

South African anchovy assessment sensitivity tests

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A number of sensitivity tests to assumptions underlying the baseline assessment of South African anchovy have been undertaken. From the results presented in this document, a sub-set of these sensitivity tests are highlighted as important for testing the robustness of OMP-18rev, some are no longer considered important for future assessments, and further development of some of the alternatives in the future is also desired.

Introduction

The most recent baseline assessment for South African anchovy, conditioned to data available from 1984 – 2019, was presented by de Moor (2020a). This document considers a number of sensitivity tests to assumptions underlying the baseline assessment. The robustness of OMP-18rev (a planned update to OMP-18 (de Moor 2018)) to uncertainty can be tested by using Operating Models developed from these sensitivity tests.

Methods

The baseline model is detailed in Appendix A and the data in de Moor *et al.* (2020c). For ease of comparison, the changes in the model since de Moor (2020b) have been highlighted. The parameters are defined in Tables A.1 and A.2.

The following sensitivity tests were considered:

A_{BH} - Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity. $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2$.

Baseline OM.

A_{2BH} - Two Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity¹, one estimated using data from 1984 to 1999 and the other from 2000 to 2019.

A_{3BH} - Three Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity¹, one estimated using data from 1984 to 1999, one from 2000 to 2012, and one from 2013 to 2019.

A_{2BHrtn} - Two Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity¹, one estimated using data from 1984 to 1999 and 2013 to 2019, and the other from 2000 to 2012.

A_R - Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity.

A_{HS} - Hockey stick stock-recruitment curve, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity.

A_{M1} - $\bar{M}_j^A = \bar{M}_{ad}^A = 0.9$ (for comparison with the baseline assessment of 2007)

A_{M2} - $\bar{M}_j^A = 1.5$ and $\bar{M}_{ad}^A = 1.2$ (alternative \bar{M}_j^A , similar to A_{BH} in terms of value of the negative log joint posterior mode)

A_{Mad} - Annually varying adult natural mortality, i.e. random effects model with $\sigma_{ad} = 0$ and 0.2 (2 alternatives²) and $\rho \sim U(0,1)$.

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¹ When carrying capacity is estimated over a shorter time period, the average maturity at age, \bar{f}_a^A , used in calculating stock recruitment parameters is calculated over the corresponding time period.

² Convergence proved difficult for this parameter when estimated and thus two alternative fixed values were used.

- A_{Mj} - Annually varying juvenile natural mortality, i.e. random effects model with $\sigma_j = 0.1$ and 0.2 (2 alternatives) and $\rho \sim U(0,1)$.
- A_{M2000+} - Natural mortality assumed to have increased in 2000; i.e. $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2 \text{year}^{-1}$ prior to 2000 and $\bar{M}_j^A = \bar{M}_{ad}^A = 1.5 \text{year}^{-1}$ from 2000 onwards³. The Beverton-Holt stock recruitment relationship was estimated to correspond to the years 2000 onwards, with no stock-recruitment relationship assumed prior to 2000⁴.
- A_{M1996+} - Natural mortality assumed to have increased in 1996; i.e. $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2 \text{year}^{-1}$ up to 1995 and $\bar{M}_j^A = \bar{M}_{ad}^A = 1.5 \text{year}^{-1}$ from 1996 onwards⁵. The Beverton-Holt stock recruitment relationship was estimated to correspond to the years 1996 onwards, with no stock-recruitment relationship assumed prior to 1996³.
- A_4 - No plus group, all remaining fish assumed to die as they reach age 5.
- A_{DD} - Density dependent natural mortality: $\bar{M}_{j,y}^A = \bar{M}_{ad,y}^A = \bar{M} + e^{-\chi B_y^A - 1}$.
- A_{sur} - Survey selectivity below 7.5^6cm was estimated to be a constant, and uniform (1) selectivity was assumed for lengths $\geq 7.5 \text{cm}$.
- A_{com} - Commercial selectivity was not estimated to decrease at higher lengths, i.e. $\delta_q = 0$.
- A_{com2} - Commercial selectivity was modelled using a double-logistic curve.
- A_{kegg} - Egg survey bias estimated with uninformative prior, i.e. $\ln(k_g^A) \sim U(-100, 0.7)$.
- A_{kegg1} - Negatively biased egg surveys, i.e., $k_g^A = 0.75$.
- A_{kegg2} - Positively biased egg surveys, i.e., $k_g^A = 1.25$.
- A_{lamR} - Additional variance (over and above the survey sampling CV) associated with the recruit survey fixed $(\lambda_r^A)^2 = 0$.
- A_{lamN} - Additional variance (over and above the survey sampling CV) associated with the November survey, estimated with $(\lambda_N^A)^2 \sim U(0, 100)$.
- A_{lamN2} - Additional variance (over and above the survey sampling CV) associated with the November survey fixed $(\lambda_N^A)^2 = 0.02$.

Results

Table 1 compares the fit to the data, the posterior (objective function) at the joint posterior mode and the posterior distributions for some key model parameters for all the sensitivity tests.

The three sensitivity tests with two or three Beverton Holt stock-recruitment curves, instead of one time-invariant relationship, all result in a slightly better fit to the data, but not substantially so such that model selection would prefer these models to A_{BH} . Relationships estimated including the early 2000s are higher than those estimated using the remaining years (Figures 2 to 4). The relationship estimated for recent years for A_{2BH} is more optimistic at higher spawner biomass values than that under A_{BH} , while that estimated for A_{2BHrtn} is more pessimistic at higher spawner biomass values than that under A_{BH} . These provide useful and realistic alternative recruitment scenarios to A_{BH} and it is **recommended** that they be used to test the robustness of OMP-18rev to uncertainty and are retained as future sensitivity tests. Similarly, A_R and A_{BH} are **recommended** to be used to test OMP-18rev robustness and retained as future sensitivity tests due to their similar fit to data, but potentially different projected outcomes.

³ It was assumed that there was an increase in natural mortality of 0.3year^{-1} . $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2 \text{year}^{-1}$ for 1984-1999 provided the best deterministic fit in the range 0.9 to 1.2.

⁴ The average maturity at age, \bar{f}_d^A , used in calculating stock recruitment parameters is calculated over the corresponding time period.

⁵ It was assumed that there was an increase in natural mortality of 0.3year^{-1} . $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2 \text{year}^{-1}$ for 1984-1995 provided the best deterministic fit in the range 0.9 to 1.2.

⁶ This length was chosen as it provided the best deterministic fit in the range 6-9cm.

While A_{3BH} is **recommended** to be retained as a future sensitivity test, it is not recommended that OMP-18rev be tested against A_{3BH} due to the difficulty in estimating the parameters of the stock recruitment relationship for the (relatively short) final time period at the joint posterior mode, together with the similarity between the ‘posterior median’ stock recruitment relationship of A_{3BH} in recent years and that of A_{2BH} (Figure 5).

A_{M1} produces a poorer fit to the historical data, while A_{M2} produces a very similar fit to the data. It is **recommended** that the robustness of OMP-18rev be tested against these two alternatives, and that future sensitivity tests include either these or other alternative time invariant natural mortality assumptions (chosen based on historical fits to data and realistic natural mortality assumptions), noting that it is now no longer necessary to maintain a comparison with the 2007 baseline assessment and that the data suggest higher, rather than lower, natural mortality rates.

Figure 6 shows the annually estimated natural mortality rates for A_{Mj} and A_{Mad} . These indicate an improved fit to the data is obtained for lower rates of $M_{ad,y}^A$ prior to 2000 and cyclically higher and lower $M_{ad,y}^A$ after 2000 than the time invariant $\bar{M}_{ad}^A = 1.2$ year⁻¹ assumed for A_{BH} . The variability in the rate of juvenile natural mortality is estimated to be higher prior to 2000 and then again higher around 2008-2013. While one needs to be cautious that these models which fit the data better do not primarily reflect a ‘fitting to noise’ (Smith *et al.* 2011), they remain informative, particularly from an ecosystem perspective and are **recommended** to be retained as future sensitivity tests. It is **recommended** that the robustness of OMP-18rev to changes in natural mortality be tested against the more extreme alternatives with σ_{ad} and $\sigma_j = 0.2$, with future cyclical patterns reflective of those in the recent past.

While the objective function at the joint posterior mode is substantially better for A_{M2000+} and A_{M1996+} than for A_{BH} , this primarily comes from estimating fewer residuals about the stock recruitment relationship and the fit to the data is actually slightly worse than A_{BH} for these two alternatives. The higher rate of natural mortality since 1996/2000 corresponds with a Beverton Holt stock recruitment relationship with a substantially higher recruitment at higher spawner biomass values. It is not yet clear if this test is accurately capturing the proposed hypothesis. It is thus **recommended** that OMP-18rev be tentatively⁷ considered under A_{M2000+} and that further investigation of whether these two sensitivity tests are adequately capturing the proposed hypothesis be undertaken during the next round of sensitivity testing.

A_4 results in a better fit to the data, but not substantially so, such that the historical time series of survey indices do not differ visibly from A_{BH} . Due to the potential difference in future projections following a large year class forming part of the plus group (or not), it is **recommended** that the robustness of OMP-18rev be tested using A_4 and that A_4 be retained as a future sensitivity test.

Assuming natural mortality is density dependent with $\bar{M}_{j,y}^A = \bar{M}_{ad,y}^A = \bar{M} + e^{-\chi B_y^A - 1}$ results in a slightly poorer fit to the data and a relatively flat relationship between natural mortality and November biomass (Figure 7). The shape of the relationship could allow for greater changes in natural mortality with biomass, but this isn’t preferred by the model. The lowest natural mortality rate is estimated above that assumed for A_{BH} . Although this is a new alternative that is still under development, there is some

⁷ Particularly considering the limitation of a fixed σ_r^A .

contrast in natural mortality estimated against November biomass. It is thus **recommended** that the robustness of OMP-18rev be tested against A_{DD} , and that A_{DD} be further investigated (including alternative relationships that allow for increases in natural mortality at both high and low stock size and/or the possibility of density dependence operating on either the juvenile or adult natural mortality, and not necessarily both) with future sensitivity testing. In particular, the more substantially productive stock recruitment relationship coupled with the substantially lower bias estimated for the May acoustic survey estimate of recruitment for A_{DD} compared to A_{BH} (Table 1) warrants further investigation.

The estimated survey selectivity curves and fits to survey proportion at length data for A_{sur} is compared to A_{BH} in Figure 8. A_{sur} produces similar results to A_{BH} . It is therefore **recommended** that OMP-18rev not be tested against A_{sur} , but that A_{sur} be retained for future sensitivity testing.

The estimated commercial selectivity curves and fits to commercial proportion at length data for A_{com} and A_{com2} are compared to A_{BH} in Figures 9 and 10. A_{com} produces a much poorer fit to the data indicating little weight to the hypothesis that if the population consists of larger fish, they will be landed by the fishery. This is consistent with our current understanding of the population structure with the fishery operating primarily on the recruits and in a different area to the primary (November) distribution of larger fish (e.g. Shabangu *et al.* 2019). However, given the global sensitivity to dome shaped selectivity assumptions, A_{com} is **recommended** to be retained for future sensitivity testing, but given the poor fit to the data it is **not recommended** that OMP-18rev need be simulation tested against A_{com} . A_{com2} produces a substantially better fit to the commercial length frequency data, although with some loss in the goodness of fit to the survey abundance indices. It is **recommended** that the robustness of OMP-18rev be tested against A_{com2} and that A_{com2} be retained for future sensitivity tests. It is further recommended that the development of OMs for OMP-22 consider whether the baseline model should rather assume double-logistic commercial selectivity relationships.

