Asset Allocation in the South African Environment

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Abstract

The aim of this paper is to find solutions to the asset allocation problem in the South African environment. These solutions look at a variety of different investor's preferences. These include an investor's age, risk aversion and required levels of returns. To do this, an analysis was done of prior research, so the most up to date mean-variance asset allocation model could be developed. Returns from 10 different indices, over different asset classes were gathered. The indices of importance were found to be: All Bond Index (ALBI), Inflation Linked All Maturities Index (ILB), Salient's Momentum Active Index Fund (MOME), Salient's Value Active Index Fund (VAL), South African Short Term Fixed Interest Index (STEFI) and South African Property Index (SAPY).

The model developed herein uses a shortfall threshold constraint, risk aversion and time horizon to compile a set of results with regard to the needs of various types of investors. These three constraints were helpful in ascertaining how an investment should be structured.

The results showed that a portfolio should normally first be filled to the maximum level, as outlined in Regulation 28, with the best performing asset class. Thereafter, the second best asset class should be used, and so on, until the portfolio is 100% invested. In South Africa, the best performing asset classes were found to be property, equity and bonds. The best distribution would thus be 25% property (SAPY), 65% equity (MOME and VAL) and 10% bonds (ALBI and ILB). The percentages allocated to each of the asset classes varied according to the investor's level of risk aversion: with regard to equity, the distribution ranges between 25.02% MOME, 39.98% VAL and 14.84% MOME, 50.16% VAL respectively, as risk aversion increases. Bonds consist of ALBI and ILB, which range between 100% investment in ALBI for investors with lower levels of risk aversion, due to its higher returns, and 100% investment in ILB for more risk averse investors, due to the better risk return profile of ILB. Property is always invested with 25% in SAPY.

According to the results of the model, only in the last few years of an investor's investment plan, does this distribution change. Generally, when investors are two years before their retirement, they tend to reduce their exposure to equity and to increase their holdings of bonds (always ILB). However, the maximum level of bonds is only 30.51% of the portfolio, which is lower than expected.

When investors are only one year before their retirement, the level of required returns must be reduced, to secure their investment. The maximum level of returns was found to be 7.61% with one year to retirement. Only 15.94% of such an investor's portfolio consisted of equity, property and bonds. The remaining 84.06% was invested in cash (STEFI).

The general practice of gradually reducing equity, while increasing bonds and then cash, while an investor was approaching retirement age, is disproved in this paper. Based on the findings of this study, portfolios should be changed abruptly in the last 2 years, rather than making a gradual change over time, as is the norm in the industry currently. This is more in line with the associated risk levels of an investor, as he or she gets older and approaches retirement age. Gradually changing the portfolio over time does reduce the risk, but it reduces it to too low a level and far too soon than would be required, based on the reasonable amount of risk that could be taken. Reducing the risk too much also reduces the returns the investor could have received if the investments had remained at a reasonable level of risk.

In this paper, the model was not constructed to forecast which indices to invest in, nor to forecast returns for each type of investor. The purpose was to calculate a set of portfolios and discover how the portfolio changes when the shortfall constraint, risk aversion and time to retirement are altered. This gives insight into the asset allocation dilemma. This model is therefore an asset allocation template, for future research.

Declaration

I, Kevin Mahoney, hereby declare that the work on which this thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other University. I empower the University to reproduce this thesis for the purpose of research, either the whole or any portion of the contents, in any manner whatsoever.

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Logarithmic Sharpe Ratio $= (\alpha_W – r_{fL})/\sigma_{LW}$

Arithmetic Annualised Sharpe Ratio $= ((G_W/T)-(r_f/T))/(\sigma_{AW}/\sqrt{T})$

Logarithmic Annualised Sharpe Ratio $= ((\alpha_W/T)-(r_f/T))/(\sigma_{LW}/\sqrt{T})$

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Logarithmic Objective Function: $(\mu_W – \lambda\sigma^2{}_W) = (\mu_W – ½\,(1-Y)\sigma^2{}_{LW})$

List of Acronyms

ALBI	All Bond Index
ALSI40	South African Top 40 Shares Index
CRRA	Constant Relative Risk Aversion
FINDI	Financial, Basic Industrials or General Industrial Sectors Index
ILB	Inflation Linked All Maturities Index
MOME	Salient’s Momentum Active Index Fund
MSCI	MSCI World Equity Index
RESI	Resources Sector Index
RRA	Relative Risk Aversion
SAPY	South African Property Index
STEFI	South African Short Term Fixed Interest Index
VAL	Salient’s Value Active Index Fund

1. Introduction

1.1. Introduction

For many years, academics and practitioners have tried to define the optimal asset portfolio mix. However, asset allocation is complicated. Markowitz's (1952) Portfolio Selection paper was one of the first to research this issue. If the problem can be solved conclusively in this paper, it will thus provide a framework that can be followed by anyone who wishes to formulate an effective investment strategy. Individual investors, mutual fund managers, pension fund managers, hedge fund managers and any other manager or individual who seeks to find the optimal weightings between asset classes could use this method.

Prior research, originally set out to identify an optimal mix in a single period context, namely, a model that only considers the present and not the investor's age or future needs. These solutions are supposed to help practitioners make their investment decisions, but they do not give convincing explanations. The problem lies in the model used, as it does not encompass all the relevant needs of individual investors. Most importantly, an investor will have different needs at various times of his or her life.

Consequently, research was undertaken to answer the asset allocation question in a multi-period situation. The objective of this was to find solutions that are unique to individual investors at various stages of their life. These methods take into account the degree of risk aversion, the time horizon and the specific needs of each investor. These additions to the model are necessary to determine the optimal mix of a portfolio. In theory, this is an easy statement to make; however, the actual implementation of these new techniques is complicated. Since the late 1960s, new variations of the basic mean-variance framework have been adapted. Most of these papers have given insight into certain aspects of the problem but it is difficult to find a single all-encompassing theory.

The aim of this paper is thus to review past research in this field and to use the best models available to address the asset allocation problem relating to investment decisions from a South African perspective. The outcome is not to forecast an investor's return, but rather to develop a model template that can be used to uncover the underlining dynamics of the asset allocation dilemma.

The paper uses data from ten different asset classes to tailor unique solutions for different types of investors. Two models will be used in this paper. The one variation uses logarithmic returns, based on the book by Stewart, Piros and Heisler (2011), titled *Running Money: Professional Portfolio Management*; this provides an up-to-date analysis of how such a model could be constructed. It also gives insight into the model's assumptions and its problems. The second model is similar in nature, although the input data differs slightly. This model's inputs are arithmetic returns rather than logarithmic returns.

Chapter 2 of this thesis contains a review of past research in this field. The literature relating to the modern portfolio theory is reviewed, followed by the research relating to the multi-period problem.

The equations required to find the most relevant and updated optimisation solution are discussed in Chapter 3.

Chapter 4 is an in-depth discussion of the data obtained and the methods used in this thesis.

All the findings are discussed in Chapter 5. Firstly, a comparative analysis is done between two types of returns, which form the basis of this paper. Secondly, the different sets of results are discussed when risk aversion is taken into account. Risk aversion is then deleted from the model and a shortfall threshold constraint is introduced instead. This model is used again to compare the different types of return input. The second last set of results includes both risk aversion and a shortfall constraint. All of these models and constraints are examined in Chapter 5. In the last set of results, some amendments are made to the models: The aim is to arrive at the most concise set of results possible, using the model that has been adopted as the most effective and accurate.

The relevant conclusions of this thesis are presented in Chapter 6, which is followed by the appendices, containing all the tables of results that have not been included in the main write-up. Lastly, the references with regard to prior research are listed.

2. Literature Review

2.1. Modern Portfolio Theory

The foundations of an optimal model of asset allocation stem from modern portfolio theory. The aim of this approach is to adjust a portfolio by using risk and return until an optimal level is reached. The basis of this theory is discussed in a paper called “Portfolio Selection” (Markowitz, 1952), which uses returns, standard deviation for risk, and covariance to construct an optimal solution for a portfolio. Other material explaining the basic methods and processes can be found in a number of books and papers, for example, Ingersoll (1987) and Litterman (2003). Other papers and theories followed the first work of portfolio construction by Harry Markowitz, effectively bringing modern portfolio theory to the forefront of finance.

> “While modern portfolio theory may be dated from 1952, it did not stir a great visible interest until after the works of Tobin (1958), Sharpe (1963, 1964), and Linter (1965)” (Markowitz, 2000: 38).

There is much detail on how investors treat risk from a liquidity preference angle. Tobin’s paper discusses the choice of splitting wealth between a group of risky assets and non-risk or low-risk assets (Tobin, 1958).

Sharpe (1963) creates a model that uses fewer parameters to calculate similar results to those produced by using Markowitz’s technique.

Asset price information, valuations and selections with capital budgets were also reviewed around this time (Sharpe, 1964) and (Linter, 1965).

This theory is primarily concerned with risk and return. The assumption is that, for a certain level of risk, an investor will want the highest possible return, or that, for a certain level of return, an investor will want the lowest possible risk. Risk is measured by either standard deviation or variance (Markowitz, 1952). This mean-variance framework has become popular because it can be used either to calculate the combined maximum returns for a group of assets when holding risk at certain levels, or to minimise the risk for a group of assets for differing levels of returns.

Repeating this process for different levels of either risk or return can produce an efficient frontier. This line shows the various maximum returns for the corresponding risk levels. This technique was developed by Markowitz (1952).

2.2. Multi-period Analysis

Although this theory is regarded as sound, its implementation is problematic. The mean-variance framework proposed by Markowitz (1952) only considers a single period investment. General theory states that a higher proportion of riskier assets should be held when an investor is young, with this holding being reduced when the investor approaches retirement. This theory is also used in practice in the financial industry. However, implementing Markowitz's mean-variance framework means that a general weighting of each asset will be calculated for all types of investors and then held in the future. As this method does not include a time horizon, multi-period frameworks were researched.

A transition model is introduced to extend an existing model methodology for portfolio selection to an intertemporal basis (Smith, 1967). Smith uses Markowitz's mean-variance model but changes it to include future expectations. The investor only moves from one portfolio to the other if the expected dollar return on the revised portfolio is larger than the dollar cost of the transition.

Another method uses the single period model but it includes two constraints to ascertain the effect of the transfer costs on revising the portfolio (Chen, Jen, and Zoints, 1971). Therefore, in each new period a single period Markowitz model containing the constraints is used. If the new allocation after transfer costs would result in higher returns, then the investor would change the weightings.

Mossin (1968) argues that, if a decision is made based on a single period model, then each period will be treated as if it were the investor's last period with myopic results. Mossin attempts to correct this disregard for the future. The study uses two assets: a riskless or non-risky asset with constant return and a risky asset with changing returns but a modicum of foresight to explain its method.

In 1969, research was undertaken, using a continuous-time model, where the income, generated by the returns or growth rates, is stochastic (Merton, 1969). A Weiner Brownian-motion process generated these rates of return.

Also in 1969 another accompanying paper uses a similar model for more general probability distributions (Samuelson, 1969).

Fama (1970) uses a consumption-investment model, in which an investor must choose how much to invest and how much to consume. This is done at the beginning of each period until the investor dies. The objective of the model is to maximise the expected utility of consumption over the investor's lifetime.

Hakansson (1971) uses arithmetic means to optimise asset allocation, resulting in a solution that does not lie on the efficient frontier. This finding makes Markowitz's model irrelevant and disproves its accuracy.

However, in a later paper Elton and Gruber (1974) find that, if the returns are log normally distributed and a geometric mean is used, then the maximisation will lie on the efficient frontier, thus making Markowitz's model useful for certain multi-period analyses.

Winkler and Barry (1975) develop a Bayesian model, which includes new information and uses it to determine when an investor revises his portfolio. This "learning effect", which is based on new information, works for single period and multi-period analyses.

Dumas and Luciano (1991) examine a different investment-consumption model, where the investor does not consume during the entire period of investment, but only when the investment is terminated and cashed in, right at the end. The objective is to maximise the utility at this terminal point.

Grauer and Hakansson (1993) compare two methods of calculating an optimal portfolio, namely, the mean-variance method and the quadratic approximations method. These are tested against a dynamic reinvestment model. The results show that, when a portfolio is revised quarterly, both methods work well. If the portfolio is only reviewed once a year, however, the results differ. Using the annual approximation method, the results vary by more than the mean-variance technique, and thus the mean-variance is found to be superior.

Li and Ng (2000) were the first to evaluate the analytical or numerical method of working out the efficient frontier for a multi-period mean-variance model. An algorithm for creating an optimal portfolio to maximise a terminal wealth utility function was also proposed.

Other papers have tried to combine asset allocation techniques with saving strategies to build a comprehensive model. Many different logarithms have been tested. Sun, Xuand and Fang (2006) attempted to use a Quantum-Behaved Particle Swarm Algorithm as the objective function in the model. Particle Swarm Optimisation aims to simulate social behaviour. The research found that the Quantum-Behaved Particle Swarm Algorithm is a promising technique for solving multi-period optimisation problems.

In general, research uses continuously compounded returns instead of the normal gross returns when dealing with the so-called multi-period problem.

3. Theoretical Framework

3.1. Introduction and Objective of the Mean-Variance Framework

This chapter is a review of the techniques used in this paper.

> "Asset Allocation is the term used to describe the set of weights of broad classes of investments within a portfolio" (Stewart, Piros and Heisler, 2011: 51).

> "Asset allocation techniques represent tools that help professionals set the optimal mix between broad classes of investments" (Stewart, Piros and Heisler, 2011: 52).

These techniques can help to solve investment strategies covering individual investors to pension funds.

> "Asset classes are typically defined as groups of securities with similar characteristics" (Stewart, Piros and Heisler, 2011: 52).

This generally means that assets within a class have high correlations to one another, while different asset classes display lower correlations. A popular technique of asset allocation is Markowitz's mean-variance framework. This model uses risk, return and other preferences in order to compute different asset weights. The framework does make some assumptions. The first two assumptions presume that investors are wealth maximising, while also being risk averse. Another assumption is that returns are normally distributed, or they can be converted into such a distribution. Often logarithmic returns are used for this purpose. These normally distributed returns are also assumed to be uncorrelated for each sub-period over time. The last assumption states that the statistical parameters and preferences are stable.

From the mean-variance framework an investor can estimate future wealth, obtain confidence intervals around this estimate, and calculate downside probabilities. An efficient frontier can also be plotted using different risk levels and their associated maximum returns. Combining this with a risk aversion coefficient, an optimal asset allocation level can be found for individual investors. Other constraints can also be included in this framework to meet the goals of individuals. These can include a long-only constraint, and varying time horizons and weightings on different asset classes to comply with regulations governing the industry.

This review will examine the theory of the mean-variance framework. Essentially, it is assumed that all investors want to maximise their wealth. In economic terms, investors want to maximise the expected utility of wealth, which is expressed below:

$$\text{Max } E[U(W)] \tag{3.1}$$

Where:

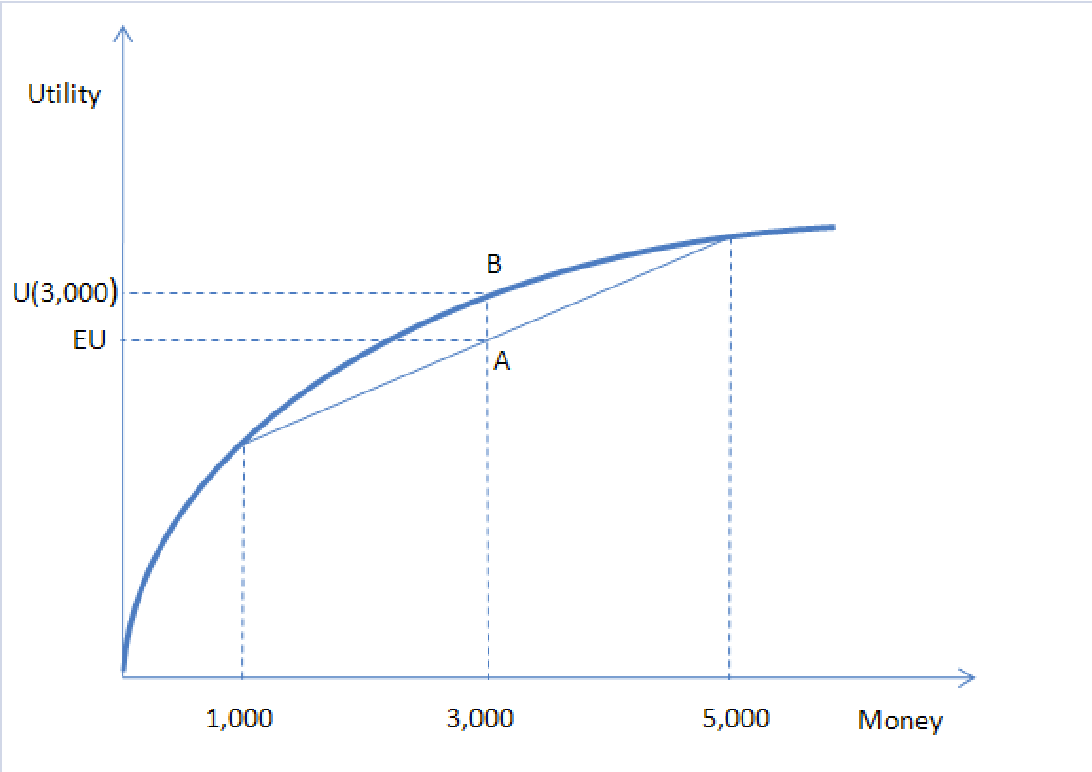
E = expected value

U = utility

W = wealth

This is the first assumption of the mean-variance framework. The next assumption, with regard to risk aversion, relates to the slope of the utility curve increasing at a decreasing rate. This means that an investor will suffer more from a decrease in wealth than he will experience joy from an equal increase in wealth. The graph below illustrates this:

Figure 3.1: Utility Curve of Wealth



The slope reflects the investor's marginal utility. According to the graph, the marginal utility decreases, while wealth increases. This means that an investor experiences diminishing benefits, the wealthier he becomes.

Risk aversion, too, can be seen in the above graph. For an investment where there is a 100% certainty of receiving R3,000, the investor will gain a utility of U, which is reflected at point B. Alternatively, if an investor has a 50% chance of making R1,000 and a 50% of making R5,000, they will only gain a utility of EU, labelled as A on the graph. The curvature of the utility curve thus reveals an investor's risk aversion. Relative risk aversion (RRA) is one of the most useful measures of aversion:

$$RRA = \frac{-\%\text{ change in marginal utility}}{\%\text{ change in wealth}} \qquad (3.2)$$

As discussed above, marginal utility decreases as wealth increases, and the negative sign in the equation reflects this. One of the most useful and widely used utility functions is called the constant relative risk aversion (CRRA) utility function:

$$U(W) = W^Y/Y \quad \text{where } Y<1 \tag{3.3}$$

Where:

Y = risk aversion parameter

Y is typically assumed to be negative. The inputs of this power utility function can be used for any quantity, such as life duration, quality of life and money. Therefore, CRRA is used to account for risk aversion in this paper. A more in-depth explanation is needed to understand how the utility function works. To do this, the first and second derivative of the utility function is needed. RRA and CRRA can also be expressed as:

$$RRA = -W\,[U''(W)] / [U'(W)] \tag{3.4}$$

Where:

U' = the first derivative of the utility function

U'' = the second derivative of the utility function

Therefore, the second derivative is divided by the first derivative and then multiplied by the negative amount of wealth. -[U''(W)] / [U'(W)] is also called the concavity index. The first and second derivatives can be found from Equation 3.3, and they are:

$$U'(W) = W^{Y-1} \quad \text{where } Y<1 \tag{3.5}$$

Therefore $U'(W) > 0$

$$U''(W) = (Y-1)W^{(Y-2)} \quad \text{where } Y<1 \tag{3.6}$$

Therefore $U''(W) < 0$

The first derivative is therefore a positive function and can be explained as the marginal change in wealth. An increase in wealth relates to an increase in utility. The second derivative is always negative and is thus a concave function. This function relates to the marginal change in utility. When risk aversion increases, the function becomes more concave. When these equations are entered into the RRA formula, it results in an investor's utility increasing at a decreasing rate when wealth increases. This is the foundation of risk aversion used in the paper.

3.1.1. Returns

An investor can only ever know the past returns of an asset, but he will not be able to determine what exactly the future returns will be. Nonetheless, the assumption of normally distributed returns is used to find expected returns. In the literature reviewed in Chapter 2, it was shown that logarithmic returns tend to follow normal distribution more closely than arithmetic returns do. The continuously compounded return of an asset is the logarithmic ratio of prices at the beginning and end of the period. The equation is thus as follows:

$$X_i(t) = \ln[P_i(t) / P_i(t-1)] \quad (3.7)$$

Where:

$X_i(t)$ = the logarithmic return of an asset

$P_i(t)$ = the price of an asset at time t

$P_i(t-1)$ = the price of an asset during the period before time t

Using the assumption that logarithmic returns during the investment horizon are normally distributed, and that they thus have a relationship, this is stated below:

$$P_i(t) = P_i(t-1)\exp[X_i(t)] \quad (3.8)$$

Therefore $P_i(t) = P_i(t-1)\exp[\ln[P_i(t) / P_i(t-1)]]$

Where:

$\exp[X_i(t)]$ = the exponential return is normally distributed with

Mean = α_i

Variance = σ_i^2

With logarithmic numbers, it is essential to understand an important relationship:

$$\mu_I = \ln(E[\exp(X_i(t))]) \quad (3.9)$$

Where:

μ_I = Alpha

Here the mean return is found first and then that figure is logged to get μ_l.

$$\alpha_l = E(\ln[\exp(X_i(t))]) \quad (3.10)$$

Where:

α_l = Mu

In this instance, the returns are logged and then the mean is found. This can be simplified to:

$$\alpha_l = E(X_i(t)) \quad (3.11)$$

but $\mu_l \neq \alpha_l$

$\mu_l > \alpha_l$

The log of the expected value does not equal the expected log value. Actually, the log of the expected value is always greater than the expected value of the log. But an approximate relationship between the two does exist.

$$\mu_i = \alpha_i + ½ \sigma_i^2 \quad (3.12)$$

Arithmetic returns are generally used for simple single period analysis, while continuously compounded returns are better for multi-period analysis, as the cumulative return is the sum of the percentage returns in each period. Multi-period analysis is thus used in this paper, and a comparison between continuously compounded returns and arithmetic returns will be conducted.

