

**A Markowitz Mean-Variance
Analysis of Hedge Fund
Investments for Multi-Asset Class
Portfolio Holders in South Africa**

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by
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Dedication

I dedicate this to my parents and sister who have shown tireless support towards me throughout my studies, and without whom it would not have been possible to complete my thesis work. I would like to also thank Dr Ryan Kruger for his supervision and guidance throughout my research. His input has been invaluable and I look forward to working with him again in the future. Lastly, I would like to thank my friends and fellow students for providing me with the necessary motivation to overcome the challenges of this paper.



A Markowitz Mean-Variance Analysis of Hedge Fund Investments for Multi-Asset Class Portfolio Holders in South Africa

by

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This research aims to provide insight into the hedge fund industry in South Africa. The focus is on retirement funds and the use of hedge funds in a multi-asset class portfolio. Diversification is an important tool for portfolio managers who make use of correlation to achieve higher risk-adjusted returns for investors. As such this paper tests whether higher risk-adjusted returns can be achieved in well diversified multi-asset class portfolios if hedge funds are included. To test for the optimal risk-adjusted returns that can be achieved, mean-variance, mean-semi variance and Omega portfolios were created. The results suggest that portfolios that include hedge fund investments outperformed those that exclude it using mean-variance, mean-semi variance and Omega analysis. Furthermore it was found that portfolios that included Pure Hedge Funds outperformed those that included Fund of Hedge Funds. The evidence suggests that hedge fund investments should be included in a well-diversified South African multi-asset class portfolio, with Pure Hedge Funds being preferred to Fund of Hedge Funds.



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Chapter 1: Introduction

The South Africa asset management industry has evolved since the 1990's, changing with the times and riding the waves of uncertainty. As the industry has evolved, so too has regulation, most prominently in response to a US-led housing bubble and increased questions about income inequality. Asset managers earn their keep by being able to circumvent these bumps whilst still managing to earn a positive return for investors. In an ever-changing market, managers need to find innovative ways to make money. The archetype of this innovation is perceived to be hedge fund managers. However, due to the high barriers to entry, these managers are often beyond the reach of the average pension fund investor. This exclusivity also comes with the expectation of higher earnings and greater levels of risk.

In South Africa institutional investors still comprise the largest proportion of market liquidity and capitalisation. That being said, many hedge fund investors look to institutional investors for capital, these institutional arms being called Fund of Hedge Funds. Given the typical investment opportunity set of equities, bonds, cash, commodities and real estate-one could argue that hedge funds should be included as a source of additional diversification and returns. This agrees with the work of Harry Markowitz who argues that there is an optimal level of risk to reward for each investor that weights assets along an efficient frontier (Markowitz 1952). The efficient frontier represents the use of diversification by taking the disparity in correlation between two or more assets to maximise the return per unit of risk. The idea is that, for taking on assets with lower or negative correlations, investors will improve their risk-adjusted returns.

This leads to the argument of this research paper, which hypothesises that a higher risk-reward ratio can be achieved by adding hedge fund investments to a South African multi-asset class portfolio. Given the unique investment strategies employed by hedge fund managers in South Africa, if institutional investors were to increase their scope to these funds, they would be enhancing the benefits of diversification. Beyond this greater market interaction and higher levels of trading also offers the opportunity for increased market liquidity, transparency and price efficiency.



1.1 Problem Statement

The aim of this study is to examine whether hedge fund investments should be included in a South African multi-asset class portfolio. The focus is on the diversification benefits of taking on a hedge fund investment in a multi-asset class portfolio.

The questions this study will endeavour to answer:

- I. Is there a diversification benefit to including hedge fund investments in a well-diversified traditional assets portfolio?

- a. *Does a traditional assets portfolio inclusive of hedge fund investments offer a greater optimal Sharpe ratio than a portfolio comprising only traditional assets?*

The Sharpe ratio represents the tool used in a Markowitz mean-variance analysis and is of particular importance as it is an illustration of risk to reward - as measured by excess return over standard deviation (Markowitz 1952). It is the backbone of portfolio optimisation. Over time, however, newer tools and flaws in the existing Markowitz mean-variance model has drawn criticism as to its trustworthiness. Hence, other models should be tested alongside the mean-variance analysis.

- b. *Does a traditional assets portfolio inclusive of hedge fund investments offer a greater optimal Sortino ratio than a portfolio comprising only traditional assets?*

The Sortino ratio is a measure used in mean-semi variance analysis, which is an extension of the mean-variance analysis. The difference between the Sharpe and Sortino is that, instead of standard deviation, the Sortino uses downside deviation (Sortino & Hopelain 1980). This addresses a major shortfall in the mean-variance analysis: using standard deviation assumes that investors feel the same way about upside gains as we do about downside losses (Sortino & Price 1994). Given the occurrences of the market crash in 2008/2009 and the impact of the major downside loss, the use of tools to manage downside risk, and their importance, has increased. Hence, the optimal Sortino portfolios will be analysed alongside the optimal Sharpe portfolios.

- c. *Does a traditional assets portfolio inclusive of hedge fund investments offer a greater optimal Omega ratio than a portfolio comprising only traditional assets?*

The Omega ratio is a measure used to probability weights gains and losses in returns, and is particularly powerful when working with hedge fund returns which exhibit high



levels of skewness and kurtosis (Keating & Shadwick 2002b). Omega makes no assumptions of returns, and is an unbiased tool that can be used to optimise portfolio returns; given its strengths it will be used to diagnose the viability of the mean-variance and mean-semi variance optimisation (Keating & Shadwick 2002b).

If hedge funds are seen to provide greater levels of risk-adjusted returns using Sharpe, Sortino and Omega, then there is compelling evidence to suggest that hedge fund investments do have a place in a South African portfolio.

The outcome of this study will be to shed light on the hedge fund industry, and expand on the limited amount of knowledge available on it. An increased understanding of the hedge fund industry's benefits for portfolio holders could lead to it being included alongside the traditional investment set. This inclusion could lead to greater regulation around the industry whilst also attempting to understand when and where hedge fund investments should be included.

Going forward, collective investment schemes or institutional investors could allow access to individuals who would normally be excluded from the hedge fund industry. The positive benefits would be an increased risk-adjusted return for investors. The outcome of this study will allow South African portfolio managers' as well as pension assets managers' valuable insight into the benefits of taking on hedge fund investments.



Chapter 2: Literature Review

2.1 Markowitz Mean-Variance

The idea behind portfolio selection goes back to the work of Markowitz (1952), where the concept behind optimal portfolio selection was formally introduced. Markowitz proposed that portfolio selection attempts to maximise the “Risk-Return traded off”. In his work, Markowitz defined the concepts of expected return and risk in statistical terms. Expected return was the “mean value of the theoretical probability distribution of returns”, and risk was defined as “variance”, later adjusted to standard deviation¹ (Markowitz 1952). Markowitz suggested that, to obtain an optimal portfolio, investors should weight investments in assets so as to maximise the offsetting relationship between co-variances. By maximising the offsetting relationship between assets, investors would be left with an efficient frontier: a plot of the expected return and risk combinations.

Tobin (1958) expanded on this work by introducing the idea of a risk-free asset, which allowed for investors’ risk preferences to be considered in portfolio selection (Tobin 1958). Tobin showed that the optimal portfolio weighting was a straight line called the “Tangency Portfolio”; consisting of risky and risk-free assets. Tobin suggested that the tangency portfolio is represented by the line where the risk-free asset and efficient frontier are tangent. It was suggested that investors move weightings across this straight-line according to their risk tolerance.

In 1964, Sharpe produced a paper that summarised the work of Tobin and Markowitz into a single-factor model. The model proposed that the return of securities are linearly related to market. This was done by introducing the assumption of homogenous expectations, and that the borrowing/lending rate is the same for all investors (Sharpe 1964). In doing this, Sharpe introduced the idea that the optimal portfolio of risky assets was in fact the market itself. From this assumption, and the introduction of “unlimited borrowing and lending at the risk-free rate”, the Capital Asset Pricing Model (CAPM) was formally derived by Sharpe-Lintner:



Capital Asset Pricing Model

$$E(R_i) = R_f + \beta[E(R_m) - R_f]$$

where:

$E(R_m)$ = Expected return on asset i

R_f = Risk-free rate of return

$E(R_m)$ = Expected return of the market

β = Beta coefficient of asset i

(Litner 1965)

Later, Black (1972) relaxed the unlimited borrowing/lending assumption, which was considered unrealistic and a flaw in the model. Black did this by showing that a similar result can be obtained by allowing for unrestricted short-selling of the risky asset. This is a common trading strategy among hedge funds, with 59.9% of the local hedge fund industry using the “equity long/short” strategy (Novare Investments 2014).

Criticism of the Markowitz model by Bowen (1984) is that it requires high volumes of data, which makes it computationally cumbersome. This is backed up by the finding of Michaud (1989), who suggests that the Markowitz-model is conceptually demanding in nature. These criticisms, however, have become less onerous over time due to the growth of computing power since these studies were conducted. A further criticism of the Markowitz model is that investors do not have a homogenous set of expectations, which formulates the notions behind behavioural finance. Hogan et al. (2004) argues that the assumptions that underpin the model are in line with the “risk-reward” analysis and control for these variables. Without these assumptions outside variables may violate the risk-return relationship that is key to this model.

Another key criticism of the Markowitz model is that it assumes implicitly that share returns are normally distributed and explained only by first and second central moments (mean and variance). Recent studies by Mangani (2007), into the normality of share returns on the JSE using the Jarque-Bera test for normality, found compelling evidence that the returns were non-normally distributed. Concerns with kurtosis and skewness² -were also

² Also known as the third (skewness) and fourth (kurtosis) central moments



investigated, with excess kurtosis in the form of leptokurtosis and skewness being found in JSE share returns. This agrees with work by Cont (2001), which found that the violations of normality have an implication on the reliability of standard deviation and variance and more, directly the Markowitz model. These findings also suggest the importance of using a model that includes the third and fourth central moments as part of a risk-reward profile. Semi-variance analysis does not treat risk as symmetrical, but looks specifically at the down-side risk. Its inherent benefit is that it considers skewness and kurtosis. Markowitz himself, upon receiving the Nobel Prize in Economics, proposed that semi-variance be considered when analysing risk (Markowitz 1990, p.286). Levy and Markowitz (1979), however, defend the ideas behind mean-variance criterion as being sufficient in that it “yields a level of utility almost equal to an investor’s expected return”.

2.2 Alternatives to Mean-Variance

Bergh & van Rensburg (2008) investigated the semi-standard deviation using the Sortino ratio³. The origin of the Sortino ratio can be traced back to an article in the Financial Executive Magazine by Sortino & Hopelain (1980), where the risk-free in the numerator of the Sharpe ratio was replaced with a target rate of return, and the standard deviation in the denominator was replaced with down-side deviation. Sortino referred to the downside deviation as the root mean-square (RMS) of the realised return’s underperformance from the target rate, with all returns above the target treated as “0” (Sortino & Hopelain 1980). This allows for users of Sortino to “distinguish between good and bad volatility” and addresses the need for a mean-semi variance analysis (Sortino & Price 1994). It is evident, that as an investor, you would prefer to reduce losses from extreme downside events and increase profits from extreme events on the upside.

Formally, Bergh & van Rensburg (2008) simplify this by stating that investors would like to maximise mean and skewness, while minimising variance and kurtosis. Empirical research has found that equities and hedge fund portfolios do not effectively lower the risk of the overall portfolio. This is due to the combined portfolios producing lower skewness, as well as increased kurtosis, which one would try to reduce when diversifying a portfolio (Bergh & van Rensburg 2008). In “economic terms, the data suggest that when things go wrong in the stock market, they also tend to go wrong for hedge funds” (Amin & Kat 2002). This

³ Sortino discussed further in Methodology



is backed up by the work of Guesmi, Khaled, Jebri & Jabri (2014), who found that during the sub-prime mortgage crisis of 2007 hedge fund performances tended to be poor, in line with the weak performance of passive asset classes (bonds and equities). Hedge funds are usually leveraged much higher than passive portfolio hence extreme tail events could have a much larger impact on their returns.

To understand properly the risk-return benefits of hedge fund investments the Omega⁴ ratio will also be used in this study. The Omega which was introduced by Keating and Shadwick (2002a; 2002b) as a more superior risk-return measure than that of Sharpe and Sortino, considers the distribution of the returns being analysed. Omega is a total probability weighted ratio of upside (higher partial moment) or downside (lower partial moment) risk. This allows practitioners to use it in analysis without worrying about the parametric assumptions in the distribution of the returns. As suggested by Keating and Shadwick, “it can be used to rank and evaluate portfolios unequivocally”. In a practical sense the ranking of portfolios using Sharpe or Sortino might not be the same when compared with Omega (Keating & Shadwick 2002a; Keating & Shadwick 2002b). The difference in ranking depends on the impact of central moments such as kurtosis and skewness on the return distribution. Where higher order moments are significant and could lead to erroneous assessments of optimal portfolios under Sharpe, the Omega acts as a correcting mechanism. Given the ability to set a preference of downside or upside potential with Omega, users have to decide which they prefer, or there will be conflicting results. The preference in this study is to reduce downside risk or to attain the lowest variance and kurtosis. This is in line with the Sortino ratio analysis and fits into the need to understand the diversification benefits of hedge fund investments in traditional portfolios.

⁴ Omega to be discussed further in Methodology



2.3 Traditional Assets and Hedge Fund Assets

The traditional asset portfolio will be constructed using equities, fixed income, cash and property. This is in line with work by Wilcox & Faobozzi (2013), who define the difference between traditional and alternative asset classes. They go further to define hedge funds as alternative investment strategies, which is in line with the rationale behind this study: that hedge funds offer diversification benefits to investors in multi-asset class portfolios.

Earlier studies into hedge funds as an asset class found defining “what hedge funds do, and what they are, to be a major challenge” (Henriksson & Kiernan III 2005), given the low level of regulation and disclosure in the industry. To limit the scope of uncertainty as to the definition of a hedge fund, the broad categories will be labelled according to a survey by Novare Investments showing the industry breakdown in South Africa by strategy:

Strategy	Market Share	
	<u>2014:</u>	<u>2013:</u>
Equity Long/Short	59.9%	52.5%
Fixed Income	15.7%	11.6%
Equity Market Neutral	7.4%	7.1%
Multi-strategy	9.3%	9.0%
Volatility Arbitrage	3.1%	3.9%
Structured Finance	2.7%	2.8%

(Novare Investments 2014)

Strategy Definitions

- Equity Market Neutral: A Market Neutral strategy uses a combination of buying and short-selling (sometimes augmented by options⁵ and future positions⁶) to offset any correlation between portfolio return and overall market returns. This is to exploit

⁵ Option: A financial contract that offers the right but not the obligation to buy (call) or sell (put) an asset at an agreed-upon price during a certain period of time or on a specific date.

⁶ Future: A financial contract obligating the buyer/seller to purchase/sell an asset at a predetermined future date and price.



perceived information advantages. This strategy has a low correlation with the market due to an attempt at market neutrality (Connor & Lasarte 2005).

- Equity Long/Short: Equity Long/Short is similar to equity market neutral but without any explicit promise to maintain a market neutral position. This increases the opportunity for managers to take net-long or net-short positions. This, however, comes at a cost of clarity as it relates to the effects of asset allocation to overall portfolio risk (Connor & Lasarte 2005).
- Fixed Income: Fixed Income refers to arbitrage trading, where managers attempt to exploit mispricing among fixed income securities. These managers rely heavily on mathematical modelling of term structures of interest rates to identify opportunities. These mispricings can be along a yield curve for treasury bills or focused on taking long/short positions within corporate and treasury bonds (Connor & Lasarte 2005).
- Multi-strategy: Multi-strategy allows managers to change investment strategies depending on market conditions, or allocate capital across different asset classes simultaneously (Connor & Lasarte 2005).
- Volatility Arbitrage: Volatility Arbitrage is a directionless strategy that sells short-term call and put options to profit from mispricing between derivatives and their theoretical values (Phillips & Surz 2003).
- Structure Finance: Structured Finance is an overarching definition that refers to customisable financial securities traded or created by hedge funds in an attempt to facilitate liquidity, funding or risk transfer needs (Fabozzi et al. 2007).

The above strategies will be the broad focus of the study while also incorporating Fund of Hedge Funds. Fund of Hedge Funds refer to investment vehicles that are directly mandated to invest across a portfolio of hedge funds (Connor & Lasarte 2005). Fund of Hedge Funds are more regulated than individual hedge funds as they are usually responsible for institutional client investments in hedge funds.

Focusing on the equities sector, Ineichen (2002) argues that long-short equity hedge funds are correlated to their respective sectors, with the added benefit of being controlled for downside risk exposure. Feldman (2002) suggests that hedge funds achieve high Sharpe ratios, at the expense of excess kurtosis and negative skewness. As mentioned earlier, this downfall opens holes in the Markowitz model. Singer et al. (2003) suggest that, in a portfolio, individual assets may have a low correlation. When they are added to a well-

diversified portfolio, however, the underlying economic fundamentals and systematic interaction become significant. In simple terms this means that the more diversified an alternative asset portfolio is, the more it behaves like the market itself. Their research suggested that institutional investors should include a 20% allocation to alternative investments and 3% to hedge funds, which they call the “appropriate” mix for mid-risk investors. Scheeweis et al. (2002) argue that, due to the boutique nature of hedge funds and the reliance on individuals, investors should be concerned with idiosyncratic and operational risk. This is attributed to the specialised nature of hedge fund investors, with their reliance on small teams. It must be noted, however, that Fund of Hedge Funds deal with this concern on an institutional level as they can diversify across hedge funds.

Lhabitant & Learned (2002) did research into how many hedge funds are needed to create a well-diversified portfolio. They found that 5 to 10 hedge fund styles are sufficient to eliminate 75% of specific risk. They also found that style-specific hedge fund diversification allows for fewer hedge funds to be used. Ackermann et al. (1999), using data from Managed Account Report, Inc. (MAR) and Hedge Fund Research, Inc. (HFR), found that, over multiple time frames, the Sharpe ratio of an average and median hedge fund index is less than that of a “60% equity/40 bonds index” portfolio. This was, however, done in the US market and focused on 1994, a year in which the hedge fund industry had been doing particularly poorly. More recent studies by Edwards & Gaon (2003) using data from Hedge Fund Research, Inc. (HFR) and The Barclays Groups between 1990 and 2002 suggested a better performing Sharpe ratio than the “60% equity/40 bonds index” mentioned earlier. They state that that differences in “reporting and collection of hedge fund returns somewhat reduce our confidence in all measures of hedge fund performance” (Edwards & Gaon 2003).

One of the main concerns with hedge fund analysis is presence of bias within return data. Firstly, Capocci (2007) found that many hedge funds report their performance on a voluntary basis, which can result in sampling biases, as only a portion of the hedge fund universe is observable. Funds tend to report only when their performance is good and tend to stop reporting when they are performing poorly. Secondly, survivorship bias can be observed as funds that no longer trade will disappear from databases. This also implies that the observed data might outperform the entire population, as poorly performing funds will stop reporting. Fung & Hsieh (2002) found that it is natural for hedge funds to go through a rapid life cycle, as hedge fund managers tend to operate where trading and



leverage gaps are available. Due to the dynamic nature of the global market where trading gaps open and close over time, operations will also change over time. This will lead to hedge funds dropping in and out of that market, and is part and parcel of the industry.

Furthermore, survivorship bias is well documented in studies of mutual funds, where survivorship bias is measured as “the difference between the population of mutual funds returns over a given period versus the returns of surviving funds at the end of the period” (Malkiel 1995). As mentioned above, within the hedge fund industry, the population of hedge funds is not directly observable since much of the data is kept private. This was, however, a larger concern in the mid 1990’s; the industry has evolved since then. Hedge fund data is easier to access and regulation is evolving to incorporate these funds (Fung & Hsieh 2002). Historical research by Brown, Goetzmann & Ibbotson (1999), Liang (2000; 2001) and Fung & Hsieh (2002) suggests that survivorship bias seems to be style-related, with estimates of the bias ranging between 60bps to 360bps. This must be considered in line with the work of Ackermann et al. (1999), who argues that the impacts of non-reporting by poorly performing defunct funds is offset by the lack of reporting by top performing funds due to self-selection (hedge funds simply choose not to report their activities) . Most recently Ibbotson, Chen and Zhu (2011) found that survivorship bias ranges from 3.16% to 5.13%, when including backfilled bias⁷.

The Mesirow Finance institute, in its 2011 White Paper, identified two concerns with backfilled bias (Mesirow Finance 2011). The first was that managers tend to hand pick only the best performing funds for reporting. This is because one hedge fund team can manage multiple funds and cherry pick which to report on. Secondly, from an index perspective, because the majority of indices only require 12-months track record to be included in an index, managers game the system and decide how many months to reveal so as to attain the best possible return. Ibbotson, Chen and Zhu (2011) state that this tends to cause an upward bias in the return data. Research by Fung & Hsieh (2002), studying the impact of that lag when reporting- formally referred to as “instant history bias”- to be approximately 1.4%. Malkiel & Saha (2005), however, suggested that backfilled bias was as high as 5% using data between 1994 and 2003. More recently, Ibbotson, Chen and Zhu (2011) found backfilled bias using an equally-weighted portfolio to be 2.97% and 0.27% for a value-weighted portfolio. Their research covered January 1995 to December 2009 which is more

⁷ Backfill bias occurs “when a manager adds a new fund to a database for the first time and backfills the historical returns, by reporting only when returns are favourable” (Mesirow Financial, 2011)



relevant for this study. One of the interesting implications was that the majority of backfilled bias in a portfolio of hedge funds appears to come from smaller funds. This can be seen in the difference between the backfilled bias for value and equally weighed portfolios.