A_{kegg} is a new sensitivity test, made possible, in part, due to the recent inclusion of an informative prior on the November survey bias (de Moor *et al.* 2020a). Interestingly, the results for A_{kegg} and A_{kegg1} are very similar, with the DEPM survey bias being estimated to be around 0.75, and the bias on the November survey being slightly greater than that estimated for A_{BH} (around 0.80 rather than 0.88 for A_{BH}). A_{kegg2} resulted in a substantially poorer fit to the data. Given the availability of an informative prior on the November survey bias, it is **recommended** that A_{kegg} be used in robustness testing of OMP-18rev. It is additionally **recommended** that A_{kegg1} and A_{kegg2} be removed from future sensitivity testing.

There is little difference between A_{lamN} and A_{BH} as A_{lamN} estimates $(\lambda_N^A)^2$ to be close to zero. This is in line with previous assessments (e.g. de Moor 2019). It is therefore **recommended** that A_{lamN} be removed from future sensitivity testing in favour of A_{lamN2} which provides some contrast to A_{BH} . While A_{lamR} (unexpectedly) provides a poorer fit to the data, it is **recommended** that the robustness of OMP-18rev be tested against both A_{lamR} and A_{lamN2} and that A_{lamR} be retained for future sensitivity testing.

Discussion

This document has provided a number of sensitivity tests to assumptions underlying the baseline assessment of South African anchovy. In addition to the below lists, these results indicate that anchovy may be subject to a higher natural mortality than that

currently assumed as a time-invariant value for the baseline model and a change to that assumption may be necessary during the next ‘benchmark’ assessment.

In summary, the list of sensitivity tests to form the basis for alternative Operating Models to be used to simulation test the robustness of OMP-18rev to uncertainty is recommended as follows:

A_{BH} , A_{2BH} , A_{2BHRtn} , A_R , A_{HS} , A_{M1} , A_{M2} , A_{Mad} with $\sigma_{ad} = 0.2$, A_{Mj} with $\sigma_j = 0.2$, A_{M2000+} , A_4 , A_{DD} , A_{com2} , A_{kegg} , A_{lamR} and A_{lamN2} .

The list of sensitivity tests for future assessments (assuming a baseline hypothesis with a Beverton Holt stock recruitment relationship and time-invariant natural mortality) is recommended as follows:

A_{2BH} , A_{3BH} , A_{2BHRtn} , A_R , A_{HS} , some alternative time-invariant natural mortality assumptions, noting that it is no longer necessary to keep $\bar{M}_j^A = \bar{M}_{ad}^A = 0.9$ for comparison with the 2007 baseline assessment, A_{Mad} , A_{Mj} , further investigation of the hypotheses underlying and modelling of A_{M2000+} and A_{M1996+} , A_4 , further investigation of the hypotheses underlying and modelling of A_{DD} , including the allowance for increases in natural mortality (at different rates) at both high and low anchovy biomass, A_{sur} , A_{com} , A_{com2} which may form a new baseline model, A_{kegg} , A_{lamR} and A_{lamN2} .

Acknowledgements

Computations were performed using facilities provided by the University of Cape Town’s ICTS High Performance Computing team: <http://hpc.uct.ac.za>.

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Table 1. The posterior median and 95% probability intervals for key parameters and outputs. The value from the deterministic fit is given in *italics* and the individual contributions to the objective function from the deterministic fit at the joint posterior mode are shown. All robustness tests are defined in the main text and all parameters are defined in Table A.1. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tons.

	A_{BH}	A_{2BH}	A_{3BH}			
Obj fn	-721.8	-723.1	-724.2			
$-\ln L$	-761.5	-761.9	-761.8			
$-\ln L^{Nov}$	0.38	0.92	1.19			
$-\ln L^{Egg}$	6.62	6.32	6.38			
$-\ln L^{rec}$	19.38	19.74	19.69			
$-\ln L^{sur pr}$	-485.2	-486.3	-486.5			
$-\ln L^{com pr}$	-302.6	-302.6	-302.6			
ε_y^A	33.59	32.59	31.45			
Growth	-1.47	-1.40	-1.37			
δ_1, δ_3	-2.04	-2.04	-2.04			
$N_{1983,a}^A$	10.79	10.78	10.78			
k_N^A	-1.19	-1.18	-1.19			
\bar{M}_j^A	1.2	1.2	1.2			
\bar{M}_{ad}^A	1.2	1.2	1.2			
σ_j	-	-	-			
σ_{ad}	-	-	-			
ρ	-	-	-			
k_N^A	<i>0.89</i> <i>0.88</i> [0.74,1.04]	<i>0.90</i> <i>0.87</i> [0.75,1.03]	<i>0.89</i> <i>0.87</i> [0.74,1.03]			
k_r^A	<i>0.74</i> <i>0.74</i> [0.58,0.92]	<i>0.74</i> <i>0.74</i> [0.58,0.92]	<i>0.74</i> <i>0.74</i> [0.57,0.92]			
k_g^A	1.00	1.00	1.00			
$(\lambda_N^A)^2$	0.00	0.00	0.00			
$(\lambda_r^A)^2$	<i>0.11</i> <i>0.25</i> [0.11,0.53]	<i>0.11</i> <i>0.25</i> [0.11,0.52]	<i>0.11</i> <i>0.25</i> [0.11,0.50]			
α^A	<i>1106</i> <i>1107</i> [577,3169]	<i>482</i> <i>621</i> [337,1792]	<i>1236</i> <i>1370</i> [704,7593]	<i>478</i> <i>644</i> [342,1963]	<i>1078</i> <i>1539</i> [773,6669]	<i>510</i> <i>1011</i> [413,15402]
b^A	<i>1616</i> <i>1121</i> [66,6750]	<i>250</i> <i>304</i> [15,3128]	<i>1697</i> <i>1216</i> [30,22056]	<i>254</i> <i>329</i> [11,4147]	<i>698</i> <i>1099</i> [24,1958]	<i>0.006</i> <i>705</i> [19,59386]
K^A	<i>2966</i> <i>3318</i> [2003,6717]	<i>1743</i> <i>2112</i> [1281,4607]	<i>3415</i> <i>4110</i> [2315,8859]	<i>1722</i> <i>2249</i> [1273,4618]	<i>3786</i> <i>4891</i> [2515,9079]	<i>2100</i> <i>3156</i> [1219,9390]
h^A	<i>0.41</i> <i>0.50</i> [0.29,0.91]	<i>0.67</i> <i>0.67</i> [0.33,0.98]	<i>0.43</i> <i>0.53</i> [0.24,0.97]	<i>0.66</i> <i>0.66</i> [0.31,0.98]	<i>0.62</i> <i>0.57</i> [0.25,0.98]	<i>0.99</i> <i>0.56</i> [0.21,0.97]
σ_r^A	<i>0.63</i> <i>0.80</i> [0.59,1.07]	<i>0.61</i> <i>0.78</i> [0.59,1.06]		<i>0.59</i> <i>0.79</i> [0.58,1.06]		
B_{2019}^A	<i>1547</i> <i>1424</i> [868,2246]	<i>1567</i> <i>1457</i> [913,2294]		<i>1534</i> <i>1426</i> [896,2317]		
$B_{2019}^{sp,A}$	<i>1456</i> <i>1343</i> [822,2079]	<i>1474</i> <i>1371</i> [869,2137]		<i>1443</i> <i>1346</i> [846,2144]		

Table 1 (continued).

	A _{BH}	A _{2BHrtn}	A _R	A _{HS}	A _{M1}	A _{M2}	
Obj fn	-721.8	-724.0	-721.3	-720.7	-707.3	-721.6	
$-\ln L$	-761.5	-761.8	-761.1	-762.5	-747.6	-761.5	
$-\ln L^{Nov}$	0.38	0.75	0.47	0.02	6.92	0.54	
$-\ln L^{Egg}$	6.62	6.40	6.73	6.28	11.02	6.61	
$-\ln L^{rec}$	19.38	20.02	19.26	20.41	19.67	19.32	
$-\ln L^{sur pr}$	-485.2	-486.4	-485.0	-486.4	-486.1	-485.2	
$-\ln L^{com pr}$	-302.6	-302.6	-302.6	-302.8	-299.1	-302.7	
ε_y^A	33.59	31.65	33.79	35.59	34.46	33.89	
Growth	-1.47	-1.37	-1.47	-1.37	-1.97	-1.55	
δ_1, δ_3	-2.04	-2.04	-2.04	-2.04	-2.03	-2.04	
$N_{1983,a}^A$	10.79	10.78	10.79	10.78	11.03	10.78	
k_N^A	-1.19	-1.18	-1.20	-1.10	-1.27	-1.19	
\bar{M}_j^A	1.2	1.2	1.2	1.2	0.9	1.5	
\bar{M}_{ad}^A	1.2	1.2	1.2	1.2	0.9	1.2	
σ_j	-	-	-	-	-	-	
σ_{ad}	-	-	-	-	-	-	
ρ	-	-	-	-	-	-	
k_N^A	0.89 0.88 [0.74,1.04]	0.90 0.87 [0.73,1.02]	0.89 0.87 [0.74,1.02]	0.91 0.89 [0.75,1.04]	0.86 0.86 [0.73,1.01]	0.89 0.87 [0.73,1.04]	
k_r^A	0.74 0.74 [0.58,0.92]	0.74 0.73 [0.56,0.91]	0.73 0.74 [0.56,0.91]	0.76 0.75 [0.58,0.92]	0.84 0.81 [0.66,0.96]	0.66 0.67 [0.51,0.84]	
k_g^A	1.00	1.00	1.00	1.00	1.00	1.00	
$(\lambda_N^A)^2$	0.00	0.00	0.00	0.00	0.00	0.00	
$(\lambda_r^A)^2$	0.11 0.25 [0.11,0.53]	0.12 0.26 [0.11,0.49]	0.11 0.24 [0.10,0.56]	0.12 0.25 [0.11,0.53]	0.11 0.30 [0.13,0.65]	0.11 0.26 [0.11,0.57]	
a^A	1106 1107 [577,3169]	533 625 [376,1822]	1079 1531 [750,7048]	0.50 0.58 [0.34,1.12]	614 690 [474,1186]	744 687 [387,2063]	1484 1641 [795,9522]
b^A	1616 1121 [66,6750]	344 308 [15,3778]	699 1191 [24,23072]	0.0002 0.0002 [0.0001,0.0005]	1074 1098 [69,3235]	1332 952 [36,7012]	1639 1437 [47,19776]
K^A	2966 3318 [2003,6717]	1856 2190 [1395,4336]	3771 4902 [2568,9452]	3259 4094 [2421,8326]	2533 2793 [1953,4953]	2929 3427 [2200,6912]	2911 3531 [2101,9373]
h^A	0.41 0.50 [0.29,0.91]	0.62 0.66 [0.31,0.97]	0.63 0.56 [0.24,0.98]	0.36 0.40 [0.26,0.67]	-	0.44 0.54 [0.30,0.95]	0.41 0.46 [0.26,0.93]
σ_r^A	0.63 0.80 [0.59,1.07]	0.60 0.76 [0.57,1.01]		0.64 0.81 [0.60,1.10]	0.67 0.25 [0.19,0.33]	0.65 0.82 [0.63,1.11]	0.64 0.80 [0.60,1.11]
B_{2019}^A	1547 1424 [868,2246]	1485 1429 [887,2217]		1556 1419 [877,2242]	1516 1427 [867,2272]	1713 1532 [966,2364]	1551 1419 [890,2272]
$B_{2019}^{sp,A}$	1456 1343 [822,2079]	1397 1346 [843,2083]		1464 1337 [832,2109]	1426 1340 [818,2127]	1566 1434 [918,2202]	1460 1336 [849,2115]

Table 1 (continued).

