

SLURRY FLOWMETERING
USING CORRELATION TECHNIQUES

by

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DECLARATION

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ABSTRACT

A review of slurry flowmeters leads to the conclusion that most are unsuited for measuring the flowrate of abrasive and corrosive slurries such as are found in the gold mining industry in South Africa. An alternative flowmeter based on cross-correlation techniques is thus proposed. Random signals generated by the flowing fluid are measured at two points a known distance apart in the direction of the flow. Assuming that this distance is not 'too' large, the downstream signal is a time-delayed replica of the upstream signal. This delay equals the time it takes flow discontinuities to travel between the two measuring locations and can be obtained by cross-correlating the two flow signals and finding the position of the correlation peak.

An analysis to establish a theoretical basis for the method is performed and an attempt is made to model the flowmeter consisting of the two transducers a known distance apart.

A review of transducers for converting time-varying spatial flow patterns into electrical signals is undertaken and leads to ultrasonic transducers being chosen as most suitable for slurry flowrate measurements. Such transducers are non-intrusive and can be of a clamp-on type. The characteristics and requirements of ultrasonic transducers for correlation flowmetering are investigated and a suitable clamp-on design is described.

Since correlation is a time-consuming operation, various methods of processing the flow signals before correlating them are analyzed. From a review of existing correlators, it is concluded that no dedicated correlator suitable for flowmetering exists. Two new correlators that have been designed and constructed are described. Both use a two point difference correlator to track a correlation peak accurately and provide an output linearly related to flow velocity. The manner in which the two correlators find the approximate position of the correlation peak is, however, different. The first correlator is micro-processor based, whereas the other sweeps the two point difference correlator over the delay range of interest until a correlation peak is found.

A model of the basic two-point difference correlator is presented in order to predict and optimize the response of this correlator.

Results indicate that both correlators locate and track a correlation peak and give an output that is linearly related to the flow velocity. Good accuracy and repeatability has been verified on a test rig. It is concluded that the micro-processor based instrument is more versatile and has a faster

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response than the serial correlator. Hardware costs of both correlators including transducers are low, of the order of a few hundred rand. It should thus be possible to manufacture and market accurate, low cost non-intrusive correlation flowmeters which are ideally suited for slurry flowrate measurements.

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CHAPTER 1

SLURRY FLOWMETERING - A Review

1. INTRODUCTION

A knowledge of the flowrate of fluids is one of the most important parameters in many industrial processes. Its value at any given moment indicates movement of system throughput making its measurement particularly critical to material balance. With the advent of computer control on many plants increased importance is being attached to the accurate determination of flowrate in order to maximize process efficiency.

The properties of fluids, the flowrates of which are to be measured, can differ widely depending on the process and they determine which flowmeters are acceptable for a particular fluid. The cleanliness, conductivity and viscosity of a fluid are the main factors affecting this choice⁽¹¹⁾. Having selected the flowmeters suitable for a specific fluid on the above basis, the next task is to identify precisely measurement requirements in terms of accuracy, cost, reliability, repeatability, installation, maintenance and required output. Each item on the list has various degrees of importance depending on the application and a suitable 'weighting' should be assigned to each factor.

Although many accurate flowmeters exist for most single phase fluids, the number of flowmeters suitable for two-phase fluids is limited. A particularly difficult problem in the flowmetering industry is measuring the flowrate of mining slurries since these slurries often are abrasive and corrosive. At a workshop attended by mining men in 1973, the following statement was made: 'Slurry flow measurement is considered to be one of the most inaccurate in the industry. A special programme should be launched to encourage development'⁽²⁾. Although progress has been made and accurate flowmeters are available for some slurry flowrate measurements^(4,11,16), situations still exist in which available flowmeters are unsuitable. An example of such a situation is measuring the flowrate of aerated mining slurries.

The aim of this chapter is to investigate current slurry flowmetering methods and to introduce an alternative technique in which flowrate is measured using cross-correlation.

2. DESCRIPTION OF A TYPICAL MINING SLURRY

Considerable confusion exists about the meaning of the term 'slurry'. A functional definition is offered by DALSTAD⁽³⁾: 'A slurry is any mixture of liquids and solids capable of causing significant abrasion, clogging or mechanical failure due to high loads or impact shocks'. 'Slurries can either be 'deliberate', such as mine tailings or chemical concentrates, or 'accidental', in which the transport of the liquid is the prime purpose - the presence of solid materials is not intended (or sometimes even recognized). An example of such an 'accidental' slurry is the supply of large quantities of water from a sandy river for cooling or irrigation.

A particularly harsh slurry is that found in the gold mining industry in South Africa. The reasons for this become clear on examining the properties of such a slurry.

In general, gold occurs in the form of minute particles embedded in quartzite. The amount of gold per ton of rock mined is approximately 4 - 30 gms/ton depending upon the location of the mine⁽⁷⁰⁾. To extract this gold with a good percentage recovery (>99,8 per cent) the quartzite is firstly ground into an extremely fine powder (particle size = 80 percent; <75 μm) before undergoing a cyanidation process to extract the gold. This sometimes is preceded by uranium leaching and pyrites flotation. In all cases the mineral-bearing quartzite is transported and processed as a slurry using water as the liquid medium.

The hardness of quartzite is approximately 7 on the Mohs scale of hardness (TALC=0; DIAMOND=10) and its specific gravity about 2,65 (Ref.182). As a result of this hardness in addition to the sharp edges of the particles, an extremely abrasive slurry results necessitating all pipe-lines, pumps, etc., to be rubber-lined for protection. Any intrusion into a pipe will rapidly be worn away and also cause solids build-up. Furthermore, pipe geometry must be carefully planned to prevent excessive wear at sharp bends and solids accumulation in dead-spots. VOCADLO and SAGOO⁽¹⁾ say the following about the selection of slurry pumps: 'The abrasive nature of slurries is a major consideration in the design and selection of slurry pumps. The life of some pump parts is sometimes measured in weeks! The same could be said about flowmeters unless special precautions are taken.

Another problem is caused by the large difference in density between water and quartzite. Unless the slurry is constantly agitated to keep the quartzite powder in suspension, settling rapidly occurs. By ensuring that the flow of

the slurry is in the turbulent region under all conditions settling can be avoided. This condition sets a lower limit on the flow velocity of slurries. Two opposing factors thus manifest themselves, namely, abrasion can be minimized by reducing the flow velocity, but this results in settling. The optimum transport velocity is hence the lowest velocity at which settling does not occur (ref. 74,126).

In certain processes, for example, cyanidation, air is pumped through the slurry to ensure proper mixing between the slurry and the reagents, and to keep the slurry in suspension. Some of this air will remain entrained and result in an aerated slurry during the subsequent flow.

From the above description it becomes clear that mining slurries are difficult fluids to transport, requiring special precautions. Measuring the flowrate of such slurries furthermore presents unique problems which, as will be shown subsequently, exclude most conventional flowmeters.

3. FLOW PARAMETERS THAT CAN BE MEASURED

Various parameters of a flowing fluid can be measured. When specifying a flowmeter it is necessary to state clearly what parameter is required for a particular application.

Firstly, the flow velocity of a fluid in a pipe is of interest. Since a velocity profile exists along any diameter of the pipe, this quantity requires further clarification. The velocity profile is caused, amongst other factors, by the viscous drag existing at the pipe-walls, and results in the flow velocity being a minimum (zero) at the pipe-walls and a maximum at the centre of the pipe. The shape of the velocity profile depends on the Reynolds number, Re , of the flow. By definition, the Reynolds number is a dimensionless quantity given by

$$Re = \frac{\rho v D}{\eta}$$

where ρ = fluid density
 v = fluid velocity (mean)
 D = pipe diameter
 η = fluid viscosity

This number characterizes the flow by giving information about its state - whether it is laminar or turbulent, and the degree of turbulence. For pure laminar flow ($Re < 2000$) the velocity profile is parabolic; as the Reynolds number increases, so the flow becomes more turbulent and the velocity profile flattens out ⁽¹⁸³⁾.

Usually the mean flow velocity needs to be measured, but sometimes point velocities must be known. The mean flow velocity can be obtained by dividing the volumetric flowrate by the cross-sectional area of the pipe or by averaging the velocity profile, giving equal weight to equal annular areas. Point velocities, as the name implies, represent the velocity of the fluid in a localized region and are useful for obtaining flow patterns. Furthermore, the mean flow velocity can be derived from a number of point velocities taken along various diameters of the pipe using integration techniques ⁽⁵⁾.

The most important parameter in many flow applications is the volume flow rate, that is, the total volume of fluid flowing through the pipe per unit of time. This can easily be obtained from the product of mean flow velocity and pipe cross-sectional area. Volume flow rate is usually measured by the suitable calibration of a velocity measuring flowmeter.

Another important quantity is the mass flow rate, that is, the total mass of fluid flowing through the pipe per unit of time. The product of fluid density and volume flowrate yields the mass flow rate.

In slurry flowmetering, the mass flow rate of the solid phase often is of interest since the liquid phase is merely used as a carrier of the solid phase. What is required is the total mass of dry solids flowing through the fluid per unit of time. This is one of the most difficult parameters to measure. Knowing the mean flow velocity and percentage solids loading, however, determines this factor.

Inclusion of various quantities of the gaseous phase in a fluid presents even greater difficulties since not only does this phase occupy some of the volume within the pipe, but this volume also depends on pressure due to the compressibility of the gaseous phase. Attempts to reduce the gaseous phase to a minimum are usually made prior to measuring the flow rate of such a fluid, since normally only the volume flow rate of the liquid phase is of interest.

In this project emphasis will be on measuring mean flow velocity, since both the volume and mass flow rates of single and two-phase fluids depend on this quantity.

4. THE 'IDEAL' SLURRY FLOWMETER

Based on the description of a typical mining slurry, it becomes possible to define some of the requirements of an 'ideal' slurry flowmeter. Such a flowmeter should be:

(a) Non-intrusive

This is an essential feature since non-intrusion into the flow eliminates abrasion and solids build-up problems. Furthermore, no pressure drop is created by the flowmeter.

(b) Clamp-on

By arranging the flowmeter to be a clamp-on device it will be more readily acceptable in existing plants and industries since pipe integrity will not be affected thus eliminating potential leak sources. The flowmeter need also not be dedicated to a single application, but can easily be relocated on another pipe. Problems caused by corrosion and scale build-up will furthermore be avoided.

(c) Robust

Since the environments in which flowmeters must operate are often harsh, the 'ideal' flowmeter should be robust so as to keep maintenance to a minimum. Such a feature will again make the flowmeter attractive to many industries since maintenance is costly and time-consuming.

(d) Reliable

Down-time on any plant is expensive, hence the 'ideal' flowmeter must be reliable and must be able to operate under all conditions normally encountered.

(e) Accurate

Depending on the particular application, the 'ideal' flowmeter must satisfy the specified accuracy requirements. In order to compete with existing slurry flowmeters, accuracies of the order of ± 2 per cent of reading are required.

(f) Cheap

The cheaper the flowmeter is the better, provided all the other conditions are satisfied.

5. REVIEW OF SLURRY FLOWMETERS

One of the main requirements of a mining slurry flowmeter is that it should be non-intrusive. This condition immediately excludes a large number of flowmeters, some of which are listed below (see references 9-14, 16, 62-68, 71):

- (a) orifice plate
- (b) positive displacement
- (c) variable area or rotameter
- (d) turbine
- (e) vortex shedding
- (f) insertion meters (e.g., pitot tubes)
- (g) venturi, and
- (h) fluidic oscillators.

The only flowmeters that can be considered are those that are totally external to the pipe or those that are mounted flush with the inside of the pipe wall. A description of some flowmeters satisfying these requirements and an evaluation in terms of the 'ideal' requirements listed previously, follows below. Before proceeding, it should, however, be noted that under certain favourable conditions such as the transport of light non-abrasive slurries, some of the abovementioned flowmeters can be used.

5.1 Electro-magnetic Flowmeter

The electro-magnetic flowmeter^(2,10,17-19) is at present the most commonly used meter for measuring the flow rate of mining slurries. It operates on the law of electro-magnetic induction which states that the emf induced in a conductor moving through a uniform magnetic field is proportional to the velocity of the conductor.

An electro-magnetic flowmeter usually consists of a non-magnetic pipe, lined with an insulating material. A magnetic field is produced across the pipe by exciting coils arranged around the outside. Liquid passing through the flowmeter becomes the conductor; the emf induced by the fluid moving through the magnetic field is proportional to the fluid velocity. Two small electrodes mounted opposite each other and flush with the lining detect this emf.

The main advantages of this flowmeter are that it is obstructionless, it has no moving parts, and it produces a pressure loss that equals that of an equivalent length of pipe. It is, furthermore, unaffected by changes in temperature, viscosity, pressure, density and conductivity. It has a typical accuracy of ± 1 per cent of actual flow, or $\pm 0,2$ per cent full scale, whichever is the greater⁽¹¹⁾. Its output is linear with flow rate and it can measure bi-directional flow.

The main limitation is that the fluid must be conductive. A minimum conductivity of about 5-20 μ Siemens/cm is claimed by most manufacturers for standard instruments although special meter construction enables fluids with conductivities as low as 0,1 μ Siemens/cm to be measured⁽¹⁷⁾. As mentioned previously, conductivity variations do not affect measurement accuracy provided the conductivities remains above the minimum for the meter.

The calibration of most electro-magnetic flowmeters is sensitive to velocity profile changes but meters have been constructed to measure the average flow rate regardless of the velocity profile⁽⁶⁹⁾.

Inaccuracies can further be caused by the inclusion of magnetic materials in the fluid since such materials will distort the magnetic field produced by the field coils. Experiments have shown that the calibration shift is approximately proportional to 'mass % magnetic material' concentration⁽²⁾. If 10 per cent magnetic material by mass is present, the meter reads 10 per cent too high. Special magnetic flowmeters which measure the quantity of magnetic material present in the fluid and correct the calibration of the flowmeter are available⁽²⁾. Since many slurries, typically gold mining slurries, are non-magnetic, standard electro-magnetic flowmeters can be used.

Another factor causing errors is fouling of the electrodes. A variety of devices for keeping the electrodes clean are offered by the manufacturers of magnetic flowmeters. These devices have various degrees of effectiveness and include ultrasonic cleaners, protruding electrodes, mechanical scrapers, pronged electrodes and 'hot' electrodes.

Further disadvantages of electro-magnetic flowmeters are that they are expensive especially in the smaller sizes, require special installation precautions and careful electrical grounding. The inclusion of air in the fluid also degrades their performance by decreasing the S/N ratio.

5.2 Nuclear Magnetic Resonance Flowmeter

A relatively new flowmeter based on the principles of nuclear induction formulated by BLOCH⁽²⁰⁾ has recently appeared on the market^(2,14,23-24).

The so-called 'badger meter' has a vitrified ceramic tube, suitably lined which is surrounded by permanent magnets and detector coils. Nuclei of one of the constituent elements of the fluid are magnetized. A radio frequency field causes magnetic vectors to precess at a high rate. A pulsed modulating signal disturbs the magnetization of part of the fluid. The downstream receiver coil detects the passage of the disturbed magnetization. The time difference between modulation and detection of each pulse is registered and computed as flow velocity. The meter works well on all fluids containing water since the nuclear magnetic resonance effect is greatest in nuclei of hydrogen.

This meter appears well suited for mineral slurry flow measurements. Unfortunately little is known about its performance in such an application. This fact in addition to its cost and complexity exclude it at this stage as a serious contender for slurry flow rate measurement. However, with continued research and development it may become a viable alternative. It is interesting to note that considerable work is being done in measuring blood flow using n.m.r. techniques^(21,22,25).

5.3 Thermal Flowmeters

A variety of flowmeters based on the transfer of energy from a heated body to the flowing fluid have been reported^(8,14,15,26-29). A typical thermal flowmeter consists of a spool piece in the pipe, a small heater coil and two temperature sensing devices - one upstream of the coil and the other downstream. Two methods of operation are possible: a variable power and a variable temperature method. In the variable power technique the temperature difference is maintained constant across the coil and the power consumption is a measure of mass flow rate. With the variable temperature method power input is kept constant and the temperature difference is a measure of the flow rate.

Another version operates by temporarily heating a small area of a tube conveying fluid and then allowing it to cool⁽²⁶⁾. The rate of cooling is a function of the fluid mass flow rate and a simple measure of the cooling rate may be obtained from the time taken for the temperature to fall between two levels fixed with respect to the normal working temperature of the pipe.

Having no moving parts and no components in contact with the flow, thermal flowmeters are well suited to most slurry flows. Their main limitations are that they are rather complex and limited to streams having sufficient thermal conductivity. Since such flowmeters depend essentially on the heat transfer coefficient at the inner surface of the tube, this surface must be kept clean and free from corrosion and deposits. Factors enhancing the normal level of turbulence at this inner surface such as bends and junctions can have an unpredictable effect on the calibration. Furthermore, the rubber linings of pipes conveying abrasive slurries will also influence the readings. Overall accuracy of thermal flowmeters is of the order of ± 3 per cent if calibrated on site, and ± 15 per cent if not.

5.4 Gyroscopic Flowmeter

A gyroscopic flowmeter^(8,34,35) is a mass-flowrate measuring device which operates by forcing fluid through a 360° loop that is perpendicular to the axis of the main flow. An oscillating vibration at the input produces a torque that is proportional to the mass flow rate. This meter can handle some slurries in the medium pressure and temperature ranges. It produces a pressure drop of the order of magnitude expected for a 360° loop. Its main disadvantage is that it is limited to low mass flow rates and is hence unsuitable for typical mining slurries.

5.5 Radioactive Tracer Flowmeter

There are several different methods of measuring flowrate using radioactive tracers⁽³⁰⁻³²⁾. In the first method a small amount of radiotracer is rapidly injected into the stream. Subsequently the time of transit between two detectors located at a known distance apart on a straight uniform section of pipe is measured. The volume flow rate is then computed by multiplying the linear velocity by the inside cross-sectional area of the pipe.

Alternatively, it is possible to use an isotope dilution technique in which a tracer of known concentration is injected into the flow at a constant rate. The concentration downstream from the point of injection is measured and related to flow rate. Various other techniques are described in references 30 to 32.

Radiotracer methods are not generally suitable for continuous flowrate measurements but they provide an accurate and precise means of making an instantaneous measurement for such purposes as checking the calibration of other flowmeters.

5.6 Ultra-sonic Flowmeters

A number of flowmeters which depend on the interaction between sound waves and the flowing fluid are available. Three basic methods underlie nearly all of the many specific techniques that have been used in ultrasonic flowmetering:

- (a) measuring the deflection of a beam
- (b) measuring the time for an ultrasonic wave to get from one point to another in the flowing fluid, or the change in phase, and
- (c) determining the Doppler shift in the frequency of the sound.

A brief description of these principles of operation follows:

- (i) Beam deflection method (refs. 38-40)

If a transducer transmits a beam perpendicular to the pipe axis through the fluid, the beam is deflected downstream at an angle θ given by (see Fig. 1.1)

$$\tan \theta = \frac{v}{c}$$

where v = velocity of fluid

c = velocity of sound in the fluid.

For a pipe of diameter D in which the beam is deflected a distance y ,

$$\tan \theta = \frac{y}{D}, \text{ and hence}$$

$$y = \frac{D}{c} \cdot v$$

Although the distance the beam is deflected is proportional to the flow velocity, assuming that c is constant, a linear output is obtained only if the deflection can be measured linearly. The usual technique of taking the voltage difference between two initially balanced receivers is inherently non-linear.

The method is attractive since the ultrasonic beam is transmitted perpendicular to the pipe axis. This allows good coupling and eliminates errors due to refraction at interfaces. The method is, however, only suitable for use with higher flow rates when a significant deflection of the beam can be achieved. A further disadvantage is that it is very susceptible to changes in sonic velocity in the fluid caused by changes in temperature, pressure or composition and, therefore, requires compensation.

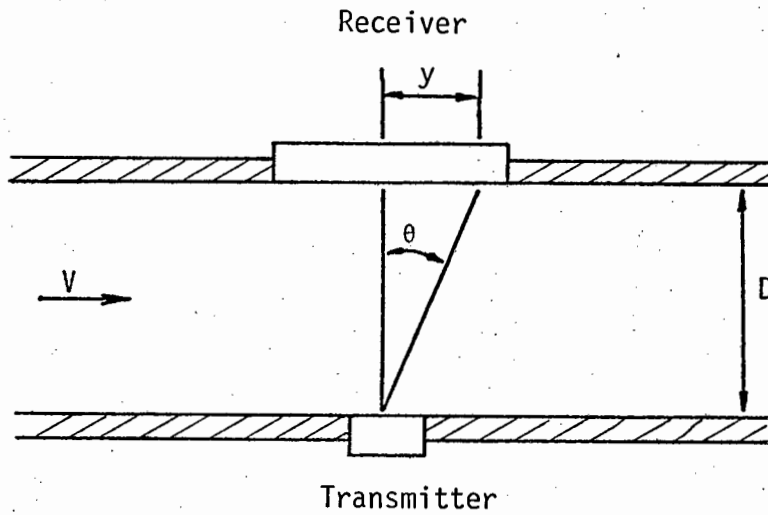


FIGURE 1.1 : Beam Deflection Method

(ii) Travel Time Difference Method (refs. 37-41)

The travel time difference method or 'leading edge' method, as it is also called, measures the difference in time it takes an ultrasonic pulse to travel upstream and downstream between two fixed points a distance L apart (see Figure 1.2). If t_u and t_d are the upstream and downstream travel times respectively, then

$$t_u = \frac{L}{c - v \cos \theta}$$

$$t_d = \frac{L}{c + v \cos \theta}$$

and their difference Δt is given by

$$\Delta t = t_u - t_d = \frac{2 v L \cos \theta}{c^2 - v^2 \cos^2 \theta} \approx \frac{2 v L \cos \theta}{c^2}$$

since $v \ll c$ for most liquid flow rates.

An output signal proportional to Δt is thus linearly related to the flow velocity provided c is constant. Since in general the sound velocity c is not constant but depends on temperature, composition and density, this technique requires compensation to eliminate this source of error. Another limitation is that in small pipes or at low flow rates, the time difference becomes very difficult to measure.

The minimum flow rate of such a meter is typically determined by a Δt of 100 n secs. The maximum flow rate is, however, well in excess of any liquid flow in a pipe.

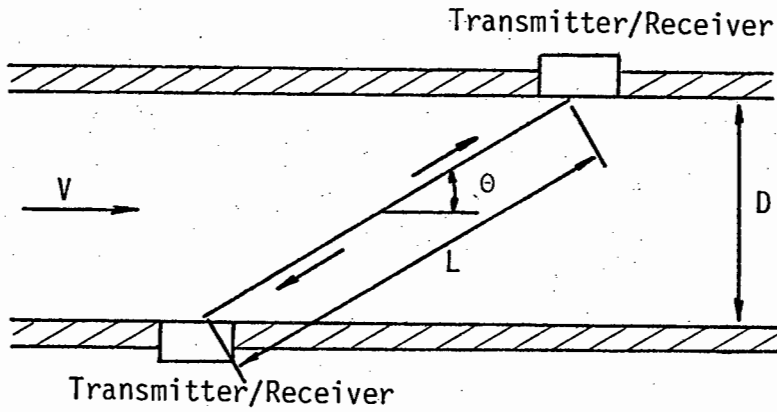


FIGURE 1.2 : Time, Phase or Frequency Difference Method

(iii) Phase Difference Method (refs. 38,39,61)

If continuous sinusoidal signals of frequency f are transmitted instead of pulses, as in the previous case, then the phase difference between the received upstream and downstream signals is given by:

$$\Delta\phi = 2\pi f \cdot \Delta t \approx \frac{4\pi f L \cos\theta v}{c^2}$$

This equation shows that the phase difference is proportional to flow velocity and can be made as large as possible by increasing the frequency. However, at high frequencies the attenuation of sound (in many fluids) is large and phase measurement becomes more difficult. The flow range of such a meter is also limited since phase can only be measured unambiguously over the range of 0° to 360° . Changes in the velocity of sound again affect the accuracy unless compensation is included.

(iv) Frequency Difference Method (refs. 37-41, 43, 52, 57)

If a received pulse in the time difference method (see Section 5.6(ii)) is used to trigger another transmit pulse, a train of pulses will be generated whose period equals the acoustic travel time between trans-

mitter and receiver. Such a system is commonly known as a 'sing-around' system. The frequencies of such pulse trains propagating upstream and downstream and their difference are given by:

$$f_u = \frac{1}{t_u} = \frac{c + v \cos \theta}{L}$$

$$f_d = \frac{1}{t_d} = \frac{c - v \cos \theta}{L}$$

$$\Delta f = f_u - f_d = \frac{2 v \cos \theta}{L}$$

The attraction of this method is that the difference frequency is linearly related to the flow velocity independent of the velocity of sound in the fluid. This advantage, in addition to the fact that the difference frequency is easily measured, has resulted in frequency difference methods being widely used. One drawback is that in practical cases the frequency difference is small and the long counting interval needed for a given resolution results in slow response time. The long counting interval, however, does provide averaging of fluctuations in t_u and t_d .

(v) Doppler Method (refs. 38, 40, 44-50, 53, 55, 59)

When an ultrasonic beam is projected into an inhomogeneous fluid, some acoustic energy is scattered back towards the transmitter or towards a second receiver (Figure 1.3). The received signal will differ in frequency from the transmitted signal because of the Doppler effect. This frequency difference depends on the velocity of the scatterers, and is given by:

$$\Delta f = f_t - f_r = \frac{2 f_t v \cos \theta}{c}$$

where f_t and f_r are the transmitted and received frequencies respectively and θ the angle between the transmitted or received beam and the flow axis.

The limitations of this technique include the need of compensation for changes in sonic velocity and the presence of scatterers. The latter requirement, however, makes the flowrate of slurries and sewage obvious applications.

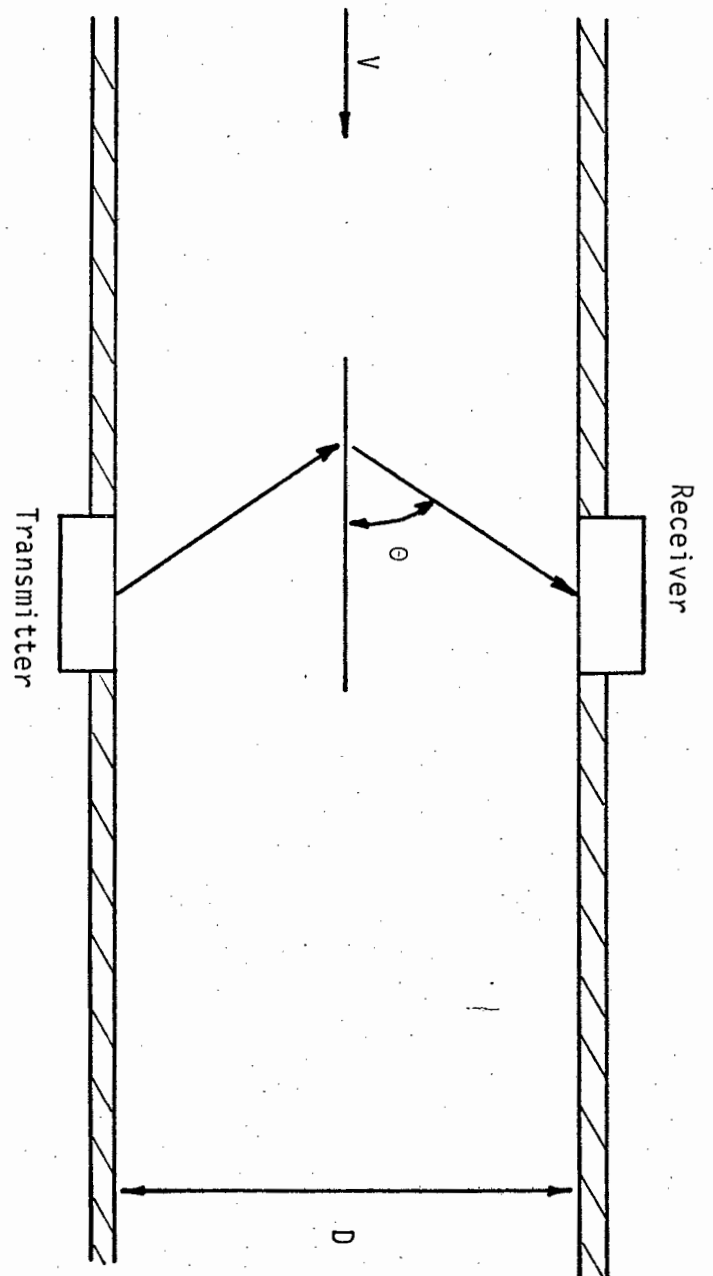


FIGURE 1.3 : Doppler Method

(vi) Electro-acoustic Feedback Method (refs. 42,54,56)

This method is based on the following principle: an ultrasonic transmitter and receiver are connected respectively to the input and output of an amplifier. The oscillation caused by the electro-acoustic feedback has a frequency which depends on the velocity of the fluid between the two transducers. If two such systems are used, in one of them the acoustic wave travelling upstream and in the other the wave travelling downstream then the frequencies of oscillation are:

$$f_u = \frac{n(c - v \cos\theta)}{l}$$

$$f_d = \frac{n(c + v \cos\theta)}{l} \quad \text{where } n = 1, 2, 3, \dots$$

Clearly various modes of oscillation are possible and one frequency must be selected using a frequency sensitive network. The difference frequency is given by:

$$\Delta f = f_d - f_u = \frac{2 v n \cos\theta}{l}$$

It is linearly related to the flow velocity independent of the sound velocity. The main limitation is that great attention must be paid when designing the filter that selects the mode of oscillation in order to obtain the flattest response in the range of frequency used. This is necessary because the linearity of the device depends on the phase distortion of the filter.

Ultrasonic flowmeters can be of the 'wetted' type in which the transducers are in contact with the fluid, or of the 'clamp-on' type, in which the transducers are bonded or clamped directly to the outside of the pipe. This latter technique is, as has been stated, particularly attractive for slurry flowrate measurements.

Unfortunately, however, for most of the measuring techniques described previously, the transducers must be inclined at an angle to the pipe wall and a serious loss of accuracy occurs due to refraction of the signal at the pipe and again at the fluid. Thus the angle at which the signal crosses the pipe axis becomes a variable depending on the temperature, the nature of the pipe material and the fluid.

The flowmeter to be described in this project also uses clamp-on ultrasonic transducers. However, they are directed at right angles to the direction of flow and hence do not suffer from refraction problems.

6. CROSS-CORRELATION FLOWMETERING

From the preceding review it is apparent that all the flowmeters described suffer from some defect which prevents them from being ideal for slurry flow measurement. At present the electro-magnetic flowmeter is the standard in most industrial applications. However, its cost (typically R7000,00 for a 6" pipe) is high. Nuclear magnetic resonance flowmeters show promise, but are not viable propositions at this stage. Thermal and gyroscopic mass flowmeters both have limited application and radioactive tracer techniques are unsuited for continuous use. Ultrasonic flowmeters satisfy most of the requirements for slurry flowrate measurements. However, their dependence on temperature and fluid composition, and the need to direct them at an angle to the flow, are their main limitations. Nevertheless, in many applications ultrasonic flowmeters are being used with good results. An alternative technique based on cross-correlation which overcomes most of the problems associated with conventional slurry flowmeters, is introduced below.

The basic principle of the cross-correlation method for measuring flowrates is best described with reference to Fig. 1.4. A pair of transducers is spaced a distance, d , apart as shown. The transducers produce electrical signals proportional to the instantaneous variations of some fluid property e.g., conductivity, density or acoustic absorption. These random variations arise, inter alia, due to turbulence effects within the flowing fluid.

Now, provided the distance, d , between the transducers is 'small', the signal at the downstream transducer will be similar to the signal at the upstream transducer, as shown in Fig. 1.5. However, a definite time delay, β , will exist between the signals. This delay is equal to the time it takes variations measured at the upstream transducer to reach the downstream transducer. Assuming that these variations travel at the mean flow velocity of the fluid or some known fraction of it, a knowledge of the transducer spacing, d , and time delay, β , uniquely determines the flow velocity, v , viz.,

$$\text{velocity } v = d/\beta$$

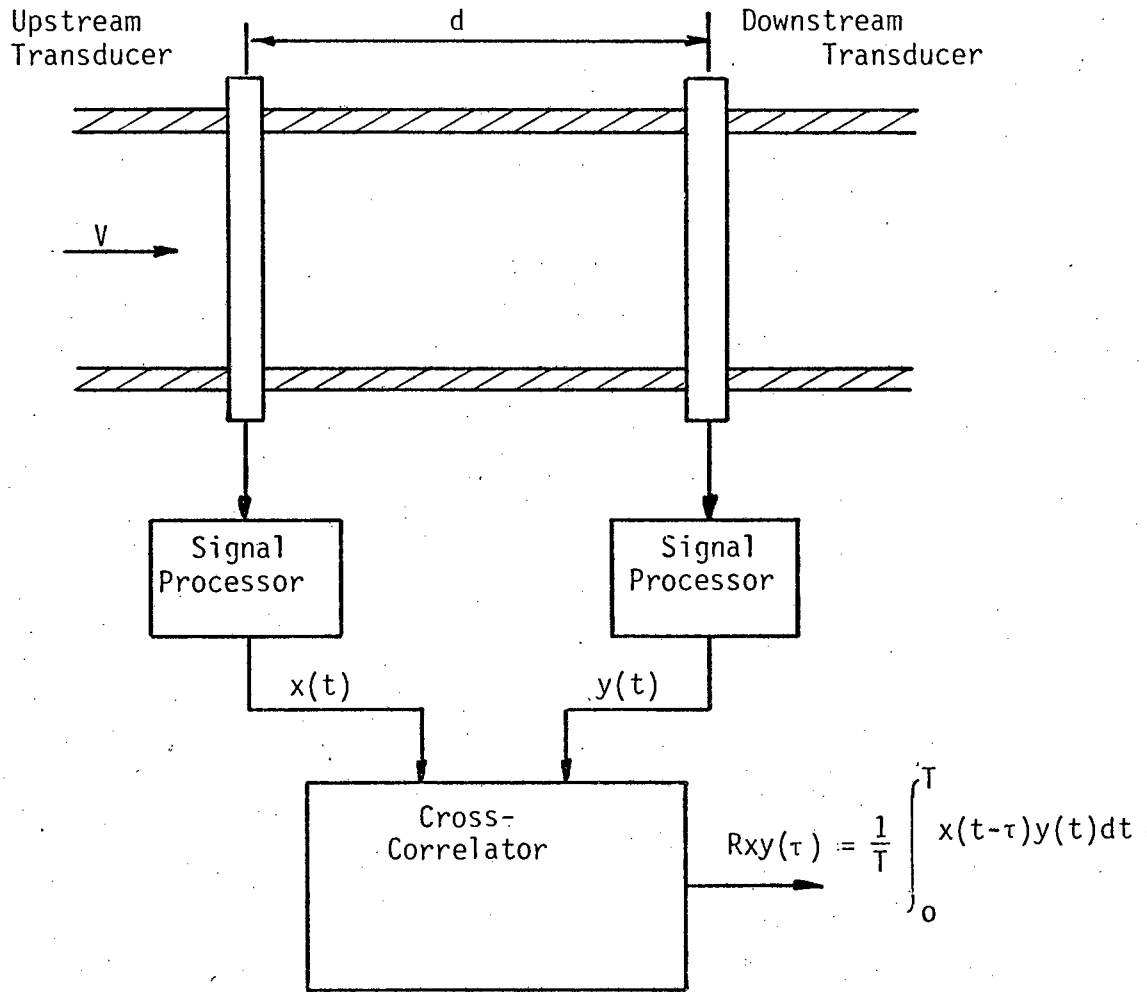


FIGURE 1.4 : Principle of Correlation Flowmetering

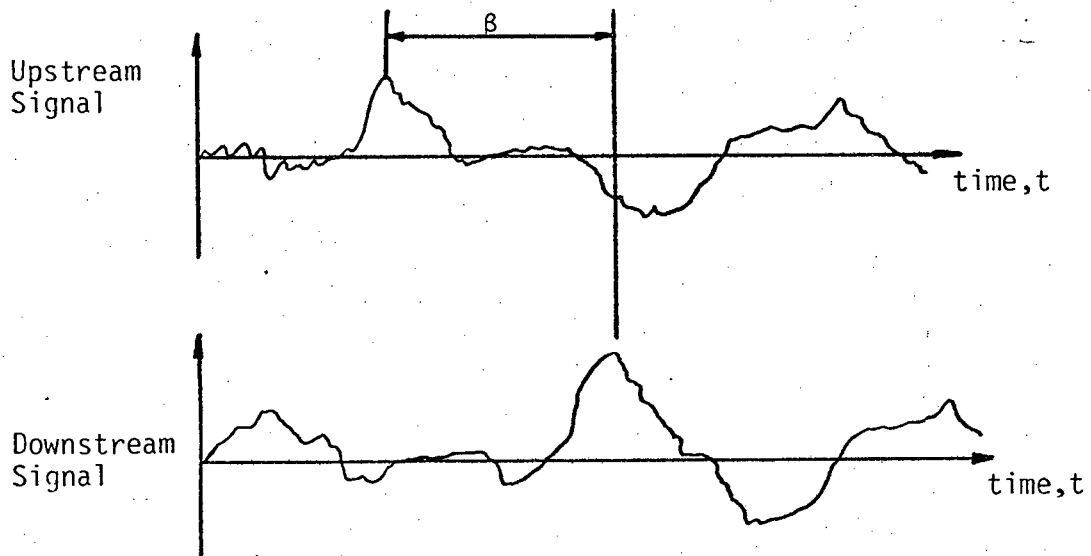


FIGURE 1.5 : Typical Upstream and Downstream Flow Signals

The time delay, β , cannot be measured directly using standard techniques employed in tracer methods since in this case the tracer is in fact a random variation produced by the flowing fluid itself. It can, however, be determined by cross-correlating the two transducer signals.

A block diagram of a correlator can be seen in Figure 1.6 and a typical correlation curve is plotted in Figure 1.7. The time delay, β , of the random variations is obtained by multiplying the output, $y(t)$, of the downstream transducer with a time delayed version of the output of the upstream transducer, $x(t-\tau)$, as shown in Figure 1.6 where τ represents an adjustable time delay inserted in the output of the upstream transducer. The product $x(t-\tau).y(t)$ is then averaged to give its mean value. Repeating this calculation for various values of delay, τ , yields the cross-correlation function $R_{xy}(\tau)$. This function is mathematically given by:

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t-\tau).y(t)dt$$

When the fluid transit time, β , and the cross-correlation time delay, τ , are unequal, the mean value of the above product is small. As the correlation time delay approaches the fluid transit time, the correlation function increases. When these time delays are equal, i.e., $\tau = \beta$, then the signals $x(t-\tau)$ and $y(t)$ will have the closest possible similarity and the mean value of their product will reach a maximum. Hence, the time delay of the maximum value of the cross-correlation function, illustrated in Figure 1.7 uniquely defines the transit time of the fluid discontinuities between the two transducers.

7. DOES CORRELATION FLOWMETER SATISFY 'IDEAL' REQUIREMENTS?

The accuracy of a correlation flowmeter depends primarily on the precise determination of the travel time of fluid discontinuities between two fixed points. A definite advantage is that both these parameters are independent of fluid properties such as temperature, composition, density and pressure.

The transducers used need cause little or no obstruction to the flow so that application to flowmetering problems with abrasive, toxic and corrosive fluids is possible. Using ultrasonic transducers can lead to a clamp-on flowmeter with all the advantages mentioned previously. The transducers need furthermore not be calibrated since their absolute characteristics do not affect the accuracy of the flowmeter. Their only purpose is to convert fluctuating spatial flow patterns into randomly varying electrical signals suitable for

correlation at two fixed locations in the direction of flow. The presence of discontinuities such as solid particles or gas bubbles does not degrade the flowmeters performance - in fact their presence improves the correlation between the two transducer signals by acting as distinct variations in some fluid property. Due to their simplicity the transducers can be made robust, simple to install and should require little routine maintenance.

Summarizing it appears that a correlation flowmeter, with clamp-on transducers does satisfy all the requirements of an ideal slurry flowmeter. Whether it is the solution to many mining slurry flowmetering problems remains to be seen.

8. AIM OF THE THESIS

Since slurry flowmetering presents unique problems especially in the mining industry in South Africa, the need for an improved flowmeter became apparent. A flowmeter based on cross-correlation techniques appeared to show most potential. It thus was decided to embark on a project to investigate the characteristics of such a flowmeter and produce a viable and accurate industrial device.

The aims of this thesis can be summarized as follows:

1. To review the state of the art of correlation flowmetering. This includes an investigation of:
 - a) Correlation flowmeter models
 - b) Flow sensors
 - c) Correlation techniques and
 - d) Hardware correlators.
2. To examine each of the above aspects and to improve, modify or propose new methods.
3. To analyse a correlation flowmeter so as to predict its performance and understand its characteristics.
4. To specify the requirements of a correlation flowmeter for slurry flows.
5. To develop a flowmeter suitable for industrial application. This involves the design of both sensors for generating flow signals and a correlator for determining the time-delay between upstream and downstream flow signals.
6. To evaluate the developed flowmeter and determine if it satisfies the specified criteria.
7. To decide whether it is feasible to produce an industrial instrument from the developed flowmeter.

