

One-dimensional wave spectrum analysis of wind waves off Cape Town.

by

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ABSTRACT.

Wind waves in the vicinity of Cape Town, South Africa, were measured by means of an N.I.O. (England) ship-borne wave recorder, in water depths varying from 12 - 140 metres. One dimensional frequency spectra were computed from the records, by power spectrum analysis, based on the method of Blackman and Tukey, using an I.B.M. 1130 Computer.

Assumptions of normality and stationarity were tested. The Gaussian assumption of the waves was found to be acceptable for the waters off Cape Town. At the 5% critical level, using the chi-square test, 3 out of 23 records tested were found to be non-Gaussian. No significant difference between the Gaussian properties of the deep and the shallow stations was found. Tests for stationarity applied to 3 selected records showed 1 record as clearly non-stationary. However, this recording was obtained under fluctuating wind conditions.

Comparison of the total variances of the power spectra obtained in shoaling water showed a systematic decrease of variance with depth. Normalised spectra did not show a systematic selective attenuation of the variances with frequency. Factors which might have caused the systematic reduction of the total variances with depth have been examined.

The bottom friction factor for this coast was estimated. The mean value of the bottom friction is 0.22. This is higher than found by other workers.

CONTENTS.

	Page
Abstract.	1
Introduction.	2
PART I.	
Chapter 1 : Theory of power spectrum analysis.	5
1.1 Amplitude line spectrum.	5
1.2 Band spectrum.	5
1.3 Power spectrum.	6
1.4 The analysis of continuous records of finite length.	8
1.5 The analysis of equally spaced records of finite length.	11
Chapter 2 : Recording and digitizing of the wave data.	14
2.1 Introduction.	14
2.2 Principle of operation of the wave recorder.	15
2.2.1 Pressure unit.	15
2.2.2 Mechano-electric transducer.	15
2.2.3 Accelerometer unit.	16
2.2.4 Bridge stabilised oscillator.	16
2.2.5 Computer circuit.	16
2.3 Calibration of the wave recorder.	17
2.4 Digitizing of the wave records.	18
2.5 Choice of the sampling interval.	18
2.6 Computation of the wave data.	19
Chapter 3 : Preliminary investigations.	20
3.1 Introduction.	20
3.2 Effect of the lag number on the resolution of a spectrum.	20
3.3 Choice of resolution.	20
3.4 Stationarity.	21
3.5 Normality.	22
3.5.1 Minimum interval of independence.	23
3.5.2 Effect of sample length.	25
3.5.3 Normality of off-shore wave data.	26
3.5.4 Normality of in-shore wave data.	26
3.6 Aliasing.	28

PART II.

	Page
Chapter 4 : Wave spectra in shoaling water.	30
4.1 Introduction.	30
4.2 Airy's wave theory.	30
4.3 Solitary and cnoidal wave theory.	33
4.4 Transformation of wave spectra.	35
4.5 Change of wave height of a progressive oscillatory wave of small steepness due to shoaling, refraction, bottom friction and bottom percolation.	37
4.6 Observed wave spectra in shoaling water near Melkboschstrand.	39
4.6.1 Description of the test area.	39
4.6.2 Experimental procedures.	39
4.6.3 Experimental results.	40
4.7 Non-preferential attenuation of the wave heights.	40
4.8 Calculation of friction factor from the wave data.	43
Chapter 5 : Discussion of results and suggestions for future work.	46
5.1 Reliability of technique.	46
5.2 Normality of data.	47
5.3 Stationarity of data.	48
5.4 Comparison of spectra.	48
5.5 Calculation of friction factor.	49
5.6 Suggestions for future work.	51
Conclusions.	
Acknowledgements.	
References.	
Appendix I.	
List of Illustrations.	
List of Tables.	
List of Notations.	

INTRODUCTION.

The purpose of this thesis is to record some wind wave studies made, using one-dimensional wave spectrum analysis, of waves recorded on the research ship "Thomas B. Davie" near Cape Town (Fig. 1.1) from 1968 until the middle of 1970. These wave studies comprise an investigation into the Gaussian properties of the waves, aspects of stationarity and modifications of the wave spectra in shoaling water.

Power spectrum analysis of sea waves has only become possible since 1952, although Tukey (1949) and Rice (1944) had, at a much earlier stage, successfully applied this technique to the analysis of noise in electronic circuits. Before 1952 the prediction and analysis of sea waves was based on a semi-empirical/semi-theoretical approach. Sverdrup and Munk, during the second world war, developed the significant wave theory, based on work by Jeffreys which made it possible to forecast the characteristics of sea waves in terms of significant wave heights and periods, for a given wind speed. Barber and Ursell (1948) using frequency spectrum analysis, showed that the frequencies of the waves generated by a storm form a continuous spectrum. As the lower frequency waves travel faster, a narrow band of low frequencies appeared suddenly and shifted towards high frequencies as time passed.

The change from the classic approach to a statistical description of the sea surface is primarily due to Pierson (1952). He strongly advocated the study of sea waves in terms of stochastic processes and spectra and the resulting wave spectrum method (Pierson, Neumann, James, 1953) has been a very important contribution to wave forecasting and wave research.

Since 1955 the discussions in the study of waves have centred upon the precise form of the wave spectrum to be associated with a given wind. Various spectral forms have been proposed by Bretschneider (1959), Roll and Fischer (1956) and many others. Darbyshire (1963) presented formulae to represent spectra of pure wind waves, measured in the Atlantic Ocean and the coastal waters. The differences between the proposed spectral forms are quite striking when the different forms are plotted together on one dia-

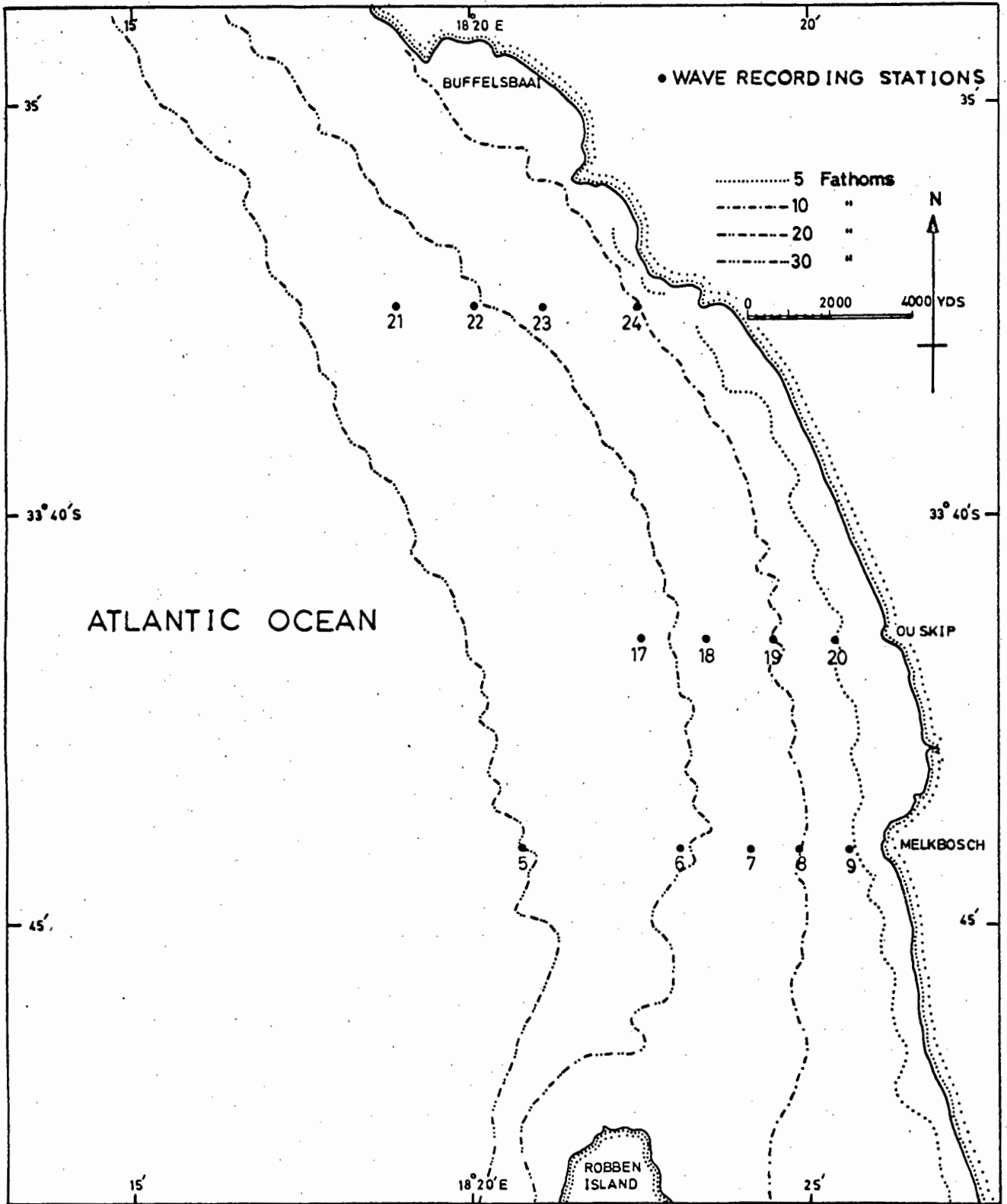


FIG 1.1

MELKBOSCH STRAND AREA

gram as Walden (1963) has done. However, Moskowitz (1964) and Pierson and Moskowitz (1964) by computing averages of selected spectra were able to show that the spectra for various wind speeds form a nested family of curves with the peak shifting toward lower frequencies with increasing wind speed.

Discussions upon the modifications necessary to adapt the linear models used to predict the wave spectra to the non-linear conditions have been in progress since 1958. Participants in the nonlinear discussions include Longuet-Higgins, Kinsman (1961) and many others.

The recording of waves off Cape Town was started by Wilson (1959) who studied range action in Table Bay Harbour.

In 1958, some wave measurements were carried out in Table Bay by the Société-Grenobloise d'Etudes et d'Applications Hydrauliques (SOGREAH) and since 1962 wave measurements have regularly been made on board the Division of Sea Fisheries research ship "Africana II" and also on other research ships.

A study of long waves on the coast of the Cape Peninsula was made by Molly Darbyshire (1963) using a Long Period Wave Recorder. Two types of wave periods were found to exist: the shorter one having a period of 30 seconds to 6 minutes and the longer one having a period of 15 minutes.

In 1964 J. and M. Darbyshire published the results of an analysis of data collected on board the "Africana II". In this paper maximum wave heights, together with mean and dominant wave periods were determined and a few examples of wave spectra were given. This paper was followed by a study by Darbyshire and Pritchard (1966) of waves recorded in South African waters also using data collected on the "Africana II". For various areas near Cape Town, the wave parameters of height, period and wind speed and direction were analysed statistically.

In 1967, a special wave study was initiated on a national basis and is now being carried out by a research team established in the C.S.I.R.'s Hydraulics Research Unit, Stellenbosch. The purpose of this national programme is to record and analyse wave conditions along the coastline of South Africa and South West Africa and to correlate these wave data with existing

meteorological conditions. Initial findings of this Unit were published in progress reports (1968 and 1969). However, no specific wave study was made in the test area covered during this survey.

In 1968, the research ship "Thomas B. Davie" of the University of Cape Town, participating in the National Oceanographic Programme, initiated by the Council for Scientific and Industrial Research (C.S.I.R.) in 1966, was equipped with a ship-borne wave recorder, developed by the National Institute of Oceanography, England.

Through the National Oceanographic Programme it became possible to systematically collect and analyse many wave records. This thesis reports some of the results of this study.

PART I.

Chapter 1. THEORY OF POWER SPECTRUM ANALYSIS.

1.1 Amplitude line spectrum.

If a wave record X (t) is statistically stationary and repeats itself exactly every interval T, the process can be described by the amplitude line spectrum of the waves.

The amplitude spectrum can be obtained by applying the Fourier series analysis to X (t) within the interval T.

$$\begin{aligned}
X(t) = & a_0 + 2 a_1 \cos 2 \pi \frac{t}{T} + 2 a_2 \cos 4 \pi \frac{t}{T} + \dots \\
& + 2 b_1 \sin 2 \pi \frac{t}{T} + 2 b_2 \sin 4 \pi \frac{t}{T} + \dots
\end{aligned}
\tag{1.1}$$

The Fourier coefficients are given by

$$\left. \begin{aligned}
a_n &= \frac{1}{T} \int_0^T X(t) \cdot \cos 2 \pi n \frac{t}{T} \cdot dt \\
b_n &= \frac{1}{T} \int_0^T X(t) \cdot \sin 2 \pi n \frac{t}{T} \cdot dt
\end{aligned} \right\} \text{for } n = 0, 1, 2 \text{ etc.}
\tag{1.2}$$

By plotting the amplitudes a_n and b_n corresponding to each frequency, a line spectrum can be obtained.

1.2 Band spectrum.

If the record is considered to be truncated and replaced by zero outside the interval T of X (t), the Fourier integral technique can be applied and the resulting spectrum A (f) would be a continuous amplitude or a band spectrum.

$$X(t) = \int_{-\infty}^{\infty} A(f) \exp(i 2 \pi f t) \cdot df
\tag{1.3}$$

$$A(f) = \int_{-\infty}^{\infty} X(t) \exp(-i 2 \pi f t) \cdot dt
\tag{1.4}$$

However, the sea surface does not repeat itself periodically, nor

may the wave record be considered zero outside an arbitrary interval T . Therefore, as neither the amplitude spectrum nor the band spectrum can ever describe the waves satisfactorily a different approach, based on the statistical properties of sea waves must be used.

1.3 Power spectrum.

The following statistical assumptions are the basis for this modern approach to wave analysis.

It is assumed that the process which generates the waves is :

- a. a random process and therefore governed by probability laws.
- b. a stationary process.
- c. a Gaussian process, and therefore governed by a Gaussian law. The distribution of the wave heights are therefore determined by the first and the second moments.
- d. an ergodic process, i.e. the time average across an ensemble is the same as the average along a single record at a given time.

Using this statistical approach, wave records can be analysed by selecting a large number of sections of length T from a record of sufficient duration, and obtaining an amplitude spectrum for each section. Although the resulting amplitude spectrum is different for each section, statistically they show the same properties. Therefore, the variability of the amplitudes of the spectra obtained is used to describe the properties of the waves. As variability is measured statistically in terms of variance and variance being proportional to wave energy, this concept makes it possible to describe waves in terms of a power spectrum.

Alternatively, instead of using a number of sections of length T from one long record, it is possible to use a large number of different recordings of length T , recorded under exactly similar conditions and obtain an amplitude spectrum for each record.

Unfortunately both methods are impractical as a very

long recording can never be made under stationary circumstances, nor can a number of wave records be obtained under exactly similar conditions. However, by introducing the concept of the autocovariance function it becomes possible to obtain the energy spectrum from a single record in a practical and reliable way.

The autocovariance function, in ensemble terms and assuming a zero mean is defined as

$$C(\tau) = \text{ave} \left\{ X(t) \cdot X(t + \tau) \right\} \quad (1.5)$$

where τ is the difference in time (lag) of the two values $X(t)$ and $X(t + \tau)$ which are considered together.

In terms of a single wave record the function can be expressed as

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t) \cdot X(t + \tau) \cdot dt \quad (1.6)$$

The normalised function $C(\tau)/C(0)$ is known as the autocorrelation function.

The autocovariance function $C(\tau)$ may be expressed by the Fourier series

$$C(\tau) = \sum \left[A(f) \right]^2 \exp i \pi n \frac{\tau}{T} \quad (1.7)$$

where $A(f)$ is the amplitude density.

The autocovariance function $C(\tau)$ can be expressed as the Fourier transform of a function of frequency $P(f)$ by

$$C(\tau) = \int_{-\infty}^{\infty} P(f) \exp i 2 \pi f \tau \cdot df \quad (1.8)$$

Comparison of expression (1.7) and (1.8) shows that the function $P(f)$ behaves like the energy density $(A(f))^2$ and that it is a set of Dirac delta functions located at intervals of $\frac{1}{T}$.

The function $P(f)$ can be obtained by taking the Fourier transform of the autocovariance function $C(\tau)$.

Since $P(f)$ and the autocovariance function $C(\tau)$ are even functions of their respective arguments, the relation between them may be expressed by the Fourier cosine transform, and because of the symmetry of the resulting spectrum, the one-sided cosine transform may be used.

$$P(f) = 2 \int_0^{\infty} C(\tau) \cos 2 \pi f \tau . d\tau \tag{1.9}$$

1.4 The analysis of continuous records of finite length.

In practice, the autocovariance function $C(\tau)$ can never be estimated for arbitrary long lags, as the longest possible lag is determined by the finite length of the record.

Following Blackman and Tukey (1958), the autocovariance function as defined in (1.6) is therefore reduced to

$$C_{\infty}(\tau) = \frac{1}{T_n - |\tau|} \int_{-(T_n - |\tau|)/2}^{(T_n - |\tau|)/2} X(t - \frac{\tau}{2}) \cdot X(t + \frac{\tau}{2}) \cdot dt \tag{1.10}$$

where $C_{\infty}(\tau)$ is called the apparent autocovariance function and T_n is the total length of the record. Since $C_{\infty}(\tau)$ is small for large values of τ , and large for small values of τ , the value of $C_{\infty}(\tau)$ for τ greater than the maximum lag which will be used are ignored. This is done by multiplying the apparent autocovariance function $C_{\infty}(\tau)$ by a lag or masking function $D_1(\tau)$

$$C_i(\tau) = D_i(\tau) \cdot C_{\infty}(\tau) \tag{1.11}$$

The resulting function $C_i(\tau)$ has been named the modified apparent autocovariance function by Blackman and Tukey (1958).

The masking function $D_i(\tau)$ is defined as follows :

$$D_i(\tau) = \begin{cases} 0 & \text{for } |\tau| > T_m \\ 1 & \tau = 0 \end{cases} \tag{1.12}$$

where T_m is the maximum lag which will be used.

The masking function $D_i(\tau)$ as defined above does not define $D_i(\tau)$ for $0 < \tau \leq T_m$. A number of alternative shapes of $D(\tau)$ within the interval have been proposed, each function having its own merits.

In the following analysis the lag window, whose use is called "hanning" will be employed. This function, named after Julius von Hann, is defined as

$$D(\tau) = \begin{cases} \frac{1}{2} \left(1 + \cos \frac{\pi \tau}{T_m} \right) & \text{for } |\tau| \leq T_m \\ 0 & |\tau| > T_m \end{cases} \tag{1.13}$$

The modified apparent autocovariance function $C_i(\tau)$ can be calculated for any wave record. However, because of the finite length of the records, $C_i(\tau)$ turns out to be a poor estimate of the true autocovariance function $C(\tau)$. Fortunately, by Fourier transforming expression (1.11) very good estimates of the smoothed values of the true power spectrum $P(f)$ are obtained. (Blackman and Tukey, 1958).

Transforming expression (1.11) yields :

$$P_i(f) = Q_i(f) * P_{oo}(f) \tag{1.14}$$

where $P_i(f)$ is the transform of $C_i(\tau)$, $Q_i(f)$ the transform of $D_i(\tau)$ and $P_{oo}(f)$ the transform of $C_{oo}(\tau)$. The asterisk indicates convolution.

Writing expression (1.14) explicitly

$$\text{Ave} \{ P_i(f_1) \} = \int_{-\infty}^{\infty} Q_i(f_1 - f) \cdot P(f) \cdot df \tag{1.15}$$

Since the function $P(f)$ is composed of discrete Dirac spikes at intervals of $\frac{1}{T}$, the function $Q_i(f_1 - f)$ may be considered as a weighing factor of spikes near each frequency, the weighing being carried out with weights proportional to $Q_i(f_1 - f)$. For this reason $Q_i(f_1 - f)$ is sometimes called a frequency window. The resulting smoothed two-sided spectrum $P_i(f)$ is therefore a smoothed average of the energy contained in the true power spectrum $P(f)$ at frequencies near f_1 .

