The impact of the FRTB on Market Risk Capital for the South African InterBank Interest Rate Market

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

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Abstract

Regulations require banks to hold a minimum amount of capital for market risk resulting from their trading operations and prescribe two approaches to calculating this minimum capital requirement: (i) a Standardised Approach (SA); and (ii) an Internal Models Approach (IMA). The global financial crisis of 2008 highlighted flaws in the Basel 2 regulatory framework used by banks to calculate market risk capital charges for trading operations. In 2009, Basel 2.5 was introduced to deal with some but not all of the flaws of Basel 2. Both Basel 2 and 2.5 use the Value at Risk (VaR) risk measure as the basis to determine IMA capital charges. From 2022 onwards, Basel 2.5 will be replaced by the Fundamental Review of the Trading Book (FRTB), a new framework for calculating market risk capital charges for trading operations. The FRTB replaces VaR with the Expected Shortfall (ES) risk measure in the IMA and introduces a new SA.

This dissertation investigates the impact the FRTB will have on market risk capital charges for portfolios of linear South African interbank interest rate products. Capital charges are calculated for these portfolios under the Basel 2, Basel 2.5 and FRTB regulatory frameworks. A comparison and analysis of the resulting capital charges is then presented.

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Chapter 1

Introduction

Banks as part of their trading operations hold portfolios of financial instruments referred to as trading books. The values of financial instruments held in the trading book are marked-to-market daily. Hence, trading books are exposed to market risk, i.e., the risk of losses resulting from changes in market prices of instruments held. The main drivers of market risk are interest rates, credit spreads, equity prices, commodity prices and foreign exchange rates. Regulations require banks to maintain a minimum capital amount to account for the market risk in their trading books.

The Basel Committee on Banking Supervision (BCBS), a committee formed to develop international standards for banking regulation, prescribes guidelines on how the minimum capital amount for market risk is determined. For this dissertation, the BCBS will be referred to as "the committee". According to BIS (2018), the committee's mandate is to "strengthen the regulation, supervision and practices of banks worldwide to enhance financial stability". Since its founding in 1974, "the committee has established a series of international standards for bank regulation, most notably its landmark publications of the accords on capital adequacy which are commonly known as Basel 1, Basel 2 and, most recently, Basel 3" (BIS, 2016). The committee prescribes two approaches to calculating the minimum capital amount for market risk: (i) a Standardised Approach (SA); and (ii) an Internal Models Approach (IMA). The SA is a model proposed by the committee, while the IMA uses a risk measure from a bank's internal risk model.

During the 2008 global financial crisis, banks incurred significant losses on their trading books, highlighting an under capitalisation of trading books and a need to address the rules of the Basel 2 accord on market risk which were in place at that time. In Basel 2, Value at Risk (VaR) is the basis for the calculation of IMA regulatory market risk capital banks are required to hold for their trading operations. To address the most pressing deficiencies exposed by the crisis (one of these being that regulatory market risk capital as calculated by VaR alone was insufficient to absorb trading book losses), in 2009 the committee introduced the Basel 2.5 market risk standards which are still in operation today. In Basel 2.5, an additional risk measure called stressed VaR (sVaR) is introduced to complement VaR and increase overall capital requirements. At that time, several structural flaws in the market risk framework remained unaddressed by Basel 2.5. In response, the committee undertook the Fundamental Review of the Trading Book (FRTB) to address the flaws that remained unaddressed by Basel 2.5 (see section 4.4). The FRTB addressed the flaws by incorporating:

- A new definition of the boundary between the trading and banking book, reducing incentives for banks to arbitrage regulatory requirement capital between the two.
- A revised, risk-sensitive Standardised Approach (SA) to calculating the minimum required capital.
- A revised Internal Models Approach (IMA) with stringent modellability criteria for risk factors, a focus on tail risk capital calculations and a stressed capital add-on for risk factors that do not meet the modellability criteria.

The latest FRTB standards published in (BIS, 2019) will be the focus of the dissertation.

The aim of this dissertation is to create a framework to analyse and compare changes in market risk capital standards for the South African interbank interest rate market. The focus is on trading books which comprise of linear interbank interest rate products used to construct the South African interbank swap curve, i.e., cash deposits, forward rate agreements (FRAs) and interest rate swaps (IRSs). An outline of the objectives of the dissertation are presented below:

1. Review the regulatory market risk capital standards prescribed by the committee in Basel 2, Basel 2.5 and the FRTB.
2. Calculate, compare and analyse the capital charges for portfolios of linear interbank interest rate products using the regulatory market risk capital standards prescribed by the committee in Basel 2, Basel 2.5 and the FRTB for both the SA and IMA.
3. Find FRTB SA parameters that recover the FRTB IMA capital charges.
4. Investigate the effect of the modellability criteria on FRTB IMA capital charges.

The structure of this dissertation is as follows. In chapter 2 we look at the interest rate derivative market, linear interest rate products and the fair valuation of linear interest rate products. In chapter 3 we look at the risk measures used to calculate IMA capital charges and the models used to estimate the risk measures. Thereafter we give an overview of the regulations prescribed by the committee on how to calculate regulatory market risk capital and explain how and why these regulations have changed over time in chapter 4. In chapter 5 we present the methodology to calculate the regulatory market risk capital charges. Chapters 6 and 7 present the results and conclude the dissertation respectively. Two appendices are included to complement the results.

Chapter 2

The Interest Rate Derivative Market

In finance, a derivative is a financial instrument whose value is derived from the value of another underlying financial instrument. The underlying financial instrument may be referred to as just the underlying. For interest rate derivatives, the underlying is an interest rate. Interest rate derivatives can be divided into two subclasses:

- Linear interest rate derivatives: interest rate derivatives whose payoffs are linearly related to the underlying interest rate. Examples are FRAs and IRSs; and

- Non-linear interest rate derivatives: interest rate derivatives whose payoffs are non-linearly related to the underlying interest rate. Examples are options written on the underlying interest rate.

In this chapter, we give a brief description of linear interest rate derivatives and provide insights into the fair valuation of these products.

2.1 Definitions and Notation

Here we summarise some key definitions used in the interest rate market as well as notation.

- Zero coupon bond: a financial contract that guarantees its holder a payment of one unit of currency at maturity, with no intermediate payments. At time t, the value of a zero coupon bond maturing at time T is denoted by $Z(t, T)$. The zero coupon bond may be referred to as the discount factor.

- Spot interest rate: an interest rate locked in today for a transaction that is taking place today. The interest rate locked in at time t for the period $[t, T]$ is denoted by $R(t, T)$.

- Forward interest rate: an interest rate locked in today for a transaction that that will take place at a predetermined date in the future. The interest rate locked in at time t for the period $[T, S]$, $t \leq T < S$ is denoted by $f(t; T, S)$.

- Simple interest rate: the rate at time t at which an investment of $Z(t,T)$ units accrues interest proportionally to the investment period to yield one unit at maturity T, denoted by $L(t,T)$ for a simple spot interest rate and $L(t;T,S)$ for a simple forward interest rate.
- Continuously compounded interest rate: the rate at time t at which an investment of $Z(t,T)$ units accrues interest continuously to yield one unit at maturity T, denoted by $r(t,T)$ for a continuously compounded spot interest rate and $r(t;T,S)$ for a continuously compounded forward interest rate.
- Yield curve: a plot of interest rates against their maturities at a given point in time.

From the definitions above, we have the following relationships:

- We can express the value of a zero-coupon bond at time t in terms of simple or continuously compounded spot rates as follows:

$$Z(t,T) = e^{-r(t,T)\tau(t,T)},$$

in terms of continuously compounded spot rate $r(t,T)$; and

$$Z(t,T) = \frac{1}{1 + L(t,T)\tau(t,T)},$$

in terms of simple spot rate $L(t,T)$. Where $\tau(t,T)$ is the difference between time points t and T in year fractions.

- We have the following relationship between simple rates and continuously compounded spot rates:

$$r(t,T)\tau(t,T) = \ln\left(1 + L(t,T)\tau(t,T)\right).$$

- Using a no-arbitrage argument, we can express the continuously compounded forward interest rate in terms of continuously compounded spot interest rate as follows:

$$r(t;T,S) = \frac{r(t,S)\tau(t,S)}{r(t,T)\tau(t,T)\tau(T,S)}.$$

2.2 Money Market Deposits

A money market or cash deposit is a transfer of money from one party (the lender) to another (the borrower) for a certain maturity. At maturity, the borrower has an obligation to return the money to the lender with interest. The interest is the money the lender charges the borrower for the use of money lent. Typically, the interest paid is determined by a reference rate. In the South African interbank market, the reference rate is JIBAR. JIBAR stands for Johannesburg Interbank Agreed Rate, it is an average rate determined from South African interbank deposit rates submitted by the leading banks in the South African interbank market. These deposit rates are simple interest rates. Other reference rates are LIBOR (London Interbank Offered Rate) and EURIBOR (Euro Interbank Offered Rate). The name of a cash deposit usually reflects its reference rate. For example, cash deposits referencing JIBAR are known as JIBAR deposits. For this dissertation, JIBAR will be our reference rate.

The value of the deposit at maturity is dependent on the rate of a given maturity. For example, if we invest N units of currency in a money market account at time t for the period $[t, T]$. At maturity, the value of this deposit is:

$$V_{DEPOSIT} = N\left(1 + \tau(t, T)L(t, T)\right), \tag{2.1}$$

where:

- $L(t, T)$ is the simple interest rate for the period $[t, T]$; and
- $\tau(t, T)$ is the difference between time points t and T in year fractions.

2.3 Linear Interest Rate Derivatives

2.3.1 Forward Rate Agreements

A FRA is an over-the-counter (OTC) agreement to earn or pay an interest rate on a deposit starting at a future point in time. A FRA allows a borrower or lender to fix the interest rate for a specific period. The fixed interest rate for a specific period is known as the FRA/strike rate. The buyer of a FRA would effectively be paying the FRA rate while receiving the referenced floating rate.

A standard FRA involves three time points: (i) the current time t, (ii) the expiry time T_{i-1}, and (iii) the maturity time T_i, with $t \leq T_{i-1} < T_i$. At any point in time $s \in [t, T_{i-1}]$, the fair value of a FRA with a FRA rate of K to the buyer is given by:

$$V_{FRA} = N\tau(T_{i-1}, T_i)[L(s; T_{i-1}, T_i) - K]Z(s, T_i), \tag{2.2}$$

where:

- N is the notional amount of the trade;
- $\tau(T_{i-1}, T_i)$ is the difference between time points T_{i-1} and T_i in year fractions;
- $L(s; T_{i-1}, T_i)$ is the simple fair forward rate at time s for the period $[T_{i-1}, T_i]$; and
- $Z(s, T_i)$ is the discount factor from time s to time T_i.

In the market, FRAs are denoted by short hand notation like 3x6. This refers to a FRA with start date 3 months from now and maturing 3 (6 - 3) months later. Thus a 3x6 FRA is a contract fixing the 3-month JIBAR rate in 3 months time.

2.3.2 Interest Rate Swaps

An IRS is an OTC agreement between two parties to exchange a series of interest rate payments. The most common type of IRS is a "plain vanilla" IRS where the two parties exchange a series of fixed interest rate payments for a series of floating interest rate payments linked to a reference rate. Economically, the actual payments between the two parties to an IRS will be the difference between the fixed and floating interest rate payments. Hence, at any time $t_s \in [t_0, t_n]$, the fair value of an IRS is equal to the sum of the present value of the difference between the fixed and floating interest rate payments. Mathematically, for an IRS with fixed rate K, this can be written as:

$$V_{IRS} = \sum_{j=\max(s,1)}^{n} N\tau(t_{i-1}, t_i)[L(t_s; t_{i-1}, t_i) - K]Z(t_s, t_i), \tag{2.3}$$

where:

- N is the notional amount of the trade;
- $\{t_0, t_1, \cdots, t_{n-1}\}$ denotes the reset times in year fractions;
- $\{t_1, t_2, \cdots, t_n\}$ denotes the payment times in year fractions;
- $\tau(t_{i-1}, t_i)$ is the difference between time points t_{i-1} and t_i in year fractions;
- K is the fixed rate of the IRS;
- $L(t_s; t_{i-1}, t_i)$ is the simple fair forward rate at time t_s for the period $[t_{i-1}, t_i]$; and
- $Z(t_s, t_i)$ is the discount factor from time t_s to time t_i.

