

**BEHAVIOUR OF REINFORCED CONCRETE DEEP BEAMS**

by J.R. GREEN, B.Sc.(Civil) Engineering, Cape Town

Submitted to the University of Cape Town in partial  
fulfilment of the requirements for the Degree of  
Master of Science in Engineering. September 1985.

The University of Cape Town has been given  
the right to reproduce this thesis in whole  
or in part. Copyright is held by the author.

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.

DECLARATION OF CANDIDATE

I, JEREMY ROBERT GREEN, hereby declare  
that this Thesis is my own work and that  
it has not been submitted for a Degree  
at another University.

Signed by candidate

J.R. GREEN

SEPTEMBER 1985

## CONTENTS

	<u>Page</u>
Synopsis	(i)
Acknowledgements	(ii)
Notation	(iii)
1. INTRODUCTION	1
2. REVIEW OF CODES OF PRACTICE AND DESIGN GUIDES	
2.1 General	2
2.2 ACI 318-83	4
2.3 PCA	7
2.4 CEB-FIP Recommendations 1970	9
2.5 BS CP110	12
2.6 IS 466	15
2.7 Kong et. al.	18
2.8 CEB-FIP 1978	21
2.9 AS 1480 1982	24
2.10 Summaries	27
3. EXPERIMENTAL PROGRAMME	
3.1 General	30
3.2 Experimental Results	31
4. CODE OF PRACTICE AND DESIGN GUIDE PREDICTIONS	
4.1 General	37
4.2 Comparison of Series 1 failure loads with predicted ultimate strength	40
4.3 Comparison of Series 2 failure loads with predicted ultimate strength	41
4.4 Comparison of Series 3 failure loads with predicted ultimate strength	42
4.5 Comparison of Series 5 failure loads with predicted ultimate strength	42

4.6	Factor of Safety	43
5.	FACTORS INFLUENCING DEEP BEAM BEHAVIOUR	
5.1	Concrete Compressive Strength	47
5.2	Reinforcement	51
5.3	Shear Span to Depth Ratio ( $a/d$ )	57
5.4	Absolute Beam Depth	61
5.5	Bond	63
5.6	Support Conditions	67
5.7	Shear Failure Modes	69
6.	CONCLUSIONS	73
7.	REFERENCES	75
8.	Courses Completed in Partial Fulfilment of the Requirements for the Degree	80

## SYNOPSIS

Twenty five model beams were progressively loaded to failure in order to investigate the influence of the following variables on the behaviour of reinforced concrete deep beams :

- i) Concrete compressive strength
- ii) Reinforcement
- iii) Geometry

### SPAN,

The model beams were all of 1500mm with a depth of 750mm. This span to depth ratio of 2 corresponds to the upper limit, to which the recommendations for deep beam design applies, as provided by many current codes of practice.

Methods currently in use for the design of reinforced concrete deep beams were reviewed and compared. The experimental results were compared with the predictions of these design methods. This comparison revealed a large lack of agreement in the predictions of the cracking and ultimate strengths of deep beams.

## ACKNOWLEDGEMENT

The author wishes to thank Professor M.O. de Kock of the Department of Civil Engineering, University of Cape Town, for extremely valuable comments, discussions and assistance during all stages of this study. Sincere thanks are also due to the staff of the Civil Engineering workshop for assistance with the experimental work.

## NOTATION

$A_b$	Area of bearing
$A_c$	Area of concrete
$A_s$	Area of main longitudinal reinforcement
$A_{sh}$	Area of horizontal web reinforcement
$A_{sv}$	Area of vertical web reinforcement
$a$	Shear span
$b$	Width of beam
$b_t$	Width of beam at level of tension reinforcement
$C$	Factor with subscript, defined where used
$d$	Overall beam depth
$d_e$	Effective depth to centroid of main tensile reinforcement
$F_{cu}$	Actual cube strength at time of testing of beam
$F_e$	Design stress of concrete, as defined where used
$F_s$	Design stress of reinforcement, as defined where used
$F_y$	Actual yield stress of reinforcement in beams tested
$f$	Factor defined where used
$f_{calc}$	Calculated tensile strength of concrete $(0,56 \sqrt{F_{cu}})^1$
$f_{cu}$	28 day characteristic cube strength of concrete
$f'_c$	28 day characteristic cylinder strength of concrete

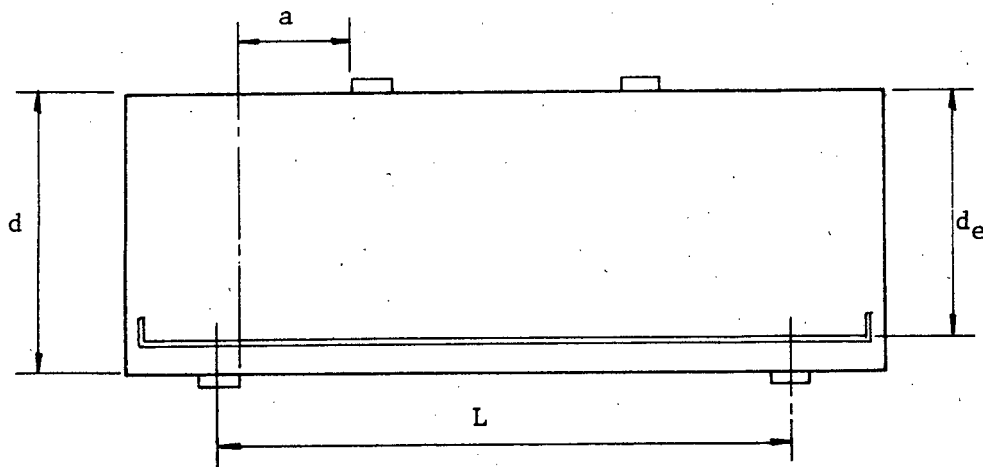


$f_t$	Concrete tensile strength, obtained by indirect tensile test
$f_y$	Characteristic yield stress of reinforcement
L	Span. Definition for design purposes, varies between codes of practice
l	Length of support
M	Bending moment
$M_a$	Applied bending moment
$M_r$	Moment of resistance of main tensile reinforcement
P	Applied point load
$R_u$	Ultimate reaction
r	Reinforcement ratio ( $A_s/bd_e$ )
$r_v$	Reinforcement ratio of vertical web reinforcement
$r_h$	Reinforcement ratio of horizontal web reinforcement
s	Factor defined where used
$s_h$	Spacing c/c of horizontal web reinforcement
$s_v$	Spacing c/c of vertical web reinforcement
T	Tensile force
V	Shear force
$V_c$	Shear capacity of concrete plus main longitudinal reinforcement. (No web reinforcement)
$V_{cr}$	Shear causing visible inclined cracking
$V_s$	Shear capacity of web reinforcement

$V_{test}$	Ultimate shear capacity of beams tested
$V_u$	Shear force due to ultimate load
$v_c$	Unit shear strength of concrete section
$v_{cr}$	Nominal stress at cracking ( $v_{cr} = V_{cr}/bd$ )
$v_u$	Nominal shear stress ( $V_u/bd_e$ )
$x$	Depth of rectangular stress block
$y$	Depth to web bar
$z$	Lever arm of main longitudinal reinforcement

GREEK LOWER CASE

$\alpha$	Angle, defined where used
$\gamma$	Safety factor
$\epsilon$	Strain
$\mu$	Coefficient of friction
$\sigma$	Stress
$\sigma_b$	Bearing stress
$\phi$	Bar diameter or capacity reduction factor, defined where used



## 1. INTRODUCTION

The customary design procedure adopted for the design of reinforced concrete sections subjected to flexure is based on Navier's assumption of a straight line strain distribution. This assumption is a reasonable reflection of the behaviour displayed by beams with fairly large span/depth ratios ( $L/d > 5$ )<sup>1</sup>. When span/depth ratios are reduced below about 3 for simple beams, strain distributions deviate considerably from a straight line.<sup>27</sup> The behaviour of deep beams is therefore significantly different from that of beams of more usual proportions.

As a result of their proportions the strength of deep beams is usually governed by shear rather than flexure, provided normal quantities of longitudinal reinforcement are used. Brittle failure of concrete due to tensile cracking is much more difficult to predict than ductile failures. Current codes of practice vary substantially in their predictions of the ultimate strength of deep beams which, expressed in terms of shear strength per unit depth, exceeds that of shallow beams by substantial margins.

The writer's purpose is firstly, to describe an investigation of the shear strength and behaviour of 25 moderately deep beams ( $L/d = 2,0$ ), and secondly, to discuss those factors which govern the behaviour of reinforced concrete deep beams. The results of the tests will then be compared with the predictions of current codes of practice and design guides.

## 2. REVIEW OF CODES OF PRACTICE AND DESIGN GUIDES

### 2.1 General

Methods used in current practice for the design of reinforced concrete deep beams, are not founded on a rational general theory or design procedure. The design procedures proposed in different codes of practice and by different research bodies, generally comprise a collection of restrictive empirical equations. This dependance on empirical equations is largely due to the absence of a consistent central philosophy or rational model on which to base the design for shear forces in reinforced concrete members.

If the empirical equations proposed were to take account of all the variables influencing the shear strength of deep beams, it would imply an understanding beyond that displayed by a comparison of code predictions with experimentally obtained test results. This is a plausible argument for the adoption of the simple restrictions on the permissible shear stress, as recommended by codes of practice such as CEB-FIP<sup>8</sup> and IS 466.<sup>4</sup> However, these were written before or without taking account of the vast quantity of experimental results obtained by researchers such as; Kong et. al. 11, 12, 13 & 14, Rawdon de Paiva, H., 32, Kani, G. 37; Smith, K. 21 & 23 and Vantsiotis, A. 23. These and other test results have made it possible for codes of practice such as ACI 318-83<sup>1</sup> and researchers such as Kong et. al. to propose guidance for the design of deep beams, which include variables such as  $a/d$  ratio which were previously frequently ignored.

The application of the empirical equations for shear design, provided by the different codes of practice, leads to safe design; however, the factor of safety is inconsistent. This inconsistency is most marked in the predictions of those codes of practice which do not take account of variables such as the  $a/d$  ratio and/or the quantity of tensile reinforcement.

There are numerous complex factors influencing the behaviour and shear strength of reinforced concrete deep beams. They include :

#### i) CONCRETE

Compressive strength

Tensile strength

Placement, curing and environment

ii) REINFORCEMENT

Tensile, compressive and transverse (web) reinforcement

Quantity and arrangement

Detailing

Characteristic strength

Bond characteristics

iii) GEOMETRY

Penetrations (web openings)

Span/depth ( $L/d$ ) ratio

Shear span/depth ( $a/d$ ) ratio

Absolute beam depth

iv) STRUCTURAL RESTRAINTS

Direct/indirect support and loading

Interaction of beam with other elements in structural system (continuity)

It is clear that a review of all the above factors is beyond the scope of this thesis. The writer therefore elected to briefly examine a few of the preceding variables. See Section 5.

---

"EMPIRICAL" versus "RATIONAL" Formula (KANI, G. 40)

EMPIRICAL FORMULA - After a relatively large number of test results have been obtained, arbitrary variables (co-ordinates) are chosen so that the results can be presented in a diagram and a mean value line can be plotted. A mathematical expression which in the same diagram produces a line reasonably close to the mean value line is an empirical formula.

RATIONAL FORMULA - If a relationship of two or more variables has been established, designed or even invented mainly by a process of reasoning and if this relationship is expressed in mathematical symbols, the result is a rational formula.

---

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.2 ACI. 318.83 BUILDING CODE REQUIREMENTS FOR REINFORCED CONCRETE

#### 2.2.1 General

Special recommendations for the design of reinforced concrete deep beams are given in the 1983 ACI Code. These provisions emphasize the importance of the capacity of the deep beam to resist shear force.

The special provisions for the design for shear forces apply to both simple and continuous beams when the span/depth ratio is less than 5 and the load is applied at the compression face. (Clause 11.8.1). The Code also recommends, when the span/depth ratio is less than 2,5 for continuous spans, or 1,25 for simple spans that the design shall take into account nonlinear distribution of strain and consider the possibility of lateral buckling. (Clause 10.7)

The design calculations for shear are carried out at the critical section, which is defined as being at 0,15 of the clear span from the face of the support for uniformly loaded deep beams and midway between the load and the face of the support for concentrated loads (Clause 11.8.4). No guidance is provided as to the position of the critical section of a beam subjected to a combination of concentrated and uniformly distributed load.

The recommendations for flexural design calculations do not differentiate between deep beams and beams of more usual span/depth ratios. Flexural strength can be predicted with sufficient accuracy using the concept of the equivalent rectangular compressive stress block. As the ultimate strength of deep beams depends on tied-arch action special attention must be paid to the anchorage of longitudinal tensile reinforcement.

The recommendations were based mainly on the experimental work carried out in America by Christ, de Paiva and Siess.

The design calculations recommended by this Code of Practice are done for the ultimate limit state. The equations provided in this summary have been modified in order to relate to concrete compressive strength as measured by the crushing of cubes. ( $f'_c = 0,8 f_{cu}$ )<sup>10</sup>

## DESIGN PROCEDURE

### 2.2.2 SHEAR DESIGN

First calculate the nominal shear stress ( $v_u$ )

$$v_u = \frac{V_u}{\phi b d_e} \quad (\phi = 0,85)$$

By suitable selection of the dimensions of the beam (b,d,) the designer must ensure that  $v_u$  does not exceed the following limits.

$$v_u \leq 0,60 \sqrt{F_{cu}} \quad \text{Span/Depth} < 2 \quad \text{CLAUSE 11.8 (EQU. 11.28)}$$

$$v_u \leq 0,05(10 + L/d) \sqrt{F_{cu}} \quad 2 \leq \text{Span/Depth} \leq 5$$

Next calculate the shear strength of the concrete section ( $v_c$ )

$$v_c = 3,52(1 - 0,71 \frac{M_u}{V_u d}) (0,14 \sqrt{F_{cu}} + 17,29R \frac{V_u d}{M_u}) \quad \text{CLAUSE 11.8 (EQU. 11.30)}$$

The term

$$3,52(1 - 0,71 \frac{M_u}{V_u d})$$

shall not exceed 2,5

And  $v_c$  shall not be taken greater than

$$0,45 \sqrt{F_{cu}}$$

A conservative approximation of  $v_c$  can be made by

$$v_c = 0,15 \sqrt{F_{cu}} \quad \text{(EQU. 29)}$$

The design calculations for shear are to be carried out for the "critical section". See Clause 11.8.4.

The Code specifies a minimum orthogonal mesh of web reinforcement when  $v_u$  is  $\leq v_c$

Minimum vertical web reinforcement is 0,15%  $b \cdot s_v$  CLAUSE 11.8.8  
 ie. 0,075% per face with a spacing not  $\geq d/5$

Minimum horizontal web reinforcement is 0,25%  $b \cdot s_h$  CLAUSE 11.8.9  
 ie. 0,125% per face, with a spacing not  $\geq d/3$

When  $v_u > v_c$  then the orthogonal mesh of web reinforcement shall also satisfy the requirements of Clause 11.8.7 (EQU. 11.31)

$$\frac{A_{sv}}{s_v} \left( \frac{1 + L/d}{12} \right) + \frac{A_{sh}}{s_h} \left( \frac{11 - L/d}{12} \right) = \frac{(v_u - v_c) b}{F_y} \quad F_y \text{ not to be taken } \geq 415 \text{ MPa}$$

### 2.2.3 FLEXURAL DESIGN

The Code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling must be considered.

The area of principal tension reinforcement ( $A_s$ ) shall be calculated as follows:

$$A_s = \frac{M_u}{0,87 \cdot F_y \cdot z}$$

The lever-arm  $z$  is the distance between the centroid of the equivalent rectangular stress block and the centroid of the main flexural reinforcement which must be distributed in the zone of flexural tension in accordance with Clause 10.6.7.

The spacing of lateral supports to the compression face of a beam is limited to  $50 b$  to ensure lateral stability. (Clause 10.4).



## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.3 PCA, CONCRETE INFORMATION ST66: DESIGN OF DEEP GIRDERS

#### 2.3.1 General

The recommendations of this publication are based on the mathematical analysis by F. Dischinger who used the classical theory of elasticity and assumed the beam to be homogeneous. Therefore Dischinger's results do not accurately predict actual deep beam behaviour. However because of the inclusion of factors of safety, the method proposed by the PCA is likely to be conservative.

The provisions apply to beams with span/depth ratios less than 1,25 for simple beams and 2,5 for continuous beams, suggesting that for higher span/depth ratios, the straight-line stress distribution is satisfactory as a basis for design.

The design calculations are carried out for the serviceability limit state with a suggested function limiting maximum shear stress which provides an adequate factor of safety. The allowable stress in the principal flexural reinforcement is left to the judgement of the designer. The recommendations suggest an orthogonal mesh of reinforcement is only required if the appearance of the faces of the beam is of importance. This reinforcement should be of small diameter bars at close centres, total amount for both faces to equal 0,25% horizontal and 0,15% vertical.

The recommendations refer the designer to other codes of practice for guidance on serviceability limit state design stresses.

### 2.3.2 FLEXURAL DESIGN

The tensile force  $T$  to be resisted by the main longitudinal steel ( $A_s$ ) is obtained from a graph with different curves for various values of  $E$ . The plot is  $d/L$  versus  $T$ , where  $T$  is expressed as a function of the applied load.  $E$  is typically the ratio of the support length to the span.

$$A_s = T/F_s$$

Where  $F_s = 0,55 F_y$

TABLE 11

BRITISH STANDARD CODE OF PRACTICE  
CP 114 : PART 2 : 1969  
REINFORCED CONCRETE IN BUILDINGS

The area of steel ( $A_s$ ) as calculated is to be detailed and fixed below a depth  $d_0$  read off relevant graph in Figures 5 or 9 of the PCA Design Guide.

### 2.3.3 SHEAR DESIGN

The applied unfactored shear force ( $V_s$ ) shall not exceed that given by the following equation:

$$V_s \nrightarrow 0,4 b.d (1 + 5 d/L) v$$

Where  $v$  is the allowable shear stress for an ordinary beam of similar quality concrete:

$$v = \frac{F_{cu}}{50} + 0,27 \nrightarrow 0,9$$

( $F_{cu} > 20$  MPa)

TABLE 10

BRITISH STANDARD CP 114:PART 2 : 1969

or

$$v = 0,1 \sqrt{F_{cu}}$$

APPENDIX B. CLAUSE B.3.1.

ACI 318.83

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.4 CEB-FIP (1970) APPENDIX 3

#### 2.4.1 General

Special recommendations for the design of reinforced concrete deep beams are given in the 1970 CEB-FIP rules. These provisions emphasise the importance of careful detailing of reinforcement to ensure calculated load carrying capacity of the beam.

The provisions apply to beams with span/depth ratios less than 2,0 for simple beams and 2,5 for continuous beams (Clause 1,0) The rules provide detailed recommendations for design and detailing, differentiating between beams loaded at the compression edge, tension edge or on the face of the beam.