The average smoothed power spectrum for a one-sided spectrum can, according to Blackman and Tukey (1958), be found from

$$\text{Ave} \{ 2 P_i(f_1) \} = \int_0^{\infty} H_i(f; f_1) \cdot 2 P(f) \cdot df$$

where (1.16)

$$H_i(f; f_1) = Q_i(f + f_1) + Q_i(f - f_1)$$

The smoothed power spectrum $P_i(f)$ is still a blurred imitation of the true power spectrum $P(f)$ because of the result of convolution. Only a finite number of Dirac spikes can be used as an estimate of the true power within each interval and this causes sampling errors. The error of the resulting weighted power spectrum therefore depends on the number of spikes.

One way to reduce error would be to increase the number

of spikes in each interval by widening the frequency window $Q_i(f_1-f)$. Unfortunately, widening the frequency window would in effect mean a narrowing of the lag window $D_i(\tau)$, and this could lead to a loss of meaningful information as waves having a relative short frequency may not be detected. The best way to reduce sample error is therefore to increase the sample length.

1.5 The analysis of equally spaced records of finite length.

If it is required to obtain a power spectrum for data in digital form, the above derived formulae must be adapted.

The process of digitizing produces uniformly spaced values of $X(t)$ at $X_0, X(\Delta t), \dots, X(n\Delta t)$ only and there is therefore no possibility of calculating the autocovariance function $C(\tau)$ other than for $C_{(0)}, C_{(1)} \dots \dots C_{(M)}$.

The effect of digitizing $X(t)$ at equally spaced intervals is effectively to multiply the autocovariance function $C(\tau)$ by a "comb" of equally spaced ordinates of value unity and intermediate values taken as zero.

Consequently, the final result of the multiplication is not the energy spectrum $P(f)$, but rather a convolution of the Fourier transform of the autocovariance function $C(\tau)$ and the "comb" of equally spaced ordinates of value unity. This can be seen in Figure 1.2 taken from Barber (1961) where the convolution is presented diagrammatically.

The figure shows that the result of digitizing can be obtained by multiplying the arbitrary autocovariance function $C(\tau)$ represented in the figure as a_1 , by a "comb" with intervals of $\Delta t(b_1)$. The result of this multiplication is shown in c_1 . The transform of a_1 shown as its cis spectrum only, is shown in a_2 while the transform of b_2 is another "comb", whose ordinates lie at frequencies $0, \pm \frac{1}{\Delta t}, \pm \frac{2}{\Delta t}$ etc. (b_2)

The convolution theory shows that the convolution of a_2 and b_2 is obtained by spreading the spectrum of a_1 out into the spectrum of b_2 . The result is shown in c_2 . It can be seen that c_2 contains essentially a sum of copies of the spectra located at frequencies $0, \pm \frac{1}{\Delta t}, \pm \frac{1}{2\Delta t}$ etc. Therefore, in order to prevent aliasing or overlapping of these spectra care must be taken

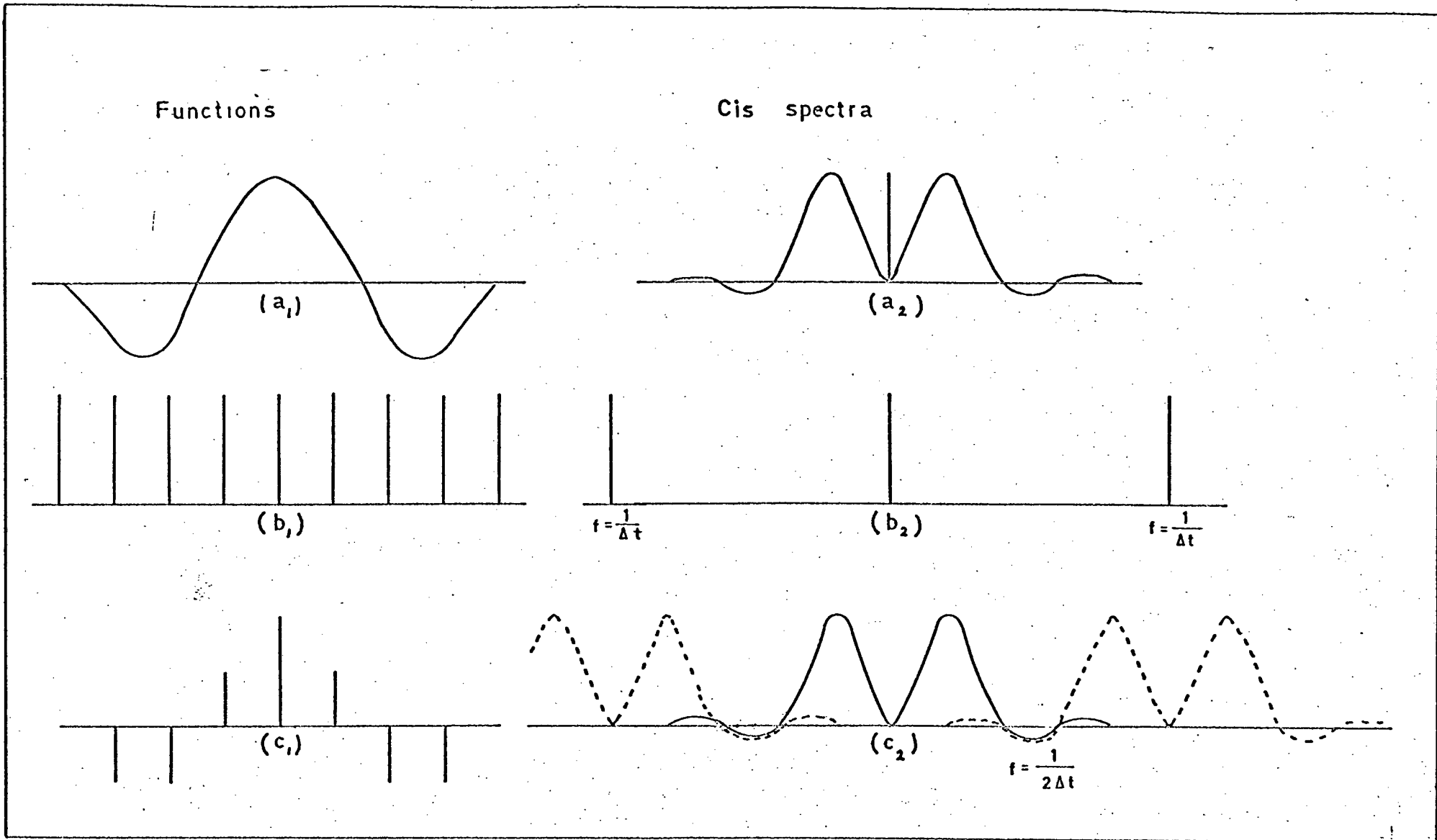


Fig 1.2

CONVOLUTION OF A DIGITIZED FUNCTION.

[from BARBER (1961)]

that the waves have no substantial energy at frequencies greater than $\frac{1}{2\Delta t}$. This frequency

$$f_N = \frac{1}{2\Delta t} \tag{1.17}$$

is known as the Nyquist frequency.

In the analysis of a continuous record the apparent autocovariance function $C_{oo}(\tau)$ is first modified by a masking function and the resulting modified apparent autocovariance function $C_1(\tau)$ is then transformed in order to get the smoothed power spectrum $\text{Ave}\{P_i(f_1)\}$.

The analysis of digital data can be carried out by first transforming the sample autocovariance function C_R and then convolving, as convolving in this case means only smoothing with certain weights (Blackman and Tukey, 1958).

The sample autocovariance function C_R , with an adjustment for the mean, can be obtained from

$$C_R = \frac{1}{N-R} \sum_1^{N-R} X_t \cdot X_{t+R} - \left(\frac{1}{N} \sum_1^N X_t \right)^2 \tag{1.18}$$

where N is the number of observations, X_t the observations, M the lag number and $R = 0, 1, \dots, M$. The variance of the observations is given by C_0 .

As a consequence of having the data in digital form, the finite Fourier cosine series transform must be used instead of the infinite Fourier integral transform. The finite cosine transform V_R , which gives the "raw" estimates of the spectral density as a function of R, can be found from

$$V_R = \frac{1}{M} \left[C_0 + 2 \sum_{q=1}^{M-1} C_q \cdot \cos \frac{qR\pi}{M} + C_M \cos R\pi \right] \tag{1.19}$$

The raw estimates V_R are smoothed for the effects of distortion

introduced by reading off the record at discrete points, and by the finite length of the record.

Several methods of smoothing the estimates are available (Blackman and Tukey, 1958). The method used in the subsequent analysis smooths with weights 0.25, 0.50 and 0.25, and its use is named "hanning". The final smoothed estimates of the spectral density as a function of R are then given by U_o , U_R and U_M .

$$\begin{aligned}
U_o &= \frac{1}{2} (V_o + V_1) \\
U_R &= \frac{1}{4} V_{R-1} + \frac{1}{2} V_R + \frac{1}{4} V_{R+1} \quad 1 \leq R \leq M-1 \\
U_M &= \frac{1}{2} V_{M-1} + \frac{1}{2} V_M.
\end{aligned}
\tag{1.20}$$

The transformation of the U values as a function of R to a function of the frequency can be found from

$$f \text{ (cps)} = \frac{R}{2M\Delta t} \tag{1.21}$$

where Δt is the sampling interval of the observations and R and M defined as before.

Tukey (1949) has shown that the errors in the estimation of U_o , U_R and U_M are distributed according to a chi-square distribution with f degrees of freedom, where df is given by

$$df = \frac{N - \frac{1}{4} M}{\frac{1}{2} M} \tag{1.22}$$

Following Kinsman (1965) and using his chi-square table, 80% confidence limits have been calculated for some of the spectra obtained.

The smoothed estimates U_o , U_R and U_M are estimates of the average energy contained in a frequency band of width $1/M\Delta t$. Therefore, the choice of M and Δt will determine the resolution of the spectrum.

Chapter 2. RECORDING AND DIGITIZING OF THE WAVE DATA.

2.1 Introduction.

The recording of sea waves is complicated by the fact that the sea surface normally consists of waves having a wide range of frequencies. Ideally, a wave recorder must therefore be equally sensitive over all the frequencies which happen to be present. No such instrument exists at present. Wave recorders normally only cover a limited frequency range. However, even within this restricted frequency range, the instrument can never give a true representation of the sea surface. Distortion will always take place because of the limitations of the instruments.

For this reason the calibration of a wave recorder presents a problem, as no absolute standard for comparison is available. As a substitute, different types of recorders are often used simultaneously and the resulting records compared with one another.

Basically, wave recorders can be divided into three groups : instruments which measure waves from below the surface, instruments which measure waves at the surface and instruments which measure waves from above the surface.

Instruments which are designed to measure waves from below the surface usually record the fluctuations of pressure induced by the surface waves. However, for any given depth of water and wave height the amplitude of the pressure fluctuation depends on the wave period in such a manner that waves of a very short period may be virtually eliminated.

Instruments designed to measure waves at the surface are mostly of the accelerometer, resistance or float type. Again the characteristics of each instrument are such that distortion of the wave records takes place.

Wave recording from above the sea surface is not very well developed yet, although it appears to be a very promising approach. For instance, stereophoto pairs have been used as a means of getting instantaneous configurations for large areas of the sea surface.

The wave recorder used for this investigation was an N.I.O. ship-borne wave recorder (Tucker, 1956), fitted on the hull of the research ship "Thomas B. Davie".

2.2 Principle of operation of the wave recorder.

The N.I.O. ship-borne wave recorder measures the height of the water surface above two pressure units, one on each side of the ship. An accelerometer near each pressure unit measures the acceleration and this is integrated twice electronically to obtain the vertical displacement from an imaginary reference level. The sum of the height of the water surface and the displacement of the accelerometers gives the height of the surface above the reference level. This height will thus be independent of the motion of the ship.

If only one measuring head was used, short waves approaching the side of the ship remote from the instrument might be partially reflected and thereby not fully measured, or approaching from the instrument side, they might have their height increased by reflection.

For this reason two measuring heads are used and the average of the outputs taken.

Following the N.I.O. Handbook for the wave recorder, a brief description of the main components of the instrument is given below.

2.2.1 Pressure unit.

The pressure unit connects with the sea through a small hole drilled in the ship's hull. An oil-filled chamber is separated from the sea water by a rubber diaphragm. The oil takes up the pressure of the sea water and applies it to the outside of a metal capsule. The resulting deflection of the capsule is measured by a mechano-electric transducer.

2.2.2 Mechano-electric transducer.

This consists of an E-type transformer of which the primary coil is wound round the base of the centre limb. This coil is supplied with 1000 C/s at 80 volts. The secondary coil, which is the pick-up coil, has a voltage induced in

it, which varies linearly with its displacement in the gap between the limbs. Thus, the transducer is in effect a transformer with variable coupling controlled by the position of the secondary coil.

The output voltage ranges from 6.3 to 13.7 volts and is 10 volts in the central position.

2.2.3 Accelerometer unit.

This consists essentially of a weight hung on a spring and coupled to a transducer such as described above.

The body of the accelerometer is filled with transformer oil to provide sufficient damping and is hung in gimbals in a bowl to keep it vertical as the ship rolls and pitches.

The full deflection of the accelerometer is reached by an acceleration of between 0.4 and 0.5 g.

2.2.4 Bridge-stabilised oscillator.

The oscillator is basically a stabilised Wienbridge oscillator. The amplitude of oscillation is controlled almost entirely by the bridge characteristics and is independent of power supply changes.

It is essential to use a high-quality bridge-stabilised oscillator because the sensitivity of the wave recorder is proportional to the oscillator voltage which has to be kept stable. Variations in the frequency are comparatively unimportant.

2.2.5 Computer circuit.

The computer receives the signals from the two accelerometers and the pressure units. The 1000 ^c/s output of the accelerometer is amplified, rectified and integrated twice. Rectification of the starboard accelerometer rectifier is different in sense to the port accelerometer rectifier.

The signals from the pressure units are rectified and electronically added to the integrated signals from the accelerometers. The resulting voltage represents the height of the water surface above the reference level. This voltage is fed to the recorder. The rate of recording, checked by stop-watch timing, was about 2 inches per minute.

2.3 Calibration of the wave recorder.

According to the hydrodynamical theory, the relative response of a pressure instrument to waves of different periods T depends on the depth of the pressure units below the water line as follows,

$$r = e^{(-4 \pi^2 h / g T^2)} \tag{2.1}$$

where h is the actual hydrostatic depth of the pressure units and g the acceleration due to gravity.

Molly Darbyshire (1961) found that the pressure units behave as though they are at a depth of more than twice the actual hydrostatic depth.

Although the precise value of h may depend on the ship's characteristics, it has been assumed, that for the "Thomas B. Davie", having a net tonnage of 53 tons, overall length of 96', beam of 23' and a draft of 9', 2.5 d instead of h must be used in the formula, where d is the hydrostatic depth obtained from the dimensions of the "Thomas B. Davie".

The overall attenuation curve must also take into account the electronic characteristics of the instrument. According to Molly Darbyshire (1961) the general expression can be given as

$$\frac{\text{true height}}{\text{recorded height}} = 0.83 \left[1 + (8.8 \times 10^4 \pi f)^{-2} \right]^{3/2} \cdot e^{-4 \pi^2 f^2 k d / g} \tag{2.2}$$

where f is the frequency, k a constant. The first term represents the electronic characteristics of the instrument, and the exponential term the hydrodynamical effect.

As the pressure units on board the "Thomas B. Davie" are normally 3 feet below the water surface, expression (2.2) reduces to

$$\frac{\text{true height}}{\text{recorded height}} = 0.83 \left[1 + (8.8 \times 10^4 \pi f)^{-2} \right]^{3/2} \cdot e^{-9.2 f^2} \tag{2.3}$$

This expression is graphically presented in Fig. 2.1.

The spectra presented in the subsequent chapters are therefore corrected accordingly.

2.4 Digitizing of the wave records.

The instrument designed to obtain readings from the wave record at equally spaced intervals can briefly be described as follows :

A piece of perspex, on which a thin line is engraved is attached to a moving carriage, the movement of which is controlled by a threaded rod.

The rod can be rotated by a knob, each complete revolution of which is indicated by a click and corresponds to the line's displacement of exactly $\frac{1}{8}$ " along the wave record.

The wave record, together with the line is sufficiently enlarged by a magnifying glass, which is attached to the carriage.

The observations were read off to the nearest 0.1' and it is estimated that the sampling interval was accurate to $\pm 5\%$. Error due to parallax was considered to be negligible.

2.5 Choice of the sampling interval.

As the readings were taken at intervals of exactly $\frac{1}{8}$ ", the corresponding sampling interval was approximately 2 seconds. This sampling interval limits the frequency range of the estimated power spectra, as a result of the Nyquist frequency, to the range 0 - 0.25 cps.

In the following analysis it is assumed that for the spectra no appreciable energy is found at frequencies greater than the Nyquist frequency and that therefore no appreciable aliasing has blurred these spectra.

The reasons for the assumption are :

- a) Spectra obtained by other workers (Walden, 1963) show that for a fully arisen sea at the energy levels obtained, most of the energy is found at frequencies lower than the Nyquist frequency resulting from the above sampling interval. If the

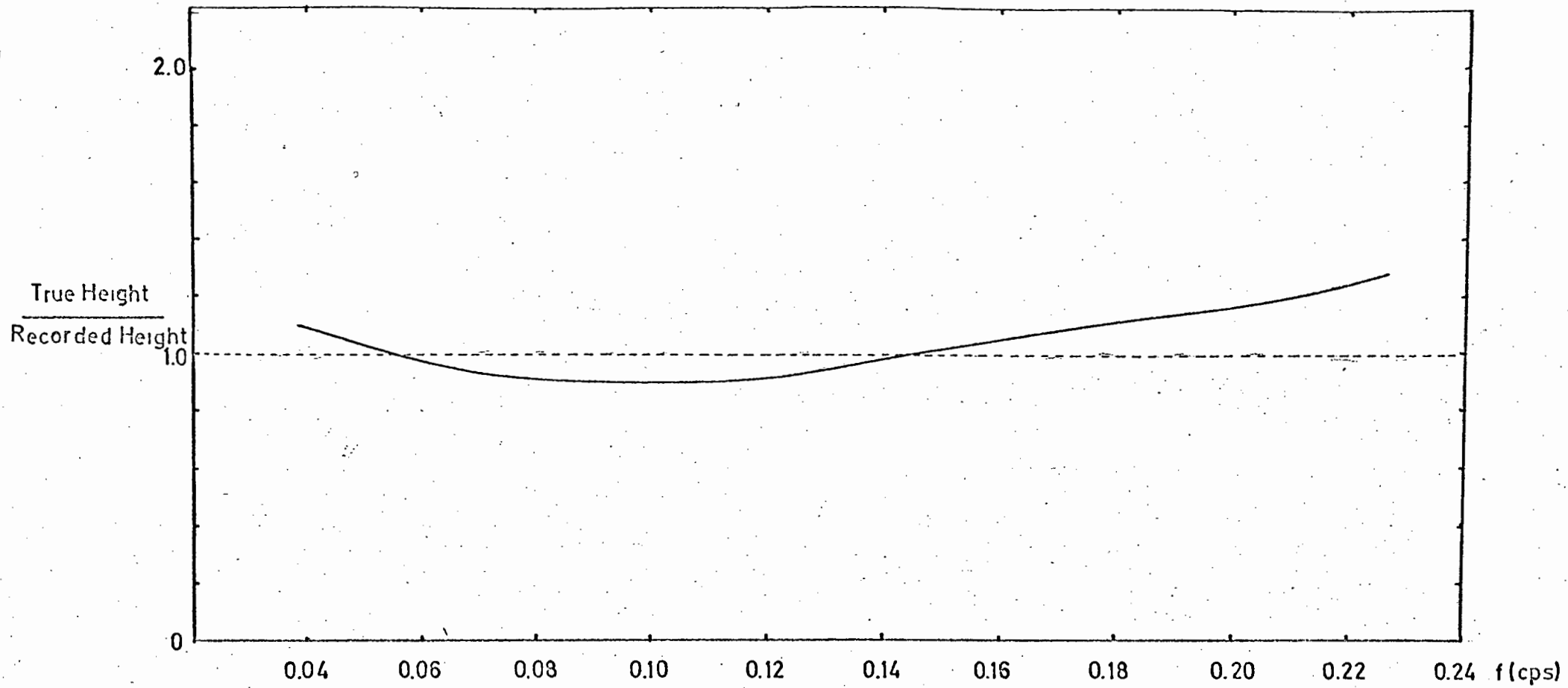


Fig 2.1 Correction curve for wave recorder
"Thomas B. Davie".

sea is not fully arisen at the energy levels obtained, relatively more energy may be expected near the Nyquist frequency (Pierson, Neumann and James, 1954) but it seems unlikely that much aliasing will occur.

b) The spectra obtained in the area show that only a small proportion of the energy is found in the neighbourhood of the Nyquist frequency.

c) Experimental data, discussed in Chapter 3 confirm the assumption. The data show that a substantial decrease in the sampling interval does not significantly alter the spectrum.

