The Representation of Geometric Concepts in Grade Six Mathematics Textbooks: A Socio-cultural Analysis

By

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Abstract

In this study Vygotsky's theory of concept formation is used to gain insight into the representation of geometric concepts in grade six mathematics textbooks. According to Vygotsky, higher order mental functioning is developed when scientific and everyday concepts are dialectically linked. The textbook as a mediating artefact has an important role to play in the process of concept formation. To determine if the geometry of space and shape is presented as scientific concepts in the text, content analysis of three randomly sampled grade six mathematics textbooks were conducted. The analysis included an investigation of what misconceptions exist in the text and what level of cognitive demand is required by the textbook tasks and activities. The initial structure for the research was derived from Valverde et al. (2002)'s methodology after which frameworks for the content analysis were developed for the distinction between everyday and scientific concepts, the identification of misconceptions and the classification of levels of cognitive demand. The results not only showed a low prevalence of scientific concepts, but also high incidences of misconceptions put forward in the textbooks. There were few textbook tasks and activities that required problem-solving, thereby limiting the learner to lower order thinking.

Declaration

I declare that:

The Representation of Geometric Concepts in Grade Six Mathematics Textbooks: A Socio-cultural Analysis

is my own work and that all the sources I have used or quoted have been acknowledged by means of complete references in APA format.

Marelize Barnard

02 April 2015

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Acknowledgements

> "Hence, we may say that we become ourselves through others and that this rule applies not only to the personality as a whole, but also to the history of every individual function"
>
> Vygotsky (1966, p. 43)

Heartfelt thanks to all the truly great teachers whom I have had the pleasure of knowing over the many years. You have inspired me and gave me the great gift of an education. A special thank you to my teacher and supervisor, Dr Joanne Hardman, who believed in me from the start, supported me and unlocked a whole new world for me. I owe my gratitude to my friends and family, who have each in their own way contributed towards what is written in these pages. Finally, thank you, JE, for being there at the end.

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shapes (Roux, 2005; Atebe, 2008; Cooper, 2010). Learners typically perform worse on geometry related tasks than on algebraic tasks (De Villiers, 2011).

It can be argued that the ubiquitous use of textbooks by mathematics teachers will play a role in how the teaching and learning of geometry takes place. Teachers have been found to follow the textbook very closely and when it comes to the teaching of geometry they rarely present topics that are not in the textbook (Suydam, 1985). How content is presented in textbooks will inform how teachers present it (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). In light of this, a pertinent issue to explore is how the textbook makes the content available. And since geometry seems to be a particular weakness amongst learners, it is appropriate to scrutinize the textbooks' handling of this topic especially.

In the South African context, mathematics textbooks have been evaluated mostly in terms of levels of distribution, compliance with the curriculum, use in the classroom and its use at home (Outhred et al., 2013). Although these issues are important, the content of mathematics textbooks needs to be explored to ascertain how the representation of the concepts assists in learning. Although having a textbook is imperative to the learning process, it is of little help if that textbook is not well written and instructionally sound to guide the learner towards understanding. In some instances the content of mathematics textbooks in South Africa has been analysed (e.g. Bowie, 2004), but analysis that focusses on geometry specifically in primary school textbooks has not been performed.

The learning of geometry leads to the development of integral skills such as visualisation, critical thinking, deductive reasoning, problem solving and logical argument (Jones, 2002). These are considered to be higher order cognitive skills. The development of these higher order functions, according to Vygotsky, can only take place through acquiring understanding of scientific concepts (Karpov, 2003). Therefore, instruction in geometry needs to be geared towards the acquisition of scientific concepts in order for learners to develop cognitively. This study therefore sets out to investigate the ways in which geometric concepts are presented in textbooks and how the textbook acts as a mediator in the learning process.

Chapter 1: Introduction

Introduction

> "Especially for early childhood, geometry and spatial reasoning, form the foundation for much learning of mathematics and other subjects."
>
> Clements, 2004, p. 267

Geometry is one of the key components of the mathematics curriculum. After its absence from the curriculum as a compulsory topic over the past few years, it has been reintroduced in the CAPS curriculum up to grade 12. In primary schools emphasis is placed on learners developing an understanding of the properties, relationships, orientations, positions and transformations of two and three-dimensional shapes (Department of Education, 2011). However, this is easier said than done and research continues to focus on the difficulties that learners experience in grasping the concepts involved with learning geometry (e.g. Feza & Webb, 2005; Atebe & Shäfer, 2013; Mhlolo & Shäfer, 2013).

Background to the study

Mathematics textbooks play an integral role in the South African educational context. For most teachers the textbook is often the primary source of learning materials as it serves as a translation of the curriculum and as a guide to the content that needs to be covered in a specific school subject (Schmidt & McKnight, 2012). In South Africa the issue of textbooks and their provision have received a lot of attention in recent years, highlighting how important a resource it is considered to be.

The issue of South Africa's poor performance in mathematics has been well established. In 1995, 1999, 2002 and 2011 South Africa participated in the Trends in International Mathematics and Science study (TIMSS). Although there has been an improvement in results, South Africa's scores remain in the bottom three countries out of 42 (Reddy, 2012). One of the content domains assessed by the TIMSS is learners' understanding of geometric shapes and their ability to use spatial visualisation skills, i.e. geometry (Mullis, Martin, Ruddock, O' Sullivan & Preuschoff, 2009). Studies have found that learners experience particular difficulty when it comes to geometric content topics such as the naming of shapes and the understanding of the properties of

Research questions

The focus of this study is to provide a description of the representations of the geometry of shapes in grade six textbooks. Textbooks have many features and this study is less concerned with physical features than with the textual content. As a result, the analysis was narrowed down to three key focus points. The first is to analyse the geometric concepts in the textbook in light of Vygotsky's theory of concept formation. Grade six is an important stage of schooling. It is the final year of the Intermediate Phase before learners progress to the Senior Phase where they will face higher cognitive demands. The acquisition of scientific concepts determines the development of higher order thinking.

Secondly, it will investigate alongside the presence of scientific concepts, whether the textbook supports any misconceptions as it is often the case that the textbook is a source of misunderstandings (Kajander & Lovric, 2009). Finally, the tasks and activities will be analysed for the level of cognitive demand they pose to the learner. Activities and tasks are main features in many textbooks and perform a valuable function in leading the learner towards understanding and application thereof.

The three focus points of the study are key aspects in the development of spatial reasoning abilities. More importantly the features of the textbook (concepts, misconceptions and activities) play a key role in how the textbook facilitates the process of mediation of learning in the indirect presence of a more knowledgeable other. The textbook as a product of the prevailing culture needs to assist the learner in progressing through the zone of proximal development towards understanding and independent functioning.

In summary, the research questions are:

1. How are the scientific concepts of the geometry of space and shape presented in the textbook in relation to Vygotsky's theory of concept formation?
2. What typical misconceptions as identified from the literature are present in the textbooks?
3. What level of cognitive demand do the activities in the textbook require so as to assist in the development of geometric concepts?

Rationale for the study

My interest in this study is twofold. Firstly, as a Mathematics educator, it has been my experience that learners find geometry particularly challenging. Secondly, I have interacted with a number of textbooks and learning materials and have often found them inadequate and in need of supplementation. I have found learners to be very reliant on the textbook as a basic resource and for use when they are outside of the classroom.

Furthermore, this study is important in that it provides a way of looking at textbooks through a socio-cultural theoretical lens, especially through the theory of concept development. What is actually written in a textbook? Do we as educators blindly follow the text, or do we sometimes stop and think critically about how the information is presented and how we in turn relay this information to our learners? If educators do follow the text closely, then what is the nature of the concepts provided in the textbook? If we want to use the text to support learning, then the text needs to be adequate in presenting concepts to learners, especially in an area such as geometry where learner confusion is so prevalent (Mhlolo & Shäfer, 2013). A study such as this one will begin to add insight into how concepts are presented in textbooks.

The reasons for learners' poor performance on geometric tasks can to some extent be attributed to contextual factors (e.g. Van der Sandt & Nieuwoudt, 2003; Van der Sandt, 2007), but the learning of geometry in the secondary school especially enjoys much attention (e.g. Bowie, 2004; Mogari, 2003; Hulme, 2012). There are very few existing studies about the teaching and learning of geometry in the primary school (e.g. Feza & Webb, 2005; Shäfer, 2010 and Khembo, 2011). To address the problem of learners' poor performance in the secondary school, one needs to take a closer look at how geometry is learnt in the primary school. De Villiers (2010, p. 7) makes a strong case in this regard:

> "It seems clear that no amount of effort and fancy teaching methods at the secondary school will be successful, unless we embark on a major revision of primary school geometry… The future of secondary school geometry thus ultimately depends on primary school geometry."

With the renewed focus on geometry in secondary school, more research in the primary school is needed to investigate how learners fare in acquiring the foundational thinking required for formal proof in the senior grades. Children in the early years acquire the concepts of space and shape – the foundations of geometric thinking – in specific ways and they have to develop the ability to recognise and categorise a wide variety of shapes (Shäfer, 2010). The role that the textbook plays in this learning has enjoyed very little attention and an analysis of mathematics textbooks can not only provide new insight into textbook learning material, but also steer teachers, authors and publishers towards developing content that leads to and supports concept formation.

Research methodology

Three textbooks for the study were selected from a catalogue provided by the Western Cape Education Department (Western Cape Education Department, 2014). Due to the extent of geometry covered in grade six and the space limitations of this study, it was decided that the content that deals with properties of two dimensional shapes will be the focus of the analysis.

There are a number of other reasons why this topic has been chosen over others. Children encounter shapes in their everyday lives and so form certain prototypes of shapes. These prototypes are culturally determined. Teaching about shapes will need to lead to conceptual changes in these prototypes and studies have shown that this is a particularly difficult task (Clements, 2004). Learning about shapes prepares learners to learn other more sophisticated geometry (Lesh & Landau, 1983) and the important concepts acquired when learning about shapes leads to building logically controlled structures that aid problem-solving and reasoning (Fischbein, 1993).

Given that the goal of the study is to determine whether geometric concepts of space and shape are presented as scientific concepts, content analysis of the text was undertaken. Valverde et al.'s (2002) TIMSS study and analysis of textbooks had a major influence on the methodology of this study. The text blocking procedure they used in their study provided a necessary structure to the content analysis. Analytical frameworks were developed to analyse the three research questions. These frameworks along with coding rules for the text were used to analyse and code the data for the identification of scientific concepts, for the misconceptions in the text and for establishing the level of cognitive demand required to complete respective activities. Quantitative and qualitative analysis was performed on the data that revealed certain findings and conclusions. A further detailed discussion of the methodology follows in Chapter 3.

Structure of the report

The report is divided into five chapters. Chapter 2 provides the theoretical framework and the literature review for this study. Vygotsky's theory of socio-cultural development, with specific focus on the theory of scientific concept formation is outlined. Alongside this, theory on the learning of geometric shapes and the development of visual spatial reasoning ability is presented.

The literature review looks at other studies that have focussed on textbook content analysis and geometry learning. Socio-cultural approaches to textbooks and other studies that have applied Vygotsky's theory of concept formation to the development of geometric ideas form part of the discussion. Chapter 3 shows the methodology for the study and in Chapter 4 the quantitative and qualitative findings are outlined. The study concludes with the discussion, limitations and conclusion in Chapter 5.

Chapter 2: Theoretical Framework and Literature Review

In this chapter I will set out the theoretical framework and related literature that guided my research. The framework is based on Vygotsky's theory of socio-cultural learning and development, and specifically on his theory of concept formation. This theory will assist in determining whether geometry is in fact presented in a scientific way in the sampled textbooks. I will further discuss the geometric concepts of shape and the development of visual spatial reasoning. I will review international as well as South African literature on textbook content analysis, the use of sociocultural theory in textbook analysis and how Vygotsky's theory of concept development has been applied to the learning of geometry.

Socio-cultural theory and the work of Vygotsky

Central to Vygotsky's theory, is the fact that human development is socially determined by the individual's interaction with the prevailing culture (Davydov & Kerr, 1995). Culture is created historically and is exemplified in different sign and symbol systems. Signs and symbols play a key role in the individual's internalisation of the culture. Signs and symbols are considered by Vygotsky as carriers of the culture and represent "the ideal" (Davydov & Kerr, 1995). So, on the one hand there is the cultural historical development of the collective consciousness and on the other the development of individual consciousness. The internalisation of the collective forms the individual's consciousness (Davydov & Kerr, 1995). Davydov suggests that this outline of Vygotsky's theory should be the theoretical basis by which education can be organised.

The individual's internalisation of the collective consciousness takes place through a process of collaboration between a child and an adult, or more knowledgeable other. The basis of this collaborative process is found in Vygotsky's general genetic law:

> "Every function in the cultural development of the child appears on the stage twice, on two planes. First, on the social plane, and then on the psychological; first, between people, and then, inside the child".
>
> (Vygotsky, 1978, p. 57).

The general genetic law is the foundation of Vygotsky's notion of mediation. All knowledge is based in the social environment, and it is through interaction with that environment that humans acquire knowledge and make it their own. According to Vygotsky, an object of knowledge is never directly approached by a child but instead, the child needs the guidance of a more skilled or knowledgeable "other".

Semiotic mediation

Mediation was seen by Vygotsky as the main determining factor in development of higher order mental functions (Karpov, 2003a). Higher order mental functions are not established spontaneously but need to be developed over the course of our lifetime. They depend on the personal mastery of culturally situated tools and the use of these tools as inner psychological functions (Bodrova, 1997). Tools can include symbolic artefacts such as signs, symbols, texts and formulae (Daniels, 2001). Mathematical symbols, diagrams and graphs can therefore also be considered signs. Psychological tools, also referred to as signs, are described by Vygotsky as devices for controlling mental behaviour, in other words *internal actions* (Daniels, 2001). Semiotic mediation (or the mediation of meaning) is essential to the development of higher order thinking: without understanding a psychological sign system such as geometry, appropriation of these signs can be compromised (Kozulin, 2003). Linked to the acquisition of signs is concept formation, which will be discussed later in more detail.

Mediation and artefacts

The process whereby the individual appropriates the socially (i.e. externally) situated signs and tools to become internal processes, is called internalisation. Mental activity is the product of the internalisation process. Outward activity on the other hand, as described by Vygotsky, is accomplished through the use of artefacts (Bartolini Bussi & Mariotti, 2008). Artefacts (also called tools) provide access to a community's knowledge, activities and practices (Jones, 1998). So, as Mariotti (2009) states, in the learning process a student will use an artefact in order to accomplish a task and in doing so, appropriate particular subject knowledge. Meaning, according to Mariotti, is derived from the use of the artefact in accomplishing the task. By using the artefact, the learner's activity in the world is transformed and changed (Jones, 1998). An artefact in an educational setting can constitute a number of objects, for example, pencils, blackboard, technology and textbooks.

the absence of an adult, can be considered as the more knowledgeable other, mediating the child's learning. A mathematics textbook as a cultural artefact will contain the written text, pictures, diagrams and so forth that constitute the signs and symbol systems of the mathematical subject area, including the area of focus for this study, namely geometry. The interaction between the student and the textbook is based on interpretations of the symbolism in the textbook- here in lies the semiotic mediation of the mathematical knowledge through the textbook and the key to a learner's development of higher order thinking. In using the textbook, the child first encounters the externally located knowledge, before appropriating the information and becoming able to use it as a tool to influence self.

It should be acknowledged that the teacher plays an important role in the process where further mediation of textbook use can take place (Rezat, 2006). However, the scope of this study is focused on only the textbook as artefact and semiotic mediator. I would like to clarify at this point that although the textbook is considered to be a mediating artefact, by some theorists referred to as a physical tool (e.g. Cole, 1996), it should not be thought of in the same way as acquiring psychological tools. Psychological tools are internally directed and will only be acquired through the learning of scientific concepts. But the textbook as mediator potentially makes these concepts available to the learner in a particular way.

Concept formation

When children learn the meaning of signs, concepts are formed (Berger, 2006). All signs have meaning attached to them and, more importantly, they embody concepts. In a typical learning situation a child will start to use mathematical signs in class, in discussion with the teacher, class mates and through interacting with textbooks. If meaning is derived from signs, and the textbook as a cultural artefact contains those signs, and acquisition of scientific concepts depends on understanding those signs, it is clear that textbooks have a cardinal role to play in the learning process. What is contained in the textbook has to be scientifically and instructionally sound for it to be of any use. What exactly Vygotsky meant by scientific concepts requires further elaboration.

Vygotsky made the distinction between everyday and scientific concepts. Everyday concepts are acquired through our natural interaction with the world and developed spontaneously through everyday activities (Hedegaard, 2007). Scientific concepts are described by Daniels (2001) as those concepts that form an ordered, logical system situated within academic subject areas. Only scientific concepts were considered true concepts by Vygotsky and all mathematical concepts fall under this label (Schmittau, 2003). Scientific concepts are consciously acquired through systematic instruction that typically takes place in schools (Karpov, 2003). Everyday (also called spontaneous) concepts lie outside of academic subject areas but are instead connected to daily, family and community life (Hedegaard, 2007). Once a child has acquired scientific concepts, they can manipulate these concepts at will and they become tools for the child's mental processes such as thinking and problem solving (Karpov, 2003b).