2.4 Regulation 28

Previous studies of portfolio allocation within South Africa have considered regulatory constraints. The South African retirement industry has to abide by the Pension Fund Act 24 of 1956. The act imposes statutory limits on the allocation to particular assets. Most recently this gave birth to Regulation 28 which came into effect on 01/07/2011:

Regulation 28 Asset Allocation Restrictions

Asset Class:	Percentage of Portfolio Limit:
Listed Equities	75%
Listed Property	25%
Hedge Fund	10%
<ul style="list-style-type: none"> • Fund of Hedge Funds 	10%
<ul style="list-style-type: none"> • Other 	2.5% per individual hedge fund
Commodities	10%
Non-Government Bonds	25%
Government Bonds	100%
Off-Shore Assets	25%

(National Treasury 2010)

Davis & Hu (2009) suggested that statutory limits or Quantitative Asset Restrictions (QAR) be put in place to protect investors against moral hazard within the investment industry. QAR refers to putting statutory limits in place for assets deemed too risky. More specifically, this refers to putting limits in place on assets with “relatively volatile nominal returns, low liquidity or high credit risk” (Davis & Hu 2009). Given the occurrences of the 2007 market crisis, proponents of QAR may argue that it is a necessary evil. At its best QAR, can be used to fill in liquidity gaps in areas that are vital for market operations, and force portfolio managers to diversify their holdings into those areas. Practically this is counterintuitive from a portfolio optimisation perspective (Chan-Lau 2004). Using the Prudent Person Rule



⁸(PPR), a portfolio manager in this case must act with his full skill to maximise risk adjusted returns for an investor. Putting limits on the weighting in asset classes that might limit the fiduciary's ability to carry out his duty could come at the cost of investors.

As Chan-Lau (2004) suggests, "investment limits may lead to suboptimal portfolio holdings and may also imply that assets are evaluated by their individual risk level rather than by their contribution to the overall portfolio risk". Furthermore, these limits are applied asymmetrically to private institutes and government institutes. As is the case with Regulation 28, that there is no limit on an investor's holding in government debt, this may lead to an "overweighting in government securities beyond what an optimal asset allocation will dictate" (Chan-Lau 2004).

⁸ Prudent Person Rule (PPR) OECD definition (Galer 2002) "...a fiduciary must discharge his or her duties with the care, skill, prudence, and diligence that a prudent person acting in a like capacity would use in the conduct of an enterprise of like character and aims."



Chapter 3: Methodology

The main emphasis of this study is to understand the diversification benefits of hedge fund investments for portfolio holders in South Africa, while also considering the legal framework in which these investors have to operate. The analysis period for this paper is 31/01/2005 to 31/01/2015 (121 observations per asset-class), using end-of month returns. Where a month-end date falls on a public holiday, business holiday or a weekend, the last trading date return will be used. Using Markowitz Portfolio theory, an efficient frontier for multiple portfolios will be constructed.

The performance of each portfolio will be analysed using the global efficient frontier, with returns on the Y-axis and risk (standard deviation) on the X-axis. These portfolios will be represented by different weightings in each asset class, varied so as to obtain the highest return per unit of risk. In line with convention, the highest Sharpe-Ratio will be used for each portfolio so as to identify the optimal portfolio/tangent portfolio.

A secondary analysis will be conducted using the Sortino-ratio to meet the requirements of mean-semi variance analysis. The optimal Sortino will be calculated for each fund by reweighting assets to attain the highest return per unit of downward deviation. This represents an area which Markowitz himself acknowledged as being one of the ways to deal with the shortfalls in mean-variance analysis (Markowitz 1990).

A third analysis will be conducted using the Omega-ratio. Omega does not require parametric assumptions of the return distribution of assets, nor does it require inputs as to an investor's utility function. This is relevant given that one of the flaws of Markowitz mean-variance is the normal distribution assumption, and the stylised fact that hedge fund returns and equity returns in general are not normally distributed. Using the Omega will allow for additional insight into the advantage of hedge fund investments in diversified portfolios. As with the previous two tests, portfolio weightings in each asset class will be optimised in order to maximise the return per unit of the first-lower partial moment.



3.1 Data Integrity

All return data used in this thesis has been sourced via Morningstar, a global data provider. The data used has also been cleaned using winsorisation, where data that is below the 1% and above the 99% level on either tail is set equal to the 1% or 99% value. Winsorisation was used instead of trimming, as trimming can be justified only if the representative data is not legitimate (Ghosh & Vogt 2012). Since the returns are entirely representative of the asset to which it pertains, trimming would be a step too far. Trimming also makes artificial changes to the dataset that could misrepresent the data, whereas Winsorisation reduces the impact of outlier on the analysis, and brings the third (symmetry) and fourth (kurtosis) central moments within workable deviations.

3.2 Portfolio Construction

The base-line portfolio will be a “Traditional multi-asset class portfolio” consisting of local equities, foreign equities, bonds, property and commodities. As mentioned in the literature review convention dictates that equities and bonds make up the majority of an investor’s portfolio, with the usual 60/40 mix. From a South African asset management perspective, as shown below in “*Balanced Funds Holdings as at 30/09/2015*”, cash, property, commodities and global equities are then included beyond equities and bonds. They are included to act as a diversification tool or to increase risk-adjusted returns. Balanced funds will be selected to illustrate this point, as they represent “a mix of stocks, bonds and other assets to provide capital appreciation, income, diversification or specific allocation based on a planned retirement date” (Morningstar 2015).

The portfolio weightings will be in line with Regulation 28, which states the maximum value that can be invested into each asset class. Where Regulation 28 does not specify the weighting limits, the average of five major asset managers’ balanced funds’ allocation as at 30/09/2015, will be used, (see table below).



Balanced Funds Holdings as at 30/09/2015

<u>Market</u>	<u>Local</u>						<u>International</u>
<u>Asset Class</u>	<u>Equities</u>	<u>Fixed Income</u>	<u>Cash</u>	<u>Property</u>	<u>Commodities</u>	<u>Other</u>	<u>Equities</u>
Prudential: Balanced Fund (2015)	40,50%	13,50%	18,60%	2,40%	0,00%	4,00%	21,00%
Allan Gray: Balanced Fund (2015) ⁹	45,00%	11,50%	8,40%	1,00%	5,60%	4,9%	23,70%
Old Mutual: Balanced Fund (2015)	37,20%	18,30%	15,30%	3,70%	1,00%	0.50%	24,00%
Kagiso Asset Management: Balanced Fund (2015)	42,30%	9,30%	6,20%	4,50%	7,90%	10,30%	19,50%
Coronations: Balanced Fund Plus (2015)	39,60%	14,00%	2,90%	12,00%	4,50%	2,40%	24,60%
Average	40.92%	13,32%	10,28%	4,72%	3,80%	5.40%	22,56%
Regulation 28 Limits	75%	100%	100%	25,00%	10,00%	0.00%	15,00%
<u>Traditional- Portfolio Weightings</u>	75%	100%	10,28%*	25,00%	10,00%	0.00%	15,00%

- **Weighting total per fund may deviate by .1% due to rounding errors as per market value movements.**

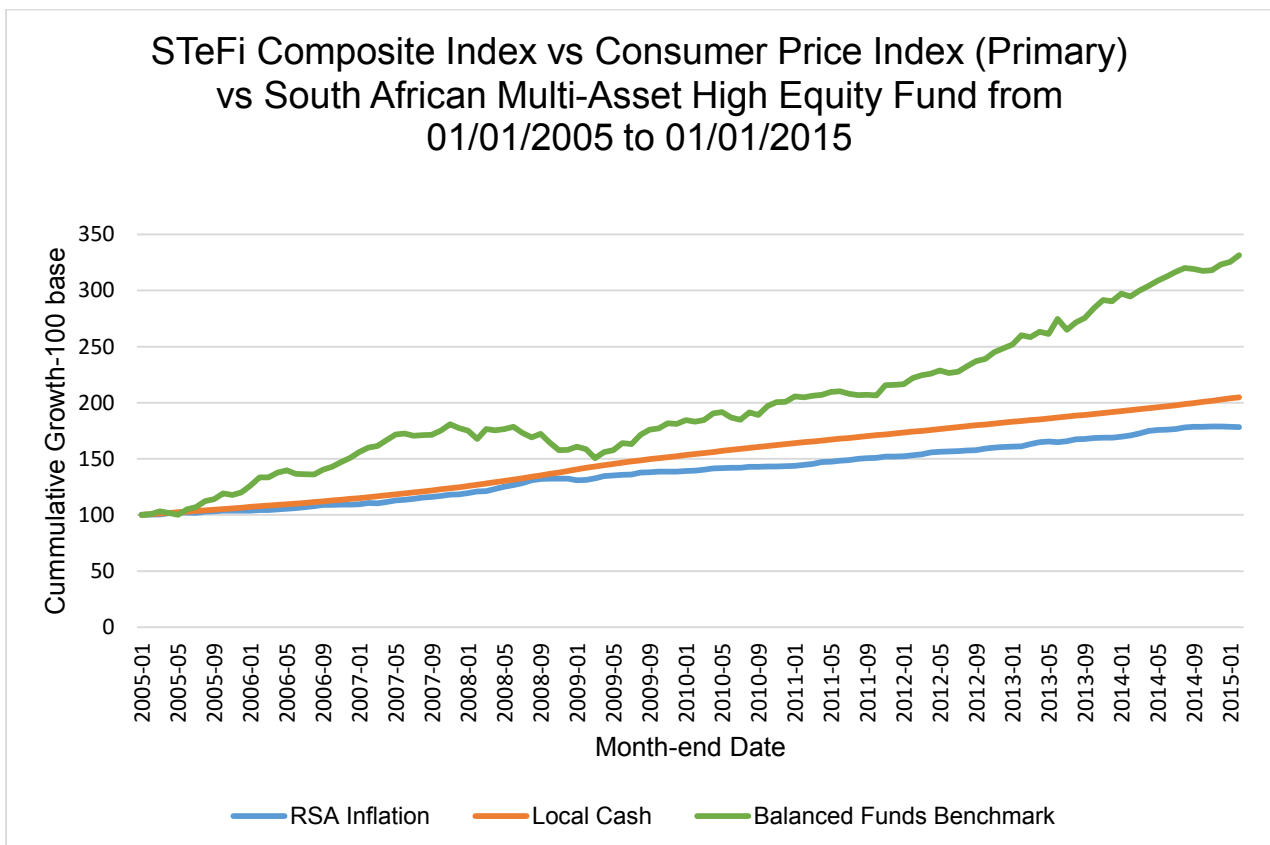
⁹ Data as per fund fact sheet at 15/01/2015



- All fund Weightings in line with Regulation 28, with the exemption of cash investments.

3.2.1 Cash Holdings

Regulation 28 allows for a 100% holding in cash assets. No multi asset portfolio manager would, however, hold 100% in cash assets. This is because cash assets represent a hedge against inflation if held over the long term and outperformed by the balanced funds benchmark shown below.



Referring to the above graph, Local Cash followed inflation between 2005/01/01 and 2008/12/01, and outperformed it between 2009/01/01 - 2015/01/01. This is in line with the low inflationary environment that persisted after the Global Finance crisis of 2008/2009. The above relationship also shows how cash can be used to offset losses due to inflation as the two basically follow each other between 2011 and 2015.

Another key point is that four out of the five balanced funds are also benchmarked against the South African Multi-Asset High Equity Fund, with the exception of the Old Mutual Balanced Fund which is benchmarked against CPI. If all the funds were to hold 100% in



cash assets, the majority would be in breach of their mandates due to their underperformance relative to their benchmarks.

Cash also represents a transitory mechanism over short periods while investors are looking for new investment opportunities. This is because cash is a low-risk asset that attains stable returns relative to other assets. In Table 1 below, Local Cash, as measured by the STeFI Composition Index, has an arithmetic mean of 7.37%, and a standard deviation of 0.59. Its excess kurtosis is only -0.073. Measuring the Sharpe ratio, Local Cash has a risk-adjusted return of 12.42. Following the same logic, the next highest risk-adjusted return is 1.27 for Local Bonds. This is a difference of 877% which, from a portfolio management standpoint, would mean that investors should over-weight cash and disregard all other securities. This is an unrealistic observation and hence needs to be constrained beyond the legal framework of Regulation 28. The maximum weighting in cash has therefore been set to 10.28%, which represents the average of the five balanced funds cash holding’.

Table 1: Asset Class First, Second, Third and Fourth Central Moments¹⁰

Asset Class	Arithmetic Mean	Standard Deviation	Skewness(Monthly)	Excess Kurtosis(Monthly)
Local Equity	16,38721767	17,90915123	-0,253425981	0,431385757
Local Bond	9,316705557	7,327451446	0,415243985	1,215252178
Local Cash	7,376731119	0,593896913	0,957632775	-0,073655821
Local Property	23,41848757	19,74117612	-0,404086672	0,405765603
Commodities	4,591184328	23,9473969	-0,144587013	0,191507316
Global Equities	15,84416715	14,68123004	0,273800984	1,104532875

*Data has been cleaned via a process of winsorisation at the 1% level.

¹⁰ Formula for first to fourth central moments provided in “Appendix 1 to 4: First, Second, Third and Fourth Moments Formula”.



3.3 Proxy for Asset Classes

Indices will be used as a proxy for the asset classes in the *Traditional Assets Portfolio*. The indices used will be selected from trusted institutes and according to how investable they are. Trustworthiness refers to an index that is managed by a well-known institute and received by the market. It must also have a published methodology paper that is publicly available. An investable index is one that is tracked by an Exchange Traded Fund (ETF). ETFs offer direct investment opportunities for investors attempting to replace the performance of wide range of asset classes. ETFs also trade like common stock on an exchange, so they are readily investable. Direct ETFs will not be used as they increase the potential tracking error of an asset class, and are prone to portfolio managers' bias when selecting securities to replace an index. At all times Total Returns indices will be used. Total Return indices track both capital gains and assume that cash distributions, such as dividends, are reinvested into the index, which more accurately reflects the performance of the index. The base-currency for all returns will be South African Rands (ZAR). The calculation to convert from a United States Dollar (USD) base index to a ZAR index is shown below:

Currency Conversion Calculation

$$R_{ZAR} = (100 + R_{USD}) \times \frac{\frac{ER_{End,ZAR}}{ER_{Start,ZAR}}}{\frac{ER_{End,USD}}{ER_{Start,USD}}} - 100$$

where:

R_{ZAR} = Total Return in ZAR;

R_{USD} = Total Return USD;

ER_{End} = Exchange Rate at the end of the observation period;

ER_{Start} = Exchange rate at the start of the observation period.

The *Currency Conversion Calculation* is a simple conversion of the Total Return in USD to the Total Return in ZAR by taking the change in the respective currency over the observation period, which was 2005/01/31 to 2015/01/31.



3.3.1 Local Equity Proxy

Index	Description	Investable Fund/ETF
<p><u>The FTSE/JSE All-Share Index Total Return ZAR</u></p>	<p>The FTSE/JSE All-Share index makes up “99% of the total pre-free float market capitalisation on the JSE” (FTSE, 2015). It is well-known and trusted market wide as a proxy for the South African equity market.</p>	<p>There is no directly investable fund that replicated the All-share index, but the Satrrix 40 (STX40) ETF does replicate the FTSE/JSE Top 40. Research by Raubenheimer (2012), however, found high levels of concentration within the JSE. Raubenheimer (2012) found that between 2005 and 2012 the effective number of shares on the JSE ranged between 21 and 25 shares. Given these findings, using either the FTSE/JSE Top 40 or the FTSE/JSE All-Share Index to track local equities will not make a material difference.</p>



3.3.2 Global Equities Proxy

Index	Description	Investable Fund/ETF
<p><u>S&P Global BMI Total Return USD¹¹</u></p>	<p>The S&P BMI has 11783 constituents representing global stock market performance using a fully free-float adjusted methodology (S&P Dow Jones Indices 2015b). The S&P Global BMI combines both the S&P Developed BMI and S&P Emerging BMI which can be expanded to generate sub-indices.</p>	<p>There is no ETF that tracks the S&P Global BMI. This is an obvious shortfall as it does not meet the “investable” criteria set above. However, the S&P Global BMI includes large, medium and small cap stocks from nearly 50 countries. Cole & Jr (2010) note that although the MSCI All-World Index and the FTSE All-World Index have ETFs that track them, the indices themselves excluded small-cap stocks. This exclusion means that, to attain complete market coverage, investors must allocate a portion of their investment to small-cap stocks. Given this finding, the S&P Global BMI is seen to be a superior measure for global equity performance.</p>

¹¹ Converted to ZAR



3.3.3 Fixed Income Proxy

Index	Description	Investable Fund/ETF
<u>All-Bond Index (ALBI)</u> <u>Total Return ZAR</u>	The ALBI is a composite index containing the top 20 vanilla bonds ranked dually by liquidity and market capitalisation (Johannesburg Stock Exchange (JSE) 2015).	The Satrix ALBI Index Fund

3.3.4 Cash Proxy

Index	Description	Investable Fund/ETF
<u>STeFI Composite Index</u> <u>Total Return ZAR (STeFI)</u>	STeFI is based on the overnight interbank call rate, and the 3-, 6- and 12-month Negotiable Certificate of Deposits (NCD). STeFI is considered to be the “industry benchmark for cash and cash equivalent” (Rodrigues 2013).	etfSA STeFI-Linked Money Market Deposit, managed by Nedbank



3.3.5 Property Proxy

Index	Description	Investible Fund/ETF
<u>FTSE/JSE SA Listed Property Total Return ZAR</u>	The SA Listed Property Index is a full market capitalisation index that tracks the 20 most liquid companies in the Real Estate Investment Trust and Real Estate Investment & Service Sector of the JSE (FTSE Russell 2015, p.18).	Stanlib SA Property ETF

3.3.6 Commodities Proxy

Index	Description	Investible Fund/ETF
<u>S&P GSCI Commodity Index Total Return USD¹²</u>	THE S&P GSCI was the first major investable commodity index. The index is production-weighted using an average of a 5 year period, and tracks the related commodity futures price across five sectors: energy, agriculture, livestock, industrial metals and precious metals (S&P Dow Jones Indices 2015a).	iShare S&P GSCI Commodity-Index Trust

¹² Converted to ZAR



3.4 South African Hedge Fund Market Index

To simulate the hedge fund industry, an index using 30 South African domicile hedge funds will be used. The data has been sourced from Morningstar covering 2005/01/31 to 2015/01/31, which is in line with the analysis period of this study. The broad categories of the hedge funds will be Pure Hedge Funds and Fund of Hedge Funds, allowing for 3 categories to be analysed:

- A. Pure Hedge Funds
- B. Fund of Hedge Funds
- C. Pure Hedge Funds + Fund of Hedge Funds

The investment strategies covered by the 3 broad categories are in line with the Novare Survey, as mentioned in the literature review:

- Hedge Fund/Fund of Hedge Funds:
 - Equity
 - I. Market Neutral
 - II. Long/short
 - Non-Equity
 - III. Fixed-Income
 - IV. Multi-strategy
 - V. Volatile Arbitrage
 - VI. Structure Finance

3.4.1 Pure Hedge Funds and Fund of Hedge Funds Returns Calculation

Morningstar reports using a total return methodology, where the percentage change of Net-Asset Value (NAV) from one period to the next represents the return of the fund. This number is assumed to be net of fees, as this is the industry standard for reporting performance.

Return Calculation

$$r_t = \frac{(NAV_t - NAV_{t-1})}{NAV_{t-1}}$$

where:

r_t = Return for period "t";



NAV_t = Net asset Value at period t;

NAV_{t-1} = Net asset Value at period t-1.

3.4.2 Weighting of Index

The index is weighted using fund size/Assets under Management (AUM), further explained in “3.5 Criteria for Inclusion in Hedge Fund Index”.

3.4.3 Fund Size Data Sources

Multiple data sources were used to collect fund size data, as this information is not readily available. Sources were rated on a scale from 1 to 4 on level of priority: if a level 1 priority was not available a level 2 priority was used, etc.

- 1) Survey Data¹³
 - a) Direct Fund Manager Data
 - b) Morningstar Survey Data
- 2) Comprehensive¹⁴
- 3) Aggregate of Share Classes¹⁵
- 4) Calculated Fund Size via Returns

If level 1 to 3 data was not available, then the fund size is calculated from a static fund size reported to Morningstar as at 31/01/2015.

Calculated Fund Size via Returns:

$$FS_{t-1} = \frac{FS_t}{(1 - r_t)}$$

where:

FS_t = Current Fund Size;

FS_{t-1} = Previous Fund Size;

r_t = Return for period “t”.

¹³ Survey Fund size: the total fund size that is reported by the fund company itself.

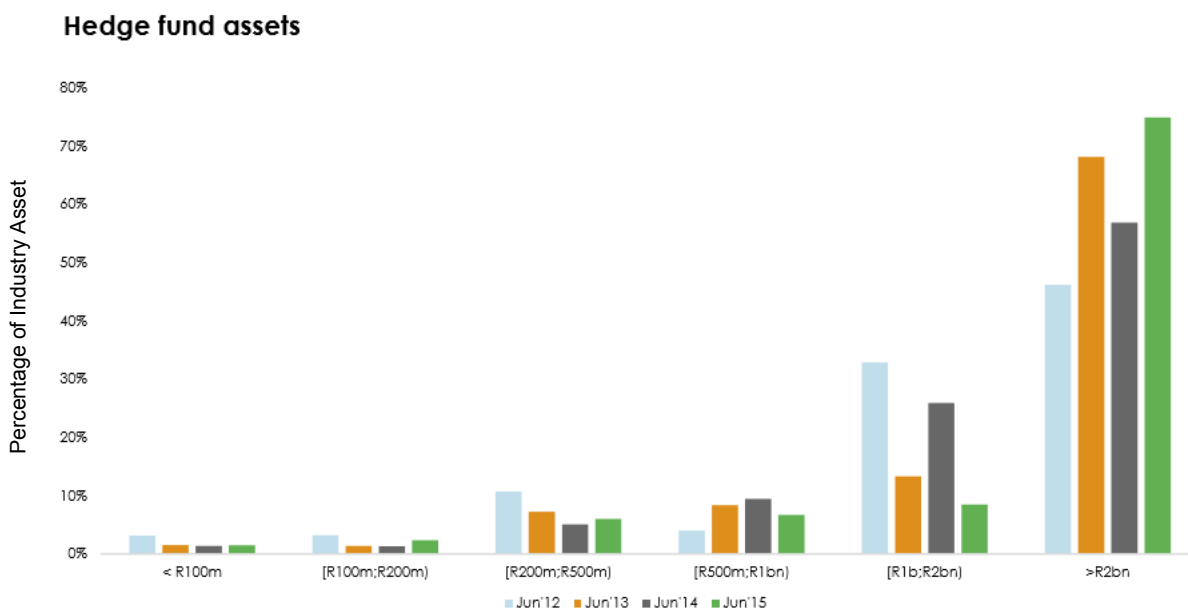
¹⁴ Comprehensive Fund Size: populated with aggregate share class if available.

¹⁵ Aggregate of Share Classes Fund Size: populated by adding up the individual share classes available in the Morningstar database.



