Novel Approaches to Performance Evaluation and Benchmarking for Energy-Efficient Multicast: Empirical Study of Coded Packet Wireless Networks

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Declaration

I declare that this thesis is my own work. Where collaboration with other people has taken place, or material generated by other researchers is included, the parties and/or materials are indicated in the acknowledgements or are explicitly stated with references as appropriate.

This work is being submitted for the Doctor of Philosophy in Electrical Engineering at the University of Cape Town. It has not been submitted to any other university for any other degree or examination.

AJIBESIN, Adeyemi Abel

Name	Date

Dedication

To the safe arrival of Nigerian **Chibok girls** and restoration of their dreams.

In April 2014, over 200 secondary school girls were kidnapped from the Government Secondary School by a group called ***boko haram*** (Western education is a sin).

&

To GOD ALMIGHTY, without whom nothing would have been possible,

I dedicated this work.

Nobel Laureate, Emeritus Professor Wole Soyinka on February 11, 2015 said:

*"When a thing like this abduction of **Chibok Girls** happens,*
a leader doesn't hesitate.
In this war the entire nation must be mobilised because we are facing enemies
of humanity whose only mission is to destroy.
The whole nation needs to be mobilised, not just the arm forces, but the citizenry as well,
*because the **boko haram** threat is existential.*
We must be practical. We will never see those girls again in the same form.
But we will never abandon them.
We can never really have closure, because of the weight of guilt
*we should feel towards the **Chibok Girls**"*

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Abstract

With the advancement of communication networks, a great number of multicast applications such as multimedia, video and audio communications have emerged. As a result, energy efficient multicast in wireless networks is becoming increasingly important in the field of Information and Communications Technology (ICT). According to the study by Gartner and Environmental Protection Agency (EPA) report presented to United State Congress in 2007, energy consumption of ICT nodes accounts for 3% of the worldwide energy supply and is responsible for 2% of the global Carbon dioxide (CO_2) emission. However, several initiatives are being put in place to reduce the energy consumption of the ICT sector in general. A review of related literature reveals that existing approaches to energy efficient multicast are largely evaluated using a single metric and while the single metric is appropriate for effective performance, it is unsuitable for measuring efficiency adequately. This thesis studied existing coded packet methods for energy efficiency in ad hoc wireless networks and investigates efficiency frontier, which is the expected minimum energy within the minimum energy multicast framework. The energy efficiency performance was based on effective evaluation and there was no way an inefficient network could reach a level of being an efficiency frontier. Hence, this work looked at the position of how true efficiency evaluation is obtained when the entire network under examination attains their efficiency frontiers using ratios of weighted outputs to weighted inputs with multiple variables.

To address these challenges and assist network operators when formulating their network policies and performing network administrations, this thesis proposed novel approaches that are based on Data Envelopment Analysis (DEA) methodology to appropriately evaluate the efficiency of multicast energy and further minimizes energy transmission in ad hoc wireless networks without affecting the overall network performance. The DEA, which was used to study the relative efficiency and productivity of systems in Economic and Operational Research disciplines, is a non-parametric method that relies on linear programming technique for optimization of discrete units of observation called the decision making units (DMUs). Thus, the main goal of this work was to design an empirical DEA architecture that incorporates Technical Efficiency (TE), Scale Efficiency (SE) and Energy Gap (EG) and Benchmarking models to extend the minimum energy multicast system. The first novel contribution of this thesis is the

adaptation of the Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) models to develop Envelopment models that are based on input-orientation approach, and assuming constant returns to scale (CRS) and variable returns to scale (VRS) in comparison with the existing techniques in literature for the implementation of TE in ad hoc wireless networks. Subsequently, the Slack models were formulated to improve the performance of the Envelopment models. Hence, the Envelopment models were only able to evaluate the TE scores (ratings) of ad hoc wireless networks thereby classifying them into efficient or inefficient networks. More so, the Slack models were able to identify the inefficient, the weak efficient and the full efficient ad hoc wireless networks and project the inefficient, the weak efficient unto their efficiency frontiers so that they also become full efficient. The SE was obtained by comparing the TE measures, and derivative parameters assumptions of CRS and VRS.

In addition, a novel Benchmarking model was proposed to establish standard of excellence among the ad hoc wireless networks. Similar to Envelopment models, the CCR and BCC models were adapted to develop variable-benchmarking models that are based on input-orientation approach, and assuming CRS and VRS are compared with existing techniques in literature for the implementation of benchmark in ad hoc wireless networks. This architecture ensures that all the weak efficient and inefficient ad hoc wireless networks that were identified by the Envelopment and Slack models performed efficiently according to the best practice meaning they are on efficiency frontier. To achieve this, the architecture considered an Efficiency Reference Set (ERS) to create peers for the weak efficient and inefficient ad hoc wireless networks. In addition to this, it considered the Lambdas to calculate the extent to which weak efficient and inefficient ad hoc wireless networks would observe or catch up with their peer networks.

Furthermore, in order to estimate the amount of energy reduction in ad hoc wireless networks and address the concerns of the ICT environmentalist, a novel energy gap (EG) model was formulated to analyse and compare energy reduction using empirical DEA architecture for minimum energy multicast and the existing architecture that was designed based on network coding technique. This is important because if the entire ad hoc wireless networks operated efficiently, then, the excess energy that could be very hazardous for environmental sustainability and global warming can be conserved.

The Envelopment, the Slack, EG and the Benchmarking models developed are implemented using the DEA tool, which technically consists of DEA Solver and Linear Programing (LP) Solver libraries available over the Internet as open source or propriety package. The data set used for the implementation of these models is obtained from the simulation results of the minimum energy multicast framework. Thus the primary basis used to validate the claims is done through simulation, and then the DEA analysis of the data produced from simulated scenarios are reported. Empirical results using DEA tool show an improvement in term of frontier performance and energy reduction when ad hoc wireless networks operated efficiently compare to the existing solutions that are implemented using simulation tool for the same data set.

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Glossary

The following terms are defined in the context of this report:

Allocative efficiency: The inputs that are used in the proportion which minimizes the cost of output (production) whether, for any level of output (production) given input prices.

Benchmarking: The process of comparing the performance of an individual network against a benchmark, or ideal level of performance. Benchmarks can be set on the basis of performance over time or across a sample of similar networks, or against some externally set standard.

Best practice: In this context, the set of network management and work practices which results in the highest potential, or optimal, quantity and combination of outputs for a given quantity and combination of inputs (*productivity*) for a group of similar organisations. Best practice can be identified at a number of levels, including network, nodes and segment of a network.

Constant returns to scale (CRS): Constant returns to scale may be assumed if an increase in a unit's inputs leads to a proportionate increase in its outputs i.e. there is a one-to-one, linear relationship between inputs and outputs. For example, if a 10% increase in inputs yields a 10% increase in outputs, the unit is operating at constant returns to scale. This means that no matter what scale the unit operates at, its efficiency will, assuming its current operating practices remain unchanged.

Convexity constraint: The convexity constraint, which forms part of the formulation of the BCC model, ensures that each composite unit is a convex combination of its reference units.

Data Envelopment Analysis (DEA): is a non-parametric technique, used for performance measurement and benchmarking. It is a *linear programming* technique which identifies *best practice* within a sample and measures *efficiency* based on differences between observed and BEST PRACTICE units. DEA is typically used to measure *technical efficiency*.

Data set: The data set is the group of units (DMU's) and the values of their inputs and outputs to be included in the analysis. The data set is usually presented in tabular form (often initially in a spreadsheet), where the unit names constitute the rows and the input and output variables constitute the columns. Zero values are not allowed in DEA and where the value of an input or output is missing, that particular unit may have to be omitted from the data set (unless a substitute value can be agreed upon).

Decision making unit (DMU): Decision making unit was the name used by Charnes et al (1978) to describe the units being analysed in DEA. The use of this term is intended to redirect the emphasis of the analysis from profit making businesses to decision making entities. In other words, the analysis which is performed can be applied to any unit based enterprise and need have nothing to do with profit.

Decreasing returns to scale (DRS): Decreasing returns to scale are operating when an increase in a unit's inputs result in a less than proportionate increase in its outputs.

Dual model: The dual model and the primal (CCR) model provide two ways of looking at the same problem and the efficiency scores calculated are the same with both. Mathematically, the dual model is much faster to solve (although its formulation looks more complex). The difference between the two is that for each unit the dual model (internally) tries to create a hypothetical composite unit, from the existing units, that will out-perform the unit being analysed. If, within the dual model, this composite unit can be created, then the original unit is found to be inefficient, otherwise the unit is efficient.

Effectiveness: Degree to which the outputs of a service provider achieve the stated objectives of that service — for example, the extent to which networks are meeting the set configurations. In the case of network services, the network administrator normally sets such configuration.

Efficiency: Degree to which the observed use of resources to produce outputs of a given quality matches the optimal use of resources to produce outputs of a given quality. This can be assessed in terms of *technical efficiency* and *allocative efficiency*.

Efficient frontier: The efficient frontier is the frontier (envelope) representing “best performance” and is made up of the units in the data set which are most efficient in transforming their inputs into outputs. The units that determine the frontier are those classified as being 100% efficient. Any unit not on the frontier has an efficiency rating of less than 100%.

Increasing returns to scale (IRS): Increasing returns to scale exist when an increase in a unit’s inputs yields a greater than proportionate increase in its outputs.

Inefficient unit: An inefficient unit is one which, when compared with the actual performance achieved by other units in the analysis, should be able to produce its current level of outputs with fewer inputs or generate a higher level of outputs given the same inputs.

Inputs: An input is any resource used by a unit to produce its outputs (products or services). This can include resources which are not a product but are an attribute of the environment in which the units operate. They can be controlled or uncontrolled.

Input minimization: Input minimization is the DEA mode adopted when the analysis tries to minimize the amount of inputs used to produce the specified outputs. (The opposite of input minimization is output maximization).

Input orientated: Input orientated is a term used in conjunction with the BCC and CCR ratio models, to indicate that an inefficient unit may be made efficient by reducing the proportions of its inputs but keeping the output proportions constant.

Linear program: A set of linear mathematical equations for which a solution can be obtained subject to an upper bound (maximization) or a lower bound (minimization).

Multiplier form: Associated with both the BCC and CCR models, the multiplier form is both a primal and a dual formulation. The multiplier form of DEA model formulation involves virtual multipliers.

Output: Outputs are the products or outcome which results from the processing and consumption of inputs (resources). An output may be physical goods or services or a measure of how effectively a unit has achieved its goals.

Output maximization: Output maximization is the DEA mode adopted when the analysis tries to maximize the outputs produced for a fixed amount of inputs.

Output orientated: Output orientated is a term used in conjunction with the BCC and CCR ratio models, to indicate that an inefficient unit may be made efficient by increasing the proportions of its outputs while keeping the input proportions constant (see also input minimization and output maximization).

Peers: In DEA studies, the peers are group of best practice networks with which a relatively inefficient network is compared.

Peer group: Another name for a Reference Set.

Primal (CCR) model: The primal model allows a set of optimal weights to be calculated for each variable (input and output) to maximize a unit's efficiency score. The weights are such that if applied to any other unit in the data set the efficiency score would not exceed 1 (or 100%).

Production frontier: The curve plotting the minimum amount of an input (or combination of inputs) required to produce a given quantity of output (or combination of outputs).

Productivity: Measure of the output produced from the use of a given quantity of inputs. This may include all inputs and all outputs (*total factor productivity*) or a subset of inputs and outputs (*partial productivity*). Productivity varies as a result of differences in *production technology*, differences in the *technical efficiency* of the network, and the *external operating environment* in which production occurs.

Ratio models: Both the BCC and CCR models are called ratio models because they define efficiency as the ratio of weighted outputs divided by weighted inputs.

Reference set: The reference set of an inefficient unit is the set of efficient units to which the inefficient unit has been most directly compared when calculating its efficiency rating. It contains the efficient units which have the most similar input/output orientation to the inefficient unit and should therefore provide examples of good operating practice for the inefficient unit to emulate.

Returns to scale (RS): Relationship between output and inputs. Returns can be constant, increasing or decreasing depending on whether output increases in proportion to, more than or less than inputs, respectively. In the case of multiple inputs and outputs, this means how outputs change when there is an equi-proportionate change in all inputs.

Scale efficiency: The extent to which a network can take advantage of returns to scale by altering its size towards optimal scale (which is defined as the region in which there are constant *returns to scale* in the relationship between outputs and inputs).

Slack(s): The extra amount by which an input (output) can be reduced (increased) to attain *technical efficiency* after all inputs (outputs) have been reduced (increased) in equal proportions to reach the *production frontier*. This is a feature of the piece-wise linear production frontier derived when using DEA.

Targets: The values of the inputs and outputs which would result in an inefficient unit becoming efficient.

Technical efficiency: Conversion of inputs such as energy into *outputs*. Technical efficiency is determined by the difference between the observed ratio of combined quantities of an entity's output to input and the ratio achieved by *best practice*. It can be expressed as the potential to increase quantities of outputs from given quantities of inputs, or the potential to reduce the quantities of inputs used in producing given quantities of outputs.

Variable: Variables are the input and output factors identified as being of particular importance to the operation of the units under consideration. Classification as inputs or outputs depends on the process being measured and the goals against which units are being measured. What may be an input when measured against one set of goals may be an output when considered under another.

Variable returns to scale (VRS): If an increase in a unit's inputs does not produce a proportional change in its outputs then the unit exhibits variable returns to scale. This means that as the unit changes its scale of operations its efficiency will either increase or decrease.

Virtual input/output: Virtual inputs are calculated by multiplying the value of the input with the corresponding optimal weight for the unit as given by the solution to the primal model. It is similar to virtual outputs. Virtual inputs/outputs define the level of importance attached to each factor. The sum of the virtual inputs for each unit always equals 1. The sum of the virtual outputs is equal to the unit's efficiency score.

Weights: Within DEA models weights are the 'unknowns' which are calculated to determine the efficiency of the units. The efficiency score is the weighted sum of outputs divided by the weighted sum of inputs for each unit. The weights are calculated to solve the linear program, in such a way that each unit is shown in the best possible light. Weights indicate the importance attached to each factor (input/ output) in the analysis.

Acronyms

BCC	Banker, Charnes and Cooper
CCR	Charnes, Cooper and Rhodes
CRS	Constant returns to scale
DEA	Data Envelopment Analysis
DMU	Decision Making Units
DRS	Decreasing returns to scale
ERS	Efficiency Reference Set
ICT	Information Communication Technologies
IRS	Increasing returns to scale
LP	Linear Programming
NC	Network Coding
NGN	Next Generation Network
MIP	Multicast Incremental Power
OR	Operations Research
RTS	Returns to scale
SBM	Slack-Based Model
TE	Technical Efficiency
TIE	Technical Inefficacy Efficiency
VRS	Variable returns to scale

Chapter One: General Introduction

The research project presented by Gartner shows that energy consumption of Information and Communication Technologies (ICTs) nodes accounts for 3% of the worldwide energy supply and also responsible for 2% of the global Carbon dioxide (CO_2) emission, which is almost equivalent to the aviation industry [1], [2], [3]. But an Environmental Protection Agency (EPA) report presented to United State Congress in 2007, emphasized that the emissions from ICT is rising faster [4]. The continuous increase in these emissions from ICT was projected to increase from 3% of total global emissions in 2009 and could reach a whopping 12% by 2020 [5]. The study further estimated the CO_2 emissions for each ICT category. For example, it was shown that telecommunication infrastructure and nodes constitute 86% of the CO_2 emissions while server farm and data centers constituted the remaining 14% in the ICT sector [6], [7], [8].

As a measure to counteract this growth, the recent reports by the National Academy of Engineering identified 13 Grand Challenges of Engineering for the twenty-first century, where three of these challenges are on energy issue and all share a common goal - to reduce high energy consumption that leads to the global warming [9], [10]. But, the telecommunication infrastructure and nodes have continued to grow exponentially [11], [12]. Most of the past research have focused on improving the performance of these telecommunication systems and how to reduce the purchasing cost [10], [13]. In these reports, scant attention was recorded on minimizing energy consumed by the telecommunication nodes. Also, there is almost little attention on the effect of telecommunication systems to the environment. Thus the contribution of current telecommunication infrastructure to increased carbon emissions has given green communication a major attention [14], [15]. Green communication is an optimal use of telecommunication systems for managing the environmental sustainability of enterprise, which includes its operations, products, services and resources [16], [17]. Its goals include achieving improved energy efficiency in the use of telecommunication systems, and to increase the utilisation of already installed resources.

The shift and emphasis toward supporting the ICT, especially the telecommunication systems needs by consuming less energy and minimising carbon emissions was the main motivations behind energy-efficient networking technologies which this thesis addressed [18], [19], [20], [21]. Therefore, it is important to discuss the recent frameworks that were proposed to enable energy-efficient networking and communications. Our focus is to address the problem of energy efficiency using ‘minimum energy multicast’ framework applied to ad hoc wireless networks. This framework for energy efficiency based its evaluation on average minimum energy rather than expected minimum energy. The next subsection discusses the concepts of minimum energy multicast framework.

1.1 Minimum Energy Multicast – an Effective Framework for Energy Efficiency

The main optimization problem for energy efficiency broadcast and multicast routing in ad hoc wireless networks is to minimise the total transmission power assigned to all nodes [22]. This is widely recognized as one of the performance challenges in wireless networking. The minimum energy multicast problem in ad hoc wireless networks is solvable as a linear program, assuming coded packet technique [23]. Compared with conventional routing solutions, coded packet technique does not only promise a potentially lower multicast energy, but also enables finding the optimal solution in polynomial time. In this thesis, the minimum energy multicast framework involving routing and coding is considered. Application to the ad hoc wireless networks is considered. Other energy efficiency frameworks that was presented in the literature include virtualization technique for energy reduction in data centers [13], [24]. However, they were all designed to achieve similar goal using effective performance evaluation approach [22], [25].

1.1.1 Energy-efficient networking approaches

Researchers have worked on energy-efficient networking for several years especially with the growth of the wireless networks such as wireless sensor networks, mesh networks and ad hoc wireless networks [22]. Many studies have explored the topic of energy efficiency of these networks [26],[27],[28]. Some of the studies that were investigated in literature include routing, coding, cross-layer designs, MAC protocols, spectrum allocation, resource allocation and scheduling. Our scope does not cover all these techniques applied to different types of networks. The goal of this section is to present the recent advances that were made specifically to improve the energy efficiency of ad hoc wireless networks. So we discuss energy efficiency in the routing and coding approaches.

An approach to energy efficiency is the exploration of the broadcast nature of the wireless links [18]. Wireless links are either omnidirectional or directed over a large area to ensure that transmissions are received by more than one node. This feature has effect on multicast networks, and it is known as Wireless Multicast Advantage (WMA) [29]. In routing, the problem of performing energy-efficient multicast considering WMA is NP-complete [30]. Thus the problem of minimum energy broadcast/multicast is solved in wireline case by various minimum weight spanning tree algorithms but the solutions are generally sub-optimal [31]. However, alternative approach using heuristics method was employed [32]. An example of this method is the Multicast Incremental Power (MIP) algorithm but this technique is also sub-optimal [32], [33]. In order to maximise energy efficiency, the coding technique was considered to further simplify the problem of minimum energy multicast [34]. This is achieved by solving the problem of minimum energy multicast in polynomial time. As a result, optimal energy is less when coding is used compared to routing technique. Hence, coding in packet networks is a promising scheme for minimum energy multicast. Simulation results have shown that coding can reduce between 13% to 49% average total multicast energy in random wireless networks of varying size over MIP technique [33], [35].

1.1.2 Energy-efficient communication protocols

There are two major characteristics of a protocol that can affect energy efficiency. The first is the energy overheads incurred to transmit the same amount of data [10]. Protocols with higher overheads experience degradation in energy efficiency. The second major factor that can affect the energy efficiency of a protocol is the time overhead. The longer the time it takes to send data, the longer a radio interface should be active increasing the energy consumption. In this work, we studied routing and coding, which are the energy-efficient protocols that were recently considered in wireless networks. There are numerous publications on energy-efficient communication protocols for wireless ad hoc networks [36], [36]. Most of these publications were focused on energy-aware routing, MAC techniques and related performance issues [37], [38]. Some surveys on energy-efficient are also provided in [39], [40], [41]. However, there is a need for improved framework to further explore energy-efficient communication so that they perform at the expected optimal value known as *efficient frontier* [42], [43]. A current framework that is well established in literature is the minimum cost (energy) multicast, and is designed to explore routing and coding protocols, and to reduce multicast energy in both wireline and wireless networks. This thesis is restricted to energy-efficient methods with extension to models development to reduce multicast energy in ad hoc wireless networks so that efficient frontier is achieved.

1.1.3 Energy efficiency Standard and performance metrics

Today, energy efficiency has become an important metric that is being increasingly used to evaluate the energy consumption of Decision Making Units (DMUs) such as devices, hardware, software, networking architectures, and communication protocols. Unfortunately, energy efficiency metrics is yet to be standardised, and this makes energy performance comparisons for networking devices hard to achieve in practice [10]. The need to standardise energy efficiency was mentioned in several literatures and expected to be resolved soon [6],[13]. However, in this thesis we consider normalised energy consumption, which is the sum of energy consumed by all the components in the network. In the literature, some works assumed absolute Power in Watts, and Power per Bit for energy efficiency evaluation [10].

1.2 Overview of Multicasting and Related Energy-Efficient Techniques

This section briefly discusses the concept of multicast, which is an important technique for the minimum energy multicast framework. It then discusses some of the related techniques that are considered for minimum energy multicast framework.

1.2.1 Multicasting Technique

Multicasting is an important component of the minimum energy multicast that was explored to achieve effective energy transmission in wireline and wireless networks. It is an effective method of reducing energy and time overheads in both wireline and wireless networks. The technique is used to support group communication than unicasting as compared to broadcasting also, because it allows transmission and routing of packets to multiple destinations using fewer resources [44]. In other words, multicast is the data delivery technique where the same data unit is transmitted from a single source to selected destination in a single invocation of service. For example, Figure 1.1 which is taken from [29] illustrates a cost and time reduction using multicasting operation. From the Figure, the intermediate nodes 2, 3, 5, and 7 are selected and their transmission ranges are determined such that a packet is forwarded from source node 1 to sink nodes 8, 9, 10, 11, and 12 through the selected intermediate nodes 2, 3, 5, and 7.

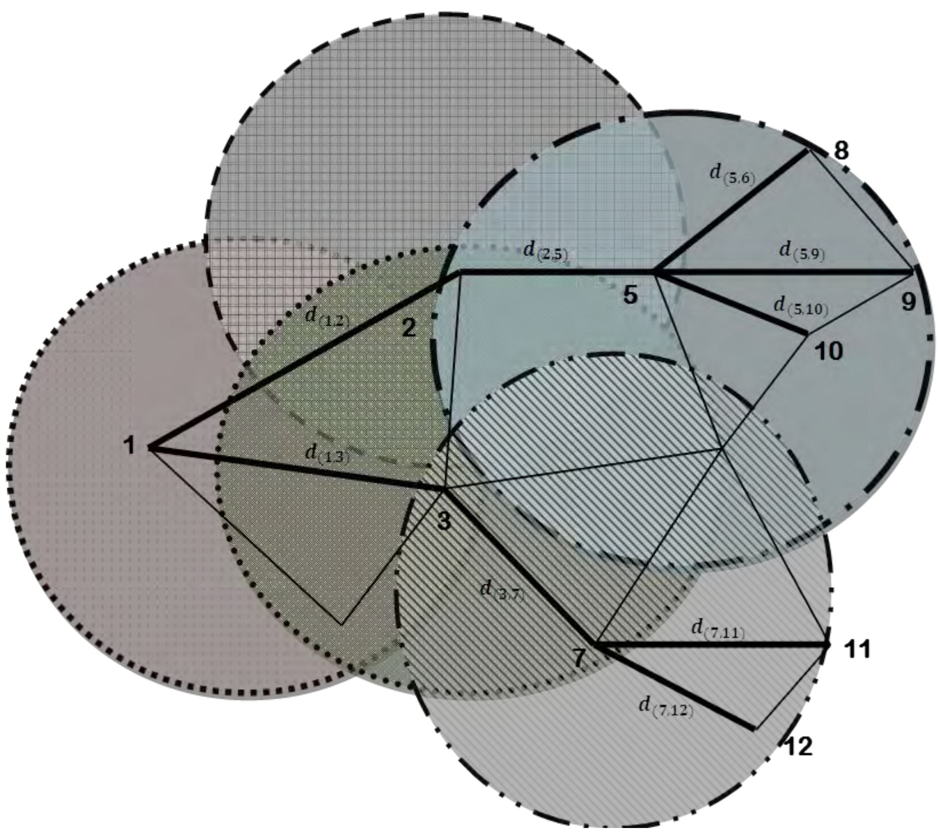

Figure 1.1: An example of minimum energy multicast tree consisting of five paths (i. 1→2→5→8, ii. 1→2→5→9, iii. 1→2→5→10, iv. 1→3→7→11, v. 1→3→7→12)

1.2.2 Wireless Multicast Advantage (WMA) Techniques

Another important technique for minimum energy multicast framework is the Wireless Multicast Advantage (WMA). Using omnidirectional antenna, wireless links are broadcast in nature. The omnidirectional antenna is applied so that the signal transmitted from a wireless node may reach several neighbouring nodes [45], as illustrated in Figure 1.2. It means that a transmission from node *i* to node *j* implies transmission to nodes *m* and *k* for free (given that $i \rightarrow k$, and $i \rightarrow m$ are within radio range of $i \rightarrow j$) and is termed the Wireless Multicast Advantage (WMA) [45]. Instead of making use of the incremental function of the energy experience with wireline networks given by $P_{i\{j,k,m\}} = P_{i,j} + P_{i,k} + P_{i,m}$; the wireless network can optimise the energy as $P_{i\{j,k,m\}} = \max\{P_{i,j}, P_{i,k}, P_{i,m}\}$. This approach is used to optimise the energy usage in wireless networks and is known as the *node based model*, while that of wireline is known as *link based model* [46]. This explains why multicasting is different in wireless networks compared to wireline networks.

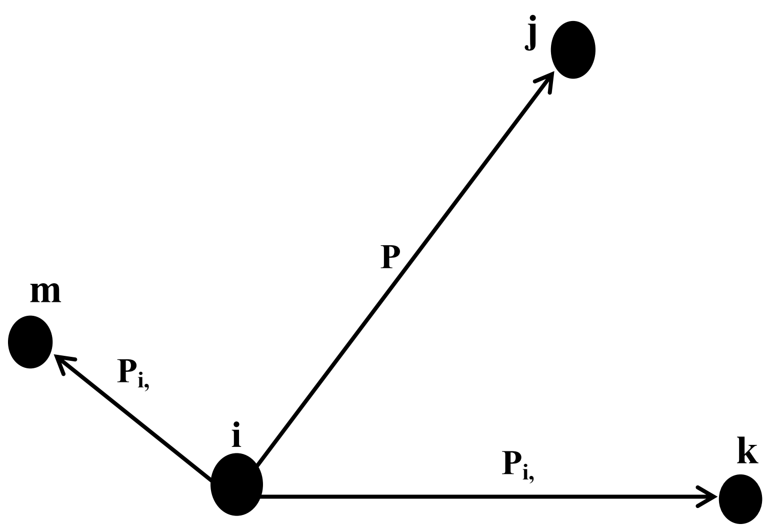

Figure 1.2: The wireless multicast advantage (WMA)

1.2.3 Wireless Network Routing and Coding Technique

In order to further explore the broadcast nature of the wireless medium the idea of network coding was later considered [47]. Network coding technique allows intermediate nodes to combine packet streams and consequently improves the overhead as well as enhancing the robustness of wireless networks [48], [49]**.** A simple example demonstrating the use of network coding at a 2-way relay-node in a wireless setting is shown in Figure 1.3 and Figure 1.4. While Figure 1.4 is used to illustrate how network coding technique reduce overheads. Figure 1.3 demonstrates the communication exchange without network coding. The Figures show the exchange of packets transmission between terminal A and terminal B via a relay. Figure 1.3 shows terminals A and B sending packets to the relay, and the relay just forwards the packets to the respective destination. The result of the packet exchange is presented in the attached table. It could be observed that the operation needed four time slots to achieve the desired transmissions, which is the traditional approach to information exchange via a relay. Figure 1.4 demonstrates the communication exchange with network coding. This method allows the relay to code the packets received from terminals A and B using XORs, and then broadcasts the coded packets. Consequently, the two terminals received their intended packets from the relay. The result as presented in the attached table requires three times slots to achieve the desired transmission. This technique improves the communication overheads in wireless networks. [47], [50], [51].

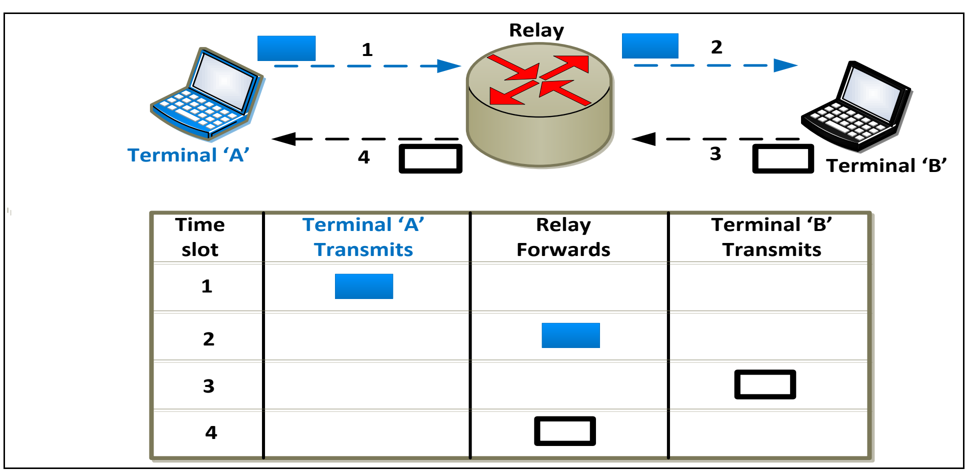

Figure 1.3: Traditional Approach to Information Exchange via a relay, Time slot = 4

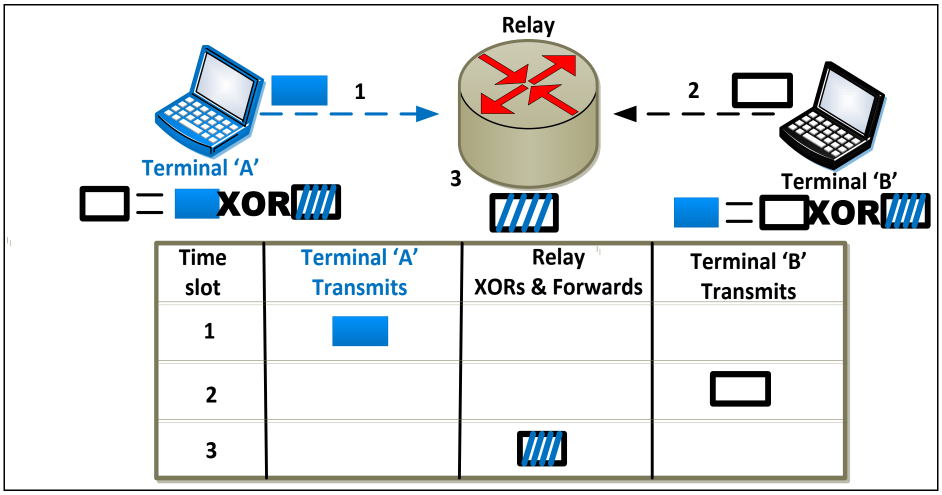

Figure 1.4: Coded packet Approach to Information Exchange via a relay, Time slot = 3

1.3 Background and Motivation

Several techniques were presented to optimise multicast operations in wireline and wireless networks, but the coded packet technique whereby relay nodes mix packets using mathematical operations was a promising method of reducing the multicast energy in ad hoc wireless networks [33], [23]. The technique has the capability to improve the overhead in the networks especially in wireless networks where the broadcast feature and the diversity of the links can be explored [46]. [47], [50]. As a results of these unique features, the energy-efficient using coded packet wireless method performed better than the traditional Steiner Tree method and other minimum energy multicast methods such as Multicast Least-Unicast-Cost (MLU) algorithm and Multicast Link-based MST (MLiMST) algorithms and multicast incremental power (MIP) [29], [33], [52] and [45]. However, the goals of these methods are similar. They seek to reduce as much as possible the multicast energy in wireless networks. But some questions remained unanswered, for example; how efficient are these methods in reducing multicast energy? Is there a method close to the best practice? These questions are impossible to answer without extensive study of the technical efficiency evaluation. The literature review shows that the existing methods for minimum energy multicast were largely based on effective performance of the algorithm rather than technical efficiency performance. Therefore, an interesting problem that this thesis addresses is how a particular ad hoc wireless network performs overall in terms of multicasting messages using expected minimum energy, and without sacrificing the output performance? In order to achieve this goal, our evaluation method considered the *best practice or efficiency frontier*. This method is necessary to adequately evaluate energy efficiency of ad hoc wireless networks. Generally, the problem is divided into two - the performance evaluation and benchmarking. In this thesis, four models are proposed to address these problems.

1.3.1 Problem Description

Given multiple set of ad hoc wireless networks that are equipped with coded packet algorithm such that each of the ad hoc network multicast messages successfully to some selected group of nodes with average minimum energy, then, a problem with these networks is that many of them are not multicast messages at the expected minimum energy or targeted energy. A similar problem that was established in literature used evaluation based on average performance. In addition, this type of evaluation is termed effective performance measurement. For example, how effective is the coded packet algorithm in multicast messages to selected group of nodes? The approach to answer this question was to calculate the average minimum multicast energy and then ranked them according to the lowest. The lowest of the average minimum multicast energy is the most effective using coded packet algorithm. However, the lowest average minimum multicast energy does not mean it is efficient or the most efficient. So what is the expected or targeted minimum multicast energy? This is the first question that motivates this work. Any ad hoc wireless network that multicast messages to a selected group of nodes using expected minimum energy is said to perform according to the best practice or attain efficient frontier). Performance according to the best practice is possible if a network makes use of combination of its multiple input and multiple output resources correctly.

However, the best practice or efficient frontier has not been evaluated quantitatively. Thus the main focus in this work is to quantitatively evaluate the technical efficiency (TE) of ad hoc wireless networks and benchmark their performance with the best practice. Given that the best practice network are those that stay on efficient frontier represented by 100%. The challenge is how other networks that fell below (inefficient networks) this benchmark reach the efficient frontier (i.e., 100%). It should be noted that networks, which fell below the benchmark even though may be effective based on the average performance, but inefficient based on best practice or frontier performance. A network may be inefficient if the inputs resources such as energy are underutilised. The excess energy for instance can generate enormous amount of CO_2 that is very hazardous for environmental sustainability and global warming. Ad hoc wireless networks that are multicasting at expected minimum energy or targeted energy are not as hazardous to the environment because they are operating according to the best-practice. In order for networks to operate according to the best-practice, first, there is a strong need to develop models that will extend the current frameworks so that networks performance level in terms of best-practice is determined. Second, a model is required so that the networks identified as inefficient (those networks that are below the efficient frontier) are projected unto the efficient frontier. Once they operate at their efficient frontier, these networks will not be as hazardous to the environment and the excess energy could be conserved. The third model is necessary to evaluate the amount of energy that is saved if the networks multicast at the expected minimum energy. The fourth model will be responsible for formation of peer group to benchmark the inefficient networks. The peer group serves as efficiency reference set (ERS) for the inefficient networks. Interestingly, optimisation in this direction has raised a number of questions, namely:

i. How efficient are the current algorithms?
ii. How do we determine the degree of efficiency for a network against the best-practice networks?
iii. How do we project the inefficient networks unto efficient frontier?
iv. How do we evaluate the amount energy that could be hazardous to the environment?
v. How do we benchmark inefficient networks with their peer group?

These questions are addressed in Chapters 3, 4, 5, 6, and 7 of this thesis respectively. Also, these questions are solved by formulating optimization tasks and appropriate models, developed at different levels of our proposed system architecture presented in Chapter 4.

1.3.2 Effectiveness vs Efficiency

The distinction between the problem that we address and the current practice is the evaluation of network performance in terms of their effectiveness and efficiency. The current methods for energy-efficient multicast networks focused on effective performance while we focus on efficiency performance. Therefore, it is important to differentiate between these two concepts. In the literature, the term "efficient" is very much confused and misused instead of the term "effective". Efficiency is a measurable concept that quantitatively evaluates the ratio of output to input. Thus efficiency is concerned with *how well the things are done*. With efficiency a larger output from the same inputs or the same output with less of one or more inputs without increasing the amount of other inputs could be achieved. In order to achieve efficiency, performance evaluation is based on correct combination of multiple inputs and multiple outputs [53]. "Effectiveness" is mainly concerned with achieving objectives. In terms of network, effectiveness is the ability of a network to attain its predetermined goals and objectives. It is the degree to which the outputs of a network achieve the stated objectives of that network. Thus objective of effective evaluation is to produce results or outcome [53]. Thus effectiveness is concerned about *doing the right things rather than how well the things are done*.

In the case of energy system for instance, energy effectiveness is concerned with comparing different ways of achieving the same objective such that the most energy effective will be preferred to the alternatives being compared. This is illustrated as: g*iven that the standard requirement of an ad hoc wireless network for instance is to reduce energy by 7 units, and if given that Network(X) reduces the energy by 8 units while Network(Y) reduces it by 6 units, then it can easily be concluded that Network(X) is effective because it meets the standard requirements. Or we say Network(X) is more effective than Network(Y) in reducing the energy because 8 > 6. It is also possible to condemn Network(Y) because it performed below the required standard.* However, this does not necessarily mean that Network(X) is efficient or more efficient than Network(Y) until when its multiple inputs and multiple outputs variables or the resources consumed in meeting the standard requirements is carefully evaluated.

1.3.3 Illustration of Best Practice for Energy Efficiency

We consider a problem whereby multiple sets of ad hoc wireless networks are multicasting with multiple inputs and multiple outputs as illustrated in Figure 1.5. Each of the networks is assumed to be equipped with coded packet network algorithm. Also, each network is assumed to successfully multicast messages to a selected group of nodes (receivers) with a certain average multicast energy of the set of networks computed according to the coded packet algorithm. For easy illustration, we assume that each of the networks uses one input resources (energy) to multicast messages to a certain number of receivers as shown in Table 1.1.

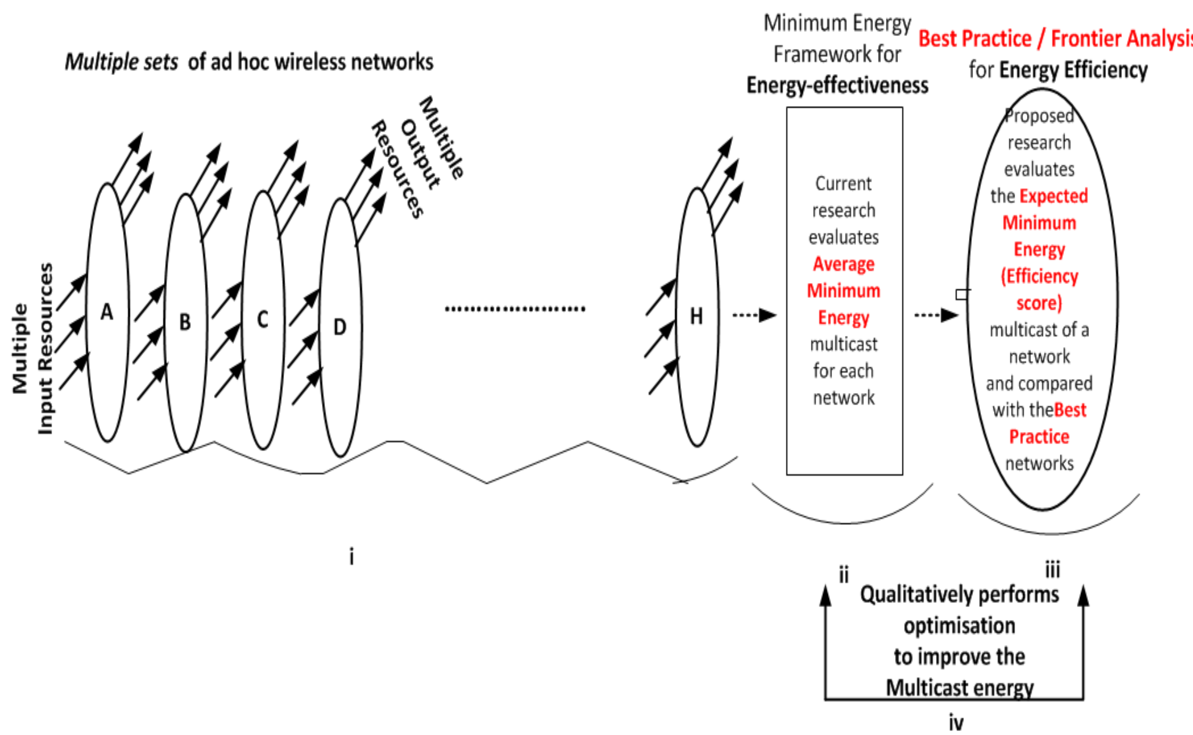

Figure 1.5: Illustrating best practice with multiple sets of ad hoc wireless networks multicasting with multiple inputs and multiple outputs

A task here is to identify the ad hoc wireless networks that are inefficient. Also, we want to know how the inefficient ad hoc networks could be projected unto the efficient frontier. Figure 1.6 represents the plots for all the ad hoc wireless networks with input on the *x* axis (the horizontal axis), and the output on the y axis (the vertical axis). Figure 1.6 shows the technical efficiency of each ad hoc wireless network with the picture of best practice frontier, as well as a regression line that predicts the average behaviour of the observe ad hoc wireless networks. The best practice frontier or efficient frontier is the one that floats on top of the data observations, indicating the best practice frontier ad hoc wireless network(s). In this work, we did not consider regression method because as shown in the Figure, it requires a specific functional relationship between inputs and outputs but efficient frontier approach does not require such *a priori* information.

Observe that only ad hoc wireless network E shows the best practice frontier with an efficiency of 1 or 100%. The line that spans from the origin to the score ad hoc wireless network E is the efficient frontier (best practice). By contrast, using the outcome based result (average) the coded packet method regards ad hoc wireless network E and H to perform at the same level. Also, using this method ad hoc wireless network H was preferred to C in terms of output produced whereas in terms of technical efficiency, ad hoc wireless network C is more close to the efficient frontier. However, the region envelop by efficient frontier is capable of improving their efficiency to become best performance like ad hoc wireless network E. The ad hoc wireless networks A, B, C, D, F, G, and H are below efficient frontier and could improve their efficiency performance [42].

