

Building a Model of “Spatial Ability”
An Analysis of Grade 5 and 6 Learners’ Strategies for Solving
“Spatial” Activities

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(Mathematics Education)

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Abstract

This study explores the notion of “spatial ability” from the perspective of mathematics education. A review of the literature on “spatial ability” is used to compile a preliminary model of the phenomenon. Certain questions related to interaction in space arising from the literature review are noted in this model. Three aspects of this interaction are the focus of the empirical study. The results of the research are used to shed light on the preliminary model of “spatial ability”.

The three themes of the empirical study can be described as follows: the visualisation of “objects” from different perspectives (in “small-scale” space); the visualisation of “objects” from different perspectives (in “large-scale” space); and the representation of a three-dimensional “object” in a two-dimensional net.

The results of the study suggest that a range of strategies can be used on the same “spatial” activity, and that a learner can adopt a variety of different strategies on a set of activities. Of the ten strategies identified in the study, some appear to rely on the manipulation of visual imagery, while others suggest that the manipulation process has been generalised resulting in a more “abstract” strategy. Interesting features related to the use of physical manipulation in solving spatial tasks and the communication of visual processes in the form of drawings and verbal responses are discussed.

These results are used to expand the preliminary model of “spatial ability”. This updated model suggests that a learner who has mastered four “abilities” and has a working knowledge of visual conventions will be able to interact successfully in the visual world. The different strategies identified in the empirical study are required during this interaction in space. The researcher also identifies areas for further research in the field of “spatial ability” and reflects on the methodology associated with the assessment of this phenomenon. The potential value of the results of the study for use in mathematics education is discussed.

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Chapter 1: Introduction

1.1 Trends in Curriculum Development: The Case of Geometry

The school geometry curriculum in South Africa has traditionally been restricted to the study of plane figures, that is, the classification of these figures and the study of their properties. In the senior grades most of the work has focused on the study of synthetic geometry as an example of an axiomatic system. Those involved in curriculum reform have, however, recognised that there is more to geometry than this, and that these additional aspects can realistically be explored at school level. One such aspect is the view that geometry is a body of knowledge which can support interaction in space.¹

Recent innovations in mathematics curricula in this country and elsewhere² have included calls for classroom activities that promote or require the use of “spatial skills”.³ For example, in Curriculum 2005 in South Africa, one of the ten Specific Outcomes for Mathematical Literacy, Mathematics and Mathematical Sciences requires that learners:

Describe and represent experiences with shape, space, time and motion, using all available senses: Mathematics enhances and helps to formalise the ability to be able to grasp, visualise and represent the space in which we live. In the real world, space and shape do not exist in isolation from motion and time. Learners should be able to display an understanding of spatial sense and motion in time.

The four Assessment Criteria for each of the three phases⁴ are:

1. *Description of the position of an object in space*
2. *Descriptions of changes in shape of an object*
3. *Descriptions of orientation of an object*
4. *Demonstrate an understanding of the inter-connectedness between shape, space and time.*

Department of Education (October 1997)

¹ Human (1998) has identified the following additional aspects of geometry as important:

- Geometry is a domain of purely mathematical activity
- Geometry is an activity that sheds light on the nature of mathematics itself
- Geometry as the study of different objects, for example, figures, movements or representations
- Geometry provides models and analogies for other areas of mathematics.

² Standard 3 of the NCTM Draft “Standards 2000” Document suggests that mathematics instruction programmes should pay attention to geometry and spatial sense so that all students, among other things, “use visualisation and spatial reasoning to solve problems both within and outside of mathematics”. One of the six strands in the Western Australian Curriculum focuses on “the visualisation, analysis, representation and interpretation of shapes and objects in space”.

³ As the aim of this study is to explore and obtain some clarity on “spatial ability”, inverted commas will be used when referring to this notion.

⁴ The three phases are the foundation (grades 1 to 3), intermediate (grades 4 to 6) and senior (grades 7 to 9) phases.

Furthermore, research in the past few decades has studied links between “spatial ability” and performance in geometry and in mathematics in general. Clements and Battista (1992 : 442-443) provide a summary of the work done in this area and quote the work of Gardner, Hadamard, Krutetskii, Fennema and Sherman, Guay and McDaniel etc. They warn that the relationship between spatial skills and the learning of non-geometric concepts is not straightforward. Results of the research do vary: Guay and McDaniel (1977), for example, suggest that high mathematics achievers at elementary school have greater spatial ability than low achievers and that there is a relationship between mathematical and spatial thinking for pupils with high as well as low spatial ability.⁵ In contrast, in a study of foundation year engineering students in Papua New Guinea, Lean and Clements (1981) found that spatial ability and knowledge of spatial conventions had less influence on mathematical performance than the literature seemed to suggest.

1.2 The Development of a Malati Geometry Curriculum

Given the task of reconceptualising the school geometry curriculum, materials developers in the Geometry Working Group at the Mathematics Learning and Teaching Initiative (Malati)⁶ undertook an extensive review of current curriculum innovations and literature regarding the teaching and learning of geometry and formulated the following objectives for the study of geometry at school (grades 1 to 12):

- To develop spatial skills
- To learn to use a number of tools to solve problems
- To develop a sense of the structure of mathematics.

The development of the Malati geometry curriculum to attain these objectives has been based within the framework of the van Hiele Theory (as well as subsequent research that has been done in relation to this theory) and Hoffer's five geometry skills, namely, visual, verbal, drawing, logical and applied (1981).

As one of the materials developers charged with the task of creating tasks for the development of “spatial skills” in the intermediate phase and, as a secondary school teacher with no experience working in this phase, I undertook an extensive review of the literature on “spatial ability”. This review revealed certain features of this field which have implications for the development of materials for the Malati curriculum.

Firstly, there is little consensus on the definition of “spatial ability” and, in fact, there is little consensus on the notion of “space” itself. The study of the field is complicated by the use of a variety of terms, for example, “spatial reasoning”, “spatial intuition”, “spatial perception”, “spatial thinking”, “spatial sense”, “spatial

⁵ Guay and McDaniel (1977 : 211) define “low-level spatial abilities” as those requiring the visualisation of two-dimensional configurations, but no mental transformations of these visual images. “High-level spatial abilities” are characterised as requiring the visualisation of three-dimensional configurations, and the mental manipulation of these images.

⁶ The Mathematics Teaching and Learning Initiative (Malati) is a three-year project of the Open Society Foundation of South Africa involving the universities of Cape Town, Stellenbosch and the Western Cape. The brief of the Project was to develop mathematics learning and teaching materials and to develop a model for both curriculum and teacher development. The Project has permission to work in seven schools administered by the Western Cape Education Department.

orientation" and "spatial visualisation". In some cases the different terminology is used to refer to the same phenomena, whereas in other cases it is used to refer to different "abilities" or "skills". This concern is also expressed by Bishop (1983 : 184) in his review of the literature on "spatial ability": He suggests that the increased interest in the field of "spatial skills" in education has encountered problems in that definitions of what we mean by "spatial ability" are inadequate.

Secondly, there is little consensus on how to test or identify "spatial ability". This dilemma is particularly pertinent in the context of South African education: Egan (1979 : 1) notes that there have been many examples in the history of science in which qualitative measures of phenomena have been used before the phenomena themselves have been understood. He notes that, although these measures have not been based on a deep understanding of the mental processes, they have been used for personal selection and classification.

Furthermore, some researchers identify different aspects of "spatial ability" with particular tasks, whereas others refer to the processes involved. For example, some researchers refer to "performance on tasks which require...", suggesting that the skills involved are task specific. Reference to "processes", however, suggests that an individual has a repertoire of skills which can be used on a variety of tasks, possibly depending on the task and / or the individual.

Lastly, there is not much research on "space" and "spatial ability" in mathematics education itself. Most work done in this area has been done by developmental psychologists and factor analysts. This is confirmed by Bishop (1983 : 175). Recent reviews of this literature (Clements (1983), Clements and Battista (1992), Bishop (1980) and Eliot and Hauptman (1981)) have attempted to synthesise this literature. The work of van Hiele, Lesh, Krutetskii, Skemp, Bishop, Mitchelmore, Wattanhawa and Fennema has been identified as making a contribution in the field of mathematics education.

This literature review suggested to me that the process of developing "spatial skills" activities as part of the Malati geometry curriculum was going to be more complicated than expected. Clements (1983 : 17) suggests what might be involved: He suggests that, before mathematics educators can devise tasks for which "spatial" strategies are likely to be used, they need to agree on criteria for deciding whether a particular solution strategy is "spatial" in character and on a procedure for determining the extent to which a strategy is "spatial". Furthermore, he indicates that the development of such tasks needs to be accompanied by the development of methods which assist in the unambiguous identification of the strategies used by individuals on the tasks. For it is only then, he suggests, that claims about the relationship between "spatial ability" and performance on mathematical tasks can be explored. Bishop (1983 : 181) describes the task for mathematics educators: "It is clear (to a mathematics educator, at least) that there can never be a 'true' definition of spatial ability: We must see definitions and descriptions of abilities and processes that help us to solve our own particular problems".

It seems, however, that simply clarifying the nature of "spatial ability" for our purposes might not be enough: Berthelot and Salin (1998) suggest that mathematics

educators also need to focus on the actual **teaching processes** used in addressing spatial abilities. For they use their research in French schools to suggest that the treatment of “spatial abilities” might, in fact, be the source of many learning difficulties experienced by secondary school learners as well as the source of many deficiencies in spatial representation needed in real life!

1.3 This Study

With this daunting task in mind, I decided to intensify my study of “spatial ability” from the perspective of mathematics education, in the hope that this would inform the development of the Malati geometry curriculum, that is, in the

- The design of materials for developing and promoting the use of “spatial skills”
- The design of assessment tools for use with these materials
- The provision of support for Malati Project school teachers when using “spatial skills” activities in the classroom.⁷

This process has involved the following:

1. A review of the literature on “spatial ability”, noting certain questions and dilemmas that have arisen during reading.
2. The formulation of a preliminary model of “spatial ability” with this literature review as a basis
3. The design and execution of a short empirical study in an attempt to shed some light on the questions that arose in (1)
4. The re-formulation of the preliminary model in the light of the empirical study
5. A reflection on the methodology of the study, possible uses of the results and directions for further research.

⁷ The Malati Project works with teachers at four primary and three secondary schools in the Cape Peninsula.

Chapter 2: Literature Review

2.1 What is “Space”? What is “Shape”?

If the topic of “spatial ability” is to be explored, the approach to the notion of “space” needs to be clarified. What is meant when one talks about “space”? Views on this notion will, of course, influence ideas on the nature of “shapes” within this “space”. This, as Hershkowitz, Parzysz and van Doormolen (1996 : 161) point out, is not an easy task: They suggest that one’s beliefs about space will depend on one’s beliefs about the mind and the external world. The question is whether “space” is a “real” thing, or whether the individual imposes “space” on reality.

The *Macmillan Dictionary of Philosophy* (1979 : 332) identifies two “extreme” views on the nature of “space”:

1. According to the “absolutist theory”, space is regarded as a real thing in which objects can be at rest or really moving (rather than appearing to move because their relationship to other things is changing)
2. In contrast, the “relationist theory” denies the existence of an “absolute space”. According to Leibniz, reality is composed of “non-extended mental items” and is not spatial. Thus, according to Kant, the experience of the world as spatial is an “act of the mind” and objects in themselves have no spatial properties.

Clements (1983 : 10) notes that it is common to use the terms “physical” and “psychological” space respectively to distinguish between space in the external world, independent of the mind, and space which exists in the minds of individuals and would not exist if the mind did not exist.

A number of writers seem to adopt a compromise between these two “extreme” views. For example, Liben (in Clements, 1983 : 10) is quoted as suggesting that “psychological” and “physical” space cannot exist independently of one another. The *Macmillan Dictionary of Philosophy* (1979 : 332) refers to a “less strenuous” relationist theory in which propositions about “space” are regarded as relations among “ordinary things”. No indication is given of the status of “ordinary things” in this framework, but what is important is the suggestion that space does not have properties of its own, but that the properties are obtained through relations between objects.

These relations between objects are also referred to in the Dutch “Realistic Mathematics Education” (as discussed in Hershkowitz *et al*, 1996 : 177): Space is not seen to have properties of its own. Rather, “experiencing space” is said to be about the relative position of objects in space as well as the relative position of the objects and the position of the observer of these objects.

This notion that the position of the observer in the space is involved in determining space is explored by a number of researchers. According to Yi-Fu Tuan (1977 : 34-36) the human body has an important role to play in “creating” this space: He refers to “space” as “humanly constructed space” and indicates that “the human being, by his mere presence, imposes a schema on space”. Reference points and landmarks in space therefore only exist in relation to the co-ordinates of the human body. This

interaction between the individual and his / her environment also seems evident in the work of Piaget and Inhelder (1956): In this framework the representation of space is not a perceptual "reading off" of the environment, but is built up through the progressive organisation of a child's motor and internalised actions. They note, for example, that sensori-motor space is "super-imposed" on various pre-existing spaces. Each new space is regarded as a new mental construct.¹

Fischbein's studies of "intuition" (1920 : 86-87) also suggest that "space" results from interaction between the individual and the "world": He suggests that intuition about space is not innate, but consists of "a complex system of **conceptions**...which exceed the data at hand and the domain of perception in general". "Subjective space" is thus only an interpretation of reality, not a reproduction of it.²

Wheatley and Cobb (as quoted in Clements and Battista, 1992 : 446) also stress the importance of interaction between objects and the individual: They claim that "individuals give meaning and structure to spatial patterns based on their experiences, conceptual structures, intentions and ongoing social interactions". Patterns and structures are not "out there", but are constructed through action.

In all the approaches discussed here, therefore, it is suggested that the "space" we explore results from the interaction between the individual and the "objects" in space.

Hershkowitz *et al* (1996) provide an interesting interpretation of the nature of "shape" and "space": They identify three "perspectives" about the possible roles of shape and space and suggest that the meaning of "shape" depends on the changing role of the term within and between the three perspectives. For example, they suggest that in the first perspective ("interaction with real shapes in space"), interest lies in "real objects" in the "real world" and relations between them. Of course, the use of these terms says a great deal about beliefs on the nature of "space". There are, however, different levels of abstraction within this perspective: For example, when objects are classified according to certain chosen properties, they come to be regarded as representatives of a whole class of shapes.

In the second perspective ("shape and space are regarded as fundamental ingredients for constructing a theory" - only the visual representations can remain), shapes which are mainly geometric figures³, can in one regard be seen to represent reality in a theory. Thus the figure can be seen as a tool for solving problems and for constructing a geometric theory. On the other hand, geometric theory can be made

¹ Clements and Battista (1992 : 426) note that this particular aspect of Piaget and Inhelder's theory has not been widely discussed by researchers (as opposed to their theory on the progressive organisation of geometric ideas from topological to projective to Euclidean).

² An interesting view from the perspective of mathematics education is Fischbein's claim (as quoted by Clements and Battista *ibid.* : 426) that these intuitions do not automatically develop into correspondence with logic or mathematics, as is suggested by Piaget and Inhelder.

³ Hershkowitz *et al* (1996 : 164) note that the use of the word "figure" is in itself ambiguous: It can refer to a geometrical object (conceptual or actually existing) or to a graphical representation of the object.

more accessible by constructing a model which can be visualised and represented in the form of a diagram.

In the third perspective ("Shapes and visual representations are regarded as a means for better understanding concepts, processes and phenomena"), shapes are regarded as visual representations of mathematical entities.

I would argue, however, that if we are to regard "space" as a construction or interpretation resulting from interaction between the learner and the physical world, "space" is in itself a form of representation and it is perhaps not necessary (or even possible) to distinguish between actual "objects" and their "representations", for example, drawings. For this reason I will be referring to "objects" in inverted commas in the discussion that follows.

From this perspective, therefore, it appears, that the distinctions made by Hershkowitz *et al* as discussed above might not be necessary. For "real shapes in space", shapes acting as representatives of a class of shapes, visual representations of geometric shapes, models and visual representations of "mathematical entities" could all be regarded as "representations" of some form.

But Hershkowitz *et al*'s classification points to two interesting aspects of space. Firstly, it suggests possibilities for the **content** of "space": Hershkowitz *et al* suggest that "spatial ability" might not only be necessary in interacting with the "physical space" and its representations, but also with mathematical entities which do not have any correspondence with the "physical world". For example, the use of a graph can be useful when working with an algebraic formula. The variety of "objects" with which a learner is required to interact is also evident in Wessels and Van Niekerk's identification of three different "models": "full-scale models" (three- and two-dimensional real life objects and representations); "conventional graphic images" (drawings, projections and diagrams); and "semiotic models" (graphs, maps, symbols, arts and systems) (1998 : 3).

Furthermore, Hershkowitz *et al*'s classification suggests that there might be different forms of interaction in space, ranging from interaction with concrete "objects" to that with abstract mathematical entities. Interestingly, Hershkowitz *et al* do not indicate that this interaction is in any way hierarchical.

2.2 Ideas on "Spatial Ability"

Ideas on the nature of "spatial ability" will not only be influenced by ideas on "space" and "shape", but also by fundamental beliefs about the nature of knowledge and the mind. Eliot and Hauptman (1981 : 63), for example, suggest that the rules and criteria for "spatial ability" and the vocabulary used are subject to change according to changes in beliefs about content and scientific knowledge. Regarding "spatial ability" as an intrinsic part of human thinking or as a set of mental operations will influence debates about and studies of the phenomenon. Such questions have been the object of study by factor analysts and developmental psychologists during the past century.

Arnheim (as quoted in Gardner, 1983 : 177), for example, claims that visual and spatial imagery is a primary source of thought.⁴ Gardner (*ibid.* : 8) recognises the importance of “spatial ability” in thinking: He suggests that this is one of several “relatively autonomous human intellectual competences” called “human intelligences” and claims that it is essential to “scientific thought”. Working from the perspective of information processing on the other hand, Egan (1979) suggests that performance on spatial tasks requires a set of distinct mental operations.

Beliefs about the nature of “spatial ability” are also reflected in the debate about whether the phenomenon is a unitary trait or whether it consists of a set of “abilities”. The early factor analysts were engaged in identifying one “spatial” factor distinct from other factors like the “verbal” factor. Later researchers have suggested, however, that the situation is more complex. Gardner (1983 : 177), for example, refers to the “spatial domain” which consists of “an amalgam of abilities required for success in the general domain”. It appears, however, that ideas on this aspect will, in the end, be determined by the interest of those involved. Bishop (as quoted in Clements, 1983 : 173) has suggested, for example, that from the perspective of mathematics education it is more useful to focus on “spatial abilities”.

2.2.1 Factor Analysis

This work is part of the psychometric tradition and can be categorised by the study of the performance of individuals on specific tasks. This performance is classified using a test score and does not focus on the methods used. With regard to “spatial ability” itself, Eliot and Hauptman (1981 : 52-53) suggest that factor analysts are interested in individual differences in performance on “spatial” tasks, and that these differences are obtained using a score obtained from a test. Egan (1979 : 2) notes that in this tradition a test score is taken as a measure of some underlying factor. The score places a learner on a dimension with respect to other learners, but gives no insight into what the dimension is, nor what difficulty the learner might have.

Researchers using this approach tend to classify the actual spatial tasks, for example as “spatial orientation” or “spatial visualisation” tasks. Eliot and Hauptman (*ibid.*) classify the work in this area roughly into three phases:

- 1904-1938: Attempting to establish the existence of a “spatial” factor over and above a general ability factor
- 1938-1957: Determining the extent to which “spatial abilities” differ from one another, and the debate on whether the skill/s should be regarded as “spatial ability” or the plural, “spatial abilities”
- 1957-1979: Designating the status of spatial abilities in relation to other abilities, for example, general mathematics ability.

2.2.2 Developmental Psychology

Researchers using this approach focus on the meanings attached to and processes involved in solving “spatial tasks”. Data is usually gathered using clinical interviews. The work of Piaget and Inhelder is usually quoted in this field, but Clements

⁴ Gardner questions, however, how Arnheim regards this feature in relation to the importance of language as a source of thought.

(1983 : 15) notes that Werner, Bruner, Mitchelmore, Siegal and White, and Yi-Fu Tuan have also attempted to describe "spatial ability" from the developmental psychology perspective.

2.2.3 Other Contributions to the Debate

Work done in the area of information processing has also contributed to the debate on "spatial ability". This work focuses on the mental **processes** used by learners when completing spatial tasks. Egan (1979 : 2) describes this work as "an attempt to describe the flow of information in an organism that occurs between the onset of a stimulus and the execution of an observed response". The flow of information is said to be represented by a sequence of hypothetical mental operations. Scores obtained from information processing models represent estimates of performance on hypothetical operations. Each spatial test can yield a number of different scores, each representing information on a different set of operations. Each measure is tied to a theoretical model of how learners typically perform a task.

Egan notes that the information processing approach is often used in the study of "individual differences". Again, psychologists and educationalists have differing interests in this area of research. Bishop (1980 : 262) notes that there is some interest in "within-individual" differences in the context of "spatial ability", but that studies of "outside-individual" differences such as cultural and environmental factors are increasingly becoming the focus of interest in the study of "spatial ability" in the field of mathematics education. I would suggest that clarity on the notion of "spatial ability" itself is imperative for the study of these differences.

Researchers working from the different perspectives described above clearly have different focuses when it comes to the study of "spatial ability" and, it seems, there has been little communication between the two approaches in the literature. It is useful for my purposes, however, to scrutinise these, be aware of problems in each, and to identify what could be useful in the field of mathematics education. Bishop (1980, 1983) suggests, for example, that the factor analysis tests can be useful in suggesting different classifications of "spatial abilities", providing information on possible ordering of and dependence between these abilities, and as examples for the development of teaching materials for classroom use. Furthermore, of particular interest in this study, the theoretical debates can be useful in conceptualising the intellectual tasks and shedding light on the skills involved when completing the tests used by the factor analysts.

Bishop (1983) stresses, too, the potential benefit of an acquaintance with the work of the developmental psychologists: This work gives ideas on what can be expected by children of different ages and what stages these children are likely to pass through in becoming "spatially mature". It should be remembered, however, that these ideas were usually obtained in a clinical context and possibly need to be adapted for the educational context.

Egan (1979 : 3) identifies three important benefits of understanding spatial tasks in terms of mental processes as is the case in the information processing approach. Firstly, it enables us to look at traditional interpretations in a new light. New

interpretations can provide a conceptual framework in contrast to the traditional statistical approach. Secondly, Egan suggests that an understanding of the processes enables further study of individual differences. Lastly, and related to the aims of this particular study, Egan suggests that an understanding of processes could improve methods for assessing abilities.

In the discussion of the literature and in my attempt to formulate my own framework on "spatial ability", I will be drawing on work from the variety of these approaches, as well as on what has already been done in mathematics education itself.

2.3 An Initial Definition of "Spatial Ability"

Results produced in the first phase of the work of the factor analysts (1904-1938) is particularly useful when attempting to establish a general definition for this phenomenon as this was the phase in which researchers attempted to distinguish a "spatial ability" factor from other factors such as the "verbal" factor or "numerical" factor.

The work of Kelley (as quoted by McFarlane Smith, 1964 : 46) in 1928 is a useful starting point. He identified two spatial factors:

- ϵ : "an ability involving sensing and retention of geometric forms"
- θ : "facility in mental manipulation of spatial relationships".⁵

In 1935, El Koussy used reports of introspection and tests to claim that there exists a factor named the "k-factor" which is "the ability to obtain and the facility for utilising visual spatial imagery" (Eliot and Hauptman, 1983 : 53). McFarlane Smith (1964:55) reports on his own research which yielded results similar to those obtained by El Koussy and stresses that the group factor was present in tests which require an ability to form and retain an exact impression of a figure or pattern.

These three definitions indicate the possibility of distinguishing two aspects of "spatial ability", these being: the ability to obtain and retain a visual image⁶, and the ability to use this image in problem solving (possibly requiring manipulation of some sort).⁷

Furthermore, McFarlane Smith (1964 : 51) notes that Emmett used El Koussy's data in 1949 to suggest there are differences in (a) the ability to perceive and retain

⁵ Interestingly, McFarlane Smith notes that other research done in the 1920's and 1930's showed no evidence of this distinction.

⁶ There is a debate in the literature regarding the nature of "visual imagery", for example, are visual images "pictures in the mind" which are isomorphic to the referents, or are they propositional in nature. I am not in a position to comment on the actual nature of these images, but I do find Presmeg's definition useful: She defines a visual image as "a mental scheme depicting visual or spatial images" (1994 : 114). Bishop's (1989 : 8) review of the literature on visualisation is also useful for my purposes: He notes that these visual images can be quite simple (eg imagining a door handle rotating), or they can be relatively abstract (for example, an imagined right-angled triangle inscribed in a circle), and that they are a "very individual matter".

⁷ Clements and Battista (1991 : 225) relate the first ability, that is the ability to form and retain a visual image to the van Hiele levels of thinking: They suggest that there may be a level below van Hiele's first visual level. On this level it appears that a learner is not able to form certain visual images.

spatial patterns "in the mind", and (b) the ability to switch attention from one item to another "when perceived in temporal succession". This latter ability could be regarded as the ability to compare different figures, for which mental manipulation might be necessary.

Thurstone's work over a number of decades yields important information on the existence of a "spatial ability" factor, as well as on the nature of different aspects of this factor. In his work of 1938 (as quoted in Eliot and Hauptman, 1981 : 54), he claims that there is a space factor defined as "facility with spatial and visual imagery". This factor was found in tests requiring "facility in holding a mental image and mentally twisting, turning, or rotating it to a different position, and then matching this transformed image with a suggested solution". Once again the distinction between having an image and moving this image in some way seems to be evident. The reference to "matching" the image with a suggested solution could be specifically linked to the use of multiple choice tests, but could perhaps occur in the solving of tasks in which no possible solutions are presented. For it is possible in the latter case that a learner might use checking mechanisms on his / her work.

This distinction between the two aspects of "spatial ability" is evident in the descriptions of other writers. For example, Gardner (1983 : 173) refers to "spatial intelligence" as follows:

Central to spatial intelligence are the capacities to perceive the visual world accurately, to perform transformations and modifications upon one's initial perceptions, and to be able to re-create aspects of one's visual experience, even in the absence of relevant visual stimuli.

This description suggests that the learner has to be able to form a visual image and then modify this by manipulating these images. This should enable a learner to obtain an "accurate" perception of the visual world. The sense of the interaction between the two "abilities" is interesting, for it suggests that the relationship is a two-way relationship in that the manipulations / transformations can be used to reflect back on the original images, allowing for modification.

Furthermore, Lean and Clements (1981 : 267) define "spatial ability" as "the ability to formulate mental images and to manipulate these in the mind", again indicating a distinction between being able to create and retain an image, and then manipulating this image. This is also suggested in the work of Lohman (as quoted in Clements, 1982 : 34) who describes "spatial ability" as "the ability to generate, retain and manipulate abstract spatial images".

Clements and Battista (1992 : 420) use the term "spatial reasoning" to describe similar abilities to the two discussed so far: This is described as a "set of cognitive processes by which mental representations for spatial objects, relationships and transformations are *constructed and manipulated*" (my emphasis). The interesting aspect of this description is that it is framed in terms of "cognitive processes", which gives an indication of these researchers' ideas on the nature of "spatial ability" itself.

Furthermore, it is also interesting to note that this explanation refers to mental representations of both “objects” **and** the relationships between these “objects”. This aspect is also referred to by Hershkowitz *et al* (1996 : 162) who identify certain “dynamic” aspects of interaction in space, two of which are the “relative position of shapes to one another” and the “relative position of the observer and the things the observer looks at”.

Eliot and Hauptman (1981 : 54) relate the two “abilities” discussed here to underlying beliefs about the nature of “spatial ability” identified in the way the term is used in the literature at the time:

- The ability to obtain and retain imagery is regarded as “an intrinsic aspect of thinking”. They claim that “spatial ability” as an aspect of intrinsic thinking is seen in “the ways we respond to the positional quality of things; both directly in our response to the distribution of words, numbers, or letter-like forms on a page; and indirectly in our organisation and representation of structured information”.
- The application of the imagery in problem solving is classified as “operations for solving problems”. “Spatial ability” as a set of mental operations for solving problems is seen in “the ways we respond to tasks which require the estimation or prediction of rotated objects, and to tasks which require judgements about object arrangements when imagined from different perspectives”.

2.4 “Spatial Ability” and “Visual Imagery”

The discussion above suggests that mental / visual imagery plays an important part in “spatial ability”. But what is the relationship between the two? Clements (1982 : 34) notes that, while there is consensus amongst psychologists that visual imagery is necessarily involved in “spatial ability”, there have been few attempts to unify the literature on the two aspects. Clements (*ibid.*) quotes Cook as claiming that mental imagery is, in fact, a type of “spatial ability”. This is clear in his definition of “mental imagery” which seems to overlap with the ability (a) above, that is, “the formation and retention of an image that involves no mental movement of the image once formed”. The work of Liben (as quoted by Clements, 1983 : 10) is also seen as important in bridging the gap between the two fields: She identifies three “spatial representations”, namely “spatial products”, “spatial thought” and “spatial storage”. Visual imagery is regarded as being an important aspect of “spatial thought”.

2.5 The Development of “Spatial Ability”

The discussion above suggests that “spatial ability” involves the use of mental processes, but what is not clear in the literature is the nature of the relationship between physical interaction in the environment or sensori-motor activity and these mental abilities. Does “spatial ability” require that a learner work only with visual images? What about a learner’s ability to manoeuvre in a particular environment?

A number of researchers have identified “stages” in the development from the concrete to the abstract, but it is often not clear to what extent each stage is regarded as being part of “spatial ability”. Clements (*ibid.*: 10), for example, makes a distinction between “spatial behaviour” which he describes as sensori-motor behaviour in the environment, and “spatial ability” which is regarded as being a measure of the quality of a learner’s spatial representations. It is not clear in this

case whether this is simply a matter of specific use of terminology or a matter of definition, that is, whether Clements is actually excluding the sensori-motor behaviour from his definition of "spatial ability".

This distinction between sensori-motor behaviour and the use of visual images is also addressed by Piaget and Inhelder (1956): They refer to "perceptual" thinking (related to sensori-motor activity) and "representational" thinking in which a subject can mentally manipulate objects. They do not, however, seem to separate the former from general "spatial ability". The process of gaining "spatial intelligence" is regarded as a developmental one and Piaget and Inhelder identify stages in the development of each form of thinking. This development is not, however, regarded as a one-way process: It is suggested that development on the representational level is reflected back on perceptual activity.

A number of other researchers suggest that there are "levels" of interaction in "space". Also working in the field of developmental psychology, Werner identifies three stages in the development of "spatial ability", namely, "sensori-motor", "perceptual" and "contemplative" stages (in Bishop, 1980 : 260).

Clements and Battista (1992 : 425) refer to work by Rosser, Lane and Mazzeo which identifies a hierarchical developmental sequence:

- (a) The reproduction of geometric figures requiring only encoding ("building and matching a configuration of shapes with the original constantly in sight")
- (b) Reproduction requiring memory ("building a matching configuration by recall")
- (c) Transformation involving rotation and visual perspective-taking ("building a matching configuration either from recall after a rotation or from another's perspective").

These approaches suggest that the sequence follows from the use of "concrete objects", to the formation of a visual image (necessarily static), to the ability to manipulate these images.