This *Calculated Fund Size via Returns* computation involves taking the fund size at 31/01/2015, and reducing it by current return to get the previous fund size at 31/12/31. This is then repeated for each period to get to the starting fund size. The downfall of this methodology is that it does not take into account explicitly the change in distributions from one period to the next. As a static fund size is available at the end of the analysis period, thus, the cumulative impact of a change in distributions is considered. The impact of distributions is also built into the return figure as total return is a net figure, that includes costs and distributions.

An equally weighted index is not used as it would incorrectly reflect the hedge fund industry in South Africa. The industry's AUM was R62.096 billion on 30/06/2015, with the 10 largest hedge funds holding 42.5% of the total hedge fund assets. The industry is broken down into two parts with hedge funds holding R26.3 billion (42%) of the R62.096 billion asset under management. The remaining R35.8 billion (58%) forms part of diversified businesses that have chosen to increase their asset bases beyond hedge Funds, and form part of institutional asset.



(Novare Investments 2015)

Looking at hedge funds with an undiversified asset base, the above shows that the industry is skewed towards funds with an asset base greater than R2 billion. On a concentration basis, the top ten hedge Funds have maintained 40% or more of industry assets since



2002 (Novare Investments 2015). Funds outside the top 30, on an asset base measure, held only 22.2% of the total industry assets as at June 2015. This suggests that the industry is skewed towards larger hedge funds and, as such, an index tracking its performances should represent this occurrence. An equally weighted index would over-represent the small funds in the data-set and lead to incorrect or misleading results.



3.5 Criteria for Inclusion in Hedge Fund Index

3.5.1 General Requirements

The data for the hedge fund indices to be constructed is sourced via Morningstar and covers the period 31/01/2005 to 31/01/2015. The hedge fund indices will be broken down into a Pure Hedge Funds index, a Fund of Hedge Funds index and a combined Pure Hedge Funds and Fund of Hedge Funds index as mentioned in “3.4- South African Hedge Fund Market Index”. For a fund to be included in any one of the three indices it must be a Republic of South African domicile fund that reports earnings using South African Rands (ZAR) as its base currency. The returns must be reported end of each month over the period 31/01/2005 to 31/01/2015. For a fund to be included in the indices it must have a return history of 1 year (12 consecutive months) within the analysis period 31/01/2005 to 31/01/01.

For the sake of prudence, and to improve the trustworthiness of this research, all funds included in the indices must have audited financial statements over the period for which returns have been included in the indices.

The indices are rebalanced on a semi-annual basis, these dates being a January month-end and July month-end¹⁶. The indices are also reconstituted (non-trading funds dropped and new funds added) on a semi-annual basis, similar to the rebalancing dates January month-end and July month-end.

3.5.2 Application of Index Criteria

Of the 30 fund data-set, 2 funds did not meet the trading length requirement mention above: they traded for under 12 consecutive months within the analysis period. Hence, the data-set was reduced to 28 funds. All other requirements were met by the remaining 28 funds.

3.5.3 Requirements under Regulation 28

The regulation for asset managers that deals with retirement funds is specific. Regulation 28 restrictions explicitly state that, as of 01/07/2011¹⁷:

- No more than 10% of a portfolio’s overall value may be allocated to Pure Hedge Funds or Fund of Hedge Funds;

¹⁶ Month-end refers to the last trading day of the month.

¹⁷ The effective date for the implementation of Regulation 28



- Of that 10% no more than 5% of a portfolio's overall value may be allocated to a single Fund of Hedge Funds;
- Of that 10% no more than 2.5% of a portfolio's overall value may be allocated to a single Pure Hedge Funds.

3.5.4 Application of Regulation 28

Given the 10% overall limit (refer above) and the specific allocation limit to a single Fund of Hedge Funds and Pure Hedge Funds, post 01/07/2011,

- No single Fund of Hedge Funds may have greater than 50% weighting in the Fund of Hedge Funds index;
- No single Pure Hedge Funds may have greater than 25% weighting in the Pure Hedge Funds index;
- No single Fund of Hedge Funds and Pure Hedge Funds may have greater than 50% and 25% weighting in the combined Pure Hedge Funds and Fund of Hedge Funds index.

This ensures that the requirements under Regulation 28 are met. If a fund is not within the 50% or 25% parameters, its weighting will be reduced to those limits. The excess allocation will then be allocated equally among the constituent funds.

3.5.6 Reweighting formula for Regulation 28 limits

If post 01/07/2011 Hedge Fund A¹⁸, a Pure Hedge Funds has a fund size weighting of 26% within the combined Fund of Hedge and Pure Hedge Funds index, the re-weighting formula would be applied according to:

$$\left[\frac{\text{Overall Weighting of Fund "A" in index} - \text{Upper Limit for the index}}{\text{Total Funds Currently Trading in index} - 1} \right]$$

where:

Overall Weighting of Fund "A" in index refers to the weighting of a Pure Hedge Funds in the combined index, in this case 26% for Hedge Fund A;

¹⁸ Reweighting per fund: $\frac{26\% - 25\%}{28 - 1} = 0.0370\%$,



Upper Limit for the index denotes to the limit of 25% for a Pure Hedge Funds and 50% for a Fund of Hedge Funds, in this case 25% as Hedge Fund A is a Pure Hedge Funds;

Total Funds Currently Trading in index denotes to the number of funds that make up the index at that time.

If we assume that all 28 funds are trading when Hedge Fund A violates the 25% limit, then the remaining 27 funds excluding Hedge Fund A will have their allocation increased by 0.037%. This 0.037% increase must not take the allocation of a Fund of Hedge Funds or a Pure Hedge Funds above 50% and 25% respectively. For a formulaic breakdown of the index composition please refer to “*Appendix Formula 5 Pure Hedge Funds/Fund of Hedge Funds Index Composition*”



3.6 Portfolio Optimisation Technique

Optimal portfolios are generated using the *Morningstar Direct Asset Allocation* tool. It is a comprehensive modelling tool that allows users to customise parameters and vary inputs according to their preference. In order to generate an efficient frontier where return is maximised per unit of risk, 30000 iterations of different portfolio weightings are run. From this, 100 equidistant points, where return is maximised per unit of risk are combined to attain the efficient frontier. The efficient frontier is graphed with a portfolio arithmetic mean on the X-axis and a standard deviation on the Y-axis. The optimal portfolio is represented by the maximum Sharpe portfolio or the tangency portfolio on the efficient frontier. Using only the Mean-Variance Optimisation (MVO) model to optimise portfolios does have a shortfall, as it assumes that assets have a skewness of 0 and excess kurtosis of 0. As *Table 2* below shows, neither of the assets meets this criteria.

In order to compensate for the impact of non-normality of asset returns on a risk-reward analysis, the maximum Sortino and Omega ratios for each portfolio are analysed. The process is similar to that of the Mean-Variance Optimisation, but for the Sortino or mean semi-variance optimisation, the frontier is graphed such that the Y-axis represents returns and the X-axis represents downside deviation. The frontier represent the maximum return per a unit of downside deviation. The Omega is graphed in a similar manner, with the Y-axis representing returns and the X-axis representing the first lower partial moment. For a formulaic breakdown of Sharpe, Sortino and Omega optimisation, refer to “*Appendix Formula 6, 7 and 8*”.

Table 2: Asset Third and Fourth Central Moments

Asset Class	Skewness(Monthly)	Excess Kurtosis(Monthly)
Local Equity	-0,253425981	0,431385757
Local Bond	0,415243985	1,215252178
Local Cash	0,957632775	-0,073655821
Local Property	-0,404086672	0,405765603
Commodities	-0,144587013	0,191507316
Global Equities	0,273800984	1,104532875
Fund of Hedge Funds + Pure Hedge Funds	-0,642456965	1,342192263
Pure Hedge Funds	-0,766621299	2,358475542
Fund of Hedge Funds	-0,427149103	0,500553887



Chapter 4: Results¹⁹

4.1 Analysis of Historical Return and Risk

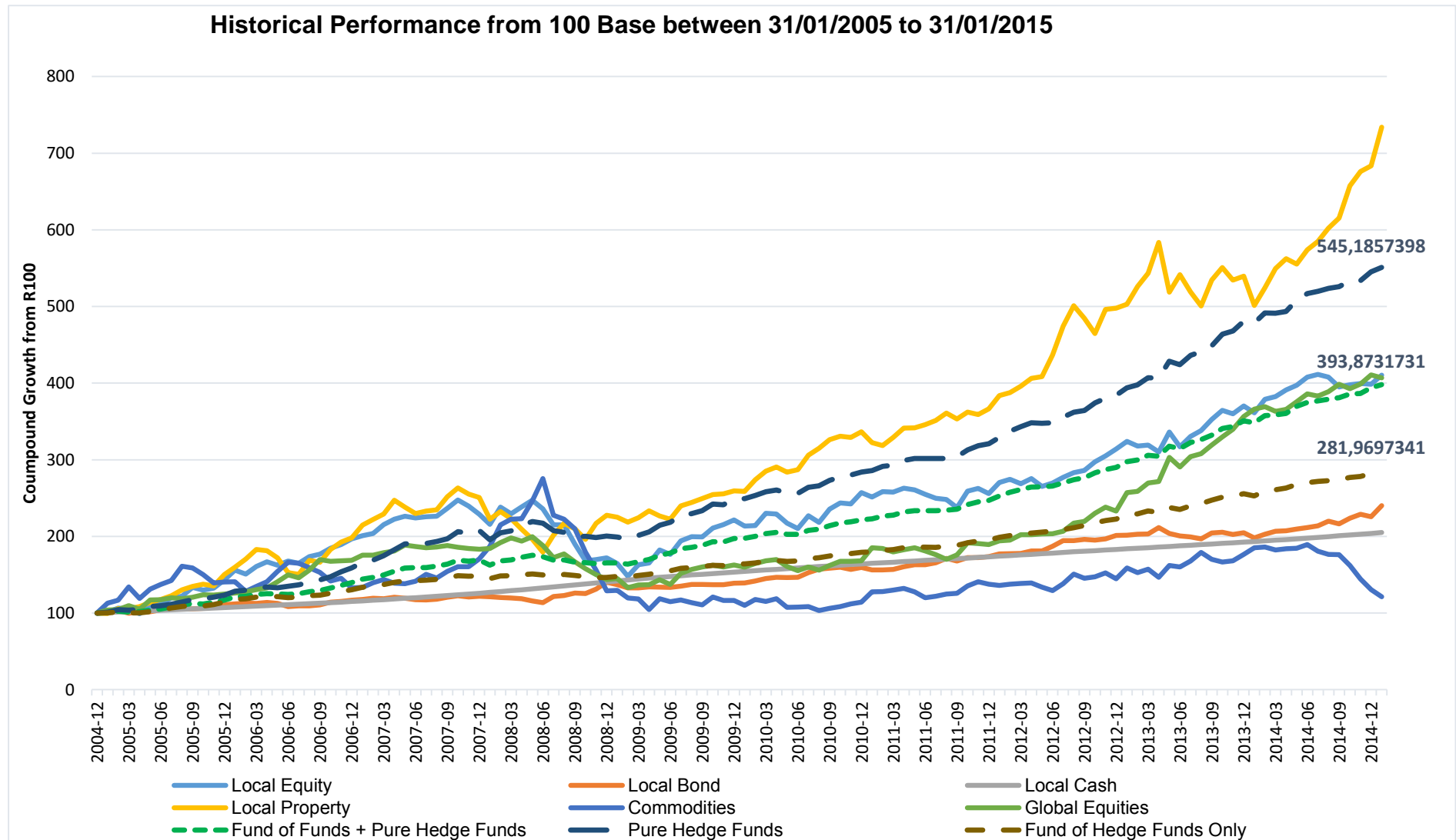
As mentioned previously, historical data spanning 31/01/2005 to 31/01/2015 was collected, translating to 121 observations. The performance of traditional assets is represented by 6 proxy indices, and the hedge fund industry by 3 composite indices. This section is broken down into an analysis of the historical performance of each asset class, the first and second central moments (mean and variance, with the latter converted to standard deviation) and the third and fourth central moments (skewness and excess kurtosis).

This is followed by a mean-variance analysis of the mix of assets to test for the optimal portfolio; in this case the portfolio with the greatest Sharpe ratio. Next, the mix of assets is optimised using the Sortino ratio to test for the impact of downside risk. This is referred to as the mean semi-variance analysis. Lastly, the asset mix is optimised using the Omega ratio to establish whether a higher Omega can be attained by adding hedge fund assets to a traditional asset portfolio.

¹⁹ I would like to acknowledge the work of (Rodrigues, 2013) for he's insight into an effective manner of presenting results on portfolio optimisation.



Figure A: Historical Performance of Traditional Asset and Hedge Fund Asset between 31/01/2005 and 31/01/2015



4.1.1 Historical Performance of Traditional Assets

Figure A represents the compounded growth of R100 invested in the above assets, assuming returns and the initial capital outlay were reinvested. Local Properties performed particularly well over the 10 year period, returning R734 (compound annual growth of 634%) for every rand invested. Local Equities and Global Equities were the next best performers with a return of R410 (compounded annual growth of 310%) and R407 (compounded annual growth of 307%) respectively. The interesting aspect is that both equity indices appear to track each other's performance to a large extent. This supports increasing evidence of globalisation and contagion between developed and international markets. During the market crash of 2008/2009, both local and foreign equity markets slumped, ostensibly due to the contagion impact of global disinvestment. Over that same period the local bond market outperformed equity markets as investors moved from equities to bonds. Local Bonds returned R240 (compounded growth of 140%) over a 10 year period, which is well below returns achieved by the riskier asset classes but is understandable in the context of quantitative easing (QE) by the Federal Reserve. The QE program was introduced in November 2008 where investors move to emerging markets for excess yield which can be seen by the rise during 11/2008 to 12/2008. The action of the Federal Reserve also led to the European Central Bank, Bank of Japan and Bank of England taking on QE programs which started a period of low interest rates globally.

Commodities performed the worst of all in the asset classes, returning only 21.32% over a 10 year period. The largest loss occurred between 06/2014 and 01/2015, where 33% of the value of the Commodities was lost. This lacklustre performance can be attributed to the fall in oil prices at the end of 2014. The S&P GSCI held nearly 50% of its value in crude oil in 2014 (S&P Dow Jones Indices 2015c, p.47). Hence, when Brent Crude oils fell from \$112 in 06/2014 to \$47.52 per barrel on 30/01/2015 (down 58%), the entire commodity market also dropped.

4.1.2 Comparison of historical performance of hedge funds to traditional assets

Figure A represents the hedge fund asset class with three composite indices: a Pure Hedge Funds index ("Pure Hedge Funds"), a Fund of Hedge Funds index ("Fund of Hedge Funds") and a combined index of both ("Fund of Hedge Funds plus Pure Hedge Funds").



The “Pure Hedge Funds” asset class performed well over the 10 year period with a compounded return of 451%. This is less than the compounded return of the Local Property index but greater than the five other “Traditional Assets”. An interesting note is that, although the Pure Hedge Funds index seems to follow the Local Equity index, over the period between 2007 and 2008, Pure Hedge Funds investments seemed more resilient to the market slump. A possible explanation for this is the ability of the hedge fund managers to take on short positions which would have allowed for positive returns in both bull and bear markets.

Fund of Hedge Funds underperformed relative to the Local and Global Equities, but still outperformed the Local Bonds. Fund of Hedge Funds had compounded returns of 184.5%, or 48% less than the Pure Hedge Funds over the 10 year period. One is to expect Fund of Hedge Funds to underperform relative to Pure Hedge Funds, however, as Fund of Hedge Funds are institutionally run. Being an institutionally run fund in South Africa comes with regulatory restrictions that reduces the amount of risk a fund may take on. If you are to reduce your appetite for risk then you must accept a lower level of absolute return. Fund of Hedge Funds also take a portfolio of hedge funds, rather than just representing one hedge fund. Given the greater exposure to multiple hedge funds means that the positive, well-returning hedge funds might be overshadowed by poorly performing hedge funds in a portfolio. Diversifying across hedge funds reduces the idiosyncratic risk to any particular hedge fund. This comes at a cost of lower returns, however, when a particular hedge fund performs well.

The combined index of Fund of Hedge Funds and Pure Hedge Funds performed similarly to Local and Global equities with a compounded return of 297% versus 310% and 307% respectively. Looking at the path of the combined index it seems to be far more stable with far fewer peaks and troughs. This translates to a lower standard deviation than that of the Local and Global equities. Due to the combined nature of this index it is understandable that the Pure Hedge Funds index outperformed it while the Fund of Hedge Funds underperformed.



Figure B: Historical Performance of Hedge Fund Asset between 31/01/2005 and 31/01/2015

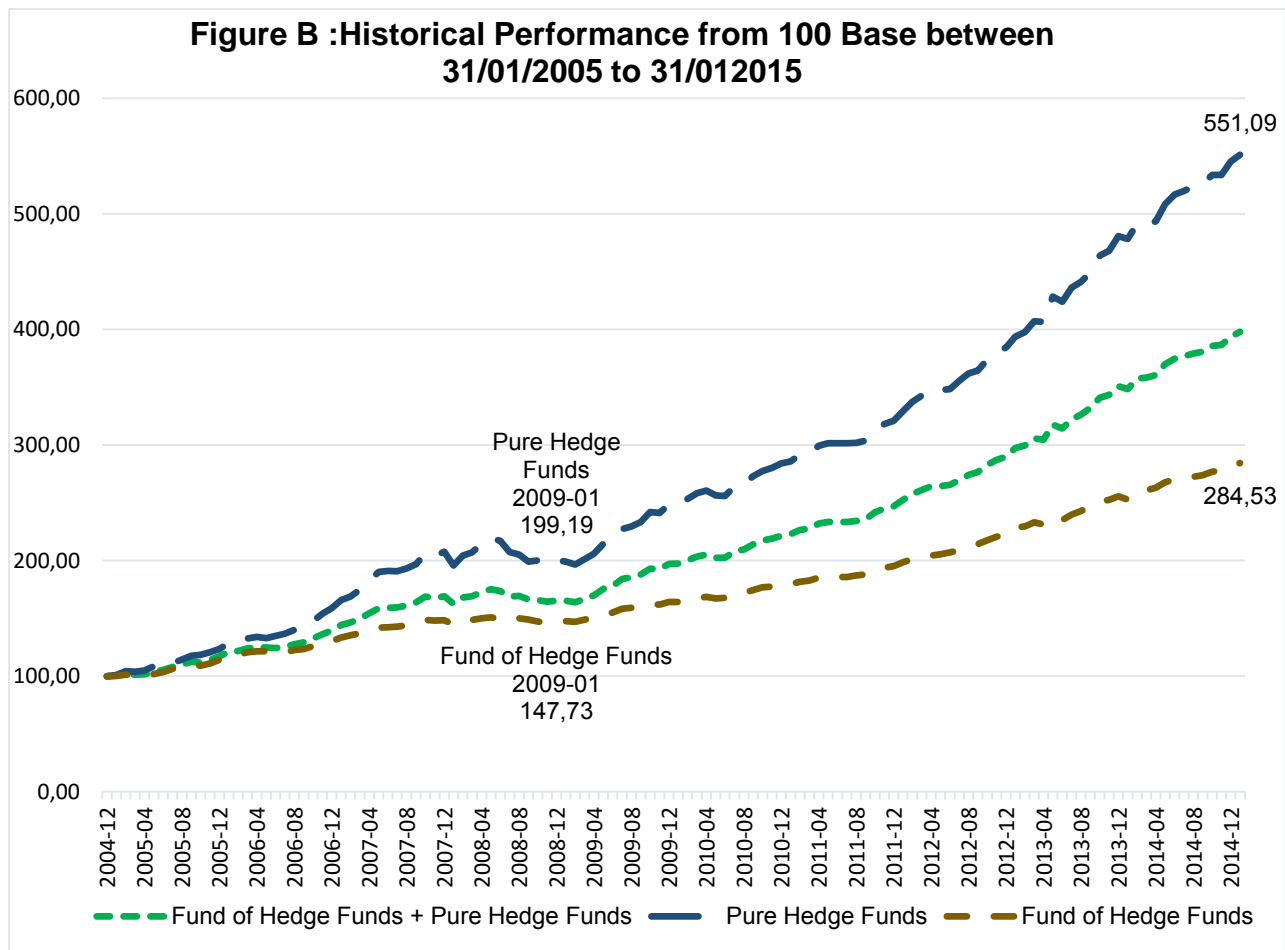


Figure B represents the compound growth of the hedge fund industry over a 10 year period. It is an extension of Figure A, although Figure B attempts to analyse the difference between the Pure Hedge Funds index and the Fund of Hedge Funds index. From 01/01/2005 the Pure Hedge Funds index outperformed that of the Fund of Hedge Funds index. A profound divergence in their performance can be seen post 2008, however, where the gap between the two graphs becomes larger. On 01/01/2009 the Pure Hedge Funds index was at 199.19 while the Fund of Hedge Funds index was at 147.73, a difference of 34.83%. By the end of the observation period that difference was 93.68%, with the Pure Hedge Funds index at 551.09 and the Fund of Hedge Funds index at 284.53. Given a larger data set, it could be useful to investigate the differences between the Pure Hedge Funds and Fund of Hedge Funds markets pre- and post-crisis.

The combined index of Pure Hedge Funds and Fund of Hedge Funds also seems to diverge from the Fund of Hedge Funds post 2009. This can be explained by a potential



bias in the composition of the index towards Pure Hedge Funds rather than Fund of Hedge Funds. As Pure Hedge Funds are better represented, the performance of the combined index follows the Pure Hedge Funds index.



4.2 Central Moment of Traditional and Hedge Fund Assets

Table 3: Assets First and Second Central Moments (annualised)

Asset Classes	Arithmetic Mean Return²⁰	Rank	Geometric Mean Return	Rank	Standard Deviation	Rank	Sharpe Ratio (Risk-free 7,3767)	Rank
Local Equity	16,3885	3 rd	14,8670	3 rd	17,9430	7 th	0,5022	6 th
Local Bond	9,3134	7 th	9,0479	7 th	7,2880	5 th	0,2657	7 th
Local Cash	7,3767	8 th	7,3749	8 th	0,5941	1 st	0,0000 ²¹	8 th
Local Property	23,4093	1 st	21,6714	1 st	19,6775	8 th	0,8148	4 th
Commodities	4,5448	9 th	1,6031	9 th	23,5342	9 th	-0,1203	9 th
Global Equities	15,8452	4 th	14,8295	4 th	14,6362	6 th	0,5786	5 th
Fund of Hedge Funds + Pure Hedge Funds	14,7952	5 th	14,6658	5 th	5,1784	3 rd	1,4326	2 nd
Pure Hedge Funds	18,6532	2 nd	18,4195	2 nd	7,0675	4 th	1,5955	1 st
Fund of Hedge Funds	10,9910	6 th	10,9199	6 th	3,7781	2 nd	0,9566	3 rd

Table 3 presents the annualised arithmetic mean, geometric mean, standard deviation and Sharpe ratio, with accompanying ranks for each asset class. The arithmetic mean return, geometric mean return and the standard deviation are all absolute measures of return and risk, while the Sharpe ratio is a relative measure for risk adjusted return. The risk-free rate used in the Sharpe calculation is the average cash assets return (annualised).

