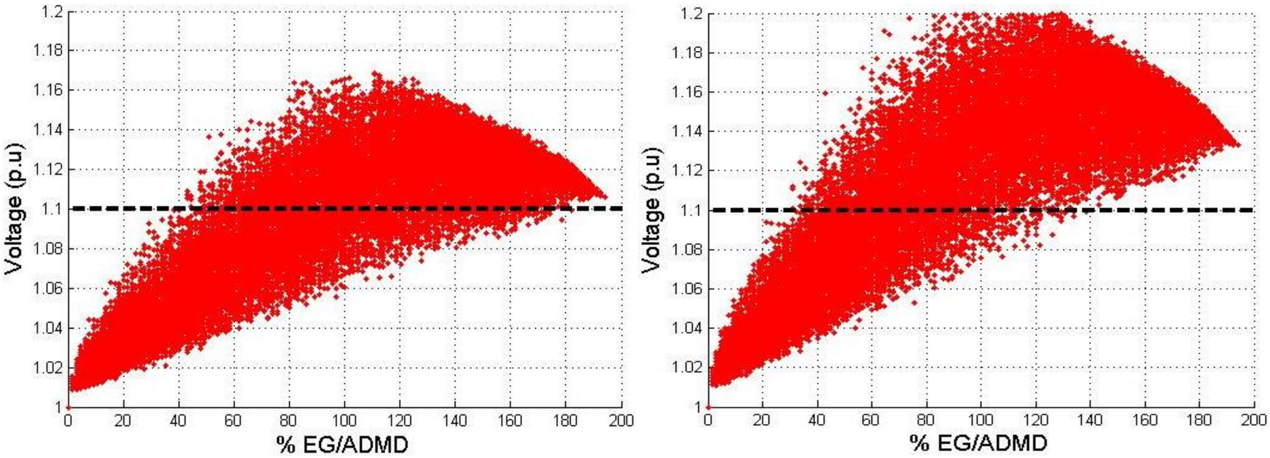


Figure C-13: Feeder over-voltage profile for penetration of single phase PV units on feeder with:

a) Lateral branch far away from the voltage source (left) *b) Lateral branch near the voltage source (right)*

APPENDIX D

Appendix D – Supplementary Work

Supplementary work done in this research is found in this chapter. In this chapter, the current calculation for passive LV feeders, in the Herman-Beta method, is validated by use of MCS and then is modified to enable planners calculate the current in active feeders with DG. This modified current calculation for active feeders with DG is also validated by use of a MCS.

D.1 Introduction

Although voltage is the main parameter used in passive feeder design in the HB method, it is important to consider the current in the feeder conductor. All conductors and cables are designed for a particular current rating which must not be exceeded for any particular design.

D.2 Current Calculation Algorithm in the Herman-Beta Method

D.2.1 List of symbols

The symbols used in the formulations were chosen to avoid Greek or other non-alphabetic symbols and, where possible, to avoid subscripts and superscripts. For example, ai is used rather than a_i

ai	is the Beta probability density function parameter, alpha at node i
bi	is the Beta probability density function parameter, beta at node i
ci	is the scaling factor in amperes (usually the circuit-breaker size) at node i
mai	is the number of consumers connected to the a-phase at node I
mbi	is the number of consumers connected to the b-phase at node i
mci	is the number of consumers connected to the c-phase at node i
N	is the total number of nodes in the radial feeder section
p	is the percentage risk in the probabilistic calculation
G	is the first statistical moment
H	is the second statistical moment
Imax	is the maximum feeder current
Imin	is the minimum feeder current
L, K	are node counters
I	is the feeder current
i	is the normalized feeder current
ac	is the alpha parameter of scaled feeder current
bc	is the beta parameter of scaled feeder current

betainv is the Beta inverse function

E() is the expected value of ()

D.2.2 Algorithm for Passive LV Feeders

i. **Step-wise procedure for calculating bi-phase and three phase system feeder phase current**

