AN INVESTIGATION
OF COLOUR RENDERING PREFERENCES
BY MEANS OF SYNTHETIC SPECTRA.

THESIS

SUBMITTED TO THE

DEPARTMENT OF ELECTRICAL ENGINEERING

OF THE

UNIVERSITY OF CAPE TOWN

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

BY

D. E. H. NAUDE

- 1963 -

S U M M A R Y

The first section describes an experiment in which seven light sources of differing spectral energy distribution, but metameric with a Plankian radiator at a colour temperature of $2850^{\circ}K$, were submitted to a population sample of 1300 in order to establish the preferred colour rendering of warm white lamps. The investigation was limited to the colour rendering of the human face although the appearance of certain foods ~~were~~ was also considered and it was established that a source with a spectrum having a higher ratio of red to yellow energy than a metameric Planck radiator would be widely preferred.

The different spectra [spectral compositions] used in this test were obtained by mixing the light from a number of filtered projectors.

The second section deals with a method for obtaining light of any desired spectral energy distribution, using a single monochromator and variable spectrum mask. The construction of the apparatus is described and the requirements of the optical components are discussed.

The third section is a paper describing the analysis and prediction of stray light in a single monochromator.

ACKNOWLEDGEMENTS.

I wish to express my gratitude to Prof.R.Guelke and Dr. H. D. Einhorn for their valuable guidance and assistance during the course of this work.

My thanks also go to the S.A.Mutual Life Assurance Society for the gratis use of their computor "Perseus" for the calculations in paragraph 5; to Mr. P, Stilborg who wrote the programme; to the young ladies who gave up their time to act as models for the preference tests, and finally to my wife, without whose encouragement, assistance and tolerance this work would not have been possible.

C O N T E N T S.

SECTION II

SECTION III

1. INTRODUCTION

In the past, when artificial light was almost invariably derived from incandescence of solids or gases, practicability, efficiency and colour temperature were adequate criteria for judging the merit of a particular light source. However, with the advent of fluorescent lighting it has become increasingly important to consider the colour rendering properties of light sources. This is especially so in the design of lighting for use in homes, for merchandizing and in industrial applications where colour discrimination is important. In such applications the differences between the spectral energy distributions (S.E.D's) of fluorescent lamps and that of a black body at the same colour temperature could have marked and sometimes undesirable effects on the colour of illuminated objects.

Many methods of specifying colour rendering have been proposed (eg 1 - 8) and some of these are at present under consideration by various national and international bodies in an attempt to find a generally acceptable method.

An essential requirement for any method of colour rendering specification is a reference source or "ideal" source. To date black body radiators (or empirical S.E.D's based on daylight measurements) have for practical reasons been accepted as sources giving the optimum colour rendering.

A separate problem is the question whether the black body radiator is always an "ideal" source, or whether for certain applications a source of different S.E.D. might not be preferable. That discrepancies exist between "correct" or "natural" colours of objects and the preferred colours has been shown by various investigators. MacAdam (9) found these discrepancies when submitting a number of colour photographs to judges for evaluation of quality of skin tones. Sanders (10) investigated colour preferences for human skin and certain food-stuffs and also found that the preferred chromaticity did not coincide with the chromaticity when illuminated by a black body radiator. Buck & Froelich (11) found that for the illumination of faces, the largest proportion of people chose a soft white (pinkish) fluorescent lamp in preference to other fluorescent lamp colours and a tungsten lamp. From the S.E.D. of their preferred lamp it can be seen that there is relatively more energy in the orange-red region than in the yellow, rather like the de luxe lamps of to-day. On the other hand, a high efficiency fluorescent lamp has its peak energy in the yellow region and is known to render the skin colour poorly. unacceptably

This investigation deals primarily with colour preferences rather than colour rendering as defined in recent C.I.E. proposals; the ideal light source shifts then from the accustomed to the desired one.

For investigating this a light source, the S.E.D. of which is variable under controlled conditions, is required. This can be accomplished in two ways:

(i) Dispersing light in a monochromator, masking the spectrum suitably, recombining the light and projecting it onto the test object.

(ii) By illuminating the test object with mixtures of coloured or otherwise suitably filtered light sources.

A third method is to manufacture fluorescent lamps containing different mixtures of phosphors, but this is out of the question in this country.

The first method was used by Crawford (7) and has the advantage of placing at one's disposal an infinite variety of spectra which can be changed smoothly from one to the other. On the debit side, however, it is difficult to achieve high levels of illumination when using the apparatus to illuminate an object and it is limited to laboratory use as the apparatus is not transportable.

The second method was used by Sanders (10) and has the advantage of being robust and hence being suitable for field tests. It is also possible to obtain high illumination levels. The main disadvantage is lack of versatility due to the limitations of available filters. This is especially the case when it is desired to change the S.E.D. of the test source while maintaining the chromaticity constant.

Both methods were, however, tried during this investigation.. Section I describes a colour rendering preference test using method 2. Public preference was to be tested for a number of light sources, the main variable being the energy balance between the red and yellow regions of the spectrum. In effect, the sources would have a range of S.E.D's similar to high efficiency fluorescent (energy peak in the yellow region), de luxe fluorescent, black body radiation (smooth S.E.D), energy deficiency in the yellow region etc., until no energy remained in the yellow, and initially the investigation would be confined to the colour rendering of the human face.

In previous tests of this nature the chromaticity of the source was changed by changes in S.E.D, giving rise to adaptation problems, unless special precautions were taken

(Cf Sanders). In this investigation, however, all the sources would be metameric with one another and the reference source, thus eliminating adaptation effects. Illuminant A was chosen as reference source, as incandescent lighting (or warm white fluorescent) is more generally used for social lighting.

In September 1961 the University of Cape Town held an exhibition in Cape Town and this opportunity to obtain a fair cross-section of the public as observers for a mass colour preference test was seized.

During the week for which accommodation was available, 1,300 members of the public of Cape Town gave their opinions on the appearance of 41 volunteer models illuminated by seven different sources.

Section II describes the construction of a variable spectrum mask for an existing single monochromator and the technique used for recombination of the light. As well as taking the studies described in Section I further in the laboratory, it was hoped to use this apparatus for work on colour tolerances and colour discrimination.

Section III is an analysis of stray light in a monochromator, necessitated by the fact that a single monochromator had to be used and stray light can cause serious errors in certain cases.

SECTION I

LARGE NUMBERS OF RAPID OBSERVATIONS
OF THE HUMAN FACE
LIT BY PROJECTORS WITH COLOUR FILTERS.

2. EXPERIMENT

2.1 Arrangement

The general arrangement of the experiment is shown in fig 1. The source consisted of five 35 mm slide projectors, fitted with suitable filters and placed side by side. They were enclosed by a box and stood on a table about three feet high. The beams from the projectors, approximately two feet square, were superimposed on the test field four feet away. The field was thus illuminated to a level of five footcandles by a light mixture, the five components of which could be varied separately to give the seven mixtures used for this test.

The model sat against a grey back-cloth (Munsell N4/) her head and shoulders illuminated by the test field, while the observers stood behind the light source with the operator on the left.

In the top left-hand corner of the test field (fig. 2) was placed an irregularly shaped piece of white cartridge paper to demonstrate that the various light sources used had essentially the same chromaticity.

FIG. 1 General Arrangement of Experiment

FIG. 2

Approximate field of view of the observer

A : White disc illuminated by $2850^{o}K$ projector.
M : White patch is illuminated by test source and looks similar to A.
G : Dark grey background.

Above and to the left of the test field was a roughly circular piece of white paper, illuminated to a level of five footcandles by a separate spotlight, the colour temperature of which was set to about 2850°K and maintained constant throughout the test. This served the double purpose of a comparison source when setting up or adjusting the apparatus and a bright white surface in the visual field of the observer, in an attempt to maintain the state of adaptation to Illuminant A.

The edges of the test field were made to fall on a strip of black material in order to suppress the effect of coloured fringes. These were due to the imperfect register of the five projector beams. Trapezoidal distortion of the images from at least four of the projectors due to angular displacement is unavoidable unless the distance from source to test object is made excessive. Alternatively masking of the beams could be effected at the projectors but this was considered to be impractical for a field experiment of this nature.

It is realized that full-face lighting with no softening of shadows by inter reflections is not ideal but was resorted to in order to minimize the effect of coloured shadows. These arose from the lateral displacement of the individual projectors, and although the spacing was kept to a minimum, the coloured shadows under the model's nose and chin were found to be most disturbing when the projectors were placed above the observers line of sight, a position which would have improved the modelling and given a more natural effect. Coloured shadows cast by the model's head were minimized by using the dark backcloth.

Prior mixing of the colours was considered but as this would have entailed losses, either by reflection or by diffusion, the idea was discarded. The maximum illumination level attainable was already severely limited by the output of the projectors and the transmission factors of the filters.

2.2 Procedure

The assignment of numerical values to the mental responses evoked by a number of stimuli is a psychometric problem and can be solved in a number of ways.

A scale based on absolute subjective evaluations is difficult to establish and inherently inaccurate when applied to multi-dimensional judgements such as these. This is more especially the case when dealing with a large number of untrained observers who might have difficulty in specifying the degree of acceptability or otherwise of the situation.

Thurstone (12) described the method of paired comparisons as a means of obtaining both the subjective order and the spacing on a scale of numerical response values. This method requires that every combination of two stimuli from the set to be evaluated be submitted for judgement by each observer, who is required to indicate his opinion as to which of the two stimuli evokes the greater response. For n stimuli this represents $\frac{n(n-1)}{2}$ judgements from each observer, i.e., 21 comparisons for the seven light sources in this case. However, to ask casual visitors to the exhibition to sacrifice the time necessary to make 21 comparisons seemed somewhat unreasonable, so only three light sources at a time were used.

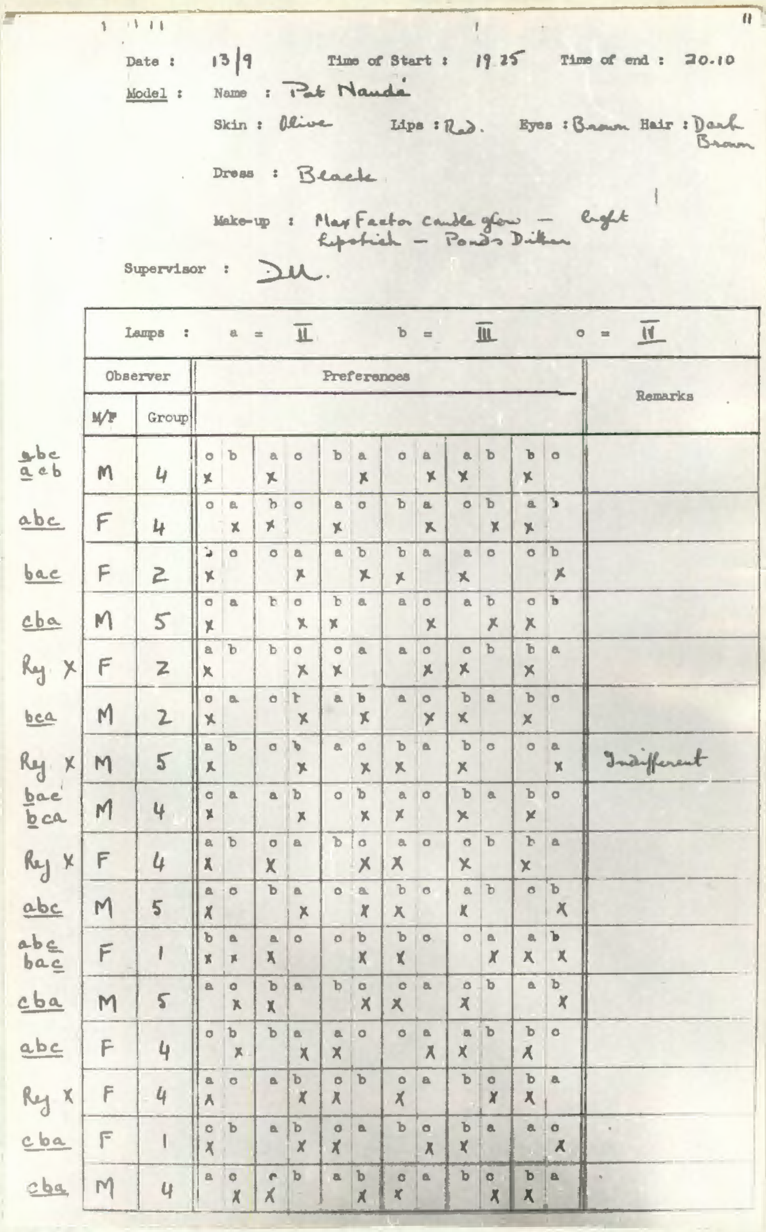
This required only three judgements per observer so one could afford to repeat the test with the order of presentation of the light sources reversed and thus obtain a check on the consistency of each observer.

Observers were given a brief explanation as to the aim of the experiment and then told that the model would be illuminated by a number of lights and that they should merely tell the operator which one of each pair they thought made the model look the most attractive. It was pointed out that although the colour of the light would not change, the skin tones, hair, lips etc. would be rendered differently.

The operator then presented the six permutations of pairs possible with three sources and entered the results on record sheets, also recording the sex and an estimate of the age of the observer. Any queries about the number or composition of the light sources etc., from interested observers were answered after completion of their test.

The order of presentation of the pairs of light sources was randomized beforehand and printed on the record sheets. (fig 3). This order was strictly adhered to by the operators.

Each light source was switched on for two to three seconds, then switched off before the next was selected, so that there was no direct transition from one light source to



Date : 13/9 Time of Start : 19.25 Time of end : 20.10

Model : Name : Pat Nandé

Skin : Olive Lips : Red. Eyes : Brown Hair : Dark Brown

Dress : Black

Make-up : Max Factor candle glow — light lipstick — Ponds Dikon

Supervisor : DM.

Lamps : a = II b = III c = IV

	Observer		Preferences												Remarks
	M/F	Group													
abc / acb	M	4	c X	b	a X	c	b	a X	c	a X	a X	b	b X	c	
abc	F	4	c	a X	b X	c	a X	c	b	a X	c	b X	a X	b	
bac	F	2	b X	c	c	a X	a	b X	b X	a	a X	c	c	b X	
cba	M	5	c X	a	b	c X	b X	a	a	c X	a	b X	c X	b	
Rej X	F	2	a X	b	b	c X	c X	a	a	c X	c X	b	b X	a	
bca	M	2	c X	a	c	b X	a	b X	a	c X	b X	a	b X	c	
Rej X	M	5	a X	b	c	b X	a	c X	b X	a	b X	c	c	a X	Indifferent
bac / bca	M	4	c X	a	a	b X	c	b X	a X	c	b X	a	b X	c	
Rej X	F	4	a X	b	c X	a	b	c X	a X	c	c X	b	b X	a	
abc	M	5	a X	c	b	a X	c	a X	b X	c	a X	b	c	b X	
abc / bac	F	1	b X	a X	a X	c	c	b X	b X	c	c	a X	a X	b X	
cba	M	5	a	c X	b X	a	b	c X	c X	a	c X	b	a	b X	
abc	F	4	c	b X	b	a X	a X	c	c	a X	a X	b	b X	c	
Rej X	F	4	a X	c	a	b X	c X	b	c X	a	b	c X	b X	a	
cba	F	1	c X	b	a	b X	c X	a	b	c X	b X	a	a	c X	
cba	M	4	a	c X	c X	b	a	b X	c X	a	b	c X	b X	a	

FIG. 3

Sample of the duplicated sheets on which results were recorded. Preferences have been worked out and entered at the extreme left.

the other. The greater majority of observers found this period adequate to make up their minds, but if they appeared to be in doubt, the same pair was presented again, in the same order as before.

The three sources used were changed at regular intervals in an attempt to keep the number of observations with particular combinations similar for models of different colouring. This was not always possible as the numbers of observers fluctuated unpredictably and except for three main models, no information about the colouring of the models was available in advance.

Modelling periods were limited to half an hour, in order to avoid undue fatigue due to looking at an uncomfortably bright source being switched on and off, but the models were unanimous in stating that there was no strain attached to it, even after more than an hour, as happened on occasion.

2.3 Light Sources

The light source consisted of five 220 volt, 100 watt 35 mm slide projectors (Romanslide Z) fitted with the following filters:

Blue	:	Strand Gelatin No. 20
Green	:	Strand Gelatin No.39
Yellow	:	Ilford Bright Spectrum Yellow No. 626
Red	:	Chance Glass OR 2 (3 mm thick)

The fifth projector was used unfiltered except for the heat absorbing filter which was left in position. The transmission curves of these filters are shown in Fig.4.

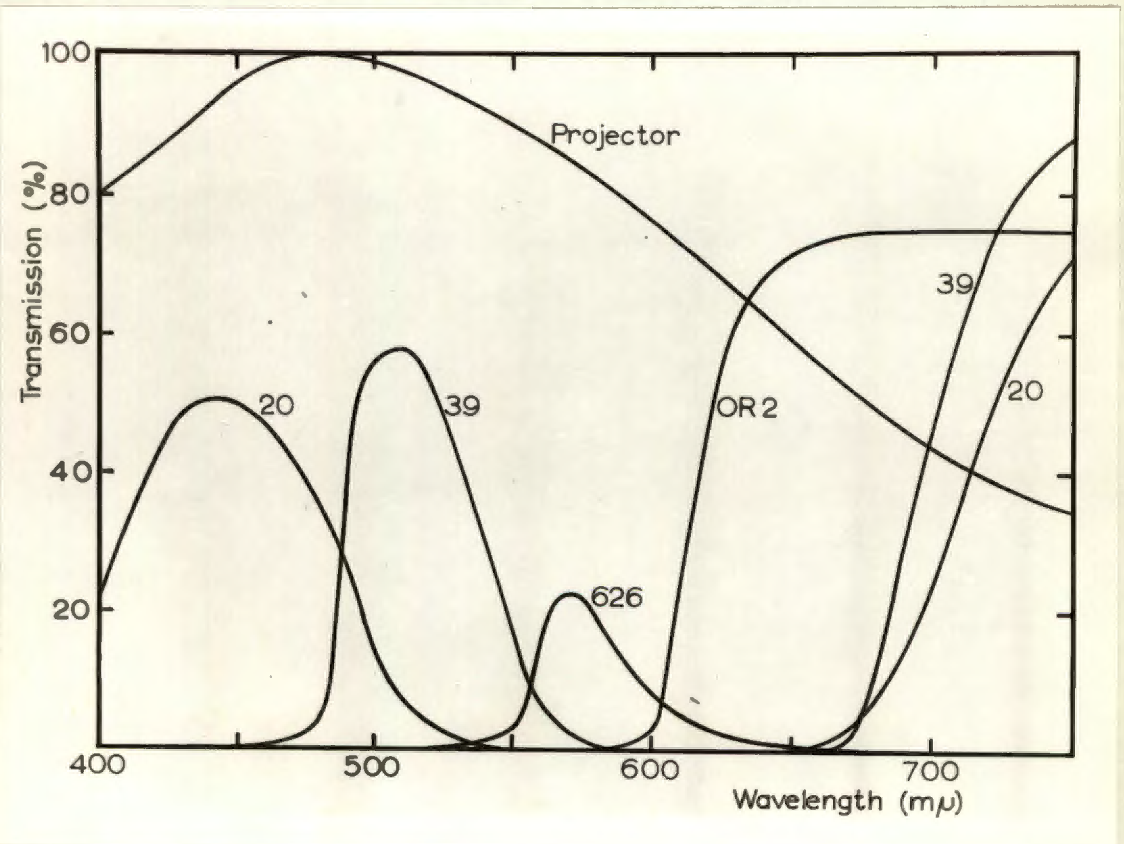


FIG. 4 Transmission of the filters & projectors used.

Also shown is a typical transmission curve for one of the projectors. As this includes the optics of the projector the curve is for relative transmission with the maximum being assigned an arbitrary value of 100%.

Power was supplied from a 500 VA constant voltage transformer and control was effected by means of a three-position rotary switch. This connected the projectors to adjustable taps on five series resistances, and by reconnecting the wires on a terminal strip, any three combinations of projector intensities could be selected to give the three light sources for a particular test. (Fig.5).

The maximum illumination obtainable was limited by the red and green projectors at the one extreme and by the yellow at the other. This was just under 6 footcandles but the figure of 5 f.c. was adopted for convenience and to allow a factor of safety by always under-running the projector lamps slightly. It was not possible to increase the illumination level by placing the projectors nearer to the model as the test field became rather streaky at a distance of less than 4 ft. Also, placing the projectors too near the model increases the angular displacement of the component colours and hence increases the effect of coloured shadows.

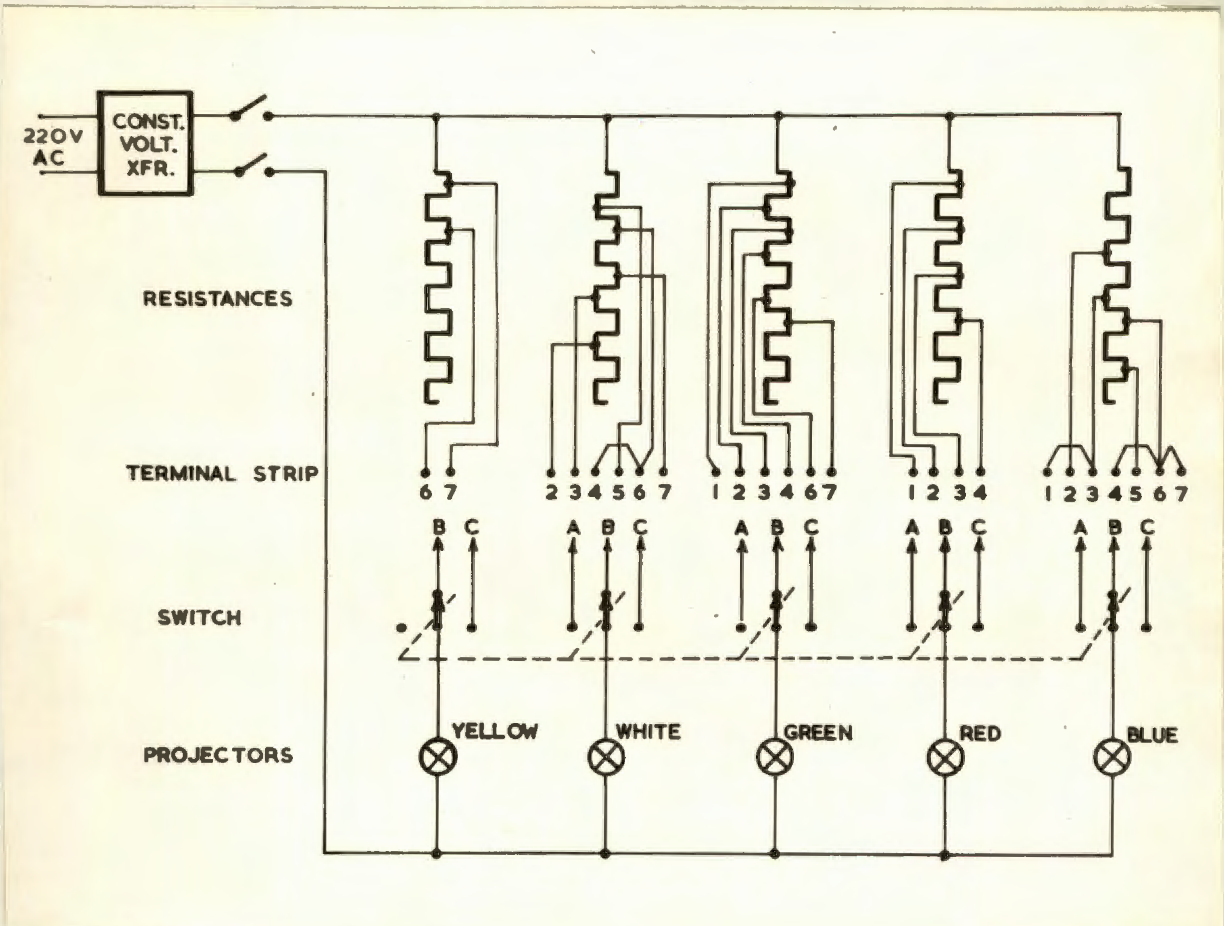


FIG. 5. Circuit Diagram

The transmission curves shown in Fig.4. were re-measured after the experiment and in addition, daily check readings were taken during the tests by means of a photovoltaic cell and narrow transmission band filters, in order to replace any filters which showed signs of fading or discoloration. No such effect was noticed.

The combined spectral energy distribution of the projectors constituting each of the seven light sources was measured spectrophotometrically (see App. I). These curves are tabulated in Table I and are shown in fig.6. with the S.E.D. for Illuminant A included for comparison. These curves have been normalized for equal Y as the illumination from the experimental sources varied by ± 3%.

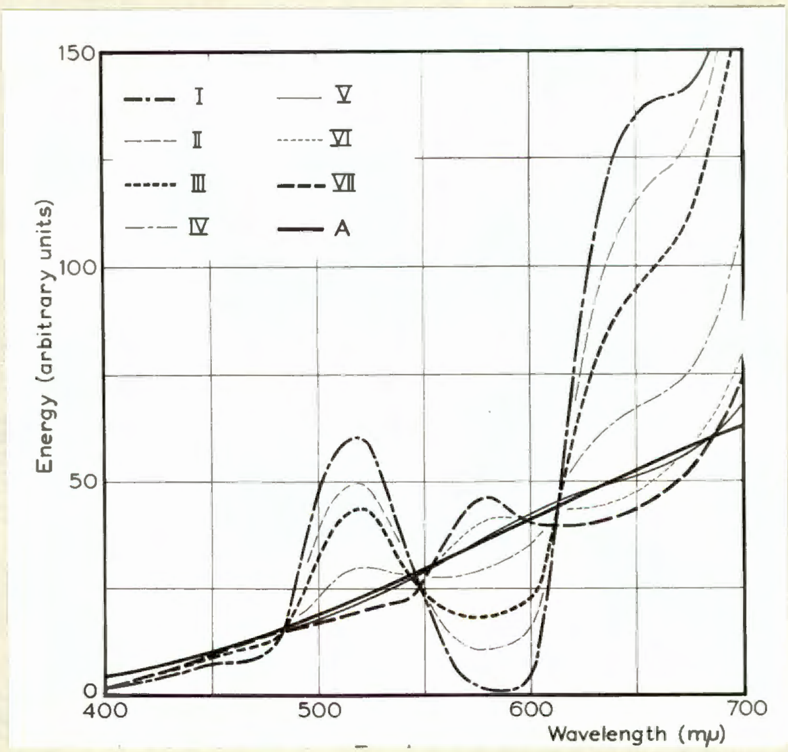


FIG. 6.
Relative Spectral Energy Distribution of the Light sources used. S.E.D. for C.I.E Illuminant A included for comparison.

TABLE I.

Spectral Energy Distribution
of the seven light sources & Illuminant A
Normalized for Y = 100.0

λ	E_λ I	E_λ II	E_λ III	E_λ IV	E_λ V	E_λ VI	E_λ VII	E_λ A
400	1.37	1.82	1.73	1.99	2.10	2.10	2.07	4.66
410	2.43	3.05	2.89	3.02	3.38	3.54	2.25	5.61
420	3.38	4.46	4.23	4.16	4.55	4.88	4.49	6.66
430	4.43	5.73	5.43	5.41	5.73	6.23	5.90	7.84
440	5.78	7.45	6.98	6.91	8.33	8.11	7.51	9.11
450	6.99	9.11	8.78	8.56	8.79	10.1	9.33	10.5
460	7.80	10.3	10.1	10.2	10.8	12.0	11.1	12.0
470	8.78	11.7	11.3	11.7	12.6	13.7	12.8	13.3
480	11.9	14.4	13.5	15.1	14.7	15.9	14.6	15.3
490	26.9	25.9	20.2	18.9	16.1	17.1	15.7	17.1
500	45.9	39.8	31.4	22.2	17.9	18.6	17.3	19.0
510	57.4	46.7	40.2	27.8	19.7	20.1	18.3	21.0
520	60.5	49.0	43.9	29.5	21.9	21.5	19.8	23.0
530	51.8	43.5	40.0	29.3	24.2	23.0	21.2	25.1
540	34.9	32.0	30.4	27.4	26.2	24.0	22.1	27.3
550	23.3	24.9	25.4	27.7	29.3	28.1	27.0	29.3
560	10.2	16.1	20.4	27.6	32.0	33.5	35.6	31.8
570	3.56	12.0	18.4	28.6	34.4	38.1	43.1	34.0
580	1.33	11.5	18.8	30.1	37.3	41.3	45.9	36.2
590	1.08	12.3	20.1	32.6	40.3	41.1	43.1	38.7
600	4.90	15.6	22.7	34.7	42.1	41.5	41.4	40.9
610	31.6	34.0	36.8	43.2	44.5	42.1	39.8	43.2
620	72.3	64.0	59.1	53.6	47.4	43.7	40.5	45.7
630	106	89.7	76.0	59.1	47.6	43.9	40.4	47.9
640	126	105	87.2	64.6	49.0	45.0	41.5	50.1
650	133	114	94.2	66.6	52.0	47.3	43.7	52.3
660	139	120	100	69.7	54.6	50.9	46.6	54.6
670	139	123	105	72.5	55.6	54.3	49.9	56.9
680	159	142	118	79.7	58.8	60.4	55.5	58.7
690	193	171	137	92.0	62.5	68.5	63.6	60.9
700	243	214	166	109	67.5	80.2	74.7	62.8

From the measured S.E.D's the chromaticity of each source was calculated and the results are shown in fig. 7.

