Sequence of return risk in South African post-retirement portfolios: The effectiveness of volatility-focused asset allocation strategies to address sequence and associated risks

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Declaration of Authorship

I, Kilian Petika SCHABORT, hereby declare that the work on which this dissertation is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other university. I hereby declare that I have read and understood the regulations governing the submission of Master of Commerce dissertations, including those relating to length and plagiarism, as contained in the rules of the University, and that this dissertation conforms to those regulations.

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Abstract

Sequence of return risk (which is the risk of unfavourable investment outcomes at the most unfavourable time) is an important consideration for efficiently funding retirement portfolio spending goals.

This study examines the sensitivity of retirement decumulation portfolios to sequence of return risk (“SOR risk”, also referred to as “sequence risk”) in a South African context and evaluates the effectiveness of five volatility-focused asset allocation strategies in addressing the risk.

Using monthly asset index returns separated into bull and bear market regimes for the period 1991 to 2020, correlated non-normal returns were simulated and combined with independently simulated inflation to generate 10 000 independent 30-year simulation trials.

The performance of each of the five strategies was measured against a benchmark set of portfolios using simulated data and was validated using partial out-of-sample historical data. Performance was assessed using the sustainable withdrawal rate (SWR) and actuarial coverage ratio (ACR) metrics.

The sensitivity results showed that SOR is greatest at the retirement date, declining asymptotically (halving within the first ten years of retirement). The geographic diversification strategy showed clear benefit to reducing SOR whereas the results for the risk parity strategy were not conclusive. The analysis and comparison of the low-risk, rising equity glidepath, and dynamic cash buffer strategies formed the focus of the study. All three showed considerable SOR potential with the dynamic cash buffer strategy outperforming the others and substantially reducing SOR relative to the benchmark.

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List of Abbreviations

ACR	Actuarial Coverage Ratio
ALBI	FSTSE/JSE All Bond Index
ALM	Asset and Liability Management
ALSI	FSTSE/JSE All Equity Index
BBEMB	Bloomberg Barclays Emerging Market Bond Index
BBGA	Bloomberg Barclays Global Aggregate Bond Index
BF	Behavioural Finance
CDAX	Deutsche Borse AG CDAX
CML	Capital Market Line
CPI	Consumer Price Index
DCB	Dynamic Cash Buffer
E&K-S	Evensky & Katz Cash Flow Reserve Strategy
EAFE	Europe Australasia, and the Far East
HHBS	Household Balancesheet
IGOV	South African government inflation-linked bond index
IPS	Investment Policy Statement
LR	Low Risk
MPT	Modern Portfolio Theory
MVO	Mean Variance Optimisation
PWA	Perfect Withdrawal Amount
REIT	Real Estate Investment Trust
REGP	Rising Equity Glidepath
REX	Deutsche Borse AG REX
RMD	Required Minimum Distribution
SOR	Sequence of Return
STeFi	Short Term Fixed interest Index
SWR	Sustainable Withdrawal Rate
TIPS	Treasury Inflation Protected Security
VaR	Value at Risk
ZAR	South African Rands

List of Symbols

P	Transition probability
π	Steady-state probability
K	Real portfolio value
K^*	Nominal portfolio value
r	Real period return
r^*	Nominal period return
w	Real annual withdrawal amount
w^*	Nominal annual withdrawal amount
ρ	Proportion of portfolio
i	Annual inflation
R_P	Cumulative portfolio return
S_P	Sequence factor
rf	Risk free discount rate
q_x	Mortality rate

Chapter 1

Introduction

1.1 Problem background

Retirement planning is a complex field with many experts and little consensus. For the individuals involved, it can be overwhelming with the different options and approaches as well as an intricate combination of financial needs and desires. At retirement, individuals have only their financial capital saved over their working lives to achieve their financial goals. These goals are subject primarily to the individual's longevity and market risk among others. Because the individuals no longer have human capital, i.e., future salary income, it is paramount that the retiree does not reach financial ruin. It is, however, also important that they do not live so far within their means, lose the enjoyment of their retirement, and leave a large financial estate having lived their last years in an excessively frugal manner. This is the basis of the retirement planning problem.

The lack of consensus stems, in part, from the differing perspectives in approaching the retirement problem in terms of portfolio design and income strategy. As described by Collins and Gadenne (2017), the portfolio design perspectives differ primarily in their degree to which the individual is at the centre of the investment plan. Collins and Gadenne (2017) argue that with a more detailed approach to the clients' needs and goals, a more sustainable total portfolio is created that has the best chance of achieving the individual's goals. Retirement investment theories also differ in their approach to income strategies that provide the means to meet spending needs. The three most popular approaches, as identified by Hopkins (2019), are systematic withdrawal, flooring retirement income, and time segmentation.[1]

The most fundamental goal, common to all individuals in retirement, is the income needed to maintain a lifestyle. While the level of income needed may vary between individuals according to the lifestyles and assets they must maintain, this need is theoretically consistent, in real terms, through time. Funding this goal is paramount to any other portfolio goals, and how efficiently this goal can be funded is the crux of the issue for advisors and individuals alike.

[1] These income strategies are defined and discussed further in §2.2.

Many risks affect retirement portfolios. Market risks, which affect the investment portfolio directly, are not limited to the returns the assets achieve. Sequence of return (SOR) risk, or sequence risk, can be devastating to an investment portfolio even if the overall returns achieved are above expectations. This is because SOR risk describes the risk of realising an unfavourable order of returns. In the case of a decumulation portfolio, as in retirement, sequence risk is the risk of realising poor returns in early retirement which have a far greater impact than late-retirement returns on achieving portfolio goals. This has been demonstrated by Pfau (2013).

The SOR risk is similar to some period-specific risks like downside or drawdown risk although the timing of returns is important to SOR risk. Unlike period-specific risk measures that have an implicit perpetuality assumption, SOR risk must also consider (extreme) event risk because the occurrence of such an event may lead to failure to meet the spending goal (absent any intervention before the portfolio is depleted). A measure of sequence risk must capture this (Nevins, 2004).

There has been much discussion on how to deal with SOR risk, considering both asset allocation strategies and income withdrawal strategies, using simpler vehicles such as guaranteed annuities to more complex ones using combinations of derivatives on a portfolio of assets. When considering a required spending goal, strategies that introduce variability into the withdrawal amounts are not appropriate and therefore asset allocation strategies addressing sequence risk are considered (Fullmer, 2007). The studies considering investment strategies to address SOR risk have also been performed to a larger extent on developed economies with lower inflation and interest rates in general in comparison to developing nations (Suarez et al., 2015; Estrada, 2016; Clare et al., 2020). Pfau (2019) identified four approaches that investment strategies can use to address the retirement income risk; they are strategies that: reduce spending, require spending flexibility, reducing portfolio volatility, and those that use buffer assets. This study will focus on considering portfolio design strategies which address sequence risk through reducing portfolio volatility as described by Pfau (2019).

The sustainable withdrawal rate (SWR), a systematic withdrawal strategy, is commonly used as a monitoring metric as well as testing the impact of portfolio strategies on retirement portfolios. Suarez et al. (2015) demonstrate the clear link between SWR and sequence risk making it appropriate in studying sequence risk mitigation. A second relevant retirement portfolio metric, adapted from the asset liability management (ALM) sphere, is the actuarial coverage ratio (ACR). This is useful as a monitoring metric to measure portfolio "*fundedness*" and SOR effects (Collins and Stampfli, 2019). Both SWR and ACR are relevant in comparing the effectiveness and efficiency different retirement strategies relating to sequence risk.

1.2 Problem statement

The sustainability of a retirement portfolio is important in funding spending goals for the duration of the retirement period. The most fundamental income needs for the individual should have the highest probability of being achieved, according to Behavioural Finance theory (Nevins, 2004). Sustainable withdrawal rates (SWR) and actuarial coverage ratios (ACR) provide a useful metric when considering sequence risk, portfolio strategies and their ability to meet retirement goals. Sequence of return risk threatens the sustainability of these portfolios of risky assets and can lead to financial ruin. Asset allocation strategies can be used to reduce sequence risk while maintaining upside potential without introducing income variability.

The purpose of this study is primarily to determine the sensitivity of post-retirement portfolio to sequence risk and to compare how effective the volatility reducing strategies identified are at achieving a reduced sequence of return risk. The specific focus is the portfolio concerned with achieving a minimum spending requirement goal in a South African context. The five strategies considered include the use of a dynamic cash buffer, using a greater proportion of lower risk assets, a rising equity glidepath, greater geographic diversification, and a risk parity approach in the portfolio construction. The comparisons of these strategies relative to each other and the benchmark through the ACR and SWR metrics is fundamental to achieving this.

1.3 Objectives

The following objectives will be pursued in this study:

1. To conduct a literature study relating to:
    a. Retirement portfolio design approaches
    b. Retirement portfolio income approaches
    c. Retirement income goals and risk management
    d. Retirement portfolio asset allocations and asset classes
    e. Sequence of return (SOR) risk
    f. Retirement portfolio performance metrics
2. To generate a set of asset returns of relevant investment asset classes using Monte Carlo simulation with macroeconomic data to be used to construct portfolios and test the efficacy of asset allocation strategies.
3. To create a set of South African benchmark portfolios with static asset allocations to be used as a comparison baseline.
4. To evaluate the performance of the benchmark portfolios in the relevant portfolio metrics for a theoretical retiree using the simulated data.

5. To assess and compare the results of the South African benchmark set of portfolios against an equivalent US set of portfolios using historical data.
6. To measure the sensitivity of retirement portfolios to sequence risk, the duration and intensity of the risk.
7. To establish and evaluate the implementation of a set of five SOR risk mitigation strategies within the investing environment described using the relevant metrics. This comparison is done relative to the benchmark as well as to the other strategies.
8. To validate the results of the SOR risk strategies using historical market and economic data.

1.4 Methodological approach

This study is executed in five stages. The first stage is the literature review on the main parts of this study: the components of portfolio design and construction, sequence of return (SOR) risk, and the appropriate portfolio metrics. The review will provide insight into the relevant parts of the retirement field as well as consider work that has already been done related to this study, what insight can be gained from this research and how this study differs from what is already in existence.

The approaches to retirement portfolio design and income techniques will first be considered which will inform the structure of the analysis. This will give a view of the retirement consumption-saving problem. Retirement income risks and goals are next discussed. Following that, asset classes and allocation methods used in the retirement investing space will be reviewed. Next, literature on sequence of return risk is considered and the strategies that have been used previously to address the risk are studied. Lastly, sequence risk metrics for retirement portfolios are discussed.

The second stage of the study will involve the creation of a set of simulation trials consisting of asset returns and inflation rates. This entails simulating return series using Monte Carlo simulation and simulating inflation rates following the method of Collins and Stampfli (2019). These data are combined to obtain the retirement period simulation trials.

The third stage of this study entails creating a benchmark set of basic portfolio asset allocations against which evaluation can be performed using simulated and historical data. The portfolios are evaluated in terms of the sustainable withdrawal rate (SWR) and actuarial coverage ratio (ACR). The South African benchmark using the historical data will be compared to a US equivalent using the SWR to make comparisons between the two nations and the SWRs identified in other studies, such as the "4% Rule" of Bengen (1994).

The fourth stage of the study deals with the sensitivity to sequence of return risk. It will consider the sensitivity of benchmark portfolios to changes in each year's portfolio returns and will also compare that sensitivity to what has been measured in other previous studies.

The final stage of the study will contain the evaluation of strategies to address the SOR risk in a retirement portfolio using simulated data. A set of five strategies identified will be implemented and their improvement in the set of portfolio SWRs and ACRs will be considered. Selected strategies will then be compared to each other and validated using historical data.

1.5 Scope

Portfolio design approach – This study intends to provide an insight that can help achieve investor goals. The focus is on the specific goal of essential spending needs, dictating that the design approach will be close to the Household Balance Sheet approaches as defined by Collins and Gadenne (2017). Thus, the study is designed to measure the relevant "*fundedness*" metrics that satisfy assumed essential spending in retirement.

Portfolio income strategy – The income strategy being considered in this study seeks to find ways to improve the probability of meeting the most basic income goal while still retaining a significant upside in the total portfolio. A systematic withdrawal strategy is required, based on fixed cash flow needs, while attempting to incorporate some of the useful traits from flooring income strategies. There are strategies that have been shown that some income variability greatly reduces risk (e.g., RMD and the mortality updating constant probability of failure model) but a fixed income requirement allows for easier comparison. This study will not consider the time segmentation strategy, discussed further in §2.2, for reasons outlined by Estrada (2019).

Theoretical modelling environment - This study considers a retirement portfolio in the context of South Africa. For modelling purposes, South African actuarial mortality data and historical macroeconomic data will be used where needed as inputs for Monte Carlo modelling and metrics. Due to the complexity of tax modelling and the numerous scenarios that would need to be considered, tax considerations have been scoped out of this study. All results are therefore provided as before-tax figures in order to standardise the results without invalidating conclusions.

Investment portfolio – This study will consider investment vehicles available to a South African individual up to and during retirement. The study will only consider a portfolio of assets, held either in a living annuity (one in which investments and withdrawals can be directed by the annuity owner) or in a personal account. The asset classes that are considered in this study are limited by the availability of data. Because of this, South African equity, bond, and cash indices will be considered primarily. Relevant alternative assets and international asset indices with sufficient history may be considered. This study will also only consider investment portfolio assets and will exclude personal property and other personal assets that can be monetised.

Annuities and derivatives – This study is excluding life, or guaranteed, annuities and derivatives from the analysis. The huge variety of derivatives and annuities and manners in which they can be implemented in the portfolio being considered are discussed by Zwecher (2010). The range of possible derivates to be applied are too numerous to be considered in this study. Reasons for not considering guaranteed annuities are given by Fullmer (2007). Fullmer (2007) shows the value of the "annuitization option" and the issues with considering guaranteed annuities, especially in a developing economy with a greater default risk, as shown by Babbel and Merrill (2006). Moreover, purchasing a life annuity also implies relinquishing financial capital in exchange for an income stream for the remaining life with no capital returned for a bequest if the individual dies earlier. Once the life annuity has been purchased, in South African law it cannot be unwound making the purchase permanent. Finally, an inflation-linked annuity (which is intended to provide a constant real income stream) is not linked to the individual's personal inflation which can differ significantly from their experienced inflation. This is not to say that an annuity has no place in a retirement portfolio, it is merely excluded here for the purpose of this investigation of sequence of return risk.

Chapter 2

Literature Review

This chapter will discuss the components of “the retirement problem”, portfolio construction, sequence of return risks, and strategies in dealing with the risk. There will be a broad review of the different prescriptions and approaches used in addressing this problem but with specific focus on the literature applicable in addressing the problem statement.

2.1 Portfolio design approaches

The design approach used for retirement portfolios informs much of the subsequent decision-making regarding how the income is drawn from the portfolio and what will constitute that portfolio. The design approaches differ by the extent of focus on the specific needs of the individual. Collins and Gadenne (2017) separated these into three categories: Modern Portfolio Theory (MPT), Behavioural Finance (BF), and Household Balance Sheet (HHBS) approaches.

2.1.1 Modern Portfolio Theory approaches

The approaches that fall under the MPT standard are drawing primarily from Markowitz (1952) of identifying the efficient frontier and the market portfolio. These approaches are also defined as risk-return or mean-variance optimising (MVO) approaches. In MVO, the optimal, or market portfolio, is tailored to the investor’s risk tolerance as opposed to the portfolio being designed bespoke for the individual. This can lead to concentrated portfolios and end up excluding certain asset classes and sub-classes.

The MPT approach is suitable for investors in their wealth accumulation phase with less complex portfolio goals. It is however problematic to do the same in the decumulation phase because the specific goals, spending requirements, and bequest motives are not aligned between investors. The proponent of the MPT, Markowitz (1991), went on to clarify that MPT was intended to be applied to institutions rather than individuals. The specific needs and situation of the individual is important when investing as individuals (Markowitz, 1991). An approach which addresses the needs of the retired investor is important, not simply focusing on the simple risk-adjusted return performance of the portfolio.

2.1.2 Behavioural Finance approaches

The behavioural finance (BF) perspective to retirement portfolio design takes into consideration the sustainability of the investor's portfolio, as described by Collins and Gadenne (2017). This is achieved by framing the wants and needs of the retiree as goals and planning based on the importance of each of these goals in a total portfolio context. In this approach, sub-portfolios are created to address each goal with the most important goals, for example the individuals annual spending needs or near-term spending needs, having the lowest risk and ensuring the highest probability of being achieved. These goals will be funded from lower risk investments such as inflation-linked and nominal government bonds or life annuities. The less important goals are placed at a higher level of portfolio risk, utilising more risky assets such as equities and corporate bonds, with a potentially lower probability of being achieved.

Nevins (2004) identified how appropriate metrics for a goals-based, BF, approach should look, reflecting the risk to the portfolio goals. Furthermore, these metrics should use event-specific measures, as opposed the period-specific measures of MPT such as standard deviation. The intention is to use richer metrics that reflect the sustainability of the goal of each sub-portfolio. Examples of event-specific risk measures would include sustainable withdrawal rates, value at risk, or potential portfolio loss in a certain period.

2.1.3 Household Balance Sheet approaches

The HHBS approach is similar to BF approaches with some subtle changes. This approach is implemented by including *fundedness* measures adapted from asset liability management (ALM) world of institutional investors. These measures compare the spending obligations related to each goal and the assets in the sub-portfolio used to address them. This is like the way in which insurers constantly balance their asset holdings to liabilities, ensuring they remain profitable. In a retirement portfolio context, however, recovering from an underfunded scenario is near impossible as the individual has lower ability to take on risk at this point and a lack of human capital to supplement their requirements. As such, the risk management is paramount, and the risk level of each sub-portfolio should reflect the *fundedness* of the goal. Collins and Gadenne (2017) argue that the chosen definition of *fundedness* should depend on the individual's needs and resources.

The HHBS is based primarily around the feasibility of the investor's goals. The result is that goals are constantly monitored, and adjustments are made with respect to the sustainability and feasibility of the goals. Based on this review of approaches to portfolio design, this study will approach the retirement problem from an HHBS perspective. This is the appropriate approach in identifying and addressing an individual's specific needs with a focus on feasibility and risk management. The HHBS approach is appropriate when considering the required spending goal and the risk management thereof, specifically with regards to sequence of return risk.

2.2 Portfolio income approaches

This section discusses retirement income approaches, describing the spectrum of income approaches as well as discussing in further detail some of the most popular techniques with which income is drawn from the portfolio. The selection of an appropriate income approach will inform the asset allocation and the asset universe considered.

2.2.1 Safety first versus probability-based approaches

The primary debate in retirement income approaches is around the safety first versus the probability-based techniques. The choice of approach should theoretically be largely informed by the risk appetite of the individual and how they view risk in their retirement portfolio.

Pfau and Cooper (2014) describe the debate in detail, defining the safety-first end of the spectrum as taking an asset-liability matching perspective overall and promoting the idea of completely de-risking the essential spending portion of their portfolio. The approach takes a prioritised goals-based perspective and supports the use of annuities and low-risk bonds to create income floors, trading upside for certainty. The approach covers strategies including completely annuitizing one's portfolio, using the "floor-leverage rule", and bond ladders with longevity insurance. The floor-leverage-rule is akin to a ALM surplus management approach and bond ladders are a cash flow matching technique.

In contrast, the probability-based approach does not separate portfolio goals, but rather seeks to withdraw enough from the overall portfolio that can cover all requirements. The approach draws from modern portfolio theory, taking a total returns perspective. Spending and asset allocation are determined by the probability of failure, i.e., depleting the portfolio while the individuals needs are not met. Sustainable withdrawal rates, variable withdrawal rates, and time segmentation approaches fall within the probability-based view.

2.2.2 Popular income approaches

Hopkins (2019) identified the most used income approaches. The popularity of the approach may be related to how easily it is understood by wealth managers, advisors, and investors. Nevertheless, a discussion of these approaches provides insight into the industry as well as investors' views on risk and informs the decision on the appropriate approach.

Systematic withdrawals The systematic withdrawal approach is based on the probability-based extreme of the income approach spectrum. This approach takes a total returns perspective and seeks to withdraw a fixed real income from a risky portfolio. This strategy aims for certainty in the value of the real amount to be received, ensuring a certain standard of living for as long as the portfolio lasts. The real income certainty is exchanged for longevity/market risk as the portfolio can be withdrawn to depletion leaving the investor destitute.

The simplest systematic withdrawal strategy is the sustainable withdrawal rate (SWR). Bengen (1994) (which made "the 4% Rule" famous) and Cooley et al. (1998) demonstrated how an inflation-linked 4% could have been withdrawn from a risky portfolio sustainably for any 30-year period in the entire available history at the time (1926-1995). This strategy is criticised for ignoring market conditions. Critics say it leads to (depending on the sequence of returns) surpluses or shortfalls (Scott et al., 2009) and that it is unsupported by economics (Fonda, 2008).

Variable withdrawal rates can address some of the criticisms of SWR by removing some of the longevity/market risk by increasing income variability. These strategies range from constant percentage withdrawal or the Required Minimum Distribution (RMD) method, to complex ones, such as the mortality updating constant failure percentage of Blanchett et al. (2012).

Essential-versus-discretionary Essential-versus-discretionary falls on the safety-first end of the spectrum. Similar to the floor-leverage rule, this flooring strategy aims to create a low-risk income level for essential spending needs and discretionary goals are funded by riskier assets. The floor can be achieved through a combination of annuities, pension income, and other low risk assets like government-backed inflation-linked or nominal bonds.

The aim of this method is to greatly improve the chances of essential goals being met. The downside is that the lower return on the portfolio implies that fewer discretionary goals can be funded by the portfolio or the available income will be lower.

The principles in this framework align with those of goals-based BF and HHBS portfolio design more closely and result in a more tailored product that meets one's individual needs. The criticisms of this framework are captured in the review of the approach recommended by Bodie and Taqqu (2011). Pfau (2012) asserts that these approaches can be overly conservative and unnecessarily sacrifice return by excessively reducing the portfolio risk.

Time segmentation The time segmentation framework separates a portfolio into sub-portfolios based on different time segments as opposed to different goals. Also known as income bucketing, this approach uses low risk bucket to fund more immediate spending needs (one to two years) with high-risk assets to fund needs in the more distant future (seven years onwards) and an intermediate risk-level sub-portfolio to address the period in between.

This approach has the primary benefit of accommodating investor's mental accounting behavioural bias by reducing volatility in the short-term bucket (Pfau, 2012). The approach allows retirees to stay the course when markets are experiencing volatility or crashes because of the separation of long-term growth assets from short-term income generating assets. Pfau (2012) further states that this strategy is dominated by a total return strategy such as a SWR strategy. Hopkins (2019) notes the issues in when and how assets are shifted to lower risk buckets and the complexity of these rules. Estrada (2019) performed an extensive analysis of bucket approaches and found that systematic withdrawals outperformed time segmentation in all four-performance metrics used.

2.2.3 Income approach trade-offs and compromise

Based on the spectrum of income frameworks, it is clear there are various prescriptions being made on the income solution for retired individuals. This variety of approaches shows the conflicting fears and desires: the desire for certainty in the level of income, the fear of portfolio depletion, the desire for maximising risk-adjusted return, the fear of market volatility and the desire for safety.

The probability-based approach is overly simplistic, paying little attention to the specific needs of the individual. Safety first's investment approach goal-based approach is perhaps overly conservative with aims of achieving a certain income floor with any excess used for discretionary goals. Safety-first is, however, more needs oriented and takes a HHBS view on retirement, as recommended by Collins and Gadenne (2017).

Some promising income approaches that trade-off these risks exist in this spectrum. For example, variable spending rate strategies consider current portfolio status and market events indirectly which reduces probability of ruin of SWR approaches. Alternatively, Fullmer (2007) uses a safety-first approach but with a probability-based portfolio called "Modern Portfolio Decumulation". Instead of annuitizing immediately at retirement, one instead gains the time value of the annuity option by only annuitizing as a last resort.

Based on an HHBS portfolio design approach, this study favours the safety-first side of the income approach spectrum. The objective of this study considers a goals-based approach, modelling required spending needs being met with a risky portfolio. This closely resembles the funded ratio management which continually compares the value of a sub-portfolio with the goals that it seeks to fund and adjusts risk accordingly. The annuity option is used only as a last resort (Pfau, 2012), like Fullmer (2007). The asset allocation is informed by both the funded status as well as the client's risk appetite and risk allocation. In this study, the total portfolio is effectively managed according to the funded ratio management framework.

2.3 Retirement income goals and risks

This study approaches the retirement problem from an investor-oriented perspective. Therefore, the portfolio that is designed to an individual's goals and risks as opposed to a product which is then tailored to the individual. A common way of documenting these goals and risks is through an investment policy statement (IPS) which compiles these aspects and allows a portfolio to be created to the individual's needs. These aspects are considered to inform the asset allocation to address the risks and goals.

This study focuses on addressing the essential spending income need. For this reason, this section discusses the general goals and needs of retired individuals with a specific emphasis on lifestyle maintenance spending. Following discussion of goals and risks, the risk allocation will be discussed based on the relevant risks and goals for individuals at retirement.

2.3.1 Goals

In terms of retirement goals, Pfau and Cooper (2014) identified general retirement goals including estate building and request motives, maximising spending power, generating a consistent income, having the flexibility to change the approach if needed, and an emergency reserve. This study is concerned with the goal of a consistent real (inflation-linked) income to support essential spending to maintain a lifestyle.

2.3.2 Risks

With the relatively simple goal, the focus of the portfolio design then becomes a trade-off of the different risks. As was mentioned in reference to income approaches in §2.2, the process of setting up goal-based portfolios requires trade-offs in which risks are accepted or mitigated. This part of the portfolio design process is informed by the individual's aversion to different risks, the income approach, and the design approach being applied. As Chhabra (2005) described the portfolio creation process for individuals, based on the risk allocation, the asset allocation can be derived. A few of the most important risks are discussed now with reference to the approach of the study and the essential spending goal.

In "Beyond Markowitz", Chhabra (2005) considers the wealth management for individuals as it differs from MPT. Chhabra (2005) categorises risks faced by individuals into personal risks, market risks, and aspirational risks. The aspirational risks are those significant risks taken consciously to enhancing the lifestyle of the individual and are not relevant to the goal. The discussion will address only the first two categories as they relate to the essential spending goal.

In this section, the definition of risk being used is comparable to the one provided by Dus et al. (2005), which is “the probability of not reaching the desired level of consumption”. The “desired level of consumption” is taken to mean maintaining a standard of living. This definition speaks to both personal and market risks which are now explored further.

Personal risks The personal risks identified by Chhabra (2005) include: cash flow, lifecycle stage, ability to weather shortfalls, and event risk. The relevant risks contained in these four fundamental components of personal risks vary for different individuals. However, for individuals in retirement and with reference to the required spending goal, the relevant risks become quite similar for all retirees. These individuals have strict cash flow needs and have low ability to weather shortfalls, in general, as they have no other income. Implicit in these risk components is inflation risk: an important return requirement to maintain the real value of cash flow requirements. Chhabra (2005) suggests asset allocations to cash, short-term and inflation-linked government bonds as they are low risk and ensure the highest likelihood of meeting this basic standard of living goal.

Market risks Following on from the basic standard of living goal, market risks are those that become relevant in maintaining a current standard of living (Chhabra, 2005). This includes systematic and specific risk. Systematic risk is inherent to financial markets and, in theory, undiversifiable risk. Specific risk is the risk associated with individual components of a portfolio. In MPT, the specific risk component can be removed by diversifying the portfolio of assets sufficiently. The asset allocation within this bucket tends to be, through mean-variance optimisation, largely comprised of equities and fixed income which can offer market returns.

Another consideration is extreme event risk which is primarily systematic. The risk of events like the 2008 financial crisis and associated global recession can be catastrophic to an investment portfolio with no income to supplement the lack of investment income. This risk of extreme events is better captured by metrics such as failure rates (Nevins, 2004).

2.3.3 Risk allocation

This study follows a funded ratio management rather than a pure safety-first approach. This includes behavioural finance components, and it is appropriate to include a basic goals-based perspective. Therefore, in looking to address and balance the risks of the essential spending goal defined, both the personal and market risk categories of Chhabra (2005) will be relevant. Furthermore, allocating between these two gives rise to risks that are defined uniquely for retirement investing, namely, longevity risk and sequence of return (SOR) risk. Longevity risk is that of outliving your wealth, i.e., risk of financial ruin. SOR risk in this case is the risk of poor portfolio returns being achieved in the early stages of retirement and its effect on longevity risk.

Milevsky and Robinson (2005) provide a useful way of visualising the trade-off of risks with the Retirement Finances Triangle in Figure 2.1 below. The area of the triangle represents the probability of depleting the portfolio where increasing withdrawal rates, reducing portfolio return (assuming constant risk), and decreasing investor age all push the vertices out and increase the risk. The age vertex is assumed in this study, while the spending and allocation vertices are variable.

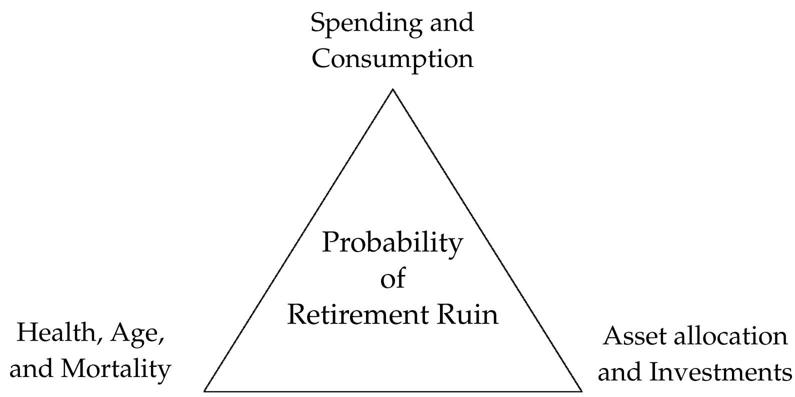

FIGURE 2.1: Retirement Finances Triangle
(Milevsky and Robinson, 2005)

Considering the goal of the portfolio, spending has an intuitive effect on ruin probability. The interactions within asset allocation are however more complex and their relationship with SOR return and consumption are the focus of this study.

2.4 Asset allocation and asset classes

The asset allocation decision is the final major step in creating an investment portfolio. In the case of a post-retirement portfolio, the previous sections all inform the way assets are allocated. This study is making use of a funded ratio management HHBS approach in considering the essential spending goal in order to maintain a lifestyle standard. The goal is addressed using a single portfolio.

This section details some of the discussion in the field around asset classes and the risks associated. This will inform the asset allocation in the benchmark portfolio, as well as assist in identifying strategies in which traditional asset allocations can be improved to improve the probability of achieving the intended goal.

2.4.1 Traditional asset classes

The set of traditional asset classes commonly thought to include cash and money market instruments, listed fixed income, and listed equities. These three classes form the majority of many investors' asset allocation due to their liquidity and differing risk-return profiles. It is traditionally assumed that equities offer the greatest return at the greatest level of risk of the three (measured in standard deviation of returns), cash equivalents with the lowest and most certain return, and fixed income falling in the middle. While the discussion around risks in a MPT framework is widely agreed, risk from a retirement income perspective is drastically different. For example, Merton (2014) explains that a T-bill is risk-free from a MPT perspective, but from a retirement income perspective is very risky. This section discusses risk from this perspective.

The role of equity

Equities serve an important role in accumulation portfolios because of the human capital availability and longer time horizons. The role in retirement portfolios is less clear because of the potentially irrecoverable damage market volatility can cause in terms of drawdown, sequence, and by implication longevity risk. In post-retirement investment, the uncertainty in horizon and the lack of human capital reduces the risk capacity of individuals.

A prominent debate in the retirement space is whether equities are safe or unsafe, whether they follow a mean reversionary process or a random walk. Siegel (1994) argues that although the volatility of equity is higher in the short-run, in the long-run the range of equity returns narrows to a range smaller than that of fixed income but with a higher expected return. Siegel (1994) adds that stocks, unlike bonds, have never over a 20-year holding period yielded a negative real return in the US. Bodie (1995) opposes this with the assertion that the long-run is an aggregation of short-runs and that it must be that stocks are therefore riskier in the long-run. Maurer et al. (2001) asserts that the finding of Siegel (1994) is based on estimation bias of highly correlated overlapping periods.

Bonds versus Equities

Another discourse that exists is the debate between allocations tilted towards fixed income versus equities respectively. Bodie (2003) argues that the only safe investments in the long-run are inflation-protected bonds and real annuities. Although equities have a higher average return, its volatility exacerbates sequence and downside risk and can deplete wealth early on. The risks on bonds are predominantly credit risk and market risks where the returns may be insufficient to support the spending required.

The role of cash

The final traditional asset class to be discussed is cash equivalents and money market instruments. This class comprises assets with the lowest traditional risk and returns that are usually insufficient to retain purchasing power (Merton, 2014). For this reason, they are usually used to some degree in time segmentation as the returns are much less affected by market volatility and therefore little to no SOR risk. One example of their use is the time segmentation Evensky & Katz Cash Flow Reserve Strategy (E&K-S) (Evensky, 2006).

2.4.2 Flooring income and annuities

It is widely agreed in the industry that income flooring is necessary. Pfau and Cooper (2014) show this can be achieved explicitly or implicitly. Explicit floors are guaranteed in some form, reducing the risk to the investor, such as an annuity. These harder floors are favoured by the safety-first proponents. The softer, implicit floors, supported by the probability-based approach, are without the guarantees or certainty but retain upside potential. The use, timing, and extent of flooring as well as annuities are discussed below.

Choosing a type of floor to implement deals with the trade-off between income guarantee and upside potential. On the hard floor extreme, the entire portfolio in a guaranteed annuity, takes out market risk and provides a certain income stream but giving up any upside of more income and lose control of the portfolio. On the implicit floor extreme, the floor has no guarantee and can lead to prosperity beyond what a guaranteed floor could offer, or demise however has the flexibility to revert to an explicit floor if need be. Collins and Gadenne (2017) describe this as the trade-off between the disutility of a decrease in aggregate expected return versus the utility of reduced uncertainty in future consumption.

According to Siegel (1994), historically the long-term range of outcomes of equities are narrower and better than of lower risk assets. The argument implies that a floor can be created using a strategy without any guaranteed income, like that of the 4% rule (Bengen, 1994). Kitces (2012) describes it as a non-guaranteed floor but shows parity in performance between the 4% safe withdrawal rate and a hard, annuity income floor. Collins and Gadenne (2017) respond to this, stating that although on average a strategy may work, "investors are stuck with only one result that unfolds over a single long-term period". To the same point, Zwecher (2010) describes each retiree as only having "one whack at the cat" and with only one life to live, the expected or average life does not mean anything.