	A_{BH}	A_4	A_{DD}	$A_{Mad0.1}$	$A_{Mad0.2}$	$A_{Mj0.1}$	$A_{Mj0.2}$
Obj fn	-721.8	-721.5	-720.3	-773.7	-753.6	-772.5	-750.3
$-\ln L$	-761.5	-760.8	-758.1	-766.0	-776.0	-761.8	-761.7
$-\ln L^{Nov}$	0.38	-0.10	0.12	-0.11	-0.81	0.37	0.28
$-\ln L^{Egg}$	6.62	5.97	5.28	6.15	5.80	6.65	6.76
$-\ln L^{rec}$	19.38	19.66	16.18	18.57	16.83	18.86	18.04
$-\ln L^{sur pr}$	-485.2	-483.7	-481.1	-488.1	-495.9	-485.0	-484.3
$-\ln L^{com pr}$	-302.6	-302.7	-298.6	-302.5	-302.0	-302.6	-302.5
ε_y^A	33.59	33.23	32.78	33.84	34.39	32.18	25.92
Growth	-1.47	-1.48	-2.44	-1.23	-0.65	-1.47	-1.52
δ_1, δ_3	-2.04	-2.04	-2.01	-2.05	-2.06	-2.04	-2.04
$N_{1983,a}^A$	10.79	10.77	10.77	10.77	10.77	10.79	10.79
k_N^A	-1.19	-1.16	-1.24	-1.15	-1.11	-1.19	-1.20
$\varepsilon_y^j, \varepsilon_y^{ad}$	-	-	-	-47.9	-19.0	-49.0	-20.5
\bar{M}_j^A	1.2	1.2	1.3-2.0	1.2	1.2	1.1-1.3	1.0-1.6
\bar{M}_{ad}^A	1.2	1.2	1.3-2.0	1.1-1.3	1.0-1.6	1.2	1.2
σ_j	-	-	-	-	-	0.10	0.20
σ_{ad}	-	-	-	0.10	0.20	-	-
ρ	-	-	-	0.41 0.41 [0.02,0.96]	0.39 0.38 [0.03,0.87]	0.14 0.49 [0.03,0.96]	0.13 0.42 [0.03,0.96]
k_N^A	0.89 0.88 [0.74,1.04]	0.90 0.88 [0.74,1.04]	0.88 0.87 [0.74,1.03]	0.90 0.88 [0.75,1.05]	0.91 0.89 [0.75,1.05]	0.89 0.88 [0.74,1.04]	0.89 0.87 [0.74,1.05]
k_r^A	0.74 0.74 [0.58,0.92]	0.73 0.73 [0.59,0.92]	0.49 0.53 [0.39,0.73]	0.74 0.74 [0.57,0.92]	0.74 0.74 [0.56,0.93]	0.74 0.74 [0.57,0.93]	0.73 0.74 [0.57,0.92]
k_g^A	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$(\lambda_N^A)^2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(\lambda_r^A)^2$	0.11 0.25 [0.11,0.53]	0.11 0.25 [0.11,0.54]	0.08 0.21 [0.08,0.47]	0.10 0.24 [0.10,0.52]	0.09 0.23 [0.09,0.49]	0.11 0.25 [0.10,0.54]	0.10 0.24 [0.09,0.53]
a^A	1106 1107 [577,3169]	1100 1119 [579,3815]	4246 3387 [2230,7932]	1153 1126 [562,4360]	1293 1168 [573,3884]	1111 1108 [563,3612]	1129 1075 [540,3052]
b^A	1616 1121 [66,6750]	1577 1084 [65,8529]	5483 3555 [993,17074]	1740 1158 [57,10404]	2116 1217 [50,9504]	1680 1060 [58,8145]	1868 1021 [36,7212]
K^A	2966 3318 [2003,6717]	3018 3439 [2090,7405]	10000 8452 [7037,9882]	3037 3369 [2003,7981]	3218 3461 [2080,7509]	2922 3276 [2070,7125]	2816 3260 [1973,6516]
h^A	0.41 0.50 [0.29,0.91]	0.42 0.50 [0.29,0.92]	0.41 0.46 [0.27,0.70]	0.41 0.50 [0.28,0.92]	0.39 0.49 [0.28,0.93]	0.41 0.51 [0.28,0.92]	0.39 0.51 [0.28,0.95]
σ_r^A	0.63 0.80 [0.59,1.07]	0.63 0.80 [0.58,1.09]	0.61 0.76 [0.58,1.03]	0.65 0.79 [0.58,1.08]	0.65 0.80 [0.60,1.08]	0.61 0.78 [0.59,1.07]	0.51 0.75 [0.55,1.04]
B_{2019}^A	1547 1424 [868,2246]	1531 1431 [892,2221]	1681 1506 [901,2372]	1514 1389 [858,2222]	1447 1394 [857,2252]	1552 1426 [902,2332]	1561 1410 [881,2270]
$B_{2019}^{sp,A}$	1456 1343 [822,2079]	1442 1348 [852,2077]	1529 1365 [827,2133]	1422 1305 [814,2104]	1352 1305 [795,2107]	1461 1346 [858,2176]	1471 1331 [831,2113]

Table 1 (continued).

	A_{BH}	A_{M2000+}	A_{M1996+}	A_{sur}	A_{com}	A_{com2}
Obj fn	-721.8	-746.4	-743.4	-719.5	-515.3	-727.8
$-\ln L$	-761.5	-760.1	-759.6	-757.3	-731.5	-769.1
$-\ln L^{Nov}$	0.38	0.57	1.20	0.36	2.27	1.11
$-\ln L^{Egg}$	6.62	5.65	5.66	6.63	6.90	6.98
$-\ln L^{rec}$	19.38	19.73	19.52	19.11	20.32	19.56
$-\ln L^{sur pr}$	-485.2	-487.36	-487.15	-483.4	-481.7	-487.3
$-\ln L^{com pr}$	-302.6	-298.72	-298.79	-300.0	-279.3	-309.5
ε_y^A	33.59	7.88	10.63	33.16	32.74	34.18
Growth	-1.47	-1.71	-2.01	-2.97	1.74	-2.44
δ_1, δ_3	-2.04	-1.98	-2.00	-2.05	172.2	-
$N_{1983,a}^A$	10.79	10.78	10.78	10.79	10.78	10.79
k_N^A	-1.19	-1.20	-1.19	-1.18	-1.25	-1.20
\bar{M}_j^A	1.2	1.2; 1.5	1.2; 1.5	1.2	1.2	1.2
\bar{M}_{ad}^A	1.2	1.2; 1.5	1.2; 1.5	1.2	1.2	1.2
σ_j	-	-	-	-	-	-
σ_{ad}	-	-	-	-	-	-
ρ	-	-	-	-	-	-
k_N^A	0.89 0.88 [0.74,1.04]	0.89 0.88 [0.73,1.03]	0.89 0.88 [0.74,1.02]	0.90 0.88 [0.74,1.03]	0.84 0.82 [0.69,0.97]	0.89 0.87 [0.73,1.02]
k_r^A	0.74 0.74 [0.58,0.92]	0.56 0.58 [0.46,0.77]	0.57 0.61 [0.46,0.80]	0.75 0.75 [0.59,0.92]	0.46 0.47 [0.36,0.61]	0.73 0.73 [0.56,0.90]
k_g^A	1.00	1.00	1.00	1.00	1.00	1.00
$(\lambda_N^A)^2$	0.00	0.00	0.00	0.00	0.00	0.00
$(\lambda_r^A)^2$	0.11 0.25 [0.11,0.53]	0.11 0.25 [0.10,0.55]	0.11 0.24 [0.11,0.53]	0.11 0.25 [0.11,0.51]	0.12 0.25 [0.11,0.55]	0.11 0.24 [0.10,0.52]
a^A	1106 1107 [577,3169]	3349 2043 [1071,41621]	2383 2029 [1108,10541]	1102 1060 [558,2986]	1584 1573 [862,5668]	1094 1152 [626,3861]
b^A	1616 1121 [66,6750]	4477 1795 [66,81311]	2379 1798 [234,18309]	1688 1148 [45,7066]	1070 672 [27,6439]	1562 1191 [87,7923]
K^A	2966 3318 [2003,6717]	2449 2668 [1658,6301]	2582 2742 [1900,5012]	3063 3343 [2087,6783]	1961 2200 [1381,4690]	3000 3395 [2086,7110]
h^A	0.41 0.50 [0.29,0.91]	0.28 0.38 [0.21,0.91]	0.34 0.39 [0.23,0.75]	0.41 0.50 [0.30,0.93]	0.41 0.51 [0.28,0.94]	0.42 0.49 [0.29,0.89]
σ_r^A	0.63 0.80 [0.59,1.07]	0.44	0.44	0.62 0.79 [0.58,1.05]	0.62 0.78 [0.59,1.06]	0.64 0.79 [0.60,1.07]
B_{2019}^A	1547 1424 [868,2246]	1540 1487 [935,2359]	1599 1524 [970,2446]	1549 1421 [870,2297]	1748 1607 [1023,2598]	1558 1460 [938,2262]
$B_{2019}^{sp,A}$	1456 1343 [822,2079]	1371 1339 [862,2113]	1435 1396 [886,2203]	1470 1345 [835,2167]	1317 1206 [777,1920]	1468 1371 [884,2120]

Table 1 (continued).