²⁰ The arithmetic and geometric mean return have the same rank while the geometric mean is always smaller than the arithmetic mean. The difference between each measure is not observed when each observation in a series is the same, rather when the observations differ (Silber 2008). The arithmetic average assumes that each return is independent of the next, while the geometric assumes that there is a relationship between prior and future returns.

²¹ Cash was used as the measure of the risk-free rate, hence the excess return = 0



4.2.1 Standard deviation

The ranking of returns according to either the geometric or arithmetic mean is the same as that of the continuously compounded return in Figure A. This is, of course, due to the continuously compounded return being an extension of the geometric mean. The standard deviations, however, rank differently from the returns. Local Cash has a standard deviation of .59 which is drastically smaller than the next lowest at 3.77 (Fund of Hedge Funds). This is due to the short-term nature of investments in cash and its use as a hedge against inflation rather than as a tool to maximum an investor's returns. Of particular interest is the standard deviation of the hedge fund indices, ranking second, third and fourth. The Pure Hedge Funds index has a standard deviation of 7.06, which is 87% larger than the Fund of Hedge Funds index at 3.77 and 37% larger than the combined index. The difference between the Fund of Hedge Funds and Pure Hedge Funds index can be explained by a difference in mandate, where the Fund of Hedge Funds has to take on less risk due to its focus on institutional investment.

Local Bonds, which are seen as less risky than Local Equities, have a standard deviation of 7.28 which is similar to that of the Pure Hedge Funds index. The Pure Hedge Funds index, however, seems to outperform on a return per unit of risk measure. Global and Local Equities are next, with a standard deviation of 14.63 and 17.94 respectively. For Global Equities this is 1.82x larger than the combined index of Pure Hedge funds and Fund of Hedge Funds; and 2.46x larger for Local Equities. This shows that, within the hedge fund, industry there is a better ability to manage and limit risk.

By contrast, Local Property which has the highest level of absolute return has the second highest level of volatility. True to form, the Commodities ranks worst when it comes to standard deviation. This is interesting because, although the returns were poor for Commodities, risk was not low. Given the standard deviation as a risk measure, with Commodities investors did not benefit from taking on extra risk.

4.2.2 Sharpe Ratio

The Sharpe ratio is a simple measure of excess return per unit of risk taken on. Within this measure the three hedge fund indices outperformed all of the traditional asset indices. The Pure Hedge Funds index returned the largest Sharpe ratio of 1.595, 2.17x greater than Local Equities. The Fund of Hedge Funds and Pure Hedge Funds index ranked second with 1.4326, only 11% smaller than the Pure Hedge Funds index. Following after was the



Fund of Hedge Funds index which, returned a Sharpe ratio 40% smaller than that of the Pure Hedge Funds index. This raises questions as to the ability of Fund of Hedge Funds managers, and their skill at managing the idiosyncratic risk of hedge funds. The overall hedge fund industry as represented by the combined index has nevertheless performed well, which suggests that the hedge fund industry overall can attain better risk adjusted returns for investors when compared with those for traditional assets.

Local Property ranked fourth with a Sharpe ratio of 0.814, due to the high volatility associated with property market returns. The standard deviation, which is the denominator in the Sharpe ratio, was large enough to undo the high returns of the property index. Even with the high standard deviation, Local Property still ranked highest on a risk adjusted return basis among traditional assets, Global and Local equities were fifth and sixth with a Sharpe ratio of 0.57 and 0.50 respectively. If investors took on a passive ETF that tracked the Local Equity or Global Equity markets, they would have a risk adjusted return that is 64% and 69% (ignoring tracking error and administration costs) lower than they would for investing in Pure Hedge Funds. This has implications for investors, especially those who are more risk averse and would like higher levels of return.

Local Bonds and Commodities performed the worst, ranking seventh and eighth with a Sharpe ratio of 0.2657 and -0.1203. The performance of Local Bonds can be explained by the cycle of low interest rates, with central banks attempting to move investors into the equity market following the market crash of 2008. Of particular interest is the negative Sharpe ratio of Commodities. A negative Sharpe ratio occurs because the numerator “excess return” is measured by the expected return less the risk-free rate. Considering that Local Cash, which is the proxy for the risk-free rate, has a higher return over the 10 year period than Commodities, there is no excess reward for taking on Commodities. This is particularly interesting, as commodities such as gold are often used as hedges during high inflationary environments or when cash assets are performing poorly. Using the US as an example, post 2008 has seen a low inflationary environment, with post 2008 inflation ranging from negative .35% to 1.62%, according to the World Bank. This could be one of the reasons, along with the drop in prices, for the poor performance of Commodities.

Analysis of Skewness and Kurtosis

To understand the skewness and kurtosis of assets they should be compared with those of a standard normal distribution. The normal distribution assumes skewness is set to 0 and kurtosis to 3, with excess kurtosis being a measure of kurtosis above or below the level of 3. The normal distribution is considered to be fully symmetrical around the mean. In simple terms, this means that there is an equal chance of both positive and negative returns around the mean. The reason such an analysis is important is because of the implications for mean-variance optimisation. One of the assumptions of mean-variance optimisation is normality of returns. A violation of this assumption directly challenges the results or outcomes of this model, which can of course be rectified by using a mean semi-variance model such as the Sortino ratio or the Omega ratio. Understanding how the standard normal assumptions have been violated, and the extent of that violation, enhances the importance of using alternative optimisation techniques.

Table 4: Third and Fourth Central Moments

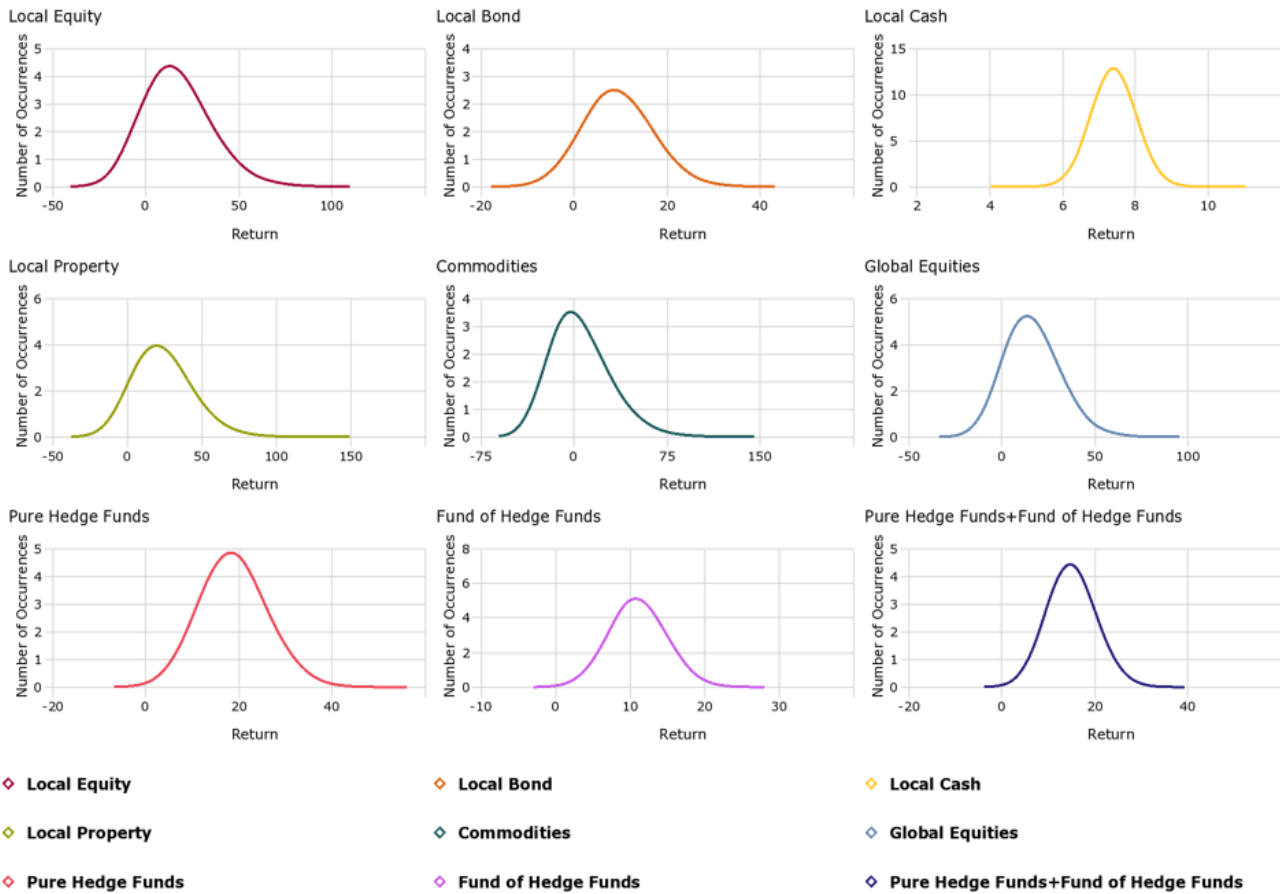
Asset Classes	Skewness²²	Excess Kurtosis
Local Equity	0,345413975	-0,001249683
Local Bond	0,374844019	0,721101173
Local Cash	0,325871867	0,317089003
Local Property	0,307363024	0,12737224
Commodities	0,462434719	0,300326509
Global Equities	0,327815109	0,184972501
Fund of Hedge Funds + Pure Hedge Funds	-0,08924138	0,208269186
Pure Hedge Funds	-0,060211072	0,376772218
Fund of Hedge Funds	-0,059191214	-0,053204879

²² All returns have gone through a process of winsorisation to reduce the impact of outliers on the skewness and kurtosis.



Diagram 1: Asset Distributions

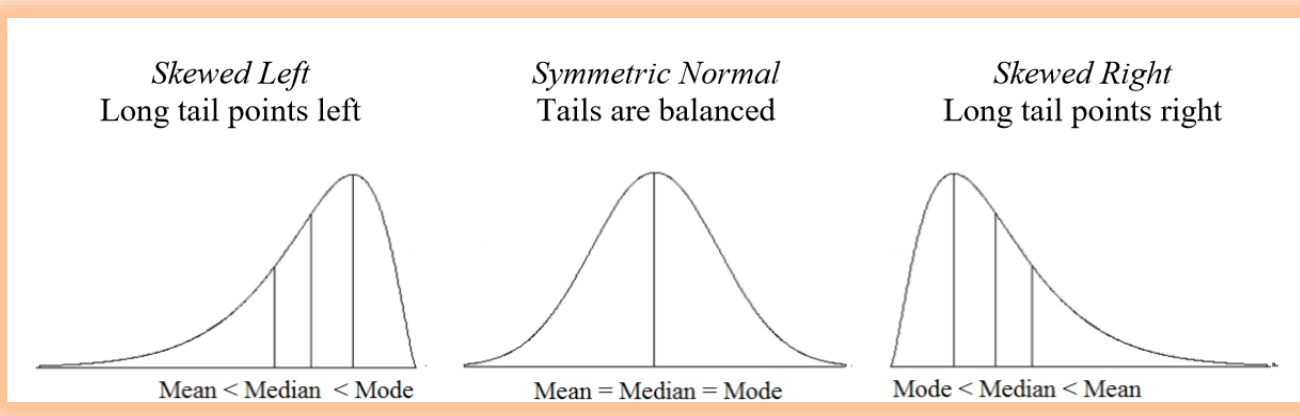
Asset Class Distribution



Source: Morningstar Direct

(Morningstar Direct)

Diagram 1.2: Visual representation of skewness for bimodal data



(Doane & Seward 2011)



4.2.3 Skewness

Table 4 and Diagram 1 show that none of the assets meet the normal distribution assumption of 0 skewness and 0 excess kurtosis. The traditional assets, however, exhibit positive skewness while the hedge fund assets all exhibit negative skewness. The positive asymmetry of the traditional assets returns is welcomed, as it indicates that the data has a long positive tail. This implies that the likelihood of a positive outlying event is greater than the likelihood of a negative outlying event. With both the Fund of Hedge Funds and Pure Hedge Funds the skewness is negative, which is a concern as it suggests that the likelihood of an extreme-loss making event is greater than that of a profit-making event.

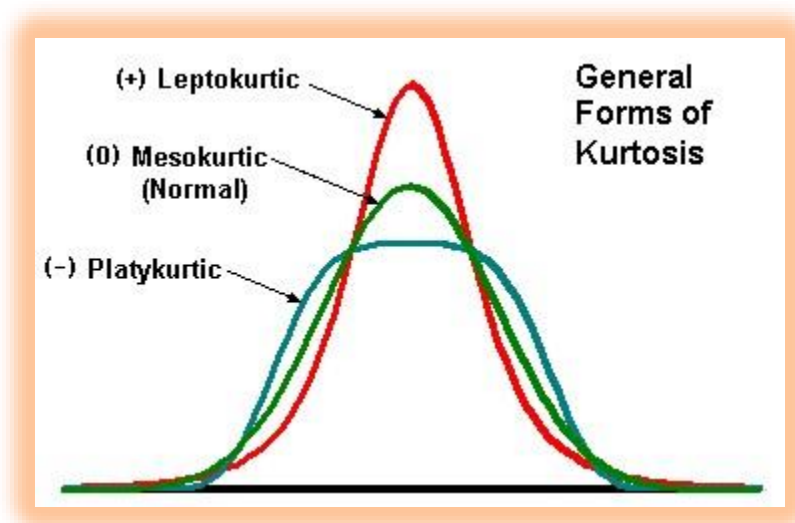
The rule of thumb is a skewness above 1 or less than -1 is a concern, as it represents a substantial violation of the symmetry associated with a normal distribution (Bulmer 1979). The largest skewness on the positive side is within commodities at 0.462, which is far below the threshold of 1. On the negative side, the negative skewness for three hedge fund assets is minor, with the skewness being -0.089. Bulmer (1979) suggests that a skewness level between $(0; 1/2)$ and $(0; -1/2)$ is approximately symmetrical. The traditional assets are all within the upper limit of $1/2$. The three hedge fund indices are close to symmetrical, with a skewness slightly smaller than 0, thus reducing the concern of regarding their negative skewness.

4.2.4 Kurtosis

When analysing the kurtosis, the understanding is that a higher/lower kurtosis implies a fatter/thinner tail and a smaller/greater chance of extreme events. The normal distribution is referred to as mesokurtic, the higher peaked distribution is leptokurtic and the lower peaked distribution is platykurtic.



Diagram 1.3: Visual representation of kurtosis²³:



That being understood, none of the assets is mesokurtic whilst Local Equities and Fund of Hedge Funds are platykurtic. The platykurtic assets are less fat-tailed than the leptokurtic assets, and are hence less of a concern, as this suggests that there is a reduced chance of an extreme outlying event. The remaining traditional assets: Local bonds, Local cash, Local Property, Commodities and Global Equities are all leptokurtic. Local Cash has low volatility with little surprise return, its returns being centered on the mean. Local Bonds have a higher kurtosis than Local Cash, it is surprising since it suggests that Local Bonds have a greater peak than Local Cash. This may, however, be due to the greater tail risk within bond returns as bonds are naturally riskier than cash. Traditional assets excluding Local Equities are all leptokurtic; however they also have a positive skewness. This is welcomed by investors as it suggests a greater chance of positive returning extreme outlying events. Investors would like a positive surprise when investing as opposed to a negative surprise.

The Pure Hedge Funds index and the combined Fund of Hedge Funds and Pure Hedge Funds index are leptokurtic with an excess kurtosis of .21 and .38. The combined index is likely to leptokurtic due to the index composition favouring the Pure Hedge Funds over the Fund of Hedge Funds. As seen in the literature review, kurtosis is a problem when dealing with hedge fund returns. The main concern is the positive kurtosis combined with the negative skewness, as this suggests that there is tail risk on the negative side. As an

²³ Image extracted from: <http://mvpprograms.com/help/mvppstats/distributions/SkewnessKurtosis>



investor one does not want to have increased chance of an extreme outlying event on the negative-side; such an occurrence would typically be characterised by a market crash.

Overall, none of the assets exhibit symmetry or a mesokurtic distribution. Skewness alone, however, suggests that they are all approximately symmetrical. Given the concern of outlying events on the ability to model risk with the Sharpe ratio, an alternative mean semi-variance optimisation is justified. Considering the downside risk allows us to understand better the impacts of extreme loss-making events in a portfolio of assets.



4.3 Correlation Analysis of Asset Class

Investigation of the correlation analysis attempt to evaluate the linear relationship between the traditional assets and the hedge fund assets. The correlation analysis is done over a 10 year period between 31/01/2005 and 31/01/2015. The correlation between assets is important for a portfolio optimisation study, because any non-perfect correlation represents an opportunity to weight assets such that overall portfolio risk is reduced. This is seen by the covariance coefficient that has a positive relationship with portfolio risk (see “Appendix Formula 5: Sharpe Ratio”). In a more practical sense, assets with a low correlation or negative correlation move in opposite directions to each another. This helps investors protect against volatile returns, as the poor performance of one asset class can be offset by a positive performance in another asset class.

Table 5: Correlation Matrix for Asset Class

Asset Class Correlation Matrix									
Time Period: 2005-01-31 to 2015-01-31									
Asset Class	1	2	3	4	5	6	7	8	9
1 Local Equity	1,00								
2 Local Bond	0,12	1,00							
3 Local Cash	-0,18	0,00	1,00						
4 Local Property	0,30	0,65	-0,09	1,00					
5 Commodities	0,35	-0,47	-0,12	-0,29	1,00				
6 Global Equities	0,56	-0,28	-0,30	-0,06	0,52	1,00			
7 Pure Hedge Funds	0,73	-0,03	-0,17	0,20	0,25	0,49	1,00		
8 Fund of Hedge Funds	0,73	0,13	-0,24	0,36	0,19	0,45	0,80	1,00	
9 Pure Hedge Funds+Fund of Hedge Funds	0,77	0,04	-0,22	0,27	0,24	0,50	0,96	0,92	1,00

■ 1,00 to 0,80 ■ 0,80 to 0,60 ■ 0,60 to 0,40 ■ 0,40 to 0,20 ■ 0,20 to 0,00
 ■ 0,00 to -0,20 ■ -0,20 to -0,40 ■ -0,40 to -0,60 ■ -0,60 to -0,80 ■ -0,80 to -1,00

(Morningstar Direct)

Interpreting correlations:

Size of Correlation	Interpretation
.90 to 1 (-.9 to -1)	Very high positive (negative) correlation
.70 to .90 (-.70 to -.90)	High positive (negative) correlation
.50 to .70 (-.50 to -.70)	Moderate positive (negative) correlation
.30 to .50 (-.30 to -.50)	Low positive (negative) correlation
0 to .30 (0 to .30)	Negligible correlation

(Mukaka 2012)



Traditional Assets

4.3.1 Local Equities

Local Equities has a strong positive relationship with the three hedge fund asset. The Pure Hedge Funds and Fund of Hedge Funds have a correlation of .73, while the combined index has a correlation coefficient of .77. The higher correlation of the combined index is due to the ancillary impact of the positive correlation between the Pure Hedge Funds and Fund of Hedge Funds. The hedge fund industry will naturally have a strong relationship with the local equity market since the majority of the investments that hedge funds make are in equities.

4.3.2- Local Bonds

Local Bonds is not highly correlated to any of the assets. It does have a moderate positive relationship with Local Properties, which makes sense as low interest rates (higher bond value) would encourage people to take on financing for home ownership or expansion projects. The interesting component is the lack of correlation between the bond market and the hedge fund industry. As a strategy, fixed income represents 14.1% of hedge fund assets (Novare Investments 2015) . The lack of correlation could be due to the sample set used. Future studies could expand the scope of hedge funds, and might return a different correlation coefficient.

4.3.3 Local Cash

Local Cash, as noted earlier, represents an alternative stable investment opportunity when the market is performing poorly. This can be seen by the negative although negligible correlation between Local Cash and all other assets. The result is predictable, and is a motive for limiting the cash investments when creating an optimal portfolio (see “3.2.1 Cash Holdings”).

4.3.4-Hedge Fund Assets

The hedge fund industry is highly correlated within the Fund of Hedge Funds and Pure Hedge Funds index. The Fund of Hedge Funds and Pure Hedge Funds index has a .8 correlation. This of course is due to the Fund of Hedge Funds managers being invested directly in Pure Hedge Funds; more specifically this study is focused on the South Africa which has a shallow hedge fund market, and subsequently high correlations. The



correlation coefficient, however, does not address the advantage a Fund of Hedge Funds has over a Pure Hedge Funds. Due to the Fund of Hedge Funds holding multiple hedge funds, they are diversified across the industry. This is advantageous because individual hedge funds have been known to go through a rapid life cycle. For an investor this is a similar type of exposure with the added benefit of a lower risk, notwithstanding a reduced reward. This can be seen with the standard deviations, where the Pure Hedge Funds index was at a level of 7.06 and the Fund of Hedge Funds index 3.78. This came at a cost of a lower return for the Fund of Hedge Funds compared with the Pure Hedge Funds index and subsequent a lower Sharpe ratio.

The combined index of Fund of Hedge Funds and Pure Hedge Funds has a very high correlation with the Pure Hedge Funds and Fund of Hedge Funds indices, with a correlation of .96 and .92 respectively. This is to be expected, as the combined index is composed of both the Pure Hedge Funds and Fund of Hedge Funds indices. As noted previously, the index is biased towards individual hedge funds as the NAVs are weighted higher for the Pure Hedge Funds. This may explain the higher correlation with the Pure Hedge Fund index. The strength of the relationship can also be observed in *Figure A: Historical Performance of Traditional Assets and Hedge Fund Assets between 31/01/2005 and 31/01/2015*. Figure A shows how, post 2009, the returns of the combined index moved towards the Pure Hedge Fund index and diverged from the Fund of Hedge Funds index.