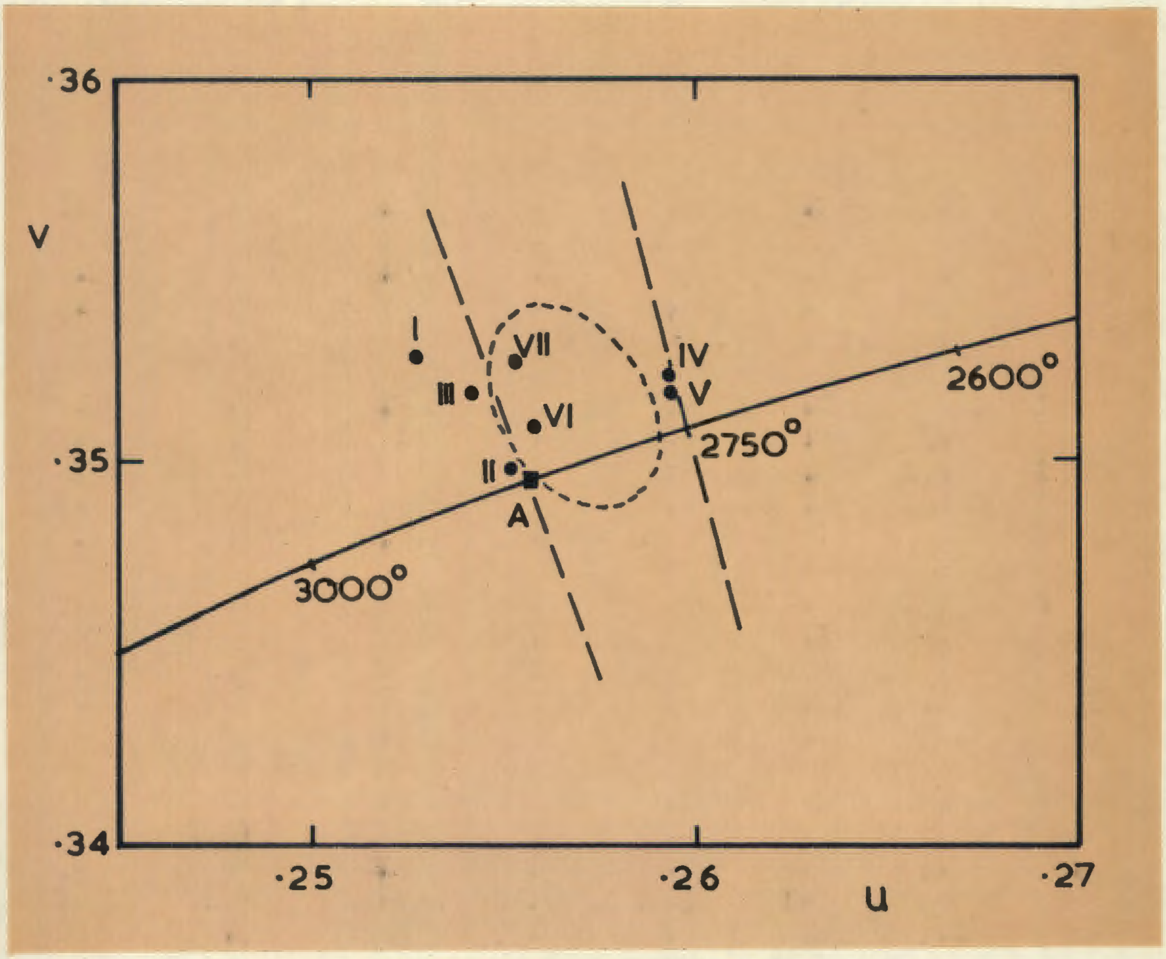


FIG. 7.

Chromaticities of the Seven Test Sources and Illuminant A (C.I.E. 1960 UCS co-ordinates)

Dashed lines - lines of correlated colour temperature

Dashed ellipse - 3x standard deviation of colour matching (MacAdam)

Although the sources are not quite metameric with Illuminant A, the deviations are random and do not correlate with the shifts in skin colour found later. Also shown are the lines of correlated colour temperature (13) for Illuminant A and 2750°K. The dashed ellipse is approximately the area covered by three times the standard deviation of colour matching as found by MacAdam (14,15) and represents one "just noticable difference" for the point u = .257, V = .351. This ellipse very nearly contains the light sources and when it is borne in mind that the JND is defined for adjacent fields in a colorimeter whereas the light sources were seen successively, i.e. a memory match had to be made, these deviations from Illuminant A are not significant.

3. RESULTS

3.1 Method of analysis

Given three stimuli a, b, c, it is possible to make three paired comparisons and hence establish a preference triad (16). In this test, as a check on the consistency of each observer, the same three pairs were again presented in reversed order, thus giving rise to another triad.

As there are six permutations of the three stimuli, six preference triads are possible, as well as two circular triads, giving eight in all. Thus there are 64 combinations of triads possible per observer. It can be shown that of these 64 pairs, eight are consistent with one another, 24 contain one reversal of preference, 24 contain two reversals and the last eight contain three reversals of preference (fig 8).

	a>b>c	a>c>b	b>a>c	b>c>a	c>a>b	c>b>a	↺	↻
a>b>c	0	1	1	2	2	3	1	2
a>c>b	1	0	2	3	1	2	2	1
b>a>c	1	2	0	1	3	2	2	1
b>c>a	2	3	1	0	2	1	1	2
c>a>b	2	1	3	2	0	1	1	2
c>b>a	3	2	2	1	1	0	2	1
↺	1	2	2	1	1	2	0	3
↻	2	1	1	2	2	1	3	0

FIG 8.
The 64 choices possible per observer. The letters a,b, & c represent the three stimuli in descending order of preference while the numbers give the number of reversals of preference. The heavy squares contain choices considered acceptable.

In determining the preferred light source it was decided to reject all observations containing more than one reversal and all those giving rise to one or two circular triads. Thus only six of the eight consistent pairs (the remaining two being consistently circular) and 12 of the 24 pairs containing one reversal were considered acceptable.

One reversal was permitted in order to allow for observers whose preference lay midway between two stimuli or who, while preferring the middle stimulus, could not decide consistently between the outer two. Also considered acceptable were observations indicating a minimum or levelling off of the individual's preference curve, as shown by a consistent preference for one extreme stimulus and confusion between the middle and the other extreme.

There is possibly some justification for excluding observers expressing a definite dislike for the middle light source on the grounds that it is unlikely, in a continuous range of stimuli such as this, that anyone would genuinely prefer both extremes to the middle, and these results probably arose by chance. However, as only 17 observers (1.7%) fall into this category, they cannot influence the results appreciably and are included.

The results thus obtained are given in Table II.

TABLE II : Test Results

Roman numerals refer to the seven light sources, while a, b & c represent the three sources used for a particular test triad.

Preference Triads	Test Triads (a)					Total (a)	Test triads (b)			Total (b)	Total (a+b)
	I II III	II III IV	III IV V	IV V VI	V VI VII		I III V	II IV VI	III V VII		
Consistent Observers											
a b c	9	37	101	55	7		3	8	18		
a c b		1	3	3				1	4		
b a c		13	18	6	1		7	2	4		
b c a	2	21	22	4			1	22	11		
c a b	1	2	1								
c b a	16	60	36	9			7	8	3		
Totals	28	134	181	77	8	428	18	41	40	99	527
Observers with one reversal of preference											
a b c , a c b	1	9	24	23	3		2	2	8		
a c b , c a b		3	1	1	1				1		
c a b , c b a	7	13	6	3	1		4	1	1		
c b a , b c a	6	39	33	10	1		4	10			
b c a , b a c		22	16	7			3	13	3		
b a c , a b c	9	23	43	21			7	13	9		
Totals	23	109	123	65	6	326	20	39	22	81	407
Total Cons. +1 Rev.	51	243	304	142	14	754	38	80	62	180	934
Rejects	29	110	95	63	11	308	12	14	29	55	363
Total Observers	80	353	399	205	25	1062	50	94	91	235	1297
% Rejects	36	31	24	31	44	29	24	15	32	23	28
% Consistent	35	38	45	38	32	40	36	44	44	42	41

It will be seen that, although they may not have agreed with one another, about 40% of all the observers were definite in their choice of light source and gave consistent observations. On the other hand, 28% of the observations were rejected due to inconsistent choices.

It cannot be said that 28% of the population is undiscriminating, but merely that the colour shift of the skin between adjacent sources is insufficient for 28% of the population to give firm decisions about their preference.

On the other hand, when alternate sources were used the reject percentage only dropped to 23%. Once can thus infer that, within the region tested, about 20% of the population hold no strong views about which light source makes the human face look most attractive.

3.2 Significance

That the results are significant can be shown by the application of the χ^2 test (16). The hypothesis to be tested is that all the results were obtained by chance. If this were the case the expected number of observers in the categories "consistent", "one Reversal", "reject" would depend on the number of choices possible within each group.

These have already been given as 6, 12 and 46 respectively. On this basis a value of $\chi^2 = 5000$ is obtained. For two degrees of freedom a probability of 0.1% corresponds to a value of 13.82 for χ^2 so that the probability of the hypothesis being correct is infinitesimally small and the results can be considered significant.

3.3 Preference Curve

Ordinate Scale.

The simplest method of obtaining the preference for each of the light sources is to total the number of observers choosing a particular source as being preferable to the other two. In the case of an inconsistency between the first and second choices, half is assigned to each.

If this is done for each test set of three light sources and the numbers expressed as percentages, the results in Table III are obtained.

The relative preference is obtained from the middle value in each test as this is the only figure representing the number of observers choosing a particular light source. The number on either side of it represents a preference for the source under which it is listed as well as any of the sources beyond it.

TABLE III

Percent First Choices.

a) Consistent observations

Light source	I	II	III	IV	V	VI	VII	
	32	7 28	61 25 57	47 22 75	21 13 88	12 12	0	
Relative Preference	32	7	25	22	13	12	0	TOTAL-111
Percentage Preference	29	6	22	20	12	11	0	

b) Consistent observations plus those containing one reversal

	28	19 25	53 36 49	39 31 65	20 23 75	12 11	14	
Relative Preference	28	19	36	31	23	11	14	TOTAL-162
Percentage Preference	17	12	22	19	14	7	9	

A number of points arise from these results. Most obvious is the fact that the totals of the relative preferences are not 100 in either case. Two possible reasons for this are that the observers for each test were different and one cannot expect different random samples of the population to behave in exactly the same way. These fluctuations will naturally be greater for the small groups of the I, II, III and V, VI, VII tests than for the other three groups. The other factor influencing the total is the fact that there appeared to be a tendency to favour the middle stimulus in spite of the careful randomization of order of presentation. This would also increase the total.

Another anomaly is the sharp rise in preference for source I. This is also noticeable for source VII among the observations with one reversal. It is possible that the range of colours chosen did not extend far enough into the red and yellow regions, (see section 4.2), but a likelier explanation is that the results of the small number of observers in the I, II, III and V, VI, VII tests were upset by a few people who preferred the rather startling extreme red and greenish yellow skin colour.

This view is supported by the figures in Table III which indicate that a greater proportion of observers chose No.I, in the I, II, III test (Column 1, row 1) than chose No.II (which includes No.I) in the II, III IV test (Column 2, row 2) and similarly for Nos. VI and VII. (Table IIIb).

As there was no discontinuity in the colorimetric shift of the skin at sources II and VI there should be no reason for these colours being particularly disliked.

It might be argued that the above method does not utilize all the information available as it ignores the second and third choices. In order to do this the results are shown in the form of observed proportions for each paired comparison in tables IV and V. The number of observers participating in each test is shown in the last column. In this form the data can be obtained directly from the recorded preferences, or from Table II if half is once more assigned to each side in the case of reversals. Considering for the moment only comparisons between adjacent light sources, the following procedure can be adopted in order to establish the relative preferences:

In any a,b comparison the number of observers choosing a in preference to b is the number liking a, or something beyond a.

TABLE IV

Observed Proportions (x 100) for Consistent Observers

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
36 32	64 39	61 68					28
	30 38	70 53	47 62				134
		58 67	42 78	22 33			181
			75 83	25 85	15 17		77
				87 100	13 100	0 0	8
17 56		83 61		39 44			18
	22 27		78 78		22 73		41
		55 65		45 82		18 35	40

TABLE V

Observed Proportions for Consistent Observers plus those with one reversal of preference

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
37 37	63 46	54 63					51
	29 39	71 58	42 61				243
		51 65	49 75	25 35			304
			66 79	34 77	23 21		142
				82 82	18 71	29 18	14
28 54		72 54		46 46			38
	23 41		77 80		20 59		80
		58 73		42 79		21 27	62

TABLE VI

Observed Proportions for all Observers

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
39 42	61 49	51 58					80
	34 41	66 57	43 59				353
		49 63	51 72	28 37			399
			63 71	37 71	29 29		205
				70 79	30 67	33 21	25
34 55		66 68		32 45			50
	28 46		72 80		20 54		94
		56 66		44 76		24 34	91

The number choosing b in preference to c, is the number liking b plus those liking a and beyond. Therefore it follows that the number actually preferring b are those choosing b in preference to c minus those choosing a in preference to b. If these subtractions are performed for each test set, relative preferences given in Table VII are obtained : (the figures obtained from the first method are given below in brackets for comparison).

TABLE VII

Percentage Preferences from Observed Proportions.

Light source	I	II	III	IV	V	VI	VII	
(a) Consistent observations								
Relative preference	36	3	23	20	10	13	0	Total: 105
% preference	34	3	22	19	9.5	12.5	0	
	(29)	(6)	(22)	(20)	(12)	(11)	(0)	
(b) Consistent observations plus those containing one reversal								
Relative preference	37	9	29	24	11	-11	29	Total: 128
% preference	29	7	22	19	8.5	- 8.5	22	
	(17)	(12)	(22)	(19)	(14)	(7)	(9)	
(c) All observations regardless of consistency								
Relative preference	39	10	23	23	8	-3	33	Total: 133
% preference	29	8	17	17	6	-2	25	

If one compares the results of the two methods for consistent observers the agreement is remarkably good. In fact, applying the χ^2 test for goodness of fit gives a probability level of 84%. If one accepts the convention that the 5% level or less indicates a significant difference then these two sets can be considered as being the same curve.

In the case of the result for consistent plus one reversals, there is an obvious anomaly in the negative observers for source VI, but too much importance cannot be attached to this as the proportions are obtained from only 14 observers. As each observer represents about 7% in the proportions it means that two facetious observers, or two completely undiscriminating observers obtaining their result by chance, could give rise to this result. The agreement over the central, more important part of the range i.e. for sources II, III, IV & V is still fair.

The preferences of all observers, regardless of consistency can be given in the same way. (Tables VI & VIIc).

It will be noticed that the maximum has now shifted slightly to midway between sources III and IV but that the main effect of including all observers is to flatten the curve somewhat due to the diluting effect of the inconsistent observers.

The preference curve given in fig. 9. has been drawn using percent preferences obtained by the first method (first choices). As it is felt that the fully consistent observers are more reliable than those with one reversal, the mean of the preferences has been taken, thus giving double weight to consistent observers.

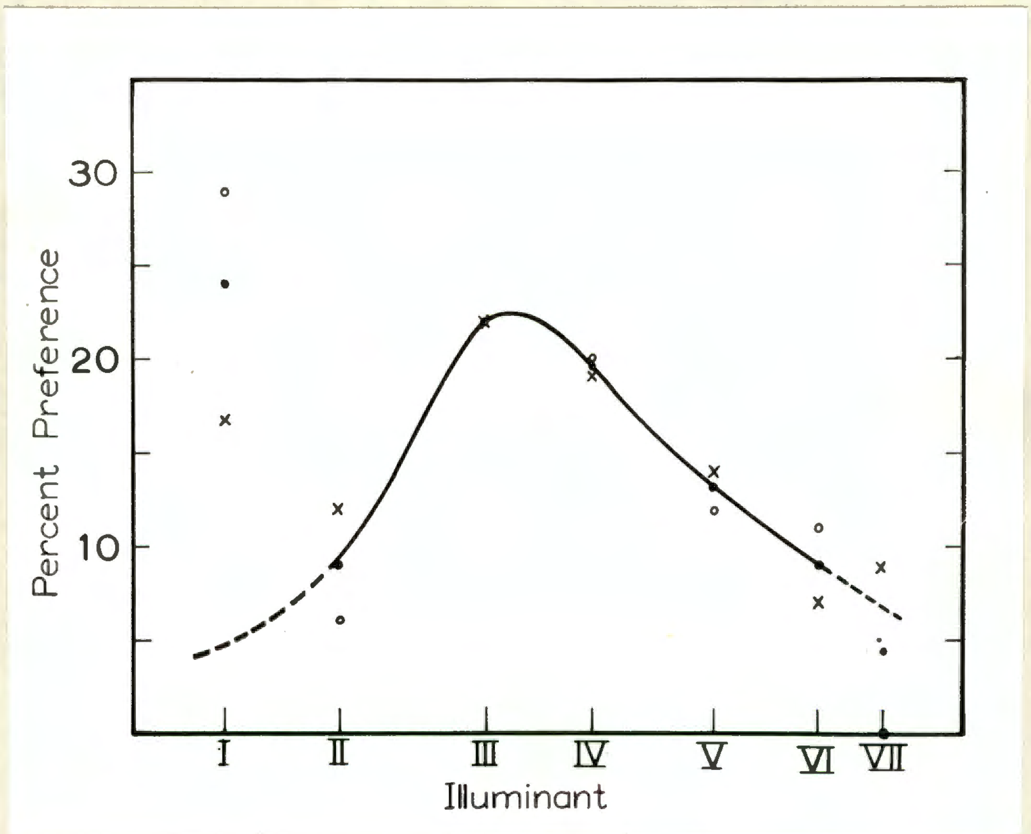


FIG. 9.
Preference Curve from first choices.

Abscissa	: Test sources spaced in accordance with colour shift of face
Open Circles	: Consistent observers only
Crosses	: Consistent observers plus those with one reversal
Full Circles	: Mean of both

3.4 Preference Curve

The Abscissa Scale.

For drawing a preference curve as in fig. 9., a number of different scales are possible to represent the abscissa - spacing of the illuminants. One could arrange the spacing on a purely physical basis, such as the percentage luminance in CIE band 6, (which was just about the region boosted or suppressed) or the ratio of luminance in band 6 to that in the adjacent bands. Alternatively it is possible to use a psychological scale, which, as the light sources are metameric, would have to be linked with the colour shift of some sample under the various sources. As this test was primarily concerned with the appearance of the human face, the skin is a logical choice for the sample. As it was not possible to measure the spectral reflectance of each model's skin, or for that matter to measure the actual colour shifts in situ, the colour shifts were calculated using the spectral reflectance curves of the average skin as given by Edwards & Duntley (17) in terms of C.I.E. (1960) UCS co-ordinates. These shifts agree tolerably well with those calculated from spectral reflectance curves given by Buck & Froelich (11).

Consideration was given to using Munsell 5 YR 8/4 as a standard sample but the agreement with the natural skin was not good enough. These curves are compared in fig. 10.

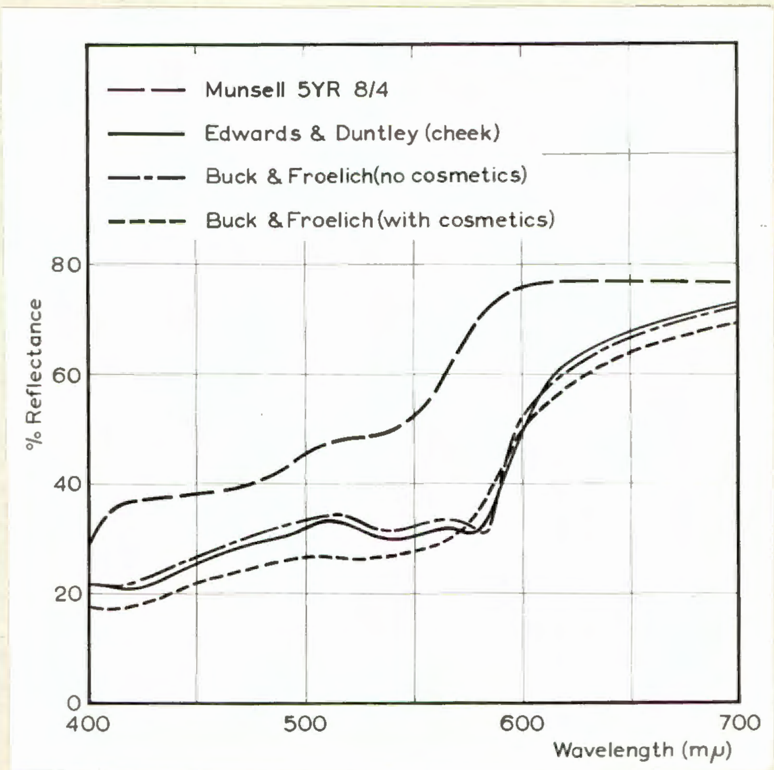


FIG. 10.

Three curves of spectral reflectance for the human skin compared with the spectral reflectance of Munsell 5 YR 8/4

Note that the colour shifts used finally are the uncorrected ones i.e. the source chromaticity has not been corrected to that of Illuminant A. It is assumed that the light sources were switched on for such a short time (2 - 3 seconds) that the observers did not have time to adapt to the slight differences in chromaticity of the sources. These colour shifts of the skin are given below and form the abscissa scale of the preference curve.

I-II	II-III	III-IV	IV-V	V-VI	VI-VII
.0062	.0079	.0057	.0067	.0057	.0034

3.5 Least-squares Method for obtaining the Ordinate Scale.

If it is desired also to utilize the information contained in the double step tests, a different approach must be used. This has been done for the case of the consistent observers plus those with one reversal as a check on the preference curves obtained in section 3.3.

The observed proportions can be written in the form of a matrix as follows :

	I	II	III	IV	V	VI	VII
I	(0.50)	0.63	0.67		0.46		
II	0.37	(0.50)	0.68	0.65		0.59	
III	0.33	0.32	(0.50)	0.46	0.37		0.27
IV		0.35	0.54	(0.50)	0.28	0.21	
V	0.54		0.63	0.72	(0.50)	0.23	0.20
VI		0.41		0.79	0.77	(0.50)	0.29
VII			0.73		0.80	0.71	(0.50)

In this matrix the proportions for each pair which has been compared in more than one test is the mean of all the tests, weighted according to the number of observers for each test.

Each of these observed proportions can be interpreted as the area under the normal frequency function $\left[y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} \right]$ from minus infinity to some finite value of the standard normal deviate. This value in units of standard normal deviate $\frac{x}{\sigma}$ is taken to be the response difference, both in magnitude and sign, between the two stimuli. Consequently a matrix of response differences can be obtained from tables of area under the normal curve vs $\frac{x}{\sigma}$

	I	II	III	IV	V	VI	VII
I	0	0.332	0.441		0.102		
II	-0.332	0	0.467	0.385		0.225	
III	-0.441	-0.467	0	-0.102	0.334		0.610
IV		-0.385	0.102	0	0.584	0.807	
V	0.102		0.334	0.584	0	0.741	0.841
VI		0.225		0.807	0.741	0	0.554
VII			0.610		0.841	0.554	0

In order to solve this incomplete matrix one has to refer to J.P. Guilford (34) as being the only worker who has discussed the incomplete case. His method is to form an n x(n-1) matrix, where n is the number of stimuli), by subtracting each column, element by element, from the one to the left of it, whenever such a subtraction is possible. He then calculates the mean for each column and offers these n-1 means as a solution to the incomplete matrix.

Morrissey (35) however, shows that the set of (n-1) values is not a unique solution in the incomplete case and proposes a method which applies the principle of least squares in order to establish a unique set of response values. These are chosen so as to minimize the sum of the squares of the differences between the observed response differences and the corresponding set of computed response differences.

This method requires writing in matrix form the m equations for the observed response differences among the n stimuli. The solution will be an n-dimensional vector whose components constitute the optimum set of n response values. This involves an m x n operational matrix, A, each row of which specifies the comparison of a single pair; an mxl data matrix d, each element of which specifies the result of such a comparison; and an n x 1 matrix, , such that

$$Ar \cong d$$

The quantity to be minimized according to the least squares principle is

$$(d-Ar)' \quad (d-Ar)$$

The least squares normal equations are

$$A'Ar - A'd = 0$$

and their solution is

$$r = (A'A)^{-1} \quad (A'd)$$

provided that $A'A$ is non-singular. For this to be so A must be augmented by an (m + 1) st row which does not specify a differencing operation. The additional row is arbitrary except for the requirement that it raise the rank of A to n and in practice a row of n ones is most convenient.

The corresponding augmentation of d is most conveniently an arbitrary constant, usually zero. These augmentations represent the addition of an equation stating the assumption that the sum of the n response values (and hence the mean) is zero. The matrices so obtained are shown below.

$$\left[\begin{array}{c|c} A & d \\ \hline A'A & A'd \end{array}\right] =$$

1	2	3	4	5	6	7	
1	-1	0	0	0	0	0	-.332
1	0	-1	0	0	0	0	-.441
1	0	0	0	-1	0	0	.102
0	1	-1	0	0	0	0	-.467
0	1	0	-1	0	0	0	-.385
0	1	0	0	0	-1	0	-.225
0	0	1	-1	0	0	0	.102
0	0	1	0	-1	0	0	.334
0	0	1	0	0	0	-1	.610
0	0	0	1	-1	0	0	.584
0	0	0	1	0	-1	0	.807
0	0	0	0	1	-1	0	.741
0	0	0	0	1	0	-1	.841
0	0	0	0	0	1	-1	.554
1	1	1	1	1	1	1	.000
4	0	0	1	0	1	1	-.671
0	5	0	0	1	0	1	-.745
0	0	6	0	0	1	0	1.954
1	0	0	5	0	0	1	1.674
0	1	0	0	6	0	0	.562
1	0	1	0	0	5	0	-.769
1	1	0	1	0	0	4	-2.005

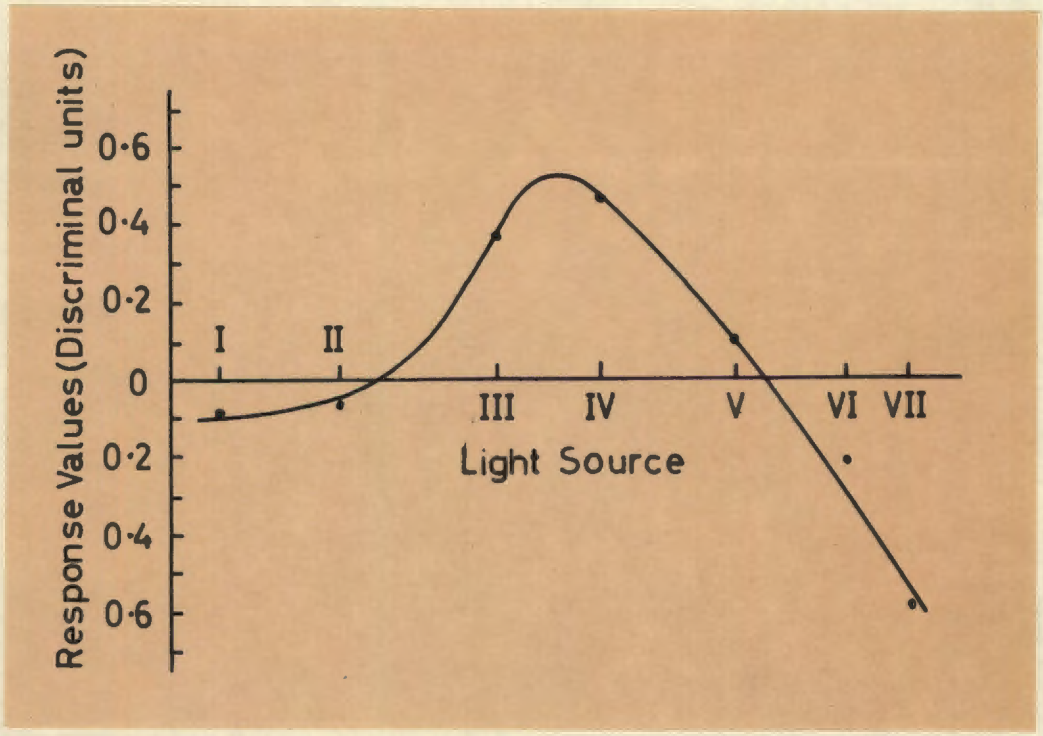


FIG 9A
Response values calculated by the least squares method.

$(A'A)^{-1}$ =

	1	2	3	4	5	6	7
1	0.2928	0.0135	0.0101	−0.0455	−0.0023	−0.0606	−0.0652
2	0.0135	0.2194	0.0005	0.0094	−0.0366	−0.0028	−0.0606
3	0.0101	0.0005	0.1728	−0.0016	−0.0001	−0.0366	−0.0023
4	−0.0455	0.0094	−0.0016	0.2182	−0.0016	0.0094	0.0455
5	−0.0023	−0.0366	−0.0001	−0.0016	0.1728	0.0005	0.0101
6-	−0.0606	−0.0028	−0.0366	0.0094	0.0005	0.2194	0.0135
7	−0.0652	−0.0606	−0.0023	−0.0455	0.0101	0.0135	0.2928

(= Response values)

	I	II	III	IV	V	VI	VII
r =	−.0087	−0.053	0.360	0.469	0.102	−0.208	−0.583

These response values are expressed in discriminal units (i.e. a unit response difference represents an expected proportion of 0.84) and are thus not directly comparable with the results obtained in section 3.3 but if they are plotted on the same horizontal axis as the % preferences the shape of the curve is similar, again indicating a preference for a source between III & IV.

