Modelling of Volatility of Stock Prices using GARCH Models & its importance in Portfolio Construction

Tinotenda Mtemeri

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Supervisor: Professor Renkuan Guo

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Abstract

This thesis is aimed at investigating the possibility to model the risk of stocks in financial markets and evaluating the adequacy and effectiveness of univariate GARCH models such as the symmetric GARCH and a few other variations such as the EGARCH, TARCH and PARCH in modelling volatility in monthly returns of stocks traded on the Johannesburg Stock Exchange. This is further used to investigate the importance of GARCH modelling in portfolio construction using Improved Sharpe Single Index Models. The data used for model estimation has been randomly selected from different sectors of the South African economy. GARCH models are estimated and validated for the data series of the randomly selected 15 JSE stocks. Conclusions are drawn regarding the different GARCH models, best lag structure and best error distributions for modelling. The GARCH (1,1) model demonstrates a relatively good forecasting performance as far as the short term forecasting horizon is concerned. However, the use of alternatives to the more common GARCH (1,1) and use of non-normal distributions is not clearly supported. Also, the use of higher order GARCH models such as the GARCH (1,2), GARCH (2,1) and GARCH (2,2) is not clearly supported and the GARCH (1,1) remains superior overall to these models. The results obtained from this thesis are of paramount importance in portfolio construction, option pricing and formulating hedging strategies. An illustration of the importance of the GARCH (1,1) model in portfolio construction is given and conclusions are drawn regarding its usefulness in improving our volatility estimations for purposes of portfolio construction.

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Chapter 1. Introduction

1.1 Background to Study

The measurement of risk on specific assets has become increasingly important in recent times. Risk can be divided into three broad subcategories namely strategic risks, operational risks and financial risks. However, in this thesis we focus on financial risks, more precisely stock market risk, which is classified under financial risks. Calculating stock market risks has become increasingly important for *portfolio selection, pricing derivatives and hedging strategies*. In a broad sense companies as well as investors want to have good control of the risk inherent in their investments. In the financial market this is of even greater importance as stock markets are increasingly volatile as a consequence of globalization. One of the inherent hazards of investing in financial markets is the risk of a major event or occurrence causing a sudden large shock to stock prices and volatilities. There are many recent examples of event risk such as the stock market crash of October 19, 1987 in which the Dow index fell by 508 points; the October 27, 1997 drop in the Dow index by more than 554 points, and the flight to quality in the aftermath of the Russian debt default where swap spreads increased on August 27, 1998 by more than twenty times their daily standard deviation, leading to the downfall of Long Term Capital Management (LTCM) and many other highly-leveraged hedge funds. Each of these events was accompanied by major increases in market volatility which had a downside on financial markets. Hence the increasing interest in the modelling of financial market risk for purposes of portfolio construction as well as many other financial applications.

The notion of efficient markets has made it increasingly difficult to forecast the volatility of stock markets with certainty. However, several models have been suggested to try and predict future volatility including GARCH models, although forecasting of volatility cannot be achieved with certainty. Furthermore, volatility models aimed at predicting stock return volatility have *lately* received a lot of attention in the literature since previous research work has shown that stock return variances as well as the covariances of stock returns are not constant over time but evolve over time (Bollerslev 1986).

1.2 Aims and Objectives

The overall objective of this thesis is:

- To investigate the possibility to model with some reasonableness the risk of stocks in financial markets and to what extent this can be achieved using univariate GARCH models for purposes of portfolio construction.

The aims of this thesis are:

- To determine whether prior to estimation, the residuals of the data sets exhibit a normal distribution, any autocorrelation, unit roots, or any conditional heteroscedasticity (chapter 4.2).
- To investigate the capability of the selected univariate GARCH models, namely the symmetric GARCH, TARCH, EGARCH and PARCH model in forecasting volatility using the out-of-sample technique and compare their performance (chapter 4.3).
- To determine which specific lag (p,q) structure of the selected GARCH models, best models the volatility of the stock returns (chapter 4.3).
- To determine the best error distribution for modeling the stock returns: normal, student's t-distribution or the generalized error distribution (GED) (chapter 4.3).
- To determine whether the residuals of the different univariate GARCH models exhibit a normal distribution, any autocorrelation or any conditional heteroscedasticity after estimation (chapter 4.4).
- To make a final decision on which model performs best on out-of-sample measures.
- To investigate the importance of GARCH modelling in Portfolio Construction with a look at the Classical Markowitz Mean-Variance Model, the Single Index Model and the Improved Single Index Model (Hossain, Troskie and Guo (2005)).

1.3 Overview of Thesis

Chapter 1- Introduction

This chapter introduces the thesis topic and outlines the background of the study and why the researcher has shown interest in the subject area. It goes on to state the main objective of the study. Following this a list of more specific aims of the thesis are stated giving a clearer view of what the researcher intends to achieve. A diagrammatic overview of the thesis is then given.

Chapter 2- Literature Review - Background to Study

The chapter is a review of available literature on the subject area the researcher intends to carry out a research on. This chapter starts of by introducing the fundamentals of financial markets from the perspective of the Efficient Market Hypothesis (EMH). The different forms of market efficiency are discussed in detail by the researcher. The researcher then discusses the empirical distribution of financial markets; high volatility, the presence of volatility clustering, fat tails and leverage effect, amongst others. Furthermore the researcher looks into the statistical properties of stock returns to be used in the data analysis namely the mean, standard deviation, skewness, and kurtosis before an in-depth look into the stochastic processes responsible for the behaviour of stock returns. This chapter is concluded by a look at the different methods in use for estimating volatility of stock returns; namely the EWMA model, ARCH model and GARCH family models.

Chapter 3 – Methodology

This chapter introduces the method of modelling to be used by the researcher in this case econometric modelling. The type of financial data to be used is discussed; continuous in nature by taking natural logarithms and the source is the Johannesburg Stock Exchange (JSE). Furthermore, the researcher discusses the hypothesis tests to be conducted on the stock returns: Arch test to investigate heteroscedasticity; Ljung-Box-Pierce Q-test – for the autocorrelation test; Unit Root test to investigate whether the data is stationary and Jarque-Bera test to investigate normality. In addition to this the researcher discusses the packages that are used to model and estimate volatility; E-views and MATLAB. The researcher also discusses how one will determine the best performing model and that is by assessing the Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error (MAPE) and Theil Inequality Coefficient (TIC).

Chapter 4 – Market Data Analysis & Results

This chapter basically presents the results of the different tests discussed in chapter 3. The moments of the return series, that is mean, standard deviation, skewness and kurtosis are presented. Plots of series data, namely price plot, squared returns plot and autocorrelation of returns and squared returns plots. Results of the Arch test, Ljung-Box test, Jarque-Bera test and Unit Root test prior to estimation are presented. Furthermore, results of the estimation procedure for GARCH, EGARCH, PARCH and TARCH as well as estimates when alternative distributions such as the student's t-distribution and generalized error distribution (GED) are presented and compared. Following this, parameter estimates for the selected GARCH model specification are then presented. The researcher then has a validation section where tests are performed to judge whether ARCH effects and autocorrelation have been removed or not. Tests are also performed to see if the given assumptions about the model are fulfilled i.e. are the standardized residuals z_t normally distributed.

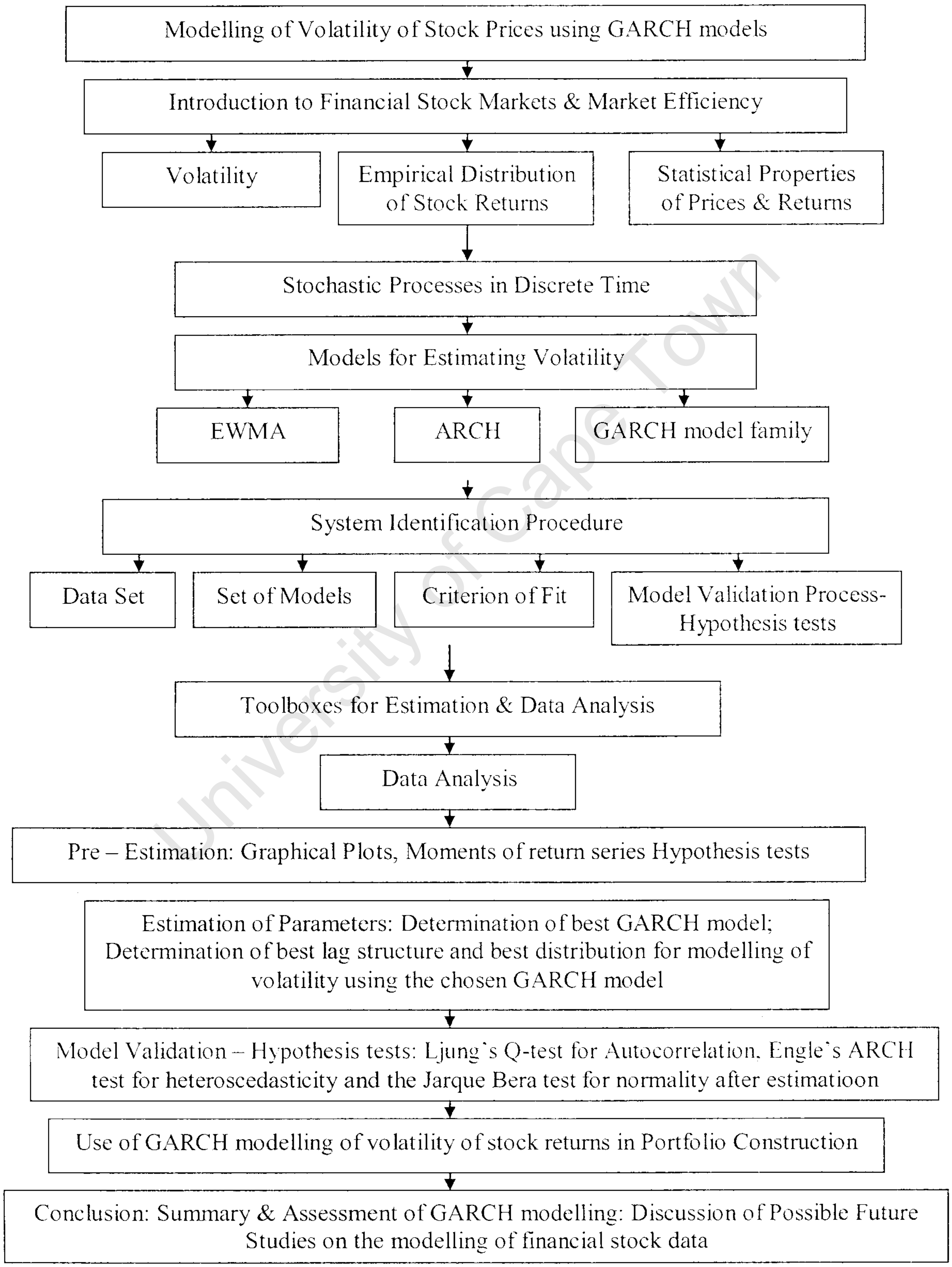
Chapter 5 – GARCH Modelling & Portfolio Construction

This chapter shows us the importance of GARCH modelling of volatility of stock returns in Portfolio Construction. The researcher makes use of Sharpe's Single Index Models together the covariance matrix envisaged by Hossain, Troskie & Guo (2005b), which results in the Improved Sharpe Single Index Models. Previous work by Hossain, Troskie & Guo (2005b) has shown that the covariance matrix they put forward produces superior results to the one suggested by Sharpe (1970). The researcher plots efficient frontiers for Model 1: Improved Sharpe Single Index Model (with volatility modelled with AR terms only) and Model 2: Improved Sharpe Single Index Model (with volatility modelled with AR terms and a GARCH (1,1) process) and compares them to the Classical Markowitz Mean-Variance Efficient Frontier. The researcher analyses the similarities and differences with the Markowitz model drawing conclusions about the usefulness of GARCH modelling in portfolio construction.

Chapter 6 – Conclusion

This chapter draws conclusions based on the different GARCH models and how well each of them models volatility of stock returns. Also the chapter offers a commentary on any visible parameter instability and states the final model chosen by the researcher. Further to this the researcher then also points towards possible further study on the subject area. The researcher also draws conclusions on the usefulness of GARCH modelling in Portfolio Construction.

Figure 1.3.1 Overview of the Thesis



Chapter 2. Background to GARCH Modelling

The modelling of volatility has been the subject of vast empirical and theoretical investigation in recent times by academics and practitioners. There are several motivations for this interest but arguably, volatility is one of the most important concepts in finance. Volatility as measured by the standard deviation is used, as a crude measure of the total risk of financial assets; Value-at-risk (VAR) models for measuring market risk require forecasts of volatility and the Black Scholes model a benchmark in financial markets, requires the input of volatility of stock market prices for option pricing. Volatility is generally unobservable and hence must be estimated. The modelling of volatility has become increasingly important in the ever volatile global asset markets hence the need for good modelling techniques. Since risk is inherently related to volatility, expected future volatility is a major factor in portfolio construction. Good models of volatility can be used to investigate any relation between current prices and expected risk. Since it is impossible to forecast volatility with any certainty, modelled conditional variances are used instead, for example, in portfolio construction, derivative pricing and hedging, risk management, market timing, and market making.

Given that the data to be used in modelling volatility is financial stock data one needs to understand how financial markets function hence in the next section the researcher gives a brief account of previous investigations into the behaviour of stock markets.

2.1 Introduction to Financial Stock Markets and Market Efficiency

Brealy and Myers (1981) attributed the concept of efficient markets to a chance discovery by Kendall, who in 1953 met with the Royal Statistical Society to discuss one of his papers concerning the behaviour of stock and commodity prices. Specifically, Brealy and Myers (1981) stated that, "*Kendall's purpose had been to separate out regular price cycles, but to his surprise he could not find any such cycles. Each series appeared to be a wandering one, almost as if once a week the Demon of Chance drew a random number...and added it to the current price to determine the next week's price.*" Kendall was in fact making the suggestion that price changes are independent. It should also be noted that Louis Bachellier (Cootner 1964) in a doctoral thesis in 1934 also detected the random behaviour of commodity prices.

In recent times numerous researchers have applied various statistical efficiency tests resulting in increased support of the random walk concept, notably Fama (1965). As evidence supporting the random walk concept accumulated, the focus of attention shifted to the kind of market process producing such results, leading to the theory of efficient markets in which prices adjust rapidly to new information.

Roberts (1967) defined three forms of market efficiency: weak form, semi-weak form and strong form efficiency.

- **Weak form** – in this case current prices fully reflect the information contained in the record of past prices. In other words it is impossible to form a trading strategy that performs better than average, using the information of historical prices.
- **Semi-strong form** – in this form of market efficiency, current prices fully reflect published information. Therefore attempts to acquire and analyse public information will not yield superior results.
- **Strong form** – in this form prices reflect not just public information but also all privileged information that can be acquired by fundamental analysts of the company and economy.

Having discussed previous literature on how financial stock markets function, the researcher goes on to define volatility and how crucial it is in today's globalised financial markets in the section below.

2.2 Volatility

Volatility of an asset is a measure of how uncertain we are about future asset price movements (Hull 2006). Also, one can define volatility as a statistical measure of the dispersion of returns for a given security or market index. In other words, volatility refers to the amount of uncertainty or risk about the size of changes in a security's value. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. Generally the rule of thumb is that, the higher the volatility, the riskier the security. A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the price of the security can change dramatically over a short time period in either direction. A lower volatility means that a security's value does not fluctuate dramatically, but changes in value at a steady pace over a period of time.

A common assumption is that the volatility of a stock for example is caused by new information reaching the market in turn resulting in investors changing their opinion about the value of the stock. However, research conducted does not support this view and the conclusion reached by researchers is that volatility is a consequence of trading itself (Roll 1984).

To be able to understand financial stock data one has to first investigate the empirical distribution of stock returns and hence this is investigated in *Section 2.3*.

2.3 Stylized Facts of Stock Returns

The empirical distribution of financial assets returns exhibit some well-known characteristics and regularities, generally referred to as stylized facts. A set of properties, common across many instruments, markets and time periods, has been observed by independent studies and classified as stylized facts. These have been unearthed as a consequence of statistical analysis of price variations in various types of financial markets. The stylized empirical facts include among others:

2.3.1 Volatility Clustering

In stock markets, periods of high risk seem to be followed by periods of high risk and for low risk periods it is the other way around (Hull 2006). This phenomenon is often referred to as volatility clustering. Volatility clustering implies a strong autocorrelation in squared returns. As noted by Mandelbrot (1963), "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." A quantitative manifestation of this fact is that, while returns themselves are uncorrelated, absolute returns $|r_t|$ or their squares display a positive, significant and slowly decaying autocorrelation function: $corr(|r_t|,|r_{t+\tau}|) > 0$ for τ ranging from a few minutes to a several weeks. For example when a shock in a stock market occurs, the volatility is high for the subsequent period. Hence, it is observed in the financial literature that the intensity of stock price changes can persist for shorter and/or longer time periods: volatility clustering. In other words, when a stock price has changed a lot today, it is likely that it will change a lot tomorrow, the day after tomorrow and perhaps even further into the future. Thus returns are conditionally heteroscedastic i.e.

$$Var\,[\,r_{t+1}\,|\,\mathrm{F}_t\ \,] \neq Var\,[\,r_{t-1}\,] \tag{2.3.1}$$

The returns r_t are not independent but their variability depends on recent changes of price as shown in *Eq. 2.3.1.*

2.3.2 Heavy tails

The financial series often exhibit leptokurtosis, indicating that their empirical distribution has *fat tails* and excess peakedness at the mean, i.e., kurtosis exceeds that of a Gaussian distribution. The non-Gaussian character of the distribution makes it necessary to use other measures of dispersion than the standard deviation in order to capture the variability of returns, more precisely higher order moments.

2.3.3 Leverage Effect

One interesting phenomenon in equity markets is that volatility tends to increase more when the return decreases, than it does, when the return increases. Black (1976) observed that volatility responds asymmetrically to the sign of the change in the price of the financial asset, i.e., volatility increases more after negative changes than after positive changes of the same magnitude, naming this phenomenon as the *leverage effect* (also referred as the *Fisher-Black effect*). As a company's equity reduces in value the leverage for the company increases. The reason for this maybe is that when the equity price falls the debt remains constant in the short term, so the debt/equity ratio increases. Hence, the company becomes more risky. On the other hand, when the company's stock price increases, the leverage decreases. The risk of a stock is therefore dependent on the sign of the return (Hull 2006). The leverage effect is also a secondary cause of the implied volatility skew in equity markets. Furthermore it implies an asymmetry in volatility clustering in equity markets: if volatility is higher following a negative return than it is following a positive return then the autocorrelation between yesterday's return and today's squared returns will be large and negative (Alexander 2001).

2.3.4 Non-Stationary

Time series of share prices X_t and other basic financial instruments are not stationary time series and possess at least a local trend (Franke 2004). In order to transform the data to a stationary form, one takes first differences, more often log returns in financial analysis.

2.3.5 Non-trading Periods

Information that accumulates when financial markets are closed is reflected in prices after the markets open. Fama (1965) and French & Roll (1986) found that information accumulates more slowly when the markets are closed than when they are open. Variances are higher following weekends and holidays than on other days, but not nearly as much as would be expected if the news arrival rate were constant. Studies by French & Roll (1986) on daily returns across all NYSE and Amex stocks from 1963-1982 for example showed that volatility is 70 times higher per hour on average when the market is open compared to when its closed.

2.3.6 Forecastable events

Forecastable events of important information are associated with high volatility. Patel and Wolfson (1984) show that individual firm's stock returns volatility is high around earning's announcements. Furthermore, Baillie and Bollerslev (1991) have documented that there are predictable changes in volatility across the day. Notably, volatility is typically much higher at the open and close of stock trading than during the middle of the day. The volatility surge at the opening of trade can be explained by accumulation of information when the markets are closed and the surge at the close of trade is not so easily explained.

2.4 Mathematical characteristics of Prices and Returns

To summarize the statistical properties of prices and returns, firstly visual inspection is done after graphing the price series and the returns series (log returns). Secondly, the relevant descriptive statistics for returns are computed (including the Jarque–Bera test which is a normality test. Following this, we carry out unit root tests: that is the augmented Dickey–Fuller (ADF) to conclude if the series are stationary. Furthermore, we discuss the autocorrelation of the returns and we test the presence of conditional heteroscedasticity.

In *Section 2.4.1* below we give the moments that will be used to describe the data in this thesis. The variable X is a discrete valued random variable.

2.4.1 Moments

The first way to describe a probability distribution function is by some measure of central value or location and the key measures of central location is the mean although the median and mode can be used as alternatives. The **mean** is the first moment and is defined as

$$\mu = E(X) = \sum_i x_i P(X = x_i) \tag{2.4.1}$$

Another measure that can help us to describe a probability distribution function is the dispersion. The most commonly used measure of dispersion is variance. The **variance** is the central second moment and is given by

$$Var(X) = \sum_i (x_i - E(X))^2 P(X = x_i) \tag{2.4.2}$$

A probability distribution may be symmetric or asymmetric around its mean. A popular measure for the asymmetry of a distribution is called its skewness. A negative skewness measure indicates that the distribution is skewed to the left and vice versa. **Skewness** is a rescaled third central moment and is defined as

$$\frac{E((X-\mu)^3)}{(Var(X))^{3/2}} \tag{2.4.3}$$

A variable with positive skewness is more likely to have its values far above the mean value than far below. Notably, for a normal distribution the skewness is zero.

Further information about a probability distribution function is provided by measuring the concentration of potential outcomes in its tails. The tails of a probability distribution function contain the extreme values. In financial applications, it is these tails that provide information about the potential for a financial ruin. The joint measure of peakedness and tail features is called kurtosis. **Kurtosis** is a rescaled fourth central moment and is

$$\frac{E((X-\mu)^4)}{(Var(X))^2} \tag{2.4.4}$$

It is well documented that the kurtosis of a normal distribution is 3. Hence notably, a distribution with a kurtosis greater than 3 has more probability mass in the tails and is described as leptokurtic.

2.4.2 Association Measures

The autocorrelation and partial autocorrelation functions for the squared process have been shown to be useful in identifying and checking time series behaviour in the conditional variance equation of the GARCH form. Granger and Anderson (1978) advocated for the use of the squared process to check for model adequacy as they unearthed the fact that some of the series modeled in Box and Jenkins (1976) exhibit auto-correlated squared residuals even though the residuals themselves do not appear to be auto-correlated. Below we define the autocorrelation function (ACF) and the partial autocorrelation function (PACF):

Autocorrelation Function (ACF)

The autocorrelation function (ACF) measures the linear dependence between past and current returns (Hamilton 1994). Its formulation is shown in *Eq. 2.4.5* below:

$$Corr(X_{t+\tau}, X_t) = \frac{Cov(X_{t+\tau}, X_t)}{\sqrt{Var(X_{t+\tau})Var(X_t)}} \qquad (2.4.5)$$

Partial Autocorrelation Function (PACF)

However, it can be observed that in some instances, the correlation between two elements of a time series with a lag h time steps is non-zero. This therefore implies that a return which has occurred one year ago still influences the return of today, but the influence decreases geometrically with the time lag. In this instance we use another method to measure the connection between X_t and $X_{t+\tau}$ which involves filtering out of X_t and $X_{t+\tau}$ the linear influence of the random variables that lie in between, $X_{t+1}, \ldots, X_{t+\tau-1}$, and then calculating the correlation of the transformed random variables (Franke 2004). This is called the partial autocorrelation. Hence the use of the partial autocorrelation function (PACF) which measures the influence of past observations on the current value of the time series (Rachev 2005). The partial autocorrelation of the *h-th* order is given by:

$$\phi_{hh} = Corr\,(X_t \mid X_{t+1,\ldots,}X_{t+h-1}), X_{t+h} - P\,(X_{t-h} \mid X_{t-1}, \ldots, X_{t-h-1})) \qquad (2.4.6)$$

where $P\ W \mid Z$ is the best linear projection of W on Z (Franke 2004).

2.5 Stochastic Processes in Discrete Time

The theory of stochastic processes in discrete time is an important tool when examining the fundamental characteristics of financial data. Stochastic processes describe the evolution of a random variable (in this instance the return of the stock) through time. In other words stochastic processes describe the relationship between the probability distribution of yesterday`s return; today`s return and tomorrow`s return (Rachev 2005). Below we discuss various stochastic processes in discrete time starting with a white noise process:

2.5.1 White Noise

The building block of all time series models is the white noise process given by a sequence $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ of independently and identically distributed $(i.i.d)$ random variables which are usually assumed to have mean zero and variance σ^2 (Hamilton 1994).

$$E(\varepsilon_t) = 0 \tag{2.5.1}$$

$$E(\varepsilon_t^2) = \sigma^2 \tag{2.5.2}$$

A popular candidate for the white noise distribution is the standard normal distribution. A plot of a white noise process should exhibit no trends or clusters in the realizations which can be explained by the fact that the observations are independent from each other. (Rachev 2005)

2.5.2 Autoregressive Processes

An autoregressive model is one where the current value of a variable, x depends upon only the values that the variable took in previous periods plus an error term (Brooks 2002). An autoregressive process of order one AR (1) is defined by the model:

$$x_t = c + \beta x_{t-1} + \sigma\varepsilon_t \tag{2.5.3}$$

where c and β can be any real constants and ε_t is a white noise process.

The name autoregressive comes from the fact that the value of x_t is obtained as an affine function (i.e. a linear function with an intercept) of x_{t-1} plus an error term. An autoregressive process of higher order AR *(n)* is defined as:

$$x_t = c + \sum_{i=1}^{n} \beta_i x_{t-i} + \sigma\varepsilon_t \qquad (2.5.4)$$

It has been observed that the path of an AR process differs substantially from that of a white noise process. Highly positive returns are followed by positive returns and vice versa, hence one can recognize trends and the reason for this is that the realization of current return depends on strongly on the previous one. Hence, the AR *(n)* process is useful in order to incorporate possible dependencies of the current return on several previous returns.

To measure the influence of past returns on current return we use the Autocorrelation function (ACF) or the Partial Autocorrelation function (PACF). However, it can be observed that in the general case of an AR *(n)* process, we obtain non-zero partial autocorrelation for the first n lags and zero partial autocorrelation otherwise. In contrast to the ACF, the PACF jumps immediately to zero when the lag exceeds the order of the autoregressive process.

It should be noted in some cases one would like to model financial data in which the ACF behaves in this way; that is, for the first 1, 2, or m lags one observes autocorrelation and then the autocorrelation jumps immediately to zero. This concept can be realized by use of a Moving-Average (MA) process.

2.5.3 Moving Average Processes

The name 'moving average' comes from the fact that the next return is given by a weighted average of today's and the q previous observations of the white noise process (Brooks 2002). The dynamics of a moving average process of order m, (MA *(q)* process) is given by:

$$x_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right)\varepsilon_t = \theta\varepsilon_t \qquad (2.5.5)$$

where L is a lag operator, and θ represents the polynomial $\theta = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right)$.

2.5.4 Choice of AR or ARMA Model

To decide which AR or ARMA model to consider in modelling a particular return series one should use the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots to determine how best to model the return series. *Table 2.5.1* below provides a general guideline on how one can decide which model to use based on the autocorrelation function (ACF) plot.

Table 2.5.1 Choice of AR or ARMA model

Shape of ACF	Indicated Model
Exponential, decaying to zero	Autoregressive model - Use the partial autocorrelation plot to identify the order of the autoregressive model.
Alternating positive and negative, decaying to zero	Autoregressive model - Use the partial autocorrelation plot to help identify the order.
One or more spikes, rest are essentially zero	Moving average model - order identified by where plot becomes zero.
Decay, starting after a few lags	Mixed autoregressive and moving average model.
All zero or close to zero	The data used is random.
No decay to zero	The series is not stationary

2.6 Discrete Stochastic Volatility Models

It is necessary in most instances, when modelling financial return data to incorporate time-varying conditional volatility and correlations. This arises because of the recognition that volatilities and correlations are not constant. During some periods, a particular volatility or correlation may be relatively low, whereas during other periods it may be relatively high. Hence the use of the exponentially weighted moving average (EWMA) model, autoregressive conditional heteroscedasticity (ARCH) model and generalised autoregressive conditional heteroscedasticity (GARCH) models.