Choosing the type(s) and degree of flooring is largely up to the individual and their utility preferences. The individual should ideally determine how much certainty they require and how important is the upside potential, the flexibility, and the control given the risk of longevity, markets, and potential failure. Although Kitces (2012) argues for a broad flooring definition, the term floor will be used as to mean a "harder" floor with more certainty, such as using annuities, or other very low risk assets such as short-term government bonds.

One of the prominent topics relating to income flooring is the "annuity puzzle", referring to the low levels of annuitisation despite the benefits of annuitizing a large portion of one's wealth (Goedde-Menke et al., 2014).[1] An annuity places one's assets under control of the insurer with the obligation to make contracted payments to the individual. Depending on the country, the annuity may be government-backed, giving a stronger guarantee to the stream of cash flows. Zwecher (2010) describes the importance of "monetising mortality", pooling risk and ensuring the cash flow stream. The act of purchasing an annuity like this is effectively exchanging financial capital for a future stream of cash flows, linked to the individual's survival.

An annuity is the closest an investor can come to a riskless real income stream. However, Kitces (2012) argues that while a floor of risky assets is subject to market risk, an annuity is subject to similar tail risks. A severe downturn or extreme event could bankrupt the insurer as well as wipe out a portfolio of risky assets and there is no true haven. Barring this, an annuity gives a great deal of certainty in exchange for a lack of flexibility offered when investing in other assets and instruments. The certainty offered must also be compared to the upside potential foregone, the acceptance that once purchased, the capital is relinquished. This raises the question of how much financial capital should be placed into an annuity. Zwecher (2010) gives good guidance on this, the answer in the end depending on investor preference.

The final important discussion related to flooring is when a floor should be implemented. One group asserts that flooring should be done immediately, as Collins and Gadenne (2017) described it floor-at-the-first-opportunity, or flooring as necessary, described in Modern Portfolio Decumulation (Fullmer, 2007). This again is a trade-off of greater mortality credits versus utilising the value of the annuitisation option, retaining the risky portfolio with a higher expected return for as long as possible, on annuitizing as a last resort. Due to guaranteed annuities being scoped out of this study, the implicit stance is that of Fullmer (2007) although Zwecher (2010) provides good support of the safety-first argument.

2.4.3 Portfolio diversification

The discussion regarding asset allocation so far has covered the asset classes that form the majority if not the entirety of retirement portfolios. Portfolio risk diversification is important to prevent excessive concentration of risk in specific financial markets, especially in downturns when return correlation increases (Campbell et al., 2002). This section will discuss two methods of addressing the market risk through risk diversification: alternative assets, and geographic diversification.

Alternative assets Investments in alternative assets gives exposure to a different risk profile. Baker and Filbeck (2013) identify the potential risk-diversification that an allocation to alternative assets can provide while still maintaining a reasonable return. Chhabra (2005) also notes these assets' diversification benefits. However,

[1] As mentioned in the scope of this study, while they do serve an important purpose in a retirement portfolio, annuities are not included in this study's portfolios.

coupled with this are the typical characteristics of relative illiquidity, limited historical data, complex structure, high due diligence and transaction costs, and difficulty in performance evaluation.

Because alternative investments are a very broad super-class, Baker and Filbeck (2013) separate them into the oxymoronic traditional alternative investments, and modern alternative investments. The latter group includes distressed equities, hedge funds, and managed futures which all require some degree of active management since performance is investment specific. Traditional alternatives include real estate, commodities, and private equity. Private equity has numerous highly diverse sub-categories and investment specific. The modern alternative assets and private equity require active management and are therefore excluded from the portfolios being analysed in this study.

The alternative assets still in consideration are the real, as opposed to financial, assets: commodities, real estate, and infrastructure. The use of real assets, value-generating physical assets, is intuitive as a diversification technique as they are linked to a lesser degree to valuations in capital markets. Real assets are also seen to have a stronger degree of inflation protection than other unprotected financial assets. Zwecher (2010) argues that the link to inflation is not guaranteed, depending on the asset, and that these assets all have their own specific risks warning that investors confuse their illiquidity with lack of correlation. The exposure to a different set of specific risks is, however, the intent of seeking portfolio risk diversification.

Real estate is the most common alternative asset with some believing it to be included in the traditional asset clique. Real estate investment trusts (REITs) are already included in listed equity indices and exhibit the financial behaviour that is closer to listed equities (Bond and Chang, 2013). Although private real estate showed very low return volatility compared to the stock-like volatility of REITs, only listed real estate is considered due to lack of private real estate data.

Infrastructure funds seek to invest in public infrastructure that provides uncorrelated stable, long-term returns. The diversification benefits, inflation-protection, and predictability of these assets explains the interest from institutions, driving down yields. Due to lack of significant historical data, infrastructure had to be excluded.

Lastly, the case for commodities is more contentious. Conover et al. (2010) show significant benefit while Daskalaki and Skiadopoulos (2011) found no benefits of including commodities in a passive portfolio. Amenc et al. (2009), from an ALM perspective, showed the inflation hedging benefits of commodities and real estate relative to traditional assets as well as showing the reduced cost of inflation protection relative to TIPS (treasury inflation protected securities) and reduced the shortfall probability. Huang and Zhong (2013), however, did find benefits in including inflation-protected bonds, commodities, and REITs together in a portfolio.

Geographic diversification While possibly more trivial than use of alternative assets, geographic diversification is a simple way to reduce the magnitude of risk of any single economy in the total portfolio (Driessen and Laeven, 2007). As Solnik (1995) showed, there are substantial benefits of international diversification in reducing portfolio risk. A significant obstacle to realising this, however, is the home bias of investors as well as legislation keeping capital invested locally despite the potential damage concentrated country risk can do to retired individuals. While there is a degree of correlated tail risk between economies, as was seen in the 2009 financial crisis, a significant portion of country risk can be eliminated with international diversification.

2.5 Sequence of return risk

This section of the literature review deals with sequence of return (SOR) risk, or sequence risk, touching on some of the associated risks in a retirement portfolio context, and the portfolio approaches to addressing the risk. Strategies to address sequence risk in the context of this study are then discussed.

2.5.1 Introduction

Sequence of return risk is the amplified impact that the volatile returns of a portfolio has on the end portfolio value when there are intermediate withdrawals from or additions to the portfolio. Cotton (2013a) explains that for two identical investors, with periodic portfolio withdrawals or investments, experiencing the same average return, can have vastly different outcomes due to the difference in the order the returns are realised. The implication is that a portfolio is most susceptible to SOR risk when portfolio values are highest.

The effect is displayed visually in Figure 2.2 below showing the effect of sequence risk on accumulation and decumulation portfolios.[2] As can be seen, the worst accumulation portfolio outcomes experience the best returns in the early periods, where the worst decumulation portfolio values have the best returns occurring in the final periods.

[2] In the figures above, a portfolio starts at a value of 100. For the decumulation portfolio five units are withdrawn at the start of each period, and for accumulation, five units are added at the end of each period. This process is repeated on all 24 permutations of a set of four periodic returns. The return set includes: -10%, -5%, +5%, +10%.

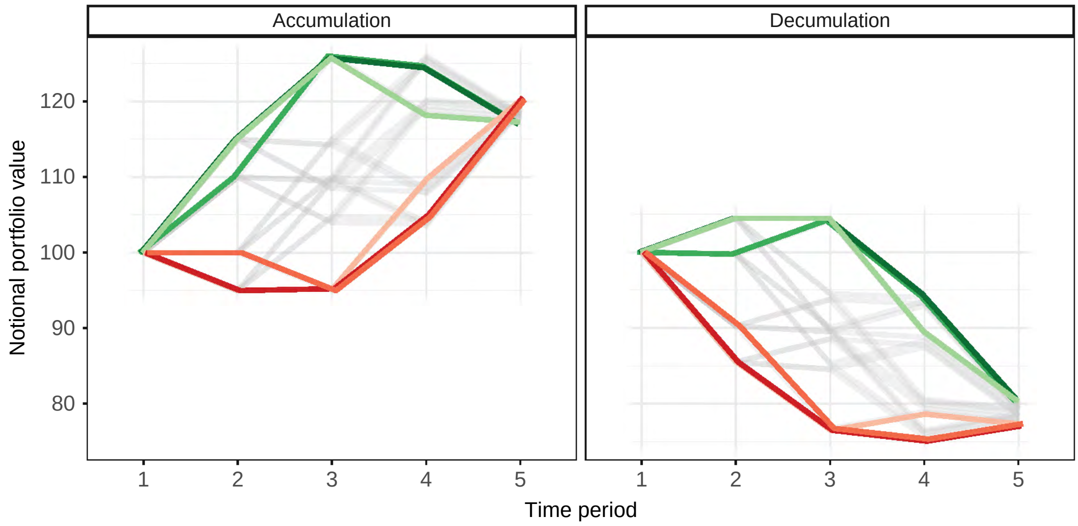

FIGURE 2.2: Sequence risk effect in accumulation and decumulation portfolios

This visually demonstrates sequence risk: for a portfolio with intermediate cash flows, the final portfolio value is most sensitive to periods where the portfolio value is greatest. For decumulation (accumulation) portfolios, it is most sensitive to the early (final) returns of the portfolio. In the most sensitive period, an investor would prefer to realise the best returns of a given set.

2.5.2 Retirement portfolios and SOR

Consider a retirement sub-portfolio with the goal of funding an investor's lifestyle spending requirements. The ideal portfolio theoretically achieves the maximum portfolio value at the retirement date and is slowly depleted for the remaining life. This portfolio is most susceptible to sequence risk at the start of the decumulation phase.

Pfau (2013) depicted the explanatory power of each year's return on a retirement portfolio (i.e., portfolio return sensitivity) shown in Figure 2.3. Considering both the accumulation and decumulation phases, the sensitivity peaks around the retirement date returns. The discontinuity in the figure is due to different explanatory power measurements being used before and after the retirement date.

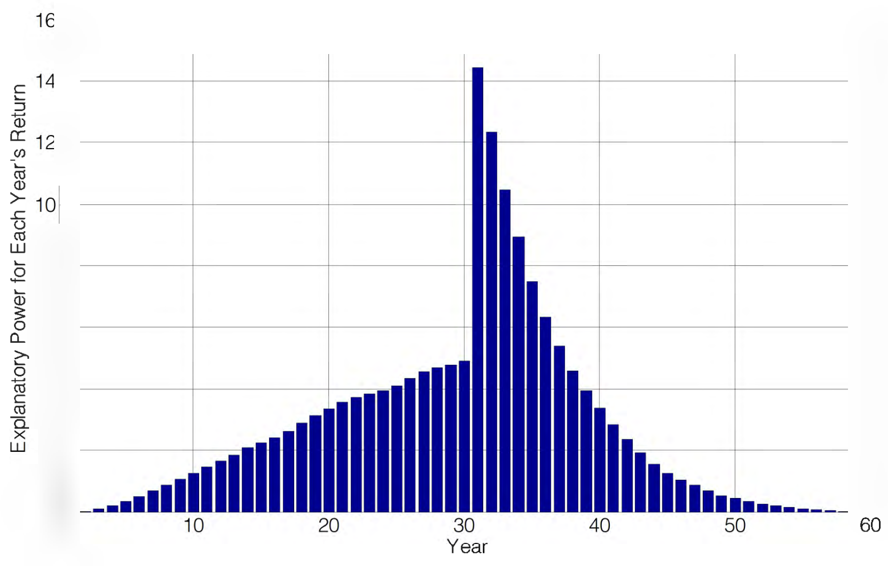

FIGURE 2.3: Explanatory power of each annual return Pfau (2013)

An interesting observation is that the curvature of the SOR risk sensitivity is opposite to the portfolio value plot of Clare et al. (2020), shown in Figure 2.4. While the portfolio value in the decumulation phase decreases at an *increasing* rate, the portfolio return sensitivity appears to decrease at a *decreasing* rate. This difference in rate of change of gradient is also apparent (in reverse) pre-retirement. This difference likely relates to the size of intermediate cash flows relative to the portfolio value.

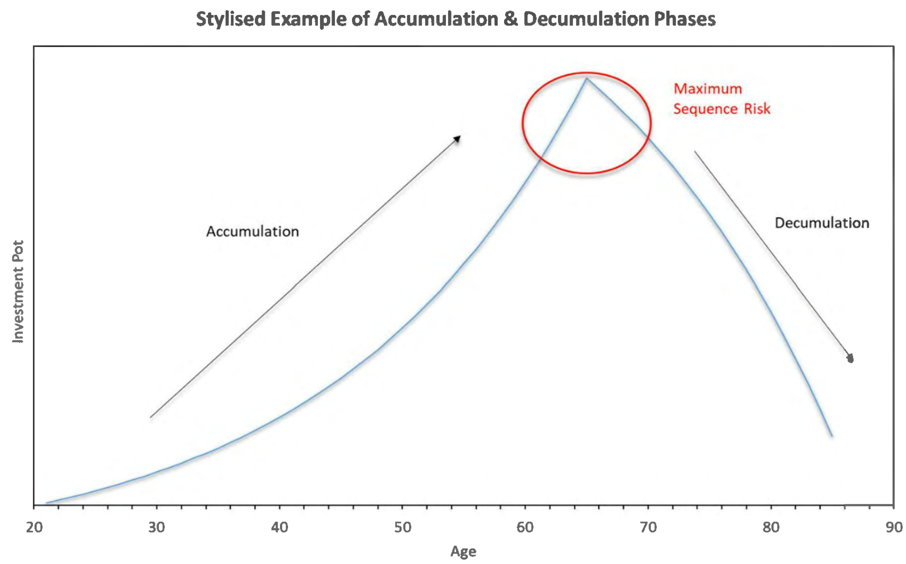

FIGURE 2.4: Investment portfolio value and sequence risk (Clare et al., 2020)

Cotton (2013c) refers to the downside risk in the beginning of the decumulation phase as early loss risk, as opposed to SOR risk. They are similar because the early loss risk, the risk of poor returns in the first few years of retirement, is the most crucial part of SOR risk, as Pfau (2013) showed.

Collins and Stampfli (2019) confirmed the observation of Pfau (2013) on sequence risk but also showed the risk peaks again at the end of retirement: "the danger smile". The SOR risk peak late in life relates to the risk metric Collins and Stampfli (2019) used – the Actuarial Coverage Ratio, discussed in §2.6.2 – which does not make a fixed portfolio horizon assumption. Instead, the portfolio is measured against the survival weighted present value of all possible future withdrawals.

Early in life, aging primarily compresses the mortality and survival curves, with minimal overall shift. However, late in life, the mortality and survival curves are less able to compress so they shift into a shorter period. This is seen in Figure 2.5, where the 65-year-old and 70-year-old mortality curves are very similar with some compression. Conversely, the 85-year-old mortality curve primarily shifts to provide the 90-year-old curve, only with slight compression.

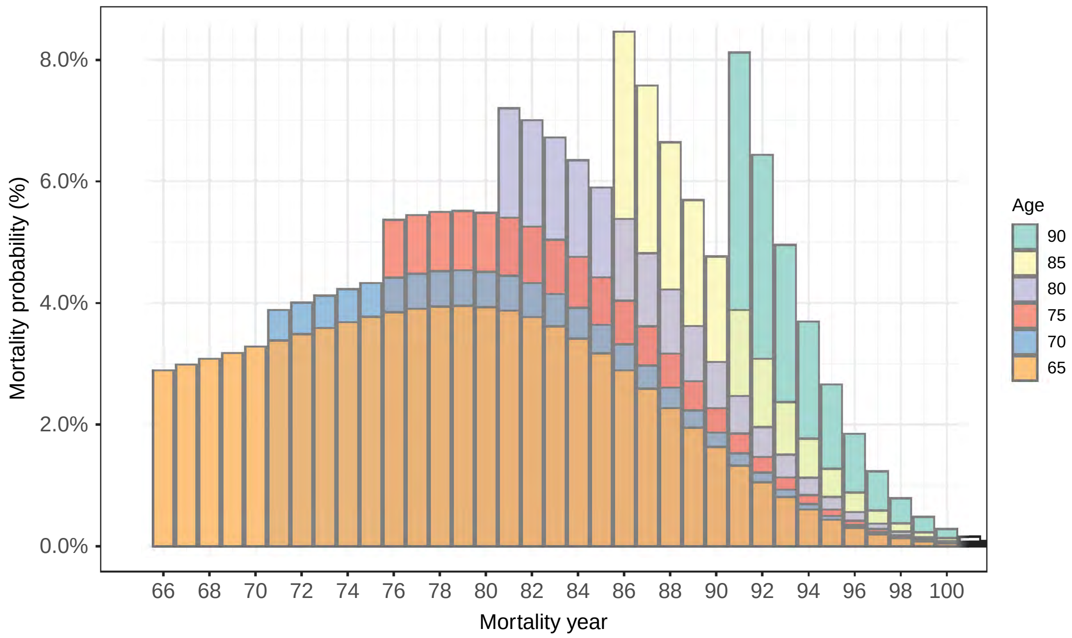

FIGURE 2.5: Danger smile and mortality curves

The result of shifting survival curves is that provisioning for previously unlikely expenses, quickly leads a portfolio that was sufficiently funded to be deemed underfunded. While this SOR risk spike is evident for real investors, the metric used by Collins and Stampfli (2019) does in a manner double count the risk, since a "safe" investor by their metric would necessarily perish with a portfolio sufficient to fund further years. The danger smile or late-in-life sequence risk are discussed further in the results of this study.

2.5.3 SOR risk mitigation strategies

While early loss risk does not capture all of sequence risk, investors have no control of returns order so preventing the early losses is important in preserving a retirement portfolio. Addressing the risks of SOR and early losses can be addressed in two manners, by optimising the withdrawal strategy or optimising the risk allocation, and asset allocation by implication. While this study focuses on the latter, a short discussion of these approaches and their components follows.

Withdrawal optimisation Pfau (2019) separated the withdrawal optimisation into spending conservatively, and spending flexibility. The first approach, spending conservatively, is intuitive. With a lower amount or percentage being withdrawn from the portfolio annually, a lower annual real return is required to maintain capital value. Also, the smaller the relative size of withdrawals in relation to portfolio value, the smaller effect returns sequences have.

Flexible spending approaches call for the income drawn from the portfolio to, at least, reflect the returns on portfolio. Cotton (2013b) showed mathematically why the constant percentage income strategy, where a fixed percentage of the portfolio value is withdrawn each year, is immune to returns path-dependency. Other similar variable withdrawal strategies, like Required Minimum Distributions (RMD), significantly reduce sequence risk. Blanchett et al. (2012) compare sustainable withdrawal rates to various flexible spending strategies, finding spending flexibility beneficial with mortality updating constant failure percentage strategy performing the best. These strategies, however, are not a safe mitigation solution to this specific problem as it merely transforms sequence risk into income risk and does not protect against early loss risk (Tomlinson, 2017). Where the strict portfolio goal is to provide a constant real income stream, an investor cannot accept the risk of less income than what is required.

While these findings mentioned above are valuable and should be applied in practice, this study considers only the essential spending goal in maintaining a lifestyle. As such, this study aims to find risk mitigation approaches that do allow variability in income.

Risk and asset allocation optimisation Asset and risk allocation can be optimised by two approaches that Pfau (2019) identified: using buffer assets and reducing portfolio volatility. The approach of using buffer assets is in theory to have an alternate source of income that can be drawn from after poor returns in one's financial portfolio, which is in effect a volatility reducing strategy. The way to manage income risk is to reduce portfolio volatility, specifically when the volatility would have the largest negative impact. It is a broad approach because of the many ways in which portfolio volatility can be considered.

Firstly, volatility can be reduced from an MPT approach. Most simply, volatility can be reduced by having a larger weighting to lower risk assets. Secondly, considering inter-asset class correlation, a strategy can take advantage of alternative asset classes, that have attractive investment characteristics but with different risk factors than traditional classes. This is easily implemented through the risk parity approach where each asset has an equal risk contribution to the portfolio, ensuring diversified asset holdings (unlike MVO which can be highly concentrated). Finally, volatility can be reduced by having a more internationally diversified portfolio (Driessen and Laeven, 2007). This diversifies the specific country risk, reducing total portfolio risk.

Considering alternate approaches, one possibility is to follow the rising equity glidepath approach explored by Pfau and Kitces (2013) among others (Estrada, 2016; Drew and West, 2021). The declining equity glidepath is a ubiquitous concept in the lifecycle portfolios, reducing the portfolio risk as time to retirement decreases. The rising equity glidepath then begins to increase the equity holding, and portfolio risk, as the individual ages past the retirement date. This keeps portfolio volatility low in the early stage of retirement when sequence risk is high, i.e., when it matters most.

Finally, a buffer of cash can be used in the early stages of retirement to reduce volatility. Pfau (2019) demonstrated the potential by showing how avoiding any one of four annual withdrawals saved an individual from reaching financial ruin. A portfolio's cash holding is effectively a strategy which reduces total portfolio volatility. The cash allocation can then be drawn on based on some decision rule depending on portfolio returns, similar to the preferential withdrawal strategies tested by Spitzer and Singh (2006, 2007).

2.6 Portfolio performance metrics

This study considers an essential lifestyle-maintenance goal and evaluates portfolios that can address sequence risk. The performance assessment requires appropriate metrics for comparison. Common metrics used in this field include measures of drawdown risk such as the value at risk (VaR), shortfall risk, and risk of ruin (Collins and Gadenne, 2017). This section considers two metrics which can be useful in evaluating the improvement in portfolio outcomes by the reduction in sequence risk.

2.6.1 Sustainable Withdrawal Rate

This study is considering a retirement portfolio that must fund a fixed real annual cash flow requirement. This describes the need for a sustainable withdrawal rate (SWR) for a portfolio to be determined. However, an expected SWR alone is not sufficient because it ignores the variability around that expected SWR. As Nevins (2004) describes, the metrics should be framed in a way to be consistent with the investor's goals. A lower-bound confidence interval of the SWR can be used as a risk measure. These two metrics together describe the distribution of SWRs and allow for comparisons.

Since sequence risk is higher early in the retirement period, a portfolio that best protects against early loss risk and SOR will have higher SWR metrics. The relationship between the SWR and sequence risk as well as the SWR calculations used in this study are discussed further in §3.4.1.

2.6.2 Actuarial Coverage Ratio

A limitation of using SWRs is that it is calculated for the entire retirement period and hides the path that the portfolios took. This is relevant because portfolios are monitored and managed continuously. The funded-ratio management approach in this study suggests that a *fundedness* metric is appropriate. Collins and Stampfli's actuarial coverage ratio (ACR) is the ideal metric (Collins and Stampfli, 2019).

The ACR metric periodically compares the portfolio value to the present value of the future expected spending requirements based on annually updated mortality requirements. Using this metric allows *fundedness* to be monitored, the path of the portfolios to be seen, and improvements to downside risk in the early stages of retirement observed. Like the SWR, the ACR should also be used in a confidence interval to capture the argument of Nevins (2004) for relevant risk metrics.

2.7 Chapter Summary

The literature study covered the relevant topics in this study. The portfolio design was first discussed, explaining why a HHBS design approach is appropriate. Following this, the issue of income approach was discussed with reference to the debate between safety-first and probability-based theories. It was also explained how hybrid approaches could gain from aspects of both extremes. Retirement goals and risks were then discussed, with a focus primarily on the essential lifestyle maintenance goal of retirees. Following this, the asset allocation and asset class decision was discussed, forming the final part of the portfolio creation process. Sequence risk and other related risks were then discussed in a retirement context, then considering on strategies which could reduce these risks. Finally, appropriate metrics were discussed, with specific reference to those that can be used to evaluate SOR risk.

Chapter 3

Data and Methodology

This chapter describes the components of this study's methodology. Firstly, a short overview of the design approach is presented, briefly discussing the approach and the reasons for it. Next, a description of the simulation methodology is provided. The portfolios and strategies employed are discussed next, followed by a discussion of the portfolio performance metrics and the SOR sensitivity analysis. Finally, the data inputs for the simulation as well as those for validation are considered.

3.1 Methodological design

The aim of this study is to measure the importance of sequence of return risk and determine the efficacy of portfolio strategies in addressing that risk. The sequence of return risk can be measured by sensitivity to early retirement returns relative to late returns, as was explained in §2.5. Once the sensitivity of retirement portfolios to sequence risk is established, the efficacy of portfolio strategies must be benchmarked using appropriate event risk metrics because the standard mean-variance optimisation (MVO) portfolio metrics are inappropriate in this environment as described in §2.1.1. For this reason, the sustainable withdrawal rate (SWR) is used to encapsulate the overall portfolio performance in achieving the minimum spending goal. Secondly, the actuarial coverage ratio (ACR) metric is used to measure the "*fundedness*" of the portfolio annually and to take longevity risk into account.

The confidence level on this study is set at 95% which determines both the simulation length and how results will be evaluated. The average retirement age in OECD countries, here considered as a sample of all nations comprising both developing and developed nations, has been trending upwards over the last 25 years from an average age of 62.9 in 2010 to a forecast of 63.5 in 2020 (Chomik and Whitehouse, 2010). With that information on trends, a retirement age of 65 years was assumed. Using the South African Annuitant Standard Mortality Tables (Dorrington and Tootla, 2007), the confidence level implies the portfolio should last 95% of lifetimes. The mortality table shows a 5% chance for a 65-year-old to live to 95 years and beyond. This supports an assumed 30-year period for which the portfolio should last; 30 years is in similar length to what others in the field (Bengen, 1994; Blanchett, 2007; Spitzer and Singh, 2007; Suarez et al., 2015; Estrada, 2016; Collins and Stampfli, 2019) have used (which enhances the comparability of this study with others).

As defined in the scope of this study, the passive approach to investing means that all portfolios will hold investments in broad asset class indices. However, the limited history on the necessary indices prevents a solely historic analysis when considering 30-year retirement periods. For this reason, Monte Carlo simulation was necessary to generate sufficient non-overlapping asset return data on which to evaluate the portfolio strategies.

Figure 3.1 below illustrates the overall methodology employed in this study with the data sources on the left, and the different results being investigated on the right. The central processes indicate the transformation of asset return data to portfolio return data to results. The line types indicate whether the data are actual historical data or simulated data, with the subscripts denoting the sections in which the processes are discussed.

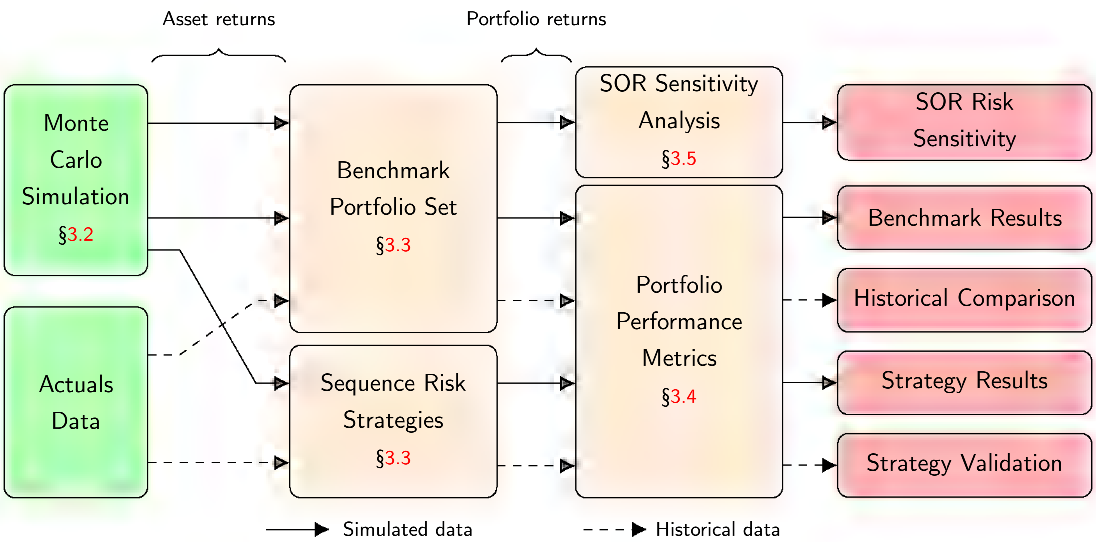

FIGURE 3.1: Methodology flow diagram

3.2 Simulation methodology

In simulating the asset class index returns it is important to capture the aspects of return series that have a significant influence on SOR risk. These are market cycles, distribution of returns within those cycles, and the correlation of asset returns, especially in bear phases when correlations are often seen to increase (Campbell et al., 2002). The method by which these criteria are achieved are described in the following sections.

The implementation of the simulation, achieved through Monte Carlo simulation, is illustrated in Figure 3.2, describing the transformation from historical returns and inflation input data to simulated returns and inflation data. The subscripts in the nodes indicate the sections where the contents are discussed.

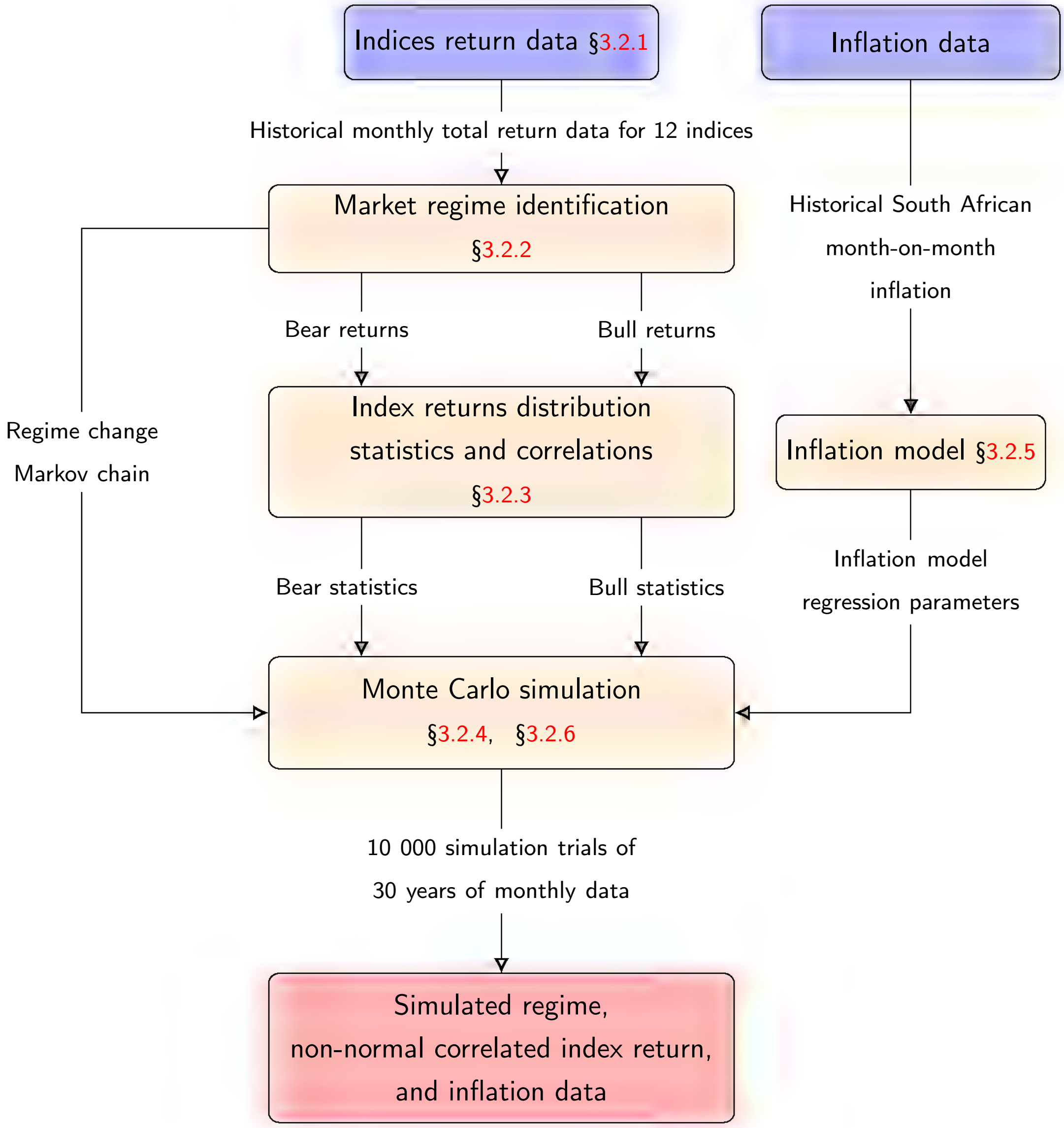

FIGURE 3.2: Monte Carlo simulation methodology flow diagram

3.2.1 Returns simulation input data

The Monte Carlo simulation requires asset return distribution parameters in generating trials. These parameters were sourced from historical monthly closing return data of the relevant total return indices. The data for most of the asset classes span from 1 January 1991 to 1 August 2020 with some having shorter histories. More discussion on the input data and the sources thereof is provided in §3.6.

All asset returns were priced in South African Rands (ZAR) and unless otherwise stated, the results are from the perspective of an individual investing in ZAR. This does mean there is implicit exchange rate return for some of the indices which was not separately modelled. As stated in the scope of this study, all fees, expenses, and transaction costs are excluded.

3.2.2 Bull and bear market regime identification

Financial markets, especially equity markets, move through substantial periods of good and poor performance. Bull and bear markets, representing general positive price and negative price trends respectively, are very important in a retirement portfolio. A poor return, bear phase early in retirement means that it is unlikely that a portfolio will be able to last as long as would be expected (Collins and Stampfli, 2019).