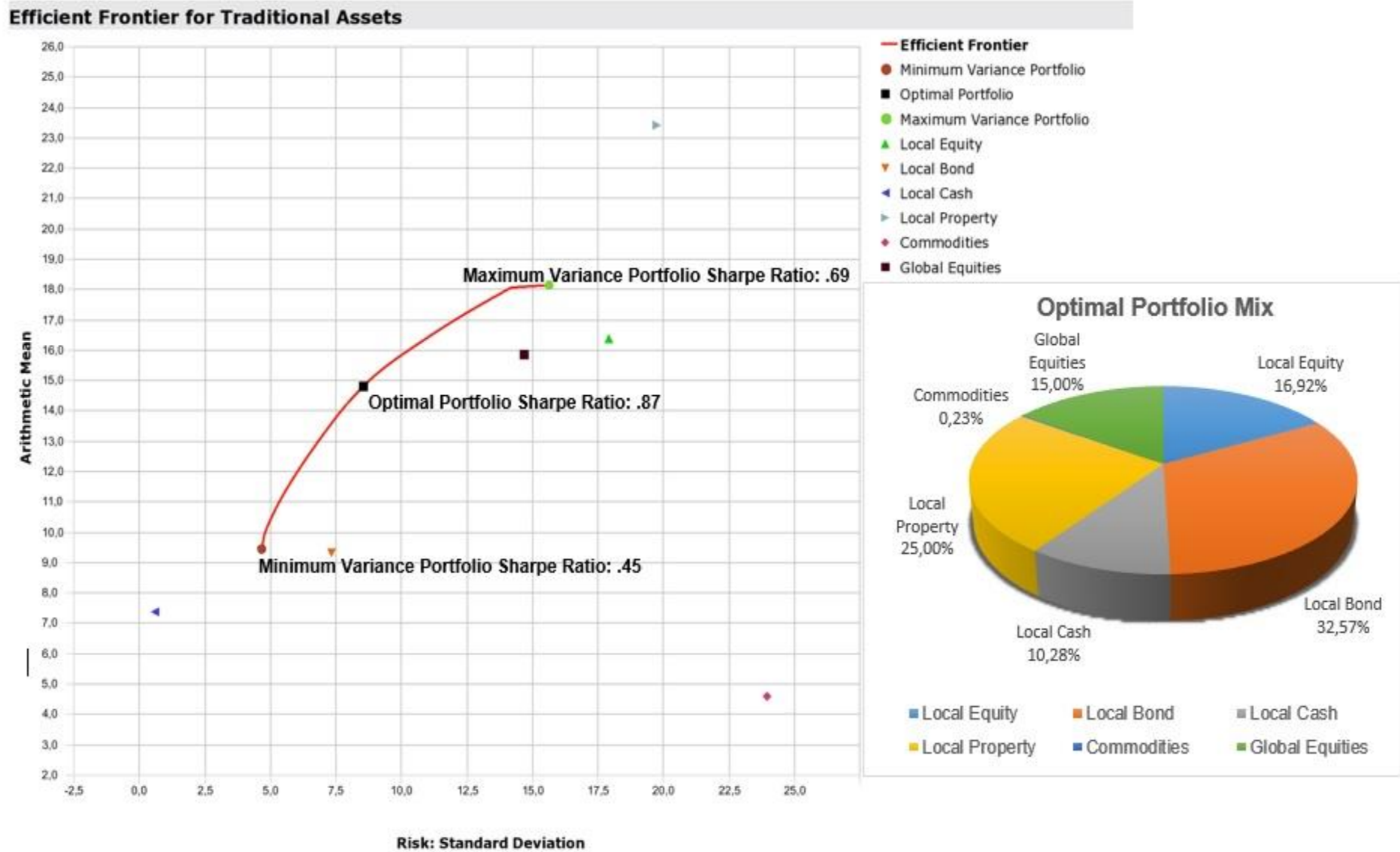
4.4 Mean-Variance Optimisation

This section deals with the mean-variance optimisation of the four portfolios, namely a Traditional Asset portfolio, a Traditional Asset plus Pure Hedge Funds portfolio, a Traditional Asset plus Fund of Hedge Funds portfolio and a Traditional Asset plus a combined Pure Hedge Funds and Fund of Hedge Funds portfolio. A log-normal model is used to find the maximum Sharpe ratio of each portfolio by re-weighting assets to produce an efficient frontier. This frontier is a graphical representation of the maximum reward per unit of risk, where a rational investor would aim to maximize his/her reward for each unit of risk. The measure of reward is the expected return or arithmetic mean return, and the measure for risk is standard deviation. Hundred equidistant points representing the maximum return per unit of risk are graphed, the combination of these points being used to produce the efficient frontier.

Below, Figure 1 represents an efficient frontier for the Traditional Assets portfolio, Figure 2 represents the Traditional Assets plus Pure Hedge Funds portfolio, Figure 3 represents the Traditional Assets plus the Fund of Hedge Funds portfolio, and Figure 4 represents the Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds. Accompanying each of the figures is a pie chart of the asset weights in each of the optimal portfolios. Table 8 then shows the weightings in the optimal portfolio in each asset class. The asset class weightings are restricted by Regulation 28 which regulates the allocation a portfolio manager may attribute to an asset class. As mentioned in the methodology, the cash assets have been limited to 10.28% so as to mitigate the risk of an unrealistic weighting due to the low deviation in cash returns. Table 9 represents the arithmetic mean return, standard deviation and Sharpe ratio for each of the four optimal portfolios. The Sharpe ratios are calculated using a risk-free rate of 7.37673%, the annualised average of cash return.



Figure 1: Efficient Frontier for Traditional Assets and Asset Weightings Graphic for the Optimal Portfolio

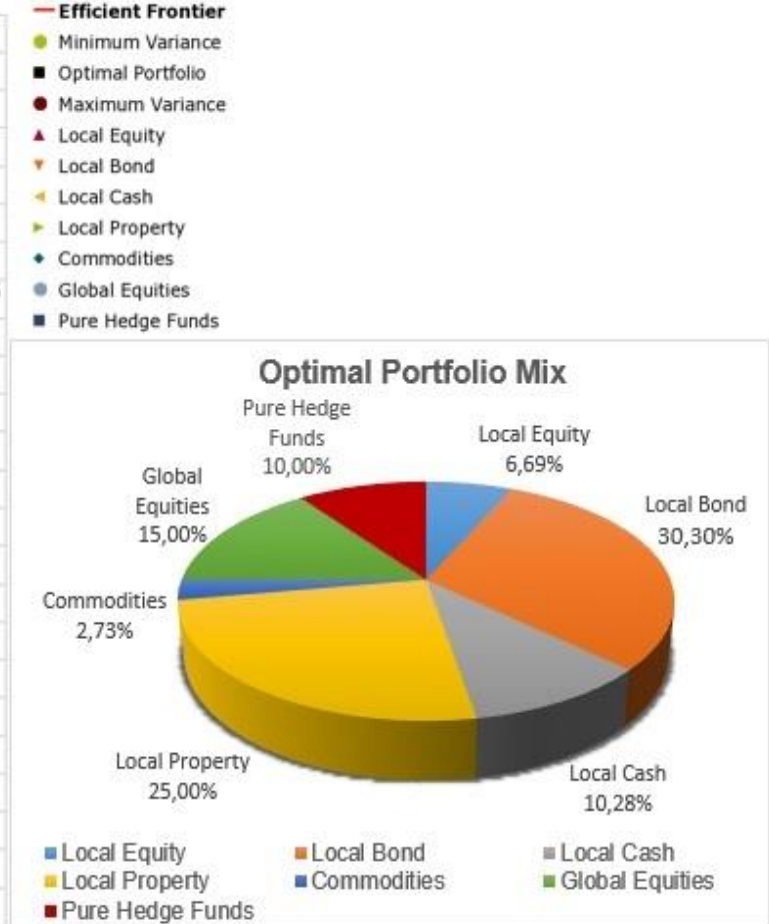
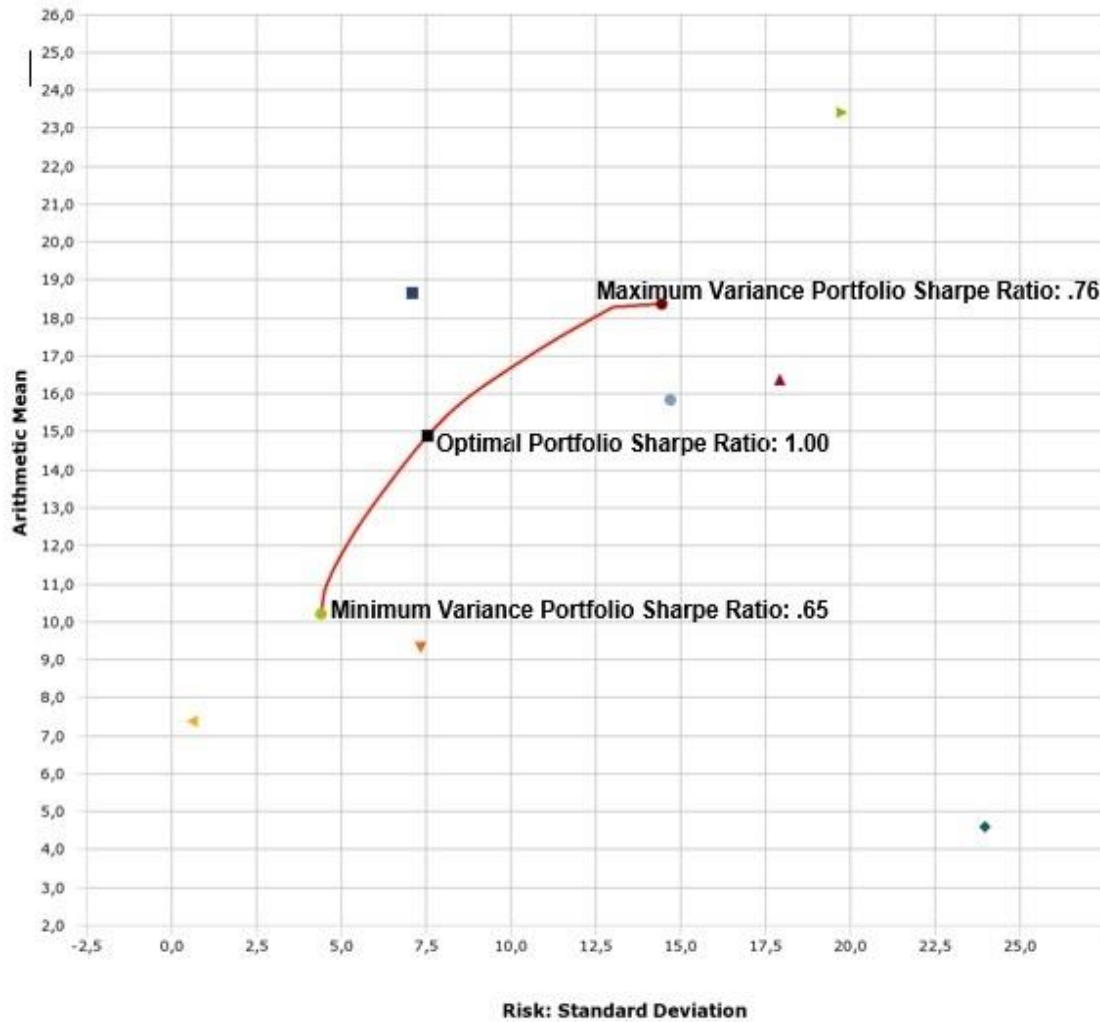


Source: Morningstar Direct



Figure 2: Efficient Frontier for Traditional Assets plus Pure Hedge Funds and Asset Weightings Graphic for the Optimal Portfolio

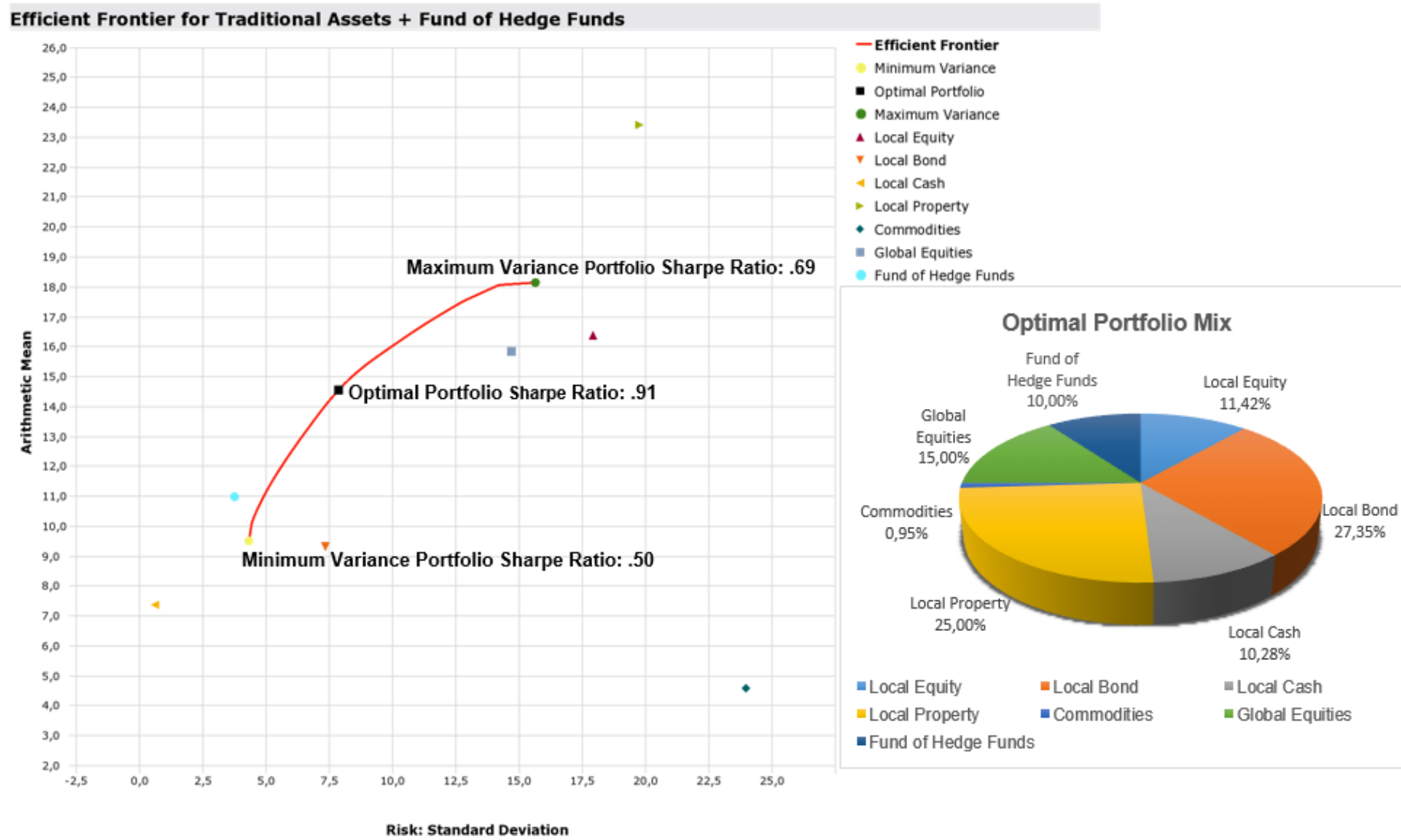
Efficient Frontier for Traditional Assets + Pure Hedge Funds



Source: Morningstar Direct



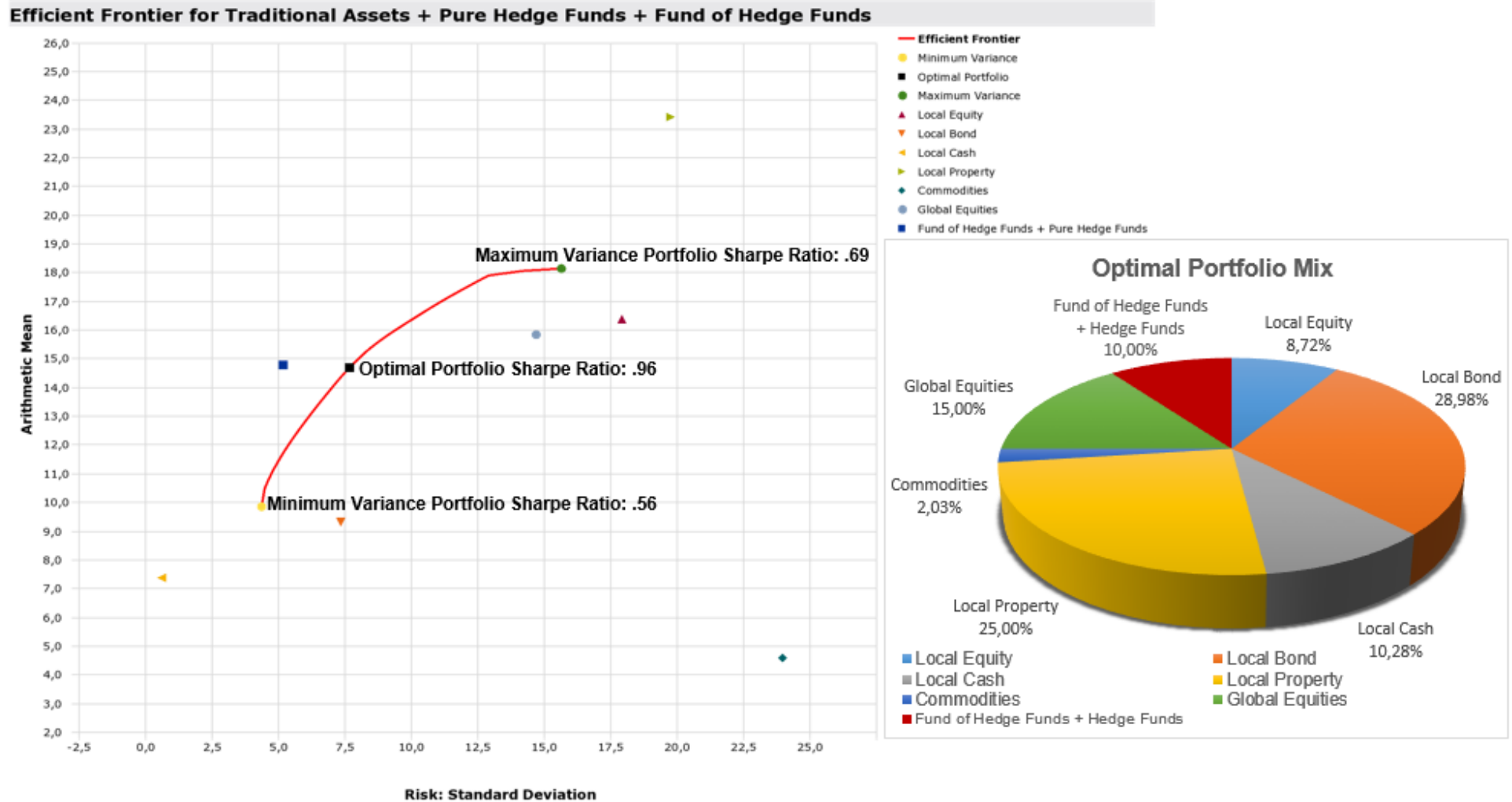
Figure 3: Efficient Frontier for Traditional Assets plus Fund of Hedge Funds and Asset Weightings Graphic for the Optimal Portfolio



Source: Morningstar Direct



Figure 4: Efficient Frontier for Traditional Assets plus Hedge Funds and Fund of Hedge Funds and Asset Weightings Graphic for the Optimal Portfolio



Source: Morningstar Direct



Table 6: Optimal Portfolio Asset Weightings

Portfolio	Local Equity	Local Bond	Local Cash	Local Property	Commodities	Global Equities	Pure Hedge Funds	Fund of Hedge Funds	Fund of Hedge Funds + Pure Hedge Funds
Traditional Assets	16,92%	32,57%	10,28%	25,00%	0,23%	15,00%	-	-	-
Traditional Assets + Pure Hedge Funds	6,69%	30,30%	10,28%	25,00%	2,73%	15,00%	10,00%	-	-
Traditional Assets + Fund of Hedge Funds	11,42%	27,35%	10,28%	25,00%	0,95%	15,00%	-	10,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	8,72%	28,98%	10,28%	25,00%	2,03%	15,00%	-	-	10,00%

Table 7: Optimal Portfolio Arithmetic Mean Return, Standard Deviation and Sharpe Ratio

Portfolio	Arithmetic Mean	Rank	Standard Deviation	Rank	Maximum Sharpe Ratio (Risk-free Rate =7,37673)	Rank
Traditional Assets	14,80754046	2 nd	8,540841877	4 th	0,870032553	4 th
Traditional Assets + Pure Hedge Funds	14,89980362	1 st	7,531025344	1 st	0,998944138	1 st
Traditional Assets + Fund of Hedge Funds	14,55177629	4 th	7,853615232	3 rd	0,913597888	3 rd
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	14,69018059	3 rd	7,655570371	2 nd	0,955311	2 nd



4.4.1 Efficient Frontier Overview

Local Property, Local cash and Global Equity investments had their allocations maximised within Regulation 28. The three assets had a 25% (Local Property), 10.28% (Local Cash) and 10% (Global Equities) investment in all four portfolios being tested. Local Equity, Local Bond and Commodity weights vary for each portfolio. The Commodities investment rose from 0% in the Traditional Assets portfolio to a marginal weighting in the three hedge fund inclusive portfolios. This increase in allocation is not significant, as the largest allocation to Commodities is 2.73% in the Traditional Assets plus Pure Hedge Funds portfolio.

Of particular interest is the 10.23% reduction on an absolute basis in Local Equities in the Traditional Assets vs the Traditional Assets plus Pure Hedge Funds portfolio, which is 2.52x smaller on a relative basis. A similar outcome can be seen in the Traditional Assets plus Fund of Hedge Funds portfolio, where the reduction in Local Equity investments is 1.48x and 1.94x for Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds. The outcome is probably due to the high correlation of the three hedge fund indices with the Local Equity index. Individually, the hedge fund investments outperformed local equities on a risk adjusted return basis. Hence, when weighting for the optimal portfolio, the hedge fund indices replaced local equities.

Going beyond the traditional assets, all three hedge fund inclusive portfolios had the maximum weighting in their respective Pure Hedge Fund and Fund of Hedge Funds indices, shown by the 10% allocation in Table 6. This suggests that there are diversification benefits for investors for taking on alternative assets beyond the standard bond and equity mix.