Although scientific concepts and everyday concepts are distinguishable, they are inextricably related. Everyday concepts form the foundation for the acquisition of scientific concepts (Karpov, 2003). Moll (1990, p.10) explains further:

> "It is through the use of everyday concepts that children make sense of the definitions and explanations of scientific concepts; everyday concepts provide the "living knowledge" for the development of scientific concepts"

We can use the example of a child taking sweets from a packet. Each time a sweet is taken out of a packet there are fewer left in the packet as more are being eaten. This enables the child to develop an everyday concept of "more" and "less" which forms the basis of the mathematical concepts of addition and subtraction. Once scientific concepts have been acquired they transform the everyday concepts in that they give structure and form to everyday concepts (Karpov, 2003b). Once the child in our example has learnt to count, add and subtract it has changes her everyday notion of "more" and "less". She might decide to eat fewer sweets at a time, saving some for later.

According to Vygotsky, scientific concepts cannot be acquired without what he called *verbalism* (Vygotsky, 1986, as cited in Karpov, 2003b). With this he meant that in order to understand scientific concepts, one needs to understand the words that signify the concept. Vygotsky contended that all scientific concepts originate from verbal definition (Miller, 2011). These verbal definitions are provided to the child during formal schooling. For example, the word "differentiate" has no mathematical meaning if one does not understand the concept of calculus that underlies this specific operation. Clarifying the meaning of words therefore has important implications for textbook content.

A further implication of Vygostky's notion of scientific concepts is that the learner should be allowed the acquisition of scientific concepts in such a way that he or she gains control over these concepts. By presenting concepts only as a series of facts to be memorised the learner will never gain control over them and will as a result become substantially handicapped in using them as tools to enhance thinking. However, this does not mean one should assume that the emphasis placed on the acquisition of scientific concepts leads to the exclusion of procedural knowledge. Often procedural knowledge is considered meaningless by learners but in most cases the procedural knowledge cannot be divorced from the scientific knowledge (Karpov, 2003b). Thus, in order to bring about generalisability and flexible use of scientific concepts by students, the textbook should present learners with both procedural and conceptual knowledge.

Theoretical and empirical knowledge

At this point it is necessary to draw a distinction between empirical and theoretical knowledge which is an important elaboration of Vygotsky's theory of concept formation. Empirical knowledge can be thought of as the learning of factual knowledge: abstract concepts are arrived at based on the obvious common characteristics of objects without considering that the common characteristics on which the classification was based are not necessarily present in all objects of a certain class (Karpov, 2003b). Schools often choose large numbers of objects, put them together in groups and then make comparison between the objects to detect common, stable characteristics and to create a general idea (Vianna & Stetsenko, 2006). In so doing, the concept that is formed is based on the characteristics instead of the objects. The characteristics become the concept. An example would be where a textbook presents quadrilaterals as an assortment of shapes to be recognised and classified without clarifying what the concept of a quadrilateral entails. The child could identify a rectangle, because this has been memorised by rote, but does not understand why it is a rectangle, or that it is in fact a special parallelogram. If then presented with another four sided shape, he or she might name that as a rectangle too believing that all four sided shapes are rectangles. As illustrated, the acquisition of empirical knowledge can lead to errors- concepts are acquired but they are not necessarily accurate scientific concepts.

In contrast, theoretical knowledge is based on the acquisition of ways to perform scientific analysis of objects in different subject areas (Karpov, 2003b). It requires scientific thought that leads to constructing theoretical abstractions and the analysis of the interconnection and internal relations between things (Vianna & Stetsenko, 2006). Learning theoretical knowledge involves identifying the *essential characteristics* of an object and representing them in the form of symbolic or graphic models. In the example of the child learning to classify shapes, if she learnt what the essential characteristics of quadrilaterals were, she would be much more adapt at sorting and classifying them because her understanding would be based on a theoretical knowledge of the concept of the shape.

Therefore, it is essential that the textbook focuses on the presentation of theoretical knowledge instead of empirical knowledge as this leads to the development of scientific concepts (Karpov, 2003b). This in turn leads to the learner being able to utilise these concepts as tools, which ultimately leads to the learner's mental development. Theoretical generalisation is essential for the appropriation of scientific concepts (Schmittau, 2003).

Linking the scientific and the everyday

Since scientific and everyday concepts are closely linked, the textbook should be written in a way that brings these together. Fleer (2009) argues that if children learn scientific concepts without establishing links to their everyday experience, these concepts become meaningless to the child. Hedegaard (1990) calls the linking of the everyday and scientific *the double move* in instruction. For Vygotsky the abstract or scientific concept should always lead the everyday concept (Hedegaard, 1990). So, the textbook can start off with presenting general laws of the subject matter to learners, in other words abstract concepts are introduced first before they are related to concrete objects that fall within the child's everyday knowledge. The concrete examples are used to demonstrate the general concepts and laws as far as possible (Hedegaard, 1990). In turn students need to be engaged in activities that are relevant to the subject matter being presented, the developmental level of the child and to his or her circumstances. Relating scientific concepts to everyday concepts makes it possible for the child to connect what they learn from the textbook to what they see in the world around them (Hedegaard, 1990). We can return to the previous example of the child eating sweets to illustrate the double move. She reads about and learns the general laws of addition and subtraction in mathematics from the textbook, after which it is related to the concrete concept of eating sweets that lie within the children's everyday knowledge. The eating of sweets can be used to demonstrate how addition and subtraction works. The double move and acquisition of scientific concepts are linked to teaching in the zone of proximal development, which will be discussed further in the next section.

To summarise so far, the textbook as a cultural artefact mediates the learning of mathematical knowledge (including geometry) by making it available to the learner through signs and symbols. Scientific concepts, as the basis for the development of higher order functions, are acquired through learning theoretical knowledge, which should be the focus of the textbook. The development of conceptual knowledge and the acquisition of mental tools play out in the process of mediation in the zone of proximal development. I will now turn to outlining how the textbook can be considered to advance learners through the zone of proximal development.

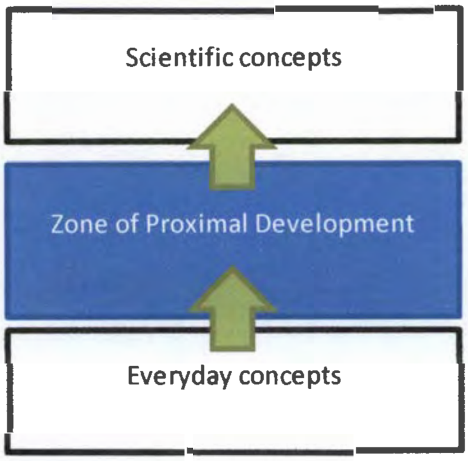
The zone of proximal development

The zone of proximal development (ZPD) is arguably Vygotsky's most influential idea (Moll, 1990). The definition of the ZPD is that it is the difference between what a child is capable of doing on his or her own, compared to what he or she can do with the assistance of a more skilled person (Chaiklin, 2003). To elaborate, it is the difference between processes and concepts that have already been developed in the child, and the concepts that are still on their way to being developed (Moll, 1990). Therefore, the ZPD can be understood as the *potential* for learning, in other words, what the child is prospectively capable of achieving if provided with the correct support and instruction.

Assistance through the ZPD requires the presence of a more knowledgeable other, providing support in relation to the child's learning and development (Chaiklin, 2003). What is important to note is the role that instruction plays in movement through the ZPD. Through instruction, children can achieve more and perform at higher levels than what they can do on their own (Miller, 2011). Just as this is true for a teaching situation where an adult is present to provide assistance, so it is true for the child engaging with a textbook. By being presented with new and novel information in the textbook, the zone of proximal development for a child will open up. The textbook, through various forms of instruction such as explanations, summaries, questioning and so forth can guide the child towards unassisted performance.

Developing the child's higher order mental functions is frequently described as a collaborative effort between teacher and child. In this case it can be argued that a child engaging with a textbook (bearing in mind that the textbook acts as the more knowledgeable other) is a similar collaborative process. The ZPD should, however, be understood as not just the transfer of skills from the more knowledgeable other to the child, but as the appropriation of tools by the individual (Moll, 1990). The goal of learning in the ZPD is to allow the child to internalise the mental tools that are presented in such a way that he or she can use it independently without the assistance of another. In the previous section it was argued that the acquisition of scientific concepts is key to the development of higher order mental functions. The learning of scientific knowledge will take place in the ZPD and the instructional goal of teaching and learning needs to be the development of scientific concepts that lie within the child's ZPD (Hedegaard, 1990). The figure below shows the progression that takes place from everyday concepts, through the ZPD to the attainment of scientific concepts:

Figure 3: Scientific concepts in the ZPD



The goal of assistance in the ZPD is the development of higher order mental functions and as a result, assistance in the ZPD has to be developmentally focused (Gallimore & Tharp, 1991). This means that it should lead to the advancement of those psychological functions that are in the *process of maturing*, instead of focusing on existing mental functions (Daniels, 2001). It is therefore the task of any textbook to direct its actions towards ensuring the attainment of this goal. It should be acknowledged that the textbook, due to its static presentation of knowledge, is limited in addressing every child's ZPD individually. In the absence of instructional conversations (Gallimore & Tharp, 1991), it cannot acknowledge each child's ZPD and customize instruction accordingly. However, it is a reasonable assumption that all knowledge presented in a particular textbook, for a particular grade level, is within that child's ZPD.

How does assistance in the ZPD take place in light of textbook usage? To illustrate, we can imagine a typical situation where a child is reading about novel information in a textbook. The textbook guides him or her towards an exercise. At first the child is confused about what is required and the zone of proximal development is opened up. The textbook in turn provides an explanation of the concept, refers back to previous knowledge, or models a solution by means of a worked example in order to guide the learner through the task. The textbook might also have reminders or side notes drawing the learner's attention to important aspects of the concept and leading her towards independent performance of the task.

Once performance has been achieved, the child can complete the task independently without the assistance of another, in other words, the child has internalised the relevant concept. Internalisation means that what is presented to the child externally, in other words, socially, becomes personally appropriated and leads to the development of higher order mental functions (Hedegaard, 1990). Evidence of internalisation presents itself when the learner can independently work with and manipulate the concept that was presented. Internalisation indicates that the child has reached the end of the ZPD for a particular concept or task (Gallimore & Tharp, 1991).

The child is an active participant in the learning that takes place in the ZPD and will be required to perform certain activities in the textbook. The performance of tasks and activities has a very important role to play in movement through the ZPD. If the tasks are too easy, learning will remain at the everyday and the ZPD will not open up. In keeping with Vygotsky's theory, tasks must be set in advance of development to enhance learning and gain understanding of the relevant scientific concepts. The discussion will now continue by looking at an analysis of the levels of cognitive demand in textbook activities.

Level of cognitive demand and textbook activities

An exercise or activity generally is thought of as products that students are expected to deliver, the operations students are expected to employ to deliver these products and the resources available to students when they do so (Stein, Grover & Henningsen, 1996). Tasks provide the learner with a chance to apply new knowledge and to develop that knowledge into higher order thinking. In order for learners to become more proficient at mathematics, they have to be engaged in activities that require high-level cognitive thinking (Henningsen & Stein, 1997). Levels of cognitive demand are frequently used in textbook content analysis as a way to determine the potential effectiveness of the learning materials (Zorin, 2011). Textbook tasks need to be geared towards the development of problem solving and critical reasoning skills (Smith & Stein, 1998). According to Stein and Smith (1998) tasks have to provoke thinking and reasoning in intricate and significant ways.

From a socio-cultural perspective, for children to acquire concepts, meaning of these concepts needs to be actively constructed. Meaning cannot merely be passed on through direct instruction. Memorising definitions, procedures and algorithms can easily be done, but will not lead to acquisition of concepts (Steele, 2001). While children engage in an activity they actively internalise the concept. This leads to concept formation and generalisation (Steele, 2001). Textbook exercises reinforce the concepts presented in the narrative and graphical sections of the book and allow the learner to internalise these concepts (Zorin, 2011).

Smith and Stein (1998) suggest that thinking, reasoning and problem-solving can only be developed by starting with complex, high-level tasks. This is in line with Vygotsky's notion of the ZPD that instruction should be in advance of development. They argue that a task that requires a learner to rely on memorising and following procedures only lead to lower levels of cognitive demand, whereas tasks that ask learners to think conceptually, lead to higher levels of thinking. As a result they suggest that tasks can be classified into four broad categories, namely:

- Memorisation;
- Procedures without connections to concepts and meanings;
- Procedures with connections to concepts and meanings;
- Doing mathematics.

They expanded the above four categories into a framework that can be used for analysing tasks to determine the kind of thinking that a learner is expected to engage in. The complete framework can be found in Appendix A and will be used in the analysis of tasks in this study. Due to the focus of this study on the cognitive aspects of geometry learning in textbook, it is necessary to outline the theory that will inform the study in this regard. This will be done in the next section of this chapter.

Geometry of shapes and the development of visual spatial reasoning

In the teaching and learning of geometry, the goal of instruction is the development of visual spatial reasoning. The importance of the development of this cognitive function cannot be underestimated, not just in our everyday world, but also in learning mathematics:

> "All cognition has a visual spatial dimension, indeed our minds and cognition are always essentially embodied and hence spatially mediated... Visualisation allows one to access spatial, embodied intuitions, and these are essential in all mathematical thinking, especially in number and algebra."
>
> Ryan and Williams (2007, p. 80)

Visual spatial reasoning is highly correlated with success in mathematics (Hegarty & Kozhevnikov, 1999). In geometry, objects such as lines and shapes are studied and spatial reasoning involves the building and manipulation of mental representations of these objects (Clements, 1998). From an early age children are introduced to shapes such as triangles, squares, rectangles and circles. In the early years, it is typical instructional practice to allow learners to compare, sort and group shapes based on surface features (Ryan & Williams, 2007). After the child has successfully achieved this, they move on to naming the features of shapes and to classify them.

Common geometry misconceptions

Much of what the child learns about shapes is based on perception and visual prototypes (Clements, 1998). This is typically brought about by, for example, showing triangles and rectangles with horizontal bases only and children often disregarding the shape if it is shown in a different orientation.

Developing prototypes is typically as a result of reasoning about shapes based on visual aspects only. For children to develop proper understanding of shapes they would need to consider the essential and distinguishing characteristics of shapes, based on concepts of sides, lengths, width, perpendicularity, parallelism and straightness (Monaghan, 2000). Here it is helpful to refer back to Karpov's notion of theoretical knowledge and Hedegaard's view that theoretical knowledge is imparted through guided discovery. Exploring shapes along these lines will lead to understanding shapes as scientific concepts.

Fischbein (1993) makes a valuable point in that much of the difficulty with geometric concepts have to do with the fact that they are not just mental concepts, but also figural concepts and that a geometric concept includes the mental representation of a space property. Often too much emphasis is placed on the visual aspects of the concept. Fischbein and Mariotti (1997) propose that a distinction needs to be made between the figural and conceptual parts of a geometric concept. The figural aspects have to do with the visual representation of the geometric concept and the conceptual aspect refers to the abstract and theoretical nature that the geometric concepts have in common:

> "What characterises a concept is the fact that it expresses an idea, a general, ideal representation of a class of objects, based on their common features. In contrast, an image (we refer here to mental images) is a sensorial representation of an object or phenomenon.
>
> Fischbein (1993, p. 140)

How can one bring across the conceptual aspects of shapes and move learners beyond the visual? Clements (1998) suggests that as a way to overcome this, children need to be shown many examples and non-examples of shapes so as to help them understand those characteristics that are mathematically relevant and those that are not. He also suggests that learners are asked to say why a shape is in a particular category or not. Often classes of shapes are misrepresented or there is a failure to show connections between different shapes and their classes (Fuys, Geddes & Tischler, 1988; Clements & Battista, 1992). This has been illustrated (especially in grade six level mathematics) where learners are not shown the connections between parallelograms, rectangles and squares (Clements, 1998).

Another frequent misconception that arises is failing to classify a square as a rectangle. This results in what Monaghan (2000) calls a standard *one-to-one object word match* that causes children to form rigid ideas about shapes that fails to allow for generalisations. De Villiers (1994) argues that if subsets of concepts are presented as disjointed from each other, then a partition definition of concepts are formed. A good example of such a partition definition is when rectangles and squares are presented as disconnected concepts and not classified as subsets of parallelograms and squares not as subsets of rectangles. When this happens, rectangles and squares are seen as static objects and the learner cannot form economical definitions of shapes. Teaching the properties inherent in the shape and related to the definition of it, will bring about the beginnings of abstraction and generalisation As a result children will be able to learn how to categorise shapes based on an understanding of the inherent class differences between the categories. Orientation of a shape and the size of a shape are not essential characteristics – a triangle with three straight sides and three angles remain a triangle regardless of it position, size or orientation.