The market cyclicality has a significant effect on retirement portfolio performance, however, a simulation drawing from the overall return distribution does not capture the cyclicality of markets. For this reason, the market regime phases need to be specifically modelled in this study, separating the distributions of the indices into bear and bull phases. The method by which this was done in this study follows a modification of the procedure provided by Pagan and Sossounov (2003) using the ALSI total return monthly index as the basis for identifying cycles. The ALSI is used as a basis because this study is based on a South African context with a focus on South African asset indices from which portfolios are formed. An explanation of the regime identification steps as well as the results are provided in Appendix A. Figure 3.3 below demonstrates the regime identification results with bear markets shown in red and bull markets shown in green.[1]

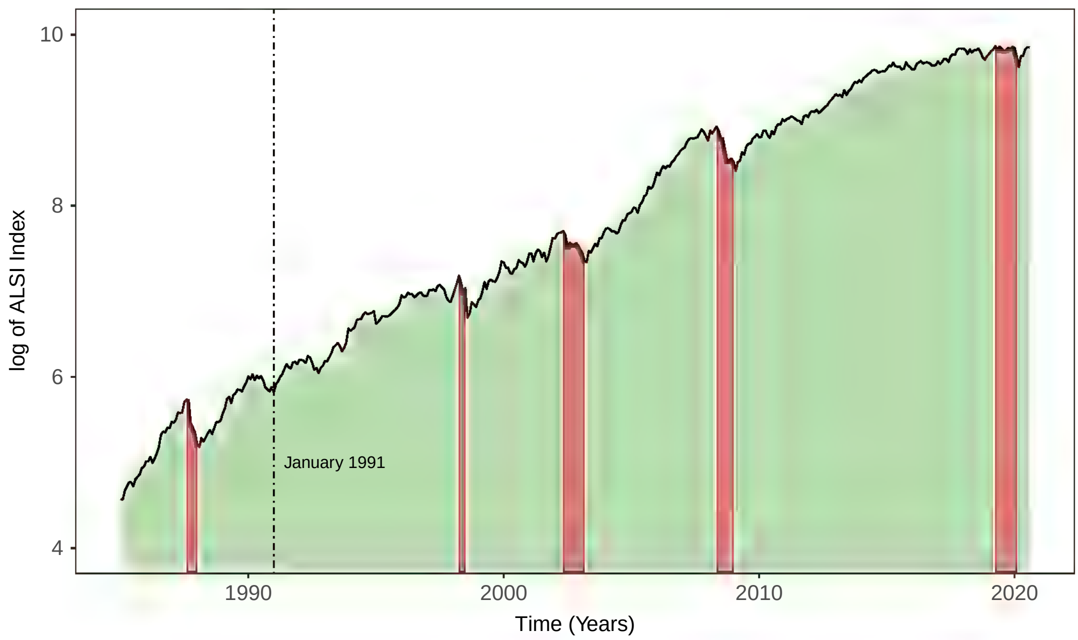

FIGURE 3.3: Bear and bull market regime identification

[1]The y-axis of Figure 3.3 is the natural log of the ALSI price index with a January 1985 basis of 100.

With the data separated into bull and bear markets, Markov chains describing the switching between bull and bear market states, or regimes, were developed, following the method of Winston and Goldberg (2004) and reflecting the modified regime identification method of Pagan and Sossounov (2003). The steady state probabilities were also derived for seeding the starting market states of each simulation trial. The calculations for the probabilities of switching as well as the steady state probabilities is given in Appendix A.3 and A.4.

Figure 3.4 below shows the steady-state vectors (denoted π_{Bull} and π_{Bear}) and the Markov chain describing the initial state selection for each simulation trial and the transitions between the bear and bull market regimes respectively. Because the number of monthly periods in a bull state far outnumbered the number of bear state periods, the steady-state vector probabilities show the initial state is far more likely (93% probability) a bull state than a bear state (7% probability). Once the initial state is determined, the Markov chain probabilities are used to determine changes between states. Due to the historically longer average bull market run length than bear market average run length, the probability of staying in a bull state is greater (98.7%) than staying in a bear state (82.6%). The complement to these probabilities is the probability of switching state i.e., the market regime.

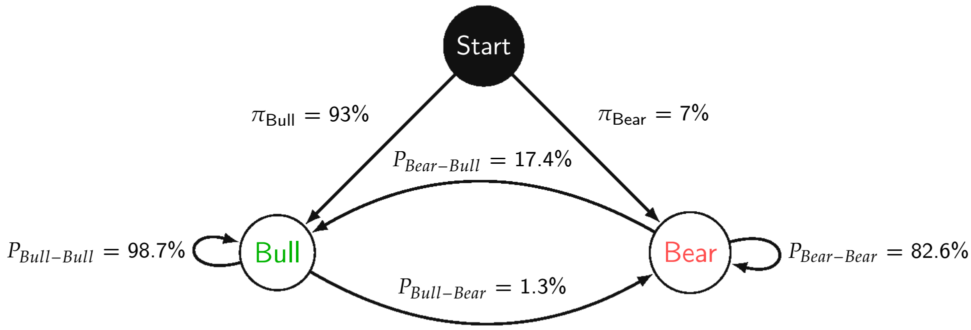

FIGURE 3.4: Market regime switching Markov chain

One feature of the Markov chain not shown in Figure 3.4 relates to how regimes were identified, as set out in in Appendix A.1. In identification, market phases needed to be at least four months long. In order to reflect this constraint in the regime simulation, when the regime or market state changed, that state would remain constant for at least four months before it could change (in which case the Markov probabilities would be used).

3.2.3 Distribution statistics

Once the returns were separated into appropriate cycles, the distribution statistics were determined for each asset for bull and bear markets separately. The parameters include the mean, standard deviation, skewness, and kurtosis for the monthly returns of each asset. When the returns history was shorter than the January 1991 to August

2020 period, that period was used. Correlation matrices were also generated for the set of assets in each regime. The correlation only considered months where return data was available for all assets so that the resulting matrix was positive definite, a requirement for the simulation method used. This resulted in some of the returns data being excluded from the correlation calculation.

Appendix B contains further information describing the distributions of the assets in each regime as well as the asset return correlation matrix for each regime.

It has been observed that asset return distributions exhibit fat-tails and non-normality (Johansen and Sornette, 1999) and different market conditions tend to have vastly different skews (Gonzalez et al., 2005). This, in combination with the fact that the sensitivity of SOR risk to extreme returns early in retirement is very high, as explained in §2.5 of the Literature review (Chapter 2), means the simulation needs to take into account the skew and kurtosis of the asset distributions in each market cycle, over and above the normal statistics of standard deviation and mean. Extreme event risk is pertinent when considering the returns sensitivity of a retirement portfolio. Having an accurate distribution with four moments will account for these outcomes and accurately represent their likelihood.

3.2.4 Multivariate non-normal distributions

The method of producing observations or simulation trials from the distributions becomes more complex when the distributions are no longer assumed to be normal but while the variables are correlated. The method used follows the definitive extension of Vale and Maurelli (1983) to the widely used Fleishman (1978) power method. Firstly, the Fleishman (1978) method describes the process to generate the necessary constants to transform standard normal variables into non-normal ones. Thereafter, an intercorrelation matrix is derived using the extension by Vale and Maurelli (1983) with Newton Raphson. From this intercorrelation matrix, a Cholesky decomposition is used to generate observations or trials from the correlated non-normal distributions. This process was implemented in R using the SimMultiCorrData package (Fialkowski, 2018).

3.2.5 Inflation model and regression

The final input to the simulation model is inflation data. Inflation was modelled separately from asset returns because of the lack of strong historic correlation between any of the assets and South African inflation in the period considered. Table 3.1 below shows the correlation matrix of the three South African assets and South African inflation.

	Inflation	ALSI	ALBI	STeFi
Inflation	1.00	-0.02	0.19	0.40
ALSI	-0.02	1.00	0.27	-0.00
ALBI	0.19	0.27	1.00	0.17
STeFi	0.40	-0.00	0.17	1.00

TABLE 3.1: Correlation matrix of South African assets and inflation

The inflation modelling method implemented here is the same as implemented by Collins and Stampfli (2019), i.e., a "serially correlated random variable with a smoothed reversionary factor". The inflation model persistence parameter was determined through an ordinary least squares linear regression of historical South African month-on-month headline CPI inflation data from June 1995 to August 2020. Due to the volatile relatively high inflation in the early 1990s and before, an inflation regime change is deemed to have occurred in mid-1995; this means that it is inappropriate to extrapolate trends from that earlier data. Figure 3.5 below shows the rolling-annual South African price inflation with the regime change indicated. For the inflation model, the regime change is selected to have occurred in June 1995.[2] The long-term average inflation and inflation standard deviation were also calculated from the inflation regime change date. The inflation model parameters are provided in Appendix C.

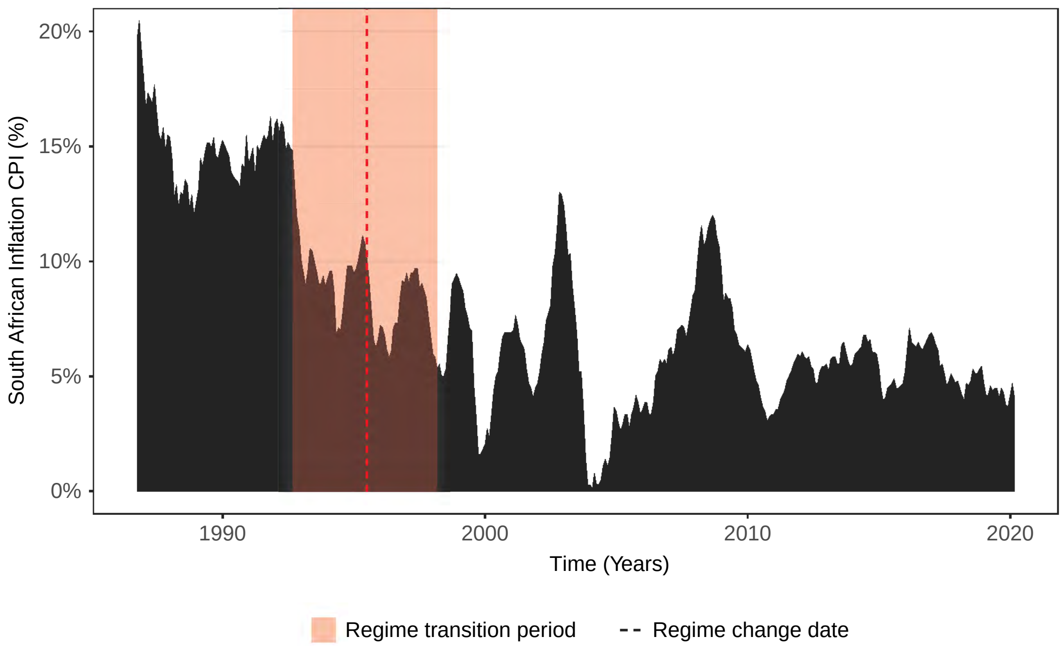

FIGURE 3.5: South African CPI and inflation regime change

[2]The regime change point was taken as the midpoint of the transitionary period between end of the regime ending in the early 1990s and the current inflation regime. The start of the transitionary period was taken as September 1992, coinciding with the sharp drop in annual inflation. The end point of the transitionary period was when annual inflation first dropped below the new regime's average annual inflation, February 1998.

The inflation generation was performed using Equations 3.1 and 3.2 below. The parameters in this model include, k, the persistence coefficient, $\text{Inflation}_{\text{LT avg}}$, the long-term average month-on-month inflation, and $\text{sigma}_{\text{infl}}$, the standard deviation of month-on-month inflation.

$$Inflation_t = \underbrace{\frac{k}{12} \times \sum_{n=t-12}^{t-1} Inflation_n}_{Persistence\ component} + \underbrace{(1-k) \times Inflation_{LT\ avg}}_{Reversion\ component} + \underbrace{\sigma_{Inflation} \times \mathcal{N}(0,1)}_{Error\ term} \tag{3.1}$$

$$Inflation_t = \begin{cases} \frac{k}{t-1} \times \sum_{n=1}^{t-1} Inflation_n + (1-k) \times Inflation_{LT\ avg} + \sigma_{Inflation} \times \mathcal{N}(0,1) & for\ 1<t\leq 12 \\ Inflation_{LT\ avg} + \sigma_{Inflation} \times \mathcal{N}(0,1) & for\ t=1 \end{cases} \tag{3.2}$$

Equation 3.2 shows the inflation model's persistence term for the first twelve periods of each trial was limited to any previous periods generated until twelve months had been generated and could continue using a rolling twelve-month inflation history. Therefore, for the first period generated, there is no persistence component.

3.2.6 Simulation trial generation

The process of generating trials was completed as follows. It was determined that 10 000 trials were to be generated to sufficiently exceed the minimum requirement for accurate distributions identified by Lee (2015), similar to Spitzer and Singh (2008), Basu et al. (2011), and Pfau (2011).

For each trial, the market state of the first period was determined using the steady state of the regime-switching Markov chain. For the remaining 359 monthly periods, the regime switching was determined using the Markov chain which captures the market phase definition of Pagan and Sossounov (2003) (see §3.2.2 above). For each period, the market regime would determine whether to draw a sample from the bull or bear distribution sets. The inflation for the period is then determined following the method detailed in §3.2.5. Figure 3.6 describes the process of generating simulation trials.

The end of this process resulted in 10 000 trials of 360 months of non-normal correlated asset class index returns and inflation data with regime changes in asset returns. The monthly returns in each trial were aggregated into consecutive, non-overlapping 12-month periods, resulting in trials of 30 annual periods.

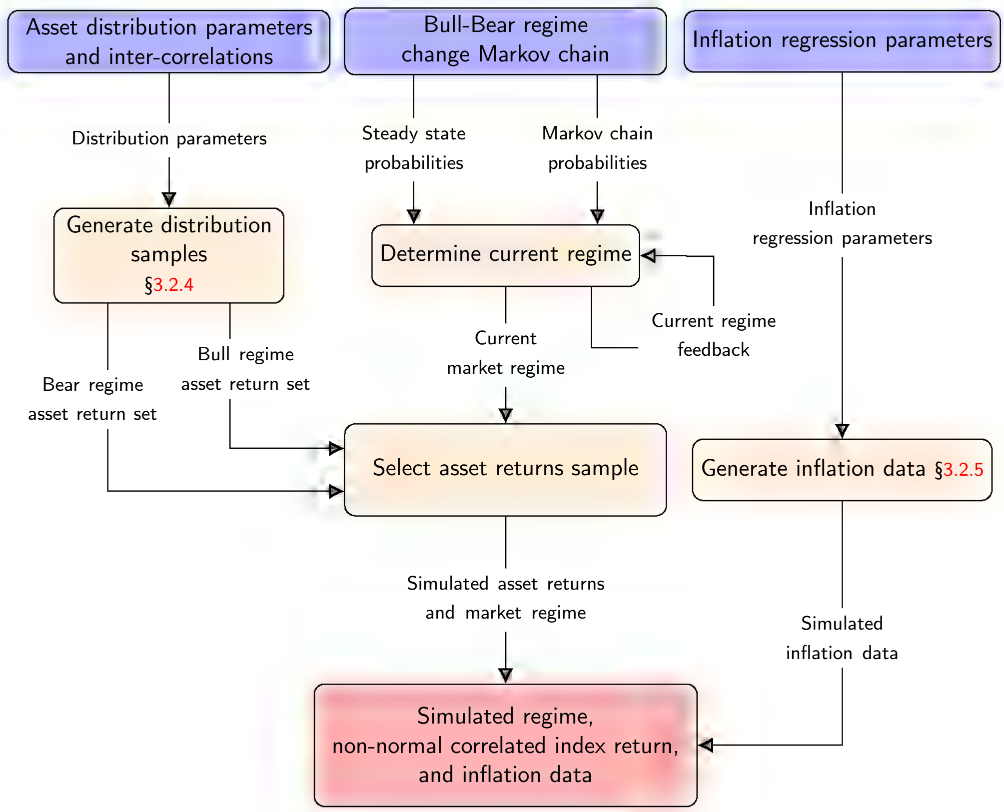

FIGURE 3.6: Simulation trial generation flow diagram

3.3 Portfolios and strategies

This section details the strategies implemented and the rationale for their inclusion as well as the benchmark set of portfolios against which the strategies are compared. The assets these portfolios hold and the proportions with which they are held, i.e., the portfolio holdings, are explained for each strategy below.

The portfolios discussed in the following sections include the benchmark set of portfolios, the historical comparison, the portfolio strategies to address SOR risk, and the validation of the strategies. The benchmark set forms a baseline for assessing the effectiveness of SOR risk strategies. Historical asset return and inflation data for South Africa and the United States was used for historical comparison purposes. Following this the sequence risk strategies are discussed. The portfolio strategies for addressing SOR risk being considered are:

- reducing portfolio volatility with a greater proportion of lower risk assets,
- geographic diversification,
- risk parity,
- rising equity glidepath, and
- dynamic cash buffer.

All these strategies are used or have theoretical benefits to reduce volatility by different metrics, not necessarily in a retirement portfolio context. These are explained for each strategy in this section. Finally, strategy validation is discussed for selected strategies, applying actual historical data to the strategies to determine how they would have performed.

Retirement portfolio construction

This study considers minimum spending requirement retirement portfolios, maintaining a constant real (inflation adjusted) cash flow stream withdrawn at the beginning of each year for 30 years. The portfolio begins with a value, K_s, or K_0. The real portfolio value at the end of period t is given by:

$$K_t = (K_{t-1} - w) \times (1 + r_t) \tag{3.3}$$

where K_t and r_t represent the real portfolio value and real portfolio return at time t. w represents the real annual withdrawal amount. When K_0 is set notionally to 1, w represents the percentage withdrawal rate, the percentage of the initial portfolio value where the real (inflation-linked) amount is withdrawn annually. Equation 3.3 describes that the portfolio value at the end of period t is equal to the previous period's value less the withdrawal amount, then grown by the real return achieved by the portfolio during the t^{th} period. Equation 3.3 and those that follow are given in real terms where possible for simplicity although they are equally valid in nominal terms. The nominal equivalents are given in Appendix D.

The real portfolio return in period t is given by:

$$r_t = \sum_{i=1}^{n} r_{i,t}\,\rho_{i,t} \tag{3.4}$$

where $r_{i,t}$ is the real return of asset i in period t and $\rho_{i,t}$ is the portfolio's holding of that asset i.e., the proportion which asset i comprises in the portfolio in period t. In Equation 3.4, n is the total number of assets. For static and pseudo dynamic portfolios, the proportion of asset holdings, $\rho_{i,t}$, are constant or at least independent of the previous periods' asset and portfolio returns. In the case of dynamic portfolio strategies, this is not the case, and is discussed in §3.3.5.

The convention in this paper for describing a portfolio's proportion each asset constitutes, or portfolio holdings, are given in the format of "X/Y" where the first number represents the equity percentage and the second number the bond percentage. Occasionally holdings will be given in the form "X/Y/Z" where the final number represents the money market (or "cash") percentage.

3.3.1 Benchmark portfolios

As a benchmark set, eleven portfolios consisting of equities and bonds in varying proportions are utilised. In this case, given the South African focus of the study and because they represent the common traditional asset classes, used in forming typical balanced and lifecycle portfolios, the FTSE/JSE All Share Total Return Index (ALSI) and the FTSE/JSE All Bond Total Return Index (ALBI) will form the benchmark set of portfolios. The benchmark set of portfolios consists of eleven portfolios ranging from 100% equity (ALSI) to 100% fixed income (ALBI), changing in 10% increments. The asset allocations of all eleven portfolios are provided in Appendix E.

3.3.2 Historical comparison

As a useful comparison, the benchmark portfolios (incorporating actual historical South African return and implicit macroeconomic data) were compared to an equivalent set of US portfolios using US historical data. This analysis also allows evaluation of the South African and US benchmarks to the "4% Rule" as was derived by Bengen (1994) as well as Maré (2016). The South African benchmark portfolio performance using historical data can also, to some extent, validate the simulated South African benchmark set to the extent that they produce similar results.[3]

The South African assets were the ALSI and ALBI while the US assets chosen to represent US equities and bonds (that also had sufficient comparable history) were the Russell 1000 (Russell 1000) Total Return Index and the Bloomberg Barclays US Aggregate Bond (BBUSAB) Total Return Index. The portfolio sets for the US and South Africa have the same holdings as the benchmark portfolios in §3.3.1. The input data are discussed further in §3.6 while the asset allocations of the historical comparison portfolios are provided in Appendix E.

The actual historical data for both countries included a history of 413 months of actual asset return and macroeconomic month-on-month inflation data from April 1986 to August 2020. Considering the 30-year portfolio goal, the historical input data resulted in 54 overlapping periods of 30 years.[4]

3.3.3 Static portfolio strategies

Lower portfolio risk

The first strategy being considered is holding a greater proportion of low-risk traditional assets, fixed income and cash equivalents or money market instruments. The money market index used in this study is the Short-Term Fixed Income (STeFi)

[3] The "4% Rule", from Bengen (1994), submits that a 4% SWR has not failed in any consecutive 30-year period between 1926 and 1994 for a US investor.

[4] The historical comparison considers the performance of the minimum spending goal portfolio, investing only in local assets between countries. In other words, a South African retiree investing in only South African bonds and equities is compared to the US equivalent, in terms of SWRs each can support.

total return index. This strategy has the purpose of reducing portfolio return volatility as defined by MVO and to determine whether this volatility reduction translates to reducing risk in a retirement context.

Seeing as the benchmark set already contains the range of equity to bonds, the strategy is implemented by first selecting four portfolios from the benchmark set: the endpoints (100/0) and (0/100), the mid-point (50/50), and a portfolio some way between the mid-point and all-bond portfolio (30/70) in order to better determine the rate at which reducing portfolio risk affects the metrics. Using these portfolios and maintaining the relative proportion between bonds and equities, cash is added in varying percentages to each to create a set of sixteen portfolios with varying proportions of low-risk assets. This set of portfolios with varied allocation allows for the analysis of the extent to which low risk assets improve the SOR risk metrics being considered. The detailed holding proportions of these portfolios is given in Appendix E while the SOR risk metrics are discussed in §3.4.

Geographic risk diversification

The diversification benefits of investing in other countries and groups of countries are proven in a traditional accumulation portfolio environment, and from a developing market nation even more so (Driessen and Laeven, 2007). This considers how SOR risk metrics, discussed in §3.4, are affected by this form of portfolio risk reduction. The geographic diversification considered here is relative to investing solely in a single emerging market, specifically South African bonds and equities. The diversification cases considered here include:

- *Diversifying into a broad index of emerging market nations*. This aims to test the extent to which emerging market investment risks are correlated to the South African market. The indices to represent emerging market bonds and emerging market equites were the Bloomberg Barclays EM bonds (BBEMB) total return index (TRI) and the MSCI Emerging Market equities TRI respectively.

- *Diversifying into a single developed nation*. Complementing the previous point, this aims to test the degree to which a single developed nation can reduce the risk metrics due to reduced correlation. The single developed selected in this study was Germany.[5] The indices used to represent German bonds and equities were the CDAX TRI and the REX TRI respectively. The CDAX (composite DAX) was chosen as it forms a more appropriate equivalent to the ALSI, where the DAX would be equivalent to the South African Top 40 index.

[5]While the selection of Germany is seemingly arbitrary, it is chosen to illustrate the potential for geographic diversification into a single developed nation. Countries like the USA, Japan, and the UK are not chosen because of their dominant weight in broad indices. Choosing any of these nations would create correlation and excessive overlap between the single developed nation case and the developed market broad index case. Germany is a developed nation that is less dominant in broad indices which reduces overlap between the cases, however Switzerland or France are equally valid choices.

- *Diversifying into a broad index of developed market nations*. Similar to the previous point, this attempts to determine how diversifying away the specific risk of any one developing nation affects the diversification potential. The indices used here are the MSCI World TRI and the Bloomberg Barclays Global Aggregate TRI (BBGA) representing developed market equities and bonds.[6]
- *Diversifying equally into developed and emerging markets.* Finally, this considers the equities and bonds of all nations and how they can reduce the retirement risk metrics. The indices used here are equal weightings of the MSCI World and MSCI Emerging Market for equities, and in equal weightings the Bloomberg Barclays Global Aggregate and Bloomberg Barclays EM for bonds.

In each diversification case, 20 portfolios are considered as a range from 0% to 100% equities transition from bonds on the one axis and 100% South African assets to 100% non- South African assets on the other. The exact portfolio holdings, for each portfolio, are given in Appendix E.

Risk parity

The last of the static portfolio strategies is using a risk parity approach that differs from conventional MVO theory. Risk parity seeks to have equal risk contribution – risk defined as the standard deviation of portfolio returns – from each asset class included in the portfolio. The premise of risk parity is the risk diversification leads to loss diversification. The risk and asset diversification are guaranteed in risk parity portfolio construction, as opposed to MVO portfolios where allocations can be extremely concentrated on singular, relatively strong performing assets. This portfolio strategy will test the efficacy of risk parity diversification on SOR risk.[7]

Five combinations of assets were considered in the risk parity strategy. These all contained South African equities and bond indices but three of the portfolios sets added other asset classes viz gold, inflation-linked bonds, and a combination of gold and inflation-linked bonds. The final risk parity portfolio contained South African equities, bonds, inflation-linked bonds, and gold as well as the MSCI World, EAFE, EM, the BBGA, and the BBEMB. In each portfolio type, the assets were held at a constant weighting as determined by the risk contribution based on the distribution statistics of each asset class. This was implemented with riskParityPortfolio package in R (Vinicius and Palomar, 2019).

In order to visualise the transition between the benchmark portfolio and the risk parity portfolios, a portfolio midway between each risk parity portfolio and the benchmark were included. The midpoint portfolio between each of the five risk parity portfolios

[6] The bond index used here is limited to investment grade bonds resulting in the index comprising over 90% developed market bonds. The MSCI EAFE is also used as an alternative to the MSCI World for this case because of the US comprises over two thirds of the World index.

[7] While the risk parity strategy is sometimes used with leverage to increase the average return (Qian, 2016), that is not considered in this study as it would likely expand the returns distribution and not improve the risk metrics at a 95% certainty level.

and the equal-weighted (50/50) benchmark portfolio were used. The holdings of all the risk parity portfolios are given in Appendix E.

3.3.4 Pseudo-dynamic strategies

Rising equity glidepath

Glidepaths are a common trait of life cycle strategies implemented in target-date funds (Schalkwyk et al., 2017). In lifecycle strategies, the glidepath implemented is a declining equity glidepath where the risk level of the asset allocation reduces over time, beginning with a traditionally "balanced" or aggressive allocation to stocks, and ending with an allocation entirely to lower risk assets such as fixed income. This view of asset allocation pre-retirement is similar to those of popular (although naïve) allocation strategies such as the "120 minus age" rule in which the portfolio's percentage equity holding should be 120 less the age of the investor (Estrada, 2016). For example, at 50 years old, an investor should be 70% invested in stocks, and 30% in bonds.

The lifecycle view, reducing portfolio risk as an investor ages, is in line with literature on SOR risk pre-retirement as investors are much more susceptible to sequence risk near the retirement date. However, applying the lifecycle view has been shown internationally (Blanchett, 2007; Estrada, 2016) and in South Africa specifically (Estrada, 2016) that declining equity strategies post-retirement are sub-optimal (when considering failure percentages in supporting a certain withdrawal rate) and that balanced portfolios outperform them. Furthermore, Spitzer and Singh (2006, 2007) show that withdrawing preferentially from bonds, was superior to some degree.[8] This is, in effect, supporting the case for a rising equity glidepath. Pfau and Kitces (2013) went on to show specifically that rising equity glidepaths were, in their analysis, superior to both declining equity glidepaths and static balanced portfolios, reasoning that the rising equity allocation avoids excessive SOR risk.

The rising equity glidepath strategy is included in this study to test its efficacy, specifically in a South African context. This pseudo-dynamic strategy steadily increases the weighting in equities while keeping the proportions of other assets (bonds and cash) fixed relative to each other. Given the significant number of possibilities for starting and ending equity holdings, the relative proportions of the remaining assets, and the rate at which the glidepath was structured, guidance was taken from previous studies. As such, based on the best performing rising equity glidepath portfolio results of Pfau and Kitces (2013), a differential between starting and ending equity holding of 30% is selected and this will be applied to different starting equity holdings.

[8] Withdrawing preferentially means to liquidate a specific asset for spending needs while leaving the other asset classes unaffected. This results in the weights of the other asset classes in the portfolio to rise until the asset being drawn preferentially is depleted.

These portfolios considered only include South African equities, bonds, and money market indices. Sixteen portfolios are considered here, with starting equity holdings of 0%, 10%, 30%, and 50% increasing their equity holding linearly each year to ending equity holdings of 30%, 40%, 60%, and 80%, respectively. The remaining proportion of assets are similar to those in the low-risk strategy; that being made up of bonds with either 0%, 5%, 10%, or 20% money market index to start.

3.3.5 Dynamic strategy

The final strategy being considered is dynamic where the holdings and returns of one period are determined by the set of returns and holdings preceding it. Based on the Evensky & Katz Cash Reserve Strategy (E&K-S) (Evensky, 2006) as well as the results of Spitzer and Singh (2006, 2007) while considering the criticisms of Estrada (2019), this strategy aims to capture the view of Pfau (2019) in "reducing volatility (when it matters most)" in addressing SOR risk. This strategy aims to utilise the stability of the cash equivalent asset class to minimise the volatility of the portfolio.

The strategy starts with initial asset holdings the same as those of the low-risk strategy. The difference is that the proportions with which assets are held, $\rho_{i,t}$, depends on the portfolio return of the previous when there is a period. In this case, in periods for which the portfolio return (excluding the cash asset) is negative, withdrawals for the next period only liquidate the cash asset. This is given by Equation 3.5 below.[9]

If $\quad r^*_{t-1} \quad \leq \quad r^*_{cash,t-1} \times \rho_{cash,t-1} \quad$ then:

$$\rho_{cash,t} \quad = \quad \begin{cases} 0 & \text{for} \quad w \geq K_{t-1} \times \rho_{cash,t-1} \\ \dfrac{K_{t-1} \times \rho_{cash,t-1} - w}{K_{t-1} - w} & \text{for} \quad w < K_{t-1} \times \rho_{cash,t-1} \end{cases} \tag{3.5}$$

This shows that only the cash holding is used for spending needs in preference to the other assets, i.e., the other assets remaining undrawn, if the nominal return contribution of cash, $r^*_{cash,t-1} \times \rho_{cash,t-1}$, is greater than the total nominal return of the portfolio, r^*_{t-1} (in other words, where the return contribution of bonds and equities together was negative). If so, the cash proportion is adjusted depending on the available cash in the portfolio and the required withdrawal amount.

After the cash holding is adjusted for spending, the bond and equity proportions are rebalanced by Equation 3.6 below.

$$\rho_{i,t} \quad = \quad \frac{1 - \rho_{cash,t}}{\rho_{bonds,t-1} + \rho_{equities,t-1}} \times \rho_{i,t-1} \qquad \text{for} \quad i = bonds, equities. \tag{3.6}$$

[9] The notation used here continues from that defined in Equations 3.3 and 3.4 in §3.3.

Notice that if $\rho_{cash,t-1} = \rho_{cash,t}$, the denominator and numerator are equal and the proportion each asset comprises in the entire portfolio will remain constant. The logic behind this strategy is that it avoids selling risky assets at depressed prices before they have a chance to return to their pre-decline level.

The dynamic nature of this portfolio strategy should now become apparent. The asset proportions in the portfolio and therefore the portfolio returns in period t are dependent on the returns of period $t-1$. Furthermore, the size of the annual withdrawal, w, determines how fast the cash buffer is depleted should negative return periods occur in the non-cash portion of the portfolio. The returns in later periods are thus, dynamically determined by the withdrawal rate.

The dynamic cash buffer aims to avoid spending from depressed assets (with higher expected returns) in periods of poor returns to support the survival of the portfolio. This is similar to what Pfau (2019) showed in the positive impact of skipping a year of distributions. Spitzer and Singh (2006, 2007) also test this to an extent by drawing preferentially from bonds (in a bond-equity portfolio). This is taken further here by using cash as the volatility absorbing asset. The strategy differs from E&K-S in that it does not replenish the cash buffer after it has been used. This avoids the criticisms of E&K-S where it is criticised for assuming that risky asset prices will have recovered in five years. Instead of replenishing cash to maintain percentage holdings, as the portfolio experiences poor returns, the cash reserve is depleted, and the portfolio moves closer to a benchmark, static portfolio.

3.3.6 Strategy validation

The strategy validation aims to compare the performance of the strategies on historical data. This considers the strategies limited to the same set of assets, namely low-risk, REGP, and DCB. The validation considered the same historical data used in the historical comparison in §3.3.2, a history of 413 months of actual South African monthly asset returns and inflation data. This resulted in 54 overlapping periods of 30 years from 1986 to 2020 with 5 years of out-of-sample data (1986-1991) included.

Using the South African historical benchmark described in §3.3.1, the performance of the three comparable strategies are assessed. Their performance will be able to validate the viability of these strategies.

3.4 Portfolio metrics

The portfolios were evaluated based on two primary metrics, the Sustainable Withdrawal Rate (SWR) and the Actuarial Coverage Ratio (ACR). An explanation of these metrics, their necessity in this study and their usage follows. While a metric reflecting early-retirement portfolio return sensitivity would measure their effectiveness in addressing SOR, the SWR and ACR more appropriately measure what is of interest to the retired individuals i.e., the sustainability and feasibility of their retirement goals.

3.4.1 Sustainable Withdrawal Rate

The derivation of the SWR formula is given below, adapted from the Suarez et al. (2015) derivation of the Perfect Withdrawal Amount (PWA), an alternate name for the SWR. Consider a retirement portfolio at $t = 30$. The future value of the initial investment must equal the future value of the withdrawals plus the terminal value. This equality is given in real terms as:

$$\underbrace{K_0 \prod_{i=1}^{30} (1 + r_i)}_{Value\,of\,Initial\,investment\,at\,t=30} = \underbrace{w \sum_{j=1}^{30} \prod_{i=j}^{30} (1 + r_i)}_{Value\,of\,withdrawals\,at\,t=30} + \underbrace{K_{30}}_{Terminal\,portfolio\,value}$$

The withdrawals term is slightly more complex; however, it merely takes each of the 30 cashflows forward to bring them all to $t = 30$.

When considering a SWR, the portfolio is assumed to be depleted at the end of the intended duration, the thirtieth period in this case. This means the terminal portfolio value is zero and can be ignored. Therefore, by solving the real withdrawal amount, w, when K_{30} is set to zero and K_0 is set to 1 such that w is expressed as a percentage, Equation 3.7 for the SWR follows:

$$SWR = \prod_{i=1}^{30} (1 + r_i) \; / \; \sum_{j=1}^{30} \prod_{i=j}^{30} (1 + r_i) \tag{3.7}$$

The SWR is calculated for each static and pseudo-dynamic portfolio for each trial using the equation above. The dynamic portfolios, the dynamic cash buffer portfolios specifically, however, cannot be calculated in this manner. Because of the dynamic nature in which asset proportions are determined, dependent on the withdrawal rate, the SWR of dynamic portfolios was determined by using a set of 30 equations as described in Equation D.3 in Appendix D. The proportional asset holdings were determined in each period according to the method in §3.3.5. The SWR was then solved for by searching for a solution for the withdrawal rate that resulted in a zero terminal portfolio value.

<u>SWR and SOR risk</u>

The SWR formula above, Equation 3.7, can be restated by simplifying the notation of the numerator and denominator. The numerator, $\prod_{i=1}^{30} (1 + r_i)$, is equivalent to the cumulative portfolio return over the 30-year period, denoted as R_P. The denominator can be simplified by representing the reciprocal of the denominator, $1 \; / \; [\sum_{j=1}^{30} \prod_{i=j}^{30} (1 + r_i)]$, as the sequence of return factor, S_P. S_P, as Suarez et al.

(2015) showed, acts as a measure of sequence of return risk for the portfolio. The SWR formula is now simplified to:

$$SWR = R_P \times S_P \tag{3.8}$$

The sequence risk factor, S_P, is the reciprocal of a sum of compound returns, where the final period returns occur in most if not all the terms and the early period returns in very few of the terms. The portfolio returns achieved in the early years act on the fewest summation terms. Therefore, to maximise the sequence factor, S_P, and in turn maximise the SWR, the greatest returns are ideally achieved at the beginning of the retirement period. This is demonstrated in Appendix F.

