PREPLANNING GPS SESSION LENGTHS FOR CADASTRAL SURVEYING IN SOUTH AFRICA

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DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in Engineering in the University of Cape Town, Cape Town. It has not been submitted before for any degree or examination in any other University.

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ABSTRACT

Local Surveyors using GPS (Global Positioning System) are particularly interested to know for what length of time they need to collect GPS data when static surveying in order to achieve results suitable for cadastral surveying purposes. At present many surveyors are observing a minimum of one hour at a set-up for static differential observations. At times suitable accuracies are achieved on twenty minutes of observations and poor results on an hours worth of observations and vice versa.

Many factors influence the length of time for which one would observe, amongst them being :-

- Geometry of satellite constellation
- Number of satellites

and * Collecting sufficient data to resolve the ambiguities.

Preplanning packages presently available have pre-planning yardsticks or "indicators" which consider some of the above factors. The shortfall of these indicators, however, is that they evaluate satellite geometry for a given instant in time, rather than over an entire observation session, making them suitable for navigation purposes, but not necessarily for survey purposes. This dissertation considers using a preplanning factor known as BDOP1 (Bias Dilution of Precision 1) as an alternative to the factors presently included on preplanning software. The BDOP1 is used not only as an indicator of accuracy but also as an indicator of how long one should observe a GPS session.

Most researchers believe that the key to accurate positioning with GPS static surveying, lies in whether or not the ambiguity integer is correctly resolved. The BDOP1 factor takes into account the length of time required to collect sufficient data to resolve the ambiguities, unlike other pre-planning indicators presently available on software packages.

In addition, this dissertation evaluates a technique of observing with GPS known as pseudo-static surveying, in an attempt to find an alternative solution to solving the issue of the length of time for which one should observe.

Distances which would be commonly used in cadastral surveying were tested; the emphasis being on distances under 10km (but extending up to 23km).

The BDOP1 indicator proved to be invaluable in pre-planning the observation session lengths of 10km and less. It was found that with a BDOP1 of ten or less, A-class distances and directions to within a second were

easily achieved on distances less than 10km. A 100% of all distances tested (under 10km) achieved the 2A limit, and 89% achieved the A limit as defined in Regulation 11 of the Land Survey Act No.9 of 1927. Repeatability of the accuracy of a given observation session proved excellent. The results of the dissertation show that on the shorter baselines the geometry of the satellites over the entire observation session is more important than determining the integer value of the ambiguities, as regards to achieving accurate results.

As distances increased there tended to be a decrease in the level of accuracy achieved. On distances of the range 10 to 23km, 45% of all distances tested achieved the A-class limit and 85% of all distances tested achieved the 2A limit; the A limit being defined in the Land Survey Act. Although the success rate warranted the use of preplanning with BDOP1, it certainly did not guarantee A-class distances. This decreased accuracy, with the occasional very large random errors, can be attributed to errors which cannot be effectively removed by differencing or computer modelling, such as atmospheric conditions. As these errors cannot be forecasted accurately the possibility of any future pre-planning packages guaranteeing accurate results is remote (unless the post-processing software improves remarkably !). Lowering the BDOP1 to be used on the longer distances, did not necessarily guarantee a high accuracy.

In conclusion, it can be said that the BDOP1 factor is a valuable pre-planning tool for GPS surveys, not only in pre-determining the length of time for which one should observe, but also as an indicator of expected accuracy, thus making static surveys more economic and productive.

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LIST OF ACRONYMS

ANARDOP	Analysis of the Relative Dilution of Precision
BDOP1	Bias Dilution of Precision 1
BDOP2	Bias Dilution of Precision 2
BDOP3	Bias Dilution of Precision 3
DGDOP	Differential Geometric Dilution of Precision
DOP	Dilution of Precision
DOS	Disk Operating System
EDM	Electronic Distance Measurement
FARA	Fast Ambiguity Resolution Approach
.FLT	Float
.FIX	Ambiguity-fixed
GDOP	Geometric Dilution of Precision
GLONASS	Global Orbiting Navigation Satellite System
GPS	Global Positioning System
HDOP	Horizontal Dilution of Precision
ION	Ionosphere
P-code	Precise/Protected Code
PDOP	Position Dilution of Precision
RDOP	Relative Dilution of Precision
RMS	Root Mean Square
TDOP	Time Dilution of Precision
.TRP	Triple-differenced
UERE	User Equivalent range Error
URA	User Range Accuracy
UTC	Universal Coordinated Time
US DoD	United States Department of Defence
VDOP	Vertical Dilution of Precision

1. INTRODUCTION

1.1 WHAT IS GPS AND HOW DOES IT WORK ?

GPS stands for Global Positioning System. This is a constellation of 24 radio navigation satellites which orbit the earth at a high altitude every 12 hours. (At present seventeen of these satellites are orbiting the earth). The United States Department of Defence (US DoD) is responsible for the development of the all-weather system to facilitate the accurate positioning of their military vehicles twenty-four hours a day. The US DoD has kindly made certain codes emitted by the GPS satellites available to the civilian sector, and left it to the hands of the civilian sector and private research to use these codes as they wish. The outcome of this, is position fixing equipment in many forms - to date, cars with GPS receivers that can position-fix on electronic maps, cheaper hand-held devices for position-fixing, position-fixing devices fitted to aeroplanes and ships, and of course the GPS receivers used for surveying purposes. As one can imagine this is a field ever ready for further research on improving existing methods for position fixing, or inventing alternative methods of position fixing with GPS. (Hurn;1989) (Cannon;1990)

Further prospects for GPS, include combining the GPS system with the Russian GLONASS satellite positioning system, to increase satellite coverage for any given point on earth at any time.

GPS satellites emit radio waves which travel roughly at the speed of light. By knowing the speed of travel of the radio wave and measuring the time from when the signal left the satellite to when it was received on earth, one can determine the distance to the satellite (distance = velocity x time difference). One can then determine a position on earth by the intersection of these distances from the known positions of the satellites (a process known as trilateration in surveying). With a minimum of four distances to four different satellites, a unique solution is determined - the fourth satellite being required to solve for the receiver clock bias. (The system has been designed, so that once the full constellation of satellites is up, there will always be a minimum of four satellites in view).

1.2 SURVEYING WITH GPS

Surveying with GPS has many advantages, among them being the potential ability to survey for twenty-four hours a day as opposed to the daylight hours normally used, weather conditions are not a restriction on working hours, and the fact that intervisibility of observed points is no longer a prerequisite as in conventional surveying.

A high accuracy of position-fixing is achievable with a technique known as "differential surveying". This entails two separate GPS receivers tracking the same satellites at the same time, and a differential vector being determined between these two points. A number of techniques exist for surveying with differential GPS and are briefly discussed in the following chapter.

At present in South Africa, static surveying is the most suitable method and the method most used by local surveyors for the distances and accuracies required in cadastral surveying.

1.3 THE PROBLEM

Local surveyors using GPS in static differential surveying, are particularly interested to know for what length of time they need to collect GPS static observations in order to achieve results suitable for cadastral surveying purposes. At times suitable accuracies are achieved on twenty minutes of observations, and poor results on an hours worth of observations and vice versa.

Many factors influence the length of time for which one would observe, amongst them being :-

* Geometry of satellite constellation

and * Number of satellites (Merminod et al;1990)

Many preplanning software packages for GPS give one an indication of the satellite geometry and its relative merits for instants in time during a given observation session. While this is suitable for navigation purposes, it has its very little application when it comes to static surveying. This dissertation considers using a preplanning factor known as BDOP1 (Bias Dilution of Precision 1) as an alternative to the factors presently included on preplanning software. The BDOP1 is not only as an indicator of accuracy, but also an indicator of for what length of time one should observe a static GPS session with a given satellite geometry (Merminod et al;1990). The outcome of this would be accurate and more economically productive surveys. Additionally, this dissertation considers the technique of pseudo-static surveying, as an alternative method of solving the issue of how long one should observe a GPS session.

1.4 OUTLINE OF DISSERTATION

This dissertation commences by mentioning the various methods of surveying with GPS, the errors in GPS positioning, and the facilities available on static GPS software packages for pre-planning and post-processing. Post-processing software packages attribute the accuracy of static positioning to whether the cycle ambiguity has been accurately resolved or not, however they do not accommodate this in their preplanning packages. Many researchers have validated the fact that ambiguity resolution is the key to accurate static surveying (Wells;1986). Present recommendations on how long one should observe a GPS session tend to have been based on trial and error experimentation, with the general philosophy that the longer one observes, the more accurate one's results are. These suggested standards draw much criticism and will be further discussed in Chapter 3.

B. Merminod proposed a BDOP1 (Bias Dilution of Precision 1) precision indicator (Merminod et al;1990) which evaluated the potential of a given satellite geometry over THE ENTIRE OBSERVATION SESSION to solve the cycle ambiguities. The merits of this indicator were tested in the field by a number of experiments, designed to test whether the BDOP1 held any weight, and if so :

- what value of BDOP1 should be used to satisfy South African cadastral standards,
- what sort of accuracies could be expected with different values of indicator,
- reliability of these indicators were investigated by considering repeatability,
- how BDOP1 performed over varying distances, and

- how BDOP1 relates to the post-processing accuracy criteria.

The limitations of this indicator are also pointed out.

This dissertation tests distances which would be commonly used in cadastral surveying when using GPS static surveying. The emphasis being on distances under 10km as these distances are expected to be used more frequently. The distances tested, however, extended up to 23km. Longer distances were not tested as it was felt that these distances would seldom be used in cadastral surveying, and that atmospheric conditions which could not be modelled in the BDOP1 program, would adversely affect results on longer baselines.

Other pre-planning indicators invented by other researchers are mentioned in this project. Unable to obtain these computer programmes, these theories were not tested.

The pseudo-static surveying technique is described and tested as an alternative method to solving the problem of the length of time to observe. The ultimate aim being to make GPS surveys more economical and productive.

This dissertation was limited by the GPS receiver equipment available in the country at the time of the experiments. Dual frequency receivers were not obtainable, nor were the software or technique for Rapid-Static surveying.

Kinematic surveying was not investigated as it was considered inappropriate for the baseline lengths tested and the travelling times between the baselines (during which time one would have to maintain satellite lock). The aim of this research was to improve the static technique.

The last few chapters draw conclusions and recommendations from the experiments performed.

2. SURVEYING WITH GPS

This chapter discusses methods of surveying with GPS, the mathematics and post-processing of GPS observations, and the errors that are found in GPS observations.

2.1 METHODS PRESENTLY AVAILABLE FOR POSITION FIXING WITH GPS

This section discusses the various methods by which one may position-fix with GPS, with particular reference to differential or relative surveying.

Position fixing with GPS can be classified into two groups :-

1) Point Positioning (also known as Absolute Positioning)

The position of a single point is independently determined with respect to a well-defined coordinate system (i.e. the coordinate system is positioned and orientated with respect to the earth).

2) Relative Positioning (also known as Differential Positioning)

A point is determined with respect to another known point on the local system. (This is analogous to the survey polar).

This method is generally used for survey purposes (Merry and Van Gysen;1985) and may use pseudo-range or carrier-phase measurements (discussed in Section 2.1.1) or a combination of both. Relative positioning with GPS can further be classified into STATIC and KINEMATIC positioning.

The types of Relative Positioning that are discussed are: Static Surveying, Kinematic Surveying, Pseudo-static Surveying and Rapid Static Surveying.

2.1.1 STATIC SURVEYING

Remondi (1988) defines static GPS surveying as follows: "A relative positioning method, based upon the carrier beat phase, which can yield millimetre accuracies after fixing cycle slips and double difference ambiguities to integers." The process of static GPS observing involves keeping a minimum of two static GPS receivers stationary at selected points for a common period of time to collect GPS measurements.

GPS navigation mode uses range measured from at least three satellites to a ground station as its basic requirement for determining the position of the ground station. The GPS system works by timing how long it takes a radio signal to reach the ground station from a satellite and then calculates the distance using this time difference and velocity of the radio-wave. The distances measured to the satellites of known position, are used in a three-dimensional trilateration to determine the position of the ground point. The equations for pseudo-ranging are given in section 3.1.2.

The chief limitations on the accuracy achievable with pseudo-ranging are the unmodelled biases in the satellite orbits, and the biases introduced by refraction. It can be assumed that two points on the earth surface will be affected equally by these biases. Consequently the difference in position of these two points will almost be completely free of these effects. In spite of this, differential pseudo-ranging still has rather limited accuracy. The standard positioning service, which is the mode freely available to civilian users, offers accuracies of 100m in the horizontal position and 162m in the vertical position (Merry;1989).

An alternative is to use the carrier beat phase generated by the satellites. The difference in phase between the received carrier signal and the signal generated by the receiver oscillator is observed to be:

$$\Phi_i^j = \Phi_i(T) - \Phi^j(t)$$

The transmitted signal is generated at time t and is received at time T. The subscript i is used to refer to terms that identify with the receiver, and the superscript j is used to identify terms that depend upon the satellite. (Merry;1989)

The total phase measured consists of a measured fractional phase part and an integer count of phase cycles for a given epoch. The difference in phase depends on the transmission time from the satellite to the receiver. This in turn is a function of range and of ionospheric and tropospheric delays. The satellite and receiver clocks are not perfectly synchronised to GPS time, which results in phase shifts. Phase observations are ambiguous by an unknown integer number of cycles between the receiver and the satellite (cycle ambiguity). Once the satellite signals have been acquired by the receiver the whole number of cycles are tracked and counted. These effects are modelled in the carrier beat phase equation (Merry;1989) :-

$$\phi_i^j = \frac{f}{c}\circ\rho_i^j + f.(dt^j-\delta t_i) + \frac{f}{c}.(-d_{ion}+d_{trop}) + N_i^j$$

where	f	is the frequency of the carrier wave
	c	is the velocity of light in a vacuum
	dt^j	is the satellite clock error
	δt_i	is the receiver clock error
	d_{ion}	is the ionospheric refraction delay
	d_{trop}	is the tropospheric refraction delay
	N_i^j	is the integer cycle count (cycle ambiguity)
	ρ_i^j	is the range from receiver to satellite

where $\rho_{ij} = \surd[(x_i-x^j)^2 + (y_i-y^j)^2 + (z_i-z^j)^2]$

This equation can be transformed to units of length by multiplying the above equation by wavelength λ (= c/f). The equation has been grouped so that the observables appears on the left-hand side and the unknowns to be solved appear on the right-hand side :

$$\lambda\circ\Phi_i^j + d_{ion} - d_{trop} = \rho_i^j + c\circ dt^j - c\circ\delta t_i + \lambda\circ N_i^j$$

Multiple observations are required to solve for this equation, with a minimum of two receivers and two satellites. The last three unknowns in the equation above are usually dealt with in one of the following manners :

i) Using multiple observations in matrices to solve for the unknowns. However, this involves a large number of unknowns to be solved for, as each satellite-receiver combination has different solutions to the unknowns.

ii) By differenced simultaneous observation equations which eliminate some of the unknowns, and makes for easier computation.

The following types of differencing can be performed on carrier phase measurements :

a) *Between Receiver Differences* (Single differencing): Simultaneous observations are made from two receivers to the same satellite. This eliminates the satellite clock error, reduces the effects of refraction and orbit bias, and results in differences of the unknowns, as can be seen in the following equation:

$$\lambda . \Delta\Phi_{21}{}^{j} + \Delta d_{ion} - \Delta d_{trop} = \Delta\rho_{21}{}^{j} - c . \Delta\delta t_{21} + \lambda . \Delta N_{21}{}^{j}$$

where the subscripts 1,2 refers to the two receivers and the superscript j refers to the satellite

b) *Between Satellite Differences* :

Simultaneous observations are made from two satellites to the same receiver. This eliminates the receiver clock error. The equation can be expressed as follows:

$$\lambda . \Delta\Phi_{i}^{21} + \Delta d_{ion} - \Delta d_{trop} = \Delta\rho_{i}^{21} + c . \Delta dt^{21} + \lambda . \Delta N_{i}^{21}$$

where the superscripts 1,2 refers to the two satellites and the subscript i refers to the receiver

A *double-difference* equation is obtained by combining the equations of between receiver differences and between satellite differences, and can be expressed as follows :

$$\lambda . \Delta\Phi_{21}{}^{21} + \Delta^{2} d_{ion} - \Delta^{2} d_{trop} = \Delta\rho_{21}{}^{21} + \lambda . \Delta N_{21}{}^{21}$$

where the subscripts 1,2 refer to the receivers and the superscripts 1,2 refer to the satellites

Double-differencing results in the satellite and receiver clock errors being entirely eliminated and the orbit and refraction errors being considerably reduced. The term ΔN_{21}^{21} contains four cycle ambiguities, but the double difference of these ambiguities can be treated as a single unknown.

c) *Between Epochs Difference* :
In this case, observations from a single receiver to a single satellite are differenced over time.

$$\lambda.\Phi_i^j(t_2-t_1) + \Delta d_{ion} - \Delta d_{trop} = \Delta\rho_i^j(t_2-t_1) + c.dt^j(t_2-t_1) - c.\delta t_i(t_2-t_1)$$

In the above equation the cycle ambiguity term has been eliminated. These initial unknown integer number of cycles are the same over a particular observing session and can be represented by a single bias term. This only holds if there is no cycle slip. If the receiver loses lock on the satellite a new bias term must be introduced.

In practice, the *triple-differenced* approach, is used to identify cycle slips. The equation can be expressed as follows:-

$$\lambda.\Delta\Phi_{21}{}^{21}(t_2-t_1) + \Delta^3 d_{ion} - \Delta^3 d_{trop} = \Delta\rho_{21}{}^{21}(t_2-t_1)$$

The left hand side of the equation contains observables, while the right hand side of the equation contains satellite and receiver coordinates. This equation is only valid if no cycle slips occur during the observation period. Triple difference residuals will show large discontinuities at points where cycle slips have occurred. Algorithms have been developed to identify these slips and repair them. Once the cycle slips have been repaired, the solution can be repeated. (Merry;1989).

Once cycle slips have been repaired it is better to solve for the cycle ambiguity, using single or double differences, than to eliminate it. After the first solution of cycle ambiguities, the nearest integers to each of these are chosen and held fixed for the second iteration (other methods of ambiguity resolution are discussed in Section 2.3.1). There is a danger in this as the wrong integer may be selected, thus introducing a bias into the observations.

2.1.2 KINEMATIC SURVEYING

Kinematic surveying is a rapid type of surveying based on carrier beat phase where data are collected for approximately a minute at a number of selected points by a roving GPS receiver. A receiver is kept at a known master station for the entire length of time that the roving receiver collects data. Both receivers MUST maintain lock on at least four (preferably more) satellites during the survey. Before this survey is carried out, the receivers must be initialized to determine the integer ambiguities (Leick;1990) (Frei and Beutler;1989). This can be done by one of the following methods:

i) determining a baseline statically;

ii) setting up on an accurately known baseline and collecting GPS data; and

iii) by a process known as antenna swap. In this method, one receiver is placed at a station of known position, and the roving receiver is placed at an unknown station a few metres away. Four to eight epochs are observed. The receivers are swapped and a further few epochs are observed. The receivers are then returned to their original positions. During this process each receiver must maintain lock to a minimum of the same four satellites during the antenna swap. The antenna swap takes little more than 5-10 minutes to complete (Trimvec-Plus Manual;1990).

The primary strengths are speed, low data requirements and accuracy (Remondi;1988). It is important that there be no cycle slip while carrying out the survey, as each time lock is lost to the satellites, the integer ambiguities need to be re-evaluated. Although there are remedies in the field for this, this is ultimately the downfall of this method (Cannon and Schwartz;1989).

The equations for kinematic surveying are relatively simple, and are based on the static carrier phase observations mentioned earlier. For the case where the integer ambiguities are solved on a known baseline, or determined statically, we have the following double-differenced equation on the baseline AB:

$$\lambda.\Delta\Phi_{21}^{21}(t_1) + \Delta^2 d_{ion} - \Delta^2 d_{trop} = \Delta\rho_{BA}^{21}(t_1) + \lambda.\Delta N_{21}^{21}$$

where the superscripts 1,2 refer to the satellites,

the subscripts 1,2 refer to the receivers, and

the subscripts A,B refer to the baseline terminals

The roving receiver is then moved to the first point that is to be determined in the kinematic survey (called C), to obtain :

$$\lambda.\Delta\Phi_{21}^{21}(t_2) + \Delta^2 d_{ion} - \Delta^2 d_{trop} = \Delta\rho_{CA}^{21}(t_2) + \lambda.\Delta N_{21}^{21}$$

If B is known then C can be determined by differencing the two equations above:

$$\lambda.\Delta\Phi_{21}^{21}(t_2-t_1) + \Delta^3 d_{ion} - \Delta^3 d_{trop} = \Delta\rho_{CA}^{21}(t_2) - \Delta\rho_{BA}^{21}(t_1)$$

This requires four satellites; five is a practical minimum and six or more recommended (Remondi;1988).

In the antenna swap technique, the equations are similar:-
At time t_1, receiver 1 is at A, and receiver 2 at B , and at time t_2, vice versa. The double-differenced equations can be expressed as follows:

for time t_1 :-

$$\lambda.\Delta\Phi_{21}{}^{21}(t_1) + \Delta^2 d_{ion} - \Delta^2 d_{trop} = \Delta\rho_{BA}{}^{21}(t_1) + \lambda.\Delta N_{21}{}^{21}$$

for time t_2 :-

$$\lambda.\Delta\Phi_{21}{}^{21}(t_2) + \Delta^2 d_{ion} - \Delta^2 d_{trop} = \Delta\rho_{AB}{}^{21}(t_2) + \lambda.\Delta N_{21}{}^{21}$$

Differencing the above two equations, one can obtain :

$$\begin{aligned}\lambda.\Delta\Phi_{21}{}^{21}(t_2-t_1) + \Delta^3 d_{ion} - \Delta^3 d_{trop} &= \Delta\rho_{AB}{}^{21}(t_2) - \Delta\rho_{BA}{}^{21}(t_1) \\ &\approx 2\Delta\rho_{AB}{}^{21}\end{aligned}$$

where B is known and A is to be determined.

A provisional value of the unknown point is determined from the triple-differenced equation immediately above (using a minimum of four observed satellites.) Once this is determined, it can be substituted into one of the double-differenced equations above to solve for $\Delta N_{21}{}^{21}$. These cycle ambiguities that have been resolved, can then be used in subsequent double-differenced equations to solve for the unknown coordinates (Remondi;1988).

2.1.3 PSEUDO-STATIC SURVEYING (also known as PSEUDO-KINEMATIC surveying)

The pseudo-static technique takes into account the fact that statically observed observations are taken for a relatively long period of time because of the slow change in satellite-receiver geometry. Pseudo-static surveying attempts to reduce the length of time for which one must observe a static session, by occupying the point to be determined for a few minutes, at least twice under different satellite geometries (Minkel;1989), (Ashkenazi and Summerfield;1989).

A master station occupies a known point and continually collects GPS measurements during the entire survey session. Other receivers collect approximately ten minutes of data at points to be fixed, and then an hour or more later (an interval of 50-115 minutes seems to be optimum (Ewing;1990)) returns to collect a further ten minutes of data at each point. According to the TRIMVEC-PLUS Manual (Revision D), it is important to observe while the PDOP (Position Dilution of Precision, see Chapter 3) is low, and that as many satellites as possible should be tracked.

The roving receiver need not maintain lock on the satellites between moves. As a result, pseudo-static surveying was developed for use when kinematic GPS was unsuitable, e.g. large amount of travelling time between points, where masking of receiver antenna would occur during transport (e.g. under trees, buildings) (Ewing;1990). As with regular kinematic, the primary strength of pseudo-kinematic surveying is that one can occupy more sites in a given period of time, than one could with static surveying. More than one roving receiver can be used. Pseudo-kinematic has an advantage over kinematic in that loss of lock is acceptable as one moves from site to site. This means that the distance between the master and roving receivers is not a limitation, as is sometimes the case with kinematic surveying. Furthermore, satellite swapping is not of concern in pseudo-static surveying, as an addition of a satellite is in many ways similar to loss of lock (Minkel;1989).

The data is processed statically (Ewing,1990; Remondi,1988; Minkel,1989) with the presumption that the computer interpolates the missing data (as if fixing cycle slips) between the two ten minute slots, or simply ignores the missing data, for that particular point. According to Remondi (1988), Ashkenazi and Summerfield (1989) and Ewing (1990) a high precision and accuracy can be achieved (centimetre accuracies) and the method proves to be cost-efficient.

Pseudo-static surveying, does not have special

equations. Pseudo-static is like regular static GPS where there are periods of data outages. For best results, the double-difference integer ambiguities need to be set to exact integers (Remondi;1988).
Alternatively, the Nottingham technique could be used, as outlined in Ashkenazi and Summerfield (1989). The post-processing would be similar to kinematic surveying. This technique allows for cycle slips by employing modified algorithms to solve for the integer ambiguities.

2.1.4 RAPID-STATIC SURVEYING

This technique is similar to kinematic surveying in that it involves a minimum of two receivers, one a master and the other rovers, which collect only a few minutes of data at each station. Unlike kinematic surveying, the roving receivers need not maintain lock onto satellite carrier phase readings while moving between stations. The length of time required to solve the ambiguities at a survey station is a function of the number of satellites and their geometry. This is determined prior to ambiguity resolution by considering the variance-covariance matrix of the initial differential position (Frei and Beutler;1990). This technique generally employs dual frequency receivers to aid the resolution of integer ambiguities. The problem with this is that dual frequency receivers are dependant on the P-code which may not always be available for public use if the US DoD denial of accuracy policy proceeds and P-code encryption is imposed.

Data is processed using a variety of statistical ambiguity search algorithms to provide baseline precisions commensurate with traditional static survey methods (Talbot;1991). One of these methods, the "FARA" (Fast Ambiguity Resolution Approach) for dual frequency observations is outlined in Frei and Beutler (1990). The "FARA" technique is an elaborate statistical technique of selecting the most likely integers in a sequential fashion. All consistent alternatives are identified for the solution vector of coordinates and integer ambiguities, and the option that yields the smallest a posteriori variance of the unit weight is selected to solve for the integer ambiguities.

An alternative approach, the Sequential Ambiguity Resolution strategy is a technique designed for resolving rapid-static single frequency observations and is described in Talbot (1991).

2.1.5 WHY OBSERVE STATICALLY ?

At present in South Africa, most local surveyors use static surveying when position-fixing for cadastral purposes. The method is generally reliable, except for the problem of determining how long one should observe to achieve accurate results. Due to this problem, surveyors tend to be on the conservative side and collect an hour or more's worth of data for a given baseline.

Kinematic surveying has been used in South Africa often enough, but tends to be unreliable. Cycle slip is a major problem, and if not picked up in the field (as is often the case) and rectified, one's observations are rendered useless. When kinematic surveying has gone smoothly, the accuracy of results has been high.

Not many people have used pseudo-static surveying in South Africa. The author found the accuracy of this method to be poor and unsuitable for cadastral purposes and felt this was due to the primitive software used. Better quality software would surely yield better results.

Rapid-static surveying has only just arrived in South Africa, and few people have had the opportunity to test it. The author found that reasonable results, though not always A-class accuracy, can be achieved up to 15km with rapid-static surveying (Cochlovius Gouws;1992). Merry (1992) claims millimetre accuracy in both height and distance on baselines of a few hundred metres. Rapid-static observations are limited to periods of low GDOP (<8) only. This technique generally employs dual-frequency receivers. The shortcoming of this, is that dual frequency receivers are dependant on the P-code which may not always be available for public use if the US DoD denial of accuracy policy proceeds and P-code encryption is imposed.

Given the present methods available and the experience that the author has had of the reliability and accuracy of these methods, the author feels that the betterment of GPS static surveying for cadastral purposes is well worth pursuing.

2.2 FACTORS AFFECTING GPS OBSERVATIONS

A number of factors affect the length of time a GPS session should be observed, as well as the accuracy that can be achieved. These factors should be considered when pre-planning GPS surveys. Many of these factors can be modelled in pre-planning packages to optimise a survey. Only some of the biases and errors mentioned below can be eliminated or reduced during the post-processing.

The length of time for which one needs to observe a GPS session is dependant on :-

- the geometry of the satellite constellation during the observation session. (Because of the inclination of the orbits, satellite coverage is a function of site latitude. Even with the full deployment of the GPS constellation the distribution of visible satellites will not be uniform.) (Santerre;1988),
- the number of satellites during the observation session,
- receiver/hardware characteristics,
- location on the earth's surface,
- the conditions prevailing at each survey site at the time of the observation session,
- the atmospheric conditions,
- the length of the "window" for which sufficient

satellites are available,

- the accuracy which is sought (it is generally assumed that the longer the session, the more accurate the result) (Mazur;1990), and
- practical constraints, such as battery capacity, operator fatigue, and travel times between stations (King and Blewitt;1989).

2.2.1 ERRORS AND BIASES IN GPS OBSERVATIONS

The precision that can be obtained from GPS observations is affected by systematic errors and biases. These errors and biases affect one's observations, regardless of the methods of GPS surveying used. Errors and biases affecting GPS measurements can be grouped as follows :

(i) SATELLITE-DEPENDENT ERRORS

- Orbit bias : Errors are present in the broadcast ephemeris. The positions of satellites are known to a limited accuracy. The effect of this bias can be reduced by orbital modelling techniques (Ashkenazi et al;1989).
- Satellite Clock Bias : Most satellite clocks show a linear drift when compared with GPS time. Most GPS post-processing packages have a facility to model or eliminate this bias.
- Selective Availability : The US DoD has reserved the right at any time, to degrade the accuracy of the GPS signals used by the civilian sector, to

protect its military vehicles from hostile forces. This is usually done by degrading the satellite broadcast ephemeris or dithering the real-time navigation positioning accuracy. The effect on static differentially determined baselines is approximately 3ppm. Techniques exist for overcoming selective availability on differentially-determined baselines (Talbot;1990).

ii) OBSERVATION-DEPENDENT ERRORS

* The atmosphere has a significant effect on GPS signals. The tropospheric refraction has a retarding effect on the signals. As refraction close to the horizon is particularly bad, these observations should not be used. Ionospheric refraction causes the signals to disperse. Tropospheric and ionospheric refraction can be reduced by modelling the atmospheric conditions in post-processing software.

* The cycle-ambiguity is an observation-dependant error in that it differs for every receiver-satellite combination and for every observation session. This is solved for in the software.

iii) STATION-DEPENDENT ERRORS

* Multi-path errors : This occurs when the signal from a satellite arrives at a receiver along two or more paths due to the signal being reflected off smooth surfaces. The remedy for this lies in site selection and antenna design.

- The antenna phase centre of the receiver (which is the point at which the radiation appears to be received) may differ for different satellites at differing azimuths and elevations. This effect can be reduced by always mounting the antenna with the same orientation and by calibrating the antenna.
- Centring of the receiver over the point.
- Receiver clock-error : This is inherent in every receiver and can usually be eliminated by computer software.
- Cycle slips occur when the signal is obstructed temporarily and is not tracked for a number of cycles. These errors can usually be corrected for by computer modelling.
- Random measurement error which is due to the limitations of the receiver's electronics.

The UERE (User Equivalent Range Error) is the satellite-receiver range error resulting from the total combination of these biases affecting GPS measurements. A poor satellite geometry will magnify the effects of biases and errors.

The accuracy of GPS results is usually dependant upon:-

- the measurement precision,
- unmodelled systematic errors present,
- the processing methods used (e.g. methods of orbital adjustment, or ambiguity resolution,

manual editing of data or fixing of cycle slips, modelling of atmospheric effects.),

- the receiver-satellite geometry at the time of the observation session, and
- receiver characteristics (e.g. whether the receiver is single or dual frequency, the number of receiver channels, etc.) (King and Blewitt;1989).

To improve the accuracy of GPS determined baselines, the above-mentioned factors should be considered when observing and processing one's results.

2.3 POST-PROCESSING OF STATIC GPS OBSERVATIONS

This section discusses some of the facilities available on software packages for post-processing GPS static observations.

The available software varies considerably as far as bias modelling (orbital biases, clock biases, range biases and atmospheric biases) is concerned. Software can process data either in network mode, as individual baselines, or both.

The modelling of atmospheric conditions in software packages varies considerably. Some software packages allow a choice of models for post-processing; others have standard models and only some allow the input of actual weather conditions for a particular time. Observations are treated differently depending on whether the observations taken were of single or dual frequency.

The rate of data reduction varies, as does the treatment of cycle slips in different programs. Each set of software produces different output, particularly in error estimates.

