Decision support for the production and distribution of electricity under load shedding

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Abstract

Every day national power system networks provide thousands of MW of electric power from generating units to consumers. This process requires different operations and planning to ensure the security of the entire system. Part of the daily or weekly operation system is the so called *Unit Commitment* problem which consists of scheduling the available resources in order to meet the system demand. But the continuous growth in electricity demand might put pressure on the ability of the generation system to sufficiently provide supply. In such case *load shedding* – a controlled, enforced reduction in electricity supply – is necessary to prevent the risk to system collapse. In South Africa at the present time, a systematic lack of supply has meant that regular load shedding has taken place, with substantial economic and social costs. In this research project we study two optimization problems related to load shedding. The first is how load shedding can be integrated into the unit commitment problem. The second is how load shedding can be fairly and efficiently allocated across areas. We develop deterministic and stochastic linear and goal programming models for these purposes. Several case studies are conducted to explore the possible solutions that the proposed models can offer.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Signed by candidate

Rosephine Georgina RAKOTONIRAINY, 7 December 2015.

Acknowledgements

"Cast all your anxiety on Him because He cares for you", 1 Peter 5:7.

I first thank God for His mercy and love. He always stands by my side in everything I do.

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Finally, I thank my Mom, my family and my love Tanjona for their prayers and encouragements. This work is dedicated to my Mother.

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Acronyms

AIMMS	Advanced Integrated Multidimensional Modelling Software
CEO	Chief Executive Officer
COPT	Capacity Outage Probability Table
DSS	Decision Support System
ED	Economic Dispatch
EENS	Expected Energy Not Served
ELNS	Expected Load Not Served
ExD	Expected Demand
GP	Goal Programming
GUI	Graphical User Interface
LOLP	Loss of Load Probability
LP	Linear Programming
MOLP	Multiple Objective Linear Programming
MW	Megawatt
PLS	Planned Load Shedding
RCUC	Reliability Constrained Unit Commitment
RV	Range of Value
SA	South Africa
SO	Single Objective
SR	Spinning Reserve
TD	True Demand
UC	Unit Commitment
ULS	Unplanned Load Shedding
VOLL	Value of Loss of Load

Chapter 1

Introduction

1.1 Background

Electricity infrastructure consists of complex systems of power generation, transmission and distribution. Sufficient quantities of electricity have to be generated from generation plants and transmitted through transmission components to meet demand at all times. This system requires a complex mix of management, operations and planning in order to safely deliver power to consumers. Part of this requirement is the so called "power system operations scheduling". It is a set of daily and/or weekly processes which encompass a variety of tasks needed to generate electric supply. Figure 1.1 provides a diagram illustrating the basic components of these operations.

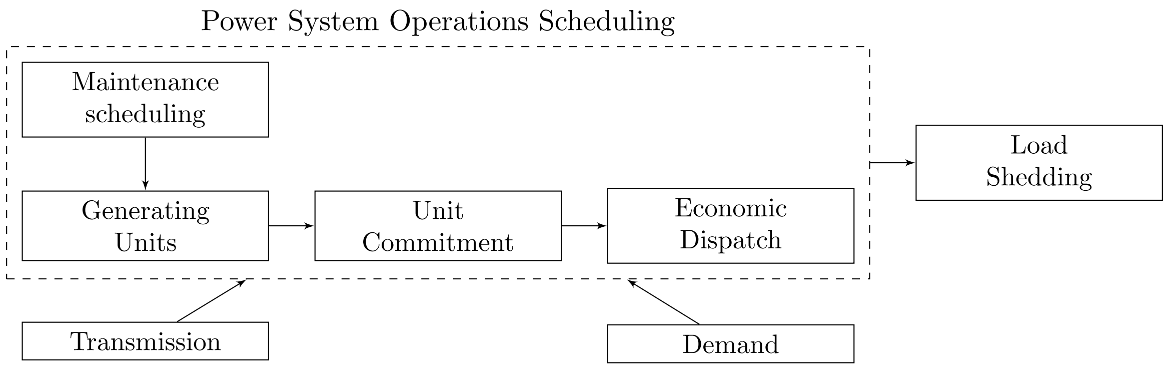


Figure 1.1: Diagram for operation scheduling in power system.

- Maintenance scheduling consists of finding a schedule for the planned outages of generating units. Depending on the utility, this task might be performed on a daily or weekly basis.
- Unit commitment (UC) is an optimization problem that schedules at what stage of the planning period different units should be turned on or off in order to meet demand at a low operating cost. This is performed on a day-to-day basis, based on load forecasting.
- The economic dispatch (ED) is the determination of the output level of the committed units induced from the UC problem. The UC and ED problems are interdependent and are modelled as a single problem (Wood and Wollenberg (2012), Carpentier et al. (1996)). From now on, when we refer to UC problem, it does involve the ED problem.

Each of these operations depend on the forecasted demand which can never be totally accurate. For example, the electricity demand may fluctuate and vary at any time depending on the weather; generating units might break down and fail to produce electricity at unknown times. This failure cannot be predicted in advance. However, each of these uncertainties can result in a shortage of electricity supply and an overload of the system. In such situations, where supply falls behind demand, operators have two options: either they enforce a reduction in supply (or demand) via load shedding or they leave the system at a high risk of total failure. The latter option is avoided if at all possible since it could take days to restore the entire system after such a failure. Thus load shedding is the last resort implemented to compensate for shortage in supply.

Eskom, the state-owned producer and distributor of electricity in South Africa, for example was forced to implement load shedding in October 2007. Several causes were reported, including insufficient coal supply, shortage in supply to meet the high electricity demand, low reserve margin and multiple trips at a number of different stations. Nevertheless, the situation became stable from May 2008 as Eskom managed to take some measures to improve and maintain plants performance. However, early in November 2014, two coal storage silos in Majuba power station collapsed which caused the shut down of the plant and loss of 4110 MW of supply. This event forced Eskom to again implement load shedding. The situation even deteriorated to such an extent that on 5 December Eskom implemented stage 3 load shedding, due to the loss of two power plants and water reserve shortages in some hydro plants. Since then periodic load shedding has taken place in South Africa [1].

Recently, the situation has alleviated, with no load shedding in the past 3 months. It was explained that "Eskom traditionally conducts maintenance on its power stations during summer when demand is lower than the winter month. This year Eskom has not put too many units on maintenance and this year's winter was relatively warm, allowing the utility to conduct some maintenance" [2]. In addition Eskom CEO Brian Molefe said that "the capacity outlook until August next year, when the next maintenance season will commence, was healthy"[3]. However the longer-term outlook remains uncertain.

Load shedding has considerable economic and social costs. According to Chris Yelland, an energy expert, "the controlled blackouts since December 2014 have had a serious negative economic impact". He estimated that "stage 1 load shedding – 10 hours of blackouts per day for 20 days a month – is R20 billion per month. Stage 2 load shedding, using the same time parameters, cost the economy R40 billion per month, while stage 3 is estimated to cost SA R80 billion per month"[4].

1.2 Statement of the problem

In view of the information described above, a key area of concern is the study of load shedding problem. In this thesis, two optimization problems related to the load shedding are addressed.

The first consists of modelling load shedding as part of the UC problem. That is we schedule load

[1] https://en.wikipedia.org/wiki/Eskom#cite$_{\text{note}}$-faq-10
[2] http://www.fin24.com/Economy/Eskom/live-eskom-state-of-the-system-update-20151116
[3] http://www.fin24.com/Economy/Eskom/molefe-we-plan-to-meet-electricity-demand-100-20151116
[4] http://mybroadband.co.za/news/energy/118479-how-much-load-shedding-costs-south-africa.html

shedding by integrating it in the UC decision process. This problem has been considered by many researchers (Ruiz et al. (2009), Parvania et al. (2010), Xiong and Jirutitijaroen (2013)). A commonly used approach consists of accommodating reliability constraints in the UC process, referred to as reliability-constrained unit commitment (RCUC). Load curtailment is allowed but limited by bounding the risk of loss load expressed by some reliability metrics. In our case we model load shedding as a dummy unit and schedule it as part of the available units used for supply. Deterministic and stochastic mixed-integer programs will be considered to model the problem. They are drawn from the proposed models in literature (Carpentier et al. (1996),Ruiz et al. (2009), Parvania et al. (2010), Xiong and Jirutitijaroen (2013)). Our objective is to compare the performance of those models with respect to the following three metrics: the expected costs induced, the amount of planned and expected unplanned load shedding.

Furthermore, should load shedding have to happen, a planned schedule is necessary to allocate areas on the required period of shedding. This constitutes the second part of the work whereby a time-based load shedding schedule is developed. The ultimate goals of the load shedding scheduling are to fairly allocate areas on the scheduling horizon and also to minimize the possible economic impacts. Defining fairness goal is not straightforward but we consider two types of fairness in this thesis. The main objective is to emphasize the trade-off between fairness and economic cost goals. Linear and goal programming models will be developed for this purpose.

1.3 Objectives

The following objectives are pursued in this thesis:

1. to conduct a literature survey related to:
    (a) mathematical optimization, specifically linear and goal programming, deterministic and stochastic; single and multi-objective,
    (b) unit commitment and load shedding,
2. to develop models for UC under load shedding and investigate a comparison of them,
3. to develop models for load shedding scheduling and explore possible solutions that the models can offer.

1.4 Limitations

- The study will not cover the other components of power system operations scheduling, such as maintenance scheduling, as well as external systems, like transmission network. As displayed in Figure 1.1 the UC (under load shedding) problem might depend on those infrastructures but for the sake of simplicity they are not incorporated in the modelling process.
- For the simulation and experimental studies, real data are used where available, otherwise hypothetical data are considered.

1.5 Thesis organisation

The outline of this work is as follows. In Chapter 2, a comprehensive literature review on mathematical optimization is presented. This comprises linear and goal programming as well as stochastic programming. The reader is also introduced to the UC problem whereby different approaches to model the problem are described.

The study of UC under load shedding is discussed in Chapter 3. The first section contains some background information concerning the same. Then the problem formulation is presented in the following section. Thereafter, the simulation study as well as the obtained results are discussed. The chapter concludes with a presentation of *decision support system* (DSS) that might be used to solve the problem.

Chapter 4 contains the load shedding scheduling problem. In the first section, the problem is placed in its context. Then the next section contains the problem formulation. Furthermore, the numerical experiments and the study results are discussed in the following sections. The last section covers the DSS for solving load shedding scheduling problem according to the proposed formulations.

The thesis closes in Chapter 5 with a summary of the entire work as well as discussion and future work.

Chapter 2

Literature review

This chapter introduces the reader to the basic mathematical optimization techniques that are in use throughout this thesis. Additionally, a literature review concerning the unit commitment problem is presented.

2.1 Mathematical optimization

Mathematical optimization encompasses a variety of techniques that deal with problems of maximizing or minimizing real function(s) subject to constraints on the variables involved. This includes linear programming, integer programming and stochastic programming (Shapiro et al. (2009), Sakawa et al. (2013), Birge and Louveaux (2011)). This section contains a concise review concerning these optimization techniques that are in use for this work.

2.1.1 Linear and Goal programming

2.1.1.1 Linear programming

Linear programming (LP) is a subfield of mathematical optimization in which the objective function and all constraints are linear. An LP problem can be expressed in the following form

$$\text{Maximize} \quad c^T x \tag{2.1}$$

$$\text{subject to} \quad Ax \leq b \tag{2.2}$$

$$x \geq 0, \tag{2.3}$$

where $c, x \in \mathbb{R}^p, b \in \mathbb{R}^m$ and A is an $m \times p$ matrix. The vector x is the decision variable, the expression $c^T x$ consists of the objective function and the inequality $Ax \leq b$ expresses the constraints. The intersection of the constraints forms a polyhedron in $\mathbb{R}^p$, and it is closed, convex with a finite number of extreme points.

The *simplex method*, published by Dantzig in 1947, was the first practical algorithm for solving LP problems. It explores the property that if the problem is feasible and bounded then the optimum is attained at a vertex. Thus, this technique is an iterative procedure attempt to find the optimal solution by moving from one extreme point to another. It uses the "pivot rules" to determine the next direction of travel once a basic point is reached [1]. The simplex method is accurate in practice and is often used in different packages and software for solving optimization problem; but in worst case it takes an exponential time (see Robere (2012), Karmarkar (1984)).

An alternative method for solving LP problems is called *primal-dual interior point* method as described in Robere (2012) [2]. This algorithm, instead of walking along the edges of the polytope from vertex to vertex as in simplex method algorithm, rather it moves inside the polytope by starting from an interior point. The problem is solved iteratively by choosing a convenient sequence which will approach the optimal solution and by taking an appropriate step direction [3].

2.1.1.2 Integer programming

In some cases, some or all decision variables are constrained to be integers as we will see in Chapter 3 and Chapter 4. The first case is known as *mixed-integer linear programming* and the second case is called *integer linear programming.*

The *Branch and Bound algorithm* is often used to solve such problem. The integer condition is relaxed and the corresponding relaxed problem is solved using LP method. If the integrality constraint is not satisfied then branch the problem, split it into two sub-problems by adding a new constraint in each case. Solve again the relaxed sub-problems and continue the process until the optimal integer solution is obtained [4].

An alternative approach to solve integer programming problems is the *cutting plane method.* This method starts by solving the continuous relaxation of the problem. Geometrically, the resulting solution lies on one of the vertex of the convex polytope consisting of all feasible points. If this solution is not an integer, then the method creates an additional constraint which cuts out the vertex found. It is performed by finding a hyperplane with the vertex on one side and all feasible integer points on the other and creating a modified linear program. The new program is then solved and the process is repeated until an integer solution is found.

2.1.1.3 Multi-objective and goal programming

In real problems, one may need to take into account more than one objective functions. In such situations, the problem is formulated in the form of *multiple objective linear programming* (MOLP) by considering all objective functions. If we denote by $z_k = c_k^T x$ the k-th objective ($k = 1, \dots, K$), the aim is to simultaneously optimize all z_k, $k = 1, \dots, K$. It arises often when solving MOLP that a point that optimizes all z_k is not feasible. Therefore the aim is to find a feasible point that satisfies all objectives as closely as possible.

One feature of the MOLP optimization is in its *interactive application*; precisely it requires an iterative solution procedure in which the decision maker investigates a variety of solutions to find

[1] More details in the key steps of the algorithm are provided in Appendix A.1.

[2] This algorithm is similar to Karmarkar‘s algorithm called the projective algorithm based on an interior point approach (in 1984).

[3] Details of the algorithm is explained in Appendix A.2.

[4] The solution to the relaxed problem consists of an upper bound on the best possible integer solution that can be obtained from the problem. Details of the branch and bound algorithm is given in Appendix A.3.

one that is most satisfactory. One simple approach to solve the problem is to optimize a composite objective of the form

$$\sum_{k=1}^{K} \omega_k z_k$$

for suitably chosen weights $\omega_k, k = 1, \ldots, K$. In this case, decision makers can adjust the value of weights in order to achieve the best compromise solution.

An alternative approach is called STEM as described in Steuer (1986) (see also Ragsdale (2010)). The following is the summary of the steps of the procedure:

1. Solve the problem by optimizing each objective function to determine the optimal value z_k^* of each objective k, $k = 1, \ldots, K$.
2. Use the optimal objective value obtained as a target value and the corresponding objectives as goals. Then, create a deviation that measures the amount by which any given solution fails to meet the goal (e.g. $\frac{z_k - z_k^*}{z_k^*}$).
3. Assign a weight to the deviation function and create an additional constraints that require the value of the weighted deviation function to be less than the MINIMAX variable Q.
4. Solve the resulting problem with the objective of minimizing Q.
5. If the solution is not satisfactory, adjust the weights and return to step 3.

Another way to formulate problems with multiple objectives is called *goal programming* as proposed by Charnes and Cooper (1957). In this case, a desirable target value for each objective z_k, say t_k ($k = 1, \ldots, K$) is known in advance. Additional variables d_k^+ and d_k^-, $k = 1, \ldots, K$, are considered to represent the deviation of each goal from its target value ($z_k - d_k^+ + d_k^- = t_k$). In addition some weights, say w_k^+ and w_k^-, are associated to the deviational variables in order to, first adjust scales as the z_k may be measured on different scales and also to penalize any undesirable deviation.

The classical method in solving goal programming is to reduce the multiple goal-achievement problem into a single objective of either minimizing a weighted sum of deviations from goals, known as "*Archimedian method*": min $\sum_{k=1}^{K}\{w_k^- d_k^- + w_k^+ d_k^+\}$, or minimizing the maximum weighted deviations, called "*Chebychev method*": min D where $D = \max_k\{w_k^- d_k^- + w_k^+ d_k^+\}$ (Deb (2001)). In this sense, the classical algorithm for solving linear programming can be applied to solve the problem [5].

It may be noted that all of the above programming problems are deterministic in the sense that all coefficients and parameters involved are fixed. However, for many practical problems, the problem data cannot be known with certainty for a variety of reasons such as measurement errors, information about the future, or unobserved events. Many ways have been proposed to model uncertain quantities, including *range modelling* by which the uncertain quantities are viewed to lie into a particular intervals or in some given set of possible values (Kleywegt and Shapiro (2001)). In such case the decision is made by hedging against the worst possible outcome. Another approach is the so called *stochastic modelling* or *stochastic programming* which will be discussed in the next section.

[5] In our context we talk only about linear goal programming. However the model has been extended to non-linear goal programming (Malhotra and Arora (1999), Deb (2001)).

2.1.2 Stochastic programming

The uncertain elements are modelled as random variables with known or estimated probability distributions. Here the probability of the random demand can be estimated from data that have been collected over time. The major class of stochastic programming consists of *stochastic programs with recourse* or *two-stage programming*: a decision has to be taken before the realization of uncertain parameters, referred to as the first-stage decision. After occurrence of the random events, a recourse decision is made in the second-stage to compensate any inconsistency resulting from the first-stage decision (Shapiro and Philpott (2007), Birge and Louveaux (2011), Shapiro et al. (2009), Sakawa et al. (2013)). Another class of stochastic programming is the so called *chance-constrained programming* (CCP). In this approach constraints violation is allowed up to a specified threshold (Prekopa (1970), Birge and Louveaux (2011), Sakawa et al. (2013)). These two models can be applied for both single and multi-objective problems. The latter case includes *stochastic goal programming*. These models are described below.

2.1.2.1 Two-stage stochastic programming

The classical two-stage stochastic programming problem can be formulated as

$$\min\ c^T x + E_\zeta[Q(x,\zeta)] \tag{2.4}$$
$$\text{s.t. } Ax = b, \tag{2.5}$$

where E_ζ denotes the expectation with respect to the distribution of ζ and $Q(x,\zeta)$ is the optimal value of the second-stage problem

$$\min \left\{ q(\zeta)^T y | Wy = h(\zeta) - T(\zeta)x; y \in Y \right\}. \tag{2.6}$$

Here $x \in X \subseteq \mathbb{R}^{n_1}$ is the first stage decision vector, $c \in \mathbb{R}^{n_1}$ and $b \in \mathbb{R}^{m_1}$ are known vectors, A and W are known matrices of size $m_1 \times n_1$ and $m_2 \times n_2$ respectively. The random vector ζ is defined on some probability space (Ξ, F, P) and for each $\zeta \in \Xi$, $q(\zeta) \in \mathbb{R}^{n_2}$, $h(\zeta) \in \mathbb{R}^{m_2}$ and $T(\zeta)$ an $m_2 \times n_1$ matrix. Finally $Y \subseteq \mathbb{R}^{n_2}_+$ [6].

Typically, to avoid computation burden, a finite number of possible realizations of ζ, called *scenarios*, is considered. We denote by (q^s, h^s, T^s) the s-th scenario with probability p^s, $s = 1, \dots, S$. In this case, the expected value $E_\zeta[Q(x,\zeta)]$ is equal to the optimal value of

$$\min \sum_{s=1}^{S} p^s q^s y^s \tag{2.7}$$
$$\text{s.t. } Wy^s = h^s - T^s x, \tag{2.8}$$
$$y^s \geq 0, \quad s = 1, \dots, S \tag{2.9}$$

[6] It might be noted that the given formulation is for linear two-stage, but one can consider a non-linear model as proposed in Beets, Kulkarni and Shanbhag (2012). Moreover, the set of variables X and Y can be constrained to be a subset of $\mathbb{Z}^{n_1}_+$ and $\mathbb{Z}^{n_2}_+$ respectively, which may result a non-convex objective function (see Klein Haneveld and van der Vlerk (1999)).

and the whole two-stage problem is equivalent to the following large-scale programming problem

$$
\begin{aligned}
&\min\ c^T x + \sum_{s=1}^{S} p^s q^s y^s && (2.10)\\
&\text{s.t. } Wy^s = h^s - T^s x, \quad s = 1, \dots, S && (2.11)\\
&\quad\ Ax = b, && (2.12)\\
&\quad\ x \geq 0, y^s \geq 0, \quad s = 1, \dots, S && (2.13)
\end{aligned}
$$

Duality-based decomposition methods are usually used to solve two-stage stochastic programs (CarøE and Schultz (1999), Klein Haneveld and van der Vlerk (1999), Birge and Louveaux (2011)). One class of such approaches is "*Lagrangian decomposition*". In this approach the problem is split into manageable sub-problems referred to as *scenario decomposition*. The solution procedure can be stated as follows:

- Given the deterministic equivalent formulation (2.10)-(2.13) of the problem, the first- stage variables are replicated for each of the scenarios and then the problem can be rewritten in the form

$$
\begin{aligned}
&\min\ \sum_{s=1}^{S} p^s (c^T x^s + q^s y^s) && (2.14)\\
&\text{s.t. } Wy^s = h^s - T^s x, \quad s = 1, \dots, S && (2.15)\\
&\quad\ Ax = b, && (2.16)\\
&\quad\ x^1 = \dots = x^S, && (2.17)\\
&\quad\ x \geq 0, y^s \geq 0, \quad s = 1, \dots, S && (2.18)
\end{aligned}
$$

  The additional equality constraint $x^1 = \dots = x^S$ is called the *non-anticipativity* condition which means that the first-stage decision should not depend on the future observation.

- From (2.14)-(2.18), form the Lagrangian relaxation with respect to the non-anticipativity condition and establish the corresponding Lagrangian dual problem [7].

- Solve the Lagrangian dual problem by using subgradient methods (see Appendix B). This can be done easily by splitting the problem into separate scenario subproblems.

- It is well known that, from the weak duality theorem (see Appendix B), the solution of the dual problem gives a lower bound of the optimal solution of the primal problem (2.10)-(2.13). The difference between the optimal solution and the solution from the dual problem is known as duality gap. The algorithm iterates until the value of the duality gap vanishes or falls below a threshold [8].

[7] Explanation of the Lagrangian duality is given in Appendix B.

[8] In the case of convex problem the duality gap always vanishes. When applied to non-convex problems, the duality gap can remain strictly positive and the algorithm stops when it is relatively small. In this case there is no guarantee that the global optimum is reached. The convergence to the global optimum has been obtained by performing this method within a branch and bound framework as explained by CarøE and Schultz (1999).