As discussed in §2.6.1, Nevins (2004) made a strong case for appropriate event metrics, especially to capture the possibility of extreme events. For this reason, the expected SWR of a portfolio will be considered with the 5th percentile SWR of that portfolio to give a fuller picture of the downside. The 5th percentile SWR is used, as was defined in the scope of this study in §1.5, to give an idea of the distribution of SWRs and a confidence level of achieving a certain SWR.

This metric is necessary in that it describes the actual rates that would have lasted the 30-year retirement period. While it can only be determined after the fact, it is useful when considering the distribution of SWRs from the simulation and selecting a withdrawal rate that suits the individual's aversion coefficient and the desired sustainability of a portfolio. It is also clear how the sequence risk factor directly affects SWR, making the expected and 5th percentile SWRs a good metric of measuring reduction in sequence risk.

3.4.2 Actuarial Coverage Ratio

The second metric being used is the ACR which considers the *fundedness* of the portfolio, similar to asset liability matching used by institutional investors. This metric is important in monitoring and managing a portfolio in that it gives continuous information about the feasibility of achieving the intended goal of the portfolio: fulfilling spending needs of the individual for the remaining life. This ability to monitor portfolio performance and goal feasibility is important for the HHBS retirement approach as was discussed in §2.1.3.

The ACR metric is a ratio of current assets, i.e., portfolio value, to the present value of expected future expenses where a metric of 1 implies the portfolio is currently exactly equal to the present value of expected expenses. Higher than one indicates the portfolio is more than sufficient, and less than one indicates the portfolio is underfunded for the spending that is expected.

In this study, the ACR is calculated annually in nominal terms using a range of data following the methodology of Collins and Stampfli (2019). This calculation can be done in real terms but due to less available real yield curve data, the nominal form was chosen.[10]

In short, the present value of assets, the investment portfolio, is weighed against the present value of liabilities, the present value of future expected expenses. The metric is calculated as follows (using the same notation as earlier in this section). The ACR in period t is given as:

$$ACR_t = \frac{Value\ \ of\ \ Assets\ \ at\ \ t}{Value\ \ of\ \ Liabilities\ \ at\ \ t} = \frac{Wealth_t}{Value\ \ of\ \ Expected\ \ Consumption_t} \tag{3.9}$$

where $0 \leq t \leq 30$. The numerator, nominal current wealth at t, is simply K_t^*, as shown in Equation D.3 in Appendix D. The denominator is more complex and requires more data to calculate the present value of future expected expenses. The requirements are mortality data, discount rate information, and an assumption of a spending rate. It is calculated by discounting the spending requirements for the remaining life and scaled by the probability that the individual will be alive to make that withdrawal. Equation 3.10 below details the calculation of the ACR denominator.[11]

$$\begin{aligned} Value\ \ of\ \ Expected\ \ Consumption_t &= \sum_{n=t}^{N} \frac{Nominal\ spending_n}{Discount\ factor_{n-t}} \times P(Alive_n \,|\, t) \\ &= \sum_{n=t}^{N} \frac{w_n^*}{\prod_{m=t}^{n}(1 + rf_{m-t})} \times \prod_{x=t-1}^{n-1}(1 - q_{x+65}) \end{aligned} \tag{3.10}$$

The nominal spending amounts are dependent on an initial withdrawal rate, the realised inflation, and the forecasted inflation. An initial withdrawal rate of 4% was selected close to that of what can be expected to last most of the time while also being too high for some portfolio simulations to support when considering unfavourable trials. The nominal spending at t, w_t^*, is calculated from the real initial withdrawal rate, compounded by realised inflation. The nominal expected spending in period n, where $n \geq t$ is given below as the current nominal spending amount extrapolated at the current rate of inflation.

[10] The ACR is the same whether the wealth, spending amounts, and discount rates in the calculation are all nominal or all in inflation-adjusted, real, terms. Due to the greater number of nominal South African government bond tenors than inflation-linked bonds, upon which the discount rates in the ACR are based, a nominal yield curve would be more accurate and require less interpolating. For this reason, the ACR was calculated in nominal terms.

[11] The upper bound on the series summation. Here N is 50 which implies the ACR considers expenses up to the 115$^{\text{th}}$ year which is deemed sufficiently long considering a 65-year-old male has a 0.0003% probability of surviving to 106 years (Dorrington and Tootla, 2007).

$$w_n^* \quad = \quad w_t^* \ \times \ (1 + i_t)^{n-t}$$

The discount rates, rf_n, used is modelled using a simplified risk-free yield curve model using the South African government (risk-free) yield curve following the method of Collins and Stampfli (2019). The yield curve was calculated using the current period's inflation, a fixed premium of short-term rates to inflation, and a fixed shape yield curve. The shape of the yield curve is in effect the premium of each longer-term rate to the shortest rate, the one-year risk free discount rate. The consequence of this is that the yield curve modelled is only dependent on the inflation generated in the simulation and the yield curve is shifted accordingly. This decision ensures that the yield curve remains upward sloping (as opposed to inverted) which is the more common of the two states (Benzoni et al., 2018) and that discounting expenses using an inverted yield curve is avoided. The link between inflation and the yield curve is important as this captures the fact that monetary policy is the main tool of the South African central bank in addressing inflation. The data used to set the yield curve shape are discussed in §3.6.

The mortality rates, q_x, used to calculate the survival rates were obtained from the South African Annuitant Standard Mortality Tables (Dorrington and Tootla, 2007) for a male individual as was assumed in the scope of this study. Here the mortality rate, q_x, is the probability of an individual not surviving one year, to period $x+1$. The use of a male individual is more prudent as the effects of longevity risk are more pronounced and ensure portfolios will be effective 95% of the time for either sex.

The ACR metric was calculated annually for each trial of each portfolio and then ranked to be analysed by quantile. This becomes a useful metric to compare the performance of portfolio strategies over the portfolio run. The ACR can visually depict the effect of SOR when portfolios are separated by ACR quantile at each period. ACR is also useful in practice since SWR is only ex post, where ACR can be used for monitoring and adjusting the portfolio strategy if necessary and reveal to what degree the portfolio goals are feasible.

This study chose a 30-year portfolio horizon based on a 95% confidence level of survival; however, the reality is that an investor may live longer than 95 years. The ACR takes these future expenses into account, thereby monitoring longevity risk and the 'danger smile' as discussed in §2.5 of the literature review.

3.5 SOR sensitivity analysis

An aim of this study was to determine the sensitivity of retirement portfolios to SOR risk and how it develops in the accumulation and decumulation phases. SOR sensitivity was measured by making use of the SWR metric. §3.4.1 describes SWR and its clear connection to SOR risk, and therefore is a suitable metric to measure sensitivity to SOR.

The method by which the sensitivity of a portfolio to SOR risk is measured considers a 60-year period from 30 years before retirement and 30 years of retirement. The sensitivity of a portfolio to SOR risk was calculated using two modelled simulation trials, back-to-back, to create a 60-year trial on which the benchmark portfolios were applied.

During the accumulation phase, it was assumed that portfolio contributions grow with inflation. The rationale was that a constant portion of annual salary was contributed to retirement savings where the salary increased by inflation. The decumulation phase was the same as previously described in the SWR methodology description. The sensitivity of a specific period was calculated as the percentage change in the portfolio's SWR assuming a change in that period's portfolio return of 1% while keeping the rest of the returns unchanged.

The SOR sensitivity to the return of period t is calculated as:

$$SOR\ \ Sensitivity_t \quad = \quad \frac{SWR_t^+ - SWR_t^-}{2 \times SWR_t^0}$$

SWR_t^0 represents the unchanged SWR of the returns set. The SWR_t^+ and SWR_t^- represent the SWR when the portfolio return in the t^{th} period is increased or decreased by 1%, respectively.

3.6 Data

This section summarises the data that were used in this study. All the returns data was obtained for total return indices on a monthly basis which were then used to calculate monthly returns for each index. Certain non-South African assets required returns to be converted to a South African Rands (ZAR) return since this is the perspective taken in this study, unless otherwise stated. Further information, detail is provided on the input data and their sources in Appendix G.

3.6.1 Simulation input data

The data in this section are those necessary for the Monte Carlo simulation which are then used in generating the benchmark results, the strategy results, and the SOR risk sensitivity.

Asset returns

The simulation considered twelve asset indices from January 1991 to August 2020. The primary three assets and focus of this study are the ALSI, ALBI, and STeFi indices representing South African equities, bonds, and money market, respectively. These data were sourced from two sources, detailed in Appendix G.

For the geographic diversification strategy, seven more indices were included: MSCI World, MSCI EM, MSCI EAFE, BBGA, BBEMB, CDAX, and REX (as described in the discussion of this strategy in §3.3.3). The first five of these indices are priced in US Dollars and were converted to ZAR returns using the USD/ZAR exchange rate. The CDAX and REX were priced in Euros but because Germany adopted the Euro in January 1999, the conversion was slightly more complicated. The CDAX and REX returns post-January 1999 were available as ZAR returns on Bloomberg L. P. (2020). The German asset returns pre-1999, where the Deutschmark (DEM) was the official currency, was converted to a ZAR return via a USD rate because no EUR/ZAR rate history was available. All exchange rate data were obtained from Bloomberg L. P. (2020).

The final two asset indices included in the simulation relate to the risk parity strategy. The IGOV index was used to represent South African (government) inflation-linked bond returns while the Gold index (priced in ZAR) was used to represent the commodity's return.

It should be noted that the available histories for the IGOV and BBEMB indices were shorter than the other indices. The BBEMB has history available from January 1993, while the IGOV has history available from February 2004.

Inflation rates

The inflation rate data required for the simulation were the South African month-on-month headline inflation consumer price index (CPI) rates for the period January 1991 to August 2020. Further detail is provided in Appendix G.

3.6.2 Actuals input data

The data discussed in this section include those used in the historical comparison and the strategy validation. The data used span the period from April 1986 to August 2020, about five years longer than the simulated input data.

Asset returns

The actual, historical input data include returns from the three primary South African assets (ALSI, ALBI, and STeFi) as well as two US assets used in the historical comparison. The US assets considered are the Russell 1000 and the BBUSAB indices. Their use is discussed in §3.3.2 and 3.3.6. The US assets here are used to compare the position of a South African retired investor to a US retired investor each investing in their local bond and equity indices. As such, US returns are not converted to ZAR equivalent but remain in USD.

Inflation rates

The calculation of the SWR portfolio performance metric for the US and South African portfolios requires inflation history for each nation. The month-on-month headline inflation (CPI) history for the April 1986 to August 2020 was used.

3.6.3 Metric input data

The data described below were necessary for the calculations of the ACR metrics. This includes data on mortality and yield curve rates for calculating the present value of future expected spending.

Yield curve rates

The yield curve rates were derived from the South African Sovereign Curve and the South African (Sovereign) Inflation-Linked Curve on five dates, in August of: 2020, 2018, 2015, 2010, and 2005. This sample of five curves was intended to remove the bias of any one curve. The curve was constructed by determining a 1-year real rate and bootstrapped by calculating each subsequent year's average premium to the shorter rate. Specific outlier rates were excluded when obviously anomalous.

The 1-year rate was determined as an average of the 1-year yield of the Inflation-Linked (real) Curve. The 2-year rate was determined by taking the 1-year rate determined and adding the average premium of the 1-year rate to the 2-year rate. This continued until the 30-year rate after which it was assumed constant. The bootstrapping was preferably done using the real (inflation-linked) curve, but the nominal curve was used when real rates were not available or inappropriate. Linear interpolation was used when there were no rates available at certain tenors. The yield curve data source and the resulting real yield curve that was used is given in Appendix G.

Mortality rates

The mortality rate data was obtained from the South African Annuitant Standard Mortality Tables (Dorrington and Tootla, 2007). Because of the confidence level assumed, a 65-year-old male was the starting point of the portfolio where mortality data extended to 110 years. From 110 years to 115 years, mortality rates were assumed constant.

Chapter 4

Results and discussion

This chapter begins with an introduction to the results followed by an analysis of the benchmark set of portfolios. After that, the historical benchmark comparison of the results are discussed with the sequence risk sensitivity results. Finally, the results of the portfolio strategies are discussed, and the strategies validated using historical data.

4.1 Introduction

For the purpose of giving context to the simulation results and their presentation, a few basic figures are presented. As an introduction to the perspective and results of the analysis, Figures 4.1 and 4.2 show a comparison of a traditional capital market line (CML) plot of the assets considered in this study alongside a graph of the same assets using the post-retirement portfolio specific SWR metric. Figure 4.2 compares the expected SWR to the 5th percentile SWR while Figure 4.1 plots expected return against standard deviation of return. The 5th percentile SWR indicates a 95% certainty level acts as an event risk measure, as described by Nevins (2004). A superior asset would lie to the top right of Figure 4.2 and on the top left of Figure 4.1, an ordinary CML plot. This is because a high 5th percentile SWR, like a low standard deviation, indicates a narrower distribution of outcomes although with specific reference to the left tail of the distribution. The 5th percentile SWR takes into account the extreme event risk and variance drain associated with SOR risk when considering a decumulation portfolio.

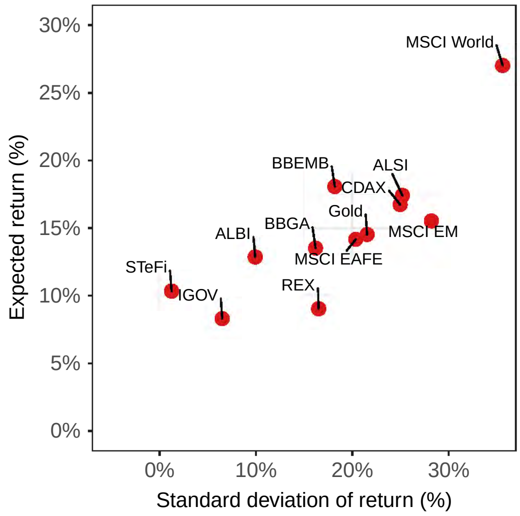

FIGURE 4.1: Investment assets CML

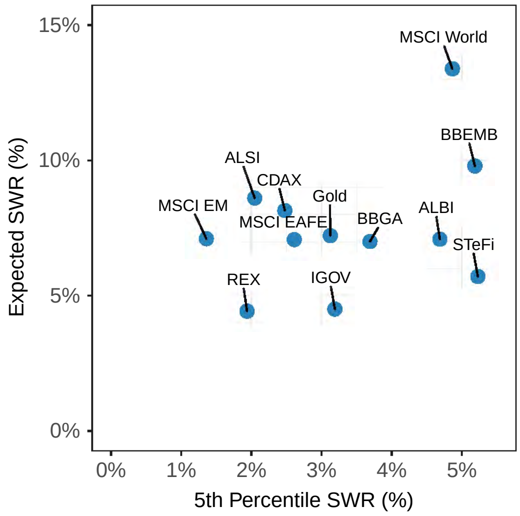

FIGURE 4.2: Investment assets SWR

As expected, the CML shows the proportional relationship between expected return and the standard deviation of the returns. According to traditional MPT, the higher the required expected return the greater the risk which an investor must assume. This is clearly different to Figure 4.2 where one would expect a negative correlation or a trade-off between expected SWR and 5th percentile SWR. However, there is very slight positive correlation and when excluding the MSCI World equity index, which seems to be an outlier, there is no discernible correlation.

When considering the three assets which are the focus of this study, (the ALSI, the ALBI, and the STeFi representing South African equities, bonds, and money market asset classes) the presumed relationship between the expected and 5th percentile SWRs is seen. Equities have a higher expected SWR but lower 5th percentile SWR. The cash asset has a much lower expected SWR but is compensated for with a much higher 5th percentile SWR. Bonds lie between these two. The other assets are discussed later in this chapter.

Considering the ACR metric, Figure 4.3 below shows the performance of a 50/50 South African bond and equity portfolio. The rows of the heatmap indicate the ACR performance percentile and the columns show the year of measurement. As was discussed in the ACR description in §3.4.2, there is an implicit withdrawal rate in the metric. In the case of Figure 4.3 below, 5% was used. The text in each cell displays the ACR for that period-percentile combination while the colour of each cell indicates the *fundedness* level with dark green being comfortably overfunded and red indicating under-*fundedness* or depletion. Beyond an ACR of 10, the table values are shown as "+10x".

In Figure 4.3 the ACR ratio begins at approximately 1.5 and diverge from there. If a portfolio ACR dips below 1 this indicates the present value of liabilities exceed assets and the portfolio is underfunded. Ideally, before reaching this point the portfolio should be annuitized as described by Fullmer (2007), although it is possible to recover from an underfunded ACR. When a portfolio reaches zero, the portfolio has been exhausted. Figure 4.3 shows that at least the 20th percentile portfolios and above can sustain a 5% withdrawal rate for 30 years. However, the 5th percentile and 10th percentile portfolios are exhausted by 22 and 30 years respectively. As is discussed in §3.1, the specific focus in this study is the 5th percentile (based on the 95% confidence level) and as such the metric results will focus on this performance percentile.

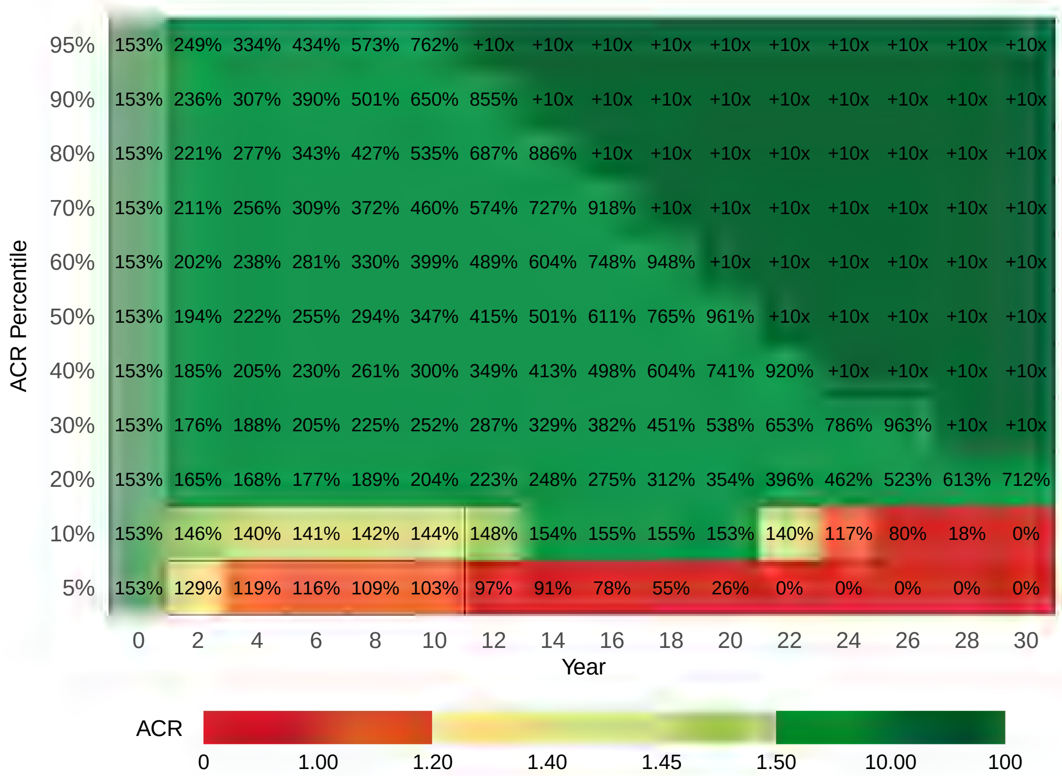

FIGURE 4.3: ACR heatmap of 50/50 benchmark portfolio at a 5% withdrawal rate

The risk associated with the sequence of returns is observed in Figure 4.3 in the 10th and 5th percentile rows. The decline from 150% ACR to about 100% in the 5th percentile portfolio in 10 years demonstrates the destructive potential of sequence risk. Similar decline is seen in the 10th percentile although it shows a recovery. The ACR of the 10th percentile begins to decline again in year 20, eventually leading to depletion. This second decline captures the 'danger smile' as Collins and Stampfli (2019) described. This effect is discussed in more detail in §2.5.2 and relates to the faster shift in mortality late in life, creating a spike in sequence risk.

4.2 Benchmark portfolios

The benchmark portfolios consist of a set of eleven portfolios of South African assets ranging from all bonds to all equities. The SWR probability density functions for the benchmark set are given in Figure 4.4 with the vertical lines on each distribution, from left to right, indicating the 5th percentile, mean, and 95th percentile SWRs respectively.

The first observation to be made is the stark difference in spread between the two assets. The ALSI has a much greater spread with a positive skew where the ALBI has a much narrower distribution with a slight positive skew. The mean and 95th percentile both generally increase with holding in equities, however the pertinent metric is the 5th percentile which is much higher in portfolios with a greater proportion of fixed income in the portfolio.

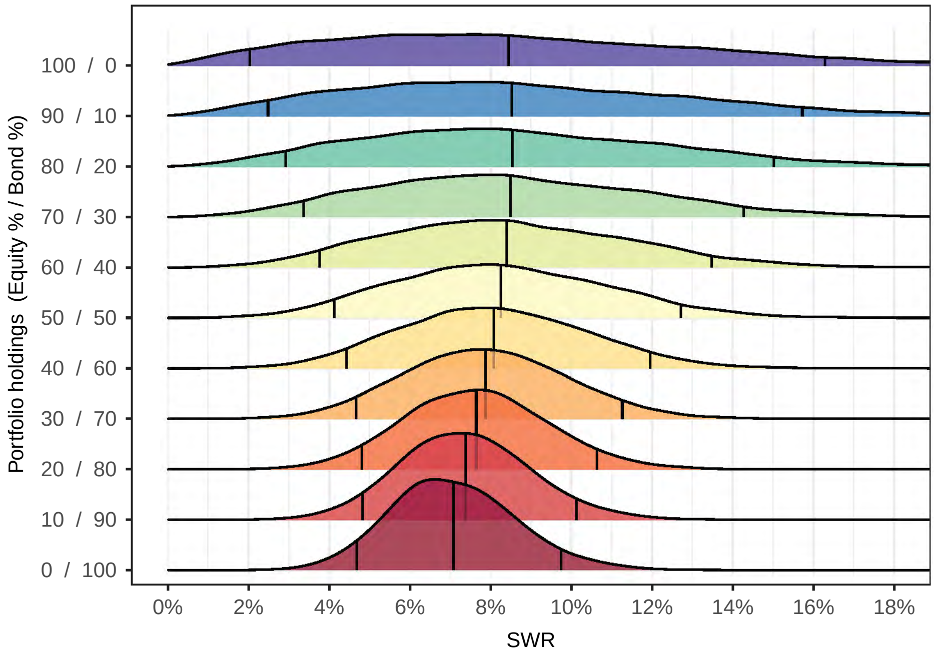

FIGURE 4.4: Benchmark portfolio SWR probability density distributions

The relationship showing the trade-off between expected SWR and 5th percentile for the benchmark set of portfolios is more clearly shown in Figure 4.5. The figure shows that, on the far left, a pure equity ALSI portfolio can support an 8.5% sustainable withdrawal rate on average with a 5th percentile SWR of just above 2%, a wide SWR distribution. For the pure bond, ALBI portfolio, the expected SWR drops to between 7% and 8% but is compensated for with the 5th percentile SWR increasing to 4.7%. The minimum downside portfolio, the portfolio with the greatest 5th percentile SWR occurs somewhere between the 10/90 and 20/80 equities-bonds portfolio with a 5th percentile SWR of approximately 4.8%.

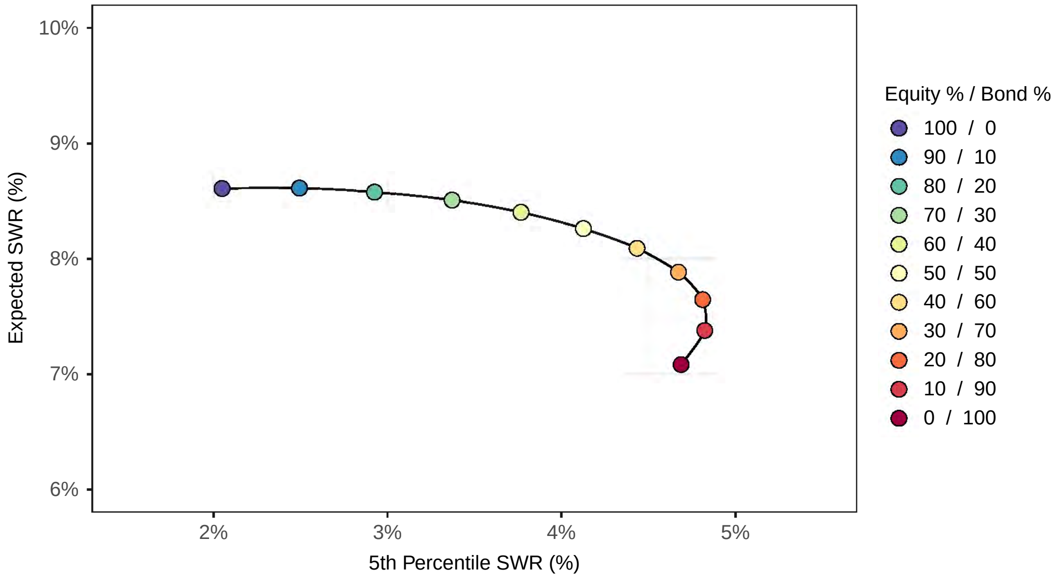

FIGURE 4.5: Benchmark portfolio SWR curve

The basic shape of this curve shows similarities to an efficient frontier in MPT. Due to imperfect return correlation, combinations of the assets diversify risk and improve the risk metric. An interesting difference is that from a MPT perspective, the expected return of a portfolio cannot be greater than the expected return of any component. While not clearly visible in Figure 4.5, the expected SWR of the 90/10 portfolio is marginally (0.004%) higher than the pure equity portfolio. This is also due to risk diversification between assets.

The general shape of the curve in Figure 4.5 corresponds with the findings of Clare et al. (2020), and shows a higher mean SWR with a wider distribution for stock-heavy portfolios in comparison to bond-heavy portfolios with a lower mean SWR but which have a narrower spread. The diversification interaction is similar when considering the 5$^{\text{th}}$ percentile SWR where the best performer seems to be the 40/60 portfolio.

For further insight into the performance of the benchmark set of portfolios consider Figure 4.6 displaying only the 5$^{\text{th}}$ percentile ACR of each portfolio in the benchmark set. The 4% withdrawal rate applied to the ACRs in this figure demonstrate that only the six portfolios, from 40/60 to pure ALBI, could successfully support a 4% withdrawal rate at a 95% certainty level. This can also be seen in Figure 4.5 where the points lying to the left of the 4% vertical are unable to support that withdrawal rate. What the ACR does show is the importance of strong stable performance early on to be at a safe funded ratio before the 'danger smile' has an effect, depicted in the 50/50 and 60/40 portfolios late decline in ACR.

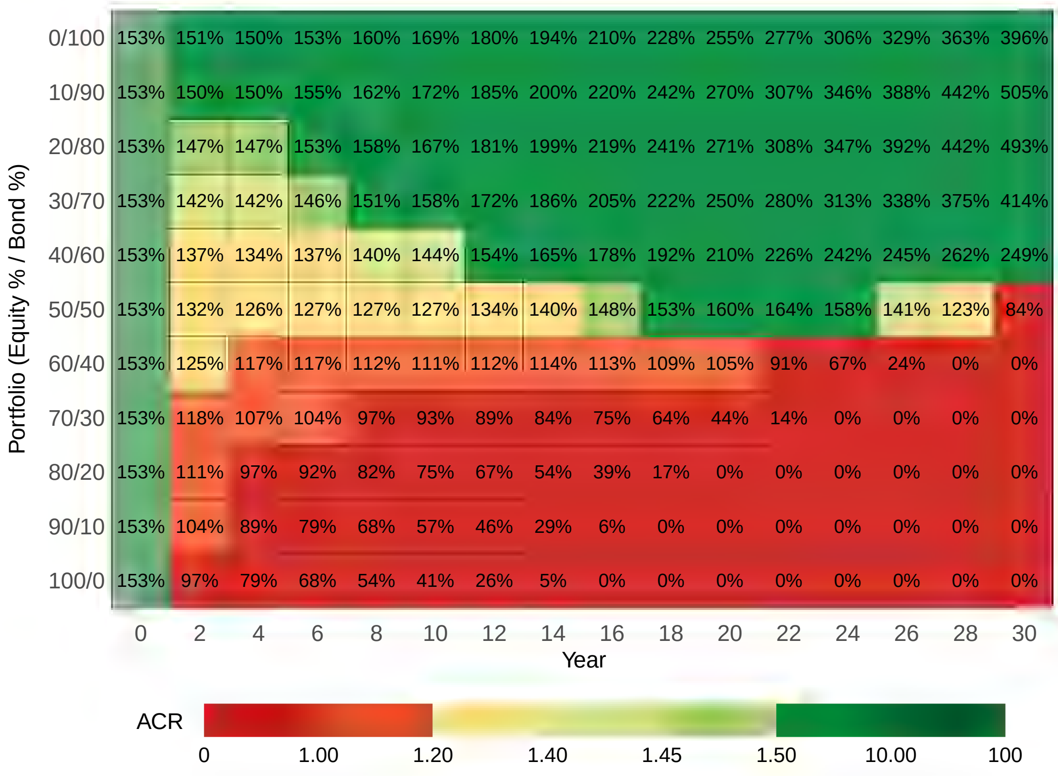

FIGURE 4.6: 5th Percentile ACR heatmap of benchmark portfolios at a 4% withdrawal rate

Seen in the figure above, the 50/50 portfolio was nearing depletion in supporting the 4% withdrawal rate at the 5th percentile after 30 years. This portfolio, with an ACR of 1.64 at 22 years and appeared to be comfortably funded, would unlikely have lasted four more years at that withdrawal rate.

4.3 Historical comparison

This section compares the experience of a retirement portfolio funding a SWR in South Africa to the US. The sustainable withdrawal rate is a popular retirement portfolio metric and has gained popularity since Bengen (1994) identified what is known as the "4% rule". The paper stated that the 4% SWR that would have lasted through all the history considered, 39 sets of 30-year periods starting from 1926 to 1965 with the final period ending in 1994. Bengen (1994) considered only US assets, so it is worth considering what the South African equivalent is and how that compares to the current US equivalent.

The data considered are the 54 partially overlapping 360-month periods between April 1986 and August 2020. It should be noted that all these periods contain at least 25 periods in common, so the results are correlated to the degree their results overlap.

Without a statistical analysis to account for this correlation, it is difficult to determine if these results are significantly different to what Bengen (1994) identified.

The comparison of the SWR between the US and South African portfolios are shown in Figure 4.7. The middle pane shows the 50/50 equity-bonds portfolios, the same weighting as the portfolios used by Bengen (1994) for the thirty-year periods. The results are similar with the United States exhibiting slightly less variation. The left and right panes show the pure bond and pure equity equivalents showing relative dominance by the South African bonds and the US equity portfolios, respectively. The high volatility and poor early performance of South African stock returns at the start of the historical period considered in this study causes the lowest SWRs of all portfolios considered, a 4.85% SWR for the period September 1987 to August 2017.

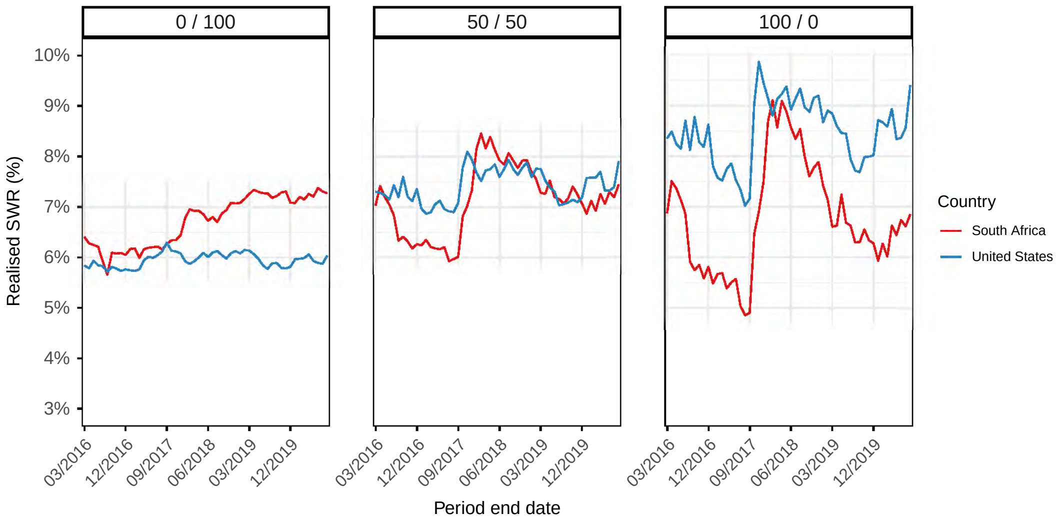

FIGURE 4.7: Historical benchmark SWR comparison between South Africa and US

Figure 4.8 below displays a boxplot for each portfolio, capturing the distribution of results from this historical sample dataset. The result from both countries show that the distribution of lower-risk bonds result in a narrower set of SWRs in comparison to equities. Furthermore, while this dataset is a small sample, the 4% threshold is not breached in any portfolios considered.

The superior performance of the US overall is expected: Estrada (2016) and Pfau (2010) have shown US retirees have experienced better investing environments than a large majority of countries. Estrada (2016), using a longer dataset, found that South African stocks outperformed South African bonds funding a 4% SWR at all percentiles. Because a shorter dataset is used in this study, some of the superior historical South African equity performance is excluded such that the SWR means are similar for all bond and all equity portfolios.