The procedures used for processing the observations will depend upon the receiver type (code correlating or codeless), observables used (carrier phase, P-code transition phase, pseudo-range, or any combination of these) (King et al;1985). Certain elements are common to all these procedures: the data must be downloaded onto the processing computer which has a software package capable of processing the data to the required accuracy and ephemeris data for each satellite must be available for the time of observations.

To process the observed data, the coordinates of each satellite in a reference frame correlated with the exact time of observation is required. This information is usually stored in the computer in the form of Keplerian elements for the mid-time of observation or X,Y,Z values at minute intervals. The first step in using ephemeris data is to transform it to the coordinate system of the station coordinates. These coordinates are computed for each observation time by an orbit generating program or polynomial interpolator. As codeless receivers cannot decipher the navigation message and recover the broadcast ephemeris, the ephemeris information must be obtained from an independent source e.g a navigation receiver.

From the approximate site coordinates and satellite ephemeris, theoretical values of observations at each epoch of observation are computed. The observed values are compared with the theoretical values, and an improved set of site coordinates is obtained by using least squares procedures and modelling equations. The time required for data processing depends on the accuracy required, the software available, the number of baselines that can be processed at a given time and the amount of data lost due to sky-obstructions and equipment malfunctions. The coordinates or baselines obtained need to be transformed to one's local system and height differences should be corrected for geoidal slope.

2.3.1 AMBIGUITY RESOLUTION

Cycle ambiguity can be defined as the unknown integer number of cycles of the reconstructed carrier phase contained in an unbroken set of measurements from a single satellite pass at a single receiver.

Carrier beat phase measurements lead potentially to the most precise information about receiver-to-satellite ranges. The problem with utilizing this potential, is that of cycle ambiguity. Achieving a high accuracy hinges on the capability of resolving the cycle ambiguity (Wells;1986). It has been found that resolving ambiguities improves the positioning accuracy by a factor of four (Frei and Beutler;1989). Cycle slips add to the difficulty of solving this problem, in that every time a cycle slip occurs, the cycle ambiguity term needs to be reevaluated for that receiver-satellite combination.

The ability to resolve integer cycle ambiguities depends on the change in satellite geometry, the length of the observation period, the type and quality of available measurements and the effects of systematic disturbances (Grant,1990; Frei and Beutler,1989). Ambiguity resolution is generally considered to be the key to accurate horizontal positioning with GPS. If the ambiguities are well ascertained during the initial double-differencing adjustment, they are held fixed for the sequential double-differencing adjustment. Grant(1990) found that the horizontal coordinates of the ambiguity-fixed solution, were generally more accurate than the ambiguity-free solution, provided, of course, that the ambiguities had been fixed to the correct integer values.

Heights, however, were not found to be greatly improved by ambiguity resolution. In addition, Grant (1990) found that ambiguity-fixed solutions yielded far better repeatability on a baseline measured several times, than an ambiguity-free solution.

This was further reinforced by findings of Bock, where the precision of baselines of 10-30km in length is improved when the cycle ambiguities were resolved (Dong and Bock;1989).

On longer baselines, atmospheric conditions tend to play a greater role in determining the accuracy of one's results. While estimating the cycle ambiguities by double-differencing, their estimation is affected by those errors that are not eliminated by double-differencing, such as: tropospheric and ionospheric refraction, and orbit errors. Departures from integer values can be attributed to measurement noise and errors in the theoretical model for the observable (King et al;1985). The effect of these errors on double-differencing, and therefore also on the estimated ambiguities, increases with an increase in baseline distance (Dong and Bock;1989). The distance at which ambiguity resolution becomes difficult depends on the size of these errors. The distances over which these ambiguities can be estimated can be increased by taking dual frequency observations, and estimating tropospheric conditions and satellite orbits (Grant,1990; Dong and Bock,1989). It is not reasonable to assume that all ambiguities can always be solved.

The vertical component is particularly sensitive to unmodelled errors in the atmospheric refraction. The dominant error for the height component is the residual tropospheric error (Rizos et al;1989). Geometrically, it has been found that low satellites are preferable for horizontal surveys, while high satellites are more appropriate for vertical surveys. (Grant;1990)

With double differences, the ambiguity biases are estimated. Departures from integer values can be attributed to measurement noise and errors in the theoretical model for the observable. Ideally values for the bias parameters will be close to integers and uncertain by less than one cycle.

There are a number of different methods used to choose the correct integer values for the ambiguities, a few of which are (Frei and Beutler;1990) (Refer to Section 2.1.1):

a) *The Nearest Integer Method* : The estimated ambiguities are rounded to the nearest integer value. The nearest integer value is not necessarily the correct value as measurement noise and errors could adversely affect the solution. (D'Arcy-Evans;1991)

b) *The General Search Method* : As many as possible sets of integer-combinations are formed around the initial estimates for the ambiguities. The set of integer-combinations that yield the smallest sum of the squares of the residuals in a subsequent adjustment run is taken as the final solution.

The chosen solution may not necessarily be the correct one. To test the reliability of the solution the following test is performed : the ratio of the second smallest sum of residuals to the smallest sum of residuals should be greater than two or three for one to assume that the correct integer-combination had been found. This test is performed in the TRIMBLE TRIMVEC software.

The disadvantage of this method is that all ambiguities are resolved, as opposed to leaving some unresolved, at the risk of solving some ambiguities incorrectly and thus biasing the solution (D'Arcy-Evans;1991).

c) *The Confidence Interval Method* : With this method, a search is performed to determine all integers within a three-sigma range of the real-valued ambiguities determined from an initial adjustment. The estimated standard deviation is used to evaluate whether or not the resolution to an integer value is feasible from a statistical point of view.

If more than one, or no integers are found within this range then the ambiguity is not resolved. If one integer is found within this range, and the difference between the estimated and integer value is less than or equal to half the carrier wavelength, then the ambiguity is resolved to this integer. The adjustment may then be re-run with these ambiguities resolved, and the test repeated on the previously unresolved ambiguities. (D'Arcy-Evans;1991).

Other techniques have been developed by Dong and Bock; Blewitt, Remondi and others. A short description of these techniques is given in Frei and Beutler (1990) and Talbot (1991).

2.3.2 CYCLE SLIPS

Cycle slips occur when the satellite tracking stops for a moment and is subsequently resumed. The fractional phase measurement after the cycle slip has occurred is the same, but the integer number of cycles is different (With dual-frequency measurements, cycle slips can occur on either frequency), (King et al;1985).

Cycle slips can be dealt with in one of the following ways :-

a) By holding the station positions fixed (obtained by differencing between epochs) and calculating the residuals.

   Cycle slips can be visually identified as discontinuities in double-difference residuals. The times of cycle slip are noted, the observations corrected and the observations reprocessed.

b) A polynomial function can be fitted to the data to identify cycle slips, which can then be edited (Beutler et al;1984).

c) Remondi (1985) describes an automated cycle slip editing procedure. Preliminary station positions are obtained by triple-differencing. Outliers are then identified in the double-differenced phases as cycle slips and rounded to the nearest integer value. The receiver channel in which the slip occurred is identified and the cycle slip removed from all subsequent observations on that channel.

d) Goad (1985) proposed a base-station base-satellite concept, whereby cycle-slips pertaining to the base-site or base-satellite are used for all subsequent data pertaining to all other sites and satellites. This incorrect transfer of cycle slip to other receiver channels cancels out during double-differencing. Cycle slips are edited from the data and station positions are recomputed.

2.3.3 ALTERNATIVES TO DIFFERENCING

Algorithms have been developed which operate on the undifferenced phase observed. In these methods, the clock errors are eliminated at each epoch. This is achieved by a method of partitioning the normal equation matrix or transforming the data acquired at a particular epoch to derived observations containing no clock terms, by means of orthogonalisation algorithms (King et al; 1985). These methods tend to be used on large networks, and do not form part of this dissertation.

2.3.4 PROCESSING WITH TRIMBLE SOFTWARE AND EVALUATION OF THE RESULTS

For the purposes of this dissertation the Trimble GPS receivers and TRIMVEC-PLUS GPS Software (Revision D) were used. Although the TRIMVEC-PLUS software is referred to in particular, that which is mentioned below is applicable to many GPS post-processing software packages.

The data that is downloaded onto the computer for any given session consists of the following files :

i) *Ephemeris File* : This file gives the time of observations in GPS time, clock correction parameters and the Keplerian elements describing the orbits of the satellites.

ii) The *Ionospheric File* : The ION/UTC parameters are described in the file for the given session i.e. time corrections for the Coordinated Universal Time and an ionospheric model.

iii) The *Message File* : This gives information such as at which station the data was collected, antenna height, date of data collection, rough coordinates of the point, the receiver used, times of data logging and amount of data collected on the respective channels.

iv) *Data File* : This file has data collected from the satellite and includes information such as :

receiver clock error, raw distances computed by GPS receiver from ground station to the satellite, the number of uninterrupted code epochs, the fractional part of carrier phase measurement and the phase measurement time-tag.

Once the data has been downloaded onto the computer, the data processing may begin. The automatic processing of a GPS baseline consists of the following steps:

1) *Triple-differencing*. This method is the fastest method of converging data as there are no integer ambiguities to solve. This method is most sensitive to cycle slips. A TRIMBLE .TRP file is the output.
2) *Cycle-slip fixing*. Each triple-differenced epoch is compared with the neighbouring epochs to check for large jumps, which would indicate cycle slip. Cycle slips are then repaired with a polynomial to bridge the cycle slip.
3) *Double-differencing*. Double-difference processing is used to estimate and fix the integer ambiguity. Clock errors are eliminated in double-differencing. A double-difference float solution in the form of a TRIMBLE .FLT file is the output.
4) *Integer bias selection*. The biases calculated in the float solution are fixed to the nearest integers, and other closely related sets of integers are sought. The sum-of-squares error is determined for each set of integers, and these

ordered from smallest to largest. The second smallest set is divided by the smallest set to determine the Quality Factor. If the Quality Factor is three and greater, the smallest set of sum-of-squares error, would be selected as having solved the integer ambiguities correctly. This has been described as the general search method in Section 2.3.1.

5) *Double-differencing*. Once again the observations are double-differenced, but this time the integers are held fixed. The solutions are in a TRIMBLE .FIX file.

Baseline processing can be done manually. This is usually done when there are problems, e.g. unhealthy satellites that need to be deleted, cycle slips that were not adequately repaired, when initial pseudoranging may improve one's results, etc.

A number of criteria are available for analyzing the precision and trustworthiness of the results of GPS observations.

According to the TRIMVEC manual, the RMS (root mean square) of fit indicates how noisy the data are, and this generally gets larger with increasing line length. According to the PoPS (Wild Processing Software) manual, the RMS values of the double differences should not differ significantly before or after ambiguity resolution. Large discrepancies indicate poor or incorrect ambiguity resolution. When the ambiguity-fixed solution does not meet the qualifiers mentioned below, the ambiguity-free solution should be used, provided the RMS of fit is better than 0.08 metres.

The TRIMVEC manual recommends the following criteria for "acceptable" results (results one could have confidence in) :

- An RMS of 0,02 + (0,004*L) metres, where L is baseline length in kilometres
- A Quality Factor (also known as the Quality Ratio) of three and greater
- The difference between fix and float solution should be small (of the order of 10cm in each coordinate component)
- The RDOP should be less than 0,1 for static surveys.

The Quality Factor is the ratio of the sum of the squares of residuals of the second best set of integers to the first best set of cycle ambiguity integers. The larger the value, the safer the estimation of the integers is believed to be. When the value is greater than three, the ambiguity-fixed solution is strongest.

The RDOP (Relative DOP) value is a measure of the strength of the survey. RDOP is defined similarly to PDOP, but uses the post-processed carrier-phase observations over the entire observation session as opposed to instantaneous pseudo-range observations at the pre-planning stage.

When processing the data, it is advisable to check the URA (User Range Accuracy), also known as the User Equivalent Range Error (UERE), for each of the observed satellites. The URA is used for navigation purposes, but it is useful in the survey context in that it enables one to detect unhealthy or problem satellites. A high value indicates a suspect satellite. The URA is a statistical indicator of the contribution of apparent clock and ephemeris prediction accuracies to the ranging accuracies obtainable with a specific satellite, based on historical data (Cochlovius Gouws and Merry;1992).

3. PRE-PLANNING STATIC GPS OBSERVATIONS

This chapter discusses :

* the pre-planning software facilities presently available on GPS post-processing packages,
* pre-planning software facilities that are documented - but not readily available, and
* factors that need to be considered in pre-planning a GPS survey.

3.1 FACILITIES PRESENTLY AVAILABLE ON GPS POST-PROCESSING SOFTWARE PACKAGES FOR PRE-PLANNING GPS OBSERVATION SESSIONS

Pre-planning is important to increase the productivity of a GPS survey. At present it is essential, as one needs to identify satellite windows where a minimum of three or four satellites with a strong geometrical configuration appear.

Software presently available includes the following facilities to assist one in pre-planning one's observations for a given session :-

* Skyplots of satellite paths,
* Number of visible satellites,
* Satellite health,
* Azimuth and elevation of satellites,

- Tables of constellations of satellites available, and
- GDOP, PDOP, HDOP, VDOP and TDOP vs Time for a receiver.

The above can be computed for a given place and time with options to select particular satellites and mask areas where satellites would not be visible from the site.

3.1.1 THE DOPS (Dilution of Precisions) AVAILABLE ON POST-PROCESSING SOFTWARE PACKAGES FOR PRE-PLANNING GPS OBSERVATION SESSIONS

Present indicators used for evaluating the geometric "strength" of a given satellite configuration when pre-planning static GPS surveys, include PDOP (Position Dilution of Precision) and GDOP (Geometric Dilution of Precision). These components can be broken down further to evaluate the potential strength of a fix in the horizontal position (Horizontal Dilution of Precision (HDOP)), or in the vertical position (Vertical Dilution of Precision (VDOP)) or to evaluate clock errors (Time Dilution of Precision (TDOP)).

The GDOP and PDOP indicators are used to evaluate the geometric strength of a satellite configuration for a given instant in time. The GDOP and PDOP are calculated for real-time navigation solutions using pseudo-range observations. To compute PDOP and GDOP, pseudo-range equations between a GPS receiver and a minimum of four satellites in view, are generated. Four pseudo-range observations made simultaneously to four satellites enables a unique solution to be found. Further observations increase the number of redundancies. The optimum estimate for the coordinates and clock offset can be obtained by a least squares solution. The PDOP and GDOP values are obtained from the trace of the cofactor matrix, which is the inverse of the normal equation matrix, and are indicators of the precision of the coordinates and clock offset estimates.

In conventional surveying, the precision of a fix is reflected by components of the error ellipse or ellipsoid. The parameters describing the error ellipse or ellipsoid are obtained from the cofactor matrix, which differs from the covariance matrix by the apriori variance factor. In GPS, the error ellipsoid is commonly approximated by an error sphere, with a radius equal to the square root of the sum of square of the ellipsoid axes. The radius is referred to as the PDOP which is calculated from the square root of the trace of the coordinate components of the cofactor matrix. From the trace of the cofactor matrix one can also determine the HDOP, the VDOP and the TDOP (Merry;1989). The accuracy of a navigational position fix can be determined by multiplying the PDOP by the UERE (see Section 2.3.4).

The GDOP only differs from the PDOP in that it includes a receiver clock offset. The PDOP and GDOP graphs show the DOP values evaluated at each epoch. The PDOP and GDOP reflect the geometric strength of an *INSTANTANEOUS* GPS position fix. However, according to Merminod et al (1990); the absence of other indicators of good geometry (and by implication, indicators of precision), result in the PDOP and GDOP factors being used to pre-plan surveys.

While GDOP and PDOP are calculated using pseudo-range observables, the relative positioning mode used for more precise surveys relies on carrier phase observables. The essential differences between the two types of observables, is the cycle ambiguity present only in carrier phase observables. Random noise affects the carrier phase observables to a lesser extent than pseudo-range observations.

There are many differing opinions on how to interpret PDOP and GDOP values as an aid to pre-planning surveys. Certain sources recommend that one plans observation sessions to occur where the PDOP is below five (Wells;1986), while other have found PDOPs above five can produce good results (Grant;1990). The author found that good results could be obtained regardless of the size of the PDOP. This will be discussed further in the Chapter 5.

3.1.2 THE MATHEMATICS OF COMPUTING DOPS

GPS uses range measured from at least three satellites to a ground station as its basic requirement for determining the position of the ground station. The GPS system works by timing how long it takes a radio signal to reach the ground station from a satellite and then calculates the distance using this time difference and velocity of the radio-wave. The distances measured to the satellites of known position, are used in a three-dimensional trilateration to determine the position of the ground point.

The range, ρ_i, from each satellite is ideally computed by the following expression for pseudo-ranges:

$$\rho_i = c.\Delta t_i$$

where c is the velocity of light in a vacuum

Δt_i is time difference between when the signal was sent and when it was received

However, in practice the satellite clock and receiver clock are not perfectly synchronised. The clock offset needs to be accounted for in our equation, and a fourth satellite would be required to solve for the clock offset. Therefore, to account for the clock offset :

$$\rho_i = c.(\Delta t_i + \delta t)$$

where δt is the clock error

The true ranges between a given satellite, i, and the GPS receiver on the ground, p, can be calculated in terms of the coordinates of the receiver and satellite:

$$\rho_i = \sqrt{[(x_p-x_i)^2 + (y_p-y_i)^2 + (z_p-z_i)^2]}$$

where x, y, z are the coordinate components

When the above two equations are combined, there are four unknowns (x_p, y_p, z_p, δt), which can eventually be solved for in four simultaneous equations to give the station coordinates and clock off-set.

The corrected pseudo-range, R_i, can be rewritten as :

$$R_i = \sqrt{[(x_p-x_i)^2 + (y_p-y_i)^2 + (z_p-z_i)^2]} - c.\delta t$$

In order to solve for the unknowns in this equation by simultaneous equations, this equation needs to be linearised by differentiation :

$$R_i = \rho_i^o + \frac{x_p^o - x_i}{\rho_i^o}.dx_p + \frac{y_p^o - y_i}{\rho_i^o}.dy_p + \frac{z_p^o - z_i}{\rho_i^o}.dz_p - c.\delta t$$

where ρ_i^o is the range computed from the provisional receiver coordinates.

The simultaneous equations may be written in matrix notation as:

$$\mathbf{A.x = L}$$

where **A** is the matrix of the coefficients of the unknowns, **x** is the matrix of the unknowns $(dx_p, dy_p, dz_p, \delta t)$, and **L** is the misclosure vector $(R_i - \rho_i^o)$. The unknowns can be solved for by the following equation:

$$\mathbf{x = (A^TPA)^{-1}.A^TPL}$$

A weight matrix is represented by the matrix P. The cofactor matrix $(A^TPA)^{-1}$, also written as N^{-1}, is used to compute the various DOP factors. The respective DOP factors are computed by taking the square root of the sum of the relevant terms on the diagonal of the cofactor matrix. e.g. PDOP would include the dx_p, dy_p and dz_p terms, VDOP would only include the dh_p term, etc.

$$\mathbf{GDOP = \sqrt{[Tr(N^1)]}}$$

3.2 OTHER PRE-PLANNING FACILITIES NOT PRESENTLY AVAILABLE ON POST-PROCESSING SOFTWARE PACKAGES.

This section discusses pre-planning facilities not presently available on commercial GPS post-processing packages. These packages tend to have been developed at various universities around the world, and are mentioned as it is felt that these packages may answer the question of "How long should one observe a static GPS session?". Although every attempt was made to obtain all of these programs, the only software program which was available was the PREDICT package with the BDOP (Bias Dilution of Precision) indicators.

3.2.1 THE BDOP1 THEORY

Integer cycle ambiguities remain unresolved when GPS carrier-phase observations are double-differenced. For short-baselines, it is the reliable resolution of carrier cycle ambiguities during the phase reduction stage that is the key to precise relative GPS positioning (Merminod et al;1990). Ideally, indicators are sought, that reflect the difference in precision between an ambiguity-fixed (where cycle-ambiguities are solved for, converted to integers and held fixed as constants when solving for baseline components) and an ambiguity-free adjustment (where cycle-ambiguity is solved for, but not held fixed when solving for baseline components). It follows that, if the productivity of GPS surveying is to be increased, one should observe only long enough to resolve the integer ambiguities.

Merminod et al (1990), felt that a reliable estimate of the integer biases could be obtained from a strong ambiguity-free solution and proposed a set of BDOP (Bias Dilution of Precision) factors for pre-planning purposes. In a double-differenced observation model with the cycle ambiguity term per satellite-receiver combination assumed constant over an observing session, (pseudo-range observations with integer ambiguity built into the equations were used. (Discussion:Merminod:1991)), the normal equation matrix can be partitioned in the following manner :-

$$N_{xx} = \begin{bmatrix} N_{cc} & N_{bc}^T \\ N_{bc} & N_{bb} \end{bmatrix}$$

where the coordinate component is contained in N_{cc}, the cycle ambiguity component in N_{bb}, with N_{bc}^T and N_{bc} including terms that pertain to both coordinates and cycle ambiguities.

The cofactor matrix , Q_{xx}, is the inverse of the above normal matrix, and is similarly partitioned :

$$Q_{xx} = N_{xx}^{-1} = \begin{bmatrix} Q_{cc} & Q_{bc}^{T} \\ Q_{bc} & Q_{bb} \end{bmatrix}$$

The BDOP factors determined by Merminod were :-

$$BDOP1=\sqrt{TRACE(Q_{cc})}$$

BDOP1 gives an indication of the precision of coordinates before resolution of the cycle ambiguities as integers.

$$BDOP2=\sqrt{TRACE(Q_{bb})}$$

BDOP2 gives an indication of the precision with which cycle ambiguities can be determined.

$$BDOP3=\sqrt{TRACE(N_{cc}^{-1})}$$

BDOP3 indicates the precision after resolution of cycle ambiguities. BDOP3 is similar to the PDOP in that the coordinate components of the respective matrices are evaluated, however BDOP3 is accumulated over the entire observation session.

To compute the BDOP factors, double-differenced observations are ACCUMULATED in the normal matrix from a specified commencement time to a specified stopping time, observing all or specified satellites that would be available during that session. BDOP factors consider the geometry accumulated over an entire observation session, whereas GDOP and PDOP only evaluate the satellite geometry INSTANTANEOUSLY for a given instant of time. Using BDOP factors as pre-planning tools one can answer the question: How long must an observing session be so as to maximise the chances of resolving ambiguities ?

Merminod et al (1990), found that:

(i) For longer observation sessions there was a dramatic improvement in BDOP1 values. The number of satellites observed affects the magnitude and variability of the BDOP1 value;

(ii) The BDOP1 and BDOP2 graphs have a similar trend, as they describe similar properties of satellite configuration;

(iii) The definition of BDOP2 depends on the method used to overcome rank defects in the solved-for ambiguity parameters, and is therefore not uniquely defined. Interest in the BDOP2 factor is therefore rather limited and thus BDOP1 would be the better indicator to use for preplanning purposes;

(iv) BDOP1 and BDOP3 describe the precision of coordinates before and after ambiguity resolution.

Their definition is unique and are therefore suited for general purposes;

(v) The size of BDOP3 is always smaller than the corresponding BDOP1 (the precision of an ambiguity-fixed solution is generally better than an ambiguity-free solution); and

(vi) The precision of baselines for BDOP1 is virtually independent of baseline length and orientation, though its accuracy is dependent on systematic errors. BDOP factors compute the geometric strength of a point position; whereas observed GPS carrier phase adjustment is used to compute the relative position of two or more receivers (Grant;1990).

(vii) Times of low BDOP values do not necessarily coincide with times of low PDOP or GDOP values.

BDOP1 is not only an indication of how "good" the satellite geometry is, but can be used for the purpose of choosing the optimum set of satellites and an optimum observation time. BDOP1 is an indicator of relative precision and accuracy (provided systematic errors are not too large), and has been used in this dissertation for pre-planning purposes.

The suggested use of the BDOP indicators is as follows:

i) Decide on the satellites to be tracked.

ii) Compute the BDOP1 factor for varying session lengths and observation start times.

iii) Select the observation scenario that gives the lowest BDOP1 subject to two planning constraints (e.g. satellites tracked and observation start time, or satellite tracked and session length, or observation start time and session length) (Merminod et al;1990).

3.2.2 THE DGDOP THEORY

Magnavox have developed a "Differential GDOP" (DGDOP) program that provides a measure of the ability to resolve the cycle ambiguities as a function of the selected satellites and the duration and timing of the data collection interval (Hatch and Avery;1989).

Differential GDOP (DGDOP) is computed from the resultant cofactor matrix of "simplified" triple-differenced carrier phase observations (i.e. the effects of cycle ambiguities, clock errors and atmospheric effects on short baselines are cancelled). Using the Differential GDOP value (which is the mathematical equivalent of BDOP1, provided there are no changes to satellite constellation during the session), one can determine the duration and time of a GPS observation session. Generally, the lower the value of the DGDOP, the better the accuracy that can be achieved.

Like the BDOP factors, DGDOP is computed for a single site. The result of triple differencing is the cancellation of cycle ambiguities, which is possible since they should not change with time. A further advantage of using triple-differencing is that the loss of lock does not pose a problem. After triple-differencing (post-processing), the position coordinates solved for can be substituted into the original equations to solve for ambiguity and clock values.

Hatch and Avery (1989) argued that the triple difference solution accuracy was directly related to whether or not the whole cycle ambiguities could be successfully determined. Therefore, they argued that the computed variance or uncertainty in the triple difference solution could be used as a measure of the probability of whole cycle resolution. The variance of the triple difference coordinates can be computed by the inverse of the normal equation matrix scaled by the measurement noise. The GDOP which is computed from the square root of the trace of the normal equation matrix (based on pseudo-range equations), now computed from this triple-differenced carrier-phase solution is called the Differential GDOP (DGDOP). Assuming the measurement noise is constant, the DGDOP is a thus a measure of the probability of successfully resolving cycle ambiguities. The double-difference BDOP1 is considered by the Hatch and Avery (1989) to be the mathematical equivalent of the DGDOP.

The DGDOP is not dependent on baseline orientation, but is dependent on baseline length through the measurement scale factor. DGDOP is not a strong function of site location, but does depend on satellite geometry. The DGDOP computed for one site is characteristic of all sites in a large adjacent region.

The choice of satellites to be used, together with the duration and time of data collection, can be judged on the relative value of DGDOP obtained. The DGDOP program computes a value for these selected factors. The smaller the value of the DGDOP, the greater the probability that the cycle ambiguities will be properly resolved. To minimise the amount of time on site, the time of day and satellite selection are important. The DGDOP program is capable of generating a one-day chart of graphs for each combination of data collection interval and satellite constellation selected.

Hatch and Avery (1989) found that when satellites are tracked from horizon to horizon, the DGDOP value drops significantly below one and that for long observation intervals, it becomes relatively less important whether whole cycle ambiguities are resolved or not.

3.2.3 THE ANARDOP THEORY

The University of Calgary have developed a program called ANARDOP (ANAlysis of the Relative Dilution of Precision) which is used to assess the accuracy, and internal and external reliability of a single point or network for a selected observation session. Accuracy pre-analysis considers the random error propagation into the adjusted results. Reliability pre-analysis is concerned with the ability to detect outliers (e.g. cycle slips) and determine the influences of the minimum undetectable errors on the final results. Evaluating these criteria enable optimal design and quality evaluation of static differential GPS surveys.
The ANARDOP program computes the Relative Dilution of Precision (RDOP) from the cofactor matrix of the adjusted parameters for accumulated single-difference, double-difference (ambiguity-fixed or ambiguity-free) or triple-difference carrier phase equations. The RDOP is computed from the square root of the coordinate components on the diagonal of the cofactor matrix, similar to PDOP.

Internal and External Reliability may be evaluated for single-difference, double-difference (ambiguity-fixed or ambiguity-free) and triple-difference equations. The formula are given in Gang Lu, et al (1990). In the ANARDOP program, graphs are generated which show the incremental effects of amount of data collected with time in the observation session on the computed RDOP or Reliability factors. The RDOP and Reliability factors improve with length of time that data is collected. Accuracy estimates of the baseline are determined by multiplying RDOP by σ, where σ is a function of several parameters such as residual atmospheric and orbital errors and internal noise.

As this program was not obtainable, it was not used in the experiments of this dissertation.

3.3 PRESENT STANDARDS AND SPECIFICATIONS FOR GPS CADASTRAL SURVEYS

Discussed below are standards and specifications that apply to single baselines observed for cadastral purposes and which have been determined statically by GPS. It has been felt by many authors (Rapatz et al;1988) that "specifications and procedures are essential in ensuring the applicability of GPS to urban environments and providing a common standard for which performance may be easily interpreted and evaluated." Section 3.3.2 discusses present standards applicable to conventional cadastral survey methods in South Africa.

3.3.1 PRE-PLANNING GPS SURVEYS

SATELLITE SELECTION

A pre-planning package should be used to evaluate the health of the satellites before surveying with GPS. The geometry of the healthy satellites should be evaluated to determine the strength of one's fix. The GDOP is presently used to evaluate the geometric strength of the satellite configuration.

A satellite window, when sufficient satellites are available needs to be selected. Bearing in mind the limitations of the GPS receiver, as many healthy satellites as are visible for a given observation session are tracked. If the number of satellites which the receiver can track is limited, the satellites which give the best geometric configuration are selected. A minimum of four satellites should be tracked from at least three arbitrary quadrants of the sky during the observing session. The precision of the GPS vector baseline results depends on the number of satellites visible simultaneously from each station during an observing session, their geometric relationships, duration of the period when the number of satellites can be observed simultaneously, and the length of the line.

Rapatz (1987) is reported as saying satellites should not be observed at elevations less than 20 degrees in urban areas. In South Africa the cut-off value is generally accepted as 15 degrees, though this value is lowered where horizons are wide open (many surveyors observe all satellite observations, but apply the 15 degree elevation cut-off when processing the observations. This gives them the option of having extra observations to process later should they need to). Although low satellites are theoretically geometrically suitable for horizontal fixes, the effect of atmospheric refraction and multipathing increases with decreasing height above the horizon.

According to Rapatz et al (1987), observing sessions should have a Horizontal Dilution of Precision (HDOP) and Vertical Dilution of Precision (VDOP) that reaches a minimum of 6 sometime during the session. In their experience DOP values were unrealistically optimistic indicators of accuracy and should not be used in an absolute sense to determine the accuracy obtainable. Instead, DOP values should be used in the relative sense, to compare the geometrical strength of the various satellite configurations or to determine the variation of the geometric strength of a particular satellite configuration over time. When DOP values vary between good (low DOP) and bad (high DOP), they can be used as an indication of a very desirable variability in satellite geometry (Greggor;1991).

Other contradicting opinions occur on how to use PDOP or GDOP as a preplanning indicator. Merminod et al (1990) states that:

- Australian specifications suggest a maximum GDOP value of between 5 and 10 at the end of an observing session,
- American specifications recommend that best results can be achieved when GDOP values are changing during the observation session, and
- that Norton reported times of high GDOP may be best for observing carrier phase measurement.

Wells (1986) recommends that observation sessions should occur when the PDOP is below five.

It has been found by the author that one cannot state absolute rules about pre-planning from PDOP and GDOP. Good results were obtained on ten minute sessions where the PDOP or GDOP graph was very steep (where the satellite geometry is changing rapidly) and poor results when the graph was flat, and vice versa. Good results have been achieved irrespective of whether the PDOP or GDOP is rapidly changing or flat. However, it has been noticed that one generally needs to observe a longer session for a flat PDOP or GDOP, than for a rapidly changing PDOP or GDOP, in order to achieve results of similar accuracy (Cochlovius Gouws and Merry;1992).

BDOP1 (Bias Dilution of Precision 1 - See section 3.2.1) promises to be a better pre-planning indicator, in that it considers the satellite geometry over the entire observation session, (not just instantaneously - as does the PDOP and GDOP) and recognises the importance of ambiguity resolution for good results.

According to Merminod et al (1990): "A simple example of the inappropriateness of GDOP for GPS carrier phase adjustment is the fact that GDOP is undefined for three satellites, and yet three satellites can be used in a baseline solution using integrated carrier phase."

SITE SELECTION AND RECONNAISSANCE

Before the survey is conducted, a plan of the survey network should be designed and some knowledge of accessibility and travelling times and distances to chosen sites should be acquired (Berquist;1988). The number of possible observing sessions per day is a function of the required survey accuracy, satellite availability, and logistical considerations such as travel and set up time required between observing sessions (Federal Geodetic Control Committee;1986).

