A DESCRIPTIVE ANALYSIS OF THE VARIOUS SOURCES OF PORTFOLIO RISK ON THE NAMIBIAN STOCK MARKET

STA5088H MASTERS RESEARCH PROJECT

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EXECUTIVE SUMMARY

We considered Portfolio Diversification Index (PDI), and Concentration to see their impact on portfolio risk on the Namibian Stock Market.

The most remarkable insights in this paper are:

- Amongst several issues we investigated, we formed portfolios within Namibian All Share Index (Market Capitalisation weights: Categories) and found that financial category is significantly more diversified than the rest of the categories, highlighting that there are far more independent factors driving returns in the Financial category (as reflected by the PDI). We further found that by forming portfolios using market capitalisation weights, reduces PDI value and increase portfolio risk, highlights the additional effect that concentration has on portfolio risk as well as on PDI measure in general.

- We considered the number of stocks needed in a typical portfolio in Namibia to achieve effective risk reduction. Our analysis using the typical levels of concentration in our markets, reveals that a bare minimum of 6 stocks are required and more likely as many as 10 stocks are needed before the marginal reduction in risk becomes of little practical advantage.

- We also show the significance of concentration in determining the risk of portfolio both empirically and theoretically. We demonstrate that if large portfolio were equally weighted then all-equity portfolios should have risks in line with the average covariance between stocks, i.e. p.a.. Contrasting this to the 27% p.a. for the Namibian All Share Index, highlights the additional effect that concentration has on portfolio risk in Namibia.

- By comparing our results on the Namibian Stock Exchange (NSX) with the similar study done on the Johannesburg Stock Exchange (JSE) by Bradfield and Kgomari (2004), we find that most of total risk on the NSX is due to concentration while in JSE was due to correlation.

- Finally comparison of our results with international studies on risk reduction highlights two key insights. Remarkably the high correlation amongst the stocks in our market, as well as the high levels of concentration of our market. These two characteristics jointly impact the potential to reduce risk by forming diversified portfolio in Namibia. We find that we are able to achieve a reduction (in single stock risk) of only 55% by forming diversified equally – weighted portfolios, but this number drops to only 26% when the concentration of our market is taken into account.

ACKNOWLEDGEMENTS

Firstly, I would like to extend my sincere appreciation and gratitude to God for giving me the ability to make the right decision to enrol on this course (MPhil in Mathematical Finance) and for me to have come this far to enjoy the fruits of my efforts. This opportunity has truly enriched my life in a way I will cherish forever. It is the greatest decision I have made to date.

Secondly, I would like to thank my supervisor, Dave Bradfield for his guidance during the preparation of this thesis. I also extend my thanks to the entire staff members of the Statistics, Mathematics and Commerce Departments at the University of Cape Town for a productive year spent with them. Let me also thank Terry Seaward for his guidance in the programming part of this thesis.

I also wish to thank Mac Hengari for the financial and moral support in my pursuit of this course.

DEDICATION

I dedicate this thesis to my lovely wife Aquilla Kaveiririrua Uzera and my mother Alexia Uazukuani for their support throughout this year. I also dedicate this to my two children, my son, Uetuuanisa Karuhengere Uzera and my daughter, Uatjiri Uzera.

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1. INTRODUCTION

The concentration of portfolio weights has a direct impact on the risk of the portfolio. One of the characteristics of small markets is that they tend to be concentrated in a few relatively large counters and the Namibian market is no exception. As a consequence the Namibian All Share Index is itself a highly concentrated index; hence portfolios based on this index tend themselves to be highly concentrated (Bradfield and Kgomari (2004)).

The risk of a portfolio is largely related to two factors, the covariance structure of the underlying assets as well as the weighting structure of the portfolio. Portfolio managers can only influence the weighting structure of their portfolio as it mainly depends on their investment choices, but does not have control over covariance structure. Significantly the weighting structure of the portfolio determines the degree of concentration of the portfolio.

Bradfield and Kgomari (2004) define concentration as "the extent to which portfolio weights skew away from an equally weighted distribution of portfolio weights". Thus the concept reflects whether portfolio weights are concentrated in few stocks by contrasted to evenly spread across the portfolio. The Namibian All Share Index for example currently has about 6 (six) of the total of 25 (twenty five) stocks in the index accounting for about 84 percent of the index weight (market capitalization). In this paper we will use two measures of concentration, namely the Herfindahl-Hirschman measure, and the measure proposed by Richard Roll.

The primary objective of this paper is to assess how the level of concentration of a portfolio impacts the risk of the portfolio, and in particular to gain insights regarding the concentration of portfolio in the Namibian setting.

A further important consideration in portfolio design is to establish the appropriate number of counters that should be held in a portfolio so that effective diversification is achieved. A further objective of this report therefore is the assessment of the number of counters that should be held in concentrated portfolios to achieve effective diversification.

In this thesis we closely followed the research paper titled "Concentration – should we be mindful of it" by Bradfield and Kgomari (2004) on South African stock market for monthly data (as at end September 2003) over the prior 3 year period. Bradfield supervised this thesis, therefore we replicated their study for Namibian stock market for monthly data (as at end September 2009) over the prior 3 year period.

1.1 Outline of the report

In section 2 we give some overview of the Namibian Stock Exchange. Section 3 will cover the portfolio diversification measure proposed in 2006. In section 4 we define concentration and discuss the concentration levels observed in the Namibian market indices. Section 5 gives some theoretical background on how concentration affects the risk of portfolios. In section 6 we present the results of simulating various scenarios of concentration levels, ranging from equally-weighted portfolios and the market capitalisation-weighted portfolios in the Namibian All Share Index. In section 7 we compare our results on risk reduction in Namibia to results of international studies (including South Africa). In the end in section 8 we conclude with the major insights in the report.

2. NAMIBIAN STOCK EXCHANGE

2.1 Structure

The Namibian Stock Exchange was established in October 1992, as a non profit making association and is licensed, in terms of the Stock Exchange Control Act, to operate the stock exchange in Namibia. It is a self-regulatory organisation with the capacity to perform functions such as approving listing applications, surveillance, licensing stockbrokers, operation of trading, clearing and settlement activities. It is a body that comprising of 43 associate members (rights holders) such as banks, listed companies, investment institutions, etc. that has sponsored the establishment of the NSX by each donating N$ 10,000, the sponsors (brokers) are also rights holders. Each year the rights holders elect 3 members to the Board to replace retiring members (Executive Committee). The Board comprises nine members representing different business sectors and all meetings are attended by a representative of Namfisa (Namibia Financial Institutions Supervisory Authority). Sub-committees are appointed from time to time as the need arises, with the Listings Committee meeting regularly. The NSX is regulated in terms of the Stock Exchanges Control Act (1985, amended 1992) and is overseen by Namfisa.

2.2 Brokerage rates

The charges for equity dealings are the following percentages of the deal amount:

1.00% on deals up to N$ 10,000
0.85% on deals up to N$ 20,000
0.65% on deals up to N$ 100,000
0.55% on deals up to N$ 500,000
0.40% on deals up to N$ 5,000,000
0.35% on deals greater than N$ 5,000,000

Source: Namibia Stocks Exchange

The charges for dealings in bonds on the NSX are the following percentage of the deal amount:

0.065% on deals where the nominal value is up to N$ 100,000
0.050% on portion up to N$ 1,000,000
0.030% on portion greater than N$ 1,000,000

Source: Namibia Stocks Exchange

Trading takes place from Monday to Friday 09h00 – 17h00 (GMT +2). There is a five minute variation at market open and close. The NSX is closed for trading on South African public holidays only.