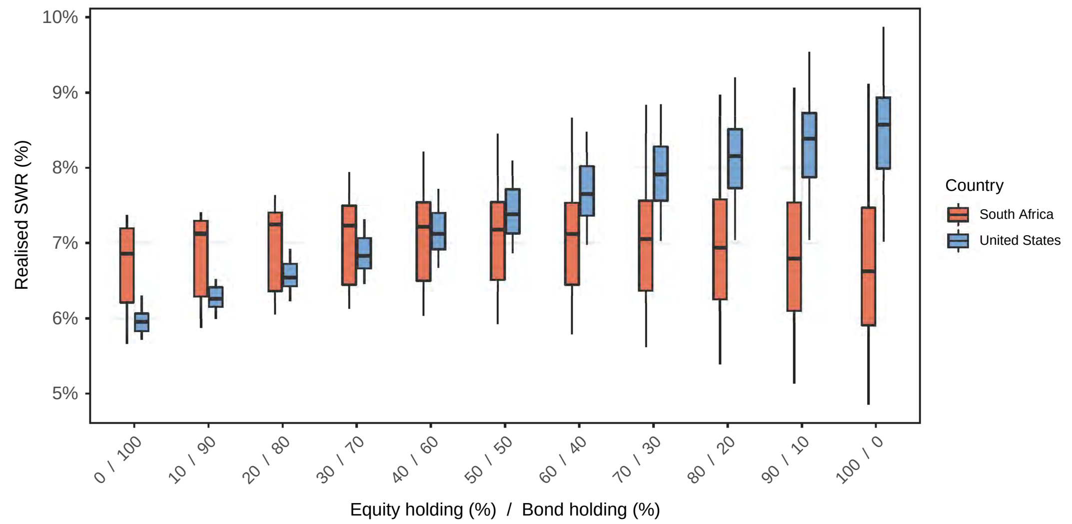

FIGURE 4.8: Historical benchmark SWR comparison boxplot

While this historical sample is likely insufficient to validate the Monte Carlo model, the improvement to the minimum SWR due to risk diversification between assets is apparent in the minimum of the South African historical SWR distributions seen in Figure 4.8, as was seen with the benchmark portfolios on simulated data in Figure 4.5.

4.4 SOR risk sensitivity

With the benchmark set established, the SOR risk of these portfolios is considered. As was established in the methodology, the sensitivity to SOR risk was measured as the sensitivity of the SWR to a 1% change in the portfolio return of each period. The sensitivity results of are presented in Figure 4.9. The sensitivity profile is similar to what Pfau (2013) showed although here shown with a continuous sensitivity profile. The sensitivity measured here halves after about ten years, double that which Pfau (2013) shows.

The sensitivity appears to correlate with portfolio value (shown in Figure 2.4 in §2.5.2), which is intuitive to the concept of SOR risk, however with opposite curvatures. Sensitivity is convex after retirement while portfolio value is concave. The sensitivity in Figure 4.9 considers the sensitivity to a 1% change in portfolio return, however the return volatility is higher in stock-heavy portfolios. As such the sequence risk sensitivity will be higher for stock-heavy portfolios, portfolios with a greater standard deviation of portfolio returns.

The continuous decrease in sensitivity to SOR after retirement demonstrates the importance of optimal portfolio management. Every period after retirement is the most sensitive or vulnerable the portfolio will be for the remaining life of the individual, at least in terms of the SWR and funding a minimum spending requirement goal. This contrasts the pre-retirement phase where the portfolio value

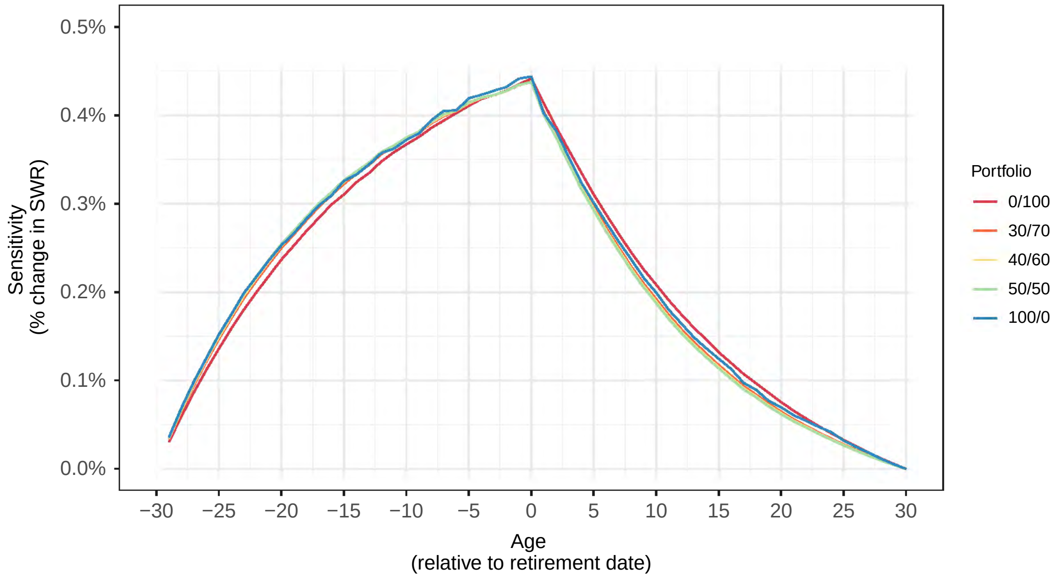

FIGURE 4.9: Sequence risk return sensitivity

is increasing and sensitivity is increasing and as such, a series of poor returns can be recoverable whereas in retirement may be devastating.

Furthering the discussion on pre- and post-retirement investing, Figure 4.9 perhaps does not capture the full picture of the reality of investing for the purposes of funding a retirement minimum spending goal and the real risks associated. This is due to the fact that once an individual has retired there is no longer any or very little flexibility. An individual who suffers significant portfolio losses pre-retirement can change their lifestyle to live more frugally and save a greater percentage of income. A retired individual does not have this luxury and perhaps an accurate representation of SOR risk of accumulation and decumulation phases should look more similar to what Pfau (2013) showed where there is a disconnect at the retirement date where sensitivity spikes to a level much higher than pre-retirement.

4.5 Portfolio strategies

With a defined benchmark set of portfolios, the following section considers the effectiveness of the identified portfolio strategies at improving the retirement metrics. The first two sections consider geographic diversification and risk parity respectively with the final three sections covering low-risk portfolios, rising equity glidepaths, and the dynamic cash buffer. This separation exists because of the first two strategies make use of other assets beside the three main South African indices considered in this study, the ALSI, ALBI, and STeFi. For that reason, these strategies are less comparable with the remainder and can be seen as more of a complementary strategy rather than an alternative. The latter three make use of the same assets and are thus significantly more comparable.

It should be noted that changes in metrics are discussed as absolute changes (i.e., difference in metric) rather than relative differences (i.e., percentage change in metric). The relative differences are however an important factor to be aware of when considering SWRs which are small percentages, generally below 10%, implying that a 1% change is extremely significant.

4.5.1 Geographic diversification portfolio

The geographic diversification strategy considers the benchmark portfolios and combining them with equivalent stock and bond indices from other geographies for diversification benefit. This benefit can be measured by their effect on the SWR metrics used. The ACR does not provide further information since the foreign assets included make direct ACR comparison between the portfolios unsuitable. The four diversification cases considered (discussed in detail in §3.3.3) include:

1. diversification into a single developed nation,
2. diversification into a broad index of developed market nations,[1]
3. diversification into a broad index of emerging market nations, and
4. diversification into developed and merging market indices equally.

A useful way to interpret the set of 25 portfolios considered in each geographic diversification case is shown in Figure 4.10 as a coordinate plane with the y-axis representing the asset class from purely bonds to purely equities and the x-axis representing the degree of geographic transition from pure South African assets to purely the diversification asset. The performance of each of the 25 portfolios is considered through expected SWR and 5th percentile SWR.

[1] The broad index of developed market nations case is also considered with the US excluded.

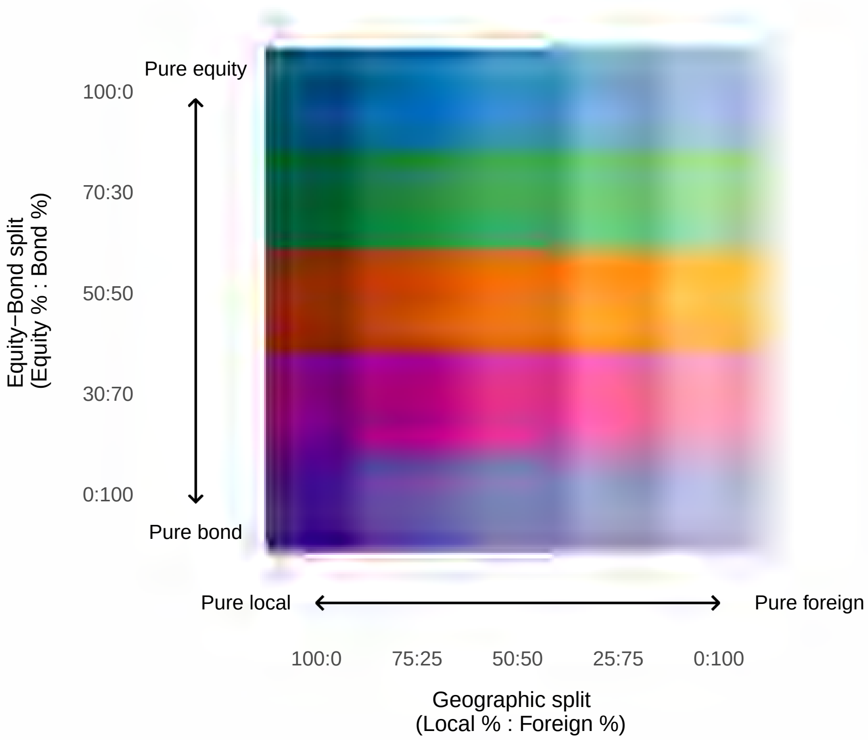

FIGURE 4.10: Portfolio geographic asset allocation map

Overall, geographic diversification proved beneficial in terms of improving the 5$^{\text{th}}$ percentile SWR for all four cases considered. (The 5$^{\text{th}}$ percentile SWR, as discussed previously, serves as an event risk metric and confidence level.) Furthermore, the optimal 5$^{\text{th}}$ percentile SWR portfolio for all four cases occurred at some geographic split between the extremes. This shows that the diversification benefit exists not only from the perspective of the South African retiree, but also for the international investor diversifying into South African assets.

As an example, the second case is considered: diversification into a broad index of developed market nations. In this case, the South African benchmark set is combined with the MSCI World and BBGA indices. The results of the 25 portfolios considering the expected SWR and the 5$^{\text{th}}$ percentile SWR are shown in Figure 4.11 below.

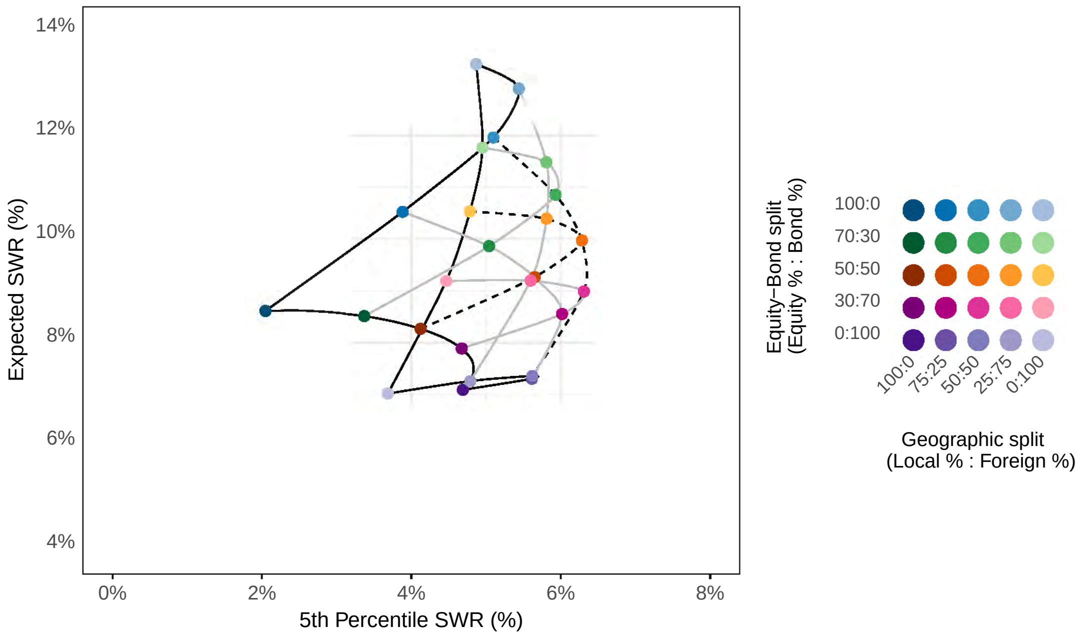

FIGURE 4.11: Broad developed market diversification SWR performance

Considering initially the geographic extremes, the pure MSCI World portfolio as well as the 70/30 MSCI World-BBGA portfolio dominate the South African benchmark set in terms of both expected and 5th percentile SWR. However, the combination of both investment geographies results in an improvement in 5th percentile SWR of about 1.6% (from the South African benchmark perspective). This is a shift in the 5th percentile SWR from 4.7% to 6.3%. This risk metric improvement is more easily observed in a heatmap of the 25 portfolios showing 5th percentile SWR in Figure 4.12 below. This figure depicts the effectiveness of geographic diversification for its potential to improve downside risk in retirement.

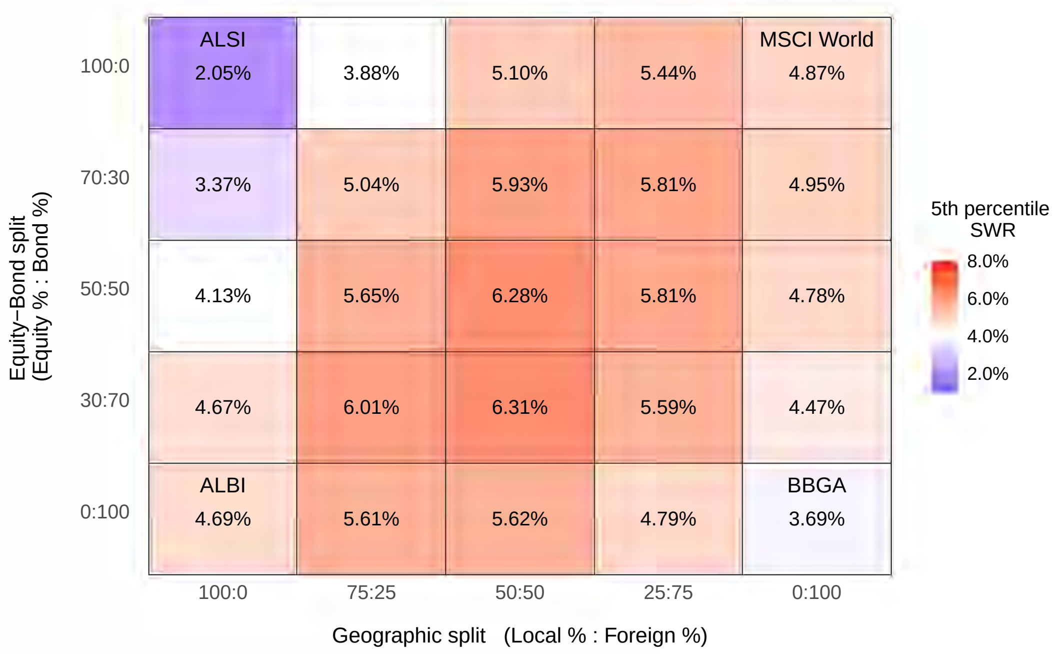

FIGURE 4.12: Broad developed market diversification
5th percentile SWR heatmap

The best performing portfolio for each geographic diversification case in comparison to the best performing benchmark portfolios is presented in the distributions in Figure 4.13. For all four geographic diversification cases there is an improvement in 5th percentile SWR (seen as the leftmost vertical line in each distribution). The best cases, those with the broadest diversification index, saw 5th percentile SWR improvement of about 1.5%. Furthermore, for the top three performing cases all show significant improvement to the expected SWR (shown as the middle vertical line in each distribution). The full results of each diversification case show are provided in Appendix H.

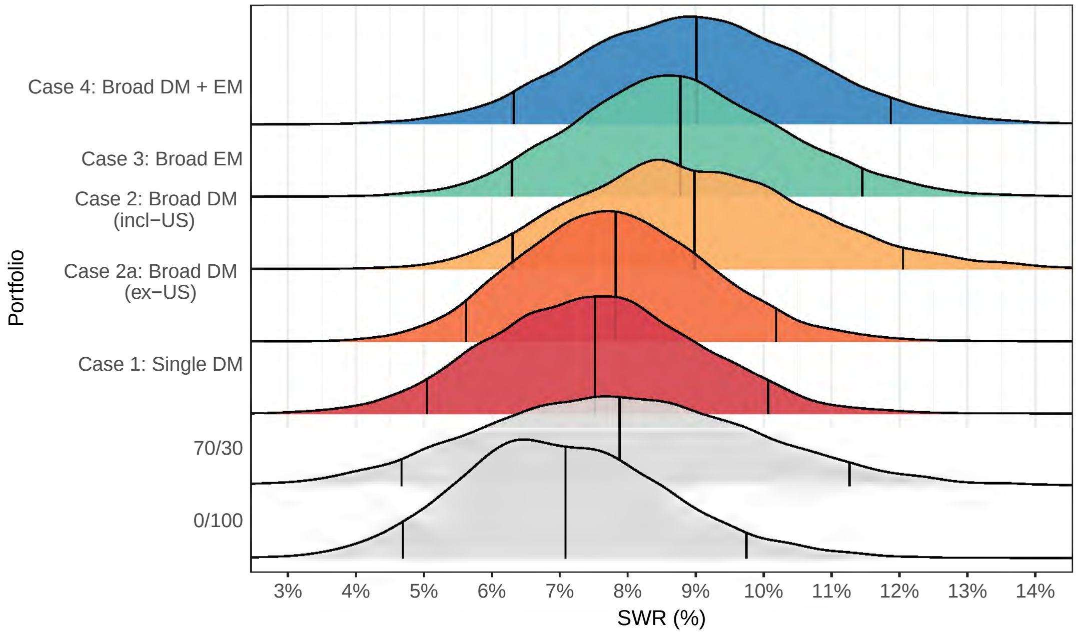

FIGURE 4.13: Geographic diversification SWR distribution

As can be seen in this section, there are therefore strong diversification benefits for a South African investor who diversifies their portfolio internationally. Diversification benefits were possible in both developed markets and emerging markets due to negative correlations between assets and strong performance. There were sufficient diversification benefits from including one other single developed market to the South African portfolio to improve the risk metric significantly.

4.5.2 Risk parity portfolio

The second of the less comparable strategies is the risk parity strategy which aims to spread the risk contribution equally among all the asset sub-classes included. Five combinations of assets were selected to test their efficacy. The assets selected for possible inclusion were South African equities and bonds, inflation-linked bonds, commodities, and international bonds and equities. The traits of these asset classes and sub-classes are generally accepted in risk parity portfolios as they are viewed as being sufficiently uncorrelated (Qian, 2016).

The most basic portfolio variation was using only South African equities and fixed income in risk parity proportions. The following three variations included the same as the basic portfolio and include gold, the South African government inflation-linked bond index (IGOV), and both, respectively. The final variation includes South African equities and bonds, Gold, IGOV, and international equities and bonds (including emerging market and developed market assets. The portfolio asset holdings are given in Appendix E.

The results for these portfolios are shown in Figure 4.14. All the variations are shown relative to a 50/50 South African equity-bond pure benchmark portfolio. The 50/50 benchmark portfolio is shown in black with the full benchmark curve in light grey for reference. The basic risk parity portfolio, shown in grey, lies on the benchmark curve because it is constructed from the same assets.

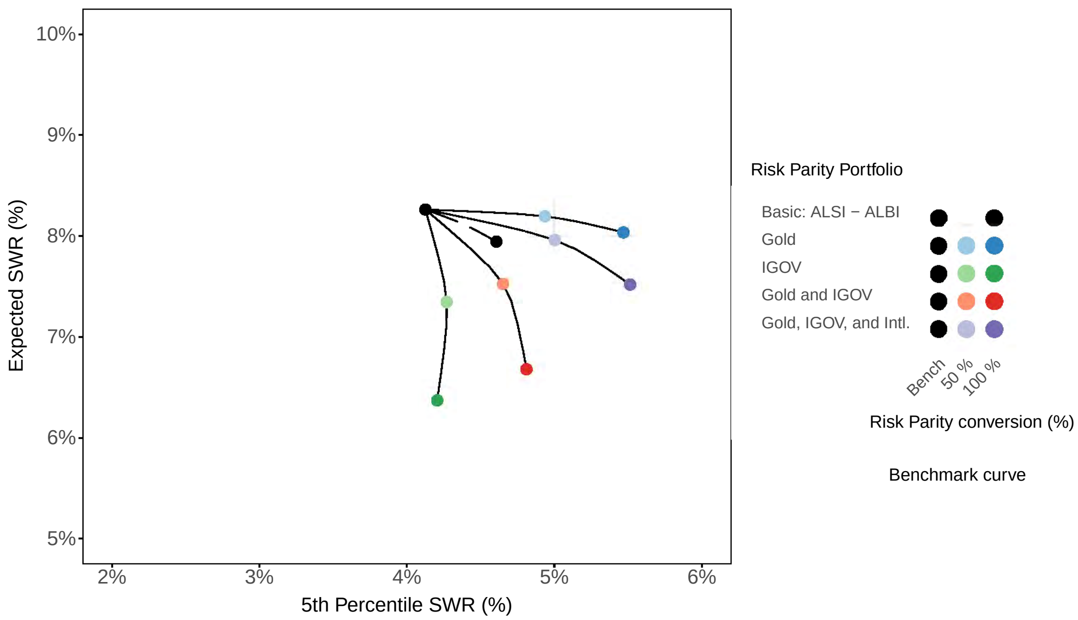

FIGURE 4.14: Risk parity strategy SWR performance

Overall, the performance of the risk parity portfolios relative to the benchmark set is mixed. The IGOV, and IGOV and Gold portfolios were dominated by the benchmark set due to the poor results of the IGOV and its high weighting due to lack of correlation with the others. The Gold portfolio performed very well and was able to improve 5th percentile SWR by 0.7% compared to the best 5th percentile SWR of the benchmark set (from 4.8% to 5.5%) while maintaining an expected SWR above 8%. The counter-cyclical performance of gold in South Africa is the likely cause of this diversification potential. The final risk parity portfolio, which includes an international allocation as well as IGOV and Gold, performs well (as would be expected based on the results of §4.5.1) in improving 5th percentile SWR but is also weighed down by poor performance of IGOV.

The IGOV asset provided no benefit to the portfolio and was clearly the worst performing of the five portfolios which is most likely due to the modelling methodology of asset results. Although the inter-asset correlation is correct, the inflation model is not linked to the asset returns model which is significant when looking at an inflation-linked asset and an inflation-linked portfolio spending goal. This is an area for future improvement.

The risk parity portfolio results do show some benefit to a risk parity approach in terms of improving the portfolio risk metric for retirement portfolios, although the selection of assets to include is vital. A more appropriate model that links the inflation-linked asset returns with the inflation model in some manner could potentially make a strong case for risk parity strategies, but in this study the benefit is less clear.

4.5.3 Low-risk portfolio

The next strategy considered is a low-risk portfolio. This considers the view that reducing overall portfolio risk will improve the portfolios robustness against SOR and early loss risk. This strategy, as well as the two that follow, are limited to South African equities, bonds, and money market indices. This allows for a more direct comparison and as such ACR metrics as well as SWR metrics are considered.

The results shown in Figure 4.15 show the effect that increasing the proportion of cash and fixed income have when deviating from four selected benchmark portfolios. Starting from the left-most all-equity portfolio, by adding either cash or bonds the 5th percentile SWR were strongly improved. The addition of fixed income was however more favourable as it did not require sacrificing as much expected SWR. The results are similar when considering the 50:50 equity-bond portfolio: adding more bonds or cash both improve the 5th percentile SWR with bonds still providing a slightly better trade-off than adding cash. From the 30:70 equity-bond portfolio, however, adding additional cash still improved the 5th percentile SWR while adding fixed income provided diminishing improvements with consistent loss in expected SWR. Due to the negative correlation, adding cash to an all-bond still increases the 5th percentile SWR while losing expected SWR due to low cash returns.

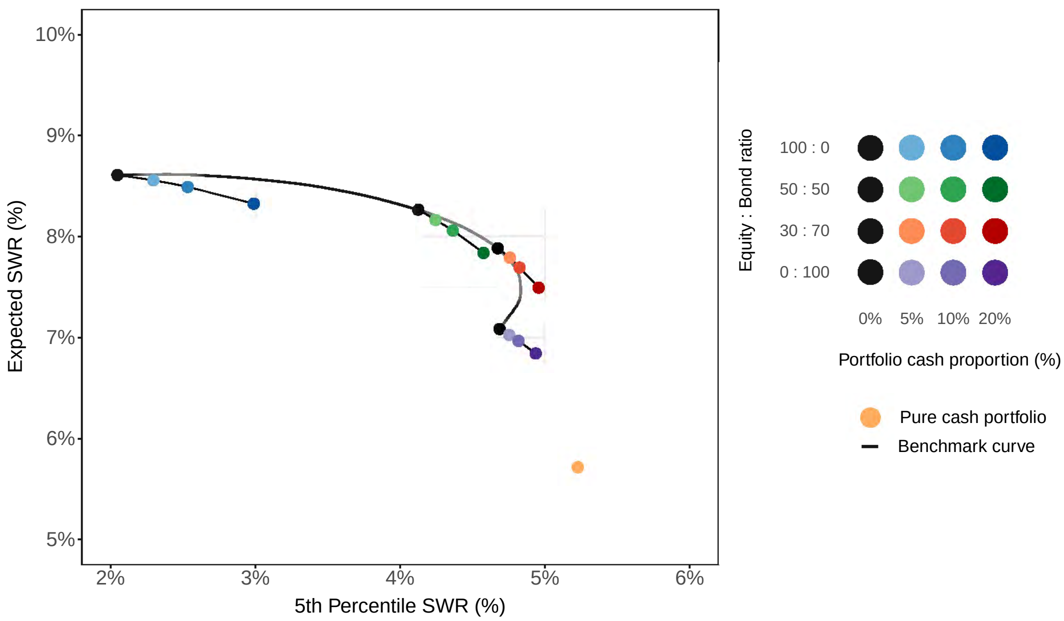

FIGURE 4.15: Low risk strategy SWR performance

Overall, adding the lower risk assets, bonds and cash, to a portfolio improved the metric of interest in this study, the 5th percentile SWR. The narrower distributions of these low-risk assets meant that while they have a lower mean SWR, they provide an improvement to the portfolio sensitivity to SOR as measured by 5th percentile SWR. Although the all-cash portfolio has a 0.3% higher 5th percentile SWR, it costs 1.3% in expected SWR. It appears a combination between bonds and cash could provide greater diversification and lower SOR risk than either asset on its own, but this is left as an area for further study.

The 5th percentile ACR results of this strategy are shown in Figure 4.16. The figure shows that at a 5% withdrawal rate, the greater the proportion of lower-risk assets, the greater the ACR is and the more likely the portfolio is to last the retirement period. It also shows the benefits of adding cash to increase resistance to SOR risk. The low correlation that equities have with cash and fixed income, as well as the negative correlation between those two, demonstrate the risk reduction from adding these low-risk assets to a risky portfolio.

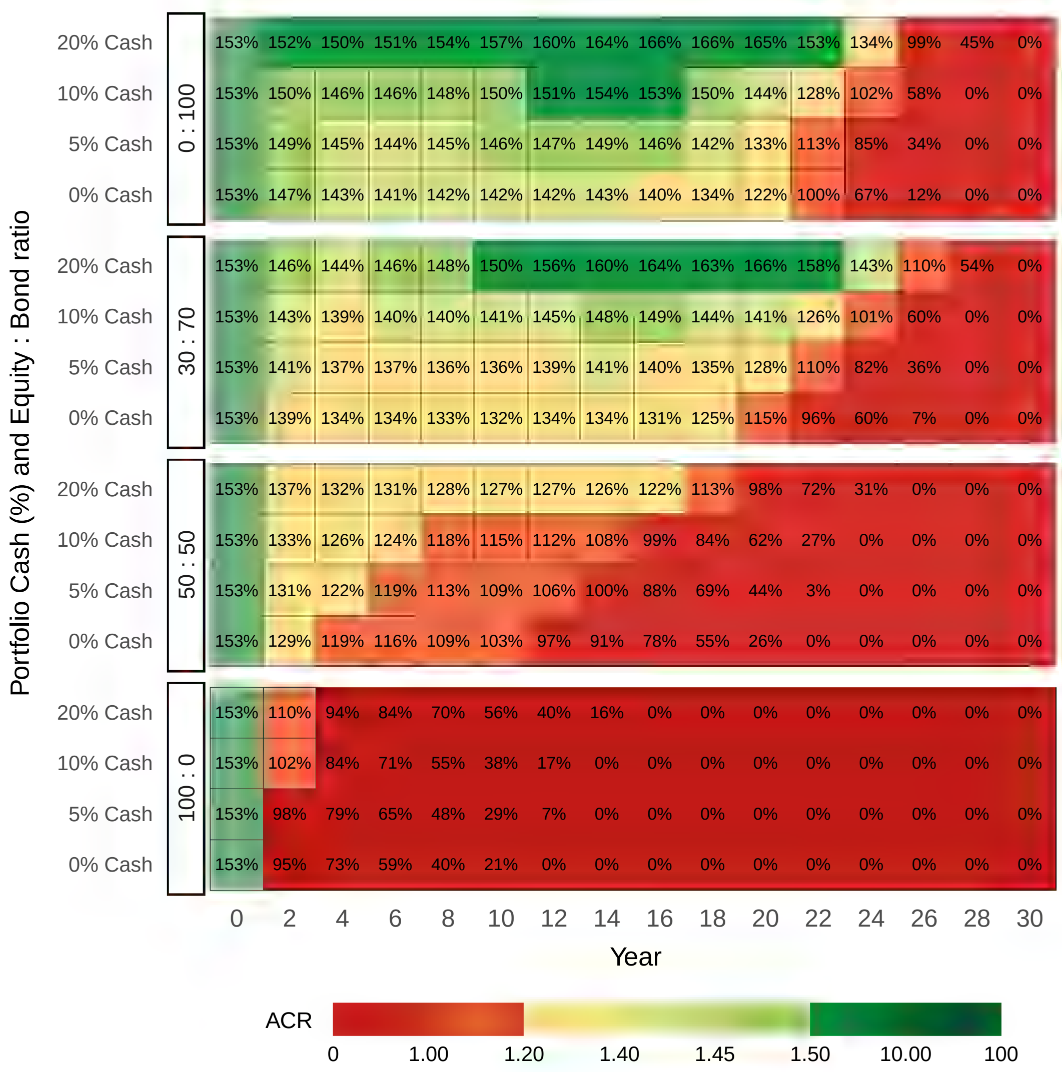

FIGURE 4.16: 5th percentile ACR heatmap of Low-risk strategy

These results show that reducing portfolio volatility by reducing conventional portfolio risk as measured by standard deviation of returns also reduces SOR risk and improves the chances of a withdrawal rate lasting for an individual's retirement period. The benefit of low-risk assets does come as a trade-off to expected SWR and there is diversification benefit of keeping some amount of equities in the portfolio.

4.5.4 Rising equity glidepath portfolio

The rising equity glidepath (REGP) strategy has been explored by others in the context of retirement portfolios with the intent of reducing sequence risk, but not in an emerging market space. Here the results are shown of using the three traditional asset classes in a pseudo-dynamic manner: the portfolios begin with a certain holding and the holding in equities increases over the life of the portfolio. In this study, REGP results are shown for four equity holding starting points, (0%, 10%, 30%, and 50%) and increase the equity proportion by 30% linearly over the 30 years of retirement. For each starting point the remainder of the portfolio is split between cash and bonds, and these relative proportions remain constant for the life of the portfolio.

The results of this strategy are shown through the SWR metrics in Figure 4.17 and through the ACR in Figure 4.18. The results in Figure 4.17 clearly show that for the portfolios beginning with higher equity holdings (30% and 50%), all REGP portfolios lie below the benchmark frontier and are dominated by it. The REGP portfolios beginning with lower equity holdings (0% and 10%) did achieve higher 5th percentile SWRs than the benchmark frontier, gaining approximately 0.2% while sacrificing an equal percentage of expected SWR. Considering this trade-off, the best REGP portfolios started with 10% equity, ending with 40%. This is, however, similar to the improvement achieved with the low-risk strategy. One notable difference relates to the presence of cash, which was less beneficial in high equity REGP portfolios than in low equity REGP portfolios. The opposite was true for low-risk portfolios where adding cash was highly effective for pure equity portfolios but less so for low-equity portfolios.

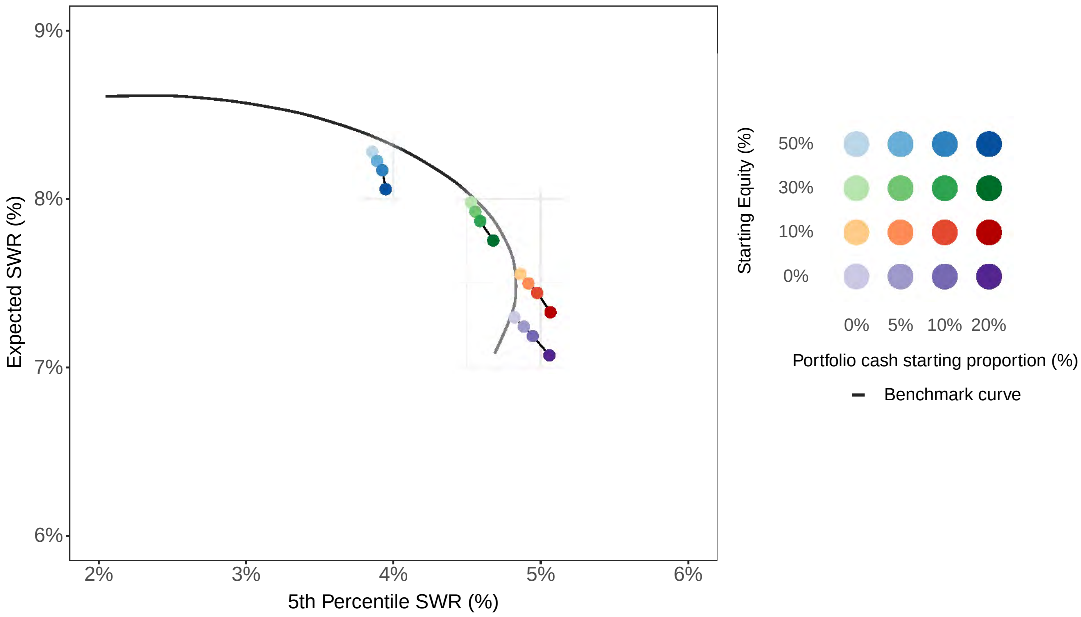

FIGURE 4.17: REGP strategy SWR performance

The ACR results in Figure 4.18 show the greater cash holding improves all the REGP portfolios. It also shows almost identical results for the portfolios starting at 0% and 10% equity. This implies that the 10% starting equity holding REGP dominates the 0% starting point, which is confirmed by the 0.25% difference in their expected SWRs in Figure 4.17.