2.6 Computation of the wave data.

The Algol Computer Programme for spectral analysis, as obtained from Professor J. Darbyshire of the Marine Science Laboratories, Anglesey, England, was translated and extended. The resulting IBM 1130 Computer Programme is listed in Appendix I.

Chapter 3. PRELIMINARY INVESTIGATIONS.

3.1 Introduction.

In this chapter an experiment to show the effect of the lag number on the resolution of the spectrum is discussed, and the results are applied to the resolution chosen for most of the following spectra.

Secondly, experiments to test the assumptions of stationarity and normality are discussed. Although the theory of power spectrum analysis assumes that the process, which describes the fluctuation of the water height at a particular point on the free surface, is also an ergodic process, this assumption will be assumed as a hypothesis, as it can never be tested.

Finally, an analysis is made of a wave record which was analysed by using different sampling intervals. The effect of aliasing is discussed.

3.2 Effect of the lag number on the resolution of a spectrum.

Increasing the lag number M for a particular set of observations, decreases the number of degrees of freedom according to expression (1.22) and increases the resolution.

As the resolution increases, the average energy contained in each frequency band decreases, because the total energy of the record must remain constant. Consequently, the confidence limits will increase.

The effect of changing the lag number M , for a particular spectrum, is shown in Fig. 3.1.

It can be seen that a high resolution brings out more detail, but it is doubtful that this increase in detail reflects a better knowledge of the spectrum, as the corresponding confidence band has also increased in width.

The only way to prove the reality of this detail for a fixed sampling interval is to considerably increase the number of observations N , thus decreasing the band width.

3.3 Choice of resolution.

According to Tukey (1961) it is often desirable to make

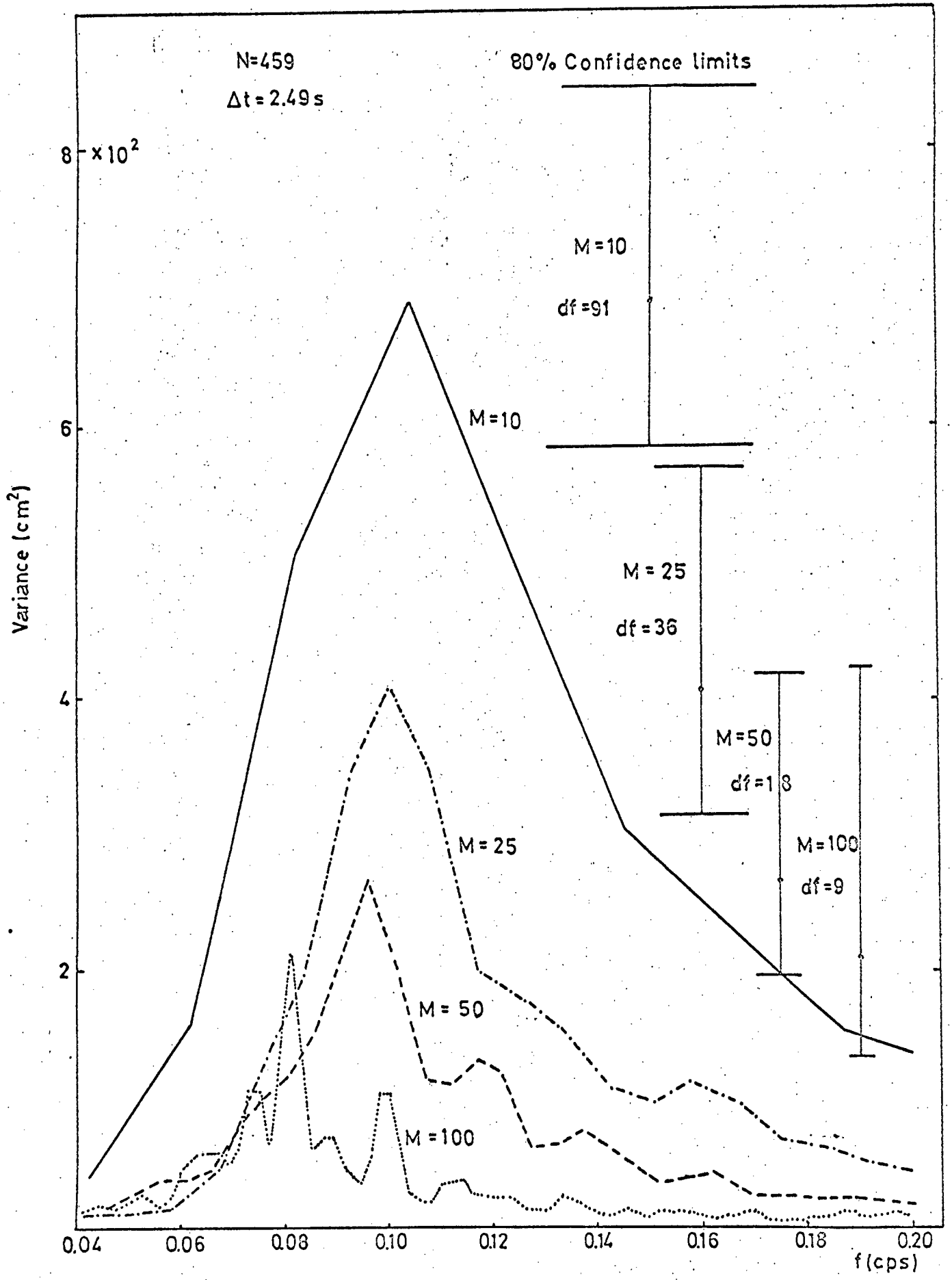


Fig 31

Effect of lag number on resolution.

analyses of the same data at different resolutions. Kinsman (1963) thinks it a good practice never to let M exceed 0.1 N and if it can be afforded, to make it even smaller.

Most of the subsequent analyses have therefore been carried out with M approximately equal to 0.1 N. Only in cases where comparison with similar spectra casted doubt on the reality of some of the peaks has the resolution been decreased.

3.4 Stationarity.

As weather conditions are always changing, no matter how little, very few records are strictly stationary.

Stationarity can be crudely tested by splitting up a long record and comparing the corresponding spectra with one another. If the process is stationary, the spectra so obtained should be similar, as a stationary process is unaffected by shifts of the time origin.

Spectra of non-stationary processes are known as average spectra. Provided the time interval over which the spectrum is averaged is not excessive, average spectra are still quite useful as will be shown below.

In order to test the stationarity of some of the records obtained, three recordings of sufficient duration were split up into parts, and the spectrum for each part was determined. The following Table shows the relevant data of each record.

Table 1.

Wave recordings used for stationarity tests.

	TBO+ 39	TBO 41	TBO 42
date.	10.9.1969	18.11.1969	18.12.1969
duration (min).	50	63	60
wind (m/s).	15	9	2
wind direction.	170°	190°	variable
wind (m/s)*.	increased from 11	unchanged	decreased from 5
wind direction.	unchanged	unchanged	-

+ Table Bay Operation (cruise number).

* Wind history over previous 6 hours.

The record obtained on cruise number TBO 39 was split up into two equal parts and the spectrum for each part was determined (Fig. 3.2). In order to prevent cluttering of the figure, only one frequency band together with the corresponding 80% confidence band is shown (at an arbitrary energy level). The lag number M, the number of observations N, and the corresponding degrees of freedom are indicated in the figure.

Computation shows that the total variance of the first part of the record is $3.68 \times 10^2 \text{ cm}^2$ and of the second part $5.44 \times 10^2 \text{ cm}^2$. Consequently it may be concluded that the record is not stationary. However, the general distribution of the energy with frequency is very similar for both spectra. The difference between the spectra is primarily found at the peaks.

The record obtained from TBO 41 was split up into 3 parts and the spectrum for each part is shown in Fig. 3.3. The total variance of the first part of the record is $1.63 \times 10^4 \text{ cm}^2$, the second part $1.76 \times 10^4 \text{ cm}^2$ and the third part $1.90 \times 10^4 \text{ cm}^2$. The difference between the total variance of the first and the last part of the record is 14%, and may suggest some degree of non-stationarity. However, the general shape of the three spectra are again very similar, and the peak energy of each spectrum is found at the same frequency.

The record obtained from TBO 42 was split up into 2 parts and Fig. 3.4 shows a good agreement between the spectra. The variance of the first part of the record is $2.05 \times 10^3 \text{ cm}^2$ and of the second part $2.17 \times 10^3 \text{ cm}^2$. The difference is only 5%. This suggests a high degree of stationarity.

Although the peak energies are slightly different, these differences are still well within the 80% confidence band and are therefore probably not significant.

3.5 Normality.

In the analysis of sea records it is generally assumed, that the wave heights measured at a particular point on the sea surface are distributed according to the Gaussian law.

However, Mac Kay (1959) found, that one record out of the sixteen tested, showed some form of non-normality.

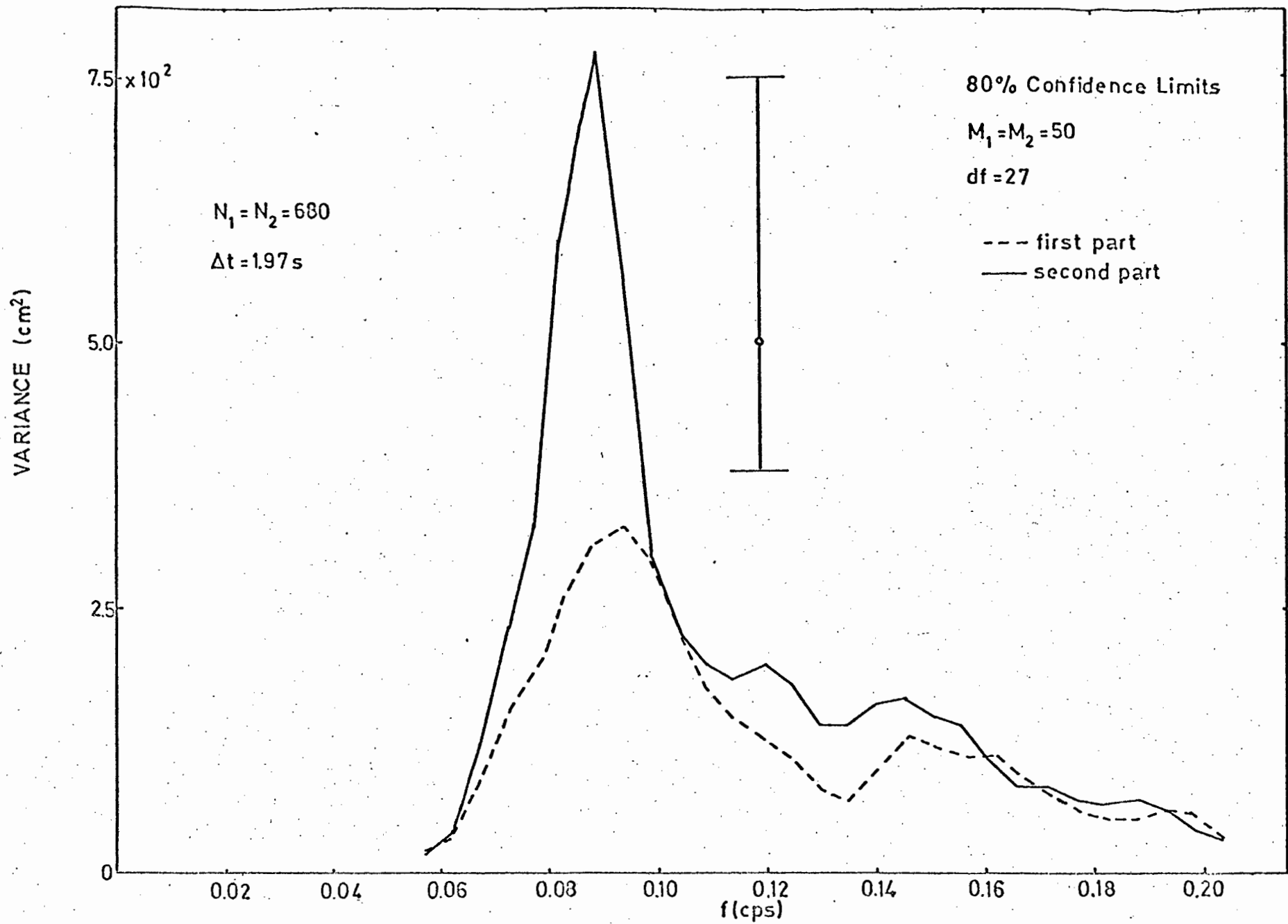


Fig 3.2 Spectra computed for first and second half of record

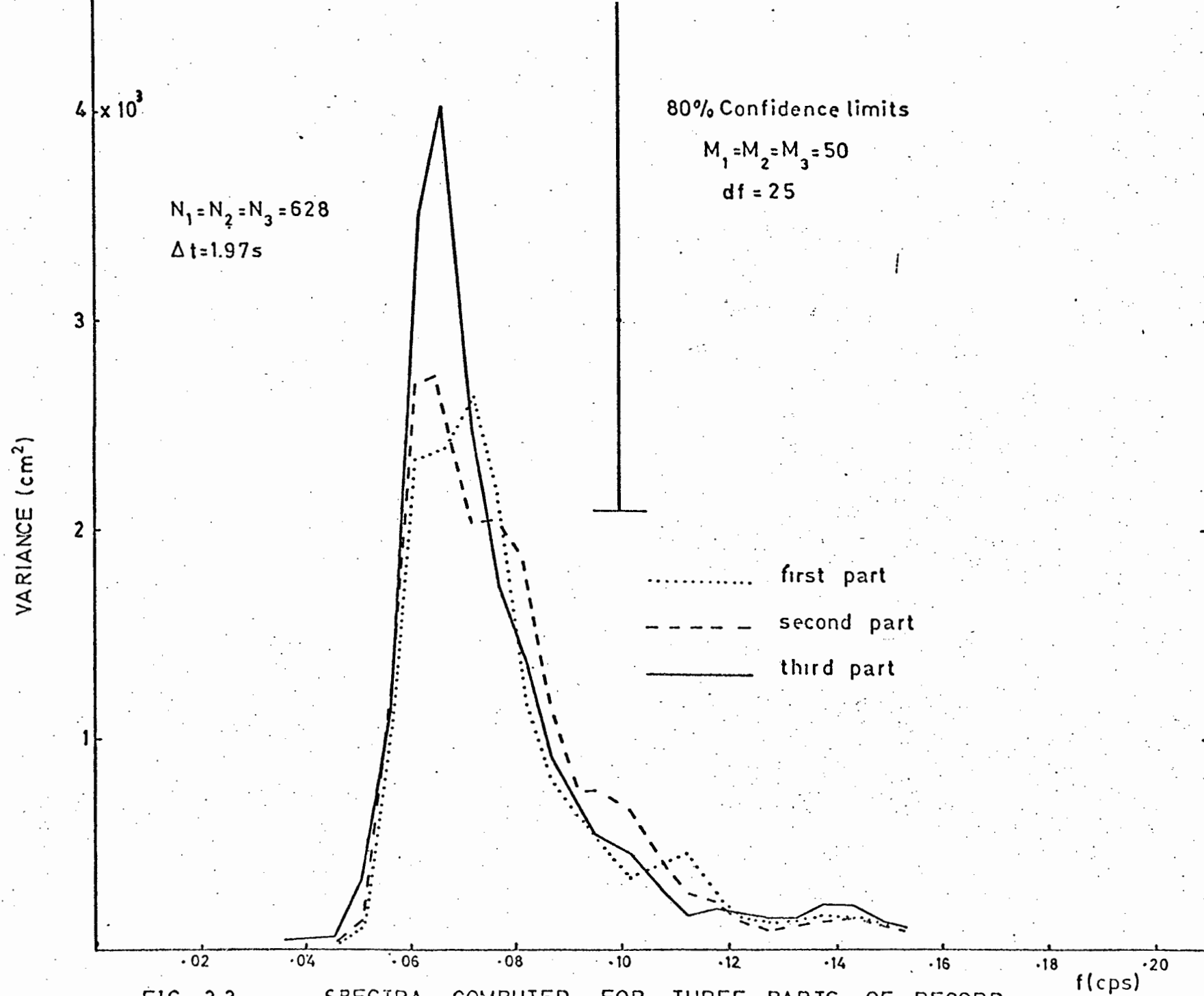


FIG. 3.3 SPECTRA COMPUTED FOR THREE PARTS OF RECORD

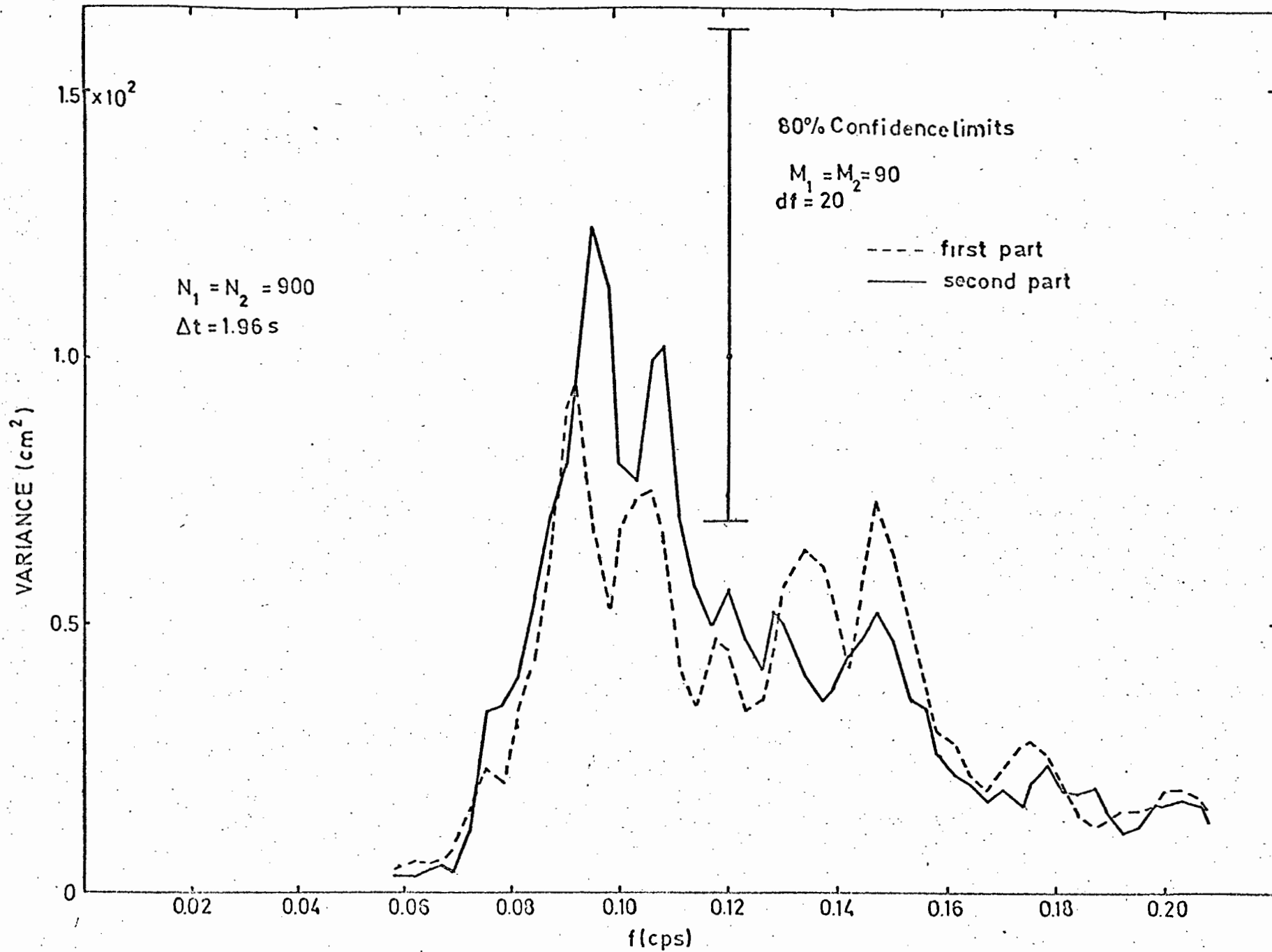


Fig3.4 Spectra computed for first and second half of record

If the distribution of the wave heights is not normal, the autocovariance, although providing a large amount of useful information, will not completely specify the process (Blackman and Tukey, 1958).