The flexural design is based on a reduced lever arm ( $z$ ) which is expressed as a function of the span and depth of the beam. The expression for  $z$  takes into account the position of the main reinforcement which is specified in the detailing recommendations. The instability or lateral buckling of the compression zone should be examined, although the compressive stress due to bending is rarely critical. (Clause 5.1)

The recommendations recognise that the state of high shear and compressive stress near the support is critical and therefore specify a maximum equivalent nominal shear stress as well as placing a limit on the maximum bearing stress. The recommendations centre on flexural design and do not give specific guidance on how to calculate the web steel required for shear strength.

These recommendations were based mainly on the tests carried out in Germany by Leonhardt and Walther, though they could have been influenced by the earlier tests carried out in Sweden by Nylander & Holst.

The design calculations to be carried out in compliance with these recommendations are for the ultimate limit state. The limits on permissible stress recommended by the original document are based on cylinder compressive strength. For ease of application the equations in this summary have been modified in order to relate to cube compressive strength. ( $f'_c = 0,8 f_{cu}$ )<sup>10</sup>

## 2.4.2 FLEXURAL DESIGN

The area of principal tension reinforcement ( $A_s$ ) shall be calculated as follows:

$$A_s = \frac{M_u}{0,87 F_y z}$$

With the lever arm ( $z$ ) being taken for simply supported beams as:

$$z = 0,2(L + 2d) \quad 1 \leq L/d < 2$$

or

$$z = 0,6L \quad L/d < 1 \quad \text{CLAUSE 3.1}$$

And for continuous beams the lever arm ( $z$ ) is defined as:

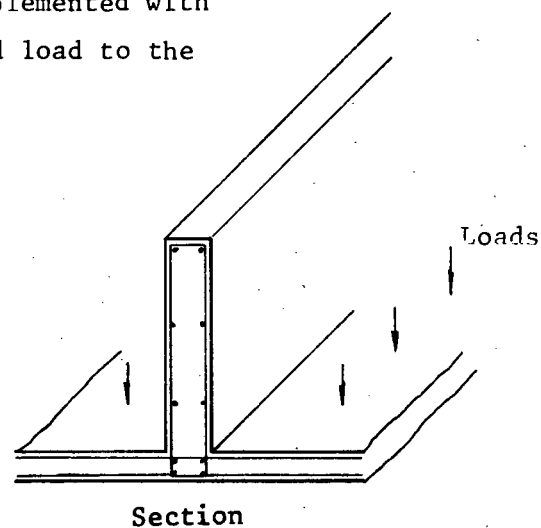
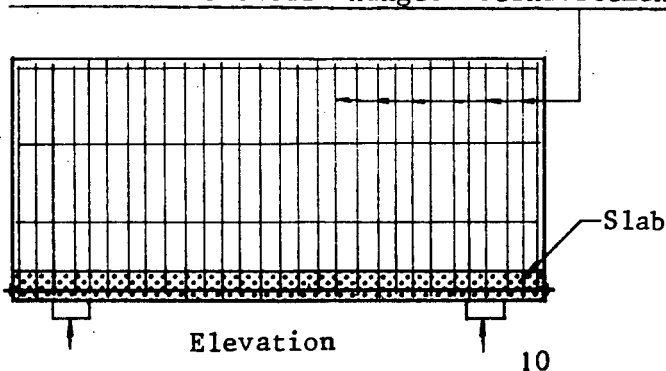
$$z = 0,2(L + 1,5d) \quad 1 \leq L/d < 2,5$$

$$z = 0,5L \quad L/d < 1$$

## 2.4.3 WEB REINFORCEMENT

The recommendations specify an orthogonal mesh of web reinforcement equal to 0,25% per face for smooth reinforcement and 0,20% per face for high-bond reinforcement. This recommendation applies to deep beams loaded at the compressive edge. When the load is applied to the tensile edge the above specified mesh shall be supplemented with additional stirrups to transmit the applied load to the upper portion of the beam.

Additional vertical "hanger" reinforcement



## DESIGN PROCEDURE

### 2.4.4 SHEAR DESIGN

First calculate the nominal shear stress ( $v_u$ )

$$v_u = \frac{V_u}{bd}$$

By suitable selection of the dimensions of the beam (b,d) the designer must ensure that  $v_u$  does not exceed the following limit.

$$v_u < 0,06 F_{cu}$$

CLAUSE 5.2

### 2.4.5 BEARING DESIGN

First calculate the nominal bearing stress ( $\sigma_b$ )

$$\sigma_b = f \left[ \frac{R_u}{A_b} \right]$$

Where :  $f = 1,0$  simple span

$f = 1,1$  continuous span

$R_u =$  Ultimate support reaction

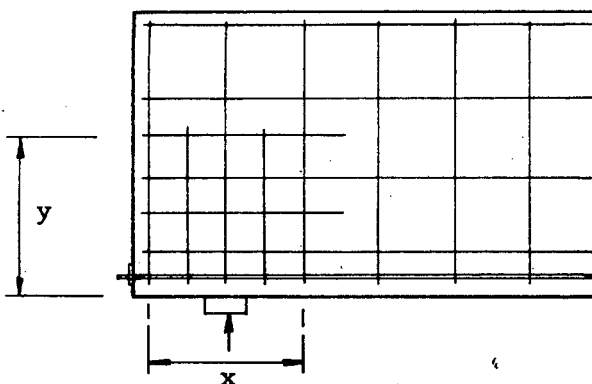
The bearing area  $A_b$  is defined in Clause 7,0 as a function of the beam width, flange depths, and length of support. See 2.6.3

The designer must ensure that  $\sigma_b$  does not exceed the following limit.

$$\sigma_b < 0,43 F_{cu} \quad \text{for end supports}$$

CLAUSE 7.0

$$\sigma_b < 0,64 F_{cu} \quad \text{for interior supports}$$



#### Additional support reinforcement

$x$  is lesser of  $0,3d$  or  $0,3L$

$y$  is lesser of  $0,5d$  or  $0,5L$

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.5 CP 110 THE STRUCTURAL USE OF CONCRETE

#### 2.5.1 General

No specific recommendations for the design of reinforced concrete deep beams are given in the 1972, CP 110 as amended in 1977, however compliance with the provisions for dealing with concentrated loads near supports will adequately cover the design of most deep beams.

The special provisions for dealing with concentrated loads near supports implicitly recognises the enhanced shear strength resulting from any shear plane being forced to be inclined at a steeper angle than for beams of greater  $L/d$  values, loaded in a more usual manner. (See Clause 3.3.6.2.). This characteristic will be common to most deep beams ( $L/d < 2,0$ ).

The code also provides guidance for the provision of additional horizontal side steel in beams of depth greater than 750mm. (Clause 3.11.8.2.(4)) which in effect stipulates a minimum proportion of horizontal (web) steel of  $0,8/f_y$  per face. A minimum percentage of vertical (web) steel is stipulated in Clause 3.11.4.3.

The design calculations for flexure recommended for beams of standard proportions will result in deep beams of adequate flexural strength. In order to ensure lateral stability the code imposes a limit on the clear distance between lateral supports which is a function of the beam proportions. (Clause 3.3.1.3.).

The design calculations recommended by this code of practice are done for the ultimate limit state.

## 2.5.2 DESIGN PROCEDURE

### SHEAR DESIGN

First calculate the nominal shear stress  $v_u$

$$v_u = \frac{V_u}{b \cdot d_e} \quad \text{EQU. (8)}$$

By suitable selection of the dimensions of the beam (b,d) the designer must ensure that  $v_u$  does not exceed the following limit :

$$v_u < 0,75 \sqrt{F_{cu}} \quad \text{(As tabulated in Table 6)}$$

After calculating the quantity of longitudinal flexural steel the quantity of vertical web reinforcement can be calculated from :

$$\frac{A_{sv}}{sv} \geq \frac{b(v_u - v_c)}{0,87 F_y} \quad \begin{array}{l} F_y \text{ not to be} \\ \text{taken} > 425\text{MPa} \end{array} \quad \text{Clause 3.3.6.1.}$$

Provided that :

$$\frac{A_{sv}}{sv} \geq 0,0012b_t \quad \text{(High yield stirrups)}$$

or

$$\frac{A_{sv}}{sv} \geq 0,002b_t \quad \text{(Mild steel stirrups)} \quad \text{Clause 3.11.4.3}$$

Where  $v_c$  is the equivalent average unit shear strength of the concrete section plus the main flexural reinforcement as tabulated in Table 5. For deep beams  $v_c$  is to be increased by the factor  $2d/a$  provided this does not exceed  $0,75 \sqrt{F_{cu}}$  (Clause 3.3.6.2.) This provision is interpreted as applying only to beams which satisfy the conditions of direct support. See 5.6

The value of  $v_c$  in Table 5, is the shear stress at which diagonal cracking in beams of normal proportions subjected to common loading patterns (eg. U.D.L.) is estimated to occur.

TABLE 5

Concrete Grade \ % $A_s$	20	25	30	40 or more
0,25	0,35	0,35	0,35	0,35
0,50	0,45	0,50	0,55	0,55
1,00	0,60	0,65	0,70	0,75
2,00	0,80	0,85	0,90	0,95

### 2.5.3 FLEXURAL DESIGN

The area of main tension reinforcement ( $A_s$ ) shall typically be calculated with equation 1, as the compression stress due to bending is never critical in a beam of  $L/d \leq 2,0$ .

$$M_u = 0,87.F_y.z.A_s \quad (A_s \geq 0,15\%b.d_e) \quad \text{EQU. 1}$$

Where  $z$  is the lever arm calculated from an equivalent rectangular compressive stress block in conjunction with the actual  $d_e$  of the beam.

Additional horizontal reinforcement as specified in Clause 3.11.8.2. must be provided over a distance of  $2/3$  of the overall beam depth measured from the tension face, for beams deeper than 750 mm.

Side steel bar diameter  $\phi \geq \sqrt{\frac{s_b \cdot b}{F_y}}$  Clause 3.11.8.2.

Where :  $s_b$  is the distance  $c/c$  of side steel  
 $b$  is the beam width  
 $F_y$  is the side steel yield stress



## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.6 IS.466 CONCRETE ELEMENTS AND STRUCTURES

#### 2.6.1 General

Special recommendations for the design of reinforced concrete deep beams are given in this 1979 Israeli Code. These provisions appear to be largely based on the 1970 CEB-FIP recommendations

The flexural design is based on a reduced effective depth which is expressed as a function of the span and the actual depth. Adequacy of shear strength is ensured by restricting the stress due to the principal compressive force to a specified level.

The provisions apply to beams with span/depth ratios less than 2,0 for simple beams and 2,5 for continuous beams.(Clause 26.1). In common with the 1970 CEB-FIP rules, detailed recommendations for the distribution and detailing of the reinforcement are provided.

The Code specifies a minimum orthogonal mesh of reinforcement, which has to be supplemented with additional reinforcement over supports. This permitted minimum is low when compared with other current codes and it is suggested that steel in excess of that specified may be required if the beam is subjected to high or widely fluctuating temperature.

The design calculations recommended by this code of practice are done at the ultimate limit state.

## 2.6.2 FLEXURAL DESIGN

The area of principal tension reinforcement shall be calculated as follows:

$$A_s = \frac{M_u}{0,87 F_y z} \quad \text{EQU. 140}$$

With the lever-arm ( $z$ ) being taken for simply supported beams as:

$$z = 0,2 (L + 2d_e) \quad \text{EQU. 141}$$

And for continuous beams the lever-arm ( $z$ ) is defined as:

$$z = 0,2 (L + 1,5d_e) \quad \text{EQU. 142}$$

Where the effective depth ( $d_e$ ) to be used is defined as:

$d_e$  = Actual effective depth, for  $d_e \leq L$

$d_e$  = Span ( $L$ ) for  $d_e > L$

CLAUSE 26.2.2

## 2.6.3 BEARING DESIGN

To ensure that crushing at a support does not occur the bearing stress  $\sigma_b$  must comply with :

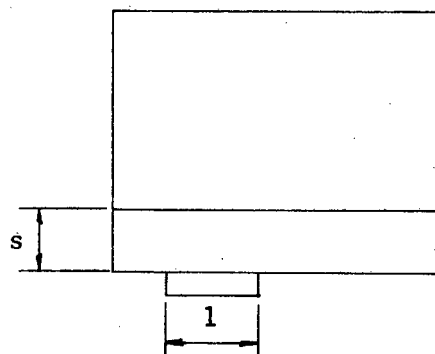
$$\sigma_b = \frac{1,25 (R_u)}{(1 + s) b} < F_e \quad \text{EQU. 138}$$

$R_u$  = Ultimate reaction

$F_e$  =  $0,4 F_{cu}$  (See Table 12, Part 1)

$l$  = Length of support

$s$  = Thickness of intermediate layer of concrete, as for example a floor.



#### 2.6.4 SHEAR DESIGN

To prevent failure of the web due to the principal compressive force, the width (b) must comply with the following condition:

CLAUSE 26.2.4

$$b \geq \frac{4 V_u}{d_e F_e}$$

EQU. 137

This is equivalent to ensuring that the nominal shear stress  $v_u$  does not exceed the limit stipulated by:

$$v_u < 0,1 F_{cu}$$

(0,1  $F_{cu}$  corresponds to an estimate of concrete tensile strength. Kong et. al.<sup>11</sup>)

Where;

$$v_u = \frac{V_u}{b d_e}$$

#### 2.6.5 WEB REINFORCEMENT

The recommendations specify an orthogonal mesh of web reinforcement near both faces of the beam.

The reinforcement ratio (r) of both the vertical and the horizontal steel in each face must equal or exceed the following:

$$r_v = r_h = \frac{0,3}{F_s} \quad (\text{Where } F_s = F_y/1,15) \quad \text{EQU. 143}$$

This recommendation applies to deep beams loaded at the compressive edge. When the load is applied to the tensile edge the above specified mesh shall be supplemented with additional stirrups to transmit the applied load to the upper portion of the beam (See 26.2.5.4.)<sup>4</sup>

See diagram in section 2.4.3.

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.7 F. Kong, P. Robins & G. Sharp<sup>11</sup>

#### 2.7.1 General

The method of design of reinforced concrete deep beams proposed by the Authors, is based on a series of ultimate load test of about 270 normal weight concrete beams. Application of the proposed formula is restricted to beams with a span to depth ratio ( $L/d$ ) less than 3 and with a shear span to depth ratio ( $a/d$ ) between 0,23 and 0,70. For a beam subjected to a uniformly distributed load a shear span to depth ratio of  $L/4d$  is suggested.

The recommendations emphasize the importance of the capacity of the deep beam to resist shear force. Unlike other design formulae the proposed method recognises that the shear span/depth ratio ( $a/d$ ) has a major influence on the shear strength, it is also suggested that the concrete cylinder splitting tensile strength ( $f_t$ ) is more directly related than the cube strength ( $F_{cu}$ ) to the ultimate shear strength. The proposed formula is also used to calculate the contribution to ultimate shear strength, made by both the vertical and horizontal web reinforcement.

It is recommended that the proposed design formula is used in conjunction with a recognised current Code of Practice, as guidance is not provided for factors such as minimum areas of steel, maximum bearing stress and reinforcement detailing. The Authors also suggest modifications to the formula which can then be used to predict the ultimate strength of deep beams with web openings of a limited size.

These recommendations are the only reference found to implicitly take account of the beneficial influence of good bond characteristics on deep beam behaviour. (See 5.5)

## 2.7.2

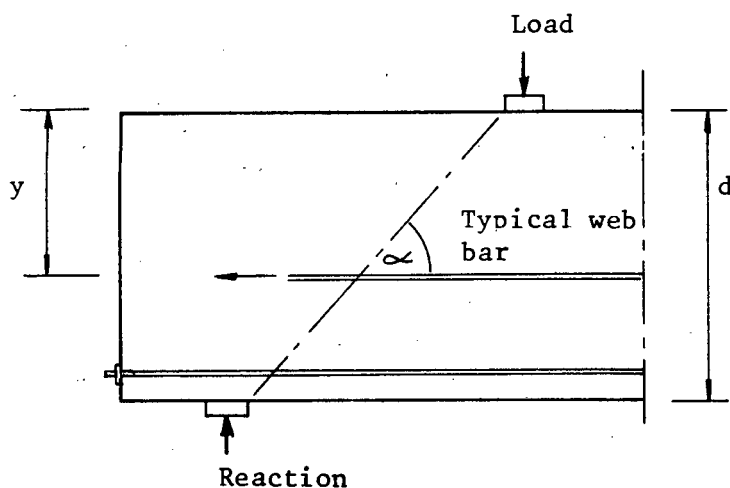
SHEAR DESIGN

The shear force  $V_u$  due to ultimate limit state loads must not exceed the ultimate shear strength ( $Q_u$ ) defined as:

$$Q_u = X_m \left[ C_a (1 - C_c (a/d)) f_t b.d + C_b \sum A(y/d) \sin^2 \alpha \right]$$

NOTATION

- $X_m$  Partial factor of safety, Adopt 0,75
- $C_a$  Empirical Coefficient, 1,4 for normal weight concrete
- $C_b$  Empirical Coefficient, 130 MPa for plain round bars,  
300 MPa for deformed bars.
- $C_c$  Empirical Coefficient, 0,35
- $f_t$  Cylinder splitting tensile strength (MPa)  
Can assume 0,1  $F_{cu}$  (Kong. et. al)  
or  $0,5 \sqrt{F_{cu}}$  (Kong. et.al)  
or  $0,56 \sqrt{F_{cu}}$  (ACI 318.83 Clause 9.5.2.3)
- $a$  Shear span
- $d$  Overall beam depth
- $b$  Beam width
- $y$  Depth from top of beam to intersection of bar being considered with the line joining the inside edge of the support and the outside edge of the load.
- $\alpha$  Angle between the bar being considered and the line described above.



## DESIGN PROCEDURE

### 2.7.3 FLEXURAL DESIGN

The area of principal tension reinforcement ( $A_s$ ) shall typically be calculated with the following equation, which should be adequate as the compressive stress due to bending is rarely critical.

$$A_s = \frac{M_u}{0,87 F_y z}$$

Where :  $z = d_e - 0,2d$

Additional horizontal side steel must be detailed in accordance with a current Code of Practice such as ACI 318-83 (Clause 11.8.8) . This reinforcement as well as the main flexural reinforcement should be included in the assessment of the shear strength of the beam.

### 2.7.4 LIMITATIONS OF THE FORMULA

- A) Applicable to deep beams subjected to static loads applied to the compression (top) face.
- B)  $0,23 < a/d < 0,70$ , which was the range of the tests on which the formula was based. The Authors believe that the  $a/d$  ratio has a greater influence than the  $L/d$  ratio on the behaviour of deep beams.
- C) The main flexural reinforcement should be detailed in such a manner as to provide positive end anchorage with no curtailment in the span.
- D)  $23,6 \text{ MPa} \leq F_{cu} \leq 43,8 \text{ MPa}$ , which was the range of the tests on which the formula was based.