2.6.1 Exponentially Weighted Moving Average (EWMA) model

The exponentially weighted moving average (EWMA) model hinges on the idea that more weight should be given to more recent data, specifically a more recent volatility observation, σ_n, with weights α_i decreasing exponentially as we move back through time. To be more precise $\alpha_{i+1} = \lambda\alpha_i$, where λ is a constant between 0 and 1 (Hull 2006). Manipulating the simple model given in *Eq. 2.6.1* will lead to the fundamental updating equation for the EWMA method shown in E*q. 2.6.2.*

$$\sigma_t^2 = \sum_{i=1}^{m} \alpha_i u_{t-i}^2 \qquad (2.6.1)$$

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)u_{t-1}^2 \qquad (2.6.2)$$

where,

α_i, is the amount of weight given to the observation i days ago and $\sum_{i=1}^{m} \alpha_i = 1$.

u_i, is defined as the continuously compounded return during day t (between the end of day $t-1$ and the end of day i), that is:

$$u_t = \ln \frac{S_t}{S_{t-1}} \qquad (2.6.3)$$

and,

$$\sigma_t^2 = \frac{1}{m}\sum_{i=1}^{m} u_{t-i}^2 \qquad (2.6.4)$$

The first term in the equation $\lambda\sigma_{t-1}^2$ determines the persistence in the volatility: irrespective of what happens in the market, if volatility was high yesterday it will still be high today and the second term $(1-\lambda)u_{t-1}^2$ determines the intensity of the reaction of volatility of market events: the smaller λ is, the more the volatility reacts to the market's information in yesterday's return (Alexander 2001). The EWMA approach has the attractive feature that only two values have to be stored for each volatility estimate, namely the current estimate of the variance rate and the most recent observation. It also adapts to market changes because of the functional form of the equation. Large changes in u_{t-1} will directly affect the volatility estimate.

However, the EWMA method is only an estimation method and additional assumptions have to be made to turn it into a forecasting model. One has to use statistical methods to estimate λ in order to minimize the root mean forecasting error, unlike in GARCH models for example where maximum likelihood estimation is an optimal method that always gives constant estimators. So consequently one can conclude that statistical methods are not necessarily the best method to estimate λ and hence the EWMA model is not the best for modeling and forecasting stock price volatility.

2.6.2 Autoregressive Conditional Heteroscedastic (ARCH) model

The ARCH model was first introduced by Engle in 1982. An ARCH model is a discrete time stochastic process $\{\varepsilon_t\}$ of the form:

$$\varepsilon_t = z_t p_t \tag{2.6.5}$$

where, the z_t's are $i.i.d$ over time, $E(z_t) = 0$, $\text{var}(z_t) = 1$, and p_t is positive and time-varying.

Usually p_t is further modeled to be an autoregressive process. In other words returns are assumed to be generated by a stochastic process with time-varying volatility. The basic idea of ARCH models is that *(i)* the asset return z_t is serially uncorrelated, but dependent and *(ii)* the dependence of z_t can be described by a simple quadratic function (Pena, Tiao & Tsay 2001).

According to Engle (1982), an autoregressive conditional heteroscedasticity model considers the variance of the current error term to be a function of the variances of the previous time period's error terms. The ARCH process introduced by Engle (1982) explicitly recognizes the difference between the unconditional and conditional variance allowing the later to change over time as a function of past errors leaving the unconditional variance constant. ARCH relates the error variance to the square of a previous period's error. This model is employed commonly in modeling financial time series that exhibit time-varying volatility clustering.

As the name suggests, the model has the following properties:

- **Auto-regression** – the use of previous estimates of volatility to calculate subsequent or future values and hence volatility values are closely related to each other.
- **Heteroscedasticity** - probability distributions of the volatility vary with the current value.

In order to clearly introduce ARCH processes, let us assume that we have a time series of asset price quotes p_t for each time step t. We calculate the fractional change in the price of the asset between time step t and $t+1$ using

$$u_t = \frac{p_t - p_{t-1}}{p_{t-1}} \qquad (2.6.6)$$

Furthermore, we are required to determine the long-running historical volatility (e.g. over several years) denoted by V_L. We have seen that the volatility rates fluctuate about this mean long-running mean volatility, therefore, it seems reasonable to incorporate this quantity in the ARCH model.

Formally, an ARCH (q) process may be expressed mathematically as

$$\sigma_t^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 \qquad (2.6.7)$$

where, σ_t is the volatility at the t^{th} time step, γ and α_i are weighting factors that satisfy

$$\gamma + \sum_{i=1}^{m} \alpha_i = 1 \tag{2.6.8}$$

Here m denotes the number of observations of ε_{t-i} used to determine σ_t.

The most common ARCH (q) process used to model asset price volatility dynamics is the ARCH (1) model where,

$$\sigma_t^2 = \gamma V_L + \alpha\varepsilon_{t-1}^2 \tag{2.6.9}$$

or,

$$\sigma_t^2 = \gamma V_L + (1-\gamma)\varepsilon_{t-1}^2 \tag{2.6.10}$$

using the above relation.

Several models using the ARCH process formulation have been successful in modelling different economic phenomena as documented in Coulson and Robins (1985) and Domowitz and Hakkio (1985), just to mention a few. However, common to most of these models, is the introduction of a rather arbitrary linear declining lag structure in the conditional variance equation to take account of the long memory found in empirical work, since the estimation of a free lag distribution will in more cases lead to violation of the non-negativity constraints (Engle 1995). Hence, this resulted in the development of GARCH models to allow for a more flexible lag structure.

Notably as the lag increases in an ARCH model, it becomes more difficult to estimate parameters because the likelihood function becomes very flat. Also, ARCH models often over-predict the volatility, because they respond slowly to isolated large shocks to the return series (Pena, Tiao & Tsay 2001). Adding to this the inadequate dynamics in an ARCH model with only a few lags, the use of standard ARCH models for financial volatility estimation is not recommended.

2.6.3 Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model Family

In light of the need to allow for both a longer memory and a more flexible lag structure Bollerslev (1986) later proposed a more generalized form of the ARCH (q) model appropriately termed the GARCH *(p,q)* model. In the ARCH (q) process the conditional variance is specified as a linear function of past sample variances only, whereas GARCH *(p,q)* process allows lagged conditional variances to be included as well. This in many ways can be said to correspond to some sort of adaptive learning mechanism. The GARCH *(p,q)* model may be written as

$$\sigma_t^2 = \gamma V_L + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (2.6.11)$$

p and q denote the number of past observations of u_{t-i} and σ_{t-j}, respectively, used to estimate σ_t. The GARCH *(p,q)* process has the following properties:

$$E(u_t) = 0 \quad and \quad E(u_t | \Im_t) = 0 \qquad (2.6.12)$$

$$Var(u_t) = E\sigma_t^2 = \frac{\gamma V_L}{1 - \sum_{i=1}^{p} \alpha_i - \sum_{j=1}^{q} \beta_j} \quad and \quad Var(u_t | \Im_{t-1}) = \sigma_t^2 = \gamma V_L + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2 \qquad (2.6.13)$$

Thus the GARCH *(p,q)* process possesses the desired properties stated below:

- Constant unconditional mean and variance.
- Constant conditional mean but time-varying conditional variance.

It should be noted that The GARCH *(p,q)* model reduces to the ARCH(q) process when $p = 0$. At least one of the ARCH parameters must be non-zero $(q > 0)$. Variations of the GARCH model family are discussed below:

GARCH (1,1) model

In the case where $p = q = 1$ the model becomes

$$\varepsilon_t \mid I_t \sim N(0, \sigma_{t-1}^2) \tag{2.6.14}$$

letting $\gamma V_L = \omega$ then,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2.6.15}$$

where

$\omega > 0$, $\alpha \geq 0$, $\beta > 0$ and $\alpha + \beta < 1$ suffices for wide-sense stationarity.

For the GARCH process to exist, $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are sufficient conditions for the conditional variance to be positive. The ARCH coefficient, α measures intensity of reaction of volatility to market movements, and the GARCH effect, β measures the contribution to long run persistence, namely $\alpha + \beta$. Several statistical properties have been established for the GARCH *(p,q)* process in order to define the moments of the unconditional shock. Ling and McAleer (2002) establish all the moment conditions for the general class of models, GARCH *(p,q)*, and relate the moment conditions to the statistical properties of the models. The implications of the non-existence of moment conditions, such as the possible inconsistency of parameter estimates and invalid inference, are frequently ignored in the empirical literature on modelling volatility using GARCH processes.

The necessary and sufficient condition for the conditional variance to exist for the GARCH (1,1) model, guaranteeing that the GARCH (1,1) process is strictly stationary is given by:

$$\alpha + \beta < 1 \tag{2.6.16}$$

The conditional variance (2^{nd} moment) is sufficient for consistency of the (quasi-) maximum likelihood estimator, while the 4^{th} moment is sufficient for asymptotic normality.

Threshold ARCH (TARCH) model

This model due independently to Zakoian (1994) and Glosten, Jaganathan and Runkle (1993) accounts for the asymmetric effect of "news". The basic idea of the TARCH models is to divide the distribution of the innovations into disjoint intervals and then approximate a piecewise linear function for the conditional standard deviation and the conditional variance respectively (Franke 2004). If there are only two intervals, the division is normally at zero: that is the influence of positive and negative innovations on the volatility is differentiated. The TARCH *(p,q)* model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \varepsilon_{t-k}^2 I_{t-k}^- \tag{2.6.17}$$

where,

$$I_{t-k}^- = \begin{cases} 1 \; if \; \varepsilon_t < 0 \\ 0 \; otherwise \end{cases} \tag{2.6.18}$$

In the TARCH model, good news; $\varepsilon_{t-i} > 0$ and bad news; $\varepsilon_{t-i} < 0$, have different effects on the conditional variance. When $\gamma_k \neq 0$, we conclude that the news impact is asymmetric and that there is a presence of leverage effects. When $\gamma_k = 0$ for all k, the TARCH model is equivalent to the symmetric GARCH model. The difference between the TARCH and the Exponential - GARCH (EGARCH) models is that in the former the leverage effect is quadratic while in the latter, the leverage effect is exponential.

It should also be noted that GARCH is a special case of the TARCH model where the threshold term is set to zero. To estimate a TARCH model, one specifies the GARCH model with ARCH and GARCH order and then changes the Threshold order to the desired value.

Exponential - GARCH (EGARCH) model

The exponential general autoregressive conditional heteroscedasticity (EGARCH) model by Nelson (1991) is another variation of the symmetric GARCH model. This is a form of an asymmetric GARCH model. One limitation of the symmetric GARCH (1,1) model is that it restricts the shocks to the model to have the same effect on the conditional variance whether the shocks are negative or positive. This may or may not be a reasonable assumption but one would like to be able to test this. The positivity constraints on the parameters can also be viewed as restrictive, since it rules out cyclical behavior in the conditional variance.

A number of researchers have found evidence of asymmetry in stock price behaviour; that is negative price shocks seem to increase volatility more than positive shocks (Pagan & Schwert 1990). The EGARCH can be used to test for the presence of the leverage effect.

The key advantage of the EGARCH is that since *Eq. 2.6.20* describes the log of σ_t^2, the variance itself σ_t^2 will be positive regardless of whether the coefficients ω, α, β are positive. This makes numerical optimization simpler and allows flexibility of dynamic models for the variance (Hamilton 1994). The idea is to loosen the positivity constraints from the standard GARCH but still keep the non-negativity constraint on the volatility for the conditional variance. Formally:

$$\log \sigma_t^2 = \omega_t + \sum_{i=1}^{p} \alpha_i g(z_{t-i}) + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 \qquad (2.6.19)$$

where,

$$g(z_t) = \theta z_t + \gamma \, | z_t | - E | z_t | \quad \text{is the conditional variance} \qquad (2.6.20)$$

$\omega, \alpha, \beta, \theta$ *and* γ are coefficients, and

$$z_t = \frac{\varepsilon_t}{\sqrt{\sigma_t^2}} \quad \text{is a standard normal variable.} \qquad (2.6.21)$$

However the model is difficult to use for volatility forecasting because there is no analytical form for the volatility term structure.

The Power-ARCH (PARCH) model

When one talks of volatility the appropriate measure is the standard deviation rather than the variance as noted by Barndorff-Nielsen and Shephard (2002). Most of the ARCH-type of models discussed so far deal with the conditional variance in the specification. A GARCH model using the standard deviation was introduced independently by Taylor (1986) and Schwert (1989). The conditional standard deviation as a measure of volatility is modeled instead of the conditional variance. This class of models is generalized by Ding, Granger & Engle (1993) with the Power - ARCH specification. The PARCH specification is given by,

$$\sigma_t^{\delta} = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^{\delta} \qquad (2.6.22)$$

where

$$\delta > 0, |\gamma_i| \leq 1 \; for\ i = 1, 2, \ldots, t \; and\ \gamma_i = 0 \; for\ i > t, \; and\ t \leq p \qquad (2.6.23)$$

It should be noted that in the PARCH model, $\gamma \neq 0$ implies asymmetric effects. The PARCH model reduces to the GARCH model when $\delta = 2$ and $\gamma_i = 0$ for all i.

Chapter 3. Financial Econometric Methodologies

3.1 Introduction

This chapter deals with the methodology used in conducting this research as well as the various research instruments used by the researcher. Methodology can be defined as an operational framework within which the facts are planned so that their meaning may be seen more clearly (Leedy 1991). This master thesis is concerned primarily with financial econometrics. The focus is on the modelling and forecasting of the volatility of stock returns hence we are interested in prediction of the first and the second moments, particularly the second moments.

3.2 System Identification

System identification deals with the problem of building mathematical models of dynamical systems based on observed data from the systems (Ljung 1987). The construction of a model from data involves three basic entities:

3.2.1 The Data Set

As is standard procedure when modeling stock returns, the continuous approach is also used in this master thesis when converting between stock prices and return. Let the stock price be denoted S then the expected drift rate in S should be assumed to be μS for some constant parameter μ. This implies that in a short period of time, Δt the expected increase in S is $\mu S \Delta t$. If volatility of stock price is always zero, then this model implies that as $t \to 0$,

$$\frac{dS}{S} = \mu dt \tag{3.2.1}$$

and by integrating the differential equation between 0 and T, we obtain

$$S_T = S_0 e^{\mu T} \tag{3.2.2}$$

To convert between return and stock prices we will use *Eq. 3.2.2* as is illustrated in Hull (2006). The data for the 15 JSE stocks is monthly data and the return is calculated by:

$$R_t = \log\left(\frac{p_t}{p_{t-1}}\right) \quad (3.2.3)$$

Natural logarithms of the monthly data were used in the analysis as taking natural logarithms linearises any type of polynomial and scales both large jumps and small jumps in data removing scale differences, hence proportional changes are modelled as compared to modelling actual changes.

Plots of the particular stock data are then presented together with tables containing interesting and important statistical facts about the 15 JSE stocks. Several hypothesis tests are performed to see: whether or not correlation exists in the squared return, whether or not the returns are stationary (unit root test), whether the returns follow a normal distribution (Jarque-bera test) and to see if heteroscedasticity exists (ARCH test). The different hypothesis tests are discussed below in *Section 3.2.4:*

3.2.2 The Set of Models

A GARCH model consists of two equations: the Conditional Mean equation and the Conditional Variance equation.

Conditional mean equation

The first is the conditional mean equation. This can be anything but since the focus of GARCH model is on the conditional variance equation it is usual to have a very simple conditional mean equation. All the second moment modeling the researcher will undertake in this thesis is done under the assumption that markets are efficient. The Efficient Market Hypothesis (EMH) states that an investor expects a risk free return and additional premium for taking on risk and that all information available up until today is already included in the price. Hence, based on the assumption that stock markets are efficient as envisaged by many researchers in the past including Fama (1965) and Roberts (1967), many of the GARCH models used in practice take the simplest possible conditional mean equation which is given by the formula:

$$R_t = \mu_t + \varepsilon_t \qquad (3.2.4)$$

where

R_t, is the return on the particular day,

μ_t, the drift is the average of all past returns,

ε_t, is the mean deviation return, i.e. compensation for unpredictability of stock markets.

$\Im_t$, denotes the information set at time t, which encompasses X_t and all the past realizations of the process X_t. It should also be noted that the errors ε_t are serially uncorrelated but dependent and the dependence of ε_t can be described by a simple quadratic function of the lagged values of ε_t, specifically $\varepsilon_{t-1}, \varepsilon_{t-2}$.....Care should be taken not to use too many parameters in the conditional mean equation as this will most likely result in convergence problems; hence the simple model above is most appropriate.

Conditional Variance equation

The second equation in a GARCH model is the conditional variance equation. To model the second moment (conditional variance) one can use any of the GARCH models discussed previously in *Section 2.5* since the conditional variance equations are specified in different forms.

3.2.3 Determination of Best Model – Criterion of Fit

The assessment of model quality is typically based on how the models perform when they attempt to reproduce the measured data and this in essence implies the criterion of fit should be based on how well the model can forecast the observed volatility of stock returns. Several criteria have been proposed for selecting time series models in the literature among them the Akaike

Information Criteria of Akaike (1979) and Bayesian Information Criteria (BIC) of Schwarz (1978). In this master thesis we look at these information criteria and also further evaluate the performance of out-of-sample volatility forecasts from different GARCH models using the error of estimates observed in the estimation to assess performance and fit of the models. The goal of this master thesis is to determine the best GARCH model for modelling volatility of stock returns and this will be achieved by evaluating the forecasting performance of various GARCH models when different error measures and utility functions are used and when the true data generating process (DGP) is in fact a GARCH process. We check whether a practitioner can have a good assessment of the accuracy of volatility forecasts using the following measures: The root mean squared error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), the Theil Inequality Coefficient (TIC) and the log-likelihood. In statistics, the MSE, MAE, MAPE and TIC of an estimator are some of the many ways to quantify the amount by which an estimator differs from the true value of the quantity being estimated. The model exhibiting relatively lower error in estimation compared to the others, best models the volatility of the stock returns. A detailed discussion of the different forecast measures is given below:

Root Mean Squared Error (RMSE)

The RMSE provides a quadratic loss function and so may be particularly useful in situations where large forecast errors are disproportionately more serious than smaller errors. A RMSE of zero means that the estimator $\hat{\theta}$ predicts observations of the parameter θ with perfect accuracy. While particular values of RMSE other than zero are meaningless in and of themselves, they may be used for comparative purposes. Two or more statistical models may be compared using their RMSEs as a measure of how well they explain a given set of observations: The unbiased model with the smallest RMSE is generally interpreted as best explaining the variability in the observations. The RMSE is calculated by

$$RMSE = \frac{1}{i+1}\sum_{t=N}^{N+i}(\hat{\sigma}_t^2 - \sigma_t^2) \qquad (3.2.5)$$

where,

i, is the number of steps ahead that we want to predict, in our case i is always equal to 1,

$\hat{\sigma}_t^2$, is the forecast volatility, σ_t^2 is the "true" volatility ε_t^2 and N is the total sample size.

Mean Absolute Error (MAE)

The Mean Absolute Error (MAE) is a quantity used to measure how close forecasts or predictions are to the eventual outcomes. The MAE is calculated by

$$MAE = \frac{1}{i+1}\sum_{t=N}^{N+i}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| \tag{3.2.6}$$

Mean Absolute Percentage Error (MAPE)

Mean absolute percentage error (MAPE) is measure of accuracy in a fitted time series value in statistics, specifically trending. The MAPE is calculated by

$$MAPE = \frac{1}{i+1}\sum_{t=N}^{N+i}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2}\right| \tag{3.2.7}$$

Theil Inequality Coefficient (TIC)

A Theil Inequality Coefficient (TIC) of 1 implies that the model under consideration and the benchmark model are equally inaccurate, while a value of less than 1 implies that the model is superior to the benchmark. It is a scale invariant measure that lies between 0 and 1 where 0 indicates perfect fit. The TIC is calculated by:

$$TIC = \frac{\frac{1}{i+1}\sum_{t=N}^{N+i}\left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2}{\sqrt{\frac{1}{i+1}\sum_{t=N}^{N+i}\hat{\sigma}_t^2} + \sqrt{\frac{1}{i+1}\sum_{t=N}^{N+i}\sigma_t^2}} \tag{3.2.8}$$

3.2.4 Model Validation

After, considering the discussed criterion of fit in *Section 3.2.3* above, one can make a choice on a particular model, which is the one that best describes the data according to the chosen criterion. The task now will be to test if the model is good enough and this is known as model validation. We assess the model performance by conducting out-of-sample forecasts and comparing to the actual observation in the volatility of the given stock over the given period. In assessing our model, several possible reasons can be sited for model deficiency and these include that maybe:

- The data set was not informative enough to provide guidance in model selection.
- The numerical procedure failed to find the best model according to the given criterion.
- The criterion was not determined properly.
- The model set was not adequate as it did not describe the system sufficiently.

In this section we conduct roughly the same hypothesis tests conducted in the pre-estimation stage to see if the autocorrelation in the squared residuals and heteroscedasticity has been removed from the data series as well as normality tests to validate the model. More accurately the ARCH test, autocorrelation test and test for normality to determine if the assumptions for the model are fulfilled, and this is discussed below:

Hypothesis tests and p -values

This section presents the hypothesis tests that will be used in conducting model validation. Statisticians in most cases want to affirm the validity of assumptions about a specific unknown parameter. This is referred to as hypotheses testing. For a specific sample, the *p-value* is the maximal level of significance for which a rejection of the null hypothesis is not possible.

All tests have a corresponding *p-value*. This p-value, under the assumption of the null hypothesis, is the probability of observing the given sample result. At the significance level of 95%, which in our notation corresponds with a critical value of $\alpha = 0.05$, then we reject the null hypothesis if the p-value is lower than this critical value. A *p-value* greater than 0.05, is insufficient evidence for rejecting the null hypothesis. A rule for rejecting the null hypothesis which is equivalent to the previously described decision function is given by the following decision function:

$$\delta\ x\ = \begin{cases} d_1\ if\ \ p < \alpha \\ d_0\ if\ \ p \geq \alpha \end{cases}$$

Below is a discussion of the different hypothesis tests to be conducted for model validation:

Test for Autocorrelation - (Ljung – Box Pierce Q - Test)

There are a large number of tests of randomness (e.g., the runs tests). Autocorrelation plots are one common method test for randomness. The Ljung-Box (1978) test is based on the autocorrelation plot. However, instead of testing randomness at each distinct lag, it tests the overall randomness based on a number of lags. For this reason, it is often referred to as a Portmanteau test.

More formally, the Ljung-Box Pierce Q - test can be defined as follows.

H_0: The data is random (no autocorrelation).

H_1: The data is not random (there is autocorrelation).

Test Statistic:

$$Q_{LB} = (T(T+2)\sum_{k=1}^{m}\frac{\hat{\tau}_k^2}{T-k}$$

where T is the sample size, τ_k is the sample autocorrelation coefficient and m is the maximum lag length.

Significance Level: α =0.05

Critical Region: The hypothesis of randomness is rejected if

$$Q_{LB} > \chi^2_{1-\alpha;m}$$

where χ^2 is the percent point function of the Chi-square distribution.

If Q_{LB} is large then the probability that the process has uncorrelated data, decreases. The null hypothesis for the test is that there exists no correlation and under that hypothesis, Q_{LB} is χ^2 with k degrees of freedom.

Test for Presence of Heteroscedasticity (Arch tests)

Engle (1982) derived a test based on the Lagrange Multiplier principle. The Lagrange multiplier (LM) test can be used to formally test the presence of conditional heteroscedasticity and the evidence of ARCH effects. The test is based on OLS regression, where the OLS residuals u_t from the regression are saved. u_t^2, is thereafter regressed on a constant and its own m- lagged values:

$$u_t^2 = \zeta + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_m u_{t-m}^2 + e_t$$

This is done for all samples $t = 1 \ldots T$. This regression has a corresponding R^2 - value. TR^2, is then asymptotically χ^2-distributed with m degrees of freedom under the null hypothesis that u_t is $i.i.d\ N(0;\sigma^2)$.

Test for Stationarity (Unit Root Test)

The Augmented Dickey – Fuller (ADF) unit root test is used to determine if the time series data is stationary or non-stationary. The test determines if there are any unit roots present in the data. The presence of unit roots is an indication of the non-stationarity of the time series data. The Augmented Dickey –Fuller test can be conducted without intercept and trend; with intercept or with intercept and trend which is the strictest of the tests. Hence, the ADF test with intercept and trend is represented by the formula:

$$\Delta X_t = (\alpha - 1) X_{t-1} + \varepsilon_t$$

If X_t is a random walk, then the coefficient of X_{t-1} is equal to zero.

The Hypothesis to be tested is as follows:

H_0 : $\sigma = 0$ - Series is Non - Stationary (Has Unit Roots)

H_1 : $\sigma \neq 0$ - Series is Stationary (No Unit Roots)

The test is done at a 5% Critical Level

Test for Normality (Jarque – Bera Test)

The Central Limit Theorem states that a sum of many independent observations behaves like a normal random variable. This result is often applied to stock market returns for example, that is a return or change in the stock price is the result of many small influences and shocks and thus the return can be treated as a normal random variable. With this in mind there is only one condition which must be fulfilled for the theorem to apply: The individual random variables must possess finite variances (Rachev 2005). Hence to answer the question whether the data is generated by a normal distribution, the Jarque – Bera test of normality was developed.

The test statistic is given by the expression below:

$$JB = \frac{n}{6}\gamma^2 + \frac{n}{24}(\kappa - 3)^2$$

where

$$\gamma = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^3 \text{, is the sample skewness}$$

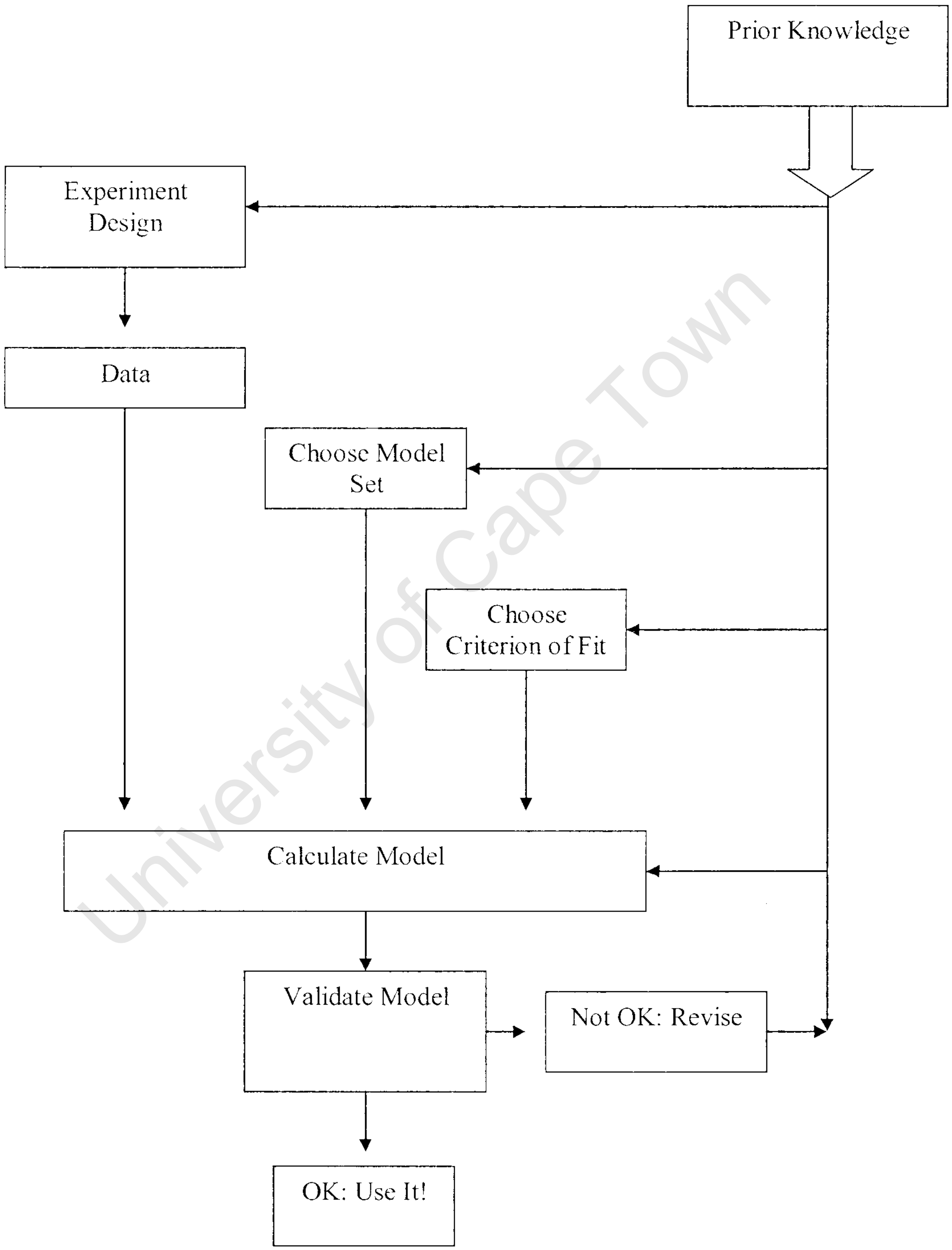
$$\kappa = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^4 \text{ , is the sample kurtosis}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \text{ , is the sample variance}$$

The intuitive feeling about this test is that the larger the Jarque-Bera value is, the lower the probability is that the given series is drawn from a normal distribution. The test statistic of the Jarque-Bera test is χ^2-distributed with 2 degrees of freedom and the null hypothesis is that the series is normally distributed. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being 3, since samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 3. As the definition of the Jarque-Bera shows, any deviation from this increases the Jarque-Bera statistic. To accept the null hypothesis of normality say at the 5% level, the *p-value* given should be greater than 0.05.