A similar approach appears in the Wattanhawa's DIPT System. Wattanhawa (in Clements, 1983 : 15-16), a Thai mathematics educator, uses her review of the factor analytic and developmental psychology to develop a classification system for spatial tasks. Each task, she suggests, can be classified according to four criteria, namely, the "Dimension" of thinking required, the amount of "Internalisation" required, the manner in which the answer has to be "Presented", and the "Type" of thinking required. It is the second category that is relevant here: A task can be done on a perceptual level; can require the construction of a visual image but no transformation; or can require that the visual image be transformed in the mind. Wattanhawa does not, however, indicate that these approaches occur in a particular sequence.

Owens and Gould (1998 : 4) have developed a framework of strategies used by students in solving spatial problems and list these in the order in which they are likely to emerge, namely, "emergent strategies", "perceptual strategies", "pictorial imagery strategies", "pattern and dynamic imagery strategies" and "efficient

strategies". These range from manipulation in the physical world and perception, to the use of strategies using visual imagery only, to the use of this imagery in problem solving.

Van Niekerk (1995) also identifies different levels of interaction in space: She distinguishes between "spatial orientation" ("basically a three-dimensional experience") and "spatial insight" ("a two-dimensional experience"). The ability to work with two-dimensional "objects" is regarded as being more advanced (two-dimensional representations of three-dimensional "objects" are included in this category): She suggests that an individual must have experiences with a three-dimensional cube before being able to describe it verbally, make a mental image of the cube, or make a two-dimensional drawing of the cube. In her more recent work with Wessels (1998 : 1), van Niekerk identifies four "skills" that are said to "enable" spatial thinking, namely, "visual", "verbal", "tactile" and "mental" skills. The latter is described as "the ability to mentally manipulate spatial images and thus to understand the interconnectedness between these four skills...". This suggests that this ability might be regarded by these two researchers as being more developed than the remaining three abilities.⁸ It is interesting to note that Wessels and van Niekerk include the notion of **verbal** competency in a discussion of "spatial ability", thus suggesting that communication is also part of this phenomenon.

Hershkowitz *et al*'s (1996) three perspectives discussed in section 2.1 ("interaction with real shapes in space", "shape and space as fundamental ingredients for constructing a theory" and "shapes and visual representations as a means for better understanding concepts, processes and phenomena"), seem to fit into this framework, too, in that this perspective suggests varying degrees of abstraction. As mentioned earlier, Hershkowitz *et al* do not, however, suggest an order in which a learner might develop the ability to interact with these different aspects.

Del Grande (1990) refers to Hoffer's identification of seven "visual perception abilities" which are thought to be relevant in the study of mathematics, these being, "eye-motor co-ordination", "figure-ground perception", "perceptual constancy", "position-in-space perception", "perception of spatial relationships", "visual discrimination" and "visual memory".⁹ Interestingly, Saads and Davis (1997) conducted a study using the van Hiele levels and these seven "abilities", the results of which confirmed the hierarchical nature of the van Hiele levels, but found no such relationship between the "spatial abilities".

⁸ The remaining three abilities are defined by Wessels and van Niekerk (1998 : 1) as follows:

- "Visual skill": "Includes all the abilities and competencies to view objects from different points (lines / angles) and understand their characteristics in coherence"
- "Verbal skill": The ability to talk about different views and interpret what is observed. This includes the mastery of the use of and understanding of the terminology
- "Tactile skill": The ability to "build, cut and paste, to sew, to construct etc according to a specific plan or manual.

⁹ Del Grande (1990) notes that Brennan, Jackson and Reeve identified nine aspects of "spatial ability" (not specifically related to mathematics), these being, "visual copying", "hand-eye co-ordination", "left-right discrimination", "visual retention", "visual rhythm", "visual closure", "figure-ground relationships" and "language and perception".

Yi-Fu Tuan (1977) distinguishes between an individual's ability to manoeuvre in space, for example walking (this is termed "spatial ability"), from "spatial knowledge" in which "the mind learns to grapple with space", for example holding a picture in the mind so that one can analyse how one moves. Yi-Fu Tuan (*ibid.* : 26) raises a question which possibly has important implications for education: It is suggested that sensori-motor achievements do not necessarily imply conceptual knowledge of spatial relations. He illustrates this by referring to Swiss children (aged 5 to 6) who could get to-and-from school, but had difficulty explaining how this was done.

Liben (in Clements, 1983 : 10-11) adopts an interesting approach in this regard: Rather than referring to different types of thought or "abilities", she makes a distinction between two aspects of one's relationship to space: She refers to "specific space", corresponding to an "individual's knowledge of and ability to manoeuvre in a particular environment", and "abstract space" referring to the "spatial abstractions" which an individual may develop. It is not clear in the former case, however, to what extent the knowledge is mental, nor is any indication given of a possible relationship between a learner's ability to work in the different "spaces".

The approach adopted in "Realistic Mathematics Education" (as discussed by Hershkowitz *et al*, 1996 : 178 - 179) used in the Netherlands suggests an additional dimension to this debate: The proponents of this approach identify the "What I See" approach which requires that the learner is an observer or identifies with the observer in a particular task (this would require the use of visual imagery). But they distinguish another aspect, namely, "How I See", requiring that the learner reflect on "how the observer sees". This is regarded as requiring reflection as the individual has to move from what s/he sees with the eye to what is seen in the "mind's eye" and it is suggested that when the learner answers questions of this type, s/he might become aware of certain mathematical ideas.

Perhaps one could suggest that the two part "definition" of "spatial ability" formulated in section 2.3, namely, that "spatial ability" is the ability to form and retain a visual image as well as the ability to manipulate the image, could describe a certain level of "spatial ability" that is attained during the process of interacting with space. Perhaps one could not indicate that a subject is "spatially mature" until these skills have been developed. I certainly would be hesitant at this stage to suggest that a learner who can successfully manoeuvre in a specific environment has no "spatial skills".

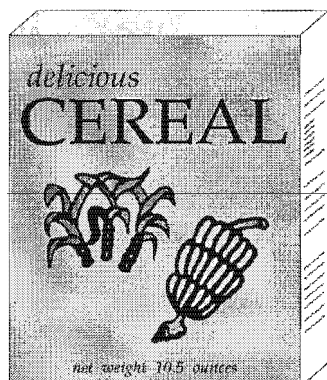
2.6 The Use of Images in Problem Solving

Although there might be some consensus on a general "definition" of "spatial ability", as discussed in section 2.3, the literature review suggests that there does not seem to be consensus on the nature of the manipulations required in problem solving. For the sake of completeness of the "spatial ability" model, I will be discussing a number of areas of the debate, but have chosen to focus in more detail on three themes which will be part of the empirical study.

2.6.1 Visualising¹⁰“Objects” from Different Perspectives-“Small-Scale” Space¹¹

I have interpreted this as being able to respond correctly to the following challenge:

Look carefully at this box⁷:



- (a) *Draw what you would see if you were looking at the box from **behind**.*
- (b) *Draw what you would see if you were looking at the box from **above**.*
- (c) *Draw what you would see if you were looking at the box from **the side**.*¹²

Reflecting on this example from the Malati materials, I would suggest that this single task could be solved in either of these two ways:

- By the viewer re-orientating his / her position in relation to the box, and attempting to view the box from a different perspective
- By the viewer mentally rotating the box so that it faces the other way.

Wattanawaha (as quoted in Clements, 1983 : 15) suggests that factor analysts and developmental psychologists have similar overall ideas regarding a definition of spatial ability: "...both groups believe that persons with well-developed spatial skills should be capable of imagining spatial arrangements of objects from different points of view and of manipulating visual images". Closer analysis of their work, suggests, however, that there are differences which require exploration.

¹⁰ "Visualising" is being used in the sense that an individual can form a mental image of an "object" as it is seen from different perspectives.

¹¹ "Small-scale" space is being used in the sense suggested by Clements (1983 : 11) to refer to space which does not surround the individual (although it might require multiple vantage points to be apprehended). This can be contrasted with "large-scale" space which surrounds the individual and requires multiple vantage points to be totally apprehended. The size of the space is discussed further in section 2.6.2.

¹² Three aspects of this presentation should be noted:

- (a) This picture is a two-dimensional representation of a three-dimensional object so is not, in fact, "a box", but a picture of a box. The issue of representing three-dimensional "objects" in two-dimensions is discussed in the section "The Understanding of Visual Representations" in section 2.6.1
- (b) The two-dimensional representations used in the Malati materials are not drawn in perspective, but using parallel projection. The initial decision to use this form of presentation in the materials is currently under review
- (c) The size of the box has been scaled down for the representation.

My confusion in this regard was sparked when reading the work of Tartre (1990b): She uses the work of McGee to distinguish between two types of **tasks**. She claims that each of the two categories are composed of several tasks and can be differentiated on the basis of what is to be moved, namely the "object" or the perspective of the viewer. Each is described as follows:

- Spatial "Visualisation" Tasks: These require that "the viewer mentally moves the representation of the indicated object, either turning it, as in mental rotation, or rearranging it, as in mental transformation"¹³
- Spatial "Orientation" Tasks: In these tasks "nothing need move at all except the perspective of the person doing the task. Spatial orientation skill requires that the subject understand a visual representation or comprehend some change that has taken place in two representations".¹⁴ She indicates that these tasks are not as easy to classify as the visualisation tasks.

Tartre thus seems to be suggesting that on some tasks, the learner will mentally move the "object", whereas on others the learner will mentally move his / her own position. This approach of classifying tasks according to the type of movement required is also evident in the work of other researchers. Senk (as quoted by Tartre, 1990a : 217), for example, classifies spatial "orientation" tasks as those in which the subject "mentally readjusts his / her perspective to become consistent with a representation of an object presented visually". Thus, once again, the *viewer* is moving and not the "object". Like Tartre, Fennema (as quoted by Tartre, 1990b : 29) refers to movement of the "object" when describing "visualisation": She classifies spatial "visualisation" as involving "visual imagery of objects and movement or change in objects themselves or change in their properties...".

Bishop (1983 : 184-185) claims to be extending Tartre's classification in his framework. He does not, however, link the different classes to specific tasks, but identifies the following "**abilities**":

- "Ability to interpret figural information" (IFI) (regarded as an extension of Tartre's "orientation"): The "understanding of visual representation and spatial vocabulary used in geometric work, graphs, charts, diagrams of all types, reading, understanding, and interpreting this information". In this case Bishop does not mention any motion on the part of the observer, but focuses rather on the representation aspect of the ability
- "Ability for visual processing" (VP) (regarded as an extension of Tartre's "visualisation"): This involves the visualisation and translation of abstract

¹³ Tartre (1990b) claims to be using the classification of forms of manipulation suggested by Kersh and Cook: They divide spatial visualisation into two categories, namely, "mental rotation" and "mental transformation". These can be distinguished by whether all or only part of an object is mentally changed. In the former case, the whole object is moved, whereas the latter involves "different operations on separate parts of the mental image". Thus it would appear that the particular use of the words "rotation" and "transformation" can cause confusion. But it is also interesting that Tartre classifies manipulation of the whole configuration **and** parts of a configuration within the "visualisation" category.

¹⁴ The suggestion that "orientation" tasks require that the learner "understand a visual representation" is an interesting one, for I would suggest that this might require that the learner learns the conventions associated with, for example, representing a three-dimensional "object" in two-dimensions. This will be explored in the section "The Understanding of Visual Representations" later in section 2.6.1.

relationships and non-figural information into visual terms. It also includes the manipulation and transformation of visual representations and visual imagery. As with the work of Tartre and Fennema, the manipulation of the "object" in this category is evident.¹⁵

McGee, on whose work Tartre bases her classification, has summarised existing factor analytic studies and concludes that there has been "strong and consistent support for the existence of at least two distinct spatial abilities - visualisation and orientation" (as quoted by Clements, 1983 : 11). As Clements (*ibid.*) notes, McGee uses the classifications suggested by Ekstrom, French, Harman and Derman:

- Spatial "Visualisation": "An ability to manipulate or transform the image of spatial arrangements; requires either the mental restructuring of a figure into components for manipulation or the mental rotation of a spatial configuration in the short term memory, and it requires performance of serial operations, perhaps involving an analytic strategy"
- Spatial "Orientation": "An ability to perceive spatial patterns or to maintain orientation with respect to objects in space; requires that a figure be perceived as a whole".¹⁶

Like those used by Bishop, these definitions refer to "abilities" rather than specific tasks as is the case with Tartre. The "visualisation" category does give an indication that this "ability" requires that the "object" itself is moved.¹⁷ Motion is not referred to explicitly in the "orientation" definition, but it is possible that the learner might have to visualise his / her own movement in order to "maintain orientation with respect to objects in space".

In his later work of 1950, Thurstone (as quoted in McFarlane Smith, 1964 : 85) also suggests the existence of three "spatial abilities", but selects specific tasks to illustrate these:

¹⁵ Bishop's suggestions in both these categories that the content of "space" also includes "non-figural" information is consistent with that of Hershkowitz *et al* (1996) as discussed in section 2.1. Bishop's ideas on these two abilities in relation to instruction are interesting: He claims that IFI is an ability of "content" and "context" as it relates particularly to the form of the stimulus material. This is thus regarded by Bishop as a "public" ability which is trainable. In contrast, Bishop refers to VP as "an ability of process" which is not related to the form of the stimulus material. This ability is regarded as "private" and Bishop has noted difficulties regarding the instruction of this ability. The suggestion that IFI is 'trainable' whereas VP is not, has important implications in mathematics education. I would argue that perhaps some of the manipulation of VP might be necessary for IFI. Is it not perhaps necessary to use transformations / manipulations of some form to "understand" visual representations? Ben-Haim *et al* (1985), in fact, refer to their own work and that of Brinkman in claiming that visualisation can be improved with appropriate instruction.

¹⁶ Other interesting aspects of these definitions are:

- The suggestion that the solving of "visualisation" tasks could involve the use of "analytic strategies"
- The suggestion that the solving of "visualisation" tasks could involve the use of "serial operations"
- The distinction made between dealing with "objects" as a whole in "orientation" and the use of the component parts in "visualisation".

¹⁷ Ben-Haim, Lappan and Houang (1985 : 390), in fact, quote McGee's work of 1979 to suggest that spatial visualisation involves "the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object".

- S₁: "the ability to recognise the identity of an object when it is seen from different angles" or "the ability to visualise a rigid configuration when it is moved into different positions". Thurstone provides the "flags activity" as an example in this case in that a subject has to decide whether or not a figure can be rotated to coincide with a given figure
- S₂: "the ability to imagine the movement or internal displacement among parts of a configuration" (paper puzzles are given as an example of the type of tasks on which this skill is required)
- S₃: "the ability to think about abstract relationships in which the body orientation of the observer is an essential part of the problem". (It is suggested that this could also involve the use of kinesthetic imagery.) This could perhaps involve the ability to deal with left / right discrimination, for example, in map reading.

Reflecting on S₁, I would suggest that, if one interprets "from different angles" as also referring to "from different perspectives" this ability could be used for the "flags activity", as well as for an activity such as this:

Which eye sees which picture?



Write your answer like this:

1. sees

2. sees

3. sees ...

4. sees

In each case, explain why you chose each picture.

In this activity the individual needs to recognise the "objects" as viewed from different perspectives.

But this would overlap with S_3 , for surely the "body orientation" of the observer plays a role in the solving of this problem? Furthermore, it seems possible that a learner might need to visualise his / her position being different in order to recognise the identity of an "object" as required in S_1 . It could be argued, therefore, that the two abilities described above are not necessarily distinct and that a learner might use different skills on the same task, for example, the "flags activity".

The distinction of three factors by Thurstone is consistent with much of the research which followed studies undertaken during World War 2. This research claimed to reveal the existence of two, and in some cases three, spatial factors.

French (as quoted in Eliot and Hauptman, 1981 : 55) used the above conclusions as well as the earlier research to claim the existence of three spatial factors, namely,

- The "Spatial" Factor: "the ability to perceive spatial patterns accurately and to compare them with one another"
- The "Orientation" Factor: "requiring a person to remain unconfused by varying orientations in which a configuration may be presented"
- The "Visualisation" Factor: "the ability to comprehend imaginary movements in three-dimensional space or to manipulate objects in the imagination".

Again, I would question whether these can be regarded as distinct. It could be argued, for example, that a learner might require mental manipulation to "remain unconfused by different orientations" and / or to compare different spatial patterns. Zimmerman's use of the Army Airforce tests (as described in Eliot and Hauptman, *ibid.*) suggests that these factors might not be distinct: He notes that, by adapting the difficulty of a test, the same test can be made to emphasise each of the three spatial factors. He suggests that each of the three factors required a somewhat different response procedure and suggested that "visualisation" required "more intellectual effort" than the ability to carry out tasks requiring spatial orientation.¹⁸

In summary, therefore, the above discussion of the literature on "spatial ability" has raised two questions. Firstly, is it possible to classify specific tasks? Bishop (1983 : 84), in fact, quotes Guay, McDaniel and Angelo's claim that the methods used by individuals are actually idiosyncratic (some will use analytic thinking, some will use "gestalt" thinking, others will work with components of an object). Clements (1983 : 9) lists a number of researchers whose work supports the idiosyncratic nature of performance on "spatial" tests. Referring to "visualisation", Bishop (1989 : 11) also stresses that the activity on a task depends on the individual.¹⁹ He suggests that what happens depends on a subject's preferences, memory of

¹⁸ The relative difficulty of "spatial" tasks and their subsequent classification has been explored by a number of researchers and will be discussed in the section "The Level of Difficulty" in section 2.6.1.

¹⁹ Bishop's (1989 : 11) use of the term "visualisation" in this context is similar to the general description of "spatial ability" described in section 2.3, that is, a process which "appears to involve the learner constructing some kind of visualisation and using it appropriately". Bishop indicates that this is an important component in his "Visual Processing" component of "spatial ability".

visualisations, ability to recall or generate appropriate visualisations, ability to choose appropriate visualisations, and ability to operate appropriately with the chosen visualisations. If this is so, it would be of little use to classify actual tasks. Rather, one should classify the **strategies** or **mental processes** (possibly different) on a set of tasks.²⁰

Connor and Serbin (as quoted in Tartre (1990a : 42)) add an interesting dimension to the "visualisation" / "orientation" classification in that they suggest that the angle of rotation determines the skill used. They classify the "card rotation" test, in both categories ("visualisation" / "closure and orientation")²¹: They claim that the greater the angle of rotation of the object, the longer it takes subjects to determine whether there is a correspondence between the two objects, particularly if the angle is greater than 90°. It appears from their research that spatial "orientation" skills might be necessary if the angle of rotation is less than 90°. If the angle of rotation is greater than 90° it seems that "multi-step" manipulations might be required.

Secondly, are the "abilities" or factors identified by the researchers discussed above actually distinct? Lohman (as quoted by Clements, 1983 : 11) seems to have developed a framework that allows for possible overlaps. Lohman, who re-analysed data from preceding factorial studies and also identified three factors, is regarded by Clements as having made a significant contribution to attempts to resolve the issue of providing a definition for "spatial ability". These three factors are described as follows:

- Spatial "Relations": This is described as "performance on tasks requiring subjects to rotate figures or objects mentally". It is suggested that this might represent both the ability to solve problems accurately as well as the amount of time required to carry out the manipulation/s
- Spatial "Orientation": This is described as "the ability to imagine how a stimulus array would appear from a different perspective"
- Spatial "Visualisation": This is not clearly defined but the tests are usually administered under different time limits and are "more complicated or difficult" than those tasks classified above.²²

Clement's criticises the vagueness of the last category. Eliot and Hauptman (1981 : 57) have also criticised this classification as they suggest that tasks classified as "orientation" tasks might, in fact, require mental manipulation (seemingly classified in the "relations" category). This is precisely the argument I made about the classifications used by McGee, Thurstone and French.

This apparent overlap does not, however, seem to be a problem in the light Lohman's specific approach to the classification as discussed below. For Lohman

²⁰ In contrast, when studying mental processes used in "visualisation", "orientation" and "kinesthetic imagery" tests, Egan (1979) concludes that all subjects use the same processing sequence, but differ on the rate of execution of the operations.

²¹ In the "card rotation" test a learner has to identify those two-dimensional figures which are rotations of one another.

²² Interestingly, the mental rotation mentioned in "spatial relations" is classified as "visualisation" by McGee and Tartre: It seems that this is just a case of different use of terminology.

seems to have adopted a different approach to that of other researchers regarding the classification of skills and this could be useful in considering my own approach. Clements (*ibid.*) notes that Lohman's categorisation described above does not provide precise descriptions of the actual mental operations involved in the different categories of tasks. Rather, Lohman emphasises that some sort of mental transformation or combination of transformations is involved in all tasks and classifies these separately (each can be required in the same task):

- "Mental movement": displacement, folding, rotation and reflection
- "Construction": physical "reproduction" of images, "mental construction".²³

This approach suggests a framework that might be useful in obtaining clarity on the notion of "spatial ability", that is, it might be possible to identify what we could call certain aspects of interaction in space, for example, the ability to visualise an "object" or group of "objects" from different perspectives, or the ability to compare configurations in different orientations, and then to identify the possible (possibly different) visual strategies used by learners during this interaction, for example, the mental movement in the position of the observer or the mental movement of a visual image.²⁴

The work of Senechal (as quoted in Hershkovitz *et al*, 1996 : 162) is useful in assisting in the identification of possible aspects of interaction in space. She does not refer specifically to tasks or specific skills, but refers to three "goals" of interaction in space:

- (a) To "discover similarities and differences between objects"
- (b) To "analyse components of form"
- (c) To "recognise shapes in different representations".

I would suggest that these are not distinct, in that it might be necessary to "analyse components of form" or to "recognise shapes in different perspectives" in order to compare "objects".

Hershkovitz *et al* add some additional aspects which they refer to as the "dynamic aspects":

- (d) "The relative position of shapes to one another"
- (e) "The relative position of the observer and the things the observer looks at"
- (f) "The processes of changing shapes"

I would argue, however, that the three aspects (a) to (c) have an implicit dynamic aspect, for in order to "discover similarities and differences" and "recognise shapes in different representation", it appears that manipulation might be necessary.

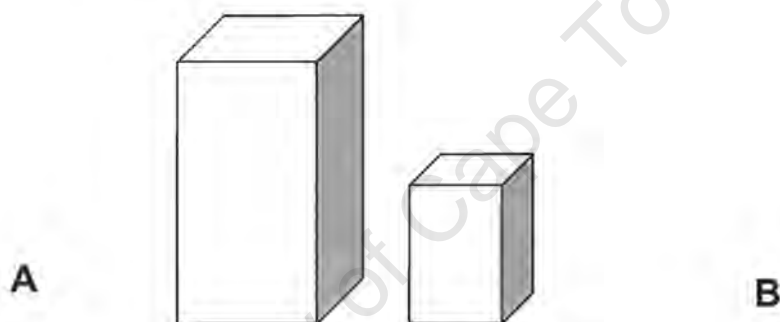
²³ It is possible that "physical reproduction" could also refer to construction in the form of drawing. The relationship between "spatial ability" and drawing skills seems to be a complicated one and will be discussed in the discussion "The Understanding of Visual Representations" in section 2.6.1.

²⁴ While I feel that Lohman's approach could be valuable, it is interesting to reflect on the use of vocabulary in the classifications and how this is interpreted. In spatial "relations", for example, Lohman is quoted as referring to "performance on tasks requiring subjects to...": This seems to suggest that the responses are task specific! I find the wording for "spatial" orientation more suitable in that it refers to "the ability to imagine...": This suggests that the individual has a particular skill which can be used in an appropriate context, rather than being linked to a particular task.

This approach of Hershkowitz *et al* is particularly valuable in that it deals with the relative position of "objects" and the observer as was discussed in section 2.3. In the context of interaction in space as discussed here, this could be interpreted as the ability to respond to a task such as this in which the learner has to consider the relative position of the "objects" as well as his / her relative position to these "objects":

Draw what you would see if you were looking at these two boxes from

- (a) *above*
- (b) *below*
- (c) *point A on the left of the boxes*
- (d) *point B on the right of the boxes*
- (e) *behind.*



The aspects of "spatial ability" or "visual perception abilities" as discussed by Del Grande (1990) and mentioned in section 2.5 could also be incorporated into a list of forms of interaction, for although these are described as "abilities", the descriptions used say a great deal about the nature of the interaction in space during which these "abilities" are likely to be employed. He lists those identified by Brennan, Jackson and Reeve ("visual copying", "hand-eye co-ordination", "left-right discrimination", "visual retention", "visual rhythm", "visual closure", "figure-ground relationships" and "language and perception"), and those of Hoffer ("eye-motor co-ordination", "figure-ground perception", "perceptual constancy", "position-in-space perception", "perception of spatial relationships", "visual discrimination" and "visual memory").

The Level of Difficulty of a Task

Some researchers distinguish between the level of difficulty of different "spatial" tasks. The work of Zimmerman in exploring the relative difficulty of different tasks has already been mentioned, as well as Lohman's claim that the "visualisation" tasks are more difficult than others. Linn and Peterson (as quoted in Tartre, 1990a : 216-217) also suggest that the tasks they term spatial "visualisation" tasks are more difficult than others. These tasks are identified as those which involve multi-step

manipulations of spatially presented information.²⁵

But Lohman distinguishes between the "orientation", "relations" and "visualisation" factors and the level of difficulty of a task: He classifies the former three factors as "major" factors and those related to the complexity of items, the use of visual memory, speed in matching visual stimuli, and speed in making left and right discrimination as "minor" factors (as noted in Clements, 1983 : 12).²⁶ Clements suggests that this distinction between "major" and "minor" factors is important for us in assisting the uninhibited study of the "major" factors identified by factor analysis. This classification is particularly useful for this study as it suggests that it is possible to explore the different forms of "interaction in space" without having to consider factors such as the level of difficulty or the speed at which tasks are performed.

The Understanding of Visual Representations

It has been noted that Tartre includes the "understanding of visual representations" and the "comprehension of some change that has taken place in two representations" in her "orientation" classification. But what is meant by these two phrases? "Changes" could suggest the following:

- (a) A two-dimensional representation of a three-dimensional "object" in the form of a drawing (orthogonal projection, parallel projection, or non-parallel projection)
- (b) A two-dimensional representation of a three-dimensional "object" in the form of a net
- (c) A two-dimensional representation of a three-dimensional "object" in the form of a cross-section
- (d) An aerial view of a scene in the form of a floorplan or map
- (e) A change in the relative position of the observer and the "object".

Bishop suggests some further possibilities in this regard: In his extension of Tartre's "orientation" category, he defines the "ability to interpret figural information" (IFI) as the "understanding of visual representation and spatial vocabulary used in geometric work, graphs, charts, diagrams of all types, reading, understanding, and interpreting this information". This thus broadens the possibilities suggested by Tartre's classification to include graphs and charts which, in fact, can represent non-spatial information.

The skill of interpreting visual information is also suggested by Senechal (quoted in Hershkowitz *et al*, 1996 : 161-162): She suggests that the following kinds of thinking are some of those necessary to interact successfully in the world:

- Understanding the visual world around us, describing it, and encoding and decoding visual information
- Interpretation of visual information.

It is important to remember that all these representations mentioned serve as a mediation for a particular spatial (or possibly non-spatial) situation. Furthermore,

²⁵ Linn and Peterson distinguish spatial "visualisation" tasks from spatial "perception" tasks and "mental rotation" tasks.

²⁶ Interestingly, Eliot and Hauptman (1981 : 57) associate the "minor" factors with "intrinsic aspects of thought" and the "major" factors with "sets of mental operations" as discussed in section 2.2.

there are certain conventions associated with the understanding of these representations and the “reading-off” is not self-evident.

A number of researchers remind us of this fact. Piaget and Inhelder (in Clements and Battista, 1992 : 423), for example, are quoted as claiming that drawing “is an act of representation” (and not perception). Bishop (1983 : 184) notes that figural stimuli have their own conventions and visual vocabulary. Referring to the different forms of representation (orthogonal projection, parallel projection, perspective) Hershkowitz *et al* (1996 : 199) indicate that a drawing can, in fact, be a compromise between the figure “as it usually can be seen” and “its essential characteristics”. Doudy (1998 : 25) suggests that the ability to read two-dimensional representations of three-dimensional objects requires both an ability to build mental images as well as some knowledge of two-dimensional geometry at a certain level.

Although in most cases references are made to representations in the form of actual drawings, I would suggest that this would be the case for all the forms of representation (a) to (e) mentioned above.

Liben’s identification of different types of representations could be useful in this context: She distinguishes between “spatial products”, “spatial thought” and “spatial storage”. The last category is a contentious one, but the remaining two are described as follows: “spatial thought” is said to be “thinking that concerns or makes use of space in some way” and “is knowledge that individuals have access to, can reflect upon or can manipulate”; “spatial products” refer to external products that represent space, such as sketch maps, models and verbal descriptions. I would argue in the latter case, however, that a distinction should be made between drawings and verbal explanations in that drawing requires a knowledge of particular conventions.

It would seem from the above discussion, that the “understanding” of visual representations in a number of different forms is regarded as an important aspect of “spatial ability”. It appears that a knowledge of the conventions used in the representation of spatial “objects”, which are not necessarily directly related to perception in the physical world, might be a prerequisite for such an understanding.

The Use of “Analytical” Strategies

McGee has been quoted as suggesting that “analytical” strategies might be necessary for spatial “visualisation”, but precisely what he means by this is not clear. The literature review suggests that there are a number of different uses of the term “analytic”. In “Realistic Mathematics Education” (as discussed in Hershkowitz *et al*, 1996 : 180), for example, it is suggested that the transition from “What I See” to “How I See” requires “visual analytic thinking”. The designers identify certain “dynamic analytic tools” such as sight-lines and sight-angles to illustrate this concept. Bishop (1983 : 184) seems to adopt a different approach: He contrasts “analytic thinking” with “gestaltist thinking”, thus suggesting that this form of thinking could refer to the breaking down of the whole into smaller parts or simpler elements to solve a problem.

Clements (1983 : 12) mentions the work of Guay, McDaniel and Angelo in which it is suggested that many of the paper and pencil tests used by factor analysts, and on which McGee bases much of his work, require "analytic" thinking rather than "spatial" thinking, but precisely what is meant by this term is not discussed.

The *MacMillan Dictionary of Philosophy* (1979 : 12) refers to the approach of Frege: "A statement is an analytic truth or falsehood if it can be proved or disproved from definitions by means of only logical laws". It seems, too, that the term "analytic" can be used to signify the use of algebra or calculus in mathematical problem solving, thus suggesting that "visual" strategies are not used.

Perhaps in the context of "spatial ability" we could refer to "analytic thinking" as the use of certain logical steps (not requiring a visual strategy) to solve a problem?

2.6.2 Visualising "Objects" from Different Perspectives - "Large-Scale" Space

In section 2.6.1 the literature on "spatial ability" relating to the visualisation of representations of "objects" such as boxes from different perspectives was discussed. Bishop (1980 : 260), however, suggests that there is more to "spatial ability" than the study of "objects" in "small-scale" space: He notes Werner's stress on the importance of paying attention to "large, full-sized space" as well as space represented in models and drawings on paper. Interestingly, this aspect does not feature much in the literature on "spatial ability", but has been studied by researchers in the field of geography education.