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.8 CEB-FIP 1978 MODEL CODE FOR CONCRETE STRUCTURES

#### 2.8.1 General

No specific recommendations for the design of reinforced concrete deep beams are provided by the 1978 CEB-FIP Code; however compliance with the provisions for dealing with concentrated loads near supports will adequately cover the design of most deep beams. The user of the 1978 CEB-FIP code is referred to Appendix 3 of the 1970 CEB-FIP Recommendations, for guidance on factors such as reinforcement detailing and diagonal compression stress near the bearing zones.

The special provisions for dealing with concentrated loads near supports implicitly recognises the enhanced shear strength resulting from any shear plane being forced to be inclined at a steeper angle than for beams of greater  $L/d$  value, loaded in a more usual manner. (See Clause 11.1.2.3.)

The recommendations for flexural design calculations do not differentiate between deep beams and beams of more usual span/depth ratios. Flexural strength can be predicted with sufficient accuracy using the concept of the equivalent rectangular compressive stress block. The effective depth to be adopted for calculation purposes is the actual depth to the centroid of the main longitudinal reinforcement, distributed in accordance with Appendix 3 of the 1970 CEB-FIP Recommendations. As the ultimate strength of deep beams depends on tied-arch action, special attention must be paid to the anchorage of longitudinal tensile reinforcement.

The design calculations recommended by this code of practice are done for the ultimate limit state. The equations and tables provided in this summary have been modified in order to relate to concrete compressive strength as measured by the crushing of cubes. ( $f'_c = 0,8 f_{cu}$ )<sup>10</sup>

This code of practice was the only reference found to implicitly recognise the influence that absolute depth has on unit shear strength (See k Factor in Clause 11.1.2.1.), and was also the only code found to provide guidance for the calculation of the width of inclined shear cracks.

#### DIRECT SUPPORT

The concentrated load and the support reaction are such as to create diagonal compression in the member. (Clause 11.1.2.3.)

## DESIGN PROCEDURE

### 2.8.2 FLEXURAL DESIGN

The Code does not contain detailed requirements for designing deep beams for flexure except that non-linearity of strain distribution and lateral buckling must be considered. (See Clause 9.2)

The area of main longitudinal tension reinforcement ( $A_s$ ) shall be calculated as follows :

$$A_s = \frac{M_u}{\phi F_y \cdot z} \quad (\phi = 1/1,15 \text{ Clause 6.4.2.3.})$$

The lever-arm ( $z$ ) is the distance between the centroid of the equivalent rectangular stress block and the centroid of the main flexural reinforcement.

### 2.8.3 WEB REINFORCEMENT

The recommendations specify an orthogonal mesh of web reinforcement with a minimum area of the cross section of mesh in each direction equal to :

0,075% per face for high yield reinforcement  
0,125% per face for mild reinforcement

CLAUSE  
18.1.8.

This recommendation applies to beams loaded at the compressive edge. When the load is applied to the tensile edge the above specified reinforcement shall be supplemented with additional stirrups to transmit the applied load to the upper portion of the beam. See 2.4.3 for diagram.



## DESIGN PROCEDURE

### 2.8.4 SHEAR DESIGN

First calculate the nominal shear stress  $v_u$

$$v_u = \frac{V_u}{bd_e}$$

By suitable selection of the dimensions of the beam (b,d) the designer must ensure that  $v_u$  does not exceed the following limits.

$$v_u < v_c \cdot k \cdot (1 + 50r) \cdot f$$

CLAUSE 11.1.2.1

CLAUSE 11.1.2.3

and

$$v_u \leq 0,16 F_{cu}$$

EQU. 11.9

where:

$v_c$  is obtained from Table 11.1 (modified)

$k = (1,6 - d_e) \leq 1,0$ ;  $d_e$  in metres (size effect)

$r$  is the reinforcement ratio ( $A_s/bd_e$ ) not to be taken greater than 0,02.  $A_s$  denotes the area of longitudinal tensile reinforcement anchored beyond the intersection of the steel and the possible inclined shear crack.

$f = 2d/a \geq 1,0$   $f$  to be taken greater than 1,0 only if conditions of direct support are satisfied

TABLE 11.1<sup>52</sup>(modified)

$F_{cu}$ (MPa)	15,0	20,0	25,0	31,3	37,5	43,8	50,0	56,3	62,5
$v_c$ (MPa)	0,18	0,22	0,26	0,30	0,34	0,38	0,42	0,46	0,50

## RECOMMENDATIONS FOR THE DESIGN OF DEEP BEAMS

### 2.9 AS1480-1982 SAA CONCRETE STRUCTURES CODE

#### 2.9.1 General

Special recommendations for the design of reinforced concrete deep beams are given in this Australian Code of Practice.

The provisions for flexural design owe much to the 1970 CEB-FIP rules on which they are based. The flexural design is based on a reduced effective depth which is expressed as a function of the span and the actual depth. AS1480-1982 Code suggests that the shear design of deep beams should be in accordance with the "shear friction" method.

The provisions apply to beams with span/depth ratios less than 2,0 for simple beams and 2,5 for continuous beams. (Clause 9.9.1). In common with 1970 CEB-FIP, the Code provides detailed recommendations for the distribution and detailing of the main flexural reinforcement.

The Code specifies a minimum orthogonal mesh of web reinforcement, which is only applicable if the beam is not subjected to high or widely fluctuating temperatures, in which case reinforcement in excess of the amount calculated on the basis of ultimate strength may be required.

In order to ensure the lateral stability of the compression face the code imposes a limit on the clear distance between lateral supports which is a function of the proportions of the beam.

(Clause 9.8.a)

The design calculations carried out in compliance with the recommendations of this Code are for the ultimate limit state. The limits on permissible concrete stress recommended are based on cylinder compressive strength. For ease of application the equations in this summary have been modified in order to relate to cube compressive strength.  $(f'_c = 0,8 f_{cu})^{10}$

### 2.9.2 FLEXURAL DESIGN

The area of main tension reinforcement ( $A_s$ ) shall be calculated as follows:

$$A_s = \frac{M_u}{\phi F_y z} \quad (\phi = 0,90)$$

CLAUSE 13.2.

The lever-arm ( $z$ ) is,  $d_e$  less half the depth of the assumed rectangular stress block. As the compressive stress due to bending is rarely critical this is generally adequate providing the  $x/d_e$  ratio is limited.

CLAUSE 14.1.

With the equivalent effective depth being defined, for simply supported beams as:

$$d_e = 0,25L + 0,5d \quad L/d \geq 1$$

$$d_e = 0,75L \quad L/d < 1$$

CLAUSE

9.9.2.a

And for continuous beams the effective depth is defined as:

$$d_e = 0,25L + 0,375d \quad L/d \geq 1$$

$$d_e = 0,625L \quad L/d < 1$$

### 2.9.3 BEARING DESIGN

The bearing stress shall not exceed

$$\phi (0,68 F_{cu})$$

CLAUSE 14.6

as modified by Clause 14.6.1

where  $\phi = 0,70$

TABLE 13.2.1

## DESIGN PROCEDURE

### 2.9.4 SHEAR DESIGN

The recommendations for shear design given in section 15 are specifically excluded from application to the design of deep beams. The Code draws attention to the "shear friction" method of design, guidance for which is provided in ACI 318.83 Chapter 11.7 (Recommended for  $a/d < 0,5$ )<sup>39</sup>

AS.1480

CLAUSE 15.2.

#### PROCEDURE

By suitable selection of the dimensions of the beam (b,d) the designer must ensure that  $v_u$  does not exceed the limit stipulated by :

$$v_u = \frac{V_u}{\phi b d_e} < 0,16 F_{cu} \quad \text{and} \quad v_u \neq 5,53 \text{ MPa}$$

ACI 318.83

CLAUSE 11.7.5

The shear resisted by concrete is taken as :

$$V_c = 2,76 b \cdot d_e \text{ SIN}^2 \alpha$$

The difference between the applied shear ( $V_u$ ) and  $V_c$  must be resisted by shear reinforcement.

ACI. 318.83

EQU. 11.27

$$V_s = V_u - V_c = A_w \cdot F_y (\mu \text{ SIN} \alpha + \text{COS} \alpha) \phi$$

Where : 
$$\frac{A_w \cdot F_y \cdot \text{SIN} \alpha}{b d_e} \geq 1,38 \text{ MPa}$$

The Code specifies an orthogonal mesh of web reinforcement which must equal or exceed the minimum permitted by Clauses 11.8.8 and 11.8.9, as well as satisfy the requirements for "shear friction" reinforcement as calculated in accordance with Clause 11.7.4.2.

#### NOTATION

$V_s$	Factored shear force resisted by shear reinforcement
$F_y$	Yield strength of reinforcement not to be taken as greater than 415 MPa.
$\mu$	Coefficient of friction (See Clause 11.7.4.3.)
$\alpha$	Angle between shear-friction reinforcement and assumed shear plane.
$\phi$	Capacity reduction factor (0,85)
$A_w$	Area of web steel crossing shear plane.

CLAUSE 9.3.2.

2.10.1 MAXIMUM SPAN/DEPTH RATIOS\*

CODE	CLAUSE	SIMPLE SPAN	CONTINUOUS SPAN
ACI 318.83	10.7	1,25	2,5
AS. 1480-1982	9.9.1.	2,0	2,5
CEB-FIP -1970	Section 1	2,0	2,5
IS 466 Part 2-1979	26.1	2,0	2,5
KONG. et. al		3,0	3,0
PCA-No. ST66-1954	Section 1	1,25	2,5
CEB-FIP 1978	18.1.8.	2,0	2,0

NOTE:

- \* The tabulated ratios are of the span/depth ratio, below which the recommendations of the Codes and Design Guides are intended to apply.

ACI 318.83, Clause 11.8, Additional recommendations governing the shear design of deep beams applies to beams with  $L/d < 5$

B.S. CP 110:1972 does not define deep beam behaviour in terms of the  $L/d$  ratio but rather emphasizes the importance of the  $a/d$  ratio.

2.10.2 MINIMUM REINFORCEMENT

CODE	MINIMUM MAIN FLEXURAL REINFORCEMENT	TOTAL VERTICAL WEB STEEL	TOTAL HORIZONTAL WEB STEEL
ACI 318.83 CLAUSE 11.8.8, 11.8.9, and 10.5.1	The lesser of $\frac{2}{F_y}^*$ or $\frac{4}{3} A_s$  Required by analysis	0,15% ( $s_v < d/5$ )	0,25% ( $s_h < d/3$ )
AS 1480.1982 CLAUSE 9.10.3 and 9.10.1.1	* $\frac{1,4}{F_y}$ or $\frac{4}{3} A_s$  Required by analysis	0,15% ( $s_v < 300$ )	0,25% ( $s_h < 300$ )
CEB-FIP(1970) CLAUSE 6.1	No guidance provided	0,25% smooth round bar 0,20% high bond bar	
B.S. CP110:1972 CLAUSE 3.11.4.3 and 3.11.4.1	0,25% mild steel 0,15% high yield steel	0,20% mild steel 0,12% high yield steel ( $s_v < 0,75d_e$ )	$0,8/F_y^*$ (See table 24 for $s_h$ )
IS 466-Part 1 EQU. 143	See table 28 Min. % $A_s$ is a function of steel type of concrete strength	$0,35/F_y^*$ ( $s_v < 330$ )	$0,35/F_y^*$ ( $s_h < 330$ )
CEB-FIP :1978 CLAUSE 18.1.8  CLAUSE 18.11.1	0,25% mild steel 0,15% high yield steel	0,25% mild steel 0,15% high yield steel  $s_v < 300$ and $s_v < 2b$ $s_h < 300$ and $s_h < 2b$	

\* All asterisked expressions are reinforcement ratios (dimensionless)

## 2.10.3

EFFECTIVE DEPTH ( $d_e$ ) & LEVER-ARM ( $z$ ) : CODE DEFINITIONS

CODE	SIMPLE SPAN	CONTINUOUS SPAN
ACI 318.83 CL. 10.0 (Notation)	$d_e$ = Actual depth to centroid of flexural reinforcement	
AS 1480-1982 CL. 9.9.2.	$d_e = 0,25L + 0,5d$ ( $d \leq L$ ) $d_e = 0,75L$ ( $d > L$ )	$d_e = 0,25L + 0,375d$ ( $d \leq L$ ) $d_e = 0,625L$ ( $d > L$ )
CEB-FIP 1970 CL. 3.1 CL. 4.1	$z = 0,2L + 0,4d$ ( $1 < L/d < 2$ ) $z = 0,6L$ ( $L/d < 1$ )	$z = 0,2L + 0,3d$ ( $1 < L/d < 2,5$ ) $z = 0,5L$ ( $L/d < 1$ )
B.S. CP 110 1972	$d_e$ = Actual depth to centroid of flexural reinforcement	
IS 466 Part 2 1979 CL. 26.2.2 CL. 26.2.5.2	$d_e$ = Actual depth to centroid of flexural reinforcement ( $d \leq L$ ) $d_e = L$ ( $d > L$ ) $z = 0,2L + 0,4d_e$	$z = 0,2L + 0,3d_e$
KONG. et. al	$d_e$ = Actual depth to centroid of flexural reinforcement  Typically take $z = d_e - 0,2d$	
PCA NO. ST66 1954	Centroid of flexural reinforcement to be at or below a depth ( $d_o$ ) read off a graph. See Fig. 5 & Fig. 9 of PCA Design Guide.	
CEB-FIP 1978 CL. 10.4.3.2 Fig. 10.3	$d_e$ = Actual depth to centroid of flexural reinforcement	

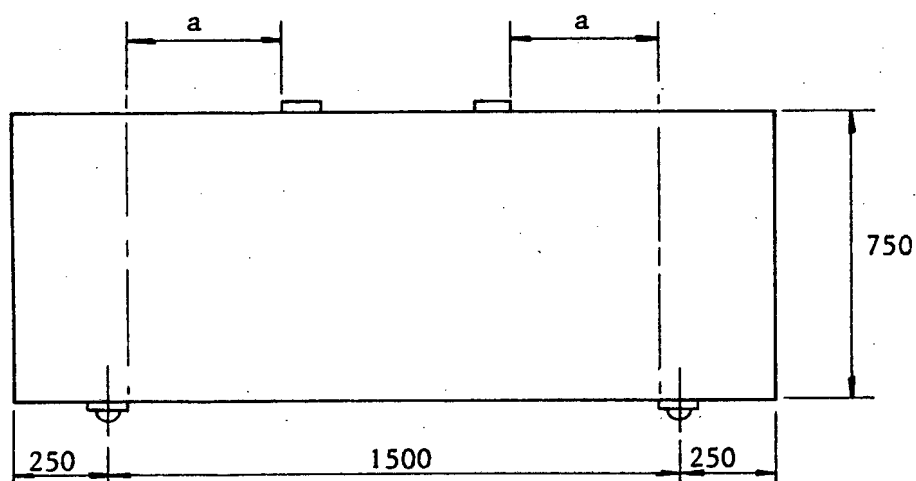
### 3. EXPERIMENTAL PROGRAMME

#### 3.1 General

The test specimens consisting of 25 simply supported reinforced concrete model deep beams were tested to failure in this laboratory investigation.

The beams were cast face down on a plywood shutter in a purpose made steel mould. The concrete was mechanically compacted with an immersion vibrator before the top face was floated off with a steel trowel. Each beam with its associated cubes was cured under wet sacking until it was load tested at 24 days. To facilitate crack observation the beams were painted white.

The beams were loaded on the top compression edge with two equal point loads, applied symmetrically on either side of the beam centreline. The loading was applied by means of a twin hydraulic system, with a capacity of 500kN (50t) per load point. The beams were loaded in discrete increases of 10kN or alternatively they were subjected to a gradually smoothly increasing load. As the actual duration of loading was essentially the same for both methods, this was not believed to influence either cracking or ultimate capacity. During loading the beams were carefully examined for new cracks and changes to existing ones. Loads in kN were written alongside the cracks to provide a record of the extent of cracking at a particular level of load.



TYPICAL BEAM



## SPECIMEN DESCRIPTION

Span	(L)	1500mm	c/c of supports
Depth	(d)	750mm	
Thickness	(b)	75mm	
Span/depth ratio		$L/d$	of 2,0 corresponds to upper limit recommended by many codes of practice for the application of recommendations for deep beam design. See 2.10.1

Overall beam length of 2000mm provided anchorage for main longitudinal reinforcement and avoided superimposing high anchorage stresses directly over the supports.

Load plates 100 x 100 x 25mm

Support plates 100 x 100 x 12mm on 30mm radius semi-circular rockers.

## 3.2 EXPERIMENTAL RESULTS

### 3.2.1 General

The experimental work was divided into five series, each investigating the influence of a specific variable on deep beam behaviour. The physical properties, cracking and ultimate loads of these five series are tabulated in 3.2.2 to 3.2.7.