Equation 2.3 shows that the value of a swap is equal to the sum of the values of a series of FRAs all with a strike rate of K.

2.4 Constructing the Nominal Swap Curve

To value any deposits or linear interest rate derivative, we require a yield curve. The yield curve can be constructed from market observable quotes using a technique called bootstrapping. According to Brugger (2018), in a bootstrap technique we construct a yield curve $Y : t \to Y(t)$ from finite market observable quotes iteratively, where we get the unknown point $Y(t_i)$ at t_i by a calculation that depends on previous points of the curve, $\{Y(t_j) : j < i\}$. From the finite points, the rest of the curve can be generated using interpolation and/or extrapolation techniques.

Two decisions are crucial to constructing a yield curve:

- the set of input instruments; and
- the construction methodology.

Ideally the set of input instruments must be liquid and the construction methodology should produce a curve that fits market observable rates and demonstrates favorable mathematical and economic characteristics (see Hagan and West (2008) for characteristics).

When using FRAs and IRSs in a bootstrapping process, the maturity dates of the FRAs and IRSs must coincide with the maturities of the yield curve tenors being bootstrapped for. Inverting equation 2.2 for a fair FRA issued at time t_0 for the period $[t_{i-1}, t_i]$ with a unit notional amount and using the relationship between simple rates and continuously compounded spot rates, we get the following expression for the continuously compounded spot rates:

$$r(t_0, t_i) = \frac{r(t_0; t_{i-1}, t_i)\tau(t_{i-1}, t_i) + r(t_0, t_{i-1})\tau(t_0, t_{i-1})}{\tau(t_0, t_i)}, \tag{2.4}$$

which allows us to iteratively build the yield curve using FRAs. Similarly, for a fair IRS with payment dates $\{t_1, \ldots, t_n\}$ and fixed rate K, we can derive the following expression for continuously compounded spot rates:

$$r(t_0, t_n) = -\frac{1}{\tau(t_0, t_n)} \ln\left[\frac{1 - K\sum_{i=1}^{n-1}\tau(t_{i-1}, t_i)Z(t_0, t_i)}{1 + K\tau(t_0, t_n)}\right], \tag{2.5}$$

which allows us to iteratively build the yield curve using IRSs.

In the South African interbank market, the yield curve is constructed from money market deposits and linear interest rate derivatives with JIBAR as an underlying. Using equations 2.4 and 2.5 and the instruments in table 2.1, JSE (2012) provides a bootsrapping algorithm used to construct the nominal swap curve which proceeds as follows:

1. At time t, select a set of input instruments consisting of money market deposits maturing at times $\{t_1, \ldots, t_k\}$, a set of FRAs maturing at times $\{t_{k+1}, \ldots, t_m\}$ and a set of IRSs maturing at times $\{t_{m+1}, \ldots, t_n\}$.

2. Guess values of the continuously compounded rates $\{r(t, t_1), \ldots, r(t, t_n)\}$.

3. Apply an appropriate interpolation method, and interpolate between $\{t_1, \ldots, t_n\}$ and $\{r(t, t_1), \ldots, r(t, t_n)\}$ to estimate the rates corresponding to each cash flow date of the input instruments.

4. Input the rates obtained in step 3 into equations 2.4 and 2.5 to get new estimates for $\{r(t, t_{k+1}), \ldots, r(t, t_n)\}$.

5. Repeat steps 3 and 4 until convergence, i.e., until the yield curve fits market observable rates.

Point	Instrument
1 day	SAFEX Overnight
1 Month	1-month JIBAR deposit
3 Month	3-month JIBAR deposit
4-24 Months	FRAs
2-30 Years	IRSs

Tab. 2.1: Inputs to the nominal swap curve.

2.5 Present Value of a Basis Point

The present value of a basis point or PV01 is the change in the present value of a portfolio of interest rate products due to a one basis point (1bp or 0.01%) change in interest rates. Mathematically this is can be written as:

$$\mathrm{PV01} = V(r + 0.0001) - V(r),$$

where V is the value of the portfolio as a function of the interest rate r. The PV01 is a measure of interest rate sensitivity for interest rate products.

PV01s are used to approximate the profit & loss ($P\&L$) of a portfolio of interest rate products due a change in an interest rate. For example, suppose the interest rate r changes by an amount equal to Δr. According to Alexander (2009), the net change in the present value of a portfolio of interest rate products may be approximated as follows:

$$P\&L \approx \mathrm{PV01} \times \Delta r \tag{2.6}$$

For small changes in the interest rate, the use of PV01s provides a good approximation of a portfolios $P\&L$ for small changes in the interest rate. In table 2.2 below, we present a comparison between a PV01 approximation and an exact calculation of the $P\&L$ for a deposit that pays R10 million in one year. The prevailing Nominal Annual Compounded Continuously (NACC) spot rate is 4% and the deposit has a PV01 of -960.74.

Δr	**PV01 Approximation of $P\&L$**	**Exact $P\&L$**	**Relative Error of PV01 Approximation compared to Exact $P\&L$**
0.01%	-960.74	-960.74	0.00%
0.10%	-9 607,41	-9 603,09	0.04%
1.00%	-96 074,14	-95 600,15	0.50%
10.00%	-960 741,40	-914 312,04	5.08%

Tab. 2.2: A comparison between a PV01 approximation and an exact calculation of the $P\&L$ for a deposit that pays R10 million in one year.

From table 2.2, we see that PV01s provide a good approximation of the $P\&L$ for small changes in the interest rate. However, the approximation gets worse as changes in the interest rate become bigger.

From equations 2.1, 2.2 and 2.3 it is clear that the PV01s of deposits, FRAs and IRSs scale linearly with the notional keeping other variables constant. In figure 2.1, we plot PV01s of deposits, FRAs and IRSs for an upward slopping yield curve. We see that for each instrument class, as the time-to-maturity increases, the PV01 of the instrument increases. Hence, all else equal; deposits, FRAs and IRSs with longer maturities have higher interest rate sensitivities as measured by the PV01s.

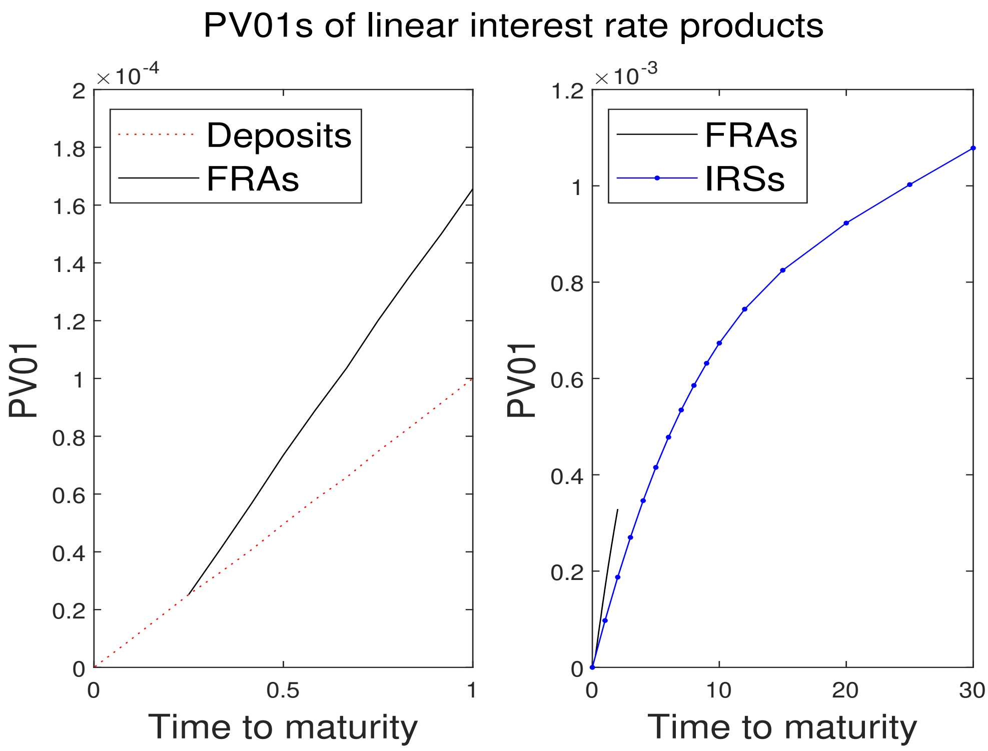

Fig. 2.1: The left panel displays the PV01s of deposit instruments with maturities ranging from one day to one year and FRAs referencing 3-month JIBAR with maturities ranging from 3-months to one year. The right panel displays the PV01s of FRAs referencing 3-month JIBAR with maturities ranging from 3-months to two years and IRSs referencing 3-month JIBAR with maturities ranging from one to thirty years. All instruments have a notional of R1.

2.6 Summary

Linear interest rate derivatives can be used to fix the interest for a specific period(s) of time, making them a popular tool in the risk management area. Additionally, linear interest rate derivatives are used to construct a yield curve which can be used to value almost any interest rate product. However, we see that as the time-to-maturity of linear interest rate products increases, so does their risk.

Chapter 3

Risk Measures and Models

In this chapter, we present definitions of the risk measures used to calculate regulatory market risk capital requirements. Two risk measures are presented: (i) VaR; and Expected Shortfall (ES). Models used to estimate VaR and ES are also presented.

3.1 Risk Measures

3.1.1 Value at Risk

Hull *et al.* (2012) define VaR as a loss that will not be exceeded at some specified confidence level, over a certain period given assumed market conditions. It is a function of two parameters: the time horizon and the confidence level. For example, a one-day VaR of R10million on a 95% confidence level means that we are 95% confident that we will not experience a loss of more than R10 million in the next day. VaR is an attractive measure because it answers the simple question "How bad can things get?". Despite this, VaR is not a coherent risk measure. According to Acerbi and Tasche (2002), a coherent risk measure is one that is:

- Monotonous: if portfolio A has weak stochastic dominance over portfolio B, then the risk measure of A is under most circumstances lower than that of B.
- Sub-additive: for a given portfolio, the risk measure of the portfolio is no greater than the corresponding weighted average of the risks of the constituents of the portfolio.
- Homogeneous: the risk measure increases as a position in the portfolio increases.
- Translation invariant: if cash is added to a portfolio, the risk measure of the portfolio decreases.

VaR does not always meet the sub-additive property (Alexander, 2009). This means that in the case of a bank made up of several branches, if VaR is used as a measure of risk for each branch, then the overall risk of the bank may turn out to be greater than the sum of the branches' risk. Hence, if regulatory capital for each branch is calculated independently based on VaR, the regulator should not be confident that the overall bank capital is adequate.

Additionally, since VaR looks at a point on the tail of the distribution, it ignores what happens beyond that point. Thus, it does not answer the question "If things do get bad, how much can we expect to lose on a portfolio?".

3.1.2 Expected Shortfall

According to Hull *et al.* (2012) a risk measure that answers the question "If things do get bad, how much can we expect to lose on a portfolio?" is ES, also known as conditional VaR (CVaR) or tail loss. ES is the average of losses that exceed VaR and does depend on the distribution beyond the VaR point. Hence, ES will differentiate between two distributions that have the same VaR, but one has fatter tails. Acerbi and Tasche (2002) show that ES is a coherent risk measure.

3.1.3 Summary

VaR as a risk measure is attractive because it indicates the loss expected for a given confidence level. However, VaR is not sub-additive and does not give insight on how huge losses can be when they exceed the VaR point. Hence it does not give a full reflection of the risk taken by a bank. ES, the average of losses beyond the VaR point, is sub-additive and gives insight on how huge losses can be when they exceed the VaR point. This makes ES a more attractive risk measure when compared to VaR.

3.2 VaR Models

3.2.1 Parametric Method

This is also known as the variance-covariance or correlation method. According to Alexander (2009), the parametric method calculates VaR and ES using analytic formulae that are based on an assumed parametric distribution for the risk factor returns of a portfolio. The parameters of the distribution for each risk factor are determined from historical data. The use of a parametric method is problematic if the assumed distribution of a risk factor does not form a suitable fit for the data used.