Below is an illustration of the how we have gone about system identification in the form of a system identification loop as proposed by Ljung (1987):

Fig 3.2.1 System Identification Loop



3.3 Toolboxes for Estimation & Data Analysis

Over the years different software has been developed to conduct GARCH model estimation. The formulations are generally consistent although different software has different features, functionality and drawbacks. Brooks, Burke & Persand (2001) present several different GARCH estimating software packages and compare them. Amongst their findings is the fact that there could be large differences between estimated GARCH parameter values from different toolboxes. Hence it is of paramount importance to choose an appropriate toolbox to use. The researcher in this thesis has decided to use basically two computer software packages: E-views 5 and MATLAB 7.

As a consequence of its simple and systematic approach to pre-estimation of the data, the researcher will use MATLAB 7 for this purpose. However, the estimation and forecasting will be done in E-views 5, a statistical computer software package geared for statistical purposes only although it has the fault of not being able to estimate MA models for GARCH estimations. It should be noted, however that AR models will be sufficient for this master thesis. Several different error distributions are available in E-views 5 including the normal, student-*t* and generalized error distribution amongst others. Also, a number of different models are available, both for the conditional mean and the conditional variance, as well as a number of tests and forecast possibilities.

Chapter 4. Market Data Analysis & Results

4.1 JSE Data Used in The Thesis

Monthly price values of 15 stocks listed on the Johannesburg Stock Exchange (JSE) are used for the empirical tests. The data represents monthly prices from January 1991 to February 2008. Estimation for two different periods is conducted, that is for a longer period; January 1991 to February 2008 and for a shorter time period; January 1991 to February 2003 to ensure consistency and reliability of results obtained. Results for the longer period; January 1991 to February 2008 are presented in this chapter whereas results for the shorter period; January 1991 to February 2003 are presented in Appendix C for purposes of comparison. The decision to use monthly price values was influenced by the fact that portfolio selection and adjustments of portfolios for risk by fund managers is not conducted on a daily basis as this would incur absurd transaction costs. Hence, the researcher assumes that on average the fund manager significantly adjusts their portfolio based on monthly changes in volatility.

The Cluster sampling technique is used to pick the fifteen JSE stocks. The JSE stocks are clustered into their respective sectors with reference to the different sectors of the South African economy. Fifteen return series from the JSE namely Tongaat Hulett, KWV Beleggings, Nuworld, Tiger Brands, Impala Platinum, Barloworld, Omnia, Sable, Liberty Group, Hudaco, SABmiller, Anglo-American, Rainbow Chicken, FirstRand and SASOL were selected by the researcher to illustrate the modelling and estimation of volatility of stock returns using GARCH models.

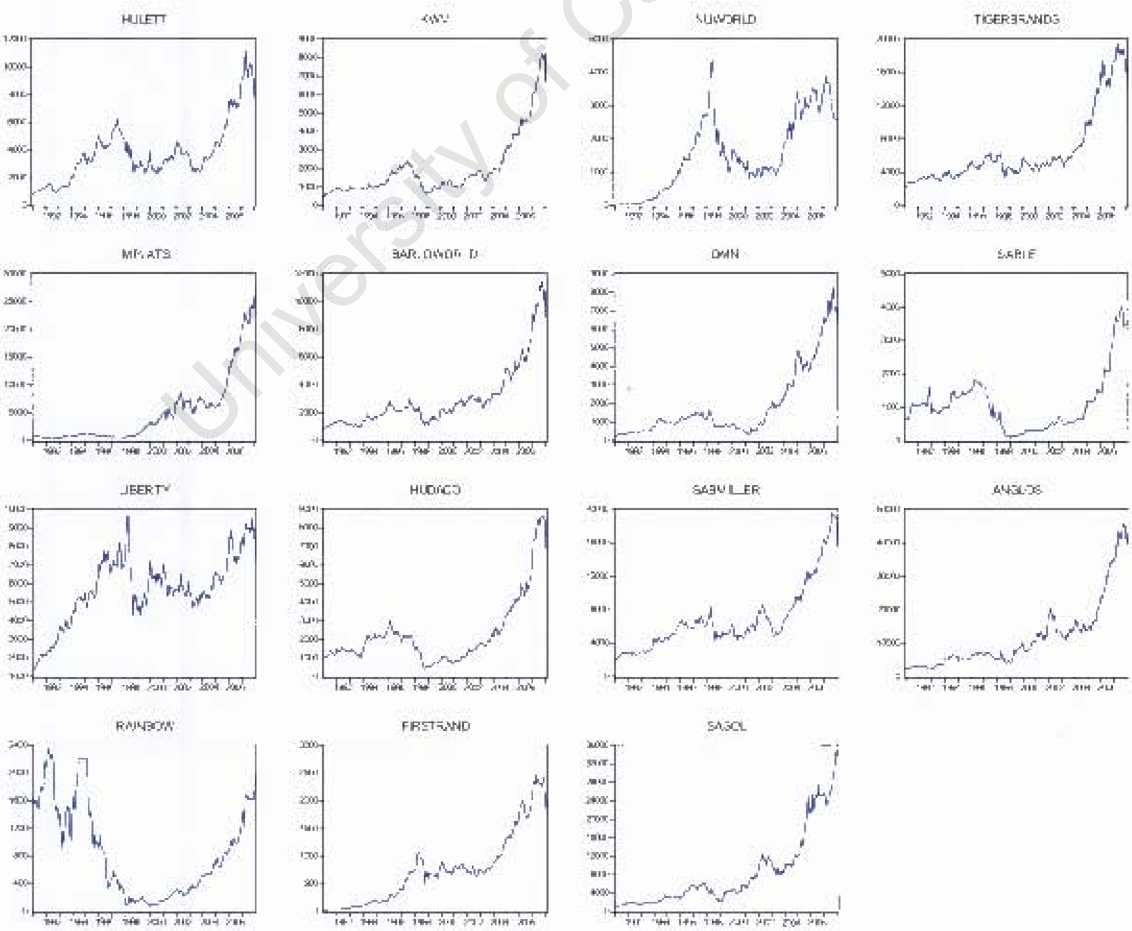
4.2 Pre-estimation

All the 15 data series for the period January 1991 – February 2008 constitute of 206 data points, each data point representing the monthly price of the given stock. For purposes of estimation and validation of the models, data points Jan 1991-February 2006 are used for estimation and data points March 2006-February 2008 are used for validation purposes.

4.2.1 Plots for the Return Series

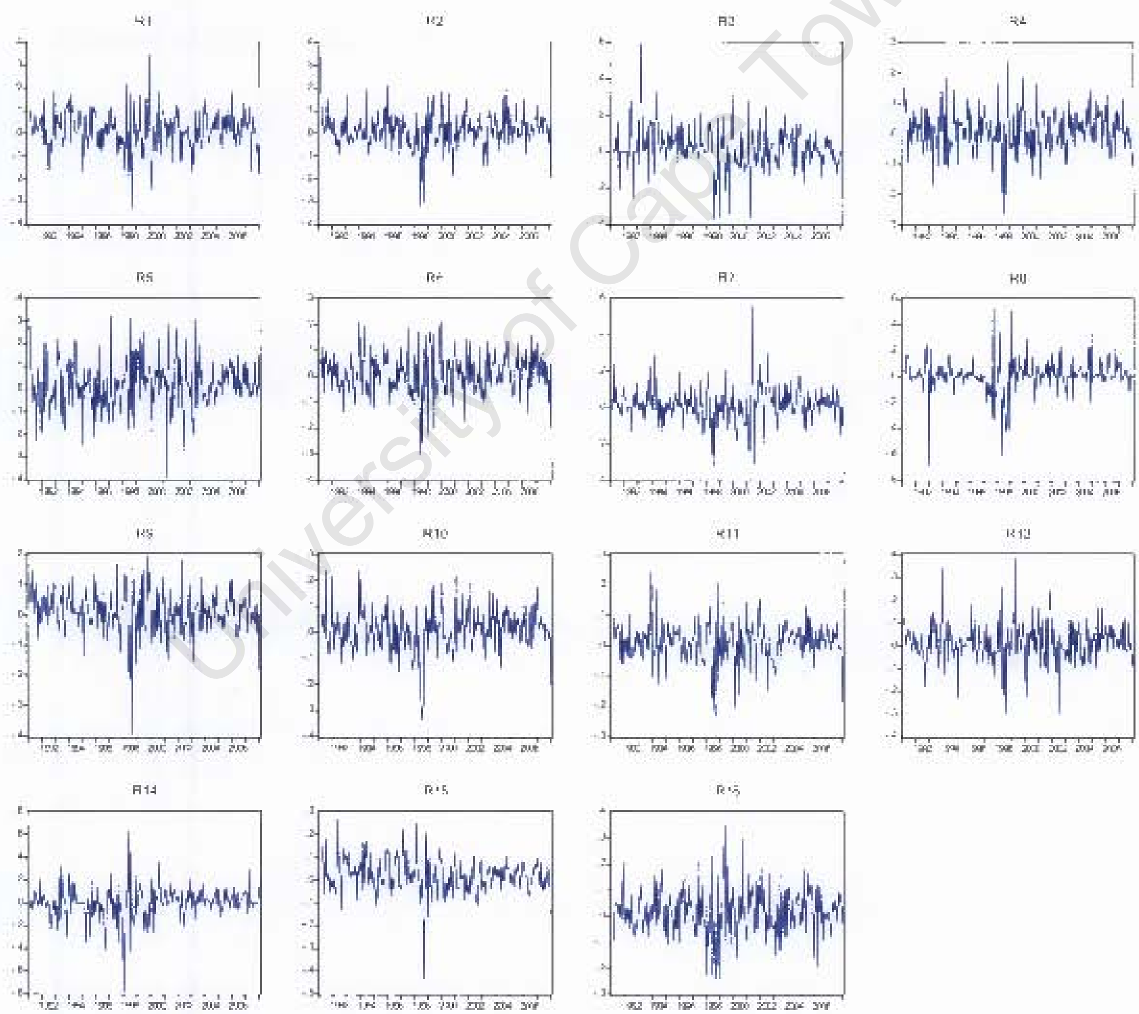
To summarize the statistical properties of prices and returns, firstly visual inspection is done after graphing the prices (levels) and the returns series. Graphical plots of the price series as well as other specific return series plots also give a visual on the behaviour of the specified data over the chosen period. *Figure 4.2.1* gives the price plots, p_t and *Fig. 4.2.2* gives the return plots r_t for the 15 JSE stocks. The figures show the long run movement of monthly p_t and r_t over the past 18 years. The price series for all the four series p_t show an upward trend. Below in *Fig. 4.2.1* are the price series for each of the 15 stocks:

Fig 4.2.1 Monthly Price Plots for the 15 JSE Stocks



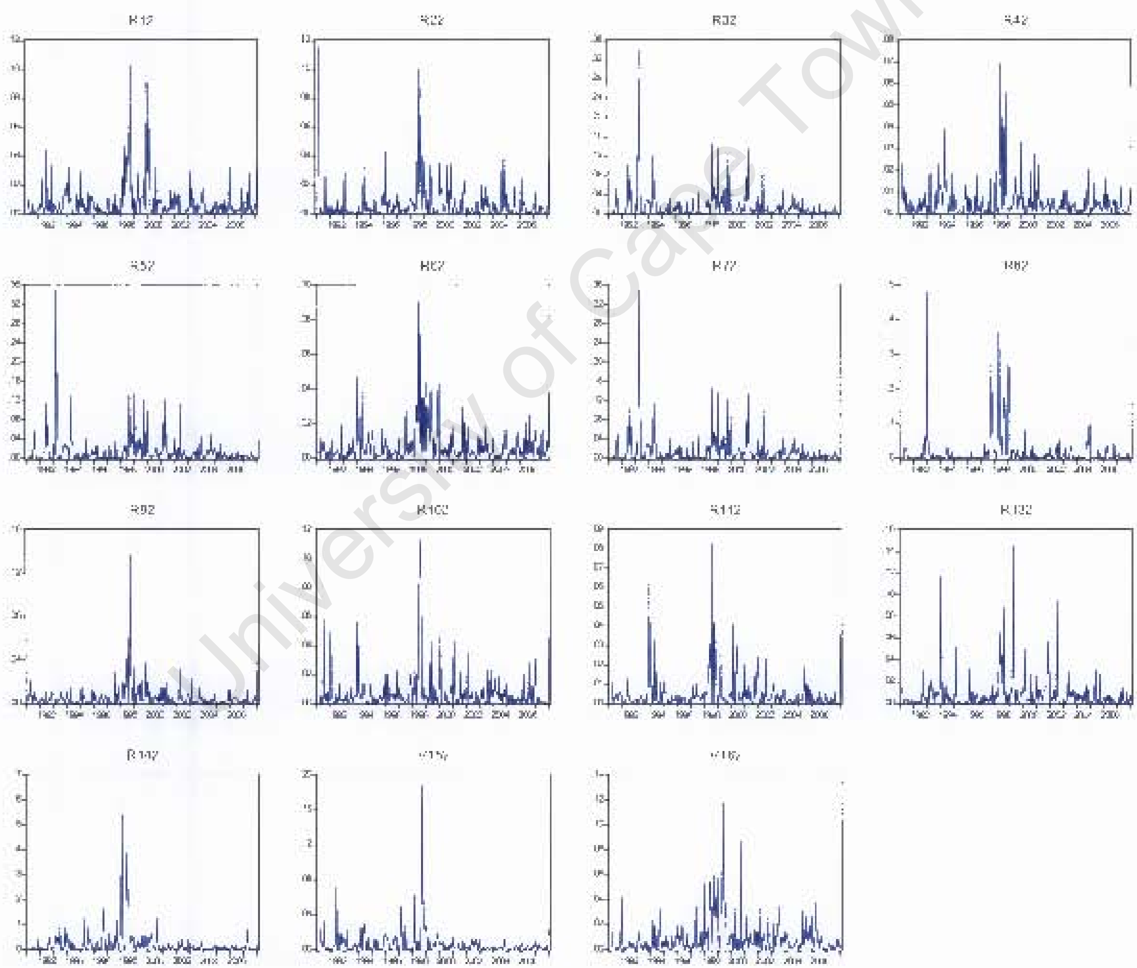
However, taking log differences of the price changes gives us the return series for each of the stocks r_t. One observes that the return series r_t is stable around the mean. Below in *Fig. 4.2.2* are plots of the return series for each of the 15 stocks, Tongaat Hulett, KWV Beleggings, Nuworld, Tiger Brands, Impala Platinum, Barloworld, Omnia, Sable, Liberty Group, Hudaco, SABmiller, Anglo-American, Rainbow Chicken, FirstRand and SASOL respectively.

Fig 4.2.2 Monthly Log-Returns for the 15 JSE Stocks



Further scrutiny into the plots for the squared returns of each of the 15 stocks shown in *Fig. 4.2.3* below reveals that the data exhibits volatility clustering which could indicate a GARCHinnovation process and further to this the data exhibits trends which indicate the presence of autocorrelation. The data exhibits long range dependence as past observations appear to have a persistent impact on future realizations. Graphical presentation of the autocorrelation of the squared residuals is presented in *Fig. 4.2.5* below providing further support to the notion as we observe that there is a high persistence in the second moment for at least 10 lags.

Fig 4.2.3 Squared Returns of the 15 JSE Stocks



4.2.2 Moments of the Return Series

Following this, the relevant descriptive statistics for the returns are computed including the Jarque - Bera test statistic. A theoretical introduction to these measures was given in *Section 3.2.4*. These moments are presented in the *Table 4.2.1* below. The descriptive statistics shown in the table below give us a general picture of what the distribution of the raw return series is like. The moments namely, the mean, standard deviation, skewness and kurtosis enable us to describe the probability distribution of the financial return series.

Table 4.2.1 Descriptive Statistics and Jarque-Bera test statistic

	Mean	Median	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
Hulett	0.010589	0.016024	0.340366	-0.32952	0.091857	-0.280519	4.108069	13.11194	0.001422
KWV	0.013613	0.005333	0.339868	-0.31845	0.085595	-0.113248	5.317026	46.06924	0.000000
Nuworld	0.020845	0.003391	0.592051	-0.38137	0.127893	0.241294	5.787984	68.04884	0.000000
Tiger Brands	0.009642	0.011679	0.237515	-0.26897	0.075442	-0.198536	3.698286	5.484785	0.064416
Impala Platinum	0.019717	0.01655	0.321981	-0.38705	0.122354	-0.004446	3.38024	1.229626	0.540742
Barloworld	0.011227	0.011103	0.218200	-0.30169	0.088095	-0.312981	3.623911	6.639305	0.036165
Omnia	0.015971	0.012528	0.584934	-0.33647	0.107687	0.453834	7.038269	145.6175	0.000000
Sable	0.008406	0.000000	0.526093	-0.69315	0.143468	-0.906664	9.17062	351.6000	0.000000
Liberty	0.008236	0.009215	0.192849	-0.39246	0.077683	-0.781494	6.12463	103.7531	0.000000
Hudaco	0.009397	0.000000	0.244197	-0.33647	0.091217	-0.244466	4.683426	26.12031	0.000002
SABmiller	0.010302	0.016324	0.256361	-0.29199	0.072901	-0.337493	4.98118	37.23577	0.000000
Anglo-American	0.014175	0.017681	0.384624	-0.30714	0.09473	0.059879	5.066306	36.41367	0.000000
Rainbow Chicken	0.000994	0.000000	0.622530	-0.78412	0.155213	-0.612481	7.279176	168.4010	0.000000
FirstRand	0.021419	0.02060	0.280302	-0.43415	0.083485	-0.541246	7.016222	147.0655	0.000000
SASOL	0.015679	0.01261	0.350052	-0.24467	0.098305	0.057985	3.506917	2.298521	0.316871

The sample moments for most the 15 stocks indicate that the empirical distributions have heavy tails and sharp peaks at the center compared to the normal distribution, with the exception of Tiger Brands, Impala Platinum and SASOL which according to the Jarque-Bera test are normally distributed. At the 5% level, the *p-value* for the Jarque – Bera test is 0.05 and anything less than this *p-value* indicates a leptokurtic distribution. Notably the kurtosis is considerably larger than three for all the stocks, which is the expected kurtosis if the return series was drawn from a normal distribution, save that of Tiger Brands, Impala Platinum and SASOL. Financial markets have been observed to exhibit this phenomenon and hence, several leptokurtic distributions have therefore been proposed as more descriptive models than the normal, a leading example being the student's t-distribution.

4.2.3 Hypothesis Tests

Before any probability distribution model is fitted to data, the underlying assumptions of the model have to be verified empirically. Most importantly one has to determine that the data being modelled is stationary to ensure that no problems with spurious regression will be experienced. To test for whether the data is stationary one has to conduct the unit root test. Furthermore, almost all of the popular models of stock returns require that returns be independent random variables, and many also require that they be identically distributed i.e. *i.i.d* .

Unit Root test

Since non- stationarity can be a possible source of nonlinearity in stock returns, we compute the Augmented Dickey Fuller (ADF) test to draw conclusions about the stationarity of the returns. The unit root test is conducted with both trend and intercept where the null hypothesis is that there is a unit root. The results of the ADF test strongly reject the null hypothesis in each of the 15 stocks. It is conclusively observed that all the return series for all the 15 stocks in *Table 4.2.2* below, do not have a unit root either at the 5% level indicating that the data is stationary.

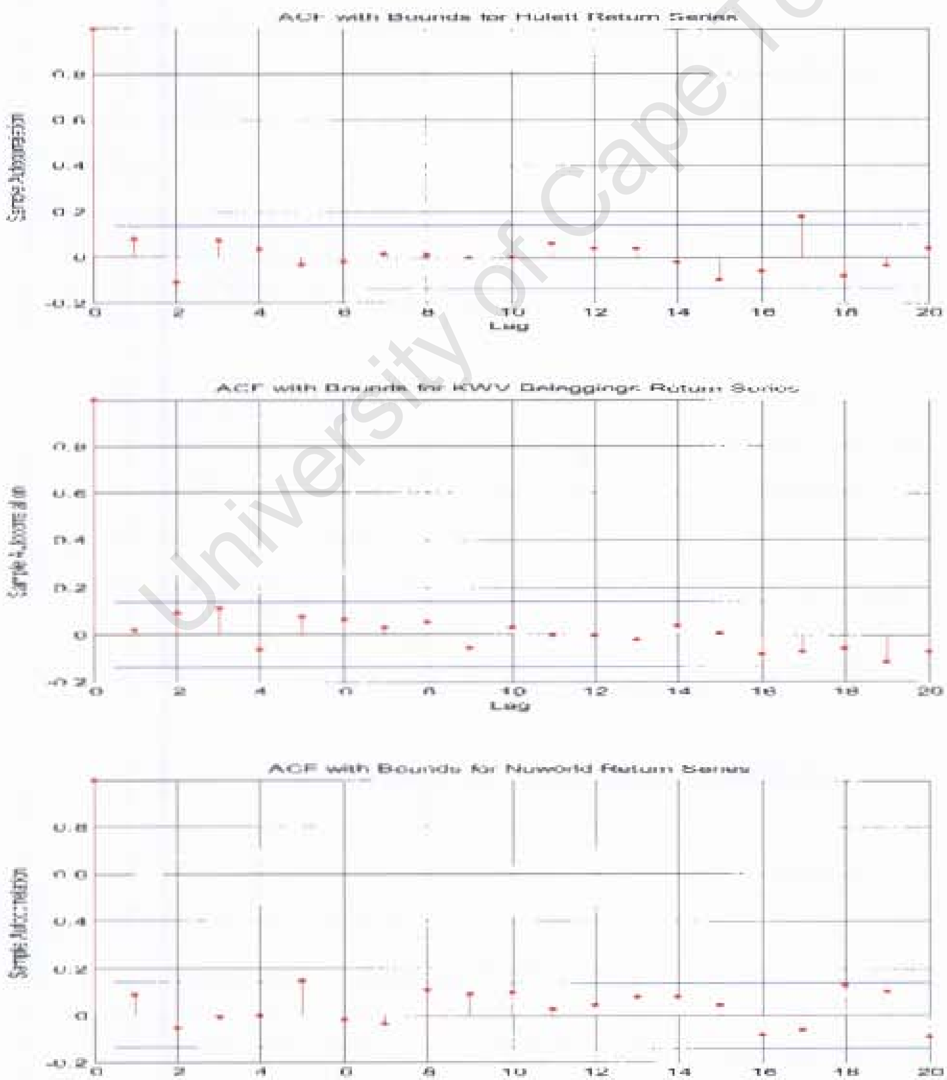
Table 4.2.2 Unit Root Test for the 15 JSE Stocks

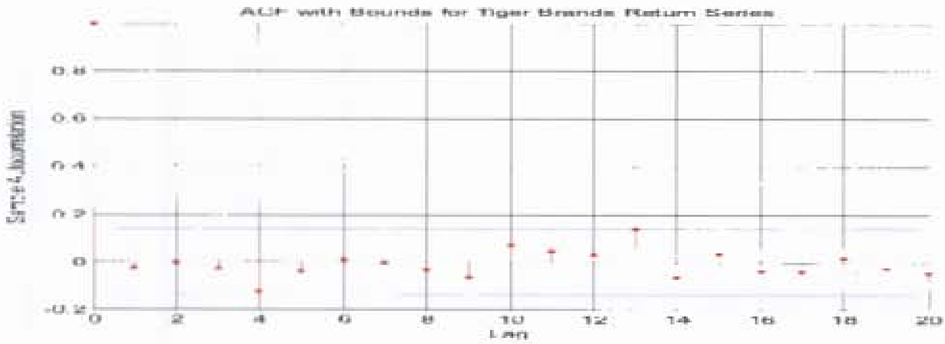
	Augmented Dickey Fuller test	
	ADF t-statistic	**Critical t-statistic at 5% level**
Hulett	-12.74392	**-3.432115**
KWV	-14.10554	**-3.432115**
Nuworld	-13.21427	**-3.432115**
Tiger Brands	-14.70943	**-3.432115**
Impala Platinum	-15.56041	**-3.432115**
Barloworld	-13.74583	**-3.432115**
Omnia	-13.00320	**-3.432115**
Sable	-15.62101	**-3.432115**
Liberty	-14.18154	**-3.432115**
Hudaco	-11.80965	**-3.432115**
SABmiller	-14.17150	**-3.432115**
Anglo-American	-14.88369	**-3.432115**
Rainbow Chicken	-14.27378	**-3.432115**
FirstRand	-10.70807	**-3.432115**
SASOL	-15.62873	**-3.432115**

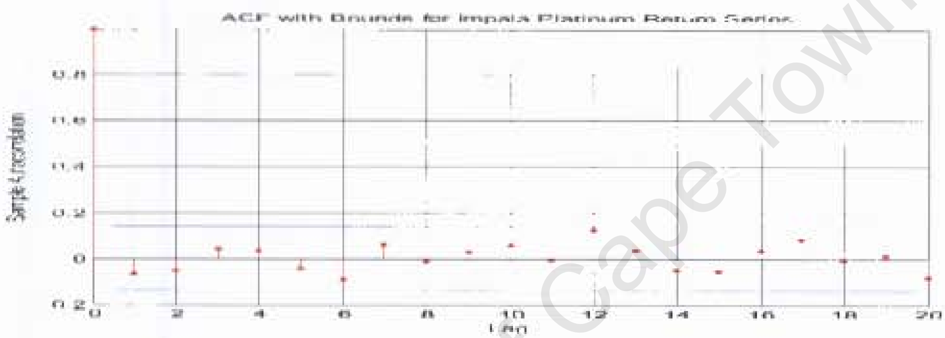
Autocorrelation Tests

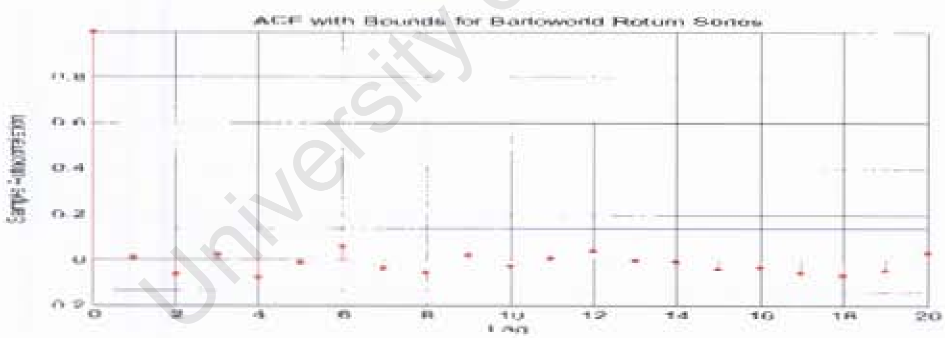
Plots of the autocorrelation correlograms are shown below starting of with plots for the return series of each of the 15 stocks. It can be noted generally that the ACF plots for the return series show no significant autocorrelation. The spikes at each lag clearly lie within the bounds on the autocorrelation plots for the return series shown below in *Fig 4.2.4*.