	A_{BH}	A_{kegg}	A_{kegg1}	A_{kegg2}	A_{lamR}	A_{lamN}	A_{lamN2}
Obj fn	-721.8	-722.4	-722.2	-714.2	-717.7	-721.8	-720.4
$-\ln L$	-761.5	-762.1	-762.1	-752.8	-756.9	-761.5	-759.7
$-\ln L^{Nov}$	0.38	-0.29	-0.60	2.52	8.64	0.38	5.00
$-\ln L^{Egg}$	6.62	5.73	5.55	8.03	8.38	6.62	6.89
$-\ln L^{rec}$	19.38	20.06	20.44	18.32	11.24	19.38	16.78
$-\ln L^{sur pr}$	-485.2	-485.0	-484.9	-482.5	-483.1	-485.2	-486.0
$-\ln L^{com pr}$	-302.6	-302.6	-302.6	-299.1	-302.1	-302.6	-302.4
ε_y^A	33.59	33.82	33.98	32.94	33.25	33.59	33.20
Growth	-1.47	-1.57	-1.61	-2.36	-1.49	-1.47	-1.41
δ_1, δ_3	-2.04	-2.04	-2.03	-2.02	-2.03	-2.04	-2.04
$N_{1983,a}^A$	10.79	10.77	10.77	10.83	10.79	10.79	10.78
k_N^A	-1.19	-1.27	-1.18	-0.78	-1.27	-1.19	-1.22
\bar{M}_j^A	1.2	1.2	1.2	1.2	1.2	1.2	1.2
\bar{M}_{ad}^A	1.2	1.2	1.2	1.2	1.2	1.2	1.2
σ_j	-	-	-	-	-	-	-
σ_{ad}	-	-	-	-	-	-	-
ρ	-	-	-	-	-	-	-
k_N^A	0.89 0.88 [0.74,1.04]	0.84 0.80 [0.66,0.97]	0.81 0.80 [0.68,0.94]	0.95 0.93 [0.79,1.10]	0.86 0.83 [0.70,0.98]	0.89 0.88 [0.73,1.04]	0.88 0.87 [0.73,1.03]
k_r^A	0.74 0.74 [0.58,0.92]	0.70 0.68 [0.51,0.85]	0.68 0.68 [0.53,0.86]	0.70 0.74 [0.56,0.95]	0.70 0.69 [0.56,0.83]	0.74 0.74 [0.56,0.92]	0.73 0.73 [0.57,0.91]
k_g^A	1.00	0.84 0.77 [0.56,1.04]	0.75	1.25	1.00	1.00	1.00
$(\lambda_N^A)^2$	0.00	0.00	0.00	0.00	0.00	0.00 0.01 ⁸ [0.00,0.01]	0.02
$(\lambda_r^A)^2$	0.11 0.25 [0.11,0.53]	0.12 0.26 [0.11,0.55]	0.12 0.26 [0.12,0.58]	0.10 0.24 [0.10,0.50]	0.00	0.11 0.24 [0.09,0.53]	0.09 0.24 [0.10,0.51]
a^A	1106 1107 [577,3169]	1161 1191 [634,3390]	1199 1206 [600,4372]	1184 1171 [609,5821]	1260 1354 [672,3170]	1106 1124 [559,4830]	1145 1125 [553,3211]
b^A	1616 1121 [66,6750]	1715 1155 [67,7352]	1780 1223 [65,9565]	1512 1158 [80,11869]	2021 1818 [153,7417]	1616 1141 [64,12141]	1725 1094 [42,7901]
K^A	2966 3318 [2003,6717]	3128 3647 [2223,7438]	3236 3670 [2196,8203]	2600 3108 [1815,8334]	2990 3659 [2260,6576]	2966 3435 [2038,8249]	3023 3313 [2027,6440]
h^A	0.41 0.50 [0.29,0.91]	0.41 0.52 [0.30,0.92]	0.41 0.51 [0.29,0.91]	0.40 0.48 [0.28,0.89]	0.38 0.43 [0.29,0.81]	0.41 0.50 [0.28,0.91]	0.41 0.50 [0.29,0.94]
σ_r^A	0.63 0.80 [0.59,1.07]	0.64 0.81 [0.61,1.11]	0.64 0.80 [0.62,1.09]	0.62 0.79 [0.60,1.06]	0.63 0.73 [0.57,0.98]	0.63 0.80 [0.59,1.07]	0.62 0.79 [0.59,1.07]
B_{2019}^A	1547 1424 [868,2246]	1633 1570 [969,2551]	1693 1574 [969,2458]	1496 1356 [816,2202]	1972 2015 [1393,2895]	1547 1480 [886,2452]	1720 1522 [910,2484]
$B_{2019}^{sp,A}$	1456 1343 [822,2079]	1540 1475 [916,2371]	1597 1482 [922,2320]	1362 1258 [763,1989]	1857 1893 [1312,2728]	1456 1383 [832,2286]	1618 1430 [853,2309]

⁸ Convergence not achieved, but as this was not considered a 'key' robustness test, a longer chain was not (yet) run.

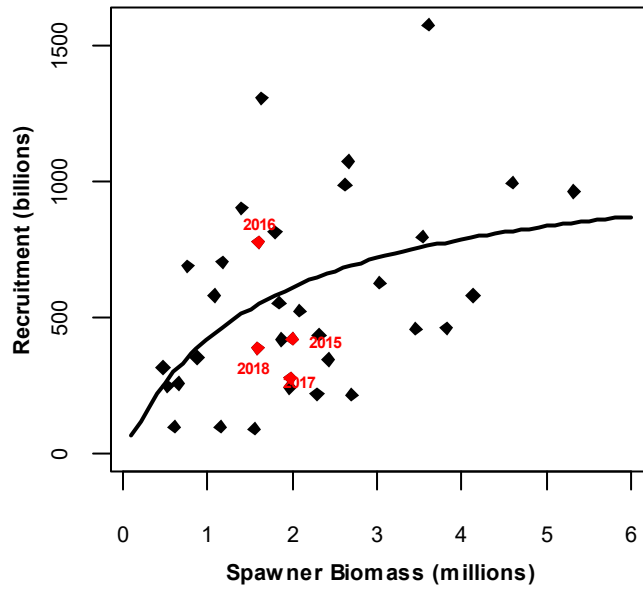


Figure 1. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2018, with the Beverton Holt stock recruitment relationship estimated for A_{BH} at the joint posterior mode.

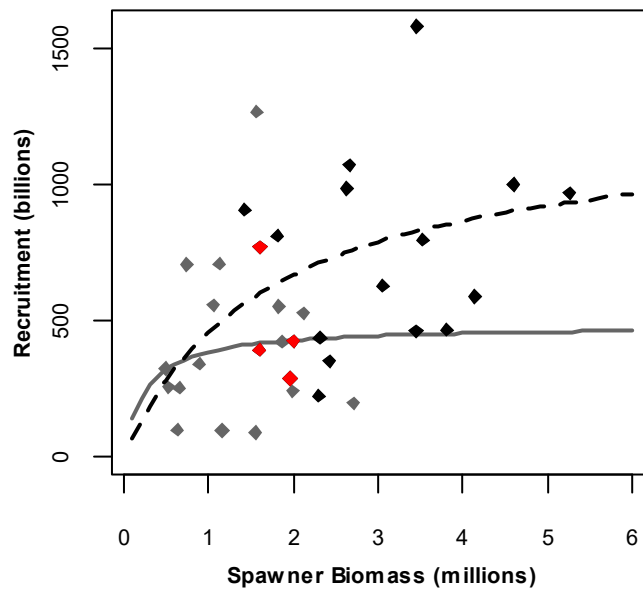


Figure 2. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to 2018, with the Beverton Holt stock recruitment relationships estimated for 1984-1999 (grey) and 2000-2018 (black) for A_{2BH} at the joint posterior mode.

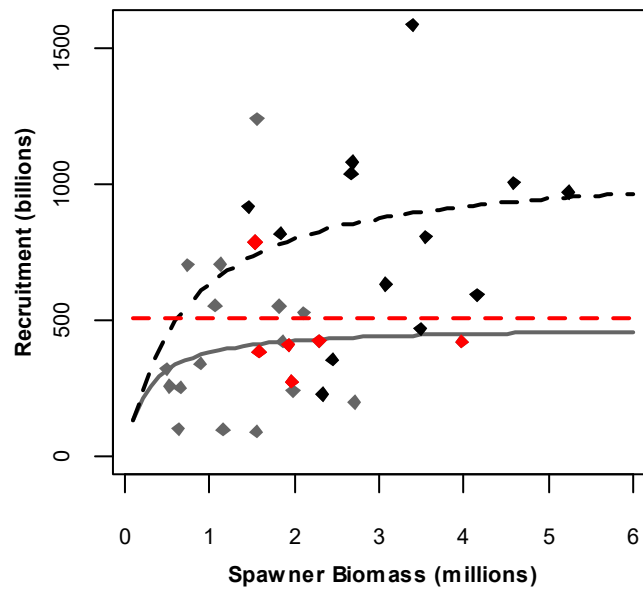


Figure 3. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to 2018, with the Beverton Holt stock recruitment relationships estimated for 1984-1999 (grey) and 2000-2012 (black) and 2013-2018 (red) for A_{3BH} at the joint posterior mode.

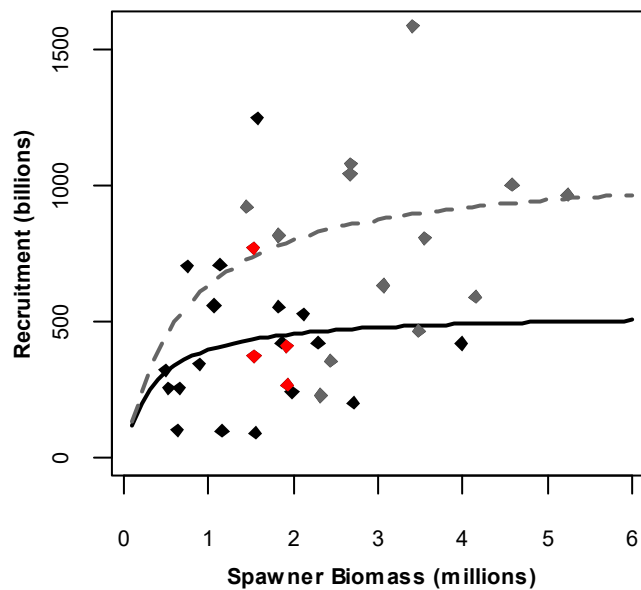


Figure 4. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to 2018, with the Beverton Holt stock recruitment relationships estimated for 1984-1999 and 2013-2018 (black) and 2000-2012 (grey) for $A_{2BHreturn}$ at the joint posterior mode.

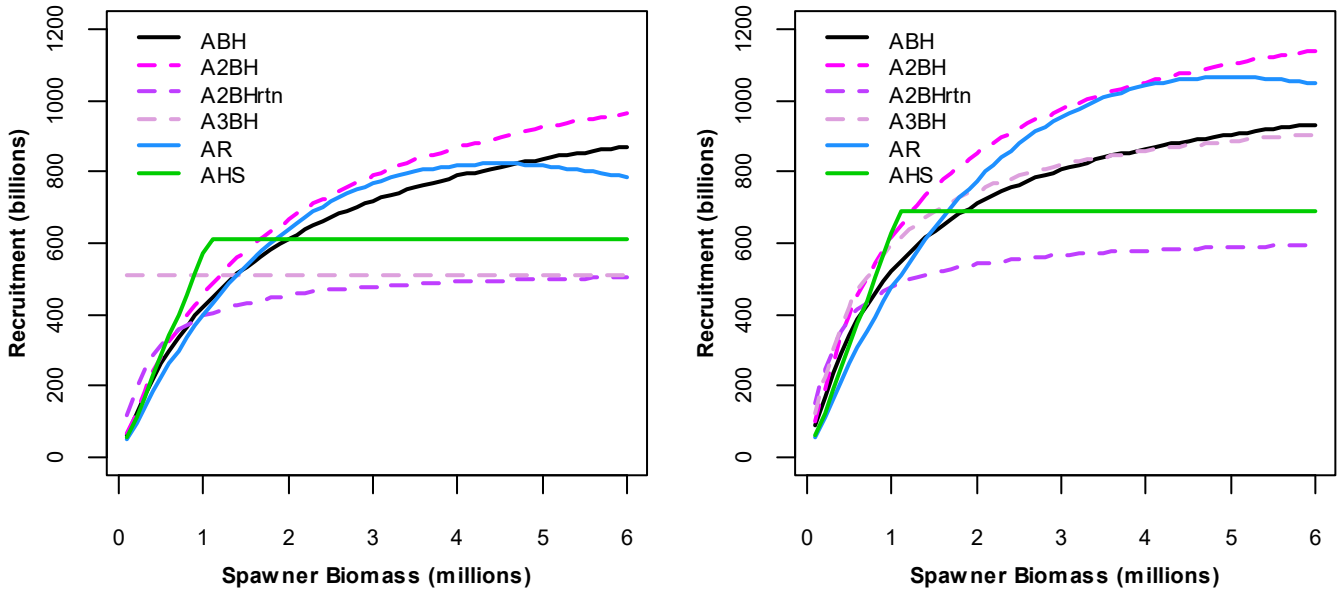


Figure 5. Estimates stock recruitment relationships corresponding to the most recent historical years, and thus to be used in future projections under A_{BH} , A_R and A_{HS} (no change over time), A_{2BH} (2000-2018), A_{3BH} (2013-2018), A_{2BHrtn} (2013-2018) as estimated at the joint posterior mode (left) and as the ‘posterior median’⁹ (right).