The series and the variable under investigation are :

Series 1, Concrete compressive strength. ( $16,5 \text{ MPa} = F_{cu} = 37,3 \text{ MPa}$ )

Series 2, Web reinforcement

Series 3, Longitudinal reinforcement

Series 4, Bond

Series 5,  $a/d$  ratio ( $0,3 < a/d < 0,8$ )

BEAM	$F_{cu}$ (MPa)	$f_{calc}$ (MPa)	$V_{cr}$ (kN)	$V_u$ (kN)	$v_{cr}$ (MPa)	$\frac{V_{cr}}{f_{calc}}$	$\frac{V_{cr}}{V_u}$
1/1	16,5	2,27	70	115	1,24	0,55	0,61
1/2	21,5	2,60	95	130	1,69	0,65	0,73
1/3	22,0	2,63	90	125	1,60	0,61	0,72
1/4	26,7	2,89	105	170	1,87	0,65	0,62
1/5	29,7	3,05	110	170	1,96	0,64	0,65
1/6	31,1	3,12	100	185	1,78	0,57	0,54
1/7	31,3	3,13	100	170	1,78	0,57	0,59
1/8	37,3	3,42	110	190	1,96	0,57	0,58

NOTE:

i) Main flexural reinforcement is 4Y10's,  $F_y = 505$  MPa,  $d_e = 685$  mm

ii) No web reinforcement

iii)  $a/d = 0,5$

3.2.2

SERIES 1, PHYSICAL PROPERTIES AND EXPERIMENTAL RESULTS

BEAM	$F_{cu}$ (MPa)	$f_{calc}$ (MPa)	$V_{cr}$ (kN)	$V_u$ (kN)	$v_{cr}$ (MPa)	$\frac{v_{cr}}{f_{calc}}$	$\frac{V_{cr}}{V_u}$
2/1	17,4	2,34	80	150	1,42	0,61	0,53
2/2	18,3	2,40	85	145	1,51	0,63	0,59
2/3	17,9	2,37	95	150	1,69	0,71	0,63
2/4	21,5	2,60	-	-	-	-	-
2/5	17,0	2,31	90	160	1,60	0,69	0,56
2/6	30,6	3,10	115	210	2,04	0,66	0,55

(Laterally  
unstable)

NOTE :

- i) Main flexural reinforcement is 4Y10's,  $F_y = 505$  MPa,  $d_e = 685$  mm
- ii) See 3.2.4 for details of web reinforcement
- iii)  $a/d = 0,5$

3.2.3

SERIES 2, PHYSICAL PROPERTIES AND EXPERIMENTAL RESULTS

BEAM	%A <sub>sv</sub>	%A <sub>sh</sub>	WEB STEEL	A <sub>sv</sub> = A <sub>sh</sub>	
				sv	sh
2/1	0,075	0,075	R3-140	0,11	
2/2	0,10	0,10	R3-105	0,15	
2/3	0,15	0,15	R3-70	0,22	
2/4	0,15	0,15	R3-70	0,22	
2/5	0,20	0,20	R3-53	0,29	
2/6	0,25	0,25	R6-150	0,38	

NOTE :

i) Web reinforcement is an orthogonal mesh of equal vertical and horizontal steel to both faces. Tabulated value is % per face in each direction.

ii) F<sub>y</sub> of web reinforcement is 290 MPa

3.2.4

SERIES 2, WEB REINFORCEMENT

BEAM	$F_{cu}$ (MPa)	$f_{calc}$ (MPa)	$V_{cr}$ (kN)	$V_u$ (kN)	$V_{cr}$ (MPa)	$\frac{V_{cr}}{f_{calc}}$	$\frac{V_{cr}}{V_u}$
3/1	17,5	2,34	80	100	1,42	0,61	0,80
3/2	23,5	2,71	90	115	1,60	0,59	0,78

NOTE :

- i) Main flexural reinforcement is 2Y10's,  $F_y = 505$  MPa,  $d_e = 715$  mm
  - ii) No web reinforcement
  - iii)  $a/d = 0,5$
- 3.2.5

SERIES 3, PHYSICAL PROPERTIES AND EXPERIMENTAL RESULTS

BEAM	$F_{cu}$ (MPa)	$f_{calc}$ (MPa)	$V_{cr}$ (kN)	$V_u$ (kN)	$V_{cr}$ (MPa)	$\frac{V_{cr}}{f_{calc}}$	$\frac{V_{cr}}{V_u}$
4/1	20,1	2,51	125	125	2,22	0,89	1,00
4/2	28,8	3,01	175	190	3,11	1,03	0,92

NOTE :

- i) Main unbonded tie reinforcement is 4Y10's,  $F_y = 505$  MPa,  $d_e = 685$  mm
  - ii) No web reinforcement
  - iii)  $a/d = 0,5$
- 3.2.6

SERIES 4, PHYSICAL PROPERTIES AND EXPERIMENTAL RESULTS

BEAM	$F_{cu}$ (MPa)	$f_{calc}$ (MPa)	$a/d$	$V_{cr}$ (kN)	$V_u$ (kN)	$V_{cr}$ (MPa)	$\frac{V_{cr}}{f_{calc}}$	$\frac{V_{cr}}{V_u}$
5/1	27,8	2,95	0,3	105	-	1,87	0,63	-
5/2	30,1	3,07	0,3	150	235	2,67	0,87	0,64
5/3	22,7	2,67	0,4	100	175	1,78	0,67	0,57
5/4	27,0	2,91	0,5	98	157	1,74	0,60	0,62
5/5	18,2	2,39	0,6	80	-	1,42	0,59	-
5/6	29,4	3,04	0,6	108	156	1,92	0,63	0,69
5/7	26,0	2,86	0,7	100	120	1,78	0,62	0,83
5/8	25,6	2,83	0,8	105	110	1,87	0,66	0,95

(Laterally unstable)

(Laterally unstable)

NOTE :

i) Main flexural reinforcement is 4Y10's,  $F_y = 505$  MPa,  $d_e = 685$  mm

ii) No web reinforcement

3.2.7

SERIES 5, PHYSICAL PROPERTIES AND EXPERIMENTAL RESULTS

#### 4. CODE OF PRACTICE AND DESIGN GUIDE PREDICTIONS

##### 4.1 General

It is important to realise when comparing the code predictions with the cracking and/or ultimate loads exhibited by the experimental beams, that the laboratory tested beams possessed that specific load - support geometry which provided the most ideal configuration for the formation of a tied-arch. The expressions provided by codes of practice are intended for the design of beams, typically with less ideal support and load geometry, subjected to long term, generally fluctuating load.

As demonstrated by Fereig, S., and Smith, K.<sup>21</sup>, as well as Taub, J., and Neville, A.<sup>35</sup>, indirectly loaded beams do not possess the same reserve of shear strength, beyond the formation of the inclined shear cracks, as displayed by those beams loaded with short duration concentrated load in the manner of the experimental work reported in this thesis.

The equations provided by codes of practice are not intended to give ultimate strength but the initiation of diagonal shear cracking. Those codes of practice which take account of the increased strength due to low  $a/d$  ratios do so by applying a factor\* to the predicted cracking load. The cracking stress is assumed the same for shallow and deep beams. Over the range of  $a/d$  tested this assumption was found to be correct. ( $0,3 < a/d < 0,8$ ).

See 5.3.2 which shown no trend of  $V_{cr}$  increasing as  $a/d$  decreased.

The equations provided by codes of practice such as ACI 318-83 and CEB-FIP 1978 provide limits for stress levels which are based on the characteristic cylinder strength of concrete. The cylinder strength was assumed equal to 0,8 times the cube strength for calculation purposes.<sup>10</sup>

Partial factors of safety for materials and loads vary between codes.

Beams tested in Series 1, 3, 4 and 5 do not satisfy the requirements of web reinforcement minima as stipulated by codes of practice such as ACI 318-83, CEB-FIP 1978, CP 110, IS 466 nor that of the 1970 CEB-FIP Recommendations.

The shear strength predictions of ACI 318-83<sup>1</sup>, BS CP 110<sup>7</sup>, CEB-FIP 1978<sup>52</sup> and Kong et. al.<sup>13</sup> take direct cognisance of the contribution of main tensile reinforcement to the shear strength.

\* Shear enhancement factor see 5.3.3

The recommendations of CEB-FIP 1970<sup>8</sup>, CEB-FIP 1978<sup>52</sup> and IS. 466<sup>4</sup> take no direct account of web reinforcement although a minimum orthogonal mesh is stipulated.

The shear friction method of design recommended by AS 1480<sup>5</sup> only applies to beams with  $a/d < 0,5$ <sup>39</sup> and hence was not calculated.

The load calculated in accordance with the recommendations of the PCA<sup>9</sup> are at serviceability limit state. The balance of the tabulated loads are for ultimate limit state.

The following factors must be borne in mind, when interpreting the tabulations of the experimental results and the predictions of the various codes of practice, namely:

The concrete compressive strength ( $F_{cu}$ ) used in the calculation recommended by the various codes of practice was the mean strength of six cubes cured alongside the respective beams.  $F_{cu}$  as used in the previous summaries of code and design guide recommendations must be substituted with the characteristic strength ( $f_{cu}$ ) for design purposes, in practice.

The reinforcement yield stress, ( $F_y$ ) used in the calculations recommended by the various codes of practice was the actual yield strength, and must be substituted with the characteristic strength ( $f_y$ ) for design purposes, in practice.

The limited number of cubes made for each concrete strength did not permit the calculation of the characteristic strength of concrete. ACI 318-83, Clause 4.3.1. calls for a minimum of 30 tests in order to calculate the characteristic strength ( $f_{cu}$  at 28 days).

The conditions under which the cubes were cured could be described as "field cured", ACI 318-83, Clause 4.7.3.4. recommends that the strength of field cured cubes be divided by 0,85 to allow a comparison with the characteristic strength of cubes made, cured and tested in accordance with a standard test method.



The following tables which contrast the actual failure load of the beams tested with loads predicted by the design guides and codes of practice must be read in conjunction with :

3.2.2 for full description of the beams tested in Series 1

3.2.3 for full description of the beams tested in Series 2

3.2.5 for full description of the beams tested in Series 3

3.2.7 for full description of the beams tested in Series 5

There are no predictions for the beams tested in Series 4, which had unbonded main longitudinal reinforcement. See 3.2.6

The unit of load in the following tables is kN

Figures in brackets are the predicted load expressed as a fraction of the actual ultimate load.

BEAM	TEST	ACI 318-83	PCA	CEB-FIP 1970	CP 110	IS 466	KONG et. al.	CEB-FIP 1978
		REF. 1.	REF. 9.	REF. 8.	REF. 7.	REF. 4.	REF. 13.	REF. 52.
1/1	115	94 (0,82)	34 (0,30)	56 (0,49)	81 (0,70)	85 (0,74)	116 (1,01)	51 (0,44)
1/2	130	104 (0,80)	55 (0,42)	73 (0,56)	102 (0,78)	110 (0,85)	128 (0,98)	62 (0,48)
1/3	125	105 (0,84)	56 (0,45)	74 (0,59)	103 (0,82)	113 (0,90)	129 (1,03)	63 (0,50)
1/4	170	114 (0,67)	63 (0,37)	90 (0,53)	112 (0,66)	137 (0,81)	142 (0,84)	72 (0,42)
1/5	170	119 (0,70)	68 (0,40)	100 (0,59)	119 (0,70)	153 (0,90)	143 (0,84)	78 (0,46)
1/6	185	121 (0,65)	68 (0,37)	101 (0,55)	119 (0,64)	154 (0,83)	149 (0,81)	80 (0,43)
1/7	170	121 (0,71)	70 (0,41)	106 (0,62)	119 (0,70)	161 (0,95)	150 (0,88)	81 (0,48)
1/8	190	130 (0,68)	80 (0,42)	126 (0,66)	119 (0,63)	192 (1,01)	162 (0,85)	91 (0,48)

4.2 Series 1, Comparison of failure loads with predicted ultimate strengths

BEAM	TEST	ACI 318-83	PCA	CEB-FIP 1970	CP 110	IS 466	KONG et. al.	CEB-FIP 1978
		REF. 1.	REF. 9.	REF. 8.	REF. 7.	REF. 4.	REF. 13.	REF. 52.
2/1	150	118 (0,79)	48 (0,32)	59 (0,39)	105 (0,70)	89 (0,59)	121 (0,81)	54 (0,36)
2/2	145	127 (0,88)	50 (0,34)	62 (0,43)	116 (0,80)	94 (0,65)	124 (0,86)	56 (0,39)
2/3	150	140 (0,93)	50 (0,33)	60 (0,40)	126 (0,84)	92 (0,61)	125 (0,83)	54 (0,36)
2/4	-	147	55	73	141	110	134	62
2/5	160	152 (0,95)	47 (0,29)	57 (0,36)	134 (0,84)	87 (0,54)	125 (0,78)	54 (0,34)
2/6	210	196 (0,93)	69 (0,33)	104 (0,50)	185 (0,88)	157 (0,75)	159 (0,76)	80 (0,38)

4.3 Series 2, Comparison of failure loads with predicted ultimate strengths

BEAM	TEST	ACI 318-83	PCA	CEB-FIP 1970	CP 110	IS 466	KONG et. al.	CEB-FIP 1978
		REF. 1.	REF. 9.	REF. 8.	REF. 7.	REF. 4.	REF. 13.	REF. 52.
3/1	100	101 (1,01)	49 (0,49)	59 (0,59)	71 (0,71)	94 (0,94)	94 (0,94)	47 (0,47)
3/2	115	117 (1,02)	58 (0,50)	79 (0,69)	81 (0,70)	126 (1,10)	108 (0,94)	59 (0,51)

4.4 Series 3, Comparison of failure loads with predicted ultimate strengths

BEAM	TEST	ACI 318-83	PCA	CEB-FIP 1970	CP 110	IS 466	KONG et. al.	CEB-FIP 1978
		REF. 1.	REF. 9.	REF. 8.	REF. 7.	REF. 4.	REF. 13.	REF. 52.
5/1	-	116	65	94	191	143	158	126
5/2	235	119 (0,51)	68 (0,29)	102 (0,43)	211 (0,90)	155 (0,66)	166 (0,71)	130 (0,55)
5/3	175	106 (0,61)	57 (0,33)	77 (0,44)	130 (0,74)	117 (0,67)	138 (0,79)	81 (0,46)
5/4	157	114 (0,73)	64 (0,41)	91 (0,58)	111 (0,71)	139 (0,89)	141 (0,90)	73 (0,47)
5/5	-	97	50	61	75	94	113	47
5/6	156	118 (0,76)	68 (0,44)	99 (0,63)	99 (0,63)	151 (0,97)	138 (0,88)	65 (0,42)
5/7	120	111 (0,93)	62 (0,52)	88 (0,73)	80 (0,67)	134 (1,12)	122 (1,02)	52 (0,43)
5/8	110	105 (0,95)	61 (0,55)	87 (0,79)	70 (0,64)	132 (1,20)	114 (1,04)	44 (0,40)

4.5 Series 5, Comparison of failure loads with predicted ultimate strengths

## 4.6 FACTOR OF SAFETY

### 4.6.1 General

Although application of the recommendations of the codes of practice and design guides under review do result in safe design, there is however an inconsistent factor of safety. At low  $a/d$  ratios the predictions appear conservative. This is possibly a reflection of the large scatter in ultimate shear strength, due to possible variations in modes of failure of beams with low  $a/d$  ratios. See 5.7.4.

Bearing in mind the factors listed in 4.1, plus the limited number of beams tested, it is not possible to make conclusive statements about the predictions of the different design methods. However, an inspection of the results tabulated in 4.1 to 4.5 provides an indication of the manner in which variable such as ;

Concrete compressive strength  
Web reinforcement  
Main longitudinal reinforcement  
Shear span to depth ratio

influence the accuracy of the predictions of the design methods under review.

The apparent factor of safety is defined as  $V_{test}/V_{predicted}$  where:

$V_{test}$  is the ultimate failure load of the beam tested

$V_{predicted}$  is the predicted failure load of the beam

hence, the apparent factor of safety is the inverse of the fraction in brackets, tabulated in 4.2, 4.3, 4.4, and 4.5.

### 4.6.2 Concrete Compressive Strength (Read in conjunction with 5.1)

The beams tested in Series 1 (See 3.2.2) were designed to investigate the manner in which concrete compressive strength influences firstly, the ultimate shear strength of deep beams and secondly, the accuracy of the predictions of the design methods reviewed.

With the exception of the predictions of ACI 318-83 and Kong et. al., no trend could be observed of the manner in which the apparent factor of safety was influenced by varying the concrete compressive strength.

#### 4.6.2.1 ACI 318-83

Examination of the apparent factor of safety indicates a trend to increase with increasing concrete strength. This implies that for beams with low  $a/d$  ratios the recommended design formula underestimates the gain in shear strength due to increased concrete strength.

#### 4.6.2.2 Kong et. al.

The strength of the concrete in beams 1/1, 1/2 and 1/3 did not fall within the range of concrete strengths of the beams tested by Kong et. al., on which their proposed formula is based. See 2.7.4. The balance of the predictions of this design method displayed the most consistent factor of safety of the methods under review.

#### 4.6.3 Web Reinforcement (Read in conjunction with 5.2)

The beams tested in Series 2 (See 3.2.3) were designed to investigate the influence of web reinforcement on firstly, ultimate shear strength and secondly, the accuracy of the predictions of the design methods reviewed.

With the exception of the design methods recommended by ACI 318-83, CP 110 and Kong et. al., the methods reviewed, while stipulating a minimum orthogonal mesh of web reinforcement do not take direct account of the web steel contribution to ultimate shear strength.

#### 4.6.3.1 ACI 318-83 and CP 110

The apparent factor of safety is substantially lower than that of the beams tested in Series 1. This implies that for beams with low  $a/d$  ratios the recommended design formulae overestimates the contribution to ultimate shear strength made by web reinforcement.

#### 4.6.3.2 Kong et. al.

As the concrete of beam 2/6 is the only concrete strength to fall within the range of strengths of the beams tested by Kong et. al. (2.7.4), no conclusions can be drawn.

#### 4.6.4 Main Longitudinal Reinforcement (Read in conjunction with 5.2)

The two beams tested in Series 3 (See 3.2.5) were designed to investigate the manner in which under provision of main longitudinal reinforcement influences firstly, the ultimate shear strength of deep beams and secondly, the apparent factor of safety.

The extremely limited number of beams in Series 3 prevents any meaningful interpretation of the manner in which the apparent factor of safety is influenced by under-reinforcement.

The design methods which take direct account of main longitudinal reinforcement in assessing shear strength are, ACI 318-83, CP 110 Kong et. al. and CEB-FIP 1978.

#### 4.6.5 Shear Span to Depth Ratio ( $a/d$ ) (Read in conjunction with 5.3)

The beams tested in Series 5, (See 3.2.7) were designed to investigate the manner in which the  $a/d$  ratio influences firstly, the ultimate shear strength and secondly, the accuracy of the predictions of the design methods reviewed.

With the exception of the recommendations of CEB-FIP 1970 and IS 466, all the design methods reviewed take account of the influence of the  $a/d$  ratio on shear strength.

##### 4.6.5.1 ACI 318-83, PCA and Kong et. al.

Examination of the apparent factor of safety indicates a trend to increase with decreasing  $a/d$  ratio. This implies that the proposed design formula under-estimates the enhanced ultimate shear strength resulting from low  $a/d$  ratios. See 5.3.3

4.6.5.2 CP 110 and CEB-FIP 1978

Examination of the apparent factor of safety indicates a trend to decrease with decreasing  $a/d$  ratio. This implies that the proposed design formula over-estimates the enhanced ultimate shear strength resulting from low  $a/d$  ratios.

---



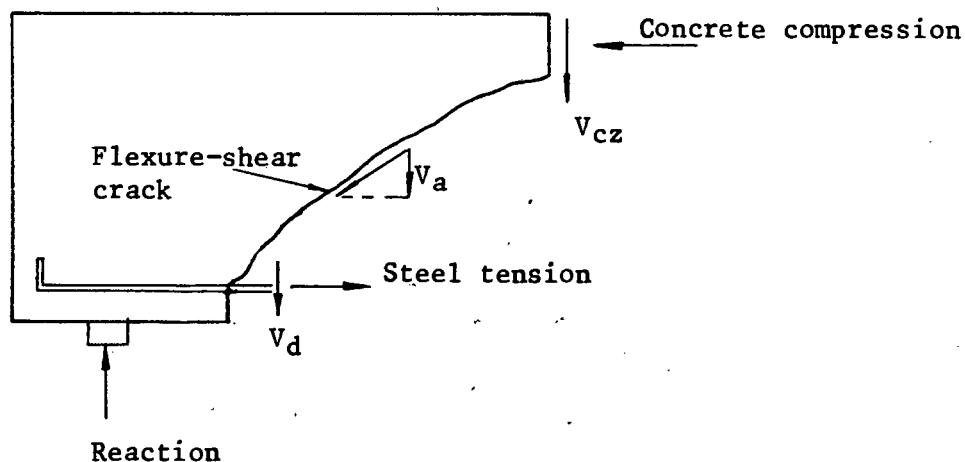
## 5. FACTORS INFLUENCING DEEP BEAM BEHAVIOUR

### 5.1 CONCRETE COMPRESSIVE STRENGTH

#### 5.1.1 General

The equations provided by most current codes of practice are intended to predict the inclined shear cracking load, which for shallow beams can be sensibly accepted as the ultimate load. See 5.7

The mechanisms of shear transfer in a shallow cracked reinforced concrete beam with no web reinforcement are illustrated in the following diagram.