But what do we mean by "small-scale" and "large-scale" space? Clements (1983 : 11) uses the work of Herman and Siegal to suggest that "large-scale" space is that which surrounds the learner and requires multiple vantage points to be apprehended completely. In contrast, "small-scale" space is used to refer to space which does not surround the learner (although multiple vantage points might be required for comprehension). This notion of "small-scale" space overlaps with Werner's reference to models and drawings on paper as these do not "surround" the learner.

Other classifications of different sized spaces have been used: Berthelot and Salin (1998 : 72-74) refer to the classification of "space" suggested by Brousseau and Galvez which distinguishes between "microspace" ("corresponding to the usual prehension relations"), "mesospace" ("corresponding to the usual domestic spatial relations"), and "macrospace" (corresponding to unknown spaces). This suggests that the "large-scale" and "small-scale" classification could be extended to distinguish between "known" and "unknown" space.

Furthermore, I would suggest that in the case of "large-scale" space, a distinction could be made between the "space" that a learner can see from his / her position in the space (for example, viewing a room in a building from a position in the room) and space that cannot be viewed from the position of the observer (for example the whole building in relation to the learner's position in one room). These two classifications could be described as "seen" and "unseen" space respectively.

But why might it be important to make such a distinction? Working in the context of "spatial ability", Herman and Siegal (1978 : 390) note that many assessments of spatial knowledge have focused on "small-scale" environments only, and that "large-scale" environments "which seem more closely to simulate the real world" are used infrequently. Furthermore, they suggest that in studies in which learners are tested on their knowledge in a particular environment, this is tested in a "small-scale" environment, that is, the solutions have to be presented in a "scaled down" form. A learner's mapping ability can thus be confounded with the ability to translate and represent his / her knowledge on a "small-scale". Herman and Siegal thus propose that these factors could be avoided by requiring individuals to construct their representations using elements in the same scale as the original environment.²⁷

Clements (1983 : 11) suggests that it is important to make a distinction between different sized spaces in mathematics education, too. He notes Bishop's claim that mathematics teachers often use "small-scale" physical objects or symbolic representations for teaching abstract mathematical concepts and focus on the development of models to predict the outcome of events in "large-scale" space. He also notes that some school mathematics textbooks require that a learner engage in "large-scale" outdoor activities and that some mathematical problems require that a learner think about "large-scale" space. These examples suggest that it is important to determine how learners actually think in the different sized spaces. Is there a difference between strategies used by learners in "large-scale" and "small-scale" space?

Interestingly, Yi-Fu Tuan (1977 : 26-28) does not distinguish between the size of the space, but focuses rather on the angle from which the space is viewed. Commenting on studies of children's understandings of aerial photographs and more oblique views, he suggests that children comprehend the environment better when there has been a 90° rotation (as in an aerial photograph), than a perspective after a 40° or 50° rotation. He suggests, therefore, that the oblique view can be more difficult to comprehend than the vertical view. His discussion seems to be restricted to "large-scale" environments, but it is interesting to consider whether this would be the case in "small-scale" environments. Furthermore, it was noted in section 2.6.1 that Connor and Serbin (as quoted in Tartre (1990a : 42)) claim that different skills are used in "small-scale" space, depending on whether the angle of rotation is greater or smaller than 90°. But what is the case if the angle is 90°?

Other researchers in mathematics education have referred briefly to the size of the space. Van Niekerk (1995), for example, distinguishes between the "geometry of

²⁷ Their research suggests that accuracy of constructions of the environment improved with an increase in the number of viewings (either by walking through the environment or by simply viewing it) and that younger children also perform better in "bounded" rather than "unbounded" space (Herman and Siegal note that in one of the experimental spaces, the layout of the model "town" was closely bounded by the walls of the classroom, whereas in the other space, the closest wall was about 8m away. The former case is thus classified as "bounded" and the latter as "unbounded". It is suggested that the proximity of the classroom walls in the "bounded" space might have provided clues which assisted in performance on the tasks).

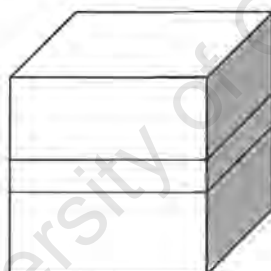
shapes and figures" and the "geometry of location". Does this mean that different skills are used? Bishop (1983 : 194) classifies the "map drawing" activity as a VP task, and notes that the subjects in his study found this task one of the easiest.²⁸ This classification is interesting for, as noted in the discussion in section 2.6.1, some researchers classify activities requiring the visualisation of "objects" from different perspectives as "orientation" activities!

It is clear from this literature survey that most researchers do not distinguish between "spatial ability" in different sized environments. The work that has been done in this regard, however, suggests that a distinction should be made between the size of the space in which a learner is working as well as the size of the space in which a representation is given or required. With reference to the discussion and the questions raised in section 2.6.1, I would ask whether a learner might use different strategies when visualising a larger scene such as a whole room or a neighbourhood from different perspectives, to those used when visualising a representation of an "object" such as a cereal box as already discussed.

2.6.3 Facility with Two- and Three- Dimensional Representations - The Case of Nets

Consider the following two problems from the Malati "spatial skills" activities:

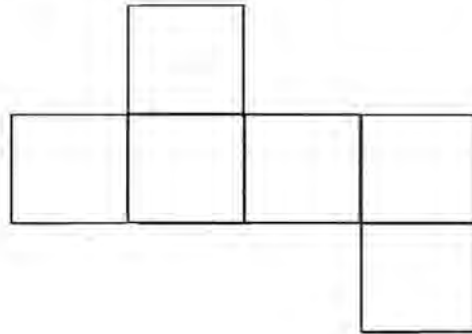
1. *Tezi made a box and painted a stripe around four of the sides:*



What net did Tezi use to make the box?

²⁸ This activity requires that a learner draw an outline of Papua New Guinea and a map of the campus.

2. When the staff at Pick 'n Pay have finished unpacking goods from boxes, they flatten the boxes so that they can be packed and easily transported to the recycling depot. This flattened box was found in the storeroom.



Draw three different nets that will make the **same** box.

In the first activity the learner is required to draw the net of a "box" (the drawing of the box is provided), whereas in the second activity the learner is required to draw a net of a "box" when one net is already given.²⁹

In section 2.6.1 I suggested that the use of a net as a two-dimensional representation of a three dimensional "object" is a particular convention. But it seems that, in addition to understanding the notion of a "net", a learner also needs certain "spatial skills" to answer these two questions. Bishop's classification (1983 : 184-185) of "spatial ability" into VP and IFI suggests that the understanding of the convention of a net and the ability to visualise the unfolding are very different, for he classifies "understanding of visual representation and spatial vocabulary" as IFI, but the actual manipulation and transformation of the visual images as VP.

Reflecting on my own mental processes when solving such tasks, it seems that these two activities could be solved in two possible ways:

- (a) A learner mentally unfolds the three-dimensional "object" to get a two-dimensional net
- (b) A learner works with his / her knowledge of the net of a cube and mentally folds it up to check whether the net is suitable for this three-dimensional "object".

Even if a learner uses one of these two strategies it is possible that the other strategy might be used as a checking mechanism. It is also possible that the form of presentation of the activity might influence the strategy for used, for example, the

²⁹ Of course the "box" in this problem is in fact a two-dimensional representation of a three-dimensional "object". So it seems that there are two stages in the transference from a "box" to its representation in a net.

fact that the learner is presented with a representation of a three-dimensional object might result in him / her using the strategy labelled (a).

Having these possible strategies in mind, my confusion was sparked once again when reading the work of Tartre (1990b : 29-32): As discussed in section 2.6.1, she uses the work of Kersh and Cook to distinguish between two categories within spatial "visualisation" tasks, namely, "mental rotation" and "mental transformation". Focusing on "transformation" itself, Kersh and Cook are quoted as identifying four types of transformations. Each task is classified according to **the number of dimensions** of the initial state and the goal state in the task:

- (a) Two-dimensional to two-dimensional transformation
- (b) Three-dimensional to three-dimensional transformation
- (c) Two-dimensional to three-dimensional transformation. Tartre provides an example of a test in which the net of a rectangular prism is given and the learner has to select one of four drawings of three-dimensional boxes
- (d) Three-dimensional to two-dimensional transformation, for example finding cross sections of three-dimensional "objects" and unfolding paper.

Reflecting on the last two categories and the examples of tasks provided, I would question whether a learner will always work from the two-dimensional representation to the three-dimensional "object" in the third category. Is it not possible that the learner will work from the three-dimensional box and mentally unfold it? And if this is so, would this approach not be classified in the fourth category of transformation?

An approach similar to Tartre's is evident in Wattanahawa's DIPT system (in Clements, 1983 : 15-16): She attempts to classify tasks according to the "Dimension" of thinking required, that is, one-; two-; or three-dimensional thought. I would question whether in the examples of net activities provided above, a learner's thinking will be restricted to thought in one dimension and whether all learners will use the same strategy.

Guay and McDaniel (1977 : 211-212) suggest a classification which might be useful in this context: They also distinguish between thinking in different dimensions, but they suggest further that thinking in three dimensions is more advanced than that in two dimensions.³⁰ They identify the following "spatial abilities":

- (a) "Low-Level" Spatial Abilities: "Requiring the visualisation of two-dimensional configurations, but no transformation of those visual images"
- (b) "High-Level" Spatial Abilities: "Requiring the visualisation of three-dimensional configurations, and the mental manipulation of these visual images".

This classification suggests that, in order to be able to successfully engage with the net tasks above, a learner would have to have "high-level" abilities. These two researchers do, in fact, classify the "Surface Development Test" as a test to assess "high-level" abilities.³¹

³⁰ In fact, Doudy (1998 : 26) claims that, in terms of mathematics, the study of three-dimensional space is more complex than that of two-dimensional space.

³¹ This test is "designed to measure the ability to visualise the development (unfolding) of three-dimensional object surfaces so that these surfaces can be super-imposed onto a single plane" (subjects are required to choose from three given nets).

Reflecting again on the strategies used for solving activities 1 and 2 mentioned above, two other interesting aspects can be noted. Firstly, it seems that when mentally folding or unfolding an "object", a learner will be working with specific parts of the whole "object", that is, the learner will have to work with individual squares or faces making up the cube. If this is the case, this would support my suggestion in section 2.3 that the classification of specific tasks by some researchers might not be appropriate. If one considers the classification by Thurstone (as quoted in McFarlane Smith (1964 : 85)), for example, one category is S_2 which is described as "the ability to imagine the movement or internal displacement among parts of a configuration". I would suggest that such a skill **as well as** the ability to mentally manipulate the parts of the "object" might be necessary in solving the "net" activities. This suggests, therefore, that the different categories of tasks might not be distinct. The issue of working with "parts" of a whole "object" will be discussed in section 2.6.4 below.

2.6.4 Working with Components of Representations and Representations as "Wholes"

Within the constraints of a masters dissertation, it is not possible to explore this aspect in detail, but as the issue has been raised in section 2.6.3, and for the sake of completeness of the model of "spatial ability", the different views reflected in the literature will be discussed briefly.

The literature review suggests that there is not consensus on whether "spatial ability" requires that an individual be able to work with parts of an "object" and / or the whole of the "object" nor on the relationship between these.

This issue first came to my attention when reading Clement's (1983 : 11) discussion of McGee's work: In his classification of spatial "visualisation", McGee refers to "spatial arrangements" and the "mental restructuring of a figure into components for manipulation or the mental manipulation of a spatial configuration". This seems to suggest that there can be manipulation of parts **or** the whole. But McGee's classification of spatial "orientation" tasks emphasises the "ability to perceive spatial patterns...requires that a figure be perceived as a whole".

Clements points out that there seems to be a contradiction in the "orientation" classification with the use of "spatial patterns" and the emphasis on "perception as a whole". He suggests that relationships between the components need to be studied to perceive patterns. He suggests that this contradiction in "orientation" could be overcome if the analysis of parts of a figure, as well as the perception of the figure as a whole, were included. This, however, would appear to overlap with the "visualisation" description! This suggests that a learner might need to work with both parts of and the whole of an "object", and that the use of the two approaches might not necessarily be distinct.

Review of the other work on this aspect of "spatial ability" suggests that approaches can be classified into four different groups:

- (a) Those researchers who specify that one of these approaches is a prerequisite for "spatial ability":

- Clements (1983 : 12), for example, notes McFarlane Smith's contention that "spatial ability" requires "perception of a configuration as a whole".
- Clements (*ibid.*) also notes the stress by Guay, McDaniel and Angelo that the essence of spatial ability is "gestalt processing" characterised by the "formation and transformation of mental visual images as organised wholes".
- Van Hiele (1986) would suggest that thinking on the "visual level" requires gestaltist thinking and that it is only once an individual has experienced tasks on this level, that s/he is able to attend to the properties of figures. Thinking on these different levels is regarded as being hierarchical.

(b) Those researchers who place these two aspects on a hierarchical scale of ability:

- Bishop (1983 : 182) refers to the work of Krutetskii in the field of "Individual Differences": This researcher notes that the "capable" learners could integrate separate parts into "significant ordered structure", whereas the "less-able" learners were "tied to the detail of the problem, seeing only disconnected facts, all of which were treated with equal significance".³²

(c) Those who classify the two different aspects as distinct abilities:

- This is seen in the distinction made between S_1 and S_2 abilities suggested by Thurstone and discussed in section 2.6.1.
- Owen's (1990 : 48) reference to work by Halpern and Bishop which suggests that "orientation" requires an individual to work with parts of a figure. This orientation factor includes "the ability to detect arrangements of elements within a pattern".
- Tartre (1990b : 34-40) makes the distinction between the two aspects in both spatial "orientation" and spatial "visualisation". In spatial "orientation" she identifies those tasks which require a "re-organised whole" strategy which involves the organisation and comprehension of "an entire pictorial representation or a perceptual change from one representation to another". She distinguishes these tasks from those that require the "part of a field" strategy which is concerned with the relationship of a part of representation to the whole field. Tartre's definition of "visualisation" (using the work of Kersh and Cook) also suggests a distinction between the two: "Mental rotation" is said to describe the mental movement of "an entire object", whereas "mental transformation" involves "different operations on separate parts of the mental image".

(d) Those who suggest that there is some form of interaction between the two types of thinking or that the use of the different approaches varies:

- Burden and Coulsen (in Clements, 1983 : 14) suggest that different strategies can be used: They suggest a learner can attend to a whole configuration, or to parts of it at a time (the "processing focus").³³ This approach still suggests

³² No indication is given in this case precisely how the "capable" and "less-able" learners were classified.

³³ Burden and Coulsen suggest that any solution strategy for a "spatial" task has three main properties, namely, "a representational mode" (the use of the "visual" mode, the "verbal" mode or a mixture of these two), "a processing focus" (as described above) and "a processing aid" (any concrete means used to "reduce the cognitive demand" of a task).

that a learner will use either one or the other approach, but this form of interpretation might be useful for my purposes: For Burden and Coulson do not seem to restrict the approaches to particular tasks, but classify them as different **strategies**. Perhaps the strategies used for one task could vary from learner to learner or perhaps a learner's strategy could vary from task to task.

- Referring specifically to "spatial visualisation", Fennema (as quoted in Tartre, 1990b : 29) expresses the view that this skill "involves visual imagery of objects and movement or change in objects themselves or changes in their properties. In other words, *objects or their properties* must be manipulated in one's mind's eye or mentally" (my emphasis).

Although this issue will not be discussed in detail in this study, instances during which certain interesting aspects arise during the empirical study will be noted.

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Chapter 3: A Preliminary Model of “Spatial Ability”

In this chapter the issues discussed in Chapter 2 will be used to build a preliminary model of “spatial ability”. As suggested by the discussion this model will, at this stage, include a number of questions which arose in the literature review. The model will be revised on the basis of the empirical research conducted in this study.

3.1 The Notion of “Space”

I would suggest that “shape” and “space” result out of an interaction between the individual and “objects” in space. These “objects” obtain their features / properties as a result of their relative position to other “objects” and to the individual. For example, if we consider the following “objects”:

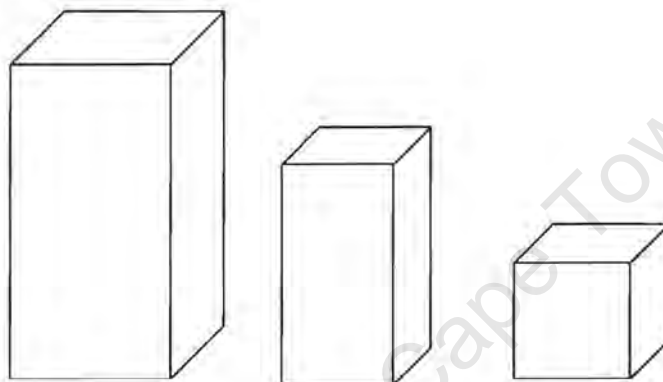


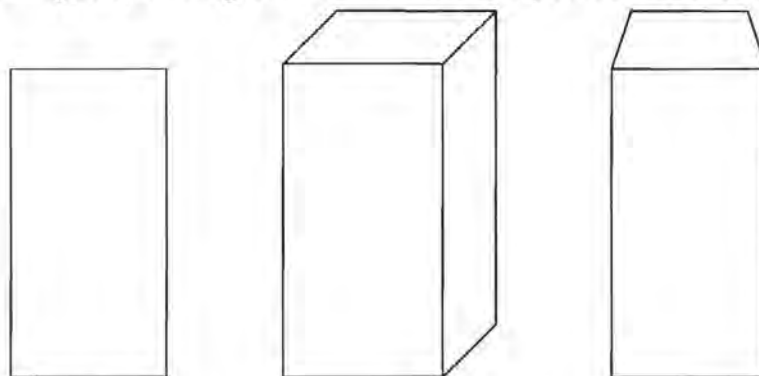
Figure 1

Figure 2

Figure 3

The “object” in figure 1 can be classified by comparing it to those in figures 2 and 3. It has the same shape as the “object” in figure 2, but is a different size. It has different properties (and different size) to the cube in figure 3.

These figures can also differ as a result of an individual’s position in relation to them. For example, an individual viewing the “object” in figure 1 from the front might see any of the following, depending on his / her relative position to the “object”:



Thus in the discussion that follows I will be referring to “objects” when discussing both three-dimensional figures as well as their two-dimensional representations. For as noted in section 2.1 if we are to regard “space” as resulting from an interaction

between the individual and the "world", then "space" and "objects" in space are, in fact, all forms of representation, rather than actual "objects".

The "objects" with which a learner must interact in "space" can be summarised as follows:

- Two-dimensional "objects", for example a square
- Three-dimensional "objects", for example a cube
- Two-dimensional representations of these three-dimensional "objects". Those forms of representation identified are drawings (parallel projection, orthogonal projection, non-parallel projection), nets and cross sections. These forms of representation have certain conventions associated with them.

Furthermore, the learner has to consider the relationship between these "objects" and the relationship between himself / herself and these "objects".

It should also be mentioned that these "objects" do not necessarily have to have a physical referent in space, but can also be in the form of a "model" of a non-geometric entity, for example, the use of a graph to represent a function in algebra.

The "space" in which these "objects" occur can also vary. I have chosen to use the following terminology to classify the different forms of "space":

- "Small-scale" space, as suggested by Clements (1983 : 11) will be used to refer to space which does not surround the individual (although it might require multiple vantage points to be apprehended). This includes all scaled-down representations in the form of models or drawings (including representations of "large-scale" space).
- "Large-scale" space as suggested by Clements (1983 : 11) will be used to refer to space which surrounds the individual and requires multiple vantage points to be apprehended. This space can be "known" (for example, a learner's neighbourhood) or "unknown" (for example, a city the learner has not visited before). It can also be "seen" (the whole scene can be seen by the learner's position in the space) or "unseen" (only some of the scene can be viewed from the particular position of the learner in the scene).

3.2 The Notion of "Spatial Ability"

The literature review suggests that there are two perspectives in this regard, namely, "spatial ability" can be regarded as

- (a) an intrinsic aspect of human thinking, or
- (b) a set of mental operations.

3.3 A General Definition of "Spatial Ability"

There does seem to be general consensus on the part of researchers in different fields that "spatial ability" involves two aspects, namely,

- (a) the ability to form and retain a visual image
- (b) the ability to use mental manipulations for problem solving.

There is a suggestion that, while ability (a) is a prerequisite for (b), there can, in fact, be a two-way interaction in that the knowledge gained from the manipulations in (b) can be used to reflect back on the accuracy of the initial visual images.

Furthermore, it is possible that there might be another aspect of "spatial ability", this being the ability to reflect on the actual mental process used during problem solving.

It appears that these skills would be necessary for successful interaction in the visual world, that is, what Senechal (as quoted in Hershkowitz *et al*, 1996 : 161-162) would call the understanding of the visual world, as well as the handling and causing of changes in this visual world.

But what is the nature of this interaction in space? The following forms of interaction can be identified in the literature:

- Interacting in physical space, for example, manoeuvring in a particular environment (finding one's way to and from school) (Del Grande's "eye-motor co-ordination")
- Visualising an "object" or "objects" (in "small-scale" and "large-scale" spaces) from different perspectives
- Related to the above visualisation, and possibly a prerequisite for this, is what Del Grande terms "perception of spatial relations", that is, the ability to see two or more "objects" in relation to oneself and in relation to one another
- Representing an "object" in a different representation, for example, making a perspective drawing or drawing the net of a three-dimensional "object". In this case the ability to use the correct visual skills such as mentally "unfolding" an "object" is being distinguished from the possession of the knowledge of the convention used for this form of representation
- Distinguishing specific components of the whole in a visual representation
- Recognising "objects" in different sizes, shadings, textures, positions in space (Del Grande's "perceptual constancy")
- Comparing "objects" in different forms (Del Grande's "visual discrimination").

3.4 The Use of Images in Problem Solving

As noted in the discussion in section 2.6, there is little consensus amongst researchers with regard to this aspect of the nature of "spatial ability". For this reason this section of the model contains only suggestions based on the literature review. Some of the questions raised are explored in the empirical study.

The discussion suggests that it might not be appropriate to classify different tasks according to different "abilities". For, as it appears that different learners might use different strategies on the same task and that a learner might use a variety of strategies on the same task, it might be more useful to identify the different strategies used by a learner on a range of tasks.

It might be useful for the purposes of mathematics education to classify the different types of interaction in space (as described in section 3.3), and then to classify the different mental strategies used by learners during such interaction.

Possible mental strategies used in problem solving in space are:

- The ability to visualise the mental movement of an "object" or "objects". This movement could be rotation, reflection, translation, enlargement, folding / unfolding etc

- The ability of a learner to visualise the mental movement of his / her own position in relation to an "object" or "objects".

It should be noted that the term "strategies" in this case is being used to refer to specific mental processes. Thus, when a learner is interacting in space and using visual imagery in problem solving, as discussed in section 3.3, s/he is likely to use one or more of these strategies.¹

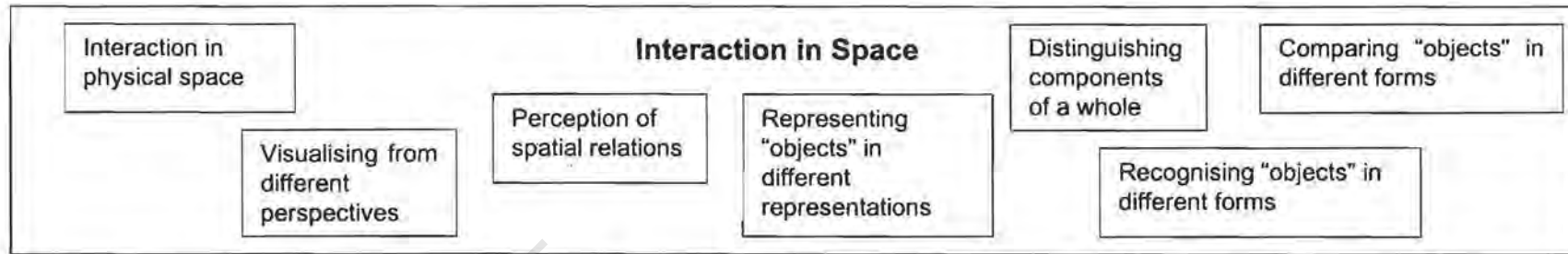
The following should be noted in relation to these "strategies":

- It is possible that the strategies used will vary according to the size of the space in which the learner is working and the form of presentation of the task.
- It has been suggested that the learner could work with parts of an "object" or with the "object" as whole, or a combination of these.
- In some cases the manipulation might require a single movement, or it could consist of multiple steps.

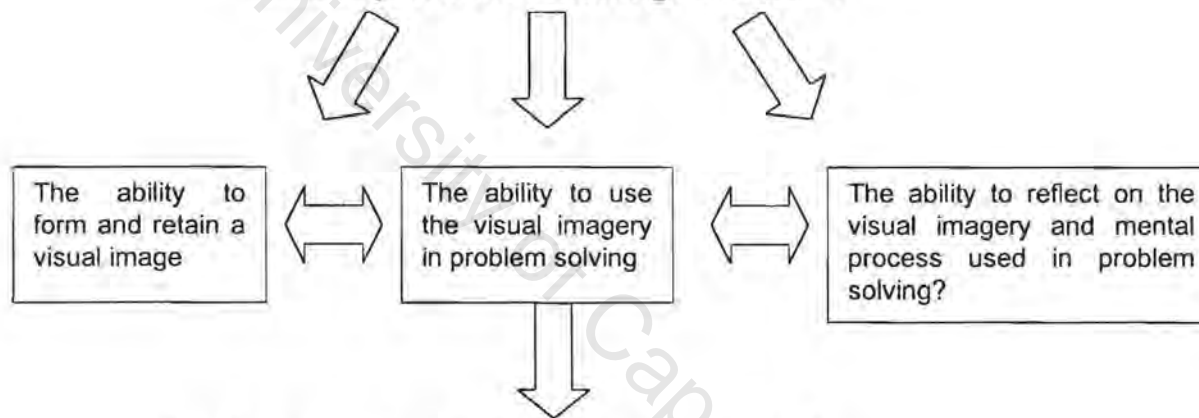
With regard to "recognising objects in different forms" a learner will also require a knowledge of the conventions required for representing three-dimensional "objects" in two dimensions, and this is not necessarily a visual skill.

A summary of this model of "spatial ability" is provided in Figure 1 below:

¹ The word "strategy" is being distinguished from the word "ability". The latter is being regarded as a more general skill, for example the "ability" to use visual imagery in problem solving. This might require the use of a number of strategies.



This requires the following "abilities":



Involves, among others, these "strategies":

(possibly single or multi-step manipulations; possibly working with parts of an "object" or an "object" as a whole)

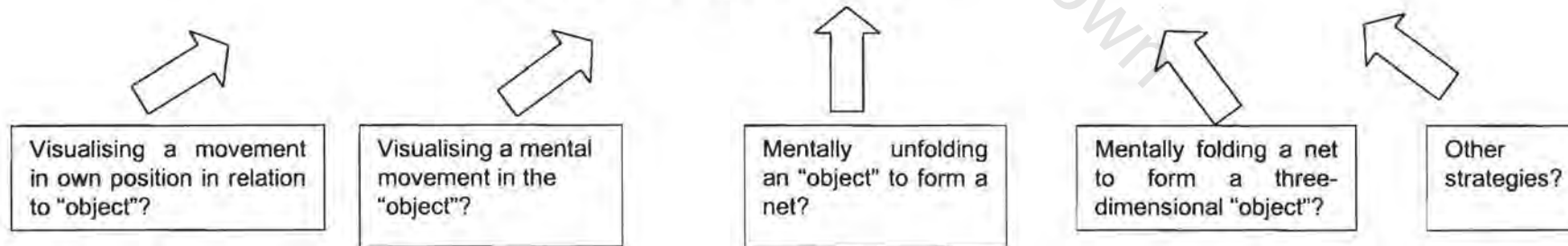


Figure 1: A Preliminary Model of "Spatial Ability"

3.5 The Development of “Spatial Ability”

There are suggestions in the literature that the development of “spatial ability” occurs in the following sequence:

- (a) interaction in “specific” space / the physical environment
- (b) the formation of visual images
- (c) the manipulation of these visual images .

But the question remains as to what the relationship is between these aspects of “spatial ability” once the learner is “spatially mature”. For example, a learner who is able to use the skills in (c), might at times need to revert back to physical interaction in space to assist with the visual processes.

Furthermore, if as suggested in section 3.3, an additional aspect of “spatial ability” is the ability to reflect on the mental processes used in problem solving, one could ask how this aspect is related to the above sequence. Is it perhaps the final “level” in the sequence?

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Chapter 4: Methodology

In Chapters 2 and 3 I have attempted to summarise some of the existing work done in trying to clarify the notion of "spatial ability" and have raised questions relating to this work. If I am to develop my own classification, the distinctions that I need to make must be supported empirically. The themes discussed in sections 2.6.1, 2.6.2 and 2.6.3 have been selected as the focus of this study and can be summarised as follows:

(a) Visualising "Objects" from Different Perspectives:

- What strategies are used by learners when visualising an "object" or "objects" from different perspectives?
- Do different learners use different strategies?
- Do the strategies used differ if the size of the "space" differs?
- In the case of "small-scale" space, do the strategies differ if the number of "objects" varies? Do the strategies differ if the demands placed on the learner in terms of the presentation of the response are varied?
- In the case of "large-scale" space, do the strategies differ if the space is "seen" or "unseen"?

(b) Representing Three-dimensional "Objects" in Two Dimensions - The Case of Nets:

- What strategies are used when learners are required to represent a three-dimensional "object" in two dimensions (in the form of a net)?
- Do different learners use different strategies?
- Do the strategies vary if the form of presentation of the activity is varied?

But how should information on these aspects of "spatial ability" be gathered? This is an important consideration for as Stea and Blau (as quoted in Eliot and Hauptman, 1981 : 52) warn, we need to consider whether the behaviours we observe are not artefacts of the particular methods we have chosen to elicit those behaviours.

In this chapter I will discuss existing methods of data collection in the field of "spatial ability" and comments made on these approaches, and will then explain the subsequent decisions regarding the process to be used in this study.

4.1 Existing Methods of Data Collection

When considering existing methods the question we need to ask is, "What are these tests or data collection methods actually measuring?"

4.1.1 The Form of Presentation

When workshopping the Malati materials with colleagues and teachers I was alerted to the fact that, in cases where possible answers are given, or when a multiple choice format is used, a learner might use different (and perhaps not spatial) strategies than those used in tasks in which no possible solution is provided.

Consider for example Activity 2 (Appendix 1): In this activity a learner is required to visualise a representation from four different perspectives and four possible drawings are provided as solutions. When workshopping this activity with adults it emerged that two strategies appear to have been used:

- The subject visualises the picture from the different perspectives indicated by the "eyes" (possibly by visualising a movement in his / her own position or by visualising a movement in the "objects"), and then looks for a similar picture from the possible solutions (a) to (d)
- The subject begins by working with the possible solutions (a) to (d), and then matches these to the possible views (1) to (4) using a process of elimination. It seems that a logical argument rather than the manipulation of visual imagery might be employed in this case.

These two strategies are possibly different to those that might be employed in answering a question that requires that a learner **draw** the picture from the four different perspectives. In the case of Activity 2 the learner is presented with possible visual images and possibly has to manipulate these to solve the problem. But in the case where drawing is required, the learner has to construct **and** manipulate the appropriate visual image as well as employ drawing skills.

