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LOW VOLATILITY ALTERNATIVE EQUITY INDICES



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By

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Dedication
To
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THE OLADELES

Abstract

In recent years, there has been an increasing interest in constructing low volatility portfolios. These portfolios have shown significant outperformance when compared with the market capitalization-weighted portfolios. This study analyses the low volatility portfolios in South Africa using sectors instead of individual stocks as building blocks for portfolio construction. The empirical results from back-testing these portfolios show significant outperformance when compared with their market capitalization weighted equity benchmark counterpart (ALSI). In addition, a further analysis of this study delves into the construction of the low volatility portfolios using the Top 40 and Top 100 stocks. The results also show significant outperformance over the market-capitalization portfolio (ALSI), with the portfolios constructed using the Top 100 stocks having a better performance than portfolio constructed using the Top 40 stocks. Finally, the low volatility portfolios are also blended with typical portfolios (ALSI and the SWIX indices) in order to establish their usefulness as effective portfolio strategies. The results show that the Low volatility Single Index Model (SIM) and the Equally Weight low-beta portfolio (Lowbeta) were the superior performers based on their Sharpe ratios.

Keywords: Low volatility portfolios, Minimum Variance (MIN VAR), Low Volatility Single Index Model (SIM), Equal Risk Contribution (ERC), Naïve Risk Parity (NRP), Maximum Diversification (MAX D), Maximum Decorrelation (MDS), Diversified Risk Parity (DRP), Equal Weighting (EWBS), Equal Weight Within each Sector (EWWS), Covariance Bi-plot.

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List of Abbreviations

ABBREVIATIONS	MEANING
MIN VAR	Minimum Variance Portfolio
SIM	Low Volatility Single Index Model Portfolio
ERC	Equal Risk Contribution Portfolio
NRP	Naïve Risk Parity
MAX D	Maximum Diversification Portfolio
MDS	Maximum Decorrelation Strategy
DRP	Diversified Risk Parity Portfolio
EWBS	Equal Weighting By Sector
EWWS	Equal Weight Within each Sector
LOWBETA	Equal Weight Low beta portfolio
HIGHBETA	Equal Weight High beta Portfolio
ALSI	All Share Index (J203T)
SWIX	Share Holder Weighted Index
CAPM	Capital Asset Pricing Model
FTSE/JSE SECTORS	Financial Times Stock Exchange/ Johannesburg Stock Exchange Sectors

1 Introduction

The Capital Asset Pricing Model (CAPM) is based on the idea that securities with high systematic risk are expected to earn higher expected returns, while low beta securities are expected to have lower returns (Markowitz, 1976; Sharpe, 1963). However, the existence of the low volatility anomaly have shown that the higher risk securities have typically underperformed compared with lower risk securities (see for example Haugen and Baker, (1991, 2012); Black, 1972). The low volatility anomaly; originally put forward by Black (1972) showed empirical evidence indicating that the security market line is flatter than what is predicted by the CAPM. He pointed that when investors are restricted from using leverage or borrowing, they tend to buy high risk stocks thereby leaving the low risk stocks under-priced. This theory was supported by the works of Haugen and Heins (1975). They showed that low risk stocks have historically outperformed higher risk stocks using the U.S. Stock Market and Bond Market over the period 1926 to 1971.

Recently, Bali, Cakici, Yan, and Zhang (2005) and Ang, Hodrick, Xing, and Zhang (2009) found evidence explaining the low volatility anomaly by focusing on idiosyncratic risk using data from U.S. and international markets. They showed that high idiosyncratic volatility stocks have lower returns. They named it the “Idiosyncratic puzzle”. However, Clarke et al. (2006a) formed minimum variance portfolios and showed that although a stock with high idiosyncratic risk may have lower weight in a portfolio, high systematic risk (beta) had the potential of making the stock inadmissible in the minimum variance long-only solution. As a result, idiosyncratic risk was unlikely to impact returns.

Baker, Bradley, and Wurgler (2011) gave some behavioral reasons as to why some investors may behave irrationally and continue to select high risk stocks that have historically underperformed and also why they cannot take advantage of the anomaly by buying low risk stocks. The reasons (Baker, Bradley, and Wurgler, 2011) were:

- Investors’ preference for lottery like payoffs; implying that investor’s accept a low probability of receiving a massive windfall.

- Representativeness bias, which suggests that an investor's preference for high risk stocks that contains a lot of news (or for more speculative stocks). Hence, they buy the stock at a high price, which in turn lowers the return of the stock.
- Portfolio managers are required to beat a specific benchmark and also to minimize the tracking error relative to that benchmark. Hence, they are averse to investing in low beta stocks, because of its high tracking error relative to the benchmark.

In addition to explaining the low volatility anomaly, Baker and Haugen (2012) found that, there are fewer opportunities for portfolio managers to earn a high performance bonus. As a result, the portfolio managers will rather not buy the low volatility stock but prefer to invest in the high risk stock. To date, low volatility alternative equity indices (which will be called low volatility portfolios throughout this study) have been constructed in order to take advantage of the low volatility anomaly and have shown significant out performance when compared with the market capitalization weighted portfolio, which is considered well diversified(see Arnott, Hsu, and Moore, 2005; Carvalho, Lu, and Moulin, 2012; Chow, Hsu, Kalesnik, and Little, 2011; R. G. Clarke, de Silva, and Thorley, 2006b; Haugen and Baker, 1991; Leclerc, L'Her, Mouakhar, and Savaria, 2013). This suggestion implies that low volatility stocks have higher returns on average.

1.1 Low volatility Anomaly in South Africa

The first evidence of the low volatility anomaly in South Africa equity markets was documented by Rensburg and Robertson (2003), where it was found that the beta of a stock is negatively related to its return. Recently, Kruger, Strugnell and Gilbert (2011) also found similar evidence of the low volatility anomaly using a more refined beta estimate. More recently, Khuzwayo (2011) found strong evidence explaining the low volatility anomaly in South Africa using the Top 100 JSE stocks from 2001-2011. He showed that the low volatility portfolios constructed have a lower drawdown and also outperform the market portfolio in falling markets. Panulo (2014) constructed risk parity portfolios and other risk based portfolios, which showed significant out performance compared with the market capitalization weighted portfolio (All share index).

1.2 Research Objectives

The objective of this study is to assess the performance of low volatility portfolios constructed from indices relative to the market capitalization weighted indices using the FTSE/JSE sectors. The use of FTSE/JSE sectors for portfolios was motivated by Leclerc et al. (2013), who created industry-based weighting schemes which they termed Alternative Equity Indices (AEIs) designed to outperform the capitalization weighting portfolio. They highlighted three reasons for using Industries namely:

- Industries help in overcoming the “curse of dimensionality” as explained by Michaud (1989). That is, the number of parameters to estimate is reduced.
- The outperformance of constituents-based alternative equity indices (AEIs) are more often exposed to a small cap factor.¹ Using industries overcomes this limitation.
- Industry tilts are important in explaining constituent- based stocks outperformance over the market capitalization indices.

Another contribution to this study is the construction of low volatility portfolios using the JSE Top 40 stocks. The low volatility portfolios examined in this study are: the Minimum Variance Portfolio (MIN VAR), the low Volatility Single Index Model (SIM), the equal-weighted low beta (Lowbeta) versus equal-weighted high beta (Highbeta), Equal weighting by sector (EWBS), Equal weighting within sector (EWWS)², the equal-weighted Risk Contribution (ERC), Naïve Risk Parity (NRP), the Maximum Diversification Portfolio (MAX D), Maximum Decorrelated Strategy (MDS), and the Diversified Risk Parity (DRP). These portfolios are rebalanced annually using a 36 months rolling window to estimate the covariance matrix (i.e. Ledoit and Wolf (2004a) shrinkage estimator) in order to reduce the effect of errors in the sample covariance matrix. The performances of the industry low volatility portfolios are compared with their cap weighted equity benchmark counterpart (ALSI) to assess which produced a better risk-adjusted performance. Their performances was also measured with respect to their individual characteristics such as to reduce

¹ Choueifaty and Coignard, (2008) found evidence of a small cap factor using constituents based alternative weighting schemes.

² EWWS will be used for the JSE Top 40 stocks.

concentration (Lee, 2011; Lohre, Opfer, and Orszag, 2012; Lohre and Zimmer, 2011). Furthermore, the robustness of the low volatility portfolios is assessed using a monthly rebalancing frequency. Specifically, the performances of the low volatility portfolios are compared for each rebalancing frequency (monthly and annually), after accounting for turnover and transaction costs. In the same vein, a robustness test is also applied to the low volatility portfolios using a larger universe of stocks (JSE Top 100 stocks).

The main innovative aspects of this project are:

- The application of different construction techniques for forming the low volatility portfolios based on sectors and stocks in the South African equity markets (FTSE/JSE sectors and JSE Top 40 stocks). This builds on previous works done by Khuzwayo (2011) and Panulo (2014), who constructed portfolios using stocks only. In addition, a broader range of techniques are applied to the sectors, including the DRP, and MDS techniques and stocks, including the DRP, MDS and EWWS techniques.
- The blending of the low volatility portfolios with typical portfolios (ALSI and SWIX index) in order to establish their usefulness as effective portfolio strategies.

1.3 Document Structure

In Chapter 2, the theory and literature review of the low volatility portfolios is introduced, including parameter estimation (shrinkage methods) used for constructing the covariance matrix. In Chapter 3, the data and a description therefore are presented, together with the back-testing methodology implemented for each low volatility portfolio. Chapter 4 starts by discussing the empirical findings for each dataset (FTSE/JSE sectors and JSE Top 40 stocks) rebalanced annually. Further, the performance measures (Sharpe ratios, information ratios, tracking error, Gini index, R-squared, and betas) of the portfolios are discussed and contrasted. In addition, a technique called the covariance bi-plot, which summarizes the risk exposures of the low volatility portfolios graphically, is utilized and discussed. Chapter 4 concludes by investigating the performances of the low volatility portfolios, rebalanced monthly for each dataset. In Chapter 5, the low volatility portfolios are blended with the Shareholder Weighted Index (SWIX index) and All Share Index (ALSI) and their performances are also assessed. Chapter 6 gives a summary of the major conclusions of the dissertation.

2 Literature Review

This chapter gives a comprehensive literature overview of each low volatility portfolio construction strategy. This review starts with a brief discussion on mean-variance optimization, then goes on to the critique the market capitalization portfolio. The chapter also reviews the literature on the relative performance aspects of the low volatility portfolios. Finally, a shrinkage estimator is discussed to solve the problem of the estimation error in the covariance matrix.

The mean-variance optimization is a model where an investor seeks to effectively allocate his/her investment by choosing a portfolio on the efficient frontier (see Markowitz, 1952, 1959, 1976). The theory uses standard deviation to measure risk. Markowitz derived a mathematical formulation of the concept of Diversification. Lintner (1965) and Sharpe (1963) extended Markowitz work on portfolio theory by introducing the Capital Asset Pricing Model (CAPM), where they found that that the market capitalization portfolio was the most diversified portfolio. In the same vein, Siegel (2003) discussed the characteristics of an index weighted by market capitalization. He noted that the market capitalization weighting is the central principle of good index construction and gave the following reasons:

- The market capitalization-weighting is the only weighting scheme consistent with the buy-and-hold strategy. That is, it doesn't require a portfolio to be constantly rebalanced.
- The market capitalization-weighted index is the only portfolio that is expected to be mean variance efficient. That is, all investors should hold the market portfolio according to the CAPM.
- Low turnover and transaction costs.

However, the market capitalization-weighted index suffers from various pitfalls including the underlying unrealistic assumptions of the CAPM. For example, Fama and French (1992) and Roll and Ross (1994) found evidence of an inefficient tradeoff of the risk/return relation of the CAPM. Another reason is that the market capitalization-weighted portfolio tends to suffer from concentration in the largest securities in the portfolio, such that the contribution of the smallest capitalization securities will not be felt (Kruger and Van Rensburg, 2008;

Leclerc et al., 2013). Bradfield and Kgomari (2004) found evidence of high concentration in the JSE All share index (ALSI). They showed that fewer companies that dominate the index have a high correlation with each other. Consequently, the risk of a market capitalization portfolio increases. Furthermore, the mean-variance optimization has been shown to be associated with “error maximizers” (Merton, 1980; Michaud, 1989; Chopra and Ziemba, 1993; Ledoit and Wolf, 2004). Small changes in the input parameters (especially estimates of expected returns) will have a significant effect on the optimal portfolio weights. Consequently, Chopra and Ziemba (1993) suggested that the performance of the portfolios could be improved by assuming that all assets have the same expected returns. In recent years, the outperformance of low volatility portfolios that do not rely on the estimate of returns have received increasing interest. These portfolios make use of either heuristic or optimization techniques to manage risks and improve diversification. In the next section, a review of the various low volatility portfolios techniques is discussed.

2.1 Varieties of Low Volatility Portfolios

A naïve approach for constructing the low volatility portfolio is based on the equal weighting portfolio (EWBS) proposed by DeMiguel, Garlappi, and Uppal (2009) and Velvadapu (2011). The portfolio assumes that the risk and return cannot be forecasted (W Lee 2011). This helps in reducing the effect of concentration of risk by influencing the weighting structure of the portfolios. Thus, investors are equally exposed to the smallest companies as well as the largest companies in the portfolio. The equal weight low beta (Lowbeta) versus high beta (Highbeta) portfolio aims at comparing the performance of portfolios based on their beta to the market portfolio (M. Baker et al., 2011; Khuzwayo, 2011; Frazzini and Pedersen, 2014). This follows from analysis of Black (1972) and Haugen and Heins (1975) who showed that low risk (beta) stocks have historically outperformed high risk (beta) stocks. Frazzini and Pedersen (2012, 2014) argued that when investors are constrained to use leverage, they prefer to bid up high beta stocks. Thus, the demand for high beta stocks increases the price of the stocks, which in turn reduces the required risk –adjusted return. This motivated them to look at the benefits of conducting a “betting versus beta” factor. The “betting versus beta” factor involves taking a long position in low beta stocks and taking a short position in high beta stocks.

The equal risk contribution portfolios (ERC) came into existence based on the argument that the traditional 60/40 portfolios³ are dominated by equity risk (Maillard, Roncalli, and Teiletche, 2008). Its return is largely driven by exposure to equity risk and less from other sources of risk. Qian (2005) found that equities contribute over 90% of the risk of a 60/40 portfolio and therefore, impacts negatively whenever equities recorded a huge loss in the portfolio. He argued that such portfolios were not well diversified. A heavy concentration of risk can be found in a well-diversified portfolio, for example an equally-weighted portfolio (Maillard, Roncalli, and Teiletche, 2008; Lee, 2011). ERC portfolios allocates weight of assets classes by its contribution to risk and thus takes into account the correlation between the assets in a portfolio. As a result, the ERC portfolio targets maximum risk diversification by focusing on assets with low volatility and low correlation with other assets (Maillard, Roncalli, and Teiletche, 2008; Lee, 2011). Unlike ERC portfolio, the naïve risk parity (NRP) assumes that all asset classes have the same pair-wise correlations. It has an advantage that the optimal portfolio weights can only be computed analytically instead of relying on numerical resolutions. Therefore, assets that are more risky than others will receive a low weight in the portfolio.

The minimum variance portfolio (MIN VAR) is the portfolio located on the left most tip of the efficient frontier, which is made up of the least volatile collection of assets (Clarke et al., 2006b; Haugen and Baker, 1991). The objective function of the minimum variance portfolio is to minimize ex-ante portfolio risk. It is considered more robust than the mean-variance portfolio because of the mean-variance portfolio's sensitivity to inputs in the covariance matrix (Chopra and Ziemba, 1993; Ledoit and Wolf, 2004; Merton, 1980; Michaud, 1989). The covariance inputs comprises of the correlations and volatilities. Hence, the risk of the MIN VAR portfolio is reduced when assets that have low volatility and low correlation are included. Similarly, a simplified version of the MIN VAR portfolio considers constructing analytic solutions as opposed to optimization routines to derive the optimal portfolio weight for the portfolio under long-only constraints. Clarke, De Silva, and Thorley (2011) first derived long-only analytic solutions using Sharpe's single-index model (Sharpe 1963) in their minimum variance portfolio construction; which assumes that the only common source

³ That is, 60 % stocks and 40% bonds.

CHAPTER TWO-LITERATURE REVIEW

of risk is a single factor (market portfolio). Khuzwayo (2011); following R. Clarke, de Silva, and Thorley, (2011), constructed similar portfolios which he termed the low volatility single index model (SIM). Like the MIN VAR, SIM portfolios are also targeting low beta assets. However, the MIN VAR portfolios have been shown to be concentrated in terms of weight and risk contribution, since they are more exposed (or biased) towards the least volatile assets (Chan and Karceski, 1999; Maillard, Roncalli, and Teiletche, 2008; Wai Lee, 2011). Jagannathan and Ma (2002) proposed using a normed constraint to limit the effect of concentration towards the least volatile asset, instead of using weight constraints. Similarly, R. G. Clarke et al. (2006a) also applied constraints on the weights to limit the effect of concentration of the MIN VAR portfolio.

In the same vein, the maximum diversification portfolio (MAX D); originally proposed by Choueifaty and Coignard (2008) is constructed to help investors maximize the benefits of diversification especially when the market conditions are not favorable. The objective function maximizes the diversification ratio, which is the weighted average of the volatilities divided by the portfolio volatility. Unlike the EWBS portfolios that allocate equal weight to each asset in the portfolio, the MAX D will rather allocate weight based on the assets correlation with the portfolio. This suggests that the MAX D portfolio will invest in assets that are less correlated to the portfolio, which in turn, increases the diversification ratio. Furthermore, the maximum Decorrelated strategy (MDS) proposed by Christoffersen, Errunza, Jacobs, and Langlois, (2011), aims to minimize the portfolio volatility under the assumption of equal volatilities between assets. This is relevant for an investor who is only interested in taking advantage of the interaction between assets (Martellini 2014). The MDS portfolio focuses on the pair-wise correlation between assets, thereby avoiding the errors in estimating returns and covariances (Merton 1980). MDS relies on the correlation as the only input source for calculating the optimal portfolio weights when compared with the MAX D and the MIN VAR. As a result, the risk of a portfolio is reduced by targeting assets that are less correlated with other assets.

Finally, the concept of allocating risk by percentage contribution of each asset to the portfolio volatility alone (that is, the risk parity approach) may not reveal the main drivers of portfolio risk (Meucci 2009). Diversified risk parity (DRP) strives for maximum diversification by

identifying the major drivers of a portfolio volatility. Meucci (2009) introduced the concept of building portfolios such that the true independent drivers of portfolio returns are extracted using Principal Component Analysis (PCA). PCA is a statistical procedure for extracting important information from a large dataset. PCA is defined as dimension-reducing technique that performs a transformation on a set of correlated observations into a set of linearly uncorrelated variables called principal components (Shlens 2005). The eigenvectors and eigenvalues are calculated from a sample covariance matrix or correlation matrix. The eigenvector associated with the largest eigenvalue explains the most variation in the data. The eigenvector associated with the second largest eigenvalue explains the next variation in the data and so on for the remaining eigenvalues. The PCA approach has been used as an estimation technique (Connor and Korajczyk, 1988), and in building principal portfolios (Rudin and Morgan, 2006; Bera and Park, 2008). Because assets are more often correlated with each other, the intuition behind constructing principal portfolios is to find a change of basis of the original assets into a set of assets with no covariation between them (Rudin and Morgan, 2006; Bera and Park, 2008; Meucci, 2009). Rudin and Morgan (2006) formed a diversification index⁴ from the PCA decomposition of securities and constructed principal portfolios. The index attempts to mitigate the approach of using the number of assets to measure the diversification of a portfolio. Within this context, it measures the relative strengths of principal components in the portfolio. Kritzman, Li, Page, and Rigobon, (2011) also used the concept of principal component analysis to monitor the fragility of the market. They called their measure the absorption ratio. The absorption ratio is defined as the proportion of the total variance of a set of assets returns that are absorbed by a small number of eigenvectors. Similarly, Bailey and Prado (2012) also observed that the risk of a portfolio increases when a basket is concentrated in the direction of an eigenvector. Like Bera and Park (2008) and Rudin and Morgan (2006), Meucci (2009) constructed principal portfolios that represent uncorrelated linear combinations of the original securities. However, Bera and Park (2008) did not consider the assets' dependence structure as observed by Meucci (2009). Meucci (2009) used a principle in information theory (entropy) and defined a well

⁴ $PDI = 2 \sum_{k=1}^N KW_k - 1$, W_k is the relative strength given by $\frac{\lambda_k}{\sum \lambda_k}$ and λ_k is the eigenvalue of each principal portfolio.

diversified portfolio as a portfolio having its overall risk is equally distributed along principal portfolios. Therefore, it is budgeting risk along principal portfolios rather than the original portfolio of assets (Meucci, 2009; Lohre and Zimmer, 2011). In the same vein, Lohre, Opfer, and Ország, (2012); and Lohre and Zimmer, (2011); Bernardi, Leippold, and Lohre (2013); Deguest, Martellini, and Meucci (2013) also used the concept of principal component analysis to extract uncorrelated factors in a portfolio by maximizing the effective number of uncorrelated bets subject to investment constraints.

In the next section, the existing evidence of the low volatility portfolios outperformance is discussed.

2.2 Existing Evidence of the Low Volatility Portfolio Outperformance

This section discusses the low volatility outperformance in recent literature starting with the naïve equal-weighting which allocates weight equally among assets. DeMiguel et al. (2009) analyzed 7 empirical datasets of monthly returns in the U.S. and compared the out-of-sample performance of 14 different models. They found that none of the models performed better than the naïve equal weighting strategy (EWBS). They showed that out-of-sample, the gains from optimal diversification are more than offset by the estimation error. Further, Velvadapu (2011) introduced a sector-based equally weighted index that equal weights constituents within a sector (EWWS). He found that equal weighting by constituents alone will allocate higher weights in some sectors than in others. This induces what he called “sector biases”. He also provided evidence of higher returns and lower volatilities in EWWS than equal weighting constituents and market cap weighting using large, medium, and small cap U.S. indices.

Khuzwayo (2011) computed low beta (Lowbeta) versus high beta (Highbeta) stocks using the Top 100 JSE shares over the period 2001 to 2011 and compared their performance with the All share index (ALSI). He found that during periods of underperformance in the market, low beta portfolios outperformed both high beta portfolios and ALSI. The intuition being that high beta portfolios should outperform low beta portfolios when the market produced positive returns is flawed as seen (see Figure 2-1 extracted from Khuzwayo (2011)) in periods 2003, 2004, 2007 and 2010. This corresponds with the criticisms of the CAPM (that

high beta stocks should be rewarded). Leclerc et al. (2013) also compared Lowbeta, EWBS, ERC, MAX D and MIN VAR portfolios using industries (Fama-French U.S. industry total return indices of NYSE, Amex and NASDAQ) and found that the Lowbeta had the lowest volatility compared with the other strategies. The industry-based low volatility portfolios outperformed the market capitalization index in terms of Sharpe ratios and lower volatility than the market.

Source: Khuzwayo (2011) Cadiz Securities

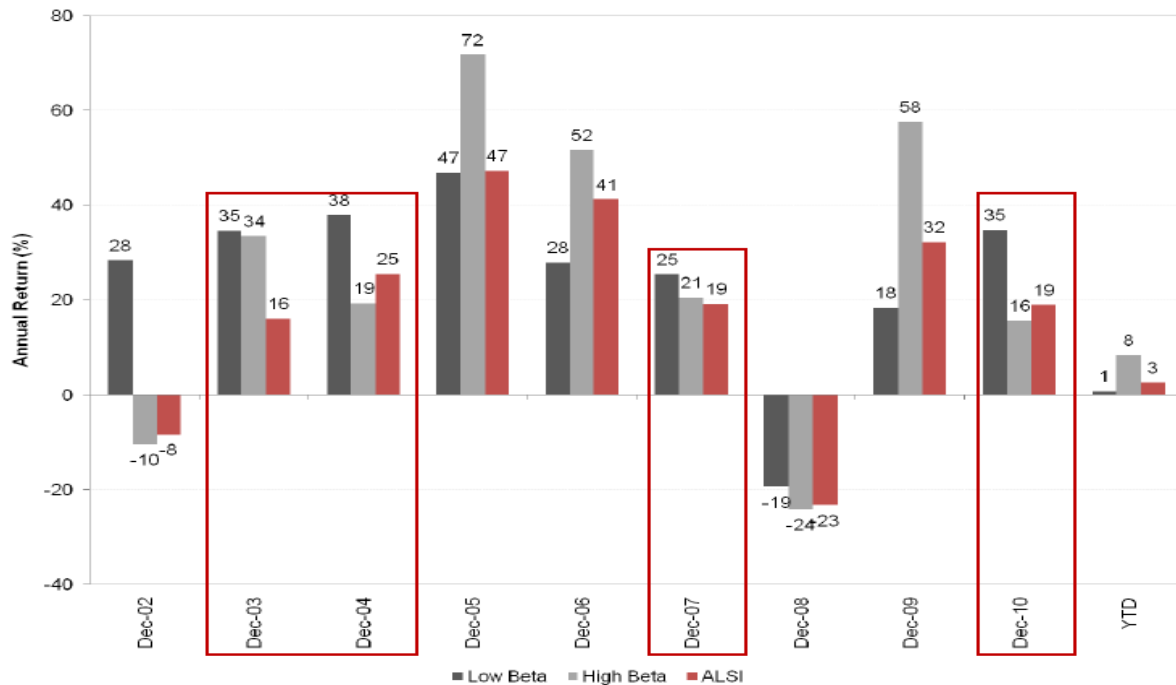


Figure 2-1: Annual performance of low beta versus high beta baskets (December 2001-May 2011) Khuzwayo (2011)

Maillard, Roncalli, and Teiletche (2008, 2010) compared the EWBS, ERC, MAX D and MIN VAR using equity U.S. sectors in the construction of portfolios. They found that all strategies, except the EWBS outperformed the market capitalization portfolio with a lower volatility. They also showed that the volatility of the ERC portfolio is located between the EWBS and the MIN VAR. Clarke et al. (2006a) constructed MIN VAR portfolios and found higher realized average return and lower risk than the market return by using shrinkage methods⁵ applied

⁵ They used both Asymptotic Principal Component by (Connor and Korajczyk, 1988) and Bayesian shrinkage estimation of (Ledoit and Wolf, 2003)

to the sample covariance matrix using Large Cap U.S. stocks over the periods 1968 to 2009. Figure 2-2 (extracted from that follows shows the cumulative returns of the 1000 largest U.S. stocks from 1968 to 2005 relative to market capitalization portfolio. The realized risk was also lower for the MIN VAR compared to the market capitalization portfolio. In the same vein, Jagannathan and Ma (2002) constructed MIN VAR by imposing weight constraints and found them equivalent to using a shrinkage estimator on the variance covariance matrix. Kritzman, Page, and Turkington, (2010) found higher Sharpe ratios in their MIN VAR portfolios constructed relative to the 1/N and market capitalization-weighted indices. Similarly, Haugen and Baker, (1991) also showed that the MIN VAR portfolio outperformed the Wilshire 5000 at a lower risk between periods 1972 to 1989.

Source: Extracted from Clarke et al (2006a)

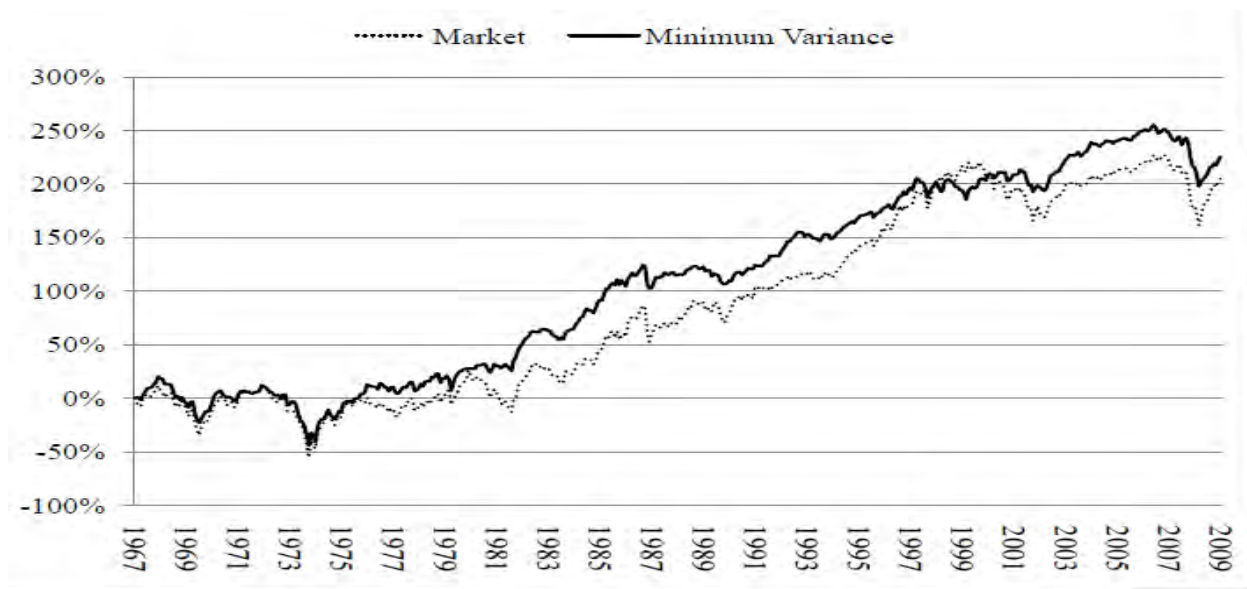


Figure 2-2: Cumulative Returns of the Minimum Variance Portfolio of the 1000 largest U.S. stocks from 1968 to 2005 relative to Market Capitalization Portfolio. Clarke et al (2006a)

Clarke et al. (2006b, 2011) derived analytic solutions in the construction of the SIM portfolios and found a higher cumulative return and a low volatility relative to the market capitalization-weighted portfolio using the largest 1000 U.S. stocks. Clarke and Thorley (2012) also compared analytic solutions for the MIN VAR, ERC and MAX D using 1,000 U.S. stocks over the period 1968 to 2012 and compared them with the market capitalization

weighted portfolio. They found that the MIN VAR posted the lowest risk and the highest Sharpe ratios. Khuzwayo (2011) showed significant outperformance on a risk-adjusted basis and lower risk relative to the All share index (ALSI 100) using the Top 100 JSE stocks. He also found higher Sharpe ratios for the SIM portfolios when compared with the performance of the MIN VAR, MAX D, EWBS and the ALSI 100 for the periods 2008 to 2011.

Choueifaty and Coignard (2008) investigated the performance of the MAX D, with the EWBS and the MIN VAR portfolios using constituents stocks from S&P 500 and Dow Jones EURO STOXX Large Cap indices over the period 1992-2007. They showed that the MAX D produced the highest return. All strategies also outperformed the market indices in terms of Sharpe ratios, with a lower risk.

Lohre and Zimmer (2011) compared the performance DRP, MIN VAR, MAX D and EWBS using Dow Jones EURO STOXX 50 constituents (January 1993- September 2011) and found the DRP strategy to be more diversified than the MIN VAR, MAX D, EWBS and the index, averaging 2.5 bets over the sample period, whereas other strategies were found to average 1-bet. The DRP portfolios also posted significant outperformance (Sharpe ratios) when compared with the index. Recently, Lohre, Opfer, and Ország (2012) compared the performance DRP, MIN VAR, MAX D and EWBS using the S&P 500 over the period October 1993-September 2011 and found the DRP to have averaged 5.1 bets.

In this section the performance of the low volatility portfolios found in literature was discussed. In the ensuing section, the focus is on reviewing the methodology in the literature that would be drawn in the later sections.

2.3 Review of Methodology of the Low Volatility Portfolios

For a portfolio of N assets (sectors or stocks), the EWBS portfolio allocates weights (w_i) given by:

$$w_i = 1/N \quad (1)$$

For the Lowbeta versus Highbeta portfolio, the betas of the assets are first ordered and classified as low or high beta baskets (Khuwayo 2011). Then the weights given to Lowbeta portfolio is:

$$w_i = 1/N_L \quad (2)$$

Consequently, the Highbeta portfolio is given by:

$$w_i = 1/N_H \quad (3)$$

Where, N_L and N_H are the number of assets in the Lowbeta and Highbeta portfolio respectively. However, for the NRP strategy, the optimal solution of portfolio weights (w_i) are equal to the inverse of each assets standard deviation. Thus, assets with low volatility will receive the highest weight in the portfolio Maillard, Roncalli, and Teiletche (2008). The weights are calculated as:

$$w_i = \frac{1/\sigma_i}{\sum_{i=1}^N 1/\sigma_i} \quad (4)$$

Where σ_i represents the asset's total risk.

However, if the correlation between assets are taken into account, the ERC portfolio can be thought of as a portfolio where the risk contribution of each asset is the same. Maillard, Roncalli, and Teiletche (2008) used the principle of Euler decomposition of portfolio risk⁶ (see Appendix 2.1) to show how an asset contributes to portfolio risk. Thus, weights are allocated to the ERC portfolio such that the total contribution between any two assets is zero. Because of the endogeneity⁷ of the solution under long-only and full investment constraint, there are no closed form solutions (Chaves, Hsu, Li, and Shakernia, 2012; Maillard, Roncalli, and Teiletche, 2008). Thus, Maillard, Roncalli, and Teiletche (2008) proposed a sequential quadratic programming algorithm that numerically solves for the optimal portfolio weights which minimizes the sum of squared risk contribution differences given below:

$$w^* = \arg \min \sum_{i=1}^N \sum_{j=1}^N (w_i \frac{\partial MCR_p(w)}{\partial w} - w_j \frac{\partial MCR_p(w)}{\partial w})^2 \quad (5)$$

Subject to

$$w'1 = 1$$

⁶ Litterman (1997) originally proposed the concept of decomposing risk using the standard deviation of a portfolio.

⁷ w_i is a function of the risk contributions, which in turn depends on w_i .

$$0 \leq w_i \leq 1.$$

where, $w_i \frac{\partial MCR_p(w)}{\partial w}$ is the total risk contribution of the portfolio.

Therefore, the ERC portfolio will allocate higher weight to assets that have low correlation and volatility with other assets. Chaves, Hsu, Li, and Shakernia (2012)⁸ also proposed an algorithm to compute risk parity portfolios without using optimization techniques.

MIN VAR portfolios are designed by minimizing the variance of a portfolio without any assumption on the return forecast. The long-only weights are found by minimizing the objective function:

$$w^* = \min \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} \quad (6)$$

SIM portfolios; originally put forward by Sharpe (1963), decomposed the asset's total risk into the systematic and idiosyncratic risk as shown below:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e,i}^2, \quad (7)$$

Where, σ_i represents the asset's total risk and σ_M is the risk of common factor (market), whereas $\sigma_{e,i}$ is the assets idiosyncratic risk and β_i is the asset's systematic risk.

R. Clarke and Thorley, (2011) derived (see Appendix 2) analytic solutions under the long-only constraint as:

$$w_i = \frac{\sigma_{mv}^2}{\sigma_i^2} \left(1 - \frac{\beta_i}{\beta_L}\right) \text{ for } \beta_i < \beta_L \text{ else } w_i = 0, \quad (8)$$

where β_L is the long-only threshold beta, β_i is the beta of the asset to the common factor (market), and σ_{mv}^2 is the ex-ante variance of the long-only MIN VAR portfolio which is a scaling parameter that enforces the budget constraints to sum to 1. The intuition behind Equation 8 is to target those assets with betas that are lower than the threshold (β_L). Equation 8 also shows that, the portfolio weights are highly dependent on the beta to the common factor (market) and idiosyncratic risk. However, the idiosyncratic risk will not drive

⁸ They used Newton's method and power method (Boyd and Vandenberghe, 2004) to solve for the optimal portfolio weights.

the asset out of the solution (Clarke and Thorley, 2011). The formula for calculating β_L is given by:

$$\beta_L = \frac{\frac{1}{\sigma_m^2} + \sum_{\beta_i < \beta_L} \frac{\beta_i^2}{\sigma_i^2}}{\sum_{\beta_i < \beta_L} \frac{\beta_i}{\sigma_i^2}}. \quad (9)$$

The method for calculating β_L is by first sorting the betas of assets in ascending order, then each asset is compared with the summation term until its beta exceeds the required threshold beta.