These brokerage rates include a transaction levy of 10% of brokerage paid to the NSX. Deals can only take place during the trading hours of the NSX. The completion of each deal – the transfer of money in exchange for the share certificate or scrip – takes place within five working days. Stockbrokers say that settlement takes place within t + 5. In this case "t" stands for trade date. Stockbrokers have strict obligations towards their clients and must obey the rules of the NSX. They are obliged to take out insurance to cover themselves for mistakes or negligence up to N$ 10 million.

2.3 Foreign Investment on the NSX

There are no capital gains, marketable securities tax or stamp duty on deals executed on the NSX. The only special tax for foreigners is Non-Resident Shareholders Tax at 10% of dividends. There are no general restrictions on foreign investment. Foreign exchange regulations are related to those in South Africa through Common Monetary Area treaties. Investors must apply through an authorized dealer (most commercial banks) for free remittance of dividends and sales.

2.4 Namibia within the CMA

Namibia belongs to a regional arrangement called the Common Monetary Area (CMA). The other members are South Africa, Lesotho, and Swaziland. Under the terms of the CMA, each member country's currency can be exchange one – for - one with that of any other and the South African Rand can be used freely as legal tender anywhere. Money can flow unhindered from one country to the other. At the same time controls regulating the flow of money into and out of the region as a whole are the same. The implication of this arrangement is that because South Africa's economy dominates in size – it is about forty times larger than Namibia's economy – interest rates and exchange rates are determine by the policies of the South African Reserve Bank (SARB).

CMA membership means that Namibia's financial system is highly integrated with that of South Africa. Namibian banks can freely lend to and borrow from South African banks. Namibian shareholders can freely buy and sell shares and bonds on the Johannesburg Stock Exchange. Anyone with an interest in the Namibian economy watches the SARB closely for changes both in its key interest rate, the repo rate, and in other instruments of monetary policy such as reserve ratios and exchange controls.

Table 1 below is a summary of all stocks in the Namibian Stock Exchange (NSX) with their closing price, market capitalisation weights as well as the average monthly volume traded (three year period) as at September 2009. The stocks labelled (**) formed Local Index.

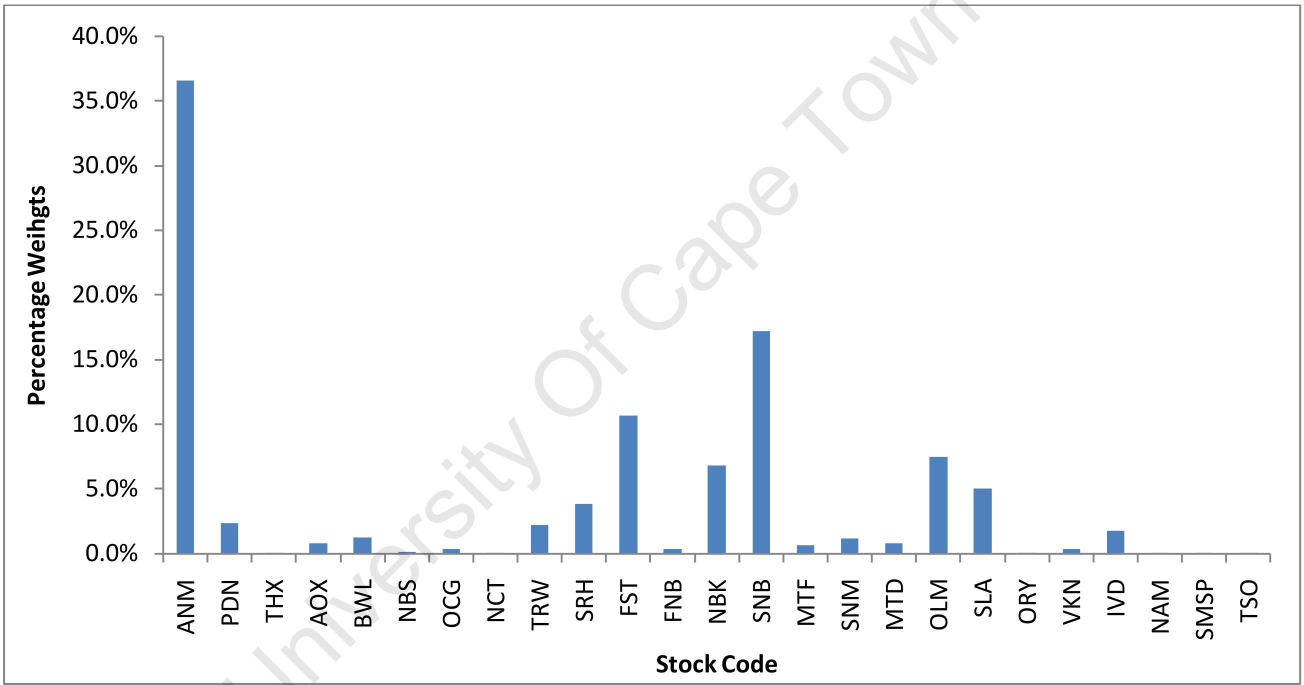
Table 1: Namibian All Share Index (NSX) as at 30 September 2009.

Categories: Section-Stock(Code)	Close	Market cap (Million's)	Weights (%)	Average monthly trading volume
Basic Materials				
Sector – Industrial Metals				
Anglo-American plc (ANM)	239.07	321,053	36.57%	586,497.06
Sector - Mining				
Paladin Energy Limited (PDN)	28.56	20,482	2.33%	2,543.89
Trans Hex Group Limited Nm (THX)	3.40	361	0.04%	59,586.58
Sector - Chemical				
Afrox (AOX)	20.50	7,028	0.80%	423,270.72
Industrials				
Sector – General Industrial				
Barloworld Limited (BWL)	49.00	11,144	1.27%	816,408.78
Consumer Goods				
Sector - Beverages				
**Namibian Breweries (NBS)	6.06	1,252	0.14%	388,738.03
Sector – Food Producers				
Oceana Group Limited (OCG)	26.25	3,113	0.35%	169,789.36
Consumer Services				
Sector – General Retailers				
**Nictus (NCT)	0.86	46	0.01%	18,977.53
Truworths (TRW)	42.50	19,336	2.20%	1,718,990.42
Sector – Food & Drug Retailers				
Shoprite Holdings (SRH)	62.00	33,696	3.84%	917,125.78
Financials				
Sector - Banks				
Firstrand (FST)	16.64	93,815	10.69%	4,095,774.97
**FNB Holdings (FNB)	11.65	3,117	0.36%	417,024.11
Nedbank Group Limited (NBK)	119.50	59,472	6.77%	324,301.47
Standard Bank Group (SNB)	97.00	151,031	17.21%	1,047,506.69
Sector – Nonlife Insurance				
Mut & Fed Insurance (MTF)	17.95	5,730	0.65%	86,904.61
Santam Limited (SNM)	88.50	10,562	1.20%	52,653.89
Sector – Life Insurance				
Metropolitan Holdings Limited (MTD)	12.60	6,945	0.79%	1,358,622.00
Old Mutual Plc (OLM)	11.93	65,789	7.49%	5,109,090.17
Sanlam Limited (SLA)	20.40	44,064	5.02%	1,031,518.89
Sector – Real Estate				
**Oryx Properties Limited (ORY)	10.50	578	0.07%	167,252.58
Vukile (VKN)	10.86	3,210	0.37%	620,161.33
Sector – General Financial				
Investec Limited (IVD)	57.00	15,316	1.74%	746,407.42
**Namibia Asset Management limited (NAM)	0.25	50	0.01%	2,530.56
**Stimulus Investments Limited – Pref (SMSP)	104.99	130	0.01%	2,032.14
**Trustco Group Holdings Limited – Par value (TUC)	0.70	495	0.06%	958,965.50
Total:		877,814	100%	20,930,399.36

