Development of a Heuristic Methodology for Designing Measurement Networks for Precise Metal Accounting

by

Paul Aaron Bepswa

B.Sc. (Hon) Metallurgical Engineering (University of Zimbabwe)

A Thesis Presented for the Degree of

DOCTOR OF PHILOSOPHY

Department of Chemical Engineering
University of Cape Town
October 2015

Declaration

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for another degree at any other university or institution.

Paul Bepswa

October 2015

Acknowledgments

I would like to express my sincere gratitude to my supervisor, David Deglon for his guidance, patience and good humour throughout my PhD study. I would also like to thank Aninda Chakraborty for his enthusiasm and valued contributions at the initial stages of the project.

The assistance of the following is acknowledged with gratitude:

- The AMIRA P754 Project, the Department of Labour National Research Foundation and the Department of Chemical Engineering at UCT for their combined assistance in financing this project.

- My colleagues Andre van der Westhuizen, Jason Waters and Kenneth Maseko for their assistance in conducting the sampling and mass measurement survey work at Namakwa Sands; Jenny Sweet and Aubrey Mainza for allowing me some time to work on my thesis under their respective supervisions.

- Namakwa Sands Northern Operations for providing the opportunity to conduct error modelling studies on an operational mineral processing plant as well as performing the required sample analyses.

- The sponsors of the AMIRA P754 Project and all the participants of the industrial survey on metal accounting practice conducted in this study.

- My lovely wife Neo, my daughter Rumbidzai and my son Tumelo. I could not have done this without your unwavering support. This is for you.

Publications

Below are peer-reviewed journal and conference papers arising from this work. In addition, technical and project management reports as well as testwork reports were produced in the course of this project.

Journal Publications

Bepswa, P. A., Deglon, D., 2013. Numerical investigation of a heuristic methodology for designing precise metal accounting measurement networks. Minerals Engineering, Volume 42, pp. 50-61.

Chakraborty, A., Deglon, D., 2008. Development of heuristic methodology for precise sensor network design. Computers & Chemical Engineering, Volume 32, Issue 3, pp. 382-395.

Conference Papers (peer-reviewed)

Bepswa, P., Deglon, D., Chakraborty, A., 2008. Development of a heuristic methodology for designing measurement networks for metallurgical accounting. Proceedings of the 24th Int. Mineral Processing Conference, Beijing, China, ISBN: 978-7-03-022711-9 (Beijing).

Bepswa, P., Deglon, D., & Chakraborty, A. (2006). Heuristic methodology for designing measurement networks for metallurgical accounting. *Proceedings of the 23rd International Mineral Processing Congress*, *Istanbul*, *Turkey*, ISBN/ISSN: 975-7946-27-3.

Chakraborty, A., & Deglon, D. (2006). Measurement network design for metallurgical accounting: A heuristic methodology. In T. Yalcin & H. Shang (Eds.), *Proceedings of Mineral Process Modelling Simulation and Control Conference,* ISBN 0-88667-066-7 (pp. 409–421).

Synopsis

This thesis investigates the development of a heuristic based methodology for designing measurement networks with application to the precise accounting of metal flows in mineral beneficiation operations. The term 'measurement network' is used to refer to the 'system of sampling and weight measurement equipment' from which process measurements are routinely collected. Metal accounting is defined as the estimation of saleable metal in the mine and subsequent process streams over a defined time period. One of the greatest challenges facing metal accounting is 'uncertainty' that is caused by random errors, and sometimes gross errors, that obtain in process measurements. While gross errors can be eliminated through correct measurement practices, random errors are an inherent property of measured data and they can only be minimised.

Two types of rules for designing measurement networks were considered. The first type of rules referred to as 'expert heuristics' consists of (i) Code of Practice Guidelines from the AMIRA P754 Code, and (ii) prevailing accounting practices from the mineral and metallurgical processing industry which were obtained through a questionnaire survey campaign. It was hypothesised that experts in the industry design measurement networks using rules or guidelines that ensure requisite quality in metal accounting.

The second set of rules was derived from the symbolic manipulation of the general steady-state linear data reconciliation solution as well as from an intensive numerical study on the variance reduction response of measurements after data reconciliation conducted in this study. These were referred to as 'mathematical heuristics' and are based on the general principle of variance reduction through data reconciliation. It was hypothesised that data reconciliation can be used to target variance reduction for selected measurements by exploiting characteristics of entire measurement networks as well as individual measurement characteristics.

It was found that experts in the industry minimise metal accounting variance by sampling and weighing key streams with high precision. Terminal streams in general, and *Feed* and *Product* streams in particular, were identified as key to metal accounting. The emphasis on the measurement and usage of terminal streams was found to be consistent with the widespread

use of the Check In-Check Out method of accounting in the minerals beneficiation industry. Of concern however is the low usage of *Tailings* stream measurements in metal accounting despite the universal employment of the Check In-Check Out system. It thus appeared that expert design philosophy advocates the precise measurement and utilisation of terminal streams in general to define corporate metal accounts through the Check In-Check Out method of accounting while internal measurements appear to be reserved for the evaluation of internal unit operations.

Mathematical heuristics developed in this study illustrate the benefits of precise measurement of internal streams so that terminal streams can experience maximum variance reduction after data reconciliation. However, the design philosophy of concentrating resources on internal streams is contrary to common expert practice where emphasis is on precise measurement of the actual input and output streams of the process. In this case, the metal accounting system will behave as a single node and will not benefit significantly from data reconciliation i.e. a Check In-Check Out type accounting philosophy will be suitable.

This thesis concludes by proposing a heuristic design strategy for constructing metal accounting measurement networks depending on quality and governance based imperatives. For accuracy requirements, the use of applicable metrology standards and the Check In-Check Out method of accounting were deemed sufficient for purpose. Adoption of additional tools such as the AMIRA P754 Code of Practice was suggested as necessary in order to achieve governance based requirements that include transparency and credibility of the metal accounting process. Data reconciliation was suggested as a 'no-cost' means for primarily improving precision of measured data beyond the capabilities of existing hardware and as a check of the integrity of measured data. Measurement network design was proposed as an additional tool for improving accounting precision by maximising variance reduction of selected measurements after data reconciliation. The use of reconciled data is however conditional on the acceptance of 'adjusted' data as valid or legitimate input to metal accounting reporting.

Statement of originality/novelty

A considerable amount of research has been done on the implications of data reconciliation based analysis of process systems, in particular its application to process design and optimisation in the pharmaceutical and chemical process industries. Data reconciliation has generally been used as an effective tool for reducing the total measurement variance associated with experimental data. There is little evidence that due attention has been paid to the effect of data reconciliation on individual measurement variances. Even less attention has been paid to the development and use of heuristics in influencing the variance reduction outcome for selected measurements through data reconciliation. This study is considered to be novel in the following two main areas.

1) Development of a heuristic methodology for designing measurement networks aimed at achieving targeted precisions on selected stream(s) through data reconciliation.

   There is currently no recorded evidence in sensor network design research on the development of heuristics for designing sensor networks that achieve targeted precisions on selected variables after data reconciliation. The rules developed are used to make *a priori* decisions for the design of measurement networks that maximise precisions on selected streams through data reconciliation. The heuristic design approach is proposed either as an alternative or complement to computational design. Computationally intensive algorithms such as MINLP-based search methods tend to explore all possible measurement network alternatives, resulting in large solution spaces that are often expensive to search. Heuristic based methods exploit the underlying 'split-and-prune' nature of heuristic selection resulting in more tractable solution spaces. The methodology developed allows metal accounting practitioners to design their own metal accounting systems from a set of general rules.

2) Secondly, in the application of heuristic design of measurement networks for precise metal accounting.

   The idea of designing measurement networks that achieve specified precisions on selected measurements after data reconciliation is relatively new in the practice of metal accounting. This allows precision targeting of not only important measurements such as concentrate tonnages but also a variety of 'downstream' performance evaluations such as routine computations of key process indicators that rely on requisite metal accounting precisions for their successful definition.

Table of Contents

List of Figures

List of Tables

Nomenclature

Abbreviations

CUSUM	cumulative sum
DR	Data Reconciliation
MA	Metal Accounting
SNR	Stream Variance to total Parent-node Variance Ratio
SSDR	Steady-state Data Reconciliation
s.t.	subject to
VRR	Variance Reduction Ratio
WLS	Weighted Least-squares

Indices

a	adjusted value
m	measured value
n	nodes
s	streams
$i(n1, n2)$	internal streams connecting nodes $n1$ and $n2$
T	transpose

Labels

n, n1, ni, nj, nk nodes

Parameters (units)

$\sigma^2_{m(n1)}$	measured variance of observed stream attached to node n_1 ([mass flow units]^2)
M_n	sum of measured variance of all streams attached to node n ([mass flow units]^2)
N_n	total number of nodes in a flowsheet (–)
N_s	total number of streams in a flowsheet (–)
Var(measured)	sum of measured variance of all streams in a flowsheet ([mass flow units]^2)
x_m	vector of measured component flow rates (mass flow units)

Variables (units)

$\sigma^2_{a(n1)}$	adjusted variance of observed stream attached to node n_1 ([mass flow units]^2)

R matrix of observed variance reduction ratios (–)

$Var(adjusted)$ sum of adjusted variance of all streams in a flowsheet ([mass flow units]^2)

x_a vector of adjusted component flow rates (mass flow units)

Matrices

A flowsheet incidence matrix

R_m measured variance–covariance matrix

R_a adjusted variance–covariance matrix

I identity matrix

Chapter 1

INTRODUCTION

Mining companies around the world devote considerable effort to metal accounting but are faced by a number of challenges that impact on the credibility and transparency of the metal accounting process. Firstly, there is no international standard for metal accounting and, other than a number of ISO standards for sampling and analysis, companies tend to develop their own internal metal accounting systems. This makes adherence to best practice and external audits of the metal accounting process impossible. Secondly, metal accounting analysts (engineers and technical personnel) tend to consider production in probabilistic terms while financial auditors work in a framework of absolute numerical precision. Financial auditors are suspicious of the use of reconciled data in preference to measured data and require a clear definition of the metallurgical accounting process. It was clear to address these challenges, a set of methodologies and best practices had to be developed and agreed upon by both technical and financial personnel across the mining industry worldwide.

This research stems from an Australian Mineral Industries Association (AMIRA P754) Project, entitled 'Metal Accounting and Reconciliation' which intends to address some of the issues facing mining companies worldwide to improve corporate governance with respect to metal accounting from mine to product and to improve the credibility and transparency of the metal accounting reporting process. Metal accounting is the estimation of (saleable) metal in the mine and subsequent process streams over a defined time period.

Metal accounting relies on measurements and these measurements contain random error, and sometimes gross error (bias). Gross errors are a major concern for any metal accounting system. They can, however, be effectively minimised or eliminated altogether through correct design, installation and operation of sampling and mass measurement equipment. Random errors occur as a result of not only the probabilistic nature of measurement processes but also of ore constitution heterogeneity. They can only be minimised and never be eliminated.

There are a number of approaches at improving the precision on metal accounting streams and associated performance measures. The default method is to improve the precision of the measurements themselves through better sampling and mass measurement (Holmes, 2004a). Improving precision through better measurement is without doubt the most preferred/dominant method of improving the precision of metallurgical accounting systems, and is of benefit to all subsequent methods aimed at variance reduction.

However, better measurements are generally obtained at higher cost. Metal accounting practitioners who follow purely measurement based systems, such as the widely used ‘Check In-Check Out’ system, have no choice in this regard as improving the precision of the metal accounting system can only be achieved through better (and likely more costly) measurements. Here, measured precisions on metal accounting streams are critical as they are used to ensure that ‘unaccounted gains or losses’ are kept within acceptable limits.

Steady state data reconciliation is frequently used to adjust measured data so that network constraints are verified while measurement variances are simultaneously reduced. Measured data seldom satisfy mass balance, energy balance and other physical constraints of the process as a result of random error obtaining in measurements. Data reconciliation is only possible if sufficient data is redundant, which is usually the case in metal accounting systems where most (if not all) relevant accounting streams are measured. Most metal accounting systems are concerned with mass or metal balances. Here, data reconciliation is generally referred to as metallurgical mass balancing or metal balancing in the mineral and metallurgical industry. Mass balanced data has the advantages that:

- Firstly, mass balancing allows one to use many additional measurements from internal nodes in the process flow sheet and a complete balance of the process is in principle attainable. Traditional metallurgical accounting systems often use measurements from terminal streams only and do not attempt to exploit the ‘information value’ contained in internal node measurements.
- Secondly, the new set of adjusted variables will be consistent and will satisfy the conservation of mass. Consequently, performance measures such as recovery

calculated using various methods (e.g. two-product formula, final product/feed) will gave the same answer.

- Thirdly, the new set of adjusted variances (or precisions), associated with the set of adjusted variables, will be smaller (or better) than the measured variances as they will be reduced through the mass balance constraints. Consequently, the precision of metallurgical accounting data will be improved at no additional cost.

Linear steady-state data reconciliation was first addressed in the seminal work of Kuehn and Davidson (1961). The problem was formulated as a Weighted Least Square optimisation problem (Equation (1)) subject to mass balance constraints (Equation (2)):

$$Objective\ Function = \frac{min}{x_a}[x_m - x_a]^T V^{-1}[x_m - x_a] \qquad \textit{(Equation 1.1)}$$

$$Constraint\ f(x_a) = 0, process\ models \qquad \textit{(Equation 1.2)}$$

Equation 1.1 outlines the objective function, where x_m and x_a are vectors of measured and adjusted (reconciled) values, and V is the variance-covariance matrix. When random errors are assumed to be normally distributed and covariances are assumed to be zero, the objective function reduces to a simple minimisation of the weighted sum of squared error (WSSE). However, the adjusted data is just one of many data sets that could arise from a set subject to random error. Hence there is a 'calculated' variance associated with the distribution of reconciled values which would be obtained if one were to repeat the data reconciliation process with numerous random measurements i.e. each measurement generated based on experimental error models. This is referred to as 'reconciled variance'.

Mass balancing offers many advantages, not the least of which is 'better precision for free', but is currently not widely used in metal accounting systems as financial auditors in particular are suspicious of the use of reconciled data in preference to measured data. A major drawback of data reconciliation is that the extents of variance reductions experienced by individual measurements are generally unpredictable. In other words, one cannot predict which measurement variances will be reduced the most and by what margin of reduction. Thus performing data reconciliation on a given data set may result in a good reduction in variance on streams that are of little importance to metallurgical accounting but leave the variance of important streams (e.g. boundary streams) relatively unchanged. This detracts from the data reconciliation process the potential of being utilised as a single step design tool for predicting improvements in precision on targeted measurements at the conceptual stages of network design.

In order to address this, a relatively new area of research called 'measurement network design' is proposed. The approach is premised on the fact that metal accounting systems rely on measurements obtained from a 'network of samples and sensors', commonly referred to as a 'measurement network'. Data reconciliation, or mass balancing, improves the precision of these measurements as the adjusted variances will always be smaller than the respective measured variances.

The study of measurement network design originates from the broader research area of sensor network design in the field of process control. Sensor network design is concerned with both the precision of sensor networks and the sensitivity of these networks to sensor failure i.e. robustness. Work done in the area of sensor network design involves the use of one of the following three methods (Narasimhan & Jordache, 2000):

- Matrix algebra: Determining an analytical expression for solving Equation 1.1 and Equation 1.2 for the case of unmeasured streams using the method of projection matrix (Crowe et al., 1983) factorization to estimate projection matrices (Sánchez & Romagnoli, 1996).
- Graph theory: The use of graph theoretic concepts to determine observability and redundancy in measurement networks and thereby ascertain which streams to measure (Kretsovalis & Mah, 1988; Madron, 1992; Meyer et al., 1993) .
- Mathematical programming: The use of Mixed Integer Non-linear Programs (Bagajewicz, 1997) and genetic algorithms (Gerkens & Heyen, 2004; 2005) to obtain optimal decisions for sensor network design.

All of the above techniques are algorithmic or numerical in nature and tend to provide optimal solutions to existing flow sheets with sensor network schemes (designs) in place, hence cannot be used to make *a priori* design decisions. Save for a few efforts (Bepswa & Deglon, 2013; Bepswa et al., 2006, 2008; Chakraborty & Deglon, 2008; Lyman, 2005), attempts to develop guidelines or heuristics that predict the reduction in variance for specific streams after data reconciliation had not been explicitly dealt with in the literature.

This research study aims to address this by developing a methodology for designing measurement networks that achieve specified precisions on selected streams. In simple terms this means how do I make informed decisions regarding the selection of measurements and their associated precisions in a flow sheet so that I will consistently know the precision of (for example) final metal product tonnages to, say for instance, 1% or better. The methodology is meant to be heuristic (rule) based rather than computationally based and should allow metal accounting practitioners to (tentatively) design their own systems from a set of general rules and a methodology for applying them. The heuristic approach to design is used as an alternative to intensive computational design for complex systems and often complements computational design. Heuristic based methods tend to exploit the underlying 'split-and-prune' nature of heuristic selection resulting in more tractable solution spaces. Two types of heuristics for designing measurement networks aimed at maximising precision on key metallurgical streams were investigated in this work.

The first type of rules refers to principles or guidelines that operations in the mineral and metallurgical industry currently use to design their metal accounting systems. These are procedures specific to site operations that have evolved through experience and eventually 'formalised' to facilitate decision-making regarding the placement of samplers and sensors on key streams in order to achieve requisite metal accounting precision. This set of rules will be referred to as 'expert heuristics'.

The second set of heuristics is based on mathematical consideration of the linear steady state data reconciliation solution for redundant measurement networks (Equation 1.1 & Equation 1.2). These heuristics are not expected to be 'hard mathematical rules' but rather sensible design principles/observations for maximising the reduction in variance on streams of metal accounting interest based on the random error reduction attributes of data reconciliation. This set of rules will be referred to as 'mathematical heuristics'.

1.1 Objectives of thesis

The objective of this study is to develop a heuristic based methodology for designing measurement networks for precise metal accounting. Firstly, current 'expert' design practices are investigated in order to establish measures currently employed to select or design measurements that meet the quality requirements of metal accounting. In this context, it is hypothesised that experts in the industry pre-select sites on process flowsheets for the placement of measurements designed for metal accounting purposes. Secondly, data reconciliation is investigated as an independent (though complementary) alternative to expert measurement design approaches. It is hypothesised that data reconciliation can be used to target variance reduction for selected measurements by exploiting characteristics of entire process measurement networks as well as individual measurement attributes. Here, 'mathematical' heuristics for designing precise measurement networks are deduced from factors derived from the manipulation of the general linear steady state data reconciliation solution. The efficacy of the mathematical design heuristics is tested on a case study from the mineral sands beneficiation industry.

1.2 Key questions

In order to meet the objectives of this study, the following questions are posed:

(1) Where do metal accounting measurements originate in process networks? What criteria do industry experts/practitioners use to place metal accounting measurements on process flowsheets?

(2) What are the requirements for metal accounting measurements? Is there a relationship between the quality of process measurements and their respective sources on process networks?

(3) How is metal accounting conducted on a routine basis? Is data adjustment or mass balancing done to improve the quality of the metal accounting function? What is best practice for sound metal accounting quality?

(4) What influences the extent of variance reduction through data reconciliation? Can measurement and/or flowsheet characteristics be used to target selected measurements for preferential variance reduction?

1.3 Scope and limitations

This work focusses on the design of linear flow networks under steady state conditions. Although flows in mineral and metallurgical operations are often bi-linear (or higher), they are linearised by considering the cross products of gross mass flow rates and assays. Errors obtaining in the resultant component flow rates are estimated based on the principle of error propagation through formulae. Measurement errors investigated in this study are assumed to be small, random, independent and Gaussian (in distribution). All measurements are considered free of systematic error, hence bias detection and removal procedures will not form part of the current design considerations. The design objectives are restricted to redundant networks in which all streams are measured.

1.4 Plan of development

Chapter 2 of this thesis contains the literature review which firstly highlights the principles of metal accounting according to the inaugural work of the AMIRA P7574 Project on metal accounting. This is followed by a review of the effects of measurement error on metal accounting variance and the current methods that are used ameliorate variance in metal accounts. Different approaches to measurement in mineral processes are highlighted with particular reference to their precision limitations. The role of metallurgical balances in metal accounting reporting is reviewed in light of their susceptibility to error in measurements. A review of the role of data reconciliation as a tool for precision improvement ensues. This is followed by a review of the role played by data reconciliation as a basis for design decision making in sensor network studies. Chapter 2 rounds off with an overview of sensor network design and its commonly reported objectives that include accuracy, observability and reliability.

Chapter 3 describes the research methodology for an industrial survey done to determine measurement design practices for metal accounting, before the results of the survey are presented in Chapters 4. The industrial survey was designed to gather evidence on rules that are currently used by experts in the minerals industry to design measurement systems that meet desired quality for use in metal accounting. The results of the survey are then reviewed in light of the recommendations and guidelines from the AMIRA P754 Project.

An in depth study of a typical metal accounting system at an operating plant is presented in Chapter 5. Salient aspects of metal accounting practice at the site were audited based on best practice recommendations from the Code as well some of the findings made in the industrial survey campaign described in the Chapter 4. A sampling and mass measurement campaign conducted at the case study site in order to determine measurement errors is described in this chapter, followed by an assessment of the impact of the errors on mineral flow and recovery estimations at the operation. The operation is unique in that the analytical method (manual grain counting) contributed the most to mineral flow rate errors. Analysis of the effects of the incorrect accounting of spillage-recovered values on the accuracy of the metal accounting function highlights practical problems encountered in the correct estimation and interpretation of key indicators such as mineral recovery on an operating site. The case study highlights the impact of measurement error on metal accounting and provides a basis for testing the efficacy of the measurement network design rules developed in this study.

Chapter 6 outlines the mathematical basis for the selection of factors that significantly influence the variance reduction of terminal streams for the case of linear steady state data reconciliation with all streams measured. The factors were derived through the symbolic manipulation of the general linear steady state solution applied to single to multi-node hypothetical process networks. The factors identified represent key network parameters that provide a basis for developing rule-based approaches to precise network design based on data reconciliation. The mathematical basis of the heuristics derived from the data reconciliation solution as part of this study is explained.

Chapter 7 presents a numerical study of the data reconciliation based mathematical factors for designing measurement networks that maximize variance reduction for terminal streams. The case study described in Chapter 5 is used as a test case. The case study flowsheet presents a complex multi-component environment featuring all stream types found on a typical mineral/metallurgical operation.

Chapter 8 summarises the key findings of the study and their implications for measurement network design for precise metal accounting. A decision process for designing measurement networks based on heuristics is proposed before recommendations for future work are made in conclusion.

Chapter 2

LITERATURE REVIEW

This chapter presents a review of literature on the main areas of research contributing to the objectives of this thesis. The metal accounting function and its reliance on measurement and sampling, metallurgical balances, the data reconciliation procedure and its applications are reviewed.

2.1 Metal accounting

Metal accounting is concerned with estimating total ore processed and total valuable metal produced, lost to waste, and held up in inventory over a specified time period. Metal accounting attempts to monitor process efficiencies and yields as well as accurately measure the amount of material undergoing transformation in process units and inventories, and hence it predicts metal availabilities in different parts of a process while highlighting areas where unexpected losses are occurring. Metal accounting is also useful as a basis for tracking by-products and consumables, such as media, reagents and power (Fuerstenau & Han, 2003).

The quality of metal accounting relies on installed sampling and mass measurement equipment from which measured data are obtained. Measurement campaigns are routinely performed on operating plants to generate metal accounting data. The data collected are rarely consistent and will almost always contain redundant information. The challenge is therefore to produce data which are both self-consistent and as accurate a representation of the plant operation as possible.

Metal accounting records are mostly used for auditing purposes and to give a clear indication of the areas of concern within the process plant. The data records obtained for metal accounting provide a basis to motivate process improvements including capital expenditure applications and forecasts of operating costs. Audits of circuit changes comparing predicted and achieved plant performance use metal accounting data as a basis for evaluation.

One of the major problems facing metal accounting is the “uncertainty” associated with the measured values used in metal accounting. Uncertainties are associated with the sampling, weighing, preparation and analysis of the material processed. As a means to manage ‘uncertainties’ in metal accounting data and ensure consistency in the manner in which data is collected, processed and interpreted, a code of practice for metal accounting was developed through the AMIRA P754 project.

2.1.1 Standard code of practice in metal accounting – AMIRA P754 Project

One of the major deliverables of the AMIRA P754 Project was the development of a Code of Practice for Metal Accounting for the Mining and Metallurgical Industry (Code). The aim of the Code was to provide standard generic procedures and guidelines for obtaining credible figures of metal quantities processed and produced, and methods for obtaining a metal balance, and to be recognised and accepted as the industry standard for best practice in this area (Gaylard et al., 2009).

Prior to the AMIRA P754 Project (P754) no attempts had been recorded concerning the development of a universally accepted standard for metal accounting and reconciliation across the mining and minerals processing industry world-wide. The conceptualisation of what later became P754 took place at a SAIMM workshop held in Cape Town on 1 August 2001. The workshop identified the lack of accepted standard procedures for metal accounting as an industry-wide problem.

The first draft was released in October 2005 (Release 1), with Release 2 and later Release 3 being published in December 2005 and February 2007 respectively. From the first release, the Code was circulated widely to core sponsors, individuals and a large number of interested companies whose comments, together with contributions from experts in sampling and statistics, assisted in modifying the Guidelines.

The Code document contains two main parts: a brief 'Code of Practice' and a set of Guidelines. The Code of Practice lists 10 basic Principles of Metal Accounting upon which the Code is based. The Guidelines prescribe standards and best practices for mass measurement, sampling, sample preparation, analysis, data management and metal balancing to enable compliance with the basic principles. The Code is clear that wherever possible, procedures used in metal accounting must be based on the appropriate International or National Standards (Release 2, 2005). Figure 1 shows a schematic of the Code structure (Gaylard et al., 2009).

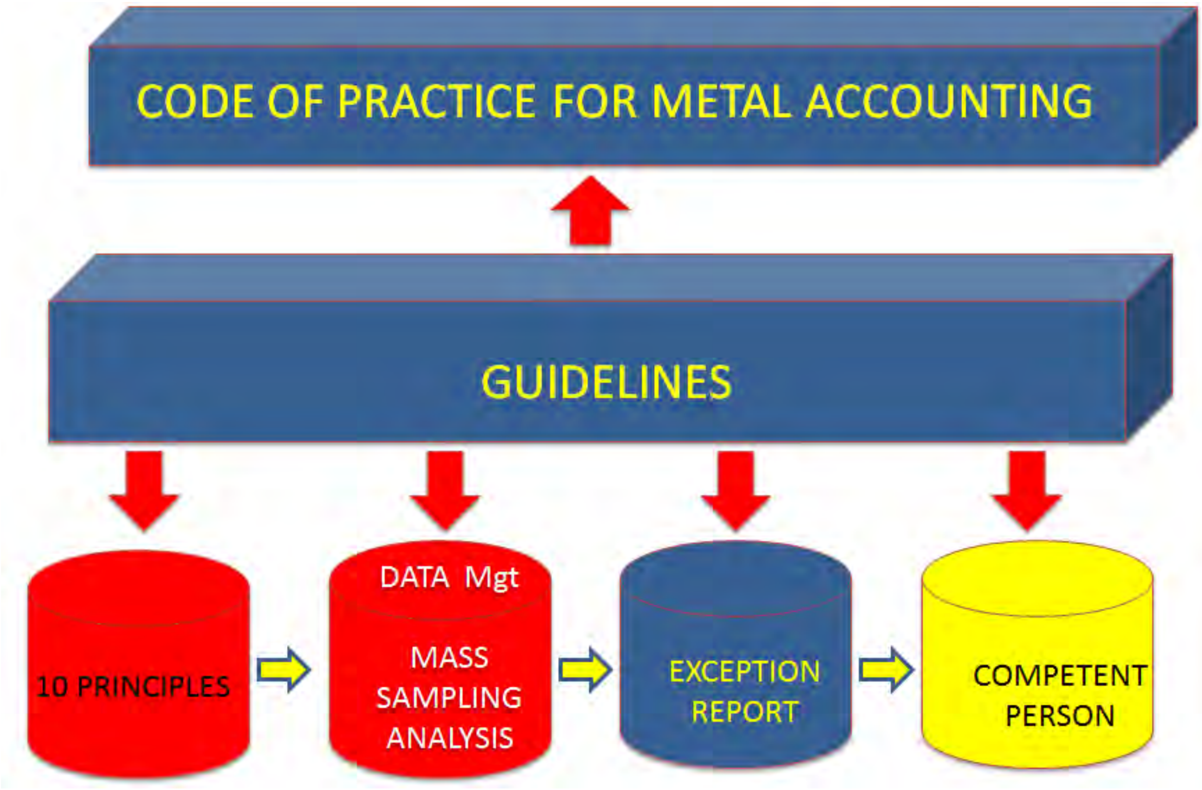


Figure 1: Code structure

Exception reporting is recommended in cases where companies or operations are unable to comply with the basic principles. The Code also provides for "signing-off" of the exception report by a Competent Person who belongs to a statutory board recognised by administrators of the Code (Figure 1) and subsequently submitted to the Company's Audit Committee.

2.1.2 Principles of metal accounting

Common problems facing the metal accounting discipline include the lack of an accounting standard; deployment of inadequately qualified personnel to work in the area as it is often seen as a subsidiary activity to production; a lack of understanding of variability in mass measurement, sampling and analysis; the use of inconsistent methods for computations of key measures such as metal yields and stocks and inventories. Often, the confidence of measured data and derived measures are not quantified despite the data being used as a basis for generating corporate financial accounts reported to shareholders.

To address these problems, the Code is based on 10 Principles of Metal Accounting (Principles) which were agreed upon by the Code development team in consultation with the Accounting Profession and P754 sponsors. The Principles as adopted from the 'AMIRA P754 Code: Release 2' are listed below for reference:

1. The metal accounting system must be based on accurate measurements of mass and metal content. It must be based on a full Check In-Check Out system using the Best Practices as defined in this Code, to produce an on-going metal/commodity balance for the operation. The system must be integrated with management information systems, providing a one-way transfer of information to these systems as required.

2. The system must be consistent and transparent and the source of all input data to the system must be clear and understood by all users of the system. The design and specification of the system must incorporate the outcomes of a risk assessment of all aspects of the metal accounting process.

3. The accounting procedures must be well documented and user friendly for easy application by plant personnel, to avoid the system becoming dependent on one person, and must incorporate clear controls and audit trails. Calculation procedures must be in line with the requirements set out in this Code and consistent at all times with clear rules for handling the data.

4. The system must be subject to regular internal and external audits and reviews as specified in the relevant sections of the Code, so as to ensure compliance with all

aspects of the defined procedures. These reviews must include assessments of the associated risks and recommendations for their mitigation, when the agreed risk is exceeded.

5. Accounting results must be made available timeously, to meet operational reporting needs, including the provision of information for other management information systems, and to facilitate corrective action or investigation. A detailed report must be issued on each investigation, together with management's response to rectify the problem. When completed, the plan and resulting action must be signed-off by the Competent Person.

6. Where provisional data has to be used to meet reporting deadlines, such as at month ends when analytical turn-around times could prevent the prompt issuing of the monthly report, clear procedures and levels of authorisation for the subsequent replacement of the provisional data with actual data must be defined. Where rogue data is detected, such as incorrect data transfer or identified malfunction of equipment, the procedures to be followed together with the levels of authorisation must be in place.

7. The system must generate sufficient data to allow for data verification, the handling of metal/commodity transfers, the reconciliation of metal/commodity balances, and the measurement of accuracies and error detection, which should not show any consistent bias. Measurement and computational procedures must be free of a defined critical level of bias.

8. Target accuracies for the mass measurements and the sampling and analyses must be identified for each input and output stream used for accounting purposes. The actual accuracies for metal recoveries, based on raw data, achieved over a company's reporting period must be stated in the report to the Company's Audit Committee. Should these show a bias that the Company considers material to its results, the fact must be reported to shareholders.

9. In-process inventory figures must be verified by physical stock-takes at prescribed intervals, at least annually, and procedures and authority levels for stock adjustments and the treatment of unaccounted losses or gains must be clearly defined.

10. The metal accounting system must ensure that every effort is made to identify any bias that may occur, as rapidly as possible and eliminate or reduce to an acceptable level the source of bias from all measurement, sampling and analytical procedures, when the source is identified.

2.1.3 Key components of a metal accounting system

A metal accounting system is comprised of all activities and equipment that contribute to the generation of metal accounting information. The Code stipulates that design specifications for a metal accounting system must include the following (Code of Practice, 2005):

- A system for data acquisition and management
- Reporting intervals, dates and timing, as well as reporting rules and procedures
- Characterisation of the process streams involved in the balance, including the nominal mass flows of balance species, heterogeneity of the solids to be sampled from the stream and usual levels of process variability
- Sample equipment, sampling frequency, sample preparation procedures, expected sampling and preparation accuracy and analyses required, as well as cross-references to the required analytical procedures and the accuracy of those analytical procedures
- Methods of monitoring the data streams of analytical results that will ensure that the target sampling, preparation and analysis variances are maintained
- The quality control and assurance systems of all areas of metal accounting must be included in the accounting system documentation
- Mass measurement points, type of measuring device and accuracy required
- Levels of authority for approval of the data and, where necessary, for handling and approving changes to the data
- Accounting battery limits
- Stock-take procedures and frequencies.

In essence, the list outlines minimum criteria that define a typical metal accounting system compliant with the provisions of the Code. Importantly, two measures that impact on the credibility of data generated from the metal accounting system receive emphasis: (i) precision and (ii) accuracy, of measured data. In order to design a system that closely monitors these measures adequate knowledge of the sources of variations observed in measured data is required.

2.1.4 Types of error in measurements

All process measurements are subject to variation, or error as it is sometimes commonly referred. Repeated measurements will rarely yield the exact same value each time the same measurement process is performed. In the metallurgical processing context, this may be the determination of mass, a sampling event or even assaying (sample analysis). If the measurement process is done correctly, the results are expected to cluster around an unknown true mean best estimated by the arithmetic average of the repeated results. A common measure of the variation associated with the measurement process is the standard deviation (or 'variance') statistic, calculable from the repeated measurement results.

Two major classes of measurement error are well documented in the literature. These are random error and systematic (or bias) error. Random errors are an inherent property of measured data which occur as a result of the precision limitations of measurement devices. For metallurgical measurements, random error is a consequence of a variety of factors that include the 'nature of the ore', process variation and the chosen methods of mass measurement and sampling. For mass measurement the error is associated with the type of instrument and its installation, calibration and maintenance. They can be minimised but never eliminated. Systematic errors are caused by systematic deviations that persist during measurement.

There is another class of error associated with measurement data that contributes to 'uncertainty' facing metals accounting. Factors affecting the handling and processing of data and/or samples such as incorrect reading of displayed results, erroneous transcription of figures onto log sheets or mixing of samples cause what may be referred to as 'illegitimate' error. This type of error in measurements is almost always caused by 'human error' and can be avoided altogether by using automated data acquisition and information handling platforms coupled with stringent sample handling procedures.

All sources of error in measurements give rise to inaccurate assessments of the true state of the process resulting in potentially erroneous decisions regarding interventions. While systematic (and illegitimate) error, once detected, can be eliminated, random error is an inherent property of measured data and can never be eliminated. Random errors degrade the precision of metal accounting and the quality of key performance measures that rely on metallurgical measurements. Moreover, because of the immense scale of most operations, the impact of error may be magnified in absolute terms (Mah & Stanley, 1976). The notion of propagation of error through formulae attests to this assertion.

2.1.4.1 Random errors

Random errors are assumed to be independent and normally distributed (Gaussian) around a central value with a mean of zero (Liebmann et al., 1992). Random error is assumed to arise from a variety of sources that include power supply fluctuations, natural variations in the material measured (e.g. material heterogeneity), ambient conditions (e.g. temperature, humidity) and fluctuations in the operation of measurement devices (e.g. analytical instrument, sampling machine). The conditions underlying random error generation cannot be controlled or reproduced. Consequently, it is impossible to eliminate their effects leading to inconsistent definitions of process models that rely on measured data for definition.

Random errors are associated with sampling, weighing and assaying of material processed. It is possible to estimate the variances arising from sampling, weighing, preparation and analysis not only using statistical techniques but also performing experimental tests, such as the heterogeneity test. By combining these variances mathematically, the confidence limits for any of the figures used in metal accounting can be obtained. An important attribute of random error is that relative random error associated with an average result diminishes with the number of results included in the combined result. This is because random errors oscillate on either of a central value with equal chance per measurement event and tend to cancel out through summation.

2.1.4.2 Systematic errors

Systematic errors tend to persist in one direction (either positive or negative) during measurement campaigns and therefore accumulate over time. They are often difficult to detect, particularly when they are similar in magnitude to random error obtaining in the measured quantity. Systematic errors are often the result of erroneous calibration of measurement equipment. Temporal based techniques such as the CUSUM control chart have found use in detecting systematic error given that relative random error ameliorates over time.

2.1.5 Precision in metal accounting

Precision refers to the magnitude of randomly distributed variations (random variations) in the measurement procedure applied to estimate the central value of the stochastic variable of interest (Merks, 2002). Precision is a qualitative concept; however quantitative measures of scatter such as confidence intervals rely on the variance measure for definition.

The variance is the fundamental measure of precision, although derived measures of precision such as confidence intervals, ranges and standard deviation are more intuitive and easier to appreciate than the variance measure. The standard deviation is the square root of the variance of a given set of measurements. A small standard deviation implies high precision indicating a high probability that the average result obtained from the sample will be close to the population mean.

A measurement operation with the highest variance will influence the total metal accounting variance the most. Therefore it is imperative to determine the variance of each operation and ensure that the values are acceptable. The additive property of variance enables resolution of variance contributions from independent operations in a measurement process to the final quantity estimated. For example, the total variance metal flow accounts obtained by multiplying mass flow rates and assays can be represented by the following equation:

$$V_T = V_M + V_S + V_P + V_A \qquad \textit{(Equation 2.1)}$$

where: V_T = total metal accounting variance

V_M = variance due to measurement of dry mass

V_S = the variance due to all sampling stages

V_P = the variance due to sample preparation

V_A = the variance due to analysis

From the determination of V_T the uncertainty in the metal accounts can be stated since:

$$\sigma_T = \sqrt{V_T}. \qquad \textit{(Equation 2.2)}$$

where: σ_T = the standard deviation of the metal accounts

For most applications in the minerals processing industry, the convention is to quote the precision measure at the 95% confidence level, although if the risk of a wrong decision is high, 99% or 99.1% probability is often considered.

2.1.6 Accuracy in metal accounting

Accuracy describes how close a measured value is to the "true" or expected value of the quantity measured. The definition shows that 'accuracy' is an abstract term. By contrast, a lack of accuracy can be measured and quantified in terms of 'bias' or 'systematic error'. The absence of bias in measurement implies therefore that the measured value is accurate. The true value is usually not known.

The uncertainties in a measurement chain can be resolved into randomly distributed variations (represented by the variance) and systematic errors (bias). Mathematically, the mean squared error (MSE) conveniently expresses this relationship. The MSE is the average of the squared differences or deviations between the estimate of a measured quantity and the true (usually unknown) value. For the case of a set of repeated measurements (X_i) with a computed average($\bar{X}$):

$$\text{MSE}(\bar{X}) = Var(\bar{X}) + \left(Bias(\bar{X}, X_i)\right)^2 \qquad \textit{(Equation 2.3)}$$

where $Var(\bar{X})$ = variance of the sample average estimator

$\left(Bias(\bar{X}, X_i)\right)$ = bias of the sample average estimator

The MSE could be viewed as a measure of accuracy or representativeness of the average result (Pitard, 1993). However if the measurement is accurate (no bias exists) then the MSE represents the variance of the sample average.

2.1.7 Data collection techniques

Successful use of the Code principles relies on strict adherence to the accurate and precise application of procedures to the generation of metallurgical measurements. Metal accounting data is derived from mass measurement, sampling, sample preparation and analysis of material processed.

2.1.7.1 Mass measurement

The main objective of mass measurement for metal accounting is to maximise precision and eliminate bias. To be considered credible, wherever possible a mass measurement system must be:

- Repeatable to within the defined error criteria
- Reproducible to within a defined error criteria using different methods or equipment
- Unbiased, or alternatively capable of being tested for bias over a period of time (Morrison, 2008).

In order to achieve this any mass measurement would have to take into consideration design and operational criteria that include selecting suitable measurement points, correct application of methods, correct design and installation of measurement equipment, regular calibration, certification , training of operators and good housekeeping.

There are a range of methods for determining wet mass that are used in industry that include weigh bins, conveyor weight meters, weighbridges and platform scales. Selection of mass measurement method invariably depends on the state of the material, amounts processed and the level of precision required for the purpose of measurement.

2.1.7.1.1 Accuracy and precision in mass measurement

To achieve acceptable accuracy in mass measurement a variety of measurement instruments and devices are available depending on the scale of the operation and type of material measured. Regular calibration and maintenance are necessary to curb the ingress of bias into measurement systems and achieve specified measurement quality.

Errors are introduced into the mass measurement chain through a number of factors that include variability of flow rate, design of installation of measuring equipment, nature of material weighed, sensing device and machine electronics (Gaylard et al., 2009). The cumulative effect of all these factors impacts on the accuracy and precision of mass measurement devices.

It is important to note that for machines that quote accuracies as a percentage of full capacity, high error can be introduced if the quantity weighed is significantly less than full capacity. This is particularly relevant for mass measurement devices operating over a wide weighing range.

Weighing of processed material falls into two general categories: (i) static measurements involving electronic load cell systems which use either resistance strain gauges or magneto-restrictive load cells, or (ii) dynamic measurements which involve classical mechanical systems that utilise levers and knife edges or in-motion weighing of moving trucks/wagons over static bridge scales. Static scales provide the more accurate and precise method of measuring mass because of the absence of dynamic effects as well as ease of calibration. The Code identifies these as the preferred method for metal accounting purposes wherever possible.

2.1.7.1.2 Static methods

Platform scales and weighbridges constitute the most common static mass measurement methods and are generally used for primary accounting and at custody transfer points. Capacities vary widely from several tonnes to a few kilograms depending on application. Hopper/bin and gantry/crane weight meters are another common group of static weighing methods. Hopper/bin weight meters include equipment such as feed hoppers, charging buckets and scale cars, usually sensed by load cells mounted on supporting frames. Gantry/crane scales use load cells fitted on supporting beams or moving crab of large bridge cranes.

When correctly installed and operated, precisions that can be obtained for platform scales vary between ± 0.05% to ±0.2%, weighbridges ±0.1% to ±0.2%, weigh bins ±0.1% to ±0.25% and for gantry scales from ±0.15% to ±0.25%. Use of this equipment in metal accounting requires certification and calibration by an approved external body with certified weights traceable to the International Unit of Mass held in France (Wortley, 2009).

2.1.7.1.3 Dynamic methods

Common dynamic mass measurement methods include (i) in-motion weighing of moving trucks/wagons (ii) electromechanical conveyor belt weight meters and (iii) electromagnetic flow meters. In instances where static weighing cannot be used, in-motion and conveyor belt weighing are recommended for metal accounting purposes ahead of magnetic flow meters, particularly for primary accounting. However, in-motion weighing is deemed unacceptable for metal accounting and certification for road trucks due to difficulty in regulating acceleration and deceleration of trucks, although the method can be calibrated and certified for rail trucks. Indicative accuracies of ±0.5% are possible for in-motion and belt scales while magnetic flow meters may be as poor as ±15% (Wortley, 2009).

In metallurgical operations electromechanical belt scales are common at the mine/plant interface and are recommended in the Code for primary accounting. Nuclear weight meters are not recommended for use because of sensitivity to fluctuations in the properties of material conveyed.

The accuracy and precision of belt scales depend on the installation and operation of the equipment. They are however certifiable and can be calibrated reliably. Calibration is done either in dynamic mode where test chains of known specific mass per unit length or bulk material of known mass are passed over a weighing system in motion; or in static mode where certified weights are placed over the weight carriage while the belt is at rest. Dynamic testing is preferable as it captures the effects of load in motion.

Assuming correct installation and design, sources of error in the use of belt scales are mostly operational, and these mainly include spillage on weigh carriage and belt speed tachometer; fluctuating load and size distribution of the material weighed.

Electromagnetic flow meters are widely used in the metallurgical industry owing to their ability to handle high flows of abrasive slurries with high contents of solids, conditions ubiquitous in the minerals processing setting. They measure mass by combining volume flow rates determined by flow meters and density measurements usually determined by nuclear (gamma) meters to calculate mass flow rate in conduits. Hence the error associated with the measurement combines uncertainties in volume and density determinations.

The accuracy of flow meters is affected by changes in the properties of the fluid or slurry transported and any conditions that hinder steady flow such as air entrainment and pump cavitation, channelling and other consequences of poor design and installation. Fluctuations in the properties of slurry/fluid may cause significant errors, indicating the need for regular calibration particularly when they are used for primary accounting purposes such as measurement of mineral concentrate slurry from thickeners (Wortley, 2009).

2.1.7.1.4 Density measurement

In many instances mass cannot be measured directly so that mass estimations have to be made by determining the volume of material and then using density to calculate the mass. Common density determinations in minerals processing include bulk density of ores/concentrates and specific gravity of slurries and molten metals. Error in the determination of density propagates to the resultant calculated mass.

Bulk density calculation takes into account the void fraction as shown in Equation 2.4, where ω is the void fraction and ρ is the true density of solids.

$$\rho_{bulk} = (1 - \omega)\rho \qquad \textit{(Equation 2.4)}$$

The void fraction depends on packing, which in turn is sensitive to particle shape and degree of compaction of the contained material. A value of 0.4 for ω is often assumed which may lead to error as high as ±15% in calculated bulk density.

For slurries, the density of the carrier liquid, which is usually water, may fluctuate due to recycling of process water which may significantly impair the precision and accuracy of the dry mass calculation when the specific gravity is assumed to be unity. The assumption of constant density for processed material potentially adds to the risk of bias in mass calculations as the nature of the material changes over time. Pulp or slurry density is measured by directly weighing samples or using nuclear density gauges. Equation 2.5 gives the relationship between weight per cent solids in pulp and true solids density in a water medium (specific gravity assumed to be unity).

$$Wt.\%\ solids = \frac{\rho_s(\rho_p - 1)}{\rho_p(\rho_s - 1)} \times 100 \qquad (Equation\ 2.5)$$

Error propagation studies using Equation 2.5 have demonstrated that most of the error (over 90%) in the calculated weight per cent solids is due to uncertainties in pulp density (ρ_p) estimates. Additional error in assumptions of solids density (ρ_s) estimates and water quality would only serve to increase uncertainty in the calculated fractional solids measure.

2.1.7.1.5 Stockpiles, stocks and inventory

Determination of metal content of stored material is made difficult by uncertainties in measuring volume, bulk density and obtaining a representative sample to analyse for metal concentration. The Code recommends running a system of parallel stockpiles that are alternatively emptied and filled. Mass measurement and representative sampling can then be performed on the moving material during draw down or filling to obtain more accurate estimations of metal content.