3.6 Effect of Age and Sex of Observers.

The observations were analysed according to the sex and approximate age of the observers. ~~This~~ These data ~~is~~ are given in Table VIII. The relative preferences for light sources II, III, IV and V below each group are given merely to indicate the position of the maximum, not the height, as no normalizing has been done (see fig. 11).

TABLE VIII

Observed Proportions analysed according to age & sex of observers
(Consistent plus one reversal)

(a) Males up to 35. (Total 378; Consistent 40%; Rejected 32%)

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
38 35	62 47	53 65					17
	26 39	74 58	42 61				85
		58 73	42 78	22 27			111
			76 80	24 72	28 20		38
				88 81	12 75	25 19	8
Rel. Pref:	9	32	20	-4			

(b) Males over 35 (Total 207; Consistent 44%; Rejected 27%)

35 40	65 50	50 60					10
	25 36	75 57	43 64				50
		48 65	52 80	20 35			59
			78 86	22 81	19 14		32
Rel. Pref:	15	32	32	3			

(c) Females up to 35 (Total 342; Consistent 36%; Rejected 26%)

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
39	61						
	42	58					18
33		67					
	34	66					
		57	43				87
	42		58				
		30	70				
			64	36			90
		57		43			
			52	48			
				77	23		52
			72		28		
				70	30		
					70	30	5
				80		20	
Rel. Pref: 3		23	34	25			

(d) Females over 35 (Total 135; Consistent 47%; Rejected 33%)

I	II	III	IV	V	VI	VII	Number of Observers
33	67						
	50	50					6
50		50					
	26	74					
		57	43				21
	38		62				
		50	50				
			83	17			44
		61		39			
			65	35			
				80	20		20
			83		17		
Rel. Pref: 17		31	33	15			

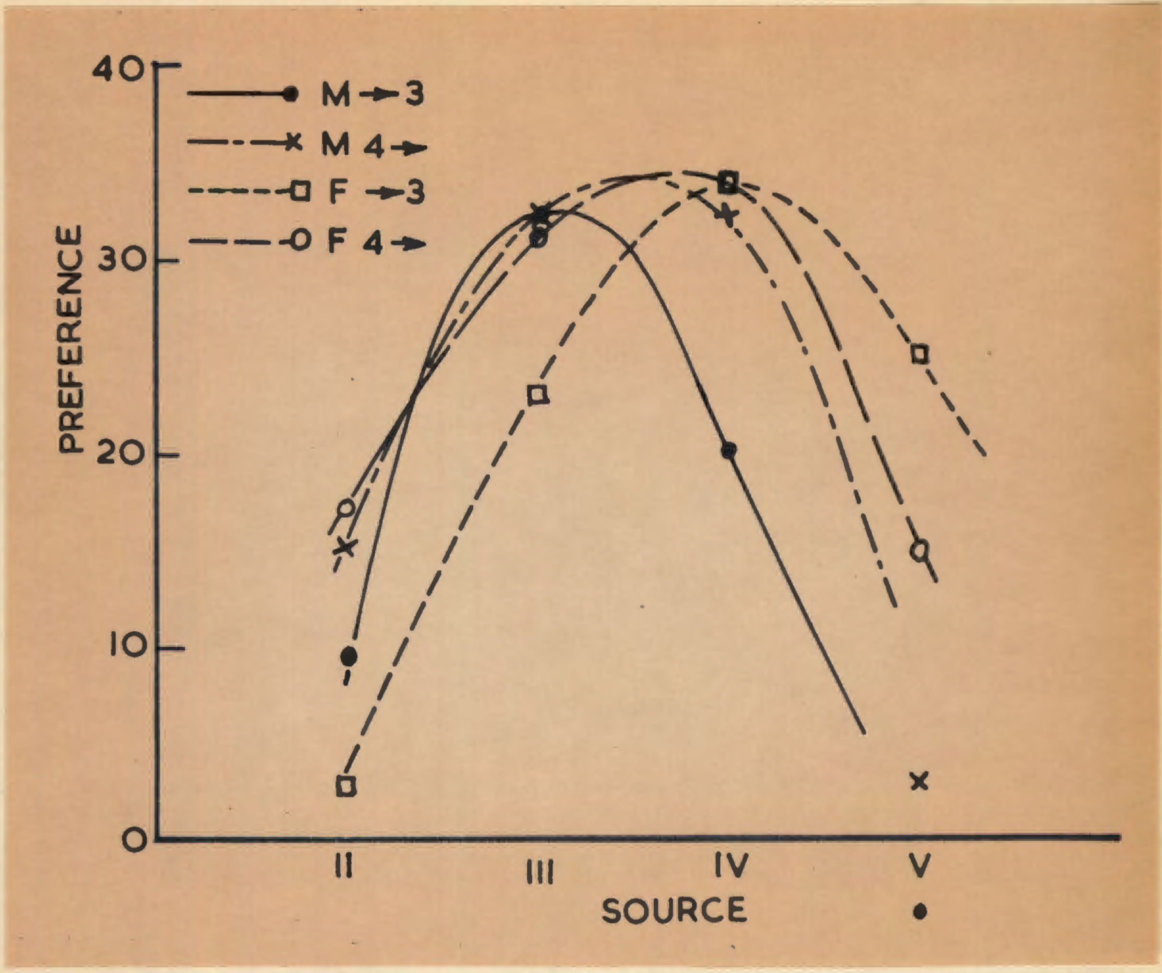


FIG. 11

Relative preferences according to age and sex of observers. (Not normalized)

M →3 : Males up to 35

M 4→ : Males over 35

F →3 : Females up to 35

F 4→ : Females over 35

The extreme sources I, VI and VII have been omitted as even for all the observers together, the numbers are inadequate to be reliable. The percentages of the four groups which gave consistent decisions and which had to be rejected are given in Table IX. It will be seen that there are no significant differences between these groups.

TABLE IX

	Consistent	Rejected	Total Obs.
Males → 35	40%	32%	378
Males 35 →	44%	27%	207
Females →35	36%	26%	342
Females 35 →	47%	35%	135

3.7 Effect of complexion colour of models

Table X gives the observations analysed according to the complexion colour of the models, the relative preferences being plotted in fig. 12. The shift of the preference to the less saturated colour under source IV for fair and pinkish complexions is understandable although not very significant. The larger amount of melanin in the darker skin presumably suppresses the colour shifts slightly in addition to lowering the reflectance. The result is a colour which does not look as flushed as in the case of a very fair skin.

TABLE X

Observed Proportions analysed according to complexions of models (Consistent plus one reversal)

(a) Dark and Olive.

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
35 32	65 41	59 68					17
	26 41	74 63	37 59				72
		55 68	45 78	22 32			74
			59 75	41 81	19 25		16
Rel. Pref:	6	37	23	23			

(b) Medium.

46 48	54 59	41 52					23
	40 49	60 67	33 51				76
		56 69	44 76	24 31			127
			70 80	30 73	27 20		67
				82 82	18 71	29 18	14
Rel. Pref:	13	27	20	3			

(c) Fair.

Light sources							Number of Observers
I	II	III	IV	V	VI	VII	
23 18	77 23	77 82					11
	32 32	68 44	56 68				25
		40 57	60 73	27 43			90
			60 83	40 90	10 17		15
Rel. Pref:	0	12	33	30			

(d) . Pinkish.

	19 31	81 49	51 69				70
		42 54	58 85	15 46			13
			68 76	32 74	26 24		44
Rel. Pref:		30	43	6			

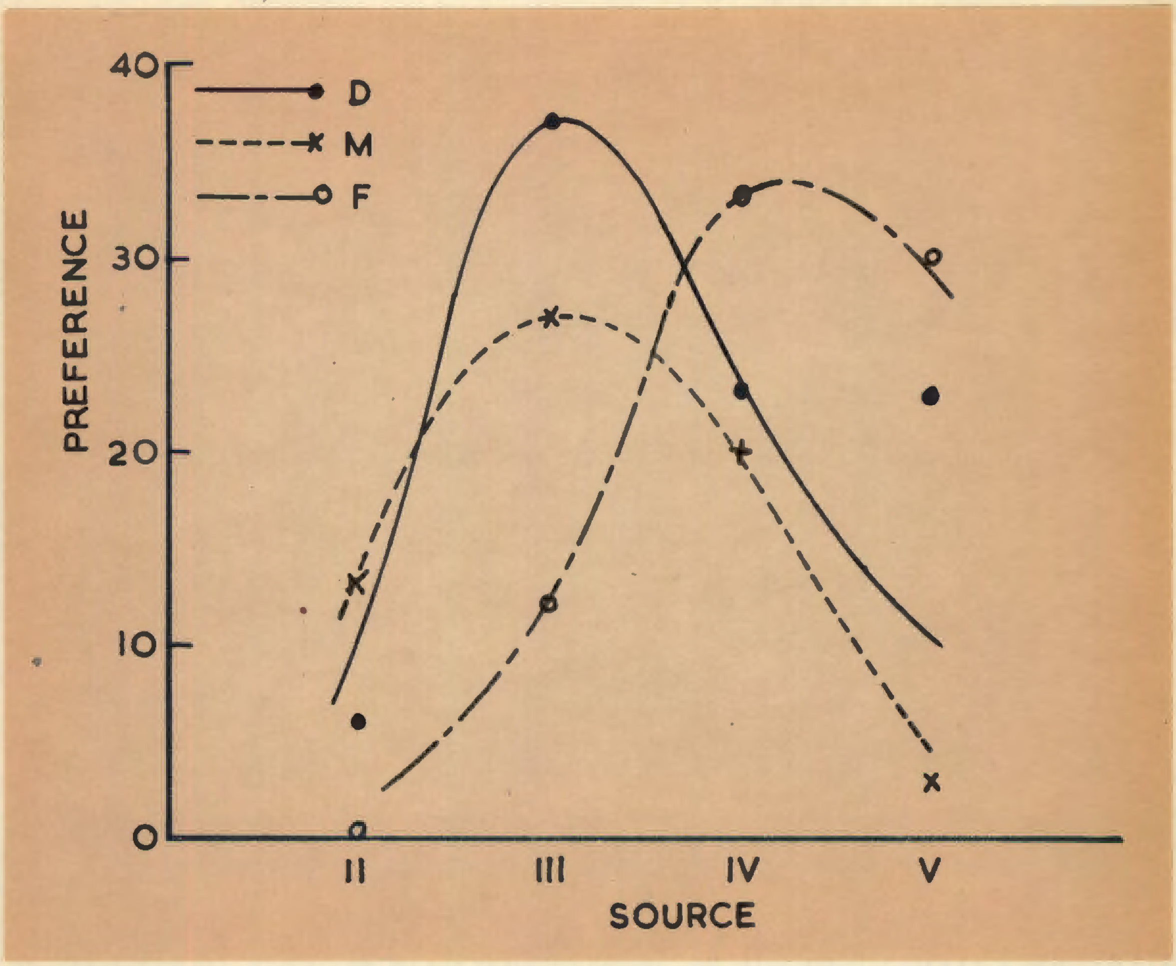


FIG 12

Relative preferences according to complexion of model
(not normalized)

D : Dark and olive

M : Medium

F : Fair

3.8 Effect of race of observers.

Only a very small proportion (about 7%) of the observers were non-Whites so it is impossible to make any statements about their preference as a group. It was noticed, however, that 40% of the observations made by this group had to be rejected as opposed to 28% of the white group. Due to the lack of social contact which exists between these groups in this country, the conditions of the experiment were possibly unfavourable to the nonwhites. To obtain a reliable indication of their preferences, the experiment would have to be repeated using non-white models as well as observers.

3.9 Foodstuffs

Although this experiment was primarily intended to establish the preferred colour of the human skin, a limited test was carried out with a few foodstuffs in order to indicate whether the preferred source would be acceptable in a restaurant for example, where the appearance of food is also important. A plate containing buttered bread, lettuce, tomato, carrot and sliced ham was viewed under the seven sources by five observers. The preference was for source III or IV but in each case the other was considered quite acceptable, for all the foods except carrot.

This underwent a rather startling change of hue from orange to red when going from source V towards I, so that in this case source V was considered preferable.

Sources III and IV were, however, tolerable. Bread and butter were the least critical as they showed the smallest change. The excess green energy of the preferred sources increased the saturation of the lettuce while the higher red content did the same for the tomato.

3.10 Tolerances

From the form of the data obtained in this experiment, no statements can be made about the tolerance of observers to deviations from the preferred chromaticity. Sanders (10) has, however, given tolerance ellipses for his preferred chromaticities which agree in shape but are larger than MacAdam's ellipses for the standard deviation of colour matching. If one assumes that his data also hold for Illuminant A, then from Ref (15) a tolerance ellipse can be calculated for the preferred skin colour as found in this experiment.

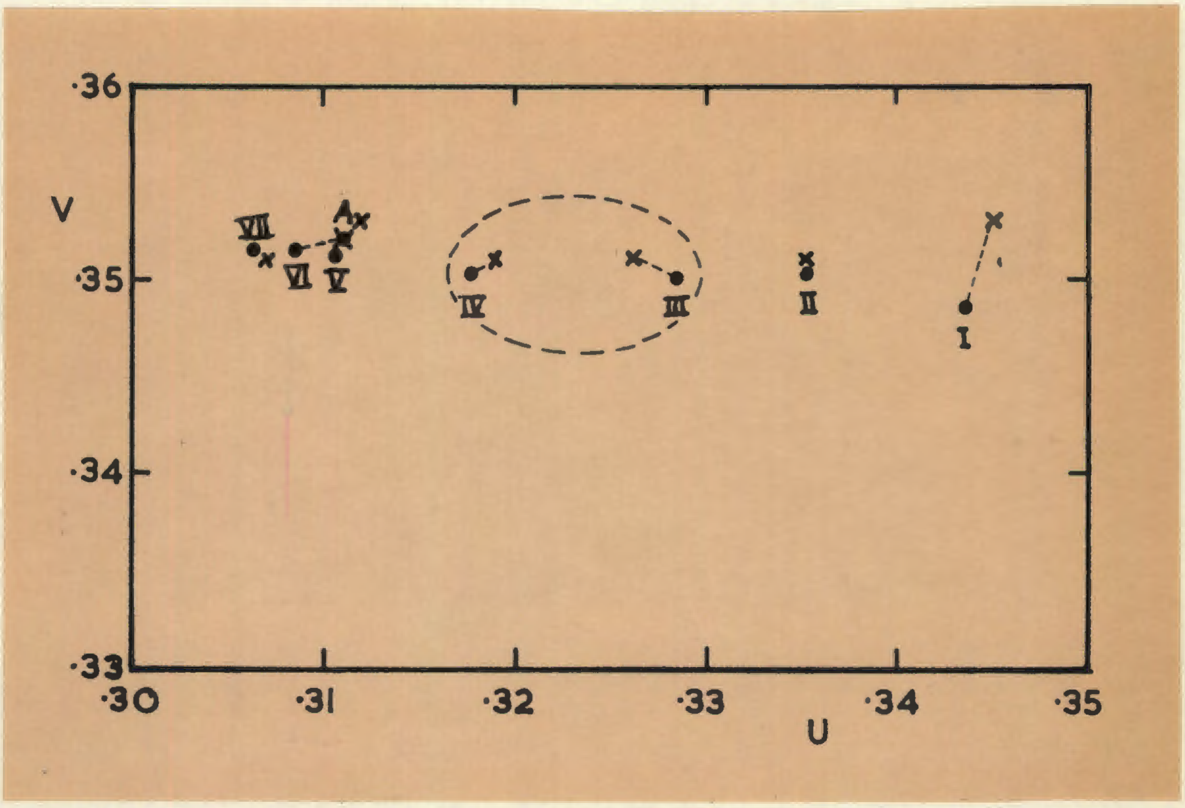


FIG 13

Chromaticity of the skin under test sources.

Full circles	: Calculated from Edwards & Duntley data (corrected for adaptation)
Crosses	: Measured by Tintometer on model P.N.
Dashed line	: 5 x standard deviation of colour matching (MacAdam)

This has been done for a point midway between the chromaticities of the skin under sources III & IV. (u = .323, v = .350) and the ellipse for five standard deviations of colour matching is shown in fig. 13. The chromaticity of the skin under both sources III & IV lie within this ellipse and thus either of these sources would presumably find general acceptance.

4. CHROMATICITIES

4.1 Comparison

As Munsell sample 5YR 8/4 is considered to be the equivalent of the human skin for the purposes of colour rendering, the chromaticity of this sample was compared with that of the skin under each of the seven test sources and Illuminant A (see bottom of Table XI). The chromaticities of the Munsell sample have been included in fig. 14 and from this it can be seen that, although the variation is systematically similar in each case, the Munsell sample is always considerably less saturated and somewhat yellower than the natural skin. Also apparent is the fact that the colour shift of the skin caused by a change of light source is about 50% greater than that of the Munsell sample. Thus any changes in colour would be considerably underestimated if Munsell 5YR 8/4 were used to represent the human skin.

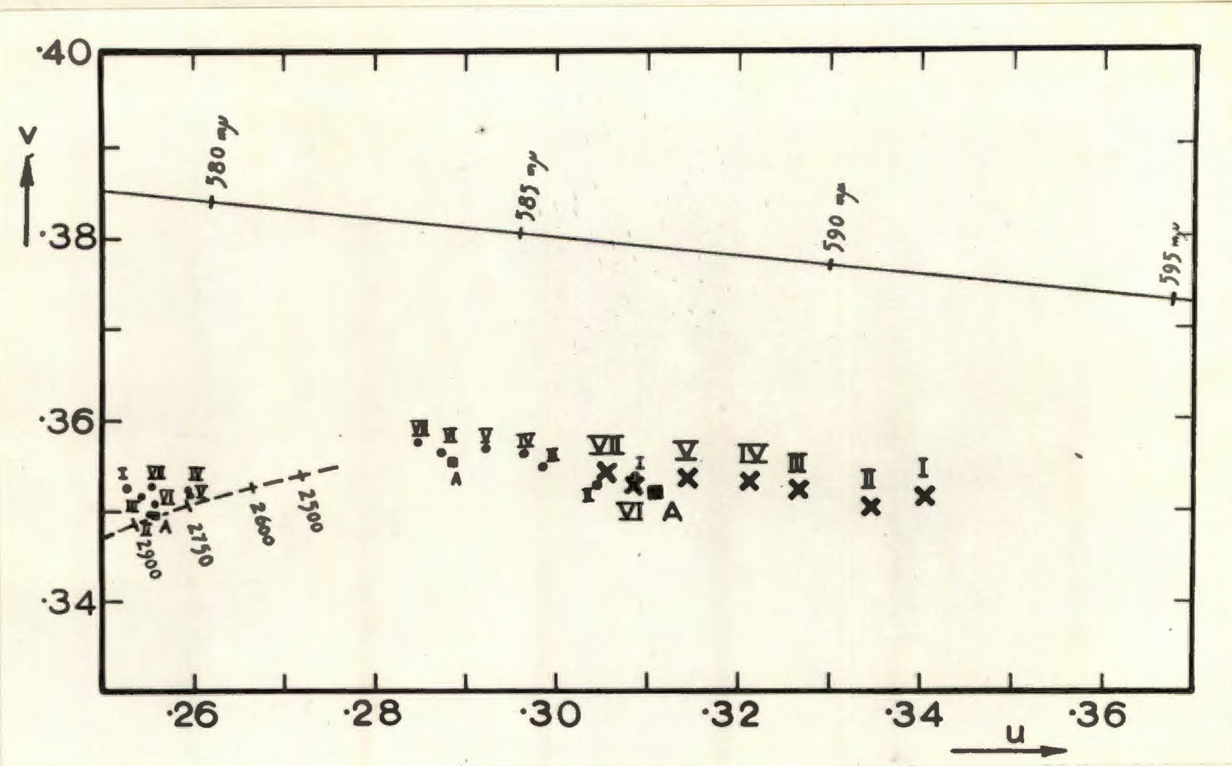


FIG 14.

Calculated Chromaticities (uncorrected for adaptation).

Full Circles	:	Light sources
Open Circles	:	Munsell 5YR 8/4 (6324) under test sources
Crosses	:	Skin under test sources.

The same problem was found when attempts were made to use photographs instead of live models for preference tests. During slack periods at the exhibition the number of observers did not warrant having a model available, so life size colour prints of the three main models were obtained and positioned in the beam from the test sources. Most observers could not distinguish between adjacent test sources, however, and these had to be abandoned in favour of tests using alternate sources only. Allowing for the fact that some difficulty was experienced in obtaining a correct colour balance on the prints, a preference in the region of source III or IV was also indicated but quite clearly photographs are not a suitable substitute for live models in tests of this nature.

Although this comparison has been made with the cheek as given by Edwards & Duntley, it will be seen from Table XI that there is virtually no difference in chromaticity between their cheek and forehead. The reflectance curves of Buck & Froelich also give results which are substantially in agreement with these. As a further check, the colour of the cheek of one of the main models for the preference tests was measured, using a Lovibond Schofield Tintometer Type IA (fig.13). Good agreement with the calculated results is found.

4.2 Range of Skin Colours Attainable.

Bearing in mind the condition that all the light sources must be metameric with Illuminant A, there is only a certain range of skin colours possible. For the particular width of yellow band used in these tests, source I represents the limit of red saturation obtainable with the skin, as the energy in the yellow band is zero. Higher saturation is only possible if the gaps in the spectrum are widened.

In the limit this would result in a spectrum, still metameric with Illuminant A, but consisting of three spectral lines. Choosing these three lines at 450, 520, and 650 nm, the relative energies of the lines must be as follows for the mixture to be metameric with Illuminant A: 450nm : 18.5; 520nm : 100; 650nm : 397 (The wavelengths of first two lines were chosen to lie at the peak energy of the blue and green projectors used, and that of the red line is as far to the right of the gap as the green is to the left).

The chromaticity of the skin (Edwards & Duntley, cheek) under this light source (Ls) is given by u = .362, v = .350. Taking the average vector shift of the skin to be

.006 between test sources, this represents about $3\frac{1}{2}$ steps beyond source I.

In a similar way three lines at 450, 520 and 580nm with relative energies of 18, 2.4 and 100 respectively would be the limit at the pale end of the range. Such a source (Ld) would give rise to a skin chromaticity of $u = .256$ $v = .355$ which represents about 8 or 9 steps beyond Source VII.

Reference to fig. 15 will show that, except for minor variations, the skin colours obtained under the seven light sources used in this experiment as well as under fluorescent lamps and the two hypothetical sources above, lie along a line of constant hue. Consequently any preference found can only be a preferred saturation and it was thought desirable to find out whether this was in fact also the preferred hue. It then became apparent that, given the condition that the light source must be metameric with Illuminant A, the chromaticity of the skin will lie somewhere very close to the horizontal line joining Illuminant A on the Planck locus to 615nm on the spectrum locus in the UCS diagram (fig. 16).

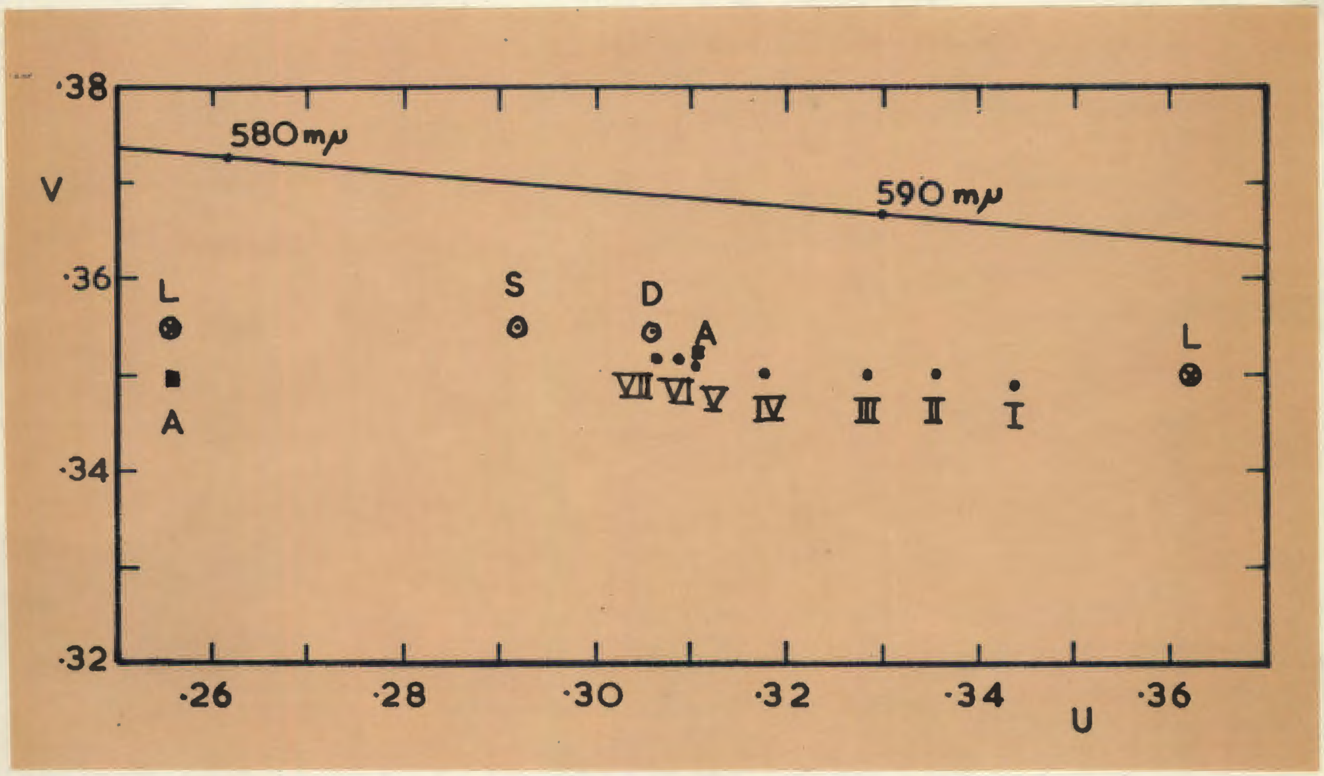


FIG. 15

Calculated chromaticities of the skin under the following sources :

Full circles : Test sources I to VII
Full square : Illuminant A
Open circles : L_D : Hypothetical source giving minimum saturation.
L_S : Hypothetical source giving maximum saturation.
S : Standard Warm White fluorescent.+
D : Deluxe Warm White fluorescent.+

+Fluorescent lamps as used by I E S Subcommittee (18)

(All chromaticities corrected for adaptation to light source)

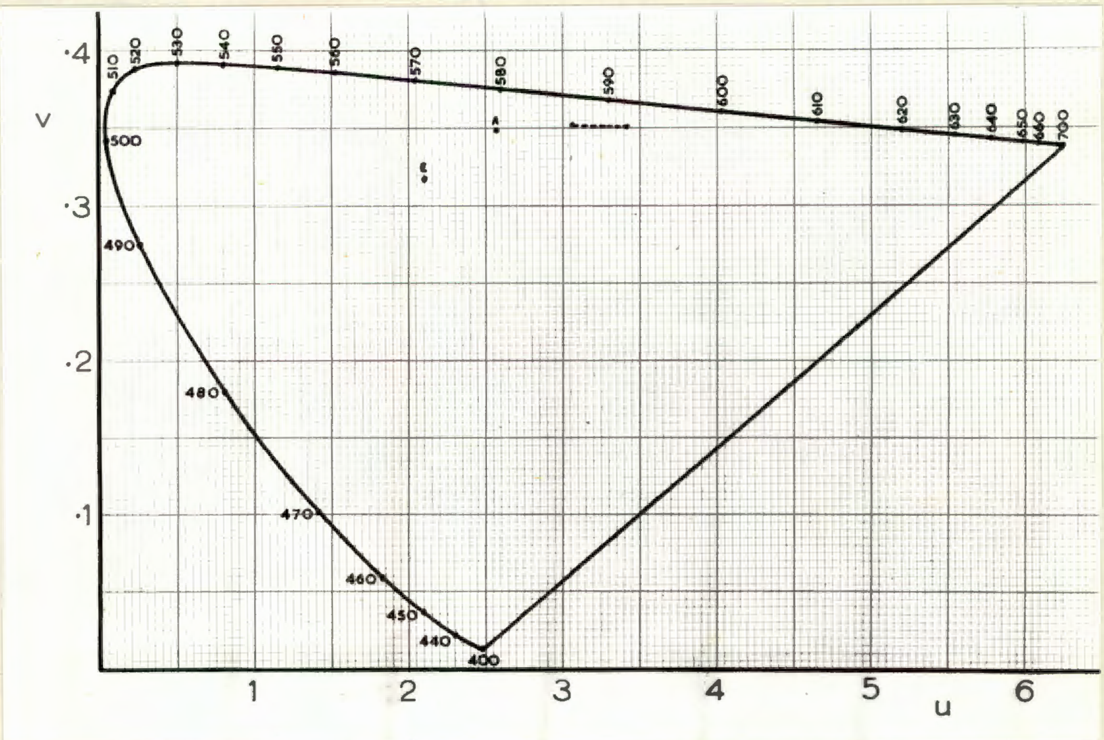


FIG 16

U.C.S. Diagram showing range of variation of skin colour under test sources.