Sites should be chosen with minimum amount of obstruction to the satellites, so as to maximise the amount of data that can be received from the satellites and to avoid multipathing errors.
It is a good idea to place receivers with the same orientation at all stations.

DATA COLLECTION

In order to observe a baseline, two GPS receivers are required to collect data simultaneously during the observation session. While code tracking receivers synchronise their clocks on the Coarse Acquisition code signals, codeless tracking receivers must be externally synchronised to UTC (Universal Coordinated Time) to within milliseconds. Meteorological readings should be taken on longer baselines.

RECEIVER REQUIREMENTS

According to Rapatz (1988), the following are the requirements of GPS receivers:-

- Receiver must be capable of measuring carrier phase,
- receiver must be able to track 4 or more satellites simultaneously,
- data sampling rate should be a minimum of one observation a minute,
- the receiver should give an indication of working condition and data quality,
- receivers should be selected for the stability of their phase centre and overall quality of their design,
- antenna at both ends of the baseline should be orientated in the same direction, and be the same model of antenna,
- data recorders should as a minimum requirement be capable of recording phase, receiver clock time and signal strength, (Federal Geodetic Control Committee;1986)
- recording media should be tested or calibrated before using, and
- an uninterruptable power source.

Rapatz et al(1987) propose that all GPS receivers should be built to pre-determined specifications and be tested and calibrated. All receivers should be tested on a network of points by their users, who must not only demonstrate their ability to perform a GPS survey, but also to process the results to the required accuracy. Both the operator and the software should be certified as meeting the necessary requirements.

POST-PROCESSED RESULTS

The following have been suggested to obtain reliable post-processed results :

- A comprehensive check for blunders must be made
- The residuals and statistical evaluation of computed baseline should be analyzed.
- The Federal Geodetic Control Committee (1986) recommends that software be certified as capable of producing results specified for the survey, and that this be proved on a certified test network.

LENGTH OF TIME FOR OBSERVING SESSIONS

According to the experience of Rapatz et al (1987), it is necessary to observe at least four satellites simultaneously, with no uncorrectable cycle slips, for a minimum of thirty minutes. They claim it could take

"as long as two hours worth of observations to a constellation of three or four satellites to resolve ambiguities in a multipath environment. To determine the duration of the observations, it is necessary to take into account the presence of possible multipath as well as the quality of the data and the strength of the satellite configuration."
Greggor (1991) reports the Federal Geodetic Control Committee of the U.S. National Geodetic Survey prescribing observation periods in excess of an hour, which he claims may place a severe strain on the economy of using GPS for conventional survey work.

Greggor (1991) claims that :"The rough equivalent of Class A cadastral surveys, the FGCC Class 2-I (Version 5) effectively ensures about 20ppm (or 1:50000). This appears, in terms of our experience, to be unnecessarily pessimistic, as it is not too difficult to achieve better than 1:100 000 (Class 1)."
For the Class 2-I, the FGCC makes the following recommendation: A minimum of 5 satellites are to be observed for one hour, four of which are observed simultaneously for at least 30 minutes (for double-differencing computations).

According to Greggor (1991), opinions of South African users favours times of between 40 minutes, as an absolute minimum, to one hour when observing five or six satellites for at least part of the session.

The need to observe for an extended period of time is threefold:

i) The need for multiple measurements,

ii) to allow unknown atmospheric conditions to vary stochastically and help stabilize the affects on either end of the baseline, and

iii) to enable the solution to determine the number of whole cycles in satellite distance and to distinguish these from other nuisance parameters such as refractive effects. To achieve this the satellite needs to be given enough time to move through a fair arc in the sky (Greggor;1991).

Frei and Beutler (1990) claim : "It is well known from static positioning that observing for one hour or even longer allows for fixing the ambiguities on short baselines."

At this stage one notices that requirements for determining the length of time for which one should observe a static GPS session are difficult to lay down. This dissertation will investigate this problem.

3.3.2 SOUTH AFRICAN LAND SURVEY ACT REGULATION REQUIREMENTS FOR CADASTRAL SURVEYS

On 2 July 1992, the Chief Surveyor General issued Circular No.2 of 1992, discussing guidelines for GPS surveys. These guidelines, although still under discussion, are being used at present by Surveyor-Generals as an aid in assessing GPS surveys. The accuracy requirement of this circular is discussed at the end of this section. At present the South African Land Survey Act regulations (No.9 of 1927) has the following to say about fieldwork for cadastral surveys:-

INSTRUMENTS

Regulation 7 states that every Land Surveyor shall ensure that the instruments and equipment used in any survey for which he is responsible is in proper adjustment. The regulation prescribes that measuring tapes and EDMs must be calibrated against a standard base, of which an official record is made. In time, this regulation will surely extend to GPS equipment being calibrated on a standard base.

FIELD MEASUREMENTS AND OBSERVATIONS

- When the position of a point is determined by intersection, the angle at the vertex of the triangle shall be between 30 and 150 degrees. A point may not be determined by a single triangle only, unless observations are taken at all three points.

- When a point is determined by resection, its position shall not be determined by less than four points favourably situated.
- When traversing between two fixed points, observations must be taken at both fixed points in order that the traverse may be properly adjusted, unless the orientation is otherwise adequately checked.
- A baseline shall be measured in two directions, in one continuous length, as well as in two sections.
- Distances must be determined in metres and reduced for slope, and all other factors which enable the correct plane distance to be determined. Measurements based on trigonometrical beacons, must also be reduced for sea level and scale enlargement factors.
- Any survey of rural land or of a new township must be based on trigonometrical beacons.
- When land being surveyed is closer than 300m from a reference mark, is must be connected to the reference mark.

The conditions mentioned in this paragraph will surely apply to points determined by GPS measurements.

LIMITS OF ALLOWABLE ERROR IN FIELD WORK

* According to Regulation 11 : "When the position of a point is determined by polars, triangulation, trilateration or a combination of these methods, the displacement between any observed ray or measured distance and the final coordinates of the point fixed shall be of the order-

for Class A - A metres

for Class B - 1,5A metres

for Class C - 3A metres, and shall not exceed two times this quantity where :

$$A=0.012+\frac{0.082S}{3S+1000}+\frac{0.15S}{100000}$$

and S is the distance in metres between the known and unknown point."

Class A surveys refer to -

i) the determination of new reference marks or fixing reference marks in previously surveyed townships.

Class B surveys refer to -

i) the survey of new townships,

ii) the resurvey or subdivision of an erf in an existing township,

iii) the resurvey for the replacement of a beacon in a township, and

iv) the survey for the preparation of a diagram required under the law relating to the registration of mining titles in respect of precious stones and minerals.

Class C refers to all surveys not included in Class A or Class B, and includes surveys for mining titles in respect of base minerals.

- When the position of a point is determined by traverse,

  A = 0.01 + (Total traverse length)/24 000

  (Provided that when the traverse closes on the starting point, the closure for Class C does not exceed that prescribed for Class B.) The error in a traverse made for the purpose of determining the position of a curvilinear boundary may not exceed one percent of the length of a traverse.
- When the position of a beacon in a township is checked by the measurement of distance from adjacent beacons, the difference between a single measured distance and the adopted final distance shall not exceed 0.06 metres.
- When the vertical position of a point is determined, the difference between the determined and finally adopted height shall be of the order of 0.03m for Class B and 0.06m for Class C and may not exceed two times this quantity.

As these are the present regulated conditions in accuracy when determining points by conventional survey methods, these conditions will be used as yardsticks for evaluating the results obtained by the GPS experiments that follow. Class A accuracy in distances will be the ultimate goal.

The Chief Surveyor General's Circular No.2 of 1992, issued on 2 July 1992, specifies the following accuracy for GPS surveys :-

- Loop closures computed from individual base lines must not exceed 5cm + 4 ppm.
- Individual base line determinations must conform to a relative positional accuracy given by :-

  $s = e + 0{,}1\, pd$

  where

  s = maximum allowable error in centimetres

  d = distance in kilometres between stations

  p = minimum geometric relative position accuracy standard in parts per million

  e = base error in centimetres

  e.g. e = 2cm (1cm centring error at each end)

  d = 5km

  p = 4 ppm

  s = 4 cm

As this accuracy standard is not yet regulated, it will only be briefly discussed when evaluating BDOP1 as a pre-planning indicator.

4. EXPERIMENTS

4.1 INTRODUCTION

Local surveyors using GPS are particularly interested to know for what length of time they need to collect GPS static observations in order to achieve results suitable for cadastral surveying purposes. At times surveyors have achieved suitable accuracies with twenty minutes of observations, and poor results with an hours worth of observations and vice versa. Being able to preplan static session lengths will increase the productivity and efficiency of GPS surveying. The experiments outlined below seek to determine the length of time for which one should observe a GPS session.

This problem was tackled from two perspectives. The first approach was to investigate the possibility of pre-planning static session-lengths for a given satellite geometry. Initially the trends and patterns of PDOP graphs were investigated, and later the BDOP1 factor was investigated as a suitable pre-planning indicator. The second approach was to investigate whether pseudo-static surveying would be a better alternative in reducing the length of observing time. As rapid-static surveying was not available in the country at the time of the experiments (1990-1991), it was not investigated as an alternative.

All experiments were performed using trigonometrical beacons in the Honeydew-Midrand area near Johannesburg. Reduced GPS results were compared with join values of the published Gauss Conform co-ordinates of the trigonometrical beacons, for evaluation purposes.

A locality map of the Honeydew-Midrand area and all baselines measured has been included (Fig. 4.1). Figure 4.1 has been taken from the 1: 50 000 map sheets 2627BB Roodepoort and 2628AA Johannesburg. The length of baselines tested extended up to 23km. The emphasis of baseline lengths tested was under 10km, as it was felt that these distances would be most frequently used in cadastral surveying. The lengths of baseline tested extended to 23km to examine whether the BDOP1 factor was independent of baseline length, as claimed by Merminod et al (1990). The lengths of baselines tested were :- 2.9km, 4.4km, 5.6km, 6.0km, 8.2km, 12.6km, 14.6km, 16.5km, 16.9km, 22.5km and 23.0km. Experiments were often repeated for the same session over a number of days to test for repeatability. Many baseline lengths with intervisible trigonometrical beacons were measured by EDM for comparison (the Zeiss ELDI10, which has an accuracy of 5mm plus 3 parts per million was used).

All experiments were performed with TRIMBLE GPS receivers. In all the experiments performed A-class accuracy distances were aimed for. The cadastral A-class accuracy as described in the Land-Survey Act No.9 of 1927, has been described in Table 4.1. In most cases, whenever A-class distances were achieved, the accuracy of the directions was one second. Therefore, the accuracy of the distances was concentrated on. In assessing the results, the S-class accuracy as outlined in the Chief Surveyor General's Circular No.2 of 1992, issued on 2 July 1992, is also briefly discussed. (Refer to Table 5.12 and Section 3.3.2 for an explanation of the S-class accuracy).

Table 4.1. Explanation of Class A, B and C Surveys

According to Regulation 11 of the Land Survey Act (No.9 of 1927), Class A surveys are called for when Reference Marks are surveyed or resurveyed, or for any case that may be called for in the regulations. Class B is used for General Cadastral surveying. Class C is the lowest accuracy allowed for farm surveys.

According to Regulation 11 :- "when the position of a point is determined by polars, triangulation, trilateration or a combination of these methods, the displacement between any observed ray or measured distance and the final coordinates of the point fixed shall be of order :

for Class A - A metres
for Class B - 1,5A metres
for Class C - 3A metres

and shall not exceed two times this quantity where A is equal to -

$$0.012 + \frac{0.082S}{3S + 1000} + \frac{0.15S}{100\,000}$$

and S is the distance in metres between the known and the unknown point."

Below is a table of the distance surveyed using GPS and the relevant class of accuracies :-

DIST	A	2A	B	C
2.9 km	0.041	0.082	0.062	0.123
4.4 km	0.044	0.088	0.066	0.132
5.6 km	0.046	0.092	0.069	0.138
6.0 km	0.047	0.094	0.070	0.141
8.2 km	0.051	0.102	0.076	0.153
12.6 km	0.058	0.116	0.087	0.174
14.6 km	0.061	0.122	0.092	0.183
16.8 km	0.064	0.128	0.096	0.192
22.5 km	0.073	0.146	0.110	0.219

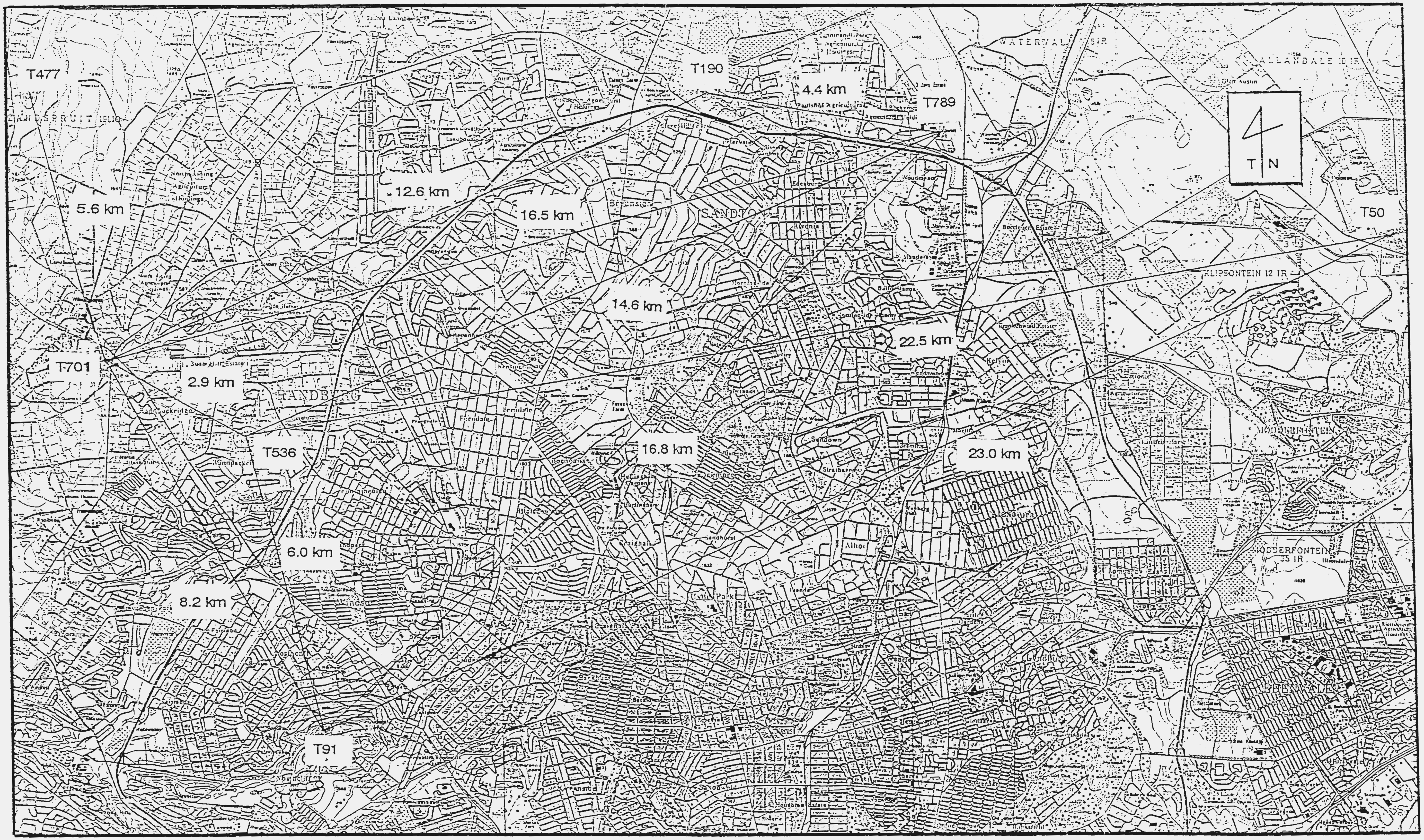


Figure 4.1. Locality map of Baselines Measured with GPS

4.2 STATIC SURVEY EXPERIMENTS

4.2.1 OBSERVING STATIC GPS OBSERVATIONS

The process of static GPS observing involves keeping a minimum of two static GPS receivers stationary at selected points for a common period of time to collect GPS measurements. Once connected to the power supply, the static program is activated and the instrument height and the point and session number is entered. The session is ended once the desired amount of data has been collected. Static observations can be pre-programmed.

All the experiments below mention the PDOP values for preplanning. The GDOP value has not been mentioned as it follows the PDOP trend. The PDOP value is a reflection of the geometric strength of a position fix for a given instant in time. Similarly the BDOP2 follows the BDOP1 trend, and has therefore not been mentioned as a pre-planning factor.

4.2.2 EXPERIMENTS PERFORMED

PROJECT : ALITIME

This project was used to get acquainted with the TRIMBLE receivers and TRIMVEC software. Data was collected for a number of sessions, varying in time from ten minutes to fifty minutes and under varying conditions of PDOP. An attempt was made to see if there was any direct relationship between length of time the session was observed, and the accuracy of results achieved. Any patterns of PDOP, and any effect they may have on the observed results was considered. All distances observed in this experiment were under 10km in length. A total of thirteen baselines were observed. Results from this experiment can be found in Table 2 and in Table 3 (observations number 10 and 11) of Appendix A. The BDOP1 factor was not considered at the pre-planning stage as it had not yet been learned of. It was however, entered into the tables in retrospect and the results evaluated against the BDOP1 factor.

PROJECT : ALITRI

The aim of this experiment was an attempt to find trends and patterns in the PDOP for pre-planning purposes. Results of the sessions were evaluated against the PDOP. All distances tested were under 10km in length. An investigation was made to determine what level of accuracy one could achieve with ten minutes of observations. Varying session lengths were also considered. A total of twenty-eight baselines were observed. The BDOP1 was evaluated in retrospect. The results from this experiment are shown in Table 3 (observations numbers 12 to 20), Table 4 and Table 5 of Appendix A.

PROJECT : 3 DAYS

At this stage, it was noticed that very little could be drawn from the PDOP patterns in the results from the above two experiments. The BDOP factors were evaluated at the pre-planning stage. The BDOP factors for the above two projects were looked at in retrospect, and it appeared that a BDOP1 of ten or less may guarantee A-class distances.

For this project, a number of sessions with a BDOP1 of ten and less were determined, each session having differing PDOP conditions and session lengths. The PDOP was still not eliminated as a preplanning factor at this stage. Figure 4.2 shows the shape of the PDOP during the selected sessions. This experiment was tested on a distance of 8.2km and over a period of three days to test repeatability of each session. Each of the six sessions observed daily was started four minutes earlier each day over the three days (as satellites can be predicted as travelling along their same paths, but appearing four minutes earlier each day). The results of this experiment were very favourable and the results were published by Cochlovius Gouws and Merry (1992). The results of this experiment are shown in Table 1 of Appendix A.

For this experiment and the following experiments, the observed sessions were pre-programmed into the receivers, to avoid inaccuracies by observers.

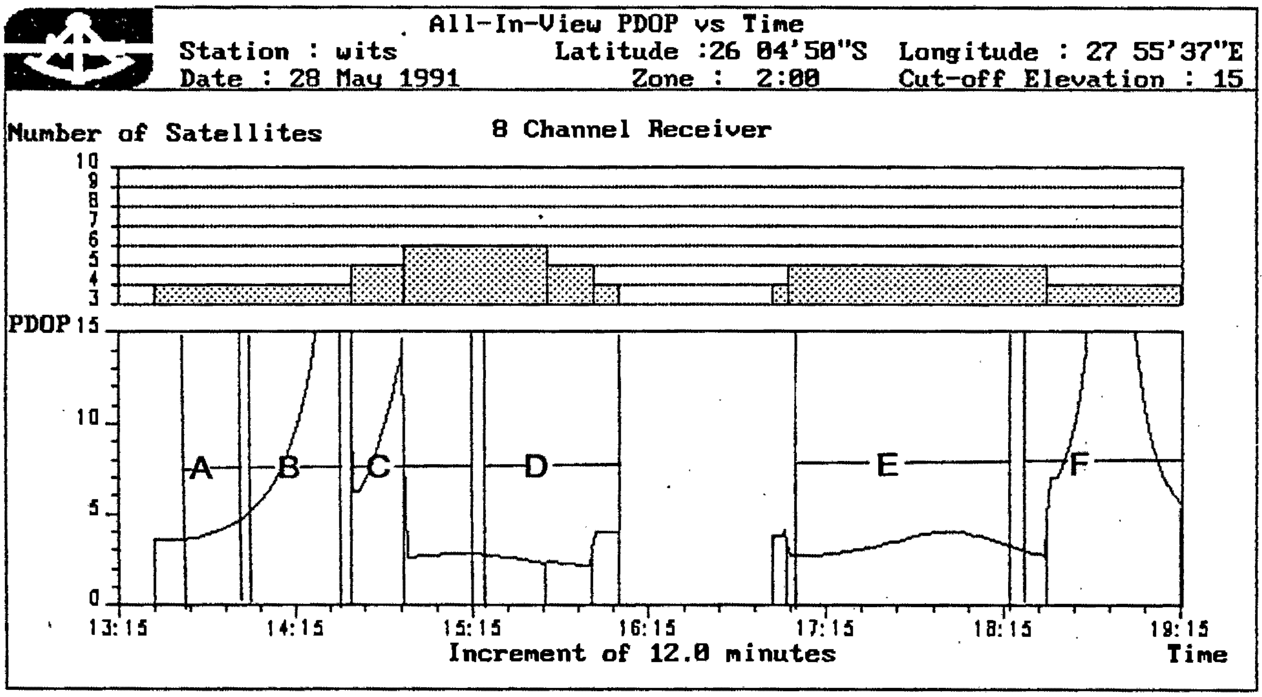


Figure 4.2 : PDOP Graph for Sessions A, B, C, D, E and F of Project 3 DAYS (Produced by Trimble Software)

PROJECT :BDOPT

This experiment was intended to be similar to the above experiment, but over varying distances under 10km and under varying BDOP conditions. The aim of the experiment being to determine just how high a BDOP1 could be used to achieve A-class results with good repeatability. Unfortunately, the GPS receivers were only available for one day, and this was discovered at short notice. For this reason, the experiment was carried out for one day only. A total of twelve baselines were measured. The results were still considered to be valuable in evaluating the BDOP factors. The Results are found in Table 6 and 7 in Appendix A.

Throughout this dissertation, GPS receivers were incredibly difficult to obtain. At the time of the experiments, there were only four GPS receivers available in the whole of South Africa. Much of the time the receivers were in use commercially. Doing the dissertation part-time meant trying to coincide the time when the receivers were available, when the author had time available and when there were financial resources available to hire the receivers.

PROJECT : 4DAYS

From the above projects, it was established that a BDOP1 of ten and less would give A-class distances on distances less than 10km. According to the theory in the determination of BDOP1 (Merminod et al;1990), the BDOP factors are independent of baseline length. This experiment tested a range of distances, largely for a BDOP1 of less than 10, and with a few sessions with a higher BDOP1. The range of distances tested included : 2.9km, 4.4km, 6km, 8.2km, 12.6km, 14.6km, 16.5km, 16.9km, 22.5km and 23km.

The sessions observed were the same for all four days (each particular session starting four minutes earlier each day).

Only the 6km baseline was tested for three consecutive days for repeatability. All other baselines were tested for one day only. The corresponding sessions were compared to identify any "problem" sessions.

On the third day of testing the 6km baseline, one of the stations had to be eccentrically observed, due to an antenna system that was rigged up on one of the trigonometrical beacons concerned. The eccentric reduction of observations might explain the slight inaccuracies on day 3 (see Table 17). The results of all other baselines, observed for one day only, are shown in Tables 8 to 16 found in Appendix A. A total of 142 baselines were measured.

Due to the difficulty in obtaining receivers, and the cost involved in hiring the equipment, a maximum amount of sessions was strived for each day (given the satellite availability at the time). This led to problems on the last few days of the experiment, as insufficient time for the batteries to recharge, meant not being able to observe all the pre-planned sessions.

4.2.3 REDUCING STATIC GPS OBSERVATIONS

The post-processing and evaluation of the GPS observations using Trimble software has been discussed in Section 2.3.4.

The GPS data were processed using the Trimble TRIMVEC software (using TRIMMBP or TRIM640 options), at the same rate as the observations were recorded (at fifteen second intervals). The ambiguity-fixed solutions are evaluated in all cases (except those expressly noted on the tables as being float-solutions). Once processed, the observations were reduced to the Gauss Conform system and listed on the tables together with their pre-planning indicators, their post-processing evaluation criteria and their comparative join values. The join values and height differences were determined from the Chief Directorate of Surveys and Land Information's published list of Gauss Conform co-ordinates for the trigonometrical beacons.

The GPS post-processed results were reduced to the Gauss Conform system as follows :- meridian convergence was applied to azimuths and distances were reduced to the Gauss Conform plane by GPS observed height differences and by applying a sea level and scale enlargement factor. Young's GPS1 program was used for this purpose.

Height differences are corrected for geoidal slope, using data taken from Merry and van Gysen (1987).

The difference between reduced GPS observations and the join values have been showed in Tables 18A, 18B, 18C, 18D and 18E of Appendix A.

The TRIMVEC post-processing evaluation criteria listed in the tables in Appendix A has been discussed in Section 2.3.4.

The conclusions of the results of static experiments has been discussed in the following chapter.

4.3 PSEUDO-STATIC SURVEY EXPERIMENTS

4.3.1 OBSERVING PSEUDO-STATIC GPS OBSERVATIONS

Pseudo-static observations are obtained by having one receiver continuously occupy a reference mark with known control coordinates during the entire survey session. Another receiver is moved between other survey marks and occupies them for up to 10 minutes, but is returned to each mark about an hour later for reoccupation (TRIMVEC-Plus Manual;1990).

To start pseudo-static surveys with TRIMBLE receivers, the Quickstart Survey option is activated once the receivers have been switched on. After the roving receiver has logged ten minutes of data, the antenna is disconnected (the receiver is left on) while one moves to the next mark. The receiver is not required to log data during move times, or to track the same satellites during each period. At each setup, the point name is entered. A minimum of two ten minute sessions separated by an hour are required for each point, although more sessions could be used.

Reoccupation after an hour allows sufficient change in geometry to occur to determine the double-difference integer bias terms. According to the TRIMVEC manual, it is important that the PDOP and RDOP are low - results with two ten-minute observation sessions with five satellites can be achieved when the PDOP ranges between three and five. The more satellites tracked, the better. The longer the interval between observations, the greater the change in satellite geometry and the greater the confidence in the results (TRIMVEC-Plus Manual;1990).

4.3.2 EXPERIMENTS PERFORMED

PROJECT : ALITRI

A number of ten minute sessions were observed statically during the project ALITRI described in Section 4.2.2. A number of these ten minute sessions, each selected pair being a minimum of an hour apart, were processed statically (the two respective files of each station being called up in the TRIMVEC static program). The two files for each station were chosen for not only being more than an hour apart, but also because each pseudo-static pair had different patterns of PDOP.

Distances of under 10km were used for this experiment. The results have been tabulated in Table A found in Appendix B, together with static post-processing evaluation criteria.

PROJECT : 3 DAYS

The observing procedure for this method is described in Section 4.3.1 above. Two ten minute sessions approximately an hour apart were observed in the same file, while a master station tracked data continuously. The experiment was repeated for three days in a row observing the same ten minute sessions (as far as possible) four minutes earlier each day, to investigate repeatability of results. The two sessions observed on all three days were observed during periods of low PDOP (less than 5) as recommended in the TRIMVEC Manual (Version C). A fourth set of two observation sessions were taken on a fourth day during periods of rapidly changing PDOP to evaluate the requirement of having a low PDOP.

The results were processed by entering the times of observation into a kinematic table, and then processing the results manually. The processing of these sessions was very primitive and awkward, but was the only TRIMVEC software available for pseudo-static observations at that point in time. A distance of 8.2km was used for this experiment.

The results of this experiment are shown in Table B of Appendix B.

PROJECT : 4DAYS

This experiment involved a master station which tracked data continuously while a roving receiver collected two ten minute sessions, at least an hour apart, statically. At the processing stage, these two static sessions were combined into one file using DOS commands. The results were then processed manually as for the project described above, using a kinematic table.

The experiment tested baselines of 2.9km, 4.4km, 6km, 12.6km, 14.6km, 16.9km, 22.5km and 23km for the same sessions over a period of four days. Each session was observed four minutes earlier each day, so similar sessions could be compared. On each baseline, six sessions were observed : four sessions with a low PDOP (less than five) and two sessions with a rapidly changing PDOP. The results of these experiments are shown in Tables C, D, E and F of Appendix B.

According to the TRIMVEC manual, distances longer than 20km could be observed pseudo-statically. This was investigated in this experiment.

4.3.3 REDUCING PSEUDO-STATIC GPS OBSERVATIONS

The pseudo-static observations were processed using TRIMMBP of the TRIMVEC software. The results are processed manually, using the KIN (kinematic) option on the menu. Once one had gone through the laborious process of identifying the exact occupation times, by stepping through the data, these were entered into a kinematic table for occupation times, and processed.

The post-processed results were reduced to the Gauss Conform System and compared with join values from the Chief Directorate of Surveys and Land Information's published list of Gauss Conform coordinates of the trigonometrical beacons used. The processed observations were reduced to the Gauss Conform System as follows: meridian convergence was applied to azimuths and distances were corrected for scale enlargement factor. A Maths-CAD package was used for this purpose. Height differences were corrected for the effect of geoidal slope, using data taken from Merry and Van Gysen (1987). The processed results in the table are tabulated against comparative join values and height differences, PDOP conditions and post-processing evaluation parameters (i.e. RMS and RDOP).

The difference between pseudo-statically observed and reduced sessions and join values has been shown in Tables G and H in Appendix B.

The conclusions of the results of pseudo-static experiments are discussed in the next chapter.

5. DISCUSSION OF RESULTS

5.1 INTRODUCTION

This chapter discusses the results (found in Appendix A and B) of the experiments described in the previous chapter. The static and pseudo-static results are evaluated in two separate sections of this chapter. Recommendations are discussed at the end of this chapter.

5.2 STATIC SURVEYING

5.2.1 DISCUSSION OF RESULTS OF PROJECTS CONDUCTED

(Tables 18A, 18B, 18C, 18D and 18E found in Appendix A, reflect differences between GPS reduced results and join values). All solutions presented in Appendix A, are the ambiguity-fixed solutions, unless stated otherwise.

PROJECT : ALITIME
(See Results in Table 2 and Table 3 (observations 10 and 11) of Appendix A).
The results from this project were evaluated as follows:
Generally, the longer one observes a session, the better the accuracy of one's results. However, longer sessions do not necessarily guarantee good results. Good results were obtained on as little as ten minutes (Table 3,

observation 10) and twenty minutes (Table 3 observation 11 and Table 2 observation 9) and poor results were obtained on longer sessions of forty minutes of observations (Table 2, observations Y and Z). It was clear from this experiment that collecting data for longer periods of time did not guarantee better results and that other factors needed to be considered. Good results were obtained, irrespective of the PDOP patterns.

Evaluating the BDOP1 factors for the various sessions at a later stage, showed that the two forty minute sessions with poor results had very high BDOP1 factors, while all the other sessions with good results had much lower BDOP1 factors. All BDOP3 factors were low, and it was difficult to see trends in results with the BDOP3 factors. Poor results were achieved with a low BDOP3 value, while good results were achieved with higher and lower BDOP3 values. For this reason, the BDOP1 was concentrated on.

Good results were generally accompanied by higher Quality Factors (refer to Section 2.3.1 and Section 2.3.4), although good results were also obtained with a Quality Factor below three. The poor results were accompanied by low Quality Factors. All results with a Quality Factor of three and more satisfied the A-class requirement.

Good results were obtained in spite of TRIMVEC RDOP or RMS requirements not being satisfied. Conversely, poor results (observations Y and Z in Table 2) were obtained when the RDOP and RMS requirement was satisfied.

At this stage it appears that the TRIMVEC requirements for short baselines (less than 10 km) are mere guidelines, and that the BDOP1 factor is useful in pre-planning and predicting "good" results.

PROJECT : ALITRI

(See Results in Table 3 (observations 12 to 20), Table 4 and Table 5, of Appendix A).

Many very short observation sessions were observed. Generally the results were poor, regardless of the PDOP. Better results were achieved with lower BDOP1 factors. To answer the question, whether A-class distances can be achieved on ten minutes worth of data - yes, it is possible (Table 5 observation 40, Table 3 observations 10 and 20). Notably these observations had lower BDOP1 factors.