Thus the cumulative compounded return over the investment horizon is:

$$R_i(T) = \Sigma_t \ln[P_i(t)/P_i(t-1)] \quad (3.13)$$

$$= \Sigma_t X_i(t)$$

And the sum of the mean equals the mean, multiplied by the time horizon:

$$\Sigma_t \alpha_i = \alpha_i T \quad (3.14)$$

3.1.2. Return Variance

In most cases, the variance of a sum is not equal to the sum of the variance. In this paper, the covariances are zero, therefore the sum of the variance equals the sum of the variance over time. This is because of the assumption that the normal distributions of assets for each sub-period are uncorrelated over time, thus:

$$\Sigma_t \sigma_i^2 = \sigma_i^2 T \quad (3.15)$$

3.1.3. Portfolio Return and Variance

Using normal gross returns, the portfolio return is simply the weighted average of all the gross returns of the assets. However, the sum of the logarithms is not equal to the logarithm of the sum. Adjusting for some volatility, an equation can approximate the return:

$$R_W(T) = \Sigma_i \omega_i R_i(T) + ½\, \Sigma_i \omega_i \sigma_i^2 T - ½\, \sigma_W^2 T \quad (3.16)$$

Where:

W = Wealth

ω_I = weights of each asset

T = time horizon

The first term is the weighted average of the returns, and the second and third terms adjust for volatility. The relationship between the first two terms was shown in Equation (3.12). Equation (3.16) is slightly changed by using the mean to explain the returns of the assets and thus, to obtain:

$$\Sigma_i \omega_i \alpha_i T + ½\, \Sigma_i \omega_i \sigma_i^2 T - ½\, \sigma_W^2 T = (\mu_W - ½\, \sigma^2{}_W) T \quad (3.17)$$

Where:

$$\mu_W = \Sigma_i \omega_i (\alpha_i + ½\, \sigma_i^2) = \Sigma_i \omega_i \mu_i \quad (3.18)$$

The variance of the portfolio must take into account the variance of each asset and the covariance between each pair of assets:

$$\sigma_W^2 T = [\Sigma_i \Sigma_j \omega_i \omega_j \sigma_{ij}] T \quad (3.19)$$

3.1.4. Objective Function

To begin an investor puts down money now and it will grow to

$$\exp[R_W(T)] \qquad (3.20)$$

at the end of the investment period. If the CRRA from Equation (3.3) is substituted into Equation (3.20), then an investor with this utility will choose the following asset weightings to maximise his criteria:

$$E\{(1/Y)\exp[YR_W(T)]\} \qquad (3.21)$$

$R_W(T)$ exhibits a normal distribution and thus the bracketed term in Equation (3.20) has a log normal distribution; the mean of this can be expressed by:

$$(1/Y)\exp[Y(\mu_W T - ½\,(1-Y)\sigma^2_W T)] \qquad (3.22)$$

Maximising Equation (3.21) is similar to maximising:

$$(\mu_W - \lambda\sigma^2_W)T = (\mu_W - ½\,(1-Y)\sigma^2_W)T \qquad (3.23)$$

Where:

λ = the coefficient of risk aversion

$\lambda = ½\,(1-Y)$

Equation (3.23) is the objective function of the mean-variance model. The ultimate goal is to maximise this equation and thus to maximise wealth and the investor's utility. In this equation, μ_W is not the expected continuous compounded return but rather the log of the expected gross return.

3.1.5. Constraints

When optimising a portfolio, a number of constraints can be added to meet the goals and strategies of specific investors. When optimising without constraints, the calculated weightings may be unrealistic, or they may differ from an investor's wants, and thus a budget constraint can be introduced. By using such a constraint, the sum of the weightings of all the assets can be made to equal 100% or $\Sigma_i\omega_i = 1$. If an investor wants to be leveraged, for instance, the weightings can be adjusted to equal a different percentage, for example $\Sigma_i\omega_i = 1,5$, which means that they are thus 50% leveraged. Another constraint that can be added is the long-only constraint. This will stop the weightings of any assets being negative or stop the short selling of the assets. This constraint can be written as $\omega_I >= 0$. A shortfall constraint can also be included. This is done if an investor wants to limit the probability of earning below a certain threshold level. The shortfall constraint is given by:

$$N(H,(\mu_W - ½\,\sigma^2_W)T,\ \sigma^2_W T) <= K \qquad (3.24)$$

Where:

N = the assumption of normal distribution

H = the shortfall threshold level

K = the probability of a shortfall

The assumption made earlier, namely, that continuously compounded returns are normally distributed, is denoted by N. H is the threshold level and K is the probability the investor wants to limit.

3.1.6. Investment Horizon

It is often said that risky assets, such as stocks, have less risk over a long investment period. This implies that a younger investor should allocate more risky assets to their portfolio than an older investor should. Although this argument is probably correct, the justification for it is often misunderstood. We have shown earlier how a simple mean-variance model can be derived. The mean of the portfolio is proportional to the time horizon of the investment. Under an assumption of this framework, the continuously compounded returns of the assets are normally distributed in each period and are uncorrelated over time. This uncorrelated assumption implies that variance of the portfolio is likewise proportional to the time horizon (see Equation (3.15)). The objective function derived earlier (Equation (3.23)) is also proportional to the time horizon. This makes the time horizon a scale factor, which means that it can be eliminated without changing anything else. It also means that the length of an investment will not change the optimal asset allocation.

Thus, the basic mean-variance framework does not support the argument that young investors should be investing in riskier assets.

To be more realistic, we should thus change the model from cumulative returns to per period returns. As log returns were used for cumulative returns, we can just divide by the total periods of the time horizon. Thus, Equation (3.17) becomes:

$$(1/T)\,(\mu_W - \tfrac{1}{2}\,\sigma^2_W)T \quad = \mu_W - \tfrac{1}{2}\,\sigma^2_W \tag{3.25}$$

And variance from Equation (3.15) becomes:

$$(1/T)^2(\sigma_W{}^2T) = \sigma_W{}^2/T \tag{3.26}$$

Therefore, the expected returns per period remain the same, but the risk decreases with time. What is actually happening here is that risk is increasing over time, but this risk is distributed over the various periods. Furthermore, risk increases at a rate of T, but this is spread out or decreases at a factor of $(1/T)^2$. The mean-variance objective function is then also changed to per period returns and variance:

$$\mu_W - \tfrac{1}{2}\,(1\text{-}Y/T)\sigma^2_W \tag{3.27}$$

The length of the time horizon also affects investor strategy. The only difference between Equation (3.23) and Equation (3.27) is that Y is divided by T. This equation thus states that, increasing the time horizon of an investment will decrease the risk aversion of the investor. This is the real reason why younger people should invest more in riskier assets and not because investing in risky assets over a long period is less risky. The shortfall constraint can also be changed to reflect per period returns and variance. Therefore, Equation (3.24) can be written as:

$$N[H\text{-}(\mu_W - \tfrac{1}{2}\,\sigma^2_W)T/(\sigma^2_W\sqrt{T})] <= K \tag{3.28}$$

This equation implies that the continuously compounded returns are normally distributed, and denoted by N. H is the threshold return level required by the investor. K is the probability limit of the shortfall.

4. Methodology

4.1. The Data and Descriptive Statistics

To research the problem of asset allocation, this paper uses the returns from ten different South African and international asset classes, namely: the South African All Bond Index (ALBI), the South African Top 40 Shares Index (ALSI40), the South African Top 30 Companies from Financial, Basic Industrials or General Industrial Sectors Index (FINDI), Barclays' South African Government Inflation Linked All Maturities Index (ILB), Salient's Momentum Active Index Fund (MOME), MSCI World Equity Index (MSCI), the South African Resources Sector Index (RESI), the South African Property Index (SAPY), South African Short Term Fixed Interest Index (STEFI), which is used to approximate money market instruments and, finally, Salient's Value Active Index Fund (VAL). Monthly returns were gathered for each asset class from January 2003 until July 2012. This is the period of time where there is available data for all the indices in this paper. The data needed for the mean-variance framework included mean returns and standard deviations for each asset, a correlation matrix and a covariance matrix.

Two separate models were used (viz. logarithmic and arithmetic), and thus different sets of data were needed.

4.1.1. Logarithmic Model

The first model, as explained in Chapter 3, uses logarithmic returns. It is summarised in Table 4.1.1 below. Starting from the top of Table 4.1.1, the first line of the results is Mu (μ), which, simply put, is the log of expected returns. From Equation (3.9):

$$\mu_I = \ln(E[\exp(X_i(t))]) \quad (4.1)$$

The second term, Alpha (α), refers to the expected returns of the log returns. From Equation (3.10):

$$\alpha_I = E(\ln[\exp(X_i(t))]) \quad (4.2)$$

Table 4.1.1: Logarithm Descriptive Statistics

	ALBI	ALSI40	FINDI	ILB	MOME	MCSI	RESI	SAPY	STEFI	VAL
μ_{annual}	0.10	0.15	0.18	0.10	0.23	0.05	0.09	0.25	0.08	0.22
α_{annual}	0.10	0.13	0.17	0.10	0.21	0.04	0.06	0.24	0.08	0.21
$\sigma_{L\ annual}$	0.06	0.18	0.16	0.04	0.20	0.13	0.27	0.17	0.01	0.16
Sharpe Ratio	0.36	0.36	0.60	0.41	0.71	-0.21	0.04	1.01	0.00	0.88

Correlation

ALBI	1.00	0.04	0.31	0.03	0.10	-0.19	-0.13	0.64	0.07	0.33
ALSI40	0.04	1.00	0.79	-0.04	0.82	0.49	0.92	0.28	-0.22	0.75
FINDI	0.31	0.79	1.00	-0.06	0.76	0.35	0.49	0.50	-0.23	0.90
ILB	0.03	-0.04	-0.06	1.00	-0.06	-0.09	-0.04	-0.06	-0.15	-0.07
MOME	0.10	0.82	0.76	-0.06	1.00	0.33	0.67	0.34	-0.22	0.74
MSCI	-0.19	0.49	0.35	-0.09	0.33	1.00	0.48	-0.02	-0.22	0.28
RESI	-0.13	0.92	0.49	-0.04	0.67	0.48	1.00	0.09	-0.16	0.50
SAPY	0.64	0.28	0.50	-0.06	0.34	-0.02	0.09	1.00	-0.07	0.55
STEFI	0.07	-0.22	-0.23	-0.15	-0.22	-0.22	-0.16	-0.07	1.00	-0.12
VAL	0.33	0.75	0.90	-0.07	0.74	0.28	0.50	0.55	-0.12	1.00

Legend

ALBI	All Bond Index
ALSI40	Top 40 Share Index
FINDI	Financial Index
ILB	Inflation Linked Bonds
MOME	Momentum Index
MSCI	World Equity Index
RESI	Resource Index
SAPY	Property Index
STEFI	Fixed Interest Index
VAL	Value Index

Covariance

ALBI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
ALSI40	0.00	0.03	0.02	0.00	0.03	0.01	0.05	0.01	0.00	0.02
FINDI	0.00	0.02	0.03	0.00	0.03	0.01	0.02	0.01	0.00	0.02
ILB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MOME	0.00	0.03	0.03	0.00	0.04	0.01	0.04	0.01	0.00	0.02
MSCI	0.00	0.01	0.01	0.00	0.01	0.02	0.02	0.00	0.00	0.01
RESI	0.00	0.05	0.02	0.00	0.04	0.02	0.07	0.00	0.00	0.02
SAPY	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.03	0.00	0.01
STEFI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VAL	0.00	0.02	0.02	0.00	0.02	0.01	0.02	0.01	0.00	0.03

The third and fourth lines of the Table are Standard Deviation (σ_L) and the Sharpe Ratio. These numbers will be explained in more detail below, in the following Section. With regard to Mu, it can be seen that SAPY had the highest average returns over the period with 25%. This is followed by the two active equity index funds, MOME and VAL, with 23% and 22% respectively. Both the bond indices ALBI and ILB had returns of 10%. Cash (STEFI) had returns of 8% and the lowest performer was MCSI over the period with only 5%.

However, although MOME has the second highest return, the associated risk or Standard Deviation is the second highest too, and this results in a Sharpe Ratio of 0.71. In contrast, VAL has a higher Sharpe Ratio of 0.88, even though its return is lower. SAPY was the most attractive property index over the period. It has the highest Sharpe Ratio coupled with the highest return. RESI had by far the highest Standard Deviation over the period, along with mediocre returns. STEFI is the benchmark risk free return and thus its Sharpe Ratio is zero. MSCI, whose return was lower than STEFI, is therefore Sharpe Ratio negative.

The next section of Table 4.1.1 shows the correlations: ALSI, FINDI, MOME, RESI and VAL are all highly correlated with each other. This is expected, as they are all equity indices. STEFI is only slightly positively correlated to ALBI and negatively correlated with the other assets. ILB has the same relationship with the other indices. This could confirm the general finding in the industry, that cash and bonds are negatively correlated to interest rates for this period.

4.1.2. Arithmetic Model

The next set of data results, presented in Table 4.1.2 below, is used to optimise the maximum growth rate. This model, although very similar to the one before, used arithmetic numbers instead of logarithmic returns. The resulting figures are close to the logarithmic model.

Table 4.1.2: Arithmetic Descriptive Statistics

	ALBI	ALSI40	FINDI	ILB	MOME	MCSI	RESI	SAPY	STEFI	VAL
$E(R)_{annual}$	0.10	0.15	0.18	0.10	0.23	0.05	0.09	0.25	0.08	0.23
g_{annual}	0.10	0.13	0.17	0.10	0.21	0.04	0.06	0.24	0.08	0.21
$\sigma_{A\ annual}$	0.06	0.18	0.16	0.04	0.20	0.13	0.26	0.17	0.01	0.16
Sharpe Ratio	0.33	0.27	0.52	0.39	0.62	-0.28	-0.09	0.93	0.00	0.80

Correlation

ALBI	1.00	0.04	0.31	0.03	0.10	-0.20	-0.12	0.64	0.07	0.33
ALSI40	0.04	1.00	0.80	-0.04	0.81	0.48	0.92	0.28	-0.20	0.75
FINDI	0.31	0.80	1.00	-0.06	0.77	0.34	0.51	0.49	-0.22	0.90
ILB	0.03	-0.04	-0.06	1.00	-0.06	-0.09	-0.03	-0.06	-0.15	-0.07
MOME	0.10	0.81	0.77	-0.06	1.00	0.32	0.66	0.34	-0.20	0.74
MSCI	-0.20	0.48	0.34	-0.09	0.32	1.00	0.47	-0.02	-0.22	0.27
RESI	-0.12	0.92	0.51	-0.03	0.66	0.47	1.00	0.09	-0.14	0.51
SAPY	0.64	0.28	0.49	-0.06	0.34	-0.02	0.09	1.00	-0.06	0.54
STEFI	0.07	-0.20	-0.22	-0.15	-0.20	-0.22	-0.14	-0.06	1.00	-0.11
VAL	0.33	0.75	0.90	-0.07	0.74	0.27	0.51	0.54	-0.11	1.00

Legend

ALBI	All Bond Index
ALSI40	Top 40 Share Index
FINDI	Financial Index
ILB	Inflation Linked Bonds
MOME	Momentum Index
MSCI	World Equity Index
RESI	Resource Index
SAPY	Property Index
STEFI	Fixed Interest Index
VAL	Value Index

Covariance

ALBI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
ALSI40	0.00	0.03	0.02	0.00	0.03	0.01	0.04	0.01	0.00	0.02
FINDI	0.00	0.02	0.03	0.00	0.03	0.01	0.02	0.01	0.00	0.02
ILB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MOME	0.00	0.03	0.03	0.00	0.04	0.01	0.04	0.01	0.00	0.02
MSCI	0.00	0.01	0.01	0.00	0.01	0.02	0.02	0.00	0.00	0.01
RESI	0.00	0.04	0.02	0.00	0.04	0.02	0.07	0.00	0.00	0.02
SAPY	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.03	0.00	0.02
STEFI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VAL	0.00	0.02	0.02	0.00	0.02	0.01	0.02	0.02	0.00	0.03

In Table 4.1.2 below the first line refers to Expected Returns (E(R)), which are average returns and are similar to Mu, with the difference being that E(R) is not logged.

$$E(R)_I = (E(X_i(t))) \qquad (4.3)$$

Where:

$E(R)_I$ = expected arithmetic returns

Growth rate (G) is E(R) minus half variance; this is thus similar to Alpha from the logarithmic model.

$$G = E(R) - \frac{1}{2}\sigma_A^2 \qquad (4.4)$$

Where:

G = the growth rate

σ_A^2 = the arithmetic variance

The Standard Deviation (σ_A) and the Sharpe Ratio will also be explained in more detail in the next section. Although these four lines in Table 4.1.2 are very similar to the first four lines of Table 4.1.1, the figures differ slightly. SAPY has the highest return and the highest Sharpe Ratio. MOME and VAL have the same returns as in Table 4.1.1 but VAL still has a lower Standard Deviation and thus a better Sharpe Ratio. RESI once again has the highest Standard Deviation. The two bond indices ALBI and ILB have the same returns as in Table 4.1.1, but ILB has lower associated risk as before. STEFI has a zero Sharpe Ratio because it has risk free returns, and MSCI is still the worst performer with a negative Sharpe Ratio.

The next section explains the differences between the logarithmic and the arithmetic models. It examines the step-by-step process of setting them up, by considering the relevant formulae and how the two models are used to calculate the resulting portfolios.

4.2. Basic Modelling

In the section, a mean-variance model is built up step-by-step for an arithmetic framework and a continuously compounded framework in turn. The main differences between the two models that will be demonstrated relate to the types of input returns.

4.2.1. Arithmetic Returns

Arithmetic returns are also called gross returns. These returns are the percentage increases in price from one period to the next. This is calculated by:

$$\text{Returns} = (P_1 - P_0)/P_0 \quad (4.5)$$

Where:

P_1 = the price of an asset at time 1

P_0 = the price of an asset in the previous period

From here, monthly-expected returns (E(R)) are calculated by finding the average or mean of the returns, while the monthly Standard Deviations are computed using the formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2} \quad (4.6)$$

These figures are then changed to reflect the annual expected returns and Standard Deviation. These figures are needed, as the objective function of this optimisation problem is the growth rate (G). The relationship between these three terms can be written as:

$$G = E(R) - \tfrac{1}{2}\,\sigma_A^2 \quad (4.7)$$

From Equation (4.7), then, it can be seen that it is more appropriate to maximise the growth rate than to maximise expected returns: focusing on maximising the latter may lead an investor to choose assets with the highest returns but also bearing the highest risks. Growth rate penalises risk by subtracting half the variance, and it thus reaches a more optimal solution.

4.2.2. Continuously Compounding Returns

Research has shown that continuously compounding returns are a better predictor of earnings, as continuously compounding returns follow normal distribution better than arithmetic returns. They also fit the normal distribution assumption in the mean-variance framework better. Continuously compounded returns can be calculated in two different ways, as discussed in the previous section. This is either by finding Alpha (the expected returns of the log) or by calculating Mu (the log of the expected returns). The log return is calculated by using the formula:

$$=\ln(1+\text{Return}). \qquad (4.8)$$

Equation (3.12) shows the relationship between Alpha and Mu:

$$\alpha_i + ½\,\sigma_i^2 = \mu_i \qquad (4.9)$$

To test this relationship, a small random sample was taken to ascertain, firstly, if Mu is always greater than Alpha and, secondly, if adding half variance brings Alpha closer to Mu. This is discussed in Chapter 5 below.

This equation can be rearranged to resemble the similar equation for the growth rate:

$$\alpha_i = \mu_i - ½\,\sigma_{Li}^2 \qquad (4.10)$$

$$G_i = E(R)_i - ½\,\sigma_{Ai}^2 \qquad (4.11)$$

It can be seen that half variance is subtracted from Mu to get Alpha, which is thus the equivalent to the growth rate when logarithms are used, just as Mu is similar to expected returns. Since Alpha is the objective function that maximises continuously compounded returns, the expected returns of the logs can be calculated. This is done by logging the monthly returns first and then finding the monthly mean. The monthly standard deviations are found using the logged returns. These figures are then annualised. Monthly and annual Mu can be found by using the formula above.

4.2.3. Correlation and Covariance Matrices

Herein, the correlation matrices are computed, for both models. From this, the covariance matrices are created, using the formula:

$$COV_{ij} = \rho_{ij}\sigma_i\sigma_j \quad (4.12)$$

These figures are then transferred to the model in order to set up the optimisation framework for both variations of the model.

4.2.4. Arithmetic Portfolio Returns

First, a hypothetical portfolio is created, where each asset class has equal weightings (ω = 10% each). The expected returns and the growth rate of the portfolio are calculated using the formulae:

$$G_W = \Sigma_i \omega_i G_i \quad (4.13)$$

$$E(R)_W = \Sigma_i \omega_i \, E(R)_i \quad (4.14)$$

4.2.5. Continuously Compounded Returns

Thereafter, Alpha and Mu are calculated in a similar manner on the other spreadsheet. The formulae used are the same as those in Equation (3.18):

$$\alpha_W = \Sigma_i \omega_i \alpha_i \quad (4.15)$$

$$\mu_W = \Sigma_i \omega_i \mu_i \quad (4.16)$$

4.2.6. Standard Deviation

Standard Deviations of the two portfolios are then calculated in the same way by means of Equation (3.19):

$$\sigma_{AW}^2 = [\Sigma_i \Sigma_j \omega_i \omega_j \sigma_{Aij}] \quad (4.17)$$

$$\sigma_{LW}^2 = [\Sigma_i \Sigma_j \omega_i \omega_j \sigma_{Lij}] \quad (4.18)$$

Where:

σ_A = the arithmetic standard deviation

σ_L = the logarithmic standard deviation

4.2.7. Sharpe Ratio

The Sharpe Ratio is also used in a variety of calculations to see which asset classes, or which combinations of asset classes, will result in the highest risk return per unit. The Sharpe Ratios used for both Arithmetic and Continuously Compounded Returns are as follows:

$$\text{Sharpe Ratio}_{art} = (G_W - r_{fA})/\sigma_{AW} \qquad (4.19)$$

$$\text{Sharpe Ratio}_{ccr} = (\alpha_W - r_{fL})/\sigma_{LW} \qquad (4.20)$$

Where:

r_{fA} = the arithmetic risk free return

r_{fL} = the logarithmic risk free return

Once this is accomplished, a user can find an optimal portfolio with no constraints. This function allows an investor to search for a portfolio that can maximise returns, minimise Standard Deviation, maximise the Sharpe Ratio, or find the highest return for each level of Standard Deviation or the lowest Standard Deviation for each level of return. This is similar to the basic mean-variance framework proposed by Markowitz (1952).

Two other ratios are included to add interest and insight. One is an Annualised Sharpe Ratio for facilitating multi-period analysis. This is necessary, as the Sharpe Ratio numerator and denominator do not increase at the same rate when time is increased. The mean increases directly with time but the Standard Deviation increases by the square root of time. Therefore, the formulae are as follows:

$$\text{Annualised Sharpe Ratio}_{art} = ((G_W/T)-(r_f/T))/(\sigma_{AW}/\sqrt{T}) \qquad (4.21)$$

$$\text{Annualised Sharpe Ratio}_{ccr} = ((\alpha_W/T)-(r_f/T))/(\sigma_{LW}/\sqrt{T}) \qquad (4.22)$$

4.2.8. Kelly Ratio

The Kelly Ratio is also used. This ratio is ideal, as it is similar to the Sharpe Ratio but the numerator and denominator increase at the same rate as time increases. This is because the Kelly Ratio's denominator is variance and not standard deviation.

Therefore, an Annualised Kelly Ratio is the same as the normal Kelly Ratio. The ratios are calculated as follows:

$$\text{Kelly Ratio}_{art} = (G_W - r_f)/\sigma_{AW}^{\;2} \quad (4.23)$$

$$\text{Kelly Ratio}_{ccr} = (\alpha_W - r_f)/\sigma_{LW}^{\;2} \quad (4.24)$$

4.3. Adding Risk Aversion and Other Constraints

Next, a new objective function can be introduced from Equation (3.23) and changed slightly for the arithmetic application.

4.3.1. Arithmetic Objective Function

$$(E(R)_W - \lambda\sigma^2_W) = (E(R)_W - \tfrac{1}{2}\,(1-Y)\sigma^2_{AW}) \quad (4.25)$$

Where $\lambda = \frac{1}{2}\,(1-Y)$

4.3.2. Continuously Compounded Objective Function

$$(\mu_W - \lambda\sigma^2_W) = (\mu_W - \tfrac{1}{2}\,(1-Y)\sigma^2_{LW}) \quad (4.26)$$

Where $\lambda = \frac{1}{2}\,(1-Y)$

By using this objective function, an investor's risk aversion is taken into account. As the risk aversion parameter (Y) becomes more risk averse (or more negative) μ_W is subtracted by a relatively higher amount of associated risk. Consequently, the optimisation is not necessarily seeking the highest returns, but the highest returns at an acceptable risk level for the investor. This kind of scenario is more aligned to real-life scenarios.

