EMPIRICAL EVIDENCES OF STOCK SPLIT MARKET EFFECTS

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Abstract

Under normal financial market circumstances (i.e., not under the shadow of financial crisis) it is common to believe that buying shares from large institutions leads to high profit. This is because the shares are of high trading value due to the solid financial foundation and superior performances of large institutions or companies. In contrast to these traders' belief, large companies often exercise "stock split" to strengthen the confidence on the company and encourage more investments in the company. A "stock split" increases the number of shares outstanding without increasing the company's capital. A conjecture is that a "stock split" action will increase the market liquidity because of the price decrease of each share; consequently, market trading activities would be intensifying such that log-return will be higher and the volatility also higher accordingly. The financial market literature shows that the impacts of "stock split" were controversial. In other words, the influences on the market of "stock split" did not always behave as the management expected. In this thesis, we intend to use limited available stock split data from NASDAQ to explore some empirical evidences on the impacts of "stock split". We also propose a DEAR-based trend analysis in log-return and market volatility measured by daily trading range for technical analysis on "stock split" impacts.

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Chapter 1. Introduction

1.1 Statistical Events Associated with Stock Split Phenomena

The stock market was originally established for attracting investments for company's further developments. Once the trading of stocks has started, the market mechanism plays its own roles with or without interference from the market traders. The greedy nature of human beings inevitably leads market traders to try every means to make profits from stock market trading activities. One of the games of trading companies is the numbers game since psychologically a larger amount of shareholding is better than small one. A common number game is *stock split* which refers to a corporate action that increases the number of shares in a public company.

A stock split occurs when the existing shares are "split" into more shares. For example, in a 3-for-1 stock split, three new shares are issued to replace each existing share. Because a stock split does not change the assets or the earning ability of a company, we should not expect it to have any effect on the wealth of the company's shareholders. All else being equal, the 3-for-1 stock split should cause the price to go down to one-third of its previous value. In general, an n-for-m stock split should cause the stock price to go down to m/n of its previous value.

Similarly, reverse stock split or reverse split, is just the same as stock split but in reverse, a reduction in the number of shares and an accompanying increase in the share price. The ration is also reversed 1-for-2, 1-for-3.

The market regulation requires that the price of the shares in a stock split should be adjusted such that the total market capitalization of the company remains unchanged before and after stock split action is taken. A stock split seems a technical adjustment for a share number increase. However, market psychology affords market traders some chances in pursuing more profit from this number game. Making profit from the stock split number game sounds impossible; however, market evidences reveal abnormal returns, which imply share-trading earning increase. This contradicts the technical fact that the total values of the company remain the same. This is what is normally referred to as the *stock split puzzle*.

According to Wulff (2002), stock splits have long shown eagerness among stock traders and recently seem to be attracting a very large group of people in America and most emerging markets. The results of a stock split are puzzling; in theory, a stock split is merely an accounting change, which should leave investors no better or worse off than they were before the split. However, stock splits are now a regular occurrence in America. This implies there must be some profit, either real or perceived, that results from a company splitting its stock. The reason why stocks are likely to outperform after a split is a mystery which we try to solve in this thesis. A range of explanations has been proposed. One of these is the signalling theory, which states that by splitting the stock, the company's management signals its confidence in the firm's bright prospects. If management believes that the company's profits will carry on growing, and that the shares will keep appreciating, it might as well go ahead and split the stock so as to reduce the share price and thus to maintain an optimal price range. If the price of the share goes above a certain price, the management feels that the share will be beyond the reach of many traders who demand the share to be illiquid.

According to Wulff (2002), the other stock split puzzle is in the increase in stock volatility that occurs on the ex-date of a stock split and it has puzzled researchers. The effective date of the split is known well in advance, and there appears to be no additional information about the firm revealed on the split day. Thus there is no obvious reason why volatilities should increase after the effective date of the split. Yet they do. Such a predictable increase in volatility in the absence of apparent information is unexplained. Before the ex-date a firm may trade when-issued shares at the post-split price level. The introduction of the when-issued trading provides an opportunity for traders to elect one of the two markets for trades, the un-split shares trading at one price level and the when-issued shares trading at the post-split price level. The introduction of lower priced when-issued shares attracts small volume traders and separates the market into two trading sets. When measuring volatility of shares before the split, the volatility is lower for both the un-split shares and the when-issued shares as compared with matching firms that do not trade when-issued shares. After the split, the small-volume traders return to trading in the regular way with a single price level and the volatility measure increases significantly. However another approach to this mystery draws on Black's (1986) conjecture that noise traders may prefer low priced stocks and suggests that a stock split may induce more noise trading by lowering the stock price. Although, even if noise trading was one part of the answer, the noise trader hypothesis does not explain why the volatility jumps so much on the ex-date of the split; the hypothesis gives no apparent reason for the number of noise traders to jump significantly on the ex-date. Such an increase would imply either that an information event occurs precisely on the ex-date of the split, which is unlikely, or that noise traders are waiting on the sidelines until the ex-date.

Another puzzling phenomenon to market practitioners and researchers has been a change in liquidity after a stock split. As Lakonishok and Lev (1987) put it, "taken at face value, such distribution is just a finer slicing of a given cake – the total market value of the firm and as such should have no effect on firms and investors". In a perfect market, the market value of a firm's equity is independent of the number of shares outstanding. Therefore the ex-date for a stock split should simply involve a change in the number of shares outstanding along with a change in the level of the stock price. According to Copeland (1979), there should be no change in the distribution of stock returns around ex-date of the stock split. One of the main reasons the literature puts forward to explain stock splits is in effect liquidity. In simple terms, it is argued that the splitting of stocks allows more investors to buy the stock, therefore creating a more liquid environment and leading to an observable abnormal return around the announcement and ex-dates. A more detailed analysis of this and other possible explanations is made in the following sections of this dissertation in order to set up a framework on which to base the hypotheses that stock split has a liquidity effect that will be tested subsequently.

Bechmann and Raaballe (2004) presented four main competing explanations that have been suggested for the stock price effect of stock splits:

1. The optimal trade range hypothesis suggests that a stock split changes the price to a more optimal trading range, for example such that the stock is affordable for a large group of investors. This, in turn, could increase the demand for the stock, leading to a positive stock price effect.
2. Market hypothesis argues that the size of the relative bid-ask spread is important for the incentives of the market maker to promote the stock. Hence, a stock split can increase the relative bid-ask spread, whereby the market maker will be more active in promoting the stock, leading to positive stock market effect.

3. The neglected firms' hypothesis suggests that stock splits are made primarily by firms that believe they are undervalued. The stock split is considered to be a way to attract analysts' attention.
4. There is a cosmetic hypothesis, which argues that stock splits are just cosmetic events. According to this hypothesis, the positive stock market reactions to stock splits can be explained by close relationship between these events and changes in the firm's payout policy.

There are three statistical problems mentioned above which are associated with the stock split puzzle. We give a brief description of the three problems below.

1.1.1 The Abnormal Returns Associated with Stock Splits

According to Charitou et al. (2005), stock splits are known to have positive abnormal returns in the short run (around the announcement and the execution dates). They showed this in their paper for the Cyprus stock market, an emerging market

Also, Wulff (1999) presented evidence of wealth increase effect around the announcement and execution dates, for his sample of German stocks and U.S stock splits. Around the announcement date, the author finds an important price run-up in the ten days leading to his date. The author also finds price increases around the execution date, though of smaller magnitude than those recorded for the announcement date. McNicholas and Davis (1990) argue that the positive reaction on the ex-date cannot be connected to known well in advance. They try to find support for this price reaction in microstructure components of stock market.

A study by Easley et al., (1998) which examined abnormal returns associated with stock splits, concluded that they noticed that stock splits attract uninformed traders and also that informed trading increases, resulting in high trading and providing a better liquidity.

Also Brennan and Copeland (1988) find that evidence on the reduction in the extent of information asymmetry following stock splits is related to the abnormal returns. Savitri and Matani (2007) concluded that there are significant abnormal returns associated with the stock splits on the date of the split. Their results revealed that stock splits have effects to the stock return, because stock splits have information content and they give the investor-signalling effect to the market. Also they concluded by saying a stock split increases investor perception about the future earnings, as corporate action will influence the stock price and finally have an impact to the stock return.

1.1.2 Liquidity Changes Associated with Stock Splits

Amihud and Mendelson (1991) defined liquidity as follows:
"An asset is liquid if it can be traded at the prevailing market price quickly and at low cost".

One can argue that the inconclusive status of liquidity may come from the vague status of the definition and indices of stock liquidity.

First, one must consider that liquidity can be measured in many different ways. For instance, Wulff (2002) uses the following measures:

a) Volume, calculated as the adjusted daily number of shares traded
b) Volume turnover which is calculated as the volume divided by the shares outstanding
c) Percentage of days with trades.

Wulff adjusted the daily number of shares by multiplying the number of shares traded by the split factor. Most of the data banks do not take care of this on the day the split is executed, but only a day later.

Another way of thinking about liquidity is by considering the cost of trading.

In this dissertation three indicators for liquidity are mainly used:

a) The price measure
b) The bid-ask spread measure
c) The volume measure.

Definition 1.1.2.1: Price Measure

Price measure is defined as the midpoint which is calculated as the product of 0.5 and the difference between the highest day price and the lowest day ask price.

Definition 1.1.2.2: Bid-ask spread measure

We define the bid-ask spread measure as the effective spread and is calculated as the modular of the difference between the trade close price and the midpoint which is defined above.

Definition 1.1.2.3: Volume measure

Here we define the volume measure the daily percentage change of the volume of shares traded per day.

The higher the turnover of a stock, the easier and faster it is to sell a stock at a given price limit. When stocks have to be sold immediately, it is not possible to command a certain price. One has to accept the higher ask price when buying and the lower bid price when selling. The spread constitutes a considerable cost component in stock trading. Therefore the narrower the spread, the more liquid and attractive a stock is for potential investors. Given the essential role of liquidity in the market place we investigate the impact of stock splits on the above-mentioned indicators.

Wulff (2002) analyzed 276 stocks splits in the Official Market of the Frankfurt Stock Exchange (FTE) from 1960 to 1996. One striking feature he documented was that the splits were highly clustered in the years 1967 to 1970 (1969 alone had 94 splits) and 1995-1996. The author reasoned that the main reason behind this clustering was connected with minimum par value rules that were applicable at the time to German companies. This restriction led the author to claim that signalling could not be the main reasons behind splits as companies did not seem to split when they found this operation to be appropriate, but only when the law changed. His analysis concerning liquidity is supportive of enhanced liquidity brought about the split.

One area where greater consensus seems to exist is that of bid-ask spread changes induced by the split. The split itself reduces the price of share while under normal circumstances the bid-ask spread in absolute terms also decreases. What also seems consensual is that liquidity *per se* lacks explanatory power for the abnormal returns associated with stock splits, especially those that have been found.

1.1.3 Stock Splits and Volatility Changes

Although most work surrounding stock splits focuses on the effects on prices and its relation to liquidity changes, some work has also been developed concerning changes in risk. Sheikh (1989) addressed this issue in the context of a study that tested the efficiency of the Chicago Board Options Exchange (CBOE), following previous authors that identified a significant increase in volatility subsequent to stock splits with a split factor larger than 25%. Even if the causes concerning this increase may not be clear, an increase in the price of calls should occur as a consequence of that increase in volatility. On the ex-date Sheikh observed a significant increase for the splitting group, with the control group showing an insignificant difference between the two groups. The author concluded that the CBOE captured the ex-date variance increase as it occurred.

Ohlson and Penman (1985) and Koski (1995) reported an increase in volatility following a stock split. Their results indicated that there is a decrease in liquidity rather than an increase after the stock split. Easley et al (1998) showed that an increase in the relative spread was not caused by an increase in the underlying volatility of the stocks by increasing the dispersion of their true values.

Dubofsky (1991) also found significant evidence of an increase in volatility following a stock split in his paper "Volatility increases subsequent to NYSE and AMEX stock split". Also Desai et al., (1998) found evidence that even after controlling for microstructure biases, stock splits still show significant increase in the volatility after the split. They concluded that changes in the volatility and its permanent component are positively related to changes in the number of trades.

In Figure 1.1.1 we present the theoretical mechanism diagram according to stock split literature. This diagram shows how this thesis is structured and how we try to solve for the three indicators of liquidity which arise from stock splits. In the diagram we show the increase in liquidity, which is discussed in Chapter 2, abnormal returns associated with stock split, in Chapter 3, increase in volatility which is discussed in Chapter 4, and finally the technical analysis (DEAR), in Chapter 5.

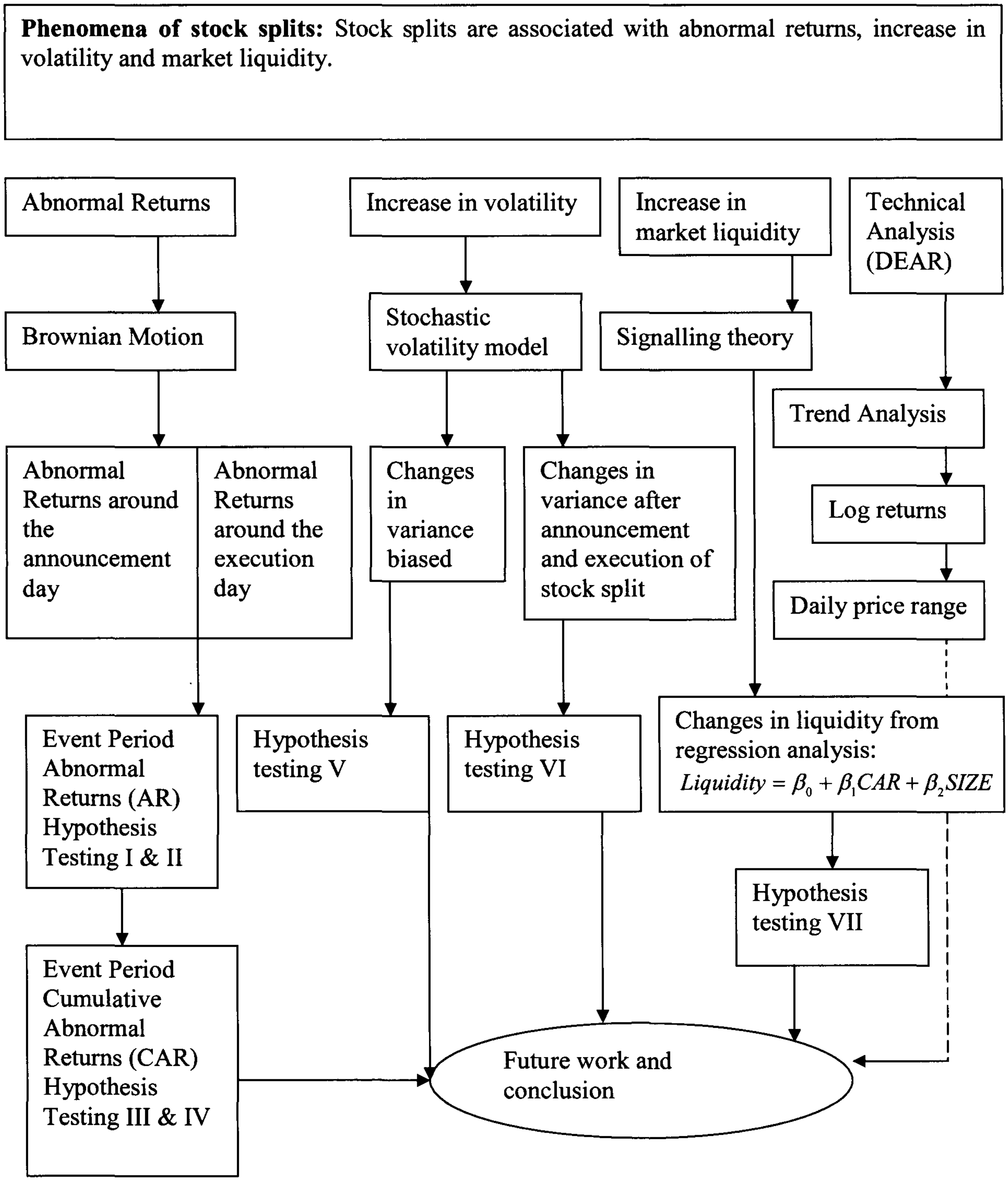

Figure 1.1.1 Three definitions of liquidity

1.2 The Nature of Stock Split

Below we present studies at RightLine Research Company which describes the "typical" life of a cycle of a splitting stock and how it is dissected into six major categories:

Pre-announcement	Stocks usually enter this stage quietly and without fanfare after a long period of healthy growth. However, in some cases emergence into the Pre-announcement stage occurs quickly, as an unexpected windfall causes a rapid increase in the stock price. **This stage of a stock split is often associated with significant appreciation in share price.** The key to profiting from this stage is being able to determine which stocks are the most likely to split and when.
Announcement	The upbeat atmosphere of a stock split often pulls in a large number of new buyers. This influx of traders and investors can lift the stock price higher, giving exceptional gains for those positioned in the stock prior to the stock split announcement. For those who are in the stock before the split announcement, this stage usually offers low-risk setups for timing short-term trading entries.
Dormancy	There is generally a return to normal price behaviour in the weeks following a split announcement as the initial interest subsides. The shorter the time between the announcement and the split date, the less subdued this stage will be.
Pre-split Run	For many stocks this is the most powerful phase of the split cycles as investors dramatically bid up the price of the limited supply of shares.
The Split	The day of the stock split provides more investor awareness of the already well-publicized stock split. Many investors who watched the stock rise at the announcement and again during the pre-split run will now buy shares at the lower split prices. These final buyers can push prices even higher.
Post-split	After the last buyers are in, investor excitement for the split stock can begin to fade. Prices will often retreat for a while as shares are sold to lock in profits. This stage of a stock split can deliver excellent shorting prospects. While some split stocks will pull back and consolidate for a while, strong performers often dip, quickly rebound and then continue to fly higher.

1.3 Aims and Objectives

In this mini-dissertation we intend to examine patterns of the stock split phenomenon on the NASDAQ from statistical standing point as well as from the DEAR-based trend analysis approach.

1.3.1 The overall objective of this dissertation is:

- To examine if stock splits are associated with abnormal returns and an increase in variance and liquidity following the ex-day.

Definition 1.3.1.1: Cross-sectional regression

A cross-sectional regression is a type of regression model in which the explained and explanatory variables are associated with one period or point in time. This is in contrast to a time-series regression or longitudinal regression in which the variables are considered to be associated with a sequence of points in time.

1.3.2 The aims of this thesis are:

- To examine the change in liquidity using regression analysis, section 2.4
- To investigate the potential of abnormal returns around the event date (announcement and execution) of stock splits, section 3.4.2 and section 3.4.3.
- To explore why, even after controlling for microstructure biases, we find a significant increase in the volatility after the split, section 4.3 and section 4.4.
- To explore the patterns surrounding stock splits via the DEAR-based trend analysis styled approach, section 5.2 and section 5.3.

1.4 The Data

Stock split firms and data were sourced from Yahoo Finance website (Ref 77) and we chose the month of June 2007. The initial sample consisted of 30 firms that had a stock split in June 2007. Of those 30 firms provided 13 firms were excluded because they do not trade on the NASDAQ and we felt that they might corrupt the calculations since the index used is weighted on the stocks traded on it. Of the 17 remaining, one firm was also excluded because the company was also having a stock split on its mirror company which was already accounted for. We extracted the close price of the stock, bid price, offer price and the trading volume was extracted for the 16 firms. Information on the split factor was collected from the Yahoo Finance website (Ref 77) and the companies' websites.

1.5 The structure of dissertation

We start this dissertation with an introduction of the stock split puzzle, coupled with aims and objectives of the dissertation. We present the three statistical problems associated with stock splits which are the abnormal returns, an increase in volatility and the market liquidity increases. We proceed into Chapter 2 where we give the theoretical background of the market efficiency and also the foundations of market liquidity. Applied regression analysis follows, and we explore the changes in liquidity around the ex-date of a general and a particular case. In Chapter 3 we introduce the abnormal returns associated with stock splits. Here, we present results of change in return for both pre- and post-split and we examine the abnormal returns for both general and particular cases. The results are presented in the form of Tables and in Chapter 4 we test the hypothesis for higher volatility between pre and post-split for both the general and the particular cases. This chapter also took the opportunity of using a slightly different but new technique called the EVARCH. In Chapter 5 we give a new technique called the Differential Equation Associated Regression (DEAR), proposed by Guo and Guo (2009). This method is still young and provided gave us some empirical evidences. Chapter 6 summarizes the dissertation and we discuss future developments.

Chapter 2. EMPIRICAL EVIDENCE OF MARKET LIQUIDITY INCREASE

2.1 Market Efficiency

In an efficient capital market, securities prices adjust rapidly to the arrival of new information; therefore the current prices reflect all information about security. Financial markets can be mimicked as an ecological system where trades intermingle with one another and act in response to information in order to determine the best price for a given product. If one is to consider the price, volume and number of transactions of a financial product, one would realize that the change is not predictable. At first sight, looking at a share price, there is an inconsistency in the price change. Hence time series, share price is indistinguishable from a stochastic process.

2.1.1 Efficiency of Markets

Market efficiency is interpreted as saying that market prices incorporate all of the relevant information. Exactly what is meant by this phrase is not entirely clear, nor is its relation to the other definitions of efficiency, but roughly speaking, it is intended to convey the ideas that since prices are not the results of the decisions of individual agents, prices should therefore depend upon the information underlying those decisions.

In this dissertation we show that even though the market efficiency reflects that superior returns are not possible to attain since all information is already incorporated into the price, one has the ability to make superior returns when a stock split announcement is made. This will show that as a matter of economic logic, markets are not perfectly efficient. If markets were efficient, then no one would act on their own information and the number of irrational traders would decrease significantly. It follows, then, that there must be a loophole in the market or insider trading, hence violation of the market efficiency to allow individuals to gather and process information. According to Ross (2004), transactions costs and information processing costs render many supposed violations of efficient market ineffective, but financial markets are as close as we have come to frictionless markets, and such imperfections seem a weak foundation for understanding how information is incorporated into prices. There are different types of efficiency. We will list only two, namely the operational efficiency and the information efficiency. The operational efficiency is a measure of how well things function in terms of speed of execution and accuracy. This is mostly used by engineers. The informational efficiency is a measure of how quickly and accurately the market reacts to new information, normally used by economists. The efficient market hypothesis (EMH) deals with information efficiency.

If all information is priced in the securities, then the issue of abnormal returns on announcement and execution becomes a puzzle. However, abnormal returns on announcement are realized, but this might be due to insider trading as market efficiency hypothesis states. Therefore we assume that our markets might be of weak form efficiency.

For this dissertation, it is most instructive to begin with the martingale or risk neutral representation of the No Arbitrage pricing framework. The price of an asset with next period payoffs of z is

$$p = \frac{1}{(1+r)} E[z], \tag{1}$$

where r is the risk-free rate.

To study efficient markets we must be explicit about the information set, S_t, which the market uses to condition expectations at time t, and that also requires us to be explicit about the timing of both when information is known and when values are determined. We will call S_t the market information set since it is the one that is used for price determination, and we will write

$$p = \frac{1}{(1+r)} E\left[z_{t+1} | S_t\right]. \tag{2}$$

Three levels of market efficiency are defined.

Definition 2.1.2: Weak Form Efficiency

According to Ross (2004), a market is said to be weak form efficient if S_t asserts that market prices reflect historical prices of the asset, that is if $\{..., p_{t-2}, p_{t-1}, p_t\} \in S_t$

This form asserts that historical prices cannot be used to make a profit as prices are presumed independent over time. Tests of weak form efficiency include:

a) Autocorrelation Tests

This investigates whether share returns are correlated in the course of time.

b) Filter Rule

This is a trading rule regarding the actions to be taken when share prices move up or down in value by a specified percentage.

Definition 2.1.3: Semi-Strong Form Efficiency

According to Ross (2004), a market is said to be semi-strong form efficient if S_t asserts that market prices reflect all publicly available information, including past prices.

We do not believe markets are semi-strong form efficient because not all publicly available information is priced in S_t hence the abnormal returns on execution.

Definition 2.1.4: Strong Form Efficiency

According to Ross (2004), a market is said to be strong form efficient if S_t asserts that market prices reflect all information, both public and private.

Strong form efficiency requires the information set that determines prices to include not only the publicly available information, but also the private information known only to some participants in the market.

Proposition 2.1.5: According to Ross (2004), if S_t denotes the market information set, then the value of any investment strategy that uses information set $A_t \subseteq S_t$ is the value of the current investment.

Proof: Suppose that there are n assets whose terminal payoff will be

$$z=(z_1,z_2,\dots,z_n), \tag{3}$$

and that an investment strategy consists of a portfolio

$$\alpha(A_t)=(\alpha_1(A_t),\alpha_2(A_t),\dots,\alpha_n(A_t)), \tag{4}$$

chosen at time t dependent upon the information set, A_t and costing $\alpha(A_t)p_t$ where p_t is the vector of the initial values of the assets. Since $A_t \subseteq S_t$ and since the current investment and the current interest rate, r_t, are elements of A_t, by the law of iterated expectations the value of the initial investment $\alpha(A_t)p_t$ in this strategy is given by

$$\begin{aligned} &\frac{1}{(1+r_t)}E[z_t|A_t] \\ &=\frac{1}{(1+r_t)}E[E[\alpha(A_t)z_{t+1}|S_t]|A_t], \quad \textit{iterated expectation law and combining } 1 \ \& \ 2 \\ &=\frac{1}{(1+r_t)}E[\alpha(A_t)E[z_{t+1}|S_t]|A_t] \\ &=\frac{1}{(1+r_t)}E[\alpha(A_t)(1+r_t)p_t|A_t] \\ &=p_t, \end{aligned} \tag{5}$$

which is the initial investment.

Proposition 2.1.6: According to Ross (2004), if S_t denotes the market information set, then investment strategy that uses an information set $A_t \subseteq S_t$, has a risk-adjusted expected return equal to the interest rate, r_t.

Proof: the return $R_{\alpha(A_t)}(t)$ on an investment strategy,

$$\alpha(A_t)=(\alpha_1(A_t),\alpha_2(A_t),\dots,\alpha_n(A_t)), \tag{6}$$

is given by

$$
\begin{aligned}
& R_{\alpha(A_t)}(t) \\
& = \frac{z_{\alpha(A_t)} - \alpha(A_t) p_t}{\alpha(A_t) p_t} \\
& = \frac{\alpha(A_t) z - \alpha(A_t) p_t}{\alpha(A_t) p_t},
\end{aligned} \tag{7}
$$

where $\alpha(A_t) p_t$ is the initial investment and $z_{\alpha(A_t)}$ is the terminal payoff.

The risk-adjusted expected return is the expectation under the martingale measure conditioning on A_t, and, from proposition 2.1.5,

$$
\begin{aligned}
& E\left[R_{\alpha(A_t)}(t) | A_t\right] \\
& = E\left[E\left[\frac{z_{\alpha(A_t)} - \alpha(A_t) p_t}{\alpha(A_t) p_t} | S_t\right] | A_t\right] \\
& = E\left[\frac{(1+r_t)\alpha(A_t) p_t - \alpha(A_t) p_t}{\alpha(A_t) p_t} | A_t\right] \\
& = E\left[r_t | A_t\right] \\
& = r_t.
\end{aligned} \tag{8}
$$

If the market is semi-strong form efficient, then proposition 2.1.5 and 2.1.6 assert that looking at past data adds no value and that their risk-adjusted returns are the same as the risk-free investment in government bonds. If the market is strong form efficient then we have the truly discouraging result that no amount of information or analysis can add value in the financial markets since it is already being used in the determination of the market prices.

Proposition 2.1.7: According to Ross (2004), a weak form efficiency implies that returns are serially uncorrelated over time and, indeed that returns are uncorrelated with any linear combination of past returns when correlations are computed using the martingale probabilities.

Proof: The result follows from proposition 2.1.6. Let $L\left(p_{t^-}\right)$ denote some linear combination of past price. From proposition 2.1.6 we know that in the martingale measure the unconditional expected return is

$$E[R_t] = r_t. \tag{9}$$

From weak form efficiency, $L(p_{t^-})$ is a subset of the market information set and, again applying Proposition 2.1.6, it yields

$$\begin{aligned} \operatorname{cov}(R_t, L(p_{t^-})) &= E[(R_t - E[R_t])L(p_{t^-})] \\ &= E\left[E\left[(R_t - r_t)\middle|L(p_{t^-})\right]L(p_{t^-})\right] \\ &= E\left[E\left(\left[R_t\middle|L(p_{t^-})\right] - r_t\right)L(p_{t^-})\right] \\ &= E\left[(r_t - r_t)L(p_{t^-})\right] \\ &= 0. \end{aligned} \tag{10}$$

As an example, if $L(p_{t^-})$ is the lagged return on the asset

$$L(p_{t^-}) = R_{t-k},$$

then proposition 2.1.7 would imply that

$$\operatorname{cov}(R_t, R_{t-k}) = 0.$$

Definition 2.1.8: Fundamental Analysis

This is an attempt to determine the present discounted value of all payments received from a share of stock, using expected future interest rate, a firm's earnings and dividend prospects, and the firm's risk evaluation.