The Quality Factor was low in all cases and the BDOP1 values were high. In all cases the RMS requirement was satisfied in spite of the results being poor. It appears that the TRIMVEC RMS requirements are of little use in evaluating the accuracy of results. Most results exceeded the RDOP requirement of being less than 0.1, which is reflected in the poor results obtained.

PROJECT : 3 DAYS

(See Results in Table 1 of Appendix A).

A number of sessions with a BDOP1 of ten and less, were repeated over a period of three days. Reduced distances generally fulfilled the A-class accuracy requirement. When comparing the reduced GPS directions and distances to the Gauss Conform directions and distances (join values) between the trigonometrical beacons, one notices that all directions are within one second of the join value and all distances are within 6cm of the join distance. If one considers the experiment was repeated for three consecutive days, we can see that the results are consistently to an A-class standard, though they may vary for the same session by a few centimetres from day to day.

Although not the main purpose of these projects, it is interesting to note that the agreement in height difference is not as good as that in distance. Although geoidal slope correction improved the accuracy of the height differences, there are still large discrepancies in height differences. This is not surprising, if one considers the accuracy of the geoid and of trigonometrical heighting.

Refer to Figure 4.2. to observe the PDOP trends during the sessions tested. Sessions were observed in all types of PDOP conditions. Low BDOP1 values can occur regardless of the shape of the PDOP curve. Accurate results can be achieved independent of the shape of the PDOP graph. There is a tendency for flatter PDOPs to require longer observation sessions than for rapidly changing PDOPs.

From Table 1, one notices that using a BDOP1 factor of approximately ten and less for pre-planning GPS surveys will generally ensure GPS distances of an A-class accuracy. The GPS-derived reduced directions generally differed only by a second from join directions. Heights on the other hand were not of suitable accuracy for general surveying purposes. These results for a BDOP1 of ten and less are also verified by the previous two projects.

The RMS requirements for accuracy were satisfied for all experiments using a BDOP1 of ten and less. Under most circumstances, the requirement that RDOP be less than 0.1 was satisfied. On the rare occasions where it was not satisfied, results were still satisfactory.

What is apparent, is the frequent failure of the results to satisfy the Quality Factor requirement of being greater than three. In spite of the Quality Factor being poor, good results were achieved. Even where the Quality Factor was consistently poor for the same session on three consecutive days, the results were within A-class accuracy. This questions the merit of the requirement for the Quality Factor being greater than three, or the importance of solving integer ambiguities on short baselines.

Generally, accurate distances and directions can comfortably be achieved from thirty minutes of observations, but this will not necessarily guarantee A-class distances. It is recommended that a BDOP1 factor of ten and less is a better criterion (Cochlovius Gouws and Merry;1992).

PROJECT : BDOPT

(See Results in Table 6 and Table 7 of Appendix A).

The A-class accuracy in distances was achieved on a BDOP1 of as high as 18.63 (Table 6, observation 41), with the 2A limit being satisfied with a BDOP1 as high as 23.13 (Table 6, observation 45). However, a BDOP1 value of 18.63 did not guarantee A-class distances (Table 6, observation 43). An observation with a BDOP1 of 13.49 missed the A-class limit on distances, satisfied the 2A limit, but had very poor accuracy in direction. An observation with a BDOP1 of 11.99 (Table 7, observation 49), satisfied the 2A limit in distance accuracy, but accuracy in direction was very poor. Observations 51 and 52, on Table 7, have BDOP1 values below 10, A-class accuracy distances and a high accuracy in directions. Referring back to Table 1, Session F has a BDOP1 of 9.98, achieved A-class accuracy in distances and a high accuracy in directions for three days in a row. At this stage, it still appears that to achieve A-class accuracy, a BDOP1 of ten and less should be used as a pre-planning factor.

PROJECT : 4DAYS

(See Results in Tables 8 to 17 of Appendix A).

Once again A-class accuracy distances and a high accuracy in directions was achieved on baselines of less than 10km for a BDOP1 less than ten (See Tables 8, 9, 15 and 17). Notably these accuracies were achieved on baselines of under 10km with a BDOP1 of 10.55. Therefore, a BDOP1 of ten and less will provide A-class accuracy distances on distances of less than 10km.

In Table 17, Day 3 (17-THURS), Sessions C to N were outside A-class accuracy. This can be explained by inaccuracies in reducing the observations that had to be observed eccentrically for those sessions on that day, due to an antenna that was erected on the trigonometrical beacon by another party. If one was to remove a constant of 5cm from all these sessions, the observations would achieve A-class accuracies well.

Once again, the repeatability of results on the sessions proved excellent (Table 17).

In most cases, the distances tested of less than 10km with a BDOP1 less than ten, achieved a Quality Factor of three and greater and satisfied RMS and RDOP requirements.

To investigate whether the BDOP1 theory is independent of baseline length, longer distances were tested. On distances of 12.6km (Table 16) and for a BDOP1 of less than ten, most distances satisfied the 2A limit rather than the A-limit. There was one outlier with a BDOP1 of 9.66 that satisfied the C-class limit. Directions were of a high accuracy. It would appear from the table that a BDOP1 of ten and less on distances of 12km would satisfy the B-class accuracy requirement. Notably, most of the observations did not satisfy the Quality Factor requirement of being greater than three, but did generally satisfy the RMS and RDOP requirements.

On the 14.6km baseline tested (See Table 10), most distances satisfied the A-class accuracy requirement for a BDOP1 of less than 10. The accuracy of directions was high. Fewer observations satisfied the Quality Factor requirement than for distances of less than 10km, but generally the RMS and RDOP requirements were satisfied. With the exception of one large outlier (observation 83), it can be said that the A-class accuracy requirement was satisfied. The large outlier occurred for this same session on three different baselines on the same day (observations 83, 97, 111). However on other days this session presented no problem at all (observations 55, 69, 125, 139, 153, 167 and Session C

(Table 17)). This phenomena was difficult to explain.

On baselines as long as 16.5km (Table 14) observed with a BDOP1 less than ten, it was generally found that distances satisfied the B-class accuracy limit and had a high accuracy in directions, with two exceptions (observations 138 and 140). Observation 138 was exceptionally poor, and hard to explain. This poor observation was not reflected in any of the other baselines observed at the same time. Fewer observations satisfied the Quality Factor requirement and the RDOP requirement, but all observations satisfied the RMS requirements. On baselines a little bit longer of 16.9km (Table 11) observed with a BDOP1 of less than ten, A-class accuracies were achieved with a high accuracy in directions, however there are two very large and unexplained outliers (observations 97 and 99). Fewer observations satisfied the Quality Factor requirement and the RDOP requirement, but all observations satisfied the RMS requirements.

In Table 12, the results of baselines of 22.5km, show C-class accuracies in the distance, and once again a high accuracy in direction. The Quality Factors and RDOP requirements are not generally satisfied, but the RMS requirements are. Once again there is a large and unexplained outlier (see observation 111). These findings are further verified in Table 13, where distances of 23km are tested.

From this project it appears that a BDOP1 of ten and less guarantees A-class distances on baselines less than 10km. On longer baselines from 10km to 15km, a B-class accuracy can be achieved in the distance. On baselines of 15km to 23km, C-class distances can be achieved. On all baselines tested (for a BDOP1 less than ten), a high accuracy was achieved in the direction - in the vicinity of one second. However, on baselines longer than 10km, there were frequent observations that were complete outliers, and that were difficult to explain (refer to Section 2.2.1). Baselines longer than 10km, observed for a BDOP1 less than ten, were less likely to satisfy the TRIMVEC post-processing requirements than baselines less than 10km. The longer the baseline, the less likely that the TRIMVEC post-processing evaluation requirements are satisfied. (All the blank areas in Tables 8 to 17, are sessions that were not observed due to the batteries going flat.)

While there was a tendency for lower BDOP1 values to yield better accuracies on longer baselines, it certainly did not guarantee A-class distances. A number of baselines longer than 10km, where the BDOP1 was less than five failed to yield A-class distances (refer to observations 116, 118 and 130 in Appendix A). This could be attributed to the definition of the A-class accuracy.

In terms of A-class accuracy, the longer the baseline, the greater the accuracy required. Further evidence that a lower BDOP1 will not necessarily satisfy A-class accuracies can be found in observations 83, 97 and 111 of Appendix A which are large outliers observed with a BDOP1 of seven.

The possibility of scale error on longer baselines was investigated. A scale error correction would have improved the accuracy on some baselines, but not all baselines. Scale error was not applied to baselines to avoid distorting the results.

5.2.2 GENERAL CONCLUSIONS IN STATIC SURVEYING

Refer to Table 5.1, for a summary of the accuracy of the results of the various baseline lengths for a BDOP1 of ten and less. Up until 15km it appears that A-class accuracies can be achieved. However due to a few outliers found after 10km, the author would recommend that baselines be limited to under 10km in length. Regardless of the baseline length there appears to be an accuracy in the vicinity of one second in the directions.

With a BDOP1 of ten and less, height accuracies are not of the same standard as distances and directions. This can be explained by the limited accuracy of trigonometrical levelling and knowledge of the geoid. Heights determined by GPS are particularly sensitive to atmospheric refraction. Whether integer ambiguities are resolved or not has little bearing on the accuracy of height determination (Rizos et al,1989). As determining height differences was not the main purpose of this project, they will not be discussed further.

The importance of solving the integer cycle ambiguity was investigated. The ability to solve the integer ambiguity is reflected in the Quality Factor which should be greater than three (Refer to section 2.3.1 and section 2.3.4). Consult Table 5.2 and Figure 5.1. One

Table 5.1 Comparison of Diferences between Reduced GPS Distances, Directions and Heights to Join Distances and Directions, and Heights (on the Gauss Conform System), for a BDOP1 of ten and less.

Distance = 2,9 km	n = 12		
	Distance (m)	Direction	Height (m)
RMS	0.015	0.6"	0.0
Mean diffs.	−0.014	3.1"	0.0
SDEV of Mean diffs	0.007	0.3"	0.0

Distance = 4,4 km	n = 7		
	Distance (m)	Direction	Height (m)
RMS	0.006	0.8"	0.0
Mean diffs.	0.001	0.7"	0.0
SDEV of Mean diffs	0.005	0.4"	0.0

Distance = 6 km	n = 33		
	Distance (m)	Direction	Height (m)
RMS	0.036	1.1"	0.3
Mean diffs.	0.014	1.0"	0.3
SDEV of Mean diffs	0.033	0.4"	0.1

Distance = 8,2 km	n = 25		
	Distance (m)	Direction	Height (m)
RMS	0.023	1.2"	0.4
Mean diffs.	0.000	1.0"	0.4
SDEV of Mean diffs	0.023	0.7"	0.1

Distance = 12,7 km	n = 7		
	Distance (m)	Direction	Height (m)
RMS	0.086	1.0"	0.2
Mean diffs.	−0.050	1.0"	0.1
SDEV of Mean diffs	0.052	0.0"	0.1

Distance = 14,6 km	n = 10		
	Distance (m)	Direction	Height (m)
RMS	0.038	0.8"	0.1
Mean diffs.	−0.035	0.3"	0.1
SDEV of Mean diffs	0.015	1.4"	0.0

Distance = 16 km	n = 16		
	Distance (m)	Direction	Height (m)
RMS	0.087	1.2"	1.1
Mean diffs.	−0.056	1.0"	0.2
SDEV of Mean diffs	0.066	0.6"	0.2

Distance = 23 km	n = 17		
	Distance (m)	Direction	Height (m)
RMS	0.123	0.9"	0.4
Mean diffs.	−0.113	0.8"	0.3
SDEV of Mean diffs	0.047	0.4"	0.2

notices that there is a dramatic improvement in RMS error in direction and distance once a Quality Factor of three has been reached. Below a Quality Factor of three, the RMS error in direction and distance is very high. Once a Quality Factor of three has been reached, the RMS error in distance and direction does not improve dramatically with an increase in Quality Factor. From this we can conclude, that the Quality Factor is a useful criterion in assessing one's results. When the Quality Factor criteria was not satisfied it did not mean that one's results were poor. This was seen very often in Tables 1 to 17 of Appendix A.

Table 5.2 Table of Quality Factor vs Error (0 – 23 km)

		Distances (metres)			Directions (seconds)		
Quality Factor	No. of Readings	Mean error/km	Sdev of error/km	RMS error/km	Mean error	Sdev of error	RMS error
1.0 – 1.2	51	−0.059	0.302	0.308	−2.6"	46.1"	46.6"
1.2 – 1.5	28	−0.077	0.359	0.367	13.8"	38.2"	38.3"
1.5 – 2.0	22	0.007	0.052	0.052	1.7"	7.9"	1.7"
2.0 – 2.5	11	0.025	0.069	0.073	38.9"	116.0"	122.4"
2.5 – 3.0	10	−0.018	0.029	0.034	45.7"	134.8"	142.3"
3 – 4	18	−0.004	0.011	0.012	1.4"	0.8"	1.6"
4 – 5	16	−0.003	0.005	0.005	1.0"	1.0"	1.3"
5 – 6	12	0.005	0.010	0.012	1.2"	1.0"	1.6"
6 – 7	8	0.000	0.002	0.002	1.4"	0.8"	1.6"
7 – 10	14	0.000	0.003	0.003	1.0"	0.7"	1.2"
10 – 13	8	0.001	0.005	0.005	1.1"	0.8"	1.4"
13 – 18	12	−0.001	0.007	0.007	1.6"	0.9"	1.8"
18 – 29	4	−0.001	0.002	0.002	1.7"	0.8"	1.9"
SUMMARY							
1 – 3	122	−0.007	0.260	0.268	8.9"	63.7"	65.3"
3 – 29	92	0.000	0.007	0.008	1.2"	0.9"	1.5"

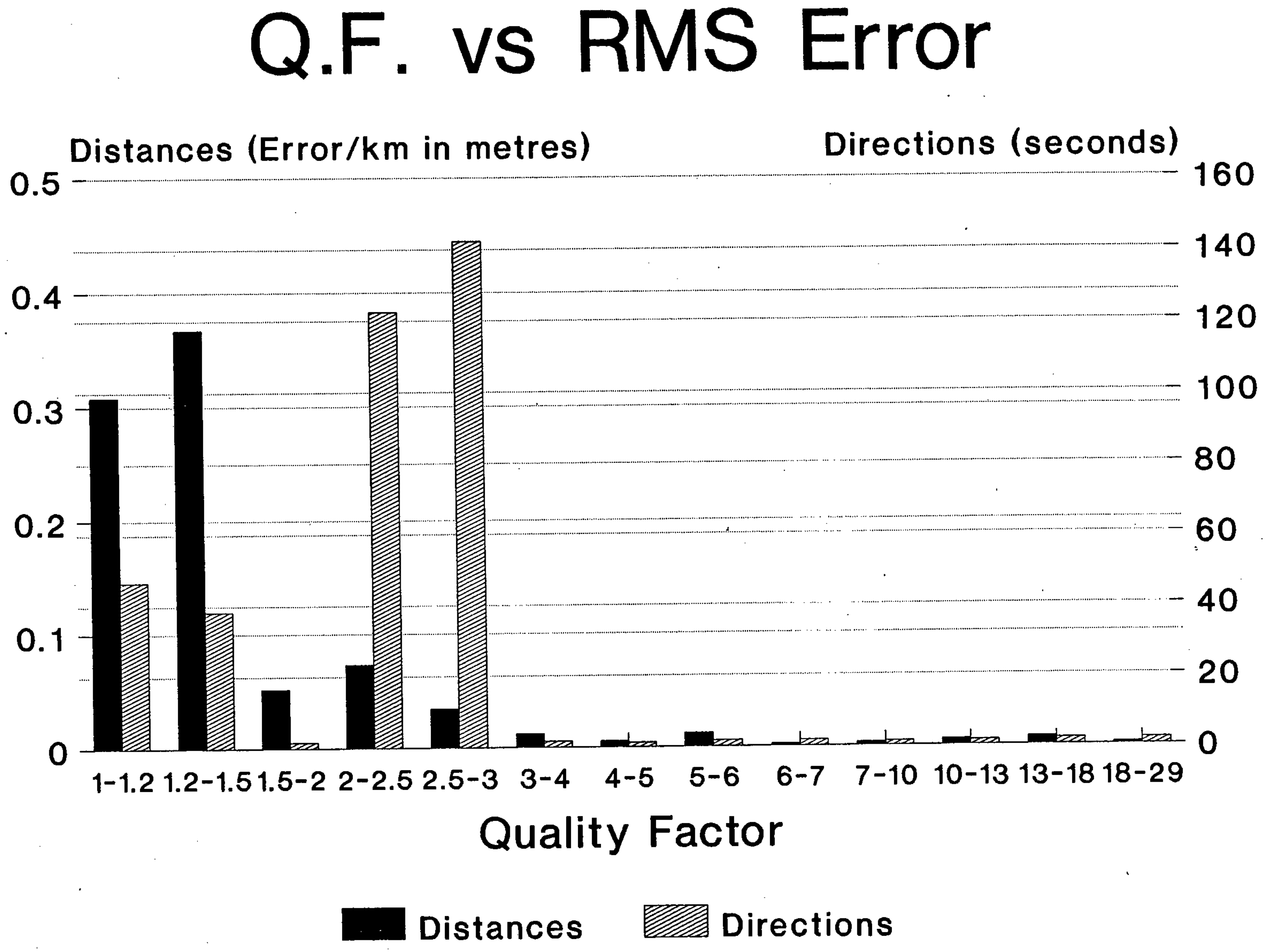
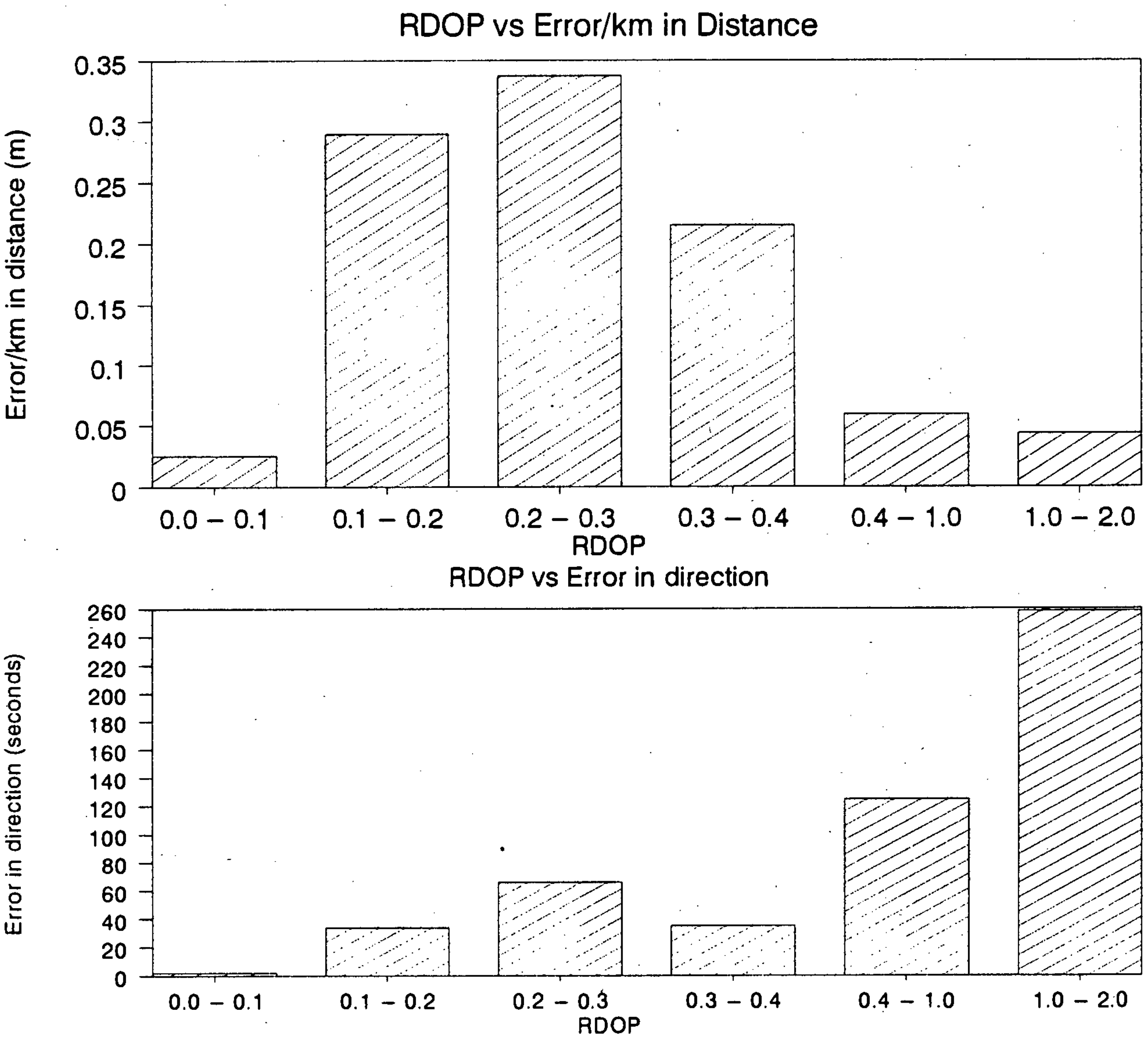


Figure 5.1 The relationship between Quality Factor and RMS Error in Distances and Directions

Table 5.3 The Relationship between RDOP and Error

		Distances (metres)			Directions (seconds)		
RDOP	No. of Readings	Average error/km	SDEV of error/km	RMS error/km	Average error	SDEV of error	RMS error
0.0 – 0.1	106	−0.007	0.025	0.025	1"	2"	2"
0.1 – 0.2	50	−0.025	0.284	0.289	7"	33"	34"
0.2 – 0.3	19	−0.082	0.327	0.337	−3"	63"	66"
0.3 – 0.4	7	0.077	0.201	0.215	−9"	34"	35"
0.4 – 1.0	12	0.011	0.048	0.059	40"	124"	125"
1.0 – 2.0	10	−0.002	0.032	0.043	156"	205"	258"
> 2.0	6	−0.261	0.486	0.560	4422"	6996"	8276"
SUMMARY							
< 0.1	106	−0.007	0.025	0.025	1"	2"	2"
0.1 – 2.0	98	−0.027	0.247	0.287	38"	128"	104"


RDOP vs Error/km in Distance

RDOP vs Error in direction

Fig 5.2 Graphs showing RDOP vs RMS error in Distance and Direction

The TRIMVEC requirement of the RDOP being less than 0.1 to achieve "acceptable" results was evaluated (Fig 5.2 and Table 5.3). It is very clear from the table and the figure that an RDOP of less than 0.1 results in a significantly improved RMS error in both direction and distance. If the requirement of the RDOP being less than 0.1 was not satisfied it did not mean that the results are poor. This was often seen in Tables 1 to 17, in Appendix A. The TRIMVEC RMS requirement was satisfied for almost all observations, regardless of the quality of the results.

Merminod et al (1990) found BDOP1 of all the BDOP factors to be the greatest use in pre-planning GPS observation sessions. The definition of BDOP2 depends on the method of dealing with the rank deficiency in the ambiguity parameters and thus is not uniquely defined. The BDOP3 is the equivalent of an accumulated PDOP. Accurate positioning is strongly related to the resolution of integer ambiguities. If the ambiguities are not well determined in a ambiguity free solution, the correct ambiguity fixed solution cannot be found. The ability of the ambiguities to be correctly estimated in an ambiguity free solution is reflected in the BDOP1. If the ambiguities can be fixed to the correct integer values, an increase in solution precision of the horizontal coordinates is assured. The BDOP3 reflects the accuracy of the ambiguity-fixed solution, therefore the BDOP3 value is always less than the BDOP1 value.

The relationship between the various ranges of BDOP1 and accuracies achieved was investigated (Table 5.4, Table 5.5, Figure 5.3 and Figure 5.4). In Table 5.5 the results of baselines of less than 10km are examined. It is very clear that excellent results are achieved with a BDOP1 of less than ten in both direction and distance, but a degradation in the accuracy of results occur with larger BDOP1 ranges. A high accuracy of results appears in the BDOP1 range of 20 to 30. This can be explained by the very small sample of data (two readings) for this particular range. Over a range of 23km (See Table 5.4 and Figure 5.3) the results are similar. The accuracy of results over a range of 23km is not as good as the range of results over 10km. From the graph and tables, one can conclude that a BDOP1 of ten and less is a good pre-planning factor for accurate results over a range of 23km. This has been further verified in Table 5.6 which evaluates the success rate of achieving A-class distances over a range of 23km for a BDOP1 of less than ten. Realising by definition that the A-class accuracy limit extends to 2A (for the minority of cases), a failure rate of only 5% over all distances tested to 23km was achieved. The success rate in the BDOP1 range of 20 to 30 was similar, except that the ratio of observations in the 2A range was considered to be too high by the author. The BDOP1 range of 0 to 10 in Table

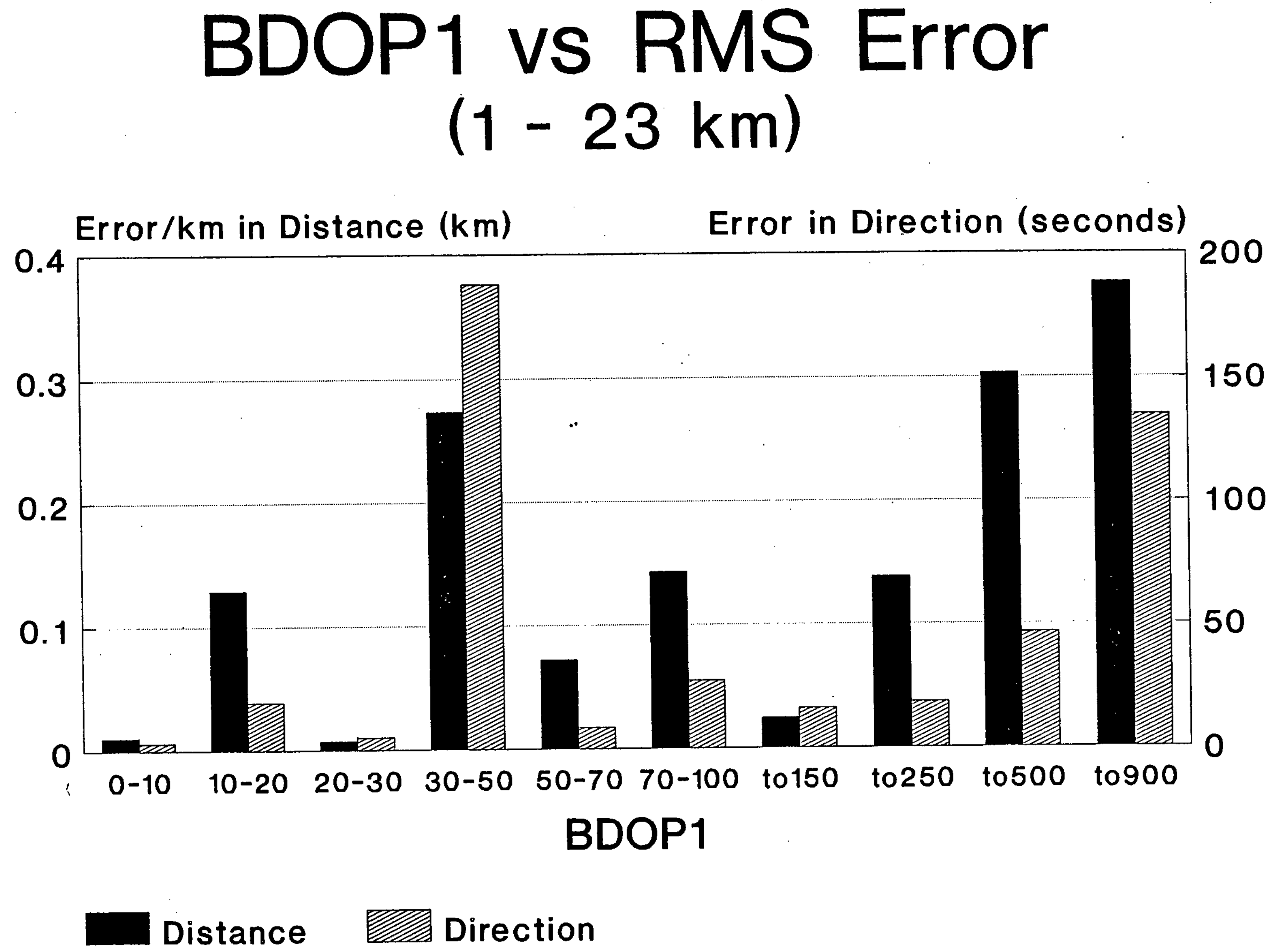


Figure 5.3 The relationship between BDOP1 Ranges and RMS Error in Distances and Directions

Table 5.4 BDOP1 vs Error (1 – 23 km)

BDOP1	No. of Readings	Distances (m)			Directions (sec)		
		Average Error/km	Sdev of Error/km	RMS Error/km	Average Error	Sdev of Error	RMS Error
0 – 10	127	−0.003	0.009	0.010	1.4"	2.6"	3.0"
10 – 20	16	0.034	0.124	0.129	−6.0"	18.2"	19.4"
20 – 30	4	0.006	0.004	0.007	0.3"	5.0"	5.0"
30 – 50	22	−0.078	0.281	0.273	83.5"	168.4"	188.0"
50 – 70	6	−0.043	0.058	0.072	5.3"	6.8"	8.6"
70 – 100	11	0.042	0.137	0.143	12.5"	24.2"	27.3"
100 – 150	5	0.003	0.023	0.024	−6.2"	14.8"	16.1"
150 – 250	8	−0.076	0.115	0.138	14.5"	11.2"	18.4"
250 – 500	8	−0.024	0.302	0.303	−1.0"	46.3"	46.4"
500 – 900	4	−0.951	0.965	0.376	18.3"	133.3"	134.6"

Table 5.5 BDOP1 vs Error (< 10 km)

BDOP1	No. of Readings	Distances (m)			Directions (sec)		
		Average Error/km	Sdev of Error/km	RMS Error/km	Average Error	Sdev of Error	RMS Error
0 – 10	74	0.000	0.005	0.005	1.3"	0.9"	1.6"
10 – 20	13	0.040	0.133	0.138	1.4"	21.9"	23.6"
20 – 30	2	0.005	0.005	0.000	−3.5"	4.5"	5.7"
30 – 50	10	0.008	0.053	0.053	169.7"	204.3"	256.6"
50 – 70	6	−0.055	0.055	0.077	5.3"	6.8"	8.6"
70 – 100	6	0.005	0.144	0.144	24.5"	23.2"	38.8"
100 – 150	5	0.003	0.023	0.024	6.6"	14.7"	16.1"
150 – 250	8	−0.076	0.115	0.138	11.6"	13.0"	17.4"
250 – 500	8	−0.024	0.302	0.303	1.7"	46.3"	46.4"
500 – 900	4	−0.951	0.965	0.376	18.3"	133.3"	134.6"

Table 5.6 Success of A Class Distances in the various BDOP1 Ranges (1 – 23 km)

BDOP1	No. of Readings	A–Class	2A	Failed
0 – 10	127	72%	23%	5%
10 – 20	16	56%	38%	6%
20 – 30	4	25%	75%	0%
30 – 50	22	23%	9%	68%
50 – 70	6	33%	0%	67%
70 – 100	11	9%	0%	91%
100 – 150	5	40%	0%	60%
150 – 250	8	12.5%	0%	87.5%
250 – 500	8	0%	0%	100%
500 – 900	4	0%	0%	100%

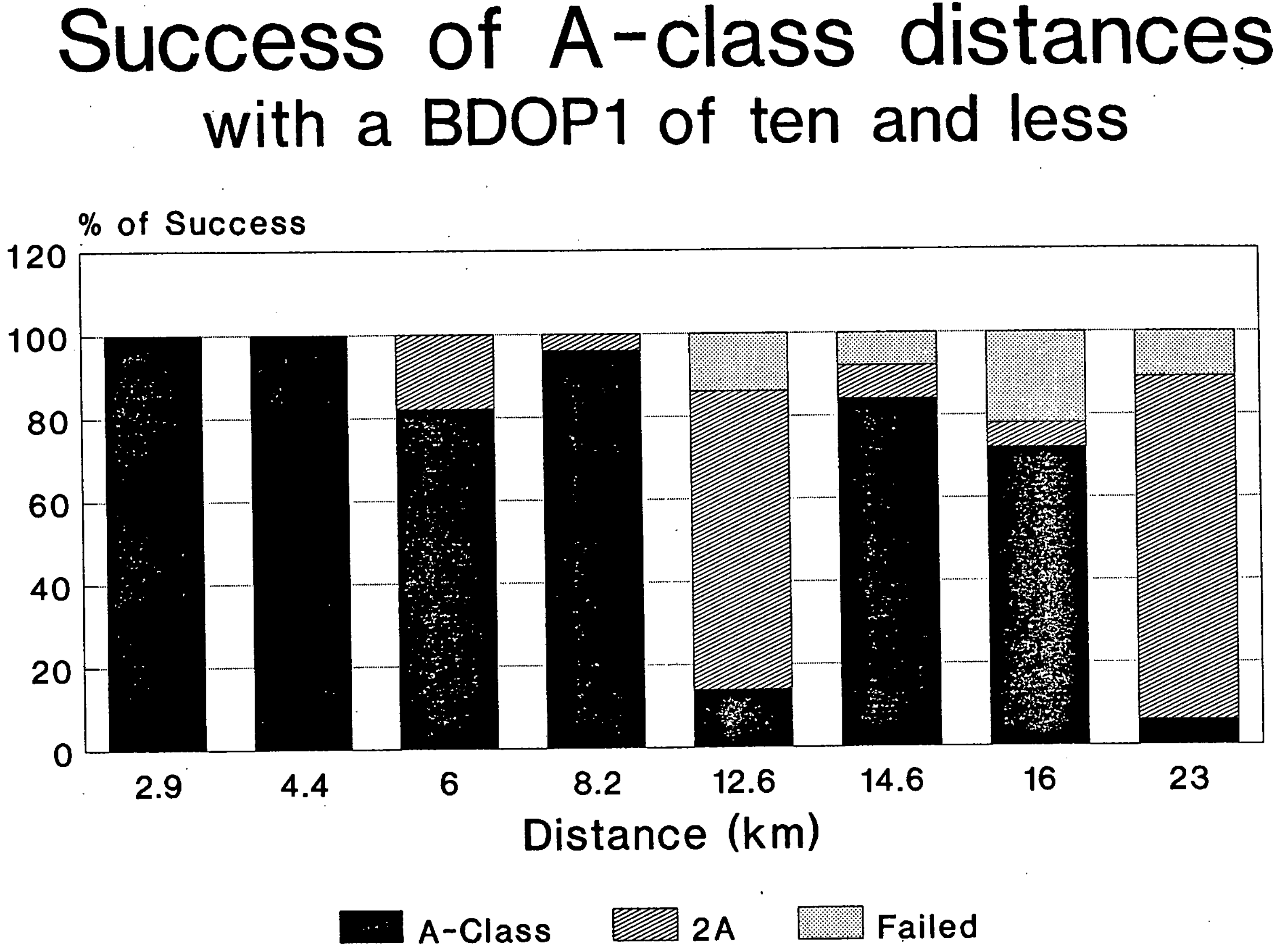


Figure 5.4 The Success of attaining A–class Distances with a BDOP1 of Ten and Less

5.6 was analyzed in greater detail in Table 5.7 and Figure 5.4. Evaluating baselines of less than 10km, there is a 0% failure rate in achieving A-class distances, and only the minority of cases fall in the 2A limit. This table gives a clear indication, that distances of less than 10km can be pre-planned to achieve A-class distances with a BDOP1 of ten and less.