3.2.2 Historical Simulation

The historical simulation approach to estimating VaR reprices the current portfolio using different historical scenarios. Each historical scenario assumes actual historical changes in the values of key market risk factors experienced during the historical sample period. Suppose today is day n and we have n consecutive days of data, the one-day i^{th} scenario in the historical simulation assumes that the value of the market risk factor tomorrow will be:

$$\text{Value under } i^{th} \text{ scenario} = v_n \frac{v_{i+1}}{v_i}, \text{ for } 1 \leq i \leq n-1,$$

where v_i is the value of the market risk factor on day i. Hence, given n consecutive days of data, we can create $n - 1$ one-day scenarios.

The $P\&L$ of the portfolio is then calculated under each scenario, and the results sorted from the largest loss to the largest gain. To estimate VaR at the $X\%$ confidence level, we choose the point on the $P\&L$ distribution beyond which $(1 - X)\%$ of the outcomes result in larger losses and ES is calculated as the average of losses beyond that point.

An advantage of the historical simulation approach is that it estimates VaR and ES based on what happened in the past. Hence, it cannot be dismissed as introducing impossible outcomes. On the other hand, many challenges have to be overcome for its successful implementation. One of them being that there can be no guarantee that a historical event will re-occur, or that it would occur in the same manner or with the same likelihood as represented by the historical data. Another is that a large sample is required to measure VaR accurately, and this sample may contain periods where markets have been through regimes that may be quite different from the current regime (Alexander, 2009). Hence, to reliably estimate VaR using Historical Simulation, we require a data sample that is expected to be representative of the future.

3.2.3 Monte-Carlo Simulation

A Monte-Carlo simulation approach to estimating VaR reprices the current portfolio using hypothetical scenarios randomly generated from a model describing the change in the market risk factors that determine the value of the portfolio. Similar to the parametric method, a distribution describing the change in the market risk factors that determine the value of the portfolio is assumed. To calculate 1-day VaR in a Monte-Carlo simulation, we proceed as follows:

1. Given the values of the market risk factors today, calculate the value of the portfolio.

2. Randomly generate n hypothetical scenarios from the model describing the change in the market risk factors that determine the value of the portfolio 1-day ahead.

3. Revalue the portfolio 1-day ahead given the generated values of the risk factors.

4. Calculate the $P\&L$, VaR and ES as in the historical simulation approach.

An advantage of the Monte-Carlo simulation is that it can use any distribution for the market risk factors comfortably, including those that have no analytical formula for VaR. However, a Monte-Carlo simulation requires a huge number of hypothetical scenarios to get a meaningful estimate of VaR, which can be computationally intensive.

3.2.4 Summary

The parametric method requires us to make assumptions about the returns distribution of the risk factors of a portfolio, but is simple to compute once we have the analytical formula for VaR. The historical simulation makes no assumptions about the returns of the risk factors of a portfolio, but assumes the data used is representative of the risk factors in the future. The Monte-Carlo simulation can incorporate almost any distribution for the risk factors, but can be computationally intensive. Each of the three VaR models has its own advantages and disadvantages, hence the committee does not prescribe any specific model to be used to compute the risk measures. For this dissertation, a historical simulation will be used to calculate the risk measures because it makes the least modelling assumptions.

Chapter 4

Basel Regulations

In this chapter, we look at a background of the Basel regulations for market risk capital requirements prescribed by the committee. An overview of the Basel 1, Basel 2, Basel 2.5 and FRTB regulations on market risk requirements are presented. Our main focus will be on the FRTB regulations for the interest rate risk class.

4.1 Basel 1

In 1988, the committee published the Basel Capital Accord (BIS, 1988). These were the first internationally recognised risk-based standards for capital requirements. The Accord came to be known as Basel 1. Basel 1 focused mainly on credit risk, the risk of loss from the failure of counterparties meeting their financial or contractual obligations when due.

In 1996, the committee published an amendment to Basel 1 aimed at addressing market risk exposures. The amendment introduced an additional capital requirement for market risk associated with trading activities. The market risks subject to these requirements were interest rate risk, equity price risk, foreign exchange risk and commodity price risk.

In the amendment, banks would have a choice between two approaches to calculating capital requirements for market risk. One alternative is the SA, which is a model proposed by the committee to determine minimum capital requirements. The SA prescribes parameters to use and rules to follow in the calculation of the minimum capital requirement. In Basel 1 two methods are prescribed under the SA for the interest rate risk class: (i) a maturity method; and (ii) a duration method. An outline of the Basel 1 SA is not presented in this dissertation, instead we refer the reader to BIS (1996) and Crouhy *et al.* (1999) for an outline and examples.

The other alternative is for banks to use a risk measure derived from their internal VaR models. This approach is known as the IMA, and its use is subject to supervisory approval. Banks wishing to use the internal models approach for capital requirements have to meet specific qualitative and quantitative criteria before the supervisory authority grants permission [1]. The risk measure used for capital calculations is VaR for a 10-day holding period based on a 99% one-sided confidence level and the sample period for calculating VaR should be one to four years.

[1] See Crouhy *et al.* (1999) for criteria to be met before supervisory approval is granted.

The minimum capital requirement, C, is then calculated as

$$C = \max(\mathrm{VaR_{t-1}}, \mathrm{m_c} \times \mathrm{VaR_{avg}}), \tag{4.1}$$

where

- $\mathrm{VaR_{t-1}}$ is the previous days VaR;
- $\mathrm{VaR_{avg}}$ is the average of VaR over the past 60 days; and
- $\mathrm{m_c}$ is a multiplicative factor with a value between 3 and 4. An 'adjustment factor' ranging from 0 to 1 is added to this multiplication factor based on the models ex-post performance as determined by a backtest[2].

Basel 1 allowed the 10-day VaR to be proxied by scaling up 1-day VaR by the square root of time rule, i.e. 10-day VaR = $\sqrt{10} \times$ 1-day VaR.

Banks that do not meet all the requirements for the internal models approach are allowed to use a combination of the SA and IMA, however, each risk category must be measured according to only one approach Crouhy *et al.* (1999). If a combination of the approaches is used, the total capital requirement is then a sum of the capital requirement for each risk category, without accounting for a possible diversification benefit between the risk categories.

4.2 Basel 2

In 2004, the second Accord, known as Basel 2 was published (BIS, 2004). Basel 2 was meant to improve capital requirement calculations for credit risk. It also considered capital requirements for operational risk. The guidelines on calculating the minimum regulatory capital requirement for market risk remained unchanged from the 1996 amendment. Hence for this dissertation, we shall refer to the Basel 1 rules as Basel 2.

4.3 Basel 2.5

During the 2008 global financial crisis, banks suffered significant losses on their trading books. This highlighted the need for an improvement of the market risk framework. As a stopgap response in 2009, a revised market risk framework known as Basel 2.5 was published to increase the minimum capital requirement (BIS, 2009). While the regulation regarding the SA remained unchanged, an additional risk measure, sVaR, was introduced to complement VaR in the IMA. sVaR refers to VaR calculated from a 12-month period of stress for the portfolio in consideration. Banks are therefore required to calculate two VaR measures: one is the VaR introduced in the Basel 1 amendment, and the other a VaR calculated from a 12-month period of stress. The capital requirement in the IMA is then calculated as:

$$C = \max(\mathrm{VaR_{t-1}}, \mathrm{m_c} \times \mathrm{VaR_{avg}}) + \max(\mathrm{sVaR_{t-1}}, \mathrm{m_s} \times \mathrm{sVaR_{avg}}), \tag{4.2}$$

where

[2] Backtesting involves testing a model on historical data to see how well it performs.

- $\mathrm{VaR_{t-1}}$, $\mathrm{VaR_{avg}}$, and $\mathrm{m_c}$ are as defined above;
- $\mathrm{sVaR_{t-1}}$ and $\mathrm{sVaR_{avg}}$ are the previous days sVaR and the average of sVaR over the past 60 days; and
- $\mathrm{m_s}$ is a multiplication factor similar to $\mathrm{m_c}$ that is applicable to $\mathrm{sVaR_{avg}}$.

Because VaR calculated from a 12-month period when a portfolio is at stress is at least equal to VaR calculated from the most recent 12-month period, the impact of the new rule is to at least double the capital requirement.

4.4 Shortcomings of Basel 2.5

Although Basel 2.5 increased the market risk capital requirements, strengthening the capital base of banks and allowing them to better withstand periods of financial stress, the committee agreed that several shortcomings with the risk measurement methodologies remained unaddressed by Basel 2.5.

The committee identified the following shortcomings with the IMA and its continued reliance on VaR (BIS, 2012, p53-55):

- By not looking beyond the 99^{th} percentile, VaR and hence capital requirements fail to capture 'tail risks'. A tail risk is an event with a small probability of happening, e.g. losses beyond the 99% VaR point. This creates perverse incentives for banks relating to tail events.
- Pro-cyclicality of market-implied measures of risk: When asset prices are rising and volatility is falling (as was the case during the pre-crisis period), capital charges based on VaR calibrated to the most recent period are low. This allows a bank to take on more risk, because extra capital is freed up. Similarly, when asset prices are falling and volatility is rising (as was the case during the crisis period), capital charges based on VaR calibrated to the most recent period are high. Banks then take on less risk by exiting their existing positions, contributing to asset price falls and market illiquidity.
- Inability to capture market liquidity risk: During the financial crisis, markets became illiquid. Moreover, the liquidity premia rose during the height of the crisis. Thus, banks were unable to exit or hedge certain trading positions in a short period, and they incurred substantial mark-to-market losses on certain positions. This proved that in some asset markets, the 10-day VaR risk metric is inadequate to capture liquidity risk in a time of severe illiquidity.

According to BIS (2012), the current SA rules fail to distinguish the riskiness of different portfolios. They are inadequate for complex or innovative products, as they force these products into simple categories. Hence, capital charges will be the same for some portfolios with different risk characteristics, and banks can arbitrage the rules by designing product features to minimise capital charges.

4.5 FRTB

The FRTB is a set of standards published by the committee in 2019. It proposes significant revisions to the methodology used to calculate regulatory market risk capital charges for the trading book. Plans are to fully implement the new set of rules by the beginning of 2023 for the South African banks.

Three major changes are central to the FRTB. Firstly, it introduces a new definition of the regulatory boundary between the trading and banking book. This is to reduce capital arbitrage from banks by transferring transactions between the two. Secondly, the SA is redesigned, introducing a risk sensitive methodology for calculating SA capital. Lastly, ES replaces VaR as the risk measure for calculating IMA capital charges.

The FRTB also proposes a more rigorous process for approval to use an internal model for calculating capital requirements, with approval granted at a trading desk level as opposed to the entire bank [3]. Hence, some trading desks will have capital requirements determined under the SA, while for others capital requirements will be determined under the IMA. Regardless of internal model approval status, the FRTB requires banks to publish regulatory capital numbers as determined by the SA for each trading desk. For this dissertation, we assume that we have a single trading desk.

4.5.1 FRTB Standardised Approach

The FRTB SA minimum capital requirement is the sum of three components: a sensitivities-based method capital requirement; a default risk capital (DRC) requirement; and a residual risk add-on (RRAO) capital requirement. The DRC is calculated for instruments subject to default risk and aims to capture jump-to-default risk, while the RRAO is introduced to account for market risks that are not captured by the new SA. To calculate the capital requirement under the sensitivities-based method, we aggregate three risk measures (BIS, 2019, p19):

- Delta: a risk measure based on the first-order sensitivity of a financial instrument to the price of the underlying.
- Vega: a risk measure based on the first-order sensitivity of a financial instrument to its implied volatility.
- Curvature: a risk measure that captures higher-order sensitivities of a financial instrument to the price of the underlying not covered by delta.

The committee requires banks to calculate a delta risk capital requirement for all instruments held in the trading book that are subject to the sensitivities-based method. Whereas only non-linear products require vega and curvature capital requirements. BIS (2019) specifies which instruments are subject to vega and curvature risk capital requirements.

[3] See BIS (2019) for criteria a trading desk has to meet for internal model approval to be granted.

The calculation of a single sensitivity (delta, vega, curvature) has the following structure: for a risk class (interest rate, credit spread, equity etc.), risk factors (variables that affect the value of an instrument) are identified and are grouped in buckets by common characteristics (e.g. market capitalisation for equities). Sensitivities to these risk factors are then calculated for each position taken in an instrument and multiplied by the prescribed risk weight for the respective bucket. The weighted sensitivities are aggregated within each bucket to determine the bucket's risk position and the risk positions of each bucket within a risk class are aggregated at the risk class level to determine the capital requirement for that risk class.