2.1.9 Market anomalies

The efficient market hypothesis is a controversial concept which is almost far from being completely accepted in the investment world. There can be three issues which can be raised in the debate and these issues are selection bias, magnitude and lucky events. The magnitude issues focus on the fact that most of the inefficiencies in the market can be exploited by large portfolios only, hence the neglect of the medium investor. The selection bias issue points to the fact that methods which generate abnormal returns are not available to the public, hence an issue of insider trading can be raised. According to the Lalm Summaries book, "techniques that are available to the public are the ones that do not produce abnormal returns. Therefore, publicly available techniques have been pre-selected as failures". The lucky-event issue considers that superior performance by some investors does not necessarily contradict market efficiency, since this performance may be a result of luck. The lucky-event issue can be mirrored to guessing the lucky six numbers in the Lotto and winning. This is purely a lucky event and has nothing to do with the quantitative and qualitative skills of the individual. However, several market anomalies have been revealed that reflect inconsistencies of market efficiency. The low price earnings (P/E) ratio suggests that portfolios with low P/E ratio have higher returns that those with high P/E ratio. The small-firm effect proposes that small firms have higher average annual returns. This is normally observed in the month of January, which brings us to the January effect. This January effect implies that returns are high in January and small firms tend to do better than large firms. The reversal effect advocates that reversals are observed in which previous winning investments perform poorly and previously losing investments perform well. These reversals suggest inefficiencies in that markets overreact to information. The weekend effects hint that security price moves tend to be relatively good on Friday and generally bad on Monday.

2.1.10 Using Efficient Market Theory

Because the efficient market hypothesis describes how information is reflected in prices, it is also the basis for some extremely useful techniques for using prices to examine the impact of particular types of news. Although the explanatory power of regressions of price changes on information is typically low, despite the noise, there is no doubt that market prices respond to information. The efficient market hypothesis argues that prices respond to information. The efficient market hypothesis argues that price movements reflect – indeed, fully reflect – the economic impact information. This has led to the development of the events study as an important tool for using market return data to parse out the effects of particular events (Fama et al., Brown and Warner 1985). Suppose, for example we wish to determine the impact of stock issues on the price of stocks.

For example, suppose that Company J announced on 17 June 2007, that it had made a promising discovery about a new Aids drug, and we wish to determine the effect of that news on the value of the company. Let us call 17 June 2007 the zero date, $t = 0$. The cumulative returns of the stock is

$$CR(t) = Cumulative\,\mathrm{Re}turn(t) = \sum_{t-10}^{t+10} R_\tau \,, \tag{11}$$

during a twenty-day window, the event window centred at that date. The cumulative returns are just the change in the price including stock splits.

2.2 Market Liquidity literature

Definition 2.2.1: Liquidity

According to the Wikipidea website liquidity is having enough financial resources to cover financial obligations in a timely manner with minimal costs.

Generally, the motivation behind splits is to improve the liquidity of the stock, so that investors can easily buy and sell the stocks at the prevailing price. If a stock were very high-priced, then buying even a small lot would require significant investment. This may reduce liquidity in the share and also affect the price discovery because the price of a liquid share is more likely to be its fair value than that of an illiquid share.

Liquidity usually depends on a number of factors, including size of the firm's market capitalization, number of shares available to general public, investor interest, the quality of corporate governance. However, three areas are of particular concern when it comes to the liquidity of a firm, that is net working capital, the current assets minus liabilities; the current ratio compares the level of the firm's most liquid asserts, current assets against that of its shortest maturity obligations, current liabilities; the quick ratio is sometimes referred to as the acid test, and is a more conservative measure than the current ratio.

$$Net\_working\_capital = current\_assets - current\_liabilities \tag{12}$$

$$Current\_ratio = \frac{current\_assets}{current\_liabilities} \tag{13}$$

$$Quick\_ratio = \frac{(Cash + Marketable\_securities + Accounts\_receivable)}{Current\_liabilities} \tag{14}$$

In order to gain more insights about the abnormal returns associated with stock split we conducted three regression analyses which analyse the abnormal returns. We examine the impact of signalling on the liquidity measures. The friction measures are:

i) Bid ask spread measure (Effective bid-ask spread)

$$Effective\_spread_t = 2\left|trade\_price_t - midpo\text{int}_t\right| \tag{15}$$

ii) Price measure (Midpoint)

$$Midpo\text{int}_t = \frac{(bid\_price_t - ask\_price_t)}{2} \tag{16}$$

iii) Returns measure (Volume change)

Pre-split 220 days are estimated over the period from day -230 to day -11 relative to the announcement day and post-split 220 trading days are estimated beginning 11 days after the event.

2.3 Applied Regression Analysis

Regression analysis deals with the forecasting of one or more dependent variables on the basis of one or more independent variable (regressors). The purpose of regression analysis is to try and fit an optimum model which can be used with the minimal possible error and most significant regressors.

Before we proceed to the next section we give some important measures and certain tests of significance that will be conducted in sections 2.4 and 2.5. R^2 is used in the regression model to measure the proportion of variation explained by the model. R^2_{adj} is adjusted for the number of explanatory variables, and it is better to use than R^2. The more variables included in your regression equation, the better R^2, while R^2_{adj} takes the number of explanatory variables into consideration. An F-statistic measures the significance of the full model, where as a t-statistic measures the significance of an individual parameter. Confidence intervals for the predicted mean value of dependent variable are constructed by using the t-statistic.

2.4 Change in liquidity from regression analysis

We perform regression analysis, in order to inquire further into the potential causes of the abnormal returns associated with stock splits announcement. We perform three regressional analyses of the abnormal returns with the following regression estimate:

$$\textit{Cumulative abnormal returns } \left(CAR_i\right) = \beta_0 + \beta_1 Liquidity_i + \beta_2 SIZE_i + \varepsilon_i, \tag{17}$$

where i is 60 days before the event to 1 day before the event, also Liquidity can be effective spread, midpoint or log change in volume and $\varepsilon \sim N\left(0,\sigma^2\right)$ and $E\left(\varepsilon_i \varepsilon_j\right) = 0, \forall i \neq j$.

The dependent variable is the cumulative abnormal returns over the period $t = -60$ to day $t = -1$ (i.e. 60days before to a day before the stock split execution). This specific period was chosen as it is known to capture the full announcement effect. Brennan and Copeland (1988) signalling model implies a positive relationship between stock splits and abnormal returns. Since some of the liquidity measures involve returns on the stock, we investigate the effects of signalling on the liquidity measures.

The null hypothesis that we test is:

$H_0 : \beta_1 \geq 0$

$H_1 : \beta_1 < 0$

The dependent variable is the cumulative abnormal return from day –60 to –1 relative to the split announcement date calculated from simple daily returns. The explanatory variables is the liquidity measured as Effective Spread, Midpoint and Change in volume and the logarithm of the market value of equity on day –10 relative to the split announcement (SIZE). We include p-values in parentheses.

Table 2.4.1: Regression analysis of the CAR and liquidity

Variable	Model (1) Effective Spread	Model (2) Midpoint	Model (3) Volume Change
β_0	0.5533 (-0.3120)	0.5441	0.5438 (0.8249)
β_1	-0.0039 (0.5753)	-0.0213 (0.2163)	0.4612 (0.0766)
β_2	-15.2414 (0.0000)	-15.06 (0.0000)	-14.9175 (0.0000)
R^2	0.96714	0.9680	0.96885
R^2_{adj}	0.96538	0.9663	0.96718
Durbin Watson	0.6853	0.6421	0.6226
F-Statistic	548.81 (0.0000)	564.54 (0.0000)	581.15 (0.0000)
M.S.E	0.0001	0.0001	0.0001

Table 2.4.1, presents the regression results where the dependent variable is the CAR and the explanatory variable is the liquidity measure. We use three measures of liquidity as described in equation 17. Before we write up any conclusion, we carry out model checking in the next two sections.

2.4.2 Model Diagnostics

Definition 2.4.1: If the error term in the regression model has a constant variance, we call it homoscedastic, but if the variance is changing, we call the error heteroscedastic. We did not compute the critical values and the cut-off values for the studentized residuals and the yhat statistics due to time constraints, but the plot of residual against fitted values was used to detect for heteroscedastistity and plot of residual against regressors is used for linearity of regressors.

Heteroscedasticity or unequal variances usually does not occur in time series studies because changes in the dependent variable and changes in one or more of the independent variables are likely to be the same order of magnitude.

We started by checking for normality on the residuals in our models presented above. We plotted 3 QQ-plots of the residuals and from the three figures presented below we see that our residuals are not normally distributed.

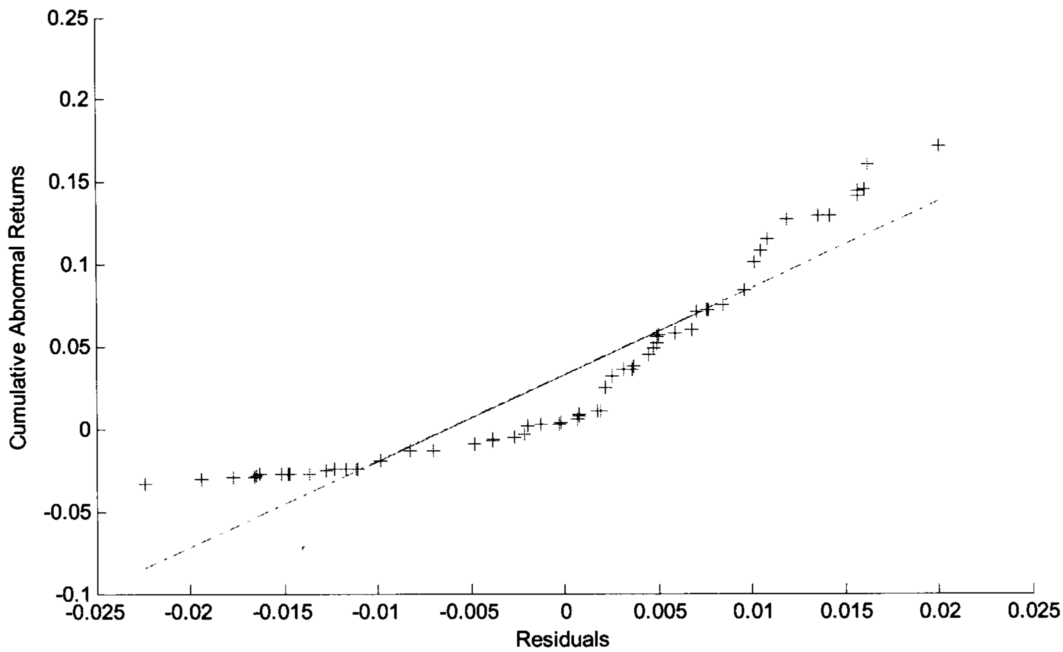

Figure 2.4.1: Effective Spread least squares residuals QQ-plot

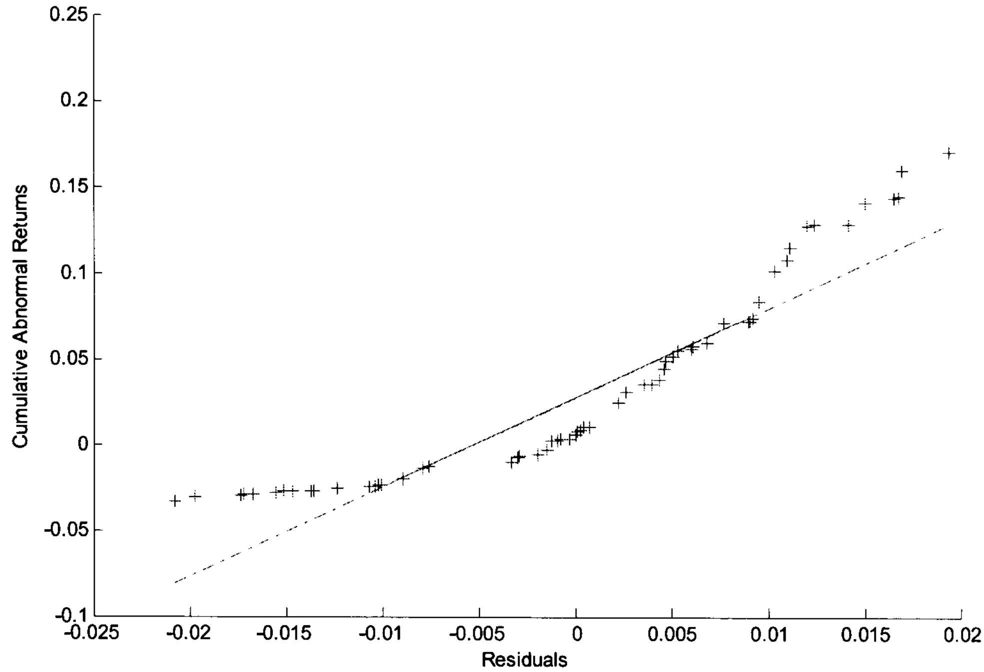

Figure 2.4.2: Midpoint least squares residuals QQ-plot

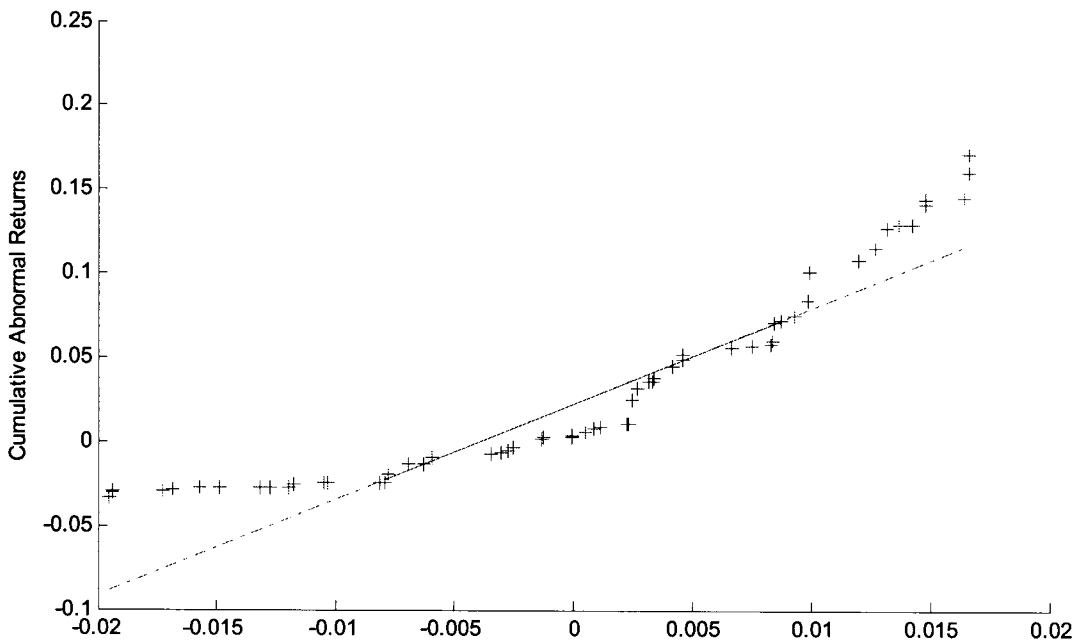

Figure 2.4.3: Change in Volume least squares residuals QQ-plot

From the graphs presented above the error distribution is skewed and the errors may come from a χ^2 distribution rather than a normal distribution. In such a case, one of the tails will be light and the other heavy. We went further to plot the raw residuals against the predicted values and we present graphs below of the three models we used to check for liquidity. We notice that the plots do not give a random scatter, which shows that our regression assumptions are not satisfied and the models do not fit.

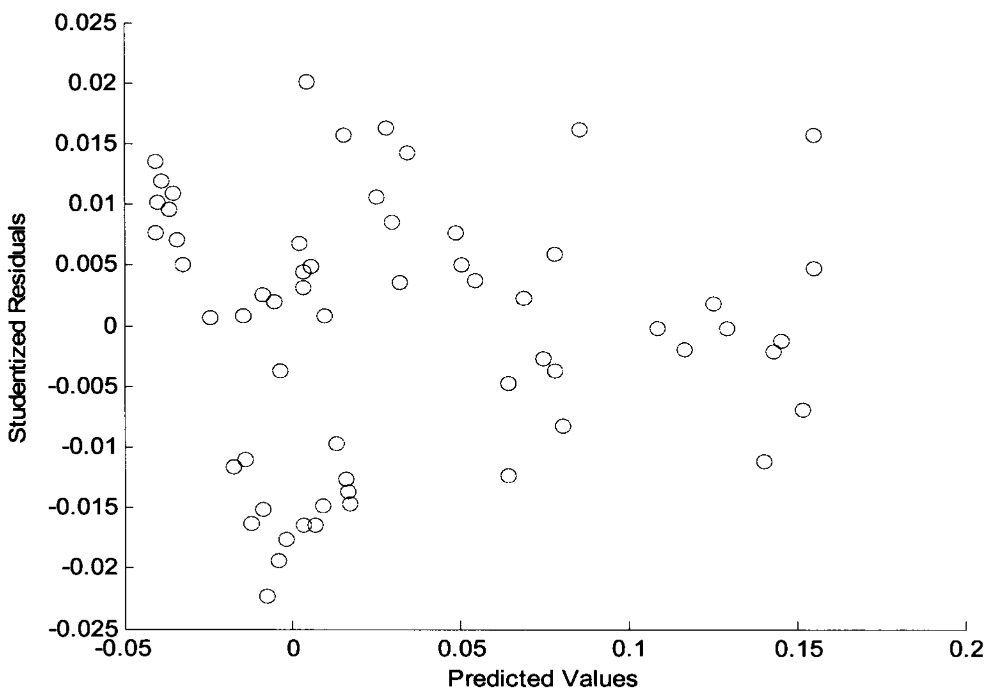

Figure 2.4.4: Effective Spread (Studentized residuals vs. yhat)

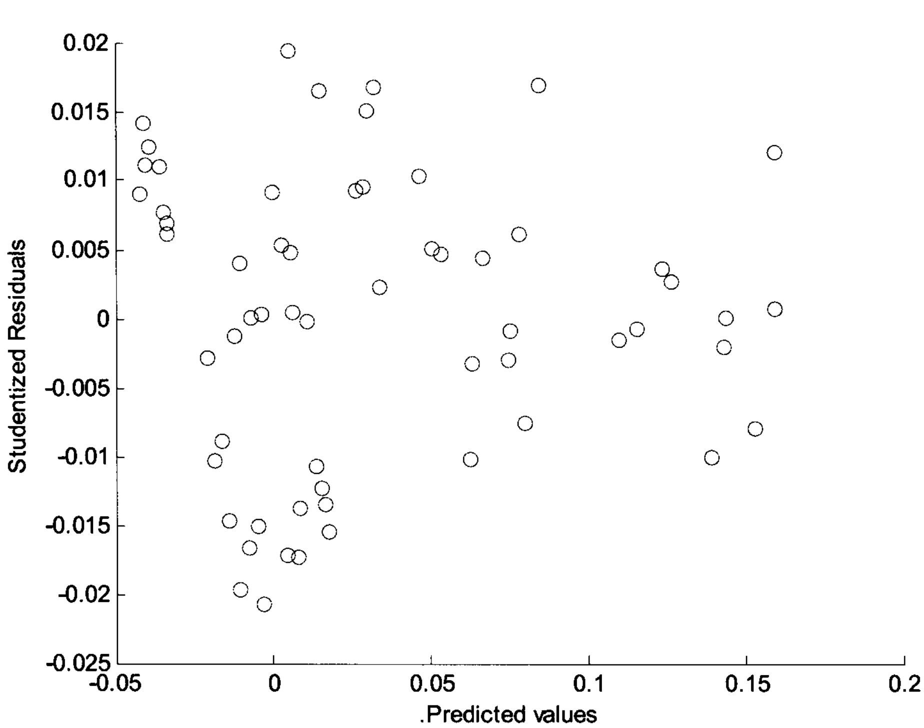

Figure 2.4.5: Midpoint (Studentized residuals vs. yhat)

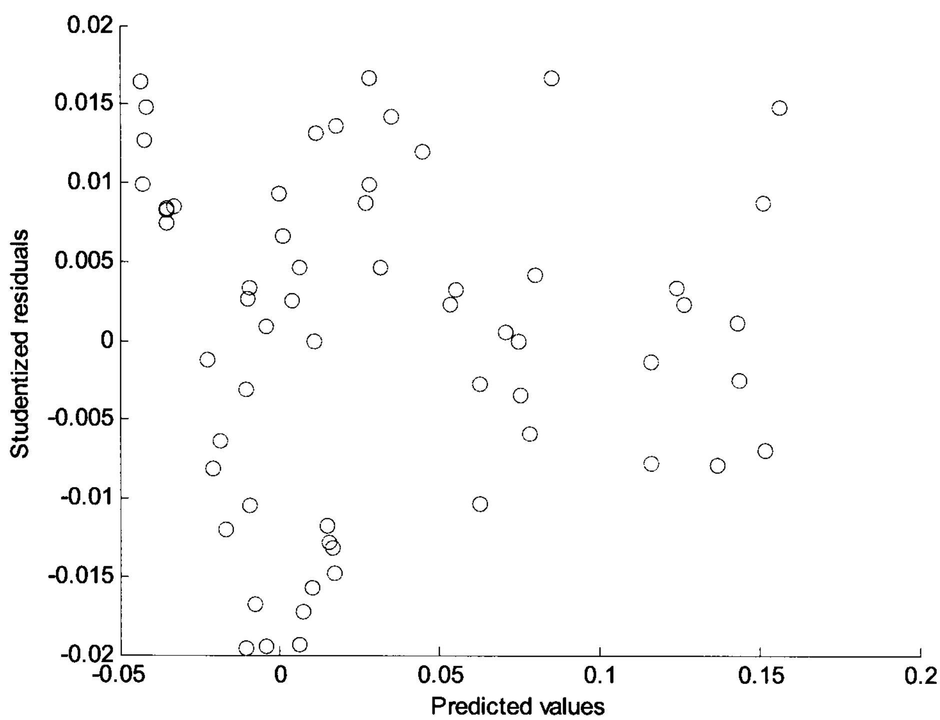

Figure 2.4.6: Change in Volume (Studentized residuals vs. yhat)

Since the change in volume is the only model which had signs of change in liquidity, we went further to plot the residual case order. This plot displays an error bar plot of confidence intervals on the residuals from regression. The figure below shows a plot of the residuals with error bars showing 95% confidence intervals on the residuals. We notice that all our error bars pass through the zero line, indicating that there are no outliers in the data.

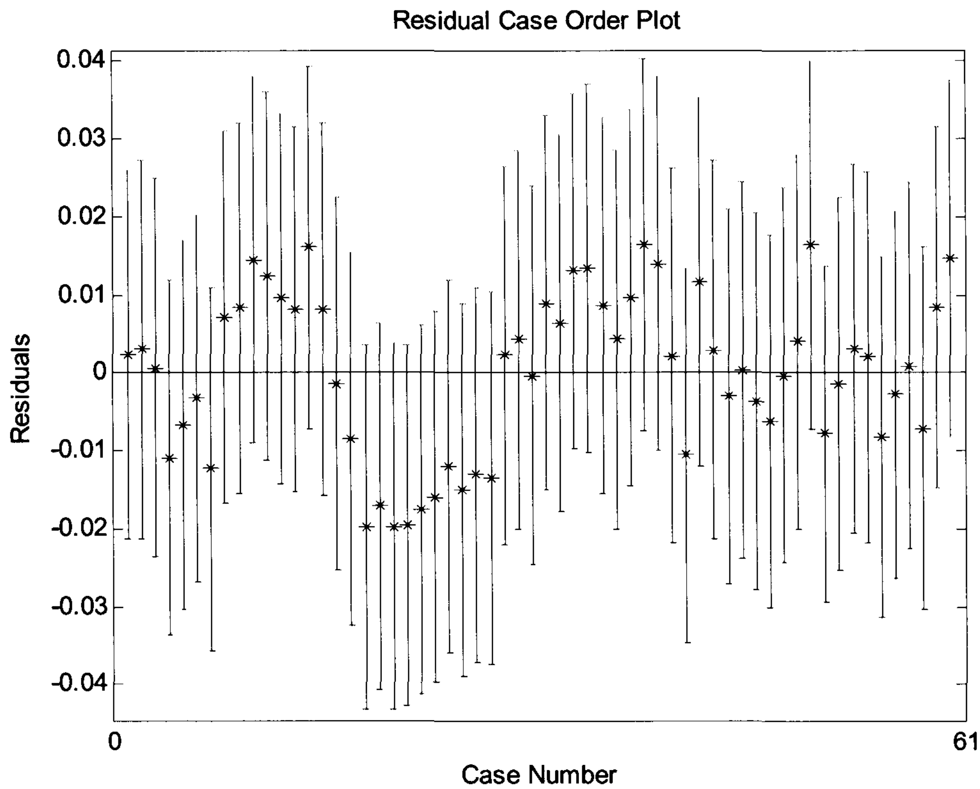

Figure 2.4.7: Residual case order plot (% change in volume)

Although we might have no outliers in the model we notice that there is a cyclical trend on the chart in Figure 2.4.7. We went further to calculate the Durbin Watson statistic to test for auto (serial) correlation and we found from the statistic that the Durbin Watson statistic is 0.6220, which means that there is positive serial correlation.

2.4.3 Test for Heteroscedasticity

There are many tests which can be used to test for heteroscedastistity and these include the Goldfeld-Quandt, Breusch Pagan and the White test among others. In this dissertation we will focus on the Breusch Pagan.

From the regression model equation (18), if heteroscedasticity is present we have a model which includes the general assumptions about the relationship between the true variance and an independent variable Z:

$$\sigma_i^2 = f\left(v + \delta Z_i\right) \tag{18}$$

$f(\ )$ represent a general function and Z can be an independent variable. For one to be able to test for heteroscedasticity, one has to calculate the test square residuals $\hat{e}_i$ from the regression model:

$$Y_i = \alpha + \beta X_i + e_i \tag{19}$$

Then one has to use the residuals to estimate:

$$\hat{\sigma}^2 = \frac{\sum \hat{e}_i^2}{N}, \tag{20}$$

and run the regression

$$\frac{\hat{e}_i}{\hat{\sigma}^2} = v + \delta Z_i + \gamma_i \tag{21}$$

According to Pindyck and Rubinfeld (1998), if the error term e in equation 19 is normally distributed and there is no heteroscedasticity, then one half of the regression sum of squares, $\frac{RSS}{2}$, provides a suitable test statistic i.e. $\frac{RSS}{2} \sim \chi_p^2$, where p are independent Z variables and χ_p^2 is a chi-square with p degrees of freedom.

We went further to test the following hypothesis for our model:

$H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_p^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2 \neq ... \neq \sigma_p^2$

We obtained the following data, from which we rejected the null hypothesis at 5% level and conclude that there is heteroscedasticity in our data.

Table 2.4.2: Breusch Pagan Test for heteroscedasticity

Variable	Model (1) Effective Spread	Model (2) Midpoint	Model (3) Volume Change
$\hat{\sigma}^2$	0.0011	0.0041	0.0001
$\frac{RSS}{2}$	10.2380	10.4463	9.2733
χ_2^2	5.99	5.99	5.99

We went further and transformed our model in equation 17 to the model below so that we correct for the heteroscedasticity in the data.

$$\frac{Cumulative \quad abnormal \quad returns \quad (CAR)}{Liquidity} = \frac{\beta_0}{Liquidity} + \beta_1 + \frac{\beta_2 SIZE}{Liquidity} + \frac{\varepsilon}{Liquidity}. \qquad (22)$$

After transforming our model, we did a model check to test whether the assumptions hold and below we present the graphs for our new model. We first checked for normality in the residuals and we saw that the errors now followed a normal distribution and we also plotted the raw residual against the predicted values and we saw that there was no pattern.