The MAX D portfolio (Choueifaty and Coignard, 2008) is a solution of the maximization of the diversification ratio, defined as the ratio of the weighted average of volatility to the portfolio volatility:

$$w^* = \operatorname{argmax} \frac{\sum w_i \sigma_i}{\sigma_p} \quad (10)$$

Where, σ_i is the volatility of an asset, and σ_p is portfolio volatility. From Equation 10, assets that have a low correlation with other assets are more often included in the portfolio.

The MDS portfolio, derived by (Christoffersen et al. 2011) focuses only on the correlations of individual assets without considering the difference in volatilities of the assets. The MDS portfolio is given by:

$$w^* = \operatorname{argmax} \left(1 - \sum_{i=1}^N \sum_{j=1}^N w_i w_j \rho_{i,j} \right)^9 \quad (11)$$

Where $\rho_{i,j}$ is the correlation between assets. The above maximization problem will give higher weight to assets with low correlation and underweight assets with high correlation.

Nonetheless, the DRP strategy, unlike the ERC portfolio; aims at allocating equal risk budget to principal portfolios. The intuition behind the DRP strategy is the use of principal component analysis to construct portfolios that are uncorrelated with each other. Meucci (2009) constructs principal portfolios by applying a principal component decomposition to the covariance matrix of returns of a portfolio. Given portfolio weights, the portfolio return

⁹ This is equivalent to minimizing the portfolio volatility under the assumption of equal volatilities i.e. minimizing $\sum_{i=1}^N \sum_{j=1}^N w_i w_j \rho_{i,j}$

is expressed in matrix form as, $R_p = w'R$. The Eigen-decomposition of the covariance matrix is given by:

$$\Sigma = B^T \Lambda B \quad (12)$$

where, $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_N)$ is a diagonal matrix representing the eigenvalues of the covariance matrix sorted in descending order ($\lambda_1 \geq \dots \geq \lambda_N$). The columns of matrix B are the eigenvectors of the covariance matrix Σ . Meucci (2009) defined the eigenvectors as a set of uncorrelated principal portfolios and its variance as λ_i for $i = 1, \dots, N$. The principal portfolios' returns are given by $\tilde{R} = B'R$ and its weights $\tilde{w} = B'w$. It follows that, the covariance between each principal portfolios is 0.¹⁰

$$\text{Var}(R_p) = \sum_{i=1}^N \tilde{w}_i^2 \lambda_i \quad (13)$$

Each principal portfolio are then normalized by the portfolio variance which gives what Meucci (2009) called a diversification distribution:

$$p_i = \frac{\tilde{w}_i^2 \lambda_i^{11}}{\sum_{i=1}^N \tilde{w}_i^2 \lambda_i} \quad i = 1, \dots, N \quad (14)$$

The p_i 's can be thought of as a percentage contribution to risk by each principal portfolio (Kind 2013). Each p_i is also positive and they sum to 1. Meucci (2009) defined a portfolio to be well-diversified, when the p_i 's are close to uniform (That is, allocating equal risk budgets to principal portfolios). He then applied the exponential of Shannon entropy on the diversification distribution to measure the number of the true independent sources of risk in the portfolio given by (Cover and Thomas, 2005):

$$N_{ENT} = \exp(-\sum_{k=1}^N p_k \ln p_k) \quad (15)$$

Where, $N_{ENT} = 1$ when $p_i = 1$ for one i and $p_j = 0$ for $i \neq j$. Also, $N_{ENT} = N$, when $p_i = p_j = \frac{1}{N}$ (That is, a portfolio is equal in terms of uncorrelated risk sources (Lohre and Zimmer 2011; Lohre et al. 2012)). The Figure 2-3 that follows show a portfolio loaded on one principal portfolio (p_i).

¹⁰ $cov(\tilde{w}_i \tilde{R}_i, \tilde{w}_j \tilde{R}_j) = 0$ for $i \neq j$.

¹¹ $\tilde{w}_i^2 \lambda_i$ is the variance due to the i^{th} principal portfolio.

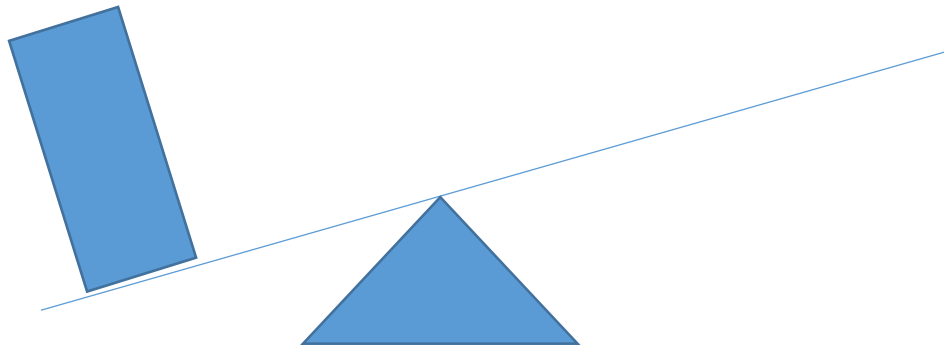


Figure 2-3: An example of loading on a single principal portfolio.

Similarly, Figure 2-4 shows an example of a portfolio with equal loadings on all $p_i \forall i$.

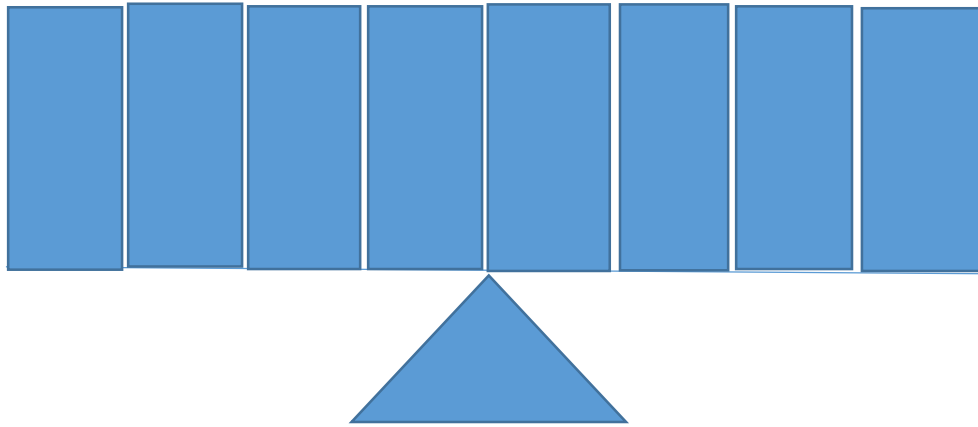


Figure 2-4: An example of an equal loadings on all principal portfolios.

Consequentially, Lohre and Zimmer (2011) and Lohre et al. (2012) build on the Meucci (2009) metric to obtain a maximum diversification portfolio and called their approach the Diversified Risk Parity (DRP). They maximized Equation 15 subject to a set of constraints:

$$w_{DRP} = \underset{C}{\operatorname{argmax}} N_{ENT}(w) \tag{16}$$

Lohre and Zimmer (2011); Lohre et al. (2012); Deguest et al. (2013); and Bernardi et al. (2013) observed that there are 2^N optimal inverse volatility solutions along the principal portfolios which do not require numerical optimization. They observed further that imposing a positive constraint for the weights does not guarantee a unique Diversified Risk Parity solution. They proposed to impose sign constraints (for not trading against its

associated historical risk premium) with respect to the principal portfolios (uncorrelated risk factors) to obtain a unique DRP (Diversified Risk Parity). Thus, they equalized the sign of each principal portfolio with the sign of its equivalent historical risk premium¹² in order to obtain a unique DRP strategy.

To build intuition on the concept of the DRP, (Lohre, 2014) utilize the following hypothetical example of imposing sign constraints on a 2 asset portfolio which is given below: Let the asset universe consist of JSE stocks, MTN and RMB. Then the first principal portfolio (PC1) might have loadings like $E_{11} = 0.4$ for MTN and $E_{21} = 0.6$ for RMB mimicking the “market portfolio”. Most likely, the historical premium to this portfolio is greater than 0, hence you want to buy PC1. For not trading against PC1 we impose the following linear constraint $E_{11} x_1 + E_{21} x_2 > 0$ (or $0.4 x_1 + 0.6 x_2 > 0$) where x_i are the final weights for MTN and RMB. Stepping on to the second principal portfolio (PC2), assume to have found loadings like $E_{12} = -0.6$ for MTN and $E_{22} = 0.3$ for RMB and the historical premium of playing MTN vs. RMB turns out to be negative, hence you want to sell PC2 which is equivalent to wanting to buy the negative of PC2. The constraint imposed first flips signs of the loadings and then reads as follows: $0.7 x_1 - 0.3 x_2 > 0$. Hence, imposing long-only constraint for the DRP strategy requires that $B^T w > 0$. The approach of allocating risks budgets to all principal portfolios¹³ seems unreasonable when allocating to higher principal portfolios. The relevance of higher principal portfolios tend to die off quickly. Consequently, Lohre, Opfer, and Orszag (2012) and Bernardi et al. (2013) proposed using Bai and Ng (2002) information criterion to cut off irrelevant principal portfolios. Having discussed all the low volatility portfolios, the Table 2-1 that follows summarizes the low volatility portfolios discussed previously.

¹² The historical risk premium is calculated by multiplying the principal portfolio weights by the historical asset returns.

¹³ For example, a portfolio of 500 assets will have 500 principal portfolios when there are only few portfolios that explains the variation of the asset.

Table 2-1: Summary of the Low Volatility Portfolios

Strategy	Authors	Targets	Required Parameter	Formula
Equal Weighting (EWS, EWBS)	(DeMiguel et al. 2009; Velvadapu 2011)	Maximum weights diversification	No risk or return parameter	$w_i = 1/N$
Lowbeta versus High beta	(Leclerc et al., 2013; khuzwayo, 2011)	Low beta assets	Beta	$w_i = 1/N_L$ $w_i = 1/N_H$
Minimum Variance (MIN VAR)	(R. G. Clarke et al., 2006a; Haugen and Baker, 1991)	Low volatility assets and Low correlation	Covariance Matrix	$w^* = \min \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij}$
Low Volatility Single Index Mode (SIM)	(Clarke et al. 2011a)	Low beta and low volatility assets	Beta and Idiosyncratic variance	$w_i = \frac{\sigma_{mv}^2}{\sigma_i^2} \left(1 - \frac{\beta_i}{\beta_L}\right)$
Equal risk contribution (ERC)	(Maillard, Roncalli & Teiletche 2008)	Maximum risk diversification	Volatility and correlation	$\min \sum_{i=1}^N \sum_{j=1}^N \left(w_i \frac{\partial \sigma_p}{\partial w_i} - w_j \frac{\partial \sigma_p}{\partial w_j}\right)^2$
Naïve Risk Parity (NRP)	(Maillard, Roncalli, and Teiletche, 2008; Edward Qian, 2006)	Assets with lower volatilities than the others	Volatility	$w_i = \frac{1/\sigma_i}{\sum_{i=1}^N 1/\sigma_i}$
Maximum Diversification (MAX D)	(Choueifaty and Coignard, 2008; Choueifaty, Froidure, and Reynier, 2011)	Low correlation with other assets	Volatility and correlation	$w^* \operatorname{argmax} \frac{w_i \sigma}{\sigma_p}$
Maximum Decorrelated (MDS)	(Christoffersen et al., 2011; Goltz and Gonzalez, 2013)	Low correlation with other assets	Correlation	$\max 1 - \sum_{i=1}^N \sum_{j=1}^N w_i w_j \rho_{i,j}$
Diversified Risk Parity (DRP)	(Lohre, Opfer, and Ország, 2012; Lohre and Zimmer, 2011; Meucci, 2010b)	Maximum risk diversification along principal portfolios	Eigenvectors (principal portfolios) and eigenvalues	$\operatorname{argmax} \left(- \sum_{k=1}^N p_k \ln p_k \right)$

In the next section, the shrinkage estimators are discussed, which will later be used in the dissertation to estimate the covariance matrix.

2.4 Shrinkage Estimators

As discussed earlier, mean-variance optimization suffers from estimation errors (Chopra and Ziemba, 1993; Michaud, 1989; Ledoit and Wolf, 2004). The covariance matrix is an important input source that is required to produce the optimal portfolio weights. Thus, small changes in input assumptions of the covariance matrix may imply large changes in the optimized portfolio.¹⁴ Michaud (1989) proposed using resampling methods to estimate the mean and covariance terms. Ledoit and Wolf (2004) derived a shrinkage transformation on the sample covariance matrix and applied it to monthly U.S. stock data. They showed that the shrinkage estimator reduces the tracking error relative to the benchmark index. Clarke et al. (2006b) used both the Ledoit and Wolf (2004) Bayesian shrinkage estimator and the Connor and Korajczyk (1988) asymptotic principal component method in implementing their minimum variance optimization. They found that the Bayesian shrinkage estimator produces a better result than the asymptotic principal component approach.

In this project, the Ledoit and Wolf (2004) bayesian shrinkage estimator will be used to estimate the covariance matrix and is described below:

Let S be a sample covariance matrix. Ledoit and Wolf (2004) found that the sample covariance matrix is unbiased but contains a lot of estimation error. They suggested using a highly structured estimator but pointed out that it may be biased. They therefore found a compromise between the sample covariance matrix and a highly structured estimator. Let F be a highly structured estimator, they found a convex linear combination of of both S and F given below:

$$\Omega^* = \mu F + (1 - \mu)S \tag{17}$$

where μ is a shrinkage constant between 0 and 1. They refer to the technique as a shrinkage technique.

¹⁴ Especially when the number of stocks is larger than the number of observations, the sample covariance matrix becomes invertible.

It follows that from Equation 17, the shrinkage estimator is dependent on the estimator with no structure (S), an estimator that has a lot of structure (F), and a shrinkage constant (μ). Ledoit and Wolf (2004) proposed a constant correlation model as the structured estimator.¹⁵The constant correlation model assumes that all pairwise correlation are identical. It is made up of the average pairwise sample covariance on the off diagonal and the sample variance on the diagonal. Ledoit and Wolf (2004) also derived the optimal shrinkage constant (μ) for the shrinkage estimator which is found by minimizing the expected loss of the difference between the shrinkage estimator and the true covariance matrix. The package *tawny* (cov.shrink function) in R software is used to implement the method.

2.5 Chapter Summary

In this Chapter, the varieties of the low volatility portfolios found in literature are reviewed. The review suggests that the low volatility portfolios can be aptly described by the exposure to assets with low volatility (EWWS, EWBS and NRP), the exposure to assets with low volatility and low correlation with other assets (ERC), the exposure to low volatility and low beta assets (MIN VAR and SIM, Lowbeta), and the exposure to assets with low correlation with other assets (MAX D and MDS). The low volatility portfolios were also found in the literature to have outperformed market capitalization-weighted portfolios typically used as benchmark proxies. The Chapter concludes by explaining the shrinkage estimators used to solve the problem of the estimation errors in the covariance matrix which will be adopted later in the thesis.

¹⁵ Ledoit and Wolf (2003) also suggested using the single index model as the structured estimator.

Appendix 2

Euler's theorem

Let $f(x_1, x_2 \dots x_n) = f(x)$ be a continuous and homogenous function of degree one function of the variables $x = (x_1, x_2 \dots x_n)'$.

$$\text{Then, } f(x) = x_1 \frac{\partial f(x)}{\partial x_1} + x_2 \frac{\partial f(x)}{\partial x_2} + \dots + x_n \frac{\partial f(x)}{\partial x_n}$$

The motivation for introducing this theorem is to show how an investor can see which asset contributes the most to the portfolio risk.

Low volatility single index model (SIM)

R. Clarke and Thorley, (2011) decomposed the covariance matrix of a single factor model in matrix form as: Let σ_m^2 represent the variance of a factor e.g. a market index and N the number of stocks. Also, let σ_i^2 variance of e_i also called the idiosyncratic variance. Then the covariance matrix is:

$$\Omega = \beta\beta' \sigma_m^2 + \text{Diag}(\sigma^2) \quad 2.1$$

where β is an N-by-1 vector of β_i , σ^2 is the vector of σ_i^2 Using the matrix inversion lemma by (Woodbury 1949)¹⁶, the inverse covariance matrix in the single-factor model is solved analytically and is given by:

$$\Omega^{-1} = \text{Diag}\left(\frac{1}{\sigma^2}\right) - \frac{(\beta/\sigma^2)(\beta/\sigma^2)'}{\frac{1}{\sigma_m^2} + (\beta/\sigma^2)'\beta} \quad 2.2$$

From the construction of the minimum variance portfolio, the optimal portfolio weights derived are: $w^{MV} = \frac{\Omega^{-1}1}{1'\Omega^{-1}1}$, where $\sigma_{mv}^2 = \frac{1}{1'\Omega^{-1}1}$ represents a scaling value, which ensures that the budget constraint of the sum of weights of assets sums to one. Therefore, substituting Equation 2.2 and $\sigma_{mv}^2 = \frac{1}{1'\Omega^{-1}1}$ in Eq.2.1, $w^{mv} = \frac{\Omega^{-1}1}{1'\Omega^{-1}1}$ becomes Equation 8.

¹⁶ The Woodbury Matrix identity is stated as: $(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$, where, A is an N by N matrix, U is N by K matrix, C is K by K matrix and V is K by N matrix.

3 Data and Methodology

This chapter first introduces the data used in constructing the low volatility portfolios. In addition the back-testing terminology is explained, with a thorough description of the implementation of the low volatility portfolios.

3.1 Description of the Data

The Data used in this study are: the FTSE/JSE sector data and the JSE Top 40 stocks; collected from DataStream and adjusted for corporate actions.¹⁷ In addition, a larger dataset (JSE Top 100 stocks) will be used to test the robustness of low volatility portfolios. The first dataset contains monthly total return indices of the 9 FTSE/JSE sectors (Oil and Gas, Health Care, Consumer Goods, Consumer Services, Financials, Industrials, Telecoms, Technology and Basic Materials). The monthly data covers the sample period from January 2003 to December 2013 (132 months). Table 3-1 that follows describes the annualized return, annualized risk and Sharpe ratio (assuming a risk free rate of 0%) for the FTSE/JSE sectors.

Table 3-1: Descriptive statistics for the 9 FTSE/JSE sectors, January 2003 -December 2013

FTSE/JSE SECTORS	Annualized Return	Annualized Risk	Annualized Sharpe (Rf=0%)
OIL&GAS	16%	25%	0.66
BASIC MATS	10%	25%	0.41
CONSUMER GDS	24%	21%	1.16
HEALTH CARE	26%	19%	1.42
CONSUMER SVS	29%	18%	1.62
TELECOM	28%	23%	1.21
FINANCIALS	18%	17%	1.06
TECHNOLOGY	24%	28%	0.85
INDUSTRIALS	20%	17%	1.17

¹⁷ The sectors and the individual stocks are adjusted for stock splits and dividend.

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From Table 3-1, it can be seen that the Financial sector had the lowest annualized risk (17%) across all sectors, while Technology had the highest annualized risk (28%). Consumer Services had the highest annualized return (29%) and the highest annualized Sharpe ratio (1.62) over the period. Basic materials had the lowest annualized Sharpe ratio (0.41) and the lowest annualized return for the sample period. The Figure 3-1 that follows depicts the cumulative return of 1rand invested in each sector over the sample period.

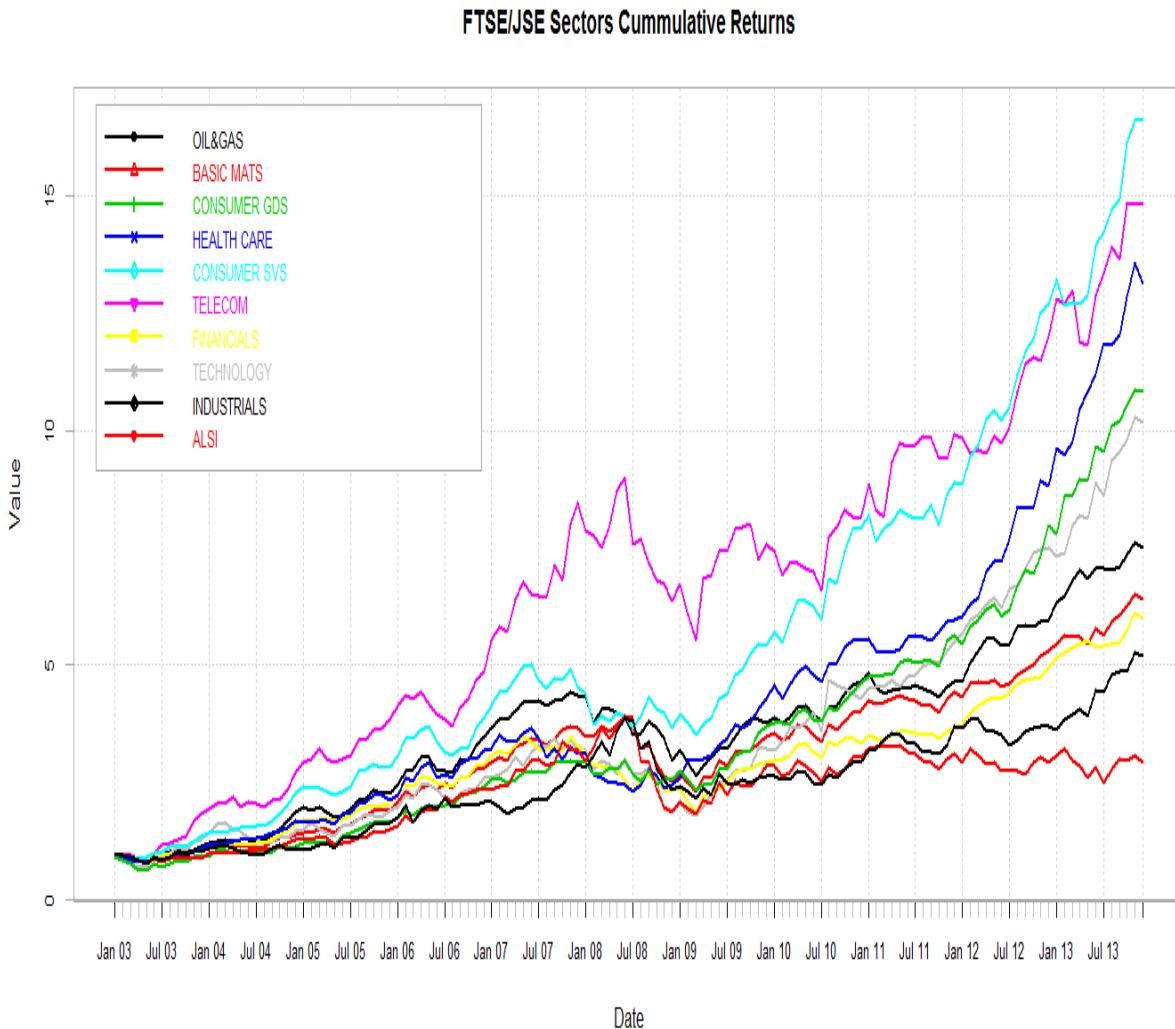


Figure 3-1: Cumulative Return of the FTSE/JSE sectors, January 2003 -December 2013

Examining the sectors dependence, the correlation matrix is also presented in Figure 3-2 for the whole period.

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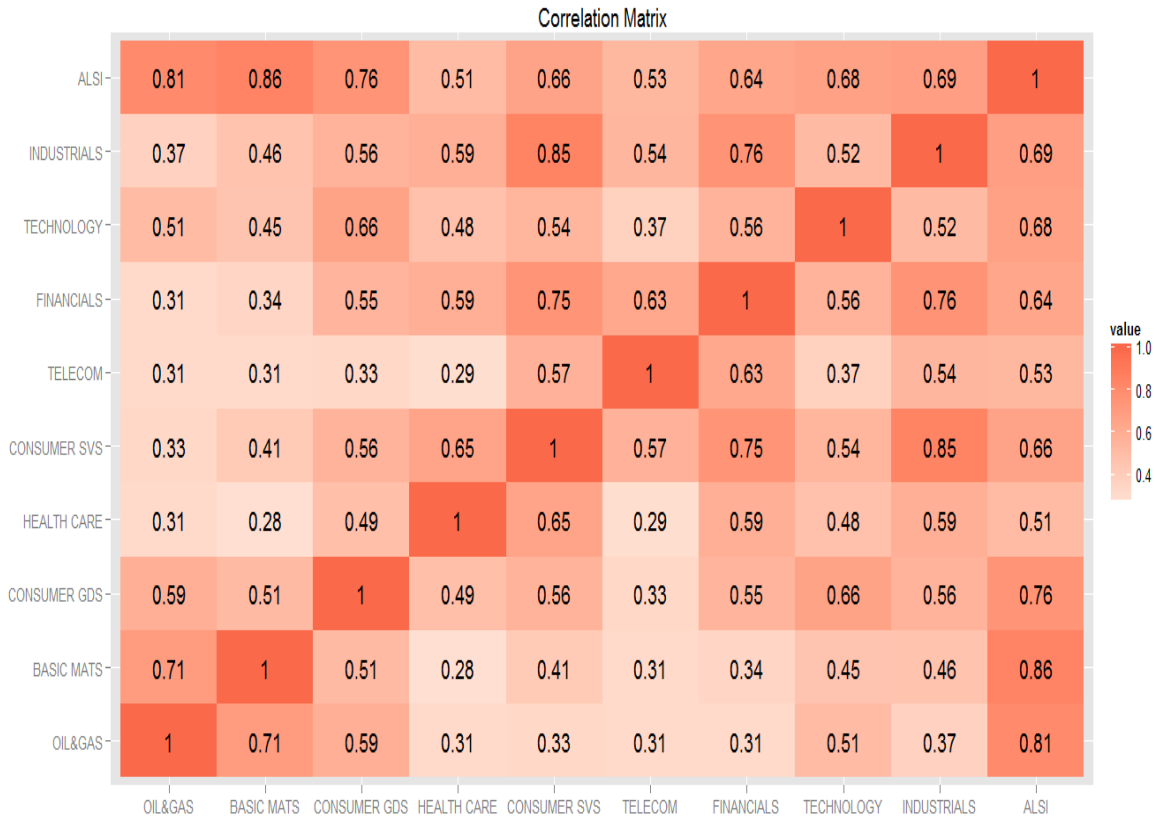


Figure 3-2: Correlation Matrix for the FTSE/JSE sectors, January 2003 –December 2013

From Figure 3-2, one notices that there is a high correlation between the Oil and Gas, Basic Materials, and Consumer Goods and the ALSI. The Table 3-2 that follows shows the maximum drawdown of the FTSE/JSE sectors over the entire period (January 2003 to December 2013).

Table 3-2: Maximum Drawdown of the FTSE/JSE sectors, January 2003-December 2013

FTSE/JSE sectors	Drawdown
OIL&GAS	0.44
BASIC MATS	0.53
CONSUMER GDS	0.38
HEALTH CARE	0.37
CONSUMER SVS	0.30
TELECOM	0.39
FINANCIALS	0.46
TECHNOLOGY	0.48
INDUSTRIALS	0.41

From Table 3-2, Basic Materials had the highest maximum drawdown of 0.53 followed by Technology 0.48 and Financials 0.46. Consumer Services had the least maximum drawdown of 0.3.

The second dataset contains the total return index of the JSE Top 40 stocks which extends from January 2004 to December 2013. The annualized return, annualized risk, annualized Sharpe ratio, average correlation and drawdown for the whole period are presented in Appendix B. Assore posted the highest annualized return (45.38%), followed by Mr Price (42.12%). AngloGold and Goldfields had a negative return of (-6.42% and -4.97% respectively). Also, Remgro had the lowest volatility (15.75%) while Exxaro posted the highest volatility (41.04%). Over the sample period, Remgro, Shoprite and Liberty Holdings had the lowest drawdown (12.4%, 19.3%, and 20.3% respectively). Conversely, Anglo American, Old Mutual and Anglo American Platinum had the highest drawdown (75.9%, 74.7% and 74.3%). In this section, the data used in this thesis was introduced. The next section expands on the methodology highlighted in the introductory section.

3.2 Back-testing Methodology

Back-testing is a process that tests the performance of a strategy over a historical period to assess its ability to produce the intended result (Schlegel 2014). A model is estimated using historical data up to a point in time. The parameters from the model are then used to assess

the quality of the model out of sample. An advantage of back-testing is that it helps the portfolio manager understand the weakness of a strategy on so called “unseen” data. To test the efficacy of the low volatility portfolios, periods of different market regimes of the All Share Index (ALSI) are included in the analysis. It is also required that there are no missing data¹⁸ in the sectors and stocks for the entire sample period (January 2003 to December 2013 for the sectors, and January 2004 to December 2013 for the stocks). The covariance matrix used for estimation must also satisfy the condition of non-missing data. In this study, the covariance matrix is estimated over 36 month rolling windows¹⁹ using the Ledoit and Wolf (2004) Bayesian shrinkage estimator. The low volatility portfolios are also rebalanced annually and monthly over the period 2003-2013 for the sectors and 2004-2013 for the individual stocks, assuming a trading cost of 25 basis points. The methodology follows Snopek (2012) approach in his construction of the minimum variance portfolio. The only difference is in the calculation of the holding period return. Before discussing the implementation of each low volatility strategies, the following constraints are imposed:

- Fully invested constraint in the risky portfolio, that is $\sum_{i=1}^N w_i = 1$.
- long-only constraint ($w_i \geq 0$).

3.3 Covariance Matrix Estimation of the Low Volatility Portfolios

The low volatility portfolios that do not require optimization are the EWBS, EWWS, Lowbeta versus Highbeta, SIM, and NRP. In contrast, optimization of MIN VAR, MAX D, MDS, is essential for computing the optimal portfolio weights. The DRP also requires optimization but employs the concept of a principal component analysis to create principal portfolios. As discussed in section 3.1, the 9 FTSE/JSE sectors monthly data cover the sample period from January 2003 to December 2013. The methodology for the annual rebalancing is given below:

The first stage (optimization and estimation of parameters) begins in January 2006 and utilizes returns of the preceding 36 months (3 years). A period of 48 months is used for the

¹⁸ Clarke et al. (2006b) also imposed non-missing historical return data for the 1000 largest U.S. capitalization-weighted stocks of the CRSP when constructing their minimum variance portfolios.

¹⁹ 36 months rolling windows is used to estimate the covariance matrix before the portfolio formation date.

estimation window, where the first 36 months (in-sample-period) are used as inputs to the covariance matrix and estimation of parameters (For example; standard deviations, betas, threshold betas and idiosyncratic variance) to derive the optimal portfolio weights. The returns from the last 12 months (out-of-sample period) are then multiplied with the weights to give the returns of the portfolio for the strategy. The Figure 3-3 gives a graphical illustration of the methodology:

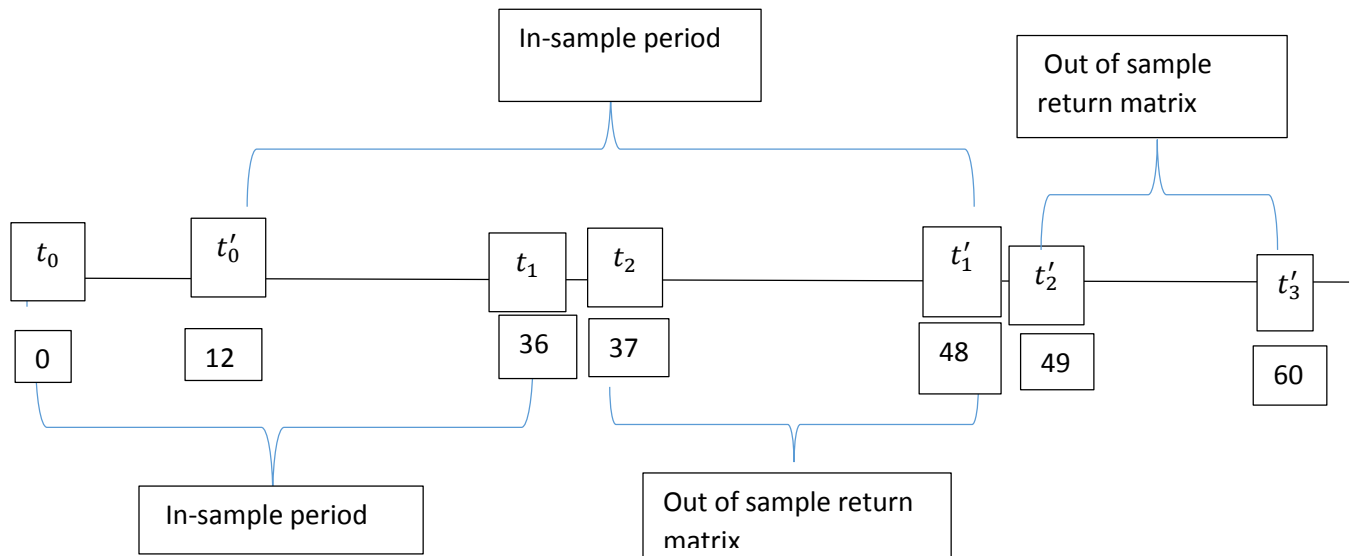


Figure 3-3: Methodology for the Annual rebalancing on the FTSE/JSE sectors adapted from Snopek (2012)

From Figure 3-3, the first covariance²⁰ estimation period takes place between t_0 = January 2003 to t_1 = December 2005. The in-sample weights are then multiplied by the out-of-sample 12 months' returns i.e. from t_2 = January 2006 to t'_1 = December 2006. However, the in-sample weights are allowed to drift²¹ in the holding period. Thus, one doesn't have to rebalance the portfolio weights back to the initial portfolio weights every month. The data is then rolled over from t'_0 = January 2004 to t'_1 = December 2006 to re-estimate the

²⁰ Using Ledoit and Wolf (2003) shrinkage estimator to estimate the covariance matrix for the MD, MIN VAR, MDS, and ERC portfolios.

²¹ The drift occurs because the market has moved the weight of stock i over the period t and thus the new weight is adjusted by all the other portfolio weights that also have been moved by the market.

parameters and obtain the in-sample weights. This is repeated until December 2013 (rebalanced annually). An illustration of the calculation of the holding period return derived by Bradfield (2014) is given below:

The portfolio return is given by:

$$R_p^t = \sum_{i=1}^N w_i^{t-1} \cdot R_i^t \quad (17)$$

R_p^t = the return on the portfolio at time t

R_i^t = the return on stock i over the period t-1 to t

w_i^{t-1} = the portfolio weight in stock i at time t-1

N = the number of stocks in the portfolio

Given an initial weight w_i^0 for stock i at the beginning of the portfolio holding period.