For the interest rate risk class, the risk factors are tenors on the risk-free yield curve and the buckets correspond to different currencies. Since we are dealing with linear interest rate products for a single currency, only the delta risk capital requirement for a single bucket is required. Below we outline the three steps to the computation of the delta risk capital requirement of a portfolio for a single bucket:

1. Calculate the risk factor sensitivities in the bucket. For the interest rate class, the delta sensitivity s_t is defined as:

$$s_t = \frac{V(r_t + 0.0001) - V(r_t)}{0.0001}, \tag{4.3}$$

   where r_t is the risk-free yield at tenor t and V is the market value of the instrument as a function of the risk-free yield curve. The numerator in equation 4.3 is the PV01 of the instrument with respect to r_t. The FRTB requires the risk-free yield curve to be constructed using money market instruments held in the trading book with the lowest credit risk or be based on a market implied swap curve.

2. Calculate the weighted sensitivities, WS_t, given the supervisory risk weights RW_t (presented in table 4.1 below) as a product of s_t and RW_t, i.e.

$$WS_t = RW_t \times s_t, \tag{4.4}$$

   where WS_t is the weighted sensitivity at tenor t and RW_t is the prescribed supervisory risk weights for tenor t. For the following currencies Euro (EUR), US dollar (USD), Australian dollar (AUD), Japanese yen (JPY), Canadian dollar (CAD), and the reporting currency of the bank, the committee allows the bank to divide the risk weights by the square root of 2.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Risk weight (%)	1.70	1.70	1.60	1.30	1.20	1.10	1.10	1.10	1.10	1.10

Tab. 4.1: FRTB delta prescribed risk weights for the interest rate risk class.

3. Calculate the delta capital requirement for the bucket, by aggregating the weighted sensitivities, using the prescribed correlation ρ_{kl}.

$$\text{Delta Capital} = \sqrt{\max\left(0, \sum_k RW_k^2 + \sum_k \sum_{l \neq k} \rho_{kl} WS_k WS_l\right)}, \tag{4.5}$$

where $\rho_{kl} = \max\left[e^{\left(-\theta\times\frac{|T_k-T_l|}{\min(T_k,T_l)}\right)}, 40\%\right]$ is the prescribed correlation between tenors T_k and T_l, and θ is set at 3%.

In order to address the risk that correlations increase or decrease in times of financial stress, the capital charges must be calculated under three correlation scenarios, and the final capital charge is the largest of these scenario-related capital charges:

- a medium correlation scenario, where the correlation parameters ρ_{kl} are as defined above;
- a low correlation scenario, where the correlation parameters ρ_{kl} are replaced by $\rho_{kl}^{\text{low}} = \max(2 \times \rho_{kl} - 100\%, 75\% \times \rho_{kl})$; and
- a high correlation scenario, where the correlation parameters ρ_{kl} are uniformly multiplied by 1.25 subject to a cap at 100%.

4.5.2 FRTB Internal Model Approach

Under Basel 2.5, calculations for IMA market risk capital charges are based on a 99% confidence level VaR. The FRTB proposes a change from a 99% confidence level VaR to a 97.5% confidence level ES. According to Hull (2015), for a normally distributed $P\&L$ profile, 97.5% confidence level ES is approximately equal to 99% confidence level VaR. In the FRTB, the ES measure must be calibrated to a 12-month period of stress for the portfolio in consideration over the observation period. The observation period must go back to at least 2007 in order to find this 12-month period of stress for the portfolio being considered.

The Basel 2 and 2.5 IMA allow for 10-day VaR to be calculated by scaling up the 1-day VaR and use a 10-day time horizon for all risk factors. Using a 10-day horizon for all risk factors does not account for the fact that market variables underlying transactions vary according to their liquidity horizon. BIS (2019) defines the "liquidity horizon (LH)" as the time required to exit or hedge a risk position without materially affecting market prices in stressed market conditions. The FRTB requires a full 10-day change to be applied to the risk factors and introduces differentiated LHs for market variables to account for the risk of market liquidity. Five different LHs are introduced: 10, 20, 40, 60, and 120 days. In calculating ES, the LHs are to be reflected by scaling ES calculated on a base horizon, which is 10-days. The differentiated LHs make holding instruments assigned to a category with higher LHs expensive from a capital point of view. The proposed LHs for the interest rate risk class are indicated in table 4.2

Market Variable	Liquidity Horizon
Interest rate: EUR, USD, GBP, AUD, JPY, CAD and domestic currency of a bank	10
Interest rate: other currencies	20
Interest rate: volatility	60

Tab. 4.2: Interest rate liquidity horizons.

Now, consider a portfolio with market variables in each of the LH categories in table 4.3. For such a portfolio, the calculation of regulatory ES is outlined in the steps below:

1. Calculate 10-day ES for the portfolio with all the risk factors in the model.
2. Calculate 10-day ES for the portfolio with risk factors that have LH ≥ 20 in the model, while risk factors with LH < 20 are kept constant.
3. Scale the result in step 2 with the square root of time ($\sqrt{T}$) rule, for the base horizon T.
4. Repeat Steps 2 and 3 for LH $\geq 40, 60, 120$.
5. Calculate regulatory ES according to equation 4.6 below

$$ES = \sqrt{(ES(P))^2 + \sum_{j \geq 2} \left(ES(P_j) \sqrt{\frac{\mathrm{LH_j} - \mathrm{LH_{j-1}}}{10}} \right)^2}, \tag{4.6}$$

where:

- P is the set of all risk factors;
- $ES(P)$ is the 10-day ES for the portfolio with all the risk factors in the model;
- LH_j is the liquidity horizon for category j;
- P_j is the set of all risk factors with LH $\geq \mathrm{LH}_j$; and
- $ES(P_j)$ is the 10-day ES for the portfolio with only risk factors in P_j in the model, keeping other risk factors constant.

Category (j)	LH
1	10
2	20
3	40
4	60
5	120

Tab. 4.3: Liquidity Horizons of market risk factors.

If we do not have a full history of data for some risk factors, the FRTB allows for the ES measure to be calibrated to a period of stress based on an indirect approach. This indirect approach uses a reduced set of risk factors which must account for at least 75% of the full ES in the internal model, and a full 10-years of historical data must be available for this reduced set of risk factors. The indirect approach then calculates ES as follows:

1. Find the 12-month period for which the ES calculated with the reduced set of risk factors, $ES_{R,S}$, is largest. These 12 months of stress must be updated at least quarterly or when there are material changes in the risk factors in the portfolio.

2. For the current 12-month period, calculate ES for the reduced set of risk factors $ES_{R,C}$ and that for the full set of risk factors $ES_{F,C}$.

3. Calculate the ES measure by scaling up $ES_{R,S}$ by the ratio of $ES_{F,C}$ to $ES_{R,C}$, this ratio is floored at 1. i.e.

$$ES = ES_{R,S} \times \frac{ES_{F,C}}{ES_{R,C}}. \tag{4.7}$$

   This ES measure is then scaled up to the relevant liquidity horizon.

The FRTB specifies risk factors to be used in an internal model as the market rates and prices that affect the value of the bank's trading position. All the risk factors that are used for pricing and specified in the SA for a corresponding risk class must be included in a bank's internal model (BIS, 2019).

To calculate ES for a set of risk factors using the methods described above, we require good data quality for the risk factors. The FRTB proposes a distinction between modellable risk factors (MRFs), for which such data exists and non-modellable risk factors (NMRFs) for which appropriate data does not exist. For a risk factor to be classified as modellable and be included in an internal model, the FRTB requires the risk factor to pass a risk factor eligibility test (RFET). This test requires identification of at least 24 real price observations per year with gaps between observations being not more than one month[4].

For risk factors that are deemed modellable, the internal model capital charge (IMCC) is calculated as a weighted average of a bank-wide ES and partial ES measures for each risk class, i.e.,

$$\text{IMCC} = \rho \times ES(C) + (1 - \rho) \times \sum_{i=1}^{n} ES(C_i), \tag{4.8}$$

where:

- C is the set of all risk factors affecting the bank's portfolio;
- $ES(C)$ is the bank-wide ES, the 10-day ES of the bank's portfolio with risk factors C in the model;

[4] See (BIS, 2019) for criteria that a price observation must meet to be considered real.

- C_i is the set of risk factors in risk class i that affect the bank's portfolio, for n risk classes;
- $ES(C_i)$ is the partial ES for risk class i, the 10-day ES of the bank's portfolio with risk factors in C_i in the model, keeping other risk factors constant; and
- $\rho = 0.5$.

For NMRFs, capital requirements are determined using stress scenarios calibrated to a 12-month period of stress as prudent as the ES calibration used for the MRFs (BIS, 2019). A common 12-month period is to be used across all NMRFs in the same risk class. "For each NMRF, the liquidity horizon of the stress scenario must be the greater of the liquidity horizon assigned to the risk factor and 20 days" (BIS, 2019, p93). Suppose we have n NMRFs, the capital requirement for the NMRFs is the aggregate of the scenario stressed losses (SSLs) for each risk factor. i.e.

$$C_{NM} = \sqrt{\left(\rho \times \sum_{i=1}^{n} \mathrm{SSL}_i\right)^2 + (1-\rho^2) \times \sum_{i=1}^{n} \mathrm{SSL}_i^2}, \tag{4.9}$$

where:

- C_{NM} is the capital requirement for the NMRFs;
- SSL_i is the scenario stressed loss for NMRF i (i.e. the loss experienced by the portfolio under the scenario when NMRF i is under stress); and
- $\rho = 0.6$.

If we have a combination of modellable and non-modellable risk factors, the capital charge for a trading desk with IMA approval is calculated as follows:

$$C = \max(\mathrm{IMCC_{t-1}} + C_{NM,t-1}, \mathrm{m_c} \times \mathrm{IMCC_{avg}} + C_{NM,avg}), \tag{4.10}$$

where

- $\mathrm{IMCC_{t-1}}$ and $C_{NM,t-1}$ are the previous days IMCC and capital requirement for NMRFs respectively;
- $\mathrm{IMCC_{avg}}$ and $C_{NM,avg}$ are the previous 60 days average IMCC and capital requirement for NMRFs respectively; and
- $\mathrm{m_c}$ is a multiplicative factor between 1.5 and 2.

If all the risk factors are modellable, the capital charge is

$$C = \max(\mathrm{IMCC_{t-1}}, \mathrm{m_c} \times \mathrm{IMCC_{avg}}), \tag{4.11}$$

while if all the risk factors are non-modellable, the capital charge is

$$C = \max(C_{NM,t-1}, C_{NM,avg}). \tag{4.12}$$

4.6 Summary

In Basel 2, internal model capital charges for market risk are calculated based on 99% confidence level 10-day VaR. Capital charges calculated according to Basel 2 proved to be inadequate to absorb trading book losses during the 2008 global financial crisis. Basel 2.5 improved on the Basel 2 calculation by introducing a sVaR measure, which is added to the Basel 2 calculation. Despite the improvement in internal model capital charges, criticism with VaR remained unaddressed by Basel 2.5. The FRTB replaces 99% confidence level 10-day VaR with 97.5% confidence level 10-day ES as the risk measure for calculating internal model capital charges for market risk. Additionally, the FRTB introduces a distinction between modellable and non-modellable risk factors.

In the SA, the FRTB introduces a risk-sensitive methodology for calculating SA capital charges.

Chapter 5

Methodology

In this chapter, we present the data set to be used and methodology to be followed to calculate minimum regulatory capital charges for hypothetical portfolios of instruments outlined in table 5.1. Our focus will be on the methodology to calculate IMA capital charges. We also present a brief outline on how we will calculate SA capital charges and find FRTB SA parameters that recover FRTB IMA capital charges.

Asset	Instrument
Cash deposits	1 Day, 1 Month and 3 Months
FRAs	1x4, 2x5, 3x6, 4x7, 5x8, 6x9, 7x10, 8x11, 9x12, 12x15, 15x18, 18x21
IRSs	2y, 3y, 4y, 5y, 6y, 7y, 8y, 9y, 10y, 12y, 15y, 20y, 25y, 30y

Tab. 5.1: Instruments held in hypothetical portfolios.