With regard to the efficient frontier for each portfolio, the minimum variance, optimal and maximum variance portfolio of the three hedge fund portfolios was greater than those of the traditional asset portfolio. Overall, the three hedge fund portfolios thus achieve a higher risk to reward ratio for investors than that of the traditional assets portfolio.

4.4.2 Optimal Sharpe Ratio Analysis:

The crux of this section is the optimal Sharpe ratio achieved by each portfolio. Breaking the Sharpe ratio down into its components, the expected return of each portfolio was quite similar, with the Traditional Assets attaining the second highest return at 14.81% vs the highest return of 14.90% by the Traditional Assets plus Pure Hedge Funds. The Traditional Assets plus the combined index of Pure Hedge Funds and Fund of Hedge Funds portfolio



ranked third with 14.69% and the Traditional Assets plus Fund of Hedge Funds ranked last with 14.55%. This ranking, however, is not significant from a risk to reward perspective as the difference between first and fourth place is only 2.4%.

From a risk perspective or observing standard deviation when dealing with the Sharpe ratio, the Traditional Assets plus Pure Hedge Funds portfolio ranked first with a standard deviation of 7.53. Next was the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio with 7.66, followed by Traditional Assets plus Fund of Hedge Funds with 7.85. Among the three portfolios that included hedge fund assets, the difference in standard deviation between the best performing portfolio and the worst performing portfolio is 4.2%. When this analysis is expanded to the Traditional Assets portfolio, we find it has a Sharpe ratio of 8.54, and is ranked last; the difference is that the riskiness of the portfolios is emphasised. The difference between the Traditional Assets plus Fund of Hedge Funds portfolio standard deviation and the Traditional Asset portfolio is 13.41%. This is significant, as the Fund of Hedge Funds portfolio was placed last among the hedge fund portfolios yet still did significantly better than the Traditional Assets portfolio.

The ranking among the optimal portfolios Sharpe ratio has the Traditional Assets plus Pure Hedge Funds placed highest, with the Traditional Assets plus the combined index of Pure Hedge Funds and Fund of Hedge Funds second. The Traditional Assets plus the Fund of Hedge Funds rank third and the Traditional Assets ranked last. This ranking is similar to that of the ranking amongst the standard deviations. The difference in Sharpe ratios can be seen in Table 8, below:



Table 8: Difference in Sharpe Ratios for Optimal Portfolio ²⁴

Portfolio	Traditional Assets	Traditional Assets + Pure Hedge Funds	Traditional Assets + Fund of Hedge Funds	Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds
Traditional Assets	0,00%	-	-	-
Traditional Assets + Pure Hedge Funds	14,82%	0,00%	-	-
Traditional Assets + Fund of Hedge Funds	5,01%	9,34%	0,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	9,80%	4,57%	4,57%	0,00%

The Traditional Assets plus Pure Hedge Funds portfolio has a 14.82% larger Sharpe ratio than that of the Traditional Assets portfolio. The Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds, and the Traditional Assets plus Fund of Hedge Funds portfolio have a 5.01% and 9.8% larger Sharpe ratio, respectively, than the Traditional Assets portfolio. The combined portfolio which incorporates Pure Hedge Funds and Fund of Hedge Funds is principally significant as it is more representative of the industry. A 9.8% difference in optimal Sharpe ratios is thus a significant observation. Such a substantial difference emphasises the need for portfolio managers to include hedge fund assets in their investment opportunity set.

Within the three hedge fund portfolios the Traditional Assets plus Hedge Funds outperformed the Traditional Assets plus Fund of Hedge Funds by 9.34%. This suggests that there is significant benefit from choosing individual hedge funds compared with a set of hedge funds available through Fund of Hedge Funds. The diversification benefits could be reduced when we look at Fund of Hedge Funds, as they are already spread across a set of hedge funds.

Overall, comparing the four optimal portfolios using an efficient frontier one has to conclude that, between the periods 31/01/2015 to 31/01/2015, portfolios inclusive of hedge fund assets outperformed those with only traditional assets. Furthermore, the risk adjusted returns of portfolios including Pure Hedge Funds outperformed those including Fund of

²⁴ Table 8 shows the difference in Sharpe Ratios as (Higher ranked Sharpe ratio/Lower Ranked Sharpe ratio)-1.



Hedge Funds. From a weighting perspective all portfolios that included Pure Hedge Fund assets and Fund of Hedge Funds asset maximized their investment allocations to the alternative assets. This allocation came at the cost of a lower investment in either Local Equities or Local Bonds; however the reduction in allocation came primarily out of Local Equities. This reiterated the need for portfolio managers to diversify their investment set.



4.5 Mean-Semi Variance Optimisation

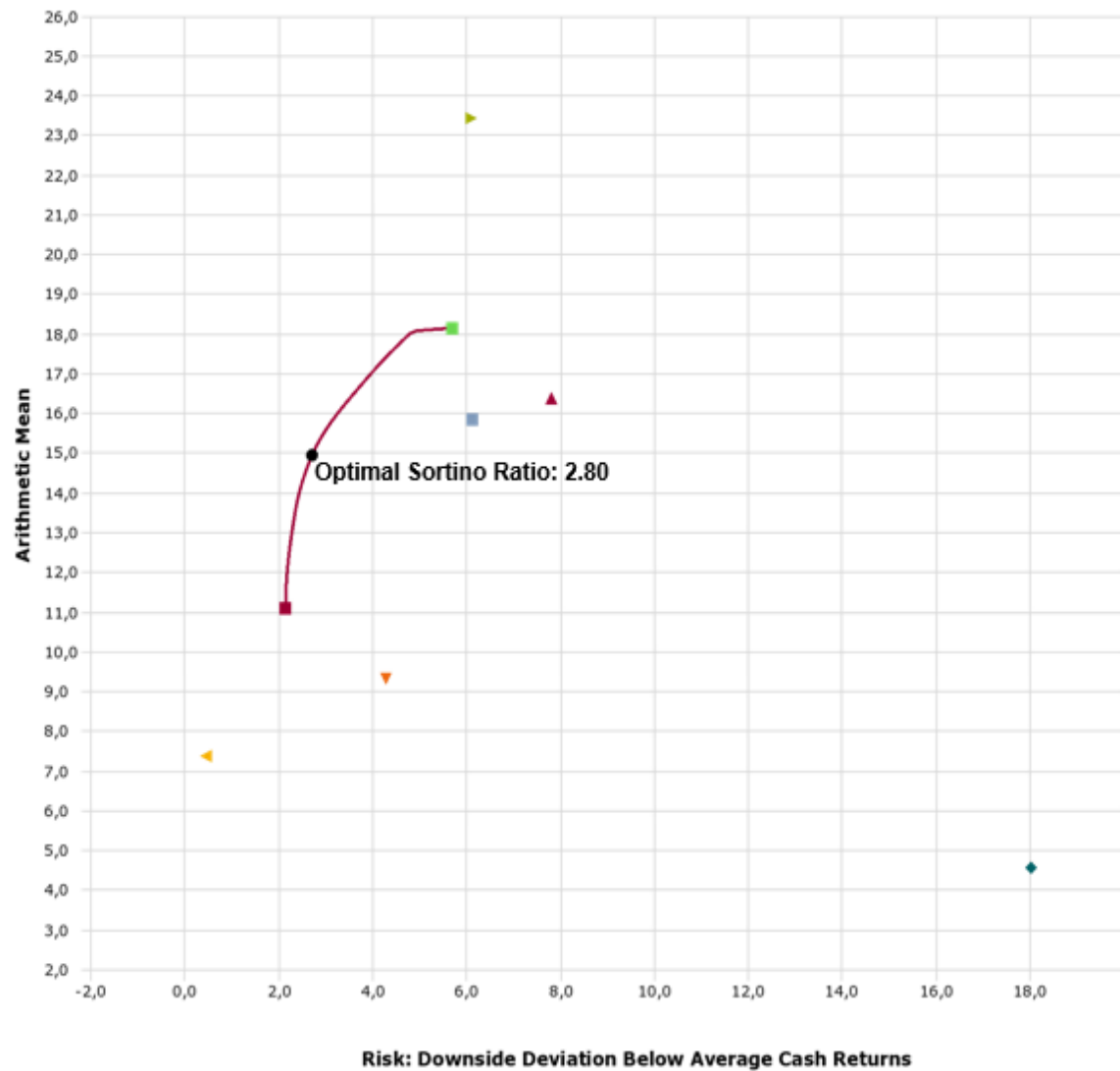
This section deals with the mean semi-variance optimisation for the four portfolios, namely a Traditional Assets portfolio, a Traditional Assets plus Pure Hedge Funds portfolio, a Traditional Assets plus Fund of Hedge Funds portfolio, and a Traditional Assets plus a combined Pure Hedge Funds and Fund of Hedge Funds portfolio. The model makes use of the Sortino ratio, by re-weighting assets to attain the maximum excess return per unit of downward deviation. The weightings are run using 30000 iterations to produce 100 equidistant points that can be joined together to yield an efficient frontier. This analysis is in addition to the mean-variance analysis; however the focus will be on the downside risk rather than standard deviation. The flaw in the previous analysis is that it assumes that risk is symmetrical. The traditional assets and the hedge fund assets have a skewness between $(-1/2; 1/2)$, or are considered approximately symmetric, which reduces the need for a semi-variance analysis. As noted by Sortino & Price (1994), however, portfolio managers should distinguish between upside and downside volatility. This section makes that distinction, and focuses the analysis on the potentially negative impact of downward moments in returns.

In the place of the risk-free rate, a target rate is used to measure excess returns, the target rate being the average cash asset return. This is the same as the risk-free rate used in the mean-variance optimisation. The cash assets represent a passive return for investors and a protection against inflation. Any return above this should be seen as excess reward for investing in a fund that is actively managed; hence it has been set as the target return for the Sortino ratio.

Figures 5 to 8 are the Sortino Frontiers for the Traditional Assets, Traditional Assets plus Pure Hedge Funds, Traditional Assets plus Fund of Hedge Funds, and the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio. These figures represent the arithmetic mean return or expected return on the Y-axis and the downside deviation below the target return on the X-axis. Accompanying each of the figures is a pie chart of the asset weights for the optimal (maximum) Sortino portfolios. Table 3 also shows the optimal Sortino portfolio weightings, while Table 4 shows the arithmetic mean, downside deviation and Sortino for the optimal portfolio. All the portfolios have a maximum asset weighting set according to Regulation 28, as per section “3.2 Portfolio Construction”.

Figure 5: Sortino Frontier for the Traditional Assets and Assets Weighting for the Optimal Portfolio

Sortino Frontier for Traditional Assets



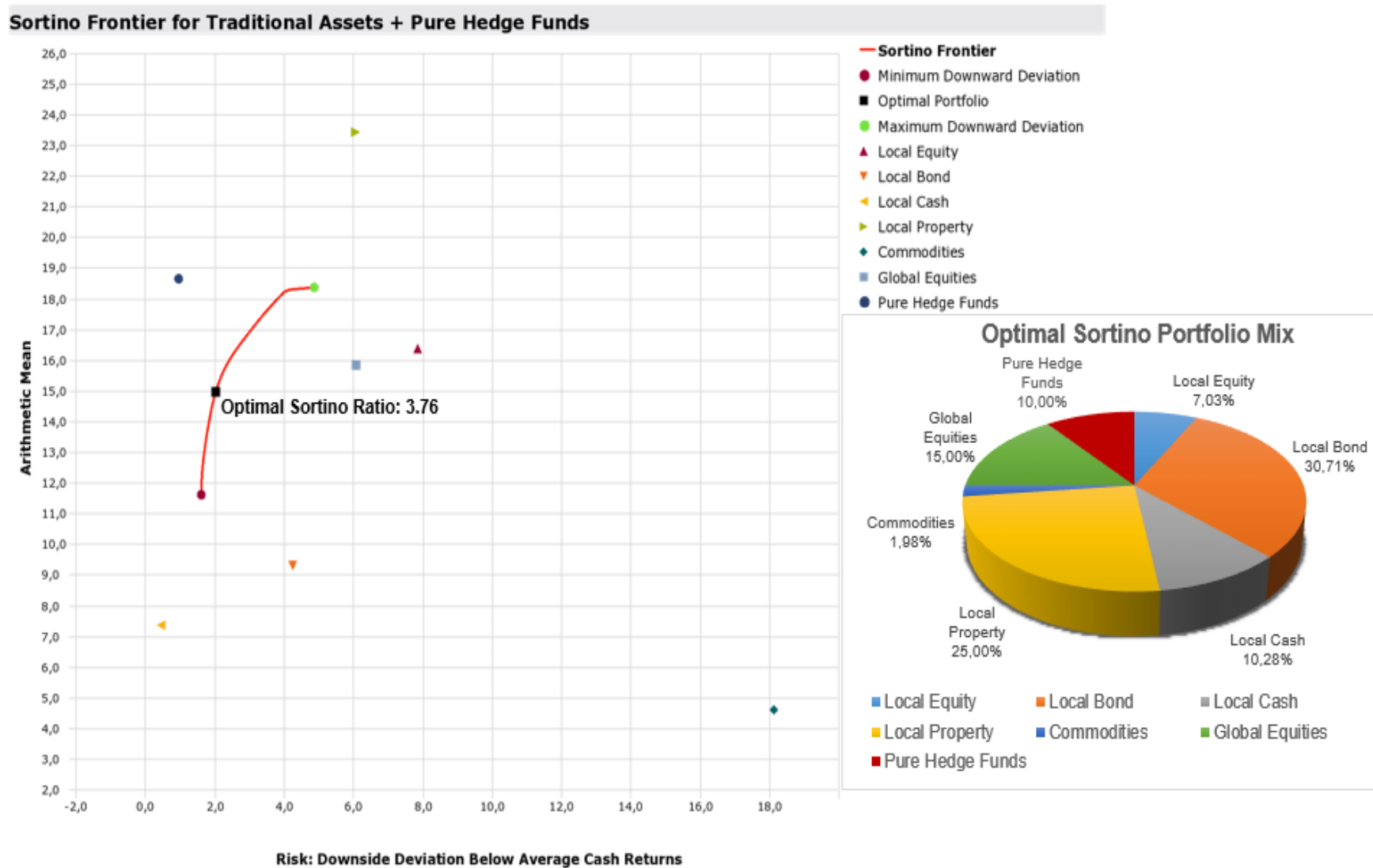
- Sortino Frontier
- Minimum Downward Deviation
- Optimal Portfolio
- Maximum Downward Deviation
- ▲ Local Equity
- ▼ Local Bond
- ▲ Local Cash
- ▲ Local Property
- ◆ Commodities
- Global Equities



Source: Morningstar Direct



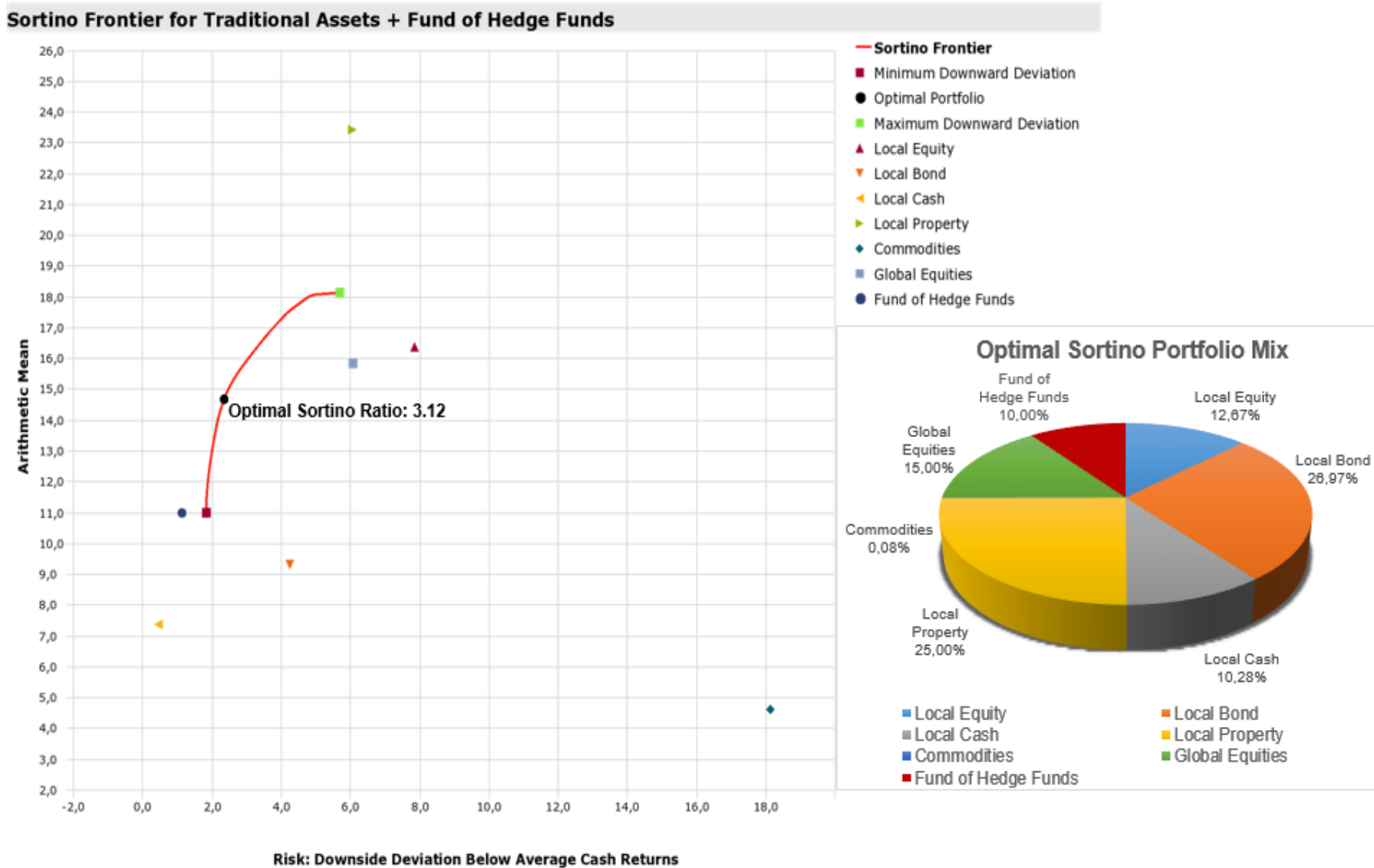
Figure 6: Sortino Frontier for the Traditional Assets plus Pure Hedge Funds and Assets Weighting for the Optimal Portfolio



Source: Morningstar Direct



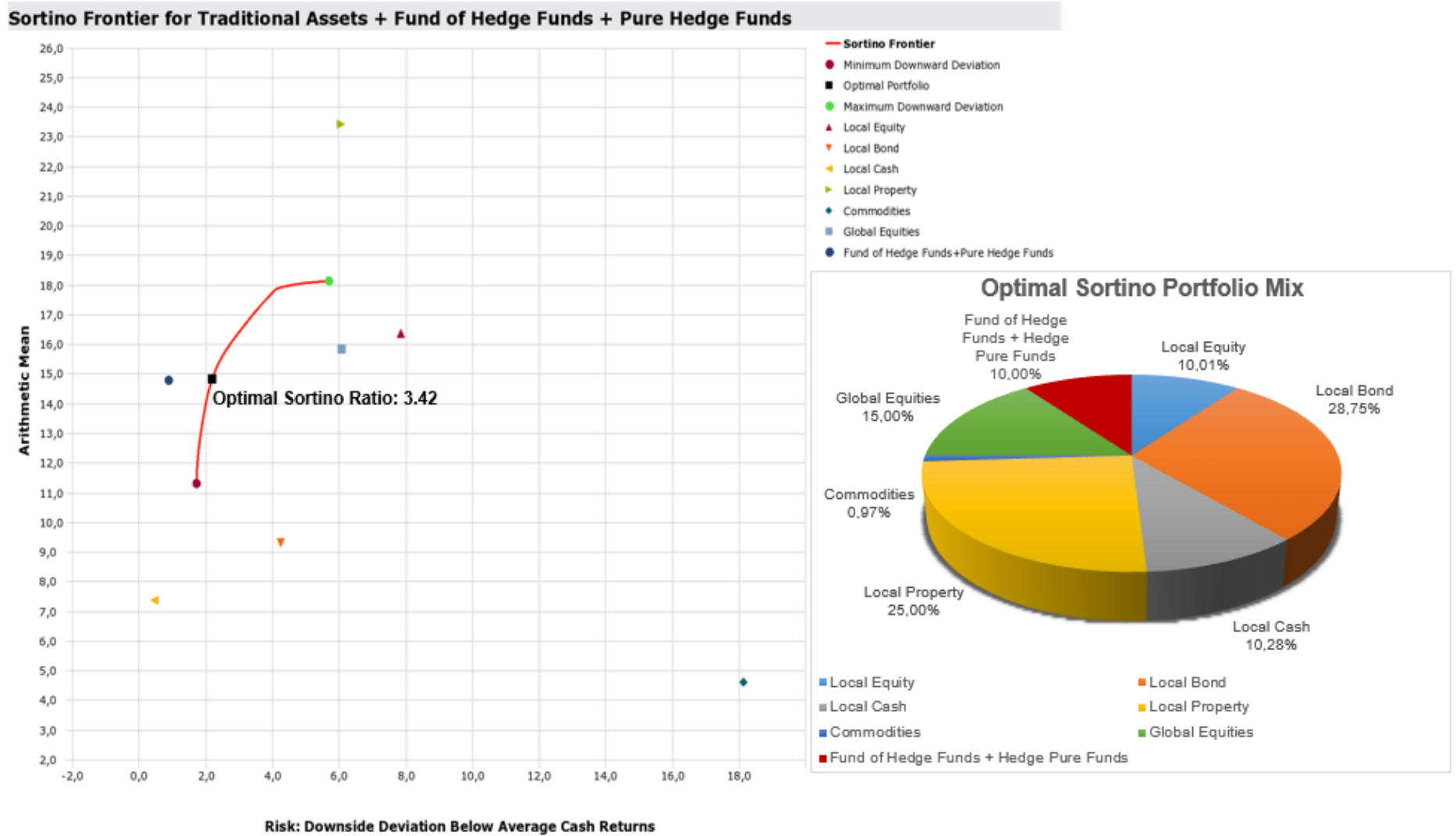
Figure 7: Sortino Frontier for the Traditional Assets plus Fund of Hedge Funds and Assets Weighting for the Optimal Portfolio



Source: Morningstar Direct



Figure 8: Sortino Frontier for the Traditional Assets plus Pure Hedge Funds and Fund of Hedge Funds and Assets Weighting for the Optimal Portfolio



Source: Morningstar Direct



Table 9: Optimal Sortino Portfolio Asset Weighting

Portfolio	Local Equity	Local Bond	Local Cash	Local Property	Commodities	Global Equities	Pure Hedge Funds	Fund of Hedge Funds	Fund of Hedge Funds + Pure Hedge Funds
Traditional Assets	18,69%	31,03%	10,28%	25,00%	0,00%	15,00%	-	-	-
Traditional Assets + Pure Hedge Funds	7,03%	30,71%	10,28%	25,00%	1,98%	15,00%	10,00%	-	-
Traditional Assets + Fund of Hedge Funds	12,67%	26,97%	10,28%	25,00%	0,08%	15,00%	-	10,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	10,01%	28,75%	10,28%	25,00%	0,97%	15,00%	-	-	10,00%

Table 10: Optimal Sortino Portfolio Arithmetic Mean Return, Standard Deviation and Sortino Ratio

Portfolio	Arithmetic Mean	Rank	Downside Deviation Below 7.37673%	Rank	Maximum Sortino Ratio	Rank
Traditional Assets	14,95056165	2 nd	2,701611725	4 th	2,803449356	4 th
Traditional Assets + Pure Hedge Funds	14,96203187	1 st	2,019079538	1 st	3,756811818	1 st
Traditional Assets + Fund of Hedge Funds	14,68416926	4 th	2,343471628	3 rd	3,118211109	3 rd
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	14,83418483	3 rd	2,182421852	2 nd	3,417054692	2 nd



4.5.1 Optimal Sortino Portfolio Asset Weightings Overview

As shown in the weighting chart alongside Figures 5 to 8, all four portfolios tested have a maximum Regulation 28 allocation to Local Property, Local Cash and Global Equities. This is similar to the observation of asset weightings in the mean-variance analysis. Commodities have a zero allocation in the Traditional Assets portfolio, but are included in all three hedge fund portfolios. The highest allocation to Commodities is in the Traditional Assets plus Pure Hedge Funds portfolio, with a 1.98% allocation. This is a small amount when considered on an overall portfolio basis, and opens up the question of whether commodities are worth holding for portfolio managers when one looks at downside risk.