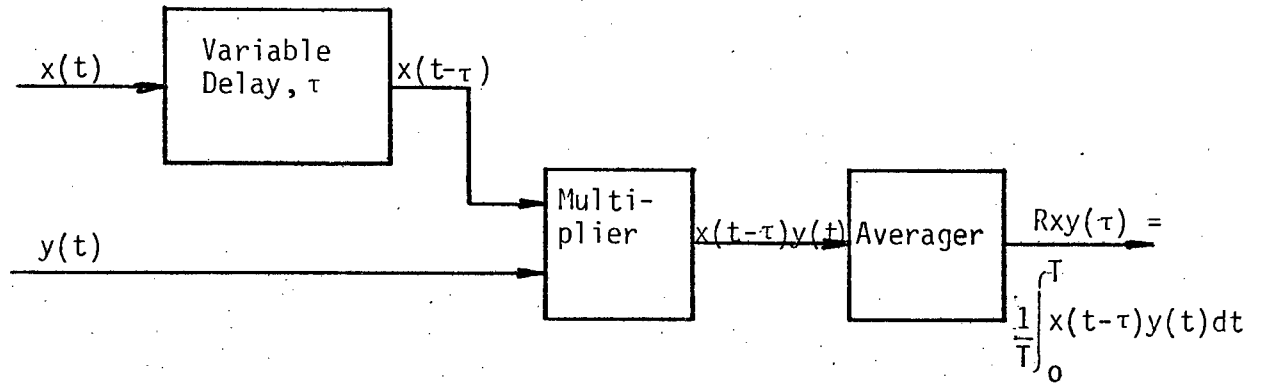


FIGURE 1.6 : Block Diagram of a Cross-Correlator

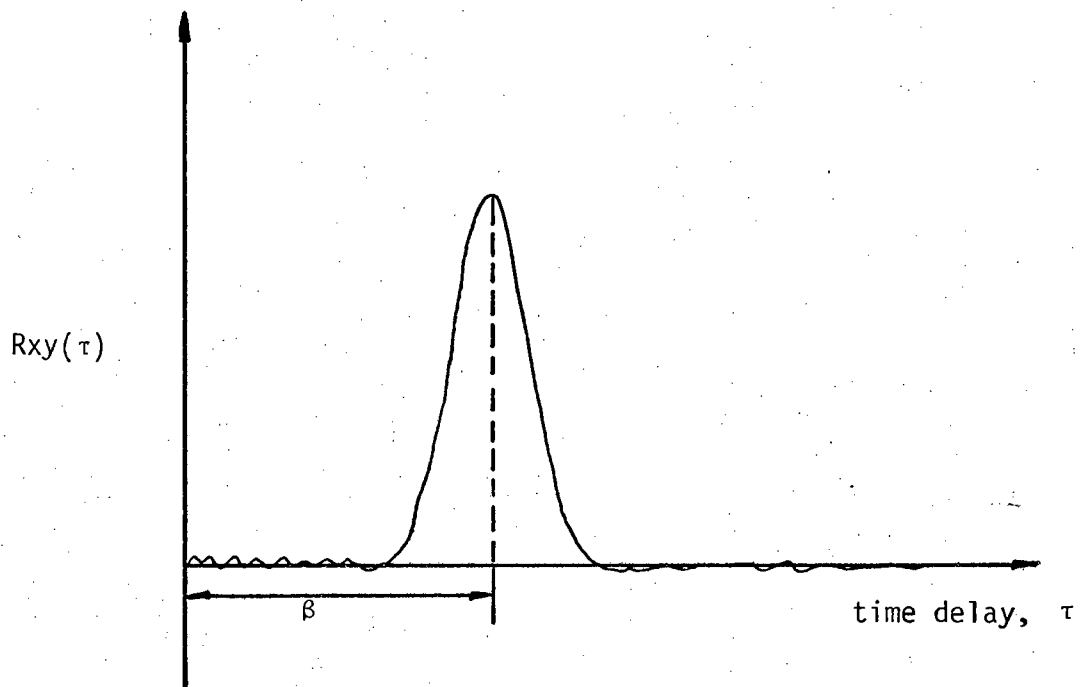


FIGURE 1.7 : Typical Cross-Correlation Plot

9. ORGANISATION OF THE THESIS

Chapter 2 commences with a review of the work done on correlation flowmetering during the past 20 years. The fundamentals of correlation are then summarized before proceeding to an analysis of correlation flowmeters. Some interesting features of these flowmeters become evident.

Sensors for generating flow signals suitable for correlation are examined in chapter 3. After specifying the requirements of sensors for slurry flowmetering, it becomes apparent that ultra-sonic sensors are most suitable.

Chapter 4 is devoted to the analysis and design of ultra-sonic flow sensors. Firstly, a simple model for an ultra-sonic correlation flowmeter is discussed before a brief investigation of the mechanism producing flow signals is undertaken. The characteristics of ultra-sonic transducers are described hereafter, this leading to the design of clamp-on transducers. The signal processing circuitry required to extract flow signals from the ultra-sonic sensors is also discussed.

Correlation techniques for simplifying the process of determining correlation functions are examined in chapter 5. The effect of amplitude quantization of the flow signals is reviewed and a new delta-modulation correlation technique is introduced. It eventually is decided to use a Polar correlation method in the design of a correlator.

A review of hardware correlators is included in chapter 6. From this review it becomes apparent that a definite lack of a dedicated cheap peak-seeking correlator exists. It is concluded that this is one of the main reasons why correlation flowmetering is not widely used in industry.

Chapter 7 describes two correlators that have been developed to possibly fill this gap. The first is a novel micro-processor based instrument which satisfies most of the requirements for slurry flowmetering. The other, is a cheaper discrete logic device with reduced potential. Both devices use a two-point difference correlator to provide the necessary accuracy and resolution. The difference between them is the manner in which a correlation peak is located.

In order to optimize the performance of a two-point difference correlator, a model of the correlator is proposed in chapter 8. Comparison with experimental results, confirms the validity of the model.

Chapter 9 is divided into two sections. In Section A, the characteristics of typical flow signals are examined in the frequency, amplitude and time domains. The effect of changing certain flow parameters on the correlation function is also investigated. Section B contains an evaluation of the developed flowmeters.

Possible sources of error are discussed in chapter 10. In particular, the calibration factor of a correlation flowmeter is derived. It is concluded, that by careful mounting of the flow sensors, many potential problems can be eliminated.

Finally, chapter 11 summarizes the results of the thesis, discusses them in relation to the original aims and suggests directions for further investigations.

CHAPTER 2

BACKGROUND

1. INTRODUCTION

The previous chapter introduced the concept of velocity measurement using correlation techniques and indicated that it may be the solution to many slurry flowmetering problems. This chapter firstly reviews the work that has been done in the field over the past 20 years. It becomes evident that although many papers have been published on the subject, no commercial instrument of general applicability has been produced. Hereafter the mathematical fundamentals of correlation are discussed briefly. In order to understand the characteristics of correlation flowmetering, various models are derived and some unusual features become evident.

2. HISTORIC BACKGROUND

The principle of velocity measurement by cross-correlating random disturbances generated at two points is not new. For a long time it has been one of the most useful techniques in the study of turbulent flows. One may cite the experiments of TOWNSEND⁽²¹¹⁾ (1947) on isotropic turbulence; the investigations of the turbulent boundary layer due to FAVRE, GAVIGLIO and DUMAS^(210,196) (1957/58) and BULL⁽²⁰⁰⁾ (1966); and the investigations of jet turbulence due to DAVIES and FISHER⁽¹⁴⁷⁾ (1963), to mention but a few. In these studies, hot-wire anemometers and pressure sensors were used to generate signals proportional to the instantaneous fluid velocity and pressure respectively at selected points within the fluid. The results obtained yielded information on factors such as turbulent scales, eddy life-times and convection velocities.

The researchers mentioned above used correlation merely as a tool for obtaining a better understanding of turbulence. One of the first reports in which velocity measurement was the main aim is that of BUTTERFIELD, BRYANT and Dowsing⁽¹³³⁾ (1961). They attempted to measure the speed of strip steel in a rolling mill by cross-correlating two signals derived from reflectance variations of the strip. Since fast measurement was necessary for control purposes and the frequency range of the fluctuations was low, the accuracy was poor when only a short measurement time was available.

A large number of papers on cross-correlation flowmetering have appeared in the last decade. In early reports thermocouples were used to detect naturally occurring temperature fluctuations^(87,94,122,136,139,145).

ASHTON and BENTLEY⁽⁸⁷⁾, for example, measured the flow rate of sodium in a sodium-cooled fast reactor using thermocouples. With an averaging time of approximately one hour, they achieved an accuracy of ± 5 per cent. Other researchers^(73,79,83,90,121,137) injected a pseudo-random binary sequence of heat pulses into the fluid because the naturally occurring temperature fluctuations were too small. BECK and ABESEKERA⁽⁸³⁾ used this method to investigate pulsating flows and found that a linear relationship exists between the velocity obtained via correlation and the mean flow velocity in both the laminar and turbulent regions, but a non-linear relationship prevails in the transitional region. In a recent report GOH and BENTLEY⁽¹¹⁹⁾ used hot thermistor anemometers to measure the velocity of flue gases from boilers. They concluded that in order to decrease the uncertainty in their velocity measurements, the bandwidth of the thermistor anemometers would have to be increased.

Optical transducers have been applied to correlation flowmetering for many years^(72,75,77,82,88,93,105,121,132,140,142,143,144,135). FISHER and KRAUSE^(143,144) pioneered the use of such transducers in their crossed-beam correlation technique for deriving local flow information in a turbulent medium. Their method has the advantage of being non-intrusive and has been applied extensively in investigations of jet exhaust gases. Optical sensors have also been used to measure the velocity of bubbles in two-phase liquid/gas flows - MATTHES et al⁽⁸²⁾, MESCH et al⁽¹⁰⁵⁾, and of solids in pneumatic conveyors, PIPLIES⁽⁸⁸⁾, MESCH et al⁽⁷⁵⁾. They have been applied further to the non-slip, non-contact velocity measurement of moving surfaces BUTTERFIELD et al⁽¹³³⁾, FRITSCHÉ et al⁽⁷²⁾.

It is generally agreed by most researchers that capacitance transducers are best suited for measuring the flowrate of solids in pneumatic conveyors^(72,76,78,80,84,89,96,98,101,102,134,141). Such transducers respond to the instantaneous volume of solid between their electrodes and this easily permits the volume flowrate of the solid phase to be determined. Capacitance sensors have also been applied to the measurement of bubble velocities with the aim of determining the 'slip-ratio' between the liquid and gaseous phases - CIMORELLI and EVANGELISTI⁽¹⁴¹⁾.

Conductivity transducers take on the rôle of capacitance transducers in two-phase liquid/solid flow provided that the liquid phase is conductive. Numerous papers by BECK et al^(74,76,85,89,100,103,117,128) describe the results obtained with such sensors. Although in most cases naturally occurring conductivity variations were used, BULLOCK⁽¹²¹⁾ describes a system in which pseudo-random salt pulses were injected into the fluid.

Recently ultrasonic sensors have been applied to correlation flowmetering. Their non-intrusive potential is an obvious advantage. In an early report⁽⁹⁶⁾ a magneto-strictive device is described; however, in most of the other papers piezo-electric crystals were employed^(81,85,91,92,101,117,120,123,125,138). In addition to the abovementioned sensors, isolated accounts have appeared in which electrostatic charge generated by the flowing fluid is used to generate flow signals^(99,124). Both naturally occurring and injected pseudo-random pressure fluctuations have also been used in correlation flowrate measurements^(129,130).

Apart from transducers, the other important component of correlation flowmeters is the device that actually cross-correlates the two signals and finds the position of the correlation peak. In many of the references listed above general-purpose digital computers were used^(87,99,102,141,143). Such computers are useful for experimental work, but their cost is prohibitive for general applicability. Numerous proprietary correlators are available on the market, e.g., HEWLETT-PACKARD 3721A, HONEYWELL SAICOR, PAR, and various researchers have used these instruments in their investigations^(83,85,92,117,142).

Cost and the fact that they are general-purpose instruments, again make them unattractive for dedicated applications. The need for a cheap peak-finding correlator has been realized by many researchers and a number of interesting devices have been reported^(73,75,77,81,82,86,93,95,105,109-112,131,132,168).

Although most have too limited performance for general flowmetering applications, a few correlators with improved potential have been developed. The most concerted effort has been made by JORDAN et al^(109,131) who have designed and produced a large-scale integrated circuit correlator. FLEMONS⁽⁸¹⁾ reports on an application of this correlator using ultrasonic transducers.

An alternate approach has been adopted by MESCH et al^(75,86) who have developed an extremum seeking correlator.

In spite of the fact that a great deal of research has already been done in the field of correlation flowmetering, the concept is still not wellknown in industry. It does, however, appear that much of the groundwork leading to widespread industrial use has been completed and the coming years should see correlation flowmeters move out of research institutions into the commercial and industrial fields. It is hoped that the correlation flowmeter to be described in this thesis will possess all the features to make it a viable industrial instrument.

3. MATHEMATICAL FUNDAMENTALS OF CORRELATION

Before proceeding to an examination of some characteristics of correlation flowmeters, a brief review is presented of the basic mathematical concepts involved.

3.1 Random Processes

Random processes, of which the mechanism that generates flow signals suitable for correlation flowmetering is a typical example, can be classified as shown in Figure 2.1 as being either stationary or non-stationary. Stationary random processes may be further categorized as being either ergodic or non-ergodic, whereas non-stationary random processes are usually described in terms of their specific type of non-stationary properties.

3.2 Ensemble

Of fundamental importance is an understanding of the term *ensemble*. An ensemble is a collection of sample functions which describes a random process. For example, assume that the output voltage of a thermal noise generator is recorded as a function of time. A specific voltage-time history will be obtained as shown in Figure 2.2. However, if a second thermal noise generator of identical construction is operated simultaneously, a different voltage time history would result. In fact every thermal noise generator, which might be constructed would produce a different voltage time history as illustrated in Figure 2.2. Hence, the voltage-time history for any generator is merely one example of an infinitely large number of time histories which might have occurred. A single time history representing a random phenomenon is called a sample function. The collection of all possible sample functions which the random process might have produced, is an ensemble.

3.3 Ensemble Average

The average value of all the sample functions describing a random process at some instant in time $t = t_1$ is called an ensemble average and is written as

$$\mu_X(t_1) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N x_k(t_1)$$

3.4 Stationarity

The value of the above ensemble will in general depend upon the instant of measurement, t_1 . If, however, it is invariant with a shift of the measuring instant, then the random process described by the ensemble is said to be stationary; that is, if

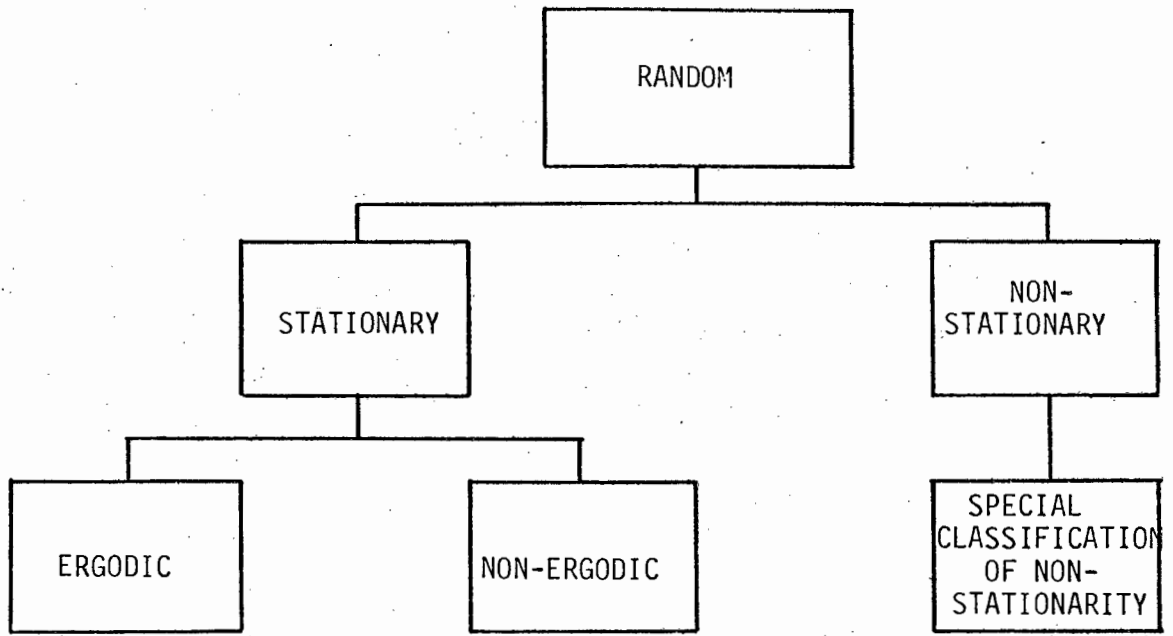


FIGURE 2.1 Classification of Random Data

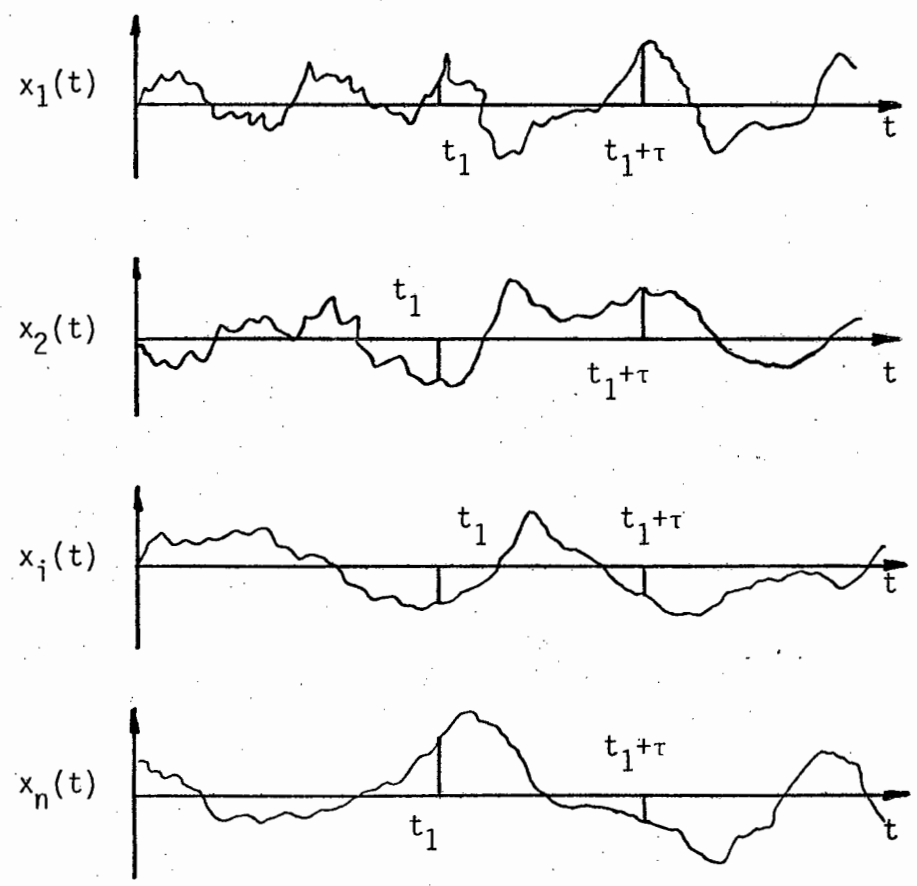


FIGURE 2.2 Ensemble of Sample Functions Forming Random Process

$$\mu_X(t_1) = \mu_X(t_2) \text{ for any } t_1 \text{ and } t_2$$

then the random process is stationary.

3.5 Time Average

In engineering practice, an ensemble of random variables from which ensemble averages can be computed is often not available. Instead a single record of long duration, i.e., a sample function is more frequently available. Another type of average called a *time average* can be computed from such a sample function

$$\mu_X(k) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_k(t) dt$$

This average gives specific information about the k-th sample function, but it is not possible to deduce anything about the time averages of other similar recordings except in special cases.

3.6 Ergodicity

Under certain conditions, the time average taken of any sample function from a stationary process is equal to the ensemble average at any instant of time. Whenever time averages are equal to ensemble average, the process described by the ensemble is ergodic. For such a process

$$\mu_X(t_i) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N x_k(t_i) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_j(t) dt = \mu_X(j)$$

for any i, j.

The ergodic hypothesis is a convenient aid in analysis and it can be justified by reasonable assumptions about the physics of the system. It allows predictions to be made about hypothetical experiments when the only information available is one recording from a single experiment.

3.7 Auto-correlation

The auto-correlation function of a random process describes the general dependence of the values of the process at one time on the values at another time. The auto-correlation function can be obtained either via an ensemble average or via a time average. Referring to figure 2.2, the ensemble average method of computing auto-correlation functions is to find the average value of the product of samples taken at two instants of time t_1 and $t_1 + \tau$ i.e.,

$$R_x(t_1, t_1 + \tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N x_k(t_1) \cdot x_k(t_1 + \tau)$$

Alternately, the auto-correlation function can be obtained by forming the product of any sample function at times t and $t + \tau$ and averaging this product with respect to time, i.e.,

$$R_x(k_1 \tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_k(t) \cdot x_k(t + \tau) dt$$

If the random process is stationary, the ensemble averaged auto-correlation function will be independent of the instant of time, t_1 but only depend on the time difference, τ , between the two samples, i.e.,

$$R_x(t_1, t_1 + \tau) = R_x(t_2, t_2 + \tau) = R(\tau) \text{ for any } t_1 \text{ and } t_2$$

Furthermore, if the process is ergodic, then the ensemble and time averaged auto-correlation functions will be equal, i.e.,

$$R_x(t_i, t_i + \tau) = R_x(j, \tau) \text{ for any } i \text{ and } j.$$

This condition again allows conclusions to be reached about a process from only a single recording. Some important properties of auto-correlation functions are listed below:

- (i) The auto-correlation functions is an even function of τ , i.e.,

$$R_{xx}(\tau) = R_{xx}(-\tau)$$

- (ii) The auto-correlation for zero delay is equal to the mean square value of the signal (assuming zero mean value)

$$R_{xx}(0) = \sigma_x^2$$

- (iii) If the signal has a finite mean value μ_x then

$$R_{xx}(\tau) \rightarrow \mu_x^2 \text{ as } \tau \rightarrow \infty$$

- (iv) A periodic signal has a periodic auto-correlation function of the same period as the signal

- (v) The auto-correlation function always has a maximum at zero delay, i.e.,

$$R_{xx}(0) \geq R_{xx}(\tau) \text{ for all } \tau$$

3.8 Cross-correlation

The cross-correlation function of two sets of random data describes the general dependence of the values of one set of data on the other set as a function of the time delay between the two sets. Expressed as an ensemble average, the cross-correlation function between two random processes $\{x(t)\}$ and $\{y(t)\}$ is given by the mean value of the product of samples taken at time t_1 from the first process and time $t_1+\tau$ from the second process, i.e.,

$$R_{xy}(t_1, t_1+\tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N x_k(t_1) \cdot y_k(t_1+\tau)$$

For stationary processes, this average will only depend on the time difference τ . The cross-correlation function can be further expressed as the time averaged product of two corresponding sample functions one from each ensemble, with one sample function shifted in time by τ , i.e.,

$$R_{xy}(\tau, k) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_k(t) \cdot y_k(t+\tau) \cdot dt$$

Again if both processes are ergodic, the ensemble and time averaged cross-correlation functions will be equal. Some properties of cross-correlation functions are listed below.

- (i) The cross-correlation function is not necessarily symmetrical in τ .
- (ii) The cross-correlation function does, however, display symmetry about the ordinate when x and y are interchanged, i.e.,

$$R_{xy}(\tau) = R_{yx}(-\tau)$$

- (iii) $R_{xy}(\tau)$ does not necessarily have a maximum at zero delay.
- (iv) When $R_{xy}(\tau)=0$, $x(t)$ and $y(t)$ are said to be uncorrelated.
- (v) If $x(t)$ and $y(t)$ are statistically independent, then $R_{xy}(\tau) = 0$ for all time delays assuming either one or both signals have zero mean values. If both $x(t)$ and $y(t)$ have non-zero mean values, then the cross-correlation function has a value $\mu_x \cdot \mu_y$ for all time displacements.

3.9 Correlation Coefficients

Both auto and cross-correlation functions may be normalized to yield auto and cross-correlation function coefficients by dividing the former with the root mean square values of the signals. The normalized auto-correlation function coefficient is hence given by:

$$\rho_{XX}(\tau) = R_{XX}(\tau)/\sigma_X^2$$

and the cross-correlation function coefficient is:

$$\rho_{XY}(\tau) = R_{XY}(\tau)/\sigma_X \cdot \sigma_Y \text{ where } \sigma_X, \sigma_Y \text{ are the r.m.s. values of } x(t) \text{ and } y(t) \text{ respectively.}$$

These correlation coefficients satisfy the following relationships:

$$-1 \leq \rho_{XX} \leq 1$$

$$-1 \leq \rho_{XY} \leq 1$$

The cross-correlation function coefficient $\rho_{XY}(\tau)$ measures the linear dependence for a time displacement τ , between the random processes of which $x(t)$ and $y(t)$ are sample functions. If $\rho_{XY} = 1$ the two signals are identical, if $\rho_{XY} = 0$ they are uncorrelated and if $\rho_{XY} = -1$ the signals are identical but in anti-phase.

3.10 Spectral Density Functions

The auto-correlation functions gives some indication of the relationship between signal samples taken τ seconds apart and it is to be expected that this will depend in some way on the bandwidth of the signals. In fact, the auto-correlation function and power density spectrum form a Fourier transform pair, i.e.,

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) \cdot e^{-j2\pi f\tau} d\tau$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} S_{XX}(f) \cdot e^{j2\pi f\tau} \cdot df$$

Similarly a cross-power density spectrum can be defined as being the Fourier transform of a cross-correlation function, i.e.,

$$S_{XY}(f) = \int_{-\infty}^{\infty} R_{XY}(\tau) \cdot e^{-j2\pi f\tau} d\tau$$

$$R_{XY}(\tau) = \int_{-\infty}^{\infty} S_{XY}(f) \cdot e^{j2\pi f\tau} \cdot df$$

Whereas the power density spectrum is real and gives no phase information about the signal, the cross-power density spectrum is a complex function and can be written in the form

$$S_{xy}(f) = C_{xy}(f) - j Q_{xy}(f)$$

where $C_{xy}(f)$ is called the co-spectrum and

$Q_{xy}(f)$ is called the quadrature spectrum

For a more detailed description of the mathematical fundamentals of correlation, the reader is referred to BENDAT and PIERSON⁽²⁴⁵⁾, LEE⁽²⁴⁷⁾, PAPOILIS⁽²⁴⁸⁾ for excellent accounts. Further reviews of correlation applied to measuring techniques can be found in references^(127, 155, 160).

4. FLOW SIGNALS: AN ERGODIC PROCESS

If the mechanism that produces flow signals suitable for correlation is an ergodic process, then it will be possible to determine the transit time of flow discontinuities between two points in the direction of flow, by time-averaging the recordings measured at the two points instead of having to use ensemble averages. This clearly would be distinct advantage since it is much easier to find time averages than ensemble averages.

Fortunately by examining the physical nature of the process it is possible to predict that the mechanism producing flow signs will be ergodic. Consider a large number of identical pipe-lines conveying the same fluid at the same flowrate, temperature, viscosity, etc. Furthermore, assume that pairs of identical transducers that convert some flow discontinuity, such as density or conductivity into electrical signals are attached to each pipe in exactly the same manner. The resulting flow signals generated at both up- and down-stream transducers will produce two ensembles. The cross-correlation function can be calculated as an ensemble average using the method described previously in Section 3.8. Clearly this average will depend only upon the time delay between pairs of samples, not on the actual instants at which the samples are taken. The two ensembles are thus stationary. Furthermore, time averaged correlation functions could be computed using corresponding pairs of recordings. They would all exhibit a peak at a time delay equal to the transducer spacing divided by the flow velocity. Since both these factors are the same for all the flow systems considered, the time-averaged cross-correlation functions will be identical and equal

to the ensemble averaged cross-correlation function. The mechanism producing flow signals is thus an ergodic process. As a result it will be possible to determine flowrates by considering merely a single pair of transducers and using time-averaging techniques.

5. CORRELATION FLOWMETER MODELS

In correlation flowmetering random signals generated by the flowing fluid are measured at two points a known distance apart in the direction of flow. The transit time of flow discontinuities between these two points is determined by cross-correlating the random signals and finding the position of the correlation peak. This then yields the flow velocity since $v = d/\beta$. Various questions now arise:

- (i) Does the cross-correlation peak position represent the mean transit time of the fluid between the two measuring points?
- (ii) If not, how can the mean transit time or preferably the mean flow velocity be obtained from the cross-correlation function?
- (iii) Is the cross-correlation function symmetrical or not?
- (iv) What factors effect the shape of the cross-correlation function?

In order to be able to answer these questions, it is necessary to understand the process that generates the flow signals. In other words a model of the flow-system, consisting of a section of pipe conveying a fluid with two transducers attached to it, is required. This can generally be described by Figure 2.3 where the moving medium between the two sensors is represented by an unknown dynamic system with input signal being the upstream transducer signal and output signal being the downstream signal. The determination of the unknown system from $x(t)$ and $y(t)$ is the problem at hand.

The standard method of describing a linear system in the time domain is by its impulse response $g(t)$. For such a system the output is related to the input by the convolution integral

$$y(t) = \int_{-\infty}^{\infty} g(\theta) \cdot x(t - \theta) d\theta \dots\dots\dots (2.1)$$

Alternatively a linear system can be described by its transfer function, $G(j\omega)$, which is the Fourier transform of its impulse response. The transfer function can be obtained by using either deterministic or random signals. A few

methods for obtaining these functions are described below since they will be used in the subsequent analysis

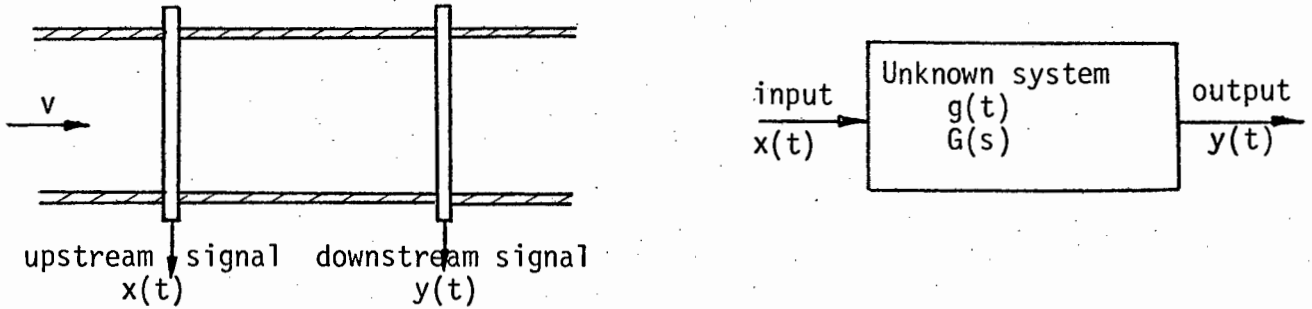


FIGURE 2.3 : Model of Correlation Flow System

5.1 Direct Measurement of Impulse Response using Deterministic Signals

The system can be excited by signals such as impulses, steps, ramps and the output recorded. The impulse response, $g(t)$, can then be obtained by deconvolving equation 2.1. If the input is a Dirac-delta impulse, evaluation of $g(t)$ is particularly simple since the output will directly equal the impulse response.

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} g(\theta) \cdot x(t - \theta) \cdot d\theta \\ &= \int_{-\infty}^{\infty} g(\theta) \cdot \delta(t - \theta) d\theta = g(t) \end{aligned}$$

The above is, however, only true if no disturbances are present. A further disadvantage of the impulse method is the large impulse amplitude necessary for reasonable signal-to-noise ratios, which can lead to saturation and other non-linear effects.

5.2 Cross-correlation of Random Signals

It can be shown that for a linear system, the cross-correlation function of the system input and output $R_{xy}(\tau)$ is the response of the system to excitation by the auto-correlation function of the input $R_{xx}(\tau)$, i.e.,

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} g(\theta) \cdot R_{xx}(\tau - \theta) d\theta$$

The random signals $x(t)$ and $y(t)$ may occur either naturally as operating signals or they may be artificially injected as test signals. A very simple solution again results if the input is chosen so as to have an impulse like auto-correlation function since then for $R_{xx}(\tau) = \delta(\tau)$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} g(\theta) \cdot \delta(\tau - \theta) d\theta = g(\tau)$$

i.e., the cross-correlation function is directly equal to the impulse response of the system.

The above relation remains unchanged even if the output is corrupted by noise provided that it is uncorrelated with the input. Furthermore low-power input signals having small crest factors can be used which do not affect the normal operation of the system and do not produce non-linear effects.

5.3 Frequency Domain Method

Instead of measuring the impulse response, the transfer function of the system can be obtained in the frequency domain. By inverse transforming the transfer function, the impulse response can be derived. This classical method of system analysis operates by applying sine waves of known amplitude and frequency to the system and measuring the amplitude and phase of the output of the system. Repeating these measurements at a number of frequencies yields the amplitude and phase spectrum of the system from which the transfer function can be determined.

6. CHARACTERISTICS OF SPACE-TIME CORRELATIONS

In order to model the flow system consisting of a section of pipe conveying fluid with two transducers attached to it, it is necessary to examine some of the characteristics of correlation flowmetering. Fortunately a large amount of work has been done on space - time correlations by TAYLOR⁽²⁰⁸⁾; TOWNSEND⁽²¹¹⁾; DAVIES and FISHER^(143, 147); FAVRE, GAVIGLIO and DUMAS^(196, 210) to mention but a few. The results of these investigations has led to a better understanding of turbulent flows. A number of these results are directly applicable to the problem at hand.

The correlation functions most commonly measured in turbulence studies are the space correlation, the auto-correlation and the cross-correlation functions, the first two merely being special cases of the more general cross-correlation function. Cross-correlation measurements involve the correlation of signals from two spatially separated measuring positions, the signal from the upstream

point being delayed by a time, τ . Thus, if $v_1(0,0)$ denotes the signal received at one point at time $t = 0$ and $v_2(x,t)$ denotes the signal received at a point distance, x , from the first at time, t , their cross-correlation coefficient $R(x,\tau)$ may be defined as:

$$R(x,\tau) = \frac{\overline{v_1(0,0) \cdot v_2(x,\tau)}}{\{\overline{v_1(0,0)^2}\}^{\frac{1}{2}} \cdot \{\overline{v_2(x,\tau)^2}\}^{\frac{1}{2}}}$$

where the overbar denotes an average over a period sufficiently long to obtain stationary values⁽¹⁴⁷⁾.

The space correlation which involves the comparison of the instantaneous signals received at two spatially separated measuring points is thus merely the cross-correlation for zero time delay, i.e., $R(x,0)$ whilst the auto-correlation coefficient (which involves the correlation of the signal received at a measuring point with the signal received at the same point a time, τ , previously) is the cross-correlation for zero separation of the measuring points, i.e., $R(0,\tau)$.

6.3 Frozen pattern

Distinction has to be made between two modes of flow. The first flow to be discussed is based on the assumption that a frozen pattern of turbulence exists, in other words, the statistical distribution of turbulent eddies does not change as the fluid propagates along the pipe. This assumption is generally called TAYLOR's hypothesis⁽²⁰⁸⁾ and is valid only if the turbulence level is low, viscous forces are negligible and the mean shear is small. Under these conditions the signal received at one measuring position will be received at a second position distance x_1 directly downstream from the first at time τ_1 later. Thus the value of the particular cross-correlation coefficient $R(x_1,\tau_1)$ will have a maximum value of unity.

The convection velocity of the pattern may thus be defined as

$$u_c = x_1/\tau_1$$

For such a frozen pattern, the auto and space-correlation coefficients are exactly related through the convection velocity by the relation

$$R(\tau) = R(x = u_c \cdot \tau)$$

It is possible further to deduce from the frozen nature of the pattern that all eddies travel with this unique velocity and hence the mean and convection velocities are equal.

6.2 Non-frozen Pattern

If shearing forces are present as is the case in most flows encountered in practice, the convected pattern changes as it travels downstream. The results of a typical series of cross-correlation measurements of such a pattern is shown in Figure 2.4. It can be seen that although each cross-correlation curve rises to a maximum at some value of the time delay, clearly indicating the presence of convection, the amplitude of this maximum is a function of the measuring point separation. Another point to note is the broadening of the cross-correlation curves as the measuring point separation increases.

FISHER and DAVIES⁽¹⁴⁷⁾ define the convection velocity no longer to be the time delay at which the maximum of a particular cross-correlation curve appears but by the time delay at which the envelope of all the cross correlation curves intersects the curve for a particular measuring point separation. The motivation for this definition is as follows: Consider an observer travelling at the velocity so defined. The auto-correlation function in his reference is the envelope of all the cross-correlation functions and geometric considerations of Figure 2.4 show that this is the auto-correlation curve having the maximum time-scale, the time-scale being defined as that time delay for which the moving axis auto-correlation function falls to a value of $1/e$. Thus, it is in this frame of reference that the temporal rate of change of turbulence is a minimum. It thus seems physically acceptable that it is this velocity which defines the mean rate of transport of the energy bearing eddies, i.e., the convection velocity.

FAVRE et al⁽¹⁹⁶⁾, FISHER et al⁽¹⁴³⁾ do not use the above definition for the convection velocity but use the time delay at which the cross-correlation curve attains a peak to give the convection and mean velocities. In most practical situations the difference between the two definitions is very small and can be neglected.

It is also apparent that the space and auto-correlation curves are no longer exactly related for a non-frozen pattern of turbulence. The distortion of the pattern as it is convected between the two measuring positions, distance x apart, causes the auto-correlation $R(\tau)$ (where $\tau = x/u_c$) to have value less than the space correlation $R(x)$.

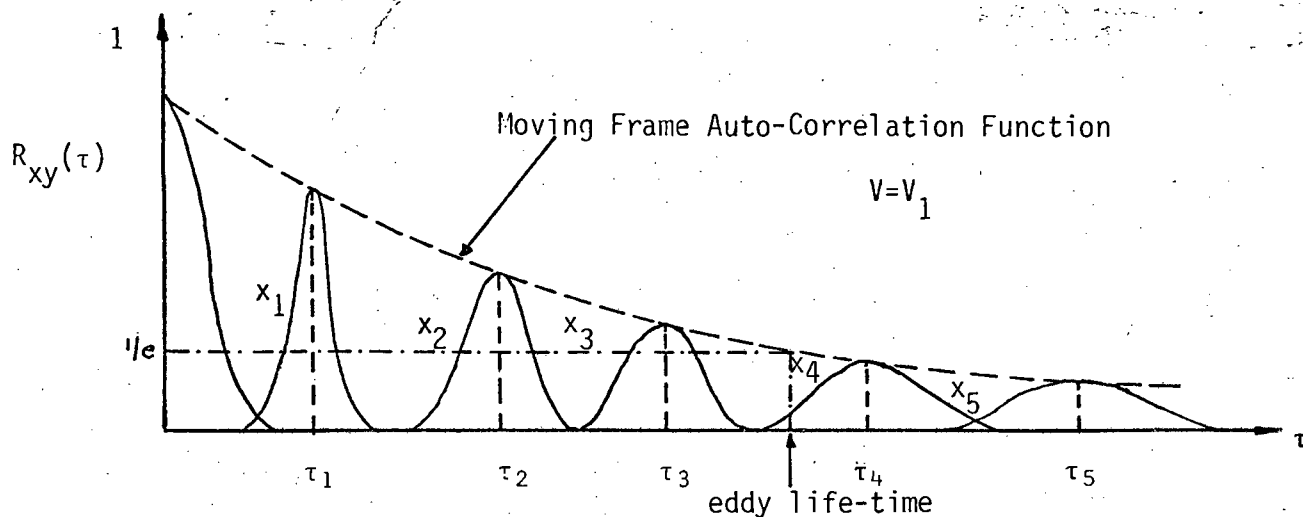


FIGURE 2.4 : Typical Cross-Correlation Curves for Different Sensor Spacings

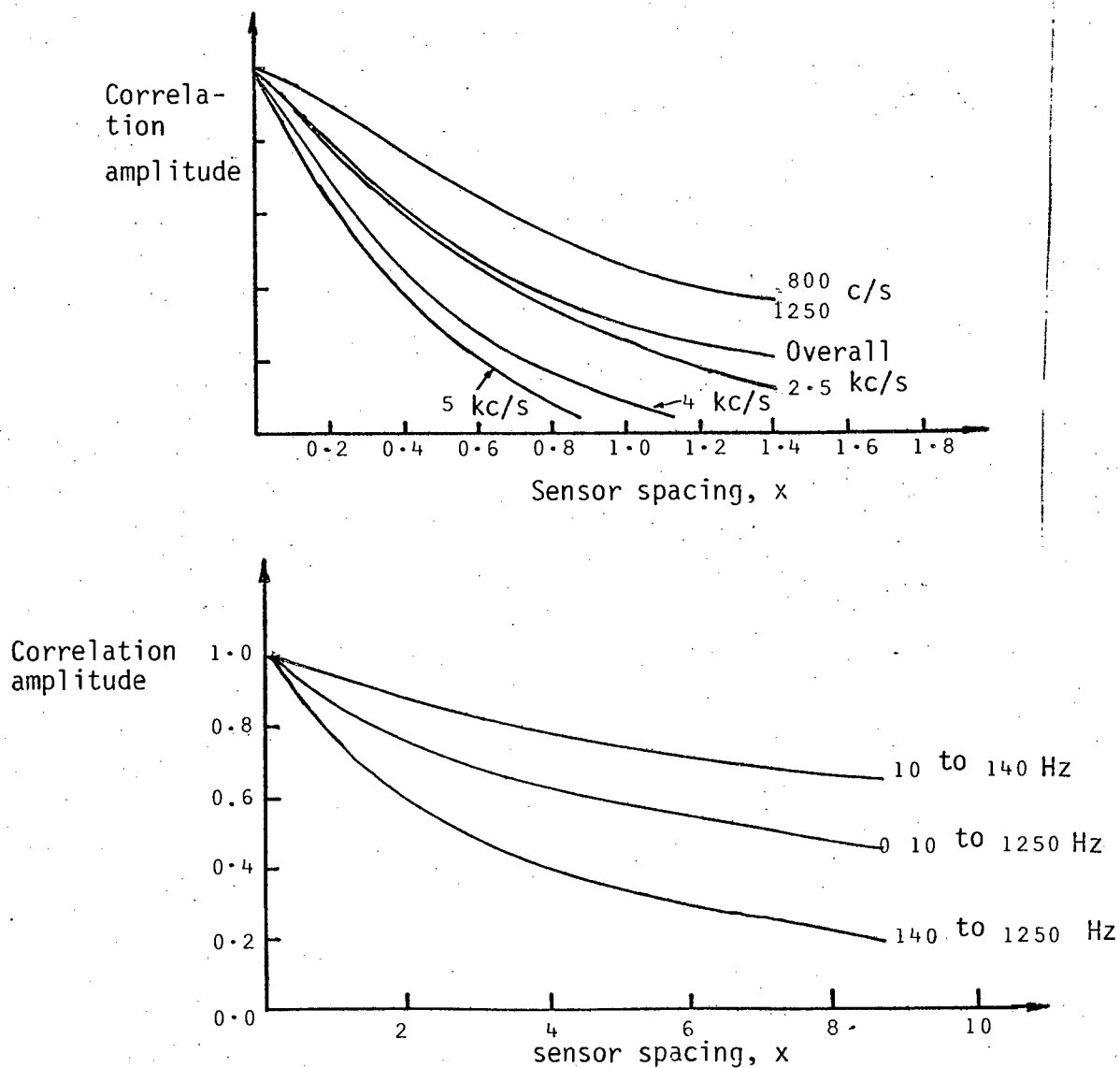


FIGURE 2.5 : Effect of Frequency Band on Space-Time Correlation with Optimum Delay

Both FISHER⁽¹⁴⁷⁾ and FAVRE⁽¹⁹⁶⁾ have investigated the effect of filtering the flow signals before cross-correlating them in order to determine the phase velocities of the different turbulent components. Results obtained by both researchers are shown in Figure 2.5.

In both cases it can be seen that the high frequency turbulent components decay more rapidly than the low frequency components. This is mainly due to the fact that the high frequency components participate in viscous dissipation effects to a much greater extent. The relatively long conservation of the overall cross-correlation coefficient is thus attributable to the large-scale eddies.

In Figure 2.6 the convection velocities are plotted as a function of frequency and it can be seen that this velocity increases with increasing frequency. This suggests that, in general, the high frequency components contain a rather higher percentage of the faster moving eddies than do those of lower frequency.