Although in-situ determination of volume can be done for stockpiles using technologies that can achieve as much as ±5% accuracy (photogrammetric or laser techniques), errors in metal content calculation still arise due to uncertainties in bulk density estimations and sampling for analysis. Determining metal in stocks held in containers and inventory in process units faces similar challenges. To compound the problem, no international standards for best practice exist other than the Code attempt at addressing the issue. Therefore differences in approaches to dealing with stored materials add to the uncertainty already present as a result of existing technical challenges.

2.1.7.2 Sampling

The basic requirement for correct sampling is that all parts of the material being sampled must have an equal probability of being collected and becoming part of the final sample for analysis (Gy, 2004; Holmes & Robinson, 2004; Pitard, 1993). Violation of this rule leads to a potentially biased result and sometimes poor sampling precision. Bias presents greater risk than poor precision and it cannot be corrected by replicate sampling or analysis.

2.1.7.2.1 Sampling protocol

In general, sampling protocols can be divided into random sampling, stratified random sampling and stratified systematic sampling. Random sampling and stratified random sampling are routinely applied to consumer products, while stratified systematic sampling is regarded as most effective for bulk material and slurry flows in mineral processing operations (Merks, 2002).

The stratified random sampling regime would be preferable for processing plant applications. Randomisation eliminates the probability for periodic production variations to synchronise with sampling intervals, a process phenomenon to which systematic sampling is susceptible. Random sampling also provides the most realistic estimates of precision for a given sampling scheme based on time series considerations of variability.

However, most systems and regimes are based on stratified systematic sampling. This is mainly because increments taken at even intervals in space or time always give the highest estimate of variance (Merks, 1985). Besides most standards using the classical variance as a reference, a major design challenge that always faces random sampling regimes is the construction of sampling equipment with suitably large secondary sampling systems capable of handling multiple primary increments simultaneously.

2.1.7.2.2 Accuracy of the sampling system

Inaccurate sampling practices are exposed by non-uniform distribution of material properties in storage (stockpiles, bins, tanks, trucks) or conveyance equipment (belts, conduits) as a result of segregation of particulates under gravity and forces acting on particles in motion (e.g. centrifugal force). These practices are characterised by non-probabilistic extraction of samples from sampling lots such as taking a sample from the edge of a stockpile, sampling from the top of a truck and taking a "bleed" sample from the side of a slurry pipe.

Testing for bias at the sampling and sample preparation stages is made difficult by the many possible sources of variation in procedures applied and therefore making it difficult to eliminate. However, the bias test procedure generally consists of using some form of t-test for means to investigate the significance of the discrepancy between results obtained from different sampling and/or sample preparation procedures on the same sample in order to test for sampling and/or preparation bias (Merks, 2002).

The best location for sampling a process stream in a mineral processing plant is at the discharge point of a conveyor belt or chute where the entire stream can be intersected at regular intervals (Holmes, 2010). This facilitates compliance with the fundamental requirement for representative sampling since all parts of the stream can be accessed with equal probability using a correctly designed, installed and operated cross-stream sampler.

It follows that sampling of stockpiles and other stationary storage or bulk transportation vessels for the purposes of metal accounting should be avoided due to the possible violation of the basic requirement for representative sampling. In practice, sampling from stationary storage cannot be avoided for convenience and cost but standards that include this procedure e.g. ISO 3082 caution that use of a spear or auger for sampling wagons for instance is permitted "only if the sampling device penetrates to the full depth of the fine ore at the point selected for sampling and the full column of fine ore is extracted" (Holmes & Robinson, 2004).

2.1.7.2.3 Precision of the sampling system

The sum of primary selection, sample preparation and analytical variances estimates the total precision associated with a sampling result. The primary selection variance (V_S) is a measure of random variations in selecting a subset of the set of all possible primary samples into which a quantity of material can be divided (the primary part of the whole); the preparation variance (V_P) is a measure for random variations in selecting a test sample of a primary sample (the secondary part), and the analytical variance (V_A) is a measure for random variations for selecting a test portion of a test sample (the tertiary part) and analysing it (Equation 2.6).

$$V_{SPA} = V_S + V_P + V_A \qquad (\textit{Equation 2.6})$$

The classical variance assumes that values in the observed set are statistically independent. It is instructive therefore to check for spatial dependence within a set of values presented for analysis. The F-test (Fisher's) can be used to achieve this by evaluating the significance of observed differences between the ordered variance (first or higher order variance) and randomised (classical) variance for a given set of measurements (Merks, 2002).

In order to determine the precision of a sampling system an appropriately designed interleaving sampling experiment is necessary. This approach is well documented in the standards (e.g. ISO/DIS 12744, AS 2884.4 - 1997) and is the most effective procedure to estimate the variance of the entire measurement process.

Using the ANOVA technique to partition the total variance (Equation 2.6) enables the optimisation of sampling protocols by identifying the dominant sources of variance obtaining in a measurement result and effecting measures to improve the precision of the entire measurement procedure.

Primary sampling variance is reduced not only by increasing the number of primary increments but also by increasing the sample mass; the sample preparation variance is effectively minimised by comminution and mixing prior to division; and the variance of the mean of replicated assays is reduced by a factor equal to the inverse of the number of replicates. A sufficiently high number of interleaving pairs improves confidence in variance estimates by increasing the degrees of freedom for the resolved variance estimates. Critically few samples reduce the ability of the test procedure to adequately determine the significance of observed differences amongst the contributing variance terms, which may invalidate the arithmetic operations in Equation 2.6.

2.2 Metallurgical (metal) balances

The term 'metallurgical balance' is normally used to refer to the overall accounting of material (and sometimes energy) entering and leaving a metallurgical process. In the mineral processing industry context, the term 'metal balance' or 'metbalance' is more frequently used as the process of producing a metallurgical balance applies to any mineral based commodity and not restricted to metallic elements (D. W. Fuerstenau & Han, 2003) .

Most balances are performed under the assumption that the processes observed are at steady state. In other words, there is no accumulation expected within the balance boundary and all unit processes are assumed to maintain the levels of in-process material present. Steady state is not achieved in practice especially over short periods such as an hour, a shift or even a day, given the size and number of equipment present in modern mineral processing operations (mill, thickeners, sumps etc.) as well as variations in material flow. Typically, balances are performed on data obtained from at least a month period of operation during which time the amount of material processed substantially exceeds internal hold ups, with perhaps the exception of feed and product stocks in periods of high stock retention.

2.2.1 Common applications of metal balances

Metal balances are performed throughout the life of mineral processes for a variety of reasons. The most common reasons for performing balances include design, optimisation, control and production accounting. Balances performed at the design stage of operations are mainly theoretical and here, minor losses are often ignored or not anticipated, and only exact balances are calculated. However during the operational stage of the process real balances are determined by sampling and measuring inputs and outputs of the process for control, optimisation and accounting purposes (Fine & Geiger, 1979).

A good understanding of material balance calculations is essential in process design (Coulson et al., 2005). A design balance is a useful tool for achieving underlying objectives of the design process such as selecting optimal process routes, specifying equipment capacities, determining operating costs, estimating economic and operational efficiencies etc. Material balances assist in the estimation of important variables such as solids, slurry and bulk densities, which are necessary for proper equipment design.

The process optimisation function also requires material balances in order to compare process efficiencies before and after the implementation of proposed improvement strategies. Process optimisation is often intertwined with the process control function. The end result of the control objective is in most cases the optimisation of process performance without having to physically modify the plant. It is common cause that processes cannot be controlled and optimised unless they are measured. In the mineral processing context, this often means performing a metal balance in order to determine prevailing efficiencies. Process efficiencies in the industry are usually expressed in terms of ‘grade’ and ‘recovery’, both of which are determined through a conceited material balancing exercise.

Metal accounting is primarily concerned with estimating total material processed and total valuable metal produced, discarded and present as part of in-process inventory. Invariably, metal balances lie at the centre of the metal accounting activity. The discerning characteristics appear to be that a metal balance has to be transformed into ‘coherent report format that is delivered in a timely fashion in order to meet specified (corporate) reporting requirements’ (Code, 2005) to satisfy the requirements of a metal account. Importantly, the Code states that a metal balance that is suitable for reporting (metal accounting) purposes should include errors associated with measurements constituting the balance calculation.

2.2.2 Types of mass balances

Despite the varied applications of mass balances in the processing industry, two categories of such balances can be discerned: non-model based and model based balances (Fuerstenau & Han, 2003). The difference is based on the level of participation of unit operations in determining the final balanced values.

2.2.2.1 Non-model based balances

The non-model based balance is premised on the conservation of mass across operations (i.e. mass in equals mass out). The non-model balance uses measured process data of the input and output streams of unit operations or entire processes. Unit operations are represented by nodes joined by a network of streams. Simple equality constraints define the balance of mass flow across each node. The implication is that the balance is reversible i.e. products of a unit operation (or entire process) can be used to reconstitute the feed.

The non-model balance approach does not make use of the sometimes high dimension system equations that are often associated with unit process models, but obtains closed balances by adjusting only the measured input and output flow data. As a result, the non-model balancing approach is commonly referred to as 'data adjustment', 'data reconciliation' or 'mass balancing'.

Two key criteria characterise the non-model balancing approach: (i) measured data should be in excess of the amount required to satisfy the balance (redundant data); and (ii) the equations defining the balance should be independent. Data redundancy implies the removal of a sufficient number of degrees of freedom resulting in a system that is over specified.

The low costs and improved reliability of plant instrumentation as well as the availability of digital data acquisition systems make it possible to collect more measurements than required to close a balance. Multiple solutions exist for such systems leading to conflicting solutions as a result of error obtaining in measured data. The challenge therefore is to determine, by adjusting measured values, the best set of consistent values that satisfy the balance. The measured values need to be 'reconciled' through a 'data reconciliation' process.

Data reconciliation is widely applied in the process industry. Software solutions such as JKMBal (Richardson & Morrison, 2003) that encapsulate the basic technique have found wide use either off-line for process accounting purposes (e.g. JKMetAccount® software) or in conjunction with on-line applications such as process optimisation and advanced process control.

2.2.2.2 Model based balances

The model based balance utilises process unit models to achieve a balance. These balances are referred to as 'simulation' and 'design' balances. They are largely irreversible i.e. one cannot reconstitute the feed to the process unit or entire operation using product values. The set of equations describing the process model predict the output (products) values based on given input (feed) values, prevailing operating conditions and unit model parameters. Hence measurements of products do not influence the balance results directly except in instances where measured data are used to fit the process unit models.

In fact, a procedure commonly utilised in process optimisation studies starts with non-model balancing (data reconciliation) of plant surveyed data, followed by the adjustment of unit model parameters (model fitting) based on the reconciled data, after which model based balances (simulations) are performed. Model based balances have found extensive use in process design, optimisation and control applications across the minerals processing industry (e.g. JKSimmet, JKSimfloat).

2.2.3 Weighted sum of squares approach to data adjustment

Data adjustments are usually made based on some measure that describes the accuracy of individual measurements. Richardson and White (1982) pointed out that although all mass balance equations must be satisfied in performing mass balances, measured data cannot be adjusted arbitrarily. The measurement variance is frequently used to assess the quality of measured data and derivatives, owing to its stochastic description of measured data.

Measurement variances have demonstrably been used to assess the reliability of performance indices calculated from measurements and have been recommended for introduction in data adjustment or reconciliation as weighting factors to account for measurement reproducibility (Hodouin et al., 1989; Simpson et al., 1991; Smith & Ichiyen, 1973).

Most data adjustment procedures attempt to minimise an objective function (in the least squares sense) comprised of the sum of the squared adjustments weighted for measurement variance (Equation 2.7).

$$S = \sum_{i=1}^{N} \omega (X_m - X_a)^2 \qquad \textit{(Equation 2.7)}$$

Where:
- S = sum of squares objective function
- X_m = the ith measured value
- X_a = the ith adjusted value
- ω = weighting factor of the ith value
- N = number of measured variables.

The reasoning behind the weighted least squares approach is that the required adjustments to the data and the statistical variations are drawn from the same population, assuming that the process is at steady state. Consequently, on average the adjustments and standard deviation should be similar in value and the expected value of the objective function should be identical to unity (Richardson & Morrison, 2003).

2.3 Data Reconciliation

SSDR was first addressed by Kuehn and Davidson (1961). The authors adjusted measured data in the weighted least squares sense (Equation 2.8) subject to linear mass balance constraints (Equation 2.9) for processes with all streams measured.

$$\min_{x} (x-y)^T W^{-1} (x-y)$$ *(Equation 2.8)*

$$Ax = 0$$ *(Equation 2.9)*

The authors used the inverse of the measurement error variances (W) as weights for the optimisation and the constraints to be satisfied were the mass balance equations that defined the process network as described by the network incidence matrix A. The adjusted and measured component flow rates are represented by the vectors x and y respectively. By applying the method of Lagrange Multipliers to solve Equation 2.8 and Equation 2.9 for the adjusted flow rates the general solution for the problem is obtainable as shown in Equation 2.10 where the resultant adjusted flow rates (x) verify Equation 2.9.

$$x = y - W^{-1}A^T (\mathrm{A}W^{-1}A^T)^{-1}\mathrm{A}y$$ *(Equation 2.10)*

Linear SSDR with all streams measured is the only case with an explicit mathematical solution (Equation 2.10). Derivation of this equation from first principles is explained in Chapter 7.

Subsequent to earlier works based on the analytical solution of the linear steady-state data reconciliation, efforts in measurement optimisation research based on data reconciliation were focussed in at least three distinct areas of study, namely non-linear and dynamic data reconciliation, bias and gross error detection, and optimal sensor placement (sensor network design).

2.3.1 Non-linear steady state and dynamic data reconciliation

Most industrial process streams are multicomponent in nature and often the reconciliation of component concentrations and overall flow rates are required. The constraints to be satisfied are bilinear or higher, rendering the classical solution to the linear case (Equation 2.10) inadequate to solve the non-linear case. Approaches that reduce the non-linear problem to linear have particularly attracted attention from researchers ostensibly due to the relative simplicity of linear-based solutions.

The bi-linear system is arguably the simplest and most common non-linear case encountered, where the constraints to be satisfied are usually comprised of products of overall stream flows and component determinations. A range of techniques have been developed to tackle bi-linear problems which constitute a wide-variety of problems in chemical engineering processes (Hodouin & Makni, 1996; Kelly, 2004; Narasimhan & Jordache, 2000). Crowe et al. (1986) modified the matrix projection method to convert the bilinear problem to linear by variable substitution. The authors linearised component flows from the products of measured stream flows and assays and adjusted these subject to component flow constraints while using Taylor series first derivative estimates of variances of the component flow rate estimates as weights for the adjustment. This approach was to be later argued as less efficient by Sanchez and Romagnoli (1996) who used the QR decomposition technique to determine the projection matrix for both linear and bi-linear problems. Simpson et al. (1991) also proposed a linear based solution to the bilinear problem by deriving expressions of component values as functions of overall stream flow ratios approximated using a first order approximation of the flow ratios around initial flow estimates. This approach enabled the use of variance estimates of the actual measured variables (component assays, moisture ratios) as weights in the new objective function compared to the derived weights proposed by Crowe et al. (1986).

Process operations usually involve complex processes that are characterised by conservation balances (mass and/or energy) as well as limitations imposed by thermodynamic, physical and equilibrium properties of materials and equipment. As a result the reconciliation model may include inequality and normalisation equations that impose bounds on individual variables and attendant operational limitations. This complexity presents challenges in solving the resulting non-linear problem that are not paused in the simple linear or bi-linear cases.

Non-linear data reconciliation has been addressed by several authors. Non-linear problems with equality constraints have been solved satisfactorily by relatively elementary approaches such as the classical Lagrange multiplier method and successive linearization approaches (Kelly, 2004; Pai & Fisher, 1988) by exploiting linear derivatives of the original non-linear constraints in an effort to convert the problems to linear (Britt & Luecke, 1973; Knepper & Gorman, 1980). In order to deal with inequalities in the constraints equations non-linear programming (NLP) methods have been proposed including gradient-based approaches such as Newton-based or Generalised Reduced Gradient (GRG) methods. Tjoa and Biegler (1991) developed an efficient hybrid successive quadratic programme (SQP) for solving the combined reconciliation and gross error problem. SQP is generally regarded as providing a more accurate solution compared to the linearization approach given that the function to be minimised (Equation 2.8) in data reconciliation is quadratic. NLP methods facilitate the use of a non-linear objective function in addition to a weighted least squares function and can handle non-linear, inequality constraints and limits imposed on variables. Stochastic search techniques like Genetic Algorithm (Wongrat et al., 2005) have also been applied to overcome the difficulty of gradient based methods in handling discontinuities and non-convex properties.

A further complication to solving practical processes is the reality that they are never at steady state although it is normally the goal of operations to maintain operations as close to steady state as possible. Processes constantly undergo changes even around intended steady state conditions. It is necessary therefore to represent these with dynamic models that are solved using dynamic data reconciliation approaches.

The reconciliation for dynamic systems has been an active field (Albuquerque & Biegler, 1996; Liebman, 1992; Mingfang et al., 2000). Process industrial application for solutions to dynamic systems is still at a relatively early stage compared to advances made for steady state systems. The most widely reported methods for solving dynamic data reconciliation problems include statistical filtering (e.g. Kalman filtering) and mathematical programming approaches.

Kalman filtering estimates state variables and simultaneously calculates their associated variances (Kalman, 1960). The method has been used to solve both linear (Gelb, 1974; Muske & Edgar, 1998; Schmidt, 1980) and non-linear (Chiari et al., 1997; Karjala & Himmelblau, 1996; Norgaard et al., 2000) dynamic systems. A variety of stochastic-based filtering methods based on the technique have been developed for various engineering applications (Bai et al., 2006; Chen et al., 2008), however, one of the drawbacks of the method is its inability to handle inequality constraints.

The general solution for dynamic data reconciliation based on the mathematical programming approach is premised on the minimisation of a weighted objective function (similar to Equation 2.8), subject to a dynamic model comprising differential constraints and sometimes non-linear algebraic and/or inequality equations. Developments in mathematical programming techniques for solving both non-linear and dynamic data reconciliation are well reported in the literature (Albuquerque & Biegler, 1996; Liebman, 1992) and have found use in various applications in the industry (Barbosa et al., 2000; de Andrade Lima, 2006; Eksteen et al., 2002).

2.3.2 Bias and gross error detection

The problem of data reconciliation and gross error detection are closely related. Ripps (1965) first recognised the importance of identifying gross errors in the data reconciliation process. If corrupt data are reconciled, the error is spread across all measurements, degrading the quality of estimates obtained through data reconciliation. A great deal of research has been focussed on developing techniques for detecting and reducing the impact of systematic errors or on removing them altogether. Gross error detection and diagnosis is generally expected to occur in three distinct stages: detection, identification and estimation.

Many detection techniques involve classical hypothesis tests on the residuals produced from a data reconciliation step. Statistics calculated on the basis of balance residual values are compared with tabulated values to confirm or reject hypotheses testing the presence of gross error (or lack thereof). Four basic statistical tests have been developed and are widely applied in gross error detection (GED) for linear steady state processes: the global test, the nodal test, measurement test and the generalised likelihood ratio test (Narasimhan & Jordache, 2000).

Reilly and Carpani (1963) first presented the Global Test (GT) based on the expected distribution (chi-square) of the data reconciliation objective function residual value under the null hypothesis and with degrees of freedom equal to the rank of matrix A (ref. Equation 2.8). The nodal test (NT) presented by Mah and Stanley (1976) evaluates the normality of the z-statistic for each constraint residual (under the null hypothesis) to decide which of the constraints contains gross error. Similarly, Mah and Tamhane (1982) proposed the measurement test (MT) under which it is hypothesised that measurement adjustments after data reconciliation follow a multivariate normal distribution with zero mean and covariance calculated from the balance residuals and first measurement variances. However, the methods described assume that measurement errors are independent and that the covariance matrix of the balance residuals is diagonal. The generalised likelihood ratio (GLR) test (Narasimhan & Mah, 1987) along with approaches such as principal component analysis (Tong & Crowe, 1995) was designed to deal with full covariance matrices. The GLR is based on the likelihood ratio statistical test.

Subsequently, several standardized statistical tests have been proposed for gross error detection (Almasy & Sztano, 1975; Mah & Stanley, 1976; Narasimhan & Mah, 1987; Romagnoli & Stephanopoulos, 1980; Tamhane et al., 1988; Tong & Crowe, 1995). Later some interesting research was done in the simultaneous data reconciliation and gross error detection using mathematical programming (Soderstrom et al., 2003) and novel definitions and treatment of gross errors (Benqlilou et al., 2005; Nguyen & Bagajewicz, 2011).

2.3.3 Sensor network design

The sensor network design problem (SND) is concerned with selecting variables to be measured in a process network in order to meet desired measurement objectives. Research in SND has led to the development of several procedures for meeting criteria such as observability, precision, accuracy, robustness and reliability for steady state and dynamic systems.

Solution procedures involved either one or a combination of the following methods: matrix algebraic, graph theoretic, mathematical programming and sometimes genetic algorithm approaches (Narasimhan & Jordache, 2000), depending on the objectives of the problem and whether the measurement systems are linear/non-linear and steady-state/dynamic.

The extension of the linear data reconciliation problem to a case where not all streams are measured was first investigated by Vaclavek and Loucka (1976). The authors devised a strategy for ensuring observability of a selected set of variables in linear processes using graph theory. The concept was later extended to bilinear systems by Ragot et al. (1992). Madron and Verveka (1992) also built on the method by using Gauss-Jordan elimination to identify the minimum number of sensors required to observe key variables thereby minimising the cost of measurement.

Kretsovalis and Mah (1987) developed linear algebraic procedures for maximising accuracy in redundant sensor networks. They achieved this by quantifying the sensitivity of estimates to sensor placement in redundant linear flow networks.

Ali and Narasimhan (1993) used graph theoretic approaches to evaluate the effects of the likelihood of sensor failure on the observability and the probability of estimation of variables in observable linear steady state processes. The work introduced the idea of reliability of estimation of variables. The authors extended the approach to the design of optimal sensor networks for redundant linear processes (Ali & Narasimhan, 1995). Bagajewicz and Sanchez (1999) later combined observability and redundancy criteria into a unified objective termed degree of estimability and sought to optimise this in the upgrade of pre-existing sensor networks. Benqlilou et al. (2001) also tackled the sensor reallocation and upgrade problem with the aim of maximising precision.

Bagajewicz (1997) proposed a solution for the minimum cost sensor network problem based on graph theory and linear algebra. The optimisation was posed as MINLP subject to gross error detectability, resilience and precision criteria.

Sen et al. (1998) developed an algorithm based on graph theory concepts and genetic algorithms to optimise both single and multiple objectives for linear nonredundant sensor networks. Methods based on genetic algorithms gained tractability in solving SND problems for non-linear systems (Heyen et al., 2002). Later Carnero et al. (2005) used a genetic algorithms to close mass balances using a localised search strategy.

Static-based methodologies were extended to dynamic systems. Chmielewski et al., (2002) simplified the SND solution approach for dynamic processes by posing the problem as NLP and subsequent linearization. Benqlilou et al. (2005) solved the SND problem subject to Kalman filtration based dynamic reconciliation.

The works cited here characterise the progress of SND research whereby sensor placement is largely viewed as a mathematical optimisation problem. The problem is generally posed as a mixed integer non-linear programming problem (MINLP), with variations in formulation largely a function of the nature of the problems encountered that render themselves into linear, bilinear or non-linear systems, as well as the subject of the optimisations. The numerical routines developed solve for existing sensor networks cannot be used to make measurement placement decisions at the conceptual stage of sensor network design.

Chapter 3

MEASUREMENT NETWORK DESIGN PRACTICE – RESEARCH METHODOLOGY

This chapter presents the methodology used to establish prevailing industrial practices in the design of measurement networks for metal accounting in the minerals and metallurgical processing industry. The mode of enquiry used to obtain the information is explained. The preparation and analysis of data collected are described.

3.1 Introduction

The approaches to designing measurement networks for metal accounting were investigated by exploring the relationship between measurement usage in metal accounting and two important measurement design attributes: firstly, the source of accounting measurements in operating mineral and metallurgical processing flowsheets and, secondly, the quality of measurement. The industrial sites visited, interviews conducted and process flowsheets audited as part of this study are listed in Table 1.

Table 1: Sites visited, interviews conducted and plant flowsheets sourced

Participants	Questionnaires	Interviews	Flowsheets
Concentrators	4	5	4
Smelters	2	2	2
Refineries	2	2	2
Evaluation	1	1	1
Consultants	-	5	-
TOTALS	**9**	**15**	**9**

The names of the participating companies and individuals are not listed since the purpose of the study is to establish trends pertaining to measurement network design practices. The operations visited belong to major global mining corporations. These included coal upgrading operations with capacities of up to 4000 tph; precious metal concentrators processing approximately 1000 tph; and smelter operations tapping to the order of 300 ton of metal per day.

The data for this analysis were obtained through a questionnaire based survey conducted on operating plants in the South African minerals and metallurgical industry. Surveys are typically used to determine the distribution of variables that are normally difficult to observe. The survey approach is a type of non-experimental research in which questionnaires are usually instrumental in obtaining data with the aim of understanding the underlying factors that determine the characteristics of a population (Durrheim, 2006; Underhill & Bradfield, 1998).

3.2 Questionnaire design and administration

The questionnaire was designed to establish the following:

- The relationship between measurement location in mineral process networks and the level of usage in metal accounting.
- The relationship between the precision levels in measurements and the level of usage in metal accounting.
- To establish the criteria used to select measurements for use in common metal accounting procedures.

The key topics in metal accounting practice that were considered in the investigation were (i) mass measurement and sampling and (ii) metal balancing and reconciliation procedures.

(i) Mass measurement and sampling

Methods used to obtain mass measurement and samples on operating mineral process plants were requested from participating sites. Attributes such as measurement frequency, usage, precision and importantly, the source of the measurements on the process flowsheet were collected and used to build a profile for each measurement identified in the survey campaign.

(ii) Metal balancing and reconciliation

The metal balance lies as the centre of the metal accounting function. Measurements taken are used to calculate the balance as well as reconcile what has been processed and what was received from raw material sources such as the mine or other suppliers. The survey investigated methodologies governing the procedures associated with metal balancing and reconciliations activities such as selection of accounting boundaries, secondary accounting, calculation of key performance indicators and protocols around custody transfer interfaces.

3.2.1 Generic flowsheet

In order to compare practices across different operations, a generic flowsheet was constructed in which streams were identified based on function as shown in Figure 2. The diagram shows all possible roles process streams can assume in a typical mineral/metal beneficiation flowsheet.

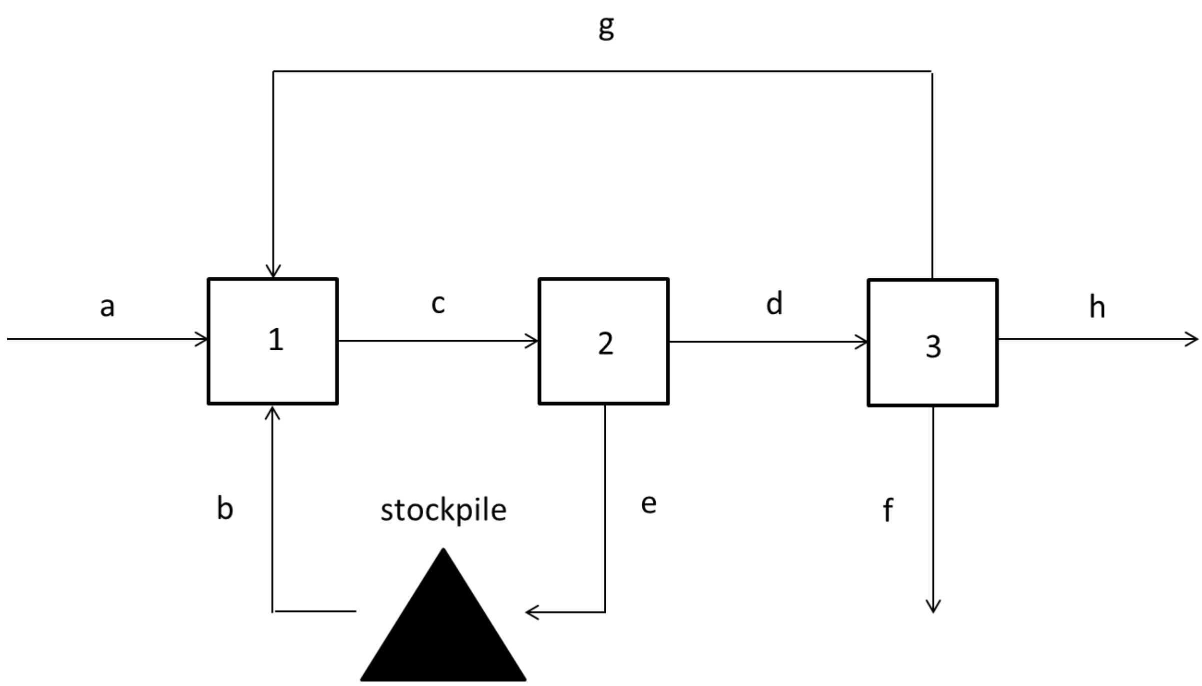


Figure 2: Generic flowsheet

In practice, common names such as feed, product etc. are often used to refer to process streams based on the extent of beneficiation of the material conveyed. Alternatively, technical nomenclature based on spatial location of streams in a process network such as internal (in-process) or terminal (feed, intermediate and final products, internal and external recycles, and tailings) stream may be used. Table 2 lists the stream types shown in Figure 2 based on these two naming schemes.

Table 2: Identification of characteristic streams

Stream ID	Nominal	Spatial
a	Fresh feed	Terminal
b	Reclaim (external recycle)	Terminal
c,d	In-process	Internal
e	Intermediate product	Terminal
f	Final Tailings	Terminal
g	Recycle (internal recycle)	Internal
h	Final product	Terminal

Internal streams can be referred to as either 'complex' or 'simple', the former indicating connection between adjacent nodes (streams *c* and *d*) and in the latter case indicating connections between non-adjacent nodes (stream *g*).

3.2.2 Distribution and administration of the questionnaire

Personal contact with participants in the AMIRA P754 Project enabled the author to gain access to a wide cross section of operations in the South African minerals industry. Most of the contacts obtained were second and higher referrals, personally unknown to the author.

On securing agreement for participation from contacts, the questionnaire document was sent in electronic form, mostly through electronic mail. The main advantage of this approach was that a large set of participants was reached.

A log of all contacts and questionnaires sent out was kept for administration and follow up purposes. After receiving a completed questionnaire, the respondent was contacted for a follow-up verification of the data submitted.

3.2.3 Survey sample

Representative sampling was viewed from two perspectives: firstly, at least one of the three different types of mineral beneficiation processes in the value chain had to be included i.e. concentration, extraction and refining; and secondly, a "large enough" number of streams and storage equipment had to be captured. This is referred to as 'sampling to redundancy' (Durrheim, 2006). In this approach, sample size is not determined in advance. An increasing number of participants are incorporated in the primary sample until the same themes or observations begin to recur. In this sense, redundancy is achieved when no new information is discovered by increasing the sample size.

3.2.4 Flowsheets

Process flowsheets detailing all routine measurement points were obtained as part of the survey. All routine measurement sources on the flowsheets were audited and verified during follow up interviews with participants.

Nodal diagrams, as first described by Smith & Frew (1983) were prepared for each operation using the audited flowsheets as a basis. In these diagrams only those streams that convey target mineral ore constituted the diagram. Hence the prepared nodal diagrams constituted 'measurement networks' as defined in this study. Further processing of the flowsheets included identifying stream types on the prepared nodal diagram according to the nomenclature listed in Table 2.

3.3 Data preparation

Raw data collected consisted of completed questionnaires and process flowsheets. Questionnaire responses were coded and stored in spreadsheet form (Microsoft EXCEL). Flowsheet data were similarly collated and presented in spreadsheet form. The spread sheets containing the raw survey data are included on a compact disc submitted with this thesis.

3.3.1 Questionnaire data

A template of the questionnaire used in this study is presented in Appendix A. Questions 1.0 – 4.0 of the questionnaire sought to establish relationships between plant equipment types (i.e. stream or storage equipment types) and measurement attributes (i.e. extent of measurement usage in MA and relative precisions). Table 3 lists all the relationships investigated.

The alphanumeric categories 'A1-A6' and 'B1-B6' referred to in Table 3 denote normative levels of measurement usage in metal accounting for mass and assay measurements (respectively) in the survey sample. The criteria used to rate measurement usage for placement in the respective categories are shown in 'List A' (mass) and 'List B' (assays) in Appendix A. The level of precision of each measurement encountered in the survey was assessed and rated on an ordinal scale comprised of simple numerical categories '1-6'. The precision assessment criteria are listed in Table 4-1 (Appendix A).

Table 3: Categorical relationships between equipment type and measurement attribute

Questionnaire section	Column variable	Row variable
1.0	Stream type	Mass measurement usage in MA (categories A1 – A6)
1.0 (Table 1)	Stream type	Mass measurement precisions (categories 1 – 6)
2.0	Storage type	Mass usage in MA (categories A1 – A6)
3.0	Stream type	Sample usage in MA (categories B1 – B6)
3.0 (Table 4)	Stream type	Sample precisions (categories 1 – 6)
4.0	Storage type	Sample usage in MA (categories B1 – B6)

Table 4 shows an example of a cross-classification between stream type (column labels) and mass measurement usage in metal accounting (row labels) for all questionnaire responses to Question 1.0. Table 4 constitutes a contingency table. Contingency tables are used to study relationships between categorical variables (Durrheim, 2006; Everitt, 1977; Underhill & Bradfield, 1998).

Table 4: Mass measurement usage in MA by stream type (Question 1)

Category	Stream ID							Totals
	Fresh feed	In-process	Internal recycle	External recycle	Intermediate Product	Final Product	Final Tailings	
A1	9	4	4	5	4	9	3	38
A2	0	1	1	1	1	0	3	7
A3	0	1	1	1	1	0	1	5
A4	0	2	2	2	2	0	0	8
A5	0	1	1	0	1	0	0	3
A6	0	0	0	0	0	0	2	2
Totals	9	9	9	9	9	9	9	63

A contingency table consists of a matrix of 'counts' where each cell value has no quantitative meaning besides indicating the occurrence of an attribute or characteristic under observation. In Table 4, for instance, each cell in the table represents a 'count' of the number of times that a stream type was selected by respondents in a given mass measurement usage category i.e. from category A1 to category A6.

Cross-tabulation of the two categorical variables was designed to test the hypothesis that the level of measurement usage in metal accounting is dependent on the type of stream from which the respective measurements originate. Further processing of the data involved collapsing categories of the row variables to obtain more descriptive categories to facilitate data analysis. The ordinal classes of High, Medium and Low were selected and used to re-classify the primary categories as shown in Table 5. Using this scheme, the Question 1.0 data listed in Table 4 is re-classified as shown in Table 6.

Table 5: Re-classification of measurement usage and precision categories

Category level	Stream measurement usage ranges		Measurement precision ranges
	Mass	Assay	
High	A1-A2	B1-B2	1-3
Medium	A3-A4	B3-B4	4-5
Low	A5-A6	B5-B6	6

Table 6: Question 1 responses summary with re–classified row categories

Measurement Usage Category	Stream type						
	Feed	In-process	Internal recycle	External recycle	Interm. Prod.	Final Prod.	Tailings
High	9 (100)	5 (56)	5 (56)	6 (67)	5 (44)	9 (100)	6 (67)
Medium	0 (0)	3 (33)	3 (33)	3 (33)	3 (56)	0 (0)	1 (11)
Low	0 (0)	1 (11)	1 (11)	0 (0)	1 (0)	0 (0)	2 (22)

The presentation of the data in Table 6 based on spatial classification of streams Table 7. Similarly, the data as presented in Table 6 and Table 7 examines the aforementioned hypothesis with re-classification of measurement usage categories and use of spatial stream notations coupled with re-classified categories respectively.

Table 7: Question 1 responses summary with re–classified row categories and spatial categorisation of stream types

Measurement Usage category	Stream type		Total
	Internal	Terminal	
High	10 (56)	35 (78)	45 (71)
Medium	6 (33)	7 (16)	13 (21)
Low	2 (11)	3 (7)	5 (8)

Section 6 of the questionnaire provided factual data that were summarised by summing the number of positive or ‘yes’ responses to either dichotomous queries which prompted yes/no answers or multiple choice type questions that required the selection of appropriate responses from a number of alternatives. The summarised data were presented in graphical form. Bar graphs are used in instances where the independent variable under observation is measured at the nominal level.

3.3.2 Flowsheet data

The flowsheets submitted include all functional streams in a given operation. The streams measured and the respective measurements used in metal accounting were marked on the flow diagrams. The detailed flowsheets provided information on the proportion of streams measured as well as the percentage of measurements used in metal accounting. These statistics were collated as shown in Table 8.

Table 8: Flowsheet D mass measurement statistics

Stream type	Total no. of streams	Weight, %	Streams weighed		Usage in MA	
			No.	Fraction, %	No.	Usage, %
Fresh Feed	1	4	1	100	1	100
In-process	17	74	3	18	1	6
Internal Recycle	3	13	1	33	0	0
External Recycle	0	0	0	-	0	-
Intermediate product	0	0	0	-	0	-
Final Product	1	4	1	100	1	100
Tailings	1	4	0	0	0	0
TOTAL	23	100	6	26	3	13

The information in Table 8 was obtained from a smelter operation (*Flowsheet D* in Appendix B) that participated in the current survey. Column 2 in Table 8 lists the total number of each characteristic stream followed by the weighted fraction of each stream type expressed as a percentage of the total number of streams in the network (Column 3). Column 4 lists the total number of streams weighed and Column 5 expresses this number as a percentage of the total number of the respective stream type identified in all flowsheets submitted. Column 6 lists the number of stream measurements used in metal accounting, while Column 7 expresses this number as a percentage of the total number of the respective stream type counted in the survey. A summary table with the same format as Table 8 was compiled by summing corresponding cell values of individual flowsheet tables. Assay measurements were similarly collated and summarised.

The designation of streams as either measured or not measured and the stream measurement usage status provided a means of investigating the association between stream type, level of measurement as well as the extent of measurement usage in metal accounting. Table 9 shows a contingency table derived from the smelter data listed in Table 8. The table describes the association between stream type and extent of mass measurement observed on the smelter operation. The frequency data presented as percentages are shown in parentheses.

Table 9: Incidence of mass measurement per stream type for a base metal smelter

Sampling category	Stream type						
	Fresh Feed	In-process	Internal Recycle	External Recycle	Intermediate product	Final Product	Tailings
measured	1 (100)	3 (18)	1 (33)	0 (0)	0 (0)	1 (100)	2 (100)
not measured	0 (0)	14 (82)	2 (67)	0 (0)	0 (0)	0 (0)	1 (0)
Total	1 (100)	17 (100)	3 (100)	0 (0)	0 (0)	1 (100)	2 (100)

Contingency tables for investigating the association between stream type and measurement usage were similarly compiled. Based on the spatial classification of streams as either internal or terminal (Table 2), the data in Table 9 becomes a 2 x 2 contingency table as shown in Table 10.

Table 10: A 2 x 2 contingency table of mass measurement incidence per stream type for a base metal smelter

Measurement category	Stream type		Total
	Internal	Terminal	
measured	4 (20.0)	2 (67.0)	6 (26.1)
not measured	16 (80.0)	1 (33.0)	17 (73.9)
Total	20 (100.0)	3 (100.0)	23 (100.0)

3.4 Data analysis

3.4.1 Contingency tables

The contingency tables described in the preceding section have been set up conventionally with ‘stream type’ as the independent variable constituting the columnar space and ‘measurement usage category’ as the dependent variable occupying the row space of the contingency matrices. The tabulated frequencies can be converted to percentages where the convention is to calculate the percentages in the direction of the independent variable by using the summed frequencies in the columnar direction as a basis for calculating percentage values.

A comparison of the differences in percentage values in the tables provides a rapid method for assessing the extent of association between the categorical variables. As a convention, comparisons are made in the direction contrary to that in which the percentages were determined. In this study, percentages were computed down the columns and therefore all comparisons will be conducted across the rows.

3.4.2 Factual data

Factual data gathered on metal accounting procedures (Section 6 of the Questionnaire document) was effectively presented in the form of frequency bar graphs for analysis as presented in Chapter 4 (Sections 4.2).

Chapter 4

METAL ACCOUNTING PRACTICE

This chapter presents the results of the questionnaire survey described in Chapter 3. The survey sought to establish prevailing practices in the minerals industry with regards to the design of measurement networks for metal accounting purposes. Indicators of design intentions were the level of usage and frequency of determination of characteristic stream and storage measurements for metal accounting. The accounting practices observed in this study are compared with best practice guidelines from the AMIRA P754 Code.

4.1 Measurement usage in metal accounting

4.1.1 Measurement usage according to stream type

Two approaches were employed in gathering data on the selection of measurements originating from different stream types for use in metal accounting. The structured questionnaire responses represent an assessment study (Durrheim, 2006; Everitt, 1977; Underhill & Bradfield, 1998) designed to measure the perceptions of industry practitioners, whereas flowsheet data provided information on observations made on actual practice ‘on the ground’.

4.1.1.1 Questionnaire responses

The association between measurement usage in metal accounting and stream type tested whether mass and assay measurements were selected on the basis of the stream types they originate from. Table 11 presents the respondents’ views based on the nominal stream type descriptions. All respondents were of the opinion that all *Feed* and *Final Product* stream measurements taken are routinely used in metal accounting. The scores for both stream types are unanimous (100%) in the high measurement usage category, strongly suggesting that *Feed* and *Final Product* measurements are integral to the metal accounting function. The rest of the streams exhibit indiscernible frequency distributions across the measurement usage levels, suggesting indifference in the way respondents view measurements generated from the respective stream types in metal accounting.

Table 11: Incidence of mass and assay usage in MA per stream type (questionnaire)

Stream type	Measurement usage level					
	Mass			Assay		
	High	Medium	Low	High	Medium	Low
Feed	9 (100)	0 (0)	0 (0)	9 (100)	0 (0)	0 (0)
In-process	5 (56)	3 (33)	1 (11)	2 (22)	6 (67)	1 (11)
Internal Recycle	5 (56)	3 (33)	1 (11)	1 (11)	6 (67)	2 (22)
External Recycle	6 (67)	3 (33)	0 (0)	2 (22)	6 (67)	1 (11)
Interm. Product	5 (44)	3 (56)	1 (0)	1 (11)	7 (78)	1 (11)
Final Product	9 (100)	0 (0)	0 (0)	9 (100)	0 (0)	0 (0)
Tailings	6 (67)	1 (11)	2 (22)	6 (67)	1 (11)	2 (22)
Total	45 (71)	13 (21)	5 (8)	30 (48)	26 (41)	7 (11)

Analysis based on the spatial description of process streams shows a more pronounced difference in measurement usage between the two stream types. Table 12 presents the same frequency data listed in Table 11 using the internal and terminal stream designations (ref. Table 1). The frequency data in Table 12 indicates that industrial practitioners rate terminal stream measurements higher than internal stream measurements in metal accounting, scoring 78% in the high usage category for terminal measurements compared to the internal stream measurement score of 56% in the case of mass measurements. Assay measurements were rated similarly, producing a 60% score for terminal streams compared to an internal stream measurement score of 17% in the high measurement usage category.

Table 12: Incidence of mass and assay usage in MA per stream type – questionnaire responses (spatial stream types)

Stream type	Measurement usage level					
	Mass			Assay		
	High	Medium	Low	High	Medium	Low
Internal	10 (56)	6 (33)	2 (11)	3 (17)	12 (67)	3 (17)
Terminal	35 (78)	7 (16)	3 (7)	27 (60)	14 (31)	4 (9)
Total	45 (71)	13 (21)	5 (8)	30 (48)	26 (41)	7 (11)

Interestingly, aggregated data suggests a positive bias towards mass measurement usage in metal accounting compared to assays. The summed frequency data (last row in Table 11 and Table 12) shows that 71% of the total mass measurement usage responses rated the use of mass as high compared to 48% in the case of assay measurement usage. In addition, assay data recorded a marginally higher score in the low usage category compared to mass data.

4.1.1.2 Flowsheet results

The association between measurement usage and nominal stream types is shown in Table 13 based on flowsheet data collected from all participating operations. The nodal diagrams and flowsheet statistics for each operation visited are listed in Appendix A for reference. The sum of the 'Used' and 'Not Used' column frequencies constitutes the total number of streams measured. For instance, of the 129 *In-process* streams in the entire flowsheet survey results in Table 13, 34 (i.e. 21 plus 13) were weighed and 95 were not weighed.

Table 13: Incidence of mass and assay usage in MA per stream type – flowsheet data

Stream type	Measurement Status					
	Mass			Assay		
	Used	Not Used	Not measured	Used	Not Used	Not measured
Feed	15 (100)	0 (0)	0 (0)	15 (100)	0 (0)	0 (0)
In-process	21 (16)	13 (10)	95 (74)	28 (22)	21 (16)	80 (62)
Internal Recycle	1 (5)	3 (15)	16 (80)	1 (5)	3 (15)	16 (80)
External Recycle	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Interm. Product	1 (100)	0 (0)	0 (0)	1 (100)	0 (0)	0 (0)
Final Product	34 (100)	0 (0)	0 (0)	34 (100)	0 (0)	0 (0)
Tailings	3 (14)	3 14)	16 (73)	5 (23)	2 (9)	15 (68)
All streams	75 (34)	19 (9)	127 (57)	84 (38)	26 (12)	111 (50)

It is seen in Table 13 that all *Fresh Feed* and *Final Product* streams are always weighed and assayed in actual practice and the measurements taken are always used in metal accounting. Notably, 74%, 80% and 73% of the total number of *In-process, Internal Recycle and Final Tailings* streams respectively were not routinely weighed, while for assay measurements 62%, 80% and 68% of the same streams respectively were not assayed, indicating a relatively lower significance in metal accounting for these stream types compared to *Fresh Feed* and *Final Product* streams.

In terms of the absolute number of measurements, more assays are performed than mass measurements in general. Overall, 43% of all streams were weighed compared to 50% that were assayed. However 80% of all mass measurements taken were used in metal accounting i.e. 75 out of 94 stream masses, while 71% of all assays performed were used in metal accounting i.e. 84 out of 110 stream assays. Although this suggests a higher utilisation of mass than assay measurements, the absolute quantities suggest that assays would tend to dominate metal accounting data sets compared to mass measurements.

Analysis based on the spatial stream categories suggests a substantive difference in the treatment of terminal and internal streams with respect to incidence of measurement and the use of these measurements in metal accounting. Table 14 shows the spatial stream based analysis of the same assay usage data presented in Table 13. The data shows that a total of 78% and 79% of all terminal streams encountered in the flowsheet survey were weighed and assayed respectively. Approximately 95% of these measurements were used in metal accounting, suggesting that terminal stream data are primarily generated for metal accounting purposes. Notably, the 5% fraction of terminal stream measurements not used in metal accounting originated from the *Tailings* stream category.