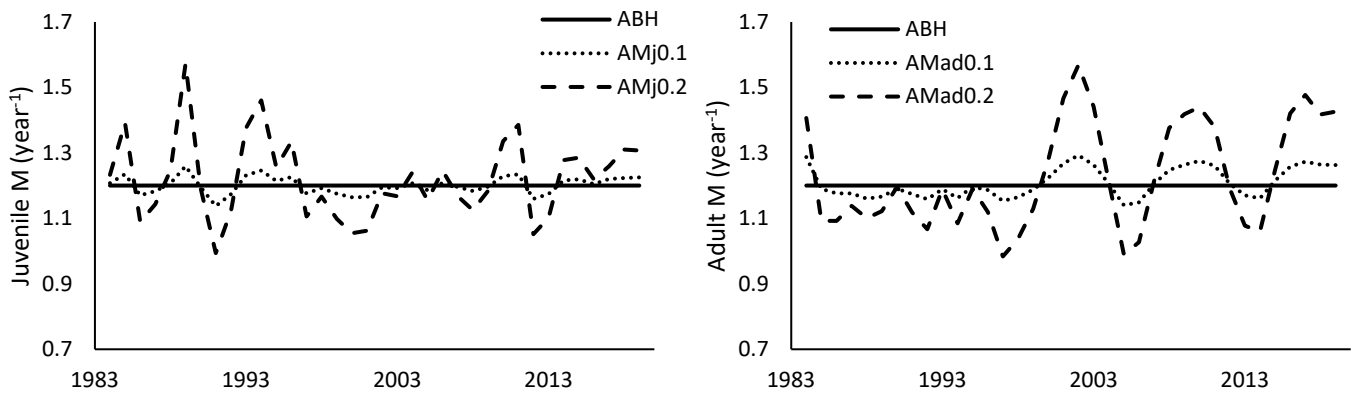


Figure 6. The annual juvenile and adult natural mortality rates estimated for A_{BH} , A_{Mj} ($\sigma_j = 0.1$ or 0.2) and A_{Mad} ($\sigma_{ad} = 0.1$ or 0.2).

⁹ These curves correspond to those with stock recruitment parameter values at the posterior median, rather than a median of all of the individual sampled stock recruitment relationships from the posterior distribution.

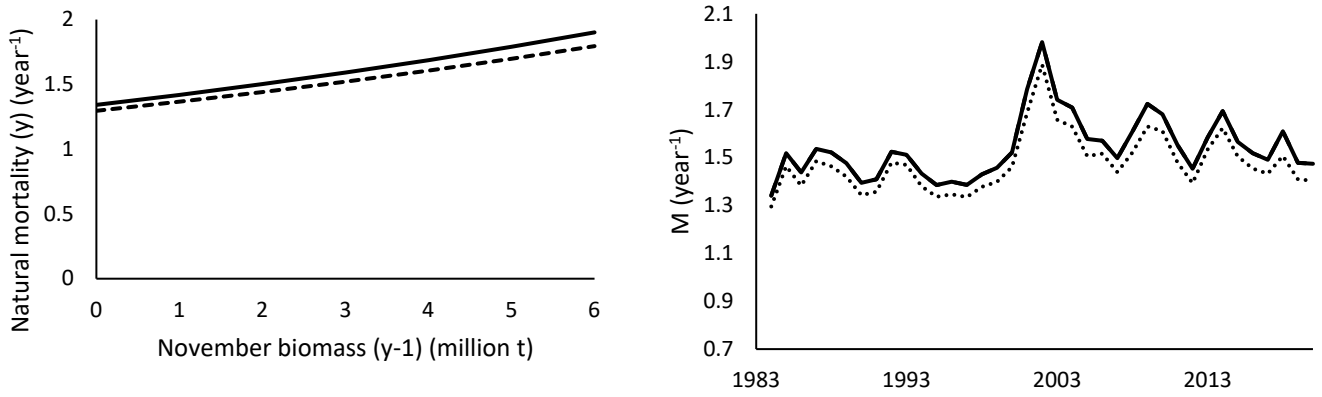


Figure 7. The estimated relationship between juvenile and adult natural mortality in year y and the November $y-1$ biomass (left) and the time series of natural mortality (right) under A_{DD} at the joint posterior mode (solid line) and posterior median (dotted line).

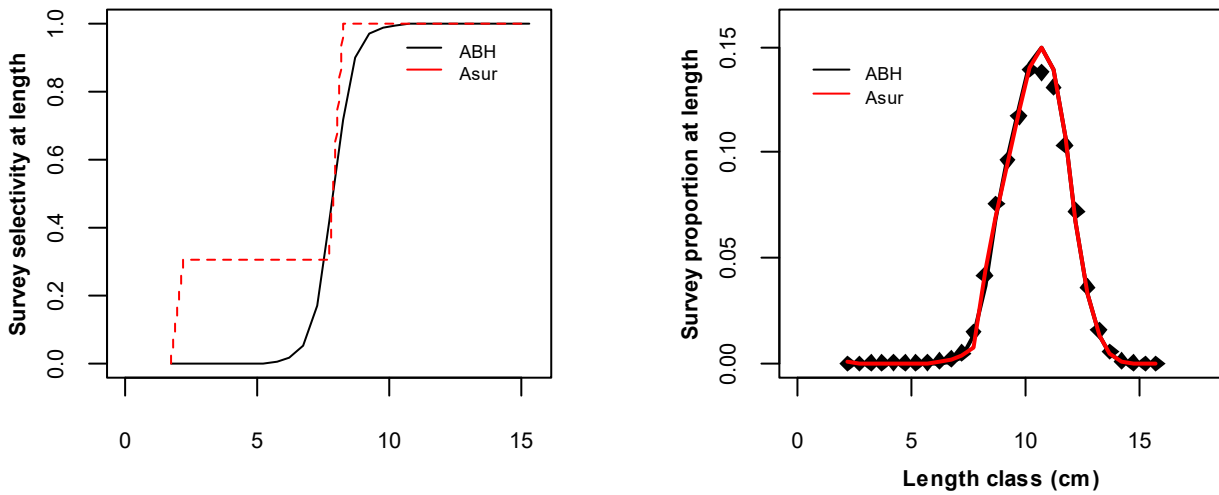


Figure 8. Model estimated trawl selectivity at length (left) and average (over all years) model predicted and observed proportions-at-length in the November survey trawls (right) for the baseline model (A_{BH}) and sensitivity test A_{sur} .

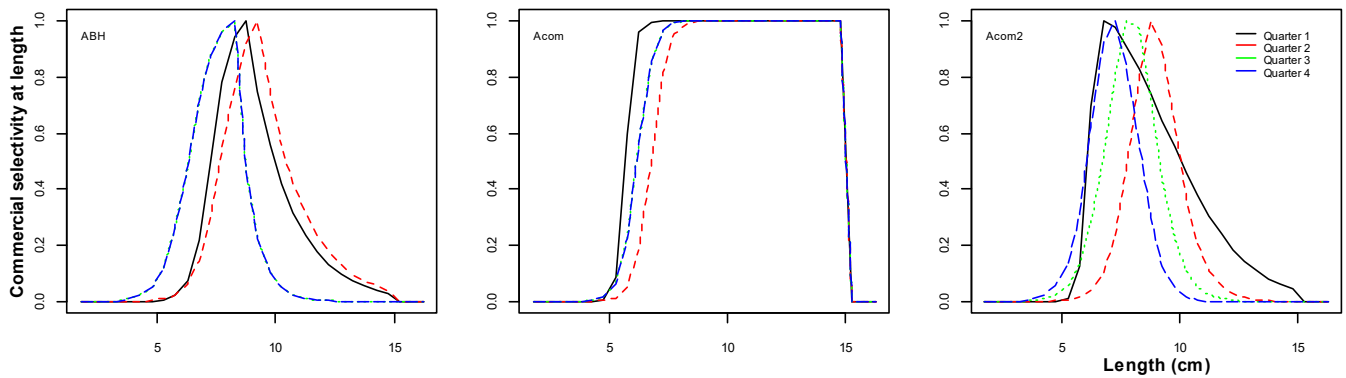


Figure 9. Model estimated quarterly commercial selectivity at length for A_{BH} , A_{com} and A_{com2} .