Table 1.1: Example of simple efficiency ratio to determine best practice or efficient frontier

	A	B	C	D	E	F	G	H
Input	2	3	3	4	5	5	6	8
Output	1	1.5	2	3	5	2	3	5

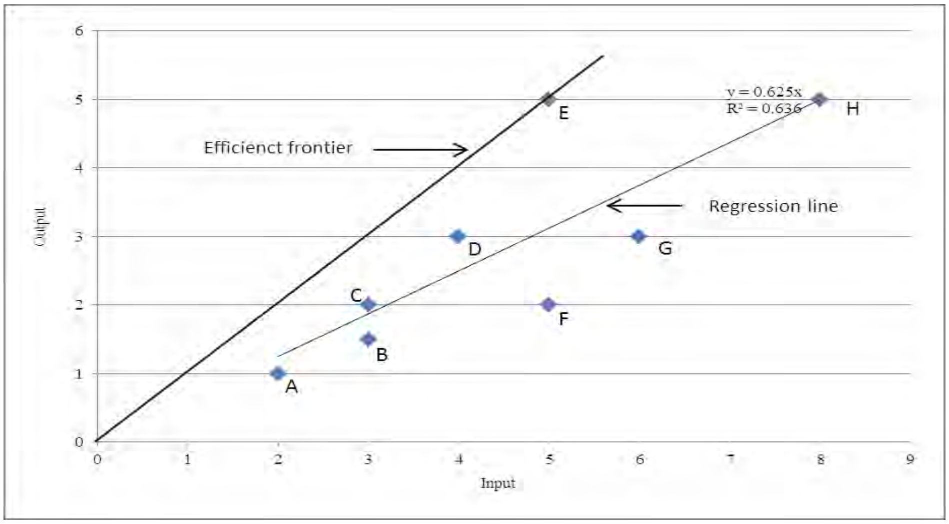

Figure 1.6: Efficient frontier vs Regression line

Note that whenever ad hoc wireless network is inefficient (sub-optimal), it is located beneath the efficient frontier. However, because these inefficient ad hoc wireless networks lies within Production Possibility Set (PPS) region they have potential to be moved unto their efficient frontier [42], [54]. Production Possibility Set (PPS) is defined as the set of all inputs and outputs of an ad hoc wireless network in which inputs can produce outputs. DEA models make use of PPS to evaluate relative efficiency of DMUs. There are two fundamental directions to achieve this move: The input-oriented and output-oriented directions. The input-oriented approach reduces the inputs while the outputs are fixed at their current levels. The output-oriented approach increases the outputs while the inputs are fixed at their current levels [55] [56]. Again, to demonstrate this concept, we consider the data in Table 1.1. This is represented in Figure 1.7. It is observed from Figure 1.7 that ad hoc wireless network F is inefficient. Considering input-oriented optimisation for F, then it will be projected onto point F'. Also, considering output-oriented optimisation for F, then it will be projected onto F". In Mathematical modelling, slack function is considered for projecting the inefficient networks unto their efficient frontier. Once the inefficient networks are projected to their efficient frontier, it is easy to evaluate the energy saved and determine the benchmark for the peer group so that standard of excellence could be achieved. It should be noted that while the DEA frontier technique is good at estimating relative efficiency, it is poor at absolute values. That is, it converges slowly to absolute efficiency not allowing a comparison to the theoretical maximum. Also the technique requires that each entity has a separate linear programming formulation, which leads to many LP iterations.

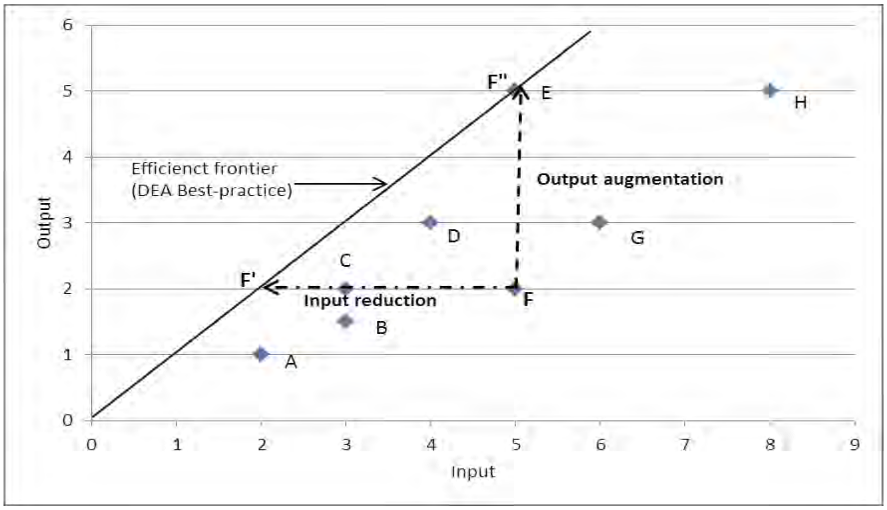

Figure 1.7: Input and Output Orientation for Inefficient Networks

Let consider a generalised theoretical frontier with some assumptions. Given that ad hoc wireless networks possessed the ability to use inputs (e.g., energy) in optimal proportions without sacrificing the output results (receiver nodes) [57], [58]. This is illustrated in Figure 1.8. Again, let us consider a network with single output *(y)*, under the assumption of Constant Returns to Scale (CRS) and using two inputs (x_1 and x_2), *EE'* represents the fully efficient network, which is a *theoretical frontier*. Also, let us assume that the efficiency score lies between *0* and *1*. As illustrated in the Figure, the network defined by Q is efficient while that of P is inefficient. Using these notations, the technical efficiency of the network is defined as ratio of O*Q* and OP. Then, it could be concluded that network Q is technically efficient because it is on EE' isoquant. In order to further analyse this network, assuming that a network's technical efficiency score is 75%, the interpretation is that OQ line is 75% of the whole line, and QP line is 25% of the whole OP line. So network P is 25% inefficient comparing to network Q, which is an efficient network. This method was presented for the evaluation of efficiency in economics and Operation Research (OR) to reduce input resources while the outputs are kept constant [57], [58].

In this work, energy efficiency evaluation using DEA tool that rely on mathematical programming, is considered [59]**.** By contrast, previous works on energy efficiency were largely evaluated based on effective performance. Energy efficiency is a way of managing and restraining the growth in energy consumption. For example an ad hoc wireless network is more energy efficient if it performs more multicast operations for the same energy input, or the same multicast operations for less energy input.

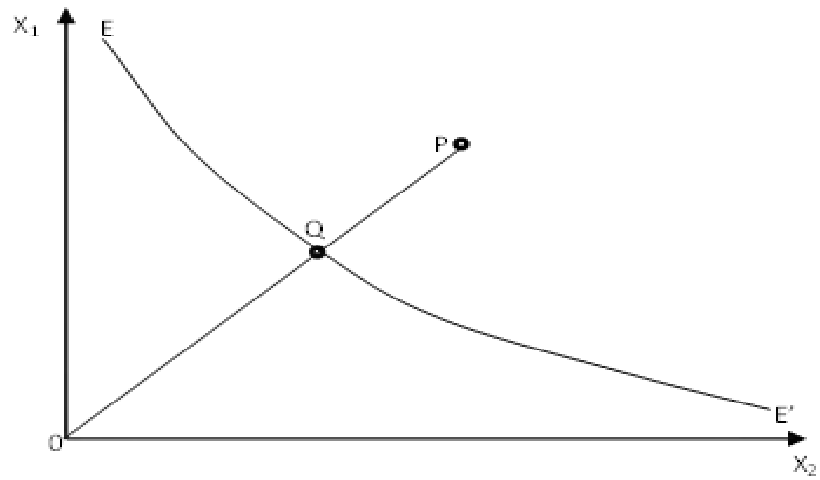

Figure 1.8: Efficiency Evaluation for Minimizing Input Resources

1.4 Particulars of Thesis

The previous sections described the importance of energy efficiency. The concept of efficient frontier was explained. The principle behind our idea is summarized as follows: In order to reduce the current energy consumption by ad hoc wireless multicast networks, our technique assumes that if a given ad hoc wireless network 'A' is capable of multicast Y(A) number of sinks (output) with X(A) expected minimum energy (input), then another ad hoc wireless networks should also be able to do the same or better. The key to this technique is to find the *best* virtual ad hoc wireless network(s) for each real ad hoc wireless network, achieving above. If the virtual ad hoc wireless network is better than the original ad hoc wireless network by multicast messages to the same number of sinks (output) with lower energy (inputs), then the original ad hoc wireless network is inefficient. Note that all the illustration provided, including graphical methods, are only useful for simple scenario. In real world, the situation often is a lot more complex as some systems have many inputs and outputs. The appropriate technique for calculating the efficiency of ad hoc wireless network is by using linear programing formulation. This is what we adopt in this thesis after investigating the current minimum energy multicast methods. Contributions of the thesis are discussed in details in the following subsection.

1.4.1 Objectives and Contributions

The beginning of the thesis is dedicated to the description of the general minimum multicast energy model, which is an efficient framework for energy-efficient communication. The major objectives and contributions of this thesis are summarized as follows.

- ***Investigate Minimum Energy Multicast Algorithms for Efficiency Performance:*** As an important part of efficiency frontier, we investigated minimum energy model for two algorithms: *the Multicast Incremental Power (MIP)* and the *coded packet algorithms*. The system model for this framework is presented in Chapter 3. We adopt multicast energy of the algorithms' output as a metric to compare the energy-efficiency between MIP and coded packet algorithms. Simulation results have shown that coding can reduce substantial amount of multicast energy in ad hoc wireless networks of varying size over MIP technique. This is expected simply because energy consumption is less when coding is used compared to routing technique. However, the expected minimum energy could not be reached using coded packet technique. Therefore, there is a need for alternative method that can meet this requirement. The central objective of this thesis is to develop empirical energy-efficient architecture for minimum energy multicast to evaluate the energy efficiency of ad hoc wireless networks and to project the inefficient ad hoc wireless networks unto their efficient frontier so that excess energy is conserved. This is done such that amount of energy consumed by ad hoc wireless networks are minimized without affecting their output performance. In order for multicast energy to meet the efficient frontier objective, we develop a family of *energy efficiency models*, which is suitable for measuring technical efficiency of networks appropriately. It is also suitable for projecting the inefficient networks unto their efficient frontier. The key difference from past work is the decisions using DEA is based on multi-criteria approach. In other words, the proposed architecture allows multiple inputs and multiple outputs. This is an econometric approach to efficiency evaluation. Specifically, four models are developed: The envelopment model, the slack model, the benchmark model and the energy gap (EG) model. These model are summarised as follows:

- ***A Novel Envelopment Models for Evaluating the Efficiency Scores (Ratings) of Multicast Energy in Ad Hoc Wireless Networks***: The envelopment model is the first model presented in Chapter 4. It is a new technique for technical efficiency evaluation to improve the energy efficiency performance in ad hoc wireless networks using multiple inputs and multiple outputs. With envelopment model, an ad hoc wireless network efficiency scores is known. In coded packet network, optimisation is done using simple linear programing technique. In technical efficiency approach, the same challenge became a ratio problem, thus making it more difficult especially when there are multiple inputs and multiple outputs variables. In essence, the model finds the maximum efficiency which an ad hoc network can achieve under a set of weights. These set of weights then provide optimal values that efficiency ratio can be achieved from n observations. By contrast, the coded packet approach does not resolve a ratio problem with multiple inputs and multiple outputs simultaneously. In other to evaluate the technical efficiency (TE) using a weighted ratio, a set of n observations on the ad hoc wireless networks where each observation, say j $\{j=1,2,\ldots,n\}$ uses n multiple inputs x_{ij} $(i=1,2,\ldots,m)$ to produce s multiple outputs y_{rj} $(r=1,2,\ldots,s)$ is considered. Then, performance in terms of efficiency ratio for ad hoc network j is expressed as: $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$, where $u_r\,(r=1,2,\ldots,s)$ and $v_i\,(i=1,2,\ldots,m)$ are unknown weights. The weights assigned to each input and each output is used as variables in the optimisation process. So if we intend to optimize a particular ad hoc wireless network, the objective is to: $max\frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{io}}$. Some constraint could be considered so that maximisation problem is bounded. For example, a set of normalisation for each ad hoc wireless network is assumed such that efficiency is less than or equal to unity. With this approach, an envelopment model was formulated using transformation technique to change the non-linear programing into a multiplier model, then the dual of multiplier was derived to obtain envelopment model. The efficiency scores of each ad hoc wireless networks were then obtained from the implementation of envelopment model. However, the result obtained from envelopment model raises the question that asks "how can we identify the inputs needed to be reduced by calculated proportions?" These input reductions are called slacks. Hence, slack model was needed to answer this question. Again, none of the current techniques for energy efficiency provide ideas on what to do so that inefficient or weak efficient networks reach efficient frontier.

- ***A Novel Slack Model to Improve the Performance of the Envelopment Models:*** The second model in this thesis, presented in Chapter 5, is slack. This improves network performance by exploiting the input resources (e.g. multicast energy). In both traditional routing as well as more recent coded packet approaches [65, 12, 18], the network performance evaluations are done without regard to their slack variables. Slack model is used to improve the performance of ad hoc wireless network based on technical efficiency scores. The slacks model is needed to push the inefficient or weak efficient ad hoc wireless networks to their real optimal efficiency. Sometimes though the technical efficient scores recorded 100% for an ad hoc wireless network but it is a weak efficient. In other to achieve slacks using Mathematical modelling, a second stage linear programming is solved and then an optimal solution obtained after solving this problem is the maximum slack solution. The slack values are optimised to achieve efficient frontier.
- ***A New Energy Gap (EG) Model to Evaluate and Analyse the Amount of Multicast Energy Saved:*** The third model presented in Chapter 6 determines the amount of energy saved if all the ad hoc wireless networks achieved efficient frontier. That is, once all the ad hoc wireless networks are projected unto their efficient frontier using slacks the inefficient and weak efficient ad hoc wireless networks would save some amount of energy. Earlier, we asked whether a coded packet network can further reduce its multicast energy beyond those that were reported. The answer to this question was yes because some ad hoc wireless networks using coded packet method were sub-optima though the problem was reduced to polynomial time solution. As a result, these sub-optimal

solutions are needed to be projected unto their efficient frontier. Once this is achieved, the difference between the average/approximate energy of the set of networks calculated by the coded packet algorithm and the expected/projected energy of the set of networks calculated by the empirical frontier function is the energy saved, and is evaluated using EG model. Note that this model rely on envelopment and slack models.

- ***A Novel Benchmark Model to Establish Standard of Excellence for Inefficient Ad Hoc Wireless Networks:*** Efficiency analysis is good and fundamental method in performance evaluation. However, the fourth model presented in Chapter 7 explores other method to improve the performance of ad hoc networks. Specifically we explore benchmarking for further performance evaluation [58], [56]. Once the frontier is established, a set of new ad hoc wireless network is then compared with the frontier so that a new frontier is generated whenever a new ad hoc wireless network outperforms the identified frontier. In both traditional and the recently proposed coded packet method, establishing standard of excellence among communication networks has yet to be reported. In contrast, using benchmark technique, an ad hoc wireless network learn how best to utilise its available resources from peers [60]. The benchmark model also provides significant additional information on how to improve the network efficiency. Such information includes the formation of Efficiency Reference Set (ERS) for each ad hoc wireless network and lambdas so each inefficient ad hoc wireless network know (i) what to learn, (ii) from which peer to learn from and (iii) the magnitude of what to be learned.

  All the models proposed were implemented and evaluated using DEA tools and our results does not only provide useful network information to the network administrators but also demonstrated a substantial amount of energy saving. Figure 1.9 shows the methodology adopted in this thesis: from basic system model definition of minimum energy multicast framework toward models development for energy efficient frontier.

To the best of our knowledge this work is the first to present the Envelopment, the Slack, the EG and the benchmark models as alternative approaches for energy efficiency in ad hoc wireless networks.

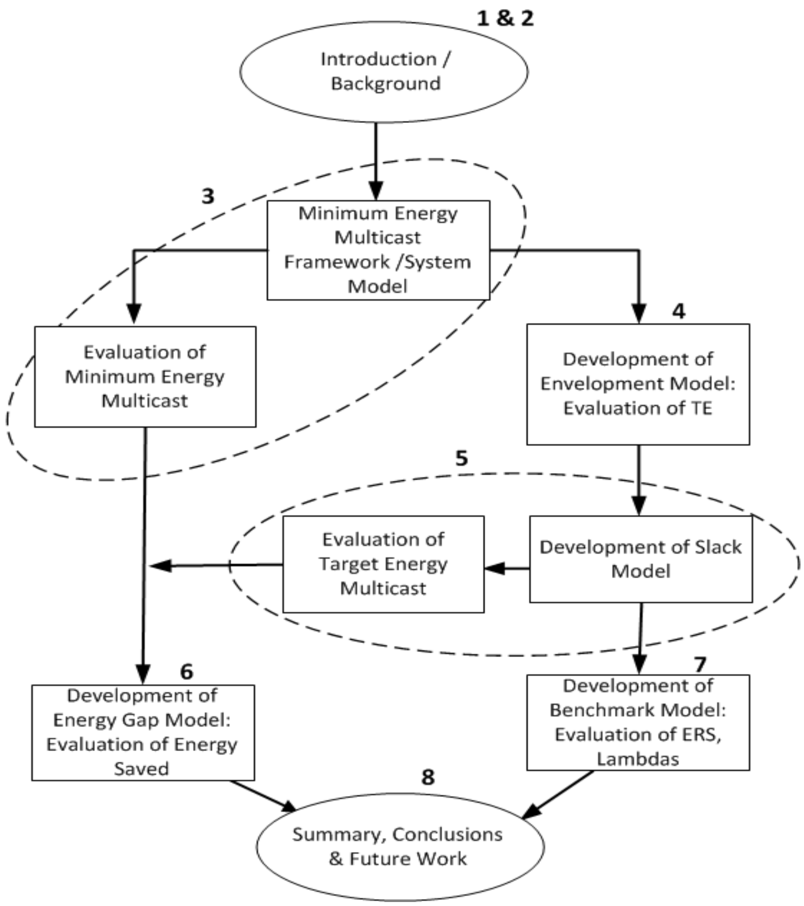

Figure 1.9: Thesis methodology

1.4.2 Significance of the Research

One of the goals of the network engineers and operators is to achieve minimum transmission energy so as to conserve the excess energy. Achieving this goal will help their networks to compete with other network because they would have achieved lower operational costs. In addition, from the perspective of network regulators, inefficient networks are riskier and have higher likelihood of failure, especially at this time that the efficiency of communication networks is linked to the productivity of the economy. Therefore, no network can afford to operate inefficiently. Furthermore, the aim of the ICT environmentalist is to reduce as much as possible the whopping grow of CO2 emission by the telecommunication networks and nodes. Thus achieving reduction in multicast energy will be an excellent contribution to the current research in green communications.

1.4.3 Scope and Limitations

In this thesis, we consider a minimum energy multicast framework and investigate the MIP and coded packet algorithms for energy efficiency. Many studies have explored the topic of energy efficiency which includes routing, coding, cross-layer designs, MAC protocols, spectrum allocation, resource allocation and scheduling but our scope does not cover all these techniques. Thus we considered energy efficiency in coded packet networks, which is the current area of research to reduce multicast energy in both wireline and wireless networks. This thesis is restricted to energy-efficient methods with extension to models development to further reduce multicast energy in ad hoc wireless networks so that efficiency is achieved. The scope covers recent advances that have been made specifically to improve the energy efficiency of ad hoc wireless networks.

Furthermore, the efficiency evaluation in this thesis was based on input-oriented approach with the aim to reduce transmission energy while the outputs are kept constant. The output-oriented aspect of the DEA where the output may be possibly maximized with the inputs kept at constant level was not covered. It is also possible to optimise both the inputs and outputs variables but this is outside the scope of this research work. In addition, this work is limited to two problems namely: performance evaluation and benchmarking problems. Other Economic problems such as super efficiency problem were not investigated [54]. Also, due to the complexity of the coded packet algorithm, this work considered up to 40 randomly generated nodes with up to 10 receiving nodes (sinks) for multicasting operations in ad hoc wireless networks. However, more than 40 randomly generated nodes, and 10 receiving nodes can computed using powerful computer. It should be noted that the proposed DEA technique has potential to accommodate larger volume of data, even with many inputs and outputs. In terms of network topology, static multicast network is assumed.

1.4.4 List of Publications

Publications that have resulted from this research are:

1 A. A. Ajibesin, et al., "*Input-oriented CCR DEA Model for Minimum Energy Multicast: Empirical Study of a Computational Method*", 6th IEEE International Conference on Intelligent Systems Modelling & Simulation (ISMS), Kuala Lumpur, Malaysia, pp. 220 - 225, Feb. 9-12, 2015 (***One of the 4 best papers selected for Award***).

2 A. A. Ajibesin, et al., "*Data Envelopment Analysis with Slacks Model for Energy Efficient Multicast over Coded Packet Wireless Networks*", Special Issue on Green Wireless Internet Technology by IET Journal of Science, Measurement and Technology, pp. 1-12, 2014.

3 A. A. Ajibesin, et al., Book Chapter title: "*Service Productivity in IT: A Network Efficiency Measure with Application to Communication Systems*", in Book (***Chapter 14***) "Managing Service Productivity: Using Frontier Efficiency Methodologies and Multicriteria Decision Making for Improving Service Performance", International Series in Operations Research & Management Science, Vol. 215, pp 241-261, Springer Berlin Heidelberg, 2014.

4 A. A. Ajibesin, et al., "*Reducing power consumption in WSNs nodes*", 6th IEEE International Conference on Adaptive Science & Technology (ICAST2014), Ota, Nigeria, pp. 266-272, 29-31 Oct. 2014 (***Best Paper Award***).

5 A. A. Ajibesin, et al., "*Energy Minimization in WSNs: Empirical Study of Multicast Incremental Power Algorithm*" Southern Africa Telecommunication Networks and Applications Conference (SATNAC), pp. 259-264, 2014.

6 A. A. Ajibesin, et al., "*Cost-efficient multicast over coded packet wireless networks using data envelopment analysis (DEA)*" in 10th IEEE Consumer Communications and Networking Conference (CCNC), pp. 546-551, 2013 (***Current best conf. paper on LINKNOVATE – Beta on Energy: online Portal for experts in network DEA***).

7 A. A. Ajibesin, et al., "*Performance Evaluation of Cooperative Relays in Rayleigh Channel Over Coded Packet Wireless Networks*" in 4th IEEE International Conference on, Intelligent Systems Modelling & Simulation (ISMS), pp. 563-568. IEEE, 2013.

8 A. A. Ajibesin, et al., "*Data envelopment analysis: Efficient technique for measuring performance of wireless network coding protocols*" in 15th IEEE International Conference on Advanced

Communication Technology (ICACT), pp. 1122-1127, 2013.

9 A. A. Ajibesin, et al., "*Energy-efficient Multicast in Wired and Wireless Networks: Analysis and Performance Measures*," 2013 IEEE Fifth International Conference on Computational Intelligence, Communication Systems and Networks (CICSyN), pp.131-136, June 2013. (***2nd best conf. paper on LINKNOVATE – Beta on Energy: online Portal for experts in Energy Efficiency***).

10 A. A. Ajibesin, et al., "*Energy-efficient for Multicast Networks: A New Approach to Efficiency Measure*," 2013 IEEE 8th EUROSIM Congress on Modelling and Simulation (EUROSIM2013), pp. 616-621, September 2013.

11 A. A. Ajibesin, et al., "*Performance of Multicast Algorithms over Coded Packet Wireless Networks*" in 14th IEEE International Conference on Computer Modelling and Simulation (UKSim), pp. 596-600, 2012.

12 A. A. Ajibesin and Neco Ventura., "*Gap Mechanism for Energy Efficiency Models in Wireless Multicast Networks*", 12th IEEE AFRICON International Conference on Green Innovation for African Renaissance, Addis Ababa, 14–17 September 2015.

13 A. A. Ajibesin, et al., "*Performance Comparison of Energy Minimization Models in Wireless Multicast Networks*" 4th iSTEAMS Research Nexus Conference, University of Ilorin, Nigeria, 11th - 15th March, 2015.

14 A. A. Ajibesin, *"Minimum Energy Multicast in Wireless Networks: Empirical Study of Coded Packet Model"* in International Journal of Computer Research, Vol.22, Issue 4, 2016.

15 A. A. Ajibesin, "*Data Envelopment Analysis Method for Energy Efficient Multicast in Wireless Networks*" in International Journal of Measurement Technologies and Instrumentation Engineering (IJMTIE), International Publisher of Progressive Academic Research, Hershey, Pennsylvania, USA (Invited paper).

16 A. A. Ajibesin, and Neco Ventura, Book Chapter Title "*Minimum Energy Multicast in Wireless Networks: Empirical Study of Coded Packet Model*" in Book (***Chapter 4***) "Network Coding and Data Compression: Theory, Applications and Challenges: Nova Science Publishers, Inc. NY, USA., 2015

1.4.5 Structure of Thesis

The structure of the thesis corresponds to the adopted methodology (Figure 1.9) proceeding from basic concepts ranges from minimum energy framework to multicasting techniques.

The general introduction of this thesis is presented in chapter 1. Chapter 2 presents overview of the coded packet model and the data envelopment analysis.

Chapter 3 consider the minimum energy multicast framework and then investigated the performance of MIP and coded packet algorithms. We then present their comparative analysis using energy-efficient as metric.

Chapter 4 is entirely dedicated to the envelopment model, which is a new model derived for Technical Efficiency evaluation. The model, which is based on linear programing technique is optimally exploits multiple inputs and multiple outputs of ad hoc wireless networks and then qualitatively calculates the degree of their efficiency.

In Chapter 5, Slack model is derived to provide opportunity for identified inefficient and weak efficient networks and to project them unto their efficient frontier. The model is the extension of the Envelopment Model and derived to improve the energy efficiency of ad hoc wireless networks.

In Chapter 6, the EG model seek to calculate the amount of energy that is saved if the networks that are examined achieved efficient frontier. These models make a strong case for network performance evaluation as an alternative model for minimum energy - a model that can deliver significant improvement in minimising multicast energy.

In Chapter 7, Benchmark Model was introduced to establish the standard of excellence among the peers so as to determine the ERS or peer group and the Lambdas.

Finally, Chapter 8 summarises the work and provides general conclusions. To ease reading of the material, we include some mathematical derivations as appendices. The appendices also contain some pseudocode, Tables and Figures.

Chapter Two: Literature Review

2.1 Introduction

This chapter presents the background related to the study and the literature associated with the work in question. It begins with the concept of multicasting in section 2.2 and goes on to explore the broadcast nature of wireless links, which leads to a significant model called Wireless Multicast Advantage (WMA). Subsequently, section 2.3 presents a review of network coding techniques, including that of the multicast technique. In section 2.4, the overview of the DEA technique is explored while section 2.5 discusses related work that gives motivation for this research. Section 2.6 then concludes the discussion of the work reviewed in this chapter.

2.2 Multicast Basic Concept

Based on a unified characteristic of group communications, a set of users for a particular service could be grouped. This type of connection is known as *multicast* [61]. Multicast group refers to a set of service users that abide by appropriate group membership criteria, or a set of rules belonging to a group that enables multicast-based services and applications [62] . In other words, multicast is the data delivery scheme, where the same data unit is transmitted from a single source to multiple destinations, in a single invocation of service [63]. An advantage of multicasting is the reduction in the transmission overhead it takes for all the network nodes in the subset to receive the information [64].

The Internet Protocol (IP) is an old network model that was estimated to grow by, factor of four between 2009 and 2015 and this would lead to a rise of nearly 64 Exabytes per month. A technique that helps to manage this growth, is the use of the multicast method [65]. Therefore, in this age of multimedia applications and high speed networks, multicasting is one of the solutions by which the internet can be made use of in an efficient manner. In the case of the Internet's challenges, IP multicasting was proposed in [66], and some works were demonstrated using an "audio cast". This describes how end systems send and receive multicast packets [67], [68]. Since the introduction of multicasting, a number of techniques were proposed to implement the multicasting concept in the internet and intranet domains [66]. Multicasting is most efficiently implemented and handled at the network layer, and was initially implemented as IP-encapsulated tunnels forming the Multicast backbone (MBONE) [69], [70]. Its data is routed over the network using either the IP-encapsulated tunnels or the multicast enabled routers [69].

2.2.1 Multicast communication: Types, Applications and Challenges

Three types of multicast communication modes are required to be supported by Next Generation Network (NGN) multicast capabilities [71]. The *Many-to-many* multicast communication mode, is applied to deliver data from multiple senders to multiple recipients. It assumes a group consisting of multiple senders and multiple recipients. *Many-to-one* multicast connection mode is applied to deliver data from multiple sources to one recipient. It assumes a group, consisting of multiple senders and one recipient. *One-to-many* multicast communication mode, is a multicast type that could be applied to deliver data from a source to multiple recipients. In this thesis, it is assumed that one source is sending, while the receiving group consist of multiple recipients [72], [73]. Whatever the type of multicast formation, multicasting within a network has many benefits. Multicasting reduces the communication costs such as that of the energy used for applications that send the same data to multiple recipients. Instead of sending via multiple unicasts, multicasting minimises the link cost, sender and router processing, and the data delivery delays [61]. In addition, multicast service plays an important role in computer or communication networks supporting NGN applications [74], [75]. In particular, as shown in Figure 2.1, Next Generation Network (NGN) services and applications will offer multicast features [63]. Currently, multicast services are increasingly used by a wide variety of applications, ranging from content broadcasting and streaming, voice and video conferencing, collaborative environments and massive multiplayer gaming, up to the self-organisation of distributed systems, services, or autonomous networks [46]. In addition, NGN applications are emerging as a mass scenario, which demands for the provision of efficient communications from multicast routing.

However, an efficient measurement of communications cost (energy), in multicast connections will adequately capture network resources so as to assess the important network components and their ratings, against the best practice is yet to be explored. According to the report in [63], the NGN multicast features are associated with a level of cost in which a solution is the use of efficient multicast communication models and this is yet to be explored. In this regard, the coded packet technique, was proposed for establishing efficient multicast connections, as a promising scheme especially when compared to the traditional Steiner Tree approach, but fails to adequately capture network resources [32]. Alternatively, this is an effective technique that is used for minimising the multicast energy but does not adequately address the evaluation of the total Technical Efficiency (TE) of the model.

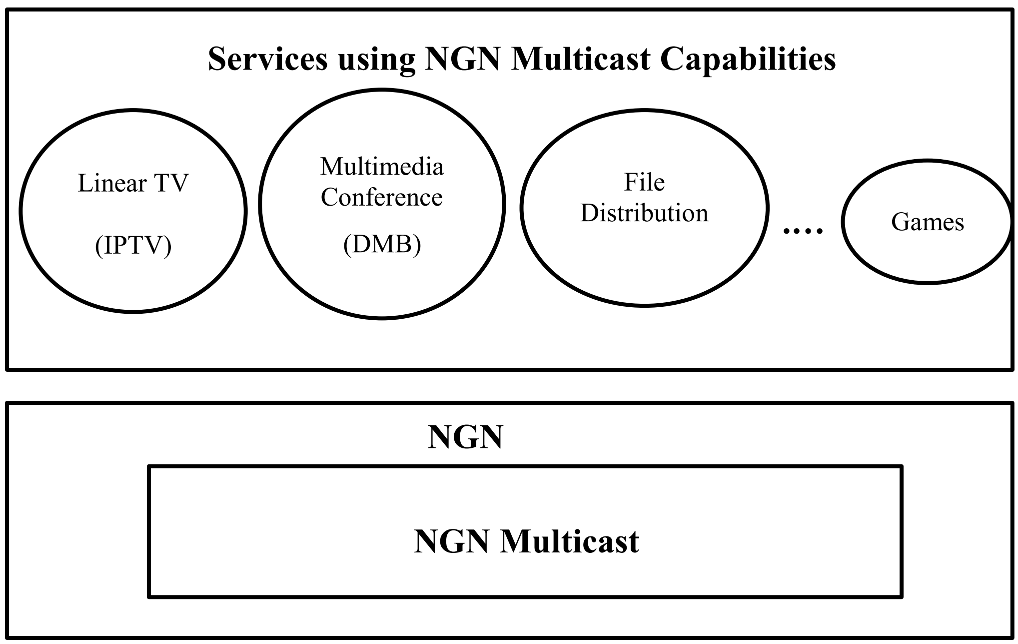

Figure 2.1: Services using NGN multicast capability

2.3 A Brief Review of Network Coding (Coded Packet) Network

In traditional computer networks, each node behaves like a switch or router. The operation of an existing router, for example, requires that each node receives information on an input (or set of input) links and the node either forwards this information to an output link (or replicates this information and sends it to a set of output links). This behaviour of nodes, where it is simply routed and replicated, is likened to a fluid flowing in a pipe. From the information theoretical view point, each node should be allowed to perform an encoding operation and then send the encoded information to all of the output links [76]. The network coding method, where the intermediate nodes are allowed to combine independent data streams together such that the packets are encoded arbitrarily by not just end nodes, but also by nodes within the network, was proposed by Ahlsewede et al. [77]. In [78], [79] the authors make use of the encoding concept but the problem of transmitting from multiple correlated sources, that is from a number of nodes to a single node, comes as a distinction in addressing the multicast problem that was done in [77]. The work of [80], [81] and [82] shows a very important contribution to the work by Ahlsewede *et al* and in their report, it also shown that the codes with a simple linear structure, are sufficient to achieve more capacity involving a multicast problems. Furthermore, the work by [80] and [82] is given as an extension by [83] and [84] to accommodate a decentralised approach in network coding implementation. Also, achieving a multicast capacity with an application to wireless networks was addressed. Since the introduction of network coding to achieve capacity and throughput in wireless multicast networks, many research approaches have been presented [85], [86], [87], [88].

It is important to discuss the network coding idea, which was originally meant for wireline networks and intended for exploiting the multicast transmissions [77], [89]. This technique was described using the butterfly network. Figure 2.2 and Figure 2.3 are presented to demonstrate the network coding technique in butterfly network. These two figures demonstrate the multicast of two packets using the butterfly network graph. They show how data transmission from a source node, to a set of receivers (sinks) in the network. In Figure 2.2, the node sends packets P_1 and P_2 to receivers' r_1 and r_2 respectively. The nodes *a* and *b* broadcast the received packets P_1 and P_2 respectively, to receivers r_1 and r_2 respectively.

If these packets were also broadcasted to node *c*, for example, the receiver r1 receives a packet P_1 from node *a,* and receiver r_2 receives a packet P_2 from node *b,* while node *c* simultaneously receives packets P_1 and P_2 from nodes *a* and *b* respectively. In a traditional routing technique as shown in Figure 2.2, link *cd* is a *bottleneck* and either packet P_1 or packet P_2, can normally be allowed to transmit at a time. The coded packet technique, as shown in Figure 2.3 allows the node *c,* to be equipped with the coding capability and therefore performs the simple XOR of the two packets P_1 and P_2, then outputs the packet $P_1 \oplus P_2$. The receiver r_1

receives packet P_1 and coded packet $P_1 \oplus P_2$, and uses the coding scheme to decode packet P_2. Similarly, the receiver r_2 uses a similar coding scheme, to recover packets P_1 from received packet P_2 and coded packet $P_1 \oplus P_2$. It can be concluded that, without the coding scheme, it would definitely be impossible to multicast two packets per unit time from the source node *s* to both the receives r_1 and r_2. Thus it is clear from this illustration, how network coding benefits during the transmission of data on a multicast wireline network. For instance, the multicast rate that can be achieved using traditional method of routing is *1.5 bits* per time unit, while *2.0 bits* per time unit can be achieved if coded packet network is used [90]. The idea of network coding was later extended to wireless networks. The technique as applied to wireless communication was demonstrated in chapter 1, section 1.3.3.

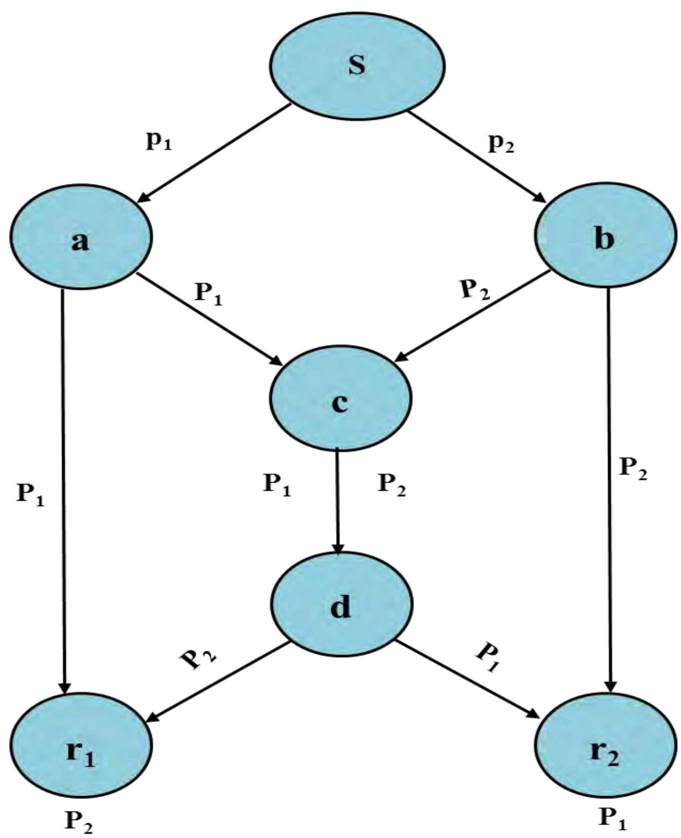

Figure 2.2: Butterfly network example: A routing based on traditional method

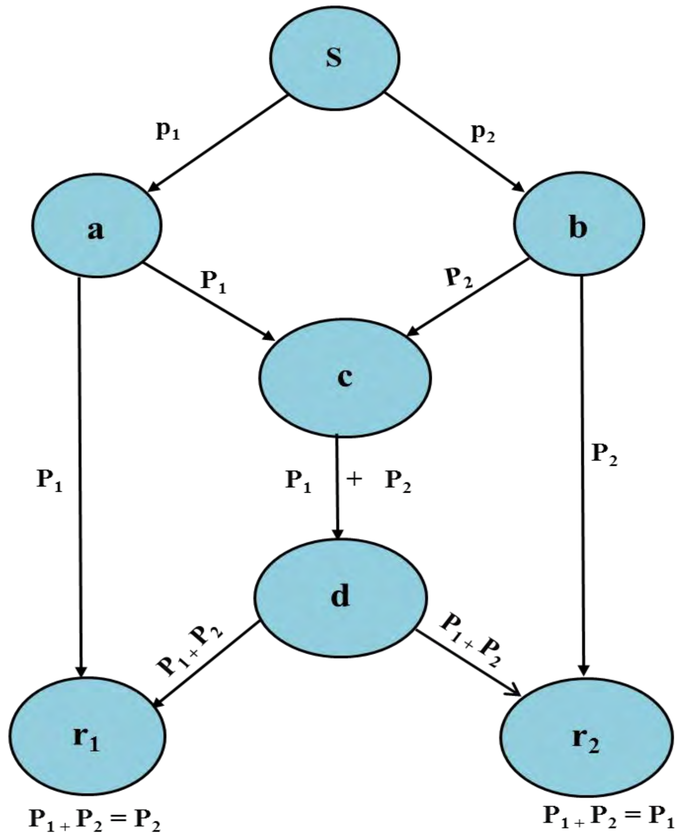

Figure 2.3: Butterfly network example: A routing based on Coded Packet (network coding)

2.4 Overview of Data Envelopment Analysis (DEA)

The Data Envelopment Analysis (DEA), is a non-parametric method that relies on a linear programming technique for optimisation. It is used to measure the relative performance of entities called Decision Making Units (DMU), where the presence of multiple inputs and outputs makes the comparisons difficult [91], [92]. The DMU in this work are the 54 ad hoc wireless networks considered for examination. This method is different from other methods that were studied because its performance evaluation, for instance is based on the actual measure of efficiency such as technical and scale efficiency evaluation rather than effective evaluation. Therefore, it provides alternative ways to stir a network into becoming one of the best performers. The DEA technique, has the capability to improve input resources such as saving transmission energy or augment the outputs such as number of receivers in a multicast network scenario. These are achieved without affecting the general performance of the ad hoc networks.

In addition, the DEA method goes beyond identification of optimal performance but can also serve as a benchmark in a normative way [92]. Thus DEA, is a state of the art benchmarking technique which is particularly useful for multi-criteria benchmarking studies [93]. In DEA, the productivity of a unit is evaluated by comparing the amount of output(s) produced in comparison to the amount of input(s) used. The performance of a unit is calculated by comparing its efficiency with the best observed performance in the data set.

2.4.1 Decision Making Units (DMUs)

In a DEA, we use the term Decision Making Units (DMUs) to represent an entity under evaluation. In this work, DMUs represent ad hoc wireless networks. We use DMU to represent the ad hoc wireless networks for the remaining discussion in this thesis. We may use them interchangeably sometimes. Generally, DMU can be used to represent any entity that converts multiple inputs into multiple outputs [94]. Figure 2.4 shows a basic DMU transformation inputs into outputs.

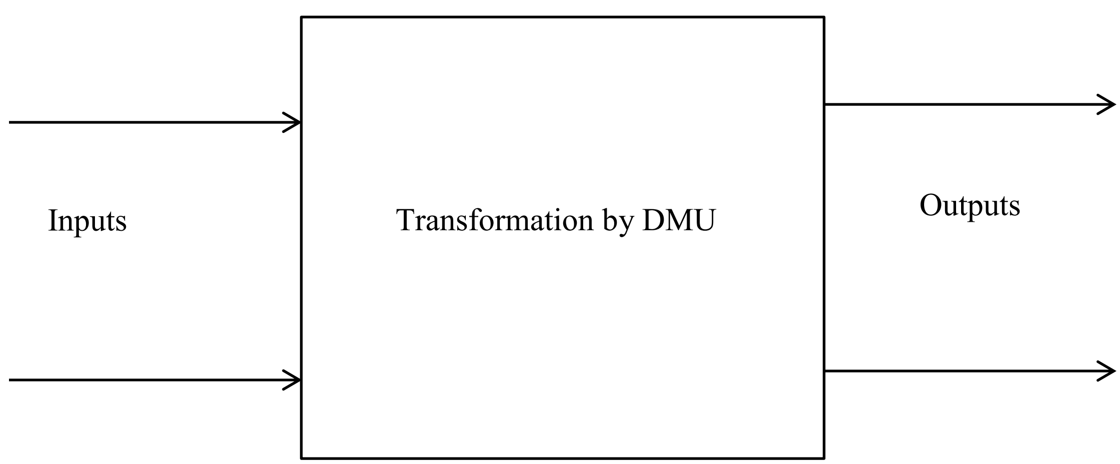

Figure 2.4: A DMU Transformations inputs into outputs

2.4.2 The Basic DEA models

There exist many basic DEA models, each with its characteristics. The two most popular DEA models that served as bases, are studied and adopted in this work are discussed as follow:

Charnes, Cooper and Rhodes (CCR) model: This model was proposed by Charnes, Cooper and Rhodes (CCR) [94]. The CCR model is based on the radial minimisation (maximisation) of all inputs (outputs) and assumes an environment of Constant Returns to Scale (CRS). The CRS assumes that an increase in the amount of inputs consumed would lead to a proportional increase in the amount of outputs produced.