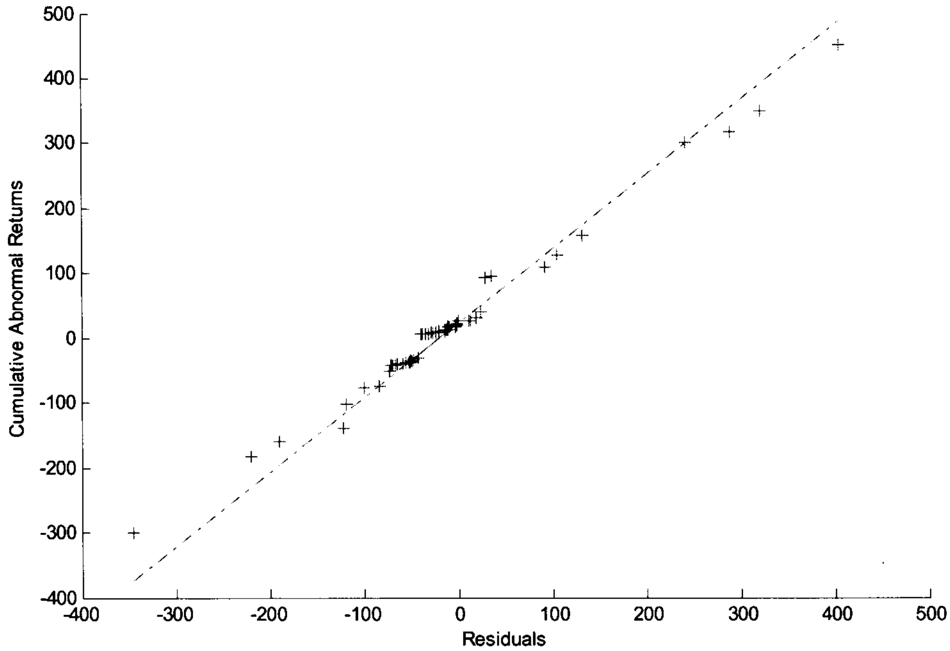

Figure 2.4.8: Transformed Effective Spread least squares residuals QQ-plot

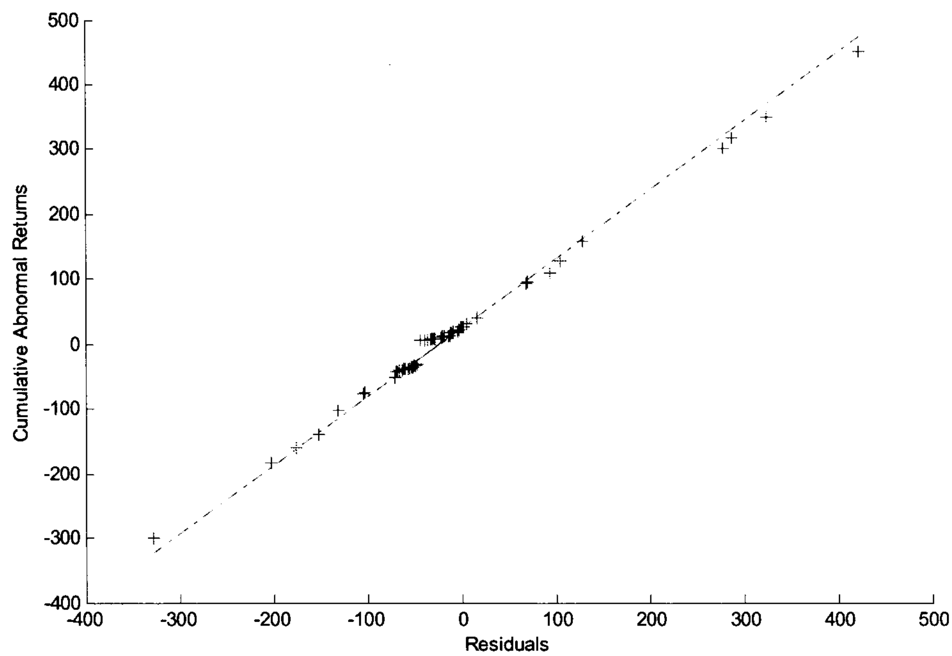

Figure 2.4.9: Transformed Midpoint least squares residuals QQ-plot

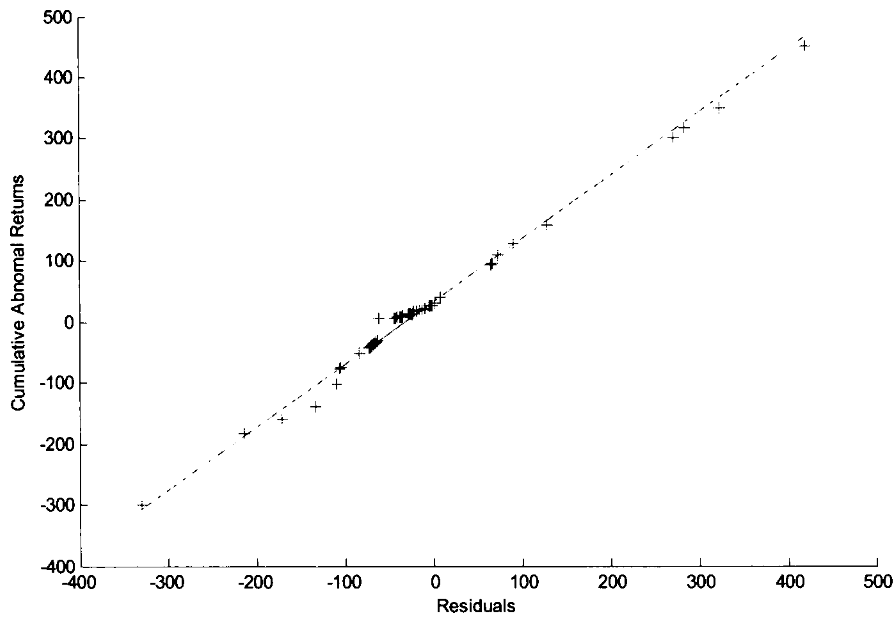

Figure 2.4.10: Transformed Change in Volume least squares residuals QQ-plot

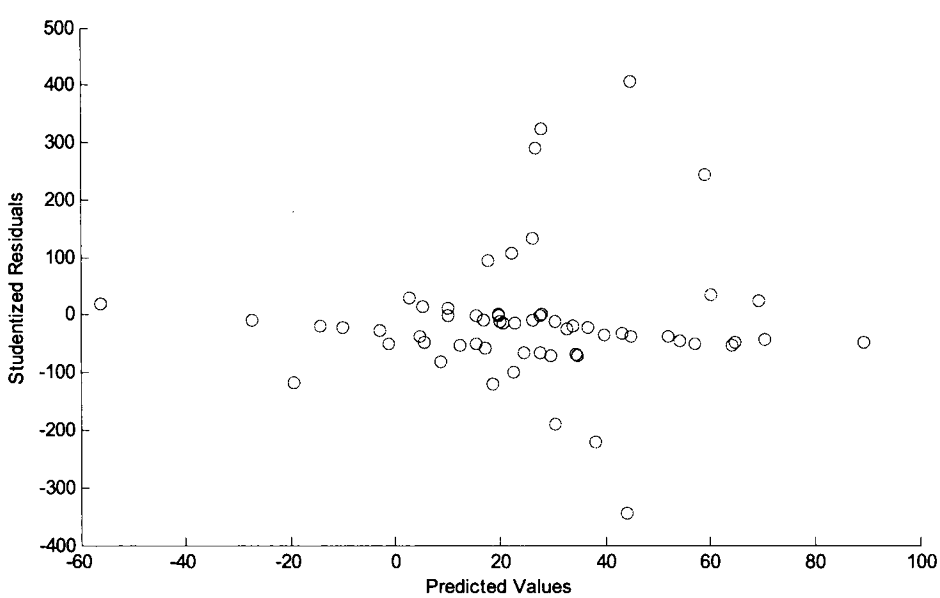

Figure 2.4.11: Transformed Effective Spread (Studentized residuals vs. yhat

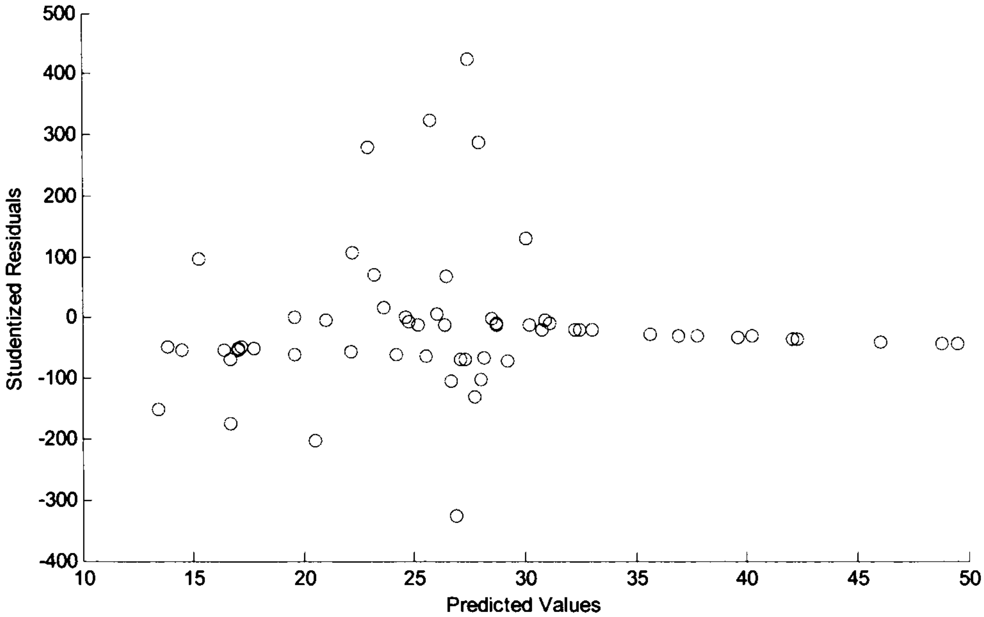

Figure 2.4.12: Transformed Midpoint (Studentized residuals vs. yhat)

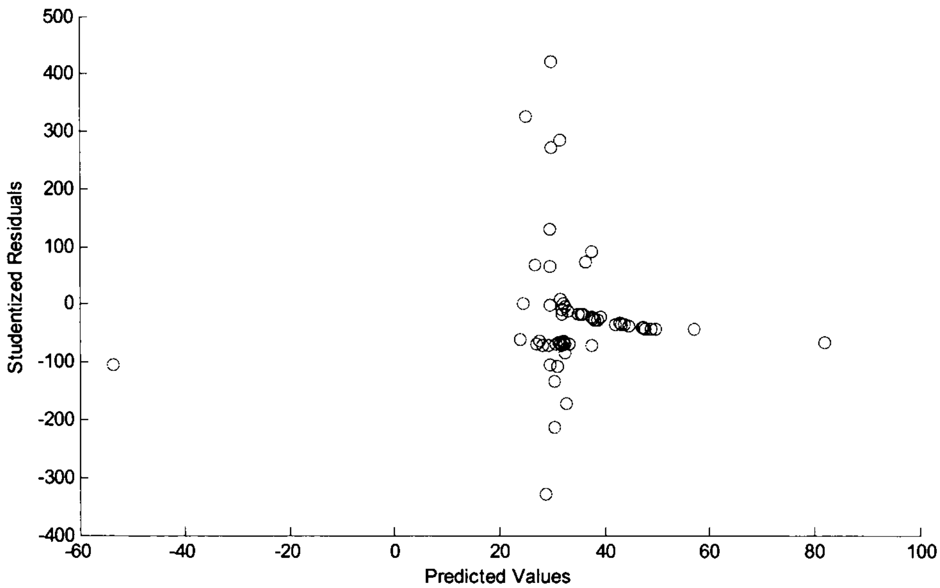

Figure 2.4.13: Transformed Change in Volume (Studentized residuals vs. yhat)

The residual plots shown above show a random pattern, indicating that our transformed model is a good fit for a linear model of liquidity.

Table 2.4.3: Regression analysis of the Transformed CAR and liquidity

Variable	Transformed Model (1) Effective Spread	Transformed Model (2) Midpoint	Transformed Model (3) Volume Change
β_0	8.1068 (0.024)	-243.12 (0.4615)	-50.6638 (0.012)
β_1	-9.3863 (0.026)	-112.32 (0.7838)	0.0094 (0.078)
β_2	1.3842 (0.469)	-3.87 (0.011)	2.2256 (0.034)
R^2	0.561	0.887	0.895
R^2_{adj}	0.534	0.845	0.856
Durbin Watson	1.48	0.708	0.82
F-Statistic	20.85 (0.000)	21.27 (0.000)	23.06 (0.000)
M.S.E	4.48	1.49	1.39

We ran the transformed regression to test whether there is explanatory power of the announcement effect on change in liquidity at the stock split, but we found none in the effective spread and the midpoint. However, the percentage change in volume did show that there is a change in liquidity. From our analysis we found three significant variables, the CAR regressed against the volume change. R^2 value is 0.895 which means 89.5% of the variables are explained by the model. We also noticed that for the models with the effective spread and the midpoint those liquidity variables are none-significant. However, we did notice a different result in the change in volume which showed a positive liquidity variable and one which is also significant. We also noticed that the total variation of CAR is small and the explained variation of CAR is large, with the explained variation of CAR being large it means that there is a large difference between the predicted value of CAR and the mean of CAR. Since total variation of CAR is measured as $\sum\left(Y_i - \bar{Y}\right)^2$, where Y_i is CAR and $\bar{Y}$ is the mean of CAR . We also have the explained variation of CAR being measured as $\sum\left(\hat{Y}_i - \bar{Y}\right)^2$, where $\hat{Y}$ is the predicted value of CAR. For us to get a high r-squared this means that $\hat{Y}_i \geq Y_i > \bar{Y}$. With the model given in equation 17 and the high values of R^2 (r-squared) which is a proportion of the total variation in CAR is explained by the regression of CAR on liquidity. We can also notice that the sample points lie almost on the estimated regression line. Surprisingly, even if all our models have a positive slope, only the last model has a positive coefficient of liquidity. The last models show that as cumulative abnormal returns increase, so does the liquidity. From our models we cannot conclude with support from Amihud & Mendelson (1986) that improvement in liquidity leads to an increase in abnormal returns.

2.5 Changes in Liquidity From A Particular Case

We looked at approaching the changes in liquidity from the new event study to test if there is significant change in liquidity of a stock split.

To test the hypothesis discussed in section 2.4, this study will use the regression model given below:

$$Liq_i = f(\Delta EPS, \Delta Div, D_1, D_2, \Delta Vol, \Delta Spread, \Delta Top40, \Delta NShares) \tag{23}$$

where Liq_i is the liquidity of a firm during the period $t = -1$ to $t = 1$.

ΔEPS is the percentage of earning per share (*EPS is adjusted for the split effect)

ΔDiv is percentage change of dividend payment per share

$$\Delta Div = \frac{Dividend^{*}_{t} - Dividend^{*}_{t-1}}{Dividend^{*}_{t-1}}$$

D_1 is a dummy variable whose value is 1 when the stock did not pay dividend the year before but pays dividend within 1 year after the split and 0 otherwise.

D_2 is the dummy variable whose value is 1 when the stock increases capital with in 1 year after the split and 0 otherwise.

ΔVol is change in trading volume after the split (the volume is already adjusted for the split effect)

$$\Delta Vol = \frac{Volume_t}{TotalStock_t} - \frac{Volume_{t-1}}{TotalStock_{t-1}}$$

$\Delta Spread$ is change in bid ask spread

$$\Delta Spread = \frac{\sum_{t=-20}^{20}\left(Ask_t - Bid_t\right)}{T_1} - \frac{\sum_{t=-20}^{180}\left(Ask_t - Bid_t\right)}{\frac{\sum_{t=-20}^{180} S_t}{T_0}}$$

$\Delta Top40$ is a percentage change of the proportion of shares held by the Top 40 large shareholders.

$\Delta NShares$ is the change in the number of shareholders

We did not carry out any analysis here because our model was short of critical data. The NASDAQ was not at liberty to part with the data without paying for it. The data we fell short of were the earning per share data, dividend payment per share data, changes in the proportion of the Top 40 shares data, and the number of shareholders data. All the data which was not provided was critical data for the analysis of the changes in liquidity. Due to time we could not secure any sponsorship to buy the data, but we believed that when funds are available we would do our analysis and add the section in the future to complete the puzzle.

Chapter 3. ABNORMAL RETURNS ASSOCIATED WITH STOCK SPLIT

We examine the price reaction to stock split by applying the methodology as described by Brown and Warner (1985). Market and risk-adjusted log returns are calculated as follows:

$$AR_{i,t} = R_{i,t} - \hat{\alpha}_i - \hat{\beta}_i R_{m,t}, \tag{24}$$

where $AR_{i,t}$ is the abnormal returns for stock *i*, where $i = 1 \ \ to \ \ 16$ at day t for $t = -230 \ \ to \ \ -31$, $R_{i,t}$ is the return for stock *i* at day t, $R_{m,t}$ denotes the return of the NASDAQ Index and $\hat{\alpha}$ and $\hat{\beta}$ are OLS estimates from the market model regression. The announcement day is the first day the information becomes publicly available. The announcement day and the execution day are denoted as day zero $(t = 0)$, the event date, and the trade-to-trade method over the same period to match the stock returns. The market model parameters for abnormal trade-to-trade returns are estimated from the trade-to-trade regression as described by Dimson and Marsh (1983):

$$\frac{R_{i,n_t}}{\sqrt{n_t}} = \frac{\alpha_i}{\sqrt{n_t}} + \beta_i \frac{R_{m,n_t}}{\sqrt{n_t}} + \mu_{i,t}, \tag{25}$$

where R_{i,n_t} is the return on security i over the period between two recorded trades, R_{m,n_t} denotes market return over the same period and n_t is the length of the return measure interval in day, ending at day t and $\mu_{i,t}$ is the error term. The abnormal returns for trade-to-trade are calculated using equation 17.

Cumulative abnormal returns (CAR) are the sum of abnormal returns over event interval period, usually two, three or more days and is calculated as follows:

$$CAR_i = \sum_{t=T-a}^{T+a} AR_{i,t} . \tag{26}$$

where a is the interval period in days over the event T.

3.1 Abnormal Returns around the announcement date

In this section we analyze the price change around stock split announcement and ex-dates. The null hypothesis that we test is:

$$H_0: \textit{There is no price change around stock splits announcement and ex-date}$$
$$H_1: \textit{There is price change around stock splits announcement and ex-date}$$

For the determination of statistical significance, we complete two statistics. The first statistic is the t-test statistic by Brown and Warner (1985) to cater for cross-sectional correlation.

The t-test statistic proposed by Brown and Warner (1985) is calculated as follow:

$$T_t = \frac{\overline{AR_t}}{S(AR_t)}, \tag{27}$$

where $\overline{AR_t} = \frac{1}{N_t}\sum_{i=1}^{N_t} AR_{i,t}$,

$\overline{AR} = \frac{1}{200}\sum_{t=-230}^{t=-31} \overline{AR_t}$,

and $S(AR_t) = \sqrt{\frac{\sum_{t=-230}^{t=-31}\left(\overline{AR_t} - \overline{AR}\right)^2}{200}}$.

The second is the standardized cross-sectional test of Boehmer et al. (1991), which controls for event, induced variance increases.

The Boehmer et al. (1991) t-test statistic is calculated as follows:

$$T_{B,t} = \frac{\frac{1}{N_t}\sum_{i=1}^{N_t} SAR_{i,t}}{\sqrt{\frac{1}{N_t(N_t-1)}\sum_{i=1}^{N_t}\left(SAR_{i,t} - \sum_{i=1}^{N_t}\frac{SAR_{i,t}}{N_t}\right)^2}} \tag{28}$$

with

$$SAR_{i,t} = \frac{AR_{i,t}}{\hat{\sigma}_i} = \frac{AR_{i,t}}{\sqrt{\frac{1}{(T_{s,i}-1)}\sum_{t=-230}^{t=-31}\left(AR_{i,t} - \overline{AR_i}\right)^2}}$$

N_t denotes the number of shares of which return data is available at day t, for $t = -230$ to -31. i is the number of firms, for $i = 1,...,N_t$, $SAR_{i,t}$ is the standard abnormal returns, $AR_{i,t}$ is abnormal returns and $\overline{AR_i}$ is the mean abnormal returns.

$\overline{AR_i}$ accounts for the cross-sectional average daily risk-adjusted and market-adjusted returns.

If the NASDAQ is of strong form efficiency, we should see no unusual price movements around the announcement date and therefore we would expect that AR and $CAR(t_1,t_2)$ fluctuate randomly around zero. However, if there is a leakage of information and trading on in side information just prior to announcement date, this should show up in the form of positive daily average abnormal returns as t approaches 0 and a corresponding build-up in the CAR_t.

3.2 Results of Changes in Return

Table 3.2.1 and Table 3.2.2 present abnormal returns around the stock split announcement of log return and trade-to-trade returns respectively. As can be seen from both Tables, abnormal returns are reported around the announcement dates.

Table 3.2.1: Abnormal returns around announcement of stock split based on log returns:

1. Event Period Abnormal Returns (AR)

Event date	AR in %	$t(BW)$	Negative AR	$t(BMP)$
-10	0.28	0.0005	20.69	1.0326
-9	0.11	0.0005	24.14	1.0254
-8	0.11	0.0009	20.69	1.9441
-7	0.20	0.0001	34.48	0.2426
-6	-0.03	0.0000	31.03	0.0003
-5	0.20	0.0009	17.24	1.9148
-4	0.04	0.0002	31.03	0.4241
-3	0.02	0.0001	24.14	0.1505
-2	0.24	0.0011	27.59	2.3304
-1	0.18	0.0008	20.69	1.7043
0	0.01	0.000	24.14	0.0902
1	0.51	0.0023	10.34	4.9396
2	-0.09	0.0004	34.48	-0.8491
3	-0.04	0.0002	24.14	-0.3794
4	0.15	0.0007	20.69	1.4647
5	-0.06	0.0001	31.03	-0.1446
6	0.02	0.0001	24.14	0.1789
7	-0.09	0.0004	17.24	-0.8808
8	0.06	0.0003	17.24	0.6376
9	-0.07	0.0003	37.93	-0.7124
10	0.04	0.0002	27.59	0.3386

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	$t(BW)$	Negative CAR	$t(BMP)$
Day -1 to day +1	0.70	0.0009	18.39	0.0421
Day – 2 to day +2	0.85	0.0016	23.45	0.1136
Day -2 to day +3	0.81	0.0006	23.56	0.2142

The Event Period Abnormal Returns Table column 1 lists the event date (i.e. the period date $t=-10$ to $t=+10$), column 2 has the abnormal returns for $t=-10$ to $t=+10$ in percentage. Also included is column 3 and column 5 which represent t-test statistic proposed by Brown and Warner (1985) to take cross-sectional correlation into account and t-test statistic of Boehmer et al. (1991), which is denoted as t (BMP) which controls for event-induced increase in variance. Column 4 contains the percentage of firms with negative abnormal returns. With returns of 0.07% and 0.08% are reported for the announcement log returns and trade-to-trade returns respectively. At the announcement date the abnormal returns are low as compared to the day before the announcement. This can raise questions if inside trading is playing a role in stock splits. Both the log returns method and the trade-to-trade method show there are 20 basis points and 22 basis points reported abnormal returns before the announcement respectively. The cumulative returns for the log returns method and trade-to-trade returns method are also reported in Table 3.2.1 and Table 3.2.2. The cumulative abnormal returns for the event window day $t=-2$ to $t=+2$ are 39 basis points and 43 basis points for the log returns and trade-to-trade returns respectively. As can be seen from the Tables the results of log returns method and trade-to-trade returns method are almost similar, this similarity shows that the price increase cannot be explained by measurement error due to thin trading but may be explained by the signalling theory. The cumulative abnormal returns are in supportive of the signalling hypothesis proposed by Grinblatt et al. (1983) which is based on the retained earning constraint.

Table 3.2.2: Abnormal returns around announcement of stock split based on trade-to-trade returns:

1. Event Period Abnormal Returns (AR)

Event date	AR in %	$t(BW)$	Negative AR	$t(BMP)$
-10	0.09	0.004	20.69	0.9128
-9	0.09	0.004	24.14	0.8433
-8	0.19	0.0008	20.69	1.7877
-7	0.02	0.0001	34.48	0.1594
-6	-0.01	0.000	31.03	-0.0852
-5	0.18	0.001	17.24	1.7256
-4	0.03	0.000	31.03	0.2473
-3	0.00	0.0010	24.14	0.0138
-2	0.23	0.0007	27.59	2.1784
-1	0.17	0.0007	20.69	1.5939
0	-0.01	0.000	24.14	-0.0989
1	0.50	0.0022	10.34	4.7939
2	-0.10	0.0005	34.48	-0.9974
3	-0.06	0.0003	24.14	-0.5529
4	0.13	0.006	20.69	1.2861
5	-0.03	0.0001	31.03	-0.2800
6	0.00	0.000	24.14	0.0395
7	-0.11	0.0005	17.24	-1.0182
8	0.05	0.0002	17.24	0.5044
9	-0.09	0.0004	37.93	-0.8359
10	0.02	0.0001	27.59	0.1752

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	$t(BW)$	Negative CAR	$t(BMP)$
Day -1 to day +1	0.65	0.0003	18.39	0.0508
Day – 2 to day +2	0.77	0.0002	22.76	0.0353
Day -2 to day +3	0.72	0.0001	23.56	0.0269

3.3 Abnormal Returns around the execution date

Initially, the stock market reaction to the announcement of a stock split was explained in section 3.2. Although some studies of stock splits have found stock price effects around the execution date. We examined the stock split pattern around the execution date using the same methods as for the announcement period that is log returns and the trade-to-trade return method.

Table 3.3.1: Abnormal returns around execution of stock split based on log returns:

1. Event Period Abnormal Returns (AR)

Event date	AR in %	$t(BW)$	Negative AR	$t(BMP)$
-10	0.09	0.0004	24.14	0.8566
-9	-0.05	0.0001	24.14	0.1173
-8	0.01	0.0000	27.59	0.0829
-7	-0.01	0.0005	34.48	-0.9553
-6	-0.07	0.0003	27.59	-0.6285
-5	-0.03	0.0002	31.03	0.3145
-4	-0.04	0.0002	24.14	-0.4037
-3	0.03	0.0001	27.59	0.2346
-2	-0.03	0.0001	27.59	-0.2896
-1	0.04	0.0002	27.59	0.3565
0	0.01	0.0000	24.14	-0.0166
1	0.00	0.0003	17.24	0.6738
2	0.19	0.0007	27.59	1.35
3	-0.07	0.0002	27.59	-0.4625
4	-0.18	0.0005	27.59	-1.1249
5	-0.11	0.0005	24.14	-0.9985
6	-0.10	0.0003	17.24	-0.6961
7	-0.06	0.0006	24.14	1.1557
8	-0.06	0.0002	20.69	-0.5035
9	-0.14	0.0005	41.38	-1.0511
10	-0.10	0.0004	34.48	-0.9169

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	$t(BW)$	Negative CAR	$t(BMP)$
Day -1 to day +1	0.05	0.0002	26.44	0.0464
Day – 2 to day +2	0.21	0.0008	24.83	0.0345
Day -2 to day +3	0.04	0.0001	24.71	0.0269

As can be seen from Table 3.3.1 and Table 3.3.2 on the event date t = 0 (the execution date) abnormal returns are reported for both methods the log returns and the trade-to-trade returns methods, with abnormal returns of 12 basis points and 17 basis points respectively. Abnormal returns significance is not attributed by all statistics on the execution date t = 0, hence the result can be attributed to event-induced variances.