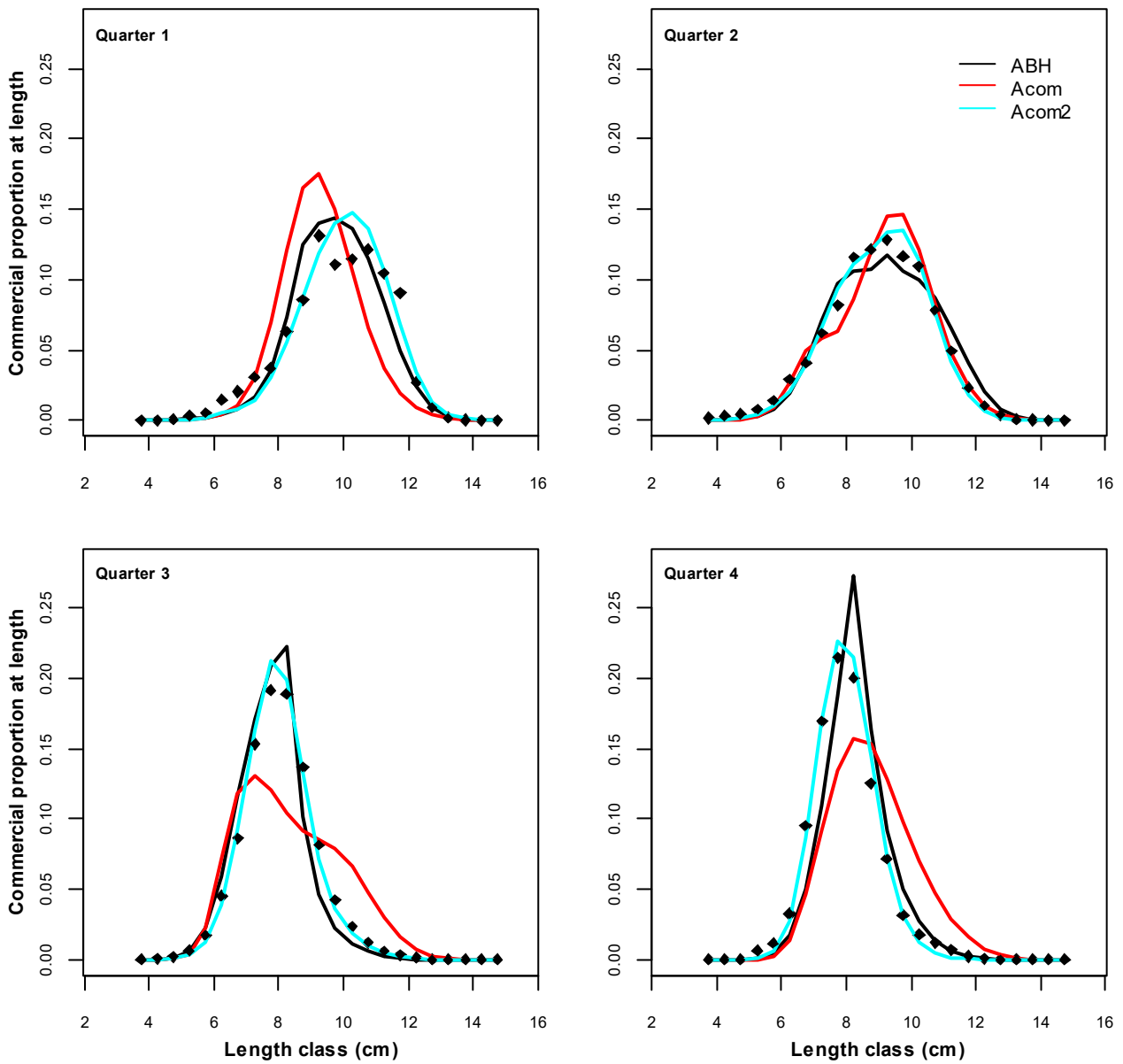


Figure 10. Average (over all years) model predicted and observed proportions-at-length in the quarterly commercial catch for A_{BH} , A_{com} and A_{com2} .

Appendix A: Bayesian operating model for the South African anchovy resource

In the below equations a “ $\hat{}$ ” is used to represent an estimate of a quantity (e.g. biomass) from a source external to this model (e.g. a survey). Model predicted quantities are represented by terms without any additional super-/sub-scripts other than dependencies on, for example, year, length etc.

Model Assumptions

- 1) All fish have a birthdate of 1 November.
- 2) Anchovy mature according to a length-based ogive with an annually varying L_{50} .
- 3) A plus group of age 4 is used.
- 4) A minus length class of 2cm and a plus length class of 16cm is used.
- 5) Natural mortality is age-invariant for fish aged 1 and older.
- 6) Two hydro-acoustic surveys are held each year: the first takes place in November and provides an index of abundance of the total biomass; the second is in May/June (known as the recruit survey) and provides an index of abundance of recruitment.
- 7) The November and recruit surveys provide relative indices of abundance of unknown bias.
- 8) The egg survey observations (derived from data collected during the earlier November surveys) provide estimates of spawner biomass in absolute terms.
- 9) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
- 10) Pulse fishing occurs four times a year, in the middle of each quarter of the assessment year (November to October).

Population Dynamics

The basic dynamic equations for anchovy, based on Pope’s approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2019$.

Numbers-at-age at 1 November

$$N_{y,a}^A = \left(\left(\left(\left(N_{y-1,a-1}^A e^{-M_{a-1,y}^A/8} - C_{y,1,a-1}^A \right) e^{-M_{a-1,y}^A/4} - C_{y,2,a-1}^A \right) e^{-M_{a-1,y}^A/4} - C_{y,3,a-1}^A \right) e^{-M_{a-1,y}^A/4} - C_{y,4,a-1}^A \right) e^{-M_{a-1,y}^A/8}$$

$$y_1 \leq y \leq y_n, 1 \leq a \leq 3$$

$$N_{y,4+}^A = \left(\left(\left(\left(N_{y-1,3}^A e^{-M_{3,y}^A/8} - C_{y,1,3}^A \right) e^{-M_{3,y}^A/4} - C_{y,2,3}^A \right) e^{-M_{3,y}^A/4} - C_{y,3,3}^A \right) e^{-M_{3,y}^A/4} - C_{y,4,3}^A \right) e^{-M_{3,y}^A/8}$$

$$+ \left(\left(\left(\left(N_{y-1,4+}^A e^{-M_{4+,y}^A/8} - C_{y,1,4+}^A \right) e^{-M_{4+,y}^A/4} - C_{y,2,4+}^A \right) e^{-M_{4+,y}^A/4} - C_{y,3,4+}^A \right) e^{-M_{4+,y}^A/4} - C_{y,4,4+}^A \right) e^{-M_{4+,y}^A/8}$$

$$y_1 \leq y \leq y_n \quad (\text{A.1})$$

Numbers-at-length at 1 November

The model estimated numbers-at-length range from a 1.5cm minus group to a 16cm plus group, denoted 1.5^- and 16^+ , respectively, in the remaining text. The length class sizes are 0.5cm and, where length is used in an equation, the mid-point of the length class is used. The model predicted numbers-at-length at the time of the survey are:

$$N_{y,l}^A = \sum_{a=0}^{4+} A_{a,l}^{sur} N_{y,a}^A \quad y_1 \leq y \leq y_n, 1.5^- cm \leq l \leq 16^+ cm \quad (A.2)$$

The model predicted numbers-at-length of ages 1+ only are given by:

$$N_{y,l}^{A,1+} = \sum_{a=1}^{4+} A_{a,l}^{sur} N_{y,a}^A \quad y_1 \leq y \leq y_n, 1.5^- cm \leq l \leq 16^+ cm \quad (A.3)$$

The proportion of anchovy of age a that fall in the length group l at 1 November matrix, $A_{a,l}^{sur}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{a,l}^{sur} \sim N(L_{\infty}(1 - e^{-\kappa(a-t_0)}), \vartheta_a^2) \quad 0 \leq a \leq 4^+, 1.5^- cm \leq l \leq 16^+ cm \quad (A.4)^{10}$$

Natural mortality

Natural mortality is modelled to vary annually around a median as follows:

$$M_{0,y}^A = \bar{M}_y^A e^{\varepsilon_{j,y}} \text{ with } \varepsilon_{1984}^j = \eta_{1984}^j \text{ and } \varepsilon_y^j = \rho \varepsilon_{y-1}^j + \eta_y^j \sqrt{1 - \rho^2}, y > y_1 \quad (A.5)$$

$$M_{1+,y}^A = \bar{M}_{ad}^A e^{\varepsilon_{ad,y}} \text{ with } \varepsilon_{1984}^{ad} = \eta_{1984}^{ad} \text{ and } \varepsilon_y^{ad} = \rho \varepsilon_{y-1}^{ad} + \eta_y^{ad} \sqrt{1 - \rho^2}, y > y_1 \quad (A.6)$$

Biomass associated with the November survey

$$B_y^A = \sum_{l=1.5^-}^{16^+} N_{y,l}^A w_l^A \quad y_1 \leq y \leq y_n \quad (A.7)$$

November spawner biomass

The spawning stock biomass is:

$$SSB_y^A = \sum_{l=1.5^-}^{16^+} f_{y,l}^A N_{y,l}^{A,1+} w_l^A \quad y_1 \leq y \leq y_n \quad (A.8)$$

where

$$f_y^l = 1 / (1 + e^{-(l-L_{50,y})/\delta^{mat}}) \quad y_1 \leq y \leq y_n \quad (A.9)$$

Commercial selectivity

Commercial selectivity-at-length is assumed to follow the logistic shape, with a dome at high lengths. Commercial selectivity is assumed to vary by quarter, but remain unchanged over time. Selectivity-at-lengths less than the smallest observed length class (3.5cm) and greater than the largest observed length class (14.5cm) are taken to be zero. Thus we have:

$$S_{y,q,l} = \begin{cases} 0 & 1.5^- cm \leq l \leq 3cm \\ 1/(1 + e^{\psi_q(l-150_q)}) & 3.5cm \leq l \leq S_q^{break} \\ S_{y,q,l-1} e^{\delta_q} & S_q^{break} \leq l \leq 14.5cm \\ 0 & 15cm \leq l \leq 16^+ cm \end{cases} \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4 \quad (A.10)^{11}$$

Commercial selectivity-at-age is given by:

$$S_{y,q,a} = \sum_{l=1.5^-}^{16^+} A_{q,a,l}^{com} S_{y,q,l} \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4, 0 \leq a \leq 4^+ \quad (A.11)$$

The proportion of anchovy of age a that fall in the length group l in quarter q , $A_{q,a,l}^{com}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{q,a,l}^{com} \sim N\left(L_{\infty}(1 - e^{-\kappa(a+(2q-1)/8-t_0)}), \left(1 - \frac{2q-1}{8}\right) \vartheta_a^2 + \frac{2q-1}{8} \vartheta_{a+1}^2\right)$$

¹⁰ The proportion is calculated as the area under the curve between the lower limit and upper limit of length class l . The lower and upper tails are included in the proportions calculated for the minus and plus groups, respectively.

¹¹ These selectivities-at-length are renormalized so that the maximum is 1.

$$1 \leq q \leq 4, 0 \leq a \leq 4^+, 1.5^- \text{ cm} \leq l \leq 16^+ \text{ cm} \text{ (A.12)}^{12}$$

Commercial catch

Anchovy quarterly pulse catches are split between ages using a model estimated selectivity:

$$\begin{aligned} C_{y,1,a}^A &= N_{y-1,a}^A e^{-M_{a,y}^A/8} S_{y,1,a} F_{y,1} \\ C_{y,2,a}^A &= \left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} S_{y,2,a} F_{y,2} \\ C_{y,3,a}^A &= \left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} S_{y,3,a} F_{y,3} \\ C_{y,4,a}^A &= \left(\left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,3,a}^A \right) e^{-M_{a,y}^A/4} S_{y,4,a} F_{y,4} \end{aligned}$$

$$y_1 \leq y \leq y_n, 0 \leq a \leq 4^+ \quad (\text{A.13})$$

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion of the available biomass from the anchovy fishery is estimated by:

$$\begin{aligned} F_{y,1} &= \frac{\sum_{m=11}^{12} \sum_{l=3.5}^{14.5} C_{y-1,m,l}^{RLF} + \sum_{l=3.5}^{14.5} C_{y,1,l}^{RLF}}{\sum_{a=0}^{4^+} N_{y-1,a}^A e^{-M_{a,y}^A/8} S_{y,1,a}} \\ F_{y,2} &= \frac{\sum_{m=2}^4 \sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4^+} \left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} S_{y,2,a}} \\ F_{y,3} &= \frac{\sum_{m=5}^7 \sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4^+} \left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} S_{y,3,a}} \\ F_{y,4} &= \frac{\sum_{m=8}^{10} \sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4^+} \left(\left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,3,a}^A \right) e^{-M_{a,y}^A/4} S_{y,4,a}} \end{aligned}$$

$$y_1 \leq y \leq y_n \text{ (A.14)}^{13}$$

A penalty is imposed within the model to ensure that $S_{y,l} F_{y,q} < 0.95$ for all l .