Source: Namibia Stock Exchange

The last column of table 1 shows the average amount of individual securities traded on monthly basis over three year period (October 2006 to September 2009). Trading activity relates to the liquidity of a security; therefore, when average monthly trading volume is high, the stock can be easily traded and has high liquidity. As a result, average monthly trading volume can have an effect on the price of the security. If trading volume isn't very high, the security will tend to be less expensive because people are not as willing to buy it. It is clear from our result that Old Mutual Plc (OLM), Firstrand (FST), Standard Bank Group (SNB) and Truworths (TRW) among others are the most liquid securities and Stimulus Investments Limited – Pref (SMSP), Namibia Asset Management limited (NAM) and Paladin Energy Limited (PDN) tend to be the least liquid securities on the Namibian Stock Market.

Figure 1: Weights percentage per stock within the Namibian All Share Index



In Figure 1 above, we give a graphical portrayal of the percentage weights of various stocks within the Namibian All Share Index as at 30 September 2009. It is evident from figure 1 that the weights are concentrated in few stocks for example 6 (six) of the total of 25 (twenty five) stocks in the index account for about 84 percent of the index weight.

Table 2: Industry and Sector Weights % within the Namibian All Share Index (NSX) as at 30 September 2009.

Categories (Industry Weights %)	Sector	Sector Weights (%)
Basic Materials: (39.75%)	Industrial Metals	**36.57%**
	Mining	**2.37%**
	Chemicals	**0.80%**
Industrials: (1.27%)	General Industrial	**1.27%**
Consumer Goods: (0.49%)	Beverages	**0.14%**
	Food Producers	**0.35%**
Consumer Services: (0.65%)	General Retailers	**2.21%**
	Food & Drug Retailers	**3.84%**
Financials: (52.44%)	Banks	**35.02%**
	Nonlife Insurance	**1.85%**
	Life Insurance	**13.31%**
	Real Estate	**0.44%**
	General Financial	**1.82%**

Table 2 above shows market capitalisation weights of various sectors as well as industries (categories) within the Namibian All Share Index as at 30 September 2009. The Financial category (industry) is highly concentrated than the rest of the categories with about 52% followed by Basic Materials with about 40%. The industrial metals sector is highly concentrated with about 37% with only one stock (Anglo-American plc) followed by bank sector with four stock and account for about 35% of the concentration measure.

3. PORTFOLIO DIVERSICATION MEASURE

Diversification in a portfolio of stocks is a primary tenet for combining assets and is a central principle of portfolio construction, but still the traditional methods for measuring diversification remain inexact. Diversification is concerned with the number of unrelated sources of return, however little thought is given to the importance of diversification. Thus, one should be able to accurately quantify the declining marginal value of diversification as assets are added to portfolios. (Bradfield et al (2006))

Bradfield et al (2006) points out that the extent to which a portfolio is diversified is measured relative to the market index traditionally. The measurement in essence involves running a regression of the portfolio returns against the returns of the market index and as a result the variance of the residuals is taken to measure the extent of diversification still achievable. Meaning, the residual variance measure the diversifiable risk remaining in the portfolio, thus traditionally the smaller the residual variance, the more diversified the portfolio.

One serious flaw of this traditional approach is when the market index itself is not appropriately diversified across industries. In smaller markets where the concentrations of market indices are potentially high, an overwhelming concentration in a few industries (categories) can swamp the diversification effect of other industries (categories).

3.1 Portfolio Diversification Index

Bradfield et al (2006) state that what one needs to measure the diversification concept cleanly is a measure which is essentially free from the influences of the overall market index. A well diversified portfolio is one where the return, and consequently the risk, arises from as many unrelated (or independent) sources as possible.

This measure was proposed by researchers Alexander Rudin and Jonathan Morgan in 2006, and the proposed measure of diversification is based on the number of independent factors observed in a portfolio. They quantify these factors using Principal Components Analysis (PCA) and this measure is called Portfolio Diversification Index (PDI).

3.2 Methodology and Data

Throughout our analysis that follows we selected some stocks within the Namibian All Share Index to form portfolios of (n) assets each (as indicated in the tables 3 and 4 below). The component weights in the portfolio were taken as at 30 September 2009 and the expected risks were computed using the historical prior 3-year covariance matrix.

The methodology adopted in this section is outlined in Portfolio Diversification Index (PDI) report by (Bradfield et al (2006)).

The following procedure is followed when calculating the PDI of a portfolio:

- The data required to establish the current PDI of a portfolio is the current compositions, on a stock level, as well as a return history of the stocks held.
- A column consisting of a time series of returns multiplied by the respective weight is constructed for each of the N stocks in the portfolio.
- A Principal Component Analysis (PCA) is then conducted on the covariance matrix of these series to quantify all the uncorrelated sources of risk and their relative magnitudes.
- The PCA produces a series of uncorrelated factors describing the portfolio's return volatility.
- The factors are ordered from most significant to least significant.
- These are then substituted into the formula:

where N is the number of assets
and   is the percentage contribution of factor k to total volatility.

3.3 Results

In table 3 and 4 below we compare the PDI and expected risk along with the concentration measure for a variety of Portfolio formed within the Namibian All Share Index. The component weights in the portfolio were taken as at 30 September 2009 and the expected risks were computed using the historical prior 3-year monthly covariance matrix. Take note the portfolios in table 3 and table 4 are the same in terms of stocks in each portfolio (A – H), but the only difference is that table 3 is using market capitalisation weights while table 4 uses equal weights among stocks in each portfolio.

It is clear that the portfolios formed using equal weights are having high PDI and lower risk as compared to those formed using market capitalisation weights that show low PDI and high risk, highlighting the additional effect that concentration has on portfolio risk as well as on PDI measure in general. Here concentration has a serious effect as it reduces the PDI and as a consequence this leads to portfolio risk increasing.

Table 3 : PDI of various Portfolio's (Namibian All Share Index – Market Cap Weights)

	Nr. of Stocks	PDI	Concentration	Annualised Expected Risk
Portfolio A	10	1.11	0.471	33.75%
Portfolio B	10	1.57	0.251	26.00%
Portfolio C	10	1.24	0.350	31.12%
Portfolio D (All Share Index)	25	1.48	0.191	27.31%
Portfolio E	10	1.04	0.603	38.53%
Portfolio F	10	1.70	0.278	25.47%
Portfolio G (Local Index)	7	1.89	0.369	10.79%
Portfolio H (Non-Local)	18	1.48	0.193	27.47%

Table 4 : PDI of various Portfolio's (Namibian All Share Index – Equal Weights)

	No Of Stocks	PDI	Concentration	Annualised Expected Risk
Portfolio A	10	4.91	0.10	17.52%
Portfolio B	10	3.79	0.10	18.41%
Portfolio C	10	3.46	0.10	18.45%
Portfolio D (All Share Index)	25	6.94	0.04	16.36%
Portfolio E	10	4.67	0.10	18.16%
Portfolio F	10	4.19	0.10	17.49%
Portfolio G (Local Index)	7	1.83	0.14	2.97%
Portfolio H (Non-local)	18	6.05	0.06	20.75%

In Figure 2 we give a graphical portrayal of the observed relationship between the PDI measure and the portfolio expected risk as in table 2.