It was therefore decided to test a number of wave records, as no tests of this nature had ever been carried out in the area before.

Testing the normality of data can be done in many ways; however the chi-square test with seven degrees of freedom was used, as a computer programme for this test was locally available.

3.5.1 Minimum interval of independence.

Firstly, the minimum interval of independence was determined. This is defined as the minimum time interval required, to obtain samples of the wave record which are independent of one another.

It might be expected that, because of basic periodicities of sea-waves, sampling intervals of the order of a few seconds will not produce samples which have this property. A larger sampling interval will usually be required.

To investigate this, two wave records were selected, and by using sampling intervals of 60, 40, 10 and 2 seconds, the minimum sampling interval which must be used for reliable testing for normality of wave data was determined.

As the sample length of the wave record is determined by the product of the sampling interval (ΔT) and the number of sampling points (N), the number of sampling points was selectively increased with smaller sampling intervals, to assure that the sample length of the record remained reasonably constant.

Table 2 shows the results of these tests.

Table 2.

Minimum sampling interval which must be used for tests for normality.

Number of sampling points. (N)	Sampling interval. (ΔT)	Sample length. (N. ΔT).	Chi-square value.
46	60 sec.	2760 sec.	8.0
68	40	2720	7.0
100	20	2000	14.7 *
150	10	1500	5.3
1360	2	2720	18.3 *
Wind speed 15 m/s. Wind direction 170° Depth of water 35 m.			TBO 39 Date 10.9.1969
Number of sampling points. (N)	Sampling interval. (ΔT)	Sample length. (N. ΔT)	Chi-square value.
64	60 sec.	3840 sec.	4.6
100	40	4000	4.5
100	20	2000	8.6
150	10	1500	19.9 *
750	2	1500	18.1 *
Wind speed 09 m/s. Wind direction 190° Depth of water 140 m.			TBO 41 Date 18.11.1969

* significant at 5% critical level of chi-square

The 5% critical level of chi-square for 7 degrees of freedom is 14.1. The table shows that if the sampling interval is less than 40 seconds, 4 out of 6 samples must be rejected at this level. The results shown in the table suggest that the minimum sampling interval which must be used for reliable testing for normality of wave data, is of the order of 40 seconds.

This result confirms the findings of Mac Kay (1959) who

found with the incorporation of a factor of safety, 37.5 seconds as the minimum interval of independence.

3.5.2 Effect of sample length.

As the sample length ($N \cdot \Delta T$) must obviously be of importance in tests for normality of wave data, the minimum sample length which must be used for reliable testing for normality was investigated.

Using a sample interval of 40 seconds the number of sampling points were gradually reduced as shown in Table 3.

Table 3.

Chi-square values for different sample lengths using a fixed sampling interval.

Number of sampling points. (N)	Sampling interval. (ΔT)	Sample length. ($N \cdot \Delta T$)	Chi-square value.
68	40 sec.	2720 sec.	7.0
60	40	2400	5.7
40	40	1600	5.5
30	40	1200	4.9
Wind speed 15 m/s. Wind direction 170° Depth of water 43 m.			TBO 39 Date 10.9.1969
Number of sampling points. (N)	Sampling interval. (ΔT)	Sample length. ($N \cdot \Delta T$)	Chi-square value.
100	40 sec.	4000 sec.	4.5
60	40	2400	3.0
40	40	1600	7.9
30	40	1200	3.7
Wind speed 09 m/s. Wind direction 190° Depth of water 140 m.			TBO 41 Date 18.11.1969

The table shows that with a fixed sampling interval of 40 seconds, a sample length of 20 minutes will still be sufficient

to determine the degree of normality of the record.

However, whenever possible in the subsequent analysis, a larger sample was used.

3.5.3 Normality of off-shore wave data.

The Gaussian properties of the sea surface in the area of interest were tested by selecting a number of recordings, obtained at stations with a water depth of about 140 metres.

The selection was based on the wind history, wind velocity and direction, and the observed swell during the recording.

The selected recordings therefore cover a wide range of wave generating conditions : wind velocities varied from 0 - 18 m/s, wind directions from 010° - 310° while on TBO 34 a heavy swell was recorded in the absence of local winds.

Table 4 shows the relevant data for each recording, and the chi-square values obtained. At the 5% critical value of chi-square for 7 degrees of freedom, two out of a total of nine recordings must be rejected.

3.5.4 Normality of in-shore wave data.

As the stations covered in this survey varied in depth from 10 - 140 metres, it was decided to investigate if factors such as differential refraction, shoaling and damping due to bottom friction and circumstances as discussed in Chapter 5, can possibly lead to a departure of the Gaussian nature of sea waves.

In order to make comparison between the deep and shallow water stations possible, only those recordings which were made during reasonably stationary circumstances were chosen.

Table 5 shows the relevant data for the four different tests, together with the chi-square values obtained.

Only one record out of sixteen must be rejected at the 5% critical level.

The Table shows clearly, that the Gaussian properties of the off-shore waves are not altered as the waves move into shallow water.

Table 4.

Chi-square values for recordings under different wave generating conditions.

TBO	34	37	40	41	44	45	47	48	47
date	28.5.69	16.7.69	8.10.69	18.11.69	23.2.70	17.3.70	7.4.70	20.4.70	8.4.70
wind m/s.	calm	3	18	9	6	6	9	7	3
wind dir.	-	310°	275°	190°	10°	140°	160°	170°	var.
swell dir.	225°	250°	290°	230°	220°	200°	220°	170°	220°
N	80	60	80	100	60	80	60	60	53
ΔT (sec)	37.5	37.5	37.5	40	37.5	37.5	37.5	37.5	37.5
wind vel. ⁺	unch'd.	unch'd.	unch'd.	unch'd.	var.	var.	var.	var.	unch'd.
wind dir.	unch'd.	unch'd.	unch'd.	unch'd.	var.	unch'd.	var.	unch'd.	var.
chi-square value	10.9	17.2*	8.3	4.5	8.5	17.9*	10.0	2.5	4.5

⁺ wind history over previous 6 hours.

* significant at 5% critical level of chi-square.

Table 5.

Chi-square values for recordings in ranges of different depths.

	TBO 39			TBO 41			
date	10 . 9 . 1969			18 . 11 . 1969			
time	12.27	10.20	17.55	14.25	18.34	10.24	9.28
wind (m/s)	15	10	15	9	10	9	9
wind dir.	170°	170°	180°	190°	210°	250°	230°
swell dir.	190°	170°	180°	230°	230°	250°	230°
N	68	60	60	100	60	60	60
Δ T (sec)	40	37.5	37.5	40	37.5	37.5	37.5
depth (m)	35	10	10	140	10	10	10
chi-square value	7.0	3.6	6.8	4.5	1.4	10.2	13.0

	TBO 42					TBO 47			
date	18 . 12 . 1969					8 . 4 . 1970			
time	12.47	11.16	10.37	09.53	09.24	7.32	9.15	10.23	11.00
wind (m/s)	2	2	3	calm	2	3	2	3	5
wind dir.	195°	330°	325°	-	295°	var.	var.	170°	190°
swell dir.	190°	210°	210°	215°	230°	220°	220°	225°	235°
N	90	44	38	50	35	53	47	44	47
Δ T (sec)	40	37.5	37.5	37.5	37.5	37.5	37.5	37.5	37.5
depth (m)	140	40	30	15	10	140	80	40	20
chi-square value	6.3	13.0	3.4	10.0	11.0	4.5	5.6	16.5*	12.0

* significant at 5% critical level of chi-square

3.6 Aliasing.

Increasing the Nyquist frequency increases the frequency interval covered by the analysis and may therefore reduce distortion caused by aliasing.

A wave record having a peak energy at 0.095 cps was analysed by using two different sampling intervals.

Fig 3.5 shows the spectra obtained for $\Delta t = 1.97$ seconds and $\Delta t = 2.49$ seconds. The variance is expressed in $\text{cm}^2 \cdot \text{sec}$. The area under the curves (variance) is $2.21 \times 10^3 \text{ cm}^2$ and $2.50 \times 10^3 \text{ cm}^2$ respectively. The difference is 8%.

Although the general shape of the graphs is similar and the peak energy is located at the same frequency in both cases, some aliasing might have taken place.

However, the energy level obtained, at the higher frequencies near the Nyquist frequency for the spectrum $\Delta t = 1.97$ seconds is low. It is therefore assumed, that for this sampling interval no appreciable aliasing has taken place.

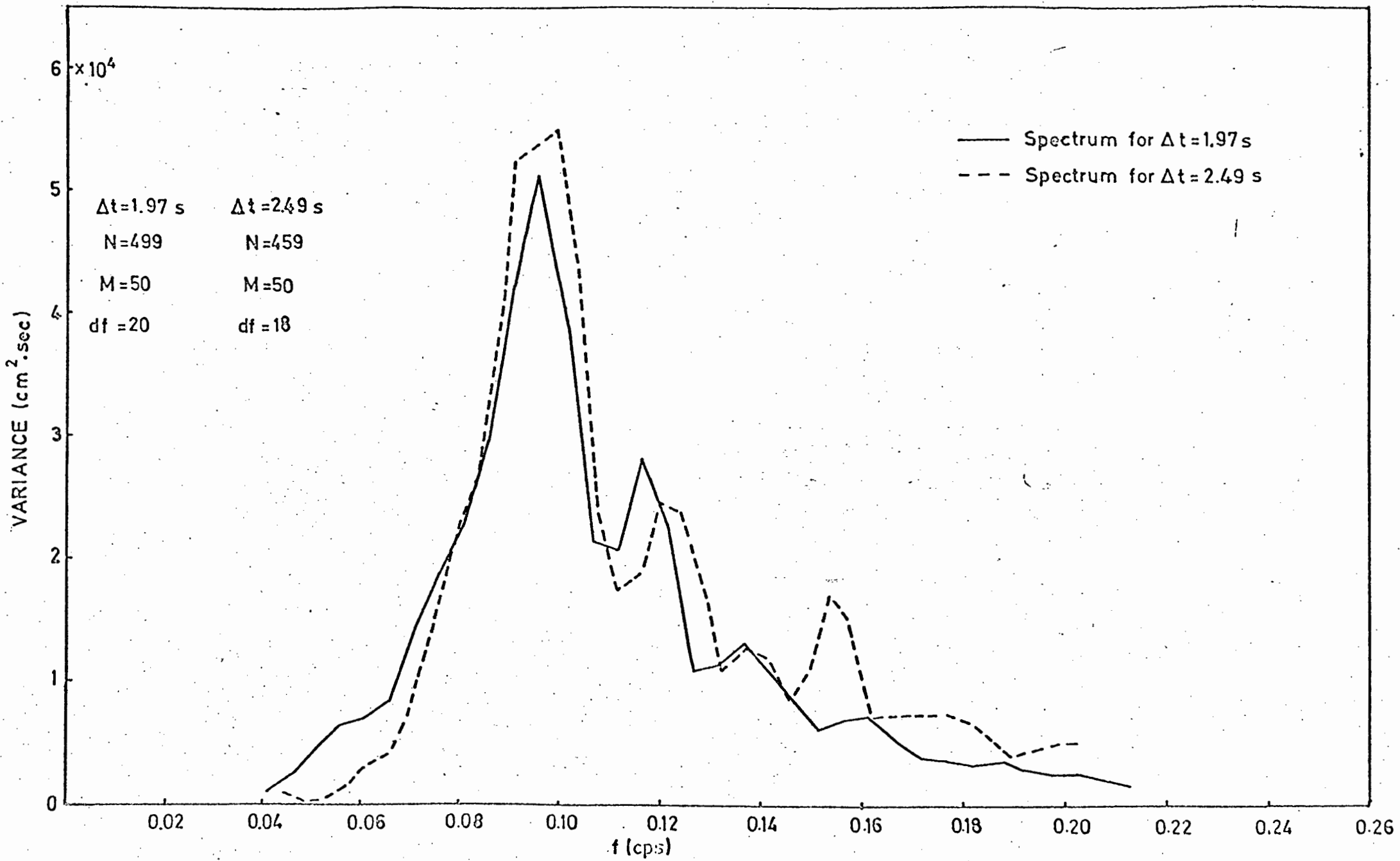


Fig 3.5 Spectra of record using different sampling intervals

PART II.

Chapter 4. WAVE SPECTRA IN SHOALING WATER.

4.1 Introduction.

When sea waves propagate into shallow water, a number of changes in the characteristics of the waves may take place.

First, the wave front will be affected by the bottom topography. As soon as the waves start to feel the bottom, which is approximately at the point where the bottom depth is equal to half the wave length, the waves are slowed down and a change in the direction of the wave front may result. Because this is a selective process, depending on the wave length, a spectrum of waves is generally selectively refracted. It is generally assumed that the wave energy contained between the orthogonals remains constant as the front progresses, and that there is no dispersion of energy laterally along the front (Ippen, 1966). Therefore as a result of refraction alone, the wave height will either increase or decrease, depending on whether the orthogonals converge or diverge.

Secondly, due to shoaling, the celerity of the wave decreases, the wave length decreases correspondingly, and assuming no change of energy otherwise, an increase in wave height must ultimately be the result.

Thirdly, if the wave front is interrupted by a structure, such as a breakwater, the portion of the waves incident to the structure will be reflected, or break. The waves moving past the structure into the region in the lee of the structure will diffract in approximately a circular arc with the amplitudes decreasing exponentially along this arc.

Lastly, the amplitudes of the waves may be decreased by bottom friction, bottom percolation, spilling of waves and turbulence effects. A brief outline of the important wave theories is given below.

4.2 Airy's wave theory.

According to Airy's wave theory, based on a small amplitude wave, the celerity C_0 may in deep water, as a first approximation be given by

$$C_0 = \frac{gT}{2\pi}$$

(4.1)

where T = period of the wave

g = acceleration due to gravity.

The wave length in deep water, L_0 , is given by

$$L_0 = \frac{gT^2}{2\pi}$$

(4.2)

It is assumed that the ratios $\frac{H}{h}$ and $\frac{H}{L}$ of the single oscillatory wave (where H = wave height, h = total depth of water), are small compared with unity.

The group velocity C_G , which is the velocity at which the wave train propagates, is given by

$$C_G = C \cdot \frac{1}{2} \cdot \left[1 + \frac{2kh}{\sinh 2kh} \right]$$

(4.3)

where C = wave velocity

k = wave number

In deep water, the ratio $C_G/C = \frac{1}{2}$ and approaches unity in very shallow water. The average energy E, which consists of equal parts of potential and kinetic energy, is given per unit surface area according

$$E = \frac{1}{8} \rho g H^2$$

(4.4)

where ρ = density of the water

g = acceleration due to gravity

When the relative depth $\frac{h}{L} < \frac{1}{20}$, the waves are termed "shallow water" waves, while if the ratio is greater than $\frac{1}{2}$, the waves are called "deep water" waves. For $\frac{1}{20} < \frac{h}{L} < \frac{1}{2}$ the waves are called "intermediate depth" waves.

In shallow water, the wave celerity C is given by

$$C = \sqrt{gh} \tag{4.5}$$

and the wave length L by

$$L = \frac{g T^2}{2\pi} \tanh \frac{2\pi h}{L} \tag{4.6}$$

The orbital motions of an Airy wave can be described by the horizontal and vertical velocity components u and w . At any depth z ,

$$u = \frac{agk}{\sigma} \frac{\cosh k(h+z)}{\cosh kh} \sin(kx - \sigma t) \tag{4.7}$$

$$w = -\frac{agk}{\sigma} \frac{\sinh k(h+z)}{\cosh kh} \cos(kx - \sigma t)$$

where a = amplitude of wave

σ = wave angular frequency $\frac{2\pi}{T}$

x = horizontal distance in direction of wave propagation

t = time

The paths of the particles are in general elliptical in shape. The horizontal (major) semi axis A of the ellipse and the vertical (minor) semi axis B of the ellipse are given by

$$A = a \frac{\cosh k(h+z)}{\sinh kh} \tag{4.8}$$

$$B = a \frac{\sinh k(h+z)}{\sinh kh} \tag{4.8}$$

As the shape of the orbit depends on the depth of the water h and the wave length L , changes in the shape of the orbits of the particles may be expected in shoaling water.

Assuming conservation of energy, the change of wave height of an Airy wave due to shoaling and refraction can be found from

$$\frac{H_1}{H_0} = \sqrt{\frac{1}{2n} \cdot \frac{c_0}{c_1} \cdot \frac{b_0}{b_1}} \tag{4.9}$$

where

- H_1 = wave height to which suffix 1 applies
- H_0 = wave height in deep water
- n = ratio of group velocity to phase velocity
- b_0 = distance between orthogonals in deep water
- b_1 = distance between orthogonals to which suffix 1 applies.

The change in wave height of an Airy wave, due to shoaling, refraction, bottom percolation and bottom friction is given by Bretschneider and Reid (1954).

4.3 Solitary and cnoidal wave theory.

In shallow water, due to the relative importance of $\frac{H}{h}$ and $\frac{H}{L}$, finite wave theories are often preferred.

The solitary wave theory applies to finite waves, lying wholly above the still water level. The motion of such a wave is not affected by preceding or following crests. The range in which the solitary wave theory applies is taken by Ippen (1966) as $\frac{h}{L} < \frac{1}{50}$

The wave celerity, as derived by Boussinesq, is given approximately by

$$C = \sqrt{g(h+a)} \tag{4.10}$$

where

a = crest height.

The average energy E, which consists approximately of equal parts of kinetic and potential energy, per unit crest width is given by

$$E = \frac{8}{3} \rho g h^3 \gamma \sqrt{\frac{\gamma}{3}} \tag{4.11}$$

where

$$\gamma = \frac{H}{h}$$

Muir Wood (1969) showed that according to Boussinesq's wave theory, the change of wave height of a solitary wave due to shoaling and refraction and assuming conservation of energy otherwise, is given by

$$\frac{H_1}{H_0} \approx \frac{H_0^{\frac{1}{3}}}{8.5 h_1} \left[\frac{L_0 b_0}{b_1} \right]^{\frac{2}{3}} \tag{4.12}$$

The cnoidal wave theory applies over the range $\frac{1}{50} < \frac{h}{L} < \frac{1}{10}$. This theory, which is not readily applicable, is described in terms of elliptic functions. The results of this theory show that limiting cases of cnoidal waves are similar to solitary and Airy waves.

Eagleson (1956), in laboratory studies, found the

small amplitude theory of Airy satisfactory for the prediction of celerity of the transforming waves, up to the point of breaking. The theory was also found to be applicable to the prediction of wave steepness. However, predictions for wave heights were found to be smaller than measured experimentally.

Munk (1949) carried out laboratory studies to test predictions of the solitary wave theory. Unfortunately, his observations exhibited a large degree of scatter. However, his overall impression was, that the solitary wave theory provides a useful tool for the study of waves in shallow water. Inman and Nasu (1956) showed, that the observed maximum horizontal orbital velocities in general compare more favourably with velocities predicted from solitary wave equations than from equations of Airy and Stokes. The measurements were made near the bottom in water depths ranging from about 5 - 15 feet and for wave heights as great as 7.5 feet. Calculations show that, apart from the Airy wave theory, the cnoidal and solitary finite wave theory sometimes apply to waves which might be present at the shallower stations in the test area.