4.3.3. Constraints

Constraints may then need to be added to the model to adjust for an investor's strategies or to meet regulations. The first constraint is to make the amount invested equal to 100%:

$$\Sigma_i\omega_i = 1 \quad (4.27)$$

This amount can be changed if the investor wishes to implement other strategies. For instance, if an investor wants to leverage his investment by 50%, then the equation will equal 1.5. A long-only constraint can also be introduced to prevent short-selling of assets:

$$\omega_i >= 0 \qquad (4.28)$$

This constraint can also be relaxed to allow a limited amount of short-selling, in which case 0 would be changed to -0.1 for example, and thus allow 10% short selling. Another very important constraint is the shortfall constraint, introduced by Equation (3.24). The shortfall constraints of the arithmetic and logarithmic models respectively are as follows:

$$N(H,(E(R)_W - ½ \sigma^2_W)T, \sigma^2_{AW}T) <= K \qquad (4.29)$$

$$N(H,(\mu_W - ½ \sigma^2_W)T, \sigma^2_{LW}T) <= K \qquad (4.30)$$

By defining a threshold, this probability function, which uses normal distribution, effectively limits the amount an investor can lose at any time. Other constraints are included to comply with the Regulation 28 of the South African Pension Funds Act, and to ensure that the weightings found by the model will reflect reality. Regulation 28 states that equity must be less than or equal to 75% of the portfolio, that property must be less than or equal to 25%, that equity and property together must be less than or equal to 90%, that debt must be less than or equal to 75%, and that foreign assets must be less than or equal to 25% of the portfolio.

4.4. Multi-Period Analysis

Thus far, all the optimal portfolios that can be found relate to a single-period optimisation. This means that the solutions found only apply to the present moment, and that they do not consider the future. At this point, it is thus convenient to begin an analysis of asset allocation in a South African context.

4.4.1. Objective Functions

Finding the optimal portfolio for a multiple period problem is an important issue. The solution proposed in this paper sets out to find an optimal portfolio for the present, while also considering the future. Now we can take the formulae set out above, and apply them within a specific time horizon. An objective function of the arithmetic and logarithmic model can thus be derived from Equations (4.25) and (4.26); it becomes:

$$(E(R) - \lambda\sigma^2_{AW})T = (E(R) - ½\,(1\text{-}Y)\sigma^2_{AW})T \qquad (4.31)$$

Where $\lambda = ½\,(1\text{-}Y)$

$$(\mu_W - \lambda\sigma^2_{LW})T = (\mu_W - ½\,(1\text{-}Y)\sigma^2_{LW})T \qquad (4.32)$$

Where $\lambda = ½\,(1\text{-}Y)$

The equations for the shortfall constraints become:

$$N[H\text{-}(\,E(R) - ½\,\sigma^2_{AW})T/(\sigma^2_{AW}\sqrt{T})] <= K \qquad (4.33)$$

$$N[H\text{-}(\mu_W - ½\,\sigma^2_{LW})T/(\sigma^2_{LW}\sqrt{T})] <= K \qquad (4.34)$$

N in these equations denotes normal distribution; H is the shortfall threshold constraint required by the investor, and K is the probability of not violating the shortfall constraint. All the equations in this section allow an investor to find a more appropriate solution for asset allocation.

4.5. Overview of Optimiser Procedures

To begin with, the continuously compounded Alpha was maximised. This objective function simply finds the highest Alpha: it does not penalise risk by using risk aversion. Determining risk aversion is problematic: First, it must be decided which risk aversion equation should be used, and then the correct risk aversion number relating to an individual investor must be identified. For example, on a scale from 1 to 10, how risk averse is an investor? How can someone have a risk aversion of say 3, and be certain about that? Risk aversion is economic theory and difficult to apply to reality. Consequently, a shortfall constraint is used, instead of risk aversion. To a certain degree, the shortfall constraint minimises the probability of losing a specified amount over a given period.

For example, the tests began by constraining the minimum return of the investment to 0% with a probability of 95% over a time horizon of one year. These tests were run to determine differing time horizons, threshold levels and probabilities. The time horizon was changed in increments of five, from 1 to 30 years, maintaining the minimum return threshold at 0% and limiting the probability of the shortfall to 95%.

After that, more tests were conducted, changing the minimum return threshold constraint. These constraints were changed as follows: (1) there was no shortfall constraint; (2) the minimum return equalled cash return or STEFI return over the relevant period; (3) the minimum return equalled cash return plus 2%; (4) the minimum return equalled cash plus 4%; (5) and the minimum return equalled cash plus 6%.

It was then deemed necessary to find results incrementally every year close to retirement. Thus results were ultimately obtained for 30; 25; 20; 15; 10; 5; 4; 3; 2 and 1 year before retirement.

These different methods were then repeated for all the different years and minimum returns, as well as by changing the shortfall probability to 90% and 99%. Although the use of risk aversion has its shortcomings, it can also provide helpful insight into portfolio construction. Thus, Alpha was also maximised by using risk aversion and no shortfall constraint. The tests were redone again, using a shortfall constraint as well as risk aversion. This provided even more insight into the model. These are discussed in the following sections. Once this had been completed for the continuously compounded Alpha, some of the tests were repeated with the arithmetic growth rate. Although these numbers were not normally distributed, which has caused problems in prior research, the differing results are nonetheless interesting. The results obtained are presented and discussed in the next chapter.

5. Results

5.1. Introduction

In general, it was found from the various calculations done in this study that logarithmic returns are more appropriate than arithmetic returns for calculating future expected returns, as these are more normally distributed than their arithmetic counterparts are.

This chapter first considers the difference between Mu and Alpha, and a random small sample of returns is taken. From this, Mu and Alpha are calculated and the relationship between the two is tested (Section 5.2). Thereafter, the results from the actual study are analysed. Section 5.3 focuses on portfolios with associated risk aversion, with the intention of maximising Alpha using risk aversion. Section 5.4 moves away from risk aversion, and instead includes a shortfall threshold constraint; this is relevant when an investor wants his annual returns to be equal to or higher than a certain amount. Section 5.5 does not use logarithmic returns; instead, a comparison is made between the portfolios of logarithmic and arithmetic returns, using a shortfall constraint. Section 5.6 again uses logarithmic returns but combines risk aversion and a shortfall constraint. Lastly, in Section 5.7, the shortfalls of the model presented in this thesis are analysed, and other solutions and amendments to the mean-variance model are proposed.

5.2. Difference Between Mu and Alpha

Before discussing the results of this paper, it is important to consider the relationship between Mu and Alpha:

$$\alpha_i + \tfrac{1}{2}\,\sigma_i^2 = \mu_I \qquad (5.1)$$

We thus first checked whether Alpha is always less than Mu, and thereafter checked whether, by adding half variance to Alpha, the result would approach closer to Mu. The results were as follows:

Table 5.2.1: Sample Test of Alpha and Mu		
Periods	$P_i(t)/ P_i(t-1)]$	$\ln[P_i(t)/ P_i(t-1)]$
1	0.2	0.182321557
2	0.15	0.139761942
3	0.05	0.048790164
4	0.8	0.587786665
5	0.27	0.2390169
6	0.4	0.336472237
7	0.35	0.300104592
8	0.3	0.262364264
9	0.25	0.223143551
10	0.15	0.139761942
11	0.2	0.182321557
Average	0.28363636	0.240167761
$\mu_I = \ln(E[\exp(X_i(t))])$	0.24969696	
$\alpha_I = E(\ln[\exp(X_i(t))])$	0.24016776	
$\mu - \alpha$	**0.0095292**	
σ	0.14070185	
$\alpha + 1/2\sigma^2$	0.25006627	
$\mu - (\alpha + 1/2\sigma^2)$	**-0.0003693**	

The returns for each period were randomly allocated. The sample size started with 5 periods and then increased by one period in turn; the results were checked each time, until 11 periods had been reached. Thereafter, the logarithmic returns were calculated, based on the formula logarithmic returns = Ln(1 + Returns). Expected returns were found by averaging the normal returns. Mu (μ), as discussed in Chapter 4: Methodology, is the logarithmic expected returns, thus resulting in 0,24969696. Alpha (α) was calculated as discussed in Chapter 4 as the average of the logarithmic returns, and was found to be 0,24016776. Based on this, the Standard Deviation of the Ln returns was computed and found to equal 0,14070185.

The above relationship can be used to calculate Mu by using Alpha and variance. This resulted in a number equal to 0,25006627. Note that the log of the expected value is always greater than the expected value of the log or Mu and greater than

Alpha. The difference between Mu and Alpha was 0,0095292; more importantly, however, it was a positive number. From the sample, it could be seen that this number was always positive in periods 5 to 11. This is inline with the formula

Next, it was ascertained whether Mu actually equals Alpha plus half variance. The last number in the table is Mu – (Alpha + half variance). Theoretically, this should be close to zero, and indeed it was found to be -0,0003693. Although this relationship has brought Alpha closer to Mu, they are not equal, as this relationship is an approximation. Nonetheless, the difference did become smaller, as the sample size increased from 5 to 11 periods. A much larger sample may be needed to approximate the relationship better.

Thus we can conclude that the approximate relationship between Mu and Alpha does hold.

5.3. Maximising Alpha Using Risk Aversion

5.3.1. Introduction

People have their own risk levels, particularly when it comes to financial investments. Risk aversion is an economic term, and models associate different numbers with different risk levels. Risk aversion is derived from the concept of utility, which was explained in Section 3. The main challenge of using risk aversion lies in determining the correct number to explain a certain level of aversion to risk. The concept is easy to explain in general terms, but it is difficult to allocate a specific number to the actual level of risk aversion. A person who seeks risk, for example a gambler, will gain utility or a unit of enjoyment from taking a risk. A risk neutral person will be indifferent to two options, as long as the expected returns are equal. A risk averse person will choose the safer option between two choices with the same expected returns.

This paper looks at investing life savings or saving for retirement, and assumes that rational people are risk averse. There are vast differences between investing money in a pension fund and gambling some disposable income on Blackjack. In this study, therefore, a risk-seeking scenario can be safely eliminated. As discussed earlier, Alpha is maximised to find the correct portfolio for each situation, and Equation (4.26) is used:

$$\alpha_W = (\mu_W - ½\,(1\text{-}Y/T)\sigma^2_{LW}) \qquad (5.2)$$

Y ranges between 0 (the least risk averse) and -2 (the most risk averse situation in this paper). The last term of the equation is a penalty for risk. -2 penalises Alpha more than 0. *T* is the time until retirement. This paper uses *T* of 1,2,3,4,5,10,15,20,25,30. The formula shows that, for a given level of risk aversion, the associated risk is lower when there is a longer time to retirement. This is in line with the theory that a younger investor can accept more risk. Tables 5.3.1 to 5.3.5 below show the results.

5.3.2. Results

Table 5.3.1 shows the resulting portfolios when risk aversion is zero (*Y*=0) or when it is risk neutral. For all the periods in this table, the optimal portfolio consists of 10% ALBI, 25.02% MOME, 25% SAPY and 39.98% VAL. The reason why this portfolio has this asset allocation can be explained by the data:

SAPY (the property index) has the highest expected returns and the highest Sharpe Ratio. It is thus predictable for property to be invested to its highest limit. Regulation 28 only allows for 25% of a portfolio to be invested in property and therefore 25% of this portfolio is in SAPY.

The next three highest return assets also have the next three highest Sharpe Ratios. These assets are VAL, MOME and FINDI, and they are all equity assets. According to Regulation 28, the maximum level of equity that one can invest in a portfolio is 75%. But Regulation 28 also states that the combined amount of equity and property can only be 90%. With property invested to its maximum, only 65% remains for equity. The two best performing equity indices in terms of both their return and their Sharpe Ratio are VAL and MOME. The solution has thus allocated the 65% of equity to these two assets. FINDI has been excluded. Of the two equity indices (VAL and MOME), VAL has been weighted more heavily, and, although MOME has a slightly higher expected return, its Sharpe Ratio is lower than that of VAL. VAL is associated with less risk for a level of return, and therefore a larger percentage is allocated to it.

The last 10% of the portfolio can only be invested in bonds or cash, which could be ALBI, ILB or STEFI. STEFI has the lowest return and a Sharpe Ratio of 0. This would make investing in bonds more appropriate. The tables show that ALBI and ILB have the same returns, although ILB has a higher Sharpe Ratio. The portfolio should thus include ILB, but it in fact allocates 10% to ALBI. On closer inspection, it can be seen that ALBI has a higher return when the decimal points are increased: ALBI is slightly over .10 and ILB is slightly under .10. In this situation, with Y=0, the higher return outweighs the higher risk. Therefore, the model's choices with regard to the different indices are intuitively sound.

The column Mu shows the returns of each portfolio during the period assigned thereto. With one year to retirement, for instance, the portfolio has a return of 21.88%; with 2 years to retirement, it has a return of 43.76%, which is double that of the return of 1 year. This supports the theory that returns increase proportionally with time. Consequently, the return of 656.40% for 30 years until retirement is 30 times the size of 21.88%.

The Standard Deviation with one year until retirement is 13.62%; however, when the period is increased to two years, the risk increases to 19.26%. This is less than two times 13.62%. In fact, the Standard Deviation increased by the square root of two. This increase is correct, as discussed previously (see Section 3). Risk increases proportionally with the square root of time. The Standard Deviation for 30 years is 74.60%, which is the square root of 30 times 13.62%. The proportional increases of returns and Standard Deviations only occur when the portfolio's assets are allocated in the same percentages. This increase of returns proportional to time and risk proportional to the square root of time corroborates the theory that a younger investor can take on a riskier portfolio.

The shortfall probability is marked N/A for these results, as this constraint was not used in this particular scenario; it will be introduced in Section 5.4 below.

Table 5.3.1 also includes the Annualised Sharpe Ratio, because the normal Sharpe Ratio is not useful when compared over different periods. This is because the expected return in the numerator is increasing proportionally with time, whereas the Standard Deviation in the denominator is only increasing by the square root of time. The Annualised Sharpe Ratios correct this discrepancy.

The Kelly Ratio was added because the denominator is squared, thus also increasing risk proportionally with time. The Annualised Mu, Alpha, Standard Deviation, Sharpe Ratio and Kelly Ratio remain the same for all periods. This is the case because the portfolio does not change over the 30-year period.

In **Table 5.3.2**, the risk aversion is increased: *Y* is changed to -0.5. As a result, the portfolio split is 10.00% ALBI, 24.58% MOME, 25.00% SAPY and 40.52% VAL, for periods 30 to 15 years. For the rest of the periods, viz. 10 to 1 years, the allocation is 2.71% ALBI, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL.

The first change to note in Table 5.3.2 is the inclusion of ILB in the portfolio. Since the investor is now more wary of risk, the portfolio setup has changed. The added asset leads to an increase in diversification. As before, the choices can be intuitively explained. Now that the Alpha has a higher risk penalty, the addition of ILB helps to bolster the solution. As discussed before, ILB has a slightly lower return than the other bond index, viz. ALBI. However, the risk in the ILB is much lower, resulting in a better Sharpe Ratio. The ILB is also negatively correlated to 8 of the other 9 indices. These factors help to increase the Sharpe Ratio of the portfolio and to counter the risk aversion. From 30 years to 15 years until retirement, however, the addition of ILB to help mitigate risk is unnecessary, as risk is not as important for a younger investor who would probably prefer to chase the higher returns from ALBI anyway.

Table 5.3.1: Maximising Alpha Using Risk Aversion and no Shortfall Constraints

Historical data has been used from January 2003 to July 2012
Maximising μ_w-1/2(1-γ/T)σ^2_w (γ=0) (No Shortfall Threshold)

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	21.88%	20.47%	13.62%	1.01	21.88%	20.47%	13.62%	1.01	7.44
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.3.2: Maximising Alpha Using Risk Aversion and no Shortfall Constraints

Historical data has been used from January 2003 to July 2012
Maximising μ_w-1/2(1-γ/T)σ^2_w (γ=−0.5) (No Shortfall Threshold)

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	21.82%	20.46%	13.32%	1.03	21.82%	20.46%	13.32%	1.03	7.75
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	328.18%	307.03%	52.72%	3.93	21.88%	20.47%	13.61%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	437.58%	409.37%	60.87%	4.54	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	546.97%	511.71%	68.06%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.3.3: Maximising Alpha Using Risk Aversion and no Shortfall Constraints

Historical data has been used from January 2003 to July 2012
Maximising μ_w-1/2(1-γ/T)σ^2_w (γ=−1) (No Shortfall Threshold)

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	21.80%	20.46%	13.21%	1.04	21.80%	20.46%	13.21%	1.04	7.87
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Table 5.3.4: Maximising Alpha Using Risk Aversion and no Shortfall Constraints

Historical data has been used from January 2003 to July 2012
Maximising $\mu_w\text{-}1/2(1\text{-}\gamma/T)\sigma^2_w$ $(\gamma=-1.5)$ (No Shortfall Threshold)

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	21.80%	20.47%	13.20%	1.04	21.80%	20.47%	13.20%	1.04	7.88
2	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	43.60%	40.94%	18.66%	1.47	21.80%	20.47%	13.20%	1.04	7.88
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.3.5: Maximising Alpha Using Risk Aversion and no Shortfall Constraints

Historical data has been used from January 2003 to July 2012
Maximising $\mu_w\text{-}1/2(1\text{-}\gamma/T)\sigma^2_w$ $(\gamma=-2)$ (No Shortfall Threshold)

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	21.80%	20.47%	13.19%	1.04	21.80%	20.47%	13.19%	1.04	7.89
2	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	43.59%	40.95%	18.65%	1.47	21.80%	20.47%	13.19%	1.04	7.89
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	N/A	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

The equity allocation in the portfolio remains at 65%, which is the maximum allowed in terms of Regulation 28, when property is invested to the maximum. However, for periods 10 to 1 year before retirement, less is allocated to MOME and more to VAL, than had previously been the case in Table 5.3.1. This is because, with the added risk aversion, VAL's higher Sharpe Ratio and lower Standard Deviation has become more favourable to the investor. The proportionally higher risk MOME index, in contrast, is penalised more, even though MOME has a higher return than VAL. Nevertheless, when the time to retirement is higher, viz. from 15 to 30 years, the risk factor is less important. During those periods, investing more in MOME than in VAL allows the investor to chase the higher potential returns.

In each case, Mu, Alpha, and Standard Deviation in this table are lower than the corresponding results in Table 5.3.1. Generally, too, the corresponding Sharpe Ratio, Annual Sharpe Ratio and Kelly Ratios are higher in Table 5.3.2. The investor has thereby minimised the portfolio risk by more than the decrease in the expected return, as behoves a more risk averse investor.

In **Table 5.3.3**, the risk aversion has been increased once more, with the risk aversion coefficient now being *Y*=-1. The portfolio split is 10.00% ALBI, 21.61% MOME, 25.00% SAPY and 43.39% for VAL for the periods of 30 to 3 years until retirement. For the remaining two periods, the portfolio consists of 10.00% ILB, 17.11% MOME, 25.00% SAPY and 47.89% VAL. These asset choices are intuitively sound, as discussed in the last two tables.

In periods 1 and 2, the percentage of ILB has increased from the percentage in Table 5.3.2, due to the 2.71% investment in ALBI being reduced to 0%. As discussed before, the lower risk, higher Sharpe Ratio and general negative correlation of the ILB makes sense when dealing with a more risk averse investor.

The equity portion of the portfolio is still 65%, but more is invested in VAL and less in MOME, than was the case in Table 5.3.2. As mentioned in Table 5.3.2, VAL is better suited to a more risk averse person. For the periods from 3 to 30 years before retirement, the 10% in ILB has been moved to ALBI and the proportion in equity has shifted from VAL to MOME.

As expected, returns outweigh risk for the younger investor. However, for the period of 3 to 10 years from retirement, the portfolio in Table 5.3.3 is more risky than that of Table 5.3.2. This is counterintuitive, but it could be due to the model's limited calculating abilities, with the solver only able to find a *reasonable* allocation rather than the completely *correct* solution. For 15 to 30 years from retirement, the model seems to correct itself. Compared to Table 5.3.2, Table 5.3.3 allocates more to VAL and less to MOME because of its minimised risk factor. The other allocations remain the same as before.

Table 5.3.4 uses *Y*=-1.5. In the periods from 30 to 3 years before retirement, the split is ALBI 2.71%, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL. The portfolios in period 2 and 1 are 10.00% ILB, 15.75% MOME, 25.00% SAPY and 49.25% VAL. When the time to retirement increases the amount of changes with regard to each asset follows the same pattern as discussed above with regard to Table 5.3.3, with this table containing a higher percentage of less risky alternatives.

Table 5.3.5 now uses *Y*=-2, which leads to a similar outcome as that described above. During periods 1 and 2, the percentages are 10.00% ILB, 14.84% MOME, 25.00% SAPY and 50.16% VAL. In period 3, MOME and VAL change to 18.48% and 46.52% respectively, whereas the others remain the same. From 30-4 years until retirement, the allocations are 2.71% ALBI, 7.29% ILB, 19.68% MOME, SAPY 25.00% and VAL 45.32%. Again, these numbers follow the same trends as discussed before. The portfolios are less risky than those of Table 5.3.4, and the portfolio is weighted in favour of higher return assets, as the years until retirement increase. For periods 30 to 4 years to retirement in Table 5.3.4 and Table 5.3.5, the portfolios are the same for each period. This could imply that a value of *Y*=-2 is nearing the upper limit of how risk averse a person can be and that thus the risk averse coefficient stops there.

5.3.3. Conclusion

These results have shown that the model can indeed explain the theory of asset allocation. The results show a logical step-by-step difference caused by decreasing time to retirement and increasing risk aversion. Quantifying an investor's level of risk aversion is difficult, but the model nonetheless does give some insight into the best asset allocations for investors with different levels of risk aversion. The solutions are always split between 25% in property, 65% in equity and 10% in bonds. The model therefore recommends that an investor should invest the maximum amount into the asset class with the best Sharpe Ratio, which in this case is property. Thereafter, the maximum amount should be invested into the next best performing asset class, which is equity. The remaining 10% is then invested into the third best asset class, viz. bonds. Investment in cash is never made, as it is the worse performing asset class. The weightings of assets in each asset class do change slightly over time and for different levels of risk aversion. It is interesting to note that the general investing practice of starting with a high percentage of equity when an investor is young and decreasing that weighting over time, while increasing the weightings in bonds and finally adding cash investments closer to retirement, is not supported by this risk aversion model.

In the next section, the model replaces risk aversion with a shortfall constraint, which is much easier to quantify. An emphasis on general investment practices and how these compare to the different models' results will continue throughout this paper.

5.4. Maximising Alpha Using a Shortfall Constraint

5.4.1. Introduction

In this section, the risk aversion coefficient is excluded and another constraint is added, namely, the shortfall threshold, which is used to align an investor's return needs to his or her investment strategy. The shortfall threshold is a level of return that the investment must meet or exceed. Therefore, when an investor stipulates a required return of at least 4% a year, the shortfall threshold is 4%. This constraint uses normal distribution to evaluate the expected returns and there must be an associated probability level of the return meeting the shortfall threshold.

In this paper, the following shortfall thresholds were analysed: no shortfall threshold, shortfall threshold equalling zero, shortfall threshold equalling cash, shortfall threshold equalling cash plus 2%, shortfall threshold equalling cash plus 4%, and shortfall threshold equalling cash plus 6%. In each case, the shortfall threshold is tested to a probability of 90%, 95% and 99%. These tests are run for logarithmic returns and arithmetic returns in turn. According to the theory presented in the reviewed literature, logarithmic returns are a better measure because they are more normally distributed and therefore a better fit for the shortfall threshold constraint. However, arithmetic returns do not differ greatly from logarithmic returns, so the arithmetic scenario is also tested. These two models are then compared. For logarithmic returns, Alpha is still maximised, but the equation differs slightly from Section 5.3 with the exclusion of the risk aversion. It is as follows:

$$\alpha_i = \mu_I - ½ \sigma_i^2 \qquad (5.3)$$

For arithmetic returns, the equation is very similar, but Alpha and Mu are exchanged for growth rate and expected returns:

$$G = E(R) – ½ \sigma^2 \qquad (5.4)$$

The shortfall constraint can be explained mathematically as:

$$N[H-(\mu_W – ½ \sigma^2{}_W)T/(\sigma^2{}_W\sqrt{T})] <= K \qquad (5.5)$$

Or

$$N[H-(E(R)_W – ½ \sigma^2{}_W)T/(\sigma^2{}_W\sqrt{T})] <= K \qquad (5.6)$$

Depending on which types of returns are used. The sections that follow discuss the results.