It is evident, from the figure below, that there is a statistically significant empirical relationship between PDI and risk. Clearly the higher the levels of PDI, the lower the expected risks of the portfolios. Meaning that there is somehow a negative relationship between the level of PDI and risk.

Figure 2: Relationship between PDI and expected risk (Portfolio formed within Namibian All Share Index)

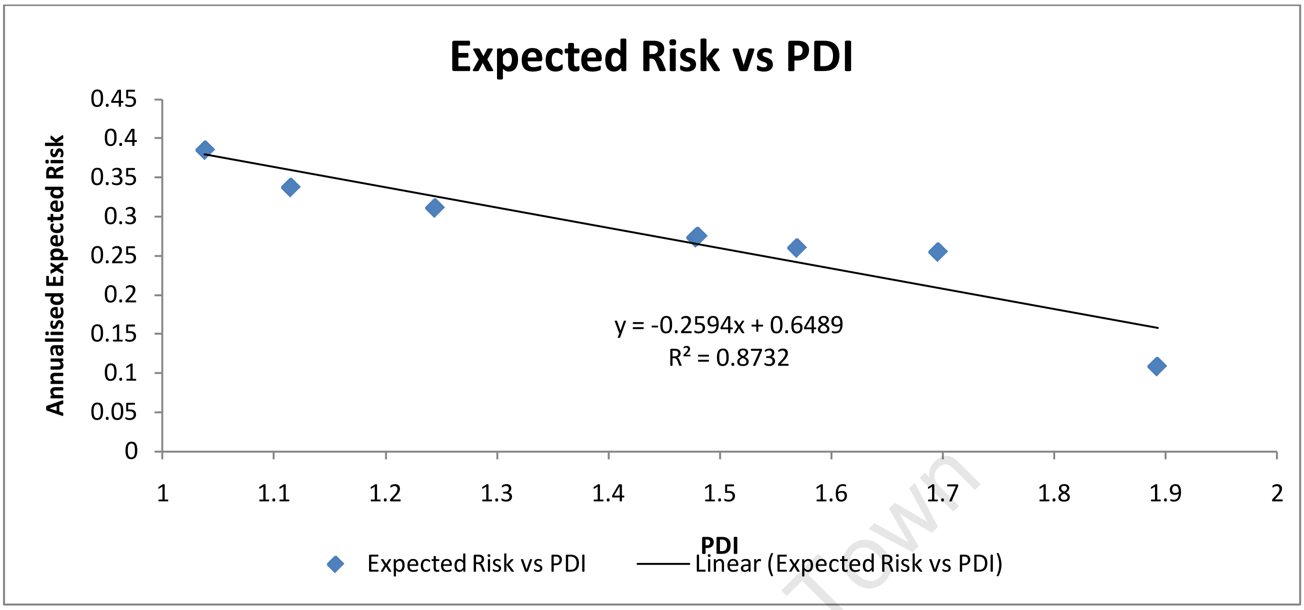


Table 5: Namibian All Share Index Categories Ranked by their PDI Measure

Categories	All Share Index (Market Cap)			
	Nr. Of Stocks	PDI	Concentration	Annualised Expected Risk
Financials	15	1.70 (1)	0.198 (4)	26.89% (3)
Consumer Services	3	1.40 (2)	0.536 (3)	24.04% (4)
Industrial & Consumer Goods	3	1.09 (3)	0.563 (2)	30.67% (2)
Basic Materials	4	1.01 (4)	0.850 (1)	45.01% (1)

Rankings are shown in parenthesis
See Table 1 for Categories

The rankings of the PDI measures across the categories are interesting, showing a large difference in the degree of diversification in each category.

Several points are worth commenting on in Table 5:

- The Financial category is significantly more diversified than the rest of the categories, highlighting that there are about 2 independent factors driving returns in the Financial category (as reflected by the PDI).
- Basic Materials are only driven by a single independent factor, resulting in a low PDI measure and extremely high concentration measure.

- Notice too that both the risk and the concentration measures are closely linked to the PDI measures across categories. The more diversified a portfolio (i.e. the higher the PDI measure), typically the lower its risk and concentration measure, thus highlighting the linkage between these concepts.

4. CONCENTRATION

4.1 Definition of Concentration

Bradfield and Kgomari (2004) define concentration as "the extent to which portfolio weights skew away from an equally weighted distribution of portfolio weights". The two concentration measure will be discussed briefly below.

The Herfindahl-Hirschman Index

Hovenkamp (1986) described Herfindahl-Hirschman Index (HHI) as a measure of concentration and is computed by first squaring and then summing the investment weights in a portfolio:

where is the investment weight in the counter.

It is clear that the HHI measure will be a minimum when the investment weights are equal and will become larger the more the investment weights are skewed (i.e. concentrated). HHI measure will be at its maximum value (i.e. unity), when there is only one counter in the portfolio.

The Roll measure of Concentration

The Richard Roll measure of concentration (RRC) was originally proposed to measure the concentration of industries and has been adapted for the purpose of measuring portfolio concentration on stock level (Divecha et al (1992)):

where N is the number of counters in the portfolio.

The RRC measure essentially encapsulates departures from equally weighted investment proportions (i.e. – in each counter). A portfolio of equally weighted stocks should thus have an RRC measure of zero. Clearly the greater the concentration of investment weights the larger the RRC measure. The measure can also be adapted for a setting where an opportunity set of stocks (i.e. the set of all stocks under consideration) is taken as N, rather than only the number of stocks in the portfolio. Hence if a smaller number of stocks are chosen, – remains the equal weights of all stocks in the opportunity set rather than the portfolio itself.

Of the two measures, the HHI measures appears to be the more well known as it is being used widely in the academic literature.

4.2 Concentration in Namibia

We compare the HHI and RRC measures of concentration along with the expected risks for a variety of benchmarks in this section. The component weights in the portfolio were taken as at 30 September 2009 and the expected risks were computed using the historical prior three year covariance matrix.

Table 6: Concentration of various benchmarks

Categories	Concentration HHI	Concentration RRC	Expected Total Risks
All Share Index	19.1%	15.7%	27.3%
Local Index	37%	26.5%	10.8%
All Share Index (Equally weighted)	4%	0%	16.4%
Non Local Stocks	19.3%	14.6%	27.5%

Table 6 above shows the two concentration measures of various benchmarks as well as their annualised expected risk. More fascinating is the perception one gets of the immense differences in concentration of the different series.

In the sections that follow we give more insights into how portfolio concentration impacts the risks of portfolio.

5. THE IMPACT OF CONCENTRATION ON PORTFOLIO RISK

5.1 Highlighting how concentration impacts the risk of a portfolio

The process by which risk is reduced through forming portfolio is central to understanding portfolio design and forms the key concept described by Markowitz (1952) on how to combine assets into efficiently diversified portfolios.

This approach assumes that variance can be used to quantify the risk of a portfolio. Representing the weight of the security in a portfolio by , the individual stock variance by , and the covariance between stocks and by , the total risk of a portfolio can be represented by equation 1 (Elton et al (2003)).

Now, if we assume (somewhat unrealistically) that the assets are uncorrelated, the second term in the above equation becomes zero and portfolio risk can be represented by equation (2) below.