Table 5.7 Table to show the success of A-Class Distances with a BDOP1 of Ten and less

Distance	No. of Readings	A	2A	Failed
2.9km	12	100%	0%	0%
4.4km	7	100%	0%	0%
6km	33	82%	18%	0%
8.2km	25	96%	4%	0%
12.6km	7	14%	72%	14%
14.6km	10	80%	10%	10%
16km	16	68%	12%	20%
23km	18	6%	83%	11%

When pre-planning A-class distances with a BDOP1 of ten and less on baselines between 10km and 23km, there is a high failure rate - between ten and twenty percent and a high percentage of observations achieving the 2A limit. A BDOP1 of less than ten, will not guarantee A-class distances on baselines over 10km. By lowering the BDOP1 value on longer baselines (say below 5), one could achieve A-class distances, but a high ratio would be in the 2A-limit.

Earlier on it was established that although the TRIMVEC manual recommends certain criteria in the way of Quality Factors, RMS and RDOP values for "acceptable" results, these are not hard and fast rules. Through analysing post-processed observations with these criteria, we established that the RDOP and Quality Factor requirements have some foundation, but cannot be stated as absolute rules. In Table 5.8 one can see to what extent these TRIMVEC requirements are satisfied for a BDOP1 of less than ten over the range of distances tested. From the table, one notices that the longer the distance the less likely the chance that the Quality Factor will be satisfied. In most cases, regardless of distance, the RMS was satisfied. However, it was pointed out that the RMS values did not relate very well to the accuracies achieved, in that the RMS requirements were met regardless of the accuracy of results. Although the RDOP requirements were generally met, it appeared that the longer the baseline the less likely the RDOP requirement would be satisfied.

Table 5.8 How observations with a BDOP1 of Ten and less satisfy RMS, Quality Factor and RDOP Requirements

Distance	No. of Readings	QF > 3	RMS Satisfied	RDOP <or= 0.1
2.9km	12	100%	58%	83%
4.4km	7	57%	100%	100%
6km	33	86%	94%	86%
8.2km	25	64%	88%	80%
12.6km	7	43%	100%	86%
14.6km	12	58%	100%	91%
16km	18	32%	100%	56%
23km	18	22%	100%	67%
Overall	132	61%	92%	78%

Having established that a Quality Factor of three and greater yields "good" results, and that a BDOP1 of ten and less yields "good" results, one should consider the relationship between Quality Factor and BDOP1. In Table 5.9 one notices that observations having a Quality Factor of three and greater and having a BDOP1 of ten and less, have a 0% failure rate in achieving A-class distances, regardless of the length of line tested. The observations falling in the 2A limit are all very long (greater than 20km) or happened to be the observations where there might have been an error in the eccentric reduction. For any BDOP1 greater than ten, but for a Quality Factor greater than three, a very small failure rate in achieving A-class distances of 3% was achieved. It appears at this stage that the ability to achieve a Quality Factor greater than three is more important in guaranteeing A-class distances, than having a pre-planning BDOP1 of less than ten. However, it is very difficult to preplan a session that can achieve a Quality Factor of three and greater. A greater possibility of solving ambiguities exists when one preplans using BDOP1. Cycle ambiguities cannot be assured of being resolved simply by selecting the observation scenario with the lowest BDOP1. The BDOP1 merely indicates what observation session has the best geometry for coordinate determination. Other factors such as the atmospheric refraction effect, multipath, etc will effect the URA (User Range Accuracy) and the ultimate results. According to Merminod et al (1990), the BDOP factors appear to be indicators of accuracy as well as precision. Periods of low BDOP indicate periods of lower sensitivity to most systematic errors. Times of low BDOP1 do not necessarily coincide with times of low PDOP or GDOP values.

Table 5.9 The Ability of a Quality Factor of Three and Greater to Guarantee A-Class Distances ?

	No. of Readings	A-Class	2A	Failed
Any BDOP1	90	81%	16%	3% *
BDOP1 of 10 & less	82	83%	17%	0%

* BDOP1 in 40's

Note : 2A-class consists of long distances (>20km) and distances (6km) eccentrically reduced.

Considering the relationship between BDOP1 and Quality Factor, as shown in Figure 5.5, it is very clear from this figure that the majority of the observations with a Quality Factor greater than three, fall in the BDOP1 range of 0 to 10. However, a BDOP1 under ten did not guarantee a Quality Factor greater than three. Referring to Table 5.10, one notices the highest BDOP1 values achieved with a Quality Factor of three and greater tend to be just under ten, for any length of baseline tested.

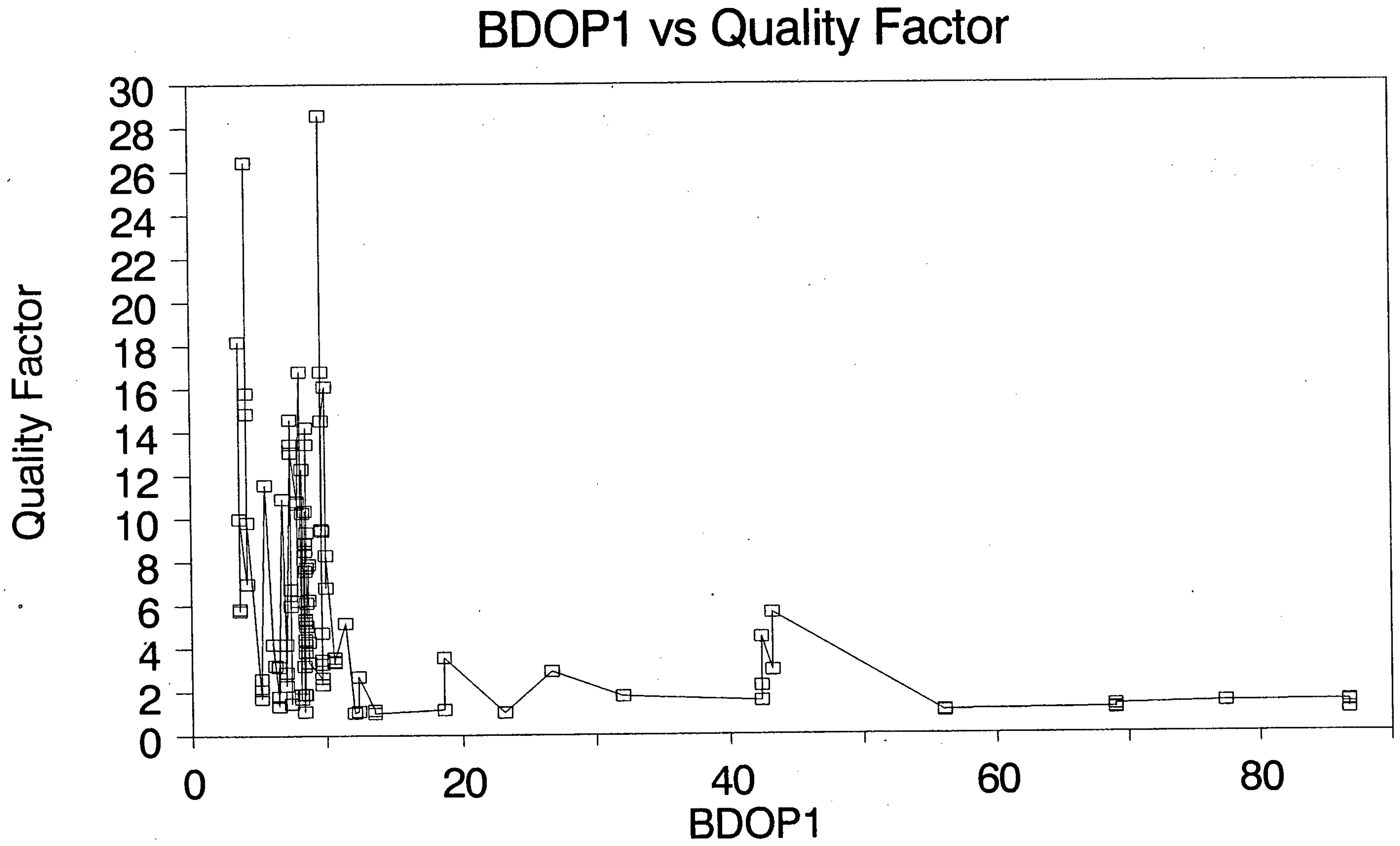


Figure 5.5 The relationship between BDOP1 and Quality Factor (for distances less than 10km)

Table 5.10 The Highest BDOP1 Obtained with a Quality Factor of Three and Greater, over varying Distances

Distance	Highest BDOP1
2.9km	18.63
4.4km	43.12 *
6km	11.39
8.2km	9.98
12.7km	8.55
14.6km	9.68
16km	10.55
23km	8.70

* A-Class was not achieved. The next highest BDOP1 with A-class accuracy, is 8.55

An attempt was made to correlate the length of time for which one observed a GPS session, and the BDOP1 value. There appeared to be no correlation. Session lengths required to obtain a BDOP1 less than ten varied between twenty minutes to an hour and twenty-five minutes, but this was in no way proportional to the BDOP1 values. However, it was noticed that the flatter the PDOP curve, the longer one needed to observe to achieve a BDOP1 less than ten.

Figure 4.2 refers to the 3DAYS project. It was clear from this experiment and the ALITIME and ALITRI projects that it was difficult, if not impossible, to use PDOP for pre-planning observation session lengths. In Section 3.3.1 previous attempts by other researchers to pre-plan observation sessions and observation session lengths were discussed. The author did not find these recommendations helpful in trying to determine the minimum session length for a given point in time for A-class accuracy distances. In Table 5.11, an attempt was made to categorise observations into PDOP trends and to evaluate the accuracies achieved on a minimum of twenty minutes of observations (it was not often that A-class accuracies were achieved on session lengths shorter than twenty minutes). From this table, it proved difficult to make any definite conclusions about pre-planning with PDOP. The author feels that A-class accuracies can be achieved, regardless of the pattern of PDOP.

Table 5.11 Investigating the Ability to Preplan an Observation Session using PDOP and a Minimum Observation Session of Twenty Minutes

Range of PDOP	No. of Obs	Distances (metres) Average error/km	SDEV error/km	RMS error/km	Directions (sec) Avg error	SDEV error	RMS error
Less than 5	54	0.005	0.059	0.059	3.4"	8.3"	8.9"
5 – 10	12	0.000	0.003	0.003	0.9"	1.0"	1.4"
Sharp upward	16	−0.014	0.038	0.040	0.3"	1.4"	0.4"
Sharp Rise and Fall	36	−0.008	0.018	0.022	1.0"	4.8"	4.9"
Downward Trend	40	−0.002	0.006	0.006	0.2"	3.0"	3.0"
Sharp Fall and rise	2	0.246	0.252	0.352	−35.5"	38.5"	52.4"

As was mentioned in Section 3.3.2, a Chief Surveyor General's Circular No.2 of 1992, issued on 2 July 1992, specified an accuracy that is recommended for GPS cadastral surveys but has not yet been legislated. Table 5.12 explains this accuracy, and the requirements for

Table 5.12 Accuracy Requirements of a Range of Distances for the Requirement s = e + 0,1.p.d, where s = maximum allowable error in centimetres
d = distance in kilometres
p = accuracy requirement in ppm (i.e. 4ppm)
e = base error in centimetres (2cm)

Distance	Accuracy Required (m)
2.9 Km	0.032
4.4 Km	0.038
5.6 Km	0.042
6.0 Km	0.044
8.2 Km	0.053
12.6 Km	0.070
14.6 Km	0.078
16.5 Km	0.086
16.9 Km	0.088
22.5 Km	0.110
23.0 Km	0.112

Table 5.13 The Success of Distances Satisfying the Accuracy Requirement : S = 2cm + 0,1.4.d for a BDOP1 of Ten and Less (where d = distance in km)

Dist.	No. of Readings	% Success	% Failed
2.9km	12	100%	0%
4.4km	7	100%	0%
6km	33	76%	24%
8.2km	25	96%	4%
12.6km	7	100%	0%
14.6km	10	90%	10%
16km	16	75%	25%
23km	18	50%	50%

Table 5.14 The Success of Distances Satisfying the Accuracy Requirement : S = 2cm + 0,1.4.d (where d = distance in km), for a Range of BDOP1 values.
(UP TO 23KM)

BDOP1	No. of Readings	% Success	% Failed
0 – 10	127	82%	18%
10 – 20	16	63%	37%
20 – 30	4	25%	75%
30 – 50	22	22%	78%
50 – 70	6	33%	67%
70 – 100	11	9%	91%
100 – 150	5	40%	60%
150 – 250	8	0%	100%
250 – 500	8	12%	88%
500 – 900	4	0%	100%

various distances more fully. This is notably a more stringent requirement than the A-class accuracy requirement. Table 5.13 evaluates the ability of a pre-planned session with a BDOP1 of ten and less to satisfy this new requirement. There appears to be a high failure rate around the 6km range. This can be explained by the problems in the reduction of eccentrically observed distances in Table 17; otherwise a 100% success rate for this distance would have been achieved. It appears that a BDOP1 of ten and less will satisfy the accuracy requirement in this Surveyor General's Circular, provided distances are kept under 13km in length. Longer distances have a much smaller chance of satisfying this requirement, although it is quite possible to do so. Table 5.14 clearly shows that the range of BDOP1 used for pre-planning this accuracy in distances should be kept under ten.

In Appendix C, a number of regression analyses were performed to test the correlation between BDOP1, some of the post-processing evaluation criteria and the accuracy of the results. On the whole correlations were poor, and it was better to evaluate data as grouped above. The best correlations for every combination of variables tested are shown in Appendix C. The correlations are poor for :-

- RDOP vs Error/km in Distance
- RDOP vs Error/km in Direction

- BDOP1 vs RDOP
- Quality Factor vs Error/km in Distance
- Quality Factor vs Error/km in Direction
- BDOP1 vs Error/km in Distance
- BDOP1 vs Error/km in Direction
- Success rate of A-class distances for a Quality Factor greater than three and a BDOP1 less than ten.
- Success Rate of RMS for a BDOP1 less than ten over a range of distances

Better correlations were achieved for :

- Success rate of A-class distances over a range of 23km for a BDOP1 less than ten
- Success rate of RDOP for a BDOP1 less than ten over a range of distances

Very high correlations were achieved for :

- Success rate of RMS for a BDOP1 less than ten over distances up to 10km
- Success rate of RDOP for a BDOP1 less than ten over distances up to 10km
- Success rate of the Quality factor being greater than three for a BDOP1 less than ten over all distances tested

5.3 PSEUDO-STATIC SURVEYING

5.3.1 DISCUSSION ON RESULTS OF PROJECTS CONDUCTED

(Tables G and H of Appendix B, show differences between GPS reduced results and join values).

PROJECT : ALITRI

(See results in Table A of Appendix B)

This experiment used two ten-minute statically observed sessions, processed statically, to evaluate this process as a method of observing pseudo-statically.

The results of this experiment were extremely poor, irrespective of the PDOP. This was reflected in the static standards of low Quality Factors and a high RDOP. The results were not suitable for surveying purposes. This method of pseudo-static surveying is therefore considered unusable for surveying purposes.

PROJECT : 3 DAYS

(See Results in Table B of Appendix B).

Sessions 1A, 2A and 3A, were not found to be repeatable over the three days tested, as was the case for Sessions 1B, 2B and 3B. The difference in directions is not as accurate as static surveying. From this project, it appeared that using this method of surveying C-class distances can be achieved during periods of low PDOP only. The sessions observed during high PDOP, Sessions

4A and 4B, proved to be poor and unusable for cadastral surveying purposes.
The RMS and RDOP values proved to be too high.

PROJECT : 4DAYS
(See Results in Tables C, D, E and F of Appendix B).
All six session were observed over the four days, but starting four minutes earlier each day. As a generalisation, the sessions tested fell outside C-class limits, and are therefore not usable for cadastral surveying. Accuracies in directions were poor.
When comparing similar sessions (See Table H), it appears that Sessions 20 produced the best results and Sessions 21 produced the worst results (see Table 5.15). Both Sessions 20 and 21 were observed during periods of low PDOP. Sessions 22 were observed during periods of rapidly changing PDOP and generally produced results better than Session 21, which paradoxically was observed during periods of low PDOP.
From this experiment, the author concluded that given the TRIMBLE software presently available, regardless of the length of baseline measured and regardless of the PDOP trends, pseudo-static surveying does not yield accuracies suitable for cadastral surveying.

Table 5.15 Comparing the Accuracy of Pseudo−static Sessions

Error	20	21	22
Distance			
RMS	0.028	0.074	0.018
Mean Difference	−0.003	0.041	−0.006
SDEV of Mean Difference	0.028	0.061	0.017
Direction			
RMS	6.9"	0.0"	1.5"
Mean Difference	4.1"	0.0"	1.0"
SDEV of Mean Difference	5.6"	6.7"	1.1"

5.3.2 GENERAL CONCLUSIONS ON PSEUDO-STATIC SURVEYING

A summary of the results of Tables B, C, D, E and F are shown in Table 5.16 for all pseudo-statically measured observations. Table 5.17 shows a summary for pseudo-statically observed sessions during periods of LOW PDOP only and Table 5.18 shows pseudo-static results for sessions observed during periods of HIGH PDOP only. It is clear from these tables that observing during periods of low PDOP does not necessarily guarantee better observations than sessions pseudo-statically observed during any type of PDOP. Accuracies achieved from pseudo-statically observed sessions do not satisfy even C-class accuracy requirements and therefore should not be used for cadastral purposes.

There is a distinct improvement in results when one observes pseudo-statically, as described in Section 5.3.1, as opposed to combining two ten minute sessions statically observed and processed. A summary of the results achieved by observing two ten minute static sessions and statically processing them is shown in Table 5.19.

The ability of pseudo-static observations to fulfil static post-processing evaluation criteria is shown in Table 5.20. Few observations satisfied the RDOP requirement of being less than 0.1. There was a tendency

Table 5.16 Pseudo–static Results: Comparison of Differences between Reduced GPS Distances, Directions and Heights to Join Distances and Directions, and Heights (Gauss Conform System)

Distance = 2,9 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.341	10"	0.1
Mean diffs.	0.143	1"	0.1
SDEV of Mean diffs	0.310	10"	0.0

Distance = 4,4 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.064	1.7"	0.1
Mean diffs.	0.031	0.0"	0.0
SDEV of Mean diffs	0.055	1.7"	0.1

Distance = 6 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.173	8.4"	0.1
Mean diffs.	−0.023	3.3"	0.0
SDEV of Mean diffs	0.171	7.8"	0.1

Distance = 8,2 km		n = 8	
	Distance (m)	Direction	Height (m)
RMS	0.231	2.8"	0.5
Mean diffs.	0.035	0.8"	0.5
SDEV of Mean diffs	0.229	3.8"	0.1

Distance = 12,7 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.412	5.9"	0.2
Mean diffs.	0.376	4.7"	0.2
SDEV of Mean diffs	0.168	3.7"	0.1

Distance = 14,6 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.975	2.1"	0.1
Mean diffs.	0.520	−0.7"	0.1
SDEV of Mean diffs	0.824	2.1"	0.1

Distance = 16 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.507	9.9"	0.2
Mean diffs.	0.166	5.7"	0.2
SDEV of Mean diffs	0.479	5.9"	0.0

Distance = 23 km		n = 6	
	Distance (m)	Direction	Height (m)
RMS	0.479	1.6"	0.7
Mean diffs.	−0.256	−0.7"	−0.2
SDEV of Mean diffs	0.405	1.5"	0.6

Table 5.17 Pseudo–Static results for Low PDOP (<5) only: Comparison of Differences between Reduced GPS Distances, Directions and Heights to Join Distances and Directions, and Heights (on the Gauss Conform System)

Distance = 2,9 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.341	10"	0.1
Mean diffs.	0.143	1"	0.1
SDEV of Mean diffs	0.310	10"	0.0

Distance = 4,4 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.075	2.0"	0.1
Mean diffs.	0.062	0.0"	0.0
SDEV of Mean diffs	0.042	2.0"	0.1

Distance = 6 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.153	10.1"	0.1
Mean diffs.	0.069	3.5"	0.1
SDEV of Mean diffs	0.136	9.5"	0.1

Distance = 8,2 km		n = 6	
	Distance (m)	Direction	Height (m)
RMS	0.114	3.1"	0.4
Mean diffs.	0.027	−0.3"	0.4
SDEV of Mean diffs	0.111	3.1"	0.1

Distance = 12,7 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.449	7.2"	0.2
Mean diffs.	0.402	7.3"	0.2
SDEV of Mean diffs	0.201	2.0"	0.1

Distance = 14,6 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	1.194	2.5"	0.1
Mean diffs.	0.731	−0.5"	−0.1
SDEV of Mean diffs	0.941	2.5"	0.1

Distance = 16 km		n = 2	
	Distance (m)	Direction	Height (m)
RMS	0.609	8.2"	0.2
Mean diffs.	0.334	7.5"	0.2
SDEV of Mean diffs	0.509	6.5"	0.0

Distance = 23 km		n = 4	
	Distance (m)	Direction	Height (m)
RMS	0.284	2.0"	0.2
Mean diffs.	−0.255	−1.0"	−0.1
SDEV of Mean diffs	0.489	1.7"	0.1

Table 5.18 Pseudo–static Results for High PDOP (>5) only: Comparison of Differences between Reduced GPS Distances, Directions and Heights Differences to Join Distances and Directions, and Height Differences

		RMS Values		
Distance	No. of readings	Distance (m)	Direction	Height (m)
2.9 km	0			
4.4 km	1	0.030	1"	0.0
6 km	1	0.208	3"	0.1
8.2 km	2	0.418	2.1"	0.7
12.7 km	1	0.324	1"	0.1
14.6 km	1	0.098	1"	0.2
16 km	1	0.172	2"	0.2
23 km	2	0.551	0"	1.1

Table 5.19 Comparison of Reduced Results obtained by Observing Two Ten Minute GPS Sessions approximately one hour apart and processed statically with Join Values and Height Differences on the Gauss Conform System

Distance = 2.9 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	6.638	6' 40"	1.1
Mean diffs.	−5.010	2' 48"	−0.9
SDEV of Mean Diffs	4.355	2' 38"	0.7

Distance = 8.2 km		n = 3	
	Distance (m)	Direction	Height (m)
RMS	0.790	41"	0.6
Mean diffs.	−0.572	14"	0.6
SDEV of Mean Diffs	0.545	39"	0.1

Table 5.20 The Ability of Pseudo-static Observations to satisfy TRIMBLE Static Requirements

Table a. The Number of Pseudo-static results with an RDOP < 0.1

Total no. of obs. = 31	RDOP	
PDOP	< 0.1	> 0.1
Low (<5)	6	16
Changing PDOP (>5)	3	6

Table b. RDOP vs RMS Error

No. of Readings	RDOP	RMS Error/km in Distance	RMS Error in Direction
9	0.0 – 0.1	0.023	5.8"
16	0.1 – 0.2	0.058	19.9"
5	0.2 – 0.6	0.028	24.9"
5	> 1.2	0.030	2.8"

Table c. The Number of Pseudo-static Results with an RMS of (0.02 + 0.004*L) or less (L = distance in km)

Total no. of obs. = 31	RMS satisfied ?	
PDOP	YES	NO
Low (<5)	9 (41%)	13 (59%)
Changing PDOP (>5)	4 (45%)	5 (55%)

for pseudo-static observations with changing PDOP to fulfil this requirement to a greater degree than observations with a low PDOP. Observations that did satisfy the requirement tended to produce much better results than those observations that did not (Table 5.20b), but the accuracy of the directions still tended to be very poor. Regardless of the PDOP trends, few pseudo-static observations satisfied the static RMS requirements (Table 5.20c).

High PDOP and low PDOP and their bearing on accuracy both in distance and direction of pseudo-static observations has been evaluated in Table 5.21. It appears that neither PDOP conditions guarantee a high accuracy in observations.

Table 5.21 The relationship between PDOP and Error for Pseudo-static Observations

	n = 23	n = 12
Error	Low PDOP	High PDOP
Distance (Error/km)		
RMS	0.048	0.029
Mean Difference	0.015	−0.003
SDEV of Mean Difference	0.046	0.029
Direction		
RMS	9.6"	24.0"
Mean Difference	2.9"	−2.8"
SDEV of Mean Difference	9.1"	23.8"

The ability of pseudo-static observations to satisfy the A-class accuracy requirement over a range of distances is shown in Table 5.22. Only 16% of the observations satisfied the A-class accuracy requirement, a further 6% satisfied the B-class accuracy requirement and 23% satisfied the C-class accuracy requirement. As only 45% of the pseudo-static observations are considered usable for cadastral purposes, the author recommends that pseudo-static observations not be used.

Table 5.22 The Success of A–Class distances with Pseudo–static observations on a Range of distances up to 23km

Total no. of observations = 31

Category	No. of Readings	
A	5	(16%)
B	2	(6%)
C	7	(23%)
2C	9	(29%)
FAILED	8	(26%)

* It was generally felt that accuracy in directions was erratic and not reliable. 35% of observations had directions that differed from join directions by 3".

6. GENERAL CONCLUSIONS AND RECOMMENDATIONS

When observing statically, it appears that accurate results can be achieved independent of the PDOP trends. From the research performed, it appears preplanning the length of a session to yield accurate results using PDOP is not possible.

The earlier discussion of results in Chapter 5 describes how A-class distances and single second accuracy directions can be achieved on distances less than 10 km, when one preplans a observation session to have a BDOP1 less than ten. Longer distances, up to 23km were tested for a BDOP1 less than ten. It was found that although A-class accuracy distances and single-second accuracy in directions was achieved with a BDOP1 of less than ten on longer baselines, there were a number of outliers that did not satisfy this accuracy requirement. The number of outliers tended to increase and the accuracy to decrease with increasing baseline length and a BDOP1 less than ten. To obtain a BDOP1 less than ten, session lengths varied between twenty minutes to an hour and twenty-five minutes.

A BDOP1 of ten and less can be said to yield an accuracy of better than 1: 150 000 in distance, and an accuracy of one second in direction for all baseline lengths under 23km. However, the results on baselines longer than 10km occasionally yield unpredictable outliers. These outliers can be attributed to errors which cannot be effectively removed by differencing or computer modelling, such as atmospheric conditions.

What should be borne in mind is that GPS measured baselines could be more accurate than the published trigonometrical data of the trigonometrical beacons used. However, it is required that cadastral GPS Surveys should conform as best as possible to the existing trigonometrical network.

With a BDOP1 of ten and less, height accuracies are not of the same standard as distances and directions, regardless of whether the cycle ambiguities are resolved or not. This can be explained by the limited accuracy of trigonometrical levelling and knowledge of the geoid. Heights determined by GPS are particularly sensitive to atmospheric refraction.

The BDOP1 factors proved to be useful in preplanning session lengths for accurate observations. The reliability of this indicator was tested, by evaluating the repeatability of the same session over a period of three days, each session starting four minutes earlier each day. (The satellite constellation can be predicted as travelling the same paths, but arriving four minutes earlier each day). The sessions tested in two different projects with two different baseline lengths, yielded results that were consistently A-class accuracy in distances and single second accuracy in directions. The BDOP1 indicator is not only accurate, but reliable and precise.

The BDOP1 is a good indicator of "good" geometry of satellites and can be used to determine the length of an observation session. Accumulated observations over an entire observation session, taking into account the cycle ambiguity result in a better cofactor matrix for evaluation of accuracy, precision and observation session length. The PDOP and GDOP only evaluate instantaneous observations for a given instant in time, making it suitable for navigation purposes, but not necessarily for surveying purposes. The GDOP and PDOP are based on pseudo-range equations which do not account for the cycle ambiguity resolution, necessary when observing carrier phase observations, as is used in surveying.

Research into both BDOP and DGDOP has shown that timing of the observation window for short data collection intervals is much more important than for long intervals and that for longer observation sessions it is less important for integer ambiguities to be solved (Hatch and Avery;1989). By moving one's starting time for an observation session, one can often shorten the length of observing time to achieve a BDOP1 less than ten, considerably. Despite a future twenty-four hour satellite coverage, there will always be some observing times that are significantly better than others (Grant;1990).

Generally, the longer the selected set of satellites were tracked, the more accurate the resulting position differences. This is due to a more extensive sampling of the atmosphere (and the resulting randomisation of atmospheric delays) and more varying geometry of the satellite set (and resulting randomisation of orbital errors) rather than more ranges being collected. Dependence of accuracy on range sampling is very weak since frequent sampling drives down only the random part of the error, but does nothing to reduce bias (Wells;1986).

Pseudo-static observations resulted in poor results, regardless of the method used in collecting data, processing the results or the conditions of PDOP under which it was observed. Pseudo-static observing is not recommended by the author for reducing the length of observing time and achieving results suitable for cadastral surveying. Future software improvements or observing technique improvements may overcome the limitations that existed in pseudo-static surveying with TRIMBLE receivers and TRIMVEC software at the time of the experiments.

A Quality Factor of three and greater will provide A-class distances. There is a high incidence of Quality Factors greater than three with a BDOP1 less than ten. Preplanning by BDOP1 will increase the possibility of the cycle integers being solved for and of a Quality Factor greater than three being achieved. However, a Quality Factor of less than three on distances under 10km, did not mean that poor results would be attained.

Other TRIMBLE criteria used proved to be useful as a guide in assessing the accuracy of the results, but are in no means a hard and fast rules in evaluating whether the results are of suitable accuracy or not.