5.4.2. Maximising Alpha and Keeping the Probability Level Constant

In this section, portfolios are optimised by maximising Alpha in the model. As stated above, no risk aversion is used, but a shortfall threshold constraint is included. This section is divided into 3 sub-sections. In Section 5.4.2.1, the portfolios are tested at a 90% probability level, in Section 5.4.2.2 at a 95% probability level and in Section 5.4.2.3 at 99%. The tests are run six times in each sub-section, with different shortfall threshold constraints, ranging from no shortfall threshold to a minimum return of cash plus 6%.

5.4.2.1. Results: Shortfall Constraint at a 90% Probability Level

Table 5.4.2.1.1 below is optimised for the scenario of no shortfall threshold. The other constraints mentioned in Section 5.3.2 remain the same, in accordance with Regulation 28. The portfolio's allocation is thus 10.00% ALBI, 25.00% SAPY and 65.00% VAL, which remains constant for all the periods until retirement. The reasoning for this is similar to that of Section 5.3.2. SAPY has the highest returns and the highest Sharpe Ratio, and thus the 25% maximum is allocated to property. VAL has the highest Sharpe Ratio and the second highest returns for the equity indices.

There is no risk aversion in this case, so the reduction in risk from diversifying is not as important as it is in Section 5.3.2. Consequently, MOME is excluded from the portfolio. VAL's investment is the maximum 65%, resulting in the combined property and equity allocations reaching the upper limit of 90% of the portfolio. The other 10% has been invested in ALBI because it has a higher return than ILB.

It is expected that this portfolio will not change over time, as there is no risk aversion coefficient included that could be affected by time, and as the shortfall threshold has not been included in this particular scenario. Therefore, the optimiser will seek to maximise this Alpha, which is the same over any period. The expected returns, Mu, are again increasing proportionally with time, while risk increases proportionally with the square root of time. The Annualised Mu, Alpha, Standard Deviation, Sharpe Ratio and Kelly Ratio are therefore constant for each period.

Table 5.4.2.1.2 introduces the shortfall threshold. The paper starts with a portfolio earning a return of at least 0%. The portfolio is divided up in the same way as in Table 5.4.2.1.1, viz. 10.00% ALBI, 25.00% SAPY and 65.00% VAL. This portfolio's investment rationale was already discussed above.

Table 5.4.2.1.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.1.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.20%	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.48%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.1.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	3.96%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.01%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.1.4: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.93%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.27%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.1.5: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.09%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.57%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.1.6: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	7.45%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.94%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

In Table 5.4.2.1.2, however, a new column is used, labelled Shortfall Probability. This refers to the probability of the portfolio not reaching the threshold for each period. It can be seen that, between 1 and 5 years until retirement, the minimum return of zero might not be reached. These probabilities are 6.20%, 1.48%, 0.39%, 0.10% and 0.03% respectively for 5 to 1 years until retirement. As the shortfall probability of these results is 90% of reaching the threshold, or – in other words – as it is allowed a 10% probability of not reaching the threshold, the model is not yet required to change the portfolio weights to align with the constraint. The probabilities are marked in green because they fall under the allowed level.

In **Table 5.4.2.1.3**, the shortfall threshold is increased to equal the return that would be earned if it were a cash investment. The expected return of STEFI (μ = 8.07%) is used to calculate this. Mu is used because that is the expected return of cash. The use of Alpha would be incorrect in this case, because Alpha is the expected return of cash minus a penalty for risk, and it thus differs from the return wanted in the strategy.

For the periods of 30 to 2 years until retirement, the portfolio is structured the same way as that in Table 5.4.2.1.2. The probability of shortfall only occurs in the period of 5 to 2 years before retirement, when it is 3.96%, 1.01%, 0.27% and 0.08% respectively. These are lower than the 10% allowed level and therefore the portfolio does not need restructuring. These probabilities are all slightly higher than when the shortfall threshold equalled 0%. This makes sense, as this threshold is more difficult to reach.

At one year before retirement, the results in Table 5.4.2.1.3 show the portfolio's allocations as 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. The portfolio is invested in the usual assets (see those of the previous test) and the reasons for this have been discussed. However, this is the first time that STEFI has been included in the portfolio and at a high weighting of 67.18%. This can be explained by the fact that this portfolio's minimum return is equal to cash itself.

The shortfall probability is 11.13%, which is higher than the maximum level allowed, therefore violating the constraint; it has thus been highlighted in red in Table

5.4.2.1.3. This violation means that the portfolio cannot be structured to reach the minimum return required and to have the right probability level. However, the model does find the minimum probability level related to the goals wanted. If the portfolio is restructured to have the same weights as in Table 5.4.2.1.2, then the probability of not reaching the threshold is 17.26%. Thus, the result in this table is a more appropriate solution.

The shortfall threshold is increased further in **Table 5.4.2.1.4** to cash + 2%. At 1 year to retirement, the portfolio is divided into 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25%. The probability of not reaching this threshold in this period is 19.03%. STEFI has been excluded due to the step up from cash to cash + 2%. This is because STEFI's expected return is not high enough. ILB is quite highly weighted in this portfolio, as the expected return from ILB is at a similar level to cash + 2%, and as it is generally negatively correlated to the other assets. The rationale for the inclusion of SAPY, MOME and VAL has been discussed before. The portfolios for the other periods remain the same. Their shortfall probabilities have increased slightly as before, and remain under the 10% level. This large amount invested in bonds and small amount invested in equity in the last year is what the industry would expect. As it follows the philosophy of lower returns and risk for an older investor.

In **Table 5.4.2.1.5**, the threshold level has been increased to cash + 4%. With 1 year to retirement, the portfolio is divided into 10.39% ILB, 12.48% MOME, 25.00% SAPY and 52.12% VAL. ILB weight has decreased, as was the case in the previous table. This happened to STEFI before. The returns expected from ILB are not high enough to make up a majority of the portfolio when searching for a return of cash + 4%. The probability of shortfall has again increased to 25.10%. For the remaining periods of 2 to 30 years until retirement, the portfolios are structured as before, and their probabilities of shortfall have increased slightly from those in Table 5.4.2.1.4.

Table 5.4.2.1.6 has a shortfall level of cash + 6%. At 1 year until retirement, the portfolio is very similar to that of Table 5.4.2.1.5. Bonds or cash have decreased to their minimum 10% level through ILB. The shortfall probability has increased to

30.17%. The portfolio's periods are the same as before and their probabilities have again increased slightly.

In the next section, these tests were re-done, using a probability level of 95% instead of 90%. This will cause some of the portfolios to shift their weightings.

5.4.2.2. Results: Shortfall Constraint at a 95% Probability Level

In this section, the constraint is tightened to a probability of 95% that the return will be equal to or higher than the required gain. The test is set up identically to that in Section 5.4.2.1, and the periods studied range from 1 to 30 years until retirement. As in the previous sections, there are 6 tables of different shortfall threshold criteria, ranging from no shortfall to cash plus 6%.

Table 5.4.2.2.1 has no shortfall threshold constraint and therefore is similar to Table 5.4.2.1.1. The figures show that both tables result in the same portfolios for every period. The portfolio thus contains 10.00% ALBI, 25.00% SAPY and 65.00% VAL. The rationale for this was explained in Section 5.4.2.1.

Table 5.4.2.2.2 now looks at a scenario where the return must equal at least 0% at a 95% probability. During the periods of 30 to 2 years until retirement, the results are the same as those in Table 5.4.2.2.1. However, the portfolio changes when there is only one year left until retirement, when the proportions are 15.01% ILB, 8.36% MOME, 25.00% SAPY and 51.63% VAL. This is the typical trend seen in this paper, where investment in ILB and MOME is used to diversify risk through negative correlation (e.g. in the case of ILB) and to increase the amount of assets held. This portfolio has been constructed to comply with the 5% probability, as seen in Table 5.4.2.2.2. It is marked in orange to show that it is on the upper allowed limit.

Table 5.4.2.2.3 summarises the results, when the shortfall threshold at least equals cash at a 95% probability level. Only the figures in period one differ from those of Table 5.4.2.2.2. In this period, the portfolio consists of 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. The shortfall probability level is violated, as can be seen by the 11.13% level in the table. The usual assets are seen again trying to reach this threshold constraint, by adding an investment of STEFI. This is because STEFI is cash and thus a high proportion is invested.

Table 5.4.2.2.4 reviews the threshold of cash plus 2% at a 95% level. Again, the only changes occur when there is one year left until retirement, and the investment is thus 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25% VAL. The probability is 19.03% that this return will not be reached. A higher proportion is invested in ILB because the expected return of this asset is similar to the required return of the investment strategy.

The next scenario is where a return of at least cash plus 4% at a 95% probability is wanted. In this case, as illustrated in **Table 5.4.2.2.5**, the portfolio alters when there are two years left until retirement, and again the next year. The portfolio changes to 15.87% ILB, 9.31% MOME, 25.00% SAPY and 49.82% VAL, when there are two more years left until retirement.

When there is one year to go, the proportion of ILB decreases, while MOME and VAL increase. The investment at this stage thus consists of 10.39% ILB, 12.48% MOME, 25.00% SAPY and 52.13% VAL. The explanation for this is that the investor is chasing the higher return offered by equity in order to reach the threshold level. There is a 25.10% chance, however, that this will not be reached.

The Annualised Sharpe Ratio increases from 1.01 to 1.06, at two years until retirement. This means that the investor has shifted to a less risky portfolio, which makes senses when reaching retirement. However, the Annualised Sharpe Ratio then decreases to 1.04 in the last year, which is due to the increase in equity and the decrease in bond weights. This increase in equity and decrease in bonds upon reaching retirement is counter to industry practices. The probability level is thus violated with one year to go, although it is in line with the 95% level with two years to go.

Table 5.4.2.2.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.2.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	15.01%	8.36%	0.00%	0.00%	25.00%	0.00%	51.63%	5.00%	21.15%	19.93%	12.39%	1.06	21.15%	19.93%	12.39%	1.06	8.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.48%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.2.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	3.96%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.01%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.2.4: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.93%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.27%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.2.5: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.87%	9.31%	0.00%	0.00%	25.00%	0.00%	49.82%	5.00%	42.08%	39.66%	17.33%	1.50	21.04%	19.83%	12.25%	1.06	8.64
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.57%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.2.6: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.51%	8.33%	0.00%	0.00%	25.00%	0.00%	36.16%	5.00%	38.37%	36.33%	14.16%	1.57	19.18%	18.16%	10.01%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.94%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

The last table in Section 5.4.2.2 (**Table 5.4.2.2.6**) considers the scenario where an investor wants a return of cash plus 6% at a 95% level. The resulting portfolios are the same as those in the tables above, until 2 years to retirement. At this stage, the portfolio changes to 30.51% ILB, 8.33% MOME, 25.00% SAPY and 36.16% VAL, at a probability of 95%. It changes again with one year to go: The investment is then divided into 10.00% ILB, 13.05% MOME, 25.00% SAPY and 51.95% VAL, at a shortfall probability of 30.17%, again decreasing the level of ILB invested and increasing the equity (MOME and VAL) levels. As mentioned before, such a change in portfolio while approaching retirement age, runs counter to financial theory.

The current section, Section 5.4.2.2, although it has generally obtained similar results to those of Section 5.4.2.1, has shown unexpected and strange results in Table 5.4.2.2.5 and Table 5.4.2.2.6. The fact that equity has increased and bonds have decreased with one year until retirement differs from both industry practice and from financial theory. The model is designed to meet a required return while not exceeding the shortfall constraint. With 1 year to retirement both variables cannot be met simultaneously. The model disregards one variable. The way this model is designed the variable that is disregarded is the shortfall constraint. This can be explained as the model attempting to "chase" the required return with one year to go until retirement. In theory, an investor should lower his expected returns at this stage, while decreasing his risk and thus guaranteeing the smaller return. There is a good reason to include a variable in this model to decrease the required return when approaching retirement. This way the shortfall constraint is never disregarded but the required return is. The variable could be similar to that of the risk aversion coefficient found in Section 5.3, where the variable for time (T) is used to adjust the overall risk appetite (*Y*), by using (1-*Y*/T) in the formula

$$\alpha_W = (\mu_W - \tfrac{1}{2}\,(1-Y/T)\sigma^2_W) \qquad (5.7)$$

Alternatively, the model's setup should be changed so that the shortfall threshold level changes and the probability of the shortfall is never violated. This model, which was set out from prior research, does not allow the shortfall threshold level to change and therefore can only be used to find the lowest possible shortfall probability level (which is violated in some cases) for an assigned required return.

5.4.2.3. Results: Shortfall Constraint at a 99% Probability Level

In this section, the probability level has been increased to 99%. All the same tests were run, in accordance with the descriptions set out in Section 5.4.2.1 and Section 5.4.2.2.

In the first case, where no shortfall constraint is included, the portfolio consists of 10.00% ALBI, 25.00% SAPY and 65.00% VAL for all periods. This can be seen in **Table 5.4.2.3.1**.

Table 5.4.2.3.2 shows the results when the required return is equal to zero. Similar results as before are found between 30 and 3 years until retirement, with the usual split of 10.00% ALBI, 25.00% SAPY and 65.00% VAL. Two years before retirement, the portfolio changes to 15.02% ILB, 8.28% MOME, 25.00% SAPY and 51.70% VAL. The investment has a 99% chance of realising this return. An investor with one year to go until retirement would have a portfolio consisting of 52.87% ILB, 5.56% MOME, 25.00% SAPY and 16.56% VAL. This portfolio's shortfall threshold has a 99% chance of being reached.

This portfolio illustrates financial theory well: It consists of 65% equity and 10% bonds for the majority of an investor's life and then, two years before retirement, the portfolio changes its proportions, with the equity versus debt split being 59.98% versus 15.02% respectively. Decreasing weights in equity and increasing weights in debt, while approaching retirement, is consistent with financial theory and practice. Debt has moved from ALBI (the higher return bond with more risk) to ILB (which has a lower return but a better Sharpe Ratio), thus reducing risk, which again is consistent with both theory and practice. Equity was 65% in VAL, but with 2 years to go, equity is split between VAL and MOME. This diversification allows an investor to reduce risk. The strategy of reducing risk closer to retirement is also consistent with the theory and with industry practice. With 1 year to go, the portfolio structure changes again: equity and debt sizes change to 22.13% and 52.87% respectively, which means that the investor has thus further increased his investment in bonds. This too is consistent with both practice and theory.

Table 5.4.2.3.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	52.87%	5.56%	0.00%	0.00%	25.00%	0.00%	16.56%	1.00%	16.35%	15.63%	6.92%	1.19	16.35%	15.63%	6.92%	1.19	17.26
2	0.00%	0.00%	0.00%	15.02%	8.28%	0.00%	0.00%	25.00%	0.00%	51.70%	1.00%	42.29%	39.87%	17.52%	1.49	21.14%	19.93%	12.39%	1.06	8.52
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	45.66%	6.45%	0.00%	0.00%	25.00%	0.00%	22.88%	1.00%	34.53%	32.89%	11.10%	1.65	17.26%	16.45%	7.85%	1.17	14.90
3	9.16%	0.00%	0.00%	0.84%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.00%	65.43%	61.78%	23.47%	1.76	21.81%	20.59%	13.55%	1.01	7.48
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.4: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	57.08%	5.88%	0.00%	0.00%	25.00%	0.00%	12.04%	1.00%	31.64%	30.29%	9.09%	1.70	15.82%	15.15%	6.43%	1.20	18.74
3	0.00%	0.00%	0.00%	11.34%	9.16%	0.00%	0.00%	25.00%	0.00%	54.51%	1.00%	64.84%	61.05%	22.46%	1.81	21.61%	20.35%	12.96%	1.04	8.05
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.5: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	44.95%	4.41%	0.00%	0.00%	18.13%	25.44%	7.06%	1.00%	27.06%	26.12%	6.36%	1.72	13.53%	13.06%	4.49%	1.21	27.01
3	0.00%	0.00%	0.00%	17.79%	8.73%	0.00%	0.00%	25.00%	0.00%	48.48%	1.00%	62.38%	58.84%	20.70%	1.84	20.79%	19.61%	11.95%	1.06	8.90
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.6: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	25.75%	2.74%	0.00%	0.00%	9.67%	58.27%	3.58%	1.00%	22.09%	21.57%	3.42%	1.74	11.04%	10.78%	2.42%	1.23	50.79
3	0.00%	0.00%	0.00%	24.60%	8.25%	0.00%	0.00%	25.00%	0.00%	42.15%	1.00%	59.79%	56.52%	18.89%	1.88	19.93%	18.84%	10.90%	1.09	9.97
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.2.3.3 changes the shortfall threshold to equal at least cash, at a 99% probability level. From 30 years to 4 years until retirement, the portfolio consists of the usual 10.00%, 25.00% and 65.00% split into ALBI, SAPY and VAL respectively. The probability levels are all above 99%. With 3 years to go, debt is divided into ALBI and ILB with 9.16% and 0.84% respectively. There is thus a small amount of diversification and a slight move to the safer alternative of ILB. There is no other change in the portfolio's structure, and the probability level has also remained 99%.

A year later, the investment has changed to 45.66% ILB, 6.45% MOME, 25.00% SAPY and 22.88% VAL, which is similar to the distribution recommended in Table 5.4.2.3.2. Equity is diversified into two indices, with the amount of equity decreasing while debt increases. Once again, this is consistent with both financial theory and industry practice. The probability level is still 99%.

One year before retirement, the investment comprises 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. The probability level is violated with an 11.13% chance that the threshold will not be reached. The Annualised Sharpe Ratios have also increased near retirement, which implies that the investment is gradually becoming less risky. This increase is due to a decrease in returns but a relatively larger decrease in risk, resulting in the Annualised Sharpe Ratio increasing.

In **Table 5.4.2.3.4**, the threshold level has been adjusted to cash plus 2% at a 99% probability. The typical portfolio is generated for 30 years to 4 years until retirement, namely 10% ALBI, 25% SAPY and 65% VAL. However, with three working years remaining, the portfolio has changed: it is now found to contain 11.34% ILB, 9.16% MOME, 25.00% SAPY and 54.51% VAL at a 99% level. This represents a change similar to the one in the previous table, Table 5.4.2.3.3, as there has been an increase in bonds, with a move to a safer bond index, viz. from ALBI to ILB. Diversification has also occurred with regard to equity, and the equity holding in total has decreased.

A year later, the portfolio breakdown is 57.08% ILB, 5.88% MOME, 25.00% SAPY and 12.04% VAL, with a probability level of 99%. There has thus been a further increase in bonds and a decrease in equity. For the last year of work, then, the portfolio has shifted to contain 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25% VAL. The chance of a shortfall occurring is 19.03%. There has been a decrease in bonds and an increase in equity for the prior period. This is counter to financial theory. The reason for this happening is that the model is chasing the required return (based on the shortfall threshold level) rather than settling on a lower return with a 99% probability.

The results suggest that a return of cash plus 2% may be too high to obtain at a 99% probability over one year. This chasing of returns has effectively increased the risk of the portfolio. The annual Mu and annual Standard Deviation have both increased from period 2 to period 1, causing the Annualised Sharpe Ratio to decrease from 1.20 in period 2 to 1.17 in period 1. Compare this change to the change observed from 3 years to 2 years, where the Annualised Sharpe Ratio was 1.04 with 3 years to go, changing to 1.20 a year later. This means that the portfolio in period 2 is safer than that of period 3. The annual Standard Deviation has decreased more than the decrease in annual Mu, thus increasing the Sharpe Ratio as mentioned. However, the portfolio becomes riskier when the investor has 1 year left until retirement, which runs counter to both financial theory and industry practice. The Annualised Sharpe Ratio is 1.01 from 30 to 4 years until retirement; the probabilities of a shortfall during these periods are also at an acceptable level.

The next group of results, summarised in **Table 5.4.2.3.5**, reflects a shortfall threshold of cash plus 4% at a 99% probability level. With 30 to 4 years until retirement, the portfolio contains the usual assets in the same proportions as before. With 3 years to go, however, the portfolio changes: it now consists of 17.79% ILB, 8.73% MOME, 25.00% SAPY and 48.48% VAL at a 99% probability level. Again, bonds have increased in magnitude from the previous period and moved to the safer bond option (ILB). Equity has decreased, and it has been diversified between two indices.

A year later, similar movements occur. ILB has increased to 44.95%, MOME has decreased to 4.41% and VAL has decreased to 7.06%. The SAPY holding remains at 25.00%. There is further diversification in the portfolio with the inclusion of STEFI

(25.44%). This asset, with its very low Standard Deviation, helps to reduce the total risk of the investment. This portfolio also complies with the 99% probability level. In the last period until retirement, the portfolio is 10.39% ILB, 12.48% MOME, 25.00% SAPY and 52.12%, with a 25.10% probability of not reaching cash plus 4%.

A similar change occurred here as in Table 5.4.2.3.4: Bonds decreased and equity increased. STEFI is excluded, thus making the portfolio less diversified. Again, the conclusion is that this portfolio is chasing the required return rather than decreasing return expectations, while guaranteeing the profit because of lower risk. The Annualised Sharpe Ratio is constant at 1.01 from 30 years to 4 years until retirement. With 3 years to go, however, the portfolio has to be readjusted to comply with the investor's required probability level. This happens again a year later. In both of these periods, the Annualised Sharpe Ratio increases. This is due to a large reduction in the annualised Standard Deviation (11.95% - 4.49%). This is how a portfolio should change. With one year to go, the model tries to find a solution to obtain the required return, while substantially increasing the risk in the portfolio. The Annualised Sharpe Ratio decreases to 1.04. This is caused by the annualised deviation increasing dramatically from 4.49% to 13.12%, combined with a relatively smaller increase in Mu.

The last test run for this Alpha section looks at a shortfall threshold of cash plus 6% with a 99% probability. In **Table 5.4.2.3.6**, periods 30 to 4 years until retirement have the same portfolio weights as before. With 3 years of work left until retirement, the portfolio distribution changes to 24.60% ILB, 8.25% MOME, 25.00% SAPY and 42.15% VAL. A year later, the portfolio changes again, to 25.75% ILB, 2.74% MOME, 9.67% SAPY, 58.75% STEFI and 3.58% VAL. These portfolios comply with the shortfall probability of only 1%. These changes are similar to those in the corresponding periods in Table 5.4.2.3.5. There is thus a progressive movement of increasing diversification while increasing bonds and decreasing equity. In the last period, the portfolio is again structured to chase the returns. This increases equity dramatically and therefore risk is high. This causes the Annualised Sharpe Ratios to increase over time and then decrease in the last period.

5.4.2.4. Conclusion

Section 5.4.2 has shown the change in results over time when the shortfall constraint increases and the probability level remains constant. It was found that the model changed the portfolios to reduce risk closer to retirement when the probability of a shortfall comes into play. However, the model does not create an appropriate portfolio when the probability of a shortfall is violated. This is a shortcoming of the model. A few results are worth noting in this regard.

For the majority of an investor's life, the investment stays the same. These asset class weights in this portfolio are the same as those in Section 5.3. First, the maximum allowed amount is allocated to the asset class that offers the best returns for risk, which, in this case, is property. Then the maximum allowed amount is allocated to the next best performing class, namely, equity. The rest of the portfolio is completed with bonds in this scenario (i.e. the third best performing class). The assets chosen in each asset class are generally chosen according to their highest return. The model in Section 5.3 diversified many of the portfolio's asset classes. This model does not do this.

The model furthermore supports the practice of decreasing equity and increasing bonds and then increasing cash, the closer the investor comes to retirement age. However, industry practices tend to implement the relevant proportions change gradually over many years, rather than only within the last few years, as was suggested by the model. In other words, the model appears to suggest that such changes should only be done in the last 1 or 2 years and, in extreme cases, during the last 3 years.