Table 3.3.2: Abnormal returns around execution of stock split based on trade-to-trade returns:

1. Event Period Abnormal Returns (AR)

Event date	AR in %	$t(BW)$	Negative AR	$t(BMP)$
-10	0.09	0.0004	24.14	0.8136
-9	0.00	0.000	24.14	0.0215
-8	0.00	0.000	27.59	-0.0372
-7	-0.12	0.0005	34.48	-1.1165
-6	-0.08	0.0003	27.59	-0.6934
-5	-0.04	0.0002	31.03	-0.4029
-4	-0.06	0.0003	24.14	-0.5205
-3	0.01	0.0001	27.59	0.1166
-2	-0.05	0.0002	27.59	-0.4547
-1	0.03	0.0001	27.59	0.2441
0	-0.01	0.0001	24.14	-0.1319
1	0.06	0.0003	17.24	0.5346
2	0.14	0.0006	27.59	1.2706
3	-0.06	0.0003	27.59	-0.5471
4	-0.13	0.0006	27.59	-1.2157
5	-0.12	0.0005	24.14	-1.1073

6	-0.09	0.0004	17.24	-0.8198
7	0.12	0.00005	24.14	1.0808
8	-0.06	0.0003	20.69	-0.5334
9	-0.12	0.0005	41.38	-1.1185
10	-0.08	0.0005	34.48	-1.0225

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	$t(BW)$	Negative CAR	$t(BMP)$
Day -1 to day +1	0.07	0.0003	49.43	0.0404
Day – 2 to day +2	0.16	0.0008	47.59	0.0082
Day -2 to day +3	0.10	0.0004	48.28	0.0322

The event-induced variances can be insider trading or signalling hypothesis effect as suggested with abnormal returns announcement from Table 3.3.1 and Table 3.3.2 there are still unusual abnormal returns reported for the next 5 days after the execution (i.e. one working week), which shows the market is not efficient with respect to stock split.

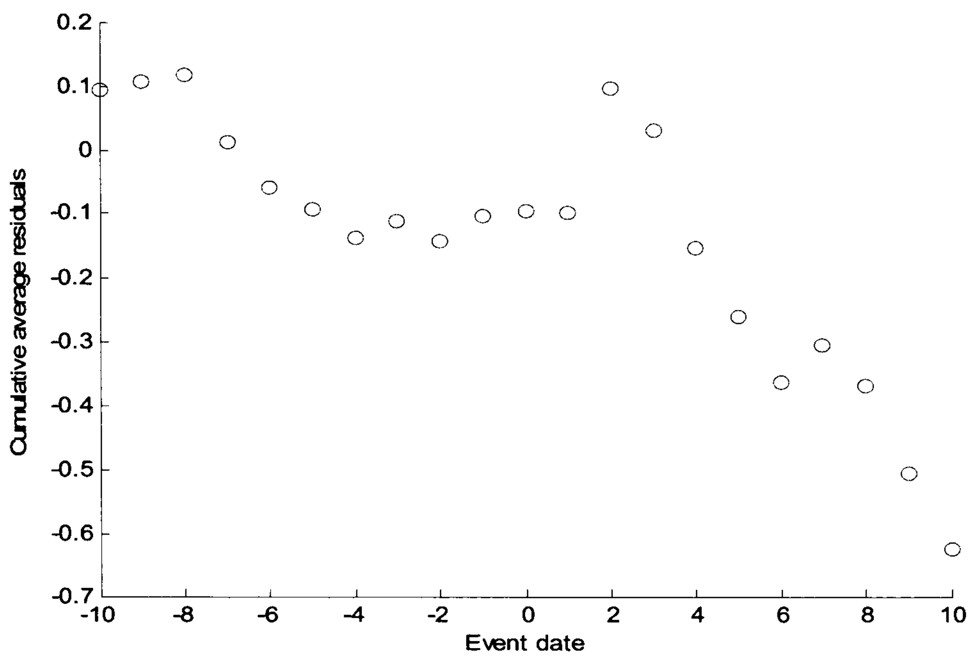

Figure 3.3.1 Cumulative execution average residuals for stock split using log returns

Looking at figure 3.3.1 i.e. cumulative execution average residuals for stock split there is a clear confirmation that the NASDAQ is of semi-strong efficiency. If we did not see the semi-strong form we would expect the return to jump on announcement and then we would see consistent returns or the cumulative returns would show a run up towards the announcement which would signal insider trading. However, here, once the announcement of a stock split is made, there are no profits to be made according to the semi-strong form efficiency of the market. There is a run-up in the cumulative abnormal returns prior to the stock split announcement.

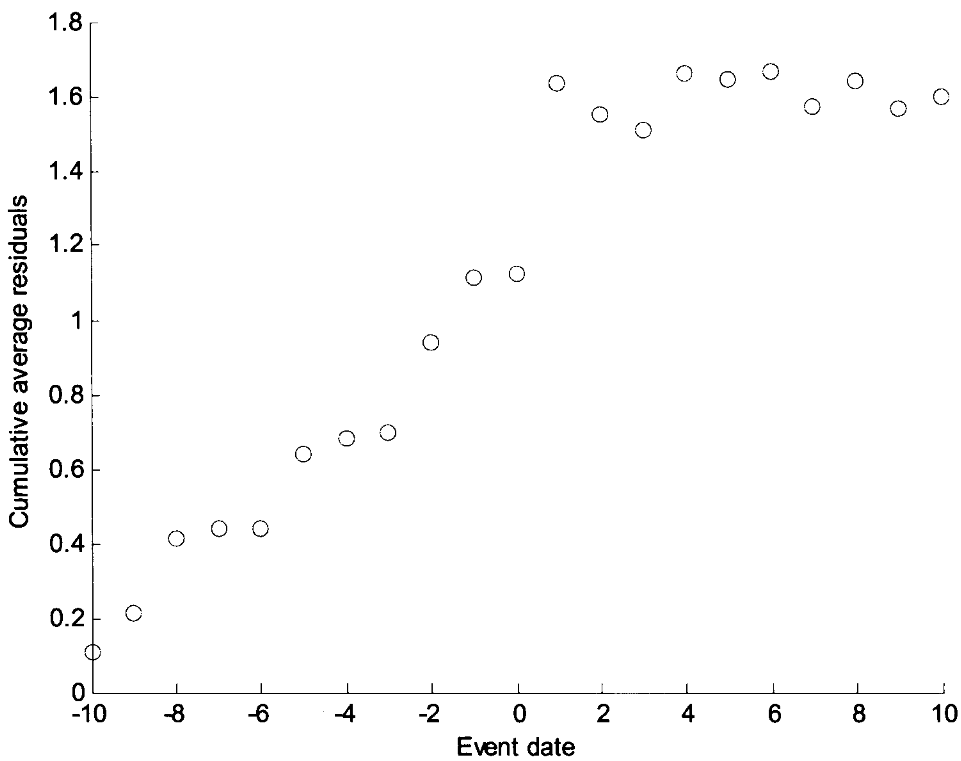

Figure 3.3.2 Cumulative announcement average residuals for stock split using log returns

This run might arise from two major causes i.e.:

a) Leakage of the news of a stock split or
b) A causal relation i.e. the optimal trading hypothesis

3.4 Changes in return: A Particular Case

We showed in the previous chapters that even though stock split is a popular practice despite the fact that it has no effect on the book value of the firm. We also showed the support of the liquidity hypothesis and found positive abnormal returns around the split date. However, we used traditional methodology and defined the event date as either the announcement date or execution date.

The event study approach is used by separating data into two windows. Pre-event is defined as (-200,-21) days before the event. The event window is defined as (-20,20) days around the event date.

The market model is estimated by the data in the estimation window

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t, \tag{29}$$

where $E(R_t)$ is actual return of the stock at time *t* and R_{mt} is actual market return at time *t*.

This study uses the percentage change of the NASDAQ Index to represent market return. The estimated model from (26) will be used to estimate expected returns during event window. The difference between actual return and expected return is presumed to be the abnormal return (AR) from the stock split.

$$AR_t = R_t - E(R_{mt}), \tag{30}$$

where AR_t is abnormal return of stock at time t.

Since the impact from event might occur before or after the event, the traditional event study also measures the impact from the event by cumulative abnormal return (CAR).

$$CAR_t = AR_t + CAR_{t-1}, \tag{31}$$

where CAR_t is cumulative abnormal return of stock at time t.

The t-test is normally used to test the significance of the existence of AR and CAR around the event date, with the null hypothesis that they are no significant abnormal return on the event date, the t-stat can be set up as equation 32. The standard deviation is estimated from data during estimation window. Note that this test assumes that the event does not induce variance.

$$\frac{AR_E}{\sqrt{\frac{1}{T-1}\left(AR_t - \frac{AR_t}{T}\right)^2}}, \qquad (32)$$

where AR_E is the abnormal return of the stock on event. T is number of days.

3.4.1 Results of Change in Return

Here we analyzed the change in price around the announcement and execution dates. We used two methods in our analysis. The first method is the traditional method which was detailed in section 3.1 and the second method is the event study approach which was detailed in section 3.4.

3.4.2 Abnormal Returns around the announcement date

Table 3.4.1 presents the abnormal returns around the stock split announcement of the log returns.

Table 3.4.1 Abnormal returns around the announcement of Microsoft

1. Event Period Abnormal Returns (AR)

Event	AR in %	t(BW)	t(BMP)
-5	-0.0067	0.0005	-0.2430
-4	-0.0035	0.0002	-0.1274
-3	0.0639	0.0045	2.3105
-2	-0.0119	0.0008	-0.4283
-1	0.0113	0.0008	0.4076
0	0.0630	0.0045	2.3056
1	0.0149	0.0011	0.5402
2	0.0019	0.0001	0.0690
3	-0.0162	0.0011	-0.5864
4	-0.0100	0.0007	-0.3608
5	0.0166	0.0012	0.6019

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	t(BW)	t(BMP)
Day -1 to day +1	0.08926	0.0056	0.1501
Day -2 to day +2	0.07932	0.0012	0.0321
Day -2 to day +3	0.06309	0.0010	-0.0273

In Table 3.4.1 above we noticed abnormal returns around the announcement date of the stock split. Almost similar results where detected in Table 3.4.2 below when we used the event study approach. We also noticed that all our t-values computed are significant at 1% level. From the hypothesis of whether stock splits are associated with abnormal returns, we accept our null hypothesis which states that stock splits are associated with abnormal returns. On the traditional approach we noticed that individuals who invested in Microsoft over the period -2 days to +2 days would have made a significant profit. However, using the event study approach we got contradicting results. The results obtained, suggest that the cumulative abnormal returns would have remained almost flat over the -2 day to +2 day period.

Table 3.4.2 Abnormal Returns Calculated from Traditional Event Study around the announcement of Microsoft

1. During Event Window (-20,19)

T	AR	t-stat
-20	-0.0025	-0.0012
-19	0.00065	0.0033
-18	-0.0911	-0.0458
-17	-0.0637	-0.0321
-16	0.0011	0.0006
-15	0.0151	0.0076
-14	0.0007	0.0003
-13	0.005	0.0025
-12	0.0068	0.0034
-11	0.0028	0.0014
-10	-0.0044	-0.0023
-9	0.0171	0.0086
-8	-0.1322	-0.0665
-7	-0.0194	-0.0097
-6	-0.0196	-0.0099
-5	0.0312	0.0157
-4	-0.0131	-0.0066
-3	0.0267	0.0134
-2	-0.002	-0.0001
-1	-0.01	-0.0051

T	AR	t-stat
0	0.023	0.0116
1	-0.0022	-0.00111
2	0.0068	0.0034
3	-0.0429	-0.0216
4	0.0017	0.0008
5	0.0182	0.0092
6	-0.0071	-0.0036
7	0.0345	0.0174
8	-0.0387	-0.0195
9	-0.0317	-0.0159
10	0.0013	0.0007
11	-0.0115	-0.0058
12	-0.0244	-0.0123
13	0.0733	0.0369
14	-0.0389	-0.00196
15	-0.0105	-0.0053
16	-0.0056	-0.0028
17	-0.0031	-0.0015
18	0.0647	0.0325
19	-0.0115	-0.0058

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	t-stat
Day -1 to day +1	0.0108	0.0108
Day -2 to day +2	0.0156	0.0156
Day -5 to day +5	0.0374	0.0017

3.4.3 Abnormal Returns around the Execution date

Table 3.4.3 presents the abnormal returns around the stock split execution of the log returns.

Table 3.4.3 Abnormal returns around the execution of Microsoft

1. Event Period Abnormal Returns (AR)

Event	AR in %	t(BW)	t(BMP)
-5	-0.0042	0.0003	-0.1523
-4	0.0093	0.0007	0.3345
-3	0.0399	0.0028	1.4310
-2	-0.0201	0.0014	-0.7199
-1	-0.0003	0.000	-0.0108
0	0.0145	0.0010	0.5196
1	-0.0331	0.0023	-1.1851
2	0.0635	0.0045	2.2769
3	-0.0042	0.0003	-0.1489
4	-0.0043	0.0003	-0.1549
5	-0.0092	0.0006	-0.3284

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	t(BW)	t(BMP)
Day -1 to day +1	-0.01886	0.0013	-0.0353
Day -2 to day +2	0.02456	0.0022	0.0579
Day -2 to day +3	0.02041	0.0042	0.1128

Just as the case for the announcement date, we also noticed similar results for the execution date. We noticed that there are abnormal returns associated with the execution of a stock split. Even though the market efficiency hypothesis states that the execution date will be priced already in the stock price, hence no reason for abnormal returns on this day. Our results, however, do suggest that the signalling hypothesis might play a role. We believe that since the execution date is well known in advance, some irrational investors play part in causing the abnormal returns. We think the irrational players will rush to the market and try to buy the shares at cheap price, but in doing so the demand of the share actually increases and hence the upward move in the price of that particular stock causing the abnormal returns on this day.

Table 3.4.4 Abnormal Returns Calculated from Traditional Event Study of execution of Microsoft 1.During Event Window (-20,19)

T	AR	t-stat
-20	0.0005	0.0031
-19	-0.0241	-0.0148
-18	0.0637	0.0391
-17	-0.0032	-0.002
-16	-0.0072	-0.0044
-15	-0.0072	-0.0044
-14	-0.0316	-0.0194
-13	-0.0046	-0.0028
-12	-0.0062	-0.0038
-11	-0.0086	-0.0052
-10	0.0077	0.0047
-9	-0.0213	-0.0131
-8	0.0052	0.0032
-7	-0.0071	-0.0043
-6	-0.0165	-0.0101
-5	-0.0176	-0.0108
-4	0.0231	0.0142
-3	-0.0038	-0.0023
-2	0.0072	0.0044
-1	0.0056	0.0034

T	AR	t-stat
0	-0.0027	-0.0017
1	-0.0571	-0.035
2	-0.0014	-0.0009
3	0.0091	0.0056
4	-0.0167	-0.0103
5	0.0101	0.0062
6	-0.0278	-0.0171
7	-0.0278	-0.0055
8	-0.009	0.0007
9	0.0011	-0.0014
10	-0.0023	-0.0301
11	-0.0491	0.0144
12	0.0234	-0.0148
13	-0.0242	0.0173
14	0.0282	0.0215
15	0.0351	-0.0762
16	-0.1243	0.0096
17	0.0156	0.0207
18	0.0338	0.0815
19	0.1333	0.0251

2. Cumulative Abnormal Returns (CAR)

Event Window	CAR	t-stats
Day -1 to day +1	-0.0542	-0.0111
Day -2 to day +2	-0.0484	-0.00596
Day -5 to day +5	-0.0112	-0.00247

The results from the traditional event study Table 3.4.4, show different results to those we achieved in Table 3.4.3. Here we see that as we are approaching the event date there some abnormal returns during the two days prior to the event; however, on the event date and the following two days we do not see abnormal returns. From the results this might suggest that, as we are approaching the event date, irrational investors might push the price as they will be hearing the news for the first time and might think they can make money. Although the cumulative results obtained actually show how the market would have already assimilated the news and would have priced in the news in the share price. Our conclusion for the negative cumulative return might be due to the investors who would have bought with the announcement news and would be cashing up their shares, anticipating that the irrational investor will continue to buy.

Chapter 4. HYPOTHESIS TESTING IN HIGHER VOLATILITY

4.1 Realized Volatility Measurement

A common model free indicator of volatility is the daily squared return. In this thesis we measure intraday volatility using intraday high frequency returns.

To set forth the notation, let $p_{i,t}$ denote the time n = 0 logarithmic price at day t. The discretely observed time series of continuously compounded returns with N observations per day is then defined by:

$$r_{i,t} = p_{i,t} - p_{i-1,t}, \tag{33}$$

where $i = 1,...,N$ and $t = 1,...,T$. If $N = 1$, for any series we ignore the first subscript n and thus r_t denotes the time series of the daily return.

We shall assume that:

$$E\left[r_{i,t}\right] = 0, \tag{34}$$

$$\operatorname{cov}\left[r_{i,t}r_{j,s}\right] = 0 \quad \forall i,j,s,t \quad but \quad i \neq j \quad and \quad s \neq t \tag{35}$$

$$\operatorname{cov}\left[r_{i,t}^2 r_{j,s}^2\right] < \infty \quad \forall i,j,s,t. \tag{36}$$

Hence, returns are assumed to have mean zero and to be uncorrelated and it is assumed that the variance and covariance of squared returns exist and are finite.

The continuous compounded daily squared returns maybe decomposed as:

$$r_t^2 = \left(\sum_{i=1}^{N} r_{i,t}\right)^2$$
$$= \sum_{i=1}^{N} r_{i,t}^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} r_{i,t}r_{j,t} \quad i \neq j$$
$$= \sum_{i=1}^{N} r_{i,t}^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} r_{i,t}r_{j-i,t}. \tag{37}$$

With equation 37 holding, the squared daily return is therefore the sum of the two components: the sample variance and twice the sum of N-1 sample auto-co variances. In this decomposition it is the sample variance that is of interest, the sample auto-co variances are measurement error and induce noise in the daily squared return measure.

From 37, 34 and 35 it therefore follows that an unbiased estimate of the daily returns volatility is the sum of intraday squared returns, the realized volatility:

$$s_t^2 = \sum_{i=1}^{N} r_{i,t}^2, \tag{38}$$

As

$$E\left[s_t^2\right] = \sigma_t^2, \tag{39}$$

Where σ_t^2 is daily market variance.

Because the realized volatility s_t^2 is an estimator, it has itself a variance which can be interpreted as measurement error. So assuming 34, 35 and 36 hold, then the variance of s_t^2 is given by:

$$V\left(s_t^2\right) = E\left[\left(\sum_{i=1}^{N} r_{i,t}^2 - \sigma_t^2\right)^2\right]$$
$$= E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\left(r_{i,t}^2 - \frac{\sigma_t^2}{N}\right)\left(r_{j,t}^2 - \frac{\sigma_t^2}{N}\right)\right] \tag{40}$$

Thus the variance of s_t^2 depends on the sum of all co-variances of the squared return process.

Upon separating the double sum for all $i \neq j$, taking expectations and rearranging terms it follows:

$$= E\left[\left(\sum_{i=1}^{N} r_{i,t}^2 - \frac{\sigma_t^2}{N}\right)^2\right] + 2E\left[\sum_{i=1}^{N}\sum_{j=i+1}^{N}\left(r_{i,t}^2 - \frac{\sigma_t^2}{N}\right)\left(r_{j,t}^2 - \frac{\sigma_t^2}{N}\right)\right]. \tag{41}$$

The first term is the variance of the intraday squared returns process and the second is the sum of all squared return auto-co variances. Upon dividing the term on the right by $\frac{1}{N}$ times the expression on the left and taking expectations one obtains:

$$E\left[\left(\sum_{i=1}^{N} r_{i,t}^2 - \frac{\sigma_t^2}{N}\right)^2\right]\left(1 + 2\sum_{i=1}^{N}\frac{N-i}{N}\rho_{N,i,t}\right), \tag{42}$$

where $\rho_{N,i,t}$ the n^{th} autocorrelation of $\left\{r_{i,t}^2\right\}$

Finally, after expanding the factor on the left and taking expectations, it follows:

$$V\left(s_t^2\right) = \frac{\sigma_t^4}{N}\left(K_{N,t} - 1\right)\left(1 + 2\sum_{i=1}^{N-1}\frac{N-i}{N}\rho_{N,i,t}\right), \tag{43}$$

where $K_{N,t}$ denotes the kurtosis of $\left\{r_{i,t}^2\right\}$.

Note that the kurtosis and autocorrelations have the subscripts N as these may change with the number of intraday returns. It follows that for any particular value of N measurement error increases with the daily population variance, with kurtosis of intraday returns and with autocorrelations of intraday squared returns.

4.2 Methodology of Change in Variance

We try to control microstructure variable measure in the volatility by estimating the bias due to price discreteness in the pre-split and post-split periods using bid to bid prices as follows:

$$\sigma_{B,D}^2 = \sigma_B^2 - \frac{d^2}{6(T_2 - T_1)} \sum_{t=T_1}^{t=T_2} \frac{1}{p_{B,t}^2}, \tag{44}$$

where B is the bid to bid price, D is the daily returns, $\sigma_{B,D}^2$ is the volatility corrected for price discreteness, σ_B^2 is the volatility estimated using bid to bid prices, $(T_2 - T_1)$ is the range of the estimation period, d is the constant equal to 0.125 and $p_{B,t}^2$ is the bid price at time t $(T_1 \le t \le T_2)$. For the examination of the change in variance we employ two different methods. The first method is according to Koski (1998) in estimating pre-split and post-split variance for each stock. A t-test is computed to test the hypothesis that paired difference have mean zero. The second test is a non parametric test initially proposed by Ohlson and Penman (1985).

The test statistic to test the null hypothesis of no variance increase after the split is:

$$z = 2(p - 0.5)\sqrt{m}, \tag{45}$$

where p is the proportion of positive squared return differences $R_2^2 - R_1^2$, where R_1 and R_2 denote pre-split and post-split returns and m is the number of observations.

Here we explain the non-parametric test proposed by Ohlson and Penman (1985) we used in section 4.3 and section 4.4 below. The binomial proportionality statistic, P, where $P = \Pr(\textit{post-split} > \textit{pre-split})$ is applied to test the hypothesis in section 4.3 and 4.4. *Pre-split* and *Post-split* are denoted by the daily stock return volatility. Here Ohlson and Penman (1985) approximates for daily return volatilities with expected squared daily returns $E^2[r]$, hence pre-split and post-split simplify to pre and post-split values of $E\left[r^2\right]$. Ohlson and Penman (1985) in their research controlled for the day of the week effects on the variables of interest in the pre- and post-split squared daily returns by matching the squared return for first trading day following the split announcement date with the squared return for the first same day of the week following the split date. The process was repeated until the day just prior to the split date during the time period between the split announcement and the split execution date. The number of comparisons for each split was equal to the number of trading days between the announcement and the execution dates.

Assuming independence across N observation, the binomial statistic z in equation 45 is distributed asymptotically as a standard normal. With this assumption in place the value of the binomial z-statistic is used for statistical significance. Since Ohlson and Penman (1985) assumed that returns in the announcement to execution period and in the period after the execution are independent and normally distributed with mean zero, follow an $F(1,1)$ distribution. This implies that:

$$\Pr\left\{R_2^2 > R_1^2\right\} = \Pr\left\{X = \frac{R_2^2}{R_1^2} > \frac{\sigma_1^2}{\sigma_2^2}\right\}. \tag{46}$$

$\Pr\left\{R_2^2 > R_1^2\right\}$ is calculated by comparing the observed squared daily returns with the matching technique described above. Then it is easy to verify that:

$$F_{1,1}^{-1}\left(1-\lambda\right) = \frac{\sigma_2^2}{\sigma_1^2}. \tag{47}$$

where $\lambda = \Pr\left\{R_2^2 > R_1^2\right\}$.

4.3 Changes in variance biased

Here, we report the daily volatility corrected for microstructure biased for the pre-split and post-split periods. Our estimates show that there is an increase in biased corrected volatility from the pre-split to the post-split period. The larger bid-ask spread may account for the increase in the volatility based on transaction prices. Roll (1984) showed that in an efficient market, if the probability of transaction price at the bid ask price is equally likely, then transaction prices for estimating the true volatility of the stock returns might induce spurious volatility equal to $\frac{s^2}{2}$ where s is the percentage bid-ask spread. This bias could be significant in the estimation of the volatility change around the stock split since the bid-ask spread would increase after the split. In this dissertation we avoid this bias due to the bid-ask bounce by estimating the volatility of the returns based on the bid-to-bid prices. Additionally, Gottlieb and Kalay (1985) and Ball and Torous (1988) conducted a research of the effect of price discreteness on the inflation in the volatility estimates. Ball, however. showed that if stock prices follow a Geometric Brownian

Motion with variance σ^2 and price p, then the bias induced by price discreteness is approximated by

$$\frac{d^2}{6p^2}, \tag{48}$$

where d is the minimum price change. We applied this correction to the volatility measure to obtain an unbiased estimate. The estimator, $\sigma^2_{B,D}$, was computed using equation 43. Here we test the hypothesis that the paired differences have a mean zero. We examine the mean variance of each security during the pre-split and post-split periods, using the bid-to-bid price. We compute the volatility of each security which is corrected for the price discreteness in equation 44. We proceed to compute the mean of the two periods that is the pre-split and the post-split. Our hypothesis shown below is to show that if volatility of the pre-split and post-split does not increase during the pre-split to post-split period then the mean of the post-split divided by the pre-split should be equal or less than 1, depending on whether volatility increases or decreases after the post-split period.

The hypothesis that we test is:

$H_0 : The \quad paired \quad difference \quad have \quad mean \quad zero$

$H_1 : The \quad paired \quad difference \quad have \quad mean \quad not \quad equal \quad to \quad zero$

Table 4.3.1: Changes in Bias Corrected Daily Volatility.

Variable	Volatility
$\sigma^2_{BD,1}$	0.2850
$\sigma^2_{BD,2}$	0.3246
$\dfrac{\sigma^2_{BD,2}}{\sigma^2_{BD,1}}$	1.1386

Subscript 1 = Pre-split and

Subscript 2 = Post-split

As can be seen from Table 4.3.1, even after we controlled for microstructure biases, there is still an increase in the volatility after the split. The post-split volatility to the pre-split volatility is 1.1386. Thus the volatility of the stock increases by 13.86% after the split. From our estimate in the changes of the bias corrected volatility we can reject our null hypothesis and accept the alternative hypothesis that the paired differences have a mean not equal to zero. If the null hypothesis was true, then the relative change in the biased corrected volatility would have been equal to one.

4.4 Changes in variance

We construct the volatility estimates as follows: let $p_{h,t}$ be the natural logarithm of the stock price for firm h on date t, where $h = 1,...,H$ and $t = 1,...,T$. The time series of continuously compounded returns per day is defined as

$$r_{h,t} = p_{h,t} - p_{h,t-1}. \tag{49}$$

Assuming that the returns are uncorrelated with finite variance, then the unbiased estimator of the population return variance σ^2 is

$$s_t^2 = \sum_{h=1}^{H} r_{h,t}^2. \tag{50}$$

We present in Table 4.4.1 our results concerning the change in variance around the pre-announcement and the post-execution of a stock split. We estimated volatility over days -211 to -11 relative to the pre-announcement and we then estimated the post-execution volatility over days 11 to 211. As in the corrected bias volatility we find that there is a significant increase in the mean post-split variance to the mean pre-split variance using the log returns methods. Variance estimates based on the trade-to-trade returns present signs of a slight decrease. Here we compute the variance using the log daily returns and the trade-to-trade returns and we would like to see if the mean variance increases from pre-split to post-split period. We expect that if there is no increase in variance then the mean pre-split variance should be equal or greater than the mean post-split variance.

The hypothesis that we test is:

$H_0 : \sigma_{post} \leq \sigma_{pre}$

$H_1 : \sigma_{post} > \sigma_{pre}$

Table 4.4.1: Change in Variance of Log Daily Returns and Trade-to-trade returns.

Method of return calculation	Mean Pre-split variance	Mean Post-split variance	z-statistic	$Pr\{\sigma_2^2 > \sigma_1^2\}$ in %	$Pr\{R_2^2 > R_1^2\}$ in %	F_{calc}
Daily	0.401E(-4)	0.413E(-4)	0.6171	48.32	43.5	0.971 (0.1)
Trade-to-trade	0.803E(-4)	0.904E(-4)	1	51.60	45.00	0.888 (0.1)

The log returns method value increase from 0.401E(-4) before split to 0.4137E(-4) after the execution of the stock split. The values of the change in variance are low due to thin trading securities, because often the last traded price of an illiquid share continues to be quoted throughout the period of non-trading leading to a row of zero returns, which cause variance estimates to be downward bias. In a nutshell, thin trading seems to affect only the level of variance estimates but not the direction of a change in variance. The F_{calc} statistic for the daily method and the trade to trade method is 0.971 and 0.888. Since $F_{calc} \prec F_{200,1}$ we reject the null hypothesis and conclude that there is an increase in variance from pre-split to post-split. The p-value is also not significant and the results of stock price volatility increase suggest that the stock split has some market microstructure related to effects on the stocks. In the next section we examine whether thin trading and volatility increase are relevant in explaining the positive market reaction to the announcement of the splits.