Table 14: Incidence of mass and assay usage in MA per stream type – flowsheet data (spatial stream types)

Stream type	Measurement Status					
	Mass			Assay		
	Used	Not Used	Not measured	Used	Not Used	Not measured
Internal	22 (15)	16 (11)	111 (74)	29 (19)	24 (16)	96 (65)
Terminal	53 (74)	3 (4)	16 (22)	55 (76)	2 (3)	15 (21)
Total	74 (34)	19 (9)	127 (57)	84 (38)	26 (12)	111 (50)

In contrast, 26% and 35% of all internal streams were weighed and assayed respectively. Of these, 58% of mass measurements taken and 54% of assays performed were used in metal accounting, indicating that in practice a significant portion of internal measurements (over 40%) are taken for purposes other than metal accounting.

4.1.2 Measurement precisions per stream type

This relationship was designed to investigate whether of precisions with which measurements were determined depended on the streams from which they originate from. Table 15 presents the respondents' views based on the nominal stream type categories.

Table 15: Incidence of mass and assay precision per nominal stream type – questionnaire responses

Stream type	Precision level					
	Mass			Assay		
	High	Medium	Low	High	Medium	Low
Feed	5 (56)	4 (44)	0 (0)	4 (44)	5 (56)	0 (0)
In-process	4 (44)	5 (56)	0 (0)	3 (33)	6 (67)	0 (0)
Internal recycle	4 (44)	5 (56)	0 (0)	3 (33)	6 (67)	0 (0)
External recycle	4 (44)	5 (56)	0 (0)	3 (33)	6 (67)	0 (0)
Interm. Prod.	5 (56)	4 (44)	0 (0)	3 (33)	6 (67)	0 (0)
Final Prod.	8 (89)	1 (11)	0 (0)	7 (78)	2 (22)	0 (0)
Tailings	3 (33)	4 (44)	2 (22)	2 (22)	7 (78)	0 (0)

The respondents rated the *Final Product* stream measurements as the most precisely determined and the *Tailings* stream measurements as the least precisely determined, suggesting that more efforts are expended in achieving high precisions for *Final Product* measurements than the rest of the stream types, with the *Tailings* stream apparently receiving the least measurement effort. In addition, the *Tailings* stream is the only stream type that registered scores in the low measurement precision category (mass data). The rest of the streams exhibited similar distributions in between the two extremes.

Table 16: Incidence of mass and assay precision per spatial stream type – questionnaire responses

Stream type	Precision level					
	Mass			Assay		
	High	Medium	Low	High	Medium	Low
Terminal	5 (56)	4 (44)	0 (0)	4 (44)	5 (56)	0 (0)
Internal	4 (44)	5 (56)	0 (0)	3 (33)	6 (67)	0 (0)

Presentation of the data with stream types designated as internal and terminal streams shows that respondents rated all internal measurements as falling in the medium to high precision categories (Table 16). Notably, as mentioned in the preceding section, over 40% of all internal measurements taken are not actually used in metal accounting.

4.1.3 Usage of storage measurements in metal accounting

Table 17 shows the frequency table of responses regarding the opinion of participants on the use of typical storage area measurements in metal accounting. The *Tailings* and *Spillage* storage areas recorded the only scores in the medium as well as the low mass measurement usage categories. All other storage area masses were placed exclusively in the high value category of mass usage in metal accounting.

Table 17: Incidence of mass and assay usage in MA per storage type – questionnaire responses

Storage type	Measurement usage level					
	Mass			Assay		
	High	Medium	Low	High	Medium	Low
Run of mine	9 (100)	0 (0)	0 (0)	2 (22)	5 (56)	2 (22)
In-process	9 (100)	0 (0)	0 (0)	4 (44)	4 (44)	1 (11)
Final Product	9 (100)	0 (0)	0 (0)	9 (100)	0 (0)	0 (0)
Tailings	6 (67)	1 (11)	2 (22)	6 (67)	0 (0)	3 (33)
Spillage	2 (22)	2 (22)	5 (55)	3 (33)	0 (0)	6 (67)
All storage types	35 (77)	3 (7)	7 (16)	24 (53)	9 (20)	12 (27)

The respondents identified the *Final Product* storage assays as the only definite accounting assay measurements followed by the *Tailings* storage assays. The *Spillage* storage area assays were considered the least in metal accounting.

On average, it appears that respondents are less likely to use storage assays in metal accounting compared to storage masses save for the *Final Product* storage assays which appear to be a definite source of metal accounting data. The aggregate frequencies score 77% for mass in the high measurement usage category and 53% for assays in the same category.

4.2 Accounting procedures

The influence of procedures on the selection of measurements used to determine common KPI indices in metal accounting were investigated based on the following common accounting processes: (i) the determination of accounting boundaries, (ii) the selection of parameters used to estimate bulk stores, (iii) the management of data across accounting boundaries and (iv) the computation of typical performance measures encountered in minerals beneficiation operations. The following critical areas in accounting were addressed:

- Primary accounting
- Secondary accounting
- Custody transfer practices
- Stock and inventory measurement
- Plant recovery
- Accountability.

Procedures defining primary and secondary accounting determine which measurements are used to fulfil the respective objectives, while custody transfer practices compound the task of selecting final data used to track material movement across defined accounting boundaries. Procedures followed in estimating stocks and inventory influence the quality of accounting and finally, the use of recovery and accountability as key measures of operational and metrological performance, respectively, is ubiquitous in the minerals beneficiation industry.

4.2.1 Primary accounting

This part of the questionnaire sought to obtain information on decision-making processes that impact the choice and use of measurements used for primary accounting. The following attributes of primary accounting were addressed:

- Accounting boundary
- Mass balance data
- Mass balance calculation

➢ Accounting period.

The survey results are summarised in Figure 3 in the form of bar graphs. The four graphs depicted in the figure show responses to alternatives provided in the questionnaire.

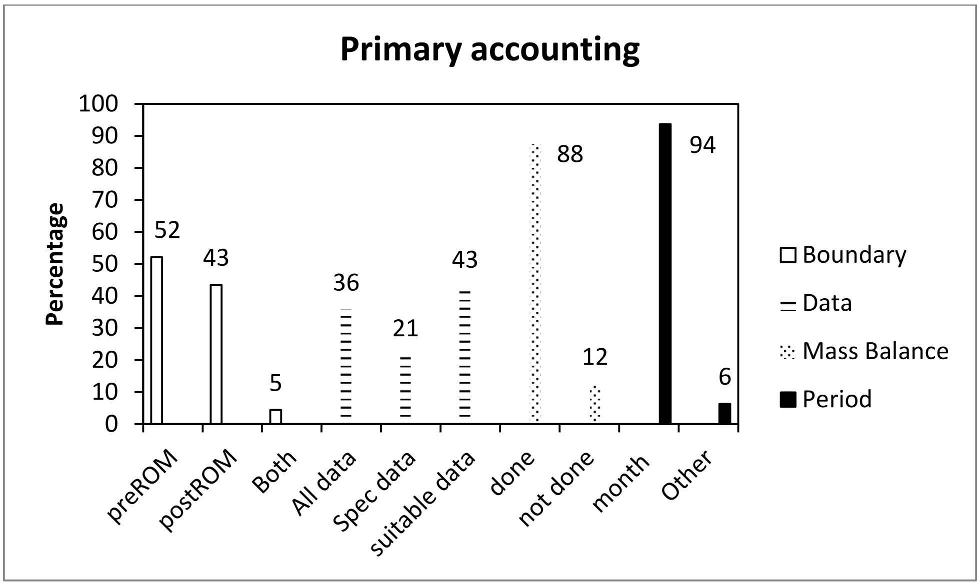


Figure 3: Primary accounting practices with respect to choice of boundaries, use of measured data, mass balance practice and accounting period

4.2.1.1 Choice of accounting boundary

Mass balances are usually calculated within specified battery limits. These limits may describe entire processes or selected sections of processes. This real or imaginary demarcation that separates the portion of the process that is of interest to the balance is normally referred to as an accounting boundary. Thus for any process several alternatives exist depending on the objectives of the balance instance. In primary accounting, the choice may be relatively straightforward given that primary balances are prepared on the basis of net plant inflow and net outflow.

Often choices have to be made whether to include the ROM or plant feed storage inside (“preROM”) or outside (“postROM”) the primary accounting boundary. Ultimately the final decision depends on site-specific procedures pertaining to custodianship that includes stock-taking, maintenance and handling costs of stored material. Given the risks associated with accurate estimation of bulk material in storage, selection of boundary limits around potentially large storage such as feed stockpiles may significantly influence the quality of mass balance computations as shown by Cutler & Eksteen (2006).

Results displayed in the first section of the graph in Figure 3 show that the difference in preference is generally insignificant i.e. setting the boundary before or after ROM storage being 52% or 43% (respectively) of the responses with 4% indicating either of the two choices. From this, it can be surmised that the choice of boundary is largely arbitrary.

4.2.1.2 Selection of mass balance data

Commonly, suitable data are selected for the precise and accurate accounting of net inflows and outflows of a process. Specified measurements are normally used to achieve this. Alternatively, the 'best' measurements available at the end of each accounting period may be used for accounting. It is also possible that all relevant measurements taken over an accounting period may be used to fulfil primary accounting objectives.

Section 6.1.4 to Section 6.1.6 of the questionnaire investigated prevalent practices around selection of data for the purpose of primary accounting. The responses are shown in the second graph of Figure 3. The option "all data" refers to instances when all relevant measurements are used for primary accounting purposes; "spec data" stands for instances when only specified data are used; and "suitable data" refers to the use of data of acceptable quality to define the primary mass balance.

The highest proportion (43%) of respondents indicated that only data deemed suitable was utilised in preparing primary accounts, suggesting an *ad hoc* approach to measurement selection for primary accounting purposes. A considerable proportion of responses (36%) indicated that all data sourced during an accounting period are used in preparing metal accounts. The lowest response rate indicated that only specific data are used to calculate the primary balance (21%).

4.2.1.3 Mass balancing

Section 6.1.7 in the questionnaire document sought to ascertain whether measured data are routinely adjusted in order to achieve consistent balances across operations. The third section of the graph in Figure 3 lists the survey responses. Approximately 88% of respondents indicated that data adjustment is "not done" when preparing primary accounts. Although 13% of responses indicated that data adjustment is "done", no methodology for systematically adjusting measurements could be put forward.

4.2.1.4 Accounting period

The choice of accounting period generally depends on corporate reporting imperatives which, in turn, are normally driven by business reporting schedules. These may vary from a single shift to a year at the most. As shown in the last section of the graph in Figure 3, most operations use the calendar month as a basis for preparing metal accounts. The period was deemed sufficiently long to account for time lags and material locked up as inventory.

4.2.2 Secondary accounting

Secondary accounting refers to the performance of mass balances over smaller sections of a process. This is normally done in order to identify areas where lock-ups may prevail, where time lags exist or where measurement inconsistencies manifest (Morrison, 2008). Invariably, secondary accounting relies on internal measurements suitable for accounting application. It was put to survey participants that secondary accounting is done for the following reasons:

- To routinely verify primary accounting results,
- To investigate large unaccounted primary accounting errors,
- To assess metallurgical performance of sub-units,
- To assess metallurgical performance of units as well as verify primary accounts,
- For process control purposes (and unsuitable for accounting application) or
- For process control and accounting purposes.

Figure 4 shows the responses in the form of a bar graph The joint use of assessing metallurgical performance ("Performance") and verifying primary accounting results ("Verification") constitutes the highest single proportion of use (26%), followed by the joint use in process control and accounting ("Control & Acct.") of 22%. Hence, decidedly, 48% of internal measurements are used to achieve metal accounting objectives.

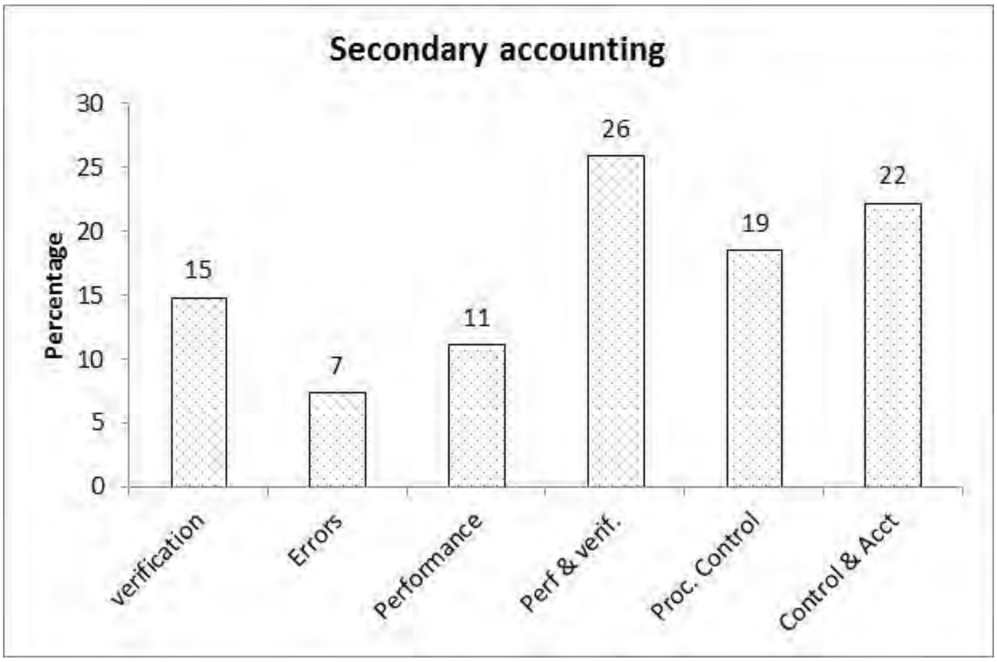


Figure 4: Secondary accounting practices with respect to routine use of internal measured data

However, since 19% of the total internal measurements taken are strictly for control purposes and deemed generally unsuitable for the more rigorous application of accounting, approximately 80% of all internal measurements taken can potentially be used in metal accounting.

4.2.3 Recovery and accountability calculation

Recovery and accountability are key measures that are used to assess the metallurgical performance and quality of measurement of metallurgical operations respectively.

4.2.3.1 Recovery

Recovery is defined as the quantity of target material (or metal) in the product stream expressed as a fraction of its proportion in the feed stream. In processes where the product stream contains several target materials, the targeted material on which the recovery calculation is based must be indicated.

There are at least three approaches that are used to calculate recovery in minerals processing. Depending on the data used for computation, the resultant measures are referred to as actual, built-up and theoretical recoveries. Using an example of a single node process serviced by a feed stream (F), a concentrate stream (C) and a tailings stream (T), Equation 4.1, Equation 4.2 and Equation 4.3 illustrate the three different approaches to calculating recovery based on the two-product mass balance model. The symbols R_A, R_B and R_T are used here to represent actual, built-up and theoretical recovery (respectively). The letters *F*, *C*, and *T* represent gross flow rates for the feed, concentrate, and tailings streams; and the lower case *f*, *c*, and *t* stand for the proportions of target material (desired metal) in the respective streams.

Actual recovery: $$R_A(\%) = \frac{Cc}{Ff} x100$$ *(Equation 4.1)*

Built-up recovery: $$R_B(\%) = \frac{Cc}{Cc+Tt} x100$$ *(Equation 4.2)*

Theoretical recovery: $$R_T(\%) = \frac{Ff-Tt}{Ff} x100$$ *(Equation 4.3)*

Respondents were requested to indicate which of the three approaches they use to calculate recovery in their respective operations. The bar graph in Figure 5 shows three different sections depicting prevalent practices in the calculation of plant recovery ('Recovery') and accountability ('Accountability') as well as typical magnitudes of 'Unaccounted balance' found in the operations that participated in the survey. Analysis of the graph will be presented in this section as well as in Section 4.2.3.2 and Section 4.2.3.3.

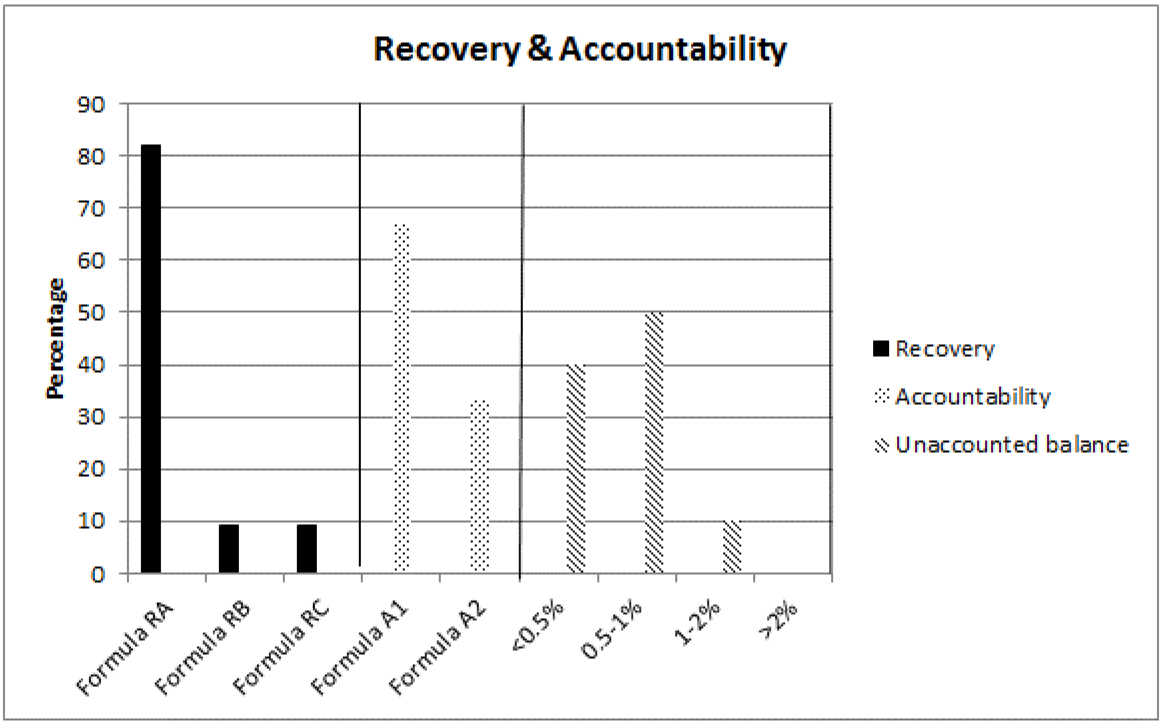


Figure 5: Recovery and accountability practices

The first section of the graph in Figure 5 shows that over 80% of respondents selected the actual recovery formula (R_A). This is significant, given that the actual recovery formula uses feed and concentrate stream data which were found to be the most precisely as well as most frequently measured and used in metal accounting (cf. Chapter 4.1). The rest of the responses appear to be equally divided between the built-up and theoretical recovery estimations.

4.2.3.2 Accountability

Mathematically, accountability is the ratio of the total output of a process to its total input, usually expressed as a percentage. The computation takes into account stock and inventory changes over the accounting period considered. Two expressions are frequently used to express this measure (see Equation 4.4 and Equation 4.5). For reference, the calculated quantities are labelled A_1 and A_2 in the two equations.

$$A_1(\%) = \frac{\sum outputs + \sum\Delta(stock\ \&inventory)}{\sum inputs} x100 \qquad \textit{(Equation 4.4)}$$

$$A_2(\%) = \frac{\sum outputs + Closing\ Stock}{\sum inputs + Opening\ Stock} x100 \qquad \textit{(Equation 4.5)}$$

The value of A_2 is profoundly influenced by opening and closing stocks particularly when stock levels are high compared to throughput. The magnitude of A_1 is relatively independent of the absolute value of stocks but is dependent on the relative magnitude of stock and inventory changes.

Invariably, the values of A_1 and A_2 measures are prone to measurement errors obtaining in stock and inventory measurements, particularly in periods of high stock retention. This is as a result of errors associated with defining stock tonnages which are directly proportional to the square of surveyed stock volumes, see Equations 4.6 and Equation 4.7 courtesy Cutler & Eksteen (2006).

The bulk density and moisture fraction variables in these equations usually do not vary widely over relatively short periods such as an accounting period.

$$M = V\rho(1 - mf) \qquad \textit{(Equation 4.6)}$$

$$var(M) = [\rho(1 - mf)]^2 var(V) + [V(1 - mf)]^2 var(\rho) + [V\rho]^2 \qquad \textit{(Equation 4.7)}$$

Where,	M	=	calculated mass of material in stock
	ρ	=	bulk density of material
	Mf	=	moisture fraction
	V	=	measured volume of material in stock

Given the possible influence of these factors on the quality of the final accountability value, respondents were requested to select which of the two equations they regularly use in their respective accounting systems. The second section of the bar graph in Figure 5 summarises the responses in terms of percentages. About two-thirds (67%) of the respondents indicated use of the A_1 measure and the rest of the participants chose measure A_2.

Use of the A_1 measure averts the direct effects of measurement error associated with estimating large stock and inventory volumes. It should be noted here that participants in the survey indicated earlier (Figure 5 and Figure 6) that the ‘difference’ method of mass measurement and sampling were the most preferred in estimating storage contents. These approaches tend to be the least prone to error as pointed out earlier. The predominant use of Equation 4.4 appears to take advantage of this.

4.2.3.3 Unaccounted balance

If all plant measurements are perfect (no measurement error) and all material movements are absolutely accounted for, Equation 4.4 would yield a deterministic value of unity. Due to random error, the value of A_1 is rarely unity and is always probabilistic in nature even when all material movement is accurately accounted for. This necessitates the introduction of an ‘error’ term in Equation 4.4 in order to ‘close’ the material balance. Equation 4.8 shows a rearrangement of Equation 4.4 and the inclusion of the error term, $\sum\Delta\varepsilon$.

$$\sum outputs = \sum inputs + \sum(stocks \,\&\, inventory) + \sum\Delta\varepsilon \qquad \textit{(Equation 4.8)}$$

The error term arising from non-closure of a material balance constraint due to random error in measured data is generally referred to as an unaccounted balance (UAB). Typically the UAB has an expected value of zero and a standard deviation that is characterised by the precisions of measurements that contribute values to the constraint relationship.

Consequently, the magnitude of the UAB error is a useful measure of the precisions of the measurements that contribute values to important key measures such as recovery. Operations regularly monitor the standard deviation of the unaccounted error according to local quality control and risk management procedures.

Respondents were requested to indicate UAB limits used for monitoring in their respective operations. The results are shown in the last section of the bar graph in Figure 5. All responses indicated levels below 2% with more than 90% of the responses selecting levels below 1%. Levels below 1% are considered normal, while those above 2% are regarded as unacceptable (Morrison, 2008). As a result, the UAB is routinely used to prompt investigations into measurement processes that contribute values to accountability assessments should a prescribed limit of the measure be exceeded.

4.2.4 Stocks, inventory and custody transfer procedures

Plant stocks are comprised of received feed material which not yet been processed as well as stored final product ready for sale. The term 'plant inventory' is used to refer to partially-processed material that is normally contained in buffer storage between process equipment or within process equipment. The term 'custody transfer' refers to the transfer of ownership of material between process plants or plant sections or between plants and customers.

Operations normally draw up procedures governing the transfer of custody of materials between parties particularly as it impacts on the accounting of material movement. Important aspects of these procedures include decisions on the use of stock/inventory measurements for metal accounting, the impact of potentially disputable key estimations such as bulk density, and the procedures governing liability of measurement accompanying custody transfer transactions.

Respondents were requested to indicate prevalent practices regarding the treatment of stocks and inventory with respect to the role of custody transfer procedures in metal accounting. The results obtained from the survey are presented in the form of a bar graph in Figure 6.

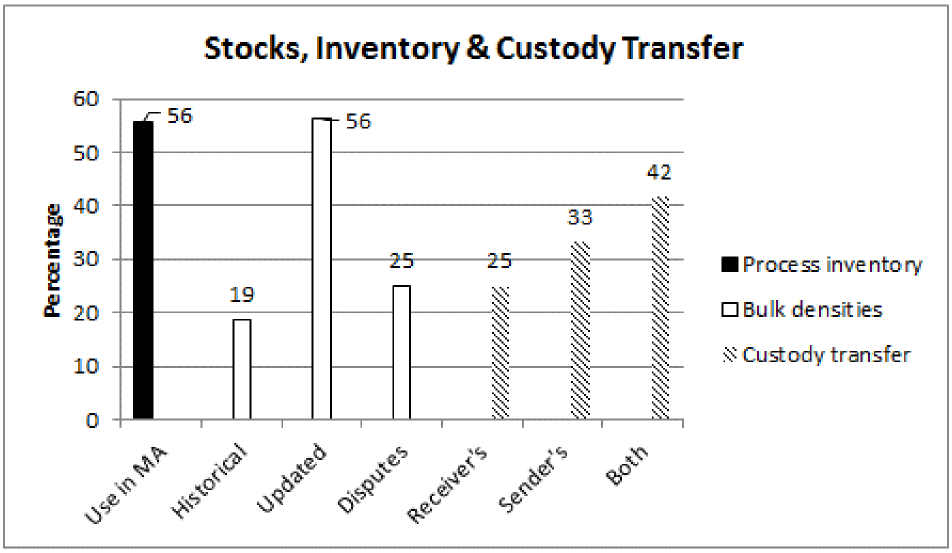


Figure 6: Treatment of stocks, inventory and custody transfer procedures

4.2.4.1 Stocks and inventory measurements

The first section of the bar graph in Figure 6 displays responses appertaining to the use of inventory measurements in metal accounting. Plant inventory is mostly stored in transit bins and silos or other regularly shaped containments such as thickeners or relatively large reactors such as leach tanks. Estimation of inventory values for metal accounting purposes is beset with requisite determinations of variables that include bulk density, assays and surveyed volumes for calculation.

Over half of the responses (56%) indicate that inventory measurements are taken for metal accounting purposes. Survey measurements suggest measured efforts to accurately determine inventory levels, a prerequisite to preparing data for metal accounting. However, inventory valuation appears to be a function of the value of material processed. Smelter and refinery operations indicated the highest use of inventory measurements in metal accounting, perhaps belying their advanced positions in the beneficiation value chain. The practice was less prevalent in mineral concentration operations.

4.2.4.2 Bulk density measurement

In this context, bulk density refers to the ratio of the mass of material stored in a containing vessel or an open stockpile to the volume occupied by the material, expressed on a dry basis.

Frequently, historical estimates of bulk density are used by operations, particularly if the nature of the material processed does not vary widely. Sometimes due to changes in ore constitution as a result of changes in feed source or pre-processing activities, updates of historical estimates are necessary. Disputes at points of custody transfer points also prompt revision of density estimates. Respondents were offered these three common motivations for bulk density measurement and requested to choose which one normally applies to their operation. The second section of the graph in Figure 6 shows how the survey participants responded.

Most respondents (56%) indicated that bulk density estimates are updated on a regular basis. Noting that a similar proportion of respondents use inventory measurements in MA (first section of graph in Figure 6), it appears that the determination of bulk density estimates for accounting purposes is the strongest motivation for regular updates. Some 19% of responses indicated use of historical estimates while 25% seem to conduct updates as a result of suspected anomalies or disputes arising.

4.2.4.3 Custody transfer

Figure 7 presents a schematic of a mine to product flow in a typical mineral beneficiation setting illustrating commonly used custody transfer points, i.e. T1, T2, T3 and T4. Transfer point T1 represents the boundary between mining (Mine shafts 1, 2 and 3) and grade control (GC) stockpiles. T2 represents a separation between GC stockpiles and the run-of-mine (ROM) storage area, while T3 separates the plant and tailings disposal activities... T4 represents the customer interface.

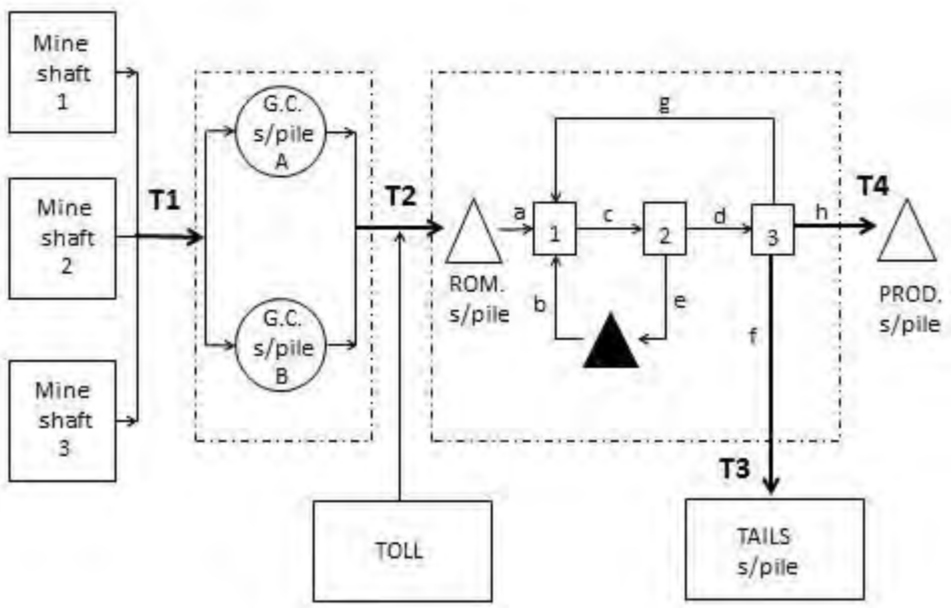


Figure 7: Common custody transfer points

Generally, transactions across T2 are subject to relevant quality-based agreements which are sometimes characterised by agreed penalties for transgressions, while deliveries across T4 are similarly governed by commercial contracts that have to be strictly adhered to. With the onset of stringent environmental requirements, movements across T3 are increasingly subjected to statutory regulations enforceable by law. T3 may also represent 'customer' interfaces in cases where third parties acquire tailings as feed to operations that beneficiate waste material. Similarly, an increasing trend in the mining industry has seen interface T2 receiving third party raw material for toll treatment.

Often, parties on either side of transfer boundaries perform their own measurements on transacted consignments which may differ significantly, giving rise to disputes. This is particularly relevant when toll material is treated for third parties (interface T2) given the potential financial gains or losses involved. Against this background, Section 6.5.3 in the questionnaire document sought to establish the prevalent practices with respect to measurements of material moved across custody transfer points.

The third graph in Figure 6 shows that 42% of respondents are of the view that parties on either side of the transfer boundary take their own measurements and compare the results. Ordinarily, this would be important at commercial interfaces i.e. transfer point T4 as well as T2 in the case of toll treatment of material.

In 33% and 25% of instances, the sender's and receiver's measurements (respectively) are accepted as final. The former predominantly applies to T3 where environmental monitoring authorities mostly do spot checks and use regular analyses reported by operations. The latter is characteristic of mining operations where process plants assay and weigh material fed to plants from GC storage, and subsequently use these values for accounting purposes.

4.3 Best Practice Metal accounting – AMIRA P754 Code of Practice

This section discusses the findings of the questionnaire survey in the context of best practice guidelines to metal accounting developed through the AMIRA P754 Code of Practice. Particular attention is paid to recommended best practice regarding the achievement of requisite precision levels for metal accounting data.

4.3.1 Metal accounting measurements

The results of this study established that terminal stream measurements are integral to the metal accounting function. Significantly, the study noted that some 80% of internal measurements taken are also suitable for accounting purposes, although a low percentage of these measurements are routinely used in accounting.

4.3.1.1 Terminal stream measurements

Feed and *Product* streams represent important custody transfer points in minerals beneficiation operations. They are characterised by high precision instrumentation and stringent sampling and assaying regimes, with certification of measurement methods often a contractual requirement at these important transfer points (Gaylard et al., 2009).

Results from the questionnaire survey indicate that *Feed* and *Final Product* streams are measured with high precisions compared to other stream types. *Final Product* streams were measured with the highest precisions and always measured and used in metal accounting. *Feed* streams were similarly valued and their precision ratings were only superseded by *Final Product* stream precision ratings.

Although the *Tailings* stream represents an important custody interface, it exhibited low usage levels in metal accounting. In addition, *Tailings* stream measurements were found to be determined with the lowest precisions compared to all other stream types. Despite this, *Tailings* stream assays experienced high levels of utilisation i.e. the measured assays enjoyed high usage in metal accounting purposes. Coupled with the low measurement precisions and low usage in metal accounting, *Tailings* stream masses were found to be the least valued metal accounting measurements.

4.3.1.2 Internal stream measurements

Internal stream measurements normally provide data for internal metallurgical balances that in turn, lie at the centre of the secondary accounting function. Amongst several applications such as process design, optimisation and control, internal balances are commonly used to measure sub-section metallurgical performances in terms of grade and recovery (Richardson & Morrison, 2003).

Approximately 80% of in-process measurements were deemed suitable for metal accounting purposes according to the questionnaire survey. However, the utilisation rates in metal accounting were found to be a low 50% on average across all sectors of the minerals beneficiation chain observed in the survey sample. The study results suggest that the 20% complement of internal measurements were considered to be more suited for process control applications (Figure 4).

4.3.1.3 Plant storage measurements

There are currently no International Standards governing in-situ estimation of metal content in plant storage areas such as stockpiles, and sometimes large tanks and bins. Direct weighing on weigh scales cannot be done in practice. Various authors describe approaches to in-situ estimation of mass in storage (Cahill, Strutt, & Wragg, 2000; Lightbody, 1983; McBride & Chambers, 1999; Ooms, 1981; Potts, n.d.), which some operations use as a basis for in-house standard operating practice.

The Code suggests weighing (and sampling) of input and output streams of storage areas and calculating changes in stored mass by difference over a given period. This approach takes advantage of credible mass measurement methods (dynamic stream measurement methods) which are less prone to inaccuracies than in-situ measurements and traceable to applicable International Standards. Representative sampling is made possible by placing suitably designed cross-stream cutters where free-falling material can be accessed.

Two exceptions are cited in the Code. Firstly, the in-situ measurement and sampling of bulk commodities (e.g. coal, iron ore, limestone) where the constitutional properties of the stored material are fairly consistent and measurements are important to plant operational practice. Even then, this is recommended as a verification measure of tonnages obtained using other measurement methods. In-situ measurement would not be appropriate for material containing heavy target species such as Platinum Group Metals where the high likelihood of segregation confounds the bulk sampling problem. Secondly, in-situ measurement may be used on ore or intermediate stockpiles in instances where content is critical to reporting or where significant changes in storage content have occurred over an accounting period.

The current survey results as shown in Figure 8. The results suggest that the survey method is the most preferred mode of mass measurement for storage areas in the minerals industry. The total counts (in percentage points) in the rightmost section of the bar graph in Figure 8 shows that approximately 54% of respondents indicated use of surveys and some 21% indicated the use of level probes to calculate mass in storage, while the remaining 25% make use of the 'difference' method i.e. measure input and output streams and determine change in mass by difference. Coupled with the observation that some 56% of respondents indicated use of inventory measurements for metal accounting (first section of bar graph in Figure 6), this strongly suggests that survey data are an important source of information in metal accounting. Interestingly, some 56% of respondents indicated regular updates of bulk density factors (see second section of the bar graph in Figure 6).

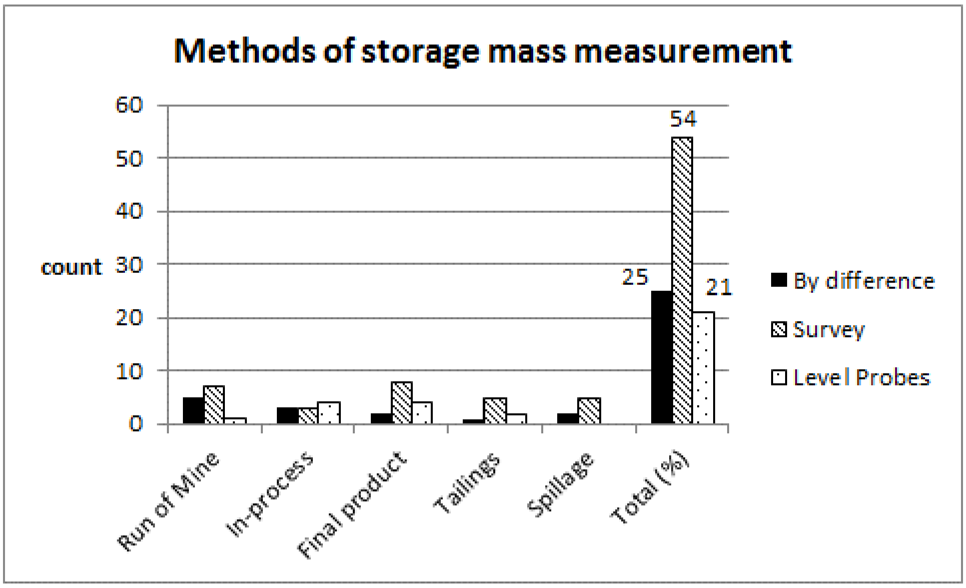


Figure 8: Methods of storage mass measurement

4.3.2 Measurement quality

The questionnaire study results showed a significant relationship between metal accounting measurements and precision of measurement. The *Feed* and *Final Product* streams were found to be determined with considerably higher precisions than other stream types, while the *Tailings* stream measurements were determined with the lowest precisions. The usage levels of the respective stream measurements in metal accounting mirrored this trend. *Feed* and *Final Product* stream measurements were rated strongly as metal accounting candidates in the questionnaire survey, achieving unit probabilities of measurement and usage. The *Tailings* stream was rated the least likely stream to be considered for measurement.

Taking the level of precision as a proxy for measurement effort, it appears that a disproportionately high amount of resources are preferred on the observed strong accounting stream types, ostensibly as a design intervention aimed at maximising measurement quality for accounting purposes.

The Code firstly recommends the identification of target quality for all data used for accounting purposes and secondly, the design of accounting systems that generate sufficient data that allows for the estimations of measurement quality and error detection (ref. Principle 7 and Principle 8, Chapter 2). Save for the recommendation that the precisions achieved in each application should be 'fit for purpose' (Morrison, 2008) the actual levels of measurement precisions for the various metal accounting applications across commodities are not expressly stated in the Code. Principle 2 of the Code however suggests that the design and specification of the (metal accounting) system must incorporate the outcomes of a risk assessment of all aspects of the metal accounting process.

The Code provides guidelines on accuracies associated with common mass measurement methods and cautions on limitations of application but largely leaves the design and selection to individual applications. References are made to relevant supplier guarantees on the reliability of values obtained from machines as a guide, with caution to consult applicable literature for verification. Generally, the Code recommends the correct design and operation of sampling systems based on common sampling theory and relevant Standards for the achievement requisite precisions for each application.

In general the Code recommends the correct installation and operation of mass measurement and sampling methods to avoid bias and achieve precision levels that allow for the detection of systematic error subject to a cost and benefit analysis of the effort involved in achieving those precisions.

4.3.3 Selection of measurements for metal accounting

Survey results suggest that plant *Feed*, and *Final Product* stream measurements experience high measurement and usage rates in metal accounting compared to other stream types. The *Tailings* stream assays were found to exhibit higher utilisation rates in metal accounting compared to all other stream types after the *Feed* and *Final Product* stream measurements.

The Code recommends the Check In-Check Out (CICO) system of accounting which states that, in "Using the Check In-Check Out system, all streams into and out of the Process or Plant for which the balance is being performed, are measured, sampled and analysed" (Morrison (Ed), 2008). The Code considers the CICO approach as the standard to be adopted by all mineral beneficiation operations in general.

The observed low value of the *Tailings* stream measurements in metal accounting may be a cause for concern with respect to CICO, particularly in the case of concentration operations where the *Tailings* stream seems to be of little relevance to metal accounting.

The Code guidelines do not pronounce on a substantive role for internal stream measurements in metal accounting. Apart from the secondary accounting function's role "to identify where lock ups are occurring, time lags are involved or where measurement problems exist" (Code, Release 2.3, 2005), direct use of internal stream measurement processes on routine computation of the primary balance is not explicit in the Code. The current study indicates that some 15% of secondary accounting practices are routinely used to verify the primary balance. An opportunity therefore exists to maximise the utilisation of all process measurements deemed suitable for metal accounting application to maximise the quality of the primary balance.

4.4 Summary

The survey has highlighted the importance of terminal stream measurements in metal accounting based on usage trends and measured data quality characteristics. Although indications are that specific measurements are pre-selected for metal accounting, particularly terminal stream data, it appears that all measured data are considered for inclusion in final metal accounts and only data that are found to be suitable are finally used. With close to 50% of routine internal measurements not being used in metal accounting despite over 80% of these being deemed of suitable quality for metal accounting purposes, an opportunity exists to utilise these measurements to augment the quality of metal accounting information. Of all the terminal streams, the *Tailings* stream was found to be the least important source of metal accounting data compared to all other terminal streams. This was noted as a concern since the Code recommends and considers the CICO system of accounting as the standard to be adopted by all mineral beneficiation operations in general. Inconsistent use of *Tailings* stream measurements weakens the case for universal application of this recommended accounting rule.

Chapter 5

INDUSTRIAL CASE STUDY

This chapter describes an audit of the metal accounting function at Namakwa Sands' Mineral Separation Plant. The objectives of the audit were twofold. Firstly, the findings of the metal accounting practice survey described in Chapter 4 are explored and subsequently compared and contrasted with relevant aspects of the Code using the case study flowsheet as a test case. Secondly, measurement error models for the plant were determined in order to demonstrate the impact of measurement error on derived quantities such as component flow rates and mineral recovery estimations in practice. The error models determined also provided realistic measurement standard deviations for conducting studies on mathematical heuristics derived in this work.

5.1 The Namakwa Sands Operations Overview

The Namakwa Sands mining and beneficiation operations are situated at Brand se Baai and Saldanha on the west coast of South Africa. The company produces titania slag and pig iron from its smelter operations situated at Saldanha. High grade zircon and rutile concentrates are produced at its mineral separation operations at Brand se Baai. A 300 km railway links Brand se Baai and Saldanha. The rail link is used for the transportation of ilmenite.

Heavy Mineral Concentrate (HMC) is transported by road from the mining and concentration facilities at the coast to the Mineral Separation Plant (MSP) situated some 60 km inland from Brand se Baai. Figure 9 gives an overview of the entire operation in the form of a schematic block diagram.

Following discovery of the mineral sands deposit in 1986, the operation was developed to full production in 1994 and currently processes over 20 Mt ROM ore per annum, delivering 450 kt ilmenite, 120 kt and 25 kt of zircon and rutile concentrate, respectively (Philander & Rozendaal, 2013).

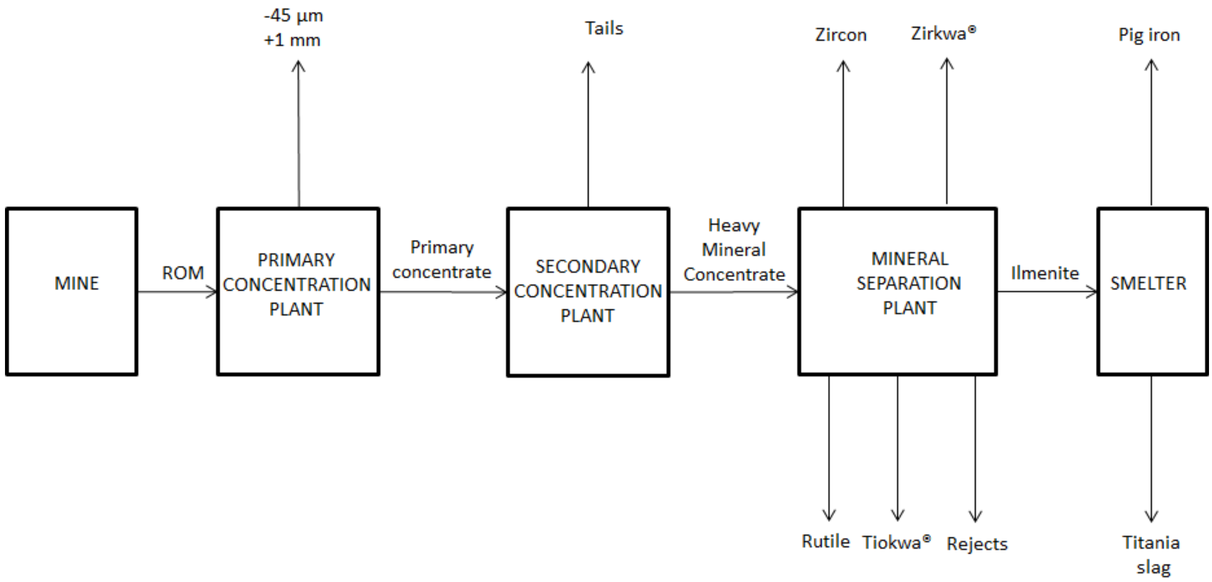


Figure 9: Schematic of the Namakwa Sands Operation

The design of the mineral beneficiation route at the Namakwa Sands operation is typically influenced by technology traditionally employed in the upgrading of mineral sands. The concentration operations exploit differences in density, size, electrostatic and magnetic properties of heavy mineral sands (Dawson, 1997). Consequently, the Namakwa Sands flowsheet includes ore preparation stages (crushing, grinding and classification), gravity concentration (spirals and shaking table separation), magnetic (high intensity induced roll and wet magnetic separation) and electrostatic (high tension roll separation and electrostatic precipitation) separation activities.

ROM ore is first treated in the Primary Concentration Plant (PCP) by removing material larger than 1mm or less than 45 µm in size using trommel screens and desliming cyclones respectively. The PCP concentrate is further upgraded by wet gravity and wet magnetic separation at the Secondary Concentration Plant (SCP) to produce an attritioned ilmenite-rich magnetic concentrate and a zircon/rutile-rich non-magnetic concentrate.

After filtration the concentrates are transported to the MSP operation, some 60 km away from Brand se Baai by road. Here, dry electrostatic and magnetic separation processes produce saleable ilmenite as well as rutile and zirconium based final products. The ilmenite product is transported by railway to the company's smelting operations for the production of pig iron and titania slag. Premium grade zircon and rutile as well as secondary zircon (zirkwa®) and secondary rutile (tiokwa®) are shipped directly to markets from the MSP.

In order to investigate the principles of metal accounting developed in the AMIRA P754 Project, the non-magnetic section of the MSP operation was chosen as the area of focus. The MSP presents a relatively high amount of interlinked metallurgical measurements therefore offering a complex platform to study interactions between measurements bound together in a closed network. In addition, the MSP despatches final product to third parties, hence manages an important custody transfer interface. The following section highlights salient features of the metal accounting practice at the Namakwa Sands.

5.2 Metal accounting at the Mineral Separation Plant

The heavy mineral concentrate (HMC) from the SCP consists of non-magnetic and magnetic (attritioned) road-trucked consignments that are processed separately in the MSP. Consequently the MSP is divided into two distinct sections, namely the non-magnetics and magnetics plants. The former produces zircon (Zr) and rutile (Rt) based products for sale and the latter produces ilmenite destined for further processing at the smelting operations.