Moreover, the change from equity to bonds is not done gradually; in some cases, in fact, the change is quite dramatic from one year to the next. In Table 5.4.2.3.4, for example, the weighting of ILB moves from 11.34% in period 3 to 57.08% in period 2. This seems to suggest that moving early into bonds and cash is an unnecessary practice in the industry, of trying to reduce risk. To the contrary, an investor could reasonably take on more risk for the last 10 to 3 years until retirement, and only increase exposure to less risky assets closer to retirement. Moving away from the risky assets too soon will result in an investor receiving lower returns and thus having less wealth for his or her retirement.

In Section 5.4.3 below, the model is further analysed in a scenario when the shortfall constraint remains constant and the probability level changes.

5.4.3. Maximising for Alpha Keeping the Shortfall Constraint Constant

This section explores the above results further, but analyses the numbers obtained, while changing the probability levels. The shortfall constraints remain constant in each sub-section.

5.4.3.1. Results: Probability Level With No Shortfall Constraint

The results for Section 5.4.3.1 are contained in **Table 5.4.3.1.1**, **Table 5.4.3.1.2** and **Table 5.4.3.1.3** below.

In this section, there is no shortfall constraint and thus all the results have the typical composition seen before: All the portfolios consist of 10.00% ALBI, 25.00% SAPY and 65.00% VAL. Thus all the annualised Mus, Alphas, Standard Deviations, Sharpe Ratios and Kelly Ratios are constant.

Table 5.4.3.1.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.1.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.1.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.4.3.2. Results: Probability Level With a Shortfall Constraint of Zero

This section adds a shortfall constraint of zero to the model. The results, as summarised in **Table 5.4.3.2.1**, reflect the usual pattern, with the portfolio consisting of 10.00% ALBI, 25.00% SAPY and 65.00% VAL.

When the probability level increases from 90% to 95% (**Table 5.4.3.2.2**), the results change, one year prior to retirement. The probability of shortfall at this stage decreases from 6.20% to 5%. The model achieves this by increasing diversification and increasing the exposure to bonds. The result shows that both Mu and Standard Deviation decrease, with a greater decrease in Standard Deviation. This results in higher Sharpe and Kelly Ratios.

The probability level is increased again to 99% in **Table 5.4.3.2.3**. Again, more is invested in bonds, ensuring a safer investment. The Sharpe Ratio increases from 1.01 to 1.06 and then to 1.19. This change can also be seen in period 2, in both Table 5.4.3.2.2 and Table 5.4.3.2.3, and it results in the Sharpe Ratio increasing from 1.01 to 1.06. These results correlate with the theory of investing in relatively safer assets closer to retirement.

Table 5.4.3.2.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.20%	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.48%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.2.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	15.01%	8.36%	0.00%	0.00%	25.00%	0.00%	51.63%	5.00%	21.15%	19.93%	12.39%	1.06	21.15%	19.93%	12.39%	1.06	8.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.48%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.2.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retiremnet	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	52.87%	5.56%	0.00%	0.00%	25.00%	0.00%	16.56%	1.00%	16.35%	15.63%	6.92%	1.19	16.35%	15.63%	6.92%	1.19	17.26
2	0.00%	0.00%	0.00%	15.02%	8.28%	0.00%	0.00%	25.00%	0.00%	51.70%	1.00%	42.29%	39.87%	17.52%	1.49	21.14%	19.93%	12.39%	1.06	8.52
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.4.3.3. Results: Probability Level With Shortfall Constraint Equalling Cash

The shortfall constraint is increased so that the return at least equals the return of cash. This is the first shortfall level that is high enough to cause the model to violate the shortfall probably level. As can be seen from the three tables below (Table 5.4.3.3.1, Table 5.4.3.3.2 and Table 5.4.3.3.3), the typical results are still obtained for most of the periods, until a few years before retirement. In all three tables, when there is one year left until retirement, the results are all the same – and all violate the shortfall constraint: they all have a shortfall probably equalling 11.13%.

In this scenario, moreover, the Annualised Sharpe Ratio increases from period 2 to period 1. In period 1, the holdings in bonds and STEFI are increased, thus greatly reducing the risk profile. This is in accordance with financial theory, although this will not always be the case when the shortfall constraint is violated. In the sub sections to follow the portfolio's risk profile does the opposite and increases in the last period. When comparing the periods of 2 and 3 years until retirement, it can be seen that none of the three tables violate the shortfall constraint. The first two tables obtain the same results. In the last table, the portfolio composition changes to equal the 99% confidence level. In period 2, the bond levels have increased and equity has decreased; this results in a safer portfolio with the Sharpe Ratio increasing from 1.01 (in Table 5.4.3.3.1 and Table 5.4.3.3.2) to 1.17 (in Table 5.4.3.3.3). In period 3, equity and bond levels remain the same, although some of the money invested in bonds has moved to ILB, thus increasing diversification. This causes the Sharpe Ratio to increase, but only by a very small amount. There are not enough decimal points to see the change but it moves from 1.012 to 1.014. In the sub sections to follow, the results show that, when the time to retirement decreases, the composition always changes to reflect a safer portfolio, as long as the shortfall constraint is not violated.

Table 5.4.3.3.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion																				
Historical data has been used from January 2003 to July 2012 Maximising α (Shortfall Threshold = Cash) at a probability of 90%																				
Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	3.96%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.01%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.3.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion																				
Historical data has been used from January 2003 to July 2012 Maximising α (Shortfall Threshold = Cash) at a probability of 95%																				
Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	3.96%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.01%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.3.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion																				
Historical data has been used from January 2003 to July 2012 Maximising α (Shortfall Threshold = Cash) at a probability of 99%																				
Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	45.66%	6.45%	0.00%	0.00%	25.00%	0.00%	22.88%	1.00%	34.53%	32.89%	11.10%	1.65	17.26%	16.45%	7.85%	1.17	14.90
3	9.16%	0.00%	0.00%	0.84%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.00%	65.43%	61.78%	23.47%	1.76	21.81%	20.59%	13.55%	1.01	7.48
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.4.3.4. Results: Probability Level With Shortfall Constraint Equalling Cash Plus 2%

The shortfall constraint is now equal to cash plus 2%. The results are shown below:

Yet again, as was the case in the previous scenario, the results violate the shortfall constraint in period 1. The results depicted in the first two tables (Table 5.4.3.4.1 and Table 5.4.3.4.2) follow financial theory. The risk profiles remain the same until period 1, when the risk is reduced.

The third table (Table 5.4.3.4.3) has an interesting result. The portfolios from 30 years to 4 years until retirement have the usual allocation, viz. 10% ALBI, 25% SAPY and 65% VAL. In period 3, however, the portfolio changes to reflect the shortfall probability of 99%. The portfolio is more risk averse than before, which is shown by the increase in the Annualised Sharpe Ratio. A year later, the same mechanisms occur: even more is invested in bonds and less in equity, and thus the Sharpe Ratio increases again. However, the year after (i.e. 1 year before retirement), the shortfall probability is violated, and a higher proportion is invested in equity than in bonds. At this point, the Annualised Sharpe Ratio decreases from 1.20 to 1.17. This runs against financial theory, and the portfolio can thus be viewed as chasing returns rather than consolidating and pursuing stable returns.

Table 5.4.3.4.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.93%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.27%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.4.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.93%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.27%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.4.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	57.08%	5.88%	0.00%	0.00%	25.00%	0.00%	12.04%	1.00%	31.64%	30.29%	9.09%	1.70	15.82%	15.15%	6.43%	1.20	18.74
3	0.00%	0.00%	0.00%	11.34%	9.16%	0.00%	0.00%	25.00%	0.00%	54.51%	1.00%	64.84%	61.05%	22.46%	1.81	21.61%	20.35%	12.96%	1.04	8.05
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.4.3.5. Results: Probability Level With Shortfall Constraint Equalling Cash Plus 4%

The results in Table 5.4.3.5.1, Table 5.4.3.5.2 and Table 5.4.3.5.3 show what happens to the portfolios when the shortfall constraint equals cash plus 4%.

In all the tables, the results of period 1 violate the probability constraint. The results in period 2 are interesting. In **Table 5.4.3.5.1**, the probability of shortfall is 6.09%, which is under the 10% level. In **Table 5.4.3.5.2**, the probability of shortfall becomes 5% in period 2 already, to reflect the level of the constraint. In **Table 5.4.3.5.3**, the portfolio changes in period 3, to reflect the 99% level.

As we have seen, moving from 90% through to 99% means that the portfolios become more diversified, decreasing their investment in equity and increasing their investment in bonds. This results in Mu and Standard Deviation decreasing in the same manner, with larger Standard Deviation decreases. This increases the reliability of the returns. The reliability of the first table in this sequence is 90% while that of the last is 99%. The Sharpe Ratios also increase, as the probability level increases, moving from 1.01 to 1.06 and finishing at 1.21, seen in period 2 on the 3 tables below. The changes in the results due to the increased probability levels are similar to the results of the increasing levels of risk aversion. This process can also be seen in period 3. Generally, the Sharpe Ratios demonstrate similar results as before. They stay the same and increase closer to retirement. In the last two tables, the Annualised Sharpe Ratio first increases and then decreases in period 1. In their respective periods, the Annualised Sharpe Ratio is either the same or higher when the 3 tables are compared, as the probability levels are higher. This aligns with financial theory. The remaining results show the typical compositions seen when the shortfall probability level is too low to cause any changes.

Table 5.4.3.5.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.09%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.57%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.5.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.87%	9.31%	0.00%	0.00%	25.00%	0.00%	49.82%	5.00%	42.08%	39.66%	17.33%	1.50	21.04%	19.83%	12.25%	1.06	8.64
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.57%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.5.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	44.95%	4.41%	0.00%	0.00%	18.13%	25.44%	7.06%	1.00%	27.06%	26.12%	6.36%	1.72	13.53%	13.06%	4.49%	1.21	27.01
3	0.00%	0.00%	0.00%	17.79%	8.73%	0.00%	0.00%	25.00%	0.00%	48.48%	1.00%	62.38%	58.84%	20.70%	1.84	20.79%	19.61%	11.95%	1.06	8.90
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.4.3.6. Results: Probability Level With Shortfall Constraint Equalling Cash Plus 6%

The results of this scenario, summarised in Table 5.4.3.6.1, Table 5.4.3.6.2 and Table 5.4.3.6.3 below, are similar to those illustrated in Section 5.4.3.5 above. The constraint is violated in period 1. The Annualised Sharpe Ratio increases, as retirement approaches, except when the shortfall constraint is violated. The Annualised Sharpe Ratio is either the same or higher when compared with its respective Annualised Sharpe Ratio from the table with a lower probability level.

5.4.3.7. Conclusion

The analysis in Section 5.4.3 has shown that an investor should indeed pursue the highest possible Alpha when retirement is still far away. The portfolio should only change when they are one or two working years remaining. The model shows that, when retirement is imminent, exposure to bonds should increase, and equity proportions should decrease. Investors should also diversify the risk by increasing the number of assets held in the portfolio. The model works well until such time, as there is no solution for a shortfall probability level, seen when the shortfall constraint is violated. The model then searches for a high result and disregards risk. This solution is problematic, as no investor wants to invest prudently over his career and then gamble everything one year before retirement.

It is interesting to compare the results for the different probability levels, done in this section. On the whole, a portfolio should only be changed in extreme cases when there are 3 years until retirement. As discussed in Section 5.4.2, nothing in this model would support the industry standard of reducing equity and increasing bonds gradually over a long time. In fact, the model suggests that risk taken with regard to risky assets in periods 10 – 4 years until retirement is reasonable. Therefore, an investor should indeed invest in higher return assets until a few years from retirement, thereby maximising the total returns that could be made for retirement.

The next section uses arithmetic returns to solve for different portfolios. This approach differs from the logarithmic models used until now.

Table 5.4.3.6.1: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	7.45%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.94%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.6.2: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.51%	8.33%	0.00%	0.00%	25.00%	0.00%	36.16%	5.00%	38.37%	36.33%	14.16%	1.57	19.18%	18.16%	10.01%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.94%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.4.3.6.3: Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	25.75%	2.74%	0.00%	0.00%	9.67%	58.27%	3.58%	1.00%	22.09%	21.57%	3.42%	1.74	11.04%	10.78%	2.42%	1.23	50.79
3	0.00%	0.00%	0.00%	24.60%	8.25%	0.00%	0.00%	25.00%	0.00%	42.15%	1.00%	59.79%	56.52%	18.89%	1.88	19.93%	18.84%	10.90%	1.09	9.97
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

5.5. Maximising the Growth Rate Using a Shortfall Constraint

5.5.1. Introduction

As stated in the literature review (see Chapter 2), Hakansson (1971) found that the use of arithmetic means to optimise returns results in a solution that does not lie on the efficient frontier. However, a subsequent paper by Elton and Gruber (1974) found that, if the returns are log normally distributed and an arithmetic mean is used, the maximisation *will* lie on the efficient frontier. Logarithmic returns are better at forecasting returns, as they follow a more normally distributed structure. Normal distribution is one of the main assumptions of the mean-variance framework. In this section, however, arithmetic returns are used, which are not necessarily normally distributed. This section's processes mimic those of Section 5.4.2 and Section 5.4.3 above. The only difference is that the model is used to maximise the growth rate, as this is the arithmetic form of Alpha. The expected return is the arithmetic form of Mu, and the Standard Deviation is calculated using arithmetic numbers.

$$G_i = E(R)_i - ½\, \sigma_{Ai}^{\ 2} \qquad (5.8)$$

The results should differ, as they are not as normally distributed as the logarithmic returns. The results are found in the Appendix.

5.5.2. Results: Shortfall Constraint at a 90% Probability Level

This section examines the results when the probability of shortfall is set at a 90% confidence level. The resulting tables can be found in Appendix 1. Table 5.5.2.1 is the arithmetic equivalent to Table 5.4.2.1.1 seen before; Table 5.5.2.2 is equivalent to Table 5.4.2.1.1 and so forth. The column headings under Portfolio and Annualised Portfolio in the table have changed. G reflects the figures for the growth rate and E(R) stands for expected returns. The other headings remain the same.

In **Table 5.5.2.1**, the first aspect of note is that the general portfolio composition is the same as in the previous sections, consisting of 10.00% ALBI, 25.00% SAPY and 65.00% VAL. Table 5.5.2.1 has the exact same solutions as Table 5.4.2.1.1 but the resulting growth rate, expected return and Standard Deviations differ slightly from their counterparts Alpha, Mu, and Standard Deviation in Table 5.4.2.1.1. The Sharpe Ratios are the same but the Kelly Ratios also differ.

In **Table 5.5.2.2**, the portfolios are the same as those of their counterparts; however, the growth rate, expected returns and Standard Deviations differ, as was the case in Table 5.5.2.1. The probability levels also differ slightly from their counterparts in the previous sections.

Table 5.5.2.3 has similar variations due to the use of arithmetic numbers. The increase in the shortfall threshold to equalling cash has resulted in the probability constraint being violated with one year until retirement. The probability level has increased from 11.13% (Table 5.4.2.1.3) to 11.18% (Table 5.5.2.3). The arithmetic optimiser has resulted in a portfolio comprising 20.91% ILB, 2.69% MOME, 7.57% SAPY, 66.34% STEFI and 2.49% VAL. Its logarithmic counterpart, discussed in Section 4.3.1, had a portfolio of 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. Both of these portfolios have the same assets, although the magnitudes differ slightly. The resulting growth rate, expected returns, Standard Deviation, Sharpe Ratio and Kelly Ratio also differ slightly but are similar.

The next table, **Table 5.5.2.4**, displays similar results. The portfolios from 30 year to 2 years until retirement are the same as those of Table 5.4.2.1.4. The last portfolio before retirement consists of 46.74% ILB, 8.92% MOME, 25.00% SAPY 19.34% VAL. The portfolio of its logarithmic counterpart is 46.74%, 8.01%, 25.00% and 20.25% respectively. Again, the two investments only differ slightly.

Table 5.5.2.5 and **Table 5.5.2.6** also have similar results. The portfolios from period 30 to period 2 are the same. In both cases, the portfolios change at 1 year before retirement to include the same assets as seen in Table 5.4.2.1.5 and Table 5.4.2.1.6. There is a slight difference in their magnitudes.

The analysis in Section 5.4.2.1 also applies to this section. The explanations, resulting returns, risks and ratios are the same, with a slight difference in the portfolio construction.

5.5.3. Results: Shortfall Constraint at a 95% Probability Level

As the probability level was increased to 95% in Section 5.4.2.2, so it has been done here. The results can be found in Appendix 2.

There are similar findings when comparing the results in Table 5.5.3.1 through Table 5.5.3.6 with their comparative logarithmic equivalents. There are some differences, however, such as the portfolios' magnitudes, probability levels, growth rates, expected returns, Sharpe Ratios and Kelly Ratios. These variations are similar to those discussed in Section 5.5.2 above.

5.5.4. Results: Shortfall Constraint at a 99% Probability Level

The probability of a shortfall being avoided is increased to a 99% level. The relevant tables are found in Appendix 3. The results are as expected and follow the same trend as discussed in Section 5.5.2 and Section 5.5.3. The arithmetic numbers change some of the results slightly, but are very similar to the logarithmic alternatives.

5.5.5. Results: Probability Level With a Constant Shortfall Threshold

The tables that were generated in this regard (see Appendix 1-3) are a cross-sectional comparison of the results when the shortfall constraints remain constant and the probability levels change from 90% through to 99%. As the results are very similar to those obtained with the logarithmic figures, the discussion for this section is the same as that contained in Section 5.4.3. The portfolio compositions, probability levels, returns, risks and ratios differ slightly, as discussed in the three sections above.

5.5.6. Conclusion

It is evident from the tables in Appendices 1 to 3 that the results do not differ significantly between the logarithmic and arithmetic models. There are only slight changes in the magnitudes of some portfolios. In general, all the arithmetic portfolios consist of the same proportions of investments, and change very similarly to the logarithmic portfolios. This could mean that the arithmetic returns are normally distributed, or that logging returns is not necessary with the assets used in this paper. However, the results could be drastically different when using other assets or different periods. This section shows that arithmetic returns can be used in certain scenarios to approximate asset allocation; nothing in this paper is to the contrary. However, prior research has shown logarithmic returns to be more appropriate for a mean-variance framework. This paper thus leaves the arithmetic model and continues with the logarithmic model in the next section. Section 5.6 combines risk aversion and the shortfall constraint in a logarithmic context.

5.6. Maximising Alpha Using a Shortfall Constraint and Risk Aversion

5.6.1. Introduction

In this section, the model combines the model used in Section 5.3 with the model used in Section 5.4 with the shortfall constraint and risk aversion coefficient used to optimise the portfolios. The objective of this is to combine the strengths of the different models. There are also two weaknesses, discussed in earlier sections, which are associated with the shortfall threshold constraint and the risk aversion coefficient. Firstly, when using risk aversion, it is difficult to allocate a particular coefficient to a certain investor; this was highlighted in Section 5.3, with a justification of the numbers used. Secondly, the shortfall threshold constraint is weakened when the confidence level is violated; however, this only happens infrequently when there is one year left until retirement.

In Section 5.7, to follow, it will be discussed why this happens, and better solutions will be found. At that stage, the model also reverts to using logarithmic figures. The previous models had a number of variables that were changed to arrive at the results. When using risk aversion, these variables were the risk aversion coefficient and time until retirement. The variables for the shortfall threshold constraint were the minimum shortfall threshold return, the probability of these returns being realised and the time until retirement. If all of these variables were used in this combined model, it would result in 120 tables. Therefore, the probability level will remain constant, to reduce the number of tables while still giving satisfactory insight. Looking at the results to follow and the results in Section 5.4 (when the probabilities were altered), the combined results for each probability level can be realised intuitively. In this section, then, the probability level is fixed at 95%, because it gives greater insight into the results than the 90% level, but it is not as strict as the 99% level. The results are examined by keeping the risk aversion ratio constant and changing the shortfall threshold level, while also making cross-sectional comparisons between the sections.

5.6.2. Maximising for Alpha

5.6.2.1. Introduction

This section sets out the results obtained when risk aversion and a shortfall threshold constraint are included in the optimisation model. The probability is fixed at a 95% level for the whole section. The coefficients of risk range from 0 to -2. Each section below compares the results for a particular level of risk aversion. Section 5.6.2.2 considers a situation where the investor is the least risk averse, and each section thereafter becomes progressively more risk averse.

5.6.2.2. Results: A Risk Aversion Coefficient of 0

This is the case where an investor is the least risk averse. The tables in this section follow the same step-by-step analysis as those in Section 5.4.2. **Table 5.6.2.2.1** portrays a situation where there is no shortfall threshold level. The shortfall threshold was gradually increased to a level of cash plus 6%. The results are found below.

Table 5.6.2.2.1 thus summarises the results in a scenario where the risk averse coefficient is zero and there is no shortfall constraint. In every period, the portfolio consists of 10.00% ALBI, 25.02% MOME, 25.00% SAPY, 39.98% VAL. This is the same portfolio as that found in Table 5.3.1, and it is also similar to the general solution in Section 5.4.2, which consisted of 10.00% ALBI, 25.00% SAPY and 65.00% VAL. These two portfolios differ due to a slight weighting in risk aversion. The Annualised Sharpe Ratio in Table 5.6.2.2.1 is 1.01 (but is 1.0136 if more decimal points are included), compared to 1.0118 in Table 5.4.2.2.1 (where the shortfall probability is 95%, with no risk aversion). This shows that diversifying the portfolio with MOME has reduced risk. Although risk aversion is low in this case, the model still factors in an investor's preference. This is apparent in every situation in this section, compared to no inclusion of risk aversion. MOME has a higher return or Mu compared with VAL, but is riskier. The Alpha, which penalises risk, is lower for MOME. Therefore, the risk adjusted return of MOME is lower; however, by including it in the portfolio, it reduces the total risk profile of the investment.

The next set of results is found in **Table 5.6.2.2.2**. The shortfall threshold level is greater than or equal to 0% return. For periods 30 to 2 years before retirement, the portfolio consists of 10.00% ALBI, 25.02% MOME, 25.00% SAPY and 39.98% VAL. As explained previously, this distribution is due to diversification. With 1 working year remaining, the portfolio consists of 14.96% ILB, 11.91% MOME, 25.00% SAPY, and 48.13% VAL, and there is a 95% probability of reaching this return. This table shows both the risk averse coefficient and the shortfall threshold constraint at work. Periods 30 to 2 years are dominated by risk aversion. The model finds the best solution for the level of risk aversion and, in every case, the probability of the shortfall is less than 5%. With one year to go, the shortfall threshold comes into play and is limited to the 5% level.

If the model were allowed to find a solution and to disregard the short probability limit, it would result in the same portfolio as in the preceding periods. The probability of shortfall in that case would be 6.20%. The solution in period 1 is very similar to that of Table 5.4.2.2.2, where no risk aversion is used. That portfolio comprises 15.01% ILB, 8.36% MOME, 25.00% SAPY, and 51.63% VAL. Both portfolios invest in the same indices, but the magnitudes differ marginally, and the portfolio with minimal risk aversion is slightly less risky. The tables do not show it but the Annualised Sharpe Ratio in Table 5.4.2.2.2 is 1.0550 and in Table 5.6.2.2.2 it is 1.0555. Therefore both the shortfall threshold and risk aversion are used to find its solution. This solution increases the holdings in bonds and decreases the amount in equity. The Annualised Sharpe Ratio is also higher for this period. This follows financial theory and industry practice.