In summary, the textbook, in its presentation of shapes as scientific concepts, needs to do the following:

- It is a reasonable expectation that most children will hold a working knowledge of shapes- in other words they will have everyday concepts of shapes. The textbook should therefore build on the everyday concepts and move beyond it to develop scientific concepts of shapes.
- Move the child beyond visual prototypes by means of examples and counter examples so as to develop the visual as well as conceptual aspects of the geometric concept.
- Present shapes as theoretical knowledge where the essential characteristics of shapes are made clear so as to build the scientific concept of shapes.
- Present properties of shapes and how the properties relate to the definition of the shape by using appropriate and correct mathematical language.

Literature review

Literature were analysed pertaining to the central themes of the study namely: difficulties with the learning of geometric concepts; the support textbooks provide in developing conceptual understanding; South African studies of mathematics textbooks; the use of sociocultural theory in textbook analysis and how Vygotsky's theory of concept development has been applied to the learning of geometry.

Learning and teaching of geometry

Historically it has been shown that learners struggle with learning geometry concepts. Of the mathematics topics assessed in the TIMSS study, South African children were found to perform the worst on geometrical tasks (Howie, 2001). A large body of international work has resulted to

The work of Smith and Stein (1998) has been influential in the analysis of the level of cognitive demand required by textbook activities. Those studies that have investigated textbook tasks using their framework have delivered similar results, showing that the majority of tasks in textbooks require a low level of cognitive demand (e.g. Jones & Tarr, 2007; Boston & Smith, 2009; Zorin, 2011).

Studies of textbooks in South Africa

In South Africa there has been some focus on textbook analysis, but mostly studies have looked at contextual issues such as how textbooks have presented diversity in terms of race, gender, social class and disability (Bowie, 2004; McKinney, 2005). In a large scale study by Outhred et al. (2013) the use of government issued workbooks in mathematics and English was analysed. The study did not perform content analysis, but evaluated issues such as how the textbook was being utilised in class and the quality of the textbook based on design, sequencing of topics, language used and representation of diversity. Exploring the same topic of the effectiveness of workbooks, Fleisch, Taylor, Herholdt and Sapire (2011) conducted a controlled experiment to see the difference between learning from a workbook as opposed to a textbook. No conclusive evidence could be found that a workbook enhances learning as opposed to a textbook, but the unavailability of textbooks was raised as a major concern.

Ensor et al. (2002) present one of the few studies where content analysis on mathematics textbooks was conducted, showing how inductive approaches that are intended to develop concepts, are often denatured by teachers' use of the text. As a result the study also considered how teachers used the text during classroom practices. These studies paint a broad picture of the research done on textbooks in the South African context and much is still left to be explored in this area.

Socio-cultural analysis of textbooks

Few studies have used a socio-cultural lens to analyse mathematics textbooks. Harries and Sutherland (2000) conducted a comparative study of books from England, France, Hungary, Singapore and the USA of how they represent the concept of multiplication. The differences found in the books proved how knowledge is culturally based and how children will construct different meanings depending on how the information is presented to them. Hornsley and Walker's (2005) analysed the mediated use of the textbook and other learning materials by complexity in which textbooks from France, Germany and England presented the topic. These differences will result in variations in teaching and learning, the level of understanding that children will gain and the quality of the concepts that will develop.

Frequently studies have suggested that textbooks do not provide sufficient presentation of mathematical ideas and as a result the textbook can become a major source of misconception. This can be due to the textbook's use of the colloquial instead of mathematical language, incorrect generalisations, misconceived diagrams and illustrations, oversimplification of concepts and discussing concepts that have not yet been defined (Kajander & Lovric, 2009). The misconceptions mentioned earlier (namely that of prototypes and incorrect classification of quadrilaterals) have been observed in a number of studies (Burger & Shaughnessy, 1986; Monaghan, 2000; Atebe & Schäfer, 2008; Clements & Sarama, 2009).

Studies by Levin (1998), Haggarty and Pepin (2002) and Dole and Shield (2008) all showed how few connections are made between concepts in mathematics textbooks. These studies found that there is a strong emphasis in textbooks on procedural rather than conceptual understanding. Similarly, Vincent and Stacey (2008) analysed nine textbooks in Australia and found that there were many repetitive problem solving activities that displayed low levels of complexity that never moved on to higher order thinking. They suggest that textbooks seldom provided learners the opportunity to gain conceptual understanding largely due to the fact that the textbooks did not support problem solving that stimulated reflection and reasoning.

Numerous other studies support this finding. In a study by Nicely (1985), an analysis of textbook activities from recent years showed that 77% of the activities were engaging lower-order thinking skills only. Textbooks with activities that required higher order skills such as explaining, justifying, proving or solving were few and far between. In the major TIMSS study by Valverde et al. (2002) it was determined that the majority of textbook activities were basic and unchallenging. Reading and understanding of factual information with continued practice of routine tasks were prevalent. Similarly, Zorin's (2011) study of transformation geometry exercises in textbooks found that lower level demand tasks were disproportionally high to higher level demand tasks. Shield and Dole (2012) concur in their study of five American textbooks that the textbook activities provided limited support for the development of "deep learning".

teachers. The study looked at how teachers select, copy and use textbook materials in their teaching and what factors influenced their decisions. The study found that in 80% of lessons observed, teachers had spent a great amount of time on selecting, preparing and photocopying learning materials. The teachers' decisions were greatly influenced by the intended use of the material in creating ZPD's and scaffolded learning. The study demonstrated the significant role teachers play in the mediation of learning from textbooks and learning materials.

Finally, Plut and Pesic (2003) provide a theoretical frame of reference whereby the key concepts of the textbook as cultural artefact can be analysed. They argue that the textbook is a cultural product that has a significant influence on the individual's development in addition to reproducing cultural knowledge and practices. Often the influence of a cultural tool such as a textbook is considered to be good, but it can also have a negative impact on an individual's development. Plut and Pesic (2003) put forward the following key aspects of a dysfunctional cultural tool:

- Relations between concepts are non-explicated, unclear and superficial or removed from the context.
- Previous knowledge and development are ignored and therefore becomes non-operable.
- Attempts at making connections between knowledge are either too simple or too complex.
- Task demands are unclear and meaningless which lead to loss of motivation and attention
- Spontaneous knowledge and experiences are not adequately related to abstract, decontextualised school knowledge.

Concept formation as a way to investigate geometry learning

The use of Vygotsky's theory of concept formation has been applied to some extent in the study of how children learn geometry. Monaghan (2000) conducted a study based on language focused mediation where learners were required to engage in concept differentiation. This involved the students being presented with pairs of related concepts and them having to analyse the differences between them. Based on the resulting cognitive conflict, learners' misconceptions were made evident to teachers and consequently the teacher could correct them.

Similarly, in the South African context Berger (2005) has argued that Vygotsky's theory of concept development has been widely neglected when considering it in relation to geometry learning. Mhlolo and Shäfer's (2013) work is of special interest. They too argued for the use of Vygotsky's theory (as opposed to that of the very popular Van Hiele theory) and applied it in their study of learner conceptual development in geometrical problems that require visual and conceptual understanding. They analysed learners' textual responses to geometry tasks and were able to determine which stage each learner was in with regards to their development of concepts. The theory proved to be useful in gaining insight into how learners formulate geometrical concepts, therefore providing a basis from which teachers can clarify misconceptions. They have suggested that further research should be done to develop this framework.

The conclusion from these studies is that there is a lack of geometric concept formation amongst children, a poor representation of concepts in textbooks and a lack of research on mathematics, and specifically geometry textbook content in South Africa. Vygotsky's theory of concept development has rarely been used to understand how children learn geometry. Research that take this view has merit and can provide new insight into this issue.

Conclusion

This chapter set out the theoretical framework that informs the study. The work of Vygotsky and his theory of concept formation, along with theory informing the learning of geometric concepts and levels of cognitive demand were set out. The chapter reviewed pertinent literature regarding the learning of geometry as well as textbook analysis. The next chapter will continue by setting out the research methodology employed during the course of the study.

Chapter 3: Methodology

The study analyses the presence of scientific concepts in grade six mathematics textbooks in relation to geometry. In this chapter the methodology and analytical frameworks will be presented. The chapter is divided into four sections. The first looks at the sample that was selected for the study, the second presents how data was collected and the third will discuss the analytical frameworks that were used to code and analyse the data. Lastly, measures of reliability are discussed.

Sample

Selection of textbooks was performed by accessing the catalogue of mathematics textbooks that are prescribed by the Department of Education (Western Cape Department of Education, no date). The catalogue prescribed eight different textbooks for grade six mathematics. Due to the limited space of the dissertation, it was decided to choose three of the prescribed textbooks for analysis.

In the absence of data on the best-selling textbooks out of the eight options, or the most frequently used, it was decided to make a random selection of textbooks. This was done through a simple random selection process conducted in Excel where each title was assigned a random number. The computer programme then performed a random sort and the first three titles from the sort were chosen. Although the textbooks are all in the public domain, it was decided not to reveal the titles and publishers of the books. This is because the sample from which the books were drawn are quite small and not representative of all books on offer (including those not on the departmental catalogue). The aim of this study is also not to compare or make recommendations, but merely to examine the text that is available to schools. Each book was assigned a number for identification. This is shown below:

Table 1: Sample selection of textbooks

	Abbreviation
Textbook 1	TB 1
Textbook 2	TB 2
Textbook 3	TB 3

Method of analysis

In order to investigate the presence of scientific concepts in the text, content analysis of the sampled textbooks were conducted. The analysis included both quantitative and qualitative aspects. The quantitative analysis was conducted so as to allow for a general overview, comparisons between aspects of the data as well as amongst the categories. The following were the main focus points:

- The number of occurrences of scientific concepts as opposed to everyday concepts were counted and grouped according to broad overall conceptual categories. Tables and graphs were used to analyse the data.
- The frequency with which misconceptions in the text occurred was recorded and analysed using percentages and graphs.
- The level of cognitive demand required by activities was classified and counted according to predetermined categories and analysed using percentages and graphical representations.

The qualitative part of the study:

- Provides a description of how geometric concepts are presented in the text.
- Depicts what common misconceptions in geometry emerged from the text.
- Explores whether the activities in the textbook promoted high levels of cognitive demand.
- Gives a general impression of the mathematical soundness of the text.

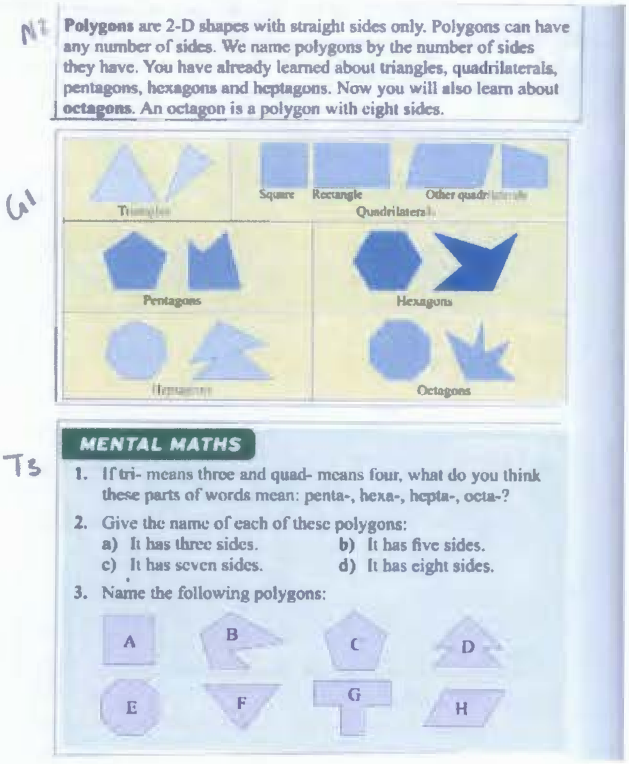
Data collection

An initial analysis of the text showed that there was variability amongst the textbooks with regards to structure and length. In a major TIMSS study of textbooks, Valverde et al. (2000) developed a method whereby the text is divided into blocks for uniform analysis of the content. In this way the variation in textbook structure, size, extent and layout is overcome and the blocks are characterised based on how the content is addressed.

In this study a total of four types of blocks were identified, in line with Valverde et al. (2000). Narrative elements were classified as all expositions, explanations, descriptions and discussions and side notes. Text that contained worked examples formed the second type of block. Task-based blocks were classified as activities, exercises, investigations and guided activities. Graphic blocks were those blocks that contained some text but mostly graphical representations such as pictures, diagrams, graphs, shapes and figures. Graphical representations were included in a separate category due to the visual nature of geometry concepts (Fischbein, 1993) and the showing of essential characteristics of shapes through pictures (Monaghan, 2000).

To ensure that all the content pertaining to the topic of the geometry of 2D shapes was covered from each textbook, the researcher examined the table of contents of each book to identify the location of each section. The size of blocks and the number of blocks were dictated by the structure of the text where changes in headings, subheadings or topic differentiated the type of block. In other words, continuous narrative parts on a specific topic were classified as such and a new block started when a change in topic took place. Exercises and activities blocks were classified as any instance where the learner was asked to perform a task. A worked example counted as a problem for which a model answer with explanations and instructions were provided. Each block was given a code based on it being narrative (N), task-based (T), graphical (G) or a worked example (WE). Blocks of each type were numbered chronologically. Figure 4 shows how the blocking procedure was done for a page from one of the textbooks:

Figure 4: Example of blocking procedure



N1

Polygons are 2-D shapes with straight sides only. Polygons can have any number of sides. We name polygons by the number of sides they have. You have already learned about triangles, quadrilaterals, pentagons, hexagons and heptagons. Now you will also learn about **octagons**. An octagon is a polygon with eight sides.

T3

MENTAL MATHS

1. If tri- means three and quad- means four, what do you think these parts of words mean: penta-, hexa-, hepta-, octa-?
2. Give the name of each of these polygons:
   a) It has three sides.
   b) It has five sides.
   c) It has seven sides.
   d) It has eight sides.
3. Name the following polygons:

A B C D E F G H

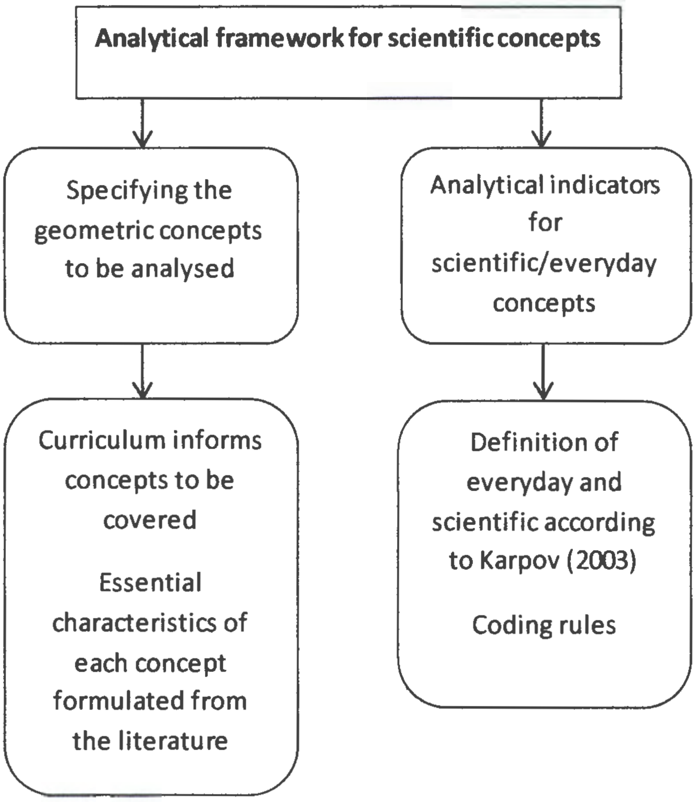
Development of an analytical framework to analyse the data

This section describes the analytical framework employed to code and analyse the data on scientific concepts, misconceptions in the text and the level of cognitive demand required in the tasks.

Scientific concepts

Analysis of the text for scientific concepts had two components to it. Two important matters had to be clarified, namely what are the specific concepts aimed at a grade six learner, the important elements of the concepts and how will they be identified as scientific concepts (or not). The process of the development of the analytical framework is shown below:

Figure 5: Development of the analytical framework



Firstly, the geometric concepts of 2D shapes learned in grade six had to be specified as these were the concepts on which the analysis would be performed. I turned to the curriculum as it guides the content of textbooks. The geometry of shapes is classified under the topic of "Properties of 2D shapes" in the CAPS curriculum for grade six (Department of Education, 2011). From there the concepts were narrowed down and the specific concepts that children have to learn in grade six were identified. These are set out in Table 2:

Table 2: Concepts to be covered according to CAPS

Properties of 2D shapes:	**Specific concepts:**
• Identify and name 2D shapes • Angles and lines in 2D shapes • Compare and sort 2D shapes • Identify and draw 2D shapes	2D shape Closed shape Triangles Squares Rectangles Circle Quadrilaterals Parallelograms Polygon: pentagon, hexagon, heptagon, octagon Angles: Acute, right, obtuse, straight, reflex, revolution

The study therefore focussed on these concepts. Although other concepts might be present in the text, only the ones set out in the table above were analysed ensuring uniformity and reliability across the textbooks. To ensure that all three textbooks do cover these concepts an initial analysis of the texts were done to check that all concepts were present. The results of the check are shown in Table 3.