Figure 10 gives an overview of the MSP operations in the form of a block process diagram. A 'Project Boundary' demarcates the non-magnetics section of the MSP operation. The rougher/IRMS (Induced Roll Magnetic Separation) section separates remnant magnetics from the non-magnetics concentrate using a series of primary, secondary and tertiary IRMS unit operations. The rougher/IRMS product is further upgraded by separating quartz in the Wet Gravity section to produce feed for the Dry Mill operation where final separation is effected.

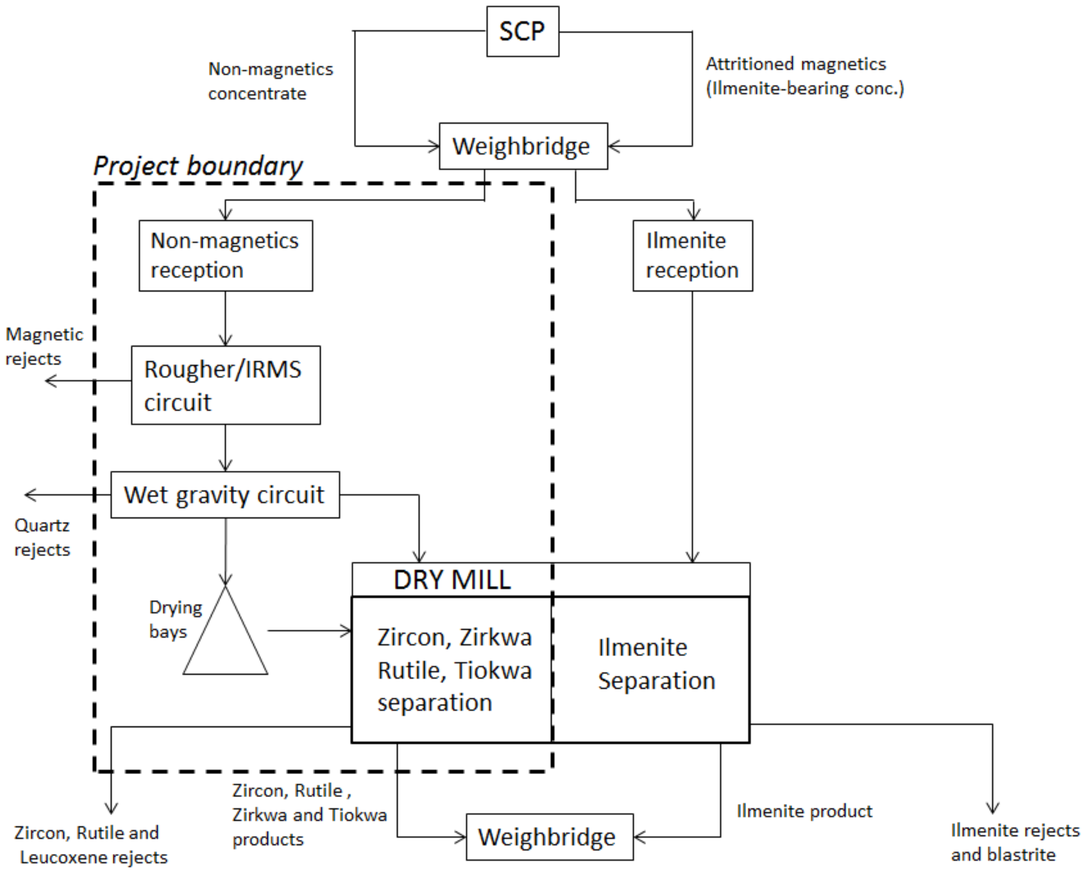


Figure 10: An overview of the MSP operations

A more detailed flowsheet of the selected project area is presented in Figure 11. The following sections describe mass measurement methods, sampling regimes and the analytical method employed at the non-magnetics section of the MSP.

5.2.1 Process flowsheet

Figure 11 presents a block process flow diagram of the MSP non-magnetics section. The operation beneficiates non-magnetic concentrate to produce saleable titanium and zirconium based products through a series of incremental beneficiation steps that include magnetic, electrostatic, and gravity separation processes as well as ancillary operations such as filtration and drying.

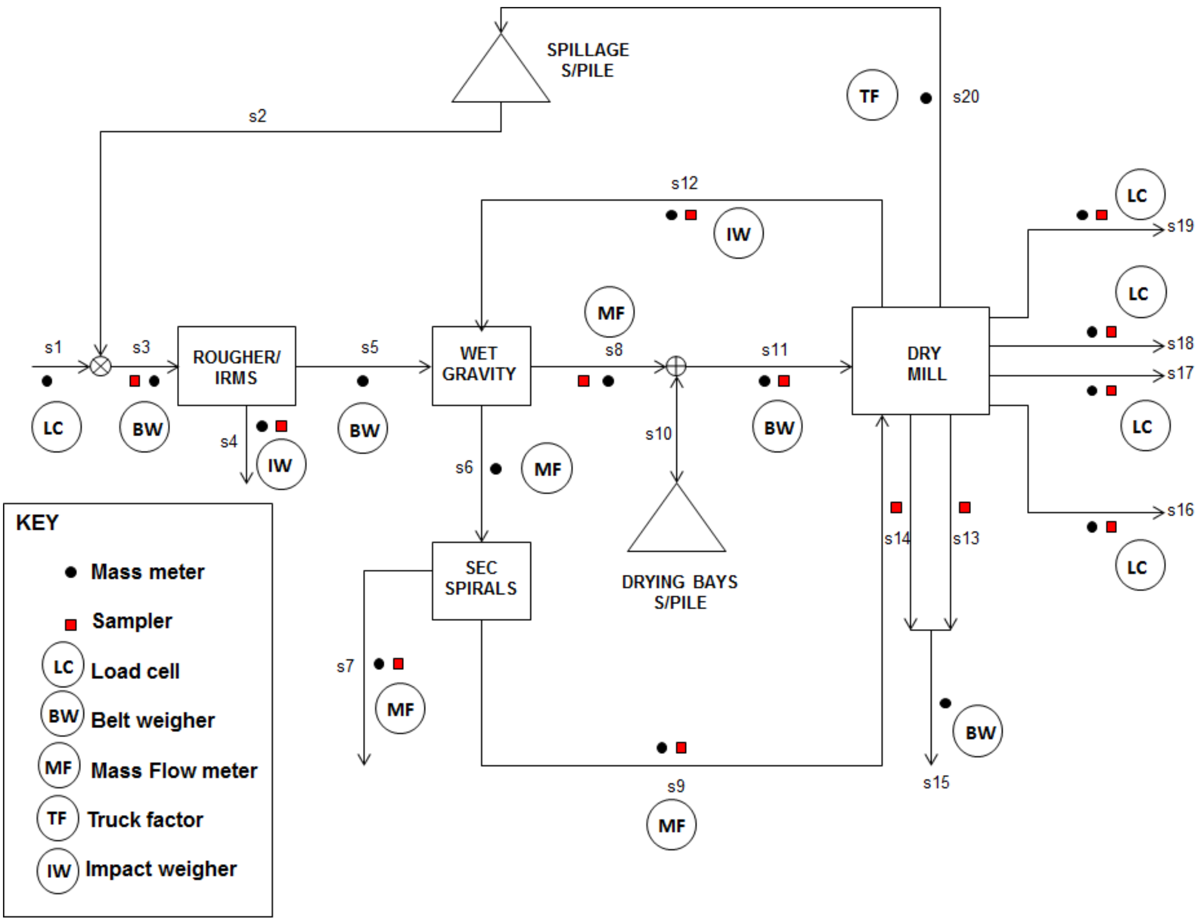


Figure 11: Process flowsheet of the MSP non-magnetics operations showing mass measurement and sampling points

Structurally, the process flowsheet consists of seven nodes, nine internal streams and eleven terminal streams. Concentrate from the SCP is supplied as fresh material through Stream *s1*. Streams *s2* and *s10* supply intermediate to the circuit as spillage retreat (Stream *s2*) dried wet gravity concentrate (Stream *s10*).

Four saleable products exit the operation through Streams *s16-s19*. Streams *s4*, *s7* and *s15* transport tailings material (magnetics, quartz and Zr/Rt rejects) destined for discard. An intermediate product (spillage) exits from the Dry Mill through Stream *s20* and re-enters the flowsheet through Stream *s2* as spillage retreat. The final products i.e. Zr, zirkwa®, Rt and tiokwa® as well as the rejects streams present a wide range of concentrations including some that exist in trace amounts in the waste streams.

5.2.2 Mass measurement

Figure 11 shows mass measurement points at the non-magnetics section of the MSP. The mass measurement method employed at each measurement point is indicated in the diagram. Concentrate delivered from the SCP reports to a load-cell based road weighbridge. The concentrate is stored in silos from which fresh feed is drawn into the plant via the main feed conveyor belt equipped with a belt weight meter (Stream *s3*).

Mass readings from the belt-scale are used for routine metal accounting purposes. Although the belt-scale is regularly calibrated, monthly reconciliations with mine/SCP receipts and despatched products are based on the road weighbridge values over the accounting period.

Mass flow rates within the plant area are based on mass flow meter readings for slurry streams and belt-scales and impact weight meters for the dry streams. The final products are stored in load-cell supported product bins that are continuously fed by conveyor belts fitted with mechanical weigh-scales for production monitoring.

The final product masses are determined using a rail weighbridge upon despatch to customers. Plant production is reconciled on the rail weighbridge readings after stock takes at the end of the accounting period, and differences arising with in-plant belt scale measurements are used to adjust plant production campaigns.

This practice resembles the CICO approach with respect to the accounting of mass movement across the MSP. The system employs static mass measurement methods (road and rail weighbridge scales) for fresh feed and product stream flow measurements, a practice which conforms to the Code recommendations of high precision and accuracy at custody transfer points (c.f. Chapter 2.1.7.1.2).

However, the rejects and spillage mass measurements are derived from lower precision measurement methods. The rougher/IRMS (Stream *s4*) rejects, secondary spiral (Stream *s7*) rejects and dry mill (Stream *s15*) rejects streams use impact weighing, electromagnetic flow meter and mechanical weigh scale mass measurement technologies respectively (Figure 11). Spillage generated from the plant is not directly measured. Truck factors are used to estimate mass outflow (Stream *s20*).

Table 18 gives some properties of the materials conveyed in the MSP non-magnetics and typical flowrates per stream relative to the main fresh feed. Stream *s7* is expected to contribute the highest absolute error to the flowsheet balance given the combination of its relatively high flowrate (compared to Streams *s4* and *s15*) and the high uncertainty normally associated with mass flow meters. Although Stream *s4* is served by an impact weight meter which is also generally imprecise, the tonnage conveyed is relatively lower than Stream *s7* mass flow rate. While Stream *s15* tonnage is comparable in magnitude to Stream *s7*, belt scale technologies are generally more precise than mass flow meters.

Table 18: Material characteristics and relative flow rates of the MSP non-magnetics circuit

Stream	Material characteristics		Relative gross mass flow rate
	Type	Moisture, %	
s1	Wet solids	5	100.00
s2		5	7.35
s3		5	107.35
s4	Dry solids	0	13.85
s5		0	93.50
s6	Slurry	90	23.43
s7		90	20.51
s8		90	75.02
s9	Wet solids	10	2.92
s10		8	3.60
s11		8	78.62
s12	Dry solids	0	4.95
s13		0	13.54
s14		0	7.52
s15		0	21.05
s16		0	37.68
s17		0	2.55
s18		0	5.17
s19		0	2.79
s20		0	4.95

5.2.3 Sampling and sample analysis

The sampling methods employed at the MSP are given in Figure 12 and listed in detail in Table 19.

5.2.3.1 Sampling methods

The main plant feed (Stream *s3*) is sampled using a parallel lip cross stream sampler (Figure 12a) located at the end of the feed belt. The choice of sampler largely meets correct sampling criteria (Holmes, 2004) that include correct geometry (parallel lips for linear cutters) to enable equiprobable sampling and a lip plane that intersects the stream trajectory perpendicularly. This sample is also used for moisture determination.

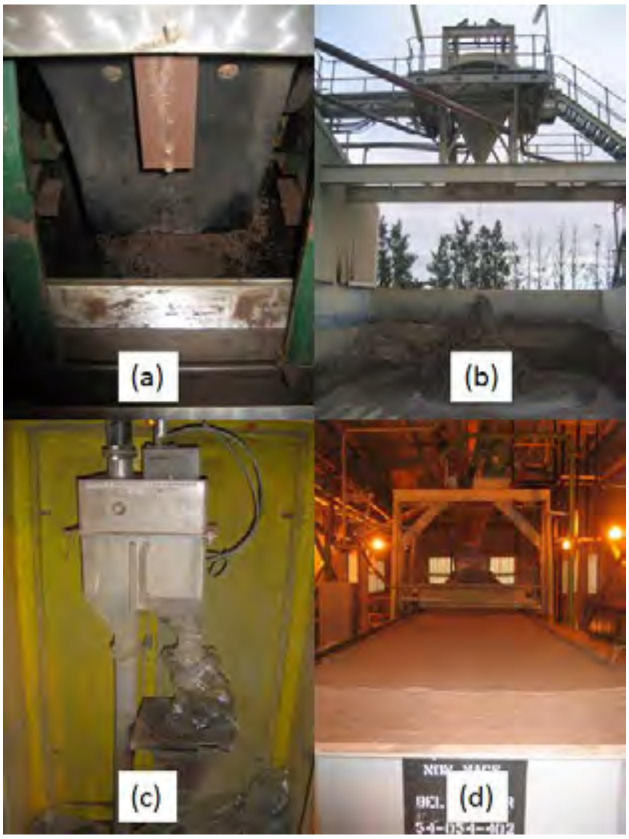


Figure 12: Sampling methods utilised at the MSP: (a) Cross-belt sampler on the main feed – Stream *s3* (b) Grab sampling point for quarts rejects - Stream *s7* (c) diverter sampler for Rutile product - Stream *s18*, and (d) Grab sampling point at wet gravity concentrate filter belt – (Stream *s8*)

The slurry streams, i.e. Stream *s7,* Stream *s8* and Stream s9 are sampled using the grab sampling technique. The quartz rejects primary sample (Stream *s7*, Figure 12b) is cut from a falling stream while the wet gravity concentrate (Stream *s8*, Figure 12d) is sampled using a manually operated cross stream slurry sampler as the concentrated cake falls from the end of a filter belt. The secondary spiral stream (Stream *s9*) is also a filter cake and is similarly sampled from the end of a smaller filter belt. All dry streams in the MSP are sampled using diverter samplers (Figure 12c). Table 19 lists the respective sampling methods for all streams identified in Figure 11. Save for the quartz rejects sampling period of four hours, all other samples are collected every two hours and composited over twenty-four hours.

Table 19: Sampling methods employed at the MSP non-magnetics flowsheet

Stream	Sampling method
s1	-
s2	-
s3	cross stream sampler
s4	diverter sampler
s5	-
s6	-
s7	grab sample
s8	grab sample
s9	grab sample
s10	-
s11	diverter sampler
s12	diverter sampler
s13	diverter sampler
s14	diverter sampler
s15	-
s16	diverter sampler
s17	diverter sampler
s18	rotary sampler
s19	rotary sampler
s20	diverter sampler

Some of the sampling practice aspects at the MSP raise concern. The ubiquitous use of the diverter sampler on dry streams increases the risk of biased samples owing to the potential systematic extraction error associated with this type of sampler's operation (Holmes, 2004b) .

A cylindrical sampling cup with a long handle is used to grab a sample from the falling stream on Stream *s7* before the material reaches the rejects stockpile (Figure 12b). The primary sampling device for taking the filter belt sample on Stream *s8* is a correct design parallel lip sampling cup fixed to a long handle that is only traversed across a small portion of the width of the stream (Figure 12d), essentially constituting a grab sample. The practice of grab sampling on Streams *s7* and *s8* is prone to error due to potentially inconsistent selection probabilities between sampling units making up the production lot (Pitard, 1993). In addition, the long sampling period for the quartz rejects stream reduces the precision of the assay result for Stream *s7*. This is consistent with one of the questionnaire study findings (c.f. Chapter 4) that indicated general negligence of tailings stream measurements as a common occurrence in the industry in general.

With respect to spillage movement and handling at the MSP, all the spillage material retreated originates from the dry mill. The spillage that is generated from the dry mill is not directly sampled. Instead, the dry mill feed assay (Stream *s11* assay) is routinely assigned to all spillage recycled.

5.2.3.2 Analytical method

The grain counting (GRC) method of analysis has been used for process control and metal accounting at Namakwa Sands for many years. The method quantifies the relative abundance of mineral species based on the point counting technique. The procedure relies on transmitted light microscopy for the identification of mineral bearing particles at a magnification of 60 times.

A 100 g aliquot is mounted on a glass slide (Figure 13) for observation under the microscope against a background of transmitted white light. A built-in regular grid superimposed on the specimen is used to map selected areas of the sample. The grid consists of 121 regularly spaced cross-haired points. Mineral species observed at each grid point are manually counted and the counts are summed for each species identified.

Figure 13: Glass mounted sample for analysis using the grain counting method

The sample area superimposed by the grid at any one time is referred to as a field. Hence a field consists of 121 potential counting points. The procedure used at the MSP requires that 450 points be counted in total for one sample. In the current practice 5 fields are presented, bringing the total number of points counted per sample to 605.

This procedure is applied to the quantification of materials that are expected to be present in quantities above 2%. Final product samples undergo a different counting regime. For product stream samples, only trace minerals are counted, and major minerals constituting the major part of the sample are not counted. This potentially introduces considerable variability in the quantification of trace elements in products and sometimes rejects.

Trace minerals are generally expected to constitute less than 2% of high value product samples. The procedure first determines the grain density of the sample. The grain density is a measure of the concentration of grains per unit area of the specimen. In contrast to the previous procedure, counting is done within the marked areas of the grid rather than points. Four fields are involved in the determination of grain density for each sample analysed. Trace elements are then quantified by selecting a total of 48 discrete fields in a 6 by 8 (fields) pattern. All trace elements encountered in the selected area are counted.

5.2.4 MSP recovery and material balance

5.2.4.1 Significant historical developments in MSP recovery formulation

Namakwa Sands historically selected between two methods for estimating mineral recoveries at the MSP. The built up recovery method was used over a long period. In this approach, plant feed grades were back calculated using assays and tonnages from all MSP output streams and reported mineral yields were based on this calculation. The calculated grades were found to be consistently lower than SCP delivered concentrate grades, resulting in higher built up recoveries than actual recovery values. Actual recovery computations were based on SCP concentrate and MSP final product grades and mass flow rates.

Although the built up recovery approach was observed to produce stable results, the apparent differences meant that the reported (i.e. calculated) MSP grades had to be adjusted downwards in order to match deliveries from the SCP. Life of mine calculations relied on actual MSP recoveries based on fresh concentrate deliveries and MSP final products.

Considered separately, the respective recovery estimates displayed stable trends, an indication of acceptable consistency in mass and/or assay measurement. However, the discrepancy observed between the recovery estimates implied systematic differences in the interpretation of mineral deportments by at least one of the recovery formulations.

5.2.4.1.1 Spillage recycling

A likely contributor to observed uncertainty in mineral recovery computations was the recycling of plant spillage. In the periods predating 2002 spillage produced at the MSP was blended with fresh concentrate while plant rejects were retreated in separate campaigns.

Thus product recovery from rejects could be separated from fresh concentrate yields. Reporting protocol at the time excluded rejects yields from MSP recovery performance. However the blending of spillage with fresh concentrate meant that the MSP performance included yields from recycled spillage, making it impossible to resolve the total plant recovery according to the different sources of feed.

This was compounded by the lack of sampling and weighing facilities for spillage. The spillage feed point was located after the MSP main feed sampler and weigh scale, resulting in built up recoveries tending to record higher values than actual recovery estimates. Actual recovery calculations were based on the combined grade of the spillage and fresh concentrate blend.

Spillage recycling practice led to a number of problems which included inaccurate accounting of material movement at the MSP. Moreover, the spillage generated was crudely measured in terms of truck factors and assays were estimated based on Dry Mill feed composition; making process control difficult. It was mainly as a result of process instability that the practice of blending spillage with fresh concentrate was discontinued in February 2002.

The new protocol combined spillage and rejects processing campaigns. Recoveries were reported separately from fresh feed campaigns. The effect of this change was an improvement in plant stability and an apparent reduction in reported MSP recoveries.

Prior to this change, historical plant data demonstrated that retreatment of spillage led to higher reported recoveries at the MSP. This was confirmed by independent plant-scale tests performed in November 2002 which concluded that spillage contributed as much as 3.5% of total Zr produced at the MSP. As a result of this, spillage re-treatment campaigns were resumed, but these were performed separately from fresh feed and rejects campaigns. The mineral recovered from spillage was reported jointly with production from fresh feed resulting in previous high levels of reported recovery.

5.2.4.1.2 Historical trends

Figure 14 compares built up and actual Zr mineral recovery trends at the MSP over the period January 2001 to December 2003. Zr mineral is the major revenue generator for the operation and is used here for illustrative purposes. Rt and Lx mineral recoveries showed similar trends.

Reported recoveries were based on the built up recovery calculation for the period leading up to December 2001. As can be seen in the figure, the values based on the actual recovery formula are lower in value.

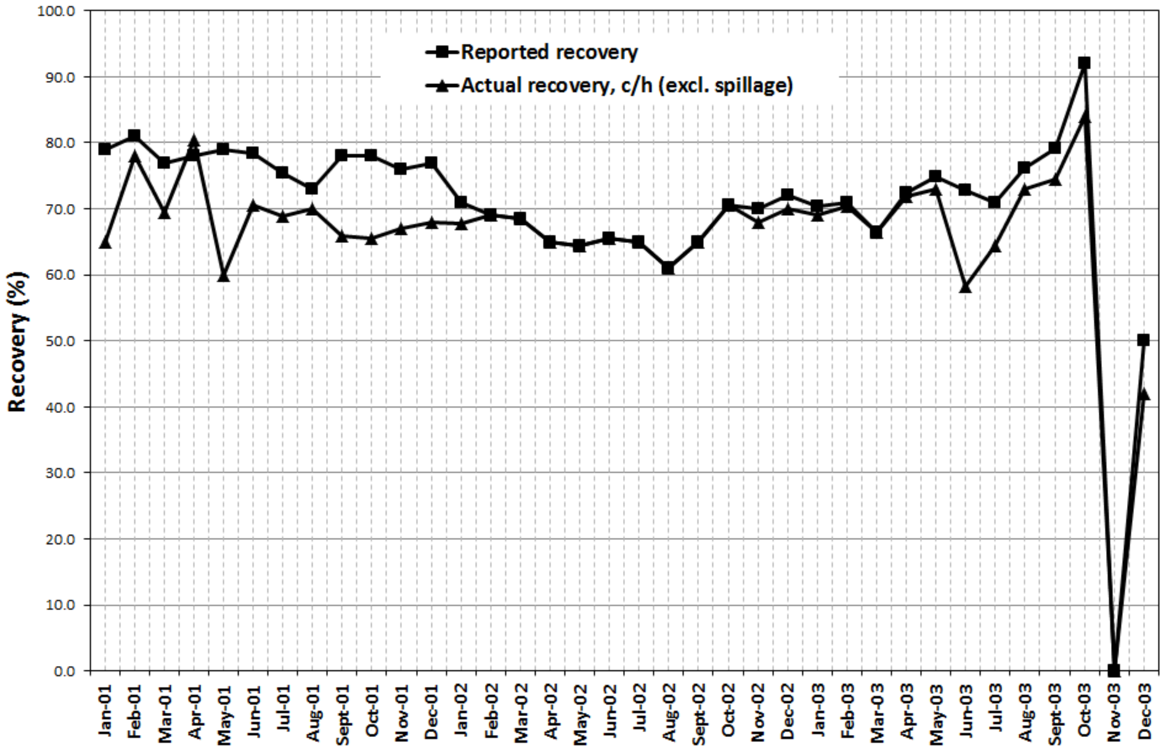


Figure 14: Namakwa Sands MSP prime Zircon monthly recoveries (Courtesy Namakwa Sands

However, from February to October 2002 the built up and actual recovery trends converged owing to the exclusion of spillage yields from reported MSP recovery. The effect of the inclusion of spillage in plant recovery calculations is observable from November 2002.

5.2.4.2 Recovery calculation and material balance methodology

The dry MSP fresh feed tonnages are based on SCP weighbridge masses and moisture (from January 2002) for monthly accounting. It should be noted here that the MSP main feed belt is equipped with a cross stream sampler and a mechanical belt scale that are used for daily mass and mineral balances as indicated earlier.

Technically, the SCP weighbridge masses are expected to be more accurate than the mechanical weigh scale, notwithstanding the fact that SCP is some 60 kilometres away from the MSP by road; and basing the MSP feed tonnage on the SCP weighbridge mass and moisture sample averts uncertainty due to moisture loss in transit. Notably, discrepancies in tonnages between the MSP and SCP have historically recorded differences as large as 4.5% of the total tonnage delivered to the MSP (Production Year 2001), prompting the need to formalise measurement procedures at this custody transfer point.

Product tonnages are based on MSP despatch tonnages. Exceptions are made in the case of ilmenite product (MSP magnetics circuit) in cases where adjustments made to sales tonnages at the smelter differ significantly from the MSP despatched mass. In such instances, Namakwa Sands accepts the smelter adjusted mass as the final measurement.

Taking the entire Namakwa Sands operation into consideration (Figure 9), measurement inefficiencies at the SCP and the smelter/market can potentially be passed on to the MSP operation. Thus it appears that the MSP is in the unenviable position where the operation accepts delivered tonnage from the feed source (SCP) and product tonnages from the customer interface (smelter, market) as final measurements. Notably, findings from the questionnaire survey indicated that in 33% and 25% of operations surveyed, the sender's and receiver's measurements (respectively) are accepted as final. The SCP/MSP and the MSP/smelter interfaces seemingly fall in the former and latter groups of the survey sample, respectively.

Following the changes outlined in the preceding section, all reported MSP recoveries are currently based on the actual recovery calculation method. Mineral yields from spillage retreatment campaigns are calculated separately and subsequently included in the reported plant recovery. Recoveries achieved from rejects retreatment runs are reported separately.

The Code does not specify which recovery calculation method to use. The questionnaire survey results suggest that the actual recovery approach is the method of choice in the industry, probably as a result of the priority given to plant feed and product (concentrate, metal) measurements in terms frequency and quality of measurement.

However it does seem prudent that the actual recovery calculation is preferred instead of the built up yield calculation given the high risk of error associated with utilising measurements from the relatively large number of outflow streams at the MSP (Figure 11). From a mine planning viewpoint, actual recovery computation at the MSP ensures consistency, particularly since the operation accepts the SCP despatch measurements as input to the MSP.

A drawback in the MSP (and the entire operation) material balance procedure is the absence of mass measurement and sampling error determination for reporting purposes. The Code guidelines (c.f. Principle 8, Section 2.1.2) suggest that computed recovery should be accompanied by actual accuracy estimates based on measured data "…in the report to the Company's Audit Committee". Apart from being a tool for gauging risks associated with key performance results, determination of precisions assists operations in deciding on the significance of changes observed in process trends.

A sampling and mass measurement error modelling campaign conducted at the MSP is described in the following section.

5.3 Determining components of variance in mineral concentrations

Metal accounting measurements consist of component (metal or mineral) concentrations and gross mass flowrates. Component flowrates are obtained by multiplying gross stream mass flowrates and component concentrations. In the absence of systematic error, random error propagated through this calculation is estimable based on the general rules of variance propagation through formulae (Equation 2.1).

While mass measurement variance is determined in practice by computing the distribution of replicated measurements, quantifying the overall assay measurement variance requires a more complex approach because this quantity consists of multiple components, namely sampling (V_S), sample preparation (V_P) and analytical (V_A) variance (Equation 2.6).

A survey was conducted in order to determine the variance components obtaining in mineral component flow rates in selected key streams on the MSP. The selected streams represented all sampling and mass measurement methods practiced at the MSP.

Five streams met this criterion. The streams covered metrological systems ranging from the ideal automatic cross-belt sampler to grab sampling for primary sample extraction; and magnetic flow meters, belt-scales, and static load cell based mass measurement systems for gross mass measurements. It was assumed that the total measurement error associated with streams with similar characteristics in terms of material constitution and measurement methods were reasonably similar.

5.3.1 Sampling variance

5.3.1.1 Sampling campaign methodology

The components of variance analysis method was used to resolve the total variance in final mineral concentrations into selected basic components namely sampling, sample preparation and sample analysis (Figure 15). The method has been extensively covered in the formal literature, research publications and some international (Bartlett, 2002; Box, Hunter, J. Stuart, & unter, 2005); AS 4433.3–2002; AS 2884.4–1997; ISO 12744:1977(E); ISO 3085:2002(E)).

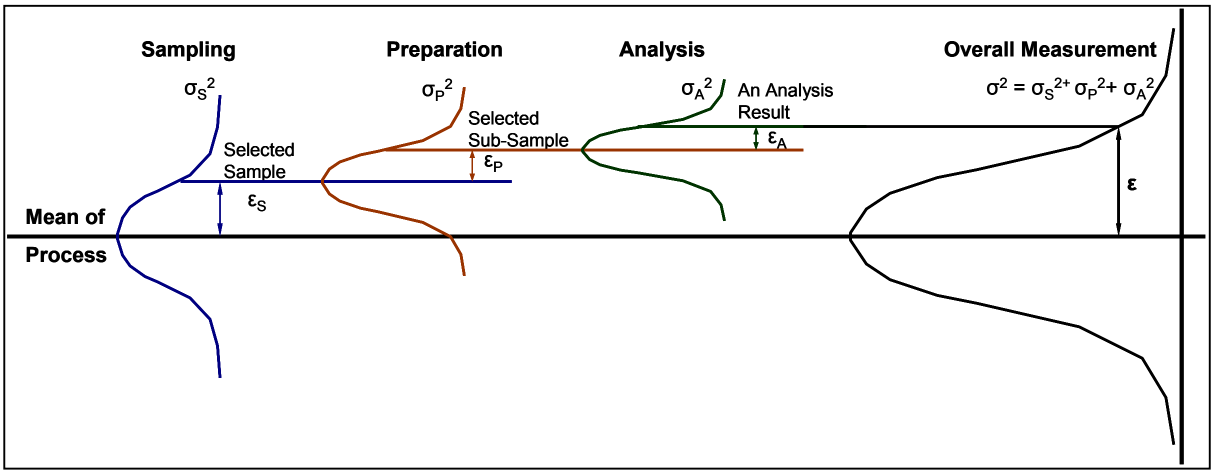


Figure 15: Overall mineral/metal concentration measurement variance and the components of sampling, preparation and analysis

The generic procedure for determining components of variance in mineral/metal concentrations is centred on a hierarchical or nested testing scheme commonly referred to as a 'sampling tree'. Different sampling tree designs are available for testing the components of variance. Here, the 'reduced' sampling tree method (ISO 3085:200(E) pp 6; AS 2884.4–1997 pp. 13) was chosen on account of the fewer number of analyses required to estimate the variance components (Figure 16).

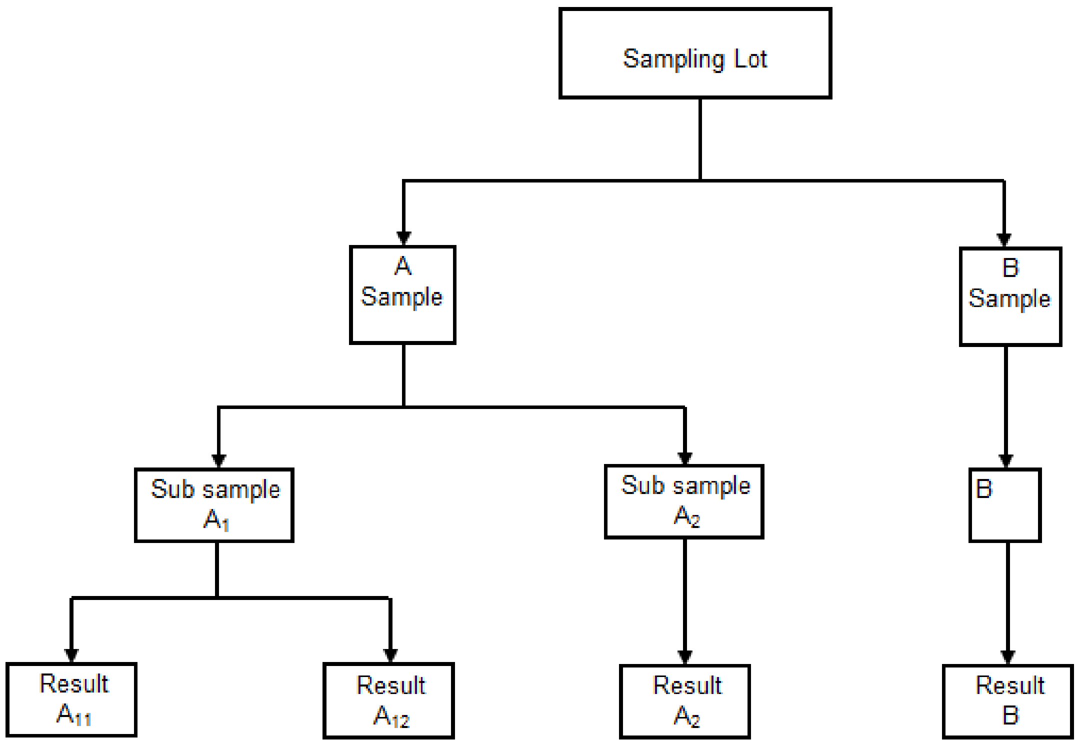


Figure 16: Reduced sampling tree for calculation of testing, preparation and sampling variance components

Each of the selected five streams was sampled over 12 sampling units (lots or sub-lots). The standard procedure followed (AS 2884.4-1997, p.10; AS 4433.3-2002 p.8) requires at least ten primary sampling units in order to get a statistically reliable value of the variance measures. Duplicate ‘A’ & ‘B’ interleaved samples were simultaneously extracted from each sampling unit. The samples were taken over a 12-hour period with each lot sampled over an hour. Sampling methods and procedures used during ‘normal plant operation’ were employed over the survey period. The number of primary increments was twice the number taken during routine plant practice to cater for the interleaved B sample. Consequently, 12 duplicate samples (Sample A and Sample B) were taken from each of the 5 streams, totalling 120 samples that were sent to the laboratory for analysis.

The samples were prepared for analysis in accordance with the ‘reduced’ sampling tree scheme. The sample preparation steps included all treatment activities done on the A and B samples upon arrival at the laboratory for analysis i.e. drying, splitting and extraction of the final test aliquot.

The Grain Counting (GRC) method was used for determining mineral concentrations. This method was in routine use at the MSP for process control and metal accounting purposes at the time the survey was conducted. As can be seen in the reduced sampling scheme presented in Figure 16, the sampling variance was calculated from Result B and Results A_{11}, A_{12} and A_2; while the preparation variance was obtained from Result A_2 and Results A_{11} and A_{12}; and the analytical variance was calculated from Result A_{11} and Result A_{12}.

5.3.1.2 Components of variance in mineral concentration measurements

The components of variance obtained for the selected streams are listed as percentages of the total absolute variance of sampling, preparation and analysis in Table 20. Total absolute variances for each mineral species, measured assays and corresponding relative standard deviations (RSD's) are also listed in the table. The RSD is numerically equal to the square root of the error variance (standard deviation) expressed as a percentage of the average measured value.

In a well-balanced sampling regime the variance of sampling and the variance of preparation are usually of the same order of magnitude (Merks, 1985). The data in Table 20, on average, attests to this for the MSP case: sampling and preparation variances constitute 12.21% and 18.65% of the total variance respectively. However, it is reasonable to expect this balance to shift for individual streams depending on the condition of the material sampled and sampling procedures employed.

Table 20: Sampling, preparation and analytical components of variance on material from selected streams on the MSP flowsheet

Mineral	Stream Name, Stream flowsheet ID	Components of variance, %			Total absolute variance ($\%^2$)	Assay (%)	RSD (%)
		Analysis	Preparation	Sampling			
Zircon	Dry Irms Feed, s11	63.12	26.76	10.12	6.23	48.93	5.10
	Quartz Rejects, s7	24.22	60.25	15.53	12.50	14.75	23.97
	Rutile Product, s18	62.77	37.23	0.00	10.46	60.33	5.36
	Rutile Rejects, s13	76.94	23.06	0.00	5.71	12.47	19.16
	Wet Irms Feed,s3	65.95	34.05	0.00	7.41	48.62	5.60
Rutile	Dry Irms Feed, s11	61.47	4.97	33.56	3.72	13.32	14.48
	Quartz Rejects, s7	96.64	0.00	3.36	7.36	12.00	22.61
	Rutile Product, s18	54.22	45.78	0.00	13.08	29.04	12.46
	Rutile Rejects, s13	100.00	0.00	0.00	5.59	22.75	10.39
	Wet Irms Feed,s3	80.57	17.94	1.48	2.65	13.13	12.38
Leucoxene	Dry Irms Feed, s11	51.98	3.64	44.38	7.80	11.82	23.63
	Quartz Rejects, s7	85.36	14.64	0.00	10.90	26.94	12.26
	Rutile Product, s18	65.78	34.22	0.00	0.42	0.97	66.60
	Rutile Rejects, s13	55.49	13.02	31.49	7.89	29.54	9.51
	Wet Irms Feed,s3	100.00	0.00	0.00	2.52	12.65	12.56
Ilmenite	Dry Irms Feed, s11	88.45	0.00	11.55	2.83	11.39	14.78
	Quartz Rejects, s7	100.00	0.00	0.00	2.35	4.69	32.68
	Rutile Product, s18	49.32	50.68	0.00	11.03	29.02	11.45
	Rutile Rejects, s13	44.37	0.00	55.63	0.11	0.20	167.16
	Wet Irms Feed,s3	92.54	7.46	0.00	3.07	10.56	16.60
Garnet	Dry Irms Feed, s11	40.50	34.08	25.41	1.23	4.95	22.41
	Quartz Rejects, s7	66.33	0.00	33.67	3.04	5.54	31.46
	Rutile Product, s18	62.82	14.83	22.34	0.06	1.21	20.63
	Rutile Rejects, s13	46.43	43.70	9.87	1.36	2.48	47.12
	Wet Irms Feed,s3	93.18	0.00	6.82	4.78	4.93	44.32
Average	-	69.14	18.65	12.21	5.37	17.29	26.59

The data in Table 21 attempts to explore the effects of material condition and sampler characteristics on the distribution of error variance amongst the three sources measured. The table lists the condition of the materials conveyed on the selected streams, primary sampling intervals, and averages of variance components for each stream across all species measured and corresponding RSD values.

Table 21: Average variance components per stream

Stream	Material	Sampling method	Sampling interval (s)	Components of variance, %			RSD, %
				A	P	S	
s11	wet	Diverter	60	60.87	11.71	27.42	11.55
s7	slurry	Grab	-	65.87	25.25	8.88	21.03
s18	dry	Rotary	35	55.38	44.58	0.04	10.98
s13	dry	Diverter	60	72.81	14.22	12.97	15.07
s3	wet	Cross-stream	800	82.42	15.80	1.79	11.24

The data shows no discernible correlation between the magnitudes of variance components and material sampled or method of sampling used, but show some correlation between the magnitude of RSD and the sampling method; as expected, the highest value resulted from grab sampling (Stream *s7*) and the lowest form value from the rotary sampling method (Stream *s18*). High sampling frequencies ameliorate the effects of the distributional component of the primary sampling variance.

It is evident from observation of the data presented in Table 20 that the cause for the low precisions (high RSD's) associated with the mineral concentration determinations at the MSP are largely due to analysis. On average approximately 70% of the total variance is accounted for by analysis. Therefore efforts to achieve significant reductions in measurement variance at the MSP should nominally be focused on improving the test method. Latter mineral counting-based technologies such as QEMSCAN® and MLA® provide more precise mineralogical assays compared to the GRC test method. The new technologies count a substantially higher number of 'fields' per test aliquot compared to the manual GRC method. The drawback however, is the capital outlay required to install, maintain and operate the superior technologies.

The MSP is characteristic of a mineral beneficiation process where the total assay variance is heavily influenced by the precision limitations of the testing method. Apart from either performing replicate analyses on a routine basis or increasing the number of fields counted, the results indicate the limitations of the GRC method on measurement precision optimization at the MSP.

The GRC analytical procedure essentially views the sample aliquot as a binomial population consisting of the mineral of interest and 'gangue', i.e. all other minerals present in the test sample. In the 121-point grid presented for analysis the rate of occurrence of the target mineral is evaluated based on distinct colours exhibited by the different minerals under transmitted light. The counting procedure is naturally susceptible to constitutional variation intrinsic to the material. Under these conditions, it is reasonable to expect compositional variance to constitute a significant proportion of the overall measurement variance. Figure 17 illustrates the sensitivity of GRC precision to the range of counting regimes that can be employed at Namakwa Sands.

In the figure, stream assays are plotted against analytical RSD's. For comparison, only the analytical variance values listed in Table 20 were used to calculate measured RSD's plotted in Figure 17. Theoretical variances for single-field and five-field GRC counting regimes were simulated based on a binomial sampling model using measured stream assays (as expected mean values) and 121 point counts per analytical counting field. Standard deviations obtained from the simulations were converted to RSD's and plotted as shown in Figure 17. Note that measured precision data are based on five-field counts in accordance with the GRC procedure outlined in Section 5.2.3.2.

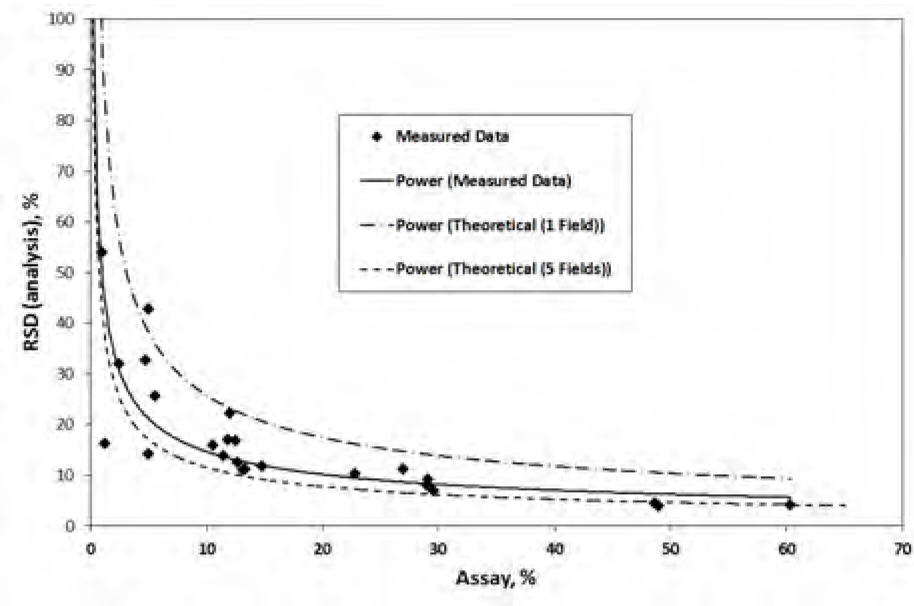


Figure 17: Comparison of theoretical and actual analytical precisions of the grain counting method as practiced at Namakwa Sands MSP

It is clear from Figure 17 that the measured data approaches the simulated five-field plot closely. However, a constant difference persists between the two plots. This is, in part, attributable to the error associated with the extraction of the final test aliquot. This error is not present in the theoretical case.

Notably, the simulated results show that the variance of the five-field data is approximately a fifth of the variance associated with the single-field data, a direct consequence of the effect of sample size on measurement variance (central limit theorem). The current test regime at the MSP takes advantage of this sampling phenomenon, ostensibly as a design limit for the local test protocol.

Suffice to mention here that there is always a trade-off between the precisions obtained and the resources available to achieve desired precisions based on sample mass. Invariably, analysing more than five fields per test aliquot entails increased labour and increased turnaround times for test results.

5.3.2 Mass measurement variance

Variances for the key mass measurement techniques were obtained from instrumentation calibration records obtained for a period spanning a year of operation before the survey was performed. Replicate results from repeatability tests performed following calibration of the mass measurement devices at the MSP provided data for estimating precisions of the respective mass measurement methods. Four mass measurement techniques were investigated, namely the belt scale, weigh flask, impact weight meter and the slurry mass flow meter.

5.3.2.1 Belt scale and weigh flask precision data

The product weighbridge at the MSP is a point of sale to third parties. It is subject to stringent calibration procedures that require certification by an approved body as part of compliance with sales agreements between Namakwa Sands and third parties. As a result the MSP production is routinely adjusted according to the weighbridge despatch masses.

Following routine calibration, repeat tests are sometimes carried out on the weigh flask and belt scale to verify the repeatability of the instruments, using the calibrated weighbridge mass as reference. The product weigh flask at the MSP is mounted on three 20 tonne 'Route' load cells (Model 11LP) and the belt scales each support an Accuweigh Series EMB200 weigh frame on four idlers.

The repeatability test procedure involves passing a known mass, as determined on the weighbridge, over the belt scale and subsequently into a weigh flask. The procedure is performed in duplicate for the same mass of material. The masses indicated on the respective equipment are recorded and kept as part of the plant's calibration records. Typical results closest to the sampling campaign were selected for analysis as listed in Table 22.

Table 22: Repeatability test results for belt scale and weigh flask measurement techniques

Sample	Mass (ton)		
	Belt scale	Weigh flask	Weigh bridge
A1	46.42	44.53	45.31
A2	44.63	45.76	45.29
B1	46.39	44.97	45.64
B2	40.81	45.11	45.57
C1	43.32	43.87	45.48
C2	44.58	44.82	45.51

The results listed in Table 22 represent duplicate tests carried out using three different masses on three separate occasions, i.e. mass/day A, B and C. Only those test instances in which the weighbridge reference masses were the same and where the belt scale and weigh flask corresponding measurements varied independently were selected for analysis. This ensured that the same quantities were used in the repeatability estimations.

The duplicate readings are statistically independent measurements of the same quantity of material. Consequently, the data are effectively independent random variables drawn from the same population (with respect to each measurement method) and can thus be used to determine the mass measurement precisions of the respective instruments by estimating the standard deviations from the respective mean ranges (Equation 5.1). The Range Method, as it is sometimes referred, is considered efficient for sample sizes of not more than a dozen (Box & Draper, 1969; Box et al., 2005; Davies & Goldsmith, 1976).

$$\sigma = \sqrt{\frac{\overline{\omega}}{d_n}} \qquad \textit{(Equation 5.1)}$$

Where,

$\overline{\omega}$ = mean range

d_n = factor corresponding to sample size *n*

Table 23: Mass measurement precision calculation for belt scale and weigh flask measurement techniques

Sample	Belt scale	Weigh flask	Weigh bridge
A	1.80	1.30	0.00
B	5.60	0.20	0.00
C	1.30	0.90	0.00
AVG	2.90	0.80	0.00
SD	1.60	0.84	0.00
RSD (%)	3.62	1.88	0.00

The results of the mass measurement precisions calculation using Equation 5.1 are shown in Table 23 for a sample size of $n = 2$ and $d_n = 1.128$. The weighbridge column contains zero values following calculation of the range since the reference masses selected were the same for each test instance. The relative standard deviations were calculated based on the columnar averages in Table 23.