7. FROZEN PATTERN CORRELATION FLOWMETER MODEL

The first model to be discussed is based on TAYLOR's frozen pattern hypothesis. Although this hypothesis in general is not valid for most pipe flows, it does hold in applications in which the velocity of moving surfaces is measured. In such applications (e.g., speed measurement of strip steel, paper or rolling-mill products), the signals measured at two positions in the direction of motion will be identical since no change in the physical structure of the surface occurs between the measuring points. For this model the downstream signal is a time-delayed replica of the upstream signal, i.e.,

$$y(t) = x(t - \beta)$$

where β is the transit time of the flow pattern between the measuring points. The impulse response is thus a Dirac-delta function given by

$$g(t, L) = \delta(t - \beta) = \delta(t - L/v_0)$$

where v_0 is the mean flow velocity at which all the fluid particles travel.

Furthermore the cross-correlation function between transducer outputs is equal to a time delayed replica of the auto-correlation function of either sensor output (see Figure 2.7).

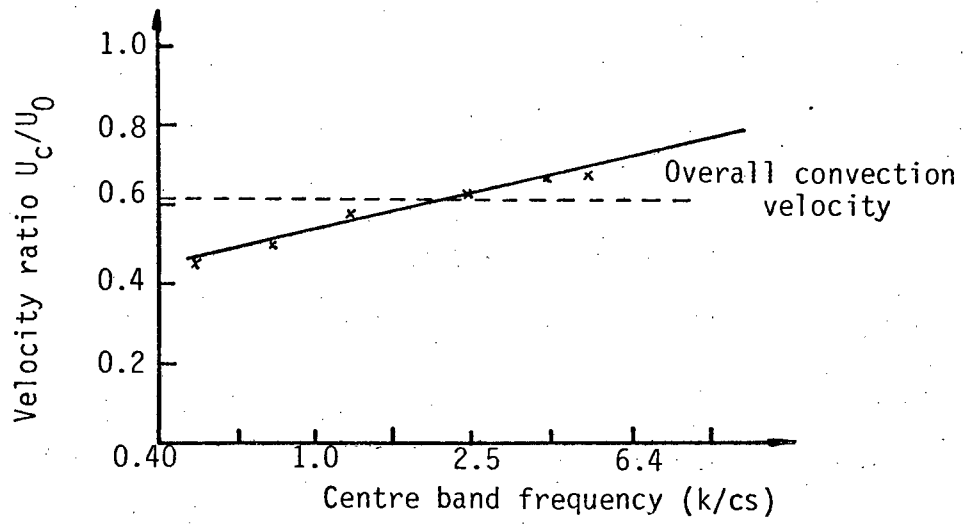


FIG. 2.6 Variation of convection velocity with filter frequency

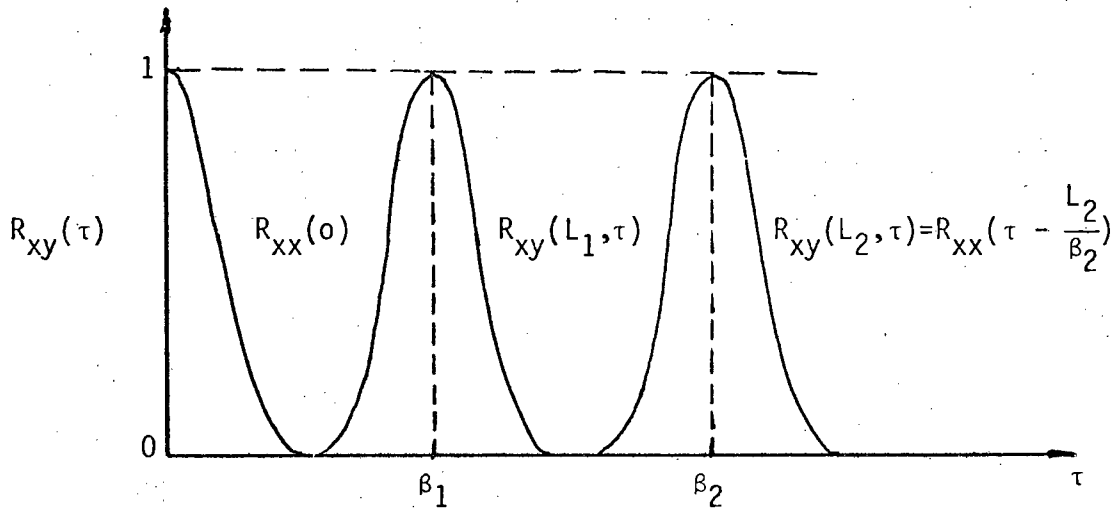


FIG. 2.7 Frozen pattern cross-correlation functions

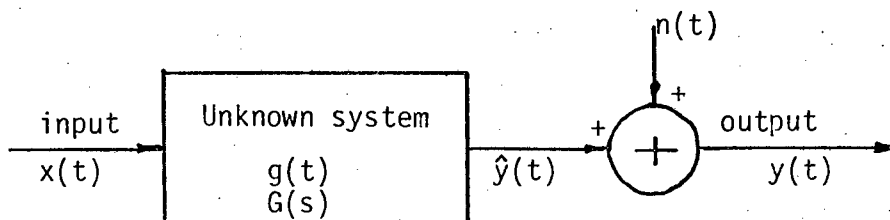


FIG. 2.8 Non frozen pattern correlation flowmeter model

$$\begin{aligned}
 R_{xy}(\tau) &= \lim_{T \rightarrow \infty} \int_0^T x(t-\tau)y(t)dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t-\tau)x(t-\beta)dt \\
 &= R_{xx}(\tau-\beta) = R_{xx}(\tau-L/v_0)
 \end{aligned}$$

There is thus no change in the shape of the cross-correlation curve with increased transducer spacing. The cross-correlation functions are symmetrical and the peak position gives the mean transit time which in turn determines the flow velocity.

This model clearly is inadequate for most flows encountered in practice and a more realistic model is required.

8. NON-FROZEN PATTERN CORRELATION FLOWMETER MODEL

From the results discussed in Sec. 6 it is apparent that for a non-frozen pattern of turbulence, the cross-correlation functions decrease in amplitude and become broader with increasing sensor spacing. A model for this system is shown in Figure 2.8 where $x(t)$ and $y(t)$ are the signals measured at the up and downstream sensors respectively⁽¹³³⁾. In this model only the macroscopic characteristics of the actual flow system are considered, no attempt is made to include directly microscopic effects such as turbulent eddies, pressure fluctuations, etc. Only the overall behaviour as represented by the sensor outputs is modelled.

Since transport of the fluid takes place only in the direction of flow, i.e., no back-flow is assumed, signal flow can be assumed to be uni-directional with no feedback present. As a result it is possible to consider the upstream signal, $x(t)$, as the input, the downstream signal $y(t)$ as the output and the unknown system as a transfer system.

The system is linear since the principle of superposition holds. Furthermore since a steady non-accelerating fluid flow is assumed, the system is both time and space invariant. It can thus be described by its impulse response $g(t)$ or its transfer function $G(s)$.

To take into account instantaneous fluctuations from the statistical mean values, a noise signal $n(t)$ is included that is uncorrelated with the input $x(t)$ since all linear relationships have already been included in the transfer function $G(s)$. The noise signal is, however, not independent of the input

and transfer function of the system. Both sensor signals must have identical statistical characteristics such as auto-correlation functions and power spectral densities since time and space stationarity have been assumed.

$$\text{Hence}^{(106)} \quad R_{xx}(\tau) = R_{yy}(\tau) \quad \text{and} \quad S_{xx}(w) = S_{yy}(w)$$

But since

$$S_{\hat{y}n}(jw) = 0 \quad (n, \hat{y} \text{ are uncorrelated})$$

$$S_{yy}(w) = S_{\hat{y}\hat{y}}(w) = |G(jw)|^2 \cdot S_{xx}(w) + S_{nn}(w) = S_{xx}(w)$$

$$\therefore S_{nn}(w) = [1 - |G(jw)|^2] \cdot S_{xx}(w)$$

The spectrum of the noise signal thus depends on both the spectrum of the input signal and the transfer function of the system. This unusual result indicates one of the peculiarities of this process.

One method of obtaining further information about the impulse response of the system is to consider experimental results. Figure 2.9 again depicts a section of pipe with sensors placed at various spacings. From the auto-correlation function of the upstream transducer signal which is the system input $R_{xx}(\tau)$ and the cross-correlation function between sensor outputs $R_{xy}(\tau)$, the impulse response of the system can be determined using the convolution integral.

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} g(\theta, L) \cdot R_{xx}(\tau - \theta) d\theta \quad \dots\dots\dots (2.2)$$

The interpretation of the impulse response $g(t, L)$ will now be attempted.

A standard method for obtaining the impulse response of a system is to apply an impulse to the system and to record its output. In a flow system this could be done by injecting a tracer at some upstream point and recording the passage of the tracer at a downstream point (see Figure 2.10). This method, if often used for determining mean flowrates, tracers varying from heat pulses and salt, to radioactive isotopes. Now assume that it is possible to inject such a tracer with very short injection time, uniformly over the cross-section of the pipe. The output of the downstream transducer will be an extended pulse since different fluid particles have different flow velocities caused by velocity profile effects and statistical speed fluctuations. This extended output pulse which equals the impulse response of the system can be interpreted as being the transit-time distribution density $w(T, L)$ for the particular flow and transducer spacing, i.e.,

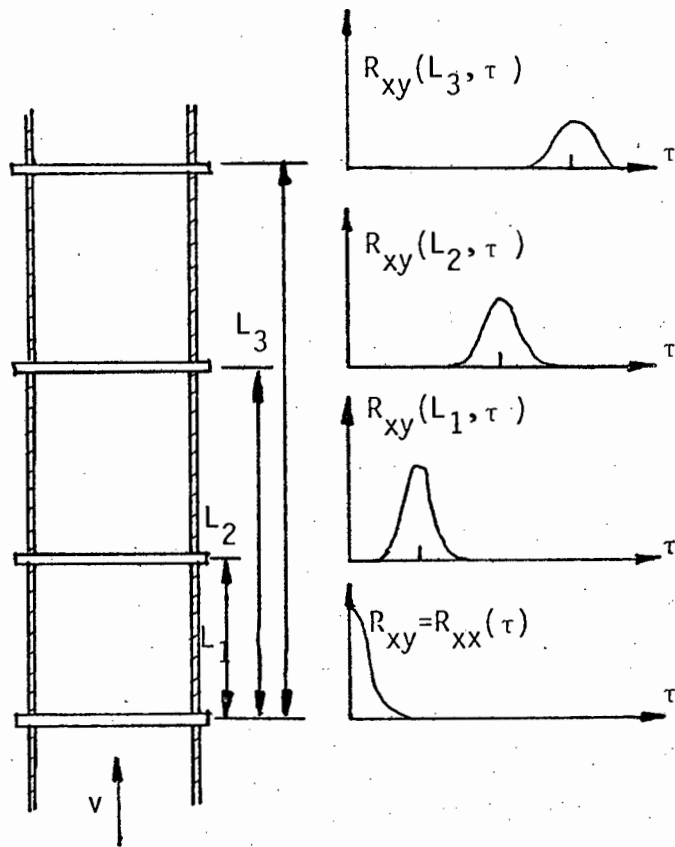


FIG. 2.9 Cross-correlation plots at different sensor spacings

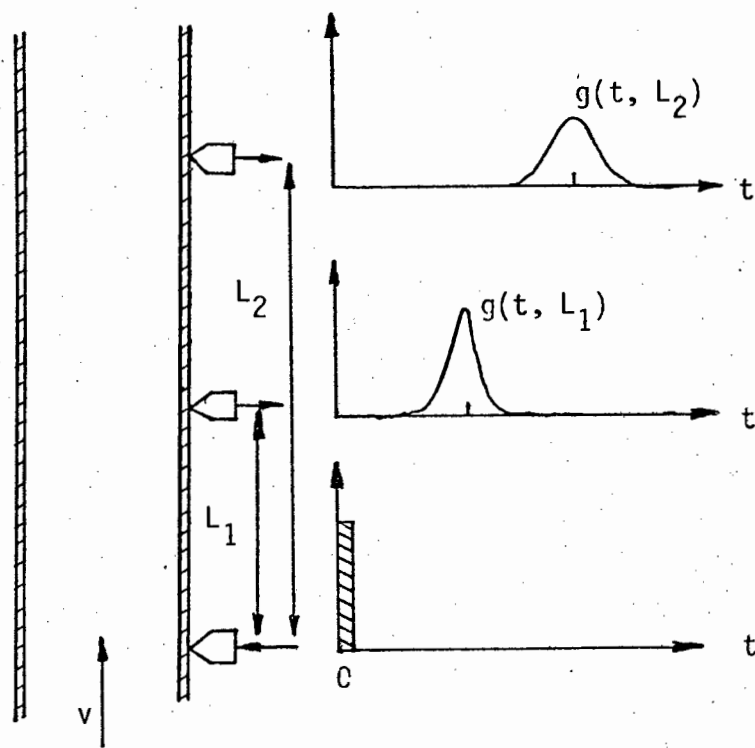


FIG. 2.10 Impulse response at different sensor spacings

$$y(t,L) = g(t,L) = w(T,L)/_{T=t>0} \dots\dots\dots (2.3)$$

The velocity distribution density $w(v)$ can now be derived easily from the transit-time distribution since⁽²⁺⁵⁾:

$$w(v) = -\frac{w(t,L)}{dv/dt} = w(t,L) \cdot \frac{t^2}{L} \text{ where } v = L/t \dots\dots (2.4)$$

It is thus possible not only to derive the mean flow velocity but also the velocity distribution density via equations (2.2), (2.3) and (2.4).

As an example assume that the velocity distribution density is Gaussian with mean value v_0 and standard deviation σ_v , i.e.,

$$w(v) = \frac{1}{\sigma_v \sqrt{2\pi}} e^{-(v-v_0)^2/2\sigma_v^2}$$

The transit time distribution density is hence given by

$$w(t,L) = \frac{t^2}{\sqrt{2\pi} \sigma_v \cdot L} e^{-(t-T_0)^2/2t^2(\frac{\sigma_v}{v_0})^2} \text{ where } v_0 = \frac{L}{T_0}$$

This is an asymmetrical function with its peak near the mean transit time $T_0 = \frac{L}{v_0}$. Its amplitude decreases and its width broadens with increasing sensor spacing. The broadening effect of cross-correlation curves with increased sensor spacing can readily be explained in terms of the velocity distribution density. Consider any two fluid particles having velocities v_1 and v_2 ($v_2 > v_1$). The transit-time of these particles over distances L_1 and L_2 ($L_2 > L_1$) are

$$\begin{aligned} T_1(L_1) &= \frac{L_1}{v_1} & T_2(L_1) &= \frac{L_1}{v_2} \\ T_1(L_2) &= \frac{L_2}{v_1} & T_2(L_2) &= \frac{L_2}{v_2} \end{aligned}$$

The difference in transit-times for the two distances is thus:

$$\begin{aligned} \Delta T(L_1) &= T_1(L_1) - T_2(L_1) = L_1 \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \\ \Delta T(L_2) &= T_1(L_2) - T_2(L_2) = L_2 \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \end{aligned}$$

But since

$$L_2 > L_1, \Delta T(L_2) > \Delta T(L_1)$$

As a result the standard deviation of the transit-time distribution density, i.e., its width, increases with increasing transducer spacing.

If velocity profile effects are taken into account, the velocity distribution density will no longer be Gaussian but have an asymmetrical shape depending on the velocity profile. This will cause further asymmetries in the transit-time distribution densities.

Since the cross-correlation function between sensor outputs equals the convolution of the auto-correlation function of the input signal $R_{xx}(\tau)$, which is symmetrical, with the impulse response of the system, which depends on both the velocity profile and statistical speed fluctuations and is hence not necessarily symmetrical, the following conclusions can be reached about the shape of the cross-correlation function (see Figure 2.11).

- (i) If the impulse response (transit-time distribution density) is narrow with respect to the auto-correlation function, then any asymmetries in the impulse response will not cause large asymmetry in the cross-correlation function.
- (ii) If the impulse response is broad in comparison with the auto-correlation function, asymmetry in the impulse response will produce asymmetry in the cross-correlation function.

The first case, i.e., a narrow, nearly symmetrical transit-time distribution density corresponds to a flat velocity profile with small statistical speed fluctuations. In the ideal case this leads to the frozen pattern hypothesis discussed earlier. Steady turbulent flow of sufficiently high Reynold's number approximates this condition. The position of the cross-correlation peak can be used confidently as an estimate of the mean transit time.

For laminar flow with a parabolic velocity profile, the transit-time distribution density will be wide and asymmetric resulting in an asymmetric cross-correlation function. In this case the position of the correlation peak does not give the mean transit time. The mean flow velocity can be calculated from the first moment, i.e., centre of gravity of the velocity distribution density, i.e.,

$$v_0 = \int_0^\infty v \cdot w(v) dv = L \int_0^\infty \frac{1}{t} w(t, L) dt$$

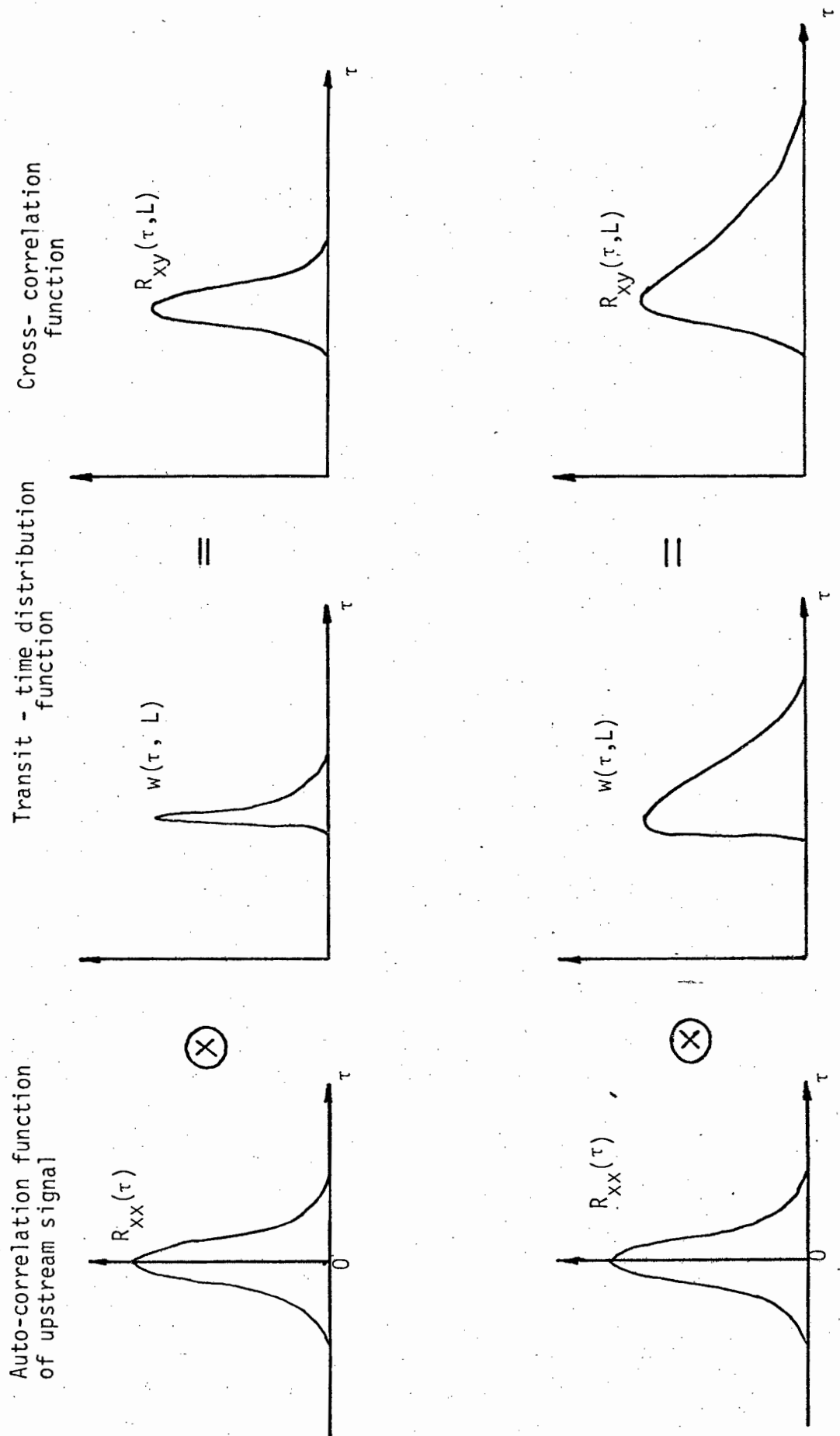


FIG. 2.11 Effect of transit-time distribution function shape on cross-correlation function shape

Another factor that influences the shape of the transit-time distribution density is the variation in convection velocity of different turbulent components. The high frequency components in general travel faster than the lower frequency components. By band-limiting the flow signals before cross-correlating them, the range of convection velocities will decrease resulting in a narrow transit-time distribution density and a wider auto-correlation function. As a result of both of these factors, the cross-correlation function will be more symmetrical.

The shape of the cross-correlation curves will further depend on the weighting effects of the sensors. Point like sensors such as hot wire anemometers and to a lesser extent, conductivity transducers are sensitive to only a small region of the velocity profile. In this case the transit-time distribution density is affected mainly by statistical speed variations about the mean value. To a good approximation these fluctuations will be Gaussian and the result discussed previously is applicable.

Other transducers such as ultrasonic and capacitance sensors, derive flow information from a greater portion of the velocity profile. This results in greater asymmetry of the transit-time distribution density since not only statistical velocity fluctuations but also velocity profile velocity variations influence the shape of the cross-correlation function.

KIPPHAN⁽⁷⁸⁾ gives results in which cross-correlation functions were predicted using the above model and compares them with experimentally derived cross-correlation curves. Theory and practice appear to compare satisfactorily.

9 . CONCLUSION

The above analysis is a starting point for further theoretical work into correlation flowmetering. It has introduced the concept of a transit-time distribution density to characterize the flow between two measuring locations. From this function, not only the mean flow velocity but also the velocity profile and the range of random velocity fluctuations about the mean value, can be determined.

CHAPTER 3

SENSORS FOR CORRELATION FLOWMETERING

1. INTRODUCTION

A correlation flowmeter consists of two equally important components, namely,

- (i) sensors that generate flow signals at two points in the direction of flow, and
- (ii) the correlator that finds the time delay between these two flow signals.

This chapter examines the first of these components; the correlator will be discussed in later chapters.

Sensors are devices that form the interface between the flowing fluid and the correlator. Their purpose is to convert time-varying spatial flow patterns into electrical signals suitable for cross-correlation. The applicability of the correlation technique to flow measurement depends on whether a physical property that is transported with the fluid can be detected and whether the measurement of the transit time of this signal can be related to the flow. As an example, a pure homogeneous fluid (such as distilled water) under ideal laminar flow will not produce flow signals since no time-varying property of the fluid that is convected with the flow is present. This situation occurs rarely in practice except, for example, in the petroleum industry. For most flows encountered in industry either one or a combination of the following properties is present:

- (i) Turbulence within the pipe producing minute variations in density, pressure and flow velocity about their respective mean values.
- (ii) Random solids distribution in two phase solid-liquid or solid-gas flows and
- (iii) Random bubble distribution in two-phase gas-liquid flows.

In mining slurry flows all three of the above properties are usually present. Such slurries contain solids within a liquid medium and due to agitation air is often present. Further in order to prevent settling of the solid phase, the flow must be turbulent. Clearly no difficulty in generating flow signals should exist for this application.

This chapter commences with a list of the requirements of flow sensors for slurry flowrate measurements before proceeding to examine and evaluate a number of sensors in view of these requirements.

2. REQUIREMENTS OF SENSORS FOR SLURRY FLOWS

(i) Sensitivity

The greater the sensitivity of the sensors, the greater will be their applicability since it will be possible to apply them to flows in which the naturally occurring tracers are very small. Furthermore, such sensors will produce more 'reliable' flow signals since they will not only detect the large variations in some fluid property but also the smaller variations that are present. This, in part, leads to the fine structure in the flow signals.

(ii) Bandwidth

Since the averaging time required to determine a cross-correlation peak depends, amongst other factors, on the bandwidth of the flow signals, the bandwidths should be as large as possible for fast response. In addition, due to the Fourier transform relationship between the time and frequency domains, a large bandwidth results in a narrow correlation peak which can be located more precisely.

(iii) Non-intrusive

For slurry flows this is one of the most important requirements since any intrusion into the pipe will result in abrasion, corrosion and clogging.

(iv) Clamp-on

Clamp-on sensors have the added advantage of not affecting pipe integrity. The flowmeter thus creates no potential leak source, installation is simple and the flowmeter can easily be re-located if required.

(v) Robust, Simple, Maintenance free

If the sensors possess these characteristics, they will be more attractive and acceptable in industry.

(vi) Calibration free

Changes in temperature, fluid composition, etc., should not result in the sensors having to be re-calibrated.

(vii) Cost

The cheaper the sensors, the better provided that all the other requirements are satisfied.

3. REVIEW OF FLOW SENSORS

A wide range of sensor concepts for use in correlation flowmetering exist, the basic difference between them being the method in which the flow disturbances are detected. This section contains a survey of some of these methods, an examination in terms of the requirements specified previously and a list of their applications reported in the literature.

3.1 Thermal (Refs. 73,79,83,87,90,94,119,121,122,145,136,137,139)

It is possible to utilize the small naturally occurring variations in the temperature of the fluid to generate flow signals. These variations can be detected by thermocouples or thermistors that are inserted into the fluid. Thermocouples are generally more attractive than thermistors since they have a faster response time and hence produce flow signals of larger bandwidth.

In applications in which the naturally occurring temperature fluctuations are too small, pseudo-random heat pulses can be injected into the fluid upstream of the measuring probes. This method works for all fluids that can tolerate externally injected heat pulses, however, this need makes it similar to conventional tracer techniques and results in additional cost and complexity.

Thermal sensors are not suited for slurry flowrate measurements since in general they are intrusive, not very robust and often adequate temperature fluctuations are not present.

3.2 Conductivity (Refs. 74,76,85,89,100,103,117,121,128)

For conducting fluids, the variations in the conductivity (resistivity) of the fluid between two electrodes placed opposite one another can be measured. These variations are produced by discontinuities such as bubbles and solid particles that possess a different conductivity to the liquid passing by the electrodes.

Both AC and DC conductivity methods can be used to detect these discontinuities. In the DC method a constant current is applied to the electrodes and the subsequent voltage fluctuations due to the varying fluid impedance between the electrodes is measured. Alternatively, a constant voltage can be applied and the current fluctuations measured. The AC method operates by applying a constant amplitude sinusoidal current or voltage waveform to the electrodes. Due to variations in the specific conductance of the fluid in the sensing field of the electrodes, the corresponding voltage and current waveforms will be modulated in amplitude. This modulation can be removed to yield a flow signal by an amplitude detector.

The AC technique is generally more suitable since the DC method can result in electrode corrosion caused by electrolytic and polarization effects. The electrodes are usually pin electrodes protruding into the pipe but can also be electrodes mounted flush with the inside of the pipe-wall. They must, however, always be in contact with the fluid.

Since most mining slurries are conductive, this requirement is not a limitation for slurry flowrate measurements. However, the fact that the conductivity sensors must be in contact with the fluid makes them prone to abrasion and corrosion problems.

3.3 Capacitance (Ref. 75,76,78,80,84,89,96,98,101,102,141,134)

The capacitance method for generating flow signals is very similar to the conductivity method except that it is mainly suitable for non-conducting fluids. Usually the capacitance variations between an insulated electrode forming part of the wall of the pipe and the earthed body of the pipe is measured; hence no obstruction is presented to the flow. Any fluctuations in the dielectric constant between the electrodes will give rise to random variations in capacitance which can easily be converted into electrical signals.

The method is attractive since it is non-intrusive. However, most industrial slurries are conductive thus limiting its use. The main application of capacitance sensors is in measuring the flowrates of solids in pneumatic conveyors. Since the sensor outputs depend on the total instantaneous volume of solids between their electrodes, it is possible also to obtain the solids loading from the flow signals.

3.4 Optical (Ref. 72,75,77,82,88,93,105,121,132,140,142-144)

Optical sensors usually consist of a light source that transmits a narrow collimated beam of radiation through the flowing fluid. The light that penetrates the fluid is converted into an electrical signal by a suitable optical detector. The intensity of the received beam will fluctuate if solid particles, bubbles or turbulent eddies are present to reflect, absorb or scatter the incident radiation. Infra-red, visible, ultra-violet light and lasers have been used as sources of light. Photo-transistors and photo-multipliers are usually employed as receivers.

The fact that the fluid must at least be partially transparent to the radiation used, is a definite limitation of optical sensors. In addition, the pipe conveying the fluid must also be transparent or otherwise windows must be inserted into the pipe-walls to transmit the radiation through the fluid.

Such windows are prone to scratching and becoming dirty and would require regular cleaning.

Since most mining slurries are not transparent, optical sensors are not suited for such flows. They have been used mainly for measuring the flowrate of bases.

3.5 Ultra-sonic (Ref. 81,85,91,92,96,101,117,120,123,125,138)

Ultrasonic sensors are in some ways similar to optical sensors. Instead of passing a beam of light through the fluid, a beam of ultrasound is transmitted through the fluid. Ultrasonic sensors usually consist of a transmitter that emits a narrowcollimated beam of ultrasonic radiation through the flowing fluid. This radiation is modulated both in amplitude and phase if acoustic impedance discontinuities exist within the fluid. Such discontinuities could again be solid particles, bubbles or turbulent eddies that reflect, scatter, refract and absorb the incident radiation. The received beam is converted back into an electrical signal and the amplitude or phase information is extracted by suitable detector circuits.

Since ultrasonic waves can propagate through solids it is possible to mount both the transmitter and receiver externally on the pipe. Non-intrusive clamp-on flow sensors with all their advantages can thus be constructed using ultrasonic transducers.

3.6 Pressure (Ref. 129,130)

In turbulent flow minute variations in pressure about its mean value are present. These fluctuations can be converted into electrical signals by suitable pressure sensors mounted flush with the inside of the pipe wall. Such a mounting arrangement, however, only detects pressure variations in and near the boundary layer and hence does not produce reliable flow signals. Alternately the pressure sensors could be mounted on probes that extend into the fluid. If the naturally occurring pressure fluctuations are too small, pseudo-random pressure pulses can be injected upstream of the measuring probes.

It appears that pressure sensors for correlation flowmetering have limited applicability and thus will not be considered for slurry flowrate measurements.

3.7 X or γ -Ray

Using X or γ -rays instead of i.r., visible or u.v. light is an obvious extension of the optical sensors described previously. By measuring the instantaneous absorption of the X or γ -rays passing through the flowing fluid, a suitable flow signal can be obtained. Radioactive sources, such as Cobalt, can be used to emit the radiation which is subsequently detected by a receiver such as a Geiger-Müller tube or photo-multiplier. Care has to be exercised in the choice of the radioactive source to ensure that it is partially absorbed by a constituent of the flowing fluid.

Since X or γ -rays can pass through the walls of most pipes, non-intrusive clamp-on flow sensors can again be constructed. However, the legal requirements concerning the use of radioactive sources apart from the cost, make them unattractive as flow sensors except in special cases.

3.8 Electrostatic (Ref. 99,124)

In certain applications it is possible to generate flow signals by measuring electrostatic charge fluctuations conveyed with the fluid. An example of such a flow is the transport of certain solids in pneumatic conveyors. Low leakage high input impedance probes are necessary to detect the electrostatic charge as the solids pass the probes. Since electrostatic charge does not exist in conducting fluids, this method cannot be used for slurry flows.

4. SENSORS FOR SLURRY FLOWMETERING

A major advantage of a cross-correlation flowmeter is its lack of dependence on the physical properties of the fluid. However, most of the transducers described in the previous section require probes in contact with the flowing fluid. This need to have a probe inserted into the flow is a definite disadvantage, especially for slurry flow measurements. It results in additional cost, a reduction in leakage integrity and in some applications, the possible requirement for frequent maintenance of the probes.

Of the non-intrusive sensors discussed previously, ultrasonic transducers are clearly superior for slurry flows. Since slurries, in general are conductive and opaque, non-intrusive capacitance and optical sensors are excluded. X or γ -ray sensors are expensive and hazardous and will thus also not be considered. The principal feature of the ultrasonic sensors is that they can be mounted externally to the pipe-wall, therefore causing no obstruction

to the flow being measured. They are hence ideally suited for measuring the flowrate of abrasive as well as toxic and corrosive fluids. Since the ultrasonic beam is passed across a diameter of the pipe, the flow signal is influenced equally by all discontinuities across that diameter. As a result, the flowmeter essentially measures a flow velocity that is the linear average across the velocity profile. The ultrasonic transducers provide a clamp-on device and the cost of the instrument does not increase much with increase in pipe diameter provided the attenuation of the ultrasonic beam in the fluid is not too large.

As it appears that the ultrasonic sensors satisfy all requirements for slurry flowrate measurements, this method was chosen. The characteristics of these sensors, such as sensitivity, bandwidth and beamwidth will be investigated in the following chapter.

CHAPTER 4

ULTRA-SONIC SENSORS FOR CORRELATION FLOWMETERING

1. DESCRIPTION OF ULTRA-SONIC CROSS-CORRELATION FLOWMETER

The basic concept of the ultrasonic cross-correlation technique is best described with reference to Figure 4.1. Two ultrasonic systems, each consisting of a transmitter and receiver are arranged to pass parallel beams of radiation through the flowing fluid. Variations in the acoustic properties of the fluid, modulate the received signals in both amplitude and phase. Signals suitable for correlation can be extracted from the received signals by amplitude or phase detectors.

The following analysis is based on the optical cross-beam correlation technique used by FISHER and KRAUSE^(144,143) in their investigations of turbulence. Assume that the flow sensors are situated in a Cartesian coordinate system (x,y,z) with the flow in the x-direction and the ultrasonic beams parallel to the y-axis (Figure 4.2). It is shown in Appendix A that the acoustic intensity of a received beam can be written in the form:

$$I(t) = I_0 \text{ EXP } \left\{ - \int_V K(x,y,z,t) dV \right\}$$

where dV is an elemental volume of the ultrasonic beam of total volume V , I_0 is the acoustic intensity at the transmitter and K is an extinction coefficient at position (x,y,z) and at time t accounting for all possible sources of acoustic attenuation. These include absorption, scattering by particulate matter and bubbles and refraction. Absorption results in a dissipation of energy whereas scattering and refraction represent a redistribution of energy away from the primary beam. All these factors depend on the wavelength of the ultrasonic radiation used. A received ultrasonic signal thus reflects the integral of acoustic disturbances along its entire path.

In the coordinate system used, the signal recorded at the upstream receiver can be written as:

$$I_1(t) = I_0 \text{ EXP } \left\{ - \int_{V_1} K(x,y,z,t) dV_1 \right\}$$

where V_1 is the volume of the upstream ultrasonic beam. If the cross-sectional area, A , of this beam is small so that the extinction coefficient is approximately constant over its area, the above equation can be rewritten as:

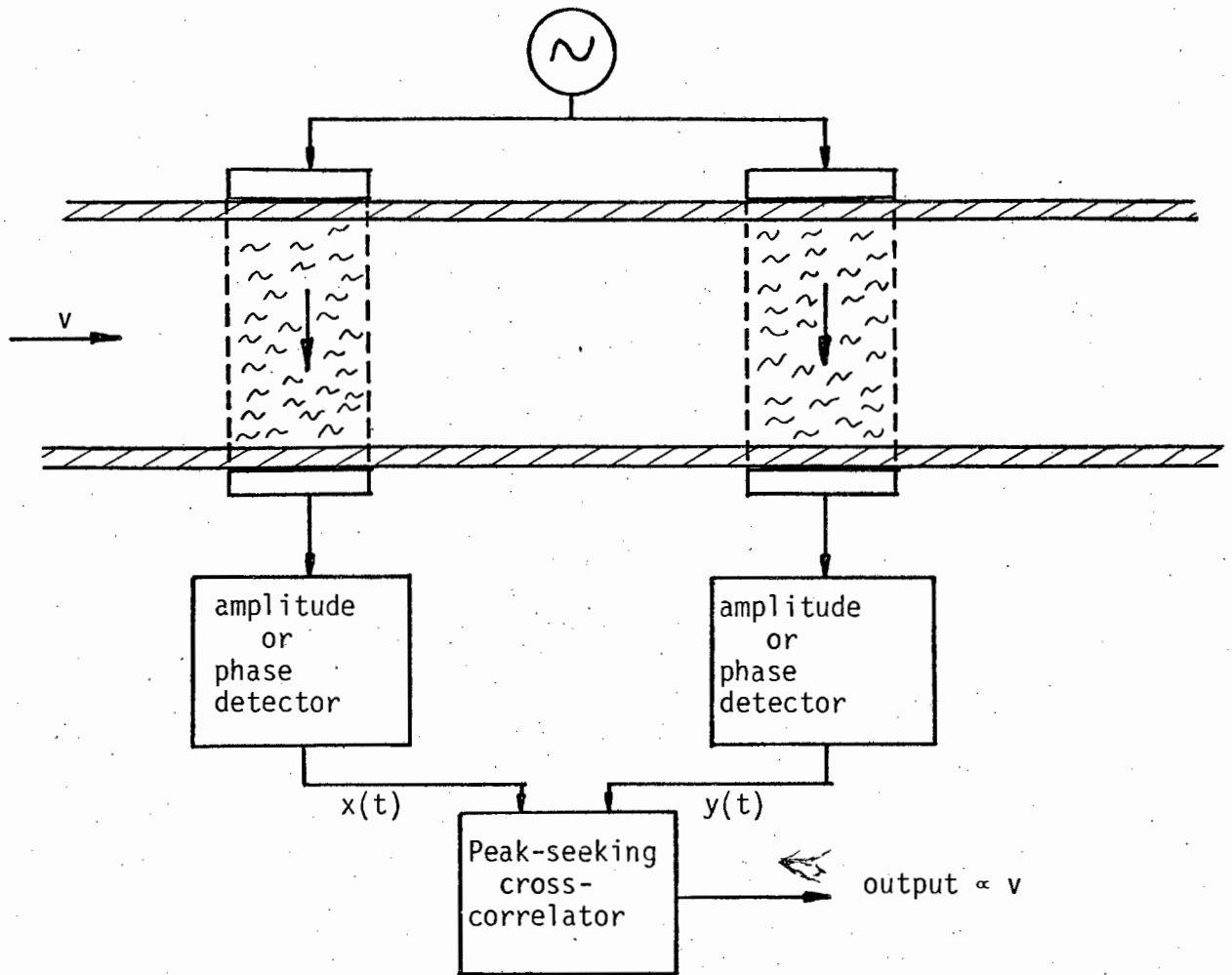


FIG. 4.1 PRINCIPLE OF ULTRA-SONIC CORRELATION FLOWMETERING

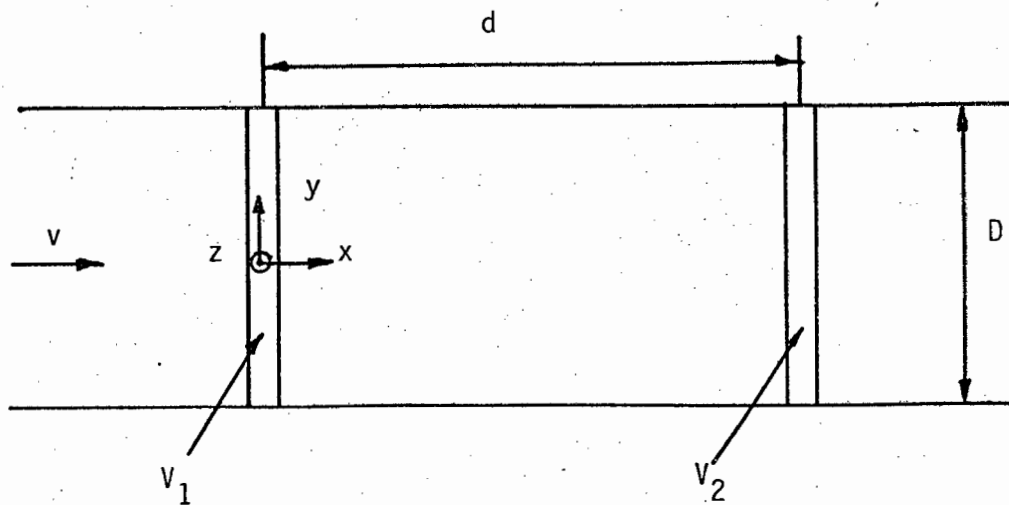


FIG. 4.2 ULTRA-SONIC BEAM CO-ORDINATES

$$I_1(t) = I_0 \text{ EXP } \left\{ -A \int_{-D/2}^{D/2} K(0,y,0,t) dy \right\}$$

The instantaneous extinction coefficient $K(x,y,z,t)$ can be expressed as the sum of its time-averaged mean value $\langle K(x,y,z) \rangle$ together with a temporal fluctuation about this value $k(x,y,z,t)$. As a result:

$$I_1(t) = I_0 \text{ EXP } \left\{ -\int_{V_1} \langle K(x,y,z) \rangle dV_1 \right\} \cdot \text{EXP } \left\{ -\int_{V_1} k(x,y,z,t) dV_1 \right\} \dots\dots\dots (4.1)$$

If the attenuation process is such that the integral of the fluctuations, i.e.,

$$\int_V k(x,y,z,t) dV$$

is small, then equation (4.1) can be linearized using a Taylor series expansion to give:

$$I_1(t) = I_0 \text{ EXP } \left\{ -\int_{V_1} \langle K(x,y,z) \rangle dV_1 \right\} \cdot \left\{ 1 - \int_{V_1} k(x,y,z,t) dV_1 \right\}$$

This requirement does not mean that all fluctuations in the acoustic properties of the fluid must be small. Only the integral of these fluctuations representing the sum of a number of statistically independent events that thus tends to reduce its value, must be small.

If the signal at the receiver is now written as the sum of its time-averaged value $\langle I_1 \rangle$ and a fluctuation relative to this value $i_1(t)$, i.e.,

$$I_1(t) = \langle I_1 \rangle + i_1(t)$$

it can easily be shown that:

$$\langle I_1 \rangle = I_0 \cdot \text{EXP } \left\{ -\int_{V_1} \langle K(x,y,z) \rangle dV_1 \right\}$$

and

$$i_1(t) = -\langle I_1 \rangle \int_{V_1} k(x,y,z,t) dV_1$$

A similar expression can be derived for the output of the downstream receiver, viz.,

$$i_2(t) = -\langle I_2 \rangle \int_{V_2} k(x+d,y,z,t) dV_2$$

where d is the transducer spacing in the flow direction, V_2 is the volume of the acoustic field of the downstream beam and $\langle I_2 \rangle$ the time-averaged value of the downstream signal. An important point to emerge from the above analysis

is that the fluctuations of the received signals $i_1(t)$ and $i_2(t)$, are proportional to the integral of temporal acoustic disturbances occurring within the volumes of their respective acoustic beams.