Wattanhawa (in Clements, 1983 : 15-16) also distinguishes between the different demands placed on learners by the form of presentation required by the question: One of the four factors in her DIPT system is "Presentation" which is regarded as having three "values", namely,

- 0: "The expected answer form does not require a final visual image to be described, identified or drawn on paper"
- 1: "The answer is a picture which has to be identified from a number of different pictures presented in diagrammatic form, or are described by words or pictures" (as in Activity 2, Appendix 1)
- 2: "The answer requires that the final visual image be represented by a drawing, or be described by words or by hand or other movements".

In terms of Wattanhawa's framework, therefore, different tasks can be categorised according to, among other things, the type of response required. I would suggest further, that these forms of presentation could, in fact, determine the strategies used.

Guay *et al* (as quoted in Clements, 1983 : 12) also refer to the possibility that the format of the test might influence the response. They suggest that many of the paper and pencil tests used by factor analysts require "analytic thinking" rather than "spatial thinking". The two or three classifications commonly suggested by factor analysts and discussed earlier thus become questionable. Precisely what is meant by "analytic thinking" is not, however, explained.

Johnson and Meade (1987 : 725) warn that, although pencil and paper tests are easier to administer than other forms, they might not be suitable for pre-adolescent children. They quote Horn who suggests that no mental ability test is, in itself, an adequate measure of "spatial ability" and proposes that the phenomenon be measured in several different ways. Johnson and Meade provide some ideas on the choice of tasks for younger children, these being that the instructions should be clear, the cognitive operations "relatively simple", and the tasks intrinsically

interesting to learners. Referring specifically to "visualisation", Bishop (1989 : 11) proposes that this be studied on a variety of tasks in a variety of contexts.

In contrast to the use of written tests by factor analysts, developmental psychologists have used clinical interviews. It is important to remember, however, that different conditions prevail in the educational context and that different methods might be more suitable for this context: Comparing clinical observations to classroom observations, Bishop (1980 : 266) questions whether the classroom context might not actually alter behaviour on "spatial" tasks.

4.1.2 The Use of "Figural Stimuli"

Referring to the "visualisation" / "orientation" classification commonly suggested in the literature and discussed in section 2.6.1, Bishop (1983 : 184) suggests that, as most of the tests involve the use of "figural stimuli", one will get a correlation between the "orientation" and "visualisation" tests. He suggests, rather, that researchers need to consider visualisation in arithmetic and algebra, too. Furthermore, Bishop (*ibid.*) warns that figural stimuli have their own conventions and visual vocabulary.

4.1.3 Time Constraints

Clements (1983 : 16) notes an increased interest in the role of the time variable in the study of "spatial ability". But it seems that "spatial ability" can be studied independent of this variable: As was noted in section 2.6.1, Clements (*ibid.*) has suggested that Lohman's distinction between "major" and "minor" factors (he classifies the use of speed in performing certain skills as a "minor" spatial factor) is actually useful in allowing researchers to focus on the study of the latter factors.

I would suggest that the inclusion of this variable in the classification of "spatial ability" depends on the specific interest of the researcher. For Bishop (1983 : 184) suggests that this type of research in which tests are usually administered to groups and completed under time constraints are of little use in determining what processes were involved in getting answers.

4.1.4 The Idiosyncratic Nature of Performance on Spatial Tasks

In section 2.6.1 it was noted that a number of researchers support the idiosyncratic nature of performance on tasks. If this is so, it would be of little use to classify actual tasks. Rather, one should classify the **strategies** (possibly different) used by learners on a set of tasks.

4.1.5 Facility with Spatial Tasks

Bishop (1989 : 11) suggests, too, that facility with "spatial" tasks might actually influence the strategies used, for example, learners might move away from visual methods. It thus seems appropriate not only to use a variety of tasks as suggested in section 4.1.2, but also to assess performance on these tasks over an extended time period.

4.1.6 Drawing Skills:

Matthews (1991 : 9) notes that a learner's drawings reflect his / her "spatial ability", but that these drawings are not always understood by teachers. This has been noted during the trialling of the Malati "spatial skills" activities and seems to be a particular problem in the case where the individual performing the assessment has poorly developed "spatial skills". Malati project workers and teachers have found it both necessary and informative to talk to the learner about the solution in an attempt to try to develop some shared meaning with regard to the drawn responses.

But Mitchelmore (1980 : 90-91) suggests that the relationship between a learner's drawings skills and "spatial ability" might be more complicated than that one simply "reflects" the other . He distinguishes between "representational ability" and "spatial-perceptual development". His research suggests that the former lags behind the latter, at times by as much as three stages.¹

Commenting on cross-cultural studies of "spatial ability", Mitchelmore (1977 : 143) notes that subjects will use different methods to solve "spatial" tasks. Referring specifically to the skill of drawing, he suggests that different cultures have different preferred artistic styles, thus a subject who is perceived to have primitive drawing skills may not have perceptual difficulties, but may be reacting to a strong cultural influence. He warns, therefore, of the dangers of using traditional paper and pencil tests for students in "developing countries", and suggests that we consider what we are actually testing and whether we are testing the same thing across samples. Furthermore, Mitchelmore's work suggests that more attention should be paid to the different forms of representation considered as appropriate in different cultures.

4.2 This Study

The data collection methods for this study were selected following consideration of three factors:

- (a) The nature of the questions that arose during my literature review and discussed in sections 2.6.1 to 2.6.3
- (b) The debate regarding methodology in this field, as discussed in section 4.1
- (c) The circumstances of my work at Malati.

4.2.1 The Activities (see Appendix 1):

My position as a materials developer in the Malati Geometry Working Group meant that I had access to activities and the opportunity to create tasks that were regarded

¹ Mitchelmore (1980 : 84-85) identifies four developmental stages for drawing and relates these to the development of a child's conception of space (from topological, to projective, to Euclidean space) as suggested by Piaget and Inhelder. The four stages are:

- Stage 1 (plane schematic): The figure is represented by a single face drawn as if viewed orthogonally (recognising the topological property)
- Stage 2 (space schematic): Several faces are shown, but either the faces are shown as if drawn orthogonally, or the hidden faces are included (starting to represent projective properties, but not yet forming a common reference point)
- Stage 3 (prerealistic): An attempt is made to represent the view from a single perspective and to depict depth (the beginning of the development of a Euclidean frame of reference)
- Stage 4 (realistic): "Parallel edges in space are represented by near-parallel lines on paper" (a Euclidean frame of reference has been established).

in the Project as "spatial activities". While conducting the literature review as discussed in Chapter 2, I, along with colleagues, was required to construct the initial draft activities focusing on "spatial skills". As I was going to be closely involved with the use of these materials in the classrooms in Malati project schools, I decided to use a selection of these activities for this study and to collect the data during my regular classroom visits. The following should thus be noted regarding the use of these activities: Firstly, they are in draft form and secondly, the assumption is being made that they are "spatial" in nature. As the focus of my work at Malati was on geometry, I did not feel equipped to include arithmetic or algebraic activities in this study.

The activities were selected from the first draft of the "Spatial Skills 1" and "Spatial Skills 2" packages.² These activities do not follow in consecutive order in the packages. Each activity is accompanied by teacher notes relating to the use of the activity in the classroom. The use of these activities in the classrooms as described below was part of the first trialling of the packages.

The activities have been numbered according to the order in which they were originally intended to be completed and in which they were completed by the majority of the learners. Reasons for any changes in this sequence are explained below.

With regard to the representation of three-dimensional "objects" in two dimensions, it should be noted that all the representations in these activities are presented using parallel projection. It must be acknowledged, therefore, that certain conventions are associated with the interpretation of the drawings in these activities.

The activities can be classified as follows:

(a) ***Visualising an "object" from different perspectives ("small-scale" space; solution required in the form of a drawing).***

Activities 1(a) and (b) (refer to Appendix 1, pages 92-94):

Activity 1(a) was designed as part of the Malati "Spatial Skills 1" package, but when this was trialled, one of the teachers involved in this study used the activity in her class without my knowledge. As a result I designed a similar activity, Activity 1(b) for use by the relevant group of learners in this class. The instructions for the activity were retained, but the representations of different "objects" were used. Of course this meant that the learners had already completed one activity of this nature when I had the opportunity to study their strategies.

Description of the activities: These activities use two-dimensional representations of three-dimensional objects in "small-scale" space. Each item uses only one "object", for example, a representation of a cereal box, a tin of beans, an egg or a bottle. The learner is required to draw what s/he thinks each "object" will look like

² "Spatial Skills 1" contains activities focusing on different forms of interaction in space, for example, viewing "objects" from different perspectives and comparing "objects" in different orientations. "Spatial Skills 2" focuses on the representation of three-dimensional objects in two dimensions, that is, in the form of pictures, nets and cross sections.

from different perspectives, these being from behind, above, and the side. Responses are to be made without the use of the actual three-dimensional "objects".³

Reason for inclusion in the study: This was included to explore whether the learner pictures a movement in his / her own position in relation to the "object" or whether s/he mentally manipulates the "object" itself or whether any other strategies are used in solving the task.

At first glance it appears that the study of relationships between "objects" will not be required, but it is possible that the learner might make use of parts of the individual "objects" in answering these questions.

Left / right discrimination can be explored in the case where learners have to draw the representation from behind, for example, the learner should note in the case of the teacup that if the handle is on the right in the given representation, this will be on the left in the representation from behind.

Activity 4 (refer to Appendix 1, page 96):

Description of the activity: This activity has an orthogonal representation of a factory. The learner is required to visualise what the factory in the picture will look like from above, below, from a point each on the left and right, and from behind. This representation is being regarded as one "object".

Reason for inclusion in the study: Again I aimed to determine whether the learner visualises movement in his / her own position, a movement in the "object", or uses any other strategy. The learner might consider the relative positions of the chimney and the rest of the building, that is, parts of the representation in making the drawings. Left / right discrimination can also be explored when the learner is required to draw the representation from behind.

(b) *Visualising two or more "objects" from different perspectives ("small-scale" space; solution required in the form of a drawing).*

Activity 3 (refer to Appendix 1, page 96):

Description of the activity: This activity requires that the learner work with two-dimensional representation of two different sized boxes. S/he must draw what s/he would see when looking at the two-dimensional representations of the boxes from different perspectives, these being, from above, below, at points each on the left and the right of the "objects", and from behind.

Reason for inclusion in the study: Because this activity gives representations of two boxes, it appears that the learner will need to reflect on the relative position of the "objects" as well as on his / her own position in relation to these "objects". It is possible that the strategies used in such a case might differ from those used in Activities 1(a) and 1(b). This is also a useful context to explore the learner's

³ In the Malati "Spatial Skills 1" package this activity is preceded by one in which learners have to draw actual "objects" from different perspectives by physically moving position in relation to the "objects".

ability to discriminate between left and right when viewing the "objects" from behind.

(c) **Visualising two or more "objects" from different perspectives: ("small-scale" space; possible solutions in the form of drawings provided).**

Activity 2 (refer to Appendix 1, page 95):

Description of the activity: This activity requires that the learner work with a number of objects in a two-dimensional representation of three-dimensional objects. The learner is presented with a picture of a flowerpot, a plate and a cup. A drawing of an eye is used to show views of these "objects" from different perspectives. The learner must match the view from these four different perspectives with four given drawings of the scene and explain his / her reasoning. S/he will thus need to consider the relative position of the objects as well as his / her own position in relation to the "objects".

Reason for inclusion in the study: In contrast to the activities described above in which the learner has to draw what s/he sees from different perspectives, possible responses showing views from the different perspectives are given. It is possible that the strategies used in this case might differ from those used in the other activities. It seems that a learner could:

- Visualise the solution for the view from eye 1 and then match this with one of the solutions (a) to (d)
- Begin working with the solutions (a) to (d) and then match these to the views (1) to (4) using logical argument rather than by mentally manipulating the visual images.

(d) **Visualising a scene from different perspectives ("large-scale" space ["seen" and "known"]; solution required in the form of a drawing).**

Activity 5 (refer to Appendix, page 97):

Description of the activity: This activity requires that the learner draws what s/he would see when looking at the classroom from above. Instructions for the task are provided by the teacher.

Reason for inclusion in the study: The classroom is being interpreted as "large scale" space ("seen" and "known"). It is possible that the skills used to perform such a task are different from those used to draw "objects" such as boxes in "small-scale" space in Activities 1 to 4. Of course it should be noted that this activity differs from Activities 1 to 4 in that the learner does have access to the "concrete" objects, that is, s/he can view the actual scene from his / her position in the classroom. In contrast, in Activities 1 to 3 the learner has to work with **representations** of the "objects".

(e) **Visualising a scene from different perspectives; "large-scale" space ["unseen" and "known"]; solution required in the form of a drawing.**

Activity 6 (refer to Appendix 1, page 97):

Description of the activity: This activity requires that the learner draws what s/he would see if s/he were looking down on the school grounds ("large-scale" space) from above. As in the case of Activity 5, the instructions are given by the teacher.

Reason for inclusion in the study: In this case it is not possible for the learner to see the entire space from one position ("unseen" space), but can view some aspects of the scene. Again there is a possibility that the skills used to perform such a task might differ from those used in Activities 1 to 4.

(f) **Visualising a representation of a three-dimensional "object" in two dimensions (in the form of a net) and vice versa.**

Activity 7 (refer to Appendix 1, page 98):

Description of the activity: The activity has two different aspects:

- The learner is given one possible net for a cube (no diagram of a cube is given) and required to draw two additional, different nets
- The learner is given a two-dimensional representation of a rectangular prism and required to draw the net of this object.

This was the first activity in which the learners involved in this study encountered nets.

Note that the notion of a net is being regarded here as a particular form of representation with its own conventions. The learner requires some knowledge of these conventions in order to be able to use the appropriate visual strategies under investigation in this study. This was taken into account when the activities were in use.

Reason for inclusion in the study: The question here is whether the learner works from the image of the three-dimensional object and mentally unfolds this to find the net, or whether s/he begins with an image of the two-dimensional net and folds this up to form a mental image of the three-dimensional "object". There is a possibility that the learner will use different strategies for the two different forms of presentation mentioned above.

The activities can thus be summarised in Table 1:

	1	2	3	4	5	6	7
"small-scale" space	✓	✓	✓	✓			
one "object"	✓			✓			
two or more "objects"		✓	✓				
response in form of drawing	✓		✓	✓			
drawings provided		✓					
"large-scale" space					✓	✓	
"seen" space					✓		
"unseen" space						✓	
representation of 3-d "object" in 2-d							✓

Table 1 : Features of Activities 1 to 7

4.2.2 The Data Collection Process

It was decided that the data would be collected in the classroom context, that is, as learners worked on the activities as part of their normal everyday learning programme. It has been noted that my position at Malati meant that I had the opportunity to gather data for this study during the trialling of the materials. But it was also felt that the classroom context would be appropriate for gathering the required data for this study because:

- (a) It was hoped that the nature of the classroom culture in the selected classes would mean that learners discussed the activities and that this would provide additional data.⁴
- (b) I would be in a position to clarify problems that arose due to the fact that the activities were in draft form and clarify problems arising from the visual conventions and vocabulary used in the activities
- (c) I would be able to record gestures and conversations
- (d) I would be able to question learners about their strategies where necessary
- (e) I would be able to question learners about their drawings in an attempt to establish some shared meanings.

The following data was collected from this setting: audiotapes of the group interactions; field notes made while observing the work on the activities; and the written responses of learners.⁵

Each of the two teachers involved in the study was given a list of the research activities and was asked to inform me of when their learners would be doing these activities (see Appendix 2). Due to the nature of my work at Malati I was also visiting these classes on a regular basis to trial the materials and to provide teacher support. I had thus built up a working relationship with these two teachers and the learners prior to the formal collection of the data.

For the research activities, I sat at the desk of the group of four learners selected from each class. The audio-recorder and conference microphones were placed on the desks. I observed the learners solving the activities, making notes on their gestures and conversations.⁶ Queries regarding the requirements of the questions were answered. Teacher interaction with these learners in the form of instructions, hints for completion of the activity or discussion with the group or individuals in the

⁴ One important aspect of the Malati philosophy of teaching and learning mathematics is the development of a classroom culture in which pupils take responsibility for their own learning. Features of such a culture are that learners work independently, work spontaneously and effectively in small groups, are motivated by finding a solution to a problem, are willing to persist and to struggle, can communicate verbally and in writing, feel safe to argue and to question, respect one another's opinions and listen to one another, and are not unduly disturbed by mistakes.

⁵ This data is being regarded as what Liben (in Clements, 1983 : 10) would call "spatial products", that is, external representations that represent space. The value that can be attached to such data is discussed in sections 5.2 and 5.3.

⁶ The following teaching approach is advocated by Malati where appropriate and was used by these two teachers for the research tasks: The learners sit in groups. When issued with a printed activity, each learner is required to read and then attempt the task as an individual (learners do discuss problems with comprehension if necessary). On completion of part or the entire activity, learners in a group discuss and compare solutions. At a time deemed as appropriate the teacher conducts a whole class discussion so that different responses can be shared and discussed.

group was noted. Responses made by the learners in class discussion were also recorded.

In two cases learners involved in the study were absent from school on the day of one of my visits. In the one instance I worked with the individual, and in the other with a pair of learners at later dates when their classmates were completing other activities. As a result these learners did in some cases complete the research activities out of the order originally intended, and without working in a group context.

In one case, as mentioned in section 4.2.1, a teacher did not inform me that a research activity was being used in the class. An additional activity as described was thus designed, and this was completed after Activities 2, 3 and 4.

The fact that the data was collected in the context of a group discussion in each case, meant that learners did discuss their strategies and they could hear the responses of their classmates. Instances in which it was felt that a learner was copying another were noted in the field notes.

4.2.3 The Sample

The choice of sample for this research project was largely determined by the context of my work at Malati. The school at which the research was conducted is one of four primary schools working in the Project and is one of the two primary schools in which instruction is conducted exclusively in English. The other English-medium school experienced a number of staffing changes during 1998 and it was felt that this might affect the data collection process.

The choice of classes in particular grades was influenced by the need for me to work closely with the teachers of the classes involved in **the classroom context** as described in section 4.2.2. Colleagues assisted me in identifying and negotiating with two teachers who were already co-operating enthusiastically in the Malati project. One class in each of grade 5 and grade 6 were selected. The teachers indicated that "spatial" activities of this nature had not been used in the school prior to the Malati intervention and this first trialling of the geometry materials.

The nature of the data collection process as described below required that I select from each class a group which was functioning well and in which all learners were participating in the discussion. Initially the two teachers were asked to identify possible groups. I subsequently spent one lesson in the grade 5 class and two in the grade 6 class observing the interaction in the pinpointed groups and testing the use of the recording equipment with each group. On the basis of this one group was selected in each class:

Grade 5: Four boys – Ashton, James, Ryan and Stuart

Grade 6: One boy – Trevor; Three girls – Belinda, Gaylene and Cindy.⁷

⁷ The names of the learners have been changed for the purposes of this study.

Chapter 5: Results and Discussion

In this chapter certain general issues related to the analysis are discussed. This is followed by comments on the results relating to the specific themes of the study.

5.1 The Analysis

The process of analysis involved a number of stages:

- (a) A review of the transcripts, the field notes and the written responses of learners on each activity was used to compile a "profile" on the performance of each of the eight learners, detailing strategies used as well as interesting features of the performance. This presentation of the initial analysis includes examples of responses to illustrate the approach used by each learner.
- (b) The next task was to analyse performance by the eight learners on the specific categories of activities related to the themes of the study, for example, those tasks that required a response in the form of a drawing, or those that dealt with "large-scale" space. The information on each learner generated in (a) was used to identify common strategies and other common features of performance on activities in each category. Again, learner responses are included as illustrations. The conditions under which each activity was performed and any teacher intervention was noted. This detailed analysis is provided in Appendix 3.
- (c) The analysis described in (b) was used to generate the discussion in this chapter. In this discussion strategies used on different tasks are described and compared. Other interesting features of performance identified during the analysis are also noted.

The discussion described in section (c) is included in this chapter as it represents the last in a number of steps undertaken in trying to make sense of the data. It thus represents a synthesis of the information generated by the processes described in (a) and (b). In this discussion the reader is referred to the detailed analysis provided in Appendix 3. It is hoped that this approach will provide the reader with some insight into both the process of analysis described above and the final results of the study.

5.2 Identifying the Use of Different Strategies

As I was collecting the data during everyday classroom interaction, I was not able to question each learner on his / her response to every item in an activity. For example, in Activity 1(a) it was not possible to discuss strategies on every "object" (the tin of beans, teacup etc) with all four learners in a group. Thus it was not always possible to determine which strategy each of the eight pupils used when solving a particular item. I am thus not able to comment on the extent to which each strategy was used. Rather, I have used the presence of a particular strategy in the data as evidence to suggest that this is **one way** in which the task in question can be solved.

5.3 Using Drawings as Data

In the discussion on "The Understanding of Visual Representations" in section 2.6.1, it was noted that drawings as obtained in the data collection process can provide some access to the "spatial thought" of learners. Further, it was noted that the use of drawing as an effective means of communication requires some knowledge of

certain drawing conventions. The difficulty a learner can have portraying his / her mental images in the form of a drawing is illustrated by Stuart's response in Activity 4: He claimed that he had a picture in his mind of what to draw, but "I couldn't like draw the shapes".

The question that needs to be asked, therefore, is what weight can be attached to these drawings in the analysis? While sitting with the learners during the data collection process, I observed them as they made their drawings. In a case where I had difficulty understanding a drawing in the context of an activity, I questioned the learner about his / her work. In this way I could attempt to establish some shared meaning with respect to the drawing and was able to obtain verbal information on the mental processes. Consequently, I relied mainly on the verbal responses in my analysis as many of these responses arose out of discussion of the drawings.

5.4 Using Verbal Reports as Data

But if, as mentioned above, I relied mainly on the visual responses in the final analysis, what weight can I attach to **these** responses? For example, if I ask a learner, "What did you picture in your mind?", what relation does the verbal reply have to the original mental processes?

Ericsson and Simon (1980) refer to doubts that have been expressed over the years about the usefulness of verbal responses in research. They note claims that intermediate processes may intervene between the internal representation of information and its verbalisation. This appears to be a particular problem when the researcher attempts to probe for specific information. A request can serve as a "hint" as was possibly the case in my interaction with Gaylene: When gathering data for Activity 1(a) I asked, "Did you think you were moving yourself or did you think you were moving the box?" I used this type of questioning as I felt that possibly by giving examples of mental processes, I might assist learners in reflecting on and describing these processes.

But what weight can I attach to a response to a question such as this? Ericsson and Simon identify different possible causes for incomplete verbal reports, related to the storage and accessibility of information in memory. They are confident, however, about the value of verbal reports as data "if elicited with care and interpreted with full understanding of the circumstances under which they were obtained".

Consequently in this particular analysis, those instances in which it was felt that my questioning, teacher input or discussion amongst learners might have influenced a response, were not considered when identifying learner strategies.

5.5 Interpreting Learner Gestures

When attempting to solve Activities 1 to 4, a number of learners lifted the page on which the activity was printed, or moved themselves in relation to the page (refer to Appendix 3, pages 102, 106, 110 and 112).

For example, when working on Activity 4, Belinda and Gaylene moved their sheets so that they could view the representation on the activity sheet from a different

angle. But when completing the same activity, Ashton and Ryan moved their own positions in their seats, holding the sheet fixed.

Furthermore in Activity 4 when using the microphone to demonstrate for Cindy what it means to view the object from above, the learners used different movements: Trevor and Gaylene stood up so that they could look down on the microphone, but Belinda stayed seated and turned the microphone to face her.

It is not possible from this data to determine how these movements can be related to the mental strategies being explored. For example, it cannot be concluded that a movement in the activity sheet is related to a strategy in which an "object" is manipulated mentally, or that the actual movement in the position of an "object" is related to a similar mental movement.

As a result it was decided not to include the data related to these gestures in the analysis. The learners would have to be interviewed further to obtain additional information on these movements and their relationship to the mental strategies used.

5.6 The Ability to Communicate Mental Processes

In this study I have attempted to gain access to learners' mental strategies by observing the learners as they work on and discuss selected activities, and by questioning them about their mental processes. The value I am attaching to the drawings and verbal responses has been discussed. It was found that, with the exception of Cindy, all the learners in the sample were able to describe their mental processes with relative ease and in a meaningful way, and I could use the discussions to gain some shared meaning with regard to their drawings (refer to Appendix 3 pages 102, 106, 110, 112, 117, 122 and 126).

Trevor, in particular, volunteered to share his thinking willingly throughout the data collection process and often before this was requested. As a result a particularly rich set of data was collected in interaction with him.

Belinda did take a while to develop this "ability" to describe her mental processes. In Activity 2 she could describe the relative position of the observer and the "objects", but did not obtain the correct answer and her explanation did not make sense (refer to Appendix 3, page 106). She also struggled drawing the "object" in Activity 4 from above and below. It should be noted that Belinda completed Activities 2, 3 and 4 before doing Activity 1(a). After completing Activity 2, however, her confidence and ability to describe her thought processes improved and valuable data on her performance was gathered. On the whole it was possible to establish some shared meaning with Belinda.

These examples illustrate how three learners were able to communicate their mental processes:

Trevor: *...Miss, I picture the box in my mind, Miss. Then I just turn the box...put the box down, Miss, and look at it from the top.*

Ashton: *I'm not standing there...I'm standing a little further...standing on that line there. I'm standing there, so I am imagining that I can see this part...so I can see the 3-d.*

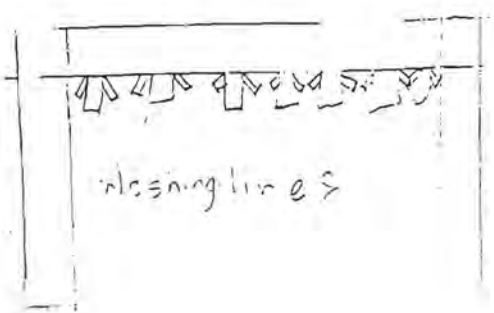
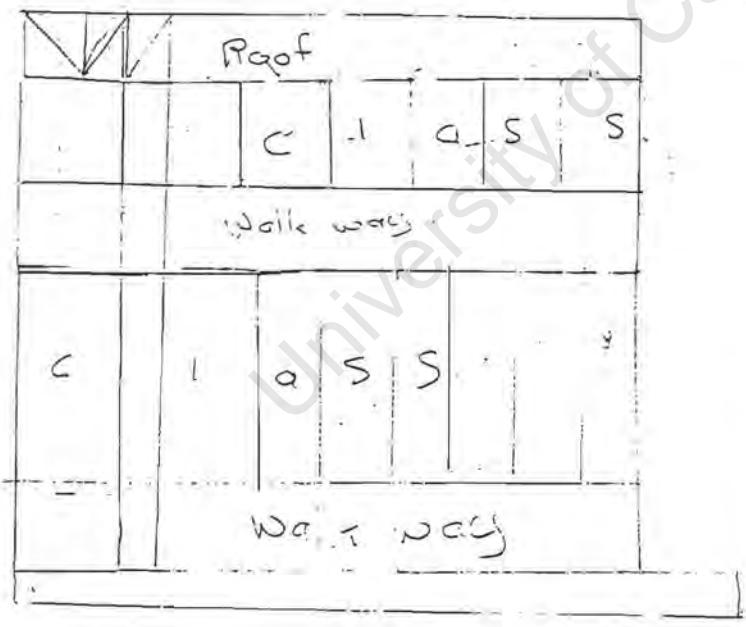
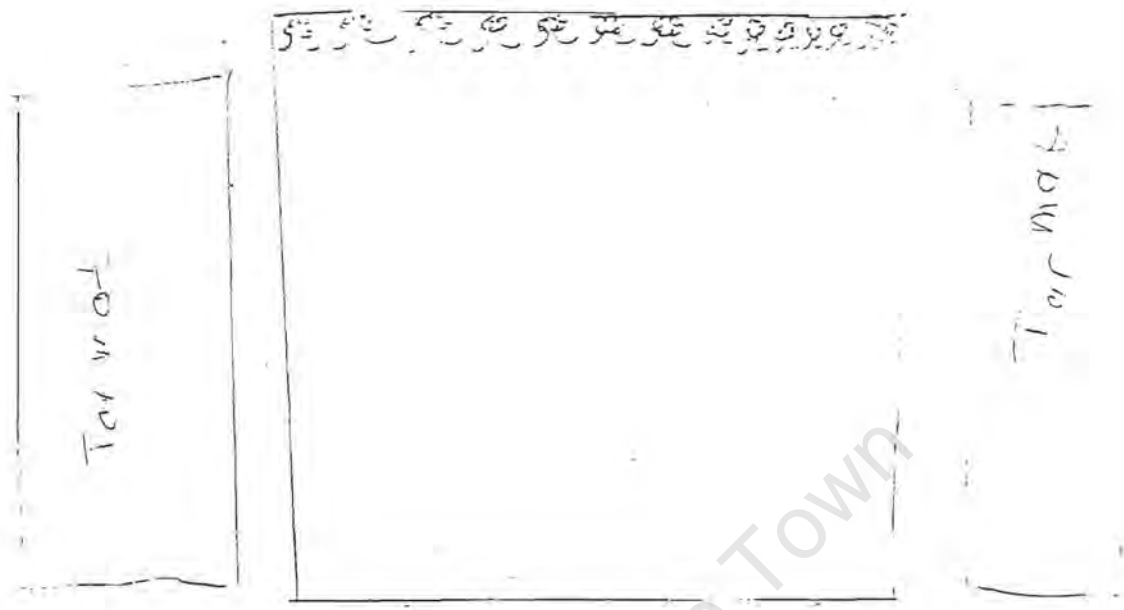
Stuart *I imagined I was, like I was walking around the factory.*

In the case of Cindy, however, problems were encountered interpreting her drawings and understanding the verbal descriptions of her mental processes. In some cases she actually claimed that she could not describe these processes (refer to Appendix 3, pages 102, 106, 110, 112, 117, 122, 126).

For example, when required to draw a tin of beans from above (in Activity 1(a)), she responded by drawing a rectangle. She changed this when she looked at Trevor's drawing. When drawing the aerial view of the classroom (Activity 5), she was not able to communicate the correct position of the desks in relation to one another. Rather, she drew these in rows along the sides of the classroom:



Similarly in the aerial view of the school grounds (Activity 6), the relative position of the buildings is incorrect and the gutters are drawn running perpendicular to the building:



Cindy's lack of confidence in her ability to perform on these spatial activities was illustrated by her frequent erasing of her drawings.

Although some of Cindy's answers were correct, she either could not describe her mental processes, or these did not make sense. For example her answers for Activity 2 were correct, but when asked to explain the mental processes she responded, "...I can't tell you what goes on in my mind, Miss". Her written explanation also did not make sense:

...because I could see sees the whole thing, ...because I could only sees the side, ...because I could only sees above, ...because I could only sees below.

In the cases where Cindy's responses did make sense it appears that she might have been copying her classmates' work and she admitted when asked about Activity 7, "I copied Trevor, Miss". The responses that did make sense, therefore, could have been learned responses that she had learned or heard during the classroom interaction, or a reaction to the questions I asked. For example, when I asked about the tin of beans: "Were you moving the tin or moving yourself?", she replied, "I was moving the tin".