5.3.2.2 Impact weight meter data

A similar repeatability assessment procedure is applied after routine calibration of impact weight meters. The impact weight meters installed at the MSP are the Ramsey Model 2106. The procedure consists of duplicate passes of a known mass, as determined on the despatch weighbridge, through a skip and conveyor arrangement over a timed period.

The average flow rate as observed on the instrument's digital output panel is recorded as the 'indicated' reading. The 'actual' flow rate is calculated from the known mass of the material recorded at the weighbridge and the time taken to run off the material over the impact plate. Table 24 lists the repeatability data obtained from the calibration data base for tests performed over the immediate period prior to the survey.

Table 24: Repeatability test results for the impact weigher measurement technique

Sample	Flow rate (ton/hr)	
	Indicated	Actual
A1	12.13	12.40
A2	11.99	12.40
B1	9.20	8.30
B2	9.10	8.30
C1	7.90	7.60
C2	7.35	7.60

Three measurement instances were selected for analysis i.e. A, B, and C. Applying the Range Method (Equation 5.1) to the test data, with $n = 2$, and $d_n = 1.128$ yields the results listed in Table 25. The 'actual' measurements display zero values on account of their function as the reference for the test. The relative standard deviations were similarly calculated based on the columnar averages in Table 25.

Table 25: Mass measurement precision calculation for impact weighing

Sample	Flow rate (ton/hr)	
	Indicated	Actual
A	0.14	0.00
B	0.10	0.00
C	0.55	0.00
AVG	0.26	0.00
SD	0.48	0.00
RSD (%)	5.03	0.00

5.3.2.3 Electromagnetic flow meter and nuclear density gauge precision test data

Slurry flow rate measurements are the product of volumetric flow rate and corresponding density measurements. The slurry flow measurement equipment at the MSP consists of an electromagnetic flow meter and densitometer combination (Figure 18).

The equipment shown in the figure consists of a typical slurry electromagnetic flow meter (Endress Hauser Promag S®, DN80 OSN PN16) and a gamma density gauge (Process Automation, Activity: 1.1GBq, 30mCi, Isotope: CS137). The principle of operation is based on the absorption of gamma rays by material flowing through a pipe. The extent of absorption is in proportion to the density of the material passing the gauge at a given instance. As a consequence, consistency of the material characteristics, in addition to steady operation, plays an important role in assessing the repeatability of gross mass flow measurement expressed in mass units per unit per time.

Figure 18: Magnetic flow meter and densitometer assembly on Stream s8

Repeatability tests are not normally carried out on either the densitometer or electromagnetic flow meter during routine calibration tests at the MSP. A test method was devised to estimate the precision of mass flow rates under normal plant operating conditions. The slurry flow equipment on Stream *s8* was used for the test.

The stream design includes a by-pass system consisting of a 4.5 m^3 drop-tank (Figure 19) that is routinely used to calibrate the volumetric flow meter. The drop-tank is equipped with a drain valve (Figure 19a) for emptying and cleaning the vessel between tests. The by-pass valve system (Figure 19d) allows for complete diversion of the entire stream flow with minimum disruption to normal plant operation.

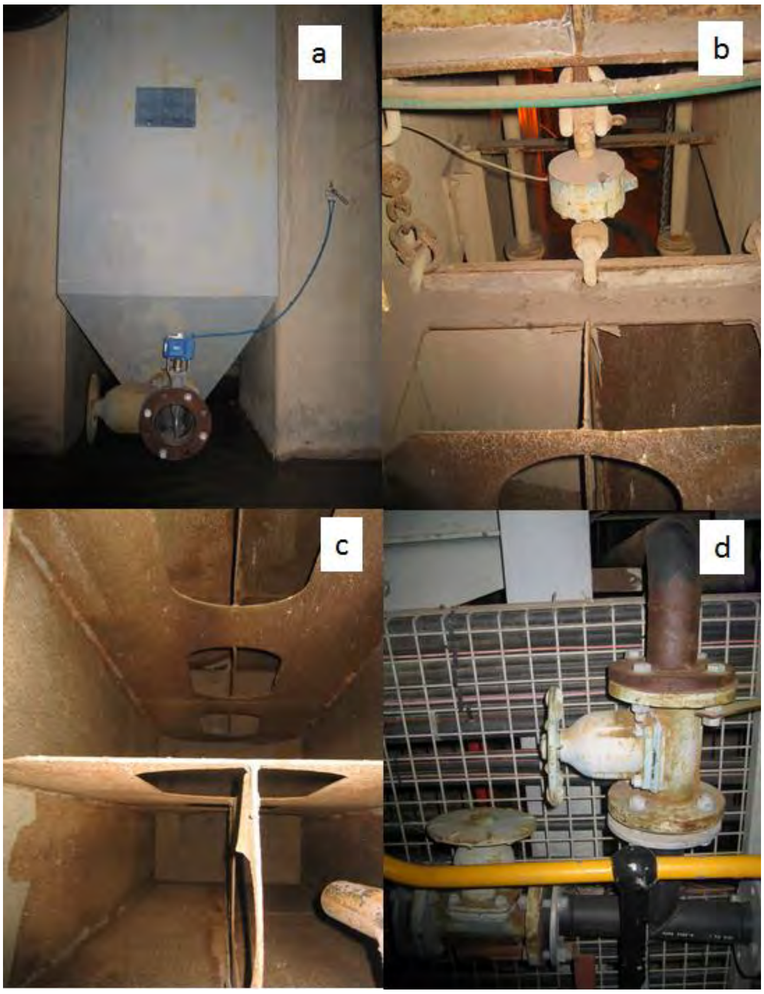


Figure 19: Concentrate drop tank on Stream s8 showing (a) the drain valve, (b) mechanical support, (c) top view and (d) by-pass valve

The tests consisted of obtaining volumetric flow rate and density measurements from two parallel systems of flow measurement: the manual drop-tank by-pass system and the automated densitometer/flow meter assembly. Material on Stream *s8* was diverted into the drop tank during normal plant operation for designated time periods per test (until 4.5 m^3 of slurry were collected) before flow was switched back to normal operation.

Random samples were collected during the flow diversion period for determining slurry density on a conventional Marcy scale. Volumetric flow rates were calculated based on the time taken to fill the nominal volume of the drop-tank. Hence each test provided a 'manually' determined volumetric flow rate as well as a slurry density value. Prior to diverting flow to the drop-tank, digital readouts of density and volumetric flow rate measurements were obtained from the densitometer and magnetic flow meter electronic display panels.

Ten tests were conducted over an eight-hour shift period. Between tests, the drop-tank was emptied and cleaned while the densitometer and magnetic flow meter were allowed to reach steady state before the next test was performed. The test data collected consisted of duplicated pairs of density and flow rate values from the 'manual' and automated systems. The results are listed in Table 26.

Table 26: Test results for the magnetic flow meter and densitometer repeatability measurements

Test no.	Time (min)	Drop tank measurements			Flow meter and densitometer readings		
		Vol flow (m^3/hr)	Density (ton/m^3)	Mass flow (ton/hr)	Vol flow (m^3/hr)	Density (ton/m^3)	Mass flow (ton/hr)
1	4.14	62.50	1.45	90.6	53.93	1.54	83.1
2	4.09	63.34	1.52	96.3	54.21	1.59	86.2
3	4.07	63.55	1.55	98.5	48.74	1.56	76.0
4	4.08	63.47	1.55	98.4	55.43	1.60	88.7
5	4.13	62.60	1.53	95.8	50.87	1.61	81.9
6	4.07	63.63	1.56	99.3	58.63	1.58	92.6
7	4.13	62.60	1.49	93.3	54.83	1.59	87.2
8	4.02	64.42	1.51	97.3	56.62	1.55	87.8
9	4.12	62.86	1.53	96.2	49.37	1.62	80.0
10	4.15	62.35	1.48	92.3	55.34	1.58	87.4

A method for estimating the repeatability of measurements obtained from instrumentation that measures transient flows was adopted from an approach proposed by Hayward (1977). The method evaluates the standard deviation of the difference between paired readings obtained from nominally identical measurement procedures that are set up in a series or parallel configuration so that they measure the same flow quantity.

The method equates the root mean square of the measurement differences to the square root of the sum of the measurement variances of the two measurement methods as illustrated in Equation 5.2 (propagation of variance).

$$\sigma_{diff} = \sqrt{\sigma_m^2 + \sigma_a^2} \qquad \textit{(Equation 5.2)}$$

Where, σ_m^2 = variance of manual measurements

σ_a^2 = variance of automated measurements

σ_{diff} = standard deviation of measurement differences

An assumption is made that since the two measurement systems employed are nominally similar, then

$$\sigma_m \cong \sigma_a = \frac{1}{\sqrt{2}}\sigma_{diff} \qquad \textit{(Equation 5.3)}$$

The current configuration simulates a parallel arrangement of the two measurement systems. The respective standard deviations of the target quantity measured i.e. solids flow rate, are nominally similar. The 5^{th} and 8^{th} columns in Table 26 list the calculated volumetric and solids flow rate values respectively. A chi-square test based on their respective variances shows that they are statistically the same. A significant difference would invalidate Equation 5.3.

Table 27 shows the arithmetic differences between the respective measurements listed in Table 26. Applying Equation 5.3 gives the measurement standard deviations (σ_a) as indicated in the penultimate row in Table 27. The last row in the table computes the relative standard deviations based on the measurement averages calculated over the ten test runs.

Table 27: Arithmetic differences between respective test results obtained from the manual and automated measurement methods

Test no.	Measurement differences (abs.)		
	Vol flow (m^3/hr)	Density (ton/m^3)	Mass flow (ton/hr)
1	8.57	0.09	7.57
2	9.13	0.07	10.08
3	14.81	0.01	22.46
4	8.04	0.05	9.69
5	11.73	0.08	13.88
6	5.00	0.02	6.63
7	7.77	0.10	6.10
8	7.80	0.04	9.52
9	13.49	0.09	16.19
10	7.01	0.10	4.84
SD, σ_a	3.36	0.03	5.83
RSD (%)	4.42	1.46	4.85

The bias observable in the flow rate measurements between the two measurement methods appears consistent throughout the measurement period. There is no apparent evidence of either value or time dependent systematic uncertainty over the ten readings taken, hence the observed bias is not expected to impact on precision determination.

5.4 Component mass flow rate and mineral recovery variance

5.4.1 Component flow rate variance

Figure 20 shows a plot of total assay RSD's versus mean stream assays obtained from survey data listed in Table 20. The plot constitutes an error model for total assay error comprising sampling, preparation and analysis for Zr, Rt and Lx. RSD values for the nominal stream assays were deduced based on the error model equation.

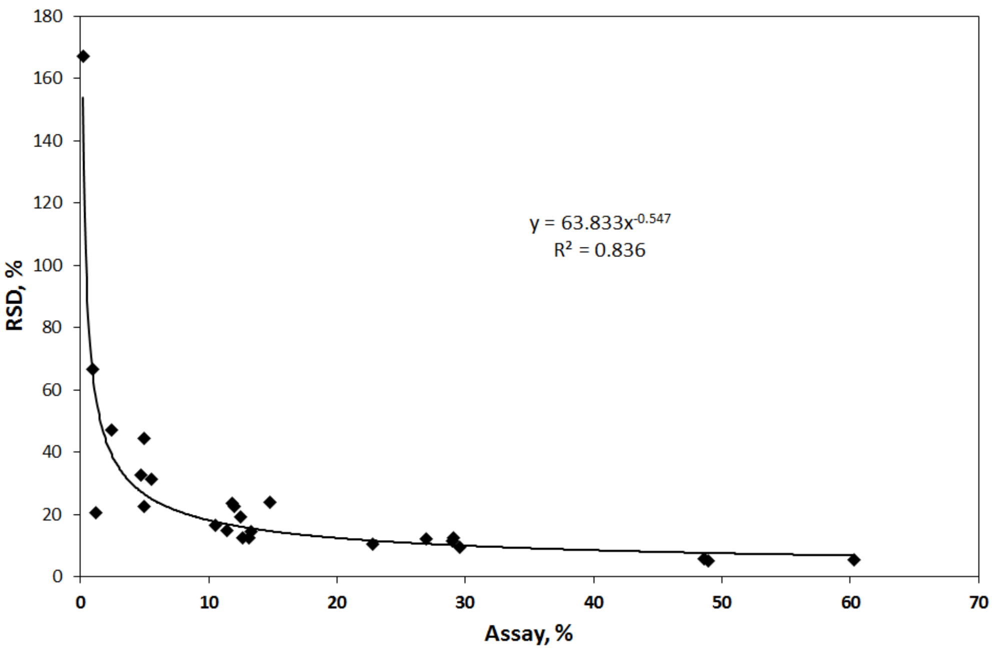


Figure 20: Total assay error model for GRC at the MSP

Since the survey was conducted over a 12-hour period, RSD values calculated from the model were converted to monthly values by dividing respective assay variances by 60 on the assumption of a 30-day month. Metal accounting at the MSP is based on a monthly production cycle. Consequently, stream mass flow rate RSD's were estimated by the propagation of variance through the product of gross flow rates and corresponding stream assays as shown in Equation 5.4 where *a* represents the stream assay and *F* represents the gross flow rate.

$$var(f) = a^2 var(F) + F^2 var(a) \qquad \textit{(Equation 5.4)}$$

Table 28 lists the monthly assay and component flow rate RSD's calculated from the survey results using Equation 5.4. It should be noted here that the total mass flow rate RSD value for Stream *s20* is a historical estimate obtained from plant records. The 'stream' consists of truckloads of spillage on which is applied a truck factor to covert the consignments to tonnages. The rest of the streams were assigned total mass flow rate RSD's determined in the preceding section as per measurement technology employed as specified Figure 11. It can be shown that Equation 5.4 is mathematically equivalent to the following:

$D_f^2 = D_a^2 + D_F^2$ *(Equation 5.5)*

Where,

D_f = component flow rate RSD

D_a = component assay RSD

DF = gross stream flow rate RSD.

Table 28: Stream assay and component flow monthly RSD's

Stream ID	Monthly RSD, %						
	D_F (Total mass)	D_a (Component assays)			D_f (Component flows)		
		Zr	Rt	Lx	Zr	Rt	Lx
s1	3.6	0.9	2.2	2.3	3.7	4.2	4.3
s2	3.6	0.8	2.0	2.7	3.7	4.1	4.5
s3	3.6	0.9	2.2	2.3	3.7	4.2	4.3
s4	5.0	2.3	4.7	3.4	5.5	6.8	6.0
s5	3.6	0.9	2.1	2.2	3.7	4.1	4.2
s6	4.9	1.5	2.2	1.5	5.1	5.4	5.1
s7	4.9	1.5	2.3	1.6	5.1	5.4	5.1
s8	4.9	0.8	2.1	2.9	5.0	5.3	5.7
s9	4.9	1.2	1.6	1.5	5.1	5.1	5.1
s10	3.6	0.8	2.0	2.6	3.7	4.1	4.4
s11	3.6	0.8	2.1	2.8	3.7	4.1	4.6
s12	5.0	0.7	3.7	6.1	5.1	6.2	7.9
s13	3.6	0.9	1.7	2.5	3.7	4.0	4.4
s14	3.6	1.2	1.5	1.5	3.8	3.9	3.9
s15	3.6	1.0	1.6	2.0	3.7	3.9	4.1
s16	1.9	0.7	25.6	58.8	2.0	25.6	58.8
s17	1.9	0.7	9.6	15.0	2.0	9.8	15.2
s18	1.9	5.3	0.8	1.3	5.6	2.1	2.3
s19	1.9	4.2	0.9	1.1	4.6	2.1	2.2
s20	8.0	0.8	2.0	2.7	8.0	8.3	8.4

From Equation 5.5 it can be surmised that the value of component flow RSD's are sensitive to the measurement determined with the higher RSD (less precisely measured). For large differences in D_a and D_F, D_f virtually assumes the magnitude of the larger RSD value.

For instance, despite a gross mass measurement RSD of 1.9% for Stream *s16*, the component flow rate RSD's for Rt and Lx are virtually equal to their stream assay precisions, i.e. 25.6% and 58.8% respectively. In comparison, while Zr assays are measured at a high precision value of 0.7% RSD, the Zr component flow rate RSD of 2.0% is closer to the gross stream mass RSD of 1.9% than the assay precision.

As a result, component flow rate precisions are improved the most by measuring the higher RSD measurement more precisely. For example, reducing the RSD of Rt assay by 50% i.e. from 25.6% to 12.8% results in a 50% reduction in the Rt flow rate RSD in Stream *s16*; whereas a similar reduction in the mass measurement RSD yields virtually no change in the Rt flow rate RSD.

Thus, for purely measurement based improvement efforts, in order to increase component flow rate precisions in low concentration streams (such as tailings streams) it is commendable to consider increasing assay precisions first before upgrading mass measurement precisions. The reverse applies for high concentration streams where more benefits are to be gained by improving mass measurement precisions ahead of assay precisions.

5.4.2 Recovery variance

Two approaches for calculating mineral recoveries have been used at the MSP, namely the actual recovery method (R_A) and the built-up recovery method (R_B) presented in Equation 4.1 and Equation 4.2 (Section 4.2.3.1). The actual recovery method is currently the preferred method as discussed in Section 5.5.

The built up recovery calculation was observed to 'create grade' at the MSP and this was attributed to insufficient accounting of total spillage produced. Spillage constitutes approximately 7% of the total MSP outflow mass (Stream *s20*). Subsequent use of the actual recovery method was seen as averting bias by removing the potentially erroneous spillage measurement from plant recovery calculations.

Despite this, the built up recovery calculation reportedly produced stable trends over time. Variance analysis based on the current study data suggests that the precision in recovery calculation obtained from the built up formula is marginally higher than the actual recovery precision.

Equation 5.6 and Equation 5.7 estimate variances of actual and built up recoveries by applying the propagation of variance rule on Equation 4.1 and Equation 4.2 respectively. RSD's for the three mineral targets were calculated based on Equation 5.5 and Equation 5.6 for all MSP outflow streams using the nominal data in Table 20 and measurement RSD values listed in Table 28.

$$var(R_A) = \left(\frac{c}{Ff}\right)^2 var(C) + \left(\frac{C}{Ff}\right)^2 var(c) + \left(\frac{-Cc}{F^2f}\right)^2 var(F) + \left(\frac{-Cc}{Ff^2}\right)^2 var(f) \qquad \textit{(Equation 5.6)}$$

$$var(R_B) = \left(\frac{Ttc}{(Cc+Tt)^2}\right)^2 var(C) + \left(\frac{TtC}{(Cc+Tt)^2}\right)^2 var(c) + \left(\frac{-Cct}{(Cc+Tt)^2}\right)^2 var(T) + \left(\frac{-CcT}{(Cc+Tt)^2}\right)^2 var(t)$$

(Equation 5.7)

For entire plant recovery calculations, the 'feed' is comprised of the three streams feeding the operation (i.e. streams *s1*, *s2* and *s10*) and the 'tailings' stream sums the remaining seven outflow streams that exclude the stream whose recovery is being assessed. This essentially reduces the MSP to a 'black-box' operation served by three streams.

The recovery precisions (RSD's) were obtained by dividing the square roots of $var(R_A)$ and $var(R_B)$ by the respective recovery values for each component per stream. Figure 21 compares the two sets of calculated RSD values in the form of a parity chart.

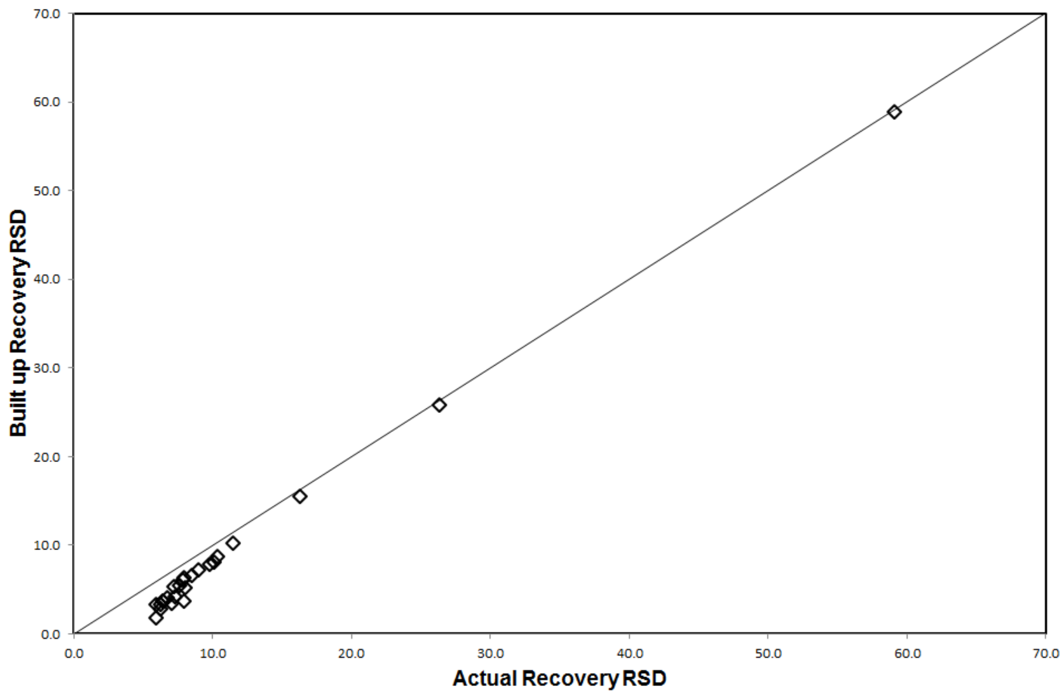


Figure 21: Comparison of actual and built up recovery precisions

Built up recovery RSD values all lie below the parity line, indicating that the precisions are higher than actual recovery values across all three mineral components. The differences in precisions are however marginal, suggesting that their response to measurement error is similar, the differences in formulas notwithstanding. A sensitivity analysis of $var(R_A)$ and $var(R_B)$ may highlight the source of the observed similarity, noting already that the only common measurements in Equation 4.1 and Equation 4.2 are the concentrate stream mass flow rate (C) and mineral concentration (c).

5.4.3 Sensitivity of mineral recovery estimates

The coefficients of the variance terms in Equation 5.6 and Equation 5.7 were calculated using the MSP nominal data. To facilitate comparison, the calculated coefficients were normalised by dividing each equation by the largest coefficient. This allows easy numerical comparison of the resultant values. Here, the higher the value of the coefficient the greater the sensitivity of the calculated recovery to the associated variance term.

The normalised coefficients for Zr component are listed in Table 29. The 'Variable term' item in the tables denotes the measurement variance terms in Equation 5.5 and Equation 5.6 and the 'normalised coefficient' refers to the calculated (and subsequently normalised) respective coefficients. Average values of the coefficients across all streams were calculated to provide a single measure of sensitivity per variance term.

Table 29: Sensitivity coefficients of actual and built up recovery precision variables for Zr component

Method	Stream	c/f	RSD, %	Variable term/normalised coefficient					
				var(F)	var(f)	var(C)	var(c)	var(T)	var(t)
R_A	s4	0.19	7.87	0.009	0.037	0.600	1.000	-	-
	s7	0.40	7.62	0.034	0.136	1.000	0.867	-	-
	s15	0.87	6.76	0.036	0.143	1.000	0.190	-	-
	s16	1.79	5.99	0.115	0.460	1.000	0.144	-	-
	s17	1.75	5.99	0.001	0.002	1.000	0.001	-	-
	s18	0.04	7.94	0.000	0.002	0.193	1.000	-	-
	s19	0.06	7.25	0.001	0.003	1.000	0.633	-	-
	s20	1.32	9.82	0.004	0.017	1.000	0.010	-	-
Average	-	0.80	7.41	0.030	0.118	1.000	0.566	-	-
R_B	s4	0.19	6.01	-	-	0.600	1.000	0.012	0.030
	s7	0.40	5.39	-	-	1.000	0.867	0.051	0.105
	s15	0.87	3.99	-	-	1.000	0.190	0.055	0.135
	s16	1.79	1.75	-	-	0.772	0.111	0.204	1.000
	s17	1.75	3.30	-	-	1.000	0.001	0.001	0.002
	s18	0.04	6.24	-	-	0.193	1.000	0.000	0.002
	s19	0.06	5.32	-	-	1.000	0.633	0.001	0.002
	s20	1.32	7.74	-	-	1.000	0.010	0.005	0.018
Average	-	0.80	4.97	-	-	1.000	0.581	0.050	0.197

The component enrichment ratio (*c/f*) is included in the tables as a proxy for actual measured concentrate assays. It is commonly used as a measure of the extent to which minerals are concentrated in product streams and is obtained by dividing the observed concentrate assay (*c*) by the corresponding feed stream assay (*f*).

The coefficients of *var(C)* and *var(c)* show the highest average values for both recovery methods in the case of Zr component (Table 29). Individual streams predominantly follow a similar trend. The coefficients of feed and tailings variable terms with respect to R_A and R_B respectively are insignificant in comparison. This suggests that $var(R_A)$ and $var(R_B)$ are sensitive to concentrate measurements the most, a likely contributing factor to the near-parity phenomenon observable in Figure 21.

Rt and Lx components show similar trends although the coefficients of feed and tailings assays (i.e. *var(f)* and *var(t)* for R_A and R_B respectively) feature more significantly than in the case for Zr as shown in Table 30 where average normalised results are compared.

Table 30: Normalised average coefficients of measurement precision variables for Zr, Rt and Lx components

Method	Component	Average c/f	Average RSD, %	Variance term/normalised coefficient					
				var(F)	var(f)	var(C)	var(c)	var(T)	var(t)
R_A	Zr	0.80	7.41	0.030	0.118	1.000	0.566	-	-
	Rt	1.87	10.60	0.006	0.533	0.784	1.000	-	-
	Lx	1.44	15.33	0.005	0.666	0.767	1.000	-	-
	Average	1.37	11.11	0.016	0.513	0.994	1.000	-	-
R_B	Zr	0.80	4.97	-	-	1.000	0.581	0.050	0.197
	Rt	1.87	8.24	-	-	0.782	1.000	0.007	0.571
	Lx	1.44	13.06	-	-	0.767	1.000	0.006	0.699
	Average	1.37	8.75	-	-	0.988	1.000	0.024	0.569

Thus recovery precisions obtained using the actual and built up methods are largely sensitive to the variance of concentrate stream measurements. Therefore measurement strategies that maximise the precision of these measurements are bound to increase recovery precisions as calculated using the two methods.

5.4.4 Check In-Check Out and unaccounted balance at the MSP

The Code recommends the CICO method of accounting which requires the measurement of all input and output streams with appropriate precisions; and recording the discrepancy due to random error as an unaccounted balance (UAB). The levels of measurement precision are not prescribed in the Code but the recommendation is made that the choice of precisions should follow a risk analysis exercise.

The measurement variance analysis conducted enables the MSP to decide on achievable UAB limits and tolerances for key information such as mineral recoveries *a priori*. This provides tools for evaluating mass balances based on the efficacy of the prevailing measurement technology and protocols.

Table 31 presents a CICO mass balance involving the production of Zr through Stream *s16*. In order to simulate a realistic data set in which the mass balance is inconsistent, the nominal data (i.e. total stream mass and assays) were perturbed according to the monthly RSD's determined in this study.

Table 31: Check In-Check Out balance of Zr through Stream s16

Stream	Total mass	Assay	Comp Mass	Distribution, %	Comp Mass Deviation	
					±σ	±2σ
Feed	1.100837	0.52	0.572501	100.00	0.0186	0.0373
Concentrate	0.378781	0.99	0.376246	65.72	0.0115	0.0231
Tailings	0.714638	0.34	0.239590	41.85	0.0062	0.0124
UAB	0.007418	-	-0.043335	-7.57	0.0364	0.0727

In Table 31, the 'Comp Mass' is the mass flow of Zr. The 'Distribution' column is the percentage of Zr mass in each stream relative to the total Zr in the feed stream and the 'UAB' is the unaccounted balance. The 'Comp Mass Deviation' is the standard deviation of Zr mass flow rate in mass units. The balance assumes that all input and output streams are measured and the measurements are free of systematic error.

The actual recovery is 65.7% and the theoretical recovery is 58.2%. The difference between the two recovery values constitutes the UAB (7.6%). The decision whether the UAB is 'too large' depends on the risk that the operation places on the acceptance of the balance. Using a 2-sigma (95% confidence) maximum deviation of 7.27% calculated on the basis of measurement precisions determined in this study, leads to the acceptance of the balance as close to prescribed limits.

Notably, the *a priori* RSD for actual recovery for Zr in Stream *s16* was calculated at 6%, i.e. 12% with 95% confidence. Here, the percentage difference between the actual recovery and theoretical is about 11.5%, providing further justification for possible acceptance of the balance at the chosen 2-sigma confidence bound.

In the absence of bias, a large UAB would indicate high imprecisions in sampling, assaying or mass measurement. As a consequence, the UAB is a random quantity that is expected to fluctuate with a mean of zero and a standard deviation defined by the precision limitations of the entire measurement scheme of an operation. The benefits of minimising the UAB are varied, not least of which are the ability of a balance to detect changes in plant efficiency that may be smaller than the 'natural' fluctuation of the UAB produced by a given measurement scheme.

5.5 Summary

The Namakwa Sands MSP is a typical mineral upgrading operation characterised by particulate sampling and weighing of bulk material. The MSP presented opportunities for this study that included a complex interaction between interlinked measurements and management of custody transfer points between second and third parties.

This chapter described salient features of the metal accounting system at the MSP in the context of the Code Guidelines, observations from the accounting practice questionnaire survey reported in this study and sound principles of sampling and mass measurement in mineral processes. The error modelling survey provided components of mass, sampling and analytical variance for use in validating mathematical heuristics derived later in this study and for conducting a numerical simulation of the effects of flowsheet configuration on variance reduction after data reconciliation.

Fresh feed and final products were found to be measured with precisions commensurate with custody transfer points. Final product masses are weighed in load-cell based weigh flasks and plant feed mass is determined on a four-idler mechanical scale. Both methods were calibrated regularly and precisions determined in this study were 1.9 % and 3.6 % (RSD) respectively. Internally, electromagnetic flowmeters are used to estimate slurry flow rates and impact weight meters serve the dry streams with estimated precisions of 5.0% and 4.9% (RSD) respectively.

The components of variance results revealed that the analytical method contributed to over 70% of total assay variance on average. Mineral grain counting using optical microscopy was used for metal accounting at the time of the survey. Grab sampling and the ubiquitous use of the diverter sampler (as in most mineral sands operations) were identified as potentially problematic and in strict non-conformance with sound sampling principles owing to the susceptibility of the two sampling methods to systematic error. However, sampling variance was substantially small compared to analytical variance.

The CICO system of accounting is practiced at the MSP and mineral yields are determined using the actual recovery method. Spillage retreatment is done separately from fresh feed, and recoveries are reported separately. Actual recovery computation was found to be most sensitive to concentrate stream measurement variance as compared to feed and tailings measurement errors. A numerical example demonstrated the use of measurement errors to evaluate the efficacy of a CICO mass balance by pre-selecting the bounds for unaccounted balance from known precision limitations of a given measurement scheme.

Barring material losses, which should generally be kept to a minimum, the reliance of the magnitude of the UAB on the aggregate precision of measurements is well-known. Measurement improvement strategies such as better hardware, multiple mass measurement equipment and increased sample sizes would serve to reduce the UAB to narrower limits thus increasing the balance confidence, albeit at a cost. Such efforts are generally limited to terminal streams as most metal balances are drawn around entire plants, thereby neglecting any additional information that internal plant measurements may offer the primary balance.

The next chapter presents data reconciliation as a no-cost option for reducing mass balance variance by making use of internal measurements that are traditionally not used in CICO. In particular, the ability to selectively maximise variance reduction of terminal streams using all measurements taken is explored.

Chapter 6

THE LINEAR STEADY STAGE DATA RECONCILIATION EQUATION

This chapter presents the general linear steady state data reconciliation solution (Kuehn & Davidson, 1961) as a basis for developing heuristics for designing precise measurement networks based on the variance reduction attributes of data reconciliation. The methodology by which the heuristics were obtained was to develop a generalised formula for variance reduction on terminal streams and to use this formula to extract design rules/principles. The heuristics were obtained from the simplest case of linear steady state data reconciliation with all streams measured but are considered to be of general relevance.

6.1 The general linear steady state data reconciliation solution

This section outlines the general linear steady state data reconciliation (SSDR) solution for problems where all streams are measured and measurement errors are considered to be independent. In this instance, the constraint equations conserve mass around process nodes although the solution is applicable to other conservable quantities such as energy.

The least squares minimisation problem presented in Equation 2.8 and Equation 2.9 can be written as follows:

$$\min_{x} \sum_{i=1}^{m} w_i (y_i - x_i)^2 \qquad \textit{(Equation 6.1)}$$

subject to,

$$\sum_{k=1}^{p} \sum_{i=1}^{m} a_{ki} x_i = 0 \qquad \textit{(Equation 6.2)}$$

Where,

- m = number of measurements
- p = number of constraint equations
- x_i = reconciled estimates
- y_i = measured variables

An analytical solution to the constrained minimisation problem as presented in Equations 6.1 and 6.2 can be derived using the method of Lagrange multipliers (Mah, 1990). The Lagrangian for the optimisation is written as follows:

$$\mathcal{L} = \sum_{i=1}^{m} w_i (y_i - x_i)^2 - 2 \sum_{k=1}^{p} \sum_{i=1}^{m} \lambda_k a_{ki} \qquad \textit{(Equation 6.3)}$$

The solution must verify the optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial x_i} = -2 \sum_{i=1}^{m} w_i (y_i - x_i) - 2 \sum_{k=1}^{p} \sum_{i=1}^{m} \lambda_k a_{ki} = 0 \qquad \textit{(Equation 6.4)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_k} = -2 \sum_{k=1}^{p} \sum_{i=1}^{m} a_{ki} x_i = 0 \qquad \textit{(Equation 6.5)}$$

Where λ_k represents Lagrange multipliers associated with the mass balance constraints at each node in the process network. From Equation 6.4,

$$-2 \sum_{i=1}^{m} w_i y_i + 2 \sum_{i=1}^{m} w_i x_i - 2 \sum_{k=1}^{p} \sum_{i=1}^{m} \lambda_k a_{ki} = 0 \qquad \textit{(Equation 6.6)}$$

$$2 \sum_{i=1}^{m} w_i x_i = 2 \sum_{i=1}^{m} w_i y_i + 2 \sum_{k=1}^{p} \sum_{i=1}^{m} \lambda_k a_{ki} \qquad \textit{(Equation 6.7)}$$

In matrix form, Equation 6.7 can be written as,

$$Wx = Wy + A^T \lambda \qquad \textit{(Equation 6.8)}$$

Where,

- W = weighting matrix of size $m \times m$
- A = connectivity matrix of size $p \times m$
- x = vector of reconciled estimates
- y = vector of measured variables

Noting that W is a diagonal matrix with non-zero elements along the major diagonal and therefore non-singular, i.e. $|W| \neq 0$, multiplying Equation 6.8 by W^{-1} yields Equation 6.9.

$$x = y + W^{-1}A^{T}\lambda \qquad \textit{(Equation 6.9)}$$

In the linear SSDR case with all streams measured, the absence of unmeasured variables implies that matrix A is full row rank; and multiplying Equation 6.9 by A gives,

$$Ax = \mathrm{A}y + \mathrm{A}W^{-1}A^{T}\lambda \qquad \textit{(Equation 6.10)}$$

In matrix form Equation 6.5 can be written as,

$$\frac{\partial \mathcal{L}}{\partial \lambda_k} = -2Ax = 0 \qquad \textit{(Equation 6.11)}$$

Combining Equation 6.10 and Equation 6.11 yields,

$$\mathrm{A}W^{-1}A^{T}\lambda = -\mathrm{A}y \qquad \textit{(Equation 6.12)}$$

and,

$$\lambda = -(\mathrm{A}W^{-1}A^{T})^{-1}\mathrm{A}y \qquad \textit{(Equation 6.13)}$$

From Equation 6.9 and Equation 6.13

$$x = y - W^{-1}A^{T}(\mathrm{A}W^{-1}A^{T})^{-1}\mathrm{A}y \qquad \textit{(Equation 6.14)}$$

If the weight matrix is chosen as the inverse of the variance matrix (Σ), then Equation 6.14 becomes:

$$x = y - \Sigma A^T(\mathrm{A}\Sigma A^T)^{-1}\mathrm{A}y \qquad \textit{(Equation 6.15)}$$

The reconciled estimates given by Equations 6.14 and 6.15 satisfy the constraints in Equation 6.2.

In this solution the estimates are unbounded. Bounds may be included as additional constraints to ensure that physically meaningful results are obtained.

6.2 Adjusted variance through steady state data reconciliation

The derivation of the analytical solution of the general SSDR for process networks with all streams measured is outlined as a precursor to the symbolic derivation of the expected variance reduction for terminal streams.

6.2.1 General solution for all stream types

This section outlines the derivation of the analytical solution for adjusted variance estimates for all streams in a given process network. Notably, Equation 6.15 shows that the adjusted flow estimates are a linear transformation of the measured flow rate values. If it is assumed that measurement errors are Gaussian then it follows that the errors in the adjusted flow values also follow a normal distribution. It is hence possible to estimate error associated with the adjusted flow values ($\boldsymbol{x}$) based on error obtaining in the measured values ($\boldsymbol{y}$).

Thus, letting

$$B = I - \Sigma A^T(\mathrm{A}\Sigma A^T)^{-1}\mathrm{A} \qquad \textit{(Equation 6.16)}$$

Equation 6.15 becomes

$$x = \mathrm{B}y \qquad \textit{(Equation 6.17)}$$

From the propagation of variance rule based on Taylor's first derivative expansion of Equation 6.17,

$$\Sigma_a = \left(\frac{\partial x}{\partial y}\right)^2 \Sigma + \left(\frac{\partial x}{\partial \boldsymbol{B}}\right)^2 Var(\boldsymbol{B}) \qquad \textit{(Equation 6.18)}$$

Σ and Σ_a represent the variance-covariance matrices of the measured and adjusted flow rate estimates respectively.

Equation 6.16 shows that matrix B is a function of measured variances only, which are constant for any measurement instance. It follows therefore that the second derivative term on the right hand side of Equation 6.18 equates to zero. Solving Equation 6.18 gives,

$$\Sigma_a = B\Sigma B^T \qquad \textit{(Equation 6.19)}$$

In order to assess the confidence improvement due to the reconciliation process, the adjusted variances can be expressed as a fraction of the corresponding measured variances to yield 'variance reduction ratios' for the respective measured variables (Equation 6.20) since the variance-covariance matrices are non-singular.

$$\frac{\Sigma_a}{\Sigma} = BB^T \qquad \textit{(Equation 6.20)}$$

Lyman (2005) produced an explicit expression of this quantity to express the average variance reduction ratio for an entire circuit based on Equation 6.20 for linear circuits with N_n nodes and N_s streams (Equation 6.21).

$$\frac{\text{trace } \Sigma_a}{\text{trace } \Sigma} = 1 - \frac{N_n}{N_s} \qquad \textit{(Equation 6.21)}$$

Equation 6.21 introduced an important decision variable, the node to stream ratio (N_n/N_s) for an entire circuit. This measure can be used to assess *a priori* the variance reduction capabilities of different network topologies based on the number of streams and nodes. A drawback in network design studies is that the expression only predicts average variance reduction for entire networks and cannot be used to predict the effects of designs on individual measurements. In addition, the formula is valid for networks with measurement variances of similar size.

6.2.2 Formulation for adjusted variance of terminal streams

This section outlines the derivation of the general solution for adjusted variance for individual terminal streams in linear steady state processes.

6.2.2.1 Single node process

Figure 22 shows a schematic of a hypothetical single node process served by one feed stream and two product streams. Measurement variances are represented by the symbols $\sigma^2_{sm(n)}$, where 'sm' represents the stream identity and 'n' denotes the parent node.

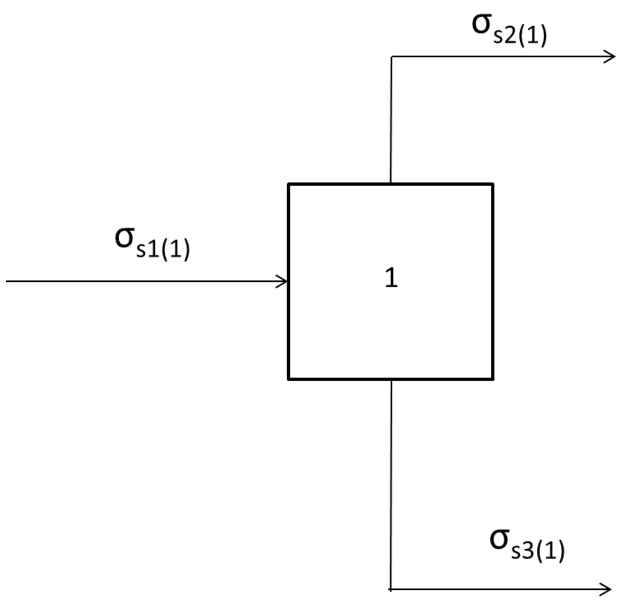


Figure 22: Hypothetical single node process

By symbolically solving Equation 6.19 the adjusted variance for the three streams depicted in the Figure 22 is as follows:

$$\Sigma_a = \begin{bmatrix} \sigma^2_{s1(1)}\left(1-\frac{\sigma^2_{s1(1)}}{\sum_n^3 \sigma^2_{sn(1)}}\right) & 0 & 0 \\ 0 & \sigma^2_{s2(1)}\left(1-\frac{\sigma^2_{s2(1)}}{\sum_n^3 \sigma^2_{sn(1)}}\right) & 0 \\ 0 & 0 & \sigma^2_{s3(1)}\left(1-\frac{\sigma^2_{s3(1)}}{\sum_n^3 \sigma^2_{sn(1)}}\right) \end{bmatrix} \qquad \textit{(Equation 6.22)}$$

where,

$$\sum_a = \begin{bmatrix} \sigma^2_{s1(1)}\big|_a & 0 & 0 \\ 0 & \sigma^2_{s1(1)}\big|_a & 0 \\ 0 & 0 & \sigma^2_{s1(1)}\big|_a \end{bmatrix}$$

$$\sum = \begin{bmatrix} \sigma^2_{s1(1)} & 0 & 0 \\ 0 & \sigma^2_{s2(1)} & 0 \\ 0 & 0 & \sigma^2_{s3(1)} \end{bmatrix}$$

A	=	$[1 \quad -1 \quad -1]$
$\sigma^2_{sm(1)}\big|_a$	=	adjusted variance of stream sm(1) and
$\sigma^2_{sm(1)}$	=	measured variance of stream sm(1)

The off-diagonal elements for the variance-covariance matrices are equated to zero based on the assumption that measurement errors are not correlated across streams.

The adjusted variance for the individual streams can be expressed in terms of their respective measured variances by equating the symbolic terms derived in Equation 6.22. For stream *s1(1)*, for instance, the expression for adjusted variance gives:

$$\sigma^2_{s1(1)}\big|_a = \sigma^2_{s1(1)}\left(1-\frac{\sigma^2_{s1(1)}}{\sum_m^3 \sigma^2_{sm(1)}}\right) \qquad \textit{(Equation 6.23)}$$

A ratio of the adjusted and measured variances gives a measure of the extent of variance reduction for the observed stream. For stream *s1(1)* this ‘variance reduction ratio’ is as follows:

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)}}{\sum_m^3 \sigma^2_{sm(1)}}\right) \qquad \textit{(Equation 6.24)}$$

Equation 6.24 applies to all single node processes regardless of the number of streams associated with the node. By observation, a number of conclusions or rules for predicting the reduction in variance experienced by the individual streams can be inferred:

(i) The extent of variance reduction for a comparatively large variance stream in relation to the other streams is likely to be high. It follows therefore that small variance streams will experience little reduction in variance.

(ii) The reduction in variance is independent of whether a stream enters or leaves a process.

(iii) The reduction in variance decreases with an increase in the number of streams across the node.

If all streams are measured with equal variance, Equation 6.24 simplifies to Lyman's (2005) conclusion (Equation 6.21) and all streams experience equal reductions in variance through SSDR. By extension, streams experience higher reduction in variance when measured with low precision and, conversely, experience lower reduction in variance when measured with high precision, although the average variance reduction for the single node instance remains constant as predicted by Equation 6.21.

6.2.2.2 Two node process

Figure 23 shows a schematic of a hypothetical two-node process. The measurement variance-covariance and connection matrices for the circuit are as follows:

$$\Sigma = \begin{bmatrix} \sigma^2_{s1(1)} & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{s2(1)} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{s3(1,2)} & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{s1(2)} & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{s2(2)} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

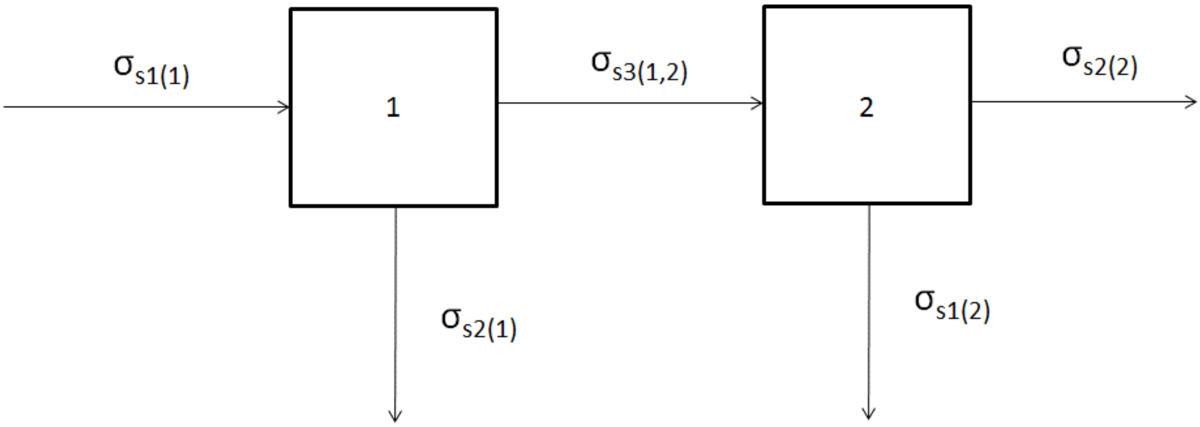


Figure 23: Hypothetical two-node process without recycle streams

The symbolic solution of Equation 6.19 for the adjusted variance of all streams shown in the two-node process example depicted in Figure 23 simplifies as follows:

$$\Sigma_a = \begin{bmatrix} \sigma^2_{s1(1)}\left(1 - \frac{\sigma^2_{s1(1)} \sum_3^5 \sigma^2_{sm(2)}}{D}\right) & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{s2(1)}\left(1 - \frac{\sigma^2_{s2(1)} \sum_3^5 \sigma^2_{sm(2)}}{D}\right) & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{s3(1,2)}\left(1 - \sigma^2_{s3(1,2)} \frac{\left(\sum_3^5 \sigma^2_{sm(2)} + \sum_1^3 \sigma^2_{sm(1)} - 2\sigma^2_{s3(1,2)}\right)}{D}\right) & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{s1(2)}\left(1 - \frac{\sigma^2_{s1(2)} \sum_1^3 \sigma^2_{sm(1)}}{D}\right) & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{s2(2)}\left(1 - \frac{\sigma^2_{s2(2)} \sum_1^3 \sigma^2_{sm(1)}}{D}\right) \end{bmatrix}$$

Where

$$D = \left(\sum_3^5 \sigma^2_{sm(2)} \sum_1^3 \sigma^2_{sm(1)} - \left(\sigma^2_{s3(1,2)}\right)^2\right) \qquad \textit{(Equation 6.25)}$$

Significantly, a discernible pattern is observable for the expressions for adjusted variance for terminal streams. Note that the element in the third row of the solution matrix pertains to the adjusted variance for the interconnecting stream in Figure 23 (i.e. stream *s3(1,2)*) and exhibits a different formulation compared to the terminal streams.