If we further assume that each asset has the same average variance, , then

using the definition of the HHI measure of concentration

Hence it is evident, under these oversimplified assumptions, that the only remaining impact on the portfolio variance is the degree of concentration of the portfolio.

For the RRC measure, after some rearranging, it can be shown that

hence

Also confirming that the RRC measure of concentration impacts directly on the portfolio risk.

5.2 How concentration affects diversification

In the above exposition we made some fairly unrealistic assumptions – in practice we are not so fortunate. In most markets covariance's between stocks are positive and the volatilities of stocks differ substantially.

We show in the appendix that under these more realistic conditions – if we had equally weighted portfolios (with – invested in each asset) then the portfolio variance essentially becomes:

This representation of the portfolio variance is a far more realistic representation of what actually happens in practice. The first term in the above expression is the average of the variances of the assets in the portfolio – divided by N. The second term is essentially the average covariance between the assets. Both of these terms convey the key concept of diversification. The second term conveys the lesson that the more assets with low covariance's (i.e. behave nearly independently) we can include in the portfolio, the lower the portfolio risk will be. The first term however conveys the notion that as the numbers of counters in the portfolio are increased, the portfolio risk will be reduced by the factor – - until for very large N the first term becomes insignificant. Hence for large N the portfolio's risk converges to the average covariance across the assets – that is assuming the portfolio is equally weighted. Interestingly it turns out that the average covariance between the stocks in the All Share Index is (15% – contrasting this to the Namibian All Share Index that has a current variance of (27% gives us some insight into extra risk we bear because of concentration.

What happens if assets are not equally weighted – but have high concentration?

If a portfolio has a high level of concentration we need to focus our attention back on equation (1) to assess the impact of weights skewing away from equal weights:

Contrasting this expression to equation (3) it is evident from the first in the above equation that if assets with larger weights also have higher than average variance, the risk of the portfolio will be higher – and will rise geometrically with the increase in weight. The second term is probably more important in the Namibian setting as it suggests that if assets with larger weights also have larger covariances with each other, the portfolio risk will be higher. This is indeed the case on the NSX where the larger stocks tend to be more highly correlated with each other. Take for example the two large stocks in the Namibian All Share Index, Firstrand and Standard Bank Group, whilst the average correlation amongst all Namibian All Share Index stocks is only 0.24, the correlation between Firstrand and Standard Bank Group is 0.85, far higher than the average correlation. Clearly the impact of concentration on risk boils down to the distribution of weights among the volatilities and correlations. It is in fact possible to increase the diversifiable risk of portfolio by adding an additional stock to a portfolio – depending on the proportion of the stock added as well as its risk characteristics. In the appendix we show what the necessary characteristics are for stocks to reduce the risk of the portfolio when added to the portfolio.

6. CONCENTRATION AND RISK – EMPIRICAL FINDINGS

Our objective is to assess empirically how various levels of concentration affect the resulting risk of the portfolio. We do this using a simulation methodology for differing portfolio sizes.

6.1 Methodology and Data

The methodology outlined below follows Bradfield and Kgomari (2004) closely. Throughout our analysis that follows we base our variance and covariance estimates of risk on the 25 stocks currently (as at end September 2009) in the Namibian All Share Index over the prior 3 year period.

Three different scenarios are considered with regard to the portfolio weighting scheme (and consequently concentration) as well as with regard to the correlation structure of the stocks:

- Scenario 1 considers equally-weighted stocks, assuming zero correlation between the stocks,
- Scenario 2 considers equally-weighted stocks, assuming their historical correlation structure
- Scenario 3 considers market capitalisation-weighted stocks, assuming their historical correlation structure.

The methodology is based on simulation run 300 times for each portfolio size (i.e. for each number of stocks.)

More specifically our process followed the following:

- Randomly select k stocks from the universe of 25 stocks
- Form a portfolio of the k stocks based on either, equal weights or market capitalisation-weights depending on the scenario under consideration
- Rebase the portfolio weights to sum to one
- Compute the risk (standard deviation) of the portfolio
- Repeat the process 300 times obtaining 300 estimates of risk for the portfolio of size k
- Average and record the 300 estimates of risk
- Thereafter increase k, the portfolio size, by one stock and repeat the entire process.
- Stop when k = 25, i.e. all 25 stocks are sampled.

6.2 Results

6.2.1 Scenario 1: Equal weights assuming independent stocks (having zero correlation with each other)

We begin with an oversimplified case where we assume that assets are equally weighted and are completely independent. Then equation (3) reduces to:

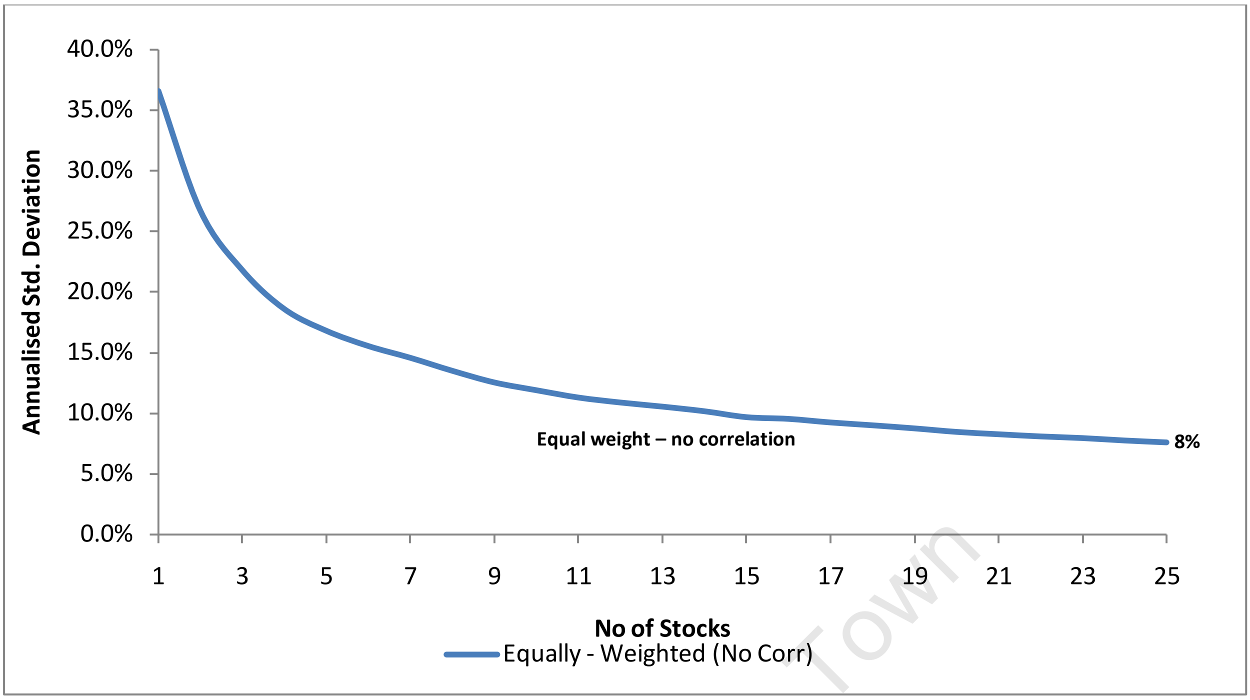
because the covariance term will be zero under the assumption of independence.

Thus

where the numerator above is the average standard deviation of stocks in the portfolio. Importantly the portfolio standard deviation decreases by a factor of .