Banker, Charnes and Cooper (BCC) model: The Banker, Charnes and Cooper (BCC) model is the Variable Returns to Scale (VRS) version of the CCR model [54]. The difference between the two models depends on the CRS and VRS assumptions. The VRS, which is formulated with additional convexity constraint over CRS is a technical property of efficiency measure whereby the observed data, exhibit a feature with changes in the outputs that are subsequent to a proportional change in all inputs [95]. The convexity property is presented as 'Property a.1' in Appendix A.

2.4.3 Frontier and DEA Orientation

As illustrated with Figure 1.6 in chapter 1, section 1.4.3, efficient frontier represents the best observed performance in the data set [96]. We define it, in this thesis as a set of ad hoc wireless networks that offers the expected minimum multicast energy. Ad hoc wireless networks that lie below an efficient level in the frontier are sub-optimal, because they do not operate at the expected minimum level of energy. The efficient frontier concept was introduced by Harry Markowitz in 1952, and is one of the modern economic theories. The next challenge is to project the sub-optimal ad hoc wireless networks into their efficient frontier, which can be achieved by the DEA orientation approach. DEA orientation is another important concept in DEA methodology, to ensure full optimisation of problem. As demonstrated in chapter 1, the type of DEA orientation approach depends on the nature of problem. For example, if the aim is to minimise input resources (such as energy levels), with the same level of output then input orientation is applied and if the aim is to improve the output results with the same input resources then output orientation is considered.

2.4.4 Returns to Scale: Constant Returns to Scale Vs Variable Returns to Scale

The Returns to Scale (RTS), is another feature of the DEA methodology. This may be either a Constant Returns to Scale (CRS), or a Variable Returns to Scale (VRS). In the case of CRS, it is assumed that an increase in the amount of inputs consumed, would lead to a proportional increase in the amount of outputs produced. That is, CRS model can be assumed whenever it is observed that a percentage increase in inputs leads to the same proportional expansion of outputs. For example, doubling all inputs leads to doubling all outputs. In such case ad hoc wireless networks are multicasting at optimal scale. Furthermore, if CRS is assumed, it means that the scale or size of the network is not a factor in assessing its relative efficiency. However, this assumption is inappropriate for network operations that have economic (or diseconomy) of scale. So if it is likely that the size, radius or dimension of network for instance will influence the ability to achieve network efficiency, the assumption of CRS is inappropriate. If these parameters will affect the efficiency of networks, the less restrictive VRS frontier, which allows the best practice level of outputs to inputs to vary with the size of the network in the sample, should be considered.

The VRS assumes that the amount of outputs produced, increase at a rate more or less than proportional to the increase in the inputs [54]. The CRS version is more restrictive than the VRS and yields a fewer number of efficient networks and also lower efficiency scores among all DMUs [97]. In order words, since the constraint set for CRS is more restrictive (i.e., convexity constraint is absent) than in the VRS formulation, then lower efficiency scores are possible and therefore more networks are declared efficient for a VRS envelop surface. In summary, CRS tends to lower the efficiency scores while VRS tends to raise efficiency scores.

These analyses show that returns to scale properties are important concept to estimate distance from the frontier. Ideally, the efficiency that could be achieved by a network is modelled with assumption of CRS. As analysed, the use of CRS assumption in the DEA model indicates that all networks under analysis are multicasting at optimal scale. In order words, the CRS assumption is only appropriate whenever all networks are multicasting at an optimal scale.

However, in the real world, it is practically impossible to achieve this optimal scale because of some circumstances. Constraints such as network size, radius of connectivity, and dimension occupied by nodes may cause a network not to multicast at optimal scale. Therefore, the CRS specification proposed by Charnes, Cooper, and Rhodes [94] did not evaluate the technical efficiency (TE) appropriately in the sense that TE scores reported under that set of constraints are biased by scale efficiency (SE). That is, the use of CRS assumption when not all networks are multicasting at the optimal scale will result in evaluation of TE, which is confounded with SE. The SE is responsible for those circumstances mentioned affecting the network performance. This significant shortcoming of this assumption is corrected by Banker Charnes and Cooper (1984) [54] who extended DEA to the case of VRS. The VRS is modelled by adding the convexity constraint $\sum_{j=1}^{n}\lambda_j = 1$, to the CRS model. This constraint guarantees that each network is only compared to others of similar size, radius of connectivity, and dimension. This mode of operation avoids the damaging effect of SE on the TE scores. It means that SE effects do not arise if VRS is assumed. This understanding had made many studies to decompose the TE scores obtained into two components: one is due to the scale inefficiency and second is due to the 'pure' technical efficiency (PTE) [98]. This may be evaluated by considering both CRS and VRS DEA models using the same data. If there is a difference in the CRS and VRS TE scores for a particular network, it indicates that the network has scale inefficiency, and that the scale inefficiency can be calculated from different between the VRS and CRS TE scores.

The two types of returns to scale are illustrated as follows using numerical example presented in Table 1.1:

Variable Returns to Scale (VRS) frontier: Consider the same data for 8 DMUs (A, B, C, D, E, F, G, and H) in Table 1.1, the VRS frontier consists of A, C, D, E, and H as shown in Figure 2.5. These are DMUs which "envelops" all the other DMU points. So, based on VRS assumption, DMU B, F, and G are inefficient. Considering DMU F, it is compared to C (a convex combination of A and D) on the VRS frontier. This means that F should reduce its input to C, or C is the efficient target for F. This calculation is based on input-orientation model. If we consider an output-orientation model, F is compared to E. That is, F should increase its output to E, or E is the efficient target for F. The VRS model is a model developed based on the properties presented in Appendix A.

Constant Returns to Scale (CRS) frontier: In the case of CRS, any point (or DMUs) on the VRS frontier (except E) are no longer efficient. That is, only DMU E is efficient in the CRS sense. Also, we can draw rays through each of the DMUs as shown in Figure 2.5. But then, we can observe that only ray OE indicates the best performance. In fact, ray OE is the CRS DEA frontier. Note that CRS has fewer efficient network (only network E is efficient) compared to VRS (5 networks are efficient) simply because VRS is less restrictive using convexity constraint.

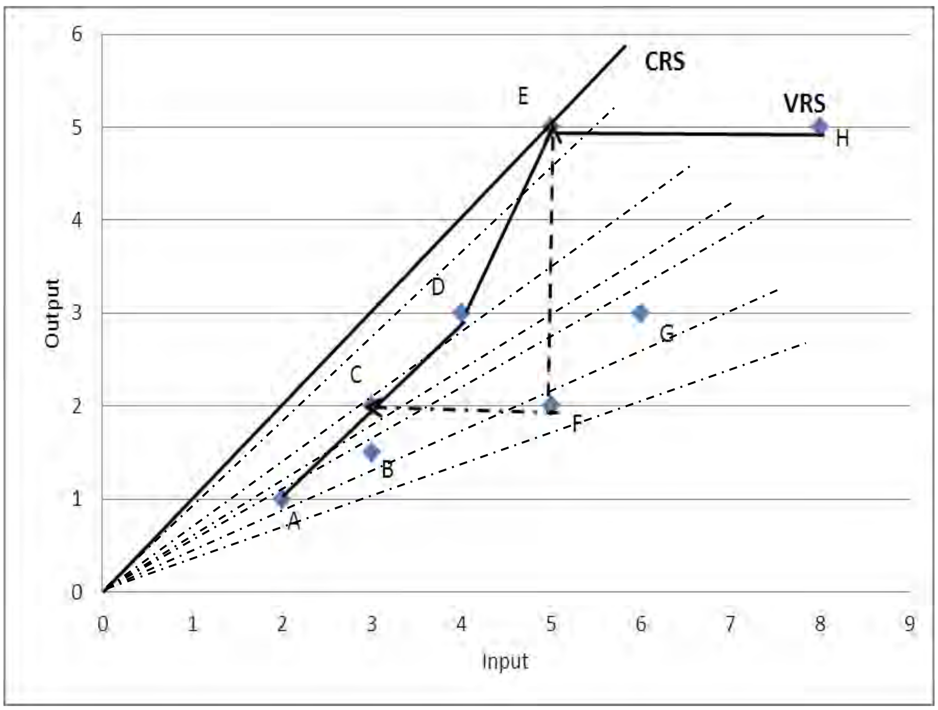

Figure 2.5: CRS and VRS frontier

2.4.5 Further Graphical Illustration of CRS, VRS and SE

Figure 2.6 represents two efficient frontiers. The first assumes CRS – represented by line OO' and the second assumes VRS – represented by line segment PABCQ. If we decide to minimise input X while hold output Y constant (that is, input-orientation), then considering the VRS, the inefficient DMU D is projected onto VRS efficient frontier, which is point E. The VRS for DMU D is calculated as X_E/X_D. Similarly, considering the CRS efficient frontier, the inefficient DMU is projected onto CRS efficient frontier, that is point F. The CRS for DMU D is calculated as X_F/X_D. Extending the above illustration to SE, the input-oriented SE is evaluated as X_F/X_E. Note that TE measure using CRS assumption represents overall technical efficiency (OTE). It evaluates inefficiencies due to the input/output configuration and as well as the size of networks. Also, note that the TE measure using VRS assumption represents pure technical efficiency (PTE). It evaluates inefficiencies due to only network administrator underperformance. The relationship SE = CRS(OTE) / VRS(PTE) evaluates scale efficiency.

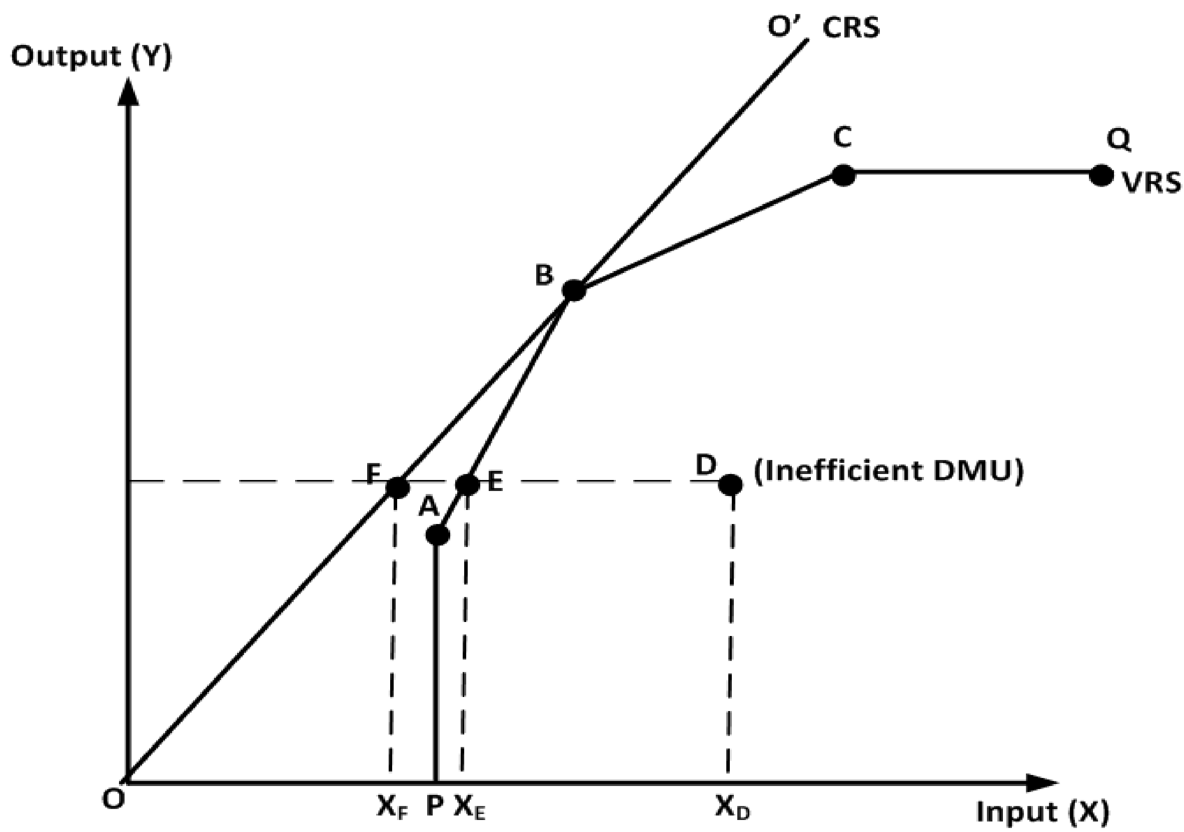

Figure 2.6: Graphical Evaluation of CRS, VRS and SE

The graphical depiction of CRS, VRS and SE evaluation can be resulted into linear programming models that can be used to optimize the efficiency of individual DMUs using actual data on input and output variables. As mentioned, the CCR linear programming model for

CRS is proposed by Charnes, Cooper, and Rhodes [94] and the BCC linear programming model is proposed by Banker, Charnes and Cooper [54]. These models are details in chapter 4,5,6 and 7 to address different types of problems. A significant point is that each of these models evaluates which of the *n* DMU exhibit the best practice or efficient frontier. The geometry of this frontier is determined by the specific DEA model used. In this thesis, we considered CCR and BCC models to evaluate efficiency under CRS and VRS assumptions, respectively. We then further analysed the efficiency results to evaluate the energy saved under each of these model/assumption.

2.5 Related Work

This section summarises existing methods of solving the minimum energy multicast problem. A popular approach in this category is Minimum Shortest Path Tree (MSPT) algorithm that has been applied to solve minimum energy network problem [90]. This algorithm builds minimum energy networks and measures the cost (energy) of an edge based on certain level [99], [100]. However, this problem is known to be NP-complete [101]. An alternative approach such as minimum spanning tree that is based on a greedy, heuristic algorithm was proposed [32]. The method used can compute the minimum energy in polynomial time, thereby reducing the cost (energy) on multicast tree at most twice than that of SMPT [100], however, the solutions provided by this approach are suboptimal. In order to achieve optimal solutions, a large number of approximation algorithms were proposed for multicasting messages in wireline and wireless networks [29], [33], and [52]. Especially, Wieselthier et al. proposed a unique method to improve the energy efficiency of multicast trees using pruned or greedy heuristics [102]. The earlier method of heuristics are designed based on a link-by-link approach, while the later method proposed by Wieselthier et al. employed a node-by-node energy increment [52]. This incremental approach to energy efficiency was used to design the Broadcast Incremental Power (BIP) algorithm. However, it should be noted that the multicast is different from a broadcast system. Multicast sends a message to a subset of nodes in the network while broadcast sends a message to all the nodes on the network. Some fundamental issues associated with energy efficient multicast were discussed in [74]. In addition, several multicast techniques were proposed and their performances were evaluated. A particular work on multicast routing was presented as the multicast incremental power (MIP) method, which was developed as an extension of the BIP method [52]. This means that the MIP multicast tree is obtained from the BIP broadcast tree using a pruning technique, to tailor the branches that do not contain the multicast destinations. Another study proposed a different approach based on an advanced localised broadcast incremental energy protocol that addresses the communication overhead problems. In their study, they compared the energy consumption of different approaches. It is also important to mention that the BIP algorithm is developed based on Prim's algorithm [103]. The Prim algorithm is used to search for Minimum Spanning Tree (MST) and this algorithm is modified for energy efficiency. This approach includes some procedure such as r-shrink [99] and Wireless Multicast Advantage (WMA). Another procedure that enhances the performance of energy efficiency is the *sweep* operation [29]. This procedure allows the transmission range for each node and allows the solution to reach a near-optimal value. Most of the previous studies on energy efficient multicast focus on configuring the energy of nodes [32] and [100]. That is, given the geometric positions of a set of nodes in a plane so as to find the transmitting energy of each mode, such that the transmission energy of the multicast tree is minimised.

Some recent work compares the performances of three greedy heuristics algorithms: Multicast Incremental Power (MIP) algorithm, Multicast Least-Unicast-Cost (MLU) algorithm and Multicast Link-based MST (MLiMST) algorithms [52] and [45]. In their implementation methodologies, networks with various nodes and various multicast group sizes were considered for the performance evaluation. As it can be observed from Figure 2.6, the MIP algorithm outperformed MLiMST and MLU for all network nodes and group sizes considered i.e. the MIP algorithm provides better performance than MLiMST and MLU over the complete range of network examples that were sampled but, this performance evaluation is based on the effective evaluation metric. However, MIP's performance is attributed to the fact that the algorithm exploits the node-based model combined with WMA properties. In contrast, MLiMST and MLU ignore these properties as their tree formations are link-based cost and this edge makes MIP algorithm to be largely considered because it has been identified as a benchmark for other algorithms using performance yardstick. A report by Lun et al. and some authors using coded packet technique, shows that this approach to minimize the energy multicast achieves better results over MIP algorithm [29], [33].

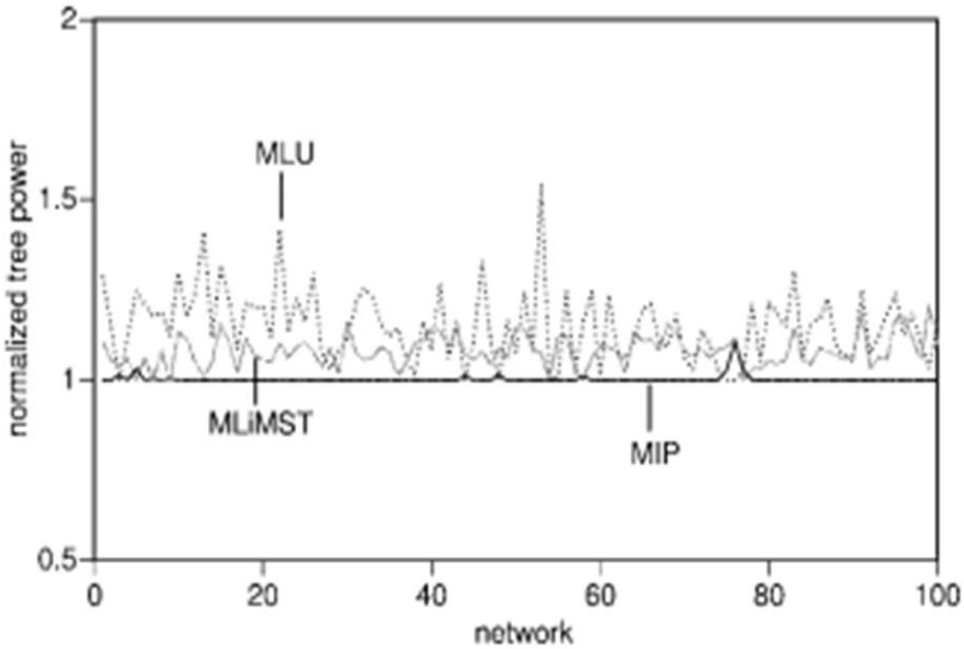

Figure 2.7 : Normalized energy of three algorithms for 100 randomly generated nodes

Other contributions by several authors, are also the reason behind this work. In this work, because of the numerous advantages that are attributed to network coding, we studied the technique and investigated its performance for energy efficiency. This is necessary because the coded packet technique is evaluated using the energy-efficient metric, which we classified as energy effectiveness because evaluation was based on average performance as explained earlier. As a result, we consider an alternative method, which is the multi-criteria decision using the DEA technique to measure TE of networks and further minimises the energy consumption in ad hoc wireless networks. Therefore the study improves the ad hoc wireless performance when we consider the best practice approach to efficiency evaluation. Specifically, the coded packet schemes implemented in literature are considered either a test bed or simulation model however, little or nothing is known about their empirical performance in terms of best practices or the efficient frontier.

2.6 Chapter Summary

This chapter exhaustively reviews the major concepts that are related to this research work. The use of Multicasting communication and its application to wireless networks are discussed. The work also discussed the current network coding techniques, which are a new paradigm and their techniques in information theory. The study of how the technique is applied to the problem of minimum energy multicast, is well documented together with the DEA technique, which is a proposed alternative approach to minimum energy multicast in ad hoc wireless networks. The literature has shown the potential of the proposed DEA technique for minimum energy multicast. In the next chapter, the work discusses the system model for minimum energy multicast and investigates two algorithms for energy efficiency in wireless ad hoc networks.

Chapter Three: Evaluation of Minimum Energy Multicast in Wireless Networks

3.1 Introduction

This chapter begins with a discussion on the system model for minimum energy multicast and proceeds to the discussion on incremental power technique for minimum energy multicast in section 3.3. Subsequently, network coding's technique to minimum energy multicasting is discussed in section 3.4. Section 3.5 compares the simulation results for both incremental power and coded packet algorithms. The conclusion is drawn in section 3.6.

3.2 System Model

3.2.1 Wireless Network Topology

The topology of the wireless network is represented using a directed hypergraph $H = (N,A)$, where N defines the set of nodes and A defines the set of hyper-arcs. A hypergraph is generally used to represent wireless networks [104] where a pair (i, K) represents a hyper-arc, where i represent the start node and it is an element of N, and K is the set of end nodes which is a non-empty subset of N. Each of (i, K) represents a loss or lossless broadcast link from node i to nodes in the set K. Loss or lossless links means that it may or may not be subject to packet erasures. In order to make the communication, packets are injected into hyper-arcs and Z_{iK} is used to denote the average rate at which codded packets are injected on hyper-arc (i, K). Therefore, the vector Z, which consists of Z_{iK} defines the rate at which packets are injected on all hyper-arcs in the network.

3.2.2 Ad hoc Wireless Multicast System and Assumptions

A Wireless multicast system assumes that there are a number of ad hoc wireless connections, whereby a single source node and more than one sink node are communicating. In a multicast ad hoc wireless connection, all of the sink nodes wish to receive the same message originating from a source node [63], [105]. These connections are associated with packets that we wish to communicate at a certain known rate. The communication system assumes that any other existing problems, such as congestion control and queue management, are separate problem i.e. for example, queues in the network are assumed to be stable. Also the system assumes static multicast networks and this is a type of network where membership of the multicast group remains constant for the duration of the connection. Figure 3.1 represents an example of a multicasting operation with source node S multicast a message at rate Z to receivers t_1 and t_2 simultaneously, using coding subgraph. The Mathematical expression for a coding subgraph and its details is provided in Appendix B.

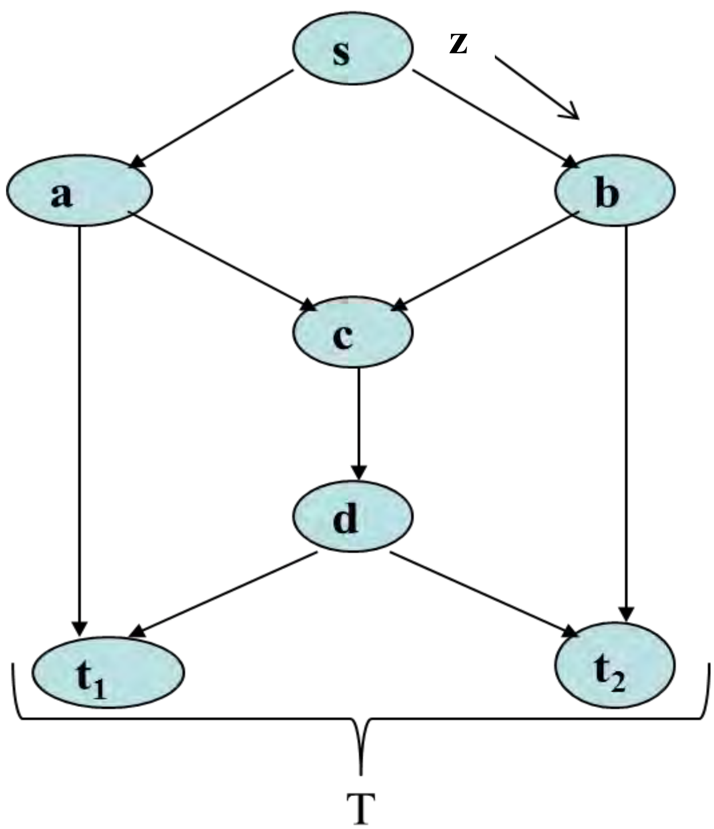

Figure 3.1: Example of a multicasting operation with source node S multicasting a message at rate Z to two receivers' t_1 and t_2 simultaneously using coding subgraph

3.2.3 Minimum Energy Multicast Framework

Figure 3.2 summarizes the minimum energy multicast model, which includes models for the MIP and coded packet algorithms. The MIP and coded packet algorithms are further explain in section 3.3 and 3.4 respectively. The framework also defines the implementation procedures for these two methods. The results of these algorithms are presented in section 3.5. Generally, as it could be observed from the Figure, the simulation is set up according to the MIP or coded packet algorithm requirements and the inputs parameters such as node and sinks are configured. The algorithm is run based on these parameters (node, sinks, etc.) set for each ad hoc wireless network and the optimal value of the multicast energy for each of these algorithm are obtained. Then, the average of these values is evaluated using a statistical mean and standard deviation.

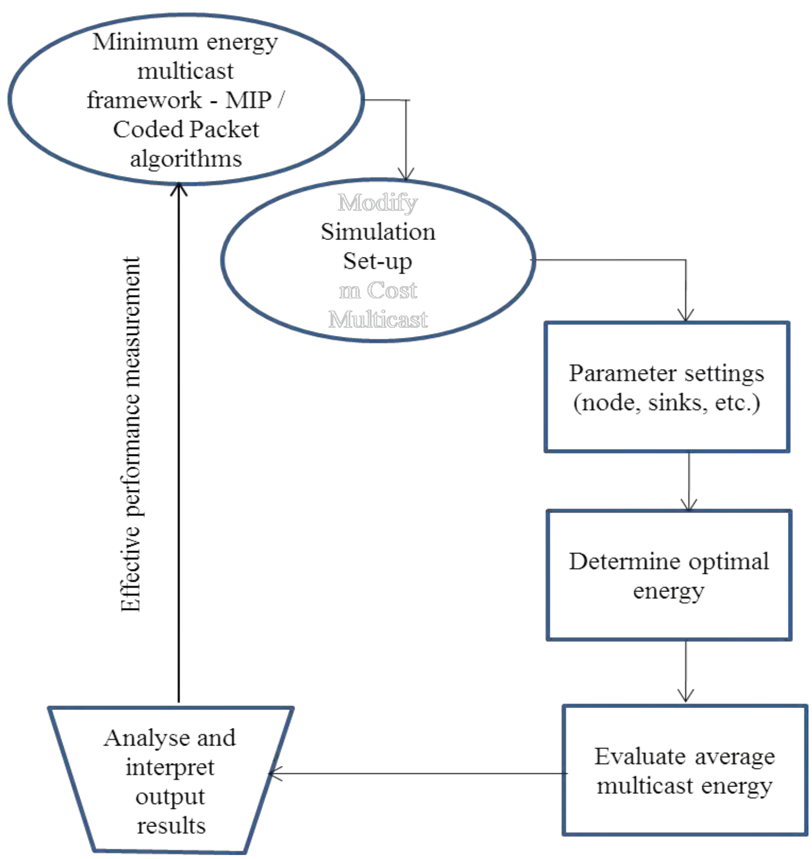

Figure 3.2: Minimum energy multicast Framework

3.3 MIP Approach and Assumptions

As mentioned earlier, the problem of minimum energy multicast using various minimum weight spanning tree algorithms is NP-complete. In order to properly address this problem, an alternative approach using heuristics method was proposed. An example of this method is the Multicast Incremental Power (MIP) algorithm, which is demonstrated in this section. As pointed out in Chapter 2, the MIP approach is largely considered in literature for a minimum energy multicast because of its performance over other existing techniques. The approach is a Source-Based Tree (SBT), where the message is routed at the sender to the selected number of receivers. The technique is designed to minimise the number of transmissions needed to reach all the members of the multicast group. This type of approach is a source-initiated, circuit-switched method where multicast sessions are established [44]. Using this technique, the network consists of N nodes that are randomly distributed in a square dimension. It is assumed that any node within this region is permitted to coordinate and initiate multicast sessions. Multicast request and session durations are generated randomly at the network nodes. It is important to mention that each multicast group consists of the source node and with at least one destination node. Sometimes, intermediate nodes that can act as relay are used to provide connectivity to all members of the multicast group. This connection makes the multicast tree to be a composition of the source node, the destination nodes, and the relay nodes. The nodes are equipped with certain level of energy, and it is assumed that each node can choose its energy level within the maximum E_{max}. A constant bit rate traffic model is assumed. Also, it is assumed that bandwidth is not a problem for the transmission. Therefore, this work focuses on the transmitter energy.

Furthermore, it is assumed that the received energy varies as r^{α}, where r represents the range between the transmitting node and the receiving node while the parameter α defines the characteristic of the communication medium, which its value ranges from 2 to 4 [106]. If we consider a particular case of node i with the minimum transmitted energy E_{ij} that enables the i^{th} nodes to multicast information to the j^{th} node, then the distance between node i and j represented by r is proportional to r^{α}. This is given as follows:

$$E_{ij} = r_{ij}^{\alpha}. \tag{3.1}$$

For the network to be fully connected, the maximum transmitted power E_{max} is required.

It is assumed that the propagation medium is uniform and if there is no interference, therefore α it is fixed for the simulation sample that we considered. Note that omnidirectional antennas are considered so as to exploit the WMA. A specialized algorithm designed for the implementation of this approach is the MIP algorithm.

3.1.1 Multicasting Incremental Power Algorithm

The MIP algorithm is a promising technique for the implementation and evaluation of the minimum energy multicast problem. The procedure for implementing MIP algorithm is summarized as follows:

- I: The first procedure is to modify the Prim algorithm by finding the minimum energy broadcast tree, and then develop the Broadcast Incremental Power (BIP) algorithm. The BIP algorithm is then modified in the next procedure.
- II: The second procedure is to prune the broadcast tree produced by BIP algorithm. This procedure transforms the broadcast tree into a multicast tree. The algorithm derived from this procedure is the expected MIP.
- III: The third procedure performs sweeping to eliminate unnecessary transmissions in the network. This procedure is required to improve the performance of the multicast algorithm.

The Pseudocode for the MIP algorithm is presented in in Appendix C.1 and the code is provided in the accompanying CD.

3.1.2 Performance Metric and Simulation Parameters

The performance metric considered for evaluation of minimum energy multicast is the energy-efficient, which is the multicast energy. Multicast energy is considered as the cost of transmission energy given a network sizes, the radius of connectivity, the dimension for the nodes, the source nodes and the receiving nodes using MIP algorithm. In order to measure its impact for different networks, simulations are conducted for each ad hoc wireless network with some specific inputs parameters specified for the simulation. The energy associated with the multicast networks are computed by the MIP algorithm. Note that network size using different nodes are generated randomly. Also, the source nodes and the receiving nodes are selected randomly, so any node may be the source node and receiving nodes. The simulation considered four sets of experimental parameters for evaluating the performance of ad hoc wireless networks. These simulation parameters considered are summarized below.

- *Multicast group size or sinks*: This parameter defines the number of multicast destinations that is the receiving nodes. It is chosen to vary from 2 to 10 nodes. As mentioned, the nodes are randomly selected by the MIP algorithm.
- *Network size or nodes:* This parameter represents the number or size of the network. It defines how large is the network and is varied from 20 to 40. These nodes are randomly generated by MIP algorithm.
- *Radius of connectivity:* - This parameter defines the small distance between the nodes. In this work, two radii are specified for connectivity. They are 30cm and 50cm.
- *The network dimensions:* This is the area containing the entire node considered for multicast. This work considered two set of dimensions, which are [10 × 10] m and [50 × 50] m.

3.4 Coded Packet Approach and Assumptions

The MIP method using WMA to minimise multicast energy was demonstrated. The procedures through designing BIP, then performing pruning or sweeping to achieve the desired algorithm were presented. However, the heuristics method used to reduce multicast energy is sub-optimal. In order to achieve improved minimum energy multicast, the coding method was introduced. This method considers the optimisation of minimum energy multicast using linear programming techniques. In coding method, optimal energy is expected to be less when compared to routing method using MIP. In this section, the performance of coded packet algorithm is investigated and compared with the MIP algorithm.

As pointed out in Chapter 2, the coding method was considered in literature to further minimise multicast energy. The popular approach that was considered is a flow-based, which is good for addressing networks with costs (e.g., energy) using linear programing framework [23], [107]. This cost is a function of coding subgraph z. Considering a situation whereby ξ represents the cost function and assumed that ξ is convex. Flow-based approach assumes that all nodes in the network are capable of coding, and focus on the problem of minimizing resources (e.g. energy) that can be expressed as a function of the coding subgraph *z*. As it relates to this study, energy is considered as the main resource to be minimized. Again, we denote energy function with ξ then the formulation of a multicast problem connection is considered to be a triplet $\left(S,T,\{R_t\}\right)_{t\in T}$, where *S* is the source of the connection, *T* is the set of receivers (sinks), and R_t is the set of rates to the sinks. Considering the multicast connections in a loss network using Random Linear Network Coding (RLNC) algorithm that was established in literature to address such problem, then the following Mathematical programing problem for loss problem is given as:

$$\min \xi(z)$$

Subject to

$$z \in Z$$

$$\sum_{j\in K} x^t_{(iJj)} \le z_{iJK} b_{iJK}, \quad \forall (i,J) \in H,\ K \subset J, t \in T, x^t \in F^t \tag{3.2}$$

Also, considering similar multicast connections in a lossless network, then the Mmathematical programing problem for lossless problem is simplified as:

$$\min \xi(z)$$

Subject to

$$z \in Z$$

$$\sum_{j\in J} x^t_{(iJj)} \le z_{iJ}, \qquad \forall (i,J) \in H,\ \ t \in T, x^t \in F^t \tag{3.3}$$

where $x^t_{(iKk)}$ represents the average rate of packets that are injected on hyper arc (i,K) and received by exactly the set of nodes K, which occurs with average rate z_{iJK} and that allocated to a particular connection. Also $b_{iJK} = \dfrac{\sum_{\{N\subset K|N \cap L 0\}} z_{iJN}}{z_{iK}}$ is the fraction of packets injected on hyper arc *(i, J)* received by a set of nodes that intersect *K*. Also, F^t is the bounded polyhedron of points x^t satisfying the conservation of flow constraints.

In order to achieve the desired results, these problems were simplified further using various techniques and assumptions [33], [108]. For example, it is assumed that when nodes transmit in a lossless network, they reach all nodes in certain regions, with cost increasing as the region expanded. This particular assumption is reasonable because it is suitable for some applications, such as minimising energy consumption problem which we are considering. As a result, the problem was analysed and reduced to the case of linear separable cost and separable constraints. Then a fixed cost such as energy can be evaluated while the constraints set for Z are dropped. That is, if we suppose $\xi(z) = \sum_{(i,K)\in H} a_{iK} z_{iK}$ where ξ_{iK} monotonically increasing while z_{iK} is varied, then, it is possible to achieve minimum energy multicast in a lossless wireless network without explicit regard for throughput or bandwidth with a_{iK} representing the energy required to transmit a packet to nodes in K from node i. This process reduces the original problem into a linear optimisation problem with polynomial number of constraints that can be solved in polynomial time. However, solving the same problem using traditional method is NP-complete [52], [108].

3.4.1 Coded Packet Algorithm

Network coding is an alternative method for solving multicast problems. The approach is used to reduce multicast problem to a polynomial-time that can be solved by optimisation technique. An optimal subgraph in polynomial time could be found using a decentralised form of computation and based on this technique, a RLNC algorithm was derived [86] [109]. The earlier algorithm, using the network coding approach is derived as Linear Network Coding (LNC) [80]. Although the LNC is sufficient for achieving the multicast capacity, however, in order to deploy network coding in a real multicast network to achieve efficient results, RLNC algorithms are used [86]. The pseudocode for RLNC algorithm is presented in Appendix C.2. Interested readers are referred to [86] for detail about RLNC algorithm. The coded packet code is provided in the accompanying CD.

3.4.2 Performance Metric and Simulation Parameters

Similar to the MIP method, the performance metric considered for evaluation of minimum energy multicast using coded packet method is the energy-efficient, which is the multicast energy. Also, multicast energy is considered as the cost of transmission energy given a network sizes, the radius of connectivity, the dimension for the nodes, the source nodes and the receiving nodes, is using the coded packet algorithm.

A performance metric in terms of energy-efficient is investigated for coded packet using RLNC algorithm. Multicast energy is considered as a cost in the multicast tree of different network sizes. In order to evaluate the multicast energy performance, simulations are set up with the parameters configured for an ad hoc network. Similar to the MIP, the source node, network size, dimension containing all nodes, radius of connectivity and the receiving nodes are specified as the input parameters. The energy associated with the multicast tree is computed using RLNC algorithm, which is designed for coded packet networks. Using these settings, multicast energy of a particular ad hoc network can be computed. The average of the multicast energy is then evaluated. In addition, the simulation considered the same sets of experimental parameters for evaluating the performance of ad hoc wireless networks. These simulation parameters, which are receiving/sink nodes, network size, and radius of connectivity and network dimensions were defined in in the previous section.

In a broader view, the simulation is performed using the minimum energy multicast framework in which communication nodes were placed randomly according to a uniform distribution over a *(10 × 10) m* and *(50 × 50) m* with a radius of connectivity of *30cm* and *50cm* to generate various network scenarios. The average energy consumed by ad hoc wireless multicast networks is evaluated. The transmissions are subject to the distance of attenuation. Therefore, when node *i* transmits, the signal-to-noise ratio (SNR) of the signal received at node J is $\mu d\left(i,J\right)^{-\alpha}$ where μ is an exponentially distributed random variable with unit mean, $d\left(i,J\right)$ represents the distance between node *i* and node *J*, and α represents an attenuation parameter that ranges between *2* and *4*, and $\alpha = 2$ is assumed. Also, a threshold for SNR is denoted with *β=0.25* such that $\mu d\left(i,j\right)^{-\alpha} \geq \beta$ implies a packet transmitted by node *i* is successfully received. In order to focus on the multicast energy, we assumed a stable condition. This means that a constant value is assumed for some parameters. Specifically, we assumed a fixed bandwidth and zero interference in the transmission system.

3.5 Simulation Results for MIP and Coded Packet Methods

After all the parameters were set for the simulation, three outputs with name and pathnames are acquired. The first output file contains information about the networks (this is the file that contains all the information about the nodes, how they are connected together and the link energy associated with each node). An example of the first output file that contains such information about each link is given in Table D.1 under Appendix. For example, the first line of information in the Table is interpreted as start-node 0 is linked to an end-node 3000 with a link-cost or link energy of 7.99970. Similarly, the next line of information in the Table is interpreted as the start-node 0 is linked to an end-node 4000 with a link-cost or link energy of 2.32269. Assuming that this output file name is “NetInfo.dat”, and then the file is directed to the next procedure of the MIP and RLNC algorithm where the link-cost (energy) are extracted and the optimal value is obtained. The readers are referred to [33] for details about the way networks are generated. Figure D1 under Appendix D.2 presents the screen shot of how the MIP algorithm computes the optimal multicast energy from the first output file while Figure D2 under Appendix D.3 shows the screen shot of how RLNC computes the optimal multicast energy. As observed from these Figures, the optimal solution when the MIP algorithm is considered is 6.73533. In the case of RLNC, the optimal solution is found and is equal to 6.35804. A process of obtaining these results is to specify the parameters for the computation of the optimal value. The format for defining parameters is given as: total number of randomly generated nodes (-- total = 30), number of receiving nodes (-- nodes = 4), dimension occupied by the nodes (-- dim = 10), radius of connectivity (-- rad= 30) and the first output file name NetInfo.dat (--file NetInfo.dat). For easy computation and analysis, scripts were written to compute all the required optimal energy. These scripts and related files are provided in accompanying CD. The next two subsections, which are 3.5.1 and 3.5.2 discuss the results obtained from the MIP and coded packet respectively.

3.5.1 Results for MIP Method

In this section, the reports of the multicast energy computed by the MIP algorithm are reported. As could be observed, Table 3.1 reports the summarized results of the average multicast energy computed by the MIP algorithm.

Table 3.1: Mean of multicast energy for sending multicast message to receiver nodes 2, 3, 4, 5, 6, 7, 8, 9, and 10 within randomly generated nodes 20, 30, and 40 connected together with a radius of [30cm and 50cm] in [(10 × 10m) and (50 × 50m)] dimensions

	Average Multicast Energy					
	Radius = 30cm, Dimension = 10m			*Radius = 50cm, Dimension = 50m*		
Sinks	20 nodes	30 nodes	40 nodes	20 nodes	30 nodes	40 nodes
2	7.33690	6.80406	6.70263	7.49550	6.61611	6.29201
3	8.19434	7.59840	6.52575	8.51317	7.82157	6.81350
4	8.98436	8.17163	7.48482	9.34969	7.66053	7.15764
5	9.04870	8.62102	7.47209	9.33436	8.30668	7.41543
6	9.48655	9.28325	8.05990	10.0200	8.60528	8.03526
7	10.4696	8.93184	8.36012	9.50838	9.78652	8.24464
8	9.92203	9.54203	8.61111	10.2374	9.73328	8.62671
9	10.7971	9.97383	8.86690	10.8043	9.93892	8.49562
10	10.8188	9.26350	9.00432	10.6641	9.60166	9.01432

The Table shows the results of 20, 30 and 40 randomly generated nodes for 54 ad hoc wireless networks. That is, each entry represents result for a network with varying network size. The graphical interpretation of the cumulative results is presented in Figure 3.3 and Figure 3.4. The Figure 3.3 represents the results with parameters {Radius = 30cm, Dimension = 10m, Nodes = 20, 30, 40, and Sinks = 2, 3, 4, 5, 6, 7, 8, 9, 10}, while Figure 3.4 represents the results with parameters {Radius = 50cm, Dimension = 50m, Nodes = 20, 30, 40, and Sinks = 2, 3, 4, 5, 6, 7, 8, 9, 10}. The network size is defined as the number of nodes randomly generated, which are 20, 30, and 40. As it could be observed from the two Figures, the interpretation is as follows: as network size increases, the performance of MIP algorithm in reducing power improved. Considering some sample from Figure 3.3, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 2} is 7.3369. Also, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 30 and Sinks = 2} is 6.80406. That is, the average multicast energy reduces from 7.3369 to 6.80406, with a difference of 0.53284.

Another parameter that we investigated is the impact of sinks. This is the receiving node involved with the multicast activities. As it could be observed from Figure 3.3 and Figure 3.4, the performance of MIP algorithm reduced as the number of sinks increases. It means that the number of sinks is one of the factors to be taken seriously because it greatly affects the performance of the MIP algorithm. Considering the sinks parameter, we take some sample from Table 3.1, the average multicast power for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 2} is 7.3369. Also, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 3} is 8.19434. As for the sinks, the performance of MIP algorithm degenerates as the number of sinks increases. The difference in average multicast energy is 0.85744.