Interestingly, the opportunity to use physical manipulation did not seem to help Cindy in responding to the tasks. In Activity 3 her classmates and I attempted to assist by setting up actual boxes on the desk for her to view (refer to Appendix 3, page 110). She was able to use these boxes to demonstrate physically to me where she would view them from, but was not able to translate these into drawings that made sense to me. In Activity 7 I encouraged her to cut and fold the given net (refer to Appendix 3, page 125). These measures did not, however, appear to assist her visual processes in any way.

Cindy's struggles suggest two possibilities. Firstly, it is possible that Cindy is not actually able to form a visual image. Secondly, she is possibly able to form a mental image, but is not able to describe or convey these images using the commonly accepted conventions for drawing. As discussed in section 4.1.5, Mitchelmore (1980 : 90-91) has suggested that a learner's ability to represent visual images can lag behind his / her ability to form these visual images.

As a result of these discrepancies between some of her responses and her inability to communicate her mental processes in a meaningful way, and the possibility that Cindy used learned responses, I feel that it is not possible to classify her strategies along with those of the other learners. Her responses have not, therefore, been included in the discussion that follows.

5.7 Visualising "Objects" From Different Perspectives - "Small-Scale" Space

5.7.1 One "Object"; Response in the Form of a Drawing (Activities 1(a), 1(b) and 4¹)

In general, the following was noted in performance on tasks of this nature:

Firstly, the two strategies suggested in 2.6.1 (mental movement of the "object" and mental movement of the position of the observer in relation to the "object") **as well as** two additional strategies were identified. Secondly, the use of the different strategies varied from learner to learner and across activities.

The four strategies noted can be described as follows:

Strategy 1: The learner visualises his / her own movement in relation to the "object", for example, s/he pictures that s/he is moving position to behind the "object". This strategy was identified in the responses of Gaylene, Ashton, James, Ryan and Stuart (refer to Appendix 3, pages 101 and 112). The following quotes suggest that these learners were visualising a movement in their own position:

James: *I imagined a real toilet roll and looked around it.*

Ryan: *That I was standing on top and I was standing and on a small table was a glass bottle.*

Ashton: *I'm not standing there...I'm standing a little further...standing on that line there. I'm standing there, so I am imagining that I can see this part...so I can see the 3-d.*

Stuart *I imagined I was, like I was walking around the factory.*

Strategy 2: The learner visualises the movement of the "object" itself, for example, s/he mentally turns the box in his / her mind. This strategy was identified in the responses of Trevor, Gaylene, Belinda and James (refer to Appendix 3, page 101):

Trevor: *...Miss, I picture the box in my mind, Miss. Then I just turn the box...put the box down, Miss, and look at it from the top.*

Belinda: *I took the ball, Miss and I turned it like this, Miss.*

Strategy 3: The learner does not necessarily visualise a movement in the particular context of this activity, but relies on visual memory of similar objects in a similar situation.² This was identified in the responses of Trevor, Belinda and Ashton (refer to Appendix 3, pages 102 and 112):

¹ (Activity 1(a) was completed by Trevor, Cindy, Gaylene and Belinda; Activity 1(b) was completed by Ashton, James, Ryan and Stuart.)

² By "visual memory" in this case, I am referring to the learner using a visual image from **another** situation that has been stored in his / her memory.

- Trevor: (Referring to the tennis ball representation in Activity 1(a)) *...once I looked up in a book that if you look at it (a sphere) from all directions, it will still be the same.*
...whenever I play cricket I look at the ball.
- Researcher: (Referring to the teacup representation in Activity 1(a)) *How did you know that it would look like that?*
- Belinda: *Miss, because I always make tea.*
- Ashton: (Referring to Activity 4) *So if you look from point A, it is like the box...*
- Ashton: (Referring to the bottle representation Activity 1(b)):*...but Miss the other day said that a round object never changes, except the words on the object...Then they say a round object when you look from above... knew that because one day they had a container on the board, so the container looked like a bottle and I drew it like this because that is smaller than this (points to parts of the bottle)...*
- Researcher: *How would you check that what Mrs Johnson says is correct?*³
- Ashton: *All...almost all bottles are round. So I used what I learnt to draw the bottle.*

Ashton's reference here to the size of the parts of the bottle suggests an additional strategy:

Strategy 4: Two learners used short arguments which do not appear to rely on the manipulation of visual imagery, but refer to the relative position and size of the parts of the "objects" (refer to Appendix 3, pages 102 and 112). Trevor used the relative position of the parts of the "object" in Activity 4 to develop a short argument in explaining the left / right discrimination when viewing the "object" from "behind": He commented, "It was on the left side, Miss, but I swopped it around so it is going to be on the right side". Ashton used the size of the parts of the "object" when working with the bottle representation in Activity 1(b): "...I drew it like this because that is smaller than this (pointing to parts of the bottle)".⁴

³ The name of this teacher has been changed for the purposes of this study.

⁴ It is interesting to note here that Ashton appears to be using the parts of the bottle to obtain a final image of the whole "object".

It should be noted with regard to the examples provided to illustrate strategies 3 and 4 that Ashton had completed activities 1(a), 2, 3 and 4 before the data on Activity 1(b) was gathered. Owing to her absence from school, Belinda completed Activities 3 and 4 before Activity 1(a). This suggests that learners might resort to the more "abstract" strategies when they have had more practice with "spatial" activities.⁵ As noted in section 4.1.4 Bishop (1989 : 11) has, in fact, suggested that increased facility with "spatial" tasks might result in a learner moving away from visual methods. He stresses that the strategy used by a learner on a task depends on, among other things, memory of visualisations and the ability to recall or generate appropriate visualisations.

But it should be noted that Trevor completed the activities in the sequence as originally planned and this was his first experience with "spatial" tasks. Furthermore, like Belinda, Gaylene completed Activities 3 and 4 before Activity 1(a), but there was no evidence of the use of strategies 3 or 4 by this learner. It certainly would be interesting to explore this further: What determines whether a learner is able to use these "abstract" strategies?

Analysis of the responses suggests that four of the seven learners adopted different strategies for the different items, that is, the different "objects":

- Trevor used strategy 2 for the cereal box and the tin of beans, but used visual memory (strategy 3) to answer the questions on the tennis ball and strategy 4 for the factory in Activity 4
- Gaylene used strategy 1 for the cereal box, but strategy 2 for the tin of beans and the sandwich
- James used strategy 1 for the toilet roll, but strategy 2 for the egg
- Belinda used strategy 1 for the tennis ball, but strategy 3 for the teacup.

Furthermore, Ashton appears to have combined strategies to solve one problem: For example, he appears to have used both strategies 3 and 4 when responding to the bottle representation:

(Referring to the bottle representation Activity 1(b))...but Miss the other day said that a round object never changes, except the words on the object...Then they say a round object when you look from above... knew that because one day they had a container on the board, so the container looked like a bottle and I drew it like this because that is smaller than this (points to parts of the bottle)...

His comments when completing the suitcase item suggest that he used strategies 1, 3 and 4! The use of a combination of strategies such as this was not, however, evident amongst the responses of the other learners in the study. It would be interesting to consider whether something particular to the different "objects" determines what strategy or combination of strategies is used by a learner.

⁵ "Abstract" in this case is being used in the sense that the learner might not be relying on the manipulation of visual image in strategies 3 and 4 to the same extent as is the case when using strategies 1 and 2. Perhaps these strategies could be classified as "analytic".

It should be noted at this point that two learners used concrete "objects" when solving certain tasks (refer to Appendix 3, page 102). Ryan, for example, looked at different parts of his pencil box to respond to the suitcase representation in Activity 1(b). When responding to the tin of beans representation, Trevor placed a toilet roll on his desk and looked at this. He indicated that the use of this "object" helped him in responding to the item. Such use of concrete "objects" in the solving of these spatial tasks is discussed further in section 5.10.

5.7.2 Two or More "Objects"; Response in the Form of a Drawing (Activity 3)

In contrast to the analysis of the above activities focusing on one "object" in which four strategies can be identified, only the use of two strategies, namely strategies 1 and 4 can be identified in performance on this activity.

The following explanations suggest that these learners visualised a movement in their own position relative to the "objects" (strategy 1) (refer to Appendix 3, page 109):

- Ashton: *So if you are looking from above, you should actually be pretending...a helicopter, whatever going, looking from above.*
- James: *I imagine I am a radar dome...but it gives you a view...round things...it walks around.*
- Ryan: *I think of a glass coffee table, and I am lying under the coffee table.*

Trevor's explanation suggests the use of strategy 4 (refer to Appendix 3, page 110): In explaining his drawing of the two boxes from "the side" he uses the relative size and the position of the "objects" to argue, "Miss, I am only drawing one box here, Miss, because if you are looking from the left this will be behind it. So I just need one box".

It is certainly necessary to gain additional data on activities such as this using two or more "objects" to determine whether additional strategies are, in fact, used in tasks of this nature.

5.7.3 Two or More "Objects"; Possible Solutions in the Form of Drawings Provided (Activity 2)

This activity deals with more than one "object" as does Activity 3, but in this case the use of all four strategies named so far can be identified. Examples of each are given here:

Strategy 1 (refer to Appendix 3, page 105):

Gaylene: *(She explains the position of different objects on her desk)...then you go and stand there and look from there...(she gets up from seat and moves position).*

Stuart: *I imagined...that I could see from that side and I could see only the handle of the cup.*

Strategy 2 (refer to Appendix 3, page 105):

Gaylene: *...turn the blank (plank) in that way you would see that way.*

Ryan: *I made like I turned it around...I turned the picture around.*

I pretended the picture was real. Then I turned it around (Written response).

Strategy 3 (refer to Appendix 3, page 106):

Ashton: *...when we did the cup in the other activity...so I used that...*

Strategy 4 (refer to Appendix 3, page 106):

Trevor: *...the plate is on the left side when you are looking at it...when you look at it from here it is on the right. So I say it is "B", Miss.*

...because the cup you can see is hiding away behind this because it is bigger and you can only see the ear (Written response).

James: *I saw that this, the plate will be on the right because that is opposite from here...because that will be on the right ...and this behind, the jug will be behind the flowers.*

The identification of the use of these four strategies is contrary to the expectation that demands placed on the learner by the required form of presentation in Activity 2 might influence the strategies used. But it seems that this form of presentation did

elicit the use of an additional strategy, as originally proposed in section 4.1.1. This strategy will be termed strategy 5 (refer to Appendix 3, page 106): Ryan appears to have used the given solutions (a) to (d) as a guide in determining his answer and was not necessarily using the manipulation of visual imagery:

First I thought number one was (c) because I looked at it from this side and so I saw the handle and then I looked at (b) and so I saw this side.

As was the case with Activities 1(a) and 1(b), it appears that some learners used a combination of strategies: From the responses it seems that Gaylene used both strategies 1 and 2 and that Ryan used strategies 2 and 5 in coming to a conclusion in Activity 2.

5.7.4 Conclusion

With regard to the visualisation of objects from different perspectives, it can be concluded therefore that, when solving problems involving one "object" in "small-scale" space, learners in this study used four different strategies. The use of these strategies was not specific to particular learners or tasks and some learners used a combination of strategies.

The results regarding the use of two or more "objects" are more varied. In the case where the response was given in the form of a drawing, only two strategies were identified, but all four strategies (as well as an additional strategy) can be observed in the case where possible drawings are provided. Again, the use of the strategies varied across learners and tasks and in some cases combinations of strategies were used on one task.

The use of the different strategies on the specific activities is summarised in Table 2:

	One "object"; Drawing	More than one "object"; Drawing	More than one "object"; Possible drawings provided
STRATEGY 1 (mental movement of position of observer)	x	x	x
STRATEGY 2 (mental movement of "object")	x		x
STRATEGY 3 (visual memory)	x		x
STRATEGY 4 (short argument)	x	x	x
STRATEGY 5 (using given answers)			x

Table 2: Summary of the Strategies used in "Small-scale" Space

5.8 Visualising "Objects" From Different Perspectives - "Large-Scale" Space

The analysis of performance on activities 5 and 6 reveals less variation in the use of strategies by the learners than on those activities that explore "small-scale" space as discussed above. Furthermore, there appears to be some overlap with the strategies identified in performance in "small-scale" space.

5.8.1 Map of the Classroom; "Known" and "Seen" Space (Activity 5)

The analysis suggests some consistency in the use of a particular two-part strategy by the learners (strategy 6). The use of the first part of the strategy can be identified in the responses of all the learners, but the second part is more difficult to identify with only three learners actually articulating the mechanics of this process.

Strategy 6(a): The learner looks at the furniture around him / her to obtain information on the relative position, and in some cases the shape of the "objects" in the classroom (refer to Appendix 3, pages 115 and 116).

In fact, all the learners in this study turned themselves around in their seats to view the classroom. (refer to Appendix 3, page 117)

These learners explained how they viewed the scene from their seats:

Researcher: *How do you know that this is what it is going to look like?*

Ashton: *If you look from here you see the desks.*

Researcher: *How did you know that this is what the classroom was going to look like from above?*

James: *I just looked at each desk, each at a time*

Researcher: *Okay, Ryan, can you tell me how did you know that is what the classroom was going to look like?*

Ryan: *The cupboard doors are open...(he matches his drawings with the furniture in the room).*

Researcher: *How do you know if you are sitting here that the desk is going to look like that?*

Ryan: *I saw it first...like ours...so I draw it the same. It was like this because it is the same like ours. I know how ours looks...that desk is the same as ours...and that other desk is the same as ours.*

This strategy is not without its problems, however, for Trevor, Ashton, Ryan and James had problems with the relative positions of the items of furniture in their drawings, either allowing too much or too little space for the different "objects". In most cases a number of attempts were made before each was happy with his drawing (refer to Appendix 3, page 116). These struggles could, perhaps, be linked to problems with drawing skills rather than an ability to visualise the scene from above.

Only Ryan actually tried to change the **angle** at which he looked at the classroom (refer to Appendix 3, page 117): He climbed onto his chair to view the classroom, whereas all the other learners stayed seated in their chairs. Thus Ryan seems to have used both his view from his seat and that obtained by standing on his chair to draw the aerial view. This behaviour is consistent with his earlier attempts at tasks related to "small-scale" space in which he began by using a "concrete" object to assist with the visualisation process.

Trevor's explanation of how he started out suggests an additional aspect of strategy 6(a) (refer to Appendix 3, page 116): He appears to have begun by establishing a reference point in the scene and then constructed the map from this point: He commented, "Okay, let's take it from this corner...there is our desk". Later he was having problems fitting in the "objects" in the drawing and changed this reference point: "Let me start in that corner".

Strategy 6(b): It appears that once a learner has obtained information on the relative position and size of the "objects" in the scene, s/he must still be able to visualise what these "objects" look like from above. Three learners made specific reference to their mental processes in this case, suggesting that they use strategy 1 mentioned in section 5.7.1, that is, the learner visualises a change in his / her own position in relation to the "objects" (refer to Appendix 3, page 116). For example,

Trevor: *Most of the time I see from the ceiling, from directly above...because I would have seen like that side of the bench if I'd had to look from here. I was looking **directly** from above.*

Gaylene: *(To Trevor): If you are looking from a helicopter above, there are some bits you won't see like the edges of the pinboards. And you can like picture yourself in a helicopter, Miss, and then you can see like down, Miss.*

Gaylene: *(She sees a problem with the way Belinda has drawn the windows) ...it depends where Belinda stands. If you are above, Miss, and you are looking straight down, then you can see...you can only see like the edges...and she first stands **on** the roof like this, and looks down like this, she will see that, but if she stands **on that side**...*

Belinda explains how her picture differs from Gaylene's drawing (Belinda has drawn some of the window panes):

Researcher: *How do you know that the board is going to look like that?*

Belinda: *I'm not standing too far, Miss.*

Belinda: *Only from the other side when you are looking...downwards, Miss.*

Gaylene was actually able to articulate the need for and the difference between strategies 6(a) and 6(b):

So when you are sitting here you can see where the desks are situated, Miss...you can also see like the size of the desks...If you look from above you can't see like the size.

5.8.2 Map of the School Grounds; "Known" and "Unseen" Space (Activity 6)

As was the case with the strategy noted in section 5.8.1, the strategy identified in performance on this activity is consistent across learners. One aspect of this strategy does in fact, correspond with parts of strategy 6. The method used on Activity 6 depends on whether the particular section of the space is "seen" or "unseen", that is, whether the learner can view the "objects" in the scene from his / her own position in the scene or not. There is a wealth of data in both these instances to suggest how the learners identify the relative position and size of the "objects" in the scene, but as was the case with Activity 5, there is little in the data to indicate how these "objects" are actually visualised from above.

Strategy 7(a): This is used for the parts of the school grounds that the learner can see from his / her own position in the scene, for example, by looking out of the classroom window: In this case s/he looks around him / her to obtain information on the relative position and size of different parts of the scene viewed (refer to Appendix 3, page 120). This corresponds with strategy 6(a) described above.

For example,

Researcher: *...so what do you do if you can't see the whole school, how do you know to draw that?*

Belinda: *pause...I can see some parts.*

Again, Ryan shows the need to move physically: He goes out of the classroom, stands in the quad and looks around him:

Researcher: *Okay, does it help you to go outside? And have a look?*

Ryan: *Yes, Miss, now I know where's the boys toilet, and where's the field and ...*(uses his hand to point to the building).

Researcher: *How did you know it was going to look like that? What did you think about?*

Ryan: *I saw from here...the roof is...and went, I went outside...I saw all the roofs.*

James also went outside:

Researcher: *What did you look at when you went out?*

James: *I wanted to see if the girls' toilet is in the same length as the end of the admin block.*

Researcher: *Is in the same...?*

James: *Has the same **length**...ends at the same place.*

Stuart claims he looked out of the classroom window:

Researcher: *How do you know the roof is going to look like this from the top?*

Stuart: *Well, when I came this morning...I could see through the window thatI could like see like the window...*

As was the case when completing Activity 5, the learners still had problems with the relative positions of the different parts of the scene (refer to Appendix 3, page 121): Talking aloud as he drew, Trevor exclaimed, "Oh, no! There won't be enough space for the garden!" Trevor also established a reference point before starting as he had done in Activity 5: He began by establishing where North is and marked this direction on his page (refer to Appendix 3, page 122).

Stuart's explanation above suggests that he recalled the view of the classroom he had seen earlier in the day. This suggests the second strategy described below.

Strategy 7(b): The learner uses visual memory for the parts of the school grounds that s/he cannot see directly from his / her position in the classroom (refer to Appendix 3, page 121).

These comments suggest that these learners are relying on visual memory:

Researcher: *How did you know the admin block looks like that?*

Ryan: *The classroom...when I still was in std...sub B, so I was the class over here (he points to classroom in corner of block)...and I could see the admin block from there. I...I was upstairs.*

Researcher: *How do you know it is going to look like that?*

Trevor: *How do I know, Miss...I am in grade 6...it is six years now that I have been in this school. Every day, Miss...I've seen it so many times.*

Researcher: *How do you know, though, that the office is that shape? (Repeated)*

Trevor: *Oh, that shape? Miss, I walk past the office when I help a teacher... and I look at the shape.*

Researcher: *So what do you do if you can't look around, if you can't run around the school?*

Trevor: *Oh, Miss, what I do is that I think hard, Miss, how the school looks, all the different buildings. I picture it in my mind, Miss, and I imagine that I am in a helicopter or something and I am looking down on the school.*

Researcher: *If you can't see something you have to draw from where you are sitting, how do you know what to draw? How do you know to draw the classrooms like that shape?*

Trevor: *I've been there before.*

Trevor's reference to the helicopter here suggests that he is using strategy 1 to visualise the scene from above, that is, he is picturing the movement of his body in relation to the scene.

Belinda also suggests that she is visualising a movement in her own position:

Researcher: *How do you know that this is what the school is going to look like?*

Belinda: *I go to the places and I draw from there.*

Researcher: *How do you know that it (the office) will look like that?*

Belinda: *I go there and see the shape.*

These are the only two comments that give some indication that the learners are visualising some movement in their own position (refer to Appendix 3, page 121).

Ashton appears to have used a combination of strategies 3 and 7 (refer to Appendix 3, page 121): It appears that he is using visual memory of the scene, but his reference to other lessons at school and other discussions suggests that he is also using strategy 3, that is, he is relying on his memory of similar activities and earlier discussions:

Researcher: *...it is the roof, why have you drawn it like that?*

Ashton: *Because the roof comes like that...yesterday when Ryan told us like that...*

Researcher: *How do you know that is what it is going to look like from above?*

Ashton: *Because, Miss, we done this in English already.*

Researcher: *How did you know when you did it in English?*

Ashton: *The place where the school is...the place where the school is determines how it looks from the top. And you can see from the quad and standing on the soccer field...you can see that.*

5.8.3 Conclusion

The analysis of performance on the two tasks in "large-scale" space indicates that there is some consistency across learners in the use of strategies as well as some correspondence with the strategies used in performance in "small-scale" space.

For the parts that the learner can see from his / her position in the space (whether "seen" or "unseen"), s/he appears to look around from his / her position in the space to obtain information on the shape and the relative position and size of the "objects" in the scene. S/he then visualises what the scene looks like from above.

For the parts that the learner cannot view directly, s/he appears to rely on visual memory of when s/he **was** able to see the scene in order to obtain information on the shape and relative size and position of the "objects" in the space.

Although there is a wealth of data suggesting how the learners identify the relative positions and sizes of the parts of these scenes in both "seen" and "unseen" space, there is certainly not sufficient information to draw conclusions as to how the learners actually visualise what these parts actually look like from above. The responses of three learners on Activity 5 suggest that strategy 1 might be used. On Activity 6 one learner appears to have used strategy 3, while two learners appear to have used strategy 1. But there is clearly a need for more research into this particular aspect of the use of strategies in Activities 5 and 6.

Interestingly, Trevor, Ashton and Ryan commented on the conventions associated with drawing an aerial view: When completing Activity 5 Ashton and Ryan had the following discussion:

Ashton: *...must we draw it like...the plan.*

Ryan: *...a floorplan.*

Ashton: *Yes, a floorplan.*

On Activity 6 Trevor commented, "I think I am going to make everything blocks, Miss...not its normal shape, like a plan, Miss".

These comments suggest that the learners have a knowledge of these conventions and can make them explicit (refer to Appendix 3, pages 118 and 122).

5.9 Facility with Two- and Three-Dimensional Objects - The Case of Nets (Activity 7)

The performance on this activity suggests the use of three different strategies.

5.9.1 Mentally Folding and Unfolding

I begin by discussing two strategies that rely on the use of mental manipulation of visual imagery. The use of these two strategies does not seem to be linked to the format of the question, as originally suggested.⁶

Strategy 8: The learner begins with the net and mentally folds it to form a cube. This was evident in descriptions in which learners described how they would fold up the net to make the box. Strategy 8 was identified in the responses of Trevor, Belinda, James, Ryan and Gaylene (refer to Appendix 3, pages 124 and 125).

For example:

Researcher: (asking about number 3): *And this one? Can you explain to me how did you work that one out?*

Trevor responds straight away by referring to different parts of the net:

Trevor: *...That one goes up and that's on the side like this...and then you fold it over so that's on that side, that one on top and that one's on the other side...and that one is there.*

Researcher: *How did you know what to draw there? How did you work it out?*

Trevor: *...Because same like this here...that one is open, two sides...it must have on the sides, because you put that one there so it works out...that one has to be there so you make a box.*

⁶ In question 3 of the activity the learner was shown one possible net of a cube and required to draw additional nets. In question 4 the learner was provided with a drawing of a rectangular prism and required to draw the net of this "object".

Explaining her response to number 3, Belinda begins by showing how the squares in the net fold up:

Researcher: *Okay, where did you start? Did you start by just drawing the squares? Or what did you picture when you started off? Or did you picture...?*

Belinda: *I start with this (pointing to the net) and then fold (she shows how she started with the "cross" and then found that something "was missing").*

Referring to number 3, the researcher asks Ryan:

Researcher: *What did you start thinking about?*

Ryan: *(Pointing to the net): I started here by this...and you put this over...then I have two sides...those are the flaps here...*

Researcher: *Okay.*

Ryan: *...I fold these two closed...and then I bring this up.*

He explains for two other nets he has drawn.

Researcher: *Did you start with a square and then add the other squares on?*

Ryan: *...I, I drew, the...the shape.*

Researcher: *Okay, which shape did you draw? Can you show me?*

Ryan: *This...cross...*

Researcher: *You drew the outline, and then you put the squares in?*

Ryan: *...just the lines.*

Strategy 9: The learner begins with the three-dimensional "object" and mentally unfolds this to get a picture of the net (refer to Appendix 3, page 125).

Gaylene explains how she went about number 3:

Researcher: *Okay, how did you picture what was going to happen? How did you picture it?*

Gaylene: *...Miss, I pictured the box like there...then you like take it apart*

Researcher: *So you picture the box and then you are going to take it apart?*

Gaylene: ...and I will see it like open.

Ashton explains quite clearly how he took the box apart in number 4:

Researcher: *Okay, so did you start thinking of the actual box? The three-dimensional box?*

Ashton: *And so I just flattened it...*

Researcher: *...and then you flattened it...*

Ashton: *Yes.*

Researcher: *Can you tell me what you were doing when you were flattening it? Can you tell me what you saw in your mind?...when you were flattening it?*

Ashton: *I saw that there was a box...but then I took like the parts of the box...parts...and then there was still another piece on the bottom so I took another piece...and there was still another piece on the side over there...so I took that one out as well. And there was a top flap on the side over there that I took out like that...*

Researcher: *Can you explain this one to me here? Did you do it the same way?*

Ashton: (emphatically): *Yes!*

Interestingly, the use of the different strategies does not appear to be specific to questions 3 or 4. Ashton's comments suggest that she used strategy 8 for question 3 and strategy 9 for question number 4. This might be expected when considering the form of presentation in each question, for example, if one possible net is given to a learner, this might encourage him / her to begin with a net and fold it up to form a box. But this does not seem to be the case with other learners. For example, James used strategy 8 for both questions. Gaylene appears to have used both strategy 8 **and** strategy 9 on question 3 of the activity.

It is also interesting to note in the responses that a number of the learners described how they would "fold" the different parts of the net or "unfold" the parts of the box: This suggests that they are working with "parts" of the "whole" in the net.

5.9.2 Generalisation

Although both Ashton and James appear to have used the two strategies mentioned above, their responses and conversation suggests that they are able to develop their thinking further by beginning to generalise the possibilities (refer to Appendix 3, Page 125). In such a case they might begin to rely less on the visual images (this strategy will be termed strategy 10). Ryan also contributed to this discussion.

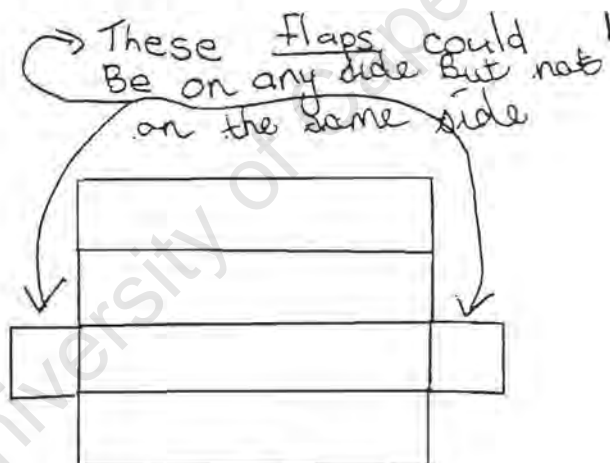
Ashton has, for example, noted that the side flaps (what he called "flaps") could be placed in a number of different positions:

Ashton: *These flaps can go anywhere...these side flaps...you can put them anywhere, actually...and it will work. But you must not put them on the same side as the other one...on a different side...*

Ryan: *...or else the other side will be open...*

Ashton: *...or these will overlap on this side and the other one will be open.*

Ashton also indicated this on his drawing:



James has also noticed a pattern:

James: *There's millions of different ways that can go there, there and there (he points to the possibilities on his drawing)...that can go there, there and there. They all work.*

Researcher: *Why can't you have the flaps on the same side?*

James: *If you have the flaps on the same side then it...but it doesn't matter if it's same together...because it will go with two flaps. But it doesn't matter, but one thing that matters is that the other side will be open.*

5.9.3 The Convention of a Net

It should be noted that both Cindy and Belinda had difficulty getting started on this activity. Belinda actually appealed to me for help (refer to Appendix 3, page 125). In response to this I encouraged the learners to cut and fold the given net. After having done this once, Belinda was able to proceed with ease with the activity. This was not, however, the case with Cindy as mentioned in section 5.6. Belinda's response suggests that there is a difference between understanding a net as a convention for representing a three-dimensional "object" in two dimensions and being able to visualise the change from one format to the other. The latter aspect requires the manipulation of visual imagery.

5.9.4 Conclusion

The analysis suggests that the learners used one or both of the two strategies on the two questions, but two learners (and possibly a third) started to generalise the results they had originally obtained using strategies 8 or 9. It certainly would be interesting to explore what precipitates such a process of generalisation on the part of a learner.

5.10 The Role of Physical Manipulation⁷

The use of concrete "objects" by Trevor and Ryan to assist in responding to certain spatial tasks was described in section 5.7.1. Reflection on the data from performance on all the activities used in this study suggests that learners use physical manipulation for a variety of purposes. These can be classified as follows:

- (a) Physical manipulation is used to begin the process of responding to the task: For example, when working on Activities 1(b) and 3, Ryan used his pencil box to begin the process. Once he had had a good look at this "object", he paid no more attention to it and, it seems, was able to proceed using mental imagery (refer to Appendix 3, pages 102 and 110). This was also the case when drawing the aerial views in Activities 5 and 6: For the map of the classroom he climbed onto his desk to look around, and to begin the school plan he asked if he could leave the classroom and went to look at the school buildings from the quad. James also asked to go outside when starting out when starting out on this activity (refer to Appendix 3, pages 117 and 122). When working on Activity 7, Belinda made several incorrect attempts before asking for assistance. Once she had cut and folded the one net provided, she was able to visualise the possibilities for questions 3 and 4 accurately and was able to describe her thinking (refer to appendix 3, page 125).
- (b) Physical movement is used to assist with the explanation: When solving Activity 2, Gaylene did not display any need to use concrete "objects" and appeared to have solved the task using the manipulation of visual imagery. But when explaining to Belinda and me she arranged actual "objects" on her desk and moved position in relation to these "objects" (refer to Appendix 3, page 106).

⁷ "Physical Manipulation" is used in this context to refer to movement of the position of the learner or to the manipulation of an "object" by the learner.

(c) Physical manipulation is used as a means of checking the visual methods. Trevor did not display any need to cut and fold the net while quickly completing the questions in Activity 7, and appeared to have completed the task using the strategies described in section 5.9, but he decided a number of times to cut and fold to check his final solution. After creating a net using five squares, Trevor added a sixth square:

Trevor: *Okay...I'm going to test this. (he starts to cut out.) I never counted this, Miss,...it was six all the time.*

Researcher: *When did you realise that you had made a mistake, Trevor?*

Trevor: *When I...when I tried to build it.*

The use of this checking mechanism seemed to be very useful as, in a number of cases in this particular activity, Trevor rapidly noted where he had made an error and was able to adapt his response accordingly.

It should be noted in this context that, in the case of Cindy the use of physical manipulation did not seem to assist her in the process of solving the tasks or in communicating the visual processes being used: As mentioned in section 5.6, it does not appear that the use of the actual "objects" in Activity 1(a) or the cutting and folding of the given net in Activity 7 assisted Cindy in responding to the task (refer to Appendix 3, pages 110 and 125). At this stage one could ask how much of this work with physical "objects" might be necessary before any change is observed in her performance.