2.1.2.2 Chance-constrained programming

A typical formulation of CCP problem is given by

$$
\begin{aligned}
&\min\ f(x) \\
&\text{s.t.}\ \ Pr\{G(x,\zeta) < 0\} > 1-\epsilon \\
&\qquad x \in X
\end{aligned}
$$

where $X \subseteq \mathbb{R}^n$, $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is the objective function, ζ is a random vector with known probability distribution, $G : \mathbb{R}^n * \mathbb{R}^d \longrightarrow \mathbb{R}^m$ is a constraint mapping and $\epsilon \in (0,1)$. The constraint $Pr\{G(x,\zeta) < 0\} > 1-\epsilon$ is called a joint probability constraint while the constraints $Pr\{G_i(x,\zeta) < 0\} > 1-\epsilon_i$, $j = 1,\ldots,m$, are called individual probability constraints. Most of the time the function $f(x)$ is treated as a linear function but it can be a non-linear convex function or even a non-convex function. Moreover, in case of linear objective function ($f(x) = c^t x$), if some or all of the elements of c are random variables then the objective function becomes an expected value function (Sakawa et al. (2013)).

In case where the random vector ζ follows a multivariate normal distribution, with mean μ and covariance matrix Σ, and the constraint is linear of the form $G(x,\zeta) = \zeta^T x - b$, the CCP problem is reduced to a deterministic convex optimization problem of the form

$$
\begin{aligned}
&\min\ f(x) \\
&\text{s.t.}\ \ \mu^T x - \phi^{-1}(1-\epsilon)\sqrt{x^T \Sigma x} < b \\
&\qquad x \in X
\end{aligned}
$$

where $\phi^{-1}(1-\epsilon)$ is the $(1-\epsilon)$- quantile of the standard normal distribution. Further discussion and study about CCP problem can be found in Ahmed and Shapiro (2008) and Birge and Louveaux (2011).

2.1.2.3 Stochastic goal programming

We have seen in Section 2.1.1.3 that two or more objectives might be required to be optimized simultaneously. In the goal programming formulation, a desirable target value t_k is associated to each objective z_k that needs to be achieved. One might consider these targets values to be random vector depending on some random parameter ζ. The problem in such case is known as stochastic goal programming (Martínez and Aguado (1998)), which can be formulated as

$$
\min\ E_\zeta[Q(x, t(\zeta))] \tag{2.19}
$$

$$
\text{s.t.}\ \ Ax = b, \tag{2.20}
$$

where

$$Q(x, t(\zeta)) = \min \left[\sum_k \omega_k y_k | z_k \pm y_k = t_k(\zeta); k = 1, \ldots, K \text{ and } y_k \geq 0 \right]. \qquad (2.21)$$

This model can be viewed as a particular case of stochastic programming with recourse, for $c = 0$ in the formulation (2.4)-(2.6). Therefore, methods to solve stochastic programming with recourse can be used to solve stochastic goal programming.

2.2 Unit commitment

2.2.1 Introduction

Every day national power system networks provide thousands of MW electricity power from generating units to consumers. The electricity demand from various consumers can be predicted throughout the day but it can also vary significantly in real time depending on the nature of human activities and different factors like weather. So meeting this demand at a minimum cost while ensuring the security of the system requires careful planning and operation. The daily and / or weekly on/off scheduling of the power generating units is called *Unit Commitment* (UC). This process consists of deciding when and which generating units at each power station to turn on and turn off and determining the optimal outputs of the scheduled units in order to meet system demand while minimizing the cost associated to the operation. It is a large-scale short-term optimization problem subject to some complicated operation and generation constraints. Moreover, this optimization problem is a stochastic problem by nature. Indeed, this task relies on demand forecast which can never be accurate. In addition, unexpected generating outages is another challenge which cannot be predicted in advance. Thus, operators need to take into account those uncertainties in the optimization problem in order to prevent instability of the system, the risk to system collapse, and also to mitigate customer outages or load shedding. Therefore, besides economic efficiency, reliability or/and security are other concerns of this short-term operation problem. Due to the complexity of the problem and the possibility of potential savings, UC has become an active research for several decades (Takriti et al. (1996), Carpentier et al. (1996), Guan et al. (1992), Wood and Wollenberg (2012), Chattopadhyay and Baldick (2002)).

Generally the UC problem is modelled in a deterministic framework which we will illustrate in the following section. However, because of the stochastic nature of the problem, instead of using a deterministic model rather some researchers adopt a stochastic programming model to tackle the problem. This is the subject of the last section. The remaining sections of this Chapter describe the existing literature on the UC problem.

2.2.2 Deterministic Unit Commitment

As mentioned earlier, the objective of operators in daily or/and weekly UC operation is to schedule the available resources and determine the output levels of the committed units so as to supply the system demand at a minimum cost. This commitment schedule takes into account generation constraints as well as uncertainties. By uncertainties we mean unexpected outages of generation units and load forecast uncertainty. In deterministic framework, the management of uncertainties is done by considering a redundant capacity which is known as "*reserve requirements*" (Guan et al. (1992), Carpentier et al. (1996)). A single point estimate for the hourly demand is used and a certain amount of generation capacity is kept to handle any unforeseen events or failures. The reserve capacity that is spinning and connected to the power system is known as "*spinning reserve*" (SR) while the extra generating capacity provided by fast-start units (gas turbine, hydro-plant) not connected or by interruptible load [9] is called "*non-spinning reserve*". Some utilities consider only the SR in the UC problem whereas others include the non-spinning reserve. A typical formulation of the problem is described below.

Consider a power system with N units or generators [10]. The study horizon is over a specified time period T (typically one day or one week) and the time unit is one hour. It is required to decide the start-up, shut-down and outputs of all units over T with the objective to minimize the total cost subject to generation constraints, system demand and spinning reserve requirements. The mathematical formulation of the problem is given by

$$\min \sum_{t=1}^{T}\sum_{g=1}^{N} C_g(u_{g,t}, p_{g,t}) + S_{g,t}(u_{g,t}) \tag{2.22}$$

$$\text{s.t. } \sum_{g} p_{g,t} = d_t, \quad \forall t \tag{2.23}$$

$$\sum_{g} r_{g,t} \geq R_t, \quad \forall t \tag{2.24}$$

$$u_{gt}\underline{p}_g \leq p_{gt} \leq u_{g,t}\bar{p}_{g,t}, \quad \forall g, t \tag{2.25}$$

$$p_{g,t} + r_{g,t} \leq u_{g,t}\bar{p}_g, \quad \forall g, t \tag{2.26}$$

$$u_{g,t} - u_{g,t-1} \leq u_{g,v}, \quad v = t+1, \ldots, \min\{t + t_i^{\text{up}}, T\}, \quad \text{for } t = 2, \ldots, T \tag{2.27}$$

$$u_{g,t-1} - u_{g,t} \leq 1 - u_{g,v}, \quad v = t+1, \ldots, \min\{t + t_i^{\text{dn}}, T\}, \quad \text{for } t = 2, \ldots, T \tag{2.28}$$

$$p_{g,t} - p_{g,t-1} \leq R_i^{\text{up}}, \quad \forall g, t = 2, \ldots, T \tag{2.29}$$

$$p_{g,t-1} - p_{g,t} \leq R_i^{\text{dn}}, \quad \forall g, t = 2, \ldots, T \tag{2.30}$$

$$u_{g,t}: \text{ binary variable }, p_{g,t} \geq 0, r_{g,t} \geq 0 \quad \forall t, g \tag{2.31}$$

The various components of this model are discussed in turn below.

2.2.2.1 Objective function

The objective function is expressed by the equation (2.22) where $C_g(u_{g,t}, p_{g,t})$ is the power production cost of unit g during time t, $t = 1, \ldots, T$. $S_{g,t}(u_{g,t})$ is the start-up cost of unit g during time t, and $u_{g,t}, p_{g,t}$ are respectively the status of unit g (1 if committed and 0 otherwise) and the power output of unit g during time t.

The cost function $C_g(.)$ is a convex quadratic function expressed as

$$C_g(u_{g,t}, p_{g,t}) = u_{g,t}[a_g(p_{g,t})^2 + b_g p_{g,t} + c_g]$$

[9] Interruptible loads refer to curtailment of consumers loads in accordance with contractual arrangements.

[10] In the case where non-spinning reserve is included in the formulation, additional units representing fast-start units or interruptible load are also included (Ruiz et al. (2009)).

where a_g, b_g, c_g are given parameters (Wood and Wollenberg (2012))[11].

The start-up cost is often given by

$$S_{g,t}(u_{g,t}) = f_g u_{g,t},$$

where f_g is a constant parameter.

2.2.2.2 Constraints

At each time period, the total generation must be equal to the demand d_t as described in equation (2.23). The constraint (2.24) is the SR requirement. A required amount of R_t is scheduled at each time t for security purpose and the total contribution of each unit g, $r_{g,t}$, should be greater or equal than R_t. Equation (2.25), (2.26) are know as capacity constraints. The solution must be such that the output of each generator is within the allowable range with $\underline{p}_g$ and $\bar{p}_{g,t}$ are respectively the minimum and maximum allowable output level of unit g at time t. Constraint (2.27) states that if unit g is on, it needs to be on for at least during a specified period t_i^{up} before it can be turned off. Constraint (2.28) states the same for the case of unit off. These two constraints are called minimum up and minimum down time. Moreover, besides all these constraints, each unit has a limited ability to change its output from one level to another during a period of time. These constraints are known as ramping limits given in equation (2.29) and (2.30) where R_i^{up} is the ramp-up limit and R_i^{dn} the ramp-down limit [12]. All of the stated constraints must be satisfied at all periods during the time horizon.

The crucial point in UC problem is the determination of SR requirements given in equation (2.24). These resources are used to withstand any sudden failure of generating units or any fluctuation of the demand to protect the system from instability or the risk of collapse. It has been long recognised that by increasing the SR provision, the system is reliable and the risk of customer outages is reduced. However, it leads to high economical cost because additional units have to be committed and some units must be operated at less than optimal output. In the opposite case, under-provision of SR would result to economic efficiency but high risk of outages. Due to these two conflicting interests, optimization of the SR provision has become a major research in the past few decades. The following subsection reports some of the available literature on this subject.

2.2.2.3 Spinning Reserve Requirements

- **Deterministic approach**

  Traditionally, a deterministic criteria is used to provide the amount of SR R_t in (2.24). A commonly used criterion is called $N-1$ criterion which guarantees that any single unit failure does not affect the balance in supply and demand (Wood and Wollenberg (2012)). In this

[11] Each unit is basically characterized by the input-output curve which represents the heat energy required per hour (in MBtu/h) in function of its electrical output (in MW). This function is ideally convex, the output increases gradually as the heat increases. The production cost each unit is mainly determined by the fuel cost times this function

[12] If we include additional units, like fast-start units or interruptible load, in the model then constraints (2.27)-(2.30) do not apply for these types of units.

case,

$$R_t = \max\,(u_{g,t}\bar{p}_g), \quad \forall t.$$

Another criteria is to combine the largest generator and a percentage of peak demand (Vazquez (2006), Wood and Wollenberg (2012)):

$$R_t = \max\,(u_{g,t}\bar{p}_g) + 10\%(\text{peak load}).$$

Most utilities have adopted these criteria as explained by Vazquez (2006) and this reserve ability enables operators to handle the most usual random disturbances. However, some criticisms are levelled at these techniques. First, the SR provision can be excessive or insufficient depending on the reliability of the committed units. In addition for the $N-1$ criteria, there is no guarantee of a positive outcome in case of multiple units outages. These shortcomings conduct some researchers to propose an alternative techniques for computing the SR requirements.

- **Probabilistic approach**

  Instead of using a fixed criteria for determining SR, rather a probabilistic approach is used. Gooi et al. (1999) were the first to consider this approach. The amount of SR is determined subject to satisfaction of a given risk level in presence of generating outage uncertainty. For each time interval, unit commitment is processed and a capacity outage probability table (COPT) is formed based on the list of units committed in this interval. The COPT summarizes all possible combinations of generating outages associated with their probability of occurrence calculated from generating outage rate [13]. This in turn accommodated with the load forecast gives a risk index value which is interpreted as the probability of failing to meet the demand. If this risk value is not less than the predefined maximum risk level then the spinning reserve is adjusted and the unit commitment process is iterated until the target risk is attained. This approach might be computationally intensive because for each time step of the time horizon, several iterations between unit commitment and adjustment of spinning reserve have to be performed before the allowable risk level is reached.

  Chattopadhyay and Baldick (2002) proposed a new approach by approximating the COPT table used in Gooi et al. (1999) with an exponential function. This approximation in turn is converted to a linear probabilistic risk constraint which allows the estimation of the SR requirements to achieve a given permissible risk level. This constraint is then incorporated in the UC problem optimization. The advantage of the method compared to Gooi et al. (1999) is that there is no need to perform an iteration between UC and determination of SR. However, the results depend on the accuracy of the approximation. The proposed method might not provide a good approximation of the exact distribution.

  Bouffard and Galiana (2004) proposed an alternative probabilistic approach for determination of SR in the UC problem. Two reliability metrics are adhered to in the UC formulation; namely LOLP (loss of load probability), which is the probability that the total generation output plus the spinning reserve is less or equal than the system demand, and ELNS (expected load not served)[14]. These authors argued that the provision of SR should be that the LOLP and the

[13] An example of computation of COPT table is given in Appendix C.1.

[14] A detail of expression of these two metrics is provided in Appendix C.2.

ELNS due to the random loss of one or two generating units are below to some specified targets, that is

$$\begin{aligned}\text{LOLP} &\leq \text{LOLP}_{\text{target}} \\ \text{ELNS} &\leq \text{ELNS}_{\text{target}}\end{aligned}$$

This approach might be computationally expensive as it requires the extra bound constraints, and also might not be economical optimal because of the arbitrary selection of the ceiling parameters.

- **SR optimization**

  Instead of using probabilistic criterion, Ortega-Vazquez and Kirschen (2007) proposed a cost/benefit analysis to optimize the SR requirement. They suggested that the reserve should be increased until its cost exceeds the benefit it provides. The benefit in this case is measured by the reduction in customer interruption cost. Thus, the cost of reserve should be compared to the cost of energy not supplied. In the model, they considered an additional term in the unit commitment objective function, the expected customer interruption, such that the optimization problem aims to find the optimal spinning reserve that simultaneously minimizes both operating cost and expected cost of outages caused by units failure. The cost of customer interruption is equal to the product of expected energy not supplied (EENS) and the average value that consumers places on unserved energy (VOLL). Details of evaluation of these two metrics can be found in Billinton et al. (1984), Ortega-Vazquez and Kirschen (2007).

  Liu et al. (2010) tackled the optimization of SR by using a different approach. They combined together in the objective function of unit commitment problem the cost of providing a reserve for each generator at each time and the expected customer interruption cost. In this case, the parameter R_t is omitted and the model automatically determine the spinning reserve capacity by making compromise between the system operation cost and expected customer outage.

2.2.3 Stochastic optimization of Unit Commitment

As stated above the UC operation is subject to two types of uncertainty: load forecast uncertainty and generation unit outages. In a deterministic framework, these two parameters are treated as fixed and the reserve requirements are set to manage any unforeseen events. In contrast, in stochastic framework these parameters are treated as (continuous or discrete) random variables. Therefore, the use of stochastic programming is an alternative approach to model uncertainty in the decision process which is emphasized in many literature, like Takriti et al. (1996), Carpentier et al. (1996), Ruiz et al. (2009), Ozturk et al. (2004) and Carøe and Schultz (1998) [15]. The stochastic techniques used in the formulation of UC problem can be classified into multi-stage stochastic programming, multi-period two-stage stochastic programming and chance-constrained programming.

[15] In some cases, the two sources of uncertainty are incorporated together in the model as in Carpentier et al. (1996), Ruiz et al. (2009), whereas in other cases they are treated separately. Takriti et al. (1996), Carøe and Schultz (1998) and Ozturk et al. (2004) considered the demand as the only uncertain parameter while Parvania et al. (2010) in their work account the random unit outages as the only source of uncertainty.

2.2.3.1 Multi-stage model for stochastic unit commitment

As argued by Takriti et al. (1996), a finite set of scenarios is considered to represent uncertainty. Each scenario, denoted by s, is assigned a probability π^s that reflects its occurrence. The decision process is made separately for each time of the study horizon alternating with observation of the random parameters. A corresponding UC formulation, in compact form, is

$$\min \sum_s \pi^s \sum_{t=1}^{T} \sum_{g=1}^{N} C_g(u^s_{g,t}, p^s_{g,t}) + S_{g,t}(u^s_{g,t}) \tag{2.32}$$

$$\text{s.t. } \sum_g p^s_{g,t} = d^s_t, \quad \forall t, s \tag{2.33}$$

$$u^s_{gt}\underline{p}^s_g \leq p^s_{gt} \leq u^s_{g,t}\bar{p}^s_{g,t}, \quad \forall g, t, s \tag{2.34}$$

$$u^s_{gt} = u_{gt}, \quad \forall t, s \tag{2.35}$$

$$(\mathbf{u}, \mathbf{p}) \in F \tag{2.36}$$

The objective function (2.32) is the expected cost over representative scenarios. The constraints (2.23)-(2.31), except constraint (2.24), in deterministic formulation are enforced for all scenarios. Constraints (2.33) and (2.34) represent respectively the demand-supply and capacity limitations for each scenario s, while constraint (2.36) comprises all other constraints such as minimum up/down time constraints and ramp constraints for each scenario [16]. Reserve requirements (Equation (2.24)) are not enforced in the formulation because it is assumed that the reserve needs are incorporated by the different scenarios. The constraint (2.35) is known as the non-anticipative bundle constraint which means that decisions depend only on the past not on the future [17].

2.2.3.2 Two-stage model for stochastic unit commitment

This model can be considered as a simpler version of the multi-stage model. In this case, the first stage decision consists of selecting units to meet the expected load for each hour during the time horizon based on the generators operating costs. This decision is carried out days or hours ahead before the actual operation. The second stage, after the realization of some uncertainties, consists of determining the output level of the committed units to meet the demand. Carpentier et al. (1996) were the first to implement such problem by considering a finite number of scenario trees to model random disturbances (demand, unit failures) and optimization consisting of minimizing the average cost over the scenario tree. Carøe and Schultz (1998) considered the system load as the only uncertainty and used a finite load profile scenarios. A typical formulation of this problem, in compact form, is given by

[16] F can be interpreted as the set of feasible region.

[17] As explained in CarøE and Schultz (1999) this condition states that two scenarios with the same history until a certain stage should result in the same decisions.

$$\min \sum_{t=1}^{T}\sum_{g=1}^{N} S_{gt}(u_{gt}) + E_s(\sum_{t=1}^{T}\sum_{g=1}^{N} C_g(p_{gt}^s)) \tag{2.37}$$

$$\text{s.t.} \sum_{g} p_{gt}^s = d_t^s, \quad \forall t, s \tag{2.38}$$

$$u_{gt}\underline{p}_g^s \leq p_{gt}^s \leq u_{gt}\bar{p}_{gt}^s, \quad \forall g, t, s \tag{2.39}$$

$$(\mathbf{u}, \mathbf{p}) \in F \tag{2.40}$$

The objective function (2.37) minimizes the sum of the cost of the first decision, start-up cost, and the expected cost over representative scenarios, which are composed by the cost of generation output and recourse actions like bringing online fast-start generating units. The constraints are similar to the multi-stage framework.

2.2.3.3 Stochastic unit commitment using chance-constrained programming

Ozturk et al. (2004) proposed an alternative approach in modelling uncertainty in the UC problem. Instead of using a scenario representation for describing uncertainty, they assumed that the random variable representing the load uncertainty is continuous. In this case, the condition that the supply meet the demand is replaced by a probability constraint that requires this constraint to be satisfied at a predetermined level. This is given by

$$P\left[\sum_{g=1}^{N} p_{g,t} \geq d_t \text{ for each } t = 1, \ldots, T\right] \geq 1 - \epsilon$$

where the demand d_t at each time t follows a normal distribution, $d_t \sim N(\mu_t, \sigma_t)$, with mean μ_t and variance σ_t. For the problem to be tractable, they replaced this constraint by a set of T separate probability constraints each of which is inverted to linear inequality and incorporated into the UC formulation.

Chapter 3

Unit commitment under load shedding

The purpose of this chapter is to describe a set of models that assess and schedule load shedding as part of the UC decision process. Three models will be considered: one deterministic and two stochastic. The main objective is to compare them, especially to quantify the benefit of using a stochastic program as compared to the deterministic one for the load shedding problem. As such the chapter is organised as follows: in section 3.1 we begin by outlining some literature concerning the same. Details about the models - deterministic, two-stage stochastic and multi-stage stochastic programs - follow in section 3.2. In section 3.3 we describe the simulation study. Models are implemented on the available data and their performance are evaluated with respect to the following three metrics: the expected costs, the expected amount of planned load shedding and the expected unplanned load shedding. The obtained results are reported in section 3.4. Finally a computerized *decision support system* (DSS) for solving each model is presented in section 3.5.

3.1 The problem in context

As seen in section 2.2, UC problem is subject to two conflicting goals: economic efficiency and security of the system. It was explained that the two sources of uncertainty, unreliability of the generation system and any unplanned growth in load, need to be taken into account to ensure the security of the system. Basically operators handle those uncertainties by scheduling enough reserve either explicitly, as a constraint in the UC formulation, or implicitly, through scenario representation. However, sometimes it is impossible to ensure a hundred percent reliability of the system because the generating reserve in place might be inadequate to compensate the unexpected events. In this situation, it is necessary to shed a portion of load. This process, named “load shedding” is used as a last resort to maintain the stability of the system, to avoid the risk of system collapse. The assessment of load shedding through UC problem has been considered by many researchers (Ruiz et al. (2009), Parvania et al. (2010), Xiong and Jirutitijaroen (2013)). A commonly used approach consists of accommodating reliability constraints in the generation scheduling and dispatching, referred to as reliability-constrained unit commitment (RCUC). Load curtailment is allowed but is limited by bounding the risk of loss of load expressed by the reliability metrics: LOLP and ELNS. In other words, the random behaviour of generating units, the likelihood of their outage states and the load uncertainty are modelled to provide reliability metrics which serve as a measure of loss of load. Load curtailment is determined such that those metrics are bounded.

To address the RCUC problem, Parvania et al. (2010) extended the UC formulation (2.2.2) by adding the cost of spinning reserve provided by each generator unit to the objective function, and imposing additional scenario-based and reliability constraints. The proposed model fixes and schedules the UC problem in advance as in the deterministic framework subject to demand-supply balance and all traditional constraints. The scenario-based constraints are considered to model the random outage of generating units. That is, for each time period, a set of scenarios is generated to represent possible states of generating units. For example, at time t, $\xi_g(s)$ represents the state of generating unit $g, g = 1, \dots, N$, in scenario $s, s = 1, \dots, S$. It is a binary number which takes the value 1 if the unit is operating and 0 otherwise. Then, for each scenario realization, the output level of each unit computed from the unit commitment does not change; instead the spinning reserve is tailored to compensate any failure in the system supply-demand balance. In case of inadequate spinning reserve, load interruption is processed. In the reliability constraints, which are incorporated in the model to promote the security and reliability concerning customer outage, the load curtailment is limited by bounding the two reliability index LOLP and ELNS. So, this approach leaves the unit commitment problem intact, and for each time period and each scenario an amount of load shedding is provided in case of insufficient reserve and failure to meet demand.