4.4 Transformation of wave spectra.

A wave spectrum, however, normally consists of many waves having random phases and different characteristics. Existing wave theories, which consider a single wave only, may therefore be inadequate, as interaction between the various waves may be present. Wave spectra, in shoaling water may therefore, under certain conditions be quite different from spectra of the corresponding waves in deep water. It is therefore of theoretical interest, as well as of practical importance, to know the relationship between the deep and the shallow water spectra. Limited experimental work has been carried out so far. The results of the following two workers is thought to be important.

Walden (1961) measured wave spectra in 10 - 15 metres of water and compared the spectra obtained with various "theoretical" spectra. He found that the shallow water spectrum was surprisingly low in energy, in comparison with the "theoretical" spectra.

Higuchi and Kakinuma (1966) studied the changes of ocean wave characteristics near the coast of Japan, by photographing the movements of anchored buoys in 14 - 2 metres of water. Their spectra show that the location of peak energy as a function of frequency does not change as a result of shoaling.

Unfortunately, no theory which takes into account all the possible causes of spectrum transformation is known at present. Longuet-Higgins (1957) gives a theoretical discussion of the refraction of a continuous spectrum of short-crested sea waves in shallow water. His results show that there is a change of the mean length of the crests. Neumann, Pierson and James (1953) showed, that by marking off the deep water spectrum into a number of frequency bands, the shallow water spectrum can be computed. However, their treatment does not incorporate the possible effects of bottom friction and percolation.

Putnam and Johnson (1949) developed a theory for the loss of wave energy as a result of friction by the oscillating motion of waves at the sea bottom. They showed numerically that the loss of energy for Airy waves could be expected to cause a reduction in the wave height amounting to as much as 30 per cent on very flat beaches (1 : 300) for wave periods commonly occurring in the ocean. The per cent reduction was found to be independent of the wave period.

A theory to account for the loss of wave energy as a result of percolation was developed by Putnam (1949) for Airy waves. He showed by numerical examples that the reduction may be of the order of ten per cent for very flat beaches (1 : 300).

Bretschneider (1954), using pressure head wave recorders, investigated field measurements in the Gulf of Mexico in 10 - 40 feet of water. From the actual reduction in wave energy, using significant waves, the friction factors were computed. Percolation losses were not considered. As a result of his study, it was concluded that to estimate the heights of waves in shallow water a friction factor $f = 0.01$ must be used.

Bretschneider and Reid (1954) investigated theoretically the transformation of waves in shoaling water due to refraction, shoaling, bottom friction and percolation. Example computations, using $f = 0.01$ are given for an actual situation in the Gulf of Mexico.

Iwagaki and Kakinuma (1963) used a pressure head wave recorder, to investigate energy losses due to bottom friction in water depth ranging from about 15 - 3 metres. These measurements were made off the Akita Coast, Japan. Using the significant height and period, the mean of the estimated bottom friction factors was found to be 0.057.

4.5 Change of wave height of a progressive oscillatory wave of small steepness due to shoaling, refraction, bottom friction and bottom percolation.

Whenever a progressive oscillatory wave of small steepness progresses from deep to shallow water, the wave height in shallow water may, based on the classical wave theory, according to Bretschneider and Reid (1954), be found from :

$$H = K \cdot K_r \cdot K_s \cdot H_0 \tag{4.13}$$

H = wave height in shallow water

K = total wave height reduction factor due to energy dissipation

K_r = refraction factor

K_s = shoaling factor

H_0 = wave height in deep water.

It is assumed that the addition of energy by surface winds, dispersion of energy by diffraction, reflection of energy by a steep slope, and loss of energy by breaking are absent. Since the wave height reduction factor K due to energy dissipation is dependant on refraction, the factors K and K_r may be combined by introducing a new factor K_{pfr} , which is the product of K and K_r . Expression (4.13) may then be written as

the wave height transformation of a simple wave train of given period due to the factors mentioned.

4.6 Observed wave spectra in shoaling water near Melkboschstrand.

4.6.1 Description of the test area.

The area lies approximately 12 nautical miles due north of Cape Town, off Melkboschstrand (Atlantic Coast). Fig. 4.1 shows the area with the stations. The area has a very regular bottom topography and the bottom contours run almost parallel to the coastline. Nautical Charts (S.A. 16 and E 4171 - PRESS 11c) show that the bottom consists mostly of sand, with isolated rock outcrops. The slope of the bottom is very gentle and uniform (1 : 150) and is shown in Fig. 4.2. It represents the profile of the bottom from Ou Skip Rock in a direction 235 degrees from that point.

4.6.2 Experimental procedures.

In order to study the properties of wave spectra in shoaling water, recordings were made along lines perpendicular to the shore as shown in Fig. 4.1. The recording time at each station was never less than 20 minutes, the time lag between the recordings was kept as small as possible. No recordings were made while the ship was under way. At each station the wind direction and velocity and the swell approach was measured from a height of about 26 feet, using a Gyro compass and a hand cup anemometer. The air and sea water temperatures were also recorded. Small differences in sea water temperatures between the stations were observed. The average temperature of the local sea water was about 13.0°C. At each station the sea current was measured and the results show that no strong tidal currents exist and that the current velocity is generally less than 25 cm/sec.

Much of the wave data collected still need to be analysed. The following analysis covers data only from 4 cruises. The selection was based on wave and weather conditions. The data recorded on TBO 29 and TBO 36 were recorded under quite calm conditions. The data from TBO 40 were obtained under fluctuating wind conditions, while slight to

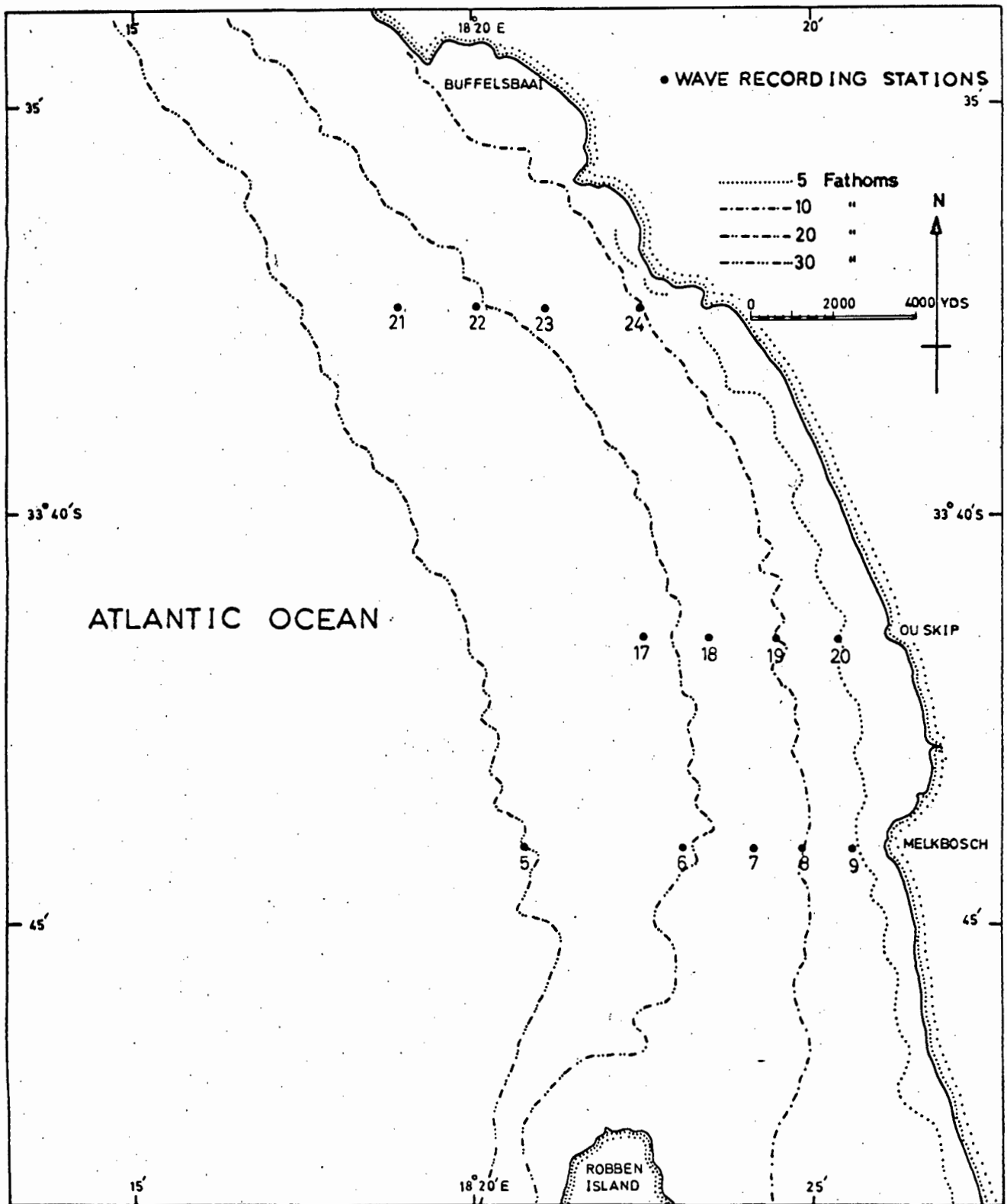


FIG 4.1

MELKBOSCH STRAND AREA

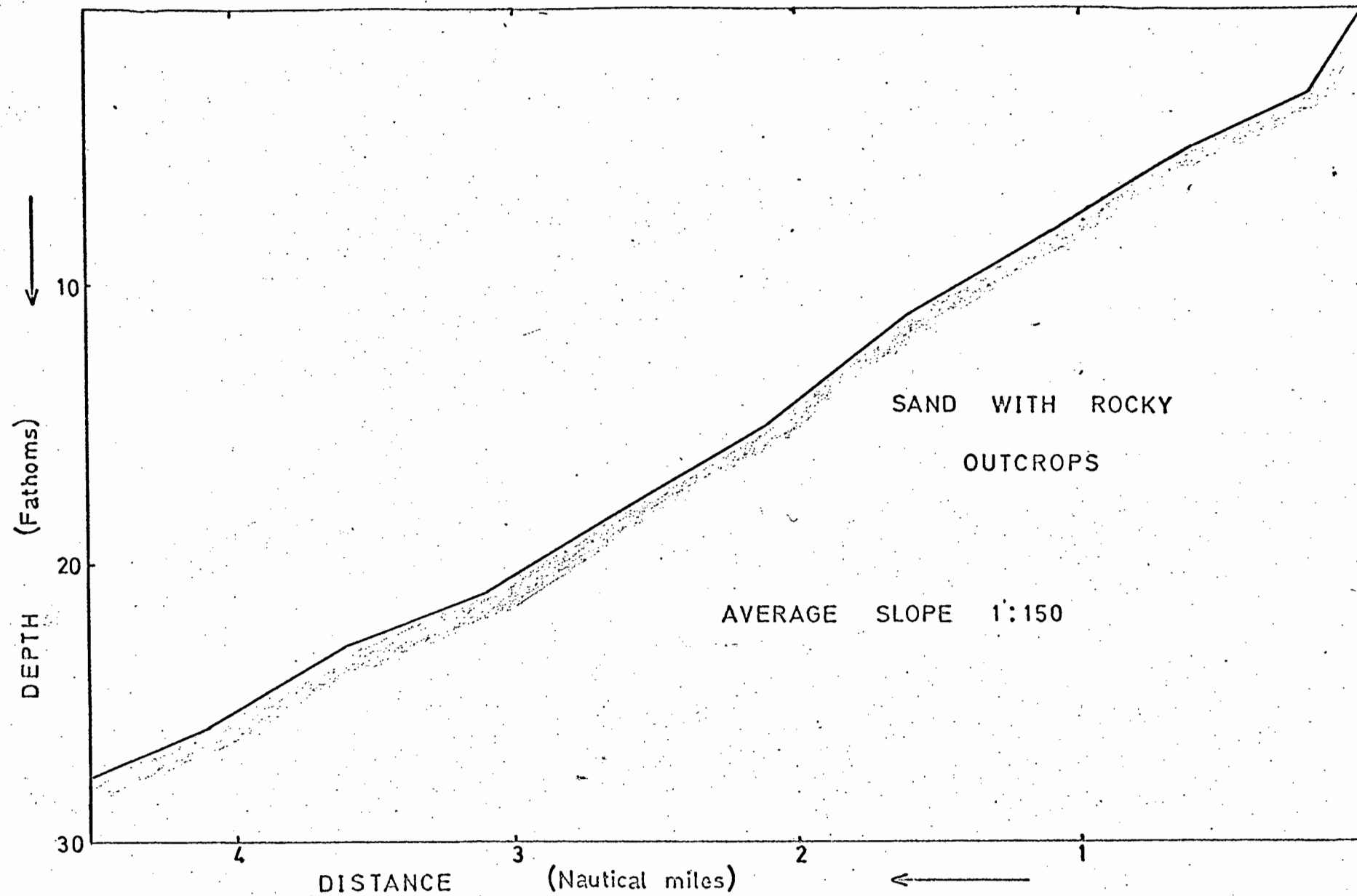


Fig. 4.2 BOTTOM PROFILE NEAR MELKBOSCH STRAND.

moderate winds prevailed on TBO 42. The conditions under which the recordings were made are tabulated in Table 6. The calculated variances and the chi-square values are also shown. The resulting spectra are graphically presented in Figures 4.3, 4.4, 4.5 and 4.6. The variances are expressed in $\text{cm}^2 \text{ sec}$, as a result of dividing the smoothed estimated variances of the frequency bands by the frequency band width $\frac{1}{2 m \Delta t}$.

4.6.3 Experimental results.

Fig. 4.3 shows that the spectra obtained from TBO 29 for stations 5 and 7 are very similar. The peak energy for both stations is found at 0.096 cps. The peak energy for station 6 is located at 0.085 cps. However, in view of the results obtained from TBO 36, 40 and 42, this difference is believed not to be significant.

Fig. 4.4 shows the spectra from TBO 36 for stations 17, 18, 19 and 20. The spectra form a nested family of curves with all the peaks located at approximately 0.07 cps. The spectra in Figures 4.3 and 4.4 are sharply peaked and were recorded under very calm conditions. These spectra are therefore thought to be swell spectra. This possibly also explains the non-Gaussian character of the waves at stations 5 and 19.

Fig. 4.5 shows doubled-peaked spectra with the primary peaks located at approximately 0.12 cps and the secondary peaks at approximately 0.08 cps. The primary peak, located at the higher frequency probably reflects the local sea waves, and the secondary peak is believed to be caused by background swell.

Fig. 4.6 shows the nested family of spectra for TBO 42. The primary peak energy is found at approximately 0.10 cps. It is thought that the spectra are basically swell spectra as the local winds were weak to moderate and variable.

4.7 Non-preferential attenuation of the wave heights.

For each line of stations, the spectra show a systematic decrease of peak energy with depth. This is reflected in the corresponding changes of the total variances of the spectra as shown in Table 6.

Table 6.

Calculations of variance and chi-square values from wave records
under stated conditions for waves in shoaling water.

station.	5	6	7	17	18	19	20	21	22	23	17	18	19	20
depth (m).	59	44	24	44	31	16	13	48	42	35	45	33	15	12
N.	619	599	499	720	720	720	560	680	680	680	720	760	760	660
M.	60	60	50	70	70	70	60	70	70	70	50	50	70	70
variance (cm ²).	3.9	3.6	2.3x10 ³	6.5	5.2	4.0	4.4x10 ³	11.0	8.6	6.0x10 ³	13.2	12.3	8.5	6.7x10 ²
chi-square value.	34.4*	6.3	10.4	3.5	5.4	35.4*	5.8	4.0	9.0	4.8	13.0	3.4	10.0	11.0
wind vel. (m/s).	nil	nil	nil	nil	nil	nil	nil	18	13	17	2	3	nil	2
wind dir.	-	-	-	-	-	-	-	310°	290°	290°	330°	325°	-	295°
wave dir.	230°	250°	260°	240°	240°	245°	240°	310°	290°	290°	210°	210°	215°	230°
time.	10.00	10.30	11.00	12.26	11.48	11.10	10.38	16.49	15.55	15.04	11.16	10.37	9.53	09.24
date.	12.12.1968			9.7.1969				8.10.1969			18.12.1968			
TBO.	29			36				40			42			
spectrum.	Fig.4.3			Fig. 4.4				Fig. 4.5			Fig. 4.6			

* Significant at 5% critical level of chi-square.

I.B.O. 29

- St. 5 (59m)
- St. 6 (44m)
- - - - St. 7 (24m)

12×10^4

Variance (cm².sec²)

f (cps)