Table 3: Checklist for concepts

Properties of 2D shapes:	**Specific concepts:**	**TB 1**	**TB 2**	**TB 3**
• Identify and name 2D shapes • Angles and lines in 2D shapes • Compare and sort 2D shapes • Identify and draw 2D shapes	2D shape	✓	✓	✓
	Triangles	✓	✓	✓
	Squares	✓	✓	✓
	Rectangles	✓	✓	✓
	Circle	✓	✓	✓
	Quadrilaterals	✓	✓	✓
	Parallelograms	✓	✓	✓
	Polygon: pentagon, hexagon, heptagon, octagon	✓	✓	✓
	Angles: Acute, right, obtuse, straight, reflex, revolution	✓	✓	✓

The second aspect of the analysis of the scientific concepts involved developing indicators for when a concept could be classified as scientific. Vygotsky's definition of a scientific concept requires that the essential characteristics of the object being study are made clear to the learner. Analysing the text for scientific concepts therefore required identifying what the essential characteristics of a particular concept are and if it was made explicit in the text. As a result, the essential characteristic of each geometric concept being studied had to be identified and these are set out in Table 4. It should be noted that more formal and expansive mathematical definitions of the geometric concepts exist, but the definitions provided below are appropriate for young learners and for school-based learning. Due to the large amount of research performed by the authors and them being considered experts in their field, the essential characteristics provided in the work of Clements and Sarama (2009) were used.

Table 4: Essential characteristics of concepts

Concept (prescribed by curriculum)	**Essential characteristics**
2D shapes	Two dimensions of width and length with both lying in the same plane.
Closed shape	A closed figure is made up of line segments that are joined together. Exactly two sides meet at every vertex and no sides cross each other.
Triangle	A figure bounded by three straight sides and three vertices.
Squares	A polygon that has four, straight equal sides and all right angles.
Rectangles	A polygon with four straight sides, four right angles. A rectangle's opposite sides are parallel and the same length.
Circle	A shape with constant curvature and consists of all points a fixed distance from the centre.
Quadrilaterals	A shape with four straight sides.
Parallelograms	Quadrilaterals with two pairs of opposite sides parallel.
Polygon: pentagon, hexagon, heptagon, octagon	A plane figure bounded by three or more straight sides.
Angles: Acute, right, obtuse, straight, reflex, revolution	Two lines that meet to make a corner. The amount of rotation around a fixed point where two lines meet.

Once the concepts for analysis were identified an analytical framework was developed to analyse the text. Based on Vygotsky's theory there is a clear distinction between spontaneous (everyday) and scientific concepts, but that good teaching moves from the everyday to the scientific. Not only was it necessary to establish the presence (or absence) of scientific concepts, but the dialectical relationship between the everyday and the scientific also had to be established. The definitions of scientific concepts and everyday concepts as described in the work of Karpov (2003b) were used in the framework. The criteria from Karpov's work served as indicators to classify concepts as either scientific or everyday. To establish if the dialectical link between the everyday and the scientific is made clear, the work of Fleer (2009) was used. The link is made through references to day-to- day and family life, but then movement beyond this has to take place in order for concept formation to happen.

Coding rules for the data were informed by the theories set out above. The coding rules ensured the consistent application of the analytical framework to the textbooks. Categories for the content analysis of the data were deductively formulated (Mayring, 2000). The definitions of scientific and everyday concepts as well as the coding rules are presented in Table 5.

Table 5: Coding rules for scientific concepts

Definition	**Example**	**Coding rules**
Scientific concept: • Verbal definitions and word meanings are made clear • Procedural *and* conceptual knowledge are taught • The essential characteristics of a class of objects are made clear • Generalisations are made possible	A parallelogram is a quadrilateral with both pairs of opposite sides parallel. All shapes with this characteristic are parallelograms. If this is the case then the opposite sides are congruent and the opposite angles are equal. This allows us to draw the conclusion that squares and rectangles are also parallelograms (Fuys et al, 1988).	• Definition is given • Even if concept has many other characteristics, the essential characteristics in line with prescribed theory is provided either through words or a graphic • Conceptual knowledge is present when an explanation as to *why* it is so is provided • Acquired knowledge will allow the child to flexibly apply it in a mathematical problem solving setting
Everyday concept: • Common salient characteristics of objects presented, but not the essential characteristics • Learning is confined to the procedural • Rote memorisation of definitions is mistaken for concepts	A square is the shape of a four sided table where all the sides are the same. A rectangle is the shape of a door. It has sharp corners and two opposite sides that are equal.	• Essential characteristic is omitted or not made clear. • Confined to the procedural if the underlying logic of the concept is not made clear. • Definitions provided but child is unlikely to be able to problem solve in a mathematical activity

The analytical framework used for text analysis is shown in Table 6. The data was analysed by using the framework and placing 1 next to either scientific or everyday depending on the criteria being met. If criteria were not met a 0 was placed. The narrative, graphical and worked example blocks were analysed for scientific concepts. Task related blocks were analysed for levels of cognitive demand and not included in this round of analysis. This framework was used alongside the "Essential characteristics of concepts" (Table 4). This was used to check that the essential characteristic is present and that it is correct. In the total column the total number of blocks analysed, the total number of concepts presented and a tally of the scientific and everyday concepts were recorded. The same was done for the number of links made between scientific and everyday concepts.

Table 6: Analytical framework for coding of scientific concepts

Block:				**Total**
Concept:				
Concept qualifies as everyday concept if: • Common salient characteristics of objects presented, but not the essential characteristics • Learning is confined to the procedural • Rote memorisation of definitions mistaken for concepts				
Concept qualifies as scientific concept if: • Verbal definitions and word meanings are made clear • Procedural *and* conceptual knowledge are taught • The essential characteristics of a class of objects are made clear • Generalisations are made possible				
Link made between everyday and scientific?				

Analysis of misconceptions

Alongside analysing the text for the presence of scientific concepts, the framework was extended to investigate what misconceptions were observed in the text. As the literature has shown, misconceptions are highly prevalent in the learning of geometry, so it is appropriate to analyse the text for these misconceptions and to see if the text assists learners in avoiding these misconceptions.

According to the literature there are common misconceptions that often occur in geometry textbooks (Clements & Batista, 1992; Clements, 1998; Monaghan, 2000). These are:

- The establishment of prototypes by showing shapes only with horizontal basis and not in different orientations.
- Triangles can only be equilateral.
- A rectangle is not a parallelogram.
- A square is not a rectangle.

These misconceptions were used in the analytical framework and the text was analysed for their presence. All blocks were analysed for misconceptions including the task blocks as tasks sometimes contain graphics that could be prototypes. Each misconception was given a code and when it was present in the text the block where it occurred was recorded. A tally of the blocks was recorded for each misconception. The framework is shown in Table 7:

Table 7: Analytical framework for coding of misconceptions

Misconceptions	**Code**	**Block (s)**	**Total occurrences**
Title of book:			
The establishment of prototypes by showing shapes only with horizontal basis and not in different orientations	PRO		
Triangles can only be equilateral	TRI		
A rectangle is not a parallelogram	REC		
A square is not a rectangle	SRE		
Total:			

Analysis of exercises

As indicated, Smith and Stein's (1998) framework for levels of cognitive demand that is required from textbook tasks, is employed in this study. First, an initial list was compiled to determine which task demands were present across the three textbooks. Coding rules for the classification of each task were devised to ensure consistency. The coding rules are shown in Table 8. Once the different task instructions were recorded they were classified according to Smith and Stein's framework. These are shown in Table 9 and the framework by Smith and Stein (1998) is available in Appendix A.

Table 8: Coding rules for levels of cognitive demand

Level of cognitive demand	**Coding rule**	**Example**
Lower level	• Memorisation type question • Giving a short answer	Count the number of sides in each of the given shapes
Lower middle level	• Repetition of steps provided earlier on in the text • No connection to meaning of concept • Deeper mathematical understanding not developed	Follow the steps in Example 1 to draw a triangle
Higher middle level	• Procedures are used but connections are made • Effort is required (Zorin, 2011).	"You are thinking of a square. What is the smallest number of clues you can give your friend so she can know what shape you are thinking of?" (Fuys et al., 1988)
Higher level	• Doing mathematics	Given four cut outs of the same quadrilateral, find the sum of the interior angles of the quadrilateral without measuring any angles (Fuys et al., 1988).

After classifying the task requirements, each textbook's task blocks were analysed for the different levels of cognitive demand. Table 9 shows the framework for this analysis. The binary system was used again, so that when a specific demand was present in a task block, it was marked by 1. Each task requirement was tallied. Sample applications of each framework can be found in Appendix B, D and C.

Table 9: Framework for the analysis of exercises

Title of book:		**Block**	**Block**	**Block**	**Block**	**Block**	**Block**	**Total**
Task requirement	**Level of demand**							
Name/Identify	1							
Fill in the missing word/vocabulary	1							
Make a drawing	1							
Follow steps to make a drawing	1							
Measure	1							
Match words to content	1							
Build a shape	1							
Cut out	1							
Arrange in descending/ascending order	1							
Categorise	2							
Sort	2							
True/False	2							
Describe	2							
Similarities and differences	2							
Draw a conclusion	3							
Explain	3							
Problem solving	4							
Total:								

Reliability

A number of measures were put in place to ensure reliability. First, in the case of the analysis of scientific concepts, it was ensured that all the same concepts were analysed across the three textbooks, as explained in the earlier sections. In the analysis the same essential criteria was used, so that there would not be variation in understanding of concepts. Furthermore, coding rules were used to analyse concepts and tasks and they were consistently applied across the three textbooks. The method of subdividing the text was consistently applied across the three textbooks and following Valeverde et al.'s (2002) criteria the different blocks were classified uniformly. As a further reliability measure the researcher performed check-coding with another researcher (Miles & Huberman, 1994). One of the textbooks was randomly selected and each researcher independently coded the text blocks and then compared results. There was 97% agreement between the coders. Each aspect on which the coders disagreed was discussed until 100% agreement was reached.

Conclusion

In this chapter the methodology for the study was set out. The method of sample collection and how the three textbooks were chosen was presented. The procedure for collecting data by blocking the text according to Valverde et al. (2002) was explained and illustrated. The analytical frameworks for the analysis of scientific concepts, misconceptions in the text and the level of cognitive demand by activities in the textbooks were presented. Lastly issues of reliability were discussed. This dissertation will now continue by presenting the findings.

Chapter 4: Findings

The purpose of this study was to analyse three grade six mathematics textbooks for how the concepts of the geometry of two dimensional shapes are presented. In this chapter the findings from quantitative and qualitative analysis of the data are presented. The quantitative analysis consisted of tallies, percentages and graphical displays to illustrate the findings pertaining to the research questions. The data from the sampled textbooks were analysed with regards to geometric concepts covered and number of instances scientific concepts were present. Similar analysis was conducted to determine the number of misconceptions in the text as well as the level of cognitive demand required in the textbook activities. The chapter is organised into three sections to address the three individual research questions.

Scientific and everyday concepts

The three textbooks were analysed to determine if the geometric concepts were presented as scientific concepts according to Vygotsky's theory of concept formation. As per the research methodology, the text was classified into narrative, worked examples, graphical and task blocks. The narrative blocks, that contained explanations, descriptions and expositions, and the graphical blocks, containing mostly a diagram, visual image or picture, and the worked examples were analysed for the presence of scientific concepts. The task blocks were used for analysis in answering the cognitive demand research question. The breakdown of the blocks per textbook is provided below:

Table 10: Blocks analysed

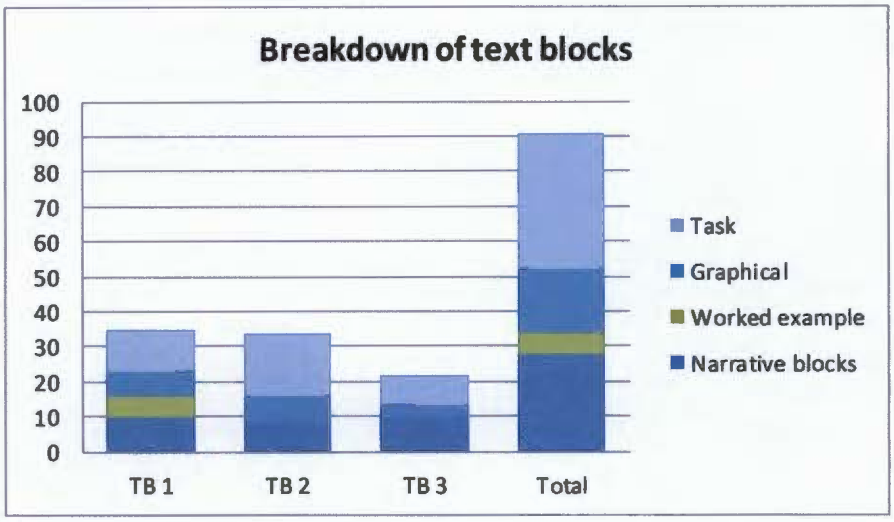
	Narrative blocks	%	Worked example	%	Graphical	%	Task	%	Total
TB 1	10	29%	6	17%	7	20%	12	34%	**35**
TB 2	8	24%	0	0%	8	24%	18	53%	**34**
TB 3	10	45%	0	0%	3	14%	9	41%	**22**
Total[*]	**28**	**31%**	**6**	**7%**	**18**	**20%**	**39**	**43%**	**91**

---

[*] Percentages are rounded off

In total 28 narrative blocks were analysed, 6 worked example blocks, 18 graphical blocks and 39 task blocks. The number of narrative blocks in the textbooks ranged from 8-10, with graphical blocks ranging from 3-8. Greater variation was observed in number of tasks, with TB 2 having the largest number at 18 (53%). All three textbooks devoted the largest percentage of blocks to tasks (43% overall) and secondly to narratives with an overall percentage of 31%. Only TB 1 had worked examples present, which were clearly indicated as worked examples in the text. The text blocks that made up the data for analysis is shown in the bar graph below:

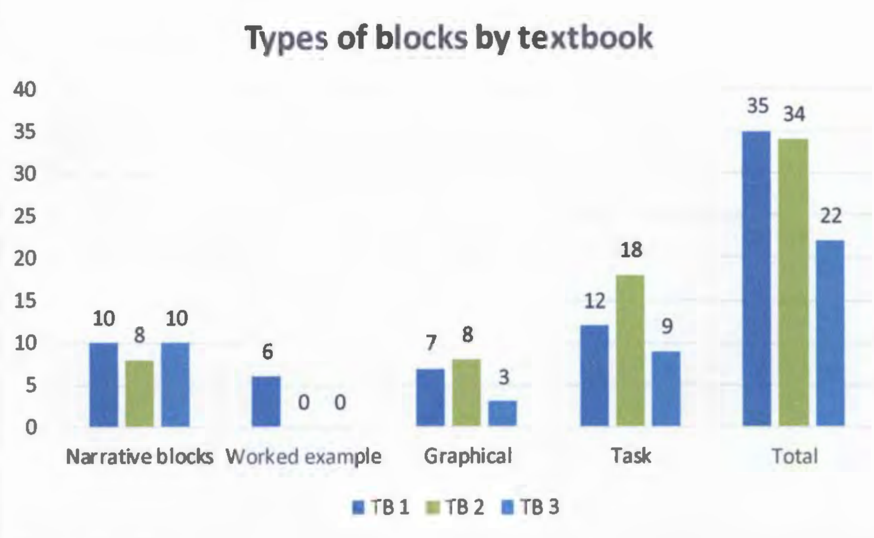
Figure 6: Breakdown of text blocks



Although the observation regarding worked examples is of some interest, it speaks more to the pedagogical approaches of the textbooks and less to the research questions under investigation. The worked examples in TB 1 were analysed regardless for a number of reasons. Firstly, although worked examples could be absent from the other two textbooks in the sample, it is a fairly common occurrence in most mathematics textbooks. Secondly, the worked examples were found to deal with important geometrical concepts, so in the interest of consistency in the design of the study, they were included in the analysis. Lastly, this study does not seek to provide qualitative analysis of the types of blocks and their use across different textbooks, but aims to analyse concepts. Since concepts are present in the worked example blocks, they must form part of the analysis.