Xiong and Jirutitijaroen (2013) considered another approach, a two-stage stochastic programming unit commitment problem where spinning reserve provided by each generator and load curtailment are part of the second stage decision subject to traditional stochastic constraints and reliability constraints. The problem formulation mimics the two-stage problem model 2.2.3.2 but with an additional term in the second stage, the cost of load curtailment. The first source of uncertainty, the unavailability of generators, is modelled by a discrete set of outage scenarios which affects the second stage decision. The second source, load uncertainty, is expressed as a continuous random variable accompanied with a distribution and incorporated in the reliability constraints. In these latter constraints, a predefined level is used to limit the probability of having power deficiency in the decision process.

In our context, we model load shedding as a dummy unit that can be interpreted as a slack variable representing any discrepancy between generation and demand. Carpentier et al. (1996) considered such model in deterministic and stochastic frameworks. In the stochastic formulation, they proposed a scenario tree to represent the demand and unit failures whereby the variables of the problem are indexed by node not by time. In our case, we consider a multi-period two-stage stochastic program and multi-stage stochastic program which mimic the formulation in 2.2.3.2 but with an additional term in the objective function, the cost of load shedding. Both deterministic and stochastic formulations will be discussed in the following section.

3.2 Problem formulation

3.2.1 Deterministic formulation

The simplest approach to model the UC under load shedding problem is the use of the deterministic method as proposed by Carpentier et al. (1996). In this framework only one demand scenario over the study horizon is considered, which is assumed to match the actual one. The amount of planned load shedding is a variable determined by the difference between supply and demand at each time. This variable can be view as a unit associated with a cost although without constraints, meaning that there is no bound in its value. The model in this case is formulated as a mixed-integer programme with the objective of minimizing total costs including production and load shedding costs. The model aims to determine the optimal production as well as the planned load shedding to meet demand. It should be noted that the possibility of shedding load may also decrease the cost. A corresponding formulation is given by

$$\min \sum_{t=1}^{T} \left(\sum_{g=1}^{N} f_g(p_{g,t}, u_{g,t}) + \nu_t(v_t) \right) \tag{3.1}$$

$$\text{s.t.} \quad \sum_{g} p_{g,t} + v_t = d_t, \quad \forall t, \tag{3.2}$$

$$\sum_{g} r_{g,t} \geq R_t, \quad \forall t, \tag{3.3}$$

$$u_{gt}\underline{p}_g \leq p_{g,t} \leq u_{g,t}\bar{p}_{g,t}, \quad \forall g, t, \tag{3.4}$$

$$p_{g,t} + r_{g,t} \leq u_{g,t}\bar{p}_g, \quad \forall g, t, \tag{3.5}$$

$$(\mathbf{u}, \mathbf{p}) \in F \tag{3.6}$$

where

- $f_g(p_{g,t}, u_{g,t}) = S_{g,t}(u_{g,t}) + C_g(p_{g,t})$ characterizes the total cost with $S_{g,t}(u_{g,t})$ representing the start up cost and $C_g(p_{g,t})$ the production cost. Recall that $p_{g,t}$ and $u_{g,t}$ represent the production level and the status of each generator at each time respectively;
- $\nu_t(.)$ is a penalty function associated with the disparity between the demand and generation output - load shedding cost;
- v_t is the difference between generation output and demand as shown in (3.2) which is considered as the generation of a dummy unit or the planned load shedding at time t;
- the remaining constraints are the same as in the traditional deterministic UC formulation described in 2.2.2. Inequality (3.3) expresses the reserve requirement which is used to cover any unforeseen event. Constraints (3-4)-(3.6) state the generating units limitations.

Recall that the planning process needs to take into consideration uncertainties like demand fluctuation and unit outages. This is assumed to be handled through the reserve requirement constraint (3.3) in this approach. This model is simple to implement. However the uncertainty is not modelled explicitly which might result in unreliability of the system in the sense that adverse events larger than the size of the reserve margin may cause unexpected outages. An alternative method is to explicitly manage those uncertainties by the use of a scenario representation. Such method can be expressed as stochastic programming, which we describe below.

3.2.2 Stochastic formulation

In the stochastic framework, uncertainties are treated as random variables. A common approach to handle the problem in this way is to consider a finite set of scenarios that is assumed to represent the uncertainty (Parvania et al. (2010),Ruiz et al. (2009), Carpentier et al. (1996), Xiong and Jirutitijaroen (2013)). As proposed by Carpentier et al. (1996), the stochastic problem consists of minimizing the average cost over a scenario tree [1] where variables are indexed by nodes. In this way, the problem may be understood as a deterministic problem but without the reserve requirement constraint. It is assumed that the reserve needs are taken into account in the scenarios. In our context we maintain some reserve requirements as suggested by Ruiz et al. (2009). This is advisable since the few scenarios cannot capture all uncertainties. Moreover, we consider a multi-period two-stage and a multi-stage program to model the problem where variables are indexed by time and scenarios. Details about the two models are described below.

3.2.2.1 Multi-period two-stage program

In this scheme, as its name suggests, the decision process consists of two stages as demonstrated in Figure 3.1. In the first stage, a decision is made about the unit commitment, that is about which units to activate during the planning horizon. This decision takes place hours ahead of the actual operation as explained by Ruiz et al. (2009). In the second stage, after some uncertain inputs have been assessed, a decision is made about the output level of each committed unit and the possible amount of load shedding for all remaining time periods. Uncertain inputs are represented by different scenarios where each scenario, denoted by s, is associated with a probability of occurrence π_s.

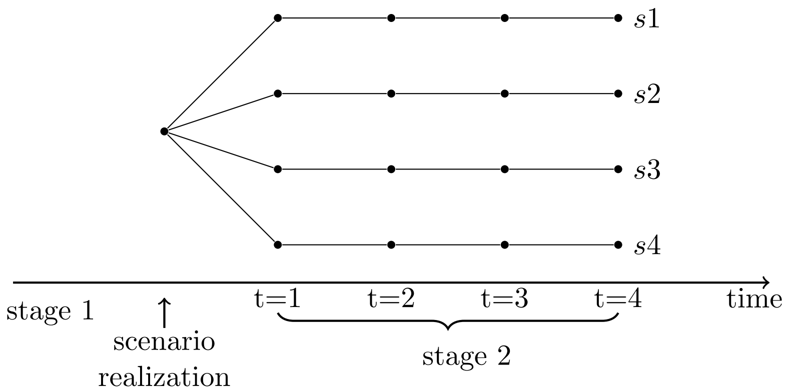


Figure 3.1: Two-stage decision model.

The corresponding model can be formulated as follows

[1]The scenario tree is understood as the description of possible futures.

$$\min \sum_{t=1}^{T}\sum_{g=1}^{N} S_{g,t}(u_{g,t}) + \sum_{s} \pi_s \left(\sum_{t=1}^{T}\sum_{g=1}^{N} C_g(p_{g,t}^s) + \sum_{t=1}^{T} \nu_t(v_t^s) \right) \tag{3.7}$$

$$\text{s.t.} \sum_{g} p_{g,t}^s + v_t^s = d_t^s, \quad \forall t, s \tag{3.8}$$

$$\sum_{g} r_{g,t}^s \geq R_t^s, \quad \forall t, s \tag{3.9}$$

$$u_{g,t}\underline{p}_g^s \leq p_{g,t}^s \leq u_{g,t}\bar{p}_{g,t}^s, \quad \forall g, t, s \tag{3.10}$$

$$p_{g,t}^s + r_{g,t}^s \leq u_{g,t}\bar{p}_{g,t}^s, \quad \forall g, t, s \tag{3.11}$$

$$(\mathbf{u}, \mathbf{p}) \in F \tag{3.12}$$

The objective function (3.7) minimizes the sum of two costs: the cost of the first-stage decision – start-up cost, and the expected cost over representative scenarios – the cost of generation and the planned load shedding. Notice that the status variables $u_{g,t}, \forall g, t$ do not depend on scenario (without index s) since they are first-stage decisions, prior to the scenario realization. Constraints (3.8)-(3.12) are similar to (3.2)-(3.6) in the deterministic model but are enforced for each scenario.

3.2.2.2 Multi-stage program

This model can be seen as a generalization of the two-stage model. In the latter case decisions for all time periods are mapped to only one stage, the second stage, whereas for the multi-stage program stages coincide with time periods and decisions are made at the end of each stage. Precisely, a study period T is associated with T stages where stages represent the different realizations of the uncertain event and the end of stages correspond to time periods where decisions are made. The timeline of the model is illustrated in Figure 3.2.

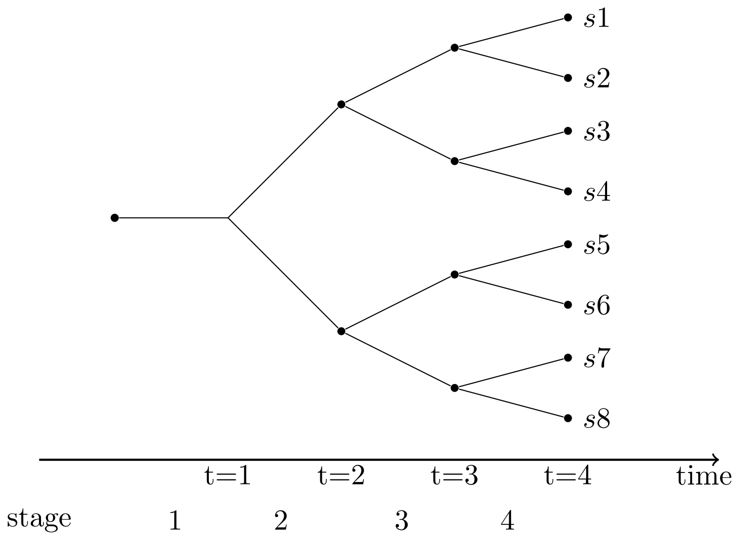


Figure 3.2: Multi-stage decision model.

Here we assume that two possible uncertain outcomes are observed at each stage, represented by the branches of the tree, and each node illustrates the state where decisions are made at a particular time. Scenarios are represented by the sequence of observations from all stages. Hence the model has the form of stochastic mixed-integer programming problem, which can be formulated as follows

$$\min \sum_s \pi_s \left(\sum_{t=1}^{T} \sum_{g=1}^{N} f_g(p^s_{g,t}, u^s_{g,t}) + \sum_{t=1}^{T} \nu_t(v^s_t) \right) \tag{3.13}$$

$$\text{s.t.} \sum_g p^s_{g,t} + v^s_t = d^s_t, \quad \forall t, s \tag{3.14}$$

$$\sum_g r^s_{g,t} \geq R^s_t, \quad \forall t, s \tag{3.15}$$

$$u^s_{g,t} \underline{p}^s_g \leq p^s_{g,t} \leq u^s_{g,t} \bar{p}^s_{g,t}, \quad \forall g, t, s \tag{3.16}$$

$$p^s_{g,t} + r^s_{g,t} \leq u^s_{g,t} \bar{p}^s_{g,t}, \quad \forall g, t, s \tag{3.17}$$

$$u^s_{g,t} = u_{g,t}, \quad \forall g, t, s \tag{3.18}$$

$$(\mathbf{u}, \mathbf{p}) \in F \tag{3.19}$$

where π_s denotes the probability associated to the occurrence of scenario s. The objective is to minimize the expected total cost, including generation and load shedding costs. Constraints (3.14)-(3.17) are similar to those in two-stage model. Equality (3.18) expresses the non-anticipative bundle constraint which ensures that the UC decisions are identical for all scenarios.

Noteworthy differences between two-stage and multi-stage models are:

- in the two-stage model: decisions in stage 2, for each time period, do not have to be the same among different scenarios.
- in multi-stage model: two scenarios that have the same information before time t must have the same decisions up to this time. For instance in Figure 3.2, decisions before time $t = 3$ for scenarios 1 and 2 have to be the same.

3.3 Simulation study

Recall the aim of the study is to compare the effectiveness of the 3 models described in Sections 3.2.1, 3.2.2.1 and 3.2.2.2 with respect to the load shedding problem. For this purpose we conduct a simulation study in which we evaluate the performance of each model. We will see below the detailed process as well as the description of the test system.

3.3.1 Data

We implement the models on the IEEE Reliability Test System (24 bus system) (Wong et al. (1999)). In this study, to reduce the computational requirements, we consider 10 generating units, all thermal units [2], and limit the scheduling horizon to be 11 hours. Therefore, computations are based on the first 11 periods of the system demand. Technical data of generating units are summarized in Table

[2]We do not include hydro units but this can be easily added to the models.

3.1 where P_{max} and P_{min} denote the maximum and minimum output of each generator respectively; C_g represents the cost of generation and C_{su} is the start-up cost.

Units	P_{max} (MW)	P_{min} (MW)	C_g (\$/MW)	C_{su} (\$)
1	152	30.4	42.32	1430.4
2	152	30.4	42.32	1430.4
3	155	54.25	40.52	312
4	155	54.25	40.52	312
5	310	108.5	43.52	624
6	350	140	40.89	2298
7	350	75	39.70	1725
8	400	100	38.11	2890
8	400	100	38.11	2890
10	591	206.85	35.93	3056.7

Table 3.1: Technical data of generating units

The ramp-up/-down limits as well as the minimum-up/-down time limits are neglected in the implementation of the models but can be easily included using additional constraints (see Parvania et al. (2010)).

3.3.2 Simulation study

The structure of the simulation is summarized in Figure 3.3 and implements the following steps:

1. Simulate the planning process: generate expected demand for each model. Then apply the UC models.

2. Simulate true demand and unit failure.

3. Estimate the amount of unplanned load shedding induced from each model.

4. Compare the performance of the three models.

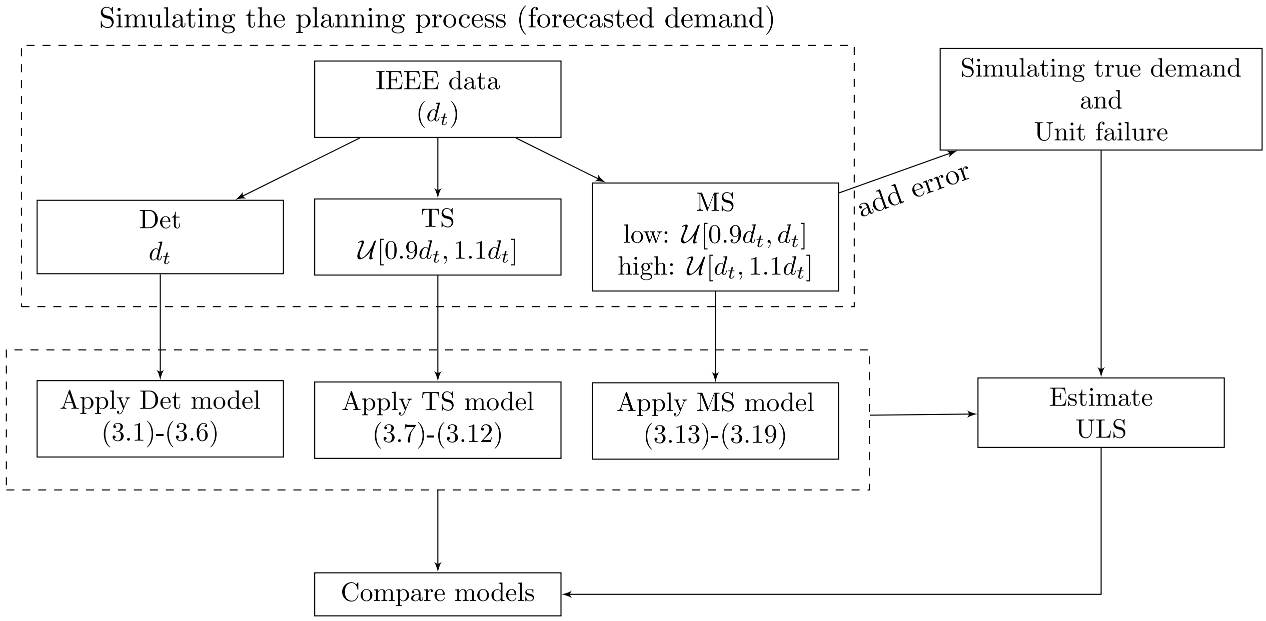


Figure 3.3: Outline of the simulation study.

3.3.2.1 Planning process

The three models differ in the way uncertainty in demand is handled.

1. In the deterministic model, the expected demand at each time is equal to the load profile (or the forecasted demand) given in the test system.

2. In the two-stage program, 1000 demand scenarios are considered in the second stage. These scenarios are generated such that the demand at each time is assumed to be uniformly distributed within a range, where this range is given in terms of lower and upper fraction of the deterministic demand. In our case study, this interval is determined by $[0.9d_t, 1.1d_t]$. Furthermore, all scenarios are each associated with a probability of occurrence, which here is equal to $\frac{1}{1000}$.

3. In the multi-stage scheme, we suppose that the stochastic demand can take values in two confidence intervals (low and high) depending on the deterministic demand. The lower and upper intervals are given by $[0.9d_t, d_t]$ and $[d_t, 1.1d_t]$ respectively. Within a range, demand is assumed to be uniformly distributed. In addition, a probability is assigned to each of these ranges, which in our context is equal to 0.5.

Remark:

- Concerning the computation, we assume that the required reserve R_t and R_t^s are identical for all scenarios at all time. Similarly, the cost of lost load ν_t are the same for all t for all models. From now we denote by R the reserve parameter and by ν the cost of lost load.

- Any subsequent failures of units are assumed to be modelled and potentially covered by the reserve requirement for all models. A separate study is conducted, which will be described below, to examine the effects of unit outages in the effectiveness of the three models.

3.3.2.2 Simulating true demand and unit failure

Typically these models give an optimal solution on the unit scheduling process, the total generation cost and the amount of planned load shedding. We will see in the study results section the comparison of these solutions. Alternatively, apart from those metrics, the amount of unplanned load shedding is also measured to compare the effectiveness and the performance of the three models. These depend critically on the relationship between expected demand (what the models see) and actual demand (the real demand). Thus we simulate a set of true demand scenarios and estimate the value of unserved load. This set is drawn from the multi-stage demand scenarios by adding a random error. Actually the multi-stage decision process makes it natural to view the generated demand scenarios as the actual load profile. However, the proposed multi-stage is limited here to two branches per time period, and so does not capture the spectrum of possible real demand. Therefore we add some random errors for each scenario. Here the random values are extracted from $[0, 0.1d_t]$.

Moreover, the possibility of unit outages is modelled and taken into account for the assessment of ULS. We only consider the failures of the three largest generators as the loss of units with small capacity has a relatively insignificant impact on the system operation. We assume that each of these three units fails for the entire period with a probability p.

3.3.2.3 Assessment of ULS

We investigate two case studies for the assessment of ULS:

First case: we assume that all units are reliable, that is no unit outages or if there are any then these are covered by the reserve requirement. The ULS for the entire scheduling horizon in this case is determined by the sum of the positive differences between the true demand (TD) and the expected demand (ED) seen by each model over time. That is

$$\mathrm{ULS} = \sum_t \max\{0, \mathrm{TD}_t - \mathrm{ED}_t\}$$

For the computation we summarize below the way we choose the expected demand for each model.

- For the deterministic model: the ED is equivalent to the single forecasted demand given in the test system.
- For the two-stage model: we consider 3 ED out of the 1000 demand scenarios. The first one, named two-stage one period ahead, is chosen by assuming that we know one period ahead of the actual demand. That is we set ED to be the demand scenario that has the smallest difference to TD at time $t = 1$. The second ED, named two-stage two periods ahead, is chosen in such a way that its difference with the true demand at time $t = 1$ and $t = 2$ are relatively small. The last ED, called two-stage three periods ahead, is the demand scenario that its values at time $t = 1, 2, 3$ are closer to the actual demand.
- For the multi-stage model: we choose one demand scenario at random.

Second case: we take into account the possibility of unit outages. As explained above we consider the failures of the three largest units where each failure is associated with a probability p. In this case the ULS is determined by

$$\text{ULS} = \sum_t \max\{0, (\text{TD}_t - \text{ED}_t)\} - \sum_k T.p.U_k$$

where $U_k = R - \bar{p}_k$ if the failure of $k - th$ unit $k = 1, 2, 3$ cannot be covered by the reserve R, otherwise $U_k = 0$. Actually U_k represents the sum of the shortfall between the reserve R and the maximum output $\bar{p}$ of each unit for the entire period T.

3.3.2.4 Comparison

Recall the main objective is to compare the three models and evaluate their performance with respect to the total expected costs, the estimated amount of planned and unplanned load shedding. This is conducted by:

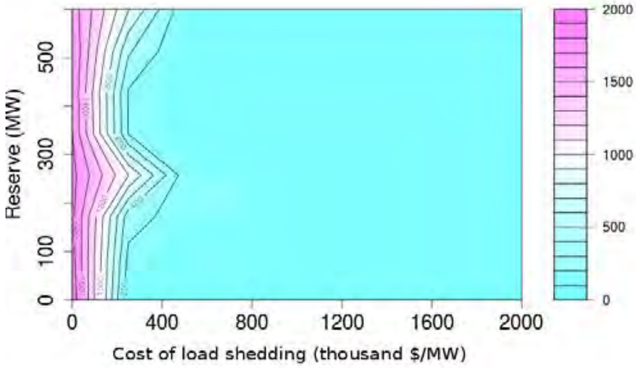
- examining the effects of varying the value of the reserve parameter R as well as the cost associated with the lost load ν,
- evaluating the influence of changing the probability p associated to the unit outages.
- estimating the total costs including the cost of unplanned load shedding.

Based on the above detailed process, study results will be demonstrated in the following section.

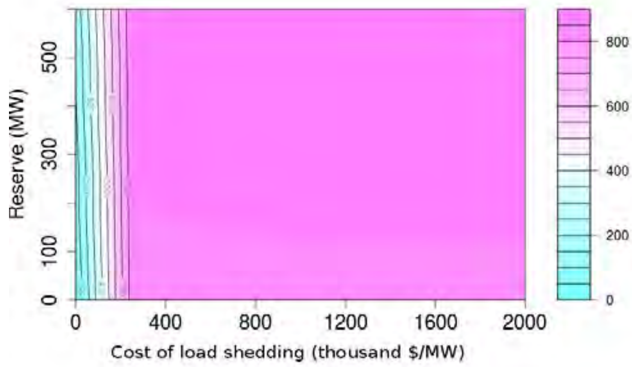
3.4 Results

3.4.1 Impact of changing values of parameters

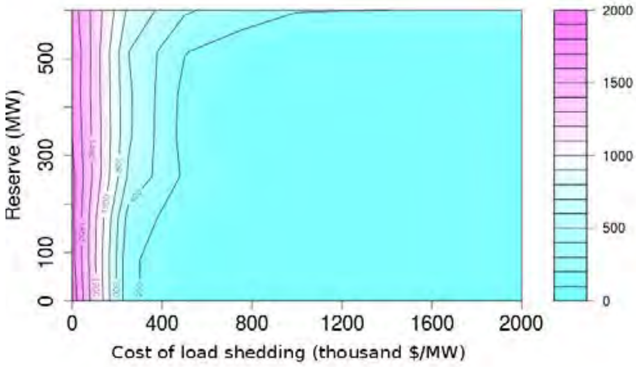
This section presents the results of parametric studies performed using the models presented in section 3.2. The parametric studies discussed here consider the impacts of changes in the value of lost load and reserve. We run the models for values of reserve between [0 MW, 600 MW] and cost of lost load between [$\$0, \$2000$].