Our simulation based on 300 samples at each "number of stocks" is displayed in the graph below for equally-weighted portfolios increasing in size until all 25 stocks in the Namibian All Share Index form the final portfolio. Note that we have also assumed no correlation between the stocks. Thus we can use equation (4) to compute the total risks of the portfolios. Hence the portfolio comprising only one stock has a standard deviation approximately equal to the average standard deviation of the 25 stocks (i.e. 38.05%) and as N increases, the risk of the portfolio declines geometrically.

Figure 3: Equally-weighted portfolios assuming independent stocks



From Figure 3 we can conjecture that the average volatility of single stocks in the Namibian All Share Index is approximately 37% p.a. (the true average standard deviation of the 25 stocks is actually 38.05%). More interestingly if our 25 stocks in the Namibian All Share Index were uncorrelated and equally weighted, Figure 3 shows we could achieve a portfolio risk (by including all 25 stocks) of only 8% p.a. contrast this to the current Namibian All Share Index risk of 27% p.a..

This significant difference in risk must to some extent reflect the impact of the high concentration of the Namibian All Share index as well as our oversimplified assumption of no correlation between the stocks.

Interestingly the above result for the 25 stock portfolio could have been estimated directly from the formula shown in equation 3, i.e.:

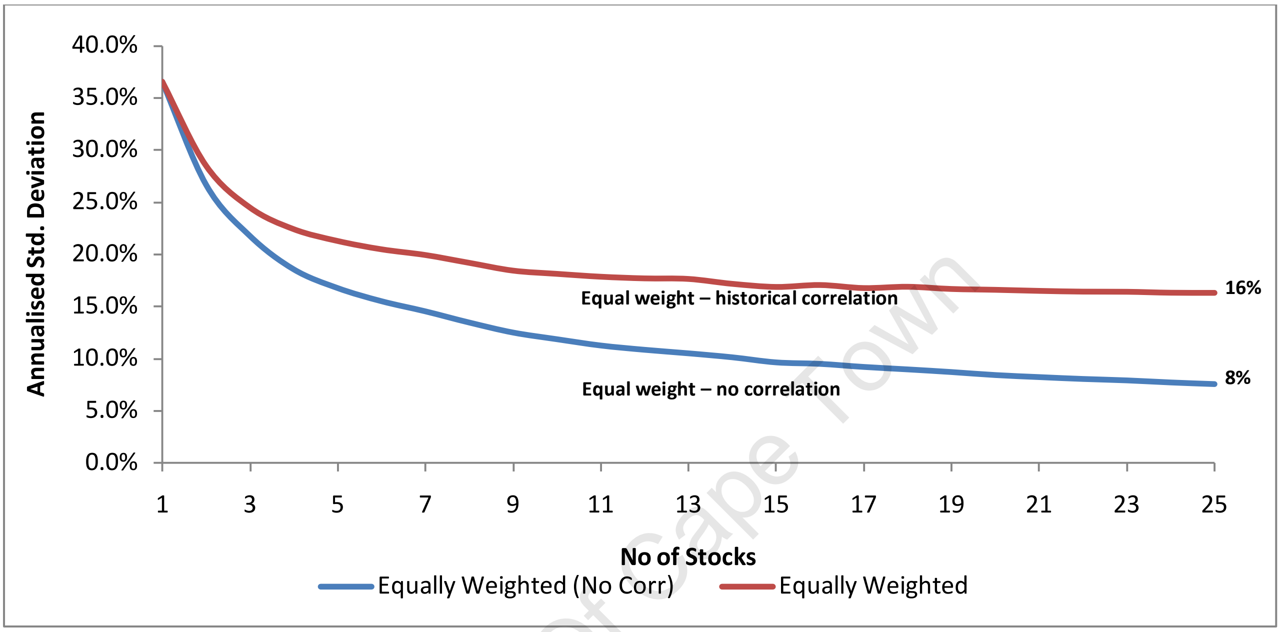
which is consistent with our simulation results of 8% p.a..

In scenario 2 and 3 below we attempt to assess the extent to which concentration is responsible for the difference between this low risk and the typically higher risk of portfolios we tend to hold.

6.2.2 Scenario 2: Equal weights – historical correlation structure assumed

In scenario 2 we simulate portfolios based on the assumptions of a normal (i.e. historical) correlation structure (estimated historically over the prior 3 years), but assuming stocks are equally weighted. This resulted in Figure 3:

Figure 4: Equally-weighted portfolios – historical correlation structure assumed



It is evident from Figure 4 that when we introduce the actual correlation between stocks the risk increases from 8% p.a. (for the 25 stock portfolio with zero correlation assumed) to 16% p.a. (for the 25 stock portfolio with actual correlation in place). Hence the substantial increase in risk of 8% p.a. (i.e. 16% - 8%) is a consequence of the fact that our stocks are fairly highly correlated. The extent to which we can find assets that behave independently is the extent to which we can reduce this component of risk.

Although these results were achieve through simulation we could just as well have generated them using the analytical derivation which assumes a normal correlation structure, but that stocks are equally weighted. Recall equation (3) effectively yields the appropriate relationship:

If we include all 25 stocks in an equally-weighted portfolio then the portfolio's risk can be analytically computed as:

which is consistent with our simulated portfolio risk of 25 stocks resulting in 16% p.a..

An important insight is that the magnitude of the average covariance between stocks seems to be a significant contributor to the volatility of the Namibian All Share Index. It was shown earlier in the report that the average risk for equally-weighted portfolios converges to the average covariance. Figure 4 depicts that the portfolio variance for 25 stocks is - this is very close to the average covariance cited above, of . This highlights the fact that the average covariance is the major determinant in portfolio risk – at least for equally-weighted portfolios.

6.2.3 Scenario 3: Market capitalisation weights – historical correlation structure assumed

In the above scenario we constructed the portfolios to be equally weighted. Whilst it turn out to be a useful starting point to assess the impact of concentration on portfolio risk, portfolios held in the Namibian setting are typically not equally weighted, but have a relatively high level of concentration.

In our simulations under this scenario we designed a weighting scheme based on the corresponding market capitalisation-weights (effectively Namibian All Share Index weights taken as at end September 2009). Our process involved randomly forming portfolios of k stocks based on their original weights in the Namibian All Share Index – these weights were then rebalanced to proportionally to sum to one. Thereafter we increased the portfolio size, k, by a further stock and repeat the process. In other words we began by selecting 300 samples of 1 stock portfolio's in the usual way. Thereafter we selected 300 samples of randomly chosen 2 – stock portfolios and assigned them their weights in the All Share Index, rebased them proportionally to sum to one, and so on, until finally we selected all 25 stocks which resulted in the original All Share Index.

For each portfolio the risk is computed using equation below:

where W' and W is weights transpose and weights respectively
and ∑ is the covariance matrix

The result of this simulation process for an increasing number of stocks is depicted graphically in Figure 5 below, along with our prior results of scenario 1 and 2.