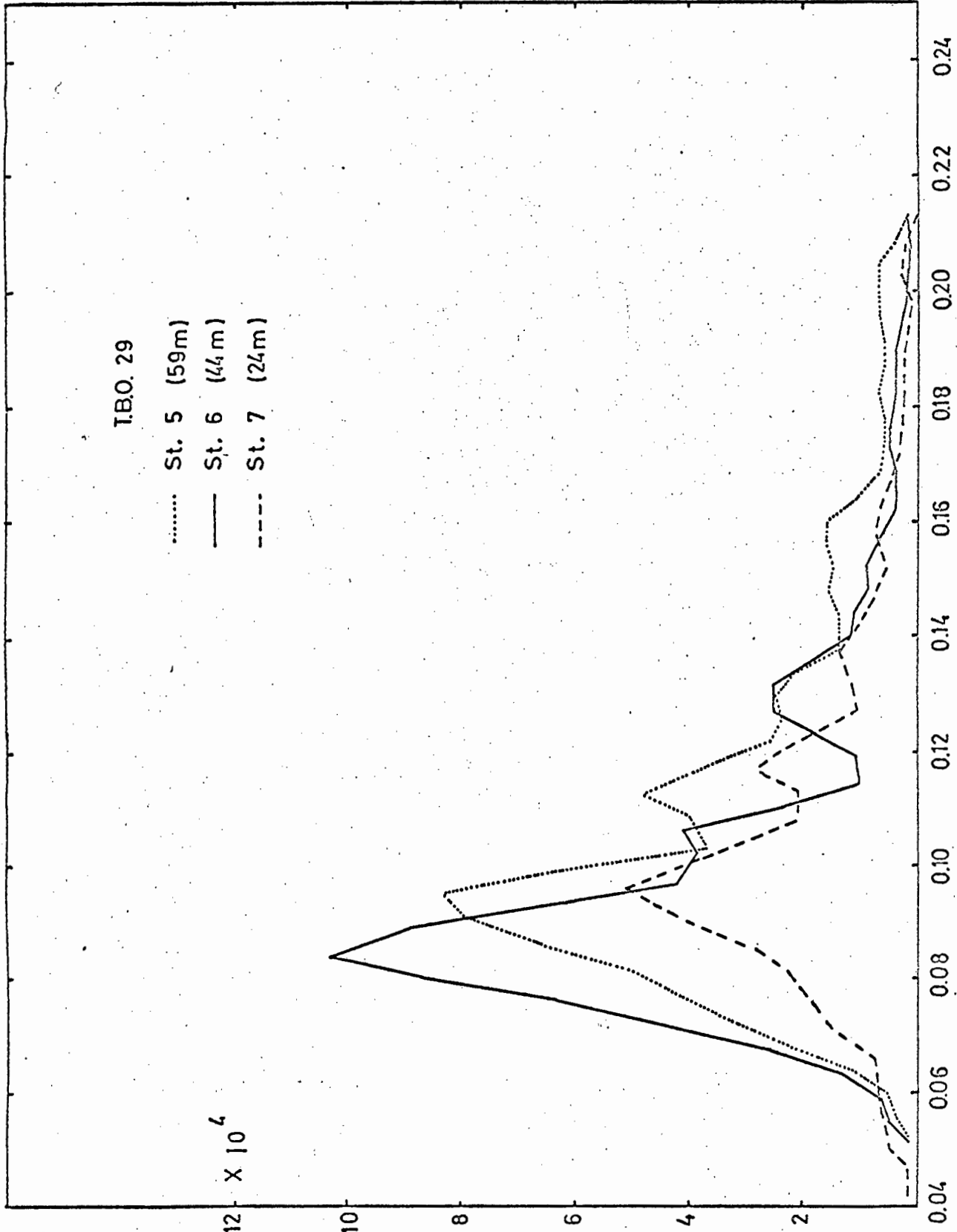


Fig 4.3 COMPUTED WAVE SPECTRA STATIONS 5, 6, 7

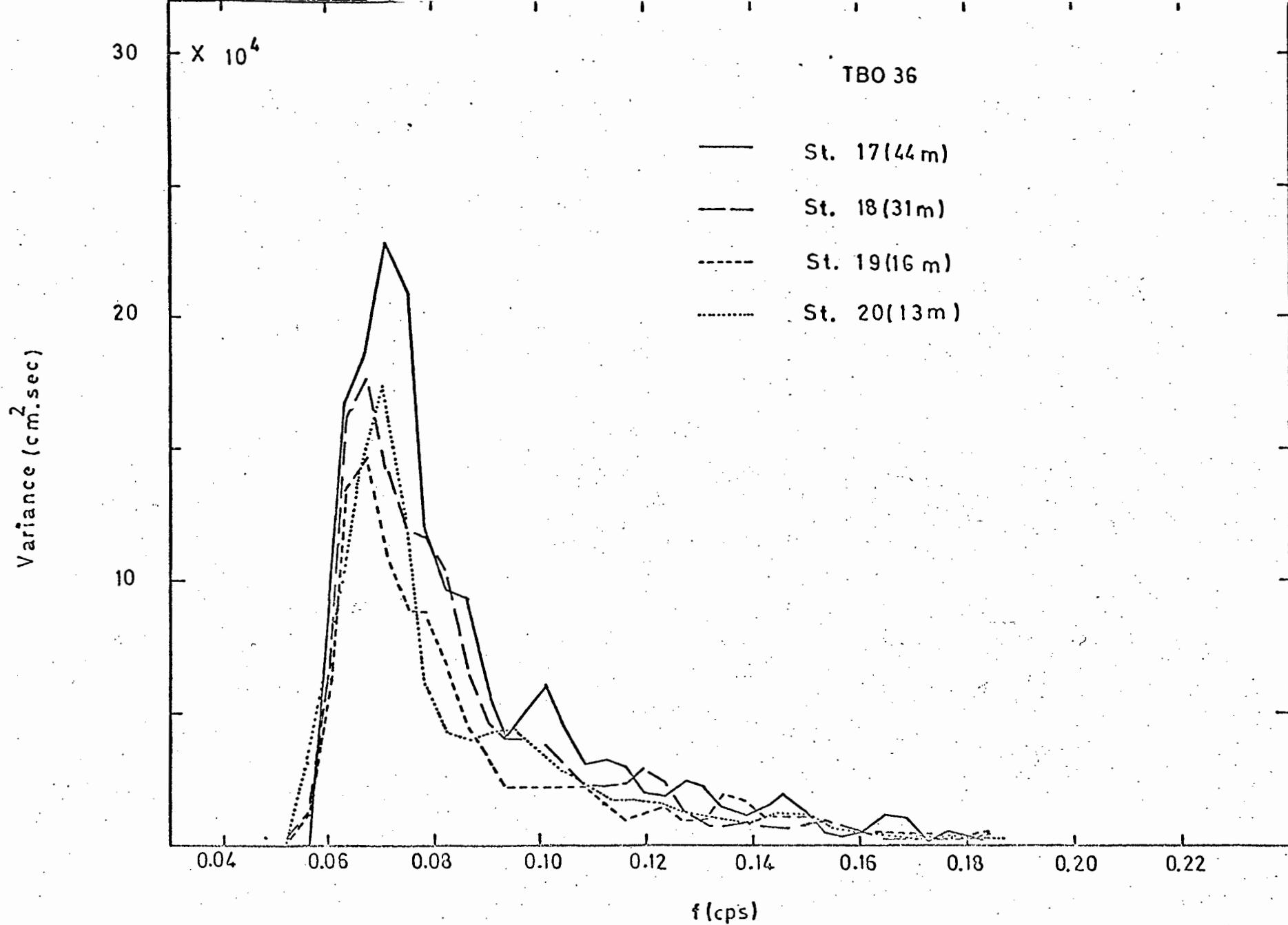


Fig 4.4 COMPUTED WAVE SPECTRA STATIONS 17, 18, 19, 20

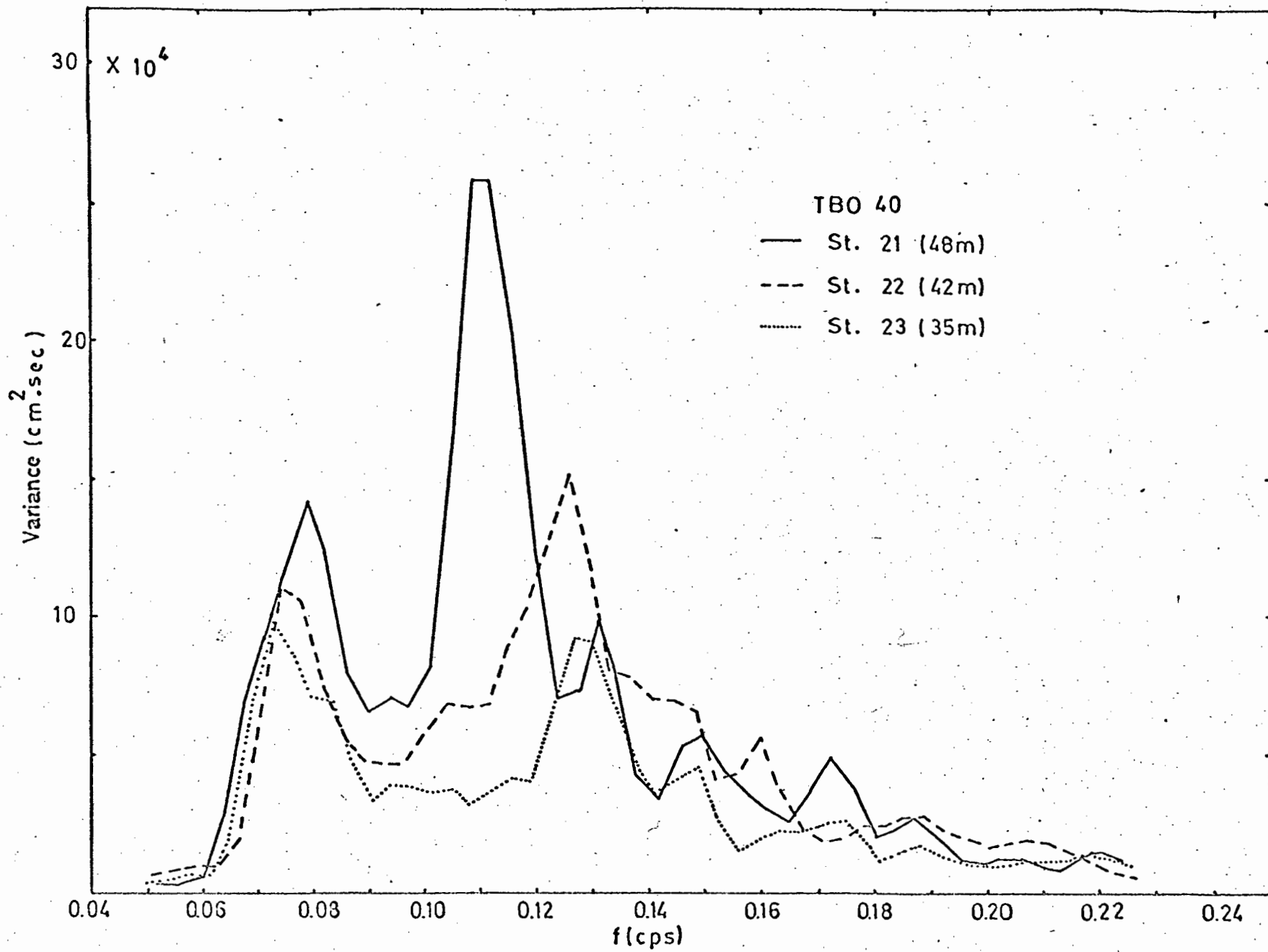


Fig 4.5 COMPUTED WAVE SPECTRA STATIONS 21, 22, 23.

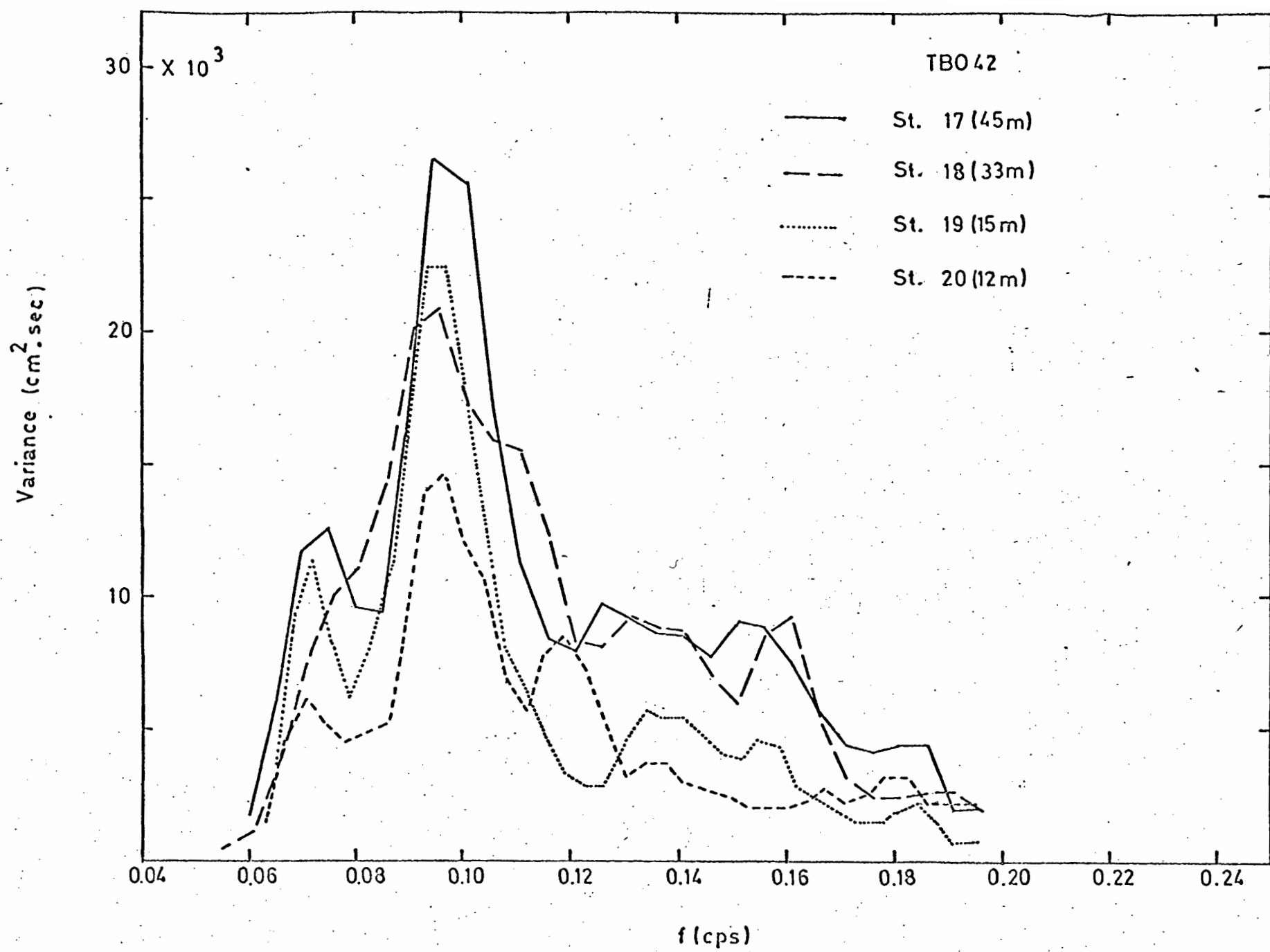


Fig 4.6 COMPUTED WAVE SPECTRA STATIONS 17, 18, 19, 20.

In order to investigate whether there is a preferential attenuation with frequency of the wave heights, the spectra obtained were normalized by dividing the smoothed estimated variances by the total variance of the spectrum (Figures 4.7, 4.8, 4.9 and 4.10). The normalized spectra shown in Figures 4.8 and 4.10 are very similar. The peak of the spectrum for station 6 (Fig. 4.7) and of the spectrum for station 21 (Fig. 4.9) show a slight shift to lower frequencies. However, in general it is concluded that the resulting spectra do not show a systematic selective attenuation of the wave heights with frequency.

In order to investigate the systematic changes in the total variances as shown in Table 6, the influence of the factors which may contribute to these changes is next considered.

a. Non-stationarity. It is thought that the waves recorded on TBO 29 and TBO 36 were reasonably stationary, as calm weather prevailed and the time differences between the recordings were always less than two hours. For TBO 40 and TBO 42 the situation is different, because of the variable winds. Some of these data might therefore be non-stationary.

b. Diffraction and reflection. Figure 4.1 shows that the effects of diffraction and reflection may be neglected, as no obstacles obstruct the path of the waves and the slope of the beach area is only 1 : 150 (Fig. 4.2).

c. Percolation. This represents the energy losses through actual infiltration of the wave motion into the semi-fluid mass of certain finely suspended sediments of the permeable sea bottom. Evidence from the charts shows the sea floor in the area to be sand with occasional rock outcrops. In situ inspection on two occasions showed the sand to be firm. It is therefore assumed that percolation will not be a significant factor under the stated circumstances.

d. Wave spilling. No evidence of wave spilling was noted on any of the cruises, except possibly on TBO 40. It is therefore unlikely that this might be a source of energy loss.

e. Shoaling, Refraction and Bottom friction. All the spectra obtained will, to an extent, depending on the wave

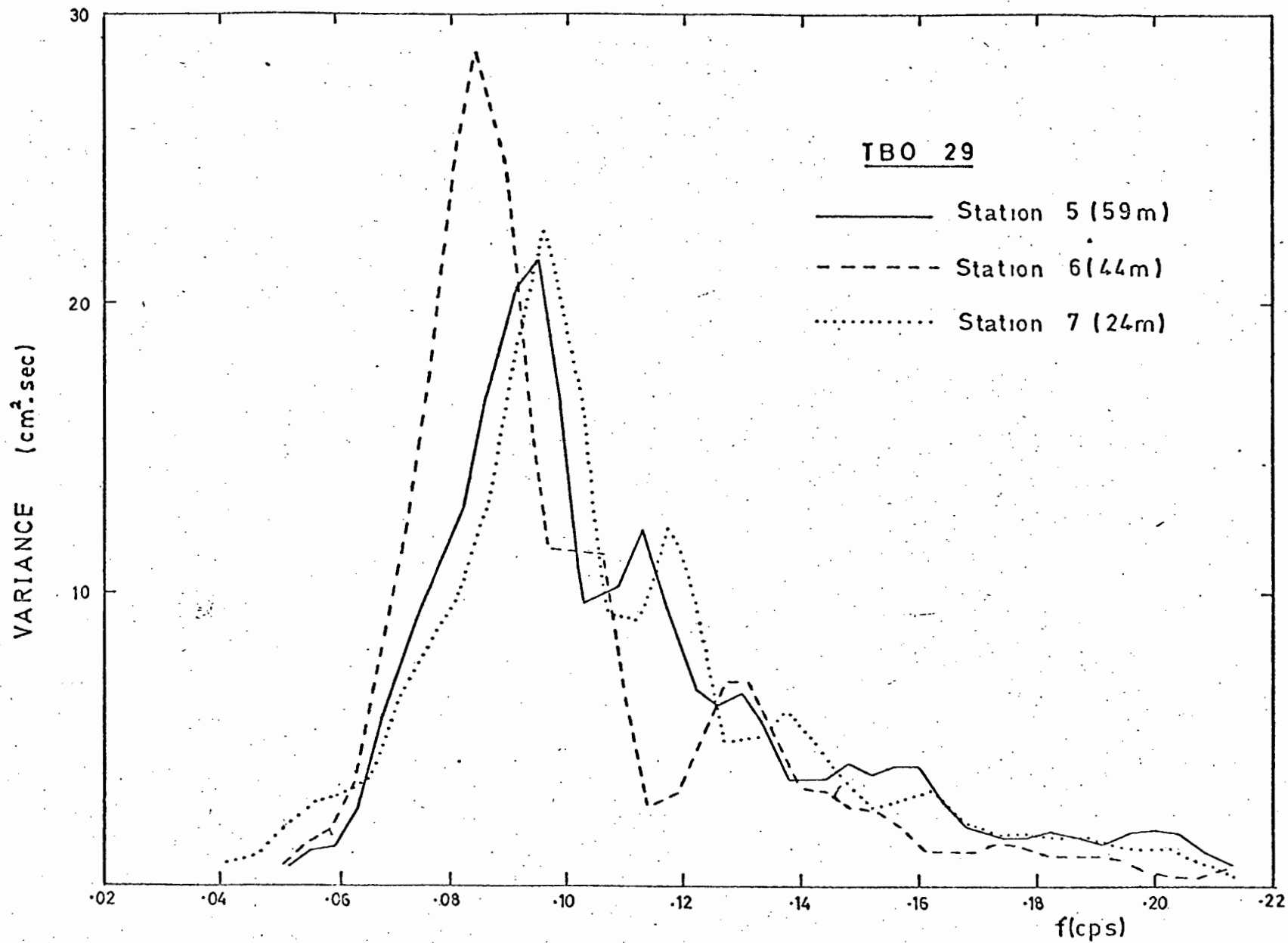


FIG. 4.7

NORMALIZED WAVE SPECTRA, STATIONS, 5, 6, 7.