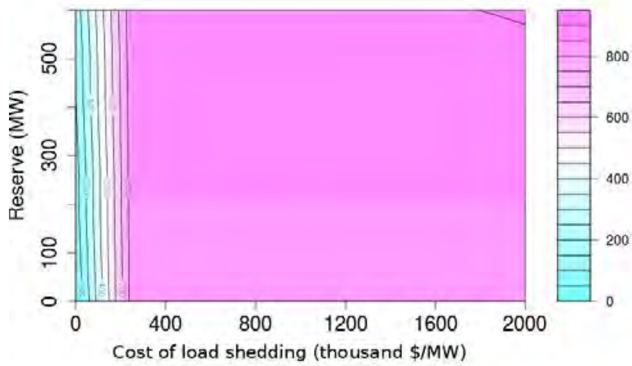
(a) Cost: deterministic model.



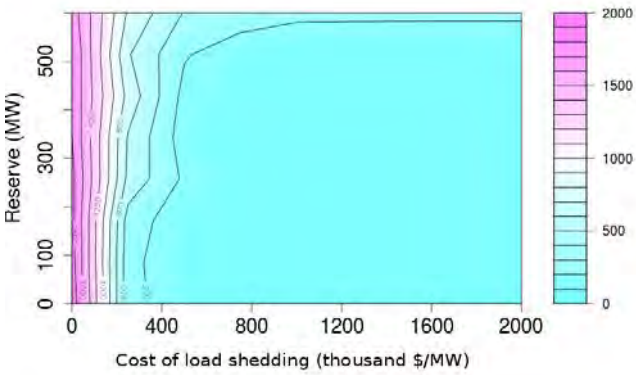
(b) PLS: deterministic model.



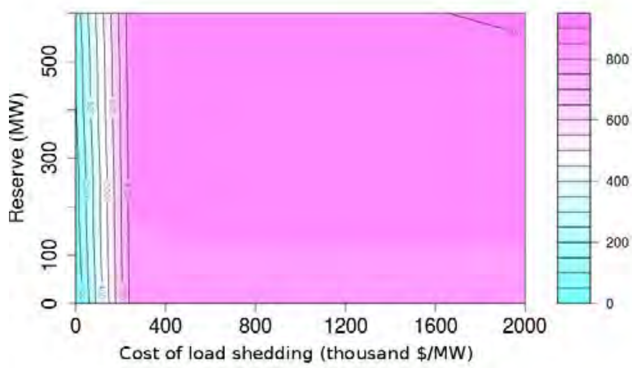
(c) Cost: two-stage model.



(d) PLS: two-stage model.



(e) Cost: multi-stage model.



(f) PLS: multi-stage model.

Figure 3.4: Models performance as a function of the amount of reserve and the cost of lost load.

Figure 3.4a, 3.4c and 3.4e are contour plots illustrating the influence of varying the two parameters on the amount of planned load shedding derived from deterministic, two-stage and multi-stage models respectively. At very low cost of load shedding the optimal solution is to shed as much load as possible. If this cost is larger than $200, load shedding is avoided if possible due to its large costs compared to the costs of generation. The deterministic solution leads to lower amount of planned load shedding than the two stochastic models for any reserve margin. Figure 3.5 for example, shows the expected amount of planned load shedding at a reserve equal to 591 MW. This is explained by the fact that the two-stage and multi-stage models account for demand uncertainty. They are more conservative, dispatching load shedding to cover any possible high demand. In contrast the deterministic case can see only one possible demand. We will see later that this feature forms a disadvantage for the deterministic model as it will suffer from high amount of unplanned load shedding.

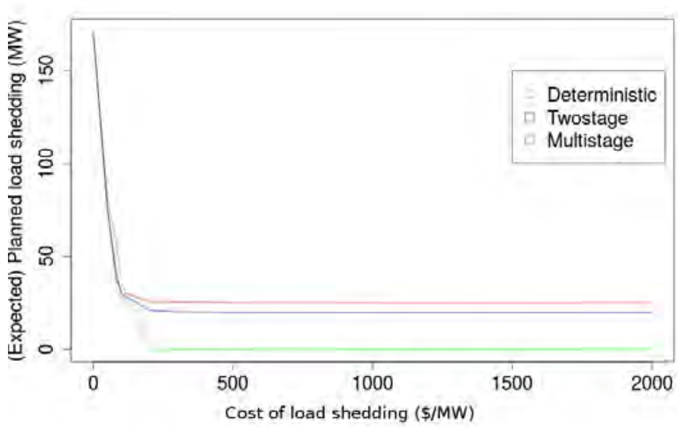


Figure 3.5: (Expected) planned load shedding under different values of lost load.

Furthermore, the amount of planned load shedding increases as the value of reserve increases (still Figure 3.4a, 3.4c and 3.4e). In fact if more reserve are needed, that requires bringing more units online. At some stage planned load shedding would be necessary. Therefore, part of the decision is to plan for a possible load shedding. One might expect the increase of the value of reserve to be associated with the decrease of load shedding, but it must be kept in mind that here we consider only *planned load shedding*.

Next, Figure 3.4b, 3.4d and 3.4f display the influence of varying the two parameters on the expected cost obtained from deterministic, two-stage and multi-stage models respectively. One might notice that the cost increases as the value of reserve increases. This is not surprising since more units are brought online in such cases. For a fixed value of reserve and at higher cost of load shedding, the cost increases linearly for the stochastic models but stays constant for the deterministic case. Figure 3.6 demonstrates the case where R = 591 MW. In fact, this is reflected by the optimal solution on the amount of planned load shedding explained above. The cost derived from the deterministic model is constant since the load shed is essentially constant but increases for the stochastic models as there is some planned load shedding.

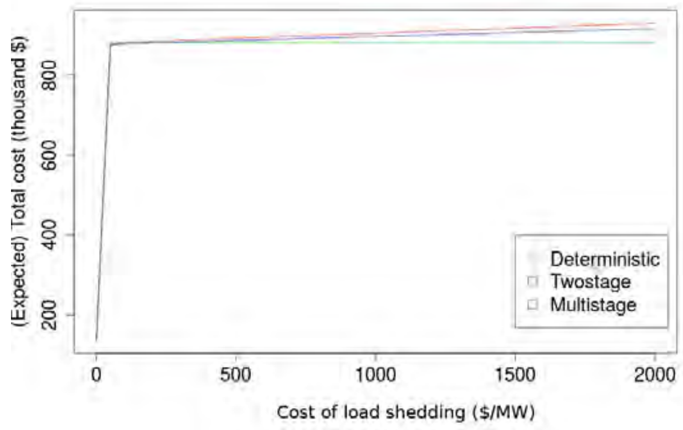


Figure 3.6: (Expected) cost under different values of lost load.

3.4.2 ULS without unit outages

This section contains result about the assessment of ULS for all models by assuming that all units are reliable. The focus is to quantify the difference between stochastic and deterministic models.

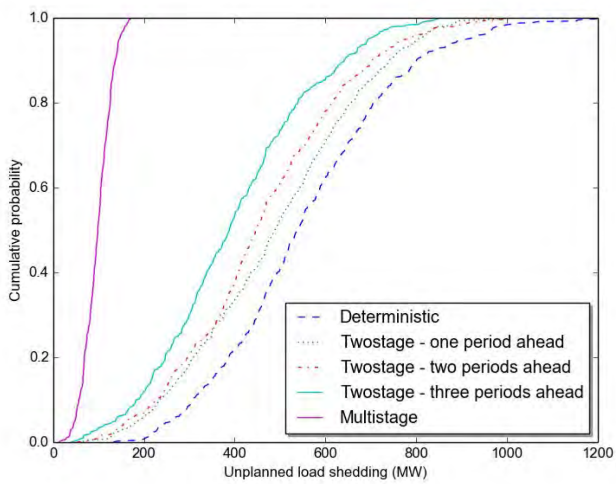
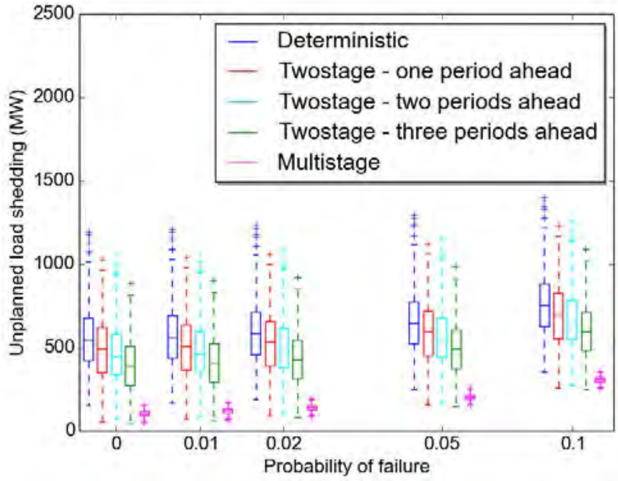


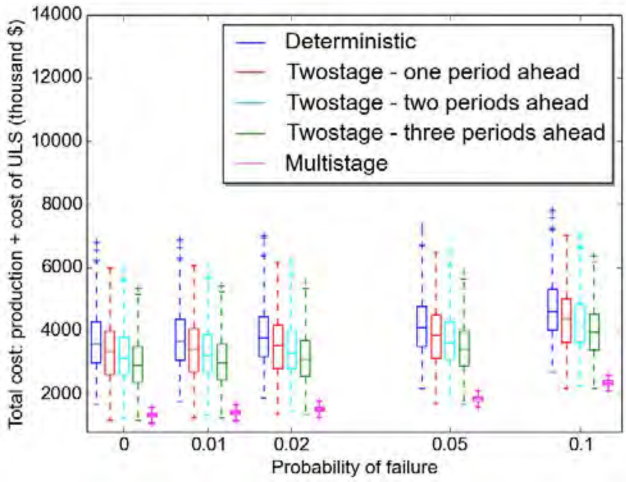
Figure 3.7: Cumulative distribution of unplanned load shedding in the case where all units are reliable.

The estimated amount of unplanned load shedding is given in terms of cumulative probability. Figure 3.7 plots the probability that the amount of unplanned load shedding is equal to or less than a given positive value. There is a 90% chance that the value of unserved load is about 850 MW or less for the deterministic model, while approximately below 750 MW for the two-stage - one period ahead and roughly equal to 120 MW or less for the multi-stage one. This tells us that it is more likely for the deterministic case to suffer from a higher amount of unserved load whereas the multi-stage model is more safe. That is the deterministic model is dominated by others.

3.4.3 Effect of unit outages



(a) Unplanned load shedding for deviations from the probability of unit outage.



(b) Total cost (generation cost plus cost of unplanned load shedding) for deviations from the probability of unit outage.

Figure 3.8: Amount of unplanned load shedding and the overall costs as a function of the probability of unit outages. The reserve is equal to 400 MW.

By taking into account the possibility of unit outages, Figure 3.8a displays the relationship between the amount of unserved load and various probability of unit failures for each model. Here the value of the reserve is set to be equal to 400 MW. This plot shows that, for any probability associated to unit breakdown, the deterministic model has a higher median than the stochastic cases. This shows that the amount of estimated unplanned load shedding is much higher for the deterministic model. Moreover, the interquartile range for the multi-stage model is less wide than the other models which suggests that the multi-stage case is more consistent. This is largely because we take true demand from the multi-stage model. One might notice that the estimated amount of unserved load increases gradually as the probability of unit failures increases. Thus we expect this value to continue increasing as the probability increases beyond 0.1, but this probably represents an upper bounds on realistic real-world problems.

Estimating a monetary cost associated with unplanned load shedding is extremely difficult, but by substituting in plausible values, one can get an idea of the economic savings potentially achievable by using a stochastic model rather than a deterministic one. For example, suppose we set the cost of unplanned load shedding to 5000\$/MW (compared to a cost of planned load shedding which we set to 2000\$/MW) and calculate the total cost yields for each model. Figure 3.8b illustrates the overall cost (generation cost + cost of unplanned load shedding) as a function of the different value of probability of unit outages for all models. Since the deterministic case has higher value of unplanned load shedding it leads to a larger cost compared to the other models. In contrast, the cost provided by the multi-stage model is much lower due to its consistency. Similar to Figure 3.8a, we expect these costs to increase as the probability of failure increases.

3.5 Decision support system

In this section, a computerized DSS is presented which may be used to solve UC problem formulated as in Section 3.2 (deterministic model, two-stage and multi-stage models). The DSS implementation is a collection of AIMMS [3] scripts with a corresponding graphical user interface (GUI). We describe below the different procedures to follow in order to solve each model via the DSS.

3.5.1 DSS for the deterministic model

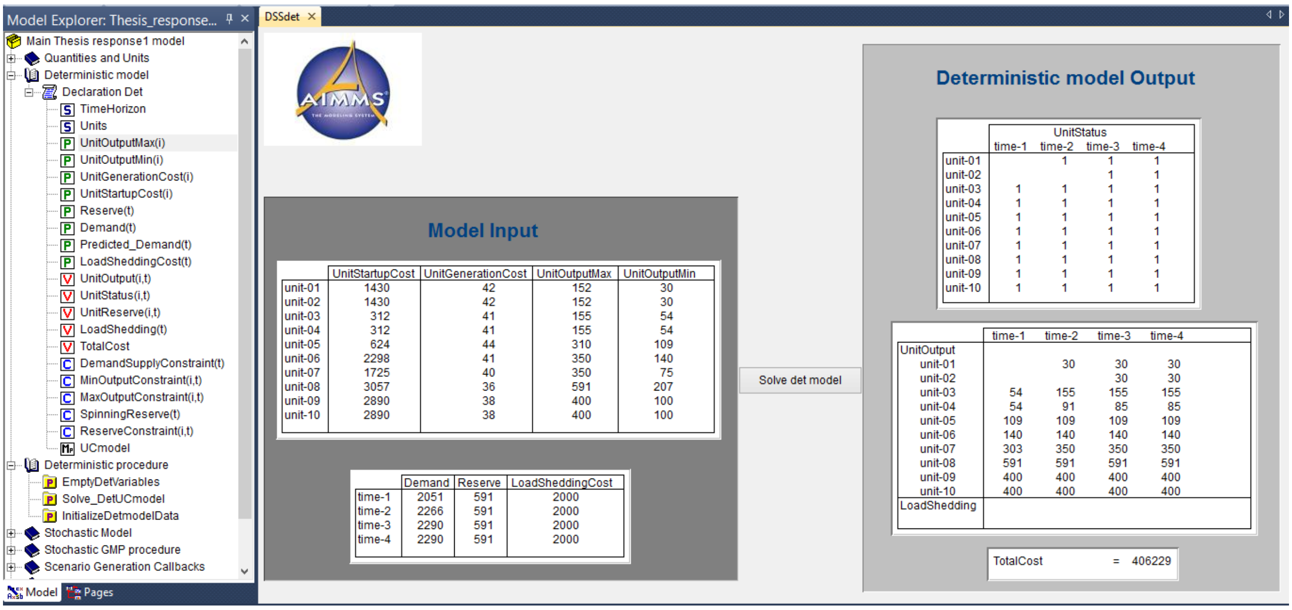


Figure 3.9: Screenshot of the GUI of the DSS deterministic model.

Upon executing the DSS for the deterministic model, the GUI shown in Figure 3.9 appears on the screen. It contains two panels – the 'Model Input' and the 'Deterministic model Output' – and a set of procedures in the left sidebar. The user first needs to enter data in the 'Model Input' panel. This can be done by clicking on the relevant parameter in the left sidebar. For example, as illustrated in Figure 3.10, the user can enter the value of the maximum output for each unit by clicking on

[3]AIMMS (Advanced Interactive Multidimensional Modeling System) is a software designed for modeling and solving large-scale optimization and scheduling-type problems, http://www.aimms.com/.

the parameter '*UnitOutputMax(i)*' and filling in the table in the open window. The value of all parameters inside the 'Model Input' panel can be entered in the same way.

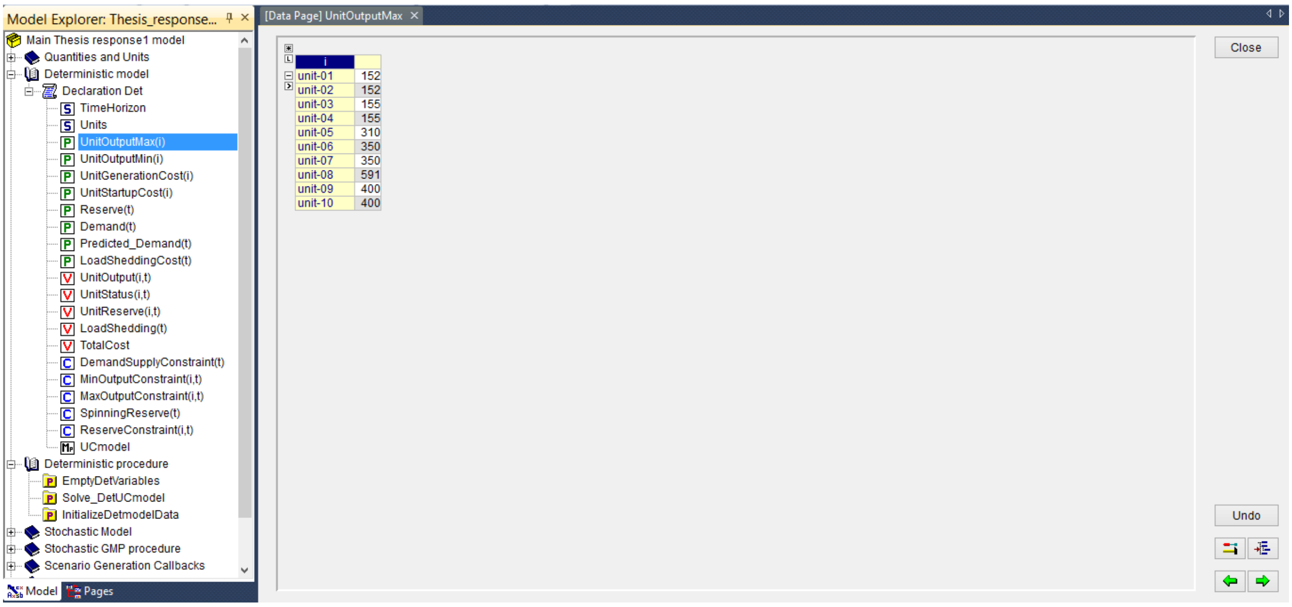


Figure 3.10: Screenshot of the window to enter the input data.

Once the user has entered all data, they can press the button '*Solve det model*' to solve the model. This provides outputs as shown in the 'Deterministic model Output' panel. These include

- '*UnitStatus*' table: which displays the status of each unit at each time – 1 if it is on and blank otherwise.
- '*UnitOutput*' and '*LoadShedding*' table: which shows the generation level of each unit and the corresponding planned load shedding at each time.
- '*TotalCost*': is the value of the objective function.

3.5.2 DSS for the stochastic models

Figure 3.11 illustrates the GUI of the DSS two-stage model. The user can input all required data in a similar way as explained in the deterministic case (Section 3.5.1). To initialize (or change) the value of the lower and upper bound fractions of the interval in which the demand scenarios are drawn from, they can click on the 'InitializeLowerUpperBoundFraction' in the left sidebar and fill in the open window.

To solve the problem, the user is prompted to sequentially click the buttons inside the '*Procedures*' option.

- Button '*Initialize branch range and chance*': is used to initialize the number of scenarios with their associated probability.
- Button '*Create Scenario Tree*': is used to create the two-stage timeline given in Figure 3.1.

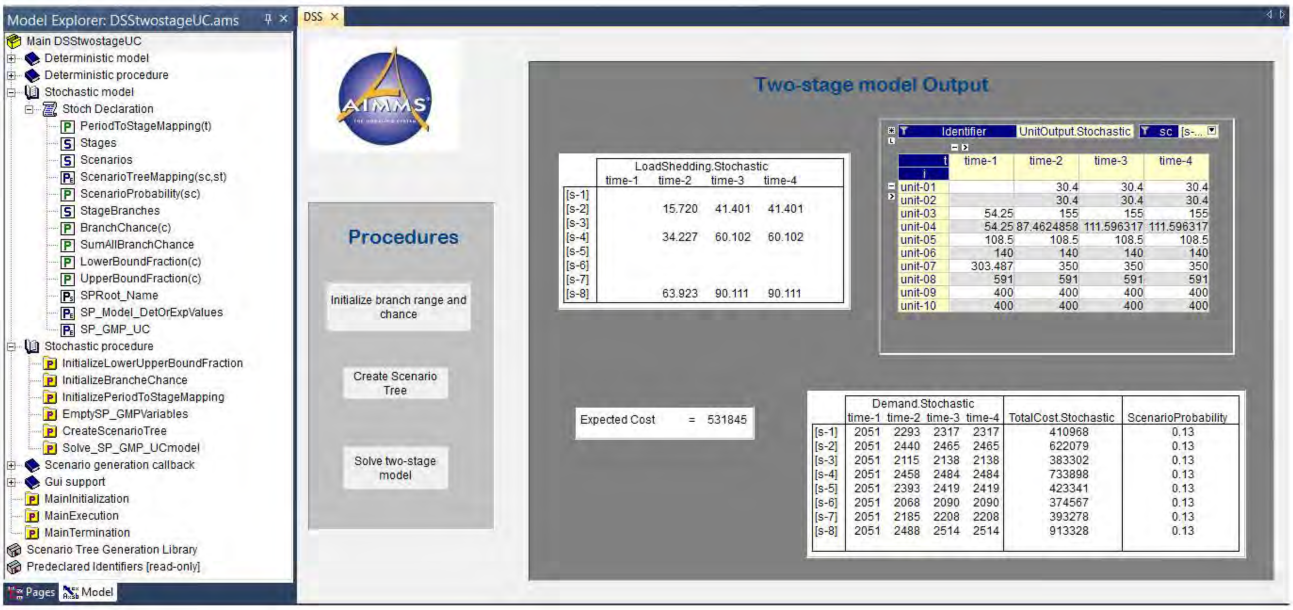


Figure 3.11: Screenshot of the GUI of the DSS two-stage model.

- Button '*Solve two-stage model*': is used to run the model.

Upon pressing the '*Solve two-stage model*' button, the model outputs are displayed in the '*Two-stage model Output*' panel. These include

- the '*UnitOutput.Stochastic*' table: which shows the output level of each unit at each time for each scenario,
- the '*LoadShedding.Stochastic*' table: which displays the amount of planned load shedding at each time for each scenario,
- the '*Expected Cost*' bar: which displays the value of the objective function,
- the '*Demand.Stochastic*' table: which shows the expected demand at each time for each scenario. It also displays the expected cost and the probability associated to each scenario.

The same procedures can be adopted and run to solve the multi-stage model (Figure 3.12). User can initialize and change data in the similar way as in deterministic case. The lower and upper bound fractions value of the confidence intervals explained in Section 3.3.2.1 can be initialized and changed by clicking on the 'InitializeBranchRangeAndChance' procedure in the left sidebar and fill in the open window. To solve the problem, the user click sequentially the buttons inside the '*Procedures*' option. The model outputs are displayed in '*Multi-stage model Output*' panel, which contains the same tables as in two-stage DSS case.