Furthermore, one learner displayed no need for physical manipulation. Referring to Activity 7, Ashton claimed that it was not necessary to cut and fold because, "...you can use your imagination". It appears that all his solutions to the activities were acquired using visual imagery (refer to Appendix 3, page 125).

This classification of the different uses of physical manipulation supports the suggestion made in section 3.4 that the relationship between interaction in physical space and the use of visual imagery might not simply be a development from the former to the latter. Rather, this analysis suggests that there might be an interaction between the different aspects: It appears that the physical manipulation might support the use of visual imagery in different ways and that learners with well-developed "spatial ability" might use physical manipulation most effectively in problem solving.

5.11 Consistency in the Use of Approaches

When analysing the data, an attempt was made to keep track of the approaches used by learners on the complete set of activities (see Appendix 3, pages 103, 107, 111, 113, 118, 123 and 126). The profiles constructed indicate that certain “trends” can be identified in the way these learners respond to “spatial” activities. Some of these are described below:

- **The Ability to Communicate Mental Processes:** As noted in section 5.6, seven of the eight learners in the study could consistently reflect on their mental processes. But one learner was unable to communicate her thoughts verbally or in the form of drawings.
- **The Use of Physical Manipulation:** As noted in section 5.10, certain learners consistently used physical movement in a particular way to assist in problem solving. For example, when working with “small-scale” space Ryan consistently used concrete “objects” to initiate his exploration of the task. In the case of “large-scale” space he had to physically move his position in relation to the scene before starting to draw.
- **Left / Right Discrimination:** Trevor consistently displayed the ability to discriminate between left and right when responding to the tasks. Furthermore, he consistently displayed careful attention to the relative size and shape of the “objects”.
- **Responses to the Work of Others:** Gaylene and Ashton displayed the ability to reflect on and challenge the responses of their classmates.
- **The Use of “Abstract” Strategies:** Although both Ashton and Trevor did use strategies which rely on the manipulation of visual imagery, it is notable that they both resorted to strategies such as using visual memory or short arguments using the shape and size of the “objects”. They are also the learners who began to generalise on the net activity.

There is certainly scope for more research in this regard. Interesting aspects to consider are:

- Why is Trevor able to use “abstract” strategies?
- At what stage will Ryan no longer require the use of physical “objects” to assist in problem solving?
- At what stage will Cindy begin to communicate her thoughts more efficiently?

Chapter 6: Conclusion

This final discussion will be divided into three sections:

- Comments on the methodology used in the study as well as recommendations for future research of this nature
- Revision of the preliminary model of "spatial ability" on the basis of the results discussed in Chapter 5, and suggestions for further research
- A discussion of the relevance of the results of this research for my work in mathematics education.

6.1 The Methodology

In this section problems as well as advantages associated with this particular data collection method will be discussed.

6.1.1 Using Verbal Reports and Drawings as Data:

As this study aimed to identify the strategies used by learners to solve particular "spatial" tasks, it was necessary to attempt to gain access to the mental processes used by learners. In Chapter 5 problems associated with the decision to collect data in the form of drawings and verbal responses were discussed. The problems associated with the analysis of data in these two forms are clearly illustrated by the case of Cindy. For it is not possible to determine from the data whether there was an absence of a mental picture or whether Cindy did have a mental picture, but was not able to communicate these in the form of drawing or a verbal description.

The possibility that intermediate processes may intervene between the mental processes and the expression of these two types of response is an enduring problem in research of this nature. It must be acknowledged that in some cases my reference to particular mental strategies when questioning the learners or the instructions provided by the teachers may have influenced learner responses.

Furthermore, the fact that drawing as a means of communication requires knowledge of certain conventions has been noted. In all of the activities used in this study (excluding Activity 2), drawing was used as a mediator for the spatial visualisations of the learners. This can be problematic in the case where a learner might be able to form the necessary visual images, but has not yet developed the required drawing skills or learnt the required conventions, as is possibly the case with Cindy. There is certainly room for further research into the relationship between "spatial ability" and drawing skills.

It should be acknowledged, however, that these possible problems were taken into account in the data collection and analysis processes:

- (a) In instances in which it was felt that my question/s or intervention by the teacher might have influenced the responses, these responses were excluded from the analysis.

- (b) The fact that I was sitting with a small group of learners while they completed the activities provided me with the opportunity to question them about their drawings in an attempt to establish some shared meaning. As noted in section 5.3, it was these discussions, together with verbal data gathered during pupil-pupil and researcher-pupil interaction that were used in the analysis.
- (c) As a result of my inability to establish meaningful communication with Cindy, the data related to her responses was not taken into consideration in the analysis.

6.1.2 The Draft Nature of the Activities

The decision to collect the data in the form of drawings and verbal responses is related to the fact that the activities used in the study were in draft form. The first trialling of the Malati "spatial skills" activities of which this study formed part was, in fact, the first time I had dealt with the field of "spatial ability". Although the literature survey had, as discussed in section 4.1, alerted me to problems associated with various forms of data collection, I was certainly not aware of the extent to which problems with verbal communication and drawing might interfere in my attempts to gain access to the mental processes. It was only when I was able to closely monitor the use of the activities in the classroom that I realised the complexity of the process. It certainly would have been more appropriate to conduct a pilot with the activities prior to the gathering of the data for this study.

I would certainly support the suggestions by Bishop (1989 : 11) and Horn (as quoted by Johnson and Meade, 1987 : 725) that "spatial ability" be measured using a range of tasks. One could, for example, include activities that require responses in the form of drawings, verbal descriptions, physical demonstrations and by selecting answers from given possibilities.

In section 5.7.2 it was noted that only one activity related to the visualisation of two or more "objects" and requiring a response in the form of a drawing was included in this study. This was certainly an oversight which possibly prevented me from obtaining clarity on whether the number of "objects" influences the strategy used. Further data of this nature thus needs to be obtained.

Owing to the draft nature of the activities, some learners did have problems interpreting some aspects of certain activities. For example, in Activity 2 Ashton admitted that he was confused by the direction in which the "eyes" appeared to be pointing. In Activity 3, a number of learners queried what it meant to view the "objects" from points A and B. However, the fact that I was sitting with the learners when these problems were encountered meant that I was able to clarify the aims of the activities where necessary.

6.1.3 Collecting Data in the Classroom Context:

The decision to collect the data for this research in the classroom context did have its problems in that

- (a) In one instance a learner appeared to have 'mimicked' responses and copied the written responses of her classmate. These responses were not, however, taken into account in the analysis.
- (b) In one instance a teacher used one of the research activities without my knowledge and an additional activity of this type, Activity 1(b), had to be designed for use with this class .
- (c) Three learners were absent from school at different times during the data collection process and I had to work with them on an activity at a later date and not in the original group context.

As a result of (b) and (c) some learners did the activities in a different sequence to that originally planned. Different learners had thus had different experiences with "spatial" activities when completing the same tasks. Such instances have been acknowledged in the discussion. This is certainly an important consideration when one notes Bishop's claim that increased facility with "spatial" tasks might lead to learners moving away from visual strategies (1989 : 11). But perhaps this is not such a problem in the light of the aim in this study to identify the different strategies used on particular topics: For the different experiences by the learners possibly revealed a wider range of strategies than might originally have been the case.

Furthermore, the fact that I had to work with these learners individually or, in one case in pairs, simply meant that I was not able to gather any additional data that might have been generated in the group discussions.

- (d) Having to follow four learners at one time during classroom activity meant that I was not able to question each learner on his / her strategy on every item. This is not, however, a problem in the light of how the data was used to identify the use of different strategies.
- (e) Some learners volunteered more information to me and initiated conversations with other learners more readily and frequently than others. Thus more information was obtained on some learners than on others.

It is felt, however, that the method of gathering data in the classroom context was useful in gaining some valuable information suited to the aims of the study:

- (a) The nature of the classroom culture in the selected classes meant that there was discussion amongst learners in the groups. This discussion was a source of additional and valuable information on learners' strategies without my having to question them. Certain learners engaged in constructive arguments with one another and after a short while openly shared their thinking with me. One should consider, however, whether this arrangement is, in fact, suitable for learners such as Cindy who is shy and has little confidence in her own ability. It appears that Cindy was

quite intimidated by her classmates and was possibly less likely to share her ideas.

- (b) As mentioned in section 6.1.2, I could assist in clarifying problems that arose owing to the fact that the activities were in draft form.
- (c) I could also address general questions related to the activities and clarify problems arising from the use of particular visual conventions and vocabulary used in the activities. For example, the requirement that a learner look at an "object" from above could be interpreted as from directly above or from an angle. In such cases we were able to discuss and demonstrate different possibilities within the group situation. In the case of Activity 7 in which knowledge of the convention of a net is necessary before visual processes can be explored, I was able to attempt to clarify the notion of a net for two learners.
- (d) I could question learners about their strategies where necessary.
- (e) The concern about learners having to use drawings as mediators for their learning was mentioned above. While sitting with the learners, however, I was able to question learners about their drawings in an attempt to establish some shared meanings.

In spite of the problems mentioned above, therefore, it is felt that this method of data collection, together with the adaptations suggested, is in fact appropriate for a study of this nature which is performed in the educational context.

6.2 A Model of "Spatial Ability":

This study has only focused on certain aspects of the model described in Chapter 3, these being,

- (a) What strategies are used by learners when visualising an "object" or "objects" from different perspectives? Do the strategies used differ if the size of the space differs? In the case of "small-scale" space, do the strategies differ if the number of "objects" varies? Do the strategies differ if the demands placed on the learner in terms of the presentation of the response are varied? In the case of "large-scale" space, do the strategies differ if the space is "seen" or "unseen"?
- (b) What strategies are used when learners are required to represent a three-dimensional "object" in two dimensions (in the form of a net)? Do different learners use different strategies? Do the strategies vary if the form of presentation of the activity is varied?

In this section the results of the empirical research will be used to comment on appropriate sections of the preliminary model. The model is summarised in Figure 2 below:

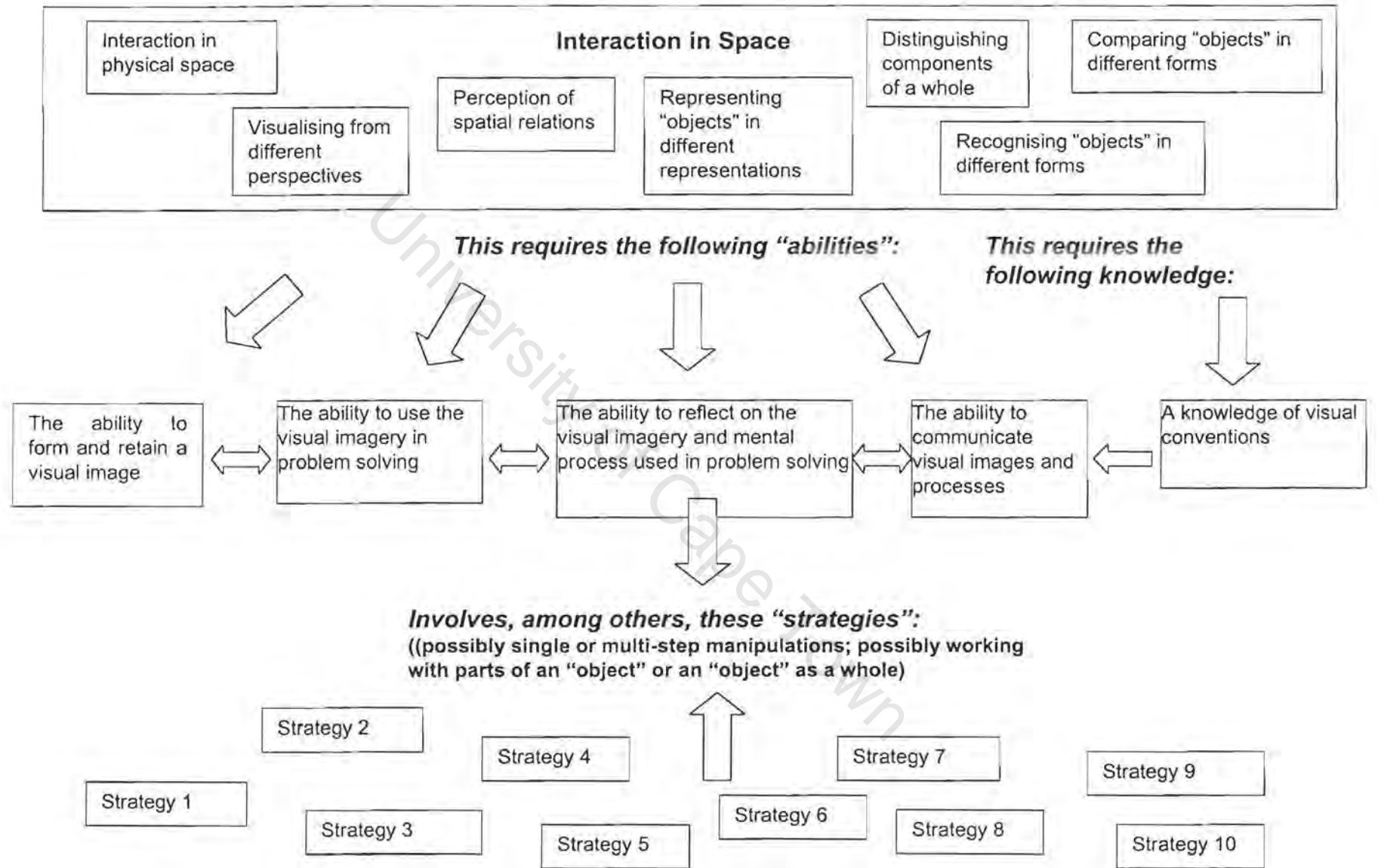


Figure 2: An Updated Model of "Spatial Ability"

6.2.1 A General Definition of "Spatial Ability" (refer to section 3.3):

As noted in Section 3.3, the following "abilities" were identified in the literature as necessary for interaction in space:

- (a) the ability to form and retain a visual image
- (b) the ability to use mental manipulations for problem solving.

After analysing the responses of the learners in this study, however, I would add a further two "abilities" (in no particular order), namely,

- (c) the ability to reflect on the mental thought processes used in problem solving
- (d) the ability to communicate visual images and processes to others in a meaningful way.

Related to (d) in respect of communication in the form of drawing is a knowledge of and ability to use the conventions for representing three-dimensional "objects" in two dimensions. The knowledge of these conventions is not being regarded here as an "ability", but if interaction in space requires that a learner can interpret visual imagery and communicate visual imagery meaningfully, it seems necessary to include it as one aspect of general "spatial ability".

This suggests therefore that "spatial ability" involves an ability to work with visual images **as well as** a knowledge of visual conventions. This supports Bishop's classification of "spatial ability" using IFI ("ability to interpret figural information") and VP ("ability for visual processing"). There is, however, certainly a need for further research into the relationship between "spatial ability" in general and the knowledge and use of drawing conventions.

I would suggest that a learner who has mastered the four "abilities" (a) to (d), and the associated visual conventions, could be considered capable of interacting efficiently in the "visual world", that is, in Senechal's words of "handling, causing and understanding changes in shapes" (in Hershkowitz *et al*, 1996 : 161-162). But what is the precise nature of such interaction?

Two aspects of interaction with the visual world explored in this study are:

- (a) Visualising an "object" or "objects" (in "small-" and "large-scale" spaces) from different perspectives
- (b) Representing an "object" in a different representation (in this case drawing the net of a three-dimensional "object").

Other aspects of interaction noted in section 3.3 are:

- (c) Interacting in physical space, for example, manoeuvring in a particular environment (finding one's way to and from school) (Del Grande's "eye-motor co-ordination")
- (d) Distinguishing specific components of the whole in a visual representation
- (e) Recognising "objects" in different sizes, shadings, textures, positions in space (Del Grande's "perceptual constancy")

- (f) Comparing "objects" in different forms (Del Grande's "visual discrimination")
- (g) Related to (a) and possibly a prerequisite for this is what Del Grande terms "perception of spatial relations", that is, the ability to see two or more "objects" in relation to oneself and in relation to one another.

At this stage these aspects are based simply on my reflection on the "spatial ability" literature: There is certainly room for more work on the actual classification of these different aspects of interaction in space.

6.2.2 Using Visual Images in Problem Solving (refer to section 3.4):

It has been noted that there is little consensus in the literature on this specific aspect of "spatial ability". The study of the strategies used by the seven learners considered in the analysis for this study has enabled me to obtain some clarity on this process of problem solving.

Learner performance on these seven "spatial" activities suggests that, in general,

- (a) Different learners use different strategies or combinations of strategies on the same task
- (b) One learner can use different strategies or combinations of strategies on a range of tasks.

These results support the suggestion made in section 2.6.1 regarding the idiosyncratic nature of performance on "spatial" tasks. Thus, rather than associating a particular strategy with a specific task as is done by some researchers, it seems more appropriate to identify **a set of strategies** used on a wide range of "spatial" tasks and needed for effective interaction in the visual world. As noted in section 2.6.1, Bishop (1989 : 11) has suggested some factors that might determine the type of response to a task, for example, a learner's preferences or memory of visualisations - it would be interesting to explore this in relation to the activities used and strategies identified in **this** study.

Some of the strategies required for interaction in space are described below:

(a) Visualising from Different Perspectives - "Small-Scale" Space

In this context the following strategies appear to be used:

Strategy 1: The learner visualises his / her own movement in relation to the "object".

Strategy 2: The learner visualises the movement of the "object" itself.

Strategy 3: The learner relies on visual memory of similar "objects" in another situation.

Strategy 4: The learner develops a short argument using the relative sizes or positions of the "objects" to explain the solution (not necessarily reliant on the manipulation of visual imagery).

Strategy 5: In the case where the learner must select from given solutions, the learner uses given strategies as a guide in determining a solution.

The following should be noted and considered in relation to the use of these strategies:

- Some learners appear to vary the strategy used when with working a variety of “objects”. Further research could be conducted into whether certain factors related to the nature of the “objects” used in the activities determine the strategy used.
- In this study strategy 4 was not used in the case where learners had to visualise two “objects” from different perspectives and provide the answer in the form of a drawing (although this strategy was identified in the activity in which possible solutions were provided). Unfortunately only one activity of this nature was included in the study. Further research using additional tasks is certainly required.
- The different demands placed on the learner in terms of the way in which the answer has to be presented (whether a drawing is required or whether a solution has to be selected from given drawings), does not appear to influence the strategy used. In the latter case, however, the presence of possible solutions appears to allow for the use of an additional strategy (strategy 5).
- It appears that strategies 1 and 2 might rely more on the manipulation of visual imagery than strategies 3 to 5. In this regard there is scope for further research on the extent to which visual imagery is actually manipulated in the last three categories. Furthermore, certain learners consistently made use of strategies 3 and 4 and it would be interesting to explore what determines whether a learner is able to use such strategies or why a learner uses such approaches.

(b) Visualising from Different Perspectives - “Large-Scale” Space

In the case of “seen” space a two-part strategy can be identified: Firstly, the learner looks around him / her from his / her position in the space to obtain information on the relative size and shape of the different “objects” (strategy 6(a)). Secondly, s/he then visualises what the “objects” in the scene look like from above (strategy 6(b)). No clarity was obtained in this study on the precise nature of the strategy / strategies used for this second part, but it appears that learners use strategy 1 as identified in “small-scale” space.

In the case of “unseen” space, on the other hand, the strategy varies according to what can be viewed from the learner’s actual position in the scene. For those parts that the learner can see, s/he uses strategy 6(a) as described above. For the “unseen” parts, the learner uses visual memory from when s/he could see the scene on a previous occasion (strategy 7(b)). S/he then visualises the parts of the scene from above. This is possibly done using strategy 1 or 3 described above.

Aspects to be noted in relation to the exploration of "large-scale" space are:

- There is clearly a distinction between the strategies used for the parts that the learner can see from his own position in the scene and those that cannot be viewed directly. This could be explored further by requiring the learner, for instance, to draw an aerial view of his / her home in which case s/he will be drawing from a position within another space.
- When it comes to visualising the parts of the scene in "large-scale" space from above, there are indications that aspects of the strategies used are the same as those used in "small-scale" space. More research is required into whether strategies 1 and 3 (and any others) are, as suggested, used in this context.
- Interestingly, when it comes to the exploration of "large-scale" space, the strategies used for determining the relative position and size of "objects" in the space appear to be used consistently across the seven learners considered. This is in contrast to the results in "small-scale" space which suggest that a greater variety of strategies is used. Of course a study using a larger sample would be required to determine whether additional strategies are, in fact used in "large-scale" space.

(c) Representing a Three-dimensional "Object" in a Two-dimensional Net

Three strategies can be identified in this form of interaction:

Strategy 8: The learner mentally unfolds the three-dimensional "object"

Strategy 9: The learner begins with the unfolded net and mentally folds this to form a three dimensional "object"

Strategy 10: The learner is able to generalise his / her initial responses (obtained using Strategy 8 or 9) to find additional forms of representation.

With regard to the use of these strategies, the following should be noted:

- (c) It does not appear that the form of presentation of the task (whether a net or three-dimensional "object" is given) determines the strategy used.
- (d) It could be said that, while strategies 8 and 9 are characterised by the mental manipulation of visual images, the learner does not appear to make use of actual mental manipulation in Strategy 10. This suggests a possible area for research in relation to the solving of "spatial" activities in general, and suggested in section 5.7.1, that is, does increased facility with activities of this type affect the strategies used by a learner?
- (e) It appears that when representing the "objects" in two-dimensional nets, a learner works with the parts of the whole object. There is certainly scope for the study of the use of parts of and whole configurations in other "spatial" activities.

6.2.3 The Development of "Spatial Ability":

The literature suggests that the development of "spatial ability" might occur in a particular sequence, from interaction with concrete "objects" to the manipulation of visual images for problem solving.

It was suggested in section 3.4, however, that the role of physical manipulation might be more complicated than indicated. The results of this study certainly suggest that physical manipulation can be used for a variety of purposes in the solving of "spatial" tasks, these being,

- Physical manipulation is used to begin the process of responding to the task, after which the learner relies only on visual images
- Visual imagery is used to solve the task, but relies on physical manipulation to assist with communicating a strategy to another individual
- Physical manipulation is used as a means of checking the responses acquired using visual methods.

The identification of these different uses of physical manipulation suggests that a learner with well-developed "spatial skills" does not discard the use of physical manipulation, but might put it to very effective use in problem solving. Thus a two-way interaction between physical interaction in space and the use of visual imagery is suggested. There is certainly room for research into these two aspects of "spatial ability".

It should also be noted that there were instances in this study in which

- the use of physical manipulation did not appear to assist in the process of solving the tasks or in communicating the visual processes being used, and
- no need to use physical manipulation was displayed.

In spite of my reservations concerning the sequential nature of the development of "spatial ability", it appears that, with the study of a learner's performance on a range of spatial skills, it might be possible to identify certain "trends" in a learner's approach to the activities. Whether these are in fact hierarchical, cannot be determined from the results of this study.

Some possible "trends" are:

- (a) The consistent use of physical manipulation for a particular purpose (possibly one of those mentioned above)
- (b) The consistent ability to discriminate or not discriminate between left and right when viewing an "object" or "objects" from different perspectives
- (c) The consistent paying attention to the scale and / or relative position of "objects" when solving "spatial" tasks
- (d) The consistent use of those strategies which appear to rely less on visual imagery than others
- (e) The consistent reflection on the representations of other learners and challenging them on these.

The comments made in this section are, of course, based on a small study with a sample of only eight learners. It is felt, however, that the initial identification of the different strategies in this study provides pointers for further research which could determine how widespread the use of these strategies is and consequently to the updating of the model of "spatial ability". Furthermore the analysis of the

data has suggested additional aspects of "spatial ability" which could be the focus of further research.

6.3 Potential Use of this Study in Mathematics Education

This study has in itself already proved useful for my work at Malati and it is felt that this could contribute to the work currently being done on the implementation of Curriculum 2005.

6.3.1 Revision of Malati "Spatial Skills" Materials:

Firstly, the reflection sparked by the literature survey and required in the building and then revision of a model of "spatial ability" has assisted in the redesign of the original Malati "Spatial Skills" packages. These are aimed at providing learners with the range of experiences thought to be part of this model of "spatial ability".

For example, when it comes to viewing "objects" from different perspectives an attempt has been made to vary the size and nature of the space explored, the number of "objects" in "small-scale" space, and the demands placed on the learner in terms of the form of presentation of the solution. Furthermore, the increased awareness of the need for the knowledge of and ability to use specific visual conventions resulting from this study has resulted in the inclusion of a set of activities specifically aimed at developing the necessary understanding and skills.

The process that I have gone through in undertaking this research and designing the Malati "Spatial Skills" packages confirms the suggestion made by Clements (1983 : 17) and noted in Chapter 1 that the development of "spatial" tasks needs to be accompanied by the development of methods which assist in the unambiguous identification of the strategies used by individuals on the tasks. Furthermore, it has become clear that before one can develop "spatial" activities, one needs some clarity on the notion of "spatial ability" itself.

Secondly, the reflection resulting from the opportunity to closely observe the use of these particular draft activities in the classroom has also been useful in the revision of the original activities.

6.3.2 Malati Curriculum Development:

One of the briefs received by Malati was to develop a "model" for curriculum development. One important aspect of our evolving "model" is that this development should be research based, that is, existing research reported in the literature as well as any additional research deemed necessary and conducted by Malati staff is used as a basis for the design of the materials. The process that I have undertaken in conducting this study is an illustration of the role of research in the curriculum development process (see Figure 3 below). This example will be used in the final write-up of the work conducted by Malati.

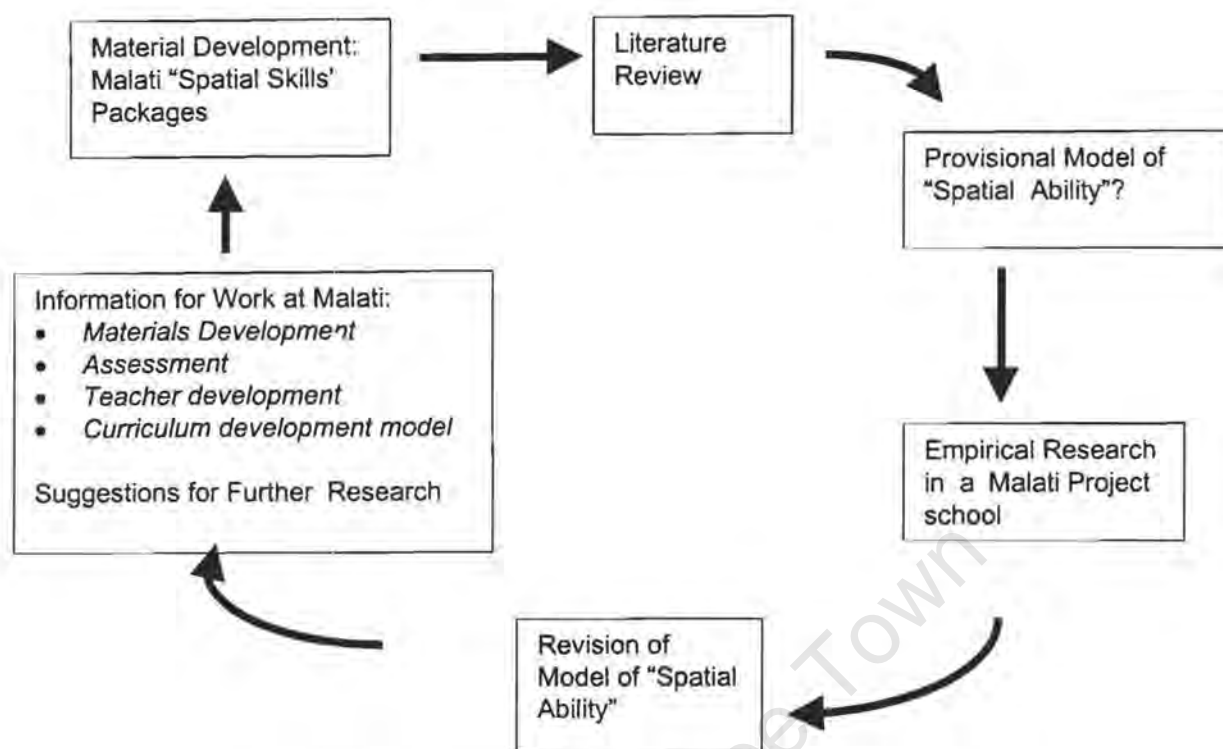


Figure 3: A Model of Curriculum Development

6.3.3 Assessment:

A year's work with Malati teachers when using the "Spatial Skills" packages has suggested that one of the most problematic issues associated with the implementation of this aspect of the curriculum is the area of assessment. How does one assess "spatial skills"? The forms of assessment currently in use at the Project schools do not seem appropriate.

An increased awareness of the different strategies used by learners resulting from this study has enabled me to compile an "assessment schedule" reflecting the different ways a learner might approach different tasks. This could be used in compiling an overall picture of a learner's "spatial ability" and in determining their needs. The use of this schedule has already been workshopped with teachers and Malati staff are currently providing support for its use.

Furthermore, the size of the sample in this study meant that I was able to compile a profile on each learner's performance on every activity and to identify "trends" in their performance. It is felt that these possibilities can be shared with teachers in assisting them to assess the needs of their individual learners. The construction of "profiles" could be useful in designing individual learning programmes for learners.

6.3.4 Teacher Development:

An important aspect of the Malati approach to teacher development is to encourage teachers to reflect on the strategies used by learners. An increased awareness of the variety of strategies used by learners has been useful in my interaction with teachers at the Malati Project schools as I have been able to alert teachers to these different approaches when they are observed in the classrooms. This is felt to be of particular importance in the case where a teacher struggles with the activities himself / herself: For it has been noted that in a case such as this, a teacher can encounter problems understanding and assessing the range of responses that arise in the classroom.

6.3.5 A Personal Response:

As a trained secondary school teacher with no experience in primary schools prior to beginning work at Malati in 1997, the opportunity to engage in detail with one aspect of the primary curriculum has been an exciting, enriching and challenging experience. The foundation established when undertaking the literature review and the engagement with the data in identifying the strategies used by learners has assisted me in establishing a basis from which I can pursue my work at Malati as a materials writer and a teacher developer with increased confidence. The study has also, as mentioned, suggested additional exciting areas for research in the field of "spatial ability".

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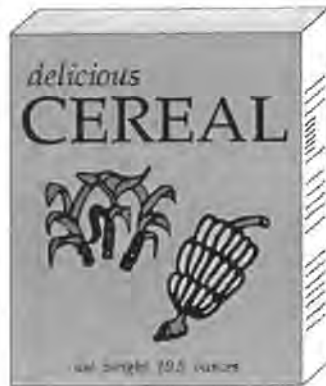
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APPENDIX 1: Learner Activities 1 to 7

ACTIVITY 1(a)

Draw What You See

1. Look carefully at this box:



- (a) Draw what you would see if you were looking at the box from **behind**.
- (b) Draw what you would see if you were looking at the box from **above**.
- (c) Draw what you would see if you were looking at the box from **the side**.

Now answer these questions for each of the objects shown below:

- (a) Draw what you would see if you were looking at the object from **behind**.
- (b) Draw what you would see if you were looking at the object from **above**.
- (c) Draw what you would see if you were looking at the object from **the side**.