Recruitment

Recruitment at the beginning of November is assumed to fluctuate lognormally about a stock-recruitment curve (see Table 1):

$$N_{y,0}^A = f(SSB_y^A) e^{\varepsilon_y^A - 0.5(\sigma_r^A)^2} \quad y_1 \leq y \leq y_n \quad (\text{A.15})$$

Number of recruits at the time of the recruit survey

The following equation projects $N_{y,0}^A$ to the start of the recruit survey, taking natural and fishing mortality into account:

$$N_{y,r}^A = \left(\left(\left(N_{y-1,0}^A e^{-M_{0,y}^A/8} - C_{y,1,0}^A \right) e^{-M_{0,y}^A/4} - C_{y,2,0}^A \right) e^{-(1/8+0.5t_y/12)M_{0,y}^A} - C_{y,0bs}^A \right) e^{-(0.5t_y/12)M_{0,y}^A}$$

$$y_2 \leq y \leq y_n \quad (\text{A.16})$$

The recruit catch from 1 May to the day before the survey is calculated as follows

¹² The proportion is calculated as the area under the curve between the lower limit and upper limit of length class l . The lower and upper tails are included in the proportions calculated for the minus and plus groups, respectively.

¹³ The range of length classes used in these summation matches the range of length classes in the observations which is a smaller range than the maximum range modelled of 2 cm to 16 cm.

$$C_{y,0bs}^A = \left((N_{y-1,0}^A e^{-M_{0,y}^A/8} - C_{y,1,0}^A) e^{-M_{0,y}^A/4} - C_{y,2,0}^A \right) e^{-(1/8+0.5t_y/12)M_{0,y}^A} S_{y,3,0} F_{y,bs} \quad y_2 \leq y \leq y_n \quad (\text{A.17})$$

where

$$F_{y,bs} = \frac{\sum_{l=3.5}^{14.5} C_{y,bs,l}^{RLF}}{\sum_{a=0}^{4+} \left((N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-(1/8+0.5t_y/12)M_{a,y}^A} S_{y,3,a}} \quad y_2 \leq y \leq y_n \quad (\text{A.18})$$

A penalty is imposed within the model to ensure that $S_{y,l} F_{y,bs} < 0.95$ for all l .

Proportion-at-length associated with the November survey

The model predicted proportion-at-length associated with the November survey is¹⁴:

$$p_{y,l}^A = \frac{N_{y,l}^A S_l^{survey}}{\sum_{l=2}^{15.5} N_{y,l}^A S_l^{survey}} \quad y_1 \leq y \leq y_n, \quad (\text{A.19})$$

where

$$S_l^{survey} = \begin{cases} 0 & l = 1.5^- \\ \frac{1}{1 + \exp(-(l - l^{sur})/\delta^{sur})} & 2cm \leq l \leq 15.5cm \\ 0 & l = 16^+ \end{cases} \quad (\text{A.20})$$

Proportion-at-length associated with the commercial catch

The commercial catch-at-length from the anchovy fishery is:

$$\begin{aligned} C_{y,1,l}^A &= \sum_{a=0}^{4+} N_{y-1,a}^A e^{-M_{a,y}^A/8} A_{1,a,l}^{com} S_{y,1,l} F_{y,1} \\ C_{y,2,l}^A &= \sum_{a=0}^{4+} \left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} A_{2,a,l}^{com} S_{y,2,l} F_{y,2} \\ C_{y,3,l}^A &= \sum_{a=0}^{4+} \left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} A_{3,a,l}^{com} S_{y,3,l} F_{y,3} \\ C_{y,4,l}^A &= \sum_{a=0}^{4+} \left(\left(\left(N_{y-1,a}^A e^{-M_{a,y}^A/8} - C_{y,1,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,2,a}^A \right) e^{-M_{a,y}^A/4} - C_{y,3,a}^A \right) e^{-M_{a,y}^A/4} A_{4,a,l}^{com} S_{y,4,l} F_{y,4} \end{aligned}$$

$$y_1 \leq y \leq y_n, 1.5^- cm \leq l \leq 16^+ cm \quad (\text{A.21})$$

The model predicted proportion-at-length by quarter in the commercial catch¹⁵ is:

$$p_{y,q,l}^{coml,A} = \frac{C_{y,q,l}^A}{\sum_{l=3.5}^{14.5} C_{y,q,l}^A} \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4, 3.5cm \leq l \leq 14.5cm \quad (\text{A.22})$$

Fitting the Model to Observed Data (Likelihood)

The survey observations of abundance are assumed to be log-normally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. A “sqrt(p)” formulation, rather than the “adjusted lognormal” (“Punt-Kennedy”, Punt and Kennedy 1997) error distribution formulation, is assumed for the estimated proportions-at-length particularly as it can deal with occasional zero observations more easily. This “sqrt(p)” formulation mimics a multinomial form for the error distribution by forcing near-equivalent variance-mean relationship for the error distributions. The negative log-likelihood function is given by:

¹⁴ Note the model predicted survey proportion of lengths 1.5 cm and 16⁺cm is zero, given a zero survey trawl selectivity in equation (A.19). This is consistent with the range of length classes in the observed trawl survey proportions-at-length.

¹⁵ Note there model predicted commercial catch of lengths <3.5cm and >14.5cm is zero, from a zero commercial selectivity in equation (A.9). This is consistent with the range of length classes in the observed commercial proportions-at-length.

$$-\ln L = -\ln L^{Nov} - \ln L^{Egg} - \ln L^{rec} - \ln L^{surpropl} - \ln L^{compropl} \quad (\text{A.23})$$

where

$$-\ln L^{Nov} = \frac{1}{2} \sum_{y=y_1}^{y_n} \left\{ \frac{(\ln \hat{B}_y^A - \ln(k_N^A B_y^A))^2}{(\sigma_{y,N}^A)^2 + (\phi_{ac}^A)^2 + (\lambda_N^A)^2} + \ln \left(2\pi \left((\sigma_{y,N}^A)^2 + (\phi_{ac}^A)^2 + (\lambda_N^A)^2 \right) \right) \right\} \quad (\text{A.24})$$

$$-\ln L^{Egg} = \frac{1}{2} \sum_{y=y_1}^{1993} \left\{ \frac{(\ln \hat{B}_{y,egg}^A - \ln(k_{gSSB}^A B_y^A))^2}{(\sigma_{y,egg}^A)^2} + \ln \left(2\pi (\sigma_{y,egg}^A)^2 \right) \right\} \quad (\text{A.25})$$

$$-\ln L^{rec} = \frac{1}{2} \sum_{y=y_1+1}^{2017,2019} \left\{ \frac{(\ln \hat{N}_{y,r}^A - \ln(k_r^A N_{y,r}^A))^2}{(\sigma_{y,r}^A)^2 + (\phi_{ac}^A)^2 + (\lambda_r^A)^2} + \ln \left(2\pi \left((\sigma_{y,r}^A)^2 + (\phi_{ac}^A)^2 + (\lambda_r^A)^2 \right) \right) \right\} \quad (\text{A.26})$$

$$-\ln L^{surpropl} = w_{propl}^{sur} \sum_{y=y_1}^{y_n} \sum_{l=2}^{15.5} \left\{ \frac{(\sqrt{\hat{p}_{y,l}^A} - \sqrt{p_{y,l}^A})^2}{2(\sigma_{sur}^A)^2} + \ln(\sigma_{sur}^A) \right\}^{16} \quad (\text{A.27})$$

$$-\ln L^{compropl} = w_{propl}^{com} \sum_{y=y_1}^{y_n} \sum_{q=1}^4 \sum_{l=3.5}^{14.5} \left\{ \frac{(\sqrt{\hat{p}_{y,q,l}^{A,coml}} - \sqrt{p_{y,q,l}^{A,coml}})^2}{2(\sigma_{com}^A)^2} + \ln(\sigma_{com}^A) \right\} \quad (\text{A.28})$$

¹⁶ Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

Table A.1. Assessment model parameters and variables.

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$N_{y,a}^A$	Model predicted numbers-at-age a at the beginning of November in year y	Billions		A.1		
Annual numbers and biomass	$N_{1983,a}^A$	Initial numbers-at-age a	Billions	$N_{1983,0}^A \sim N(51, 30^2)$ $N_{1983,1}^A \sim N(143, 20^2)$ $N_{1983,2}^A \sim N(349.6, 5^2)$	$N_{1983,3}^A =$ $N_{1983,2}^A e^{-M_{2,1}^A}$ $N_{1983,4+}^A =$ $N_{1983,3}^A \frac{e^{-M_3^A}}{1 - e^{-M_3^A}}$	Assumed $M_{ad,1983}^A = M_{ad,1984}^A$
	$N_{y,l}^A$	Model predicted numbers-at-length l at the beginning of November in year y	Billions		A.2	
	$N_{y,l}^{A,1+}$	Model predicted numbers-at-length length l at the beginning of November in year y of anchovy ages 1+ only	Billions		A.3	
	B_y^A	Model predicted total biomass at the beginning of November in year y	Thousand tons		A.7	
	w_l^A	Mean mass of anchovy of length l^{17} (in cm) during November	Grams	$w_l^A = 0.0079 \times l^{3.0979}$		Using model viii) in November from de Moor and Butterworth (2015)
	SSB_y^A	Model predicted spawning biomass at the beginning of November in year y	Thousand tons		A.8	
	$f_{y,l}^A$	Proportion of anchovy of length l (in cm) that are mature in year y			A.9	de Moor (2020a)
$L_{50,y}$	Length at 50% maturity in year y	Cm	Table A.3		de Moor (2020a)	
δ^{mat}	Rate of increase in maturity at length	-	0.34245		de Moor (2020a)	
Catch	$C_{y,q,a}^A$	Model predicted number of anchovy of age a caught during quarter q^{18} from 1 November $y-1$ to 31 October y	Billions		A.12	
	$F_{y,q}$	Fished proportion in quarter q of year y for a fully selected length class l	-		A.13	
	$C_{y,obs}^A$	Number of recruits caught between 1 May and the day before the start of the recruit survey in year y	Billions		A.16	
	$F_{y,bc}$	Fished proportion between 1 May and the day before the start of the recruit survey in year y	-		A.17	

¹⁷ Where length is required in an equation, the mid-point of the length class is used.

¹⁸ The quarters are $q = 1$: November-January; $q = 2$: February-April; $q = 3$: May-July; $q = 4$: August-October.