At the end of month 1 the portfolio return would be:

$$R_p^1 = \sum_{i=1}^N w_i^0 \cdot R_i^1 \quad (18)$$

It follows that, month 2 portfolio return would be:

$$R_p^2 = \sum_{i=1}^N [w_i^1 \cdot R_i^2] \quad \text{where } w_i^1 = \frac{w_i^0 \cdot (1 + R_i^1)}{\sum_{i=1}^N w_i^0 \cdot (1 + R_i^1)}$$

At the end of month 3 the portfolio return would be:

$$R_p^3 = \sum_{i=1}^N [w_i^2 \cdot R_i^3] \quad \text{where } w_i^2 = \frac{w_i^0 \cdot (1 + R_i^1)(1 + R_i^2)}{\sum_{i=1}^N w_i^0 \cdot (1 + R_i^1)(1 + R_i^2)}$$

The end of the month 12 month portfolio return becomes:

$$R_p^{12} = \sum_{i=1}^N [w_i^{11} \cdot R_i^{12}] \quad \text{where } w_i^{11} = \frac{w_i^0 \cdot \prod_{t=1}^{11} (1 + R_i^t)}{\sum_{i=1}^N w_i^0 \cdot \prod_{t=1}^{11} (1 + R_i^t)}$$

At the end of the 12 month holding period, the portfolio will need to be rebalanced, which will incur transaction cost at each rebalancing date. As a result, the turnover of the portfolio may affect the return of the portfolio strategy. Thus, one has to adjust the portfolio return at the end of the holding period (at the 12 month return). Maillard, Roncalli, and Teiletche

CHAPTER THREE –DATA AND METHODOLOGY

(2008) calculated the portfolio turnover between two consecutive rebalancing dates. Victor DeMiguel et al. (2009) computed their portfolio turnover as the average sum of absolute number of trades over the asset in the portfolio. The portfolio turnover computed in this study considers the sum of the magnitude of trades in each asset given below:

$$\sum_i |w_i^{t,new} - w_i^{t,bef}| \quad (19)$$

$w_i^{t,bef}$ = the portfolio weight in sector i at time t immediately before rebalancing.

$w_i^{t,new}$ = rebalanced (new) portfolio weight of the portfolio at time t .

Hence, the portfolio weights immediately before rebalancing at the end of the holding period will have drifted to:

$$w_i^{12,bef} = \frac{w_i^0 \cdot \prod_{t=1}^{12} (1+R_i^t)}{\sum_{i=1}^N w_i^0 \cdot \prod_{t=1}^{12} (1+R_i^t)} \quad (20)$$

Thus, the readjusted portfolio return at the end 12 month holding period when the portfolio is rebalanced must account for turnover and transaction cost. The rebalancing cost incurred from the trade for all asset in the portfolio is $c * \sum_i |w_i^{t,new} - w_i^{t,bef}|$ (DeMiguel et al. 2009). The readjusted return is derived from the Money Value (in Rands) immediately before rebalancing at time t which is:

$$MV_p^{t-1} * (1 + R_p^t) \quad (21)$$

It follows that, the Money Value of the portfolio immediately after rebalancing ($MV_p^{t,adj}$) is given by:

$$MV_p^{t,adj} = MV_p^{t-1} * (1 + R_p^t) - \text{total portfolio rebalancing cost} \quad (22)$$

where the total rebalancing cost is the trading cost (c) and the money value of the portfolio at time t are multiplied by the portfolio turnover. Hence, the money value becomes:

$$MV_p^{t-1} * (1 + R_p^t) - c * MV_p^{t-1} * (1 + R_p^t) * \sum_i |w_i^{t,new} - w_i^{t,bef}| \quad (23)$$

Denoting the percentage increase in money value of the portfolio over the period $t-1$ to t immediately after rebalancing as:

$$R_p^{t,adj} = \frac{MV_p^{t,adj} - MV_p^{t-1}}{MV_p^{t-1}} = \frac{MV_p^{t,adj}}{MV_p^{t-1}} - 1 \quad (24)$$

Rearranging Equation 23 and substituting it in Equation 24 becomes:

$$R_p^{t,adj} = R_p^t - c * \sum_i |w_i^{t,new} - w_i^{t,bef}| - c * R_p^t * \sum_i |w_i^{t,new} - w_i^{t,bef}| \quad (25)$$

The second term on the right hand side RHS of Equation 25 ($c * \sum_i |w_i^{t,new} - w_i^{t,bef}|$) represents the cost of rebalancing the portfolio over the period while the term ($c * R_p^t * \sum_i |w_i^{t,new} - w_i^{t,bef}|$) is the cost of rebalancing the portfolio that has either grown or shrunk.

Thus, the end of 12 month portfolio return is adjusted as:

$$R_p^{12,adj} = R_p^{12} - c * \sum_i |w_i^{12,new} - w_i^{12,bef}| - c * R_p^{12} * \sum_i |w_i^{12,new} - w_i^{12,bef}| \quad (26)$$

However, if the portfolio is rebalanced monthly the portfolio weight of sector i at time t immediately before rebalancing ($w_i^{t,bef}$) is:

$$w_i^{t,bef} = \frac{w_i^{t-1} * (1 + R_i^t)}{\sum w_i^{t-1} * (1 + R_i^t)} \quad (27)$$

And the readjusted return becomes:

$$R_p^{t,adj} = R_p^t - c * \sum_i |w_i^{t,new} - w_i^{t,bef}| - c * R_p^t * \sum_i |w_i^{t,new} - w_i^{t,bef}|.$$

In the next section, the methodology for the equal-weighted portfolio is explained including the EWWS and the EWBS portfolios used for both the sectors and the stocks.

3.4 Methodology for the Equal-weighted Portfolio (EWBS, EWWS)

For the FTSE/JSE sectors, equal weights (EWBS) are only assigned to the 9 sectors. However, the composition of the equal weighting for the JSE Top 40 stocks exploits two methods (EWWS and EWBS). The first method involves Velvedapu (2011) construction of sector equal weighted indices (EWWS) discussed in Chapter 2. At the end of the annual rebalancing period, equal weights are not only assigned to the 9 sectors in JSE Top 40 stocks but also the stocks within the 9 sectors. However; the second method (EWBS) assigns equal weights by stocks only, where the in-sample weights $(1/N)^{22}$ are generated at the end of the rebalancing

²² Where N is the number of stocks.

period. The next section describes the methodology for obtaining the betas that are required for the SIM portfolio and Lowbeta versus Highbeta portfolio.

3.5 Methodology for the betas of the Lowbeta versus Highbeta Portfolio and the SIM portfolio

The methodology used for obtaining the betas is calculated in a similar manner to Clarke et al., (2011a, 2013b). The historical betas are estimated using the standard ordinary least square estimates of the previous 36 months, where the common market factor is the ALSI. The betas are then adjusted by $+1/2$ towards 1 similar to Clarke et al., (2011a, 2013b). Given that there are 9 sectors, equal weights are assigned to a sector betas lower than 50th percentile beta ($\leq 50\%$) while also equal weighting sectors betas equal and above the 50th percentile beta ($> 50\%$). Thus, Lowbeta portfolios would contain 5 out of the 9 sectors. Conversely, 4 out of 9 sectors will make up the Highbeta portfolio. However, for the JSE Top 40 stocks, the Lowbeta versus Highbeta portfolio will be formed by equal weighting stocks based on their betas lower than the 20th percentile beta ($< 20\%$) and greater than the 80th percentile beta ($> 80\%$) respectively similar to Khuzwayo (2011).

3.6 Chapter Summary

In this Chapter, the data used for the analysis of the low volatility portfolios was introduced. This includes the 9 FTSE/JSE sectors and the JSE Top 40 stocks. Additionally, the back-testing methodology was discussed here, showing a step-by-step guide into the low volatility construction process. The covariance matrix estimation procedure was also described in this methodology section. Furthermore, the methodology for constructing the Equal-weighting for both the FTSE/JSE sectors and the JSE Top 40 stocks was presented. Similarly, the methodology for calculating the betas that will be used for constructing the Lowbeta versus Highbeta portfolio, and the SIM portfolios are described. Finally, the Table 3-3 below summarizes the strategies implemented for each dataset.

Table 3-3: Summary of Low volatility portfolios implemented for each dataset

Dataset	Low Volatility Strategies
FTSE/JSE sectors	EWBS, Lowbeta versus Highbeta, MIN VAR, SIM, ERC, NRP, MAX D, MDS, and DRP
JSE Top 40 stocks	EWBS, EWWS, Lowbeta versus Highbeta, MIN VAR, SIM, ERC, NRP, MAX D, MDS, and DRP

4 Empirical Results

This Chapter starts with a preliminary descriptive in order to build intuition on the beta and the DRP strategy. Then the next section presents the central empirical results of this thesis. The risk and diversification characteristics of the low volatility portfolios are also presented and discussed here. The central objective in this chapter is to build and analyze low volatility portfolios using sectors and stocks. Additionally, a covariance bi-plot is presented to graphically portray the low volatility portfolios.

4.1 Preliminary Descriptive explained for the Beta and the DRP strategy.

This section is included here so as to build intuition on the two techniques (betas for the SIM, and principal portfolios for the DRP) giving more insight in the later sections. In section 4.1.1, the preliminary results for the sectors betas are described. Section 4.1.2 presents the preliminary results required for the DRP.

4.1.1 Preliminary Results for the FTSE/JSE Sector Betas

Similar to Khuwayo (2011), the betas of the FTSE/JSE sectors (obtained monthly) are depicted in Figure 4-1, together with the threshold beta in order to understand the selection procedure employed by the SIM portfolio.

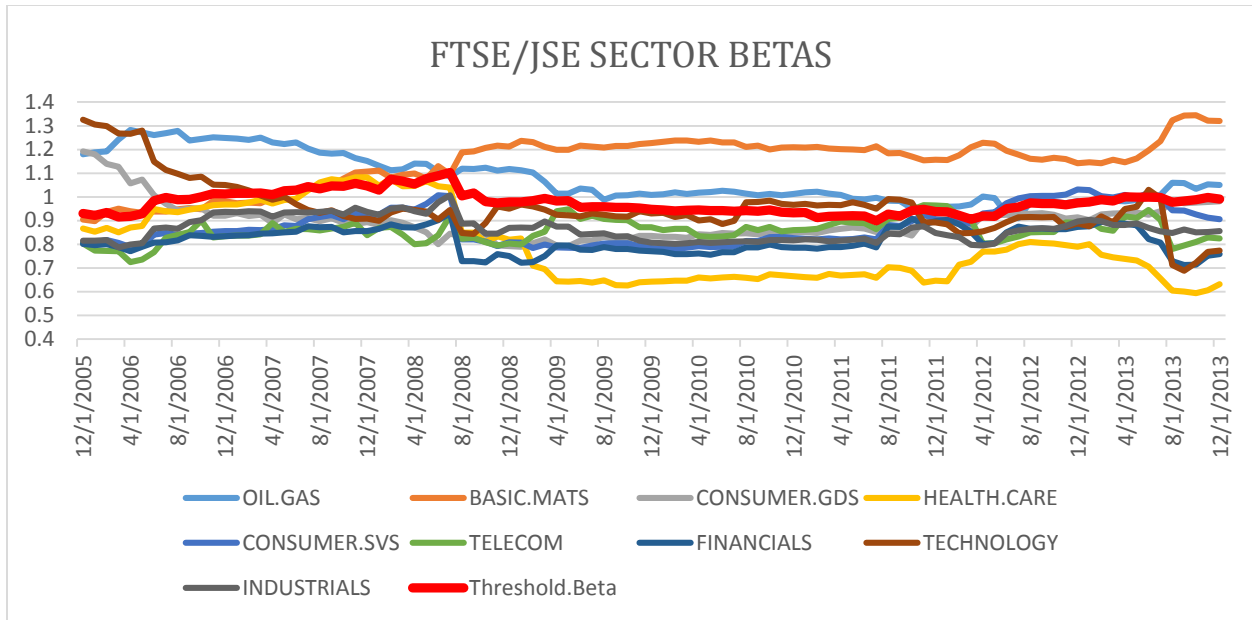


Figure 4-1: FTSE/JSE sector betas and the Threshold beta, January 2005-December 2013

From Figure 4-1, one notices that the Oil and Gas and the Basic Material sector were above the threshold beta (thick red line), over the sample period. This perhaps is not surprising given that it has been shown in South Africa that high betas have been historically associated with the Resources sector (Basic material and Oil and Gas) (Khuzwayo 2011). Consequently, one will expect that during periods of outperformance of the Resource sector over the other sectors, the Highbeta portfolio will tend to outperform the other low volatility portfolios. Khuzwayo (2011) attributed the Highbeta’s outperformance to the significant underweight of the Resources sector. However, it is suffice to say that the outperformance of the low volatility portfolios can also be attributed to the significance overweight of the low beta assets in South Africa, as shown by Panulo (2014).

4.1.2 Preliminary Results Required for the Diversified Risk Parity (DRP)

In order to investigate the relevance of principal portfolio variance over time, a 36 months’ rolling windows is used to extract the principal portfolio variance every month in each sector. The Figure that follows show the variation of principal portfolio variances (see section 2.3) over time and the average variance over time respectively.

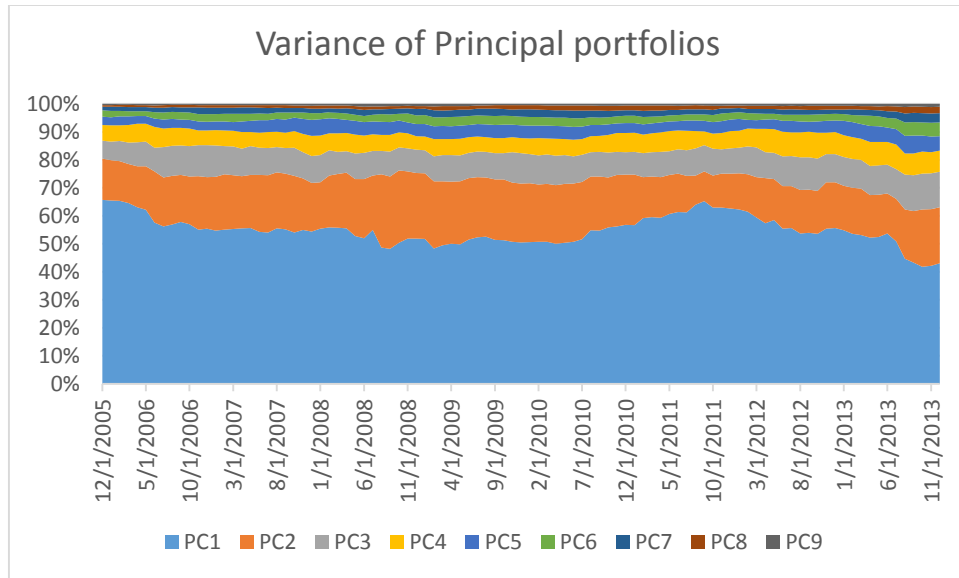


Figure 4-2 Principal portfolio variances over time

From Figure 4-2, one notices that the first component (PC1) explains on average about 55% variation of the sectors, whereas the other components explain on average less than 10%. In line with similar works of Lohre, Opfer, and Ország (2012), the principal portfolios for the JSE Top 40 stocks was also extracted from PCA estimation of 36 month rolling windows. This gives rise to 40 principal portfolios. The Figure 4-3 below depicts the barplot of average principal portfolio variances over the sample period (January 2004 to December 2013) explained by each of the 40 principal portfolios.

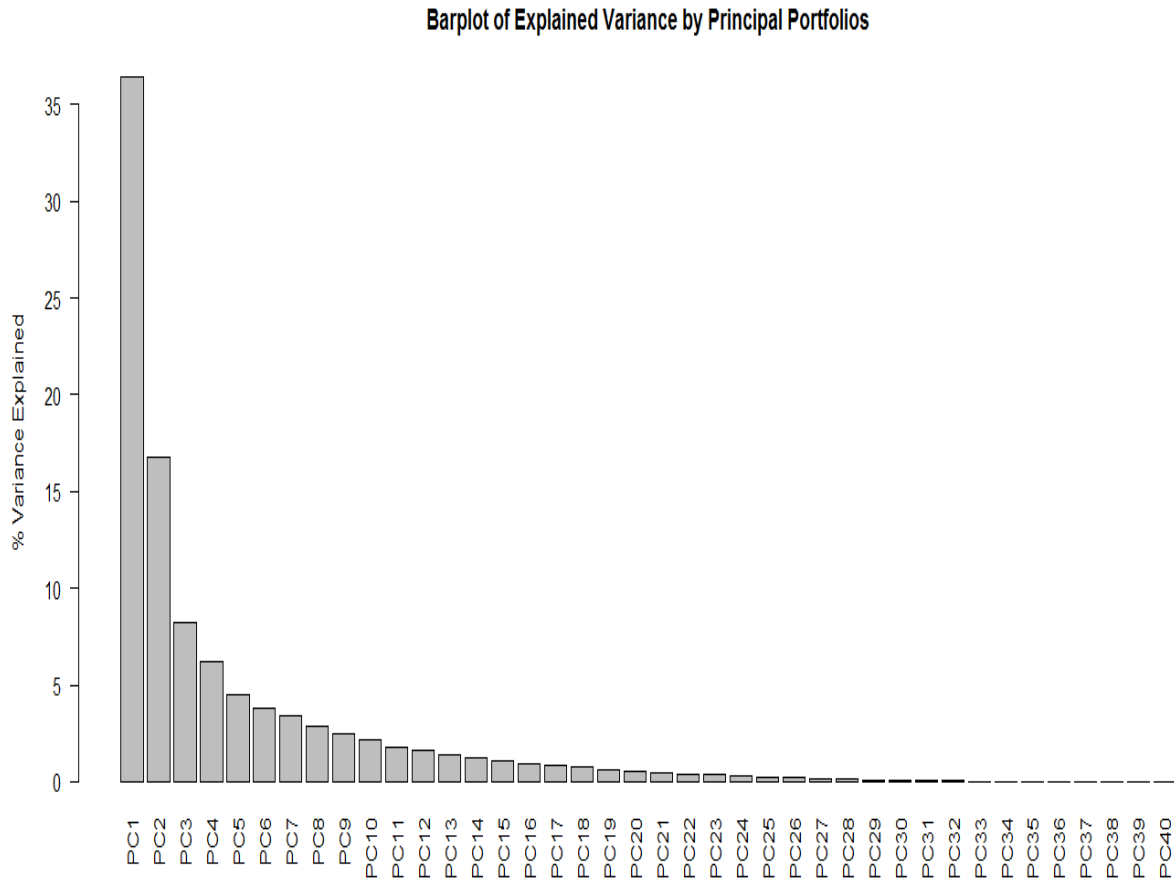


Figure 4-3: Barplot of Explained Variance by Principal Portfolios over time

From Figure 4-3, one notices that PC1 represents on average about 36% variation of the stocks, PC2 explains 17%, while the other principal portfolios represent single digit figures (for example, PC3, 8% and PC4, 6%). As discussed in Chapter 2, it seems irrelevant to allocate risk budgets to irrelevant principal portfolios. Similar to Bernardi et al. (2013); Lohre and Zimmer (2011); and Lohre (2012), the ($PCp1$ and $PCp2$) criteria of Bai and Ng (2002) is used to determine the number of relevant principal portfolios. The minimum on average of $PCp1$ and $PCp2$ method for determining the number of factors in the JSE Top 40 stocks obtained is 5. Thus, the constant number of 5 principal portfolios is used as the number of

relevant factors for the JSE Top 40 stocks.²³ In the next section, the central empirical results of the low volatility portfolios using sectors are presented to assess the performance of the low volatility portfolios.

4.2 Performance of the Low Volatility Portfolios on FTSE/JSE sectors

The Table 4-1 below reports the performance statistics (annualized return, annualized risk, Sharpe ratios, annualized drawdowns and the number of sectors) for the low volatility portfolios after accounting for rebalancing cost (25bps) and turnover at each rebalancing date, from January 2006-December 2013 (rebalanced annually). The low volatility portfolios are also ranked from the highest Sharpe ratios to the lowest Sharpe ratios.

Table 4-1: Performance summary of the low volatility portfolios using the FTSE/JSE sectors, January 2006 – December 2013

	Lowbeta	SIM	NRP	ERC	MAX D	MIN VAR	EWBS	MDS	DRP	Highbeta	ALSI
Annualized Return	22%	22%	20%	20%	21%	21%	20%	21%	23%	17%	16%
Annualized Risk	15%	15%	15%	15%	16%	15%	15%	17%	19%	18%	17%
Annualized Sharpe (Rf=0%)	1.48	1.41	1.37	1.36	1.35	1.35	1.33	1.28	1.2	0.95	0.95
Drawdown	0.31	0.33	0.32	0.32	0.38	0.35	0.32	0.39	0.39	0.37	0.41
Skewness	-0.41	-0.35	-0.09	-0.08	-0.29	0.28	-0.04	-0.30	0.09	0.18	-0.08
Kurtosis	3.44	3.25	3.30	3.30	3.51	3.28	3.36	3.59	3.18	3.96	3.62

Acronyms of Each Techniques used.

Lowbeta-Equal weight Low-beta SIM-Low beta Single Index model NRP- Naïve Risk Parity ERC-Equal Risk Contribution
 MAX D- Maximum diversification MIN-VAR- Minimum variance EWBS- Equal weight by Sector MDS- Maximum Decorrelated Strategy
 DRP- Diversified Risk Parity ALSI- All share Index

From Table 4-1, it is evident that all of the portfolios, bar the Highbeta portfolio, outperformed the ALSI in terms of lower risk (except DRP), and higher Sharpe ratio. Furthermore, the portfolios also showed a lower drawdown than the ALSI. More salient is

²³ (Lohre and Zimmer, 2011) used a constant number of 6 principal portfolios as the relevant number of factors in their DRP strategy for the Dow Jones EURO STOXX 50 total return index.

CHAPTER FOUR–EMPIRICAL RESULTS

the high risk-adjusted (Sharpe ratio) of the Lowbeta (1.48) and the SIM (1.41) over the period. The DRP had the highest return of (23%) over the period. This however came with a high annualized risk of (19%) resulting in a Sharpe ratio of 1.20 but was still much higher than the Sharpe ratio of the ALSI (0.95). The Lowbeta portfolio also posted the lowest drawdown of 0.31, whereas the Highbeta portfolio had a drawdown of 0.37. However, the highest drawdown resulted from the MDS, DRP and MAX D portfolios (0.39, 0.39, and 0.38 respectively). In the same vein, Table 4-2 illustrates the other performance measures (Beta, Beta+, Beta-, Timing Ratio, R-squared, Correlation, Tracking Error and Information Ratio) of the low volatility portfolios relative to the ALSI.

The Timing ratio = $\frac{\text{Beta}+}{\text{Beta}-}$, where the Beta + is the beta for positive market returns and the Beta- is the beta for negative market returns (Bacon 2011). Preferably, one will expect a beta greater than 1 in a rising market and a beta less than 1 in a falling market. The timing ratio will be used to assess whether the low volatility portfolios will be a good timer of assets allocation decisions, see Bacon (2011).

Table 4-2: Performance Measures of the Low Volatility Portfolios relative to the ALSI, January 2006 – December 2013

	Lowbeta	SIM	NRP	ERC	MAX D	MIN VAR	EWBS	MDS	DRP	Highbeta
Beta	0.69	0.68	0.8	0.8	0.87	0.72	0.82	0.9	0.94	0.98
Beta+	0.65	0.63	0.76	0.76	0.79	0.68	0.78	0.81	0.99	0.96
Beta-	0.48	0.46	0.68	0.7	1.02	0.53	0.72	1.08	0.79	0.98
Timing ratio	1.34	1.35	1.12	1.1	0.77	1.27	1.08	0.75	1.24	0.98
R-squared	0.62	0.58	0.84	0.85	0.89	0.64	0.87	0.84	0.69	0.87
Correlation	0.79	0.76	0.92	0.92	0.93	0.8	0.93	0.92	0.83	0.93
Tracking Error	0.11	0.11	0.07	0.07	0.06	0.1	0.06	0.07	0.11	0.06
Information Ratio	0.54	0.49	0.59	0.61	0.86	0.43	0.59	0.75	0.64	0.1

From Table 4-2, it is evident that the MIN VAR, SIM, Lowbeta and the DRP had a high tracking error relative to the ALSI, whereas, the ERC, NRP, MAX D, MDS, Highbeta, and EWBS posted

a low tracking error relative to the ALSI. Unsurprisingly, are the betas of SIM (0.68), MIN VAR (0.72), and Lowbeta (0.69) relative to the ALSI given that are composed of low beta sectors. The betas of the other strategies are also less than 1 even though, they don't directly target low betas like the as the MIN VAR, SIM and Lowbeta portfolios. Furthermore, the bull beta (Beta+) for the DRP has the highest beta (0.99) among all strategies and still did considerably well together with the bear beta (Beta-) of 0.79 when the market was down, with a timing ratio of 1.24. Also, the SIM, Lowbeta and MIN VAR portfolios posted the highest timing ratios (1.35, 1.34, and 1.27 respectively), which tells us the ability of these portfolios in making good market timing decisions (especially when markets are unfavorable). However, MAX D and MDS showed lower timing ratios given that their bear beta is higher than their respective bull beta. The proportion of variance (R-squared) of the MAX D, ERC, NRP, MDS, EWBS portfolios explained by the variance of the ALSI is high compared with the other portfolios. All portfolios also show a high correlation with the market (ALSI).

Figure 4-4 depicts the risk-return analysis of the portfolio strategies over the period.

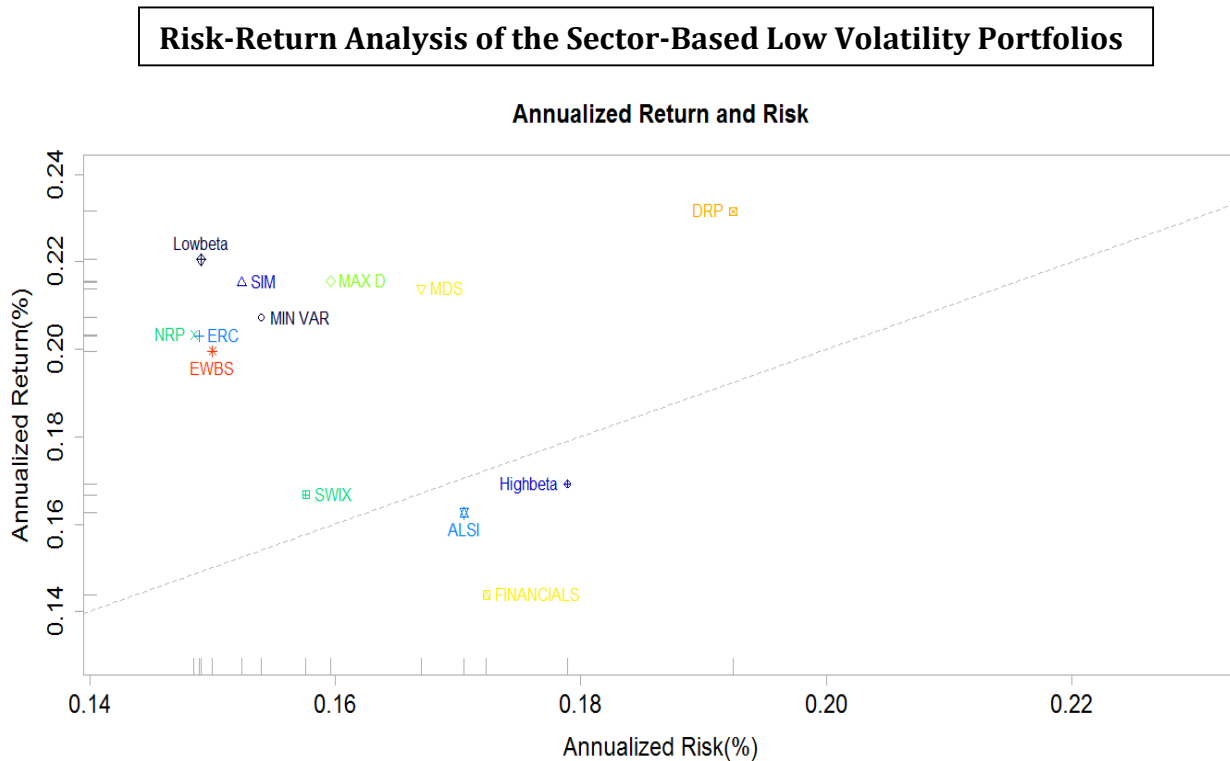


Figure 4-4: Risk-Return Analysis of the sector-based Low Volatility Portfolios, January 2006- December 2013

From Figure 4-4, it is evident that the Lowbeta, MIN VAR, SIM, MAX D, and the ERC portfolio, posted lower risks but also higher returns over the sample period. This is similar to the works by Clarke et al. (2011b); Clarke and Thorley (2012b); Maillard, Roncalli and Teiletche (2008); Choueifaty and Coignard (2008); Baker et al. (2011); Khuzwayo (2011); and Velvadapu (2011), who all found that low volatility portfolios have outperformed the market capitalization portfolio over time. These portfolios have also outperformed the Basic Materials and Financial sectors over the sample period.

In the same vein, Figure 4-5 depicts the out-of-sample cumulative returns of the portfolio strategies. The left subplot (A) depicts the out-of-sample performance of the MIN VAR, ERC, SIM, NRP, MAX D, MDS, DRP, SWIX, and the ALSI. The right subplot (B) shows the out-of-sample cumulative return of the EWBS, Lowbeta, Highbeta, SWIX and the ALSI. All strategies fared better than the SWIX and the ALSI over the sample period.

Out-of-Sample Cumulative Returns of the Sector-based Low volatility portfolios.

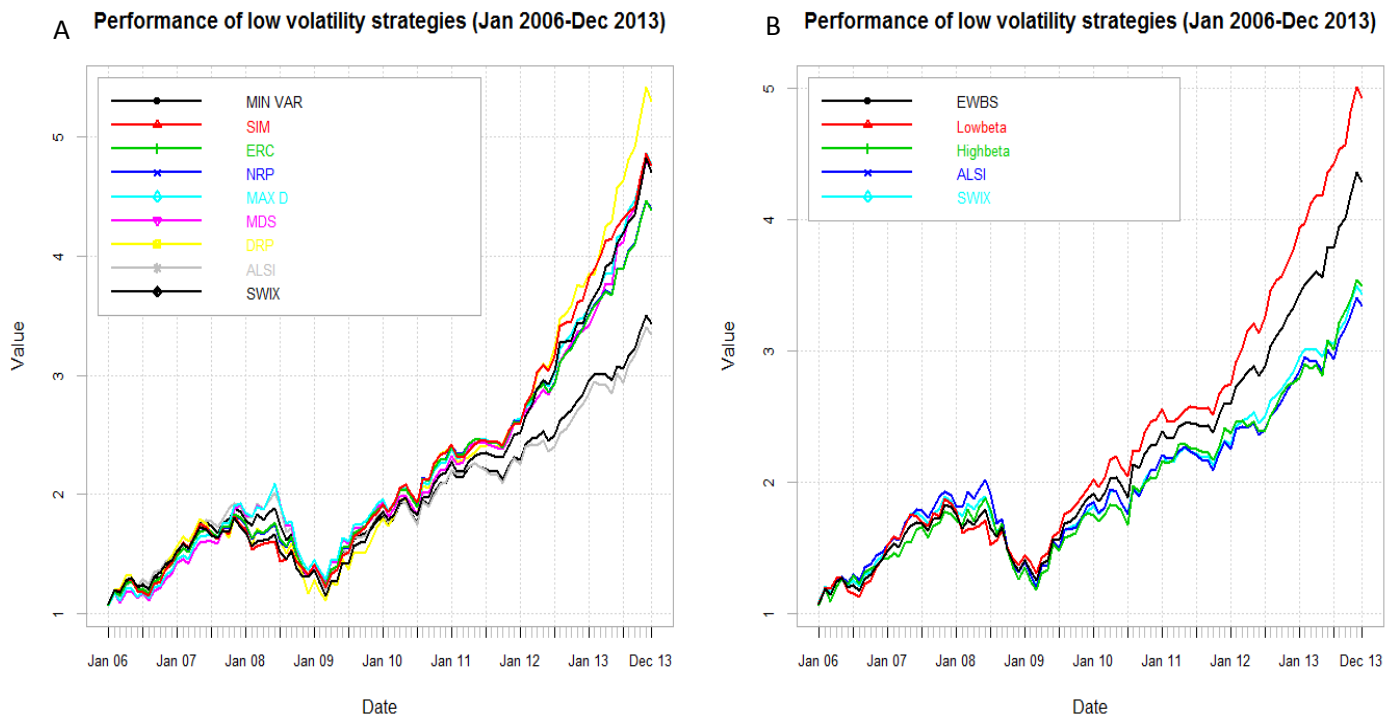


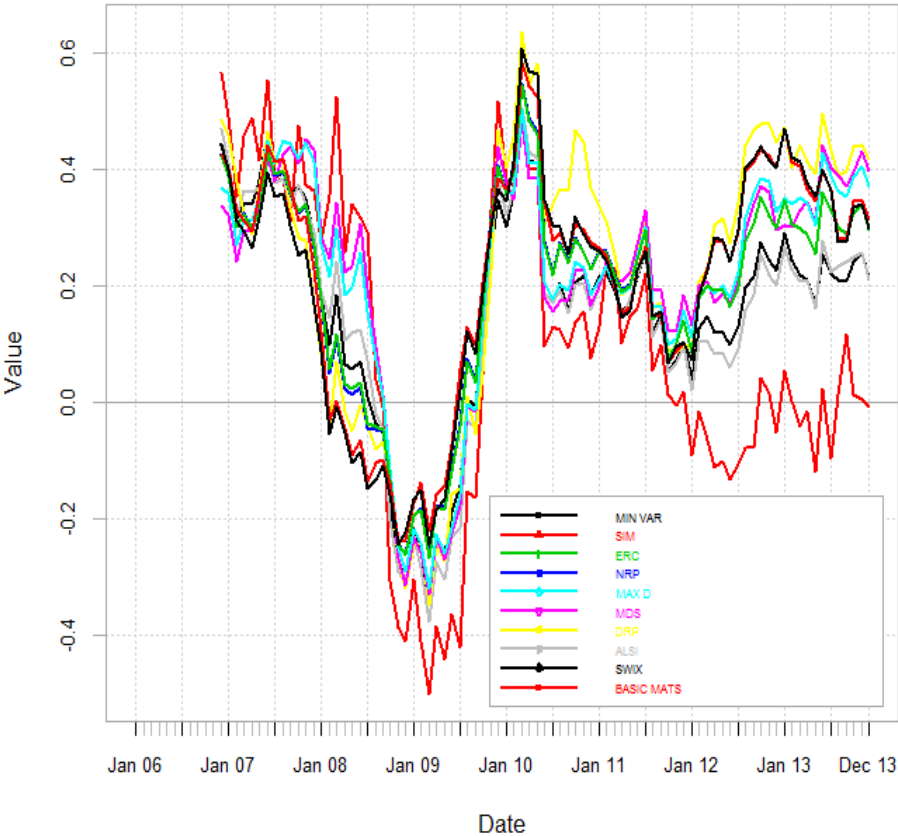
Figure 4-5: Out-of-Sample Performance of the FTSE/JSE sector-based low volatility portfolios rebalanced annually, January 2006- December 2013

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From Figure 4-5, one notices that most strategies tend to move in sync with each other. For example the MIN VAR and the SIM, the ERC and NRP, the MDS and the MAX D respectively. Not surprising is the performance of the Highbeta portfolio closely following both the SWIX and the ALSI (B) due to the fact the beta for the Highbeta portfolio is close to 1 (see Table 4-2). In addition to the cumulative return over time (Figure 4-5), the sector-based Low volatility portfolio outperformance (or under performance) over time is shown in Figure 4-6, together with the rolling 12 month performance of the ALSI. Additionally, the Basic Materials sector is also shown on the same chart. The low volatility portfolios have outperformed the ALSI in most periods bar 2008, where the Basic Materials, Highbeta portfolio and the ALSI outperformed the portfolios. In the next section, the performance attribution (sector allocation effect) in 2008 is examined to explain the underperformance of the low volatility portfolios in 2008.