Step 1 – Select the network parameters

1. Load description in Beta pdf form: ***ai, bi, ci***. Where *c*i is the scaling factor – usually the circuit-breaker size.
2. Specify the number of consumer connections at each load node, i: ***mai*** or ***mbi*** or ***mci***
3. Specify total number of nodes in the radial section, ***N***.
4. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$Gi = \frac{ai}{(ai+bi)} \qquad Hi = \frac{ai(ai+1)}{(ai+bi)(ai+bi+1)}$$

Step 3 – Calculate the expected values: E (I) and E (I²)

$$E(Ii) = |mai| \times Gi \times ci$$

$$E(I) = \sum_{i=1}^{N} E(I)$$

$$E(I^2 i) = ci^2 \left[\, |mai| \times Hi + |mai|(|mai|-1) \times Gi^2 \right]$$

$$E(I^2) = \sum_{i=1}^{N} E(I^2 i) + \sum_{K=1}^{N} \sum_{\substack{K=1 \\ L \neq K}}^{N} E(I_K) \times E(I_L)$$

Step 4 – Calculate maximum current Imax

$$I = mai \times ci$$

$$Imax = \sum_{i=1}^{N} I$$

Step 5 – Calculate the scaled values of E (i) and E (i²)

$$E(i) = \frac{E(I)}{Imax}$$

$$E(i^2) = \frac{E(I^2)}{(Imax)^2}$$

Step 6 – Calculate the Beta parameters of feeder phase current: ac and bc

$$ac = \frac{E(i^2) - E(i)}{E(i) - \frac{E(i^2)}{E(i)}} \qquad bc = \frac{ac}{E(i)} - ac$$

Step 7 – Select a risk percentage p and calculate percentile value current %

Use the Beta inverse function:

$$\text{current \%} = \text{betainv}[\left(1 - \frac{p}{100}\right), |av|, |bv|]$$

A percentile value of 10 % (less commonly 5 %) is used for passive LV feeders.

Step 8 – Rescale the feeder phase current, CURRENT %

$$\text{CURRENT \%} = \text{current \%} \times (\text{Imax})$$

D.2.3 Algorithm for Active LV Feeders

Note: It should be noted that for all formulae apply to all nodes (load and generator nodes) except symbols denoted with *'symbol'*$_{LOAD}$ for load nodes and *'symbol'*$_{DG}$ for generator nodes

i. **Step-wise procedure for calculating bi-phase and three phase system feeder phase current**

Step 1 – Select the network parameters

1. Load description in Beta pdf form: ***ai, bi, ci***. Where *c*i is the scaling factor – usually the circuit-breaker size.
2. Specify the number of consumer connections at each load node, i: ***mai*** or ***mbi*** or ***mci*** (loads in load nodes and embedded generators in generator nodes).
   A positive number represents a load and a negative number is an embedded generator.
3. Specify total number of nodes in the radial section, ***N***.
4. Specify a design risk value: ***p***, in percent.

Step 2 – Calculate constants Gi and Hi

$$\text{Gi} = \frac{\text{ai}}{(\text{ai}+\text{bi})} \qquad \text{Hi} = \frac{\text{ai}(\text{ai}+1)}{(\text{ai}+\text{bi})(\text{ai}+\text{bi}+1)}$$

Step 3 – Calculate the expected values: E (I) and E (I²)

$$\text{E(Ii)} = |\text{mai}| \times \text{Gi} \times \text{ci}$$

$$\text{E}(\text{I}) = \sum_{i=1}^{N} \text{E(I)}$$

$$\text{E}\left(\text{I}^2\text{i}\right) = \text{ci}^2\ [\ |\text{mai}| \times \text{Hi} + |\text{mai}|(|\text{mai}|-1) \times \text{Gi}^2]$$

$$\text{E}(\text{I}^2) = \sum_{i=1}^{N} \text{E}(\text{I}^2\text{i}) + \sum_{K=1}^{N} \sum_{\substack{K=1 \\ L\neq K}}^{N} \text{E}(\text{I}_K) \times \text{E}(\text{I}_L)$$