5.1 Data

A full data set of daily South African interbank interest rate data consisting of cash deposit, FRA and IRS rates for the instruments in table 5.1 for the period 1 January 2004 to 31 October 2018 was collected from Bloomberg. All the FRAs and IRSs reference 3-month JIBAR. With these rates, daily interbank NACC swap curves are bootstrapped using a raw interpolation method. According to Hagan and West (2008), raw interpolation produces bootstrapped yield curves that demonstrate favorable mathematical and economic characteristics, while also being simple to implement; hence it is used here. We do not explain how to bootstrap using the raw interpolation method. Instead, we refer the reader to Hagan and West (2008).

For this dissertation, the interbank NACC swap curve is considered to be a proxy for the risk-free yield curve, while the risk factors of the portfolios are the bootstrapped NACC spot rates corresponding to the maturities of the instruments in table 5.1. For example, the 1-month cash deposit is used to bootstrap the 1-month NACC spot rate, a 3x6 FRA is used to bootstrap the 6-month NACC spot rate and a 5y IRS is used to bootstrap the 5-year NACC spot rate.

5.2 Calculating the Internal Model Capital Charge

To calculate the internal model capital charge for a portfolio, the following steps are taken:

1. Create a hypothetical portfolio consisting of the instruments in table 5.1.
2. Perform a check to determine whether the portfolio is realistic. If the portfolio is realistic, move to step 3, otherwise, go back to step 1.
3. Calculate the risk measures of the portfolio, i.e. the 10-day VaR and ES.
4. Using the risk measures calculated in step 3, calculate the capital charge.

Below, we outline each of the steps above and any assumptions made.

5.2.1 Creating a Hypothetical Portfolio

To create a hypothetical portfolio consisting of the instruments in table 5.1, we require a position (long or short) on an instrument and a notional amount for the position taken. To create a position, we generate a Bernoulli random variable [1] with the outcomes 1 and -1, each equally likely. The outcome 1 represents a long position and -1 a short position. To generate a notional amount of a position, we generate a uniform random number in the range 0 to 1 and round it to two decimal places. We then multiply the rounded number by $10^8 \times t^{-h}$, a power-law scale notional amount, rounded to the nearest one million. Where t = maturity date of the instrument in year fractions and $h \in [0, 1]$. Since PV01s of linear interest rate products for the same nominal increase with time-to-maturity, a power-law scale notional is used to reduce the PV01s on long-dated linear interest rate products. The value of h is chosen to ensure the following:

$$\frac{\text{sum of averages of the absolute PV01s of risk factors determined by FRAs}}{\text{sum of averages of the absolute PV01s of risk factors determined by IRSs}} \approx 1. \tag{5.1}$$

Since most of the risk factors are determined by the FRAs and IRSs, this ensures that neither the risk factors determined by the FRAs nor those determined by the IRSs dominate the risk of the hypothetical portfolios.

To ensure the portfolio created is realistic, we perform a check on the portfolio. The check is based on the PV01s of the portfolio with respect to its predetermined risk factors. The PV01s are calculated as follows:

$$\text{PV01}_t = V(r_t + 0.0001) - V(r_t),$$

where PV01_t is the PV01 of the portfolio with respect to the spot rate for tenor t and V is the market value of the portfolio as a function of the risk-free yield curve. PV01_t represents the change in the value of the portfolio due to a 1bp increase in the spot rate for tenor t, assuming all other spot rates remain constant. The PV01s

[1] A Bernoulli random variable is a discrete random variable with two outcomes.

of the portfolio with respect to each of the risk factors are aggregated to get the total PV01 of the portfolio, i.e.,

$$\text{total PV01} = \sum_{t \in T} \text{PV01}_t,$$

where T is the set of tenors corresponding to the maturities of the instruments in table 5.1. The total PV01 is a measure of the risk of a portfolio, and it represents the change in the value of a portfolio due to a 1bp parallel shift in the risk-free yield curve.

To reflect realistic limits that may be imposed on a trader, we put two constraints on the PV01s of the hypothetical portfolio. The first is a limit of 10000 on the absolute values of each of the PV01s, PV01_t, of the portfolio with respect to the risk factors. The second a limit of 100000 on the absolute value of the total PV01. If the constraints are met, the portfolio is considered to be realistic, and we proceed to calculate its minimum regulatory capital charge. Otherwise, we discard the portfolio.

5.2.2 Calculating the Risk Measures

Basel 2 requires us to use a period of one to four years for the calculation of VaR. While Basel 2.5 and the FRTB require us to find a 12-month period of stress for the portfolio in consideration. Hence for consistency, we use a 12-month period to compute all risk measures.

For this dissertation, to generate scenarios for the 10-day changes in the risk factors, we use overlapping periods. Assuming we have 250 trading days in a 12-month period, to generate 250 scenarios of the risk factors 10-days ahead using overlapping periods, we require a subset of 260 trading days. The first scenario in each historical simulation considers the change in the risk factor between day 1 and day 11; the second scenario considers changes in the risk factor between day 2 and day 12; and so on, the last simulation scenario considers the change between day 250 and day 260. i.e.

$$\text{Value of risk factor under } i^{th} \text{ scenario} = v_t \frac{v_{i+10}}{v_i}, \text{ for } 1 \leq i \leq 250,$$

where v_t is the value of the market variable on the day t (i.e. today) and v_i is the value of the market variable on the day i. Since changes in successive sample points of a historical sample are not independent, the use of overlapping periods is less than ideal. However, according to Hull (2015), this does not bias the results, but reduces the effective sample size, thus making results noisier than they would otherwise be. Additionally, the FRTB allows the use of overlapping periods to determine scenarios for the changes in the risk factors.

From the simulated scenarios of the risk factors 10-days ahead, we calculate the 10-day change under each scenario. Then, using the PV01s of the portfolio with respect to its risk factors, an approximation of the 10-day profit & loss ($P\&L_{10d}$) of the portfolio under each scenario is calculated using a PV01 based valuation as follows:

$$P\&L_{10d} \approx \sum_{i}^{n} PV01_i \times \Delta v_{10d,i},$$

where $\Delta v_{10d,i}$ is the 10-day change in risk factor i on the day t, i.e.

$$\Delta v_{10d,i} = \text{value of risk factor under } i^{th} \text{ scenario} - v_t.$$

Provided $\Delta v_{10d,i}$ is small, this should provide a good approximation for the $P\&L_{10d}$.

From our 250 scenarios, we sort the $P\&L_{10d}$ values from the largest loss to largest gain and compute the 10-day VaR and ES risk measures. We calculate the 99% confidence level VaR as the third largest loss of the sorted $P\&L_{10d}$. 97.5% ES is calculated as the average of the 7 largest losses of the sorted $P\&L_{10d}$. Since our portfolios consist of South African Rand (ZAR) interest rate products, we only have risk factors with a 10-day liquidity horizon [2]. Since a full data set is available for all of the risk factors, and we only have risk factors with a 10-day liquidity horizon, we only require a single 10-day ES number.

To calculate the VaR introduced in Basel 1, historical scenarios are created from the most recent 12-month period. Since Basel 2.5 and the FRTB require us to go back in time to at least 2007 to find a 12-month period when our portfolio is at stress, for this dissertation, we go back to 2004 so as to get a more conservative estimate. Between 1 January 2004 and day t, we get subsets of 260 consecutive trading days and create 250 scenarios from each subset. We then compute the $P\&L_{10d}$ values for the scenarios and calculate VaR and ES for each subset of scenarios using the PV01s calculated on day t. We identify the 12-month period of stress as the period producing the largest VaR and ES measure over the subsets of scenarios. The largest values for VaR and ES are taken as our sVaR and ES measures for the purpose of calculating capital charges. This approach to calculating sVaR and ES is proposed by EBA (2012).

5.2.3 Calculating the Minimum Capital Charge for the Portfolio

Assuming all the risk factors are modellable, to calculate the minimum capital charge, we require the values of the risk measures on 60 consecutive trading days. Hence to simplify computations, we assume that the trader will carry out any necessary re-balancing each day to keep the PV01s of the portfolio constant over the next 59 trading days. Then on each day $t + 1d$ to $t + 59d$, we compute the 10-day VaR calibrated to the most recent 260 trading days and the 10-day sVaR and sES as outlined above. The only change is that we use the rates on the days $t + jd$, for $j = 1, 2, \ldots, 59$ to create the scenarios of the changes in the risk factors. Minimum capital charges for the portfolio on day $t + 60d$ are then calculated according to equations 4.1, 4.2 and 4.11. We set day t to be 8 August 2018 and day $t + 60d$ is 31 October 2018.

All capital charges will be calculated at a single point in time. Even though the calculations will be done at a single point in time, we believe by doing this for

[2] ZAR is the domestic currency for South African banks.

many randomly created hypothetical portfolios, we will be able to get meaningful results on what the impact of the change in regulation will have on capital charges.

5.3 Calculating Capital Charges for Non-Modellable Risk Factors in the FRTB

For this dissertation, no test is performed to check if a risk factor meets the modellability criteria. Instead, we shall assume scenarios where a particular portion of the risk-free yield curve is non-modellable. The capital charge under each scenario is then compared with that assuming all risk factors are modellable. The following scenarios are assumed for the non-modellable risk factors:

A. All risk factors are non-modellable.

B. Only the risk factors corresponding to the maturities of the cash deposits are non-modellable.

C. Only the risk factors corresponding to the maturities of the FRAs are non-modellable.

D. Only the risk factors corresponding to the maturities of the IRSs are non-modellable.

When a particular portion of the risk-free yield curve is deemed to be non-modellable, the instruments that determine that portion are still used to construct the risk-free yield curve. We also calculate PV01s for all risk factors using the risk-free yield curve constructed. The PV01s for the modellable risk factors are then used to calculate the IMCC as outlined above, while those for the non-modellable risk factors are used to calculate C_{NM} as outlined below.

No framework exists in practice or the academic literature on how to calculate the capital charges for non-modellable risk factors. For this dissertation, for each risk factor deemed non-modellable, we calculate the SSL for a 12-month period as follows:

$$\mathrm{SSL}_i = \sqrt{2} \times |\mathrm{PV01}_i \times \max(\Delta_{10d,r_i})|, \tag{5.2}$$

where

- SSL_i is the stress scenario capital charge for NMRF i;
- $\mathrm{PV01}_i$ is the PV01 of the portfolio with respect to the spot rate for tenor i;
- $\max(\Delta_{10d,r_i})$ is the maximum 10-day change in spot rate, r_i, during the 12-month period being considered;
- the $\sqrt{2}$ is a multiplication factor to account for a 20-day liquidity horizon for each NMRF (the liquidity horizon of the domestic interest rate is 10, which is less than 20); and
- the absolute value sign accounts for an upward or downward move in the spot rate.

Here $|PV01_i \times \max(\Delta_{10d,r_i})|$ represents the largest 10-day loss our portfolio would have experienced over the 12-month period being considered for a 10-day change in non-modellable risk-free rate i. The capital charge for the NMRFs is calculated according to equation 4.9, and this is calculated for subsets of 12-months between 1 January 2004 and day t. We take the C_{NM} as the largest capital charge for the NMRFs over all of the aforementioned subsets.

To calculate the minimum capital charge when we have non-modellable risk factors, we require the value of the C_{NM} on 60 consecutive trading days. Similar to when all risk factors are modellable, we assume that the trader will carry out any necessary re-balancing each day to keep the PV01s of the portfolio constant over the next 59 trading days. Then for each day $t + 1d$ to $t + 59d$, the C_{NM} measure is calculated as outlined above.

5.4 Calculating the Standardised Approach Capital Charge

Since the portfolios considered consist of ZAR denominated linear interest rate products assumed to be default-free, we only require a delta risk capital charge for a single bucket under the FRTB SA. The risk factors used are the risk-free rates with tenors corresponding to the maturities of the instruments held in our portfolios. These are the same risk factors used to calculate the IMA capital charges. Since we assume the PV01 structure of the portfolio is kept constant, the PV01s used in the internal model are also used to calculate SA capital charges. Given the PV01 structure of the portfolio, we calculate the delta risk capital charge according to the steps described in section 4.5.1. For tenors not in table 4.1, the corresponding risk weight is taken as a linear interpolation of the risk weights of the tenors either side of it. To ensure we get a conservative estimate of the minimum capital charges, we do not divide the prescribed risk weights by the square root of 2.