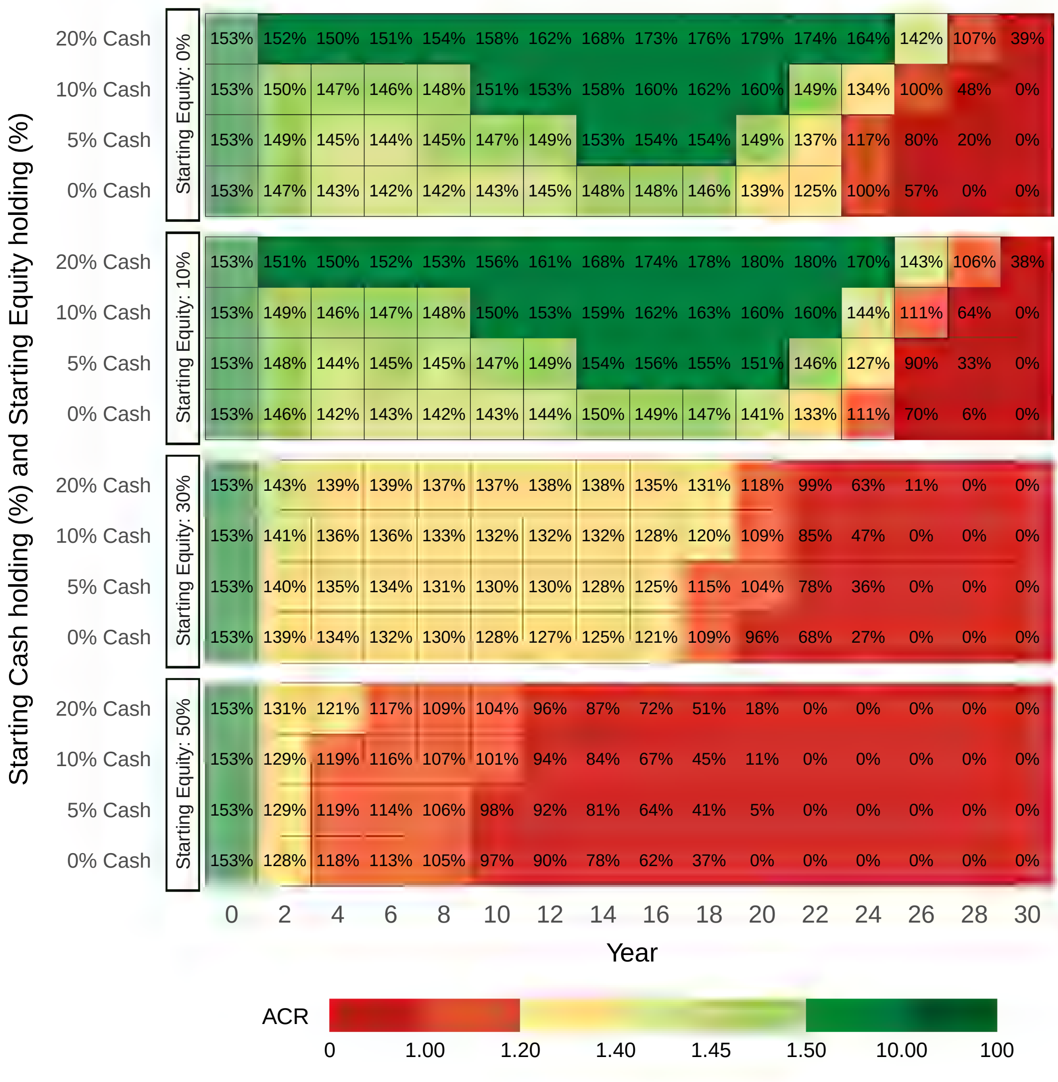

FIGURE 4.18: 5th percentile ACR heatmap of REGP strategy

Considered together, these results show minimal differences in comparison with the low-risk strategy but confirm that there can be benefit in a REGP portfolio to reduce SOR risk and early loss. Further study can consider a greater range of cash holdings to determine the limit to its benefit when added to a REGP portfolio.

4.5.5 Dynamic cash buffer portfolio

The final strategy considered in this study is a dynamic one based on the Evensky & Katz Cash Flow Reserve Strategy. Instead of replenishing the cash reserve, as described by Evensky (2006), the cash holding is merely reduced. This has the effect of allowing a portfolio to absorb the shock of poor market returns in its most stable asset, while increasing the proportion of traditional risky assets in the portfolio. The purpose of this strategy is to minimise portfolio volatility when it matters most and has the biggest impact on portfolio outcomes.

The results of the dynamic cash buffer (DCB) strategy were promising. Figure 4.19 shows the SWR metrics when adding an initial cash holding in 5% increments to four of the benchmark portfolios considered, while keeping the equities and fixed income in fixed relative proportions. The strategy improved both the expected SWR and the 5th percentile SWR in all portfolios by some degree when compared to the benchmark. The improvements in 5th percentile SWR were greater for portfolios with higher equity holding as they could benefit more from the cash buffer during bear periods. This allowed for higher expected SWRs for high-equity portfolios. The portfolios with higher proportions of bonds did also benefit from some expected SWR improvement but primarily saw improvement in the 5th percentile SWR.

The 30:70 equity-bond portfolio performed best after the application of this strategy. The 30:70 portfolio before the strategy was applied had 5th percentile SWR of 4.7%, the same as the all-bond portfolio, but with an expected SWR of 7.9%, 0.8% above that of the all-bond portfolio. After the strategy was applied with 25% initial cash, the 30:70 portfolio's expected and 5th percentile SWR shifted up to 8.1% and 5.8%, (improvements of 0.2% and 1.1%) respectively, dominating the all-bond portfolio which achieved a still impressive 5.6% 5th percentile SWR and 7.3% expected SWR. This demonstrates that this portfolio gained extra benefit from the riskier assets by using this dynamic buffer strategy.

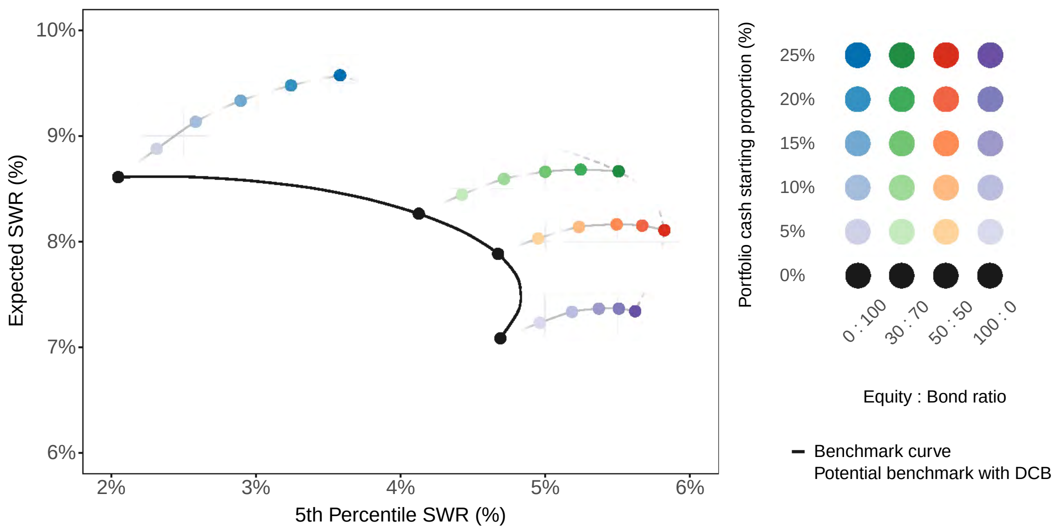

FIGURE 4.19: DCB strategy SWR performance

The ACR results of the dynamic cash buffer strategy at a 5% withdrawal rate are shown in Figure 4.20. The improvement due to this strategy can clearly be seen where a 30/70 portfolio cannot support a 5% withdrawal rate, the same portfolio while applying a 10% cash buffer has an ACR above 1.45 throughout the portfolio's life. With 20% cash buffer, the portfolio reaches an ACR above 4.

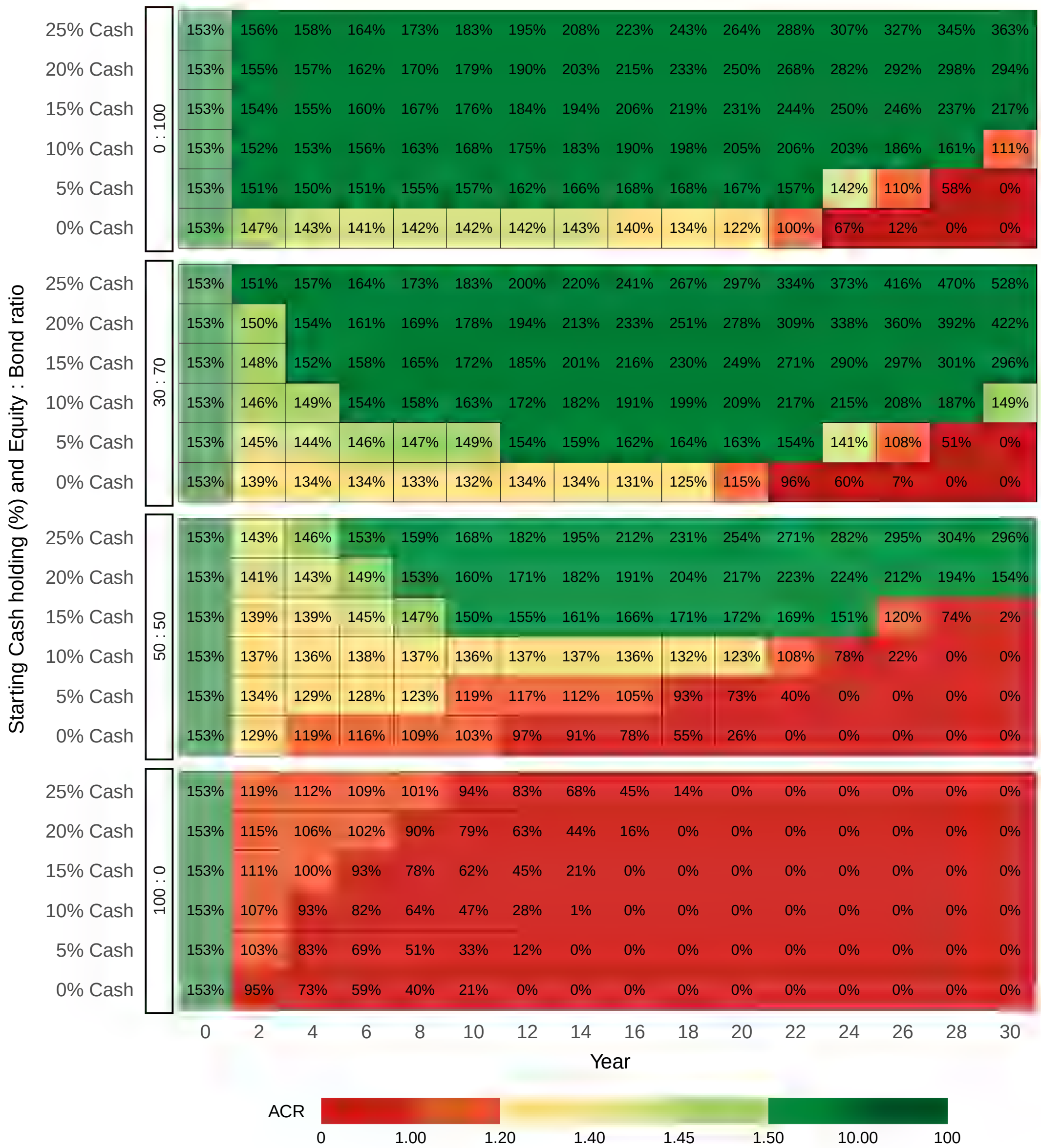

FIGURE 4.20: 5th percentile ACR heatmap of DCB strategy

These results show the strong case for a dynamic cash buffer strategy, being supported by the improvements to both the expected SWR and 5th percentile SWR metrics and demonstrated by the comparative performance in the ACR results.

4.6 Strategy comparison

As stated in this chapter's introduction, a comparison is now made between three strategies and with the benchmark.[2]

In this study, the low-risk, rising equity glidepath (REGP), and dynamic cash buffer (DCB) strategies are comprised of the three traditional asset classes (equities, bonds,

[2]The geographic diversification and risk parity do not form part of this comparison as they are formed from a different investing universe.

and cash) and are therefore more suited to comparison. The REGP strategy has, due to its nature, a greater change in its equity-bond ratio and makes comparisons with the other strategies complex.

The comparison of the low-risk and dynamic cash buffer strategies with the appropriate benchmark portfolios are shown below in Figure 4.21 displaying the 5th percentile ACRs at a 5% withdrawal rate. Each panel indicates the strategy with each row therein showing the strategy applied to a certain equity-bond ratio. Therefore, the first row of each pane should be compared and so on. The REGP portfolios are not directly row-by-row comparable although the best of each strategy can be compared.

The results in Figure 4.21 show that the DCB strategy performed the best. REGP performed second best, while the low-risk strategy still showed significant improvement compared to the benchmark set.

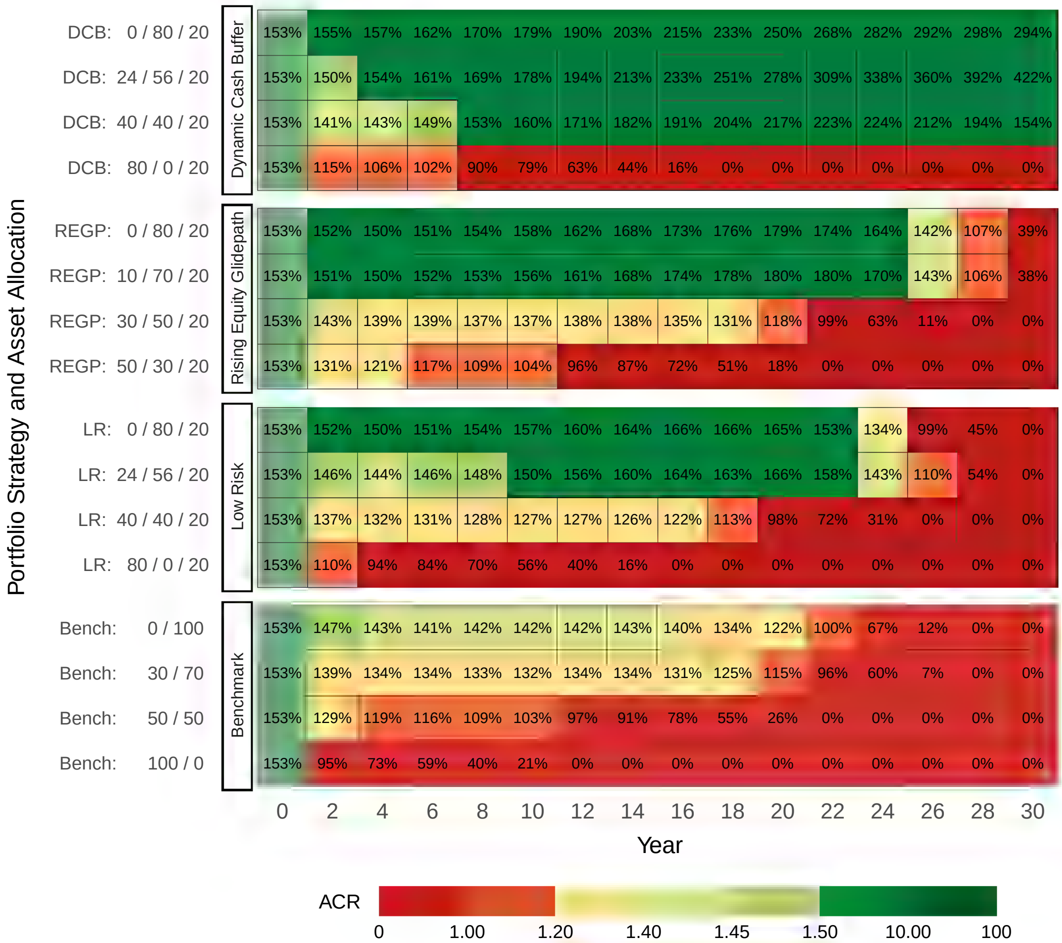

FIGURE 4.21: 5th percentile ACR heatmap strategy comparison

These results show clear cases in improving the 5th percentile SWR for all three strategies. Overall, the DCB strategy is superior to the other two comparable strategies considered in this study.

4.7 Strategy validation

The final component of the results discussion entails the validation of the strategy performance on simulated data by applying the strategies to historical data. The strategies validated here are the three focused on in this study: low-risk, REGP, and DCB strategies. The results of these strategies in comparison to the historical results benchmark is given in Figures 4.22, 4.23, and 4.24.

The validation set of actuals data is limited to 413 months, including five years of out-of-sample data, which results in the 54 overlapping retirement periods highly correlated. This is relative to 10 000 independent trials used in the simulation. As such, the SWR distributions, and therefore the 5$^{\text{th}}$ percentile SWR results, are unlikely representative of long-term performance. Furthermore, the first five years of the historical data used were out-of-sample when sequence risk has the greatest influence in the portfolio results, which explains some of the difference between the simulated and actual results.

The first observation to be made is the shape of the benchmark curve where the performance of portfolios with a mix of bonds and equities performed better than either asset in terms of expected SWR. This is likely due to the low correlation between the ALBI and the ALSI (0.1) in the historical results in comparison to the relatively higher correlation (0.26) in the simulated results. The 5$^{\text{th}}$ percentile SWR for portfolios between 50/50 and 0/100 also outperformed either individual asset, showing the risk diversification effect, especially when allocated more towards the lower-risk bond asset.

The results of the low-risk strategy are shown in Figure 4.22 where the low-risk strategy portfolios with cash holdings were dominated by the benchmark set in terms of both metrics. However, the benchmark set does show that a bond-heavy portfolio, e.g., the 30:70 equity-bond portfolio, did perform the better than stock-heavy portfolios, showing some benefit to the low-risk approach.

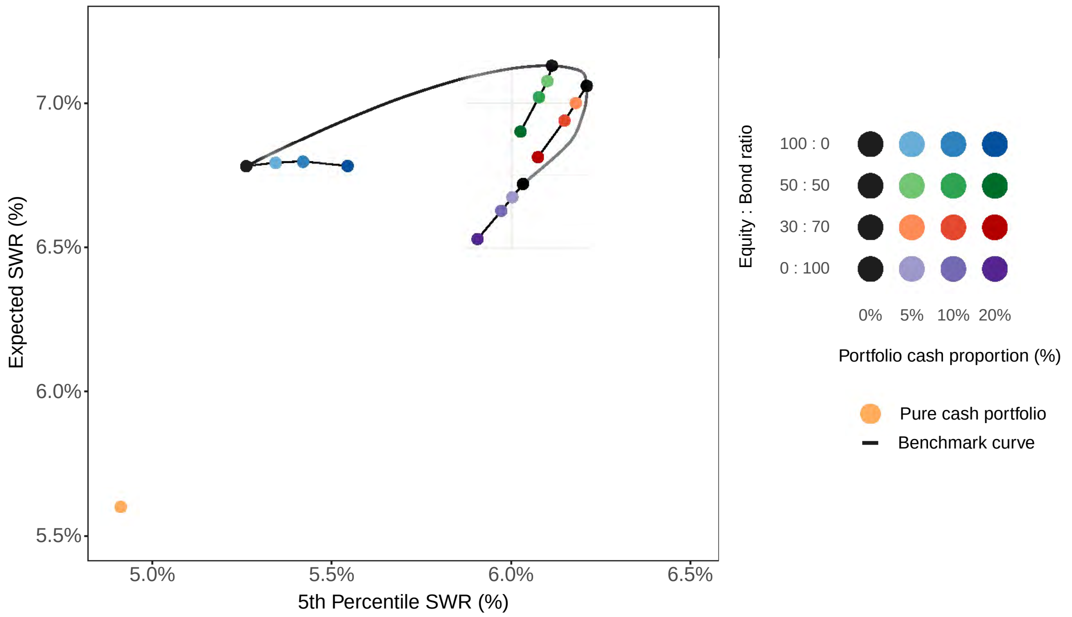

FIGURE 4.22: Low risk strategy actual SWR performance

The REGP strategy showed more promise than low risk and outperformed the benchmark for certain portfolios, presented in Figure 4.23. The portfolios with greater cash holdings suffered the same drag on performance as the low-risk portfolios, however, the cashless REGP portfolios performed well with the cashless 30% starting equity REGP portfolio dominating the entire benchmark set.

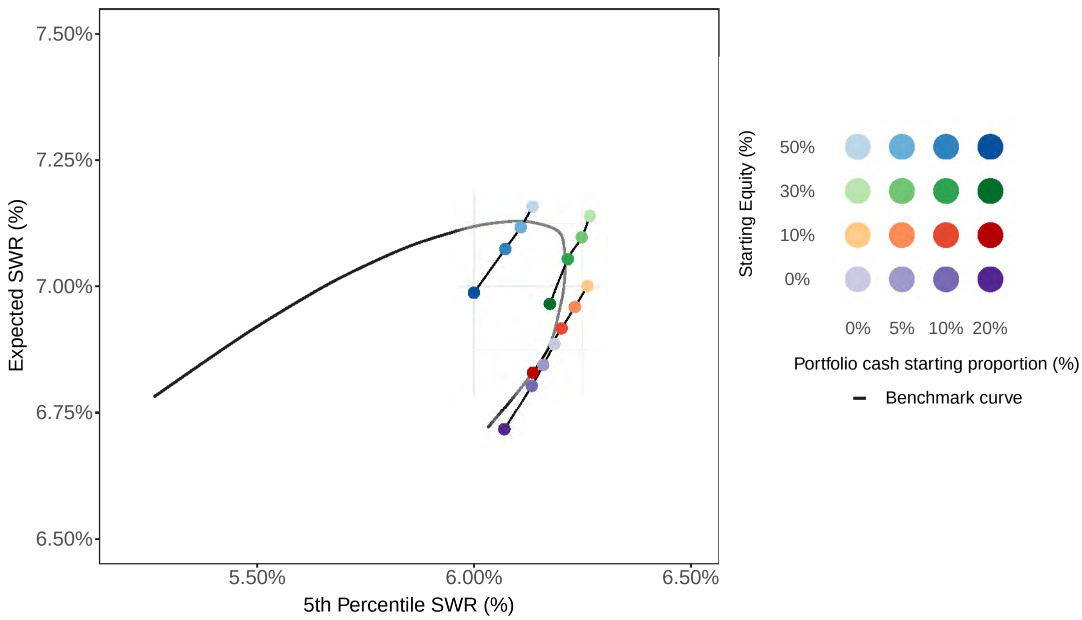

FIGURE 4.23: REGP strategy actual SWR performance

The validation results of the DCB, presented in Figure 4.24, showed highly volatile performance. The strategy did, however, improve upon each benchmark portfolio it was applied to. The requirement of the DCB to spend cash preferentially is based on the non-cash portion of the portfolio experiencing a negative nominal return. Because of this, the DCB performed best when applied to the most volatile, pure equity, portfolios as it provides the greatest number of opportunities for the preferential spending to come into effect. The ability of portfolios to increase portfolio risk after poor performance resulted in stellar performance from pure equity portfolios starting with significant cash holdings. This encapsulates the view of Pfau (2019) in reducing sequence risk by reducing portfolio risk when it matters most, i.e., in periods of poor performance.

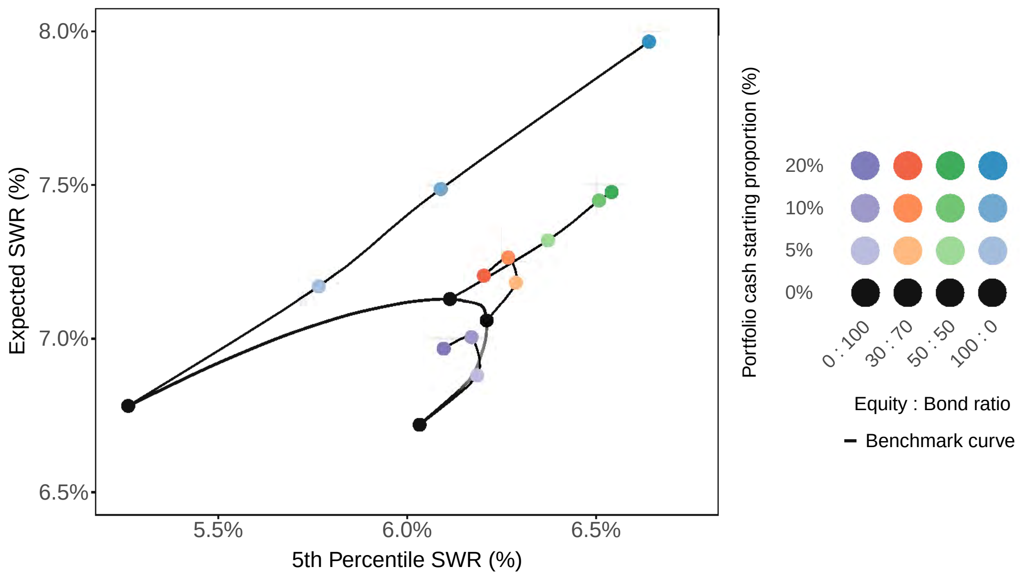

FIGURE 4.24: DCB strategy actual SWR performance

The results of the strategy validation show the same relative ranking in performance as in the simulated results, with low-risk performing the worst and DCB, the best. The performance of the strategies compared to the benchmark was, however, less impressive than the simulated results. This is likely due to the low and negative real returns of the cash asset in the out-of-sample portion of the data which dragged down performance of these strategies which all rely to an extent on cash's ability to yield stable returns above or near inflation. Both the low-risk and REGP strategies make extensive use of cash and bonds, the REGP strategy does so predominantly early on in a retirement period when SOR risk has the greatest influence. The DCB suffers this same performance drag but is offset by the benefit of avoiding selling assets at low prices offsets this.

Considering how the strategies performed over the historical period, Figure 4.25 shows the SWR performance for all 54 of the overlapping historical periods for the best performing portfolios in the simulated results from each strategy as well as the best benchmark (30/70) and the pure cash portfolio. It is clear from Figure 4.25 that cash underperformed throughout. While cash yielded stable returns, the high inflation rate in South Africa between 1986 and 2000 meant that cash returns were below the inflation rate at points, eroding a minimum spending portfolio. The cash in the portfolios of the three strategies can be seen to drag the performance of those portfolios down. The DCB's preferential cash spending however does cause sporadic and significant outperformance while the other two strategies remain underperforming throughout.

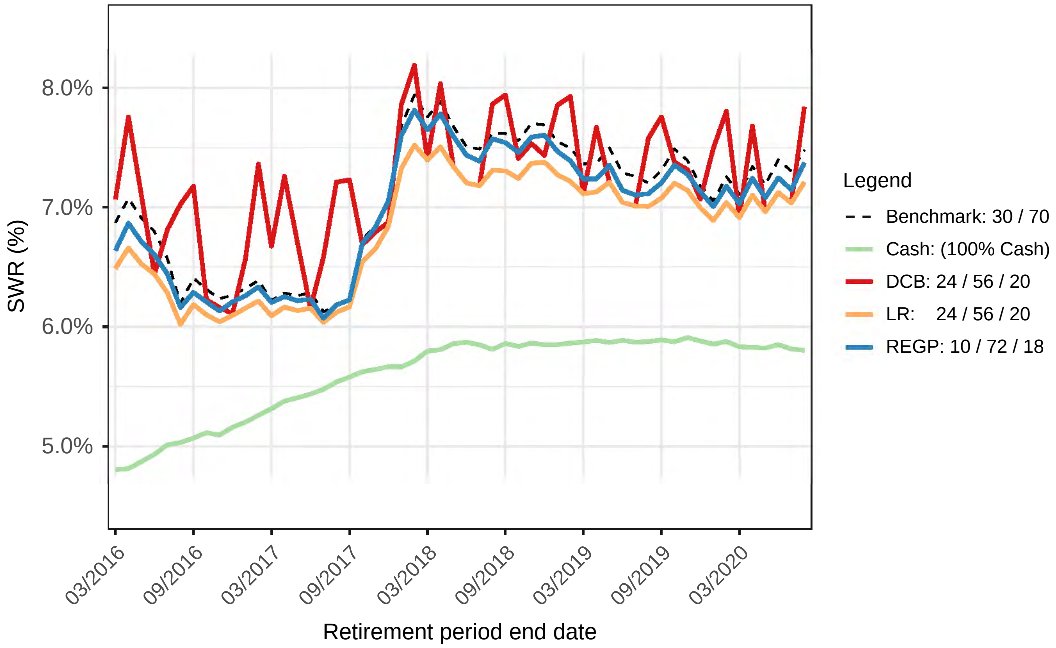

FIGURE 4.25: Actual strategy SWR performance

Another observation from the figure above is a cyclicality of the DCB portfolio although the equity-bond mix in the DCB portfolio makes this less apparent. Figure 4.26 below demonstrates this more clearly. The bond-and-cash DCB portfolios appear to achieve a SWR at a consistent margin above the benchmark but then drop below the benchmark occasionally. This is caused by the month retirement begins and how the year is defined according to that date.

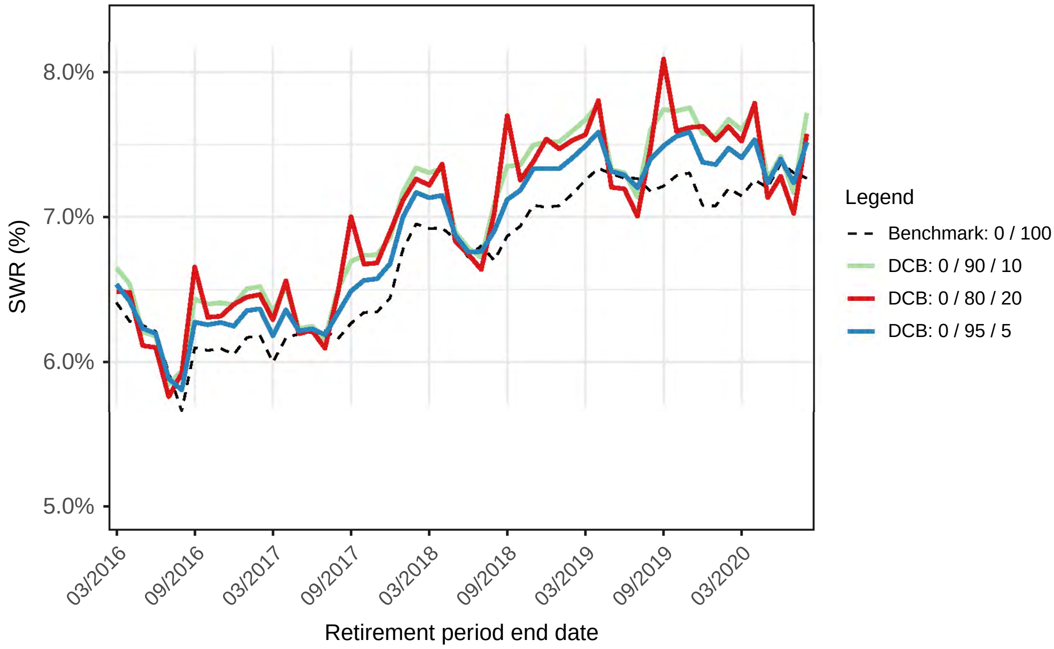

FIGURE 4.26: Actual DCB strategy SWR cyclicality

The DCB strategy aggregates each set of twelve monthly returns into an annual return. If the aggregation contains a sequence of negative returns, a portfolio loss could be recognised, and cash could be spent selectively. However, if the aggregation splits the negative returns sequences such that the yearly return for both can be above zero, and the DCB would not recognise the year as a portfolio loss and the portfolio would not spend selectively from cash. This could cause the cash to remain in the portfolio and act as a drag on investment performance. The result is that some retirees would benefit from DCB and withdraw preferentially from cash while other retirees would not have had the same benefit, all depending on the retirement date and how the annual grouping is defined.

Table 4.1 below provides a count of the number of times the bond return was below zero for each retirement period end date. Retirees who retired in August and whose retirement periods end in July would never experience a below zero bond return, never spend cash selectively, nor benefit from stability of the cash holding in the DCB strategy. The results of Table 4.1 explain the cyclicality seen in Figure 4.26.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2016	-	-	2	3	1	1	0	1	2	1	1	3
2017	3	3	2	3	1	1	0	1	2	1	1	3
2018	3	3	2	3	1	1	0	1	2	1	1	3
2019	3	3	2	3	1	1	0	1	2	1	1	3
2020	3	3	3	3	1	1	0	1	-	-	-	-

TABLE 4.1: Number of DCB selective spending opportunities by retirement period end date

The results of the DCB strategy validation show potential issues with the identification of underperforming years although it did still demonstrate improvement to the benchmark set. While workable as defined herein, the DCB identification method should be revised to offer greater benefit to the investor. The low-risk and REGP have shown to be less effective than the DCB strategy in the historical validation, however, should still prove somewhat effective in ordinary bull and bear cycles with stable inflation. The extraordinary and lengthy period in the late 1980s and early 1990s of high inflation and below inflation money market returns related to an abnormally volatile socio-political situation should perhaps be taken as such. These strategies using cash as a stable return asset should perform better when excluding this anomaly.

4.8 Chapter summary

The initial results of the simulation showed the difference between the retirement metrics and conventional MVO metrics where stability was more important than the expected return in producing high SWRs.

The benchmark set showed that bonds improved in the risk metric significantly (gained significant 5^{th} percentile SWR) for a relatively small loss in expected SWR relative to equities. The diversification effects from lack of perfect correlation resulted in the lowest risk benchmark portfolio lying between 10/90 and 20/80 equities-bonds portfolio.

Historical comparison showed the differing SWR performance between South African and US assets. The comparison, however, shows that a 4% was easily achieved by all portfolios in both countries, supporting the 4% Rule of Bengen (1994).

Analysis on the SOR risk showed that SOR risk did peak with maximum portfolio values at retirement. This sensitivity dropped off significantly after retirement implying the years surrounding the retirement date are very influential on the outcomes of the retirement portfolio and what is possible.

The geographic diversification strategy showed strong benefits for a South African investor irrespective of the type although the greatest benefit was achieved by diversifying into the broader indices. The risk parity showed some benefit in improving 5^{th} percentile SWR. Due to the way the assets were modelled, this study could be expanded such that the inflation and returns models are integrated to give a better representation of reality for the inflation-linked assets.

The low-risk, REGP, and DCB strategies all showed benefits. The DCB strategy did out-perform the other two as it allowed the portfolio to gain from the extra risk being assumed with a higher equity holding while mitigating some of the downside. This resulted in improvements to the expected SWR as well as reducing SOR risk.