The cross-correlation function between these two flow signals can now be written as:

$$\begin{aligned} R(d, \tau) &= \frac{1}{T} \int_0^T i_1(t+\tau) \cdot i_2(t) dt \\ &= \langle I_1 \rangle \langle I_2 \rangle \frac{1}{T} \int_0^T \int_{V_1} k(x, y, z, t+\tau) dV_1 \int_{V_2} k(x+d, y, z, t) dV_2 dt \end{aligned}$$

where T denotes a period of integration of sufficient length to yield a statistically stationary value for $R(d, \tau)$. By reversing the order of the spatial and temporal intergration, the above expression can be rewritten as:

$$R(d, \tau) = \langle I_1 \rangle \langle I_2 \rangle \int_{V_1} \int_{V_2} \frac{1}{T} \int_0^T k(x, y, z, t+\tau) \cdot k(x+d, y, z, t) dt \cdot dV_1 \cdot dV_2$$

The interpretation of the term within the double volume integral can be simplified by assuming that the cross-sectional areas of the beams A_1 and A_2 are small so that the extinction coefficients are approximately constant over these areas at any point. Hence:

$$R(d, \tau) = \langle I_1 \rangle \langle I_2 \rangle \cdot A_1 \cdot A_2 \int_{-D/2}^{D/2} \int_{-D/2}^{D/2} \frac{1}{T} \int_0^T k(o, y_1, o, t+\tau) \cdot k(d, y_2, o, t) dt dy_1 dy_2$$

The inner integral represents the space-time correlation at the points (o, y_1, o) and (d, y_2, o) for the value of time delay τ . Non-zero contributions to this double space integral will be obtained only from those pairs of points that experience a common fluctuation.

Assuming steady flow, these points will be restricted to a local region around the line of minimum beam separation, i.e., for $y_1 = y_2$ where a fluctuation incident on the upstream beam subsequently passes through the downstream beam. Furthermore the integral will only have a non-zero value when the time delay τ is equal to the time it takes for fluctuations at the upstream point to reach the downstream point.

Rewriting the inner time average in terms of the root mean square values of the fluctuations at the two points $(0, y_1, 0)$ and $(d, y_2, 0)$ and their space-time correlation coefficient gives

$$R(d, \tau) = \langle I_1 \rangle \langle I_2 \rangle A_1 A_2 \int_{-D/2}^{D/2} \int_{-D/2}^{D/2} \frac{\{k(0, y_1, 0, t + \tau)^2 \cdot k(d, y_2, 0, t)^2\}^{1/2} \cdot \rho(d, y_1 - y_2, 0, \tau) dy_1 \cdot dy_2}{\dots}$$

The two-point space-time correlation coefficient $\rho(d, y_1 - y_2, 0, \tau)$ only has non-zero values for $y_1 \approx y_2$ and $\tau \approx d/v$ and in general decreases with increasing sensor spacing, d , due to the non-frozen nature of the turbulence pattern. As a result the cross-correlation function $R(d, \tau)$ between the two flow signals $i_1(t)$ and $i_2(t)$ will have a peak at a time delay $\tau = d/v$ and can be used to determine the flow velocity of the fluid.

2. MODULATION OF AN ULTRA-SONIC BEAM BY TURBULENCE

In the previous section it was shown that the modulation of two parallel ultrasonic beams directed through a flowing fluid can be cross-correlated to yield flow-velocity. No attempt, however, was made to understand the mechanism producing the amplitude and phase modulation of the received beam. This requires an investigation of acoustic wave propagation in random media. The random medium to be discussed in this section consists of a homogeneous fluid in turbulent flow. The following section considers the propagation of an ultrasonic beam through suspensions. Considerable work, both experimental and theoretical, has been done in the field of wave propagation through turbulent media^(184, 185, 193, 195, 197, 198, 202-204, 212, 213) and only a brief summary of some of the main results that are applicable directly to correlation flow-metering using ultrasonic sensors will be presented.

When a constant-amplitude sinusoidal acoustic plane wave propagates through a region of random velocity fluctuations, the received wave will exhibit random amplitude and phase fluctuations. The acoustic wave is a propagating pressure disturbance so that the speed of sound is constant relative to the fluid itself and not relative to the fixed sensor coordinates. The turbulent velocity fluctuations in the direction of sound propagation will cause the sound wave-front to be advanced or retarded. Also in the turbulent flow-field, the spatially random distribution of turbulent velocities in the plane perpendicular to the wave propagation will redistribute the sound energy fluctuating amplitude. The first effect produces the phase modulation and the second the amplitude modulation of the received beam.

LIGHTHILL⁽¹⁸⁶⁻¹⁸⁹⁾ has shown that the equation for a sound wave having passed through a turbulent fluid is given by

$$\frac{\partial^2 \rho}{\partial t^2} - C_0^2 \nabla^2 \rho = \sum_{i,j=1}^3 \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

where ρ is the density, C_0 the velocity of sound in the stationary medium and T_{ij} a stress tensor that describes the interaction between the sound wave and the turbulent flow. If a low intensity sound wave is assumed as well as a flow velocity very much less than the sound velocity and if viscous and heat conduction effects are ignored, then it can be shown that

$$T_{ij} = 2 \rho_0 \tilde{v}_i \cdot v_j$$

where ρ_0 is the density of the stationary fluid

$\tilde{v}_i$ the sound velocity

v_j the instantaneous flow velocity

WITTEK^(184,185) uses the above equations and shows that the total signal arriving at a point r at time t is:

$$S^T(r,t) = S^0(r,t) + S^1(r,t)$$

It consists of a high frequency carrier $S^0(r,t)$ arriving at r at time t in the absence of turbulence to which a low frequency turbulence dependent signal $S^1(r,t)$ is added. The average signal $G(t)$ detected by a receiver at time t is the integral of $S^T(r,t)$ over the cross-sectional area, A , of the receiver i.e.,

$$G(t) = \frac{1}{A} \int_A S^T(r,t) dA$$

WITTEK then calculates that $G(t)$ is of the form

$$G(t) = A(t) \cdot \cos\{w \cdot t + B(t)\}$$

where $A(t)$ and $B(t)$ are dependent on the turbulence of the flow. In other words, an ultrasonic beam propagating through a turbulent medium is modulated in both amplitude and phase by the turbulence. Furthermore information about the turbulence can be obtained from these modulations.

3. PROPAGATION OF ULTRA-SOUND THROUGH SLURRIES

An ultrasonic beam propagating through a fluid will be attenuated due to number of factors. In a homogeneous fluid the main losses are viscous losses, heat conduction losses and losses associated with molecular exchanges of energy. The viscous losses result from the relative motion occurring between various portions of the medium during the compressions and expansions that accompany the transmission of a sound wave. Based on the assumption that the pressure variations produced by a sound wave are adiabatic and consequently are accompanied by changes in temperature, there is a tendency for heat to be conducted from regions of condensation where the temperature is raised to neighbouring regions of rarefaction where the temperature is lowered. In the process of this heat transfer there is a move towards pressure equalization which reduces the amplitude of a wave as it propagates through the medium. The dissipation of acoustic energy that is associated with the changes in the molecular structure of the medium results from the finite time that is required for these changes to take place.

If solid particles or gas bubbles are present in the fluid, additional losses occur. These can be divided into non-dissipative losses such as scattering, reflections and refraction and dissipative losses caused by viscous and thermal transport processes at the interfaces of the inhomogeneities.

The attenuation of an ultrasonic beam propagating through an aerated slurry increases with frequency and depends on fluid properties such as viscosity, thermal conductivity, density and molecular structure; on particle properties such as size, shape, concentration, density and elasticity and bubble properties such as size, compressibility and surface tension^(194, 199, 201, 205-207, 214-219) give both experimental and theoretical results in which the above loss mechanisms are investigated. Due to the complex dependence of the acoustic attenuation on fluid, particle and bubble properties, no attempt will be made to summarize these results. Suffice it to say that in the design of ultrasonic sensors for correlation flowmetering, care must be exercised in the choice of sensor frequency and power for any application in order for the received signal to be of sufficient amplitude for flow signals to be recovered.

4. ULTRA-SONIC SENSOR DESIGN CONSIDERATIONS

Ultrasonic sensors for obtaining flow signals consist of a transmitter and receiver mounted on opposite sides of a pipe. This section investigates the requirements of both the transmitter and receiver and describes some of the characteristics of ultrasonic transducers. Factors which need investigation are:

- (i) beam-width
- (ii) transducer size
- (iii) frequency of operation
- (iv) sensitivity
- (v) bandwidth
- (vi) impedance matching (electrical and acoustical)

4.1 Ideal Ultra-sonic Sensors

To commence with, a description of ideal ultrasonic sensors for correlation flowmetering is given. Such sensors should transmit a narrow collimated beam of ultrasound through the flowing fluid. The narrower the beam, the easier it will be to determine the distance between the upstream and downstream transducers. The sample of fluid being interrogated by the beam is also smaller resulting in reduced spatial averaging of the acoustic discontinuities and, hence a sharper cross-correlation peak. In addition, cross-coupling between upstream and downstream transducers will be reduced. The sensors should be of a clamp-on type and have good sensitivity. Reflections at interfaces should be minimized and unwanted transmission paths from transmitter to receiver such as feedthrough along the pipewall should be eliminated. The wavelength of the ultrasonic radiation in the fluid must be such that turbulence and other acoustic disturbances modulate the ultrasonic beam. Care must also be taken to ensure that the attenuation at the frequency of the radiation used is not too large to cause the signal to noise ratio at the receiver to be too small.

4.2 Types of Sensors

Of the numerous ultrasonic transducers available amongst which piezo-electric, magnetostrictive, electromagnetic and electrostatic devices are most common⁽²⁴¹⁾, circular piezo-electric discs operated in their thickness mode of vibration are best suited for correlation flowmetering. In this mode the electric polarization is parallel to the principal axis of expansion and contraction as shown in figure 4.3. Such discs are physically small and strong, are relatively easy to use and are available in various diameters and thicknesses. In addition, the characteristics of circular sensors is wellknown.

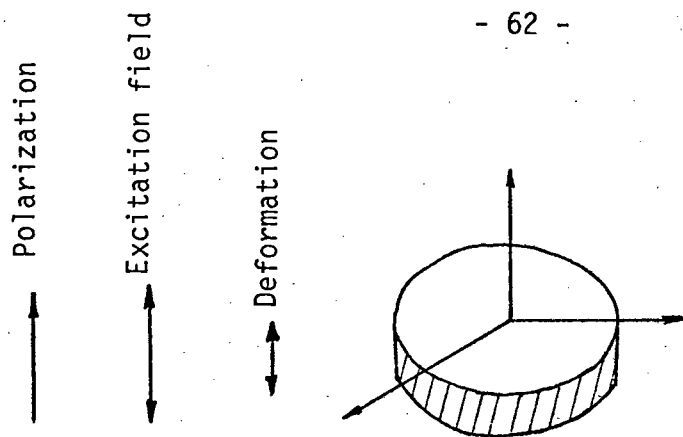


FIGURE 4.3 : Thickness Mode Piezo-electric Discs

4.3 Sound Intensity Distribution

Assume that the disc operates with piston-like motion, i.e., it has a uniform velocity distribution over its radiating surface. The axial intensity of the resulting radiation is: (Ref. 239.)

$$I = 2 \rho c U_p^2 \sin^2 \left\{ \pi \left(\frac{R}{\lambda} \right) \left[\sqrt{\left(\frac{D}{R} \right)^2 + 1} - \frac{D}{R} \right] \right\}$$

where ρ is the density of the medium into which the disc is radiating, c is the velocity of sound in the medium, U_p is the maximum velocity when the velocity of the piston is given by $u = U_p e^{j\omega t}$, R is the transducer radius, D the distance from the transducer face and λ the ultrasonic wave-length in the medium. The function $I/I_{\max}$ where $I_{\max} = 2 \rho c U_p^2$ is shown in Figure 4.4 for $R/\lambda = 10$.

Two distinct zones can be seen. In the region near the surface of the piston, the intensity undergoes large variations. With increasing distance from the surface the intensity runs through a series of maxima of constant amplitude with intervening nulls. The last null occurs in the vicinity of $D = R^2/2\lambda$. At still greater distances the intensity goes through its final maximum and then when $D \gg R^2/\lambda$, decreases inversely as the square of the distance. The first region is generally known as the nearfield of the piston source whereas the latter region is called the far-field.

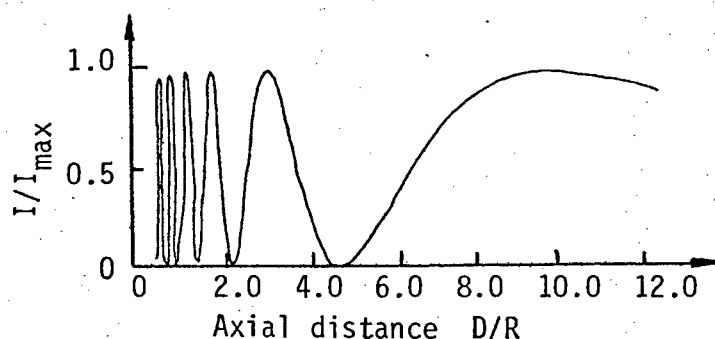


FIGURE 4.4 Axial intensity variations for a circular piston source

4.4 Beam Width and Radiation Patterns

The directivity of the beam in the farfield of a piston source oscillating in an infinite baffle has been determined to be⁽²³⁹⁾:

$$d = 2J_1\{2\pi(\frac{R}{\lambda}) \sin \theta\} / 2\pi(\frac{R}{\lambda}) \sin \theta$$

where J_1 indicates a first-order Bessel function of the first kind, θ is the angle between the piston axis and a line through the centre of the crystal and R and λ are respectively the piston radius and wavelength of the radiation. It is seen that the directivity is simply a function of θ for any given transducer and medium (R and λ fixed). The equation for the directivity is plotted in Figure 4.5. The first zero occurs at an angle given by:

$$\frac{2}{\pi}(\frac{R}{\lambda}) \sin \theta = 3.83$$

$$\text{i.e., } \sin \theta = 0.61.(\frac{\lambda}{R})$$

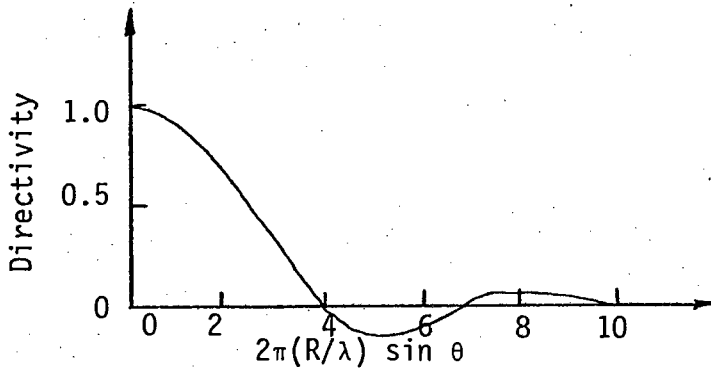


FIGURE 4.5 : Directivity Plot for a Circular Piston Source

Therefore, the farfield main-lobe of a piston source is theoretically confined within a cone of half-angle θ as determined by the above equation.

Figure 4.6 shows polar plots illustrating the manner in which the radiation pattern of a piston of constant radius R and source strength Q varies with frequency in the far field. It is observed that the axial intensity is much greater at high frequencies than it is at low frequencies. The directivity of the radiation thus increases and side-lobes appear as the frequency is increased.

The beam-shape for a piston is shown in Figure 4.7. The point A indicates the position of the last axial intensity maximum. It is seen that the beam does not diverge beyond the piston diameter until well beyond the last axial

maximum. In the nearfield (the FRESNEL zone) the beam is approximated by a cylinder of radius equal to the radius of the piston whereas in the farfield (the FRAUENHOFER zone) the beam diverges with a half beam angle given by:

$$\theta = \sin^{-1}(0,61 \frac{\lambda}{R})$$

The dividing point between the far and near fields is thus not situated at the position of the last axial intensity maximum R^2/λ although it is commonly assumed to be at this point. Ref.(223,232,235) provide further details on the beam behaviour in the near field of an oscillating piston.

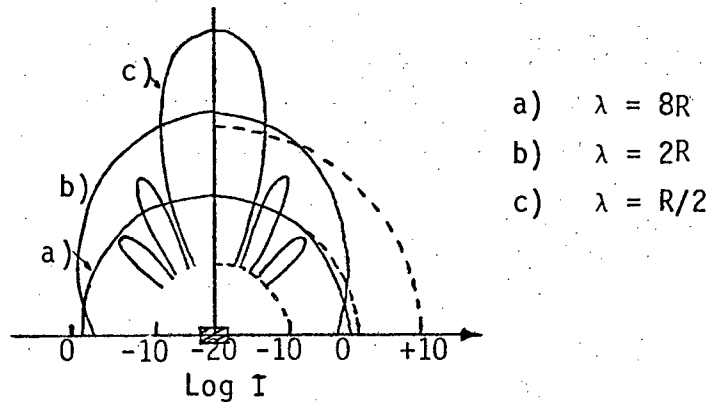


FIGURE 4.6 : Polar Plots for a Circular Piston Source

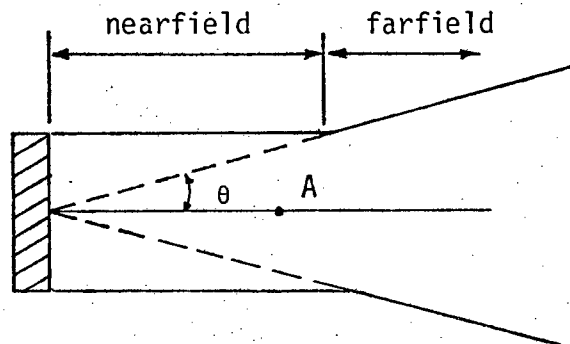


FIGURE 4.7 : Beam-shape for a Piston Source of Radius R for $R/\lambda = 5$ ($\theta=7^\circ$). Nearfield drawn as cylinder. 'A' - Position of last axial maximum

4.5 Sensitivity

The sensitivity of the receiving transducer depends amongst other factors on the area of its receiving face. Similarly, the power radiated by the transmitting transducer is also dependent on its surface area. Hence the larger the transducers, the better the overall sensitivity. In addition the far field beam width is reduced. However, by increasing this area the diameter of the nearfield beam is also increased and hence a compromise between these various factors is required.

Returning to the ultrasonic sensor requirements for correlation flowmetering, it can be seen that the narrow beam requirement can be met by choosing $R \gg \lambda$. However, care must be taken to ensure that the pipe diameter, d , at least exceeds the approximate distance from the radiating transducer to the nearfield/farfield transition point, i.e., $d \geq R^2/\lambda$. This is so because in the nearfield the acoustic intensity varies considerably as shown by Figure 4.4. The optimum pipe diameter is given by $d = R^2/\lambda$ since then the last intensity maximum will exist at the receiving transducers. Table 4.1 summarizes the above parameters for a piston radiating into water.

4.6 Non-uniform Velocity Distributions

In some instances a piezo-electric disc may not move as a piston but may have a velocity distribution that is a minimum at the edges and a maximum at the centre. This alters both the nearfield and farfield radiation patterns. DEKKER et al⁽²³⁰⁾ calculated the radiation patterns for the following two cases.

(i) a disc with simply supported edges having a velocity distribution:

$$u = u_p \cdot \{1 - (r/R)^2\}$$

(ii) a disc with clamped edges and a velocity distribution:

$$u = u_p \{1 - (r/R)^2\}^2$$

The results obtained for these velocity distributions are strikingly different from those of a piston-like source. The axial intensity has no zeroes although there are axial maxima and minima. The ratio of maximum and minimum axial intensity is considerably less than the axial oscillations of a piston-like source. The angle of divergence of the main lobe of the radiation is greater than that of a piston. However, the side-lobes are less. The position of the last axial maximum also moves progressively closer to the transducer face. It has been calculated that for a Gaussian velocity distribution on the radiating face the nearfield axial intensity ripple will be eliminated. In the farfield the half beam angle is, however, considerably increased (approximately 60 per cent for $R/\lambda = 10-20$).

4.7 Matching and Damping Layers

The method of mounting the transducers to the pipe wall affects the total power radiated by the transmitter into the fluid and the sensitivity and band width of the receiving transducer. Wherever there is a boundary between two media having different characteristic impedances, reflections of acoustic energy incident on the boundary occur. If Z_1 and Z_2 are the characteristic

Radius R mm	Area $A = \pi R^2$ cm ²	R / λ	NF - FF Transition PT R^2/λ cm	Half Beam angle $\theta_0 = \text{SIN}^{-1}$ (,61 λ /R)	
2,5	0,196	0,833	0,208	44,1	$f = 500 \text{ kHz}$ $\lambda = 3 \text{ mm}$
5,0	0,786	1,666	0,833	21,5	
7,5	1,767	2,500	1,875	14,1	
10,0	3,142	3,333	3,333	10,5	
15,0	7,068	5,000	7,500	7,0	
20,0	12,566	6,666	13,333	5,3	
2,5	0,196	1,666	0,417	21,5	$f = 1 \text{ MHz}$ $\lambda = 1,5 \text{ mm}$
5,0	0,786	3,333	1,666	10,5	
7,5	1,767	5,000	3,750	7,0	
10,0	3,142	6,666	6,666	5,3	
15,0	7,068	10,000	15,000	3,5	
20,0	12,566	13,333	26,666	2,6	
2,5	0,196	3,333	0,833	10,5	$f = 2 \text{ MHz}$ $\lambda = 0,75 \text{ mm}$
5,0	0,786	6,666	3,333	5,3	
7,5	1,767	10,000	7,500	3,5	
10,0	3,142	13,333	13,333	2,6	
15,0	7,068	20,000	30,000	1,7	
20,0	12,566	26,666	53,333	1,3	
2,5	0,196	8,333	2,083	4,2	$f = 5 \text{ MHz}$ $\lambda = 0,3 \text{ mm}$
5,0	0,786	16,666	8,333	2,1	
7,5	1,767	25,000	18,750	1,4	
10,0	3,142	33,333	33,333	1,0	
15,0	7,068	50,000	75,000	0,7	
20,0	12,566	66,666	133,333	0,5	

TABLE 4.1 : Beam Characteristics of Piston-like Sources
Radiating into Water

impedances of the two media, then the reflection coefficient for an ultrasonic wave propagating from Z_1 to Z_2 is

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Thus, if $Z_2 = Z_1$, $r = 0$ and no reflection occurs. In correlation flowmetering such boundaries exist between the

- (i) transducer and pipewall, and
- (ii) the pipewall and fluid.

Many papers are devoted to the analysis of single and multiple impedance matching layers for increasing the energy transmitted through these interfaces (222, 226, 227, 229). Figure 4.8 shows a variety of methods for improving the acoustical coupling between the transducer and the fluid. In Figure 4.8(a) no coupling or backing layers are used. The transducer is air-backed and assuming it is made of the wellknown material PZT5 and a steel-pipe is used, approximately 25 per cent of the energy incident on the transducer pipewall/ interface will be reflected. Similarly, about 98 per cent of the energy incident on the pipewall/water interface is reflected. A high quality factor, Q , transducer having a narrow bandwidth results with low power transfer efficiency into the fluid. If the transducer is backed by an absorbing medium with similar characteristic impedance to the transducer as shown in Figure 4.8(b), a reduction in transducer, Q , will be observed but, the power transmitted into the fluid will not be increased. If an airbacked transducer is matched to the fluid with a quarter wavelength layer having a characteristic impedance equal to the geometric mean of the characteristic impedances of the transducer and fluid, considerably more power will be transmitted into the fluid (neglecting the pipewall). By suitably backing the above arrangement as shown in Figure 4.8(d), the Q of the transducer can be decreased while still maintaining good power transfer into the load.

Further improvements in the acoustic power transferred from the transducer into the fluid and vice versa can be achieved by using two or more matching layers of definite thickness and characteristic impedance as shown in Figure 4.8 (e) and (f). The references listed previously provide further information regarding such arrangements.

To summarize, a transducer radiating a continuous wave into a medium should be air-backed and matched to the medium in order to maximize the power radiated into the load. A receiving transducer should also be matched to the medium

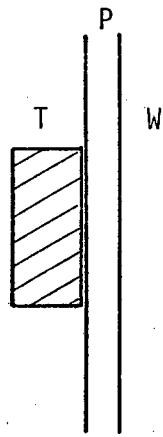


FIG. 4.8a

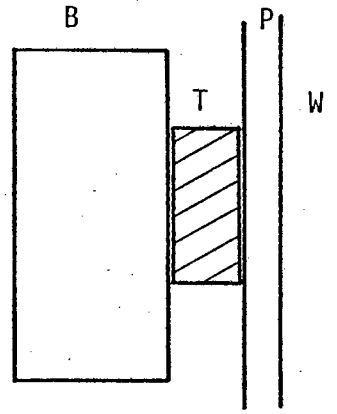


FIG. 4.8b

T: Transducer
P: Pipe-wall
W: Fluid
M: Matching layer
B: Backing layer

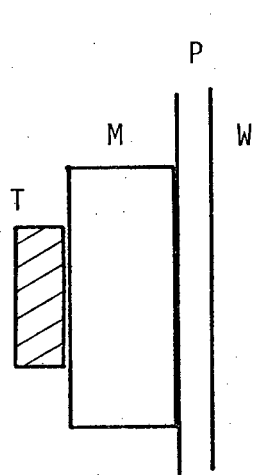


FIG. 4.8c

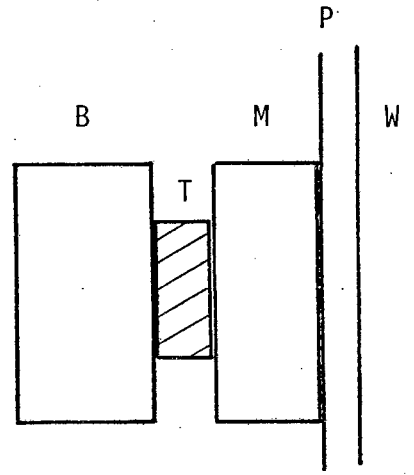


FIG. 4.8d

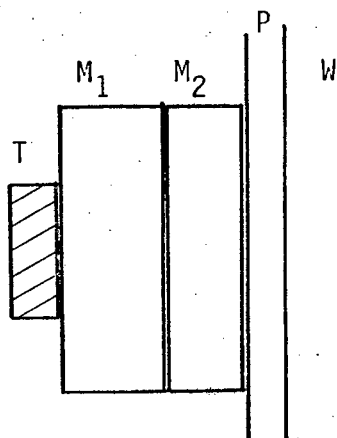


FIG. 4.8e

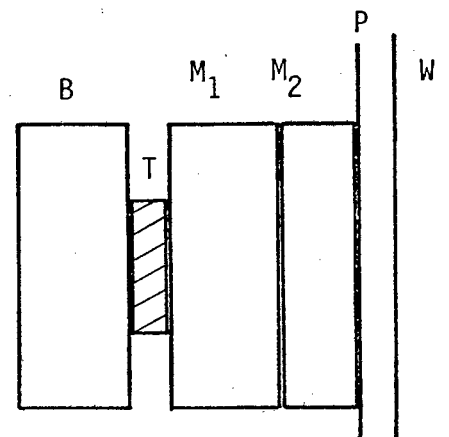


FIG. 4.8f

FIG. 4.8 VARIOUS METHODS OF ACOUSTIC IMPEDANCE MATCHING

from which it is being irradiated. If a narrow band, high-sensitivity receiver is required, no matched backing layer is necessary. If, however, a wide-bandwidth receiver is required, an absorbant damping layer of the same characteristic impedance as the transducer can be used as a backing.

4.8 Electrical Matching

The performance of piezo-electric transducers can be further improved by electrically matching the transducers to their transmitting or receiving circuitry. The power transferred from the transmitter circuitry to the transmitting transducer and from the receiving transducer to the receiving circuitry can be maximized by such electrical matching. The equivalent circuit for a piezo-electric transducer near resonance, is shown in Figure 4.9 where L_m , C_m and R_m represent the mechanical vibrator which is characteristic of the transducer itself^(238, 239). Since the transducer consists of a dielectric between two electrodes, it appears as a capacitor C_0 in the equivalent circuit. These parameters are constant (independent of frequency) only if the vibrator has no other mode of motion near the resonance considered.

Only part of the current supplied to the transducer is used to maintain its vibrations. The remainder is short-circuited through the capacitance C_0 . This may be prevented by connecting a coil of inductance L_0 across the sensor to tune out C_0 . The value of L_0 is chosen such that

$$\omega_0 = 1/\sqrt{L_0 \cdot C_0}$$

where ω_0 is the frequency of operation of the sensor. Maximum efficiency is obtained by matching the impedance of the driving source R_s to the impedance R_m of the transducer by means of a transformer. The number of turns, n_p , of the primary winding is chosen to give $(n_p/n_s)^2 = R_s/R_m$ where n_s is the number of turns of the secondary coil (Figure 4.10).

The input impedance of the receiving circuitry should be similarly matched to the transducer.

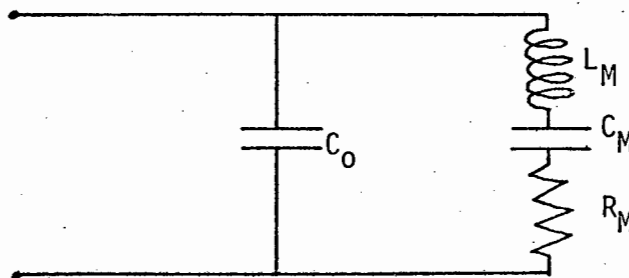


FIGURE 4.9 : Equivalent Circuit of Piezo-electric Transducer near Resonance

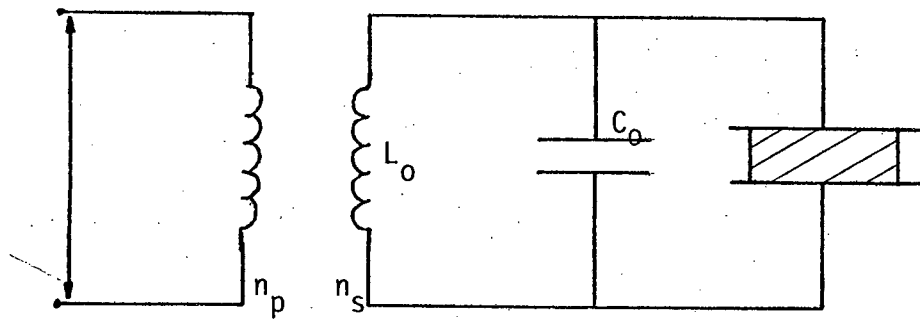


FIGURE 4.10 : Electrical Matching of Piezo-electric Transducer

5. DESIGN OF ULTRA-SONIC FLOW SENSORS

The sensors described in this section consist essentially of three components, namely:

- (i) the primary electro-acoustic device that generates and receives ultrasonic waves,
- (ii) the case that supports this device, and
- (iii) the mount that clamps the case onto a pipe.

Although the transmitting and receiving sensors have different functions and hence design criteria, identical sensors for both transmitter and receiver are used for simplicity. In general, the requirements of flow sensors for correlation flowmetering are not critical, so the design of such flow sensors is simplified and satisfactory results should be obtainable for non-optimized sensor designs. This is one of the major advantages of correlation flowmetering since the performance of the flowmeter is independent of sensor characteristics. The only requirement of the sensors is that they convert a random flow discontinuity into an electrical signal.

5.1 The Piezo-electric Transducer

Piezo-electric transducers operated in their fundamental thickness mode of vibration are used for transmitting and receiving longitudinal waves into the fluid. The piezo-electric effect occurs naturally in certain materials (e.g., quartz, Rochelle salt) but can be induced in various ceramics during production. The effects can be defined as electrical polarization produced by mechanical strain. In certain classes of crystal the polarization

is proportional to the strain and changes sign with it. The piezo-electric effect is reversible, that is, electric polarization produces mechanical strain (see ref. 233,234,238).

A large number of ceramics exhibiting piezo-electric properties are available. Depending on their composition, they have different characteristics and some are more suitable for a given application than others. Circular lead-zirconate-titanate discs 10 mm diameter and 1 mm thick were used in the design that follows because they were readily available from ROSENTHAL TECHNIK A.G. This material is very similar to the wellknown PZT-5 produced by CLEVITE CORPORATION, USA. It is suited to low-power transducers and its low quality factor ($Q_m = 90$) permits wide-band application. Its high coupling factor and piezo-electric coefficients make it attractive for uses such as hydro-acoustic, solid-conducted sound systems, air-borne sound and electro-mechanical transducers. In addition, SONOX^R P5 has a Curie temperature of 340°C, above which the piezo-electric effect disappears. This high temperature (c.f. 120°C for Barium-titanate) enables it to be used in most industrial environments. Further details on the properties of SONOX^R P5 can be found in APPENDIX B.

The discs are operated in their fundamental thickness mode of resonance in which the disc is half a wavelength thick. The resonant frequency is thus given by:

$$t = \frac{\lambda}{2} = \frac{c}{2 \cdot f}$$

$$\therefore f = \frac{c}{2 \cdot t} = \frac{4000}{2 \times 1 \times 10^{-3}} = 2,0 \text{ MHz}$$

since the velocity of sound in SONOX^R P5 is approximately 4000 m/sec.

Using the results of the previous sections and assuming piston - like motion, the polar radiation pattern of this disc radiating directly into water is shown in Figure 4.11 on a linear scale. The beamwidth of the major lobe is about 4,4° and hence the sensor requirement of a narrow collimated beam is satisfied. The position of the last axial maximum is at a distance of approximately 13 mm from the transducer face (see Table 4.1). The pipe diameters on which the sensors are to be used should preferably exceed this size, although since the preceding axial minimum is at about 7 mm, pipes of diameter greater than 8 mm are acceptable. The above results have neglected the pipewall, the transducer mounting arrangement and possible non-uniform velocity distributions, and are merely approximate values serving as guidelines.

FIG. 4.10 POLAR RADIATION PATTERN OF FLAT PISTON TRANSDUCER

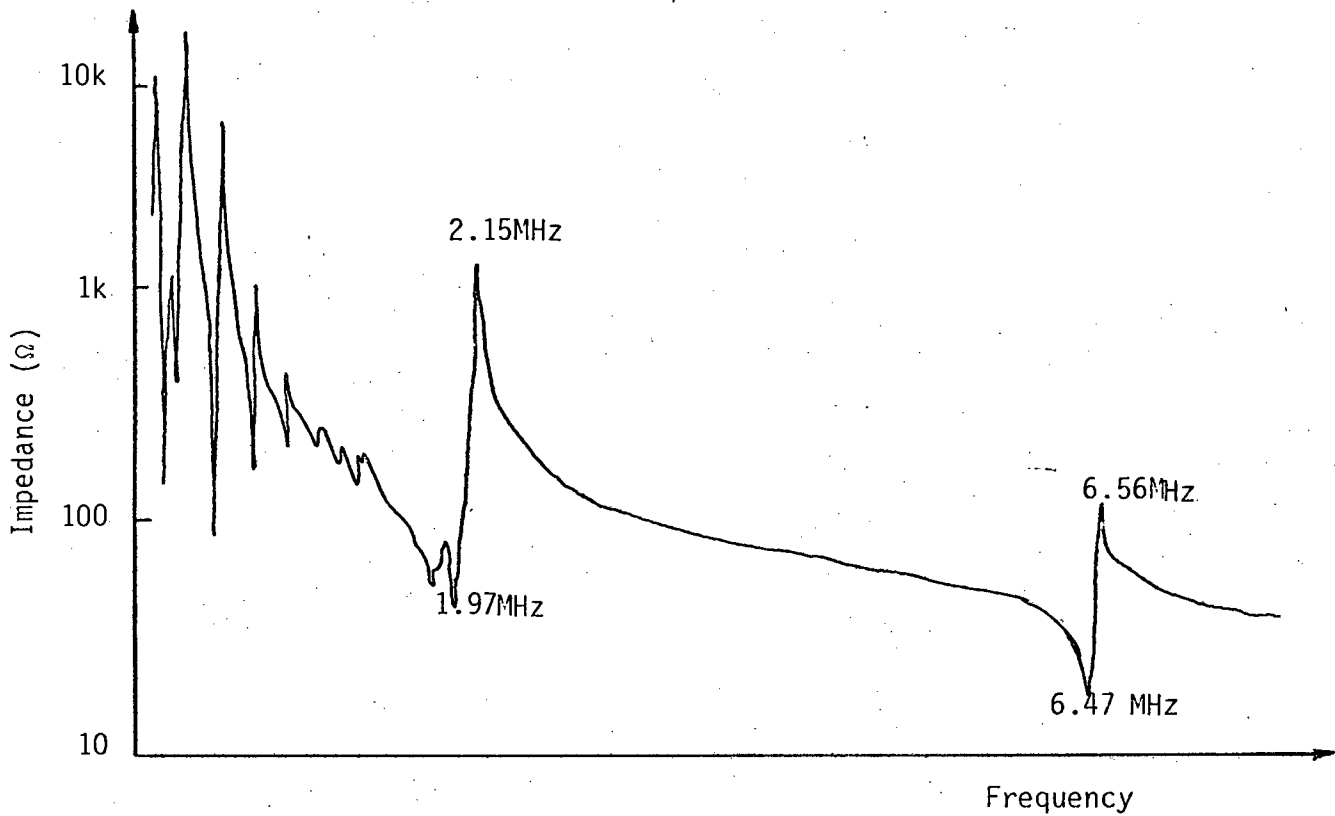
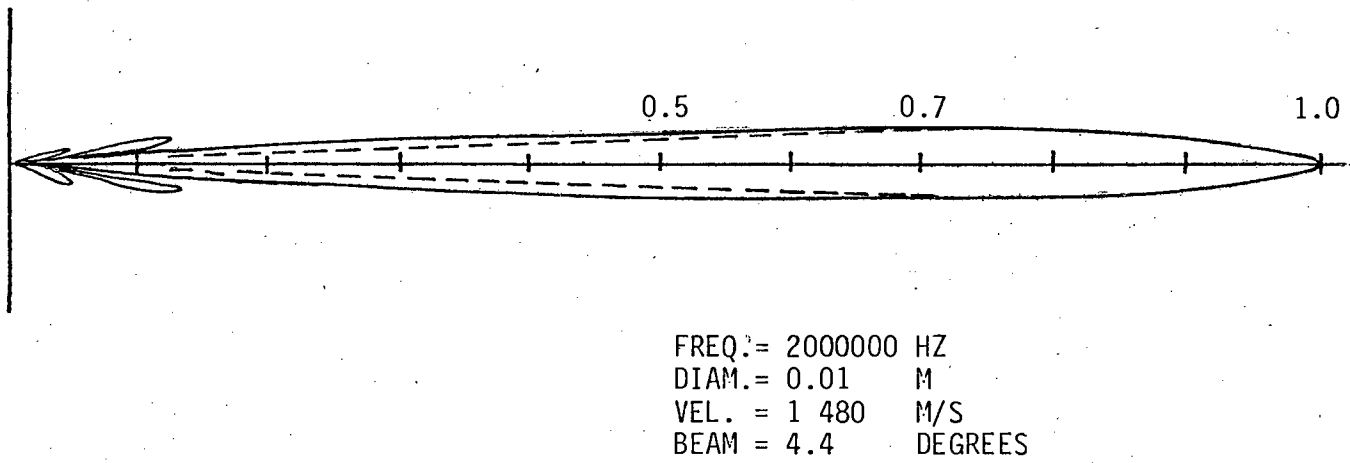


FIG. 4.12 IMPEDANCE PLOT OF PIEZO-ELECTRIC TRANSDUCER IN AIR

The impedance of a disc in air was measured using a network analyser and is shown in Figure 4.12. It exhibits the typical resonance/anti-resonance characteristic at the fundamental thickness mode of vibration (2MHz) and the higher harmonics (6 MHz, 10MHz, etc.) The resonant point yields an impedance minimum and occurs at the series resonance of the motional branch of the equivalent circuit. This is the desired frequency of operation since the acoustic output will be a maximum at this point. The anti-resonance frequency yields an impedance maximum and occurs at the parallel resonance of the two branches of the equivalent circuit. In addition to the thickness mode, various other low frequency modes of vibration are evident from Figure 4.12. These are due to radial and flexural modes, amongst others (see Ref. 234).

5.2 The Case

The piezo-electric disc on its own is not suited for mounting onto a pipe and is enclosed within a holder which protects the transducer from the environment and contains suitable backing and matching layers as well as making electrical contact. A cross-sectional diagram of a transducer case is shown in Figure 4.13. It consists of a copper tube of about 15 mm diameter. The air-backing is provided by placing a piece of foam rubber on its rear surface.

A piece of silicone rubber of 2 mm thickness is placed on the front surface of the transducer. This protects the front surface and acts as a soft layer for making contact with the pipe-wall. Silicone rubber is suitable for this application since it can stand up to severe environmental conditions, it is acoustically not very lossy and is reasonably well matched to water (ref. 236). Thin sheets of aluminium foil are glued onto both electrodes, care being taken to ensure that no airgaps are present. Electrical leads are subsequently attached to these sheets of foil and brought out at the back. RTV silicone rubber is then poured into the copper tube from the rear and allowed to set.

A short plunger is inserted as shown in Figure 4.13, and a B.N.C. co-axial connector attached to the electrical leads. The copper tube, in addition to providing a protective case also ensures adequate electrical and magnetic shielding.