Table 5.6.2.2.1: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (No Shortfall Threshold) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	21.88%	20.47%	13.62%	1.01	21.88%	20.47%	13.62%	1.01	7.44
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.6.2.2.2: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_w-1/2(1-\gamma/T)s^2_w$ (Shortfall Threshold = 0) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	14.96%	11.91%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.48%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.39%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.10%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.03%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.6.2.2.3: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (Shortfall Threshold = Cash) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	3.95%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.01%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.27%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.07%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.6.2.2.4: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	4.92%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.26%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.34%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.09%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.6.2.2.5: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.57%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.42%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.12%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.6.2.2.6: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.50%	10.00%	0.00%	0.00%	25.00%	0.00%	34.50%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.93%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.52%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.15%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

The two roles played by these variables are interesting. The model suggests that, when a person wants to invest in a pension fund, for example, there are 3 main areas to address. The first is to find a suitable risk aversion coefficient. This will help the fund manager to decide the client's portfolio composition for the majority of his working life. The next step is to determine the level of return the client requires. This indicates to the fund manager how aggressive the investments should be. The last decision is the probability level of achieving that return. This will indicate to the manager how strictly he must reach the required return. This is more important when the client is nearing retirement.

Next, the model is used to find a solution for a shortfall threshold of cash. In **Table 5.6.2.2.3**, a portfolio of 10.00% ALBI, 25.02% MOME, 25.00% SAPY and 39.98% VAL is found for 30 to 2 years until retirement. This is the general solution for this level of risk aversion. The portfolio changes to 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL, with one remaining working year. This solution violates the probability limit of 5%. The Annualised Sharpe Ratio is higher in this period compared to prior periods, which was discussed in previous sections but is not always the case. In response, the portfolio increases investment in bonds and decreases equity, while also diversifying the holding by including STEFI. This has been discussed in the previous sections. It is interesting to note that the solution in period 1 is the same as that in Table 5.4.2.2.3, which does not include a risk aversion coefficient. This would suggest that when portfolios in this section violate the shortfall probability limit, the model also disregards risk aversion.

Table 5.6.2.2.4 shows the results when the shortfall threshold equals cash plus 2%. For 30 years to 2 years until retirement, the solution found is usually the same portfolio as above, which consists of 10.00% ALBI, 25.02% MOME, 25.00% SAPY and 39.98% VAL. The portfolio composition was justified for Table 5.6.2.2.1. When there is one remaining working year, however, the portfolio changes to comprise 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25% VAL. This portfolio breaks the shortfall threshold probability. The chance of not achieving cash plus 2% is 19.03%. The Annualised Sharpe Ratio is higher in period 1 than in the others. The solution found in Table 5.4.2.2.4 is the same as this portfolio, showing that the model has totally disregarded risk aversion when the shortfall constraint is violated.

When the shortfall threshold level is increased to cash plus 4%, **Table 5.6.2.2.5** refers. The usual portfolio breakdown is found for periods 30 to 3 years. In period 2, however, the portfolio changes to 15.84% ILB, 11.81% MOME, 25.00% SAPY and 47.35% VAL. This portfolio probability is at the upper limit of 95%. The comparative solution where risk aversion is excluded can be found in Table 5.6.2.2.5. At this stage, the portfolio comprises 15.87% ILB, 9.31% MOME, 25.00% SAPY and 49.82% VAL. It also has a 5% chance of shortfall. Again, both solutions include the same assets, albeit differing slightly in magnitudes. The portfolio when risk aversion is included is once again found to be safer. The Annualised Sharpe Ratios are 1.0581 and 1.0582 for Table 5.4.2.2.5 and Table 5.6.2.2.5 respectively. Although the difference is small, this shows that the model has combined the shortfall threshold constraint with the risk aversion coefficient.

At one year until retirement, the portfolio consists of 10.39% ILB, 12.48% MOME, 25.00% SAPY and 52.13% VAL. This breaches the shortfall limit with a probability of 25.10%, and this solution is identical to Table 5.4.2.2.5. Yet again, the model has omitted risk aversion when the probability constraint is violated.

The last table, **Table 5.6.2.2.6**, searches for solutions when the shortfall threshold is cash plus 6%. Periods 30 to 3 years are the same as before. The portfolio of period 2 contains 30.50% ILB, 10.00% MOME, 25.00% SAPY and 34.50% VAL. The threshold lies at the upper probability limit. Its comparative solution in Table 5.4.2.2.6 has a portfolio consisting of 30.51% ILB, 8.33% MOME, 25.00% SAPY and 36.16% VAL. The portfolio also has a 5% chance of shortfall. Again, the solution in Table 5.6.2.2.6 has a better risk profile. The Annualised Sharpe Ratios are 1.1095 and 1.1096 for Table 5.4.2.2.6 and Table 5.6.2.2.6 respectively. The portfolio in period 1 contains 10.00% ILB, 13.05% MOME, 25.00% SAPY and 51.95% VAL, which is the same portfolio as that in period 1 for Table 5.4.2.2.6. Both have violated the shortfall probability level with 30.17%.

5.6.2.3. Results: A Risk Aversion Coefficient of -0.5

In this section, the risk aversion coefficient is changed to -0.5. This implies that the investor has become more risk averse. This is clear from the objective function used in the model:

$$\alpha_W = (\mu_W - ½ (1-Y/T)\sigma^2_W) \qquad (5.9)$$

Y has been changed from 0 to -0.5. This will make the bracketed term (1-*Y/T*) larger for each period than it was in Section 5.6.2.2. Thus the term $(½ (1-Y/T)\sigma^2_W)$ will be greater and will penalise μ_W more, thus resulting in a smaller α_W. This shows the investor is more risk averse. The analysis of the results follows the same process as that of Section 5.6.2.2.

Table 5.6.2.3.1 shows the results when the risk aversion coefficient is -0.5 and there is no shortfall constraint. With these parameters, the portfolio comprises 10.00% ALBI, 24.58% MOME, 25.00% SAPY and 40.32% VAL for periods 30 to 15 years. When the risk aversion coefficient was 0, the portfolio consisted of 10.00% ALBI, 25.02% MOME, 25.00% SAPY, and 39.98% VAL. The holding in MOME has thus decreased and VAL has increased. MOME does have a higher Mu but is also riskier. The reason for the changed portfolio distribution is that the investor now has a higher penalty on risk, and thus some of the holdings in MOME have been swapped for VAL. This has resulted in a less risky portfolio, which is confirmed by the Annualised Sharpe Ratio, which has increased from 1.0136 to 1.0412. For periods 10 to 1 year before retirement, the portfolio contains 2.71% ALBI, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL. Between period 15 and period 10, some of the bond holdings have been transferred from ALBI to ILB. ILB has a lower Mu but is less risky and as a result, ILB has a better Sharpe Ratio. The holding in MOME has also decreased and moved to VAL. The resulting Annualised Sharpe Ratio for this portfolio is 1.03.

As an investor gets closer to retirement age, he puts a higher weight on risk. This mechanism is included in the objective function

$$\alpha_W = (\mu_W - ½ (1-Y/T)\sigma^2_W) \qquad (5.10)$$

Time until retirement (*T*) in the equation reduces, as the investor ages. The smaller (*T*) is the higher risk of penalties. The risk becomes too high in period 10 and so the portfolio switches. The comparative portfolio in Table 5.6.2.2.1 still has the same weights as identified at the beginning of the paragraph. Its Annualised Sharpe Ratio is 1.01. This shows that the portfolio in Table 5.6.2.3.1 has become less risky, which is in line with the investor's higher risk aversion.

Table 5.6.2.3.2 illustrates the portfolio weights when the shortfall threshold constraint is 0% return. Between 30 and 20 years before retirement, the portfolio weights are 10.00% ALBI, 24.69% MOME, 25.00% SAPY and 40.31% VAL. In Table 5.6.2.2.2, by comparison, when the risk aversion coefficient was 0, the portfolio comprised 10.00% ALBI, 25.02% MOME, 25.00% SAPY and 39.98% VAL. Due to the investor becoming more risk averse, the holdings in MOME have decreased and VAL has increased. As discussed before, VAL has a lower return and a better Sharpe Ratio, and is therefore less risky. This is in line with financial theory.

In periods 15 to 4 years until retirement, the portfolio consists of 10.00% ALBI, 23.51% MOME, 25.00% SAPY and 41.49% VAL. Again, the holding of MOME has decreased and VAL has increased. The magnitude of this change is larger than the change seen in periods 30 to 20 years, resulting in the Sharpe Ratio increasing from 1.01 to 1.02. This increase in the Sharpe Ratio or decrease in the portfolio's risk profile is a result of the investor approaching retirement, and this attaches a higher penalty to risk. This is also in line with financial theory.

Moving to periods 3 and 2, a similar change is seen. The portfolio now consists of 10.00% ALBI, 22.29% MOME, 25.00% SAPY and 42.71% VAL. MOME has increased even more, and VAL has decreased, resulting in a less risky portfolio. With 1 working year to go, the portfolio comprises 14.79% ILB, 11.98% MOME, 25.00% SAPY and 48.06% VAL and is at the probability level limit of 95%. The holdings in bonds have increased and equity has decreased. The holding in MOME has decreased, while VAL has increased. This results in an Annualised Sharpe Ratio of 1.06. All these findings are consistent with financial theory and industry practice.

Table 5.6.2.3.1: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (No Shortfall Threshold) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	21.82%	20.46%	13.32%	1.03	21.82%	20.46%	13.32%	1.03	7.75
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	328.18%	307.03%	52.72%	3.93	21.88%	20.47%	13.61%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	437.58%	409.37%	60.87%	4.54	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	546.97%	511.71%	68.06%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.3.2: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	14.97%	11.98%	0.00%	0.00%	25.00%	0.00%	48.06%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.45%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	0.37%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.10%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.03%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.3.3: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (Shortfall Threshold = Cash) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	3.90%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	0.99%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.27%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.07%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.3.4: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	4.86%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.24%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.33%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.09%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.3.5: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.83%	11.80%	0.00%	0.00%	25.00%	0.00%	47.37%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.54%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.42%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.11%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.3.6: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.00%	0.00%	0.00%	25.00%	0.00%	34.51%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	22.57%	0.00%	0.00%	25.00%	0.00%	42.43%	1.90%	65.62%	61.44%	23.52%	1.76	21.87%	20.48%	13.58%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	22.57%	0.00%	0.00%	25.00%	0.00%	42.43%	0.51%	87.49%	81.92%	27.15%	2.03	21.87%	20.48%	13.58%	1.02	7.49
5	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.14%	109.38%	102.36%	30.40%	2.27	21.88%	20.47%	13.60%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	218.77%	204.73%	43.00%	3.21	21.88%	20.47%	13.60%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	328.15%	307.09%	52.66%	3.93	21.88%	20.47%	13.60%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	437.54%	409.45%	60.81%	4.54	21.88%	20.47%	13.60%	1.02	7.47
25	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	546.92%	511.81%	67.98%	5.08	21.88%	20.47%	13.60%	1.02	7.47
30	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	656.31%	614.18%	74.47%	5.56	21.88%	20.47%	13.60%	1.02	7.47

Table **5.6.2.3.3** shows the results for a situation where the shortfall threshold level equals cash and the risk aversion coefficient is -0.5. The portfolio compositions for 30 to 2 years until retirement are the same as those in Table 5.6.2.3.2. All these portfolios have a lower risk profile than Table 5.6.2.2.3, when the risk aversion coefficient is 0. In the last period, the portfolio contains 20.45% ILB, 2.47% ILB, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. The probability of the shortfall is 11.13%, which violates the upper limit. The Annualised Sharpe Ratio is increased from 1.2 to 1.23. This is in line with financial theory; however, as has been shown before, when a portfolio violates the probability constraint, this decrease in the risk profile does not always occur. It is interesting to note that the portfolio in year 1 in Table 5.6.2.2.3 is exactly the same.

Next, the portfolio shortfall threshold constraint is increased to cash plus 2%. The resulting portfolios are set out in **Table 5.6.2.3.4**. Again, the portfolios in years 30 – 2 are the same as those in the previous table, and they have a higher Annualised Sharpe Ratio compared to when the risk aversion coefficient was 0. The portfolio in year 1 contains 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25% VAL. The Annualised Sharpe Ratio has increased from 1.02 to 1.17, but the probability of shortfall is 19.03%. This portfolio is exactly the same as the portfolio in Table 5.6.2.2.4, when the risk aversion coefficient was 0.

In **Table 5.6.2.3.5**, the shortfall threshold level is cash plus 4%. The portfolios for 30 – 3 years until retirement are the same as the portfolios in the previous table. Less has been invested in MOME and more in VAL than in Table 5.6.2.2.5. This has reduced the risk profile of the investment in line with the increase of the risk aversion coefficient. In period 2, the portfolio consists of 15.83% ILB, 11.80% MOME, 25.00% SAPY and 47.37% VAL. This is at the limit of a 5% chance of a shortfall. The Annualised Sharpe Ratio has increased from 1.02 to 1.06 from the prior period. The Annualised Sharpe Ratio has also increased slightly, since the risk aversion coefficient was 0 in Table 5.6.2.2.5.

In period 1, the portfolio contains 10.39% ILB, 12.48% MOME, 25.00% SAPY and 52.13% VAL. The probability level has been violated by a percentage of 25.10%. This violation has changed the portfolio exactly in the same way as occurred with the portfolio found in year 1 of Table 5.6.2.2.5. The Annualised Sharpe Ratio has also decreased between period 2 and period 1, from 1.06 to 1.04. This change in the portfolio's composition when the probability level has been violated has been shown before. The change opposes financial theory and logic.

The shortfall threshold has been increased to cash plus 6% in **Table 5.6.2.3.6**. For periods 30 – 5 years, the portfolios comprise 10.00% ALBI, 23.78% MOME, 25.00% SAPY and 41.22% VAL. The Annualised Sharpe Ratio is 1.02 for these results, and it has increased from 1.01 in Table 5.6.2.2.6. In periods 4 – 3 years until retirement, the portfolios consist of 10.00% ALBI, 22.57% MOME, 25.00% SAPY and 42.43% VAL. For periods 30 – 3 years, the risk profiles have decreased compared to when the risk aversion coefficient was 0. Two years before retirement, the portfolio has changed to contain 30.49% ILB, 10.00% MOME, 25.00% SAPY and 34.51% VAL. This is at the limit of the 5% shortfall probability. The Annualised Sharpe Ratio is 1.11, which represents a slight increase from Table 5.6.2.2.6, although the 2 decimal points do not show this. Again, in period 1, the portfolio violates the shortfall constraint probability level. The portfolio is the same as it was when the risk aversion coefficient was 0. The Annualised Sharpe Ratio has decreased from 1.11 to 1.04 from period 2 to 1. This is because the holdings in bonds have decreased and equity has increased. These results are due to the investor chasing returns rather than consolidating his or her position. This approach, as has been discussed before, runs counter to financial theory and practice.

5.6.2.4. Results: A Risk Aversion Coefficient of -1

In Section 5.6.2.4, the risk aversion coefficient has changed to -1. This causes the investor to become more risk averse. The coefficient exposes the associated risk in the model to a greater penalty. The shortfall probability level is kept at 95%. The results in Tables 5.6.2.4.1 to 5.6.2.4.6 below follow the same process as before.

Table 5.6.2.4.1: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (No Shortfall Threshold) (γ = -1) at a probability of 95%

Years Until	ω										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	21.80%	20.46%	13.21%	1.04	21.80%	20.46%	13.21%	1.04	7.87
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Table 5.6.2.4.2: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_{w}-1/2(1-γ/T)s^{2}_{w} (Shortfall Threshold = 0) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	14.96%	11.90%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.31%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.32%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.08%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.02%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.4.3: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	3.63%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.88%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.22%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.06%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.4.4: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	4.56%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.11%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.28%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.07%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.6.2.4.5: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	1.53%	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.41%	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.11%	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Table 5.6.2.4.6: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.48%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	1.89%	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.51%	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.14%	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

The first table, labelled **Table 5.6.2.4.1**, shows the situation when there is no shortfall threshold level, with a risk aversion coefficient of -1 at a probability level of 95%. For periods 30 – 3 years, the solution reflects a portfolio of 10.00% ALBI, 21.61% MOME, 25.00% SAPY and 43.39% VAL. This portfolio has an Annualised Sharpe Ratio of 1.02. In periods 2 and 1, the portfolio consists of 10.00% ILB, 17.11% MOME, 25.00% SAPY and 47.89% VAL. The Annualised Sharpe Ratio is 1.04, which is higher than the 1.03 level when risk aversion was -0.5.

In **Table 5.6.2.4.2**, the shortfall threshold level has been changed to equal at least 0%. The portfolio in period 30 years comprises 10.00% ALBI, 24.58% MOME, 25.00% SAPY and 40.42% VAL. This is very similar to the portfolio in Table 5.6.2.3.2, although the investment in MOME has decreased slightly and in VAL has increased marginally. This has made the investment less risky, thus aligning it with the investor's increased risk aversion. In periods 25 – 2 years, the portfolios contain 2.71% ALBI, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL. Compared to Table 5.6.2.3.2, this portfolio is more diversified thanks to the inclusion of ILB, the holdings in the riskier MOME are smaller, and the less risky VAL is more heavily invested. This makes this portfolio's profile less risky, which is confirmed by the increase in the Annualised Sharpe Ratio from 1.02 to 1.03. The profile in the last period includes 14.96% ILB, 11.90% MOME, 25.00% SAPY and 48.13% VAL. The probability level is at the 95% limit. This portfolio has also decreased its holding in MOME and increased it in VAL, thus making it less risky than its 0.5 counterpart. This is all consistent with financial theory.

Table 5.6.2.4.3 has increased the shortfall threshold level to at least equal cash. The portfolios in periods 30 – 2 years are the same as those in Table 5.6.2.4.2, and therefore have the same justifications. The portfolio in period 1 has 20.45% ILB, 2.47% MOME, 7.40% SAPY, 67.18% STEFI and 2.50% VAL. This violates the probability level of shortfall by 11.13%. This portfolio is the same as its Table 5.6.2.3.3 counterpart. The Annualised Sharpe Ratio has increased to 1.23. This increase is not always the case when the shortfall threshold constraint is violated.

Table 5.6.2.4.4 looks at the situation where the shortfall threshold level equals cash plus 2%. Again, the portfolios from 30 – 2 years until retirement are the same as those in the previous two tables, and therefore the rationale is also the same. The portfolio in period 1 consists of 46.74% ILB, 8.01% MOME, 25.00% SAPY and 20.25% VAL. The probability of shortfall is violated with a 19.03% chance. This is the identical result seen in the counterparts of this period, which are found in Table 5.6.2.2.4 and Table 5.6.2.3.4.

Table 5.6.2.4.5 analyses the situation where the shortfall threshold level is cash plus 4%. The portfolio for period 30 – 3 years comprises 10.00% ALBI, 21.61% MOME, 25.00% SAPY and 43.39% VAL. The increase of MOME and the decrease in VAL compared to that of the previous table, can be explained by the higher level of return required by the investor in this scenario. This portfolio still has a lower risk profile than that of Table 5.6.2.3.5, which is in line with the increase in the risk aversion coefficient. The investment in period 2, at the 95% probability level, contains 15.84% ILB, 11.81% MOME, 25.00% SAPY and 47.35% VAL. Its risk profile is also slightly less than its -0.5 counterpart. With one year until retirement, the portfolio breaks the probability constraint, which is the same as in its Table 5.6.2.2.5 and Table 5.6.2.3.5 counterparts. The Annualised Sharpe Ratio increases from period 3 to period 2, which is consistent with financial theory. However, the Annualised Sharpe Ratio then decreases from period 2 to period 1.

The last set of results for risk aversion coefficient -1 is shown in **Table 5.6.2.4.6**. The portfolios with 30 – 3 years until retirement are the same as those of the previous table, and thus have the same justifications. In period 2, the portfolio contains 30.49% ILB, 10.03% MOME, 25.00% SAPY and 34.48% VAL and is at the 95% threshold probability level. This portfolio also has a slightly higher Annualised Sharpe Ratio compared to its -0.5 risk aversion counterpart. The portfolio in the last period violates the shortfall probability level and is exactly the same as its counterparts in Table 5.6.2.2.6 and Table 5.6.2.3.6. The Annualised Sharpe Ratio increases from 1.02 to 1.11 in period 2, but then decreases to 1.04 a year later. This violation of the probability level means that the solution proposed is contrary to financial theory once more.

5.6.2.5. Results: A Risk Aversion Coefficient of -1.5

In this section, the risk aversion coefficient is changed to -1.5, and the probability level is kept at 95%. The shortfall threshold levels are the same as those of the previous sections.

Table 5.6.2.5.1 below shows the results for a situation with no shortfall threshold and a risk coefficient of -1.5. The results for period 30 – 3 years are 2.71% ALBI, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL. The results when the risk aversion coefficient was -1 were 10.00% ALBI, 21.61% MOME, 25.00% SAPY and 43.39% VAL. The investor has now diversified his portfolio more by including ILB. Investment in the riskier equity index of MOME has decreased, whereas the safer VAL index has increased in magnitude. This has increased the Annualised Sharpe Ratio from 1.02 to 1.03. This is intuitively sound, as an investor is more likely to be risk averse at this stage. In the last two periods until retirement, the portfolio comprises 10.00% ILB, 15.75% MOME, 25.00% SAPY and 49.25% VAL. The counterpart portfolio for the risk aversion of -1 consisted of 10.00% ILB, 17.11% MOME, 25.00% SAPY and 47.89% VAL. The investment has thus decreased its holdings in MOME and increased in VAL, which has resulted in a slightly safer strategy. The Annualised Sharpe Ratio to two decimal points does not show this, but the figures have changed from 1.0392 to 1.0399.

Table 5.6.2.5.2 has included a shortfall threshold level of 0%. The results for period 30 – 3 years until retirement are the same as those in Table 5.6.2.5.1, and offer a safer investment compared to its -1 risk aversion coefficient counterpart. With two years until retirement, the portfolio changes to 10.00% ILB, 18.09% MOME, 25.00% SAPY and 46.91% VAL. This portfolio is a less risky investment than that of the previous periods. The shorter time horizon has caused a greater risk penalty. Its risk profile is also less risky than its counterpart found in Table 5.6.2.4.2; this is in line with the change in the risk aversion coefficient. This is evident from the Annualised Sharpe Ratio, which has increased from 1.03 to 1.04. The portfolio in the last period consists of 14.96% ILB, 11.92% MOME, 25.00% SAPY and 48.12% VAL. This is at the 5% shortfall limit. The Annualised Sharpe Ratio has increased from 1.04 to 1.05 from

Table 5.6.2.5.1: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (No Shortfall Threshold) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	21.80%	20.47%	13.20%	1.04	21.80%	20.47%	13.20%	1.04	7.88
2	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	43.60%	40.94%	18.66%	1.47	21.80%	20.47%	13.20%	1.04	7.88
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.5.2: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	1.26%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.32%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.08%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.02%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.5.3: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (Shortfall Threshold = Cash) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	3.54%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.88%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.22%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.06%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.5.4: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	4.45%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.11%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.28%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.07%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.5.5: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.39%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.36%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.09%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.5.6: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.48%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.73%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.44%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.12%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

period 2 to period 1 due to increasing the investment in bonds, while decreasing equity. VAL's total investment has actually increased from the period before, whereas the riskier MOME Index has decreased by a large amount.

The results in **Table 5.6.2.5.3** illustrate the situation when the shortfall threshold level has been increased to cash. The results for period 30 – 2 years are the same as those in Table 5.6.2.5.2. The rationale and comparison are therefore the same as those contained in the previous paragraph. The portfolio for 1 year until retirement is the same as its counterparts in Table 5.6.2.2.3, Table 5.6.2.3.3 and Table 5.6.2.4.3. When the shortfall probability constraint is violated, the same findings occur across the different risk aversion coefficients.

Table 5.6.2.5.4 demonstrates the results for a shortfall threshold level of cash plus 2%. The portfolios in period 30 – 2 years until retirement are the same as the results of the previous table. The justifications for these results have been discussed and are financially sound. The portfolio in the last year is the same as the portfolios for the different risk aversion coefficients, because the probability level of the shortfall is being violated.