The validation of the three strategies, using a short historical data range with some out-of-sample data, showed that DCB was the superior strategy, followed by REGP, and the worst performer was the low-risk strategy. The under-performance of cash in this period caused all three strategies to experience cash-drag relative to the benchmark set. This lengthy period of the underperformance by cash and high inflation is however believed unlikely to repeat itself and implies all three strategies should show risk reduction benefit.

Chapter 5

Conclusion

This chapter provides a conclusion for the study into sequence of return (SOR) risk, early loss, and downside risk in the context of a South African retirement portfolio. An overview of the study is first provided, followed by the methodological execution and a summary of the results. Finally, areas for future study are suggested and concluding remarks are made.

5.1 Overview

The purpose of this study was to consider sequence risk and associated risks when funding a minimum spending requirement goal for a South African retiree and how to structure a portfolio to minimise the effect of these risks. The portfolio considered in this study has a goal of addressing the minimum annual spending requirements a retired investor requires to maintain a basic lifestyle. These annual amounts continue for the life of the investor and are kept constant in real terms.

The specific risks relevant to the analysis of these portfolios are primarily sequence risk (SOR), early-loss risk, and extreme event downside risk. These risks are similar and, in the context of this study, describe the sensitivity of a portfolio to poor returns in the early stages of retirement and the potential for the portfolio goal to become unobtainable. The aim of this study was to determine the extent to which these risks together impact this portfolio and then to consider asset allocation strategies to mitigate this risk.

The portfolio strategies considered in this analysis had the intent of reducing the volatility by some measure and by extension the relevant risks. This allows an investor to retain upside potential while minimising downside risk. These strategies included geographic diversification, risk parity, a low-risk strategy, a rising equity glidepath strategy, and a dynamic cash buffer strategy. The strategies were evaluated in isolation and compared where possible.

5.2 Summary of results

The benchmark set of portfolios was created as the baseline for comparison. The historical comparison between a US and South African investor showed similar SWRs between the two nations for the periods considered and validated the simulation as the South African historical results were similar to those in the benchmark set, generated by the Monte Carlo simulation. The results of the sequence risk sensitivity and the sensitivity of a portfolio's SWR to changes in period returns showed sequence risk peaked around the retirement date dropping off sharply in the years thereafter, similar to the findings of Pfau (2013).

The strategies that were tested all showed at least some benefit to a hypothetical investor with a 95% confidence requirement. Geographic diversification was shown to be useful in reducing downside risk. The risk parity approach showed muted results, but an experimental design change would be useful to perform a more robust test of the strategy. The low-risk strategy showed solid improvement of the SWR at a 95% confidence level but required some reduction in expected SWR. The rising equity glidepath showed mixed results but when applied to a portfolio with a low-equity starting point performed better than the benchmark and the low-risk strategy. The dynamic cash buffer strategy performed the best of all strategies considered and offered significant improvement to the 5^{th} percentile SWR as well as, at least some improvement to the expected SWR.

The results were validated using historical data, some of which was out-of-sample. The validation showed the same relative performance between the strategies, with the dynamic cash buffer strategy performing the best. The out-of-sample data did include an anomalous situation of higher-than-normal inflation with underperforming cash assets that caused the improvement of these strategies to be muted in comparison to the simulated results. The DCB still showed improvement to the expected and 5^{th} percentile SWRs.

5.3 Areas for future study

Limitations were implemented on the scope of this study in some ways to provide sufficient methodological simplicity so that the topics of interest could be explored.

- *Taxation, fees, and trading expenses*: Further research could relax these restrictions for a more realistic analysis. The inclusion of taxation and trading expenses would make the results more accurate and a similar study with those aspects included would be of interest. Differing international and local trading costs could erode, to a degree, the benefits of geographic diversification. The amount of rebalancing required when pairing negatively correlated assets or a volatile asset with stable ones could also impact the trading costs significantly.

- *Confidence interval*: A 95% confidence interval was used in this study but for a more risk averse investor, perhaps a 99% interval is more appropriate. The results of this may be of interest.
- *Optimal asset allocations*: This study was concerned more with identifying strategies that reduced sequence risk than the specific optimal allocations where the greatest reduction was achieved. Finding these optimal asset points for each strategy is left as an area for future study.
- *Asset class considerations*: Expanding the range of assets and strategies considered in this study would be another important expansion in this field. The inclusion of uncorrelated asset classes that meet a retired investor's needs, such as real estate and infrastructure, could provide significant benefits to an investor seeking to address SOR and related risks.
- *Integrated asset-inflation model:* A model that included inflation simulation with the rest of the asset result simulation so that the relationships between explicitly inflation-linked assets as well as implicit relationships are accurately represented. The inflation interaction with interest rates could also be captured considering how important fixed income and money market assets are in this field of investing.
- *Asset return generation model*: This study used non-normal continuous distributions which have non-zero probabilities for returns less than -100%. This study limited returns to between -100% and 300% artificially. A different distribution on which base the model could be chosen. Monte Carlo simulation could also be replaced with bootstrapped return sequences, sampling with replacement, to avoid having to assume a distribution. Lastly, a more complex expansion on the model would be to include time-varying correlations by implementing a multivariate GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model or a dynamic conditional correlation model.
- *Market regimes*: A model that captures the autocorrelation of returns or the tendency of asset prices to return quickly to pre-bear market levels after a trough could improve the accuracy of this model. This could be achieved with more than two market cycle phases, such as a bear, bull, and recovery phases. Another expansion on this study would be to separate out an international market cycle and a South African market cycle with correlated binary (or ternary) variables with a hazard function for switching probabilities (Durland and McCurdy, 1994).
- *Dynamic cash buffer strategy*: Further research could be done into the dynamic cash buffer strategy. The strategy could be tested on a developed market nation, the cash buffer could be expanded as a percentage of initial portfolio value, or applied with the geographic diversification or other strategy. The most important alteration that should be considered is the identification method of underperforming periods which is based on a fixed year definition and a

nominal zero return. Further study could consider a real return benchmark for identifying underperforming periods. Another improvement to the method could be a reduction in the cash buffer after a certain number of years to reduce cash drag if no underperforming years come to pass. Lastly, a rolling twelve-month period could be considered for identifying underperforming periods.

5.4 Conclusion

This study investigated sequence of return risk and other associated risks as they apply to a retired investor in South Africa. The results showed that an investor is most vulnerable to these risks when portfolio values are maximised, around the point of retirement and for a few years thereafter. With this risk in mind, a risk-averse investor looking to address a minimum, inflation-linked spending goal can implement asset allocation strategies to mitigate this risk to a degree. Diversifying one's asset allocation geographically was shown to increase the real amount an investor can withdraw annually from their portfolio. Reducing portfolio volatility through investing in lower risk assets also increased the withdrawal amount as did a rising equity glidepath. Finally, the dynamic cash buffer strategy showed promise and improved on all the metrics considered, both expected outcomes and event risk measures.

The best strategy identified to address sequence risk using asset allocation was the dynamic cash buffer which managed to absorb downside volatility better than the others considered. The strategy provided upside potential to investors supporting a minimum spending goal while also reducing downside risk to spending ability. While the results of the strategies considered here are promising, they are only indicative and are not intended to replace flooring strategies. Flooring is still important for reducing risk in retirement and retirement planning should be done from a holistic perspective, personalised to individual household balance sheets.

This study expanded on the simulation methodology of Collins and Stampfli (2019) to incorporate the non-normality of return distributions. Using the proposed approaches of Pfau (2019) to reduce sequence risk, asset allocation-based strategies affecting portfolio volatility were developed to address sequence risk. The application of this analysis on an emerging market is uncommon in this topic and contributes to the body of knowledge.

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Appendix A

Market regime switching model

A.1 Market regime identification procedure

The procedure in identifying bull and bear regimes, based on Pagan and Sossounov (2003), was as follows:

1. Identify potential turning points.

   Potential peak at time t when:

   $$ln(P_t) > ln(P_{t-8}), \ldots, ln(P_{t-1}), ln(P_{t+1}), \ldots, ln(P_{t+8})$$

   Potential trough at time t when:

   $$ln(P_t) < ln(P_{t-8}), \ldots, ln(P_{t-1}), ln(P_{t+1}), \ldots, ln(P_{t+8})$$

   where $ln(P_t)$ is the natural log of the price, P, at time t.

2. Where multiple potential peaks (troughs) exist between troughs (peaks), the highest peak (lowest trough) is selected and the additional potential turning point(s) are eliminated. This ensures peaks and troughs alternate correctly.

3. Eliminate peaks (troughs) where the price is less than 20% higher (lower) than the preceding trough (peak). This is equivalent to eliminating peaks and troughs if the following inequalities do not hold.

   Peaks:

   $$ln(P_{peak}) - ln(P_{trough}) \geq 0.182$$

   Troughs:

   $$ln(P_{trough}) - ln(P_{peak}) - \geq -0.223$$

4. Eliminate potential turning point pairs where the cycle length (peak-to-peak or trough-to-trough) is less than 16 months.

5. Eliminate potential turning point pairs where the phase length (peak-to-trough or trough-to-peak) is less than 4 months.

A.2 Market regime identification results

Below is a summary of the market regime identification results.

	Bear market	Bull market	Total
Number of phases	4	5	9
Total number of periods	35	321	356
Average phase length	8.75	64.2	39.6

A.3 Markov chain calculations

The calculations below follow the methodology of Winston and Goldberg (2004) using their notation.

Firstly, let $Bull = 1$ and $Bear = 0$. Then, let $P_{ij} = P(X_t = j|X_{t-1} = i)$ = probability from switching from state i to state j. For example, P_{10} is the probability of switching from a bull market state to a bear market state each period. The transition probability can be calculated from the observed data by:

$$P_{ij} = \frac{\sum ST_{ij}}{\sum ST_{ij} + \sum ST_{ii}}$$

Where $\sum ST_{ij}$ is the total number of state transitions from state i to state j in the historical data set considered (January 1991 to August 2020). Below is a summary of the counts of state transitions.

	(To) Bull	Bear	Total
(From) Bull	$321^{*} - 5 = 316$	4	320
Bear	4	33	37
Total	320	37	357

$*$ Excluding final 5 months of partial bull cycle

Using the information above, an initial Markov chain was developed, shown below.

$\mathbf{P_{Initial}}$ =

	(To) Bull	Bear
(From) Bull	$316/320 = 98.75\%$	$4/320 = 1.25\%$
Bear	$4/37 = 10.81\%$	$33/37 = 89.19\%$

However, recall Step 5 in the above identification procedure in which phases must be a minimum of 4 periods in a certain regime. The first 4 months in a state are therefore not possible to be transitioned from. The corrected transition state summary is given below.

	(To) Bull	Bear	Total
(From) Bull	$321 - 4 \times 4 - 5^{*} = 300$	4	$321 - 16 - 5 + 4 = 304$
Bear	4	$35 - 4 \times 4 = 19$	23
Total	304	23	326

$*$ Excluding final 5 months of partial bull cycle

Using the above table, the corrected Markov chain is given below.

P =

	(To) Bull	Bear
(From) Bull	$300/304 = 98.684\%$	$4/304 = 1.316\%$
Bear	$4/23 = 17.391\%$	$19/23 = 82.609\%$

or more simply:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{10} \\ p_{01} & p_{00} \end{bmatrix} = \begin{bmatrix} 0.987 & 0.013 \\ 0.174 & 0.826 \end{bmatrix}$$

A.4 Markov chain steady state

Winston and Goldberg (2004) defined π_i and πj as the steady state probability of the Markov chain being in state i and j after a sufficiently long time, respectively. For a two state Markov chain, the steady state probabilities are given by:

$$\pi_i = P_{ii}\,\pi_i + P_{ji}\,\pi_j \tag{A.1}$$

$$\pi_j = P_{jj}\,\pi_j + P_{ij}\,\pi_i \tag{A.2}$$

Where:

$$\pi_i + \pi_j = 1 \tag{A.3}$$

Now substituting (A.3) into (A.1) where $i = 1$ and $j = 0$:

$$\pi_1 = P_{11}\,\pi_1 + P_{01}\,(1 - \pi_i)$$

$$\pi_1 = \frac{P_{01}}{1 + P_{01} - P_{11}}$$

$$\pi_1 = \frac{0.17391}{1.17391 - 0.98684} = 92.9663\%$$

Substituting back into (A.3)

$$\pi_0 = 7.0337\%$$

Thus, the steady state probabilities for being in bull and bear states, respectively, are:

$$\begin{aligned}\pi_1 &= 92.9663\% \\ \pi_0 &= 7.0337\%\end{aligned}$$

Appendix B

Investment asset return statistics

B.1 Asset bear and bull return probability kernel density functions

The figures below show the kernel density functions for each asset in each market regime: bull or bear. Although the number of bear periods was limited, making the distribution easily biased, there appears to be a left skew of asset return distributions in bear cycles with greater kurtosis in comparison to the distribution in bull cycles.

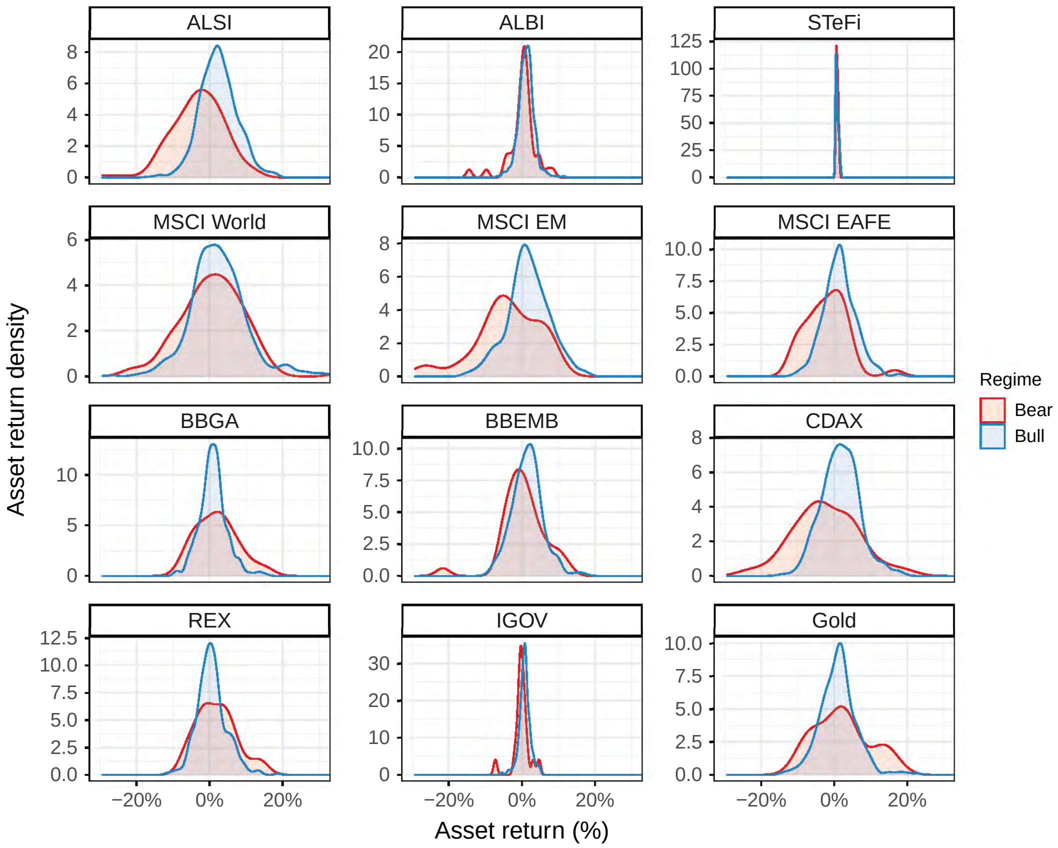

FIGURE B.1: Asset return kernel density function by regime

B.2 Asset bear and bull (annualised) distribution statistics

The table below shows the annualised distribution parameters of each asset's returns in each regime.

		Market Regime	
Asset	Distribution Moment	Bear	Bull
ALSI	Mean	-40.69 %	24.76 %
	Standard Deviation	15.05 %	19.75 %
	Skewnesss	-0.3801	0.0694
	Excess Kurtosis	0.2458	0.0293
ALBI	Mean	2.44 %	14.03 %
	Standard Deviation	14.65 %	8.73 %
	Skewnesss	-0.3886	0.0832
	Excess Kurtosis	0.3179	0.1882
STeFi	Mean	11.28 %	10.25 %
	Standard Deviation	0.90 %	1.23 %
	Skewnesss	-0.0605	0.1723
	Excess Kurtosis	-0.0766	-0.0458
MSCI World	Mean	-2.26 %	30.33 %
	Standard Deviation	35.90 %	34.37 %
	Skewnesss	0.3438	0.2485
	Excess Kurtosis	0.2473	0.1903
MSCI EM	Mean	-38.10 %	22.23 %
	Standard Deviation	20.10 %	24.34 %
	Skewnesss	-0.218	-0.0197
	Excess Kurtosis	0.0426	0.0237
MSCI EAFE	Mean	-22.42 %	18.49 %
	Standard Deviation	15.88 %	18.01 %
	Skewnesss	0.1923	0.0944
	Excess Kurtosis	0.1193	0.067
BBGA	Mean	26.41 %	12.16 %
	Standard Deviation	24.77 %	14.70 %
	Skewnesss	0.1319	0.1378
	Excess Kurtosis	-0.0229	0.1128
BBEMB	Mean	4.89 %	19.51 %
	Standard Deviation	22.14 %	17.21 %
	Skewnesss	-0.2521	0.1103
	Excess Kurtosis	0.289	0.0792
CDAX	Mean	-24.68 %	21.67 %
	Standard Deviation	24.05 %	21.78 %
	Skewnesss	0.0255	0.0563

		Market Regime	
Asset	Distribution Moment	Bear	Bull
	Excess Kurtosis	0.0096	0.0401
REX	Mean	24.69 %	7.39 %
	Standard Deviation	23.42 %	15.13 %
	Skewnesss	0.1421	0.166
	Excess Kurtosis	-0.0181	0.1303
IGOV	Mean	-1.48 %	9.39 %
	Standard Deviation	7.74 %	5.77 %
	Skewnesss	-0.2992	-0.0846
	Excess Kurtosis	0.3305	0.1764
Gold	Mean	28.62 %	13.03 %
	Standard Deviation	33.56 %	19.61 %
	Skewnesss	0.0995	0.1269
	Excess Kurtosis	-0.0641	0.1657

B.3 Asset return correlation heatmaps

The figures below show the correlation between the returns of the different assets considered in bull and bear market regimes separately.

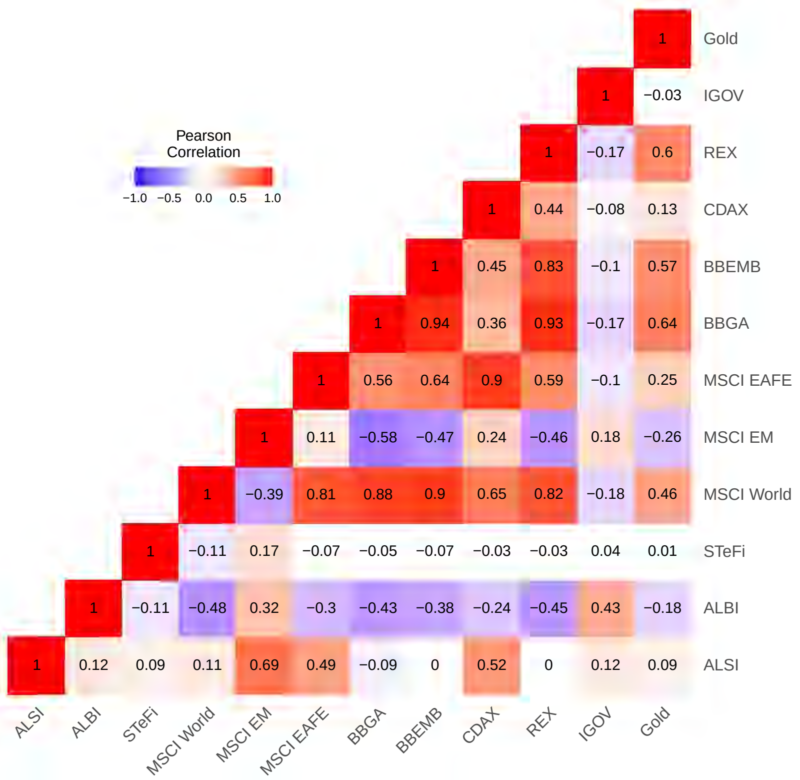

FIGURE B.2: Bear market correlation heatmap

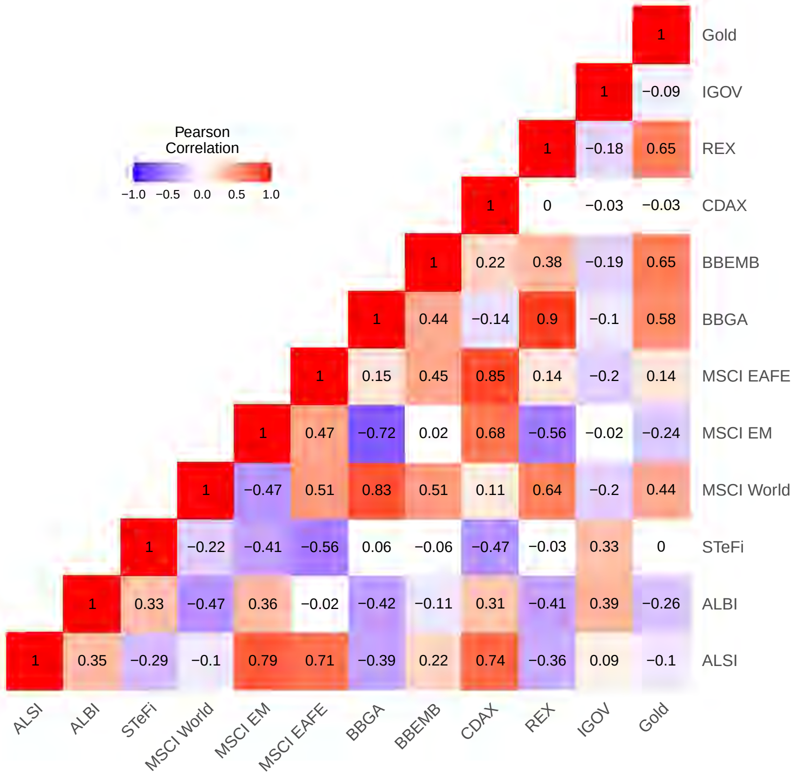

FIGURE B.3: Bull market correlation heatmap

Appendix C

Inflation generation model

C.1 Inflation model regression parameters

The regression of the inflation model persistence parameter was performed according to the method Collins and Stampfli (2019). The regression was in the form of a least squares linear regression of historical South African month-on-month headline CPI inflation data from June 1995 to August 2020 (Bloomberg L. P., 2020). The standard deviation and long-term average inflation were calculated over the same period. The parameters are provided below.

Parameter	Value
Monthly inflation standard deviation	0.004578
Persistence coefficient	0.045645
Long-term average monthly inflation	0.00476226

Appendix D

Portfolio construction calculations

The conversion between real and nominal returns is given by:

$$r_t^* \quad = \quad (1 + r_t) \quad \times \quad (1 + i_t) \quad - \quad 1 \tag{D.1}$$

where r_t^* and it are the nominal return and the annual inflation for period t respectively. The nominal annual withdrawal amount at time t, w_t^*, unlike the real annual withdrawal amount, w, is dependent on previous inflation and is calculated as:

$$w_t^* \quad = \quad w \quad \times \quad \prod_{n=1}^{t} (1 + i_t) \tag{D.2}$$

The nominal portfolio value can thus be calculated as:

$$K_t^* \quad = \quad (K_{t-1}^* - w^*) \quad \times \quad (1 + r_t^*) \tag{D.3}$$

Appendix E

Portfolio asset allocations

All the tables below detail the proportional asset holdings of all the portfolios tested in this study. The proportions are provided for only the relevant assets included in each portfolio, given in decimal form.

E.1 Benchmark portfolios

	ALSI	ALBI
Port. 1	0.00	1.00
Port. 2	0.10	0.90
Port. 3	0.20	0.80
Port. 4	0.30	0.70
Port. 5	0.40	0.60
Port. 6	0.50	0.50
Port. 7	0.60	0.40
Port. 8	0.70	0.30
Port. 9	0.80	0.20
Port. 10	0.90	0.10
Port. 11	1.00	0.00

E.2 Historical comparison portfolios

SA Benchmark Portfolios - uses the same 11 portfolios as in the benchmark portfolios in E.1 above.

US Comparison Portfolios

	Russel 1000	BBUSAB
Port. 1	0.00	1.00
Port. 2	0.10	0.90
Port. 3	0.20	0.80
Port. 4	0.30	0.70
Port. 5	0.40	0.60
Port. 6	0.50	0.50
Port. 7	0.60	0.40
Port. 8	0.70	0.30
Port. 9	0.80	0.20
Port. 10	0.90	0.10
Port. 11	1.00	0.00

E.3 Sequence risk sensitivity portfolios

The portfolios used in measuring sequence of return risk sensitivity were the same as Portfolios 1, 4, 5, 6, and 11 from the benchmark portfolio set in E.1 above.

E.4 Geographic diversification portfolios

Diversifying to a single developed nation

	ALSI	ALBI	CDAX	REX
Port. 40	0.75	0.00	0.25	0.00
Port. 41	0.50	0.00	0.50	0.00
Port. 42	0.25	0.00	0.75	0.00
Port. 43	0.00	0.00	1.00	0.00
Port. 44	0.52	0.22	0.17	0.07
Port. 45	0.35	0.15	0.35	0.15
Port. 46	0.17	0.07	0.52	0.22
Port. 47	0.00	0.00	0.70	0.30
Port. 48	0.38	0.38	0.12	0.12
Port. 49	0.25	0.25	0.25	0.25
Port. 50	0.12	0.12	0.38	0.38
Port. 51	0.00	0.00	0.50	0.50
Port. 52	0.22	0.52	0.07	0.17
Port. 53	0.15	0.35	0.15	0.35
Port. 54	0.07	0.17	0.22	0.52
Port. 55	0.00	0.00	0.30	0.70
Port. 56	0.00	0.75	0.00	0.25
Port. 57	0.00	0.50	0.00	0.50
Port. 58	0.00	0.25	0.00	0.75
Port. 59	0.00	0.00	0.00	1.00

Diversifying into a broad index of developed market nations (using MSCI World)

	ALSI	ALBI	MSCI World	BBGA
Port. 60	0.75	0.00	0.25	0.00
Port. 61	0.50	0.00	0.50	0.00
Port. 62	0.25	0.00	0.75	0.00
Port. 63	0.00	0.00	1.00	0.00
Port. 64	0.52	0.22	0.17	0.07
Port. 65	0.35	0.15	0.35	0.15
Port. 66	0.17	0.07	0.52	0.22
Port. 67	0.00	0.00	0.70	0.30
Port. 68	0.38	0.38	0.12	0.12
Port. 69	0.25	0.25	0.25	0.25
Port. 70	0.12	0.12	0.38	0.38
Port. 71	0.00	0.00	0.50	0.50
Port. 72	0.22	0.52	0.07	0.17
Port. 73	0.15	0.35	0.15	0.35
Port. 74	0.07	0.17	0.22	0.52
Port. 75	0.00	0.00	0.30	0.70
Port. 76	0.00	0.75	0.00	0.25
Port. 77	0.00	0.50	0.00	0.50
Port. 78	0.00	0.25	0.00	0.75
Port. 79	0.00	0.00	0.00	1.00

Diversifying into a broad index of developed market nations (using MSCI EAFE)

	ALSI	ALBI	MSCI EAFE	BBGA
Port. 80	0.75	0.00	0.25	0.00
Port. 81	0.50	0.00	0.50	0.00
Port. 82	0.25	0.00	0.75	0.00
Port. 83	0.00	0.00	1.00	0.00
Port. 84	0.52	0.22	0.17	0.07
Port. 85	0.35	0.15	0.35	0.15
Port. 86	0.17	0.07	0.52	0.22
Port. 87	0.00	0.00	0.70	0.30
Port. 88	0.38	0.38	0.12	0.12
Port. 89	0.25	0.25	0.25	0.25
Port. 90	0.12	0.12	0.38	0.38
Port. 91	0.00	0.00	0.50	0.50
Port. 92	0.22	0.52	0.07	0.17
Port. 93	0.15	0.35	0.15	0.35
Port. 94	0.07	0.17	0.22	0.52
Port. 95	0.00	0.00	0.30	0.70
Port. 96	0.00	0.75	0.00	0.25
Port. 97	0.00	0.50	0.00	0.50
Port. 98	0.00	0.25	0.00	0.75
Port. 99	0.00	0.00	0.00	1.00

Diversifying into a broad index of emerging market nations

	ALSI	ALBI	MSCI EM	BBEMB
Port. 100	0.75	0.00	0.25	0.00
Port. 101	0.50	0.00	0.50	0.00
Port. 102	0.25	0.00	0.75	0.00
Port. 103	0.00	0.00	1.00	0.00
Port. 104	0.52	0.22	0.17	0.07
Port. 105	0.35	0.15	0.35	0.15
Port. 106	0.17	0.07	0.52	0.22
Port. 107	0.00	0.00	0.70	0.30
Port. 108	0.38	0.38	0.12	0.12
Port. 109	0.25	0.25	0.25	0.25
Port. 110	0.12	0.12	0.38	0.38
Port. 111	0.00	0.00	0.50	0.50
Port. 112	0.22	0.52	0.07	0.17
Port. 113	0.15	0.35	0.15	0.35
Port. 114	0.07	0.17	0.22	0.52
Port. 115	0.00	0.00	0.30	0.70
Port. 116	0.00	0.75	0.00	0.25
Port. 117	0.00	0.50	0.00	0.50
Port. 118	0.00	0.25	0.00	0.75
Port. 119	0.00	0.00	0.00	1.00

Diversifying equally into developed and emerging markets

	ALSI	ALBI	MSCI World	MSCI EM	BBGA	BBEMB
Port. 120	0.75	0.00	0.12	0.12	0.00	0.00
Port. 121	0.50	0.00	0.25	0.25	0.00	0.00
Port. 122	0.25	0.00	0.38	0.38	0.00	0.00
Port. 123	0.00	0.00	0.50	0.50	0.00	0.00
Port. 124	0.52	0.22	0.09	0.09	0.04	0.04
Port. 125	0.35	0.15	0.17	0.17	0.07	0.07
Port. 126	0.17	0.07	0.26	0.26	0.11	0.11
Port. 127	0.00	0.00	0.35	0.35	0.15	0.15
Port. 128	0.38	0.38	0.06	0.06	0.06	0.06
Port. 129	0.25	0.25	0.12	0.12	0.12	0.12
Port. 130	0.12	0.12	0.19	0.19	0.19	0.19
Port. 131	0.00	0.00	0.25	0.25	0.25	0.25
Port. 132	0.22	0.52	0.04	0.04	0.09	0.09
Port. 133	0.15	0.35	0.07	0.07	0.17	0.17
Port. 134	0.07	0.17	0.11	0.11	0.26	0.26
Port. 135	0.00	0.00	0.15	0.15	0.35	0.35
Port. 136	0.00	0.75	0.00	0.00	0.12	0.12
Port. 137	0.00	0.50	0.00	0.00	0.25	0.25
Port. 138	0.00	0.25	0.00	0.00	0.38	0.38
Port. 139	0.00	0.00	0.00	0.00	0.50	0.50

E.5 Risk parity portfolios

Each set of two portfolios below consist of a pure Risk Parity portfolio followed by that portfolio combined with an 50/50 benchmark portfolio, in equal weights.

Portfolio limited to:
ALSI, ALBI, IGOV, and Gold

	ALSI	ALBI	IGOV	Gold
Port. 180	0.15	0.31	0.38	0.15
Port. 181	0.33	0.41	0.19	0.08

Portfolio limited to:
ALSI, ALBI, and Gold

	ALSI	ALBI	Gold
Port. 182	0.24	0.54	0.22
Port. 183	0.37	0.52	0.11

Portfolio limited to: ALSI, ALBI, and IGOV

	ALSI	ALBI	IGOV
Port. 184	0.19	0.35	0.46
Port. 185	0.34	0.42	0.23

Portfolio limited to: ALSI, ALBI

	ALSI	ALBI
Port. 186	0.33	0.67
Port. 187	0.41	0.59

Portfolio including all South African and international assets

	ALSI	ALBI	MSCI World	MSCI EM	MSCI EAFE	BBGA	BBEMB	IGOV	Gold
Port. 188	0.06	0.27	0.04	0.09	0.06	0.10	0.07	0.25	0.06
Port. 189	0.28	0.38	0.02	0.04	0.03	0.05	0.04	0.13	0.03

E.6 Low risk portfolios

The tables below show the portfolio holdings of the low risk strategy portfolios.

	ALSI	ALBI	STeFi
Port. 12	0.95	0.00	0.05
Port. 13	0.90	0.00	0.10
Port. 14	0.80	0.00	0.20
Port. 15	0.47	0.47	0.05
Port. 16	0.45	0.45	0.10
Port. 17	0.40	0.40	0.20
Port. 18	0.28	0.67	0.05
Port. 19	0.27	0.63	0.10
Port. 20	0.24	0.56	0.20
Port. 21	0.00	0.95	0.05
Port. 22	0.00	0.90	0.10
Port. 23	0.00	0.80	0.20

E.7 Rising equity glidepath portfolios

The tables below show the initial (left) portfolio holdings and final (right) portfolio holdings of the rising equity glidepath portfolios. The initial equity, ALSI, holding increases linearly by 30% over the 30 periods while the relative proportion of the other two assets remains constant.

REGP portfolio starting allocation

	ALSI	ALBI	STeFi
Port. 24	0.00	1.00	0.00
Port. 25	0.00	0.95	0.05
Port. 26	0.00	0.90	0.10
Port. 27	0.00	0.80	0.20
Port. 28	0.10	0.90	0.00
Port. 29	0.10	0.85	0.05
Port. 30	0.10	0.80	0.10
Port. 31	0.10	0.70	0.20
Port. 32	0.30	0.70	0.00
Port. 33	0.30	0.65	0.05
Port. 34	0.30	0.60	0.10
Port. 35	0.30	0.50	0.20
Port. 36	0.50	0.50	0.00
Port. 37	0.50	0.45	0.05
Port. 38	0.50	0.40	0.10
Port. 39	0.50	0.30	0.20

REGP portfolio ending allocation

	ALSI	ALBI	STeFi
Port. 24	0.30	0.70	0.00
Port. 25	0.30	0.67	0.04
Port. 26	0.30	0.63	0.07
Port. 27	0.30	0.56	0.14
Port. 28	0.40	0.60	0.00
Port. 29	0.40	0.57	0.03
Port. 30	0.40	0.53	0.07
Port. 31	0.40	0.47	0.13
Port. 32	0.60	0.40	0.00
Port. 33	0.60	0.37	0.03
Port. 34	0.60	0.34	0.06
Port. 35	0.60	0.29	0.11
Port. 36	0.80	0.20	0.00
Port. 37	0.80	0.18	0.02
Port. 38	0.80	0.16	0.04
Port. 39	0.80	0.12	0.08

E.8 Dynamic cash buffer portfolios

The initial portfolio holdings for the dynamic cash buffer portfolios are given below. Due to the dynamic nature, the holdings diverge after the first period for each trial.