The breakdown of the text blocks by textbook is graphically shown in Figure 7:

Figure 7: Breakdown of text blocks by textbook



One will note that TB 3 had fewer blocks analysed than the other two textbooks. TB 3 is smaller (printed on narrow crown) than the other two (printed on A4) which resulted in fewer blocks. However, this study is not concerned with structure of the textbook or how much space is dedicated to a section. The main concern of this study is the quality of the scientific concepts conveyed *in* the text, making variation in number of blocks a less important issue to consider. It is possible for scientific concepts to be accurately presented even if it is in fewer blocks.

Distribution of concepts across blocks in the text

The geometric concepts in the three textbooks were analysed for scientific concepts, everyday concepts and whether the link is made between scientific concepts and everyday concepts. The number of blocks dedicated to each concept and the type of block per textbook is shown in Table 11. The number of blocks serves as an indication of how many times the concept, or an element of the concept, was presented.

There are a number of matters to clarify especially with regards to number of blocks and the number of times a concept was presented. Firstly, in TB 1 two of the narrative blocks dealt with more than one concept. These have therefore been counted separately and explain the discrepancy between total narrative blocks for TB 1 as indicated in Table 10 and in Table 11. A similar situation occurred in TB 2 where one block dealt with both the concepts of square and rectangle. Where this occurred it was marked with an asterisk in the table.

Table 11: Distribution of concepts by block per textbook

	TB 1			**TB 2**			**TB 3**		
Geometrical concept	**Narrative**	**Worked example**	**Graphical**	**Narrative**	**Worked example**	**Graphical**	**Narrative**	**Worked example**	**Graphical**
2D shapes	2*			1			1		2
Closed shape									
Triangle		1	1						
Squares				1*					
Rectangles		1		1*					
Circle	2*			2			2		
Quadrilaterals			1						
Parallelograms	3	1		1		1	1		
Polygon: pentagon, hexagon, heptagon, octagon	3*	1	1	1		1	2		1
Angles: Acute, right, obtuse, straight, reflex, revolution	1	1	4	2		6	4		
Total	**11**	**5**	**7**	**9**	**0**	**8**	**10**	**0**	**3**

Secondly TB 1 dealt with two concepts (parallel lines and vertex) that were not present in the curriculum and also not in the other two textbooks. The importance of these two concepts cannot be overlooked and will be discussed later on, but in terms of the current analysis the three blocks pertaining to these concepts were removed from the analysis to maintain reliability.

The analysis regarding the presence of scientific concepts in the text is shown in Table 12. The table displays (for each textbook) if the various geometric concepts were presented as scientific, everyday, or as both. Listed in the first column is the number of blocks dedicated to the concept. The next two columns show how the blocks were classified in terms of being scientific or everyday, with the third column indicating when the two occurred together. If both were present a link had to be made between the everyday and the scientific. No link can be made if only one or the other was present. For instance, the concept of a two dimensional shape was presented in TB 1 in one instance as an everyday concept and in another instance as a scientific concept, but the everyday and scientific did not occur together in the same instance. In TB 2 the circle concept was presented with both the everyday and scientific present in the same instance, with a link being made between the two.

Table 12: Everyday and scientific concepts

	TB 1					TB 2					TB 3				
Concepts	**Number of blocks**	**Scientific concept**	**Everyday concept**	**Everyday and scientific**	**Link**	**Number of blocks**	**Scientific concept**	**Everyday concept**	**Everyday and scientific**	**Link**	**Number of blocks**	**Scientific concept**	**Everyday concept**	**Everyday and scientific**	**Link**
2D shapes	2	1	1	0	0	1	0	1	0	0	3	0	3	0	0
Closed shape	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Triangle	2	-	2	0	0	-	-	-	-	-	-	-	-	-	-
Squares	-	-	-	-	-	1	0	1	0	-	-	-	-	-	-
Rectangles	1	0	1	0	0	1	0	1	0	-	-	-	-	-	-
Circle	2	0	2	0	0	2	0	1	1	1	2	0	1	1	1
Quadrilaterals	1	0	0	1	0	-	-	-	-	-	-	-	-	-	-
Parallelograms	4	3	1	0	0	2	0	2	0	0	1	0	1	0	0
Polygon: pentagon, hexagon, heptagon, octagon	5	1	3	1	1	2	2	0	0	0	3	2	1	0	0
Angles: Acute, right, obtuse, straight, reflex, revolution	6	0	6	0	0	8	3	3	2	2	4	0	4	0	0
Total	**23**	**5**	**16**	**2**	**1**	**17**	**5**	**9**	**3**	**3**	**13**	**2**	**10**	**1**	**1**

To illustrate how the everyday and scientific were presented and how the coding rules were applied, examples from the text pertaining to angles are provided[2]:

TB 1 (Angle concept of a revolution)

"A revolution is a full turn"

Next to the definition of a revolution is a picture of a ballerina

The above concept was coded as everyday. Although a definition of a revolution is provided, there is no conceptual knowledge present and the mathematics pertaining to the concept of a revolution is essentially absent. By showing the picture of a ballerina no connection is made as to how this relates to the angle concept.

TB 2 (Acute angle concept)

"An acute angle is an angle that is smaller than a right angle."

This definition is followed by a number of sketches showing acute angles. They are drawn with two line segments intersecting and the angle is indicated by an arrow.

The above shows an example where only the scientific is present. The line segments and arrow indicating the angle is abstract and the essential characteristic of an acute angle is made clear by providing a number of examples of acute angles in different orientations as well as a definition. However, the everyday is absent and no link between the scientific and the everyday is made.

TB 2 (Angle concept)

"An angle is the corner that is formed when two straight lines meet or cross each other. We can also think of an angle as the amount of turn that one of the lines must move through so that it lies on top of the other line."

This description is followed by a graphic of two line segments (AB and BC) that intersect to form an angle.

---

[2] Full references to textbook examples quoted are not provided in order to maintain the anonymity of the books.

Here the concept was coded as a scientific concept, with the everyday present and a link between the two. The definition of an angle is made clear, both conceptual and procedural knowledge is present and the concept is related to the everyday concept of "turn." It goes further to mathematise the concept by showing graphically the relationship between the two straight lines that intersect. This contrasts sharply with the example from TB 1 where the learner could not possibly form a scientific concept of a revolution based on what is provided.

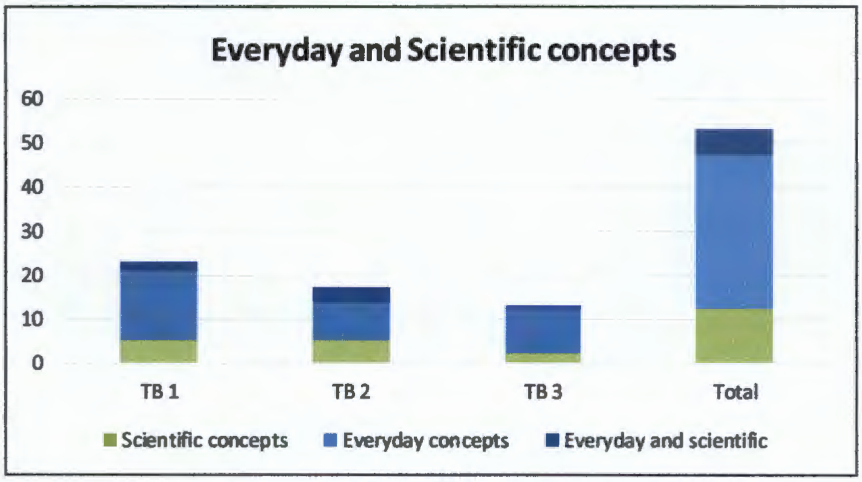
Table 13 and Figure 8 show the summary of the results by textbook as well as the overall result. Of the 53 times the various concepts were presented across the three textbooks, only 23% of the time it was done so as scientific concepts. 66% of the time the concepts were presented as everyday concepts with the scientific and the everyday linked together 11% of the time.

Although the results for scientific concepts were low overall, TB 2 showed to have the highest percentage of scientific concepts present along with making the most links between the everyday and the scientific. TB 3 showed scientific concepts only 15% of the time and made the fewest links (8%) between the scientific and everyday.

Table 13: Summary of scientific and everyday concepts

	TB 1		TB 2		TB 3		Total	
Total number of times concepts were presented	**23**	**%**	**17**	**%**	**13**	**%**	**53**	**%**
Scientific concepts	5	22%	5	29%	2	15%	**12**	**23%**
Everyday concepts	16	70%	9	53%	10	77%	**35**	**66%**
Everyday and scientific	2	9%	3	18%	1	8%	**6**	**11%**

Figure 8: Everyday and scientific concepts



Geometry misconceptions

Coding of the text showed that certain misconceptions were present. The text was analysed for establishing prototypes, showing triangles as equilateral only, stating that a square is not a rectangle and for failing to identify a rectangle as a parallelogram. The comparison of the number of misconceptions observed with the total number of blocks per textbook is shown below:

Table 14: Overall prevalence of misconceptions

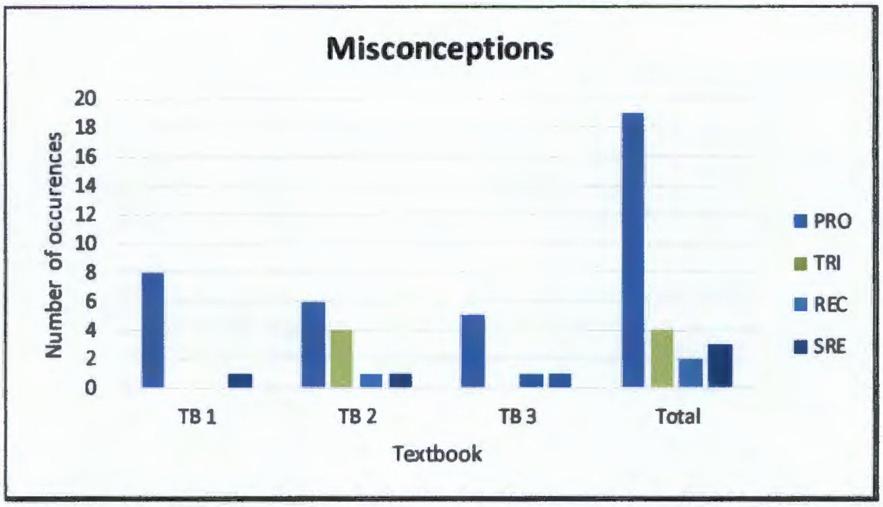
	TB 1	TB 2	TB 3	Total
Total number of misconceptions observed	9	12	7	**28**
Total number of blocks	35	34	22	**91**
% Total	**26%**	**35%**	**32%**	**31%**

Overall 28 blocks of the total 91 were recorded to show misconceptions as listed in the analytical framework. This is a prevalence of misconceptions of 31%. Further analysis was performed to reveal how the misconceptions were spread across the three textbooks. The results are shown in Table 15 and Figure 9:

Table 15: Breakdown of misconceptions in the text

		Number of occurrences				
Misconception	**Code**	**TB 1**	**TB 2**	**TB 3**	**Total**	**%**
The establishment of prototypes by showing shapes only with horizontal basis and not in different orientations	PRO	8	6	5	**19**	**68%**
Triangles can only be equilateral	TRI	0	4	0	**4**	**14%**
A rectangle is not a parallelogram	REC	0	1	1	**2**	**7%**
A square is not a rectangle	SRE	1	1	1	**3**	**11%**
Total		**9**	**12**	**7**	**28**	**100%**

Figure 9: Misconceptions in the text



All three textbooks had some misconceptions present. The most common misconception was providing the learner with prototypes and not exposing the learner to many examples of shapes in varying orientations. Most of the time shapes are shown with a horizontal base, as per the example below:

Figure 10: Shapes in a typical horizontal orientation

To avoid having the learner develop prototypes that prevent them from flexible thinking and generalization, shapes need to be shown in varying orientations. The same three shapes above are used to illustrate in the example below:

Figure 11: Showing shapes in different orientations

Across the three textbooks this occurred 19 times (68% of the total number of misconceptions observed). Showing triangles as only equilateral occurred four times, but only in TB 2. TB 1 showed different triangles (acute, obtuse, right angled and scalene) and so did TB 3. TB 2 and TB 3 both failed to identify a rectangle as a parallelogram whereas TB 1 made this clear. However, all three textbooks failed to show the learner that a square is a "special" rectangle.

The way quadrilaterals, and specifically parallelograms were handled in the textbooks raised concerns. Only TB 1 managed to explain that squares and rectangles are parallelograms and pointed out some of the defining characteristics of these shapes. The other two textbooks, however, failed to establish the relationships between the figures and therefore did not establish that parallelograms are a specific class of shapes.

One textbook started with rectangles and squares, showing their common characteristics and emphasising their differences. By showing the learner that a square is different from a rectangle due to having all sides equal, it is teaching the learner to define the shape by what it is not. The textbook then goes on to speak of parallelograms as "another type of quadrilateral" thereby failing to establish the relationship between rectangles, squares and parallelograms *as all being parallelograms.* The characteristics of parallelograms are then listed, but the essential feature (two pairs of opposite sides are parallel and equal) is completely omitted from the list.

The third textbook followed a very similar learning trajectory where squares and rectangles were presented first with a parallelogram being a completely different shape in its own category. This textbook also failed to identify them as being in the same class although later on mention is made of squares and rectangles being "special" parallelograms. However, no further explanations for this were provided.

Other omissions and misconceptions

Although the text was analysed using the framework derived from the literature, it is worthwhile to note a number of other issues were observed as the researcher analysed the text. In one such instance inconsistencies were detected, such as a contradiction as to what the definition of a polygon is. TB 1 defined it as a shape with more than four sides in one place, and then as a shape with more than three sides in another.

From all the textbooks measurement of angles were missing. Angles were only presented as acute, right, straight, reflex and revolution. Other magnitudes of angles were not presented and the measurement of degrees was omitted. Although this could be driven by CAPS that prescribes no measurement of angles with a protractor in grade six, it is possible to still explore magnitude of angles in other ways (Fuys et al., 1988). Limited mathematising of angles in TB 1 and TB 3 was also noted, with very little mention made of lines meeting to form angles. The text often presented angles as the ones mentioned before, but then went on to ask learners about angles in shapes. The salient characteristic of a shape is that angles are formed when sides meet. Not presenting the line relationship to the learner in connection to angles can hamper their understanding of angles. It is also noted that overall the text lacks richness of mathematical information as there is a general lack of notation (especially with regards to angles and line segments) and technical terms. An overall colloquial use of language was observed.

In one textbook the concept of a vertex was briefly discussed, but it was completely disconnected from the angle discussion and the relationship was not established. A further omission that raised concern is that of the idea of parallelism. In order to learn about parallelograms, children need to have a basic understanding of parallel lines, since this is an essential feature of that class of shapes. One textbook briefly dealt with the topic, but made a technical error in the example it used to explore it. Both the concepts of vertices and parallelism are not explicitly prescribed in CAPS, and with textbooks closely following CAPS it again could be a possible reason for the omission. These issues will be explored further in Chapter 5.

Level of cognitive demand in textbook activities

The activities in each textbook was analysed and coded to reveal the cognitive demand placed on the learner in completing the activities. The framework presented by Smith and Stein (1998) was used and the task requirements were classified according to the four levels presented in the framework. Cognitive demand increases with the levels, so Level 1 will be the lowest and Level 4 will be the highest. The outcomes for each textbook are shown in Table 16.

There was a broad range of activities across the textbooks and in total 87 instances where the child had to perform a task were identified. The task demands were concentrated in Level 1, with "Naming/Identifying" and "Make a drawing" being the most popular. Below is an example from TB 1:

TB 1 (Naming shapes)

"Name each of the shapes shown below."

Table 16: Level of cognitive demand of textbook activities

Level of demand	**TB 1**	**TB 2**	**TB 3**	**Total**
Level 1	**18**	**29**	**15**	**62**
Arrange in descending/ascending order		1		1
Build a shape		2	1	3
Cut out	1	1		2
Fill in the missing word/vocabulary	1	2	1	4
Follow steps to make a drawing	2	1	1	4
Make a drawing	6	8	6	20
Match words to content		1	1	2
Measure	2		2	4
Name/Identify	6	13	3	22
Level 2	**5**	**8**	**2**	**15**
Categorise	2	2		4
Describe	1	1		2
Similarities and differences	2	1		3
Sort		3	2	5
True/False		1		1
Level 3	**5**	**3**	**0**	**8**
Draw a conclusion	2	1		3
Explain	3	2		5
Level 4		**2**		**2**
Problem solving		2		2
Total	**28**	**42**	**17**	**87**

Drawing activities were highly prevalent and were typically like the one given as an example below:

TB 3 (Drawing circles)

"Draw three circles with the same centre, but make the radius of the first circle 3 cm, the second 4 cm and the third 5 cm."

It has to be noted that the high incidence of drawing activities is partly curriculum driven as CAPS dedicates a section of work on drawing 2D- shapes (Department of Education, 2011). The drawing of shapes has an important role to play in the learning of geometry, but drawing without creating understanding remains meaningless. Drawing activities will be discussed further in Chapter 5.