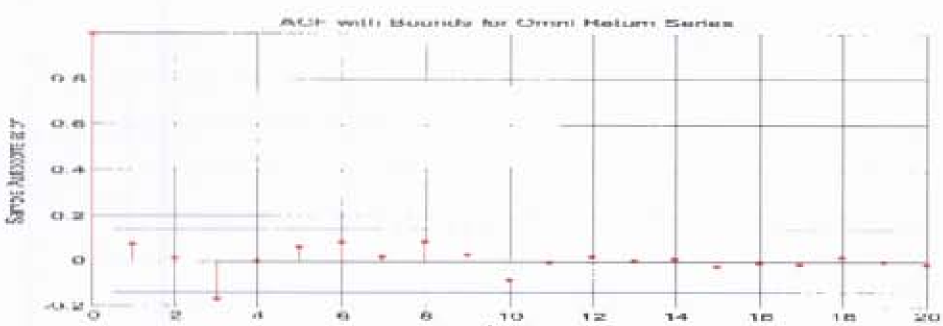
Fig 4.2.4 Autocorrelation Plots for the Return Series of the 15 JSE Stocks

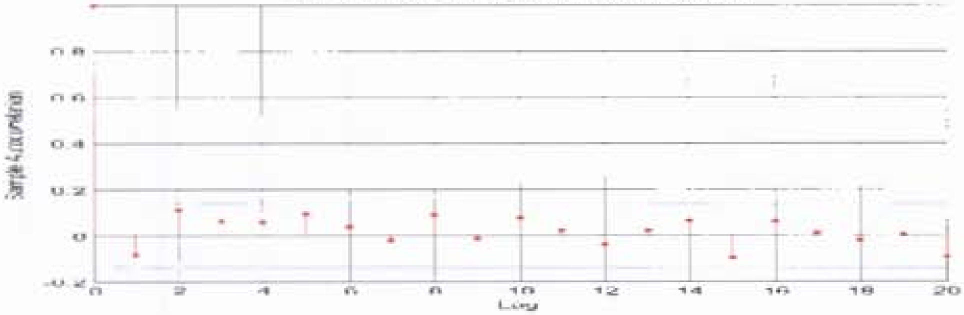


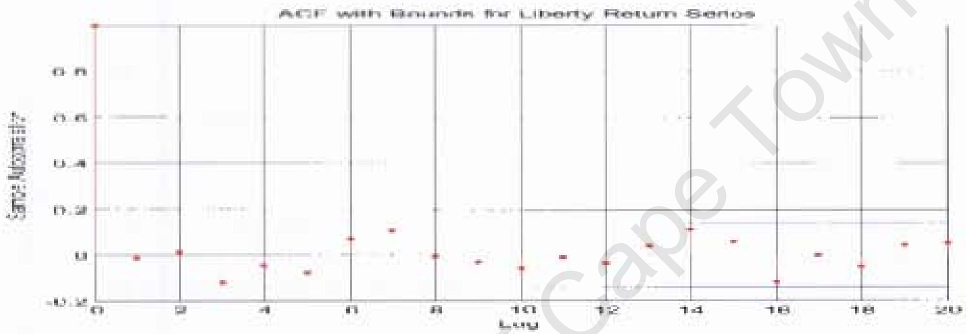

ACF with Bounds for Tiger Brands Return Series
Lag

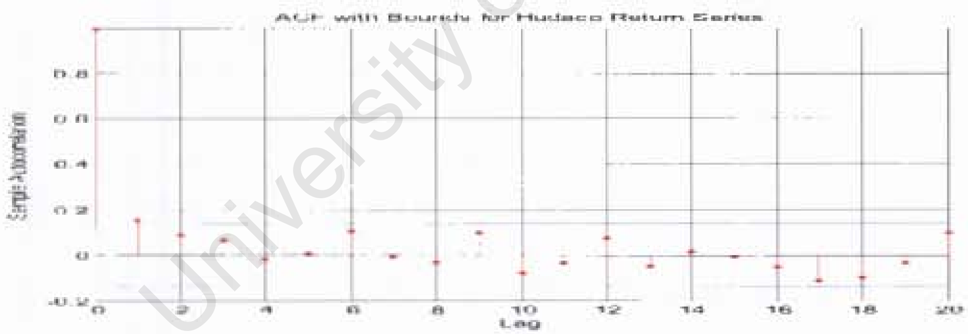

ACF with Bounds for Impala Platinum Return Series
Lag

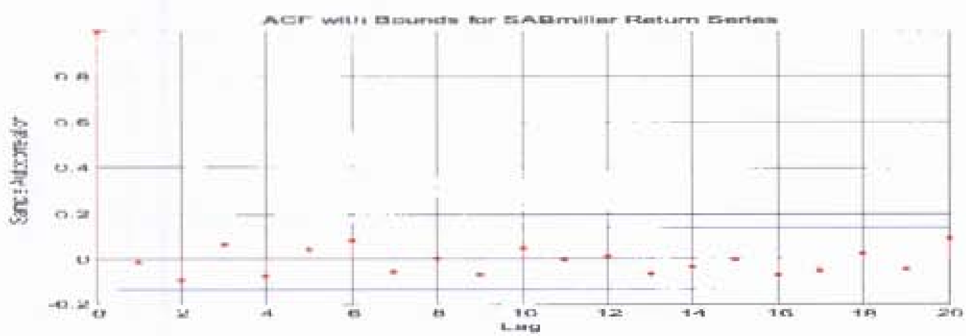

ACF with Bounds for Barloworld Return Series
Lag

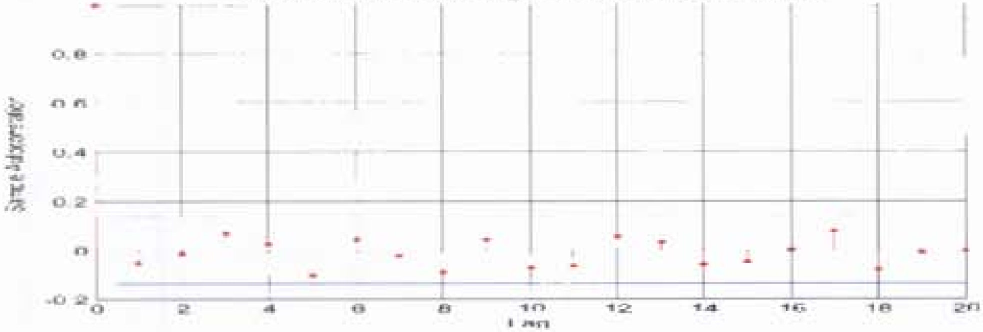

ACF with Bounds for Omni Return Series
Lag

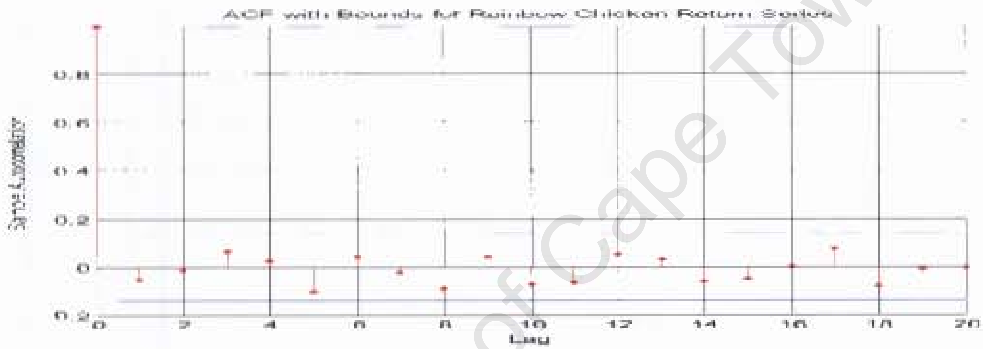

ACF with Bounds for Sabio Return Series
Sample Autocorrelation
Lag

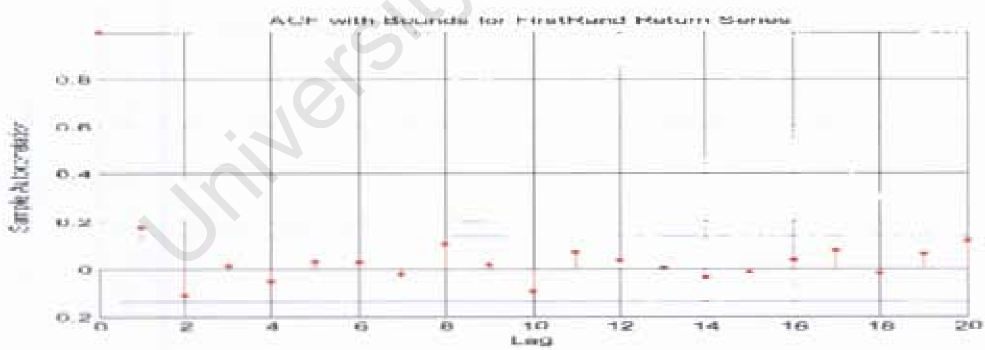

ACF with Bounds for Liberty Return Series
Sample Autocorrelation
Lag

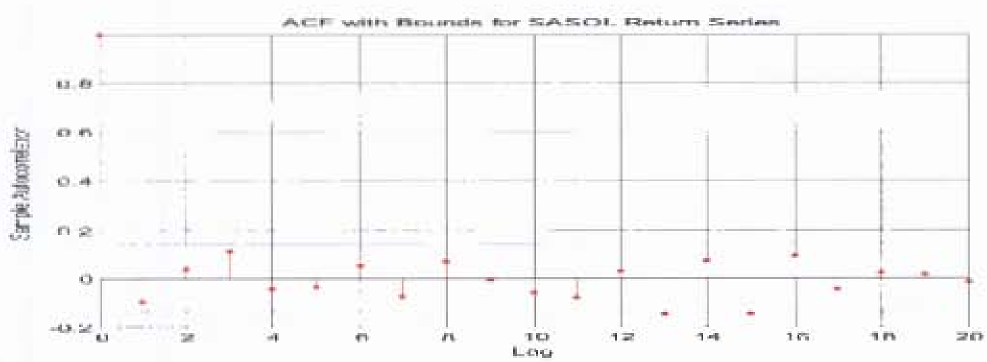

ACF with Bounds for Hudaco Return Series
Sample Autocorrelation
Lag


ACF with Bounds for SABmiller Return Series
Sample Autocorrelation
Lag


ACF with Bounds for Anglo-American Return Series
Sample Autocorrelation
Lag

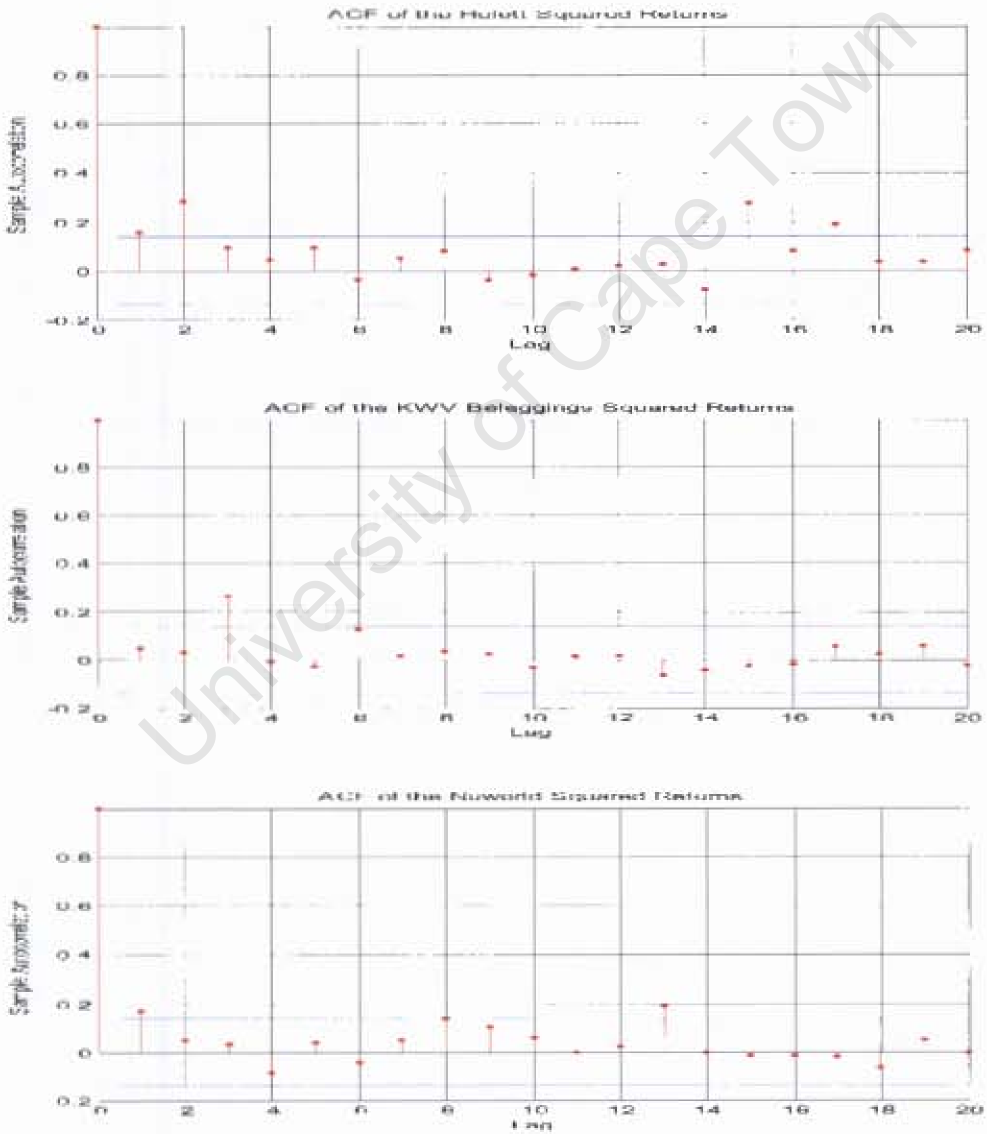

ACF with Bounds for Rainbow Chicken Return Series
Sample Autocorrelation
Lag

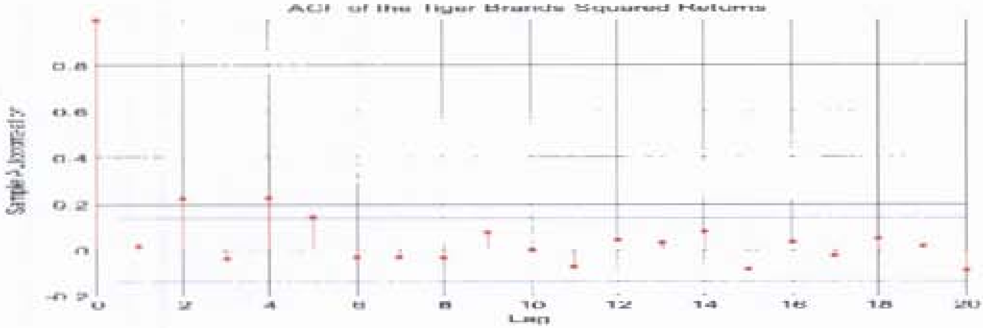

ACF with Bounds for FirstRand Return Series
Sample Autocorrelation
Lag

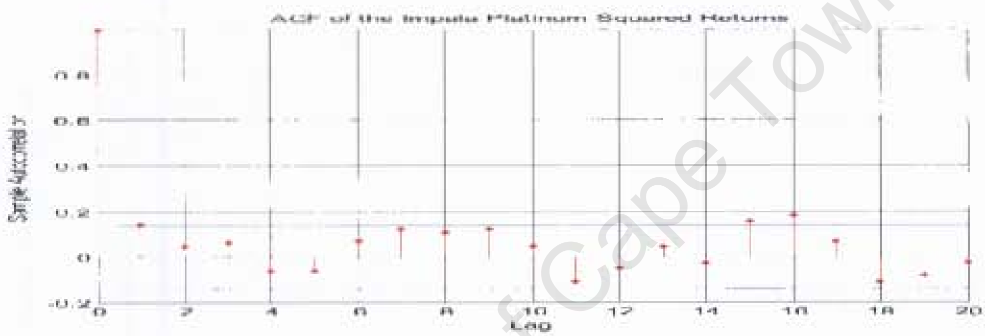

ACF with Bounds for SASOL Return Series
Sample Autocorrelation
Lag

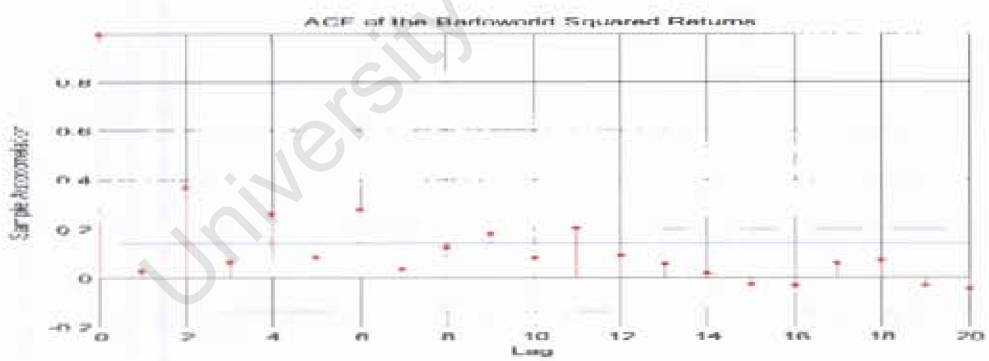
However, the autocorrelation plots for the squared return series of each of the 15 stocks in *Fig 4.2.5* below show significant autocorrelation. This indicates heteroscedasticity, which in other words can be described as the variance of the errors not being constant but varying with time hence the need to implement GARCH modelling.

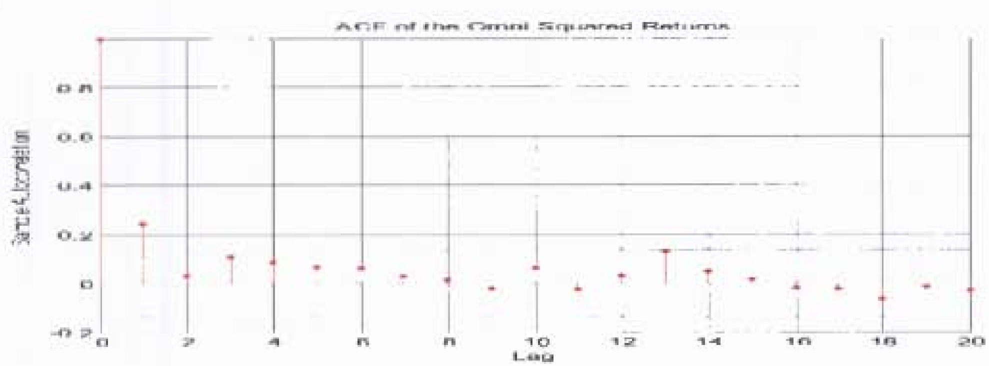
Fig4.2.5 Autocorrelation plots for the Squared Return series of the 15 JSE Stocks

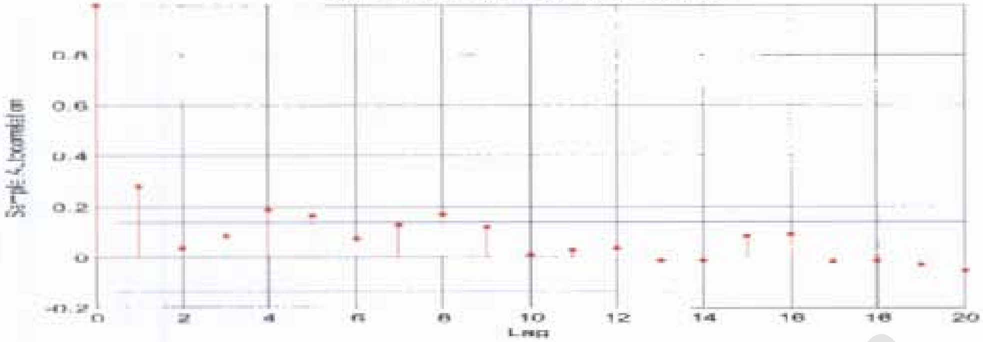


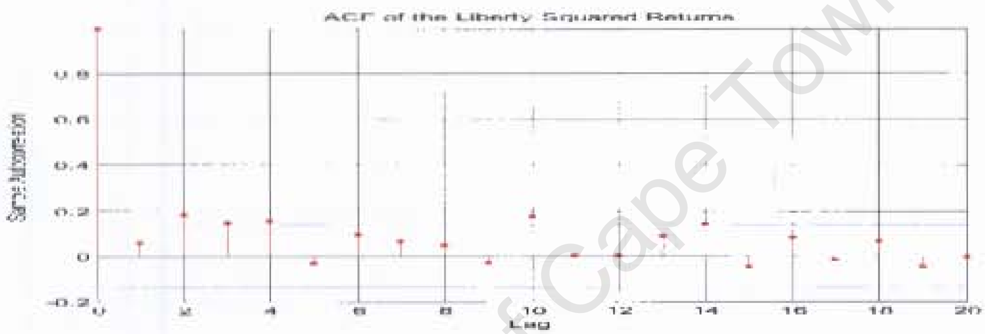

ACF of the Tiger Brands Squared Returns
Sample Autocorrelation
Lag

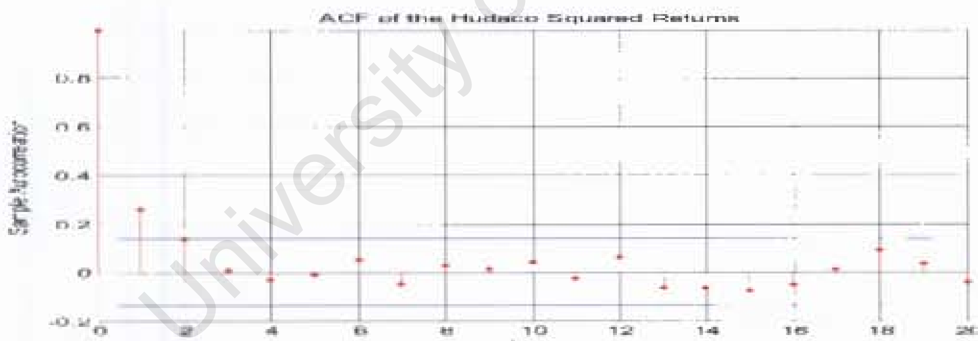

ACF of the Impala Platinum Squared Returns
Sample Autocorrelation
Lag

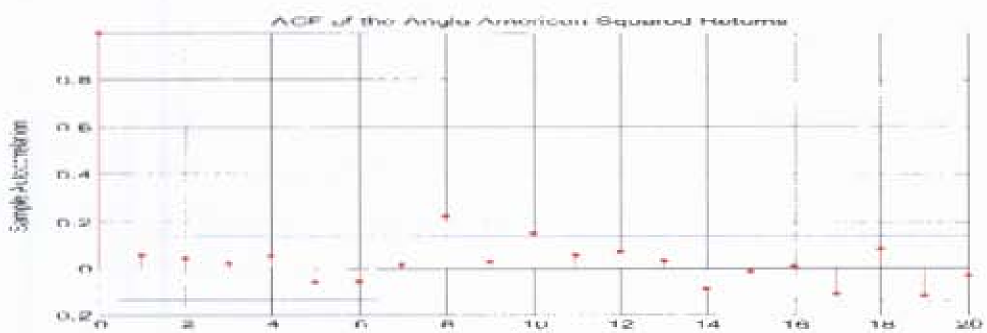

ACF of the Barloworld Squared Returns
Sample Autocorrelation
Lag

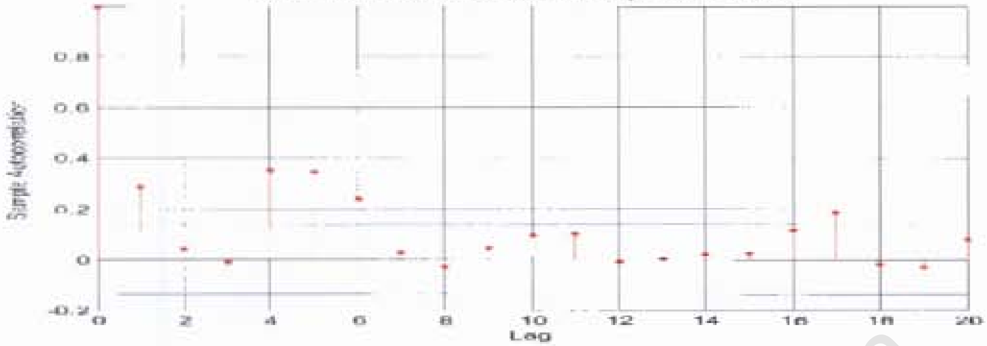

ACF of the Omni Squared Returns
Sample Autocorrelation
Lag

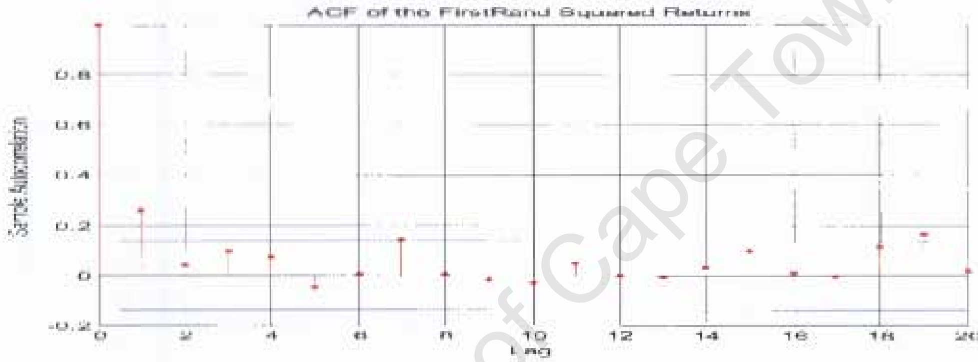

ACF of the Sable Squared Returns
Sample Autocorrelation
Lag

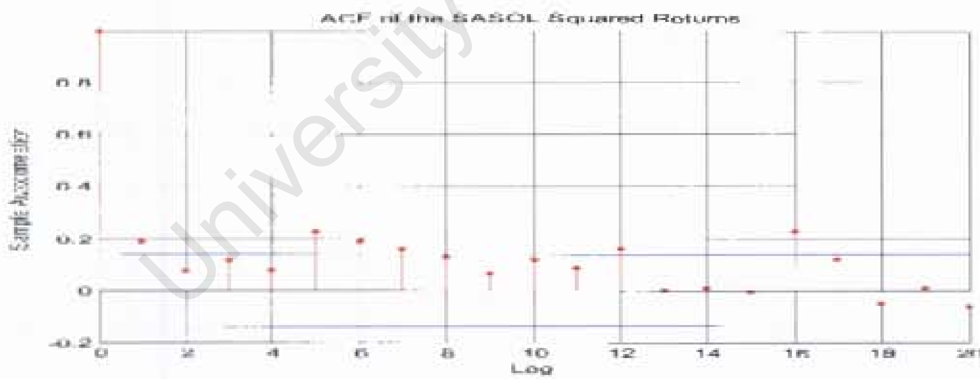

ACF of the Liberty Squared Returns
Sample Autocorrelation
Lag


ACF of the Hudaco Squared Returns
Sample Autocorrelation
Lag


ACF of the Anglo American Squared Returns
Sample Autocorrelation
Lag







For a true white noise process, the ACF of the empirical residuals $\varepsilon_t, t = 1, 2, ...$ and the ACF of the squared empirical residuals should both be zero; if there are moving average components left in the data, the first would significantly different from zero whereas in the presence of GARCH effects, the latter would be significant. Visual inspections of the autocorrelation plots for the squared returns support the notion of heteroscedasticity as they show the presence of GARCH effects whereas the ACF plots for the empirical residuals are not significantly different from zero. The Ljung-Box tests for the empirical residuals of the return series as well as the empirical residuals of the squared returns further confirm the results observed in the plots in *Fig 4.2.4* and *Fig 4.2.5*. The test statistic is calculated for up to 20 lags as reported in *Tables 4.2.3 and 4.2.4*.