	ALSI	ALBI	STeFi
1	0.95	0.00	0.05
2	0.90	0.00	0.10
3	0.85	0.00	0.15
4	0.80	0.00	0.20
5	0.75	0.00	0.25
6	0.47	0.47	0.05
7	0.45	0.45	0.10
8	0.42	0.42	0.15
9	0.40	0.40	0.20
10	0.38	0.38	0.25
11	0.28	0.66	0.05
12	0.27	0.63	0.10
13	0.26	0.59	0.15
14	0.24	0.56	0.20
15	0.22	0.52	0.25
16	0.00	0.95	0.05
17	0.00	0.90	0.10
18	0.00	0.85	0.15
19	0.00	0.80	0.20
20	0.00	0.75	0.25

E.9 Strategy validation portfolios

The portfolio asset holdings for the strategy validation section are the same as those used in the benchmark set (E.1) and the portfolios from the strategies in E.6, E.7, and E.8.

Appendix F

Sequence risk and the SWR

This appendix demonstrates the relationship between the SWR and the SOR risk directly. A portfolio is constructed equal weighted equities and bonds using 30 years of returns between September 1990 and August 2020.

In considering SOR risk and the SWR, the analysis compares the set of returns in the order they were realised as well as in ascending and descending order, in real (inflation-linked) terms. The SWR is calculated for each scenario. Equation 3.7, for calculating the SWR, is included below for ease of reference.

$$SWR = \prod_{i=1}^{30}(1+r_i) \;/\; \sum_{j=1}^{30}\prod_{i=j}^{30}(1+r_i) = R_P \times S_P$$

The real returns for the three scenarios are shown in the left-hand table below. Recall from §3.4.1, that if the returns are the same, irrespective of their order, the cumulative portfolio return, R_P, is constant.[1] Therefore, the only factor affecting the SWR is the sequence risk factor, S_P.

The compound factors are calculated as:

$$Compound factor_t = \prod_{i=t}^{30}(1+r_i)$$

where t is the current period. The compound factors are then summed to calculate the inverse of the sequence factors:

$$\frac{1}{S_P} = \sum_{i=1}^{30} Compound factor_i$$

The compound factors are shown in the right-hand table below, with the inverse of the sequence factors given in the final row.

[1] The value of R_P for all three scenarios is 6.4288.

	Real returns, r_i			Compound factors		
t	Realised	Ascending	Descending	Realised	Ascending	Descending
1	0.34 %	-23.44 %	36.18 %	6.42	6.42	6.42
2	0.29 %	-7.57 %	25.16 %	6.40	8.39	4.72
3	14.34 %	-5.51 %	23.36 %	6.38	9.08	3.77
4	13.38 %	-0.68 %	20.19 %	5.58	9.61	3.06
5	-0.60 %	-0.60 %	17.30 %	4.92	9.67	2.54
6	13.29 %	-0.53 %	14.34 %	4.95	9.73	2.17
7	5.98 %	-0.33 %	14.18 %	4.37	9.78	1.90
8	-23.44 %	0.13 %	13.38 %	4.13	9.82	1.66
9	36.18 %	0.29 %	13.29 %	5.39	9.80	1.46
10	23.36 %	0.34 %	11.39 %	3.96	9.77	1.29
11	10.17 %	0.96 %	10.17 %	3.21	9.74	1.16
12	3.82 %	3.13 %	9.37 %	2.91	9.65	1.05
13	-0.33 %	3.57 %	9.07 %	2.81	9.36	0.96
14	17.30 %	3.82 %	7.02 %	2.81	9.03	0.88
15	25.16 %	5.43 %	5.98 %	2.40	8.70	0.83
16	20.19 %	5.98 %	5.43 %	1.92	8.25	0.78
17	14.18 %	7.02 %	3.82 %	1.60	7.79	0.74
18	-7.57 %	9.07 %	3.57 %	1.40	7.28	0.71
19	-5.51 %	9.37 %	3.13 %	1.51	6.67	0.69
20	7.02 %	10.17 %	0.96 %	1.60	6.10	0.67
21	9.07 %	11.39 %	0.34 %	1.49	5.54	0.66
22	9.37 %	13.29 %	0.29 %	1.37	4.97	0.66
23	5.43 %	13.38 %	0.13 %	1.25	4.39	0.66
24	11.39 %	14.18 %	-0.33 %	1.19	3.87	0.65
25	-0.68 %	14.34 %	-0.53 %	1.07	3.39	0.66
26	-0.53 %	17.30 %	-0.60 %	1.07	2.96	0.66
27	3.57 %	20.19 %	-0.68 %	1.08	2.53	0.66
28	3.13 %	23.36 %	-5.51 %	1.04	2.10	0.67
29	0.13 %	25.16 %	-7.57 %	1.01	1.70	0.71
30	0.96 %	36.18 %	-23.44 %	1.01	1.36	0.77
			Inverse of Sequence factor	86.28	207.47	44.21

Using Equation 3.8, the resulting SWRs for the realised, ascending, and descending real returns, respectively, are provided below. The realised SWR was 7.45%, however, should the real returns have occurred in ascending order this would have less than halved to 3.10%. Furthermore, if the real returns had occurred in descending order, the SWR would have almost doubled the realised SWR, to 14.53%. This demonstrates the clear link between SWR and SOR risk and supports the use of SWR as a proxy.

	Realised	Ascending	Descending
SWR	7.45 %	3.10 %	14.53 %

Appendix G

Data sources

This appendix details the sources of all external numerical data that was used in this study. This includes the asset return data, exchange rate data, inflation data, and yield curve data. The pro forma yield curve used in the ACR calculations is also provided.

G.1 Investment assets data

The two tables below provide information on the investment assets used in this study.

Name	Short name	Description
FTSE/JSE ALSI TRI	ALSI	South African equities all share index
FTSE/JSE ALBI TRI	ALBI	South African top vanilla bonds
STeFi TRI	STeFi	South African money market
MSCI World	MSCI World	Developed world markets equities
MSCI EM	MSCI EM	Large and midcap equities in EM
MSCI EAFE	MSCI EAFE	Developed market equity in EAFE
Bloomberg Barclays Global Aggregate TRI	BBGA	Global investment grade debt index
Bloomberg Barclays EM Aggregate TRI	BBEMB	Emerging markets debt index
Deutsche Borse AG CDAX	CDAX	Composite, German equities index
Deutsche Borse AG REX	REX	German bond benchmark index
FTSE/JSE IGOV TRI	IGOV	Government inflation-linked bond index
Gold commodity	Gold	Gold spot – ZAR cross rate
Russell 1000 TRI	Russell 1000	Large-cap US equities index
Bloomberg Barclays US Agg. Bond TRI	BBUSAB	Investment grade USD debt index

Short name	Availability	Source	Code	Denomination available
ALSI	April 1986 – June 2010	Dimson et al. (2020)	AJ203	ZAR
	June 2002 – August 2020	Bloomberg L. P. (2020)	JALSHTR Index	ZAR
ALBI	April 1986 – December 2003	Dimson et al. (2020)	BNDTR5 TRI	ZAR
	January 2001 – August 2020	Bloomberg L. P. (2020)	ALBTR Index	ZAR
STeFi	April 1986 - November 2011	Dimson et al. (2020)	GMC1 Alex Forbes MMI	ZAR
	June 2003 - August 2020	Bloomberg L. P. (2020)	STEFI Index	ZAR
MSCI World	January 1991 – August 2020	Bloomberg L. P. (2020)	M1WO Index	USD
MSCI EM	January 1991 – August 2020	Bloomberg L. P. (2020)	MXEF	USD
MSCI EAFE	January 1991 – August 2020	Bloomberg L. P. (2020)	MXEA Index	USD
BBGA	January 1991 – August 2020	Bloomberg L. P. (2020)	LEGATRUU Index	USD
BBEMB	January 1993* – August 2020	Bloomberg L. P. (2020)	EMUSTRUU Index	USD
CDAX	January 1991 – August 2020	Bloomberg L. P. (2020)	CDAX Index	EUR
	January 1999 – August 2020	Bloomberg L. P. (2020)	CDAX Index	ZAR**
REX	January 1991 – August 2020	Bloomberg L. P. (2020)	REX Index	EUR
	January 1999 – August 2020	Bloomberg L. P. (2020)	REX Index	ZAR**
IGOV	April 2004* – August 2020	Bloomberg L. P. (2020)	IGOVTR	ZAR
Gold	January 1991 – August 2020	Bloomberg L. P. (2020)	XAUZAR	ZAR
Russell 1000	April 1986 – August 2020	Bloomberg L. P. (2020)	RU10INTR	USD
BBUSAB	April 1986 – August 2020	Bloomberg L. P. (2020)	LBUSTRUU	USD

* Asset has shorter available history than base period: January 1991 to August 2020
** Quoted in EUR but available as ZAR on Bloomberg L. P. (2020)

G.2 Exchange rate data

The table below provides information on the exchange rate data used in this study.

Exchange rate	Description	Source	Bloomberg code	Dates used
USD/ZAR	US Dollar to South African Rand	Bloomberg L. P. (2020)	USDZAR Curncy	January 1991 – August 2020
EUR/USD	Euro to US Dollar	Bloomberg L. P. (2020)	EURUSD Curncy	January 1991 – August 2020

G.3 Inflation data

The table below provides information on the inflation rate data used in this study.

Name	Description	Source	Bloomberg code	Dates used
US CPI Urban Consumers MoM	US consumer price inflation	Bloomberg L. P. (2020)	CPI CHNG	January 1991 – August 2020
South Africa CPI MoM	South African consumer price inflation	Bloomberg L. P. (2020)	SACPIMOM	January 1991 – August 2020

G.4 Yield curve input data

The table below provides information on the yield curve data used in this study.

Name	Bloomberg code	Date	Source
South African Sovereign Curve (I90)	YCGT0090	31 August 2005, 31 August 2010, 31 August 2015, 31 August 2018, 31 August 2020.	Bloomberg L. P. (2020)
South African Inflation-Linked Curve (I688)	YCGT0688	31 August 2005, 31 August 2010, 31 August 2015, 31 August 2018, 31 August 2020.	Bloomberg L. P. (2020)

G.5 Pro-forma yield curve

The table below contains the inflation-linked yield curve used in this study.

Tenor	Real rate (%)
1	1.00
2	1.38
3	1.71
4	1.97
5	2.14
6	2.26
7	2.38
8	2.50
9	2.55
10	2.60
11	2.62
12	2.65
13	2.67
14	2.70
15	2.72
16	2.74
17	2.75
18	2.77
19	2.78
20	2.80
21	2.81
22	2.82
23	2.83
24	2.84
25	2.85
26	2.86
27	2.87
28	2.88
29	2.89
30	2.90

Appendix H

Geographic diversification strategy results

H.1 Geographic diversification case 1:

The figures below show the results for single developed market nation diversification. The strategy makes use of the following geographic diversification investment assets: CDAX and REX.

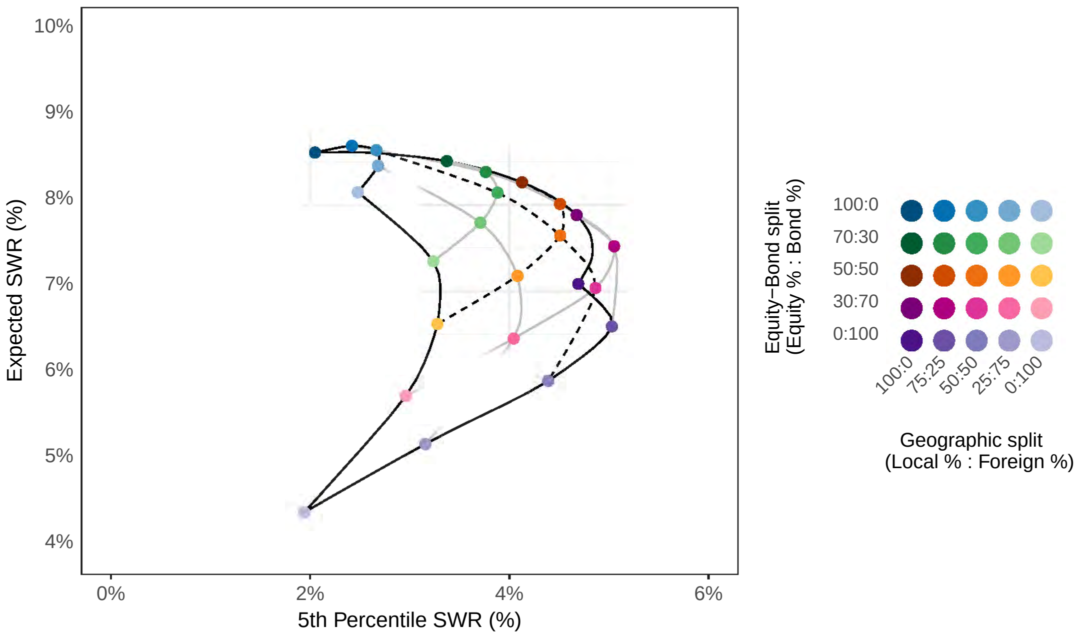

FIGURE H.1: Single developed market diversification SWR performance

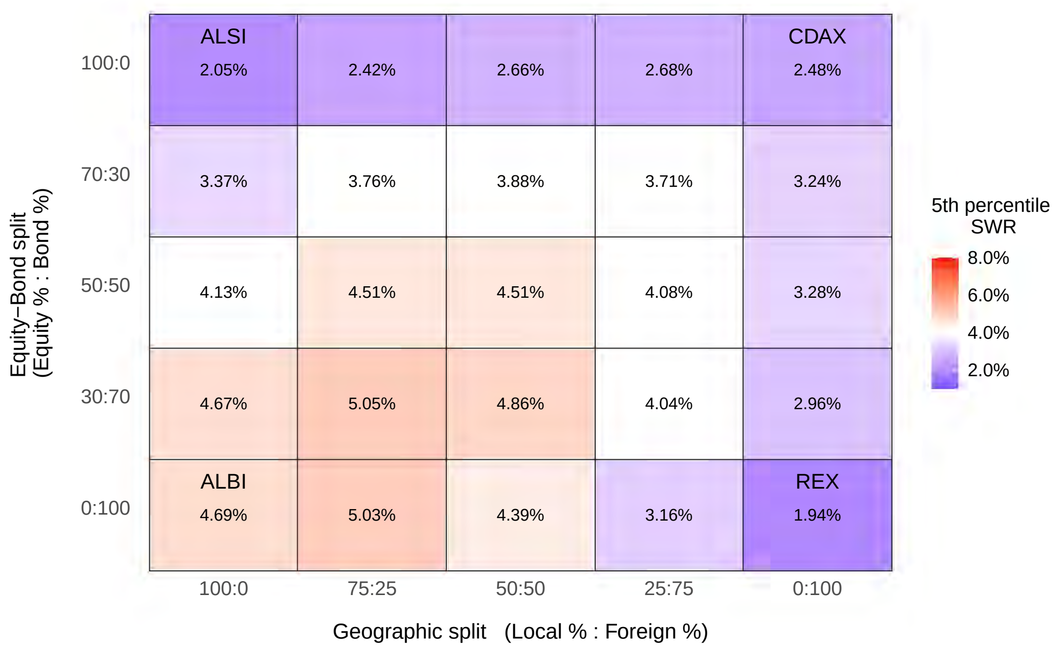

FIGURE H.2: Single developed market diversification
5^{th} percentile SWR heatmap

H.2 Geographic diversification case 2:

The figures below show the results for broad index of developed market nations (including US equity) diversification. The strategy makes use of the following geographic diversification investment assets: MSCI World and BBGA.

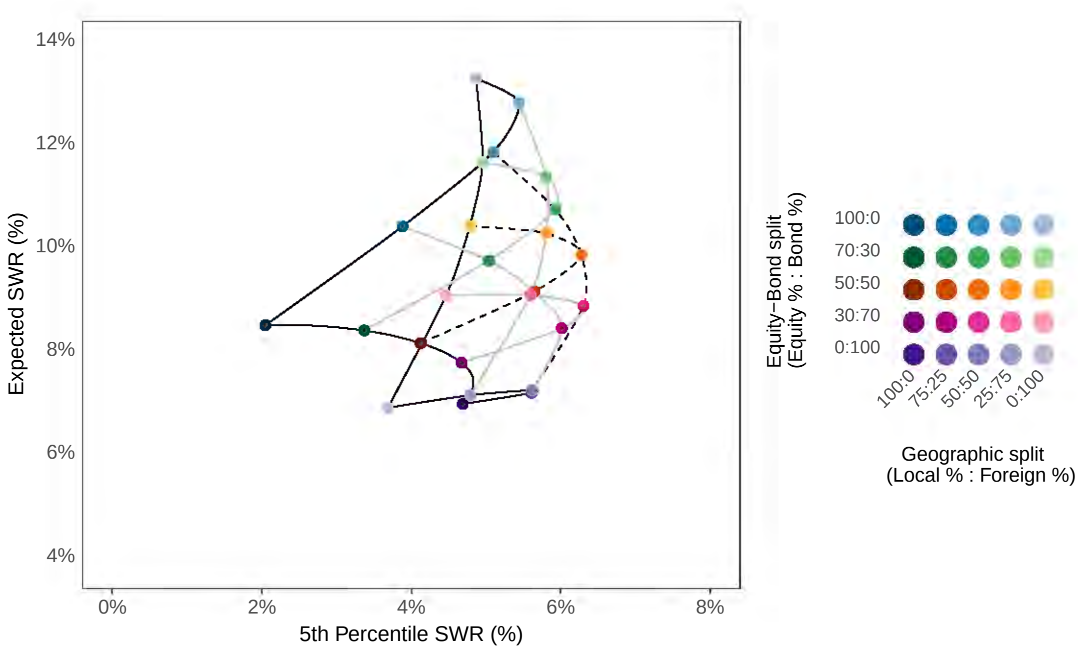

FIGURE H.3: Broad developed market diversification
SWR performance

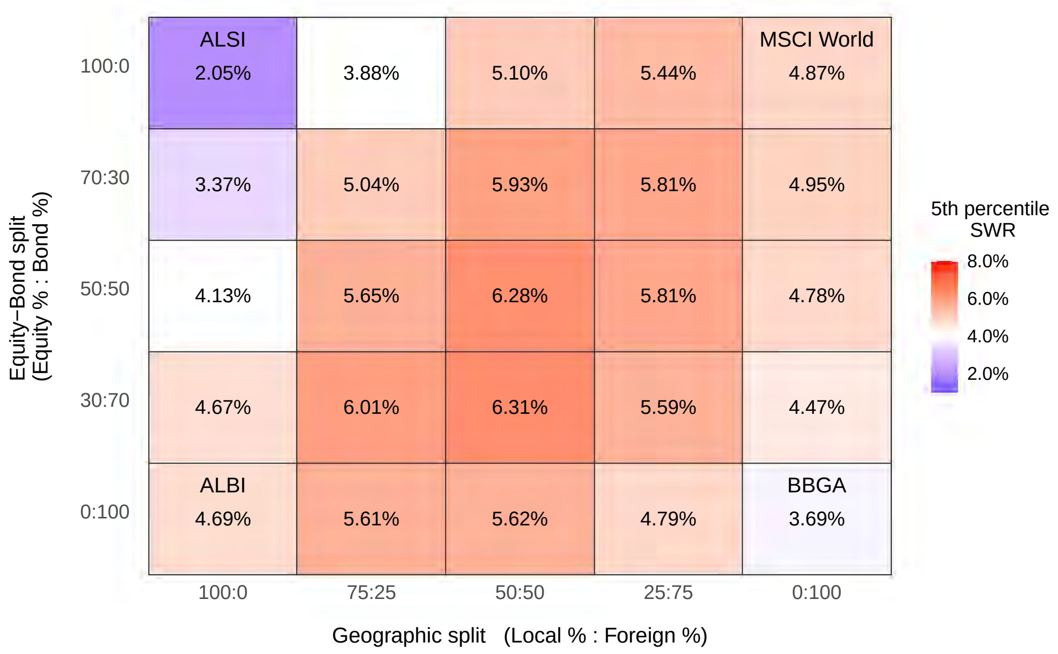

FIGURE H.4: Broad developed market diversification
5th percentile SWR heatmap

H.3 Geographic diversification case 2a:

The figures below show the results for broad index of developed market nations (excluding US equity) diversification. The strategy makes use of the following geographic diversification investment assets: MSCI EAFE and BBGA.

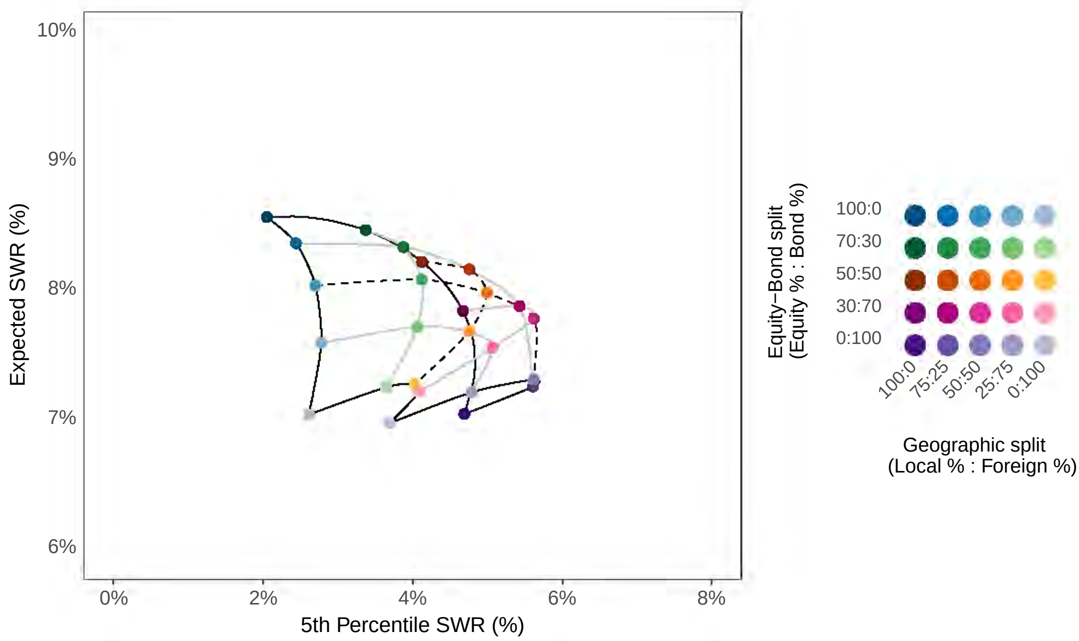

FIGURE H.5: Broad developed market diversification excl. US
SWR performance

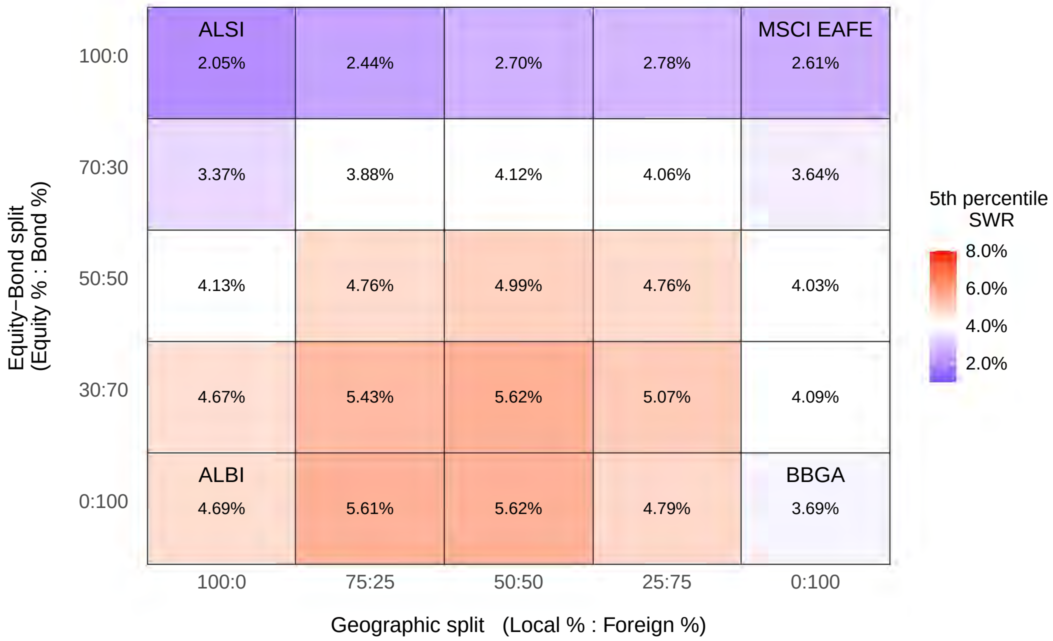

FIGURE H.6: Broad developed market diversification excl. US
5^{th} percentile SWR heatmap

H.4 Geographic diversification case 3:

The figures below show the results for broad index of emerging market nations diversification. The strategy makes use of the following geographic diversification investment assets: MSCI EM and BBEMB.

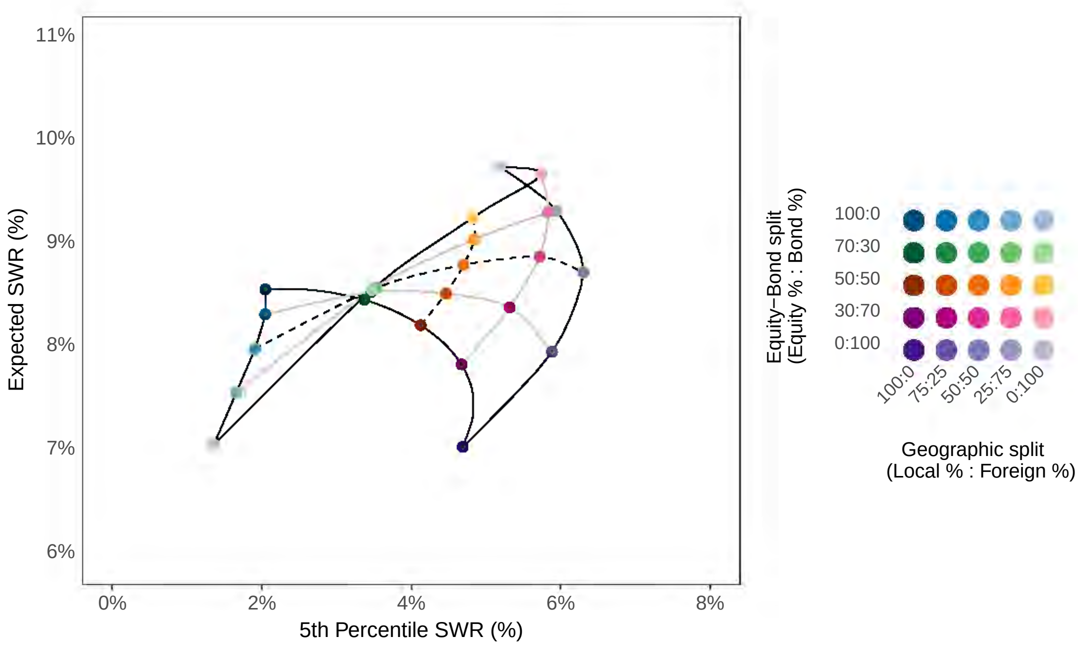

FIGURE H.7: Broad emerging market diversification
SWR performance

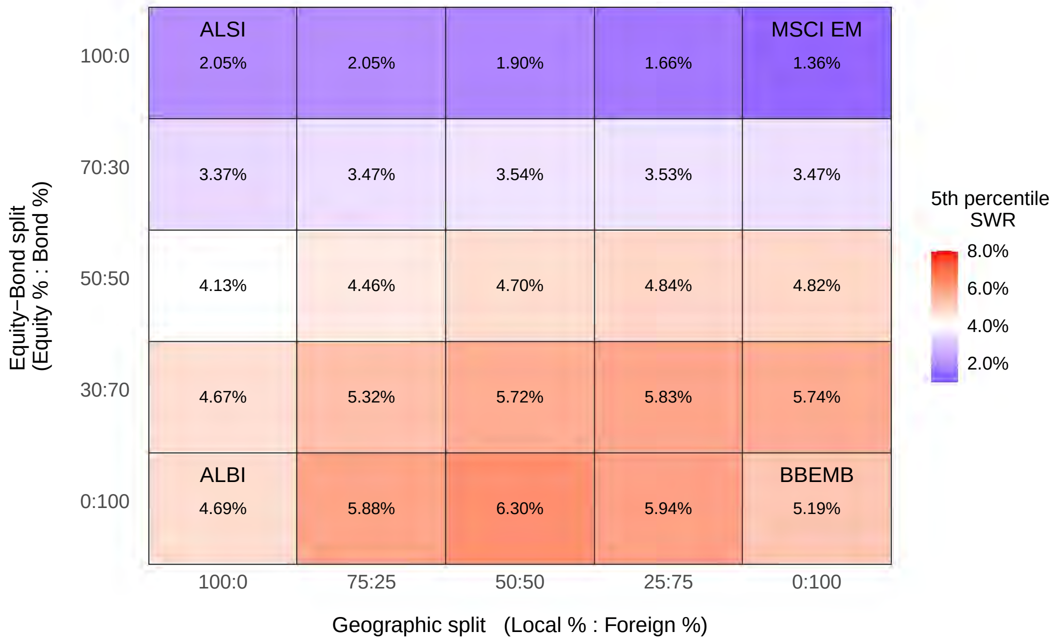

FIGURE H.8: Broad emerging market diversification
5th percentile SWR heatmap

H.5 Geographic diversification case 4:

The figures below show the results for broad index of developed and emerging market nations diversification. The strategy makes use of the following geographic diversification investment assets: MSCI World, MSCI EM and BBGA, BBEMB.

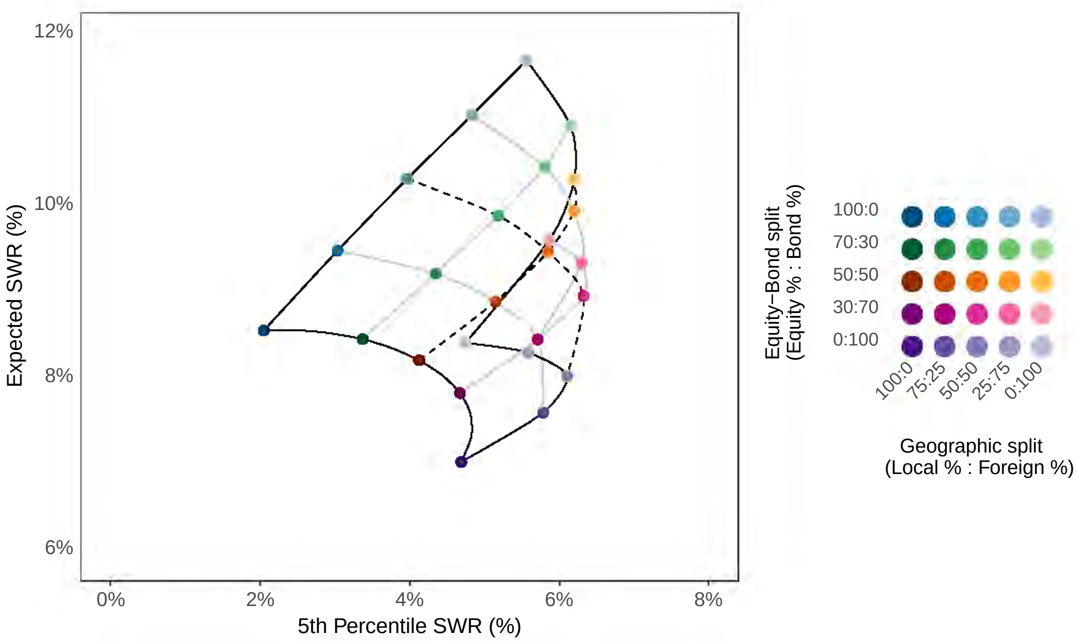

FIGURE H.9: Broad international diversification
SWR performance

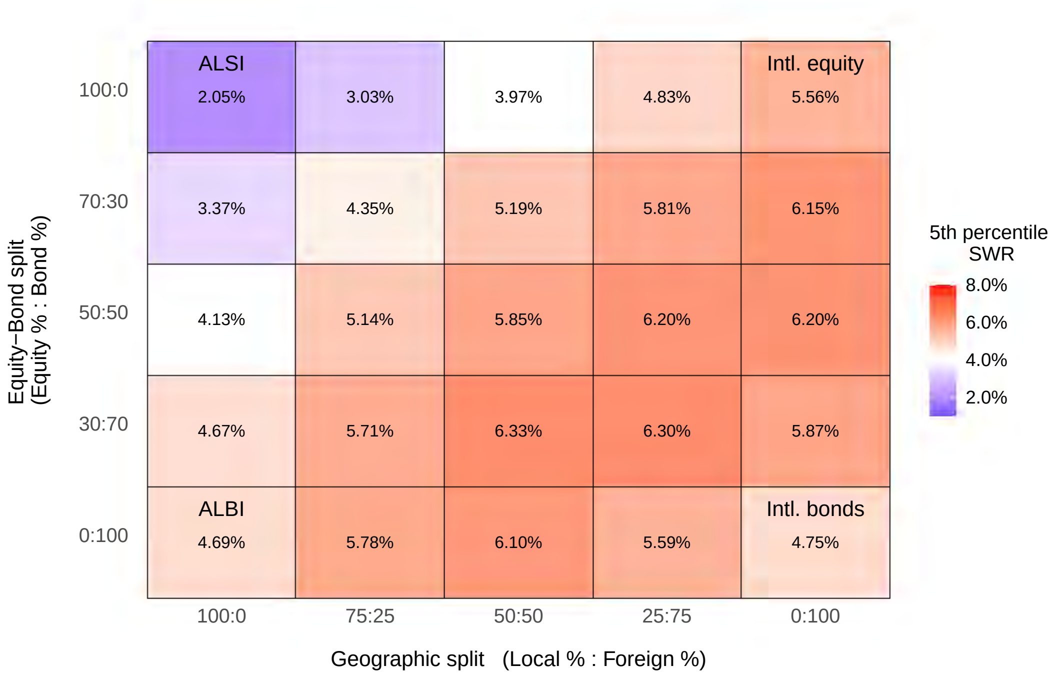

FIGURE H.10: Broad international diversification
5th percentile SWR heatmap