5.3 The Mounting

Since one of the main requirements is to have clamp-on flow sensors that easily can be attached to and removed from a pipe and will fit pipes of various diameters, a mounting arrangement, as shown in Figure 4.14, is

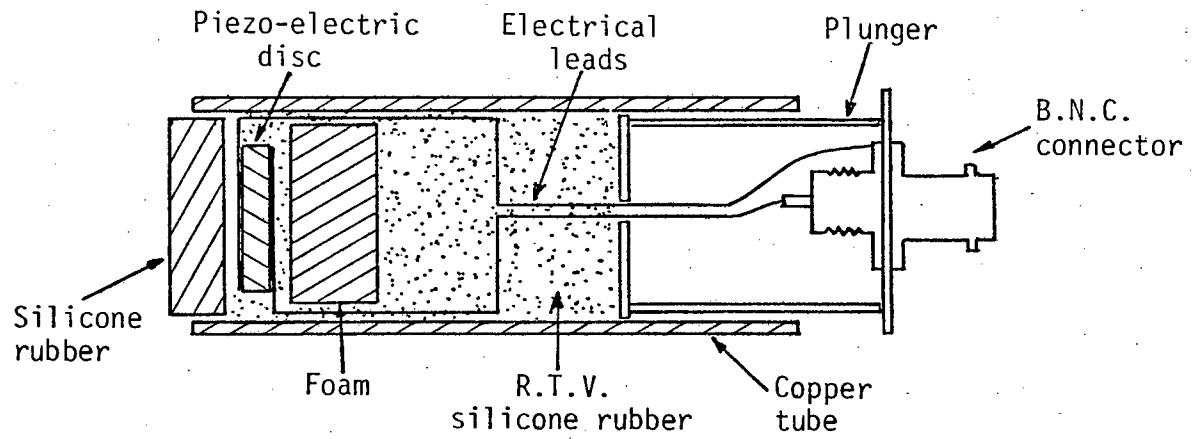


FIG. 4.13 CROSS-SECTIONAL VIEW OF TRANSDUCER CASE

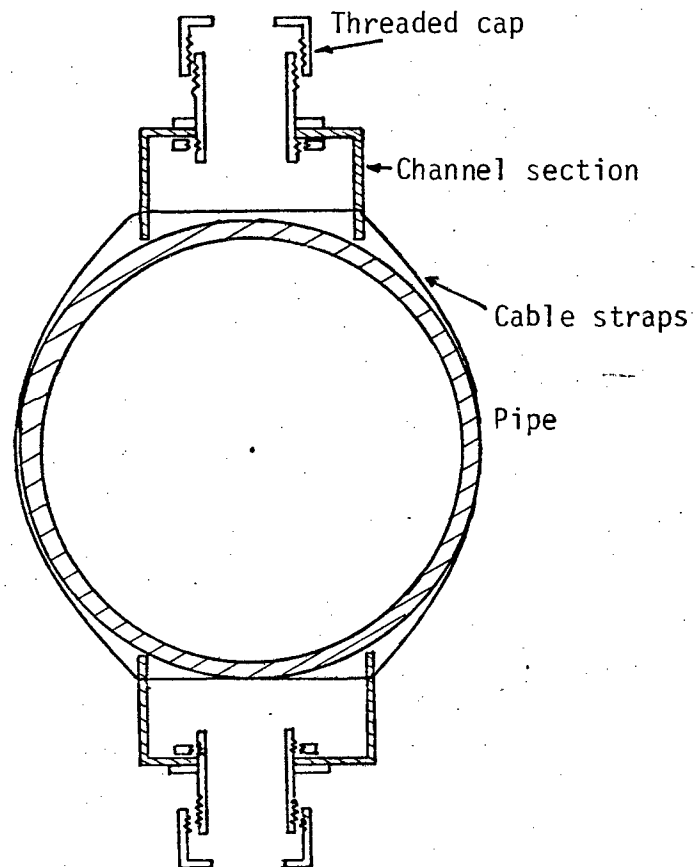


FIG. 4.14 DIAGRAM OF TRANSDUCER MOUNTING

adopted. The mount consists of a section of aluminium or steel channel through which a threaded bulkhead connector is passed. The internal diameter of this pipe-fitting equals the outer diameter of the copper tube used for the transducer case. Two similar channel sections are strapped onto opposite sides of a pipe using cable ties. The transducer case is passed through the pipe fittings and a threaded cap presses the silicone rubber front surface of the case onto the pipe. The contact pressure can be controlled by advancing the cap. A layer of silicone grease is placed between the surface of the pipewall and the front surface of the case to fill in any irregularities that may exist on the pipewall. This is necessary to ensure good coupling and eliminate any air-gaps. Since the characteristic impedance of air is very different from the impedance of the other materials, the slightest air gap will result in virtually total reflection of the ultrasonic energy incident on this gap.

The impedance of the mounted transducer is shown in Figure 4.15. It still exhibits the typical resonance/anti-resonance characteristic at approximately 2MHz, however, the Q is reduced. In addition, some of the other low-frequency modes of vibration have been damped.

The voltage transfer function between transmitter and receiver for a 10 mm diameter water filled perspex pipe is shown in Figure 4.16. The responses of both pairs of sensors are similar and both have a Q of about 5 and a bandwidth of 400 kHz. The multiple minor peaks are attributable to additional resonances caused by reflections at the pipewall.

Clearly the flow sensors described above have not been optimized. Additional damping of the receiver can be achieved by backing it with a matched absorbing layer. No care has further been taken in matching the front of the sensors to the pipewall or fluid. Since this would vary with pipe material and wall thickness, different matching layers would be required for different pipes, limiting the applicability of a specific flow sensor. Electrical matching has also not been included at this stage. It is, however, possible to include a transformer of suitable turns ratio into the plunger of the transducer case, should electrical matching be found necessary.

6. DESIGN OF SIGNAL-PROCESSING CIRCUITRY

This section describes the electronic circuitry for extracting flow signals from the received ultrasonic radiation. Suppose the transmitted ultrasonic signal is of the form:

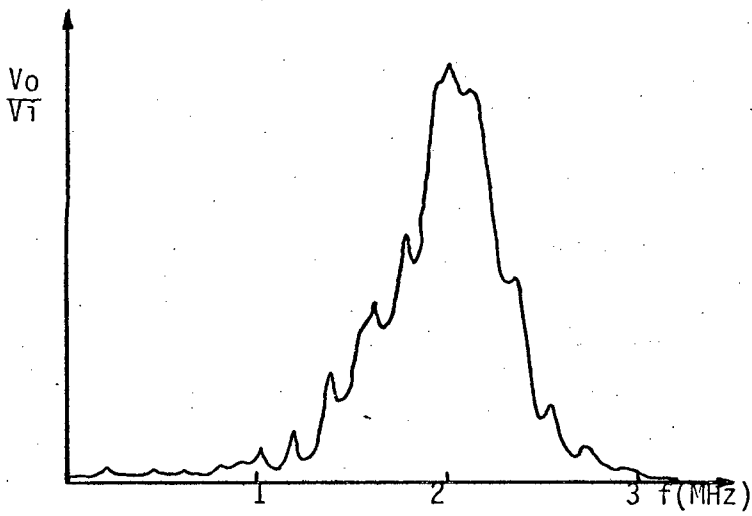


FIG. 4.16a VOLTAGE TRANSFER FUNCTION FOR UPSTREAM TRANSDUCER PAIR

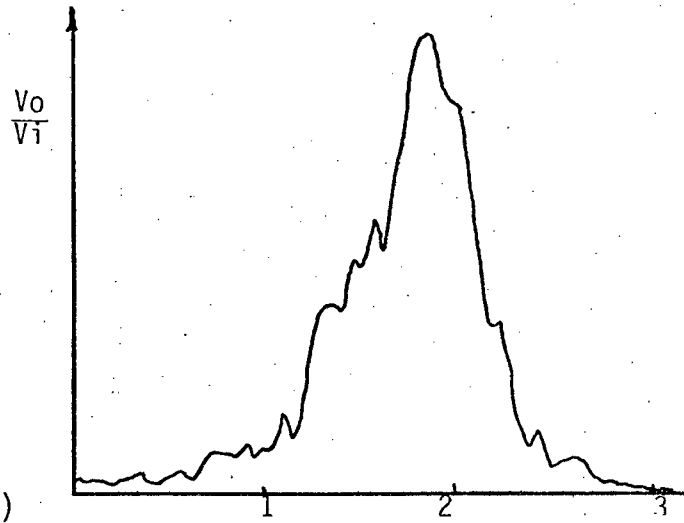


FIG. 4.16b VOLTAGE TRANSFER FUNCTION FOR DOWNSTREAM TRANSDUCER PAIR

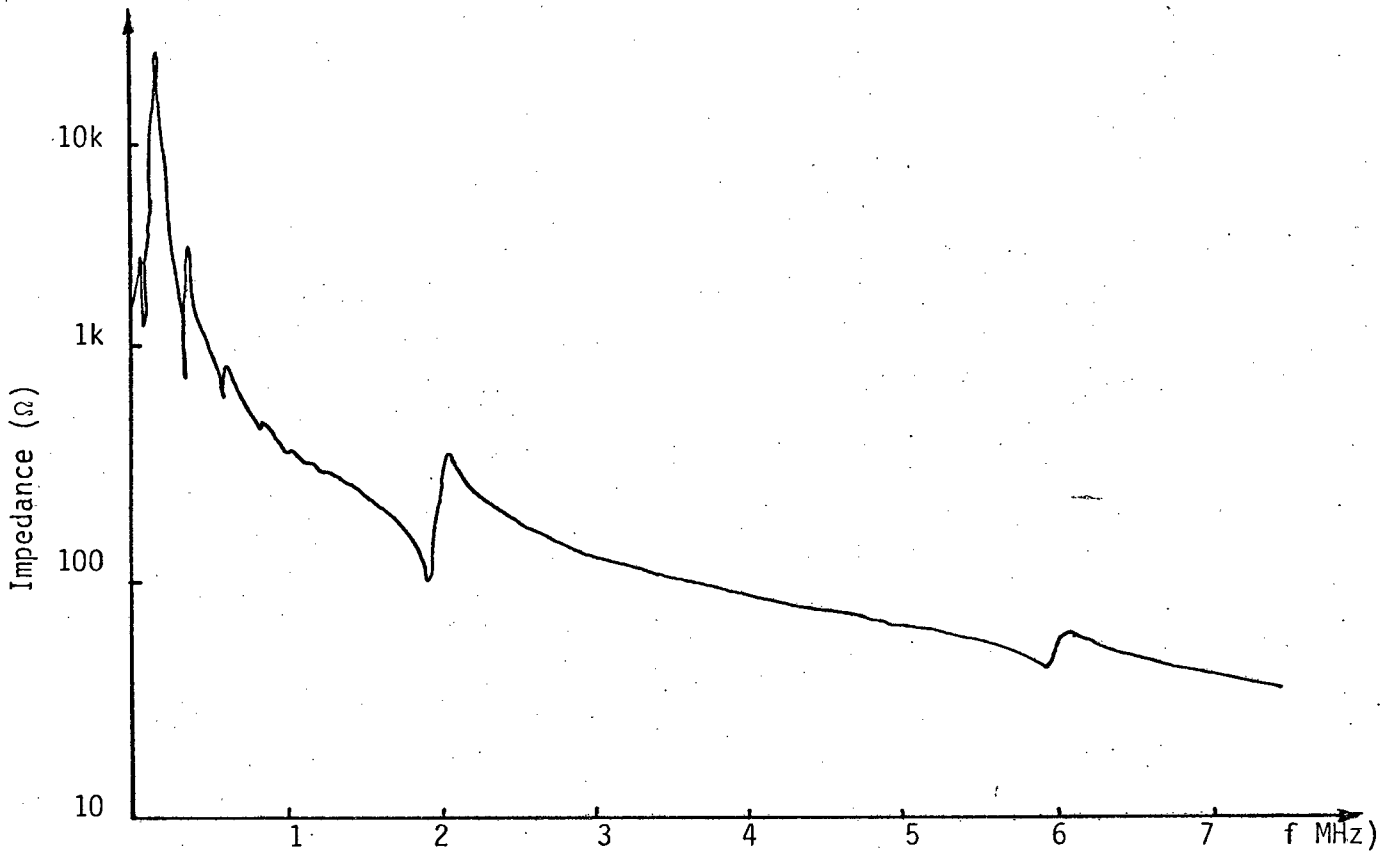


FIG. 4.15 IMPEDANCE OF MOUNTED PIEZO-ELECTRIC TRANSDUCERS

$$s(t) = A_0 \cos w_0 t = A_0 \cos 2\pi f_0 t$$

where f_0 is the carrier frequency.

The beam will be modulated in both amplitude and phase or frequency by the flowing fluid. The received signal thus has the general form:

$$r(t) = (k(t) + k_0) \cdot A_0 \cos \{w_0 t + \theta(t)\}$$

where k_0 is the mean attenuation, $k(t)$ the fluctuating component of the attenuation about k_0 and $\theta(t)$ the term representing the phase modulation. This is an amplitude modulated double side-band signal with the carrier not suppressed. The possibility of over-modulation, however, does not exist. As the total attenuation increases, the amplitude of the received signal merely tends to zero.

By definition, frequency modulation occurs when the carrier frequency is made to vary about its unmodulated value in such a way that the instantaneous deviation is directly proportional to the instantaneous amplitude of the modulating signal, i.e.,

$$w(t) = w_0 + k.f(t)$$

where $f(t)$ is the modulating signal.

The instantaneous phase is thus given by the integral of the frequency and equals

$$\phi_{FM}(t) = \int w(t) dt = w_0 \cdot t + k \cdot \int f(t) dt$$

In phase modulation, the instantaneous phase deviation of the carrier wave is directly related to the modulating signal amplitude, that is

$$\phi_{PM}(t) = w_0 \cdot t + k.f(t)$$

This shows the similarity between frequency and phase modulation. If the modulating signal is firstly intergrated and then allowed to phase modulate a carrier, a frequency modulated signal results. Since various factors contribute to the modulation of an ultrasonic beam passing through a fluid, the received signal can be considered to be modulated in either frequency or phase.

6.1 The Transmitter

The electrical input impedance of the pieze-electric disc is mainly capacitive and has a value given by

$$C = \frac{\epsilon_o \cdot \epsilon_r \cdot A}{d}$$

where A is the surface area, d the thickness and ϵ_r the relative permittivity of the di-electric of which the disc is made ($\epsilon_r = 1600$ for SONOX^R P5).

Substituting for the disc used gives:

$$C \approx 1nF$$

At a frequency of 2MHz, this represents an impedance of:

$$X_C = \frac{1}{w \cdot C} = 80 \Omega$$

which agrees well with that measured in Figure 4.12.

Two types of transmitter circuitry are possible. In the first case, the pieze-electric discs form part of a self-resonant oscillator that is designed to oscillate at the fundamental thickness mode of the discs. The second possibility is a tunable signal generator that drives the ultrasonic sensors as loads. The self-resonant system is more suitable for practical applications since it is cheap, automatically adjusts itself to the resonant frequency of the transducers and thus maintains maximum output even if this frequency drifts due to environmental changes.

During the course of this project, however, the second approach was adopted. A suitable function generator with a maximum output of 20 Vp-p was used to drive the transmitters. This is sufficient since the signal after passing through a water-filled perspex pipe of 10 mm diameter was 2Vp-p at the receiver, representing a loss of only about 20 dB. In situations in which the nett attenuation is higher, larger drive voltages of the order of 100's Vp-p can be used. The limitation on this voltage is electrical breakdown of the di-electric forming the ultrasonic sensors.

6.2 The AM Detector

In order to eliminate low frequency effects caused by variations in temperature, composition and time ageing, an automatic gain control circuit (A.G.C.) is necessary to maintain the mean level of the received signal approximately constant. Such a circuit will also be useful when the sensors are attached to different pipes conveying different fluids in which the overall attenuation changes. An intergrated circuit that satisfies the requirements of an AM detector is the LM374. This is a cheap, general

purpose, wide-band AM/FM/SSB circuit that operates off a single power-supply. It has a maximum gain of 30dB, a wide-range AGC (+30dB/-50dB) and an active peak detector. Furthermore its sensitivity is good ($10 \mu \text{Vrms}$), its input impedance is $1 \text{ k}\Omega$ and its bandwidth extends from DC to 200 MHz.

A circuit diagram of the AM detector that has been designed is shown in Figure 4.17. Since the $1 \text{ k}\Omega$ input impedance of the LM374 is sufficiently large in comparison with the impedance of the piezo-electric sensors (80Ω at 2 MHz), no buffer is required. A tuned 2MHz tank circuit of variable Q is used to reject wide-band noise prior to peak detection. The AGC circuit has a time-constant of about 1 second, thus eliminating unwanted low frequency components of the flow signals. The AM detected output is AC coupled (1Hz) to a low-pass amplifier that suppresses the high-frequency carrier from the detected signal as well as amplifies the flow signal. This amplifier is a LF 356 operational amplifier used in a non-inverting configuration. A cut-off frequency of about 5 kHz is used since most flow signal components are well below this frequency (see Chapter 9). In addition this provides over 50dB attenuation of the 2 MHz carrier frequency. Care is taken to ensure that the phase shift is about zero over the frequency range of interest and that it is the same for both receivers in order to eliminate possible errors when determining correlation peaks (see Chapter 10).

6.3 The FM Detector

The circuit that was designed to extract the frequency modulation from the received signal, again makes use of the LM 374. In this case, the AGC is adjusted for minimum attenuation in order to clip the applied high-frequency signal, thereby eliminating the amplitude modulation that is on the carrier. The resulting constant amplitude, varying frequency signal is applied to a tuned L-C-R circuit of variable Q that transforms frequency fluctuations into corresponding amplitude fluctuations. This is accomplished by detuning the tuned circuit so that the unmodulated carrier frequency f_0 lies on the sloping part of the amplitude-frequency characteristic. Assuming that the frequency variations occur within a small region about the unmodulated carrier, the amplitude of the output wave will follow the instantaneous frequency of the input. These amplitude fluctuations are subsequently peak detected to yield flow signals proportional to the frequency modulation of the carrier.

A circuit diagram of the frequency discriminator used, is shown in Figure 4.18. The slope and thereby both the sensitivity and range of the discriminator is controlled by varying the Q of the tuned circuit with a potentiometer. As in

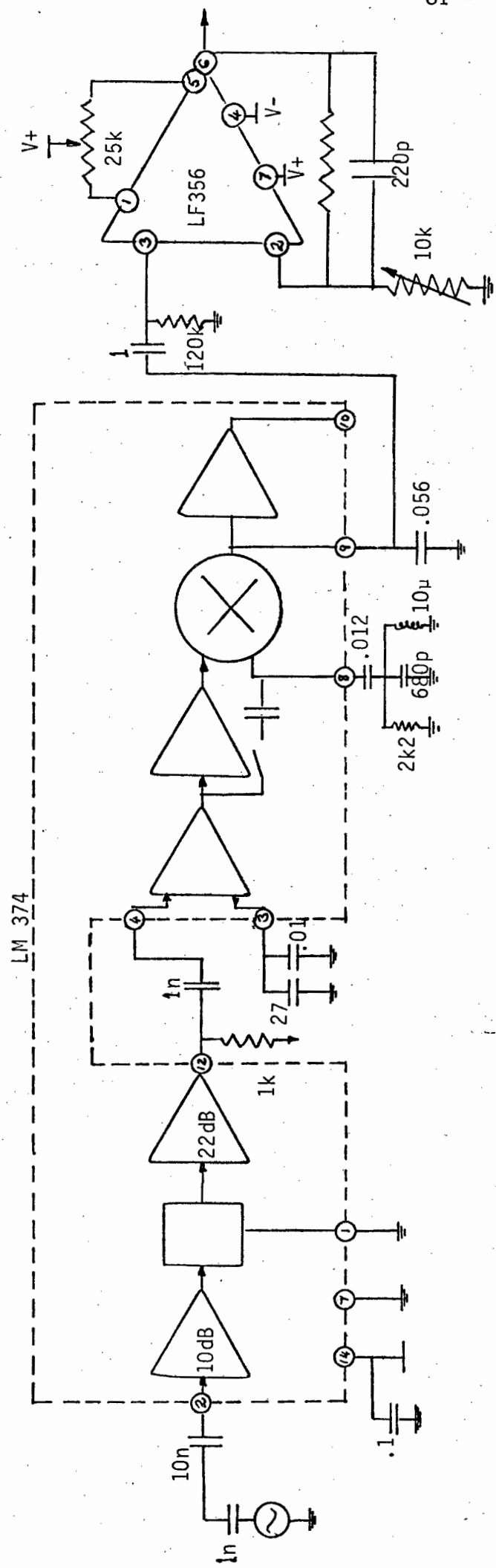


FIG. 4.18 Circuit Diagram of Frequency discriminator

the previous section, the output of the LM 374 is AC coupled to a low-pass amplifier in order to suppress the carrier and amplify the low- frequency flow signal.

7. CONCLUSION

This chapter has investigated the requirements of ultrasonic sensors for correlation flowmetering and a clamp-on design is described. The design possesses all the specified features and is particularly easy to install. All that is required for mounting the sensors is for the pipe to be cleaned on the outside so as to remove rust, scale and dirt. The sensors can then be clamped onto the pipe, the whole process taking only a few minutes.

CHAPTER 5

CROSS-CORRELATION TECHNIQUES

1. INTRODUCTION

Determining cross-correlation functions can be a time-consuming and cumbersome process requiring extreme computational effort, elaborate hardware and large amounts of data. It involves time delaying one signal with respect to the other, multiplying the two signals on a point-for-point basis and averaging the product. This operation yields the correlation function for a certain time delay and must be repeated for the other delays of interest.

Analogue correlation techniques are, in general, likely to remain the most efficient in terms of operating speed and possibly simplicity, but they are usually designed for special applications. Their versatility is often limited and their dynamic range and stability cannot compete with digital correlation techniques.

Despite the advantages of digital correlators, the high cost of digital delay lines, multiplier units and accumulators make them unattractive. The hardware involved amounts to the equivalent of a small computer and in fact a small computer equipped with fast digitizer can often be used as quite a versatile correlator. It cannot compete in speed with a specially designed correlator, however, because it has only a single processor; a full correlator generally contains a multiplier unit for each channel, presenting simultaneously the correlation function at a number of delay values.

Many of the complexities of correlator design are very significantly reduced if the signals to be correlated are suitably processed before they are correlated. Several methods exist for computing correlation functions from data that is related but not identical with the signals to be correlated. The aim of this chapter is to investigate the effect of such pre-processing on correlation functions and, in particular, to determine if a shift in the position of the correlation peak is produced. This is important since it is primarily the cross-correlation peak position that determines the flow velocity in correlation flowmetering. The methods to be discussed include coarse amplitude quantization of one or both signals, the effect of adding random independent noise to the signals before quantizing them and delta-modulating the signals before they are correlated.

2. EFFECT OF AMPLITUDE QUANTIZATION ON CORRELATION FUNCTIONS

It can be shown (Appendix C) that amplitude quantization can be considered as an addition of noise to the unquantized signals. This noise, however, is not independent of the input signals since in fact it equals the difference between the quantized and original signals.

It is thus causally related to the unquantized signals. The quantized signals x' and y' , can be written as:

$$x' = x + n_x$$

$$y' = y + n_y$$

where n_x is the quantization noise associated with the variable x and n_y the quantization noise associated with y .

The cross-correlation coefficient between the quantized variables x' and y' (with zero means) may thus be expressed as:

$$\begin{aligned} \overline{x'.y'} &= \overline{(x+n_x).(y+n_y)} \\ &= \overline{x.y} + \overline{x.n_y} + \overline{y.n_x} + \overline{n_x.n_y} \dots\dots\dots (5.1) \end{aligned}$$

where $\overline{x.y}$ is the correlation coefficient between the signals x and y , $\overline{y.n_x}$ is the correlation coefficient between the signal y and quantization noise n_x , $\overline{x.n_y}$ is the correlation coefficient between signal x and quantization noise n_y and $\overline{n_x.n_y}$ is the correlation coefficient between the quantization noises. The latter three terms are error terms produced by quantizing the original signals.

It is possible to identify these various correlation coefficients by using the fact that the first joint moment (i.e., cross-correlation) between two variables is given by the second partial derivative of the joint characteristic function of the two signals evaluated at the origin, i.e.,

$$\overline{x.y} = - \frac{\partial^2}{\partial u \partial v} W_{x,y}(u,v) /_{u=v=0} \dots\dots\dots (5.2)$$

where $W_{x,y}(u,v)$ is the joint characteristic function of the variables and equals the Fourier transform of the joint probability density function $w_{x,y}(x,y)$, i.e.,

$$W_{x,y}(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_{x,y}(x,y) \cdot e^{-j(x \cdot u + y \cdot v)} dx \cdot dy$$

Watts⁽¹⁴⁹⁾ has derived an expression for the characteristic function of two signals quantized by general quantizers (a_1, q_1, r_1, c_1) and (a_2, q_2, r_2, c_2) respectively:

$$\begin{aligned} W_{x',y'}(u,v) = & \sum_{k=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \text{EXP}[-j u r_1 (c_1 - a_1)] \cdot \text{EXP}[-j v r_2 (c_2 - a_2)] \\ & \cdot \text{EXP}(-j 2 \pi k a_1) \cdot \text{EXP}(-j 2 \pi \ell a_2) \cdot W_{x,y} \left(\frac{u r_1}{q_1} - \frac{2 \pi k}{q_1}, \frac{v r_2}{q_2} - \frac{2 \pi \ell}{q_2} \right) \\ & \cdot \frac{\sin \frac{1}{2} (u r_1 - 2 \pi k)}{\frac{1}{2} (u r_1 - 2 \pi k)} \cdot \frac{\sin \frac{1}{2} (v r_2 - 2 \pi \ell)}{\frac{1}{2} (v r_2 - 2 \pi \ell)} \dots\dots\dots (5.3) \end{aligned}$$

where $W_{x,y}(u,v)$ is the joint characteristic function of the unquantized signals and the factors a, c, q and r can be identified in Figure 1 of Appendix C.

Operating on equation (5.3) according to equation (5.2) with the assumption that the means have been subtracted from the input signals and no shift of the origin between quantized and unquantized signals has occurred, i.e.,

$$\begin{aligned} a_1 &= \frac{\bar{x}}{q_1} + A_1 ; C_1 = A_1 \\ a_2 &= \frac{\bar{y}}{q_2} + A_2 ; C_2 = A_2 \end{aligned}$$

gives the cross-correlation coefficient between the quantized variables x' and y' as:

$$\overline{x'y'} = - \left[\frac{\partial^2}{\partial u \partial v} W_0 \right]_{u=v=0} +$$

$$\begin{aligned}
 & \sum_{k=-\infty}^{\infty} \frac{r_1}{2} \cdot \frac{\cos k\pi}{k\pi} \cdot \text{EXP}(-j2\pi k A_1) \cdot \left[\frac{\partial W_0}{\partial v} \right]_{\substack{u=v=0 \\ \ell=0}} \\
 & + \sum_{\ell=-\infty}^{\infty} \frac{r_2}{2} \cdot \frac{\cos \ell\pi}{\ell\pi} \cdot \text{EXP}(-j2\pi \ell A_2) \cdot \left[\frac{\partial W_0}{\partial u} \right]_{\substack{u=v=0 \\ k=0}} \\
 & + \sum_{k=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \frac{r_1}{2} \cdot \frac{\cos k\pi}{k\pi} \cdot \text{EXP}(-j2\pi k A_1) \cdot \frac{r_2}{2} \cdot \frac{\cos \ell\pi}{\ell\pi} \cdot \text{EXP}(-j2\pi \ell A_2) [W_0]_{u=v=0} \\
 & \dots\dots\dots (5.4)
 \end{aligned}$$

The prime on the summation means 'omit the value zero' and

$$W_0 \equiv W_0 \left(\frac{ur_1}{q_1} - \frac{2\pi k}{q_1}, \frac{vr_2}{q_2} - \frac{2\pi \ell}{q_2} \right)$$

is the characteristic function of $w_{x,y}(x-\bar{x}, y-\bar{y})$.

The terms can be identified as $\overline{x \cdot y}$, $\overline{n_x \cdot y}$, $\overline{n_y \cdot x}$ and $\overline{n_x \cdot n_y}$ in that order. If the statistics of the input signals and the characteristics of the quantizers are known, equation (5.4) can be used to calculate the correlation coefficient of the quantized variables. It is interesting to note that if the input characteristic function $W_{xy}(u,v)$ is zero for $|u| \geq \frac{2\pi}{q_1}$ and $|v| \geq \frac{2\pi}{q_2}$ then all the terms except the first vanish and the quantization noises are completely uncorrelated (but not independent) with the signals and each other.

3. SPECIAL CASES OF CORRELATORS WITH AMPLITUDE QUANTIZATION

In order to apply the previous analysis to the various existing correlation techniques, use is made of the general form of correlator shown in Figure 5.1. If the input signals x and y are ergodic functions of time, the correlation coefficient $\overline{xy}$ can be obtained via a time average:

$$R_{xy} = \overline{x \cdot y} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) \cdot y(t) dt$$

When the averaging is done over a finite time interval, the output of the correlator is:

$$C_{xy} = \frac{1}{T} \int_0^T x(t) \cdot y(t) dt$$

which is an estimate of the correlation coefficient $\overline{x \cdot y}$. If the effect of finite averaging time may be neglected, it is possible to analyze all conventional correlators as special cases of the general configuration.

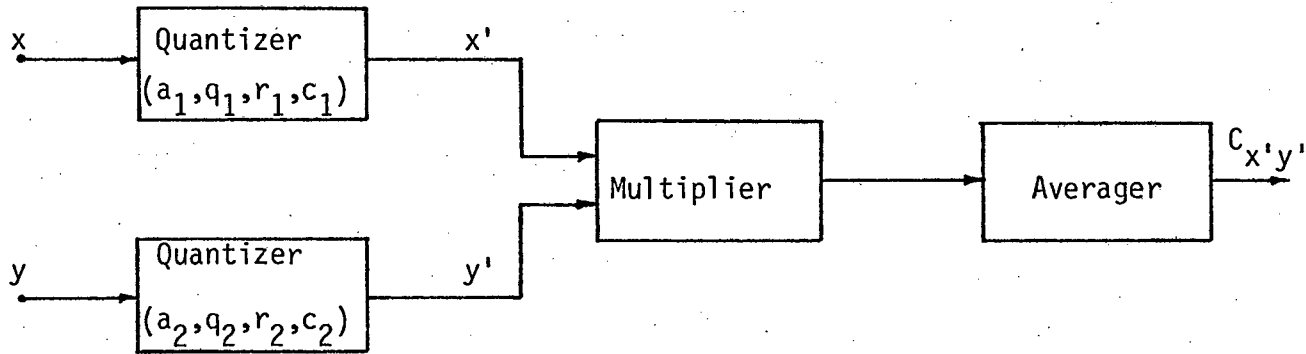


FIGURE 5.1 : General Form of Correlator

3.1 Analogue Correlator

A correlator composed of a perfect analogue multiplier and averager is a special case of the general configuration in which neither signal is quantized, i.e.,

$$x' = x \quad \text{and} \quad y' = y$$

The output of the correlator is thus:

$$C_{x'y'} = C_{xy} = \overline{xy}$$

and no change in the correlation function is produced.

3.2 Digital Computer as Correlator

In a digital computer the signals to be correlated are usually finely quantized (typically 8 bits). In this case the correlator output is:

$$C_{x'y'} = C_{xy} + C_{xn_y} + C_{yn_x} + C_{n_x n_y}$$

Reference⁽¹⁵⁰⁾ shows that even if the quantization interval is fairly large, e.g., the x and y ranges are divided into 8 levels, the terms C_{xn_y} and C_{yn_x} are negligible and the term $C_{n_x n_y}$ is also negligible except when x equals y in which case $C_{n_x n_y} = q^2/12$.

Hence for fine quantization:

$$C_{x'y'} = \begin{cases} \overline{xy} + q^2/12 & \text{for } x = y \\ \overline{xy} & \text{for } x \neq y \end{cases}$$

The cross-correlation coefficients are thus only affected if the signals are identical. In this case, the correlation coefficient in fact equals the mean square value of the signal and the difference between the quantized and original mean square value is $q^2/12$. This is simply the mean square value of the quantization noise that is uniformly distributed between plus and minus half a quantization interval.

3.3 Stieltjes Correlator

This is a form of the general correlator in which one signal is not quantized and the other is coarsely quantized to a few (3 to 6) levels. Since only one of the variables is quantized, the output of the correlator is given by:

$$C_{xy'} = C_{xy} + C_{xn_y} = \overline{xy} + \overline{xn_y}$$

assuming x to be the unquantized variable.

The only error terms is C_{xn_y} which even for quite coarse quantization of y can be extremely small. The magnitude of the error term for Gaussian input signals is given in Table 5.1 for several quantization sizes. It is clear from Table 5.1 that accurate correlation coefficients may be computed with the Stieltjes correlator in which the total range of one of the input signals is divided into as few as three or four quantization intervals.

Quantization size q_2	Intervals into which range of y is divided	Time signal is in range	True correlation coefficient	Error % of $\rho \cdot \sigma_x \sigma_y$
$\sigma_y (a_2=c_2=0)$	5	98,8%	$\rho \cdot \sigma_x \sigma_y$	$-1,5 \times 10^{-7}$
$\frac{3}{2} \cdot \sigma_y (a_2=c_2=\frac{1}{2})$	4	99,8%	$\rho \cdot \sigma_x \sigma_y$	$+1,6 \times 10^{-2}$
$2\sigma_y (a_2=c_2=1)$	3	99,8%	$\rho \cdot \sigma_x \sigma_y$	-1,0

TABLE 5.1 : Errors in Correlation Coefficient for Stieltjes Correlator

3.4 Relay Correlator

The relay correlator is the limiting case of the Stieltjes correlator in which one signal is unquantized whereas only the sign of the other is retained, i.e., the second signal is quantized to two levels or a single bit. For Gaussian input signals having a joint probability density function of the form:

$$w_{x,y}(x,y) = \frac{1}{2\pi\sqrt{(1-\rho^2)\sigma_x^2 \cdot \sigma_y^2}} \cdot \text{EXP} \left[-\frac{1}{2} \cdot \frac{\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2}}{1 - \rho^2} \right]$$

where ρ is the normalized correlation coefficient of the unquantized variables x and y , i.e.,

$$\rho = \frac{\overline{x \cdot y}}{\sigma_x \cdot \sigma_y}$$

and σ_x and σ_y are the r.m.s. values of x and y respectively, it can be shown that (see Appendix C)

$$C_{x,y} = x(\text{sgn} y) = \sqrt{\frac{2}{\pi}} \cdot \rho \cdot \sigma_x$$

The normalized correlation coefficient of the unquantized Gaussian signals can thus be determined exactly via a relay correlator.

3.5 Polar Correlator

A polar correlator is the simplest of all the correlators discussed since only the signs of both input signals are retained. All amplitude information is discarded and both signals are quantized to a single bit.

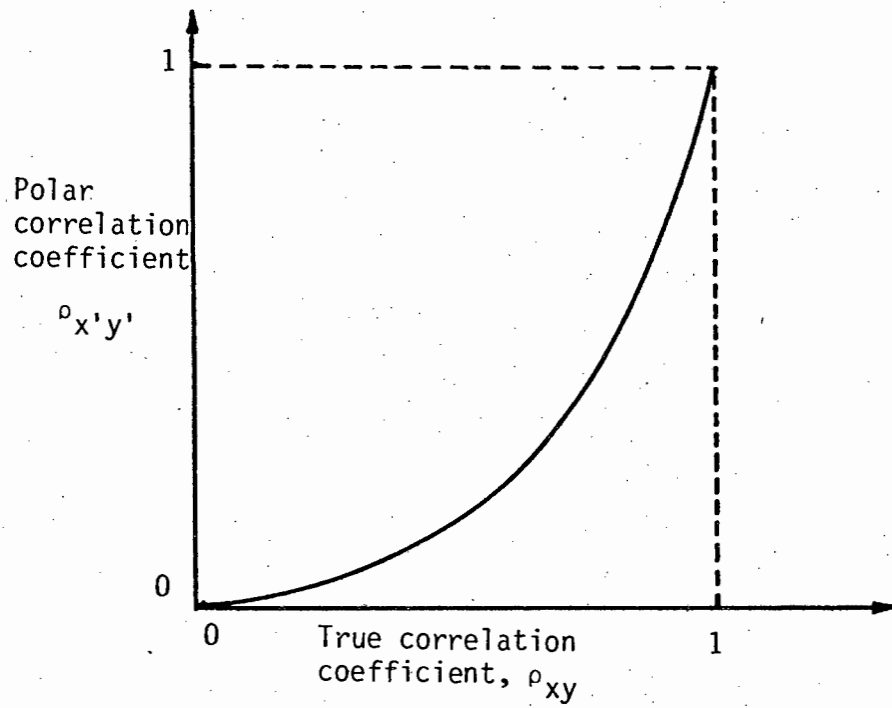


FIG. 5.2 Relation between direct and polar correlation coefficients

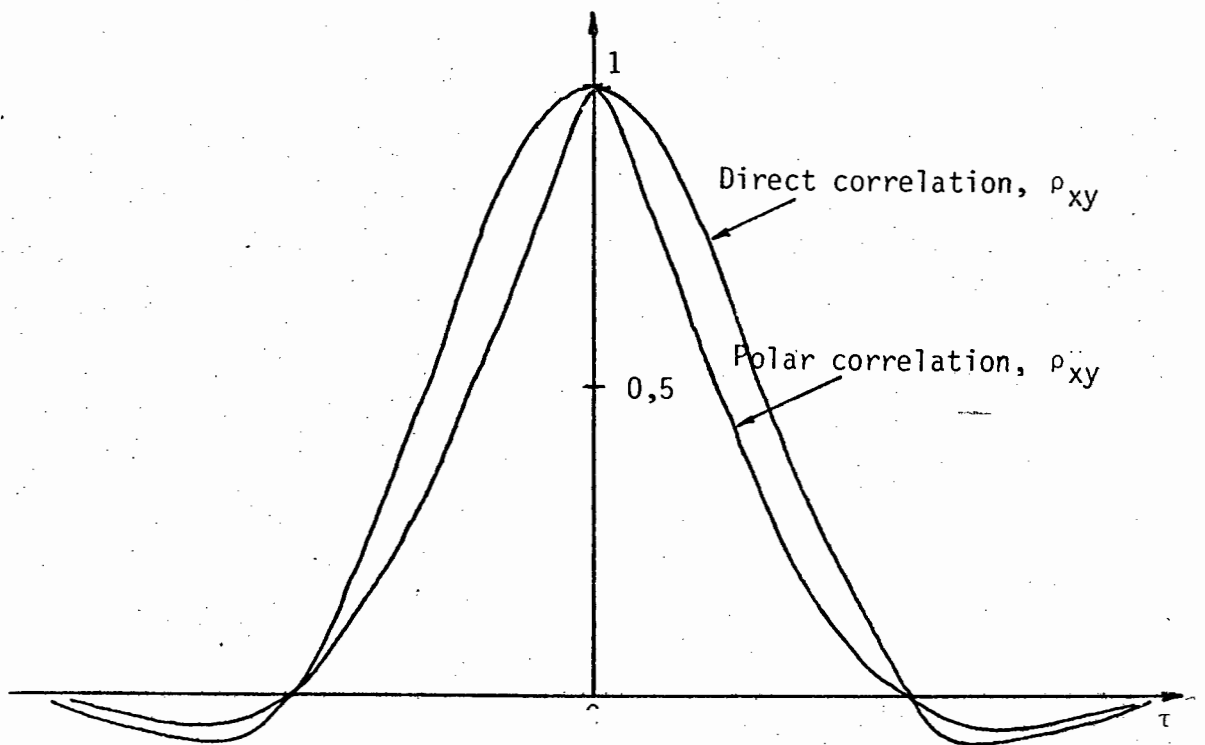


FIG. 5.3 - Direct and polar auto-correlation function of Gaussian noise signal

For Gaussian input signals, the output of the correlator is given by:
(see Appendix C).

$$\begin{aligned} C_{x,y} &= (\text{sgn } x) \cdot (\text{sgn } y) \\ &= \frac{2}{\pi} \sin^{-1} \rho \end{aligned}$$

where $\rho = \frac{\overline{xy}}{\sigma_x \cdot \sigma_y}$ as before.

A non-linear arcsine relationship thus exists between the polar correlation coefficient and the unquantized Gaussian correlation coefficient as shown in Figure 5.2. This produces a change in the shape of a correlation function but does not shift the position of a cross-correlation peak. In Figure 5.3 the auto-correlation function of a Gaussian noise signal for both direct and polar correlation is plotted. The arcsine relationship results in a reduction of the Polar correlation coefficient except when the true correlation coefficient is unity or zero.

4. MODIFIED POLAR CORRELATOR

It has been shown by WIDROW^(150,151) (see Appendix C) that if the two dimensional quantizing theorem is satisfied, that is if the joint characteristic function of the input signals is bounded in two dimensions and the quantizing step size is small enough, the correlation function of the quantized signals is equal to the true correlation function; otherwise it will deviate from the true correlation function by a bias error term. Unfortunately, no physical signals can satisfy the conditions of the quantizing theorem. As an example the bias error terms for the Stieltjes correlator and Gaussian input signals have been calculated in the previous section. Gaussian signals cannot satisfy the quantizing theorem since their characteristic functions are also Gaussian and hence not bounded.

If statistically independent noise signals satisfying certain requirements are added to the input signals before they are quantized, the quantized correlation coefficients will no longer be biased^(152,153,163 and 165).

A diagram of such a modified correlator is shown in Figure 5.4, where $x(t)$ and $y(t)$ are sample functions of stationary random processes of which the correlation function is desired (zero means assumed for simplicity). Two independent noises $a(t)$ and $b(t)$ are added to the input signals $x(t)$ and $y(t)$ respectively. The combined noise and signal waveforms $x_1(t)$ and $y_1(t)$ are converted into digital form by quantizers before being cross-correlated.

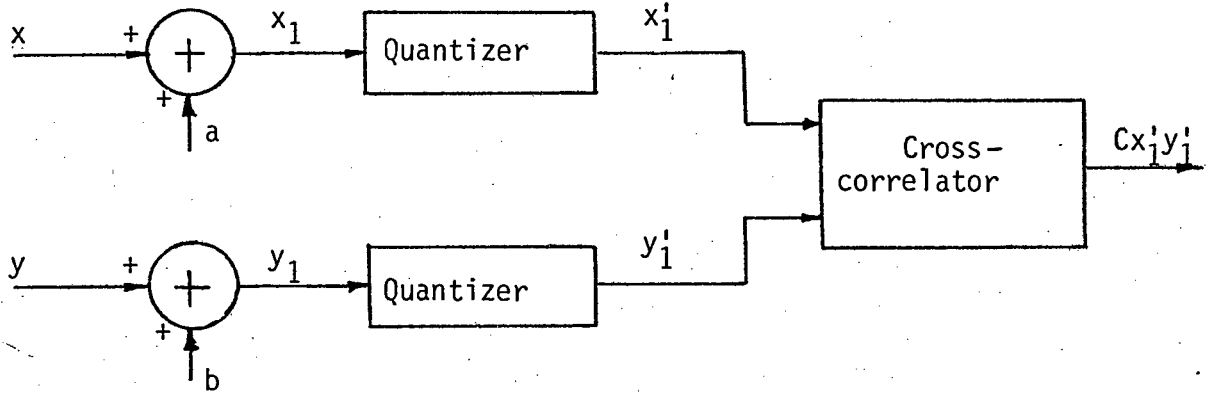


FIGURE 5.4 : Diagram of Modified Correlator

It can be shown (Appendix C, Section 9) that the correlation function of the quantized signals $x'_1(t)$ and $y'_1(t)$ is equal to the true correlation function, $\overline{xy}$ for arbitrary inputs if and only if the auxiliary noises $a(t)$ and $b(t)$ are such that:

$$W_a\left(\frac{2\pi k}{q_1}\right) = 0 \text{ for } k \neq 0, k \text{ an integer}$$

$$\text{and } W_b\left(\frac{2\pi \ell}{q_2}\right) = 0 \text{ for } \ell \neq 0, \ell \text{ an integer}$$

where $W_a(u)$ and $W_b(u)$ are the characteristic functions of the noises $a(t)$ and $b(t)$ respectively and q_1 and q_2 the quantizer intervals. In other words, for the expected value of the output of the modified correlator to equal the true correlation function of the input signals, it is necessary and sufficient that the auxiliary noise waveforms:

- (i) have zero mean value;
- (ii) are independent of the input signals and each other and;
- (iii) are so distributed that their characteristic functions have periodic zeros with periods $2\pi/q_1$ and $2\pi/q_2$, no zero lying at the origin.