Table 4.2.3 Ljung-Box Q test for the Return Series of each of the 15 JSE Stocks

	Q(10) Critical Value=18.3070		***Q(15) Critical Value=24.9958***		***Q(20) Critical Value=31.4104***	
	p-value	**Stat**	**p-value**	**Stat**	**p-value**	**Stat**
Hulett	0.8481	5.5948	0.8640	9.2524	0.4962	19.3963
KWV	0.5407	8.9092	0.8607	9.3108	0.6495	17.0528
Nuworld	0.1876	13.6902	0.2970	17.3733	0.1157	27.7418
Tiger Brands	0.8003	6.1753	0.6738	12.0690	0.8509	13.5848
Impala Platinum	0.8001	6.1781	0.7468	11.0886	0.8081	14.4284
Barloworld	0.9314	4.3266	0.9920	5.0135	0.9932	7.7877
Omnia	0.2485	12.5738	0.6179	12.7974	0.8777	12.992
Sable	0.3649	10.9065	0.4777	14.6397	0.6109	17.6434
Liberty	0.4949	9.3970	0.5641	13.4952	0.5737	18.2706
Hudaco	0.2093	13.2633	0.4275	15.3364	0.2547	23.7232
SABmiller	0.6292	7.9967	0.8572	9.3728	0.8418	13.7339
Anglo-American	0.6393	7.8925	0.7469	11.0805	0.8347	13.9172
Rainbow Chicken	0.8234	5.9018	0.4166	15.4919	0.0678	30.1303
FirstRand	0.1498	14.5393	0.3688	16.2016	0.3446	21.9244
SASOL	0.5454	8.8608	0.1290	21.2504	0.2486	23.8592

Table 4.2.4 Ljung-Box Q test for Squared returns of the 15 JSE Stocks

	Q(10) Critical Value=18.3070		***Q(15) Critical Value=24.9958***		***Q(20) Critical Value=31.4104***	
	p-value	**Stat**	**p-value**	**Stat**	**p-value**	**Stat**
Hulett	0.0004	31.9057	0.0000	47.2829	0.0000	59.3611
KWV	0.0053	25.0222	0.0319	26.6314	0.1031	28.2741
Nuworld	0.1145	15.5120	0.0750	23.4497	0.1575	26.2574
Tiger Brands	0.0005	31.3488	0.0020	35.6719	0.0103	37.4475
Impala Platinum	0.0442	18.7035	0.0438	25.4878	0.0035	41.2475
Barloworld	0.0000	75.9418	0.0000	87.8773	0.0000	90.6816
Omnia	0.0129	22.4568	0.0306	26.7807	0.1041	28.2267
Sable	0.0000	45.6623	0.0000	47.6564	0.0002	50.293
Liberty	0.0022	27.4417	0.0049	32.854	0.0190	32.2154
Hudaco	0.0019	27.8979	0.0089	30.9483	0.0293	33.5551
SABmiller	0.0056	24.8531	0.0372	26.0787	0.0242	34.2943
Anglo-American	0.0150	22.0135	0.0405	25.7739	0.0411	32.2083
Rainbow Chicken	0.0000	83.8200	0.0000	86.2672	0.0000	98.8399
FirstRand	0.0084	23.7300	0.0508	24.938	0.0405	32.2729
SASOL						

Even though the empirical residuals of the returns of the stocks seem to be weakly correlated and not significantly different from zero over time as shown in *Table 4.2.3*, the Ljung–Box statistics for up to the 20^{th} lag of the empirical residuals of the squared returns are highly significant as shown in *Table 4.2.4* suggesting the presence of strong non-linear dependence. The null hypothesis of strict white noise is rejected in all cases, for the squared returns. The conclusion must be that monthly return series are not independently distributed but show dependence in the squared residuals supporting the notion of GARCH effects being present in stock return data.

ARCH Test

Engle's Lagrange Multiplier (LM) test is conducted to formally test for the presence of conditional heteroscedasticity and the presence of ARCH effects. The null hypothesis H_0 is that no ARCH effect exists. Evidently if p-values are ≤ 0.2 we reject the hypothesis. The LM test is conducted for 10 lags, 15 lags as well as 20 lags. The results in *Table 4.2.5* below suggests that all the 15 stock returns exhibit ARCH effects as the null hypothesis of no ARCH is rejected for at least two particular lags.

Table 4.2.5 ARCH test for the Return Series of the 15 JSE Stocks

	ARCH 1-10 test		ARCH 1-15 test		ARCH 1-20 test	
	Prob.	F(10,183)	Prob.	F(15,173)	Prob.	F(20,163)
Hulett	0.009507	2.435368	0.000259	3.011566	0.000241	2.738907
KWV	0.000003	4.902678	0.000115	3.197587	0.000134	2.852285
Nuworld	0.220232	1.324277	0.178226	1.348646	0.068634	1.559538
Tiger Brands	0.00069	3.251592	0.004585	2.335594	0.035029	1.717837
Impala Platinum	0.093427	1.658956	0.044708	1.755167	0.016912	1.879974
Barloworld	0.000001	5.351751	0.000001	4.219717	0.000004	3.517913
Omnia	0.041469	1.947559	0.118747	1.475782	0.297849	1.157572
Sable	0.000481	3.361324	0.00538	2.2967	0.037945	1.699513
Liberty	0.018693	2.215562	0.081379	1.587443	0.236242	1.229657
Hudaco	0.000936	3.158765	0.002274	2.504091	0.010276	1.986957
SABmiller	0.027666	2.085149	0.144619	1.41515	0.068149	1.561264
Anglo-American	0.024392	2.127318	0.116039	1.482762	0.093364	1.483299
Rainbow Chicken	0.000000	7.474416	0.000000	4.973332	0.00000	4.682177
FirstRand	0.011282	2.38023	0.059897	1.674473	0.07678	1.532079
SASOL	0.000665	3.262911	0.001828	2.555982	0.003624	2.203367

Therefore having established: that the data is stationary, presence of autocorrelation in the squared returns and the presence of heteroscedasticity through the ARCH test one can go on to model the volatility of the data. Such behavior can be captured by incorporating GARCH structures in the model, allowing conditional heteroscedasticity by conditioning the volatility of the process on past information.

4.3 Estimation

This section focuses on estimating model parameters and examining the estimated GARCH model. The presence of heteroscedasticity, shown in the previous analysis, indicates that GARCH modeling is appropriate. In order to evaluate in-sample and out-of-sample results, the data sample is divided in two parts. One part is used for the in-sample models' estimation; the other part is for the models' performance out-of-sample evaluation. As stated earlier we consider data points Jan. 1991 – Feb. 2006 to estimate the models for the 15 stocks. For the out-of-sample analysis we use the remaining observations from March 2006 – February 2008.

4.3.1 Determining the Best GARCH model from symmetric GARCH, TARCH, EGARCH and PARCH.

To determine the best model we compare all four models, namely the symmetric GARCH, TARCH, EGARCH and PARCH with the ARMA p and q orders being equal, that is $p=1$ and $q=1$, using the same error distribution i.e. the normal distribution. To compare unconditional and conditional in-sample fitted models, we employ three likelihood based goodness-of-fit criteria. The first is the maximum log-likelihood value obtained from ML estimation. This value allows us to judge which model is more likely to have generated the data. The second is the Akaike information criteria (AIC) and the third is the Schwarz Bayesian Criterion. The distribution with a lower value for these information criteria is judged to be preferable. Based on the estimation performance measures such as the R^2, AIC and SIC together with the forecasting performance measures such as the MAPE, MSE, MAE, etc we determine the best model. Below, the performance measures for the 4 different models for each of the 15 stocks are presented in tabular form *(Table 4.3.1 – 4.3.15)*. The superior performance measures are highlighted in bold.

Table 4.3.1 GARCH model selection for Tongaat Hulett

Hulett	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	186.4823	186.6122	186.3834	187.4897
AIC	-2.203486	-2.192743	-2.189919	-2.191231
SIC	-2.051012	-2.02121	-2.018386	-2.000638
Adj. R^2	**0.275383**	0.270639	0.273377	0.262617
MSE	0.076789	0.076639	0.076667	**0.076605**
MAE	0.062974	0.062987	0.062974	**0.062742**
MAPE	174.6769	173.9575	**172.9172**	174.3674
TIC	0.646027	0.646352	0.649746	**0.644939**

Table 4.3.2 GARCH model selection for KWV Beleggings

KWV Beleggings	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	173.5348	176.1268	177.7016	176.366
AIC	-2.233797	-2.255024	-2.276021	-2.24488
SIC	-2.113372	-2.114528	-2.135525	-2.084313
Adj. R^2	**0.186176**	0.181204	0.180908	0.175787
MSE	0.055953	0.055711	**0.055432**	0.055672
MAE	**0.041661**	0.041871	0.041912	0.041734
MAPE	243.8976	214.5748	**212.431**	221.9693
TIC	**0.535014**	0.556494	0.555086	0.551118

Table 4.3.3 GARCH model selection for Nuworld

Nuworld	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	119.0266	123.4867	125.1231	112.071
AIC	-1.387832	-1.431084	-1.451539	-1.275887
SIC	-1.234074	-1.258106	-1.278561	-1.083689
Adj. R^2	0.102346	0.110213	**0.111383**	0.109249
MSE	0.083721	0.079828	0.077597	**0.07439**
MAE	0.070211	0.066397	0.06448	**0.06069**
MAPE	1390.841	1345.147	1202.75	**1072.47**
TIC	0.697038	0.694928	**0.692158**	0.711684

Table 4.3.4 GARCH model selection for Tiger Brands

Tiger Brands	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	181.791	186.4864	184.844	-
AIC	-2.378108	-2.428386	-2.40604	-
SIC	-2.235707	-2.265642	-2.243296	-
Adj. R^2	**0.091286**	0.081097	0.074448	-
MSE	**0.056131**	0.057185	0.060255	-
MAE	0.049204	0.046858	**0.04617**	-
MAPE	107.4075	**99.21949**	117.0701	-
TIC	**0.661438**	0.69014	0.672092	-

Table 4.3.5 GARCH model selection for Impala Platinum

Impala Platinum	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	117.5419	117.6784	116.6806	120.2007
AIC	-1.460558	-1.449045	-1.435741	-1.469343
SIC	-1.299991	-1.268407	-1.255103	-1.268634
Adj. R^2	**0.186374**	0.180511	0.180176	0.170957
MSE	0.084762	0.084192	**0.081938**	0.086679
MAE	0.068767	0.068018	**0.066254**	0.070062
MAPE	319.9657	315.9537	**302.3008**	332.1272
TIC	0.687167	0.691557	**0.683503**	0.697032

Table 4.3.6 GARCH model selection for Barloworld

Barloworld	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	180.3548	183.2348	179.0855	180.0279
AIC	-2.298065	-2.323131	-2.267807	-2.267038
SIC	-2.137497	-2.142493	-2.087169	-2.066329
Adj. R^2	**0.310041**	0.298441	0.304703	0.299267
MSE	0.082139	**0.08199**	0.082741	0.082274
MAE	**0.070169**	0.070615	0.07148	0.070912
MAPE	**167.4161**	197.9977	198.9947	196.683
TIC	0.624538	**0.602839**	0.606193	0.605212

Table 4.3.7 GARCH model selection for Omnia

Omnia	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	157.2999	160.0237	158.5031	161.8241
AIC	-1.709603	-1.729082	-1.7119	-1.738125
SIC	-1.601937	-1.603471	-1.586289	-1.59457
Adj. R^2	**0.0543**	0.047731	0.048859	0.040383
MSE	0.067388	0.067578	**0.067344**	0.06788
MAE	0.051026	0.051023	**0.050893**	0.05113
MAPE	176.7087	173.8494	**170.5058**	175.9197
TIC	0.687301	0.689786	0.695119	**0.686158**

Table 4.3.8 GARCH model selection for Sable

Sable	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	115.7609	116.3899	115.5861	116.8807
AIC	-1.359512	-1.313817	-1.304074	-1.307645
SIC	-1.224973	-1.163225	-1.153483	-1.13823
Adj. R^2	**0.011975**	-0.007188	0.003805	-0.021255
MSE	**0.08261**	0.084712	0.084386	0.083344
MAE	**0.055158**	0.057037	0.057939	0.056407
MAPE	**47.40151**	55.12572	58.95444	52.83875
TIC	0.729479	0.738111	**0.71771**	0.721358

Table 4.3.9 GARCH model selection for Liberty Group

Liberty Group	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	231.0262	232.1254	231.5168	232.8757
AIC	-2.514259	-2.515368	-2.508568	-2.512577
SIC	-2.40742	-2.390721	-2.383922	-2.370124
Adj. R^2	**0.168189**	0.165141	0.164954	0.159941
MSE	0.059185	**0.058552**	0.058781	0.058641
MAE	0.045196	**0.045036**	0.045063	0.045092
MAPE	148.2132	148.0383	**147.0674**	149.4846
TIC	0.603428	0.5977	0.601059	**0.596344**

Table 4.3.10 GARCH model selection for Hudaco

Hudaco	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	168.3663	171.7468	173.4796	172.8403
AIC	-2.122435	-2.153552	-2.176204	-2.154775
SIC	-2.003595	-2.014905	-2.037556	-1.996321
Adj. R^2	**0.204572**	0.19749	0.194831	0.191023
MSE	**0.071445**	0.072305	0.072787	0.072439
MAE	**0.056784**	0.057319	0.0573	0.057181
MAPE	147.0846	143.7733	**138.6511**	140.679
TIC	**0.562158**	0.579617	0.595778	0.587436

Table 4.3.11 GARCH model selection for SABmiller

SABmiller	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	196.8159	197.2979	193.7356	198.7953
AIC	-2.514117	-2.507257	-2.460075	-2.513844
SIC	-2.374244	-2.347402	-2.300219	-2.334007
Adj. R^2	**0.188112**	0.182934	0.181701	0.175909
MSE	0.049914	0.049812	**0.04946**	0.050153
MAE	0.03608	0.035892	**0.034926**	0.036405
MAPE	233.0444	226.2287	**197.3068**	234.4957
TIC	0.569769	0.57109	0.582361	**0.565003**

Table 4.3.12 GARCH model selection for Anglo-American

Anglo-American	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	153.7573	158.0448	154.4932	155.7173
AIC	-1.996697	-2.041426	-1.993105	-1.996154
SIC	-1.854296	-1.878681	-1.83036	-1.813066
Adj. R^2	**0.272451**	0.24554	0.266302	0.257628
MSE	0.068871	**0.0658**	0.069292	0.068664
MAE	0.056972	**0.053633**	0.05774	0.056971
MAPE	218.3718	**189.117**	227.0171	213.8075
TIC	0.643799	0.657728	**0.636161**	0.645939

Table 4.3.13 GARCH model selection for Rainbow Chicken

Rainbow Chicken	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	87.08226	102.8859	101.5179	94.61627
AIC	-0.994874	-1.176514	-1.159729	-1.062776
SIC	-0.880994	-1.043654	-1.026868	-0.910936
Adj. R^2	**0.11202**	0.080623	0.085414	0.10313
MSE	0.101059	**0.099602**	0.100237	0.102847
MAE	0.077495	**0.076215**	0.076919	0.079123
MAPE	197.8877	**150.3432**	158.6468	192.7316
TIC	**0.767798**	0.838786	0.823289	0.784976

Table 4.3.14 GARCH model selection for FirstRand

FirstRand	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	195.8775	195.9774	195.8329	195.0801
AIC	-2.556157	-2.54391	-2.541944	-2.518097
SIC	-2.393412	-2.360822	-2.358857	-2.314666
Adj. R^2	**0.380308**	0.371931	0.379725	0.367906
MSE	**0.051647**	0.054092	0.052905	0.054486
MAE	**0.039889**	0.041792	0.040996	0.041927
MAPE	**389.4368**	405.276	410.0387	399.1397
TIC	0.476635	**0.473483**	0.466436	0.478357

Table 4.3.15 GARCH model selection for SASOL

SASOL	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	160.7161	162.9382	160.8786	
AIC	-1.851104	-1.865918	-1.840953	
SIC	-1.700512	-1.696503	-1.671537	
Adj. R^2	**0.130354**	0.112221	0.122593	
MSE	0.077065	**0.074613**	0.076296	
MAE	0.053648	**0.052702**	0.053582	
MAPE	142.8048	**129.8759**	128.316	
TIC	**0.721494**	0.746571	0.741512	

Above in *Tables 4.3.1 - 4.3.15* the Log-likelihood together with Akaike's information criterion (AIC) as well as the Schwarz Criterion for $p=q=1$ are presented. Four different forecast measures are also presented to determine which model performs best on out-of-sample data. It is observed that 6 out of the 15 stocks, namely KWV Beleggings, Tiger Brands, Barloworld, Sable, Hudaco and FirstRand are best modelled by the symmetric GARCH; 4 stocks namely Liberty Group, Anglo-American, Rainbow Chicken and SASOL by the TARCH model; 3 namely Impala Platinum, Omnia and SABmiller by the EGARCH model and the remaining 2 namely Tongaat Hulett and Nuworld by the PARCH model. Evidently, from these results the superior model for modelling volatility of the stock returns from the selected sample, amongst the chosen GARCH models, is the symmetric GARCH model as it best models the volatility of the most stocks in the sample.

4.3.2 Determining Best Lag Structure for the Symmetric GARCH model

Having determined that the symmetric GARCH model best describes most of the stocks in the sample's return volatility we discard of the 3 other GARCH models and further investigate the best lag structure for the standard GARCH model. The best lag structure can be obtained by altering the ARMA p and q orders of the symmetric GARCH (p,q). Hence we investigate the symmetric GARCH under different lag structures namely the GARCH (1,1), GARCH (1,2), GARCH (2,1) and GARCH (2,2) for each of the 15 stocks. Based on the forecasting ability of each of these 4 lag structures, we determine the best lag structure for our modelling purposes. Below are the performance measures for the 15 different stocks based on forecasting ability for each of the 15 stocks presented in tabular form *(Table 4.3.16 – 4.3.30)*.

Table 4.3.16 Symmetric GARCH model selection for Tongaat Hulett

Hulett	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	186.4823	186.7646	186.5187	186.7657
AIC	-2.203486	-2.194625	-2.191588	-2.182293
SIC	-2.051012	-2.023092	-2.020055	-1.9917
Adj. R^2	**0.275383**	0.267413	0.270157	0.26272
MSE	0.076789	**0.076678**	0.076774	0.07668
MAE	0.062974	**0.062921**	0.062957	0.062925
MAPE	174.6769	174.81	**174.4967**	174.8182
TIC	0.646027	**0.643981**	0.646144	0.64401

Table 4.3.17 Symmetric GARCH model selection for KWV Beleggings

KWV Beleggings	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	173.5348	175.1661	173.5487	175.2271
AIC	-2.233797	-2.242215	-2.22065	-2.229695
SIC	-2.113372	-2.101719	-2.080154	-2.069128
Adj. R^2	**0.186176**	0.181136	0.180671	0.175386
MSE	0.055953	0.055634	0.05594	**0.055612**
MAE	**0.041661**	0.041661	0.041663	0.041586
MAPE	243.8976	231.7571	243.2467	**229.6805**
TIC	**0.535014**	0.543877	0.535262	0.545571

Table 4.3.18 Symmetric GARCH model selection for Nuworld

Nuworld	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	119.0266	119.1581	119.656	120.2471
AIC	-1.387832	-1.376976	-1.3832	-1.378088
SIC	-1.234074	-1.203997	-1.210222	-1.18589
Adj. R^2	**0.102346**	0.095718	0.094288	0.087856
MSE	0.083721	0.083493	**0.082641**	0.082669
MAE	0.070211	0.070048	**0.069368**	0.069368
MAPE	1390.841	1373.532	1307.044	**1295.069**
TIC	0.697038	0.69675	**0.696596**	0.697168

Table 4.3.19 Symmetric GARCH model selection for Tiger Brands

Tiger Brands	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	181.791	181.9461	181.7911	182.8576
AIC	-2.378108	-2.366614	-2.364505	-2.365409
SIC	-2.235707	-2.203869	-2.20176	-2.182322
Adj. R^2	**0.091286**	0.085086	0.084759	0.078236
MSE	**0.056131**	0.06017	0.060258	0.060376
MAE	0.049204	**0.049125**	0.049206	0.049402
MAPE	107.4075	107.8504	**107.3863**	110.3851
TIC	**0.661438**	0.661787	0.661467	0.663276

Table 4.3.20 Symmetric GARCH model selection for Impala Platinum

Impala Platinum	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	117.5419	121.1679	120.7547	121.3583
AIC	-1.460558	-1.495572	-1.490063	-1.484778
SIC	-1.299991	-1.314934	-1.309425	-1.284069
Adj. R^2	**0.186374**	0.154842	0.17794	0.169909
MSE	0.084762	**0.077547**	0.087407	0.088524
MAE	0.068767	**0.063178**	0.070949	0.071607
MAPE	319.9657	**266.5945**	336.6269	339.5463
TIC	0.687167	**0.675795**	0.691885	0.696677

Table 4.3.21 Symmetric GARCH model selection for Barloworld

Barloworld	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	180.3548	182.4066	181.8849	182.8621
AIC	-2.298065	-2.312088	-2.305131	-2.304828
SIC	-2.137497	-2.13145	-2.124493	-2.104119
Adj. R^2	**0.310041**	0.305864	0.304998	0.299453
MSE	**0.082139**	0.082345	0.082443	0.082457
MAE	**0.070169**	0.070591	0.070756	0.070716
MAPE	**167.4161**	185.2082	182.2717	186.9408
TIC	0.624538	0.629506	**0.614549**	0.621495

Table 4.3.22 Symmetric GARCH model selection for Omnia

Omnia	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	157.2999	158.3245	157.8139	158.4961
AIC	-1.709603	-1.709882	-1.704112	-1.700521
SIC	-1.601937	-1.584271	-1.578502	-1.556966
Adj. R^2	**0.0543**	0.047184	0.048056	0.041837
MSE	**0.067388**	0.067477	0.067391	0.067502
MAE	**0.051026**	0.051298	0.051108	0.0513
MAPE	**176.7087**	181.4447	178.5878	181.8815
TIC	0.687301	0.681492	0.68496	**0.680983**

Table 4.3.23 Symmetric GARCH model selection for Sable

Sable	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	115.7609	120.7505	116.1256	118.8095
AIC	-1.359512	-1.409382	-1.35157	-1.372619
SIC	-1.224973	-1.255623	-1.197812	-1.19964
Adj. R^2	0.011975	0.006549	**0.014642**	0.008835
MSE	0.08261	**0.082511**	0.082662	0.082748
MAE	0.055723	**0.055158**	0.056394	0.056612
MAPE	47.40151	**47.37679**	52.26628	49.40442
TIC	0.729479	0.716693	0.716592	**0.712242**

Table 4.3.24 Symmetric GARCH model selection for Liberty Group

Liberty Group	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	231.0262	231.0584	231.1188	231.1188
AIC	-2.514259	-2.503446	-2.50412	-2.492947
SIC	-2.40742	-2.3788	-2.379474	-2.350494
Adj. R^2	**0.168189**	0.163247	0.162604	0.157832
MSE	**0.059185**	0.059191	0.059194	0.059195
MAE	0.045196	0.045185	**0.045129**	0.045138
MAPE	148.2132	147.6008	**145.2358**	145.5833
TIC	**0.603428**	0.604522	0.608307	0.607796

Table 4.3.25 Symmetric GARCH model selection for Hudaco

Hudaco	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	168.3663	172.7982	168.9899	171.2569
AIC	-2.122435	-2.167297	-2.117515	-2.134077
SIC	-2.003595	-2.02865	-1.978868	-1.975623
Adj. R^2	**0.204572**	0.19831	0.193676	0.177389
MSE	**0.071445**	0.071482	0.072347	0.073051
MAE	0.056784	0.056686	0.056854	**0.056616**
MAPE	147.0846	146.2782	141.4673	**137.1637**
TIC	**0.562158**	0.564104	0.584598	0.600153

Table 4.3.26 Symmetric GARCH model selection for SABmiller

SABmiller	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	196.8159	197.5852	196.8363	197.5939
AIC	-2.514117	-2.511062	-2.501144	-2.497932
SIC	-2.374244	-2.351206	-2.341288	-2.318095
Adj. R^2	**0.188112**	0.180814	0.182438	0.174963
MSE	**0.049914**	0.050085	0.049916	0.050104
MAE	**0.03608**	0.036274	0.0361	0.036277
MAPE	**233.0444**	237.7858	233.6912	237.6286
TIC	0.569769	0.568039	0.56967	**0.567798**

Table 4.3.27 Symmetric GARCH model selection for Anglo-American

Anglo-American	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	153.7573	156.0152	153.9278	156.2903
AIC	-1.996697	-2.013813	-1.985412	-2.003949
SIC	-1.854296	-1.851068	-1.822667	-1.820861
Adj. R^2	**0.272451**	0.263012	0.266474	0.256805
MSE	**0.068871**	0.069919	0.069221	0.069496
MAE	**0.056972**	0.058275	0.057216	0.057896
MAPE	**218.3718**	231.185	218.7322	230.3692
TIC	0.643799	0.634802	0.646262	**0.632384**

Table 4.3.28 Symmetric GARCH model selection for Rainbow Chicken

Rainbow Chicken	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	87.08226	87.33721	87.28852	87.45064
AIC	-0.994874	-0.985733	-0.985135	-0.974855
SIC	-0.880994	-0.852872	-0.852275	-0.823014
Adj. R^2	**0.11202**	0.103912	0.103821	0.096039
MSE	0.101059	0.100359	0.100342	**0.099857**
MAE	0.077495	0.0769	0.076888	**0.076468**
MAPE	197.8877	194.8325	194.7103	**192.3053**
TIC	0.767798	**0.767643**	0.767682	0.767841

Table 4.3.29 Symmetric GARCH model selection for FirstRand

FirstRand	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	195.8775	196.1425	198.7265	196.3976
AIC	-2.556157	-2.546157	-2.581312	-2.536022
SIC	-2.393412	-2.363069	-2.398225	-2.332591
Adj. R^2	**0.380308**	0.376909	0.378179	0.372242
MSE	**0.051647**	0.052091	0.052724	0.053418
MAE	**0.039889**	0.040265	0.040498	0.041444
MAPE	**389.0851**	396.3654	390.513	389.4368
TIC	0.476635	0.475205	0.475454	**0.470321**

Table 4.3.30 Symmetric GARCH model selection for SASOL

SASOL	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	160.7161	160.7326	159.7898	160.7453
AIC	-1.851104	-1.839183	-1.827756	-1.827216
SIC	-1.700512	-1.669768	-1.65834	-1.638977
Adj. R^2	**0.130354**	0.124872	0.118371	0.119174
MSE	0.077065	0.07701	0.078239	**0.077005**
MAE	**0.053648**	0.053719	0.055258	0.053683
MAPE	142.8048	**142.5458**	155.6793	142.5626
TIC	0.721494	0.722276	**0.700523**	0.722215

Although the GARCH (1,2) model provided better estimation for 3 of the stocks namely, Tongaat Hulett and Impala Platinum, Sable; whilst the GARCH (2,1)model best described 1 stock, Nuworld and the GARCH (2,2) model also provided better estimation for 1 stock, Rainbow Chicken, the results obtained were consistent with what is commonly observed in modelling volatility of financial data and that is the fact that the GARCH (1,1) specification is adequate to capture the correlation in the squared returns of the stocks. In the selected sample of JSE listed stocks used for this research, the GARCH (1,1) model best models 10 out of the 15 stocks, namely, KWV Beleggings, Tiger Brands, Barloworld, Omnia, Liberty Group, Hudaco, SABmiller, Anglo-American, FirstRand and SASOL.