#### MECHANISM OF SHEAR TRANSFER ( $a/d < 2,5$ )<sup>26</sup>

For a typical shallow reinforced concrete beam the shear force  $V$  is carried in the following proportions.<sup>26</sup>

compression zone shear	$V_{cz}$	=	20 - 40%
dowel action	$V_d$	=	15 - 25%
aggregate interlock	$V_a$	=	35 - 50%

These three mechanisms are all obviously influenced by concrete compressive strength, there are however different opinions as to what the relationship between  $v_u$  and  $f_{cu}$  is. The current ACI 318-83 and CEB-FIP 1978 codes of practice assume that the nominal shear capacity ( $v_u$ ) of shallow beams is essentially proportional to  $f_{cu}^{0,5}$  while some investigators<sup>47</sup> have concluded it is proportional to  $f_{cu}^{0,33}$ .

For concrete compressive strengths ( $f_{cu}$ ) up to 85 MPa experimental study indicates that as  $f_{cu}$  increases, shear capacity increases at a slower rate than the  $f_{cu}^{0,5}$  proportionality would indicate.<sup>36</sup> This would serve to explain BS CP 110 not recognizing any further increase in shear strength when  $f_{cu}$  is increased beyond 40 MPa.

The addition of transverse web reinforcement (stirrups) and/or bent up bars will serve to increase the shear strength to a point where the ultimate shear capacity may be governed by the crushing strength of the concrete in the web. This places an upper limit on the strength that can be provided by adding shear reinforcement.

The equations provided by codes of practice, such as ACI 318-83, BS CP 110 and CEB-FIP 1978 for the design of beams with low  $a/d$  ratios, are of the form :

$$v_u < f \times (\text{expression derived for predicting shallow beam shear strength})$$

where the multiplier  $f$  is typically expressed as a function of the  $a/d$  ratio. See 5.3.3

The multiplier  $f$  is intended to reflect the capacity of beams with low  $a/d$  ratios ( $a/d < 2,0$ ) to carry load in excess of that load causing inclined shear cracking. This property is due to the capacity of deep beams to redistribute internal forces after cracking and to then carry the load as a tied-arch. Sufficient increase of applied load will result in shear compression failure due to the extension of the upper end of the diagonal tension crack into the compression zone until crushing of concrete occurs. Concrete compressive strength will therefore clearly influence the ultimate strength of deep beams to a greater degree than is reflected by the relationship  $v_u \propto f_{cu}^{0,5}$  as commonly adopted for the design of shallow beams.

This is substantiated by Smith, K, and Vantsiotis, A.<sup>23</sup>, who state:

"Results from a linear regression analysis indicates that plots of  $P_u$  versus  $f'_c$  result in higher correlation coefficients than plots of  $P_u$  versus  $\sqrt{f'_c}$  especially at low  $a/d$  ratios".

Where :

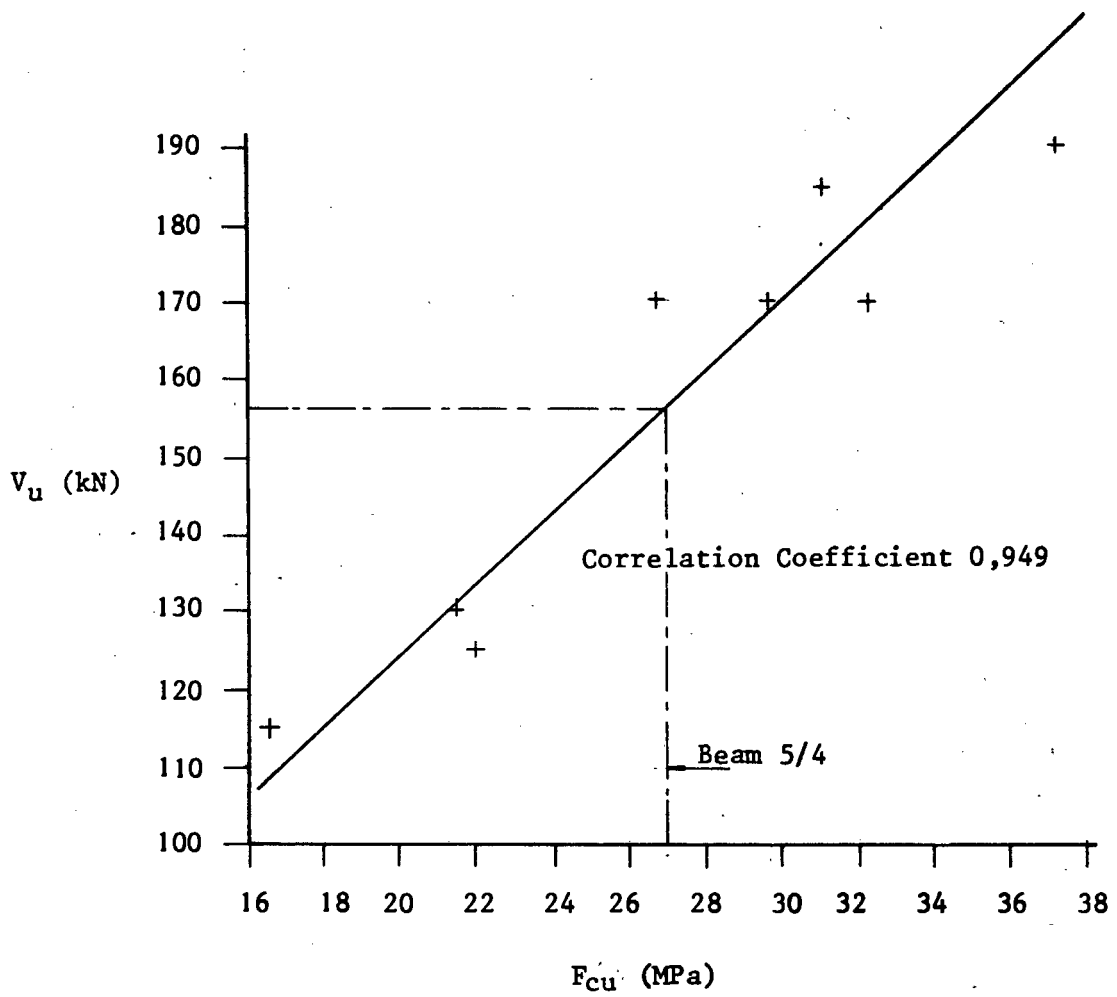
$P_u$  is the sum of two equal loads applied symmetrically about the beam centreline.

$f'_c$  is the concrete compressive strength as determined by the crushing of cylinders.

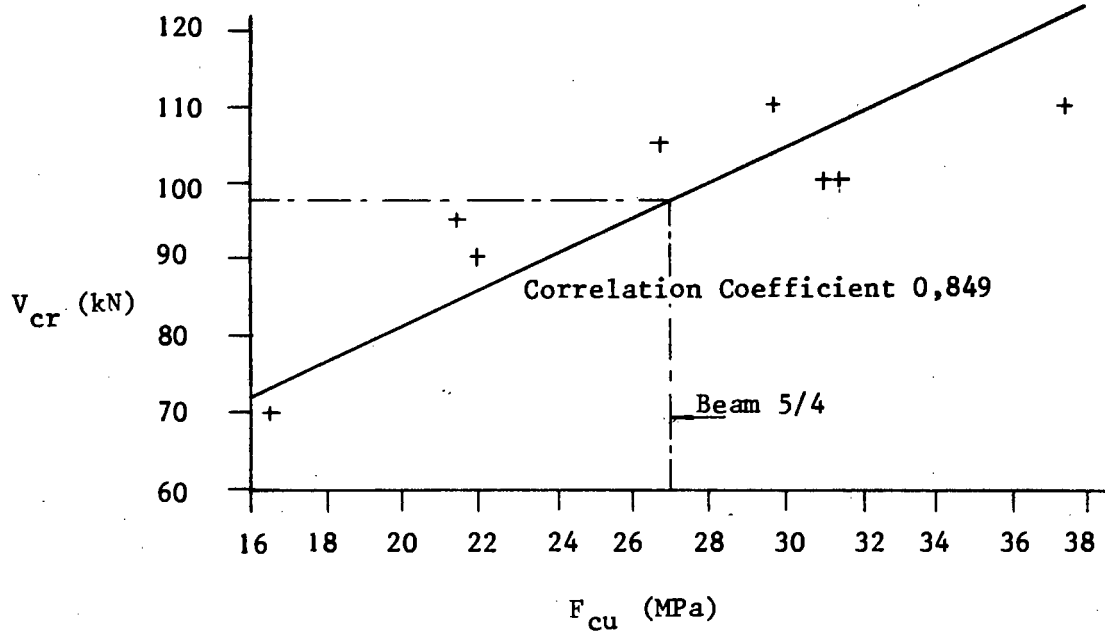
The preceding statement was made on the strength of an analysis of the results obtained from the testing of 52 deep beams with  $0,77 < a/d < 2,01$ .

A statistical analysis of the results obtained in the Series 1 tests yields a correlation coefficient of 0,949 for  $V_u$  versus  $F_{cu}$  and 0,959 for  $V_u$  versus  $\sqrt{F_{cu}}$ . This limited comparison provides an inconclusive indication that  $V_u$  is more closely proportional to  $\sqrt{F_{cu}}$  than to  $F_{cu}$  (at  $a/d = 0,5$ ) See 5.1.2 for plot of  $V_u$  versus  $F_{cu}$  for Series 1 beams.

---



5.1.2 PLOT OF  $V_u$  VERSUS  $F_{cu}$  FOR SERIES 1 BEAMS



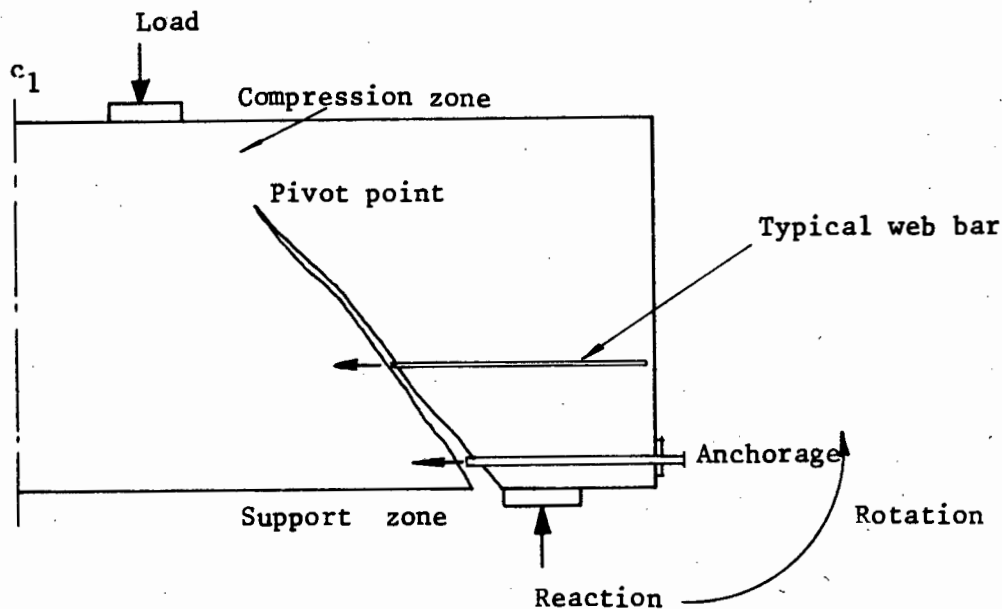
5.1.3 PLOT OF  $V_{cr}$  VERSUS  $F_{cu}$  FOR SERIES 1 BEAMS

5.2.1 General

In order to simplify the calculations of reinforcement requirements, it is common practice to distinguish between flexural and shear reinforcement. The laws of equilibrium are however unaware of the designers discrimination between the reinforcement labelled "flexural" or "shear", with the result that any flexural reinforcement intersecting the critical diagonal path will form an integral part of the shear reinforcement and similarly web reinforcement will contribute to flexural strength.

A common characteristic of reinforced concrete deep beams is that they contain an orthogonal mesh of web reinforcement, consequently the usual method of calculating moment and shear reinforcement requirements may be overly conservative. A common factor in all the recommendations provided for the design of deep beams, is the emphasis on providing full anchorage to all the longitudinal reinforcement, with curtailment corresponding to the bending moment diagram being proscribed.

The capacity to carry load after the formation of the inclined shear crack depends on whether these cracks penetrate into the compression zone at the loading position. The tensile stress in the main longitudinal reinforcement at the bottom of the inclined crack will be governed by the moment at a section at the top of the crack. Large strain in the reinforcement at this point, (which would be greatly increased if the reinforcement was curtailed in accordance with a bending moment diagram) would encourage the extension of the critical inclined shear crack into the support zone. This large strain would also increase the rotation at the end portion of the beam with the result that the inclined shear crack would penetrate the concrete compression zone adjacent to the load point, leading to collapse.

ROTATION OF BEAM END

Design methods typically proposed by current codes of practice for calculating the ultimate shear strength of deep beams, assume this to be the sum of the concrete and web steel capacity. For design purposes the yield strength of the web reinforcement is commonly restricted to  $415 \text{ MPa}^1 - 425 \text{ MPa}^7$ , in order to restrict shear crack widths. The ultimate shear capacity of a member may be governed by the crushing strength of the concrete in the web, this places an upper limit on the strength that can be provided by adding shear reinforcement.

The orientation of the principal stresses in deep beams, when diagonal cracking occurs will exceed  $45^\circ$  in most cases. As the  $a/d$  ratio decreases, the role of vertical web reinforcement changes from carrying shear primarily by tension to that of shear friction reinforcement preventing a sliding failure along the inclined crack. Consequently as the  $a/d$  ratio decreases the effectiveness of vertical stirrups decrease and that of horizontal stirrups increase.

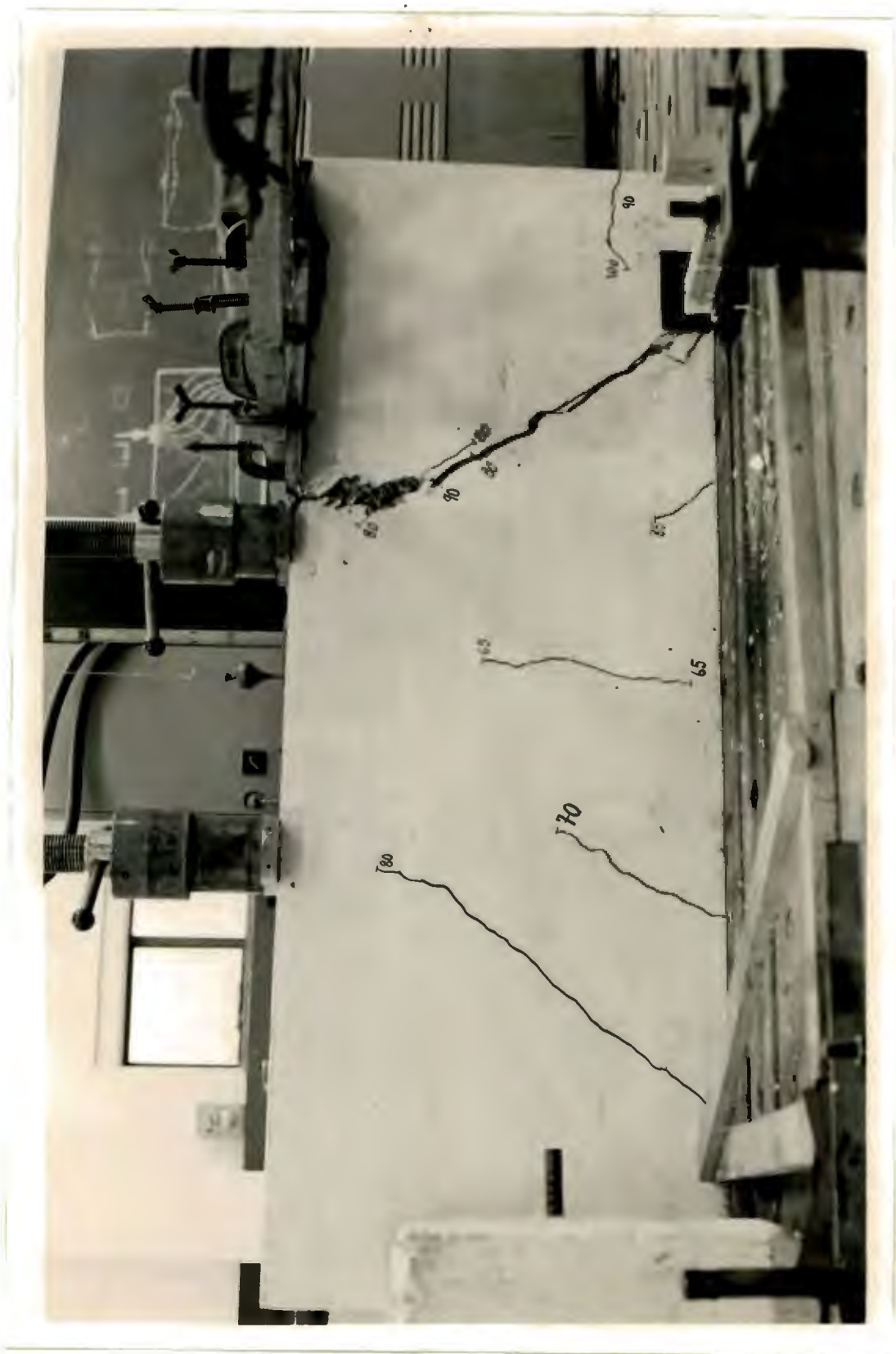
ACI 318-83 is the only code of practice found to implicitly take account of the manner in which the  $a/d$  ratio influences the effectiveness of the vertical and horizontal web reinforcement.

The Equation proposed; (EQU. 11.31)<sup>1</sup>

$$\frac{A_{sv}}{sv} \left( \frac{1 + L/d}{12} \right) + \frac{A_{sh}}{sh} \left( \frac{11 - L/d}{12} \right) = \frac{V_s}{F_y \cdot d}$$

provides a weighting factor for the relative effectiveness of the vertical and horizontal web reinforcement which in terms of the equation are deemed equally effective at  $L/d = 5$ . Thus for deep beams it is more efficient to add web reinforcement, if required, in the form of horizontal web reinforcement.

The behaviour, ultimate capacity and failure mode of a reinforced concrete deep beam is greatly influenced by the reinforcement details at supports, under concentrated loads, and at anchorages. The layout and spacing of reinforcement are also an important influence on deep beam behaviour. The CEB-FIP<sup>8</sup> provide comprehensive recommendations for the design and detailing of reinforcement for deep beams.