Rolling 12-Month Return of the Low Volatility Portfolios

Rolling 12-Month Return of the sector Low volatility portfolios



Rolling 12-Month Return of the sector Low volatility portfolios

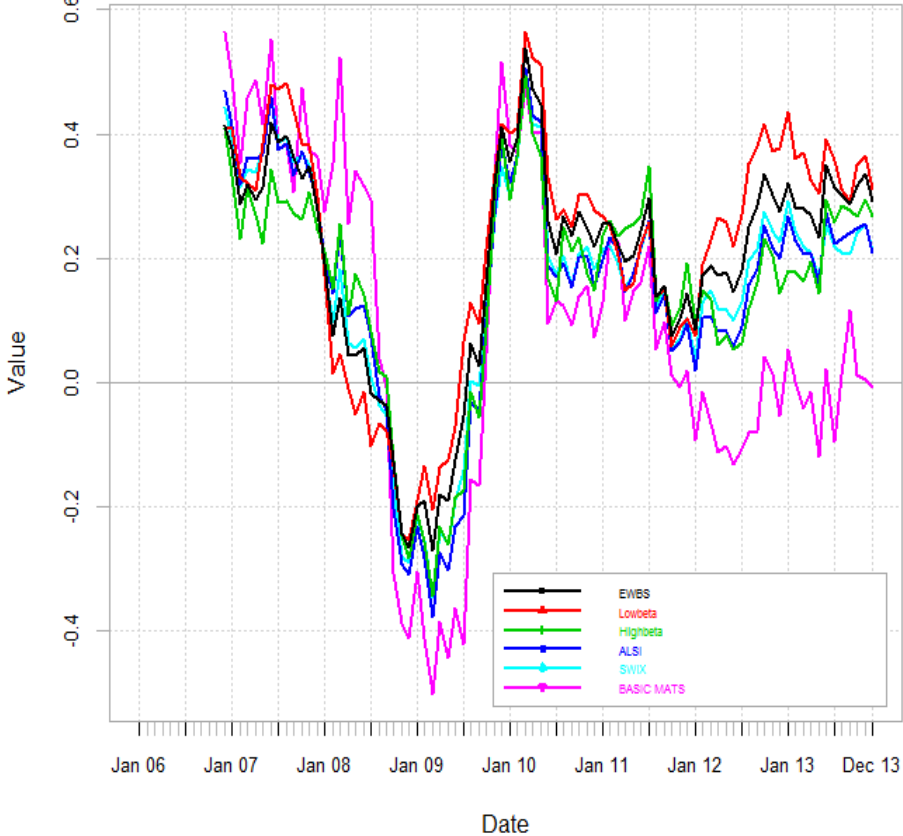


Figure 4-6: Rolling 12-Month Return of the FTSE/JSE Low volatility Portfolios, January 2006 – December 2013

4.3 Sector Allocation Effect of the Low Volatility Portfolios in 2008

As discussed previously, there was a significant outperformance of the Resource sector in 2008 relative to the ALSI, however the low volatility portfolios underperformed in 2008. Khuzwayo (2011) suggested that an underweight position in the Resources sector (Oil and Gas and Basic Materials) in the low volatility portfolios in 2008 is the reason for the underperformance of the low volatility portfolios relative to the ALSI. In order to examine the underperformance of the low volatility portfolios in 2008, the concept of performance attribution proposed by Brinson and Fachler (1985) and Brinson, Hood, and Beebower (1986), is used by performance analysts to explain the sources of active return²⁴ in a portfolio is discussed. Brinson and Fachler (1985) and Brinson, Hood, and Beebower (1986) decomposed the active returns of a portfolio into the asset allocation, and security selection. They suggested that a portfolio manager can add value by allocating weights to a group of assets in the portfolio that is different from the groups benchmark weight (Bacon, 2011). In that regard, a successful portfolio strategy (assets) would be to overweight top performing assets and underweight poor performing assets (Bacon, 2011). In this study, the sector allocation effect of the low volatility portfolios is examined to explain the outperformance of the Basic Materials sector in 2008. Brinson and Fachler (1985) suggested that an overweight position in a positive market will lead to a positive contribution relative to the benchmark whereas, an overweight position in a negative market will result in a negative contribution. Still, a positive effect could still be achieved if the portfolio strategy is overweight in a negative market that has outperformed the overall benchmark return (Bacon 2011). The portfolio weights of the Basic Materials sector generated for the low volatility portfolios are depicted in Figure 4-4, together with the benchmark (ALSI) weight as at 2008.

²⁴ The portfolio return minus the benchmark return.

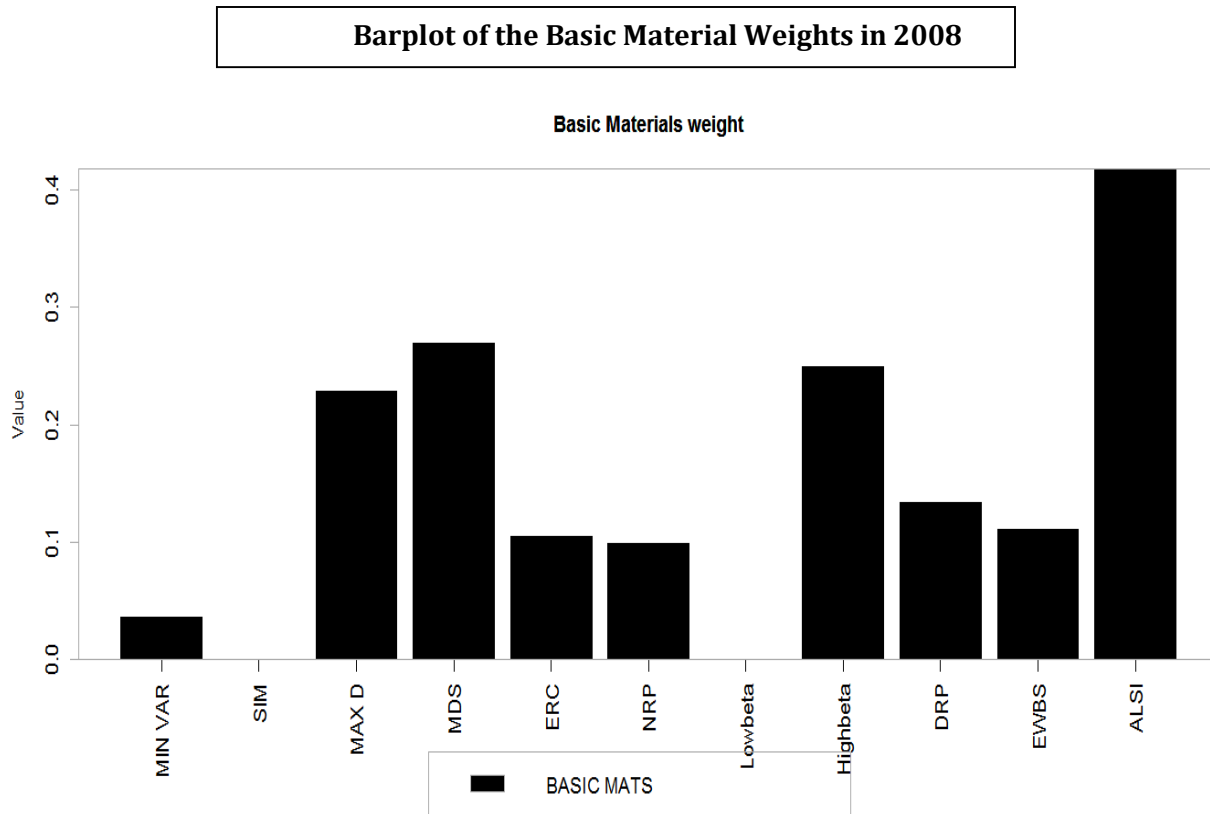


Figure 4-7: Low Volatility Portfolio weights for the Basic Materials sector and its ALSI weights in 2008

Notes: The figure above depicts the Basic Material weights generated in 2008 for the low volatility portfolios and the ALSI. The weights were derived from the back-testing of the low volatility portfolios rebalanced annually, over the period January 2006-December 2013.

From Figure 4-7, it is evident that the weight assigned to the Basic Material sector is the highest (about 0.42), whereas the Basic material weights in the SIM and the Lowbeta portfolios are 0. The sector allocation effect (or contribution) for the basic materials sector is calculated by multiplying the difference between the weights of the basic materials in the low volatility portfolios (w_{bmp}) and the benchmark (w_{bmb}), with the difference between the basic materials benchmark return (b_{bmb}) and the overall portfolio benchmark return (b) in 2008:

$$\text{Section Allocation} = (w_{bmp} - w_{bmb}) * (b_{bmb} - b)$$

Thus, the objective is to examine the performance due to the allocation of weights (Basic Materials weights) in the low volatility portfolios relative to the benchmark (ALSI). The Basic

Material benchmark return in 2008 was found to be -0.189, conversely, the overall ALSI return was -0.308. The Table 4-3 that follows shows the sector allocation effect for the Basic Material sector.

Table 4-3: Sector Allocation Effect for Basic Materials

Low volatility portfolios	Sector Allocation Effect
MIN VAR	-0.045
SIM	-0.050
MAX D	-0.022
MDS	-0.017
ERC	-0.037
NRP	-0.038
Lowbeta	-0.049
Highbeta	-0.019
DRP	-0.033
EWBS	-0.036

Table 4-3 reveals that the low volatility portfolios resulted in a negative sector allocation effect due to the portfolios' underweight position in the Basic Material sector (Figure 4-7) in 2008 even though its returns (basic material sector) in the benchmark outperformed the overall benchmark return in period 2008. Thus, one can attribute the poor performance of the low volatility portfolios in 2008 to an underweight position in the Basic Materials sector of these portfolios relative to the benchmark.

4.4 Risk and Diversification characteristics of the FTSE/JSE sector Low Volatility Portfolios

The low volatility strategies are constructed in order to achieve different characteristics. They are to reduce portfolio risk (MIN VAR), reduce concentration in the portfolio (EWWS and EWBS), improve diversification in the portfolio (MAX D, MDS and DRP), reduce systematic risk (lowbeta), and allocate equal risk budget (ERC and NRP). Lee (2011) proposed to examine each portfolio based on their risk contribution profile. (Lohre and

Zimmer 2011; Lohre et al. 2012) went further by evaluating each strategy based on their risk contribution by principal portfolios and the number of uncorrelated bets (Meucci, 2009). Assessing the principal portfolios’ risk profile aims at identifying the degree of concentration in terms of the effective number of bets (Meucci, 2009). Similar to Lohre and Zimmer (2011) and Lohre et al. (2012), Table 4-4 reports the portfolio strategies concentration (using the Gini index), and the number of uncorrelated bets (Equation 40).

The Gini index is a measure of dispersion using the Lorenz curve (Maillard, Roncalli, and Teiletche, 2008). The Lorenz curve is a graphical representation of the cumulative distribution of the distribution of wealth in a society, where the statistics of interest may be the income of a population. Mathematically, the Gini index G is computed as:

$$G = 1 - 2 \int_0^1 L(x)dx, \text{ where, } L(x) \text{ is the Lorenz curve.}$$

Applying this concept to the low volatility portfolios, the statistics of interest become the weights and risk contributions of a portfolio. The Gini Weight (GW) measures the average sector weight concentration while the Gini Risk (GR) measures the average sector risk decomposition for the low volatility portfolios. Appendix A.1-A.13 depicts the time series of the FTSE/JSE sectors weights and their risk contribution over the period 2005-2013. Interestingly, the risk contributions for each strategy tends to mirror the evolution of weights over the whole period. Below are the GWs, and GRs for the ALSI, SIM, MIN VAR, MAX D and MDS:

Table 4-4: FTSE/JSE sector weight and risk characteristics (January 2006 – December2013)

	Lowbeta	SIM	NRP	ERC	MAX D	MIN VAR	EWBS	MDS	DRP	Highbeta	ALSI
GW	0.44	0.55	0.11	0.10	0.48	0.63	0.00	0.58	0.79	0.56	0.49
GR	0.50	0.54	0.08	0.06	0.47	0.62	0.11	0.59	0.80	0.60	0.57
GPPR	0.81	0.80	0.88	0.88	0.85	0.78	0.88	0.85	0.71	0.85	0.85
N_{ENT}	2.04	2.27	1.23	1.22	1.53	2.37	1.10	1.59	3.29	1.52	1.57

From Table 4.4, it is evident that:

- The GW for the ALSI had a weight of 0.49 and GR of 0.57. Basic Materials, Financials and Consumer Goods were the dominant sectors in the index while the Technology, Health care, Oil and Gas, Consumer Services and Industrials were the least dominant sectors (see Appendix A.1).
- The GWs for the SIM, and MIN VAR (0.55 and 0.63) are also concentrated (see Appendix A.2 and A.3), with a GR of 0.54 and 0.62 respectively. This perhaps is not surprising, given that the minimum variance strategy assigns weights to low beta sectors specifically the Consumer Goods, Health care, Financials and Industrials.
- The GW for MAX D and MDS are 0.48 and 0.58, while their respective GR values are 0.47 and 0.59 (see Appendix A.6 and A.7). The DRP seems to be rather concentrated (see Appendix A.11) in some sectors before 2008 (Financials, Industrials and Basic Materials) and after 2008 (Health Care, Consumer Services and Technology) with a GW and GR of 0.76 and 0.80 respectively.

Furthermore, for the Lowbeta versus Highbeta portfolio, the GW and GR for the Lowbeta (0.44 and 0.50) are lower than the Highbeta (0.56 and 0.60). The Lowbeta portfolio is dominated by low beta sectors. Over the sample period, financials, health care, consumer goods, consumer services, and industrials were mostly included in the equal weight low beta portfolio (see Appendix A.9). However, the Highbeta portfolio (see Appendix A.10) is more often dominated by Oil and Gas, Basic Materials and Technology. Nonetheless, for the EWBS, ERC and NRP strategies, the GW (0, 0.10 and 0.11) and GR (0.11, 0.06 and 0.08) do not show any signs of weights or risk concentration and are more stable²⁵ compared with the other strategies (See details in Appendix A.4, A.5 and A.8).

Lastly, the average Gini Principal Portfolio Risk (GPPR) and the average number of uncorrelated bets (N_{ENT})²⁶ over time examines the sector risk decomposition by principal portfolios and the uncorrelated risks sources embedded in each strategy. Most strategies had a GPPR of more than 80% (see Appendix C.1-C.6). In terms of the number of uncorrelated

²⁵ This is similar to results obtained by Leclerc et al. (2013)

²⁶ Note that a portfolio that has an effective number of bets of 9 for the FTSE/JSE sectors allocates equal risk budget along principal portfolios.

bets, the DRP is a more diversified portfolio (3.29)²⁷ than the other strategies. The ALSI, ERC, NRP, Highbeta, EWBS, MAX D, MDS are also considered as 1-bet strategies. However, the SIM, MIN VAR and Lowbeta show on average a 2-bet strategy, with the MIN VAR (2.37) a more diversified portfolio when compared with SIM (2.27) or the Lowbeta (2.04).

Having analyzed the risk, return, and diversification characteristics of the low volatility portfolios, a covariance bi-plot can also be plotted to help one visualize the performance of the portfolios relative to each other, the benchmark, and other securities (Khuzwayo, 2011). The next section defines the covariance bi-plot and applies it to the low volatility portfolios and the FTSE/JSE sectors.

4.5 Covariance Bi-plot of the Low Volatility Portfolios using the FTSE/JSE sectors

The concept of the covariance bi-plot²⁸ was proposed by Barr, Underhill, and Kahn (1990); Gabriel (1971); and Underhill (1990). The covariance bi-plot is a technique for displaying multivariate data, where the length of the bi-plot vectors from the origin is equal to the standard deviation, and the angle between two bi-plot vectors is the correlation between them. The concept can also be used to analyze the risk measures of a portfolio. On the covariance bi-plot, one can view on a single plot, the beta, volatility, unique risk, correlation, exposure to a benchmark, and tracking error of a portfolio relative to the benchmark (Khuzwayo, 2011). In that regard, similar to reported risk measures in Table 4-1, Figure 4-8 also depicts the covariance bi-plot of the low volatility portfolios, depicted alongside the FTSE/JSE sectors, the JSE small-, and mid-cap, value and growth indices, while using the ALSI as the benchmark.

²⁷ However, Lohre, Opfer, and Ország (2012) and Lohre and Zimmer (2011) noted that the DRP still did not fulfill the targets of equal risk contribution along principal portfolios due to the imposition of the long-only constraint.

²⁸ See Appendix for an explanation of how the covariance bi-plot is plotted and how one can interpret the different risk measures from the plot.

Covariance Bi-plot of the Sector-based Low volatility

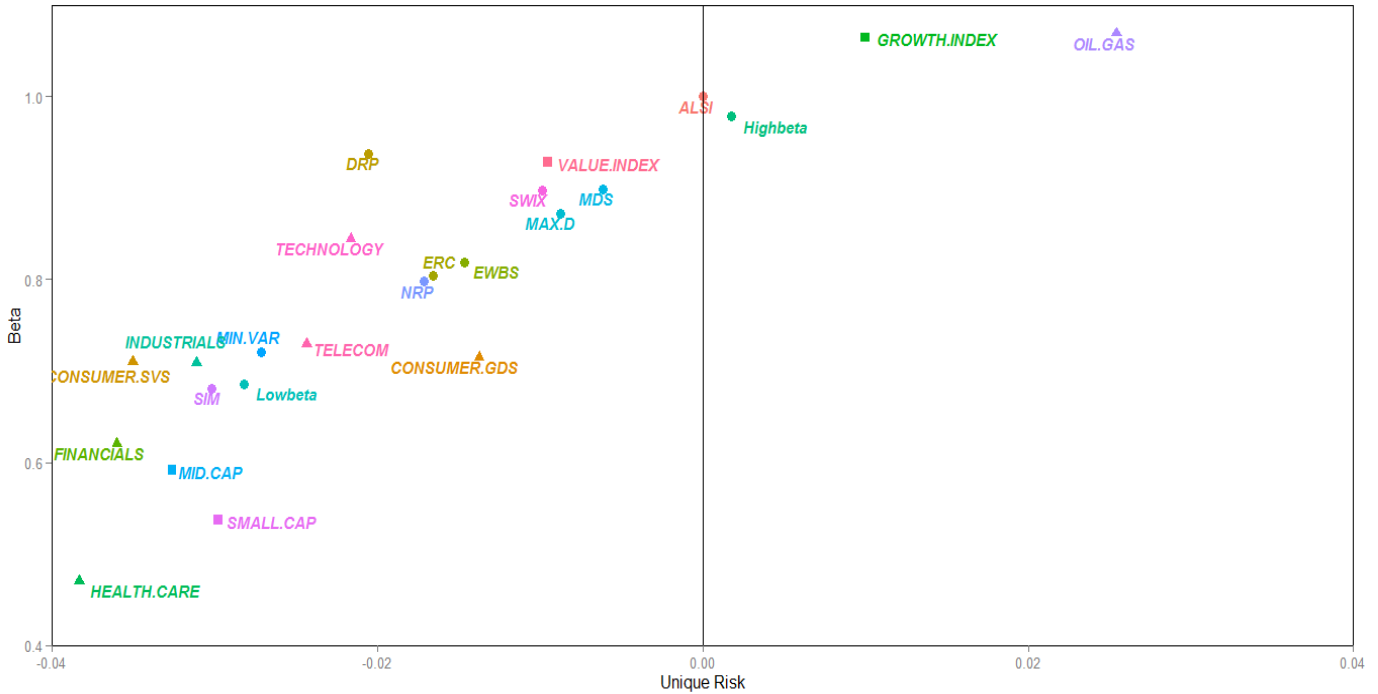


Figure 4-8: Covariance Bi-plot of Low Volatility Portfolios, with the ALSI and the FTSE/JSE sectors, January 2006-December 2013

From Figure 4-8, the low volatility portfolios, ALSI and cash are depicted as a solid circle, whereas the sectors are represented as a solid triangle and the other JSE indices, denoted as solid squares. It is interesting to observe the contrast between the basic material sector and the oil and gas sector (on the right) and the other sectors positioned on the left. In addition, one notices the exposure of the Financial, Consumer Services and Industrial sector to the Lowbeta, SIM and MIN VAR portfolios. Similarly, the Technology and Telecoms sector are more exposed to the ERC, NRP, and the EWBS portfolios. Furthermore, one notices the beta of the low volatility portfolios positioned on the bottom left of the covariance bi-plot relative to the benchmark (ALSI). This is similar to betas calculated in Table 4-2. Moreover, the volatility of the low volatility portfolios depicted as the distance from cash to each portfolio is also lower compared with the ALSI. Interestingly, the MD and MDS located close to the value index suggests that they have high exposure the value index. In the next section, the performance summary of the low volatility portfolios is assessed using the JSE Top 40 stocks.

4.6 Performance summary of the Low Volatility Portfolios using the JSE Top 40 stocks

The Table 4-5 and Figure 4-9 depict the performance summary of the low volatility portfolios rebalanced annually using the JSE Top 40 stocks adjusted for turnover and transaction costs. Boxplots of the portfolio weights generated are also depicted in Appendix D.

Table 4-5: Performance summary of the low volatility portfolios using the JSE Top 40 stocks, January 2007- December 2013

	MIN VAR	SIM	Lowbeta	EWWS	NRP	ERC	EWBS	MAX D	DRP	MDS	Highbeta	ALSI
Annualized Return	19%	20%	21%	19%	18%	17%	17%	14%	17%	12%	12%	16%
Annualized Risk	12%	13%	15%	15%	14%	14%	15%	15%	22%	18%	26%	17%
Annualized Sharpe	1.57	1.52	1.43	1.32	1.25	1.23	1.15	0.98	0.79	0.68	0.44	0.95
Drawdown	0.17	0.24	0.27	0.23	0.27	0.27	0.27	0.29	0.42	0.38	0.50	0.41
Beta	0.47	0.52	0.59	0.77	0.73	0.75	0.80	0.75	0.90	0.88	1.39	
Beta+	0.29	0.38	0.42	0.71	0.70	0.71	0.78	0.64	0.80	0.75	1.48	
Beta-	0.17	0.19	0.38	0.51	0.40	0.45	0.48	0.60	1.15	0.85	1.47	
Timing ratio	1.75	2.03	1.11	1.39	1.77	1.57	1.62	1.07	0.69	0.88	1.01	
R-squared	0.42	0.42	0.45	0.79	0.75	0.79	0.82	0.72	0.50	0.66	0.81	
Kurtosis	3.83	3.67	3.10	2.98	2.97	2.92	3.06	2.17	3.78	3.53	5.32	
Skewness	-0.32	-0.42	-0.23	0.11	0.10	0.13	0.22	-0.06	-0.29	-0.14	-0.23	
correlation	0.65	0.65	0.67	0.89	0.87	0.89	0.91	0.85	0.71	0.81	0.90	
Tracking Error	0.13	0.13	0.13	0.08	0.08	0.08	0.07	0.09	0.15	0.11	0.13	
Information Ratio	0.51	0.60	0.66	0.88	0.64	0.64	0.64	0.23	0.34	-0.01	-0.07	
No of stocks	13	20	8	40	40	40	40	16	4	11	8	
GW	0.83	0.72	0.80	0.41	0.15	0.15	0.00	0.75	0.94	0.82	0.80	
GR	0.84	0.72	0.85	0.50	0.16	0.09	0.22	0.74	0.95	0.86	0.82	
GPPR	0.63	0.59	0.54	0.66	0.76	0.76	0.78	0.69	0.21	0.65	0.70	
<i>N</i>_{ENT}	2.26	2.60	2.78	2.10	1.37	1.37	1.20	1.84	4.54	2.14	1.72	

Low Volatility Techniques

Lowbeta-Equal weight Low-beta SIM-Low beta Single Index model NRP- Naïve Risk Parity ERC-Equal Risk Contribution

MAXD- Maximum diversification MIN-VAR- Minimum variance EWBS- Equal weight by Sector MDS- Maximum Decorrelated Strategy

DRP- Diversified Risk Parity EWWS- Equal weight within Sector ALSI- All share Index

Out-of-Sample Cumulative Returns of the Stock-based Low volatility portfolios.

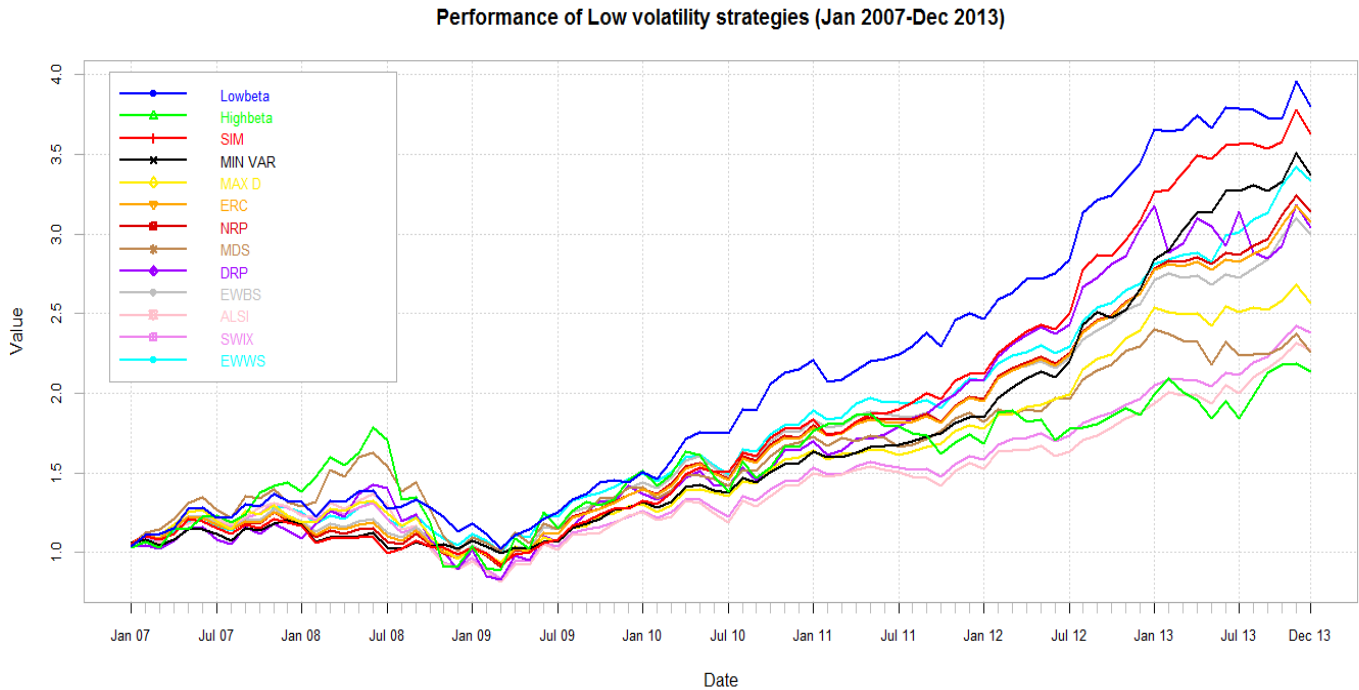


Figure 4-9: Out-of-Sample Performance of the JSE Top 40 stocks low volatility portfolios rebalanced annually, January 2007- December 2013

From Table 4-5 and Figure 4-9, it is evident that all portfolio strategies have outperformed the ALSI, except the Highbeta portfolio, which is not surprising given its preference for high beta or high risk stocks (see Appendix D.8). The MIN VAR, SIM, Lowbeta portfolios and EWWS had the highest Sharpe ratios (1.57, 1.52, 1.43, and 1.32 respectively) and the lowest drawdowns (0.17, 0.24, 0.27 and 0.23). The MIN VAR and SIM portfolios posted the lowest annualized risk (12% and 13%), and are predominantly comprised of the low beta and low volatility stocks like Remgro, Mediclinic, Growthpoint and Sabmiller (see Table 4-5, and Appendix D.1-D.2). However, High volatility stocks like Exxaro and Goldfields were given a low weight in the ERC and NRP portfolios, whereas Remgro posted the highest weight (Appendix D.3-D.4). Interestingly, the volatility of the MIN VAR is less than the ERC, which in turn is less than EWBS in line with Maillard, Roncalli, and Teiletche (2008), who showed that the volatility of the ERC portfolio is between the MIN VAR portfolio and the EWBS portfolio. Further, Intu, Golfields and Anglogold had the highest weights in the MDS and MAX D

portfolios (see Appendix D.6 and D.7), since they have a low average correlation with other stocks (see Appendix B). Moreover, the tracking errors for the DRP, MIN VAR, SIM, Lowbeta and Highbeta (0.15, 0.13, 0.13, 0.13, and 0.131 respectively) are higher than the other strategies. Furthermore, the timing ratio (see section 4.2) for the SIM was 2.03, which is the highest compared with the other strategies. In order to assess the risk and diversification characteristics of the JSE Top 40 stocks, Table 4-5 also presents the GW, GR, GPPR, and the N_{ENT} . The GWs for the EWBS, NRP, and ERC (0, 0.15, and 0.15) have the lowest concentration in terms of weight. Similarly, the GR for these portfolios is less concentrated in terms of risk. All strategies tend to be concentrated in terms of the stocks risk decomposition by principal portfolios except the DRP strategy (0.21). However, the DRP strategy posted a higher realized risk (0.22), which resulted in a lower risk-adjusted returns. Similarly, the risk-return analysis of the low volatility portfolios is depicted in Figure 4-10.

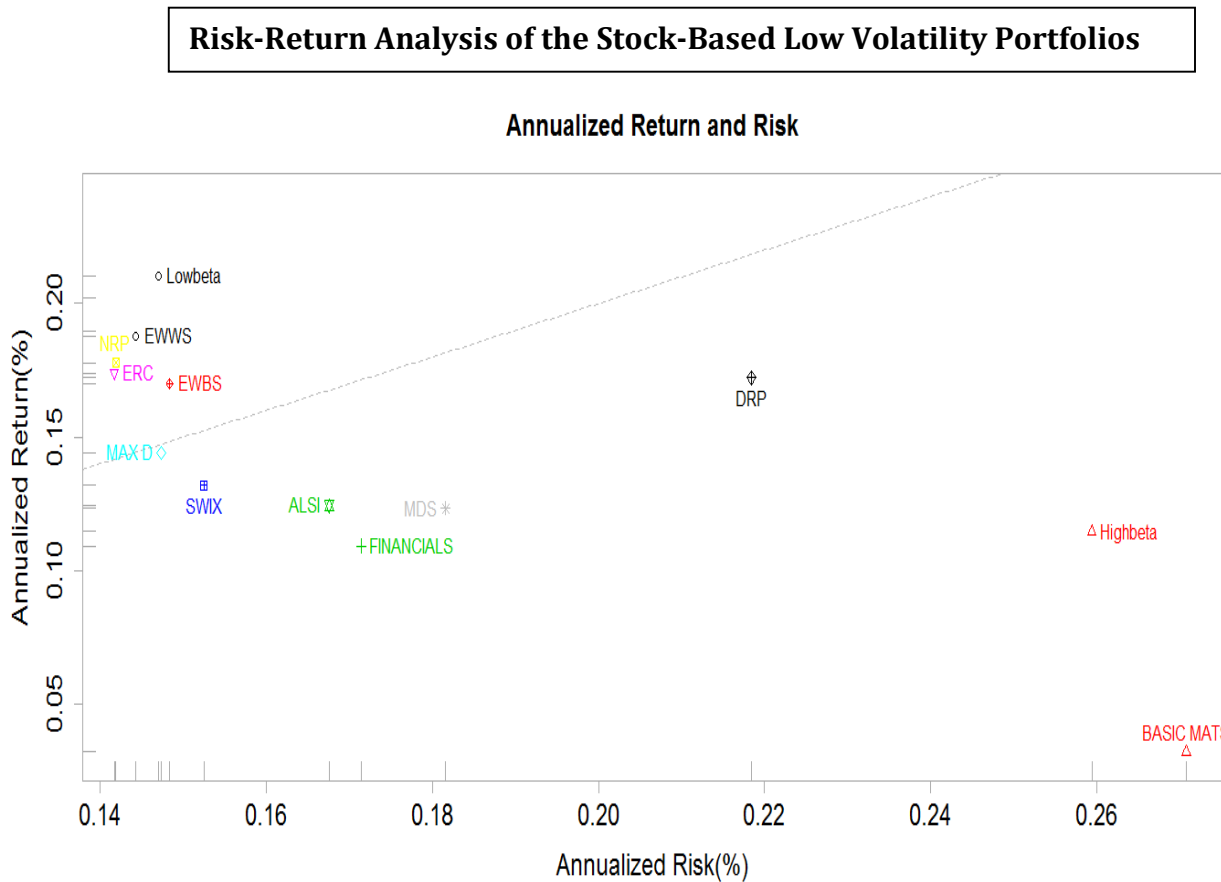


Figure 4-10: Risk-Return Analysis of the Low Volatility Portfolios, January 2007- December 2013

From Figure 4-10, it is interesting to observe the performance of the EWWS portfolio when compared with the EWBS portfolio. The EWWS portfolio was able to outperform the EWBS portfolio with almost the same amount of risk. This is as a result of the EWWS portfolio allocating equal weights within sectors, thereby avoiding the inherent sector bias in the EWBS portfolios (Velvadapu 2011). In the next section, the covariance bi-plot is depicted to show the relationships between the low volatility portfolios and the

4.7 Covariance Bi-plot of the Low Volatility Portfolios Using JSE Top 40 stocks

Similar to section 4.5, the covariance bi-plot of the Low volatility portfolios is also depicted in Figure 4-8, together with the value, small cap-, mid cap-, and growth JSE indices. The low volatility portfolios are depicted as solid points, whereas the JSE Indices are represented as solid triangles.

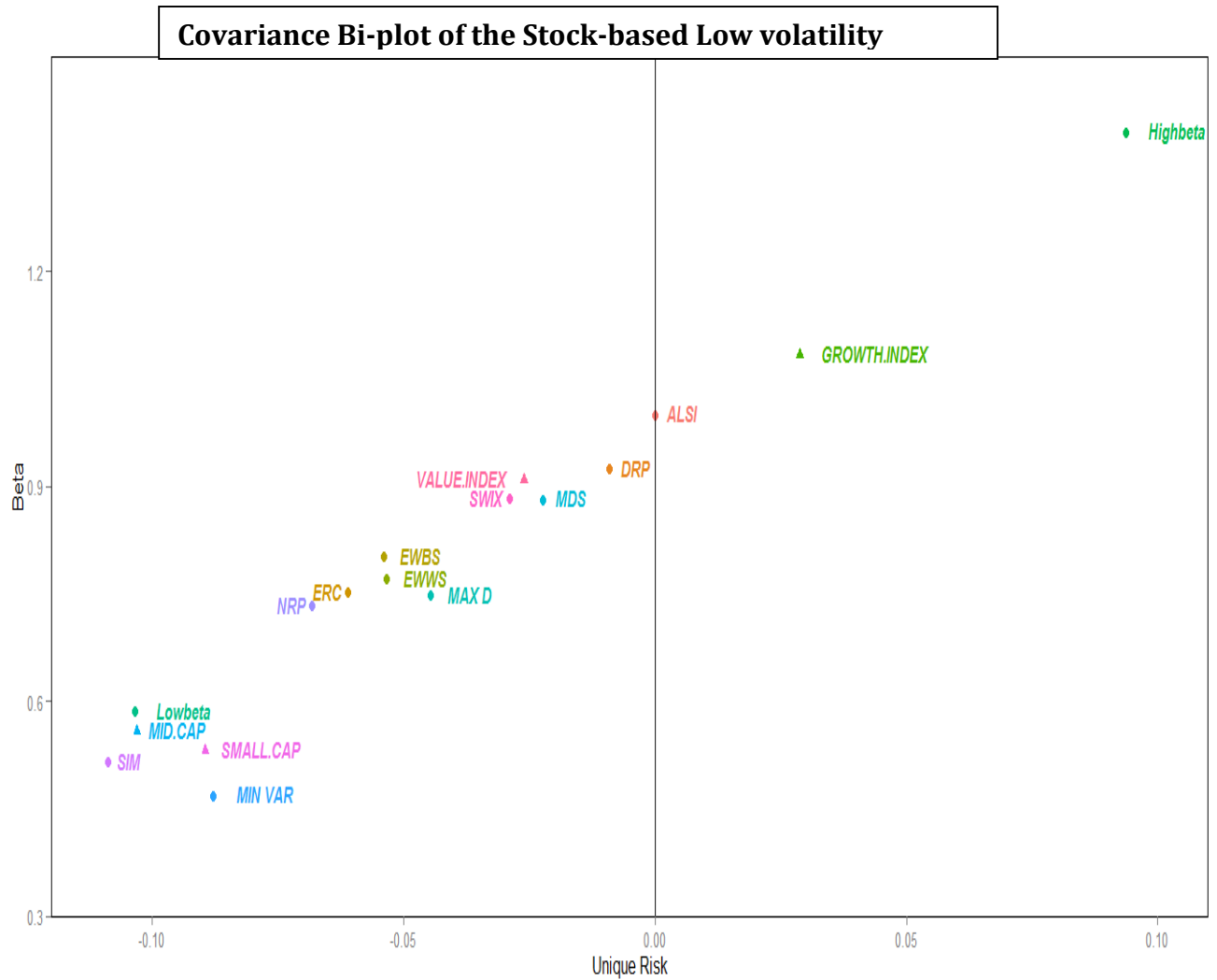


Figure 4-11: Covariance Bi-plot of Low Volatility Portfolios, with the ALSI and the JSE Indices, January 2007-December 2013

From Figure 4-11, one notices that the low volatility portfolios are positioned close to the mid-, and small-Cap indices (bottom left of the covariance bi-plot), which implies that they have a high exposure to these indices. This is consistent with works of Choueifaty and Coignard, (2008) and Chow, Hsu, Kalesnik, and Little (2011), who showed that the outperformance of constituents based AEIs are more than often driven by the exposure to the small cap factor. In contrast, the Highbeta portfolio has a high exposure to the Growth Index (top right of the covariance bi-plot). Furthermore, one also notices that the MAX D, ERC, EWWS, EWBS, and the NRP portfolios are closely related, implying a high correlation between these portfolios.