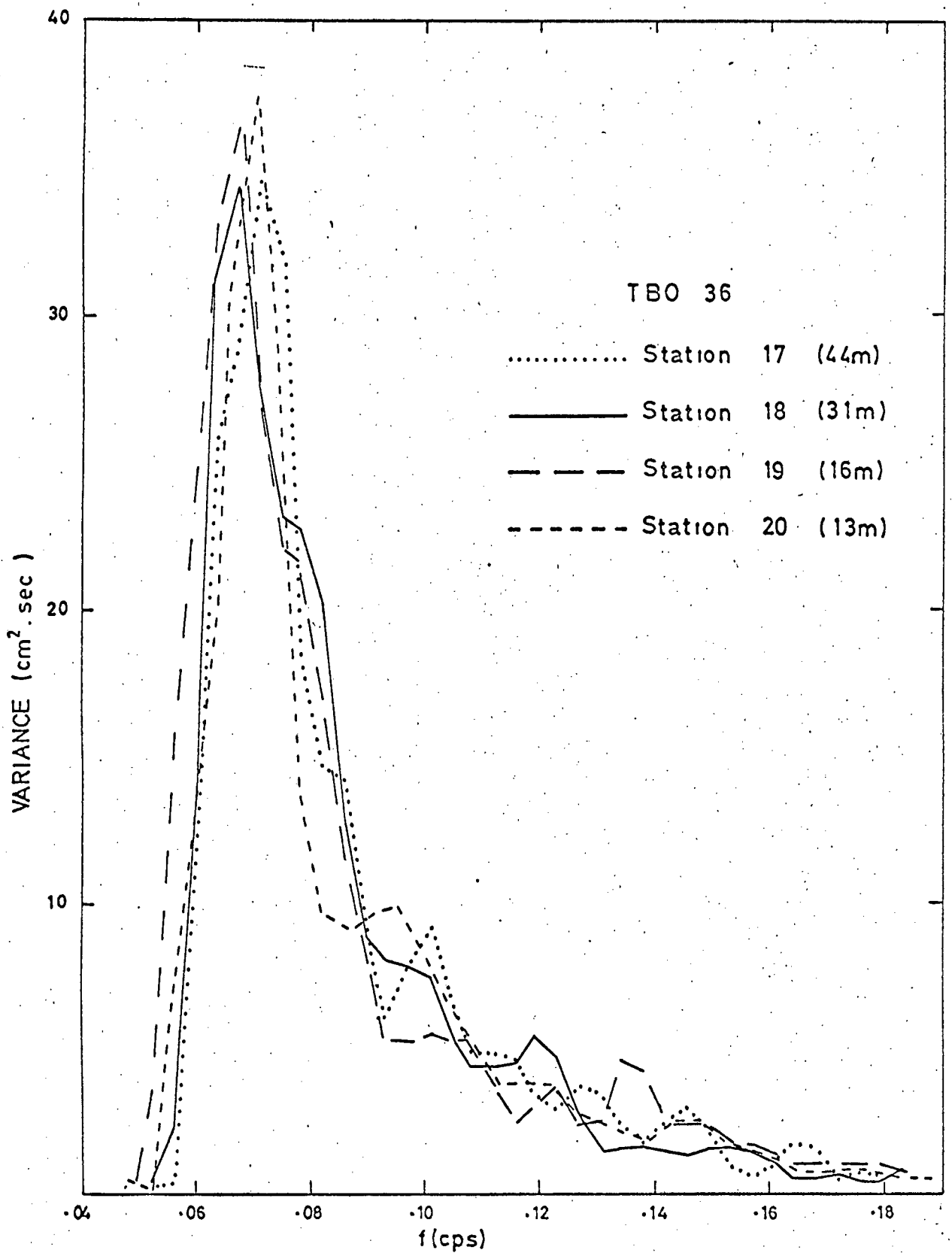


FIG 4.8 NORMALIZED WAVE SPECTRA.
STATIONS, 17, 18, 19, 20,

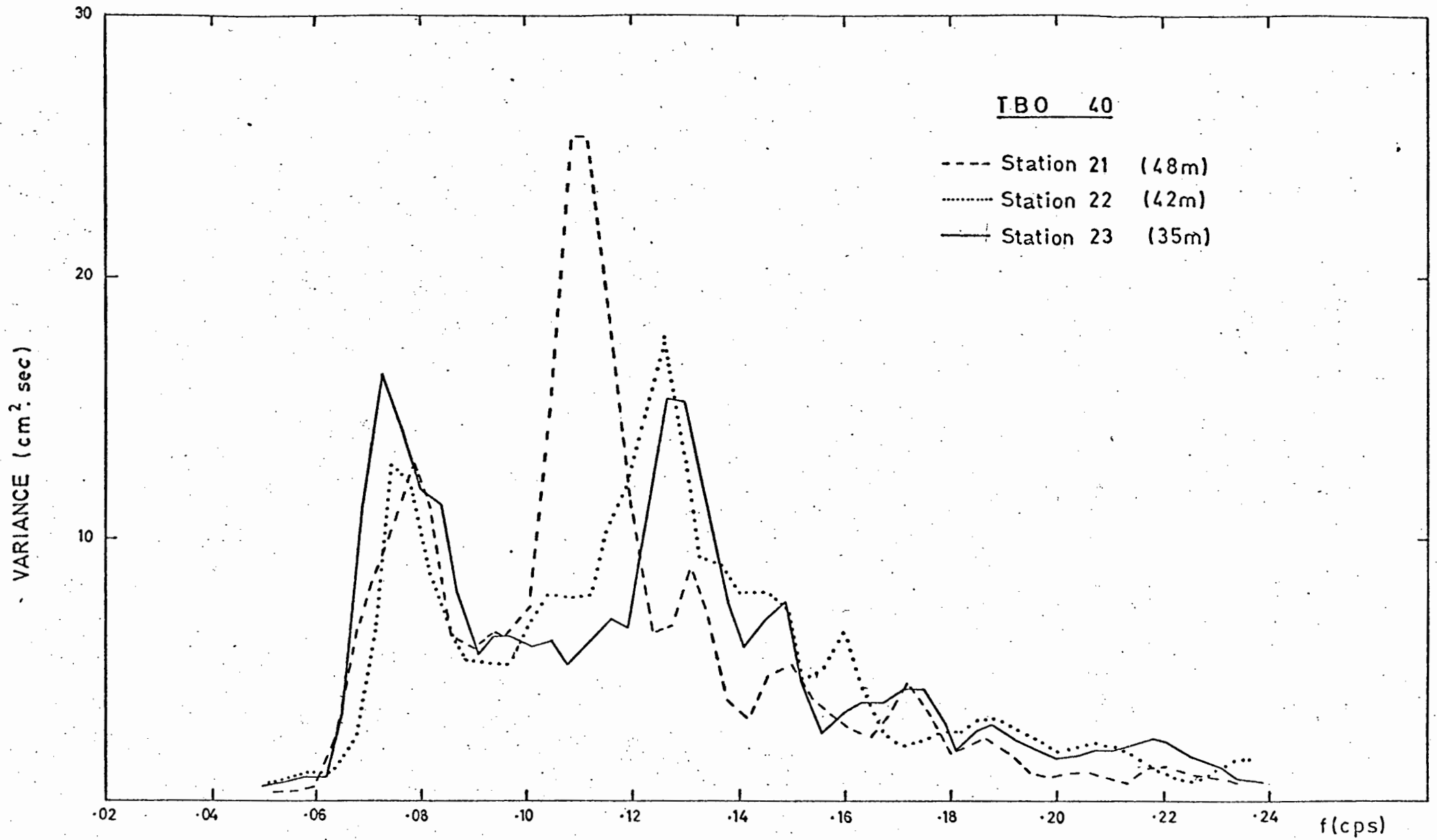


FIG 4.9 NORMALIZED WAVE SPECTRA Stations 21,22,23.

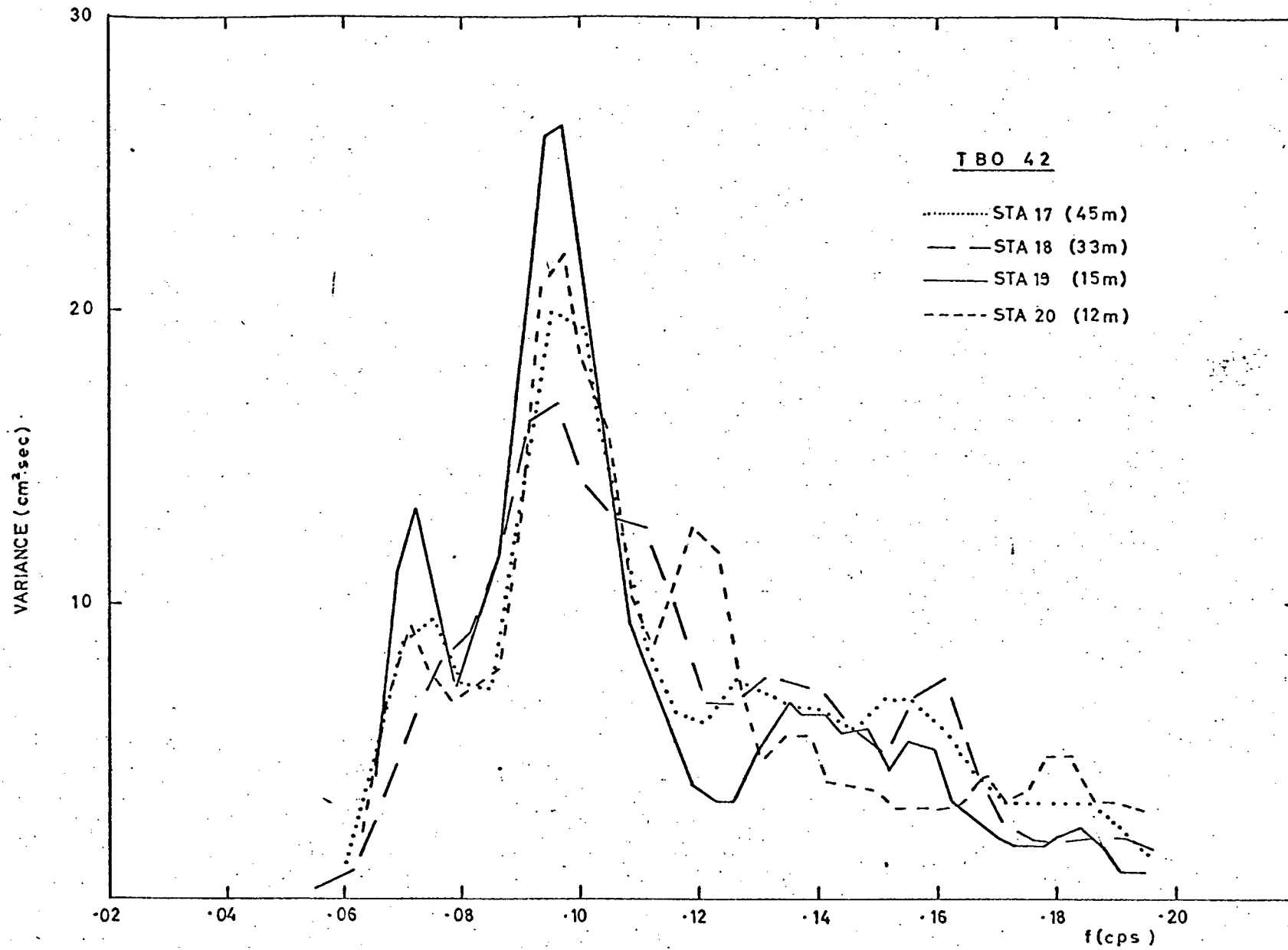


FIG 4.10 NORMALIZED WAVE SPECTRA STATIONS 17, 18, 19, 20.

characteristics, the wave approach and the bottom, be affected by these factors.

In view of the above mentioned considerations it is therefore concluded that for the spectra obtained from TBO 29 and TBO 36 shoaling, refraction and bottom friction are the only important factors which may account for the changes in variances as listed in Table 6. From a knowledge of the shoaling factor K_s , the refraction factor K_r , the wave height H , the period T at two or more positions along the wave path, and the bottom slope m , the friction factor f can be calculated.

As wide differences exist between the theoretical values of the bottom friction, based on the small amplitude wave theory and the experimental values measured directly and indirectly (Iwagaki, Tsuchiya and Sakai, 1965), calculations to determine the friction factor for the area were undertaken.

4.8 Calculation of friction factor from wave data.

According to Iwagaki, Tsuchiya and Sakai (1965), in the absence of percolation, the bottom friction factor f can be approximated by

$$f = \frac{\left\{ H_1 (K_r K_s)_2 / H_2 (K_r K_s)_1 \right\} - 1}{\frac{H_1}{(K_s)_1 m T^2} \cdot \frac{1}{2} \left\{ 1 + \frac{(K_r)_2}{(K_r)_1} \right\} \left(\frac{h}{T^2} \right)_1 \int \left(\frac{h}{T^2} \right)_2 \phi_f d \left(\frac{h}{T^2} \right)} \quad (4.16)$$

Theoretically, (4.16) is only valid for a small amplitude wave, having a single wave height H and a period T . In practice, the significant wave height and period are often used.

Unfortunately, no single sine wave can be extracted from a wave spectrum, as the estimation of the height of such a wave could be grossly in error due to the width of the confidence band of the spectrum. For this reason, a different approach was used. For the periods T , the periods corresponding to the peaks of the spectra were used. If the spectra differed slightly in the position of the peak energies, an average value for T was taken.

About the chosen periods, small period bands were selected in such a manner that they contained most of the energies of the peaks. By measuring the areas under the spectrum within the selected period band, the integrated variance for a particular peak of a spectrum was obtained. From the variance obtained, the average wave height \bar{H} was calculated by making use of the relationship for a narrow spectrum (Dorrestein, 1963).

$$\bar{H}^2 = 8.0 \text{ variance}$$

(4.17)

In order to obtain the refraction factors K_r , refraction diagrams, using the wave front method, were constructed for TBO 29 and 36 (Fig. 4.11, 4.12). The periods corresponding to the peaks of the spectra were used. Table 7 shows, among other relevant data, the average periods T , the period bands, the variances and the corresponding average wave heights \bar{H} for TBO 29 and 36. The data listed in Table 7 were used in expression (4.16) and the integral part of the expression was evaluated by making use of the graphical solution by Bretschneider and Reid (1954).

The friction factors f obtained from data from stations 5 and 7, 17 and 19, 17 and 20 are respectively 0.38, 0.19 and 0.10, having an average value of $f = 0.22$. No attempt was made to calculate the friction factors for adjacent stations as better results may be expected by using stations for which the differences in variances are as large as possible.

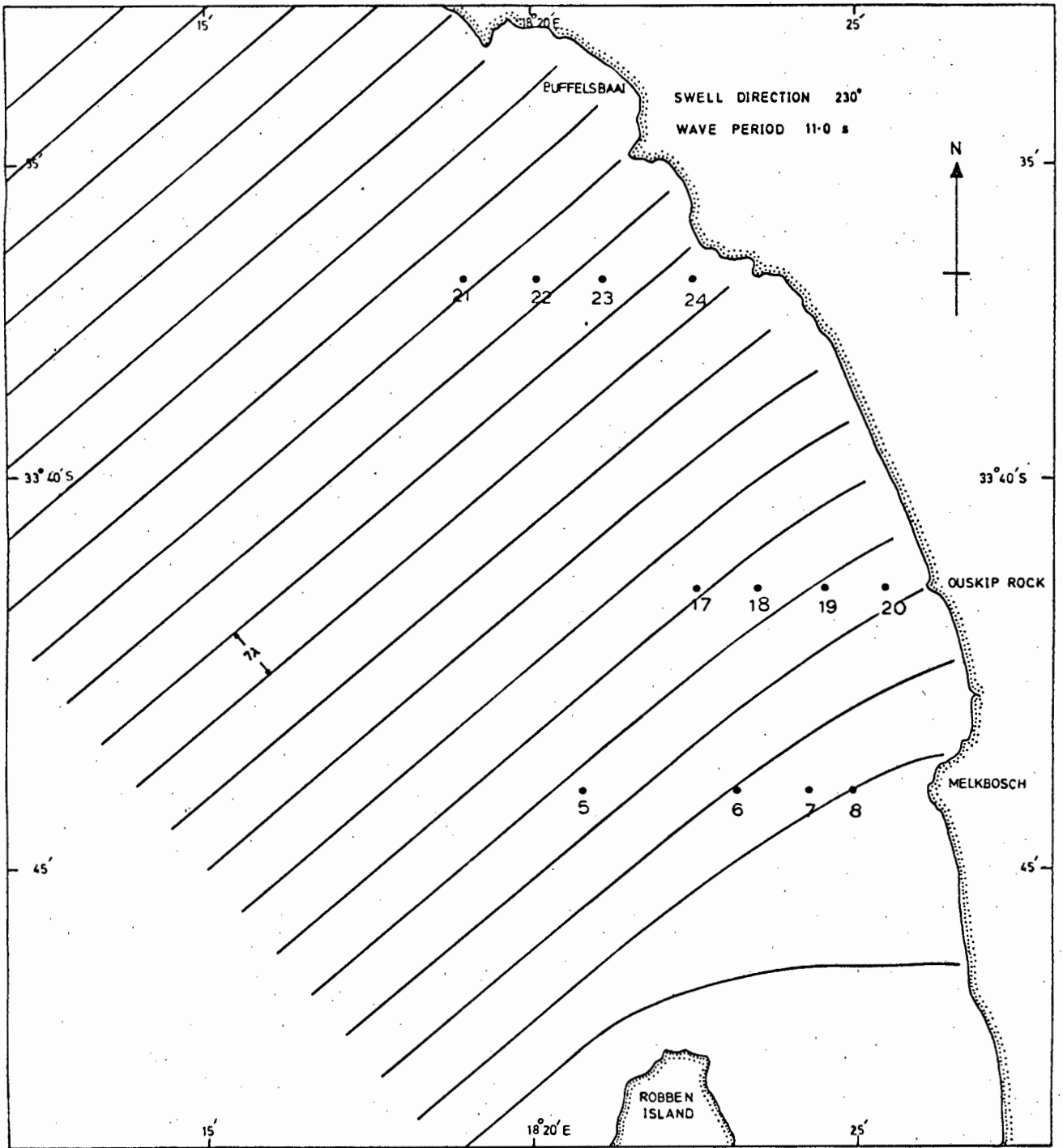


FIG 4.11 WAVE REFRACTION DIAGRAM (ORTHOGONALS)

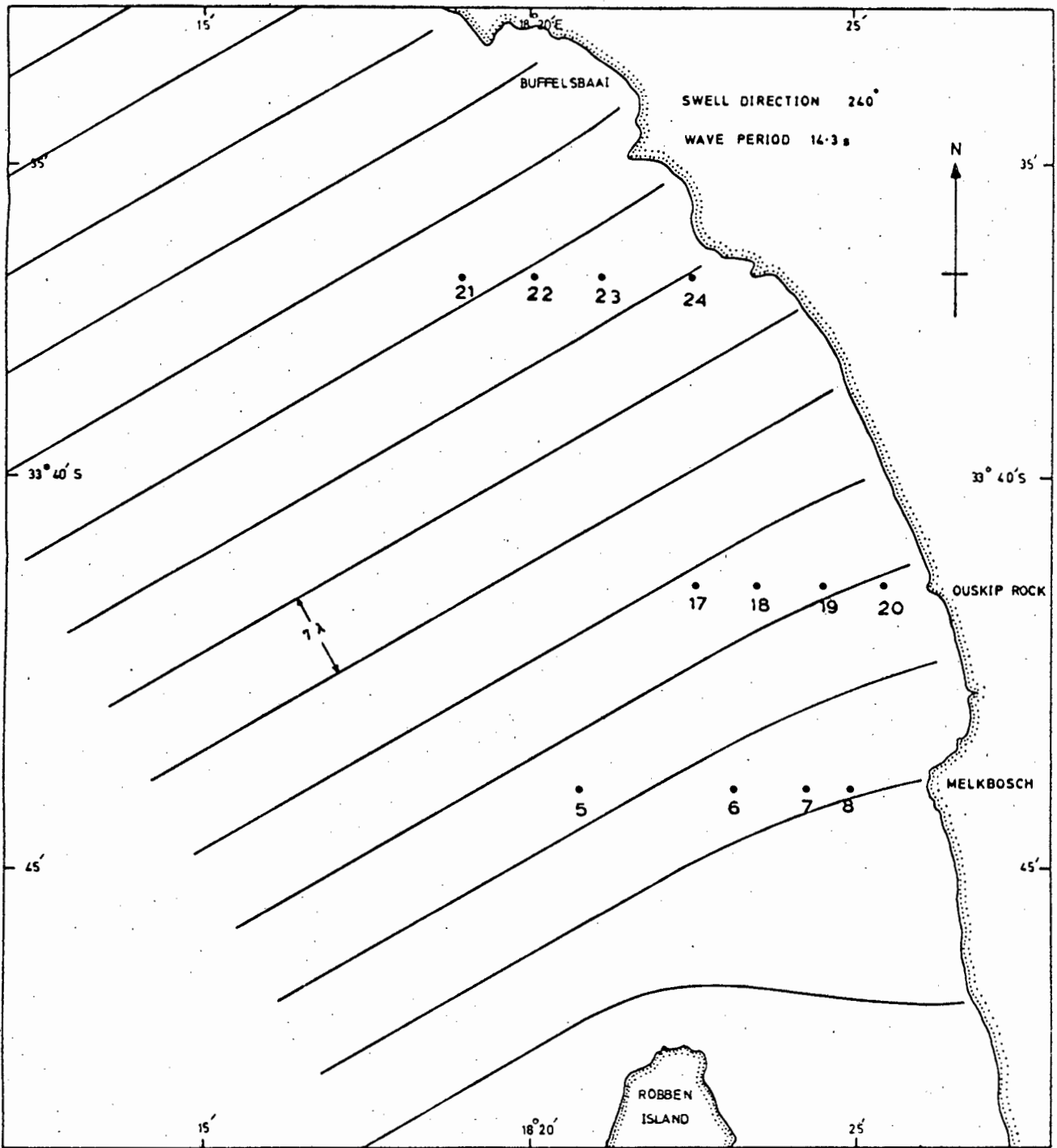


FIG 4.12 WAVE REFRACTION DIAGRAM (ORTHOGONALS)

Table 7.