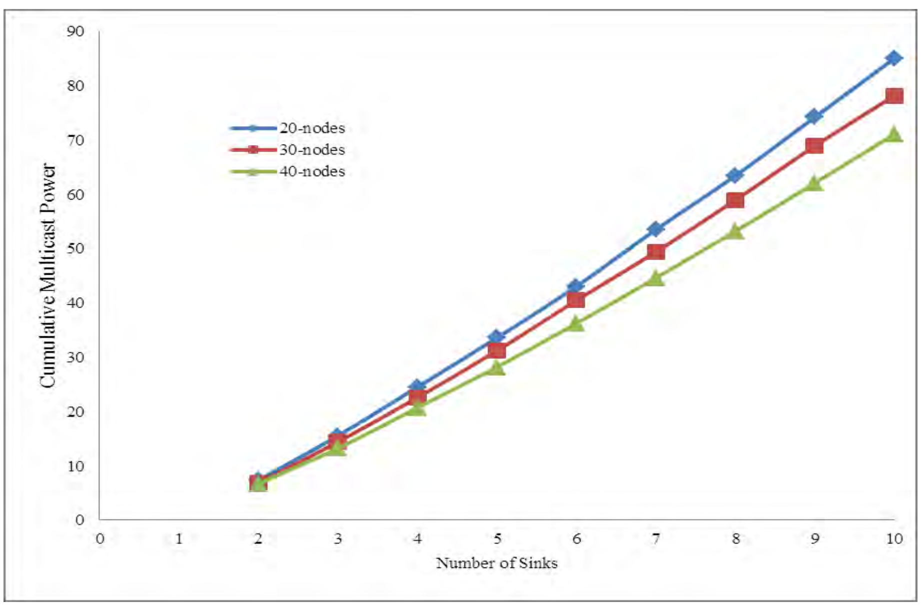

Figure 3.3: Cumulative average multicast energy against the number of sinks sending multicast message to receiver nodes 2, 3, 4, 5, 6, 7, 8, 9, and 10 within randomly generated nodes 20, 30, and 40 connected together with a radius of 30cm in 10 × 10m dimension square using Multicast Incremental Power (MIP) algorithm.

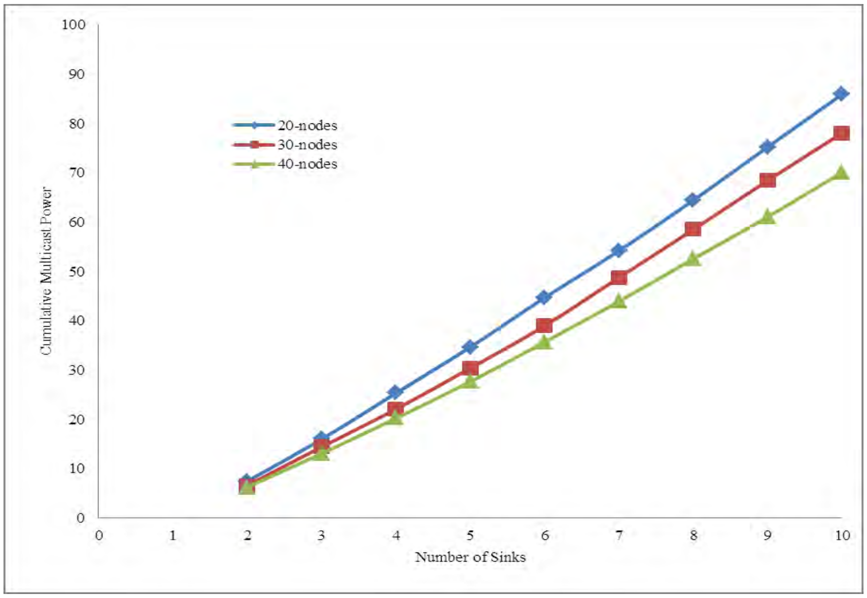

Figure 3.4: Cumulative average multicast energy against the number of sinks sending multicast message to receiver nodes 2, 3, 4, 5, 6, 7, 8, 9, and 10 within randomly generated nodes 20, 30, and 40 connected together with a radius of 50cm in 50 ×50m dimension square using Multicast Incremental Power (MIP) algorithm.

3.5.2 Results for Coded Packet Method

In this section, the results of the coded packet for minimum energy multicast computed by the RLNC algorithm are reported. As could be observed, Table 3.2 presents the summarised results of the multicast energy computed by the RLNC algorithm. Similar to the MIP algorithm, the Table shows the results of 20, 30 and 40 randomly generated nodes for 54 ad hoc wireless networks. That is, each entry represents results for an ad hoc network with varying network size. The graphical interpretation of the cumulative results is presented in Figure 3.5 and Figure 3.6. The Figure 3.5 represents the results with the following parameters: {Radius = 30cm, Dimension = 10 × 10m, Nodes = 20, 30, 40, and Sinks = 2, 3, 4, 5, 6, 7, 8, 9, 10}, while Figure 3.6 represents the results with the following parameters: {Radius = 50cm, Dimension = 50m, Nodes = 20, 30, 40, and Sinks = 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Table 3.2: Mean of multicast energy used to send multicast message from a source node to receiver nodes {2, 3, 4, 5, 6, 7, 8, 9, 10} within randomly generated nodes {20, 30, 40} connected together with radius {30cm, 50cm} in {(10 × 10) m , (50 × 50) m} dimensions square.

	Average Multicast Energy					
	Radius = 30cm, Dimension = 10m			*Radius = 50cm, Dimension = 50m*		
Sink	20 nodes	30 nodes	40 nodes	20 nodes	30 nodes	40 nodes
2	4.50027	4.15490	3.14390	5.19479	3.60785	3.62417
3	5.46086	5.30356	4.60581	5.55607	5.1002	4.31278
4	6.22791	5.35979	4.75666	6.28641	5.56776	5.06807
5	6.81511	6.07549	4.75814	6.85942	5.87098	5.12135
6	7.32855	6.18796	5.56181	7.12087	6.09464	5.45237
7	7.23365	6.37327	5.58696	7.18488	6.76687	5.74148
8	8.10404	6.57230	6.25809	7.73925	6.62772	6.43736
9	8.81448	7.34824	6.29795	8.56634	7.14271	6.42996
10	8.45438	6.74705	6.30145	8.33395	7.12791	6.50145

We define the network size as the number of nodes that are randomly generated, which are 20, 30, and 40. It can be noted from the two figures that as the network size increases, the performance of RLNC algorithm in minimising energy improved. We take some sample from Figure 3.5, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 2} is 4.50027. Also, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 40 and Sinks = 2} is 3.14390. That is, for the sampled 20 nodes and 40 nodes, the average multicast energy reduces from 4.50027 to 3.14390, with a difference of 1.35637.

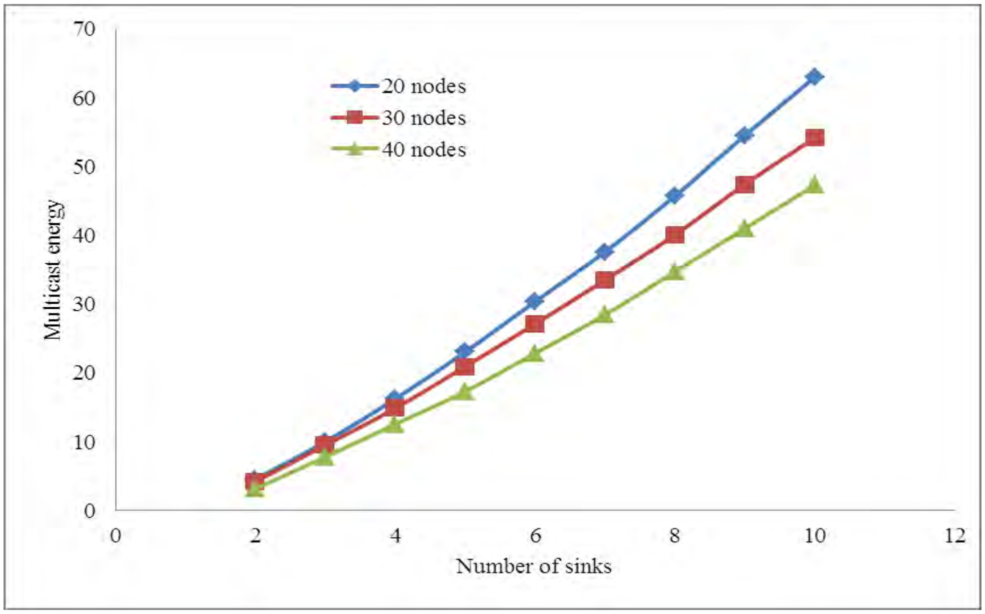

Figure 3.5: Cumulative average multicast energy against the number of sinks for sending multicast message from a source node to receiver nodes 2, 3, 4, 5, 6, 7, 8, 9, and 10 within randomly generated nodes 20, 30, and 40 connected together with radius of 30cm in 10 × 10m dimension square using RLNC algorithm for computation.

Similar to the MIP algorithm, we observe that the the number of sinks have a serious impact in the network so it is considered an important output variable in the DEA implementation. As could be observed from Table 3.2, the performance using RLNC algorithm reduces as the number of sinks increases. It means that the number of sinks is ultimately one of the factors that has to be taken seriously because it greatly affects the performance of the RLNC algorithm. The correlation value of approximately 0.75 with the multicast energy also suggests the importance of these parameters. Considering the sinks parameter, we take some sample from Table 3.2, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 2} is 4.50027. Also, the average multicast energy for {Radius = 30cm, Dimension = 10m, Nodes = 20 and Sinks = 4} is 6.22791. As for the sinks, the performance using RLNC algorithm degenerates as the number of sinks increases. The difference in average multicast energy for 2 samples and 4 sinks is 1.72764.

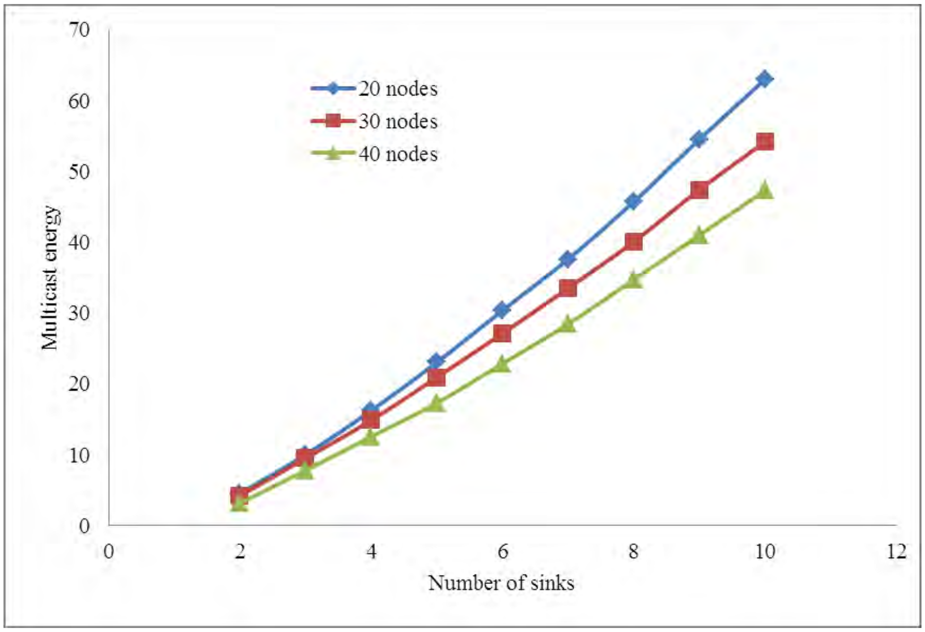

Figure 3.6: Cumulative average multicast energy against the number of sinks for sending multicast message from a source node to receiver nodes 2, 3, 4, 5, 6, 7, 8, 9, and 10 within randomly generated nodes 20, 30, and 40 connected together with radius of 50cm in 50 × 50m dimension square using RLNC algorithm for computation.

3.5.3 Performance Comparison of MIP and Coded Packet

In this section, the performance of the MIP and coded packet algorithms are compared using the simulation results presented in Table 3.1 and Table 3.2. Two parameters are considered for comparison, the network size or number of nodes generated and the multicast group size or number of receivers.

1) Varying the Network Size: Network size is the number of randomly generated nodes, which ranges from 20 to 40 nodes. In Figure 3.7**,** first, the result of the performance for the multicast energy is presented. Observing the effects when the network size increases, both MIP and coded packet algorithm performances also improved. The improvement means that their multicast energy reduced as the network size increases. Also observe that the simulation results show the superiority of coded packet over MIP algorithm in terms of energy reduction. The dotted ovals are used to separate MIP and coded packet on the same graph area. This performance gain of coded packet over MIP has been established in the literature [33].

2) Varying the Multicast Group Size: This is the number of sinks that successfully received multicast message from a source. Figure 3.7 also presents a striking difference between MIP and coded packet algorithm performance variations in terms of multicast group size (number of sinks). An increase in the multicast group size leads to the degradation of both the MIP and coded packet algorithms performances. The degradation means that their multicast energy increased as the multicast group size increases. Overall, in terms of multicast energy consumptions, the performance of the coded packet algorithm is better compared to the MIP algorithm. This is evidence from the graph, because the coded packet algorithm consumed lesser multicast energy. As a result, we proceed with the exploration of coded packet for further reduction of multicast energy using empirical method. Therefore, the remainder of this thesis is about coded packet, which is the recent paradigm in information theory.

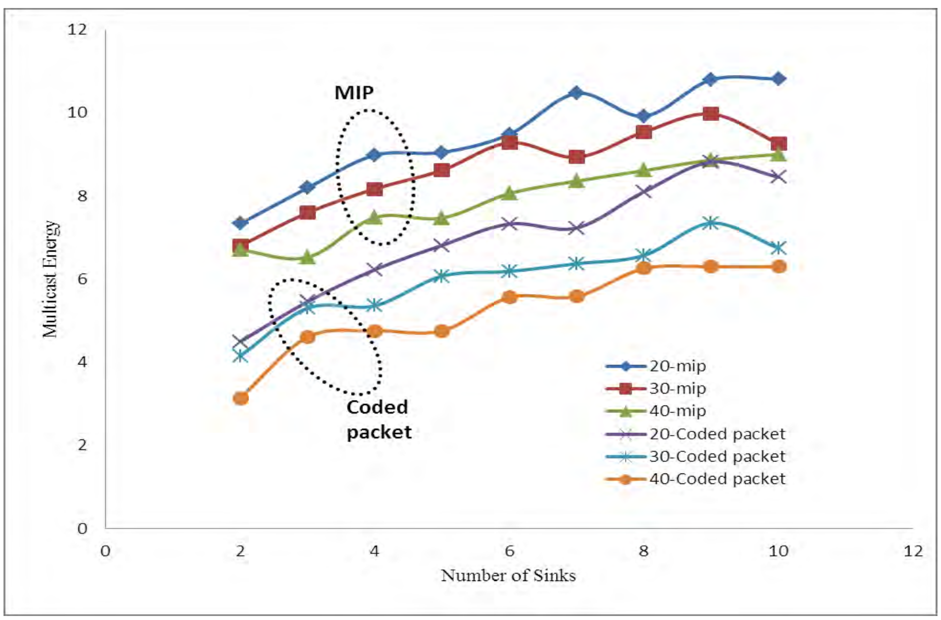

Figure 3.7: Comparing the multicast energy of MIP and NC technique for randomly generated nodes = 20, 30 and 40 with different number of receivers = 2, 3, 4, 5, 6, 7, 8, 9 and 10 of an ad hoc wireless network

3.6 Chapter Summary

This chapter investigated the performance of the two popular algorithms for minimum energy multicast and compared their performance levels. Simulation results have shown that coded packet method outperformed the MIP method. Furthermore, we found that the MIP and coded packet algorithms have tried to minimise the multicast energy, however the attempts made were based on the effective performance, which are generally sub-optimal. Thus the chapter was able to show that the performance of these algorithms was evaluated based on effective performance only. While effective performance is a good evaluation tool, it is not enough to measure the efficiency of networks appropriately. In addition, these algorithms can only provide information based on single metric meaning that the current approaches could not evaluate the efficiency of network appropriately. The next Chapter will address this problem using a multi-criteria decision that considers multiple inputs and multiple outputs. With this approach the expected optimal solution for minimum energy multicast could be achieved. Thus the minimum energy is expected to further reduce beyond the capacity of coded packet algorithms without affecting the output results. A better approach for the system evaluation is one that is based on the TE forms of measurement. It is important to mention that the remainder of this thesis will focus on more techniques to further explore the coded packet performance especially on how to further reduce multicast energy.

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Chapter Four: Envelopment Models for Evaluation of Energy Efficiency

4.1 Introduction

Chapter 3 investigated two current techniques for minimum energy multicast. The evaluation was performed using statistical average to determine the effectiveness of multicast energy. In this chapter, the evaluation will be performed using relative input and output simultaneously. However, evaluating network performance using relative input and output weights is a ratio problem, which requires economic and Operational Research approach. This of course may be a reason why the MIP solution using heuristic approach is sub-optimal, even the coded packet technique using linear programing approach could not yield the expected minimum energy. We discovered that handling minimum energy multicast in a conventional engineering method mitigate against TE evaluation whereby multiple inputs are transformed into multiple outputs irrespective of their volume. Therefore, TE evaluation approaches are based on non-parametric techniques whereby a *priori* information is not required. In the next section, a generalized proposed empirical architecture is discussed for energy efficiency. A Mathematical approach to the model's development is presented in sections 4.3 and 4.4, followed by the Envelopment model in section 4.5. A simulation set up and evaluation of Envelopment model is given in section 4.6 and section 4.7 respectively, and to finish off, the scale efficiency (SE) models is presented in section 4.8. The conclusion is drawn in section 4.9.

4.2 The Empirical Architecture for Energy Efficiency

Figure 4.1 summarizes the generalized and proposed empirical architecture, which is the combination of the existing minimum energy multicast (coded packet model) and the DEA model. As it could be observed from Figure 4.1, the first part, which is the coded packet model, requires that the simulation is set up according to the minimum energy multicast framework for coded packet networks using RLNC algorithm. The simulation is set with the algorithm requirements and the inputs parameters preconfigured. Also, note that the result obtained from the simulation set up was presented in Section 3.5. It should be noted also that the multicast energy evaluated is based on effective performance. The second part of Figure 4.1 presents the

DEA models. The architecture consists of different components and the first component converts the multiple inputs and output data by the DMU transformation. Then using envelopment or multiplier model, the technical and scale efficiency scores of each DMU are evaluated. Furthermore, with the aid of the Slack model, the inefficient and weak efficient ad wireless hoc networks (DMUs) are identified and projected unto their efficient frontier. Also, using the Benchmark model, the Efficient Reference Set (ERS) or peers group are identified for inefficient and weak efficient ad hoc networks. In addition, the expected multicast energy of each ad hoc network is evaluated and compared with the average multicast energy computed by the RLNC algorithm. The differences in energy are computed by the Energy Gap (EG) model and this difference is the energy saved.

In terms of model development, different DEA models assuming input-orientation are developed. These models include Envelopment, Slack, Benchmark and Energy Gap models. The models were built upon CCR and BCC with the assumption of CRS and VRS. The basic CCR and BCC models with their respective assumptions have been discussed in sections 2.4.2, 2.4.3, 2.4.4 and 2.4.5. In this thesis, the models that are developed for energy efficiency are summarized as follows:

(i) Envelopment Model based on CCR – This model evaluates technical efficiency of each ad hoc wireless network assuming

CRS.

(ii) Envelopment Model based on BCC – This model evaluates technical efficiency of each ad hoc wireless network assuming VRS.

(iii) The Slack Model based on CCR – This model projects both inefficient and weak efficient ad hoc wireless networks unto their efficient frontier assuming CRS.

(iv) The Slack Model based on BCC – This model projects both inefficient and weak efficient ad hoc wireless networks unto their efficient frontier assuming VRS.

(v) Benchmarking Model based on CCR - This model determines ERS (peer group) and calculates lambda for each ad hoc wireless network assuming CRS.

(vi) Benchmarking Model based on BCC – This model determines ERS (peer group) and calculates lambda for each ad hoc wireless network assuming VRS.

(vii) Energy Gap (EG) Model based on CCR – This is the mechanism that calculates the energy saved by each ad hoc wireless network assuming CRS.

(viii) Energy Gap (EG) Model based on BCC – This is the mechanism that calculates the energy saved by each ad hoc wireless network assuming VRS.

In addition, these models seek to evaluate TE, project the inefficient and weakly efficient, determine ERS, calculate lambdas, and most important minimise multicast energy through linear programing techniques obtained from ratio problem. Note that the minimal energy achieved does not affect the output results. In other words, these models keep the current empirical level of outputs constant and minimizes the inputs (energy). To the best of our knowledge, there are no energy-efficient models based on input-orientation developed upon CCR/BCC using CRS/VRS assumptions for multicasting in ad hoc wireless network. That is, there is no work in energy efficiency that explores the relative input and output weights using DEA method to minimise the transmission energy in ad hoc wireless networks.

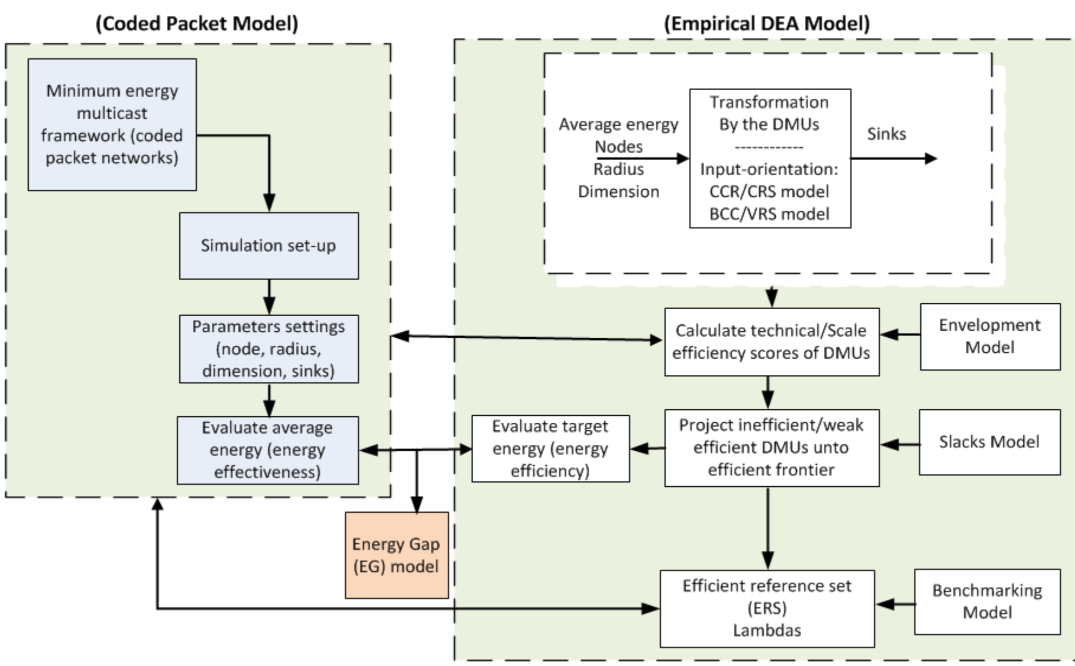

Figure 4.1: Generalised architecture of the extended minimum energy multicast combining coded packet and DEA models

4.3 Mathematical Representation of Ratio Problems

This section explores the technical efficiency (TE) evaluation through Mathematical approach. Since technical efficiency evaluation is classified as ratio problem, there is a need to express the variables in this form. This approach to ratio problems optimisation is known as efficiency ratio from which Envelopment model will be derived [110], [111]. Efficiency ratio approach using weights is accomplished through multiplier model approach or using a convex combinations that satisfy *convexity* and *inefficiency properties* [112]. See properties a.1 and a.2 in Appendix A for the definition of convexity and inefficiency properties respectively. In this thesis, we consider five indexes with four inputs and one output. If the efficiency ratio approach is applied to these indexes, then the inputs and outputs in terms of ratio problem could be defined. Thus the weights required for model development is constructed through the following Mathematical representation that relates all the variables together:

$\frac{u(g)}{v_1(e)+v_2(d)+v_3(r)+v_4(z)}$, where u denotes the weight for the output, and v_1, v_2, v_3, v_4 are weight for the inputs. Also, the sinks is denoted with letter *g*, the multicast energy with letter *e*, the dimension with letter *d*, and radius with letter *r* while *z* represents the number of nodes in the network. This ratio is evaluated for each of the ad hoc wireless network, which is denoted with notation DMU. This technique performs optimisation by finding the maximum efficiency that an ad hoc network can achieve under a set of weights. These set of weights then provide optimal value that efficiency ratio achieved from *n* observations. In this case, 54 ad hoc wireless networks are considered. The expectation is to determine the *efficient frontier* or *best practice* for these 54 ad hoc wireless networks. It is assumed that the input and the output are all non-negative data.

In order to develop appropriate model for all the 54 ad hoc networks (*n=54)*, we first express the efficiency ratio by considering a set of n observations on the DMUs where each observation, DMU_j $\{j=1,2,\ldots,n\}$ uses m multiple inputs x_{ij} $(i=1,2,\ldots,m)$ to produce s multiple outputs y_{rj} $(r=1,2,\ldots,s)$. The variable x_{ij} represents the vector of inputs into DMU_{ij} and y_{rj} represents the corresponding vector of outputs. Then, the efficiency ratio (performance) for DMU_j can be expressed as:

$$Performance = \frac{Virtual\,output}{Virtual\,input}$$

$$Performance = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{u_1 y_{1j} + u_2 y_{2j} + \ldots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \ldots + v_m x_{mj}} \tag{4.1}$$

Where $u_r \left(r = 1, 2, \ldots, s\right) and\ v_i \left(i = 1, 2, \ldots, m\right)$ are unknown weights. This ratio accounts for all outputs and inputs. The weights assigned to each input and each output is used as variables in the optimisation process. Furthermore, if a particular ad hoc network (DMU_0) is considered, the objective is to maximise the efficiency:

$$max \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{io}} = \frac{u_1 y_{1o} + u_2 y_{2o} + \ldots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \ldots + v_m x_{mo}} \tag{4.2}$$

However, the maximisation problem (4.2) is unbounded meaning that additional constraints need to be established. One of the constraints is to assume a set of normalisation, one for each DMU. Also, a condition that the virtual output to virtual input ratio of every DMU must be less than or equal to unity is necessary.

4.4 Model Development for Ratio Problem

This section presents the background about the models that are considered for minimisation of energy problem. The DEA approach to efficiency evaluation is a ratio problem that needs to be converted into a linear form. Subsections 4.4.1 and 4.4.2 present the problem in fractional and linear form respectively.

4.4.1 CCR/CRS Model in Fractional Form

In dealing with ratio problem, an effective technique to consider is the Linear Programming (LP). It is a framework that is paramount to optimisation problems. The goal here is to develop models that are appropriate for minimisation of energy problem using LP framework [111]. However, the DEA technique uses the efficiency ratio approach to determine a set of weights that yields the maximum efficiency ratio. This leads to the following Mathematical programming relation:

$$maximise \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \; j = 1, 2, \ldots, n$$

$$u_r, v_i \geq 0 \; (r = 1, 2, \ldots, s) \text{ and } v_i \, (i = 1, 2, \ldots, m) \tag{4.3}$$

The DMU_o is then maximised using efficiency ratio model (4.3), and can be generalised for all the DMU_s. In other words, model (4.3) can be used to calculate the efficiency of a specific member of a given set of n DMU_s subject to a condition that the efficiency ratings or scores of each member should not exceed 1. However, model (4.3) is in fractional form meaning that it is a non-linear programing model. Consequently, model (4.3) in its present form is difficult to solve. Hence, a transformation is required.

4.4.2 CCR/CRS Model in Linear Form

The approach to convert model (4.3) into an equivalent linear optimisation form was established and presented in [54]. Considering this transformation technique, model (4.3) is rewritten as:

$$Max\, z = \sum_{r=1}^{s} \mu_r y_{r0}$$

Subject to

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \;\; j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\mu_r, v_i \geq 0 \tag{4.4}$$

Where μ_r *and* v_i are decision variables and denoted output and input multipliers respectively. This type of formation as represented by model (4.4) is a multiplier, which its objective function is the weighted sum of outputs for DMU_o under evaluation. Again, the first set of *n* constraints specifies that each efficiency rating cannot be greater than 1. The constraint $\sum_{i=1}^{m} v_i x_{i0} = 1$ is a normalisation condition.

4.5 Envelopment Model Development for Energy Efficiency

The first model that is derived using ratio efficiency approach is Envelopment. This section presents two type of Envelopment model namely the input-oriented CCR/CRS and BCC/VRS.

4.5.1 Input-oriented CCR/CRS Envelopment Model

Model (4.4) appears computationally intensive, and for this reason it could be solved in its dual form. In addition, we consider the type of orientation suitable for the optimisation problem and carefully modelled with that assumption. That is, the goal of this work is to minimise the inputs while the outputs are kept at their current levels. A suitable approach for such scenario is the input-orientation. With this approach, inputs variables are controlled. In order to achieve input minimisation (e.g. energy), the dual of model (4.4) is formed and the derivation is given below.

$$\theta^* = min\ \theta$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i0}, \qquad i = 1, 2, \ldots, m;$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{r0}, \qquad r = 1,2,\ldots,s;$$

$$\lambda_j \geq 0, \qquad j = 1,2,\ldots,n, \qquad (4.5)$$

Where λ_j are unknown weights with $j = 1,2,\ldots,n$ and they correspond to the DMU numbers. DMU_0 is one of the *n* DMU under evaluation, and θx_{i0} and y_{r0} are the i^{th} input and r^{th} output for DMU_0 respectively. Model (4.5) is the expected input-oriented CCR with Constant Returns to Scale (CRS) envelopment model and its interpretation is summarized as follows:

Definition (4.1): If $\theta^* = 1$ *, then the DMU under evaluation is a frontier point (efficient), that is, no other* DMU_s *operates more efficiently than this DMU. Otherwise if* $\theta^* < 1$*, then the DMU under evaluation is inefficient, that is, this DMU can either increase its output levels or decrease its input levels.*

However, θ^* represents the efficiency score of DMU_o based on input-orientation. This model assumes a CRS, meaning that all observed variables combinations can be scaled up or down proportionally. In this case, the model assumes that DMU_s are able to linearly scale down the inputs to improve efficiency.

4.5.2 Input-oriented BCC/VRS Envelopment Model

The type of Envelopment model developed in the last section was formulated upon CCR model and then assumed CRS. In this section, another Envelopment model is developed. This new model is formulated using the BCC with the assumption of VRS. The CCR/CRS Envelopment model developed in previous section is derived from efficiency ratio, and then the linear programming technique is considered for input minimisation. We adopt similar procedure to Envelopment model developed based on BCC/VRS and discuss the relationship between the two models. So, we simply modify the input-oriented CCR/CRS Envelopment model presented in (4.5). The modification is given as following:

$$\theta^* = min\theta$$

Subject to

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{i0}, \qquad i = 1,2,\ldots,m;$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{r0}, \qquad r = 1,2,\ldots,s;$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0, \qquad j = 1,2,\ldots,n, \qquad (4.6)$$

where λ_j are unknown weights with $j = 1,2,\ldots,n,$ corresponds to the DMU numbers (DMU_0 is one of the n DMU under evaluation), and θx_{i0} and y_{r0} are the i[th] input and r[th] output for DMU_0 respectively. Model (4.6) is the expected input-oriented BCC assuming variable returns to scale (VRS).

The difference between CCR/CRS Envelopment model (4.5) and BCC/VRS Envelopment model (4.6) are:

(i) The $\sum_{j=1}^{n}\lambda_j = 1$, which is additional constraint means it is BCC/VRS model, and the omission means it is CCR/CRS model.

(ii) The treatment of returns to scale is another difference. The CCR/CRS model bases the evaluation on Constant Returns to Scale (CRS), whereas the BCC/VRS version is more flexible and allows Variable Returns to Scale (VRS).

Furthermore, we lay emphasis on the definition of θ^* that represents the efficiency ratings of DMU_o considering input-orientation. It means that the input values are minimised while maintaining the current output levels. Application of this to the network assumes that an ad hoc wirreless network can maintain its outputs while decreasing its input resources to become a better networks. Readers are referring to definition (4.1) for more information concerning efficiency ratings (θ^*). Also, the BCC/VRS multiplier version of the CCR/CRS model is derived and presented in Appendix E.

4.6 Simulation Set up, Envelopment Model Implementation and Results

This section discusses the simulation setup, the source of data, the architecture and model implementation, and the results of envelopment model.

4.6.1 Simulation Set up

The simulation was conducted using the coded packet framework. The first part of Figure 4.2 shows the procedures of how multicast energy was evaluated using coded packet algorithm. The details about the configuration and how results are obtained are already presented in Chapter 3. So the focus in this section is the implementation of Envelopment models, which is the second part of Figure 4.2.

4.6.2 Envelopment Model Architecture and Analysis of Data Used

Figure 4.2 represents the architecture of the Envelopment model for CCR/CRS and BCC/VRS to evaluate the TE in ad hoc wireless networks. Note that the architecture forms part of the generalized architecture presented in Figure 4.1. Observe that the first part of the architecture was discussed in Chapter 3. So this section focuses on the second part. The second part of Figure 4.2 represents the Envelopment model for the TE evaluation. The model assumed input orientation and developed upon CCR using CRS assumption. The derivation of Mathematical expression for this Envelopment model was presented in section 4.5. The main parameter that is evaluated using this model is the TE ratings, which determines the actual efficiency of the ad hoc wireless networks.

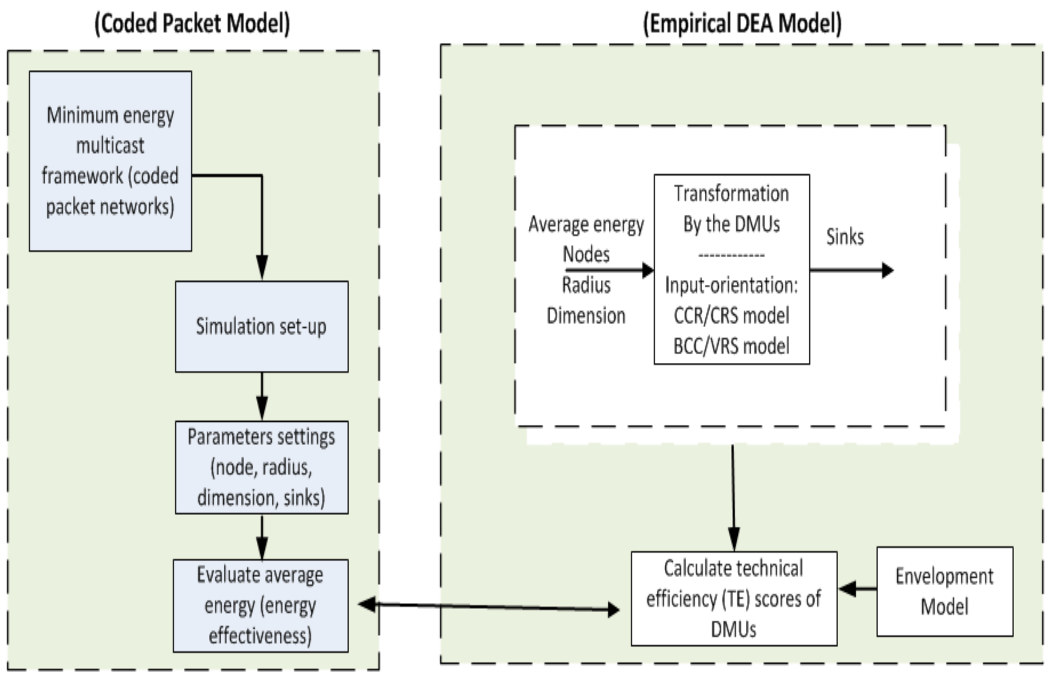

Figure 4.2: Architecture of the envelopment form for the evaluation of technical efficiency (TE) using input-oriented CCR/CRS / input-oriented BCC/VRS models

The main method of gathering the data used for Envelopment models is through the implementation of the coded packet model as shown earlier in chapter three. So using Envelopment model, the same data are transformed from inputs to outputs. Then the efficiency of each of the ad hoc wireless network is compared against the best practice networks. One of the unique characteristics of DEA models is the consideration for resources and how they impact the entire network. These resources are carefully examined then classified into inputs and outputs for the DEA solver. In the coded packet approach, the output result is the average value of the multicast energy consumed by the network nodes. In order to compare the efficiency performance with the type of efficiency evaluated by coded packet technique, the same data set presented in Chapter 3, Table 3.2 is considered and classified into inputs and output as shown in Table 4.1. This data set represents the resources or variables available for each ad hoc wireless network (DMU). Similar to the simulation set up for coded packet, each of the DMU takes four input variables and one output variable in order to multicast a message successfully from a source node to some selected nodes.

Table 4.1: Input and Output variables classification of the sampled 54 ad hoc wireless multicast networks

	Inputs				Output
DMU	**Ave. energy (e)**	**Dimension (d)**	**Radius (r)**	**Nodes (z)**	**Sinks (g)**
DMU_1	4.50027	10	3	20	2
DMU_2	5.46086	10	3	20	3
DMU_3	6.22791	10	3	20	4
DMU_4	6.81511	10	3	20	5
DMU_5	7.32855	10	3	20	6
DMU_6	7.23365	10	3	20	7
DMU_7	8.10404	10	3	20	8
DMU_8	8.81448	10	3	20	9
DMU_9	8.45438	10	3	20	10
DMU_{10}	5.19479	50	5	20	2
DMU_{11}	5.55607	50	5	20	3
DMU_{12}	6.28641	50	5	20	4
DMU_{13}	6.85942	50	5	20	5
DMU_{14}	7.12087	50	5	20	6
DMU_{15}	7.18488	50	5	20	7
DMU_{16}	7.73925	50	5	20	8
DMU_{17}	8.56634	50	5	20	9
DMU_{18}	8.33395	50	5	20	10
DMU_{19}	4.15490	10	3	30	2
DMU_{20}	5.30356	10	3	30	3
DMU_{21}	5.35979	10	3	30	4
DMU_{22}	6.07549	10	3	30	5
DMU_{23}	6.18796	10	3	30	6
DMU_{24}	6.37327	10	3	30	7
DMU_{25}	6.57230	10	3	30	8
DMU_{26}	7.34824	10	3	30	9
DMU_{27}	6.74705	10	3	30	10
DMU_{28}	3.60785	50	5	30	2
DMU_{29}	5.10020	50	5	30	3
DMU_{30}	5.56776	50	5	30	4
DMU_{31}	5.87098	50	5	30	5
DMU_{32}	6.09464	50	5	30	6
DMU_{33}	6.76687	50	5	30	7
DMU_{34}	6.62772	50	5	30	8
DMU_{35}	7.14271	50	5	30	9
DMU_{36}	7.12791	50	5	30	10
DMU_{37}	3.14390	10	3	40	2
DMU_{38}	4.60581	10	3	40	3
DMU_{39}	4.75666	10	3	40	4
DMU_{40}	4.75814	10	3	40	5
DMU_{41}	5.56181	10	3	40	6
DMU_{42}	5.58696	10	3	40	7
DMU_{43}	6.25809	10	3	40	8
DMU_{44}	6.29795	10	3	40	9
DMU_{45}	6.30145	10	3	40	10
DMU_{46}	3.62417	50	5	40	2
DMU_{47}	4.31278	50	5	40	3
DMU_{48}	5.06807	50	5	40	4
DMU_{49}	5.12135	50	5	40	5
DMU_{50}	5.45237	50	5	40	6
DMU_{51}	5.74148	50	5	40	7
DMU_{52}	6.43736	50	5	40	8
DMU_{53}	6.42996	50	5	40	9
DMU_{54}	6.50145	50	5	40	10

The function of Envelopment model is to further optimise these resources without affecting the general performance. In this thesis, the main resource to be optimized is energy, which is considered as the most important resources. However, the characteristics of all the variables are discussed and presented in Table F1 under Appendix F. Note that the results presented in Table 4.1 are the same with the simulation results presented in Table 3.2. The only difference is the serial DMU with the numbers that is assigned for identification purpose and the classification for transformation by DEA solver. With the identification number, there is no need for grouping as shown in Table 3.2. We did not consider the results evaluated by the MIP algorithm because the coded packet technique has already shown superiority over MIP technique.

Despite the variables or resources being classified into inputs and outputs, it is important to state that the data considered for this work is reliable to produce sufficient results in DEA analysis. In DEA method, the basic requirement for the number of DMUs is expected to exceed two times the number of input plus output items. Cooper *et al.* further presented a rough rule of thumb that provides guidance for DMU formation [91]. In order to have a good result using DEA analysis, the following relationship must be considered: $n > max \{m \times s, 3(m + s)\}$, where n = number of DMUs, m = number of inputs and s = number of outputs. Note that the data set considered in this thesis which are $n = 54, m = 4, and\ s = 1$ satisfied the above conditions.

In chapter three, the simulation was set-up to sample 54 ad hoc wireless networks (see the results in Table 3.2 and 4.1). These ad hoc wireless networks that were sampled assumed had different resources to achieve a successful transmission. The simulation results in Table 3.2 and 4.1 show different levels of energy required by ad hoc wireless networks. The Tables shows different 54 network scenarios for performance evaluation. The statistical analysis of data in Table 4.1 is evaluated and the summary of the findings are presented in Table 4.2. As shown in Table 4.2, the minimum of the average energy multicast (e) required is computed, which is equal to 3.1439 and the maximum of the average energy multicast required is equal to 8.8145. From this distribution, the mean and Standard Deviation (STD) is determined. The value for the mean and STD are 6.1069 and 1.2627 respectively. Similarly, the statistics for the minimum, the maximum, the mean and the STD for other network resources that is network dimension (d), radius of connectivity (r), the total number of nodes (z), and the number of sink are summarised in Table 4.2.

Table 4.2: Summary statistics of the sampled 54 ad hoc wireless multicast networks

Input /Output	Minimum	Maximum	Mean	STD
Average energy consumption, ***e***	3.1439	8.8145	6.1069	1.2627
Square dimension contains nodes, ***d***	10	50	30	20
Radius of connectivity, ***r***	3	5	4	1
Nodes (number of nodes, ***z***)	20	40	30	8.165
Sinks (receiving nodes, ***g***)	2	10	6	2.582

4.7 Technical Efficiency (TE) Evaluation and Results

The technical efficiency (TE) is evaluated using DEA software. It is the tool that was packaged to solve the Envelopment model and other types of DEA models. The DEA library includes the DEA-Solver and LPsolver (linear programming solver) to perform the network optimisations. Examples of the DEA tool are software like DEAP (DEA program), Warwick DEA, ON-Front and DEAOS (Data Envelopment Analysis Online Software). In addition, DEA tool is available as open source software online. However, this research makes use of DEAOS for the implementation of the data set discussed. This DEAOS is available as online software [113]. The readers are referred to [113] for details about the DEAOS package and user's documentation. The DEA evaluation procedures are highlighted as follows:

- *Formulating / Creating the DEA Problem/model* – First the DEA problem/model must be created as it was presented earlier in section 4.5. The data set for this problem is organised as in Table 4.1, where data were classified into input and output.
- *Setting the variables* – DEA variables must be set for DEA solver. Variables are normally set into input and output types for the DMU transformation.