TYPICAL BEAM

The optimum design of a deep beam utilizes the capacity of beams with low  $a/d$  ratios to carry load in excess of that load causing inclined shear cracks. It would therefore follow that the quantity of flexural reinforcement would influence the shear capacity of a deep beam to a much greater extent than that of a shallow beam. As the quantity of flexural reinforcement in a deep beam is unlikely to be large, the benefits in terms of shear capacity would encourage a fairly conservative calculation of this reinforcement. This is reflected by the low values for the lever-arm ( $z$ ) proposed by many codes of practice.

### 5.2.2 FLEXURAL REINFORCEMENT EFFECTS

BEAM	$A_s$ (mm <sup>2</sup> )	$F_{cu}$ (MPa)	$V_{cr}$ (KN)	$V_u$ (KN)	$\frac{V_{cr}}{V_u}$
3/1	157	17,5	80	100	0,80
3/1	157	23,5	90	115	0,78
1/*	314	17,5	76	113	0,67
1/*	314	23,5	89	141	0,63

**NOTE:**

- i) See 3.2.5 for full description of Series 3 beams.
- ii) 1/\* obtained from plot of  $F_{cu}$  versus  $V_{cr}$  and  $V_u$ . See 5.1.2 and 5.1.3

A comparison of the ultimate load capacities of the two beams tested in Series 3 (2Y10's) with equivalent beams from Series 1 (4Y10's) highlights the large gain in ultimate shear strength resulting from a moderate increase in "flexural" reinforcement. The load causing initial inclined shear cracking remained virtually unchanged by the increase in reinforcement.



### 5.2.3 WEB REINFORCEMENT

#### 5.2.3.1 General

The following equation which is adopted by many codes of practice was assumed correct for deep beams with an orthogonal mesh of web reinforcement.

$$V_u = V_c + V_s$$

Rearranging this equation enables one to calculate the contribution to the ultimate shear strength provided by the web reinforcement.

#### By Experiment

$$V_{s1} = V_u - V_c, \text{ where}$$

$V_{s1}$  Contribution to ultimate shear strength provided by web reinforcement, as derived from test results of Series 2

$V_u$  is the ultimate strength of the beams with web reinforcement. See Series 2.

$V_c$  is the ultimate strength of an equivalent beam without web reinforcement, and is assessed by referring to the results obtained in the Series 1 Tests.

See 5.1.2 for a plot of  $V_c$  versus  $F_{cu}$  ( $V_c = V_u$  for Series 1)

#### By Calculation

$$\frac{A_{sv}}{sv} = \frac{b(V_u - V_c)}{\phi F_y}$$

Taking  $\phi = 1$  (Typically 0,87 for design purposes) and rationalising :

$$V_{s2} = \frac{A_{sv}}{sv} F_y d_e, \text{ where}$$

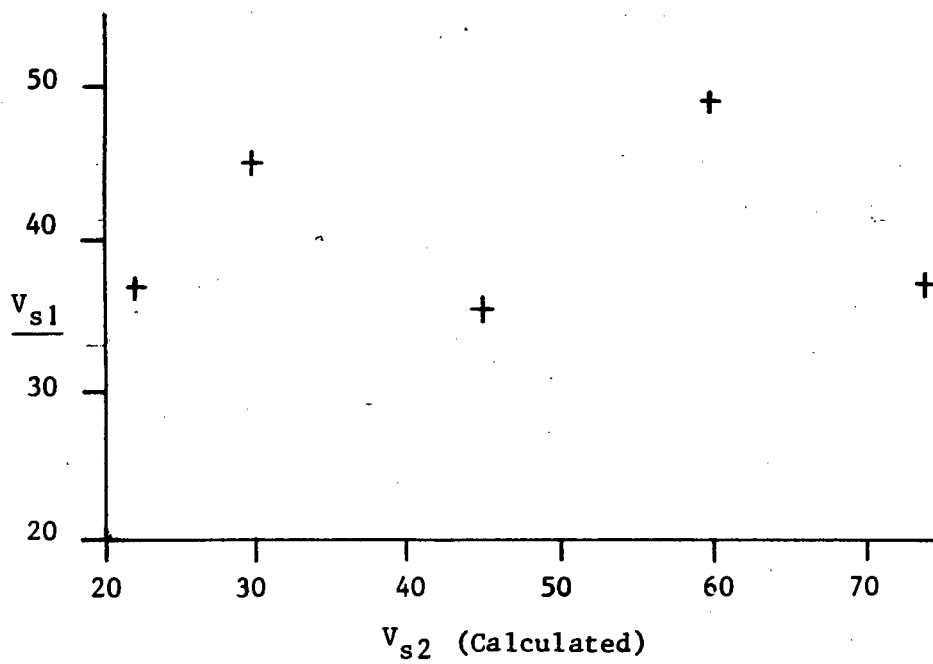
$V_{s2}$  web steel contribution to shear strength calculated with the preceding familiar equation.

$A_{sv}$  in the above equation refers to vertical web reinforcement. The beams tested in Series 2 were reinforced with an orthogonal mesh of web reinforcement, equal in both directions.

5.2.3.2 SERIES 2. WEB REINFORCEMENT EFFECTS

BEAM	F <sub>cu</sub> (MPa)	V <sub>u</sub> (KN)	V <sub>c</sub> (KN)	V <sub>s1</sub> (KN)	V <sub>s2</sub> (KN)
2/1	17,4	150	113	37	22
2/2	18,3	145	117	28	30
2/3	17,9	150	115	35	45
2/4	21,5	-	132	-	45
2/5	17,0	160	111	49	60
2/6	30,6	210	173	37	74

Read in conjunction with 3.2.3 for full description of Series 2 beams.



5.2.3.3

PLOT OF CALCULATED VERSUS EXPERIMENTAL WEB STEEL CONTRIBUTION TO SHEAR CAPACITY

#### 5.2.3.4

#### Web Reinforcement Capacity ( $V_s$ )

Those design methods which take direct account of the contribution of web reinforcement to ultimate shear strength typically adopt the following familiar equation.

$$V_u = V_c + V_s$$

This equation reflects a confidence in the reliability of web reinforcement contribution to the ultimate shear strength of deep beams which is not substantiated by the experimental work reported here.

The plot of  $V_{s1}$  versus  $V_{s2}$  in 5.2.3.3 displays the very poor correlation between the calculated and actual contribution made by web reinforcement to the ultimate shear strength of the beams tested in Series 2. (See 3.2.3)

Other researchers such as;

Kong et. al.

Fereig, S., and Smith, K.<sup>21</sup>

Smith, K., and Vantsiotis, A.<sup>23</sup>

de Paiva, H., and Siess, C.<sup>32</sup>

similarly conclude that;

while web reinforcement is highly effective in controlling crack widths, its contribution to the ultimate strength of deep beams with normal proportions of main longitudinal reinforcement is not a contribution that is either consistent or reliable.

---

## 5.3 SHEAR SPAN TO DEPTH RATIO ( $a/d$ )

### 5.3.1 General

The term  $a/d$  is equivalent to  $\frac{M}{Vd}$  for a single span, simply supported beam, subjected to symmetrical two point loading or central point load.

The equations developed to predict the shear strength of shallow beams ( $a/d > 2,5$ )<sup>1</sup> & <sup>35</sup> only recognise the following four contributions to shear strength :

- i) Aggregate interlock
- ii) Dowel action
- iii) Compression zone shear strength
- iv) Web reinforcement (Only effective post cracking)

The ultimate shear strength of deep beams will far exceed the load predicted using expressions which only take account of these four mechanisms of shear transfer. This is due to the ability of deep beams ( $a/d < 2,5$ ) to carry load in excess of that load which causes inclined shear cracking. This property is due to the special capacity of deep beams to redistribute internal forces and develop mechanisms of load transfer quite different to that of shallow beams.

The behaviour of a deep beam loaded on the top or compression face will, after the formation of the inclined shear cracks, approximate that of a tied arch. This tied arch action can be related to the tensile strains of the reinforcement over the supports. The higher the strain, the more developed the tied arch action. Research by Manuel, R. et. al.<sup>17</sup>, verifies this increased tendency to tied arch action with reducing  $a/d$  ratio.

The ratio of the ultimate shear stress of deep beams to the shear stress causing inclined cracking in shallow beams, is generally acknowledged as to be some function of the  $a/d$  value. This shear strength increase was found by Fereig, S., and Smith, K.<sup>21</sup>, to be greater for direct than indirect loading.

The following table lists some of the shear strength enhancement factors recommended by various codes of practice and design guides, and highlights the divergence of opinion as to the relationship between ultimate shear strength and the  $a/d$  ratio.

### 5.3.2 EXPERIMENTAL RESULTS

In order to investigate the relationship between the  $a/d$  ratio and both the cracking and ultimate loads of the beams tested in Series 5, it is necessary to adjust the experimental results to a common concrete grade. The cracking ( $V_{cr}$ ) and ultimate ( $V_u$ ) loads were assumed directly proportional to the concrete compressive strength over the range of  $a/d$  tested. ( $0,3 < a/d < 0,8$ ).

The experimental results were therefore modified to a concrete strength, selected to be that of beam 5/4. The properties of beam 5/4 are fictitious, corresponding to the mean concrete compressive strength of the beams tested in Series 1.

See 5.1.2 for plot of  $V_u$  versus  $F_{cu}$  of Series 1 beams.

#### SERIES 5, MODIFIED ULTIMATE LOAD ( $V_{u \text{ mod}}$ )

BEAM	$F_{cu}$ (MPa)	$a/d$	$V_{test}$ (kN)	$V_{u.mod}$ (kN)
5/1	27,8	0,3	-	-
5/2	30,1	0,3	235	211
5/3	22,7	0,4	175	208
5/4	27,0	0,5	157	157
5/5	18,2	0,6	-	-
5/6	29,4	0,6	156	143
5/7	26,0	0,7	120	125
5/8	25,6	0,8	110	116

(Laterally unstable)

Reference beam

(Laterally unstable)

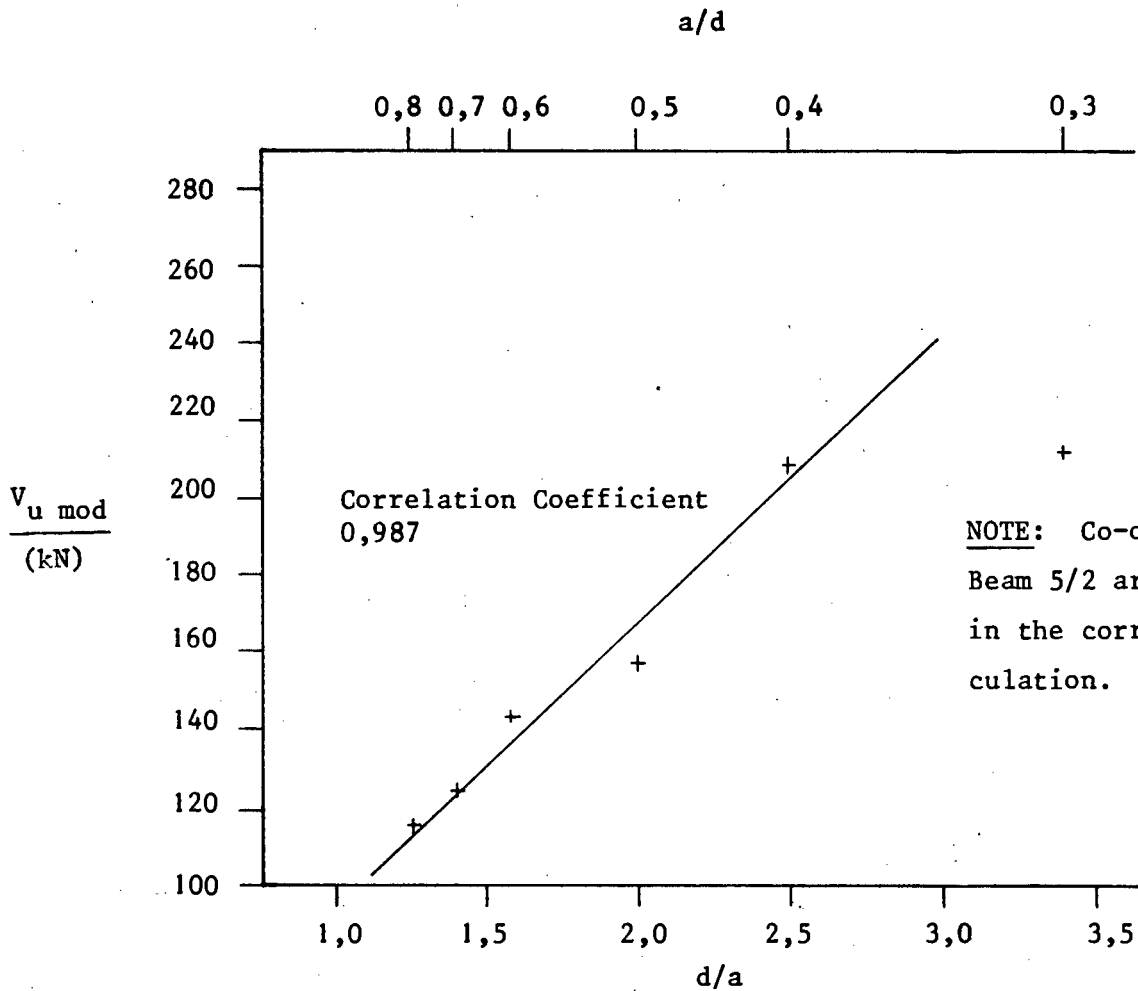
#### SERIES 5, MODIFIED CRACKING LOAD ( $V_{cr \text{ mod}}$ )

BEAM	$F_{cu}$ (MPa)	$a/d$	$V_{cr}$ (kN)	$V_{cr \text{ mod}}$ (kN)	$\frac{V_{cr}}{V_{u \text{ mod}}}$
5/1	27,8	0,3	105	102	-
5/2	30,1	0,3	150	135	0,64
5/3	22,7	0,4	100	119	0,57
5/4	27,0	0,5	98	98	0,62
5/5	18,2	0,6	80	119	-
5/6	29,4	0,6	108	99	0,69
5/7	26,0	0,7	100	104	0,83
5/8	25,6	0,8	105	111	0,94

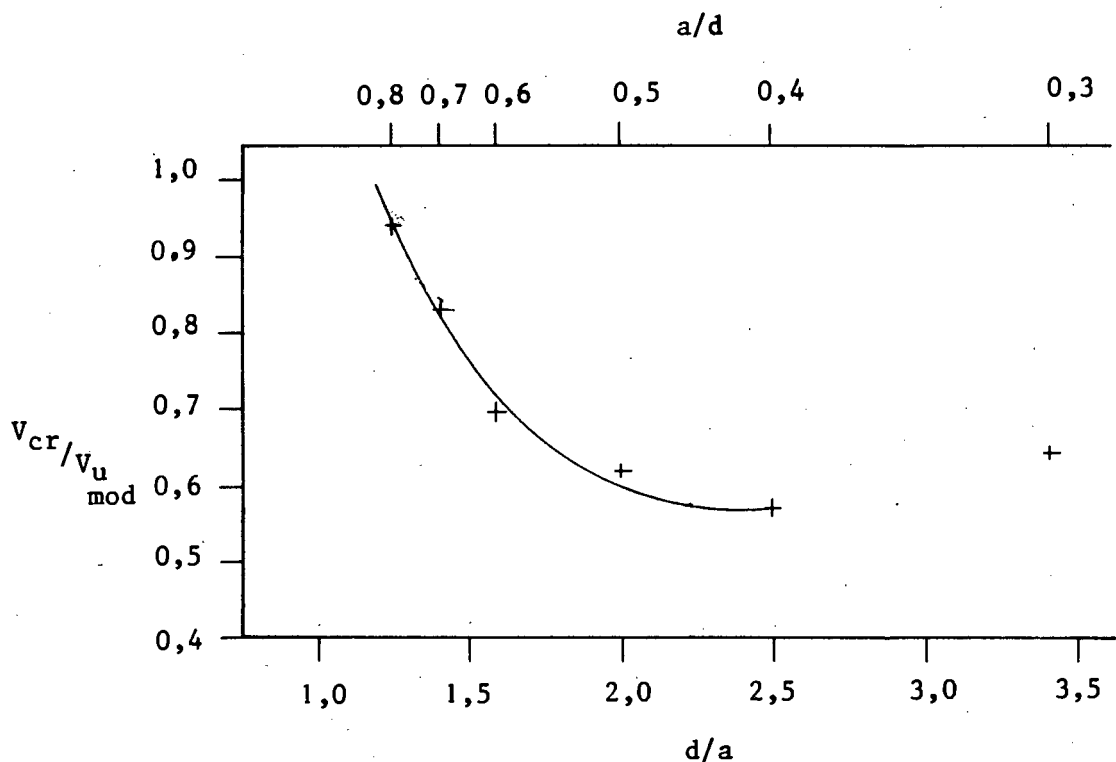
(Laterally unstable)

Reference beam

(Laterally unstable)



PLOT OF MODIFIED  $V_u$  VERSUS  $d/a$



PLOT OF MODIFIED  $V_{cr}/V_u$  VERSUS  $d/a$

### 5.3.3 SHEAR ENHANCEMENT FACTORS

<u>REFERENCE</u>	<u>FACTOR</u>
ACI 318-83 <sup>1</sup>	$3,5 - 2,5 \frac{M}{Vd} \geq 2,5$ which for a simply supported beam subjected to symmetrical point loading, simplifies to $(3,5 - 1,25 a/d) \geq 2,5$ . NOTE: Design check done at defined critical section. (CL. 11.8.4.)
CP 110 <sup>7</sup>	$\frac{2d}{a}$ Applies to concentrated loads only. $(a < 2d)$ (CL. 3.3.6.2.)
CROSS, M. <sup>43</sup>	$\frac{1,7d}{a}$ Derived from a series of tests on punching shear tests on slabs.
DIN 1045 <sup>44</sup>	$\frac{2d}{a}$ A serviceability state check.
KONG, F. et. al. <sup>13</sup>	$(1 - 0,35 a/d)$ . Enhancement factor applies only to contribution of concrete to shear strength. $(0,23 < a/d < 0,70)$ <sup>19</sup>
PCA <sup>9</sup>	$(1 + \frac{5d}{L}) / 3$ , $(L/d \leq 2,5)$ . A serviceability state check.
ZSUTTY, T., <sup>36</sup>	$2,5d/a$
CEB-FIP 1978 <sup>52</sup>	$\frac{2d}{a}$ Applies to concentrated loads only. $(a < 2d)$ (Clause 11.1.2.3.)

## 5.4 ABSOLUTE BEAM DEPTH

The empirical equations proposed by codes of practice are based on data obtained by testing to failure beams typically less than 400 mm deep.<sup>37</sup> Few beams tested compare in size to deep beams commonly in excess of 2 metres deep designed for full scale structures.