4.8 Robustness Test

This section examines the analysis of whether the previous results obtained in section 4-2, 4-3, and 4-5 will hold by rebalancing using a monthly frequency and using a larger universe of stocks (JSE Top 100 stocks instead of the JSE Top 40 stocks). The Table 4-6 and Table 4-7 shows the detailed statistics related to the rebalancing frequency (monthly) of the FTSE/JSE sector and the JSE Top 40 stocks.

Table 4-6: Performance of the FTSE/JSE sectors low volatility portfolios rebalanced monthly, January 2007- December 2013

	Lowbeta	SIM	NRP	ERC	MAX D	EWBS	MIN VAR	MDS	DRP	Highbeta	ALSI
Annualized Return	23%	21%	21%	21%	20%	20%	20%	21%	18%	16%	16%
Annualized Risk	15%	15%	15%	15%	15%	15%	15%	16%	19%	17%	17%
Annualized Sharpe (Rf=0%)	1.51	1.42	1.39	1.38	1.31	1.35	1.31	1.28	0.94	0.89	0.95
Drawdown	0.31	0.33	0.32	0.32	0.37	0.31	0.37	0.38	0.44	0.36	0.41
Skewness	-0.41	-0.35	-0.09	-0.08	-0.29	-0.28	-0.04	-0.30	0.09	0.18	-0.08
Kurtosis	3.44	3.25	3.30	3.30	3.51	3.28	3.36	3.59	3.18	3.96	3.62
Beta	0.7	0.69	0.79	0.8	0.76	0.81	0.76	0.89	0.85	0.94	
Beta+	0.67	0.65	0.76	0.77	0.68	0.79	0.68	0.83	0.82	0.92	
Beta-	0.49	0.53	0.64	0.68	0.74	0.66	0.74	1.03	1.04	0.89	
R-squared	0.62	0.62	0.83	0.84	0.7	0.85	0.7	0.85	0.61	0.86	
Correlation	0.79	0.79	0.91	0.92	0.84	0.92	0.84	0.92	0.78	0.93	
Tracking Error	0.11	0.11	0.07	0.07	0.09	0.07	0.09	0.07	0.12	0.07	
Information Ratio	0.63	0.48	0.61	0.62	0.41	0.6	0.41	0.72	0.11	-0.12	
Timing Ratio	1.37	1.23	1.19	1.13	0.92	1.18	0.92	0.8	0.79	1.03	
GW	0.44	0.52	0.11	0.1	0.63	0	0.63	0.56	0.79	0.56	
GR	0.49	0.51	0.08	0.06	0.63	0.11	0.63	0.57	0.8	0.6	
<i>N_{ENT}</i>	2	2.07	1.19	1.19	2.24	1.08	2.24	1.61	3.16	1.55	

Low Volatility Techniques

Lowbeta-Equal weight Low-beta SIM-Low beta Single Index model NRP- Naïve Risk Parity ERC-Equal Risk Contribution
 MAX D- Maximum diversification MIN-VAR- Minimum variance EWBS- Equal weight by Sector MDS- Maximum Decorrelated Strategy
 DRP- Diversified Risk Parity EWWS- Equal weight within Sector ALSI- All share Index

Table 4-7: Performance of the JSE Top 40 low volatility portfolios rebalanced monthly, January 2007- December 2013

	SIM	Lowbeta	EWWS	MIN VAR	NRP	ERC	MAX D	EWBS	MDS	DRP	Highbeta	ALSI
Annualized Return	19%	19%	19%	16%	18%	17%	17%	17%	14%	11%	11%	16%
Annualized Risk	13%	14%	14%	13%	14%	14%	14%	15%	16%	16%	25%	17%
Annualized Sharpe (Rf=0)	1.42	1.37	1.35	1.29	1.26	1.23	1.21	1.16	0.86	0.7	0.42	0.95
Drawdown	0.24	0.21	0.2	0.27	0.26	0.26	0.27	0.26	0.26	0.27	0.5	0.41
Skewness	-0.32	-0.42	-0.23	0.11	0.10	0.13	0.22	-0.06	-0.29	-0.14	-0.23	
Kurtosis	3.83	3.67	3.10	2.98	2.97	2.92	3.06	2.17	3.78	3.53	5.32	
Beta	0.5	0.54	0.77	0.53	0.71	0.73	0.71	0.77	0.75	0.75	1.34	
Beta+	0.34	0.29	0.8	0.31	0.71	0.71	0.71	0.8	0.71	0.73	1.49	
Beta-	0.13	0.32	0.37	0.31	0.33	0.4	0.33	0.37	0.56	0.56	1.39	
R-squared	0.41	0.43	0.76	0.51	0.7	0.76	0.7	0.76	0.61	0.61	0.8	
Correlation	0.64	0.66	0.87	0.71	0.84	0.87	0.84	0.87	0.78	0.78	0.89	
Tracking Error	0.13	0.13	0.08	0.12	0.09	0.08	0.09	0.08	0.11	0.11	0.13	
Information Ratio	0.48	0.49	0.58	0.31	0.6	0.59	0.53	0.58	0.13	-0.11	-0.15	
No of stocks	13	8	40	20	40	40	16	40	11	4	8	
Timing Ratio	2.64	0.9	1.75	1.01	2.17	1.78	2.16	2.14	1.26	1.29	1.07	
GW	0.72	0.8	0.44	0.83	0.15	0.15	0.75	0	0.82	0.95	0.8	
GR	0.72	0.85	0.5	0.83	0.17	0.09	0.79	0.23	0.87	0.95	0.82	
<i>N_{ENT}</i>	2.51	2.75	1.29	2.25	1.35	1.37	2.09	1.2	2.31	4.09	1.67	

Low Volatility Techniques.

Lowbeta-Equal weight Low-beta SIM-Low beta Single Index model NRP- Naïve Risk Parity ERC-Equal Risk Contribution
 MAX D- Maximum diversification MIN-VAR- Minimum variance EWBS- Equal weight by Sector MDS- Maximum Decorrelated Strategy
 DRP- Diversified Risk Parity EWWS- Equal weight within Sector ALSI- All share Index

From Table 4-6 and Table 4-7, one notices that the performance of the low volatility portfolios is not sensitive to the rebalancing frequencies (annually or monthly). The ranking of the low volatility portfolios by Sharpe ratios is similar to the ranking of the Sharpe ratios in the annual rebalancing frequency (Table 4-1 and Table 4-2). In addition, the weights and

CHAPTER FOUR-EMPIRICAL RESULTS

risk concentration of the low volatility portfolios shows similar characteristics as the annual rebalancing frequency. Thus, these results show that the low volatility strategies are robust to the rebalancing frequency and continue to outperform the market capitalization portfolio. In the same vein, Table 4-8 also reports the low volatility portfolios rebalanced annually using the JSE Top 100 stocks.

Table 4-8: Performance of the JSE Top 100 low volatility portfolios rebalanced annually, January 2007- December 2013

	SIM	Lowbeta	MIN VAR	EWWS	ERC	NRP	EWBS	MAX D	MDS	DRP	Highbeta	ALSI
Annualized Return	17%	19%	14%	19%	17%	18%	17%	14%	17%	14%	12%	16%
Annualized Risk	10%	12%	9%	14%	13%	13%	13%	12%	15%	18%	21%	17%
Annualized Sharpe (Rf=0%)	1.70	1.56	1.52	1.40	1.36	1.36	1.29	1.23	1.15	0.76	0.55	0.95
Drawdown	0.18	0.25	0.11	0.28	0.29	0.29	0.30	0.25	0.29	0.43	0.48	0.41
Beta	0.34	0.44	0.26	0.71	0.61	0.60	0.66	0.52	0.58	0.85	1.17	
Beta+	0.21	0.28	0.20	0.52	0.44	0.43	0.49	0.39	0.48	0.75	1.11	
Beta-	0.17	0.23	0.11	0.62	0.44	0.40	0.49	0.46	0.49	1.17	1.33	
R-squared	0.31	0.37	0.22	0.77	0.65	0.62	0.69	0.55	0.43	0.60	0.86	
Correlation	0.56	0.61	0.47	0.88	0.81	0.78	0.83	0.74	0.66	0.77	0.93	
Tracking Error	0.14	0.13	0.15	0.08	0.10	0.10	0.09	0.11	0.13	0.12	0.08	
Information Ratio	0.35	0.49	0.12	0.85	0.49	0.49	0.53	0.15	0.38	0.12	-0.11	
Timing Ratio	1.25	1.23	1.84	0.83	1.00	1.09	1.01	0.84	0.97	0.64	0.83	
GW	0.86	0.80	0.93	0.46	0.18	0.17	0.00	0.83	0.89	0.78	0.80	
GR	0.88	0.86	0.95	0.57	0.14	0.21	0.26	0.78	1.00	0.85	0.83	
N_{ENT}	2.07	2.18	1.73	1.15	1.33	1.36	1.22	1.39	1.77	3.00	1.73	

Low Volatility Techniques.

Lowbeta-Equal weight Low-beta SIM-Low beta Single Index model NRP- Naïve Risk Parity ERC-Equal Risk Contribution
 MAXD- Maximum diversification MIN-VAR- Minimum variance EWBS- Equal weight by Sector MDS- Maximum Decorrelated Strategy
 DRP- Diversified Risk Parity EWWS- Equal weight within Sector ALSI- All share Index

CHAPTER FOUR–EMPIRICAL RESULTS

From Table 4-8, one notices that increasing the number of stocks improves the performance of the low volatility portfolios on the risk-adjusted basis. Intuitively, this result is not surprising given that the opportunity set of the JSE Top 100 stocks is larger than the JSE Top 40 stocks. The next section shows the covariance bi-plot of the JSE Top 100 plotted with the JSE indices (the small-cap, mid-Cap, value Index, and the growth Index).

4.9 Covariance Bi-plot of the Low Volatility Portfolios using the JSE Top 100 stocks

Similar to section 4.5 and 4.7, a covariance bi-plot showing the low volatility portfolios and the JSE Top 100 stocks is illustrated in Figure 4-12, together with the Small-Cap, Mid-Cap, Value Index, and Growth Index.

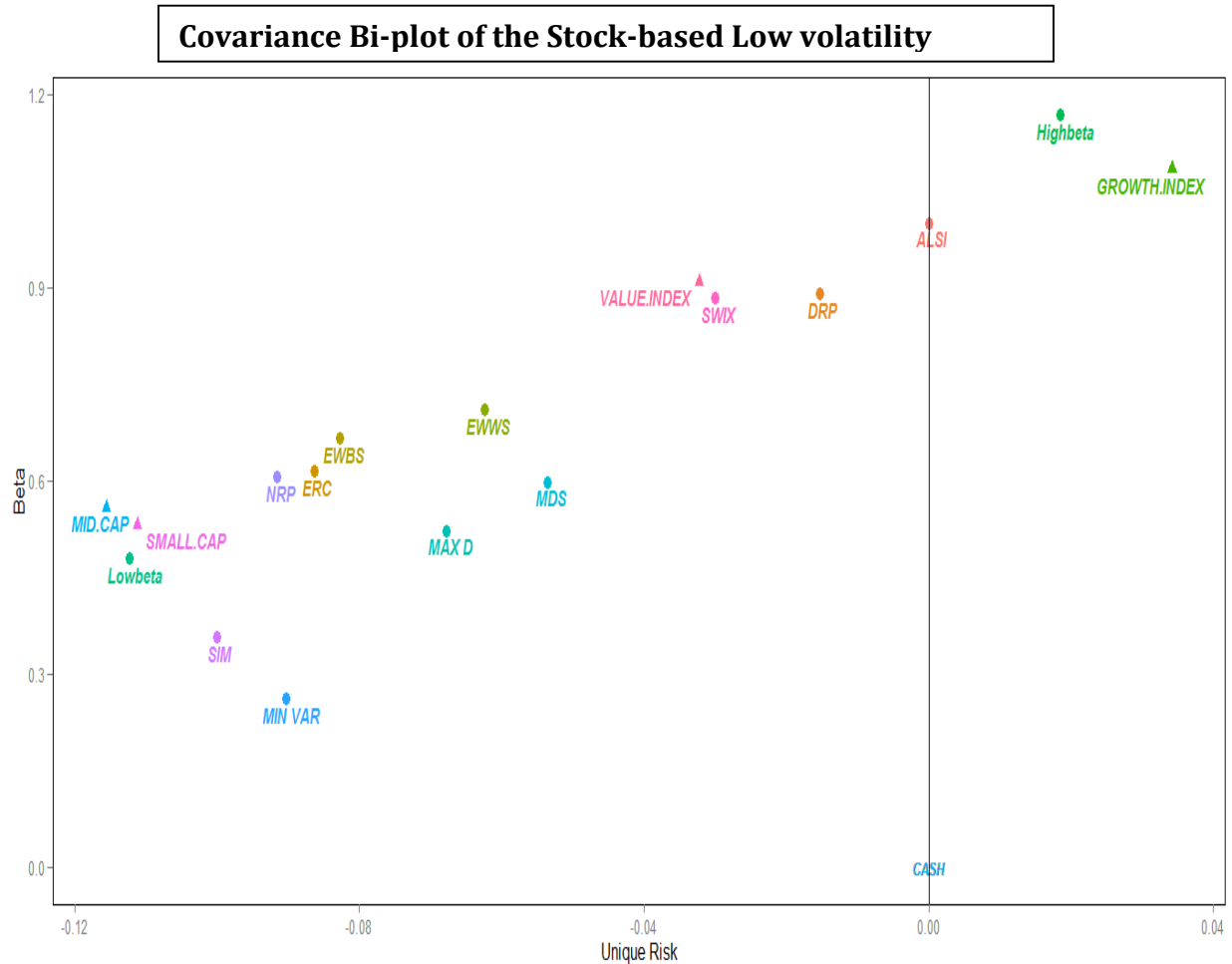


Figure 4-12: Covariance Bi-plot of Low Volatility Portfolios, with the ALSI and the JSE Top 100 stocks, January 2007-December 2013

From Figure 4-12, it is evident that the low volatility portfolios are exposed to the Small-cap, and Mid-cap JSE indices. One also notices that the low volatility portfolios are closely related to each other due to their close proximity to one another. Specifically the ERC, NRP and EWBS portfolios.

4.10 Chapter Summary

In this Chapter, the performance of the low volatility portfolios was assessed using both the FTSE/JSE sectors and the JSE Top 40 stocks. The risk and diversification characteristics of the low volatility portfolios were also examined. In addition, a covariance bi-plot was depicted to help one visualize the relationships of the low volatility portfolios relative to the ALSI. From the analysis of the low volatility portfolios using both the sectors and stocks, it was evident that:

- The MIN VAR, Lowbeta and SIM portfolios have consistently outperformed other portfolios and the market capitalization portfolio. This is due to their superior Sharpe ratios throughout the sample period.
- The MIN VAR, Lowbeta and SIM portfolios have also produced the lowest risk and the least drawdown over the period.
- The MIN VAR, Lowbeta and SIM portfolios were also exposed to low beta and low volatility stocks.

Furthermore, from the analysis of the rolling 12-month return of the low volatility portfolios, it was found that the low volatility portfolios have outperformed the ALSI in most periods bar 2008, where the Basic Materials, the Highbeta portfolio and the ALSI outperformed the portfolios. Consequently, the sector allocation effect was used to explain the underperformance of the low volatility portfolios. The analysis of the sector allocation effect suggests that the low volatility portfolios resulted in a negative sector allocation effect due to the portfolios' underweight position in the Basic Material sector.

In addition, the analysis of the covariance bi-plot of the low volatility portfolios using the sectors and stocks suggest that:

- The MIN VAR, Lowbeta and SIM portfolios were most often exposed to the Small-Cap index.
- The MIN VAR, Lowbeta and SIM portfolios were closely related due to their close proximity with one another bi-plot.

5 Blending of Low Volatility Portfolios with the ALSI and SWIX Indices

This chapter presents the empirical results from the blending of the low volatility portfolios with the ALSI and SWIX indices. This is in line with the second objective of this study, which is to investigate the performance of the blended portfolios and to assess their usefulness as effective portfolio strategies.

Having assessed the performance of the low volatility portfolios under the constraint of full investment, one could imagine that most portfolio managers (or investors') may not be willing to invest all their capital in these portfolios (i.e. the low volatility portfolios). In light of that, the portfolio manager will instead make a decision to invest a smaller amount of capital in the low volatility portfolios, and a larger portion in a portfolio which is proxied here by an index. In addition, for portfolio managers who are mandated to beat the benchmark or to follow the benchmark closely, investing in the blended portfolio can offer an interesting alternative for portfolio managers to beat the benchmark at a lower risk, and hence produce a high risk-adjusted return (Baker et al. 2011). As a result, the blended portfolios will also have a lower tracking error. The blended portfolios are constructed by taking a combination of X% in the benchmark (in this case, the ALSI and the SWIX indices) and (1-X)% in the low volatility portfolios. Empirical results of these blended portfolios include investing 10%, 40%, and 50% in the low volatility portfolios while investing 90%, 60%, and 50% respectively in the ALSI and SWIX indices.

5.1 Empirical Results of the Blending of Low Volatility Portfolios with the ALSI and the SWIX Indices using the FTSE/Sectors

Figures 5-1, 5-2 and 5.3 which follows depict the risk-return analysis of the 50-50, 40-60, and 10-90 blended portfolios and the ALSI. The MIN VAR, SIM, ERC, NRP, EWBS, MAX D, MDS, Lowbeta, and Highbeta portfolio are denoted as MV, SIM, ERC, NRP, EWBS, MD, MDS, Lowb, and Highb appended with its respective blends (for example MV50.50 denotes the minimum variance portfolio blended with the ALSI).

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

Sector-based Low Volatility Portfolios blended with the ALSI. Figure 5-1 is the 50-50 blend, Figure 5-2 is the 40-60 blend, while Figure 5-3 is the 10-90 blend, January 2006- December

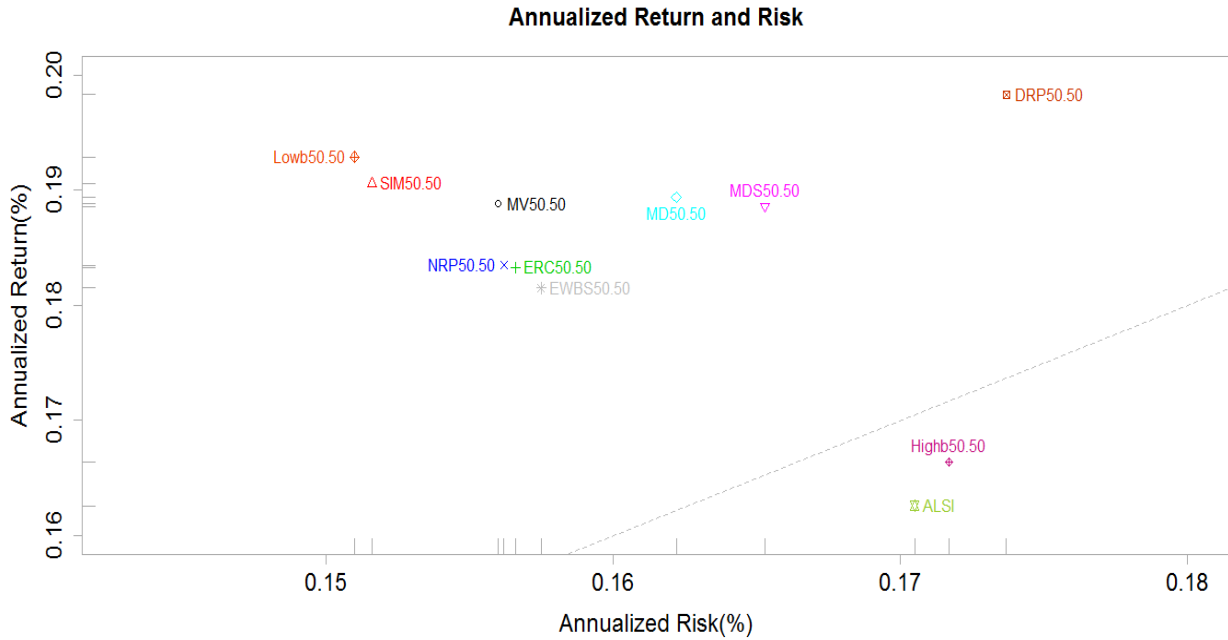


Figure 5-1: Sector-based 50-50 blended with the ALSI

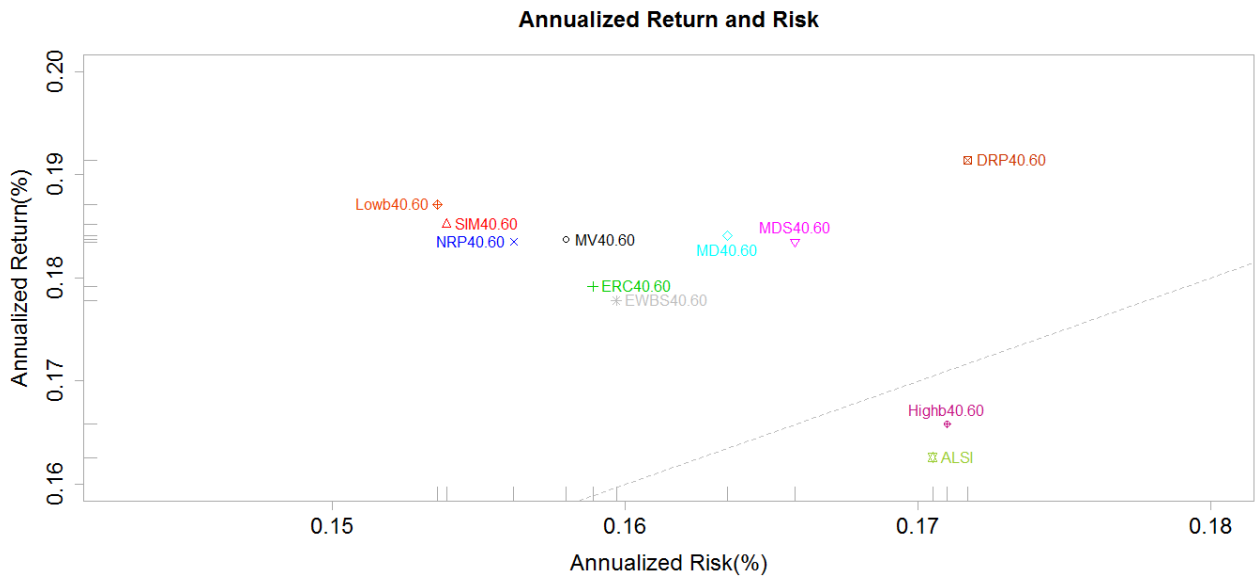


Figure 5-2: Sector-based 40-60 blended with the ALSI

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

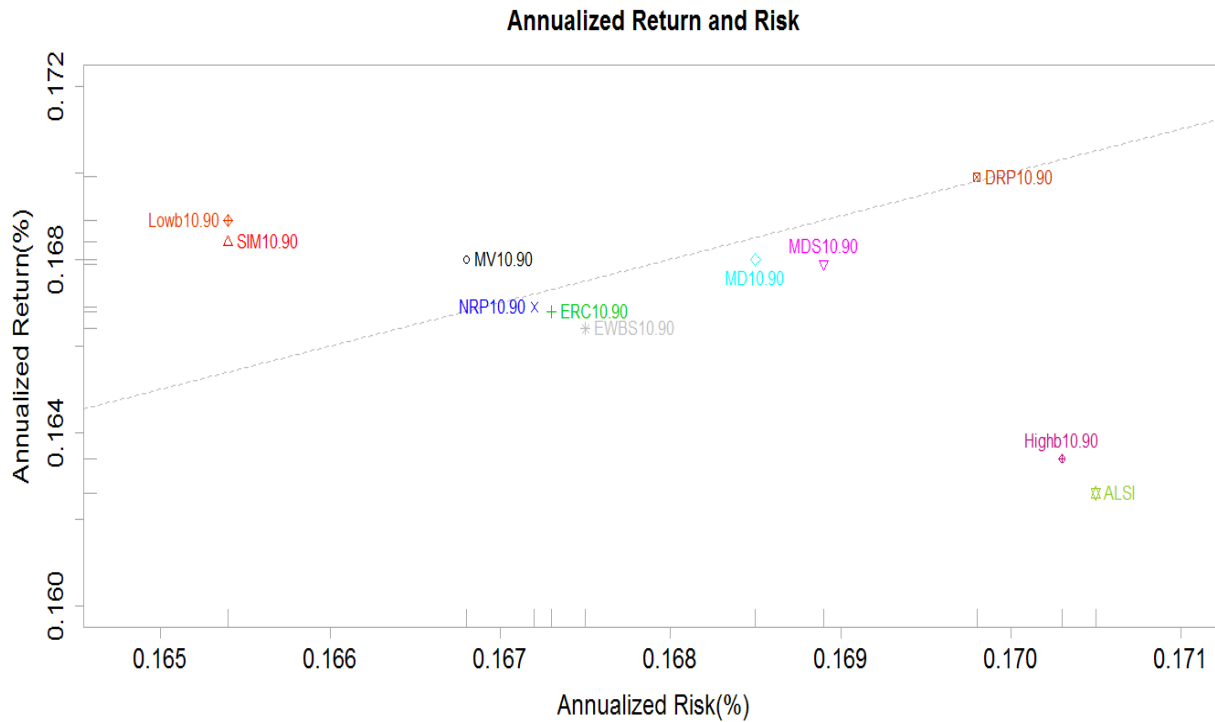


Figure 5-3: Sector-based 10-90 blended with the ALSI

From Figures 5-1, 5-2 and 5-3, one notices that the 40-60, 50-50 and 10-90 blended portfolios have outperformed the ALSI and SWIX, as indicated by the points above the diagonal, and with a lower risk. More notable is the outperformance of the Lowbeta and SIM blended portfolios. In that light, the out-of-sample cumulative returns of the top two blended portfolios with the highest return-to-risk ratios (which are the Lowbeta and the SIM blended portfolios) are depicted in Figure 5-4, together with the ALSI to assess their outperformance.

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Sector-based Top 2 blended portfolios with the ALSI

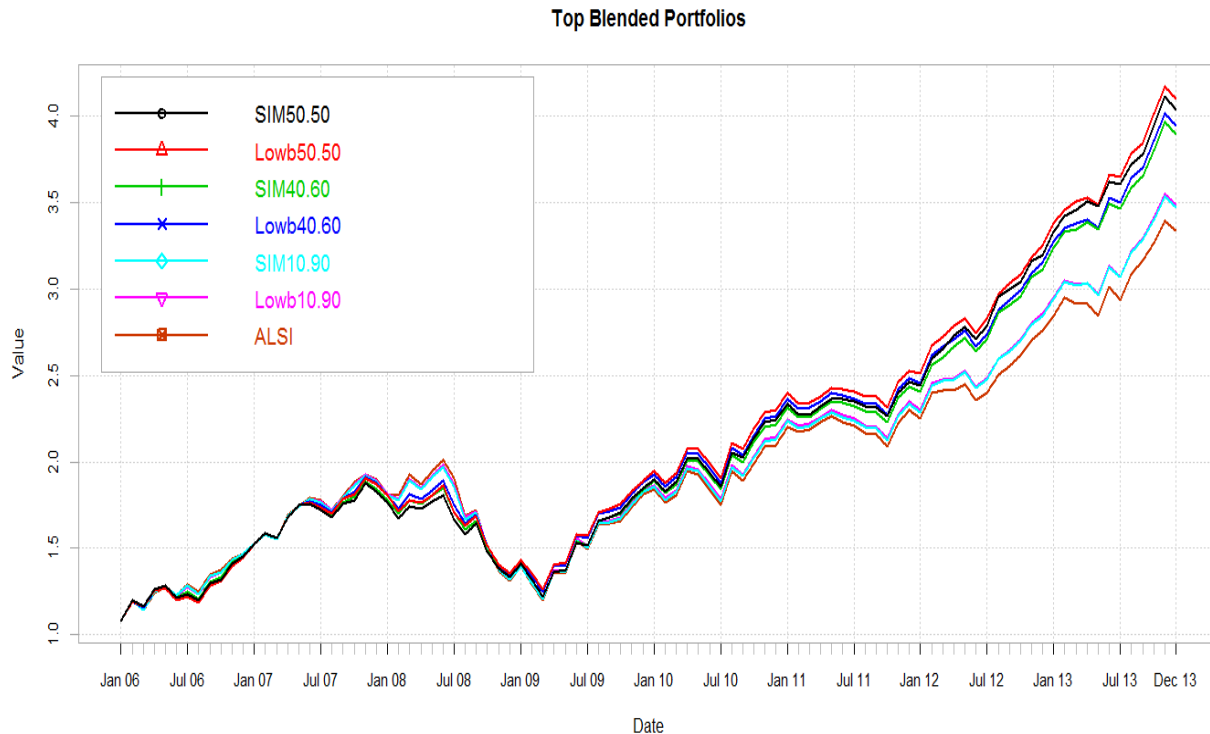


Figure 5-4: Out-of-Sample Performance of the Top 2 Blended Portfolios, January 2006- December 2013

From Figure 5-4, one notices that the blended portfolios (Lowb40.60, Lowb50.50, SIM40.60, and SIM50.50) tends to move in sync with the ALSI before July 2010 and starts moving away from ALSI after July 2010. However, the Low10.90 and the SIM10.90 still moves in sync with the ALSI. This is not surprising given that 90% of the Low10.90 and the SIM10.90 portfolios is invested in the ALSI. Similarly Figure 5-5, Figure 5-6, and Figure 5-7 depict the low volatility portfolios blended with the SWIX index.