- *Configuring the DEA Problem* – The DEAOS solver provides user friendly environment for DEA optimization problems. Such environment allows users to set the type of orientation for the models.
- *Saving the problem to* Excel *file* – DEA provide its internal Excel file, which is also compatible with MS Excel. The DEA data can be saved on this file for immediate and future references.
- *Solving the problem* – After all the necessary configurations are done, the DEA problem is solved to produce the results.
- *Exporting and Extracting the solutions from the* Excel *files* – The solution file can be exported in Excel format and extracted for analysis. The solution objectives include the following: efficiency scores of each ad hoc wireless network, the weights, the slacks, the lambdas, the peers group (efficiency reference set), and projections. In this section, we apply the DEAOS to solve the Envelopment model that was developed in section 4.5 and evaluate the TE for the ad hoc wireless networks. Two types of model were evaluated and analysed: The input-oriented CCR/CRS model (4.5), and the input-oriented BCC/VRS model (4.6). In Appendix G.1, an instance of convex combination and LP for TE formulation of 54 ad hoc wireless networks is analytically presented.

4.7.1 Input-oriented CCR/CRS Envelopment Results

In this section, model (4.5) is solved using the DEAOS and the TE scores are extracted from the "Efficiency" sheet provided. The DEAOS implementation details and the raw data of efficiency are provided on the CD accompanying this thesis. The model results are analysed and evaluated for TE in ad hoc wireless networks. The model evaluates the efficiency of each ad hoc wireless network (DMU) and compared with other ad hoc wireless network (DMUs). It identifies those DMUs that are operating inefficiently as compared with other DMUs' actual operating results. It accomplishes this by locating the best practice DMUs and then evaluates the magnitude of inefficiency of the inefficient DMUs compared to the best practice DMUs. The best practice DMUs are relatively efficient and are identified by a DEA efficiency rating of $\theta = 1$. The inefficient DMUs are identified by an efficiency rating of less than 1

($\theta < 1$). Table H.1 under Appendix H is the extraction of the efficiency of all the DMU from the "Efficiency" sheet computed by the DEA solver. Column two of Table H.1 gives the results of DEA technical efficiency (TE) ratings (scores) of the 54 ad hoc wireless networks (DMUs). Column three of Table H.1 also presents the TE scores in their percentages.

Table H.1 presents the results where only DMU_9, DMU_{18}, DMU_{27} and DMU_{45} have efficiency score of $\theta = 1$ (i.e. 100%) and thus they are identified as efficient. Other DMUs have efficiency scores of less than 1 ($\theta < 1$) but greater than 0, and thus they are identified as inefficient. Figure 4.3 presents the efficiency scores against each of the DMUs. Note that the inefficient DMUs can improve their technical efficiency scores. For example, DMU_1 can improve its technical efficiency score by reducing certain inputs up to 70% (100 - 30). Similarly, DMU_2 can do so with approximately 59.9% input reduction. However, DMU_{36} is closer to an efficient frontier, and needs only a 3.2% reduction of its input resources. This analysis is the technical inefficiency of the DMUs and the results is presented as a percentage in column four of Table H.1.

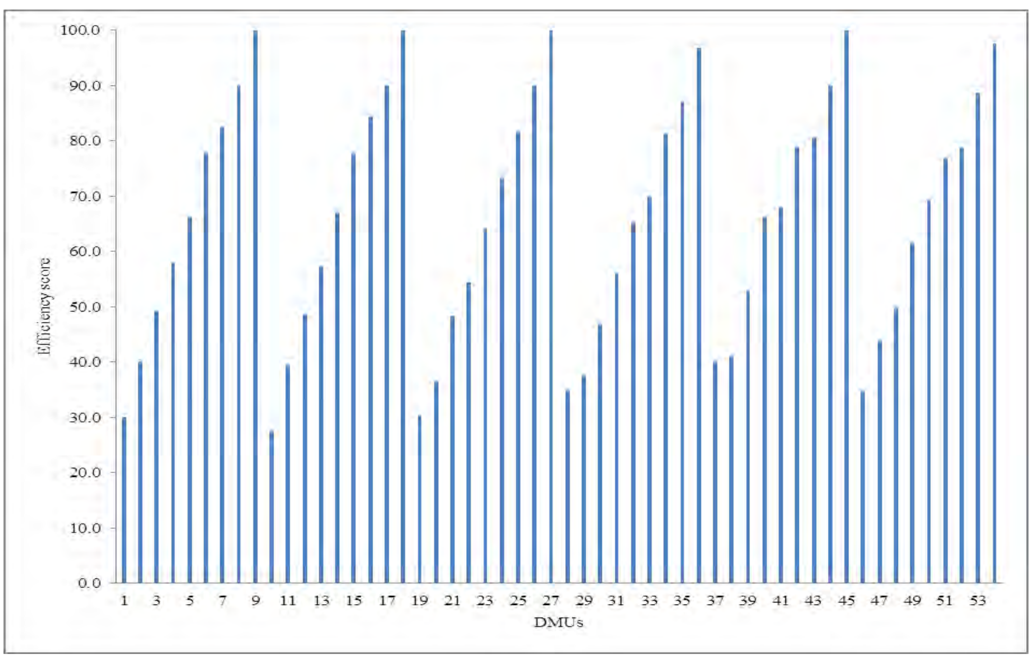

Figure 4.3: Distribution of technical efficiency scores of 54 ad hoc wireless networks computed by envelopment form using the input-oriented CCR/CRS model.

4.7.2 Input-oriented BCC/VRS Envelopment Results

In this section, the second model that was developed is also evaluated for technical efficiency in ad hoc wireless networks. Specifically, model (4.6), which is input-oriented BCC/VRS Envelopment model is implemented using the DEAOS. The TE scores are extracted from the "Efficiency" sheet provided. The DEAOS implementation details and the raw data of efficiency are provided in the CD accompanying this thesis. Similar to model (4.5), the DEA compares each DMU with all other DMUs, and identifies those DMUs that are operating inefficiently compared with other DMUs' actual operating results. Again, it accomplished this by locating the best practice DMUs. It also evaluates the magnitude of inefficiency of the inefficient DMUs compared to the best practice DMUs. Also, the best practice DMUs are relatively efficient and are identified by a DEA efficiency rating of $\theta = 1$. The inefficient DMUs are identified by an efficiency rating of less than 1 ($\theta < 1$). Table H.2 under Appendix H presents the extraction of the efficiency of all the DMU from the worksheet reports which are computed by the DEA solver. Column two of Table H.2 reports the results of DEA efficiency ratings of 54 ad hoc wireless networks (DMUs).

Also from Table H.2, DMUs 1 to 28 and DMU_{37} to DMU_{45} have efficiency scores of $\theta = 1$ and thus they are identified as efficient. DMU_{29} to DMU_{36} and DMU_{46} to DMU_{54} have efficiency scores of less than 1 ($\theta < 1$) but greater than 0, and thus they are identified as inefficient. In order to better understand how the efficiency scores are distributed Figure 4.4 presents the efficiency scores against each of the DMUs. Again, note that the inefficient DMUs can improve their efficiency, or reduce their inefficiencies proportionately, by reducing their inputs (since we run an input-oriented DEA model). For example, DMU_{29} can improve its efficiency score by reducing certain inputs up to 14.1% (1.0 - 0.859). Similarly, DMU_{31} can do so with approximately 12.7% (1.0 - 0.873) input reduction. Observe form the Table that DMU_{36} is closer to an efficient frontier and needs only a 3.2% (1 - 0.968) reduction of its input resources.

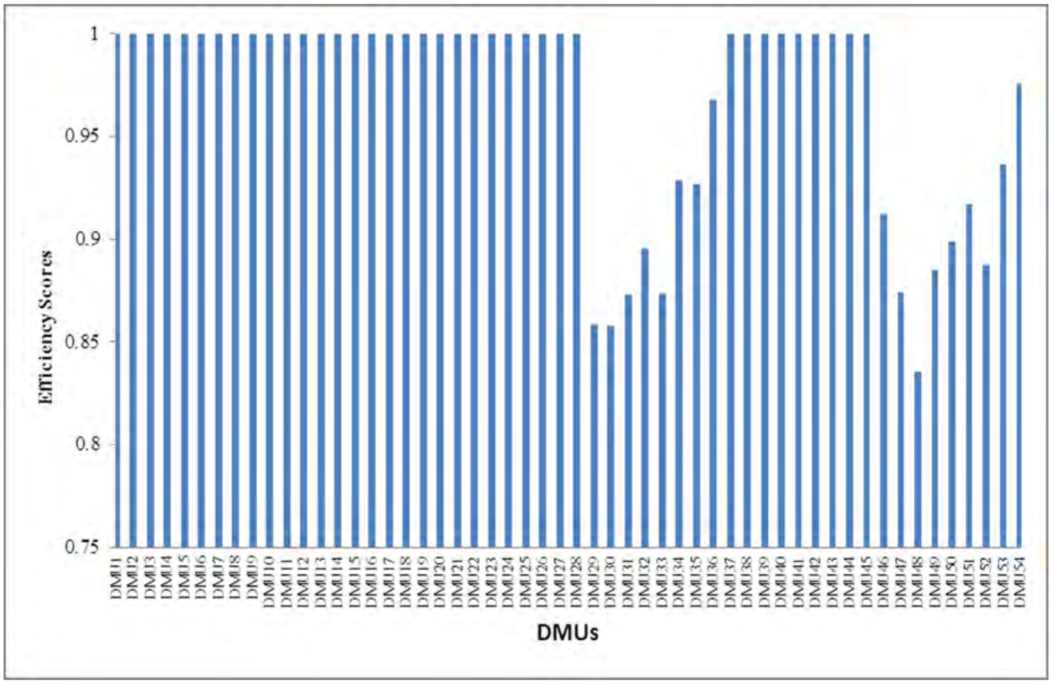

Figure 4.4: Distribution of technical efficiency scores of 54 ad hoc wireless networks computed by envelopment form using the input-oriented BCC/VRS model.

4.8 The Scale Efficiency

Scale Efficiency (SE) model architecture is discussed in this section. The SE model, which can be used by the network managers to decide the nature of returns to scale, is related to the Technical Efficiency (TE). Figure 4.5 represents the architecture of the scale efficiency. Again, the two models developed upon input-oriented CCR/CRS and BCC/VRS are considered. The architecture forms part of the generalized architecture (Figure 4.1). The concept is how the combination of both CCR/CRS and BCC/VRS scaled to achieve the expected efficiency. So, with this architecture, the nature of returns to scale is determined. The remainder of this section presents the appropriate Mathematical model formulation for the architectural requirements so that scale efficiency based on CCR/CRS and BCC/VRS model are evaluated.

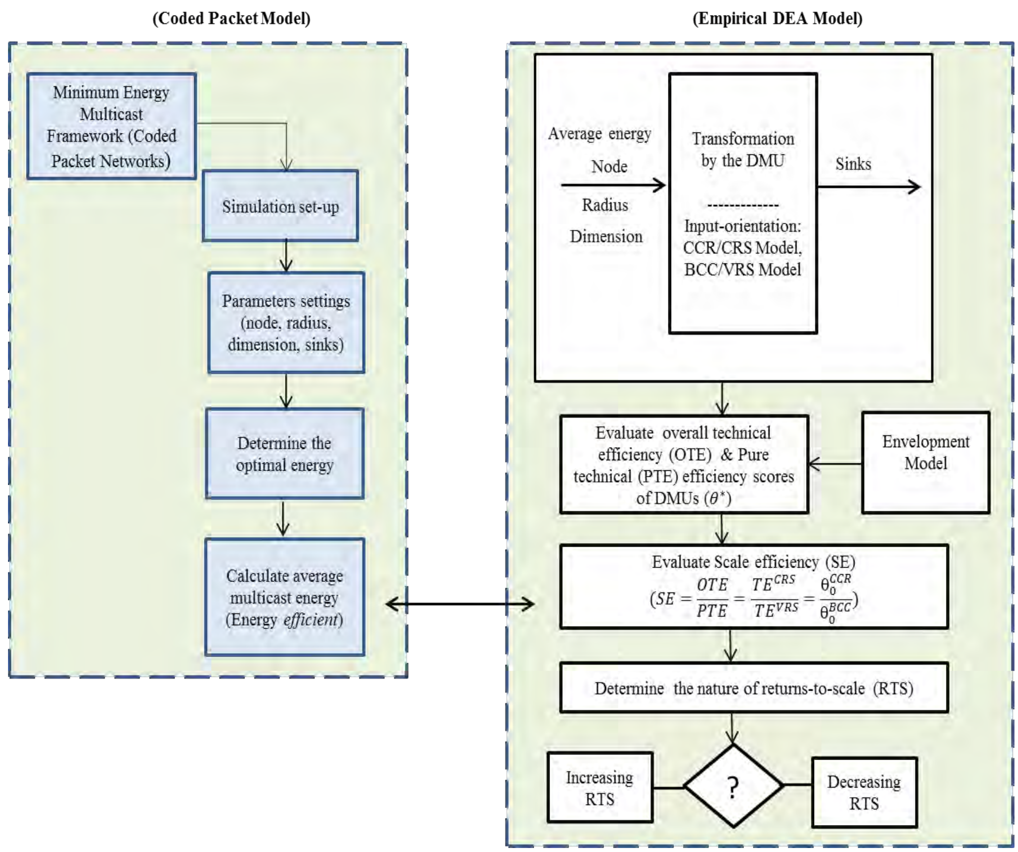

Figure 4.5: Architecture of the Scale Efficiency model for minimum energy multicast that is designed upon input-oriented CCR/CRS and BCC/VRS DEA model

In order to formulate expression for the SE, we consider the technique of DEA and apply it to evaluate individual ad hoc wireless network (DMU). As analysed in sections 2.4.2, 2.4.3, 2.4.4 and 2.4.5, a measure of SE can be obtained by comparing TE measures derived under the assumptions of CRS and VRS. The TE measure corresponding to CRS assumption represents Overall Technical Efficiency (OTE) which measures inefficiencies due to the input/output configuration and as well as the size of networks. The efficiency measure corresponding to VRS assumption represents Pure Technical Efficiency (PTE) which captures inefficiencies due to the network's managerial underperformance. The following relationship provides a measure of SE.

$$SE = OTE / PTE \text{ or } SE = TE^{CRS} / TE^{VRS} \tag{4.7}$$

Note that the formal Mathematical model for TE^{CRS} and TE^{VRS} was presented in the previous section. The TE^{CRS} is formulated based upon CCR/CRS model while the TE^{VRS} is formulated based upon BCC/VRS model. Note that the measure of an efficiency ratings provided by CCR/CRS model is the OTE and denoted as θ_0^{CCR}. The measure of efficiency ratings provided by BCC/VRS model is PTE and denoted as θ_0^{BCC}. Again, the ratio below provides a measure of SE.

$$SE = \left(\frac{\theta_0^{CCR}}{\theta_0^{BCC}} \right) \tag{4.8}$$

Remember that efficiency measures are bounded between one and zero. This measure of SE does not indicate whether the DMU in question is operating in the area of increasing or decreasing returns to scale. Thus nature of returns to scale can be determined from the magnitude of optimal $\sum_{j=1}^{n} \lambda_j^* = 1$ in the CCR/CRS model with the following three cases [60]:

Theorems 4.1

i. If $\sum_{j=1}^{n} \lambda_j^* = 1$ in any alternate optima, then returns-to-scale (CRS) prevail on DMU_0;

ii. If $\sum_{j=1}^{n} \lambda_j^* < 1$ in any alternate optima, then increasing returns-to-scale (IRS) prevail on DMU_0; and

iii. If $\sum_{j=1}^{n} \lambda_j^* > 1$ in any alternate optima, then decreasing returns-to-scale (DRS) prevail on DMU_0

The CCR/CRS and BCC/VRS models need to be solved *n* times, once for each DMU to obtain the optimal values for θ_0, à $_1$, à $_1$,….. ,à $_n$, S_i^-, S_r^+ that is $\theta_0^*, \lambda_1^*, \lambda_2^*, \ldots\ldots, \lambda_n^*, S_i^{-*}, S_r^{+*}$.

4.8.1 Scale Efficiency Results

The process of evaluating SE is similar to that of TE. The simulation set up is the same and therefore same data set is considered. The case of SE is simple because we have already obtained results for the input-oriented CCR/CRS and input-oriented BCC/VRS models. So the next procedure is to apply the Mathematical expression given as (4.7) or (4.8). From these relationship, in order to evaluate the SE, the input-oriented efficiency scores obtained from the OTE, that is $TE^{CCR/CRS}$ and PTE, that is $TE^{BCC/VRS}$ models are considered. Table 4.3 presents the overall technical efficiency (OTE) scores of 54 ad hoc networks (Column two), along with the magnitude of Overall Technical Inefficiency (OTIE) scores (Column three). Note that (OTIE% = (1-OTE) × 100). For example, for DMU_1, the OTE score is 0.29991256, and the OTIE% is 70%. Similarly, Table 4.3 presents the PTE scores of 54 ad hoc wireless networks (Column four), along with the magnitude of Pure Technical Inefficiency (PTIE) scores (Column five). Also, the (PTIE% = (1-PTE) × 100). Taking for instance, DMU_1 has PTE score = 1, and the PTIE% = 0. Using the OTE and PTE, the SE and SIE% are computed, and the results presented in column six and column seven of Table 4.3 respectively. For example, consider DMU_1, the SE is 0.299913 while the SIE% is 70%.

In determining the returns-to-scale (RTS), their mean values are significant. Thus the mean value for the CRS, VRS and SE are 0.65925, 0.96860 and 0.68089. Using theorem 4.1, the RTS of the ad hoc wireless networks are determined. Column eight of Table 4.3 shows the nature of RTS where 6 DMUs are CRS, 21 DMUs are DRS and 27 DMUs are IRS. The column eight of Table 4.3 shows that 27 ad hoc wireless networks with IRS can increase the size of their network while 21 ad hoc wireless networks with DRS can decrease the size of their networks. The remaining 6 ad hoc wireless networks with CRS need not alter the size of their network. This is useful information to network administrators, as the size of the network operators can be reduced or increased to achieve efficient frontier. For instance, the radius of connectivity or network dimensions could be adjusted depending on the nature of RTS.

Table 4.3: The OTE, PTE and SE scores for ad hoc wireless networks

DMU	OTE Score	OTIE (%)	PTE Score	PTIE (%)	SE Score	SIE (%)	RTS
DMU_1	0.29991256	70.0	1	0.0	0.299913	70.0	IRS
DMU_2	0.40084141	59.9	1	0.0	0.400841	59.9	IRS
DMU_3	0.49207254	50.8	1	0.0	0.492073	50.8	IRS
DMU_4	0.57988741	42.0	1	0.0	0.579887	42.0	IRS
DMU_5	0.66270102	33.7	1	0.0	0.662701	33.7	IRS
DMU_6	0.78002225	22.0	1	0.0	0.780022	22.0	DRS
DMU_7	0.82426848	17.6	1	0.0	0.824268	17.6	DRS
DMU_8	0.9000000	10.0	1	0.0	0.900000	10.0	DRS
DMU_9	1	0.0	1	0.0	1	0.0	**CRS**
DMU_{10}	0.27502244	72.5	1	0.0	0.275022	72.5	IRS
DMU_{11}	0.39546122	60.5	1	0.0	0.395461	60.5	IRS
DMU_{12}	0.48657482	51.3	1	0.0	0.486575	51.3	IRS
DMU_{13}	0.57348239	42.7	1	0.0	0.573482	42.7	IRS
DMU_{14}	0.67070144	32.9	1	0.0	0.670701	32.9	IRS
DMU_{15}	0.77764976	22.2	1	0.0	0.777650	22.2	DRS
DMU_{16}	0.84359551	15.6	1	0.0	0.843596	15.6	DRS
DMU_{17}	0.9000000	10.0	1	0.0	0.900000	10.0	DRS
DMU_{18}	1	0.0	1	0.0	1	0.0	**CRS**
DMU_{19}	0.30332619	69.7	1	0.0	0.303326	69.7	IRS
DMU_{20}	0.36521439	63.5	1	0.0	0.365214	63.5	IRS
DMU_{21}	0.48286367	51.7	1	0.0	0.482864	51.7	IRS
DMU_{22}	0.54530044	45.5	1	0.0	0.545300	45.5	IRS
DMU_{23}	0.64458003	35.5	1	0.0	0.644580	35.5	IRS
DMU_{24}	0.73393562	26.6	1	0.0	0.733936	26.6	DRS
DMU_{25}	0.81767584	18.2	1	0.0	0.817676	18.2	DRS
DMU_{26}	0.9000000	10.0	1	0.0	0.900000	10.0	DRS
DMU_{27}	1	0.0	1	0.0	1	0.0	**CRS**
DMU_{28}	0.34931885	65.1	1	0.0	0.349319	65.1	IRS
DMU_{29}	0.37675237	62.3	0.85857208	14.1	0.438813	56.1	IRS
DMU_{30}	0.46831949	53.2	0.85747639	14.3	0.546160	45.4	IRS
DMU_{31}	0.56077253	43.9	0.87296449	12.7	0.642377	35.8	IRS
DMU_{32}	0.65267431	34.7	0.89543998	10.5	0.728887	27.1	DRS
DMU_{33}	0.69879645	30.1	0.87356832	12.6	0.799933	20.0	DRS
DMU_{34}	0.81198616	18.8	0.92848238	7.2	0.874530	12.5	DRS
DMU_{35}	0.87008471	13.0	0.92650257	7.3	0.939107	6.1	DRS
DMU_{36}	0.9679643	3.2	0.96796430	3.2	1	0.0	**CRS**
DMU_{37}	0.40086835	59.9	1	0.0	0.400868	59.9	IRS
DMU_{38}	0.41044572	59.0	1	0.0	0.410446	59.0	IRS
DMU_{39}	0.52990544	47.0	1	0.0	0.529905	47.0	IRS
DMU_{40}	0.66217577	33.8	1	0.0	0.662176	33.8	IRS
DMU_{41}	0.67979129	32.0	1	0.0	0.679791	32.0	IRS
DMU_{42}	0.7895197	21.0	1	0.0	0.789520	21.0	DRS
DMU_{43}	0.80554291	19.4	1	0.0	0.805543	19.4	DRS
DMU_{44}	0.90050016	9.9	1	0.0	0.900500	9.9	DRS
DMU_{45}	1	0.0	1	0.0	1	0.0	**CRS**
DMU_{46}	0.34774583	65.2	0.91235901	8.8	0.381150	61.9	IRS
DMU_{47}	0.43833328	56.2	0.87412338	12.6	0.501455	49.9	IRS
DMU_{48}	0.49734514	50.3	0.83544318	16.5	0.595307	40.5	IRS
DMU_{49}	0.61521376	38.5	0.88530417	11.5	0.694918	30.5	DRS
DMU_{50}	0.69343607	30.7	0.89914054	10.1	0.771221	22.9	DRS
DMU_{51}	0.76827142	23.2	0.91704928	8.3	0.837765	16.2	DRS
DMU_{52}	0.78677236	21.3	0.88743011	11.3	0.886574	11.3	DRS
DMU_{53}	0.88591647	11.4	0.93629069	6.4	0.946198	5.4	DRS
DMU_{54}	0.97585664	2.4	0.97585664	2.4	1	0.0	**CRS**

OTE = Overall technical efficiency, OTIE% = Overall technical inefficiency = (1-OTE)*100, PTE = Pure technical efficiency, PTIE = (1-PTE)*100, SIE% = Scale inefficiency = (1-SE)*100, RTS = returns-to-scale, IRS = increasing returns-to-scale, CRS = constant returns-to-scale, DRS = decreasing returns-to-scale

4.8.2 Performance Comparison of Technical and Scale Efficiency

SE results are particularly useful to determine the size of a network. SE performance is discussed and compared with other types of efficiency evaluations. The statistics derived from the SE are summarised in Table 4.4. For instance, 4 ad hoc networks have OTE because their efficiency score is 1. These ad hoc networks together define the best practice and thus, form the reference set for inefficient ad hoc networks. This means that the resource utilisation process in these ad hoc wireless networks is functioning well i.e. the operation of these ad hoc networks is not characterised by any waste of inputs. The remaining 40 ad hoc networks have an OTE score less than 1, which means that they are technically inefficient. These results, thus, indicate a presence of marked deviations of the ad hoc networks from the best practice. These inefficient ad hoc wireless networks can improve their efficiency by reducing certain inputs. But how the inputs could be reduced is another challenge that this work address in the next chapter. The statistics of OTE scores ranges from 0.29991 for DMU_1 to 0.96796 for DMU_{36}. This finding implies that DMU_1 and DMU_{36} can potentially reduce their current input levels by 70% and 3.2%, respectively while leaving their output levels unchanged.

For the PTE scenario, 37 ad hoc wireless networks have pure technically efficiency (PTE) out of 54. This is possible PTE score is 1. It means that 37 ad hoc wireless networks define the best practice. The remaining 17 PTIE ad hoc wireless networks have PTE score of less than 1, meaning that they are technically inefficient. The PTE scores among the inefficient ad hoc networks range from 0.83544 for DMU_{48} to 0.97586 for DMU_{54}. This implies that DMU_{48} and DMU_{54} can potentially reduce their current input levels by 16.5% and 2.4%, respectively, while leaving their output levels unchanged. The statistic in Table 4.4 shows that PTE has more efficient DMUs than OTE. As presented in section 2.4.2, the reason is because PTE assumed VRS model while OTE assumed CRS model. Also, from Table 4.4, the SE shows that 6 DMUs are efficient and 48 DMUs are inefficient.

Table 4.4: Statistics of the findings from OTE, PTE and SE

Statistics	**OTE**	**PTE**	**SE**
Number of DMU	54	54	54
Efficient DMU	4	37	6
Inefficient DMU	50	17	48
Ave. of all DMU	0.65924813	0.968592	0.6808806
Ave. of all efficient DMU	1	1	1
Ave. of all Inefficient DMU	0.63198798	0.9002334	0.64099068
Ave. of all DMU (%)	34.1	3.1	31.9
Ave. of all efficient DMU (%)	100	100	100
Ave. of all Inefficient DMU (%)	36.8	10.0	35.9
Min. of all DMU	0.299913	0.8354432	0.27502244
Min. of all efficient DMU	1	1	1
Min. of all inefficient DMU	0.299913	0.8354432	0.27502244
Max. of all DMU	1	1	1
Max. of all efficient DMU	1	1	1
Max. of all inefficient DMU	0.967964	0.9758566	0.9461981

Furthermore, the average of all the efficient and inefficient ad hoc wireless networks is evaluated. While the average of efficient DMUs is 1, the averages of DMUs and inefficient DMUs have different values. For example, the averages of all DMUs for OTE, PTE, and SE are 0.65924813, 0.968592, and 0.6808806 respectively. The PTE model recorded the highest averages of all DMUs while the OTE recorded the lowest averages of all DMUs. Similarly, the averages of inefficient DMUs are 0.63198798, 0.9002334, and 0.64099068 respectively. Table 4.4 also has a record of some important findings such as the minimum and maximum values of entire DMUs, efficient and inefficient DMUs.

4.9 Chapter Summary

This Chapter has presented a new empirical architecture for energy efficiency. The chapter evaluated the degree of efficiency of each ad hoc wireless network against the best-practice frontier networks. This was achieved through the Envelopment model, which is the first model of the proposed architecture. The Envelopment model was developed and implemented to achieve Technical Efficiency (TE). It is the main performance metric for this model. The TE was evaluated for ad hoc wireless networks and results revealed the efficiency ratings of each network. The efficiency scores were analysed and it was concluded that some ad hoc wireless networks are efficient because their efficiency scores is 100% (that is they are on efficient frontier) while some have their efficiency scores less than 100% (that is they are below efficient frontier). It was discovered that, the input-oriented BCC/VRS envelopment model recorded higher number of efficient ad hoc wireless network over the input-oriented CCR/CRS envelopment model. Furthermore, the derivation of scale efficiency and its evaluation has shown how each of the ad hoc wireless network scale in terms of returns to scales. By contrast, the coded packet network investigated in chapter 3 could not evaluate the efficiency score of ad hoc wireless networks adequately using multi-criteria decision of multiple inputs and multiple outputs, but is good at comparing the effectiveness of different network. The next chapter provides answer to the question: How do we project the inefficient (sub-optimal) ad hoc wireless networks unto their efficient frontier?

Chapter Five: Slack Models for Evaluation of Target Multicast Energy

5.1 Introduction

This Chapter describes the proposed Slack model for energy efficiency in ad hoc wireless networks. The slack model is incorporated into the general architecture to achieve the goal of minimising energy consumed by ad hoc wireless networks and to enhance the performance of the Envelopment models. The efficiency ratings evaluated in chapter 4 only identified those networks that are efficient and those that are inefficient. Thus, we need another model to show how the inefficient or sub-optimal networks could be moved to their efficient frontier or become one of the best performing networks. In order to do this their slack values are required. By contrast, none of the existing minimum energy multicast techniques suggested how such inefficient networks could become efficient. Therefore, DEA technique is required. Sometimes, the Envelopment model identified a network as efficient but the Slack model confirms whether such a network is a weak efficient or full efficient. The next section presents the input-slack CCR/CRS model derivation followed by the model that identifies weak and full efficient ad hoc wireless networks presented in section 5.3. An extension of this model is presented in section 5.4 while simulation set up is presented in section 5.5. The conclusion is drawn in section 5.6.

5.2 The Input-oriented Slack Model based on CCR/CRS

In addition to model (4.5), the Slack's model is needed to push the inefficient DMUs to their real optimal efficiency. This is necessary provided that a DMU cannot reach its efficient frontier that is optimal point after proportional reductions in inputs using model (4.5). In order to obtain the slacks in DEA analysis, a second stage linear programming model is required to be solved after the dual linear programming model (4.5), which is presented in chapter 4. This second stage linear programming model formulated for slack values is as follows:

$$max \sum_{i=1}^{m} s_j^- + \sum_{r=1}^{s} s_r^+$$

Subject to

$$\sum_{j=1}^{n}\lambda_j x_{ij} + s_j^- = \theta^* x_{io}, \qquad i = 1,2,\ldots,\text{m};$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0}, \qquad r = 1,2,\ldots,\text{s};$$

$$j \geq 0 \qquad j = 1,2,\ldots,n \qquad (5.1)$$

where s_j^- *and* s_r^+ represent input and output slacks respectively. The superscripts (-) and (+) represent input reduction and output augmentation respectively. Here, θ^* is the efficiency score resulted from the initial run of model (4.5).

It is possible to solve this LP problem in two phases, the first phase model (4.5) aiming at minimising θ^*, and a second phase model (5.1) maximises s_j^- *and* s_r^+. An optimal solution θ^*, s_j^{-*} and s_r^{+*} obtained after solving phase two is the maximum slack solution that could be obtained. Note that the value for θ^* is calculated from model (4.5) while the value of s_j^- *and* s_r^+ are derived from model (5.1), but model (4.5) and (5.1) can be combined and rewritten as:

$$min\, \theta_0 - \varepsilon\left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+\right)$$

Subject to

$$\theta_0 x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + S_i^-, \qquad i = 1,2,\ldots,m;$$

$$y_{ro} = \sum_{j=1}^{n}\lambda_j y_{rj} - S_r^+, \qquad r = 1,2,\ldots,s;$$

$$\lambda_j, S_i^-, S_r^+ \geq 0, \qquad \forall i, r, j, \qquad (5.2)$$

where $\varepsilon > 0$ in the objective function of (5.2) and is a non-Archimedean element defined to be smaller than any real number and effectively allow the minimisation over θ to pre-empt the optimisation involving the slacks, s_j^- *and* s_r^+. Based on the previous explanation, model (5.2) is calculated in a two-stage process with maximal reduction of inputs being achieved first. That is model (5.2) first obtains optimal efficiency score (θ^*) from model (4.5) and then calculates them. In the second stage, it uses model (5.1) to obtain the slack values and optimises them to achieve the expected value.

5.3 Model for Identifying Fully and Weakly Efficient DMUs

It is possible that the solution obtained from model (4.5) and (5.1) or model (5.2) contains weak efficient DMUs that give rise to multiple optimal solutions. This also affects the performance of DMUs. Therefore, the following conditions help to identify full and weak efficiency status of DMUs [95], [55]:

Definition (5.1) (Full Efficient DMUs): (*DEA Efficiency*) DMU_0 is fully (100%) efficient if and only if both (i) θ^* = 1 and (ii) all slacks $s_i^{(-*)} = s_r^{(+*)} = 0$. In other words, Fully (100%) efficiency is attained by any DMU if and only if none of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

Definition (5.2) Weakly Efficient DMUs: (*Weakly DEA Efficient*) DMU_0 is weakly efficient if and only if both (i) θ^* = 1 and (ii) $s_i^{-*} \neq$ 0 and/or $s_r^{+*} \neq 0$ for some *i* and r in some alternate optima.

The presence of inefficient DMUs, as well, as weakly efficient DMUs leads to another Mathematical expression to calculate the set of input and output targets that would make an inefficient or weak efficient DMUs to become full efficient. The level of efficient target for inputs and outputs can be calculated using the following relations:

$$\begin{cases} X^*_{i0} = \theta^* x_{io} - S_i^{-*}, & i = 1, 2, \ldots, m \\ Y^*_{r0} = y_{ro} + S_r^{+*}, & r = 1, 2, \ldots, s \end{cases} \tag{5.3}$$

Efficient target can be calculated taking for instance the target for the input values, then the input values are multiplied with an optimal efficiency score (θ^*), and slack amounts are subtracted.

5.4 The Input-oriented Slack Model Based on BCC/VRS

In a similar way that was used to formulate the CCR/CRS based Slack model, we modify model (4.6) as following.

$$max \sum_{i=1}^{m} s_j^- + \sum_{r=1}^{s} s_r^+$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_j^- = \theta^* x_{io}, \qquad i = 1,2,\ldots,\text{m};$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0}, \qquad r = 1,2,\ldots,\text{s};$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$j \geq 0 \qquad j = 1,2,\ldots,n \qquad (5.4)$$

Observe that model (5.4) is similar to model (5.1). The only difference is the presence of $\sum_{j=1}^{n} \lambda_j = 1$, which characterise the model as BCC/VRS type and that it assumes VRS. Therefore, the definitions of the variables are the same. Again, similar to model (5.2), model (5.4) can be rewritten as:

$$min\, \theta_0 - \varepsilon \left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \right)$$

Subject to

$$\theta_0 x_{io} = \sum_{j=1}^{n} \lambda_j x_{ij} + S_i^-, \qquad i = 1,2,\ldots,m;$$

$$y_{ro} = \sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+, \qquad r = 1,2,\ldots,s;$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j, S_i^-, S_r^+ \geq 0, \qquad \forall i, r, j, \tag{5.5}$$

Note that definition (5.1) and (5.2) also apply to the input-oriented BCC/VRS model. It also make use of model (5.3) to determine weak and full efficient DMUs.

5.5 Simulation Set up, Slack Model Implementation and Results

The simulation was conducted using the coded packet framework. The first part of Figure 4.1 shows the procedures of how multicast energy was evaluated. The details about the configuration and how results are obtained is already presented in chapter 3. Also, an aspect of the second part (Envelopment model) was implemented and presented in chapter 4 with the results analysed for efficiency scores. The focus in this section is the implementation of Slack models, a model that aids the projection of ad hoc wireless networks unto efficient frontier. In this section, the energy efficiency is evaluated for two models. The first model is the input-oriented slack based on CCR/CRS and the second model is the input-oriented slack based on BCC/VRS. These two models are evaluated and their results compared with the coded packet model. To solve the models, the DEAOS is considered. The slack values are extracted from the "Slack" sheet that is provided by the DEAOS. The DEAOS implementation details and the raw data for slack sheet are provided in the CD accompanying this thesis. Appendix G.2 presented an analytical LP for slack formulation of 54 ad hoc wireless networks. The next two subsections analyse the results obtained from the "Slack" sheet for the two models.

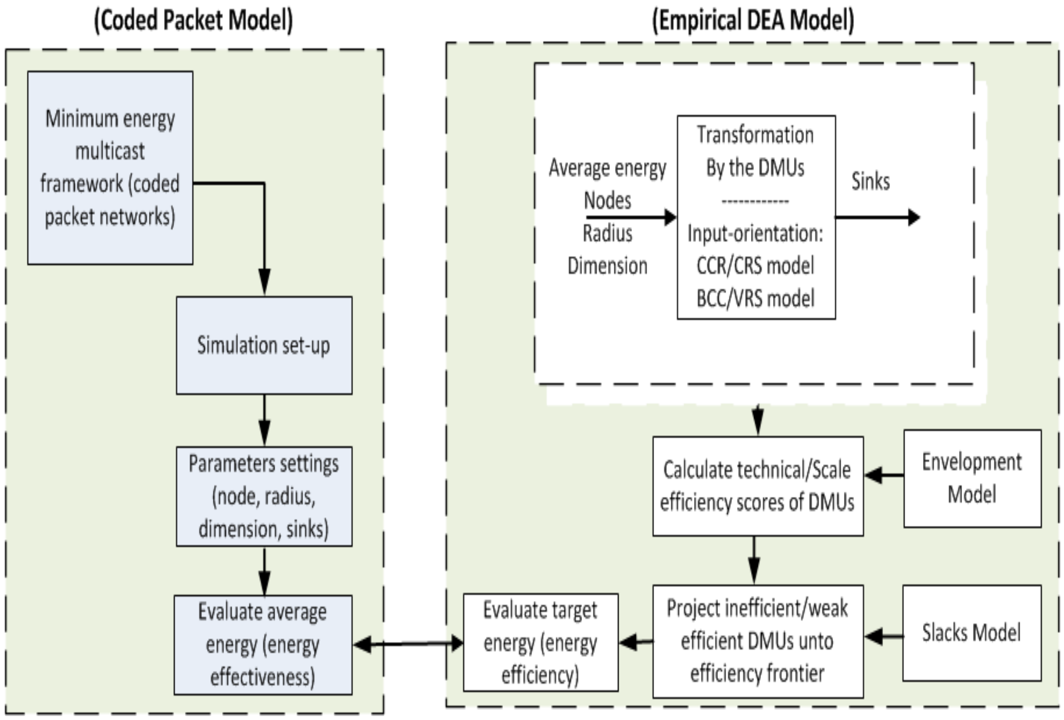

Figure 5.1: Architecture of the Slack model for projecting the inefficient and weak efficiency wireless ad hoc networks unto their efficient frontier

5.5.1 Input-oriented CCR/CRS Slack Results

Model (5.1) represented the Mathematical derivation of slacks with CRS assumption. In this section, the results of the model are analysed and evaluated for projection (expected) in ad hoc wireless networks. The aim of the model is to use the slack values as a function to determine those ad hoc wireless networks that will be projected unto efficient frontier and compared it with the coded packet model. The slacks computed by the DEA solver, and which is extracted from the "Slack" sheet, are reported in columns five, six, seven and eight of Table H.1 under Appendix H. Important properties needed for the evaluation of Slacks are definition (5.1) and (5.2)**.** Using these definitions, it is observed that none of the efficient DMUs have a slack, meaning that slacks exist only for those DMUs identified as inefficient. The Slack model is used to complement the Envelopment model. Thus, slacks are obtained after proportional reductions of inputs or outputs i.e. after the Envelopment model was executed. Slack computation is necessary especially when a DMU cannot reach its efficient frontier. Slacks are needed to project such inefficient and weak efficient DMUs. For instance, DMU_8 is required to reduce its average energy consumption using slack value of 0.32409. Similarly, DMU_{17} and DMU_{26} required reducing its average energy consumption using Slack value of 0.209151 and 0.541071 respectively. However, DMU_1 requires two input reduction with the following Slack values: the dimension (0.9921306), and radius (0.2993879) to become efficient. Readers are referred to Appendix G.2 and H.1 for slack formation and reports respectively.

The projection analysis is carried out for each DMU, where target input levels are prescribed. These projections are the function of respective Slack values that are added to inputs. The target calculation is computed by model (5.3). Numerically, to calculate the target values, the input value is multiplied with an optimal efficiency score, and then Slack amounts are subtracted from this amount. Results of the projected energy together with the results obtained from simulated coded packet model are presented in Table 5.1. As shown in the Table, the projected energy represented the expected optimal energy if all the DMUs operated efficiently. In order to evaluate the expected energy, weak efficient and inefficient ad hoc wireless networks (DMUs) are identified together with their efficiency scores and slacks. As an example, the projected energy for DMU_8 and DMU_{17} are computed using the following Mathematical relationship:

For DMU_n using $X^*_{i0} = \theta^* x_{io} - S_i^{-*}, \qquad i = 1, 2, \ldots, m$

$X^* DMU_8$ = 0.9 (*efficiency score*) × 8.81448 (*average multicast energy*) – 0.32409 (*input slack*)

= 7.608942 (*projected energy*).

Similarly, $X^* DMU_{17}$ = 0.9 (*efficiency score*) × 8.56634 (*average multicast energy*) – 0.209151 (*input slack*)

= 7.500555 (*projected energy*).

Table 5.1: Results of the average multicast energy computed by coded packet (RLNC) model, and the projected multicast energy computed by the proposed input-oriented CCR/CRS model

DMU	Average energy (Coded packet Model)	Projected energy (CCR/CRS DEA Model)
DMU_1	4.50027	1.349687509
DMU_2	5.46086	2.188938801
DMU_3	6.22791	3.064583502
DMU_4	6.81511	3.951996460
DMU_5	7.32855	4.856637563
DMU_6	7.23365	5.642407952
DMU_7	8.10404	6.679904730
DMU_8	8.81448	7.608942000
DMU_9	8.45438	8.454380000
DMU_{10}	5.19479	1.428683796
DMU_{11}	5.55607	2.197210195
DMU_{12}	6.28641	3.058808829
DMU_{13}	6.85942	3.933756586
DMU_{14}	7.12087	4.775977768
DMU_{15}	7.18488	5.587320197
DMU_{16}	7.73925	6.528796565
DMU_{17}	8.56634	7.500555000
DMU_{18}	8.33395	8.333950000
DMU_{19}	4.1549	1.260290000
DMU_{20}	5.30356	1.936936409
DMU_{21}	5.35979	2.588047851
DMU_{22}	6.07549	3.312967372
DMU_{23}	6.18796	3.988635421
DMU_{24}	6.37327	4.677569864
DMU_{25}	6.57230	5.374010935
DMU_{26}	7.34824	6.072345000
DMU_{27}	6.74705	6.747050000
DMU_{28}	3.60785	1.260290000
DMU_{29}	5.1002	1.921512433
DMU_{30}	5.56776	2.607490509
DMU_{31}	5.87098	3.292284287
DMU_{32}	6.09464	3.977814978
DMU_{33}	6.76687	4.728664738
DMU_{34}	6.62772	5.381616903
DMU_{35}	7.14271	6.214762736
DMU_{36}	7.12791	6.899562380
DMU_{37}	3.14390	1.260290000
DMU_{38}	4.60581	1.890435000
DMU_{39}	4.75666	2.520580000
DMU_{40}	4.75814	3.150725000
DMU_{41}	5.56181	3.780870000
DMU_{42}	5.58696	4.411015000
DMU_{43}	6.25809	5.041160000
DMU_{44}	6.29795	5.671305000
DMU_{45}	6.30145	6.301450000
DMU_{46}	3.62417	1.260290000
DMU_{47}	4.31278	1.890435000
DMU_{48}	5.06807	2.520580000
DMU_{49}	5.12135	3.150725000
DMU_{50}	5.45237	3.780870000
DMU_{51}	5.74148	4.411015000
DMU_{52}	6.43736	5.064736940
DMU_{53}	6.42996	5.696407480
DMU_{54}	6.50145	6.344483131
	329.77019	***227.5317618***

5.5.2 Input-oriented BCC/VRS Slack Results

Model (5.4) represented the Mathematical derivation of Slacks with VRS assumption. The model is solved similar to the previous model using the DEAOS. Also, in a similar manner, in this section, the results are analysed for projection in ad hoc wireless networks. The analysis and evaluation is carried out for fully and weakly efficient DMUs. Furthermore, the Slacks computed by the DEA solver and extracted from the worksheet are reported in Columns three, four, five, and six of Table H.2 under Appendix H. Table H.2 extracted from the "Slack" sheet of the DEA run results. Again, considering definition (5.1) and (5.2), it is observed that none of the efficient DMUs have any Slacks. Thus Slacks exist only for those DMUs identified as inefficient. Here, DMU_2 is required to reduce its average multicast energy using Slack function of 0.466326. Similarly, DMU_3 and DMU_4 required reducing their average multicast energy using slack functions of 0.739113 and 0.832049 respectively. However, DMU_{13} requires the reduction of three inputs using the following Slack functions: The average multicast energy (0.92152), dimensions (25) and radius (1.25) so that they become efficient. Readers are referred to Appendix G.2 and H.2 for slack formation and reports respectively.