To calculate the Basel 2 SA capital charge, the maturity method is used.

5.5 Finding FRTB SA Parameters that recover FRTB IMA Capital Charges

Recall from section 4.1 that the SA is a model proposed by the committee to determine the minimum capital charge. It prescribes parameters to use and rules to follow in the calculation of the minimum capital charge. In the FRTB, the parameters prescribed are the risk weights, the correlations between risk weights and the correlations between buckets. Since we have a single bucket, our parameters of interest are the risk weights and correlations between risk weights. For this dissertation, our focus is on the risk weights. Two approaches are used to find a set of SA risk weights that ensure that SA capital charges are approximately equal to IMA capital charges.

The first approach is to find the set of minimum risk weights such that the absolute difference between the calculated SA and IMA capital charges is less than a given tolerance level for each portfolio. Mathematically, this approach can be written as:

$$\min_{\omega} : |C_{SA,i}(\omega) - C_{IMA,i}| < tol, \tag{5.3}$$

where:

- ω is the set of SA risk weights we are after;
- $C_{SA,i}(\omega)$ is the FRTB SA capital charge for portfolio i as a function of ω;
- $C_{IMA,i}$ the FRTB IMA capital charge for portfolio i; and
- tol is the tolerance level for the absolute difference between the capital charge of the SA and IMA.

The alternative approach is to find the set of SA risk weights such that the absolute difference between 1 and the average of the ratios of SA and IMA capital charges is less than a given tolerance level for all the portfolios in consideration. For n portfolios this approach can be written mathematically as:

$$\min_{\omega} : \left| 1 - \frac{1}{n} \left(\sum_{i}^{n} \frac{C_{SA,i}(\omega)}{C_{IMA,i}} \right) \right| < tol. \tag{5.4}$$

Since we have a single inequality and more than one unknown (the risk weights for each tenor) for 5.3 and 5.4, we may have more than one solution for the inequalities. The presence of the $\max$ operator complicates the possibility of finding analytical solutions to the inequalities. Hence we use a numerical approach to find a set of SA risk weights that meet the tolerance level by trial and error. Two starting points are used, these are set at the weights in tables 5.2 and 5.3. For each successive trial, the weights are increased by 0.001% until the tolerance level is met. The first starting point is a simple parallel shift of the risk weights prescribed by the FRTB SA, while the second starting point assumes the risk weights for all tenors are equal.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Risk weight (%)	0.6	0.6	0.5	0.2	0.1	0.0	0.0	0.0	0.0	0.0

Tab. 5.2: The first starting point used to find a set of SA risk weights that ensure our tolerance level is met.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Risk weight (%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Tab. 5.3: The second starting point used to find a set of SA risk weights that ensure our tolerance level is met.

5.6 Summary

We calculate capital charges for hypothetical portfolios consisting of linear interest rate products at a single time point. The first step is to randomly create a hypothetical portfolio. The next step is to calculate the PV01s of the hypothetical portfolio with respect to its risk factors and perform a check to ensure the portfolio meets certain realistic constraints. Finally, capital charges for the hypothetical portfolio are calculated according to the IMA and SA of the regulations outlined in chapter 4.

Chapter 6

Results

To ensure we get meaningful results, ten thousand hypothetical portfolios were created for each of the values of $h \in \{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. For each of the values of h, ratios of PV01s are calculated according to 5.1. The ratios and corresponding h values are presented in table 6.1. The portfolios generated by $h = 0.8$ give a ratio closet to one. Thus, capital requirements for each of the portfolios generated for $h = 0.8$ were calculated according to the SA and IMA in Basel 2, Basel 2.5 and the FRTB. The minimum values of the multipliers were used to calculate IMA capital charges.

h	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3
ratio	1.4490	1.1860	1.0026	0.8773	0.7992	0.7506	0.7093	0.6934

Tab. 6.1: Ratios of $\frac{\text{sum of averages of the absolute PV01s of risk factors determined by FRAs}}{\text{sum of averages of the absolute PV01s of risk factors determined by IRSs}}$ of the ten thousand hypothetical portfolios for each value of h.

Scatter plots of the capital requirements against the total PV01 for the portfolios are plotted. The scatter plots exhibit an increasing linear trend, as the absolute value of the total PV01 increases, the minimum capital requirement for a portfolio increases. Hence, the scatter plots are complemented by simple linear regression lines of the capital requirements against the total PV01 for the portfolios. For each approach to calculating capital requirements, two regressions are performed. The first for portfolios with a total PV01 greater than zero and the other for portfolios with total PV01 less than or equal to zero. In Appendix B, we present summary statistics of the capital requirements and the regressions. With p-values close to 0, the regression lines are a reasonable fit for the data. We also present ratios of selected capital requirements and risk measures to complement the results.

6.1 Moving from Basel 2.5 to FRTB

6.1.1 Basel 2.5 Capital Charges

Under the current regulatory regime (i.e. Basel 2.5), on average the adoption of the IMA results in a capital charge that is close to 4 times that resulting from adopting the SA. The difference is quite large when compared to the difference in the two approaches under the FRTB.

Ratio	Average of Ratio
FRTB SA / FRTB IMA	1.1534
Basel 2.5 IMA / Basel 2 SA	3.8287
Basel 2.5 IMA / FRTB IMA	2.9329
FRTB SA / Basel 2 SA	1.3938
FRTB IMA / Basel 2 IMA	1.3403
Basel 2.5 IMA / Basel 2 IMA	3.7848
FRTB SA / Basel 2.5 IMA	0.3927
FRTB IMA / Basel 2 SA	1.3237

Tab. 6.2: Ratios of the capital requirements for market risk calculated according to the internal models and standardised approaches in Basel 2, Basel 2.5 and the FRTB, assuming all risk factors are modellable.

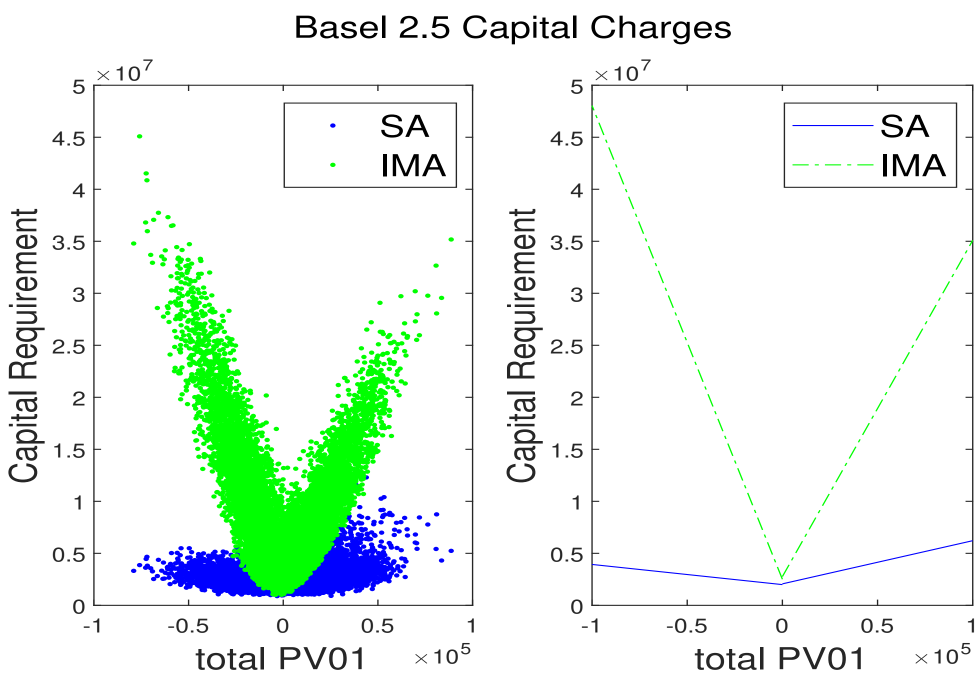

Fig. 6.1: The left panel displays the scatter plots of the Basel 2.5 capital requirements against total PV01 for the hypothetical portfolios calculated according to the IMA and SA. The right panel displays the corresponding regression lines.

6.1.2 FRTB Capital Charges

From the plots, we see that on average for portfolios with a total PV01 greater than zero, the SA results in a higher capital charge. While for portfolios with total PV01 less than or equal to zero the IMA results in a higher capital charge. For the portfolios in consideration, on average, the adoption of the SA results in a capital charge that is 15% higher than that from adopting the IMA. This difference is small when compared to the difference in the two approaches under Basel 2.5.

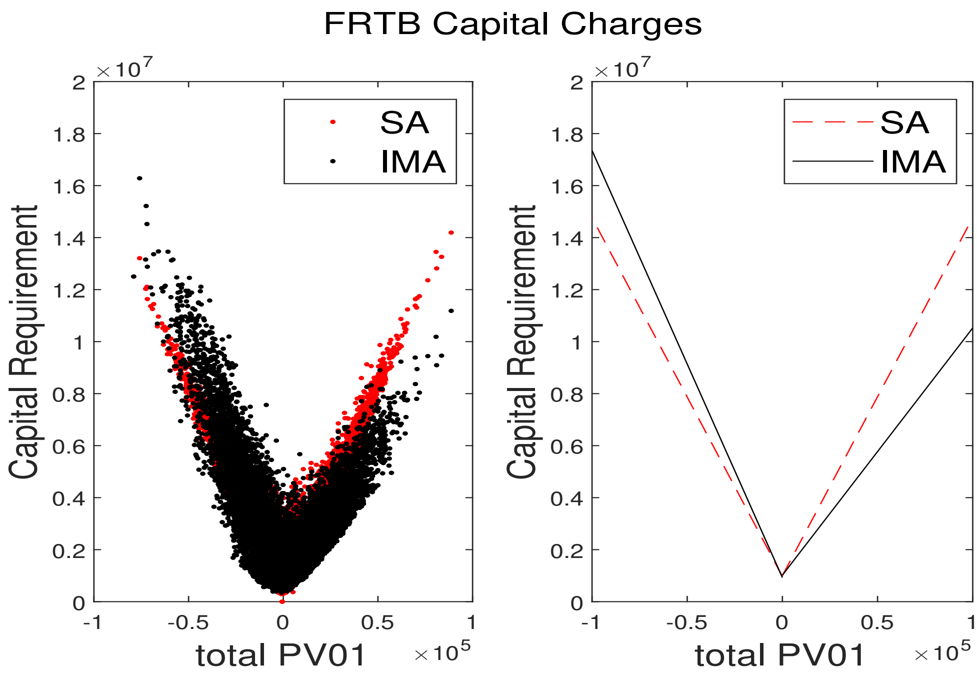

Fig. 6.2: The left panel displays the scatter plots of the FRTB capital requirements against the total PV01 for the hypothetical portfolios calculated according to the IMA and SA. The right panel displays the corresponding regression lines.

6.1.3 Standardised Approach Capital Charges

From the plots in 6.3 and the summary statistics of the regressions, we see that the FRTB SA scatter plot exhibits a steeper increasing trend compared to the Basel 2 SA scatter plot. The steeper trend shows that FRTB SA is indeed more risk-sensitive than the Basel 2 SA. On average, the FRTB SA results in a capital charge that is close to 40% higher than the capital charge for the Basel 2 SA.