Level 2 activities were less frequent with "Sorting" occurring most often. Level 3 and Level 4 were the least prevalent with problem solving tasks only occurring in TB 2. TB 3 had no Level 3 and Level 4 activities. An example of a problem solving task from TB 2 is shown below:

> **TB 2 (Problem solving)**
>
> *"Draw a hexagon with at least one reflex angle. Can you draw a pentagon with all its angles acute?"*

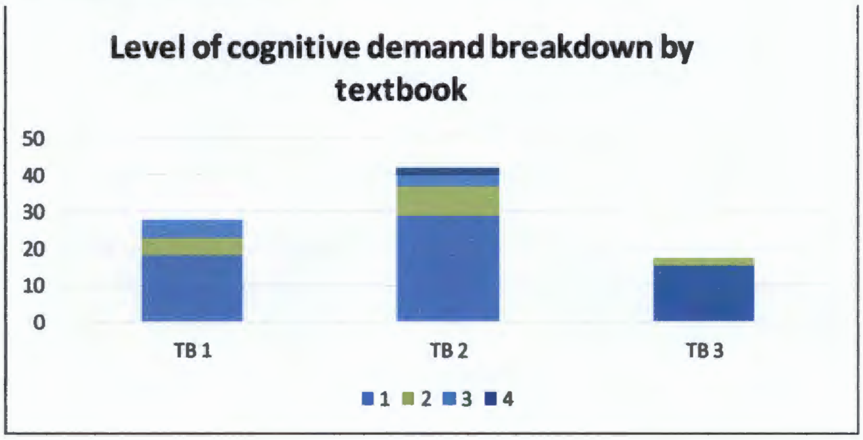
The results for the textbook activities are summarised in Table 17 below:

Table 17: Summary of textbook activities

Level of demand	TB 1	TB 2	TB 3	Total	%
1	18	29	15	62	71%
2	5	8	2	15	17%
3	5	3		8	9%
4		2		2	2%
Total	28	42	17	87	100%

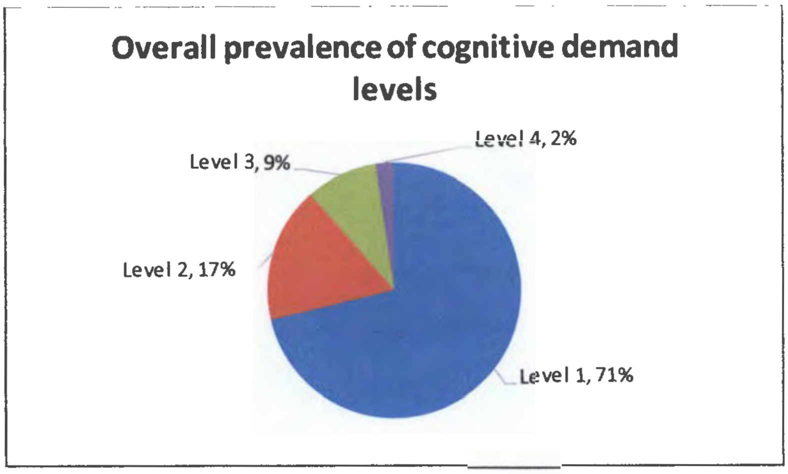
In the study I was less interested in relative number of exercises, but more in what the existing exercises require from the learner. Even though there are a varying number of task demands across the three textbooks, *within* each textbook the results are very similar: the highest number of activities is classified at Level 1 with few to none classified at Level 3 and 4. These results are illustrated in Figure 12.

Figure 12: Level of cognitive demand by textbook



This produces the overall result of 71% of activities being in Level 1, 17% at Level 2, 9% at Level 3 and 2% at Level 4. These results are displayed in the pie chart below:

Figure 13: Overall prevalence of cognitive demand levels



The small number of tasks that make high-level demands on learners found in the three textbooks that were analysed indicates that there are very limited opportunities for learners to be challenged and develop sophisticated mathematical skills.

Conclusion

In this chapter the findings from the data analysis were presented. The main research question was to determine if scientific concepts were present across the three textbooks. The analysis showed that scientific concepts were only present 23% of the time with 66% of the concepts being classified as everyday. The link between the scientific and the everyday was only made 11% of the time.

Certain misconceptions were present in the text. The most commonly occurring one was the establishment of prototypes (68%). Other misconceptions, especially with regards to angle concepts and parallelograms emerged from the text. Lastly, the tasks in the textbooks were analysed to determine what level of cognitive demand they make on the learner. The results show that 71% of the textbook tasks were Level 1 activities with only 2% of task classified at Level 4. In Chapter 5 a discussion of these findings will follow.

Chapter 5: Discussion, recommendations and limitations

This study aimed to explore geometric concepts in mathematics textbooks in light of Vygotsky's theory of concept formation. In addition it explored the misconceptions present in the text as well as the level of cognitive demand required by textbook activities. The research questions were:

1. Is the geometry of space and shape presented in the textbook as scientific concepts according to Vygotsky's theory of concept formation?
2. Are typical misconceptions as identified from the literature present in the textbooks?
3. What level of cognitive demand do the activities in the textbook require so as to assist in the development of geometric concepts?

This chapter will continue with a discussion of the findings for each of the three questions, after which a synthesis of the three aspects of the study will be provided. Limitations of the study and recommendations for further research will conclude this chapter.

Scientific concepts

The main research question was, if a Vygotskian lens were applied to three existing textbooks and their treatment of geometric concepts, can these concepts be defined as scientific? The findings showed that of the 53 times concepts were presented, only 12 of them could be classified as scientific (23%) and a link between the scientific and everyday was made only 11% of the time. Everyday concepts were present 66% of the time. This finding is in support of other research that has found that conceptual development is seldom supported by textbooks (e.g. Flanders, 1987; Levin, 1998; Dole & Shield, 2008 and Stacey & Vincent, 2009).

The low prevalence of scientific concepts in the textbooks, have implications for the efficacy of the textbook as mediational artefact and for the development of higher order functioning. Firstly, as argued in Chapter 2, geometry is a socially established scientific knowledge area. The child will be exposed to this knowledge through engaging with the textbook. In the absence of scientific concepts, the learning that is mediated through the textbook is confined to the everyday. As Kozulin (2003) points out, such empirical learning does not allow for the distinction between everyday experiences and scientific concepts as the child fails to develop scientific logic. The knowledge that is acquired in the absence of scientific concepts, remain at the everyday level. In this study concepts were classified as everyday if:

- Common salient characteristics of objects are presented, but not the essential characteristics
- Learning is confined to the procedural
- Rote memorisation of definitions is mistaken for concepts.

The textbook as mediating artefact that only exposes the child to the everyday, is withholding the true complexities of the subject area from the child.

This brings about the second important implication. The child will not have an opportunity to truly learn geometry, devise meaning from what is learnt or appropriate the concepts to be used as psychological tools. If the child cannot acquire the necessary scientific concepts from the textbook, he or she cannot internalise these concepts or use them as tools, which in turn will prevent the development of higher order thinking. The lack of higher order thinking skills amongst learners when studying geometry is frequently observed (Van der Sandt & Nieuwoudt, 2003; Feza & Webb, 2005; Atebe, 2008).

The presentation of the geometric concepts in the textbooks frequently leant towards direct reproduction of definitions of shapes and their characteristics meant to be memorised. The delivery of this kind of pre-packaged mathematical knowledge without involving learners in the mathematical activity that led to its development, is frequently observed (Human & Nel, 1987). As Bartolini-Bussi (1998) points out, meaning of mathematical content cannot be taught through memorising definitions or procedures as the child cannot connect them to existing knowledge. It is often suggested by theorists (e.g. Hedegaard, 1998; Daniels, 2001; Kozulin, 2003; Karpov, 2003a) that this is not a successful strategy to follow, but still it seems to prevail.

De Villers (1998) observes that textbooks are often rife with definitions, theorems and classifications. However, the definition of a concept *does not guarantee that the concept will be understood*. Central to Vygotsky's theory of concept formation is the fact that scientific concepts are launched from definitions, but that it is only the beginning of the development of concepts (Miller, 2011). In this study it was frequently observed that definitions of shapes are given without providing the substructure of the concept. As part of the coding rules for a scientific concept in the text, a definition is an important aspect, but during the analysis of the text, it was observed this is often where the textbook stopped. Below is an example from TB 1 to illustrate:

TB 1

"A rhombus is a quadrilateral with two pairs of opposite sides parallel and all sides equal."

The textbook goes no further (apart from providing a diagram of a rhombus) in getting the child to explore this concept extensively. It also does not provide any relation to the substructure of a rhombus, namely that of a parallelogram. De Villiers (2011) strongly argues that if learners are provided only with straight forward definitions of geometric concepts, they will fail. He suggests that learners have to be actively involved in constructing definitions, and the textbook can support this kind of learning by providing more opportunities to explore, reflect and conclude.

For optimal development of higher order functions, the everyday and scientific concepts need to both be present and dialectically linked (Fleer, 2009). This occurred only six times in the textbooks analysed (11%). Subject- matter concepts can only be transformed into personal concepts if the child is able to relate, and use them in their day-to-day life (Hedegaard, 1998). Learning is most powerful when "the double move" takes place, in other words, when the everyday and scientific are linked (Hedegaard, 1998). Fleer (2009) supports this in her study on concept development and shows how everyday concepts are the foundations of scientific concepts. Successful implementation of such learning has been performed by Hedegaard (1990) and allowed for the development of generalised methods of problem solving. Kilpatrick, Swafford and Findell (2001, p. 80) sums up the importance of linking the everyday and the scientific:

> "Students with a conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organised their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know."

To illustrate how the link between the scientific and everyday plays a role in the development of concepts, the treatment of angles in the three textbooks will be used. In all three textbooks everyday concepts were used to explore angles. These included a handheld fan (TB 3), a right angle defined as the corner of a page (TB 1) and a ballerina turning (TB 1). From these concepts, the abstract concept of an angle needs to be formed, namely that an angle is two lines with a common endpoint (Mitchelmore & White, 2000). In TB 1, this abstraction was never shown. Although it was shown in TB 2 and TB 3, links were not made to the everyday. This could lead to failure to develop the concept of an angle, which is often the case amongst young children (Battista, 2007).

Role of the teacher

There is no denying that the textbook is an essential part of the learning process, but in light of the findings from this study it is clear that the textbook alone will not be sufficient in teaching the child the required geometric concepts. In this study I have only focused on the text, but given the lack of scientific concepts in the text, the teacher's role in facilitating learning will become all the more important. If the textbook is consistently used as the principal guide to learning, then it will define the boundaries of what is learnt in mathematics (and geometry), and those boundaries will be inadequate. Rezat (2006) has suggested that the teacher is a key mediator of the text, but that the teacher is ultimately the mediator of mathematical knowledge. Others have also suggested that teachers are the ones to implement the knowledge presented in textbooks (e.g. Valverde et al., 2002; Schmidt & McKnight, 2012).

However, if the teacher were to instil conceptual understanding in learners, it would involve them adjusting, supplementing and adding to the material in the textbook (provided that the textbook is lacking in linking scientific and everyday concepts, like the ones analysed). The links between the scientific and everyday will need to be made by the teacher. Hornsley and Walker (2005) have found that teachers often use photocopied materials from a wide variety of sources to supplement textbooks and that these are chosen based on the teacher's pedagogical content knowledge. However, "teaching by the book" is often observed as teachers report a lack of time to prepare lessons (Haggarty & Pepin, 2002; Schmidt & McKnight, 2012).

The textbook and the curriculum

The researcher made an important observation in the analysis of the text. The content of the textbook was largely driven by what was prescribed in the CAPS curriculum. The concepts covered were exactly in line with the CAPS document and deviation from this happened only twice and in one of the textbooks with the inclusion of a discussion of parallel lines. The prescriptive nature of CAPS was seen especially in the omission of dealing with measurement of angles, the concepts of parallelism and the prevalence of drawing activities in the exploration of shapes.

Although this study is not focused on comparing the textbook with the curriculum, the observation of how CAPS informed the textbook is an important one. The strict following of the curriculum implies that the curriculum itself has to be scientifically sound. However, this is not the case. An example is used to illustrate. CAPS prescribes the following with regards to angles and shapes (Department of Basic Education, 2011, p. 229):

> "Learners use angles, in particular right angles to distinguish shapes. This is the case when distinguishing between rectangles and parallelograms."

This statement is problematic for a number of reasons. Firstly, a distinction between a rectangle and a parallelogram is made. This is incorrect as a rectangle is a parallelogram with the special feature of having all right angles. This connection is never made and the child cannot grasp that parallelograms are a specific class of shapes, with a rectangle as a part of that class. Disconnected mathematical knowledge starts to form if concepts are presented in this way. By using right angles as the measure for classifying shapes, the essential characteristic of a parallelogram is missed. The essential characteristic of this class of shape is the fact that the opposite sides are parallel and equal. If this is understood, the remaining classifications become obvious.

It was seen that the above was carried forward into two of the textbooks and the classification of shapes was performed following the above instruction. It was a general trend across the three textbooks, but it is unsurprising as there is more often than not a strong relationship between textbook and curriculum (Valverde et al., 2002; Fan, 2011; Fan, et al., 2013). It does however imply that the concepts and their definitions are gained from the curriculum, which could in some cases be problematic.

Returning to the treatment of angles in the textbooks, a few other observations can be made. In grade six the learner is required to only be able to distinguish between different types of angles, namely, acute, obtuse, right, reflex, straight and revolution. Measurement of angles is omitted from the textbook, which is also dictated by the curriculum. This approach is worth reconsidering for a number of reasons. First, by only suggesting that angles can be classified as mentioned, the learner could become rigid in their thinking, failing to realise that an angle is a dynamic concept. Turning, which is the underlying idea of an angle, has a unit and magnitude attached to it (Clements & Sarama, 2009). Leaving this important fact out (especially at this stage of schooling) again leaves a gap in knowledge and a lack of mathematising of the concept.

Teaching young children to work with a compass is particularly difficult (Mitchelmore & White, 2000) and this is introduced in grade seven according to the curriculum. As a building block to measuring with a protractor, children in grade six can be taught to measure angles in other ways. Angles, if understood as divisions of a circle, can be measured informally using increments and angle "slices" (Fuys et al., 1988; Clements & Sarama, 2009). It is clear in the textbooks that children are expected to make the connections between angles and shapes, a difficult task that should be started early (Clements, 1998; Clements & Sarama, 2009). Introducing some form of angle measurement earlier on could help accomplish this goal. Japan, Taiwan and Russia already introduce children in grade 1 to 3 to these concepts and have achieved great success in geometry learning (Burger & Shaugnessy, 1985; Wu & Ma, 2006).

This section of the discussion has focused on the findings around everyday and scientific concepts. The implications of the low prevalence of linking everyday and scientific concepts were highlighted. The role of the teacher in mediating the text was briefly discussed. It was pointed out that CAPS has played a role in determining how concepts are defined and which concepts are included in the textbooks. The angle concept in particular was discussed to highlight how it can be expanded to make for a richer experience of geometry. The discussion will now continue by looking at the geometry misconceptions found in the text.

Geometry misconceptions

In light of the above discussion of scientific concepts, it is unsurprising that there will be misconceptions in the text. This is in accordance with Kajander and Lovric (2009) who have argued that textbooks often support the development of misconceptions. In Vygotskian terms these misconceptions can be called *pseudo-concepts.* A pseudo-concept has the appearance of a proper concept, but the thinking behind it is non-logical and empirical (Miller, 2011). Pseudo-concepts are often based on templates, imitation and association (Berger, 2006). For example, a child is presented with a red triangle. When asked to pick out all the triangles from a variety of multi-coloured shapes, she chooses a red circle (because of its colour) and a triangle because of its form. She has not made a selection based on the essential features of the shape, but has done so based on arbitrary factors. She may have chosen a triangle from the shapes, but she did so based on imitation rather than understanding. Even though she appears to be working with the concept of a triangle as if it were a real one, her thinking is illogical and experiential (Berger, 2005). I would like to argue that the misconceptions present in the textbooks in this study are based on and can lead to the development of pseudo-concepts.

This study specifically looked for common misconceptions provided in the literature, namely establishing prototypes, showing triangles as equilateral only, stating that a square is not a rectangle and failing to identify a rectangle as a parallelogram. In the three textbooks analysed all of these were present, with establishing prototypes being the most common (68%). Showing only equilateral triangles occurred 14% of the time. Similarly, Shaughnessy and Burger (1985) found that children only saw equilateral triangles with horizontal bases as true triangles, failing to include scalene and right angled triangles in their categorisation. Other studies have shown that children fail to identify shapes when they are shown in different orientations, therefore pointing to the establishment of prototypes (Burger & Shaughnessy, 1985; Monaghan, 2000; Atebe & Schäfer, 2008; Clements & Sarama, 2009).