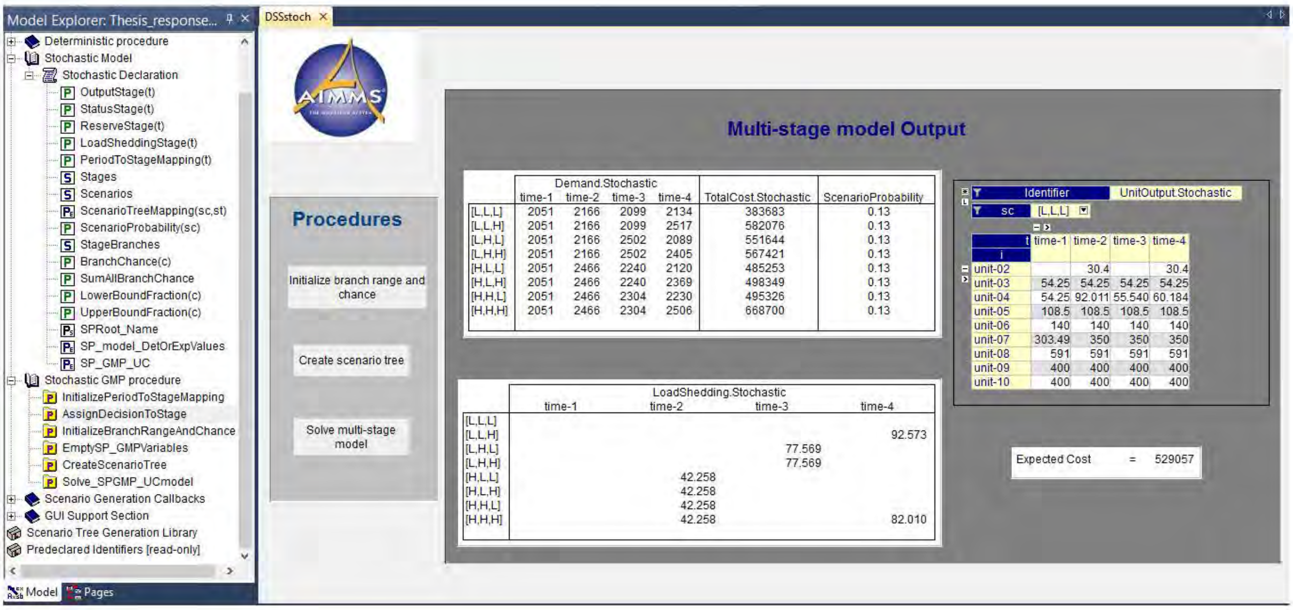


Figure 3.12: Screenshot of the GUI of the DSS multi-stage model.

Chapter 4

Load shedding scheduling

Whenever there is insufficient power to supply demand and available reserve margins have been exhausted, a planned and controlled load shedding is necessary. Parts of the network are switched off according to a load shedding schedule to protect the power system from a total blackout. The objective of this chapter is to design linear integer and goal programming models to generate such schedules while emphasizing the trade-off between social and economic impacts. We conduct intensive numerical experiments studies in order to explore and expose the possible solutions that the models can offer. This is achieved by first giving an introduction and a problem description in section 4.1. Then mathematical formulation of the problem follows, in section 4.2, in which the proposed models are described. In section 4.3 we present an illustrative example and the implementation of the models. The obtained results are shown and discussed in section 4.4. The chapter concludes with a presentation of a computerized *decision support system* (DSS) for solving each model.

4.1 Problem description

In a power system, one needs to ensure that at all times a sufficient supply is available to meet demand. Electricity demand, however, is variable both over the short term (e.g. due to peak periods when demand is high) and the long term (because of continuous growth in the number of customers requiring electricity services). Therefore, a careful management of supply is required. This task includes the unit commitment and economic dispatch problems discussed in Chapter 3, and the necessary maintenance of plant and facilities. However, when the demand is high, there is a pressure on the supply system as all infrastructures are used at levels which are close to their full capacity. Namely, continued overloading can cause plants or other facilities to fail. If one plant fails, then it is disconnected from the system, and other plants near it have to spin up to cover the extra load. Those plants might not able to handle the increased load, resulting another failure. Failures can cascade through the supply system in this way, leaving large areas without power. Therefore all measures have to be taken to avoid such occurrence. The necessity of a controlled load shedding arises in order to protect the power system from a total blackout.

Load shedding is considered as a last resort after several steps have failed to restore the balance between supply and demand. Figure 4.1 summarizes the load shedding process for the case of South

Africa.

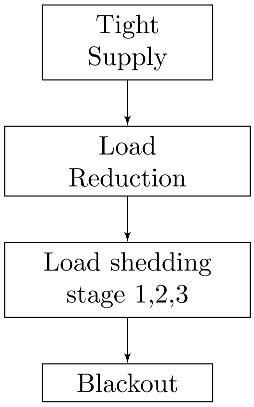


Figure 4.1: Load shedding process.

The power system is referred to as in a state of "tight supply" when all units are run at full capacity and all available reserves have been used whilst the high demand is not satisfied. In this emergency state, to help balance demand and supply, demand reduction from voluntary or contracted customers are called on to decrease their demand ("load reduction"). When these steps have been depleted to restore the power stability, as a preventive measure, customers are cut off or load shedding is processed to prevent the network from collapse. This process is executed in stages depending on the severity of the system's state. The National Control Centre (System Operator) determines the capacity shortfalls and instructs the relevant stage of shedding with a given amount or quotas to all distribution centres, municipalities and metros. Those centres in turn execute the instruction by creating their schedule for their corresponding areas. Three stages are defined for the scheduling process:

- Stage 1: about 5% of national load (around 1000 MW) shall be scheduled for shedding with customers impacted for predefined intervals.
- Stage 2: 10% of the national load (about 2000 MW) is scheduled to be shed.
- Stage 3: 20% of the national load (an additional 2000 MW, totalling 4000 MW) shall be scheduled.

In cases where all these measures are insufficient, a system blackout will happen. A national blackout could take days to restore and would have massive economic and social implications to the country, so all measures have to be taken to avoid such an occurrence.

In a case where load shedding is necessary, schedules are built in advance to indicate which areas are to be shed in each time and day slot. These schedules are published in different media to make customers aware of their possible time of shedding. They are basically drawn in a rotational way by assigning each area over the time and day slots. An example of a monthly schedule for the city of Cape Town is shown in Figure 4.2.

One might notice that this schedule is *fair* in the sense that each area is shed almost the same number of times and without repetition in the allocation within the same day or the same hour. However another factor that should also be considered is the nature of use of electricity in various areas, which will determine the economic impacts of load shedding. If this factor is taken into account, there might be a variant schedule that provides lower economic impact than the one proposed in Figure 4.2, but that might break the rotation allocation or the fairness seen in Figure 4.2. From this point, it might be advisable to treat this scheduling problem as an optimization problem while seeking schedules that guarantee a high level of fairness between areas and also minimizes the impact such as economic costs caused by the outages for all areas. This requires that decision makers explicitly consider trade-off between these two conflicting goals.

STAGE 1

DAYS OF THE MONTH		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
		17th	18th	19th	20th	21st	22nd	23rd	24th	25th	26th	27th	28th	29th	30th	31st	
TIME FROM	TIME TO																
0:00	2:30	1	13	9	5	2	14	10	6	3	15	11	7	4	16	12	8
2:00	4:30	2	14	10	6	3	15	11	7	4	16	12	8	5	1	13	9
4:00	6:30	3	15	11	7	4	16	12	8	5	1	13	9	6	2	14	10
6:00	8:30	4	16	12	8	5	1	13	9	6	2	14	10	7	3	15	11
8:00	10:30	5	1	13	9	6	2	14	10	7	3	15	11	8	4	16	12
10:00	12:30	6	2	14	10	7	3	15	11	8	4	16	12	9	5	1	13
12:00	14:30	7	3	15	11	8	4	16	12	9	5	1	13	10	6	2	14
14:00	16:30	8	4	16	12	9	5	1	13	10	6	2	14	11	7	3	15
16:00	18:30	9	5	1	13	10	6	2	14	11	7	3	15	12	8	4	16
18:00	20:30	10	6	2	14	11	7	3	15	12	8	4	16	13	9	5	1
20:00	22:30	11	7	3	15	12	8	4	16	13	9	5	1	14	10	6	2
22:00	0:30	12	8	4	16	13	9	5	1	14	10	6	2	15	11	7	3

Figure 4.2: Example of load shedding schedule for the city of Cape Town. The number 1, . . . , 16 in the table denote the area numbers.

In this thesis we propose two mathematical models, a linear integer programming model and a goal programming model, that provide a time-based load shedding schedules. The focus is to emphasize the trade-off between fairness and economic cost goals. We address the description of the proposed models in the following section.

4.2 Problem formulation

Generally, scheduling problems concern the optimal allocation of resources to activities over time. An example is jobs that must be scheduled on machines subject to certain constraints to optimize some objective function. The goal is to establish a schedule that specifies when and on which machine each job is to be executed. In this case a good schedule would be the one that minimizes the average completion time and its makespan (Méndez and Cerdá (2003)). Another type of scheduling problem is staff scheduling. In this context, the schedule consists of assigning employees to working shifts over a given period subject to several constraints like personnel policies and individual preferences, while minimizing objectives like costs (Trilling et al. (2006), Blöchliger (2004)).

In our context, the scheduling process consists of allocating a set of shedable areas over time and day slots according to the given amount of load to shed (i.e. stage) subject to some required constraints. The objectives are to minimize the economic impact on one hand and to maximize the allocation fairness on the other. Two types of fairness are considered in our models:

1. *Rotational fairness*: trying to avoid successive shedding times; meaning if one area is shed at a specific time or day it shouldn't be shed until the other areas have been shed. For example, if one area is shed in the interval of time 6:00 - 8:00 on Monday, it should not be shed in the

following time intervals or at the same time interval for the following days until all other areas have been shed.

2. *Cumulative fairness*: trying to balance the number of time each area is shed in the overall schedule horizon. For instance, say we set the schedule horizon to be J hours, K days and with a total number of areas to be shed equal to R. If we assume that one area has to be shed in each slot, then each area should be shed $\frac{JK}{R}$ times in total. In the case where $\frac{JK}{R}$ is not an integer, then the total number of times shed should range in a totally fair schedule $\{\lfloor\frac{JK}{R}\rfloor, \lfloor\frac{JK}{R}\rfloor + 2\}$ where $\lfloor x \rfloor$ denotes integer part.

To validate the use of these objectives, and how they are measured quantitatively, we held an informal workshop with eight energy experts working at the Energy Research Center at the University of Cape Town. The workshop was held in April 2015 and took two hours.

The workshop took the form of a structured discussion of the load shedding scheduling problem. It began with a brief presentation of the load shedding scheduling problem, including some preliminary models that used economic costs and fairness as objectives. A worked example was shown to the group to make these models as clear as possible. Following this, the group was asked to evaluate and discuss what objectives are important in the scheduling problem. The group agreed that economic cost and fairness were important. To this, some members of the group added an additional objective, predictability. For a given period of shedding time, areas that should be shed in these slots should be known in advance. At the present time, however, all real-world schedules are entirely "predictable" or "memoryless" and so we did not include this objective in our later work.

The group was then asked whether the notion of rotational fairness (per day and per time slot), and cumulative fairness adequately captured the main aspects of fairness. The group agreed that these were suitable attributes, and did not add any further suggestions on this topic.

Furthermore, the problem is formulated under the following notation and assumptions:

- We denote by R the total number of areas to be shed, by J and K the number of time and day slots respectively which all are assumed to be fixed. We define by i, j, k the indexes for area, time and day respectively. We denote also by jk a period defined by time j and day k. In addition, we assume that $R \geq J$ and $R \geq K$.
- For a given stage, L and S represent the amount of load to be shed and the required number of area to be shed in each period respectively [1].
- The demand of each area i within period jk is denoted by D_{ijk}.

As mentioned in previous section, two types of optimization model are designed to address the problem. One consists of linear integer programming with a single objective and a set of hard constraints while the second is a goal programming where two conflicting goals are set to be simultaneously optimized. These models are described below.

[1] $S = 1, 2, 3$ for stage 1, stage 2, stage 3 respectively in currently used schedules.

4.2.1 Single objective (SO) formulation

The first model consists of an integer linear programme, where decision variables are 0-1 integer variables defined for each area, at each time and for each day as X_{ijk}. Those binary variables can take two values

$$X_{ijk} = \begin{cases} 1 & \text{if area } i \text{ is allocated on time } j, \text{day } k \\ 0 & \text{otherwise} \end{cases}$$

The only objective is to minimize the economic impact $\sum_{i,j,k} C_{ijk}X_{ijk}$ where C_{ijk} denotes the economic cost caused by the outage of area i in period jk. The optimization model is subject to the following constraints:

- C1: Load shedding requirement for each specific period must be satisfied.
- C2: Cumulative fairness – make sure that all areas are assigned in the schedule at least a user-specified number of times.
- C3: Rotational fairness – make sure that no area is shed more than a user-specified multiple of the average, for each time and day slot.

The corresponding formulation is given by

$$\min \sum_{i,j,k} C_{ijk}X_{ijk} \tag{4.1}$$

$$\text{s.t.} \sum_{i} D_{i,j,k}X_{ijk} \geq L, \quad \forall j,k \tag{4.2}$$

$$\sum_{j,k} X_{ijk} \geq \lambda, \quad \forall i \tag{4.3}$$

$$\sum_{k} X_{ijk} \leq \alpha \left\lceil \frac{SK}{R} \right\rceil, \quad \forall i,j \tag{4.4}$$

$$\sum_{j} X_{ijk} \leq \beta \left\lceil \frac{SJ}{R} \right\rceil, \quad \forall i,k \tag{4.5}$$

$$X_{ijk} \text{ binary}, \quad \forall i,j,k \tag{4.6}$$

where $\lambda \in \{1, \dots, \lfloor \frac{SJK}{R} \rfloor\}$, $\alpha \in \{1, \dots, 5\}$ and $\beta \in \{1, \dots, 5\}$.

Constraint C1 is expressed by (4.2). The inequality (4.3) demonstrates constraint C2. The parameter λ determines the level of fairness of the generated schedule with respect to the total number of times each area is shed. The minimum value of λ is equal to 1 which says that all areas must be shed at least once in total. In this case there is no restriction in the maximum number of time each area should be shed on overall. The maximum value of λ is equal to $\lfloor \frac{SJK}{R} \rfloor$ which is the maximum number to make sure that all areas are shed with the same number of time on average. That is, all areas should be shed within the range $\{\lfloor \frac{SJK}{R} \rfloor, \lfloor \frac{SJK}{R} \rfloor + 2\}$.

Constraint C3 is illustrated by (4.4)-(4.5). Inequality (4.4) expresses the day of week rotation fairness: an area that is shed in a particular time slot should not be shed again in the same slot for the following days before the remaining areas are shed. That is for a scheduling horizon of K days and with total number of areas equal to R, the earlier statement means each area would be shed $\lceil \frac{SK}{R} \rceil$ in each slot, in a totally fair schedule. In that case the value of α is equal to 1. Note that this parameter is used to adjust the level of fairness. A high value of α implies relaxation on the constraint ($\alpha \in \{1, \ldots, 5\}$). For instance, if $\alpha = 3$ areas are allowed to be shed 3 times at the same hour slot over the K days. Similarly, inequality (4.5) expresses the time of day rotation fairness where the parameter β is used to adjust the level of fairness in this respect and $\lceil \frac{SJ}{R} \rceil$ is the bound [2].

It is noteworthy that constraints C2-C3 can be handled as soft constraints, hence converted to goals. In fact, these two constraints represent the two types of fairness mentioned above and can be set as goals to be achieved instead of hard constraints. This suggests a goal programming formulation which is subject to the following subsection.

4.2.2 Goal programming (GP) formulation

The aim of the scheduling model is to balance social and economic impacts. Social impacts are defined in terms of fairness, meaning all areas are equally shed if possible both overall and within each time and day. One approach to model the problem is the one proposed above in Section 4.2.1 with a single objective, the economic cost, and fairness as constraints. An alternative approach is to consider a set of goals (economic cost and fairness) that need to be simultaneously achieved. In this case, each goal is associated with a target value and the objective is to minimize the weighted sum of deviations of all goals to their corresponding targets. This provides a flexible model in which decisions can be updated according to different goals and weights that different decision makers might hold.

Precisely, here we consider 4 different set of goals:

- G_1: consists of minimizing the total economic cost determined by $\sum_{ijk} C_{ijk} X_{ijk}$,
- G_2: concerns the maximization of the set of cumulative fairness goals defined by the number of times each area is shed - $\{\sum_{jk} X_{ijk}, \forall i\}$,
- G_3: consists of maximizing the set of time of day rotation fairness goals given by $\{\sum_k X_{ijk}, \forall i, j\}$,
- G_4: consists of maximizing the set of day of week rotation fairness goals determined by $\{\sum_j X_{ijk}, \forall i, k\}$.

In addition, we denote by $E, F_i^{(\text{overall})}, F_{ij}^{(\text{day})}$ and $F_{ik}^{(\text{time})}$ the targets associated to each element of G_1, G_2, G_3 and G_4 respectively. We assume that goals which belong to the same set have equal target values. Hence, the corresponding formulation is given by

[2]Parameters α and β are limited here to the range $\{1, \ldots, 5\}$ for practical purpose.

$$\min\ \omega d + \sum_i \omega_i^{(\text{overall})} d_i^{(\text{overall})} + \sum_{i,k} \omega_{ik}^{(\text{time})} d_{ik}^{(\text{time})} + \sum_{i,j} \omega_{ij}^{(\text{day})} d_{ij}^{(\text{day})} \tag{4.7}$$

$$\text{s.t.}\ \sum_i D_{i,j,k} X_{ijk} \geq L, \quad \forall j,k \tag{4.8}$$

$$\sum_{j,k} X_{ijk} \geq 1, \quad \forall i, \tag{4.9}$$

$$\sum_{i,j,k} C_{ijk} X_{ijk} - d \leq E, \tag{4.10}$$

$$\sum_{j,k} X_{ijk} - d_i^{(\text{overall})} \leq F_i^{(\text{overall})}, \quad \forall i \tag{4.11}$$

$$\sum_k X_{ijk} - d_{ij}^{(\text{day})} \leq F_{ij}^{(\text{day})}, \quad \forall i,j \tag{4.12}$$

$$\sum_j X_{ijk} - d_{ik}^{(\text{time})} \leq F_{ik}^{(\text{time})}, \quad \forall i,k \tag{4.13}$$

$$X_{ijk}\ \text{binary}, \quad \forall i,j,k \tag{4.14}$$

$$d, d_i^{(\text{overall})}, d_{ij}^{(\text{day})}, d_{ik}^{(\text{time})} \geq 0 \tag{4.15}$$

where d represents the deviation from the economic cost goal and its target value as illustrated in (4.10). Similarly, $d_i^{(\text{overall})}$ denotes the difference between the number of time shed fairness goal to its target $F_i^{(\text{overall})}$ (4.11), and $d_{ij}^{(\text{day})}, d_{ik}^{(\text{time})}$ express the deviations of the rotation fairness with respect to time of day and day of week goals to their target values respectively, (4.12)-(4.13). Moreover, $\omega, \omega_i^{(\text{overall})}, \omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ are positive weights associated to the goals. We add the constraint (4.9) to make sure that all areas are assigned for the entire schedule.

4.3 Implementation

As mentioned in Section 4.1, the aim of this study is to explore the possible solutions that the proposed models can provide. For this purpose we conduct some numerical experiments to evaluate the performance of each model. This is achieved by examining the impact of varying the value of parameters like $\lambda, \alpha, \beta, \omega, \omega_{ij}^{(\text{day})}$ and $\omega_{ik}^{(\text{time})}$ on the cost and fairness of the schedules obtained. Before giving details about the simulation run we describe first the data that we are using.

4.3.1 Data

We apply the proposed models on hypothetical data generated for the City of Cape Town. Figure 4.4 shows the load shedding area map of Cape Town [3]. We consider the 16 coloured areas in this map for our study, that is $R = 16$. The scheduling horizon is defined by 12 hours $\times$ 7 days, i.e. $J = 12, K = 7$. Information about the economic of each sub-region are given in Figure 4.3 [4], which provides data on the proportion of mixed use, industrial and commercial sectors for each sub-region. Moreover, data about population and households are extracted from households data [5].

[3] $https://www.capetown.gov.za/en/electricity/LS2015/EG2014_02_AllAreas-s.jpg$

[4] Andrew Janisch, City of Cape Town, personal communication, 5 June 2015

[5] http://census2011.adrianfrith.com/place/199

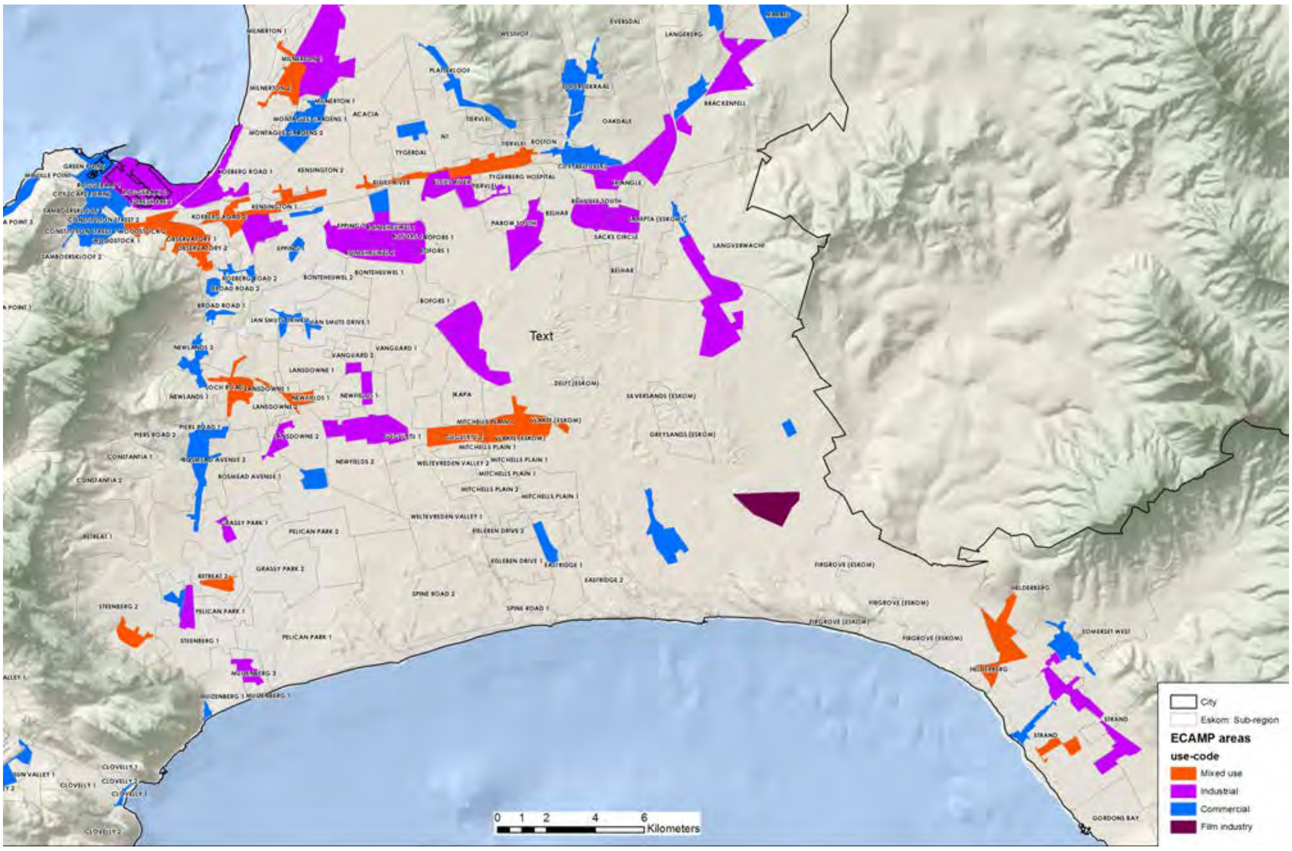


Figure 4.3: Economic areas.