4.3.3 Determining Best Distribution for Modelling with GARCH (1,1)

Further to the findings above we would also like to determine if modelling the volatility of the stock returns using other error distributions which are fat-tailed, instead of the standard normal distribution, such as the student's *t*-distribution and the generalized error distribution (GED) would improve the model. The real difficulty when looking at financial data is that we do not know what the exact distribution of the underlying population is. The normal distribution is sometimes adequate, but can also miss important features. To evaluate whether modelling the volatility of the stock prices with distributions other than the normal distribution will improve the model, we use two other distributions namely the student's *t*-distribution and the generalized Error distribution (GED) on each of the 15 stocks. If indeed there is an improvement in the forecasting ability of the model then we can conclude for the purposes of this research that the specific error distribution is superior to the normal distribution in modelling the volatility of the selected sample of JSE stocks.

As we are testing non-Gaussian distributions for the underlying process of returns, we apply out-of-sample analysis in density forecasts. Thus, rather predicting first and second moments as is common in the GARCH literature, we focus on density predictions and the overall density forecasting performance of competing models can be compared by evaluating their conditional densities at the future observed values. We tabulate the results of the investigation in *Tables 4.3.31 - 4.3.45* below and using the same forecasting performance measures as before we select the best distribution to model the volatility of the stock returns.

Table 4.3.31 Error Distribution selection for Tongaat Hulett

Tongaat Hulett	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	186.4823	186.593	186.6018
AIC	-2.203486	-2.192506	-2.192615
SIC	-2.051012	-2.020973	-2.021082
Adj. R^2	**0.275383**	0.269012	0.272068
MSE	0.076789	0.076923	**0.076704**
MAE	0.062974	0.062998	**0.062904**
MAPE	174.6769	176.2102	**173.3607**
TIC	0.646027	**0.643183**	0.648625

Table 4.3.32 Error Distribution selection for KWV Beleggings

KWV Beleggings	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	173.5348	173.5344	173.5528
AIC	-2.233797	-2.220459	-2.220704
SIC	-2.113372	-2.079962	-2.080208
Adj. R^2	**0.186176**	0.180485	0.179899
MSE	**0.055953**	0.055955	0.056044
MAE	**0.041661**	0.041668	0.041688
MAPE	243.8976	**243.8918**	247.5773
TIC	**0.535014**	0.535018	0.532632

Table 4.3.33 Error Distribution selection for Nuworld

Nuworld	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	119.0266	121.0732	121.9584
AIC	-1.387832	-1.400915	-1.41198
SIC	-1.234074	-1.227936	-1.239002
Adj. R^2	0.102346	**0.11173**	0.111295
MSE	0.083721	**0.078775**	0.080797
MAE	0.070211	**0.065202**	0.066927
MAPE	1390.841	**1279.188**	1370.654
TIC	**0.697038**	0.705577	0.704519

Table 4.3.34 Error Distribution selection for Tiger Brands

Tiger Brands	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	181.791	181.8309	182.0546
AIC	-2.378108	-2.365046	-2.368089
SIC	-2.235707	-2.202301	-2.205345
Adj. R^2	**0.091286**	0.084651	0.084094
MSE	**0.056131**	0.060325	0.0605
MAE	**0.049204**	0.049281	0.049502
MAPE	**107.4075**	107.5997	108.9967
TIC	0.661438	**0.660567**	0.659174

Table 4.3.35 Error Distribution selection for Impala Platinum

Impala Platinum	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	117.5419	118.6297	118.5853
AIC	-1.460558	-1.46173	-1.461137
SIC	-1.299991	-1.281092	-1.280499
Adj. R^2	**0.186374**	0.175193	0.177367
MSE	**0.084762**	0.086715	0.085913
MAE	**0.068767**	0.069591	0.069227
MAPE	**319.9657**	324.134	321.4673
TIC	**0.687167**	0.703033	0.697574

Table 4.3.36 Error Distribution selection for Barloworld

Barloworld	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	180.3548	180.3467	180.4054
AIC	-2.298065	-2.284622	-2.285405
SIC	-2.137497	-2.103984	-2.104767
Adj. R^2	**0.310041**	0.305178	0.304807
MSE	0.082139	0.082143	**0.082061**
MAE	0.070169	0.70176	**0.7004**
MAPE	167.4161	167.5727	**163.2104**
TIC	0.624538	0.624545	**0.62364**

Table 4.3.37 Error Distribution selection for Omnia

Omnia	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	157.2999	158.8906	158.7368
AIC	-1.709603	-1.716278	-1.71454
SIC	-1.601937	-1.590668	-1.588929
Adj. R^2	**0.0543**	0.048827	0.047862
MSE	0.067388	0.067262	**0.067178**
MAE	0.051026	0.050686	**0.050518**
MAPE	176.7087	161.9469	**155.675**
TIC	**0.687301**	0.707287	0.716592

Table 4.3.38 Error Distribution selection for Sable

Sable	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	115.7609	123.0425	265.8484
AIC	-1.359512	-1.438031	-3.223104
SIC	-1.224973	-1.284273	-3.069346
Adj. R^2	**0.011975**	0.010746	-0.04609
MSE	**0.08261**	0.08296	0.090126
MAE	0.055723	**0.055007**	0.055012
MAPE	**47.40151**	50.428	62.50007
TIC	**0.729479**	0.747034	0.999999

Table 4.3.39 Error Distribution selection for Liberty Group

Liberty Group	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	231.0262	231.065	231.0496
AIC	-2.514259	-2.503519	-2.503348
SIC	-2.40742	-2.378873	-2.378701
Adj. R^2	**0.168189**	0.163824	0.163653
MSE	0.059185	**0.059156**	0.059173
MAE	0.045197	**0.045196**	0.045199
MAPE	148.2132	**147.8912**	148.035
TIC	**0.603428**	0.604277	0.60402

Table 4.3.40 Error Distribution selection for Hudaco

Hudaco	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	168.3663	168.6495	168.3995
AIC	-2.122435	-2.113065	-2.109798
SIC	-2.003595	-1.974417	-1.97115
Adj. R^2	**0.204572**	0.199393	0.199079
MSE	**0.071445**	0.071691	0.071547
MAE	**0.056784**	0.056895	0.056823
MAPE	147.0846	**144.4211**	146.2442
TIC	**0.562158**	0.570397	0.565123

Table 4.3.41 Error Distribution selection for SABmiller

SABmiller	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	196.8159	196.8713	196.9145
AIC	-2.514117	-2.501606	-2.502179
SIC	-2.374244	-2.34175	-2.342323
Adj. R^2	**0.188112**	0.182746	0.18253
MSE	0.049914	**0.049837**	0.049861
MAE	0.03608	**0.036041**	0.036066
MAPE	**233.0444**	233.058	234.0179
TIC	**0.569769**	0.570975	0.570765

Table 4.3.42 Error Distribution selection for Anglo-American

Anglo-American	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	153.7573	157.6584	157.4787
AIC	-1.996697	-2.036169	-2.033724
SIC	-1.854296	-1.873424	-1.870979
Adj. R^2	**0.272451**	0.264448	0.264389
MSE	0.068871	0.067182	**0.065123**
MAE	0.056972	0.055573	**0.054301**
MAPE	218.3718	208.6363	**205.8242**
TIC	0.643799	0.64368	**0.634037**

Table 4.3.43 Error Distribution selection for Rainbow Chicken

Rainbow Chicken	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	87.08226	89.7688	92.13666
AIC	-0.994874	-1.015568	-1.044622
SIC	-0.880994	-0.882708	-0.911761
Adj. R^2	**0.11202**	0.084056	0.04489
MSE	0.101059	0.097092	**0.093964**
MAE	0.077495	0.074269	**0.071649**
MAPE	197.8877	186.4907	**164.1259**
TIC	0.767798	**0.754436**	0.76655

Table 4.3.44 Error Distribution selection for FirstRand

FirstRand	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	195.8775	194.772	199.1844
AIC	-2.556157	-2.52751	-2.587543
SIC	-2.393412	-2.344422	-2.404455
Adj. R^2	0.380308	0.378946	**0.380669**
MSE	**0.051647**	0.053129	0.051854
MAE	**0.039889**	0.04111	0.040316
MAPE	**389.0851**	395.7434	415.8904
TIC	0.476635	0.47419	**0.461392**

Table 4.3.45 Error Distribution selection for SASOL

SASOL	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	160.7161	160.7119	161.195
AIC	-1.851104	-1.838932	-1.844787
SIC	-1.700512	-1.669517	-1.675372
Adj. R^2	**0.130354**	0.124782	0.123532
MSE	0.077065	**0.077062**	0.07766
MAE	**0.053648**	0.053649	0.053673
MAPE	142.8048	142.8173	**138.6652**
TIC	**0.721494**	0.721498	0.724182

The Normal distribution provides a much closer approximation to the empirical unconditional density of 8 out of the 15 stocks. The stocks KWV Beleggings, Tiger Brands, Impala Platinum, Sable, Hudaco, SABmiller, FirstRand and SASOL are best modelled by the normal distribution whilst 2 of the stocks; Nuworld and Liberty Group are best modelled by the student's *t*-distribution and the other 5 stocks; Tongaat Hulett, Barloworld, Omnia, Anglo-American and Rainbow Chicken are best modelled using the generalized error distribution. Though there has been so much research into the use of other error distributions which are better equipped to capture the fat-tailed nature of stock returns and evidence to support the use of these distributions, there was no significant incentive to use alternative distributions as the majority of the stocks in the sample were best modelled using the normal distribution.

4.3.4 Estimated Parameters

Having determined that the symmetric GARCH (1,1) modelled with the Normal distribution is the best model for modelling the volatility of the sample of JSE listed stocks used for this research for the period Jan 1991 to Feb 2008, we proceed to estimate the variance equation parameters. The technique used in estimating the GARCH models involves maximization of a likelihood function constructed under the assumption of a *i.i.d* distribution for the standardized innovations. The estimated parameters are represented in *Tables 4.3.46 - 4.3.60* below.

Table 4.3.46 Parameters for the Variance Equation of Tongaat Hulett

Tongaat - Hulett	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000495	0.000716	0.690579	0.4898
RESID(-1)^2	0.065564	0.070729	0.926983	0.3539
GARCH(-1)	0.85588	0.172748	4.954486	0.0000

Table 4.3.47 Parameters for the Variance Equation of KWV Beleggings

KWV Beleggings	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001059	0.00102	1.038795	0.2989
RESID(-1)^2	0.152156	0.098953	1.537666	0.1241
GARCH(-1)	0.679235	0.232395	2.922757	0.0035

Table 4.3.48 Parameters for the Variance Equation of Nuworld

Nuworld	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000532	0.000529	1.00541	0.3147
RESID(-1)^2	0.205492	0.068305	3.008434	0.0026
GARCH(-1)	0.770087	0.071197	10.81633	0.0000

Table 4.3.49 Parameters for the Variance Equation of Tiger Brands

Tiger Brands	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000617	0.000632	0.975616	0.3293
RESID(-1)^2	0.108047	0.078241	1.380954	0.1673
GARCH(-1)	0.760596	0.177895	4.275522	0.0000

Table 4.3.50 Parameters for the Variance Equation of Impala Platinum

Impala platinum	Coefficient	Std. Error	z-Statistic	Prob.
C	0.005282	0.00642	0.822737	0.4107
RESID(-1)^2	0.116962	0.146238	0.799805	0.4238
GARCH(-1)	0.459377	0.580176	0.79179	0.4285

Table 4.3.51 Parameters for the Variance Equation of Barloworld

Barloworld	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000419	0.000596	0.703841	0.4815
RESID(-1)^2	0.090088	0.070571	1.276553	0.2018
GARCH(-1)	0.829932	0.156157	5.314745	0.0000

Table 4.3.52 Parameters for the Variance Equation of Omnia

Omnia	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000857	0.000759	1.128773	0.259
RESID(-1)^2	0.154422	0.066085	2.336712	0.0195
GARCH(-1)	0.776406	0.118973	6.525904	0.0000

Table 4.3.53 Parameters for the Variance Equation of Sable

Sable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001804	0.000518	3.485015	0.0005
RESID(-1)^2	0.332475	0.10424	3.189511	0.0014
GARCH(-1)	0.572896	0.074904	7.648391	0.0000

Table 4.3.54 Parameters for the Variance Equation of Liberty Group

Liberty Group	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000365	0.000376	0.97178	0.3312
RESID(-1)^2	0.118837	0.057947	2.050796	0.0403
GARCH(-1)	0.802664	0.123604	6.493835	0.0000

Table 4.3.55 Parameters for the Variance Equation of Hudaco

Hudaco	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001338	0.001646	0.813328	0.416
RESID(-1)^2	0.083765	0.056463	1.483543	0.1379
GARCH(-1)	0.707877	0.287978	2.458089	0.014

Table 4.3.56 Parameters for the Variance Equation of SABmiller

SABmiller	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000506	0.000474	1.066358	0.2863
RESID(-1)^2	0.182382	0.127287	1.432846	0.1519
GARCH(-1)	0.712737	0.188747	3.776149	0.0002

Table 4.3.57 Parameters for the Variance Equation of Anglo-American

Anglo-American	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000726	0.00072	1.008287	0.3133
RESID(-1)^2	0.061027	0.056009	1.089576	0.2759
GARCH(-1)	0.832723	0.122954	6.772648	0.0000

Table 4.3.58 Parameters for the Variance Equation of Rainbow Chicken

Rainbow Chicken	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000124	0.000421	0.293616	0.7691
RESID(-1)^2	0.11946	0.038566	3.097524	0.002
GARCH(-1)	0.879918	0.045147	19.49009	0.0000

Table 4.3.59 Parameters for the Variance Equation of FirstRand

FirstRand	Coefficient	Std. Error	z-Statistic	Prob.
C	8.11E-05	0.000247	0.327565	0.7432
RESID(-1)^2	0.078691	0.060554	1.299522	0.1938
GARCH(-1)	0.901982	0.09798	9.205743	0.0000

Table 4.3.60 Parameters for the Variance Equation of SASOL

SASOL	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000402	0.00047	0.853766	0.3932
RESID(-1)^2	0.069975	0.059339	1.179246	0.2383
GARCH(-1)	0.886009	0.091653	9.667003	0.0000

The estimated GARCH parameters are significant within the 5% range with the exception of the parameters estimated for Impala Platinum.

4.4 Model Validation

This section focuses on the validation of the chosen GARCH model, which in this case is the GARCH (1,1). Most of the tests performed in the pre-estimation stage are performed again and these include the Ljung Box Q - test to see if the autocorrelation in the squared return has successfully been removed; the Jarque-Bera test to see whether or not the distribution of the normalized residuals are normally distributed and the LM Arch test to see whether the heteroscedasticity has been removed. All these tests will be presented in the following section.

4.4.1 Ljung-Box Q - test

In this section we test the residuals of each of the 15 stocks to observe whether we have removed all the autocorrelation in the squared residuals. In the pre-estimation stage we observed that there was significant autocorrelation in the squared residuals and now we aim to find out whether there still is a significant correlation in the standardized innovations or not. Below in *Table 4.4.1* we tabulate the results observed for 10, 15 and 20 lags where the null hypothesis H_0 states that there is no autocorrelation.

Table 4.4.1 Ljung – Box Q test for Squared Residuals after Estimation

	Q(10) Critical Value=18.3070		*Q(15) Critical Value=24.9958*		*Q(20) Critical Value=31.4104*	
	p-value	Stat	p-value	Stat	p-value	Stat
Hulett	0.975	1.6893	0.9900	3.6023	0.9160	9.7013
KWV	0.473	8.6229	0.801	9.4486	0.825	13.268
Nuworld	0.217	10.7380	0.2630	14.622	0.459	16.933
Tigerbrands	0.207	13.312	0.2370	16.2320	0.217	23.468
Impala Platinum	0.216	11.958	0.346	13.319	0.308	22.551
Barloworld	0.2360	10.702	0.2020	15.927	0.3930	17.929
Omni	0.209	14.733	0.336	15.638	0.602	16.679
Sable	0.4800	7.5318	0.5030	12.303	0.612	15.727
Liberty	0.021	19.4860	0.03	25.455	0.1200	26.366
Hudaco	0.878	4.4684	0.774	9.8323	0.446	19.177
SABmiller	0.002	24.228	0.013	26.887	0.013	33.837
Anglo-American	0.3050	9.46	0.715	9.7366	0.908	10.666
Rainbow Chicken	0.2300	14.5820	0.2220	17.68	0.2660	15.693
FirstRand	0.886	2.9850	0.483	11.543	0.414	17.6
SASOL	0.5460	5.9425	0.7990	7.8179	0.3760	18.206

Only Liberty Group and SABmiller still show autocorrelation in the squared residual after estimation by the GARCH (1,1) model. However, H_0 is accepted in 13 of the 15 cases indicating that GARCH modeling does remove autocorrelation in the squared residuals.

4.4.2 Engle's ARCH test for the residuals after Estimation

We conduct Engle's ARCH test on the residuals after estimation using the GARCH (1,1) model to observe whether the conditional heteroscedasticity observed in the pre-estimation stage is still present. The results of Engle's ARCH test for the residuals after estimation are shown in *Table 4.4.2* below:

Table 4.4.2 Engle's ARCH test for residuals after estimation

	ARCH 1-10 test		ARCH 1-15 test		ARCH 1-20 test	
	Prob.	F(10,183)	Prob.	F(15,173)	Prob.	F(20,163)
Hulett	0.998925	0.146974	0.999511	0.19902	0.987858	0.410208
KWV	0.53935	0.895605	0.84837	0.626306	0.859355	0.656977
Nuworld	0.102961	1.633919	0.199263	1.320029	0.332344	1.126654
Tigerbrands	0.289846	1.211486	0.352282	1.113448	0.254707	1.217755
Impala Platinum	0.501858	0.936976	0.777547	0.702753	0.540981	0.938458
Barloworld	0.501928	0.936897	0.536313	0.927264	0.732132	0.779807
Omni	0.188798	1.391436	0.362944	1.098283	0.669935	0.833259
Sable	0.528391	0.907317	0.639526	0.833068	0.679786	0.82463
Liberty	0.046119	1.919503	0.123279	1.470307	0.352544	1.103774
Hudaco	0.885305	0.503226	0.792829	0.687334	0.459378	1.007604
SABmiller	0.108393	1.617383	0.260257	1.228227	0.120105	1.437884
Anglo-American	0.524188	0.912292	0.806	0.673208	0.906182	0.598385
Rainbow Chicken	0.164213	1.450745	0.340171	1.125308	0.08342	1.528437
FirstRand	0.982489	0.289662	0.846551	0.628218	0.785017	0.732092
SASOL	0.862588	0.535686	0.948377	0.477615	0.802074	0.717438

We observe that in most of the stocks, 11 to be precise, the conditional heteroscedasticity is removed. Only 4 stocks namely, Nuworld, Liberty Group, SABmiller and Rainbow Chicken still show the presence of conditional heteroscedasticity. However, we can conclude that the GARCH (1,1) comprehensively removes conditional heteroscedasticity from the stock data.

4.4.3 Are the residuals z_t normally distributed

As discussed earlier in this master thesis, we assume that z_t is supposed to be normally distributed. We can investigate this using several methods, but the Jarque-Bera test is the main method for investigating normality of the residuals z_t. Other means to investigate this include, the estimated moments for the distribution. Results from the Jarque – Bera test as well as the estimated moments of the distributions for each of the 15 stocks are show in *Table 4.4.3* below:

Table 4.4.3 Jarque–Bera Test for Normality after Estimation

	Mean	Median	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
Hulett	-0.021488	0.016161	2.243197	-3.78409	1.000672	-0.307814	3.218206	2.879625	0.236972
KWV	-0.042827	-0.128531	2.483609	-2.28109	1.000254	0.210952	2.771665	1.43837	0.487149
Nuworld	-0.076843	-0.066025	3.330455	-2.82219	1.004903	-0.037448	3.542961	2.002776	0.367369
Tigerbrands	-0.041608	-0.02541	2.584378	-2.59729	1.008099	-0.114852	3.066075	0.349918	0.839492
Impala Platinum	-0.003351	-0.036984	3.021952	-2.53958	1.003246	0.275508	3.512683	3.540394	0.170299
Barloworld	-0.021231	-0.035136	2.958217	-2.44112	1.007905	0.127638	2.863774	0.523271	0.769792
Omni	-0.031556	-0.103992	3.755603	-2.81924	1.000641	0.285203	3.792833	7.035369	0.029668
Sable	-0.062354	-0.110383	3.226733	-3.19333	1.009919	-0.025564	4.500011	15.0177	0.000548
Liberty	-0.029199	-0.001071	2.690865	-3.0833	1.002066	-0.047455	3.083662	0.119387	0.942053
Hudaco	-0.048117	-0.029778	2.46697	-3.38353	1.004466	-0.006812	3.353887	0.799562	0.670467
SABmiller	-0.027531	0.031331	2.494775	-2.80015	1.004717	-0.141843	3.102781	0.572803	0.750961
Anglo-American	0.018973	0.081061	4.122628	-2.52014	1.002364	0.176887	4.561488	15.70083	0.000390
Rainbow Chicken	-0.07289	0.029272	2.512331	-3.648	1.005515	-0.503401	3.72721	10.4760	0.005311
FirstRand	0.017038	-0.10791	2.89545	-3.00845	1.011616	0.00302	3.212291	0.276262	0.870985
SASOL	-0.011643	0.041754	2.624246	-2.24399	0.999221	0.136672	2.698178	1.139968	0.565535

The Jarque –Bera test states that at the 5% level if the p-value obtained is ≤ 0.05 then we reject the hypothesis that z_t is normally distributed. Therefore from the results in *Table 4.4.3* above we observe that 11 out of the 15 stocks show that z_t is normally distributed. Notably results for Omni, Sable, Anglo-American and Rainbow Chicken reject the hypothesis that z_t is normally distribution. It is also observed that for the 10 stocks which accept the hypothesis that z_t is normally distributed for the Jarque – Bera test, their first, second, third and fourth moments are close to their corresponding normal values. The corresponding normal value for the first moment (mean) is 0; second moment (standard deviation) is 1; third moment (skewness) is 0 and fourth moment (kurtosis) is 3 as discussed in *Section 2.3.1*.

4.5 Summary and Assessment: Modelling of Volatility of Stock Prices using GARCH models

In this master thesis we have discussed alternative unconditional and conditional distributional models for the monthly returns of 15 randomly selected JSE stocks considering various GARCH models driven by normal, student's t and generalized error distributed innovations.

The behavior of returns is consistent with the well documented stylized facts common to financial data series. Firstly, the return series exhibit a considerable level of excess kurtosis and departure from normality. Furthermore, the empirical evidence presented in this paper indicates that time series of monthly stock returns exhibit significant levels of dependence. The probability distribution of r_t, is not independent of $r_{t-\tau}$. Despite the small autocorrelation of returns, the serial dependence in the square returns is clearly not rejected pointing towards the existence of volatility clusters. The conditional heteroscedastic processes allow for autocorrelation between the first and second moments of return distributions over time, and consequently they fit to data very satisfactorily. Also notably, the volatility of returns tends to be highly persistent.

The GARCH models applied in this research more importantly provide improved forecasts of volatility as compared to normal regression techniques. Within the class of such models, the GARCH (1,1) model shows the best fit and forecast accuracy. The forecasting capabilities of GARCH models constitute further evidence as to their overall usefulness as practical models of stock returns and also about their relative merits as such.

The fit of the GARCH (1,1) model is further evaluated by the fact that the Ljung Box Q-test and LM ARCH test suggest that the autocorrelation in the squared residuals and heteroscedasticity have been removed in the post-estimation stage. Further to this, the standardized residuals have a standard normal distribution, as they should and the additional assumption of conditional normality in the Jarque-Bera tests cannot reject the hypothesis of normality. The GARCH models reduce the excess kurtosis to around zero. Skewness is also no longer present. These results are expected but still they provide supporting evidence of good fit to data. They show that large returns (of either sign) are more frequently observed in more volatile periods, and vice versa. This is a realistic description of stock-market behaviour.

Chapter 5. GARCH modelling & Portfolio Construction

For decades, Sharpe Single Index models have been used to try and provide accurate estimates of the Portfolio Mean-Variance Efficient Frontier Optimization solution. Sharpe Single Index models basically use Linear Regression Models to estimate the input parameters, i.e. the means and variances for the optimal portfolio construction. The Sharpe Single Index Models use estimated regression coefficients (betas), estimated covariance matrix between the chosen index and the conditional unbiased estimates of the residuals. There has been considerable evidence that regression index models induce both autocorrelation in the residuals and volatility in the squared residuals, hence autocorrelation (AR) processes and GARCH (heteroscedastic) processes play an important role in mean-variance construction of efficient frontiers.

In recent times AR and GARCH processes have been used to construct Efficient Portfolios with superior Efficient Frontiers and these are called Dynamic Time Series Models. These models are supposedly more superior to the traditional Markowitz and Sharpe Index models as they have significant risk reduction

5.1 The Classical Markowitz Mean-Variance Model

Let p be the number of stocks in our portfolio, then the vector of return values of the stocks will be given by the vector:

$$\boldsymbol{R} = \begin{pmatrix} R_1 \\ R_2 \\ \cdot \\ \\ R_p \end{pmatrix} \tag{5.1.1}$$

and we assume these to be log returns with mean

$$E\ \boldsymbol{R} = \mu \tag{5.1.2}$$

and the covariance matrix contains a historic covariance structure of the portfolio given by

$$\Sigma = E\left[(\boldsymbol{R}-\mu)(\boldsymbol{R}-\mu)' \right] \quad (5.1.3)$$

Further to this, a portfolio is a proportional investment, say w_i, in each stock, so that

$$\sum_{i=1}^{p} w_i = 1$$

Let

$$\boldsymbol{W} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_p \end{pmatrix} \quad (5.1.4)$$

Then our portfolio is given by $\boldsymbol{P} = \boldsymbol{W}'\boldsymbol{R}$

$$= \sum_{i=1}^{p} w_i R_i$$

Subject to

$$\sum_{i=1}^{p} w_i = 1$$

Hence the Expected return of the portfolio is given by

$$E(\boldsymbol{P}) = \boldsymbol{W}'E(\boldsymbol{R}) = \boldsymbol{W}'\mu = \sum_{i=1}^{p} w_i \mu_i = \boldsymbol{\mu_p}$$

And the variance is

$$\text{var}(\boldsymbol{P}) = \boldsymbol{W}'\Sigma\boldsymbol{W}$$

$$= \sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j \sigma_{ij}$$

$$= \boldsymbol{\sigma_p^2}$$

The Classical Markowitz Mean-Variance formulation in essence sought to choose the weights, w_i, such that the Expected Return $E\ P = \mu_p$ is maximised but in the same instance minimising the risk σ_p^2.

In addition to this, Markowitz introduced the concept of an Efficient Frontier. A portfolio is called efficient if:

- For a given amount of risk, the expected return is maximised, or:
- For a given amount of return, the expected risk is minimised

With this concept in mind, the solution of this portfolio optimization problem will lie on the efficient frontier line. Specifically the efficient frontier is a line plot in the 2 - dimensional mean-variance spaces. Finding the efficient frontier is a non-linear quadratic programming (QP) problem and can be solved using well-known numerical techniques. To solve the problem one needs either to fix the return μ_p and then minimize the variance σ_p^2, or fix the variance and then maximize the return.