4.5 Change in variance from a particular case

In this section, we analyze a univariate case of the stock split. We pick one of the nine firms analyzed in the general case of the previous chapters. We shall use the traditional method and incorporate a new approach called the EVARCH to analyze our data. We shall also employ the

GARCH method to do our volatility estimates and see if we come up with similar results to the general case.

Cyree (1999) extended traditional event study by relaxing the strict assumptions of subjective fixed window and constant variance around event date. They even allowed the systematic risk in the market model to change around the event date.

The new method defined by equation 51 to estimate the returns:

$$R_t = \beta_0 + \beta_1 R_{mt} + \beta_2 R_{mt}(T_1 - t)(t - T_2) D_{1t} + \beta_3 R_{mt}\left[(t - T)_1 D_{1t} + (T_2 - T)_1 D_{2t}\right] + \varepsilon . \tag{51}$$

We first define T_1 and T_2 as the beginning and end of the event period, respectively. We also define two indicator variables, $D_{1t}=1$ if $T_1 < t < T_2$ and zero otherwise and $D_{2t}=1$ if $t > T_2$ and zero otherwise. Thus $D_{1t}=1$ during the event period and $D_{2t}=1$ after the event period. β_0 and β_1 are the intercept and systematic risk before the event period, β_2 and β_3 permit changes in systematic risk during and after the event period and $\varepsilon \sim N(0,\sigma^2)$ and $E(\varepsilon_i \varepsilon_j) = 0, \forall i \neq j$

They relaxed the assumption of constant variance by defining variance of error term in 51 to follow an ARCH(1) process.

$$Var(\varepsilon_t) = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 . \tag{52}$$

They named the model "Event-ARCH" or EVARCH, which can be estimated by the maximum likelihood estimate (MLE).

Note that the estimation of 51 & 52 needs the whole data during the estimation and event windows. Model 51 also enables the systematic risk to change around the event. If β_2 is negative, it implies that the systematic risk declines during the event. The adjustment can be either temporary or permanent depending on β_3. Also note that if $\beta_2 = \beta_3 = \alpha_1 = 0$, the equation 51 and 52 reduces to the standard market model.

4.5.1 Empirical results on changes in volatility

We first start with an analysis from a traditional method and we proceed to do an analysis from the EVARCH model given in equation 51 & 52. The traditional methodology we use here was detailed in Chapter 4 and the Event-ARCH model was detailed above in section 4.5 of this chapter. The hypothesis that we test is:

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

Table 4.5.1: Changes in Bias Corrected Daily Volatility of Microsoft

Variable	Volatility
$\sigma^2_{BD,1}$	0.2300
$\sigma^2_{BD,2}$	0.1750
$\frac{\sigma^2_{BD,2}}{\sigma^2_{BD,1}}$	0.7610

Subscript 1 = Pre-split and

Subscript 2 = Post-split

From Table 4.5.1. we see that even after controlling for microstructure biases, there still is an increase in the volatility after the split. The relative strength of the post-split volatility to the pre-split volatility is 0.7610. Thus the volatility of the stock increases by 7.61% after the split. From our estimate in the change of bias corrected arising from bid-ask spread cannot alone account for the increase in the volatility after a stock split.

We proceed to do an analysis of the changes in variance. We present our empirical results in Table 4.5.2. Our aim is to see if there is a change in the variance after the stock split or not.

The hypothesis that we test is:

$$H_0 : No \quad \text{var}iance \quad increase \quad after \quad stock \quad split$$
$$H_1 : V\text{ar}iance \quad increase \quad after \quad stock \quad split$$

Table 4.5.2: Change in Variance of Log Daily Returns Of Microsoft

Method of return calculation	Mean pre-split variance	Mean post-split variance	z-statistic	$Pr\{\sigma_2^2 > \sigma_1^2\}$ in %	$Pr\{R_2^2 > R_1^2\}$ in %	F_{calc}
Daily	0.0011	0.0013	0.9995	49.7	42.88	0.846 (0.1)

From Table 4.5.2, the daily log returns value increase after the execution of the stock split. We think that, due to thin trading, this might be the reason we obtained low values. Thin trading plays a major role in this because the last traded price of an illiquid share continues to be quoted throughout the period of non-trading leading to a row of zeros, which cause the variance estimate to be downward-biased. However, these results which show an increase in volatility, they are believed to suggest that the stock split has some market microstructure related to the effects on the stock.

After we had analyzed traditional method we moved on to do an analysis of the estimation of the EVARCH model. The EVARCH model was mainly used for comparison purpose.

Table 4.5.3 The Estimation Results Of EVARCH Model Pre-split Event Window

Beta	Value	Confidence Interval	t-stat
β_0	-0.3580		0.1356
β_1	0.0578		0.0782
β_2	-5.3437		0.6627
β_3	-0.0463		0.1838
α_0	-0.0087	(-0.0242;0.0403)	0.9398
α_1	0.0485	(0.0068;0.0628)	0.4272

Table 4.5.4 The Estimation Results Of EVARCH Model Post-split Event Window

Beta	Value	Confidence Interval	t-stat
β_0	0.0374		0.9334
β_1	0.0198		0.4068
β_2	-2.1284		0.9474
β_3	-0.2339		0.0991
α_0	0.0009	(-0.0088;0.0252)	0.0997
α_1	0.0303	(0.0106;0.0392)	0.6464

Systematic risk is the market risk and this risk cannot be diversified away as opposed to idiosyncratic risk, which is specific to individual stocks. Systematic risk may rise or fall around stock split. The model in 44 permits our betas to follow a continuous concave or convex function and exit the sample event period at different levels. From our analysis in Table 4.5.4 we noticed that we have positive intercept, i.e. β_0. However, the systematic risk which is measured by β_1 is seen as falling, meaning that an investor who invested during the period of this stock split would have made a positive return. We also noted that β_2 and β_3 which permit changes in systematic risk during and after the event period were both positive in both scenarios i.e. pre-split and post-split event window. β_2 being positive, this also shows that the systematic risk is concave and this suggests that our earlier hypothesis of risk actually falling. The non-zero β_3 coefficient implies a permanent change in systematic risk which might actually suggest that not only irrational investors push the prices up by creating artificial demand. Even rational investors will be attracted to the market when the risk of the market is falling, implying that the demand for this share would be unusual.

Chapter 5. DEAR-TREND ANALYSIS

In Chapters 2 to 4, we performed statistical analysis based on traditional statistical hypothesis testing in order to seek empirical evidences of the changes in a stock pre-split and post-split. However, it should be fully aware that the hypothesis testing performed here relies mostly on sample size and even on the normality assumptions. We noticed many mathematically simple technical analysis indices, for example. adv (accumulation distribution volume), adx (average distributional movement index), ama (average moving adaptive), atr (average true range), are using smaller sample size ranging from 10 to 14.

It is a well-known fact that during certain sensitive periods, a stock price may change dramatically due to many reasons, even a rumour that the company CEO is facing health problems or is about to resign. Therefore the trend and strength of a stock should be evaluated in both "short" and "intermediate" levels of time scale. Stock split is a decision of a company's management to show to the public that the company is in an excellent financial position and expect the confident investments will be followed after a split action. Hence the stock-split impacts analysis should be a short-term to intermediate-term analysis.

Traditional statistical methodologies are mostly large-sample-based. Technical analysis on the other hand, small sample based but often ignores the distributional assumption as well as the independence assumption. The former (i.e., statistical analysis) is rigorous in theoretical foundation but market circumstances often do not facilitate the standard analysis, say, the time period from split announcement to split execution is usually a month, 24 to 25 trading days. A standard time series analysis requires 50 or more data points. The later (i.e., the technical analysis) is aware of the market's quick-changing feature and thus creates many indices utilizing small data ranging from 10 to 14 for extracting "local" trend" information but these approaches are mostly rejected by statisticians because there is no mathematical rigor.

In this chapter, we will use the newly created DEAR (Differential Equation Associated Regression) theory to explore the trend in log-return and pattern in volatility. We are fully aware that regression analysis appears in the set of technical analysis indices but the usage is based on large-sample data size. Although regression models are used in both technical analysis and DEAR-pattern analysis the mathematical foundations are different.

Further, we explore the applicability of a DEAR model to certain technical analysis indices and see whether we can create DEAR-technical analysis.

5.1 An Introduction to DEAR Modelling

DEAR is an abbreviation of Differential Equation Associated Regression. DEAR theory merges differential equation theory, regression theory, and fuzzy credibility measure theory into a new small-sample-based modelling and analysis. DEAR theory and modelling are still under development. For stock-split analysis based on daily records, we introduce one simple DEAR model.

Verbally, we can state that a pair of differential equations with a closed form of solution and a regression model sharing the same parameters with the differential equation is called a DEAR model.

For the stock-split analysis, the first-order linear constant coefficient differential equation of the form

$$\frac{df}{dt} + \beta f = \alpha_0 + \alpha_1 t, \tag{53}$$

will have a particular value because this equation has a general solution with closed form:

$$f(t) = ce^{-\beta t} + \frac{\alpha_0 \beta - \alpha_1}{\beta^2} + \frac{\alpha_1}{\beta} t, \tag{54}$$

which is a sum of a linear function of time t and an exponential function $ce^{-\beta t}$ (which is nonlinear).

The motivation for using DEAR models lies in the fact that the associated regression modelling can facilitate the estimators $\left(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta}\right)$ and therefore the estimated nonlinear function

$$\hat{f}(t) = c_0 e^{-\hat{\beta}t} + \frac{\hat{\alpha}_0\hat{\beta} - \hat{\alpha}_1}{\hat{\beta}^2} + \frac{\hat{\alpha}_1}{\hat{\beta}}t, \tag{55}$$

is expected to be a better (nonlinear) predictor for future values of $f(t)$ or related quantities.

It is necessary to point out that the error term ε in DEAR models is no longer simply random error, and typically is assumed zero mean and constant variance. The error in DEAR models comes from random sampling and approximations, which can be treated as fuzzy numbers. Therefore, the regression in DEAR theory is in nature a random fuzzy regression, although we still use least-square approach to treat the associated regression in Equation (54). For more theoretical details, see paper Guo and Guo.

5.2 DEAR-trend analysis on log-return

We analyzed the log trend which should divulge the movements or patterns of the market reaction with regard to the stock split announcement and execution periods. My supervisor proposed a scheme comprising four periods (i.e. Pre-Announcement, post-announcement, pre- execution and post-execution) on which each DEAR curve is constructed, based on 8-day daily log-returns.

We discuss the mathematical reasoning behind the DEAR trend analysis. Let the daily (close) price be P_t, then the log-price, $\ln(P_t)$ can be easily calculated. The first-order difference can be calculated as (by notice the time changes by unit)

$$lr_t = \text{log-return}(t) = \ln(P_t) - \ln(P_{t-1}). \tag{56}$$

Further, take the first-order log-return,

$$y_t = lr_t - lr_{t-1} \tag{57}$$

Let us perform regression model

$$y_t = \alpha_0 + \alpha_1 t + \beta(-lr_t) + \varepsilon_t \tag{58}$$

Then we perform the associated regression modelling for the four periods: Pre-Announcement, post-announcement, pre-execution and post-execution, the following Table collects the fitted coefficients and R-Square of the regression models.

Table 5.2.1: Estimated Parameters for ANSS Index Log-returns. The p-values are given in parentheses.

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.2309	-0.0191	1.8300	1.4500
α_0	-0.0130 (0.0164)	0.0073 (0.0163)	-3.6404 (1.6612)	-1.6809 (0.6751)
α_1	0.0002 (0.0036)	-0.0028 (0.0035)	0.2770 (0.2177)	0.0156 (0.0705)
β	-1.1264 (0.5293)	-1.0364 (0.4726)	-1.3761 (0.5197)	-1.2258 (0.4785)
R^2	0.5368	0.6774	0.6572	0.6322

Note the DEAR modelling on log-returns leads to the approximate function with respect to log-return, i.e.,

$$lr(t) = c_0 e^{-\hat{\beta}t} + \frac{\hat{\alpha}_0\hat{\beta} - \hat{\alpha}_1}{\hat{\beta}^2} + \frac{\hat{\alpha}_1}{\hat{\beta}}t. \qquad (59)$$

Table 5.2.2: Trend Functions for ANSS Index Log-returns

Time-period	Estimated Trend Function (8-days)
Pre-Ann	$lr(t) = 0.0230 \times e^{-1.1264t} + 0.0113 - 0.0001t$
After-Ann	$lr(t) = -0.0191 \times e^{-1.0364t} - 0.0044 + 0.0027t$
Pre-Exec	$lr(t) = 1.83e^{-1.37615t} + 2.4991 - 0.20134t$
After-Exec	$lr(t) = 1.45e^{-1.22586t} + 1.360787 - 0.01276t$

One might not be able to notice the trend in the equations provided above. We present the graphs below to show a picture of the trend and how they differ in each time period.

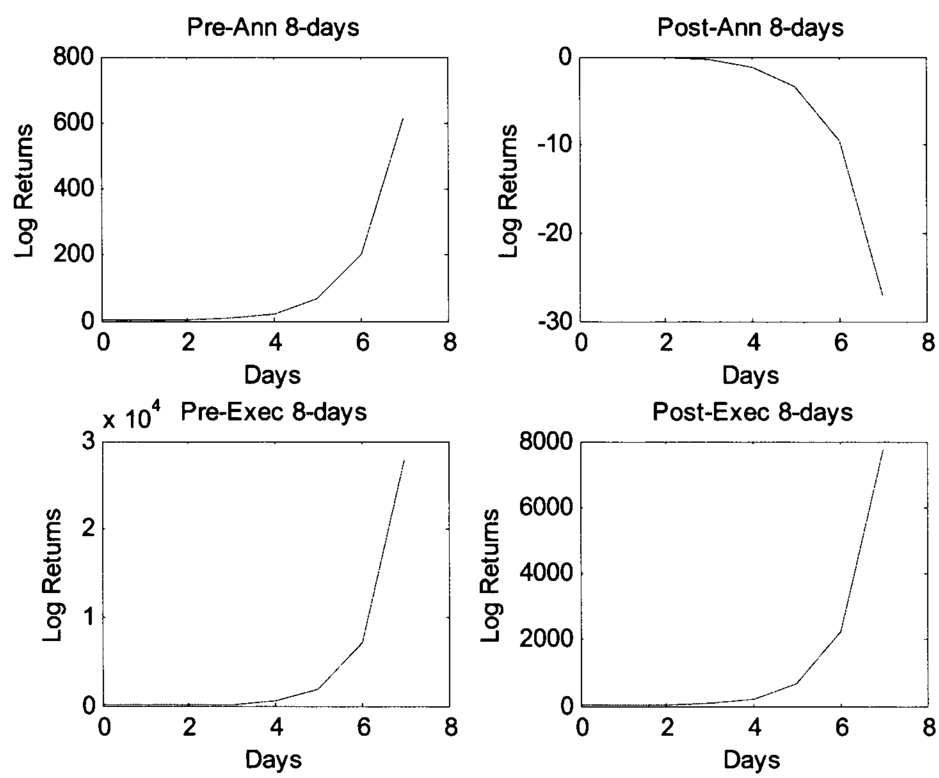

Figure 5.2.1: Trend Curves (plots) for ANSS Index Log-returns

From the graphs presented above we can notice that short-term log-return trend analysis shows no empirical evidences of abnormal log-returns in ANSS index. As shown in the graphs we see that the pre- and post-announcement graphs have different shapes, although the execution graphs show a similar trend. The pre-announcement model had a low R-squared slightly above 0.5, which makes one wonder whether the model is giving accurate forecasts.

We performed the analysis for the other indices (see Appendix B), but did not find empirical evidence of abnormal log returns. Although we could not detect empirical evidence of stock splits in the short term, we believe this could be due to the discreteness of the data used (daily data). However, if one could get the intra-day data and repeat the above procedure, then different results might be obtained.

5.3 DEAR-trend analysis on daily price range

The major measurement of stock volatility is the variance or standard deviation. However, if one is given the intra-day trading record, the daily volatility of the stock may be easily calculated because the intra-day trading shows the movement of the stock over the period of the day – challenging in case no intraday trading records were available. The only choice we are left with is to use the daily high price and low price to calculate the daily range of price, more like using the bid-ask spread of the day. According to the order statistic theory, the range of a sample from a random variable links to the standard deviation in some way. Therefore, we use the daily high price and low price to calculate the daily range of price and then investigate the trend in daily price trend. This approach maintains the local feature of the range of daily price.

The range of daily price is calculated by

$$r_t = high_t - low_t . \tag{60}$$

Then, we calculate the first-orders difference as

$$z_t = r_t - r_{t-1} . \tag{61}$$

The associated regression takes the form

$$z_t = \alpha_0 + \alpha_1 t + \beta(-r_t) + \varepsilon_t. \tag{62}$$

The regression modelling is performed within Excel for the short-term daily price range in BAM index for the four periods: Pre-Announcement, post-announcement, pre-execution and post-execution, the following Table collects the fitted coefficients and R-Square of the regression models.

We therefore obtain the approximate function for daily range

$$\hat{r}(t) = c_0 e^{-\hat{\beta}t} + \frac{\hat{\alpha}_0\hat{\beta} - \hat{\alpha}_1}{\hat{\beta}^2} + \frac{\hat{\alpha}_1}{\hat{\beta}} t. \tag{63}$$

The initial values and coefficients for the four periods: Pre-Announcement, post-announcement, pre-execution and post-execution, the following table collects the fitted coefficients and R-Square of the regression models are collected in the following Table.

Table 5.3.1: Estimated Parameters for ANSS Index Log-returns. The p-values are given in parentheses.

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.001046	0.014363	0.99	1.06
α_0	0.020168	0.009776	-1.30599	-0.67704
	(0.009979)	(0.015249)	(0.489191)	(0.492012)
α_1	-0.00431	-0.00152	0.035357	0.060434
	(0.002197)	(0.00336)	(0.04873)	(0.03218)
β	-1.62387	-1.72648	-1.08966	-0.59749
	(0.448864)	(0.427693)	(0.386118)	(0.658422)
R^2	0.767031	0.812838	0.679761	0.469724

It is noticeable that R^2 decreases after stock ANSS split announcement and the R^2-value after-execution reduces almost half of that of the Pre-Announcement. This indicates the trend pattern is diminished (for the specific model assumed, in other words, the daily price range shows no strong trend.

Table 5.3.2: Trend Functions for ANSS Index Log-returns

Time-period	Estimated Trend Function (8-days)
Pre-Ann	$lr(t) = 0.001046 \times e^{-1.62387t} - 0.01079 + 0.002655t$
After-Ann	$lr(t) = 0.014363 \times e^{-1.72648t} - 0.00515 + 0.00088t$
Pre-Exec	$lr(t) = 0.99 \times e^{-1.08966t} + 1.168748 - 0.03245t$
After-Exec	$lr(t) = 1.06 \times e^{-0.59749t} + 0.963853 - 0.10115t$

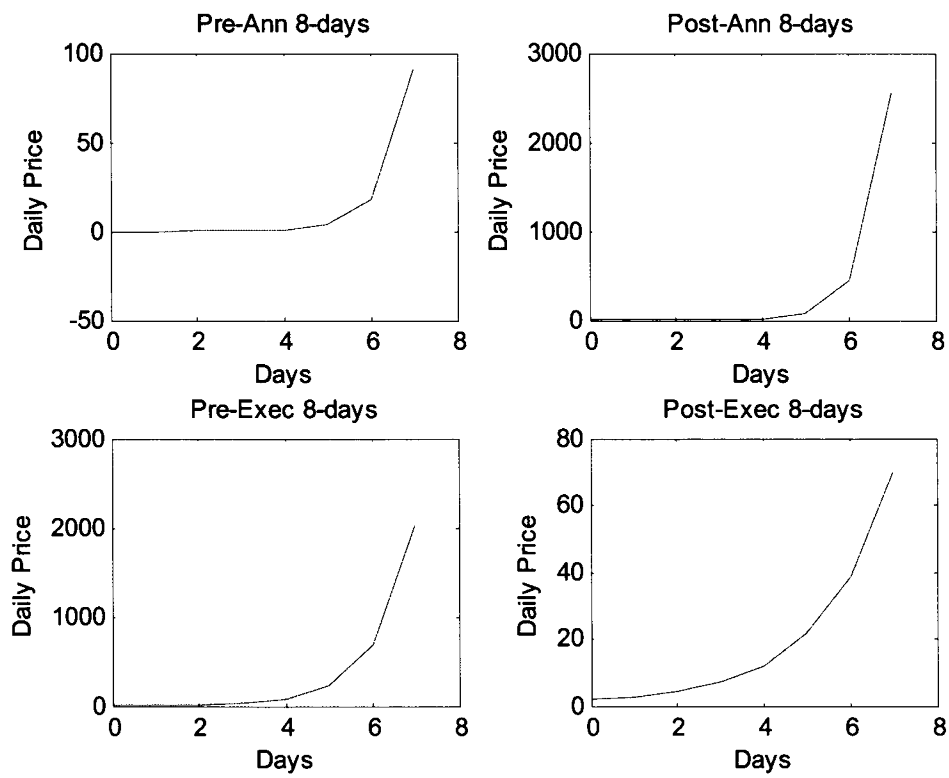

Figure 5.3.1: Trend Curves (plots) for ANSS index daily price

From the plots, it is evident that basic trend pattern is maintained as similar to each other for all four periods. However, considering the associated regression model R^2-values as well as the coefficients statistically significant, it is logical to say the stock split does vary the volatility of the stock, although the pattern change is still very vague.

We present the results of the remaining indices in Appendix B and we noticed that although there is no evidence of abnormal returns in the pre- and the post-stock split. However, we noticed that there is a change in volatility and the change in volatility cannot be conclusive whether it is increasing or decreasing as some of the graphs show different patterns.

5.4 Summary

After analysing each of the stock on its own we noticed some interesting results. We noticed the r-squared of some indices halving, doubling or remaining constant during a period. This was an indication of a diminishing trend or an increasing trend. We also notice significant coefficients, including consistent trend curve shapes and some different curve shapes. Also it is logical to consider the associated regression model r-squared values as well as the statistically significance. We noticed in the trend curves and the r-squared values that it would be logical to say stock split have abnormal returns associated with them and that also stock split does vary the volatility of the stock, although the pattern changes are still vague. Also since pattern changes are not showing significant changes in the announcement period to the execution period, we can say that this shows that no information leakages or insider trading does exist and that the NASDAQ is an efficient market. Although the DEAR approach we used here is still young, it gave us a meaningful tool to technical analysis. From the DEAR trend analysis on log return we can conclude that there are abnormal returns following a stock split and that also volatility does vary as shown in the DEAR trend analysis on daily price range. This is in support to the results we saw in section 4.3 and 4.4 that volatility does vary after a stock split.

Chapter 6. CONCLUSION

6.1 Discussion

This dissertation has examined the stock split puzzle. Our results show that although stock splits seem to be a purely cosmetic event, ample empirical evidences exist from the NASDAQ that stock splits are associated with abnormal returns on both the announcement and execution day, additionally with an increase in variance after the ex-day. Our results indicate that market reaction implies that managers and investors perceive the stock split as good news event regarding their company. Also our results show that the increase in volatility cannot be attributed solely to microstructure biases arising from the bid-ask bounce and price discreteness. Even after correcting for these biases, we find a significant increase in the volatility after the split. Finally our results suggest that our analysis of the impact of stock on traditional measures of liquidity (like volatility and spread) must first examine why different firms seem to be more or less successful in attracting additional trades to their security. The subsequent consequences for liquidity then seem to be consistent with existing theories on the way in which a change in trading activity affects liquidity. We also find that all our results are consistent with the liquidity hypothesis, which states that the stock split takes place in order to stabilize the price in a more attractive trading range. This optimal trading range is the result of the dispute between small and wealthy investors. In other words, small investors want a lower share price and wealthy investors want more shares in order to minimize the odd-lot brokerage costs.

6.2 Critical Assessment

The advantage of the returns methods used in this dissertation is that both return a method eliminates the potential influence of infrequent share trading on the detection of abnormal returns. We also calculate the bias corrected volatility so that we avoid any contamination due to information effects around the announcement day and the transient microstructure effects around the ex-split date. The effective spread measure used in this dissertation circumvents two weakness of the quoted spread. It is based on the notion that the trade is only costly to the investor to the extent that the trade price deviates from the true price. If all trades take place at the prevailing bid and ask quotes, the effective spread is equal to the quoted spread.

The dissertation was also coupled with a few setbacks. Firstly, we had to request data from the J.S.E SENS department for several weeks and the J.S.E could not provide all the announcement dates as the companies which had provided announcement dates could not be found on the software because the ticker symbol had changed or the company name had change due to a merger or acquisition. This effectively left us with only nine firms, which we believe was too few and that our results would not reflect stock split properly on the J.S.E. Also, only after extracting the JSE data did we realize that the splits were not shown on the stock price, and when we contacted the J.S.E we could not get a proper explanation for this data. This effectively left us with no option but to extract data on the American market, which was used in this thesis. Also, no access to some of the current research papers was available to the writer as the journals were unavailable, hence the use of old research papers and methodologies.

6.3 Future Developments

Further work on stock split would include the stock sector index also in the calculation of abnormal returns, because on each date there may well be news about the particular industry that would affect the returns. To isolate the impact of the announcement we should also eliminate that effect. If it is the return at the date t on the index of the industry stocks then the abnormal returns on the date would be changed to:

$$AR_{i,t} = R_{i,t} - \alpha_i - \beta_i R_{m,t} - \beta_j R_{I,t} \tag{64}$$

Under the semi-strong hypothesis of the market efficiency, conditional on the information at any date t, we should have:

$$E\left[\varepsilon_t \middle| R_{mt}, R_{It}\right] = E\left[\left(R_t - \beta_0 - \beta_m R_{mt} - \beta_I R_{It}\right)\middle| R_{mt}, R_{It}\right] = 0 \tag{65}$$

We could test this statistically since, assuming efficiency, the cumulative residual is a sum of uncorrelated random variables. Also one can examine irrational investor response to stock split and the impact of irrational traders, as they cause a lot of noise in the market. Furthermore, investigation of long-run performance can be examined to see if the stock split puzzle can be unbundled further as to what the market tells us of the ex-ante. Lastly, one can measure abnormal performance under conditions of induced variance. In Chapter 5 we saw that using the log return and daily price showed that the volatility of a security would increase for most of the models. We also noted that the use of the DEAR approach model to small sample-based technical analysis was useful for models with r-squared greater than 0.7 and for models with r-squared less than 0.7, we believe the use or exploration of other regression models might be necessary. Future work should also pay special attention to the DEAR model, even though it is still young in the field of mathematics at the time of writing this thesis.

6.4 Summary

After the stock split, the number of shares will increase, while the total capital will remain unaffected, but the price of the stock split will decrease according to the split factor. At this lower price the number of small investors will probably increase, since now more can afford to buy the specific stock, driving the stock's liquidity (marketability) upwards. The theories around the stock split depend on the conditions and the strategic objective of each company. Each stock split does give different signals from the managers to the investors. Hence, the hypotheses of signalling, liquidity, neglected firm or even optimal tick size have their implementation under different conditions. Irrespective of the firm's conditions and purposes there is a positive market reaction to the announcement and execution of a stock split.