Figure 4.4: Load shedding area map.

In our model, each area i is associated with an economic cost C_{ijk} at each period jk. In order to estimate the economic cost parameters we consider the following steps:

- For each sector, we assume estimated costs for each period jk that measures the impact of load shedding. Table 4.1 provides the corresponding amounts that are used in the numerical experiments. We denote by $C_M(jk)$(in R/km^2), $C_I(jk)$(in R/km^2), $C_C(jk)$(in R/km^2) and $C_H(jk)$(in R/household) the corresponding costs for mixed use, industrial, commercial sectors and households respectively. Here we assume that the costs are identical for all days and the households costs are equal for all households in all areas.

Hours	Mixed use C_M (R/km^2)	Industrial C_I (R/km^2)	Commercial C_C (R/km^2)	Households C_H (R/household)
00:00 - 02:00	600	550	550	5
02:00 - 04:00	600	550	550	5
04:00 - 06:00	600	580	550	5
06:00 - 08:00	620	600	560	1
08:00 - 10:00	750	700	650	1
10:00 - 12:00	800	750	700	1
12:00 - 14:00	790	750	700	1
14:00 - 16:00	810	760	700	1
16:00 - 18:00	740	700	700	5
18:00 - 20:00	700	660	650	5
20:00 - 22:00	650	620	620	5
22:00 - 00:00	610	600	580	5

Table 4.1: Estimated costs caused by load shedding for each sector and households.

  We also associate an economic cost of load shedding to households. Measuring and interpreting these costs is more difficult than in the case of industrial, commercial, or mixed uses. Households costs can be partly interpreted as direct monetary costs, because load shedding means that substitutes must be found for electricity-consuming goods and services, and these will often be more expensive (e.g. boiling water on a gas stove). However load shedding also causes an inconvenience to households that is not directly measurable in monetary terms. In principle, one could estimate a "willingness to pay" (to avoid load shedding) associated with households, either in aggregate or individually. In this thesis we do not attempt this, but rather set household costs in each time period so that total household costs are roughly of the same magnitude as the sum of costs across other sectors (commercial, industrial, mixed). That is, we select household costs so that these neither swamp, nor are swamped by, the costs of the other sectors.

- for each area i, we calculate the surface occupied by each sector as drawn in Figure 4.3. We denote by $S_M(i)$(in km^2), $S_I(i)$(in km^2) and $S_C(i)$(in km^2) the corresponding surfaces for mixed use, industrial and commercial sectors respectively.

- the estimated economic cost for each area at a specific period jk is given by $C_{ijk} = S_M(i)C_M(jk) + S_I(i)C_I(jk) + S_C(i)C_C(jk) + \text{households}(i)C_H(jk)$.

Moreover, we consider some hypothetical data about the electricity consumption of each area i at each period jk. We investigate two case studies:

First case: we assume that areas consume the same amount of electricity and equal to the required amount of load shedding L at each period jk. That is $D_{ijk} = L, \forall i, j, k$.

Second case: we assume that the 16 areas have different electricity demand which can be less or greater than L. Since we don't have real data about the average demand of each area, we generate demand such that the difference between the maximum and minimum demand across areas is relatively small. We do first an increasing ranking of the areas according to the total surfaces occupied by all sectors (mixed use, industrial and commercial); then associate accordingly the generated demand from lower to higher. For $i = 1, \ldots, 16$, the demand for each area i (in increasing order) is determined by

$$\frac{0.9^i}{\sum_i 0.9^i}.$$

4.3.2 Description of the numerical experiments

The process of the numerical experiments is summarized in Figure 4.5 for both SO and GP formulations.

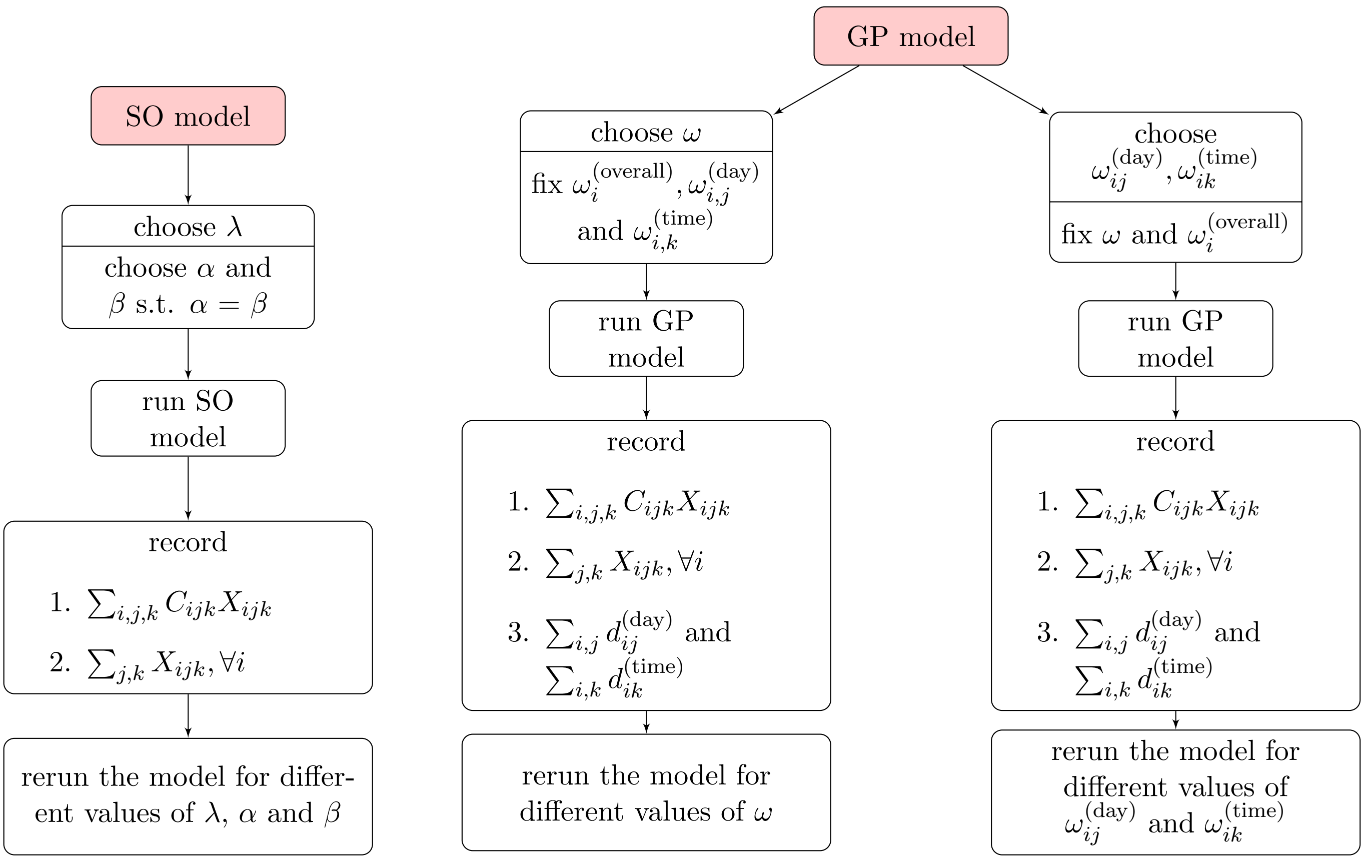


Figure 4.5: Flowchart of the numerical experiments.

4.3.2.1 SO model

For each $\lambda \in \{1, \ldots, \lfloor \frac{SJK}{R} \rfloor\}$ and for each $\alpha, \beta \in \{1, \ldots, 5\}$ such that $\alpha = \beta$, run the SO model (4.1)-(4.6). Record the corresponding cost objective function value $\sum_{i,j,k} C_{ijk}X_{ijk}$ and the number of times each area is shed $\sum_{j,k} X_{ijk}, \forall i$.

4.3.2.2 GP model

Two separate parametric studies are considered

- First, we vary the value of the weight associated to G_1, keeping the others fixed. For each ω, run the GP model (4.7)-(4.15). Record the corresponding cost value $\sum_{i,j,k} C_{ijk}X_{ijk}$, the number of times each area is shed $\sum_{j,k} X_{ijk}, \forall i$ and the total deviation from G_3, $\sum_{i,j} d_{ij}^{(\text{day})}$ and G_4, $\sum_{i,k} d_{ik}^{(\text{time})}$.
- Second, we vary $\omega_{ij}^{(\text{day})}$ and $\omega_{ik}^{(\text{time})}$, keeping the other weights fixed. For each value of $\omega_{ij}^{(\text{day})}$ and $\omega_{ik}^{(\text{time})}$ with $\omega_{ij}^{(\text{day})} = \omega_{ik}^{(\text{time})}$, run the GP model. Record the corresponding cost value $\sum_{i,j,k} C_{ijk}X_{ijk}$, the number of time each area is shed $\sum_{j,k} X_{ijk}, \forall i$ and the total deviation form G_3, $\sum_{i,j} d_{ij}^{(\text{day})}$ and G_4, $\sum_{i,k} d_{ik}^{(\text{time})}$.

One might notice that the different goals in the GP formulation (4.7)-(4.15) are measured on different scales. In order to make these scales comparable we set

$$\begin{aligned}
\omega &= \frac{1}{RV}, \\
\omega_i^{(\text{overall})} &= \frac{1}{RV_i}, \quad \forall i, \\
\omega_{ij}^{(\text{day})} &= \frac{1}{RV_{i,j}}, \quad \forall i,j, \\
\omega_{ik}^{(\text{time})} &= \frac{1}{RV_{i,k}}, \quad \forall i,k,
\end{aligned}$$

where $RV, RV_i^{(\text{overall})}, RV_{ik}^{(\text{time})}$ and $RV_{ij}^{(\text{day})}$ are the range of values (max-min) obtained from the SO model. In that way we have a dimensionless objective function and this weighting places roughly equal importance on the outcome of each objective. In addition we fix the targets values to be

$$\begin{aligned}
E &= 250000, \\
F_i^{(\text{overall})} &= \left\lfloor \frac{SJK}{R} \right\rfloor \quad \forall i, \\
F_{ij}^{(\text{day})} &= \left\lceil \frac{SK}{R} \right\rceil, \quad \forall i,j, \\
F_{ik}^{(\text{time})} &= \left\lceil \frac{SJ}{R} \right\rceil, \quad \forall i,k.
\end{aligned}$$

Fairness goals were set to values that would be obtained under a maximally fair schedule i.e. without considering economic costs. These are as described in Section 4.2. The cost goal was set so as to be moderately but not excessively demanding. For comparative purposes, the cheapest schedule, without any fairness considerations, costs R175 000; a maximally fair schedule costs R356 000.

4.4 Results

This section contains results about the experimentation studies. The focus is on examining the effect of changing the values of the different parameters on the cost and fairness of the generated schedules and report the different solutions that the models provide.

4.4.1 Case 1: Areas have equal electricity consumption

4.4.1.1 SO model

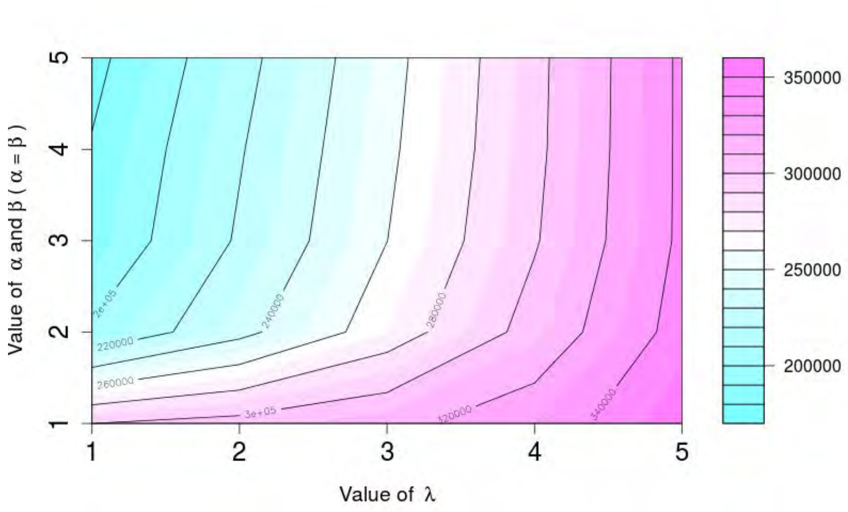


Figure 4.6: The economic cost value for different values of λ, α and β for case 1.

Figure 4.6 shows the impact of varying the values of λ, α and β on the total economic costs obtained from the schedule. For a high value of λ, say equal to 5, the resulting objective function value is high since all areas are shed a similar number of times regardless of their economic costs. Although the cost decreases as we increase the value of α and β. This is explained by the fact that increasing the value of α and β allows succession in the period of shedding time. For example, if $\alpha = \beta = 3$, an area can be shed 3 times per day and 3 times in the same period per week. Thus the optimal solution would shed areas with low economic costs more often to reduce costs.

In contrast, with value of λ less than 3, the total economic cost is low. Actually, for instance if $\lambda = 1$ then areas are allowed to be shed only once while others can be shed much more often. Therefore, an optimal solution is to shed areas with low economic impact more times than the others. The lowest objective function is obtained for $\lambda = 1$ and $\alpha = \beta = 5$.

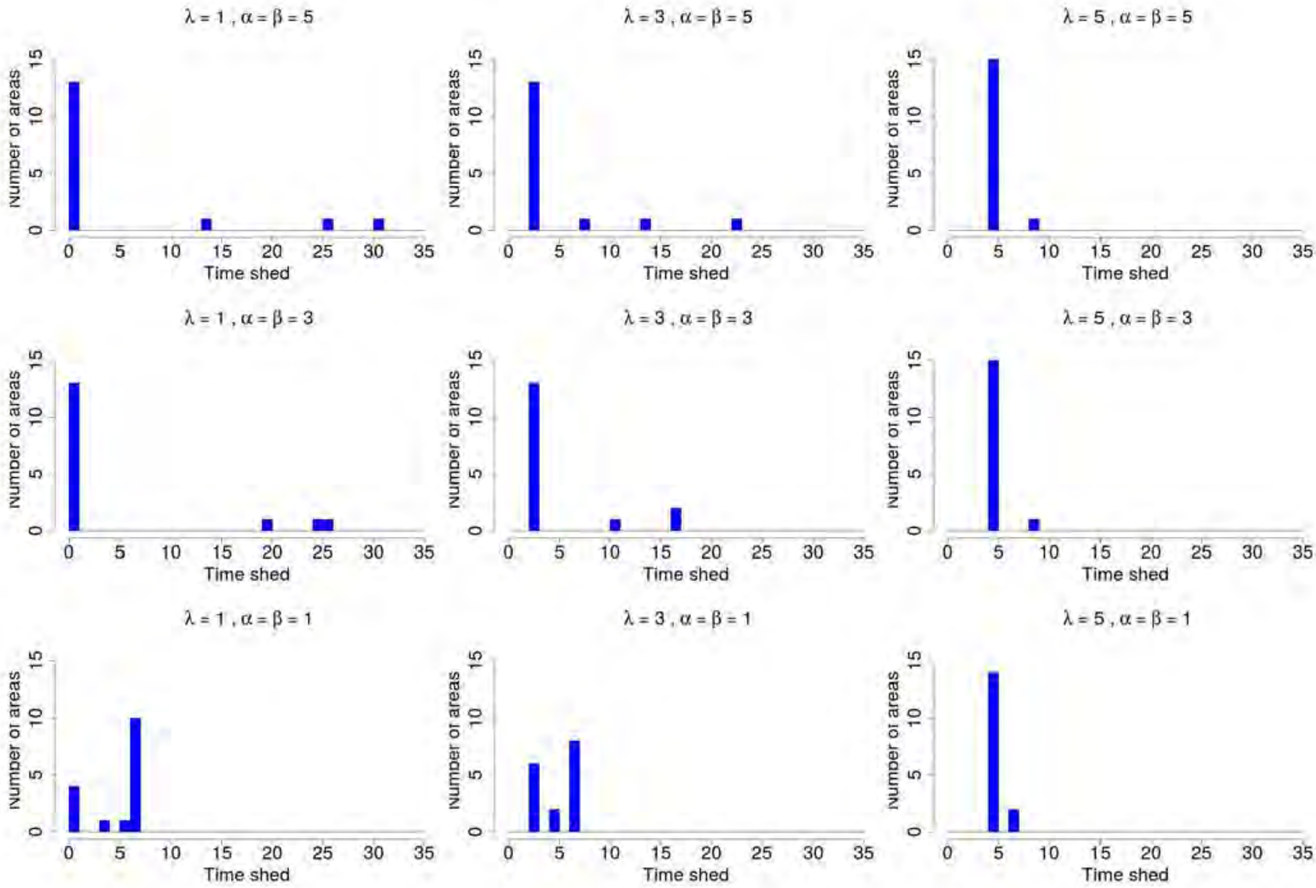


Figure 4.7: Total number of times each area is shed for different values of λ, α and β for case 1.

Figure 4.7 shows histograms of the total number of times each area is shed for various values of λ, α and β. We can see that for $\lambda = 5, \alpha = \beta = 1$, 87% of the total number of areas are shed 5 times in the schedule. This suggests a fair schedule in the sense that all areas are shed almost equally. But for $\lambda = 1$ and $\alpha = \beta = 5$, 19% of the total areas have to be shed more than 14 times whilst 81% are shed only once. This shows that there is a significant difference between the number of time each area is shed, which may well be interpreted as unfair.

If we relate Figure 4.6 and Figure 4.7, one can say that the fairness of the schedule leads to high economic cost. This fairly obvious point is important because it highlights the trade-offs that decision makers must necessarily confront when creating a schedule. The city of Cape Town for example is likely to be expensive because it is so fair.

4.4.1.2 GP model

Recall that the focus in the GP simulation is to change the value of weights $\omega, \omega_{ij}^{(\text{day})}$ and $\omega_{ik}^{(\text{time})}$ and examine the impact on the cost and fairness goals. As explained in Section 4.3.2.2 we need to adjust scales to have a dimensionless objective function. Here we take

$$RV = 181356$$
$$RV_i^{(\text{overall})} = 30, \quad \forall i$$
$$RV_{ij}^{(\text{day})} = 5, \quad \forall i, j$$
$$RV_{ik}^{(\text{time})} = 5, \quad \forall i, k.$$

First case: We vary the value of ω and keep other weights fixed.

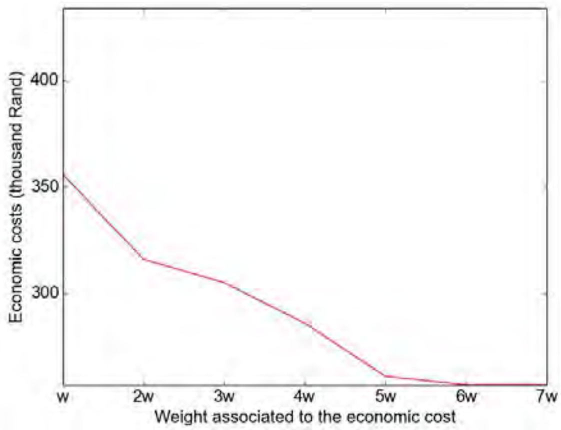


Figure 4.8: Value of the economic cost goal for different values of ω for case 1.

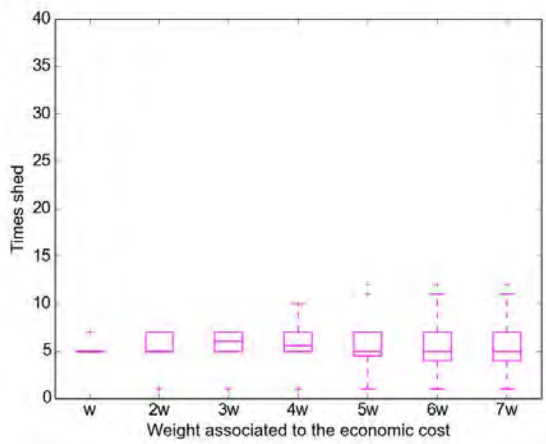


Figure 4.9: Number of times areas are shed for different values of ω for case 1.

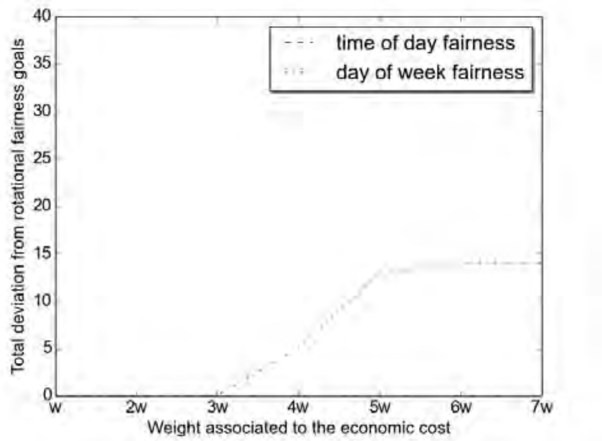


Figure 4.10: Value of $\sum_{i,j} d_{ij}^{(\text{day})}$ and $\sum_{i,k} d_{ik}^{(\text{time})}$ under various values of ω for case 1.

As shown in Figure 4.8, the increase of the value of ω results in a decrease in the amount of total economic cost from R356000 to R257000. A high value of ω puts more importance on achieving the economic cost goal, thus resulting in the convergence of the cost goal to its target value.

This also leads to an increase in the range of values on the number of times areas are shed (see Figure 4.9, which shows boxplots [6]). As we increase the value of ω, here from ω to 7ω, the range of

[6] Here the boxplots are determined by upper whisker /quartile, median, lower whisker /quartile and flyers.

number of time shed increases from 2 to 11. This implies an imbalance of the number of times areas are shed which can be interpreted as unfair schedule. Moreover, the total deviation from the day of week rotation fairness goal increases as displayed in Figure 4.10, some areas are shed more often than others. Here, the time of day fairness is equal to 0 for each weight. Whether these tradeoffs are acceptable would depend on the preferences of decision makers.

However if we look back on the results obtained from the SO formulation, for a value of economic cost between [R257000, R356000], the range of the number of time shed of all areas is between 2 to 18. This suggests that the GP formulation provides more balanced solution.

Second case: We vary the value of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ and keep other weights fixed.

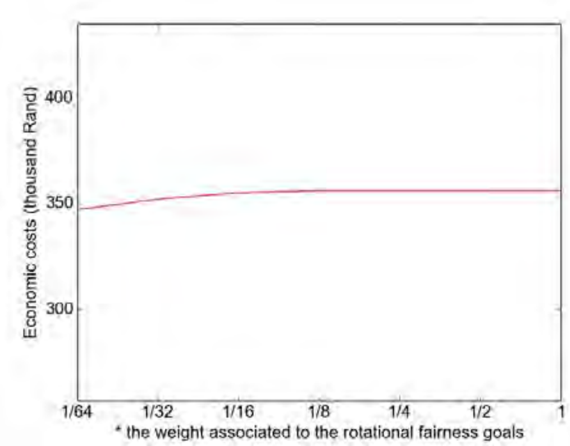


Figure 4.11: Value of the economic cost goal under the variation $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 1.