The reason for this behaviour of the skin will become clear if the simplifying assumption is made that the skin has a spectral reflectance as shown in fig 17.

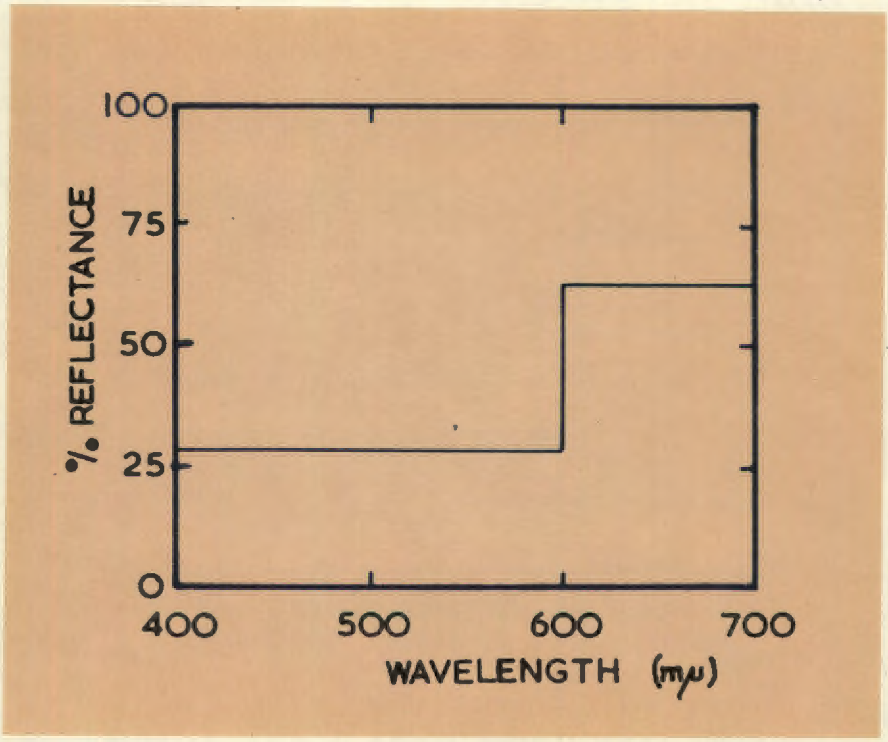


FIG 17.

Simplified spectral reflectance of skin.

It will then be seen that, provided all the components of the light source have wavelengths less than 600 nm, the chromaticity of the skin under that source will coincide with that of the source. However, as soon as one of the components has a wavelength greater than 600nm, there is a pull in the direction of that component, due to the higher reflection factor of the skin above 600nm, and a chromaticity change occurs.

This change consists almost entirely of a change in u, as v changes only from .352 to .348 for a spectral change from 600nm to 760nm, so that only very small deviations from the horizontal are possible. In fact, the centre of gravity for the energy beyond 600nm in Illuminant A lies at 615nm, where v = .350 resulting in a horizontal pull.

In practice, of course, deviations from this line occur mainly because of the deficiency in the blue reflectance and the excess at 520nm over the level reflectance assumed. The presence of powerful blue lines in the region of low reflectance is probably primarily responsible for increasing the "v" value of the skin chromaticity under the two fluorescent lamps and the hypothetical source giving the lower limit of saturation. However, as the change in reflectance between 400 and 600 nm is not very great, appreciable shifts from the line v = .350 are not possible.

5. COLOUR RENDERING OF THE LIGHT SOURCES

5.1 Colour Shift Method

In this section a colour rendering index is established for each light source, using the interim method as recommended by the IES subcommittee on colour rendering (18).

TABLE XI

C.I.E. 1960 U.C.S. Data for 20 Munsell Samples and four curves for skin under the various test sources used.

Munsell Notation	No	C I E Source A			Source I			Source II			Source III			Source IV			Source V			Source VI			Source VII		
		Y	u	v	Y	u	v	Y	u	v	Y	u	v	Y	u	v	Y	u	v	Y	u	v	Y	u	v
Light Source		1.0000	0.2557	0.3495	1.0000	0.2527	0.3526	1.0000	0.2551	0.3496	1.0000	0.2541	0.3516	1.0000	0.2593	0.3522	1.0000	0.2594	0.3519	1.0000	0.2558	0.3508	1.0000	0.2552	0.3525
7.5 R 6/4	47	.3353	.3037	.3524	.3240	.3334	.3510	.3280	.3264	.3500	.3295	.3187	.3521	.3355	.3132	.3536	.3375	.3069	.3542	.3362	.3016	.3536	.3363	.2986	.3551
5 Y 6/4	48	.3135	.2771	.3602	.2966	.2885	.3584	.3014	.2871	.3577	.3051	.2826	.3596	.3114	.2831	.3606	.3148	.2803	.3612	.3153	.2763	.3609	.3163	.2743	.3620
10 Y 6/6	49	.3156	.2629	.3668	.3248	.2662	.3645	.3317	.2676	.3645	.3383	.2642	.3658	.3472	.2674	.3666	.3528	.2661	.3671	.3532	.2630	.3671	.3557	.2616	.3678
5 GY 6/8	50	.3127	.2375	.3675	.2896	.2275	.3664	.2950	.2327	.3658	.3008	.2326	.3671	.3078	.2397	.3676	.3128	.2408	.3680	.3134	.2389	.3678	.3153	.2389	.3684
2.5 G 6/6	51	.2769	.2011	.3596	.2959	.1627	.3633	.2893	.1747	.3605	.2868	.1807	.3619	.2784	.1971	.3614	.2750	.2049	.3610	.2755	.2039	.3599	.2748	.2061	.3611
10 B 6/4	52	.3089	.2270	.3417	.3409	.2067	.3503	.3315	.2136	.3452	.3254	.2162	.3470	.3134	.2269	.3462	.3070	.2310	.3449	.3067	.2282	.3431	.3045	.2292	.3451
5 PB 6/8	53	.2793	.2248	.3321	.3189	.2053	.3446	.3080	.2118	.3380	.2997	.2143	.3397	.2852	.2246	.3383	.2773	.2288	.3363	.2770	.2258	.3339	.2739	.2269	.3362
2.5 P 6/8	54	.3057	.2770	.3348	.3351	.2942	.3425	.3278	.2907	.3377	.3207	.2862	.3397	.3117	.2849	.3399	.3056	.2811	.3387	.3039	.2759	.3369	.3014	.2743	.3392
5 P 6/8	55	.3140	.2906	.3359	.3586	.3202	.3429	.3474	.3138	.3386	.3368	.3068	.3404	.3234	.3013	.3407	.3140	.2946	.3397	.3115	.2888	.3379	.3092	.2865	.3401
10 P 6/8	56	.3474	.3102	.3402	.4004	.3495	.3444	.3873	.3407	.3413	.3744	.3318	.3431	.3591	.3229	.3437	.3480	.3140	.3432	.3443	.3077	.3418	.3402	.3045	.3438
4.5 R 4/13	58	.1700	.4667	.3476	.2579	.5267	.3443	.2356	.5157	.3445	.2147	.5040	.3456	.1897	.4837	.3473	.1715	.4673	.3483	.1643	.4618	.3483	.1574	.4561	.3490
5 YR 8/4	59	.6285	.2888	.3552	.5953	.3087	.3537	.6056	.3045	.3527	.6119	.2986	.3548	.6255	.2964	.3561	.6319	.2921	.3567	.6318	.2873	.3562	.6341	.2849	.3575
5 Y 8/10	60	.6531	.2814	.3665	.5957	.3042	.3625	.6115	.2999	.3631	.6239	.2925	.3647	.6449	.2893	.3657	.6569	.2843	.3667	.6567	.2804	.3669	.6609	.2778	.3675
5 GY 4/4	61	.1192	.2346	.3648	.1113	.2246	.3645	.1132	.2298	.3634	.1150	.2297	.3649	.1174	.2367	.3654	.1193	.2378	.3656	.1195	.2361	.3652	.1201	.2361	.3661
4.5 G 5/8	62	.1804	.1739	.3598	.2215	.1283	.3658	.2088	.1410	.3626	.2019	.1481	.3637	.1859	.1671	.3623	.1781	.1777	.3614	.1775	.1773	.3601	.1750	.1806	.3611
6 B 4/7	63	.0951	.1756	.3214	.1259	.1345	.3421	.1175	.1454	.3330	.1110	.1523	.3339	.0997	.1692	.3294	.0938	.1794	.3255	.0935	.1777	.3720	.0913	.1806	.3244
3 PB 3/11	64	.0455	.1450	.2728	.0702	.1063	.3185	.0638	.1172	.3016	.0582	.1229	.3001	.0493	.1364	.2891	.0441	.1467	.2788	.0443	.1461	.2730	.0424	.1485	.2754
10 PB 3/11	65	.0650	.2733	.3033	.0787	.2955	.3249	.0755	.2904	.3150	.0721	.2861	.3163	.0676	.2829	.3145	.0648	.2780	.3104	.0643	.2725	.3068	.0632	.2714	.3100
5.5P 3.3/14	66	.0916	.3646	.3059	.1276	.4243	.3260	.1187	.4097	.3181	.1099	.4001	.3184	.0993	.3845	.3163	.0918	.3701	.3120	.0894	.3615	.3082	.0863	.3586	.3104
2.5RP3.3/15.5	67	.1134	.4576	.3219	.1974	.5183	.3347	.1763	.5066	.3308	.1563	.4972	.3305	.1307	.4784	.3289	.1137	.4612	.3257	.1084	.4540	.3231	.1029	.4501	.3241
E & D Cheek		.3986	.3110	.3520	.4368	.3406	.3516	.4272	.3345	.3503	.4182	.3268	.3521	.4082	.3212	.3531	.4001	.3145	.3535	.3957	.3088	.3529	.3917	.3057	.3542
E & D Forehead		.3436	.3110	.3522	.3767	.3411	.3517	.3683	.3350	.3504	.3605	.3271	.3522	.3519	.3213	.3533	.3449	.3144	.3537	.3411	.3087	.3531	.3377	.3055	.3544
B & F No Cosm.		.4067	.3064	.3521	.4405	.3336	.3518	.4321	.3281	.3504	.4239	.3208	.3523	.4153	.3161	.3532	.4082	.3099	.3536	.4040	.3043	.3529	.4002	.3015	.3543
B & F Cosm.		.3790	.3170	.3532	.3932	.3535	.3513	.3907	.3452	.3505	.3864	.3361	.3525	.3844	.3281	.3539	.3813	.3202	.3546	.3780	.3144	.3541	.3758	.3109	.3554

TABLE XII

VECTOR LENGTH BETWEEN CHROMATICITY OF SAMPLES UNDER TEST SOURCES AND CHROMATICITY UNDER ILLUMINANT A

Sample Number & Munsell Notation.	Vector Shift from Illuminant A. (corrected for differences in chromaticity of Source)						
	I	II	III	IV	V	VI	VII
47 7.5R 6/4	.0330	.0234	.0168	.0061	.0008	.0022	.0046
48 5Y 6/4	.0152	.0109	.0076	.0033	.0014	.0011	.0026
49 10Y 6/6	.0083	.0058	.0042	.0030	.0022	.0010	.0022
50 5GY 6/8	.0081	.0046	.0041	.0030	.0019	.0019	.0028
51 2.5G6/6	.0354	.0258	.0188	.0076	.0010	.0029	.0058
52 10B 6/4	.0181	.0132	.0097	.0041	.0009	.0011	.0027
53 5PB 6/8	.0190	.0137	.0104	.0041	.0018	.0010	.0028
54 2.5P 6/8	.0207	.0146	.0131	.0049	.0015	.0015	.0060
55 5P 6/8	.0326	.0240	.0179	.0074	.0014	.0020	.0038
56 10P 6/8	.0423	.0311	.0232	.0091	.0006	.0026	.0052
58 4.5R4/13	.0631	.0497	.0389	.0137	.0035	.0050	.0102
59 5YR 8/4	.0234	.0165	.0117	.0044	.0010	.0016	.0035
60 5Y 8/10	.0268	.0194	.0133	.0054	.0023	.0014	.0037
61 5GY 4/4	.0078	.0045	.0039	.0026	.0017	.0017	.0026
62 4.5G 5/8	.0428	.0324	.0243	.0104	.0008	.0035	.0074
63 6B 4/7	.0418	.0301	.0240	.0113	.0017	.0021	.0055
64 3PB 3/11	.0556	.0396	.0325	.0183	.0041	.0015	.0040
65 10PB3/11	.0312	.0212	.0181	.0104	.0048	.0024	.0040
66 5.5P3.3/14	.0649	.0475	.0385	.0180	.0046	.0034	.0057
67 2.5 RP 3.3/15.5	.0644	.0504	.0417	.0177	.0014	.0037	.0070
Mean Vector Length	.0328	.0239	.0186	.0083	.0020	.0022	.0046
Rc	30	49	60	82	96	95	90
Mean for 8 Samples	.0338	.0247	.0191	.0083	.0020	.0022	.0048
Rc	28	47	59	82	96	95	90

The chromaticities of a set of samples are calculated in terms of CIE 1960 U.C.S. co-ordinates[+] ($u = 2x/(6y - x + 1.5)$, $v = 3y/6y - x + 1.5))$ for each of the sources and for a reference source, in this case CIE Illuminant A. (Table XI).

From these data the vector shift of each sample is calculated and corrected for differences in the chromaticities of the light sources by adding a vector equal to the vector shift required to make the source the same chromaticity as Illuminant A. The average vector shift for each source is then obtained. (Table XII). The set of samples used in the calculations are 20 of the set of 21 selected by Nickerson for study by members of CIE committee E 1.3.2. For ease of programming the computer No.57 (Munsell N6/) was omitted. Spectral reflectance data on the 21 samples were obtained from Miss Nickerson and ~~is~~ are given in Table XIII. These data also appear in Ref. (18) with the addition of Sample No. 69[x] which was not available when these computations were done.

From the average vector shifts an index of colour rendering is established by assigning an arbitrary rating of 50 to a warm white halophosphate fluorescent lamp, in this case the lamp referred to as "std White Houston 2" in ref (16), and 100 to the reference source.

+ Provisional recommendation by committee E 1.3.1.see ref (19)

x 10 BG 6/4

TABLE XIII

Spectral reflectance data on 21 Munsell samples used in colour rendering calculations.

No.	58	59	60	61	62	63	64	65	66	67	68
WAVE-LENGTH MY	MUNSELL BOOK NOTATION AND PRODUCTION NUMBER										
	4^5R 4/13	5YR 8/4	5Y 8/10	5GY 4/4	4^5G 5/8	6B 4/7	3PB 3/11	10PB 3/11	5^5P 3^3/14	2^5RP 3^3/15^5	5YR 8/4
	4785	6324 *	T4991	6157 *	3943	4802	2312	1508	2315	2316	6433 *
400	5.3	28.0	6.8	4.0	13.0	14.9	7.8	28.1	27.8	19.3	24.0
10	5.2	35.0	7.0	4.1	11.9	13.9	6.6	28.5	27.6	19.1	28.7
20	5.1	36.8	7.1	4.3	11.1	13.0	7.7	28.5	27.9	19.1	30.0
30	4.9	37.3	7.4	4.4	10.7	13.5	12.6	28.2	29.5	20.4	31.2
40	4.7	37.6	7.8	4.5	10.8	16.0	21.4	27.3	32.8	23.1	33.0
450	4.3	38.1	8.5	4.6	11.3	21.8	30.8	25.4	33.5	23.4	35.0
60	3.9	38.6	9.7	4.8	12.6	29.4	35.5	22.4	28.8	19.0	38.1
70	3.4	39.4	11.6	5.1	15.2	34.9	35.0	18.6	22.0	13.3	42.1
80	3.1	40.7	14.6	5.6	19.7	36.0	31.5	15.4	15.5	8.7	46.0
90	2.9	42.7	19.4	6.4	25.8	33.3	26.4	12.3	11.0	5.8	49.0
500	2.9	45.4	26.9	7.7	33.3	28.4	20.9	9.5	7.9	4.1	50.5
10	3.1	47.3	37.4	9.4	36.5	23.5	15.8	7.8	6.7	3.1	50.5
20	3.2	48.1	47.7	11.1	35.5	19.0	11.2	6.7	4.6	2.6	49.9
30	3.3	48.6	56.0	13.6	32.2	14.9	7.7	5.7	4.3	2.5	49.8
40	3.4	49.5	62.6	15.4	27.8	11.5	5.2	5.2	4.0	2.4	49.6
550	3.6	51.9	67.0	15.9	23.3	9.1	3.6	5.0	3.7	2.4	48.1
60	4.2	56.7	69.6	15.1	19.3	7.5	2.6	4.8	3.6	2.4	46.2
70	4.9	63.4	71.1	13.6	15.7	6.6	2.0	4.7	3.8	2.6	46.4
80	6.1	69.7	71.9	12.1	12.8	6.1	1.7	4.7	4.4	3.0	54.5
90	10.5	73.5	72.3	10.9	10.9	5.7	1.6	5.2	6.0	4.1	66.8
600	19.5	75.5	72.4	10.0	9.8	5.6	1.6	6.0	9.8	7.9	73.6
10	34.5	76.4	72.5	9.5	9.2	5.5	1.6	7.3	16.0	16.7	76.1
20	51.8	76.7	72.6	9.1	8.7	5.5	1.8	8.4	21.8	32.0	77.0
30	65.7	76.7	72.8	8.8	8.2	5.5	1.8	9.2	24.8	48.2	77.1
40	73.5	76.7	73.0	8.6	8.0	5.5	1.9	10.1	25.9	59.0	77.3
650	77.7	76.7	73.4	8.6	8.0	5.6	2.0	11.2	29.0	65.9	77.4
60	80.1	76.6	73.8	8.7	8.3	5.6	2.4	12.7	31.8	70.9	77.5
70	81.7	76.6	74.4	9.4	9.0	5.8	2.7	14.7	35.0	74.5	77.6
80	83.0	76.6	75.0	10.5	10.5	5.9	3.6	17.1	39.8	77.5	77.7
90	84.0	76.6	75.8	12.6	12.8	6.1	5.7	20.3	45.0	79.7	77.9
700	84.9	76.5	76.5	15.6	16.5	6.6	9.9	23.9	52.0	81.5	78.0

* NOTE: No. 59, 5YR 8/4 (6324) REPRESENTS COMPLEXION COLOR; DUPLICATE COLOR, No. 68, 5YR 8/4 (6433).
No. 61, 5GY 4/4 (6157) REPRESENTS FOLIAGE COLOR.

TABLE XIII (Contd.)

No.	47	48	49	50	51	52	53	54	55	56	57
WAVE-LENGTH mμ	MUNSELL BOOK NOTATION AND PRODUCTION NUMBER										
	7^5R 6/4 4277	5Y 6/4 6329	10Y 6/6 6431	5GY 6/8 4385	2^5G 6/6 6212	10B 6/4 4167	5PB 6/8 4892	2^5P 6/8 3837	5P 6/8 6052	10P 6/8 6432	N 6/ 4755
400	26.3	11.4	6.0	7.5	11.9	37.5	42.0	56.5	37.5	34.5	29.6
10	25.8	12.1	6.1	7.6	12.7	42.5	50.5	57.3	55.4	47.4	31.3
20	25.0	12.4	6.2	7.6	13.1	43.5	53.0	57.5	59.5	50.2	31.0
30	24.3	12.5	6.4	7.5	13.8	43.9	54.5	57.0	58.2	49.4	30.8
40	23.6	12.6	6.5	7.5	14.8	44.9	55.8	55.8	54.9	47.4	30.5
450	23.1	13.0	6.8	7.6	16.5	45.8	57.0	53.5	50.9	45.0	30.1
60	22.6	13.4	7.1	7.9	19.1	46.4	56.8	50.1	46.7	42.4	30.0
70	22.1	14.1	7.8	8.7	23.5	46.5	55.5	46.0	42.3	39.2	30.0
80	22.0	15.4	9.0	11.2	28.8	46.3	53.2	41.8	38.6	36.1	29.9
90	22.2	17.8	11.9	15.2	34.0	45.6	50.0	37.2	34.9	33.3	30.1
500	22.9	21.2	17.9	20.3	37.9	44.5	46.2	33.2	31.7	30.7	30.3
10	23.2	24.8	25.5	24.7	40.0	43.1	42.5	30.9	29.8	29.0	30.4
20	23.1	26.7	32.3	28.5	40.5	41.1	38.7	29.0	28.4	27.7	30.4
30	23.3	27.4	35.2	34.8	39.5	38.7	35.0	27.2	26.6	26.3	30.5
40	24.2	27.9	36.5	40.2	37.6	36.1	31.7	26.4	25.7	25.6	30.5
550	25.9	28.9	37.4	41.0	35.0	33.3	28.6	26.6	26.0	26.0	30.6
60	27.9	30.7	38.6	39.0	32.0	30.0	25.9	26.7	26.7	27.1	30.9
70	30.6	33.0	39.3	35.8	28.7	28.0	24.0	26.3	26.5	27.9	30.9
80	35.0	34.4	38.9	32.3	25.3	26.8	23.1	26.1	25.8	28.5	30.7
90	40.0	35.0	37.7	29.2	21.9	26.1	22.7	27.7	26.5	30.3	30.6
600	43.5	35.1	36.1	27.1	19.0	25.6	22.6	31.0	30.3	35.7	30.5
10	45.3	35.1	34.9	25.8	17.3	25.4	22.6	35.3	36.7	44.5	30.3
20	46.1	35.0	34.2	24.7	16.4	25.5	22.9	38.7	43.5	54.2	29.9
30	46.3	34.8	33.6	23.5	15.8	26.0	23.9	41.0	48.7	62.0	29.5
40	46.3	34.7	33.2	22.6	15.5	26.7	25.0	43.1	52.2	66.5	29.3
650	46.2	34.5	32.8	22.2	15.2	27.2	26.5	44.9	55.2	69.3	29.0
60	46.3	34.3	32.7	22.5	15.2	27.7	27.5	46.4	58.0	71.1	29.1
70	46.5	34.1	32.9	23.6	15.5	27.9	28.5	47.4	61.0	72.3	29.3
80	46.7	33.9	33.2	30.7	16.2	28.0	29.0	48.0	64.2	73.0	29.4
90	47.0	33.7	33.5	29.5	16.9	28.3	29.8	48.5	67.1	73.5	29.7
700	47.4	33.6	33.8	34.9	17.4	29.5	31.0	49.5	70.3	73.9	29.9

Then Rc = 100 - f x (vector shift) where f is a factor depending on the particular set of test colours chosen and given as being equal to 2220 for the 20 sample set. This value for f is for a reference source of 3000° K but as all other sources in this paper have been adjusted to Illuminant A, the given data for the fluorescent lamps were recalculated using Illuminant A as reference. The factor f then becomes 2130 for the 20 sample set. This adjustment only increases the ratings for Sources I, II and III by about 2, the ratings for the others being affected only fractionally.

In the sub-committee report it is stated that the choice of test colours is not highly critical and this is borne out by the correlation between the Rc obtained by using the 20 samples and that obtained for 8 samples selected by Ouweltjes (20). Even for extremely distorted sources such as I there is only a difference of 2 in the rating obtained.

5.2 Spectral Band Method

In Table XIV are shown the percentage luminance contained in each of the eight spectral bands proposed by Bouma and provisionally accepted by the CIE in 1948. The eight band values for the seven test sources are given and,

TABLE XIV

Percent Luminous Flux in C I E Bands (1948 provisional recommendation)

Band No.	Band Limits(nm)	Light Source									
		I	II	III	IV	V	VI	VII	A	Std.WW	deL.WW
1	380 - 420	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.005	0.006	0.006
2	420 - 440	0.036	0.047	0.045	0.045	0.050	0.050	0.049	0.061	0.261	0.327
3	440 - 460	0.165	0.215	0.210	0.208	0.221	0.229	0.226	0.253	0.161	0.065
4	460 - 510	11.0	9.75	8.09	6.31	5.15	5.30	4.96	5.46	2.29	2.57
5	510 - 560	48.0	43.0	41.0	35.3	32.6	31.5	30.0	33.5	30.3	38.5
6	560 - 610	6.24	16.9	24.2	36.4	43.6	45.8	48.9	42.5	57.0	43.8
7	610 - 660	30.5	26.5	23.5	19.7	16.9	15.6	14.4	16.7	9.80	14.2
8	660 - 780	4.08	3.58	2.98	2.04	1.53	1.52	1.40	1.53	0.183	0.476

TABLE XV

Vector shift of skin (Edwards & Duntley, cheek) from chromaticity under Illuminant A

(C I E 1960 U C S co-ordinates, corrected for adaptation to light source)

Light Source	I	II	III	IV	V	VI	VII	A	Std.WW	deL.WW
Vector	0.0327	0.0242	0.0175	0.0068	-0.0009	-0.0023	-0.0049	0.0000	-0.0189	-0.0056

(The minus sign indicates a decrease in saturation) for comparison the values for Illuminant A, and the standard warm white and de Luxe warm white fluorescent lamps used by the I.E.S subcommittee are included.

From Table XIV it will be seen that, although the original intention was to imitate a standard fluorescent lamp with source VII, Bands 6 and 7 were not sufficiently depressed to do this. This was due to the lack of intensity of the yellow projector and the increase in the transmission factors of the green and blue filters beyond 660nm, combined with the steeply rising energy curve of the projector lamps burning at temperatures less than 2800° K. However, as the preference lay in the direction of increased saturation from Illuminant A, the pale end is of less interest.

In section 4.2 it was shown that almost the entire colour shift obtained with the skin under various light sources is determined by the relative energy content at wavelengths beyond 600nm. This is borne out by the correlation (fig.18) between the percent luminance contained in the band 605 - 655 nm, and the vector shift of the skin chromaticity. These shifts are given in Table XV and are calculated from the skin chromaticities corrected for adaptation to the light sources.

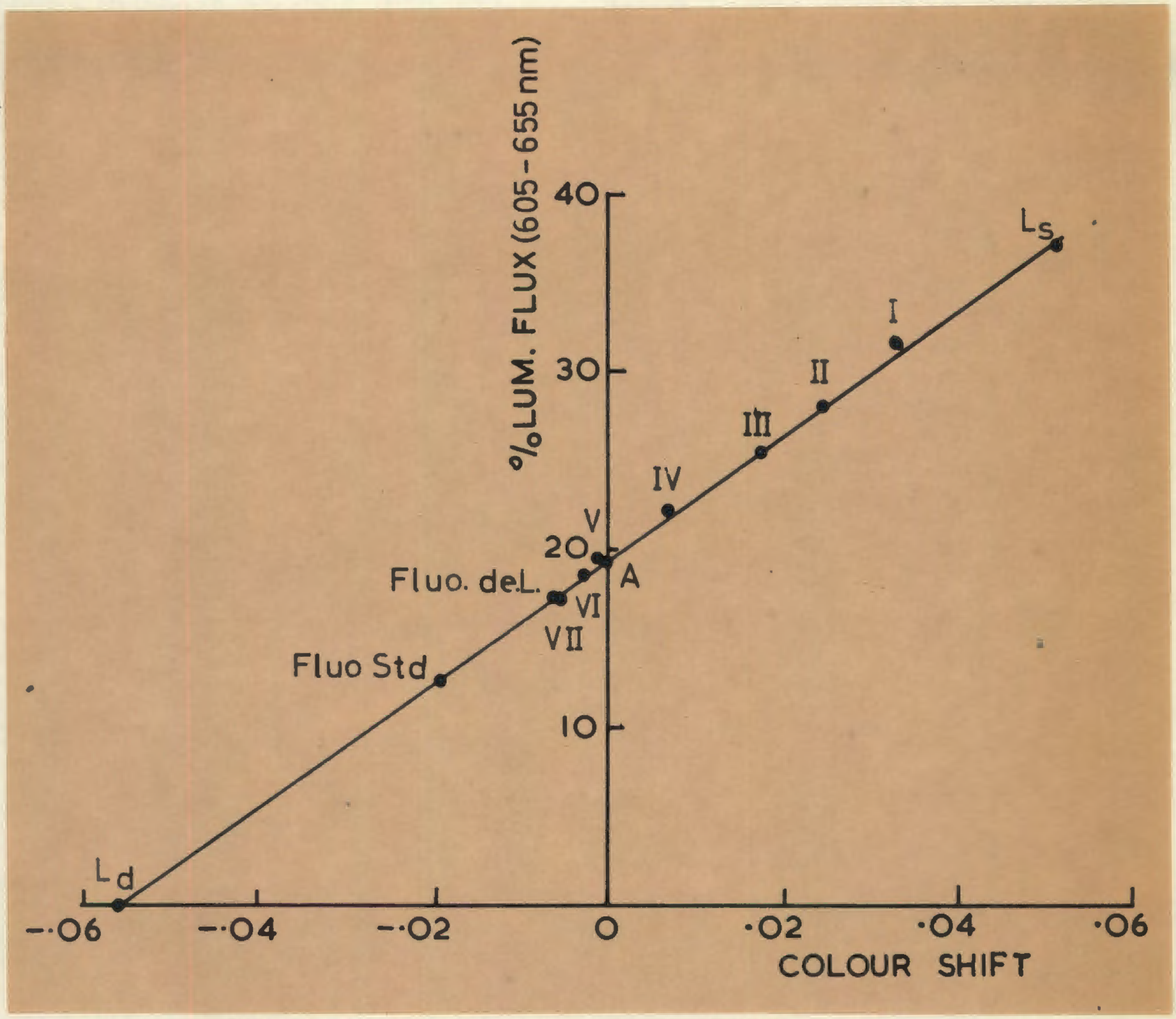


FIG 18

Correlation between percent luminous flux in the band 605 - 655 nm and the colour shift of the skin. (The minus sign indicates a colour less saturated than under illuminant A.)