Step 4 – Calculate maximum and minimum currents, **Imax$_{DG}$** ***and*** **Imax$_{LOAD}$**

For load nodes, $\text{I}_{DG} = 0$ $\qquad$ $\text{I}_{LOAD} = \text{mai} \times \text{ci}$

For generator nodes, $\text{I}_{DG} = \text{mai} \times \text{ci}$ $\qquad$ $\text{I}_{LOAD} = 0$

$$\text{Imax}_{DG} = \sum_{i=1}^{N} \text{I}_{DG} \qquad \text{Imax}_{LOAD} = \sum_{i=1}^{N} \text{I}_{LOAD}$$

Step 5 – Calculate the scaled values of E (i) and E (i²)

$$\text{E(i)} = \frac{\text{E}(\text{I}) - \text{Imax}_{DG}}{|\ (\text{Imax}_{LOAD} - \text{Imax}_{DG})\ |}$$

$$E(i^2) = \frac{E(I^2) - 2 \times Imax_{DG} \times E(I) + Imax_{DG}^{\ 2}}{(Imax_{LOAD} - Imax_{DG})^2}$$

Step 6 – Calculate the Beta parameters of feeder phase current: ac and bc

$$ac = \frac{E(i^2) - E(i)}{E(i) - \frac{E(i^2)}{E(i)}} \qquad bc = \frac{ac}{E(i)} - ac$$

Step 7 – Select a risk percentage p and calculate percentile value current %

Use the Beta inverse function:

$$current\ \% = betainv[\left(1 - \frac{p}{100}\right), |av|, |bv|]$$

A percentile value of 90 % (or 95 %) is used for active feeders with embedded generation.

Step 8 – Rescale the feeder phase current, CURRENT %

$$CURRENT\ \% = current\ \% \times |(Imax_{LOAD} - Imax_{DG})| + Imax_{DG}$$

D.3 Validating the current calculation algorithm in the HB method using MCS

D.3.1 Introduction

In this section of testing the current calculation in the HB method, the MCS is used to iteratively calculate the total phase current in the given test feeder using relevant electrical circuit theory. This is similar to the testing performed in chapter 4 with the voltage calculation in the HB method. The procedure I_{MCS} is a MCS based method that produces a set of resultant currents, on which a beta PDF is fit and compared to the beta PDF of resultant currents from the current calculation in the HB method, I_{HB}. A comparison of the two beta PDFs should give an indication of the validity of the I_{HB}. In addition, the first and second statistical moments, E (I) and E (I^2), can be obtained through calculation and analysis of the raw set of resultant currents from the I_{MCS}. These values are compared to first and second statistical moments of the feeder current from the I_{HB}.

The current calculation in the HB method for passive LV feeders and active LV feeders with DG, is denoted by $I_{p\text{-}HB}$ and $I_{a\text{-}HB}$ respectively. Current calculation using the Monte Carlo Simulation (MCS) in passive LV feeders and active LV feeders with DG, is denoted by $I_{p\text{-}MCS}$ and $I_{a\text{-}MCS}$ respectively. It should be noted that $I_{p\text{-}MCS}$ and $I_{a\text{-}MCS}$ are contained within the $VD_{p\text{-}MCS}$ and $VD_{a\text{-}MCS}$ as the total phase currents are simply the sum of sampled currents for that phase.

A snippet of the Matlab code used for the bi phase network, for example, can be shown for the feeder configuration in figure D-1.

	No Consumers		Load Parameters			Conductor	
Node	Red	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	α	β	[A]	[m]	Code
1	3	3	1.500	4.000	60	200	35mmCu
2	3	3	1.500	4.000	60	200	35mmCu

	No Consumers on phases			Load Parameters			Conductor Details	
Node	Red	White	Blue	Alpha	Beta	Cb	Length	Cable
	ma	mb	mc			[A]	[m]	Code
1	3	3	3	1.500	4.000	60	200	35mmCu
2	3	3	3	1.500	4.000	60	200	35mmCu