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Sector-based Low Volatility Portfolios blended with the ALSI. Figure 5-5 is the 50-50 blend, Figure 5-6 is the 40-60 blend, while Figure 5-7 is the 10-90 blend, January 2006- December 2013

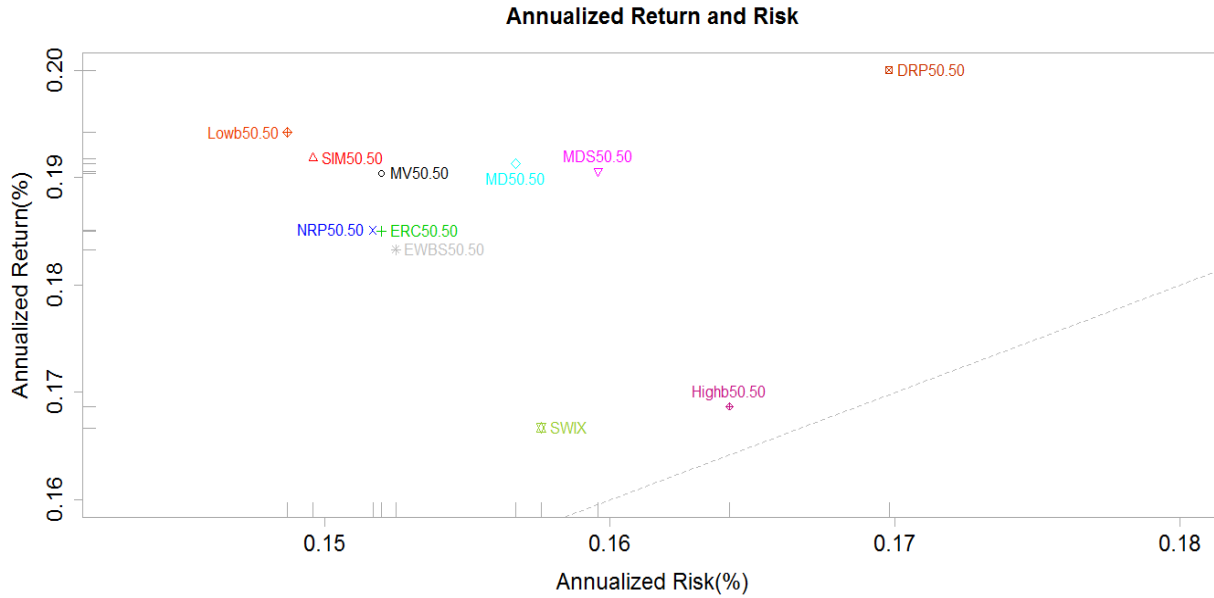


Figure 5-5: Sector-based 50-50 blended with the SWIX index

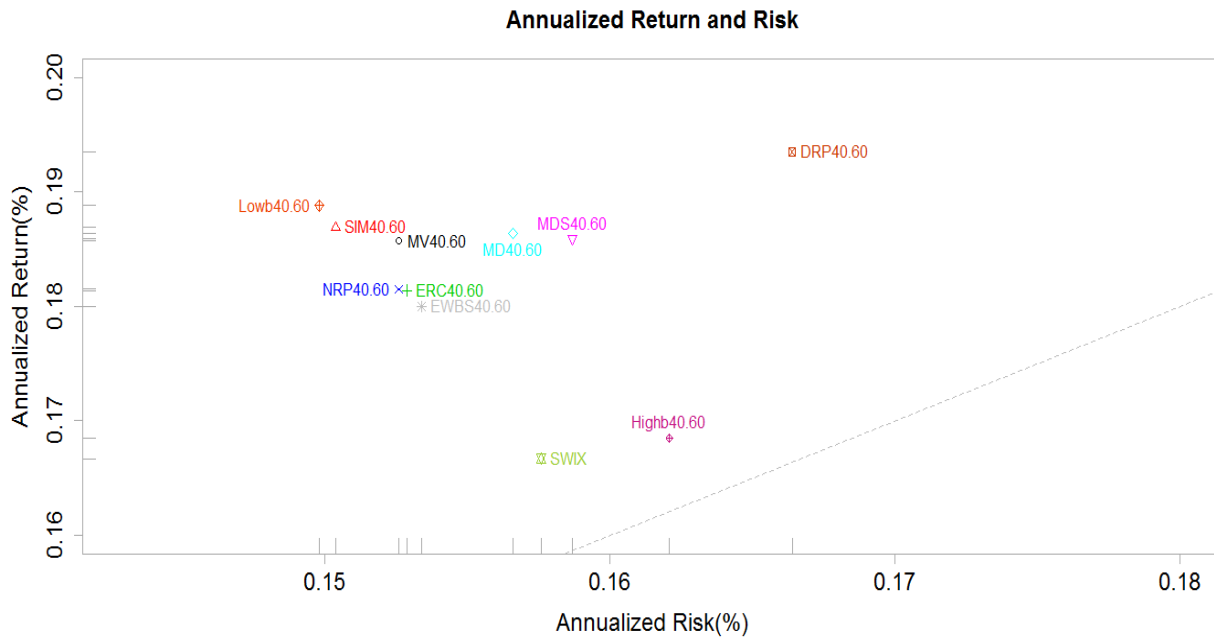


Figure 5-6: Sector-based 40-60 blended with the SWIX Index

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

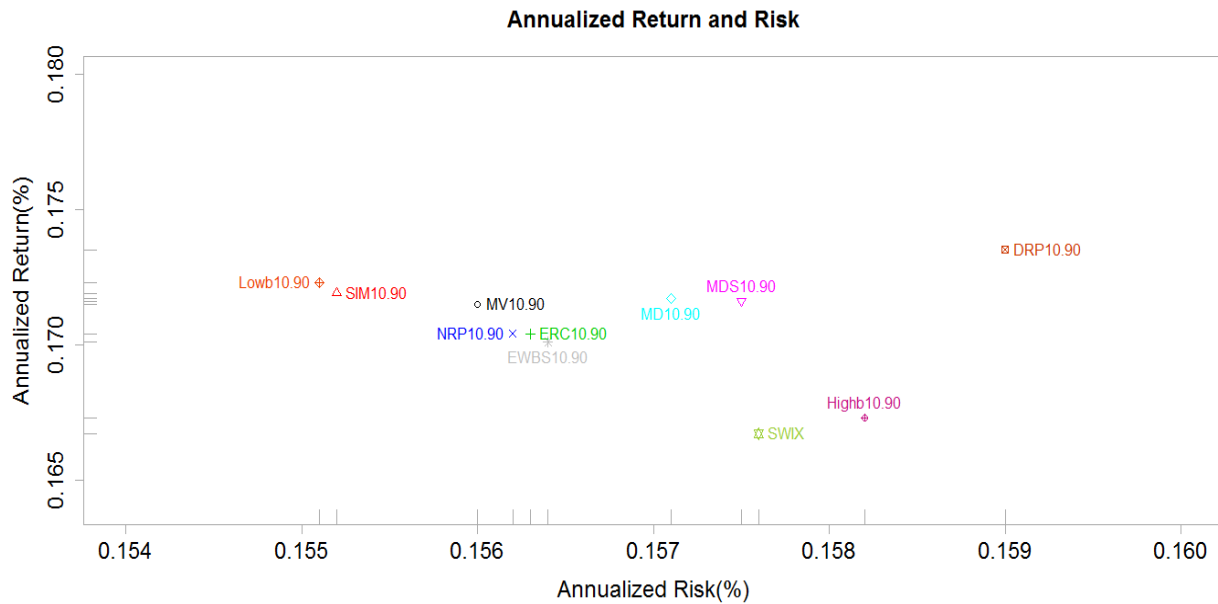


Figure 5-7: Sector-based 10-90 blended with the SWIX Index

From Figures 5-5, 5-6 and 5-7, one notices that all the blended portfolios have outperformed the SWIX, bar the Highb10.90, High40.60, and the High50.50 blended portfolio. Similar to the results obtained by the blending of the low portfolios with the ALSI, the Lowbeta and SIM blended portfolios also posted the highest return-to-risk ratios. In Figure 5-8, the out-of-sample cumulative returns of the Lowbeta and the SIM blended portfolios is depicted together with the SWIX index.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

Sector-based Top 2 blended portfolios with the SWIX

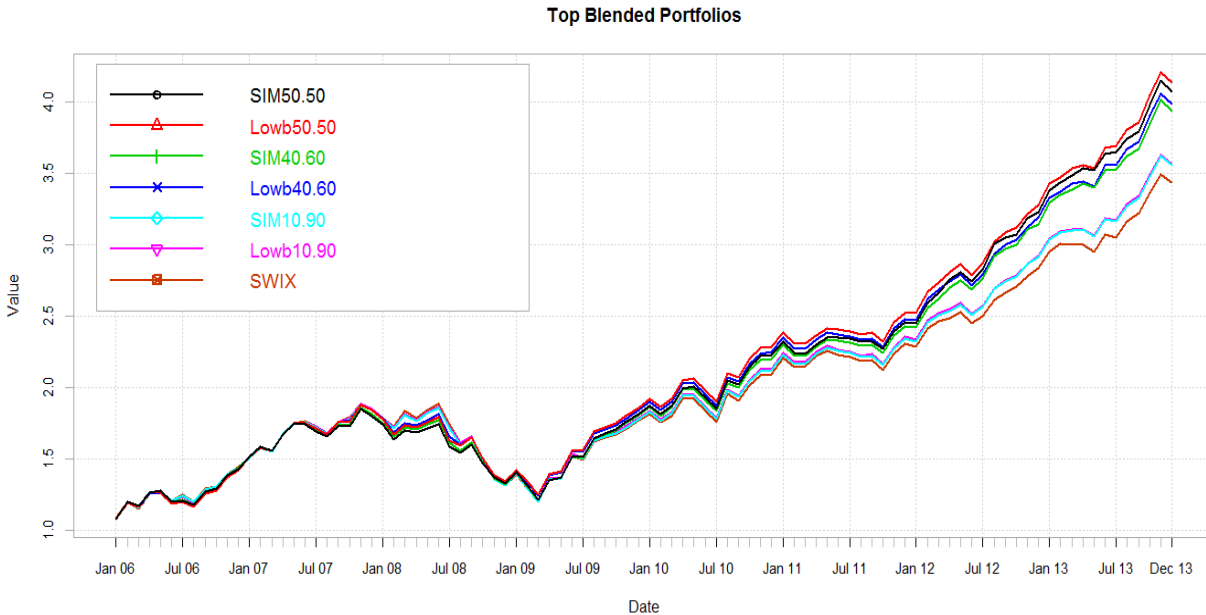


Figure 5-8: Out-of-Sample Performance of the Top 2 Blended Portfolios, January 2006- December 2013

From Figure 5-8, one notices that the blended portfolios moves in sync with the SWIX until after July 2011. The SIM50.50, Lowb50.50, SIM40.60, and the SIM50.50 also fared better than the SWIX index over the sample period. In the next section, the empirical results from blending the low volatility portfolios with the ALSI and SWIX index using the Top 100 stocks is analyzed.

5.2 Empirical Results of the Blending of Low Volatility Portfolios with the ALSI and the SWIX Indices using the JSE Top 100 stocks

In section 4.8, it was shown that using the JSE Top 100 stocks gave superior performance when compared with the JSE Top 40 stocks. Similar to section 5.1, the empirical results of low volatility portfolios, blended with the ALSI and the SWIX indices using the Top 100 stocks are assessed. Figures 5-9, 5-10 and 5-11 depict the risk–return analysis of the 50-50, 40-60, and 10-90 blended portfolios with the ALSI.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

Stock-based Low Volatility Portfolios blended with the ALSI. Figure 5-9 is the 50-50 blend, Figure 5-10 is the 40-60 blend, while Figure 5-11 is the 10-90 blend, January 2007-December 2013

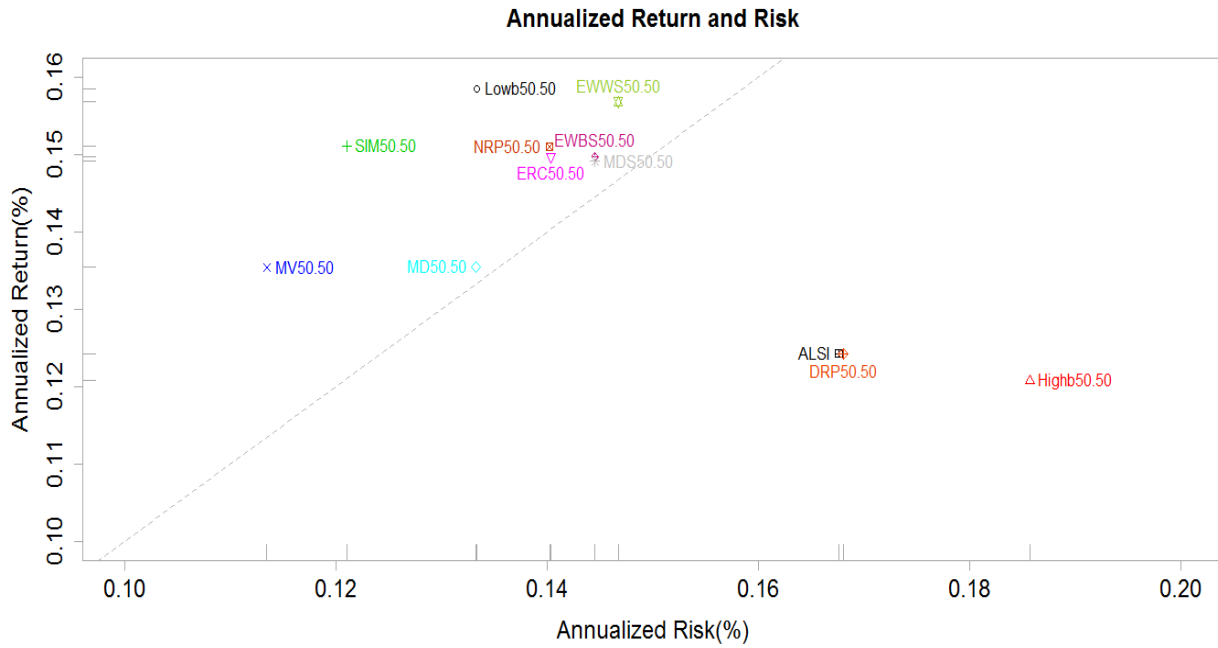


Figure 5-9: Stock-based 50-50 blended with the ALSI

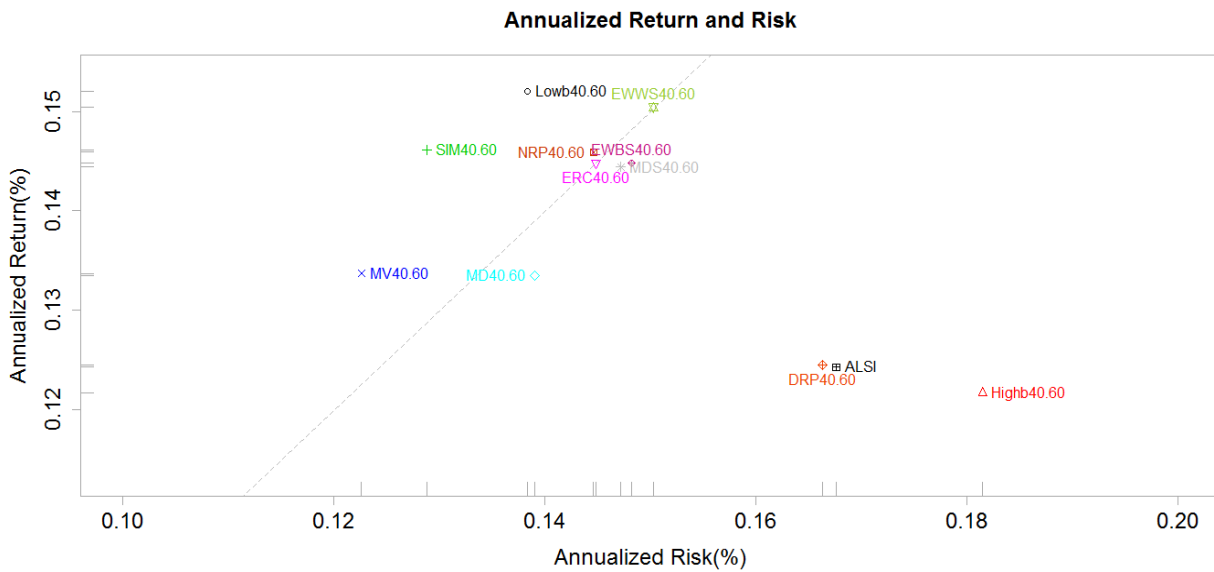


Figure 5-10: Stock-based 40-60 blended with the ALSI

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

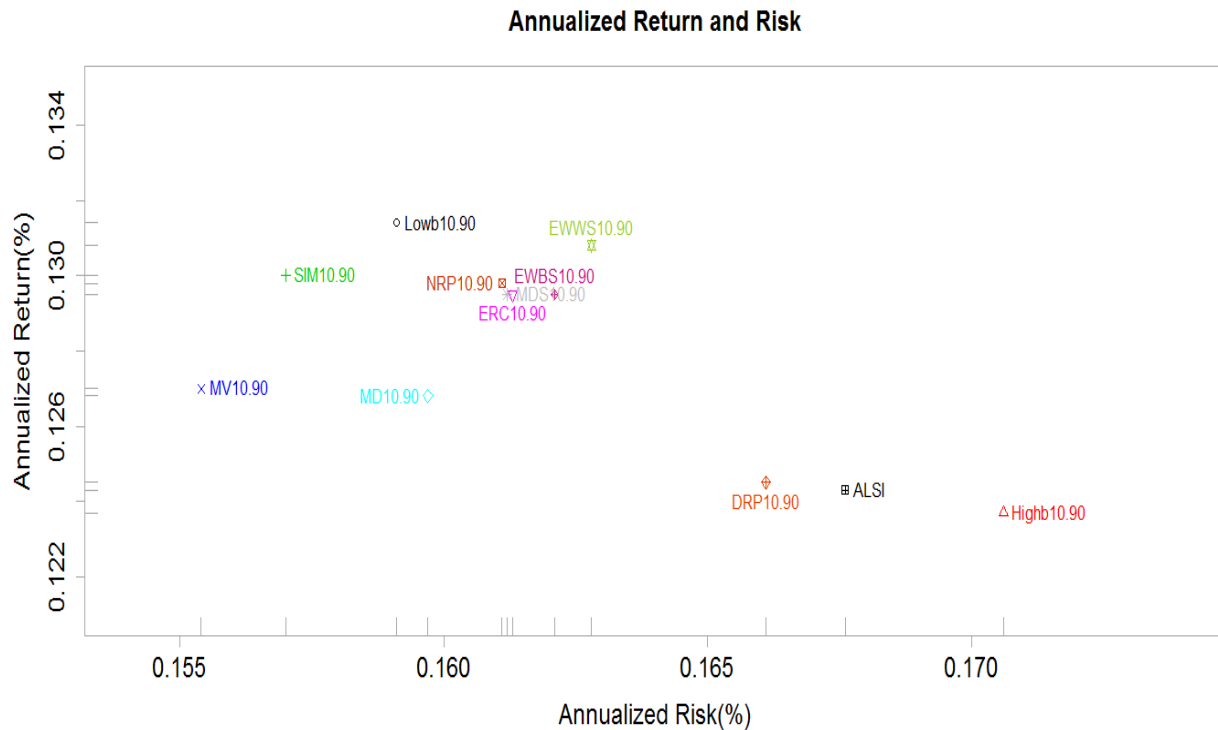


Figure 5-11: Stock-based 10-90 blended with the ALSI

From Figures 5-9, 5-10 and 5-11, it is evident that the 40-60, 50-50 and 10-90 blended portfolios have outperformed the ALSI based on their higher annualized returns and lower annualized risks, except for the DRP and the Highbeta blended portfolio. Further inspection in the figures above suggests that the MIN VAR blended portfolio (MV10.90, MV40.60, MV50.50) also posted the lowest annualized risk over the sample period. In addition, the Lowbeta and SIM blended portfolios posted the highest return-to-risk ratios over the sample period. The Figure 5-12 that follows depicts the out-of-sample cumulative returns of the Lowbeta and SIM blended portfolios.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

Stock-based Top 2 blended portfolios with the ALSI

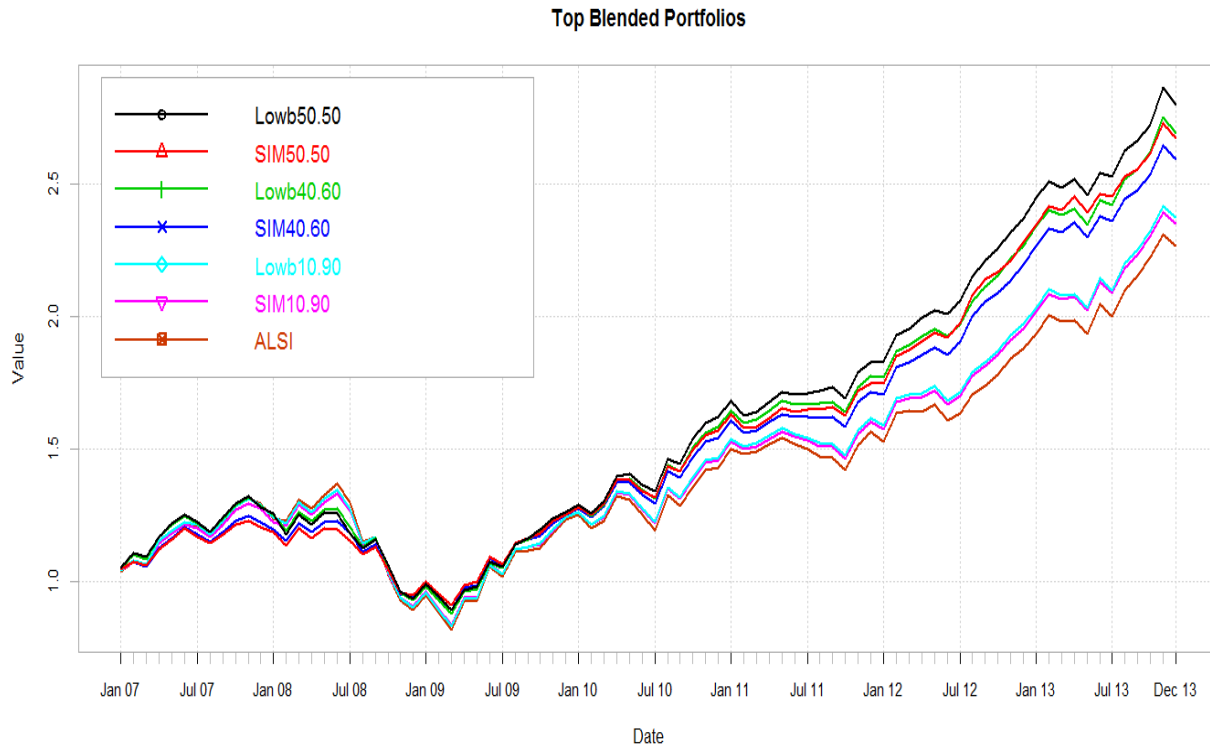


Figure 5-12: Out-of-Sample Performance of the Top 2 Blended Portfolios, and the ALSI January 2007- December 2013

From Figure 5-12, it is evident that the blended portfolios have outperformed the ALSI over the sample period. Similarly Figure 5-13, Figure 5-14, and Figure 5-15 depict the low volatility portfolios blended with the SWIX index.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

Stock-based Low Volatility Portfolios blended with the SWIX. Figure 5-13 is the 50-50 blend, Figure 5-14 is the 40-60 blend, while Figure 5-15 is the 10-90 blend, January 2007- December 2013

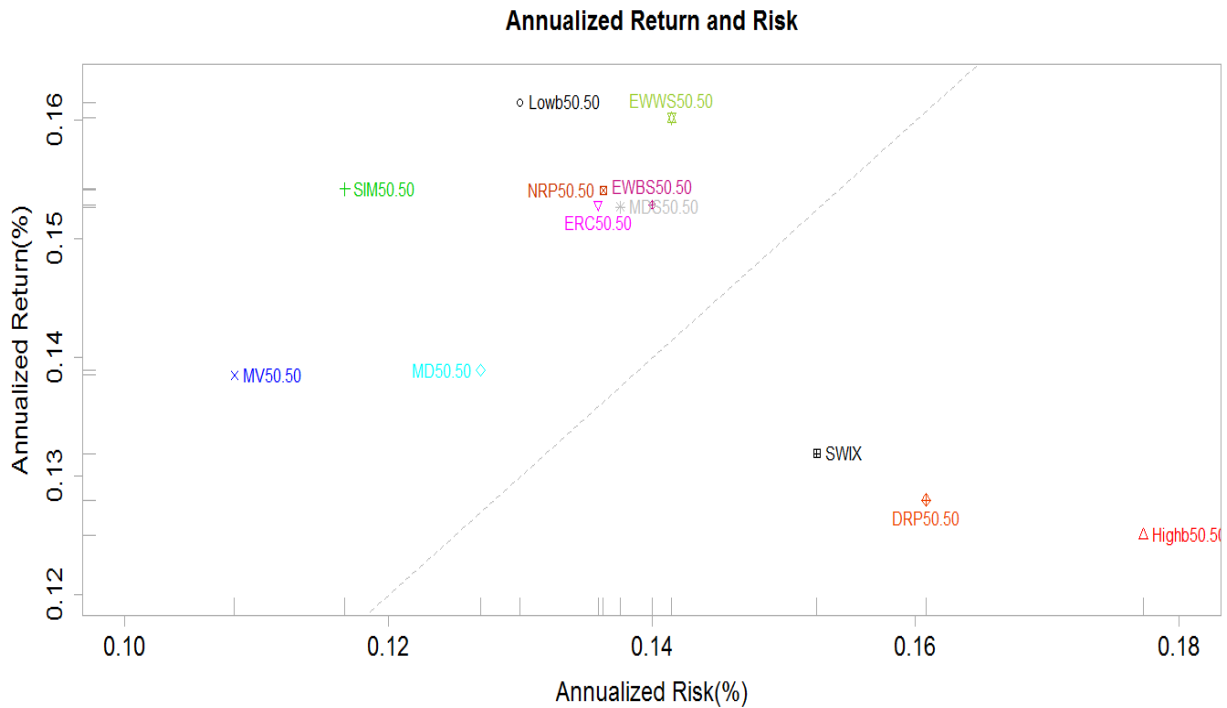


Figure 5-14: Stock-based 50-50 blended with the SWIX

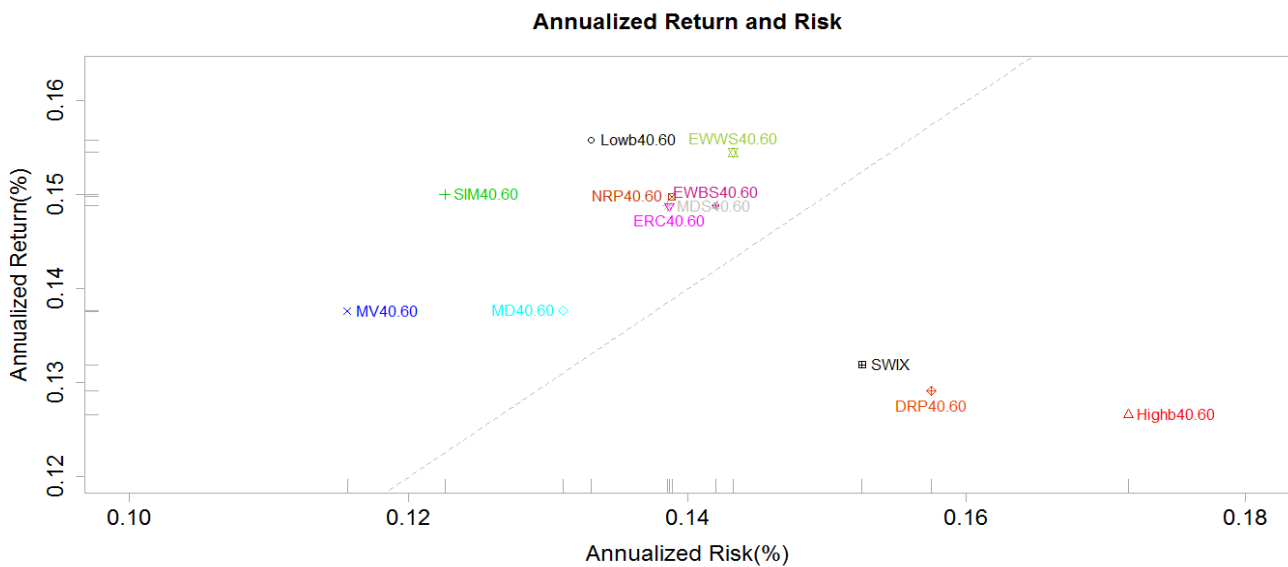


Figure 5-13: Stock-based 40-60 blended with the ALSI

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

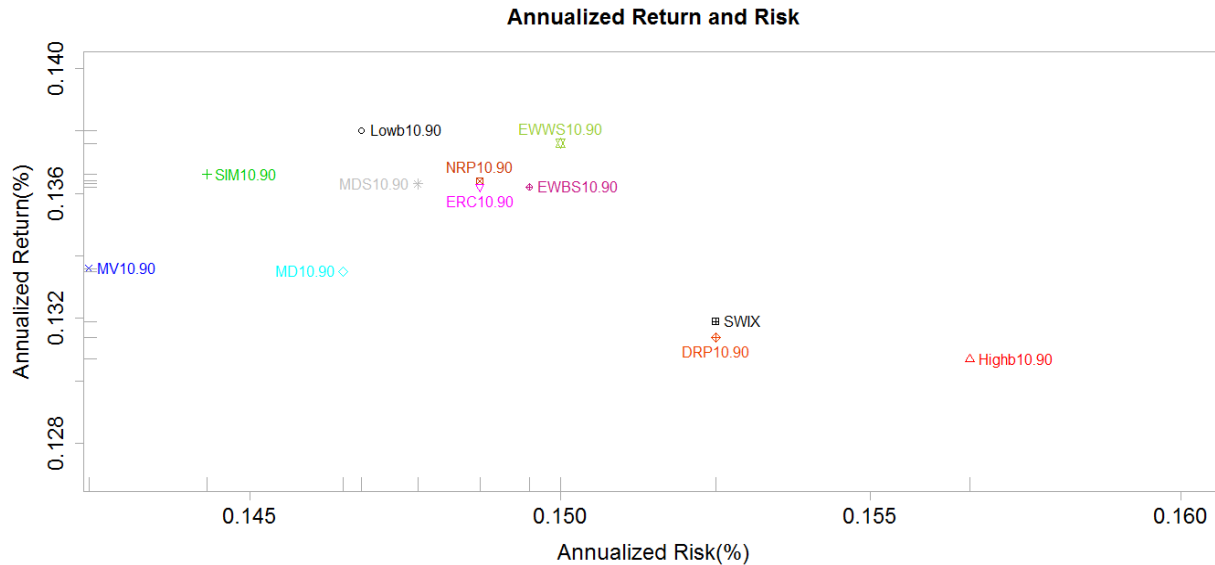


Figure 5-15: Stock-based 10-90 blended with the SWIX

From Figures 5-13, 5-14 and 5-15, it is evident that the 40-60, 50-50 and 10-90 blended portfolios have outperformed the SWIX based on their higher annualized returns and lower annualized risks, except for the DRP and the Highbeta blended portfolio. The MIN VAR blended portfolio (MV10.90, MV40.60, MV50.50) also posted the lowest annualized risk. Similarly, Figure 5-16 above shows outperformance of the top two blended portfolios with the highest return-to-risk ratios.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

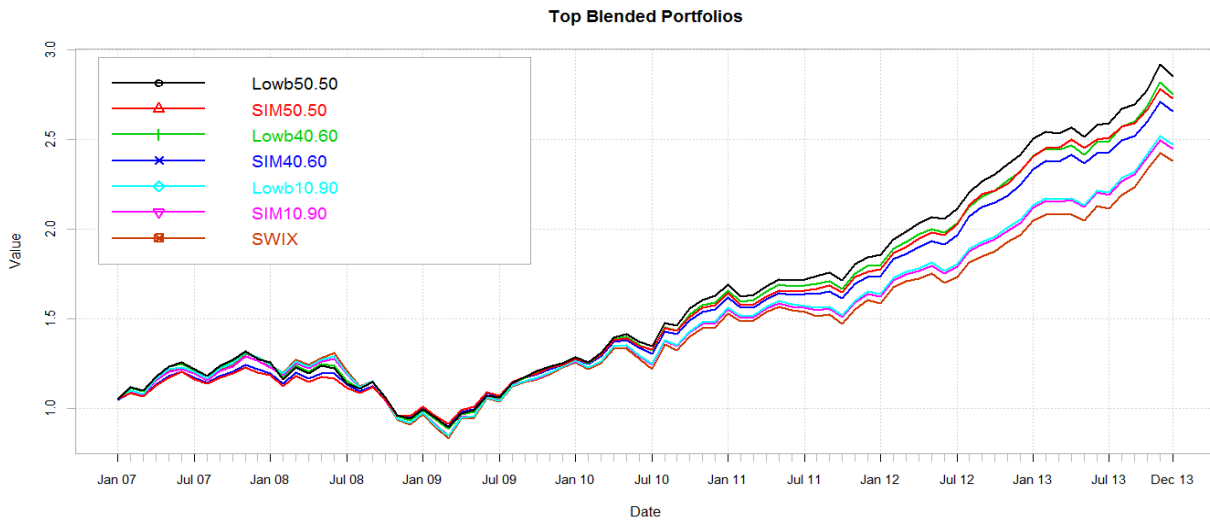


Figure 5-16: Out-of-Sample Performance of the Top 2 Blended Portfolios and the SWIX, January 2007- December 2013

5.3 Chapter Summary

In this Chapter, the low volatility portfolios was blended with the ALSI and the SWIX indices to assess their usefulness as effective blended portfolio strategies. The construction of the blended portfolios was motivated by the fact that most portfolio managers (or investors') may be unwilling to invest all their capital in the low volatility portfolios. Hence, the blended portfolios provides an alternative for portfolio managers to still beat the market, even with a lower risk. The results of the analysis of the blended portfolios using both the FTSE/JSE sectors and the Top JSE 100 stocks suggests that:

- The SIM and Lowbeta blended portfolio have consistently produced the highest return-to-risk ratios. Specifically, the Lowb40.60, Lowb50.50, SIM40.60, and the SIM50.50 portfolios.
- The SIM and Lowbeta blended portfolio have also significantly outperformed the ALSI and the SWIX indices.

CHAPTER FIVE-BLENDING THE LOW VOLATILITY PORTFOLIOS

- The MIN VAR blended portfolio posted the lowest annualized risk when the Top 100 stocks was used in constructing the blended portfolios.

Thus one can conclude that constructing these blended portfolios can be useful as effective portfolio strategies, given their superior risk-adjusted returns when compared to the ALSI and SWIX indices. It is however important to note that there is no theory that predicts, ex ante, that any of these blended portfolios will be more efficient than other portfolios (Lee, 2011).

6 Summary and Conclusion

In this study, the construction of low volatility portfolios have largely focused on sectors (industries) and stocks in South Africa. A back-testing technique was employed to test the efficacy of the low volatility portfolios and their performances were compared to the ALSI, after accounting for turnover and transaction cost. Consequently, the low volatility portfolios have also been assessed using a monthly rebalancing frequency. Additionally a larger universe of stocks were analyzed. The low volatility portfolios were also blended with the ALSI and SWIX indices to assess their usefulness as effective portfolio strategies. Furthermore, the low volatility portfolios have been assessed graphically using a covariance bi-plot to better understand their relationships and risk exposures. Performance analysis of the low volatility portfolios, constructed using Industries (the FTSE/JSE sectors), over the period January 2003-December 2013 suggests that:

- The Lowbeta, SIM, NRP, ERC, MAX D, MIN VAR, EWBS outperformed the ALSI at significantly lower risk, which resulting high risk-adjusted returns. However, the DRP and MDS had a higher realized risk, but still outperformed the ALSI.
- These portfolios also posted a lower drawdown when compared with the ALSI, which implies that they can recover from losses quicker than the ALSI.
- The low volatility portfolios constructed were found to be robust to either annually or monthly rebalancing frequencies.

Nonetheless, delving into their risk and diversification characteristics, it was found that the Financial, Consumer Services and Industrial sector were more often found in the Lowbeta, SIM and MIN VAR portfolios. Consequently, these portfolios were dominated by those sectors, which is not surprising given that they are constructed to target low beta and low volatility assets respectively. However, the Technology and Telecoms sector are more exposed to the ERC, NRP, and the EWBS portfolios.

A performance analysis of the low volatility portfolios using the Top 40 stocks was conducted and was further extended to the Top 100 stocks as a robustness check. The results on a stock level reveal that:

- All the low volatility portfolios also outperformed the ALSI over the period.

CHAPTER SIX-SUMMARY AND CONCLUSION

- The MIN VAR and the SIM portfolio posted the lowest risk and the lowest drawdown. Stocks with low betas and low volatilities were given significant weight in these portfolios.
- The EWWS portfolio had a similar realized risk with the EWBS. However, it outperformed the EWBS portfolio on a risk-adjusted basis, based on its significant higher return. This is in line with the work of Velvadapu (2011), who showed that the EWBS (equal weighting by stocks only) induces sector bias.
- The MAX D and MDS portfolio were predominantly comprised by stocks that have a high average correlation with other stocks.

This results found in this thesis is largely consistent with previous academic research, which showed that low volatility portfolios have superior risk-adjusted returns over-and-above the market capitalization portfolio. Examining the risk and diversification analysis for the stocks, it was shown that the performance of the low volatility portfolios is largely driven by exposure to the small capitalization index. In addition, the MAX D, ERC, EWWS, EWBS, and NRP portfolios were found to be highly correlated, as supported by their close proximity on the covariance bi-plot. Furthermore, the results from the blending of the low volatility portfolios with the ALSI and the SWIX indices suggests that these blended strategies can still offer a superior risk-adjusted return, with a slightly lower risk than the ALSI and SWIX indices.

Remarkably, it was evident that the Lowbeta, MIN VAR and SIM portfolios have consistently been the superior performer of the low volatility portfolios and the market capitalization-weighted index throughout the sample period. This is due to the fact that they were able to deliver superior return-to-risk ratios. They also posted the lowest drawdown when compared with the other low volatility portfolios. This was found to be the case in both the sector and stock datasets.

Furthermore, it was also evident that the DRP portfolio underperformed most of the other low volatility portfolios. Regarding directions for further research, possible reasons for the DRP's underperformance have been explained recently by Meucci, Santangelo, and Deguest, (2014). They found that the principal components betas are statistically unstable, especially the ones relative to the smallest eigenvalues. Other reasons are that the principal component

CHAPTER SIX-SUMMARY AND CONCLUSION

bets may not be easy to interpret, thus making the decision process difficult, and the principal component bets are not scale invariant. They however suggested a new measure called the Minimal torsion bets, which deals with the aforementioned issues surrounding the principal components bets. Further research may be worth considering regarding this measure of the minimal torsion bets in South Africa.