2.



3.



4.



5.



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ACTIVITY 1(b)

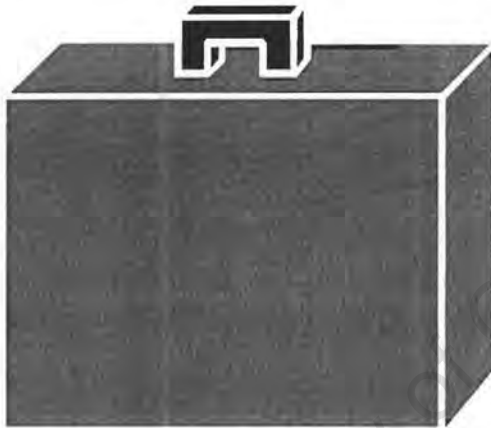
Draw What You See 2

Look carefully at each of these objects:

In each case:

- (a) Draw what you would see if you were looking at the object from **behind**.
- (b) Draw what you would see if you were looking at the object from **above**.
- (c) Draw what you would see if you were looking at the object from the **side**.

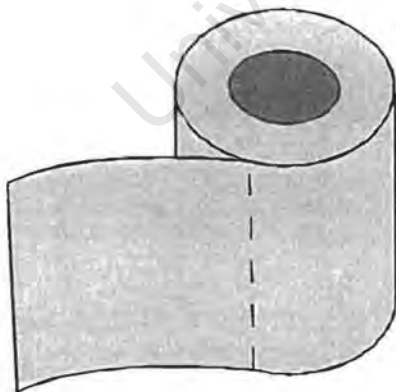
1.



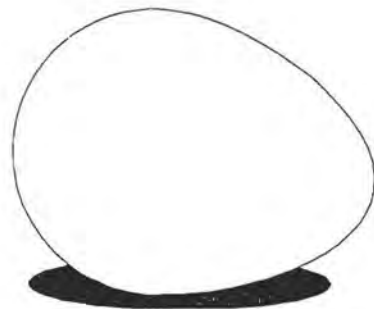
2.



3.



4.



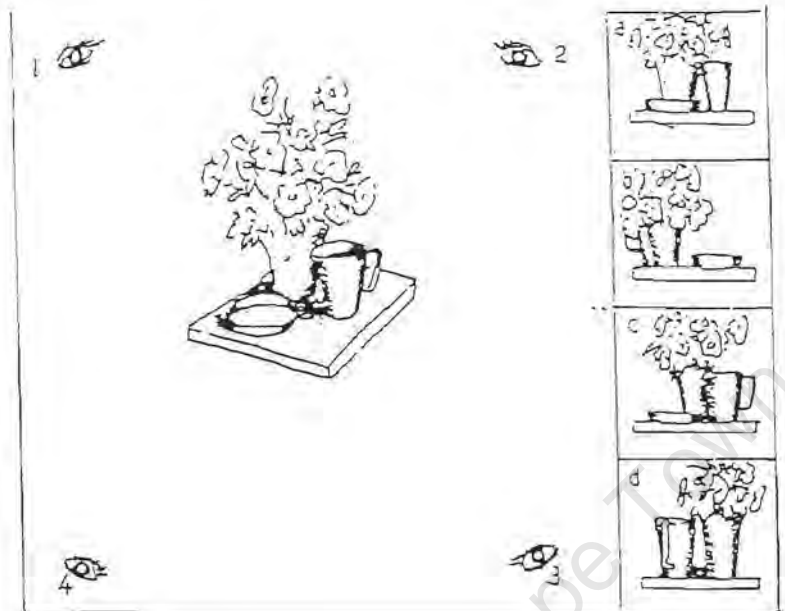
5.



ACTIVITY 2

What Do You See 1?

Which eye sees which picture?



Write your answer like this:

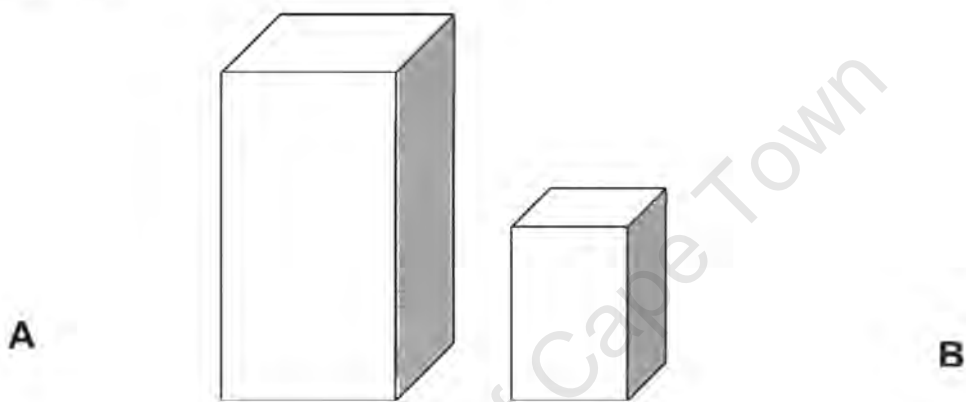
1. sees
2. sees
3. sees
4. sees

In each case, explain why you chose each picture

ACTIVITY 3

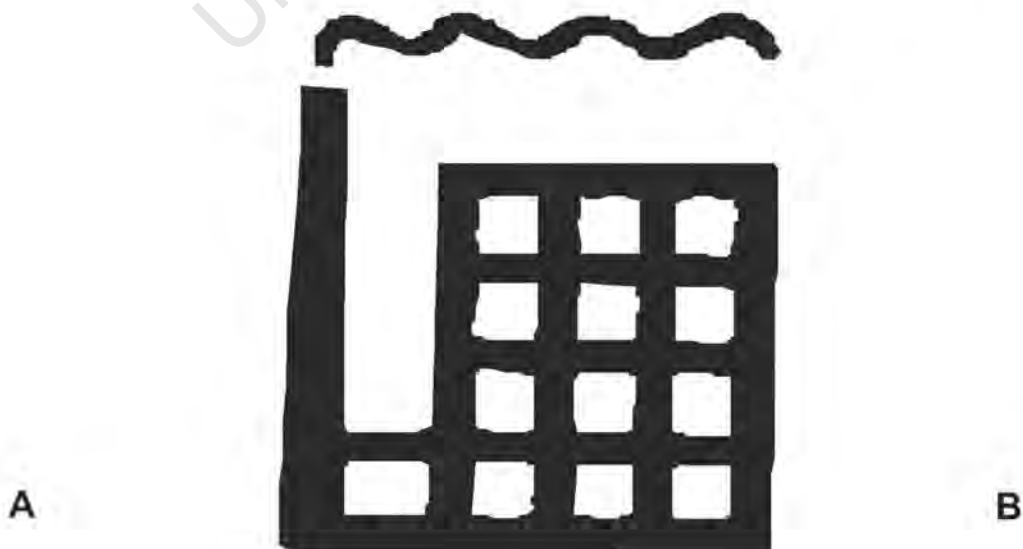
Draw what you would see if you were looking at these two boxes from

- (a) above
- (b) below
- (c) point A on the left of the boxes
- (d) point B on the right of the boxes
- (e) behind.



ACTIVITY 4

Now answer questions (a) to (e) above, this time using this picture of a factory:



ACTIVITY 5

Looking from Above (Teacher Document)

Instructions for the Activity:

Ask the pupils to imagine that they are looking at your classroom from above. Each pupil should draw a sketch of what s/he sees. Pupils should be encouraged to compare their sketches.

Notes:

This activity requires pupils to place themselves in a different position in relation to the classroom and to sketch the scene. They will have to visualise what the classroom looks like from above.

Further Activities:

The teacher might like to use as a short project, for instance, drawing an aerial plan of a different room or of the entire school or of the neighbourhood. Such an activity could also be used for assessment (see **Giving Directions**).

Pupils can also be required to draw scale plans of the classroom.

Source of Ideas:

Du Toit, D., Human, P., Murray, H. and Olivier, A. (1996) **Mathematics at Work, Standard 1** Cape Town: Nasou.

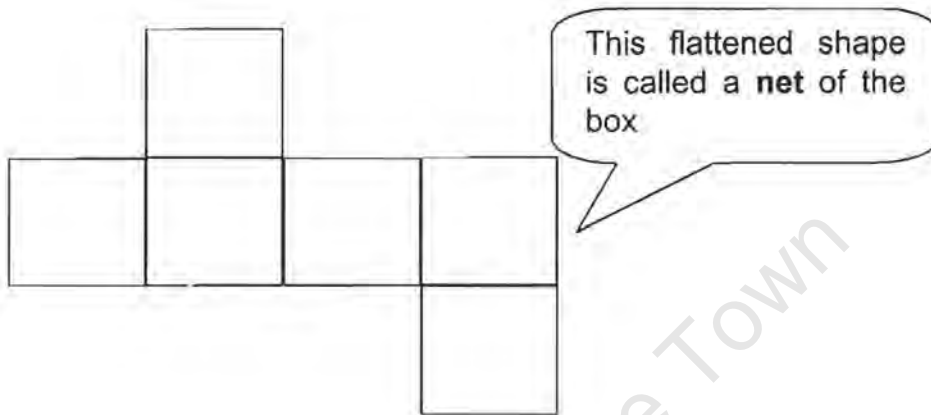
ACTIVITY 6

This refers to the "Further Activities" above. The teacher instructed the learners to draw what the school grounds would look like from above.

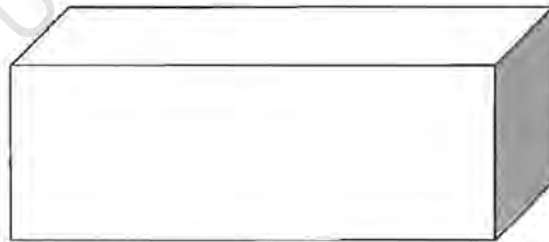
ACTIVITY 7

Flattened Boxes

When the staff at Pick 'n Pay have finished unpacking goods from boxes, they flatten the boxes so that they can be packed and easily transported to the recycling depot. This flattened box was found in the storeroom.



1. What figures is this net made of?
2. What kind of box does this net make? Describe the box.
3. Draw three different nets that will make the **same** box.
4. Pick 'n Pay also gets goods packed in boxes like this:



What will this box look like when it is flattened? Draw the net of this box.

5. Check your work in Question 4 by cutting out your drawing and fold it to make the box.
Will your paper stay together as a box? If not, add what you think you need.

APPENDIX 2: Teacher Information Sheet

Spatial Skills: Research

My aim is to explore the strategies used by the children in solving activities designed to develop spatial skills. So the focus is on the **pupils** and not on the teacher!

This is what I would like for the research.

One group of approximately four pupils in a class. This must be a group which is functioning well, that is, the pupils are talking to one another. I will need to collect the data by doing the following:

- Observing how they do the activities and writing down field notes as I go.
- Audiotaping all their interactions
- Making copies of their solutions.

Of course the pupils will need to get used to me being around and I will need to test the taping equipment in the different classrooms. It would thus be useful if I could come to one or two maths lessons prior to the geometry lessons.

The following activities have been selected:

- Draw What You See
- What Do You See 1?
- Same or Different 1?
- Creating Patterns 1
- Looking From Above
- Giving Directions
- Making Boxes (from Spatial Skills 2)
- One other visualisation activity to be provided.¹

It is important that I am present for all of these activities so we will have to plan carefully! I would of course like to be present for the other activities, too!

Thank you for your co-operation. I am looking forward to working with you learning about how the pupils work.

¹ Some of these activities were excluded from the study during the data collection process when it was noted that the scale of the study was too large. This was negotiated with the two teacher involved.

APPENDIX 3: Preliminary Analysis of Activities 1 to 7

This analysis was the second part of the three part process of analysis described in section 5.1. It followed an initial analysis in which the transcripts, field notes and drawings were used to compile a "profile" of each learner's performance on the seven activities. In this analysis the data on each learner was analysed to identify common strategies used for each category of activity. Additional aspects of interest were also noted. In each case examples of learner responses are provided to illustrate learner performance. A "profile" of each learner's performance on each activity was also compiled. The discussion in Chapter 5 was based on this analysis.

In this appendix the results are presented for each activity, in each case followed by learners' profiles of responses to the activity. Instances where prompts were used have **not been included** in this part of the analysis, for example, where I asked learners, "Did you move yourself or the 'object'?" The reasons for these exclusions are discussed in section 5.4. My questions to the learners have been placed in italics. Descriptions of learner actions are indicated in brackets.

Activities 1(a) and 1(b):

Points to consider regarding the use of the activities:

1. The sequence in which the activities were done: Gaylene and Belinda did Activity 1(a) after Activities 3 and 4; Ashton, James, Ryan and Stuart did Activity 1(b) after having completed Activities 2, 3 and 4 (although they had done Activity 1(a) prior to these without my presence).
2. In the classroom situation I was also not able to interview each learner about each item. Consequently I am not able to report on and compare responses on individual items, for example, the cereal box or the tin of beans, but will discuss responses in general on items of this nature.
3. Some learners used the strategy of lifting or turning the page on which the activity is printed or moving their own position in relation to the page. I did not question the learners about these movements and consequently am not able to comment on their relationship to the mental strategies used.
4. The pupils gave drawn responses: What value can I attach to these representations assuming that drawing involves a particular transformation of spatial information?
5. It appears that one learner, Cindy, copied from others in the group.

6. Some learners had problems interpreting what was meant by viewing from above or the side - in such case I or other learners demonstrated this to them.

Introduction from teachers:

- Activity 1(a): "You must imagine that that same cereal box was in front of you."
- Activity 1(b) (Instructions from Researcher): "This is similar to the one you did the other day (referring to Activity 1(a))."

Comments	Learner	Example of Response
Visualising a movement in the position of learner (strategy 1)	Gaylene: James: Ryan: Stuart:	Miss, I was looking at it like if I was on that side so I would only see the side. I imagined a real toilet roll and looked around it. <i>What did you picture in your mind?</i> That I was standing on top and I was standing and on a small table was a glass bottle. If you go around the suitcase like this you get the same as you have here. ...I stand so I look from here, so can draw it like this.
Visualising a movement in the "object" itself (strategy 2)	Trevor: Gaylene: Belinda: James: Stuart: Gaylene:	...Miss, I picture the box in my mind, Miss. Then I just turn the box...put the box down, miss, and look at it from the top. Turn the box on its side, Miss. When you turn the tin you will see the same. If I turned it around...the tin of beans is round, so if I turn it around it will be the same... Miss, if this is like the front, Miss, and now if you put it on the side, Miss, you can see it look like that... I took the ball, Miss and I turned it like this, Miss. ...if you turn it around then it will look like this. This was more difficult...I stand so I look from here, so I can draw it like this. Miss, I was looking at it like if I was on that side so I would only see the side. If I turned it around...the tin of beans is round, so if I turn it around it will be the same... Miss, if this is like the front, Miss, and now if you put it on the side, Miss, you can see it look like that...
Some pupils use different strategies for different items.	James: Trevor: Gaylene: Cindy: Belinda: Ashton: James: Ryan:	I imagined a real toilet roll and looked around it. ...if you turn it around then it will look like this. (Teacup correct.) (Teacup correct.) (No Left/Right discrimination.) (Left/Right by turning page over?) (Initially Left/Right not correct for toilet roll, corrects after discussion with James, Left/Right correct on egg.) (Left/Right correct for toilet roll from behind (but not from above), both correct for egg.) (Left/Right incorrect from behind and above on toilet roll, Left/Right correct for egg from behind (but not from above))
Problems distinguishing Left and Right when viewing from behind (and in some cases from above - this might be an interpretation of the question)	Stuart:	(Left/Right correct for toilet roll from behind (but not from above), Left/Right correct for behind and above.)

<p>Some learners seem to develop a strategy using visual memory (not necessarily a spatial strategy?) (strategy 3)</p>	<p>Trevor: Belinda: Ashton:</p>	<p>2(c) will be the same as (a), Miss. When you turn the tin you will see the same...because it is a cylinder. ...once I looked up in a book that if you look at it (a sphere) from all directions, it will still be the same. ... whenever I play cricket I look a the ball. <i>How did you know that it would look like that?</i> Miss, because I always make tea. You know the other one we did with the vase? The vase and the...cup, miss. The handle changed side as the eye changes. ...but Miss the other day said that a round object never changes, except the words on the object... Then they say a round object when you look from above... knew that because one day they had a container on the board, so the container looked like a bottle and I drew it like this because that is smaller than this (points to parts of the bottle)...<i>How would you check that what Mrs Johnson says is correct?</i> All...almost all bottles are round. So I used what I learnt to draw the bottle.</p>
<p>Some learners develop a short argument using the relative size of parts of the "object", the relative position and the shape of the "object" (strategy 4)</p>	<p>Trevor: Ashton:</p>	<p>From behind this is just going to be a square or a rectangle...3 across and 4 down...so it must look like this. ...and I drew it like this because that is smaller than this (points to parts of the bottle)... ...really the same as the one with the bottle because it has a flat surface with a handle.</p>
<p>Some learners still need physical "objects", need to move physically, or need to move the page</p>	<p>Trevor: Cindy: Gaylene: Belinda: Ryan : James: Cindy:</p>	<p>(For the tin of beans he puts a toilet roll in front of him, says this helps him.) (Stands up to look at picture of "object" from above.) (Moves page for sandwich) Miss, I wanted to see what it looks like from the back, but I never looked properly, so I turned it like this... (She peers over the drawing of the teacup). (Turns page over for teacup from behind, for above leans forward towards page, moves page to demonstrate from the "side".) (Uses pencil box for suitcase picture) If I use this box, this looks like the handle on top. (Moves the page to look at the egg from different perspectives.) (Drawings not correct, then copies from Trevor.)</p>
<p>Some learner do not seem to be able to communicate visual images or mental processes Some learners can reflect on the processes they are using</p>	<p>Trevor: Gaylene: Ashton: James: Ryan: Stuart:</p>	<p>I use my head, miss, I picture the box in my mind, Miss. Then I just turned the box... I always use my mind, I think hard. Miss, I was looking at it like if I was on that side so I would only see the side. I pictured that I looked at it from the other side, Miss. I was looking directly down. I looked at it on the right hand side...standing on the other side, so it will be on the left. I imagined a real toilet roll and looked around it. Miss, I was thinking like this...this is from the back, and then this is from above... <i>What did you picture in your mind?</i> That I was standing on top and I was standing and on a small table was a glass bottle. I see from above...I see only this part and the handle <i>How did you know that is all you would see?</i> I just see the top, I imagine it with my mind. This was more difficult...I stand so I look from here, so I can draw it like this....</p>

General profiles of learners based on Activities 1(a) and 1(b):

Trevor:

- Uses strategies 1, 2, 3 and 4
- Has a mental picture, aware of it and can describe processes used
- Identifies and names geometrical figures
- Concern for scale in drawings
- Draws two-dimensional and three-dimensional drawings - can reflect on reasons for differences
- Does find concrete useful at times (eg toilet roll)
- Adequate Left/Right discrimination.

Cindy:

- Cannot communicate visual images or the mental processes used / Has no mental image?
- Does try to move physical position
- Assistance provided in the form of concrete "objects" does not appear to help
- Copies Trevor (drawings and verbal responses)
- No Left/Right discrimination.

Gaylene:

- Uses both strategies 1 and 2
- Recognises and adapts incorrect responses quickly
- Can reflect on mental processes of moving herself or the object
- Adequate Left/Right discrimination
- Uses physical movement of the page as an explanation.

Belinda:

- Uses strategy 3
- Finds physical movement of her body and page useful
- Can reflect on the processes she is using
- Pictures not the same as others - perhaps looking at more of an angle

Ashton:

- Uses strategies 1, 2, 3 and 4
- Adapts incorrect responses after discussion with others
- Has a mental picture, aware of it and can describe processes used
- Left/Right discrimination corrected after discussion with James.

James:

- Uses strategies 1 and 2
- Has a mental picture, aware of it and can describe processes used
- Can comment on drawings of others
- Good drawing skills.

Ryan:

- Uses strategy 1
- Can describe processes being used
- Finds concrete object useful.

Stuart:

- Uses strategy 1
- Has a mental picture, aware of it and can describe processes used.

University of Cape Town

ACTIVITY 2:

Points to consider regarding the use of the activity:

1. Ashton indicated that he had problems interpreting the direction of the "eyes" in the activity: These do not appear to be looking in the required direction and can cause confusion regarding the actual view required. It is possible that other learners in the study had similar problems but did not articulate these.
2. The inclusion of the "eyes" in this activity might influence the kind of response received - perhaps it might force pupils to use strategy 1 (moving own position)?
3. I interviewed Ashton, James, Ryan and Stuart on this activity before they did Activity 1(b) (although they had already done Activity 1(a) with their mathematics teacher).
4. Cindy has the correct answers, but some of her responses make sense. It is possible that she has copied from others and is using learned responses.
5. Belinda obtained the incorrect answer for this activity.
6. A number of learners turned the page.

Introduction from teachers:

- For learners Trevor, Cindy, Belinda and Gaylene: "Imagine in your mind...what will I see if I look from behind?"
- For Ashton, James, Ryan and Stuart: The teacher read through the activity, indicated that the "eye" is looking from different places and looking at the objects.

Comments	Learner	Example of Response
Visualising a movement in the position of learner (strategy 1)	Gaylene: Ryan: Stuart:	(Explains position of objects on her desk)...then you go and stand there and look from there...(gets up from seat and moves position). ...if you stand up and face that direction you would see... ...if you face that way... ...I looked at it from this side. I imagined...that I could see from that side and I could see only the handle of the cup.
Visualising a movement in the "object" itself (strategy 2)	Gaylene: Ryan :	...turn the blank (plank) in that way you would see that way. <i>Explain to me what was going on in your mind?</i> I made like I turned it around...I turned the picture around. (Written) I pretended the picture was real. Then I turned it around.
Some learners use different strategies for same question	Gaylene: Ryan:	(See above.) (See above.)

<p>One learner seems to develop a strategy using visual memory (not necessarily a spatial strategy?) (strategy 3)</p> <p>Some learners develop a short argument using the relative size of parts of the "object", the relative position and the shape of the "object" (strategy 4)</p>	<p>Ashton: Trevor: James:</p>	<p>...when we did the cup in the other activity...so I used that...</p> <p>...because the cup you can see is hiding away behind this because it is bigger (pointing to the pot) and you can only see the ear. ...because the cup is now in front of the flowers and you see the ear the right side. ...because you look at the cup and the flower next to each other, but you do not see the ear. ...the plate is on the left side when you are looking at it...when you look at it from here it is on the right. So I say it is "B", Miss. I saw that this, the plate will be on the right because that is opposite from here...because that will be on the rightand his behind, the jug will be behind the flowers. (Written) I new the cup was behind the flowers. (Written) I new the plate was behind the flowers. (Written) I new the cup was on the right hand side.</p>
<p>One learner seems to use given solutions to guide the process (strategy 5)</p> <p>Some learners still need physical "objects", need to move physically, or need to move the page</p>	<p>Ryan: Gaylene: Ashton: Ryan: Stuart: Trevor:</p>	<p>First I thought number one was (c) because I looked at it from this side and so I saw the handle and then I looked at (b) and so I saw this side. (Explains position of objects on her desk)...then you go and stand there and look from there...(gets up from seat and moves position) (only as an explanation?) (Moves own position in desk and moves paper.) (Moves own position in relation to the sheet.) ...because the cup is now in front of the flowers and you see the ear the right side. ...the plate is on the left side when you are looking at it...when you look at it from here it is on the right. So I say it is "B", Miss. I saw that this, the plate will be on the right because that is opposite from here...because that will be on the rightand this behind, the jug will be behind the flowers. ...I can't tell you, Miss. (Answers correct, but answers do not make sense.) (Written)...because I could see sees the whole thing, ...because I could only sees the side, ...because I could only sees above, ...because I could only sees below. I sort of...I just sort of looked at the picture, Miss. (Has incorrect answers, does not explain) (Can relate position of eyes to original drawing)...(a) is from the front, ...this is from the back (referring to (b)), ...(c) is from the side; but cannot match these to the given solutions.</p>
<p>Some learners do not seem to be able to communicate visual images or mental processes</p>	<p>Cindy: Belinda:</p>	<p>(Starts wrong, but quickly rectifies this - problems with the "eyes"?) (Works quickly, answers initially incorrect, but corrects these after discussion with others.)</p>
<p>Some learners struggle initially, but proceed as soon as can relate to problem</p> <p>Some learners work quickly and get correct answers straight away</p> <p>Some learners can reflect on thought processes</p>	<p>Ashton: Ryan: James, Stuart, Trevor: Trevor: Gaylene:</p>	<p>...because the cup you can see is hiding away behind this because it is bigger (pointing to the flower pot) and you can only see the ear. ...if you take a pot of flowers and a jug and a plate on a piece of blank (plank) and stand up and look, one eye will see the handle, bucket of flowers...</p>

James:	I saw that this, the plate will be on the right because that is opposite from here...because that will be on the rightand this behind, the jug will be behind the flowers.
Ryan:	First I thought number one was (c) because I looked at it from this side and so I saw the handle and then I looked at (b) and so I saw this side.
Stuart:	I imagined...that I could see from that side and I could see only the handle of the cup.

General Profile of learners based on Activity 2:

Trevor:

- Uses strategy 4
- Adequate Left / Right discrimination
- Has mental image and can reflect on parts within the whole.

Cindy:

- Gets answers correct
- Does not have visual image / cannot describe mental picture or explain reasoning?
- Describes her own movement (learned responses? / reacting to prompt?)

Gaylene:

- Uses strategies 1 and 2
- Finds physical movement of herself and use of concrete "objects" useful in describing processes, but appears to obtain answers using visual imagery
- Can describe mental movement of her own body.

Belinda:

- Incorrect answers, does not explain
- Can relate to the position from which the different eyes are looking - but does not seem to be able to visualise the scene from the different perspectives and cannot match to the given answers.

Ashton:

- Uses strategy 3
- Finds physical movement of himself and the page useful
- Has problems with the "eyes" - can reflect on the problem he experienced.

James:

- Uses strategy 4
- Does not need to move himself - just looks at the picture
- Correct answer on first attempt
- Adequate Left / Right discrimination.

Ryan:

- Uses strategies 1, 2 and 5
- Moves own position in relation to page
- Works quickly, but initial answer incorrect, corrects this after discussion with others
- Can reflect on relative position of himself to the objects.

Stuart:

- Uses strategy 1
- Moves page
- Gets correct answer rapidly
- Can reflect on thought process.

ACTIVITY 3:

Points to consider regarding the use of the activity:

1. Stuart and Ryan seem to have had difficulty in seeing the two boxes in relation to one another, and treated them as separate entities.
2. Some learners seemed to have had difficulty interpreting the question which required that they view the "objects" from points A and B. I attempted to clarify this in the group setting: When asked by Ashton, James, Ryan and Stuart about viewing from below, I suggested that it was as if the boxes were on a glass table. Could this have affected the strategy used?
3. For Gaylene and Belinda this was the first activity in this sequence of spatial activities.
4. I interviewed Ashton, James, Ryan and Stuart on this activity before they did Activity 1(b) (although they had already done Activity 1(a) with their mathematics teacher).
5. It seems that some learners might have had problems drawing representations of these boxes.
6. Again, some learners moved the page when responding to the questions.

Introduction from teachers:

- For learners Trevor, Cindy, Gaylene and Belinda: "...drawing must be as accurate as possible in the sense that you look at the size of the box".

Comments	Learner	Example of Response
Visualising a movement in the position of the learner (strategy 1)	<p>Ashton:</p> <p>James:</p> <p>Ryan:</p>	<p>So if you are looking from above, you should actually be pretending...a helicopter, whatever going, looking from above. (From below) I imagine it is like that machine on the TV programme, in the comics, that machine that goes underground. Then it stands under and looks from below. I imagine I am a radar dome...But it gives you a view...round things...it walks around. <i>Where is the machine? Is it on the ground, or...the ground...it walks around...I see it on the TV programme.</i> I think of a glass coffee table, and I am lying under the coffee table. (Interjects when Ashton explains about the helicopter) A bird's eye view.</p>

<p>One learner develops a short argument using the relative size of parts of the "object", the relative position and the shape of the "object" (strategy 4)</p>	<p>Trevor:</p>	<p>...so that one is going to be in front. Miss, I am only drawing one box here, Miss, because of you are looking from the left this will be behind it. So I just need one box. I looked at the sizes of the box. See, that is right next to the box. If I look on the left side, it will be behind it. <i>How did you picture it in your mind?</i> Uhm, I took the big one first, then the smaller one. Miss, the big one was on the left side, Miss, and the small one was on the right. So if you look from behind the small one will be on the left side and the big one's going to be on the right side, Miss...</p>
<p>Some learners need physical "objects", need to move physically, or need to move the page</p>	<p>Cindy: Gaylene: Belinda: Ryan:</p>	<p>(Can explain her relative position to the boxes when Researcher places objects on desk.) (Wants to use box initially.) (Moves page to demonstrate.) (Wants to use box initially.) (Keeps paper fixed, and moves herself to look from an angle.) (Uses pencil box - stands up to look down, moves box to look from below.) (Does turn page to help - but can explain why this happens.) (Adequate.)</p>
<p>Some learners have adequate Left / Right discrimination</p>	<p>Trevor: Belinda: Cindy:</p>	<p>(Can explain where she "stood" in relation to actual boxes on desk, but this does not relate to the drawing she has made.)</p>
<p>Some learners do not seem to be able to communicate visual images or mental processes Some learners can reflect on thought processes</p>	<p>Trevor: Belinda: Ashton:</p>	<p>...I'm drawing it directly. (Trevor has drawn it from above, but at a 90° orientation to the given picture - but can explain that he has done this (turns the page).) I am looking from on top...so that one is going to be in front. (Discussion with Belinda about parallelogram and rectangle) I looked at it straight. I looked from far. (Discussion with Trevor about parallelogram and rectangle) ...looking from an angle (but draws two boxes from above - one as parallelogram and one as rectangle) So if you are looking from above, you should actually be pretending...a helicopter, whatever going, looking from above. ...it is like that machine on the TV programme, in the comics, that machine that goes underground. Then it stands under and looks from below. Left/Right discrimination wrong initially, but changes after moving page and listening to Trevor.</p>
<p>Some learners make mistakes initially - but correct after discussion, help with concrete object</p>	<p>Belinda:</p>	<p>Left/Right discrimination wrong initially, but changes after moving page and listening to Trevor.</p>

General profile of learners based on Activity 3:

Trevor:

- Uses strategy 4
- Can reflect on position from which he is viewing the representations
- Adequate Left/Right discrimination.

Cindy:

- Can explain relative position when Researcher places boxes on the desk, but this does not relate to what is drawn
- Drawings show boxes in variety of orientations - possibly not paying attention to relative position
- Does not have visual image / cannot communicate visual images or mental processes?

Gaylene:

- Initially wants to use a box
- Moves page to demonstrate to Researcher
- Has not understood relative position of objects?

Belinda:

- Initially wants to use a box
- Physically moves herself in relation to page
- Can reflect on position from which looking (at an angle), but drawing has combination (rectangle and parallelogram)
- L/R discrimination wrong initially, but changes after moving page and listening to Trevor
- Has problems from points A and B.