The author recommends that to reduce the length of time to observe a static session and to achieve A-class accuracy in distances and a high accuracy in directions, that one preplan sessions with a BDOP1 of ten and less. Sessions should be preplanned for a given starting time, and the session lengths incremented at selected intervals until a BDOP1 of ten and less is obtained. Wherever possible, baselines measured should be kept under 10km in length, as random errors often occur on longer baselines that cannot be effectively removed by computer modelling.

If the "S-class" accuracy as laid out in Chief Surveyor General's Circular No. 2 of 1992 becomes a legal requirement, the BDOP1 preplanning factor of ten and less should still be used. This should be restricted to distances less than 10km.

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APPENDICES

Appendix A : Results of Static Observations

Appendix B : Results of Pseudo-static Observations

Appendix C : Regression Analyses

APPENDIX A : RESULTS OF STATIC OBSERVATIONS

TABLE 1 : BDOP EXPERIMENTOVER 3 DAYS

	PEROID	BDOP1	BDOP3	PDOP RANGE
A	22'	12.33	1.14	< 5
B	27'	8.34	2.51	SHARP RISE 6 TO 25
C	37'	7.01	0.72	RISES 7 TO 14 & DROPS TO 3
D	42'	5.17	0.50	< 5
E	77'	7.86	0.50	< 5
F	47'	9.98	0.97	RISES 2 TO 32 & DROPS TO 6

28–TUES	29–WED	30–THUR
HEIGHT DIFFS. = 160.2		
160.804	160.606	160.622
160.642	160.750	160.626
160.596	160.600	160.643
160.635	160.617	160.647
160.639	160.639	160.648
160.624	160.600	160.624

	28–TUES	29–WED	30–THUR
	EDM SLOPE DIST. = 8200.783		
A	8200.699	8200.717	8200.749
B	8200.757	8200.836	8200.756
C	8200.738	8200.716	8200.756
D	8200.751	8200.712	8200.746
E	8200.771	8200.737	8200.764
F	8200.781	8200.750	8200.773

28–TUES	29–WED	30–THUR
JOIN DISTANCE = 8197.861		
8197.800	8197.823	8197.855
8197.862	8197.939	8197.862
8197.844	8197.823	8197.861
8197.857	8197.819	8197.851
8197.877	8197.842	8197.870
8197.887	8197.856	8197.878

28–TUES	29–WED	30–THUR
JOIN DIRECTION = 151.50.23		
151.50.16	151.50.24	151.50.24
151.50.24	151.50.21	151.50.24
151.50.24	151.50.24	151.50.25
151.50.24	151.50.24	151.50.24
151.50.24	151.50.24	151.50.24
151.50.24	151.50.24	151.50.24

	28–TUES	29–WED	30–THUR
	QUALITY FACTORS		
A	1.1	1.1	2.7
B	1.9	1.1	3.2
C	2.3	2.9	4.2
D	2.1	1.7	2.6
E	6.2	10.7	10.8
F	6.8	8.3	16.1

28–TUES	29–WED	30–THUR
FIXED SOLN RMS		
0.049	0.041	0.019
0.032	0.046	0.023
0.068	0.050	0.039
0.040	0.057	0.032
0.035	0.040	0.025
0.026	0.024	0.015

28–TUES	29–WED	30–THUR
RDOP		
0.108	0.108	0.116
0.247	0.229	0.230
0.072	0.069	0.070
0.049	0.048	0.047
0.048	0.049	0.049
0.087	0.088	0.101

* REDUCED EDM DISTANCE : 8197.904

TABLE 2 : BDOP TESTS ON SHORT BASELINES OF VARYING LENGTHS

SESSION	PEROID	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
1	35’	6.45	0.85	< 5	160.2	160.530	1.35	0.084	0.077
2	29’	6.03	0.74	6 down to 2	118.4	118.549	4.19	0.034	0.091
3	39’	6.20	0.90	< 5	118.4	118.520	3.23	0.041	0.093
4	49’	6.80	0.50	< 5	118.4	118.554	10.91	0.030	0.062
5	41’	5.52	0.59	< 5	169.2	169.520	11.54	0.018	0.065
6	34’	6.45	0.85	< 5	8.9	8.918	3.16	0.033	0.082
7	29’	8.13	1.00	SHARP INCR & DECR	8.9	8.924	16.79	0.017	0.146
8	28’	8.13	1.00	SHARP INCR & DECR	160.2	160.520	1.88	0.048	0.214
9	18’	11.39	2.31	10 down to 2	118.4	118.572	5.17	0.038	1.302
Y	42’	56.00	0.64	< 5 TO 36	160.2	160.442	1.03	0.030	0.064
Z	40’	56.00	0.64	< 5 TO 36	169.2	169.437	1.08	0.021	0.068

SESS.	SLOPE DIST – EDM	SLOPE DIST – GPS	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
1	8200.793	8200.707	8197.861	8197.857	331.50.23	331.50.24
2		5560.417	5558.318	5558.336	345.23.03	345.23.04
3		5560.416	5558.318	5558.337	345.23.03	345.23.04
4		5560.408	5558.318	5558.327	345.23.03	345.23.04
5	5994.593	5994.506	5991.192	5991.175	347.12.30	347.12.31
6	2895.672	2895.605	2895.180	2895.159	298.34.45	298.34.48
7	2895.672	2895.600	2895.180	2895.154	298.34.45	298.34.48
8	8200.793	8200.709	8197.861	8197.829	331.50.23	331.50.25
9		5560.408	5558.318	5558.328	345.23.03	345.23.03
Y	8200.793	8200.308	8197.861	8197.418	151.50.23	151.50.39
Z	5994.593	5994.411	5991.192	5991.064	347.12.30	347.12.39

* EDM REDUCED DISTANCES : 8197.904, 5991.273, 2895.226

TABLE 3

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
10	9’	31.94	5.22	15 down to 10	118.4	118.685	1.77	0.026	0.790
11	19’	11.39	2.31	9 down to 6	118.4	118.575	5.17	0.038	1.302
12	6’	841.50	1.20	< 5	169.2	168.313	1.03	0.007	0.211
13	26’	69.00	1.80	SHARP INCR & DECR	169.2	169.462	1.10	0.012	0.179
14	10’	121.40	3.80	13 down to 7	169.2	169.203	1.06	0.015	0.390
15	8’	168.20	2.64	7 down to 5	169.2	169.719	1.01	0.012	0.289
16	8’	284.27	2.26	approx. 5	169.2	169.591	1.25	0.008	0.253
17	8’	X	2.21	approx. 5	169.2	168.710	1.07	0.007	0.253
18	7’	490.80	2.50	approx. 5	169.2	169.205	1.00	0.011	0.304
19	7’	212.90	3.14	6 to 8	169.2	169.027	1.13	0.009	0.393
20	7’	137.84	5.15	8 to 19	169.2	169.535	1.26	0.009	0.674

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
10		5560.389	5558.318	5558.305	345.23.03	345.23.02
11		5560.408	5558.318	5558.326	345.23.03	345.23.03
12	5994.593	5996.681	5991.192	5993.367	167.12.30	167.08.08
13	5994.593	5994.499	5991.192	5991.152	167.12.30	167.12.31
14	5994.593	5994.344	5991.192	5991.003	167.12.30	167.12.47
15	5994.593	5994.376	5991.192	5991.020	167.12.30	167.12.59
16	5994.593	5994.572	5991.192	5991.221	167.12.30	167.12.24
17	5994.593	5994.109	5991.192	5990.782	167.12.30	167.14.52
18	5994.593	5994.391	5991.192	5991.050	167.12.30	167.13.32
19	5994.593	5994.318	5991.192	5990.983	167.12.30	167.13.45
20	5994.593	5994.516	5991.192	5991.166	167.12.30	167.12.31

* EDM REDUCED DISTANCES : 5991.273

TABLE 4

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
21	10'	841.50	1.20	< 5	160.2	161.12	1.13	0.009	0.199
22	19'	207.50	1.83	SHARP INCR 4 to 16	160.2	160.78	1.11	0.020	0.183
23	28'	69.00	1.80	SHARP INCR & DECR	160.2	160.56	1.29	0.019	0.177
24	9'	121.40	3.80	13 down to 7	160.2	160.23	1.03	0.021	0.379
25	19'	168.15	2.64	7 down to 5	160.2	160.77	1.15	0.011	0.295
26	9'	284.27	2.26	< 5	160.2	160.58	1.04	0.011	0.232
27	9'	X	2.21	approx. 5	160.2	160.08	1.02	0.012	0.222
28	9'	490.80	2.50	approx. 5	160.2	161.11	1.03	0.008	0.246
29	9'	212.90	3.14	6 to 8	160.2	160.15	X	0.011	25.126
30	9'	137.84	5.15	8 to 19	160.2	164.66	1.15	0.010	1.424

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
21	8200.793	8198.384	8197.861	8195.479	331.50.23	331.52.06
22	8200.793	8200.278	8197.861	8197.381	331.50.23	331.50.40
23	8200.793	8200.730	8197.861	8197.837	331.50.23	331.50.25
24	8200.793	8201.029	8197.861	8198.143	331.50.23	331.50.06
25	8200.793	8200.283	8197.861	8197.386	331.50.23	331.50.48
26	8200.793	8200.647	8197.861	8197.754	331.50.23	331.50.29
27	8200.793	8199.775	8197.861	8196.892	331.50.23	331.51.24
28	8200.793	8200.918	8197.861	8198.015	331.50.23	331.50.09
29	8200.793	8200.361	8197.861	8197.260	331.50.23	331.51.15
30	8200.793	8201.024	8197.861	8198.051	331.50.23	331.50.49

* EDM REDUCED DISTANCES : 5991.273

TABLE 5

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
31	6'	841.5	1.20	< 5	8.9	8.02	1.32	0.013	0.158
32	18'	207.5	1.80	SHARP INCR. 3 to 16	8.9	8.82	1.02	0.018	0.195
33	28'	69.00	1.80	SHARP INCR & DECR	8.9	8.94	1.14	0.018	0.176
34	42'	56.00	0.64	SHARP INCR. 4 to 36	8.9	9.00	1.04	0.019	0.067
35	9'	168.15	2.64	7 down to 5	8.9	8.80	1.01	0.009	0.290
36	8'	284.27	2.26	approx. 5	8.9	9.39	1.03	0.010	0.248
37	10'	X	2.21	approx. 5	8.9	8.42	1.13	0.008	0.253
38	9'	490.80	2.50	approx. 5	8.9	8.16	1.03	0.009	0.304
39	9'	212.90	3.14	6 to 8	8.9	8.97	1.07	0.019	0.393
40	9'	137.84	5.15	SHARP INCR. 8 to 19	8.9	8.62	1.91	0.007	0.674

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
31	2895.672	2890.054	2895.180	2889.611	298.34.45	298.38.00
32	2895.672	2895.123	2895.180	2894.681	298.34.45	298.35.02
33	2895.672	2895.383	2895.180	2894.937	298.34.45	298.34.56
34	2895.672	2895.166	2895.180	2894.719	298.34.45	298.35.02
35	2895.672	2894.710	2895.180	2894.265	298.34.45	298.35.26
36	2895.672	2894.319	2895.180	2893.872	298.34.45	298.36.18
37	2895.672	2891.814	2895.180	2891.371	298.34.45	298.37.16
38	2895.672	2897.346	2895.180	2896.903	298.34.45	298.33.06
39	2895.672	2895.906	2895.180	2895.460	298.34.45	298.34.47
40	2895.672	2895.606	2895.180	2895.160	298.34.45	298.34.39

* EDM REDUCED DISTANCES : 2895.226

TABLE 6

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
41	32'	18.63	0.88	SHARP DECR & INCR	8.9	8.904	3.58	0.035	0.102
42	5'	269.07	2.89	< 5	8.9	9.213	1.32	0.009	0.283
43	35'	18.63	0.88	SHARP DECR & INCR	160.2	159.134	1.18	0.022	0.186
44	20'	13.49	1.35	SHARP DECR	160.2	161.017	1.01	0.089	0.131
45	14'	23.13	1.40	< 5	169.2	169.538	1.05	0.027	0.137
46	10'	77.34	1.48	< 5	8.9	8.926	1.40	0.009	0.501
47	5'	269.07	2.89	< 5	160.2	160.306	1.20	0.010	0.308

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
41	2895.672	2895.611	2895.180	2895.164	298.34.45	298.34.48
42	2895.672	2895.523	2895.180	2895.077	298.34.45	298.34.53
43	8200.793	8204.797	8197.861	8201.945	331.50.23	331.49.09
44	8200.793	8200.839	8197.861	8197.949	331.50.23	331.50.15
45	5994.593	5994.599	5991.192	5991.255	347.12.30	347.12.22
46	2895.672	2895.096	2895.180	2894.651	298.34.45	298.35.09
47	8200.793	8200.577	8197.861	8197.703	331.50.23	331.50.35

* EDM REDUCED DISTANCES : 8197.904, 5991.273, 2895.226

TABLE 7

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
48	16'	37.00	1.43	SHARP decrease	169.2	169.50	***	0.000	0.000
49	25'	11.99	2.31	SHARP decrease	169.2	169.76	1.04	0.082	0.296
50	20'	13.49	1.35	SHARP decrease	169.2	169.72	1.14	0.065	0.131
51	33'	6.45	0.83	SHARP decrease	169.2	169.46	1.79	0.053	0.080
52	30'	8.13	1.00	SHARP incr. & decr.	169.2	169.51	1.70	0.046	0.113

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
48	5994.593	5993.772	5991.175	5990.433	347.12.30	347.13.12
49	5994.593	5994.467	5991.175	5991.122	347.12.30	347.12.19
50	5994.593	5994.590	5991.175	5991.245	347.12.30	347.12.19
51	5994.593	5994.513	5991.175	5991.176	347.12.30	347.12.30
52	5994.593	5994.484	5991.175	5991.147	347.12.30	347.12.31

* EDM REDUCED DISTANCES : 5991.273

TABLE 8

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
53	25’	86.72	1.11	< 5	8.9	8.98	1.06	0.042	0.110
54	45’	9.68	1.76	3 TO 34 TO 5	8.9	8.95	16.78	0.019	0.169
55	85’	7.36	0.70	3 TO 27 TO 4	8.9	8.92	14.53	0.028	0.070
56	25’	9.66	0.77	13 DOWN TO 2	8.9	8.91	4.74	0.026	0.077
57	30’	8.45	0.92	3 TO 8	8.9	8.93	5.21	0.017	0.090
58	30’	8.43	0.67	9 TO 2	8.9	8.91	8.83	0.024	0.066
59	30’	8.55	0.83	< 5	8.9	8.90	14.20	0.015	0.091
60	60’	4.16	0.70	< 6	8.9	8.92	9.83	0.020	0.091
61	40’	8.70	0.80	< 5	8.9	8.89	6.26	0.028	0.079
62	60’	3.61	0.88	4 TO 9 TO 5	8.9	8.93	18.17	0.014	0.112
63	30’	8.24	0.95	< 5	8.9	X	X	X	X
64	30’	10.55	0.97	< 5	8.9	8.93	3.37	0.020	0.097
65	10’	42.34	12.83	18 TO 76 TO 32	8.9	X	X	X	X
66	10’	43.12	8.75	26 TO 32 TO 15	8.9	X	X	X	X

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
53	2895.672	2896.223	2895.180	2895.777	298.34.45	298.34.44
54	2895.672	2895.599	2895.180	2895.153	298.34.45	298.34.48
55	2895.672	2895.609	2895.180	2895.164	298.34.45	298.34.48
56	2895.672	2895.614	2895.180	2895.168	298.34.45	298.34.48
57	2895.672	2895.613	2895.180	2895.168	298.34.45	298.34.49
58	2895.672	2895.612	2895.180	2895.166	298.34.45	298.34.48
59	2895.672	2895.614	2895.180	2895.169	298.34.45	298.34.48
60	2895.672	2895.617	2895.180	2895.172	298.34.45	298.34.48
61	2895.672	2895.618	2895.180	2895.172	298.34.45	298.34.48
62	2895.672	2895.622	2895.180	2895.177	298.34.45	298.34.48
63	2895.672	X	2895.180	X	298.34.45	X
64	2895.672	2895.621	2895.180	2895.175	298.34.45	298.34.48
65	2895.672	X	2895.180	X	298.34.45	X
66	2895.672	X	2895.180	X	298.34.45	X

* EDM REDUCED DISTANCES : 2895.226

TABLE 9

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
67	25’	86.72	1.11	< 5	160.2	X	X	X	X
68	45’	9.68	1.76	3 TO 34 TO 5	160.2	X	X	X	X
69	85’	7.36	0.70	3 TO 27 TO 4	160.2	160.60	6.74	0.037	0.068
70	25’	9.66	0.77	13 DOWN TO 2	160.2	160.62	3.29	0.033	0.077
71	30’	8.45	0.92	3 TO 8	160.2	160.62	4.20	0.019	0.093
72	30’	8.43	0.67	9 TO 2	160.2	160.63	3.85	0.033	0.066
73	30’	8.55	0.83	< 5	160.2	160.65	7.74	0.021	0.089
74	60’	4.16	0.70	< 6	160.2	160.62	6.97	0.026	0.091
75	40’	8.70	0.80	< 5	160.2	160.66	3.37	0.039	0.079
76	60’	3.61	0.88	4 TO 9 TO 5	160.2	160.60	5.74	0.027	0.112
77	30’	8.24	0.95	< 5	160.2	X	X	X	X
78	30’	10.55	0.97	< 5	160.2	X	X	X	X
79	10’	42.34	12.83	18 TO 76 TO 32	160.2	X	X	X	X
80	10’	43.12	8.75	26 TO 32 TO 15	160.2	X	X	X	X

SESS.	EDM SLOPE DISTANCE	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
67	8200.793	X	8197.861	X	331.50.23	X
68	8200.793	X	8197.861	X	331.50.23	X
69	8200.793	8200.731	8197.861	8197.841	331.50.23	331.50.24
70	8200.793	8200.745	8197.861	8197.853	331.50.23	331.50.25
71	8200.793	8200.755	8197.861	8197.864	331.50.23	331.50.24
72	8200.793	8200.760	8197.861	8197.867	331.50.23	331.50.24
73	8200.793	8200.855	8197.861	8197.863	331.50.23	331.50.24
74	8200.793	8200.757	8197.861	8197.867	331.50.23	331.50.24
75	8200.793	8200.768	8197.861	8197.877	331.50.23	331.50.24
76	8200.793	8200.765	8197.861	8197.874	331.50.23	331.50.24
77	8200.793	X	8197.861	X	331.50.23	X
78	8200.793	X	8197.861	X	331.50.23	X
79	8200.793	X	8197.861	X	331.50.23	X
80	8200.793	X	8197.861	X	331.50.23	X

* EDM REDUCED DISTANCE : 8197.904

TABLE 10

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
81	25'	86.72	1.11	< 5	91.2	91.41	1.01	0.089	0.113
82	45'	9.68	1.76	3 TO 34 TO 5	91.2	91.09	3.73	0.049	0.170
83	85'	7.36	0.70	3 TO 27 TO 4	91.2	91.06	1.19	0.044	0.084
84	25'	9.66	0.77	13 DOWN TO 2	91.2	91.07	1.53	0.051	0.078
85	30'	8.45	0.92	3 TO 8	91.2	91.06	1.07	0.047	0.090
86	30'	8.43	0.67	9 TO 2	91.2	91.12	4.90	0.038	0.065
87	30'	8.55	0.83	< 5	91.2	91.15	3.93	0.028	0.090
88	60'	4.16	0.70	< 6	91.2	91.16	4.05	0.041	0.069
89	40'	8.70	0.80	< 5	91.2	91.16	3.56	0.036	0.079
90	60'	3.61	0.88	4 TO 9 TO 5	91.2	90.13	2.08	0.049	0.088
91	30'	8.24	0.95	< 5	91.2	91.13	1.18	0.023	0.097
92	30'	10.55	0.97	< 5	91.2	91.15	1.05	0.027	0.095
93	10'	42.34	12.83	18 TO 76 TO 32	91.2	90.82	1.11	0.076	1.682
94	10'	43.12	8.75	26 TO 32 TO 15	91.2	91.76	2.01	0.044	0.902

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
81		14590.122	14588.360	14588.053	64.26.25	64.26.18
82		14590.399	14588.360	14588.332	64.26.25	64.26.25
83	* flt soln used	14590.000	14588.360	14587.933	64.26.25	64.26.21
84		14590.380	14588.360	14588.313	64.26.25	64.26.25
85	* flt soln used	14590.356	14588.360	14588.289	64.26.25	64.26.26
86		14590.404	14588.360	14588.339	64.26.25	64.26.26
87		14590.405	14588.360	14588.329	64.26.25	64.26.26
88		14590.407	14588.360	14588.340	64.26.25	64.26.26
89		14590.389	14588.360	14588.322	64.26.25	64.26.26
90		14590.396	14588.360	14588.322	64.26.25	64.26.26
91		14590.407	14588.360	14588.339	64.26.25	64.26.25
92		14590.396	14588.360	14588.329	64.26.25	64.26.26
93		14590.086	14588.360	14587.888	64.26.25	59.55.11
94		14591.438	14588.360	14589.233	64.26.25	59.55.14

* flt–fix too large, therefore flt solution used

TABLE 11

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
95	25'	86.72	1.11	< 5	260.3	261.15	1.12	0.111	0.124
96	45'	9.68	1.76	3 TO 34 TO 5	260.3	260.61	3.68	0.048	0.171
97	85'	7.36	0.70	3 TO 27 TO 4	260.3	260.40	1.18	0.081	0.088
98	25'	9.66	0.77	13 DOWN TO 2	260.3	260.57	1.54	0.066	0.080
99	30'	8.45	0.92	3 TO 8	260.3	260.63	1.15	0.038	0.319
100	30'	8.43	0.67	9 TO 2	260.3	260.63	4.23	0.037	0.066
101	30'	8.55	0.83	< 5	260.3	279.84	1.01	0.054	0.104
102	60'	4.16	0.70	< 6	260.3	260.69	2.97	0.056	0.067
103	40'	8.70	0.80	< 5	260.3	X	X	X	X
104	60'	3.61	0.88	4 TO 9 TO 5	260.3	X	X	X	X
105	30'	8.24	0.95	< 5	260.3	X	X	X	X
106	30'	10.55	0.97	< 5	260.3	260.80	1.12	0.032	0.095
107	10'	42.34	12.83	18 TO 76 TO 32	260.3	256.01	1.00	0.014	2.275
108	10'	43.12	8.75	26 TO 32 TO 15	260.3	260.47	1.33	0.012	2.732

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
95		16878.407	16873.882	16874.134	224.06.31	224.06.31
96		16878.113	16873.882	16873.845	224.06.31	224.06.33
97	* flt soln used	16877.261	16873.882	16872.996	224.06.31	224.06.23
98		16878.089	16873.882	16873.822	224.06.31	224.06.31
99		16877.898	16873.882	16873.629	224.06.31	224.06.31
100		16878.128	16873.882	16873.861	224.06.31	224.06.32
101		16878.117	16873.882	16873.849	224.06.31	224.06.32
102		16878.127	16873.882	16873.858	224.06.31	224.06.32
103		X	16873.882	X	224.06.31	X
104		X	16873.882	X	224.06.31	X
105		X	16873.882	X	224.06.31	X
106		16878.153	16873.882	16873.880	224.06.31	224.06.33
107		16882.390	16873.882	16882.066	224.06.31	224.07.16
108		16879.067	16873.882	16874.588	224.06.31	224.06.44

* flt–fix too large, therefore float solution used

TABLE 12

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
109	25'	86.72	1.11	< 5	46.2	46.80	1.38	0.045	0.130
110	45'	9.68	1.76	3 TO 34 TO 5	46.2	46.64	1.83	0.057	0.183
111	85'	7.36	0.70	3 TO 27 TO 4	46.2	46.79	1.43	0.090	0.071
112	25'	9.66	0.77	13 DOWN TO 2	46.2	46.82	1.42	0.065	0.076
113	30'	8.45	0.92	3 TO 8	46.2	46.83	1.11	0.069	0.091
114	30'	8.43	0.67	9 TO 2	46.2	46.73	1.50	0.069	0.067
115	30'	8.55	0.83	< 5	46.2	46.75	1.24	0.035	0.123
116	60'	4.16	0.70	< 6	46.2	46.76	3.72	0.030	0.071
117	40'	8.70	0.80	< 5	46.2	46.74	5.34	0.031	0.080
118	60'	3.61	0.88	4 TO 9 TO 5	46.2	46.77	1.40	0.051	0.102
119	30'	8.24	0.95	< 5	46.2	X	X	X	X
120	30'	10.55	0.97	< 5	46.2	X	X	X	X
121	10'	42.34	12.83	18 TO 76 TO 32	46.2	20.25	1.01	0.010	3.006
122	10'	43.12	8.75	26 TO 32 TO 15	46.2	47.44	2.84	0.016	2.527

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
109		22536.382	22533.765	22533.600	79.18.24	79.18.25
110		22536.414	22533.765	22533.632	79.18.24	79.18.25
111		22535.576	22533.765	22532.794	79.18.24	79.18.24
112		22536.397	22533.765	22533.614	79.18.24	79.18.25
113		22536.412	22533.765	22533.629	79.18.24	79.18.24
114		22536.436	22533.765	22533.650	79.18.24	79.18.25
115		22536.438	22533.765	22533.656	79.18.24	79.18.25
116		22536.460	22533.765	22533.677	79.18.24	79.18.25
117		22536.439	22533.765	22533.657	79.18.24	79.18.25
118		22536.433	22533.765	22533.651	79.18.24	79.18.25
119		X	22533.765	X	79.18.24	X
120		X	22533.765	X	79.18.24	X
121		22506.866	22533.765	22503.430	79.18.24	79.18.10
122		22535.452	22533.765	22531.971	79.18.24	79.18.24

TABLE 13

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
123	25'	86.72	1.11	< 5	122.8	122.95	1.49	0.063	0.118
124	45'	9.68	1.76	3 TO 34 TO 5	122.8	122.85	2.12	0.081	0.171
125	85'	7.36	0.70	3 TO 27 TO 4	121.4	121.26	1.97	0.062	0.071
126	25'	9.66	0.77	13 DOWN TO 2	121.4	121.25	1.77	0.059	0.080
127	30'	8.45	0.92	3 TO 8	121.4	121.25	1.04	0.079	0.091
128	30'	8.43	0.67	9 TO 2	121.4	120.94	1.13	0.079	0.067
129	30'	8.55	0.83	< 5	121.4	120.91	1.81	0.029	0.126
130	60'	4.16	0.70	< 6	121.4	121.29	4.52	0.023	0.076
131	40'	8.70	0.80	< 5	121.4	121.33	4.02	0.036	0.080
132	35'	9.03	1.13	4 TO 9 TO 8	121.4	121.39	1.13	0.042	0.111
133	30'	8.24	0.95	< 5	121.4	X	X	X	X
134	30'	10.55	0.97	< 5	121.4	X	X	X	X
135	10'	42.34	12.83	18 TO 76 TO 32	121.4	121.50	2.59	0.048	1.877
136	10'	43.12	8.75	26 TO 32 TO 15	121.4	121.38	1.78	0.026	0.889

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
123		23017.883	23013.907	23014.495	64.14.13	64.14.18
124		23017.158	23013.907	23013.772	64.14.13	64.14.14
125		23022.169	23018.949	23018.790	64.13.58	64.13.59
126		23022.183	23018.949	23018.803	64.13.58	64.13.59
127		23022.206	23018.949	23018.826	64.13.58	64.13.58
128		23022.146	23018.949	23018.769	64.13.58	64.13.58
129		23022.369	23018.949	23018.993	64.13.58	64.13.59
130		23022.248	23018.949	23018.869	64.13.58	64.13.59
131		23022.227	23018.949	23018.848	64.13.58	64.13.59
132		23022.448	23018.949	23018.858	64.13.58	64.13.59
133		X	23018.949	X	64.13.58	X
134		X	23018.949	X	64.13.58	X
135		23021.577	23018.949	23017.401	64.13.58	64.13.58
136		23022.312	23018.949	23018.138	64.13.58	64.13.59

TABLE 14

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
137	25’	86.72	1.11	< 5	100.0	100.35	2.23	0.048	0.115
138	45’	9.68	1.76	3 TO 34 TO 5	100.0	100.32	2.07	0.066	0.492
139	85’	7.36	0.70	3 TO 27 TO 4	100.0	100.05	3.74	0.049	0.072
140	25’	9.66	0.77	13 DOWN TO 2	100.0	100.14	1.83	0.038	0.083
141	30’	8.45	0.92	3 TO 8	100.0	99.95	1.50	0.036	0.097
142	30’	8.43	0.67	9 TO 2	100.0	100.03	1.89	0.039	0.072
143	30’	8.55	0.83	< 5	100.0	100.05	4.98	0.024	0.111
144	60’	4.16	0.70	< 6	100.0	100.03	6.20	0.035	0.068
145	17’	26.67	1.06	< 5	100.0	100.31	1.27	0.058	0.103
146	60’	3.61	0.88	4 TO 9 TO 5	100.0	100.02	5.14	0.034	0.111
147	30’	8.24	0.95	< 5	100.0	100.01	1.35	0.020	0.117
148	30’	10.55	0.97	< 5	100.0	100.04	3.94	0.014	0.115
149	10’	42.34	12.83	18 TO 76 TO 32	100.0	100.68	1.42	0.054	1.801
150	10’	43.12	8.75	26 TO 32 TO 15	100.0	99.90	1.77	0.027	0.911

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
137		16490.012	16484.289	16487.630	73.24.36	72.25.00
138		16485.904	16484.289	16483.552	73.24.36	72.24.37
139		16486.593	16484.289	16484.211	73.24.36	73.24.37
140		16486.501	16484.289	16484.120	73.24.36	73.24.38
141		16486.609	16484.289	16484.229	73.24.36	73.24.37
142		16486.624	16484.289	16484.241	73.24.36	73.24.37
143		16486.613	16484.289	16484.228	73.24.36	73.24.37
144		16486.638	16484.289	16484.257	73.24.36	73.24.37
145		16486.784	16484.289	16484.402	73.24.36	73.24.39
146		16486.640	16484.289	16484.266	73.24.36	73.24.37
147		16486.636	16484.289	16484.331	73.24.36	72.24.37
148		16486.640	16484.289	16484.255	73.24.36	73.24.37
149		16486.220	16484.289	16483.837	73.24.36	67.57.29
150		16486.686	16484.289	16484.308	73.24.36	67.57.25

TABLE 15

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
151	25'	86.72	1.11	< 5	25.5	25.46	2.65	0.028	0.109
152	22'				25.5	25.43	10.95	0.020	0.472
153	85'	7.36	0.70	3 TO 27 TO 4	25.5	25.47	13.43	0.026	0.070
154	25'	9.66	0.77	13 DOWN TO 2	25.5	25.49	2.64	0.024	0.078
155	30'	8.45	0.92	3 TO 8	25.5	25.50	8.51	0.013	0.106
156	30'	8.43	0.67	9 TO 2	25.5	25.48	7.59	0.024	0.069
157	30'	8.55	0.83	< 5	25.5	25.48	6.12	0.019	0.099
158	60'	4.16	0.70	< 6	25.5	25.48	26.42	0.016	0.067
159	17'	26.67	1.06	< 5	25.5	25.50	2.94	0.024	0.107
160	60'	3.61	0.88	4 TO 9 TO 5	25.5	X	X	X	X
161	30'	8.24	0.95	< 5	25.5	X	X	X	X
162	30'	10.55	0.97	< 5	25.5	X	X	X	X
163	10'	42.34	12.83	18 TO 76 TO 32	25.5	25.68	3.00	0.022	2.287
164	10'	43.12	8.75	26 TO 32 TO 15	25.5	25.46	3.17	0.011	1.692

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
151		4371.396	4370.874	4370.879	97.20.36	97.20.37
152		4371.385	4370.874	4370.868	97.20.36	97.20.36
153		4371.387	4370.874	4370.869	97.20.36	97.20.37
154		4371.391	4370.874	4370.872	97.20.36	97.20.37
155		4371.399	4370.874	4370.881	97.20.36	97.20.37
156		4371.392	4370.874	4370.875	97.20.36	97.20.36
157		4371.397	4370.874	4370.879	97.20.36	97.20.37
158		4371.400	4370.874	4370.883	97.20.36	97.20.37
159		4371.393	4370.874	4370.875	97.20.36	97.20.37
160		X	4370.874	X	97.20.36	X
161		X	4370.874	X	97.20.36	X
162		X	4370.874	X	97.20.36	X
163		4371.378	4370.874	4370.721	97.20.36	93.02.36
164		4371.392	4370.874	4370.739	97.20.36	93.02.30