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Appendix A: The Dates

Company Name	Announcement Date	Execution Date
1. Brookfield	02-May-07	04-Jun-07
2. ANSYS	14-May-07	05-Jun-07
3. CIGNA	25-Apr-07	05-Jun-07
4. Timberland Bancorp	25-Apr-07	06-Jun-07
5. EPIQ Systems	10-May-07	08-Jun-07
6. Peerless Manufacturing	04-May-07	08-Jun-07
7. N. American Galvanizing	15-May-07	11-Jun-07
8. VSE Corp	01-May-07	12-Jun-07
9. Crocs	03-May-07	15-Jun-07
10. WMS Industrials	07-May-07	15-Jun-07
11. Advanta Corp	03-Apr-07	18-Jun-07
12. Benihana	21-May-07	18-Jun-07
13. Buffalo Wild Wings	18-May-07	18-Jun-07
14. Middleby	04-May-07	18-Jun-07
15. Peoples Namk	20-Apr-07	18-Jun-07
16. Califonia Pizza	23-May-07	19-Jun-07
17. Marathon Oil	25-Apr-07	19-Jun-07
18. Questar	14-May-07	19-Jun-07
19. Penn Virginia	08-May-07	20-Jun-07
20. SEI Investments	23-May-07	22-Jun-07
21. Allegan	02-May-07	25-Jun-07
22. Express Scripts	23-May-07	25-Jun-07
23. Gilead Sciences	08-May-07	25-Jun-07
24. Omnicom	23-May-07	26-Jun-07
25. Yum Brands	17-May-07	27-Jun-07
26. Chase Corp	30-May-07	28-Jun-07
27. Spartan Motors	04-Jun-07	29-Jun-07
28. Flowers Foods	01-Jun-07	02-Jul-07
29. Petrobras Brasilerio	11-May-07	02-Jul-07

Appendix B: Chapter 5 results: Tables and Figures

Table 5.2.3: Estimated Parameters for TSBK Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0084	0.0275	0.3500	0.92
α_0	-0.0034 (0.0105)	-0.0210 (0.0068)	-0.7084 (0.4064)	-0.6710 (0.2698)
α_1	0.0012 (0.0023)	0.0039 (0.0013)	0.0512 (0.0654)	0.0633 (0.0604)
β	-1.5568 (0.4326)	-0.6705 (0.4631)	-1.2904 (0.5048)	-1.3108 (0.7433)
R^2	0.7651	0.6808	0.6341	0.6302

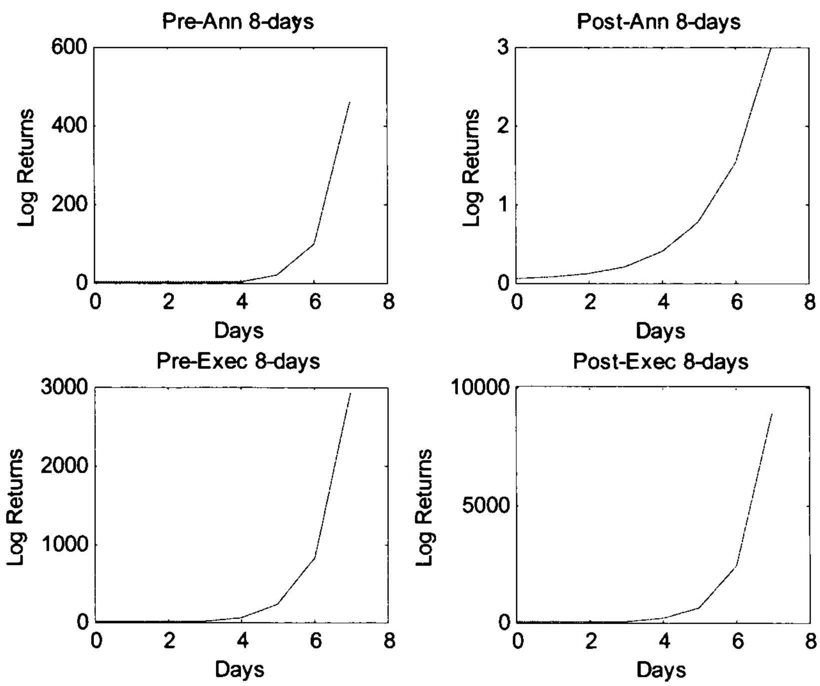

Figure 5.2.2: Trend Curves (plot for TSBK Index Log-returns

Table 5.2.4: Estimated Parameters for CPKI Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0715	0.0079	2.3400	0.8600
α_0	-0.0101 (0.0440)	-0.0015 (0.0055)	-1.9683 (0.9430)	-1.0617 (0.3026)
α_1	0.0003 (0.0100)	-0.0003 (0.0012)	0.1520 (0.1277)	0.0731 (0.0262)
β	-1.2377 (1.0931)	-0.9183 (0.4749)	-1.2248 (0.7350)	-1.5057 (0.5138)
R^2	0.3573	0.4840	0.4768	0.7528

Figure 5.2.3: Trend Curves (plots) for CPKI Index Log-returns

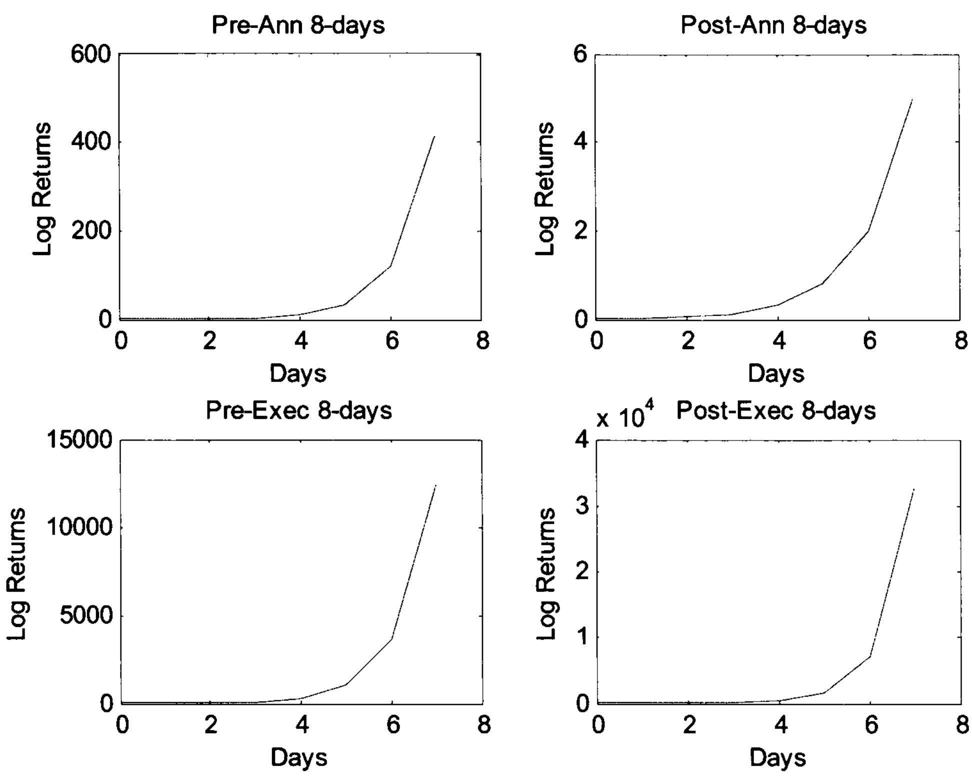

Table 5.2.5: Estimated Parameters for SEIC Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0078	0.00570	0.8800	1.3100
α_0	-0.0001 (0.0075)	-0.0051 (0.0090)	-0.9301 (0.3730)	-1.3313 (0.7345)
α_1	0.0004 (0.0015)	-0.0001 (0.0019)	0.0235 (0.0302)	0.1024 (0.1056)
β	-1.0442 (0.7171)	-0.8389 (0.4035)	-1.0251 (0.3697)	-0.9626 (0.4965)
R^2	0.4319	0.5950	0.7094	0.4893

Figure 5.2.4: Trend Curves (plots) for SEIC Index Log-returns

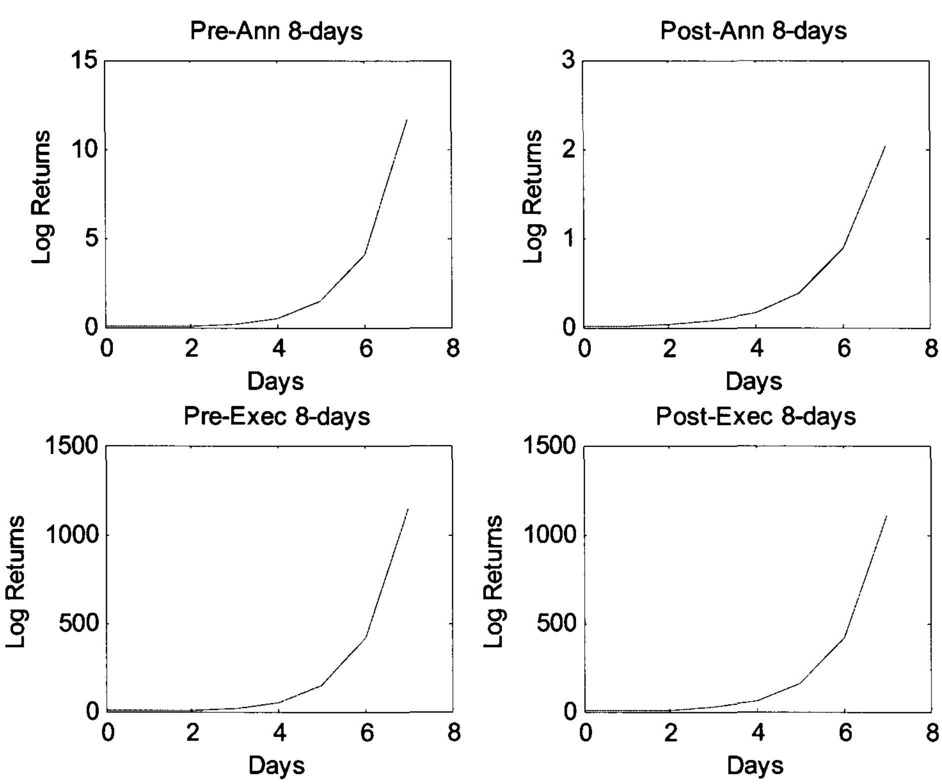

Table 5.2.6: Estimated Parameters for PMFG Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0012	0.0537	0.0400	1.0700
α_0	-0.0033 (0.0239)	-0.0254 (0.0245)	0.1407 (1.0382)	-0.9849 (0.6863)
α_1	-0.0044 (0.0049)	0.0016 (0.0058)	-0.3828 (0.2238)	-0.0659 (0.1245)
β	-1.1321 (0.4032)	-1.3804 (0.6618)	-1.0661 (0.4197)	-1.2077 (0.4721)
R^2	0.7034	0.5776	0.6588	0.6220

Figure 5.2.5: Trend Curves (plots) for PMFG Index Log-returns

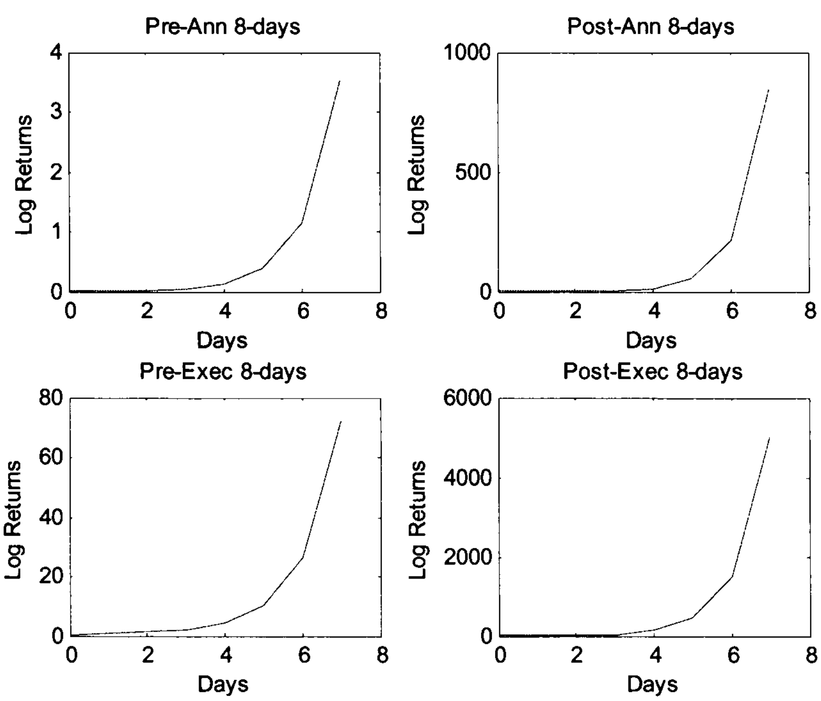

Table 5.2.7: Estimated Parameters for EPIQ Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0059	0.0418	0.7300	0.8900
α_0	0.0096 (0.0246)	0.0042 (0.0242)	-1.0914 (0.2266)	-0.7071 (0.3173)
α_1	-0.0017 (0.0055)	-0.0044 (0.0058)	-0.0398 (0.0349)	-0.0739 (0.0558)
β	-1.3280 (0.4854)	-1.6268 (0.6359)	-1.7786 (0.3076)	-1.3593 (0.4138)
R^2	0.6595	0.6375	0.8954	0.7306

Figure 5.2.6: Trend Curves (plots) for EQIP Index Log-returns

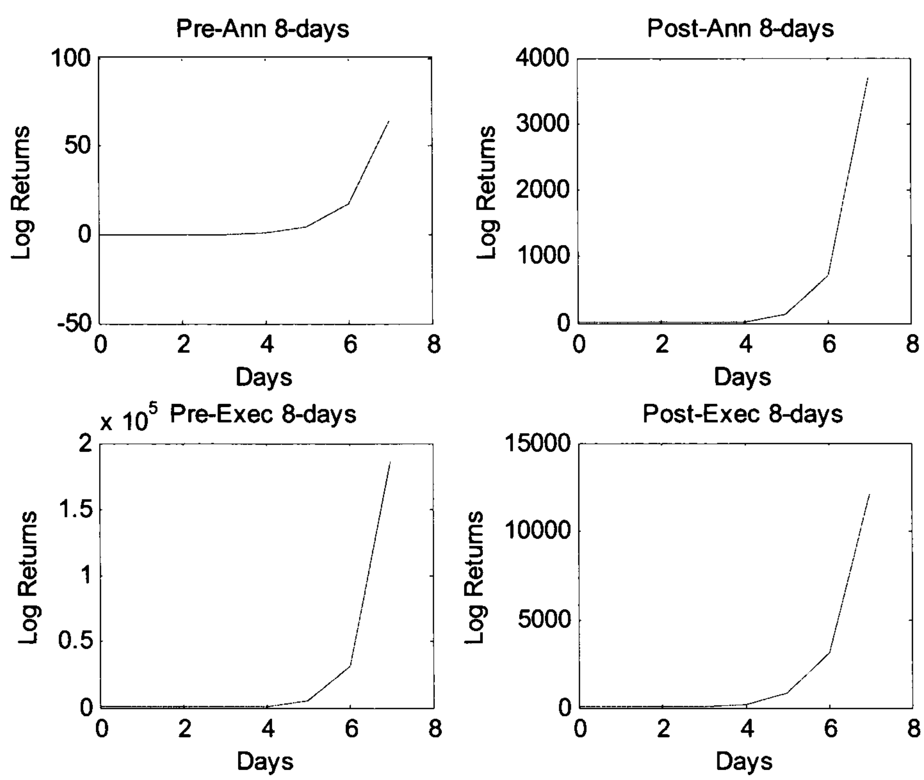

Table 5.2.8: Estimated Parameters for NGA Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.1197	0.0778	1.9000	1.7100
α_0	-0.1161 (0.0670)	-0.0852 (0.0522)	-1.7405 (0.6307)	-1.1527 (0.8030)
α_1	0.0148 (0.0140)	0.0205 (0.0120)	-0.1512 (0.0985)	-0.2258 (0.1864)
β	-1.2639 (0.4859)	-1.4776 (0.4521)	-1.5079 (0.4814)	-1.4139 (0.4895)
R^2	0.6503	0.7336	0.7110	0.6771

Figure 5.2.7: Trend Curves (plots) for NGA Index Log-returns

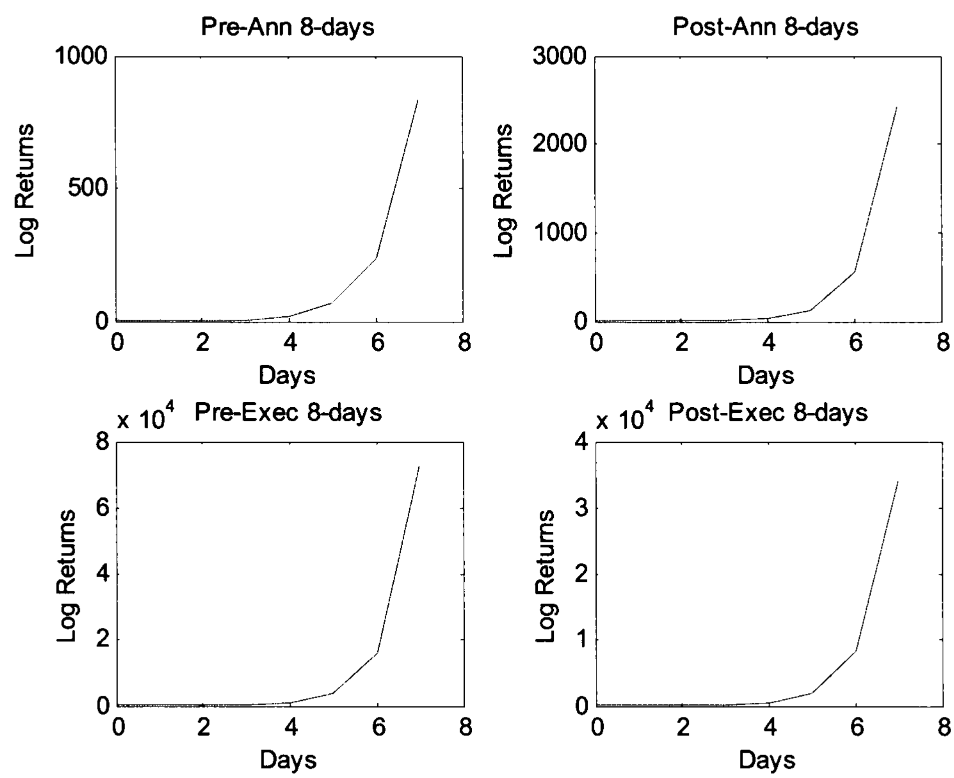

Table 5.2. 9: Estimated Parameters for VSEC Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0019	0.0688	0.3800	4.2800
α_0	0.0230 (0.0201)	-0.0540 (0.0302)	2.2479 (2.6516)	-2.6083 (1.3132)
α_1	-0.0101 (0.0047)	0.0124 (0.0067)	-1.0552 (0.7282)	0.1413 (0.4126)
β	-1.5309 (0.3436)	-0.5790 (0.4103)	-1.2312 (0.5001)	-0.9810 (0.6036)
R^2	0.8353	0.5948	0.6033	0.6391

Figure 5.2.8: Trend Curves (plots) for VSEC Index Log-returns

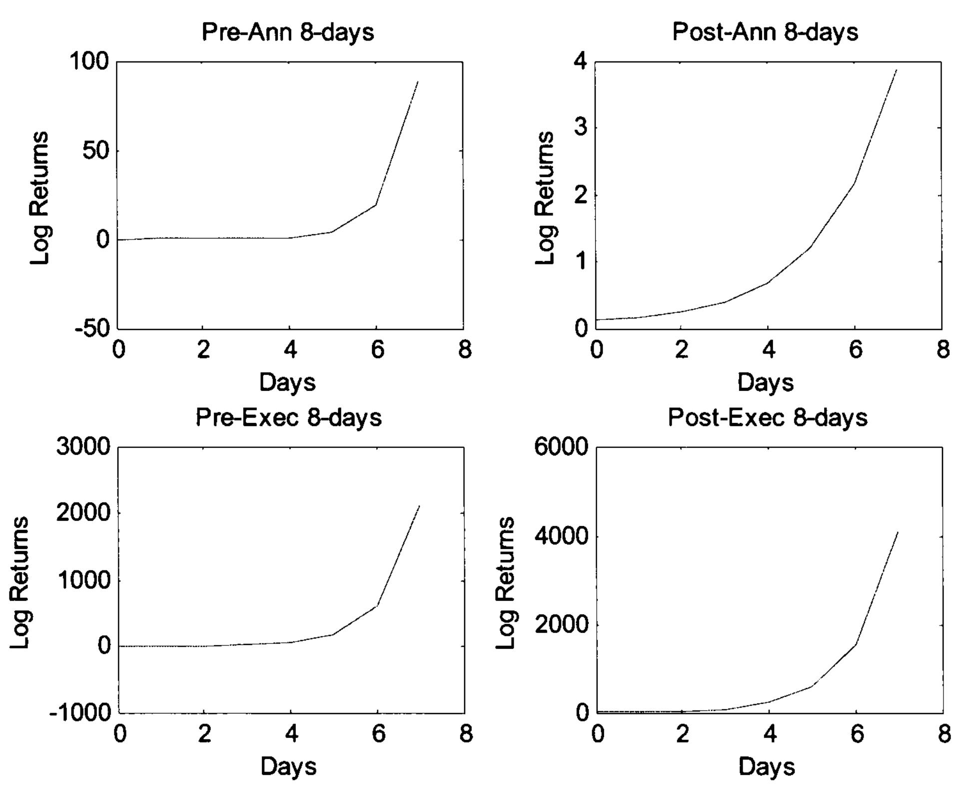

Table 5.2.10: Estimated Parameters for CROX Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0033	0.1817	1.7200	4.6500
α_0	0.0083 (0.0138)	-0.1090 (0.0579)	-1.2217 (0.8071)	-4.4269 (1.6146)
α_1	-0.0042 (0.0034)	0.0187 (0.0131)	-0.1324 (0.1631)	0.3674 (0.2076)
β	-1.5642 (0.3557)	-1.4146 (1.1195)	-1.0111 (0.5389)	-1.1788 (0.5538)
R^2	0.8386	0.5174	0.4824	0.6444

Figure 5.2.9: Trend Curves (plots) for CROX Index Log-returns

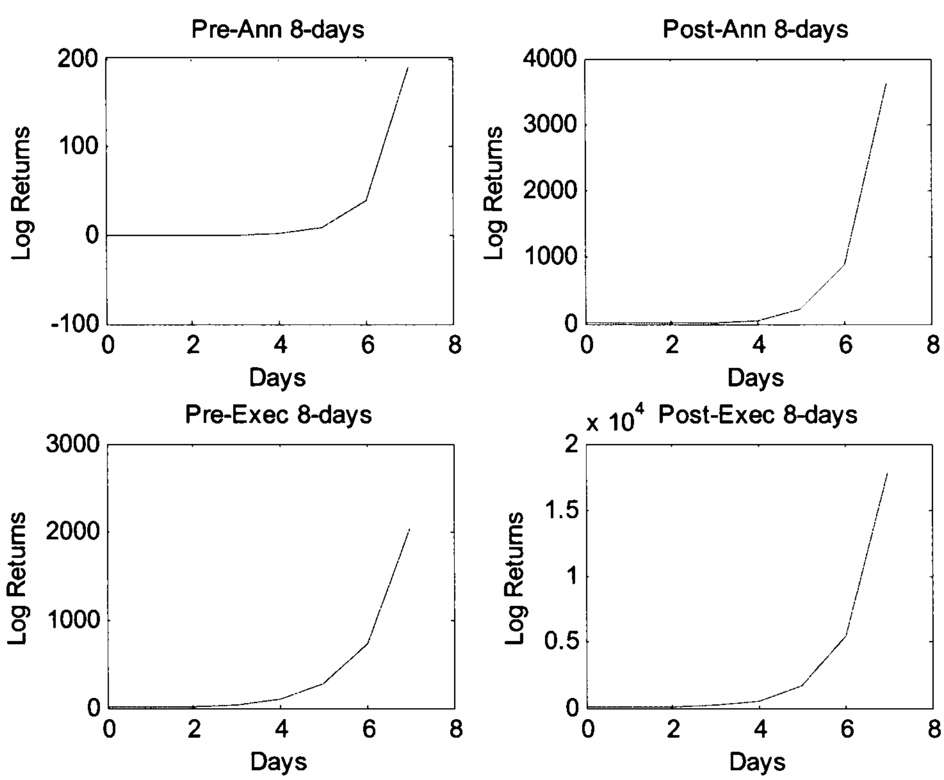

Table 5.2.11: Estimated Parameters for PEBK Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0014	0.0069	0.0200	0.3200
α_0	0.0095 (0.0104)	-0.0038 (0.0055)	0.0470 (0.3962)	-0.4139 (0.3284)
α_1	-0.0030 (0.0024)	0.0008 (0.0012)	-0.0484 (0.0725)	-0.0423 (0.0607)
β	-1.2728 (0.4636)	-1.1728 (0.5093)	-0.4558 (0.4635)	-1.2760 (0.4757)
R^2	0.6545	0.6167	0.3648	0.6455

Figure 5.2.10: Trend Curves (plots) for PEBK Index Log-returns

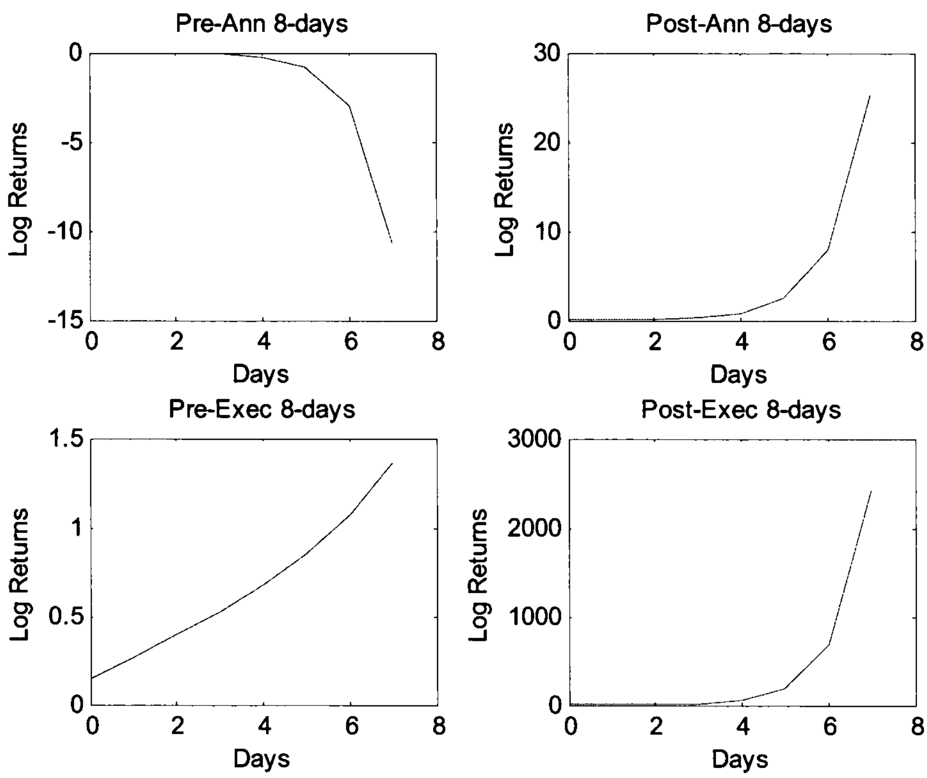

Table 5.2.12: Estimated Parameters for ESRX Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0059	0.0096	1.6900	2.1300
α_0	-0.1016 (0.0160)	-0.0078 (0.0086)	-3.2588 (0.8483)	-2.7566 (0.7315)
α_1	0.0008 (0.0030)	0.0007 (0.0018)	0.1970 (0.0842)	0.0839 (0.1015)
β	-1.2000 (0.7382)	-1.0496 (0.4765)	-1.6175 (0.3975)	-1.6288 (0.4140)
R^2	0.5172	0.5514	0.8053	0.8061

Figure 5.2.11: Trend Curves (plots) for ESRX Index Log-returns

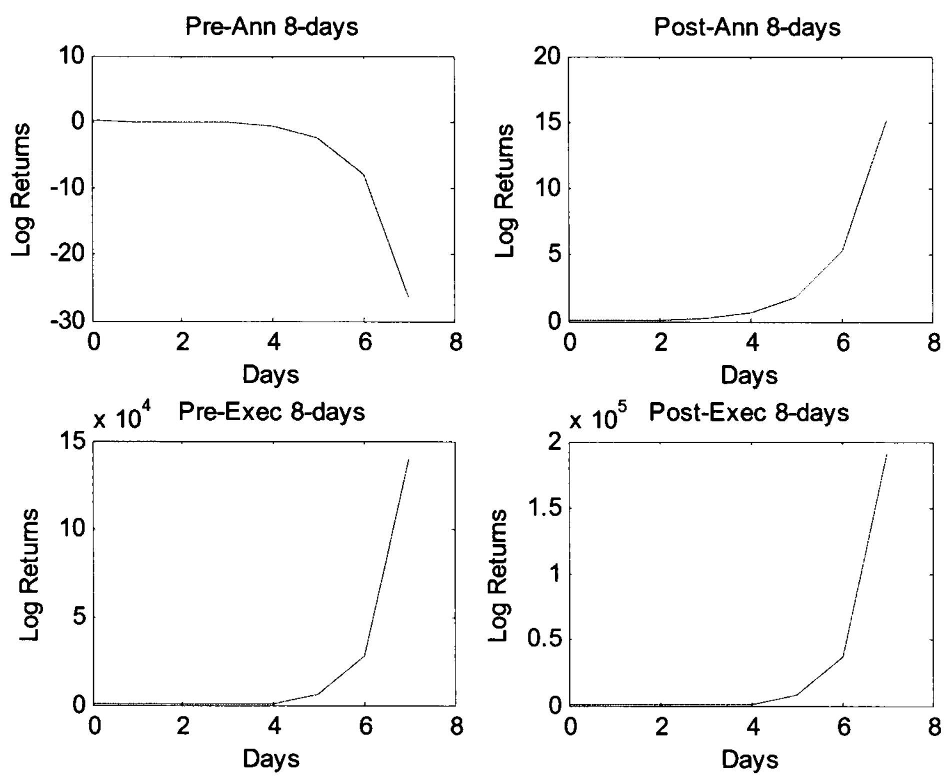

Table 5.2.13: Estimated Parameters for BNHN Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.0090	-0.0187	0.0000	2.0300
α_0	0.0031 (0.0094)	0.02401 (0.0058)	0.0518 (0.2375)	-1.6559 (0.7243)
α_1	-0.0041 (0.0036)	-0.0053 (0.0013)	-0.12013 (0.0767)	0.1759 (0.1251)
β	-1.1663 (0.5558)	-1.8934 (0.2815)	-1.1807 (0.4648)	-1.1471 (0.6479)
R^2	0.5967	0.9195	0.6228	0.5526