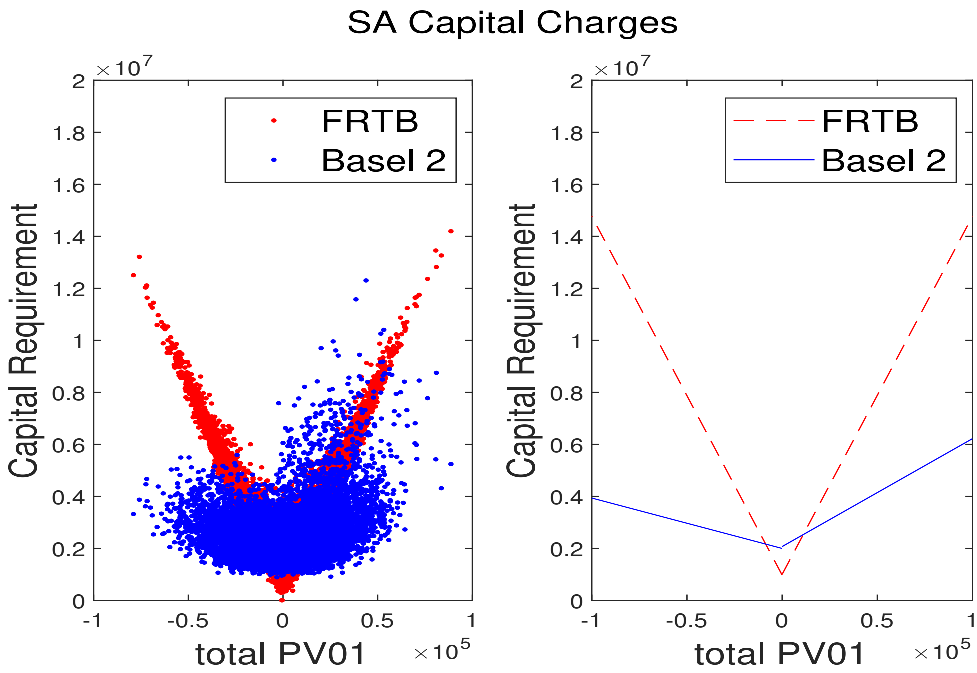

Fig. 6.3: The left panel displays the scatter plots of the SA capital requirements against the total PV01 for the hypothetical portfolios according to Basel 2 and the FRTB. The right panel displays the corresponding regression lines.

6.1.4 Internal Model Capital Charges

The Basel 2.5 internal model capital charge is close to 3 times that of the FRTB. This difference mostly results from the different values of the multipliers used, with the minimum multiplier for Basel 2.5 being equal to twice that of FRTB. Recall that for a normal distribution, 99% confidence level VaR is approximately equal to 97.5% confidence level ES. Given the minimum values of the multipliers, if our $P\&L_{10d}$ has a normal distribution, we should expect the Basel 2.5 internally modelled capital charge to be at least twice that of the FRTB.

In table 6.3 below, we present the averages of the ratios of VaR to ES for the stressed periods of each portfolio. The average is close to 1. Hence, to ensure the average internal model capital charge for the two regulatory regimes is close, we could use a multiplier of just over 3 for the FRTB IMA. Alternatively, we could use a VaR and sVaR multipliers of less than 1.5 for Basel 2.5.

On the other hand, the FRTB IMA results in a capital charge that is close to 35% higher than that of Basel 2. The move from Basel 2 to Basel 2.5 resulted in a considerable increase in the internally modelled capital charge for market risk. Moving from Basel 2.5 to FRTB should provide a relief, assuming all risk factors are modellable.

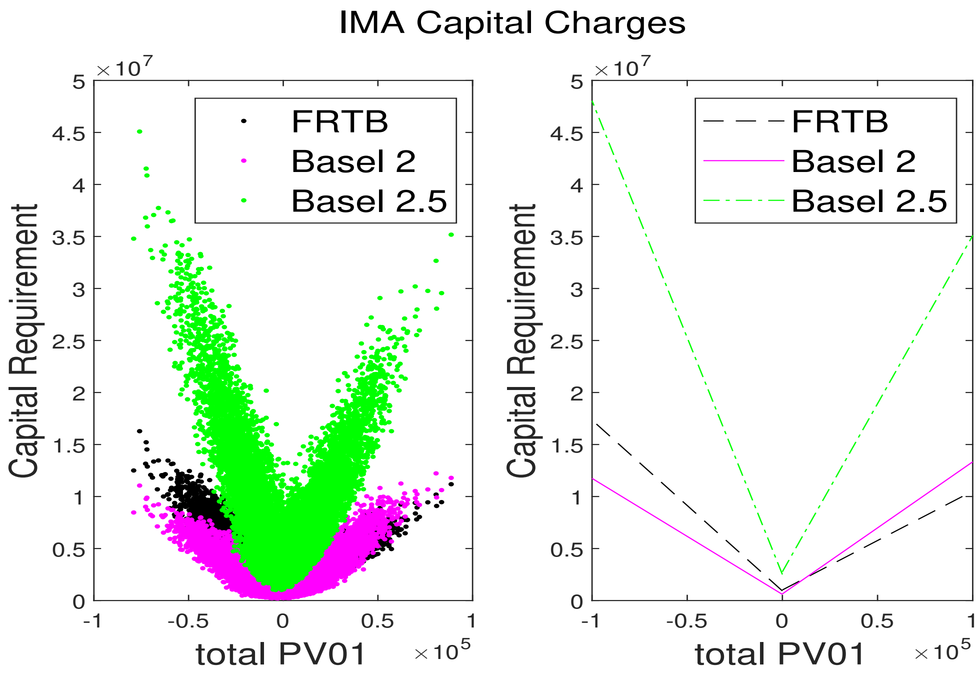

Fig. 6.4: The left panel displays the scatter plots of the IMA capital requirements against the total PV01 for the hypothetical portfolios according to Basel 2, Basel 2.5 and the FRTB. The right panel displays the corresponding regression lines.

Maximum	1.0720
Minimum	0.9642
Average	1.0300

Tab. 6.3: Summary statistics of the averages of the ratios of VaR to ES for the stressed periods.

6.1.5 Summary

On average, banks moving from the SA under Basel 2.5 to the IMA under the FRTB should expect a decrease in their market risk capital charge of around 32%. While banks moving from the IMA under Basel 2.5 to the SA under the FRTB should expect a decrease in their market risk capital charge of around 60%. For banks that do not change the approach used, those continuing with the use of the SA should on average expect a 39% increase in their capital charges, while those continuing with the IMA should expect a decrease in capital charges of more than 60%.

Given the qualitative and quantitative prerequisites of FRTB IMA, a bank moving from the SA under Basel 2.5 to the IMA under the FRTB is very unlikely even though it may result in a decrease in the market risk capital charge. While a bank currently using the IMA under Basel 2.5, moves to the SA or IMA under the FRTB are both likely however the move to the SA is more likely than that to the IMA.

6.2 Effect of Non-Modellability Criteria on the FRTB Internal Model Capital Charge

In this section, we examine the impact of the non-modellability criteria on the internally modelled capital charge for market risk under the FRTB. We assumed four scenarios for the modellability of the risk-free yield curve. In table 6.4 the average of the ratios of the capital charge under each scenario to the FRTB IMA capital charge are presented. The scatter plots and corresponding regression lines are presented in Appendix A.

Scenario	Average of Ratio
A	5.7420
B	1.4959
C	3.7678
D	3.6978

Tab. 6.4: The average of the ratios of the capital charge under each scenario with respect to the FRTB IMA capital charge.

Assuming all risk factors are non-modellable, the capital charge is on average close to 6 times what it would have been if all risk factors were modellable. Looking at the scatter plots and regression lines in figure A.1 of the appendix, we see that this is mostly due to a significant difference in the capital charges for portfolios with low absolute values of the total PV01.

When we assume only the risk factors corresponding to the maturities of the cash deposits are non-modellable, the internal model capital charge is close to 50% higher than what it would have been if all risk factors were modellable. This scenario results in the lowest capital charges for the portfolios compared to the other three. This is likely because only three of the risk factors correspond to the maturities of the cash deposits.

For the risk factors corresponding to the maturities of the FRAs, the capital charge is over 3.5 times what it would have been if all risk factors were modellable. Similarly, when we assume only the risk factors corresponding to the maturities of the IRSs are non-modellable, the capital charge is over 3.5 times what it would have been if all risk factors were modellable.

The risk factors corresponding to the maturities of the FRAs and IRSs seem to have the most significant impact on the internal model capital charge, with their effect as determined by the average ratios being approximately equal on average. This is due to the risk factors corresponding to the maturities of the FRAs and IRSs having an equal contribution to the risk of our portfolios. Hence, risk factors that contribute the largest portion of the risk of our portfolios as measured by the PV01s will have the most significant impact on the FRTB IMA capital charges when they are considered to be non-modellable.

6.3 FRTB SA Parameters recovering IMA Capital Charges

For the absolute difference between the calculated SA and IMA capital charges for each portfolio, the tolerance level is set at 10 000 which is less than 5% of the minimum value of the FRTB IMA capital charges. The set of risk weights which met the tolerance level of 10 000 is portfolio dependent. The summary statistics for the first and second starting points are presented in tables 6.5 and 6.6 respectively. The set of risk weights prescribed by the FRTB SA fall within the bounds of the weights in tables 6.5 and 6.6. On average, the set of risk weights prescribed by the FRTB SA is 0.09% higher than the weights recovering IMA capital charges. This suggests that the set of risk weights prescribed by the FRTB SA may be a little excessive. This should be expected as we see from table 6.2 that on average, the FRTB SA results in a higher capital charge than the FRTB IMA. Hence a lower set of SA risk weights should be prescribed to reconcile the two.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Minimum (%)	0.60	0.60	0.50	0.20	0.10	0.00	0.00	0.00	0.00	0.00
Maximum (%)	5.61	5.61	5.51	5.21	5.11	5.01	5.01	5.01	5.01	5.01
Average (%)	1.61	1.61	1.51	1.21	1.11	1.01	1.01	1.01	1.01	1.01

Tab. 6.5: Summary statistics for FRTB SA risk weights recovering IMA capital charges for each portfolio for the first starting point.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Minimum (%)	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51
Maximum (%)	5.61	5.61	5.61	5.61	5.61	5.61	5.61	5.61	5.61	5.61
Average (%)	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60

Tab. 6.6: Summary statistics for FRTB SA risk weights recovering IMA capital charges for each portfolio for the second starting point.

For the absolute difference between 1 and the average of the ratios of SA and IMA capital requirements, the tolerance level is set at 0.01%. The set of SA risk weights meeting this tolerance level are presented in tables 6.7 and 6.8, these are less than the set of risk weights prescribed by the FRTB SA.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Risk weight (%)	1.47	1.47	1.37	1.07	0.97	0.87	0.87	0.87	0.87	0.87

Tab. 6.7: Risk weights minimising the absolute difference between 1 and the average of the ratios of SA and IMA capital requirements for the first starting point.

Tenor (years)	0.25	0.5	1	2	3	5	10	15	20	30
Risk weight (%)	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47

Tab. 6.8: Risk weights minimising the absolute difference between 1 and the average of the ratios of SA and IMA capital requirements for the second starting point.

Overall, we see that the risk weights prescribed by the FRTB SA result in capital charges that are on average higher than those of the FRTB IMA. Thus, to ensure that on average, the two approaches result in similar capital charges for linear interest rate products, we require a lower set of SA risk weights.

Chapter 7

Conclusion and Further Research

This dissertation aimed to examine the impact of the FRTB on the minimum regulatory market risk capital a South African bank is required to hold for its trading book consisting of linear South African interbank interest rate products. The committee prescribes two alternatives to calculating minimum regulatory capital for market risk: (i) the SA; and (ii) the IMA. In the FRTB, the committee proposes changes to both the current SA and IMA. In the SA, a risk-sensitive methodology is introduced to replace the current SA. In the IMA, ES replaces VaR as the basis for calculating IMA capital charges. For this dissertation, minimum regulatory capital requirements for market risk were calculated under both alternatives for Basel 2, Basel 2.5, and the FRTB.

For linear interest rate products, under the current regulatory regime, the IMA capital charge is significantly higher than that of the SA. Under the FRTB, there is a significant reduction in the difference between the IMA and SA capital charges, with the SA resulting in a larger capital charge than the IMA on average. The sensitivities based SA of the FRTB results in a higher capital charge than the SA of Basel 2 and is more risk sensitive. For a 10-day liquidity horizon for the interest rate risk factors, the Basel 2.5 IMA capital charge is significantly higher than that of the FRTB mainly due to the different values of the multipliers used.

When specific risk factors are considered to be non-modellable, the risk factors that dominate the risk of the portfolio result in the largest increase in capital charges. Hence, banks should ensure that risk factors dominating the risk of a portfolio do meet the modellability criteria. Otherwise, banks will have to reduce trading in portfolios for which the non-modellable risk factors dominate the risk of the portfolio so as to reduce capital charges.