Figure 5: Market capitalisation – weighted portfolios – historical correlation structure assumed

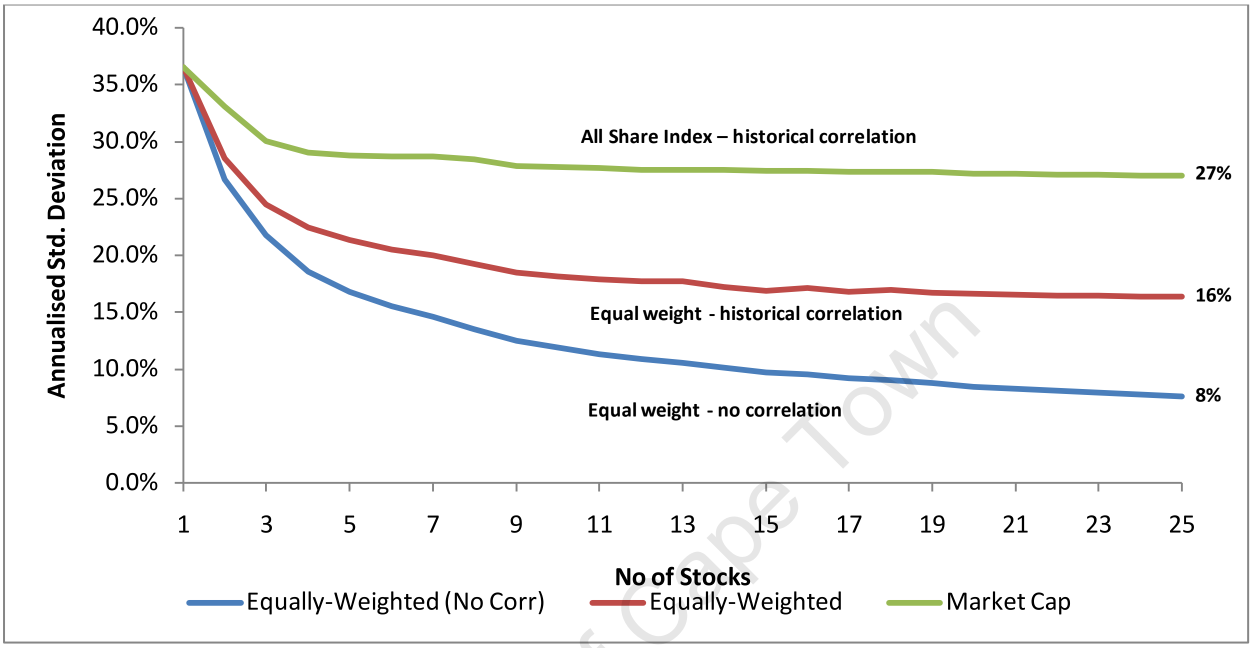


Figure 5 confirms that our risk estimate of the 25 stock portfolio is identical to that of the Namibian All Share Index, at 27% p.a. because the simulation process effectively reconstructs the Namibian All Share Index at 25 stocks.

More importantly however, we are now able to assess the impact that the concentration of our market (as reflected in the Namibian All Share Index) has on portfolio risk. Figure 5 reveals that the difference in risk between equally weighted portfolios and market capitalisation-weighted portfolios on the NSX remain fairly constant at 11% p.a. and for the final 25 stock portfolios the difference is also 11% p.a. (i.e. 27%-16%). Hence we can conclude that about 40% of the Namibian All Share Index risk is due to the concentration effect.

6.2.4 How many stocks are needed in Namibian portfolio?

One might want to know how many stocks are needed in Namibian portfolio to achieve effective risk reduction. This question would be poorly answered if we based our analysis on simulation of equally weighted portfolios. From inspection of Figure 6 showing the portfolio simulated from the Namibian All Share Index weights, we are now able to give a more realistic answer to the question by taking account of the typical levels of concentration in our local portfolios.

By focusing on Figure 6 and establishing where the risk tends to "flatten off", we note that the non-diversifiable risk of the aggregate Namibian All Share Index tends to be about 27% p.a. – and at 6 stocks we find that the aggregate total risk is still 29%. Thus at 6 stocks there remains 2% p.a. of risk that can still be diversified away. Only at 10 stocks does the remaining diversifiable risk drop to within 1% p.a. (i.e. the total risk drops to 28% p.a. at 10 stocks). Hence it could be argued that as many as 10 stocks are needed before the marginal reduction in risk becomes of little practical advantage.

Figure 6: Number of stocks needed for Namibian All Share Index weights

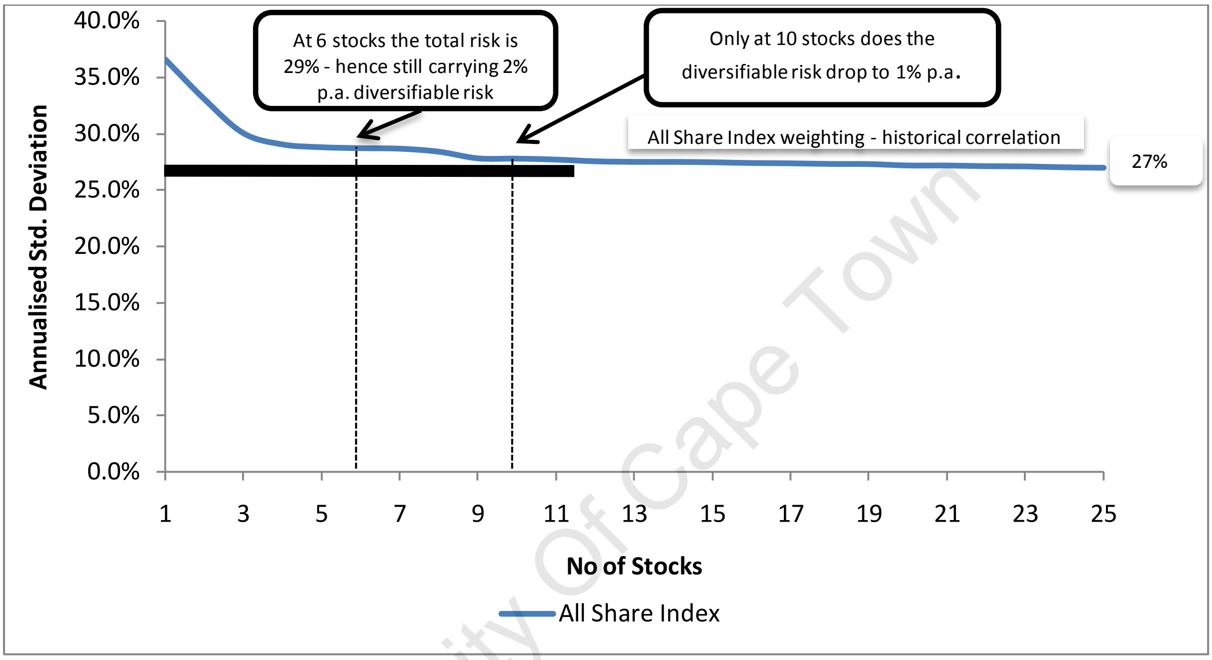


Table 7: Comparing our results to Bradfield and Kgomari (2004) studies

Stock market:	**JSE**	**NSX**
Data Collection Period: (monthly)	October 2000 to September 2003	October 2006 to September 2009
No of Stocks:	165	25
Concentration measure (HHI):	5.3%	19.1%
Average Correlation between Stocks	0.19	0.24
Average Covariance between Stocks	$(15.5\%)^2$	$(15.0\%)^2$
Annualised Expected Total risk	22%	27%
Equal weights – no correlation (Simulation)	3%	8%
Equal weights – historical correlation (Simulation)	16%	16%
ALSI weight – historical correlation (Simulation)	22%	27%

JSE – Johannesburg Stock Exchange, NSX – Namibian Stock Exchange

By comparing our results on the Namibian Stock Exchange (NSX) with the similar study done on the Johannesburg Stock Exchange (JSE) by Bradfield and Kgomari (2004), as shown on the table above, we find that most of total risk on the NSX is due to concentration while in JSE was due to correlation. This can be calculated by getting the difference of the last three rows of table 7.