Ashton:

- Using strategy 1 (helicopter, machine)
- Problems with Left/Right discrimination
- Problem from point B (correct from point A).

James:

- Using strategy 1 (radar dome)
- Problems with Left/Right discrimination
- Has problems from points A and B.

Ryan:

- Using strategy 1 (under glass coffee table, a bird's eye view)
- Needs concrete objects at first (moves position **and** box)
- Not seeing relationship between boxes.

Stuart:

- Misunderstood relationship between boxes (doing separately)
- Says has mental picture, but says has difficulty drawing figures.

ACTIVITY 4:

Points to consider regarding the use of the activity:

1. As in Activity 3 pupils had problems interpreting what was meant by viewing the "object" from points A and B.
2. I interviewed Ashton, James, Ryan and Stuart on this activity before they did Activity 1(b) (although they had already done Activity 1(a) with their mathematics teacher).
3. Again, some pupils moved the page.

Introduction from teachers:

- This activity followed immediately on Activity 3 so no additional instructions were given by the teachers.

Comments	Learner	Example of Response
Visualising a movement in the position of learner (strategy 1)	Ashton: Stuart:	I'm not standing there...I'm standing a little further...standing on that line there. I'm standing there, so I am imagining that I can see this part...so I can see the 3-d. I imagined I was, like I was walking around the factory. I imagined I was, uh, looking from the other side.
One learner develops a short argument using the relative size of parts of the "object", the relative position and the shape of the "object" (strategy 4)	Trevor:	(Behind) It was on the left side, Miss, but I swopped it around so it is going to be on the right side.
One learner seems to develop a strategy using visual memory (not necessarily a spatial strategy?) (strategy 3)	Ashton:	...I did it like that, Miss, because you can't see the funnel, Miss. So if you look from point A...it is like the box.
Some learners need physical "objects", need to move physically, or need to move the page	Cindy: Gaylene: Belinda: Ashton: Ryan:	(Asks about looking from above - Researcher gets Trevor, Gaylene and Belinda to demonstrate how to look from above using microphone on desk.) (For below: picks up paper.) (For from point A: turns page.) (Above: moves page.) (Moves position in seat to draw point from which looking.) (Shifts own position in seat.)
Some learners have adequate Left / Right discrimination	Trevor, Cindy?, Gaylene, Ashton, James, Ryan, Stuart.	
Some learners do not seem to be able to communicate visual images or mental processes	Cindy: Belinda:	(Cannot visualise from above - explanation does not make sense.) (Drawings for above and below incorrect.)

Some learners can reflect on thought processes:	Trevor: Ashlon: Stuart:	(Behind) It was on the left side, Miss, but I swopped it around so it is going to be on the right side. (Has drawn at 90° orientation to given drawing, but can explain this.) I'm not standing there...I'm standing a little further...standing on that line there. I'm standing there, so I am imagining that I can see this part...so I can see the 3-d. I done...this time I did not turn the page around. I...I imagined I had the page with a cup on exactly like that...so I turned it around. And you imagined you turned the page around. Mmm. I imagined I was, like I was walking around the factory. I imagined I was, uh, looking from the other side.
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General profile of learners based on Activity 4:

Trevor:

- Uses strategy 4
- Can reflect on processes and position
- Adequate Left/Right discrimination (no longer doing it visually?)
- Drawings from points A and B correct.

Cindy:

- Needs to have demonstration using concrete objects – but drawing is still incorrect (cannot form a mental image / cannot communicate visual processes and visual imagery?). Explanation of what she did mentally does not make sense, but can demonstrate with concrete objects where she is viewing the "object" from

- Pictures for below and behind correct

Gaylene:

- Finds it useful to move page in relation to herself
- Above: Orientation in picture incorrect
- From point B: Seems to have seen difference in sizes of objects, but not reflected in drawing
- Adequate Left/Right discrimination.

Belinda:

- Moves the page
- Drawings for "above" and "below" incorrect – has not completed the rest.

Ashlon:

- Using strategy 1 and strategy 3
- Can reflect on processes and movements
- Moves own body in relation to page
- Adequate Left/Right discrimination
- Problems with orientation from point B?

James:

- All drawings correct (detailed – above and below drawn in 3-d).

Ryan:

- Physically moves his own body
- Drawing correct.

Stuart:

- Using strategy 1
- Can reflect on processes and movements
- Problems from points A and B.

ACTIVITY 5:

Introduction from teachers:

- For learners Trevor, Cindy, Gaylene and Belinda: The teacher wrote the instructions on the blackboard: *Imagine that you are looking at the classroom from above. Draw what you will see.* She also said, "Draw what you see in your mind how the class looks from above...picture it in your mind".
 - For learners Ashton, James, Ryan and Stuart: The teacher read the instructions: "Imagine that you are looking at the classroom from above. Draw what you will see". Then she said, "So imagine that you are an eagle soaring at the top of the classroom, and you are looking down...so from **that** angle, from the top down...what we call a bird's eye view or an aerial view. You must draw what you see".
- Also instructions from the researcher: "Picture you are a fly on the roof and you look down on the classroom".
(Could these instructions from the teacher and researcher encourage the use of strategy 1?)

Comments	Learner	Example of Response
Learners drawing from own position in the classroom - using this for information on the relative positions and sizes of the "objects" (strategy 6(a))	<p>Trevor:</p> <p>Gaylene:</p> <p>Ashton:</p>	<p>(Turns around in seat to look at classroom (twice).) (In explanation matches objects in classroom to parts of his drawing.) <i>How did you know where to draw those desks? I looked at the corners, Miss. I looked at the board (pinboard)... and if it is close to the board.</i> (Holds ruler in the air and looks at it with one eye closed) Just seeing whether the classroom is a square or a rectangle, Miss. (Looks at gaps between desks and concludes "1cm".) (Looks around the classroom from her seat.) So when you are sitting here you can see where the desks are situated, Miss...you can also see like the size of the desks..if you look from above you can't see like the size. <i>Can you tell me, how did you know that that is what the classroom looks like from above?</i> Miss...that is where you come in (points to the door)...it is exactly as the classroom is right now...(points to own desk and where each member of the group is sitting...continues to match up parts). <i>How do you know that this is what it is going to look like?</i> If you look from here you see the desks. <i>How do you know what it is going to look like from directly above?</i> Because the shape of the...the bench, that makes you think it is going to look like that, because here you have the two things here (matches up parts of the desk). <i>So you are looking at the shape of the bench?</i> Yes, that's right, I look at the bench.</p>

	James:	<p>(Looking around classroom.)</p> <p>(Looks at his work) How can you be such an idiot? (Starts filling in the middle desks) Better! What is much better? I forgot about the middle. The middle also. What reminded you that there was the middle still? There was a big space, but the class is so full.</p> <p>How did you know that this is what the classroom was going to look like from above? I just looked at each desk, each at a time. How did you know that the desk looks like that from the ceiling, from above? (points to his own desk) I drew this round stuff...this round stuff helps to show which way the desk is pointing. And how do you know it is going to be shaped like that? (Looks at his own desk) I know that this is a square, and if I halve the square I get a rectangle. (Stands on chair to look around classroom) Does it help when you get on the desk? Yes, Miss. How does it help you? So here you can see it, from above, but from more of an angle...and you can see where the desks should be.</p> <p>(After discussion with Ashton) I thought it (the cupboard) must be bigger, because it is the overhead view, because the cupboard is taller than this. But the overhead view, you can't see the height, you can only see that it must be smaller.</p> <p>Okay, Ryan, can you tell me how did you know that is what the classroom was going to look like? The cupboard doors are open. (he matches drawing to furniture in the room). How do you know if you are sitting here that the desk is going to look like that? I saw it first...like ours...so I draw it the same. It was like this because it is the same like ours. I know how our looks...that desk is the same as ours...and that other desk is the same as ours.</p> <p>(Draws "objects" on teacher's desk - turns around to look.)</p> <p>(Holds picture up and looks at classroom) Stuart, why are you holding your paper like that? I was just looking if I was doing the right drawing, Miss. But I made this wrong, I must turn this desk.</p> <p>How did you know that it will look like that? (matches parts of drawing to furniture in the classroom).</p> <p>How do you know what you will see when you look from above? (continues to match picture and furniture). And the cupboard? How do you know the cupboard is going to look like that? Because the cupboard...the doors are open like that.</p>
One learner establishes a reference point in the classroom	Trevor:	Okay, let's take it from this corner...there is our desk.
Some learners have problems "tilting" in the different "objects"	Trevor: Ashton: James:	<p>(Changes this later) Let me start in that corner.</p> <p>(Has to rub out a lot to get relative position of desks correct, has problems fitting things in)</p> <p>There is something wrong here, Miss.</p> <p>I made it too small. (he rubs out, but then too big!)</p> <p>(Looks at his work): How can you be such an idiot? (starts filling in the middle desks) Better!</p> <p>What is much better? I forgot about the middle. The middle also. What reminded you that there was the middle still? There was a big space, but the class is so full.</p>
Some learners appear to use strategy 1 to visualise what the "objects" look like from above (visualising movement in own position)	Trevor: Gaylene:	<p>Most of the time I see from the ceiling, from directly above...because I would have seen like that side of the bench if I'd had to look from here. I was looking directly from above.</p> <p>(To Trevor) If you are looking from a helicopter above, there are some bits you won't see like the edges of the pinboards.</p> <p>And you can like picture yourself in a helicopter, Miss, and then you can see like down, Miss.</p>

	Belinda:	(She has drawn some of the window panes) <i>How do you know that the board is going to look like that?</i> I'm not standing too far, Miss. Only from the other side when you are looking... downwards, Miss.
Some learners do not seem to be able to communicate visual images or mental processes / Do not have mental image?	Cindy:	(Relative position of objects incorrect – desks placed in rows, draws panes in the window, draws cupboard lying down.)
Some learners draw picture in same orientation to way in which sitting in classroom	Trevor, Cindy (begins), Gaylene, Belinda.	
Some learners change orientation by moving paper	Cindy, Ashton, Ryan, Stuart:	(Rotates paper and holds it up vertically).
Some learners move own position to look at classroom	Trevor, Gaylene, Cindy, Belinda, Ashton, Ryan: Stuart:	(Climbs on to seat to look at classroom (twice), lifts himself out of seat to look around.) (Draws "objects" on teacher's desk – turns around to look.) (Turns around to look at classroom.)
Some learners can reflect on thought processes	Trevor: Gaylene: Belinda : Ashton: James:	(Can distinguish between what he sees from his position in the classroom and the aerial view.) Most of the time I see from the ceiling, from directly above...because I would have seen like that side of the bench if I'd had to look from here. I was looking directly from above. (Can distinguish between what she sees from her position in the classroom and the aerial view.) So when you are sitting here you can see where the desks are situated, Miss...you can also see like the size of the desks...if you look from above you can't see like the size. (To Trevor) If you are looking from a helicopter above, there are some bits you won't see like the edges of the pinboards. And you can like picture yourself in a helicopter, Miss, and then you can see like down, Miss. (Draws some window panes) <i>How do you know that the board is going to look like that?</i> I'm not standing too far, Miss. Only from the other side when you are looking...downwards, Miss. <i>How do you know what it is going to look like from directly above?</i> Because the shape of the...the bench, that makes you think it is going to look like that, because here you have the two things here (matches up parts of the desk). So you are looking at the <i>shape of the bench</i> ? Yes, that's right, I look at the bench. <i>How did you know that this is what the classroom was going to look like from above?</i> I just looked at each desk, each at a time. <i>How did you know that the desk looks like that from the ceiling, from above?</i> (points to his own desk) I drew this round stuff...this round stuff helps to show which way the desk is pointing. <i>And how do you know it is going to be shaped like that?</i> (looks at his own desk) I know that this is a square, and if I halve the square I get a rectangle.

	Ryan:	<p>Okay, Ryan, can you tell me how did you know that is what the classroom was going to look like? The cupboard doors are open... (matches to furniture in the room). How do you know if you are sitting here that the desk is going to look like that? I saw it first... like ours... so I draw it the same. It was like this because it is the same like ours. I know how ours looks... that desk is the same as ours... and that other desk is the same as ours.</p> <p>(But does not relate this to aerial view.) How did you know, though, that it was going to be that shape? (Does not reply.)</p> <p>Does it help when you get on the desk? Yes, Miss. How does it help you? So here you can see it, from above, but from more of an angle... and you can see where the desks should be.</p>
Some learners can reflect on drawings done by others	Gaylene:	<p>(She reflects on Belinda's work. She sees a problem with the way Belinda has drawn the windows.) ...it depends where Belinda stands.</p> <p>If you are above, Miss, and you are looking straight down, then you can see... you can only see like the edges.</p> <p>...and she first stands on the roof like this, and looks down like this, she will see that, but if she stands on that side.</p>
Some learners can adapt their work after discussion with others	Ashton:	<p>(He looks at Ryan's drawing) Huh... the cupboard is smaller than the desk. The height doesn't matter because it is an overhead view. You can't make the cupboard bigger than the desk.</p>
Some learners relate what they are doing to other activities	Ryan: Ashton: Ryan:	<p>(After discussion with Ashton) I thought it (the cupboard) must be bigger, because it is the overhead view, because the cupboard is taller than this. But the overhead view, you can't see the height, you can only see that it must be smaller.</p> <p>...must we draw it like... the plan (Ryan says, "...a floorplan") Yes, a floorplan.</p> <p>...a floorplan.</p>

General profile of learners based on Activity 5:

Trevor:

- Draws in same orientation to the way he is sitting
- Moves own position to look around classroom
- Begins by a obtaining reference point in classroom
- Concerned about relative position of objects, size and shape - has difficulty getting this accurate
- Drawing from own position in classroom, but is aware of difference between what he sees from this position and what he would see from above (appears to use strategy 1 for this)
- Can reflect on processes.

Cindy:

- Does not have visual image / cannot communicate visual images or mental processes? Draws panes in the window, draws cupboard lying down
- Relative position of objects incorrect – desks placed in rows
- Very unsure of herself, rubs out a lot
- Begins by drawing in same orientation to way sitting – then turns page
- Moves own position to look around classroom.

Gaylene:

- Draws in same orientation to the way she is sitting
- Moves own position to look around classroom
- Concerned about relative position of objects, size and shape
- Drawing from own position in classroom (uses this for shape and size), but is aware of difference between what she sees from this position and what she would see from above (appears to use strategy 1 for this)
- Can reflect on own processes and drawings of others – can argue with them.

Belinda:

- Draws in same orientation to the way she is sitting
- Moves own position to look around classroom
- Drawing – shows broad windows with some panes – but says this is because she is not looking from directly above (can reflect on this).
- Appears to be using strategy 1 for visualising "objects" from above.

Ashton:

- Moves own position to look around classroom, also moves page
- Can reflect on drawings of others
- Pays attention to shape, scale and relative position of objects (struggles with the latter!)
- Can reflect on the conventions associated with an aerial view: a floorplan.

James:

- Works quickly
- Looking around the classroom
- Paying attention to shape, size and relative position of objects in classroom – has problems getting this accurate.

Ryan:

- Finds it useful to stand on seat, and lift himself out of his seat
- Relating objects to those in his immediate vicinity, for example, his own desk
- Adapts drawings after discussion with Ashton
- Pays attention to relative sizes and positions of objects
- Turns the page when drawing
- Drawing at 90° to the way he is sitting
- Can reflect on the conventions associated with an aerial view: a floorplan.

Stuart:

- Draws at different orientation (90° to the way he is sitting)
- Turns around in desk to look at classroom
- Moves paper (rotating and lifting vertically to compare)
- Matches objects in the classroom to parts of his drawing (does not say how this is related to aerial view).

ACTIVITY 6:

Introduction from teachers:

- For learners Trevor, Cindy, Gaylene and Belinda: Instructions from the researcher: "Draw what the school would look like if you were looking at it from above".
- For learners Ashton, James, Ryan and Stuart: The researcher indicated that they must do the same as for Activity 5, but this time drawing what they would see if they looked at the **school grounds** from above.

Comments	Learner	Example of Response
<p>For parts that the learner can see ("seen" space): looks at relative position and size of "objects" from position in the classroom (strategy 7(a))</p>	<p>Gaylene: Belinda: James: Ryan:</p>	<p>...so what do you, do if you can't see the whole school, how do you know to draw that? (pause)...I can see some parts. How do you know that this is what the school is going to look like? I go to the places and I draw from there. How do you know that it (the office) will look like that? I go there and see the shape. What did you look at when you went out? I wanted to see if the girls' toilet is in the same length as the end of the admin block. Is in the same? Has the same length...ends at the same place. The water tower? How do you know that the water tower looks like that? (Has drawn crosses on top.) I could see on all sides...there are crosses, so I thought the top would also have these crosses. Ryan, Ryan, what did you go and look at outside? I went to look at the pattern on the roof...Miss, this is how I saw...Miss it comes almost...the roof pattern...it comes almost like a triangle here, Miss, and then it goes straight down, and then it comes like another triangle at the end. Okay, does it help you to go outside? And have a look? Yes, Miss, now I know where's the boys' toilet, and where's the field and ... (uses his hand to show to point to the building). (Ashton comments on Ryan's triangles in the drawing) On all the roofs, even if I look at it...but it does jut out. How did you know it was going to look like that? What did you think about? I saw from here...the roof is...and went, I went outside...I saw all the roofs. (Looks out of classroom window when explaining to Researcher.)</p>

<p>For parts the learner cannot see ("unseen" space): Bases drawing on what s/he remembers from when s/he did see the scene (for relative position and shape of "objects" in the scene) (strategy 7(b))</p>	<p>Trevor: How do you know it is going to look like that? How do I know, Miss? ... I am in grade 6...it is six years now that I, I have been in this school. Every day, Miss...I've seen it so many times. How do you know, though, that the office is that shape? (Repeated) Oh, that shape? Miss, I walk past the office when I help a teacher - and I look at the shape. (Researcher asks whether the classroom or school plan is more difficult.) This one, Miss. (referring to the school, emphatically) Cause there are so many things, Miss, to remember. Sometimes you do not know what size...it doesn't quite fit in. In the class it is much easier because you just look around you and see, but the school you can't just run out and look. So what do you do if you can't look around, if you can't run around the school? Oh, Miss, what I do is that I think hard, Miss, how the school looks, all the different buildings. I picture it in my mind, Miss, and I imagine that I am in a helicopter or something and I am looking down on the school. How did you know what to draw? I've seen it before, Miss. Have you seen it before, actually from above? Only when standing on the balcony here, Miss, ...I see the roofs, the girls' lavatory... If you can't see something you have to draw from where you are sitting, how do you know what to draw? How do you know to draw the classrooms like that shape? I've been there before. ...How did you know when you did it in English? The place where the school is...the place where the school is determines how it looks from the top. And you can see from the quad and standing on the soccer field...you can see that. How do you know that the tarmac will look like that? You are sitting here in the classroom and you can't see it. How do you know it looks like that? I remember when I walk to school, then I walk all the way along here...How do you know the roof looks like that? I can see...when I drink water then I can see. How did you know the admin block looks like that? The classroom...when I still was in std...sub B, so I was the class over here. (points to the classroom in the corner of the block)...and I could see the admin block from there. I...I was upstairs. How do you know it is going to be that shape?...because we use it for hockey and netball. How do you know the roof is going to look like this from the top? Well, when I came this morning...I could see through the window that ...I could like see like the window... I picture it in my mind, Miss, and I imagine that I am in a helicopter or something and I am looking down on the school. How do you know that this is what the school is going to look like? I go to the places and I draw from there. How do you know that it (the office) will look like that? I go there and see the shape.</p>
<p>Some learners appear to use strategy 1 to visualise what they will see from above (visualising movement in own position)</p>	<p>Gaylene: Ashton: James: Ryan: Stuart:</p>
<p>One learner seems to use strategy 3 to visualise what he will see from above (visual memory)</p>	<p>Trevor: Belinda: Ashton:</p>
<p>One learner has problems "fitting" in the different "objects" correctly</p>	<p>Trevor: Oh, no, there won't be enough space for the garden!</p>

<p>Some learners do not seem to be able to communicate visual images or mental processes / Do not have mental image?</p>	<p>Cindy:</p>	<p>(Buildings not in correct positions.) (Shape of whole buildings correct, for example rectangles, but details not drawn from above, for example clothes on washing line, gutter.) <i>Why did you draw it like this? How did you work it out? Miss, if you look from above, this is what you will see. (Does not relate to drawing, a learned response??)</i> (Shakes head when asked to reflect on the difficulty of the two activities.)</p>
<p>Some learners obtain a reference point in the school Some learners need to move physically so that can look at part of scene</p>	<p>Trevor: James: Ryan:</p>	<p>Identifies North in the school and indicates this on his page. Miss, can I have a look outside? Sorry, Miss, can I go out quickly, Miss? (Ryan goes outside and stands in quad – looks towards the roof and uses his hand to point towards the roof.) <i>So what do you do if you can't look around, if you can't run around the school? Oh, Miss, what I do is that I think hard, Miss, how the school looks, all the different buildings, I picture it in my mind, Miss, and I imagine that I am in a helicopter or something and I am looking down on the school.</i></p>
<p>Some learners can reflect on thought processes</p>	<p>Gaylene:</p>	<p>(Researcher asks whether the classroom or school plan is more difficult.) Because there is more stuff...and you have to like draw, like the class, from where you are sitting you can like see the whole class, but when you draw the school you have to draw when you can't see the whole school. <i>So what do you, do if you can't see the whole school, how do you know to draw that? (pause)...I can see some parts. If you can't see something you have to draw from where you are sitting. How do you know what to draw? How do you know to draw the classrooms like that shape? I've been there before.</i> <i>How do you know that this is what the school is going to look like? I go to the places and I draw from there.</i> <i>How do you know that it (the office) will look like that? I go there and see the shape.</i> <i>The water tower? How do you know that the water tower looks like that? (has drawn crosses on top) I could see on all sides...there are crosses, so I thought the top would also have these crosses.</i> <i>How do you know that the tarmac will look like that? You are sitting here in the classroom and you can't see it. How do you know it looks like that? I remember when I walk to school, then I walk all the way along here... How do you know the roof looks like that? I can see...when I drink water then I can see.</i> (Ashton comments on Ryan's triangles in the drawing) On all the roofs, even if I look at it...but it does jut out. <i>How did you know the admin block looks like that? The classroom... when I still was in std...sub B, so I was the class over here (points to the classroom in the corner of the block)...and I could see the admin block from there. I...I was upstairs.</i> <i>How do you know the roof is going to look like this from the top? Well, when I came this morning...I could see through the window that ...I could like see like the window...</i></p>
<p>Some learners can reflect on drawings done by others Some learners relate what they are doing to other activities</p>	<p>Ashton: Trevor:</p>	<p>(Comments on the triangles in Ryan's drawing.) I think I am going to make everything blocks, Miss...not its normal shape. Like a plan, Miss.</p>

ACTIVITY 7:

Points to consider regarding the use of the activity:

1. In Question 3 one possible net is given: Could this determine the strategy used?
2. Similarly for Question 4: Could the drawing of the box influence the response?
3. Stuart was absent when the activity was completed - he thus completed it on his own after having done a number of other net activities.

Introduction from teachers:

- For learners Trevor, Cindy, Gaylene and Belinda: The teacher read through the written activity.
- For learners Ashton, James, Ryan and Stuart: The teacher read through the written activity and stressed the notion of "net" as discussed in the speech bubble.
- Stuart read through the activity on his own.

Comments	Learner	Example of Response
Some learners start with the net and fold it up (strategy 8)	Trevor:	<p><i>How did you work it out? Because...a cube has sides like that...they can't...now they must be on each side...so there's it, Miss, on the sides...and I put that one on top, on the side and that one another way at the bottom.</i></p> <p><i>And this one? Can you explain to me how did you work that one out? (responds straight away)...That one goes up and that's on the side like this...and then you fold it over so that's on that side, that one on top and that one's on the other side...and that one is there.</i></p> <p><i>How did you know what to draw there? How did you work it out? ...Because same like this here...that one is open, two sides...it must have on the sides, because you put that one there so it works out...that one has to be there so you make a box.</i></p> <p><i>(For number 3) Can you tell me how did you know that that is what you had to draw? (points to original net on activity) You had this here like this, Miss, ...and so I wanted to change this one...and so...and so that's how I figured out that this one...so I put this one over here...that one will be on the side, and I put that one over there, and then...uhm...you sort of fold this one up and then over. Okay, so you worked from here? (pointing to the original net) Yes, Miss.</i></p> <p><i>How did you picture that that one was going to look like that? (she shows how the parts fold up) Okay, where did you start? Did you start by just drawing the squares? Or what did you picture when you started off? Or did you picture...? I start with this (pointing to the net) and then fold (she shows how she started with the "cross" and then found that something "was missing").</i></p> <p><i>(Says he folded up net for numbers 3 and 4, shows rolling-up motion.)</i></p>
	Gaylene:	
	Belinda:	
	James:	

General profile of learners based on Activity 6:

Trevor:

- Identifies reference point from which to work
- Concerned about relative position of objects
- Using strategy 1 to picture from above?
- Can reflect on mental processes
- Can reflect on the conventions associated with an aerial view.

Cindy:

- Buildings are correct shape, but in wrong position relative to one another
- Details are not drawn from above, for example clothes on washing line and gutter
- Says she is looking from 'above', but drawing not correct, possibly a learned response
- Shakes head when asked to reflect on the difficulty of the two activities. Cannot verbalise processes, no mental picture?

Gaylene:

- Works quickly
- Can reflect on processes used.

Belinda:

- Concern about the shape and size
- Using strategy 1 to picture from above?

Ashton:

- Uses strategy 3 for drawing from above?
- Can describe processes
- Comments on H's drawing (triangles).

James:

- Needs to move physically to look at parts of scene
- Can reflect on processes.

Ryan:

- Needs to move physically to look at parts of scene
- Can discuss and explain own drawing
- Works slowly
- Can reflect on process.

Stuart:

- Explains how he looks out of the window.

	Ryan:	<p>What did you start thinking about? I started here by this... and you put this over... then I have two sides... those are the flaps here. Okay... I fold these two closed... and then I bring this up. (he explains for two others). Did you start with a square and then add the other squares on? ... I drew, the shape. Okay, which shape did you draw? Can you show me?</p> <p>This... cross... You drew the outline, and then you put the squares in? ... just the lines.</p> <p>(For number 3) Okay, how did you picture what was going to happen? How did you picture it? ... Miss, I pictured the box like there... then you like take it apart. So you picture the box and then you are going to take it apart? And I will see it like open.</p> <p>(For number 4) Okay, so did you start thinking of the actual box? The three-dimensional box? And so I just flattened it... and then you flattened it. Yes. Can you tell me what you were doing when you were flattening it? Can you tell me what you saw in your mind?... when you were flattening it? I saw that there was a box... but then I look like the parts of the box... parts... and then there was still another piece on the bottom so I took another piece... and there was still another piece on the side over there... so I took that one out as well. And there was a top flap on the side over there that I took out like that... Can you explain this one to me here? Did you do it the same way? Yes (emphatically).</p>
Some learners start with the "box" and unfold it to form the net (strategy 9)	Gaylene: Ashton:	<p>See above.</p> <p>These flaps can go anywhere... these side flaps... you can put them anywhere, actually... and it will work. But you must not put them on the same side as the other one... on a different side... (Ryan says, "...or else the other side will be open")... or these will overlap on this side and the other one will be open.</p> <p>(Joins discussion with James.) ... it will be open there, but you will have double flaps on the other side.</p> <p>There's millions of different ways that can go there. There and there... that can go there, there and there. (he points to the different positions on the net he has drawn.) They all work. Why can't you have the flaps on the same side? If you have the flaps (flaps) on the same side then it... but it doesn't matter if it's same together... because it will go with two flaps (flaps). But it doesn't matter, but one thing that matters is that the other side will be open... (Ashton interrupts).</p>
One learner appears to use both strategies Some pupils start to generalise, can form short arguments (strategy 10)	Gaylene: Ashton: James:	<p>Okay... I'm going to test this. (He starts drawing a net that he can cut out. He realises his mistake)... I never counted this, Miss... it was six all the time. When did you realise that you had made a mistake, Trevor? When I... when I tried to build it.</p> <p>(He cuts out copies of answers for number four.)</p>
Some learners use cutting and folding as a checking mechanism	Trevor:	
Some learners need to cut and fold: This seems to help	Belinda	
Some learners need to cut and fold: This does not seem to help	Cindy	
Some learners say they do not need to cut	Ashton:	(Referring to question 4, Researcher says they do not have to cut out. They might need to do so at the end of the activity.) You can use your imagination.

Some learners can adapt responses quickly after noticing an error	Trevor:	(Draws a net with 7 squares.) One is for the top of the other...just to stick. (Researcher asks him to shade side that will overlap on each net. He has forgotten to add an extra square for one net, but he notices his mistake as soon as he is challenged) No, Miss, I made a mistake...I made a mistake. (Gaylene asks, "Now, how's that going to stick on top of the other when there's only six sides?") Okay...I'm going to test this. (He starts drawing a net to cut out. He realises his mistake.) I never counted this, Miss...it was six all the time. <i>When did you realise that you had made a mistake, Trevor?</i> When I...when I tried to build it. <i>And how did you know it was going to look like that?</i> That you were going to arrange them like that? Miss, I put that one there...and then I packed this one on top of that one. (Pointing to picture of box in number 4, shows with ruler.) (Challenges Trevor about the number of squares on each net.)
Some learners do not seem to be able to communicate visual images or mental processes / Do not have mental image?	Cindy	
Some learners can challenge one another's solutions	Gaylene	
Can reflect on mental processes	All learners except Cindy	

General profile of learners based on Activity 7:

Trevor:

- Works quickly
- Folds up from the net (strategy 8)
- Cuts and folds net to check himself and demonstrate to Researcher
- Corrects himself after discussions with Gaylene
- Draws additional nets for question 4.

Cindy:

- Has to cut and fold: This does not seem to help
- Unsure of herself - wants to rub out original drawing
- Copying Trevor?
- Possibly folding up the net?
- Draws additional nets for question 4.

Gaylene:

- Evidence of both strategies 8 and 9 on question 3
- Can challenge others
- Works quickly
- Develops own numbering system for sides
- Draws additional nets for question 4.

Belinda:

- Admits to Researcher that she does not understand
- Needs to cut and fold - but seems fine after that
- Starts with net and folds up (strategy 8)

Ashton:

- Works quickly
- Want to try on scrap paper to see if it "works"
- Can discuss drawings with others
- Unfolds the box (strategy 9)
- Starts to generalise the solution (strategy 10)
- Draws additional nets, but has a problem with one drawing in question 4.

James:

- Starts to generalise the solution (strategy 10)
- Draws accurate three-dimensional diagram to help him explain
- Concerned about accuracy of drawing
- Can explain how net folds up
- Does additional drawings for number 4
- Starts by folding up net (rolling action) (strategy 8).

Ryan:

- Seems confused at first - asks about number of squares required: solves this after discussion with A (Does seem to have a mental picture)
- Explains how starts with squares and folds up net
- Joins in discussion on generalisation (strategy 10)
- Draws additional nets for question 4.

Stuart:

- Drawing - draws side flaps "skew" in question 4 (side flaps are parallelograms): "I saw it skew so I drew it skew like that"
- Folds up from the net (strategy 8)
- Can recognise that some of his nets in question 3 are congruent: "...just flip it or turn it around"
- Reflecting on what he is doing - notices that one net is missing a square.