Kani, G.,<sup>37</sup> (1967) carried out a series of tests on 16 beams, designed to clarify the manner in which absolute beam depth influences ultimate shear strength. The beams tested all contained the same nominal percentage tensile reinforcement, the same concrete strength and no web reinforcement. In his conclusion, Kani states; "Increasing the beam depth must result in considerable reduction of the relative beam strength".

A more recent study by Bazant, Z., and Kim, J.,<sup>41</sup> (1984) includes a statistical analysis of existing data obtained by testing beams without web reinforcement. A comparison is made of the ultimate shear strength predictions of ACI 318-77, CEB-FIP (1978) and a proposed equation, which takes account of the influence of absolute beam depth. The predictions of this formula compared to experimental results have a correlation coefficient far greater than that of the predictions of both ACI 318-77 and CEB-FIP (1978). However as no account was taken of web reinforcement it would be unwise to extrapolate these results to the design of beams including web reinforcement. Furthermore Bazant, Z., and Kim, J.,<sup>41</sup> state: "No meaningful experimental evidence seems to be available for the size effect in presence of shear reinforcement. It is nevertheless theoretically evident that the reduction in the loss of safety margin with the increasing size, may be considerably milder or even insignificant when shear reinforcement is present."

The only references found to take account of absolute depth permit a shear enhancement factor which with the exception of CEB-FIP 1978<sup>52</sup> applies only to slabs.

REFERENCE	FACTOR
CP 110 <sup>7</sup>	$(1,6 - 0,002d)$ as tabulated in Table 14 <sup>7</sup> ( $150 < d < 300$ )
BS 0000 <sup>51</sup> (Draft revision for CP 110)	$4\sqrt{\frac{500}{d}}$ $d < 500$
CROSS, M., <sup>43</sup>	$3\sqrt{\frac{500}{d}}$ (No limit on d)
CEB-FIP 1978 <sup>52</sup>	$(1,6 - d_e) \geq 1,0$ ( $d_e$ in metres)



Deep beams designed in accordance with current codes of practice such as ACI 318-83, CP110 and CEB-FIP will be designed with a minimum orthogonal mesh of web reinforcement, and hence no modification to take account of absolute depth is required. Even when no web reinforcement is specified it is doubtful if the accuracy with which current design methods predict the ultimate strength of deep beams, warrants the inclusion of a factor to allow for the influence of absolute beam depth.

---

## 5.5 BOND

### 5.5.1 General

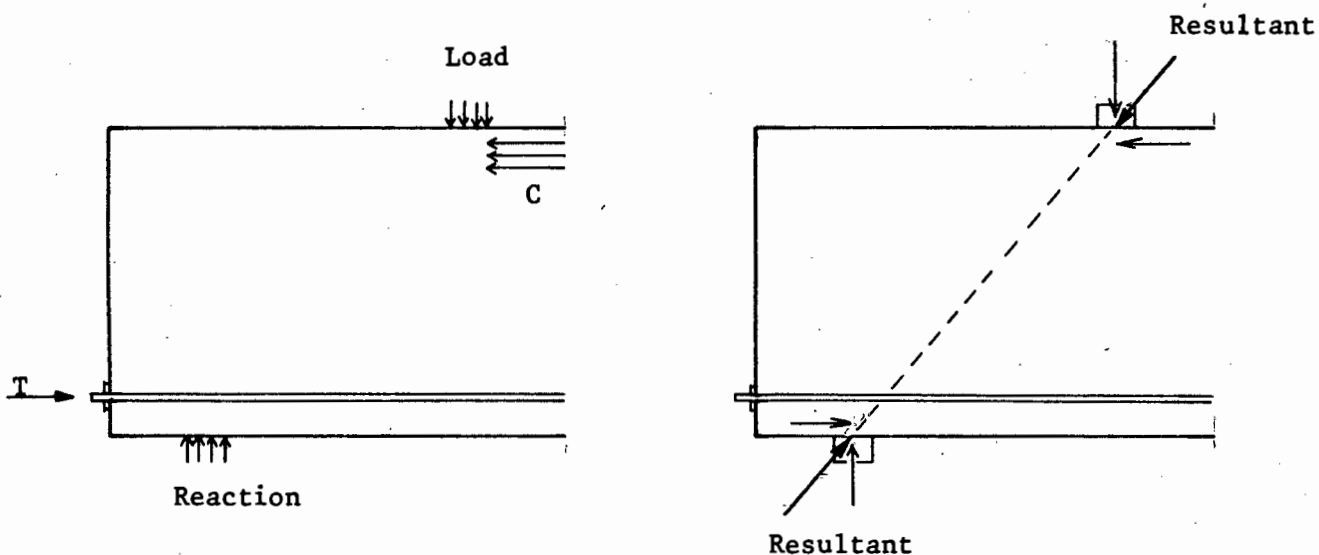
Bond is that interaction between concrete and reinforcement that ensures that they perform together so as to have the same displacement and strain.

A beam with adequately anchored but totally unbonded main tensile reinforcement is, after the concrete section fails in flexure, statically a tied arch. Due to the absence of bond the tensile force in the (tie) reinforcement is uniform and is not added as a distributed load to the concrete body but as a concentrated force at the anchorage.

The stress condition in the shear span of an unbonded beam is one of direct forces, being essentially that of a concrete body under longitudinal compression. This stress state is ideally suited to the load carrying properties of concrete.

#### 5.5.1.1

#### REINFORCED CONCRETE BEAM WITHOUT BOND



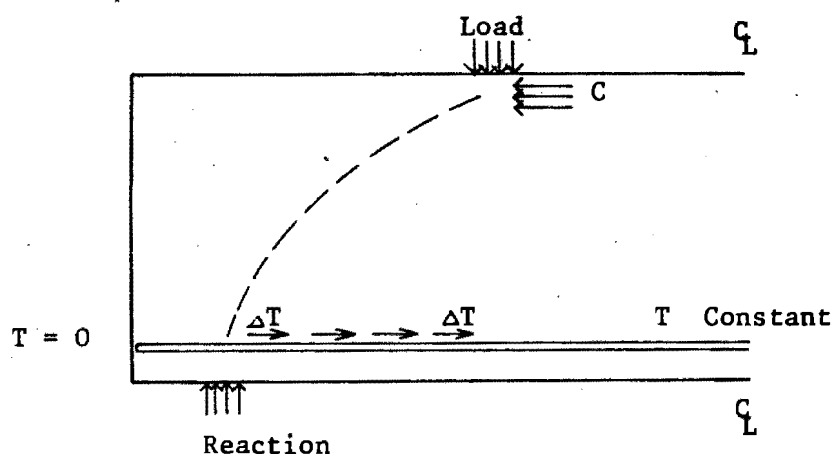
Kani, G.<sup>38</sup>, states; "It can be seen that the stress conditions in such a concrete body is rather favourable so that diagonal failure of a reinforced concrete beam without bond cannot be expected."

The behaviour of the unbonded beams tested contradicts this assertion. Both beams 4/1 and 4/2 failed due to inclined cracking. The diagonal tension cracks due to the principal tensile stress, formed at or just short of the load causing ultimate failure. (See 3.2.6 ) The failure mode exhibited by both beams 4/1 and 4/2, substantiates the opinion expressed by Brock, G.<sup>40</sup>, "The strength of the arch if adequately tied, is limited by the ability of the concrete to resist the inclined thrust. When concrete is subjected to a localised thrust, it usually fails by splitting under tensile stresses normal to the line of thrust".

The preceding comparison of the cracking and ultimate loads of the unbonded beams with that of equivalent beams with bonded reinforcement is largely a comparison of arch action versus a combined beam/arch action. Total absence of bond is one easily defined extreme of an entire spectrum of bond characteristics, the other extreme is perfect bond, which does not lend itself to precise definition or achievement in practice. The manner in which bond characteristics influences the behaviour of reinforced concrete deep beams is not only dependant on the surface roughness of the reinforcement, but is also greatly influenced by the number, diameter and distribution selected to provide the desired area of the main flexural reinforcement.

Research by Moody, K. et. al.<sup>49, 50</sup> has shown that when an equal area of reinforcement is provided by more than one bar there appears to be an increase in both the cracking and ultimate capacity. However increasing the number of bars beyond an optimum in any one layer, dependant on beam width, will result in a reduction of strength. Taub, J., and Neville, A.<sup>35</sup> offer the following explanation, "The use of a large number of smaller bars for the same area of steel means that the bond stress is smaller, and hence the resistance to bond failure is higher. On the other hand, the larger the sum of the diameters of the bars, the smaller the net cross section of the concrete (through the plane of the bars). Thus the area of concrete resisting its splitting is smaller".

#### 5.5.1.2 REINFORCED CONCRETE BEAM WITH BOND



5.5.1.3 EXPERIMENTAL WORK

The load causing inclined cracking in the two unbonded beams tested (4/1 and 4/2) is substantially more than the load which would result in inclined shear cracks in equivalent beams with fully bonded reinforcement. This gain is at the expense of an extremely large "flexural" midspan crack at a load far less than that which would cause any visual sign of distress in an equivalent beam with fully bonded reinforcement.

Beam with unbonded reinforcement			
BEAM	$V_{cr}$	$V_u$	Midspan cracking load
4/1	125	125	35
4/2	175	190	45

Beam with bonded reinforcement	
$V_{cr(bond)}$	$V_u(bond)$
82	125
102	165

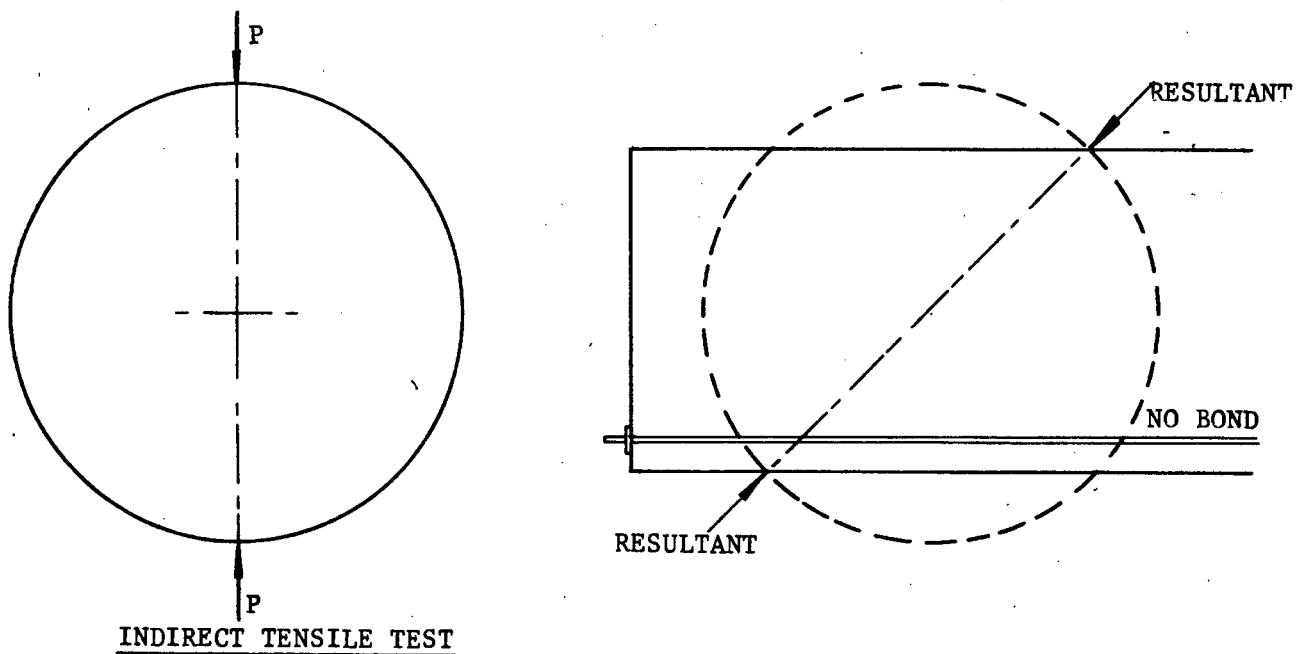
NOTE:

- i)  $V_{cr}$  and  $V_u$  are loads in kN respectively causing inclined cracking and ultimate failure of beams 4/1 and 4/2. See 3.2.6
- ii)  $V_{cr(bond)}$  and  $V_u(bond)$  are loads in kN respectively causing inclined cracking and ultimate failure of equivalent beams with bonded reinforcement. See 5.1.2 for plot of  $V_{cr}$  and  $V_u$  versus  $F_{cu}$  for Series 1 beams.

A measure of bond efficiency is the size and spacing of cracks in the concrete. The better the bond the more closely spaced and smaller the cracks at a given load level. Poor or non existent bond will result in few large cracks at wide spacings. These cracks will be obvious at load levels below that load leading to visible cracking in beams with effectively bonded reinforcement. Both the two unbonded beams tested, displayed obvious midspan cracking at load levels of less than half that load causing visible cracking in equivalent beams with bonded reinforcement. As a result of the early formation and unacceptable widths of the midspan cracks in beams with unbonded reinforcement the apparent gain in ultimate capacity cannot be utilised for practical application.

The only design guide found to implicitly take account of the beneficial influence of good bond characteristics on deep beam behaviour is that of Kong, et. al.<sup>11 & 13</sup>. See 2.7 The empirical equation proposed is based on the experimental results obtained by testing deep beams with smooth round reinforcement, yield strength 300 MPa and deformed reinforcement with a yield strength of 400 MPa. The ratio of the proposed values for the empirical coefficient  $C_b$ , 300 MPa for deformed reinforcement and 130 MPa for smooth round reinforcement, exceeds the ratio of yield strengths, reflecting the advantage of good bond characteristics.

5.5.1.4 SPLITTING ANALOGY FOR DEEP BEAMS WITH UNBONDED REINFORCEMENT



INDIRECT TENSILE TEST

BEAM	$v_{cr}$ (MPa)	$f_{calc}$ (MPa)	$v_{cr}/f_{calc}$
4/1	2,22	2,51	0,89
4/2	3,11	3,01	1,03

NOTE:

- i)  $v_{cr} = \frac{V_{cr}}{bd}$  ( $V_{cr}$  is shear at which diagonal cracking was first observed)
- ii)  $f_{calc} = 0,56 \sqrt{F_{cu}}$ , ACI 318-83 Clause 9.5.2.3
- iii) See 3.2.6 for full description of Series 4 beams.

## 5.6 SUPPORT CONDITIONS

### 5.6.1 General

The shear enhancement<sup>\*</sup> factors recommended by codes of practice and design guides to take account of post-cracking load bearing capacity of beams with low  $a/d$  ratios are typically empirical functions of the  $a/d$  ratio.

See 5.3.3 The experimental work on which these factors were based, as well as the configuration of the beams load-tested in the experimental work reported in this thesis satisfy the condition of direct support. The restraint due to the semi-circular rockers (below the support points of the beams tested in this thesis) which were not free to move until sliding friction was overcome also resulted in an increase in beam capacity.

The beams load-tested in this study were all loaded through bearing plates on the top surface of the beam, the supports were similarly directly below the bottom surface of the beam. This load-support condition which can be described as direct support will be such as to create diagonal compression in the member and is also the configuration which provides the maximum arch rise after initial inclined cracking.

Deep beams in full scale structure are often loaded through secondary beams or slabs which transfer load to the main beam. The end of the beam in turn commonly frames into another deep beam, wide column or concrete wall. This load-support condition does not fully satisfy the requirement of direct support. As the end reaction does not act at the bottom of the beam but is spread over its full depth the resulting arch will have a reduced rise and hence lowered ultimate capacity.

Research by Fereig, S., and Smith, K.,<sup>21</sup> has led them to conclude :

"For directly loaded beams without web reinforcement the nominal shear stress at failure increases as the shear span-to-depth ratio decreases below approximately 2,5. Indirectly loaded beams exhibit a much smaller gain in strength and the increase only occurs below an  $a/d$  of about 1,5".

The above conclusions are compatible with the findings of Taub, J., and Neville, A.<sup>35</sup> The results of their tests on a series of beams ( $a/d = 2,09$ ) revealed that when the loads were applied indirectly, the ultimate load was between 87 and 93% of the capacity of similar beams subjected to direct loading.

\* See 5.3.3

In the interest of uniform factors of safety it is important that the designer shall ensure that the conditions of direct support are satisfied, before taking advantage of the increased shear capacity resulting from low  $a/d$  ratios.

See 5.3.3

---

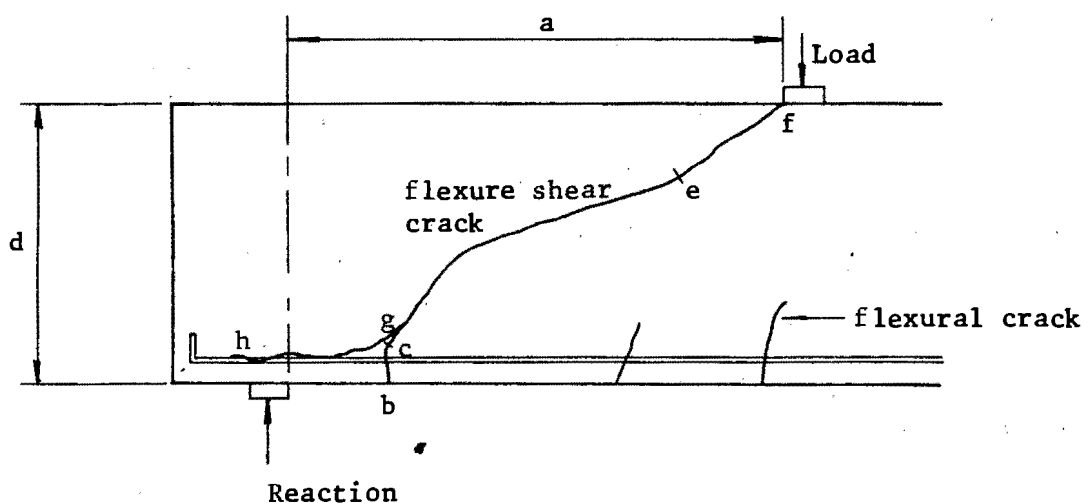
## 5.7 SHEAR FAILURE MODES

### 5.7.1 General

The strength of deep beams is usually controlled by shear, rather than flexure, provided normal amounts of longitudinal reinforcement are used. Shear failure is characterized by small deflections and lack of ductility. An orthogonal mesh of web reinforcement which can contribute substantially to the ultimate capacity increases the ductility and reduces the probability of sudden drastic failure. The actual shear failure mode is largely determined by the  $a/d$  ratio, which can be broadly grouped into three categories as follows:

### 5.7.2 Shallow Beams

- i)  $a/d > 6$ ; seldom fail due to shear.
- ii) Shallow reinforced concrete beams ( $2,5 < a/d < 6$ )<sup>26</sup> tend to fail in shear. With regard to the following diagram, as the applied load is increased the flexural crack b-c nearest the support will extend towards the point of load application. This crack gradually becomes inclined with an increase of load and is known as a flexure-shear crack. If the  $a/d$  ratio is relatively high this crack will rapidly progress to f resulting in failure by splitting the beam in two. This mode of failure is known as diagonal-tension failure. If the  $a/d$  ratio is relatively low the flexure-shear crack will tend to stop at e. Further increase of the applied load will result in the destruction of the reinforcement bond and anchorage resulting in splitting along gh. This failure is known as shear-tension failure.