TABLE 16

SESSION	TIME	BDOP1	BDOP3	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	QUALITY RATIO	RMS	RDOP
165	25’	86.72	1.11	< 5	125.4	125.82	1.58	0.037	0.116
166	45’	9.68	1.76	3 TO 34 TO 5	125.4	125.59	2.86	0.060	0.170
167	85’	7.36	0.70	3 TO 27 TO 4	125.4	125.53	7.33	0.040	0.073
168	25’	9.66	0.77	13 DOWN TO 2	125.4	125.63	1.42	0.037	0.075
169	30’	8.45	0.92	3 TO 8	125.4	125.45	1.81	0.033	0.091
170	30’	8.43	0.67	9 TO 2	125.4	125.53	1.50	0.056	0.067
171	30’	8.55	0.83	< 5	125.4	125.52	4.74	0.022	0.092
172	60’	4.16	0.70	< 6	125.4	125.51	8.96	0.030	0.067
173	17’	26.67	1.06	< 5	125.4	125.81	1.38	0.052	0.105
174	60’	3.61	0.88	4 TO 9 TO 5	125.4	X	X	X	X
175	30’	8.24	0.95	< 5	125.4	X	X	X	X
176	30’	10.55	0.97	< 5	125.4	X	X	X	X
177	10’	42.34	12.83	18 TO 76 TO 32	125.4	125.71	2.30	0.038	2.302
178	10’	43.12	8.75	26 TO 32 TO 15	125.4	125.38	1.57	0.029	0.876

SESS.	COMMENTS	GPS SLOPE DISTANCE	JOIN DIST.	GPS RED. DIST.	JOIN DIRECT.	GPS RED. DIRECT.
165		12655.837	12650.534	12653.576	65.04.25	65.05.03
166		12652.729	12650.534	12650.471	65.04.25	65.04.26
167		12652.720	12650.534	12650.463	65.04.25	65.04.26
168		12652.617	12650.534	12650.359	65.04.25	65.04.26
169		12652.728	12650.534	12650.471	65.04.25	65.04.26
170		12652.727	12650.534	12650.470	65.04.25	65.04.26
171		12652.730	12650.534	12650.472	65.04.25	65.04.26
172		12652.754	12650.534	12650.552	65.04.25	65.04.26
173		12652.878	12650.534	12650.618	65.04.25	65.04.30
174		X	12650.534	X	65.04.25	X
175		X	12650.534	X	65.04.25	X
176		X	12650.534	X	65.04.25	X
177		12652.246	12650.534	12649.986	65.04.25	65.04.28
178		12652.835	12650.534	12650.580	65.04.25	65.04.24

TABLE 17 : BDOP EXPERIMENTOVER 3 DAYS

	PEROID	BDOP1	BDOP3	PDOP RANGE
A	25'	86.72	1.11	< 5
B	45'	9.68	1.76	3 TO 34 TO 5
C	85'	7.36	0.70	3 TO 27 TO 4
D	25'	9.66	0.77	13 DOWN TO 2
E	30'	8.45	0.92	3 TO 8
F	30'	8.43	0.67	9 TO 2
G	30'	8.55	0.83	< 5
H	60'	4.16	0.70	< 6
I	40'	8.70	0.80	< 5
J	60'	3.61	0.88	4 TO 9 TO 5
K	30'	8.24	0.95	< 5
L	30'	10.55	0.97	< 5
M	10'	42.34	12.83	18 to 76 to 32
N	10'	43.12	8.75	26 to 32 to 15

	15–TUES	16–WED	17–THUR
	HEIGHT DIFFS. = 169.2		
A	169.64	170.31	169.59
B	169.53	169.52	169.55
C	169.51	169.52	169.45
D	169.53	169.51	169.48
E	169.56	169.54	169.48
F	169.55	169.51	169.47
G	169.55	169.54	169.48
H	169.54	169.54	169.45
I	169.54	X	169.48
J	169.53	X	169.46
K	169.54	X	169.45
L	X	169.53	169.45
M	169.54	170.02	169.57
N	169.50	169.13	169.50

	15–TUES	16–WED	17–THUR*
	EDM SLOPE DIST. = 5994.593		
A	5993.664	5993.616	5994.147
B	5994.525	5994.519	5994.522
C	5994.510	5994.498	5997.243
D	5994.515	5994.507	5997.240
E	5994.523	5994.518	5997.253
F	5994.539	5994.522	5997.260
G	5994.530	5994.526	5997.270
H	5994.532	5994.531	5997.251
I	5994.535	X	5997.257
J	5994.530	X	5997.259
K	5994.531	X	5997.263
L	X	5994.521	5997.263
M	5994.542	5994.419	5997.635
N	5994.536	5994.290	5997.258

	15–TUES	16–WED	17–THUR
	JOIN DISTANCE = 5991.192		
A	5990.294	5990.298	5990.808
B	5991.186	5991.182	5991.184
C	5991.171	5991.161	5991.246
D	5991.224	5991.169	5991.245
E	5991.183	5991.180	5991.255
F	5991.198	5991.184	5991.262
G	5991.189	5991.187	5991.270
H	5991.192	5991.192	5991.230
I	5991.196	X	5991.258
J	5991.190	X	5991.262
K	5991.191	X	5991.265
L	X	5991.183	5991.266
M	5991.263	5991.185	5991.761
N	5991.171	5991.082	5991.386

	15–TUES	16–WED	17–THUR
	JOIN DIRECTION = 167.12.30		
A	167.13.47	167.14.08	167.12.48
B	167.12.31	167.12.31	167.12.31
C	167.12.31	167.12.30	167.12.32
D	167.12.31	167.12.31	167.12.31
E	167.12.31	167.12.31	167.12.31
F	167.12.31	167.12.31	167.12.31
G	167.12.31	167.12.31	167.12.31
H	167.12.31	167.12.31	167.12.31
I	167.12.31	X	167.12.31
J	167.12.30	X	167.12.32
K	167.12.31	X	167.12.32
L	X	167.12.31	167.12.32
M	167.12.31	166.19.28	166.19.16
N	167.12.31	166.20.00	166.19.42

* eccentrically observed distances

TABLE 17 (continued): BDOP EXPERIMENT OVER 3 DAYS

	15–TUES	16–WED	17–THUR
	QUALITY FACTORS		
A	1.39	1.43	1.42
B	9.44	14.52	28.59
C	5.97	1.46	13.04
D	9.51	3.47	2.34
E	8.89	1.89	4.43
F	5.38	10.34	5.32
G	9.35	5.05	13.43
H	14.85	6.97	15.79
I	7.89	X	4.32
J	9.97	X	5.83
K	10.27	X	12.29
L	X	3.57	3.40
M	1.54	4.47	2.23
N	18.49	2.95	5.62

	15–TUES	16–WED	17–THUR
	FIXED SOLN RMS		
A	0.055	0.035	0.035
B	0.032	0.024	0.015
C	0.034	0.029	0.027
D	0.016	0.034	0.033
E	0.014	0.028	0.020
F	0.031	0.024	0.029
G	0.019	0.022	0.010
H	0.020	0.025	0.020
I	0.027	X	0.026
J	0.022	X	0.031
K	0.013	X	0.013
L	X	0.020	0.019
M	0.036	0.030	0.033
N	0.006	0.009	0.012

	15–TUES	16–WED	17–THUR
	RDOP		
A	0.113	0.217	0.110
B	0.180	0.170	0.172
C	0.068	0.088	0.070
D	0.079	0.079	0.079
E	0.093	0.091	0.091
F	0.065	0.065	0.065
G	0.089	0.099	0.146
H	0.069	0.084	0.072
I	0.079	X	0.084
J	0.087	X	0.088
K	0.092	X	0.092
L	X	0.095	0.097
M	1.761	1.681	1.765
N	0.916	1.243	0.891

* REDUCED EDM DISTANCE : 5991.273

TABLE 18A : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS & JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM

FROM TABLE 1:

	DIFFERENCES IN DISTANCE		
SESSION	28 – TUES	29 – WED	30 – THUR
A	−0.061	−0.038	−0.006
B	0.001	0.078	0.001
C	−0.017	−0.038	0.000
D	−0.004	−0.042	−0.010
E	0.016	−0.019	0.009
F	0.026	−0.005	0.017

FROM TABLE 1:

	DIFFERENCES IN HEIGHT		
SESSION	28 – TUES	29 – WED	30 – THUR
A	0.6	0.4	0.4
B	0.4	0.6	0.4
C	0.4	0.4	0.4
D	0.4	0.4	0.4
E	0.4	0.4	0.4
F	0.4	0.4	0.4

FROM TABLE 3:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
10	−0.013	−1"	0.3
11	0.008	0"	0.2
12	2.175	−4' 22"	0.1
13	−0.040	1"	0.3
14	−0.189	17"	0.0
15	−0.172	29"	0.5
16	0.029	−6"	0.4
17	−0.410	22"	0.5
18	−0.142	2"	0.0
19	−0.209	15"	−0.2
20	−0.026	1"	0.3

FROM TABLE 1:

	DIFFERENCES IN DIRECTION		
SESSION	28 – TUES	29 – WED	30 – THUR
A	−7"	1"	1"
B	1"	−2"	1"
C	1"	1"	2"
D	1"	1"	1"
E	1"	1"	1"
F	1"	1"	1"

FROM TABLE 2:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
1	−0.004	1"	0.3
2	0.018	1"	0.1
3	0.019	1"	0.1
4	0.009	1"	0.1
5	−0.017	1"	0.3
6	−0.021	3"	0.0
7	−0.026	3"	0.0
8	−0.032	2"	0.3
9	0.010	0"	0.2
Y	−0.443	16"	0.2
Z	−0.128	9"	0.2

TABLE 18B : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS AND JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM

FROM TABLE 4:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
21	−2.382	1' 43"	0.9
22	−0.480	17"	0.6
23	−0.024	2"	0.4
24	0.282	−17"	0.0
25	−0.475	25"	0.6
26	−0.107	6"	0.4
27	−0.969	1"	−0.1
28	0.154	−14"	−0.1
29	−0.601	52"	0.0
30	0.190	26"	0.5

FROM TABLE 5:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
31	4.431	3' 15"	−0.9
32	−0.499	17"	−0.1
33	−0.243	11"	0.0
34	−0.461	17"	0.1
35	−0.915	−19"	−1.1
36	−1.308	1' 33"	0.5
37	−3.809	2' 31"	−0.5
38	1.723	−1' 39"	−0.7
39	0.280	2"	0.1
40	−0.020	−6"	−0.3

FROM TABLE 6:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
41	−0.016	3"	0.0
42	−0.103	8"	0.3
43	4.084	−74"	−1.1
44	0.088	−8"	0.8
45	0.063	−8"	0.3
46	−0.529	24"	0.0
47	−0.158	12"	0.1

FROM TABLE 7:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
48	−0.742	−18"	0.3
49	−0.053	−11"	0.6
50	0.070	−11"	0.5
51	0.001	0"	0.3
52	−0.028	1"	0.3

TABLE 18C : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS & JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM

FROM TABLE 8:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
53	0.597	−1"	0.1
54	−0.027	3"	0.1
55	−0.016	3"	0.0
56	−0.012	3"	0.0
57	−0.012	4"	0.0
58	−0.014	3"	0.0
59	−0.011	3"	0.0
60	−0.008	3"	0.0
61	−0.008	3"	0.0
62	−0.003	3"	0.0
63	X	X	X
64	−0.005	3"	0.0
65	X	X	X
66	X	X	X

FROM TABLE 9:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
67	X	X	X
68	X	X	X
69	−0.020	1"	0.4
70	−0.008	2"	0.4
71	0.003	1"	0.4
72	0.006	1"	0.4
73	0.002	1"	0.4
74	0.006	1"	0.4
75	0.016	1"	0.5
76	0.013	1"	0.4
77	X	X	X
78	X	X	X
79	X	X	X
80	X	X	X

FROM TABLE 10:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
81	−0.307	−7"	0.2
82	−0.028	0"	−0.1
83	0.427	−4"	−0.1
84	−0.047	0"	−0.1
85	−0.071	1"	−0.1
86	−0.021	1"	−0.1
87	−0.031	1"	0.0
88	−0.020	1"	0.0
89	−0.038	1"	0.0
90	−0.038	1"	−0.1
91	−0.021	0"	−0.1
92	−0.031	1"	0.0
93	−0.472	−14"	0.6
94	0.873	−11"	0.6

FROM TABLE 11:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
95	0.252	0"	−0.1
96	−0.037	2"	0.3
97	−0.886	−8"	0.1
98	−0.060	0"	0.3
99	−0.253	0"	0.3
100	−0.021	1"	0.3
101	−0.033	1"	0.5
102	−0.024	1"	0.4
103	X	X	X
104	X	X	X
105	X	X	X
106	−0.002	2"	0.5
107	−1.816	45"	−4.3
108	0.706	13"	0.2

TABLE 18D : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS AND JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM

FROM TABLE 12:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
109	−0.165	1"	0.6
110	−0.133	1"	0.4
111	−0.971	0"	0.6
112	−0.151	1"	0.6
113	−0.136	0"	0.6
114	−0.115	1"	0.5
115	−0.109	1"	0.6
116	−0.088	1"	0.6
117	−0.108	1"	0.5
118	−0.114	1"	0.6
119	X	X	X
120	X	X	X
121	−30.335	−14"	−25.9
122	−1.794	0"	1.2

FROM TABLE 13:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
123	0.588	5"	0.2
124	−0.135	1"	0.1
125	−0.159	1"	−0.1
126	−0.146	1"	−0.1
127	−0.123	0"	−0.1
128	−0.180	0"	−0.5
129	0.044	1"	−0.5
130	−0.080	1"	−0.1
131	−0.101	1"	−0.1
132	−0.091	1"	0.0
133	X	X	X
134	X	X	X
135	−1.548	0"	0.1
136	−0.811	1"	0.0

FROM TABLE 14:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
137	3.341	24"	0.4
138	−0.737	1"	0.3
139	−0.078	1"	0.1
140	−0.169	2"	0.1
141	−0.060	1"	0.0
142	−0.048	1"	0.0
143	−0.061	1"	0.1
144	−0.032	1"	0.0
145	0.113	3"	0.3
146	−0.023	1"	0.0
147	0.042	1"	0.0
148	−0.034	1"	0.0
149	−0.452	−5.27.07	0.7
150	0.019	−5.27.11	−0.1

FROM TABLE 15:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
151	0.005	1"	0.0
152	−0.006	0"	−0.1
153	−0.005	1"	0.0
154	−0.002	1"	0.0
155	0.007	1"	0.0
156	0.001	0"	0.0
157	0.005	1"	0.0
158	0.009	1"	0.0
159	0.001	1"	0.0
160	X	X	X
161	X	X	X
162	X	X	X
163	−0.153	−4.18.00	0.2
164	−0.135	−4.18.06	0.0

TABLE 18E : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS & JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM

FROM TABLE 16:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
165	3.042	38"	0.4
166	−0.063	1"	0.2
167	−0.071	1"	0.1
168	−0.175	1"	0.2
169	−0.063	1"	0.1
170	−0.064	1"	0.1
171	−0.062	1"	0.1
172	0.018	1"	0.1
173	0.084	5"	0.4
174	X	X	X
175	X	X	X
176	X	X	X
177	−0.548	3"	0.3
178	0.046	−1"	0.0

FROM TABLE 17:

	DIFFERENCES IN DISTANCE		
SESSION	15 – TUES	16 – WED	17 – THUR
A	−0.898	−0.894	−0.384
B	−0.006	−0.010	−0.008
C	−0.021	−0.031	0.054
D	0.032	−0.023	0.053
E	−0.009	−0.012	0.063
F	0.006	−0.008	0.070
G	−0.003	−0.005	0.078
H	0.000	0.000	0.038
I	0.004	X	0.066
J	−0.002	X	0.070
K	−0.001	X	0.073
L	X	−0.009	0.074
M	0.071	−0.007	0.569
N	−0.021	−0.010	0.194

FROM TABLE 17:

	DIFFERENCES IN DIRECTION		
SESSION	15 – TUES	16 – WED	17 – THUR
A	17"	38"	18"
B	1"	1"	1"
C	1"	0"	2"
D	1"	1"	1"
E	1"	1"	1"
F	1"	1"	1"
G	1"	1"	1"
H	1"	1"	1"
I	1"	X	1"
J	0"	X	2"
K	1"	X	2"
L	X	1"	2"
M	1"	−53' 02"	−53' 14"
N	1"	−52' 30"	−52' 48"

FROM TABLE 17:

	DIFFERENCES IN HEIGHT		
SESSION	15 – TUES	16 – WED	17 – THUR
A	0.4	1.1	0.4
B	0.3	0.3	0.4
C	0.3	0.3	0.3
D	0.3	0.3	0.3
E	0.4	0.3	0.3
F	0.3	0.3	0.3
G	0.3	0.3	0.3
H	0.3	0.3	0.3
I	0.3	X	0.3
J	0.3	X	0.3
K	0.3	X	0.3
L	X	0.3	0.3
M	0.3	0.8	0.4
N	0.3	−0.1	0.3

APPENDIX B: RESULTS OF PSEUDO-STATIC OBSERVATIONS

TABLE A : PSEUDO – KINEMATIC: BY COMBINING TWO 10 MINUTE SESSIONS OBSERVED APPROX. ONE HOUR APART AND PROCESSED STATICALLY

SESSION	PDOP RANGE FOR 2 SESSIONS	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP	QUALITY RATIO
99A	BOTH < 5	8.9	7.47	0.013	0.152	1.04
99B	BOTH < 5	160.2	160.65	0.009	0.164	1.00
89A	ONE < 5 & ONE INCR	8.9	7.47	0.013	0.152	1.04
89B	ONE < 5 & ONE INCR	160.2	160.76	0.022	0.132	1.13
79A	ONE INCR & ONE DECR	8.9	8.94	0.044	0.365	2.05
79B	ONE INCR & ONE DECR	160.2	160.93	0.021	0.129	1.21

SESS.	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
99A	298.34.45	298.39.25	2895.672	2887.531	2895.180	2887.091
99B	331.50.23	331.50.59	8200.793	8199.970	8197.857	8197.078
89A	298.34.45	298.39.25	2895.672	2887.531	2895.180	2887.091
89B	331.50.23	331.51.10	8200.793	8199.648	8197.857	8196.753
79A	298.34.45	298.33.50	2895.672	2897.115	2895.180	2896.669
79B	331.50.23	331.49.43	8200.793	8200.934	8197.857	8198.036

* EDM REDUCED DISTANCES : 8197.904, 2895.226

TABLE B : PSEUDO–KINEMATIC OBSERVATIONS : ONE OBSERVATION FILE

SESSION	TIMES	PDOP RANGE	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP
1A	10' 30" & 10' 30"	< 5	160.2	160.543	0.016	0.137
1B	10' 00" & 6' 15"	< 5	151.1	151.496	0.017	0.096
2A	9' 45" & 9' 45"	< 5	160.2	160.637	0.025	0.104
2B	9' 30" & 6' 45"	< 5	151.1	151.437	0.163	0.121
3A	9' 30" & 10' 00"	< 5	160.2	160.756	0.208	0.105
3B	9' 45" & 9' 45"	< 5	151.1	151.561	0.283	0.102
4A	10' 00" & 9' 00"	SHARP INCR & DECR	160.2	160.864	0.032	1.632
4B	9' 45" & 10' 00"	SHARP INCR & DECR	151.1	151.682	0.252	0.155

SESSION	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
1A	331.50.23	331.50.28	8200.793	8200.658	8197.861	8197.768
1B	330.16.02	330.15.59		8250.595	8247.799	8247.878
2A	331.50.23	331.50.25	8200.793	8200.769	8197.861	8197.877
2B	330.16.02	330.15.58		8250.736	8247.799	8248.020
3A	331.50.23	331.50.23	8200.793	8200.646	8197.861	8197.751
3B	330.16.02	330.16.00		8250.567	8247.799	8247.848
4A	331.50.23	331.50.24	8200.793	8201.055	8197.861	8198.333
4B	330.16.02	330.16.10		8250.163	8247.799	8247.442

* EDM REDUCED DISTANCES : 8197.904

TABLE C : PSEUDO–KINEMATIC: BY COMBINING TWO 10 MINUTE SESSIONS IN DOS AND PROCESSING PSEUDO–STATICALLY

SESSION	PDOP RANGE FOR THE 2 10' SESSIONS	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP
20A	BOTH < 5	169.2	169.402	0.052	0.137
		8.9	9.043	0.138	0.092
21A	BOTH < 5	169.2	169.247	0.067	0.140
		8.9	9.042	0.138	0.092
22A	BOTH SHARPLY INCR & DECREASE	169.2	169.091	0.118	1.288

SESS.	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
20A	347.12.30	347.12.43	5994.593	5994.460	5991.192	5991.125
	118.34.45	118.34.56	2895.672	2895.533	2895.180	2895.013
21A	347.12.30	347.12.24	5994.593	5994.728	5991.192	5991.397
	118.34.45	118.34.36	2895.672	2896.111	2895.180	2895.632
22A	347.12.30	347.12.33	5994.593	5994.310	5991.192	5990.984

* EDM REDUCED DISTANCES : 5991.273, 2895.226

TABLE D : PSEUDO–KINEMATIC: BY COMBINING TWO 10 MINUTE SESSIONS IN DOS AND PROCESSING PSEUDO–STATICALLY

SESSION	PDOP RANGE FOR THE 2 10' SESSIONS	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP
20B	BOTH < 5	260.3	260.469	0.094	0.124
		91.2	91.131	0.036	0.131
21B	BOTH < 5	260.3	260.541	0.099	0.136
		91.2	91.155	0.102	0.134
22B	BOTH SHARPLY INCR & DECREASE	260.3	260.481	0.122	1.417
		91.2	91.394	0.156	1.417

SESS.	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
20B	224.06.31	224.06.32		16878.188	16873.882	16873.708
	64.26.25	64.26.22		14590.352	14588.360	14588.150
21B	224.06.31	224.06.45		16879.205	16873.882	16874.725
	64.26.25	64.26.27		14591.234	14588.360	145890.032
22B	224.06.31	224.06.33		16878.190	16873.882	16873.710
	64.26.25	64.26.24		14590.660	14588.360	14588.458

TABLE E : PSEUDO–KINEMATIC: BY COMBINING TWO 10 MINUTE SESSIONS IN DOS AND PROCESSING PSEUDO–STATICALLY

SESSION	PDOP RANGE FOR THE 2 10' SESSIONS	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP
20C	BOTH < 5	122.8	122.688	0.497	0.152
		46.2	46.001	0.026	0.228
21C	BOTH < 5	122.8	122.630	0.121	0.330
		46.2	46.196	0.035	0.503
22C	BOTH SHARPLY INCR & DECREASE	122.8	121.306	0.107	0.087
		46.2	46.836	0.084	0.086

SESS.	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
20C	64.14.13	64.14.13		23017.219	23013.106	23013.079
	79.18.24	79.18.24		22536.985	22533.055	22533.492
21C	64.14.13	64.14.09		23015.796	23013.106	23012.358
	79.18.24	79.18.24		22535.868	22533.055	22532.374
22C	64.13.58	64.13.58		23021.833	23018.146	23017.769
	79.18.24	79.18.24		22536.407	22533.055	22532.913

TABLE F : PSEUDO–KINEMATIC: BY COMBINING TWO 10 MINUTE SESSIONS IN DOS AND PROCESSING PSEUDO–STATICALLY

SESSION	PDOP RANGE FOR THE 2 10' SESSIONS	HEIGHT DIFFS	GPS H. DIFFS	RMS	RDOP
20D	BOTH < 5	125.4	125.663	0.149	0.092
		25.5	25.434	0.093	0.095
21D	BOTH < 5	125.4	125.549	0.034	6.096
		25.5	25.584	0.111	0.093
22D	BOTH SHARPLY INCR & DECREASE	125.4	125.532	0.073	0.091
		25.5	25.521	0.036	0.102

SESS.	JOIN DIRECTION	GPS RED DIRECT.	EDM SLOPE	GPS SLOPE	JOIN DIST.	GPS RED DIST.
20D	64.11.42	64.11.51		12653.411	12650.534	12651.137
	277.20.36	277.20.38		4371.426	4370.752	4370.772
21D	64.11.42	64.11.47		12653.005	12650.534	12650.735
	277.20.36	277.20.34		4371.511	4370.752	4370.856
22D	64.11.42	64.11.42		12652.718	12650.534	12650.858
	277.20.36	277.20.37		4371.376	4370.752	4370.722

TABLE G : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS & JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM FOR PSEUDO-KINEMATIC OBSERVATIONS

FROM TABLE A:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
99A	−8.089	4' 40"	−1.4
99B	−0.783	36"	0.5
89A	−8.089	4' 40"	−1.4
89B	−1.108	47"	0.6
79A	1.489	−55"	0.0
79B	0.175	−40"	0.7

FROM TABLE B:

	DIFFERENCES IN		
SESSION	DIST	DIRECT	HEIGHT
1A	−0.093	5"	0.3
1B	0.079	−3"	0.4
2A	0.016	2"	0.4
2B	0.221	−4"	0.3
3A	−0.110	0"	0.6
3B	0.049	−2"	0.5
4A	0.472	1"	0.6
4B	−0.357	8"	0.6

TABLE H : TABLES OF DIFFERENCES IN OBSERVATIONS BETWEEN REDUCED GPS OBSERVATIONS & JOIN DISTANCES AND DIRECTIONS, AND HEIGHT DIFFERENCES ON THE GAUSS CONFORM SYSTEM FOR PSEUDO-KINEMATIC OBSERVATIONS

FROM TABLE C:

		DIFFERENCES IN		
SESSION	DIST	DIST	DIRECT	HEIGHT
20A	6 km	−0.067	13"	0.2
	2.9 km	−0.167	11"	0.1
21A	6 km	0.205	−6"	0.0
	2.9 km	0.452	−9"	0.1
22A	6 km	−0.208	3"	−0.1

FROM TABLE D:

		DIFFERENCES IN		
SESSION	DIST	DIST	DIRECT	HEIGHT
20B	16.9 km	−0.174	1"	0.2
	14.6 km	−0.210	−3"	−0.1
21B	16.9 km	0.843	14"	0.2
	14.6 km	1.672	2"	0.0
22B	16.9 km	−0.172	2"	0.2
	14.6 km	0.098	−1"	0.2

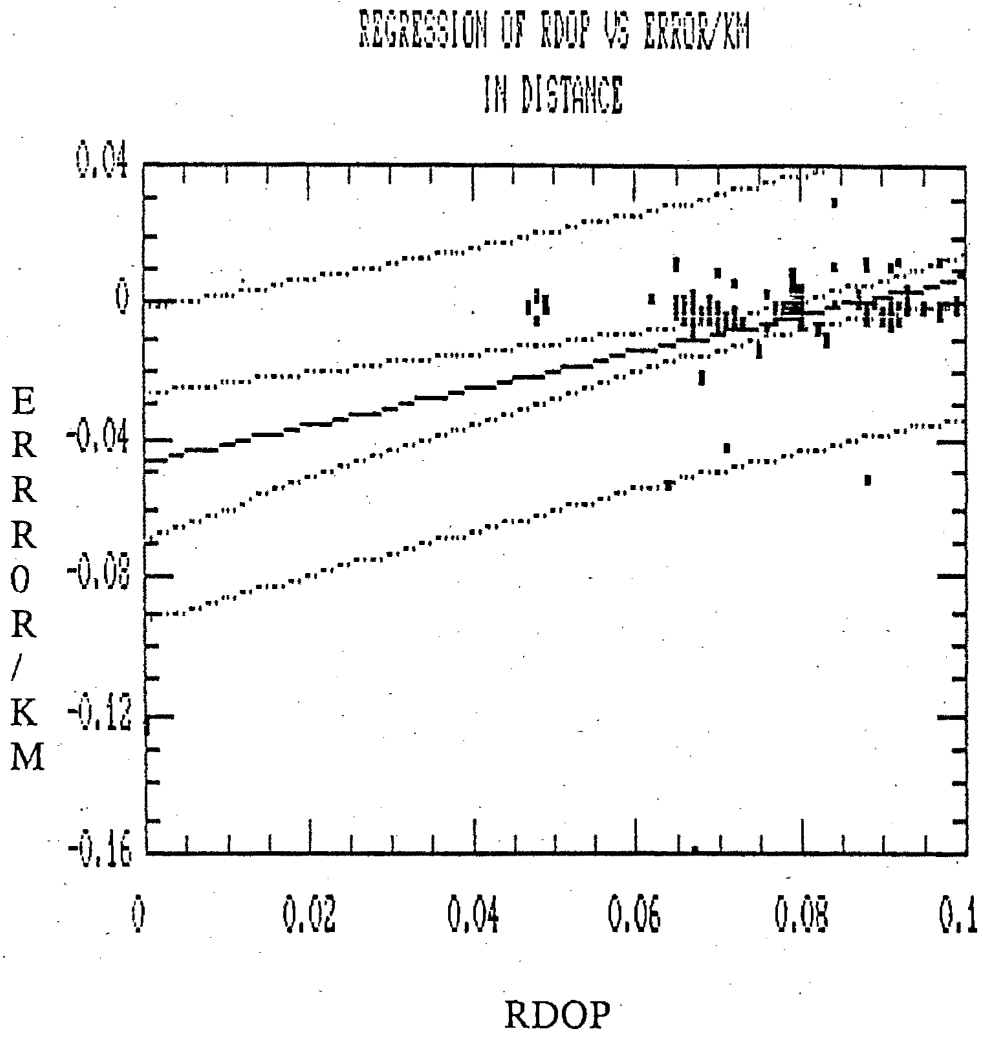
FROM TABLE E:

		DIFFERENCES IN		
SESSION	DIST	DIST	DIRECT	HEIGHT
20C	23 km	−0.027	0"	−0.1
	22.5 km	0.437	0"	−0.2
21C	23 km	−0.748	−4"	−0.2
	22.5 km	−0.681	0"	0.0
22C	23 km	−0.377	0"	−1.5
	22.5 km	−0.142	0"	0.6

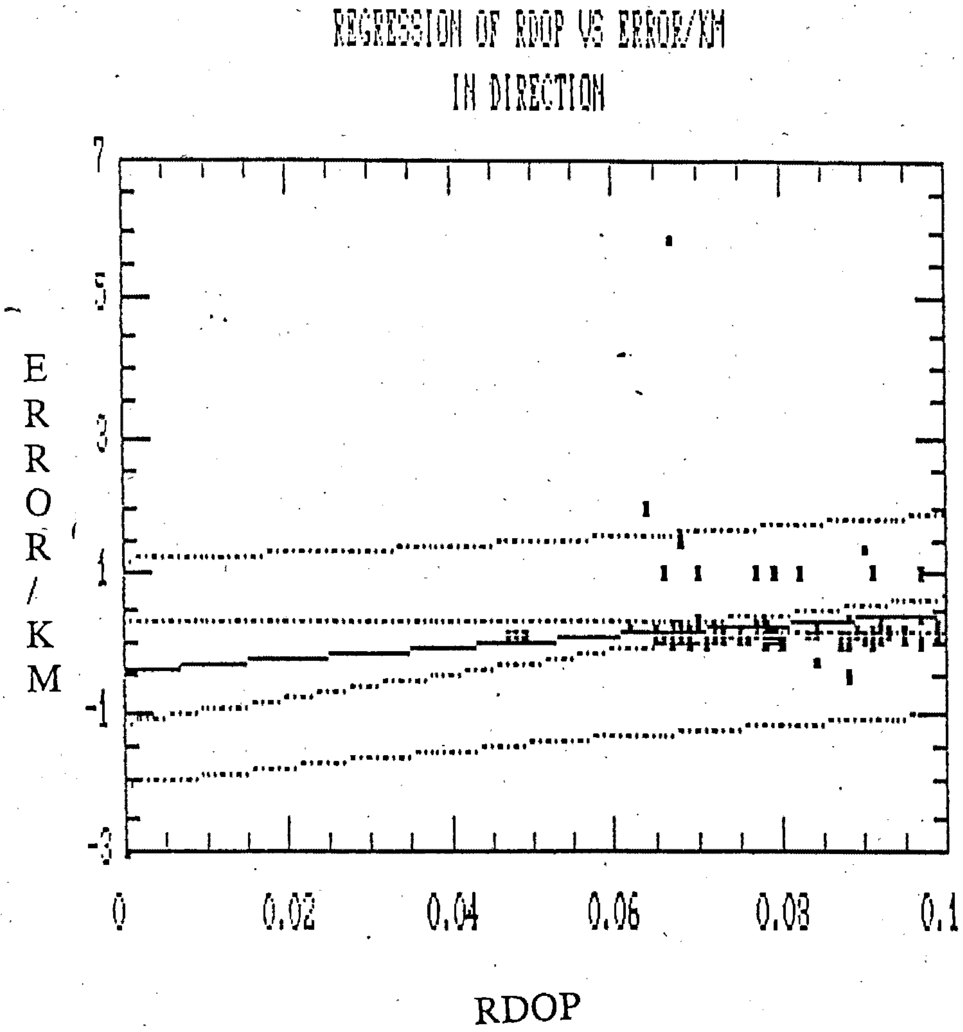
FROM TABLE F:

		DIFFERENCES IN		
SESSION	DATE	DIST	DIRECT	HEIGHT
20D	12.6 km	0.603	9"	0.3
	4.4 km	0.020	2"	−0.1
21D	12.6 km	0.201	5"	0.1
	4.4 km	0.104	−2"	0.1
22D	12.6 km	0.324	0"	0.1
	4.4 km	−0.030	1"	0.0

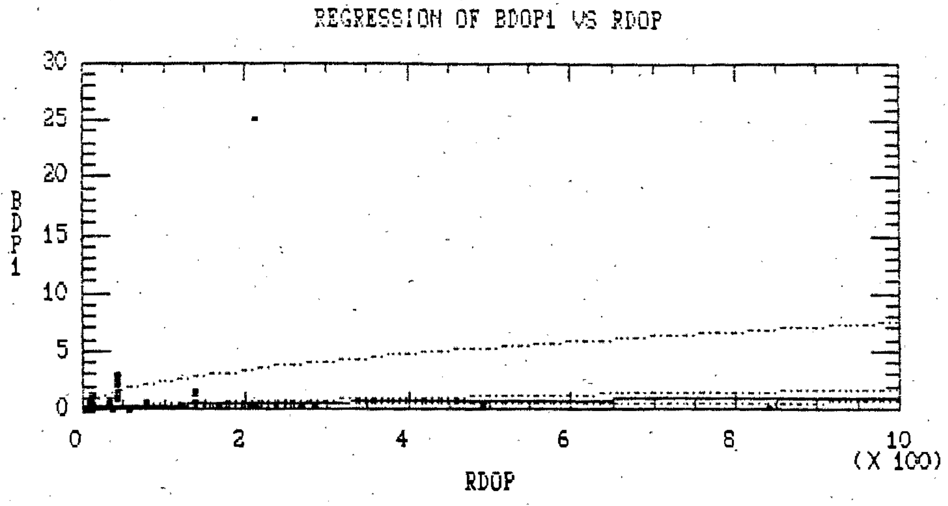
APPENDIX C: REGRESSION ANALYSES



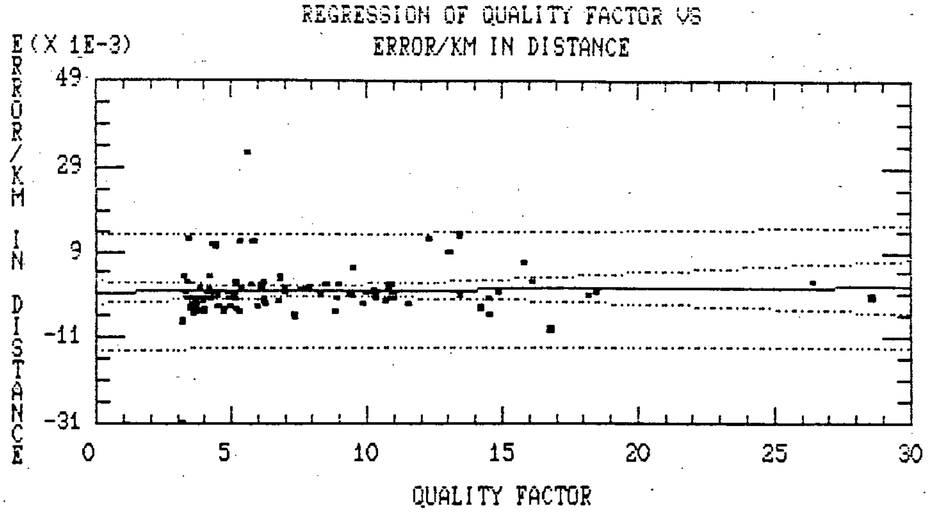
```
Regression Analysis - Linear model: Y = a+bX
=============================================================================
Dependent variable: A:FILE1.VAR1                 Independent variable: A:FILE1.VAR0
-----------------------------------------------------------------------------
                                 Standard            T             Prob.
Parameter         Estimate         Error           Value           Level
-----------------------------------------------------------------------------
Intercept       -0.0471459      0.0105773        -4.45727      2.11931E-5
Slope            0.547499        0.134675         4.06533      9.39633E-5
-----------------------------------------------------------------------------

                           Analysis of Variance
-----------------------------------------------------------------------------
Source                  Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                          .006797      1     .006797   16.526872       .00009
Error                          .04236     103     .00041
-----------------------------------------------------------------------------
Total (Corr.)                  .04915     104

Correlation Coefficient = 0.371846              R-squared =  13.83 percent
Stnd. Error of Est. = 0.0202791
```



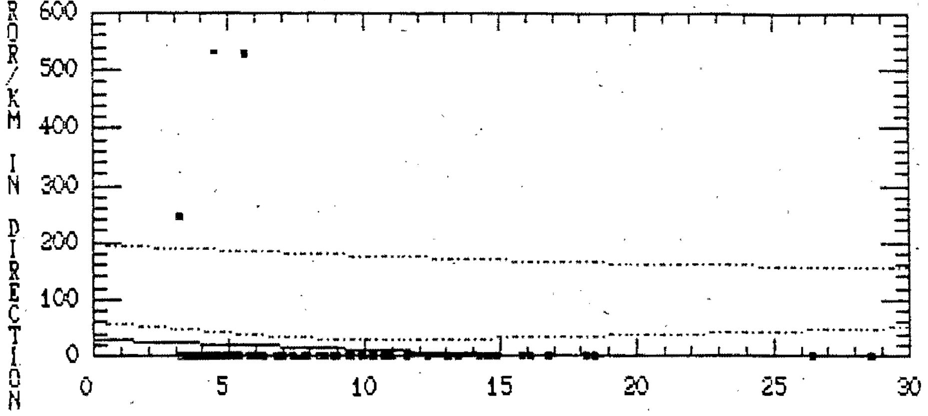
```
Regression Analysis - Linear model: Y = a+bX
--------------------------------------------------------------------------------
Dependent variable: A:FILE1.VAR2              Independent variable: A:FILE1.VAR0
--------------------------------------------------------------------------------
                                Standard            T            Prob.
Parameter       Estimate          Error           Value          Level
--------------------------------------------------------------------------------
Intercept      -0.404905        0.377811        -1.07171       0.286354
Slope           8.43307         4.81047          1.75307       0.0825661
--------------------------------------------------------------------------------

                            Analysis of Variance
--------------------------------------------------------------------------------
Source               Sum of Squares    Df  Mean Square    F-Ratio  Prob. Level
Model                     1.6124667     1   1.6124667   3.0732374      .08257
Error                      54.04206   103      .52469
--------------------------------------------------------------------------------
Total (Corr.)              55.65452   104

Correlation Coefficient = 0.170214          R-squared =   2.90 percent
Stnd. Error of Est. = 0.724348
```



```
Regression Analysis - Multiplicative model: Y = aX^b
==================================================================================
Dependent variable: A:FILE2.VAR1                 Independent variable: A:FILE2.VAR0
----------------------------------------------------------------------------------
                                 Standard            T             Prob.
Parameter        Estimate          Error           Value           Level
----------------------------------------------------------------------------------
Intercept*       -3.22838        0.169108        -19.0906              0
Slope            0.477637       0.0566609         8.42975     5.77316E-15
* NOTE: The Intercept is equal to Log a.
----------------------------------------------------------------------------------

                           Analysis of Variance
----------------------------------------------------------------------------------
Source                Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                      65.260269      1    65.260269  71.060611       .00000
Error                      181.83819    198       .91837
----------------------------------------------------------------------------------
Total (Corr.)              247.09846    199

Correlation Coefficient = 0.513913           R-squared =  26.41 percent
Stnd. Error of Est. = 0.958319
```

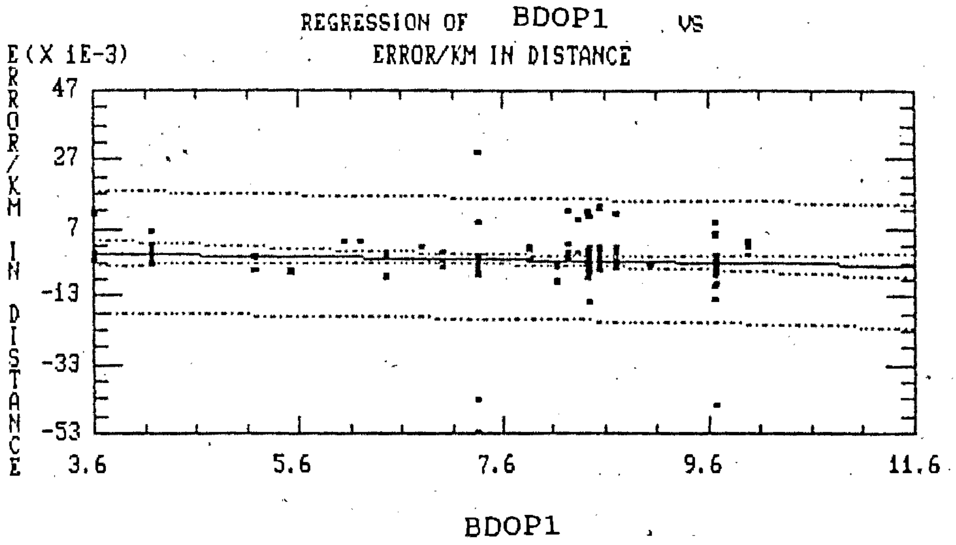


```
Regression Analysis - Linear model: Y = a+bX
==============================================================================
Dependent variable: A:FILE4.VAR1                 Independent variable: A:FILE4.VAR0
------------------------------------------------------------------------------
                                Standard            T              Prob.
Parameter         Estimate        Error           Value            Level
------------------------------------------------------------------------------
Intercept      -4.73591E-4     1.31271E-3      -0.360772         0.719144
Slope           4.88499E-5     1.39566E-4       0.350013         0.727175
------------------------------------------------------------------------------

                            Analysis of Variance
------------------------------------------------------------------------------
Source                  Sum of Squares    Df  Mean Square    F-Ratio  Prob. Level
Model                         .0000053     1     .0000053   .1225089       .72718
Error                          .003798    87      .000044
------------------------------------------------------------------------------
Total (Corr.)                  .003803    88

Correlation Coefficient = 0.0374989             R-squared =    .14 percent
Stnd. Error of Est. = 6.60701E-3
```

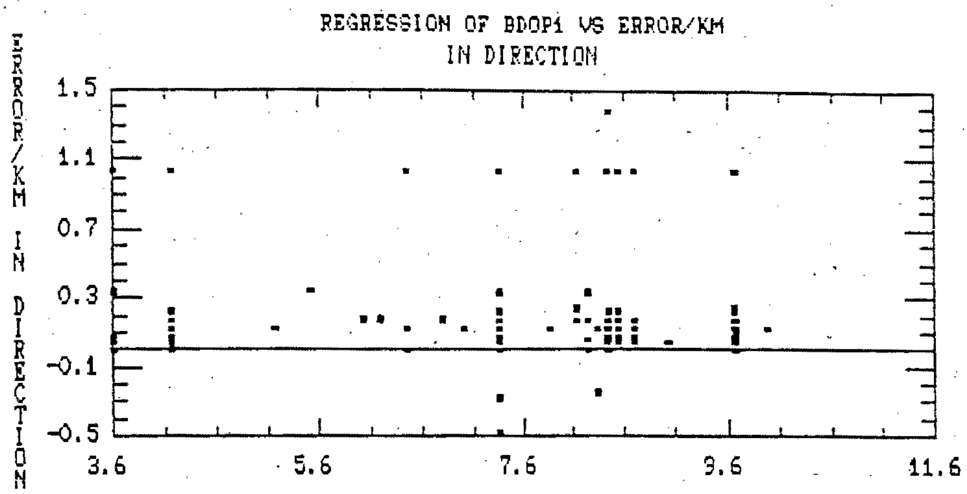


REGRESSION OF QUALITY FACTOR VS
ERROR/KM IN DIRECTION

```
Regression Analysis - Linear model: Y = a+bX
==============================================================================
Dependent variable: A:FILE4.VAR2                Independent variable: A:FILE4.VARO
------------------------------------------------------------------------------
                                 Standard              T             Prob.
Parameter          Estimate        Error             Value           Level
------------------------------------------------------------------------------
Intercept           29.9621       16.4052           1.82638        0.0712232
Slope              -1.88954       1.74418          -1.08334         0.281652
------------------------------------------------------------------------------

                            Analysis of Variance
------------------------------------------------------------------------------
Source                 Sum of Squares     Df  Mean Square      F-Ratio  Prob. Level
Model                       8001.2933      1    8001.2933       1.1736       .28165
Error                       593132.53     87      6817.62
------------------------------------------------------------------------------
Total (Corr.)               601133.82     88

Correlation Coefficient = -0.11537              R-squared =   1.33 percent
Stnd. Error of Est. = 82.5689
```

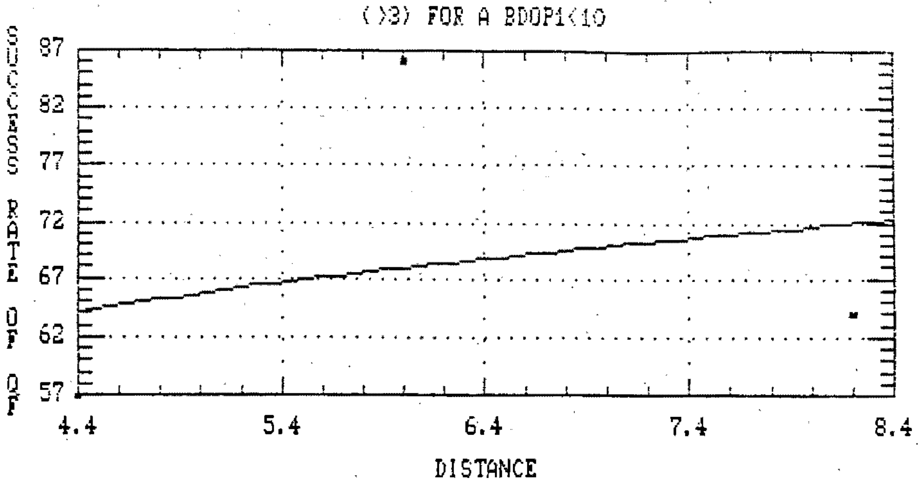


```
Regression Analysis - Linear model: Y = a+bX
==========================================================================================
Dependent variable: A:FILE5.VAR1                  Independent variable: A:FILE5.VAR0
------------------------------------------------------------------------------------------
                                   Standard              T                Prob.
Parameter           Estimate         Error             Value              Level
------------------------------------------------------------------------------------------
Intercept         1.77523E-3      3.43024E-3          0.517524           0.60572
Slope             -5.1159E-4      4.32649E-4          -1.18246          0.239305
------------------------------------------------------------------------------------------

                                   Analysis of Variance
------------------------------------------------------------------------------------------
Source                  Sum of Squares     Df    Mean Square    F-Ratio   Prob. Level
Model                         .0001106      1      .0001106   1.3982064        .23930
Error                           .00973    123        .00008
------------------------------------------------------------------------------------------
Total (Corr.)                   .00984    124

Correlation Coefficient = -0.106018              R-squared =   1.12 percent
Stnd. Error of Est. = 8.89349E-3
```

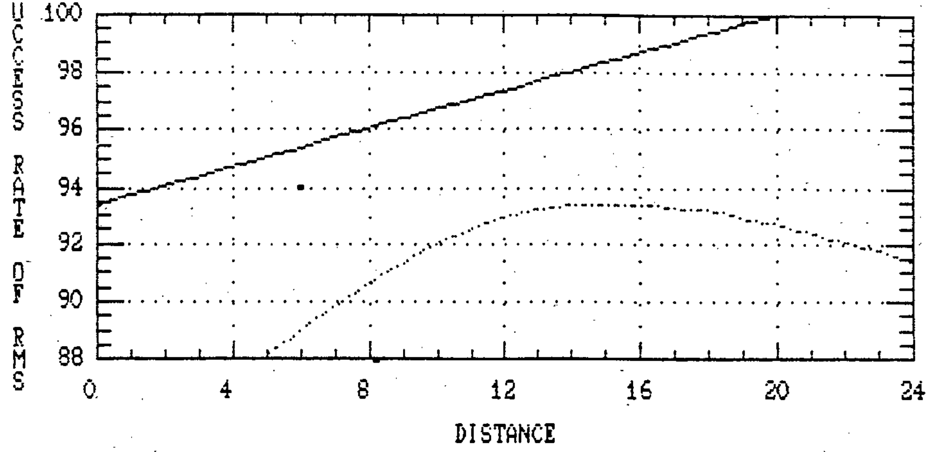


```
Regression Analysis - Reciprocal model: 1/Y = a+bX
=====================================================================================
Dependent variable: A:FILE5.VAR2                   Independent variable: A:FILE5.VAR0
-------------------------------------------------------------------------------------
                                Standard              T               Prob.
Parameter         Estimate        Error             Value             Level
-------------------------------------------------------------------------------------
Intercept          3630.01        114168          0.0317953          0.974687
Slope                12082        14399.8          0.839039          0.403075
-------------------------------------------------------------------------------------

                               Analysis of Variance
-------------------------------------------------------------------------------------
Source                  Sum of Squares     Df  Mean Square     F-Ratio  Prob. Level
Model                      6.1681E0010      1 6.1681E0010 7.0399E-001       .40308
Error                      1.0777E0013    123 8.7616E0010
-------------------------------------------------------------------------------------
Total (Corr.)              1.0838E0013    124

Correlation Coefficient = 0.075438           R-squared =     .57 percent
Stnd. Error of Est. = 296000
```

REGRESSION OF DISTANCE VS QUALITY FACTOR
(>3) FOR A BDOP1<10

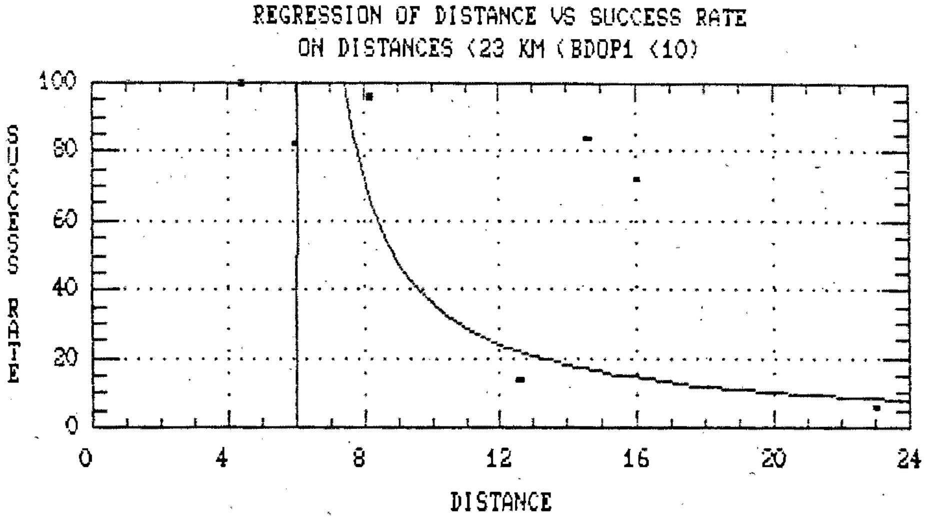


```
Regression Analysis - Multiplicative model: Y = aX^b
==========================================================================
Dependent variable: A:FILE8.VAR1             Independent variable: A:FILE8.VAR0
--------------------------------------------------------------------------
                                Standard           T              Prob.
Parameter       Estimate          Error          Value            Level
--------------------------------------------------------------------------
Intercept*       3.88766        1.18737        3.27419         0.188708
Slope           0.184716       0.655848       0.281645         0.825227
* NOTE: The Intercept is equal to Log a.
--------------------------------------------------------------------------

                          Analysis of Variance
--------------------------------------------------------------------------
Source                Sum of Squares    Df  Mean Square    F-Ratio  Prob. Level
Model                       .0066115     1     .0066115   .0793237       .82523
Error                       .0833487     1     .0833487
--------------------------------------------------------------------------
Total (Corr.)               .0899602     2

Correlation Coefficient = 0.271098          R-squared =   7.35 percent
Stnd. Error of Est. = 0.288702
```

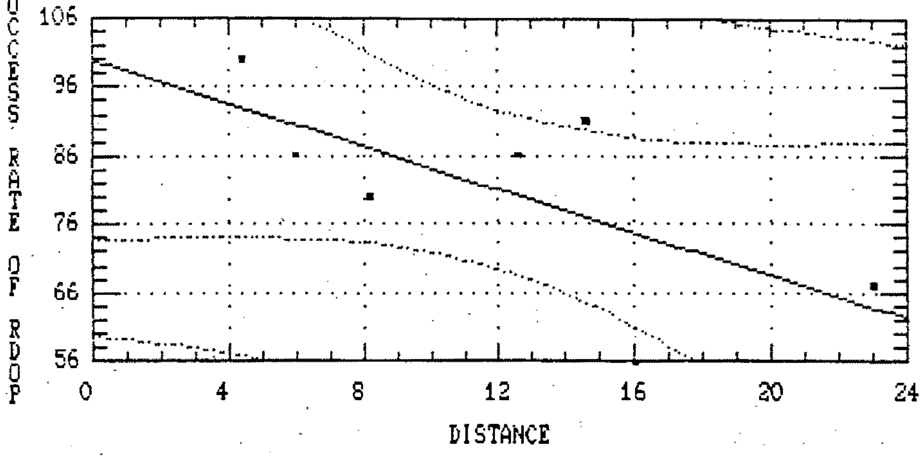


REGRESSION OF DISTANCE VS SUCCESS RATE
OF RMS FOR A BDOP1 <10

```
Regression Analysis - Linear model: Y = a+bX
=============================================================================
Dependent variable: A:FILE9.VAR2                Independent variable: A:FILE9.VAR0
-----------------------------------------------------------------------------
                           Standard             T              Prob.
Parameter       Estimate     Error             Value           Level
-----------------------------------------------------------------------------
Intercept        93.4106      3.91903         23.8351      2.42137E-6
Slope           0.331672     0.289871         1.14421        0.304341
-----------------------------------------------------------------------------

                           Analysis of Variance
-----------------------------------------------------------------------------
Source              Sum of Squares    Df  Mean Square    F-Ratio  Prob. Level
Model                    27.746728     1    27.746728   1.309209       .30434
Error                    105.96756     5     21.19351
-----------------------------------------------------------------------------
Total (Corr.)            133.71429     6

Correlation Coefficient = 0.45553           R-squared =  20.75 percent
Stnd. Error of Est. = 4.60364
```



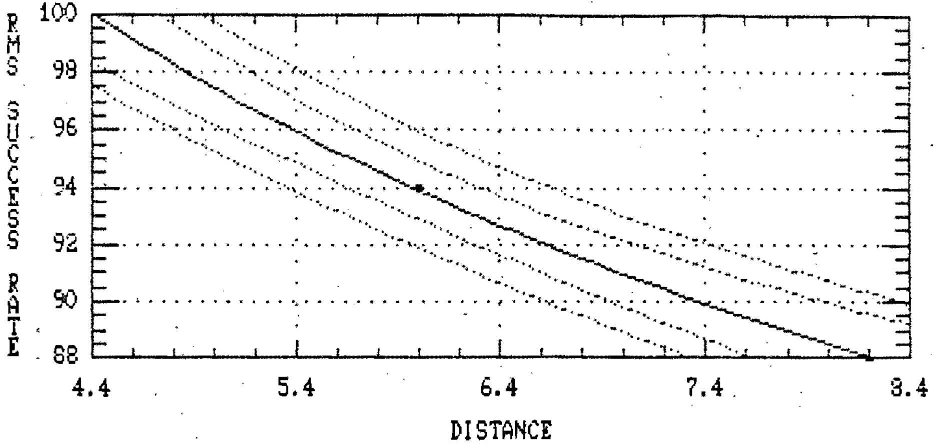
```
Regression Analysis - Reciprocal model: 1/Y = a+bX
==================================================================================
Dependent variable: A:FILE7.VAR1                 Independent variable: A:FILE7.VAR0
----------------------------------------------------------------------------------
                                Standard              T              Prob.
Parameter          Estimate       Error             Value            Level
----------------------------------------------------------------------------------
Intercept         -0.041216     0.0361342         -1.14064         0.305693
Slope            6.89874E-3    2.67266E-3          2.58122         0.049359
----------------------------------------------------------------------------------

                           Analysis of Variance
----------------------------------------------------------------------------------
Source                 Sum of Squares     Df  Mean Square     F-Ratio  Prob. Level
Model                        .0120042      1     .0120042   6.6627000       .04936
Error                        .0090085      5     .0018017
----------------------------------------------------------------------------------
Total (Corr.)                .0210127      6

Correlation Coefficient = 0.755833             R-squared =  57.13 percent
Stnd. Error of Est. = 0.0424465
```



REGRESSION OF DISTANCE VS SUCCESS RATE
OF RDOP FOR A BDOP1 <10

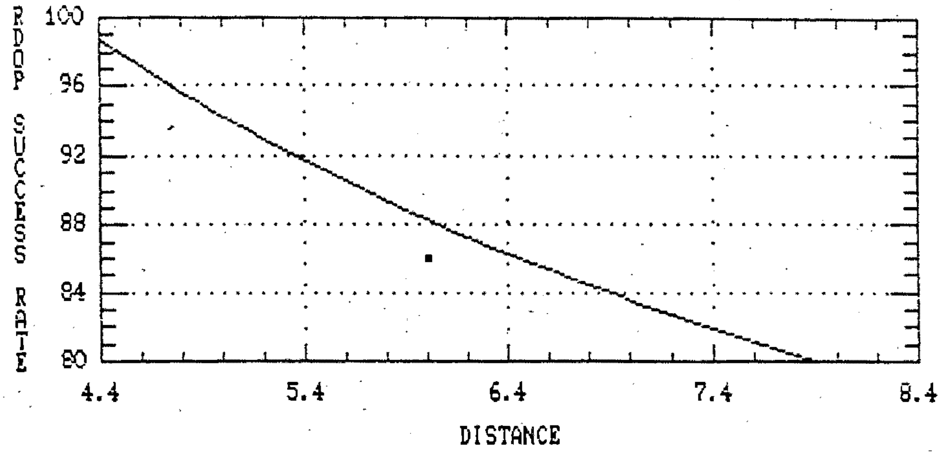
```
Regression Analysis - Linear model: Y = a+bX
=====================================================================================
Dependent variable: A:FILE9.VAR3                 Independent variable: A:FILE9.VARO
-------------------------------------------------------------------------------------
                                Standard              T              Prob.
Parameter        Estimate         Error             Value            Level
-------------------------------------------------------------------------------------
Intercept         99.8519         10.1639          9.82417       1.86116E-4
Slope            -1.56797        0.751772         -2.08569        0.0913987
-------------------------------------------------------------------------------------

                              Analysis of Variance
-------------------------------------------------------------------------------------
Source                 Sum of Squares     Df   Mean Square     F-Ratio  Prob. Level
Model                       620.10798      1     620.10798     4.35011       .09140
Error                       712.74916      5     142.54983
-------------------------------------------------------------------------------------
Total (Corr.)               1332.8571      6

Correlation Coefficient = -0.68209          R-squared =  46.52 percent
Stnd. Error of Est. = 11.9394
```

REGRESSION OF DISTANCE VS RMS SUCCESS
RATE FOR A BDOP1 <10

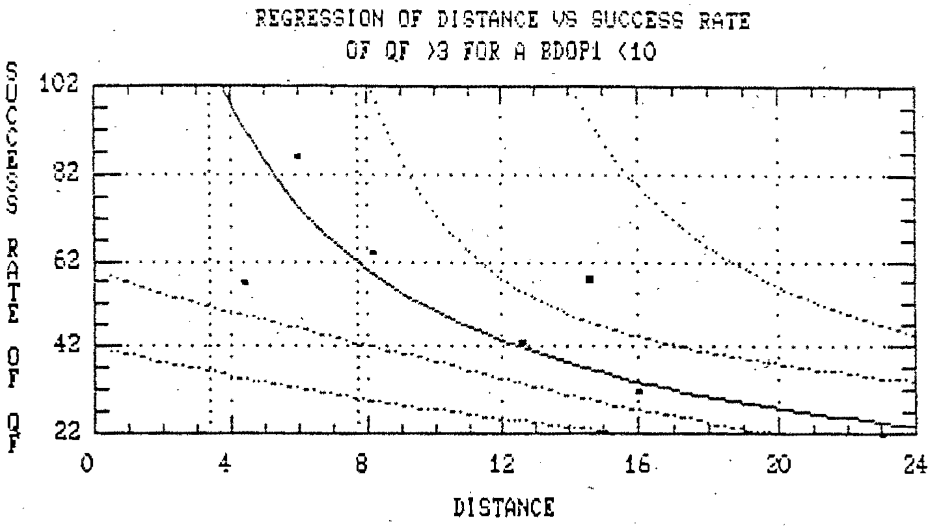


```
Regression Analysis - Multiplicative model: Y = aX^b
=====================================================================================
Dependent variable: A:FILE8.VAR2                    Independent variable: A:FILE8.VAR0
-------------------------------------------------------------------------------------
                                Standard              T               Prob.
Parameter          Estimate       Error             Value             Level
-------------------------------------------------------------------------------------
Intercept*          4.91003     6.08942E-3         806.321        7.89536E-4
Slope             -0.205352     3.36353E-3        -61.0526         0.0104265
* NOTE: The Intercept is equal to Log a.
-------------------------------------------------------------------------------------

                             Analysis of Variance
-------------------------------------------------------------------------------------
Source                 Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                           .0082      1        .0082  3727.4167        .01043
Error                        .0000022      1     .0000022
-------------------------------------------------------------------------------------
Total (Corr.)                .0081735      2

Correlation Coefficient = -0.999866            R-squared =  99.97 percent
Stnd. Error of Est. = 1.48061E-3
```

REGRESSION OF DISTANCE VS RDOP SUCCESS
RATE FOR A BDOP1 <10



```
Regression Analysis - Multiplicative model: Y = aX^b
=====================================================================================
Dependent variable: A:FILE8.VAR3                   Independent variable: A:FILE8.VAR0
-------------------------------------------------------------------------------------
                              Standard               T               Prob.
Parameter        Estimate       Error              Value             Level
-------------------------------------------------------------------------------------
Intercept*        5.12276     0.133144            38.4755         0.0165424
Slope           -0.358295    0.0735426           -4.87194          0.128381
* NOTE: The Intercept is equal to Log a.
-------------------------------------------------------------------------------------

                          Analysis of Variance
-------------------------------------------------------------------------------------
Source                Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                        .024876      1      .024876  23.735773        .12888
Error                       .0010480      1     .0010480
-------------------------------------------------------------------------------------
Total (Corr.)               .0259236      2

Correlation Coefficient = -0.979578              R-squared =  95.96 percent
Stnd. Error of Est. = 0.0323732
```



```
Regression Analysis - Reciprocal model: 1/Y = a+bX
==========================================================================
Dependent variable: A:FILE9.VAR1              Independent variable: A:FILE9.VAR0
--------------------------------------------------------------------------
                                Standard            T             Prob.
Parameter         Estimate        Error            Value          Level
--------------------------------------------------------------------------
Intercept        3.79699E-3     5.04506E-3       0.752615       0.485584
Slope            1.59693E-3     3.73157E-4        4.27951      7.86719E-3
--------------------------------------------------------------------------

                          Analysis of Variance
--------------------------------------------------------------------------
Source                  Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                          .000643      1      .000643  18.314197        .00787
Error                         .0001756      5     .0000351
--------------------------------------------------------------------------
Total (Corr.)                 .0008188      6

Correlation Coefficient = 0.886306           R-squared =  78.55 percent
Stnd. Error of Est. = 5.92637E-3
```