From fig.18 it would appear that for preferred rendition of the skin, the band 605 - 655nm should contain about 25% of the total luminance. (Except for the sources Ls and Ld, the correlation also holds if C.I.E. band 7 is used). This statement is obviously an over-simplification of the problem, as the rest of the spectrum is completely ignored. What must be remembered, however, is that the energy distribution of the rest of the spectrum has very little effect on the skin chromaticity, as was pointed out in Section 4.2. This view is supported by the fact that the correlation holds for the two fluorescent lamps as well as the test sources, notwithstanding marked differences in the spectral distribution below 600nm. In any case, to have a reasonably good general colour rendering index the S.E.D. of a lamp should not differ too radically from the reference source, so that large shifts in the hue of the skin should not occur in practice.

Had better computing facilities been more readily available it would have been interesting to see how well the relationship holds for a large number of practical and hypothetical fluorescent lamps such as those used by Ouweltjies (20).

6. LUMINOUS EFFICIENCY OF PREFERRED SOURCE

The luminous efficiency of a light source can be defined as the ratio of luminous flux to energy flux (Ie: $\eta = \int E_\lambda \bar{y}_\lambda d\lambda \Big/ \int E_\lambda d\lambda$). From the S.E.D's of the test sources in fig 1, it will be obvious that sources III and IV will have lower efficiencies than Illuminant A, due to their energy deficiency in the yellow ragion where V_λ (or $\bar{y}_\lambda$) has a high value and a large amount of energy in the red region where $\bar{y}$ is low.

It is, however, of no interest to compare the efficiencies of these sources, as it would be impractical to use an incandescent lamp to imitate source III or IV. A comparison of the luminous efficiencies of the preferred test sources with those of typical fluorescent lamps would be more relevant, but the totally different characters of incandescent and fluorescent radiation in the red region presents a problem. While the energy radiated from an incandescent lamp is still increasing at 700nm, that from most fluorescent lamps has dropped to less than 10% of the maximum value. The luminous flux, however, is hardly affected by energy differences at and beyond this wavelength as the $\bar{y}$ function is down to 0.4% of its maximum value. Thus the upper limit of integration of the energy when considering incandescent sources has a profound effect on the luminous efficiency.

An estimate of comparitive efficiencies can be made as follows :

a) The upper integration limit of energy for the test sources is set at 670nm. This decreases the luminous flux one or two percent, depending on the source concerned and changes the chromaticity slightly. It does, however, make the energy content of the red region from say 620nm onwards, comparable with that of a fluorescent lamp, the integration limit for the latter remaining at 700nm.

b) Another approach to the problem is to alter the red end of the S.E.D. of a test source in such a way that it could possibly have arisen from the fluorescence of phosphors used in practice. Ouweltjes gives the relative S.E.D's of a number of phosphors in reference 20. Using these figures it was found that a mixture of Magnesium Arsenate and Strontium-Magnesium orthophosphate activated by tin gave a spectrum from 640nm upwards which could replace the spectra of the test sources. This portion of the spectrum can be chosen in such a way that Y changes by less than 0.1% and neither x nor y shifts by more than 0.0003 in the case of the source or the skin illuminated by the source.

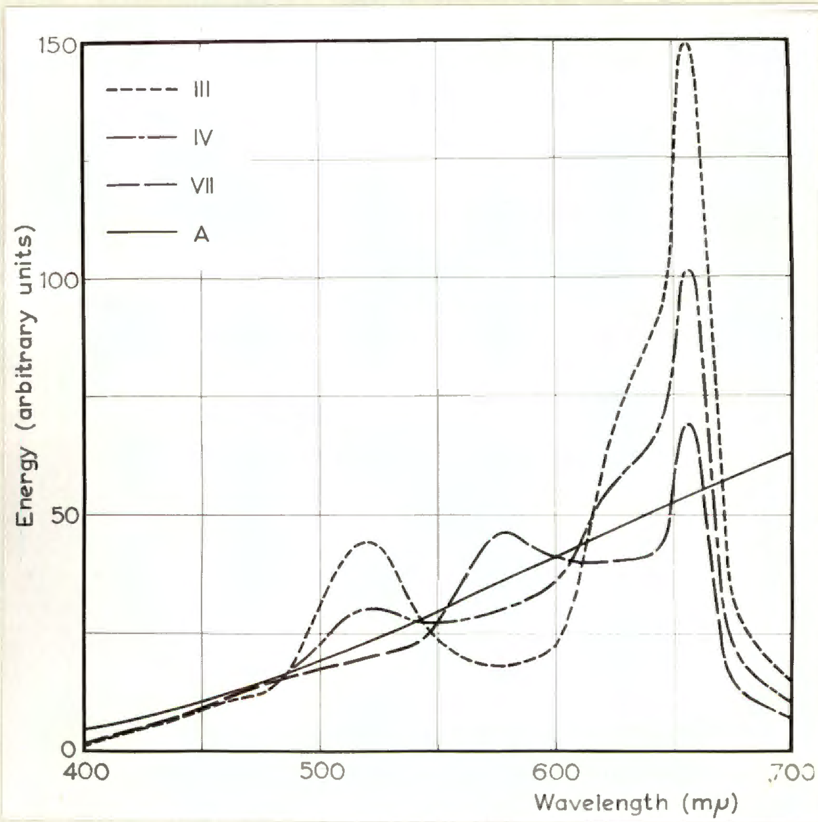


FIG 19

Modified spectral energy distributions
for sources III, IV and VII.

Such modified S.E.D's are shown in fig (19) for sources III, IV and VII. It must be emphasized here that the design of a practical fluorescent lamp does not fall within the scope of this work and the S.E.D's shown are merely suggested as a means of obtaining a basis for comparing the efficiencies of three of the test sources with existing fluorescent lamps.

If now the efficiency is calculated for light sources III, IV and VII and for the standard and de Luxe warm white fluorescent lamps used by the I.E.S. subcommittee on colour rendering (18), the following figures are obtained. (Integration limit is 700nm for all the sources)

Light source	III	IV	VII	Std. WW.	deL.WW.
Efficiency (a)	69	85	98	111	97
Efficiency (b)	67	78	95	111	97

One can thus infer that if a fluorescent lamp were constructed to give optimum rendition of the skin, it would have a luminous efficiency some 20 - 30% lower than a de luxe warm white fluorescent lamp.

The actual efficiency of this hypothetical lamp (in lumens/watt input) is a different problem, requiring a knowledge of the quantum efficiencies of available phosphors, their absorption characteristics etc. An improvement in efficiency may be possible with different, more efficient phosphors than those used in the fluorescent lamps considered.

7. CHROMATIC ADAPTATION

When calculating a colour rendering index for a light source, it seldom happens that the chromaticity of the test source agrees exactly with that of the reference source and the problem of chromatic adaptation arises i.e. the colour appearance of the sample must be determined with the observer adapted to the test source. In the I.E.S. subcommittee report (18) it is recommended that the reference source should have a colour temperature within $\pm$ 50°K of the colour temperature of the test source for sources below 6000°K. Any residual chromaticity difference between the sources is then compensated for by adding to the sample chromaticity a vector equal in length and direction to the vector from the test to the reference source.

In this experiment five of the seven light sources used had correlated colour temperatures (13) within $\pm$ 25°K of Illuminant A. Sources IV and V, however, corresponded to 2750°K and it seemed desirable to check whether the simple vector-addition method would be adequate to compensate for a discrepancy of 100°K, as from the subcommittee report it was not clear how the tolerance of $\pm$ 50°K was arrived at.

Systematic studies of the shift in perceived colour caused by adaptation of the observer's eye to various light sources have been made by a number of people, (e.g. 21 - 29) but to date the problem has not been solved satisfactorily as no formula has yet been evolved which fits the experimental results accurately. From the I.E.S. report it would appear that the subcommittee favours the Helson, Judd & Warren formula because the results were obtained by a memory method of specifying the colours. Also, from the results given in Table XII of the report, it will be seen that this formula gives the smallest average deviations of hue, value and chroma for 14 Munsell samples. Consequently, this formula was used in the following calculations.

The formulas for predicting the result of an adaptation shift from Illuminants A - C given by Helson, Judd &Warren (25) are based on the von Kries coefficient law and the co-ordinate system used is based on a set of primaries known as the dichromatic co-punctual chromaticities.

For the same set of primaries G.L. Howett has worked out formulas for the general case in which any source S may be converted to Illuminant "C" and vice-versa. These are given in Appendix I of the I.E.S. sub-committee report (18), but cannot be used here as the adaptation shift takes place between a general source S and Illuminant A. Wyszecki (30) has evolved a graphical method of predicting the change in colour caused by an adaptation shift between any two sources, based on the Judd formulas. However, a graphical method did not seem to offer adequate accuracy for the small shifts being dealt with here so equations similar to Howett's, but with Illuminant A as reference, were worked out.

If, for any colour an observer determines tristimulus values P, D, T when adapted to Source 1 then when the adapting illumination only is changed to source 2 the responses P', D', T' of the observer are given by :

$$P' = (P_1/P_2) \quad P$$
$$D' = (D_1/D_2) \quad D \quad \ldots\ldots\ldots \quad (1)$$
$$T' = (T_1/T_2) \quad T$$

where P_1, D_1, T_1 and P_2, D_2, T_2 are the tristimulus values of sources 1 & 2.

The relation between the tristimulus values P,D,T and the tristimulus values X, Y, Z of the C.I.E. co-ordinate system is given by

$$P = 1.000\,Y$$
$$D = -0.460X + 1.359Y + 0.101\,Z \;.. \quad (2)$$
$$T = 1.000\,Z$$

By means of these equations, equations (1) may be written in terms of the C.I.E. co-ordinate system, and when the known values of P_a, D_a, T_a are substituted the following relations are obtained :

<u>Adapting illuminant changes from A to S</u>

$$X' = \left[0.88963\,P_s/D_s\right]X + \left[2.9543 - 2.6283\,P_s/D_s\right]Y + \left[0.07813\,P_s/T_s - 0.19533\,P_s/D_s\right]Z$$
$$Y' = 1.0000\,Y \quad \ldots\ldots \quad (3)$$
$$Z' = \left[0.35582\,P_s/T_s\right]Z$$

Adapting illuminant changes from S to A

$$X' = \left[1.1241 \; D_s/P_s\right] X + \left[2.9543 - 3.3209 \; D_s/P_s\right] Y$$
$$+ \left[0.6171 \; T_s/P_s - 0.2444 \; D_s/P_s\right] Z \quad \ldots\ldots \quad (4)$$
$$Y' = 1.0000 \; Y$$
$$Z' = \left[2.8104 \; T_s/P_s\right] Z$$

X, Y, Z are the tristimulus values of the sample when viewed under adaptation to the "old" source ("A" in equations 3), X', Y', Z' are the tristimulus values of the sample when the observers eye is adapted to the "new" source, (S in equations 3) while Ps, Ds and Ts are the tristimulus values of the general source "S". The sample is illuminated by a source which does not change colour.

On comparing equations 3 and 4 with those given by Howett, it will be seen that equations for adaptation shifts in opposite directions are comparable. This stems from his definition of X' Y' Z', which "represent the specification of the colour perceived under adaptation to the "new" source that has the appearance that the sample with values X Y Z has under adaptation to the old source".

Equations 3 were used to calculate the colour shifts of a set of eight samples (CIE set 1) of medium saturation, due to an adaptation shift from Illuminant A to source V with results as given in Table XVI. It will be seen that the average shifts in u & v compare favourably with the u and v shift in the adapting source. It therefore seems reasonable to assume that the vector addition method of correcting for residual chromaticity differences between the test and reference sources holds adequately for differences as large as the above. It must be pointed out that although the average vector shift is the same for the two methods, due to the variation of the individual shifts the colour rendering index R_c can be affected. C.I.E. Sample Set 1 gives an R_c for Source V of 95 when the vector addition method is used to allow for adaptation whereas if equation 3 is used an R_c of 98 is obtained. The difference in the other indices will be smaller mainly because the indices are lower and hence the correction for source chromaticity is proportionately less.

TABLE XVI

U.C.S. Data for CIE sample set 1 illuminated by Source V. and corrected for an adaptation shift from Illuminant A. to source V as predicted by Equation 3

Sample No.	Munsell Notation	u	v	u'	v'	Δu	Δv
47	7.5R 6/4	.3069	.3542	.3036	.3526	.0033	.0016
48	5 Y 6/4	.2803	.3612	.2768	.3601	.0035	.0011
50	5 GY 6/8	.2408	.3680	.2370	.3673	.0038	.0007
51	2.5G 6/6	.2049	.3610	.2012	.3590	.0037	.0020
52	10 B 6/4	.2310	.3449	.2277	.3414	.0033	.0035
53	5 PB 6/8	.2288	.3363	.2257	.3318	.0031	.0045
54	2.5P 6/8	.2811	.3387	.2777	.3351	.0034	.0036
56	10 P 6/8	.3140	.3432	.3106	.3404	.0034	.0028
					Means	.0034	.0025

Illuminant A : u = .2557; v = .3495; Δ u = .0037; Δ v = .0024

Source V : u = .2594; v = .3519;

8. COMPARISON WITH OTHER RESULTS

In order to compare the preferred face colours as found by MacAdam (9) (for photographs) and Sanders (10) with the colour found in this experiment it is desirable to show the deviations from the true face colour on a diagram which would take account of the differences in light source colours. To date, no such diagram has been developed. According to Nickerson (31) the only way in which some sort of comparison can be made is by using Adam's Chromatic-Value Diagram (32) which, by virtue of the von Kries type of transformation which places the light source at the origin, allows to some extent for adaptation. It must be borne in mind that this correction is only approximate, as indicated by Judd the discussion of ref.31, where he points out that the transformation is almost certainly wrong as it is based on the CIE primaries, which fall far outside the range of primaries found by Helson (26), Burnham (27 & 28) and MacAdam (24). Burnham (33) compared ten co-ordinate systems to determine which plotted the Munsell renotation network for value 5 most accurately and found that the Adams Chromatic-Value diagram ranked top for hue but only sixth for Chroma or saturation.

Notwithstanding the fact that only the chromaticity co-ordinates of the skin as found by MacAdam and Sanders were known, and an average value of 40% was assumed for Y, fig.20 should serve to give an approximate idea of the shifts found between true and preferred skin colour. Unfortunately it is not possible to include the results of Buck and Froelich (11) as no information is given about adaptation conditions.

From fig.20 it will be seen that both Sanders' and the results of this experiment show a preference for increased saturation. It is likely that Buck & Froelich's results would agree with this as their preferred light source had a S.E.D. similar to the spectral reflectance of the average skin. MacAdam's results on the other hand, show a slight decrease in saturation, which could be due to differences of the conditions of observation, or of the observers' attitudes when observing pictures and the real object. The main discrepancy is in the hue shift.

In the case of MacAdam and Sanders there is a shift towards yellow whereas in our case, if anything, the shift is to the red.

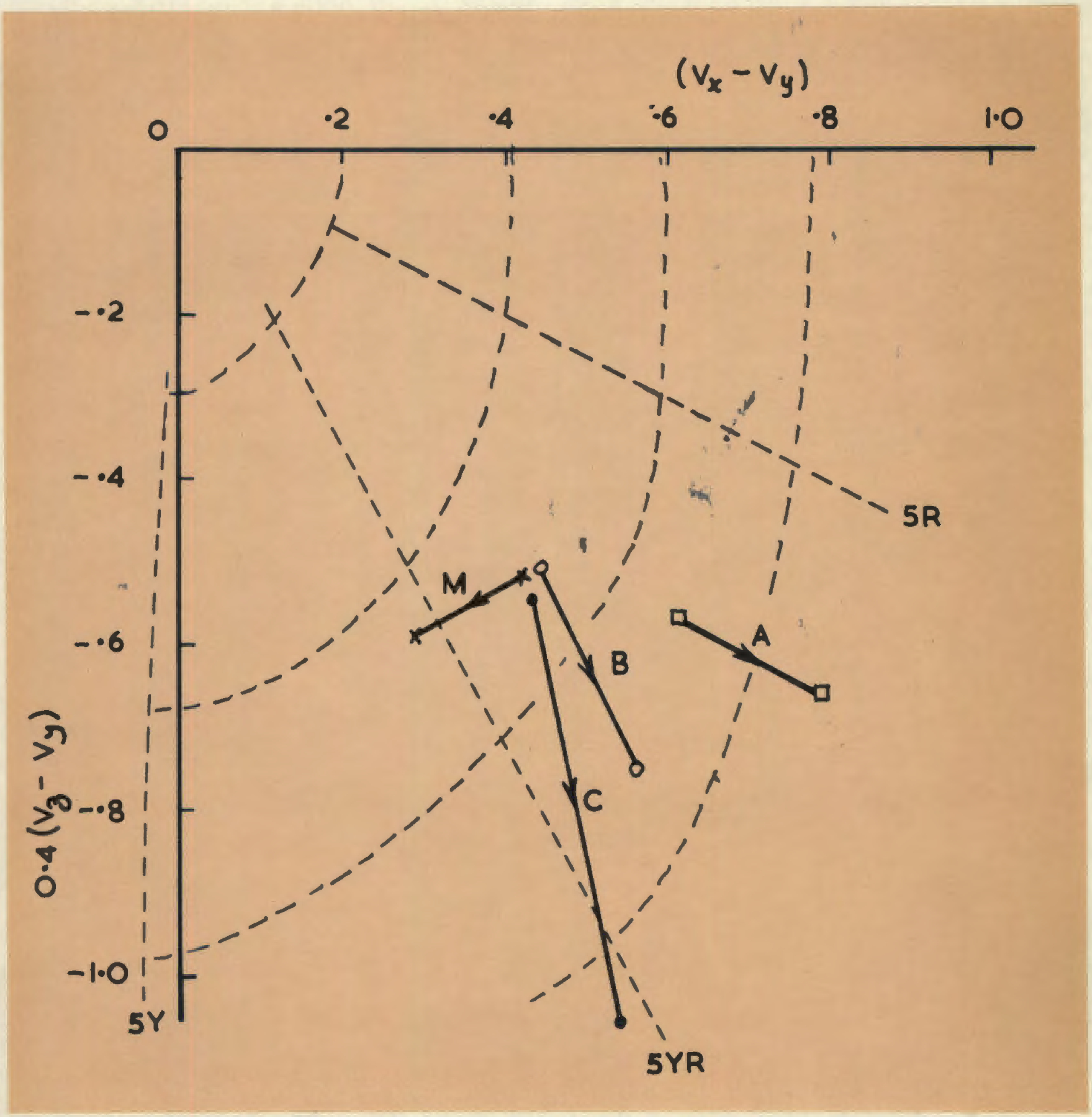


FIG 20.
Discrepancies between true & preferred skin colours shown on Adams' Chromatic-Value Diagram. Arrows point towards preferred colour.

M : MacAdam - photographs (4000°K)
C : Sanders (Illuminant C)
B : Sanders (Illuminant B)
A : This experiment (Illuminant A)

This experiment was designed to determine the preferred energy balance between the red region of the spectrum and the yellow region where the manganese band of calcium halophosphate has its peak. In section 4.2 it was shown that the experimental conditions dictated that the colour shifts lie very nearly along a line of constant hue. Thus it cannot be said with certainty that the same preferred chromaticity would be found if, for example, Sanders' experiment were repeated for Illuminant A, offering the observers changes in hue. The indication is, however, that the result would not differ widely from the preferred chromaticity found in this experiment.

9. CONCLUSIONS

a) A Planck radiator, while being accepted as a reference source for the determination of colour rendering index, is not an ideal source for applications such as social lighting. Here most attractive or pleasant appearance controls the choice of light source, not the most natural appearance. This determination was based on the appearance of the human face and certain foods, and the preferred light source was found to contain an excess of red and green energy at the expense of the yellow energy.

It cannot be stated with certainty that the same results would apply in, for example, European countries. The melanin content of the skin of the model does affect the position of the optimum slightly and it is possible that people in other countries may not concur with the choice of South Africans who are accustomed to a higher melanin content due to exposure to sunlight.

b) Although this work is not concerned with the specification of colour rendering, it does suggest that a good system of colour rendering specification should allow for this fact in some way. It seems anomalous that a light source, chosen by a majority of people, should have a rating some 30% lower than Illuminant A.

c) A fluorescent lamp simulating the preferred source would possibly have a luminous efficiency some 20% lower than one which renders colours "correctly" i.e. one which simulates Illuminant A. This reduction in efficiency is visually inherent, but may be modified either way by the radiation efficiency of the fluorescent powders chosen.

d) Certain foodstuffs may limit the permissible deviation of spectral energy from that of a Planck radiator as excessive distortion could make the food unappetizing. More comprehensive tests on foods would be useful, although the indications are that an S.E.D. similar to that of a source between III & IV would be generally acceptable.

e) No significant differences in taste were shown by various age groups of the population nor did the sex of the observers affect the choice.

On the other hand, due to differences in skin colour, the tastes of different racial groups might be investigated to some advantage.

f) A final point arising from this investigation is the lack of agreement between colour shift of Munsell 5YR 8/4 and that of the skin. It would seem that this colour is not an adequate representation of skin colour as it underestimates the colour shift which would be experienced by the skin under various light sources.

S E C T I O N II.

DEVELOPMENT OF EQUIPMENT
GIVING A CONTROLLED SPECTRUM USING A MONOCHROMATOR
WITH VARIABLE SPECTRUM MASK

SECTION II

10. INTRODUCTION

This section describes the design and construction of a light source, the S.E.D. of which can be predetermined and varied continuously. Lack of funds precluded the construction of apparatus for this purpose alone, so an existing single monochromator (described in App. 1.1) was modified by replacing the detector and exit slit assembly with a variable mask in the spectrum plane. No permanent structural or optical modifications could, however, be effected as the monochromator also had to be used for lamp spectrophotometry. The design of the complete system was further complicated by the limited availability of suitable lenses, lamps, etc. and a number of compromises had to be made.

11. CONSTRUCTION OF THE VARIABLE SPECTRUM MASK.

Fig. 21 shows the construction of the mask with the supporting structure removed for clarify. It consists of 15 sliding shutters - positioned in the spectrum plane, the width of each shutter being arranged to cover a band of wavelengths 20nm wide over the range from 380nm to 620nm.

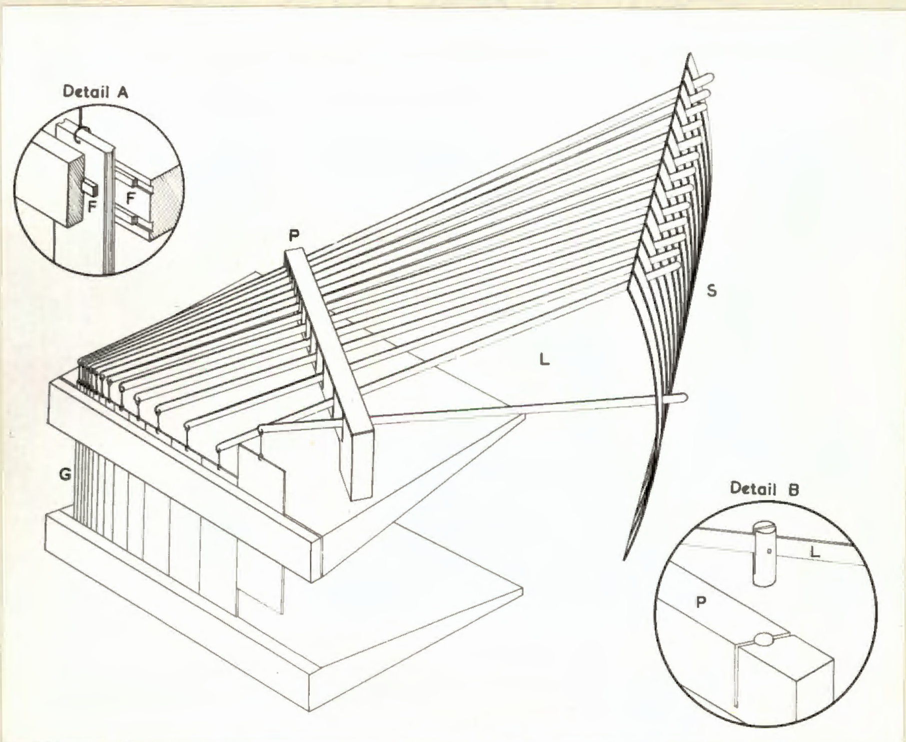


FIG. 21.

Construction of the Variable Spectrum Mask.

The last three shutters are made to cover bands of 40, 40 and 60 nm due to the lower dispersion of the spectrum towards the red region and the consequent practical difficulties involved in making very narrow shutters.

The shutters are made of 1/16" brass and, as a simple butting of the adjacent shutters was found to be ineffective, the adjacent edges have been given shallow tongue and groove joints (see detail A, fig 21). A measure of inaccuracy, amounting in the worst case to a wavelength error of 10% was thus introduced, but this is more than compensated for by the improved mechanical stability of the system. Also shown in detail A are the felt pads (F) which hold the shutters in a vertical plane and, being oil-soaked, lubricate them.

The raising and lowering of the shutters is effected by means of levers (L) pivoted at the fulcrum bar (P). This is positioned a third of the way along the levers to give a scale length of 4" for a shutter movement of 2".

The plane of the scales S is placed at 45° to the plane of the shutters in order to make them more accessible to the operator as well as to avoid obstructing the light beam between the exit lens and the spectrum plane. This necessitated pivoting each lever individually and detail B shows how this was accomplished.

Above the scales is mounted a track to which an electric motor can be fitted when it is required to change the position of a shutter (or group of shutters) at a constant, slow rate.

The motor drives a continuous vertical loop of chain at the desired speed and the levers are attached to the chain at any point by means of small S-shaped hooks.

As the spectrum mask could not be fitted permanently to the monochromator, installation was facilitated by scribing the main mercury and cadmium spectral lines on the backs of the shutters. Positioning of the mask is thus done with the aid of a mercury-cadmium lamp at the entrance slit.

12. RECOMBINATION OF THE SPECTRUM

The spectrum obtained from a monochromator may be recombined either by passing it through another identical monochromator to reverse the process of dispersion, or by reflecting it back through the same monochromator. The latter method was attempted in this case but was not very successful. Considering the layout of the monochromator and the availability of optical components, the most convenient and efficient method appeared to be the use of a large lens immediately behind the spectrum mask. Provided the diameter is larger than the spectrum length, the focal length of this lens can be chosen to give a test field of recombined light of a suitable size at a convenient distance from the monochromator.

12.1 Optical Arrangement

Fig. 22 is a schematic diagram of the optical arrangement of the monochromator and extra lenses as set up for recombination of the spectrum. For the sake of clarity the prisms have been omitted and the ray diagram could be considered as representing a vertical section along the axis or a horizontal section for light of one wavelength. All lenses have been considered as thin lenses as a simplifying assumption which is justified even in the case of the collimator lens for which the nodal points and chief planes were found by the rotation method (37. p.85) These lie ⅛" on either side of the physical centre of the lens system.

Examination of Fig. 22 will show that the system contains two separate and independent sets of conjugate planes. The first consists of the source (F) and the entrance and exit slits S_1 & S_2. The second comprises the planes L_1, L_2 and L_3, the last of which contains the recombined test field.

In order to obtain a clearly defined test field having no coloured fringes it is necessary to place a mask in the plane L_2. Once the position and dimensions of this mask have been established this mask can simply be considered to be an extended source of white light and the test field a projected image of this source.