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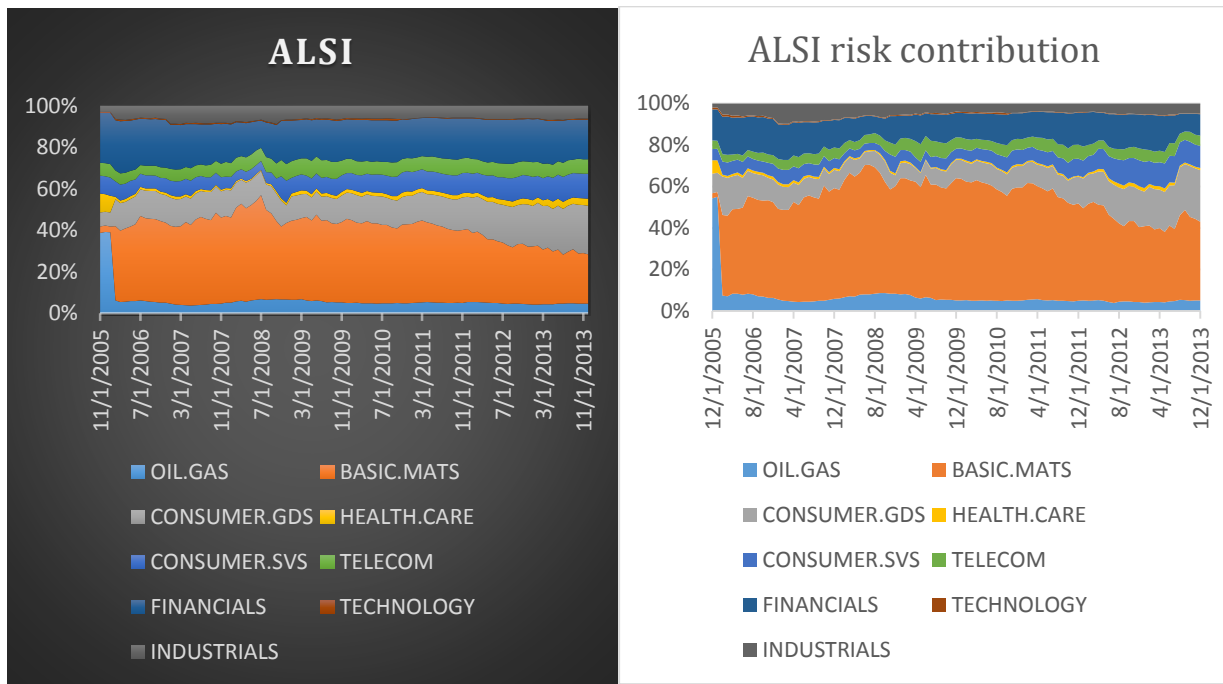
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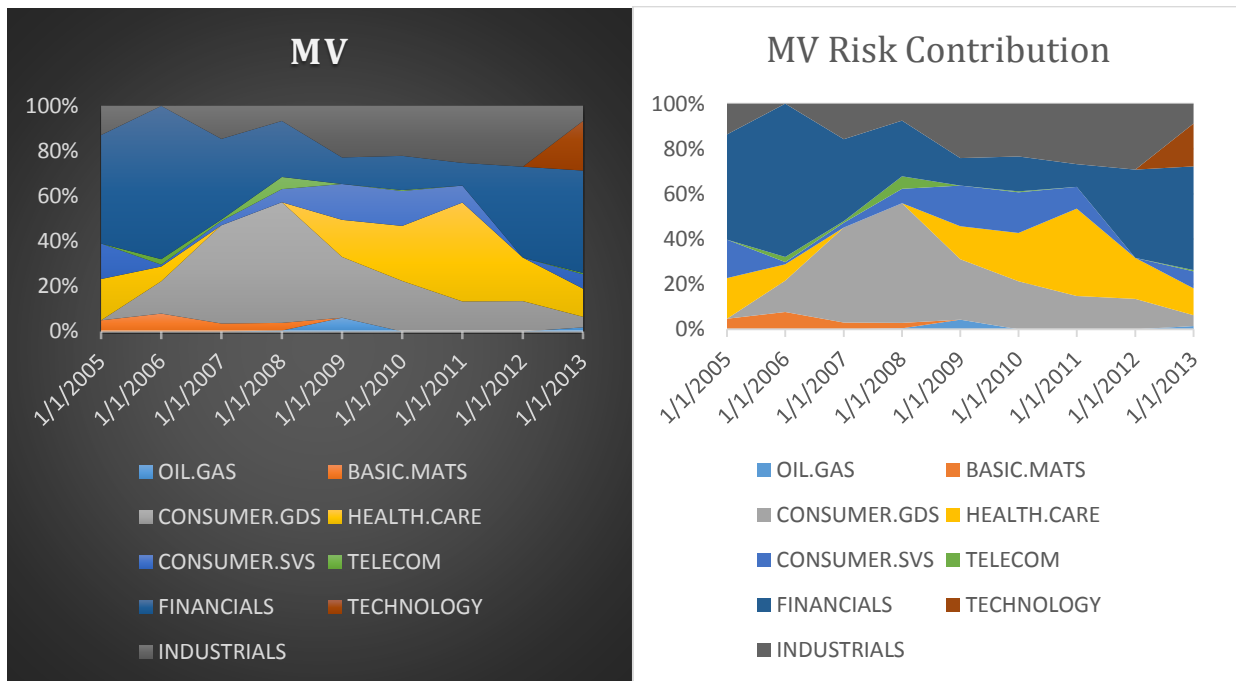
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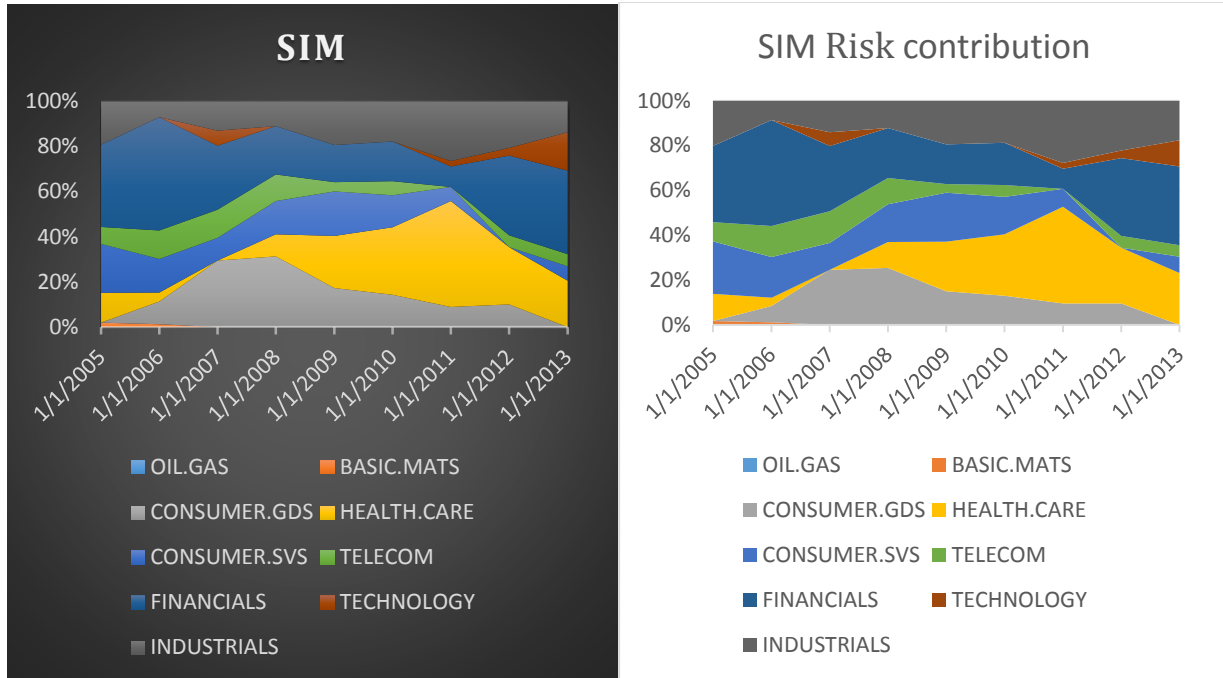
Appendix A Evolution of FTSE/JSE sector weights and risk contributions



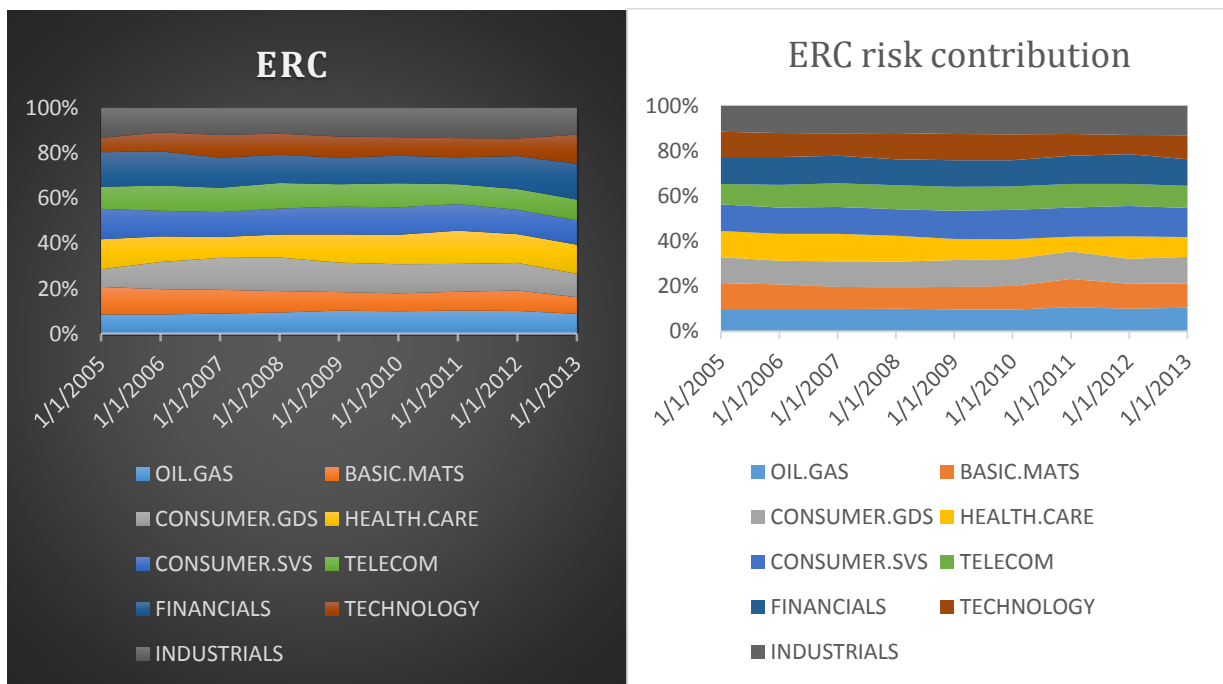
A.1: FTSE/JSE weights (left) and risk contribution (right) for the ALSI



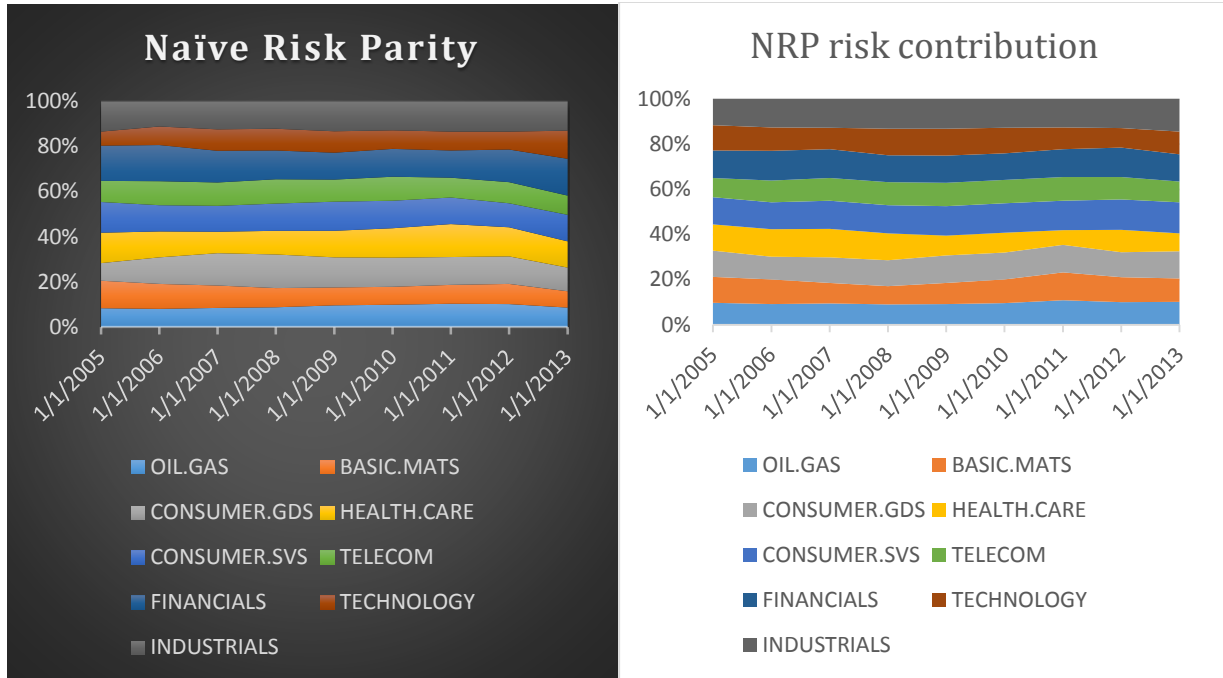
A2: FTSE/JSE weights (left) and risk contribution (right) for the Minimum Variance portfolio



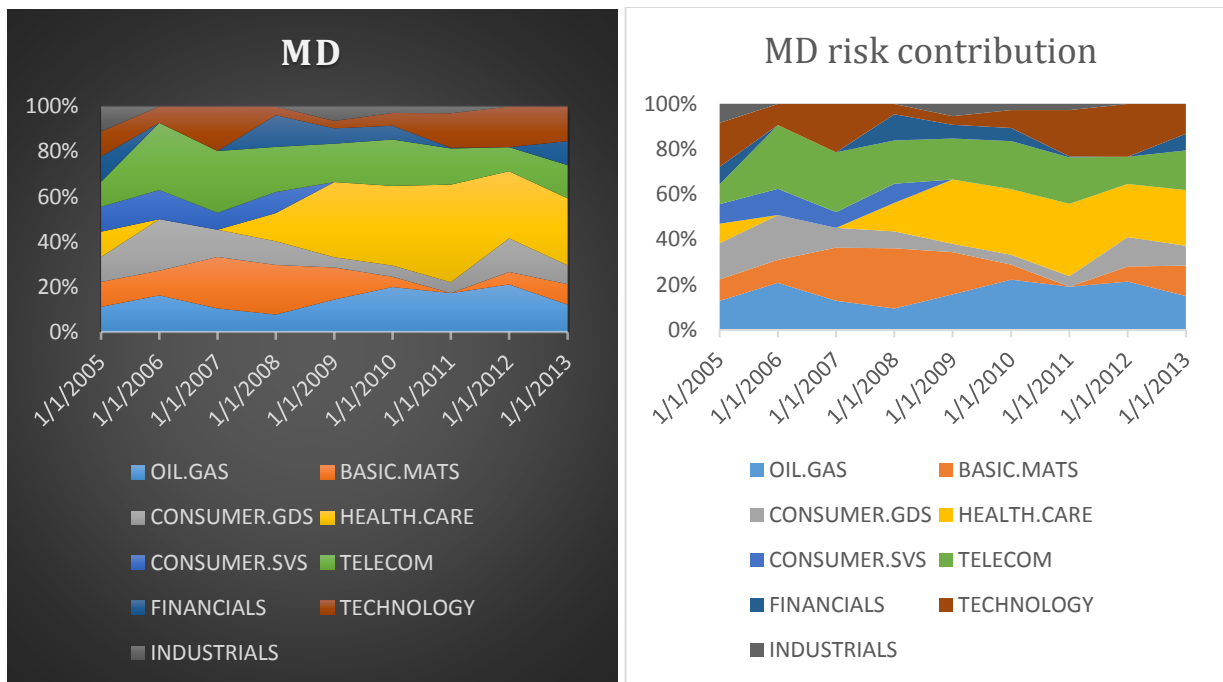
A.3: FTSE/JSE weights (left) and risk contribution (right) for the Single Index Model.



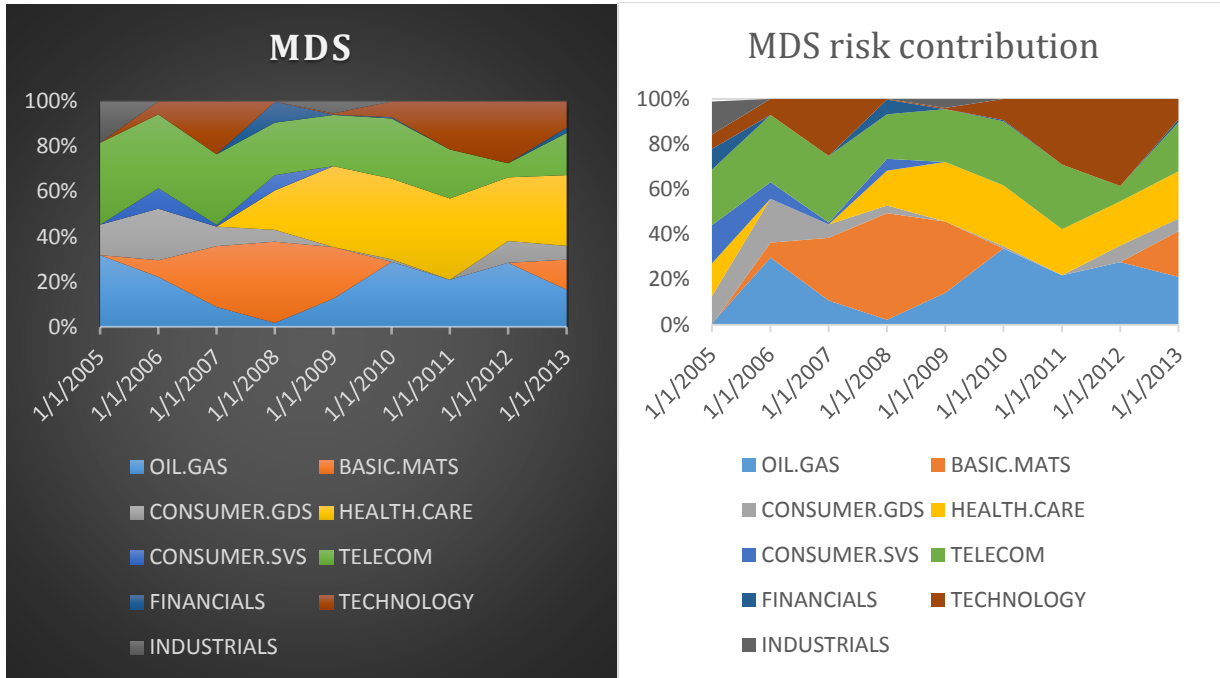
A.4: FTSE/JSE weights (left) and risk contribution (right) for the Equal risk contribution portfolio.



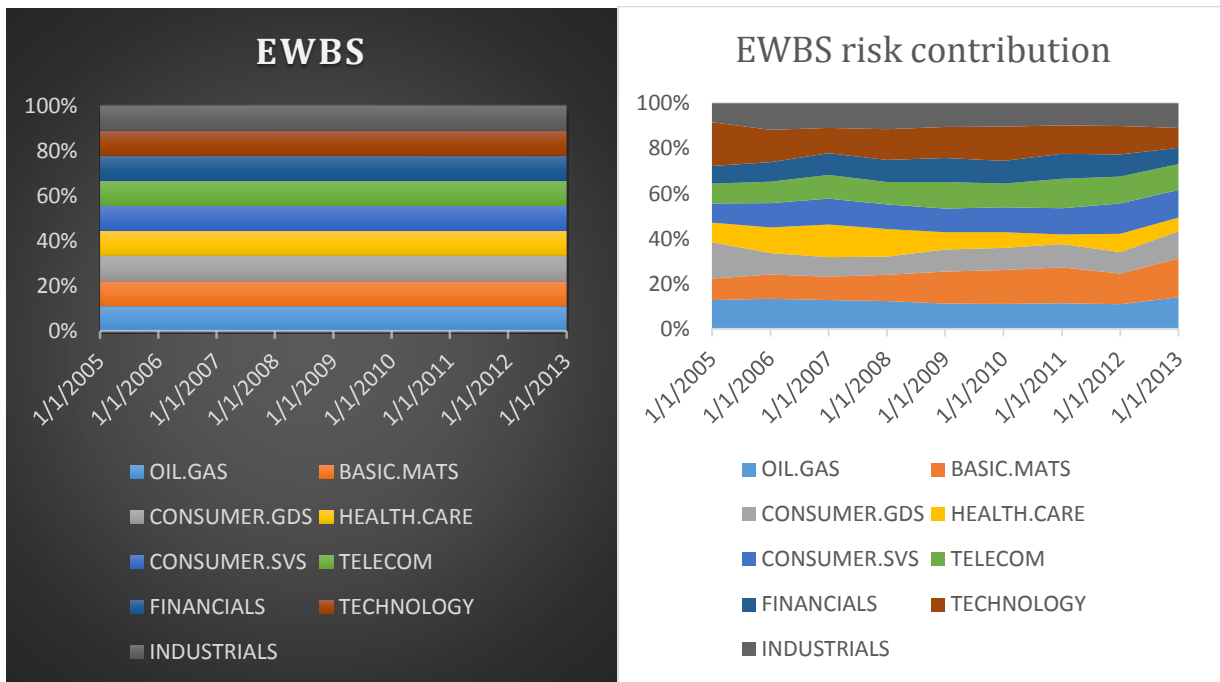
A.5: FTSE/JSE weights (left) and risk contribution (right) for the Naïve risk parity portfolio.



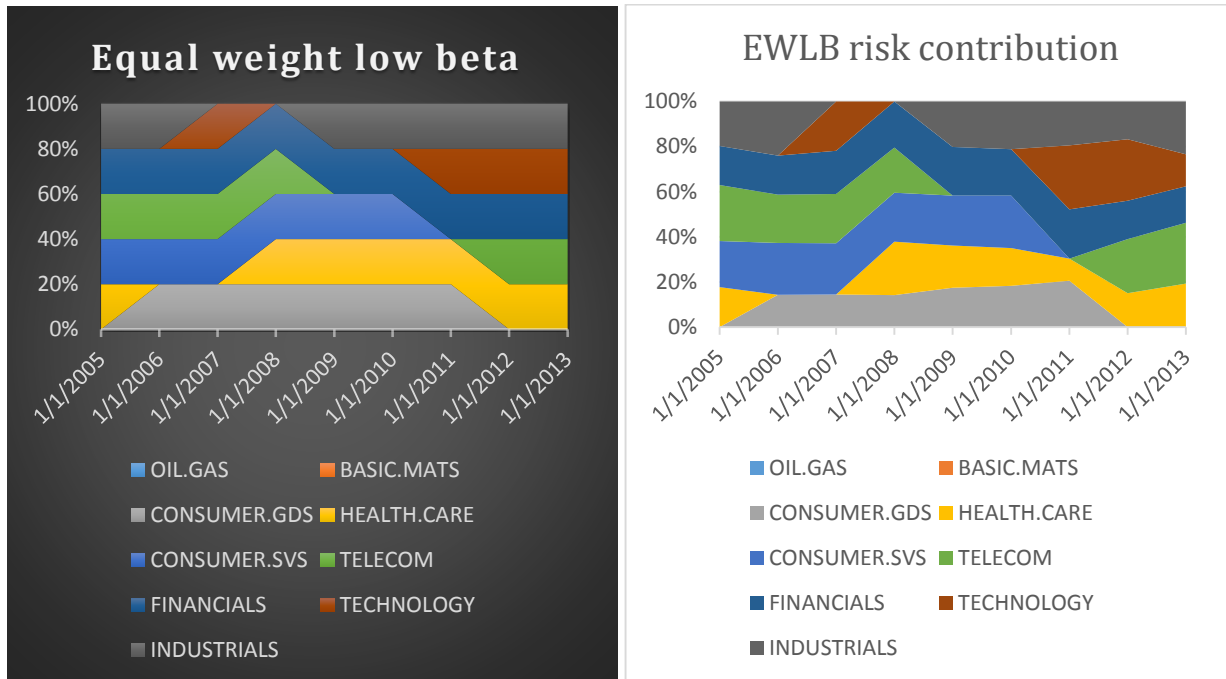
A.6: FTSE/JSE weights (left) and risk contribution (right) for the Maximum Diversification portfolio.



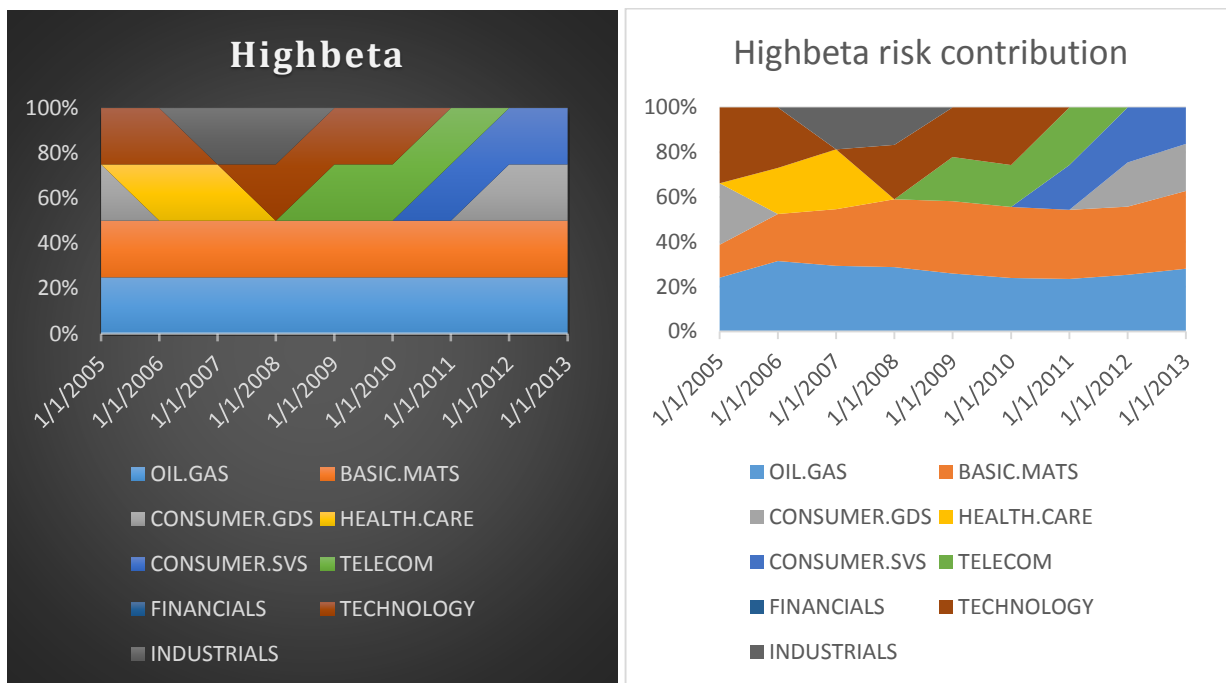
A.7: FTSE/JSE weights (left) and risk contribution (right) for the Maximum Decorrelated portfolio.



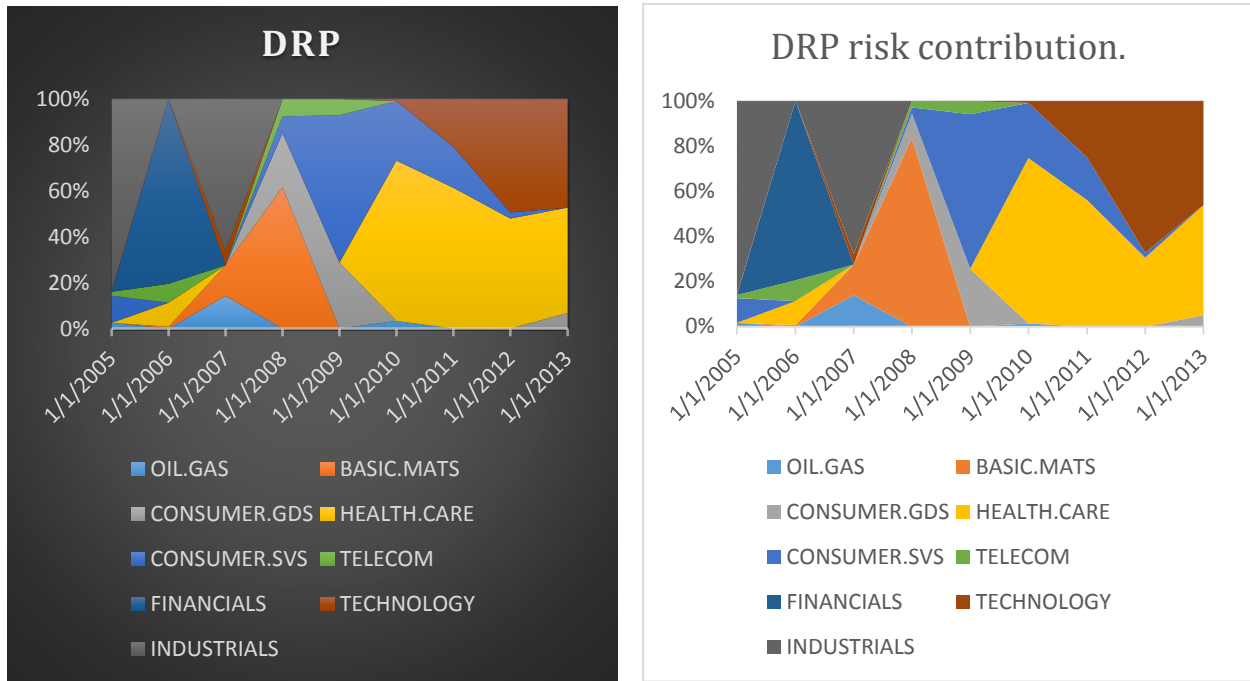
A.8: FTSE/JSE weights (left) and risk contribution (right) for the Equal weight by sectors portfolio.



A.9: FTSE/JSE weights (left) and Risk contribution (right) for the Equal weight Lowbeta portfolio



A.10: FTSE/JSE weights (left) and Risk contribution (right) for the Highbeta Portfolio.



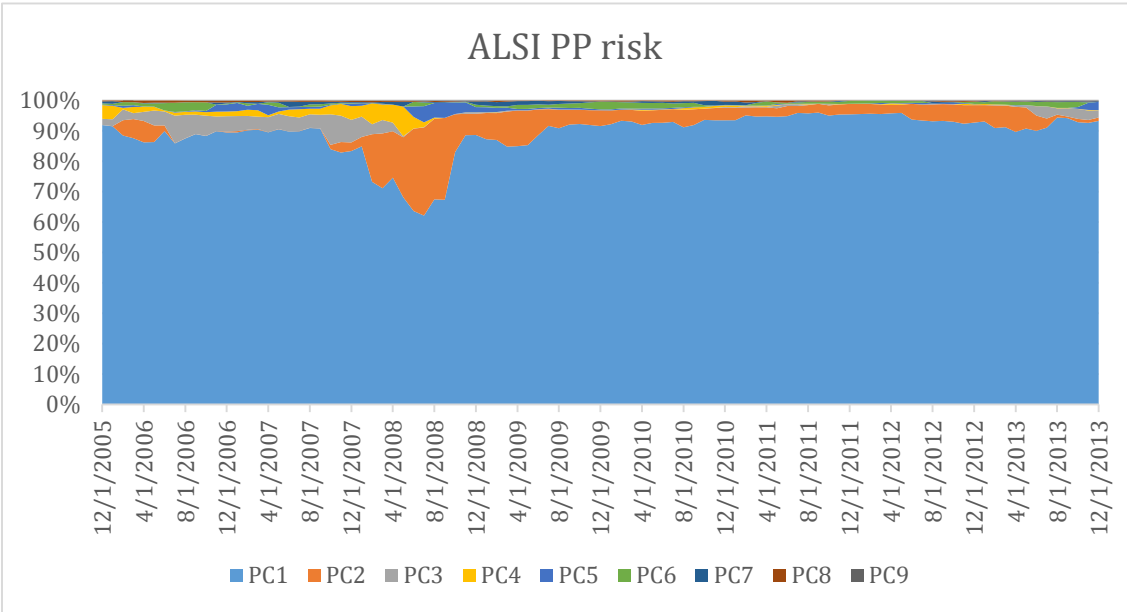
A.11: FTSE/JSE weights (left) and risk contribution (right) for the Diversified Risk parity Portfolio

Appendix B Annualized return, Volatility, Sharpe ratio, Average Correlation and Drawdown of the JSE TOP 40 stocks, Jan 2004-Dec 2013

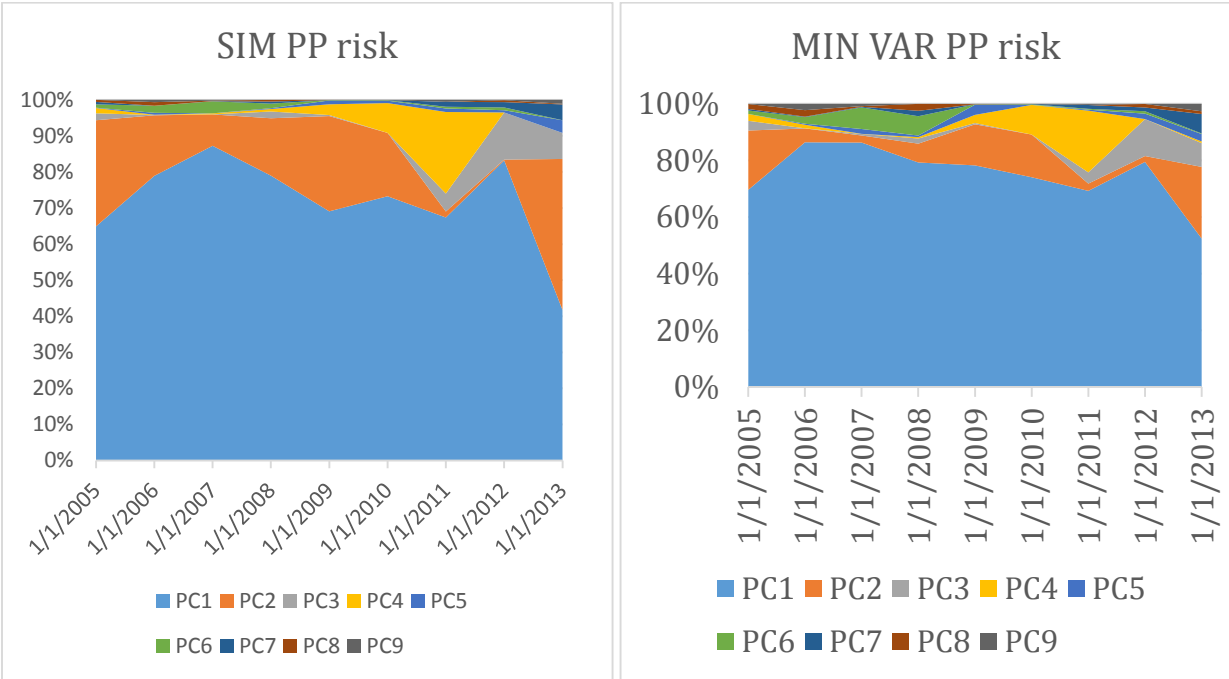
JSE Top 40 stocks	Annualized Return	Annualized Risk	Annualized Sharpe (Rf=0%)	Average Correlation to other stocks	Drawdown
SABMILLER	0.262	0.181	1.446	0.282	0.253
BHP.BILL	0.231	0.291	0.794	0.252	0.453
NASPERS	0.394	0.261	1.509	0.36	0.293
MTN	0.252	0.251	1.002	0.325	0.401
ANGLO.AMER	0.077	0.35	0.22	0.256	0.743
SASOL	0.237	0.25	0.95	0.263	0.442
FIRSTRAND	0.219	0.238	0.918	0.382	0.482
STANDARD.BK	0.17	0.229	0.74	0.393	0.417
OLD.MUTUAL	0.157	0.268	0.587	0.339	0.747
BARCLAYS.AFRICA	0.184	0.234	0.784	0.363	0.415
ASPEN.PHMCR	0.378	0.274	1.382	0.233	0.282
ANGLO.AMERICAN.PLAT	0.057	0.393	0.145	0.223	0.759
NEDBANK	0.173	0.232	0.746	0.352	0.484
SANLAM	0.249	0.205	1.216	0.32	0.318
SHOPRITE	0.385	0.228	1.687	0.248	0.193
REMGRO	0.298	0.158	1.889	0.271	0.124
BIDVEST	0.235	0.213	1.101	0.392	0.441
STEINHOFF	0.231	0.308	0.749	0.357	0.587
RMB	0.244	0.227	1.075	0.377	0.489
IMPALA.PLAT	0.083	0.373	0.222	0.225	0.684
MEDICLINIC	0.271	0.183	1.479	0.218	0.328
WOOLWORTHS	0.325	0.257	1.265	0.339	0.56
ASSORE	0.454	0.388	1.171	0.215	0.582
EXXARO	0.348	0.41	0.848	0.251	0.588
TIGER.BRANDS	0.232	0.204	1.134	0.355	0.304
ANGLOGOLD	-0.064	0.344	-0.187	0.128	0.661
INTU	0.035	0.243	0.142	0.18	0.729
GROWTHPOINT	0.236	0.201	1.173	0.235	0.309
DISCOVERY	0.244	0.202	1.21	0.272	0.363
IMPERIAL	0.201	0.301	0.67	0.366	0.643
TRUWORTHS.INTL	0.298	0.268	1.112	0.336	0.429
AFN.RAINBOW.M	0.176	0.372	0.474	0.268	0.685
NETCARE	0.212	0.246	0.862	0.279	0.53
MMI.HOLD	0.246	0.215	1.141	0.315	0.329
MR.PRICE	0.421	0.267	1.575	0.265	0.491
INVESTEC	0.166	0.294	0.565	0.375	0.678
MASSMART	0.213	0.254	0.839	0.279	0.353
GOLD.FIELDS	-0.05	0.378	-0.132	0.096	0.691
INVESTEC.JSE.	0.167	0.31	0.54	0.361	0.707
LIBERTY.HOLDINGS	0.175	0.179	0.976	0.236	0.203

Table B.1: Annualized return, volatility, Sharpe ratios, average correlation and drawdown of the JSE Top 40 stocks over the period January 2006- December 2013.