Local Equities and Local Bonds have their weightings reduced when Traditional Assets are combined with hedge fund investments. The reduction in Local Bond investments ranges from .32% (Traditional Assets plus Pure Hedge Funds), to 4.06% (Traditional Assets plus Fund of Hedge Funds) and 2.28% (Traditional Assets plus Pure Hedge Funds and Fund of Hedge Funds), as compared with the Traditional Assets portfolio. This reduction is small when compared with the reduction in Local Equities, most likely reflecting the size of the fixed income hedge funds in the data set. The difference in allocation to Local Equities between the Traditional Assets portfolio is 2.65x, 1.47x and 1.86x when compared with the Traditional Assets plus Pure Hedge Funds, Traditional Assets plus Fund of Hedge Funds and the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolios. On an absolute basis, Local Equities have an 18.69% allocation in the Traditional Assets portfolio, and the large reductions in allocations can be attributed to the impacts of correlation. When two assets are highly correlated, the higher performing asset- which in this case is the Pure Hedge Fund and Fund of Hedge Funds assets- will take the place of the weaker performing asset. Overall, from an asset allocation perspective, the outcome and changes are the same as those observed in the mean-variance optimisation. The nature of the changes and magnitudes are comparable.

The arithmetic returns, downward deviation and the Sortino ratio produced the same ranking as that for the mean-variance analysis. The individual portfolio returns were similar; however the downside deviations were largely different. This fed into the Sortino ratio where the Traditional Assets plus Pure Hedge Funds ranked first, the Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds ranked second, Traditional Assets plus Fund of Hedge Funds ranked third, and last was the Traditional Assets

portfolio. Similar outcomes were to be expected as the assets all exhibit approximately symmetrical distributions, reducing the benefit of using a mean-semi variance model.

4.5.2 Optimal Sortino Ratio Analysis

As per Table 9, the arithmetic mean returns differ at most by 1.8% for the Traditional Asset (fourth ranked) vs Traditional Assets plus Fund of Hedge Funds (third ranked). This is a minute difference and do not justify an advantage for one portfolio over the other. The largest difference in the risk adjusted returns can be observed in the downside deviation or downside risk.

The portfolio with the lowest downside risk (first ranked) was the Traditional Assets plus Pure Hedge Funds portfolio with 2.01. The second ranked portfolio was the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds with 2.18, a difference of 8.08% with the first ranked portfolio. The third ranked portfolio was the Traditional Assets plus Fund of Hedge Funds with 2.34, a difference of 16.06% from that of the first ranked portfolio. The lowest ranked portfolio, or the portfolio with the highest downside risk, was the Traditional Assets portfolio with a downside deviation of 2.70. This is 33.80% larger than the first ranked portfolio, 23.7% larger than the second ranked portfolio, and 15.28% larger than the third ranked portfolio. This is a significantly higher level of risk given the similarities in return.

Table 11: Difference in Sortino Ratios

Portfolio	Traditional Assets	Traditional Assets + Pure Hedge Funds	Traditional Assets + Fund of Hedge Funds	Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds
Traditional Assets	0,00%	-	-	-
Traditional Assets + Pure Hedge Funds	34,01%	0,00%	-	-
Traditional Assets + Fund of Hedge Funds	11,23%	20,48%	0,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	21,89%	9,58%	9,94%	0,00%

The ranking of the portfolios using the optimal Sortino produced a similar result to that of the mean-variance analysis. The highest ranked portfolio using the Sortino is the Traditional Assets plus Pure Hedge Funds, followed by the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio. In third and fourth place



was the Traditional Assets plus the Fund of Hedge Funds and the Traditional Assets portfolio.

As per Table 11, the differences in the Sortino ratios of the Traditional Assets portfolio and the hedge fund portfolios ranges from 34.01% to 11.23%. If we scrutinise the combined Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds portfolio because it better represents the industry in South Africa, it outperformed the Traditional Assets portfolio by 21.89%. This is a significant difference and suggests that on a risk adjusted basis, investors should be including hedge fund investments in their portfolios.

Within the hedge fund portfolios the Traditional Assets plus Pure Hedge Funds portfolio outperformed the other two portfolios by 9.58% and 20.48% respectively. This suggests that investors should favour individual hedge funds when attempting to enter the industry. This is of course as an alternative to looking to Fund of Hedge Funds when diversifying, as Fund of Hedge Funds may already be spread across hedge funds. As they would have already had their idiosyncratic risk reduced, this would also reduce the benefits of diversification.

Overall, the results are similar to those in the mean-variance analysis with identical observations in asset weighting characteristics and when looking at risk-adjusted returns. The overall outcome is that traditional asset portfolios that include hedge fund assets, be it the Fund of Hedge Funds, Pure Hedge Funds or a combination of the two, outperform those that constituted only traditional assets. Beyond this, when hedge fund investments²⁵ are included in the investment set, an optimal Sortino portfolio will maximise its allocation to the hedge fund investments at the expense of local equity and local bond investments.

²⁵ Pure Hedge Funds, Fund of Hedge Funds and the combined Pure Hedge Fund and Fund of Hedge Funds.



4.6 Omega Optimisation

This section analyses the four portfolios namely, a Traditional Assets portfolio, a Traditional Assets plus Pure Hedge Funds portfolio, a Traditional Assets plus Fund of Hedge Funds portfolio and a Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio, using the Omega ratio. The Omega represents an analysis that eliminates the bias due to the omission of higher order moments when observing return distributions (Keating & Shadwick 2002b). From a risk perspective, the Omega function takes on the return distribution itself without omitting any information due to the assumptions of normality or investor preferences (Keating & Shadwick 2002b). To avoid the implications of model bias, the Omega has been included to expand the scope of this research. It is particularly useful, as skewness and kurtosis are seen as challenges when dealing with hedge fund assets.

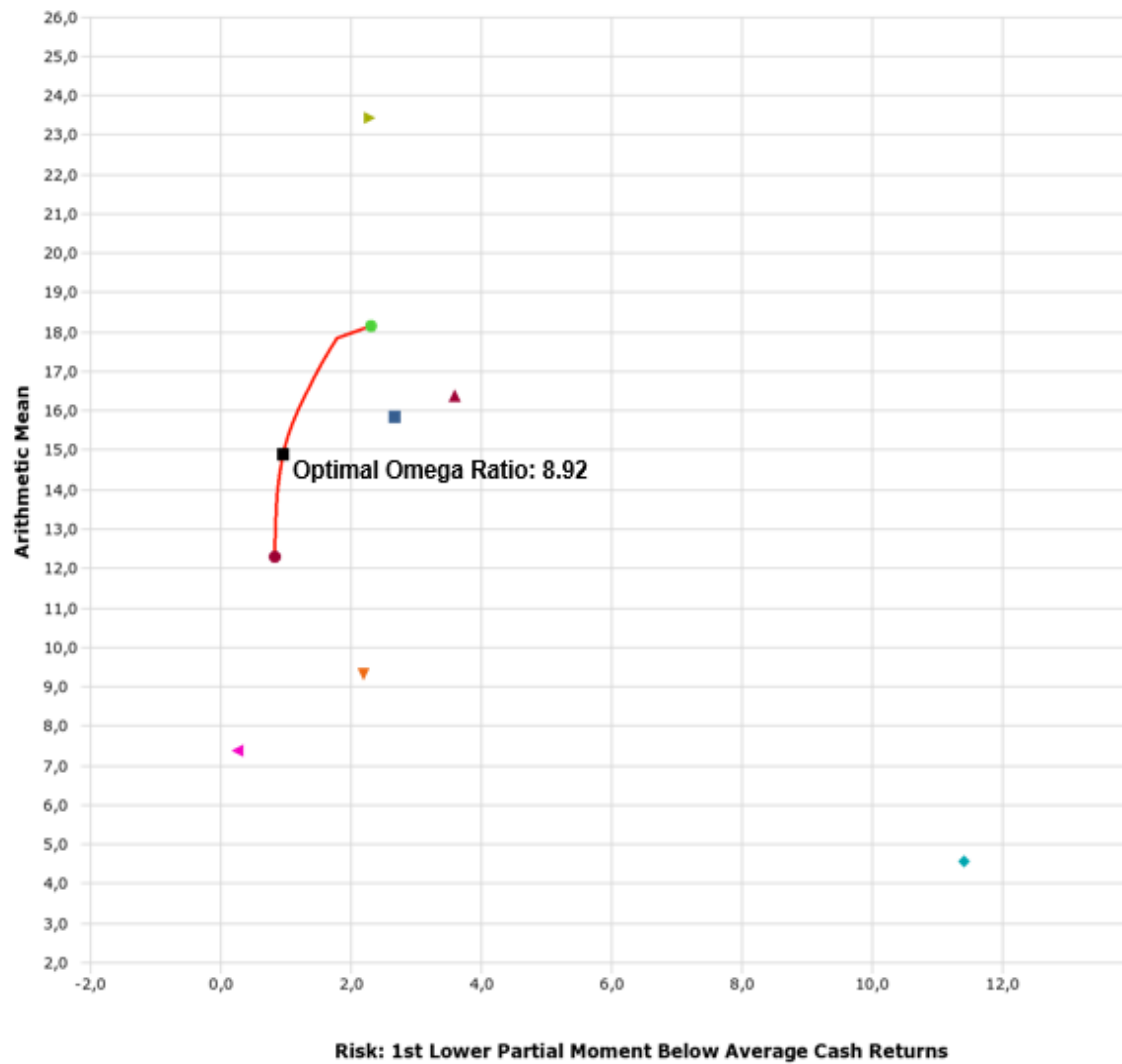
The returns of individual assets are seen to be approximately symmetrical, as shown in section “4.2.3- Skewness”. This reduces the concern of non-normality in asset returns which, in turn, reduces the importance of an Omega analysis. Furthermore, the mean-variance analysis and the mean-semi variance analysis yield similar results, which seems to suggest that the assets distribution or higher order moments is not a concern. Keating & Shadwick (2002b), however, suggest that the ranking using Omega may differ from that of an optimal Sharpe portfolio. In an attempt to emphasise prudence, the optimal Omega for each portfolio is analysed.

Figures 9 to 12 graph the arithmetic return or expected return on the y-axis and the first lower partial moment on the x-axis. These are used as inputs into the generation of the optimal or maximum Omega function for the four respective portfolios. Accompanying each figure is a pie chart illustrating the weights of assets in the optimal omega function. The lower partial moment was used in place of the higher partial moment as this study has focused on downside risk. Portfolio managers should be concerned with the instance of large losses, and should attempt to reduce the impact of such an event. The target level of return is the average cash assets return (annualised); since this was the level used in the mean-semi variance analysis. As suggested earlier, the cash assets represent a hedge against inflation-related losses and a level portfolio manager’s attempt to beat, in order to maintain their client’s wealth. Tables 12 and 13 represent the weights of each optimal portfolio and the accompanying Omega ratio for each portfolio.

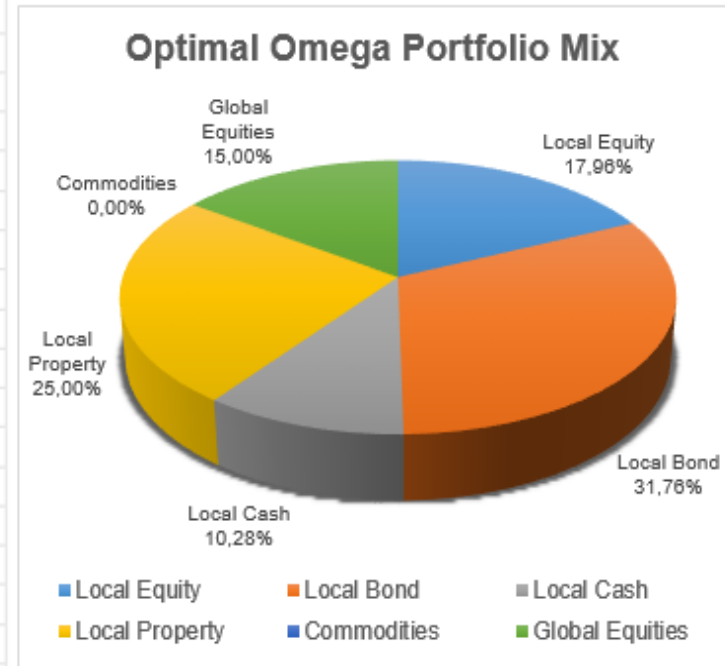


Figure 9: Omega Frontier-Traditional Assets and Asset Weighting for the Optimal Omega Portfolio

Optimal Omega Mix for Traditional Assets



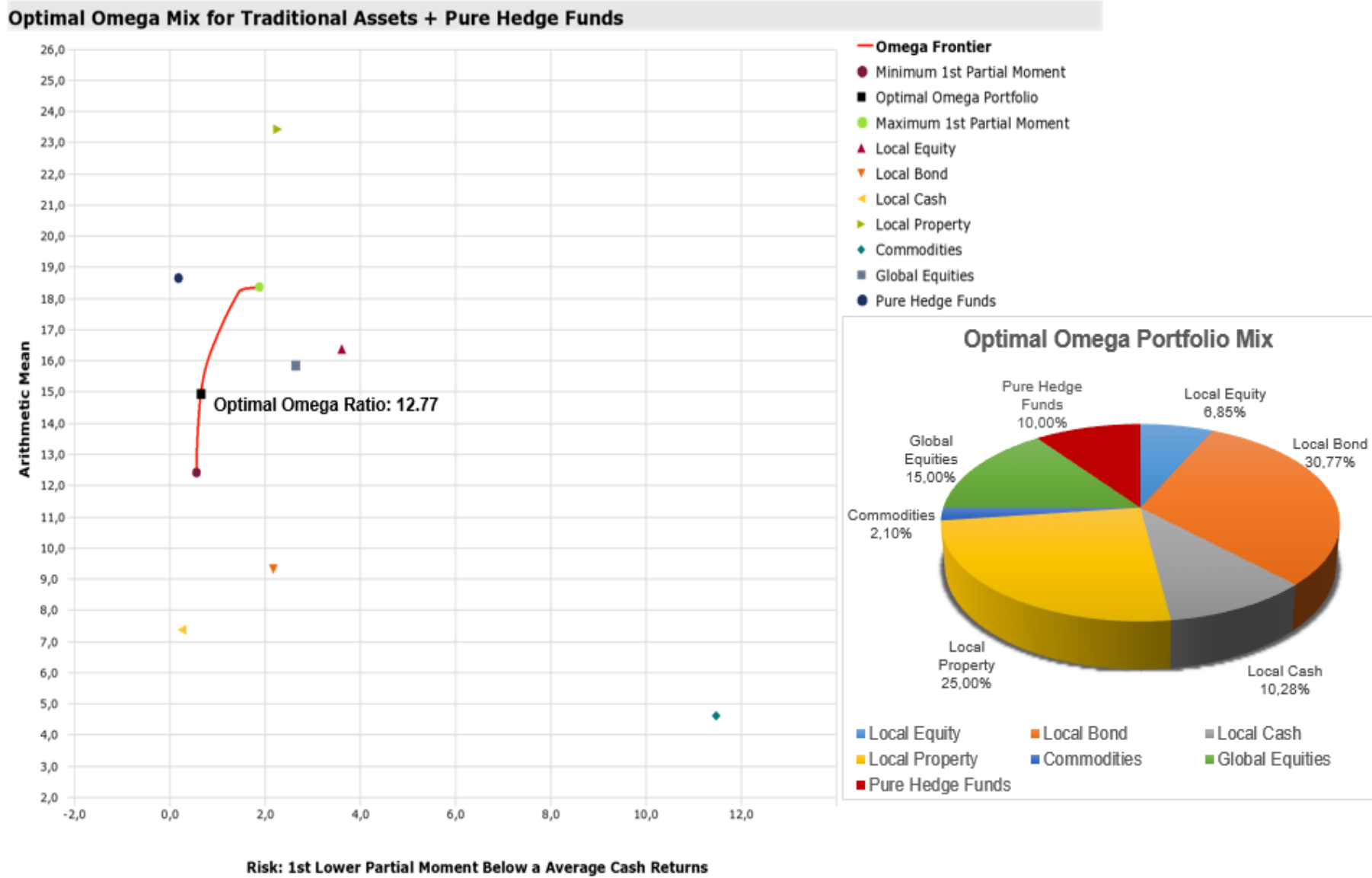
- Omega Frontier
- Minimum 1st Lower Partial Moment
- Optimal Omega Portfolio
- Maximum 1st Lower Partial Moment
- ▲ Local Equity
- ▼ Local Bond
- ◆ Local Cash
- ▲ Local Property
- ◆ Commodities
- Global Equities



Source: Morningstar Direct



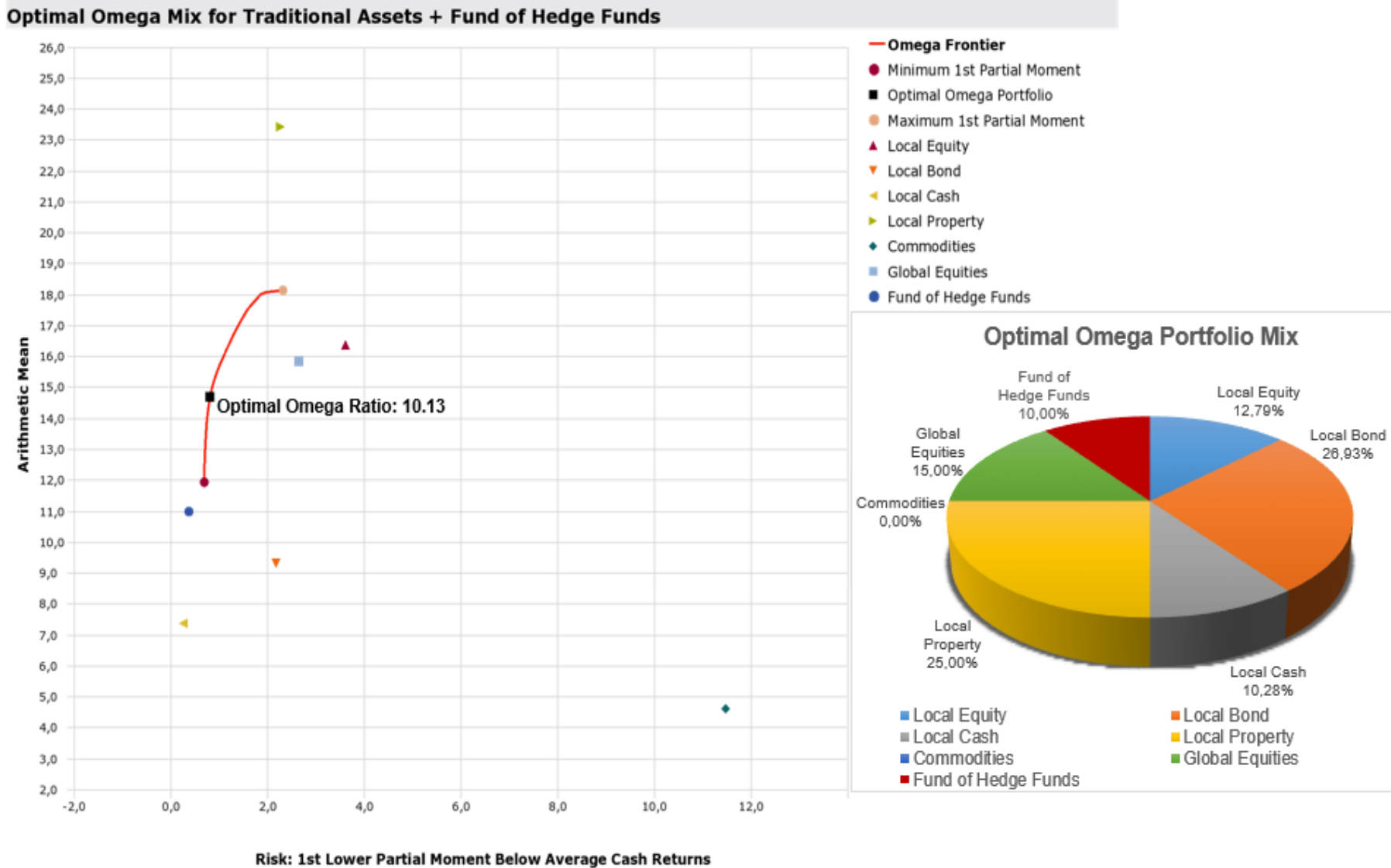
Figure 10: Omega Frontier-Traditional Assets plus Pure Hedge Funds and Asset Weighting for the Optimal Omega Portfolio