Overall, we can conclude that on average when all risk factors are modellable, the FRTB results in a significant reduction in the IMA capital charge and an increase in the SA capital charge. This results in a significant reduction in the difference between the IMA and SA capital charges, thus creating a more consistent set of rules.

For this dissertation, the risk factors considered have a liquidity horizon of 10-days, hence the impact of introducing differentiated liquidity horizons in the FRTB IMA was not examined. Additionally, our portfolios were made up of ZAR denominated default-free linear interest rate products, hence only a delta risk capital requirement for a single currency under the FRTB SA was required. Hence, the impact of vega and curvature sensitivities; default risk; and residual risk add on capital requirements were not examined. For further research, the study can be replicated for portfolios of non-linear financial instruments which have risk factors that have liquidity horizons beyond 10-days. This will allow us to fully explore the new features introduced in the FRTB such as the differentiated liquidity horizons in the IMA and the extra sensitivities in the SA.

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Appendix A

Figures

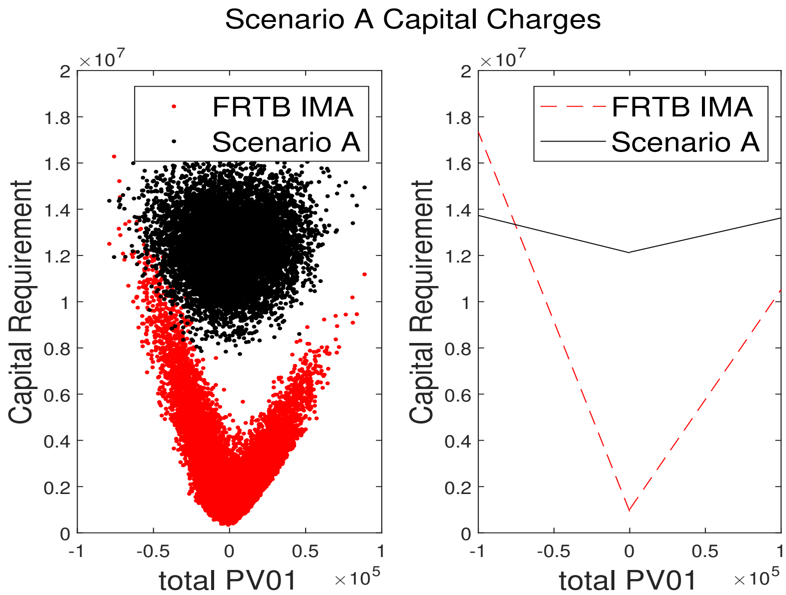

Fig. A.1: The left panel displays the scatter plots Capital Charges against the total PV01 for the hypothetical portfolios calculated under Scenario A and when all risk factors are modellable. The right panel displays the corresponding regression lines.

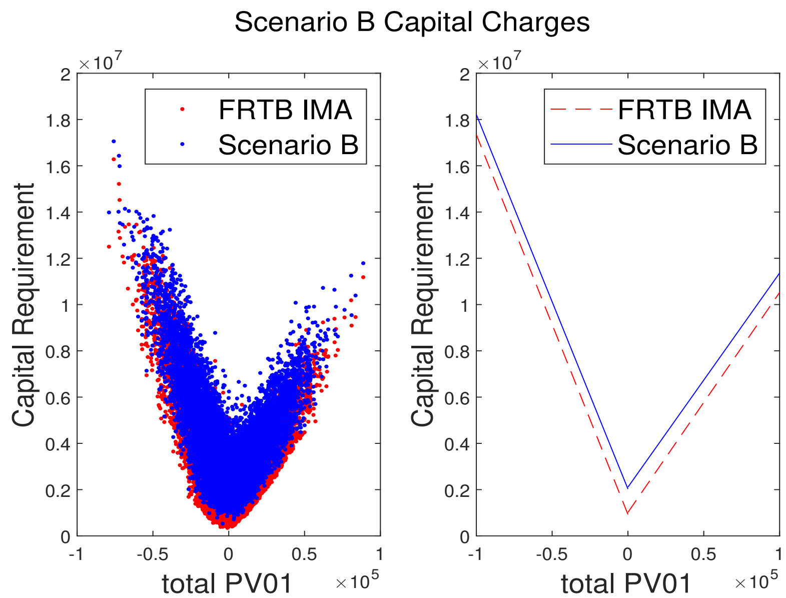

Fig. A.2: The left panel displays the scatter plots Capital Charges against the total PV01 for the hypothetical portfolios calculated under Scenario B and when all risk factors are modellable. The right panel displays the corresponding regression lines.

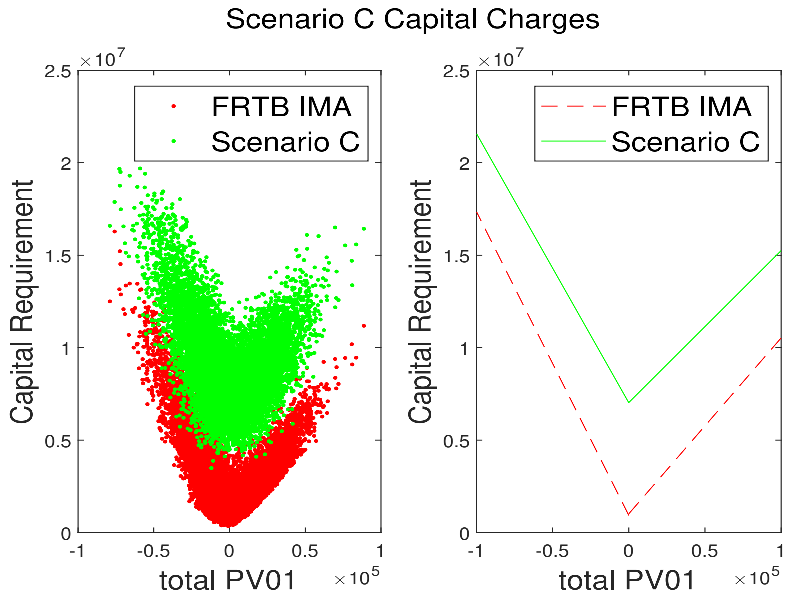

Fig. A.3: The left panel displays the scatter plots Capital Charges against the total PV01 for the hypothetical portfolios calculated under Scenario C and when all risk factors are modellable. The right panel displays the corresponding regression lines.

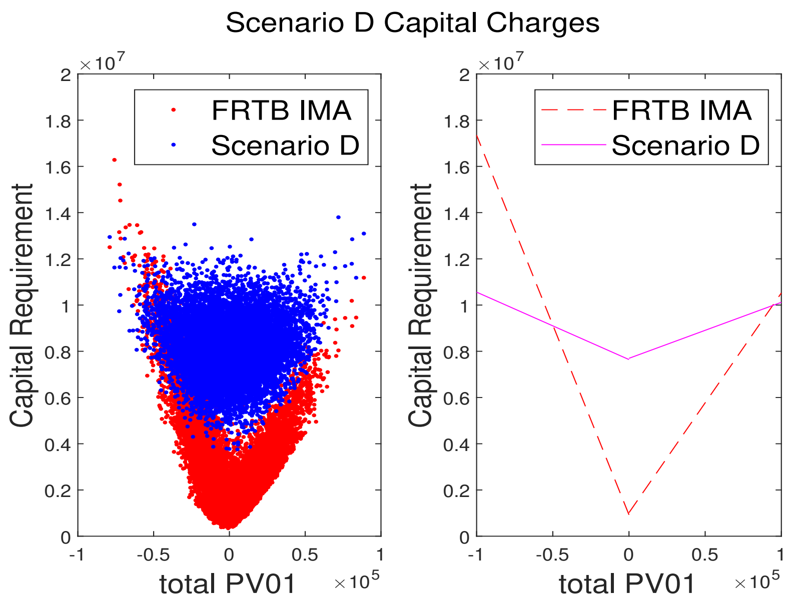

Fig. A.4: The left panel displays the scatter plots Capital Charges against the total PV01 for the hypothetical portfolios calculated under Scenario D and when all risk factors are modellable. The right panel displays the corresponding regression lines.

Appendix B

Tables of Summary Statistics

B.1 Capital Charges

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	11 183	88.702	16 281	-75.868
Minimum	404	2.209	344	-0.872
Average	2 702	17.711	3 826	-17.659

Tab. B.1: Summary statistics for FRTB IMA Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	14 192	88.702	13 208	-75.868
Minimum	300	0.051	0	-0.692
Average	3 437	17.711	3 405	-17.659

Tab. B.2: Summary statistics for FRTB SA Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	12 225	80.718	11 053	-75.868
Minimum	236	0.051	197	-2.879
Average	2 888	17.711	2 560	-17.659

Tab. B.3: Summary statistics for Basel 2 IMA Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	35 173	88.702	45 088	-75.868
Minimum	1 080	0.264	1 009	-4.025
Average	8 489	17.711	10 548	-17.659

Tab. B.4: Summary statistics for Basel 2.5 IMA Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	12 299	43.859	7 579	-2.308
Minimum	909	25.253	885	-3.227
Average	2 801	17.711	2 336	-17.659

Tab. B.5: Summary statistics for Basel 2 SA Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	17 745	38.182	17 923	-31.146
Minimum	7 735	6.787	7 839	-2.451
Average	12 403	17.711	12 400	-17.659

Tab. B.6: Summary statistics for Scenario A Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	11 787	88.702	17 057	-75.868
Minimum	711	3.355	529	-4.025
Average	3 748	17.711	4 883	-17.659

Tab. B.7: Summary statistics for Scenario B Capital Charges.

	total $\mathbf{PV01} > 0$		total $\mathbf{PV01} \leq 0$	
	Capital Charge(000's)	**Corresponding PV01 (000's)**	**Capital Charge(000's)**	**Corresponding PV01 (000's)**
Maximum	16 500	69.703	20 310	-48.409
Minimum	4 101	17 651	3 490	-11.987
Average	8 488	17.711	9 587	-17.659

Tab. B.8: Summary statistics for Scenario C Capital Charges.

	total PV01 > 0		total PV01 ≤ 0	
	Capital Charge(000's)	Corresponding PV01 (000's)	Capital Charge(000's)	Corresponding PV01 (000's)
Maximum	13 798	71.927	13 493	-23.172
Minimum	3 758	5.336	3 774	-0.043
Average	8 125	17.711	8 157	-17.659

Tab. B.9: Summary statistics for Scenario D Capital Charges.

B.2 Regression Statistics

	total PV01 > 0	total PV01 ≤ 0
Intercept	1 017 966	925 688
Slope	95	-164
R-squared	0.730	0.741
p-value	0.000	0.000

Tab. B.10: Summary statistics for regression of FRTB IMA Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	998 807	968 781
Slope	138	-138
R-squared	0.900	0.902
p-value	0.000	0.000

Tab. B.11: Summary statistics for regression of FRTB SA Capital Charges against the total PV01.

	total PV01 > 0	$PV01_{total} \leq 0$
Intercept	2 066 058	1 993 786
Slope	41	-19
R-squared	0.191	0.118
p-value	0.000	0.000

Tab. B.12: Summary statistics for regression of Basel 2 SA Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	639 570	590 769
Slope	127	-112
R-squared	0.748	0.732
p-value	0.000	0.000

Tab. B.13: Summary statistics for regression of Basel 2 IMA Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	2 763 784	2 506 773
Slope	323	-455
R-squared	0.759	0.739
p-value	0.000	0.000

Tab. B.14: Summary statistics for regression of Basel 2.5 IMA Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	1 2140 093	12 114 141
Slope	15	-16
R-squared	0.019	0.021
p-value	0.000	0.000

Tab. B.15: Summary statistics for regression of Scenario A Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	2 108 746	2 024 771
Slope	93	-162
R-squared	0.638	0.699
p-value	0.000	0.000

Tab. B.16: Summary statistics for regression of Scenario B Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	7 032 687	7 016 721
Slope	82	-146
R-squared	0.370	0.501
p-value	0.000	0.000

Tab. B.17: Summary statistics for regression of Scenario C Capital Charges against the total PV01.

	total PV01 > 0	total PV01 ≤ 0
Intercept	7 696 364	7 642 345
Slope	24	-29
R-squared	0.065	0.081
p-value	0.000	0.000

Tab. B.18: Summary statistics for regression of Scenario D Capital Charges against the total PV01.