Similar to the previous evaluation, model (5.3) is used for projection. The results of the projected energy are presented in Table 5.2. Observe from the Table that the projected energy for efficient DMUs did not change. This is because they were already being operated efficiently. However, for the inefficient DMUs the projections are required so that they become efficient. For example, the projection for DMU_2 and DMU_3 are calculated as follows:

For DMU_n using $X^*_{i0} = \theta^* x_{io} - S_i^{-*}, \quad i = 1, 2, \ldots, m$

X^*DMU_2 = 1 (*efficiency rating*) × 5.46086 (*average multicast energy*) – 0.46632625 (*slack*)

= 4.9945338 (*projected energy*).

Similarly,

X^*DMU_3 = 1 (*efficiency rating*) × 6.22791 (*average multicast energy*) – 0.73911250 (slack)

= 5.4887975 (*projected energy*).

Table 5.2: Results of the average multicast energy computed by coded packet (RLNC) model, the projected multicast energy computed by the proposed input-oriented BCC/VRS model, and energy saved calculated by EG Model

DMU	Average energy (Coded Packet Model)	Projected energy (BCC/VRS Model)
DMU_{1}	4.50027	4.5002700
DMU_{2}	5.46086	4.9945338
DMU_{3}	6.22791	5.4887975
DMU_{4}	6.81511	5.9830613
DMU_{5}	7.32855	6.4773250
DMU_{6}	7.23365	6.9715888
DMU_{7}	8.10404	7.4658525
DMU_{8}	8.81448	7.9601163
DMU_{9}	8.45438	8.4543800
DMU_{10}	5.19479	4.5002700
DMU_{11}	5.55607	4.9794800
DMU_{12}	6.28641	6.2864100
DMU_{13}	6.85942	5.9379000
DMU_{14}	7.12087	6.4171100
DMU_{15}	7.18488	6.8963200
DMU_{16}	7.73925	7.3755300
DMU_{17}	8.56634	7.8547400
DMU_{18}	8.33395	8.3339500
DMU_{19}	4.1549	3.8220850
DMU_{20}	5.30356	5.3035600
DMU_{21}	5.35979	5.0619650
DMU_{22}	6.07549	5.3428125
DMU_{23}	6.18796	5.6236600
DMU_{24}	6.37327	5.6501881
DMU_{25}	6.5723	6.0158088
DMU_{26}	7.34824	6.3814294
DMU_{27}	6.74705	6.7470500
DMU_{28}	3.60785	3.6078500
DMU_{29}	5.1002	4.3788893
DMU_{30}	5.56776	4.7742228
DMU_{31}	5.87098	5.1251571
DMU_{32}	6.09464	5.4573843
DMU_{33}	6.76687	5.9113233
DMU_{34}	6.62772	6.1537213
DMU_{35}	7.14271	6.6177392
DMU_{36}	7.12791	6.8995624
DMU_{37}	3.1439	3.1439000
DMU_{38}	4.60581	4.6058100
DMU_{39}	4.75666	4.7566600
DMU_{40}	4.75814	4.4950813
DMU_{41}	5.56181	4.9454750
DMU_{42}	5.58696	5.1173688
DMU_{43}	6.25809	5.5120625
DMU_{44}	6.29795	5.9067563
DMU_{45}	6.30145	6.3014500
DMU_{46}	3.62417	3.3065442
DMU_{47}	4.31278	3.7699018
DMU_{48}	5.06807	4.2340845
DMU_{49}	5.12135	4.5339525
DMU_{50}	5.45237	4.9024469
DMU_{51}	5.74148	5.2652201
DMU_{52}	6.43736	5.7127071
DMU_{53}	6.42996	6.0203117
DMU_{54}	6.50145	6.3444831
	329.77019	***304.62626***

5.6 Chapter Summary

This Chapter presented another component of the new empirical architecture for projecting the weak and inefficient ad hoc wireless networks into their efficient frontier. In this chapter, the Slack model was developed. It was implemented to identify ad hoc wireless networks that could not perform according to the best practice network. It then projected them to their efficient frontier. The next chapter shows how to evaluate the amount of energy that could be hazardous to the environment. This evaluation is possible after the Slack model identify the inefficient and weak efficient ad hoc wireless networks.

Chapter Six: Energy Gap (EG) Model and Performance Evaluation

6.1 Introduction

In chapter five, the Slack model was proposed for projecting inefficient and weakly efficient ad hoc wireless networks unto their efficient frontier so that they become expected optimal. In this chapter, an Energy Gap (EG) model that evaluates the difference between the average energy computed by the coded packet and the projected energy computed by DEA is proposed to account for the energy saved if ad hoc wireless networks operated efficiently. This model is important to determine the potential of a model in moving an ad hoc wireless network unto their expected optimal. For instance, the ad hoc wireless network below the efficient frontier are expected to save energy depending on their distance from the frontier, while those ad hoc wireless network that are on the efficient frontier are not expected to save energy, because they are already operating efficiently. The next section discusses the EG model follow by the simulation set up in section 6.3. The EG result, which is obtained in section 6.4 are discussed in section 6.5. Section .6.6 summarises the chapter.

6.2 Energy Gap Model

This section presents the EG model that is used to calculate the energy difference between the average energy obtained from coded packet model and the projected energy obtained from empirical DEA models. The EG architecture is equipped with two models: The first model is based on input-oriented CCR/CRS and the second is based on input-oriented BCC/VRS. Note that EG model are part of the generalised architecture presented earlier (Figure 4.1). It is a simple model that serve as comparator between the two techniques.

6.3 Simulation Set up, EG Model Implementation and Results

The simulation was conducted using the coded packet framework. The first part of Figure 6.1 shows the procedures of how multicast energy was evaluated. The details about the configuration and how results are obtained is already presented in Chapter 3. The second part of

Figure 6.1 shows the procedures of how projected energy was evaluated. The next subsections present the EG architecture for two different models - the input-oriented CCR/CRS and the input-oriented BCC/VRS models. These models show different capability in terms of how they preserve energy in ad hoc wireless networks.

6.3.1 Energy Gap (EG) Model based on CCR/CRS

Figure 6.1 is another important architecture that shows how EG model is being implemented. As can be observed from Figure 6.1, the EG model is implemented at the base of the Slack model and serves as a bridge between coded packet technique and DEA technique. It is designed to evaluate the energy difference between the coded packet model and the empirical DEA model. The difference is the energy saved if the ad hoc wireless networks operate efficiently. The architecture is designed upon input-oriented CCR/CRS.

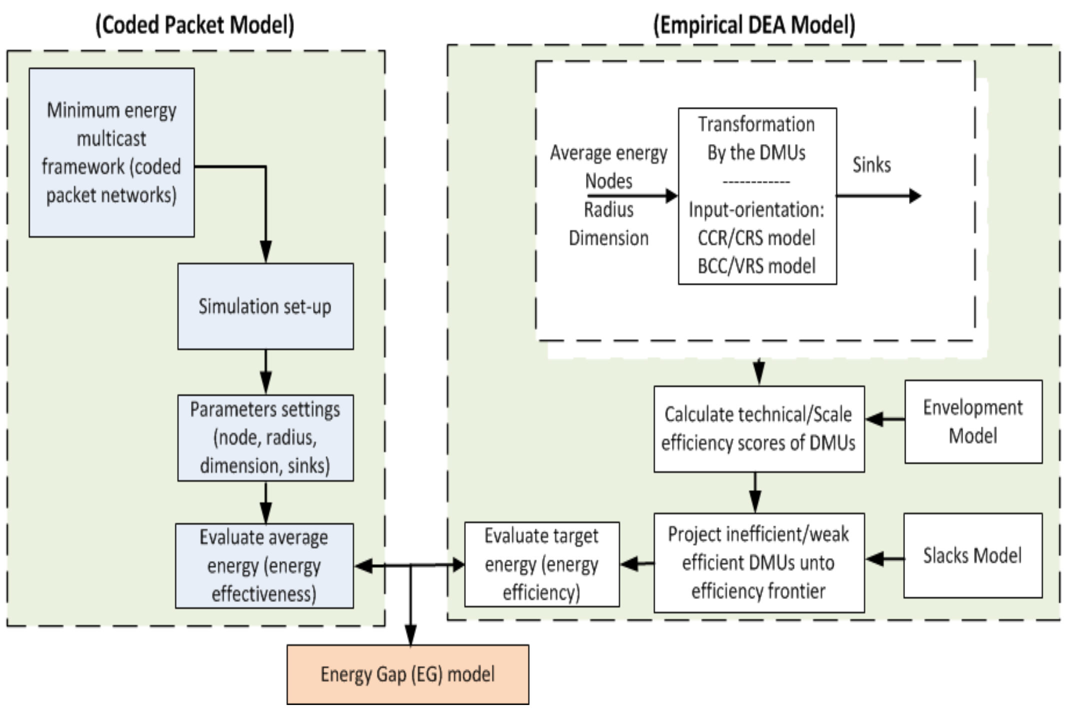

Figure 6.1: Architecture of the Energy Gap (EG) designed based on input-oriented CCR/CRS and input-oriented BCC/VRS models

6.3.2 Energy Gap (EG) Model Based on BCC/VRS

In this section, architecture based on input-oriented BCC/VRS model is considered. It is similar to the input-oriented CCR/CRS architecture. In Figure 6.1, the models are containing within the same architecture.

6.4 EG Model Evaluation and Results

The results of two models are evaluated and discussed. The first is the input-oriented CCR/CRS and the second is the input-oriented BCC/VRS.

6.4.1 Results of EG for Input-oriented CCR/CRS

The EG model considers the result obtained from the coded packet and DEA models, and then finds the difference between the two techniques.

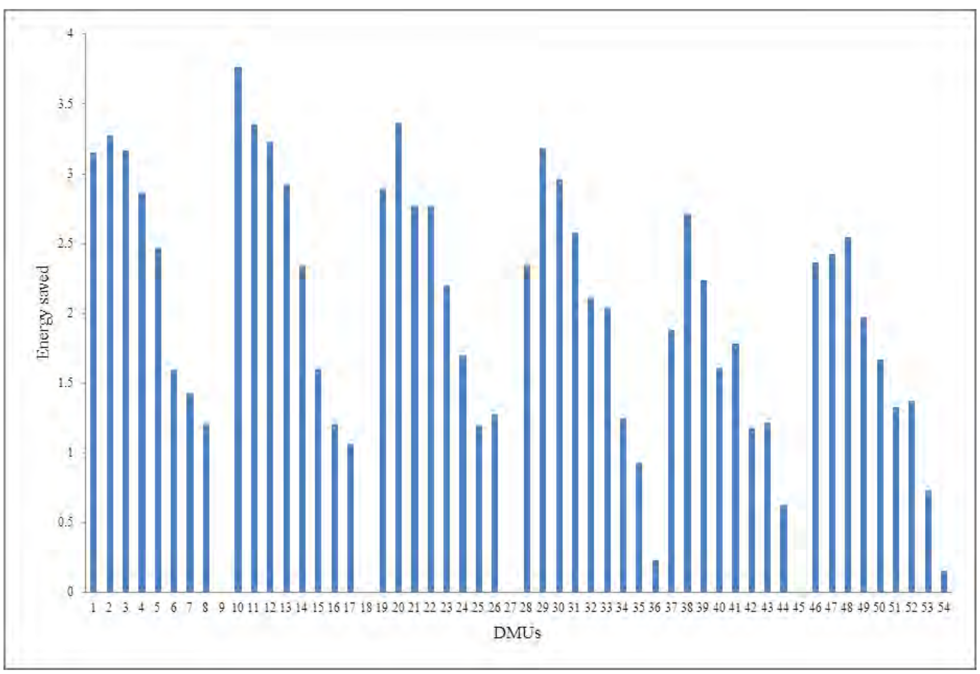

Figure 6.2: Energy saved by individual ad hoc wireless networks using input-oriented CCR/CRS DEA model

Table 6.1: Results of the average multicast energy computed by coded packet (RLNC) model, the projected multicast energy computed by the proposed input-oriented CCR/CRS model, and energy saved calculated by EG Model

DMU	**Average energy (Coded packet Model)**	**Projected energy (CCR/CRS DEA Model)**	**EG Model (Energy saved)**
DMU_1	4.50027	1.349687509	3.1505825
DMU_2	5.46086	2.188938801	3.2719212
DMU_3	6.22791	3.064583502	3.1633265
DMU_4	6.81511	3.951996460	2.8631135
DMU_5	7.32855	4.856637563	2.4719124
DMU_6	7.23365	5.642407952	1.5912420
DMU_7	8.10404	6.679904730	1.4241353
DMU_8	8.81448	7.608942000	1.205538
DMU_9	8.45438	8.454380000	0
DMU_{10}	5.19479	1.428683796	3.7661062
DMU_{11}	5.55607	2.197210195	3.3588598
DMU_{12}	6.28641	3.058808829	3.2276012
DMU_{13}	6.85942	3.933756586	2.9256634
DMU_{14}	7.12087	4.775977768	2.3448922
DMU_{15}	7.18488	5.587320197	1.5975598
DMU_{16}	7.73925	6.528796565	1.2104534
DMU_{17}	8.56634	7.500555000	1.0657850
DMU_{18}	8.33395	8.333950000	0
DMU_{19}	4.1549	1.260290000	2.8946100
DMU_{20}	5.30356	1.936936409	3.3666236
DMU_{21}	5.35979	2.588047851	2.7717421
DMU_{22}	6.07549	3.312967372	2.7625226
DMU_{23}	6.18796	3.988635421	2.1993246
DMU_{24}	6.37327	4.677569864	1.6957001
DMU_{25}	6.57230	5.374010935	1.1982891
DMU_{26}	7.34824	6.072345000	1.2758950
DMU_{27}	6.74705	6.747050000	0
DMU_{28}	3.60785	1.260290000	2.3475600
DMU_{29}	5.1002	1.921512433	3.1786876
DMU_{30}	5.56776	2.607490509	2.9602695
DMU_{31}	5.87098	3.292284287	2.5786957
DMU_{32}	6.09464	3.977814978	2.1168250
DMU_{33}	6.76687	4.728664738	2.0382053
DMU_{34}	6.62772	5.381616903	1.2461031
DMU_{35}	7.14271	6.214762736	0.9279473
DMU_{36}	7.12791	6.899562380	0.2283476
DMU_{37}	3.14390	1.260290000	1.8836100
DMU_{38}	4.60581	1.890435000	2.7153750
DMU_{39}	4.75666	2.520580000	2.2360800
DMU_{40}	4.75814	3.150725000	1.6074150
DMU_{41}	5.56181	3.780870000	1.7809400
DMU_{42}	5.58696	4.411015000	1.1759450
DMU_{43}	6.25809	5.041160000	1.2169300
DMU_{44}	6.29795	5.671305000	0.6266450
DMU_{45}	6.30145	6.301450000	0
DMU_{46}	3.62417	1.260290000	2.3638800
DMU_{47}	4.31278	1.890435000	2.4223450
DMU_{48}	5.06807	2.520580000	2.5474900
DMU_{49}	5.12135	3.150725000	1.9706250
DMU_{50}	5.45237	3.780870000	1.6715000
DMU_{51}	5.74148	4.411015000	1.3304650
DMU_{52}	6.43736	5.064736940	1.3726231
DMU_{53}	6.42996	5.696407480	0.7335525
DMU_{54}	6.50145	6.344483131	0.1569669
	329.77019	***227.5317618***	***102.23843***

Therefore, in order to evaluate the EG model, Table 6.1 is required. Column four in Table 6.1 presents the EG results for the input-oriented CCR/CRS model. Again, The EG model computes the difference between the average multicast energy of the coded packet model and the proposed empirical DEA model. This difference is the amount of the energy saved if the ad hoc wireless networks operate efficiently. For example, as could be observed from Table 6.1 the energy saved if DMU_1 operates efficiently is 3.1505825 and the energy saved if DMU_2 operates efficiently is 3.2719212. Figure 6.2 presents the graphical results of the energy saved by individual ad hoc wireless network with the DMU_{10} has the highest energy saved which is 3.7661062 followed by DMU_{20} which is equal to 3.3666236. The DMUs that satisfy definition (5.1) and (5.2) are not needed to save energy because they have already fully operated efficiently. These DMUs are DMU_9, DMU_{18}, DMU_{27}, and DMU_{45}.

6.4.2 Results of EG for Input-oriented BCC/VRS

Similarly, Table 6.2 presents the results of gap analysis for the input-oriented BCC/VRS model. This is recorded in column four of Table 6.2. Column two and column three recorded the average multicast energy and the projected energy. Taking from the Table, the EG for DMU_2 is 0.4663263 and the energy gap for DMU_3 is 0.7391125. In order to extend this analysis, Figure 6.3 presents the graphical representation of the energy saved by individual ad hoc network. As shown, DMU_{26} has the highest energy saved which is 0.9668106 followed by DMU_{13} with the value equal 0.92152. The DMUs that satisfy definition (5.1) and (5.2) are not needed to save energy because they have already fully operated efficiently. These DMUs are DMU_1, DMU_9, DMU_{12}, DMU_{18}, DMU_{20}, DMU_{27}, DMU_{28}, DMU_{37}, DMU_{38}, DMU_{39}, and DMU_{45}.

Table 6.2: Results of the average multicast energy computed by coded packet (RLNC) model, the projected multicast energy computed by the proposed input-oriented BCC/VRS model, and energy saved calculated by EG Model

DMU	Average energy (Coded Packet Model)	Projected energy (BCC/VRS Model)	EG Model (Energy Saved)
DMU_1	4.50027	4.5002700	0
DMU_2	5.46086	4.9945338	0.4663263
DMU_3	6.22791	5.4887975	0.7391125
DMU_4	6.81511	5.9830613	0.8320487
DMU_5	7.32855	6.4773250	0.8512250
DMU_6	7.23365	6.9715888	0.2620613
DMU_7	8.10404	7.4658525	0.6381875
DMU_8	8.81448	7.9601163	0.8543638
DMU_9	8.45438	8.4543800	0
DMU_{10}	5.19479	4.5002700	0.6945200
DMU_{11}	5.55607	4.9794800	0.5765900
DMU_{12}	6.28641	6.2864100	0
DMU_{13}	6.85942	5.9379000	0.9215200
DMU_{14}	7.12087	6.4171100	0.7037600
DMU_{15}	7.18488	6.8963200	0.2885600
DMU_{16}	7.73925	7.3755300	0.3637200
DMU_{17}	8.56634	7.8547400	0.7116000
DMU_{18}	8.33395	8.3339500	0
DMU_{19}	4.1549	3.8220850	0.3328150
DMU_{20}	5.30356	5.3035600	0
DMU_{21}	5.35979	5.0619650	0.2978250
DMU_{22}	6.07549	5.3428125	0.7326775
DMU_{23}	6.18796	5.6236600	0.5643000
DMU_{24}	6.37327	5.6501881	0.7230819
DMU_{25}	6.5723	6.0158088	0.5564913
DMU_{26}	7.34824	6.3814294	0.9668106
DMU_{27}	6.74705	6.7470500	0
DMU_{28}	3.60785	3.6078500	0
DMU_{29}	5.1002	4.3788893	0.7213107
DMU_{30}	5.56776	4.7742228	0.7935372
DMU_{31}	5.87098	5.1251571	0.7458229
DMU_{32}	6.09464	5.4573843	0.6372557
DMU_{33}	6.76687	5.9113233	0.8555467
DMU_{34}	6.62772	6.1537213	0.4739987
DMU_{35}	7.14271	6.6177392	0.5249708
DMU_{36}	7.12791	6.8995624	0.2283476
DMU_{37}	3.1439	3.1439000	0
DMU_{38}	4.60581	4.6058100	0
DMU_{39}	4.75666	4.7566600	0
DMU_{40}	4.75814	4.4950813	0.2630588
DMU_{41}	5.56181	4.9454750	0.6163350
DMU_{42}	5.58696	5.1173688	0.4695913
DMU_{43}	6.25809	5.5120625	0.7460275
DMU_{44}	6.29795	5.9067563	0.3911938
DMU_{45}	6.30145	6.3014500	0
DMU_{46}	3.62417	3.3065442	0.3176258
DMU_{47}	4.31278	3.7699018	0.5428782
DMU_{48}	5.06807	4.2340845	0.8339855
DMU_{49}	5.12135	4.5339525	0.5873975
DMU_{50}	5.45237	4.9024469	0.5499231
DMU_{51}	5.74148	5.2652201	0.4762599
DMU_{52}	6.43736	5.7127071	0.7246529
DMU_{53}	6.42996	6.0203117	0.4096483
DMU_{54}	6.50145	6.3444831	0.1569669
	329.77019	***304.62626***	***25.143931***

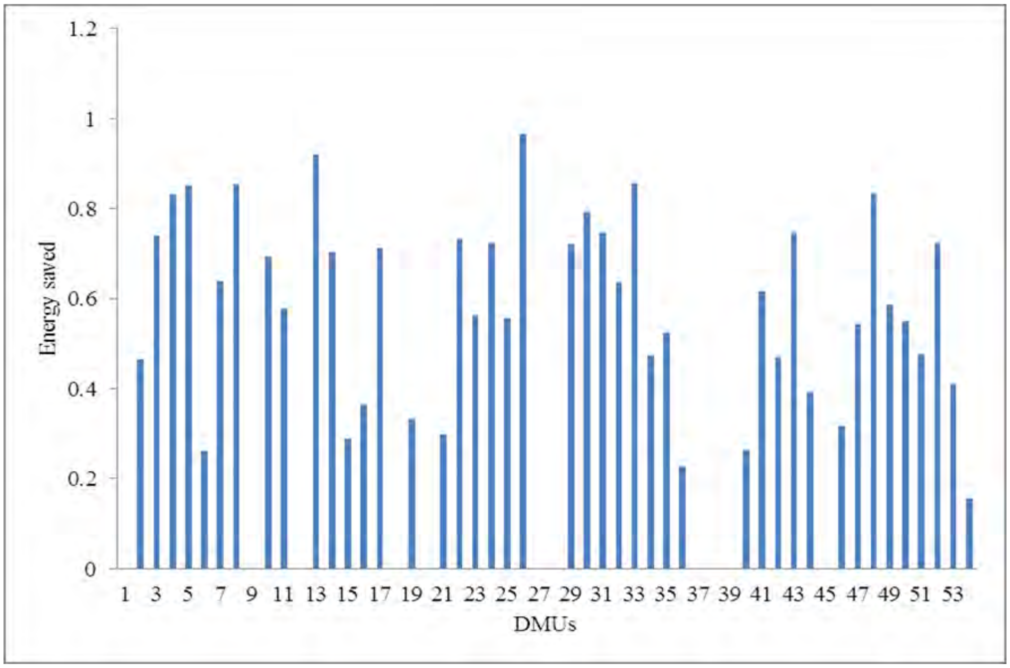

Figure 6.3: Energy saved by individual ad hoc wireless network using input-oriented BCC/VRS DEA model

6.5 Performance of Technical Efficiency and Energy Saved

This section discusses the results of two techniques by which multicast energy is minimised in ad hoc wireless networks if they operate efficiently. The two models have demonstrated the ability to further save multicast energy in ad hoc wireless networks. Remember that performances of these models are based on different assumptions. For example, the input-oriented CCR/CRS model assumed a CRS while the input-oriented BCC/VRS assumed VRS.

6.5.1 Envelopment Model (CCR/CRS) vs Coded Packet Model (RLNC)

In this section, we compare the performance of input-oriented CCR/CRS model in terms of energy saved with the coded packet model. First, we calculate the percentage of the total energy saved by the input-oriented CCR/CRS model. This is obtained by finding the difference between the total average multicast energy and the total projected energy. As could be observed from Table 6.1, the total energy saved is 329.77019 (average multicast energy) minus 227.5317618 (projected multicast energy). The difference which is equal to 102.23843 is equivalent to 16%. Figure 6.4 represents the pie chart of the total energy saved, the total projected energy and the total average energy. This reduction in energy as demonstrated by the CCR/CRS DEA model is huge compared to the coded packet model. Furthermore, the cumulative results of the proposed CCR/CRS and the coded packet models are evaluated and the difference between the two models is shown in Figure 6.5. In other words, Figure 6.5 presents the cumulative average multicast energy using coded packet model and cumulative projected multicast energy using empirical DEA model. As shown, the blue line represents the cumulative average energy computed by the RLNC algorithm (coded packet model) while the red line represents the cumulative projected energy computed by the frontier production function (empirical DEA model). The gap between the two lines represents the amount of energy saved if the ad hoc wireless networks function efficiently.

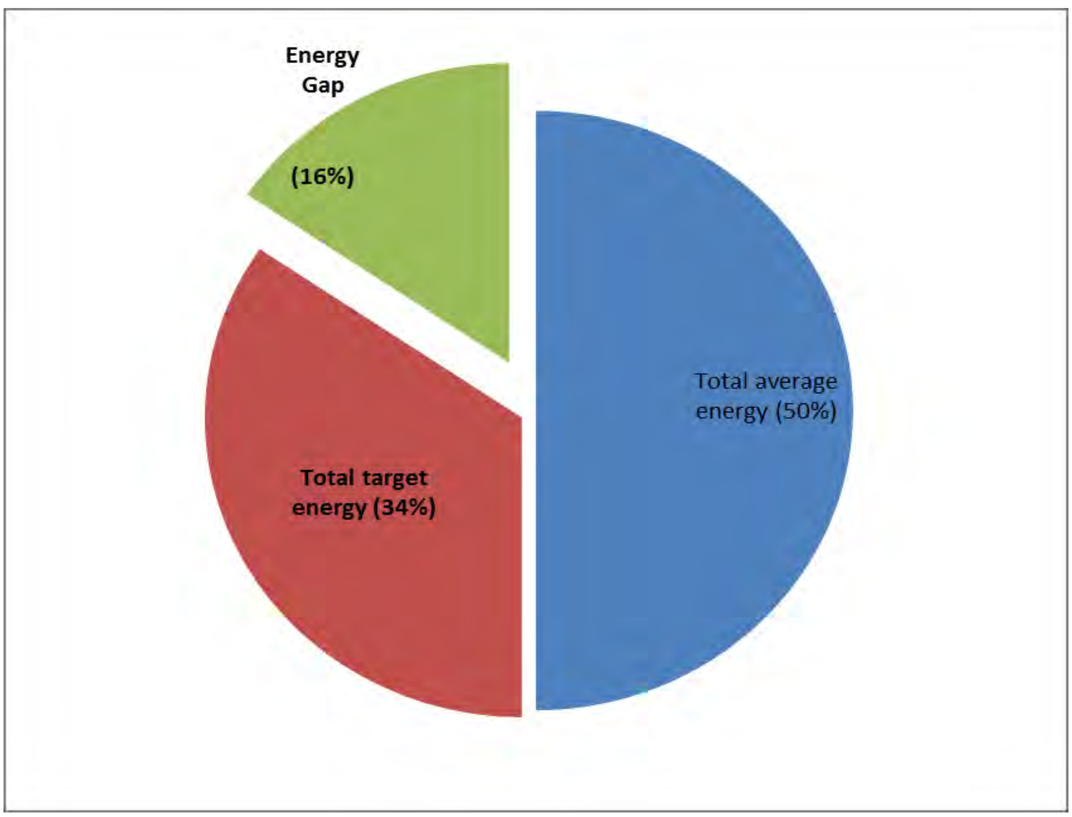

Figure 6.4: Percentage of the total projected energy, the total average energy and the total energy saved using input-oriented CCR/CRS DEA model

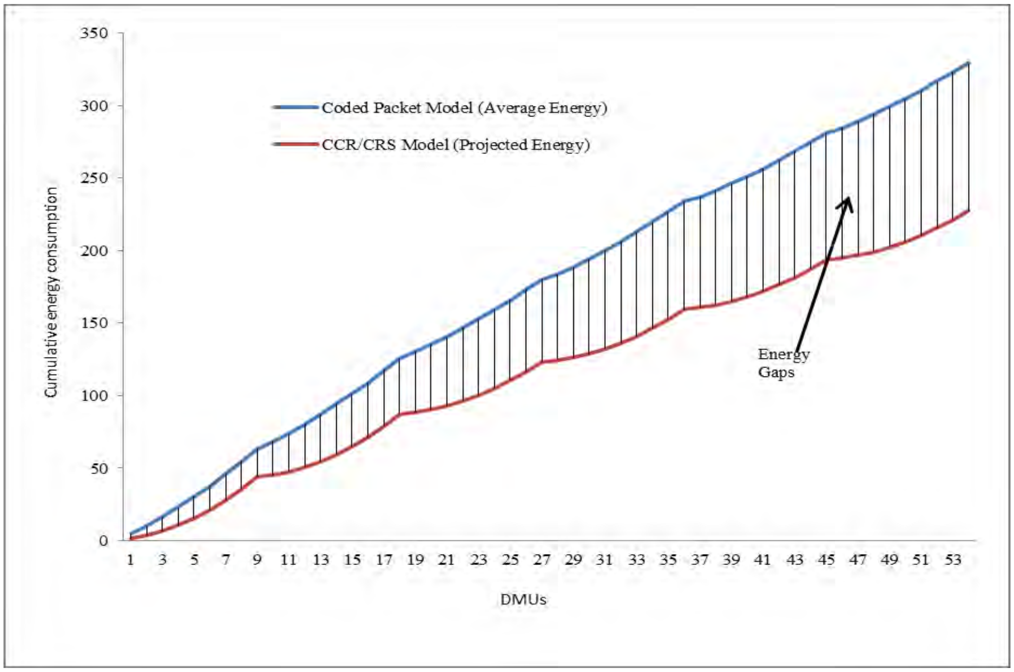

Figure 6.5: Performance comparison of coded packet model using RLNC algorithm and the input-oriented CCR/CRS DEA model using frontier production function

6.5.2 Envelopment Model (BCC/VRS) vs Coded Packet Model (RLNC)

In this section, we compare the performance of input-oriented BCC/VRS model with coded packet model. Similar to the CCR/CRS model, we calculated the percentage of the total energy saved by the input-oriented BCC/VRS model. This is obtained by finding the difference between the total of the average multicast energy and the total of the projected energy. We compute and extracted these values from Table 6.2, which is 329.77019 (average multicast energy) minus 304.62626 (projected multicast energy). The difference, which is the total energy saved (25.143931), is equivalent to 4%. Figure 6.6 represents the pie chart of the total energy saved, the total average multicast energy and the total projected multicast energy. It is also observed that the DEA method performed better than the coded packet model in terms of the energy saved. Furthermore, the cumulative results of the proposed BCC/VRS model and the coded packet model are evaluated and the difference between the two models is shown in Figure 6.7 with the blue line represents the cumulative average multicast energy computed by the RLNC algorithm (coded packet model) while the red line represents the cumulative projected energy computed by the frontier production function (empirical DEA model). The gap between the two lines represents the amount of energy saved if the ad hoc wireless networks function efficiently.

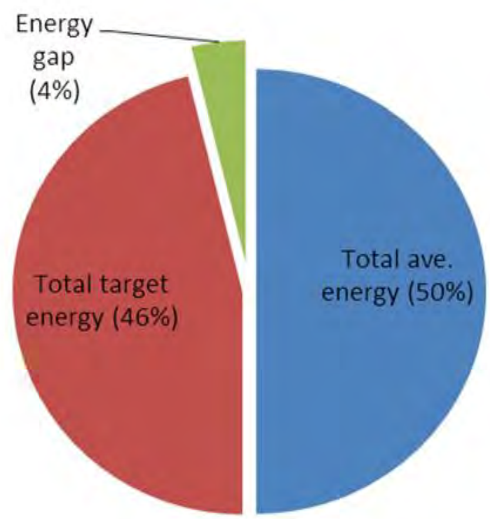

Figure 6.6: Percentage of the total projected energy, the total average energy and the total energy saved using input-oriented BCC/VRS DEA model

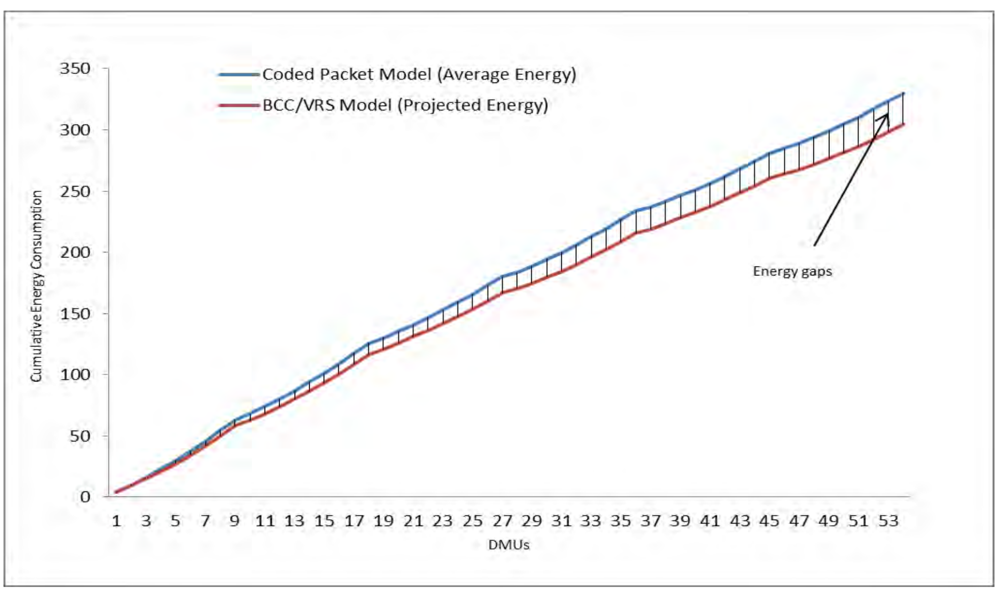

Figure 6.7: Comparison of coded packet model using RLNC algorithm and the input-oriented BCC/VRS DEA model using frontier production function

6.5.3 Envelopment Model (CCR/CRS and BCC/VRS) vs Coded Packet Model (RLNC)

The previous sections of this chapter presented the results of the two proposed models separately in relation to the current model in terms of energy saved. This section compares the performance of the three models with one another. It compares performance of the two proposed models (CCR/CRS (DEA) and BCC/VRS) and the current model (coded packet). The multicast energy using coded packet model is simulated according to the RLNC algorithm. The multicast energy using the proposed CCR/CRS and BCC/VRS models are evaluated based upon DEA frontier production function. As observed from Figure 6.8, the two proposed models perform better than the current coded packet model. However, the input-oriented CCR/CRS model saves most energy followed by the input-oriented BCC/VRS model and the least performed is the coded packet model. The variation in performance between the input-oriented CCR/CRS model and the input-oriented BCC/VRS model is due to the returns to scale assumption. As expected, more energy will be saved if constant returns to scale is assumed but this type of returns to scale does not allow flexibility. It assumed that all the observed ad hoc wireless networks operate at an optimal scale.

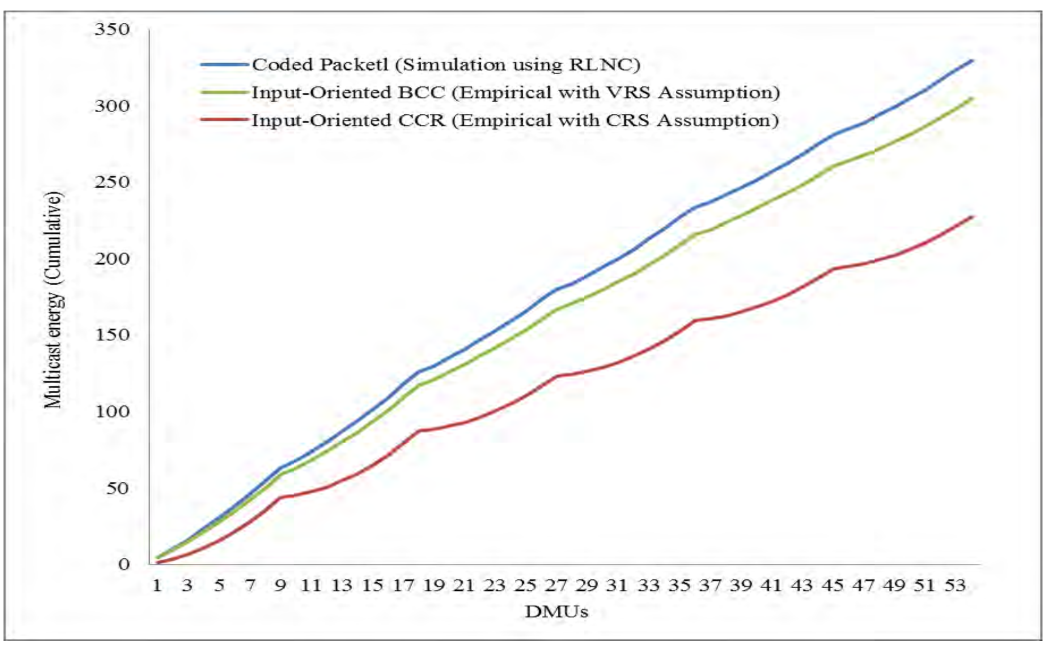

Figure 6.8: Performance comparison of coded packet (RLNC) model, CCR/CRS (DEA) model and BCC/VRS (DEA) model

6.6 Chapter Summary

This Chapter has shown how to model and evaluate the excess energy experienced by the ad hoc wireless networks in multicasting messages from a source to some selected group of nodes. The evaluation shows the extent at which different models that are considered could reduce energy in ad hoc wireless networks. The results have shown that the input-oriented CCR/CRS model saves more energy compared to the input-oriented BCC/VRS model. However, these two models save energy better than the current coded packet model. The next chapter presents the Benchmark model which provides Efficiency Reference Set (ERS) or peers group to inefficient ad hoc wireless networks. The chapter also determine the lambdas, which is the amount require by each inefficient ad hoc wireless network to catch up with their peers.

Chapter Seven: Benchmarking Model for ERS and Lambdas Evaluation

7.1 Introduction

Efficiency and gap analysis are good and fundamental methods in performance evaluation. However, we explore other methods for performance evaluation. As a result, some multifactor based gap analysis methods such as Spider charts and Z chart were developed [58], [56]. However, there are challenges in combining multiple measures in the final stage. Therefore, Benchmarking models that can deal with multiple performance measures and provide an integrated benchmarking measure are needed. Benchmarking process is ultimately establishing a standard of excellence. If ad hoc wireless networks are used in this regard, then the frontier identified can be regarded as empirical standard of excellence [93]. In the process of best practice benchmarking, once the frontier is established, we can then compare a set of new ad hoc wireless networks to the frontier. The idea is that whenever a new ad hoc wireless network outperforms the identified frontier, a new frontier is generated by DEA solver. Because of this, the benchmark for an ad hoc wireless network is different from other new ad hoc wireless networks. With the benchmark idea, an ad hoc wireless network learn how best to utilise its available resources [60]. Furthermore, this model provides significant additional information about where efficiency improvements can be achieved and the magnitude of these potential improvements. By contrast, none of the existing minimum energy multicast technique possess the features of establishing standard of excellence in this manner. In this chapter, the Benchmark model is derived in two versions. These are evaluated, compared, and conclusion is drawn.

7.1.1 Input-oriented Variable-benchmark CCR/CRS Model

Figure 7.1 represents the architecture of the Variable-benchmark model proposed for minimum energy multicast. It is designed upon input-oriented CCR/CRS DEA model. The architecture forms part of the generalized architecture (Figure 4.1). The architecture, unlike Envelopment model that evaluates the efficiency scores, the Benchmark model determined Efficiency Reference Set (ERS) and, the amount require by each ad hoc wireless network to catch up with their peers (Lambdas). The remainder of this section presents the appropriate

Mathematical model formulation for the architectural requirements so that benchmarking solution based on input-oriented CCR/CRS DEA model are determined. Furthermore, this section presents input-oriented variable-benchmark model where a new DMU is evaluated against a set of given benchmarks (standards). We develop Benchmark models based upon the CCR Envelopment model and assumed CRS.

Considering variable-benchmark approach, the Envelopment model derived for CCR/CRS is modified for the benchmark optimisation problem as follows:

$$Minimise\, \alpha^{CCR/CRS}$$

Subject to

$$\sum_{j\in E^*} \lambda_j x_{ij} \le \alpha^{CCR/CRS} x_i^{new}$$

$$\sum_{j\in E^*} \lambda_j y_{rj} \ge y_r^{new}$$

$$\lambda_j \ge 0, j \in \mathrm{E}^*, \tag{7.1}$$

where $\alpha^{CCR/CRS}$ represents the optimal value to model (7.1), E^* represents the set of benchmarks identified by the DEA. The new observation is represented by DMU^{new} with inputs $x_i^{new}\left(i=1,2,\ldots,m\right)$ and outputs $y_r^{new}\left(r=1,2,\ldots,s\right)$. The superscript of CCR/CRS indicates that the benchmark composed by benchmark DMUs in set E^* is based on CCR/CRS model. Model (7.1) represents the performance of DMU^{new} with respect to benchmark DMUs in set E^*, when outputs are fixed at their current levels.

Furthermore, model (7.1) is capable of yielding a benchmark for DMU^{new}. Thus the *ith* input and the *rth* output for the benchmark can be expressed as:

$$\begin{cases} \sum_{j\in E^*} \lambda_j^* x_{ij} & \left(ith\ input\right) \\ \sum_{j\in E^*} \lambda_j^* y_{rj} & \left(rth\ ouput\right) \end{cases} \tag{7.2}$$

The expression (7.2) indicates that although the DMUs associated with set E^* are given, the resulting benchmark may be different for each new DMU under evaluation. That is why model (7.2) represents a variable-benchmark scenario. It is possible to have situations where the same benchmarks are fixed. For example, the measurement probably based on experience or taking hard decisions may indicate that a certain DMU should be used as a fixed benchmark. However, in this work, we consider variable-benchmarks. The level of performance by the DMUs is determined by the following three cases:

Theorems 7.1

i. $\alpha^{CCR/CRS} < 1$ indicates that the performance of DMU_0^{new} is dominated by the benchmark in (7.2).

ii. $\alpha^{CCR/CRS} = 1$ indicates that $\mathrm{DMU}^{\mathrm{new}}$ achieve the same performance level of the benchmark in (7.2). It is implies that there is no input savings.

iii. $\alpha^{CCR/CRS} > 1$ indicates that input savings exist in DMU_0^{new} when compared to the benchmark in (7.2).