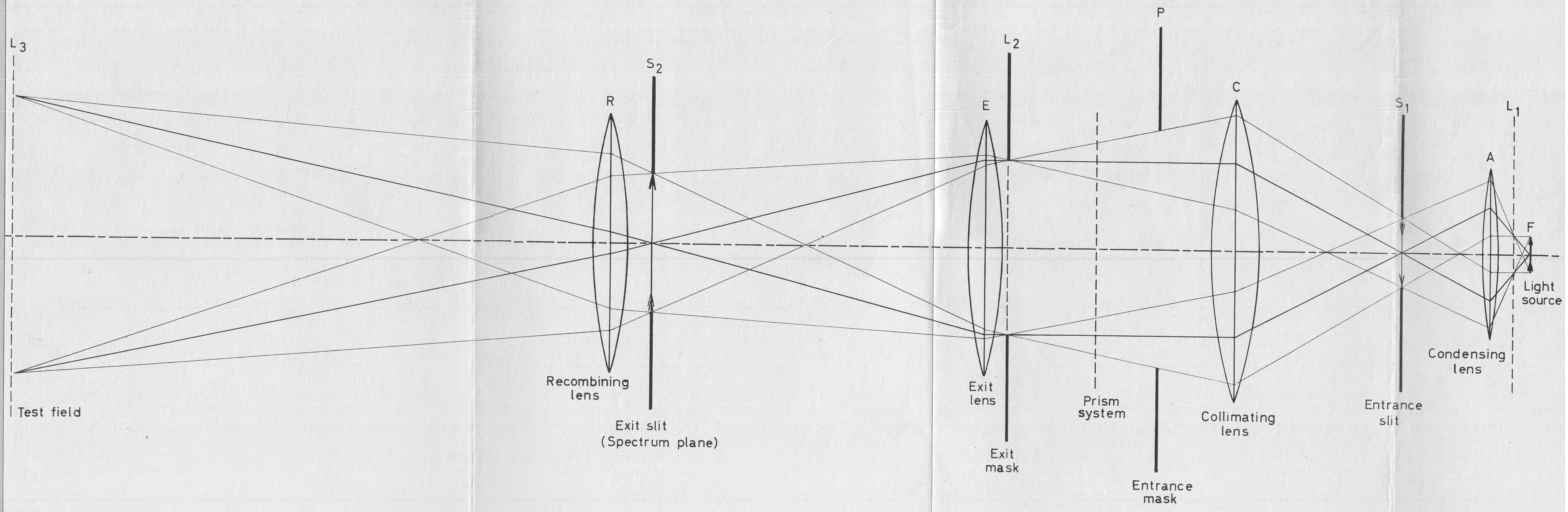

L3
Test field
R
Recombining lens
S2
Exit slit (Spectrum plane)
E
L2
Exit lens
Exit mask
Prism system
P
Entrance mask
C
Collimating lens
S1
Entrance slit
A
Condensing lens
L1
F
Light source

FIG 22

The positions of the shutters in the spectrum plane S_2 will only affect the spectral energy distribution and not the spatial distribution of light over the test field, provided that each point on the light source illuminates the aperture of the mask in L_2 evenly. The criteria for this to be the case are discussed in the following sections.

12.2 Maximum Width of the Mask in L_2

In the horizontal plane the entrance slit is virtually a point source so the beam emerging from the collimator is effectively parallel. As the diameter of the collimating lens is large, the width of the parallel bundle of light entering the prism system is determined by the projected width of the mask on the entrance face of the first prism i.e. 2.5". At the first prism surface, however, refraction takes place and at the position of the focussing lens E, the light beams of wavelengths 400 nm and 700 nm have become linearly dispersed by slightly more than one inch. As it is only in the central region of the field, where these two beams overlap, that all wavelengths are present it follows that for the maximum mask width and the minimum loss of light through masking, the mask should be as close to the prisms as possible.

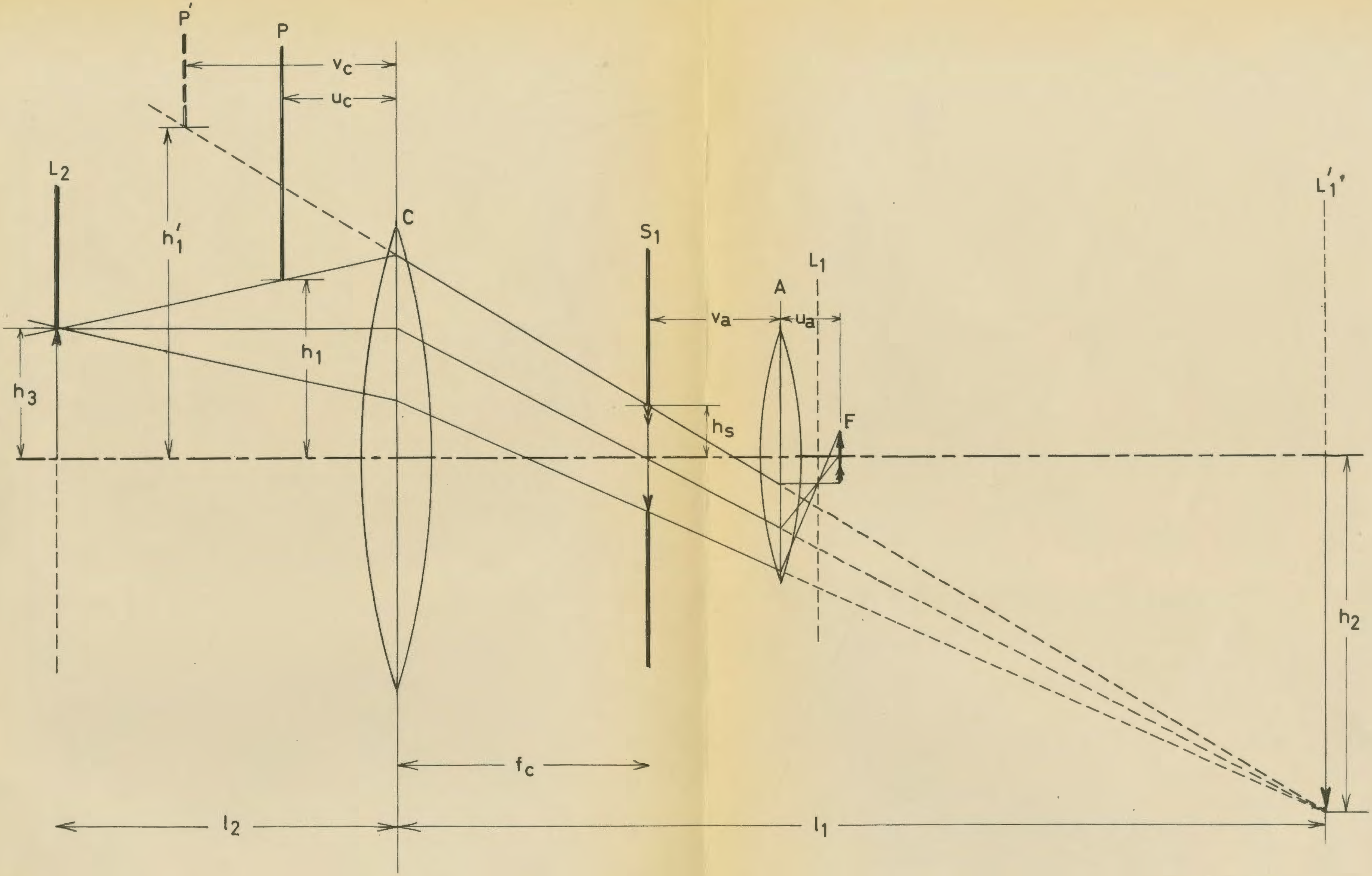


FIG 23

In this case the most suitable position for the mask was found to be 1.5" ahead of the exit lens. At this point dispersion is 1" giving rise to a maximum mask width of 1.5".

12.3 <u>Maximum height of the Mask in L_2</u>

The maximum height of the mask is determined either by the height of the entrance face of the first prism or by the aperture ratio of the condenser lens i.e. that lens which images the light source on the entrance slit. Fig. 23 shows the system as far as the plane L_2 and should be referred to for a key to the symbols used in the following discussion. If for the moment the condensing lens A is ignored, L_1' and L_2 can be considered as the object ahd image planes respectively of the collimating lens C, while the entrance slit S, acts as an aperture stop and the mask at P is the field stop for this system. h_3 is then the maximum height of image which is evenly illuminated by an object at L_1', assuming this to be an extended source of uniform Luminance. For points further than h_3 from the axis vignetting will take place and the illumination drops off (The illumination in fact decreases slightly without vignetting, being proportional to $\cos^4 \alpha$ where α is the angle made by the rays with the axis of the system (38,p.192).

In this case, however, α is less than 5° resulting in a decrease of less than 2% in the illumination),

From Fig. 23 h_3 can be obtained by simple geometry and shown to be : (App. III).

$$h_3 = \frac{h_s\,(u_c - l_2)}{f_c} + h_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

If the mask in L_2 is made larger than this, the aperture will no longer be evenly illuminated and consequently changing the height of a shutter will not affect the test field uniformly.

12.4 <u>Effect of the Relative Aperture of the Condensing Lens A.</u>

In Section 12.3 the condensing lens was ignored and hence the assumption was made that it would be of adequate diameter not to act as a field stop. The minimum diameter required for this to be so can be shown to be : (AppIII).

$$D_a = 2 \ f_a \ (m+1) \ \frac{h_3 f_c - h_s(l_2 - f_c)}{f_c^{\,2}} + h_s \quad \ldots\ldots \quad 2$$

For a given monochromator this reduces to

$$D_a = k \ f_a(m + 1) + 2h_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3$$

Equation 3 was used to calculate the curves shown in Fig. 24 relating required relative aperture ($= \frac{1}{\text{aperture ratio}}$) to the focal length with magnification as parameter for the U.C.T. monochromator.

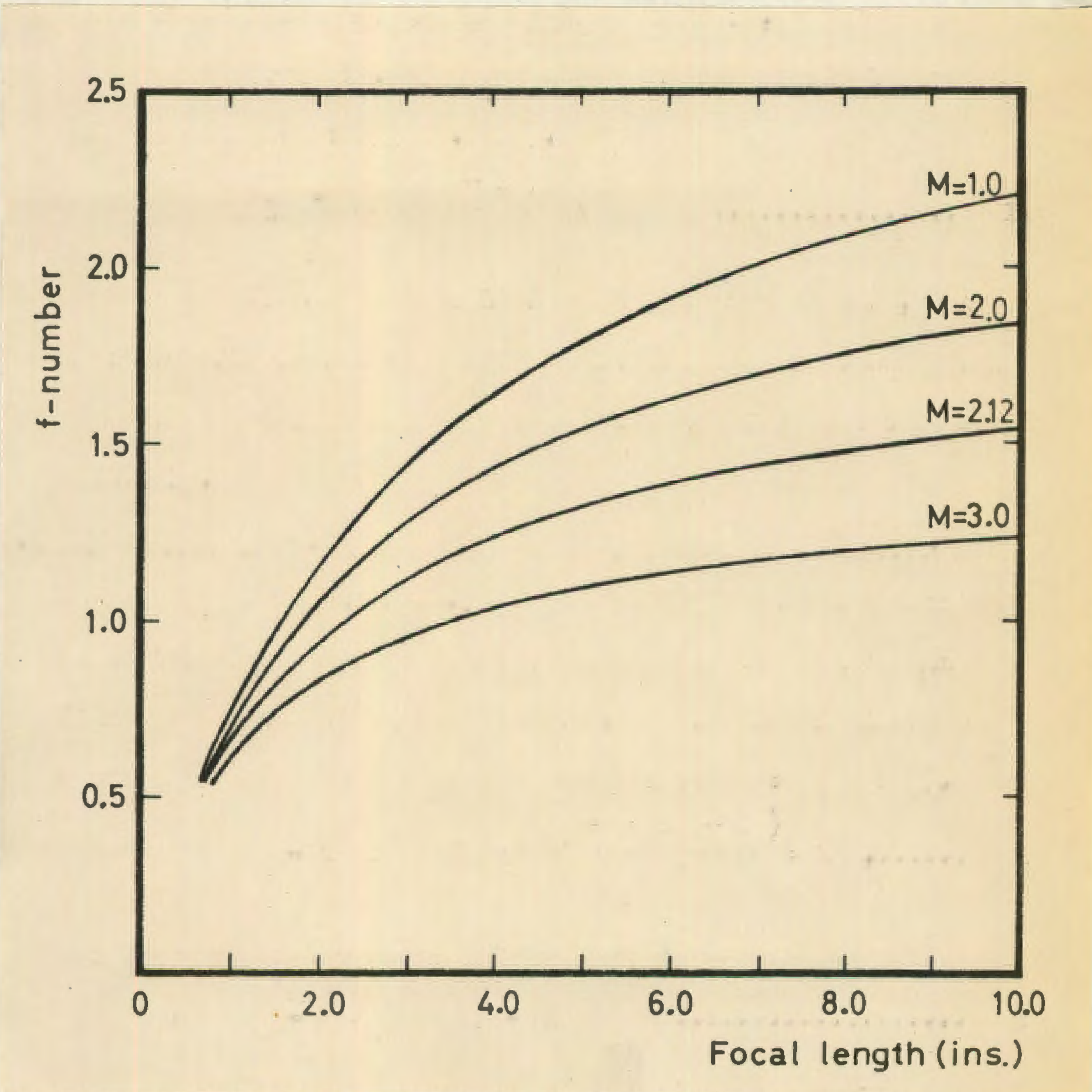


FIG 24.

Maximum Relative aperture (f- number) required of the condensing lens for it not to act as a field stop in the system, given for various values of magnification of the condenser.

If the condenser lens has to be the field stop, the maximum available height h_3 can be obtained by rewriting equation 2 so

$$h_3 = \frac{(\frac{1}{2} D_a - h_s) f_c}{f_a (m + 1)} - \frac{h_s (l_2 - f_c)}{f_c} \quad \ldots\ldots\ldots\ldots \quad 4$$

12.5 Effect of Magnification

Fig. 24. shows that the magnification of the image of the source at the entrance slit has a considerable effect on the aperture ratio required of the condenser lens. In order to reduce this aperture ratio the lamp which acts as a light source should have the longest filament possible. Equation 2 shows that the aperture ratio for a given magnification and focal length can also be reduced by increasing l_2, but apart from decreasing h_3, this is incompatible with the requirements in the horizontal plane i.e. that l_2 should be as small as possible in order to reduce the amount of light lost through dispersion.

12.6 Diameter of the Collimating Lens

If the collimating lens has an insufficiently large aperture, it will become the field stop instead.

The minimum diameter of this lens can be given as: (App III)

$$D_c = 2 h_1 + \frac{h_s U_c}{F_a} \qquad \text{.....................} \quad 5$$

13 THE COMPLETE SYSTEM

13·1 LAMP

The most suitable lamp was found to be a Philips Type No. 6215 E Locomotive headlight lamp (Even these had to be specially obtained from Eindhoven). This is a 32 - 36 volt 250 watt lamp emitting 5200 lumens at a rated life of 500 hours. The filament is a straight coiled coil, 4 mm in diameter and 12 mm long (Ref. Philips catalogue No. 03.512 BE 10/62). These lamps were found to operate reliably at 40 volts, usually exceeding the 50 hour life estimated for operation at the higher voltage. Apart from almost doubling the lumen-output, over-running them increased the colour-temperature to 3100°K, thereby to some extent compensating for the high blue-absorption of the glass components in the monochromator. It was hoped that a suitable iodine cycle quartz lamp might become commercially available in order to increase the light output still further (and to reduce the required magnification by means of a longer filament) but this did not turn out to be the case. As the entrance slit height is fixed at 1" to obtain a 2" high image in the spectrum plane, the magnification required of the collecting lens was 2.12.

13.2 Condensing Lens.

Reference to Fig. 24. shows that a magnification of 2.12 entails using a collecting lens with a relative aperture in the region of 1 to 1.5. A suitable one was found in the form of an f:1 lens with a focal length of 50mm. The lens thus became the field stop (a relative aperture of 0.94 is actually required) and the maximum height of h_3 can be found from Equation 4.

The brightness of the image thus formed at the entrance slit was further increased by a spherical concave mirror with a 4" radius of curvature and 4" in diameter, positioned with the filament at its centre of curvature. The mirror image of the filament on the slit was arranged to fill in the gaps caused by the spacing of the filament coils, thus reducing the striations of the image in the spectrum plane. The image is wide enough (8 mm) to cover the entrance slit completely, even though the slit is curved to allow a straight exit slit

13.3 The Position of L_2

The most suitable position for the mask in the plane L_2 was found to be as close as possible to the exit face of the second prism.

14 MONOCHROMATOR TRANSMISSION

On recombining the spectrum, it was immediately obvious that the transmission factor of the monochromator is not uniform throughout the spectrum. It was thus necessary to establish the spectral transmission curve.

The simplest and most obvious method for doing this was to establish the S.E.D. of the recombined light from a source of known colour temperature by means of a second monochromator. Alternatively, as the transmission curve can be assumed to be smooth, abridged spectrophotometry by means of filters would have offered adequate accuracy. However, neither a second monochromator, nor a set of calibrated filters was available. The method eventually used was in effect a modification of the latter technique. By means of a set of narrow transmission band filters the intensity of the recombined light from the monochromator and the direct light from the source (directed by an achromatic camera lens) were measured. The photomultiplier tube normally used with the spectrophotometer was used as a detector, with suitable neutral filters added to prevent overloading and extra screening added to reduce the effect of stray light. The ratio of the two intensities was taken to be the transmission factor at a wavelength corresponding to the peak of the transmission curve of each filter (Table XVII).

These were adjusted to a maximum of 1.00 as only relative transmission is of interest, and the transmission curve is shown in fig. 25.

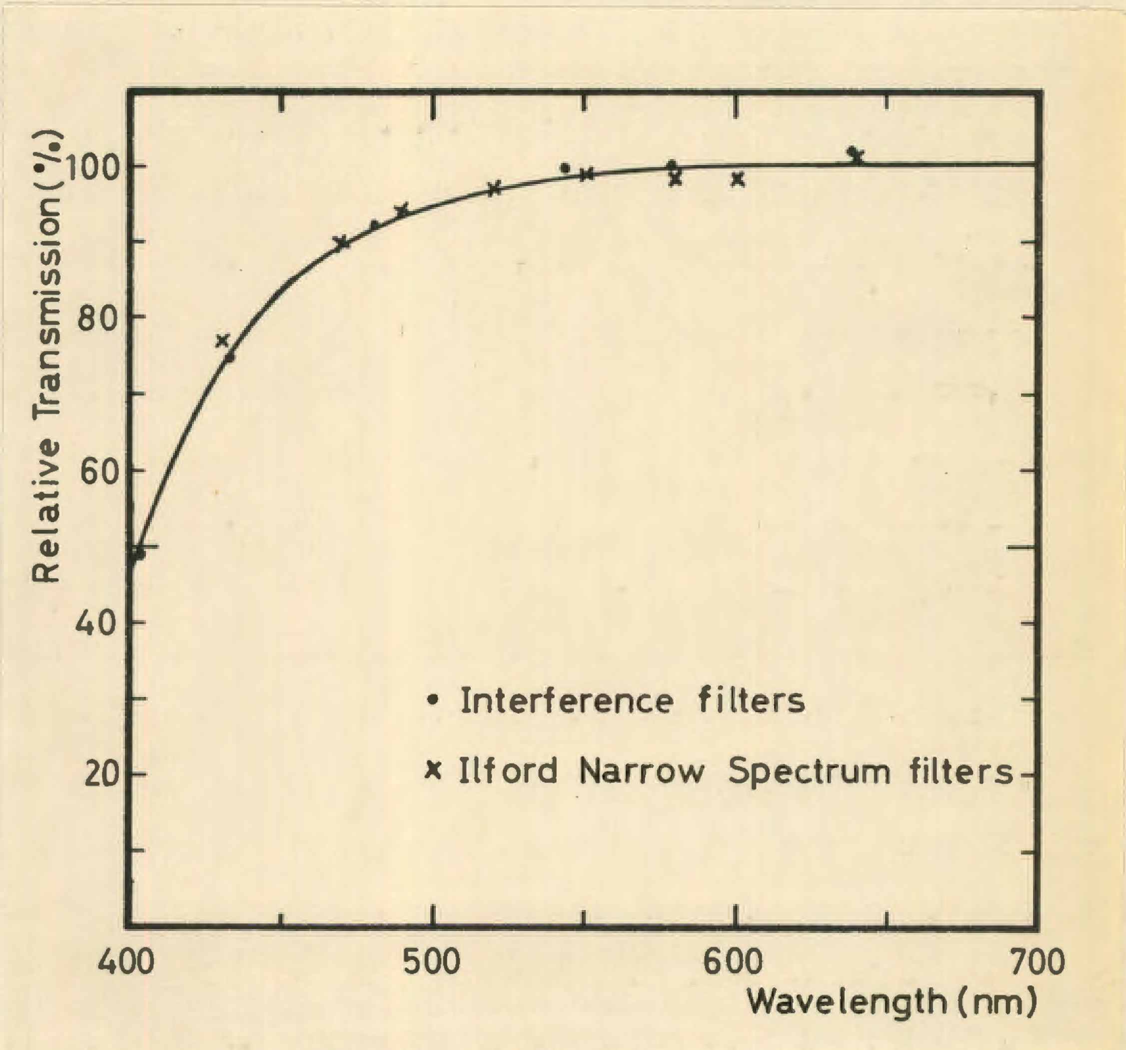


FIG. 25.

Spectral transmission of the Monochromator.

TABLE XVII

Wavelength	Filter	Transmission
406 nm	Interference	0.49
430 "	Ilford 601	0.77
431 "	Interference	0.75
470 "	Ilford 602	0.90
481 "	Interference	0.92
490 "	Ilford 603	0.94
520 "	" 604	0.97
545 "	Interference	1.00
550 "	Ilford 605	0.99
578 "	Interference	1.00
580 "	Ilford 606	0.98
600 "	" 607	0.98
639 "	Interference	1.02
640 "	Ilford 608	1.01

TABLE XVIII

Light Source	Direct		Recombined	
Calculated	x = .447	y = .407	x = .456	y = .416
Measured	x = .446	y = .412	x = .455	y = .422
Std.Deviation	= .004	= .004	= .003	= .003
Corrected for zero error	x = .446	y = .407	x = .456	y = .417

As a check on the results so obtained, the recombined light from a standard 2850°K lamp was allowed to fall on a magnesium-oxide coated surface adjacent to a patch of direct light of the same intensity. The chromaticities of the two patches were measured by means of a tintometer and compared with the calculated chromaticities in Table XVIII.

Each measured point is the mean of 15 observations by three observers and the standard deviations are given. The Tintometer was subsequently found to have a zero error equal to the discrepancy between the measured and the known colour of the direct patch. Allowing for this, the calculated and observed colours for the recombined light agree to within .001 in the C.I.E. co-ordinate system.

Knowing the transmission of the monochromator, and the colour temperature of the light source, it is possible to set the shutters to simulate any required spectral composition.

15. ILLUMINATION OF THE TEST FIELD.

As a result of the lack of information about the lamp, optical components in the monochromator, etc, the expected test field illumination cannot be calculated accurately.

By making a number of arbitrary assumptions, however, it was possible to establish the order to magnitude of this quantity as follows :

Assume the filament to be a uniformly radiating source of colour temperature 3100°K. Then to a first approximation the illumination of the image formed by the condensing lens on the entrance slit can be given by (38, p.190).

$$E = \frac{B\pi}{(m+1)^2} \frac{r}{f}^2$$

where E = illumination in lumens/cm^2

B = Object luminance in stilb

m = Magnification of the image

r = radius of the condensing lens

f = focal length of the condensing lens

Thus the luminous flux passing through the entrance slit is 25 lumens when the slit dimensions are 1.0 mm x 25.4mm. This radiates into a solid angle slightly greater than that subtended by the condensing lens A at the slit (0.08 steradians) while the solid angle subtended by the exit mask L_2 at the entrance slit is 0.0234 steradians. Thus about 7.5 lumens pass through the exit mask and reach the test field as there is no further attenuation through masking.

The above takes no account of losses caused by reflection at the surfaces, and absorption by the glass of the optical components. Martin (38) gives a figure of about four percent reflection loss at each glass-air interface for normal incidence and a transmission loss in the visible region of five percent per centimeter glass thickness for the poor quality glass usually used in condenser lenses. As most of the components in this system are of good quality, however, a transmission loss of three percent per centimeter, and a path length of 15 cm in glass is assumed.

The system contains six components excluding the collimator, which is taken to consist of four lenses (there are probably more, but as they are coated lenses, reflection losses will be considerably lower), making a total of 20 glass-air surfaces. The overall transmission factor will thus be of the order of 0.28. An illumination level of 4.5 footcandles can therefore be expected on a test field of $6\frac{3}{4}$" x 10" when all the shutters are fully open. In practice a value of 4 f.c. was measured when the spectrum was adjusted to resemble that of Illuminant A. i.e. when some of the shutters were partly closed.

16 TESTS USING THE VARIABLE MASK

The apparatus was used to determine whether the band tolerances as found by Crawford (7) for the human face lit by Illuminant B also held for Illuminant A.

The observers looked at their own faces in a mirror, being illuminated by the test field to a level of 4 f.c. Bands selected at random were shut down slowly, the total period of change always being more than three minutes. Observers were asked to indicate the nature of the colour change and, except on two occasions, were always correct. The results given are the means of three tests for each observer, the first test in each case being considered a practice run.

A sufficient number of observations are available from only four observers (fig. 26) so these results are not conclusive (Crawford suggests ten or more observers). The fact that the tolerances for all four observers are considerably lower than those found by Crawford would, however, bear further investigation.

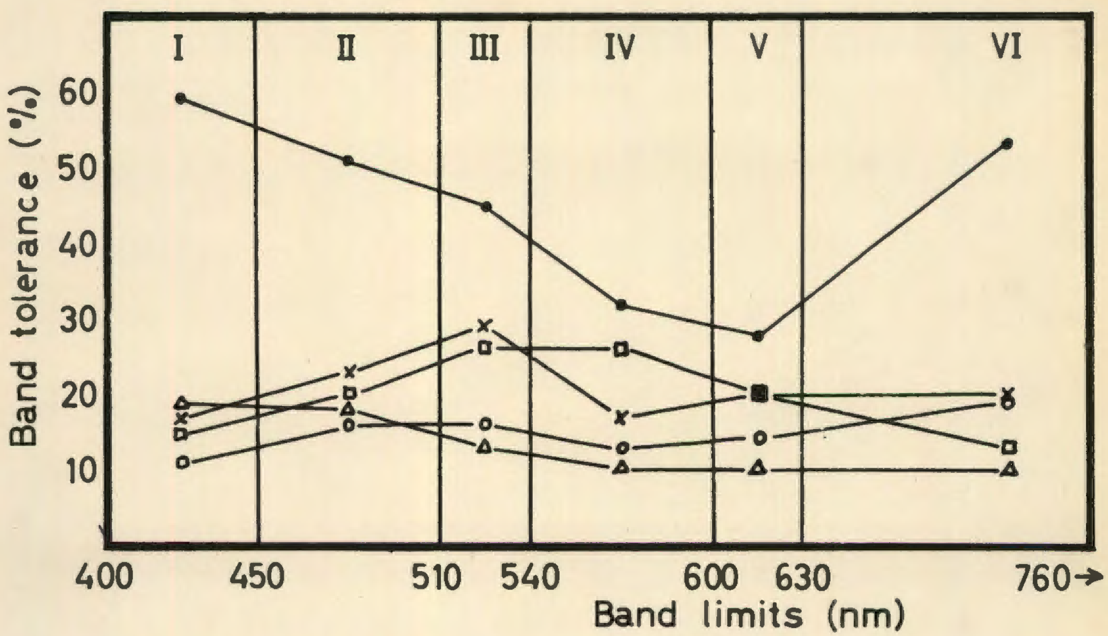


FIG. 26.

RESULTS OF BAND TOLERANCE TESTS FOR FOUR OBSERVERS.

Open circles - P.N. Open Squares - T.B.
Crosses - D.N. Open triangles- M.B.
Full circles - Crawford's mean for all observers.

17. CONCLUSIONS.

An inexpensive and fairly convenient solution was found to the problem of obtaining light of variable S.E.D. The apparatus comprising a light source, monochromator, variable spectrum mask and recombining lens, promises to be very useful for investigations concerning colour rendering, as it is a simple matter to produce any desired spectral distribution of energy in the test source. Metameric sources are easily available and, by means of suitable linking of the operating levers, it is possible to change the colour rendering of the source smoothly while maintaining the chromaticity constant.

S E C T I O N III.

A STUDY OF STRAY LIGHT IN A SINGLE MONOCHROMATOR USING NARROW TRANSMISSION BAND FILTERS AND DISCHARGE LAMPS

Reprinted from JOURNAL OF THE OPTICAL SOCIETY OF AMERICA, Vol. 53, No. 6, 721–728, June, 1963
Printed in U. S. A.

Stray Light in a Monochromator (Analysis and Prediction Based on Measurements by Means of Discharge Lamps and Filters)

H. D. EINHORN AND D. E. H. NAUDÉ
Department of Electrical Engineering, University of Cape Town, Rondebosch, South Africa
(Received 4 January 1962)

Stray light in a monochromator is analyzed. Stray coefficients are defined which allow the assessment, prediction, and correction of stray light errors. A method of measuring these stray coefficients by means of a discharge lamp and filters is described. The effect of slit dimensions is discussed. Measurements on the University of Cape Town single monochromator illustrate the method and indicate the possibilities and limitations of a single monochromator for lamp spectrophotometry.

1. INTRODUCTION

STRAY light is always present in a monochromator and can lead to noticeable errors in spectrophotometric measurements, especially in a single monochromator.

A single monochromator has inherent merits for many applications. It has a higher energy efficiency, which is particularly beneficial for the spectrophotometry of low luminance sources such as fluorescent lamps. It allows simpler, more robust, and cheaper detectors or visual observations at higher luminance levels. Moreover, the single monochromator itself is relatively simple and cheap and can be applied where a double monochromator might be considered uneconomical, hence leading to a wider application of spectrophotometry. If these stray light errors can be assessed for an instrument and its limitations in this respect become known, it could be used with confidence; for a double monochromator too, stray light merits checking in parts of the spectrum where the detector is insensitive.

External stray light, reaching the photometer or other detecting device behind the exit slit from outside can be eliminated by careful screening, but internal stray is inherent and unavoidable. It arises by scattering on glass surfaces, and by internal reflection in prisms, etc. In the case of a 60° prism with all three sides polished, this internally reflected stray appears as a single line[1] but when the third face of the prism is unpolished, light reflected by it appears as a diffused haze over the exit surface; scatter due to dust or scratches on lenses will also create diffused stray light.