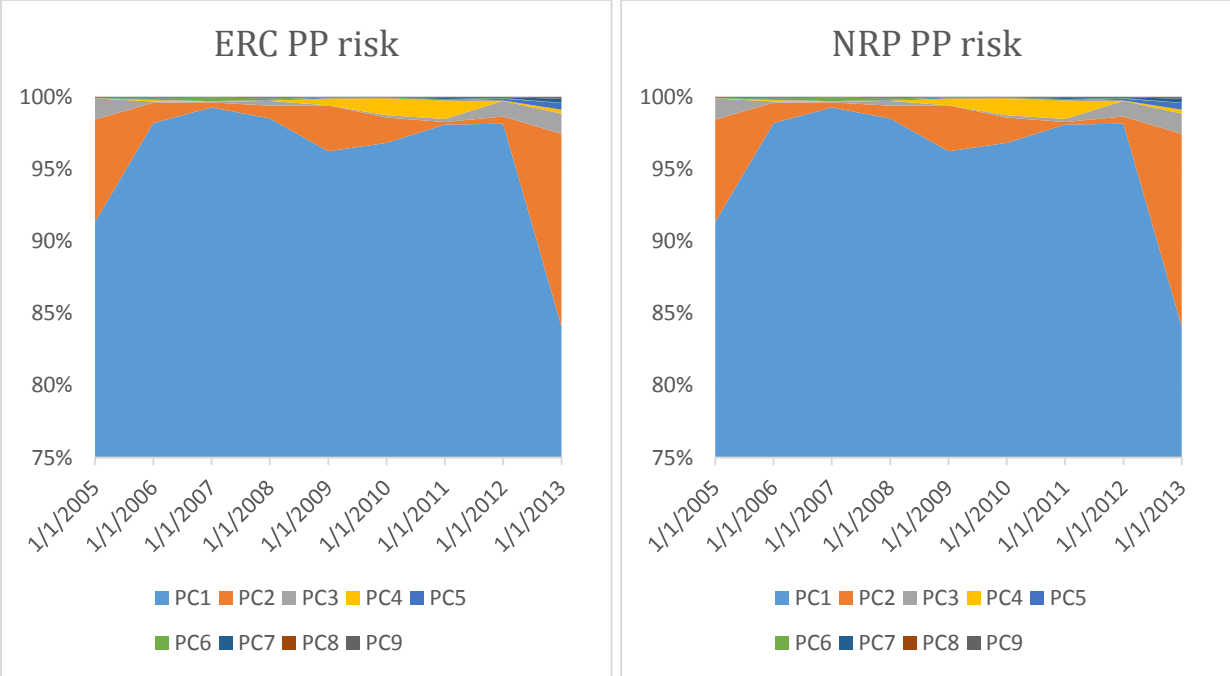
Appendix C Risk decomposition of FTSE/JSE sectors by Principal Portfolios



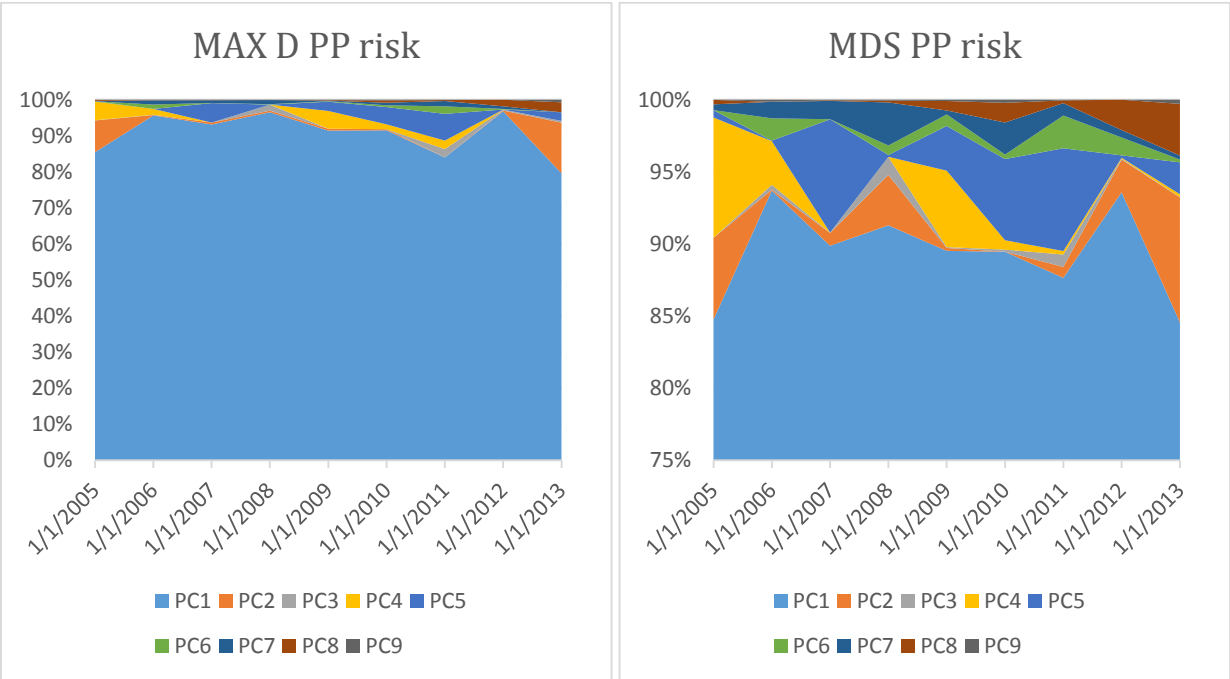
C.1 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the ALSI.



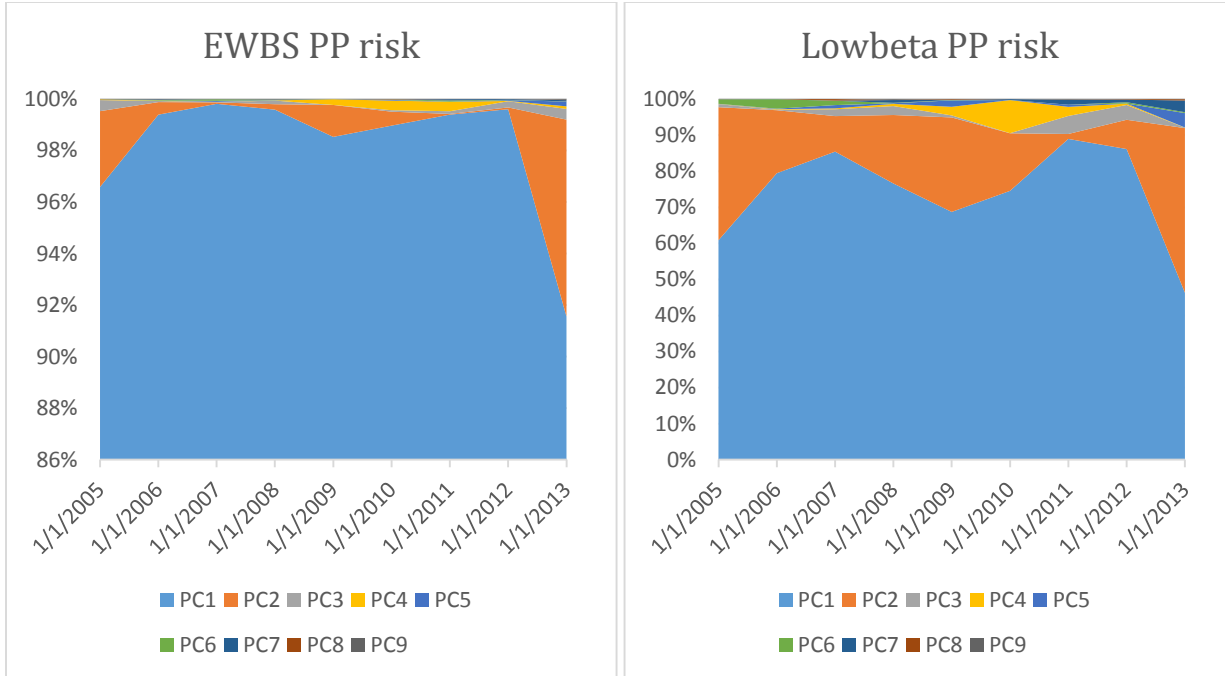
C.2 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the SIM and MIN VAR.



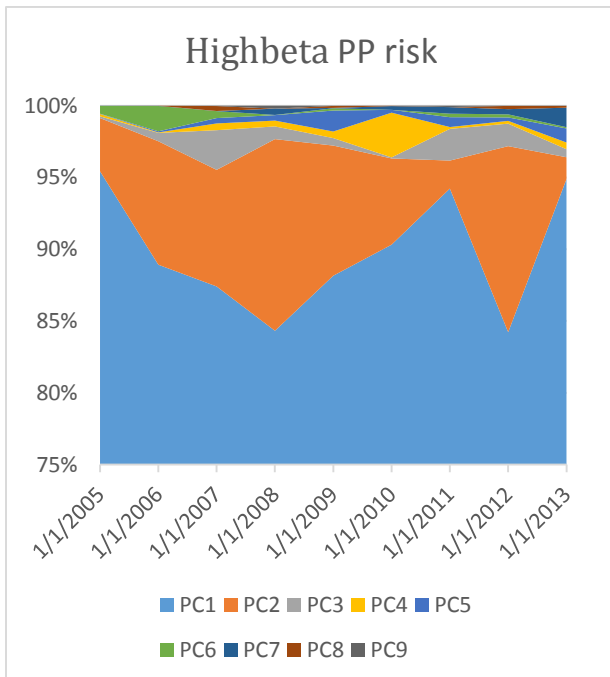
C.3 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the ERC and NRP.



C.4 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the MAX D and MDS.

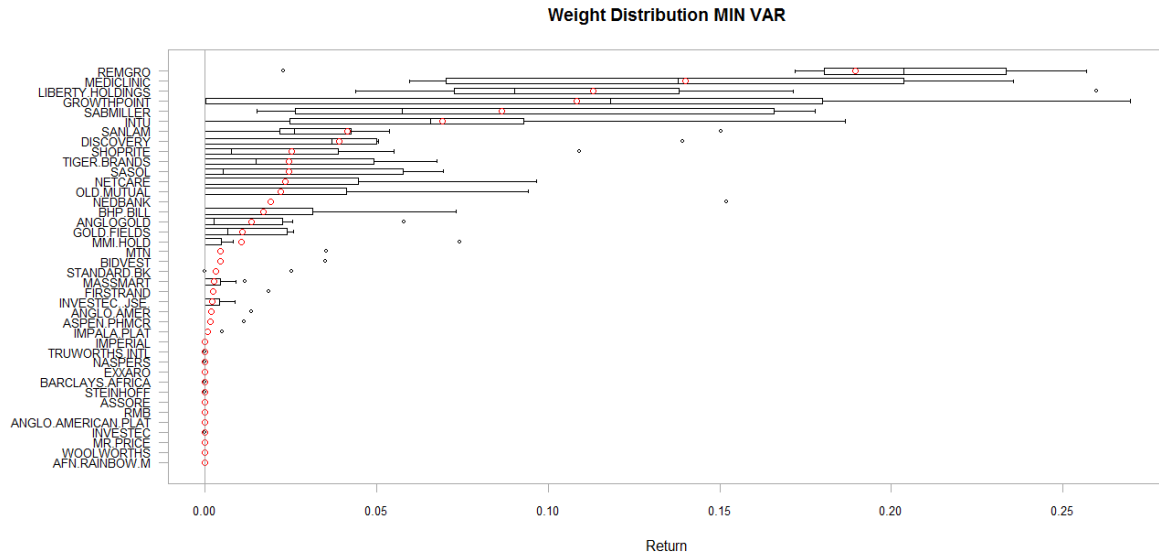


C.5 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the EWBS and Lowbeta.

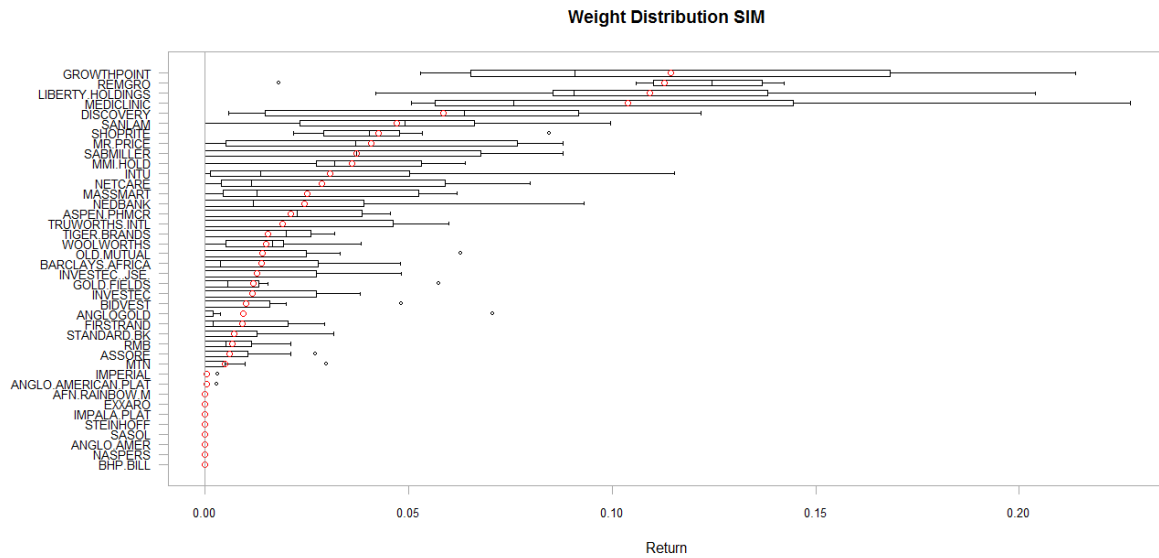


C.6 Risk Decomposition of FTSE/JSE sectors by Principal Portfolios for the Highbeta.

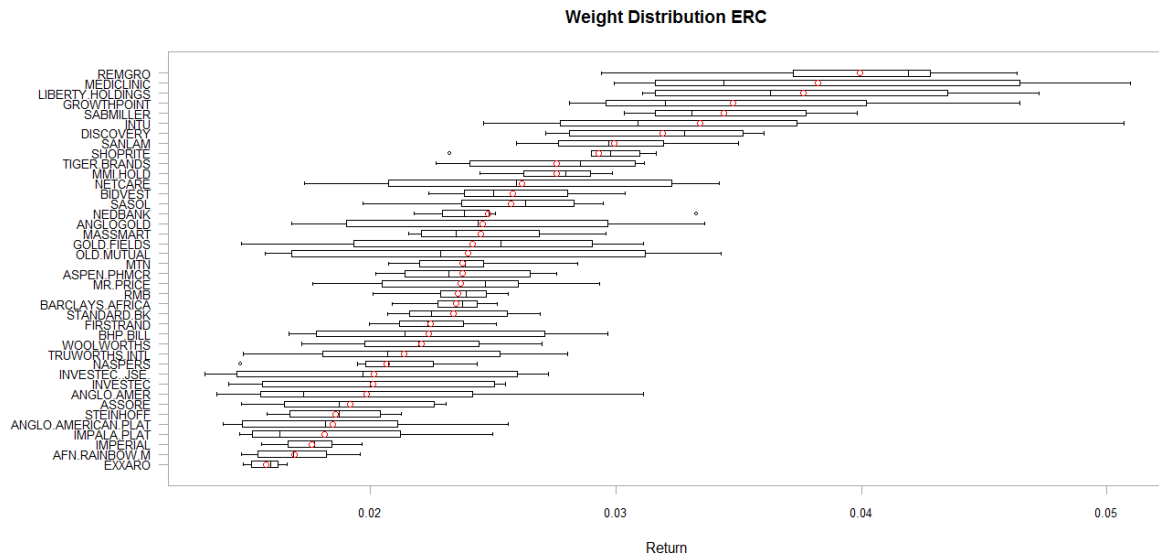
Appendix D Boxplot of Portfolio weight for the JSE Top 40 Stocks Low Volatility Portfolios



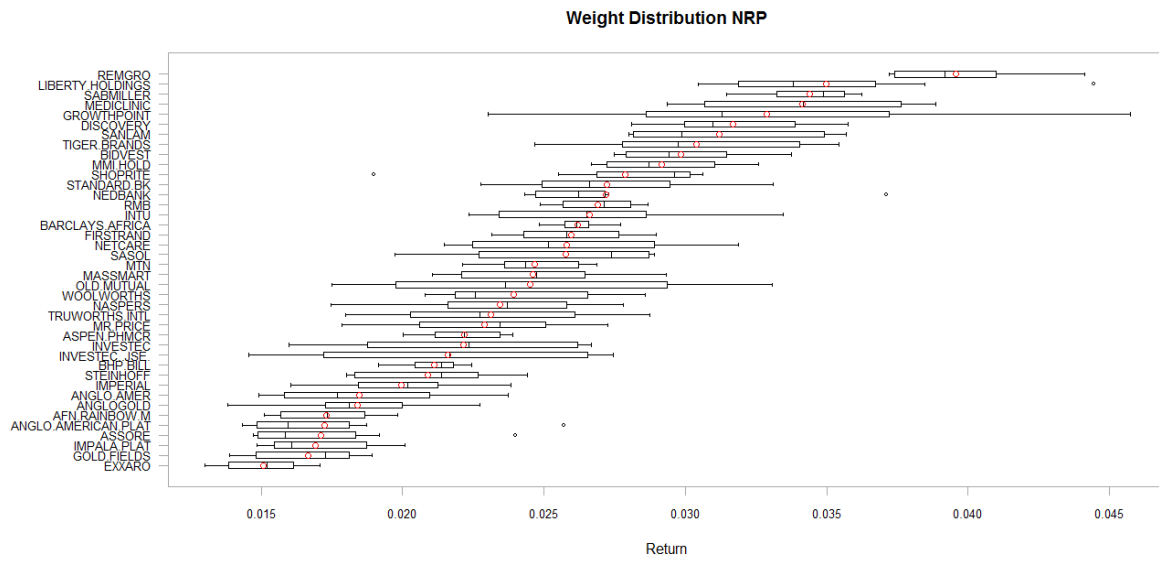
D.1 Boxplot of MIN VAR Portfolio weight



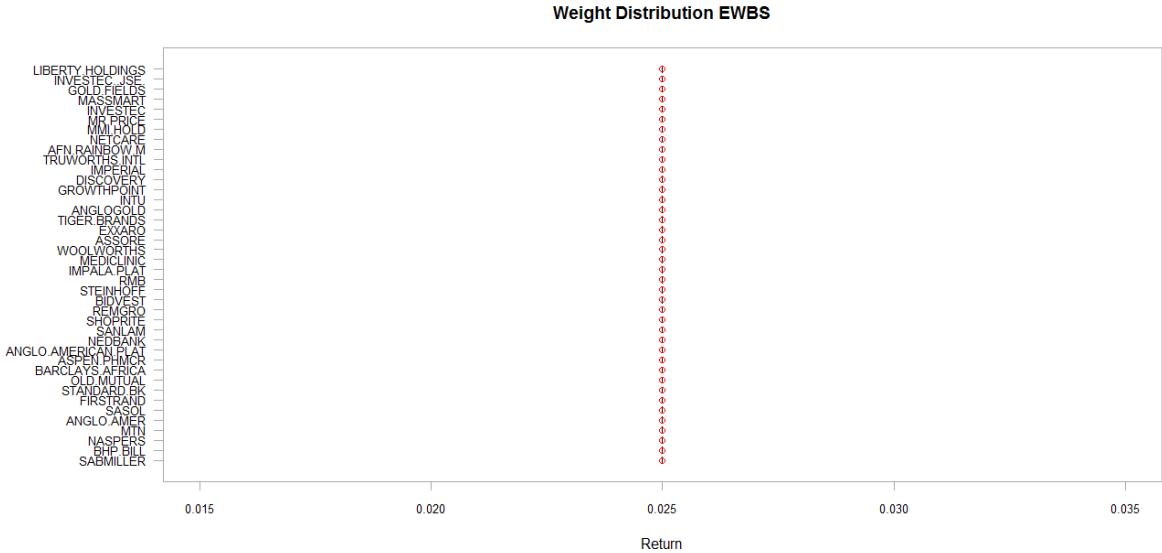
D.2 Boxplot of SIM Portfolio weight



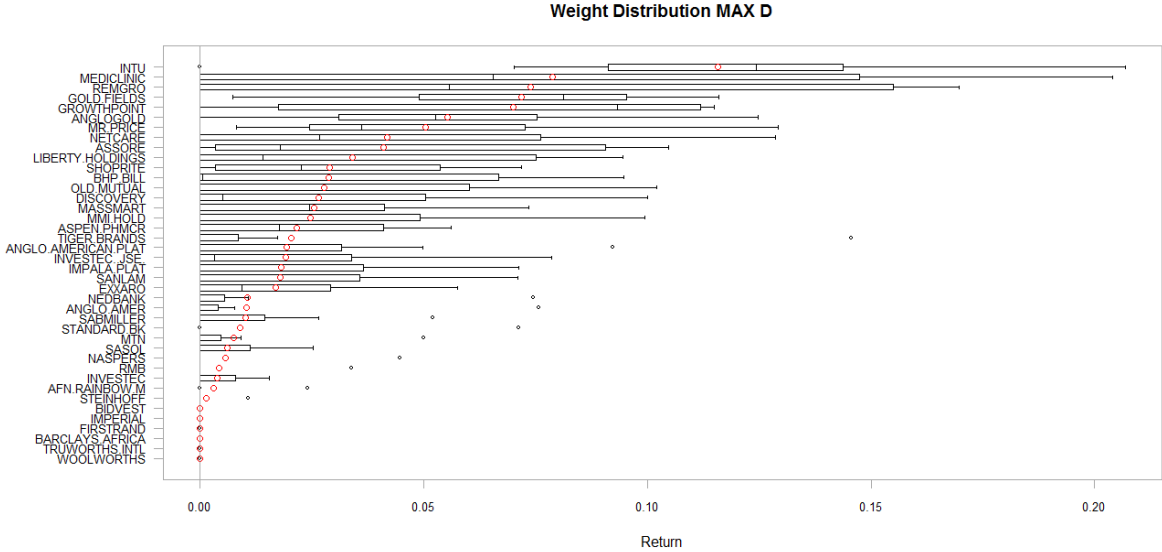
D.3 Boxplot of ERC Portfolio weight



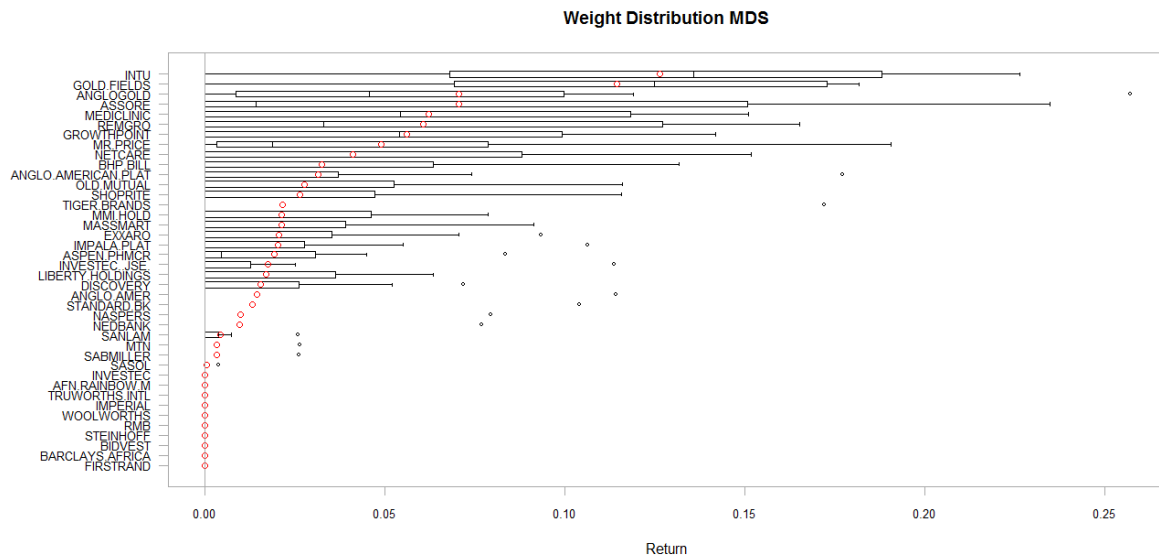
D.4 Boxplot of NRP Portfolio weight



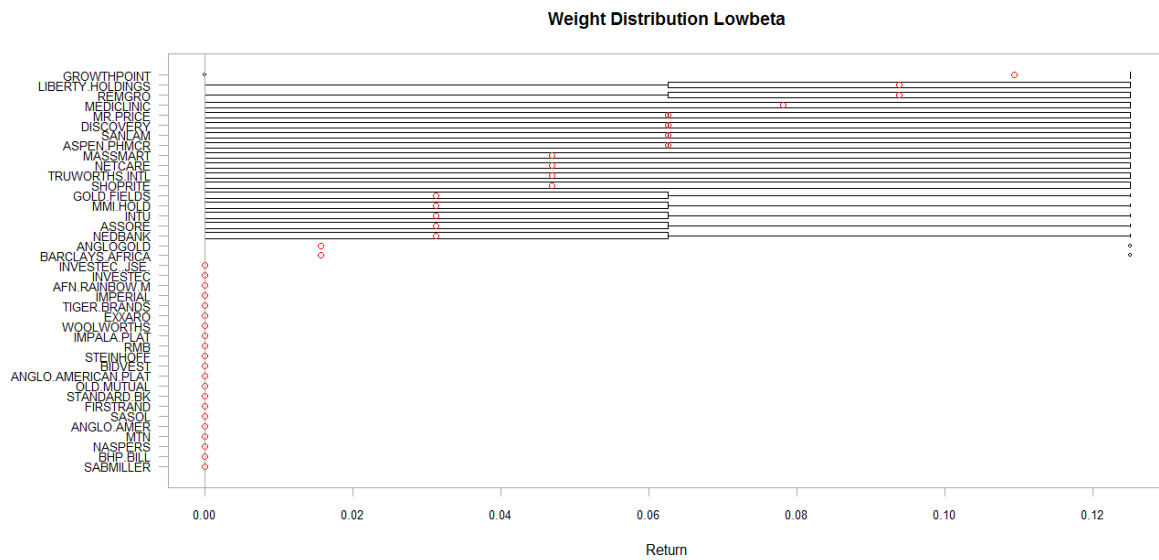
D.5 Boxplot of EWBS Portfolio weight



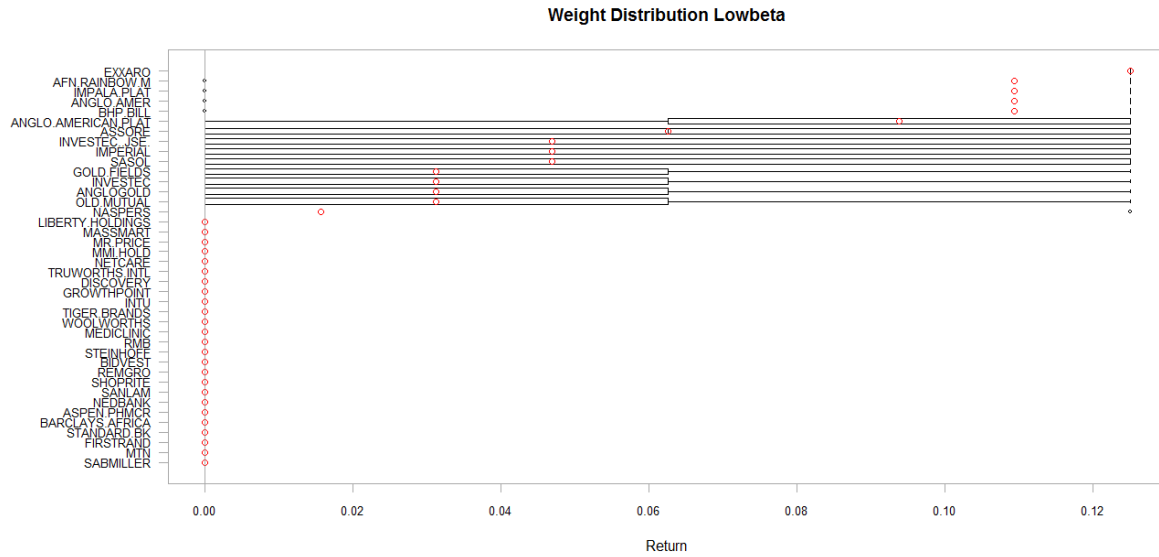
D.6 Boxplot of MAX D Portfolio weight



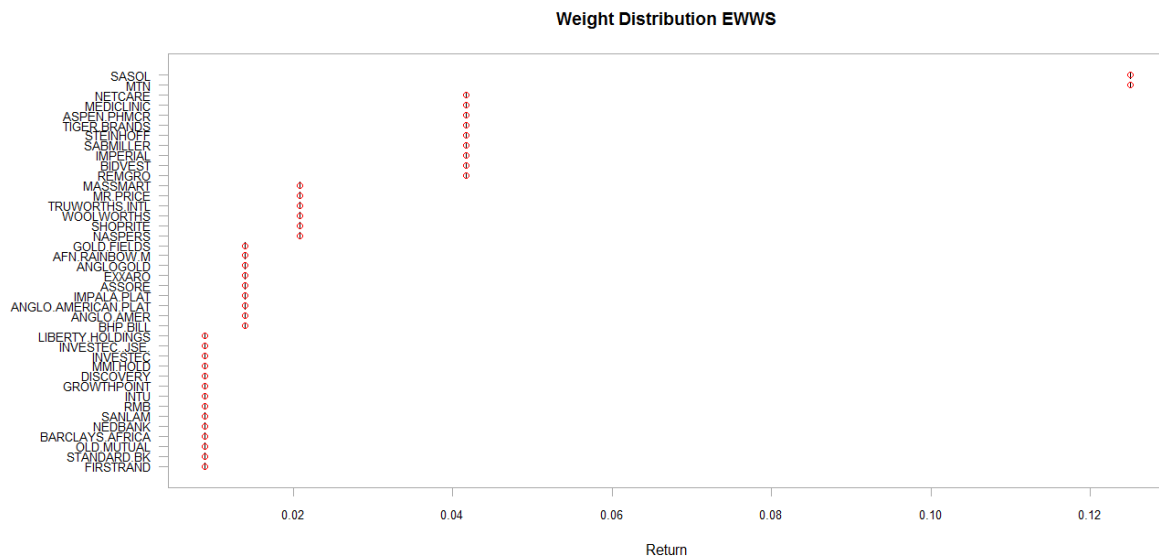
D.7 Boxplot of MDS Portfolio weight



D.8 Boxplot of Lowbeta Portfolio weight



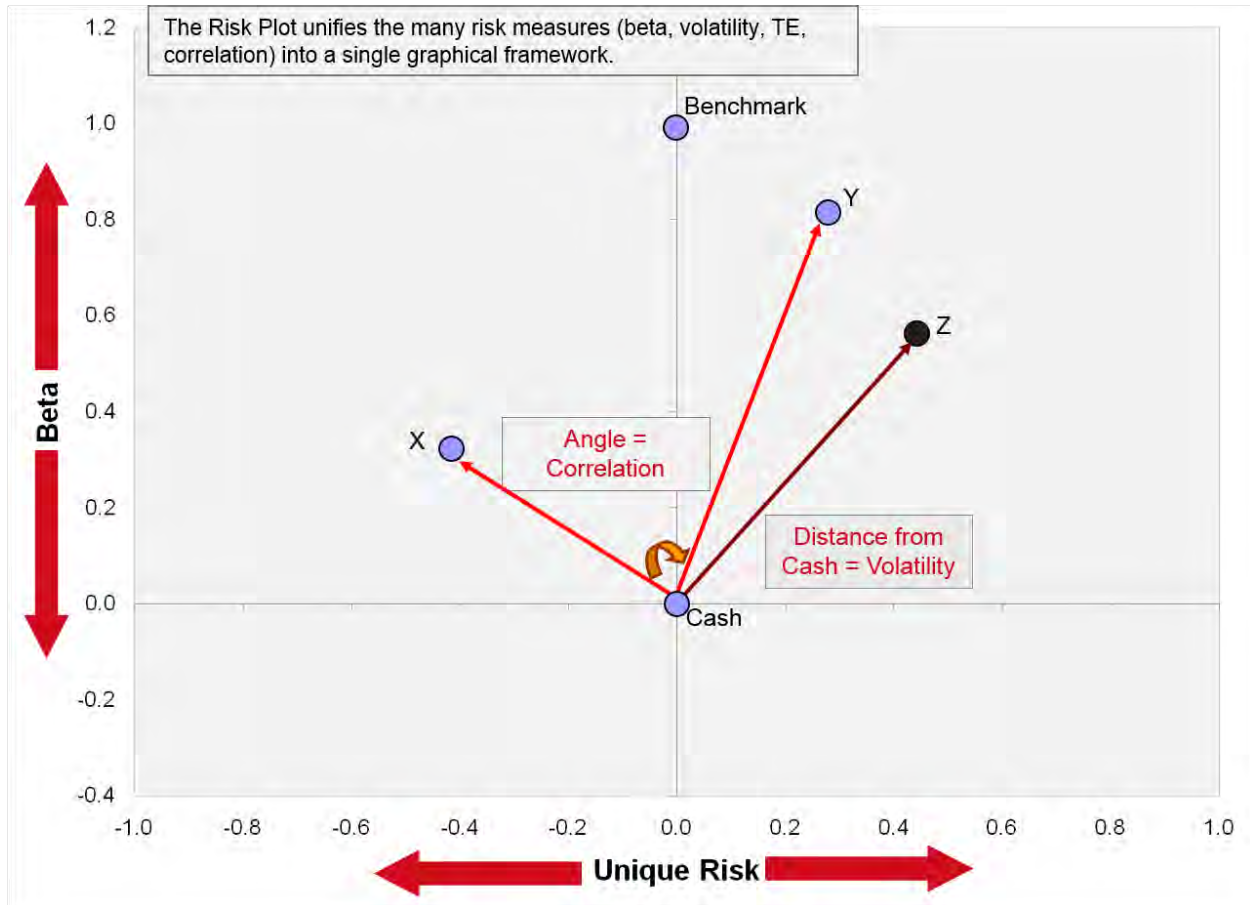
D.8 Boxplot of Highbeta Portfolio weight



D.9 Boxplot of Equal weighting within sector (EWWS) Portfolio weight

Appendix E Analysis of Covariance Bi-plot

The concept of the covariance bi-plot is used to visualize the risk characteristics of a portfolio relative to its benchmark. An illustration of how one can interpret the different measures on the covariance bi-plot is depicted below (Khuzwayo 2011):



E1: Graphical illustration of the Covariance bi-plot, Khuzwayo (2011)