A prototype is formed based on visual-perceptual limitations. These limitations are then used as a model for judgments of other instances, influencing the identification of shapes and problem solving independent of the concepts (Hasegawa, 1997). In other words, the mental image of a shape becomes separated from its mathematical definition. If the textbook only focusses on getting the learner to recognise and classify shapes based on how they look, the learner has not managed to develop true scientific concepts of geometric shapes. The focus is on what the shapes look like instead of its proper mathematical characteristics (Clements, Swamanithan, Hannibal & Sarama, 1999). Classifying shapes in this way, or failing to recognise shapes in different orientations, are typical pseudo-concepts. The child is basing their decision on non-essential attributes or mathematically irrelevant characteristics and for this reason cannot recognise shapes in different orientations. Although a child may be using the concepts of shapes, he or she has not appropriated the concept correctly.

All three textbooks in this study were visually orientated, frequently asking the learner to identify shapes by how they look. In some instances the textbook moved beyond visual recognition and started to focus on characterising shapes through their properties. It is to be expected that children will classify shapes based on its visual descriptions, but they should be encouraged to make classifications based on properties of shapes too (Clements, 1998).

Children should not only learn classification, but the properties of shapes need to be understood (Shäfer, 2010). In order to address prototypes further learners can be shown many different examples and non-examples of shapes so that they can focus on the essential characteristics of the shape (Clements et al., 1999) thereby forming true concepts of shapes and not pseudo-concepts.

In addition to prototypes, failing to show that a rectangle is a parallelogram occurred 7% of the time and failing to show a square as a rectangle happened 11% of the time. These misconceptions have been noted in the literature numerous times (e.g. Clements & Battista, 1992; Monaghan, 2000; Renne, 2004; Atebe & Schäfer, 2008). More often than not it is the convention to start off teaching learners basic shapes such as triangles, circles, squares and rectangles. But as Clements (1998) observes, this way of teaching that squares and rectangles are different shapes, solidifies children's idea that a square is not a rectangle when they are young making it difficult to change as they grow older. Failing to show that a square is a rectangle is in line with what De Villiers (1994) calls the partition definition of shapes. He warns against this as it causes rigid thinking about shapes that does not allow for generalisations. Partition thinking has been found to be particularly persistent and have been observed frequently in studies involving the learning of geometry (e.g. Burger & Shaughnessy, 1985).

De Villiers (1994) suggests that teaching children about shapes requires a hierarchical classification instead of a partition classification like it is observed in the textbooks. In a hierarchical classification of parallelograms it is shown how rectangles, squares and rhombi form subsets of the general concept of a parallelogram (De Villiers, 1994). Such an approach is far more useful in teaching children about these shapes as they come to understand them as interconnected concepts. Earlier on the example of a rhombus was used to show how the text focusses on providing definitions only. It can be argued that the hierarchical classification will also help to overcome the one dimensionality of definitions. The hierarchical classification is typically seen in high school textbooks, but by introducing it earlier the current misconceptions that become entrenched over time can be prevented. Fuys et al. (1988) have shown how this can be successfully done with young children.

The text was initially analysed only for the four misconceptions mentioned, but as the study progressed other omissions and gaps emerged. A notable omission from the text was the topic of parallelism, which is the final point of discussion for this section. Parallelism, being lines that remain the same distance apart without ever crossing (Clements & Sarama, 2009), is a foundational concept when it comes to the teaching and learning of parallelograms. Without the concept of parallelism, a learner is unable to grasp the essential characteristic of parallelograms like squares and rectangles. Grasping the properties of these shapes, such as opposite angles being equal, is based on an understanding of parallel lines and angles. Before being able to study shapes like parallelograms a learner not only has to have knowledge of angles, shapes and lines of equal length, but parallelism is part of the conceptual structure. Without this knowledge the concept of a parallelogram again becomes disjointed and meaningless.

It can be argued that a parallelogram is a shape that is formed when two sets of parallel lines intersect. All the other concepts (such as rhombi and rectangles) originate from this foundational concept. De Villiers (1994), Jones and Bush (1996) and Fuys et al. (1988) have all suggested that parallelism is an initial concept that has to be understood first before parallelograms are tackled. The omission of this concept from two of the three textbooks is concerning. Again it has to be noted that it is also omitted from the curriculum, only being covered in detail in high school. A high level of detail is not necessary for children in grade six, but as Fuys et al. (1988) have shown, the parallel line concept can be explored by young children in informal ways. Such an omission has serious implications for early concept development of geometric shapes.

This section discussed the misconceptions that occurred in the three textbooks. Most notable was the establishment of prototypes, which has been argued, can lead to the development of pseudo-concepts. The further mishandling of square and rectangle concepts were discussed and a hierarchical structure of parallelograms was suggested as an alternative. Parallelism is a foundational concept in the learning of parallelogram shapes, but it was omitted from two of the three textbooks. These misconceptions and omissions will have a significant impact on how geometric concepts are presented to the learner and the effectiveness of the textbook becomes questionable. The final part of the discussion will look at the textbook tasks and activities.

Tasks and level of cognitive demand

Activities and tasks are one of the main features of textbooks. The textbooks in this study were no different with 43% of the blocks used for analysis dedicated to tasks. This required that specific focus was placed on the analysis of tasks in order to determine the level of cognitive demand it required. It emerged that 71% of the tasks analysed in the textbooks were at Level 1 where the focus is on memorising, performing procedures, not making connections and reproducing material that has been seen before (Smith & Stein, 1998). Therefore, the activities from the textbooks in this study were mostly of low cognitive demand. This is in accordance with other studies that have investigated textbook tasks (Nicely, 1985; Valverde et al., 2002; Stacey & Vincent, 2009; Zorin, 2011; Shield & Dole, 2012).

The activities that were at Level 3 and 4 made up 11,5% of the activities. Problem solving activities were almost non-existent across the three textbooks indicating that the learner will seldom have the opportunity to engage in higher order thinking. Higher level tasks, according to Smith and Stein (1998) involve complex thinking where the approach to the problem is not immediately clear and not indicated in the task instructions or in the worked example. Learners are required to explore and link mathematical concepts and processes in completing such tasks. We can return to the example from TB 2 that was shown in Chapter 4 to illustrate:

TB 2 (Problem solving)

"Draw a hexagon with at least one reflex angle. Can you draw a pentagon with all its angles acute?"

This example has various elements of a problem solving task for a grade six learner. In the text no worked example was provided for the learner to make the drawing. The learner has to draw on various concepts to solve the problem, namely angles, shapes (polygons) and lines. Mentally the child will need to form an image of the shape first before making a representation (drawing) of it. One can imagine that the last task might involve some trial and error – the answer will not be clear immediately. No procedure has been provided beforehand for making the drawing and engaging in this activity will allow the learner to explore the links between mathematical ideas.

Along with the lack of scientific concepts in the textbook, the lack of higher levels of cognitive demand compounds the difficulty children will have in developing higher order thinking skills. Smith and Stein (1998) suggest that children only develop higher order thinking if they are involved in tasks with high levels of cognitive demand. Levels of cognitive demand of exercises are intended to stimulate learners' thinking and provide them with the opportunity to learn geometry in a meaningful way (Zorin, 2011). Following repetitive steps or continuously being asked to name and identify shapes and angles will hardly get learners excited about learning geometry.

It stands out that drawing is the second highest activity after naming and identifying shapes. In this study 20 of the 87 (23%) tasks analysed were drawing activities. In geometry teaching and learning drawing plays a very important role (Burger & Shaughnessy, 1985). In the Vygotskian tradition, drawing is considered equally important as it is a way for young children to display thoughts and symbolic thinking (Schäfer, 2010). Drawings also make children's misconceptions apparent. Drawings in geometry are meant to show classes of shapes and represent geometric properties and relationships, but this often proves problematic for learners due to not understanding the geometric object being considered (Battista, 2007). Therefore, a geometric drawing cannot be considered as a "picture" (Clements & Battista, 1992) but as the representation of a concept.

The focus that drawing enjoys in the textbooks can therefore be a way for learners to show their understanding of shapes and concepts. As Clements (1998) argues: through drawing children get to explore many representations of shapes and it is intended to develop visual-spatial reasoning. However, in the textbooks children were typically asked to follow steps to make drawings or were given a straightforward instruction to draw. Below is an example:

TB 3

"Use a pair of compasses and follow the instructions in each case below... Colour in your drawings."

The textbook goes on to instruct the learner step by step to draw two tangent circles.

It can be argued that activities such as the one above provide learners with the opportunity to explore shapes like Clements suggests. However, if the above activity is performed the child will end up with two tangent circles that he or she will colour in. It could result in something that is more of a picture than a representation of a geometric concept. It can be argued that the above activity becomes almost meaningless if there is no opportunity provided to reflect, explain and conclude about properties of geometric shapes. If provided this opportunity, the child will not just merely be drawing and following steps, but actively start to derive meaning from the drawings.

De Villiers (1994) agrees that constructions and drawing activities are very important in developing geometric concepts, but such activities can only be meaningful if conclusions are drawn, causal relationships are observed and logical structures of concepts are formed. We can return to the example provided earlier of the problem solving activity identified in TB 2. It is also a drawing activity, but here the child is given the chance to actively engage with a concept. He will have to start off by thinking of the properties of hexagons and pentagons, exploring how these properties can be changed and manipulated to produce the desired result. He might make a few drawings in attempts to figure out the problem, thereby practicing and demonstrating his understanding of the shape through drawing. The drawing becomes, in Vygotskian terms, a tool that the child is using to aid their thinking. It is not just merely a multi coloured picture on a page, but a symbolic representation of a concept.

In the analysis of the activities, we see a culmination of a number of the ideas that this study set out to explore and it illustrates the importance of providing learners with higher order activities. Low level activities such as memorising and naming, it can be argued, does not move the child towards conceptual development, but learning remains at the everyday level. It is through higher order activities, such as the one in the example, that children acquire concepts and develop higher order thinking. Tools, such as drawing, plays a major role in this development, but the use of tools is facilitated through the acquisition of concepts. If the cardinal relationship between higher order activities and cognitive development is understood, why is it that textbooks still have such a low number of high level thinking tasks?

This concludes the discussion of the findings of this study. The results in relation to the three research questions have highlighted some of the concerns around the learning and teaching of geometry. Most of all, it emerged that in terms of scientific concepts and cognitive demand of tasks, the textbooks analysed leave a lot to be desired. This chapter will conclude with a discussion of the limitations of the study and recommendations for future research.

Limitations

This study has a number of limitations. The first is the relatively small sample of textbooks that were analysed. The national catalogue, from which the sample was chosen, consisted of eight textbooks. Three were chosen, but this does not represent all the textbooks that are available in the market place, including those that are used by independent and private schools. Nevertheless, one can assume that because these textbooks are on the national catalogue, they will be widely used in schools. However, the results from this study may not be generalisable to all grade six textbooks in South Africa.

Secondly, a small section of the years' work in geometry was analysed. The geometry of two-dimensional shapes was analysed, but the mathematics curriculum (including geometry) is very dense in grade six. It is possible that applying the analytical framework to other sections (such as number and operations) show different findings. Typically textbooks are written by multiple authors, so one could observe a different approach to presenting concepts in another section of the book. Geometry as a whole (including three dimensional shapes and transformations) could also be given an integrated view.

A third limitation is that the study only focused on the textbook for the establishment of scientific concepts. Although the textbook plays a definite role in the learning process and is the primary source of learning material for most learners, what is presented in the class room is dependent on various other factors. For instance, the teachers' guides for the textbooks were not analysed so it is not possible to account for how concepts actually were presented there or what guidance it provides to the teacher. Also, the textbook does not account for all learning materials in the classroom as teachers could be supplementing the textbook with exercises or chapters from other textbooks.

The above touch on the fourth limitation and it has been mentioned in the discussion. The study only focused on textbooks, but does not account for how the teacher mediates the use of the textbook in the development of concepts. By analysing the textbook only, the research has only elucidated one part of a very intricate learning process. It cannot be said that how the concepts in the textbook are presented will ultimately and solely be what the learner acquires. For that the teaching and learning process will need to be viewed holistically.

The final limitation relates to the frameworks used for the analysis of scientific concepts, misconceptions and levels of cognitive demand. It is possible that if different frameworks are applied, the results for the study will be different. However, I have taken all measures to ensure the reliability of the frameworks used here.

Recommendations

Future research might consider analysis of a large sample of school textbooks – not just for grade six but also for other grades. Given the problems experienced with the teaching and learning of geometry, such an investigation might see role players rethink how it is taught from primary school to high school, providing a complete image of how the subject currently is handled in South African schools. The Vygotskian theory of concept formation, and some of the other frameworks in this study, could provide a meaningful way to perform analysis of the text. Researchers in South Africa are more and more calling for the consideration of Vygotsky's theory to be used in the study of mathematics education (Berger, 2006). Mhlolo and Schäfer (2013) illustrated in their study how the theory is very meaningful in understanding how children form geometric concepts. Textbook exercises and the inclusion of more higher order tasks is a definite possibility for further investigation. The framework provided by Smith and Stein (1998) can be used in a larger sample of textbooks. Research can be conducted into how geometric tasks, that engage the learner and that lead to the development of higher order thinking skills, can be developed.

An important observation from the study was the pertinent role the curriculum plays in dictating textbooks content. This was highlighted in the findings and the discussion and included the omission of certain key foundational concepts, the sequencing of topics strictly according to the curriculum and the close following of concept definitions even though these may be incorrect. If textbooks are to be included on the national catalogue they have to be CAPS compliant, but this may hamper the quality of the content in textbooks. Research around this issue could look at how textbooks are influenced by the curriculum. Analysis of the curriculum itself around the presentation of geometric concepts could also make for meaningful research.

Conclusion

Textbook analysis is a complex task, particularly because the learner does not always act alone with the text, but also at times with the teacher. However, the textbook is a powerful artefact for learning and the role it plays in the educational environment remains undeniably prominent. As such, an analysis of how the information is represented in the text is a good way to raise questions and gain insight into how mathematical knowledge, and in this case geometric knowledge, is presented. In this study textbooks were researched removed from the class room and the teacher, but it aimed to provide a snapshot of how geometrical thinking is addressed in the primary source of learning material.

The matter of why learners struggle with geometry in particular has been the topic of a large body of research. The complex nature of the geometry of shapes, seems to stem from the fact that these concepts have a theoretical element to it as well as a visual, concrete one (Fujita & Jones, 2003). The ability to generalise and understand all shapes as being dynamic is an essential skill – there are no prototypes in geometry. The study illustrated how prototypes are often entrenched in the textbook, making it difficult for learners to move past them.

The literature on geometry learning informed the analysis of the textbooks. The geometric concepts were scrutinised through a Vygotskian lens in order to observe whether the scientific nature of these concepts is brought across in the textbooks. It is believed that presenting concepts in this way will lead to the development of the higher order thinking skill of visual-spatial reasoning, a skill that is paramount in the learning of other mathematical topics. If textbooks are examined in this way (for scientific concepts) it can alert one to strength and weaknesses of textbooks and inform the selection process of textbooks.

What is present in the text generally dictates how the textbook will influence pedagogical practices (Shield & Dole, 2012). A study such as this one can spark the analysis of how other topics in mathematics such as algebra, functions and trigonometry are presented. In concurrence with the literature, textbooks in this study created an overall impression of knowledge that is presented as haphazard, disconnected and without making links to other mathematical concepts (Clements & Battista, 1992). Pseudo-concepts and misconceptions occurred frequently in the three textbooks that were analysed. Activities did little to engage learners and to instil a love of geometry. One would have to agree with Valverde et al. (2002) that the complexity of textbooks require drastic change if one is to see an improvement in student performance. If the results from this study are an indication of the current state of affairs in textbooks, there is work to be done.

Appendices

Appendix A: Levels of cognitive demand (Smith & Stein, 1998)

Lower-level demands (memorisation):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be re- produced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlies the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used .

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and non-algorithmic thinking - a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the un- predictable nature of the solution process required.

Appendix B: Sample coding of scientific concepts TB 3

Block:	**N1**	**N7**	**G2**	**Total**
Concept:	**2D shape**	**Polygon**	**Octagon**	
Concept qualifies as everyday concept if: • Common salient characteristics of objects presented, but not the essential characteristics. • Learning is confined to the procedural. • Rote memorisation of definitions mistaken for concepts.	1	0	0	**1**
Concept qualifies as scientific concept if: • Verbal definitions and word meanings are made clear • Procedural *and* conceptual knowledge are taught • The essential characteristics of a class of objects are made clear • Generalisations are made possible	0	1	1	**2**
Link made between everyday and scientific?	0	0	0	**0**

Appendix C: Sample of misconceptions coding for TB 3

Misconceptions	**Code**	**Block (s)**	**Total occurrences**
Title of book:			
The establishment of prototypes by showing shapes only with horizontal basis and not in different orientations	PRO	T1, T2, T4, T5, T6 and N8	**5**
Triangles can only be equilateral	TRI	0	**0**
A rectangle is not a parallelogram	REC	N8	**1**
A square is not a rectangle	SRE	N8	**1**
Total:			**7**

Appendix D: Sample of levels of cognitive demand coding for TB 3 activities

Title of book:					
Task requirement	**Level of demand**	**Block T1**	**Block T4**	**Block T6**	**Total**
Name/Identify	1			1	1
Fill in the missing word/vocabulary	1	1			1
Make a drawing	1		1	1	2
Follow steps to make a drawing	1				
Measure	1		1	1	2
Match words to content	1			1	1
Build a shape	1				
Cut out	1				
Arrange in descending/ascending order	1				
Categorise	2				
Sort	2	1			
True/False	2				
Describe	2				
Similarities and differences	2				
Draw a conclusion	3				
Explain	3				
Problem solving	4				
Total:		1	2	4	7

References

Atebe, H. O. (2008). *Students' Van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa.* Unpublished doctoral thesis, Grahams town: Rhodes University.