Table 5.6.2.5.5 shows what happens when the shortfall threshold has been increased to cash plus 4% return. The results are the same for periods 30 – 3 years. With two years until retirement, however, the portfolio changes, and now comprises 15.84% ILB, 11.81% MOME, 25.00% SAPY and 47.35% VAL. It is at the upper probability level. The Annualised Sharpe Ratio has increased from 1.03 to 1.06, due to the increased investment in bonds, the decrease in holdings of the MOME and increase in the VAL. In the last year, however, the portfolio violates the probability level and the Annualised Sharpe Ratio thus decreases from 1.06 to 1.04. The portfolio is the same as its counterparts from the different risk aversion coefficient scenarios.

The last table, labelled **Table 5.6.2.5.6,** contains the results for an investor requiring a return of cash plus 6%. The results from periods 30 – 3 years are the same as those in the previous tables. In period 2, however, the composition changes to increase the investment in bonds and to decrease investment in equity. This results in the Annualised Sharpe Ratio increasing from 1.03 to 1.11. In the last period, the probability level is violated, which is the same as in the counterpart portfolios from the previous risk aversion coefficient scenarios.

5.6.2.6. Results: A Risk Aversion Coefficient of -2

Section 5.6.2.6 presents the final set of results for the shortfall threshold constraint and risk averse coefficient model. In this scenario, the risk aversion coefficient is set at -2, and the probability level is kept at 95%. The same 6 different shortfall threshold levels are examined.

Table 5.6.2.6.1 shows the results when no shortfall threshold is included. The portfolio for periods 30 – 4 years comprise 2.71% ALBI, 7.29% ILB, 19.68% MOME, 25.00% SAPY and 45.32% VAL. This same portfolio composition was seen when risk aversion was fixed to a -1.5 coefficient level. This suggests that the risk averse level of -2 may be approaching the upper limit of risk aversion. Thus, it seems that the full spectrum of risk aversion has been examined. In period 3, the portfolio is made up of 10.00% ILB, 18.46% MOME, 25.00% SAPY and 46.52% VAL. The model has created a safer portfolio profile due to the shorter time horizon, which is evident from the increase in the Annualised Sharpe Ratio from 1.03 to 1.04. The portfolio becomes even safer in periods 2 and 1, when it consists of 10.00% ILB, 14.84% MOME, 25.00% SAPY and 50.16% VAL.

In **Table 5.6.2.6.2**, the shortfall threshold constraint is equal to 0%. For periods 30 – 5 years the portfolio comprises 8.27% ALBI, 1.73% ILB, 20.94% MOME, 25.00% SAPY and 44.06% VAL. This changes to 10.00% ILB, 17.11% MOME, 25.00% SAPY and 47.89% VAL in the last 3 years before retirement. These allocations have slightly smaller risks than their counterparts found in Table 5.6.2.5.2. In the last year before retirement, the portfolio consists of 14.96% ILB, 11.92% MOME, 25.00% SAPY and 48.12% VAL. This portfolio becomes less risky closer to retirement, as illustrated by the increase in the Annualised Sharpe Ratio from 1.02 to 1.04 and then to 1.06.

Table 5.6.2.6.1: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (No Shortfall Threshold) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	21.80%	20.47%	13.19%	1.04	21.80%	20.47%	13.19%	1.04	7.89
2	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	43.59%	40.95%	18.65%	1.47	21.80%	20.47%	13.19%	1.04	7.89
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	N/A	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.6.2: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_w-1/2(1-\gamma/T)s^2_w$ (Shortfall Threshold = 0) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	1.25%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.30%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.08%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.03%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.6.2.6.3: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^2_{\omega}$ (Shortfall Threshold = Cash) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	3.53%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.84%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.21%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.07%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.6.2.6.4: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	4.44%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	1.06%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.27%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.09%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.6.2.6.5: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	1.34%	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.36%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.09%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.6.2.6.6: Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.47%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	1.67%	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.44%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.12%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

In **Table 5.6.2.6.3**, the shortfall threshold level is increased to cash. The results for periods 30 – 2 years are the same as those in the previous table. The results with one year remaining are the same as the findings from its counterparts in the previous sections. This is due to the probability level being violated.

The results of **Table 5.6.2.6.4** results are the same as those of the previous table. The portfolio is identical for periods 30 – 2 years. The results in period 1 match those of its different risk aversion coefficient counterparts.

All the results from **Table 5.6.2.6.5** are the same as the results in Table 5.6.2.5.5, except for the 3 years until retirement. This again suggests that the risk aversion coefficient of -2 may be at the upper limit. Increasing risk aversion further will not affect the results in any significant manner. Period 3 is slightly different to Table 5.6.2.5.5 due to the additional risk aversion.

The last table labelled **Table 5.6.2.6.6** has similar results to those of the previous table, with the exception of period 3, which differs slightly. Finally, the Annualised Sharpe Ratio increases while nearing retirement, but due to the violation of the probability level in the last period, the portfolio changes contrary to financial theory and practice.

5.6.2.7. Conclusion

The analysis in the above sections has demonstrated that combining a risk aversion coefficient and a shortfall threshold constraint in a portfolio optimiser model gives interesting results. For the majority of the time, every portfolio contains the following weightings with regard to the asset classes: 25% in property, 65% in equity and 10% in bonds. As discussed in the previous sections, the model fills the portfolios with the best performing asset class first, in this case, 25% in SAPY. Equity in general was the second best performing asset class, and the maximum allowed amount of 65% is used. The remainder is invested into the third best asset class, namely, bonds. These weightings give an investor a good starting point when deciding where to invest.

The model recommends investing in a combination of the 2 best performing equity indices for extra diversification. The choice of the proportion to be allocated to MOME and VAL depends on the investor's level of risk aversion: the percentages range from 25.02% MOME and 39.98% VAL for the least risk averse investor, to 17.11% MOME and 44.06% VAL for a more risk averse investor. This strategy only works for the periods further away from retirement, where risk aversion plays a dominant role in the model.

This is also the case when choosing bonds. The 10% invested in bonds is split between ALBI and ILB. In general, when the investor is not risk averse, the full amount is invested in ALBI; the more risk averse an individual becomes, the more the weight moves away from ALBI and towards ILB. Either a combination is chosen or the entire percentage is moved into ILB.

Having a maximum and minimum range for every asset class gives investors a reasonable basis, when trying to understand and approximate their clients' risk aversion profile. The model's results are financially sound when the shortfall probability levels are not violated. In general, the model shows that an investor's reliance on VAL and ALBI should decrease in the last few years before retirement; these assets have the highest returns for equity and bonds respectively. The holdings thus shift to increase weightings in MOME and ILB, which are the second best performing equity and bond instruments respectively. In certain situations, we have seen STEFI being included in these late portfolios, but only when the shortfall threshold return is low enough given STEFI's small returns.

The allocations to MOME range between 0% and 25%, while those of ILB range between 0% and 46.74%. It is interesting that the allocation to bonds never increases higher than 46.74%. This is an important finding for investors who generally have positions in bonds that make up the majority of a portfolio for a client nearing retirement.

The portfolios have adjusted correctly in response to changes in risk aversion, shortfall threshold level and time until retirement. Problems in this and the other models in Section 5.4 occur, when the probability levels are disregarded. Due to the probability levels not being adhered to for some scenarios, it is difficult to conclude on the shortfall threshold constraint and the portfolios near to retirement, which are dominated by this constraint in this model. The model should therefore be changed until all the results are correct in terms of financial theory and industry practice. In Section 5.7 below, the model's inputs are adjusted slightly to ensure that the portfolios are appropriately and optimally structured.

5.7. Amendments to the Mean-Variance Model

5.7.1. Introduction

The aim of this paper is to find various portfolio mixes, using a range of asset allocations, for different types of investors in South Africa. A model was built, based on resources from prior research. The majority of the findings from the model have given insight into structuring portfolios that meet investor's needs and been financial sound. However, some of the results, particularly in the last period before retirement, have violated the shortfall constraint. This has occurred because the shortfall threshold level required by the investor is too high to obtain in one year, with reasonable certainty. In response to such an excessive shortfall threshold level, the portfolios would have switched to a high-risk high return investment in this period, thus chasing a high minimum return. Effectively, as a gamble in the final year before retirement, this approach is contrary to financial theory and common sense. In such a situation, the portfolio must be amended, so that an optimal solution can be found. A new model that can automatically decrease the shortfall constraint level, in the last period, to a more reasonable return, would be the goal. However, in this paper, the shortfall return was decreased manually until the results fell within the allotted probability level. Thus, a maximum shortfall level was found in each portfolio, for the last year that would fit the constraint. The portfolios that were re-done in this paper were maximising Alpha with a shortfall threshold constraint at 95% probability and no risk aversion, and maximising Alpha with a shortfall threshold constraint at 95% and risk aversion, ranging for Y=0 to Y=-2. This gives a good indication of the last year's portfolios for analysis purposes.

5.7.2. Amendments to Maximising Alpha Using a Shortfall Constraint and No Risk Aversion

As explained in the section above, the shortfall level was reduced in the last period, so that the shortfall constraint is not violated. In practical terms, it means that an investor has lowered the required annual return in the last year before retirement. This insures that the investment is safe and that there is no gambling with assets just before retirement. The shortfall threshold was decreased from the different levels used in the paper. In this section, the highest required return, which fits in with the shortfall constraint, was found to be a return of 7.61%. The results of this approach are found in Appendix 4; the tables show the original portfolios and have an extra line for the amendments made in year 1.

There are no changes for Table 5.5.2.1 and Table 5.5.2.2 (see Appendix 4). For the rest of the tables, the amended portfolio in period 1 consists of 10.64% ILB, 1.46% MOME, 3.01% SAPY, 84.06% STEFI and 0.83% VAL. This portfolio setup has a shortfall threshold level of 7.61% and always abides by the shortfall constraint probability level. The portfolio investments are distributed among the best performing asset (SAPY), the two best equity assets (MOME and VAL) and the bond asset with the best risk return – but all these only make up 15.94%. The rest is invested in STEFI, because the returns on cash are known for a year ahead, which makes the last year's return almost certain. The Annualised Sharpe Ratios always increase from year 2 to year 1, which is also in agreement with financial theory. The investor lowering their required return has almost guaranteed their retirement savings. This is a more appropriate investment strategy for the last year before retirement, than the other models, which were gambling on achieving high returns.

From this model, it can be concluded that an investor should invest in the highest return assets from different asset classes for the majority of their life. In the last two to three years, however, the portfolio should be diversified slightly by means of equity and by increasing the holdings in bonds. However, bonds should never make up more than 50% of the portfolio at any time. In the last year, around 85% of the portfolio should be invested in the money market to secure retirement funds.

5.7.3. Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion

The set of results for this section are found in Appendix 5 – 9. The same approach was followed here as in Section 5.7.2; however, the model also includes risk aversion. The results are only for a shortfall probability of 95%. The results include all the different shortfall constraint levels of this paper and risk aversion from 0 to -2.

5.7.3.1. Amended Results: A Risk Aversion Coefficient of 0

The results of this section can be found in Appendix 5. The tables are set up in the same way as those in Section 5.7.2 to show the two portfolios in year 1, namely, the old portfolio and the amended portfolio. The shortfall threshold level was reduced until the shortfall constraint was met. The shortfall level was again found to be a return of 7.61%. The portfolio setup in year 1 was not changed on the first two tables (viz. Table 5.7.3.1.1 and Table 5.7.3.1.2), because the shortfall constraint was not violated. However, the portfolios did change for the last four tables. The amended portfolios all consisted of 10.62% ILB, 1.48% MOME, 3.01% SAPY, 84.08% STEFI and 0.81% VAL. This distribution was found in for every amended portfolio in year 1, throughout all the different risk aversion levels, thus confirming that the shortfall constraint totally dominates the risk aversion constraint in year 1.

The portfolios for the majority of the investor's life are dominated by risk aversion. The portfolios consist of 10.00% ALBI, 25.02% MOME, 25.00% SAPY and 39.98% VAL. The shortfall constraint only comes into effect when the probability of the shortfall reaches 5%. When the shortfall constraint comes into play in period 2, this causes the level of equity to decrease, the level of bonds to increase, and the bond investment to switch from ALBI to ILB. ILB has a lower return and Standard Deviation than ALBI, and ILB thus has a better risk return profile and a higher Sharpe Ratio. When period 1 is reached, all the shortfall threshold levels are reduced to 7.61%. The equity levels decrease (they decrease further if the shortfall constraint was activated in period 2), the bond holding switches from ALBI to ILB the level in ILB is further reduced, if the shortfall constraint was activated in period 2), and the large amount is moved to STEFI. This is because STEFI offers very stable returns and because the shortfall threshold is low enough for STEFI to reach it. An important aspect to notice is the Annualised Sharpe Ratio either stays the same or it increases, as retirement nears. This was not always the case without the amendments.

5.7.3.2. Amended Results: A Risk Aversion Coefficient of -0.5

The results of this section are found in Appendix 6. The tables illustrate the amendments when the shortfall probability is violated. The shortfall threshold level needed for the amendments is again found to be 7.61%. The portfolios for the majority of the time are similar to those in the section above, but differ slightly in magnitude, which is caused by the higher level of risk aversion. The portfolios change again closer to retirement, in accordance with the shortfall constraint. Very similar changes occur as in the section above. Exposure to equity decreases, bonds change from ALBI to ILB, and there is a large movement to STEFI in the last period. The Annualised Sharpe Ratio remains the same or increases as retirement nears.

5.7.3.3. Amended Results: A Risk Aversion Coefficient of -1, -1.5 and -2

The last three sets of tables are found in Appendix 7, 8 and 9; these present the results of the model, when the risk aversion coefficient changes from -1 to -1.5 and to -2 respectively. The appropriate shortfall threshold level is found to be 7.61%. The movements and changes in these tables are very similar to those discussed in the two previous sections. The portfolios differ slightly over much of the time, due to differing risk aversion levels, has discussed in the section before. When the investor nears retirement, equity levels are reduced, bonds switch from ALBI to ILB, and a large amount is moved to STEFI in the last period. As was the case in the previous scenarios, the Annualised Sharpe Ratio remains the same or increases nearer to retirement.

6. Conclusion and Suggested Extensions

6.1. Introduction

This section discusses the main conclusions for each set of results. The conclusions set out a general platform according to which investments should be made in the South African context, given current legislation and investor's needs. Limitations of this paper and suggestion for further research are also discussed.

6.2. Conclusion: Maximising Alpha with Risk Aversion Only

Quantifying risk aversion can be a difficult task, but a full range was included in this paper. Essentially, a portfolio must comply with Regulation 28. This means that all portfolios should be filled up with the best risk to return asset class first, then the second best and so on, until the entire portfolio has been invested. In a South African context, property (SAPY) is the best performing asset class, and thus 25% of the portfolio is invested in SAPY; this is also the maximum allowed in terms of Regulation 28. Generally equity is the second best asset class based on returns to risk, and thus 65% of the portfolio is invested in equity. Bonds are the third best asset class, and thus take up the remaining 10%. This is the general make-up of the portfolio.

With regard to equity, there is a choice between MOME and VAL. In this regard, the investment magnitudes range between 25.02% MOME and 39.98% VAL, and 14.84% MOME and 50.16% VAL respectively. The more risk averse an investor is, the higher the percentage invested into VAL is, because of its relatively low-risk risk return profile. Bonds, similarly, are split between ALBI and ILB. These range between being invested entirely in ALBI for lower levels of risk aversion, due to its higher returns, and being invested entirely in ILB for more risk averse investors, due to the better risk return profile of ILB. SAPY is always invested in totally, whereas Cash (STEFI) is never invested in. There is no evidence to suggest that a younger investor should invest more in riskier assets than should an older investor.

6.3. Conclusion: Maximising Alpha with a Shortfall Constraint Only

Quantifying the required return on an investor is simple, compared to estimating risk aversion. For the majority of an investor's life, his or her portfolio should consist of 25% property, 65% equity and 10% bonds. This general profile has been found and confirmed by using two different model variations. However, equity will always be entirely invested in VAL, while bonds will always be entirely invested in ALBI. Both of which are the higher returns assets to its counterpart MOME and ILB. The portfolio structure should only change when an investor is 1 or 2 years before retirement (occasionally 3 years before retirement in extreme cases). This differs from financial practice, where investments tend to change gradually over time.

The changes in the last few years, with regard to equity, consist of decreasing VAL and increasing MOME, and thus diversifying the portfolio slightly. With regard to bonds, ALBI is reduced to zero, while ILB increases to more than the 10% level. The highest percentage of investment in bonds is only 57.07% in an extreme case and 30.51% for more general cases. This maximum level of 30.51% in bonds comes as a surprise, as, in financial practice, this level is much higher for investors nearing retirement. However, the model does violate its shortfall constraint from time to time, and thus these results will be left out of the discussion for now and will be introduced in Section 6.6 below, when the amendments to the model are described.

6.4. Conclusion: Maximising Growth with Risk Aversion Only

The results obtained with the arithmetic returns were every similar to their logarithmic counterpart. This could imply that the arithmetic returns were also normally distributed in this case, similarly to the logarithmic returns. However, prior research has shown that logarithmic returns are more appropriate for a mean-variance framework, such as the one analysed in this paper.

6.5. Conclusion: Maximising Alpha with Risk Aversion and a Shortfall Constraint

For the majority of an investor's life, the risk aversion constraint dominates the portfolio allocation. The portfolio should thus be set up in accordance with Section 6.2, consisting of 25% property, 65% equity and 10% bonds. With regard to equity, there can be a split between MOME and VAL, and with regard to bonds, between ALBI and ILB, with exactly the same percentages as stated in Section 6.2.

For the last few years until retirement, however, the shortfall constraint dominates the portfolio setup. These portfolios consist of the proportions as stated in Section 6.3, namely: MOME and VAL are reduced close to retirement, while ILB is included or increased compared to ALBI.

6.6. Conclusion: Amendments to the Model and Final Portfolio Allocation

As explained in the previous sections, the portfolios in the different scenarios consist of the 25/65/10 split most of the time, and for almost the entire duration of a 30-year-long investment. However, the model must be amended when the shortfall probability constraint is violated. This is done manually by changing the shortfall threshold return level; this level is found to be 7.61% in this paper.

With regard to asset distribution, it can be concluded from the various scenarios tested herein, that for most of an investor's life, the portfolios should consist of 25% property, 65% equity and 10% bonds. Equity comprises MOME and VAL, ranging between 25.02% MOME, 39.98% VAL and 14.84% MOME, 50.16% VAL respectively, as risk aversion increases. Bonds comprise ALBI and ILB, which range between being invested entirely in ALBI for lower levels of risk aversion, due to its higher returns, and being invested entirely in ILB in the case of more risk averse investors, due to its better risk return profile. 25% is always invested in SAPY. As retirement gets closer, the shortfall constraint comes into play, and this causes the amount of equity to be reduced significantly. Moreover, with 2 years left before retirement, the investment shifts away from ALBI towards ILB. A maximum of 30.51% is invested in bonds, which is much lower than the industry standard. In the final year before retirement, the shortfall threshold is reduced to 7.61%. At this point, ILB, MOME, SAPY and VAL only make up 15.94% of the portfolio, whereas the remaining portfolio is invested in cash (STEFI), which secures the investment.

In view of the findings discussed herein, this paper does not support the industry practice of gradually increasing bonds and eventually cash, while decreasing equity. Although this does reduce the risk profile of a portfolio, it reduces it too much and too early in an investor's life. An investor should have a portfolio of 25/65/10 until two year before retirement. Equity should then be reduced, and bonds increased a year later. In the last year, an investor should lower the level of return and invest the majority of the portfolio in cash.

6.7. Limitations and Suggestions for Further Research

This paper does have certain limitations. The returns used are constant expected returns, which do not resemble true life. Only a few indices were used in the model. The model does violate a constraint from time to time, although this problem was manually dealt with, and lastly, risk aversion is hard to quantify.

There are some points of interest that require further research. Arithmetic returns can be used in a South African context; this would require research about the distribution of different assets' arithmetic returns. The model should be altered, so that the shortfall threshold constraint is automatically reduced when the probability level is violated. A James-Stein Estimator can be included to better approximate expected returns. This estimator is centred on the idea of "shrinking" individual sample. Reducing high estimates and increase low estimates towards a common value does this. This is known as the Grand Mean. The model could also be used with asset returns from different countries and the results compared.

7. List of Appendices

Appendix 1 – Maximising Growth Rate at 90% Probability

Appendix 2 - Maximising Growth Rate at 95% Probability

Appendix 3 - Maximising Growth Rate at 99% Probability

Appendix 4 - Amendments to Maximising Alpha Using a Shortfall Constraint and No Risk Aversion

Appendix 5 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of Y=0

Appendix 6 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of Y = -0.5

Appendix 7 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of Y = -1

Appendix 8 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of Y = -1.5

Appendix 9 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of Y = -2

Appendix 1 – Maximising Growth Rate at 90% Probability

Table 5.5.2.1: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (No Shortfall Threshold) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	20.75%	22.00%	13.74%	1.01	20.75%	22.00%	13.74%	1.01	7.36
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.2.2: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = 0) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.27%	20.75%	22.00%	13.74%	1.01	20.75%	22.00%	13.74%	1.01	7.36
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.51%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.40%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.11%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.2.3: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.91%	2.69%	0.00%	0.00%	7.57%	66.34%	2.49%	11.18%	10.29%	10.50%	1.96%	1.23	10.29%	10.50%	1.96%	1.23	62.73
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.00%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.03%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.28%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.06%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.2.4: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 2%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.92%	0.00%	0.00%	25.00%	0.00%	19.34%	18.92%	16.42%	17.26%	7.79%	1.18	16.42%	17.26%	7.79%	1.18	15.11
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.97%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.29%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.35%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.2.5: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 4%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	11.57%	14.19%	0.00%	0.00%	25.00%	0.00%	49.24%	24.95%	20.46%	21.78%	13.05%	1.05	20.46%	21.78%	13.05%	1.05	8.03
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	6.12%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.59%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.43%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.2.6: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 6%) at a probability of 90%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	14.86%	0.00%	0.00%	25.00%	0.00%	50.14%	29.96%	20.63%	21.98%	13.30%	1.04	20.63%	21.98%	13.30%	1.04	7.84
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	7.47%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.96%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.54%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Appendix 2 - Maximising Growth Rate at 95% Probability

Table 5.5.3.1: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (No Shortfall Threshold) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	20.75%	22.00%	13.74%	1.01	20.75%	22.00%	13.74%	1.01	7.36
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.3.2: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	15.18%	10.26%	0.00%	0.00%	25.00%	0.00%	49.56%	5.00%	20.06%	21.31%	12.48%	1.06	20.06%	21.31%	12.48%	1.06	8.48
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.51%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.40%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.11%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.3.3: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.91%	2.69%	0.00%	0.00%	7.57%	66.33%	2.49%	11.18%	10.29%	10.50%	1.96%	1.23	10.29%	10.50%	1.96%	1.23	62.73
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.00%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.03%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.28%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.06%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.3.4: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 2%) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.92%	0.00%	0.00%	25.00%	0.00%	19.34%	18.92%	16.42%	17.26%	7.79%	1.18	16.42%	17.26%	7.79%	1.18	15.11
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.97%	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.29%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.35%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.3.5: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 4%) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	11.57%	14.19%	0.00%	0.00%	25.00%	0.00%	49.24%	24.95%	20.46%	21.78%	13.05%	1.05	20.46%	21.78%	13.05%	1.05	8.03
2	0.00%	0.00%	0.00%	15.63%	11.28%	0.00%	0.00%	25.00%	0.00%	48.10%	5.00%	40.00%	42.50%	17.55%	1.50	20.00%	21.25%	12.41%	1.06	8.54
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.59%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.43%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.3.6: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 6%) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	14.86%	0.00%	0.00%	25.00%	0.00%	50.14%	29.96%	20.63%	21.98%	13.30%	1.04	20.63%	21.98%	13.30%	1.04	7.84
2	0.00%	0.00%	0.00%	30.14%	9.83%	0.00%	0.00%	25.00%	0.00%	35.03%	5.00%	36.67%	38.78%	14.38%	1.57	18.33%	19.39%	10.16%	1.11	10.93
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.96%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.54%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Appendix 3 - Maximising Growth Rate at 99% Probability