There are two methodical approaches to the measurement of stray light:

(i) The total radiation is assessed for a particular measurement. Preston[2] did this by half covering of slits, other authors[3,4] by using a filter with a narrow *absorption* band.

(ii) The stray properties of a particular instrument are established in the form of data which permit the prediction of stray light for any spectrum to be measured. Donaldson[5] and Pritchard[6] used a double monochromator (in addition to the monochromator to be tested).

The new method described in this paper makes use of relatively inexpensive equipment: of filters each of which has a *narrow transmission* band, together with a lamp which has predominantly a line spectrum, to establish a stray coefficient which is a property of the monochromator and independent of the spectrum tested.

σ	stray coefficient defined as stray light received from a particular spectral line relative to the reading of this line, in *percent per cm*.^{2} of nonlimiting slit area.
σ^*	stray coefficient weighted by relative sensitivities of instrument in stray sending and receiving regions, as per Eq. 3, in percent per cmμ.
σ'	stray coefficient for fixed slit height, in percent per cm.
σ''	stray coefficient for fixed slit height and width in percent.
M	measurement or reading in galvanometer divisions.
e	power of a continuous spectrum, in *W per* μ *per cm*.^{2}.
E	power of a spectral line, in *W per cm*.^{2}.
ψ	instrumental coefficient, dependent on sensitivity of photo cell and on optics of monochromator, in galvodivisions per W.
ψE	reading per unit slit area.
D'	limiting slitwidth in cm.
D''	nonlimiting slitwidth in cm.
H'	limiting slit height in cm.
H''	nonlimiting slit height in cm.
λ	wavelength in μ.
x	scale setting in spectrum, in cm (on the UCT spectrophotometer, in scale divisions of 0.212 cm).

[1] H. D. Einhorn and A. E. Z. Cohen, J. Opt. Soc. Am. 44, 232 (1954).
[2] J. S. Preston, J. Sci. Instr. 13, 368 (1936).
[3] H. H. Cary and A. O. Beckman, J. Opt. Soc. Am. 31, 686 (1941).
[4] T. R. Hognes, F. P. Zscheile, and A. E. Sidwell, J. Phys. Chem. 41, 379 (1937).
[5] R. Donaldson, J. Sci. Instr. 29, 150 (1952).
[6] B. S. Pritchard, J. Opt. Soc. Am. 45, 356 (1955).

$p = dx/d\lambda$ dispersion in cm/μ.
$K = \psi/p$ sensitivity factor for continuous spectrum in relative deflection per (W/μ).

Subscripts

- u refers to "useful" spectrum region, where a measurement is made. (It corresponds to the "stray receiving" region in the stray measurement tests.)
- s refers to stray-sending spectrum region.
- su refers to stray light from "S" to "U" region.
- c refers to a continuous spectrum.
- l refers to a spectral line.
  E.g., M_{sl}—stray sending line reading.

2. DEFINITION OF STRAY LIGHT

Considering the experimental spectrum as the distribution of energy or light on a spectral surface, we can define stray light as "light out of place," and can distinguish between the "useful energy" which is that of the wavelength range to be measured for any particular λ setting (x) of the instrument, and the "stray energy" or "stray light" which is that of other wavelengths received at position x.

This distinction is not easy and is to a certain extent arbitrary. For instance, in the case of lamp-spectrophotometry, the width of one of the eight CIE bands might be taken as the "useful" region with the implication that stray light from within a band would have no serious effect on conclusions regarding color rendering, etc. The ± 10- to ± 25-mμ tolerance based on this reasoning is of the same order as the stray boundaries of ± 20 mμ proposed by Donaldson[5] and implied by Pritchard.[6]

The magnitude of the error due to stray light depends on:

(i) the relative amounts of spectral energy in the spectrum;
(ii) the spectral response curve of the photocell;
(iii) the degree of scattering, diffusion, and unwanted reflection taking place in the optical system which we express by a *stray coefficient* σ;
(iv) the slit dimensions.

Stray light is likely to lead to large errors in parts of the spectrum where the energy to be measured is relatively low, the sensitivity of the detector is relatively low, and where the stray coefficient is high for powerful radiations in the same spectrum to which the detector is sensitive. For continuous spectrum readings, stray light is most likely to be serious where the dispersion p_u is large.

Generally, stray light is most likely to be serious where the reading is relatively small compared with readings in the rest of the spectrum. The seriousness of stray light in a particular spectral region can be reduced by inserting a filter mainly transmitting in this region.

3. INFLUENCE OF SLIT DIMENSIONS

The relative effect of stray light is influenced by slit dimensions. It is useful to distinguish "limiting" slitwidth D' and limiting slit height H' and "nonlimiting" dimensions D'' and H''. Either entrance *or* exit dimensions can be limiting, whichever are optically smaller. ("Optically" means that the entrance slit dimensions must be multiplied by the optical magnification to become comparable with the exit slit dimensions.) The following statements will show the significance of these concepts:

(a) The reading obtained from a spectral line is entirely a function of the limiting dimensions H' and D'.

(b) A reading of a continuous spectrum section, is a function of the limiting height H' and both widths D' and D''.

(c) The stray light component in any reading is a function of all four slit dimensions H', H'', D', D''.

(d) The *relative* effect of stray light increases, therefore, with the nonlimiting height H''. Reducing stray light and obtaining high sensitivity are conflicting requirements since both increase with slit height.

(e) The *relative* effect of stray light on a line reading increases with D'' and H''. Equal entrance and exit slits minimize stray light error, but can lead to errors due to imperfect focusing.

4. THE STRAY COEFFICIENT AND ITS MEASUREMENT

To assess and predict stray light, *stray coefficients* σ can be defined as stray light received from a particular spectral line relative to the power of this line, in *percent per cm*^{2} of nonlimiting slit area. Stray coefficients are the property of the instrument and independent of the spectrum tested. Since a stray coefficient establishes relations between two spectral regions, there is an infinite number of stray coefficients for each spectrophotometer, which are best represented by curves.

To understand the measurement of σ, consider a purely monochromatic beam; a line S of wavelength λ_s, is sent into a spectrophotometer. Spectral measurements should then be zero at all wavelengths except at a setting corresponding to λ_s. If readings are obtained elsewhere they are evidently stray light from the line S, and by comparing them with the reading at λ_s, the stray coefficient σ can be assessed.

The method proposed for measuring σ requires (i) a light source which has a *line spectrum* and (ii) a number of *narrow band filters*, sharp enough for each to isolate a single line, or perhaps a few lines close together. For most of the experiments performed a mercury–cadmium lamp was used with a number of interference filters, or other filters which had a narrow transmission range. A helium lamp would probably have been equally suitable. A sodium lamp gave an additional useful set of readings (see Appendix 3). It is advisable to measure stray light with unequal slits, whereby the stray light effect is magnified and errors due to imperfect focusing minimized.

5. EVALUATION OF STRAY COEFFICIENTS FROM MEASUREMENTS

To evaluate σ we require the following readings:

(i) a stray light reading M_{su} in the spectrum position λ_u, in the "useful" or "receiving" R region measured with a filter transmitting only in the "sending" S region, which includes a powerful line S;

(ii) the sending line reading itself M_{sl} with the same filter, augmented by

(iii) additional readings in its vicinity M_{sc} which allow for the continuous spectrum background passed by the filter and which should be relatively small. The stray coefficient in %/cm² is then:

$$\sigma = 100M_{su} \Big/ D''H''\left(M_{sl} + \int M_{sc}dx/D''\right). \quad (1)$$

A graph of readings against setting x gives a curve of the type shown in Fig. 4. The integral in Eq. (1) can be evaluated either by adding readings taken at spacing D'' or by graphical integration of the area under the curve in the S region (see Fig. 5).

$D''H''$ are the nonlimiting slit dimensions. If the slit heights are fixed, H'' need not be known and we can work with a modified stray coefficient $\sigma' = H''\sigma$ (in % per cm). If slitwidths as well as heights are fixed, a further modification may be convenient: $\sigma'' = H''D''\sigma$ (in %). The nonlimiting width D'' is still required, however, for evaluating the correction integral in Eq. (1).

If the transmission of the stray measurement filter at λ_u is not negligible a correction has to be made given in Appendix 2. This should only be necessary for assessing "near-stray", i.e., if λ_s and λ_u are relatively close, with a filter which is not narrow enough.

6. STRAY SENDING AND STRAY RECEIVING CURVES

With any one filter a curve, called a "stray sending curve" since it shows stray sent from a particular region of the spectrum, can be measured.

Some of these curves measured for the University of Cape Town, single monochromator type, spectrophotometer (see Appendix 3) may serve as examples and are shown in Figs. 1 and 2. The stray coefficient σ is plotted against receiving spectrum position, with the sending positions as parameter, namely for mercury blue and green and the cadmium red in Fig. 1, and for the mercury violet and yellow and two cadmium turquoise lines combined in Fig. 2.

It can be seen that stray light decreases with distance from its source. With the filters available, the correction method of Appendix 1 was only required for assessing near-stray in the vicinity of the stray source, e.g., for $36 < x < 49$ for the green line. Whether correction is necessary can be checked by observations at a known line in the stray receiving region: if the line disappears, any energy measured can be diagnosed as stray light.

From the stray sending curves we can derive a more useful set of curves shown in Fig. 3, called "stray receiving curves," since they show for any position of the spectrum stray received from any other part. In Fig. 3 the abscissa refers to sending position with arbitrarily selected receiving positions as parameters. Each receiving curve obtains from each sending curve one point; it seems adequate to measure about a half-dozen sending curves to establish the receiving curves.

7. STRAY LIGHT ASSESSMENT BY MEANS OF σ CURVES

The relative stray light at any wavelength (λ_u), for which a stray receiving curve (such as shown in Fig. 3) has been drawn, can now be assessed for any particular test spectrum, e.g., that of a particular lamp for which

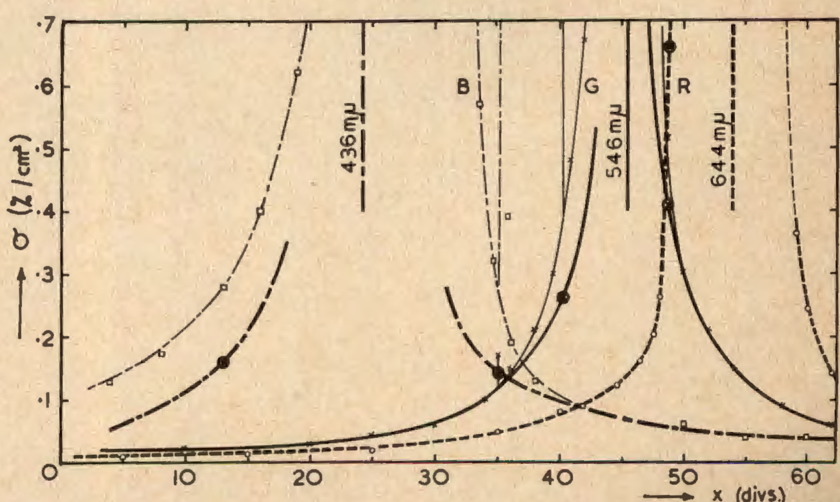


FIG. 1. Stray sending curves. Stray received in position x, sent from lines B ($x=24.5$: $\lambda=436$ mμ); G(45.3: 546 mμ); R(53.7: 644 mμ). x is in scale divisions of 0.212 cm. Thick lines are corrected curves (see Appendix 2), thin lines uncorrected from readings.

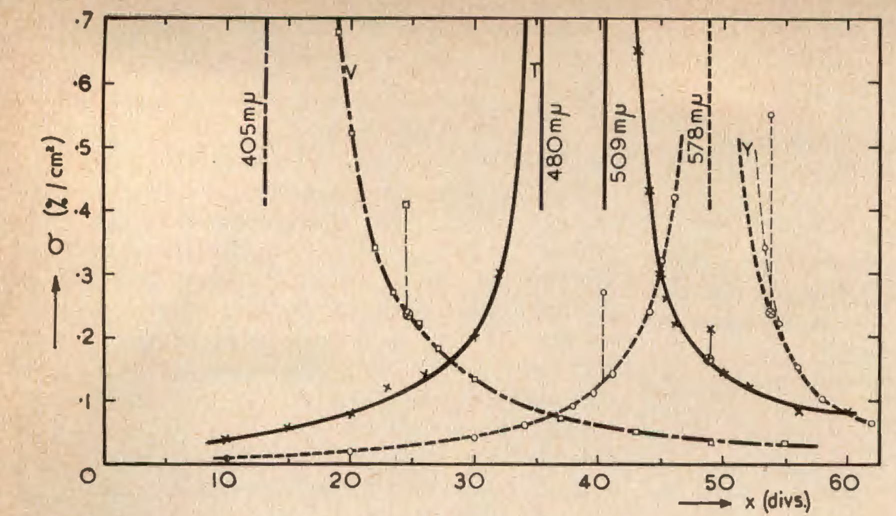


FIG. 2. Stray sending curves. Stray received at x, sent from lines V(13.5: 405 mμ and 14.6: 409 mμ); Y(48.7: 478 mμ); T(35.2: 480 mμ and 40.2: 509 mμ with nominal stray origin at center of energy 38.4).

an uncorrected set of spectral measurements, M_{sc} and M_{sl}, has been obtained.

These spectral test readings are simply weighted (multiplied) by the stray coefficient σ and suitably added up or graphically integrated:

$$M_{su} = H'' \int \sigma M_{sc} dx + D''H'' \sum \sigma M_{sl}. \qquad (2)$$

Here M_{sc} and M_{sl} are readings over the whole test spectrum with constant slit.

The integral part refers to a continuous spectrum and can be replaced by a suitable summation of equidistant ordinates; very few will suffice, since great accuracy is not usually required.

As before, H'' and D'' refer to the nonlimiting slit dimensions. And since σ is a percent value by definition, the ratio of M_{su}/M_{uc} gives the relative stray light content in *percent*.

Figure 3 is used when *readings* have been taken with the spectrophotometer. They are multiplied by the values of one of the σ curves. The area under a weighted curve thus obtained represents the stray light for λ_u.

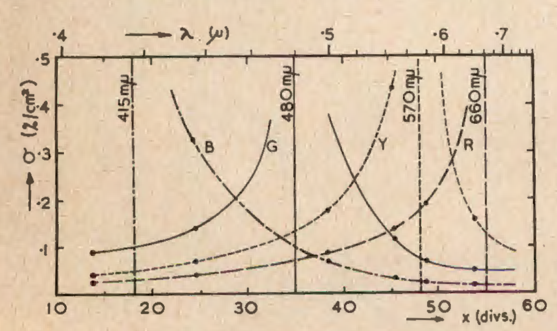


FIG. 3. Stray receiving curves (σ). Stray received at positions corresponding to 415, 480, 570, and 660 mμ from other spectrum regions of position x (in divisions of 0.212 cm).

If all slit dimensions are fixed:

$$M_{su} = \int \sigma'' M_{sc} dx / D'' + \sum \sigma'' M_{sl}. \qquad (2a)$$

8. STRAY LIGHT PREDICTION BY MEANS OF σ^* CURVES

If measurements have not been taken yet, but it is desired to predict the stray light for a spectrum which is approximately known, e.g., a published spectral energy curve for a particular lamp type, a modified stray coefficient σ^* would be more useful which allows for the relative sensitivity of the spectrophotometer.

Figure 5 shows, plotted against wavelength λ, this modified stray coefficient.

$$\sigma^* = \sigma p_s K_s / K_u \text{ in } \%/(\text{cm } \mu), \qquad (3)$$

where p_s is the dispersion (in cm/μ) and K_s and K_u are relative calibration factors. They are obtained from readings M_{st} with a standard lamp (which has a known continuous spectrum e_{st}), namely, $K = M_{st}/e_{st}$. Only relative values of K need be known.

The σ^* curves in Fig. 4 sometimes cross in a peculiar manner, e.g., the curve for a receiving position 660 mμ lies in parts above the 570-mμ curve. This is due to the low red sensitivity of the particular phototube outweighing the proximity effect.

The relative effect of stray light for a continuous spectrum reading at λ_u is then:

$$M_{su}/M_{uc} = H'' \int \sigma^* (e_s/e_u) d\lambda + H'' \sum \sigma^* E_s/e_u (\%). \quad (4)$$

If lines in a mixed spectrum are plotted as rectangles, as is commonly done for fluorescent lamps, the second term in Eq. (4) can be omitted.

Figure 5 is used when spectral *energies* are known. They are multiplied by the σ^* values and the stray reading at λ_u will be proportional to the area under the weighted curve thus obtained.

The graphical integration can be carried out by splitting the area under the weighted curve into broad wavelength strips. If a stripwidth of 40 mμ is chosen the method becomes somewhat similar to Prichard's K-factor method. If all slit dimensions are fixed,

$$M_{su}/M_{uc}=\int \sigma^{*\prime\prime}(e_s/e_u)d\lambda/D''+\sum \sigma^{*\prime\prime}E_s/(e_uD''). \tag{4a}$$

9. EXPERIMENTAL COMPARISON OF METHODS

The results of the σ method proposed were checked experimentally against the Preston method[2] for the mercury–cadmium lamp, and for a "4500°K" fluorescent lamp. The latter results are given in Table I.

Agreement within 20% of the stray coefficient value was achieved in most cases checked; the occasional bigger discrepancies were never in excess of 50%. Considering the quite different principle of assessment when using Preston's half-covered slits and the σ method, the agreement is good.

The greatest uncertainty in the σ method is due to difficulties in the assessment of "near-stray," i.e., from nearby regions of the spectrum. The importance of near-stray varies with applications.

10. APPLICABILITY OF SINGLE MONOCHROMATOR

The method was originally developed to assess the limitations of a single monochromator, particularly when used for lamp spectrophotometry. The results indicate that a single monochromator can be used for this purpose if stray light is checked by measurement and if reasonable precautions are taken. Measurements before and after showed that cleaning the glass surfaces reduced stray light to about one-half for the middle regions of the spectrum, to less than one-third in the more critical end regions.

When measuring a fluorescent lamp with the University of Capetown spectrophotometer, stray light is negligible in the center region of the spectrum (see Table 1), but a red filter is used to keep the stray light error reasonable for measurements above say 600 mμ where the spectral energy of the lamp is weak and the sensitivity of the photocell used low.

TABLE I. Comparison of results by two methods.[a]

λ (mμ)	415	480	570	660
x (Div.)	18	35	48	55
Stray light[b] by σ method	5.8%	1.1%	0.4%	5.6%
Stray light[b] by Preston method	5.2%	1.3%	0.5%	4.0%

[a] Second significant figures are uncertain. Test lamp: 4500°K standard fluorescent lamp.
[b] In percent of the "useful" reading.

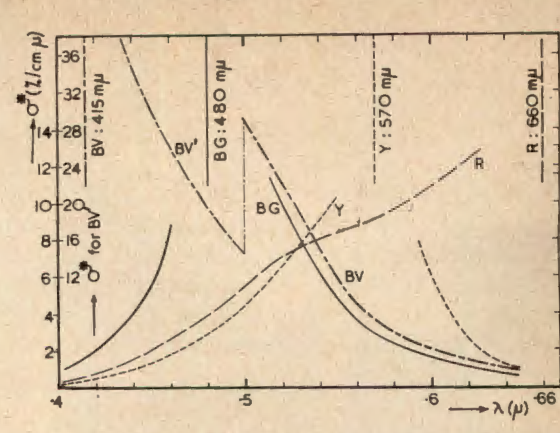


FIG. 4. Sensitivity-weighted stray receiving curves (σ^*). Stray received at wavelengths 415, 480, 570, and 660 mμ from other spectrum regions of wavelength λ_s.

11. CONCLUSIONS

The results of the investigations described can be summarized as follows:

Stray light can be assessed and predicted by means of stray coefficients which are a measure of the stray light from one region of the spectrum to another.

These stray coefficients can be measured by relatively inexpensive means: discharge lamps and a number of filters.

Stray coefficients have been defined for two different tasks, one (σ) referring to readings, the other (σ^*) to expected energies.

The effect of slit dimensions is important when comparing line and continuous spectra and especially when dealing with a mixed spectrum. Modified definitions of stray coefficient for instruments where slit dimensions are fixed are also given (in Sec. 5.)

A single monochromator can be used for lamp-spectrophotometry; filters may be required to isolate regions of the spectrum, where the test source is weak or the detector sensitivity is low, and stray coefficients can indicate whether this is necessary for achieving a desired degree of accuracy.

ACKNOWLEDGMENT

A grant for running expenses by the South African Council for Scientific and Industrial Research is gratefully acknowledged.

APPENDIX 1. QUANTITATIVE TREATMENT

A1.1. Quantitative Formulation

Let us call the power which we wish to measure at a particular wavelength setting λ_u "useful", and denote it by e_u (in W per μ per cm^2) for a continuous spectrum, and by E_u (in W per cm^2) for a line. The reading which would correspond to this useful power is for a continuous spectrum:

$$M_{uc}=D'D''H'\psi_u e_u/p_u. \tag{5a}$$

For a line measurement it would be:

$$M_{ul}=D'H'\psi_u E_u. \tag{5b}$$

D', H' are limiting, D'', H'' nonlimiting slit dimensions.

The reading actually obtained will be larger than it should be, because it contains stray light; let us denote by M_{su} the measurement component of stray light at λ_u originating from the "stray sending" spectrum position λ_s.

Then, for stray from a continuous part,

$$dM_{suc}=\sigma D'D''H'H''\psi_s e_s d\lambda_s, \tag{6a}$$

and for stray from a line,

$$M_{sul}=\sigma D'D''H'H''\psi_s E_s. \tag{6b}$$

In these equations, σ is the stray coefficient for radiation of wavelength λ_s straying into spectrum position λ_u.

Equations (5b) and (6b) allow a determination of the dimension of the stray coefficient.

If the line power is given in radiant W per cm², ψ in divisions of instrument deflection per W, M in divisions, then σ has the dimension cm^{-2} and can be expressed as "fractional per cm²" or more conveniently in "*percent per cm²*."

One can verify that this satisfies not only Eqs. (5b), (6a), and (6b) dimensionally but also Eq. (5a) if p_u is the dispersion in (cm/μ).

A1.2. Relative Magnitude of Stray Light

For any particular measurement the relative error due to stray is of interest, namely, the ratio

$$\frac{M_{su}}{M_u}=\left(\int dM_{suc}+\sum M_{sul}\right)\bigg/M_u. \tag{7}$$

Preston's method[2] yields this error. As seen in Eqs. (5) and (6) it depends partly on the test spectrum itself and partly on instrumental constants.

The slit dimensions and spectral sensitivity are known or can be ascertained. The stray coefficient σ can be measured by the method explained in Secs. 4 to 6.

By substituting (5) and (6) into (7) relations can be obtained which verify the statements made in Sec. 3.

A1.3. Line as Sole Stray Source

In the ideal stray measurements, a powerful line at λ_s is perfectly isolated by the filter and forms the source of all stray light. The stray sending energy E_s is then determined from a single reading at the setting λ_s,

$$M_{sl}=D'H'\psi_s E_s. \tag{8a}$$

Since the reading for any other setting λ_u of the monochromator is in this case M_{sul}, as given in Eq. (6b), the stray coefficient is simply,

$$\sigma=100M_{sul}/(D''H''M_{sul})\ \text{(in \%/cm}^2\text{)}. \tag{9}$$

A1.4. Line Plus Continuous Spectrum as Stray Source

In practice, a certain component M_{suc} of the stray light originates from continuous spectrum energy which the filter usually transmits. This stray sending energy can be obtained from readings in the vicinity of λ_s,

$$M_{sc}=D'D''H'\psi_s e_s/p_s. \tag{8b}$$

If the filter is narrow this continuous energy is small compared with the line energy and the stray coefficient pertaining to it is the same as for the line. The stray reading at any setting U can now be considered as a combination of two readings,

$$\begin{aligned}M_{su}&=M_{sul}+M_{suc}\\&=\sigma D'D''H'H''\psi_s\left(E_s+\int e_s d\lambda\right)\\&=\sigma D'D''H'H''\psi_s\left(E_s+\int e_s dx/p_s\right).\end{aligned} \tag{10}$$

By substituting measured values for energies, from (8), and $dx=pd\lambda$, we obtain Eq. (1) for the stray coefficient,

$$\sigma=100M_{su}\bigg/D''H''\left(M_{sl}+\int M_{sc}dx/D''\right)\ \text{(in \%/cm}^2\text{)}.$$

The integral in the denominator looks more alarming than it is in practice. A graph of readings against setting x in the spectrum plane gives a curve of the type shown in Fig. 5. The area under the curve in the vicinity of x_s represents the continuous spectrum component $\int M_{sc}dx$, and either by graphical integration and dividing by D'', or by adding readings of M_{sc} taken at spacings D'', the second term of the denominator is obtained. If, with very narrow filters, the height M_{sc} is not obvious, it can be obtained from measurements without any filter multiplied by the transmission factor of the latter.

The integration limits can be decided by inspection of the curve in Fig. 5 or from a knowledge of the filter used. High accuracy of the assessment is not required, since the integral term in Eq. (1) is only a fraction of the line reading M_{sl} if filter and lamp are chosen suitably for the test.

A1.5. Check on Filter Density

A σ curve is easily measured by a single set of readings if the filter used has negligible transmission over the wavelength range (λ_u) explored and a reasonable transmission factor at λ_s.

This is checked by observing readings in the vicinity of a powerful known line in the stray receiving region; if the line disappears, any energy measured can be safely diagnosed as stray light originating at S. If not,

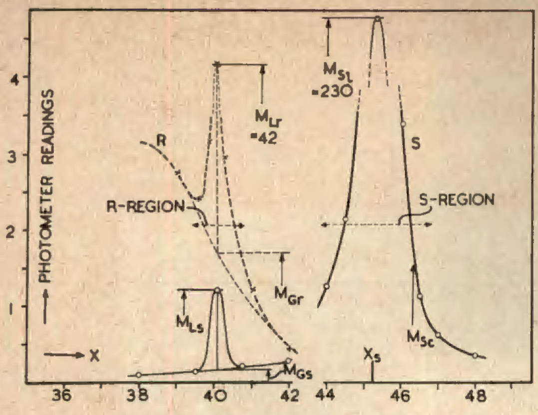


FIG. 5. Typical stray measurements. Full line: readings with green filters to obtain stray from mercury 546-mμ line. Note: $M_{sl} \gg M_{sc}$. Broken line: readings with blue-green filters to correct at 509 mμ (see Appendix 3). Note: $M_{Lr} \gg M_{Gr}$. From tests with $D''=0.6$ divisions.

correction of the M_{su} reading is possible by means of additional readings, as shown in Appendix 2.

APPENDIX 2

If, in a stray-light test, a line L in the stray receiving region is observable, it indicates that the filter used is not perfectly dense there, and the magnitude of the line's reading relative to the gap readings on either side can be utilized for correcting the stray light measurement. Any line in the mercury–cadmium lamp can therefore serve either of two purposes, namely, to act as a stray source or to disclose and correct imperfection of a filter used.

This correction method consists in taking five readings by means of *two* filters; the one (let us call it "S" filter) transmits mainly in a region of a powerful line which forms the *source* of stray radiation, the other (let us call it "R" filter) transmits mainly in the region where the stray light *received* is to be measured.

The term "G" reading means here a value obtained from readings in the spectrum gaps between lines and is defined as the reading which would be obtained in the wavelength position of the line if the line energy were eliminated; it may be obtained graphically by plotting readings taken on both sides of the line, drawing a curve through these points and reading off the value at the wavelength of the line (see Fig. 5).

Let us adopt the following additional symbols for the purpose of this appendix (see Fig. 5):

"R" region is the stray receiving region containing line L.
"S" region is the stray sending region containing line S.
M_{Gr} "G" reading with "R" filter (which transmits mainly in "R" region).
M_{Lr} "L" line reading with "R" filter. Similarly M_{Gs} and M_{Ls} readings with "S" filter (which transmits mainly in "S" region).
M_{Ss} reading of stray source with "S" filter.
F_G $D_r'H_r'\psi_G e_G$, the continuous spectrum energy at R (as evaluated by instrument).
F_{SR} $\sigma D_s'D_s''H_s'H_s''\psi_s E_s$, stray light from S to R region.
F_S $D_r'H_r'\psi_s E_s$, stray source energy, which consists of the energy of a powerful line S plus some energy of the continuous spectrum in its vicinity transmitted by the "S" filter.