The Markowitz portfolio problem is then formally:

$$\max_{w_i} E\ \boldsymbol{P} = \boldsymbol{W}'\boldsymbol{\mu} = \sum_{i=1}^{p} w_i \mu_i \qquad (5.1.5)$$

$$\min_{w_i} = \boldsymbol{W}'\boldsymbol{\Sigma}\boldsymbol{W} = \sum_{i,j=1}^{p} w_i w_j \sigma_{ij} \qquad (5.1.6)$$

subject to

$$\sum_{i=1}^{p} w_i = 1,$$

$$0 \le w_i \le 1, i = 1, \ldots, p$$

5.2 The Sharpe Single Index Model

The Sharpe Single Index Model is formulated as:

$$\boldsymbol{R}_{it} = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \boldsymbol{I}_t + \boldsymbol{e}_{it} \text{ where } i = 1, \ldots, q; t = 1, \ldots, N \qquad (5.2.1)$$

The Sharpe Single Index Model is based on the main assumption that shares move together only because of a common co-movement with the market, or simply that the only correlation that any two stocks share is explained via the stock correlation with the market index. Further to this a Single Index Model can be defined as a one-factor regression model of stock returns. It is a response to the observation that stocks follow the movements of the market.

The Sharpe Single Index Model assumes that:

$$E\left(\boldsymbol{e}_{it}^{2}\right)=\sigma_{ei}^{2}=\sigma_{i}^{2}=\sigma_{ii},\ i=1,\ldots,q. \tag{5.2.2}$$

$$E\left(\boldsymbol{e}_{it}\boldsymbol{e}_{is}\right)=\mathbf{0},\ t\neq s=1,\ldots,N. \tag{5.2.3}$$

$$E\left(\boldsymbol{e}_{it}\boldsymbol{I}_{t}\right)=\mathbf{0},\ t=1,\ldots,N. \tag{5.2.4}$$

$$E\left(\boldsymbol{e}_{it}\boldsymbol{e}_{jt}\right)=\mathbf{0},\ t=1,\ldots,N \tag{5.2.5}$$

where $i,j=1,\ldots,q, i\neq j$

In vector notation, the Sharpe Single Index Model is represented by

$$\boldsymbol{R}_t=\begin{pmatrix}R_{1t}\\ \cdot\\ \cdot\\ R_{pt}\end{pmatrix},\quad \alpha=\begin{pmatrix}\alpha_1\\ \cdot\\ \cdot\\ \alpha_p\end{pmatrix},\quad \beta=\begin{pmatrix}\beta_1\\ \cdot\\ \cdot\\ \beta_p\end{pmatrix}\quad \text{and}\quad \boldsymbol{e}_t=\begin{pmatrix}e_{1t}\\ \cdot\\ \cdot\\ e_{pt}\end{pmatrix}$$

and dropping the index t, we get

$$E(\boldsymbol{R})=\alpha+\beta_{\mu I} \tag{5.2.6}$$

And

$$\operatorname{cov} \boldsymbol{e} = \begin{pmatrix} \sigma_{e1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{e2}^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \sigma_{ep}^2 \end{pmatrix} \tag{5.2.7}$$

Implying that

$$\operatorname{cov} \boldsymbol{R} = \sigma_1^2 \beta\beta' + \begin{pmatrix} \sigma_{e1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{e2}^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \sigma_{ep}^2 \end{pmatrix} \tag{5.2.8}$$

So in essence the Single Index Model assumes that all $e_{it,} i = 1,\dots,q, t = 1,\dots,N$ are uncorrelated. However, empirical evidence has proven this not to be the case. The errors e_{it} of different stocks have been observed to be correlated and the JSE is no exception. Hence the development of the Improved Sharpe Single Index Model.

5.3 The Improved Sharpe Single Index Model

The Improved Sharpe Single Index Model was introduced by Troskie, Hossain and Guo (2005) and unlike the simple Sharpe Single Index Model which assumes that the errors e_{it} of the different stocks are uncorrelated, it assumes the opposite. The model is an attempt to improve on the Sharpe Index Model by including the covariance structure of the residuals. The improved Sharpe Index model has the advantage of including all the information used in the Classical Markowitz model and is still easily extended to include AR and GARCH components in the model which will capture the autocorrelation and heteroscedasticity in the stock returns therefore improving the Efficient Frontier. The Improved Sharpe Single Index model is formulated in the same way as the Sharpe Single Index Model:

$$R_{it} = \alpha_i + \beta_i I_t + e_{it} \text{ where } i = 1,\ldots,q; t = 1,\ldots,N$$

But

$$\begin{aligned} E\ e_{it}e_{jt} &= \sigma_{ij}, i \neq j, t = 1,\ldots,N, \\ &= \sigma_{ei}^2\ , \quad i = j \\ &= \sigma_{ii} = \sigma_i^2 \end{aligned} \tag{5.3.1}$$

which can also be represented as:

$$E\ ee' = \Omega = \begin{pmatrix} \sigma_{e1}^2 & \sigma_{12} & \ldots & \sigma_{1p} \\ \sigma_{21} & \sigma_{e2}^2 & \ldots & \sigma_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_{p1} & \ldots & \ldots & \sigma_{ep}^2 \end{pmatrix} \tag{5.3.2}$$

and hence

$$\operatorname{cov}\ R = \sigma_1^2 \beta\beta' + \Omega = \Phi \tag{5.3.3}$$

For Portfolio $P = W'R$,then,

$$E\ P = W'\ \alpha + \beta\mu_1 = \mu_p \tag{5.3.4}$$

and

$$\begin{aligned} \operatorname{var}(P) &= W'\ \sigma_1^2 \beta\beta' + \Omega\ W \\ &= \sigma_1^2 W'\beta\beta'W + W'\Omega W \\ &= \sigma_p^2 \end{aligned} \tag{5.3.5}$$

5.4 The Models & Results

The determination of the importance of GARCH modelling in portfolio construction was done with the use of Single Index models. When Index models are extended to include autocorrelation (AR) terms in the estimated residuals and heteroscedasticity that appears in the estimated squared residuals they are called Dynamic Time Series Models. The researcher uses 3 models in the analysis of the use of GARCH models in portfolio construction represented as

Model 1, Model 2 and Model 3 and compares them to the Classical Markowitz Mean-Variance Model. It should be noted however, that the covariance matrix suggested by Hossain, Troskie & Guo (2005) in *Eq. 5.3.2*, which incorporates correlations amongst the various shares giving a better variance approximation compared to the traditional Sharpe Index model which assumes zero correlation amongst the shares themselves is used. The Index used for our Single Index Models is the JSE index.

Model 1

This is a straightforward AR (k) process. The model involves fitting AR terms to each of the return series. This involves assessing the Autocorrelation plots for each of the 15 stocks and then fitting appropriate AR terms.

$$\boldsymbol{R}_{it} = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \boldsymbol{I}_t + \boldsymbol{e}_{it} \tag{5.3.6}$$

$$\boldsymbol{e}_{it} = \phi_i e_{it-1} + \phi_2 e_{it-2} + \ldots + \phi_2 e_{it-k} + v_{it} \tag{5.3.7}$$

$$i = 1, \ldots, q, t = 1, \ldots, N, \tag{5.3.8}$$

where, v_{it} has a distribution with mean zero and variance σ_{iv}^2.

Model 2

This model fits only a GARCH (1,1) process.

$$\begin{aligned} \boldsymbol{R}_{it} &= \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \boldsymbol{I}_t + \boldsymbol{e}_{it} \\ \boldsymbol{\sigma}_{it}^2 &= \theta_i + \alpha_i e_{it-1}^2 + \gamma_i \sigma_{it-1}^2 \\ i &= 1, \ldots, q, t = 1, \ldots, N, \end{aligned} \tag{5.3.9}$$

where, v_{it} has a distribution with mean zero and variance σ_{iv}^2.

Model 3

This is an AR (k)-GARCH (1,1) process. This unlike Model 1 fits a GARCH (1,1) process together with the AR terms .

$$\boldsymbol{R}_{it} = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \boldsymbol{I}_t + \boldsymbol{e}_{it}$$

$$\boldsymbol{e}_{it} = \phi_i e_{it-1} + \phi_2 e_{it-2} + \ldots + \phi_2 e_{it-k} + v_{it} \;, v_{it} = \sigma_{it} \varepsilon_{it}$$

$$\boldsymbol{\sigma}_{it}^2 = \theta_i + \alpha_i e_{it-1}^2 + \gamma_i \sigma_{it-1}^2$$

$$i = 1,\dots,q, t = 1,\dots,N,$$

where ε_{it} has a distribution with mean zero and variance one.

The objective of this study is to investigate the importance of using GARCH modelling in portfolio construction. This can be determined by observing the mean-variance response as represented by the efficient frontier. The returns, regression, covariance and expected return estimates for each of the models were calculated using a program written in EVIEWS 5. The expected return (μ) and covariance were then exported to MATLAB 7 and the quadratic programming problem was solved producing the efficient frontiers.

Efficient frontiers for the Classical Markowitz Mean – Variance model *(Fig 5.4.1)*, the Improved Sharpe Single Index Model (with AR terms only – Model 1) *(Fig 5.4.2)*, the Improved Sharpe Single Index Model (with a GARCH (1,1) only - Model) *(Fig 5.4.3)* and Improved Sharpe Single Index Model (with AR terms and GARCH (1,1) processes – Model 2) *(Fig 5.4.4)* are illustrated below:

Fig 5.4.1

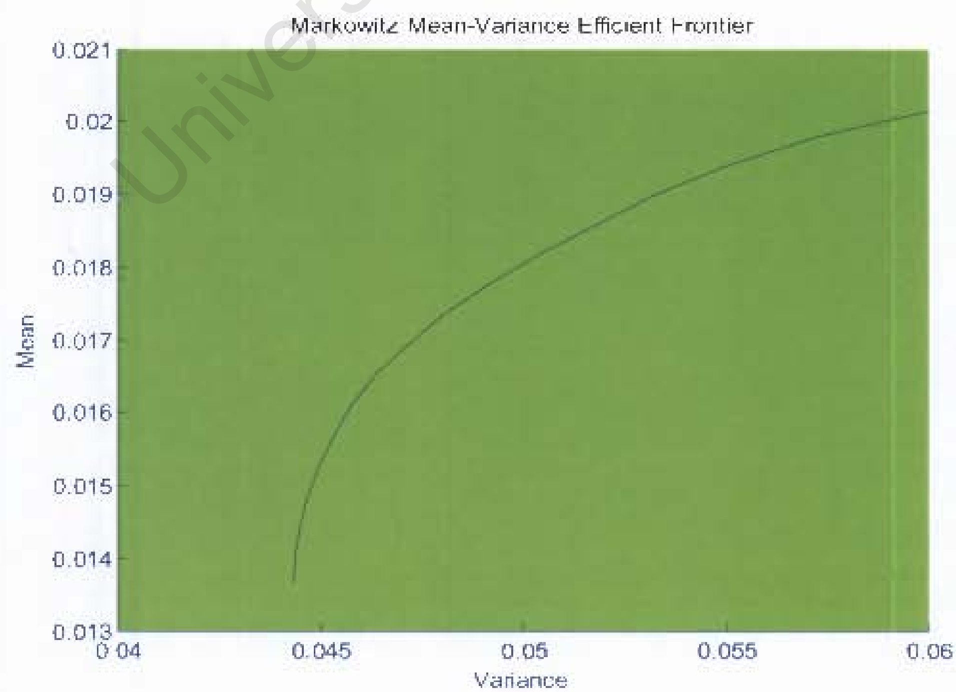


Fig 5.4.2

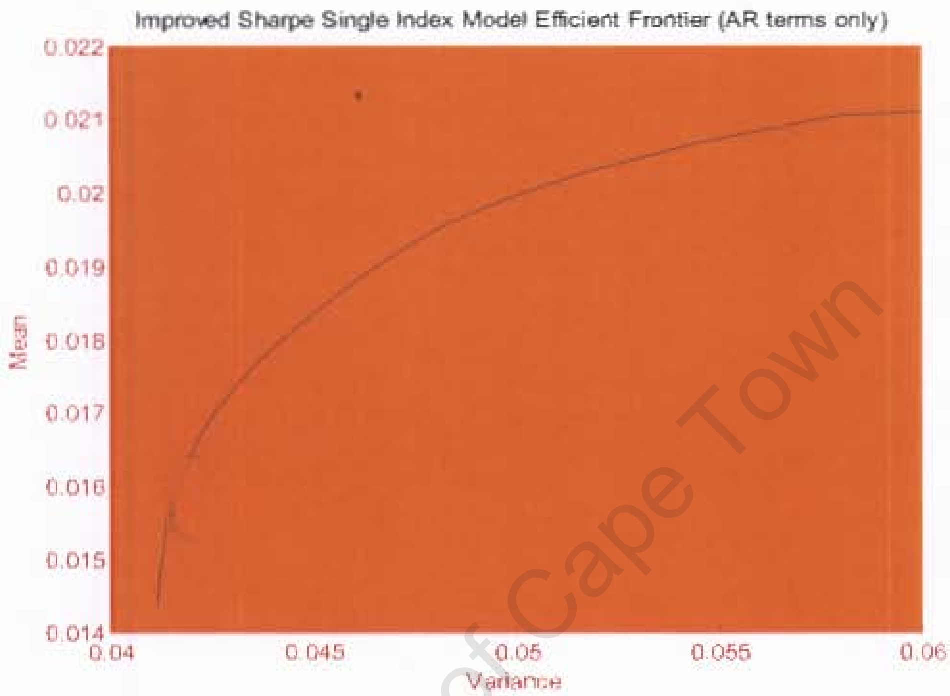


Fig 5.4.3

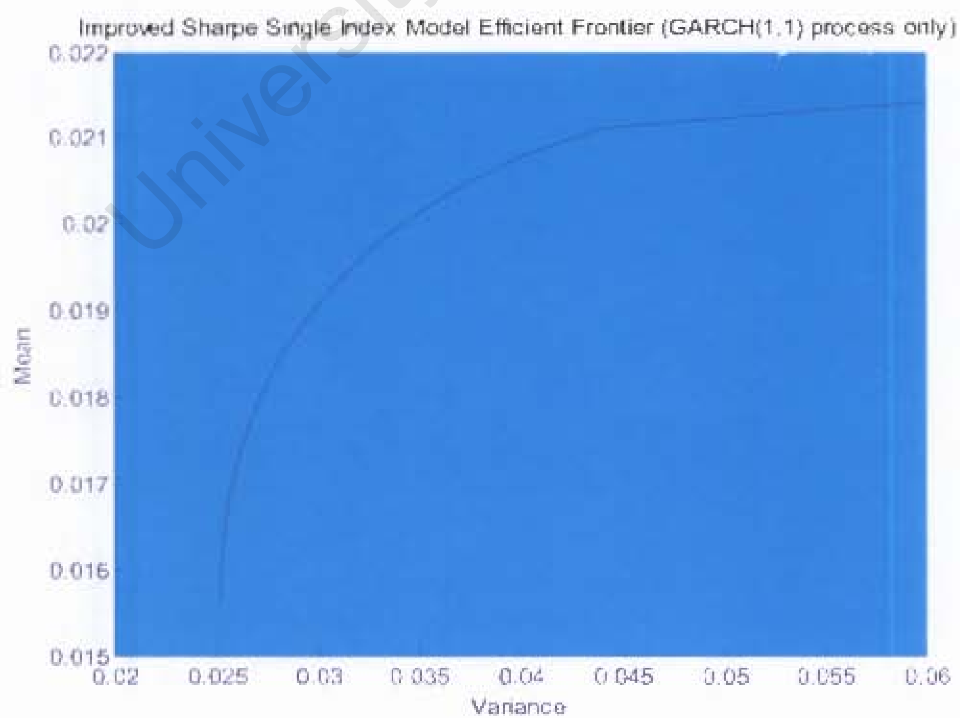


Fig 5.4.4

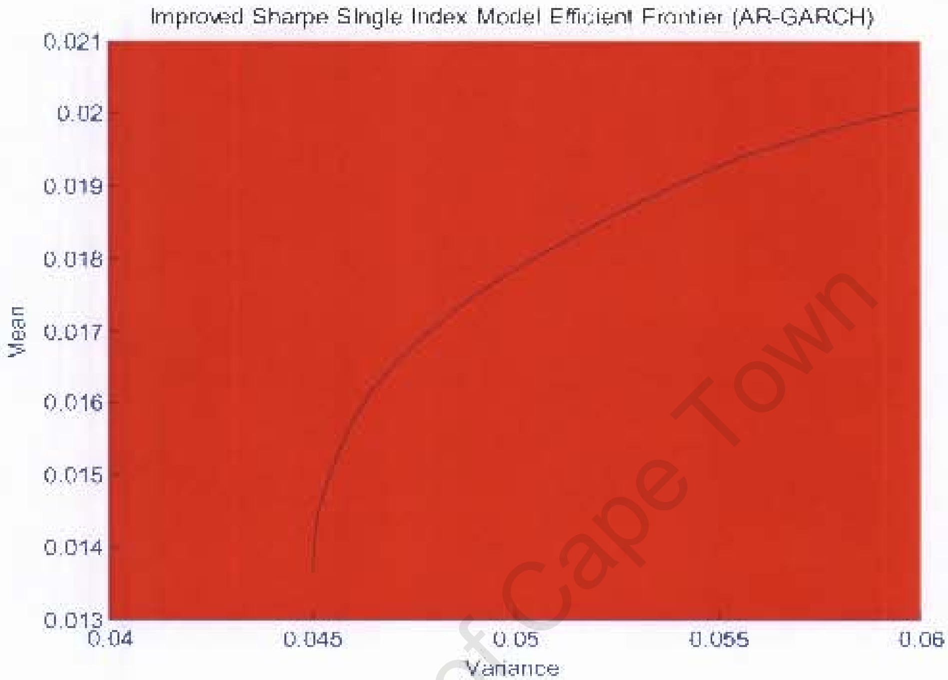


Fig 5.4.1, Fig 5.4.2 and *Fig 5.4.3* and *Fig 5.4.4* show plots of four efficient frontiers. They correspond to the Markowitz Model, Improved Sharp Single Model – (modelled with AR terms only), Improved Sharp Single Model – (modelled with a GARCH (1,1) process only) and Improved Sharpe Single Index Model – (modelled with both AR and a GARCH (1,1) process). Close inspection will show that the Improved Sharpe Single Index Model – (modelled with both AR and a GARCH (1,1) process) is the one which best approximates the Markowitz Model efficient frontier of the 3 models. These results show that Improved Sharpe Single Index Model – (modelled with AR terms only) and the Improved Sharp Single Model – (modelled with a GARCH (1,1) process only) under-estimate the risk of the portfolio compared to the traditional Markowitz model. Evidently, this is due to the fact that a model which incorporates both AR terms and a GARCH (1,1) process better approximates the variance of stock returns than a model which only incorporates AR terms or a GARCH (1,1) process only. This brings to attention the significance of GARCH modelling in portfolio construction.

Chapter 6: Conclusions

Choice of best GARCH Model

Forecasts of volatility generated by the models are compared using the root mean squared error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), the Theil Inequality Coefficient (TIC) and the log-likelihood.

Four different GARCH models are tested with the *(p,q)* specification (1,1) for the period January 1991 – February 2008. The models are namely the symmetric GARCH, TARCH, EGARCH and Power-ARCH. It is observed that 6 out of the 15 stocks, namely KWV Beleggings, Tiger Brands, Barloworld, Sable, Hudaco and FirstRand are best modelled by the symmetric GARCH; 4 stocks namely Liberty Group, Anglo-American, Rainbow Chicken and SASOL by the TARCH model; 3 namely Impala Platinum, Omni and SABmiller by the EGARCH model and the remaining 2 namely Tongaat Hulett and Nuworld by the PARCH model. Therefore it can be concluded that the symmetric GARCH is a good choice in modelling stock returns although there is evidence that the asymmetric coefficient included in the EGARCH and TARCH and power coefficient included in the Power-ARCH are good to implement in the GARCH model as shown by the stocks best modelled by the parallel models. However, the symmetric GARCH proves to be the superior of the 4 models as it best models the returns of more stocks in the sample than any of the other models.

Choice of best symmetric GARCH Model Lag Structure

Having determined that the symmetric GARCH model is superior to other models under investigation above, the new task was to determine the best lag structure *(p,q)* for modelling the stock returns. It should be noted that p is the number of GARCH coefficients and q is the number of ARCH coefficients. To take into account higher order GARCH lag structures, four different p and q specifications are tested namely (1,1), (1,2), (2,1)and (2,2). The lag structure (1,1) is by far the most used specification in GARCH research today and the results in this master thesis are also consistent with this. In the selected sample of JSE listed stocks used for this research, the GARCH

(1,1) model best models 10 out of the 15 stocks, namely, KWV Beleggings, Tiger Brands, Barloworld, Omni, Liberty Group, Hudaco, SABmiller, Anglo-American, FirstRand and SASOL. The GARCH (1,2) model provided better estimation for 3 of the stocks namely, Tongaat Hulett and Impala Platinum, Sable; whilst the GARCH (2,1) model best described 1 stock, Nuworld and the GARCH (2,2) model also provided best estimation for 1 stock, Rainbow Chicken clearly showing the superiority of the symmetric GARCH (1,1) in modelling stock returns on the JSE.

Choice of best Error Distribution

To take into account the fat-tailed nature of stock returns the researcher further modelled the volatility of the stock returns using alternative error distributions to the normal distribution, which are better equipped to capture the fat-tailed nature of stock returns to assess the significance of the choice of error distribution used to model the stock returns. Therefore three different distributions are tested: normal distribution, student-t and the generalized error distribution (GED). 8 out of the 15 stocks, namely KWV Beleggings, Tiger Brands, Impala Platinum, Sable, Hudaco, SABmiller, FirstRand and SASOL are best modelled by the normal distribution whilst 2 of the stocks; Nuworld and Liberty Group are best modelled by the student's *t*- distribution and the other 5 stocks; Tongaat Hulett, Barloworld, Omni, Anglo-American and Rainbow Chicken are best modelled using the generalized error distribution (GED). It is clear that the normal distribution is adequate for modelling the volatility of stock returns this sample of JSE listed stocks as there is no evidence that the alternative fat-tailed error distributions significantly improve the models.

Researcher's Best GARCH Model Choice

It should be noted that the final decision of what model to use is in the end the user's choice. The symmetric GARCH (1,1) is almost always a good choice for the JSE data series that we have investigated. Of course this cannot be taken as a general result as different GARCH models can be used depending on the nature of the stock data but, it is also worth noting that the symmetric GARCH (1,1) model has the advantage that it does not have convergence problems as observed in some of the alternative GARCH models such as the EGARCH model. The estimated parameters of the symmetric GARCH (1,1) are also observed to be stable.

With all this in perspective, it should be noted that the fundamental valuation theories in finance, such as the capital asset pricing models and the option pricing models for example, are based on some hypothesized risk-return relationship. Most of these models hold for the average security but they do not explain the full valuation mechanism for non-average securities. As such GARCH modelling holds for most but not all stocks and therefore there is need to explore several other avenues that can be used to model volatility of stock returns. Several extensions to this study can be suggested namely Stochastic Volatility models and Regime Switching models.

Currently, research is under way in the modelling of volatility through the use of **stochastic volatility methods** where the evolution of the volatility is explained by a stochastic differential equation. This can be seen as one possible way of handling the problem that volatility is not constant over time. Unlike in the GARCH modelling technique, it might be more useful to model stochastic phenomena on a continuous-time scale. These models are appealing mostly because if trading time is modelled as a continuum, much more powerful hedging strategies are possible than in the discrete time setup as that used in GARCH modelling. Further to this, these models are based on the notion that stock prices follow a Brownian motion or in some instances the Poisson process and both can be categorized as Levy processes. The advantage with these processes is that they follow different paths at different instances offering the use of 'what if' analysis, a very useful tool given that it is impossible to determine future stock returns.

Regime switching models on the other hand are based on the assumption that economic and financial markets seem to behave differently in different periods of time. This is used to specify different models for different periods. **Markov models** can be used for switching between these different models (Hamilton 1994).

Importance of GARCH Modelling in Portfolio Construction

The researcher observed that when a GARCH (1,1) process is included together with an AR (k) process, the efficient frontier plotted best resembles the Markowitz Mean-Variance Efficient Frontier compared to the other 2 models. There is a marked improvement to when an AR (k) term only (Model 1) is used or when a GARCH (1,1) process only (Model 2) is used. One can therefore conclude that GARCH modelling is of significant importance in portfolio construction.

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Appendix A: Sensitivity Analysis

Estimation Results for the period January 1991 to February 2003

In assessing the consistency and validity of the estimation results for the period January 1991 to February 2008, we conduct a similar estimation process but with data for a shorter period; January 1991 to February 2003. All the 15 data series used for this period constitute of 146 data points, each data point representing the monthly price of the given stock. For purposes of estimation and validation of the models, data points January 1991-February 2001 are used for estimation and data points March 2001-February 2003 are used for validation purposes. The process is identical to the one used before in *Chapter 4* of this master thesis, starting with determining the best GARCH model, followed by determining the best lag structure, determining the best error distribution and then the actual estimation having determined the best model.

Determining the Best GARCH model from symmetric GARCH, TARCH, EGARCH and PARCH.

Below, the performance measures for the 4 different models for each of the 15 stocks are presented in tabular form. As in *Section 4.3.1* above we analyze the performance of each of the models based on the estimation performance measures such as the R^2, AIC and SIC together with the forecasting performance measures such as the MAPE, MSE, MAE and TIC to determine the best model. The superior measure once again is highlighted in bold.