Table 5.5.4.1: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (No Shortfall Threshold) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	20.75%	22.00%	13.74%	1.01	20.75%	22.00%	13.74%	1.01	7.36
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	41.51%	44.01%	19.43%	1.43	20.75%	22.00%	13.74%	1.01	7.36
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.4.2: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = 0) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	53.17%	6.48%	0.00%	0.00%	25.00%	0.00%	15.36%	1.00%	15.69%	16.43%	6.96%	1.20	15.69%	16.43%	6.96%	1.20	17.20
2	0.00%	0.00%	0.00%	15.19%	10.26%	0.00%	0.00%	25.00%	0.00%	49.56%	1.00%	40.12%	42.61%	17.65%	1.50	20.06%	21.31%	12.48%	1.06	8.48
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.40%	62.26%	66.01%	23.80%	1.75	20.75%	22.00%	13.74%	1.01	7.36
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.11%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.4.3: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	20.91%	2.69%	0.00%	0.00%	7.57%	66.33%	2.49%	11.18%	10.29%	10.50%	1.96%	1.23	10.29%	10.50%	1.96%	1.23	62.73
2	0.00%	0.00%	0.00%	45.80%	7.51%	0.00%	0.00%	25.00%	0.00%	21.69%	1.00%	33.08%	34.76%	11.19%	1.66	16.54%	17.38%	7.91%	1.17	14.82
3	7.87%	0.00%	0.00%	2.13%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.00%	62.23%	65.98%	23.67%	1.76	20.74%	21.99%	13.67%	1.02	7.44
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.28%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.06%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.4.4: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 2%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	46.74%	8.92%	0.00%	0.00%	25.00%	0.00%	19.34%	18.92%	16.42%	17.26%	7.79%	1.18	16.42%	17.26%	7.79%	1.18	15.11
2	0.00%	0.00%	0.00%	57.32%	6.57%	0.00%	0.00%	25.00%	0.00%	11.11%	1.00%	30.43%	31.81%	9.15%	1.71	15.22%	15.90%	6.47%	1.21	18.64
3	0.00%	0.00%	0.00%	11.35%	11.21%	0.00%	0.00%	25.00%	0.00%	52.44%	1.00%	61.49%	65.40%	22.66%	1.81	20.50%	21.80%	13.09%	1.05	8.00
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.35%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.4.5: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 4%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	11.57%	14.19%	0.00%	0.00%	25.00%	0.00%	49.24%	24.95%	20.46%	21.78%	13.05%	1.05	20.46%	21.78%	13.05%	1.05	8.03
2	0.00%	0.00%	0.00%	44.65%	4.84%	0.00%	0.00%	17.92%	25.84%	6.75%	1.00%	26.23%	27.19%	6.40%	1.72	13.11%	13.60%	4.53%	1.21	26.84
3	0.00%	0.00%	0.00%	17.76%	10.59%	0.00%	0.00%	25.00%	0.00%	46.64%	1.00%	59.28%	62.93%	20.91%	1.85	19.76%	20.98%	12.07%	1.07	8.83
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.43%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Table 5.5.4.6: Maximising Growth Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising g (Shortfall Threshold = Cash + 6%) at a probability of 99%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	g	E(R)	σ	Sharpe Ratio	g	E(R)	σ	Sharpe Ratio	Kelly Ratio
1	0.00%	0.00%	0.00%	10.00%	14.86%	0.00%	0.00%	25.00%	0.00%	50.14%	29.96%	20.63%	21.98%	13.30%	1.04	20.63%	21.98%	13.30%	1.04	7.84
2	0.00%	0.00%	0.00%	25.60%	2.97%	0.00%	0.00%	9.57%	58.44%	3.42%	1.00%	21.66%	22.19%	3.45%	1.73	10.83%	11.10%	2.44%	1.23	50.26
3	0.00%	0.00%	0.00%	24.52%	10.05%	0.00%	0.00%	25.00%	0.00%	40.43%	1.00%	56.95%	60.33%	19.09%	1.89	18.98%	20.11%	11.02%	1.09	9.89
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.54%	83.01%	88.02%	27.48%	2.02	20.75%	22.00%	13.74%	1.01	7.36
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	103.77%	110.02%	30.73%	2.26	20.75%	22.00%	13.74%	1.01	7.36
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	207.53%	220.05%	43.45%	3.20	20.75%	22.00%	13.74%	1.01	7.36
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	311.30%	330.07%	53.22%	3.92	20.75%	22.00%	13.74%	1.01	7.36
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	415.07%	440.09%	61.45%	4.52	20.75%	22.00%	13.74%	1.01	7.36
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	518.83%	550.11%	68.71%	5.06	20.75%	22.00%	13.74%	1.01	7.36
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	622.60%	660.14%	75.26%	5.54	20.75%	22.00%	13.74%	1.01	7.36

Appendix 4 - Amendments to Maximising Alpha Using a Shortfall Constraint and No Risk Aversion

Table 5.7.2.1: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (No Shortfall Threshold) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
1	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	21.81%	20.60%	13.58%	1.01	21.81%	20.60%	13.58%	1.01	7.45
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	N/A	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.7.2.2: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = 0) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	15.01%	8.36%	0.00%	0.00%	25.00%	0.00%	51.63%	5.00%	21.15%	19.93%	12.39%	1.06	21.15%	19.93%	12.39%	1.06	8.52
1	0.00%	0.00%	0.00%	15.01%	8.36%	0.00%	0.00%	25.00%	0.00%	51.63%	5.00%	21.15%	19.93%	12.39%	1.06	21.15%	19.93%	12.39%	1.06	8.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.48%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.39%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.10%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.03%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.7.2.3: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.64%	1.46%	0.00%	0.00%	3.01%	84.06%	0.83%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	3.96%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.01%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.27%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.08%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.7.2.4: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 2%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.64%	1.46%	0.00%	0.00%	3.01%	84.06%	0.83%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	4.93%	43.63%	41.19%	19.20%	1.43	21.81%	20.60%	13.58%	1.01	7.45
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.27%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.34%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.09%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.7.2.5: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 4%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.64%	1.46%	0.00%	0.00%	3.01%	84.06%	0.83%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.87%	9.31%	0.00%	0.00%	25.00%	0.00%	49.82%	5.00%	42.08%	39.66%	17.33%	1.50	21.04%	19.83%	12.25%	1.06	8.64
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.57%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.42%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.12%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Table 5.7.2.6: Amendments to Maximising Alpha Using Shortfall Constraints and no Risk Aversion

Historical data has been used from January 2003 to July 2012
Maximising α (Shortfall Threshold = Cash + 6%) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.64%	1.46%	0.00%	0.00%	3.01%	84.06%	0.83%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.51%	8.33%	0.00%	0.00%	25.00%	0.00%	36.16%	5.00%	38.37%	36.33%	14.16%	1.57	19.18%	18.16%	10.01%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	1.94%	65.44%	61.79%	23.52%	1.75	21.81%	20.60%	13.58%	1.01	7.45
4	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.53%	87.26%	82.38%	27.16%	2.02	21.81%	20.60%	13.58%	1.01	7.45
5	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.15%	109.07%	102.98%	30.36%	2.26	21.81%	20.60%	13.58%	1.01	7.45
10	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	218.15%	205.96%	42.94%	3.20	21.81%	20.60%	13.58%	1.01	7.45
15	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	327.22%	308.94%	52.59%	3.92	21.81%	20.60%	13.58%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	436.29%	411.92%	60.73%	4.52	21.81%	20.60%	13.58%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	545.36%	514.91%	67.90%	5.06	21.81%	20.60%	13.58%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	0.00%	65.00%	0.00%	654.44%	617.89%	74.38%	5.54	21.81%	20.60%	13.58%	1.01	7.45

Appendix 5 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of *Y*=0

Table 5.7.3.1.1: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (No Shortfall Threshold) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	21.88%	20.47%	13.62%	1.01	21.88%	20.47%	13.62%	1.01	7.44
1	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	21.88%	20.47%	13.62%	1.01	21.88%	20.47%	13.62%	1.01	7.44
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	N/A	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.7.3.1.2: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	14.96%	11.91%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
1	0.00%	0.00%	0.00%	14.96%	11.91%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.48%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.39%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.10%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.03%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.7.3.1.3: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	3.95%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.01%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.27%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.07%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.7.3.1.4: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	4.92%	43.76%	40.93%	19.26%	1.43	21.88%	20.47%	13.62%	1.01	7.44
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.26%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.34%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.09%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.7.3.1.5: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.57%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.42%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.12%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Table 5.7.3.1.6: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = 0) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	Momentum	MSCI	RESI	SAPY	STEFI	Value	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.50%	10.00%	0.00%	0.00%	25.00%	0.00%	34.50%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	1.93%	65.64%	61.40%	23.59%	1.76	21.88%	20.47%	13.62%	1.01	7.44
4	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.52%	87.52%	81.86%	27.24%	2.03	21.88%	20.47%	13.62%	1.01	7.44
5	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.15%	109.40%	102.33%	30.45%	2.27	21.88%	20.47%	13.62%	1.01	7.44
10	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	218.80%	204.66%	43.07%	3.21	21.88%	20.47%	13.62%	1.01	7.44
15	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	328.20%	306.99%	52.75%	3.93	21.88%	20.47%	13.62%	1.01	7.44
20	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	437.60%	409.32%	60.91%	4.53	21.88%	20.47%	13.62%	1.01	7.44
25	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	547.00%	511.65%	68.10%	5.07	21.88%	20.47%	13.62%	1.01	7.44
30	10.00%	0.00%	0.00%	0.00%	25.02%	0.00%	0.00%	25.00%	0.00%	39.98%	0.00%	656.40%	613.98%	74.60%	5.55	21.88%	20.47%	13.62%	1.01	7.44

Appendix 6 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of *Y* = -0.5

Table 5.7.3.2.1: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (No Shortfall Threshold) (γ = -0.5) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	21.82%	20.46%	13.32%	1.03	21.82%	20.46%	13.32%	1.03	7.75
1	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	21.82%	20.46%	13.32%	1.03	21.82%	20.46%	13.32%	1.03	7.75
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	328.18%	307.03%	52.72%	3.93	21.88%	20.47%	13.61%	1.01	7.45
20	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	437.58%	409.37%	60.87%	4.54	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	546.97%	511.71%	68.06%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	N/A	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.2.2: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_w - 1/2(1-\gamma/T)s^2_w$ (Shortfall Threshold = 0) (γ = -0.5) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	14.97%	11.98%	0.00%	0.00%	25.00%	0.00%	48.06%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
1	0.00%	0.00%	0.00%	14.97%	11.98%	0.00%	0.00%	25.00%	0.00%	48.06%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.45%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	0.37%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.10%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.03%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.2.3: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash) (γ = -0.5) at a probability of 95%

Years Until Retirement	Weights: ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Shortfall Probability	Portfolio: μ	α	σ	Sharpe Ratio	Annualised Portfolio: μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.61%	1.48%	0.00%	0.00%	3.01%	84.09%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	3.90%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	0.99%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.27%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.07%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.2.4: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 2%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.61%	1.48%	0.00%	0.00%	3.01%	84.09%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	4.86%	43.75%	40.96%	19.19%	1.44	21.87%	20.48%	13.57%	1.02	7.49
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.24%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.33%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.09%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.2.5: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 4%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.61%	1.48%	0.00%	0.00%	3.01%	84.09%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.13%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.83%	11.80%	0.00%	0.00%	25.00%	0.00%	47.37%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	22.29%	0.00%	0.00%	25.00%	0.00%	42.71%	1.54%	65.62%	61.44%	23.51%	1.76	21.87%	20.48%	13.57%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.42%	87.50%	81.90%	27.18%	2.03	21.88%	20.47%	13.59%	1.02	7.47
5	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.11%	109.38%	102.37%	30.39%	2.27	21.88%	20.47%	13.59%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	218.76%	204.74%	42.98%	3.21	21.88%	20.47%	13.59%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.51%	0.00%	0.00%	25.00%	0.00%	41.49%	0.00%	328.14%	307.11%	52.64%	3.93	21.88%	20.47%	13.59%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	437.59%	409.36%	60.88%	4.53	21.88%	20.47%	13.61%	1.01	7.45
25	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	546.98%	511.70%	68.07%	5.07	21.88%	20.47%	13.61%	1.01	7.45
30	10.00%	0.00%	0.00%	0.00%	24.69%	0.00%	0.00%	25.00%	0.00%	40.31%	0.00%	656.38%	614.04%	74.56%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.2.6: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 6%) (γ = -0.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.61%	1.48%	0.00%	0.00%	3.01%	84.09%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.05
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.00%	0.00%	0.00%	25.00%	0.00%	34.51%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	22.57%	0.00%	0.00%	25.00%	0.00%	42.43%	1.90%	65.62%	61.44%	23.52%	1.76	21.87%	20.48%	13.58%	1.02	7.49
4	10.00%	0.00%	0.00%	0.00%	22.57%	0.00%	0.00%	25.00%	0.00%	42.43%	0.51%	87.49%	81.92%	27.15%	2.03	21.87%	20.48%	13.58%	1.02	7.49
5	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.14%	109.38%	102.36%	30.40%	2.27	21.88%	20.47%	13.60%	1.02	7.47
10	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	218.77%	204.73%	43.00%	3.21	21.88%	20.47%	13.60%	1.02	7.47
15	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	328.15%	307.09%	52.66%	3.93	21.88%	20.47%	13.60%	1.02	7.47
20	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	437.54%	409.45%	60.81%	4.54	21.88%	20.47%	13.60%	1.02	7.47
25	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	546.92%	511.81%	67.98%	5.08	21.88%	20.47%	13.60%	1.02	7.47
30	10.00%	0.00%	0.00%	0.00%	23.78%	0.00%	0.00%	25.00%	0.00%	41.22%	0.00%	656.31%	614.18%	74.47%	5.56	21.88%	20.47%	13.60%	1.02	7.47

Appendix 7 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of *Y* = -1

Table 5.7.3.3.1: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (No Shortfall Threshold) (γ = -1) at a probability of 95%

Years Until					ω						Shortfall		Portfolio					Annualised Portfolio		
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	21.80%	20.46%	13.21%	1.04	21.80%	20.46%	13.21%	1.04	7.87
1	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	21.80%	20.46%	13.21%	1.04	21.80%	20.46%	13.21%	1.04	7.87
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	N/A	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	N/A	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Table 5.7.3.3.2: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = -1) at a probability of 95%

Years Until					Weights						Shortfall		Portfolio					Annualised Portfolio		
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	14.96%	11.90%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
1	0.00%	0.00%	0.00%	14.96%	11.90%	0.00%	0.00%	25.00%	0.00%	48.13%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.31%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.32%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.08%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.02%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.3.3: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash) (γ = -1) at a probability of 95%

Years Until					Weights						Shortfall		Portfolio					Annualised Portfolio		
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	3.63%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.88%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.22%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.06%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.3.4: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 2%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	4.56%	43.65%	40.92%	18.83%	1.46	21.82%	20.46%	13.32%	1.03	7.75
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.11%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.28%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.07%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	10.00%	0.00%	0.00%	0.00%	24.58%	0.00%	0.00%	25.00%	0.00%	40.42%	0.00%	656.37%	614.05%	74.55%	5.55	21.88%	20.47%	13.61%	1.01	7.45

Table 5.7.3.3.5: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 4%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	1.53%	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.41%	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.11%	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Table 5.7.3.3.6: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 6%) (γ = -1) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.48%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	1.89%	65.61%	61.45%	23.49%	1.76	21.87%	20.48%	13.56%	1.02	7.50
4	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.51%	87.48%	81.94%	27.12%	2.03	21.87%	20.48%	13.56%	1.02	7.50
5	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.14%	109.36%	102.42%	30.33%	2.27	21.87%	20.48%	13.56%	1.02	7.50
10	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	218.71%	204.84%	42.89%	3.22	21.87%	20.48%	13.56%	1.02	7.50
15	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	328.07%	307.26%	52.53%	3.94	21.87%	20.48%	13.56%	1.02	7.50
20	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	437.42%	409.68%	60.65%	4.55	21.87%	20.48%	13.56%	1.02	7.50
25	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	546.78%	512.10%	67.81%	5.09	21.87%	20.48%	13.56%	1.02	7.50
30	10.00%	0.00%	0.00%	0.00%	21.61%	0.00%	0.00%	25.00%	0.00%	43.39%	0.00%	656.14%	614.52%	74.28%	5.57	21.87%	20.48%	13.56%	1.02	7.50

Appendix 8 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of *Y* = -1.5

Table 5.7.3.4.1: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (No Shortfall Threshold) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	21.80%	20.47%	13.20%	1.04	21.80%	20.47%	13.20%	1.04	7.88
1	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	21.80%	20.47%	13.20%	1.04	21.80%	20.47%	13.20%	1.04	7.88
2	0.00%	0.00%	0.00%	10.00%	15.75%	0.00%	0.00%	25.00%	0.00%	49.25%	N/A	43.60%	40.94%	18.66%	1.47	21.80%	20.47%	13.20%	1.04	7.88
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.4.2: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
1	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	1.26%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.32%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.08%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.02%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.4.3: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash) (γ = -1.5) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	3.54%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.88%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.22%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.06%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.4.4: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 2%) (γ = -1.5) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	10.00%	18.09%	0.00%	0.00%	25.00%	0.00%	46.91%	4.45%	43.61%	40.91%	18.69%	1.47	21.80%	20.46%	13.22%	1.04	7.86
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.11%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.28%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.07%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.4.5: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 4%) (γ = -1.5) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.39%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.36%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.09%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.4.6: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash + 6%) (γ = -1.5) at a probability of 95%

Years Until Retirement	Weights										Shortfall Probability	Portfolio				Annualised Portfolio				
	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL		μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.48%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	1.73%	65.47%	61.38%	23.06%	1.79	21.82%	20.46%	13.32%	1.03	7.75
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.44%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.12%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Appendix 9 - Amendments to Maximising Alpha Using a Shortfall Constraint and Risk Aversion of *Y* = -2

Table 5.7.3.5.1: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (No Shortfall Threshold) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	21.80%	20.47%	13.19%	1.04	21.80%	20.47%	13.19%	1.04	7.89
1	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	21.80%	20.47%	13.19%	1.04	21.80%	20.47%	13.19%	1.04	7.89
2	0.00%	0.00%	0.00%	10.00%	14.84%	0.00%	0.00%	25.00%	0.00%	50.16%	N/A	43.59%	40.95%	18.65%	1.47	21.80%	20.47%	13.19%	1.04	7.89
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	N/A	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	N/A	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.5.2: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising α_w-1/2(1-γ/T)s^2_w (Shortfall Threshold = 0) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
1	0.00%	0.00%	0.00%	14.96%	11.92%	0.00%	0.00%	25.00%	0.00%	48.12%	5.00%	21.16%	19.92%	12.40%	1.06	21.16%	19.92%	12.40%	1.06	8.51
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	1.25%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.30%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.08%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.03%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.7.3.5.3: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_\omega - 1/2(1-\gamma/T)\sigma^2_\omega$ (Shortfall Threshold = Cash) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	20.45%	2.47%	0.00%	0.00%	7.40%	67.18%	2.50%	11.13%	10.38%	10.18%	1.88%	1.23	10.38%	10.18%	1.88%	1.23	65.52
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	3.53%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.84%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.21%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.07%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.7.3.5.4: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 2%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	46.74%	8.01%	0.00%	0.00%	25.00%	0.00%	20.25%	19.03%	17.13%	16.31%	7.71%	1.17	17.13%	16.31%	7.71%	1.17	15.24
2	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	4.44%	43.60%	40.92%	18.68%	1.47	21.80%	20.46%	13.21%	1.04	7.87
3	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	1.06%	65.40%	61.38%	22.88%	1.80	21.80%	20.46%	13.21%	1.04	7.87
4	0.00%	0.00%	0.00%	10.00%	17.11%	0.00%	0.00%	25.00%	0.00%	47.89%	0.27%	87.21%	81.84%	26.42%	2.08	21.80%	20.46%	13.21%	1.04	7.87
5	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.09%	109.30%	102.40%	30.18%	2.28	21.86%	20.48%	13.50%	1.02	7.56
10	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	218.59%	204.79%	42.69%	3.23	21.86%	20.48%	13.50%	1.02	7.56
15	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	327.89%	307.19%	52.28%	3.95	21.86%	20.48%	13.50%	1.02	7.56
20	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	437.19%	409.59%	60.37%	4.57	21.86%	20.48%	13.50%	1.02	7.56
25	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	546.48%	511.98%	67.49%	5.11	21.86%	20.48%	13.50%	1.02	7.56
30	8.27%	0.00%	0.00%	1.73%	20.94%	0.00%	0.00%	25.00%	0.00%	44.06%	0.00%	655.78%	614.38%	73.94%	5.59	21.86%	20.48%	13.50%	1.02	7.56

Table 5.7.3.5.5: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 4%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.39%	12.48%	0.00%	0.00%	25.00%	0.00%	52.12%	25.10%	21.74%	20.44%	13.12%	1.04	21.74%	20.44%	13.12%	1.04	7.94
2	0.00%	0.00%	0.00%	15.84%	11.81%	0.00%	0.00%	25.00%	0.00%	47.35%	5.00%	42.10%	39.64%	17.34%	1.50	21.05%	19.82%	12.26%	1.06	8.63
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	1.34%	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.36%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.09%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

Table 5.7.3.5.6: Amendments to Maximising Alpha Using Shortfall Constraints and Risk Aversion

Historical data has been used from January 2003 to July 2012

Maximising $\alpha_{\omega}-1/2(1-\gamma/T)\sigma^{2}_{\omega}$ (Shortfall Threshold = Cash + 6%) (γ = -2) at a probability of 95%

Years Until	Weights										Shortfall	Portfolio				Annualised Portfolio				
Retirement	ALBI	ALSI40	FINDI	ILB	MOME	MSCI	RESI	SAPY	STEFI	VAL	Probability	μ	α	σ	Sharpe Ratio	μ	α	σ	Sharpe Ratio	Kelly Ratio
1 (Amendments)	0.00%	0.00%	0.00%	10.62%	1.48%	0.00%	0.00%	3.01%	84.08%	0.81%	5.00%	9.09%	9.00%	0.90%	1.13	9.09%	9.00%	0.90%	1.13	126.06
1	0.00%	0.00%	0.00%	10.00%	13.05%	0.00%	0.00%	25.00%	0.00%	51.95%	30.17%	21.79%	20.48%	13.18%	1.04	21.79%	20.48%	13.18%	1.04	7.90
2	0.00%	0.00%	0.00%	30.49%	10.03%	0.00%	0.00%	25.00%	0.00%	34.47%	5.00%	38.38%	36.32%	14.17%	1.57	19.19%	18.16%	10.02%	1.11	11.08
3	0.00%	0.00%	0.00%	10.00%	18.48%	0.00%	0.00%	25.00%	0.00%	46.52%	1.67%	65.42%	61.36%	22.90%	1.80	21.81%	20.45%	13.22%	1.04	7.85
4	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.44%	87.30%	81.84%	26.63%	2.07	21.82%	20.46%	13.32%	1.03	7.75
5	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.12%	109.12%	102.30%	29.78%	2.31	21.82%	20.46%	13.32%	1.03	7.75
10	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	218.24%	204.60%	42.11%	3.27	21.82%	20.46%	13.32%	1.03	7.75
15	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	327.36%	306.90%	51.57%	4.00	21.82%	20.46%	13.32%	1.03	7.75
20	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	436.48%	409.20%	59.55%	4.62	21.82%	20.46%	13.32%	1.03	7.75
25	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	545.60%	511.50%	66.58%	5.16	21.82%	20.46%	13.32%	1.03	7.75
30	2.71%	0.00%	0.00%	7.29%	19.68%	0.00%	0.00%	25.00%	0.00%	45.32%	0.00%	654.72%	613.80%	72.94%	5.66	21.82%	20.46%	13.32%	1.03	7.75

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