The simplest form of noise signal satisfying these requirements is one that is uniformly distributed, i.e.,

$$w_a(a) = \begin{cases} \frac{1}{A} & \text{for } |a| \leq A/2 \\ 0 & \text{for } |a| > A/2 \end{cases}$$

The characteristic function of such a signal is:

$$W_a(u) = \frac{\sin(uA/2)}{(uA/2)}$$

and is shown in Figure 5.5, with zeros at $u = \frac{2\pi}{A} k$; $k = \pm 1, \pm 2, \dots$

Another example is a signal having the triangular distribution shown in Figure 5.6. This is merely the convolution of two identical rectangular distributions:

$$w_a(a) = \begin{cases} \frac{1}{A}(1 - \frac{|x|}{A}) & \text{for } |a| \leq A \\ 0 & \text{for } a > A \end{cases}$$

Its characteristic function is

$$W_a(u) = \left[\frac{\sin(uA/2)}{(uA/2)} \right]^2$$

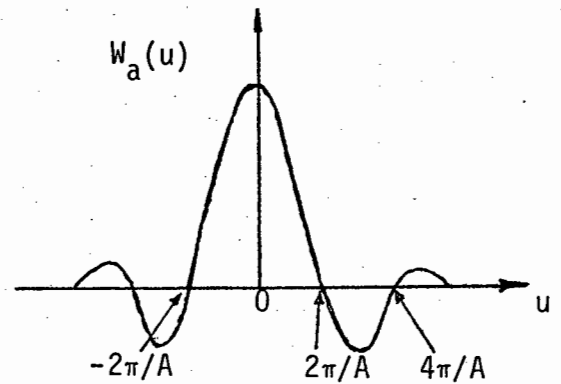
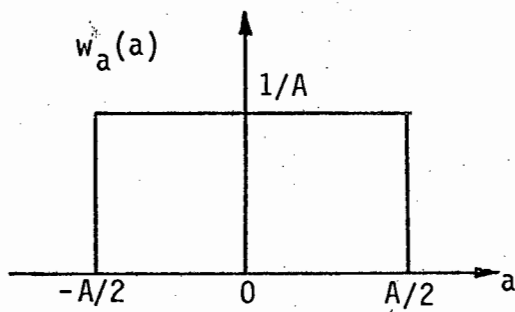


FIGURE 5.5 : Uniformly Distributed Noise Signal

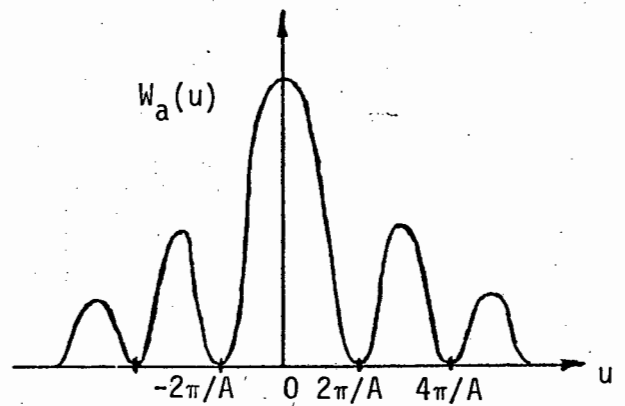
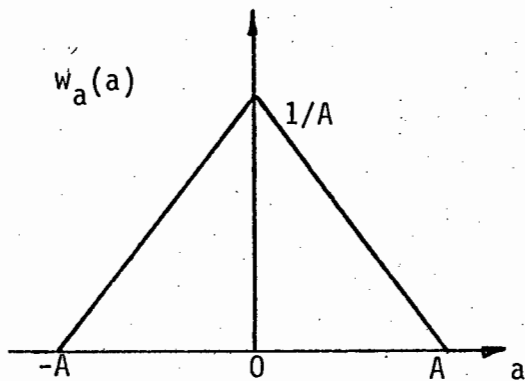


FIGURE 5.6 : Noise Signal with Triangular Probability Density Function

and again has zeros at

$$u = \frac{2\pi}{A} \cdot k \quad ; \quad k = \pm 1, \pm 2, \dots$$

Reference 152 discusses the general form of signals that satisfy the requirements of auxiliary noise signals.

If the input signals are bounded in amplitude, i.e., $w_{xy}(x,y) = 0$ for $|x| \geq A$ and $|y| \geq B$ and if suitable noise signals $a(t)$ and $b(t)$ with

$$w_a(a) = \begin{cases} \frac{1}{2A} & \text{for } |a| \leq A \\ 0 & \text{for } |a| > A \end{cases} \quad \text{and} \quad w_b(b) = \begin{cases} \frac{1}{2B} & \text{for } |b| \leq B \\ 0 & \text{for } |b| > B \end{cases}$$

are added to the input signals then

$$\overline{x \cdot y} = \overline{A \cdot B \cdot \text{sgn}(x + a) \cdot \text{sgn}(y + b)}$$

The Polar correlation coefficient in this case is equal to the true correlation coefficient apart from a change in amplitude.

Since Gaussian signals are not bounded in amplitude, the above equality does not hold. However, if the range of the auxiliary noise signals is much greater than the standard deviation, σ of the Gaussian signals, only a small error exists. Both experimental and theoretical results showing the relationship between the true correlation coefficient for Gaussian signals and the modified Polar correlation coefficients are reproduced in Figure 5.7, for various values of σ/A ⁽¹⁷⁴⁾. These results were obtained during the course of an undergraduate thesis supervised by the author.

For $\sigma/A < 1$ a linear relationship exists but as this ratio increases, a departure from linearity occurs. In the limiting case as $\sigma/A \rightarrow \infty$, the effect of the noise signal disappears and the relationship tends to the arcsine relationship of the Polar correlator.

An interesting point to note is that NORMA MESSTECHNIK GmbH of Austria market a correlator using a stochastic-ergodic measuring technique^(166, 167 and 175).

In this method the instantaneous amplitude of the input signals is compared with the instantaneous amplitude of uniformly distributed noise signals.

Depending on which is greater, either a 1 or a 0 is generated. This is equivalent to the addition (or subtraction) of the noise signals from the input signals and quantizing the sum (or difference) to a single bit.

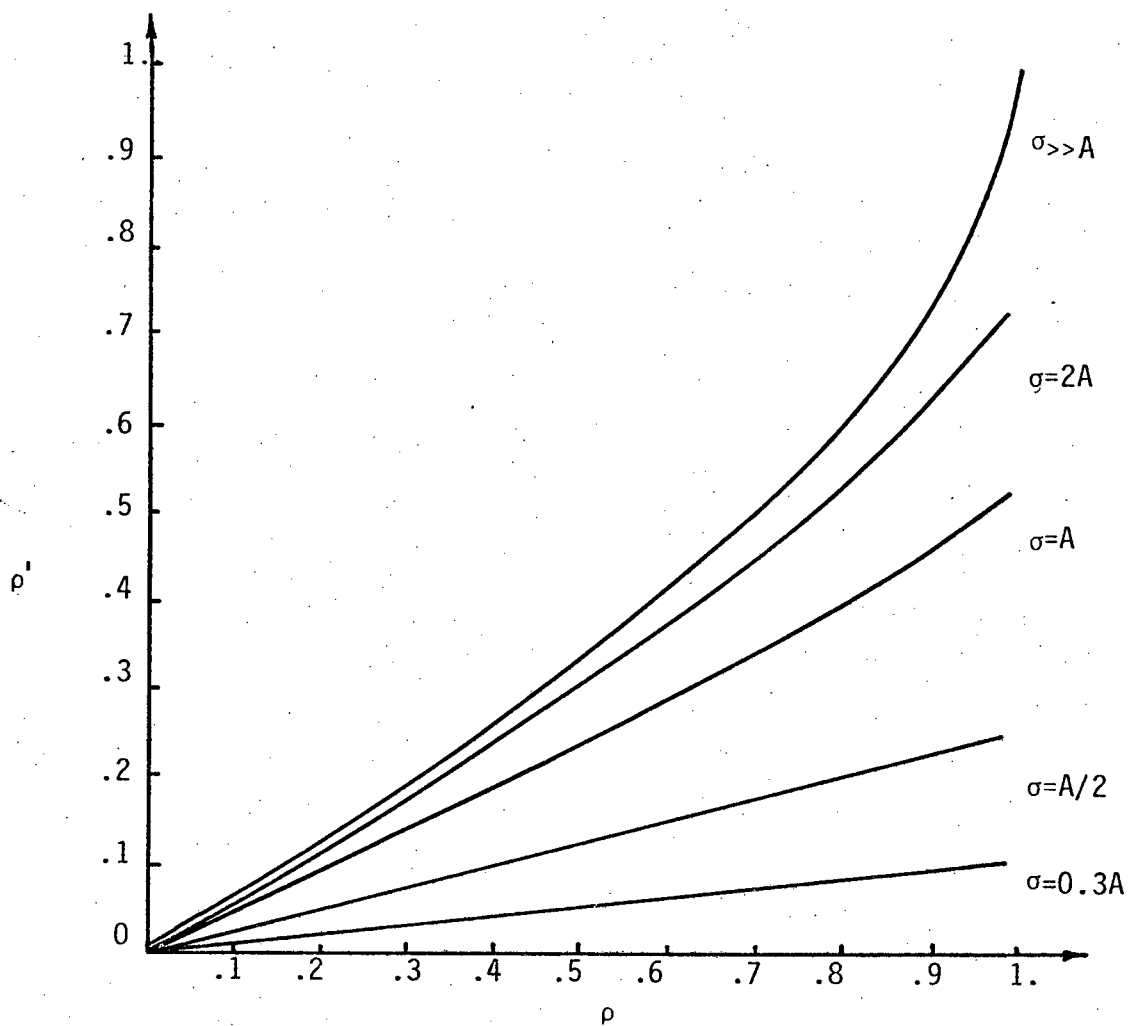


FIG. 5.7a Relationship between the true and modified polar correlation coefficient for Gaussian input signals

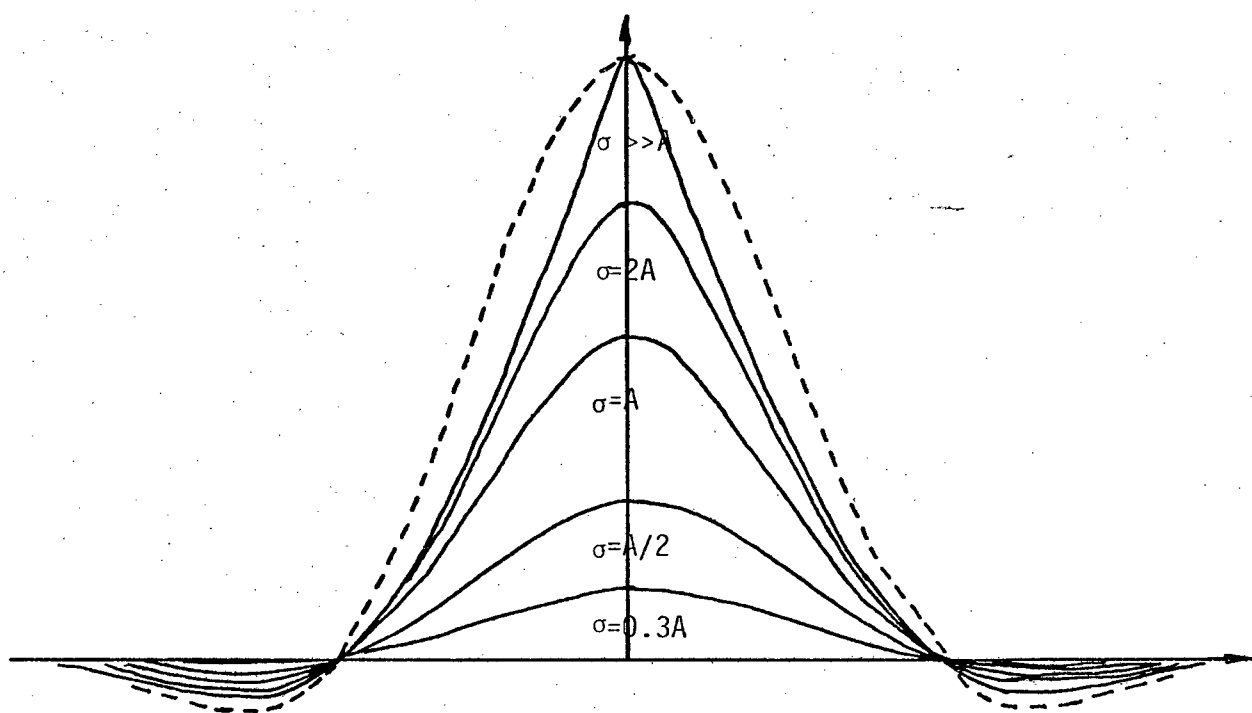


FIG. 5.7b Auto-correlation functions of modified polar correlator

5. DELTA - MODULATION CORRELATION

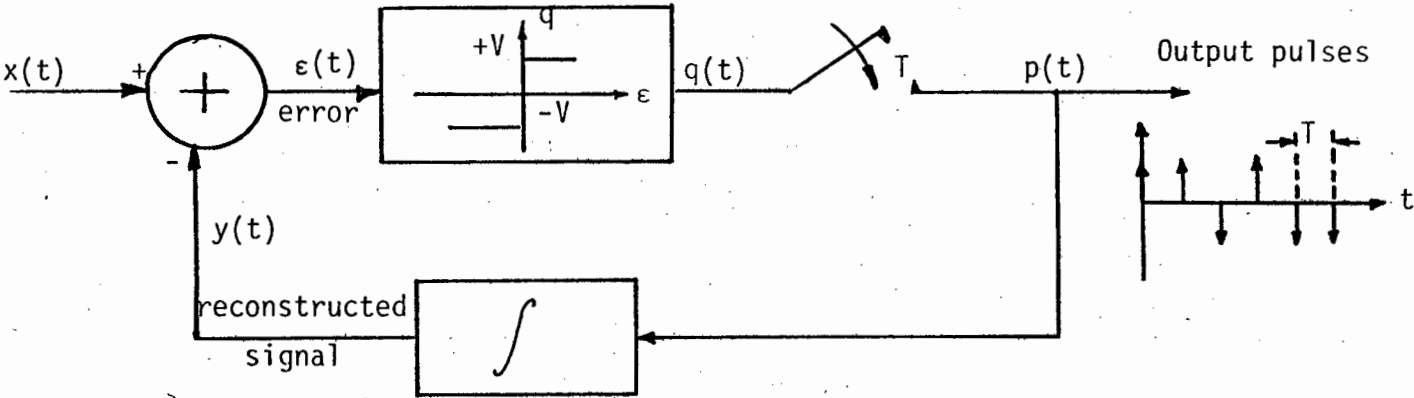
In a further attempt to reduce the complexity of determining cross-correlation functions, the signals were encoded in delta-modulated form before being cross-correlated. The attraction of this method is that the encoded signals can have only one of two values, i.e., they can be represented by a single bit as in the case for Polar correlation. However, in the latter case all amplitude information is discarded (since only the signs are retained), whereas the original signals can be reconstructed from the delta-modulated encoded signals by integration without much error. Hence it appears that cross-correlating two delta-modulated encoded signals should yield better results than coarse two-level quantization.

Reference 169 describes a correlator using delta-modulation to provide a multiplier and integrator all in one process, but the method here differs considerably from that method.

The principle of a linear delta-modulator is best described with reference to Figure 5.8. An analogue input signal is encoded by the delta-modulator into binary pulses which represent the output of the modulator. These pulses are locally decoded back into an analogue waveform by an integrator in the feedback loop and subtracted from the input signal to form an error which is quantized to one of two possible levels depending on its polarity. The output of the quantizer is sampled periodically to produce the output binary pulses. This closed loop arrangement ensures that the polarity of the pulses are adjusted by the sign of the error to track the input signal as shown in Figure 5.9. Stated another way, the delta modulator produces binary pulses which represent the sign of the difference between the input and feed-back signals - hence the name delta. The original signal can readily be recovered from these binary pulses by an integrator followed by a low-pass filter.

Although delta-modulation is fairly simple conceptually, it is very difficult to analyze theoretically. No such attempt will thus be made; the reader is referred, however, to Reference 242 for a deeper insight into the problem.

If the signal to be encoded is integrated before being delta-modulated, the signal is said to be encoded by a delta-sigma modulator (Figure 5.10). The main advantage of this method of encoding is that the decoder need only consist of a simple low-pass filter.



DECODER

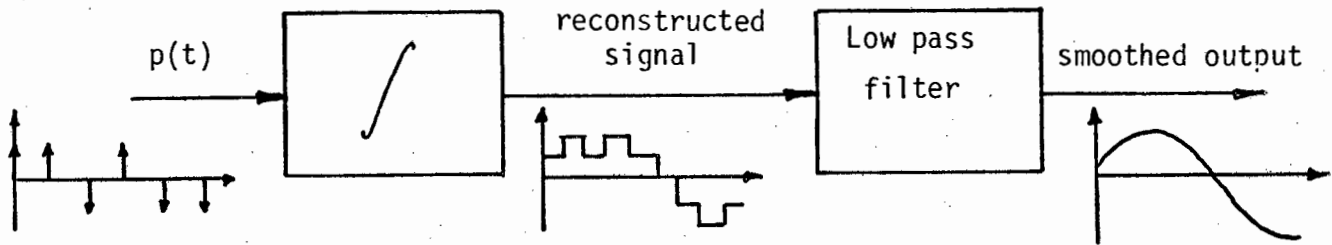


FIG. 5.8 Linear Delta-modulator

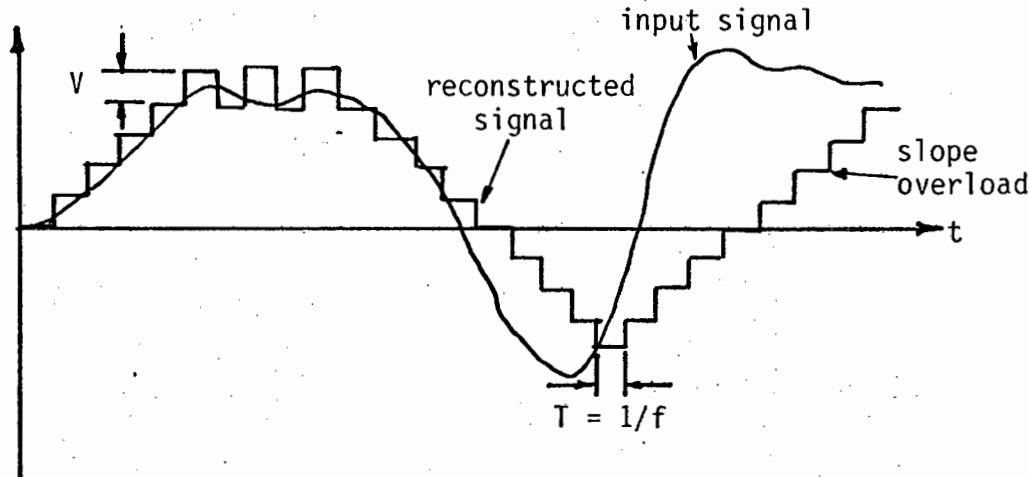
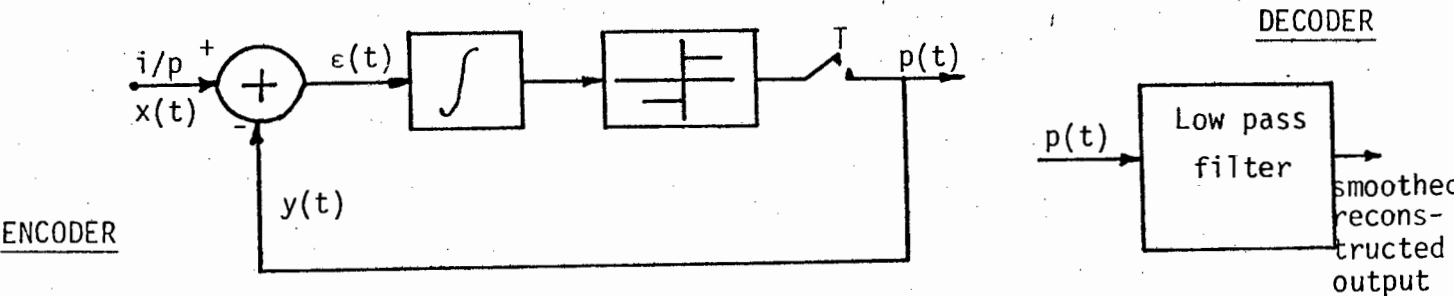


FIG. 5.9 Operation of Delta modulator



ENCODER

DECODER

FIG. 5.10 Delta-Sigma modulator

Of prime importance for accurate reconstruction of delta and delta-sigma modulated signals is a suitable choice of sampling frequency and step height. This is necessary in order to avoid the condition known as slope overload that occurs when the rate of change of the input signal exceeds the maximum possible rate of change of the reconstructed signal (Figure 5.9). For a sampling frequency, f , and step height, V , the maximum rate of change of the reconstructed signal is $V.f$ volts/sec. Hence for a signal $x(t)$ with time derivative $\dot{x}(t)$, the condition for no slope overload is:

$$\dot{x}(t) = \frac{dx(t)}{dt} \leq V.f$$

From existing literature it appears that a sampling frequency that exceeds the maximum signal frequency by a factor of ten is satisfactory provided the step height is chosen appropriately. This high sampling rate (c.f. Nyquist sampling rate) appears to be a major disadvantage of delta-modulation.

Delta-sigma modulation has a further advantage. As above, for no slope overload, the slope of the signal applied to the delta-sigma modulator must not exceed the maximum slope the modulator can track. However, since the signal applied to the delta modulator is the integral of the input signal, the condition for no slope overload is:

$$x(t) \leq V.f$$

Amplitude limiting the input signal (via A.G.C. for example) hence ensures no overload. This is better than limiting the slope of the input signal by low-pass filtering, since no signal information is lost.

Since the aim is to cross-correlate two signals encoded in delta or delta-sigma modulated form, it would be of interest to determine theoretically the relationship between the correlation functions of the encoded and original signals.

To date no reference to such an analysis has been found and no attempt was made to investigate this aspect since it appears to be extremely complex. In order to establish the feasibility of these concepts, however, a computer simulation was performed.

Briefly, the analogue signals to be correlated were applied to two channels of an A/D convertor linked to a VARIAN 620/L minicomputer. The channels were sampled sequentially at a rate determined by clock pulses applied externally

to the priority interrupt module of the computer. These signals were encoded into delta - (sigma) modulated form and stored in memory before being correlated. Plots of the resulting correlation functions were displayed on a visual display unit (VDU) for examination.

To evaluate the above methods, signals with known auto-correlation functions such as sine, triangular and square waves were used (see Figure 5.11). Results obtained by applying such signals to the delta and delta-sigma modulator correlators for various sampling frequencies and step heights are shown in Figure 5.12. The basic shape ('fundamental') of the auto-correlation function of delta-modulated sine waves show the desired cosine characteristic (Figure 5.12(a-d)). However, instead of exhibiting a smooth cosine function, considerable fluctuations are superimposed on this function. In Figure 5.12a the sampling frequency appears to be too slow, resulting in a very poor auto-correlation plot. The higher sampling frequency and smaller step height of Figure 5.12b gives an improved auto-correlation plot; however, considerable 'noise' is still present. This so called 'noise' component is reduced if the step height and hence the factor $V.f$ is further reduced (Figure 5.12c and d). However, a still further reduction leads to slope overload conditions and a distorted auto-correlation function (Figure 5.12e).

In Figure 5.12f the auto-correlation plot of a delta-modulated square wave is shown. Such a signal will always suffer from slope overload, since a delta-modulator cannot track a step change. The envelope of the auto-correlation function is triangular but a periodic high frequency component is also present.

The auto-correlation function of triangular signals is shown in Figure 5.12 (g and h) for two conditions. In the first case, the delta-modulator accurately tracks the triangular wave whereas in the second figure, the modulator is just capable of tracking the slope of the triangular wave. The auto-correlation function should consist of parabolic sections as shown in Figure 5.11, but the results obtained are clearly not of this shape.

It was further reasoned that since the delta-modulated signals have to be integrated in order to reconstruct the original signals, they are related to the derivatives of the input signals. It can readily be shown that the correlation function of the derivatives of two signals is equal to the negative of the second derivative of the correlation function of the signals, i.e.,

$$R_{xy}''(\tau) = -\frac{\partial^2}{\partial \tau^2} R_{xy}(\tau)$$

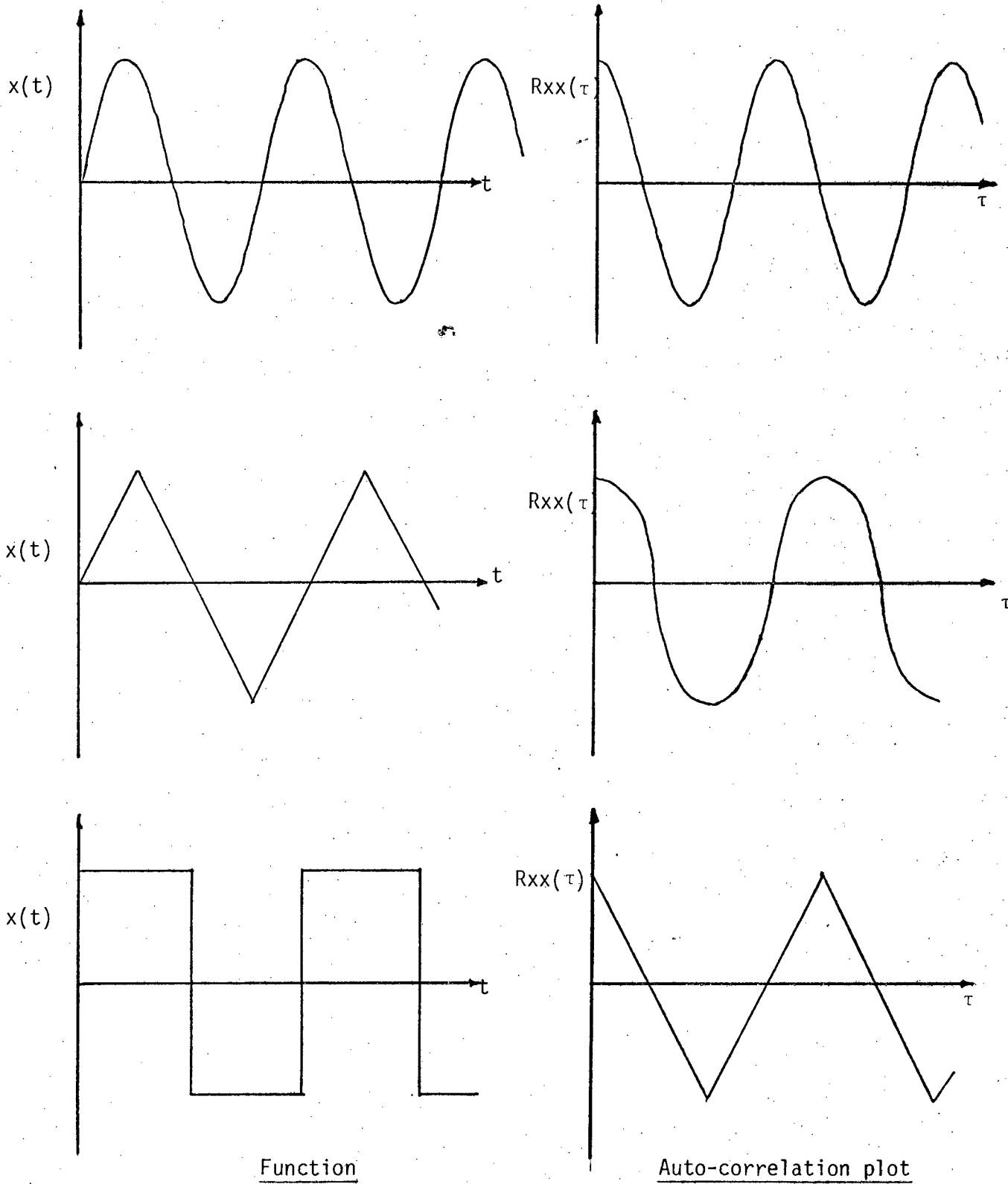


FIG. 5.11 Typical functions and their auto-correlation plots

If the signals are thus integrated before being encoded in delta-modulated form, the resulting signals should be related to the original signals. This form of encoding is merely delta-sigma modulation described previously.

Auto-correlation plots of sine, triangular and square waves that have been delta-sigma modulated are shown in Figure 5.12(i to k). The results agree with the previous reasoning since:

- (i) The auto correlation plot of sine waves is a cosine function;
- (ii) the auto-correlation plot of triangular waves consists of parabolic sections and
- (iii) the auto-correlation plot of square-waves is a triangular function.

In these diagrams the integrated input signals are also shown together with their delta-modulated reconstructions. The poor result for the auto-correlation of a delta-modulated square wave (Figure 5.12f) can now be explained by the fact that the auto-correlation function of the derivative of a square wave consists of a series of Dirac-delta impulses which may cause problems in the computer simulation. Otherwise, the auto-correlation function of the derivative of a sine wave is again a cosine function and of a triangular wave it is a triangular function. This appears to be verified by Figure 5.12(a-d,h).

Finally, a band-limited white noise signal was applied to one input of the correlator. The same signal was delayed via an external delay line before being applied to the other input of the correlator. The resulting cross-correlation functions for delta-modulation do exhibit a peak at the correct delay value (Figure 5.12(l)). However, this peak is not clearly defined.

An actual hardware correlator based on delta and delta-sigma modulation techniques was constructed as part of an undergraduate thesis (Reference 176), under the supervision of the author. The results obtained confirm the validity of the conclusions reached in the above computer simulation of the correlators.

6. DISCUSSION

The assumption that the position of a cross-correlation peak is the primary factor determining the flow velocity in correlation flowmetering makes it unnecessary to reproduce accurately the shape of a cross-correlation function.

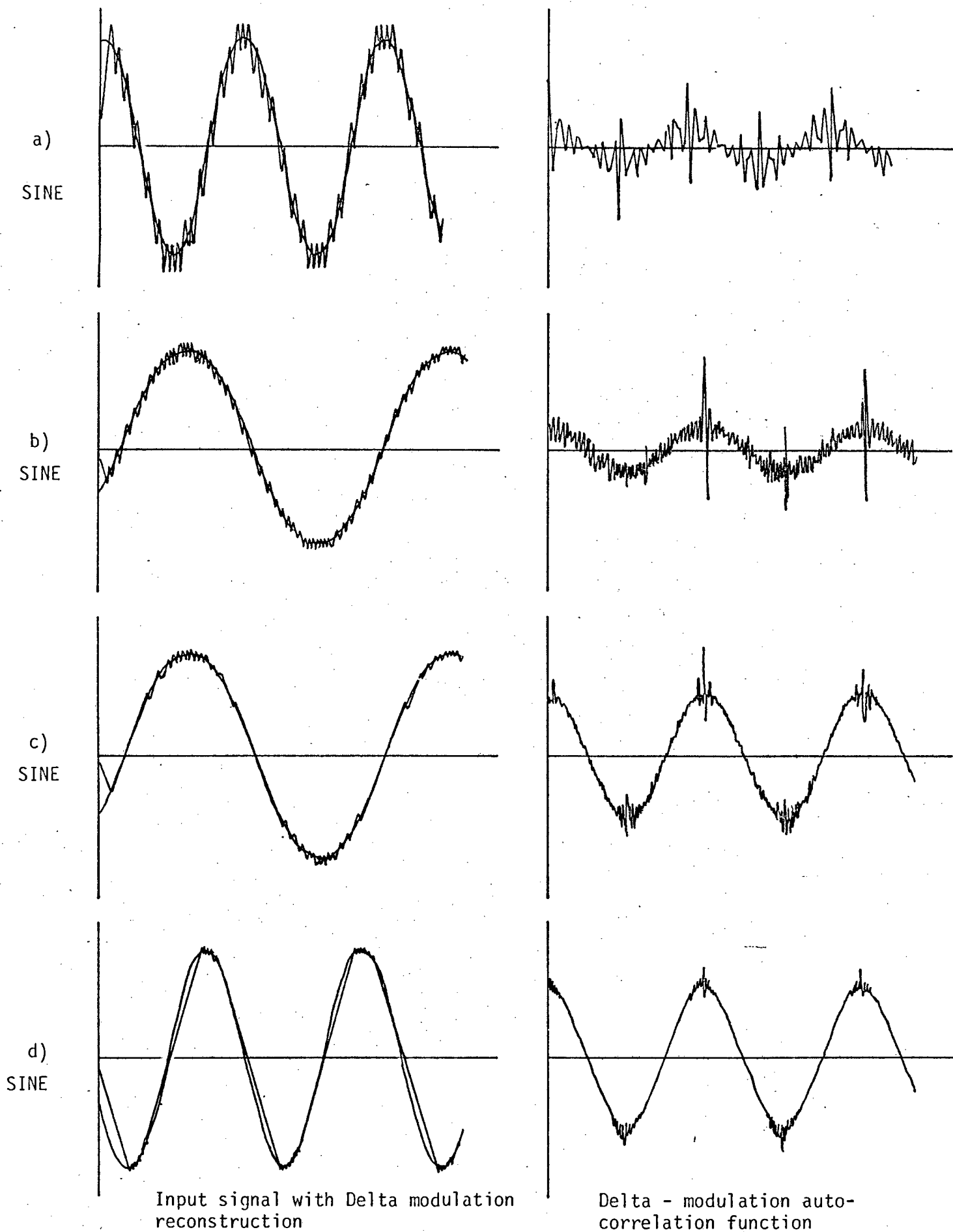


FIG. 5.12 Auto-correlation functions of deterministic signals

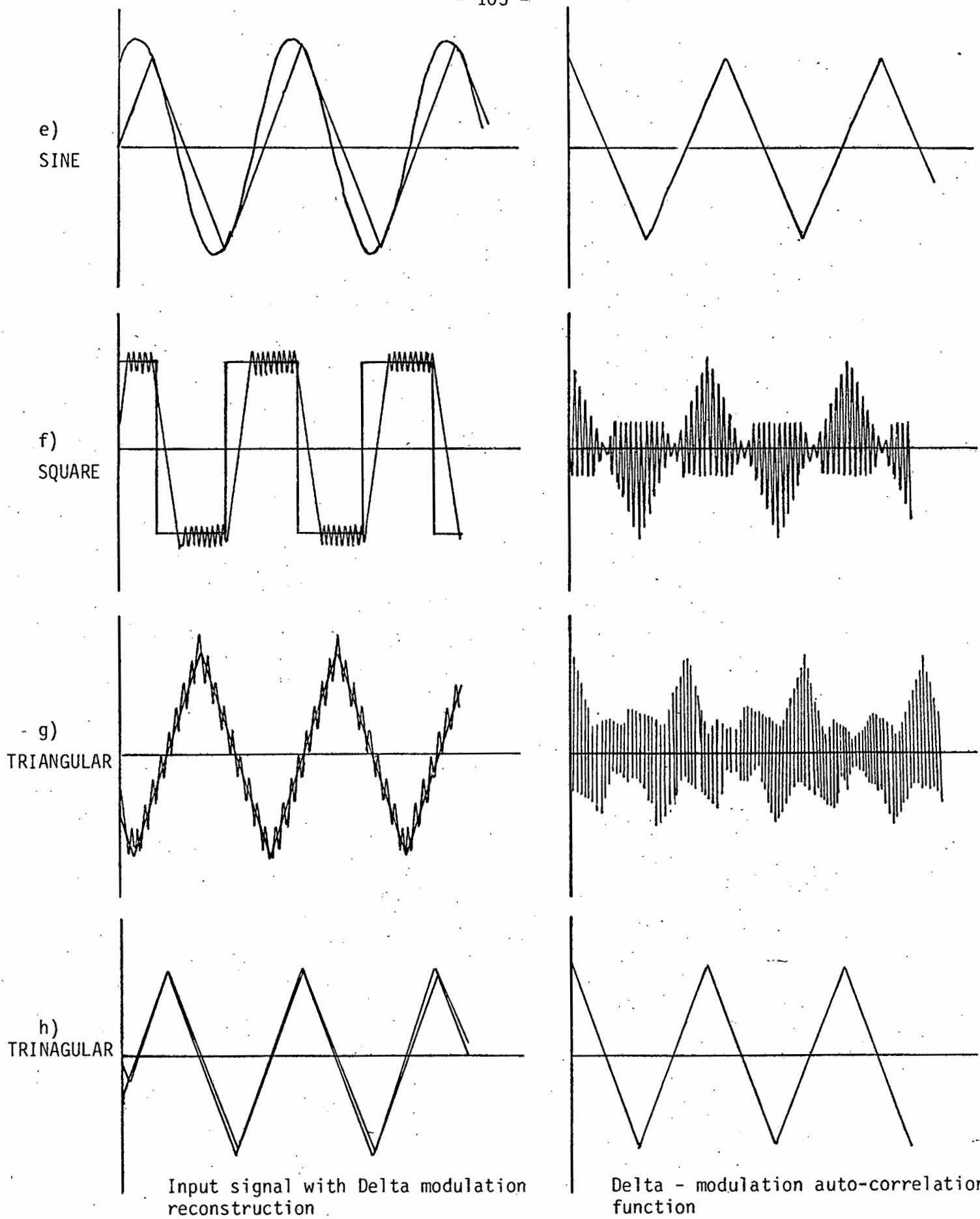


FIG. 5.12 Auto-correlation functions of deterministic signals

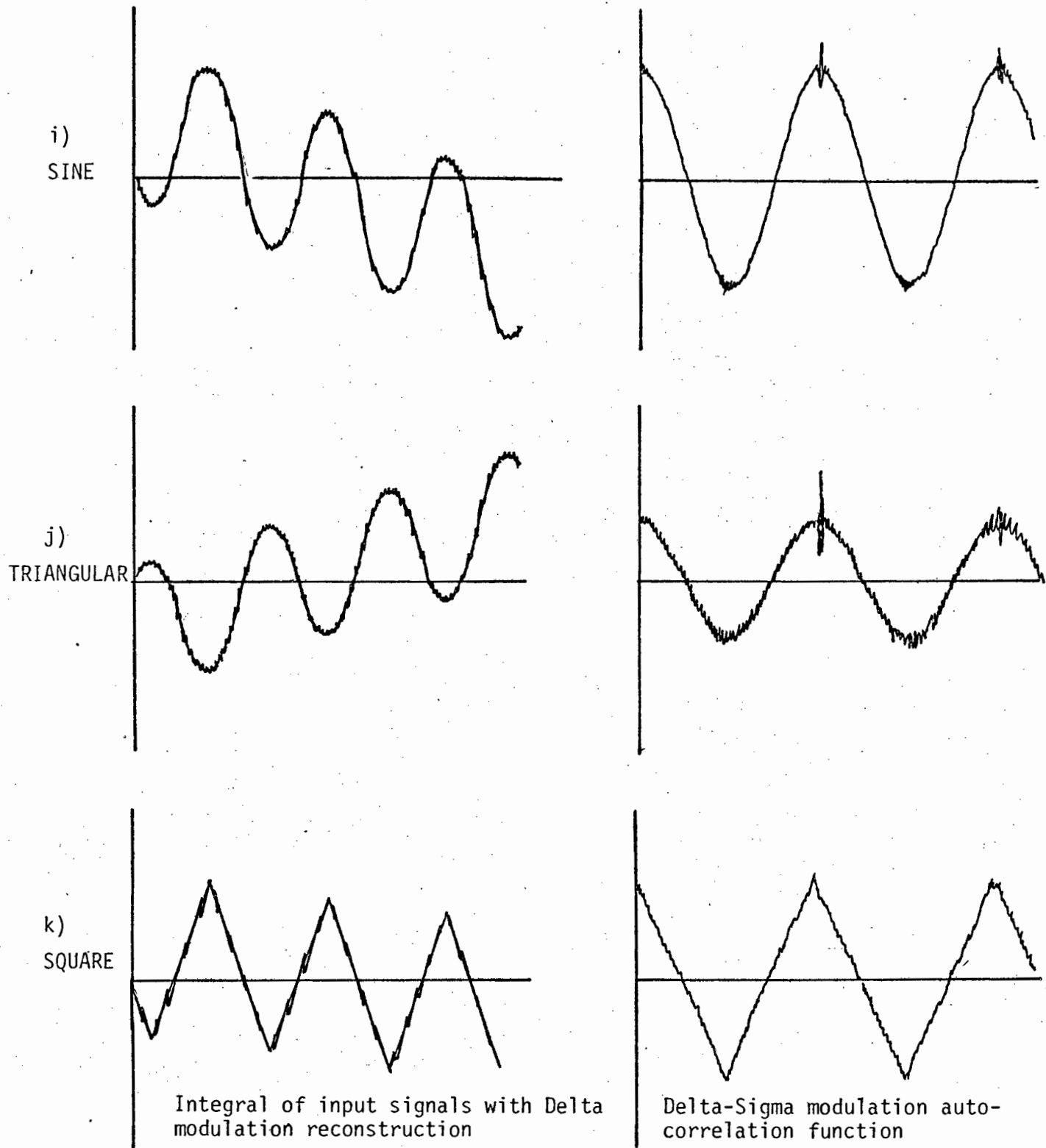
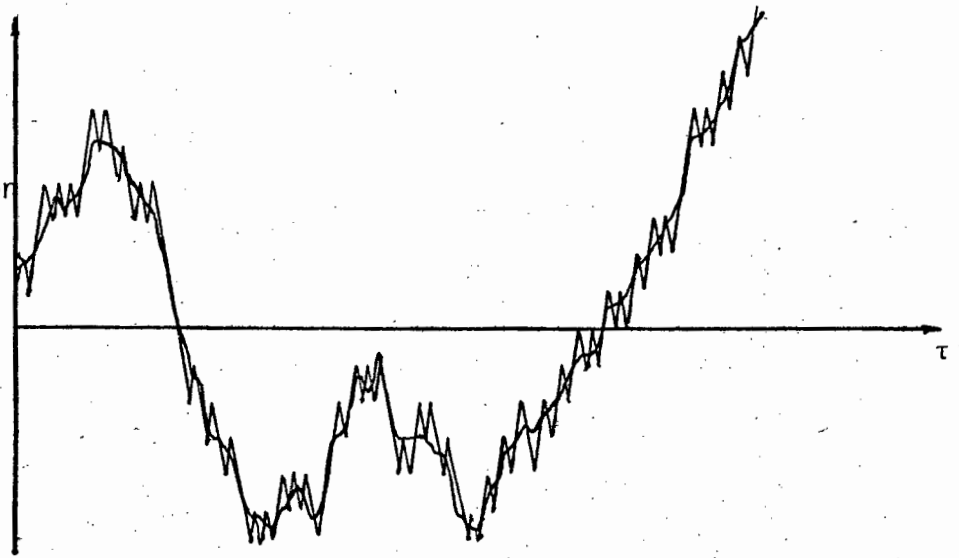
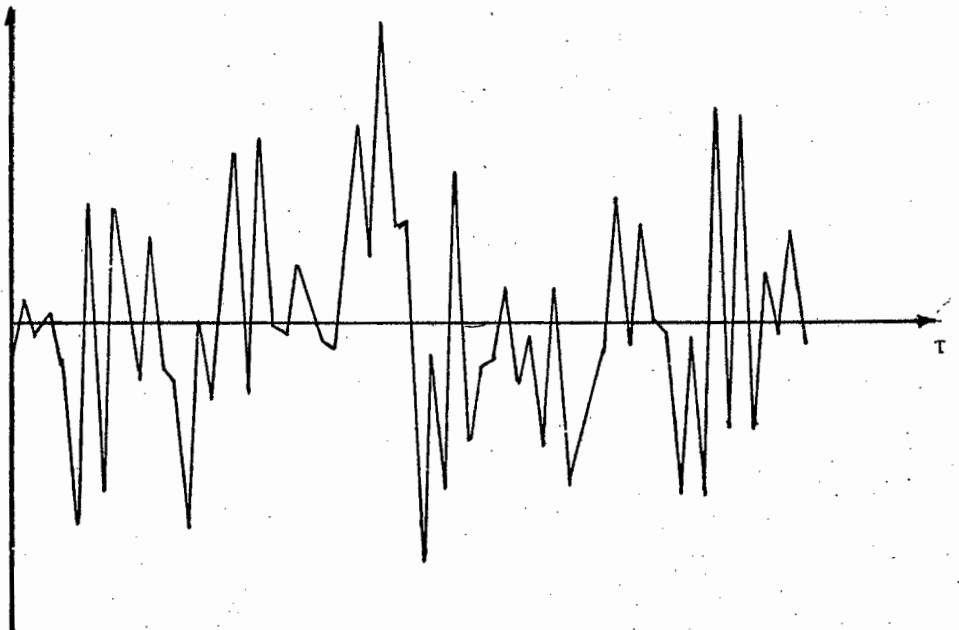


FIG. 5.12 Auto-correlation functions of deterministic signals

Typical input
signal with
Delta modulation
reconstruction



Cross-
correlation
plot



Cross-
correlation
plot

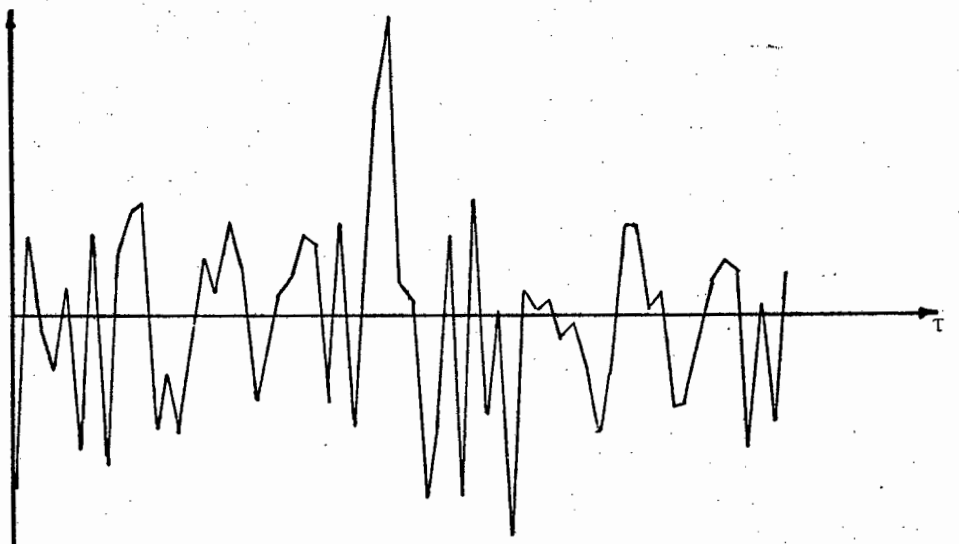


FIG. 5.12 ϵ Cross correlation functions of random signals

The simplest correlators are thus those in which both signals are quantized to one of two levels. A single bit shift-register can be used as a delay line for such signals, multiplication can be accomplished via an exclusive - or logic function and an accumulator can be used as an integrator.