FAILURE OF SHALLOW REINFORCED CONCRETE BEAM ( $2,5 < a/d < 6$ )<sup>26</sup>



### 5.7.3 Moderately Deep Beams

Moderately deep reinforced concrete beams ( $1,0 < a/d < 2,5$ ) typically fail in shear as a result of the extension of the upper end of the diagonal tension crack, which reduces the compression zone adjacent to the load point resulting in compressive failure of the concrete there. This mode of failure is known as shear-compression failure. Smith, K., and Vantsiotis, A.,<sup>23</sup> reporting the results of load tests of 52 beams ( $0,77 < a/d < 2,01$ ) state:

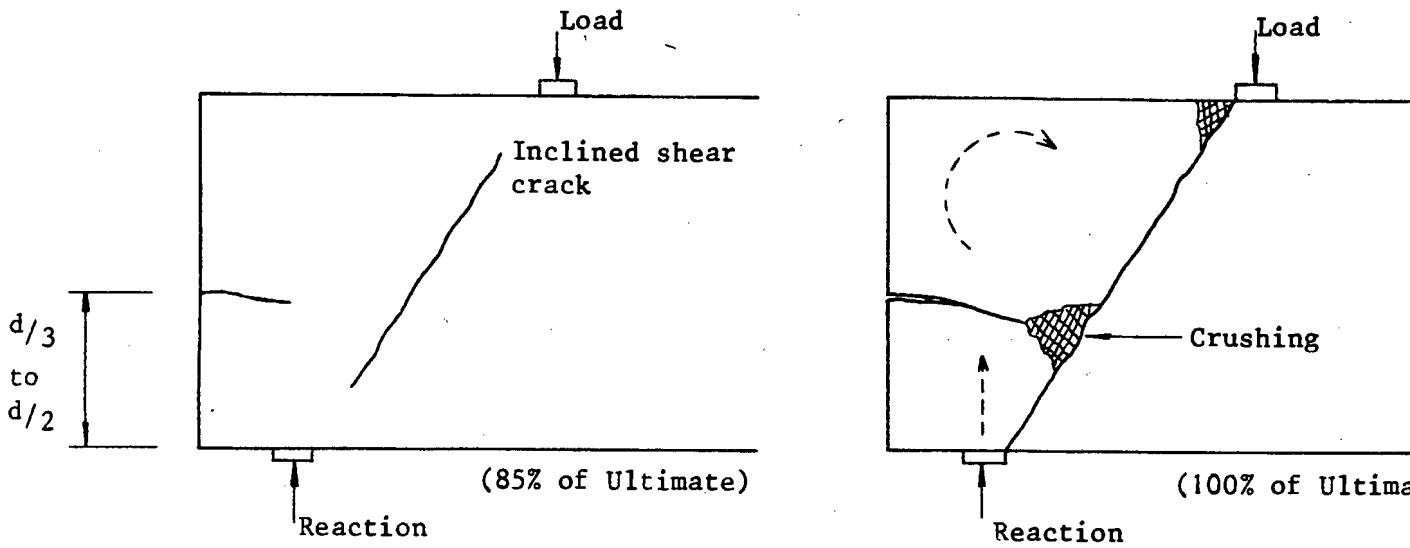
"Crushing always occurred at a position other than the region of maximum moment". The beam will be stable after the formation of the initial crack, and will be capable of carrying load well in excess of the load causing initial inclined cracking. This reserve capacity is acknowledged by codes of practice such as CEB-FIP 1978 and BS CP 110 which recommend a shear enhancement factor of  $2d/a$  for  $a/d < 2$ . See 5.3.3

### 5.7.4 Deep Beams

Deep reinforced concrete beams  $a/d < 1,0$  typically fail as a result of the splitting action of the compressive force transmitted directly from the load point to the support. The initial diagonal crack forms as a result of the primary tensile stress exceeding the tensile strength of the concrete. The ultimate failure occurs when the initial diagonal crack (which typically starts at a point  $d/4$  to  $d/3$  above the beam soffit) extends so deeply into the compression zone that failure occurs by crushing at the load and/or support. With few exceptions this mode of failure was responsible for the collapse of the beams load tested in this study.

A mode of failure reported by Kong et. al.<sup>14</sup> is the occurrence of a second inclined shear crack roughly parallel to the first, followed by crushing of the strut-like portion of concrete between these two diagonal cracks.

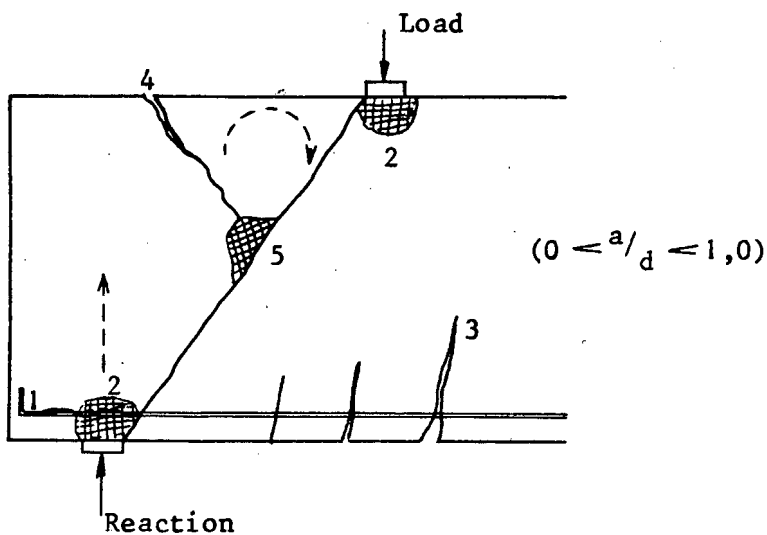
Eight of the 25 beams tested, developed horizontal cracks at  $d/3$  to  $d/2$  above soffit level at load levels between 85% and 100% of ultimate. None of these eight beams contained any web reinforcement. The formation of these cracks, (of which the writer finds no reference in the literature surveyed) is understood to be due to an increase of tensile strain resulting from the projection of the beam beyond the support. This projection was required in order to provide anchorage for the main longitudinal reinforcement. In three of the eight beams the extension of these horizontal cracks, on intersecting the existing diagonal crack, resulted in ultimate failure.



TENSION FAILURE BY HORIZONTAL AND INCLINED CRACKING

A similar mode of failure was described by Bresler, B.<sup>34</sup> who states:

"... tension failure of the "arch-rib" by cracking over the support at point 4 (in the following diagram), followed by crushing along the crack at point 5. This is the result of the eccentricity of the thrust which essentially acts along the inclined crack".



MODES OF FAILURE IN DEEP BEAMS<sup>34</sup>

Three further modes of failure, illustrated in the preceding diagram are :

- 1) Anchorage failure of the main longitudinal reinforcement. The extension beyond the supports of the beams tested provided adequate anchorage, with the result that this mode of failure did not occur.
- 2) Crushing over support or under local point, previously discussed as occurring as a result of the penetration of the inclined shear crack into this zone; however at very low  $a/d$  ratios the very high bearing stress at these points may result in local crushing. By suitable detailing of local reinforcement the writer prevented this mode of failure, except where it occurred as a result of eccentricity of load application which resulted in overall lateral instability.
- 3) Flexural failure due to under-reinforcement.  
The two beams tested in Series 3 were both under-reinforced, both still failed in shear. This would indicate that only extreme under-reinforcement would result in a flexural type failure by yielding of the main longitudinal reinforcement in the region of maximum bending moment. See 5.2.1 for discussion of the manner in which the quantity of main longitudinal reinforcement influences crack propagation.

The range of alternative possible modes in which reinforced concrete deep beams may fail contributes to the difficulty of predicting the cracking and/or ultimate load.

## 6. CONCLUSIONS

### 6.1 General

From the experimental work and literature reviewed in this thesis, the following conclusions which pertain only to single span simply supported deep beams subject to direct loading, can be drawn.

### 6.2 Concrete Compressive Strength

6.2.1 The inclined cracking load and ultimate shear capacity of reinforced concrete deep beams are significantly influenced by the concrete compressive strength.

### 6.3 Reinforcement

6.3.1 The effectiveness of horizontal web reinforcement increases and of vertical web reinforcement decreases with decreasing  $a/d$  ratio.

6.3.2 The ultimate shear capacity of deep beams is influenced to a significantly greater extent than the shear capacity of shallow beams by changes to the quantity of adequately anchored main longitudinal reinforcement.

### 6.4 Shear Span to Depth Ratio ( $a/d$ )

6.4.1 The inclined shear cracking load of reinforced concrete deep beams is not significantly influenced by the  $a/d$  ratio.

6.4.2 The ultimate shear capacity of reinforced concrete deep beams is significantly influenced by the  $a/d$  ratio.

### 6.5 Support Conditions

6.5.1 The ultimate shear capacity of reinforced concrete deep beams is significantly influenced by load support conditions.

## 6.6 Failure Modes

- 6.6.1 Reinforced concrete deep beams with usual amounts of main longitudinal reinforcement, fail in shear, typically as a result of the penetration by the inclined shear crack into the compression zone.
- 6.6.2 Deep beam shear failure is not a ductile failure mode and occurs at a load less than the flexural capacity of the beam.
- 6.6.3 Maximum flexural and diagonal crack widths were not a serviceability problem at design working loads in the beams tested.

## 6.7 RECOMMENDATIONS

- 6.7.1 Nominal shear stress ( $v_u$ ) must not exceed a permissible stress defined in terms of the following factors.

Concrete compressive strength	( $f_{cu}$ )
Main longitudinal reinforcement	( $A_s$ )
Absolute beam depth	( $d$ )
Shear span to depth ratio	( $a/d$ )

- 6.7.2 An orthogonal mesh of web reinforced must be provided in both faces and taken into account when assessing ultimate shear strength.
- 6.7.3 The method proposed by Kong et. al. should be used for the design of reinforced concrete deep beams.
- 6.7.4 Reinforcement must be detailed in accordance with the recommendations of CEB-FIP 1970<sup>8</sup>.
- 6.7.5 The recommendations of PCA and AS 1480 were found to be the least realistic and useful.

REFERENCES

1. Building Code Requirements for Reinforced Concrete.  
(ACI 318-83)
2. Commentary on Building Code Requirements for Reinforced Concrete. (ACI 318-83)
3. Israel Standard, Concrete Code : General Principles.  
SI 466 : Part 1 (1975)
4. Israel Standard, Concrete Code : Concrete Elements and Structures. SI 466 : Part 2 (1979).
5. Australian Standard. The Use of Reinforced Concrete in Structures. AS 1480-1982
6. British Standard. Reinforced Concrete in Buildings.  
CP 114 : Part 2 : 1969
7. British Standard. The Structural Use of Concrete.  
CP 110 : Part 1 : 1972
8. International Recommendations for the Design and Construction of Deep Beams. CEB-FIP (1970)
9. Design of Deep Girders. PCA No. ST66 - 1954
10. F. Fulton. Concrete Technology. PCI. 1977
11. Kong, F., Robins, P., Singh, A. and Sharp, G. "Shear Analysis and Design of Reinforced Concrete Deep Beams"  
THE STRUCTURAL ENGINEER, Vol. 50, October 1972, p. 405 - 409
12. Kong, F. and Sharp, G. "Shear Strength of Lightweight Reinforced Concrete Deep Beams with Web Openings"  
THE STRUCTURAL ENGINEER, Vol. 51, August 1973, p. 267 - 275.

REFERENCES - continued

13. Kong, F., Robins, P. and Sharp, G. "The Design of Reinforced Concrete Deep Beams in Current Practise".  
THE STRUCTURAL ENGINEER, Vol.53, April 1975, p. 173 - 180.
14. Kong, F., Robins, P., and Cole, D. "Web Reinforcement Effects on Deep Beams." ACI Journal, Vol. 67, Dec. 1970, p.1010 - 1017
15. N.V. Arunachalam, "Discussion of Paper by Kong, et. al"  
(Ref. 14). ACI Journal, Vol. 68, June 1971, p.476 - 480.
16. Kong, F., and Robins, P., "Web Reinforcement Effects on Lightweight Concrete Deep Beams". ACI Journal, Vol. 68, July 1971, p. 514 - 520.
17. Manuel, R., Slight, B., and Suter, G., " Deep Beam Behaviour Affected by Length and Shear Span Variations". ACI Journal, Vol. 68, Dec. 1971, p. 954 - 958.
18. N.V. Arunachalam, "Discussion of Paper by Kong, et. al".  
(Ref. 14) Dec. 1971, p. 954 - 958.
19. Kong, F., Robins, P. Kirby, D. and Short, D., " Deep Beams with inclined Web Reinforcement". ACI Journal, Vol. 69, March 1972, p. 172 - 176.
20. Kong, F., and Singh, A., "Diagonal Cracking and Ultimate Loads of Lightweight Concrete Deep Beams". ACI Journal, Vol. 69, August 1972, p. 513 - 521.
21. Fereig, S., and Smith, K., "Indirect Loading on Beams with Short Shear Spans". ACI Journal, Vol. 74, May 1977, p.220 - 222.
22. Taner, N., Fazio, P. and Zielinski, Z., "Strength and Behaviour of Beam Panels - Tests and Analysis". ACI Journal, Vol. 74, Oct. 1977, p. 511 - 520.
23. Smith, K. and Vantsiotis, A., "Shear Strength of Deep Beams"  
ACI Journal, Vol. 79, May - June 1982, p. 201 - 213.

REFERENCES - continued

24. ASCE "Structural Design of Tall Concrete and Masonry Buildings". Vol. CB, Edited by MacGregor, J. 1978.
25. UNESCO "Reinforced Concrete : An International Manual" Published by Butterworths, London, 1971.
26. Kong, F., and Evans, R., "Reinforced Concrete and Prestressed Concrete". Published by Nelson, 1977.
27. Regan, P., and Yu, C., "Limit State Design of Structural Concrete". Published by Chatto & Windus, London, 1973.
28. Winter, G., and Nilson, A. "Design of Concrete Structures" 9th Edition. Published by McGraw-Hill, New York, 1979.
29. Kotsovos, M., "Behaviour of Reinforced Concrete Beams with a Shear Span to Depth Ratio between 1,0 and 2,5" ACI JOURNAL, Vol. 81, May-June 1984, p. 279-286.
30. Kong, F. "Shear Strength of Reinforced Concrete Deep Beams". CONCRETE, Vol. 6, March 1972, p. 34-36.
31. Ramakrishnan, V, and Ananthanarayana, Y., "Ultimate Strength of Deep Beams in Shear" ACI JOURNAL, Vol. 65, Feb. 1968, p. 87-98.
32. Rawdon de Paiva, H. and Siess, C., "Strength and Behaviour of Deep Beams in Shear". Journal of the Structural Division. ASCE, Vol, 91, ST, Oct. 1965, p. 19-41
33. Leonhardt, F., "Strength and Behaviour of Deep Beams in Shear" Journal of the Structural Division. ASCE, Vol. 92, ST, Apr. 1966.
34. Bresler, B., and MacGregor, J., "Review of Concrete Beams Failing in Shear" Journal of the Structural Division. ASCE, Vol. 93, ST, Feb. 1967, p. 343-372.



REFERENCES - continued

35. Taub, J. and Neville, A., "Resistance to Shear of Reinforced Concrete Beams". ACI Journal, Vol. 57, Aug. 1960.
36. Mphonde, A. and Frantz, G., "Shear Tests of High and Low Strength Concrete Beams without Stirrups". ACI Journal, Vol. 81, July - August 1984, p. 350-357.
37. KANI, G., "How Safe are our Large Reinforced Concrete Beams?" ACI Journal, Vol. 64, March 1967, p.128-141.
38. KANI, G., "The Riddle of Shear Failure and Its Solution." ACI Journal, Vol. 61, April 1964, p. 441-466.
39. Joint ASCE-ACI Committee 426, "The Shear Strength of Reinforced Concrete Members". Journal of the Structural Division. ASCE, Vol. 99, June 1973., p. 1148-1157.
40. Discussion of The Paper by G. KANI. (Ref. 38). ACI Journal, Vol. 61, Dec. 1964, p. 1587-1636.
41. Bazant, Z. and Kim, J. "Size Effects in Shear Failure of Longitudinally Reinforced Beams". ACI Journal, Vol. 81, Sept. 1984, p. 456-468.
42. Collins, M., and Mitchell, D., "Shear and Torsion Design of Prestressed And Non-Prestressed Concrete Beams." PCI Journal, Sept.-Oct. 1980, p. 32-100.
43. Cross, M., "Punching Shear in Reinforced Concrete Flat Plate Structures - Code Predictions and Test Results". Die Siviele Ingenieur in Suid-Afrika, Feb. 1985.
44. Concrete and Reinforced Concrete Structures. Design and Construction. DIN 1045.

REFERENCES - continued

45. Franz, G., and Niedenhoff, H., "The Reinforcement of Brackets and Short Deep Beams".  
Translation No. 114. C & CA 1964.
46. Kong, F. and Sharp, G., "Structural Idealization for RC Deep Beams with Web Openings". University of Cambridge, Report CUED/C-STRUCT/TR.62 (1977).
47. Zsutty, T., "Beam Shear Strength Predictions by Analysis of Existing Data". ACI Journal, Vol. 65, Nov. 1968, p. 943-951
48. Zsutty, T., "Shear Strength Predictions for Separate Categories of Simple Beam Tests". ACI Journal, Vol. 68, Feb. 1971, p. 138-143.
49. Moody, K., Viest, I., Elnster, R., and Hognestad, E., "Shear Strength of Reinforced Concrete Beams". ACI Journal, Vol. 51, Dec. 1954, p. 417-434.
50. Moody, K., Viest, I., Elnster, R., and Hognestad, E., "Tests of Restrained Beams without Web Reinforcement". Vol. 52, Jan, 1955, p. 697-730.
51. BS 0000. Draft revision of CP 110. Section 3.
52. CEB-FIP-1978. Model Code for Concrete Structures.

8. COURSES COMPLETED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE  
M.Sc.(ENG) DEGREE AT THE UNIVERSITY OF CAPE TOWN

---

<u>COURSE</u>		<u>DATE CREDITED</u>	<u>CREDIT VALUE</u>
CE 538	Advanced Foundation Design	1980	5
CE 551(a)	Frame Analysis	1981	2
CE 552(b)	Introduction to the Theory of Elasticity	1981	1
CE 533	Bridge Engineering	1982	4
CE 553	Structural Loading	1982	2
CE 506	Properties of Concrete	1983	4
CE 535	Engineering Economics	1983	3
CE 556	Contract Law	1983	3
			<hr/>
		TOTAL	24
			<hr/>

This thesis represents half of the credit requirements for the Degree.