Hulett	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	117.6817	117.2304	115.4318	-
AIC	-2.150621	-2.122164	-2.086899	-
SIC	-1.944741	-1.890549	-1.855284	-
Adj. R^2	**0.346662**	0.339969	0.329165	-
MSE	0.090175	0.090478	**0.088973**	-
MAE	**0.079091**	0.079445	0.079664	-
MAPE	**339.8388**	344.7128	352.0376	-
TIC	0.634513	0.634186	**0.619535**	-

KWV Beleggings	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	97.22238	101.9043	98.7832	100.3199
AIC	-2.027164	-2.108985	-2.039627	-2.051552
SIC	-1.86051	-1.914555	-1.845197	-1.829347
Adj. R^2	**0.146299**	0.144765	0.145135	0.135139
MSE	**0.060295**	0.063729	0.064474	0.06296
MAE	**0.047807**	0.050118	0.051647	0.050351
MAPE	263.4349	**230.0248**	241.8924	251.4479
TIC	**0.590761**	0.635379	0.63586	0.617909

Nuworld	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	72.96854	81.58813	71.44228	66.79002
AIC	-1.299371	-1.451763	-1.248846	-1.1358
SIC	-1.090957	-1.217297	-1.01438	-0.875283
Adj. R^2	0.072004	**0.091652**	0.010847	0.076989
MSE	0.138303	**0.133894**	0.139633	0.134064
MAE	0.107054	**0.101701**	0.10349	0.103479
MAPE	175.6573	157.5854	**103.4292**	173.148
TIC	**0.649847**	0.666495	0.770241	0.690428

Tiger Brands	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	96.9143	98.64963	99.82846	-
AIC	-2.066995	-2.0839	-2.110999	-
SIC	-1.868589	-1.857149	-1.884249	-
Adj. R^2	**0.069909**	0.061204	0.046469	-
MSE	**0.063756**	0.063972	0.064906	-
MAE	**0.050812**	0.051998	0.054601	-
MAPE	**90.01608**	94.42124	102.0918	-
TIC	**0.650162**	0.662342	0.684175	-

Impala Platinum	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	68.69502	69.02592	69.56152	-
AIC	-1.348778	-1.333909	-1.345812	-
SIC	-1.126573	-1.083928	-1.095831	-
Adj. R^2	**0.093052**	0.078575	0.07986	-
MSE	**0.13334**	0.134164	0.133541	-
MAE	**0.108437**	0.108499	0.109087	-
MAPE	85.19486	**82.53677**	85.03677	-
TIC	0.620611	0.625611	**0.620464**	-

Barloworld	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	102.7179	103.1551	101.7089	100.9993
AIC	-2.104842	-2.092335	-2.060198	-2.022206
SIC	-1.882637	-1.842354	-1.810217	-1.744449
Adj. R^2	0.295447	**0.297896**	0.269969	0.288129
MSE	0.063776	**0.061111**	0.068383	0.061962
MAE	0.050878	**0.049755**	0.054492	0.050076
MAPE	160.6497	**157.581**	179.8258	159.3535
TIC	**0.443178**	0.445891	0.450023	0.444636

Omnia	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	119.4172	119.4305	116.8593	120.1703
AIC	-1.938756	-1.921889	-1.877936	-1.917441
SIC	-1.797106	-1.756631	-1.712678	-1.728575
Adj. R^2	**0.071139**	0.062706	0.064943	0.051244
MSE	**0.19319**	0.193284	0.193947	0.193512
MAE	**0.143279**	0.143388	0.144295	0.143697
MAPE	205.597	205.5186	**185.3744**	214.2933
TIC	0.849579	0.850042	0.880265	**0.844198**

Sable	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	58.85311	59.59658	59.10501	60.49887
AIC	-1.037062	-1.00184	-0.973429	-0.980931
SIC	-0.8547	-0.824909	-0.771222	-0.753448
Adj. R^2	**-0.032069**	-0.042909	-0.041583	-0.07922
MSE	0.094183	0.092779	**0.090921**	0.091576
MAE	0.066606	0.065821	**0.065612**	0.066151
MAPE	61.19095	61.57155	**57.17603**	59.36606
TIC	0.878418	0.879828	**0.802542**	0.84723

Liberty Group	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	148.5596	152.1826	150.0036	-
AIC	-2.39596	-2.440044	-2.403421	-
SIC	-2.255836	-2.276566	-2.239943	-
Adj. R^2	**0.171436**	0.156335	0.167242	-
MSE	0.077663	**0.076612**	0.076701	-
MAE	0.060148	0.059583	**0.059387**	-
MAPE	150.6522	**140.5894**	141.535	-
TIC	0.675757	**0.663934**	0.671327	-

Hudaco	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	98.12016	101.1632	101.6395	101.5911
AIC	-1.981079	-2.025016	-2.035259	-2.012712
SIC	-1.817685	-1.83439	-1.844633	-1.794854
Adj. R^2	**0.27958**	0.269117	0.271584	0.262183
MSE	0.090457	**0.088603**	0.09002	0.09318
MAE	0.076674	**0.073727**	0.075241	0.079546
MAPE	138.8511	**127.2381**	132.2425	146.1305
TIC	**0.721592**	0.746614	0.733015	0.725346

SABmiller	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	109.2919	109.5827	109.2962	110.3366
AIC	-2.248173	-2.232587	-2.226291	-2.227179
SIC	-2.05503	-2.011852	-2.005556	-1.978852
Adj. R^2	**0.146523**	0.136228	0.137843	0.120958
MSE	0.056975	0.056566	0.057317	**0.055607**
MAE	0.050175	0.049876	0.050458	**0.04897**
MAPE	102.9927	102.6622	**102.5178**	105.7344
TIC	0.519853	0.512942	0.527532	**0.487571**

Anglo-American	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	86.47329	86.98757	93.05754	86.88663
AIC	-1.826972	-1.815806	-1.955346	-1.790497
SIC	-1.628566	-1.589056	-1.728595	-1.535403
Adj. R^2	**0.225336**	0.212463	0.169635	0.201362
MSE	0.090042	**0.089946**	0.093001	0.090837
MAE	0.073814	**0.073307**	0.075768	0.074132
MAPE	132.619	127.4693	152.5681	**124.2007**
TIC	0.547233	0.553119	**0.511874**	0.567587

Rainbow Chicken	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	32.50166	40.52541	42.6103	-
AIC	-0.514595	-0.650979	-0.691462	-
SIC	-0.361116	-0.47192	-0.512403	-
Adj. R^2	**0.123748**	0.086344	0.106773	-
MSE	0.118487	0.118274	**0.113049**	-
MAE	0.102475	0.096382	**0.09448**	-
MAPE	**140.4888**	171.128	144.3295	-
TIC	**0.686338**	0.738008	0.705404	-

FirstRand	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	106.5361	107.1858	108.5391	109.2437
AIC	-2.265198	-2.257145	-2.288256	-2.281465
SIC	-2.038448	-2.002051	-2.033162	-1.998027
Adj. R^2	**0.45683**	0.449259	0.448719	0.441217
MSE	0.076673	0.076423	**0.075864**	0.07733
MAE	0.055485	0.055288	**0.054978**	0.055748
MAPE	254.0293	256.7974	253.4601	**253.2169**
TIC	0.557736	0.557101	**0.555708**	0.556823

SASOL	GARCH(1,1)	TARCH(1,1)	EGARCH(1,1)	PARCH (1,1)
Log Likelihood	99.71843	101.4103	97.84192	-
AIC	-1.747018	-1.760197	-1.692227	-
SIC	-1.544811	-1.532715	-1.464745	-
Adj. R^2	**0.037052**	-0.001897	0.001381	-
MSE	**0.082558**	0.085535	0.08533	-
MAE	**0.066959**	0.069665	0.068972	-
MAPE	**90.46595**	94.11542	91.25631	-
TIC	**0.698751**	0.731892	0.757771	-

In the tables above the Log-likelihood together with Akaike's information criterion (AIC) as well as the Schwarz Criterion for $p=q=1$ are presented. Four different forecast measures are also presented to determine which model performs best on out-of-sample data. It is observed that 7 out of the 15 stocks, namely Tongaat-Hulett, KWV Beleggings, Tiger Brands, Impala Platinum, Omnia, Rainbow Chicken and SASOL are best modelled by the symmetric GARCH; 5 stocks namely Nuworld, Barloworld, Liberty Group, Hudaco and Anglo-American by the TARCH model; 2 namely Sable and FirstRand by the EGARCH model and the remaining 1 namely SABmiller by the PARCH model. Evidently, from these results the superior model for modelling volatility of the stock returns from the selected sample, amongst the chosen GARCH models, is the symmetric GARCH model as it best models the volatility of the most stocks in the sample. This is consistent with the findings in *Section 4.3.1* above, further supporting previous findings for the JSE listed stocks.

Determining Best Lag Structure for the symmetric GARCH model

Having determined that the symmetric GARCH model best describes most of the stocks in the sample's return volatility we discard of the 3 other GARCH models and further investigate the best lag structure for the standard GARCH model. The best lag structure can be obtained by altering the ARMA p and q orders of the symmetric GARCH (p,q). Hence we investigate the symmetric GARCH under different lag structures namely the GARCH (1,1), GARCH (1,2), GARCH (2,1) and GARCH (2,2) for each of the 15 stocks. Based on the forecasting ability of each of these 4 lag structures, we determine the best lag structure for our modelling purposes. Below are the performance measures for the 15 different stocks based on forecasting ability for each of the 15 stocks presented in tabular form.

Hulett	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	117.6817	116.4173	113.6942	113.4122
AIC	-2.150621	-2.106222	-2.052828	-2.027691
SIC	-1.944741	-1.874607	-1.821213	-1.77034
Adj. R^2	0.346662	0.338883	**0.347223**	0.34102
MSE	**0.090175**	0.090376	0.100988	0.098593
MAE	**0.079091**	0.079907	0.087577	0.085275
MAPE	**339.8388**	343.6702	377.7209	368.3404
TIC	0.634513	**0.63315**	0.659438	0.655893

KWV Beleggings	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	97.22238	98.20566	96.42924	98.66356
AIC	-2.027164	-2.027094	-1.987174	-2.014912
SIC	-1.86051	-1.803396	-1.763476	-1.763252
Adj. R^2	**0.146299**	0.136853	0.144989	0.124981
MSE	0.060295	0.063488	0.065301	**0.060037**
MAE	**0.047807**	0.050982	0.052676	0.047957
MAPE	**263.4349**	287.8876	266.9773	273.9554
TIC	0.590761	0.598164	0.609534	**0.571084**

Nuworld	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	72.96854	73.42453	78.41547	78.01159
AIC	-1.299371	-1.288491	-1.388309	-1.360232
SIC	-1.090957	-1.054025	-1.153844	-1.099715
Adj. R^2	**0.072004**	0.05274	0.013858	0.022022
MSE	**0.138303**	0.139038	0.141694	0.139551
MAE	**0.107054**	0.108168	0.110961	0.108345
MAPE	**175.6573**	182.5359	196.4174	181.8922
TIC	0.649847	0.64751	**0.645198**	0.648091

Tiger Brands	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	96.9143	99.34745	96.99726	103.2965
AIC	-2.066995	-2.099941	-2.045914	-2.167735
SIC	-1.868589	-1.873191	-1.819164	-1.912641
Adj. R^2	**0.069909**	0.033747	0.058166	0.020054
MSE	0.063756	0.064611	**0.063745**	0.06954
MAE	0.050812	**0.049756**	0.050746	0.058148
MAPE	90.01608	**89.5683**	89.74933	109.5925
TIC	0.650162	**0.625025**	0.648814	0.665422

Impala Platinum	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	68.69502	70.60677	70.46101	73.1199
AIC	-1.348778	-1.369039	-1.3658	-1.402664
SIC	-1.126573	-1.119058	-1.115819	-1.124908
Adj. R^2	**0.093052**	0.071277	0.071974	0.0593
MSE	0.13334	0.140886	**0.128063**	0.132492
MAE	0.108437	0.11237	**0.105443**	0.107944
MAPE	85.19486	**83.23371**	85.12769	84.90632
TIC	0.620611	0.674233	**0.578484**	0.606114

Barloworld	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	102.7179	97.16265	100.2084	99.93377
AIC	-2.104842	-1.95917	-2.026854	-1.998528
SIC	-1.882637	-1.709189	-1.776873	-1.720772
Adj. R^2	**0.295447**	0.295302	0.290135	0.289053
MSE	0.063776	0.061293	0.063312	**0.060756**
MAE	0.050878	**0.048994**	0.050346	0.049748
MAPE	160.6497	**151.6033**	165.6635	156.5248
TIC	0.443178	0.442379	**0.440303**	0.450519

Omnia	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	119.4172	121.1624	119.8556	121.5042
AIC	-1.938756	-1.951494	-1.929155	-1.940243
SIC	-1.797106	-1.786236	-1.763897	-1.751376
Adj. R^2	**0.071139**	0.060963	0.06301	0.050249
MSE	0.19319	0.193535	**0.193083**	0.193481
MAE	0.143279	0.143763	**0.142899**	0.143178
MAPE	205.597	212.9879	**195.8724**	202.2073
TIC	0.849579	**0.8462**	0.854251	0.849199

Sable	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	58.85311	59.62261	58.86116	59.88257
AIC	-1.037062	-1.032452	-1.017223	-1.017651
SIC	-0.8547	-0.824039	-0.80881	-0.783186
Adj. R^2	**-0.032069**	-0.073166	-0.046864	-0.076422
MSE	0.094183	**0.092452**	0.094125	0.092811
MAE	0.066606	**0.065759**	0.066499	0.065796
MAPE	61.19095	**57.91842**	61.16594	58.81192
TIC	0.878418	**0.848432**	0.88357	0.862164

Liberty Group	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	148.5596	151.0609	148.5685	151.3694
AIC	-2.39596	-2.421192	-2.379303	-2.40957
SIC	-2.255836	-2.257714	-2.215825	-2.222738
Adj. R^2	**0.171436**	0.161928	0.163703	0.154212
MSE	**0.077663**	0.078166	0.077771	0.078191
MAE	**0.060148**	0.060754	0.060293	0.060865
MAPE	**150.6522**	155.0702	151.5262	155.1395
TIC	0.675757	0.672983	0.674823	**0.671117**

Hudaco	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	98.12016	101.1602	98.31267	101.3083
AIC	-1.981079	-2.02495	-1.963713	-2.006631
SIC	-1.817685	-1.834325	-1.773088	-1.788772
Adj. R^2	**0.27958**	0.263219	0.27113	0.265308
MSE	0.090457	0.092262	0.091831	**0.090166**
MAE	0.076674	0.077248	0.078029	**0.076253**
MAPE	138.8511	137.1465	142.2537	**136.3275**
TIC	0.721592	**0.718436**	0.722126	0.732722

SABmiller	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	109.2919	111.4461	109.3668	111.915
AIC	-2.248173	-2.27354	-2.227842	-2.261869
SIC	-2.05503	-2.052806	-2.007107	-2.013542
Adj. R^2	**0.146523**	0.062272	0.136286	0.032733
MSE	**0.056975**	0.060717	0.057067	0.060874
MAE	**0.050175**	0.051723	0.050265	0.051558
MAPE	102.9927	117.7364	**102.7102**	118.8373
TIC	0.519853	0.523269	0.522353	**0.517459**

Anglo-American	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	86.47329	92.89027	83.80547	95.36473
AIC	-1.826972	-1.9515	-1.742654	-1.985396
SIC	-1.628566	-1.72475	-1.515904	-1.730302
Adj. R^2	**0.225336**	0.214756	0.198107	0.182608
MSE	**0.090042**	0.090344	0.090828	0.090722
MAE	0.073814	0.074119	**0.073475**	0.074148
MAPE	132.619	**131.0431**	134.1657	143.4153
TIC	0.547233	0.551943	0.57317	**0.51559**

Rainbow Chicken	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	32.50166	32.52567	32.55164	32.56698
AIC	-0.514595	-0.495644	-0.496148	-0.477029
SIC	-0.361116	-0.316585	-0.317089	-0.27239
Adj. R^2	**0.123748**	0.114345	0.114032	0.104356
MSE	0.118487	0.117771	0.117209	**0.116911**
MAE	0.102475	0.101498	0.100732	**0.100257**
MAPE	**140.4888**	168.4483	166.4808	165.0649
TIC	**0.686338**	0.686931	0.687103	0.687983

FirstRand	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	106.5361	111.8891	108.1422	108.0381
AIC	-2.265198	-2.365266	-2.279132	-2.253749
SIC	-2.038448	-2.110172	-2.024038	-1.970311
Adj. R^2	**0.45683**	0.438759	0.445897	0.434199
MSE	0.076673	**0.068308**	0.072855	0.071564
MAE	0.055485	**0.05074**	0.05369	0.052659
MAPE	254.0293	**230.0773**	243.7469	234.681
TIC	0.557736	**0.545211**	0.554882	0.552666

SASOL	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH (2,2)
Log Likelihood	99.71843	98.80172	95.58143	99.2025
AIC	-1.747018	-1.710509	-1.64917	-1.699095
SIC	-1.544811	-1.483027	-1.421688	-1.446337
Adj. R^2	**0.037052**	0.024245	-0.039507	0.025021
MSE	0.082558	-	0.090819	**0.082451**
MAE	0.066959	-	0.071404	**0.0662**
MAPE	90.46595	-	111.752	**89.04041**
TIC	0.698751	-	**0.64427**	0.696651

Although the GARCH (1,2) model provided better estimation for 4 of the stocks namely, Tiger Brands, Barloworld, Sable and FirstRand; whilst the GARCH (2,1) model best described 2 stocks, Impala Platinum and Omnia and the GARCH (2,2) model also provided better estimation for 2 stocks, Hudaco and SASOL, the results obtained were consistent with what is commonly observed in modelling volatility of financial data and that is the fact that the GARCH (1,1) specification is adequate to capture the correlation in the squared returns of the stocks. In the selected sample of JSE listed stocks used for this research, the GARCH (1,1) model best models 7 out of the 15 stocks, namely, Tongaat-Hulett, KWV Beleggings, Nuworld, Sable, Liberty Group, SABmiller and Anglo-American.

Determining Best Distribution for Modelling with GARCH (1,1)

Further to the findings above we would also like to determine if modelling the volatility of the stock returns using other error distributions which are fat-tailed, instead of the standard normal distribution, such as the student's *t*-distribution and the generalized error distribution (GED) would improve the model. If indeed there is an improvement in the forecasting ability of the model then we can conclude for the purposes of this research that the specific error distribution is superior to the normal distribution in modelling the volatility of the selected sample of JSE stocks. We tabulate the results of the investigation below for each of the 15 JSE stocks, for the period Jan 1991 to February 2003 and using the same forecasting performance measures as before we select the best distribution to model the volatility of the stock returns.

Tongaat Hulett	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	117.6817	117.3968	117.1356
AIC	-2.150621	-2.125428	-2.120305
SIC	-1.944741	-1.893813	-1.88869
Adj. R^2	0.346662	**0.347382**	0.343993
MSE	**0.090175**	0.096467	0.097454
MAE	**0.079091**	0.083919	0.085831
MAPE	**339.8388**	366.3698	365.6466
TIC	**0.634513**	0.649953	0.649836

KWV Beleggings	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	97.22238	96.34513	96.34993
AIC	-2.027164	-1.985284	-1.985392
SIC	-1.86051	-1.761586	-1.761694
Adj. R^2	**0.146299**	0.1448	0.14478
MSE	**0.060295**	0.065371	0.065393
MAE	**0.047807**	0.052576	0.052592
MAPE	263.4349	251.965	**251.8351**
TIC	**0.590761**	0.618124	0.61834

Nuworld	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	72.96854	75.26521	76.17566
AIC	-1.299371	-1.325304	-1.343513
SIC	-1.090957	-1.090839	-1.109048
Adj. R^2	0.072004	**0.094584**	0.089823
MSE	0.138303	0.133959	**0.133677**
MAE	0.107054	0.102567	**0.10212**
MAPE	175.6573	147.6948	**142.497**
TIC	**0.649847**	0.69515	0.71578

Tiger Brands	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	96.9143	96.91288	97.25023
AIC	-2.066995	-2.043974	-2.051729
SIC	-1.868589	-1.817224	-1.824979
Adj. R^2	**0.069909**	0.058168	0.056559
MSE	**0.063756**	0.063759	0.064129
MAE	**0.050812**	0.050819	0.051199
MAPE	90.01608	90.01385	**89.55274**
TIC	**0.650162**	0.650257	0.652477

Impala Platinum	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	68.69502	69.57596	69.46562
AIC	-1.348778	-1.346132	-1.34368
SIC	-1.126573	-1.096151	-1.093699
Adj. R^2	**0.093052**	0.077553	0.075793
MSE	0.13334	**0.132409**	0.133557
MAE	0.108437	**0.10774**	0.107825
MAPE	85.19486	84.05792	**82.64743**
TIC	0.620611	**0.607891**	0.61531

Barloworld	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	102.7179	102.1148	105.4062
AIC	-2.104842	-2.069219	-2.142361
SIC	-1.882637	-1.819238	-1.89238
Adj. R^2	**0.295447**	0.294113	0.290202
MSE	0.063776	**0.062545**	0.063681
MAE	0.050878	**0.05025**	0.051033
MAPE	160.6497	**158.1504**	163.534
TIC	**0.443178**	0.443248	0.443691

Omnia	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	119.4172	117.9528	119.5793
AIC	-1.938756	-1.896629	-1.924433
SIC	-1.797106	-1.731371	-1.759175
Adj. R^2	**0.071139**	0.057574	0.065292
MSE	**0.19319**	0.195104	0.193675
MAE	**0.143279**	0.145552	0.143719
MAPE	205.597	**174.4764**	190.5604
TIC	**0.849579**	0.902181	0.868006

Sable	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	58.85311	61.11294	76.2875
AIC	-1.037062	-1.062259	-1.36575
SIC	-0.8547	-0.853845	-1.157336
Adj. R^2	-0.032069	-0.034368	**-0.027856**
MSE	0.094183	**0.093596**	0.093639
MAE	0.066606	**0.066117**	0.066483
MAPE	61.19095	59.68526	**57.98469**
TIC	0.878418	0.86039	**0.812117**

Liberty Group	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	148.5596	148.5595	148.5624
AIC	-2.39596	-2.379151	-2.379201
SIC	-2.255836	-2.215673	-2.215723
Adj. R^2	**0.171436**	0.164043	0.163853
MSE	0.077663	**0.077662**	0.077697
MAE	0.060148	**0.060147**	0.060184
MAPE	150.6522	**150.6449**	150.9117
TIC	0.675757	0.675763	**0.675668**

Hudaco	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	98.12016	98.87888	98.51625
AIC	-1.981079	-1.97589	-1.968091
SIC	-1.817685	-1.785264	-1.777466
Adj. R^2	**0.27958**	0.272348	0.272327
MSE	**0.090457**	0.090682	0.090557
MAE	0.076674	0.076693	**0.076468**
MAPE	138.8511	138.1535	**137.362**
TIC	**0.721592**	0.724633	0.724956

SABmiller	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	109.2919	107.8784	107.9648
AIC	-2.248173	-2.19513	-2.197028
SIC	-2.05503	-1.974395	-1.976293
Adj. R^2	**0.146523**	0.127258	0.127052
MSE	0.056975	**0.056388**	0.056429
MAE	0.050175	0.049889	**0.049836**
MAPE	**102.9927**	104.7714	104.4066
TIC	**0.506768**	0.519853	0.507399

Anglo-American	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	86.47329	-	90.16153
AIC	-1.826972	-	-1.888771
SIC	-1.628566	-	-1.662021
Adj. R^2	**0.225336**	-	0.212786
MSE	**0.090042**	-	0.091734
MAE	**0.073814**	-	0.074763
MAPE	132.619	-	**124.8678**
TIC	**0.547233**	-	0.587027

Rainbow Chicken	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	32.50166	33.70823	34.39453
AIC	-0.514595	-0.518606	-0.531933
SIC	-0.361116	-0.339547	-0.352873
Adj. R^2	**0.123748**	0.114107	0.110429
MSE	0.118487	0.1158	**0.111638**
MAE	0.102475	0.09977	**0.094577**
MAPE	**140.4888**	164.5388	150.7081
TIC	0.686338	0.677725	**0.67678**

FirstRand	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	106.5361	106.5336	106.54
AIC	-2.265198	-2.242151	-2.242298
SIC	-2.038448	-1.987057	-1.987204
Adj. R^2	**0.45683**	0.449861	0.450052
MSE	**0.076673**	0.076679	0.076862
MAE	**0.055485**	0.055489	0.055606
MAPE	**254.0293**	254.051	255.0159
TIC	**0.557736**	0.557755	0.558346

SASOL	Normal Distribution	Student's t-distribution	Generalized Error Distribution (GED)
Log Likelihood	99.71843	99.7305	100.5564
AIC	-1.747018	-1.7282	-1.743931
SIC	-1.544811	-1.500718	-1.516449
Adj. R^2	**0.037052**	0.030533	0.035587
MSE	0.082558	0.082303	**0.081788**
MAE	0.066959	0.066647	**0.066038**
MAPE	90.46595	89.90774	**88.75999**
TIC	0.698751	0.696981	**0.693768**

As noted in *Section 4.3.3* the Normal distribution once again provides a much closer approximation to the empirical unconditional density of 8 out of the 15 stocks. The stocks

Tongaat-Hulett, KWV Beleggings, Tiger Brands, Omnia, Hudaco, SABmiller, Anglo-American and FirstRand are best modelled by the normal distribution whilst 3 of the stocks; Impala Platinum, Barloworld and Liberty Group are best modelled by the student's *t*- distribution and the other 4 stocks; Nuworld, Sable, Rainbow Chicken and SASOL are best modelled using the generalized error distribution (GED). Though there has been so much research into the use of other error distributions which are better equipped to capture the fat-tailed nature of stock returns and evidence to support the use of these distributions, there was no significant incentive to use alternative distributions as the majority of the stocks in the sample were best modelled using the normal distribution. This again is consistent with the findings in *Section 4.3.3* above.

Estimated Parameters

Having determined that the symmetric GARCH (1,1) modelled with the Normal distribution is the best model to implement for modelling the volatility of the sample of JSE listed stocks used for this research for the period January 1991 to February 2003, we proceed to estimate the variance equation parameters below. The technique used in estimating GARCH models involves the maximization of a likelihood function constructed under the assumption of an *i.i.d* distribution for the standardized innovations. The estimated parameters are tabulated below.

Tongaat - Hulett	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000224	1.81E-06	123.7861	0
RESID(-1)^2	-0.091129	0.000897	-101.612	0
GARCH(-1)	1.068084	0.000608	1755.695	0.0000

KWV Beleggings	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000825	0.001038	0.794203	0.4271
RESID(-1)^2	0.245788	0.161802	1.51907	0.1287
GARCH(-1)	0.671084	0.211238	3.176912	0.0015

Nuworld	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000391	0.000581	0.671701	0.5018
RESID(-1)^2	0.27355	0.10056	2.720254	0.0065
GARCH(-1)	0.739221	0.090674	8.152486	0.0000

Tiger Brands	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000878	0.001224	0.717303	0.4732
RESID(-1)^2	0.124404	0.116997	1.063305	0.2876
GARCH(-1)	0.736593	0.266358	2.765426	0.0057

Impala platinum	Coefficient	Std. Error	z-Statistic	Prob.
C	0.005031	0.023804	0.211346	0.8326
RESID(-1)^2	0.041481	0.164742	0.251795	0.8012
GARCH(-1)	0.566177	1.979401	0.286035	0.7749

Barloworld	Coefficient	Std. Error	z-Statistic	Prob.
C	0.014573	0.002475	5.888527	0
RESID(-1)^2	-0.049043	0.007674	-6.39093	0
GARCH(-1)	-0.961573	0.015538	-61.88468	0.0000

Omnia	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000867	0.001203	0.720223	0.4714
RESID(-1)^2	0.09191	0.092423	0.994456	0.32
GARCH(-1)	0.802888	0.213353	3.763197	0.0002

Sable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001699	0.000595	2.85763	0.0043
RESID(-1)^2	0.345028	0.150232	2.296634	0.0216
GARCH(-1)	0.597309	0.098672	6.053488	0.0000

Liberty Group	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000459	0.000516	0.889301	0.3738
RESID(-1)^2	0.1538	0.087924	1.749249	0.0802
GARCH(-1)	0.761556	0.169251	-4.499566	0.0000

Hudaco	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001776	0.002853	0.622637	0.5335
RESID(-1)^2	0.096893	0.118081	0.820562	0.4119
GARCH(-1)	0.655056	0.485504	1.34923	0.1773

SABmiller	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000533	0.000657	0.810831	0.4175
RESID(-1)^2	0.246556	0.201492	1.223656	0.2211
GARCH(-1)	0.682828	0.23619	2.891012	0.0038

Anglo-American	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000511	0.000641	0.796856	0.4255
RESID(-1)^2	0.106122	0.081963	1.294753	0.1954
GARCH(-1)	0.828052	0.106476	7.776891	0.0000

Rainbow Chicken	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003711	0.004595	0.807737	0.4192
RESID(-1)^2	0.106825	0.101914	1.048195	0.2945
GARCH(-1)	0.786813	0.194443	4.0465	0.0001

FirstRand	Coefficient	Std. Error	z-Statistic	Prob.
C	2.85E-03	0.009357	0.305018	0.7604
RESID(-1)^2	-0.043986	0.115672	-0.380264	0.7037
GARCH(-1)	0.482256	1.816031	0.265555	0.7906

SASOL	Coefficient	Std. Error	z-Statistic	Prob.
C	-2.16E-05	0.000207	-0.104279	0.9169
RESID(-1)^2	-0.050366	0.045146	-1.115633	0.2646
GARCH(-1)	1.065958	0.062927	16.93968	0.0000

The estimated GARCH parameters are significant within the 5% range for all the 15 stocks with the exception of the parameters estimated for Tongaat Hulett, Impala Platinum, Barloworld, FirstRand and SASOL. This, as is observed in *Section 4.3.4*, is solved by including more data points as was done in the estimation for the period January 1991 to February 2008 in which 206 data points were used instead of the 146 data points used in this estimation which is for the period January 1991 to February 2003. Clearly it is observed that, the GARCH models are sensitive to the number of data points used in the estimation, as more data points make the models more stable. However, it can still be concluded that the GARCH 1,1 modelled with the normal distribution estimates the best GARCH parameters amongst the models examined for JSE listed stocks.

Hence, given the consistency of these results with those obtained in *Chapter 4* one can conclude that the results obtained for the estimation of volatility of JSE listed stocks are reliable for the period under investigation.

Appendix B:

Matlab Program for Stock Data Analysis

Appendix C:

E-views Program used in modelling of volatility in the Stock Prices

Appendix D:

E-views Program used for determining the Covariance Matrices for plotting the Efficient Frontiers

Appendix E:

Matlab Program used for plotting the Efficient Frontiers