Figure D-1: Bi phase (above) and three phase (below) network test feeder configuration

```
ma1 = 3;     ma2 = 3;     ma = ma1+ma2;       %No. of customers (red phase)
mb1 = 3;     mb2 = 3;     mb = mb1+mb2;       %No. of customers (blue phase)

 %Randomly sample from the beta distribution described by the alpha, beta and c
parameters
Y1 = 60*betarnd (1.500, 4.000, ma,1);      Y2 = 60*betarnd (1.500, 4.000, mb,1);
```

```
Ia1 = sum (Y1 (1:ma1));  %Total current in node 1 - red phase
Ia2 = sum (Y2 (1:ma2));  %Total current in node 2 - red phase

Ib1 = sum (Y1 (((ma1+1):(ma1+mb1))));       %Total current in node 1 - blue phase
Ib2 = sum (Y2 (((ma2+1):(ma2+mb2))));       %Total current in node 2 - blue phase
```

D.3.2 Current Calculation in Passive LV feeder

i. Test Feeder Configuration

The test feeder configuration used in this section is shown in figure D-2 and the feeder parameters/characteristics are shown in table D-1.

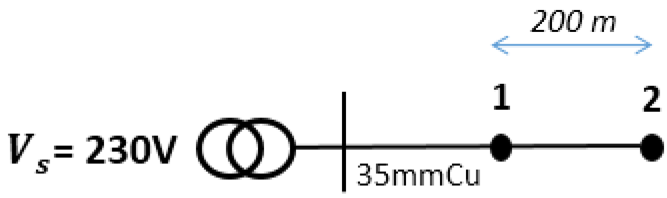


Figure D-2: $I_{p\text{-}MCS}$ Test Feeder

Table D-1: $I_{p\text{-}MCS}$ Test Feeder Configuration

FEEDER PARAMETER	Bi phase/Three phase Network Topology		
Number of nodes	2 [3 customers per node]		
Inter-node length [m]	200		
Load (abc parameters)	α = 1.5	β = 4.0	c = 60
Temperature [°C]	T_1=20; T_2=40		
Feeder Conductor	35 mm² Copper conductor (35mmCu)		

ii. Result

The results of the current calculation for both bi phase and three phase networks is shown in table D-2 comparing $I_{p\text{-}HB}$ and $I_{p\text{-}MCS}$.

Table D-2: Result data Comparison for $I_{p\text{-}HB}$ and $I_{p\text{-}MCS}$ tests

Topology	Current Calculation	α	β	I_{min}	I_{max}	E(I)	E(I²)
Bi phase	$I_{p\text{-}HB}$	10.364	27.636	0	360	98.182	10298.792
	$I_{p\text{-}MCS}$	9.4285	19.197	0.77871	296.03	98.0260	10259
Three phase	$I_{p\text{-}HB}$	10.364	27.636	0	360.00	98.182	10298.792
	$I_{p\text{-}MCS}$	11.432	30.288	-5.066	371.69	98.1805	10299

The beta PDFs from the I_{p-HB} and I_{p-MCS} are compared in each topology using the beta PDF parameters shown in table D-2. From table D-2, it can be observed straight away that the values of E (I) and E (I^2) are within very small error of each other.

Using Matlab, the beta PDFs are compared and are shown in figure D-3. Figure D-3 shows that in both bi phase and three phase networks, the beta PDFs overlap each other and therefore both methods of current calculation agree. This means that the current calculation used in the HB method is valid.