Figure 5.2.12: Trend Curves (plots) for BNHN Index Log-returns

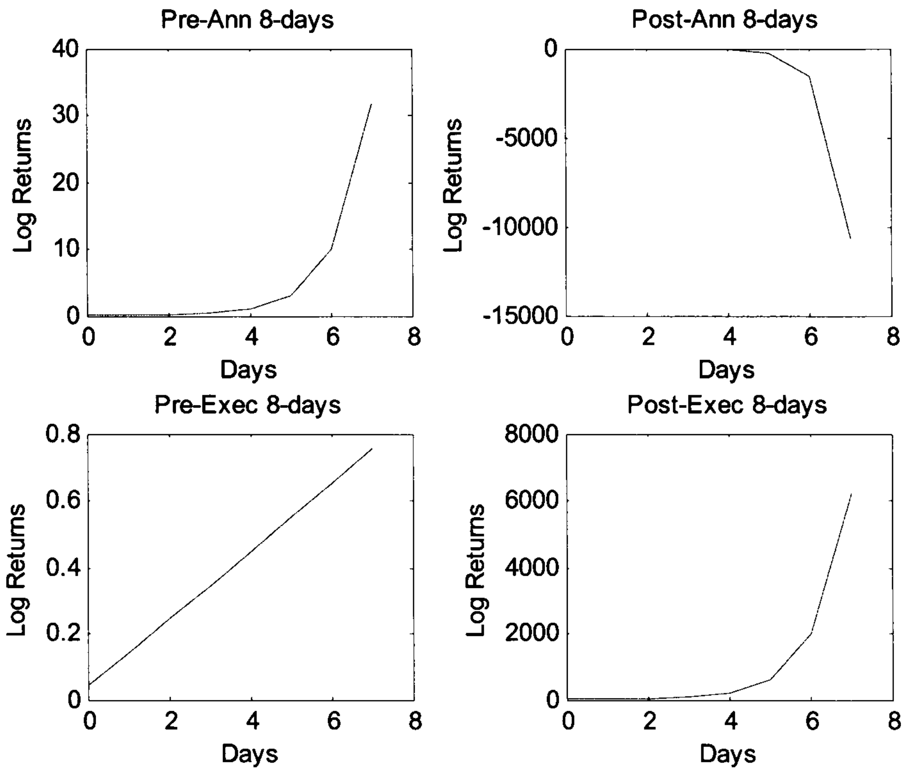

Table 5.2.14: Estimated Parameters for BWLD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0054	0.0254	1.4300	2.2900
α_0	-0.0106 (0.0206)	-0.0247 (0.0156)	-3.1213 (1.4306)	-2.1933 (2.5056)
α_1	0.0031 (0.0046)	0.0057 (0.0035)	-0.0225 (0.1548)	0.1346 (0.2769)
β	-1.4515 (0.3980)	-1.7589 (0.3283)	-1.4431 (0.5091)	-0.6358 (0.5955)
R^2	0.7712	0.8811	0.6730	0.2561

Figure 5.2.13: Trend Curves (plots) for BWLD Index Log-returns

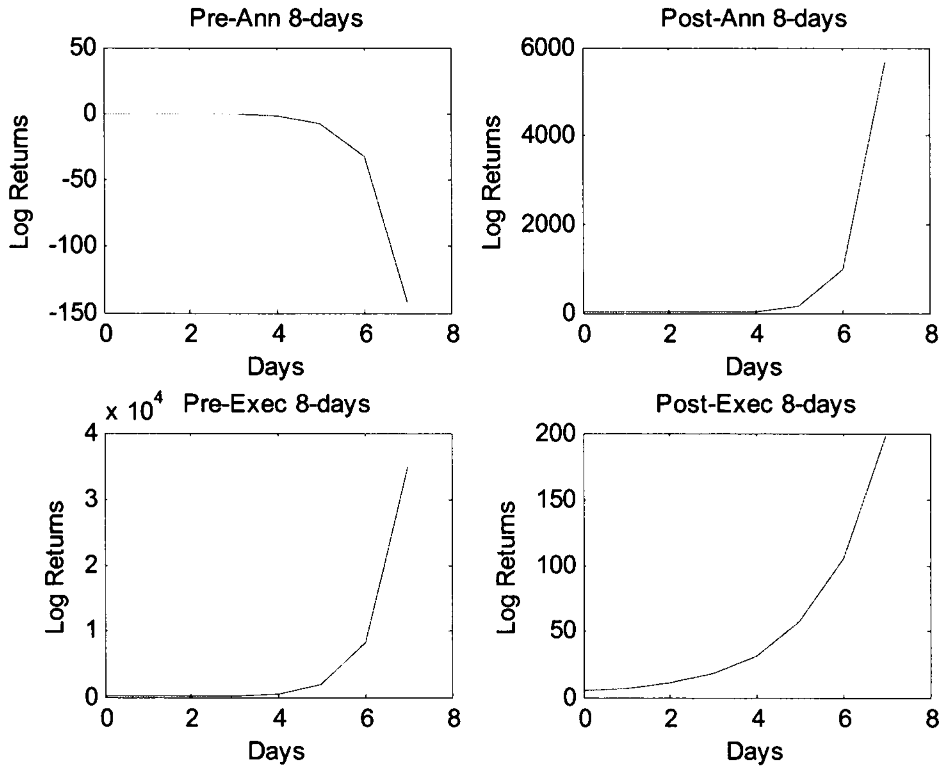

Table 5.2.15: Estimated Parameters for MIDD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0080	-0.0195	2.3600	4.7900
α_0	0.0082 (0.0154)	0.0294 (0.0246)	-1.2538 (0.9510)	-10.1560 (3.6597)
α_1	-0.0013 (0.0036)	-0.0008 (0.0052)	-0.3732 (0.3578)	0.6327 (0.3678)
β	-0.7046 (0.4652)	-1.5869 (0.3815)	-0.9843 (0.6115)	-1.5700 (0.5089)
R^2	0.3903	0.8139	0.4594	0.7189

Figure 5.2.14: Trend Curves (plots) for MIDD Index Log-returns

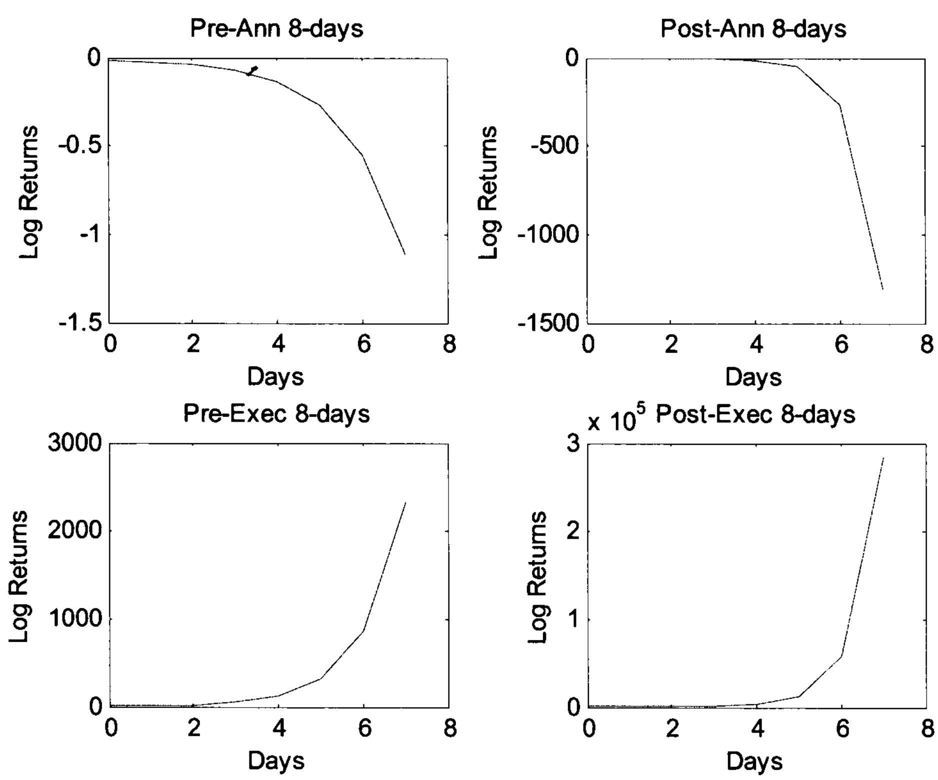

Table 5.2.16: Estimated Parameters for GILD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0028	0.0030	0.7800	1.3700
α_0	-0.0259 (0.0274)	0.0105 (0.0145)	-3.8958 (1.2385)	-3.3289 (0.8304)
α_1	0.01700 (0.0173)	-0.0026 (0.0032)	0.1905 (0.1348)	-0.0045 (0.0449)
β	-0.0174 (0.0095)	-1.5656 (0.4233	-2.0114 (0.5070)	-1.8828 (0.4273)
R^2	0.5328	0.7740	0.8184	0.8342

Figure 5.2.15: Trend Curves (plots) for GILD Index Log-returns

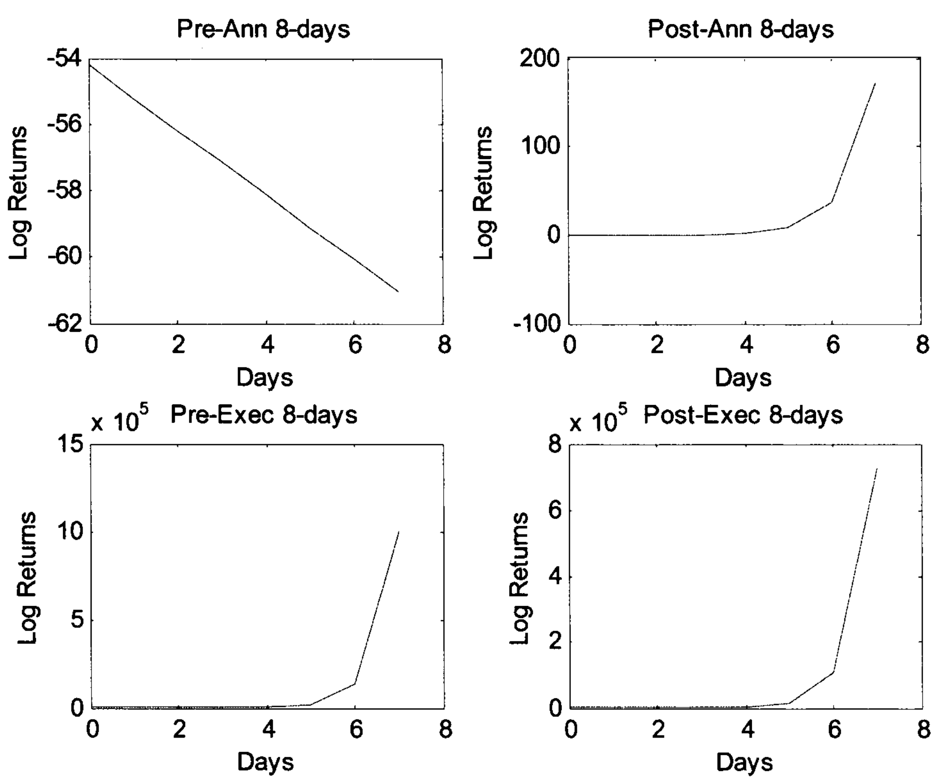

Table 5.2.17: Estimated Parameters for SPAR Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.01998	0.07017	1.53	3.10
α_0	-0.00198 (0.012465)	-0.00522 (0.05909)	-2.02817* (0.867594)	-4.99805* (1.92716)
α_1	-0.00144 (0.002368)	0.002171 (0.013014)	0.175714 (0.110974)	0.475919 (0.296502)
β	-1.59948* (0.744944)	-1.35863* (0.577772)	-1.18826* (0.490381)	-1.39508** (0.460754)
R^2	0.562217	0.619675	0.594823	0.696288

Figure 5.2.16: Trend Curves (plots) for SPAR Index Log-returns

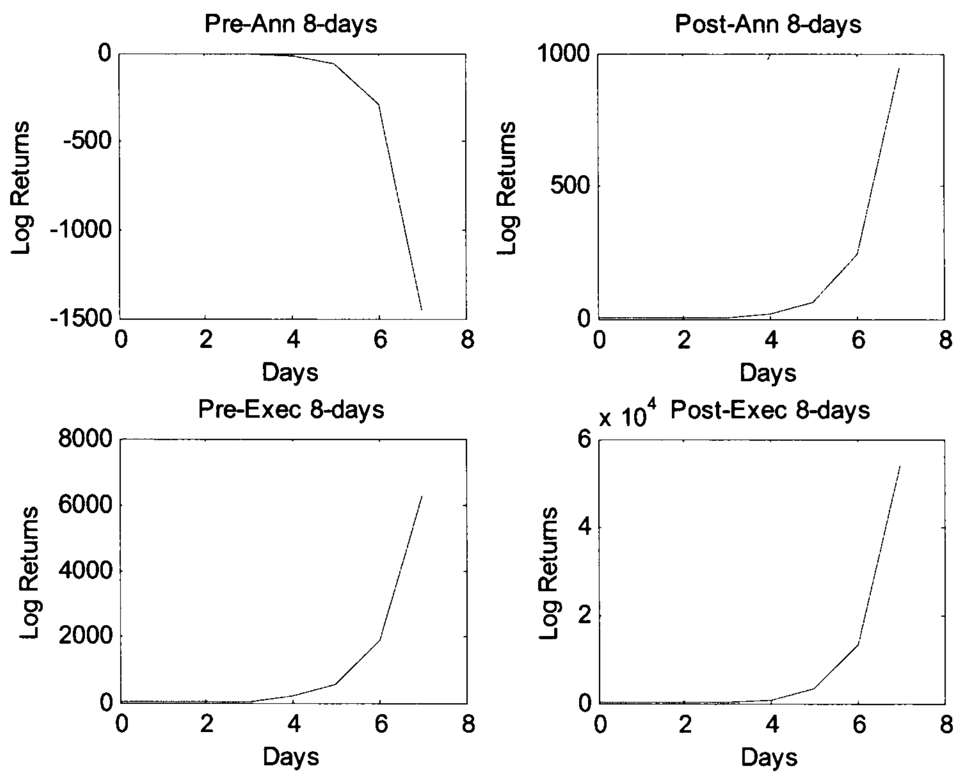

Table 5.3.3: Estimated Parameters for VSEC Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.008778	0.089682	1.22	3.48
α_0	-0.02096 (0.013047)	-0.02596 (0.042)	-0.37821 (0.320897)	-5.52218 (1.87024)
α_1	0.008997 (0.003416)	-0.00662 (0.010276)	-0.47333 (0.087544)	0.34381 (0.157134)
β	-1.61347 (0.381473)	-1.67794 (0.535043)	-0.83164 (0.189414)	-1.41874 (0.472936)
R^2	0.818174	0.721026	0.88644	0.69922

Figure 5.3.2: Trend Curves (plots) for VSEC index daily price

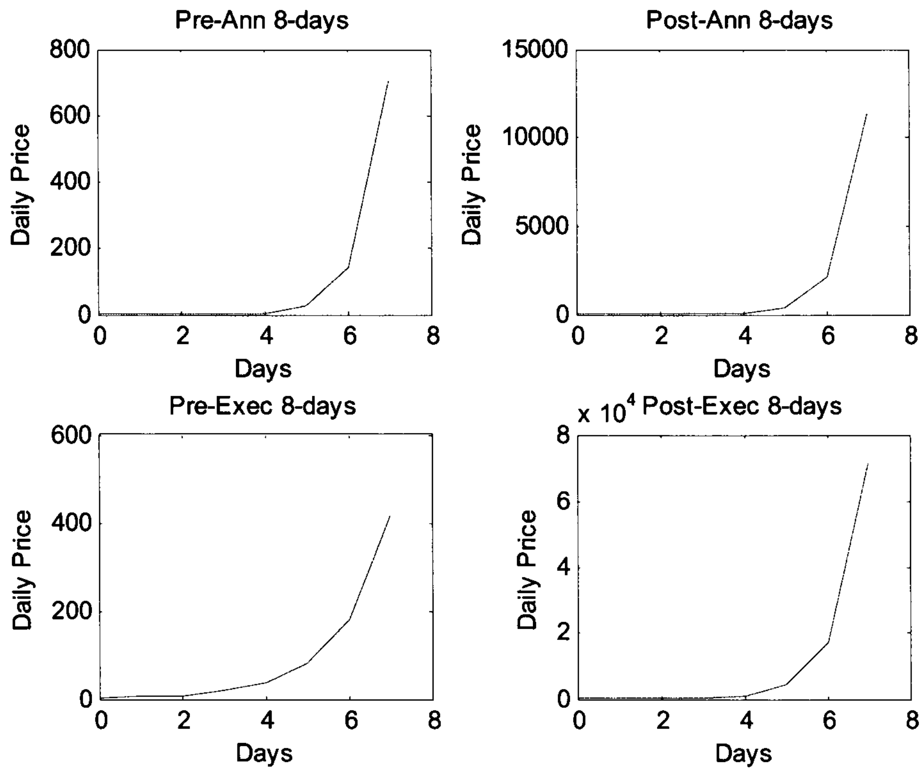

Table 5.3.4: Estimated Parameters for MIDD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.003492	0.033956	2.26	2.45
α_0	0.003326	0.007068	-5.28571	-2.30327
	(0.010316)	(0.036107)	(1.271939)	(1.327433)
α_1	-0.00014	-0.00029	0.346058	0.011883
	(0.002441)	(0.007759)	(0.157855)	(0.150562)
β	-1.19689	-1.24387	-1.40961	-0.98651
	(0.275934)	(0.659285)	(0.297713)	(0.498045)
R^2	0.867769	0.553029	0.852013	0.495311

Figure 5.3.3: Trend Curves (plots) for MIDD index daily price

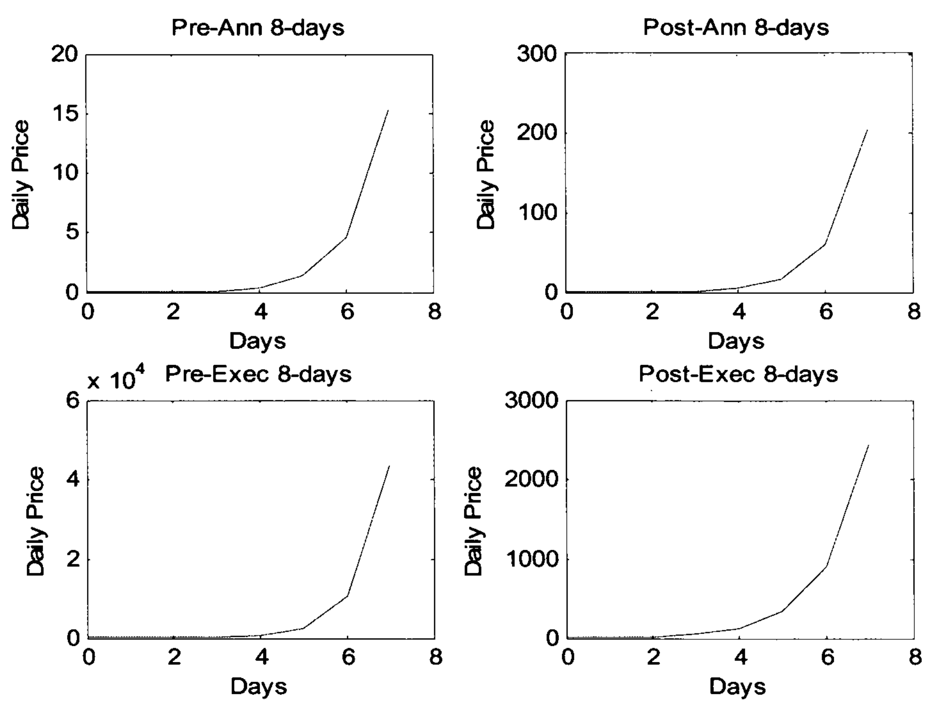

Table 5.3.5: Estimated Parameters for ESRX Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.019135	0.001622	2.10	1.20
α_0	-0.02209 (0.007844)	-0.00488 (0.014499)	-1.09706 (0.363378)	-1.68029 (0.51616)
α_1	0.004489 (0.002044)	-0.00247 (0.003203)	0.193272 (0.092821)	-0.05526 (0.072445)
β	-1.09481 (0.40552)	-1.46158 (0.442497)	-0.28495 (0.298276)	-1.61567 (0.399338)
R^2	0.657859	0.733315	0.781101	0.803753

Figure 5.3.4: Trend Curves (plots) for ESRX index daily price

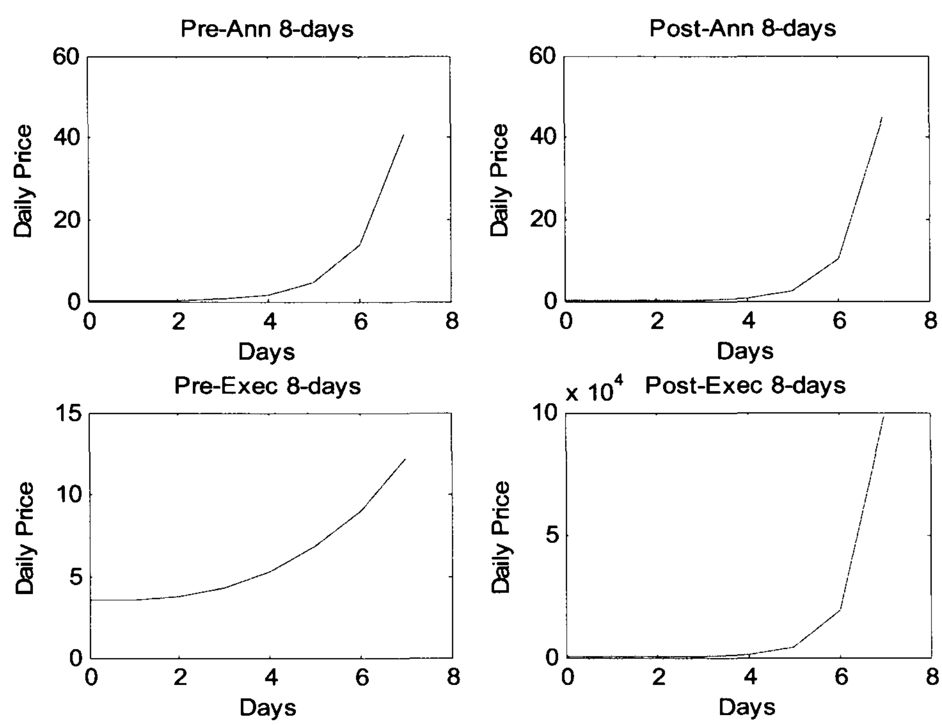

Table 5.3.6: Estimated Parameters for GILD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.017833	0.002064	1.49	0.80
α_0	-0.0178 (0.007572)	-0.01204 (0.014107)	-2.33489 (0.549745)	-2.06516 (0.467852)
α_1	0.004861 (0.001851)	0.002665 (0.003145)	-0.00145 (0.064493)	0.142345 (0.057752)
β	-1.72267 (0.416316)	-1.35944 (0.455579)	-1.55417 (0.338107)	-1.821 (0.352692)
R^2	0.815195	0.690519	0.846487	0.871231

Figure 5.3.5: Trend Curves (plots) for GILD index daily price

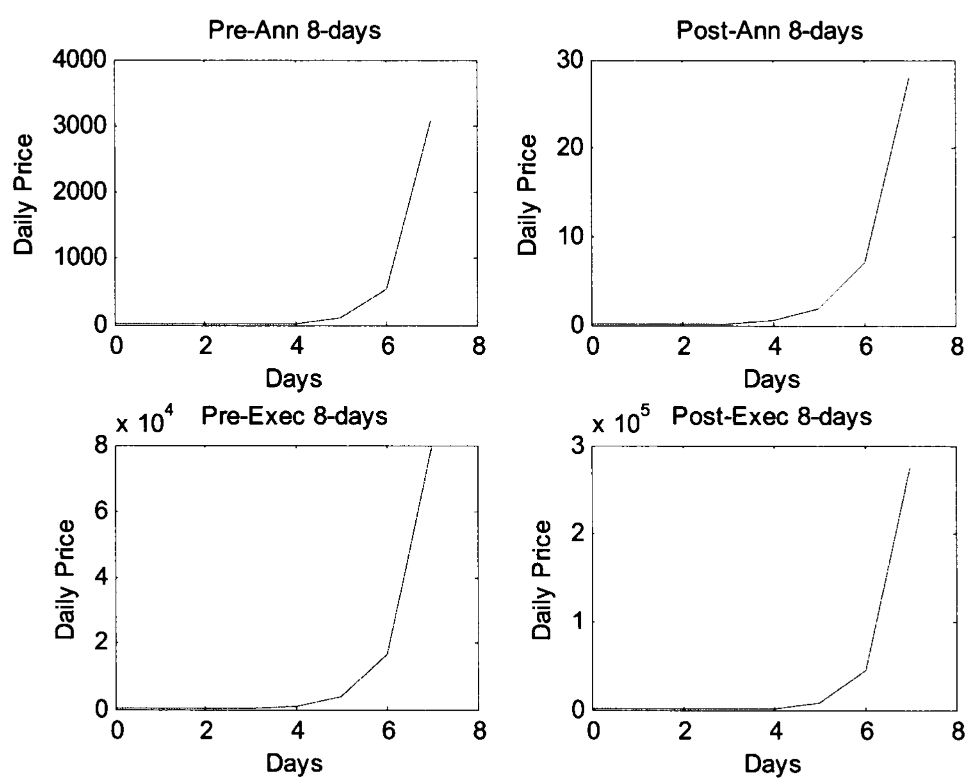

Table 5.3.7: Estimated Parameters for NGA Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.029525	-0.09092	0.79	0.79
α_0	0.003123 (0.046442)	-0.20828 (0.105475)	-1.60075 (0.76583)	-3.42487 (1.38345)
α_1	0.002957 (0.010087)	0.046499 (0.023584)	-0.00095 (0.115571)	0.407748 (0.235903)
β	-1.34963 (0.492107)	-2.19179 (0.636979)	-1.34094 (0.46185)	-1.63539 (0.480796)
R^2	0.67186	0.770435	0.679173	0.748176