Taking stream *s1(1)* as an example representing all terminal streams (the element in the first row of the solution matrix), the expression for adjusted variance for this stream is as follows:

$$\sigma^2_{s1(1)}\Big|_a = \sigma^2_{s1(1)}\left(1 - \frac{\sigma^2_{s1(1)}\sum_3^5 \sigma^2_{sm(2)}}{\sum_3^5 \sigma^2_{sm(2)}\sum_1^3 \sigma^2_{sm(1)} - \left(\sigma^2_{s3(1,2)}\right)^2}\right) \qquad \textit{(Equation 6.26)}$$

The variance reduction ratio can be written as,

$$\frac{\sigma^2_{s1(1)}\Big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)}\sum_3^5 \sigma^2_{sm(2)}}{\sum_3^5 \sigma^2_{sm(2)}\sum_1^3 \sigma^2_{sm(1)} - \left(\sigma^2_{s3(1,2)}\right)^2}\right) \qquad \textit{(Equation 6.27)}$$

The summed factors in Equation 6.27 represent the total variance associated with each node in the circuit. These can be denoted by M_1 and M_2 for the first and second node respectively. Equation 6.27 can then be rewritten as:

$$\frac{\sigma^2_{s1(1)}\Big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)} M_2}{M_1 M_2 - \left(\sigma^2_{s3(1,2)}\right)^2}\right) \qquad \textit{(Equation 6.28)}$$

If a recycle stream further interconnects the two nodes in Figure 23 as shown in Figure 24, Equation 6.28 retains the same format after solving for Σ_a save for a change to the second term in the denominator on the right hand side of the equation, i.e.

$$\frac{\sigma^2_{s1(1)}\Big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)} M_2}{M_1 M_2 - \left(\sigma^2_{s3(1,2)} + \sigma^2_{s4(1,2)}\right)^2}\right) \qquad \textit{(Equation 6.29)}$$

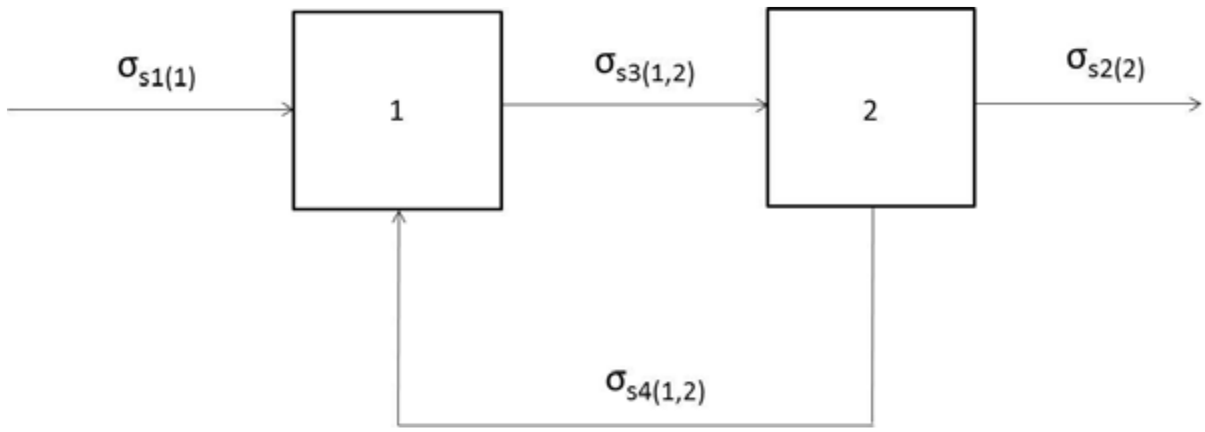


Figure 24: Hypothetical two-node process with recycle stream

The new term gathers the effect of internal streams on the variance reduction ratio and it is found to be common for all terminal streams in a given two-node circuit.

6.2.2.3 Three and four node processes

The form of the variance reduction ratios for all terminal streams in the three-node flowsheet in Figure 25 is represented by the variance reduction ratio for stream *s1(1)* after solving Equation 6.19 for this circuit (Equation 6.30).

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)}\left(M_2M_3 - \left(\sigma^2_{si(2,3)}\right)^2\right)}{M_1M_2M_3 - \left(\left(\sigma^2_{si(1,2)}\right)^2 M_3 + \left(\sigma^2_{si(2,3)}\right)^2 M_1\right)}\right) \qquad \textit{(Equation 6.30)}$$

Where

si = internal stream measurement.

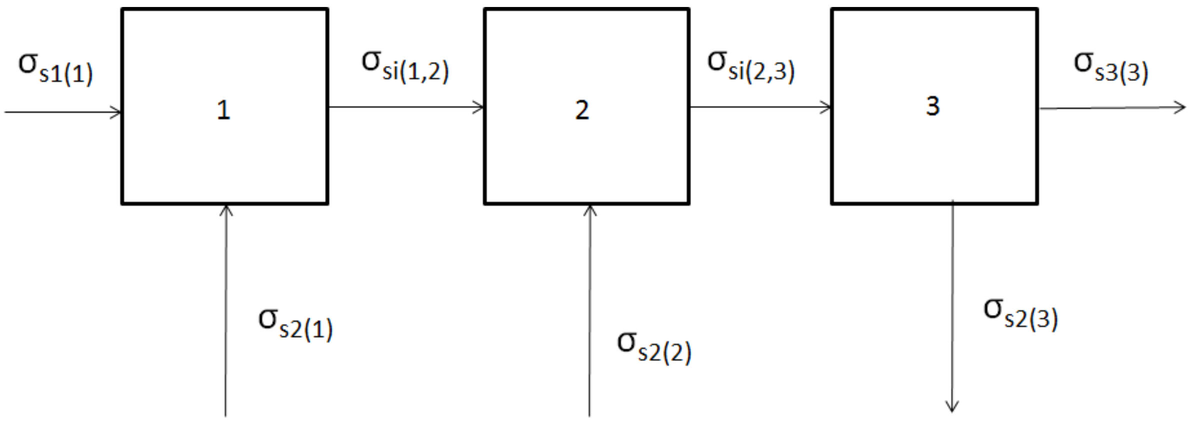


Figure 25: Hypothetical three-node process

Figure 25 contains simple interconnecting streams only. The introduction of a simple recycle stream as shown in Figure 26 increases the complexity of the variance reduction ratio measure as the effects of the additional stream are taken into account after solving Equation 6.19 for the new circuit. The variance reduction ratio measure for stream *s1(1)* is used as an example to illustrate the additional effect of simple interconnections (Equation 6.32).

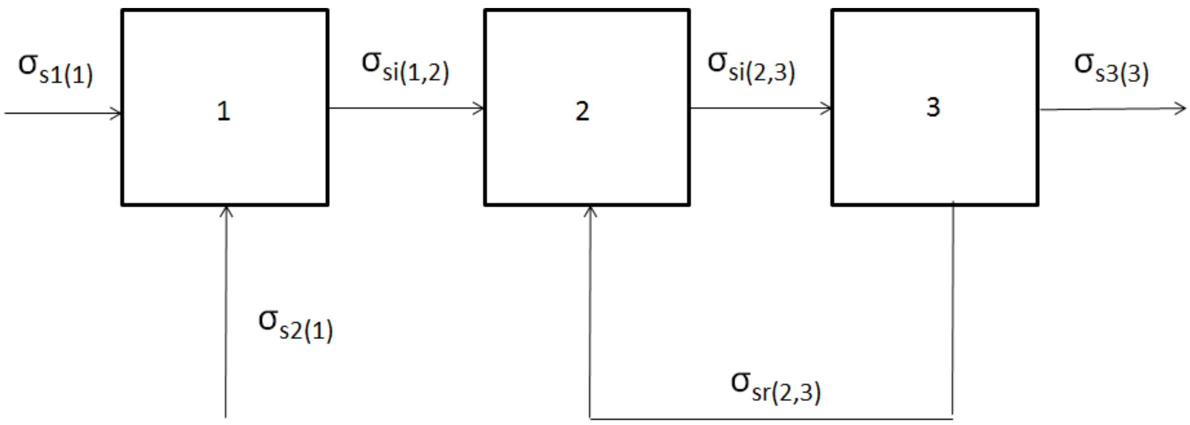


Figure 26: Hypothetical three-node process with simple recycle stream

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)}\left(M_2M_3 - \left(\sigma^2_{si(2,3)} + \sigma^2_{sr(2,3)}\right)^2\right)}{M_1M_2M_3 - \left(\left(\sigma^2_{si(1,2)}\right)^2 M_3 + \left(\sigma^2_{si(2,3)} + \sigma^2_{sr(2,3)}\right)^2 M_1\right)}\right) \qquad \textit{(Equation 6.31)}$$

Where

sr = recycling stream measurement.

The effects of complex interconnecting streams can be tested by introducing stream *sr(1,3)* that joins node 1 and node 3 (Figure 27). Solving Equation 6.19 for stream s1(1) in Figure 27 yields Equation 6.32.

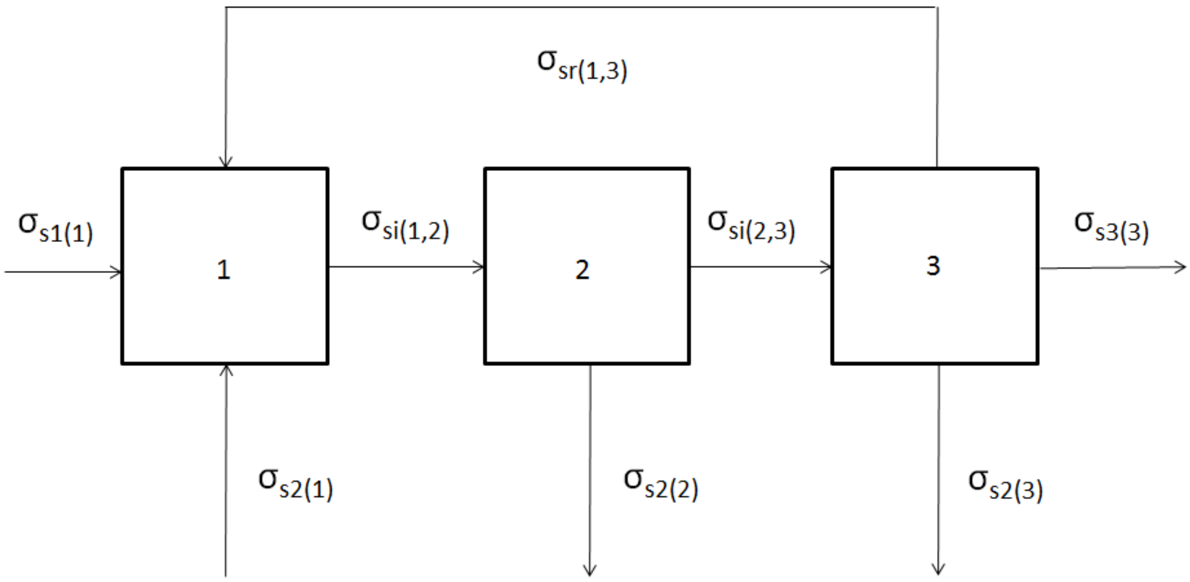


Figure 27: Three-node process with complex recycle stream

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = \left(1 - \frac{\sigma^2_{s1(1)}\left(M_2M_3 - \left(\sigma^2_{si(2,3)}\right)^2\right)}{M_1M_2M_3 - \left(\left(\sigma^2_{si(1,2)}\right)^2 M_3 + \left(\sigma^2_{si(2,3)}\right)^2 M_1 + \left(\sigma^2_{sr(2,3)}\right)^2 M_2\right)}\right) \qquad \textit{(Equation 6.32)}$$

Observation of Equations 6.30 to 6.32 shows consistent structures of the variance reduction ratio term emerge.

Firstly, in the numerator term the squared sum of the internal stream variances not linked to the parent node of the stream under observation is subtracted from the product of the summed variances of the respective nodes other than the parent node of the stream under observation.

Notably, the numerator is identical for Equations 6.30 and 6.32 because the complex recycle stream is linked to the parent node of stream *s1(1)*.

Secondly, in the denominator terms the summed product of the squared internal stream variances and total variances of nodes remote from the respective internal streams are negated from the product of the total node variances of the entire circuit.

Derivations of the variance reduction ratio for terminal streams in four node systems with simple interconnections yielded the following general formulation for stream *s1(1)* :

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = \left(1 - \sigma^2_{s1(1)} \cdot \frac{\left(\prod_{\forall n \neq n_1} M_n - \sum_{\forall p,q}\left[\left(\sigma^2_{si(p,q)}\right)^2 \prod_{\forall k \notin n,p,q}\left(\sum_{\forall si(k)} \sigma^2_{si(k)}\right)\right]\right)}{\left(\prod_{m=1}^{N} M_m - \sum\left[\left(\sigma^2_{si(p,q)}\right)^2 \prod_{\forall k \notin p,q}\left(\sum_{\forall si(k)} \sigma^2_{si(k)}\right)\right]\right)}\right) \qquad \text{(Equation 6.33)}$$

Where p, q and k represent nodes in the circuit.

It should be noted that Equation 6.33 is a good approximation for simple four node circuits. The introduction of recycle streams leads to ternary, quaternary and higher interactions of interconnected streams in the denominator and numerator terms.

To capture the effects of complex circuits, the following expressions which closely approximate internal stream interactions (Equation 6.34 and Equation 6.35) are deducible from observation of the progressive build-up of the variance reduction ratio formula as the number of nodes is increased (Equations 6.27, 6.30, 6.31 and 6.33).

$$t_{int}^{numerator} = \sum_{\forall Pairs(n_i,n_j) \notin n}\left[\left(\sigma^2_{si(n_i,n_j)}\right)^2 \prod_{\forall n_k \neq n,n_i,n_j} M_{n_k}\right] \qquad \text{(Equation 6.34)}$$

$$t_{int}^{denominator} = \sum_{\forall Pairs(n_i,n_j)}\left[\left(\sigma^2_{si(n_i,n_j)}\right)^2 \prod_{\forall n_k \neq ,n_i,n_j} M_{n_k}\right] \qquad \text{(Equation 6.35)}$$

Where

n_i, n_j = connected pairs of nodes.

The expressions are exact for systems with up to three node structures with simple recycle streams. As will be demonstrated in the following chapter (Chapter 7) using the industrial case study example presented in Chapter 5, for three node circuits with complex recycle streams, four node circuits and higher systems the expressions offer a good estimation of the variance reduction ratio measure.

The accuracy of the calculation diminishes with increasing circuit complexity as a result of ternary, quaternary and higher order internal stream interactions that are not accounted for in Equation 6.34 and Equation 6.35.

6.2.2.4 Multi-node process – general case

After solving Equation 6.19 for several flowsheets involving one, two and multiple nodes with simple to complex recycle structures, a generalised formula (Equation 6.36) for determining the variance reduction ratio of any terminal stream in a process network after data reconciliation is derived.

$$\frac{\sigma^2_{s1(1)}\big|_a}{\sigma^2_{s1(1)}} = 1 - \sigma^2_{s1(1)} \cdot \frac{\left(\prod_{\forall n \neq n_1} M_n - t_{int}^{numerator}\right)}{\left(\prod_{m=1}^{N} M_n - t_{int}^{denominator}\right)} \qquad \textit{(Equation 6.36)}$$

Here, the reconciled or adjusted variance ($\sigma^2_{s1(1)}\big|_a$) of an observed terminal stream (attached to node n1) is expressed as a fraction of its measured variance, $\sigma^2_{s1(1)}$. As mentioned earlier, M_{n1} is the sum of variances of all streams associated with a node *n1* in the given circuit, *N* is the total number of nodes in the circuit and the *t* terms are measures that gather network 'stream effects' on the numerator and denominator expressions of the quotient term in the equation as described for Equations 6.34 and 6.35.

6.3 Guidelines for minimising terminal stream variance

Observations made from the inspection of Equations 6.34 – 6.36 based on single to multiple node examples characterised by simple to high connectivity structures resulted in the development of simple design heuristics for minimising measurement variance for terminal streams through data reconciliation.

The full set of heuristics derived in this work is presented in this section for reference. The heuristics are classified in terms of the influence of stream and flowsheet characteristics on the variance reduction experienced by terminal stream measurements after data reconciliation.

6.3.1 Heuristics based on stream characteristics

(a) Rule S1. Measured variance: a high/good reduction in variance will be obtained if the stream's measured variance is high with respect to other measured streams in general.

(b) Rule S2. Direction of flow: the reduction in variance is independent of whether streams enter or leaves nodes.

(c) Rule S3. Stream type: a higher/better reduction in variance will be obtained if interacting (interconnecting) streams are measured precisely than if other terminal streams are measured precisely, i.e. interacting streams have a higher weighting.

(d) Rule S4. Location: a higher/better reduction in variance will be obtained in terminal streams which are near highly interconnected regions in the flowsheet than in terminal streams 'upstream or downstream' from the interconnected regions.

6.3.2 Heuristics based on flowsheet characteristics

(a) Rule F1. Stream/node ratio: a high/good reduction in variance will be obtained in flowsheets with a high ratio of interacting streams to nodes. This should not be confused with the principle for single node (black box) flowsheets where a high/good reduction corresponds to a low ratio of streams to 'the node' as these are all terminal streams.

(b) Rule F2. Interconnections: a higher/better reduction in variance will be obtained in highly interconnected flowsheets than in simpler flowsheets, even with equivalent ratios of interacting streams to nodes.

.

6.4 Summary

This chapter described the analytical basis of the mathematical heuristics derived in this study. The heuristics are based on the generalised equation for variance reduction of terminal streams through data reconciliation. The generalised equation was obtained from the symbolic manipulation of the general linear steady state solution applied on single to multi-node hypothetical process networks. Particular attention was paid to precision improvement of terminal streams owing to their custodial importance in material handling operations and to applications such as metal accounting. The mathematical derivation resulted in heuristics based on stream and flowsheet characteristics. The following chapter validates the heuristics based on data obtained from the case study presented in Chapter 5.

Chapter 7

NUMERICAL VALIDATION OF MATHEMATICAL HEURISTICS FOR PRECISE MEASUREMENT NETWORK DESIGN

Three sources of heuristics for obtaining suitable measurements for metal accounting have been presented in this work. Firstly, the Code provided recommendations on what needs to be done in order to obtain credible and accurate measurements for metal accounting purposes; and secondly, the prevailing practices in the minerals beneficiation industry showed what practitioners regard as suitable attributes of metal accounting measurements. Both sets of heuristics were tested using an audit conducted on the case study flowsheet presented in Chapter 5. The mathematical heuristics derived in Chapter 6 constitute the third source of heuristics. Mathematical heuristics advocated the direct and precise measurement of internal streams in order to maximise precision of terminal streams through data reconciliation. This chapter presents a numerical study aimed at testing the mathematical heuristics using data from the case study presented in Chapter 5.

7.1 Overview

Figure 28 shows a flow diagram highlighting the activities performed in the numerical validation study. The study required a large number of flowsheets to provide sufficient data to conduct a numerical based analysis. Starting with the case study flowsheet as a basis a total of 858 unique flowsheets that conserve the base flowsheet terminal stream structure were generated. A thousand random data sets per flowsheet were generated using Monte Carlo simulation on a consistent set data from the MSP and error models determined for the flowsheet to produce 858 000 inconsistent data sets.

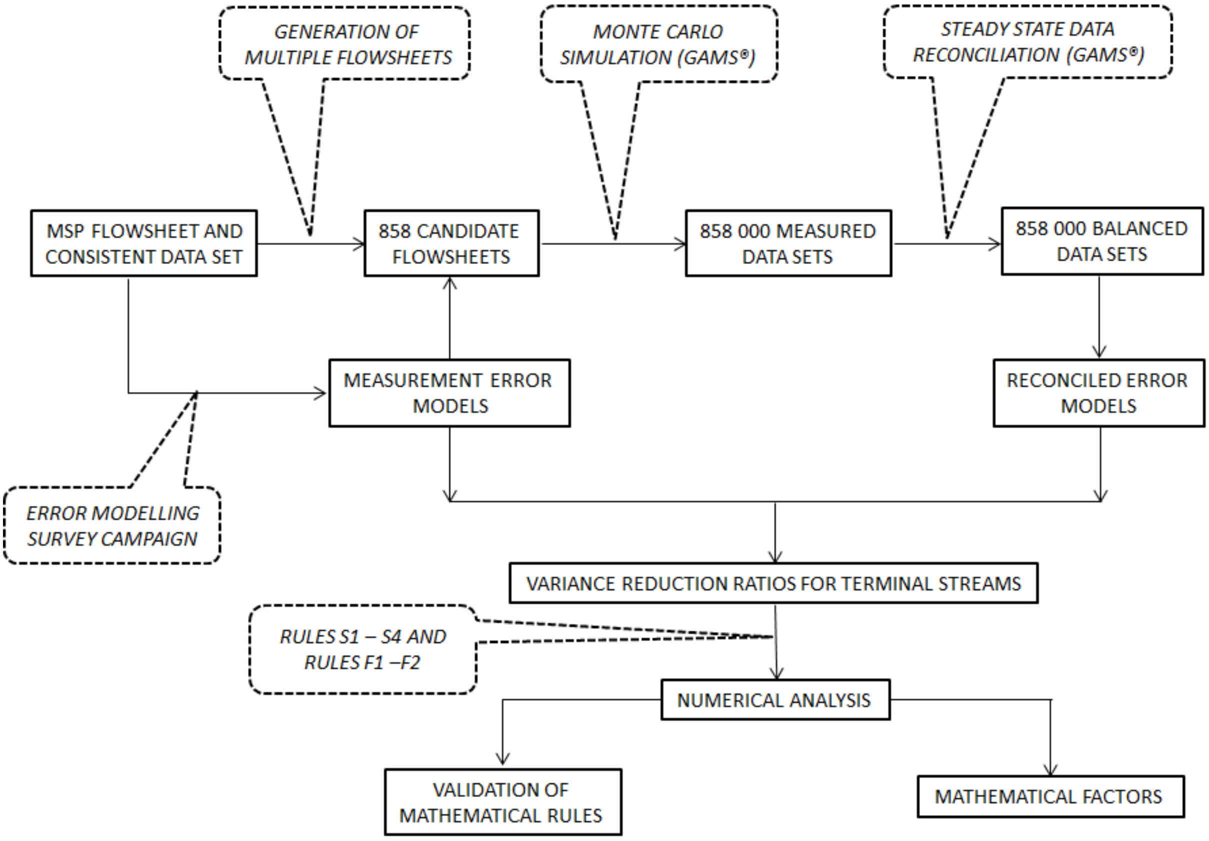


Figure 28 Overview of numerical study

Generation of the data sets and subsequent re-balancing of the resultant inconsistent balances were done using the COINIPOPT non-linear solver in GAMS®. After least squares based data reconciliation of the simulated data sets, reconciled error models were calculated from the distributions of the balanced data. The reconciled variances were then expressed as ratios of corresponding measured variances to produce variance reduction coefficients for all data points on the respective flowsheet configurations. Mathematical rules developed in this work were tested using this information. One of the outcomes of the test process was the emergence of some flowsheet parameter based factors which exhibited significant prediction capabilities of variance reduction for terminal measurements through data reconciliation. The following sections describe the numerical validation process in more detail.

7.2 Generation of candidate flowsheets

Starting with the case study flowsheet as a basis, numerous flowsheets were generated by systematically deleting different combinations of internal streams from the base flowsheet configuration. A total of 858 unique flowsheets that conserve the base flowsheet terminal stream structure were generated. This approach is similar to the 'reduced balance scheme' method where unmeasured streams are eliminated by merging adjacent units or by deleting nodes entirely serviced by unmeasured streams (Václavek, 1969). Recent approaches include matrix projection method (Crowe et al., 1983) and QR decomposition techniques (Sánchez & Romagnoli, 1996) used to ensure redundancy in networks containing non-redundant variables in data reconciliation-based bias detection procedures. In the current work however, all streams are measured and only internal streams are considered for deletion. Nodes linked by deleted streams are merged into new 'composite nodes'. The resultant networks always contain redundant measurements as a result of their origination from the base flowsheet with all streams measured.

One of the three node systems derived in this work will be used to illustrate the method used to generate the multiple flowsheets (c.f. Figure 29). In the general three-node case, the problem generating new flowsheet configurations simply reduces to finding the number of ways of constructing three node flow sheets from the base seven-node flowsheet without duplication.

Firstly, one may combine three nodes by eliminating all common internal streams. Secondly, two nodes from the remaining four nodes may be merged to form a second composite node while the remaining two nodes form the third node. This example represents a typical 'feasible transition' from the base flow sheet and it is referred to as an *N3_223* system configuration i.e. a three-node flowsheet ("*N3_*") comprised of two nodes each having two original nodes each ("*_22*") followed by a third composite node consisting of three original nodes ("*3*"). The number of candidate networks in a particular system configuration was determined by using statistical counting rules that describe the number of ways of choosing, for example k combinations (composite nodes in this case) of sizes $n_1, n_2, \ldots, n_k$ from a given set of n objects (where, $n=\sum n_k$). Equation 7.1 summarises this in the form of an appropriately designed multinomial coefficient. The parameter q caters for composite nodes that combine identical numbers of original nodes. This serves to prune permutations of identical network combinations.

$$S = \binom{n}{n_1 n_2 \ldots n_k} \frac{1}{q!}$$ ……………………………................................ Equation 7.1

In the *N3_223* combination system example, $q=2$ (two merged nodes consisting of two original nodes each), $n_1 = 3$, $n_2 = n_3 = 2$, $k=3$, and consequently $\sum n_k = 7$. Hence from Equation 7.1, a total of 105 flow sheets consisting of three nodes each can be generated for this combination system alone. Figure 29 gives an example of one of the 105 unique flow sheets generated through the *N3_223* combination system. Here, the transition eliminates three internal streams, namely *s3*, *s9* and *s11*. Similar procedures for other combination systems yielded results that are listed in Table 32.

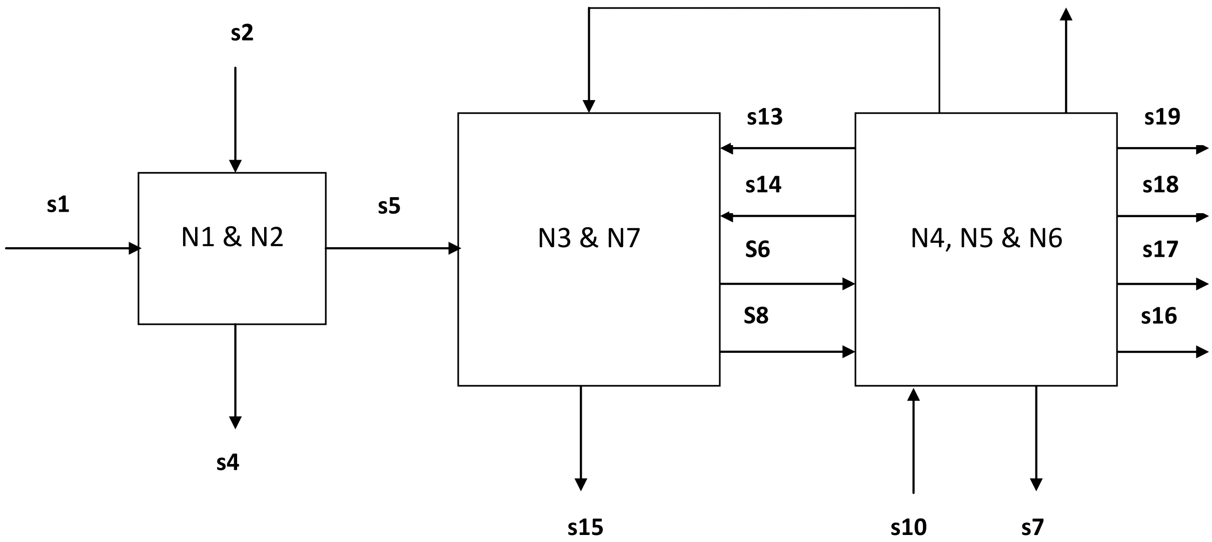


Figure 29: Example of a three node flowsheet (N3_223 system)

The second column in Table 32 lists the numbers of original nodes comprising a composite node for the respective system combination. The full candidate space, *S*, comprises 858 unique flow sheets. However, as can be seen in columns 1 and 2 in the table, the *N3_223* system is only one of four ways of forming three merged nodes from the original seven nodes. The same procedure applies to the remaining three node system combinations.

Table 32: A summary of the combinations of multi-node networks generated

Combination System	Nodal Combinations	Number of nodes	Number of flow sheets
N1	7	1	1
N2_16	1 & 6	2	7
N2_25	2 & 5	2	21
N2_34	3 & 4	2	16
N3_115	1,1 & 5	3	21
N3_124	1,2 & 4	3	105
N3_133	1,3 & 3	3	70
N3_223	2,2 & 3	3	105
N4_1114	1,1,1 & 4	4	35
N4_1123	1,1,2 & 3	4	210
N4_1222	1,2 ,2 & 2	4	105
N5_11113	1,1,1,1 & 3	5	35
N5_11122	1,1,1,2 & 2	5	105
N6_1111112	1,1,1,1,1,1 &2	6	21
N7	Original flow sheet	7	1
TOTAL			858

It is important to note that each flowsheet presents a unique balance instance as can be deduced from Equation 6.15 where the reconciled flow rates and precisions depend on flowsheet connectivity (*A* matrix) and the corresponding structure of the variance-covariance matrix (∑).

Prior to data reconciliation, the nominal flow rates were perturbed according to their respective relative standard deviations, determined from the Namakwa Sands MSP error modelling campaign, by using the Monte Carlo simulation technique. The perturbed values represent ‘measured values’. A total of 1000 experimental data points were simulated per measurement. This resulted in a set of 1000 ‘realistic’ balances which do not close exactly due to induced measurement error. Experimental precisions for each flow variable were determined from the resultant distributions.

The distributions were assumed to be Gaussian as well as independent, hence in-stream and cross-stream covariance values were not considered. The assumption of Gaussian error distributions is relatively uncontroversial. The assumption of uncorrelated measurements is routinely made in material flow circuits. This assumption is untrue in general as measurements are always correlated to some extent but inclusion of covariances does not greatly influence the quality of data reconciliation (Hodouin *et al.*, 1998). This simplifying assumption ensures that the variance-covariance matrix (Σ in Equation 6.15) is diagonal, hence off-diagonal co-variance terms are assumed to be zero. It should be noted that although the variance-covariance matrix changes with each flowsheet configuration, the terminal flow structure is conserved in all configurations since only internal streams are deleted during network transitions.

Re-balancing each of the 1000 data sets in the weighted least squares sense (cf. Equation 6.1 & 6.2) resulted in distributions of balanced/adjusted data from which adjusted precisions were determined. While adjusted precisions can be calculated from theory (cf. Equation 6.19) the Monte Carlo approach leads to the same result when comparing adjusted and measure precisions, although it is not entirely necessary (requires more computational capacity) because the weighted least squares problem is essentially linear. The numerical route was chosen because the Taylor Series based solution results in conservative estimates of measured precisions (Xiao & Vien, 2003). Repeating this procedure for each flowsheet configuration in the candidate space resulted in a similar sample space of reconciled data.

7.3 Investigation of heuristics based on stream characteristics

7.3.1 Rule S1 - The effect of measured variance ($\sigma^2_{m(n1)}$)

To demonstrate the effect of variance magnitude for the general case, Figure 30 shows a graph of variance reduction ratios for the terminal streams *s1*, *s4*, *s7* and *s15* for all flowsheets in *S*. The streams adequately cover the entire range of measured variances presented by the test flowsheet.

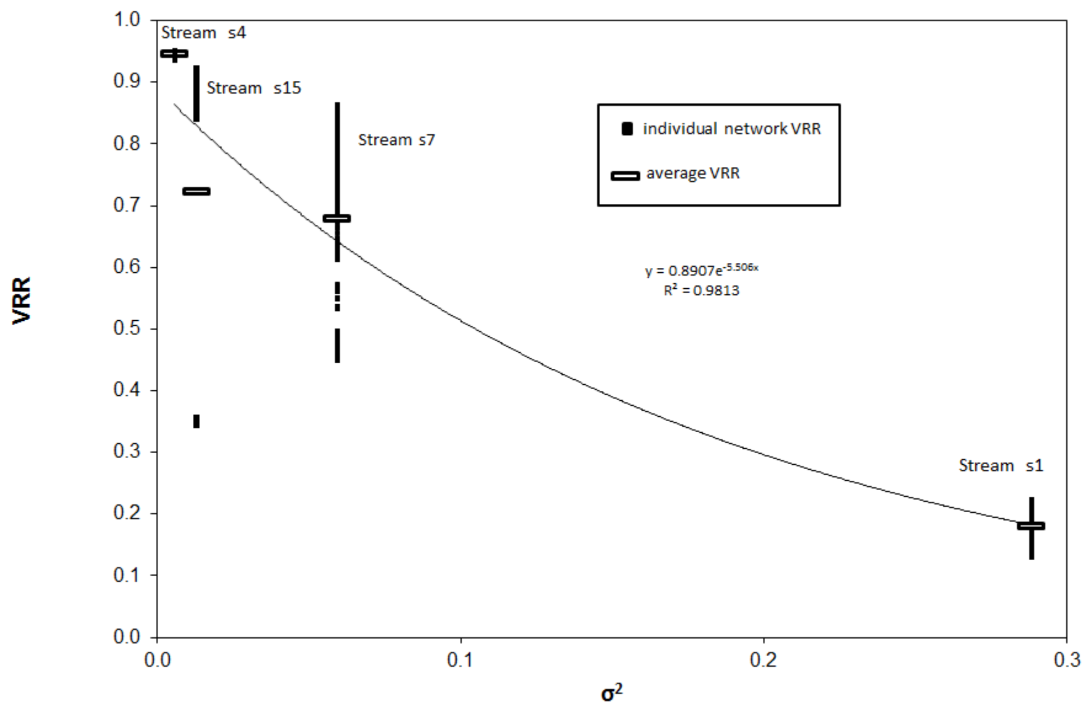


Figure 30: The relationship between measured variance and variance reduction ratio

Figure 30 also plots the relationship between the average variance reduction ratio values and measured variances for the four streams. The variance reduction ratio values for streams *s7* and *s15* can be seen to vary over a wider range compared to stream *s4* and stream *s1*, which are measured with the lowest and highest variances respectively. The average variance reduction ratio values show a trend (plotted) that implies an inverse relationship between variance reduction and measured variance as implied by *Rule S1*: "Measured variance: a high/good reduction in variance will be obtained if the stream's measured variance is high with respect to other measured streams in general" (ref. Section 6.3.1).

7.3.2 Rule S2 - The effect of direction of flow

This rule is self-evident in that measurement variances are directionless quantities (scalar). It is clear from observing Equation 6.16 and Equation 6.20 that estimation of the expected reduction in variance due to data reconciliation involves only variance quantities which are not reliant on flow direction.

7.3.3 Rule S3 - The effect of stream type

Rule S3 states that "a higher/better reduction in variance will be obtained if interacting (interconnecting) streams are measured precisely than if other terminal streams are measured precisely, i.e. interacting streams have a higher weighting". For complex networks this rule is best explained by highlighting a key flowsheet factor referred in this study as the 'stream to node ratio' (i.e. $\sigma^2_{m(n1)}/Mn1$). This factor is derived from the generalised variance reduction equation for terminal streams (Equation 6.36).

7.3.3.1 The stream to node ratio measure ($\sigma^2_{m(n1)}/Mn1$)

The stream to node ratio measure (SNR) is obtained by rearranging Equation 6.36 in order to yield Equation 7.2. In the single node case, Equation 7.2 simplifies to the general equation for single node systems (Equation 6.24) where the *t* terms disappear since there are no internal streams and the quotient term on the right hand side of the equation becomes $1/Mn_1$. Inspection of Equation 6.24 shows that an inverse relationship exists between the extent of variance reduction and the SNR as alluded to by *Rule S1*.

$$\frac{\sigma^2_{s1(n1)}\big|_a}{\sigma^2_{s1(n1)}} = 1 - \left(\frac{\sigma^2_{s1(n1)}}{M_{n1}}\right)\left[\frac{\prod_{\forall n \neq n_1} M_n - t_{int}^{numerator}}{\prod_{\forall n \neq n_1} M_n - \frac{t_{int}^{denominator}}{M_{n1}}}\right] \qquad \textit{(Equation 7.2)}$$

However, Equation 7.2 suggests that this relationship persists even for individual nodes in multi-node configurations. Thus a change in the total variance associated with a terminal node is expected to alter the extent of variance reduction of attached terminal streams. Reducing the variance of internal streams attached to the observed node will invariably increase the SNR for all terminal streams attached to the node, leading to an increase in the extent of variance reduction experienced after data reconciliation.

7.3.3.2 Reducing total terminal node variances

To illustrate this, Figure 31 shows the relationship between variance reduction ratio and stream to node ratio values for streams *s1*, *s4*, *s7*, *s15* for all 858 flowsheets in *S*. A trend of the plot of the average terminal variance reduction ratios (y-axis) with respect to the combined terminal stream to node ratio values (x-axis) for every flowsheet in *S* is also plotted. It is important to note here that there are two ways that the SNR for terminal streams can be altered in *S*; this can be achieved by either increasing/decreasing the number and/or the magnitude of variances of streams attached to its parent-node. Both events resulted in the abscissae values of the graph shown in Figure 31.

Increasing internal stream precisions invariably lead to a reduction in associated total terminal node variances resulting in increasing SNR values for attached terminal streams. It follows therefore that if internal streams attached to an observed terminal node are measured more precisely, the total variance associated with the observed node decreases, thereby increasing the value of the SNR for the terminal stream(s) attached.

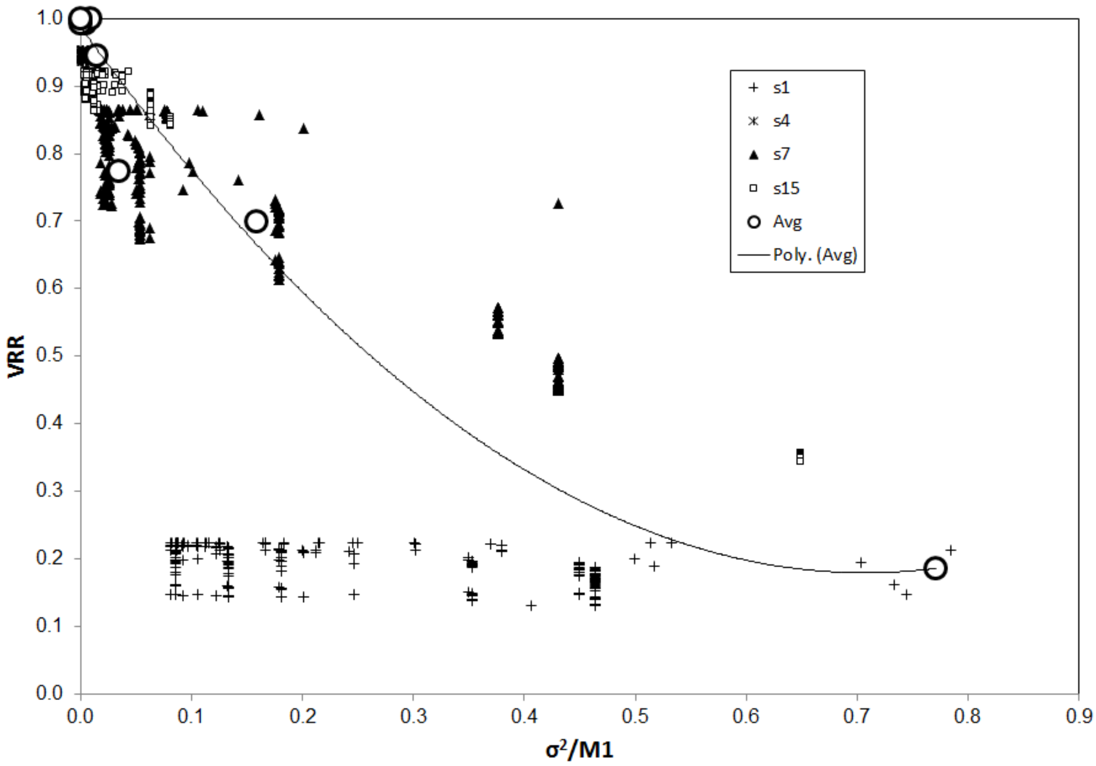


Figure 31: The relationship between stream to node ratio and variance reduction ratios for streams s1, s4, s7, s15 as well as the average for all terminal streams

The plot in Figure 31 shows an inverse relationship between SNR and variance reduction ratio for streams *s7* and *s15* as well as the average trend for all terminal streams. However streams *s1* and *s4* variance reductions appear insensitive to changes in the SNR measure. It is relevant to note here that while stream *s1* variance is relatively large and stream *s4* measured variance is small in comparison; streams *s7* and *s15* variances lie midway between these extremes. Given this, and the direct relationship between variance reduction ratio and SNR (Equation 7.2), variance reductions for streams *s7* and *s15* are expected to vary over a wider range of values compared to streams *s1* and *s4* in the closed set *S*. Hence the respective variance reduction ratio values for streams *s1* and *s4* can be observed to vary over a relatively narrow range i.e. 0.13 to 0.23 for stream *s1*, and 0.93 to 0.96 for stream *s4*, compared to ranges of 0.45 to 0.89 for stream *s7* and 0.34 to 0.95 for stream *s15* (ref. Figure 30 and Figure 31).

7.3.3.3 Effectiveness of the SNR measure

The efficacy of the SNR factor can be investigated further by observing the distribution of variance reduction values for *s15* and *s7* in Figure 31. In this graph, stream *s15* shows two distinct groupings of variance reduction ratio values while stream *s7* appears more evenly distributed across the range of possible SNR values in *S*. Stream *s7* measured variance is approximately five times in magnitude compared to that of stream *s15* (ref. Table 28). According to *Rule S1*, one would expect the set of stream *s7* variance reduction ratio values to show a lower average value (higher reduction in variance) than stream *s15*. This is in fact the case as is evident in Figure 30. It is reasonable to expect the set of stream *s7* variance reduction ratio values to have a smaller lower bound compared to that of stream *s15* for similar reasons. However, as shown in Figure 31, stream *s15* achieves a smaller lower bound of 0.34 compared to 0.45 for stream *s7*; and stream *s15* achieves this at peak SNR values which occur at a significantly higher level than that of stream *s7*.

In general, an assessment of the combined variance reduction ratios for all terminal streams per flowsheet is useful in indicating the universal effectiveness of the SNR measure. Figure 32 shows the average variance reduction ratio values for all terminal streams for every flowsheet plotted against the corresponding entire network SNR average values. In general, a consistent inverse relationship is apparent. However, it can be observed in this figure as well as in Figure 31 that SNR values account for multiple values of variance reduction ratios indicating that there are other factors besides measured variance and SNR that have a significant influence on the reduction experienced by terminal streams. The next section investigates the impact of relative positioning of terminal streams in a measurement network.

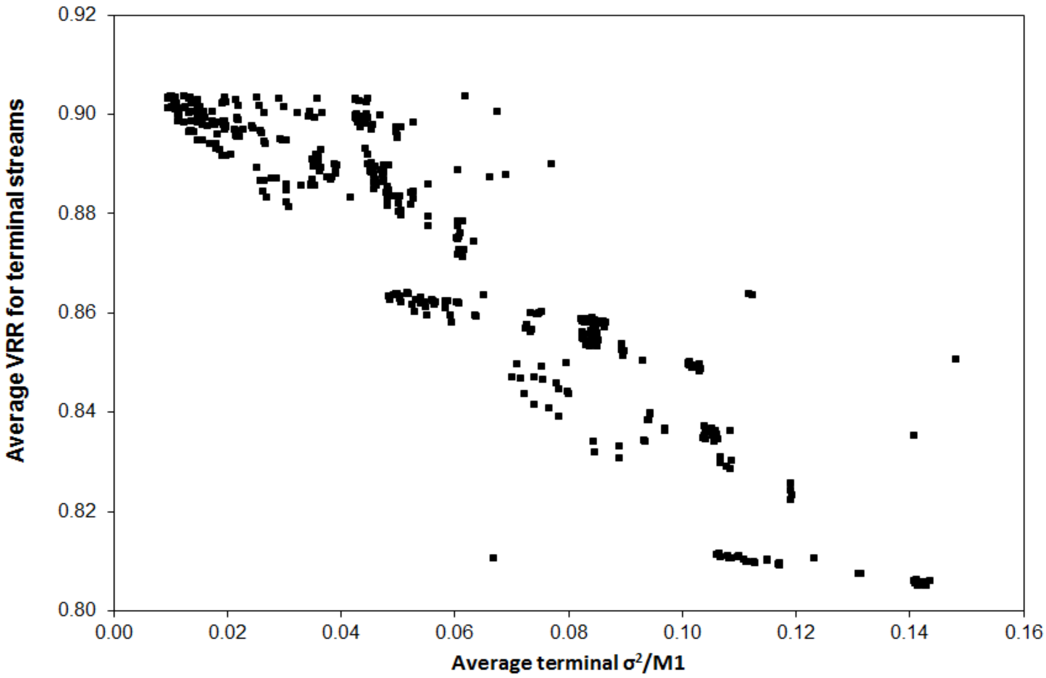


Figure 32: The relationship between flowsheet average SNR and average terminal VRR

7.3.4 Rule S4 - The effect of stream location

Rule S4 states that "a higher/better reduction in variance will be obtained in terminal streams which are near highly interconnected regions in the flowsheet than in terminal streams 'upstream or downstream' from the interconnected regions". The $t^{numerator}$ and $t^{denominator}$ terms (Equation 6.34 and Equation 6.35 respectively) gather the effects of stream interactions on the extent of variance reduction for terminal streams through data reconciliation (ref. Equation 7.2).