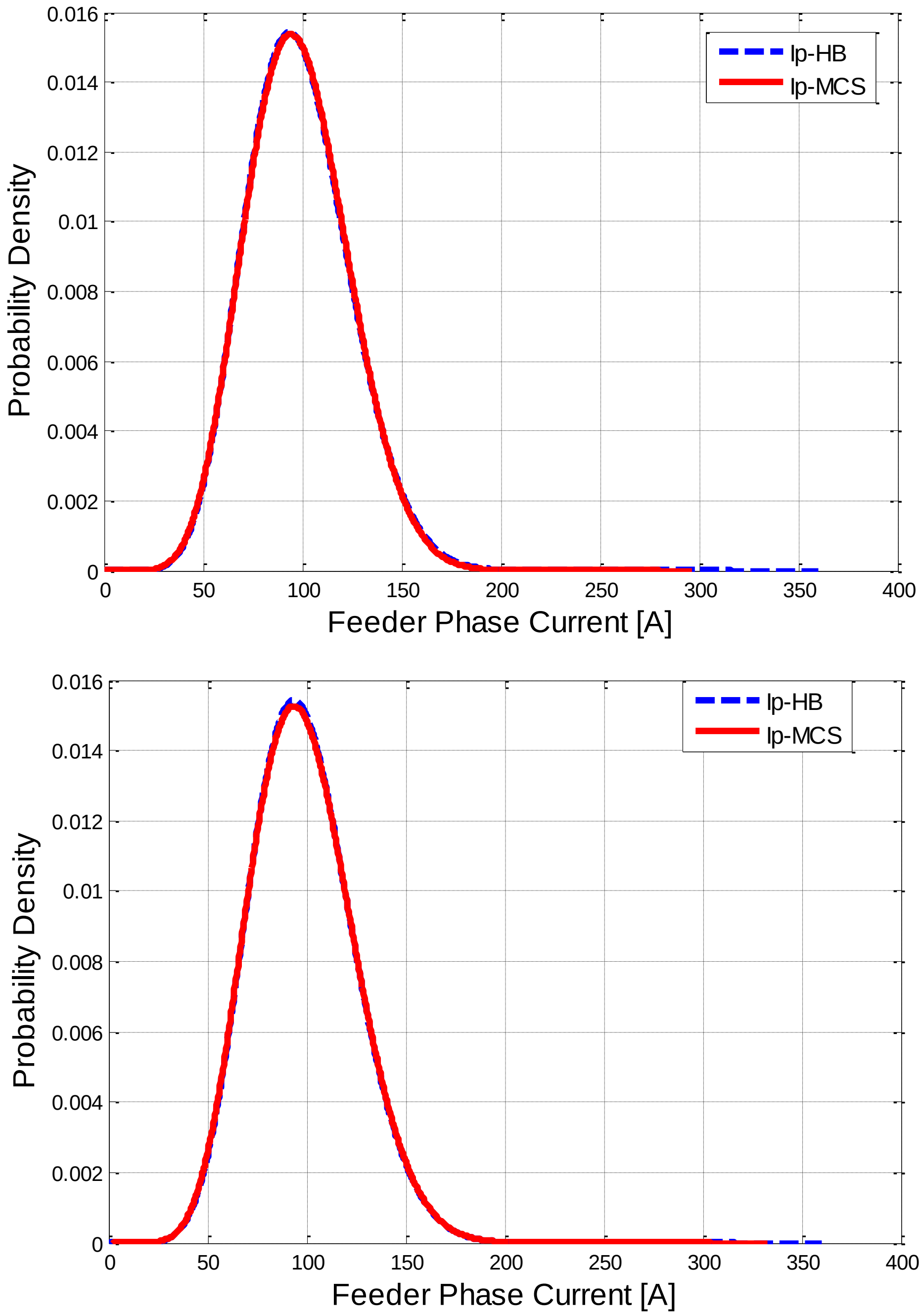


Figure D-3: Beta PDF comparison for in bi phase (above) and three phase (below) network topologies

D.3.3 Current Calculation in active LV feeders with DG

i. Test Feeder Configuration

The test feeder configuration used in this section is shown in figure D-4 and the feeder parameters/characteristics are shown in table D-3. The test feeder configuration, as seen in the HB spreadsheet is shown in figure D-5.