Table A.1 (continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
M_a^A	Rate of natural mortality of age a	Year ⁻¹		A.5 and A.6	From de Moor (2016)	
Natural Mortality	\bar{M}_j^A	Median rate of natural mortality for age-0 anchovy	Year ⁻¹	1.2		
	\bar{M}_{ad}^A	Median rate of natural mortality for 1+ anchovy	Year ⁻¹	1.2		
	ε_y^j	Annual residuals about natural mortality rate for age-0 anchovy	-		A.5	
	ε_y^{ad}	Annual residuals about natural mortality rate for 1+ anchovy	-		A.6	
	η_y^j	Normally distributed error in calculating ε_y^j	-	$\sim N(0, \sigma_j^2)$		
	η_y^{ad}	Normally distributed error in calculating ε_y^{ad}	-	$\sim N(0, \sigma_{ad}^2)$		
	σ_j	Standard deviation in the annual residuals η_y^j	-	0		From de Moor (2016)
	σ_{ad}	Standard deviation in the annual residuals η_y^{ad}	-	0		From de Moor (2016)
ρ	Annual autocorrelation coefficient	-	0		From de Moor (2016)	
S_l^{survey}	November survey trawl selectivity-at-length l	-		A.19		
Selectivity	l^{sur}	Length class number at which the survey selectivity-at-length is 50%	Length class	$U(3,21)$ ¹⁹		
	δ^{sur}	Steepness of the survey selectivity-at-length relationship		$U(0.005,5)$		
	$S_{y,q,l}$	Commercial selectivity-at-length l during quarter q of year y	-		A.9	
	$S_{y,q,a}$	Commercial selectivity-at-age a during quarter q of year y	-		A.10	
	ψ_q	Steepness of ascending limb of logistic part of commercial selectivity curve during quarter q	-	$\sim U(-10,0), \psi_2 = \psi_3 = \psi_4$		Uninformative
	$l50_q$	Length at which ascending limb of logistic part of commercial selectivity is 50% during quarter q	Cm	$\sim U(3,10), l50_3 = l50_4$		Uninformative
	δ_q	Rate of exponential decrease in commercial selectivity at large lengths during quarter q	-	$\delta_1 = \delta_2 \sim N(-0.38, 0.5^2)$ $\delta_3 = \delta_4 \sim N(-0.75, 0.04^2)$		See Appendix B
	S_q^{break}	Length at which commercial selectivity starts to decrease during quarter q	Length class	$S_1^{break} = 15; S_2^{break} = 16;$ $S_3^{break} = S_4^{break} = 14$		From de Moor (2016)

¹⁹ Length class 3 corresponds to 2.5cm, length class 21 corresponds to 11.5cm and length classes 14-16 correspond to 8.5-9.5cm.

Table A.1 (Continued).

Parameter/ Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
h^A	Steepness associated with the stock-recruitment curve ²⁰	-	$\sim U(0.2,1)$			
K^A	Carrying capacity	Thousand tons	$K^A/1000 \sim U(0,10)$			
Recruitment	α^A	Stock-recruitment curve parameter, related to h^A and K^A , for Beverton Holt and Ricker curves	-	$\alpha^A = \frac{4h^A K^A}{(5h^A - 1) \left(\sum_{a=1}^3 \bar{f}_a^A w_a^A e^{-\bar{M}_j^A - (a-1)\bar{M}_{ad}^A} + \bar{f}_{4+}^A w_{4+}^A e^{-\bar{M}_j^A - 3\bar{M}_{ad}^A} / (1 - e^{-\bar{M}_{ad}^A}) \right)}$ $\bar{f}_a^A = \sum_{y=y_1}^{y_n} \sum_{l=1.5^+}^{16^+} f_{y,l}^A A_{a,l}^{sur} \text{ and } w_a^A = \sum_{l=1.5^+}^{16^+} w_l^A A_{a,l}^{sur}$		
	β^A	Stock-recruitment curve parameter, related to h^A and K^A , for Beverton Holt and Ricker curves	Thousand tons	$\beta^A = \frac{(1 - h^A)K^A}{(5h^A - 1)}$		
	ε_y^A	Annual lognormal deviation of recruitment	-	$\sim N(0, (\sigma_r^A)^2), y_1 \leq y \leq 1999$ $\sim N(0, (\sigma_{r,2000+}^A)^2), 2000 \leq y \leq y_{n-1}$		
	$(\sigma_r^A)^2$	Variance in the residuals (lognormal deviation) about the stock recruitment curve	-	$\sim U(0.16,10)$		Lower bound chosen to restrict the influence of the stock recruitment curve on the assessment results
	$N_{y,r}^A$	Model predicted number of recruits at the time of the recruit survey in year y	Billions		A.15	

²⁰ The proportion of the median virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass, K^A .

Table A.1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
Multiplicative bias	k_N^A				de Moor <i>et al.</i> (2020a)	
		Multiplicative bias associated with the November acoustic survey	-	$\ln(k_N^A) \sim N(-0.158, 0.112^2)$		
	k_g^A	Multiplicative bias associated with the November egg survey	-	1.0	From de Moor <i>et al.</i> (2020b)	
					Recruit survey assumed to cover less of the recruits than the November survey covers of the total biomass	
	k_r^A	Multiplicative bias associated with the recruit survey	-	$k_r^A / k_N^A \sim U(0,1)$		
Proportions-at-length and growth	$p_{y,l}^A$	Model predicted proportion-at-length l associated with the November survey in year y	-		A.18	
	$A_{a,l}^{sur}$	Proportion of anchovy-at-age a that fall in the length group l in November	-		A.4	
	$p_{y,q,l}^{com,A}$	Model predicted proportion-at-length l in the commercial catch during quarter q of year y	-		A.21	
	$A_{q,a,l}^{com}$	Proportion of anchovy-at-age a that fall in the length group l in quarter q	-		A.11	
	L_∞	Maximum length (in expectation) of anchovy	Cm	$\sim N(11.05, 1.105^2)$		From de Moor (2016)
	κ	Annual somatic growth rate of anchovy	Year ⁻¹	$\frac{\kappa \times L_\infty}{10} \sim N(2.915, 0.292^2)$		From de Moor (2016)
	t_0	Age at which the length (in expectation) is zero	Year	$\sim N(0.112, 0.1^2)$		From de Moor (2016)
	ϑ_a	Standard deviation of the distribution about the mean length for age a	-	$\vartheta_0 \sim N(2.0, 0.15^2)$ $\vartheta_1 \sim N(1.2, 0.18^2)$ $\vartheta_{2+} \sim N(1.0, 0.1^2)$		From de Moor (2016)
Further output	s_{cor}^A	Recruitment serial correlation	-	$\frac{\sum_{y=y_1}^{y_n-1} \varepsilon_y^A \varepsilon_{y+1}^A}{\sqrt{\sum_{y=y_1}^{y_n-2} (\varepsilon_y^A)^2 \sum_{y=y_1}^{y_n-2} (\varepsilon_{y+1}^A)^2}}$		
	$\eta_{y_n-1}^A$	Standardised recruitment residual value for final year	-		$\frac{\varepsilon_{y_n-1}^A}{\sigma_{r,2000+}^A}$	

Table A.1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$-\ln L^{Nov}$	Contribution to the negative log likelihood from the model fit to the November total survey biomass data	-		A.23		
$-\ln L^{Egg}$	Contribution to the negative log likelihood from the model fit to the November egg survey spawner biomass data	-		A.24		
$-\ln L^{rec}$	Contribution to the negative log likelihood from the model fit to the recruit survey data	-		A.25		
$-\ln L^{surpropL}$	Contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data	-		A.26		
$-\ln L^{compropL}$	Contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data	-		A.27		
Likelihood	$(\lambda_N^A)^2$	Additional variance, over and above $(\sigma_{y,N}^A)^2$, associated with the November survey	-	0		See robustness tests
	$(\lambda_r^A)^2$	Additional variance, over and above $(\sigma_{y,r}^A)^2$, associated with the recruit survey	-	$\sim U(0,100)$		Uninformative
	$(\phi_{ac}^A)^2$	Additional hydro-acoustic survey variance	-	0.197 ²		de Moor <i>et al.</i> (2020a)
	w_{prop}^{sur}	Weighting applied to the survey proportion-at-length data	-	0.2		To allow for autocorrelation ²¹
	σ_{sur}^A	Standard deviation associated with the survey proportion-at-length data	-	$\sqrt{\sum_{y=y_1}^{y_n} \sum_{l=7}^{13} (\sqrt{\hat{p}_{y,l}^A} - \sqrt{p_{y,l}^A})^2} / \sum_{y=y_1}^{y_n} \sum_{l=7}^{13} 1}$		Closed form solution ²²
	w_{prop}^{com}	Weighting applied to the commercial proportion-at-length data	-	0.05		To allow for autocorrelation ²³
	σ_{com}^A	Standard deviation associated with the commercial proportion-at-length data	-	$\sqrt{\sum_{y=y_1}^{y_n} \sum_{q=1}^4 \sum_{l=5}^{12} (\sqrt{\hat{p}_{y,q,l}^{A,coml}} - \sqrt{p_{y,q,l}^{A,coml}})^2} / \sum_{y=y_1}^{y_n}$		Closed form solution ²⁴

²¹ Based upon data being available ~5 times more frequently than annual age data which contain maximum information content on this

²² A shorter range of lengths ($7cm \leq l \leq 13cm$) is used given the near absence of data outside this range, resulting in small/zero residuals, which would negatively bias this estimate.

²³ Based upon data being available ~4x5 times more frequently than annual age data which contain maximum information content on this

²⁴ A shorter range of lengths ($5cm \leq l \leq 12cm$) is used given the near absence of data outside this range, resulting in small/zero residuals, which would negatively bias this estimate.

Table A.2. Assessment model data, detailed in de Moor *et al.* (2020c).

Quantity	Description	Units / Scale	Shown in Figure
$C_{y,m,l}^{RLF}$	Observed number of anchovy in length class l caught during month m of year y ²⁵	Billions	
$C_{y,bs}^{RLF}$	Observed number of anchovy in length class l caught from 1 May to the day before the start of the recruit survey in year y	Billions	
t_y	Time lapsed between 1 May and the start of the recruit survey in year y	Months	
\hat{B}_y^A	Acoustic survey estimate of total biomass from the November survey in year y	Thousand tons	Figure 1
$\sigma_{y,Nov}^A$	Survey sampling CV associated with \hat{B}_y^A that reflects survey inter-transect variance	-	Figure 1
$\hat{B}_{y,egg}^A$	Egg survey estimate of spawner biomass from the November survey in year y	Thousand tons	Figure 2
$\sigma_{y,egg}^A$	Survey sampling CV associated with $\hat{B}_{y,egg}^A$ estimated from inter-transect variance		Figure 2
$\hat{N}_{y,r}^A$	Acoustic survey estimate of recruitment from the recruit survey in year y	Billions	Figure 3
$\sigma_{y,r}^A$	Survey sampling CV associated with $\hat{N}_{y,r}^A$ that reflects survey inter-transect variance	-	Figure 3
$\hat{p}_{y,l}^A$	Observed proportion (by number) of anchovy in length group l in the November survey of year y	-	
$\hat{p}_{y,q,l}^{A,com}$	Observed proportion (by number) of anchovy commercial catch in length group l during quarter q of year y		

Table A.3. Annual length (in cm) at which maturity is 50% (de Moor 2020a).

Year	$L_{50,y}$	Year	$L_{50,y}$	Year	$L_{50,y}$
1984	9.1134	1996	8.9407	2008	8.9242
1985	9.0202	1997	9.2542	2009	8.7953
1986	9.3520	1998	9.5677	2010	9.1743
1987	9.2447	1999	9.4205	2011	8.0616
1988	8.3041	2000	9.2733	2012	8.5990
1989	8.1260	2001	9.1262	2013	8.4963
1990	8.3041	2002	8.9790	2014	9.5811
1991	8.4710	2003	8.7764	2015	8.4884
1992	8.3760	2004	8.5737	2016	8.6887
1993	8.3482	2005	9.2778	2017	9.7874
1994	8.3205	2006	8.7027	2018	8.0245
1995	8.6273	2007	8.3006	2019	8.4571

²⁵ This is the observed length-frequency adjusted such that the expected mass calculated using the weight-at-length relationship matches the observed catch in tons (de Moor *et al.* 2020c). The weight-at-length relationship applied to these commercial data is taken to vary by month, as obtained from fitting an inverted normal distribution for the “a parameter” to monthly commercial data from 1984 to 1996 (de Moor and Butterworth 2015).