7.3.4.1 A measure of stream location – the $t^{numerator}$ term

The $t^{numerator}$ term expresses the level of interconnectivity associated with regions distant from observed terminals nodes; while the $t^{denominator}$ term gives a measure of stream interactions associated with the entire flowsheet. As a result, the $t^{numerator}$ term can be used to rank the impact of interconnectivity on specific sites in a given measurement network. Low values of

$t^{numerator}$ are expected to improve variance reduction for observed terminal streams (ref. Equation 7.2).

7.3.4.2 The effect of terminal stream location

Figure 33 shows a schematic of a three node flowsheet in which the SNR for streams *s7* and *s15* are the same (see *Flowsheet 1* data in Table 33). An inspection of the flowsheet structure shows that streams *s5*, *s8* and *s11* contribute to the $t^{numerator}$ value for stream *s15* while only stream *s3* contributes to that of stream *s7* (ref. Equation 6.34).

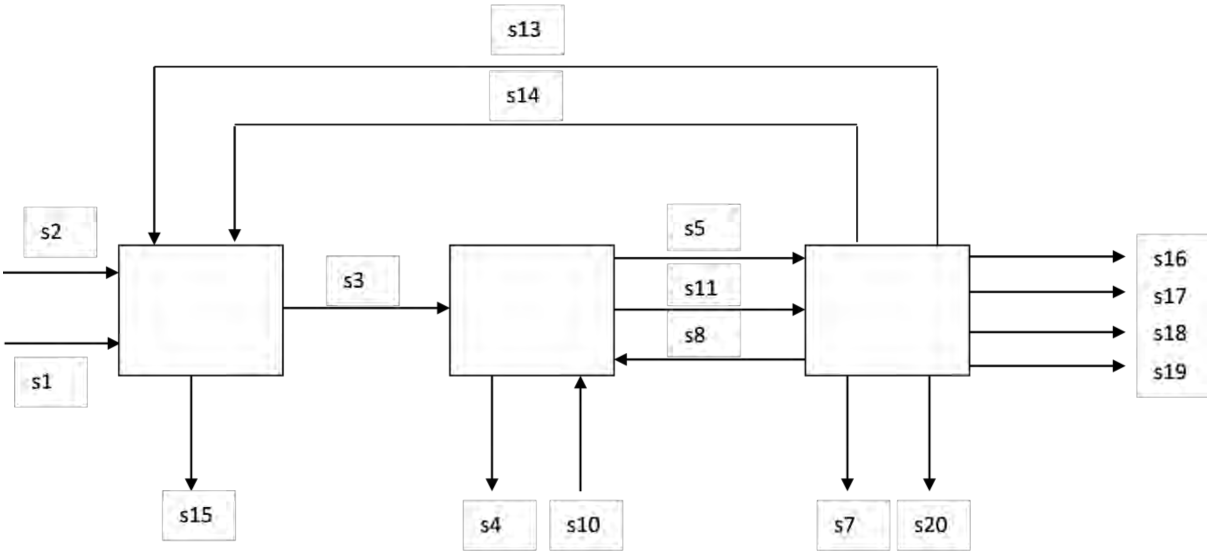


Figure 33: Flowsheet No. 1 in Table 33 showing the relative positions of streams s7 and s15

Thus, using $t^{numerator}$ as a proxy, stream *s7* is adjudged to be located in a more 'interconnected' region of the flowsheet compared to stream *s15*. Therefore stream *s7* is expected to experience a higher reduction in variance. The current study explicitly uses $t^{numerator}$ as a proxy for quantifying regional interconnectivity within a given measurement network. In *Flowsheet 1*, the $t^{numerator}$ value for *s7* is relatively low at 0.1102 and that of stream *s15* is higher at 8.0148. The variance reduction ratio values for streams *s7* and *s15* are 0.734 and 0.920 respectively. However observations of *Flowsheets 1-3* (Table 33) indicate that the influence of $t^{numerator}$ on variance reduction ratio is non-linear. This can be concluded independently from observing

Equation 7.2. In this sense, the $t^{numerator}$ factor essentially serves as an indicator of the tendency of a given location to influence variance reduction based on stream connectivity.

Table 33: Stream data for streams s7 and s15

Flowsheet	Nn	N_s	N_n/N_s	Stream 7 data			Stream 15 data		
				VRR	σ^2/M_1	$t^{numerator}$	VRR	σ^2/M_1	$t^{numerator}$
1	3	17	0.176	0.734	0.020	0.1102	0.920	0.020	8.0148
2	4	19	0.211	0.727	0.022	0.1276	0.920	0.020	4.9560
3	5	19	0.263	0.727	0.022	0.0658	0.920	0.020	1.1187
4	3	19	0.158	0.771	0.022	0.2601	0.894	0.005	0.2601
5	4	18	0.222	0.566	0.376	0.7458	0.848	0.081	0.7458
6	5	20	0.250	0.536	0.376	0.6105	0.843	0.081	0.6105
7	3	19	0.158	0.809	0.025	0.6476	0.907	0.005	0.1110
8	4	19	0.210	0.804	0.025	0.0130	0.353	0.649	7.3903
9	5	19	0.263	0.803	0.022	0.0005	0.349	0.649	1.5985

Flowsheets 2 and *3* illustrate the same observation albeit for higher node structures. It is important to note that as is the case in *Flowsheet 1* all other factors are the same for both streams since they are incident on the same flowsheets. *Flowsheets 4, 5* and *6* are included in Table 33 as controls showing that when $t^{numerator}$ values are equal (situated on the same node), the extent of variance reduction is largely dependent on the SNR factor.

The precedence of the SNR factor over the $t^{numerator}$ measure in determining variance reduction ratio is evident in *Flowsheets 7-9*. For instance in *Flowsheet 7* stream *s7* experiences a higher reduction in variance than stream *s15* apparently as a result of a higher SNR value, even though its $t^{numerator}$ value (which is higher than that of stream *s15*) puts it at a disadvantage with respect to the extent of variance reduction achieved due to stream location. For *Flowsheets 8* and *9*, the variance reduction ratio values for both streams appear insensitive to their respective $t^{numerator}$ quantities as they show a strong correlation with SNR values.

Results generally indicate that stream effects measures have reduced influence on variance reduction ratios particularly in higher node flowsheets. An example of this is *Flowsheet 8* in Table 33 where despite the $t^{numerator}$ value for *s7* being over 3000 times higher than that of stream *s7* while the SNR value for *s15* is only some 30 times higher, stream *s15* experiences over twice the variance reduction compared to stream *s7*.

It can be hypothesised that as the number of nodes increases, the SNR factor increases and stream effects decrease in magnitude. Figure 34 shows the variation of stream effects, terminal stream variance to total node variance ratio (SNR) and variance reduction ratios (VRR) with the number of nodes for stream *s15*. The values plotted here are averages from flowsheets with the same number of nodes. The graphs show that effects decrease almost exponentially as the number of nodes increase. It is important to take note of the direct correlation of variance reduction ratio with SNR for three-node and higher node networks.

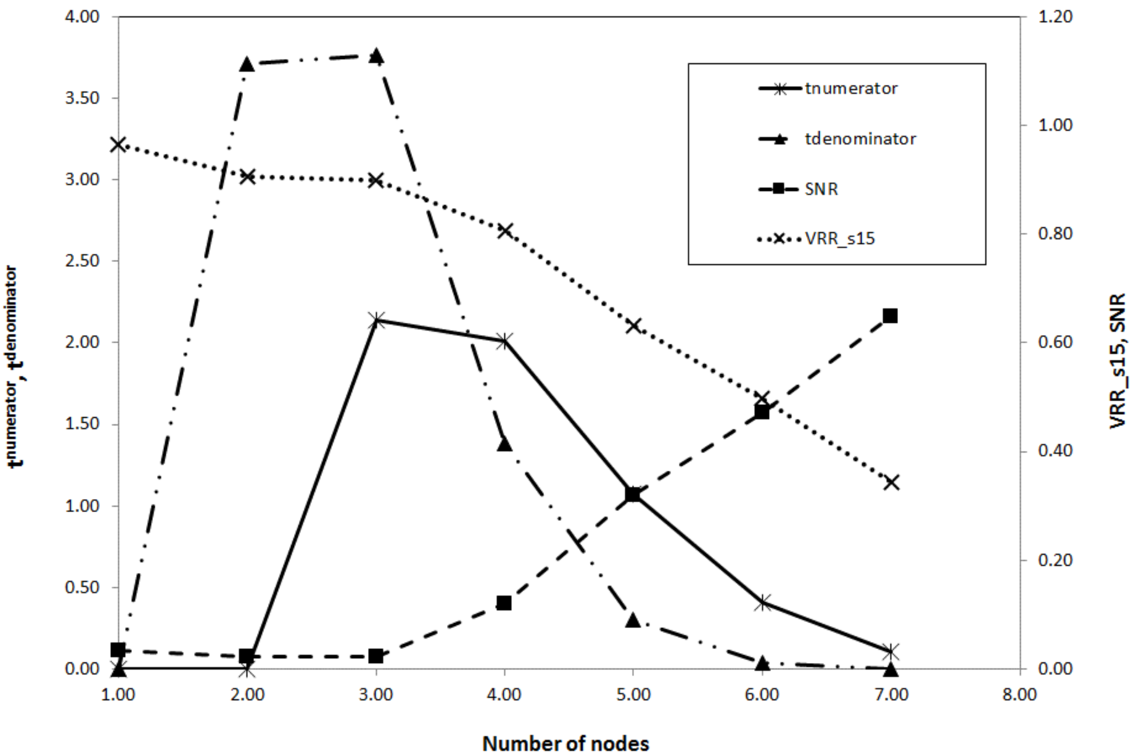


Figure 34: Graphs showing the variation of stream effects, terminal stream variance to total node variance ratio (SNR) and variance reduction ratios (VRR) for stream s15 with the number of nodes

7.4 Investigation of heuristics based on flowsheet characteristics

The two most common descriptors of flowsheet structure are the number of nodes and streams in a network. While for entire networks Lyman (2005) has shown that on average variance reduction improves with increasing node to stream ratios, *Rules S1* to *S4* demonstrate that the effects on individual streams vary depending on the factors proposed and discussed in this study.

The case study data attests to this as shown in Figure 35. In this graph, the variance reduction ratio (VRR) is plotted against the ratio of nodes to streams in accordance with Lyman's finding. Although the equation relates the effect of node to stream ratio on average network variance reduction, corresponding plots of variance reduction ratio values for streams *s1*, *s4*, *s7*, and *s15* show virtually no correlation between the network node to stream ratio values and variance reduction for individual terminal streams after data reconciliation. As a guiding design principle the expression is accurate at the macroscale but is incapable of assisting *a priori* decisions regarding the response of individual measurement sites to the precision improvement effects of data reconciliation.

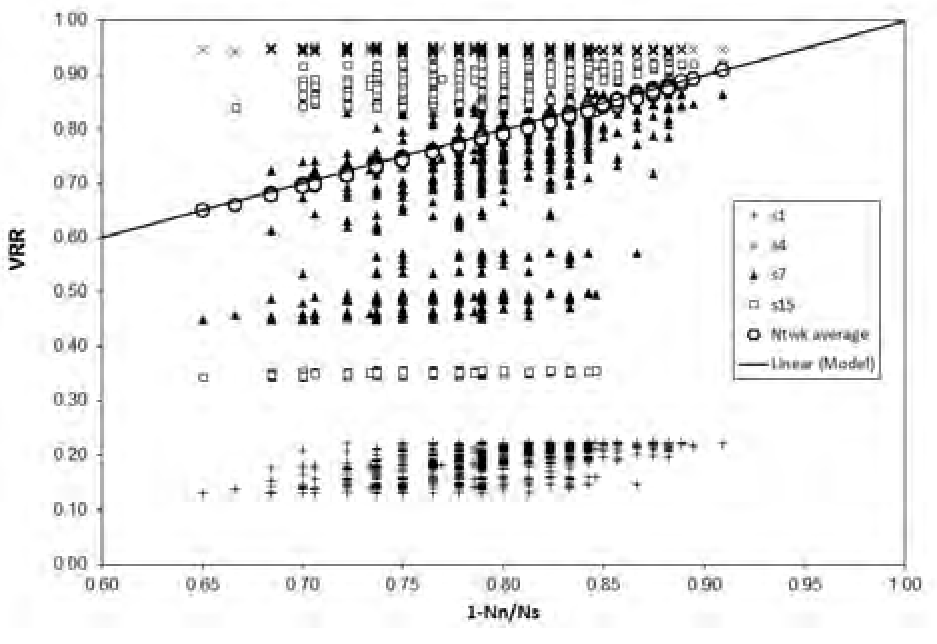


Figure 35: Graph showing the relationship between node to stream ratio and variance reduction ratio for total network average and Streams s1, s4, s7 and s15 as well as predictions from Lyman's (2005) model

With respect to the effects of data reconciliation on individual streams, an important descriptor of the influence of network interconnection which takes into account spatial relationships between nodes and streams is the $t^{denominator}$ term derived from Equation 7.2.

7.4.1 A measure of network interconnection – the $t^{denominator}$ term

The $t^{denominator}$ term gives a measure of stream interactions associated with an entire network (Equation 6.35) and is therefore well suited to describe network interconnectivity based on the number of streams and nodes in a given circuit. The value of $t^{denominator}$ is the same for all terminal streams in a given network but varies across different circuit configurations. By inspection of Equation 7.2, higher values of $t^{denominator}$ are expected to improve variance reduction for circuits where the values of this parameter increase substantially ahead of the magnitude of other parameters in Equation 7.2.

7.4.2 Rule F1 - The effect of internal stream to node ratio

Rule F1 states that "a high/good reduction in variance will be obtained in flowsheets with a high ratio of interacting streams to nodes". For simple circuits higher values of internal stream to node ratios can result in higher reductions in variance for terminal streams based on significant changes in the values of the $t^{denominator}$ factor.

However, in general, increasing the number of internal streams is expected to decrease the stream to node variance ratio (SNR) for terminal streams as discussed in Section 7.3.3, leading to the concomitant deterioration of variance reduction as illustrated in Figure 32. To illustrate the effect of increasing internal stream to node ratio on SNR, Figure 36 shows a plot of the variation of SNR and VRR with changes in the internal stream to node ratio (x-axis).

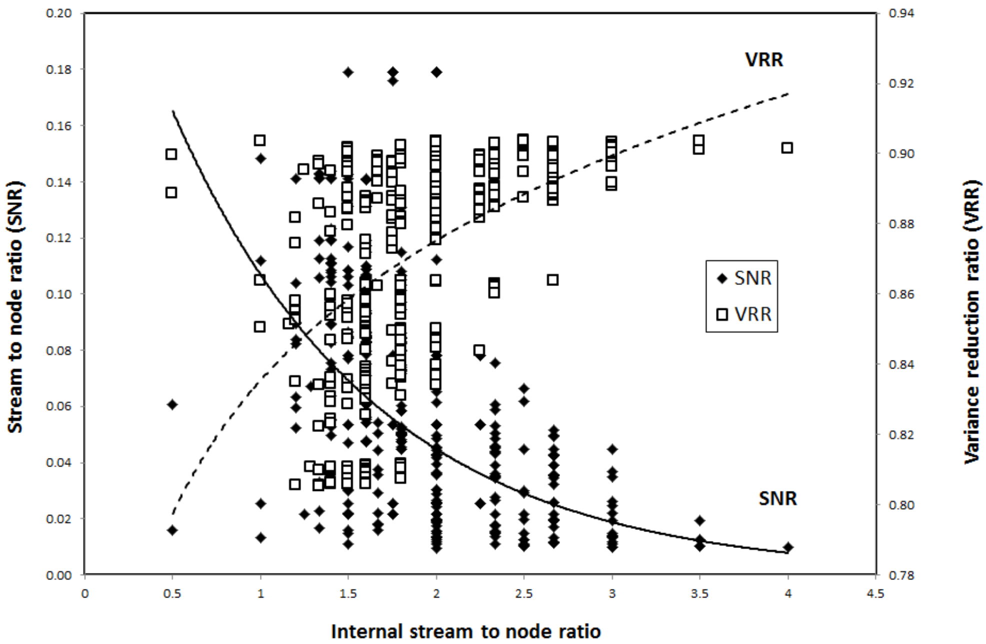


Figure 36: Variation of terminal stream to node ratio and variance reduction ratrio with internal stream to node ratio

The graph shows that SNR decreases with increasing internal stream to node ratio while the average terminal VRR deteriorates (i.e. increases). The direct relationship between these two parameters has been plotted and discussed previously (Figure 32). Hence it is highly probable that the effects of *Rule F1* would be superseded by the higher impact of changes in SNR with changes in the internal stream configurations, although for simpler circuits the rule may hold based the effects of the $t^{denominator}$ factor on VRR.

7.4.3 Rule F2 - The effect of interconnections

Rule F2 states that “a higher/better reduction in variance will be obtained in highly interconnected flowsheets than in simpler flowsheets, even with equivalent ratios of interacting streams to nodes”. Again, the $t^{denominator}$ term best describes the effect of network transitions on individual stream variance reduction through SSDR. As illustrated in the previous section, the effects of *Rule S1*, *S3* or *S4* are likely to dominate although for simpler flowsheets the rule may hold.

7.5 Factors affecting variance reduction for terminal streams

The numerical validation study has resulted in the emergence of factors that exhibit significant influence on the variance reduction for terminal streams through data reconciliation. The magnitude of measured variance ($\sigma^2_{m(n1)}$) and the SNR measure ($\sigma^2_{m(n1)}/Mn1$) significantly predicted variance reduction for terminal streams while the $t^{numerator}$ factor performed weakly in this regard particularly for structures beyond three nodes.

Although the magnitude of measured variance emerged as a good indicator of expected variance reduction through steady-state data reconciliation in general as referred to by *Rule S1,* the study indicated that for multi-node flowsheets, the SNR factor is a more robust predictor of variance reduction than measured variance alone.

In order to test the accuracy of the generalised variance reduction equation, a comparison of variance reduction ratio values as predicted through Equation 7.2 and values obtained from least squares steady-state data reconciliation optimization are plotted on a parity chart shown in Figure 37. Although the chart describes the general trend for most values, there is considerable uncertainty at predicting values for the larger streams. For instance, stream *s1* variance reduction ratio values vary between 0.13 and 0.25 according to the numerical optimization data whilst the formula predicts values between 0.01 and 0.98; a phenomenon that was primarily attributed to the dominant effects of the magnitude of individual measured variance according to *Rule S1*.

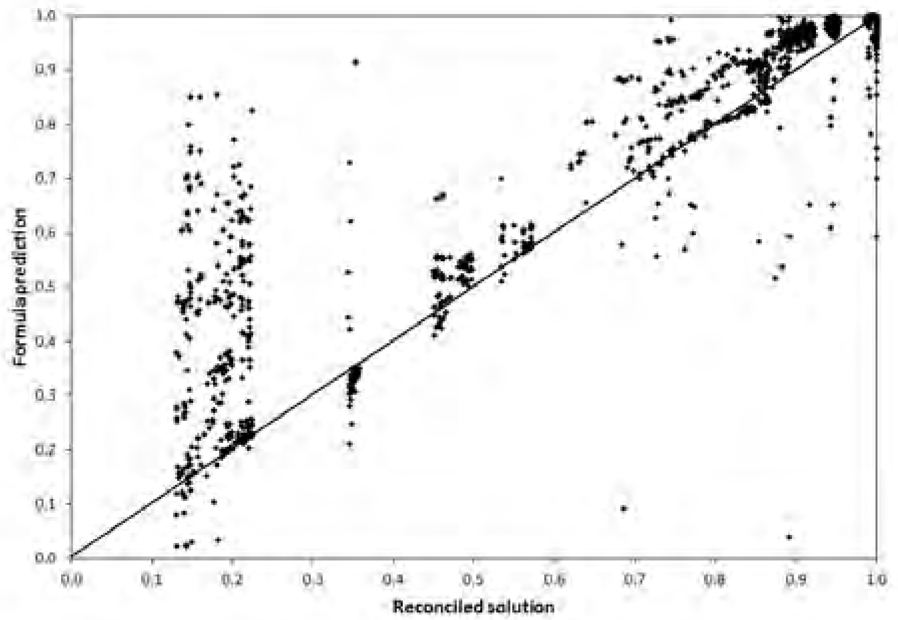


Figure 37: Comparison of variance reduction ratios obtained from theory (formula prediction) and weighted least squares solution (reconciled data)

In part this is expected as the formula is considered accurate for three node structures with simple recycle streams. In the case of four-node and higher structures, the predictions are considered reliable when internal streams are measured precisely. Thus the results obtained on factors derived from Equation 7.2 can be expected to serve as the most appropriate guidelines for network design.

7.6 Summary

This chapter presented a numerical study of mathematical rules for the design of precise measurement networks based on the random error reduction attributes of data reconciliation. Particular attention was paid to precision improvement of boundary streams owing to their custodial importance in material handling operations. The study validated *Rules S1* to *S4* based on the case study data. However, the study suggests that while *Rules F1* and *F2* may hold in some instances for simpler networks, their effects are likely to be superseded by the influence of *Rules S1* to *S4* in complex flowsheets such as the case study.

Moreover, the study identified the following network factors whose values showed significant correlations with variance reduction of terminal streams through data reconciliation: (i) magnitude of measured variance (ii) stream to node variance ratio and (iii) stream interaction effects. The study concluded that:

(i) The reduction in variance of terminal streams is dependent on the magnitude of measured variance in general. According to current findings, this appears to apply strictly to the largest and smallest variance streams in a given single or multi-node measurement scheme without modification. Thus, for relatively large or small variance streams this trend seems to be a global phenomenon. This factor was the basis for the validation of *Rule S1*.

(ii) The reduction in variance of terminal streams in multi-node measurement schemes depends on the ratio of the observed stream variance and total variance associated with the respective parent-node (SNR factor). This result supersedes the effect of variance magnitude. The SNR factor provided a basis for the mechanism behind *Rule S3* propositions.

(iii) The $t^{numerator}$ factor was found to be a non-linear indicator of the influence of stream location on variance reduction through data reconciliation. The factor also provided a basis for validating *Rule S4*. However the study also showed that the effects of measured variance and the SNR factor supersede the effects of relative location as characterised by the $t^{numerator}$ factor.

Chapter 8

A HEURISTIC METHODOLOGY FOR PRECISE METAL ACCOUNTING NETWORK DESIGN

This thesis presented an investigation into the development of heuristic based methodologies for designing measurement networks with particular application to the precise accounting of metal flows in mineral beneficiation operations. In this context, measurement network design referred to placing measurements in an accounting network so that selected stream(s) can be targeted for achieving maximum precisions.

Two types of rules for designing measurement networks were investigated. The first type of rules referred to as 'expert heuristics' consists of (i) Code of Practice Guidelines from the AMIRA P754 Code, and (ii) prevailing accounting practices from the mineral and metallurgical processing industry which were obtained through a questionnaire survey campaign. Sourcing of expert heuristics was based on the hypothesis that experts in the industry design measurement networks using rules or guidelines that ensure prescribed quality requirements in metal accounting.

The second set of rules was derived from symbolic manipulation of the general steady-state linear data reconciliation solution as well as from an intensive numerical study on the variance reduction response of measurements after data reconciliation. These were referred to as 'mathematical heuristics' and are based on the general principle of variance reduction through data reconciliation. Derivation of mathematical heuristics was premised on the hypothesis that specified measurements can be targeted for maximum variance reduction after data reconciliation through rule based design of suitable measurement networks.

This work has provided some insights into the efficacy of heuristics in achieving the objectives of measurement network design as well as delving into the philosophy underlying expert measurement design practices for metal accounting. This chapter summarises the key findings of the study, before making concluding remarks on the implications of heuristic design on metal accounting practice, after which a heuristic design procedure is proposed.

8.1 Summary of key findings

8.1.1 Location of metal accounting measurements in mineral process networks

The industrial survey on metal accounting practice conducted in this study showed that the *Fresh Feed* and *Final Product* stream measurements were the highest rated sources of metal accounting measurements. In addition, the *Final Product* stream emerged as the most likely stream to be weighed and assayed with the highest precision. In general, terminal stream measurements were rated higher than internal measurements with respect to the probability of being measured as well as frequency of use in metal accounting.

Over 40% of all internal measurements taken were indicated as not of regular use in metal accounting. Of all the terminal streams, the *Tailings* stream was considered the least significant source of metal accounting data. In addition, the *Tailings* stream was found to be weighed and assayed with the lowest precision compared to all other terminal streams. However, the survey results show that *Tailings* stream assays exhibited the highest utilisation rates after the *Feed* and *Final Product* terminal stream types. This contrasts a general trend observed where mass measurements were more widely used in metal accounting compared to assays although more assays than mass measurements tended to constitute overall measured data.

Tailings and *Spillage* storage mass measurements were found to be weak metal accounting candidates while *Final Product* storage assays were always selected for use in metal accounting. *Tailings* storage assays were ranked second with respect to assay measurement usage in metal accounting.

8.1.2 Influence of metal accounting procedures on measurement selection

The industrial survey has found that metal accounting measurements are pre-selected based on data input requirements for routine key performance evaluations such as product recovery and accountability, although it appears that operations evaluate all measurements taken in an accounting period and only utilise those measurements whose quality and integrity is deemed suitable for compiling current metal accounts. The high significance placed on *Feed* and *Final Product* measurements is consistent with the finding made that most mineral beneficiation operations use the actual metal/mineral recovery computation which only requires feed and product measurements for determination.

Despite operations rating internal measurement precisions as medium to high in terms of precision, the non-use of internal measurements in the CICO based computation of the primary balance detracts from the potential of utilising internal balances to reduce primary accounting variance through approaches such as data reconciliation. As a result, keeping the unaccounted balance below 1% as generally indicated would rely on employing high precision measurement technologies and procedures in actual practice.

However, it was found that systematic adjustment of data to achieve consistent mass balances across operations is not common practice in the industry. A small proportion of sites indicated that data are adjusted in order to achieve consistency although none of the operations in the survey indicated that systematic methods were used to make the necessary data adjustments.

It emerged that the common use of the survey approach to stock and inventory measurement makes bulk density factors and accompanying moisture analyses important metal accounting measurements. Over 50% of operations regularly update bulk densities for metal accounting purposes while approximately 20% appear to rely on historical estimates and about 25% update estimates in response to disputes.

8.1.3 Factors affecting variance reduction for terminal streams through SSDR

8.1.3.1 Magnitude of measured variance

The magnitude of measured variance emerged as a good indicator of expected variance reduction through steady-state data reconciliation in general. It was found that the reduction in variance for terminal streams depends on the magnitude of measured variance in general. The numerical study conducted in this work found that an inverse relationship exists between variance reduction for individual streams and their respective measured variance. This finding encapsulates what has been referred to as *Rule S1* in this work.

8.1.3.2 Terminal stream to parent-node variance ratio

It was found that for multi-node flowsheets, the SNR factor is a better predictor of variance reduction for individual streams compared to the measured variance alone. This observation was found to assist in understanding the underlying mechanisms of *Rule S3* which asserts that measuring internal streams more precisely increases the variance reduction of terminal streams after data reconciliation. Measuring internal streams more precisely increases terminal stream to parent-node variance ratio for terminal streams thereby increasing the variance reduction potential of terminal stream measurements. This finding alters the hierarchy of variance reduction based on the magnitude of measured variance since it was found that smaller terminal stream variances can experience higher reduction in variance than larger variances if the stream to parent-node variance ratio value of the former is larger.

8.1.3.3 Stream interaction effects

The $t^{numerator}$ stream interaction factors described in this study gather the effects of stream interactions on the extent of variance reduction for terminal streams according to position in a given network. These were found to provide a useful mechanism for ranking the impact of network interconnectivity on variance reduction for terminal streams according to relative location in a given measurement network. Low values of $t^{numerator}$ indicated higher variance reduction while high values predicted lower reductions in variance for the observed terminal streams after data reconciliation. However the numerical study showed that the influence of stream location on variance reduction as measured by the absolute values of $t^{numerator}$ is essentially non-linear. Notwithstanding, the $t^{numerator}$ factor was demonstrably efficacious as a qualitative predictor of the extent of variance reduction for terminal streams according to location.

The second stream effects factor, the $t^{denominator}$ term, on the other hand is the same for all terminal streams in a given network but different across network configurations. In contrast to the effects of $t^{numerator}$, high values of $t^{denominator}$ suggest high variance reduction for all terminal streams. It is hence possible to rank the variance reduction capabilities of entire flowsheets by comparing the magnitudes of their respective $t^{denominator}$ values. However, the $t^{denominator}$ measure appears to be of little effect for complex circuits such as the case study flowsheet.

8.2 Conclusions

8.2.1 Expert approach to measurement network design

This work has shown that the prevailing practice in the industry is to minimise metal accounting variance by sampling and weighing key streams with high precision. Terminal streams in general, and *Feed* and *Product* streams in particular, have been identified as key to metal accounting. These streams were found to enjoy higher incidences of measurement, determined with higher precisions and used more frequently in metal accounting compared to internal streams. The observed attention to terminal measurements in terms of usage and measurement is consistent with the widespread use of the CICO method of accounting practiced in the minerals beneficiation industry.

Of concern, however, is the low usage of *Tailings* data in metal accounting despite the universal employment of the CICO system. The common use of the actual recovery indicator perhaps belies the high significance placed on *Feed* and *Product* stream data and low regard for the *Tailings* stream as a regular source of accounting data.

Use of the CICO system entails the non-participation of internal measurements in defining the primary balance. An opportunity is missed to exploit internal measurement attributes to enhance the precision of primary accounting through methods that utilise all measurements simultaneously to define a single balance for the entire process such as data reconciliation.

Hence the expert design philosophy advocates the precise measurement and utilisation of terminal streams in general to define corporate metal accounts through CICO accounting method while internal measurements are reserved for internal unit operation evaluations and secondary accounting.

8.2.2 Mathematical based approach to measurement network design

The mathematical design approach is based on the variance reduction of terminal stream measurements through data reconciliation. It has been demonstrated that terminal streams, which are metal accounting streams of interest, can be targeted for preferential variance reduction through data reconciliation if metal accounting practitioners follow the mathematical design rules developed in this study.

In general, mathematical heuristics illustrate the benefits of precise measurement of internal streams so that terminal streams can experience maximum variance reduction after data reconciliation. It can be asserted that more precise terminal measurements provide a better platform for further improvement through data reconciliation but the mathematical rules suggest that this could potentially waste valuable resources. For instance, it may not be necessary to concentrate resources on measuring large variance terminal streams with high precision as these are likely to experience large reductions in variance after data reconciliation (*Rule S1*).

Where secondary accounting is routinely practiced, there is often a need to choose internal streams to measure in order to compute relevant unit parameters such as metal recovery. The possible measurement schemes available for selection may offer a range of values of design parameters such as the SNR measure. In practice, the final measurement choices are also influenced by extraneous factors such as cost and ergonomics, giving rise to a complex matrix of options. The mathematical guidelines developed in this research assist the design decision process by informing on network choices that maximise the precisions of key accounting measurements through steady-state data reconciliation.

These design principles will be of benefit to metal accounting systems based on data reconciliation or mass balancing. However, the design philosophy of concentrating resources on internal streams is contrary to common expert practice where emphasis is on precise measurement of the actual input and output streams of the process. In this case, the metal accounting system will behave as a single node and will not benefit significantly from data reconciliation i.e. a CICO type accounting philosophy will be suitable.

8.3 A heuristic methodology for precise measurement network design

Two types of heuristics for designing measurement networks for metal accounting have been presented in this thesis. The first type originated from two expert sources, namely the AMIRA P754 Code of Practice put together by experts from the industry as well as findings of the questionnaire survey conducted across operating mineral beneficiation plants in the South African mining industry. The second type consists, firstly, of rules that were drawn from the symbolic expression of variance reduction for terminal streams through data reconciliation; and secondly, network parameters that were derived from a numerical study done on the reduction of variance for terminal streams through data reconciliation.

The following sequence constitutes a proposed heuristic design decision process for constructing measurement networks for metal accounting systems based on rules presented in this thesis:

(i) If the system requirements are based on achieving requisite accuracy, then the use of applicable international standards on mass measurement, sampling, sample preparation and sample analysis should adequately address these demands. In this scenario, precision is limited by current technology.

(ii) If credibility and transparency are required (in addition to accuracy), then a design based on the Code Principles should meet these demands. The Code provides guidelines on achieving credibility and transparency in accounting that are not included in metrology standards. These include protocols around essential activities such as data acquisition, handling and reporting that address credibility and transparency of the accounting process.

(iii) If the requirements include the achievement of precision levels that are beyond the capabilities of existing hardware and procedures, then data reconciliation should be considered. Spatial redundancy is a necessary condition for data reconciliation in general and observability of all terminal streams is a minimum requirement in particular so that boundary measurements constitute part of the adjusted data set. Moreover, policy should include the acceptance of adjusted data as valid input to metal accounting reporting.

(iv) If, however, higher precision requirements are placed on metal accounting data i.e. terminal stream measurements, then the mathematical heuristics developed in this work can be used to target boundary measurements for maximised variance reduction through data reconciliation.

It should be noted that although the expert approach uses the CICO method of accounting where data are not adjusted to achieve self-consistency, data reconciliation can still be used in this instance as a diagnostic tool to check the integrity measured data (Morrison, 2008). Hence in design options (i) and (ii) above, data reconciliation would only serve as a 'go/no go' test for accepting measured data for final accounting without having to use the reconciled estimates for reporting purposes.

8.4 Recommendations for future work

In general heuristic approaches are meant to simplify design procedures such as measurement network or other optimisation routines. Depending on computational capacity and training of practitioners, computationally intensive design procedures can be used to deal with large search spaces in reasonable time frames. However, one of the motivations of this work was to develop a simple methodology that ordinary plant metal accounting practitioners can use to design their own systems without the need for expensive resources and advanced training. In light of this it is recommended that:

- the heuristics developed in this work should be compared with purely computational approaches to measurement network design in terms of consistency in arriving at the optimal solutions.
- A hybrid approach is tested where the heuristic selection is used to reduce the solution spaces of conventional computational approaches to save on computational time and effort.

References

Albuquerque, J. S., & Biegler, L. T. (1996). Data reconciliation and gross-error detection for dynamic systems. *AIChE Journal*, *42*, 2841–2856. doi:10.1002/aic.690421014

Ali, Y., & Narasimhan, S. (1993). Sensor network design for maximising reliability of linear processes. *AIChE Journal*, *39*(5), 820–826.

Ali, Y., & Narasimhan, S. (1995). Redundant sensor network design for linear processes. *AIChE Journal*, *10*, 2237–2249.

Almasy, G. A., & Sztano, T. (1975). Checking and correction of measurements on the basis of linear system model. *Problems of Control and Information Theory*, *4*, 57–69.

Bagajewicz, M. J. (1997). Design and retrofit of sensor networks in process plants. *AIChE Journal*, *43*, 2300–2306. doi:10.1002/aic.690430915

Bagajewicz, M. J., & Sánchez, M. C. (1999). Design and upgrade of nonredundant and redundant linear sensor networks. *AIChE Journal*, *45*, 1927–1938. doi:10.1002/aic.690450910

Bai, S., Thibault, J., & McLean, D. (2006). Dynamic data reconciliation: an alternative to Kalman Filter. *Journal of Process Control*, *16*, 485–498.

Barbosa, V. P. J., Wolf, M. R. M., & Maciel, R. F. (2000). Development of data reconciliation for dynamic non-linear system: application the polymerlzatlon reactor. *Computers and Chemical Engineering*, *24*, 501–506.

Bartlett, H. E. (2002). Design of primary samplers for slurries in concentrators and statistical methods for measuring components of variance in sampling. *The Journal of The South African Institute of Mining and Metallurgy*, (December), 485–490.

Benqlilou, C., Espuna, A., Bel, S., & Puigjaner, L. (2001). Improved data reconciliation by redundant model updating. In *51st Can. Chem. Eng. Conf.* Halifax, Nova Scotia, Canada.

Benqlilou, C., Musulin, E., Bagajewicz, M., & Puigjaner, L. (2003). Intrumentation design and upgrade for optimal kalman filtering. *Computer Aided Chemical Engineering*, *14*, 371–376.

Benqlilou, C., Musulin, E., Bagajewicz, M., & Puigjaner, L. (2005). Instrumentation design and upgrade for optimal kalman filtering. *Journal of Process Control*, *15*(6), 629–638.

Bepswa, & Deglon. (2013). Development of heuristic methodology for designing precise metal accounting networks. *Minerals Engineering*.

Bepswa, P., Deglon, D., & Chakraborty, A. (2006). Heuristic methodology for designing measurement networks for metallurgical accounting. In *Proceedings of the 23rd International Mineral Processing Conference*. Istanbul, Turkey.

Bepswa, P., Deglon, D., & Chakraborty, A. (2008). Development of a heuristic methodology for designing measurement networks for metallurgical accounting. In *Proceedings of the 24th Int. Mineral Processing Conference*. Beijing, China.

Box, G. E. P., & Draper, N. R. (1969). *Evolutionary Operation* (First.). John wiley & Sons, Inc.

Box, G. E. P., Hunter, J. Stuart, & unter, W. G. (2005). *Statistics for experimenters* (2nd ed.). Hoboken, N.J. : Wiley-Interscience, c2005.

Britt, H. I., & Luecke, R. H. (1973). The estimation of parameters in non-linear implicit models. *Technometrics*, *15*, 233–247.

Cahill, P., Strutt, H., & Wragg, D. (2000). Application of on-line laser scanning technology to improve unit bunker management in coal-fired power stations. *Bulk Solids Handling*, *20*(3), 289–294.

Carnero, M., Hernandez, J., Sanchez, M., & Bandoni, A. (2005). On the solution of the instrumentation selection problem. *Industrial and Engineering Chemical Research*, *44*, 358–367.

Chakraborty, A., & Deglon, D. (2008). Development of a heuristic methodology for precise sensor network design. *Computers and Chemical Engineering*, *32*, 382–395. doi:10.1016/j.compchemeng.2007.02.008

Chen, T., Morris, J., & Martin, E. (2008). Dynamic data rectification using particle filters, *32*, 451–462. doi:10.1016/j.compchemeng.2007.03.012

Chiari, M., Bussani, G., Grottoli, M. G., & Pierucci, S. (1997). On-line data reconciliation and optimisation: refinery applications. In S. Skogestad (Ed.), *7th European Symposium on Computer Aided Process Engineering* (pp. 1185–1190).

Chmielewski, D. J., Palmer, T., & Manousiouthakis, V. (2002). On the theory of optimal sensor placement. *AIChE Journal*, *48*(5), 1001–1012.

Code of Practice, A. P. (2005). Guidelines for metal balancing and reconciliation – Code Development, Release 2.3, 2005. Brisbane: JKMRC.

Coulson, J. M., Sinnott, R. K., & Richardson, J. (2005). *Coulson & Richardson's Chemical Engineering Volume 6: Chemical Engineering Design*. *Book* (Vol. 2). Retrieved from http://books.google.com/books?id=kg8dJgAACAAJ&printsec=frontcover

Crowe, C. M., Campos, Y. A. G., & Hrymak, A. (1983). Reconciliation of process flow rates by matrix projection. Part I: Linear case. *AIChE Journal*, *29*, 881–888. doi:10.1002/aic.690290602

Cutler, C. J., & Eksteen, J. J. (2006). Variance propagation in toll smelting operations treating multiple concentrate stockpiles. *JSAIMM*, *106*, 222–227.

Davies, O. L., & Goldsmith, P. L. (1976). *Statistical Methods in Research and Production: With Special Reference to the Chemical Industry*. Imperial Chemical Industries, Limited. Retrieved from https://books.google.co.za/books?id=qm_IQgAACAAJ

De Andrade Lima, L. R. P. (2006). Non-linear data reconciliation in gold processing plants. *Minerals Engineering*, *19*(9), 938–951. doi:10.1016/j.mineng.2005.10.018

Durrheim, K. (2006). Research design. In *Research in practice: applied methods for the social sciences* (pp. 33–59).

Eksteen, J. J., Frank, S. J., & Reuter, M. A. (2002). Dynamic structures in variance based data reconciliation adjustments for a chromite smelting furnace. In *Minerals Engineering* (Vol. 15, pp. 931–943). doi:10.1016/S0892-6875(02)00131-0

Everitt, B. S. (1977). *The analysis of contingency tables*. London, Chapman and Hall.

Fine, A. H., & Geiger, G. H. (1979). *Handbook on material and energy balance calculations in metallurgical processes*. The Metallurgical society of AIME.

Frew, J. A., & Smith, H. W. (1983). Design and Analysis of sampling experiments – A sensitivity approach. *Int. J. Miner. Process*, *11*, 267–284.

Fuerstenau, D. W., & Han, K. (2003). *Principles of Mineral Processing*. (M. C. Fuerstenau & Han K. N., Eds.). SME.

Gaylard, P. G., Morrison, R. D., Randolph, N. G., Wortley, C. M. G., & Beck, R. D. (2009). Extending the application of the AMRA P754 Code of Practice for metal accounting. In *SAIMM Base Metals Conference*. SAIMM.

Gelb, A. (1974). *Applied Optimal Estimation*. MIT Press, Cambridge, Mass.

Gerkens, C., & Heyen, G. (2004). Sensor network design using genetic algorithm. In D. Sauter & D. Theilliol (Eds.), *Proceedings of MMM IFAC Symposium* (pp. 149–154). Nancy, France: Elsevier.

Gerkens, C., & Heyen, G. (2005). Use of parallel computers in rational design of redundant sensor networks. *Computers & Chemical Engineering*, *29*(6), 1379–1387. doi:10.1016/j.compchemeng.2005.02.012

Gy, P. (2004). Sampling of discrete materials—a new introduction to the theory of sampling. *Chemometrics and Intelligent Laboratory Systems*. doi:10.1016/j.chemolab.2004.05.012

Hayward, A. T. J. (1977). *Repeatability and accuracy*. Mechanical Engineering Publications Limited, PO Box 24, Northgate Avenue, Bury St, Edmunds, Suffolk IP32 6BW.

Heyen, G., Dumont, M., & Kalitventzeff, B. (2002). Computer aided design of redundant sensor networks. In *12th European Symposium On Computer aided Process Engineering* (pp. 685–690). Elsevier Science.

Hodouin, D., Flament, F., & Bazin, C. (1989). Reliability of material balance calculations a sensitivity approach. *Minerals Engineering*. doi:10.1016/0892-6875(89)90037-X

Hodouin, D., & Makni, S. (1996). Real-time reconciliation of mineral processing plant data using bilinear material balance equations coupled to empirical dynamic models. *International Journal of Mineral Processing*, *48*, 245–264. doi:10.1016/S0301-7516(96)00025-7

Hodouin, D., Mirabedini, A., Makni, S., & Bazin, C. (1998). Reconciliation of mineral processing data containing correlated measurement errors. *International Journal of Mineral Processing*, *54*(3-4), 201–215. doi:10.1016/S0301-7516(98)00012-X

Holmes, R. J. (2004a). Correct sampling and measurement - The foundation of accurate metallurgical accounting. In *Chemometrics and Intelligent Laboratory Systems* (Vol. 74, pp. 71–83). doi:10.1016/j.chemolab.2004.03.019

Holmes, R. J. (2004b). Correct sampling and measurement—the foundation of accurate metallurgical accounting. *Chemometrics and Intelligent Laboratory Systems*, *74*(1), 71–83. doi:10.1016/j.chemolab.2004.03.019

Holmes, R. J. (2010). Sampling mineral commodities — the good , the bad , and the ugly. *The Journal of The South African Institute of Mining and Metallurgy*, *110*, 269–276.

Holmes, R. J., & Robinson, G. K. (2004). Codifying the principles of sampling into mineral standards. In *Chemometrics and Intelligent Laboratory Systems* (Vol. 74, pp. 231–236). doi:10.1016/j.chemolab.2004.03.011

Karjala, T. W., & Himmelblau, D. M. (19967). Dynamic rectification of data via recurrent neural network and the extended Kalman Filter. *AIChE Journal*, *42*, 2225–2239.

Kelly, J. D. (2004). Formulating large-scale quantity-quality bilinear data reconciliation problems. *Computers and Chemical Engineering*, *28*, 357–362. doi:10.1016/j.compchemeng.2003.07.003

Knepper, J. C., & Gorman, J. W. (1980). Statistical analysis of constrained data sets. *AIChE Journal*, *26*, 260–264. doi:10.1002/aic.690260208

Kretsovalis, A., & Mah, R. S. H. (1987). Effect of redundancy on estimation accuracy in process data reconciliation. *Chemical Engineering Science*. doi:10.1016/0009-2509(87)85032-7

Kretsovalis, A., & Mah, R. S. H. (1988). Observability and redundancy classification in generalized process networks—II. Algorithms. *Computers & Chemical Engineering*, *12*(7), 689–703. doi:10.1016/0098-1354(88)80008-5

Kuehn, D. R., & Davidson, H. (1961). Computer Control. II. Mathematics of Control. *Chemical Engineering Progress Symposium Series*, *57*, 44–47.

Liebman, M. J. (1992). Efficient Data Reconciliation and Estimation for Dynamic Processes Using Non-linear Programming Techniques. *Computers & Chemical Engineering*, *16*(10), 963–986.

Liebmann, J., Edgar, T. F., & Lasdon, L. S. (1992). Efficient data reconciliation and estimation for dynamic processes using non-linear programming techniques. *Comput. Chem. Eng.*, *16*(196).

Lightbody, G. (1983). How British Steel manage stockpiles. *Iron and Steel International*, 178–182.

Lyman, G. (2005). Sampling, metallurgical accounting and reduction of balance estimation variance. In *Proceedings of the 2nd international conference on sampling and blending* (pp. 127–135). Brisbane.

Madron, F. (1992). Process plant performance: measurement and data processing for optimization and retrofits. *Journal of Process Control*, *4*(4), 297. doi:10.1016/0959-1524(94)80051-0

Madron, F., & Veverka, V. V. (1992). Optimal selection of measuring points in complex plants by linear models. *AIChE Journal*, *38*(2), 227–236.

Mah, R. S. . (1990). *Chemical process structures and information flow*. Butterworths, Boston.

Mah, R. S. ., & Stanley, G. . (1976). Reconciliation and rectification of process flow and inventory data. *Ind. Eng. Chem. Process Des. Dev.*, *15*, 175. Retrieved from http://www.gregstanleyandassociates.com/ReconciliationRectificationProcessData-1976.pdf

Mah, R. S. ., & Tamhane, A. C. (1982). Detection of gross errors in process data. *AIChE Journal*, *28*, 828.

McBride, W., & Chambers, A. J. (1999). Bulk density measurement using a shear box. *Bulk Solids Handling*, *19*(4), 501–505.