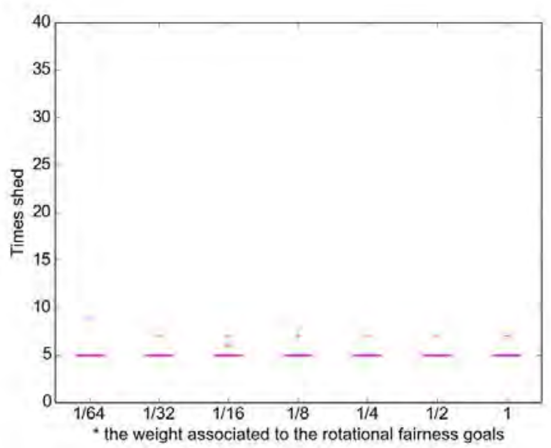


Figure 4.12: Number of times areas are shed for different values of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 1.

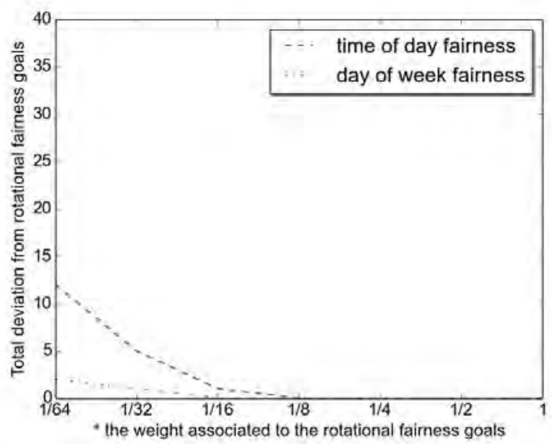


Figure 4.13: Value of $\sum_{i,j} d_{ij}^{(\text{day})}$ and $\sum_{i,k} d_{ik}^{(\text{time})}$ for different values of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 1.

As shown in Figure 4.11, an increase in the weights $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ leads to very small change in the total cost as compared to Figure 4.8. This might be interpreted as the cost is insensitive to the variation of the rotation fairness weights. Nevertheless, as these weights increase, the schedule becomes fair in the sense that rotation is avoided (Figure 4.13) while the gap between the number of time shed of all areas decreases (Figure 4.12).

4.4.2 Case 2: Areas have different electricity consumption

4.4.2.1 SO model

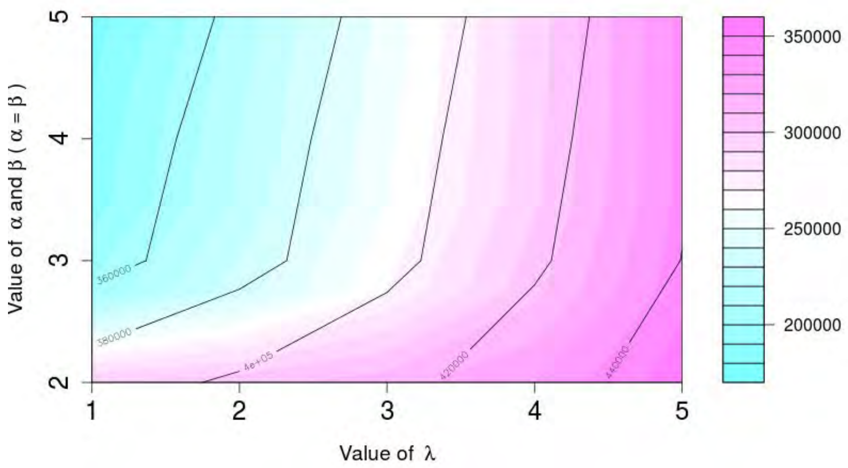


Figure 4.14: The economic cost value for different values of λ, α and β for case 2.

We conduct the same study as in Case 1 (Section 4.4.1). Figure 4.14 plots the total cost obtained from varying the values of λ, α and β. Similar to the Case 1 study, the cost decreases if we relax the rotation constraints and/or the constraint on the number of times shed. Notice that there is no feasible solution for $\alpha = \beta = 1$. Actually, α and β are parameters that express the level of rotation fairness in the formulation (4.1)-(4.6). For $\alpha = \beta = 1$, the model requires each area to be shed only once which is not possible here since 56% of the total areas have a demand below the average. In addition, the cost ranges from R341000 to R450000 which is relatively high compared to the case where demand is equal for all areas. This is not surprising since two or three areas might be shed in the same slot here. This implies that for a given slot, the associated economic cost in this case might be double the cost of the first case.

Figure 4.15 shows the distribution of the number of times each area is shed for some values of λ, α and β. As expected there is a significant difference in the number of time shed of all areas for value of λ less than 3 and α, β greater than 3. We can infer, similar to the first case study, that fairness and total economic cost are competitive and conflicting. A fair schedule requires a high cost whereas low cost can only be achieved with some sacrifices to the fairness of the schedule.

4.4.2.2 GP model

We rerun the GP model for the same value of weights as in Case 1 (Section 4.4.1) but with different demand for each area.

First case: We vary the value of ω and keep other weights fixed.

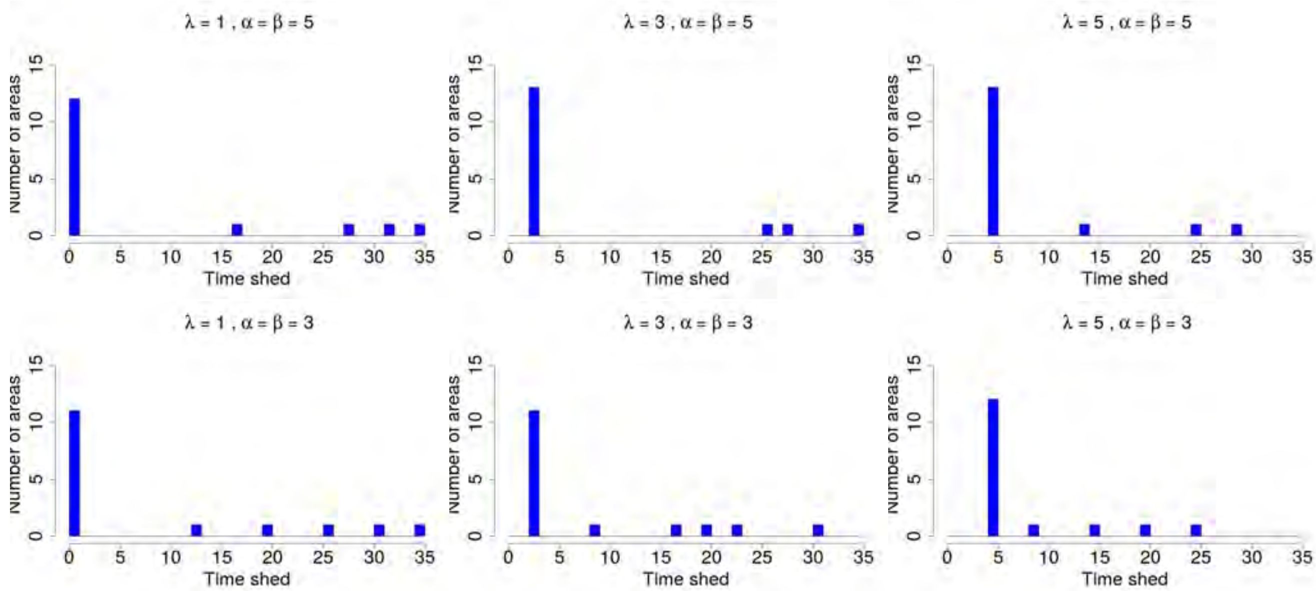


Figure 4.15: Total number of times each area is shed for different values of λ, α and β for case 2.

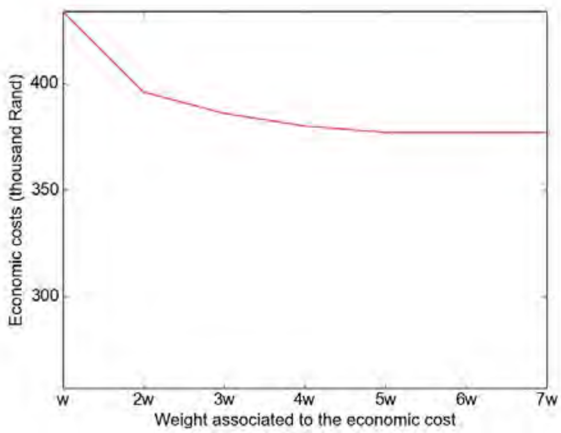


Figure 4.16: Value of the economic cost goal for different values of ω for case 2.

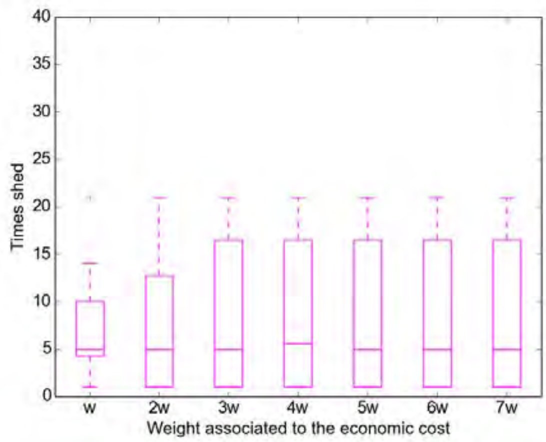


Figure 4.17: Number of times areas are shed for different values of ω for case 2.

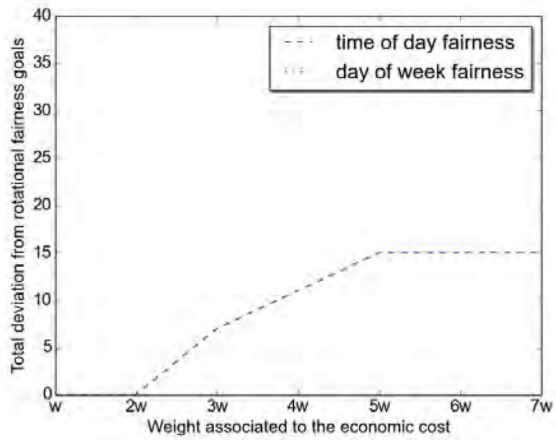


Figure 4.18: Value of $\sum_{i,j} d_{ij}^{(\text{day})}$ and $\sum_{i,k} d_{ik}^{(\text{time})}$ under various values of ω for case 2.

The increase of weight associated to the cost goal leads to a decrease in the amount of economic cost (Figure 4.16) while resulting an increase in the total deviation from the time of day rotation fairness goal (Figure 4.18) [7] as well as the range of total number of time shed (Figure 4.17). This is explained by the fact that as we put more weight on one goal, we enhance its importance. Notice that the overall range of the total number of times areas are shed is greater here as compared to the case where electricity consumptions are equal for all areas since the difference in demand might cause some areas to be shed more often. In addition the decrease in the economic cost is less than case 1: in this case it decreases from R434000 to R377000 while from R356000 to R257000 in case 1.

Second case: We vary the value of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ and keep other weights fixed.

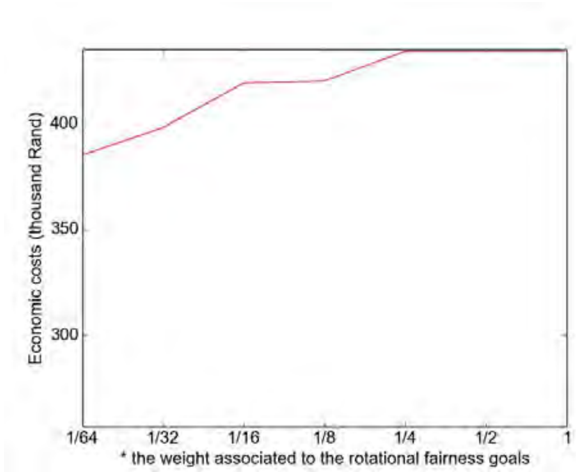


Figure 4.19: Value of the economic cost goal under the variation $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 2.

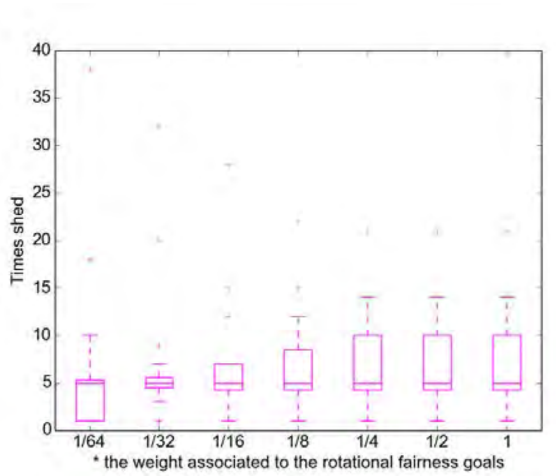


Figure 4.20: Number of times areas are shed for different values of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 2.

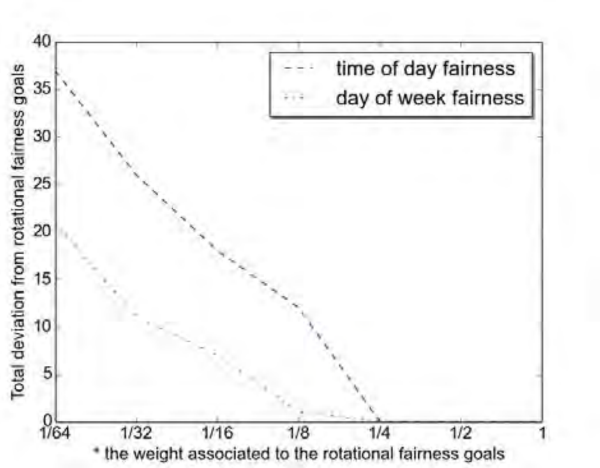


Figure 4.21: Value of $\sum_{i,j} d_{ij}^{(\text{day})}$ and $\sum_{i,k} d_{ik}^{(\text{time})}$ for different values of $\omega_{ij}^{(\text{day})}, \omega_{ik}^{(\text{time})}$ for case 2.

Figure 4.19 tells us that as we increase the weights associated with the rotation fairness goals, the cost increases. The schedule becomes fair as expected but at a relatively slow rate (Figures 4.20, 4.21). A 64-fold increase in $\omega_{ij}^{(\text{day})}$ only causes a 15% increase in costs. Notice that the value of

[7]Here, the day of week fairness is equal to 0 for each weight.

economic costs, whether in the first case where we increase the value of ω (Figure 4.16) or here, range from R434000 to R377000. But they provide different solutions in terms of the total number of times shed and deviations from the rotation fairness goals. For example, in the first case the total number of times shed for all areas ranges from 1 to 21 whereas in the second case it ranges from 1 to 37 (Figures 4.17, 4.20). Moreover the total deviation from the time of day fairness goal increases from 0 to 15 in the first case (Figure 4.18) whereas it decreases from 37 to 0 in the second case (Figure 4.21). So it is left for decision makers to choose their preferences.

One might notice also that the values of the total deviation from time and day fairness goals (Figure 4.21) as well as the number of times areas are shed (Figure 4.20) are double the values of the same in case 1 (Figure 4.13 and Figure 4.12). This is because the electricity consumptions in some areas are less than the required amount of load shedding L at each period jk in case 2 which implies that more than one area have to be shed in some slots.

4.5 Decision support system

In this section, we present a computerized DSS that can be used to solve the load shedding scheduling problem formulated as in Section 4.2 (single objective model or goal programming model). Similar to the DSS for the UC under load shedding problem (Section 3.5), the implementation is a collection of AIMMS scripts with a corresponding graphical user interface (GUI). We describe below the different steps to follow in order to solve each model via DSS.

4.5.1 DSS for the single objective model

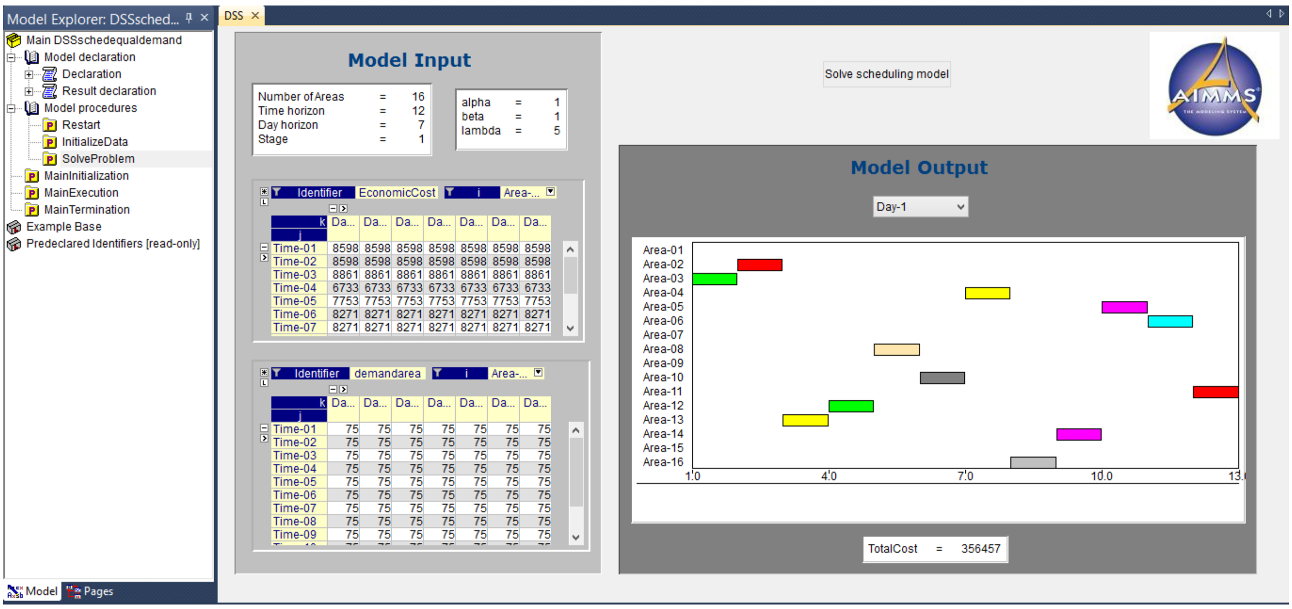


Figure 4.22: Screenshot of the GUI of the DSS SO model.

Upon executing the DSS for the SO model, the GUI displayed in Figure 4.22 appears on the screen. It contains two panels – the 'Model Input' and the 'Model Output' – and a set of procedures in the left sidebar. The 'Model Input' panel contains data and parameters of the model. The user needs to enter the value of these parameters. This can be done by clicking on '*declaration*' in the left sidebar and selecting the relevant parameter. For example, as illustrated in Figure 4.23, the user can input the data on economic cost for each area at each period by clicking on the parameter '*EconomicCost(i,j,k)*' and filling in the corresponding table in the open window. The value of all parameters inside the 'Model Input' panel can be entered in the same way. Once the user has entered all data, they can press the button '*solve scheduling model*' to solve the model. This provides outputs shown in the 'Model Output' panel. These include the value of objective function ('*TotalCost*') and the generated schedule for a specific day, represented by the gantt chart. The user can click on the drop-down list to change the day.

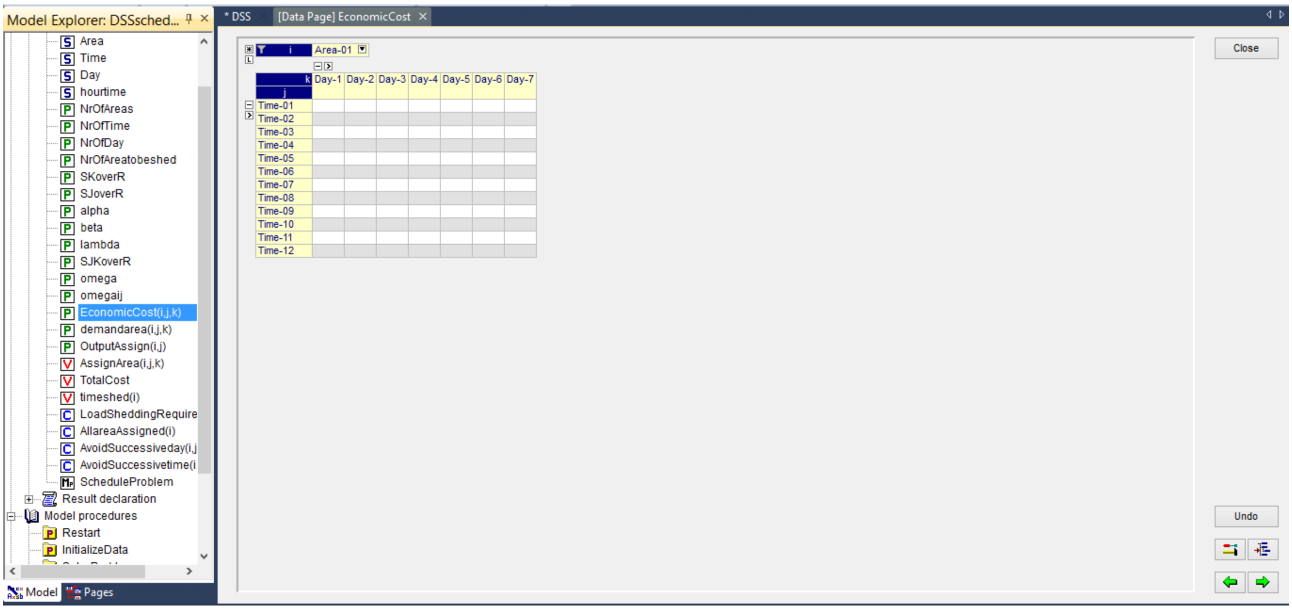


Figure 4.23: Screenshot of the window to enter the input data.