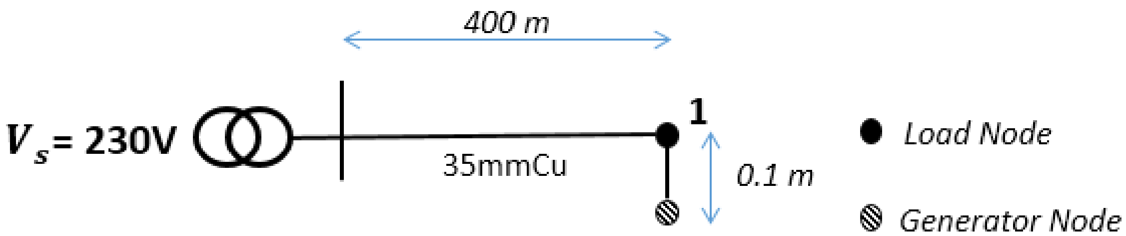


Figure D-4: $I_{a\text{-}MCS}$ *Test Feeder*

Node	No Consumers		Load Parameters			Conductor	
	Red ma	Blue mb	Alpha	Beta	Cb [A]	Length [m]	Cable Code
1L	3	3	1.500	4.000	60	400	35mmCu
1G	- 3	- 3	1.500	4.000	60	0.1	35mmCu

Node				Load Parameters			Conductor Details	
	Red ma	White mb	Blue mc	Alpha	Beta	Cb [A]	Length [m]	Cable Code
1L	3	3	3	1.500	4.000	60	400	35mmCu
1G	- 3	- 3	- 3	1.500	4.000	60	0.1	35mmCu

Figure D-5: Bi phase (above) and three phase (below) network test feeder configuration

Table D-3: $I_{a\text{-}MCS}$ *Test Feeder Configuration*

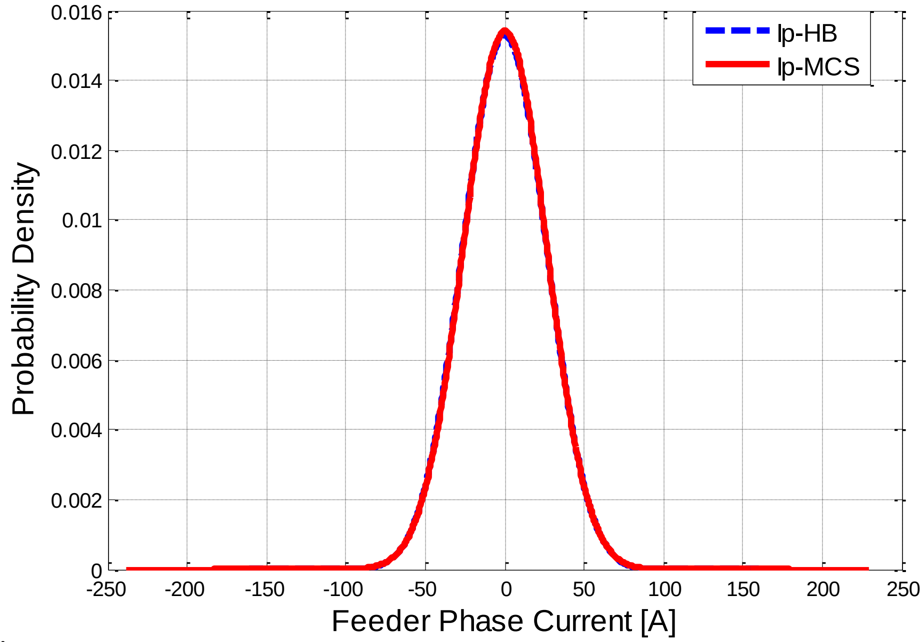
FEEDER PARAMETER	Bi phase/Three phase Network Topology		
Number of nodes	2 [3 customers and 3 generators per node]		
Inter-node length [m]	200		
Load (abc parameters)	α = 1.5	β = 4.0	c = 60
Temperature [°C]	T_1=20; T_2=40		
Feeder Conductor	35 mm^2 Copper conductor (35mmCu)		

ii. Result

The results of the current calculation for both bi phase and three phase networks is shown in table D-4 comparing $I_{a\text{-}HB}$ and $I_{a\text{-}MCS}$. The beta PDFs from the $I_{p\text{-}HB}$ and $I_{p\text{-}MCS}$ are compared in each topology using the beta PDF parameters shown in table D-4. From table D-4, it can be observed straight away that the values of E (I) and E (I^2) are within very small error of each other.