Source: Morningstar Direct



Figure 11: Omega Frontier-Traditional Assets plus Fund of Hedge Funds and Asset Weighting for the Optimal Omega Portfolio

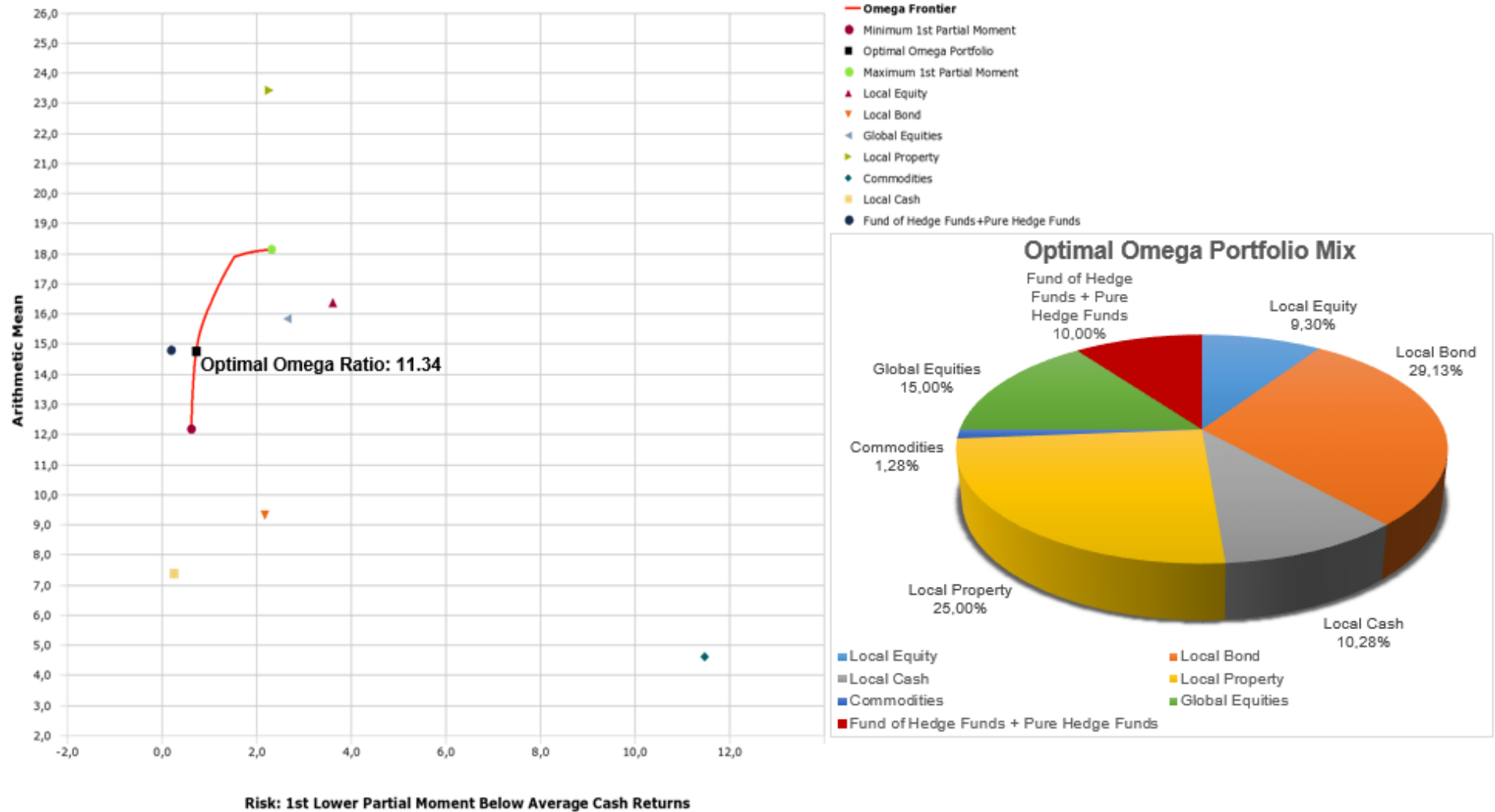


Source: Morningstar Direct



Figure 12: Omega Frontier-Traditional Assets plus Hedge Funds and Fund of Hedge Funds and Asset Weighting for the Optimal Omega Portfolio

Optimal Omega Mix for Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds



Source: Morningstar Direct



Table 12: Optimal Omega Portfolio Asset Weighting

Portfolio	Local Equity	Local Bond	Local Cash	Local Property	Commodities	Global Equities	Pure Hedge Funds	Fund of Hedge Funds	Fund of Hedge Funds + Pure Hedge Funds
Traditional Assets	17,96%	31,76%	10,28%	25,00%	0,00%	15,00%	-	-	-
Traditional Assets + Pure Hedge Funds	6,85%	30,77%	10,28%	25,00%	2,10%	15,00%	10,00%	-	-
Traditional Assets + Fund of Hedge Funds	12,79%	26,93%	10,28%	25,00%	0,00%	15,00%	-	10,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	9,30%	29,13%	10,28%	25,00%	1,28%	15,00%	-	-	10,00%

Table 13: Optimal Omega Ratio Ranking

Portfolio	Optimal Omega Ratio	Rank
Traditional Assets	8,926809832	4 th
Traditional Assets + Pure Hedge Funds	12,77102633	1 st
Traditional Assets + Fund of Hedge Funds	10,12548395	3 rd
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	11,33455293	2 nd



4.6.1 Optimal Omega Portfolio Asset Weightings Overview

As per the mean-variance and mean-semi variance analysis, the allocation to Local Property, Local Cash and Global Equities was maximised for all four portfolios. The weighting in Commodities increased to 2.1% and 1.28% for the Traditional Assets plus Pure Hedge Funds portfolio, and the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio, from a 0% allocation in the Traditional Assets portfolio. This allocation is comparable to the mean-semi variance analysis, where the allocation was 1.98% (Traditional Asset plus Pure Hedge Funds) and .97% (Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds) respectively.

The allocation to Local Bond and Local Equities was reduced for the three hedge fund portfolios when compared with the Traditional Assets portfolio. This is a similar phenomenon to those of the mean-variance and mean-semi variance analysis. Looking specifically at Local Bonds, the reduction in allocation from the Traditional Assets portfolio was .99% for the Traditional assets plus Pure Hedge Funds portfolio, as compared with .32% for the mean-semi variance analysis. Similarly, the reduction in bond investments for the Traditional Assets portfolio to the Traditional Assets plus Fund of Hedge Funds and Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds portfolio was 4.83% and 2.62%. The mean-semi variance analysis produced a reduction of 4.06% and 2.28% for comparable portfolios. Although the reduction in allocation to Local Bonds is smaller for the optimal Omega vs Optimal Sortino, this difference is minute.

In the Traditional Assets portfolio, Local Equity had an allocation of 17.96% which fell by 11.11% to 6.85% in the Traditional Assets plus Pure Hedge Funds portfolio. This was greater than the 5.17% and 8.66% reduction when compared with the Traditional Assets plus Fund of Hedge Funds and the Traditional Assets plus combined Pure Hedge Funds and Fund of Hedge Funds portfolio. This is, however, a reduction of 2.63x, 1.40x and 1.93x on a relative basis. This reduction in allocation to Local Equity between the traditional assets and hedge fund portfolios is similar to the mean-semi variance analysis which sat at 2.65x, 1.47x and 1.86x for comparable portfolios. This change can be attributed to the high correlation in the Local Equity index to the hedge fund indices as seen in the mean-semi variance analysis. The similarities between the



observations is due to the approximately symmetrical nature of the assets' distributions, as well as the focus on the downside risk for the Sortino and Omega analysis. This is to be expected as the importance of the assets' higher order moments was reduced due to their approximately symmetrical natures.

4.6.2 Optimal Omega Ratio Analysis:

The Omega rankings of the portfolios has the Traditional Assets plus Pure Hedge Funds placed first, followed by the Traditional Assets plus the combined Pure Hedge Funds and Fund of Hedge Funds in second place, the Traditional Assets plus Fund of Hedge Funds in third, and the Traditional Assets portfolio in fourth place. This is the same outcome when compared with the mean-variance and mean-semi variance analysis. Among all the methods tested, assuming normality of distribution as well as taking into account non-normality, portfolios including hedge fund investments outperformed those that excluded them.

Table 14: Difference in Omega Ratios

Portfolio	Traditional Assets	Traditional Assets + Pure Hedge Funds	Traditional Assets + Fund of Hedge Funds	Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds
Traditional Assets	0,00%	-	-	-
Traditional Assets + Pure Hedge Funds	43,06%	0,00%	-	-
Traditional Assets + Fund of Hedge Funds	13,43%	26,13%	0,00%	-
Traditional Assets + Pure Hedge Funds + Fund of Hedge Funds	26,97%	12,67%	11,94%	0,00%

The difference in Sortino to Omega optimisation, although not comparable on a magnitude basis, nevertheless shows that the nature of the differences in values of each portfolio is the same. The Traditional Assets plus Pure Hedge Funds portfolio outperformed the Traditional Assets portfolio by 43.06%. This is significant, and suggests that portfolio managers should have taken on hedge fund assets if they wanted superior risk-adjusted returns for their clients between 31/01/2005 and 31/01/2015.



Within the hedge fund portfolios the Traditional Assets plus Fund of Hedge Funds portfolio was outperformed by the Pure Hedge Funds portfolio by 26.13%. This is a similar observation to those noted in the Sharpe and Sortino analysis. This could be due to the hedge funds adding greater diversification benefits as compared with Fund of Hedge Funds. Individual hedge funds exhibit greater idiosyncratic risk than Fund of Hedge Funds, as Fund of Hedge Funds attempt to diversify away that risk.

Overall, the allocations to the alternative assets, namely Pure Hedge Funds and Fund of Hedge Funds investments, is maximised within Regulation 28 to attain the optimal Omega ratio. This is the same occurrence observed in the optimal Sortino and Sharpe ratios for comparable portfolios. Further, from an Omega optimisation perspective all portfolios which included hedge fund investments outperformed those that only had traditional asset investments. Within the hedge fund industry, traditional asset portfolios that included Pure Hedge Funds outperformed those that included Fund of Hedge Funds. This suggests that Pure Hedge Funds offer greater diversification benefits to investors and given a choice should be adopted on in place of Fund of Hedge Funds. As an institutional investor, considering the period between 31/01/2005 and 31/01/2015, had one followed a mean-variance, mean-semi variance or an Omega strategy, and invested in traditional assets plus hedge fund portfolio, one would have outperformed an investor who had focused on traditional assets only, *citrus paribus*.



Chapter 5: Conclusion

Investments for retirement portfolios can be a complex business and, in South Africa, with the regulatory tightening brought on by Regulation 28, this is proving to be ever more challenging. This study has sought to offer an alternative investment perspective for the South African portfolio manager. The use of diversification and Markowitz mean-variance is not innovative; however, within the context of South Africa notion of using hedge fund investments as a tool for diversification is still in its infancy. The development of this industry, coupled with an increased understanding of its influence overall, represents an opportunity for increased market liquidity, transparency and price efficiency. Hence, this study has aimed to understand how hedge fund investments can be used to increase the risk-adjusted returns in well-diversified portfolios.

Contextualising this within the South African market, this study found that between 31/01/2005 and 31/01/2015, well-diversified portfolios that included hedge fund investments outperformed those that excluded them. Investigating the individual asset without the benefits of diversification, the hedge fund industry, considering Pure Hedge Funds and Fund of Hedge Funds as the two primary alternatives, offered a 76% higher risk-adjusted return (Sharpe ratio) than the best performing traditional asset (Local Property). From a mean-variance perspective, the Sharpe ratio of portfolios that accounted for the hedge fund industry outperformed those that had excluded hedge funds by 9.8%.

When taking into account the non-normality of asset returns, which is a concern with the mean-variance model, the outperformance of hedge fund inclusive portfolios was accentuated. The Sortino ratio, which calculates excess return per unit of downward deviation, is used in mean-semi variance analysis. Mean-semi variance is used to overcome the normality assumption embedded in the mean-variance model. Subsequently, the Sortino ratio of traditional asset portfolios which included hedge fund investments outperformed those that excluded hedge fund investments by 21.89%. Similarly, when looking at the Omega for each portfolio, which is considered to be an unbiased tool for portfolio optimisation, the previous findings were emphasised. The optimal Omega for traditional asset portfolios that included hedge



fund investment outperformed portfolios that excluded hedge fund investment by 26.97%.

The superior performance of the hedge fund inclusive portfolios can be attributed to the correlation between the local equities industry and the hedge fund industry. As the majority of hedge funds are invested in local equities, the correlation between the two is as high as .77. The high positive correlation coupled with the superior risk-adjusted returns of the hedge fund industry over local equities meant that, when the portfolios were optimised, local equities were substituted by hedge funds.

Within hedge fund industry portfolios, Pure Hedge Funds offered better stand-alone risk-adjusted returns with a 37% greater Sharpe ratio than that of Fund of Hedge Funds. The difference can be understood by the nature of each asset class. Pure Hedge Funds are naturally riskier than Fund of Hedge Funds, as Fund of Hedge Funds are diversified across multiple hedge funds, reducing the idiosyncratic risk of individual hedge funds. The lower risk comes at a cost of lower returns. Typically, however, one would not expect such a large difference in the risk-adjusted returns. The magnitude of the difference could be influenced by the data-set which overweighs Pure Hedge Funds; representing an area on which for future studies could be focused.

From a portfolio perspective, traditional asset portfolios that included Pure Hedge Funds had a 9.34% greater Sharpe ratio, 20.48% greater Sortino, and 26.13% greater Omega ratio than traditional asset portfolios which included Fund of Hedge Funds. This difference can again be attributed to the nature of Fund of Hedge Funds which is diversified across individual hedge funds. The diversification reduces the ability to use the off-setting correlations to optimise risk-adjusted returns, which subsequently leads to poorer performance. Given the choice to include either Fund of Hedge Funds or Pure Hedge Funds in a well-diversified portfolio, looking at returns between 31/1/2005 and 31/012015, an investor who chose to include Pure Hedge Funds would produce superior risk-adjusted returns for his/her clients, *citrus paribus*.

Of greater importance is the increased risk-adjusted returns achieved by well-diversified traditional asset portfolios if they include hedge funds between 31/01/2005 and 31/01/2005, *citrus paribus*. This superior performance coupled with accommodating legislation suggest that portfolio managers should include hedge fund investments in their portfolios. Including these portfolios would also encourage greater



oversight and legislation around the hedge fund industry, as retirement funds are themselves highly regulated. In summary this research has found that conventional asset managers can make use of hedge funds for the betterment of their clients' portfolios and the industry overall, using the existing guidelines and standard investment tools.



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Appendix

Formulae 1 to 4 First, Second, Third and Fourth Central Moments

$$\bar{x}_{\text{arithmetic}} = \frac{\sum_{i=1}^n x_n}{n}$$

Arithmetic mean(\bar{x}) is the average of a set of numerical values, as calculated by adding them together and dividing by the number of terms in the set.

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

Sample standard deviation(**S**) is a measure of dispersion of a set of data from its mean.

$$\text{Skewness} = \frac{n}{(n - 1) \cdot (n - 2)} \cdot \sum_{i=1}^n \frac{(x - \bar{x})^3}{S}$$

Skewness is a measure of the asymmetry of the probability distribution of real-valued random variable around its means. The skewness can be positive or negative, or even undefined. A normal distribution is assumed to have a skewness of zero. Positive skewness is characterised by a distribution long or fatter right tail, while a distribution with a negative skew is characterised by a long or fatter left tail. According to Bulmer (1979) the rule of thumb is:

- skewness less than -1 or greater than $+1$, the distribution is highly skewed;
- skewness between $(-\frac{1}{2}, 1)$ or between $(+\frac{1}{2}, +1)$, the distribution is moderately skewed;
- skewness between $-\frac{1}{2}$ and $+\frac{1}{2}$, the distribution is approximately symmetric.



$$\text{Excess Kurtosis} = \left[\frac{n \cdot (n + 1)}{(n - 1) \cdot (n - 2) \cdot (n - 3)} \cdot \sum_{i=1}^n \frac{(x - \bar{x})^4}{S} \right] - \frac{3 \cdot (n - 1)^2}{(n - 2) \cdot (n - 3)}$$

Kurtosis is a measure of the “tailedness” of the probability distribution of a real-valued random variable. Kurtosis for a standard normal distribution is assumed to be 3, and is referred to as being mesokurtic. Excess kurtosis then expresses kurtosis above or below a level of 3. When a distribution has a higher/lower peak than the normal distribution, it is referred to as leptokurtic/platykurtic. This means that the distribution has fatter/thinner tails, and that more/less of the variability is due to a few extreme deviances from the mean. This is, of course, a concern for a leptokurtic distribution as the impact of extreme outlying events is greater.

$$\text{Correlation}^{A,B} = \frac{\sum_{i=1}^n (x_i^A - \bar{x}^A) \cdot \sum_{i=1}^n (x_i^B - \bar{x}^B)}{\sqrt{\sum_{i=1}^n (x_i^A - \bar{x}^A)^2} \cdot \sqrt{\sum_{i=1}^n (x_i^B - \bar{x}^B)^2}}$$

The **Correlation coefficient** is a measure of the linear relationship between two assets. The value of a correlation coefficient ranges between (-1;1) with :

- “1” – perfect correlation or perfect positive linear relationship;
- “ 0 to 1 ” – Imperfect positive correlation or the variables A,B tend to increase or decrease together;
- “ 0 ” – no linear relationship exists between the two variable;
- “ -1 to 0 ” – Imperfect negative correlation or as one variable increases the other variable decreases
- “-1” – Inverse correlation or perfect negative relationship between the assets.

The correlation coefficient represents the instance of a relationship between two assets, and does refer to a change in variable “A” leading to a change in variable “B”, or vice versa. This relationship can be spurious or be caused by an outside variable that leads to a change in both. Hence, one must be cautious when interpreting the correlation coefficient.



Formula 5- Pure Hedge Funds/Fund of Hedge Funds Index Composition

Fund Size Weighting at time t,

$$FSW_t = \frac{FS_i}{\sum_{i=1}^n FS_i}$$

where:

FS_i = Fund Size of Pure Hedge Fund or Fund of Fund;

$\sum_{i=1}^n FS_i$ = Sum of Fund Size of Pure Hedge Fund or Fund of Fund.

Subject to:

FSW_t – Rebalanced and Reconstituted at $t + 6$,

$$\sum_{i=1}^n FSW_i = 1,$$

Pure Hedge Funds Limit:

$$0 \leq FSW_i \leq 25\%$$

Fund of Fund Only Limit:

$$0 \leq FSW_i \leq 50\%$$

Return of Pure Hedge Funds and/or Fund of Hedge Funds:

$$r_t = \frac{(NAV_t - NAV_{t-1})}{NAV_{t-1}}$$

Fund Size Weighted Return of Pure Hedge Funds and/or Fund of Hedge Funds:

$$WR_t = FSW_t \times r_t$$



Formula 6 Sharpe Ratio

General Sharpe Ratio

$$SR_p = \frac{\mu_p - R_f}{\sigma_p}$$

where:

μ_p = the expected return on the portfolio

R_f = the risk-free rate – set equal to average cash asset return over period;

σ_p = the standard deviation of the portfolio

and where:

$$\mu_p = \sum_{i=1}^n x_i u_i ,$$

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} ,$$

x_i = represents the proportion of the portfolio invested in asset i;

u_i = expected return of asset i;

σ_{ij} = covariance between asset i and j;

n = number of assets in the portfolio.

Maximum Sharpe:

$$\text{Max}_{x_1, x_2, \dots, x_n} SR_p = \frac{\mu_p - R_f}{\sigma_p} = \frac{\sum_{i=1}^n x_i u_i - R_f}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij}}}$$

Subject to $\sum_{i=1}^n x_i = 1$ and $x_i \geq 0$ for all i

$x_i \geq 0$ denotes the no short-selling constraints



Formula 7 Sortino Ratio

$$Sortino_p = \frac{\mu_p - \tau}{TDD(\tau)}$$

where:

μ_p = the expected return on the portfolio;

$TDD(\tau)$ = the total downside deviation of return series;

τ = the target rate of return or Minimum Acceptable Rate (MAR) - set equal to R_f ;

and where:

$$\mu_p = \sum_{i=1}^n x_i u_i,$$

$$TDD(\tau) = \sqrt{\frac{\sum_{i=1}^n \max(\tau - x_i, 0)^2}{n}}$$

Maximum Sortino

$$Max_{x_1, x_2, \dots, x_n} \quad Sortino_p = \frac{\mu_p - \tau}{TDD(\tau)} = \frac{\sum_{i=1}^n x_i u_i - \tau}{\sqrt{\frac{\sum_{i=1}^n \max(\tau - x_i, 0)^2}{n}}}$$

Subject to $\sum_{i=1}^n x_i = 1$ and $x_i \geq 0$ for all i



Formula 8 Omega Ratio

$$\Omega(\tau) = \frac{\int_{\tau}^{\infty} [1 - F(x)] dx}{\int_{-\infty}^{\tau} (\tau - x)^2 dx}$$

where:

$F(x)$ = the cumulative density function (cdf) for total return on an investment

$$\int_{\tau}^{\infty} [1 - F(x)] dx = \text{Upside Potential}$$

$$\int_{-\infty}^{\tau} (\tau - x)^2 dx = \text{Downside potential}$$

τ = the target rate of return or Minimum Acceptable Rate (MAR) - set equal to R_f

(Keating & Shadwick 2002b).

Kaplan and Knowles (2004) proved that Omega can be expressed as:

$$\Omega(\tau) = \frac{(\mu - \tau)}{FLPM(\tau)} + 1$$

where:

$$FLPM(\tau) = \frac{\sum_{i=1}^n \max(\tau - R_i, 0)}{n}$$

Maximum Omega

$$\text{Max}_{x_1, x_2, \dots, x_n} \Omega(\tau) = \frac{(\mu - \tau)}{FLPM(\tau)} + 1 = \frac{\sum_{i=1}^n x_i u_i - \tau}{\frac{\sum_{i=1}^n \max(\tau - R_i, 0)}{n}} + 1$$

Subject to $\sum_{i=1}^n x_i = 1$ and $x_i \geq 0$ for all i