From the theorems, we can define $\alpha^{CCR/CRS} - 1$ as the performance gap between $\mathrm{DMU}^{\mathrm{new}}$ and the benchmark. Also, based upon $\alpha^{CCR/CRS*}$, a ranking of the benchmarking performance can be obtained.

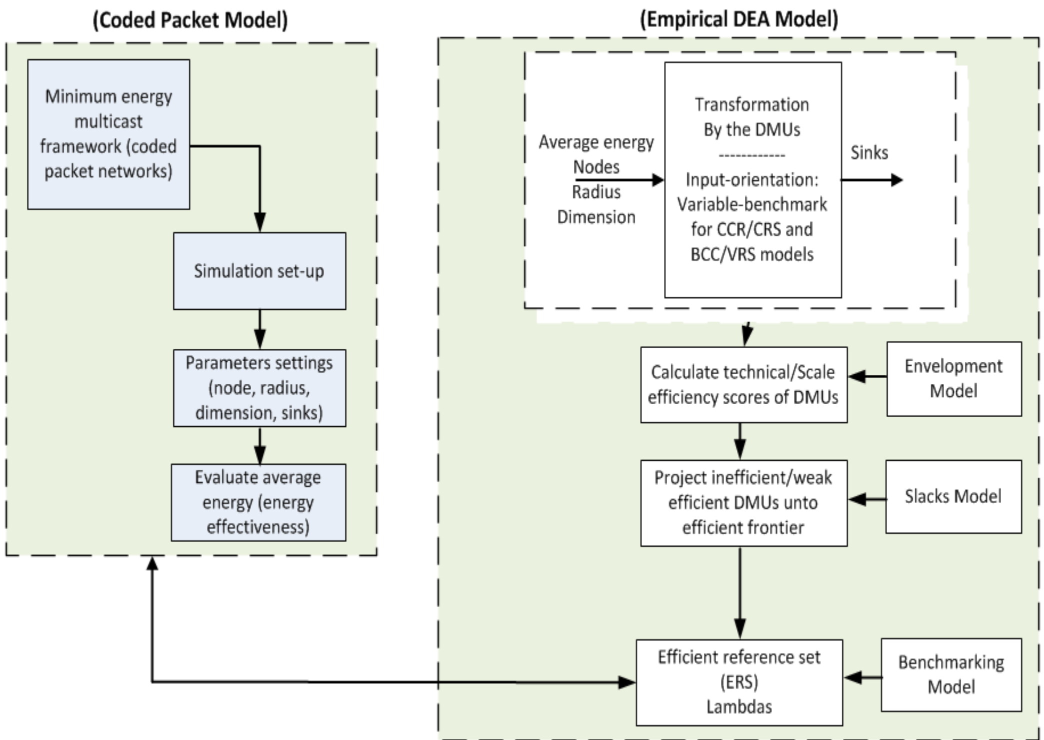

Figure 7.1: Architecture of the variable-benchmark model for minimum energy multicast that is designed upon input-oriented CCR/CRS and input-oriented BCC/VRS models

7.1.2 Input-oriented Variable-benchmark BCC/VRS Model

Figure 7.1 also represents the architecture of the Variable-benchmark model for minimum energy multicast that is designed based on input-oriented BCC/VRS DEA model. Also, the Mathematical model formulation for the architectural requirements of benchmarking solution based on input-oriented BCC/VRS DEA model is considered. In order to derive benchmark that is based upon BCC model, we modify model (7.1) by assuming VRS. Then, the benchmark optimization problem is given below:

$$Minimise\,\alpha^{BCC/VRS}$$

Subject to

$$\sum_{j \in E^*} \lambda_j x_{ij} \leq \alpha^{BCC/VRS} x_i^{new}$$

$$\sum_{j\in E^*}\lambda_j y_{rj} \geq y_r^{new}$$

$$\sum_{j\in E^*}\lambda_j = 1$$

$$\lambda_j \geq 0, j \in E^*. \tag{7.3}$$

It should be noted that model (7.3) also satisfy theorem (7.1) and that for both models to be feasible, we assume that it is possible to use all the data sets available. Table 7.1 summarises the variable- benchmark model for both CCR/CRS and BCC/VRS.

Table 7.1: Summarised variable-benchmark model for both CCR/CRS and BCC/VRS

Frontier Type	Input-orientation
CCR/CRS	$Minimise\,\alpha^{Frontier}$ Subject to $\sum_{j\in E^*}\lambda_j x_{ij} \leq \alpha^{Frontier} x_i^{new}$ $\sum_{j\in E^*}\lambda_j y_{rj} \geq y_r^{new}$ $\lambda_j \geq 0, j \in E^*$
BCC/VRS	Add $\sum_{j\in E^*}\lambda_j = 1$

7.2 Simulation Set up, Benchmark Model Implementation and Results

The benchmark model addresses the benchmark problem. It is a model for establishing the standard of excellence. The model is able to determine the ERS and lambdas of the inefficient ad hoc wireless networks. Lambdas define the amount of inputs to be reduced or outputs to be augmented for inefficient ad hoc wireless network to catch-up with their peers that are already operating efficiently. First, the simulation was conducted using the coded packet framework. The first part of Figure 7.1 shows the procedures of how multicast energy was evaluated. The details about the configuration and how results are obtained is already presented in chapter 3. The second part of the architecture began with the Envelopment model presented in chapter 4 and then with the Slack model presented in chapter 5. In this chapter, we present the Benchmark model for identification of ERS and Lambdas evaluation. The Variable-benchmark models are operated using the same data set used for Envelopment and Slack. The implementation procedures are also similar. Thus the same DEA solver is used for Benchmark model. In order to perform these benchmark analysis, the Excel report with "References" and "Lambdas" sheets are exported for ERS and lambdas respectively. The details for the implementation of the Benchmark models are found on the accompanying CD. The analysis of results and evaluation for the two models, which are input-oriented Variable-benchmark CCR/CRS model and input-oriented Variable-benchmark BCC/VRS model are presented in the following sub-sections.

7.2.1 ERS and Lambdas Using Input-oriented Variable-benchmark CCR/CRS Model

Table H.3 under Appendix H is extracted from the "References" DEA output sheet. Here, ad hoc network administrators whose network is inefficient can observe the benchmark ad hoc networks that they need to catch up to using model (7.1). From Table H.3, full efficient ad hoc network may consider itself to be its own "benchmarks". This is because efficient ad hoc network already achieved 100% efficiency. So, benchmark for DMU_9 is DMU_9, and for DMU_{18} is DMU_{18}. The same apply to DMU_{27} and DMU_{45}. However, for inefficient ad hoc networks, their benchmarks are one or many of the efficient ad hoc networks. For example, a benchmark for DMU_2 and DMU_3 are DMU_9, DMU_{18} and DMU_{27} (observe from the Table that DMU_9, DMU_{18} and DMU_{27} are efficient). This means, DMU_2 and DMU_3 must use a combination from DMU_9, DMU_{18} and DMU_{27} (virtual ad hoc networks) to become efficient.

Another benchmark analysis is the Lambda values. This benchmark analysis for CCR/VRS model calculates the amount of benchmark needed from a DMU to achieve efficiency. These values are reported as magnitude (Lambda) next to each benchmark DMUs on Table H.3 under Appendix H. Furthermore, they are λ weights obtained from the dual version of the linear program (4.5) that is solved to estimate these values. For instance, as could be observed from Table H.3 and as shown in portion (Figure 7.2), DMU_{16} will attempt to become like DMU_{18} (blue bar) more than DMU_{27} (red bar) as observed from their respective λ weights of DMU_{18} and DMU_{27} (λ_{18} = 71.3 and λ_{27} = 8.7). We also present the number of occurrence of all the efficient DMUs to determine which of the efficient ad hoc wireless networks occur the most benchmarked and least benchmarked for the inefficient DMUs. This frequency of efficient DMU is presented in Figure 7.3 with DMU_{27} occurs most while DMU_9 has the least occurrence. Specifically, DMU_{27} serves as benchmark to 33 of DMUs while DMU_9 serve as benchmark to 10 of DMUs.

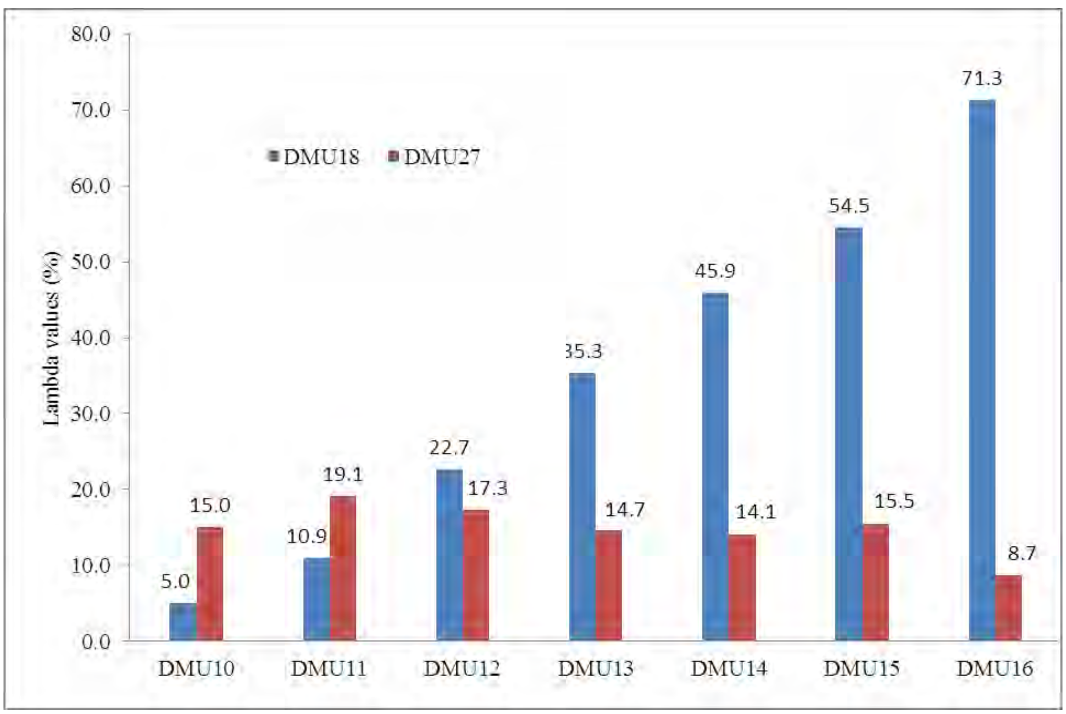

Figure 7.2: Benchmarks and Lambdas of the input-oriented variable-benchmark CCR/CRS model

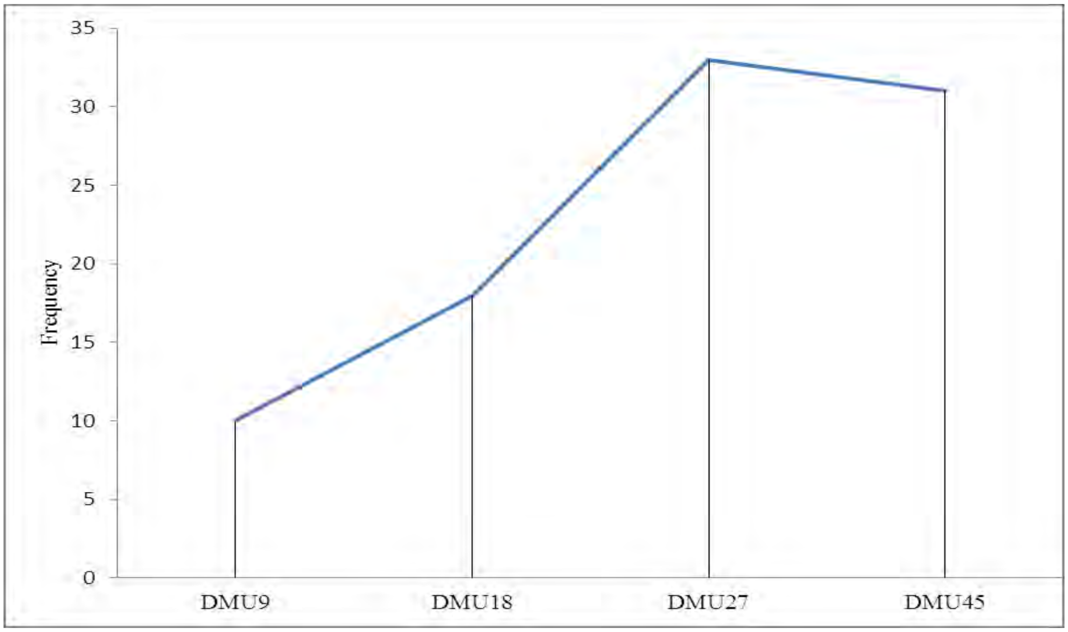

Figure 7.3: The number of occurrence DMUs that are served as benchmark for inefficient DMUs using input-oriented variable-benchmark CCR/CRS model.

7.2.2 ERS and Lambdas Using Input-oriented Variable-benchmark BCC/VRS Model

Table H.4 under Appendix H is also taken from the "References" DEA output sheet. Similar to the variable-benchmark CCR/VRS model, an ad hoc network administrator whose ad hoc network is inefficient can observe the benchmark ad hoc networks needed to catch up to using model (7.2). Again, as it could be observed form Table H.4, efficient ad hoc network may consider itself to be its own "benchmarks." So, the benchmark for DMU_1 is DMU_1, and for DMU_9 is DMU_9. The same applies to DMU_{18}, DMU_{27}, DMU_{28}, DMU_{37}, and DMU_{45}. However, for inefficient ad hoc networks, their benchmarks are one or many of the efficient ad hoc wireless networks. For example, a benchmark for DMU_2 and DMU_3 are DMU_1 and DMU_9 (observe that DMU_1 and DMU_9 are efficient). This means, DMU_2 and DMU_3 must use a combination from both DMU_1 and DMU_9 (virtual ad hoc networks) to become efficient.

The input-oriented variable-benchmark BCC/VRS model can also be used to calculate the level of benchmark needed from a DMU to achieve efficiency. These values are reported as magnitude (Lambda) next to each benchmark DMUs in Table H.4. In other words, they are known as λ weights and obtained from the dual version of the linear program (4.6). The model is solved to estimate these lambda values. For example, Table H.4 shows in portion (Figure 7.4) how DMU_2 will catch up with DMU_1 and DMU_9. Observe that DMU_2 will need to imitate DMU_1 more than DMU_9. This is evident as shown from their respective λ weights of DMU_1 and DMU_9 ($\lambda_1 = 87.5$ and $\lambda_9 = 12.5$). We also analyse the number of occurrence of all the efficient DMUs serve as benchmark. This is necessary to determine which of the efficient ad hoc networks most occurred and the least occurred benchmark. The frequency of these benchmarks ad hoc wireless networks are presented in Figure 7.5.

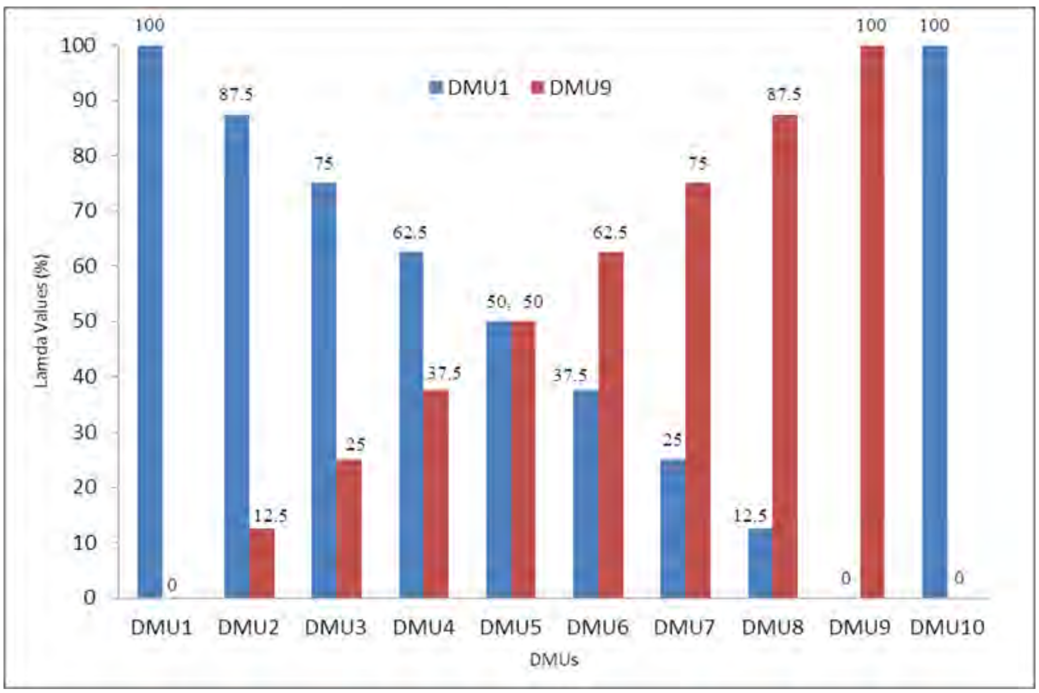

Figure 7.4: Benchmarks and Lambdas of the input-oriented variable-benchmark BCC/CRS model

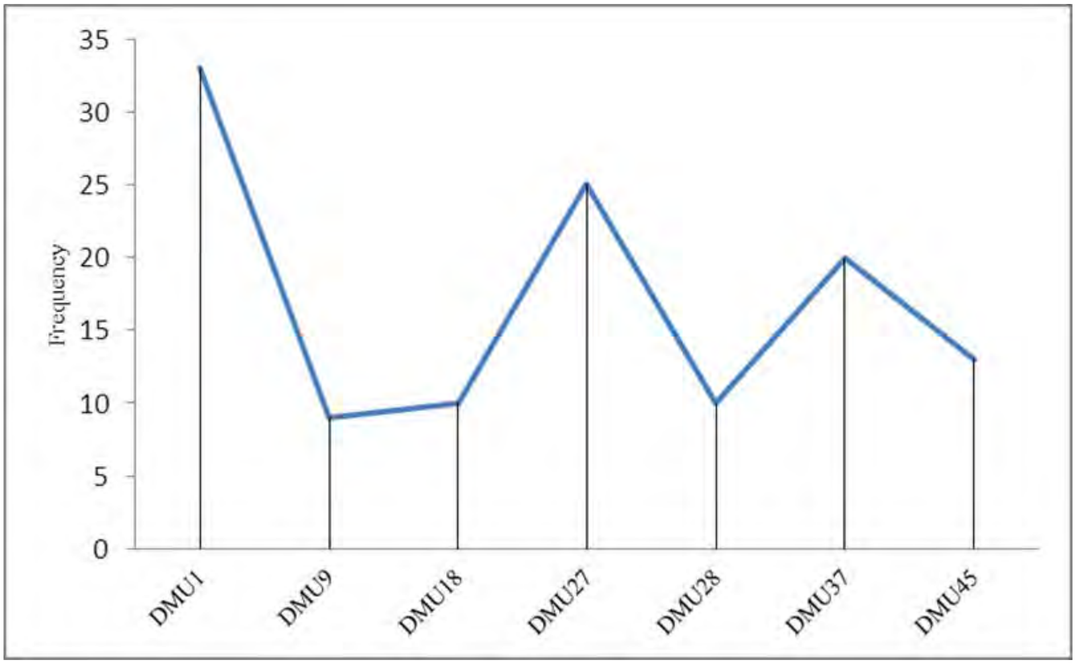

Figure 7.5: The number of occurrence DMUs that are served as benchmark for inefficient DMUs using input-oriented variable-benchmark BCC/VRS model.

7.3 Performance Comparison of Benchmarking in Ad Hoc Wireless Networks

This section discusses the results obtained from the Benchmark models and from these, two important forms of evaluations are discussed, the ERS and the Lambdas. These two forms of evaluation were analysed in the previous section. The summary of the statistics of the ERS and the lambdas values for the ad hoc wireless networks are presented in Table 7.2. These results are used to compare the performances of the coded packet, CCR/CRS, and BCC/VRS models. As it could be observed from Table 7.2, 4 DMUs serve as ERS for other DMUs if input-oriented variable-benchmark CCR/CRS model is considered while, 7 DMUs serve as ERS if input-oriented variable-benchmark BCC/VRS model is used. However, the coded packet models did not provide any indication of how the ERS could be obtained. These ERSs serve as the benchmark for inefficient DMUs. That is, they define the standard of excellence for other DMUs because their configurations and resource utilisation process is functioning well. Figure 7.6 shows the DMUs that serve as ERS for input-oriented variable-benchmark CCR/CRS and input-oriented variable-benchmark BCC models. The Figure also shows their respective capacity as ERS i.e. for example, DMU_1 appears for ERS in 33 times when input-oriented variable-benchmark BCC/VRS model is used but it does not serve as an ERS when input-oriented variable-benchmark CCR/VRS model is used. Observe that DMU_9, DMU_{18}, DMU_{27} and DMU_{45} serve as ERS for input-oriented variable-benchmark CCR/CRS and input-oriented variable-benchmark BCC models.

Table 7.2: Statistical reports of the ERS and Lambdas

Statistics	MODELS		
	Coded packet	**CCR/CSR**	**BCC/ VRS**
Number of DMU	54	54	54
Number of ERS	-	4	7
Number of Lambdas Values	-	92	121
% Ave. of ERS	-	23	17
% Ave. of Lambdas Values	-	35.22	44.63
Min. of ERS	-	10	8
Min. of Lambdas Values	-	0.02	0.4
Max. of ERS	-	33	33
Max. of Lambdas Values	-	100	100

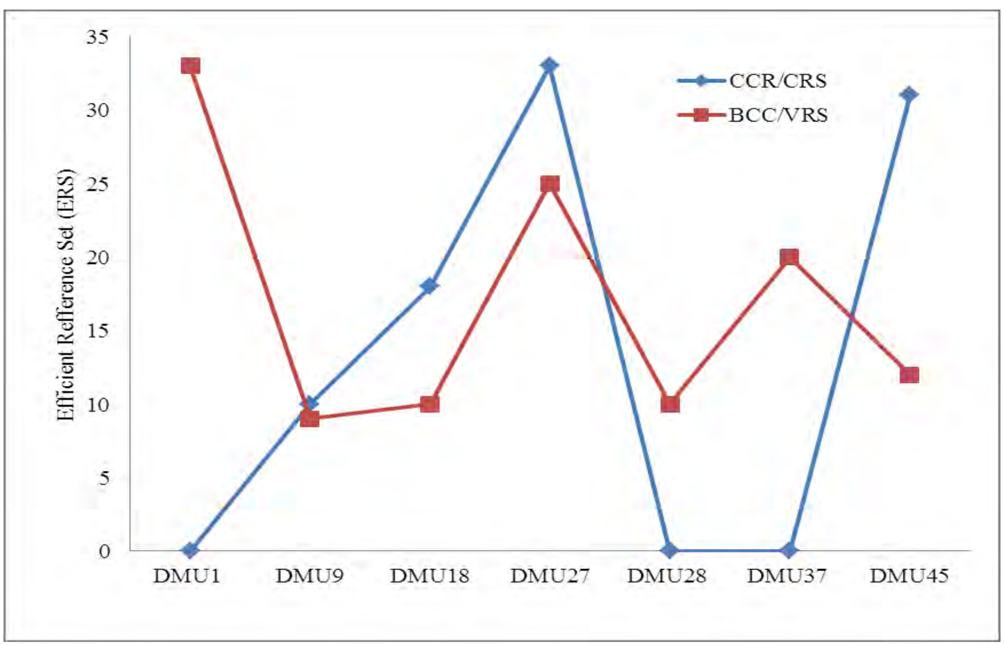

Figure 7.6: Efficiency Reference Set (ERS) of CCR/CRS and BCC/VRS models

Furthermore, these ERS represents a good operating practice especially for inefficient DMUs to emulate. The remaining 50 DMUs in the case of input-oriented variable-benchmark CCR/CRS and 47 DMUs in the case of input-oriented variable-benchmark BCC/VRS have efficiency score less than 1. This means that they are technically inefficient. This scenario indicates that these DMUs are not operating based on best practice. In the case of Lambdas, it could be observed from Table 7.2, 92 lambdas are reported against input-oriented variable-benchmark CCR/CRS model while 121 lambdas are reported for input-oriented variable-benchmark BCC/VRS model. The coded packet model did not provide any information about the lambdas. Other statistical records presented in Table 7.2 include average of ERS and average lambdas, the minimum of ERS and lambdas (also the maximum of ERS and lambdas) evaluations.

7.4 Chapter Summary

This Chapter has presented Mathematical models that are based on Data Envelopment Analysis (DEA) for benchmarking ad hoc wireless network performance. These models were designed as alternative models to address the benchmarking problem in ad hoc wireless networks. The two models developed based on CCR/CRS and BCC/VRS models were used to determine the lambdas and ERS for inefficient ad hoc wireless networks.

Chapter Eight: Summary, Conclusion and Future Work

8.1 Summary

The summary of the statistics for the energy efficiency performance of the ad hoc wireless networks that were implemented using models developed based on coded packet (RLNC), CCR and BCC are presented in Table 8.1. In terms of the number of efficiency a model produced, observe from the Table that 4 of the ad hoc networks are technically efficient using the proposed CCR/CRS based model. Also from the Table, 37 of ad hoc wireless networks are technically efficient using the proposed BCC/VRS model. This is in order as analysed in section 2.4 that CRS tends to produce lower technical efficiency while VRS tends to produce higher technical efficiency. This is because the constraint set for CRS is more restrictive (i.e., convexity constraint is absent) than in the VRS formulation, then lower efficiency scores are possible and therefore more networks are declared efficient for a VRS envelop surface. However, the CRS assumption is only applicable when it is observed that ad hoc wireless networks are operating at optimal scale. That means the scale or size of the network is not a factor in assessing its relative efficiency, otherwise, the VRS assumption should be considered. The coded packet model provides no information about degree of network technical efficiency. These technically efficient ad hoc wireless networks are the best practice networks. And they also serve as ERS for the inefficient ad hoc networks. In other words, technical efficiency means that their resources utilisation is functioning well and their operation is not characterized by any waste of energy. The ad hoc wireless networks that serve as ERS set example of good operating practices for inefficient ad hoc networks. The 50 ad hoc networks in the case of CCR/CES and 17 in the case of BCC/VRS models have efficiency score less than 1. It means that they are technically inefficient. So they deviated from the best practice use of energy. However, this research work has shown how the inefficient ad hoc networks can improve their energy utilisation and become efficient. Other details about the statistical summary of efficient and inefficient ad hoc networks are also presented in Table 8.1. The details are summarized for 54 ad hoc networks that were sampled.

Table 8.1: Summary statistics of sample 54 ad hoc wireless networks computed by coded packet, CCR/CRS and BCC/VRS models

Statistics	**Coded packet Model**	**CCR/CRS Model**	**BCC/VRS Model**
<u>Sample Space</u>			
Number of DMU	54	54	54
Efficient DMU	-	4	37
Inefficient DMU	-	50	17
<u>Average / % Average</u>			
Ave. of all efficient and inefficient DMU	-	0.65925	0.96860
Ave. of all efficient DMU	-	1	1
Ave. of all Inefficient DMU	-	0.63199	0.90023
% Ave. of all efficient and inefficient DMU	-	65.93	96.86
% Ave. of all efficient DMU	-	100	100
% Ave. of all Inefficient DMU	-	63.20	90.02
<u>Minimum / Maximum</u>			
Min. of all efficient and inefficient DMU	-	0.27502	0.85748
Min. of all efficient DMU	-	1	1
Min. of all inefficient DMU	-	0.27502	0.85748
Max. of all efficient and inefficient DMU	-	1	1
Max. of all efficient DMU	-	1	1
Max. of all inefficient DMU	-	0.97586	0.97586
<u>Other Information</u>			
Ave. Energy utilized by all DMU	6.10685537	4.21355	5.641227028
Total Energy saved	-	102.23843	25.14393
% of Energy saved	-	31.00	7.63
Efficiency Reference Set (ERS)	-	4	7
Lambdas Values	-	92	121

In summary, the average energy utilized by coded packet model was recorded as 6.10686, while the equivalent projected energy recorded by CCR/CRS and BCC/VRS models are 4.21355 and 5.64122 respectively. So in terms of energy saved, observed from Table 8.1, that the CCR/CRS models reported a reduction of 157.25219, which is an equivalent to 16%, the BCC/VRS models reported a reduction of 22.23064, which is an equivalent to 4%. Again, the same reasons analysed in section 2.4 is true concerning these results. Ideally, the energy reduction that could be achieved by ad hoc wireless networks if modelled with assumption of CRS is 157.25219. This result implied that all networks under analysis are multicasting at optimal scale. However, in the real world, it is practically impossible to achieve this optimal scale because of constraints such as network size, radius of connectivity, and dimension which may cause a network not to multicast at optimal scale. In that case the energy reduction by VRS is considered practical and achievable if the focus is to evaluate the inefficiencies due to the network's administrator underperformance, which is the focus of this work. However, considering these analysis, we concluded that the proposed models saves multicast energy in ad hoc wireless networks better than coded packet model. It provides better alternative approach to coded packet model in terms of energy saving.

8.2 Conclusions

The main objective of this work was to develop empirical-based, minimum energy multicast architecture to evaluate the energy efficiency of ad hoc wireless networks. The aim of the design is to minimise the amount of energy consumed by the networks nodes without affecting their performance. As a result, DEA methodology was explored so that the following performance evaluation and benchmark are properly addressed:

- Technical Efficiency (TE) evaluations
- Scale Efficiency (SE) evaluations
- Projection or Target evaluations
- Energy Gap (EG) evaluations
- Benchmark evaluations

These performance evaluations which are mostly economic concepts were difficult to achieve using engineering methodologies only. Therefore, approaches from different discipline were necessary since the prospect of obtaining actual energy efficiency evaluations and benchmark in the usual simulation approach does not seem feasible. In essence, the DEA approach was able to resolve multiple inputs and multiple outputs variables irrespective of their volume or unit. Specifically, the proposed models developed were able to adequately evaluate the energy efficiency performance of ad hoc wireless networks. The models have shown their ability to reduce the current energy consumed by the ad hoc wireless networks nodes while the outputs are kept at constant level. Furthermore, the results obtained have shown that if an ad hoc wireless network is capable of achieving 100% efficiency through the correct usage of average multicast energy optimally weighted, then, other inefficient ad hoc wireless networks should be capable of doing the same if they operate efficiently. Therefore, if an ad hoc wireless network is identified as efficient based on best practice, those that are inefficient can be benchmarked and their efficiency improved so that they also become efficient. In order to achieve this goal, we have proposed the following models for energy efficiency evaluations:

- Envelopment Models based on input-oriented CCR/CRS and BCC/VRS for technical efficiency evaluation.
- Scale Efficiency Models based on input-oriented CCR/CRS and BCC/VRS for exploring the nature of returns to scale and determining the size of networks.

However, these models were only adequate for technical and scale efficiency evaluations. They are not capable of minimising the multicast energy in ad hoc wireless networks on their own. Since the main aim of this thesis was to minimise the amount of energy consumed by the networks nodes without affecting their performance, another model was required to identify the inefficient ad hoc wireless networks. This model was able to project the inefficient ad hoc wireless networks unto their efficient frontier. As a result, the following models were added to the Envelopment Model so that the goal of minimising multicast energy is achieved.

- Slack models based on input-oriented CCR/CRS and BCC/VRS for the identification of inefficient ad hoc wireless networks and projecting them unto efficient frontier.

Furthermore, maintaining an efficient frontier required a Benchmarking model for establishing a standard of excellence. This would allow the inefficient ad hoc wireless networks to catch up with their peers that are efficient. As a result, the following models were proposed:

- Benchmark Models based upon input-oriented variable-benchmark CCR/CRS and BCC/VRS for establishing the standard of excellence for inefficient ad hoc wireless networks.

These models were able to establish standard of excellence and provide benchmark for inefficient ad hoc wireless networks. The benchmark models made use of the following information to function effectively:

- Efficiency Reference Set (ERS)
- Lambdas

This information is very useful for the inefficient ad hoc wireless networks. While the ERS serves as peer ad hoc wireless networks for those that are under performing, the lambdas provide the level or amount of imitation to be copied from the ERS.

Another important evaluation that this research work provided is the ability of the proposed models to evaluate the 'energy gap' using EG model. The EG is the difference between average multicast energy and the projected energy. The projected energy is evaluated from ad hoc wireless networks that are operating efficiently. The proposed models were used to calculate the energy gaps and compared them with coded packet model. It was found that the proposed models achieved substantial energy saving over existing coded packet model. From these results, it was concluded that performance evaluation based on empirical DEA approach is better if the ad hoc wireless networks operate efficiently. It is also a better alternative if energy is needed to be saved in communication networks.

As evidence for the forgoing, considering the reports that were summarised in the previous section, it is concluded that model based on CCR/CRS provides outstanding performance in terms of energy saving over other methods. This is because the model operates with CRS assumption, which is stricter, compared to VRS. As a result, the model had shown excellent energy saving in ad hoc wireless networks. Sometimes, it is difficult to realise overall technical efficiency offer by the CCR/CRS models. In that case, a model that tolerates flexibility is considered. As a result, model based on BCC/VRS was developed in this thesis. Despite this flexibility, the BCC/VRS model with VRS assumption has demonstrated good performance in terms of efficiency and energy savings over coded packet models. In terms of energy saved, the performance evaluation of CCR/CRS model is better than that of BCC/VRS model which outperformed the coded packet model. However, the two proposed empirical DEA models shows different level of superiority over the current coded packet model for minimum energy multicast in ad hoc wireless networks.

8.3 Recommendation and Future Work

In view of the findings that empirical DEA models provide better performance over the current coded packet models, then the input-oriented CCR/CRS and input-oriented BCC/VRS models are recommended for appropriate and adequate evaluation of energy efficiency. Also, the input-oriented variable-benchmark CCR/CRS and input-oriented variable-benchmark BCC/VRS models are recommended for adequate evaluation of benchmarking solution. Specifically, for energy minimisation in ad hoc wireless networks, these models are recommended. The applications of these models to energy efficiency in ad hoc wireless networks have shown good results. Thus, our approach is a good step for achieving the expected (projected) minimum energy in communication networks. It is also a good step for the realisation of multicast communication in the Next Generation Networks (NGN). Moreover, the excess energy that could be hazardous for environmental sustainability and global warming will be conserved.

It is important to mention that the results presented in this work were based on the input-oriented approach. Therefore, in the future work, models based on output-oriented approach could be considered and the results compared with our models. It is also possible to expand our idea beyond the technical and scale efficiency and in order to achieve this, more economic concepts such as price efficiency, super-efficiency could be explored. Furthermore, in this work, some assumptions were made, for instance, static ad hoc wireless network is assumed. It is also assumed that no interference and delays occur among others. Future work may consider these assumptions in their analysis to see these effects on DEA-based model performance. In this work, ad hoc wireless networks are considered for the application of our models. Future work may consider other types of network and architectures. In addition, due to simulation complexity, up to 40 nodes were able to be generated for defining the network scenarios, and up to 10 nodes for defining the sinks. It could be a good idea if this complexity is reduced and more nodes are generated to study the trend of energy efficiency using our models. It should be noted that more than 40 randomly generated nodes, and 10 receiving nodes can be computed using powerful simulation computer such as parallel processors computers. However, the proposed DEA technique has demonstrated potential to accommodate larger volume of data with many inputs and outputs.

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List of Appendices

A: Convexity and Inefficiency Properties for efficient frontier

The following two properties are required to develop empirical (piecewise linear) approximation to the efficient frontier:

Property a.1: *Convexity*. $\sum_{j=1}^{n} \lambda_j x_{ij} \left(i = 1,2,,\ \text{m}\right)$ and $\sum_{j=1}^{n} c y_{rj} \left(r = 1,2,,\ \text{s}\right)$ are possible achievable inputs and outputs (of virtual ad hoc wireless networks), where the $\lambda_j (j\ =\ 1,\ 2, \ldots\ldots, n)$ are nonnegative scalars such that $\sum_{j=1}^{n} \lambda_j = 1$.

Property a.2: *Inefficiency*. The same y_{rj} can be obtained by using x'_{ij}, where $x'_{ij} \geq x_{ij}$ (i.e., the same outputs can be produced by using more inputs); the same x'_{ij}, can be used to obtain y_{rj} , where $y'_{ij} \geq y_{ij}$ (i.e., the same inputs can be used to produce less outputs).

B: Multicast Connection and Subgraph Selection

To demonstrate a multicast connection, consider a set of receivers *T* rather than a single receiver *t* as in the case of unicast connection (Figure 3.1). The sink nodes are allowed to operate at different rates given a coding subgraph *z*. Suppose that sink $t \in T$ wishes to achieve a connection with a rate arbitrarily close to R_t where

$$R_t \ \leq \ min$$

$$Z \in Z\left(s,t\right) \left\lfloor \sum_{(i,K) \in \text{ú}+(Z)} \sum_{L \text{Ç} Z} z_{iKL} \right\rfloor \tag{B.1}$$

for all $t \in T$, where $z \in Z$, *(s,t)* represents the set of all cuts between *s* and T*,* and $_{+}$*(Z)*: = (*i, K)* $\in H \setminus i \in Z, K \setminus Z \neq \phi$ represents the set of forward hyper arcs of the cut *Z.* Therefore, by the max flow/min theorem [77], there exists, for each $t \in T$, a flow vector *x(t)* satisfying

$$\sum_{\{k|(i,K)\in H\}}\sum_{k\in K} y^t_{(iKk)} - \sum_{\{k|(k,I)\in H, i\in I\}} y^t_{(kIi)} = \beta^t_i$$

$$\forall i \in N, \quad t \in T$$

$$y^t_{(iKk)} \geq 0 \qquad \forall (i,K) \in H, k \in K, t \in T \tag{B.2}$$

and

$$\sum_{k\in L} y^t_{iKk} \leq \sum_{\{J \subset k | J\ L \neq 0\}} z_{iKJ}$$

$$\forall (i,K) \in H, \ \ and\ L \subset J, \tag{B.3}$$

where

$$\beta^t_i = \begin{cases} R & if \quad i = s, \\ -R & if \quad i = t, \\ 0 & if \quad Otherwise \end{cases} \tag{B.4}$$

The capacity can be achieved if coding subgraph z and the random linear network coding (RLNC) algorithm are considered. The capacity region is determined by z_{iKL} . One of the solutions to achieve subgraph selection is to use the *flow-based approaches*, which its objective is to establish a set of efficient multicast connections given the flow rates.

C. Pseudocode for Minimum Energy Multicast

C.1 Pseudocode for Multicast Incremental Power (MIP) Algorithm

The pseudocode for the Broadcast Incremental Power (BIP) algorithm is the first procedure for the MIP algorithm and is given below:

```
Broadcast_Incremental_Power_Algorithm ( )
{
S = {source};
TREE[source's_nearest_neighbor] = source;
for (i = 1; i <= N, i++)
        for (j = 1; j <=N, j++)
                link_cost_matrix[i][j] = P[i][j];
```

```
while (|S| < N)
        {
        (I,J) = compute_nearest_link (TREE, link_cost_matrix);
        S = S U {J};
        TREE[J] = I;
        while (j is not in S)
                link_cost_matrix[I][j] = P[I][j] − P[I][J];
        }
}
```

compute_nearest_link (TREE, link_cost_matrix)

```
{
link_cost_min = infinity;
for (all i on TREE)
        for (all j outside TREE)
                if (link_cost_matrix[i][j] < link_cost_min)
                        {
                        link_cost_min = link_cost_matrix[i][j];
                        I = i;
                        J = j;
                        }
return (I,J);
}
```

The pseudocode for sweep operation is second procedures for achieving MIP algorithm and is given below:

Sweep (TREE)

```
{
tree_cost_min = cost (TREE);
SWEPT_TREE = TREE;
        for (i = 1; i <= N; i++)
        if (i == transmitting node)
        {
        TEST_TREE = SWEPT_TREE;
        update upstreams of i;
        for (j=1; j <= N; j++)
                {
                if (j is within the transmission range of i and j is not an upstream of i)
                TEST_TREE[j] = i;
                }
        tree_cost = cost(TEST_TREE);
        if (tree_cost < tree_cost_min)
                {
                tree_cost_min = tree_cost;
                SWEPT_TREE = TEST_TREE;
```

```
            }
        }
return SWEPT_TREE;
}
simplified_sweep (TREE)
{
for (i = 1; i <= N; i++)
        if (i == transmitting node)
        {
        update upstreams of i;
        for (j = 1; j <= N; j++)
                {
                if (j is within the transmission range of i and j is not an upstream of i)
                 TREE[j] = i;
                }
        }
return TREE;
        }
```

C.2 Pseudocode for Coded Packet Networks

The pseudocode for coded packet using the Random Linear Network Coding (RLNC) algorithm is given below:

***Initialization (Source node** s*): *s* forms the message packets $w_1, w_2 ...., w_h$ according to the same rules that the intermediate nodes use. *h* corresponds to the min-cut of the network.

***Operation at intermediate node** v):*

If packet received ***then***

Gaussian elimination is performed with the packets already in the buffer.

for all outgoing edges *i* ***do***

Node *v* chooses all the packets $p_1, p_2, ...p_L$ that are in his buffer.

Form packet $x_1 = \sum_{l=1}^{L} 5 \propto_l \; p_{l,}$ where $\propto_l$ is chosen according to a uniform distribution over the elements of F_q. The packet's global encoding vector μ, which satisfies $x = \sum_{k=1}^{K} 5\, \mu_k \; p_{k,}$ is placed in its header.

Send packet x_i.

end for

end if

Decoding (sink nodes):

if packet received **then**

Gaussian elimination is performed with the packets already in the buffer.

if inverse of the matrix M^l exist **then**

the sink node applies the inverse to the packets to obtain $w_1, w_2...., w_K$;

otherwise, a decoding error occurs

end if

end if

D: Reports of the Minimum Energy Multicast Models

D.1: Example of the first Output

Example of first Output recording the Architectural Information about a Network where Nodes are Randomly Generated with Multicast energy associated with each of the Links

Table D1: Example of the first output

Start Node	End Node	Link Energy
Node 0	Node 3000	7.999703767855737
Node 0	Node 4000	2.3226909086337972
Node 0	Node 6000	8.998671602738929
Node 1000	Node 3000	0.6944542320686703
Node 1000	Node 4000	5.701885762596066
Node 1000	Node 5000	7.091841769898454
Node 1000	Node 6000	8.193081803429669
Node 1000	Node 9000	1.7872074556812065
Node 2000	Node 3000	5.301143792898993
Node 2000	Node 5000	0.28518664810277233
Node 2000	Node 6000	0.10629332279128176
Node 2000	Node 7000	0.4904149453237252
Node 2000	Node 8000	3.278725772577366
Node 3000	Node 0	7.999703767855737
Node 3000	Node 1000	0.6944542320686703
Node 3000	Node 2000	5.301143792898993
Node 3000	Node 4000	4.171349355658838
Node 3000	Node 5000	4.129679453055862
Node 3000	Node 6000	4.430197920493657
Node 3000	Node 7000	7.915587189280515

D.2: Example of the second output

Screen shot of the second output file shows the configuration example of how the Multicast Incremental Power (MIP) algorithm compute the optimum energy from a randomly generated 30 nodes multicasting messages from a source to 3 nodes (receivers) in a 10 x 10m dimension square with radius of connectivity 30cm.