Data used in the calculation of friction factors.

	TBO 29		TBO 36		
	230°		240°		
swell approach.	230°		240°		
average period (sec).	11.0		14.3		
period band (sec).	9 - 13		11.1 - 17.5		
bottom slope.	1 : 150		1 : 150		
station.	5	7	17	19	20
variance (cm ²) X 10 ² .	19.5	11.5	36	22.5	24.5
average wave height (m).	1.25	0.96	1.70	1.34	1.40
refraction factor.	1	0.900	1	0.973	0.932
shoaling factor.	0.9526	0.9176	0.9151	1.023	1.059

Chapter 5. DISCUSSION OF RESULTS AND SUGGESTIONS FOR FUTURE RESEARCH.

5.1 Reliability of technique.

For the interpretation of the results obtained, it is of importance to know the limitations of the recording instrument, and the extent of possible errors introduced by the processing of the data.

The limitation of the N.I.O. recorder can be seen from the computed correction curve for the instrument (Fig. 2.1). The period range for which the instrument apparently records most satisfactorily is from about 5 - 20 seconds. Therefore, if higher or lower periods are of importance, a different type of recorder must be used. As the actual performance of the instrument may be influenced by the ship's characteristics, it is possible that the correction curve used, may be slightly inaccurate. The instrument was regularly tested electronically and in situ, and on the whole, it has performed in a very satisfactory manner. However, on several occasions, it was noted that the speed of the recorder may vary slightly, and because the analysing technique is rather sensitive for small errors in timing, a stopwatch was always used to determine the correct time interval of the recordings. The wave heights were read off to the nearest 0.1' and it is estimated that the sampling interval obtained by using the simple read-off device was accurate to approximately 5%. As the read-off line during the digitizing of the data almost touched the record, error due to parallex was found to be very small. The overall error involved in digitizing the data was therefore small.

The data were not prewhitened. Prewhitening is sometimes applied to reduce error introduced by the recording and digitizing of the data. The nature of this error is thought to be "white" and can therefore be removed by selecting a filter with a carefully chosen frequency response function. However, as the nature of the error was unknown under the circumstances, no attempt at prewhitening has been made.

Due to roughness introduced into the calculations from the digital form of the data, the "raw" estimates were smoothed by "hanning" as shown by Blackman and Tukey. Other ways of smoothing

the data are available and the best way to smooth the data is still open to discussion (Jenkins, 1961).

5.2 Normality of data.

The wave generating areas for South African waters usually lie to the south of the continent. They are mainly associated with the passing of cyclones, which according to weather maps and satellite pictures move in quite a regular path from the South Atlantic to the Indian Ocean. In winter, the waters around the coast reflect the passing of many cold fronts, which are associated with the cyclones. In summer, fewer cyclones are formed and very few actually pass over the continent. This is reflected in findings by Molly Darbyshire (1966). She showed that the most frequent swell along the south coast of South Africa is from the south west.

In view of the fact that the generating areas often lie a great distance from the coast, it is expected that, particularly under calm summer conditions, the local waves will be stable, consisting of a narrow band of frequencies.

To ascertain that the technique of power spectrum analysis will, under these conditions, be applicable in the local waters of South Africa, tests for the Gaussian properties of the waves have been carried out.

First, the minimum sampling interval for this test was determined and found to be of the order of 40 seconds. This result is in good agreement with the result obtained by Mac Kay (1959). His value is 37.5 seconds.

Secondly, the minimum sampling length to be used for a reliable outcome of the chi-square test was determined (20 minutes).

The total percentage of the records found significant at the 5% critical level of chi-square (3 out of 23), is slightly higher than observed by Mac Kay (1 out of 16). The Gaussian properties of the deep water stations compared with those of the corresponding shallow water stations were found not to be significantly different.

Further tests for normality (Table 6.) showed that out of the 14 records tested, 2 must be rejected at the 5% critical

level. Tables 4, 5 and 6 show that the records rejected are associated with calm to moderate winds.

Although the technique of spectral analysis appears to be applicable in the local waters, it seems that caution is necessary, especially under calm weather conditions.

However, sea waves can, of course, never be strictly Gaussian. Kinsman (1965) points out, that waves are limited in height and therefore very high waves cannot exist. Secondly, from theoretical considerations, modifications of the Gaussian structure must occur in the high frequency components.

5.3 Stationarity of data.

Tests carried out for stationarity of the data (Table 1) showed that the data from TBO 42, recorded under almost calm conditions, are stationary while the data from TBO 39 (wind velocity 15 m/s) are non-stationary.

Although a locally generated sea may, of course, be stationary, the data suggest that conditions for stationarity are more favourable under calm conditions. (For this reason, the friction factors f have only been calculated from data recorded under quite calm conditions (Table 6)).

5.4 Comparison of spectra.

The presence of a spectrum of waves, instead of a single wave, makes comparison of the experimental results, with any theoretical predictions based on single wave theory, difficult.

The purpose of obtaining the spectra was therefore primarily to study possible changes in the location of the peaks of the spectra, and to investigate if selective attenuation of wave height with frequency takes place under the conditions stated.

The calculation of the friction factor must be seen as an attempt to measure this factor directly from actual wave spectra.

It is thought, that sufficient evidence has been presented to conclude that the location of the peaks of the spectra of waves in shoaling water does not change under the conditions stated. Although each spectrum has a confidence band associated with it, based on an average of about 20 degrees of freedom, the positions

of the peaks are thought to be real in view of the fact that they are based on a large number of estimates of variance. A total number of 14 spectra were computed and the largest deviation of a peak from the mean position was found to be 0.011 cps. The slight variations in the location of these peaks might have been caused by the observed non-normality of the data.

Although no attempt has been made to provide statistical proof, it is thought that the normalized spectra show sufficient evidence to conclude that there is no apparent selective attenuation of wave height with frequency under the conditions stated. Although the normalized spectra show differences, these are thought to be primarily due to the statistical nature of the analyzing technique.

The reason for non-preferential attenuation of the wave heights with frequency may possibly be that according to refraction diagrams, the spectra (especially from TBO 29 and 40) were not seriously affected by refraction. Calculations by Putnam (1949), based on Airy's wave theory, and assuming a bottom friction factor $f = 0.01$ are of interest. He calculated the change in wave height of waves due to shoaling and bottom friction on flat beaches for waves commonly found in the ocean. His calculations showed that there was no preferential attenuation of wave heights, because the effect of shoaling was roughly cancelled out by the effect of friction.

When a spectrum of waves is strongly refracted, differential refraction of waves of different periods may lead to areas of relative predominance of waves of certain periods and under these circumstances, the spectrum will be affected.

5.5 Calculation of friction factor.

The values for the friction factor obtained for the test area are by no means thought to be conclusive. The following factors may have caused the estimates to be in error :

- a) the simple sine wave theory was applied to a part of the spectrum
- b) slight variations existed in the location of the peak energies

- c) the average wave height associated with a particular period band has been used and the refraction factor K_r and the period T used, are therefore not strictly valid
- d) the existence of confidence bands associated with the spectra. Also, if the power spectrum is rapidly varying over a range of frequencies, the degrees of freedom as given by expression (1.22) will be too large. (Neumann and Pierson, 1966).

On the other hand, the following circumstances favour an accurate and realistic estimate of the friction factor. Firstly, refraction was very small, therefore an accurate estimate of the refraction factors could be made. Secondly, the effect of local winds on the waves did not have to be considered. Thirdly, the estimates are obtained from experimental values obtained from spectra in the field.

Preliminary calculations, using less reliable data, showed the friction factor to lie between 0.10 - 0.20.

Inspection of the sea bottom in 60 and 90 feet of water was carried out on one occasion under calm weather conditions. Sand ripples, having a pitch of about 8 cm and a general crest direction east - west were found to exist at both stations. The fine white sand at the bottom was thrown in suspension to a height of about 4 feet at the passage of each surface wave.

Eagleson (1962) performed laboratory measurements of bottom shearing stress due to oscillatory waves, and found that the experimental values of the bottom friction coefficient were twice to fourteen times as large as the theoretical ones. According to Eagleson's study, the average bottom friction coefficient \bar{f} can be obtained from

$$\bar{f} = 9.37 R^{-\frac{1}{2}} \tag{5.1}$$

where R = Reynolds number.

A later laboratory study by Iwagaki, Tsuchiya and Sakai (1965) obtained $\bar{f} = 6.39 R^{-\frac{1}{2}}$ instead. This relationship agrees well

with theoretical values from linearised laminar boundary layer theory.

The high values obtained for the friction factors may be partly due to non-linear interaction effects of the orbital paths of the water particles near the bottom. For an Airy wave, interaction between the various orbits of the water particles is thought to be small. However, when a spectrum of waves is considered non-linear interaction effects of the water particles may become important.

A survey of the most important non-linear interaction theories is given by Kinsman (1965).

5.6 Suggestions for future work.

It must be stressed that the conclusions presented in this thesis are of necessity based on the analysis of limited data only. Therefore, more confirmatory experimental work needs to be done. Circumstances which would favour the experimental work are : stable weather conditions, a narrow-peaked swell, absence of refraction, longer recording times to reduce the confidence limits, and smaller time lags between the recordings to exclude possible non-stationarity effects. To meet the latter requirements, simultaneous wave recordings by a number of research ships may be considered.

The collection and processing of wave data may be considerably improved by feeding the signal from the wave recorder to a digitizer. The results can then be recorded on magnetic or punched paper tapes, compatible with the IBM 1130 Computer. The calculations may be speeded up by using the Fast Fourier Transform.

Because of the variability of the coasts of South Africa, other types of coastal wave research can possibly be carried out. The coasts vary from very flat and sandy beaches to steep and rocky cliffs. Wave reflection studies could possibly be made off the latter coastal types.

The propagation and decay of the waves generated in the South Atlantic and Indian Ocean may be studied by siting a number of recording stations along the coast.

As a number of South African research ships have been

equipped with wave recorders, it is strongly felt that, after proper simultaneous calibration of the recorders, combined studies into the changes of wave spectra, due to the Agulhas Bank and the Agulhas Current, should be made. It has been noted that the Agulhas Bank seems to have a quieting effect on the heavy seas that roll up to it.

The results of the suggested studies will be of theoretical interest and of great practical importance for the coastal engineer, shipping and off-shore drilling operations.

It seems obvious that the waters off South Africa offer a wide range of interesting possibilities for basic wave research. Not only do the wave generating conditions suggest promising and rewarding opportunities for wave studies, but the characteristics of the very long and varied coastline combined with the uniqueness of the geographic location also seem to offer almost unlimited scope for wave research.

Conclusions.

In view of the possibility that wave conditions in the waters off the coast of South Africa may be different from those found elsewhere, the assumptions of normality and stationarity of waves off Melkboschstrand, near Cape Town were tested. Under the circumstances stated, it was found that

- a. the minimum sampling interval which must be used for reliable testing for normality of wave data is of the order of 40 seconds.
- b. the minimum sample length which must be used for reliable testing for normality is of the order of 20 minutes.
- c. the number of wave records rejected for normality (3 out of 23 records) is slightly higher compared with findings elsewhere. The records rejected were recorded under calm to moderate wind conditions.
- d. no significant difference in normality was found between waves recorded at the deep and the shallow water stations.
- e. tests for stationarity applied to 3 selected records, showed that one record was clearly non-stationary. However, this record was obtained under variable wind conditions. The average spectra obtained were still quite useful.
- f. In view of a, b, c, d and e above the applicability of the technique of power spectrum analysis in the local waters of South Africa may, with suitable caution, be accepted.

One-dimensional wave spectra computed from wave records obtained in deep and shoaling water, using power spectrum analysis based on the method of Blackman and Tukey, showed that

- a. the total variances obtained from recordings along lines perpendicular to the shore decreased systematically with depth. The normalized spectra did not show a systematic selective attenuation of the variances with frequency.
- b. the mean bottom friction factor, estimated from corresponding values of the heights, averaged about the peaks of the wave spectra, was 0.22. This is higher than found by other workers. As this factor is based on the average of 3 estimates only, ranging from 0.11 - 0.38, the value obtained should be treated with caution.

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Appendix I.

The IBM 1130 Computer Programme, used for the spectral analyses of the wave data, is listed on the following pages.

```

// JOB 001

LOG DRIVE CART SPEC CART AVAIL PHY DRIVE
0000 01CC 01CC 0000

V2 M06 ACTUAL 16K CONFIG 16K

// FOR 002
*NAME SPEC 003
*ONE WORD INTEGERS 005
*EXTENDED PRECISION
*LIST SOURCE PROGRAM 004
*IOCS(CARD)
*IOCS(DISK,1132 PRINTER,TYPEWRITER)
DIMENSION X(1000),C(100),V(100),U(100) 007
1000 READ(2,1)N,M 008
1 FORMAT(2I3) 009
SUMB=0.0 010
I=1 011
4 K=I+19
2 READ(2,3)(X(J),J=I,K)
3 FORMAT(20F4.1)
DO 100 J=I,K
IF(X(J))105,8,105
105 CONTINUE
100 SUMB=SUMB+X(J)
I=I+20
GO TO 4
8 IR=0
10 SUMA=0.0 021
I=1 022
9 L=I+IR
11 SUMA=SUMA+X(I)*X(L) 024
IF(I-N+IR)12,13,13 025
12 I=I+1 026
GO TO 9
13 R=IR 028
XN=N 029
C(IR+1)=(SUMA-(XN-R)*((SUMB/(XN))*2))/(XN-R)
WRITE(3,101)C(IR+1) 031
101 FORMAT(12H C(R)-VALUES ,E20.8)
IF(IR-M)14,15,15 033
14 IR=IR+1 034
GO TO 10 035
15 IR=0 036
A=SUMB/XN
WRITE(3,5)A
5 FORMAT(E20.8)
PI=3.141592654
XM=M 038
16 R=IR 039
SUMC=C(1)+C(M+1)*COS((R)*PI) 040
IQ=2 041
17 Q=IQ 042
SUMC=SUMC+2.0*C(IQ)*COS((Q-1.0)*(R)*(PI/XM))
IF(IQ-M)18,19,19 044
18 IQ=IQ+1 045
GO TO 17 046
19 V(IR+1)=SUMC 047
WRITE(3,102)V(IR+1) 048

```

```

102  FORMAT(12H V(IR)-VALUES ,E20.8)
      IF(IR-M)20,21,21
20   IR=IR+1
      GO TO 16
21   IR=1
31   IF(IR-1)26,27,26
17   U(IR)=(3.5*V(1)+0.5*V(2))/XM
      GO TO 25
26   IF(IR-M-1)28,24,28
14   U(IR)=(0.5*V(M+1)+0.5*V(M))/XM
      GO TO 25
8    U(IR)=(0.25*V(IR)+0.5*V(IR+1)+0.25*V(IR+2))/XM
25   WRITE(3,103)IR,U(IR)
03   FORMAT(3H U(I3,5H )= ,E20.8)
      IF(IR-M-1)29,30,30
29   IR=IR+1
      GO TO 31
30   WRITE(3,104)
104  FORMAT(16H THIS IS THE END)
      PROVISION SHOULD BE MADE TO PROCESS SEVERAL RECORDS
      EACH RECORD COULD BE IDENTIFIED BY SOME ID READ AT STATEMENT 1000
      CALL EXIT
      END

```

050
051
052
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072
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UNREFERENCED STATEMENTS
1000 2 11

FEATURES SUPPORTED
ONE WORD INTEGERS
EXTENDED PRECISION
IOCS

MORE REQUIREMENTS FOR SPEC
COMMON 0 VARIABLES 3948 PROGRAM 604

END OF COMPILATION

/ XEQ

List of Illustrations.

- Fig. 1.1 Melkboschstrand area.
- Fig. 1.2 Convolution of a digitized function.
- Fig. 2.1 Correction curve for wave recorder "Thomas B. Davie".
- Fig. 3.1 Effect of lag number on resolution.
- Fig. 3.2 Spectra computed for first and second half of record.
- Fig. 3.3 Spectra computed for three parts of record.
- Fig. 3.4 Spectra computed for first and second half of record.
- Fig. 3.5 Spectra of record using different sampling intervals.
- Fig. 4.1 Melkboschstrand area.
- Fig. 4.2 Bottom profile near Melkboschstrand.
- Fig. 4.3 Computed wave spectra stations 5, 6, 7.
- Fig. 4.4 Computed wave spectra stations 17, 18, 19, 20.
- Fig. 4.5 Computed wave spectra stations 21, 22, 23.
- Fig. 4.6 Computed wave spectra stations 17, 18, 19, 20.
- Fig. 4.7 Normalized wave spectra stations 5, 6, 7.
- Fig. 4.8 Normalized wave spectra stations 17, 18, 19, 20.
- Fig. 4.9 Normalized wave spectra stations 21, 22, 23.
- Fig. 4.10 Normalized wave spectra stations 17, 18, 19, 20.
- Fig. 4.11 Wave refraction diagram showing orthogonals.
- Fig. 4.12 Wave refraction diagram showing orthogonals.

List of Tables.

- Table 1 Wave recordings used for stationarity tests.
- Table 2 Minimum sampling interval which must be used for tests for normality.
- Table 3 Chi-square values for different sample lengths using a fixed sampling interval.
- Table 4 Chi-square values for recordings under different wave generating conditions.
- Table 5 Chi-square values for recordings in ranges of different depths.
- Table 6 Calculations of variance and chi-square values from wave records under stated conditions for waves in shoaling water.
- Table 7 Data used in the calculation of friction factors.

LIST OF NOTATIONS. *

$A(f)$	= continuous amplitude spectrum.
$\left. \begin{matrix} a_n \\ b_n \end{matrix} \right\}$	= Fourier coefficients.
b	= distance between orthogonals.
a	= amplitude of wave.
C	= wave velocity.
C_G	= group velocity.
$C(\tau)$	= autocovariance function.
$C(\tau) / C(0)$	= autocorrelation function.
$C_{oo}(\tau)$	= apparent autocovariance function.
$C_i(\tau)$	= modified apparent autocovariance function.
C_R	= sample autocovariance function.
$D_i(\tau)$	= masking function.
d	= hydrostatic depth of pressure units.
e	= base of natural logarithms = 2.7183.
f	= frequency or dimensionless friction coefficient for the bottom (as defined).
f_N	= Nyquist frequency.
g	= acceleration due to gravity.
H	= wave height.
h	= total depth of water.
K	= total wave height reduction factor due to energy dissipation.
K_r	= refraction factor.
K_s	= shoaling factor.
K_{pfr}	= product of K and K_r .
k	= wave number $\frac{2\pi}{L}$.
L	= wave length.
M	= lag number.
N	= number of sampling points.
n	= ratio between wave and group velocity.
P	= wave energy transmitted in the direction of wave propagation per unit time through a vertical section of unit width.
$P(f)$	= true power spectrum.
$P_i(f)$	= approximation of true power spectrum.

* unless otherwise specified.

$P_{oo}(f)$	= Fourier transform of $C_{oo}(\tau)$.
$Q_i(f)$	= Fourier transform of $D_i(\tau)$.
R	= Reynolds number.
r	= relative response of wave recorder.
T	= wave period.
t	= time.
T_m	= maximum lag used.
T_n	= total length of record.
U_o, U_R, U_M	= smoothed estimates of spectral density.
V_R	= "raw" estimates of spectral density.
$X(t)$	= value of time function.
x	= horizontal distance in direction of wave propagation.
z	= vertical distance from free water surface.
ν	= kinematic viscosity of water.
π	= 3.1416.
ρ	= density of water.
σ	= wave angular frequency $\frac{2\pi}{T}$.
τ	= difference in time of two values $X(t)$ and $X(t + \tau)$ which are considered together.
ϕ_f	= function of $\frac{h}{T_1}$ related to the frictional damping process.
ϕ_p	= function of $\frac{h}{T_1}$ related to the percolation damping process.
ΔT	= sampling interval used for chi-square tests.
Δt	= sampling interval of observations.

Suffix o applies to deep water.

Suffix 1 applies to any other point.