7. COMPARING OUR RISK REDUCTION RESULTS TO INTERNATIONAL STUDIES

We have pointed out that the two characteristics of markets that ultimately underpin the risk of portfolios constructed on them are:

- the levels of concentration of the stocks in the market capitalisation-weighted Index and
- the average covariance amongst the stocks

Prior international research has tended to focus on equally weighted portfolio in studies on diversification – hence ultimately the average covariance's of stocks in each particular market plays the central role in the results of these studies. Table 8 below (extracted from Bradfield and Kgomari (2004) and amended with Namibian results) shows the percentage of risk that can be eliminated by holding a widely diversified equally-weighted portfolio in each of several countries as well as an internationally diversified portfolio. More specifically, the percentage of risk reduction shown in the table is based on the difference in risk between the risk of an average single stock portfolio and that of a fully diversified portfolio.

Table 8 shows that the effectiveness of diversification in reducing the risk of portfolio varies from country to country. From our prior discussions we know that this variation exists because the average covariance varies across countries.

Table 8: Percentage of risk that can be eliminated by holding diversified equally–weighted portfolios

United States	73.0
U.K.	65.5
France	67.3
Germany	56.2
Italy	60.0
Belgium	80.0
Switzerland	56.0
Netherlands	76.1
International stocks	89.3
South Africa (equally-weighted)	60.0
South Africa (market cap-weighted stocks)	46.0
Namibia (equally-weighted)	55.2
Namibia(market cap-weighted stocks)	26.2

From Table 8 it is evident that in Germany, Italy, Switzerland, South Africa and Namibia, less risk reduction is achieved by forming diversified portfolio, thus these markets tend to have higher average covariances. In other words these markets have stocks with relatively higher correlations amongst their stocks, indicating that in these countries stocks tend to move together to a greater extend. On the other hand, the stock markets in the United States, Belgium and the Netherlands tend to have stocks with relatively low covariance, indicating that diversification yields greater risk reduction benefits in these markets.

The final row in the table shows the percentage of risk reduction for the market capitalisation-weighted portfolio in Namibia (i.e. the Namibian All Share Index) revealing that only 26% of risk is reduced from the average single stock risk. Hence this result clearly shows the additional effect of the concentration in our market.

CONCLUSION

By forming portfolios within All Share Index (Market Cap: Categories), we found that financial category is significantly more diversified than the rest of the categories, highlighting that there are far more independent factors driving returns in the Financial category (as reflected by the PDI). We further found that by forming portfolios using market capitalisation weights, reduces PDI value and increase portfolio risk, highlights the additional effect that concentration has on portfolio risk as well as on PDI measure in general.

We also showed the significance of concentration in determining the risk of portfolios – both empirically and theoretically. We demonstrated that if large portfolios were equally weighted then all-equity portfolios should have risks in line with average covariance between stocks, i.e. (15% p.a. – furthermore the additional risk, that is, 27% p.a. for the Namibian All Share Index, are a consequence of our high levels of concentration. Hence nearly two fifths of the Namibian All Share Index risk is due to the concentration effect.

We also considered the number of stocks needed in portfolios in Namibia to achieve effective risk reduction. This question has been poorly answered in the past because almost always analysis has been based on simulated equally-weighted portfolios. Our analysis showed that a bare minimum of 6 stocks are required for effective risk reduction – and more likely as many as 10 stocks are needed before the marginal reduction in risk becomes of little practical advantage.

By comparing our results on the Namibian Stock Exchange (NSX) with the similar study done on the Johannesburg Stock Exchange (JSE) by Bradfield and Kgomari (2004), we find that most of total risk on the NSX is due to concentration while in JSE was due to correlation.

Finally comparison of our results with South African and International studies on risk reduction highlighted two key insights. The risk reduction that is achievable through equally weighting portfolios (relative to the average single stock risk) in our market is amongst the lowest (along with South Africa, Germany, Italy and Switzerland). This suggested that the correlations amongst our stocks are relatively high (along with above-mentioned markets). We found that we are able to achieve a reduction of only 55% by forming diversified equally-weighted portfolio – but this number drops to only 26% when the concentration of our market is taken into account. Hence the high correlations amongst the stocks in our market as well as the high levels of concentration jointly impact the potential to reduce risk by forming diversified portfolios in Namibia.

APPENDIX A

Proof that equally weighted portfolios converge to the average covariance of stocks in the market.

Now if we assume that assets are correlated as they usually are, but are equally-weighted, then portfolio risk will converge to the average covariance of the assets. We begin with the usual definition of portfolio variance and follow the proof as shown in Gruber (2003).

For equally-weighted portfolios the investment weight for each stock become –, hence

For large values of N, the above expression reduces to the average covariance.

APPENDIX B

Conditions for stocks to increase diversifiable risk – rather than reduce it.

Mathematically, taking the first differential of a portfolio's unique risk with respect to weights would indicate the limit to which addition of an asset to a portfolio would reduce the portfolios unique risk.

So, if we denote a portfolios diversifiable or residual risk by , and an individual asset residual risk by , it can be shown (under the assumption uncorrelated error terms) that:

The analysis can be carried out without loss of generality in a two asset framework. Viz, we assume a portfolio of N – 1 asset to be a single asset and establish the effect on the portfolios residual risk of adding another asset into the portfolio.

where

The critical weight is obtained when the change in unique risk of the portfolio is zero.

At this point,

Hence,

The maximum level of unique risk reduction is achieved at this point. If the weight of the stock exceeds this level, the addition of the new stock will increase the unique risk of the portfolio. Interestingly, this weight turns out be the ratio of the original portfolio unique risk to the sum of the individual unique risks. We do note that the higher the level of unique risk of the additional stock the less the weighting of that stock should be added for diversification purposes. It should also be noted that the above formulation applies even when the additional asset has zero correlation to the existing portfolio.

APPENDIX C

TWO APPROACHES TO MEASURING THE PDI

The paper by Rudin and Morgan describes two PDI measures: one for a portfolio and another for a collection of assets without considering any particular portfolio. The difference between the calculations is highlighted in the table below:

	PDI MEASURE	
	PORFOLIO	**PORTFOLIO-FREE**
Step 1	For each asset in the portfolio find a time-series of asset returns times portfolio weight.	For each asset find a time series of returns.
Step 2	Find the covariance matrix of the time-series column in step 1.	Find the correlation matrix of the time-series columns in step 1.
Step 3	Perform a PCA of the covariance matrix	Perform a PCA of the correlation matrix
Step 4	Order the eigen-values from largest to smallest	Order the eigen-values from largest to smallest.
Step 5	Substitute the values into the formula shown in section 3.	Substitute the values into the formula shown in section 3.

The main difference between the two definitions is that for the PDI of a portfolio we use the covariance matrix, whereas when finding the PDI of a collection of assets, we use the correlation matrix. The reason is that when calculating the PDI of a portfolio we want to decompose the portfolio variance and so we need to take account of the fact that different assets have different volatilities. In the case of the portfolio-free definition we are only interested in the potential diversification that is available. For this reason the variance of each asset is less important than the correlation between assets. This is because it is always possible to down-weight highly volatile assets when constructing a portfolio. However it is the correlation between assets that ultimately determines the maximum possible diversification. For this reason we perform the analysis on the correlation matrix.

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