Atebe, H. O. & Schafer, M. (2008). "As soon as the four sides are all equal, then the angles must be 90° each"- Children's misconceptions in geometry. *African Journal of Research in Mathematics, Science and Technology Education, 12*(2), 47 – 65.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester, Jr. (Ed.) *Second Handbook of Research on Mathematics Teaching and Learning*, (843 – 908). Charlotte, NC: Information Age Publishing.

Battista, M. T. & Clements, D. H. (1988). *Learning Geometry with Logo: Two Research Studies.* Paper presented at the NCTM research precession, Chicago, April 1988.

Bartolini Bussi, M. B. (1996). Mathematical Discussion and Perspective Drawing in Primary School. *Educational Studies in Mathematics, 31*(1), 11 – 41.

Bartolini - Bussi, M. B., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In L. English. (Ed.), *Handbook of International Research in Mathematics Education.* (pp. 746 – 783). New York: Routledge.

Berger, M. (2005). Vygotsky's theory of concept formation and mathematics education. In *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Bergen, Norway* (Vol. 2, pp. 153 – 160).

Berger, M. (2006). Making mathematical meaning: from preconcepts to pseudoconcepts to concepts. *Pythagoras*, 63, 14 – 21.

Bodrova, E. (1997). Key concepts of Vygotsky's theory of learning and development. *Journal of Early Childhood Teacher Education, 18*(2), 16 – 22.

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, 119 – 156.

Bowie, L. (2004). *The interplay of the social, pedagogical and mathematical in a Mathematics textbook*. Available from: http://www.mes7.uct.ac.za/Bowie Paper.pdf Accessed: 10 November, 2014.

Burger, W. F & Shaughnessy, J. M. (1985). Characterising the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31 – 48.

Clements, D. H. (1998). *Geometric and Spatial Thinking in Young Children.* New York: US Department of Education.

Clements, D. H. (2004). Geometric and spatial thinking in early childhood education. In D. H. Clements and J. Sarama (Eds.), *Engaging young children in Mathematics.* (pp. 267 – 297). New Jersey: Lawrence Erlbaum.

Clements, D. H. & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws, (Ed), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics.* (pp. 420 – 464). New York, NY: Macmillan Publishing Co.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach.* New York: Routledge.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education,* 192 – 212.

Cole, M. (1996). *Cultural psychology: A once and future discipline.* Cambridge: Cambridge University Press.

Cooper, P. (2010). Problem solving in collaborative small group settings. In M. D. De Villiers (Ed.) *16th Annual AMESA National Congress, 28 March – 1 April 2010 George Campbell School of Technology Durban, KwaZulu-Natal* (269 – 277). Johannesburg: AMESA.

Daniels, H. (2001). *Vygotsky and pedagogy.* London: Routledge Falmer.

Davydov, V.V. & Kerr, S. (1995). The Influence of L. S. Vygotsky on Education Theory, Research, and Practice. *Educational Researcher, 24*(3), 12 – 21.

Department of Education (2011). *Curriculum and Assessment Policy framework: Mathematics (Grade 4 – 6).* Pretoria: Department of Basic Education. Available from: http://www.education.gov.za/LinkClick.aspx?fileticket=7AByaJ8dUrc%3D&tabid=672&mid=1885 Accessed: 4 June 2013.

De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics*, 11 – 18.

De Villiers, M. (1998). To teach definitions in geometry or teach to define? In A. Olivier & K. Newstead (Eds.), *Proceedings of the twenty-second International Conference for the Psychology of Mathematics Education:* 2, 248 – 255. University of Stellenbosch: Stellenbosch, 12 – 17 July 1998.

De Villiers, M. (2010). Some reflections on the Van Hiele theory. *Paper presented at the 4th Congress of teachers of mathematics of the Croatian Mathematical Society, Zagreb.*

De Villiers, M. (2011). Reflecting on the van Hiele Theory. In H. Venkat and A. A. Essien (Eds.), *17th Annual AMESA National Congress, 11 – 15 July 2011, Johannesburg, Gauteng. (171 – 179). Johannesburg: AMESA.*

Dole, S., & Shield, M. J. (2008). The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning. *Research in Mathematics Education, 10*(1), 19 – 35.

P. Ensor, T. Dunne, J. Galant, F. Gumedze, S. Jaffer, C. Reeves & G. Tawodzera (2002) Textbooks, teaching and learning in primary mathematics classrooms. *African Journal of Research in Mathematics, Science and Technology Education, 6*(1), 21-35.

Fan, L. (2011). Textbook research as scientific research: towards a common ground for research on mathematics textbooks. *Paper presented at the 2011 International Conference on School Mathematics Textbooks, Shanghai, China.*

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM Mathematics Education, 45*(5), 633 – 646.

Feza, N., & Webb, P. (2005). Assessment standards, van Hiele levels, and Grade 7 learners' understandings of geometry. *Pythagoras, 62*, 36–47.

Fischbein, E. (1993). The theory of figural concepts. *Educational studies in mathematics, 24*(2), 139 – 162.

Flanders, J. R. (1994). Textbooks, teachers, and the SIMS test. *Journal for Research in Mathematics Education, 25*(3), 260 – 278.

Fleer, M. (2009). Understanding the dialectical relations between every day concepts and scientific concepts within play-based programs. *Research in Science Education*, 39, 281 – 306.

Fleisch, B., Taylor, N., Herholdt, R., & Sapire, I. (2011). Evaluation of Back to Basics mathematics workbooks: a randomised control trial of the primary mathematics research project. *South African Journal of Education, 31*(4), 488-504.

Fujita, T., & Jones, K. (2003). The place of experimental tasks in geometry teaching: learning from the textbook designs of the early 20th century. *Research in Mathematics Education, 5*(1), 47 – 62.

Fuys, D., Geddes, D. & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for research in mathematics education, 3*, 1 – 196.

Gallimore, R. & Tharp. R (1991). A theory of teaching as assisted performance. In P. Light, S. Sheldon & M. Woodhead (Eds). *Learning to think*. London: Routledge/Open University.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal, 28*(4), 567 – 590.

Harries, T., & Sutherland, R. (2000). The representation of mathematical concepts in primary mathematics textbooks: a focus on multiplication. *Mathematics for living: The mathematics education into the 21st century project, Amman, Jordan, 142 – 146.*

Hasegawa, J. (1997). Concept formation of triangles and quadrilaterals in the second grade. *Educational Studies in Mathematics, 32*(2), 157 – 179.

Hedegaard, M. (1990). The zone of proximal development as basis for instruction. In L.C. Moll (Ed). *Vygotsky and education: Instructional implications and applications of socio-historical psychology* (pp. 348 – 370). Cambridge: Cambridge University Press.

Hedegaard, M. (1998). Situated learning and cognition: Theoretical learning and cognition. *Mind, Culture and Activity, 5* (2), 114 – 126.

Hedegaard, M. (2007). The development of children's conceptual relations to the world, with focus on concept formation in preschool children's activity. In H. Daniels, M. Cole and J. V. Wertsch (Eds.), *The Cambridge companion to Vygotsky.* (pp. 246 – 275). New York: Cambridge University Press.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology, 91*(4), 684 – 702.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education,* 524 – 549.

Hornsley, M., & Walker, R. (2005). Textbook pedagogy: A sociocultural analysis. *Has Passed Passed,* 47 – 69.

Howie, S. J. (2001). *Mathematics and science performance in grade 8 in South Africa 1998/1999 : TIMSS-R 1999 South Africa.* Pretoria: Human Sciences Research Council

Hulme, K. (2012). *The role of technology in the zone of proximal development and the use of van Hiele levels as a tool of analysis in a Grade 9 module using Geometer's Sketchpad.* Unpublished Masters thesis. Johannesburg: Wits University.

Human, P. G. & Nel, J. H. (1987). Alternative instructional strategies for geometry education: A theoretical and empirical study. *Final report of the University of Stellenbosch Experiment in Mathematics Education.* Retrieved from: http://mzone.mweb.co.za/residents/profmd/usewo1.htm Accessed 21 January, 2015.

Jones, K. (1998). Theoretical Frameworks for the learning of geometrical Reasoning. *Paper presented at the proceedings of the British Society for research into learning Mathematics, 28 February, 1998.*

Jones, K. (2002). Issues in the teaching and learning of geometry. In L.Haggarty (Ed), *Aspects of teaching secondary mathematics: Perspectives on practice.* (pp. 121 – 139). London: RoutledgeFalmer.

Jones, D., & Bush, W. S. (1996). Mathematical structures: Answering the "Why" questions. *The Mathematics Teacher*, 716 – 722.

Jones, D. L., & Tarr, J. E. (2007). An examination of the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks. *Statistics Education Research Journal, 6*(2), 4 – 27.

Kajander, A. & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology, 40*(2), 173 – 181.

Karpov, Y. V. (2003a). Development through the lifespan: A Neo-Vygotskian approach. In A. Kozulin, B. Gindis, V. Ageyev and S. Miller (Eds.), *Vygotsky's educational theory in cultural context.* (pp. 138 – 155). Cambridge: Cambridge University Press.

Karpov, Y. V. (2003b). Vygotsky's doctrine of scientific concepts. In A. Kozulin, B. Gindis, V. Ageyev and S. Miller (Eds.), *Vygotsky's educational theory in cultural context.* (pp. 65 – 82). Cambridge: Cambridge University Press.

Khembo, E. (2011). *An investigation into Grade 6 teachers' understanding of geometry according to the Van Hiele level of geometric thought*. Unpublished Masters thesis, Johannesburg: Wits University.

Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding it up: Helping children learn mathematics*, Washington DC: National Research Council.

Kozulin, A. (2003). Psychological tools and mediated learning. In A. Kozulin, B. Gindis, V. Ageyev and S. Miller (Eds.), *Vygotsky's educational theory in cultural context.* (pp. 18 – 38). Cambridge: Cambridge University Press.

Lesh, R. A., & Landau, M. (Eds.). (1983). *Acquisition of mathematics concepts and processes*. New York: Academic Press.

Levin, S. W. (1998). *Fractions and division: Research conceptualizations, textbook presentations, and student performances.* Unpublished doctoral dissertation, University of Chicago.

Mariotti, M. A. (2009). Artefacts and signs after a Vygotskian perspective: the role of the teacher. *ZDM Mathematics Education, 41*(4), 427 – 440.

Mariotti, M. A., & Fischbein, E. (1997). Defining in classroom activities. *Educational Studies in Mathematics, 34*(3), 219 – 248.

Mayring, P. (2000). Qualitative Content Analysis. *Forum Qualitative Sozialforschung/ Forum: Qualitative Social Research, 1*(2). Available from: http://www.qualitative-research.net/index.php/fqs/article/view/1089/2385. Accessed 9 January, 2015.

McKinney, C. (2005). *Textbooks for diverse learners: A critical analysis of learning materials used in South African schools*. Cape Town: HSRC Press

Mhlolo, M. K. & Schäfer, M. (2013). "The area of a triangle is 180°- An analysis of learners' idiosyncratic geometry responses through the lenses of Vygotsky's theory of concept formation. *African Journal of Research in Mathematics, Science and Technology Education, 17* (1-2), 83 – 93.

Miller, R. (2011). *Vygotsky in perspective*. Johannesburg; Cambridge university press.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Mitchelmore, M. C., & White, P. (2000). Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics, 41*(3), 209 – 238.

Mogari, D. (2003). A relationship between attitude and achievement in Euclidean geometry of Grade 10 pupils. *African Journal of Research in Mathematics, Science and Technology Education, 7*, 63 – 72.

Moll, L. C. (1990). Introduction. In L. C. Moll (Ed), *Vygotsky and education: Instructional implications and applications of socio-historical psychology.* (pp. 1 – 31). Cambridge, USA: Cambridge University Press.

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics, 42*(2), 179 – 196.

Mullis, I. V. S., Martin, M. O., Ruddock, G. J., O' Sullivan, C. & Preuschoff, C. (2009). *TIMSS assessment frameworks.* Boston: TIMSS & PIRLS International Study Center.

Nicely, R. F, Jr. (1985). Higher-order thinking skills in mathematics textbooks. *Educational Leadership, 42*(7), 26 – 30.

Outhred, R., Beavis, A., Stubberfield, C., Wilkinson, J., Murphy, M., & Kelly, D. (2013). *Formative Evaluation of Textbooks and Workbooks in South Africa.* Australian Council for Educational Research.

Plut, D. L., & Pešić, J. M. (2003). Toward a Vygotskian theory of textbook. *Psihologija, 36*(4), 501 – 515.

Reddy, V (Ed). (2012). *Highlights from TIMSS 2011: The South African Perspective.* Cape Town: HSRC Press.

Renne, C. G. (2004). Is a rectangle a square? Developing mathematical vocabulary and conceptual understanding. *Teaching children Mathematics,10*(5), 258 – 263.

Rezat, S. (2006). A model of textbook use. *Proceedings of the 30th PME Conference, 4, 409 – 416.*

Roux, A. (2005). The influence of language proficiency on geometric thinking. *South African Journal of Education, 2*(1), 1 – 9.

Ryan, J. & Williams, J. (2007). *Children's Mathematics: Learning from errors and misconceptions.* Berkshire: McGraw- Hill.

Schmidt, W. H., & McKnight, C. C. (2012). *Inequality for all: The challenge of unequal opportunity in American schools.* New York: Teachers' College Press.

Schmittau, J. (2004). Vygotskian theory and mathematics education: Resolving the conceptual-procedural dichotomy. *European Journal of Psychology of Education, 19*(1), 19 – 43.

Shäfer, J. (2010). *An Investigation of how Visual Arts can be used to teach Mathematical Concepts of Space and Shape in Grade R.* Unpublished Masters thesis, Grahamstown: Rhodes University.

Smith, M. S., & Stein, M. K. (1998). Selecting and Creating Mathematical Tasks: From Research to Practice. *Mathematics Teaching in the Middle School, 3*(5), 344 – 50.

Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics, 72*(3), 271 – 288.

Steele, D. (2001). Using Sociocultural Theory to Teach Mathematics: A Vygotskian Perspective. *School Science and Mathematics, 101*(8), 404 – 416.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455 – 488.

Stein, M. K., & Smith, M. S. (1998). Mathematical Tasks as a Framework for Reflection: From Research to Practice. *Mathematics Teaching in the Middle School, 3*(4), 268 – 75.

Suydam, M. N. (1985). The Shape of Instruction in Geometry: Some Highlights from Research. *The Mathematics Teacher, 78*(6), 481 – 486.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry.* Chicago: University of Chicago.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H. & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* London: Kluwer Academic Publishers.

Van der Sandt, S (2007). Pre-service Geometry Education in South Africa: A Typical Case? *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal, 1,* 1 – 9.

Van der Sandt, S. & Nieuwoudt, H. D. (2003). Grade 7 teachers' and prospective teachers' content knowledge of geometry. *South African Journal of Education, 23*(3), 199 – 205.

Vianna, E. & Stetsenko, A. (2006). Embracing history through transforming it: contrasting Piagetian versus Vygotskian (Activity) theories of learning and development to expand constructivism within a dialectical view of history. *Theory and Pedagogy, 16*(1), 51 – 80.

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal, 20*(1), 82 – 107.

Vygotsky, L.S. (1966) Development of higher mental functions. In A.N. Leontyev, A.R. Luria and A. Smirnov (Eds.) *Psychological Research in the USSR.* Moscow: Progress Publishers

Vygotsky, L. S. (1978). *Mind in Society: The development of higher psychological functions.* Cambridge, MA: Harvard University Press.

Western Cape Department of Education (2014). *Grade 4 – 6 textbook catalogue.* Available from: http://wced.school.za/branchPS/LTSM/lgsp.html#Grade4-6-Catalogue.pdf*inf.html Accessed: 10 November, 2014.

Wu, D. B., & Ma, H. L. (2006). The distributions of van Hiele levels of geometric thinking among 1st through 6th graders. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, 5, 409 – 416.*

Zorin, B (2011). *Geometric Transformations in Middle School Mathematics Textbooks.* Unpublished doctoral thesis, University of South Florida.