Figure 5.3.6: Trend Curves (plots) for NGA index daily price

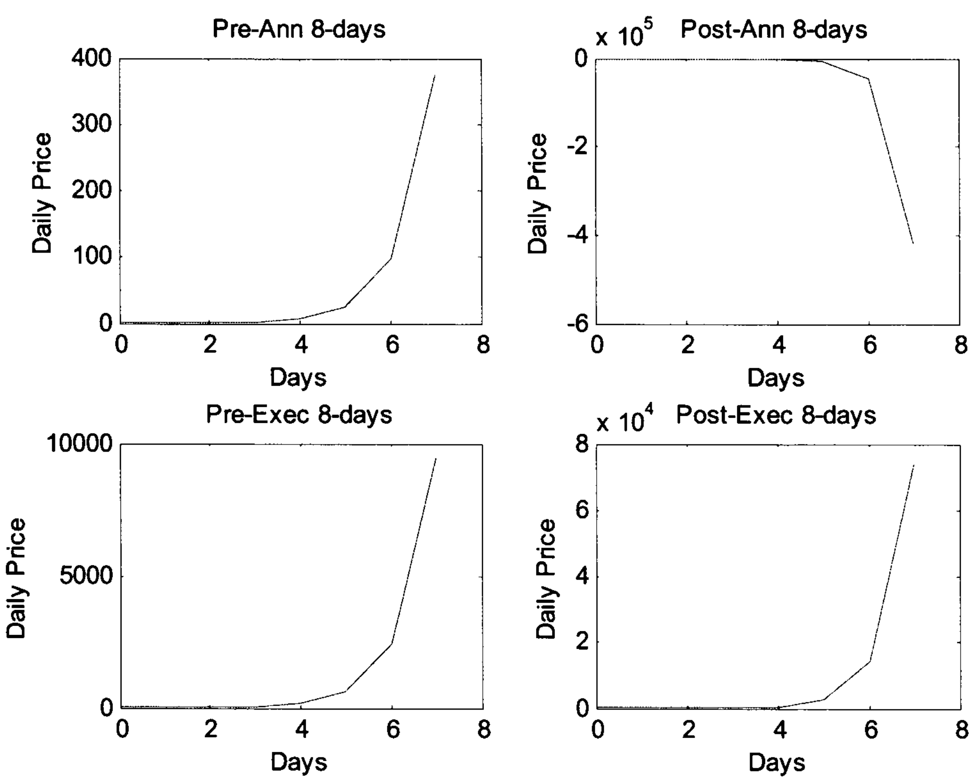

Table 5.3.8: Estimated Parameters for CROX Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.034449	-0.02781	4.00	1.89
α_0	-0.02879 (0.020387)	-0.00889 (0.037893)	-4.85299 (1.223259)	-1.96711 (0.396513)
α_1	0.00293 (0.004294)	0.00256 (0.009891)	-0.10457 (0.125702)	0.100033 (0.083993)
β	-1.27779 (0.508441)	-1.07513 (0.798571)	-1.60376 (0.373447)	-0.91899 (0.17207)
R^2	0.615619	0.573581	0.821878	0.917166

Figure 5.3.7: Trend Curves (plots) for CROX index daily price

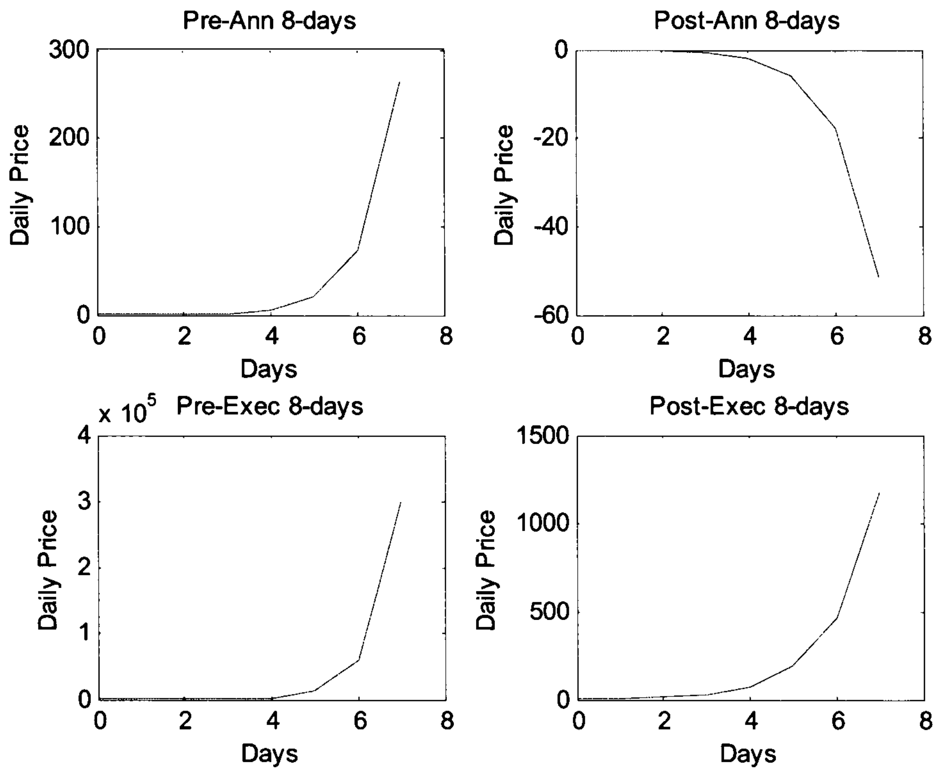

Table 5.3.9: Estimated Parameters for SEIC Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.01697	0.002091	0.98	0.59
α_0	-0.000455 (0.010186)	-0.003509 (0.007891)	-1.259544 (0.26446)	-0.972944 (0.287751)
α_1	0.0011175 (0.002397)	-0.00026 (0.001747)	0.036729 (0.02851)	0.039497 (0.032181)
β	-1.205097 (0.695783)	-1.316559 (0.338542)	-1.3947 (0.31279)	-1.459413 (0.397441)
R^2	0.4471295	0.8122351	0.859174	0.774149

Figure 5.3.8: Trend Curves (plots) for SEIC index daily price

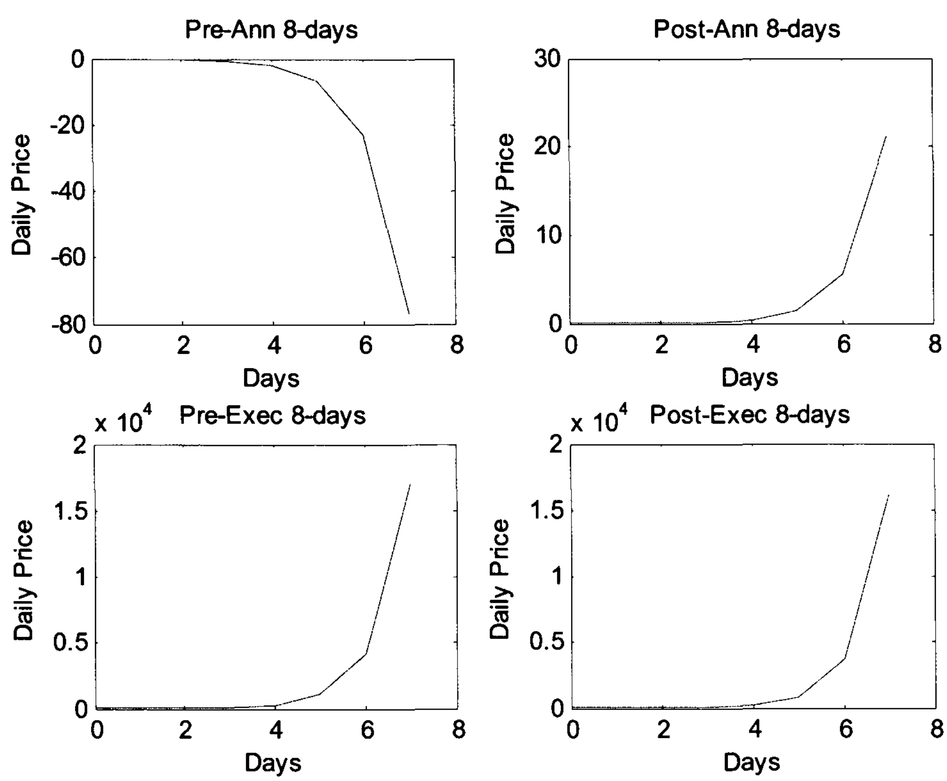

Table 5.3.10: Estimated Parameters for CPKI Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.0411	-0.01125	1.37	0.51
α_0	0.027763 (0.011915)	0.039242 (0.014808)	-0.63871 (0.638856)	-1.594 (0.274353)
α_1	-0.00441 (0.002643)	-0.00764 (0.003181)	0.07747 (0.059511)	0.092963 (0.029732)
β	-1.98695 (0.570271)	-1.54733 (0.470415)	-0.32135 (0.710977)	-1.90741 (0.28764)
R^2	0.786686	0.731848	0.363504	0.916785

Figure 5.3.9: Trend Curves (plots) for CPKI index daily price

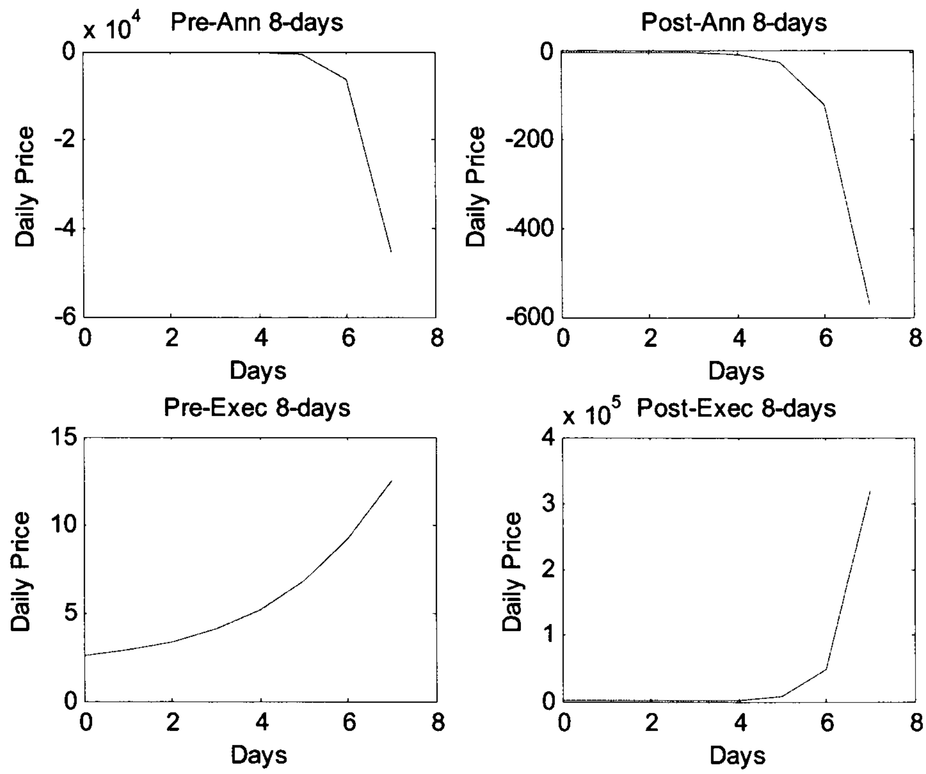

Table 5.3.11: Estimated Parameters for PMFG Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.00479	-0.02492	0.89	0.89
α_0	0.003668 (0.011993)	0.014879 (0.014014)	-0.78445 (0.757626)	-1.84517 (0.996361)
α_1	-0.00203 (0.00266)	-0.00332 (0.003176)	-0.08041 (0.115469)	0.139213 (0.155338)
β	-1.11496 (0.369508)	-1.07174 (0.49917)	-1.11279 (0.533551)	-1.30052 (0.493418)
R^2	0.743748	0.652817	0.540794	0.670138

Figure 5.3.10: Trend Curves (plots) for PMFG index daily price

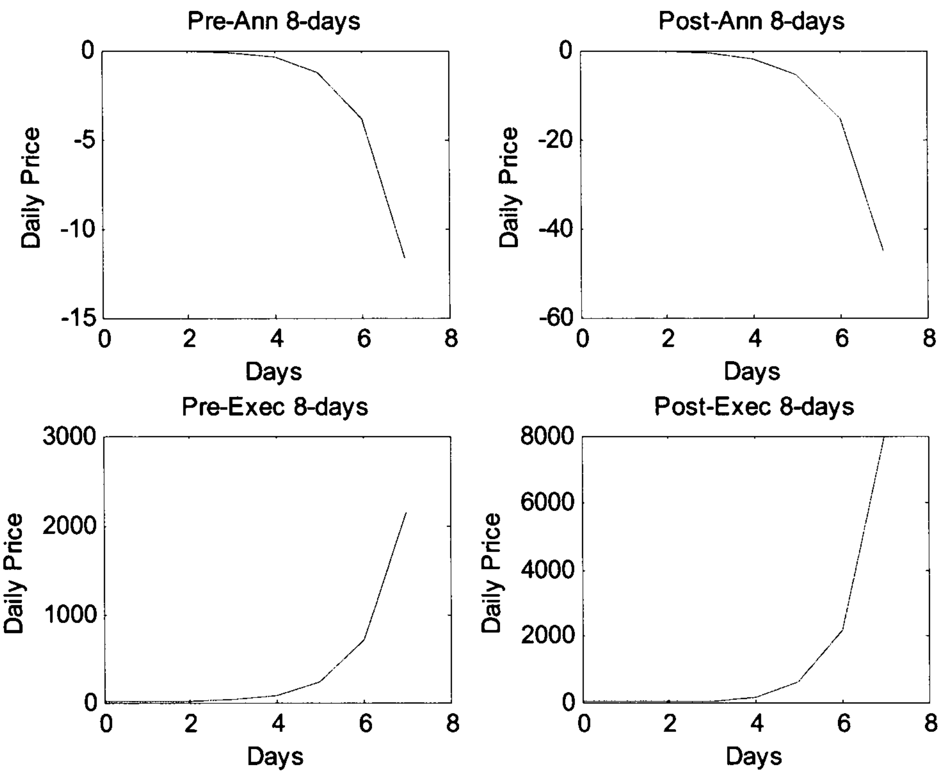

Table 5.3.12: Estimated Parameters for EPIQ Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.01255	-0.04623	0.49	0.99
α_0	-0.05986 (0.004588)	0.058766 (0.022631)	-1.7448 (0.778788)	-1.75313 (0.575525)
α_1	0.010178 (0.000917)	-0.00938 (0.004836)	0.169331 (0.106627)	0.159719 (0.077087)
β	-2.47666 (0.115299)	-1.59546 (0.36876)	-1.41769 (0.520605)	-1.48852 (0.446674)
R^2	0.991979	0.832761	0.651006	0.736551

Figure 5.3.11: Trend Curves (plots) for EQIP index daily price

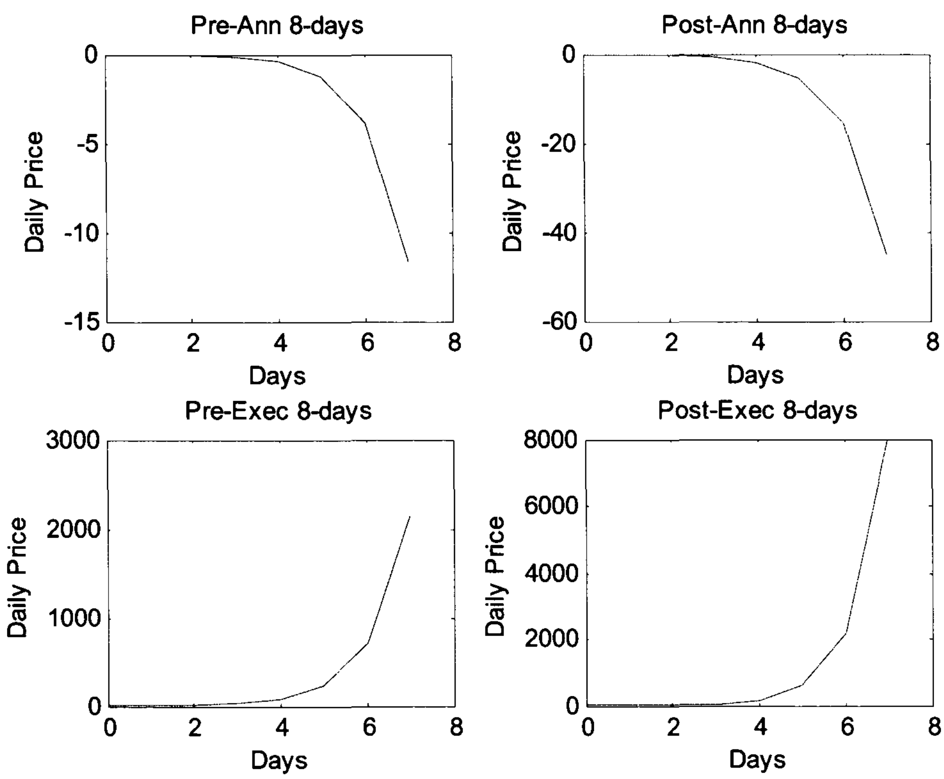

Table 5.3.13: Estimated Parameters for TSBK Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.017879	0.0076	0.93	0.30
α_0	0.004303 (0.01662)	0.025701 (0.028374)	-0.57175 (0.45759)	-0.72011 (0.465119)
α_1	-0.004553 (0.004225)	-0.006931 (0.006588)	-0.0246 (0.071501)	-0.00983 (0.078867)
β	-0.966804 (0.034422)	-1.018646 (0.512698)	-0.079379 (0.487578)	-1.41489 (0.459748)
R^2	0.997065	0.49696	0.398561	0.715979

Figure 5.3.12: Trend Curves (plots) for TSBK index daily price

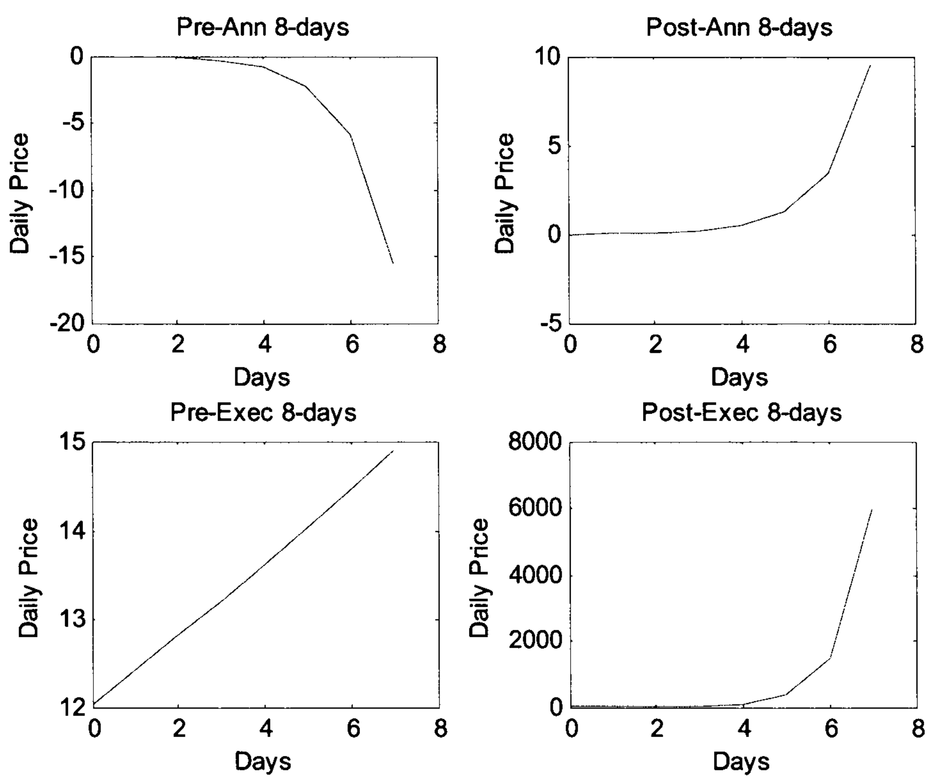

Table 5.3.14: Estimated Parameters for BNHN Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.02082	0.01469	0.34	1.24
α_0	0.024553 (0.021269)	-0.00317 (0.034254)	0.127042 (0.288836)	-1.33135 (0.79234)
α_1	-0.00499 (0.004715)	0.003498 (0.007238)	-0.14229 (0.072483)	0.103548 (0.131883)
β	-1.1464 (0.453723)	-0.73483 (0.517283)	-0.8761 (0.347772)	-1.03574 (0.499439)
R^2	0.635939	0.356442	0.64121	0.519789

Figure 5.3.13: Trend Curves (plots) for BNHN index daily price

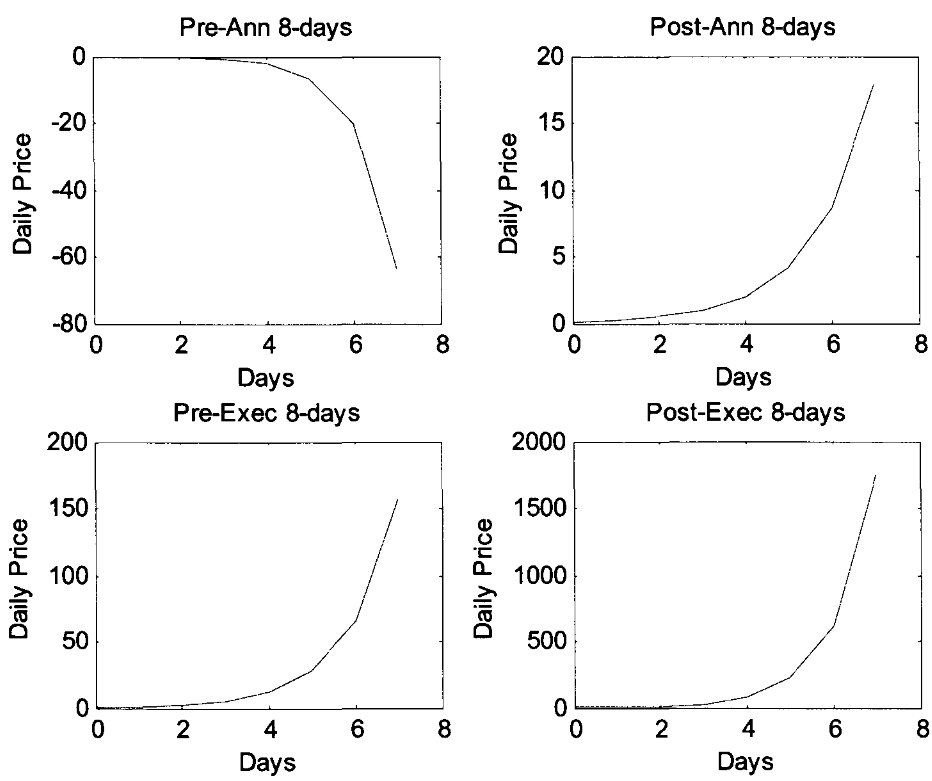

Table 5.3.15: Estimated Parameters for BWLD Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.01395	0.093715	3.01	0.003492
α_0	0.058201 (0.01755)	-0.06182 (0.04828)	-4.85847 (1.909759)	0.003326 (0.010316)
α_1	-0.01166 (0.003895)	0.012838 (0.010215)	0.311371 (0.165869)	-0.00014 (0.002441)
β	-1.78002 (0.378028)	-0.17761 (0.694818)	-1.37621 (0.521415)	-1.19689 (0.275934)
R^2	0.850124	0.350883	0.670608	0.867769

Figure 5.3.14: Trend Curves (plots) for BWLD index daily price

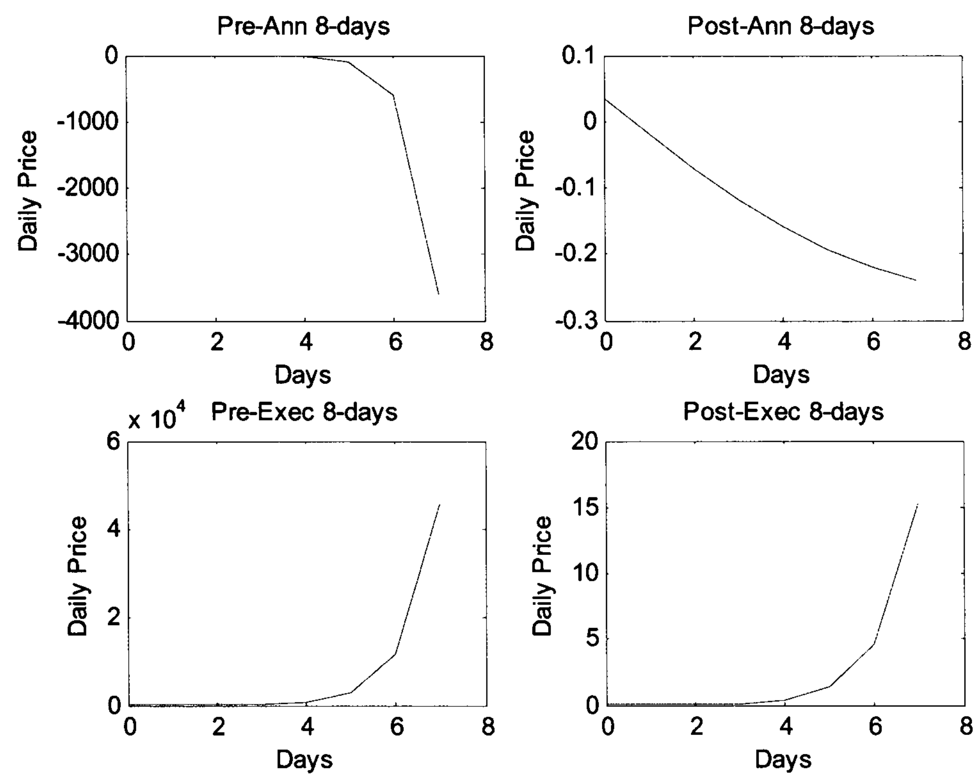

Table 5.3.16: Estimated Parameters for PEBK Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	0.00436	-0.02349	0.40	0.89
α_0	-0.00791 (0.011987)	-0.00122 (0.011772)	-0.02988 (0.303408)	-0.95403 (0.360914)
α_1	0.00262 (0.002832)	0.000057 (0.002571)	-0.0952 (0.070829)	0.101561 (0.05479)
β	-0.20511 (0.435717)	-1.80535 (0.446617)	-1.5277 (0.389168)	-1.26143 (0.524895)
R^2	0.543371	0.805853	0.794439	0.625446

Figure 5.3.15: Trend Curves (plots) for PEBK index daily price

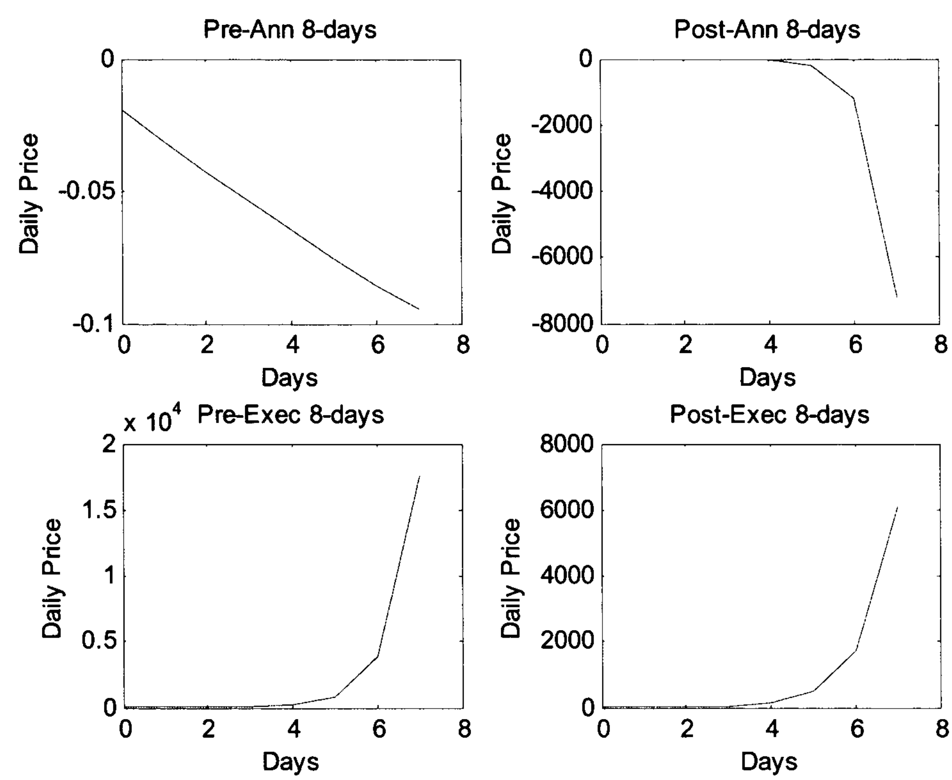

Table 5.3.17: Estimated Parameters for SPAR Index Log-returns

	Pre-Ann	After-Ann	Pre-Exec	After-Exec
c_0	-0.00853	0.055983	0.79	1.11
α_0	-0.01474 (0.048205)	-0.04241 (0.017361)	0.543469 (1.062176)	-1.34716 (0.467713)
α_1	0.006138 (0.008292)	0.009753 (0.003857)	-0.27286 (0.120888)	0.118319 (0.053622)
β	-0.21846 (0.516504)	-0.43177 (0.456398)	-0.26829 (0.351621)	-1.28008 (0.489519)
R^2	0.388826	0.642557	0.685702	0.654614

Figure 5.3.16: Trend Curves (plots) for SPAR index daily price