Table D-4: Result data Comparison for $I_{p\text{-}HB}$ and $I_{p\text{-}MCS}$ tests

Topology	Current Calculation	α	β	I_{min}	I_{max}	E(I)	$E(I^2)$
Bi phase	$I_{a\text{-}HB}$	24.0781	24.0781	-180	180	0	659.12
	$I_{a\text{-}MCS}$	41.77	40.171	-238	229.17	0.14	657.65
Three phase	$I_{a\text{-}HB}$	24.0781	24.0781	-180	180	0	659.12
	$I_{a\text{-}MCS}$	245.45	206.62	-592.93	499.06	-0.05	659.17



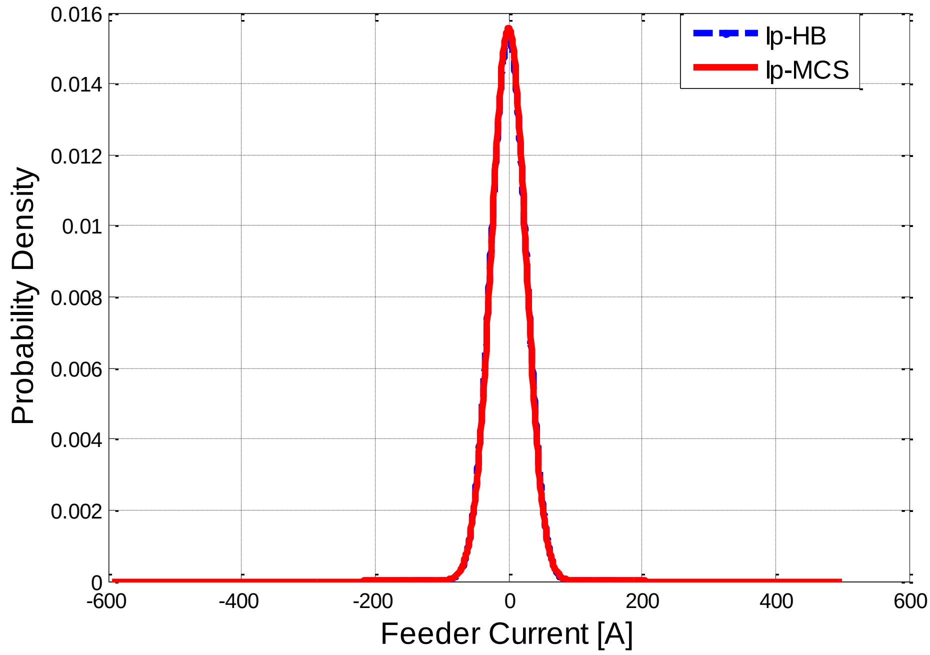


Figure D-6: Beta PDF comparison for in bi phase (above) and three phase (below) network topologies

The results from figure D-6 show the overlapping of the beta PDFs from the $I_{a\text{-}HB}$ and $I_{a\text{-}MCS}$. This means that the current calculations for both topologies are valid as both $I_{a\text{-}HB}$ and $I_{a\text{-}MCS}$ agree.

D.3.4 Conclusion

The current calculation in the HB method is a valid approach for both bi phase and three phase network feeders – for both passive LV feeders and active LV feeders with DG. $I_{p\text{-}HB}$ and $I_{a\text{-}HB}$ provide a useful check to ensure the current rating in feeder conductors is not exceeded, as a measure of caution. This allows the feeder designer know when a larger feeder conductor is needed instead of a smaller conductor i.e. when there is a high chance of exceeding the current rating of the smaller conductor for a particular feeder design. Further extension of the ideas used in this study can be explored such as analysis of DG penetration with the feeder current as a parameter using the HB method. However, unlike the voltage parameter used in this study, this analysis will involve the use of derating factors, temperature and other important factors that affect feeder conductor thermal capacity. A similar output scatter plot can be produced like in the voltage analysis.