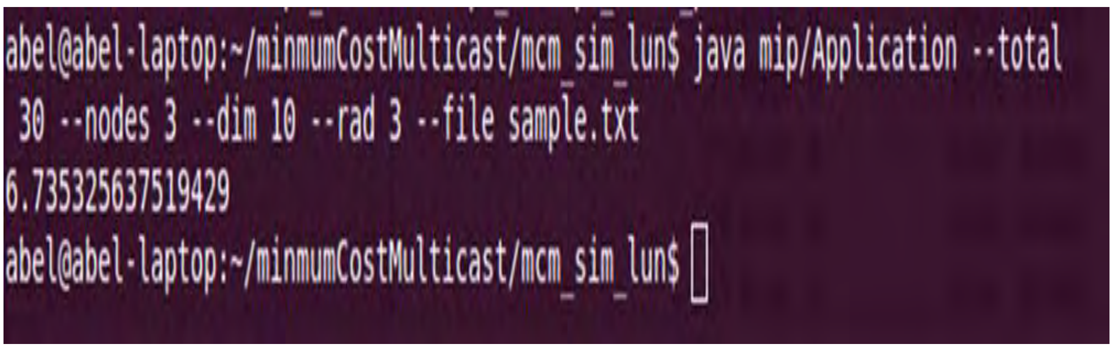

Figure D1: Example of second output

D.3: Example of the third output

Screen shot of the second output file containing the optimal solution of a multicast energy computed according to the RLNC algorithm with the following parameter settings: randomly generated nodes = 30, number of receiving nodes (sinks) = 4, dimensions occupied by the nodes = (10×10)m, radius of connectivity = 30cm.

```
No. Nodes:30 No. Edges: 554 No. Sinks: 4
[Souce Index, Node#]= [3 3000]
[Sink Index, Sink Nodes#]=[1 1000][6 6000][22 22000][29 29000]

Performing wireless optimization:


e:554 t:4, cols:2336, row:4986     0: obj =   0.000000000e+00  infeas =  8.000e+00 (2336)
    200: obj =   0.000000000e+00  infeas =  8.000e+00 (2143)
    400: obj =   0.000000000e+00  infeas =  8.000e+00 (1958)
    600: obj =   0.000000000e+00  infeas =  8.000e+00 (1771)
    800: obj =   0.000000000e+00  infeas =  8.000e+00 (1584)
   1000: obj =   0.000000000e+00  infeas =  8.000e+00 (1411)
   1200: obj =   0.000000000e+00  infeas =  8.000e+00 (1261)
   1400: obj =   1.613375720e+01  infeas =  1.162e-02 (1088)
*  1403: obj =   1.619137844e+01  infeas =  6.261e-15 (1085)
*  1600: obj =   1.264252331e+01  infeas =  1.239e-14 (984)
*  1800: obj =   1.203790289e+01  infeas =  1.718e-14 (909)
*  2000: obj =   1.184378409e+01  infeas =  1.033e-14 (838)
*  2200: obj =   1.155489959e+01  infeas =  5.795e-15 (776)
*  2400: obj =   1.047826908e+01  infeas =  9.589e-15 (735)
*  2600: obj =   9.993285544e+00  infeas =  7.315e-15 (672)
*  2800: obj =   9.631000581e+00  infeas =  1.246e-14 (625)
*  3000: obj =   8.819477233e+00  infeas =  1.390e-14 (578)
*  3200: obj =   8.444464738e+00  infeas =  3.364e-15 (542)
*  3400: obj =   8.076211151e+00  infeas =  7.751e-14 (502)
*  3600: obj =   7.723413810e+00  infeas =  1.018e-14 (477)
*  3800: obj =   7.182126785e+00  infeas =  0.000e+00 (457)
*  4000: obj =   7.182126785e+00  infeas =  0.000e+00 (434)
*  4200: obj =   6.737113718e+00  infeas =  0.000e+00 (416)
*  4400: obj =   6.358039212e+00  infeas =  0.000e+00 (404)
*  4485: obj =   6.358039212e+00  infeas =  2.142e-14 (397)
OPTIMAL SOLUTION FOUND
Z= 6.358039; x1 = 0.000000 x2 = 0.000000 x3 = 0.000000 x4 = 0.000000 x5=0.000000
x6 = 0.000000 x7 = 0.000000 x8 = 0.000000 x9 = 0.000000 x10= 0.000000
```

Figure D2: Example of the third output

E: Multiplier for BCC/VRS Model in Linear Form

The CCR/CRS multiplier model presented in chapter 4 was developed based upon efficiency ratios. In a similar way BCC/VRS multiplier model could be developed. In this section we accomplish this and demonstrate a pair of DEA dual models based upon the linear programming duality. Therefore, the input-oriented CCR/CRS multiplier model (4.4) can be modified as

$$Max\, z = \sum_{r=1}^{s} \mu_r y_{r0} + \mu_0$$

Subject to

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \mu_0 \le 0, \quad j = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1,$$

$$\mu_r, v_i \ge 0,$$

$$\mu_0 \ \textit{free in sign} \qquad \text{(E.1)}$$

where μ_r and v_i are decision variables and represented output and input multipliers respectively. The objective function in model (E.1) is the weighted sum of outputs for DMU_0 under evaluation. The first set of n constraints specifies that each efficiency rating cannot be greater than 1. The constraint $\sum_{i=1}^{m} v_i x_{i0} = 1$ is a normalization condition. The 'free in sign' variable μ_0 is associated with the convexity constraint $\sum \lambda_j = 1$ in the model.

F: Description and classification of network resources/variables

F 1: Description of network resources

Variables	Descriptions
***Input**:* Nodes or Network size (z)	This variable is arbitrarily selected and randomly generated by a simulated method. The simulation method can only present the relationship between this variable and the average power consumed. For instance, Table 3.2 shows that the average power consumed decreases as the network size (nodes) increase. However, the simulation method has fails to present the real optimal value of nodes that is required for evaluation. The numbers of nodes that are randomly generated for this work are 20, 30 and 40 nodes. In DEA, their real optimal values that will maintain the same outputs are determined.
Input: Network dimensions (d)	This variable specifies the dimensions to be occupied by the network nodes. That is, (d) contains the network size (z). In the simulation method, network dimensions are selected arbitrarily as the impact caused by the power consumption was not established. In order to experiment the level of impact with other variables, two types of dimensions are considered: $10 \times 10m$ and $50 \times 50cm$. The DEA method is able to determine the optimal value of this variable that contributes to the overall performance of the network.
***Input**:* Radius of connectivity (r):	This variable is the distance among the nodes. That is, (r) is the radius that connects each of the elements in (z) together. Also, the simulation method assumed arbitrary radius of connectivity and failed to establish any impact with the power consumed. In order to determine the (r) impact with other variables, we consider two types of radii of connectivity, which are *30cm* and *50cm*. DEA method find the optimal value of (r) and its contribution to the overall optimal performance of the networks.
***Output**:* Sinks or Multicast group (g)	This variable specifies the exact number of nodes involved with the multicast communication. Also (g) can be defined as a subset of (z). We intend to maintain the number of multicast nodes while minimising average power consumed by the nodes. As it could be observed from Table 3.2, the simulation method can only show the relationship between the multicast nodes and the average power consumption by multicast group for example, the table shows that the average power increases as the multicast group (sink) increases. The multicast group considered in this work ranges from 2 to 10.
***Input**:* Multicast energy (e):	This variable presents the average power needed by the network to multicast a message from a source node to a certain multicast group. That is, it is the average power required by network size (z) for multicast group (g) to be successful. We assume that each of the network size (z) aware of the average power to consume so as to achieve (g) operation. The DEA then attempts to minimise (e) and other variables using appropriate empirical DEA models.

G: Convex combination and LP for TE and Slack of 54 ad hoc wireless networks

G.1 TE: Weights Assumptions for 54 Ad Hoc Wireless Networks

In order to formulate appropriate model for 54 DMUs, the following unknown weights are assumed: λ_{A1}, λ_{B1}, λ_{C1}, $\lambda_{D1,}$ λ_{E1}, λ_{F1}, λ_{G1}, $\lambda_{H1,}$ λ_{I1}, λ_{J1}, λ_{K1}, $\lambda_{L1,}$ λ_{M1}, λ_{N1}, λ_{O1}, $\lambda_{P1,}$ λ_{Q1}, λ_{R1}, λ_{S1}, $\lambda_{T1,}$ λ_{U1}, λ_{V1}, λ_{W1}, $\lambda_{X1,}$ λ_{Y1}, λ_{Z1}, λ_{A2}, λ_{B2}, λ_{C2}, $\lambda_{D2,}$ λ_{E2}, λ_{F2}, λ_{G2}, $\lambda_{H2,}$ λ_{I2}, λ_{J2}, λ_{K2}, $\lambda_{L2,}$ λ_{M2}, λ_{N2}, λ_{O2}, $\lambda_{P2,}$ λ_{Q2}, λ_{R2}, λ_{S2}, $\lambda_{T2,}$ λ_{U2}, λ_{V2}, λ_{W2}, $\lambda_{X2,}$ λ_{Y2}, λ_{Z2}, λ_{A3}, and λ_{B3}. In order to construction the virtual DMUs, these unknown weights must satisfy the following condition: $\lambda_{A1} + \lambda_{B1} + \lambda_{C1} + \lambda_{D1} + \lambda_{E1} + \lambda_{F} + \lambda_{G1} + \lambda_{H1} + \lambda_{I1} + \lambda_{J1} + \lambda_{K1} + \lambda_{L1} + \lambda_{M1} + \lambda_{N1} + \lambda_{O1} + \lambda_{P1} + \lambda_{Q1} + \lambda_{R1} + \lambda_{S1} + \lambda_{T1} + \lambda_{U1} + \lambda_{V1} + \lambda_{W1} + \lambda_{X1} + \lambda_{Y1} + \lambda_{Z1} + \lambda_{A2} + \lambda_{B2} + \lambda_{C2} + \lambda_{D2} + \lambda_{E2} + \lambda_{F2} + \lambda_{G2} + \lambda_{H2} + \lambda_{I2} + \lambda_{J2} + \lambda_{K2} + \lambda_{L2} + \lambda_{M2} + \lambda_{N2} + \lambda_{O2} + \lambda_{P2} + \lambda_{Q2} + \lambda_{R2} + \lambda_{S2} + \lambda_{T2} + \lambda_{U2} + \lambda_{V2} + \lambda_{W2} + \lambda_{X2} + \lambda_{Y2} + \lambda_{Z2} + \lambda_{A3} + \lambda_{B3} = 1$. Note that virtual DMU is a convex combination of existing DMUs and the number of weights required is determined by the number of observed DMUs. Therefore, the inputs and outputs of the virtual DMU are convex combination of related inputs and outputs for observed DMUs. So when the efficient DMUs are identified, the best virtual DMU or the target is then a convex combination of the identified efficient DMUs.

Convex Combination of four Inputs and one Output Values

Average energy (Input): $= 4.50027\lambda_{A1} + 5.46086\lambda_{B1} + 6.22791\lambda_{C1} + 6.81511\lambda_{D1} + 7.32855\lambda_{E1} + 7.23365\lambda_{F1} + 8.10404\lambda_{G1} + 8.81448\lambda_{H1} + 8.45438\lambda_{I1} + 5.19479\lambda_{J1} + 5.55607\lambda_{K1} + 6.28641\lambda_{L1} + 6.85942\lambda_{M1} + 7.12087\lambda_{N1} + 7.18488\lambda_{O1} + 7.73925\lambda_{P1} + 8.56634\lambda_{Q1} + 8.33395\lambda_{R1} + 4.15490\lambda_{S1} + 5.30356\lambda_{T1} + 5.35979\lambda_{U1} + 6.07549\lambda_{V1} + 6.18796\lambda_{W1} + 6.37327\lambda_{X1} + 6.57230\lambda_{Y1} + 7.34824\lambda_{Z1} + 6.74705\lambda_{A2} + 3.60785\lambda_{B2} + 5.10020\lambda_{C2} + 5.56776\lambda_{D2} + 5.87098\lambda_{E2} + 6.09464\lambda_{F2} + 6.76687\lambda_{G2} + 6.62772\lambda_{H2} + 7.14271\lambda_{I2} + 7.12791\lambda_{J2} + 3.14390\lambda_{K2} + 4.60581\lambda_{L2} + 4.75666\lambda_{M2} + 4.75814\lambda_{N2} + 5.56181\lambda_{O2} + 5.58696\lambda_{P2} + 6.25809\lambda_{Q2} + 6.29795\lambda_{R2} + 6.30145\lambda_{S2} + 3.62417\lambda_{T2} + 4.31278\lambda_{U2} + 5.06807\lambda_{V2} + 5.12135\lambda_{W2} + 5.45237\lambda_{X2} + 5.74148\lambda_{Y2} + 6.43736\lambda_{Z2} + 6.42996\lambda_{A3} + 6.50145\lambda_{B3}$.

Dimension (Input): $= 10\lambda_{A1} + 10\lambda_{B1} + 10\lambda_{C1} + 10\lambda_{D1} + 10\lambda_{E1} + 10\lambda_{F1} + 10\lambda_{G1} + 10\lambda_{H1} + 10\lambda_{I1} + 50\lambda_{J1} + 50\lambda_{K1} + 50\lambda_{L1} + 50\lambda_{M1} + 50\lambda_{N1} + 50\lambda_{O1} + 50\lambda_{P1} + 50\lambda_{Q1} + 50\lambda_{R1} + 10\lambda_{S1} + 10\lambda_{T1} + 10\lambda_{U1} + 10\lambda_{V1} + 10\lambda_{W1} + 10\lambda_{X1} + 10\lambda_{Y1} + 10\lambda_{Z1} + 10\lambda_{A2} + 50\lambda_{B2} + 50\lambda_{C2} + 50\lambda_{D2} + 50\lambda_{E2} + 50\lambda_{F2} + 50\lambda_{G2} + 50\lambda_{H2} + 50\lambda_{I2} + 50\lambda_{J2} + 10\lambda_{K2} + 10\lambda_{L2} + 10\lambda_{M2} + 10\lambda_{N2} + 10\lambda_{O2} + 10\lambda_{P2} + 10\lambda_{Q2} + 10\lambda_{R2} + 10\lambda_{S2} + 50\lambda_{T2} + 50\lambda_{U2} + 50\lambda_{V2} + 50\lambda_{W2} + 50\lambda_{X2} + 50\lambda_{Y2} + 50\lambda_{Z2} + 50\lambda_{A3} + 50\lambda_{B3}$.

Radius of connectivity (Input): $= 3\lambda_{A1} + 3\lambda_{B1} + 3\lambda_{C1} + 3\lambda_{D1} + 3\lambda_{E1} + 3\lambda_{F1} + 3\lambda_{G1} + 3\lambda_{H1} + 3\lambda_{I1} + 5\lambda_{J1} + 5\lambda_{K1} + 5\lambda_{L1} + 5\lambda_{M1} + 5\lambda_{N1} + 5\lambda_{O1} + 5\lambda_{P1} + 5\lambda_{Q1} + 5\lambda_{R1} + 3\lambda_{S1} + 3\lambda_{T1} + 3\lambda_{U1} + 3\lambda_{V1} + 3\lambda_{W1} + 3\lambda_{X1} + 3\lambda_{Y1} + 3\lambda_{Z1} + 3\lambda_{A2} + 5\lambda_{B2} + 5\lambda_{C2} + 5\lambda_{D2} + 5\lambda_{E2} + 5\lambda_{F2} + 5\lambda_{G2} + 5\lambda_{H2} + 5\lambda_{I2} + 5\lambda_{J2} + 3\lambda_{K2} + 3\lambda_{L2} + 3\lambda_{M2} + 3\lambda_{N2} + 3\lambda_{O2} + 3\lambda_{P2} + 3\lambda_{Q2} + 3\lambda_{R2} + 3\lambda_{S2} + 5\lambda_{T2} + 5\lambda_{U2} + 5\lambda_{V2} + 5\lambda_{W2} + 5\lambda_{X2} + 5\lambda_{Y2} + 5\lambda_{Z2} + 5\lambda_{A3} + 5\lambda_{B3}$.

Nodes (Input): $= 20\lambda_{A1} + 20\lambda_{B1} + 20\lambda_{C1} + 20\lambda_{D1} + 20\lambda_{E1} + 20\lambda_{F1} + 20\lambda_{G1} + 20\lambda_{H1} + 20\lambda_{I1} + 20\lambda_{J1} + 20\lambda_{K1} + 20\lambda_{L1} + 20\lambda_{M1} + 20\lambda_{N1} + 20\lambda_{O1} + 20\lambda_{P1} + 20\lambda_{Q1} + 20\lambda_{R1} + 30\lambda_{S1} + 30\lambda_{T1} + 30\lambda_{U1} + 30\lambda_{V1} + 30\lambda_{W1} + 30\lambda_{X1} + 30\lambda_{Y1} + 30\lambda_{Z1} + 30\lambda_{A2} + 30\lambda_{B2} + 30\lambda_{C2} + 30\lambda_{D2} + 30\lambda_{E2} + 30\lambda_{F2} + 30\lambda_{G2} + 30\lambda_{H2} + 30\lambda_{I2} + 30\lambda_{J2} + 40\lambda_{K2} + 40\lambda_{L2} + 40\lambda_{M2} + 40\lambda_{N2} + 40\lambda_{O2} + 40\lambda_{P2} + 40\lambda_{Q2} + 40\lambda_{R2} + 40\lambda_{S2} + 40\lambda_{T2} + 40\lambda_{U2} + 40\lambda_{V2} + 40\lambda_{W2} + 40\lambda_{X2} + 40\lambda_{Y2} + 40\lambda_{Z2} + 40\lambda_{A3} + 40\lambda_{B3}$.

Sinks (Output): $= 2\lambda_{A1} + 3\lambda_{B1} + 4\lambda_{C1} + 5\lambda_{D1} + 6\lambda_{E1} + 7\lambda_{F1} + 8\lambda_{G1} + 9\lambda_{H1} + 10\lambda_{I1} + 2\lambda_{J1} + 3\lambda_{K1} + 4\lambda_{L1} + 5\lambda_{M1} + 6\lambda_{N1} + 7\lambda_{O1} + 8\lambda_{P1} + 9\lambda_{Q1} + 10\lambda_{R1} + 2\lambda_{S1} + 3\lambda_{T1} + 4\lambda_{U1} + 5\lambda_{V1} + 6\lambda_{W1} + 7\lambda_{X1} + 8\lambda_{Y1} + 9\lambda_{Z1} + 10\lambda_{A2} + 2\lambda_{B2} + 3\lambda_{C2} + 4\lambda_{D2} + 5\lambda_{E2} + 6\lambda_{F2} + 7\lambda_{G2} + 8\lambda_{H2} + 9\lambda_{I2} + 10\lambda_{J2} + 2\lambda_{K2} + 3\lambda_{L2} + 4\lambda_{M2} + 5\lambda_{N2} + 6\lambda_{O2} + 7\lambda_{P2} + 8\lambda_{Q2} + 9\lambda_{R2} + 10\lambda_{S2} + 2\lambda_{T2} + 3\lambda_{U2} + 4\lambda_{V2} + 5\lambda_{W2} + 6\lambda_{X2} + 7\lambda_{Y2} + 8\lambda_{Z2} + 9\lambda_{A3} + 10\lambda_{B3}$.

The virtual DMUs are obtained by varying the values of λ_{A1}, λ_{B1}, λ_{C1}, λ_{D1}, λ_{E1}, λ_{F1}, λ_{G1}, λ_{H1}, λ_{I1}, λ_{J1}, λ_{K1}, λ_{L1}, λ_{M1}, λ_{N1}, λ_{O1}, λ_{P1}, λ_{Q1}, λ_{R1}, λ_{S1}, λ_{T1}, λ_{U1}, λ_{V1}, λ_{W1}, λ_{X1}, λ_{Y1}, λ_{Z1}, λ_{A2}, λ_{B2}, λ_{C2}, λ_{D2}, λ_{E2}, λ_{F2}, λ_{G2}, λ_{H2}, λ_{I2}, λ_{J2}, λ_{K2}, λ_{L2}, λ_{M2}, λ_{N2}, λ_{O2}, λ_{P2}, λ_{Q2}, λ_{R2}, λ_{S2}, λ_{T2}, λ_{U2}, λ_{V2}, λ_{W2}, λ_{X2}, λ_{Y2}, λ_{Z2}, λ_{A3}, and λ_{B3}.

Since the interest is whether a real DMU can still increase its output(s) or decrease its input(s) when compared to the best virtual DMU, then we assumed the following:

(i) The virtual input levels for the virtual DMU are always less than or equal to the real input levels for the real DMU.

(ii) The virtual output levels are always greater than or equal to the real levels.

LP Formulation for each of the ad hoc wireless network

This section presents the virtual formulation procedures for all the DMUs beginning with DMU_1 (See Table 4.1 for the data set). So if we apply model (4.6) to DMU_1, we have:

Min θ

Subject to

$4.50027\ \lambda_{A1} + 5.46086\lambda_{B1} + \cdots + 6.42996\lambda_{A3} + 6.50145\ \lambda_{B3} \leq 4.50027 \times \theta$

$10\lambda_{A1} + 10\lambda_{B1} + \cdots + 50\lambda_{A3} + 50\lambda_{B3} \leq 10 \times \theta$

$3\lambda_{A1} + 3\lambda_{B1} + \cdots + 5\lambda_{A3} + 5\lambda_{B3} \leq 3 \times \theta$

$20\lambda_{A1} + 20\lambda_{B1} + \cdots + 40\lambda_{A3} + 40\lambda_{B3} \leq 20 \times \theta$

$2\lambda_{A1} + 3\lambda_{B1} + \cdots + 9\lambda_{A3} + 10\lambda_{B3} \geq 2$

$\lambda_{A1} + \lambda_{B1} + \cdots + \lambda_{A3} + \lambda_{B3} = 1$

$\lambda_{A1}, \lambda_{B1}, \cdots, \lambda_{A3}, \lambda_{B3} \geq 0.$

Similarly, for DMU_2 we have:

Min θ

Subject to:

$4.50027\ \lambda_{A1} + 5.46086\lambda_{B1} + \cdots + 6.42996\lambda_{A3} + 6.50145\ \lambda_{B3} \leq 5.46086 \times \theta$

$10\lambda_{A1} + 10\lambda_{B1} + \cdots + 50\lambda_{A3} + 50\lambda_{B3} \leq 10 \times \theta$

$3\lambda_{A1} + 3\lambda_{B1} + \cdots + 5\lambda_{A3} + 5\lambda_{B3} \leq 3 \times \theta$

$20\lambda_{A1} + 20\lambda_{B1} + \cdots + 40\lambda_{A3} + 40\lambda_{B3} \leq 20 \times \theta$

$2\lambda_{A1} + 3\lambda_{B1} + \cdots + 9\lambda_{A3} + 10\lambda_{B3} \geq 3$

$\lambda_{A1} + \lambda_{B1} + \cdots + \lambda_{A3} + \lambda_{B3} = 1$

$\lambda_{A1}, \lambda_{B1}, \cdots, \lambda_{A3}, \lambda_{B3} \geq 0.$

and for DMU_{30} we have:

Min θ

Subject to

$4.50027\,\lambda_{A1} + 5.46086\lambda_{B1} + \cdots + 6.42996\lambda_{A3} + 6.50145\,\lambda_{B3} \leq 5.56776 \times \theta$

$10\lambda_{A1} + 10\lambda_{B1} + \cdots + 50\lambda_{A3} + 50\lambda_{B3} \leq 50 \times \theta$

$3\lambda_{A1} + 3\lambda_{B1} + \cdots + 5\lambda_{A3} + 5\lambda_{B3} \leq 5 \times \theta$

$20\lambda_{A1} + 20\lambda_{B1} + \cdots + 40\lambda_{A3} + 40\lambda_{B3} \leq 30 \times \theta$

$2\lambda_{A1} + 3\lambda_{B1} + \cdots + 9\lambda_{A3} + 10\lambda_{B3} \geq 4$

$\lambda_{A1} + \lambda_{B1} + \cdots + \lambda_{A3} + \lambda_{B3} = 1$

$\lambda_{A1}, \lambda_{B1}, \cdots, \lambda_{A3}, \lambda_{B3} \geq 0.$

Similar formulation can be constructed for other DMUs. Note that λ_{A1}, λ_{B1}, λ_{C1},$\cdots$, λ_{A3}, λ_{B3} are decision variables and θ is the objective function. These models are solved using DEA solver.

G.2: Slacks Formulation for Input-oriented DMUs

The models that are considered in this work attempt to reduce the current levels of inputs while maintain the current output levels. This concept known as input-orientation is analytically presented in this section. In order to achieve this, slacks procedures are required. Note that the inefficient DMUs need a target to make them efficient. Let us consider the case of DMU_2 which is an example of inefficient DMU. Then, the procedure is as follows:

DMU_2:

$\max s_1^- + s_2^- + s_3^- + s_4^- + s_1^+$

Subject to

$4.50027\,\lambda_{A1} + 5.46086\lambda_{B1} + \cdots + 6.42996\lambda_{A3} + 6.50145\,\lambda_{B3} + s_1^- = 5.46086 \times \theta^*$

$10\lambda_{A1} + 10\lambda_{B1} + \cdots + 50\lambda_{A3} + 50\lambda_{B3} + s_2^- = 10 \times \theta^*$

$3\lambda_{A1} + 3\lambda_{B1} + \cdots + 5\lambda_{A3} + 5\lambda_{B3} + s_3^- = 3 \times \theta^*$

$20\lambda_{A1} + 20\lambda_{B1} + \cdots + 40\lambda_{A3} + 40\lambda_{B3} + s_4^- = 20 \times \theta^*$

$2\lambda_{A1} + 3\lambda_{B1} + \cdots + 9\lambda_{A3} + 10\lambda_{B3} + s_1^+ = 3$

$\lambda_{A1} + \lambda_{B1} + \cdots + \lambda_{A3} + \lambda_{B3} = 1$

$\lambda_{A1}, \lambda_{B1}, \cdots, \lambda_{A3}, \lambda_{B3}, s_1^-, s_2^-, s_3^-, s_4^-, s_1^+ \geq 0$

Similar formulation could be constructed for other DMUs that are inefficient or weak efficienct.

H: Reports of DEA Models Extracted from the Excel Sheets

H.1: Efficiency scores and input slacks reports for input-oriented CCR/CRS DEA model

H 1: Efficiency scores and slacks reports for input-oriented CCR/CRS DEA model

	Measurements			Input Slacks			
DMU	Efficiency Scores	TE (%)	TIE (%)	Ave. energy	Dimension	Radius	Nodes
DMU_{1}	0.2999126	30.0	70.0	0	0.9921306	0.2993879	0
DMU_{2}	0.4008414	40.1	59.9	0	0	0	0
DMU_{3}	0.4920725	49.2	50.8	0	0	0	0
DMU_{4}	0.5798874	58.0	42.0	0	0	0	0
DMU_{5}	0.6627010	66.3	33.7	0	0	0	0
DMU_{6}	0.7800223	78.0	22.0	0	0	0	0
DMU_{7}	0.8242685	82.4	17.6	0	0	0	0
DMU_{8}	0.900000	90.0	10.0	0.32409	0	0	0
DMU_{9}	1	100.0	0.0	0	0	0	0
DMU_{10}	0.2750224	27.5	72.5	0	9.7529166	0	0
DMU_{11}	0.3954612	39.5	60.5	0	12.409958	0	0
DMU_{12}	0.4865748	48.7	51.3	0	11.254727	0	0
DMU_{13}	0.5734824	57.3	42.7	0	9.5527109	0	0
DMU_{14}	0.6707014	67.1	32.9	0	9.1911873	0	0
DMU_{15}	0.7776498	77.8	22.2	0	10.094469	0	0
DMU_{16}	0.8435955	84.4	15.6	0	5.6674165	0	0
DMU_{17}	0.9000000	90.0	10.0	0.209151	0	0	0
DMU_{18}	1	100.0	0.0	0	0	0	0
DMU_{19}	0.3033262	30.3	69.7	0	0	0.3099786	0
DMU_{20}	0.3652144	36.5	63.5	0	0	0.1956432	0
DMU_{21}	0.4828637	48.3	51.7	0	0	0.2485910	0
DMU_{22}	0.5453004	54.5	45.5	0	0	0.1359013	0
DMU_{23}	0.6445800	64.5	35.5	0	0	0.1337401	0
DMU_{24}	0.7339356	73.4	26.6	0	0	0	0
DMU_{25}	0.8176758	81.8	18.2	0	0	0	0
DMU_{26}	0.9000000	90.0	10.0	0.541071	0	0	0
DMU_{27}	1	100.0	0.0	0	0	0	0
DMU_{28}	0.3493188	34.9	65.1	0	15.465942	0	2.4795654
DMU_{29}	0.3767524	37.7	62.3	0	0	0	0
DMU_{30}	0.4683195	46.8	53.2	0	19.415974	0	0
DMU_{31}	0.5607725	56.1	43.9	0	0	0	0
DMU_{32}	0.6526743	65.3	34.7	0	26.633716	0	0
DMU_{33}	0.6987965	69.9	30.1	0	27.795397	0	0
DMU_{34}	0.8119862	81.2	18.8	0	32.599308	0	0
DMU_{35}	0.8700847	87.0	13.0	0	30.914400	0	0
DMU_{36}	0.9679643	96.8	3.2	0	34.553930	0	0
DMU_{37}	0.4008683	40.1	59.9	0	0	0.6026050	8.0347339
DMU_{38}	0.4104457	41.0	59.0	0	0	0.3313372	4.4178288
DMU_{39}	0.5299054	53.0	47.0	0	0	0.3897163	5.1962175
DMU_{40}	0.6621758	66.2	33.8	0	0	0.4865273	6.4870306
DMU_{41}	0.6797913	68.0	32.0	0	0	0.2393739	3.1916516
DMU_{42}	0.7895197	79.0	21.0	0	0	0.2685591	3.5807881
DMU_{43}	0.8055429	80.6	19.4	0	0	0.0166287	0.2217162
DMU_{44}	0.9005002	90.1	9.9	0	0	0.0015005	0.0200065
DMU_{45}	1	100.0	0.0	0	0	0	0
DMU_{46}	0.3477458	34.8	65.2	0	15.387291	0	5.9098331
DMU_{47}	0.4383333	43.8	56.2	0	0	0	5.5333312
DMU_{48}	0.4973451	49.7	50.3	0	20.867257	0	3.8938057
DMU_{49}	0.6152138	61.5	38.5	0	25.760688	0	4.6085505
DMU_{50}	0.6934361	69.3	30.7	0	28.671803	0	3.7374426
DMU_{51}	0.7682714	76.8	23.2	0	31.413571	0	2.7308569
DMU_{52}	0.7867724	78.7	21.3	0	31.338618	0	0
DMU_{53}	0.8859165	88.6	11.4	0	35.295824	0	0
DMU_{54}	0.9758566	97.6	2.4	0	0	0	0

H.2: Efficient scores and input slacks report for input-oriented BCC/VRS DEA model

H 2: Efficient scores and slacks report for input-oriented BCC/VRS DEA model

	Measurements	Input Slacks			
DMUs	Efficiency Scores (θ)	Ave. energy (e)	Dimension (d)	Radius (r)	Nodes (z)
DMU_1	1	0	0	0	0
DMU_2	1	0.46632625	0	0	0
DMU_3	1	0.73911250	0	0	0
DMU_4	1	0.83204875	0	0	0
DMU_5	1	0.85122500	0	0	0
DMU_6	1	0.26206125	0	0	0
DMU_7	1	0.63818750	0	0	0
DMU_8	1	0.85436375	0	0	0
DMU_9	1	0	0	0	0
DMU_{10}	1	0.69452000	40.000000	0	0
DMU_{11}	1	0.57659000	35.000000	0	0
DMU_{12}	1	0	40.000000	0	0
DMU_{13}	1	0.92152000	25.000000	1.250000	0
DMU_{14}	1	0.70376000	20.000000	0	0
DMU_{15}	1	0.28856000	15.000000	0	0
DMU_{16}	1	0.36372000	10.000000	0	0
DMU_{17}	1	0.71160000	5.0000000	0.250000	0
DMU_{18}	1	0	0	0	0
DMU_{19}	1	0.33281500	0	0	0
DMU_{20}	1	0	0	0	9.465288
DMU_{21}	1	0.297825000	0	0	7.500000
DMU_{22}	1	0.732677500	0	0	6.250000
DMU_{23}	1	0.564300000	0	0	5
DMU_{24}	1	0.723081880	0	0	0
DMU_{25}	1	0.556491250	0	0	0
DMU_{26}	1	0.966810630	0	0	0
DMU_{27}	1	0	0	0	0
DMU_{28}	1	0	0	0	0
DMU_{29}	0.858572	0	0	0.391428	0
DMU_{30}	0.857476	0	0	0.642524	0
DMU_{31}	0.872964	0	23.892485	0.877036	0
DMU_{32}	0.895440	0	27.319201	0	0
DMU_{33}	0.873568	0	33.506615	0	0
DMU_{34}	0.928482	0	35.006233	0	0
DMU_{35}	0.926503	0	32.505438	0	0
DMU_{36}	0.967964	0	34.553930	0	0
DMU_{37}	1	0	0	0	0
DMU_{38}	1	0	0	0	15.736360
DMU_{39}	1	0	0	0	12.140820
DMU_{40}	1	0.263058750	0	0	3.7500000
DMU_{41}	1	0.616335000	0	0	5
DMU_{42}	1	0.469591250	0	0	0
DMU_{43}	1	0.746027500	0	0	0
DMU_{44}	1	0.391193750	0	0	0
DMU_{45}	1	0	0	0	0
DMU_{46}	0.912359	0	21.595392	0.860667	0
DMU_{47}	0.874123	0	18.565910	0.613604	0
DMU_{48}	0.835443	0	15.443069	0.360761	0
DMU_{49}	0.885304	0	30.913876	1.258954	0
DMU_{50}	0.899141	0	34.957027	0	0
DMU_{51}	0.917049	0	35.852464	0	0
DMU_{52}	0.887430	0	34.371505	0	0
DMU_{53}	0.936291	0	36.814535	0	0
DMU_{54}	0.975857	0	38.792832	0	0

H.3: Efficiency reference set (ERS) and Lambdas calculated by input-oriented variable-benchmark CCR/CRS DEA model

H 3: ERS and Lambdas by input-oriented variable-benchmark CCR/CRS DEA model

DMUs	Efficiency Reference Set (ERS)	Lambdas Values (%)		
DMU_1	DMU18, DMU27	0.020	19.98	
DMU_2	DMU9, DMU18, DMU27	7.310	2.520	20.17
DMU_3	DMU9, DMU18, DMU27	19.28	2.300	18.41
DMU_4	DMU9, DMU18, DMU27	32.03	2.000	15.98
DMU_5	DMU9, DMU18, DMU27	45.89	1.570	12.54
DMU_6	DMU9, DMU18, DMU27	51.99	2.000	16.00
DMU_7	DMU9, DMU18, DMU27	74.54	0.610	4.850
DMU_8	DMU9	90.00		
DMU_9	DMU9	100.0		
DMU_{10}	DMU18, DMU27	5.000	15.00	
DMU_{11}	DMU18, DMU27	10.91	19.09	
DMU_{12}	DMU18, DMU27	22.69	17.31	
DMU_{13}	DMU18, DMU27	35.30	14.70	
DMU_{14}	DMU18, DMU27	45.86	14.14	
DMU_{15}	DMU18, DMU27	54.47	15.53	
DMU_{16}	DMU18, DMU27	71.28	8.720	
DMU_{17}	DMU9	90.00		
DMU_{18}	DMU18	100.0		
DMU_{19}	DMU45	20.00		
DMU_{20}	DMU27, DMU45	10.44	19.56	
DMU_{21}	DMU27, DMU45	15.14	24.86	
DMU_{22}	DMU27, DMU45	36.41	13.59	
DMU_{23}	DMU27, DMU45	46.63	13.37	
DMU_{24}	DMU27, DMU45	59.82	10.18	
DMU_{25}	DMU27, DMU45	74.70	5.300	
DMU_{26}	DMU27, DMU45	31.69	58.31	
DMU_{27}	DMU27	100.0		
DMU_{28}	DMU45	20.00		
DMU_{29}	DMU27, DMU45	6.970	23.03	
DMU_{30}	DMU27, DMU45	19.50	20.50	
DMU_{31}	DMU27, DMU45	31.77	18.23	
DMU_{32}	DMU27, DMU45	44.20	15.80	
DMU_{33}	DMU18, DMU27	0.360	69.64	
DMU_{34}	DMU27, DMU45	76.40	3.600	
DMU_{35}	DMU18, DMU27	8.970	81.03	
DMU_{36}	DMU18, DMU27	9.610	90.39	
DMU_{37}	DMU45	20.00		
DMU_{38}	DMU45	30.00		
DMU_{39}	DMU45	40.00		
DMU_{40}	DMU45	50.00		
DMU_{41}	DMU45	60.00		
DMU_{42}	DMU45	70.00		
DMU_{43}	DMU45	80.00		
DMU_{44}	DMU45	90.00		
DMU_{45}	DMU45	100.00		
DMU_{46}	DMU45	20.00		
DMU_{47}	DMU45	30.00		
DMU_{48}	DMU45	40.00		
DMU_{49}	DMU45	50.00		
DMU_{50}	DMU45	60.00		
DMU_{51}	DMU45	70.00		
DMU_{52}	DMU27, DMU45	5.290	74.71	
DMU_{53}	DMU27, DMU45	5.630	84.37	
DMU_{54}	DMU27, DMU45	9.660	90.34	

H.4: Efficiency reference set (ERS) and Lambdas calculated by input-oriented variable-benchmark BCC/VRS DEA model

H 4: ERS and Lambdas by input-oriented variable-benchmark BCC/VRS DEA model

	Peer group	Lambda Values (%)		
DMU_1	DMU_1	100.0		
DMU_2	DMU_1, DMU_9	87.50	12.5	
DMU_3	DMU_1, DMU_9	75.00	25.0	
DMU_4	DMU_1, DMU_9	62.50	37.5	
DMU_5	DMU_1, DMU_9	50.00	50.0	
DMU_6	DMU_1, DMU_9	37.50	62.5	
DMU_7	DMU_1, DMU_9	25.00	75.0	
DMU_8	DMU_1, DMU_9	12.50	87.5	
DMU_9	DMU_9	100.0		
DMU_{10}	DMU_1	100.0		
DMU_{11}	DMU_1,DMU_{18}	87.50	12.5	
DMU_{12}	DMU_1, DMU_9	75.00	25.0	
DMU_{13}	DMU_1,DMU_{18}	62.50	37.5	
DMU_{14}	DMU_1,DMU_{18}	50.00	50.0	
DMU_{15}	DMU_1,DMU_{18}	37.50	62.5	
DMU_{16}	DMU_1,DMU_{18}	25.00	75.0	
DMU_{17}	DMU_1,DMU_{18}	12.50	87.5	
DMU_{18}	DMU_{18}	100.0		
DMU_{19}	DMU_1, DMU_{37}	50.00	50.0	
DMU_{20}	DMU_1, DMU_{45}	18.70	81.3	
DMU_{21}	DMU_1, DMU_{27}	75.00	25.0	
DMU_{22}	DMU_1, DMU_{27}	62.50	37.5	
DMU_{23}	DMU_1, DMU_{27}	50.00	50.0	
DMU_{24}	DMU_1, DMU_{27}, DMU_{37}	18.80	62.5	18.8
DMU_{25}	DMU_1, DMU_{27}, DMU_{37}	12.50	75.0	12.5
DMU_{26}	DMU_1, DMU_{27}, DMU_{37}	6.300	87.5	6.30
DMU_{27}	DMU_{27}	100.0		
DMU_{28}	DMU_{28}	100.0		
DMU_{29}	DMU_1, DMU_{27}, DMU_{28}	42.40	12.5	45.1
DMU_{30}	DMU_1, DMU_{27}, DMU_{28}	42.80	25.0	32.2
DMU_{31}	DMU_1, DMU_{27}, DMU_{28}	38.10	37.5	24.4
DMU_{32}	DMU_1, DMU_{27}, DMU_{28}	31.40	50.0	18.6
DMU_{33}	DMU_1, DMU_{18}, DMU_{27}	37.50	0.40	62.1
DMU_{34}	DMU_1, DMU_{27}, DMU_{28}	21.50	75.0	3.50
DMU_{35}	DMU_1, DMU_{18}, DMU_{27}	12.50	9.50	78.0
DMU_{36}	DMU_{18}, DMU_{27}	9.600	90.4	
DMU_{37}	DMU_{37}	100.0		
DMU_{38}	DMU_1, DMU_{37}, DMU_{45}	78.70	8.80	12.5
DMU_{39}	DMU_1, DMU_{37}, DMU_{45}	60.70	14.3	25.0
DMU_{40}	DMU_{27}, DMU_{37}	37.50	62.5	
DMU_{41}	DMU_{27}, DMU_{37}	50.00	50.0	
DMU_{42}	DMU_{37}, DMU_{45}	37.50	62.5	
DMU_{43}	DMU_{37}, DMU_{45}	25.00	75.0	
DMU_{44}	DMU_{37}, DMU_{45}	12.50	87.5	
DMU_{45}	DMU_{45}	100.0		
DMU_{46}	DMU_{28}, DMU_{37}	35.10	64.9	
DMU_{47}	DMU_{27}, DMU_{28}, DMU_{37}	12.50	37.9	49.6
DMU_{48}	DMU_{27}, DMU_{28}, DMU_{37}	25.00	40.8	34.2
DMU_{49}	DMU_{27}, DMU_{28}, DMU_{37}	37.50	8.40	54.1
DMU_{50}	DMU_{27}, DMU_{37}, DMU_{45}	40.30	50.0	9.70
DMU_{51}	DMU_{27}, DMU_{37}, DMU_{45}	33.20	37.5	29.3
DMU_{52}	DMU_{27}, DMU_{37}, DMU_{45}	45.00	25.0	30.0
DMU_{53}	DMU_{27}, DMU_{37}, DMU_{45}	25.50	12.5	62.0
DMU_{54}	DMU_{27}, DMU_{45}	9.700	90.3	

F: Accompanying CD-ROM

The thesis submission includes a CD-ROM that contains the following information:

- Source code for the MIP, the coded packet networks and related tools for its complete installation.
- GLPK Open Source for Linear Programing Optimization.
- README document on how to install and use the open source software.
- README document on DEA and how to use the DEA Solver with some screen shoot illustration.
- Source code for the scripts to perform important actions and to mine the data used in DEA solver.
- Evaluation Results – Raw data collection during performance evaluations.
- Published Articles – Collection of published papers resulting from this work.
- Thesis Documents –Portable Document Format (PDF) copies of the main thesis document.