The Polar, modified Polar, delta and delta-sigma modulator correlators all operate using only a single bit for representing the signals. Of these the latter two will not be considered further since extensive theoretical work must first be undertaken in order to predict the performance of such correlators and secondly, the results obtained to date show no improvement over other correlation techniques (see Figure 13).

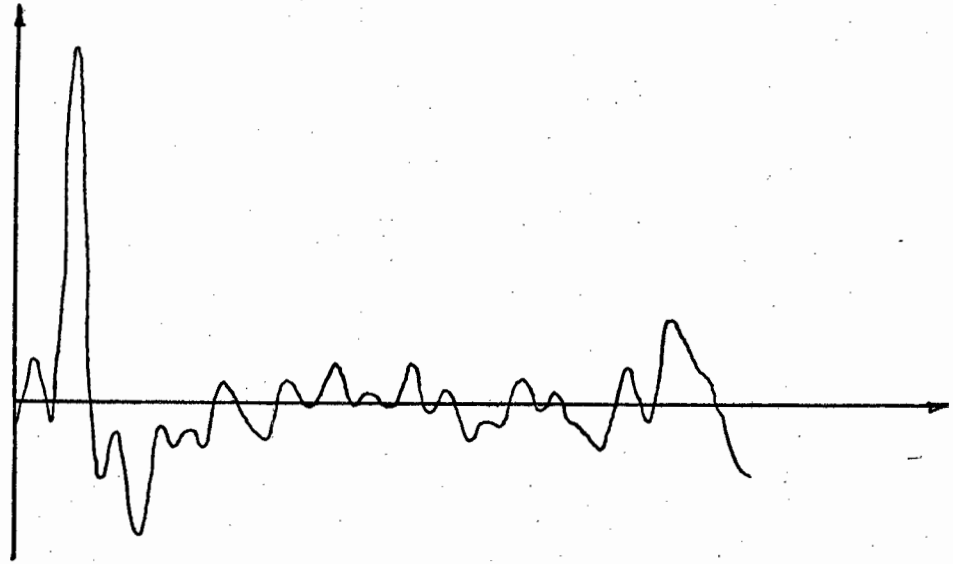
Of the remaining two correlators the modified Polar correlator has a linear relationship between quantized and unquantized signals provided the input signals are bounded in amplitude. Since flow signals have Gaussian probability density functions (see Chapter 9), they are not bounded. However, under certain conditions (viz., $\sigma/A < 1$), the linear relationship between quantized and unquantized signals still holds. For the Polar correlator and Gaussian signals a non-linear arcsine relationship exists. Since only the cross-correlation peak position is of interest, this non-linearity is not a disadvantage, so the simplest and cheapest correlator that gives adequate results in correlation flowmetering is a Polar correlator. It does not require additional noise signals that comply with certain requirements as does the modified Polar correlator. Furthermore, since only the polarity of the signals is used, any amplitude or frequency non-linearity in previous electrical stages, will not produce an error, provided the signals have zero mean value.

In Reference 107 the performance of the Polar correlator is compared with that of the relay and true correlators. The conclusion reached is that for steady-state applications such as locating a correlation peak, there is little advantage to going beyond the Polar correlator.

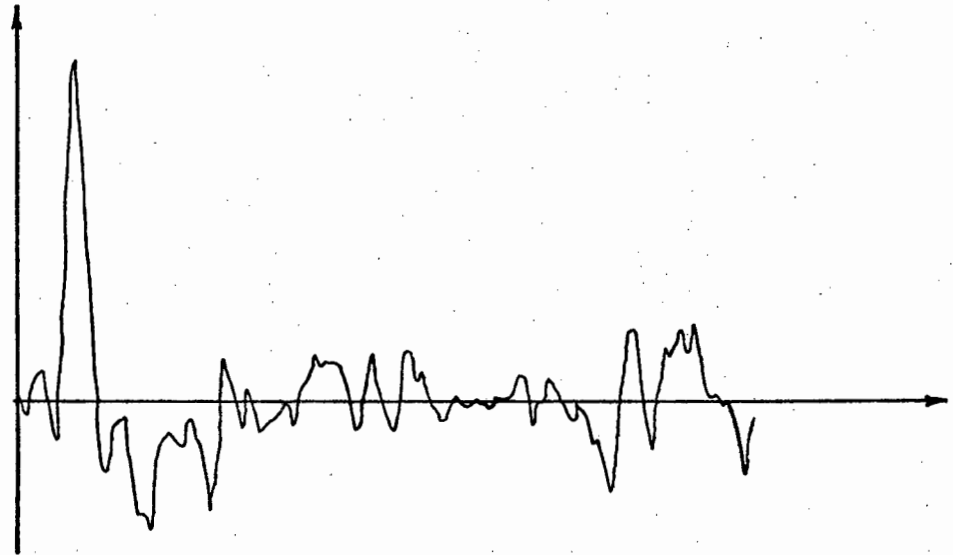
A disadvantage of the Polar correlator is that its averaging time has to be increased with respect to that of a true correlator in order to obtain results of the same statistical accuracy.

Another technique using pulse frequency signals has been proposed by OWEN⁽¹⁵⁷⁾. Like the Polar correlator, it encodes the signals into a binary sequence but it is sensitive to the shape of the input signals. However, as is the case with the delta-modulator correlator, a rigorous theoretical analysis of the pulse frequency modulator has not yet been obtained. In addition, experimental results show no major improvements over the Polar correlator.

Direct
correlation



Polar
correlation



Delta
modulation
correlation

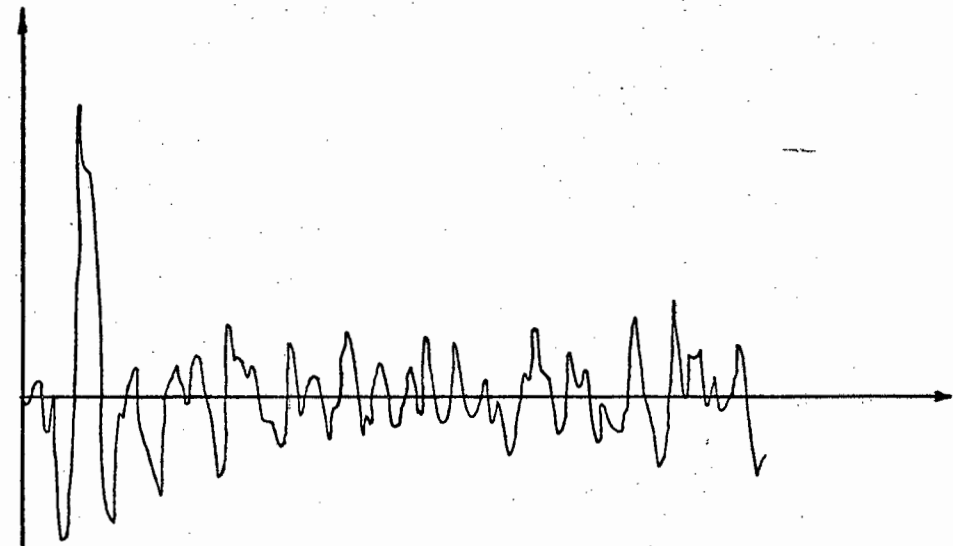


FIG. 5.13 Comparison of direct, polar and Delta modulation correlators

CHAPTER 6

REVIEW OF HARDWARE CORRELATORS

1. INTRODUCTION

One of the main factors that has prevented correlation flowmetering from being widely used in industry, has been the lack of a cheap dedicated peak-seeking correlator. In many experiments, on-plant digital computers and general-purpose proprietary correlators have been used for determining cross-correlation functions. Such instruments are satisfactory for experimental work but they are far too expensive and elaborate for widespread industrial application.

In the preceding chapter it was shown that in order to simplify the process of correlation function evaluation, it is possible to quantize the amplitude of both flow signals to only two levels. This is one method of reducing the cost of a correlator. Further cost reduction can be achieved by a suitable hardware implementation of a peak-finding correlator. The aim of this chapter is to specify the requirements of a correlator for slurry flowmetering and to examine existing correlator configurations in terms of these requirements to see, which, if any are suitable.

2. REQUIREMENTS OF CORRELATOR FOR SLURRY FLOWMETERING

(i) Range

The flowrate limits of slurries used in the mining industry are generally taken as follows⁽²⁴⁹⁾.

(a) Minimum velocity determined by velocity at which settling occurs.

This varies from 1,2 m/sec (4 ft/sec) to 3,8 m/sec (12 ft/sec) depending on the product.

(b) Maximum velocity is usually determined by the velocity at which

abrasion becomes excessive and lies between 6 m/sec and 8m/sec.

Based on these limits, the flowmeter to be designed should be capable of measuring flow velocities ranging from 1 m/sec to 10 m/sec so as to include all possibilities. In order to estimate the total delay range of a correlator recall that

velocity $u = d/\beta$ where d is the sensor spacing and β the mean transit time of the fluid between the sensors.

Hence, transit time $\beta = d/u$

Now the optimum transducer spacing, d , lies between one and three pipe diameters depending on the application (Chapter 10). Typical pipe diameters vary from $D_{\min} = 2.5 \text{ cm (1")}$ to $D_{\max} = 50 \text{ cm (20")}$. To estimate the maximum time delay, assume minimum flow velocity and maximum sensor spacing. Suppose this maximum sensor spacing is one pipe diameter for the largest pipe, then

$$\tau_{\max} = \frac{d_{\max}}{u_{\min}} = 500 \text{ m secs}$$

Similarly to estimate the minimum time delay assume maximum flow velocity and minimum transducer spacing. Suppose this minimum spacing is two pipe diameters for the smallest pipe, then

$$\tau_{\min} = \frac{d_{\min}}{u_{\max}} = 5 \text{ m secs}$$

A correlator delay range of 5 m secs - 500 m secs would thus cover most flowrates of interest, giving for a fixed sensor spacing a flowrange of 100:1. Such a range far exceeds the flowrate limits of most industrial and mining processes.

(ii) Resolution

Ideally the correlator should have infinite resolution over the entire delay range of interest. In practice, finite resolution

is acceptable provided the errors due to resolution are small (<1% say) over the entire delay range. As an example, suppose the correlation function is evaluated at N equally spaced points, Δt seconds apart. A correlation peak detected at $\beta = n \cdot \Delta t$ could be in error by as much as $\pm \frac{1}{2} \Delta t$. The fractional error in peak position is thus:

$$\epsilon = \frac{\pm \frac{1}{2} \Delta t}{n \cdot \Delta t} = \pm \frac{1}{2n}$$

Now for N = 100 (say), an error of only $\pm 0.5\%$ of reading will exist at the upper end of the delay range (n=100). However, this error increases to a totally unacceptable value of $\pm 50\%$ of reading at the lower end of the range (n=1).

(iii) Accuracy

At present electro-magnetic flowmeters are used mainly for slurry flowrate measurements. Under ideal conditions their accuracy is of the order of ± 1 per cent of actual flow or ± 0.2 per cent full-scale deflection whichever is greater⁽¹¹⁾. However, in typical industrial applications, factors such as fluid impurities and nearby electrical cabling can degrade the performance of electro-magnetic flowmeters. Accuracies of about ± 5 per cent of reading are more realistic especially at reduced flowrates⁽⁶⁾. Since a correlation flowmeter is totally independent of external factors, it should be possible to obtain accuracies of better than ± 1 per cent of reading.

(iv) Linearity

A correlation flowmeter is an inherently linear device if flow velocity is plotted against $1/\beta$ where β is the cross-correlation peak position for fixed sensor spacing.

(v) Response Time

Since, except in rare instances, the time constants associated with flowrate changes in a mining environment, are of the order of seconds, the response time of the correlator to a change in peak position need also only be of this magnitude.

(vi) Repeatability

The repeatability, that is the maximum variation in correlator output for fixed flowrate should be within $\pm 0,1$ per cent.

(vii) Peak-finding Capability

Since it is only the time delay of the position of the cross-correlation peak that is of interest, the correlator must possess peak-finding and tracking capabilities and provide an output that is inversely proportional to peak position. In addition, if minor peaks due to spurious effects (see Chapter 10) are present, the correlator must be able to distinguish between the peaks and select the largest.

(viii) Cost

Finally, the cost of the correlator should be as low as possible provided that all the other requirements are satisfied.

3. REVIEW OF EXISTING CORRELATORS

It is possible, in principle, to determine the cross-correlation function between two stationary processes $x(t)$ and $y(t)$ in one of three ways:

- (i) By using the time domain definition of the cross-correlation function

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t-\tau) \cdot y(t) \cdot dt$$

- (ii) By determining the joint probability density function $w_{xy}(x, y, \tau)$ for a particular time delay τ and using

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \cdot y \cdot w_{xy}(x, y, \tau) dx \cdot dy$$

- (iii) Or by finding the cross-spectral density function $S(\omega)$ and using the Wiener-Khinchine relationship

$$R_{xy}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cdot e^{j\omega\tau} d\omega$$

Of these only the first method using the time integral is suitable for direct measurement. Determining the joint probability density function or cross-power spectral density is in itself a complicated procedure and as a result, these methods will not be considered further.

A cross-correlation function can be evaluated in either digital or analogue form via

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t - \tau) \cdot y(t) dt$$

$$\text{or } R_{xy}(k \cdot \Delta t) = \lim_{N \rightarrow \infty} \frac{1}{2N} \sum_{n=-N}^N x[(n-k) \cdot \Delta t] \cdot y[n \cdot \Delta t]$$

In practice the correlation functions are estimated by averaging over a finite period of time T or a finite number of samples N . Such estimates of the correlation function have variances that are inversely proportional to the averaging times. By choosing T or N sufficiently large, this variance can be made arbitrarily small.

The main elements required by all correlators are variable delay units, multipliers and averagers. Different methods of implementing these elements to obtain cross-correlation functions exist, however, and some will be described below.

3.1 Serial Correlator

A serial correlator (Figure 6.1) directly implements the definition of the correlation function by delaying one signal by an amount τ , multiplying the delayed signal with the undelayed signal and averaging the product. This yields an estimate of the correlation function for the particular delay τ . In order to generate the correlation function over the entire delay range of interest, the above process is repeated sequentially for other values of delay - hence the name serial.

A typical method for obtaining a variable delay is to use a tape-loop with variable displacement between the record and playback heads. This can be done either for a continuous variation of delay using an RC averager or for discrete delay intervals by stepping the playback head along and averaging linearly for a fixed period of time at each delay point^(116,133).

Alternatively a bucket-brigade or charge-coupled delay line can be used^(114,115 and 178). Such a delay line consists essentially of an input sampling stage followed by a series of capacitors interconnected by transistor switches. The signal to be delayed is sampled and the samples held in the form of charge that can be transferred from capacitor to capacitor via the switches. Each stage consists of two switches and a sample moves through one complete stage in one clock cycle. The delay thus produced is proportional to the number of delay line stages and inversely proportional to the clock frequency. By sweeping the clock frequency, a continuously variable delay is obtained.

Of the commercial correlators, the DISA correlator operates using the serial method⁽¹⁷⁹⁾.

Advantages of Serial Correlator

- (a) Hardware costs low depending on delay element since only a single multiplier and averager are used.
- (b) Infinite resolution possible since the delay can be varied infinitely.
- (c) Large delay range possible.

Disadvantages of Serial Correlator

- (a) For large delay range and good repeatability (i.e., long averaging time) the method is very slow.
- (b) Real-time operation not possible.
- (c) No inherent peak detection and tracking or peak discrimination

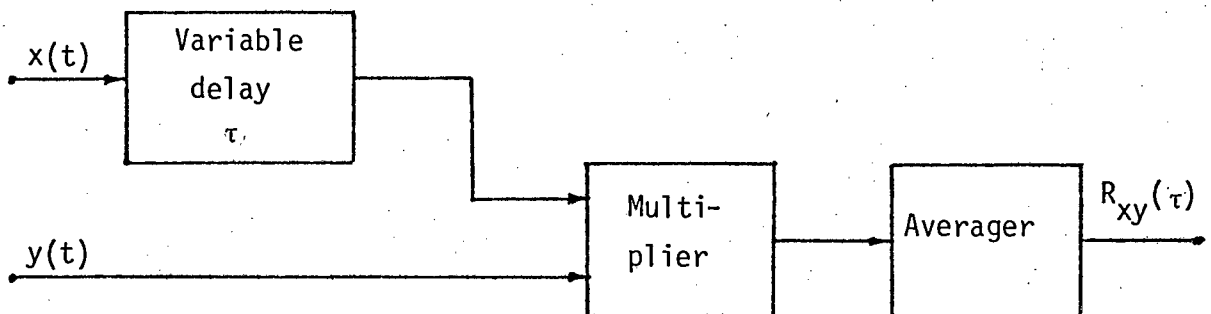


FIGURE 6.1 : Serial Correlator

3.2 Parallel Correlator

A parallel correlator consists, in effect, of a number of serial correlators operating in parallel at fixed time delays (Figure 6.2). It thus evaluates the correlation function simultaneously at a number of points. At each time delay, a multiplier and an averager is required making this correlator very expensive, if the correlation function is to be evaluated with good resolution over a large delay range.

Both tapped analogue delay lines or digital shift-registers can be used as delay elements. For these devices, the time delay at any point is equal to the number of delay stages preceding the point divided by the clock rate.

The proprietary correlator of Princeton Applied Research, PAR 101, is based on the parallel correlator principle. It is a hybrid correlator in which one signal is unquantized whereas the other is coarsely quantized to three bits. The quantized signal is delayed via a three-bit digital shift register and controls the gain of amplifiers which act as multipliers. Averaging is done via an R-C network with variable time-constant. The correlator thus uses the Stieltjes quantization method described in Chapter 5.

An even simpler implementation is achieved if the relay method is used⁽¹¹³⁾. In this case only a simple one bit shift register is required and multiplication reduces to an inversion/non-inversion of the unquantized signal.

Parallel correlators in which both signals are encoded to a single bit are described in References(104,158 and 175).

Advantages of Parallel Correlator

- (a) Very fast response possible since the correlation function is estimated simultaneously at a number of delay values.
- (b) Real-time operation is possible.
- (c) If cost is not a limitation, large delay range and good resolution possible.

Disadvantages of Parallel Correlator

- (a) High hardware cost for good resolution over large delay range.
- (b) Finite resolution due to finite number of delay stages.
- (c) No inherent peak detection and tracking or peak discrimination.

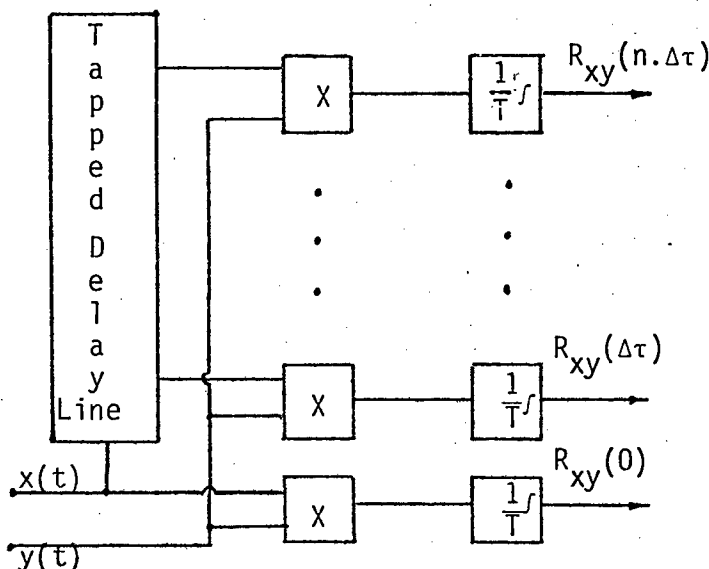


FIGURE 6.2 : Parallel Correlator

3.3 Serial-parallel Correlator

A serial-parallel correlator calculates a correlation function simultaneously at a number of delay values. However, it requires only a single multiplier and averager. Its operation is best described with reference to Figure 6.3. Input waveforms $x(t)$ and $y(t)$ are sampled and the current sample of $x(t)$ is loaded into location 0 of a delay store. Locations 1 to 99 (say, for a 100 point correlator) contain previous values of $x(t)$ all stacked in order. To compute its estimate of the correlation function, the correlator calculates:

$$R_{xy}(k.\Delta t) = \overline{x(t - k.\Delta t).y(t)}$$

for 100 values of k , i.e., for $k = 0, 1, 2, \dots, 98, 99$.

For this operation switch S (see Figure 6.3) is positioned so that the contents of the delay store rotate until they arrive back in their original positions. During this operation, each delayed value of $x(t)$ passes the multiplier where it is multiplied with the stored value of $y(t)$ to produce a series of 100 products $x(t - k.\Delta t).y(t)$; $k = 0, 1, \dots, 99$. The products are fed into an averager where they interact with the contents of the serial main store which are also undergoing a rotate operation. The contents of the main store are the 100 previously calculated values of $R_{xy}(k.\Delta t)$. In the averager the new product $x(t).y(t)$ updates $R_{xy}(0)$, $x(t-\Delta t).y(t)$ updates $R_{xy}(\Delta t)$ and so on up to $R_{xy}(99.\Delta t)$. The main store, therefore, always contains the most recent estimate of the correlation function. When this cycle of rotation, multiplication and averaging is complete, a new sample of both signals $x(t)$ and $y(t)$ is taken. The new value of $x(t)$ is put into location 0 of the delay store and all others are shifted down one place. The sample at $x(t-99.\Delta t)$ is discarded since it is of no further value. This correlator thus updates the correlation function at a series of delay values between sampling intervals.

The above method only works at sampling rates at which the time interval between samples is long enough for all the required operations to be performed. It is thus only suited for low frequency applications. However, since the input signals are assumed to be stationary, a batch sampling mode can be used. Instead of entering one sample of $x(t)$ into the delay store at the instant when $y(t)$ is sampled, the sampling of $y(t)$ is preceded by taking 99 fresh samples of $x(t)$ as before, but now the speed at which samples can be taken is independent of the time required to process them. After performing the operations of

rotation, multiplication and averaging, all the old samples are discarded and a new set of samples is acquired. For good statistical accuracy, a large number of samples of $y(t)$ must be taken and this makes the measurement less efficient than the method described previously.

Both the HONEYWELL SAICOR⁽¹⁸¹⁾ and HEWLETT-PACKARD 3721A⁽¹⁸⁰⁾ correlators are based on the serial-parallel technique. The HP3721A correlator, for example, quantizes one signal to seven bits and coarsely quantizes the other to only three bits. This, in effect, is identical to the Stieltjes method except that fine quantization of the one signal is used. The signal quantized to three bits can have the values of 0, ± 1 , ± 2 or ± 4 . The virtue of this sample encoding is that to multiply the quantized output by another binary word involves only shift operations which saves hardware in the multiplier.

Advantage of Serial-parallel Correlator

- (a) Cheaper than the parallel correlator, but not by much.
- (b) Real-time operation, i.e., fast response for low sampling rate or small delay range.
- (c) Large delay range with good resolution possible in batch mode.

Disadvantages of Serial-parallel Correlator

- (a) Still relatively expensive and complex since electronics must be capable of performing a large number of operations between sampling intervals.
- (b) Finite resolution since correlation function is evaluated at a finite number of time delays.
- (c) Response slow in batch mode.
- (d) No inherent peak location and tracking or peak discrimination.

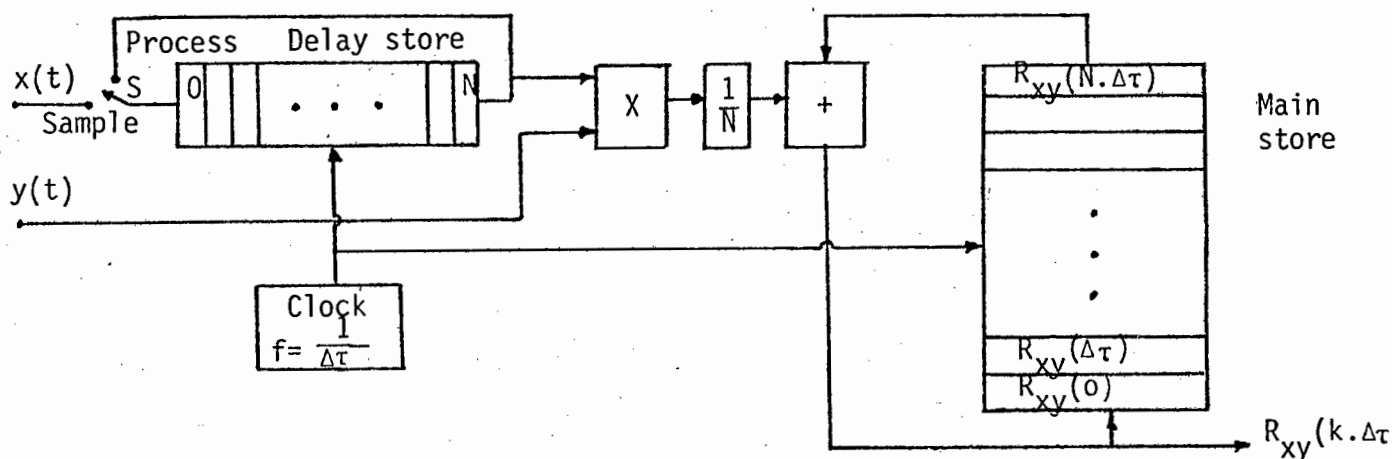


FIGURE 6.3 : Serial-parallel Correlator

3.4 Two-point Difference Correlator

A two-point difference correlator (see Figure 6.4) differs considerably from the correlators described above. As the name implies, it evaluates the cross-correlation function, $R_{xy}(\tau)$ simultaneously at two delay values τ_1 and τ_2 given by

$$\tau_1 = n \cdot \Delta t = n/f \quad \text{and} \quad \tau_2 = (n + m) \cdot \Delta t = (n + m)/f$$

where f is the shift rate of the signal to be delayed through n and m -stage delay lines.

From their difference, viz., $R_{xy}(\tau_1) - R_{xy}(\tau_2)$ is derived the input to a voltage controlled oscillator (V.C.O.) that governs the shift rate, f , of the delay lines and hence the delay values τ_1 and τ_2 . These delays can thus automatically be moved into the balance position shown in Figure 6.5, in which the difference is zero. With a symmetrical correlation function, the clocking rate in the balance position is directly related to the flow rate since in this position:

$$\text{delay, } \beta = \frac{\tau_1 + \tau_2}{2} = \frac{2n + m}{2} \cdot \Delta t = \frac{2n + m}{2} \cdot \frac{1}{f}$$

$$\text{and flow velocity, } v = d/\beta = \frac{2d}{2n + m} \cdot f$$

that is, $v \propto f$

A correlator based on a similar method has been reported as early as 1961 by BUTTERFIELD et al⁽¹³³⁾. They used a tape-recorder with two closely spaced play-back heads as their delay element. A servo-mechanism controlled the position of the playback heads so as to bring the correlator into the balance position. Recently further reports on the two-point difference method have appeared^(77, 93, 112 and 137). For example, TOMPKINS et al⁽⁷⁷⁾ describe a correlator in which the correlation function is evaluated at three equally spaced delay values. The two extreme delays are used in a two-point difference configuration whereas the centre delay value lies at the correlation peak position. The correlator only operates if a broad correlation peak is assumed to exist and is unsuited for a general slurry flowmeter.

Advantages of Two-point Difference Correlator

- (a) Less complex and costly than multi-point correlators.
- (b) In the balance position, the clock rate is directly proportional to the flow velocity - a suitable output that easily can be measured is thus available.
- (c) The device tracks a correlation peak once such a peak has been located.

- (d) Infinite resolution possible, since the delays τ_1 and τ_2 can be varied infinitely.
- (e) Fast response possible to changes in peak position once the correlator has locked onto a peak.

Disadvantages of Two-point Difference Correlator

- (a) The device cannot discriminate between correlation peaks.
- (b) It will not lock onto a peak if situated far from a peak. This can easily occur if narrow correlation peaks exist and a large delay range is used.
- (c) Error if correlation functions are assymmetric.

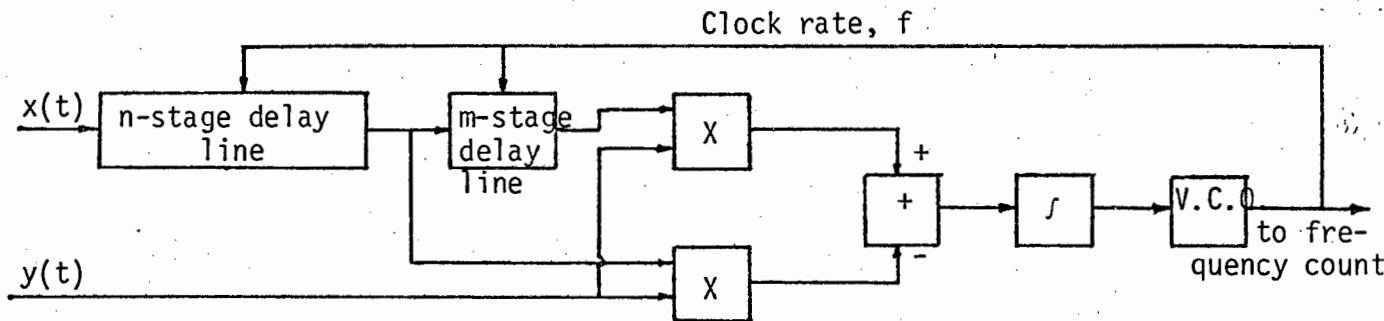


FIGURE 6.4 : Basic Two-point Difference Correlator

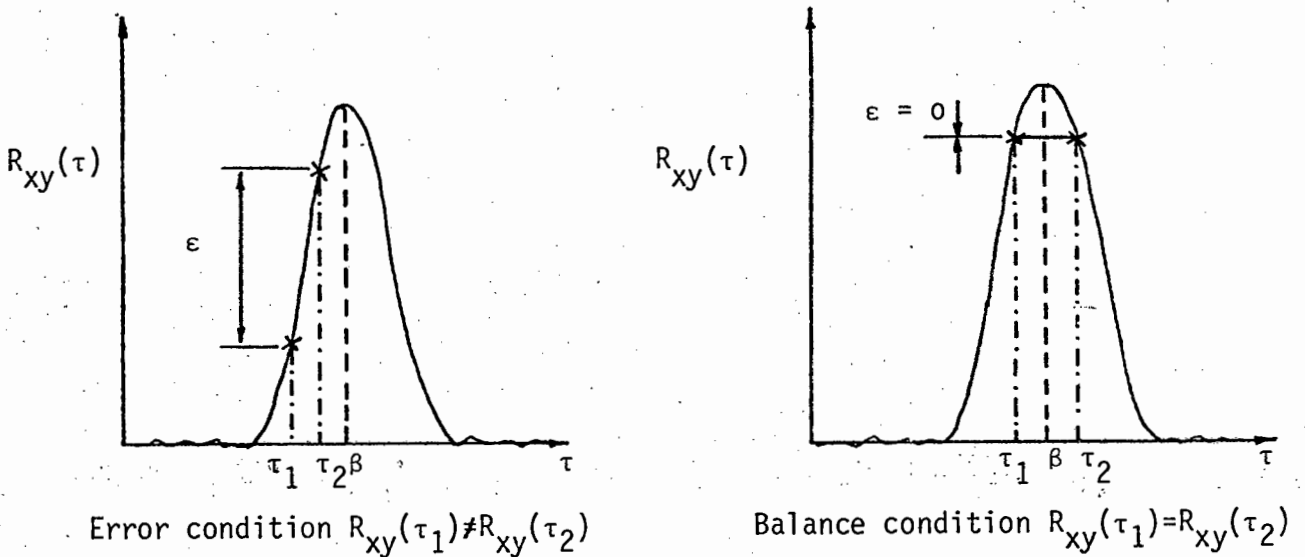


FIGURE 6.5 : Principle of Operation of Two-point Difference Correlator

3.5 Optimal Tracking Correlator

An optimal tracking correlator is in some ways similar to the two-point difference correlator in that it also tracks a correlation peak once such a peak has been located. However, its operation is different in that it does not find the correlation peak time-delay by straddling the peak; instead it finds this time delay by minimizing the mean square difference between upstream and downstream flow signals by varying the time delay.

Suppose the two signals to be correlator are $x(t)$ and $y(t)$, then after delaying the upstream signal their difference is given by:

$$\delta(t) = x(t - \tau) - y(t)$$

The correlation peak is now found by changing the delay τ such that the mean square difference between the signals is minimized, i.e.,

$$\overline{\delta(t)^2} = \text{minimum}$$

A necessary condition for this is:

$$\frac{d}{d\tau} \left[\overline{\delta(t)^2} \right] = 0$$

or

$$\overline{2\{\delta(t) \cdot \frac{d\delta(t)}{d\tau}\}} = - \overline{2\{[x(t-\tau)-y(t)] \cdot \dot{x}(t-\tau)\}} = 0$$

i.e.,

$$\overline{[x(t - \tau) - y(t)] \cdot \dot{x}(t - \tau)} = 0$$

It follows that the optimal tracking correlator should differentiate one of the signals, multiply this with the difference signal, $\delta(t)$ and form the average. Figure 6.6 shows a block diagram of this correlator. The averaged signal is applied to a voltage controlled oscillator which in turn controls the shift rate of an n-stage delay line. The clock rate of the V.C.O. is thus automatically adjusted so that the mean square difference between the signals is minimized. Once the correlator has locked onto a correlation peak, the clock frequency applied to the delay line is directly proportional to the velocity to be measured. Like the two-point difference correlator, this again results in an output that can easily be measured.

A correlator based on this principle has been analysed and constructed by MESCH et al^(75,86 and 105). To further simplify the hardware, both signals were quantized to a single bit.

The advantages and disadvantages of an optimal tracking correlator are similar to those of the two-point difference correlator. It is cheap, provides a suitable output, has infinite resolution and can track a correlation peak. However, it cannot discriminate between multiple peaks and will not lock onto a peak if situated far from such a peak. If the peak is assymetric it still locates the peak position but this is no longer directly related to the mean flow velocity.

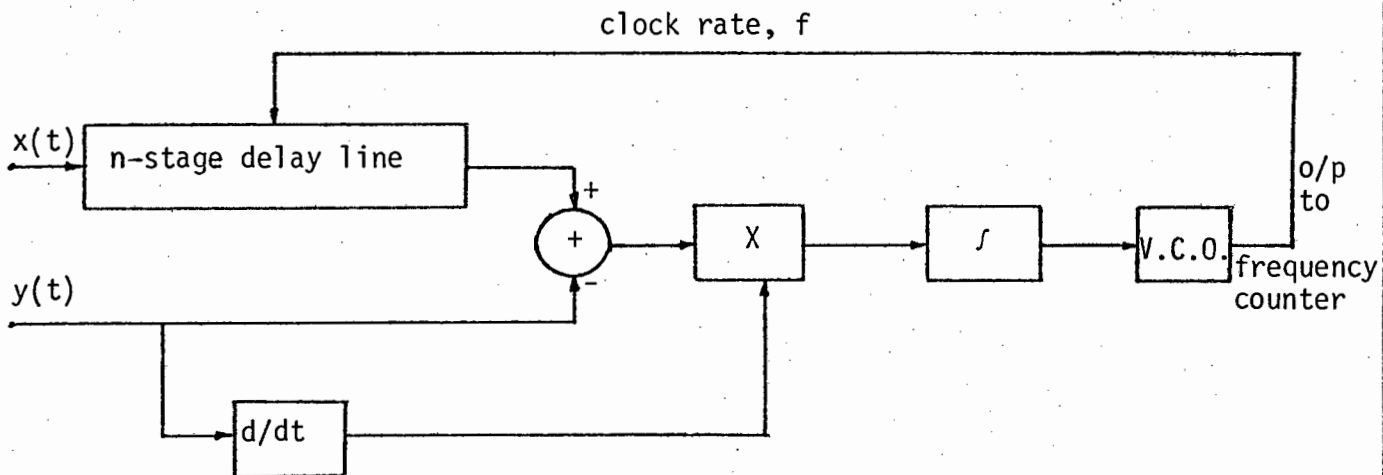


FIGURE 6.6 : Optimal Tracking Correlator

3.6 Slope Correlator

The slope method of determining flow velocity⁽⁹³⁾ is based on the assumption that the derivative of the cross-correlation function at zero time delay depends on the position of the correlation peak (see Figure 6.7). The cross-correlation coefficient is evaluated at a small time delay on either side of zero delay. The magnitude of the difference between these values is a measure of the slope at zero delay and is proportional to the flow velocity. Clearly the method only operates if a broad cross-correlation peak that has a non-zero value at zero time delay is assumed to exist. In addition although the correlator is cheap, the accuracy will, in general, be poor since the slope at zero delay will also depend on the shape of the cross-correlation function. The method is unsuited for a correlation flowmeter and will not be considered further

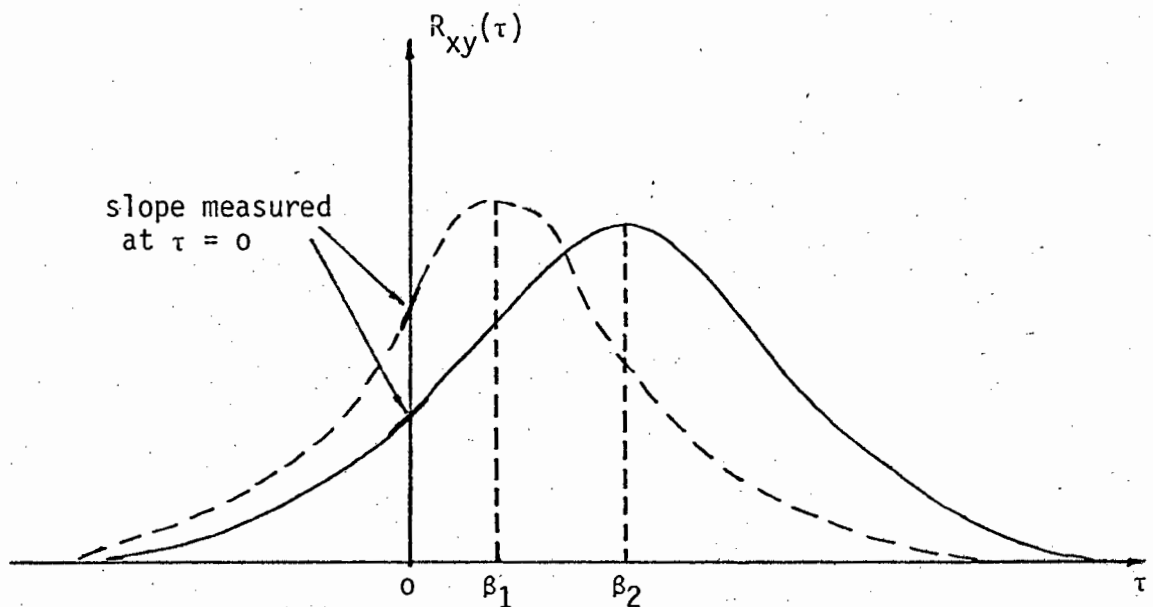


FIGURE 6.7 : Principle of Operation of Slope Correlator

4. SPECIAL CORRELATORS

One of the main drawbacks of all the correlators described in the previous section is the inability to locate a cross-correlation peak. Although the two-point difference and optimal-tracking correlators will locate and track such a peak, this is only possible if they are situated sufficiently close to a peak, in other words, these correlators have a limited capture range. Numerous reports have appeared in which special correlators possessing peak-finding capabilities are described. Some of the more significant of these correlators are discussed below.