4.5.2 DSS for the goal programming model

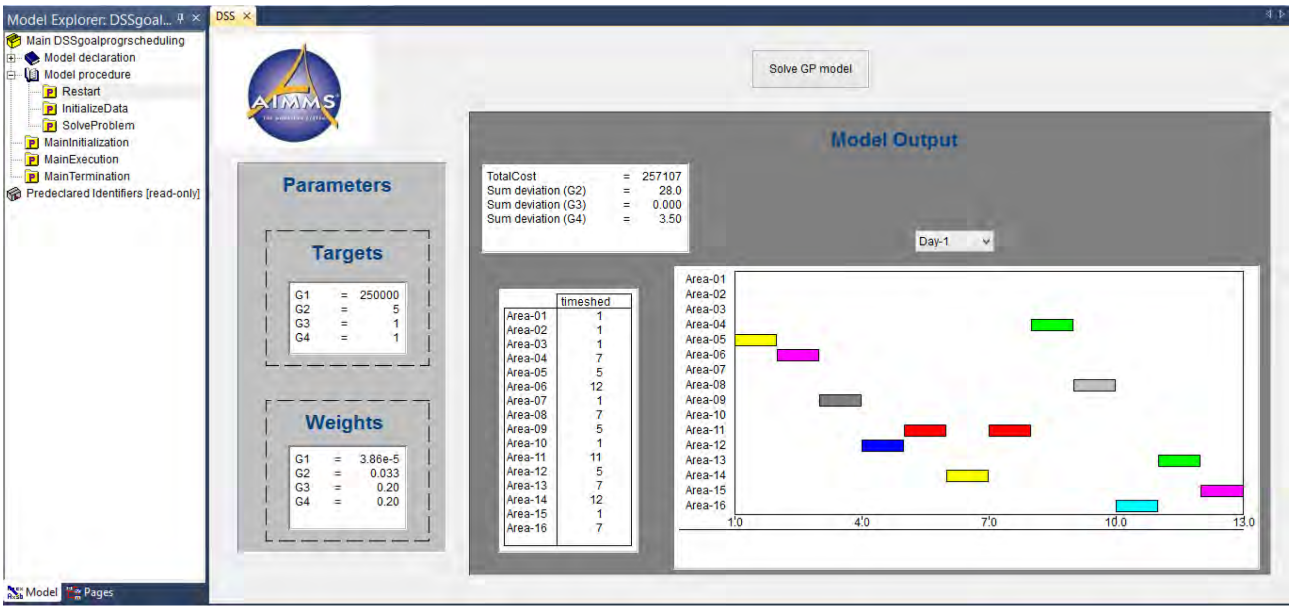


Figure 4.24: Screenshot of the GUI of the DSS GP model.

Figure 4.24 illustrates the GUI for the DSS GP model. The user can input data in the same way as explained in the DSS SO model (Section 4.5.1). The value of parameters like '*Targets*' associated to each goal can be entered by clicking on '*Model declaration*' and selecting the corresponding parameter. This will bring up a window that can be modified in the usual way. Once all data are entered, the model can be run by clicking on the '*Solve GP model*'. This creates the following outputs:

- '*TotalCost*': the value of the cost goal,
- '*Sum deviation (G2)*': total deviation from the overall fairness goals,
- '*Sum deviation (G3)*': total deviation from the time of day rotation fairness goals,
- '*Sum deviation (G4)*': total deviation from the day of week rotation fairness goals,
- '*TimeShed*': total number of times each area is shed during the scheduling horizon,
- the corresponding schedule for a specific day represented by the gantt chart.

Chapter 5

Conclusion

5.1 Thesis summary

Load shedding – a planned and controlled reduction in power supply – is inevitable in cases where there is shortage in the power production levels and insufficient reserve margins to satisfy demand. In this work, two optimization problems related to load shedding have been studied. The first was integrating load shedding into the UC process and the second consisted of generating a time-based load shedding schedule that includes fairness modelling. Deterministic and stochastic programming models were developed for these purposes. In order to fully develop these models, some background information regarding mathematical optimization as well as UC problem are required. These necessary information were presented in Chapter 2. Some basic optimization techniques and a concise survey of literature with respect to UC problem were outlined.

The assessment of load shedding as part of the UC decision process was discussed in Chapter 3. Load shedding was modelled as a dummy unit representing any discrepancy between generation output and demand. Three models – one deterministic and two stochastic – were described to represent the problem. The objective was to evaluate the performance of the models and compare them in terms of the expected costs induced, the amount of planned and expected unplanned load shedding. It was shown that the two stochastic programs (two-stage and multi-stage) are more conservative, dispatching load shedding to cover any possibility of high demand. Furthermore the results indicated that the deterministic model is more likely to suffer from a higher amount of unserved load whereas the multi-stage model is more safe. As a consequence, the overall cost (generation cost + cost of unplanned load shedding) is much higher for the deterministic model as compared to the stochastic ones. For the hypothetical (but realistic) data considered in the IEEE problem, under the assumption that the cost of unplanned load shedding is 5000$/MW, the deterministic model is about 23% more expensive than the multi-stage one.

The second problem, which consists of designing linear integer and goal programming models to generate load shedding schedules, was presented in Chapter 4. The objective was to emphasize the trade-off between fairness and economic impacts of load shedding. Two fairness goals were considered: rotational fairness, which aims to avoid successive shedding times for all areas, and cumulative fairness, which aims to balance the number of shedding times among areas. Several case studies were conducted to explore the possible solutions that the proposed models provide. It was found that a fair schedule requires a high cost whereas low cost can only be achieved with some sacrifices to the fairness of the schedule. It would be left for decision makers to choose the schedule that best fits with their preferences.

For each of these optimization problems, a decision support system was created to assist users should they want to replicate and solve the proposed models (see Sections 3.5 and 4.5). All AIMMS scripts are available online at AIMMS files [1].

5.2 Future work

This section contains ideas and suggestions for future work. A number of future developments can be addressed for both the UC under load shedding problem and the load shedding scheduling problem.

5.2.1 Suggestions regarding the study of UC under load shedding

1. In our experimentation study, we did not include the ramp-up/ -down limits as well as the minimum-up/ -down time limits for each unit to reduce the computational requirement. A further step is to incorporate these constraints in order to get more realistic results.

2. In the planning process within the simulation study (Section 3.3.2.1), we generated demand scenarios for the two-stage model by assuming that the demand at each time is uniformly distributed within the range $[0.9d_t, 1.1d_t]$. One might study the sensitivity of changing the value of the lower fraction 0.9 and the upper fraction 1.1.

3. In the multi-stage case, we assumed that the stochastic demand can take values in two confidence intervals determined by $[0.9d_t, d_t]$ and $[d_t, 1.1d_t]$. So an idea is to consider a number of intervals so that the model is more robust.

4. An idea of further study is also to include other types of units like hydro and pumped storage units.

5.2.2 Suggestion regarding the study of load shedding scheduling

1. Since we did not have real data concerning the costs of load shedding for each area, we considered estimated costs (see Section 4.3.1). A further step may be to conduct a sensitivity analysis with respect to these costs or to attempt to gather empirical data on these quantities.

2. As exposed in Section 2.1.1.3, two methods can be used to represent goal programming problems, namely Archimedian and Chebychev. In our case study, we only used Archimedian method. So an idea for future work is to consider the Chebychev formulation.

3. For the two proposed formulations, we assumed that all parameters involved are deterministic. A further step is to include uncertainty in the formulations so that the models reflect more the reality.

[1] http://dx.doi.org/10.5281/zenodo.49199"

4. An idea for further study is also to develop model that includes predictability; that is creating a schedule that would look back at the past distribution of load shedding (the number of times each area was shed) then correct any imbalances over some user-specified future time interval.

Appendix A

Algorithm for solving linear programming problem

A.1 Simplex method (SM)

To illustrate the steps of the SM algorithm, we rewrite the formulation (2.1)-(2.3) in the form

$$\text{Maximize} \quad c^t x$$
$$\text{subject to} \quad Ax + s = b$$
$$x \geq 0,$$

with s the vector of slack variables. We can refine also this formulation and get

$$\text{Maximize} \quad c^t x$$
$$\text{subject to} \quad Ax = b$$
$$x \geq 0,$$

with $c, x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $n > m$ and A $m \times n$ matrix. The decision variable x includes the slack variables. One can split A, c, x into a basis and non-basis part i.e. $A = (B, N)$, $c = (c_B, c_N)$, $x = (x_B, x_N)$. Therefore we obtain a basic representation of constraints

$$(B, N)(x_B, x_N)^T = b \Longleftrightarrow x_B = B^{-1}(b - N x_N).$$

and of the objective function

$$\begin{aligned} (c_B^T, c_N^T)(x_B, x_N)^T &= c_B^T x_B + c_N^T x_N \\ &= c_B^T B^{-1} b - (c_B^T B^{-1} N - c_N^T) x_N. \end{aligned}$$

The vector $c_B^T B^{-1} N - c_N^T$ is called the vector of *reduced costs* [1].

[1] A basic feasible solution is optimal if and only if this vector is positive. The reduced costs for a basic variable is zero and the reduced costs for non-basic variables can be interpreted as the change in the optimal objective function per unit change in the variable resulting from forcing it to take a strict positive value. By setting $x_N = 0$, $x_B = B^{-1}b$ is called *basic solution* and if $x_B \geq 0$ then it is called *basic feasible solution.*

From this we can state the outline of the algorithm as the following:

- Step 1: It begins its search from the origin (the slack variables are considered to be the initial basic variables). Denote by $z - c$ the vector of reduced costs. If none of the element of $z - c$ is negative then the current solution is optimal.
- Step 2: Otherwise, select one negative element in the vector say $z_j - c_j$ with x_j as the associated variable. The new solution is given by $x_B - y_j x_j$ with $y_j = B^{-1} a_j$ and a_j the j-th column of A corresponds to the variable x_j.
    1. If all elements in y_j are negative then x_j and the value of the objective function can be increased without bound and this indicates error in the formulation of the problem.
    2. Otherwise, if there exists one or more elements in y_j with positive value then identify the k-th element in the basis x_B which is determined by the smallest ratio $\min_i\{\frac{x_{Bi}}{y_{ij}}\}$ and replace it by x_j.
- Step 3: Compute the vector reduced costs and go to Step 1.

A.2 Interior point method

To illustrate the interior point method, we need to introduce the duality theory for LP, the Newton search direction and the Karush-Kuhn-Tucker conditions.

A.2.1 Duality theory for LP

Given the LP formulation (2.1)-(2.3), one can associate a dual programming of the form

$$\begin{aligned}&\text{Minimize} \quad u^T b\\ &\text{subject to} \quad A^T u \geq c\\ &u \geq 0,\end{aligned}$$

with dual variable $u \in \mathbb{R}^m$. We can reformulate the problem by changing the inequality into equality, by adding a non-negative *excess variables*, as follows

$$\begin{aligned}&\text{Minimize} \quad u^T b\\ &\text{subject to} \quad A^T u - e = c\\ &u \geq 0,\end{aligned}$$

with e the excess variables.

The following three points are the main theorems on duality in linear programming:

- **Weak duality**: if u and x are respectively feasible solutions for the DP and LP problems then $c^T x \leq u^T b$.
- **Strong duality**: if feasible solution exists for both LP and DP problems then there exists optimal solutions x and u such that $c^T x = u^T b$.
- **Complementary slackness**: $x^T e = 0$ and $u^T s = 0$.

A.2.2 Newton search direction

Suppose we solve the equation $f(x) = 0$ with $f : \mathbb{R}^n \to \mathbb{R}^m$. The Newton search direction is a method in which we perform a line search where the Newton direction p is obtained from the solution of the system

$$J(x)p = -f(x)$$

where $J(x)$ is the Jacobian of $f(x)$.

A.2.3 Karush-Kuhn-Tucker (KKT) conditions

KKT conditions are necessary conditions for a solution of a nonlinear programming to be optimal. Consider the following problem

$$\begin{aligned} \min\ & f(x) \\ \text{s.t. } & g_i(x) = 0, \quad i = 1, \ldots, n \\ & h_j(x) \leq 0, \quad j = 1, \ldots, m \\ & x \in X \end{aligned}$$

where f, g_i, and h_j: $\mathbb{R}^p \longrightarrow \mathbb{R}$ and $X \subset \mathbb{R}^p$. If a point x^* is an optimal solution such that the gradient of the constraints are linearly independent at x^* then there exists μ^* and λ^* for which the following hold:

1. $\nabla f(x^*) + \sum_{i=1}^{n} \mu_i^* \nabla g_i(x^*) + \sum_{j=1}^{m} \lambda_j^* \nabla h_j(x^*) = 0$
2. all constraints are satisfied
3. $\lambda_j^* h_j(x_j^*) = 0$, $\lambda_j^* \geq 0$ for $j = 1, \ldots, m$. This condition is called *complementary slackness*.

A.2.4 Outline of interior point method

The original LP problem can be reformulated as

$$
\begin{aligned}
\max\ & c^T x \\
\text{s.t.}\ & Ax + s - b = 0 \quad \text{(primal feasibility)} \\
& A^T u - e - c = 0 \quad \text{(dual feasibility)} \\
& x^T e = 0 \quad \text{(complementary slackness)} \\
& x \geq 0
\end{aligned}
$$

The algorithm makes use of a Barrier function defined by

$$
B(x, \rho) = c^T x - \rho \sum_{i=1}^{n} \ln(x_i)
$$

for ρ non-negative constant. The barrier function is introduced in order to prevent reaching the boundary of the feasible set and penalize terms for violation of constraints. At this stage, instead of considering the original maximization problem (max $c^T x$), we consider the maximization of the above barrier function and by using the Karush-Kuhn-Tucker conditions in A.2.3, we can reformulate the LP problem as the following barrier subproblem

$$
\begin{aligned}
\max\ & B(x, \rho) \\
\text{s.t.}\ & Ax + s - b = 0 \\
& A^T u - e - c = 0 \\
& x^T e - \rho = 0
\end{aligned}
$$

It shows that as we drive ρ to 0, the optimal solution to this barrier subproblem is the optimal solution to the original linear problem. So, as ρ goes to 0, a maximum solution to the barrier suproblem will be a maximum solution to the LP problem. We solve this subproblem iteratively, by choosing a convenient sequence which will approach the optimal solution in the limit and by taking a step in the direction that results an increase in $B(x, \rho)$ and then decreasing the value of ρ in the next iteration. The following is the key steps of the algorithm

- Start with a central interior point, i.e. choose a point (x_0, u_0, e_0, s_0) feasible for the primal and dual problem (non zero values). Choose also a value of the parameter $\nu \in (0, 1)$ and $\rho_0 > 0$ sufficiently large.

- At each step k, define $\rho_k = \nu \rho_{k-1}$ and compute the search direction obtained from the Newton search direction as described in A.2.2. Here the function f in A.2.2 is replaced by $F : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$
F(x, u, e, s) = \begin{pmatrix} Ax + s - b \\ A^T u - e - c \\ x^T e - \rho \end{pmatrix}.
$$

  Denote by $p_B = (x_B, u_B, e_B, s_B)$ the Newton direction obtained.

- Conducting a linear search in the direction p_B, solve the step-size program

$$\max_{\alpha} B(x_k, \rho_k)$$

  where $x_k = x_{k-1} + \alpha x_B$ subject to $Ax_k + s - b = 0$.

- With step-size α^* optimal, let $x_k = x_{k-1} + \alpha^* x_B$.

- Continue until ρ_k is sufficiently close to 0.

A.3 Branch and Bound algorithm

The basic principles of this method can be presented as the following

1. Relax the integrability constraints of the problem and solve it. If the solution satisfies the integrability constraint then stop (the resulting solution is optimal).

2. Otherwise, select a non integer variable say x_i with a non-integer value b_i, and *branch* the problem i.e. split it into two sub-problems one of which by appending the constraint $x_i \leq \lfloor b_i \rfloor$ and the other the constraint $x_i \geq \lceil b_i \rceil$. If these sub-problems are not yet solved then we call them *unfathomed* and denote by Z_{best} the objective function value of the best known integer solution.

3. Select an unfathomed sub-problem and solve its continuous relaxation.

   - If there is no feasible solution for the sub-problem or it has an optimal value not better than Z_{best} then eliminate this sub-problem and backtrack to its parent problem.
   - If this sub-problem is feasible, integer and has better value than Z_{best} then this sub-problem is fathomed and the resulting optimal value becomes the current Z_{best}. Backtrack to the parent problem.
   - If this sub-problem is feasible but do not satisfy the integrability condition and if the objective function value is better than Z_{best} then create two more unfathomed sub-problems as in step 2 and repeat step 3.

4. Continue until all sub-problems have been fathomed or eliminated. The current Z_{best} is the optimal solution.

Appendix B

Lagrangian duality

Given an optimization problem with constraints, called *primal problem*, there exists a problem associated with it, named the *Lagrangian dual problem*. These two problems are defined as follows

Primal problem:

$$\begin{aligned} \min\ & f(x) \\ \text{s.t.}\ & g_i(x) = 0, \quad i = 1, \ldots, n \\ & h_j(x) \leq 0, \quad j = 1, \ldots, m \\ & x \in X \end{aligned}$$

where f, g_i, and h_j: $\mathbb{R}^p \longrightarrow \mathbb{R}$ and $X \subset \mathbb{R}^p$.

Lagrangian dual problem:

$$\begin{aligned} \max\ & \theta(\mu, \lambda) \\ \text{s.t.}\ & \lambda \geq 0 \end{aligned}$$

where $\theta(\mu, \lambda) = \inf\{f(x) + \sum_{i=1}^{n} \mu_i g_i(x) + \sum_{j=1}^{m} \lambda_j h_j(x); x \in X\}$. Here the scaling variables μ_i and λ_j are referred to as the Lagrangian multiplier associated with the constraints $g_i(x) = 0$ and $h_j \leq 0$ respectively, $i = 1, \ldots, n, j = 1, \ldots, m$. The function $f(x) + \sum_{i=1}^{n} \mu_i g_i(x) + \sum_{j=1}^{m} \lambda_j h_j(x)$ is called the Lagrangian function or Lagrangian relaxation with respect to all constraints.

We discussed in A.2.1 the weak duality and strong duality theorem for LP problem. For non-linear problem the weak duality holds, but the strong duality requires that all functions involved are convex.

The Lagrangian dual program is often solved by using the subgradient method. It is an iterative method using a gradient adjustment by which the Lagrangian multipliers are adjusted so as the dual function $\theta(\mu, \lambda)$ moves from its initial value to one which is larger.

Appendix C

Reliability metrics

C.1 Construction of COPT

As the first step to construct the COPT, the uncertainties of the problem should be modelled appropriately. Each unit g is considered as having two-state: operating or failed at a rate μ_g and λ_g respectively. As described in Billinton et al. (1984), the time-dependent probabilities of the two-state unit are calculated as follows

$$P^g_{\text{failed}}(T) = \frac{\lambda_g}{\lambda_g + \mu_g} - \frac{\lambda_g}{\lambda_g + \mu_g} \exp^{-(\lambda_g+\mu_g)T} \tag{C.1}$$

$$P^g_{\text{operating}}(T) = 1 - P^g_{\text{failed}}(T) \tag{C.2}$$

If the time considered is very large, the second term in (C.1) vanishes and he first term is known as the forced outage rate (FOR). In cases where the study period is relatively short, like the UC problem, the failed unit may not be repaired. In such case the repair rate is neglected, i.e. $\mu_g = 0, \forall g$, and (C.1), (C.2) can be reformulated, respectively, as

$$P^g_{\text{failed}}(T) = 1 - \exp^{-(\lambda_g)T} \tag{C.3}$$

$$P^g_{\text{operating}}(T) = 1 - P^g_{\text{failed}}(T) \tag{C.4}$$

Finally if $\lambda_g T << 1$, $P^g_{\text{failed}}(T) \simeq \lambda_g T$. This last equality is known as the outage replacement rate (ORR_g) that represents that a unit g fails and is not replaced during the time T.

By using the outage replacement rate, we illustrate the computation of COPT through a simple example. Consider a committed generating system consisting of two 10MW and one 20MW having each an outage replacement rate 0.002 for 1 hour. The two identical units can be combined to give the COPT shown in Table C.1

Table C.1: COPT for the two identical units

Capacity out (MW)	Probability
0	0.998*0.998 = 0.996
10	0.998*0.002 = 0.00199
20	0.002*0.002 = 0.000004

The 20 MW unit can be added to this table by considering that it can be in service with probability 1-0.002 = 0.998 or it can be out of service with probability 0.002. The resulting COPT is given in Table C.2.

Table C.2: COPT for the three units

Capacity out (MW)	Probability
0	0.996*0.998 = 0.994
10	0.00199*0.998 = 0.00198
20	0.000004*0.998 = 39.92 10^{-7}
40	0.002*0.002*0.002 = 8 10^{-9}

One can add an additional column to represent the cumulative probability which is the probability of finding a quantity of capacity on outage equal to or greater than the indicated amount. The corresponding table is Table C.3. [1]

Table C.3: COPT (including cumulative probability) for the three-unit system

Capacity out (MW)	Probability	Cumulative probability
0	0.996*0.998 = 0.994	1
10	0.00199*0.998 = 0.00198	0.00198
20	0.000004*0.998 = 39.92 10^{-7}	39.92 10^{-7}
40	0.002*0.002*0.002 = 8 10^{-9}	8 10^{-9}

C.2 Computation of LOLP and ELNS

C.2.1 Loss of load probability LOLP

The LOLP is the probability that the total generation output plus the spinning reserve is less than the system demand. It can be defined as, for a system of N generating units,

$$\text{LOLP} \; = P\left[\sum_{g=1}^{N}(p_g + r_g) < p_d\right]$$

where p_g, r_g, p_d, $g = 1, \ldots, N$, are the output level of unit g, the spinning reserve contribution of unit g and the system demand respectively.

In practice, as considered in Bouffard and Galiana (2004), a set of binary indicator variables, $\sigma_j, j = 1, \ldots, N$ and $\sigma_{jk}, j = 1, \ldots, N, k = 1, \ldots, N$ are considered, satisfying the following inequalities

[1] In practice, the table is truncated by omitting all capacity outages for which the cumulative probability is less than a specified value (Billinton et al. (1984)).

$$\frac{p_d - \sum_{g=1,g\neq j}^{N}(p_g + r_g)}{\sum_{g=1}^{N} \bar{p}_g} \leq \sigma_j \leq 1 + \frac{p_d - \sum_{g=1,g\neq j}^{N}(p_g + r_g)}{\sum_{g=1}^{N} \bar{p}_g}$$

$$\frac{p_d - \sum_{g=1,g\neq j,k}^{N}(p_g + r_g)}{\sum_{g=1}^{N} \bar{p}_g} \leq \sigma_{j,k} \leq 1 + \frac{p_d - \sum_{g=1,g\neq j,k}^{N}(p_g + r_g)}{\sum_{g=1}^{N} \bar{p}_g}$$

These $\sigma_j, j = 1, \ldots, N$ and $\sigma_{jk}, j = 1, \ldots, N, k = 1, \ldots, N$ model the presence or absence of loss of load caused by single unit outage and double unit outages respectively [2]. In other words, σ_j (resp. $\sigma_{j,k}$) is equal to 1 if the unavailability of unit j (resp. units j and k) results loss of load, and equal to 0 otherwise. From this, the LOLP is expressed as

$$LOLP = \sum_{j=1}^{N} \sigma_j \pi_j + \sum_{j=1}^{N} \sum_{k>j}^{N} \sigma_{j,k} \pi_{j,k}$$

where π_j and $\pi_{j,k}$ are respectively the probability of the random event that the unit j and units j, k on outage.

C.2.2 Expected load not served ELNS

The ELNS is the average load shed under loss of load expressed by

$$\begin{aligned} ELNS &= \sum_{j=1}^{N} \sigma_j \pi_j \left(p_d - \sum_{g=1,g\neq j}^{N} (p_g + r_g) \right) + \sum_{j=1}^{N} \sum_{k>j}^{N} \sigma_{j,k} \pi_{j,k} \left(p_d - \sum_{g=1,g\neq j,k}^{N} (p_g + r_g) \right) \\ &= \sum_{j=1}^{N} \sigma_j \pi_j (p_j + r_j - R) + \sum_{j=1}^{N} \sum_{k>j}^{N} \sigma_{j,k} \pi_{j,k} (p_j + r_j + p_k + r_k - R) \end{aligned}$$

where R is the schedule amount of reserve, π_j and $\pi_{j,k}$ are respectively the probability of the random event that the unit j and units j, k on outage.

[2]Only single and double unit outages are considered most of the time as it is unlikely to have more than 2 units in outage in reality.

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