Merks, J. W. (1985). *Sampling and weighing of bulk solids* (First.). Published by Trans Tech Publications, Germany, 1985.

Merks, J. W. (2002). Sampling in mineral processing. In M. A.L., H. D.N., & Barratt D.J. (Eds.), *Mineral processing plant design, practice, and control proceedings*. Society of Mining Engineers.

Meyer, M., Koehret, B., & Enjalbert, M. (1993). Data reconciliation on multicomponent network process. *Computers & Chemical Engineering*, *17*(8), 807–817. doi:10.1016/0098-1354(93)80065-U

Mingfang, K., Bingzhen, C., & Bo, L. (2000). An Integral approach to dynamic data rectification. *Computers & Chemical Engineering*. doi:10.1016/S0098-1354(00)00378-1

Morrison, R. D. (Ed). (2008). *An introduction to metal balancing and reconciliation* (First.). Brisbane: JKMRC University of Queensland.

Muske, K., & Edgar, T. F. (1998). *Non-linear State Estimation*. (M. A. Henson & D. E. Seborg, Eds.). Prentice Hall, Englewood Cliffs, NJ.

Narasimhan, S., & Jordache, C. (2000). *Data reconciliation and gross error detection : an intelligent use of process data*. *Annals of the New York Academy of Sciences* (Vol. 195). doi:10.1111/j.1749-6632.1972.tb54778.x

Narasimhan, S., & Mah, R. S. H. (1987). Generalized likelihood ratio method for gross error identification. *AIChE Journal*, *33*, 1514–1521. doi:10.1002/aic.690330911

Nguyen, D., & Bagajewicz, M. J. (2011). New Sensor Network Design and Retrofit Method Based on Value of Information. *AIChE Journal*, *57*, 2136–2148. doi:10.1002/aic

Norgaard, M., Poulsen, N., & Ravn, O. (2000). New developments in state estimation for non-linear systems. *Automatica*, *36*, 1627–1638.

Ooms, M. (1981). Determination of contents in storage bins and silos. *Bulk Solids Handling*, *1*(4), 651–665.

Pai, C. C. D. ., & Fisher, G. D. (1988). Application of broyden's method to reconciliation of non-linearly constrained data. *AIChE Journal*, *34*, 873–876.

Philander, C., & Rozendaal, A. (2013). The application of a novel geometallurgical template model to characterise the Namakwa Sands heavy mineral deposit, West Coast of South Africa. *Minerals Engineering*, *52*, 82–94. doi:10.1016/j.mineng.2013.04.011

Pitard, F. F. (1993). *Pierre Gy's Sampling Theory and Sampling Practice, Second Edition: Heterogeneity, Sampling Correctness, and Statistical Process Control* (2, illustr.). CRC Press, 1993.

Potts, A. (n.d.). High tech surveying and measuring. *Mining Magazine*, 270–275.

Ragot, J., Maquin, D., & Bloch, G. (1992). Sensor positioning for processes described by bilinear equations. *Diagnostic et Surete de Fonctionment*, *2*, 115–132.

Reilly, P. M., & Carpani, R. E. (1963). Application of statistical theory to adjustment of material balances. In *13th Can. Chem. Eng. Conf.* Montreal, Quebec.

Richardson, M. J., & Morrison, R. D. (2003). Metallurgical balances and efficiency. In M. C. Fuerstenau & H. K. N. (Eds.), *Principles of Mineral Processing* (pp. 363–389). SME.

Richardson, M. J., & White, J. W. (1982). Mass balance calculations. In *Design and installation of comminution circuits*. AIME, New York.

Ripps, D. L. (1965). Adjustment of experimental data. *Chemical Engineering Progress Symposium Series*, *61*(55), 8–13.

Romagnoli, J., & Stephanopoulos, G. (1980). A general approach to classify operational parameters and rectify measurement errors for complex chemical processes. *Computer Application to Chemical Engineering*, 53–174.

Rosenberg, J., Mah, R. S. H., & Jordache, C. (n.d.). Evaluation of schemes for detecting and identification of gross errors in process data. *Industrial and Engineering Chemistry Process Design Development.*, *26*, 555–564.

Sánchez, M., & Romagnoli, J. (1996). Use of orthogonal transformations in data classification-reconciliation. *Computers & Chemical Engineering*, *20*(5), 483–493. doi:10.1016/0098-1354(95)00212-X

Schmidt, S. (1980). The Kalman Filter: its recognition and development for aerospace applications. *Journal of Guidance and Control*, *4*(1), 4–10.

Sen, S., Narasimhan, S., & Deb, K. (1998). Sensor network design of linear processes using genetic algorithms. *Computers & Chemical Engineering*, *22*(3), 385–390. doi:10.1016/S0098-1354(97)00242-1

Simpson, D. E., Voller, V. R., & Everett, M. G. (1991). An efficient algorithm for mineral processing data adjustment. *International Journal of Mineral Processing*, *31*(1-2), 73–96. doi:10.1016/0301-7516(91)90006-5

Smith, H. W., & Ichiyen, N. (1973). Computer adjustments of metallurgical balances. *Can. Inst. Min. Metal. Bulletin*, *66*(737), 97–100.

Soderstrom, T. A., Himmelblau, D. M., & Edgar, T. F. (2003). A mixed integer non-linear optimization based approach to simultaneous data reconciliation and bias identification. In *Proceedings of the FOCAPO-2003 conference*. Coral Springs, Florida.

Tamhane, A. C., Jordache, C., & Mah, R. S. H. (1988). A bayesian approach to gross error detection in chemical process data. Part I. *Chemometrics and Intelligent Laboratory Systems*, *4*(33).

Tong, H., & Crowe, C. M. (1995). Detection of gross errors in data reconciliaton by principal component analysis. *AIChE Journal*, *41*(7), 1712–1722.

Tong, H., & Crowe, C. M. (1995). Detection of gross erros in data reconciliation by principal component analysis. *AIChE Journal*, *41*, 1712–1722. doi:10.1002/aic.690410711

Underhill, L., & Bradfield, D. (1998). *Introstat*. Juta Academic. Retrieved from https://books.google.co.za/books?id=f6TlVjrSAsgC

Václavek, V. (1969). Studies on system engineering—III optimal choice of the balance measurements in complicated chemical engineering systems. *Chemical Engineering Science*, *24*(6), 947–955. doi:10.1016/0009-2509(69)87003-X

Václavek, V., & Loučka, M. (1976). Selection of measurements necessary to achieve multicomponent mass balances in chemical plant. *Chemical Engineering Science*. doi:10.1016/0009-2509(76)85029-4

Wongrat, W., Srinophakun, T., & Srinophakun, P. (2005). Modified genetic algorithm for non-linear data reconciliation. *Computers & Chemical Engineering*. doi:10.1016/j.compchemeng.2004.11.005

Wortley, C. M. G. (2009). Mass measurement for metal accounting — principles , practice. *Fourth World Conference on Sampling Blending*, 121–128. Retrieved from http://www.saimm.co.za/Conferences/SamplingBlending2009/121-128_Wortley.pdf

Xiao, Z., & Vien, A. (2003). Limitations of variance analysis using propagation of variance. *Minerals Engineering*, *16*, 455–462. doi:10.1016/S0892-6875(03)00045-1

APPENDIX A

Questionnaire

METALLURGICAL ACCOUNTING PRACTICE QUESTIONNAIRE

INTRODUCTION

The following questions are designed to prompt qualitative and quantitative responses from participants based on current metallurgical accounting practice at respective location(s). Responses can be done by marking the appropriate check boxes provided for each question. Generic terms for stream types and equipment have been used in the questionnaire to identify groups of streams or equipment which conform (closely) to the description connoted by the terms.

Where appropriate, multiple choices are provided, denoted by the numeric signatures e.g. 2.1, 2.2., 2.3 …etc. The lists of choices provided are designed to cater for a broad base of opinion on the topics presented. Responses can be augmented by additional comments from respondents.

Tables are provided for filling in specific information pertaining to actual practice (Tables 1 – 5). The tables in this WORD document are exactly the same as the EXCEL tables (attached EXCEL file) and are primarily included here for the purpose of completeness of this document (respondents may use the word document tables if they so wish, although it would be more convenient to fill in the specific information directly into the EXCEL tables). If there is more information that respondents feel needs to be added to the tables, the EXCEL spreadsheets can be extended as required.

Paul Bepswa
University of Cape Town
Mineral Processing Research Unit
E-mail: bep@chemeng.uct.ac.za
Tel: +27 (0)21 650 5503
Web: www.chemeng.uct.ac.za/people/bep.php

TABLE OF CONTENTS

1.0 MASS MEASUREMENT – STREAM EQUIPMENT

List A

A1 Always measured and always used for metallurgical accounting

A2 Always measured and sometimes used for metallurgical accounting

A3 Always measured but not used for metallurgical accounting

A4 Sometimes measured and used for metallurgical accounting

A5 Sometimes measured but not used for metallurgical accounting

A6 Not measured and not considered for metallurgical accounting

With reference to choices in List A (above), indicate the current practice with regard to measurement and use of mass flow rates determined on the following streams.

1.1 Fresh feed stream

1.2 In-process streams

1.3 Internal recycle streams

1.4 External recycle streams

1.5 Final product streams

1.6 Final tailings streams

No.	A1	A2	A3	A4	A5	A6
1.1						
1.2						
1.3						
1.4						
1.5						
1.6						

Fill in the details for all streams that are currently measured in the following table (Table 1). The options for material type, type of weighing system, precision of weighing, method of calibration, and purpose of measurement columns are given in Table 1-1. The first line of the table is filled in as an example. The stream/equipment ID's can be descriptive or if numbered conventionally as in the plant, a simplified block flow sheet should accompany the document for reference. Please note that Table 1 is provided in the attached EXCEL document for easier filling in of the detailed information.

<u>TABLE 1</u>

STREAM MASS MEASUREMENT

Stream /Equipment ID	Material weighed				Weighing System		Method of calibration	Purpose of measurement
	Material type	Transport	Throughput tph	Upper size mm	Type of Weigh System	Precision of weighing %		
ROM reclaim	1	Belt	500	100	1	4	2	1

TABLE 1-1

No	Entry				
	Material Type	Type of weighing system	Precision of weighing, %	Method of calibration	Purpose of measurement
1	Crushed	Electro-mechanical weight meters (load cells)	<0.5%	Static weights	Met Accounting - Primary
2	Milled	Nuclear belt weight meters	0.5 - 1.0%	belt cuts	Met Accounting - Secondary
3	Classified	Electro-magnetic flow meter	1-2%	material run bulk tests	Process control
4	Dust	Coriolis mass meter	2-5%	test chain	Custody transfer
5	Off-gas	Impact flow meter	5-10%		
6		Weigh bridge or platform scale	> 10%		
7		In-motion weight meter			
8		Draft survey			

2.0 MASS MEASUREMENT – STORAGE AREAS

List A

A1 Always measured and always used for metallurgical accounting

A2 Always measured and sometimes used for metallurgical accounting

A3 Always measured but not used for metallurgical accounting

A4 Sometimes measured and used for metallurgical accounting

A5 Sometimes measured but not used for metallurgical accounting

A6 Not measured and not considered for metallurgical accounting

With reference to the choices in **List A** indicate the current practice with regard to measurement and use of mass measurements determined on the following storage area types.

2.1 Run of mine stockpiles

2.2 In-process material in tanks/bins/silos

2.3 Final product stockpiles/bins/silos

2.4 Tailings stockpiles/slurry dams

2.5 Spillage stockpiles

No.	A1	A2	A3	A4	A5	A6
2.1						
2.2						
2.3						
2.4						
2.5						

In Table 2 below indicate for **all** storage area types in the plant whether mass on each is determined by difference (feed minus reclaim), by surveying or any other method of measurement that may be in use. Also indicate whether the result is used in metallurgical accounting. The first line of the table is filled in as an example. The equipment ID's can be descriptive or if numbered conventionally as in the plant, a flow sheet should accompany the document for reference.

Please note that Table 2 is provided in the attached EXCEL document for easier filling in of the detailed information.

TABLE 2

STORAGE AREAS

Equipment ID	SAMPLING					MASS MEASUREMENT				
	Sampling method			Use in met accounting		Method used			Use in met accounting	
	From feed stream	Reclaim stream	In-situ	Yes	No	By difference	Survey	Other	Yes	No
ROM S/PILE		x		x			X			x

3.0 SAMPLING – STREAM EQUIPMENT

List B

B1 Always sampled and always used for metallurgical accounting

B2 Always sampled and sometimes used for metallurgical accounting

B3 Always sampled but not used for metallurgical accounting

B4 Sometimes sampled and used for metallurgical accounting

B5 Sometimes sampled but not used for metallurgical accounting

B6 Not sampled and not considered for metallurgical accounting

With reference to choices in **List B** (above), indicate the current practice with regard to sampling and use of samples and measurements determined on the following streams.

3.1 Fresh feed stream

3.2 In-process streams

3.3 Internal recycle streams

3.4 External recycle streams

3.5 Final product streams

3.6 Final tailings streams

No.	B1	B2	B3	B4	B5	B6
3.1						
3.2						
3.3						
3.4						
3.5						
3.6						

Please fill in the details for **all** streams that are currently sampled in the following tables (Table 3, Table 4). In Table 3 options for the columns 'Type of primary sampler' and 'Mode of operation' are listed in Table 3-1. The options for 'Sampling Precision' in Table 4 are listed in Table 4-1.

The first entry for each table is filled in as an example. The equipment ID's can be descriptive or if numbered conventionally as in the plant, a flow sheet should accompany the document for reference.

Please note that Tables 3 and 4 are provided in the attached EXCEL document for easier filling in of the detailed information.

TABLE 3

SAMPLER DETAILS

Sampler ID	Stream ID	Method of transport	Material sampled			Primary Sampler		
			Type	Throughput tph	Upper size mm	Mode of operation	Cutter size mm	Type of primary sampler
1	ROM reclaim	Belt	Crushed	500	100	3	500	1

TABLE 3-1

No	Mode of operation	Type of primary sampler
1	Random - time series	Mechanical - Linear chute sampler
2	Random - mass series	Mechanical - Linear bucket sampler
3	Systematic - constant time	Mechanical - Vertical swing arm sampler
4	Systematic - constant mass	Horizontal swing arm sampler
5		Mechanical - Rotating hammer sampler
6		Manual - belt cut
7		Manual - falling stream
8		Other (specify)

TABLE 4

SAMPLE DETAILS

Sampler No	Stream ID	Sample details				Sampling Precision
		Mass of increment g	No of increments per sample	Sampling Interval h	Composite Period h	
1	ROM reclaim	150	5	2	8	5

TABLE 4-1

No	Sampling Precision
1	less than 0.5%
2	0.5 - 1.0%
3	1-2%
4	2-5%
5	5-10%
6	Greater than 10%

4.0 SAMPLING – STORAGE AREAS

List B

B1 Always sampled and always used for metallurgical accounting

B2 Always sampled and sometimes used for metallurgical accounting

B3 Always sampled but not used for metallurgical accounting

B4 Sometimes sampled and used for metallurgical accounting

B5 Sometimes sampled but not used for metallurgical accounting

B6 Not sampled and not considered for metallurgical accounting

With respect to the response options in **List B** what is the current practice on sampling of the following storage areas (either in-situ or reclaim/feed stream sampling)?

4.1 Run of mine stockpiles

4.2 In-process material in tanks/bins/silos

4.3 Final product stockpiles/bins/silos

4.4 Tailings stockpiles/slurry dams

4.5 Spillage stockpiles

No.	B1	B2	B3	B4	B5	B6
4.1						
4.2						
4.3						
4.4						
4.5						

In Table 2 (pp 6 and in attached EXCEL file) indicate for **all** storage equipment in the plant whether sampling is done either on reclaim/feed streams or in-situ (i.e. sampling directly from a stockpile/bin/silo), and also indicate whether the analyses done on the samples are used in metallurgical accounting. The equipment ID's can be descriptive or if numbered conventionally as in the plant, a flow sheet should accompany the document for reference.

6.0 METAL BALANCING AND RECONCILIATION

6.1 Primary Accounting

Primary accounting is defined as reconciliation of mass flows across an entire plant, whereas the objective of mass balancing is to determine, by adjusting individual measured data values, the best set of consistent values to solve a balance. With reference to primary accounting as defined, what is the practice in your plant with regards to the following assertions?

6.1.1 The accounting boundary is from ROM feed to despatch.

6.1.2 The accounting boundary is from ROM reclaim to despatch.

6.1.3 The primary accounting period of one month is standard, and is long enough to account for time lags and lock ups in the process (inventory)

6.1.4 All plant data collected is utilised in the reconciliation of mass over the accounting period.

6.1.5 Only data from specified streams is used for reconciliation.

6.1.6 Only data deemed accurate for that accounting period is used for primary accounting purposes.

6.1.7 Is mass balancing done (as defined above)?

No.	yes	no	sometimes
6.1.1			
6.1.2			
6.1.3			
6.1.4			
6.1.5			
6.1.6			
6.1.7			

6.2 Secondary Accounting

Secondary accounting is defined as reconciliation of mass flows across smaller sections of the plant circuit. With reference to secondary accounting as defined, what is the practice in your plant with regards to the following assertions?

6.2.1 Secondary accounting is done to verify primary accounting results.

6.2.2 Secondary accounting is only done when primary accounting data produces large unaccounted errors.

6.2.3 Secondary accounting is done to assess plant sub-section performance only.

6.2.4 Secondary accounting is done to assess plant sub-section performance and to verify primary accounting results.

6.2.5 Plant sub-section data is collected for process control and is not suitable for metal accounting.

6.2.6 Plant sub-section data is collected for process control and secondary/primary accounting.

No.	yes	no	sometimes
6.2.1			
6.2.2			
6.2.3			
6.2.4			
6.2.5			
6.2.6			

6.3 Plant recovery calculations

Which of the following expressions is currently used to calculate circuit recovery in your plant?

6.3.1 Recovery % = (<u>sum of all useful or desired outputs</u>) x 100

(sum of all inputs)

6.3.2 Recovery % = $\frac{\text{(sum of all useful or desired outputs)}}{\text{(sum of all outputs)}} \times 100$

6.3.3 Recovery % = $\frac{\text{(metal input – metal losses)}}{\text{(metal input)}} \times 100$

6.3.4 Other (specify)

No.	yes
6.3.1	
6.3.2	
6.3.3	
6.3.4	

6.4 Accountability

Accountability can be defined as the comparison of the total output of a plant to its total input, expressed as a percentage. Of the following expressions for accountability, which one closely resembles the calculation method currently is use at your plant?

6.4.1 Accountability % = $\frac{\text{(sum of outputs + stock \& inventory change)}}{\text{(sum of all inputs)}} \times 100$

6.4.2 Accountability % = $\frac{\text{(sum of all outputs + closing stock)}}{\text{(sum of all inputs + opening stock)}} \times 100$

6.4.3 Unaccounted balances (either gains or losses) always occur when accountability calculations are done. Plants may accept certain levels of such balances expressed as percentages of plant total feed. What level of unaccounted loss is acceptable at your operation?

No.	yes
6.4.1	
6.4.2	

6.4.3	<0.5 %	0.5-1%	1-2%	>2%

6.5 Plant stocks, inventory and custody transfers

With reference to stocks, inventory and custody transfers, what is the practice in your plant with regards to the following assertions?

6.5.1 Process inventory forms part of the data used to produce the metal balance at the end of each accounting period.

6.5.2 Bulk density factors used to determine plant stocks are:

6.5.2.1 Historical estimates

6.5.2.2 Updated periodically

6.5.2.3 Updated in response to disputes.

6.5.3 On transferring material in bulk from one plant section to another,

6.5.3.1 Receiver's mass, sample and analysis will apply

6.5.3.2 Sender's mass, sample and analysis will apply

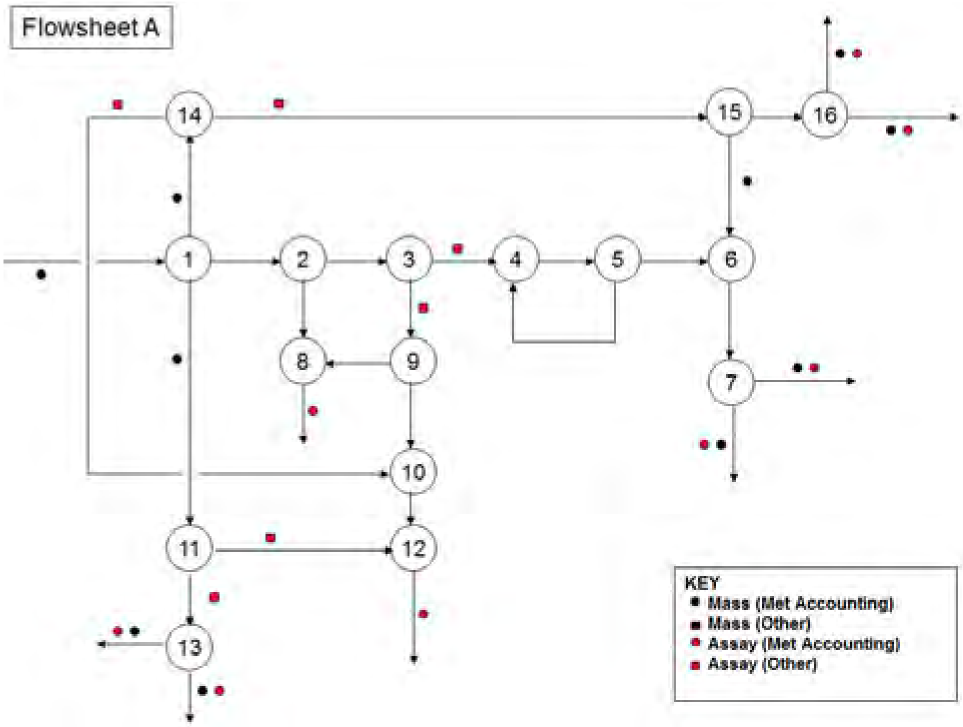
6.5.3.3 Receiving and despatching operations independently weigh, sample and analyse the material being transferred.

No.	Yes	No
6.5.1		
6.5.2.1		
6.5.2.2		
6.5.2.3		
6.5.3.1		
6.5.3.2		
6.5.3.3		

APPENDIX B

Plant flowsheets

A.1 Flowsheet A nodal diagram and measurement statistics

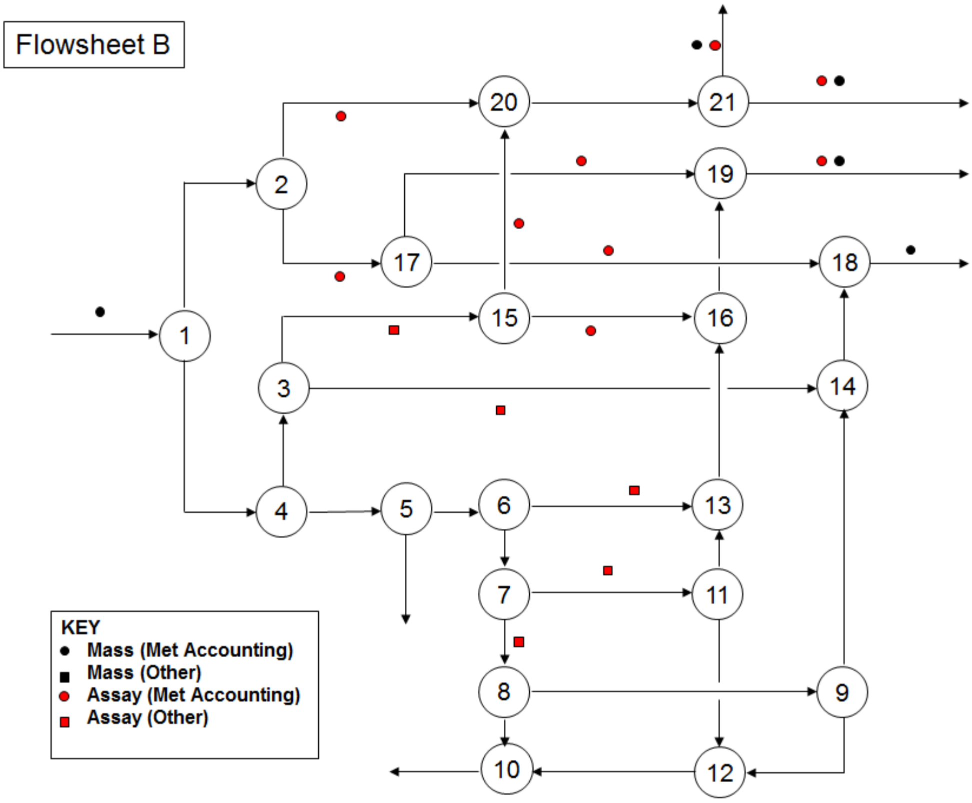


(Flowsheet A nodal diagram)

(Flowsheet A stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	1	1	1	0	0
In-process	20	3	3	6	0
Internal. Recycle	1	0	0	0	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	6	6	6	6	6
Final Tailings	2	0	0	2	2
TOTAL	30	10	10	14	8

A.2 Flowsheet B nodal diagram and measurement statistics

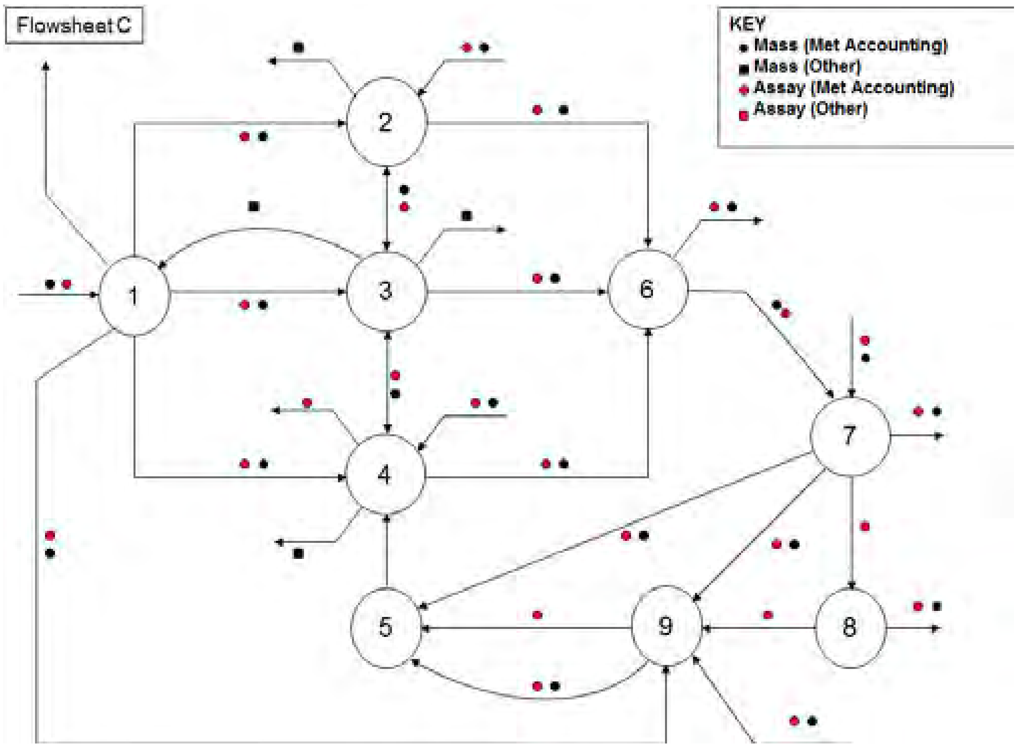


(Flowsheet B nodal diagram)

(Flowsheet B stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	1	1	1	0	0
In-process	25	4	0	12	4
Internal. Recycle	0	0	0	0	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	3	3	3	3	3
Final Tailings	3	1	0	0	0
TOTAL	32	9	4	15	7

A.3 Flowsheet C nodal diagram and measurement statistics

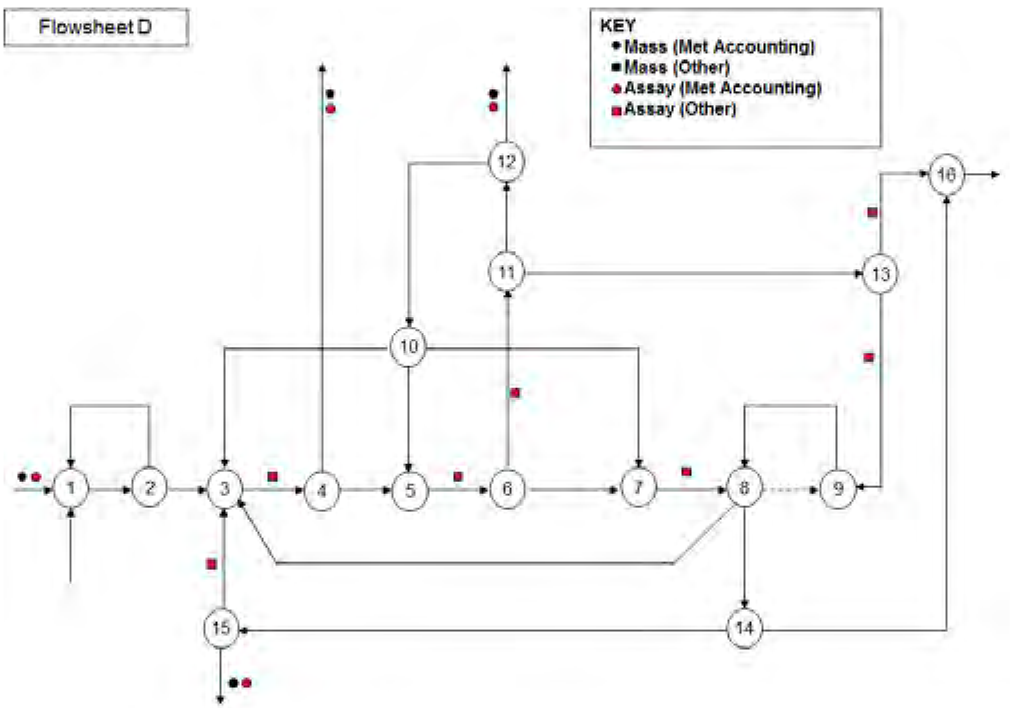


(Flowsheet C nodal diagram)

(Flowsheet C stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	5	5	5	5	5
In-process	16	13	13	16	16
Internal. Recycle	2	1	0	0	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	6	6	6	6	6
Final Tailings	2	0	0	0	0
TOTAL	31	25	24	27	27

A.4 Flowsheet D nodal diagram and measurement statistics

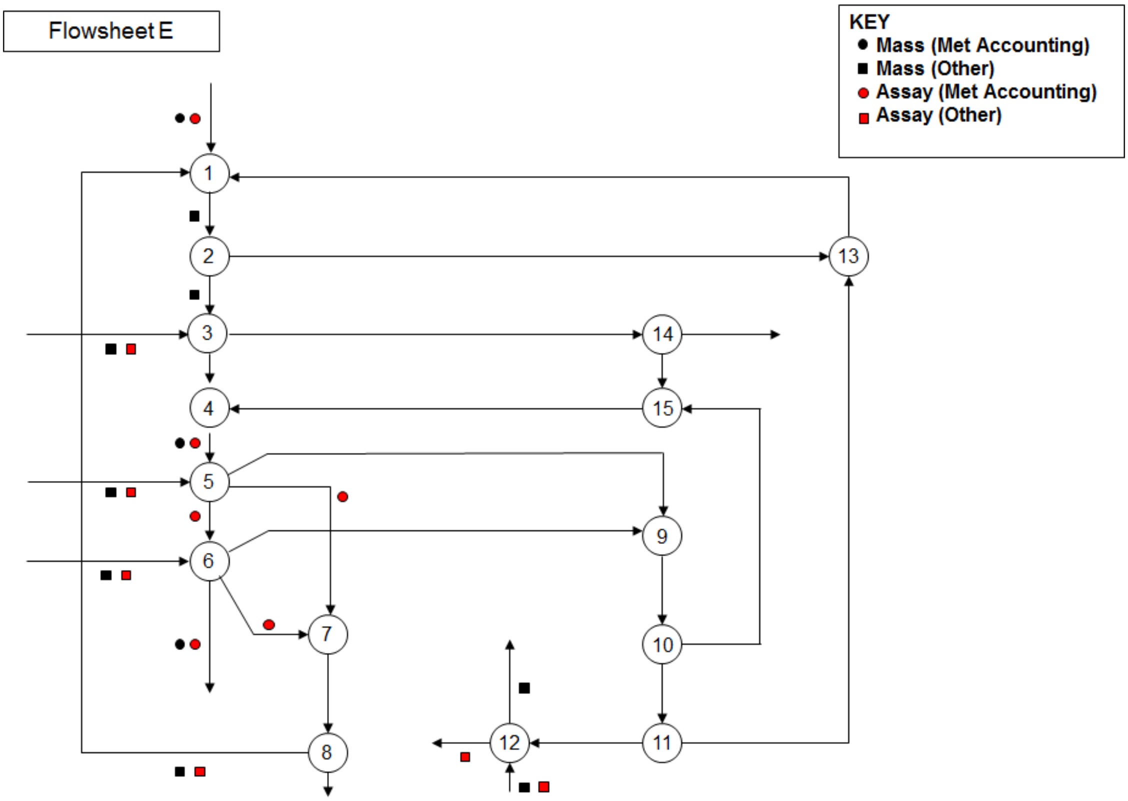


(Flowsheet D nodal diagram)

(Flowsheet D stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	2	1	1	1	1
In-process	16	0	0	6	0
Internal. Recycle	7	0	0	1	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	3	3	3	3	3
Final Tailings	1	0	0	0	0
TOTAL	29	4	4	11	4

A.5 Flowsheet E nodal diagram and measurement statistics

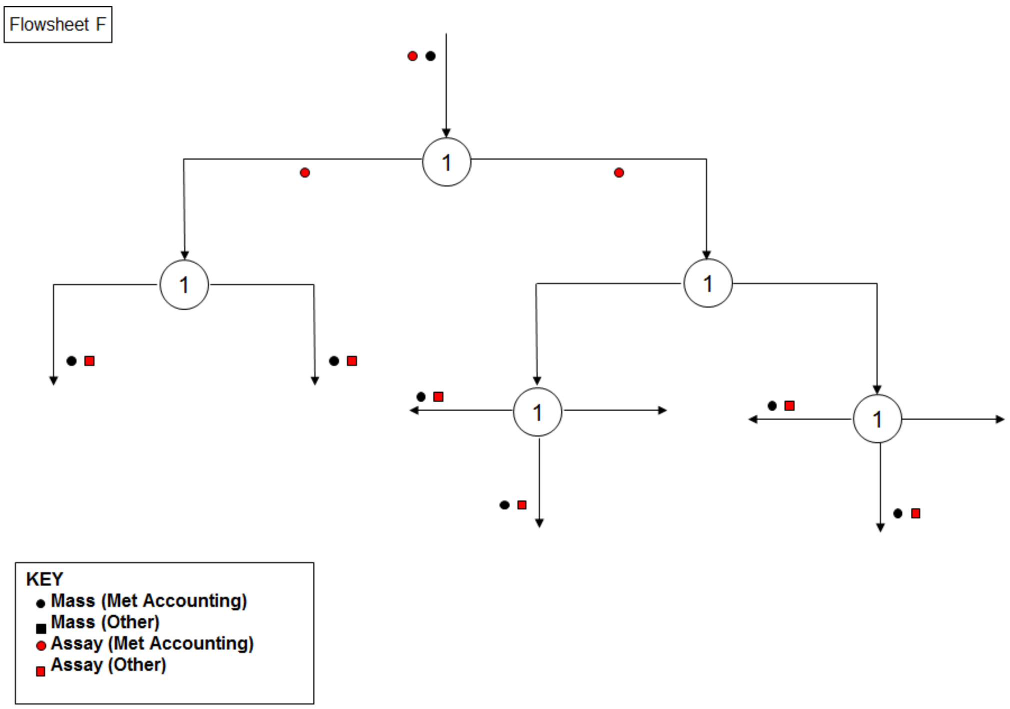


(Flowsheet E nodal diagram)

(Flowsheet E stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	1	1	1	1	1
In-process	14	3	1	4	4
Internal. Recycle	4	1	0	1	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	1	1	1	1	1
Final Tailings	3	0	0	0	0
TOTAL	23	6	3	7	6

A.6 Flowsheet F nodal diagram and measurement statistics

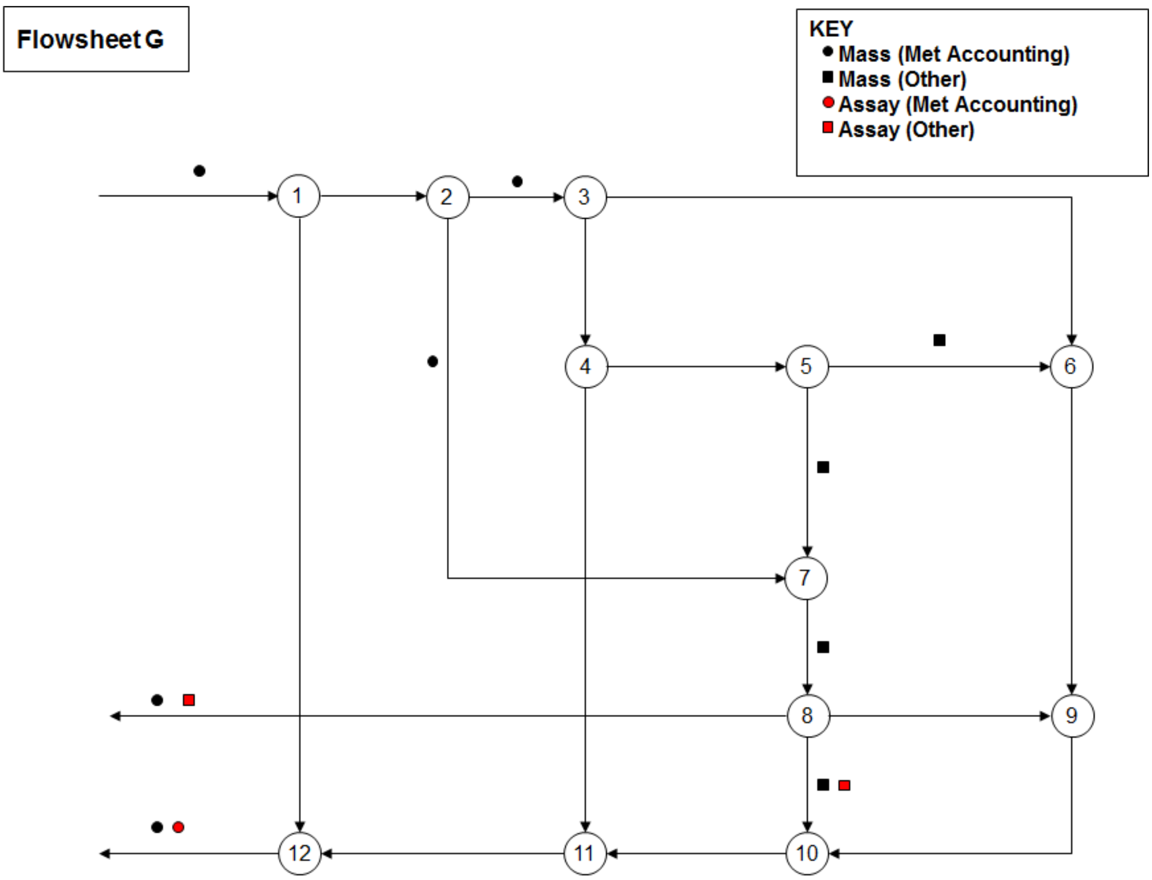


(Flowsheet F nodal diagram)

(Flowsheet F stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	1	1	1	1	1
In-process	2	0	0	2	2
Internal. Recycle	0	0	0	0	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	6	6	6	6	6
Final Tailings	2	0	0	0	0
TOTAL	11	7	7	3	3

A.7 Flowsheet G nodal diagram and measurement statistics

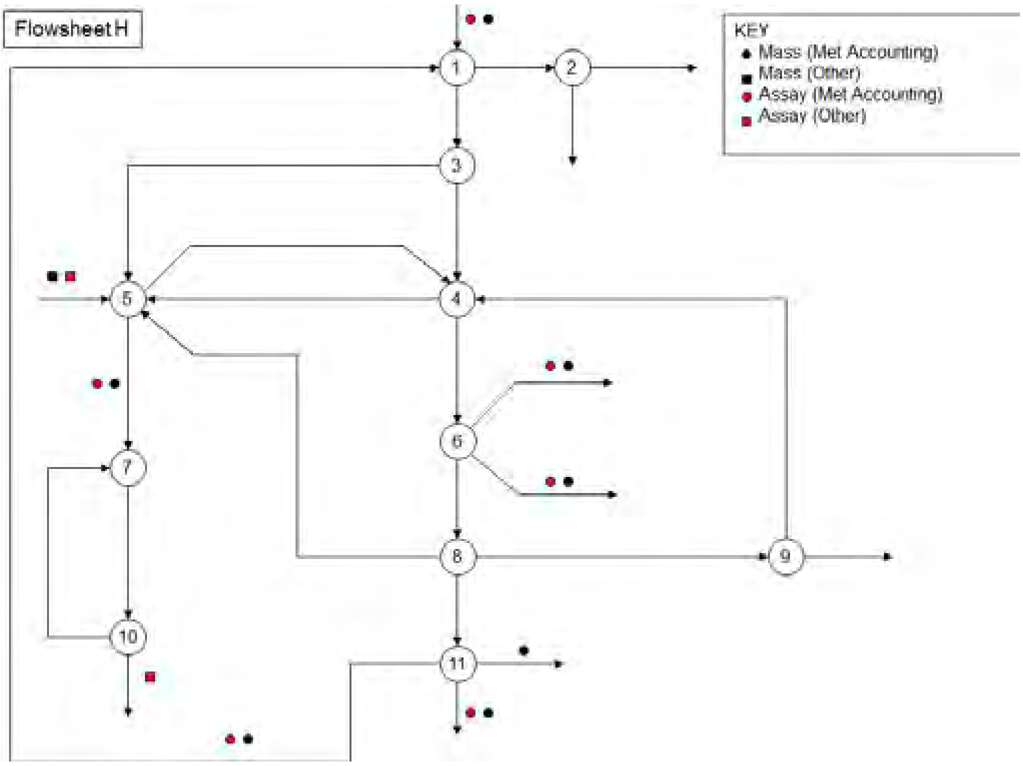


(Flowsheet G nodal diagram)

(Flowsheet G stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	1	1	1	0	0
In-process	17	6	2	1	0
Internal. Recycle	0	0	0	0	0
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	1	1	1	1	1
Final Tailings	1	1	1	1	0
TOTAL	20	9	5	3	1

A.8 Flowsheet H nodal diagram and measurement statistics

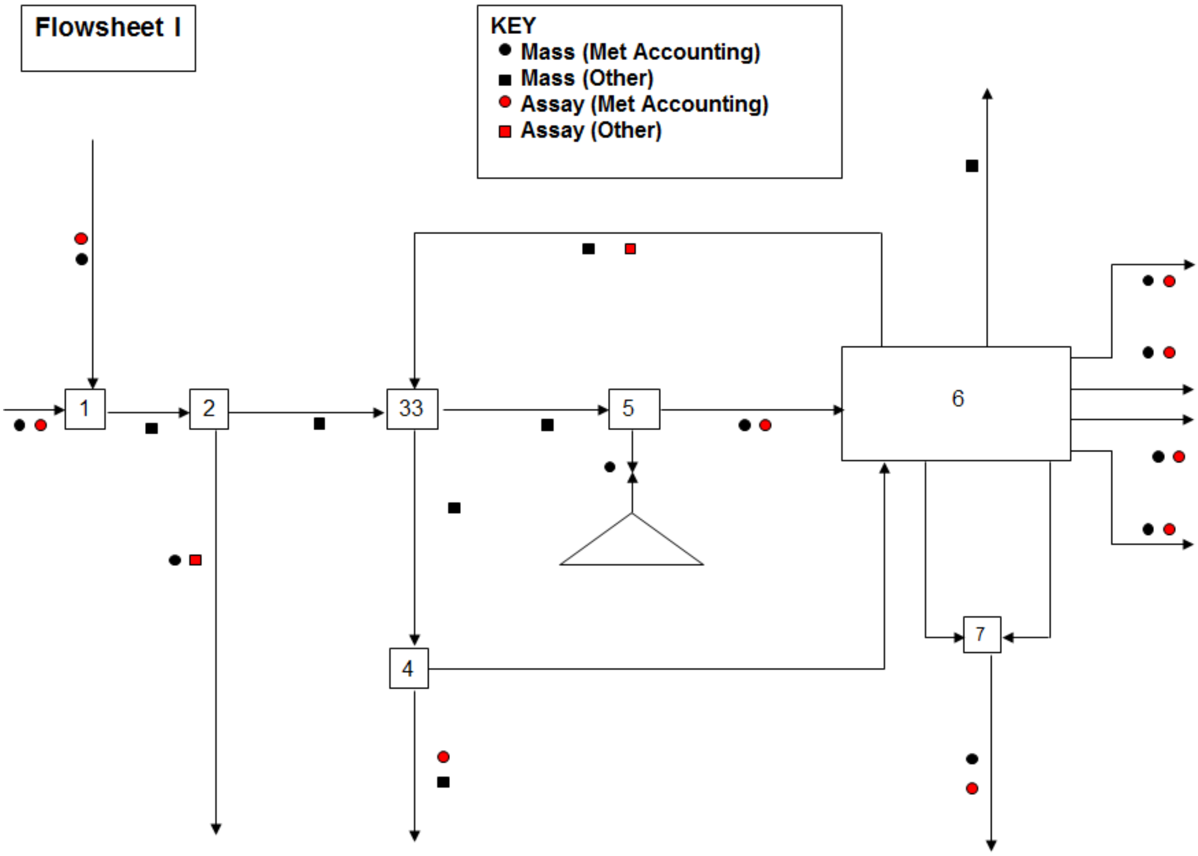


(Flowsheet H nodal diagram)

(Flowsheet H stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	2	2	1	2	1
In-process	11	1	1	1	1
Internal. Recycle	5	1	1	1	1
External Recycle	0	0	0	0	0
Intermediate. Product	0	0	0	0	0
Final Product	4	4	4	4	4
Final Tailings	4	0	0	1	0
TOTAL	26	8	7	9	7

A.9 Flowsheet I nodal diagram and measurement statistics



(Flowsheet I nodal diagram)

(Flowsheet I stream mass and assay measurement statistics)

Stream type	Total No. of stream type	Mass		Assay	
		No of streams weighed	No, of stream masses used in MA	No of streams assayed	No, of stream assays used in MA
Fresh Feed	2	2	2	2	2
In-process	8	4	1	1	1
Internal. Recycle	1	1	0	1	0
External Recycle	0	0	0	0	0
Intermediate. Product	1	1	1	1	1
Final Product	4	4	4	4	4
Final Tailings	4	4	2	3	3
TOTAL	20	16	10	12	11