All F values are energies as evaluated by instrument, i.e., components of readings.
t_r is the transmission factor of R filter in R region.
t_s is the transmission factor of S filter in S region.
t_{sr} is the transmission factor of S filter in R region.

$$(t_s \gg t_{sr})$$

Then the following relations express the quantities measured:

$$M_{Lr}=t_rF_L+t_rF_G, \tag{11}$$

$$M_{Gr}=t_rF_G, \tag{12}$$

$$M_{Ls}=t_{sr}F_L+t_{sr}F_G+t_sF_{SG}, \tag{13}$$

$$M_{Gs}=t_{sr}F_G+t_sF_{SG}, \tag{14}$$

$$M_{Ss}=t_sF_S. \tag{15}$$

From Eqs. (11–15) follows:

$$\frac{F_{SG}}{F_S}=\frac{M_{Gs}}{M_{Ss}}-\frac{M_{Ls}-M_{Gs}}{M_{Lr}-M_{Gr}}\frac{M_{Gr}}{M_{Ss}}; \tag{16}$$

$$\therefore \quad \sigma=\frac{100}{D''H''M_{Ss}}\left(M_{Gs}-\frac{M_{Ls}-M_{Gs}}{M_{Lr}-M_{Gr}}M_{Gr}\right). \tag{17}$$

If the slit settings for the tests with different filters were different, namely D_s'', H_s'' for the "S" filter test, D_r'', H_r'' for the "R" filter test, then

$$\sigma=\frac{100M_{Gs}}{D_s''H_s''M_{Ss}}-\frac{100M_{Gr}}{D_r''H_r''M_{Ss}}\frac{M_{Ls}-M_{Gs}}{M_{Lr}-M_{Gr}}. \tag{18}$$

These equations are approximations because general stray light from parts of the spectrum outside the "S" and "R" regions is neglected. Moreover, the precision of the gap values M_{Gr} obtained by interpolation is moderate. For these reasons the accuracy of the results depends on the choice of filters; the S filter should have a fairly narrow transmission region, but filters like this are quite easily obtainable. A combined use of an interference filter and ordinary absorption filters has sometimes been found useful. (See Appendix 3.)

The stray sending reading M_{Ss} includes line and continuous energy, i.e., (after Eq. (1),

$$M_{Ss}=M_{sl}+\int M_{sc}dx/D''.$$

Equation (17) or (18) of this appendix is of particular use in evaluating "near by" stray with imperfect filters.

APPENDIX 3

UCT Spectrophotometer and Filters Used

The spectrophotometer tested was a single monochromator built in the Illuminating Engineering laboratory of the University of Cape Town, mainly for lamp spectrophotometry but also for measuring filters and surface colors. Referring to Fig. 6, a large anastigmat L_1 of about 11 cm diam and 30.5 cm focal length serves as collimating lens. Two 60° prisms (11×9 cm) in tandem are followed by an achromatic lens L_2 of 12.7 cm diam and 61 cm focal length. The exit slit S_2 can move by means of a micrometric setting (x) along the spectral surface which is a plane for practical purposes over the range of interest, between 380 mμ at zero position and 780 mμ at position $x=60$ divisions$=12.7$ cm. All slit dimensions are adjustable. An EMI photomultiplier tube P with stabilized power supply serves as detector. Figure 7 shows the sensitivity curve, i.e., the deflection of the output meter for an equal energy spectrum; it is a function of dispersion, absorption, and phototube response. The dispersion is shown at the bottom of Fig. 7.

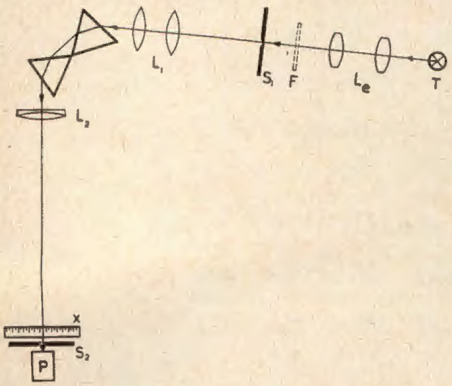


FIG. 6. The UCT spectrophotometer (see Appendix 3).

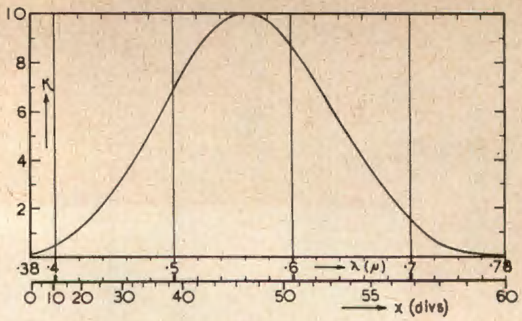


FIG. 7. Energy calibration curve and dispersion nomogram of UCT spectrophotometer. $K=\psi/p$=relative sensitivity for continuous spectrum.

Measurements were taken with a mercury cadmium lamp and a sodium lamp and the following filters:

- red interference (for 644 mμ),
- (yellow) Ilford 606 (for 578 mμ),
- green interference+Chance OGr1 (for 546 mμ),
- (green-blue) Ilford 603 (for 509 and 480 mμ),
- (blue) Ilford 601+blue interference (for 436 mμ, towards 700 mμ),
- (violet) Chance OV1 (for 405 and 408 mμ),
- Chance OY1+Ilford 626 (for 589 mμ).

APPENDIX 1.

MEASUREMENT OF THE SPECTRAL ENERGY DISTRIBUTION. OF THE TEST SOURCES.

A1.1 The Monochromator.

The monochromator was built at the University of Cape Town mainly for the spectrophotometry of lamps, especially low luminance sources like fluorescent lamps. With attachments it is also used to measure filter transmissions and the spectral reflectance of surface colours.

Referring to fig. 27. the light source B and condenser lens A are mounted on rails. The adjustable entrance slit S_1 is curved in order to counteract the curvature of the image introduced by the prisms, thus allowing the exit slit to be straight. The collimating lens C is a coated aircraft - camera objective lens of relative aperture 2.8 and focal length 12". This is followed by two 60° flint-glass prisms in series, the faces of which measure 4½" wide by 3½" high. The camera or focussing lens E is an achromat 5" in diameter and having a 24" focal length. Both the height and width of the exit slit S_2 are adjustable, and, by means of a micrometer screw, can be moved along the spectral plane, which, for practical purposes, can be considered to have a flat field over the region of interest. This extends from 0 to 60 divisions on the scale X (380 - 780 nm) and represents a distance of 5".

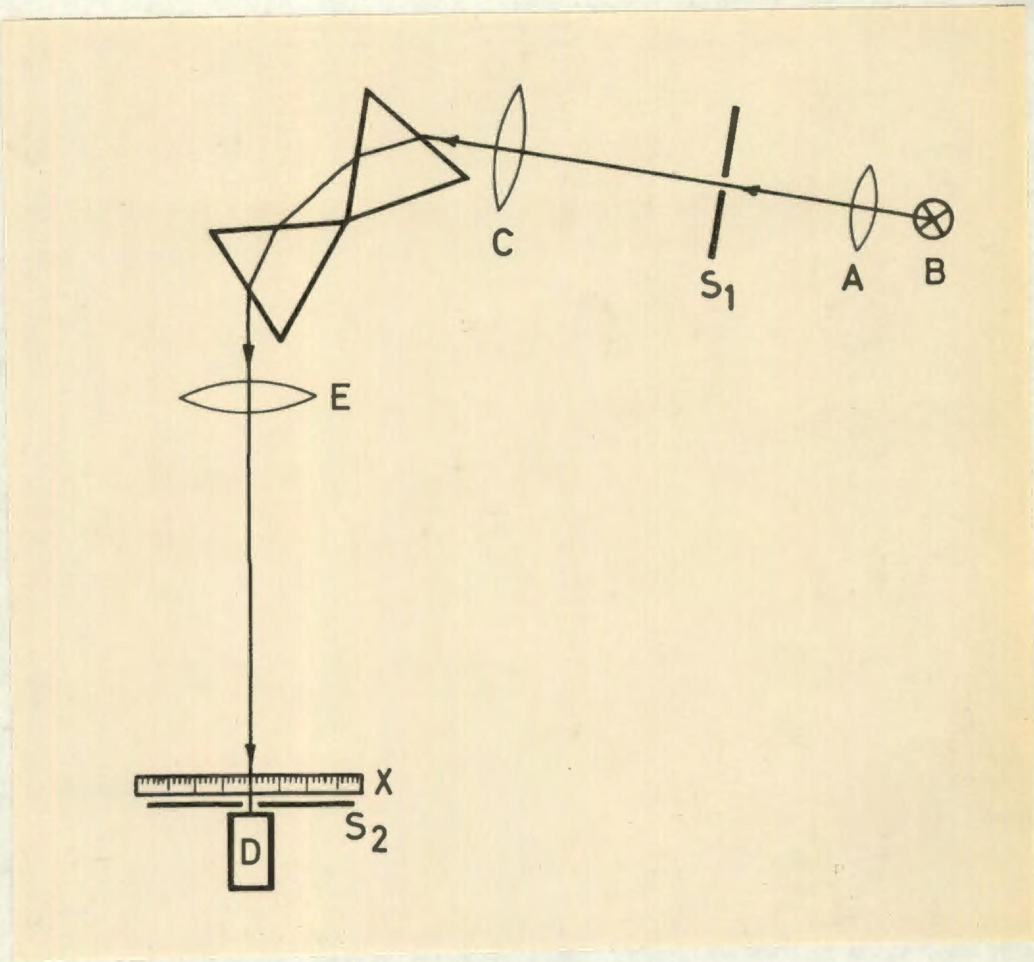


FIG 27

Schematic diagram of the U.C.T. monochromator

The detector D, which moves with the slit is an EMI photomultiplier type 6095C and has associated with it a 2kV stabilized power supply. The dynode voltage can be switched to seven values to adjust the sensitivity of the photomultiplier by a factor of 100 and the collector current is read directly on a 5 micro amp f.s.d. Cambridge light spot galvanometer via a universal shunt which allows a further adjustment of sensitivity by a factor of 20 without exceeding the rating of the p.m. tube.

A1.2 Reference lamps.

Wavelength calibration of the spectrophotometer was carried out using a high pressure Mercury-Cadmium discharge lamp as a standard. A further point was obtained at 668 nm with a helium lamp. For the energy calibration a 6v 108 watt, single coiled tungsten filament lamp was used as a colour-temperature standard and assumed to have the SED of a black-body radiator. This lamp was calibrated by the National Physical Research Laboratory in Pretoria at 5 different colour temperatures and was run at 2870°K for the calibration of the spectrophotometer.

The SED of a black body at this temperature was interpolated from tables (36) based on a value of 1.438 cm. deg. for the constant C in Planck's formula.

A1.3 S.E.D. Measurement of Sources.

The first method attempted was to measure each projector individually and then combine the various spectra. The advantage of this method lay in the brighter image available at the entrance slit but was discarded due to difficulties of positioning and focussing the projectors in a reproducible manner.

Also slow drifts in the sensitivity of the detector influenced the results appreciably.

It was then decided to mix the light from the projectors before measurement. This was first done by means of a ground-glass screen about 3" in front of the entrance slit, illuminated from the other side by the projectors. Although the results were somewhat better, the diffuser could not be made bright enough to give adequate readings in the blue part of the spectrum, as the projectors could not be focussed down to a small, bright area.

Finally an 8" diameter integrating sphere was constructed with four holes for the beams from the projector to enter. The interior was coated thickly with magnesium oxide and the sphere was placed against the entrance slit of the spectrophotometer.

Great care had to be taken when positioning the projectors and more especially the undirected light from the standard lamp in order to avoid illuminating the slit directly,as there were no internal screens.

Due to the low cost of the projectors, the illumination varied considerably over the beam area giving rise to colour shifts near the edges of the test field. This was not very noticable during the preference tests as these coloured areas fell on the dark grey backcloth, but for the S.E.D's it was felt that only the useful part of the beam should be considered.

Masks were therefore made and carefully registered, which permitted only a central rectangle, large enough to cover the face and hair of a model, to be measured. Although day-to-day changes in the sensitivity of the detector caused the absolute value of the measured energy to vary, the relative energy was now reproducible to within 3%. The S.E.D's of the seven sources have already been given in Table 1.

APPENDIX II

AN ALTERNATIVE METHOD FOR OBTAINING THE ORDINATE SCALE OF THE PREFERENCE CURVE.

As a check on the validity of the methods of analysis used to establish the preference curves in Section 3.3, the Department of Psychology was requested to assist in the use of standard psychometric techniques. The following is a letter received from Mr. J.G. Taylor in this connection.

"Ordinary psychological scaling techniques do not apply here

a) because the stimulus is not an unidimensional continuum, and

b) because the psychological continuum is not a monotonic function of the stimulus, but has a maximum near the middle of the series of stimuli.

Thurstone's law of comparative judgement assumes that each stimulus projects a normal (Gaussian) distribution ($y = \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$) on the psychological continuum. The standard deviation ($\sigma = \sqrt{\frac{\Sigma x^2}{n}}$) is unknown and may vary from one stimulus to the next.

If d_j and d_k are the values of the psychological variable produced when stimuli j and k are presented simultaneously, they may be correlated, and the correlation is represented by the symbol r_{jk}. When the stimuli are presented repeatedly, the distributions of d_j and d_k are normal, with means s_j and s_k, and standard deviations σ_j and σ_k. The differences, $d_k - d_j$, are also normally distributed, with standard deviation σ_{k-j}. It is known that

All we know, experimentally, about this distribution is the proportion of trials in which k was judged greater than j, say $p_{(k-j)}$. The rules of the experiment imply that $p_{(j-k)} = 1 - p_{(k-j)}$. Thurstone argued that the proportion $p_{(k-j)}$ may be considered as the integral of the distribution function from $-x$ to ∞ (where x is measured from the mean). Since the mean of the distribution of differences is equal to the difference between the distributions of d_j and d_k, i.e. $s_k - s_j$, x may be regarded as measuring this difference in terms of σ_{k-j} as unit. That is

$$s_k - s_j = x_{jk} \sigma_{k-j}$$

$$= x_{jk} (\sigma_j^2 + \sigma_k^2 - 2 r_{jk} \sigma_j \sigma_k)^{1/2}$$

x_{jk} may be got from the table of the normal probability integral, using the observed proportion, $p_{(k-j)}$, as argument.

If there are n stimuli, there are $\frac{1}{2}n\ (n-1)$ equations of the above form, but the number of unknowns is always in excess of the number of observations, even if we arbitrarily put one $s = 0$ and one $\sigma = 1$. But if we assume that all the terms r_{jk} are zero and σ's are all equal, the number of observations exceeds the number of unknowns. However, the observations are subject to error, and some procedure must be adopted to minimise error in the estimates of s_j, s_k. If each stimulus is compared with every other one we can write an $n \times n$ matrix, P, with elements $p_{(k-j)}$, and transform this to a matrix, X, with elements x_{jk}, derived from the table of the normal probability integral. It can be shown that the mean of the elements in column j of X gives a least squares estimate of the value of s_j. But this procedure is not applicable if $p_{(k-j)} = 1$ or 0, since in that case x_{jk} is indeterminate.

In the present problem the matrix P is incomplete and there is ground for objecting to the averaging procedure, particularly at the ends of the series. However, averaging will give us the <u>relative</u> positions of the stimuli on the scale.

Strictly speaking, the technique described here is applicable only to the scaling of a psychological continuum that is a monotonic function of some physical continuum. But our function has a maximum. So let us make another simplifying assumption, viz, that preference is normally distributed along the stimulus scale. (It will in any case be distributed according to some probability function, but as we have no direct means of determining what that function is, and in addition there is a certain arbitrariness in our stimulus scale, the simplest thing is to assume the probability function to the Gaussian).

This assumption implies that stimuli to both right and left of the mean of the distribution will be judged inferior to that nearest the mean. This enables us to say that if j is to the left of k on the stimulus scale,

$p_{(k-j)} < 0.5$ if both are to the right of the mean, and

$p_{(k-j)} > 0.5$ if both are to the left of the mean.

Consequently, if the origin of the x scale is at the mean, x_{jk} will be positive in the former case and negative in the latter. The values of $_{jk}$ derived in this way are shown in Table XVII, and the arithmetic means are shown at the bottom.

TABLE XVII

I	II	III	IV	V	VI	VII
38.8	61.2					
	43.9	56.1				
37.8		62.2				
	28.8	71.2				
		57.6	42.4			
	39.3		60.7			
		51	49			
			75.2	24.8		
		65		35		
			66.2	33.8		
				77.1	22.9	
			78.8		21.2	
				80.8	19.2	
					76.9	23.1
				84.6		15.4

Matrix Px100

TABLE XVII

(continued)

	I	II	III	IV	V	VI	VII
	-.2845	-.2845					
		-.1535	-.1535				
	-.3107		-.3107				
		-.5592	-.5592				
			+.1917	+.1917			
		-.2715		-.2715			
			+.0251	+.0251			
				+.6808	+.6808		
			+.3853		+.3853		
				+.4179	+.4179		
					+.7388	+.7388	
				+.7995		+.7995	
					+.8705	+.8705	
						+.7356	+.7356
					+1.1094		+1.1094
	-.5952	-1.2687	-.4213	+1.8435	+3.1444	+3.1444	+1.8450
x	-.2976	-.3172	-.0702	+.3072	+.5338	+.7861	+.9225
y	.38	.38	.40	.38	.34	.29	.26

Matrix X

The corresponding ordinates, y, of the normal curve are shown in the next row. The resulting section of the curve covers only a very narrow range of frequencies. But we can argue as follows :

Supposing that the subjects in our experimental population were required to express firm preferences within the range of stimuli actually used, without reference to any values beyond that range, the ordinal arrangement of the stimuli would be unaltered, but their distances apart from one another on the x scale would be greater. This effect would be obtained by multiplying the derived xs by a constant, say 3. This gives us :

	1	2	3	4	5	6	7
3x	.863	-.952	-.211	+.922	+1.601	2.358	+2.768
y	.275	.254	.390	.261	.111	.025	.009

The frequency at $x = 0$ is $\frac{1}{\sqrt{2\pi}} = .399$

If we now plot these frequencies against the stimuli, assuming them to be equally spaced, this is what we get.

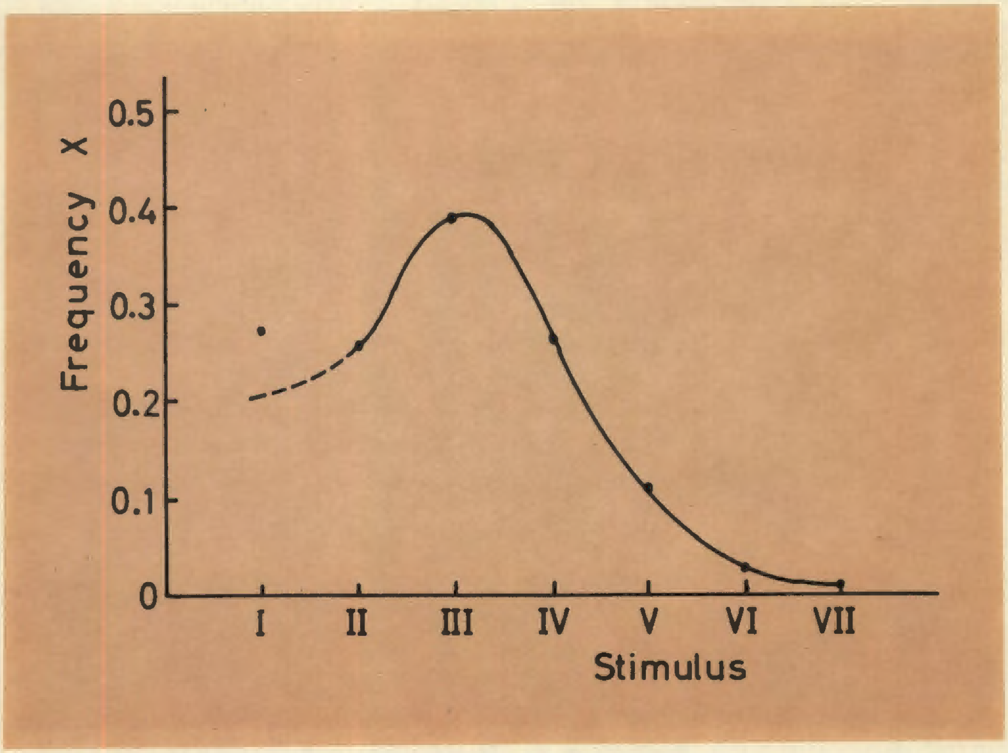


FIG. 19.

The first plotted point may be considered to be seriously in error because of the small number (2) of estimates and the small sample of the population tested. Neglecting this, the curve approximates pretty closely to those obtained by other means".

APPENDIX III

DERIVATION OF EXPRESSIONS USED IN PARA. 12

(For the symbols used, refer to Figure 23)

Determination of h_3

$$h_2 + h_s = \frac{(l_1 - f_c)(h_1' - h_s)}{f_c + v_c} \qquad \text{............ 1}$$

but

$$v_c = \frac{f_c u_c}{u_c - f_c} \qquad \text{............ 2}$$

$$h_1' = - \frac{f_c h_1}{u_c - f_c} \qquad \text{............ 3}$$

$$l_1 = \frac{f_c l_2}{l_2 - f_c} \qquad \text{............ 4}$$

Substituting 2, 3, & 4 in 1,

$$h_2 + h_s = \frac{f_c h_1 + h_s (u_c - f_c)}{l_2 - f_c} \qquad \text{............ 5}$$

$$h_2 = \frac{h_s (u_c - l_2) + f_c h_1}{l_2 - f_c} \qquad \text{............ 6}$$

but also

$$h_2 = \frac{f_c}{l_2 - f_c} h_3 \qquad \text{............ 7}$$

$$h_3 = \frac{h_s (u_c - l_2)}{f_c} + h_1 \qquad \text{............ 8}$$

Determination of D_a

The diameter of the condenser lens A, such that it just does not limit h_3, can be found as follows :

$$r_a - h_s = \frac{v_a}{l_1 - f_c}(h_2 - h_s) \qquad \ldots\ldots\ldots\ldots 9$$

$$v_a = f_a (m + 1) \qquad \ldots\ldots\ldots\ldots 10$$

where m is the magnification of the filament at the entrance slit.

Substituting 10 & 4 in 9

$$r_a - h_s = \frac{f_a (m + 1) \ (l_2 - f_c)}{f_c^{\ 2}} \ (h_2 - h_s)$$

substituting for h_2 from 7

$$r_a - h_s = \left[\frac{f_a (m+1)(l_2 - f_c)}{f_c^{\ 2}}\right]\left[\frac{f_c h_3 - h_s(l_2 - f_c)}{l_2 - f_c}\right]$$

$$D_a = 2\left[\frac{f_a (m+1)\left[f_c h_3 - h_s(l_2 - f_c)\right]}{f_c^{\ 2}} + h_s\right] \ldots\ldots 11$$

Determination of D_c

$$r_c = h_1' - h_s)\left[\frac{f_c}{f_c - v_c}\right] + h_s$$

Substituting for h_1' & v_c from 2 & 3

$$r_c = \left(-\frac{f_c h_1}{u_c - f_c} - h_s\right)\left(-\frac{u_c - f_c}{f_c}\right) + h_s$$

$$= h_1 + \frac{h_s (u_c - f_c)}{f_c} + h_s$$

$$D_c = 2\left[h_1 + \frac{h_s u_c}{f_c}\right] \qquad \text{........ 12}$$

R E F E R E N C E S.

1. BOUMA, P.J. "Two Methods of characterizing the colour Rendering Properties of a Light Source". C.I.E. Proceedings (1939) V.II. p.57.

2. HARRISON, W. "Assessment of Colour Rendering Properties of Fluorescent Lamps. Light and Lighting. V.44. (1951)p.148

3. BARR, A.C., CLARKE C.N, and HESSLER J. "Evaluation of Colour Rendition by Fluorescent Lamps". Illuminating Engineering. V.47. (1952) p.649.

4. JEROME, C.W. and JUDD, D.B. "Specification of Colour Rendering Properties of Fluorescent Lamps".Illuminating Engineering. V.48. (1953) p.259

5. BARNES, B.T. "Band Systems for Appraisal of Colour Rendition". Journal of the Optical Society of America. V. 47 (1957) p. 1124

6. SANDERS, C.L. "Colour Rendering using Colour Tolerances of Natural Objects". Illuminating Engineering. V.54 (1959) p.640

7. CRAWFORD, B.H. "Measurement of Colour Rendering Tolerances" Journal of the Optical Society of America. V.49 (1959) p.1147

8. AZUMA, T. and MORI L. "Direct Measurement of Colour Rendering of Fluorescent Lamps with a New Photoelectric Colorimeter. (Paper presented at C.I.E. meeting in Brussels 1959. Preprint 59 - 1).

9. MACADAM, D.L. "Color Reproduction": Journal of the Society of Motion Picture and Television Engineers. V.56 May (1951) p. 487

10. SANDERS, C.L. "Color Preferences for Natural Objects". Illuminating Engineering. V.54. No.7. (1959) p. 452.

11. BUCK, G.B. II and FROELICH, H.C. "Color Characteristics of Human Complexions". Illuminating Engineering. V.43 (1948) p.27.

12. THURSTONE, L.L. "A Law of Comparative Judgement",Psychological Review. V.34. (1927) p.273.

13. JUDD, D.B. "Estimation of Chromaticity Differences and nearest Color Temperature on the Standard 1931 I.C.I. Colorometric Co-ordinate System" Journal of the Optical Society of America. V.26. (1936) p.421

14. MACADAM, D.L. "Visual Sensitivities to Colour Differences in Daylight". Journal of the Optical Society of America. v.32 (1942) p.247.

15. MACADAM, D.L. "Specification of Small Chromaticity Differences" Journal of the Optical Society of America. v.33 (1943) p.18.

16. MORONEY, M.J. "Facts from Figures" 2nd Edition Penguin Books. (1956)

17. EDWARDS, E.A. and DUNTLEY, S.Q. "Pigments and Color of Living Human Skin". American Journal of Anatomy. v.65 (1939) p.1.

18. Report of the I.E.S. Subcommittee on Colour Rendering - Interim method of specifying Colour Rendering of Light Sources; Illuminating Engineering. v.57 (1962) p.471

19. MACADAM, D.L. "Projective Transformations of C.I.E. Colour Specifications". Journal of the Optical Society of America. v. 27. (1937) p.294.

20. OUWELTJES, J.L. "Specification of Colour Rendering Properties of Fluorescent Lamps". Die Farbe. v.9. (Dec.1960) p.207.

21. BOUMA, P.J. and Kruithof, A.A. "Chromatic Adaptation of the Eye". Philips Technical Review. v.9. (1947) p.257.

22. WRIGHT, W.D. "Measurement and Analysis of Colour Adaptation phenomena". Proceedings of the Royal Society (London). v. B115 (1934) p.49.

23. MACADAM, D.L. "Influence of Chromatic Adaptation on Colour Discrimination and Color Perception". Die Farbe. v.4.(1955) p.133

24. MACADAM, D.L. "The Color of Televized Pictures". Journal of the Society of Motion Picture and Television Engineers". v.65 (1956) p.455

25. HELSON, H. JUDD, D.B., and WARREN M.H "Object - color Changes from daylight to Incandescent Filament Illumination" Illuminating Engineering. v.47. (1952) p.221.

26. HELSON, H. JUDD, D.B. WILSON, M. "Color Rendition with Fluorescent Sources of Illumination". Illuminating Engineering. Vol. 51 (1956) p.329.

27. BURNHAM, R.W., EVANS, R.M. NEWHALL, S.M. "Influence on Color Perception of Adaptation to Illuminants". Journal of the Optical Society of America. v.42 (1952) p.597

28. BURNHAM, R.W., EVANS, R.M., NEWHALL, S.M. "Prediction of Color Appearance with different Adapting Illumination"; Journal of the Optical Society of America. v.47.(1957) p.35

29. BURNHAM, R.W. "Prediction of Shifts in Color Appearance with a Change from Daylight to Tungsten Adaptation". Journal of the Optical Society of America. v.49 (1959) p.254.

30. WYSZECKI, G. "A Graphical Interpretation of a Three - Components Theory in Terms of the C.I.E. Chromaticity diagram". Journal of the Optical Society of America. v.44. (1954) p.787.

31. NICKERSON, D. "Measurement and Specification of Color Rendition Properties of Light Sources". Illuminating Engineering. v.53. (1958) p.77.

32. ADAMS, E.Q. "X - Z Planes in the 1931 I.C.I. System of Colorimetry". Journal of the Optical Society of America. v.32 (1942) p.168.

33. BURNHAM, R.W. "Comparison of Color System with Respect to Uniform Visual Spacing". Journal of the Optical Society of America. v.39. (1949). p.387

34. GUILFORD, J.P. "Psychometric Methods". McGraw-Hill. New York. (1936).

35. MORRISSEY, J.H. "New Method for Assignment of Psychometric Scale Values from Incomplete Paired Comparisons". Journal of the Optical Society of America. v.45 (1955) p.373.

36. HARDING, H.G.W., and VICKERS T. "The Spectral Distribution of Power from a Plankian Radiator (c = 1.4380cm deg)" National Physical Laboratory, Teddington.

37. CURRY, C. "Geometrical Optics" Edward Arnold & Co.London (1953)

38. MARTIN, L.C. "Technical Optics" 2nd ed. Pitman & Sons Ltd. London (1960)

BIBLIOGRAPHY

MURRAY, H.D. "Colour in Theory and Practice". Chapman & Hall. London (1952)

JUDD, D.B. "Colour in Business, Science & Industry". John Wiley & Sons. New York. (1952).

ALDER, H.L. and ROSSLER, E.B. "Introduction to Probability and Statistics." W.H. Freeman & Co. San Francisco.(1960).

WRIGHT, W.D. "Researches on Normal and Defective Colour Vision". Henry Kimpton. London. (1946).

HUNT, R.W.G. "The Reproduction of Colour" Fountain Press. 1957

Le GRAND, Y. "Light, Colour and Vision". (Trans. by R.W.G.Hunt J.W.T. Welsh, F.R.W. Hunt). Chapman & Hall. 1957

BOUMA, P.J. "Physical Aspects of Colour". Philips Technical Library.

ZWIKKER, C. "Fluorescent Lighting". Philips Technical Library (1952).