4.1 Jordan's Integrated Circuit Correlator

The development of an integrated circuit (I.C.) correlator^(109 and 110) for flowmetering applications is one of the main developments in recent years. The correlator essentially is a parallel correlator with both signals quantized to a single bit. Each L.S.I. chip contains 12 channels complete with signal delays, polarity coincidence detectors and integrating counters. A number of these chips can be connected in series if a larger delay range is to be covered. They are constructed using the p-channel metal gate MTNS process of GENERAL INSTRUMENT MICRO-ELECTRONICS LIMITED and require +5V and -12V power supplies.

A novel method for locating a cross-correlation peak has been devised. The integrating counters that count the coincidences between upstream and downstream signals are arranged to indicate when a certain preset count is reached, i.e., when a counter overloads through its preset capacity a logic level change is

detected. Clearly the first counter to overload indicates the position of the most significant peak of the polarity correlation function. When this condition occurs, the pattern of overloads is loaded into a parallel input/serial output shift register. This shift register can be used to generate a frequency proportional to the flow velocity if the logic 1, which corresponds to the peak position, controls the selection of the parallel load operating mode.

Advantages of I.C. Correlator

- (a) Cheaper than discrete parallel correlator since less hardware is required.
- (b) Can locate a correlation peak.
- (c) Can discriminate between correlation peaks and select the largest peak.

Disadvantages of I.C. Correlator

- (a) Finite resolution.
- (b) If large delay range with good resolution is required, the correlator will still be expensive.
- (c) Relatively slow response to changes in peak position since the correlator must complete an entire averaging cycle before an updated correlation peak position estimate is outputed.

4.2 Multi-point Correlator with Controlled Sampling Frequency

In order to overcome the resolution problems of a parallel correlator which exist especially at short delay times, the shift rate of the data through the delay line is made variable. A feedback loop is used to hold the average overflow position of the integrating counters constant, in a chosen channel⁽¹¹²⁾. If the overflow actually occurs in a lower numbered channel (indicating a time delay which is too short), the clock frequency is increased so that each stage of the shift register represents a shorter delay and more stages are required to match the transit delay. If the overflow occurs in a higher numbered channel the opposite control action occurs and the clock frequency is lowered until the overflow occurs in the chosen channel. If the flow velocity changes, the correlator adjusts the clock frequency to follow the change in transit time. Suppose the sampling frequency, f , is adjusted so that the measured time delay, β , is always a pre-chosen fixed number N of sampling intervals, Δt ($\Delta t = 1/f$), then in the steady-state condition

$$\beta = N \cdot \Delta t = N/f$$

$$\text{and } f = N/\beta \propto 1/\beta$$

i.e., the clock rate is inversely proportional to the peak position and hence directly related to the flow velocity. An output that can be measured easily is thus available. Suppose N is chosen to equal 50, then the maximum fractional error due to resolution is ± 1 per cent which is acceptable.

A correlator based on this technique and using the I.C. correlator chips described in Section 4.1 has been constructed by FLEMONS⁽⁸¹⁾ with fair success.

Advantages of Correlator

- (a) Resolution problems of parallel correlator have been reduced.
- (b) Direct output of flow velocity available.
- (c) Using I.C. correlator, cost is reduced.
- (d) Peak discrimination possible since correlator will locate the largest peak.

Disadvantages of Correlator

- (a) Very expensive if discrete parallel correlator is used.
- (b) Since I.C. correlator chips are unavailable in South Africa, they cannot be used to reduce the cost.
- (c) Slow response since an entire averaging cycle of the correlator must be completed before a correction to the clock rate is made. If actual overflowing counter is far from chosen counter, considerable time will be required till the overflow occurs in the desired channel.

4.3 Further Correlation Methods

INTAGLIETTA et al⁽¹³²⁾ modified a commercially available correlator to act as a peak-seeking correlator. Using a PAR 101 one hundred channel parallel correlator to calculate the cross-correlation function, they added additional circuitry to automatically calculate the correlation peak position. This results in a time-saving device for determining flow velocities in experimental work, but due to the cost of proprietary correlators, it is unsuitable for industrial application.

A correlator based on the serial/parallel technique but including a peak detector is described by TAI et al⁽¹¹¹⁾. An extrapolation method is used to improve the estimate of the peak position, this being especially useful when the correlation function is flat and noisy about the peak. The cost and complexity of the serial/parallel technique, however, still makes this correlator unattractive.

TOMKINS et al⁽⁹⁵⁾ describe a circuit using sample and holds with swept sampling frequency. Although an output proportional to velocity is provided, the correlator only operates over a small delay range and has limited potential.

BULLOCK⁽¹²¹⁾ and HILL et al⁽⁷³⁾ have both designed peak-finding correlators. However, their devices are based on measuring the transit time of injected tracers and cannot be used with naturally occurring tracers.

5. CONCLUSION

It appears that if the two-point difference or optimal tracking correlators could be arranged to have a greater capture range, all the requirements specified for a peak-finding correlator for slurry flowmetering would be satisfied. These correlators are simple, cheap, have infinite resolution and a fast response and can track a correlation peak.

BUTTERFIELD et al⁽¹³³⁾ used a roller in contact with the moving surface to drive a tachogenerator and so provide an output that gives a rough estimate of the velocity of the surface. This rough estimate was used to direct their peak-seeking correlator to the approximate position of the correlation peak. This correlator then accurately locked onto the correlation peak and provided an output proportional to the velocity of the surface.

A similar system could be used for the two-point difference or optimal tracking correlators. For example, a standard low-cost flowmeter having poor accuracy could be installed to provide a rough estimate of flow velocity. The estimate could then be used to control the two-point difference or optimal-tracking correlators so that the correlation peak will be within their capture ranges. The need of an additional flowmeter, its installation costs and maintenance requirements, however, make this method unattractive. This especially is the case for slurry flowmetering for which a non-intrusive clamp-on flowmeter is of primary importance.

In order to maintain a clamp-on flowmeter, two new correlators are proposed. The first is a microprocessor-based device whereas the second uses discrete logic. Both locate the approximate position of a correlation peak over a wide delay range and direct a two-point difference correlator to this approximate position. The latter correlator can then lock onto and accurately track the correlation peak. The following chapter will give a detailed description of both correlators.

CHAPTER 7

DESIGN OF PEAK-SEEKING CORRELATORS

1. INTRODUCTION

This chapter is divided into two parts. Part I describes a micro-processor controlled two-point difference correlator and Part II a serial two-point difference correlator. In the discussion, the characteristics of two correlators are compared and it is decided that the former instrument has greater potential.

PART I : MICRO-PROCESSOR CONTROLLED TWO-POINT DIFFERENCE CORRELATOR

2. PRINCIPLE OF OPERATION

The micro-processor controlled two-point difference correlator shown in Figure 7.1 consists, in effect, of two separate correlators operating in parallel, each having its own particular function but with the micro-processor correlator controlling the two-point difference correlator.

The micro-processor correlator evaluates the cross-correlation function at a number of equally spaced points over the delay range of interest and finds the approximate position of the main correlation peak. (Figure 7.2). The resolution of this correlator is poor and it thus directs the two-point difference correlator to the approximate peak position by suitably controlling the shift rate of the delay lines. The latter correlator has good resolution and can lock onto and track the peak accurately.

The main drawback of the two-point difference correlator, namely its inability to locate the main correlation peak has thus been overcome. The total delay range is determined by that of the micro-processor correlator and can be made large. The response time of the micro-processor correlator is of the order of seconds, whereas the two-point difference correlator has a very much faster response. The response time of the composite correlator thus depends on whether the two-point difference correlator is locked onto a correlation peak or not. If a peak has been located, small changes in peak position can be tracked rapidly. Otherwise one cycle of the micro-processor correlator must be completed before the two-point difference correlator is again directed to a correlation peak and tracking can take place.

The composite correlator still possesses all the advantages of the two-point difference correlator but has greater potential. It appears to satisfy all the requirements specified for a correlation flowmeter in the previous chapter.

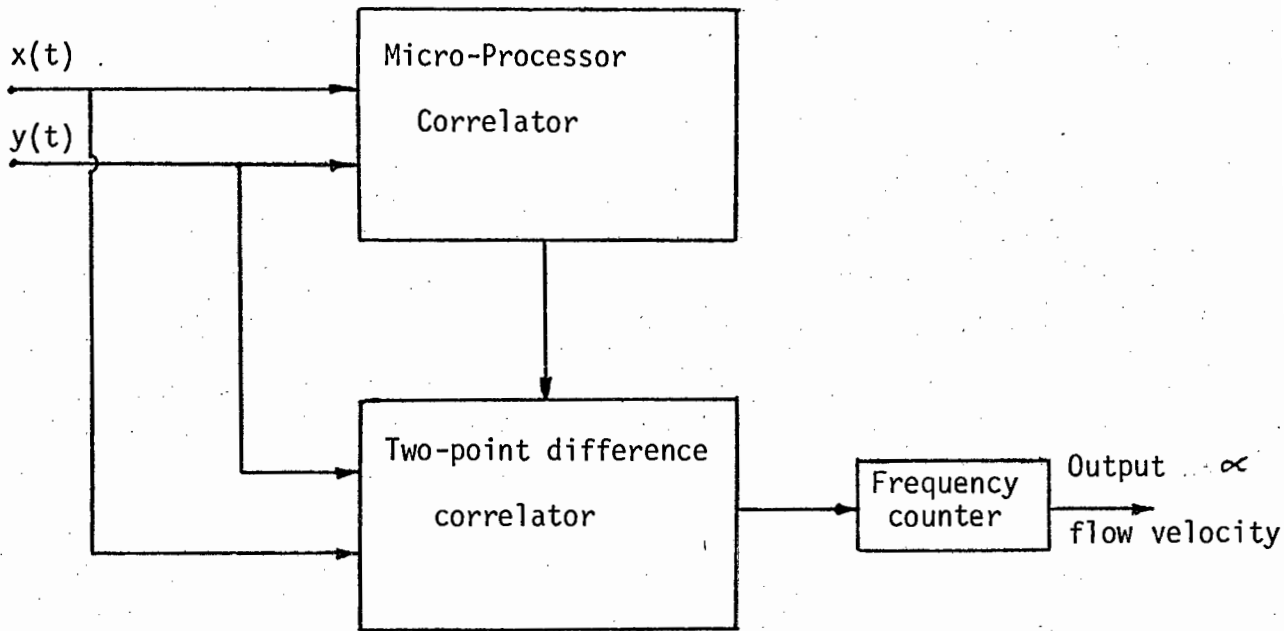


FIGURE 7.1 : Micro-processor Controlled Two-point Difference Correlator

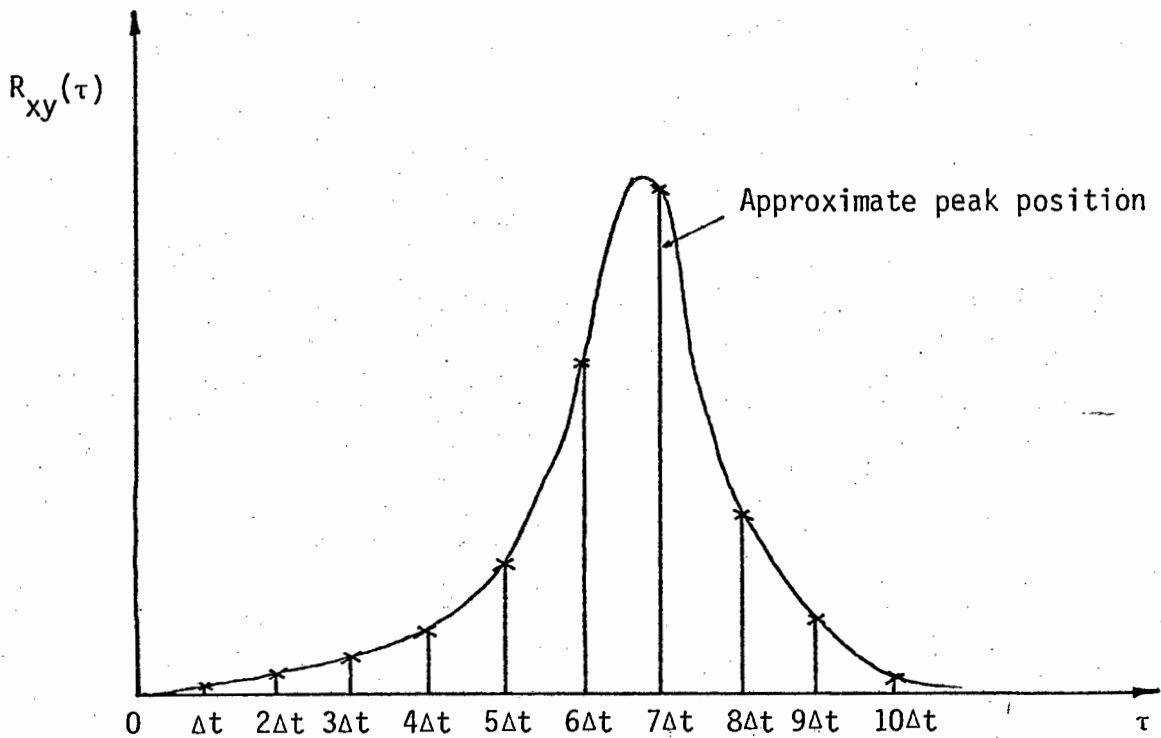


FIGURE 7.2 : Delays at which Micro-processor Correlator Evaluates the Correlation Function

3. MICRO-PROCESSOR CORRELATOR

3.1 Method of Operation

The micro-processor correlator must locate the approximate position of the main correlation peak over the total delay range of interest and output this approximate position to the two-point difference correlator. It consists of two clocked comparators that convert the analogue input signals into a series of ones and zeroes depending upon the sign of the input signals, a suitable micro-processor system and an interface to the two-point difference correlator as shown in Figure 7.3.

The micro-processor correlator is initially designed to operate in the serial-parallel mode. Other methods of operation, such as the batch mode (see Chapter 6), could be developed later using the same hardware, if found necessary. This highlights one of the major advantages of a microprocessor-based system since changes in the operation of the correlator can readily be achieved by appropriate changes in the software. No alterations to the hardware are necessary since irrespective of the software, the same sign detectors and interface to the two-point difference correlator can be used. Furthermore, parameters such as the total delay range and the averaging time can easily be tailored to suit any application.

The serial-parallel method, previously described in Chapter 6, was based on a hardware system. The same method, however, can be adapted to a microprocessor-based system. In this mode of operation, N samples of the upstream signal, $x(t)$, and a single sample of the downstream signal, $y(t)$ are stored at any time. The one downstream sample is taken at the same instant as the last upstream sample was taken. The other $N-1$ upstream samples have been taken previously, i.e., earlier in time. Figure 7.4 shows the relationship between the samples of both signals; the time interval between samples being Δt . The present samples of the two signals are x_0 and y_0 and x_1 to x_{N-1} are the $N-1$ previous samples of the signal $x(t)$.

During the time interval between samples Δt , the correlation function $R_{xy}(\tau)$ is updated at N delay values. This is done by first forming the products $x_0 \cdot y_0, x_1 \cdot y_0, \dots, x_{N-1} \cdot y_0$ i.e., the one sample of the signal $y(t)$ is multiplied with all the stored samples of the signal $x(t)$. This operation reduces to a sample logic function for polar correlation. These products are then added to the previous estimates of the correlation function $R'_{xy}(k \cdot \Delta t)$ to yield the updated estimates $R_{xy}(k \cdot \Delta t)$ for $k = 0, 1, \dots, N-1$, i.e.,

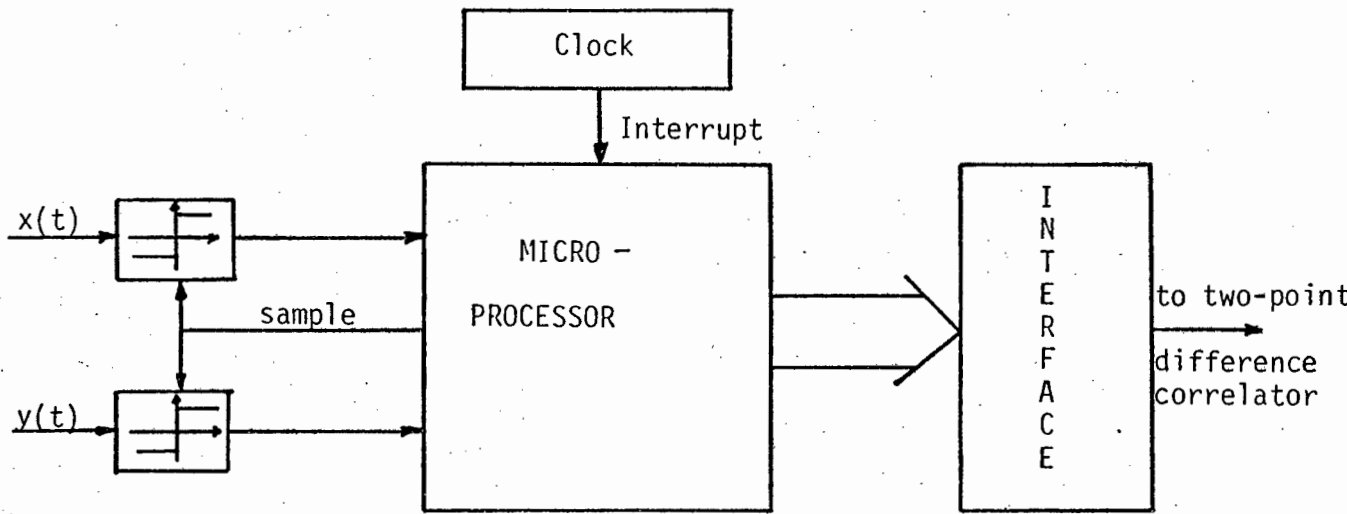


FIG. 7.3 Micro-Processor correlator

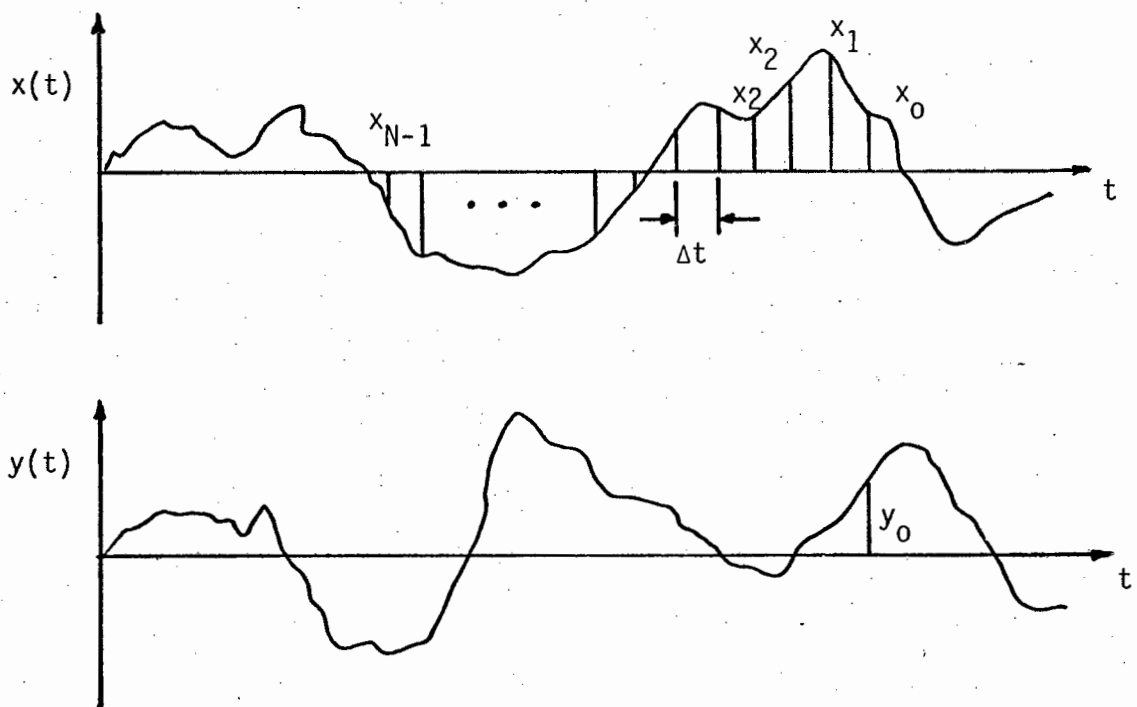


FIG. 7.4 Relationship between signal samples

$$R_{xy}(0) = R_{xy}'(0) + x_0 y_0$$

$$R_{xy}(\Delta t) = R_{xy}'(\Delta t) + x_1 y_0$$

$$\vdots \quad \quad \quad \vdots$$

$$R_{xy}[(N-1) \cdot \Delta t] = R_{xy}'[(N-1) \cdot \Delta t] + x_{N-1} \cdot y_0$$

After completing the updating of all the correlation estimates, a new sample of both signals is taken; the last sample of $x(t)$, i.e., x_{N-1} is discarded as is the previous sample of $y(t)$, viz., y_0 . The sample x_{N-2} becomes x_{N-1} , x_{N-3} becomes x_{N-2} and so forth until x_0 becomes x_1 and the fresh sample replaces x_0 .

Hereafter an update cycle is again entered. When the above process has been repeated for a specified number of times, the microprocessor enters a peak finding routine in which it compares the accumulated estimates of the correlation function and finds the delay yielding the approximate peak position. This approximate peak position will be correct to $\pm \frac{1}{2} \Delta t$ where Δt is the sampling interval.

The proposed peak finding correlator must cover the delay range 5 m secs - 500 m secs. Suppose samples are taken every 5 m secs, then 100 samples of the signal to be delayed must be stored in order to cover the specified delay range. The correlation function hence is calculated at 100 points, at 5 m secs intervals. The position of the correlation peak will thus be found to within 2,5 m secs by the microprocessor correlator.

3.2 Requirements of Micro-processor

The main requirement of the microprocessor used for this application is that it be sufficiently fast to update 100 correlation estimates in less than 5 m secs. A microprocessor, like all digital computers, is a sequential device, that is, it can only execute one instruction at a time. In the present application, the microprocessor must take a sample of both signals, shift all the samples of the signal to be delayed by one position, form 100 products and update 100 correlation function estimates, all this within 5 m secs. Even with microprocessors having instruction cycle times of 2 μ secs available, this requires considerable optimization of the software.

The memory requirements for the microprocessor correlator can be estimated to be as follows:

1. 100 Locations of RAM are required for storing the 100 samples of the delayed signal.
2. 200 Locations of RAM are required for the correlation function estimates (2 locations per estimate). The total amount of RAM is hence approximately 300 locations. The actual correlation programme will be stored in ROM and should be less than 1 K in length. A microprocessor system with 1 K of ROM and 512 locations of RAM should hence be adequate.

In addition, the input-output requirements consist of two lines for inputting the polarity quantized up and downstream signals and a parallel output for the two-point difference correlator interface.

3.3 Micro-processor Correlator Hardware

The SDK-80 evaluation kit based on the INTEL 8080A microprocessor was used to implement the microprocessor correlator.

Figure 7.5 shows a detailed diagram of the SDK-80 microprocessor kit. The only components required for the microprocessor correlator in addition to those supplied with the SDK-80 kit are:

1. An external clock to generate interrupts, and
2. sign detectors (1 bit A/D's) for both inputs.

Whenever the microprocessor receives an interrupt, it samples both input channels before commencing the updating of the correlation function estimates. These interrupts are generated by an external clock, the frequency of which can be adjusted if required. Initially a frequency of 200 Hz corresponding to a period of 5 m secs was used.

The sign detectors convert the analogue input signals into a string of ones and zeroes, depending upon the sign of the input signal. Each consists of a comparator with hysteresis and a D-type latch as shown in Figure 7.6. On receiving an interrupt, the microprocessor simultaneously clocks both D-type latches before reading the output of the latches. This ensures that the data into the microprocessor is stable during the read operation.

The interface between the two correlators requires an 8-bit binary word from the microprocessor. The 8255 Programmable Peripheral Interface (P.P.I.) provides 24 input-output lines which can be grouped into various configurations by the software.

The P.P.I. essentially consists of three ports (A, B and C) each having 8 input-output lines. Two lines of Port A are used for inputting the signals

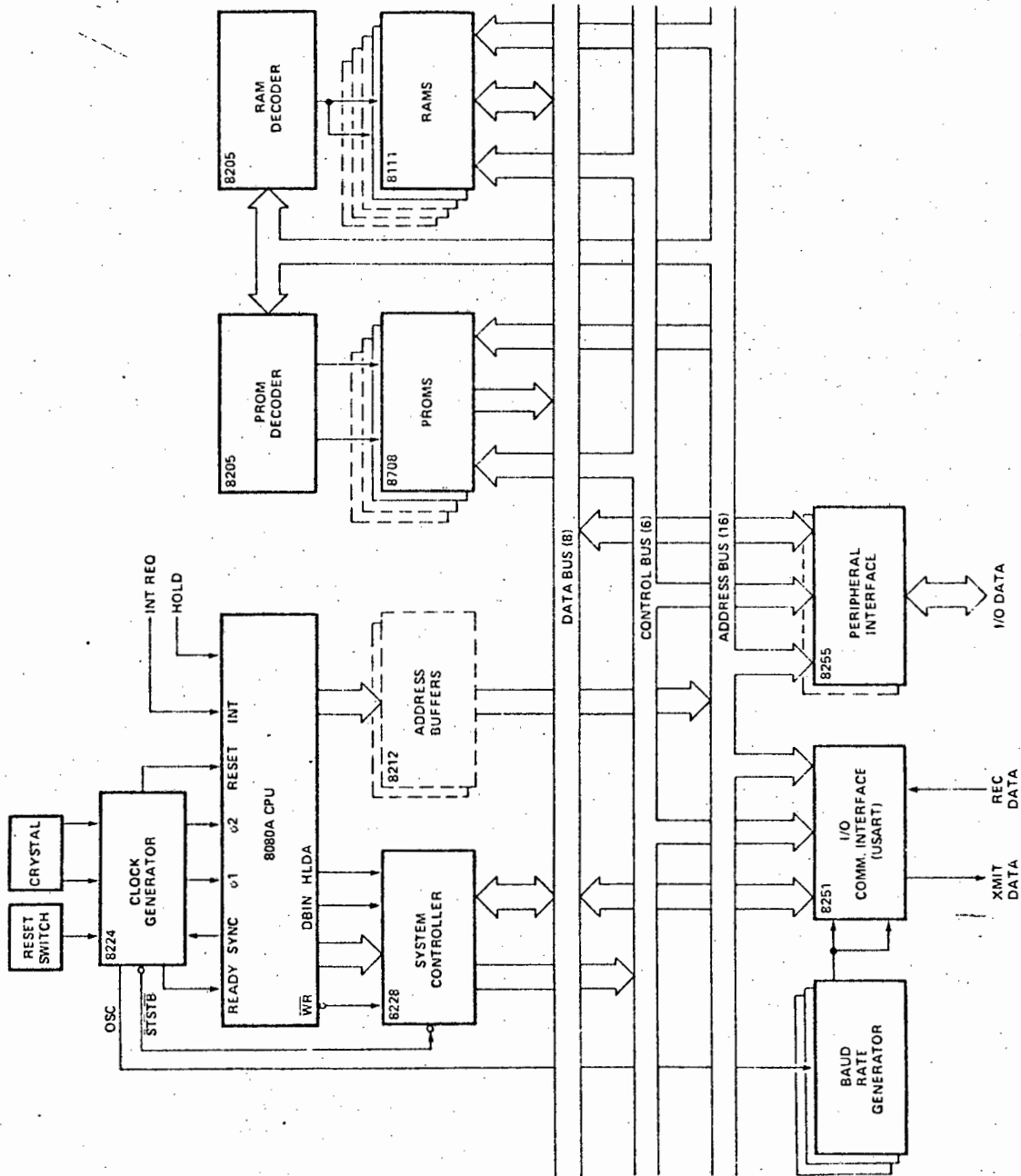


FIG. 7.5 - Diagram of SDK-80 Micro-Processor kit

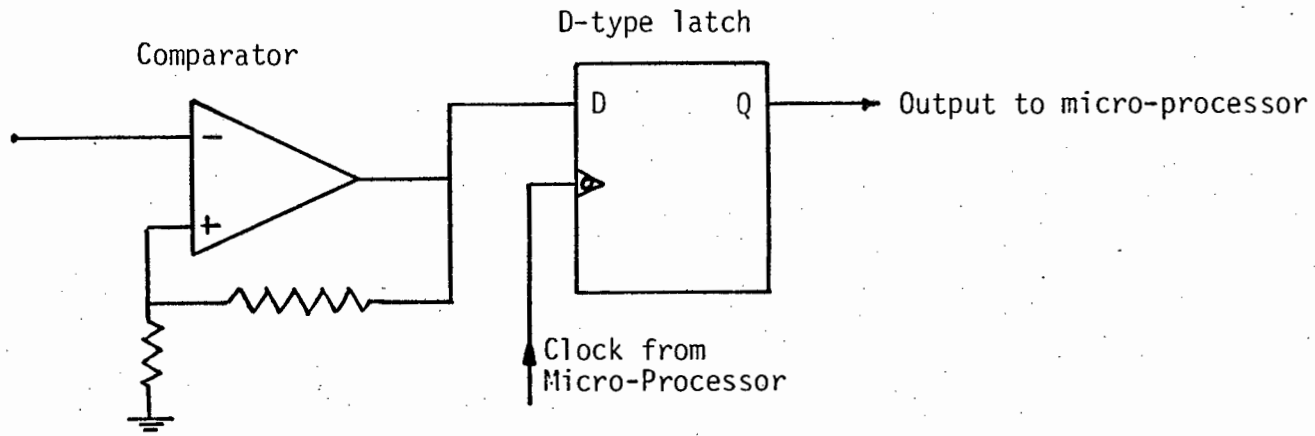


FIG. 7.6 Sign detectors

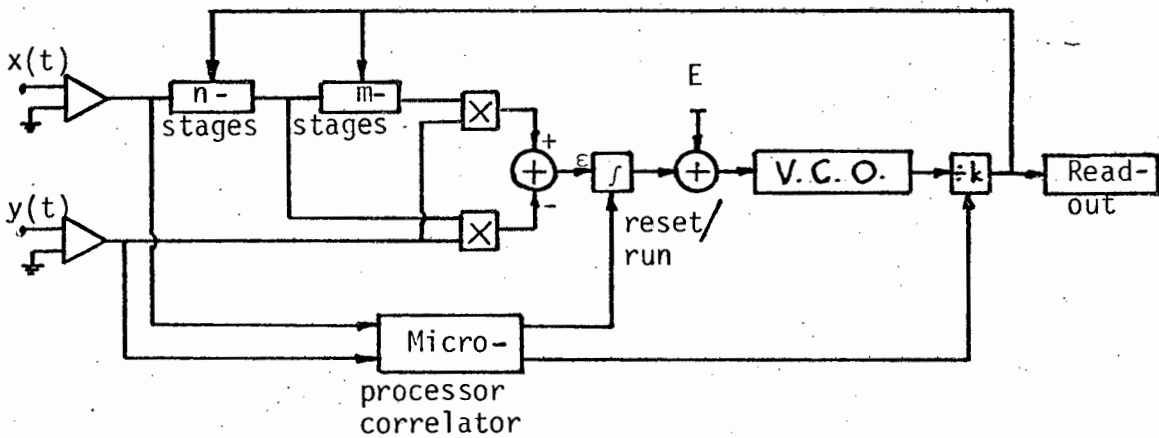
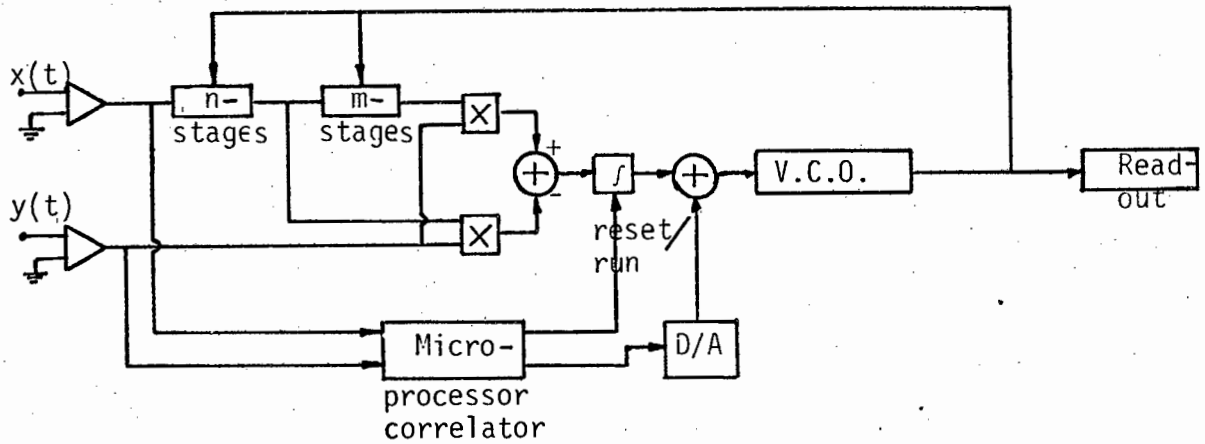


FIG. 7.7 Micro-processor controlled two-point difference correlator - two methods of operation

to be correlated, the other six lines of this port are ignored by connecting them to ground. Port A is hence programmed as an input port. Port C is programmed to be an output port and one of its lines is used to clock the D-type latches of the sign detectors. Port B is also programmed as an output port. Its 8 lines are used to output and latch the binary word required by the two-point difference correlator interface.

Two methods of interfacing the two correlators are possible. The method shown in Figure 7.7a uses a digital-to-analogue convertor to control the voltage applied to the voltage controlled oscillator of the two-point difference correlator. The micro-processor, after having found the approximate peak position, calculates the voltage that must be applied to the V.C.O. so as to bring the shift rate of the two-point difference correlator to approximately the correct value. It then resets the difference correlator voltage applied to the V.C.O. to zero and disables this correlator before applying the voltage calculated previously via a D/A convertor to the V.C.O. Hereafter the two-point difference correlator is re-enabled and it should then lock onto the correlation peak.

The second method shown in Figure 7.7b uses a programmable frequency divider. The microprocessor again finds the approximate peak position and then resets and disables the two-point difference correlator. A reference voltage is thereby applied to the V.C.O. to bring its clock rate to a specified value f_0 . Since the shift rate, f , corresponding to a peak position, $\beta = k \cdot \Delta t$, is given by:

$$f = \frac{2n + m}{2} \cdot \frac{1}{k \cdot \Delta t} = \frac{2n + m}{2 \cdot \Delta t} \cdot \frac{1}{k} = \frac{f_0}{k}$$

where $f_0 = \frac{2n + m}{2 \Delta t}$

the micro-processor need only control the frequency divider to bring the actual shift rate, f , applied to the two-point difference correlator to approximately the correct value. After outputting the correct division factor, k , to the programmable divider, the difference correlator is enabled and should then lock onto the correlation peak.

3.4 Micro-processor Correlator Software

The software for the microprocessor correlator is based on the serial-parallel method described earlier. A flowchart of the programme is shown in Figure 7.8.

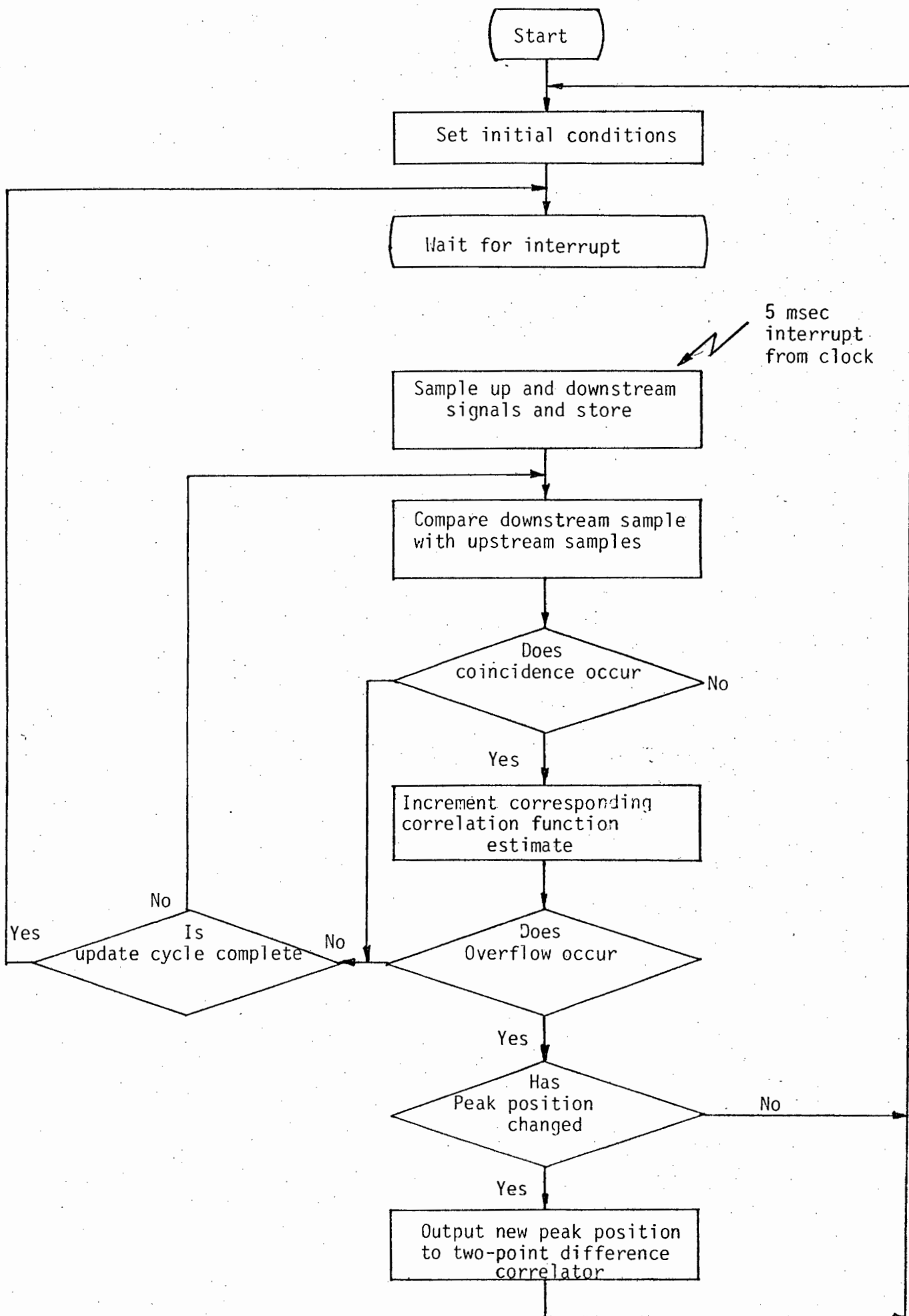


FIG 7.8 : Flowchart of micro-processor correlator software

The programme is situated in ROM starting at location 0 and is entered by resetting the microprocessor. First an initialization routine is entered in which the mode of operation of the 8255 Programmable Peripheral Interface is specified, viz., Port A - input port, Ports B and C - output ports; the locations in RAM used for storing the signal samples are cleared and the locations used for storing the correlation function estimates are loaded with a specified number. The reason for this latter operation will be discussed later. After completing this initialization, the programme enters a WAIT routine in which it idles until an external interrupt occurs. On receiving such an interrupt, the microprocessor clocks the D-type latches of the sign detectors by setting and resetting line Co of Port C. It then inputs the signal samples by reading Port A. The signal samples are the two lowest-order bits, viz., A₀ and A₁ of this 8-bit word, the upper 6 bits being ignored. The two samples are then separated; that of the signal to be delayed is placed in position 0 of the signal store. The processor then commences updating the correlation function estimates starting at the longest delay value. It takes the sample of the delayed signal and compares its sign with the one sample of the undelayed signal. If a coincidence occurs, i.e., if both signals have the same signs, i.e., both are ones or both zeroes, then the corresponding correlation function estimate is incremented, otherwise the estimates are left unaltered. The delayed sample is then immediately shifted by one position. This process is hereafter repeated for the 99 other correlation function estimates. At the end of this compute and update cycle, the programme again enters a WAIT state until the next interrupt occurs.

The above averaging cycle continues until the first pair of memory locations storing the correlation function estimates overflows. By pre-loading these locations during initialization, the number of coincidences required before an overflow occurs can be controlled. The position of the first overflowing store terminates the averaging cycle and immediately gives the correlation peak position. No further peak-finding routine is hence necessary.

The above method of locating the correlation peak saves processing time and simplifies the correlation programme. It also results in a variable averaging time since a definite number of coincidences between the delayed and undelayed signals must take place before an overflow can occur. The total number of samples used till this event happens will, however, be variable. If a high degree of correlation exists between the signals at some time delay, a fewer total number of samples will be necessary to produce an overflow - hence a shorter averaging time. On the other hand for poor correlation between the signals, a larger number of samples will have to be used till the specified

number of coincidences has taken place. The averaging time hence depends on the degree of correlation between the signals being longer for weakly correlated signals and shorter for highly correlated signals.

After the averaging cycle is terminated, the programme enters an output routine. The present estimate of peak position is first compared with the peak position obtained during the preceding averaging cycle. If no change has occurred, no further action is taken and a new averaging cycle is started. If a change in peak position has occurred, the new peak position estimate is stored before the two-point difference correlator is reset and disabled. Hereafter the approximate peak position is output to the two-point difference correlator via Port C of the 8255. The difference correlator is then re-enabled and a new averaging cycle is commenced.

During the development of the correlation programme considerable effort was spent in minimizing the number of instructions required to perform the specified operations between sampling intervals. This number of instruction cycles is variable depending upon the number of coincidences that occur during an update cycle. The maximum occurs when each correlation function estimate must be incremented and is 3270 cycles. With the 8080A having an instruction cycle time of 2 μ secs, this gives a worst case update cycle time of 6,54 m secs which exceeds the specified 5 m secs time interval between samples. The faster version of the 8080A, viz., 8080A-1 has an instruction cycle time of 1,3 μ secs, resulting in an update cycle time of 4,25 msecs that satisfies the 5 m sec criterion under all conditions.

A further method of satisfying the 5 m secs requirement is to reduce the number of points at which the correlation function is evaluated. If 75 instead of 100 points are used, thus covering a delay range of 5 m secs to 375 m secs, i.e., and 75:1 range instead of a 100:1 range, then the update cycle time using the 8080A is 4,9 m secs which is satisfactory.

4. TWO-POINT DIFFERENCE CORRELATOR

4.1 Method of Operation

The basic principle of operation of the two-point difference correlator has been described previously in Chapter 6. A block diagram of the correlator including the interface to the microprocessor correlator is shown in Figure 7.9. It essentially consists of two delay lines of n and m stages. These could be, for example, charge coupled delay lines in which case the signals to be correlated

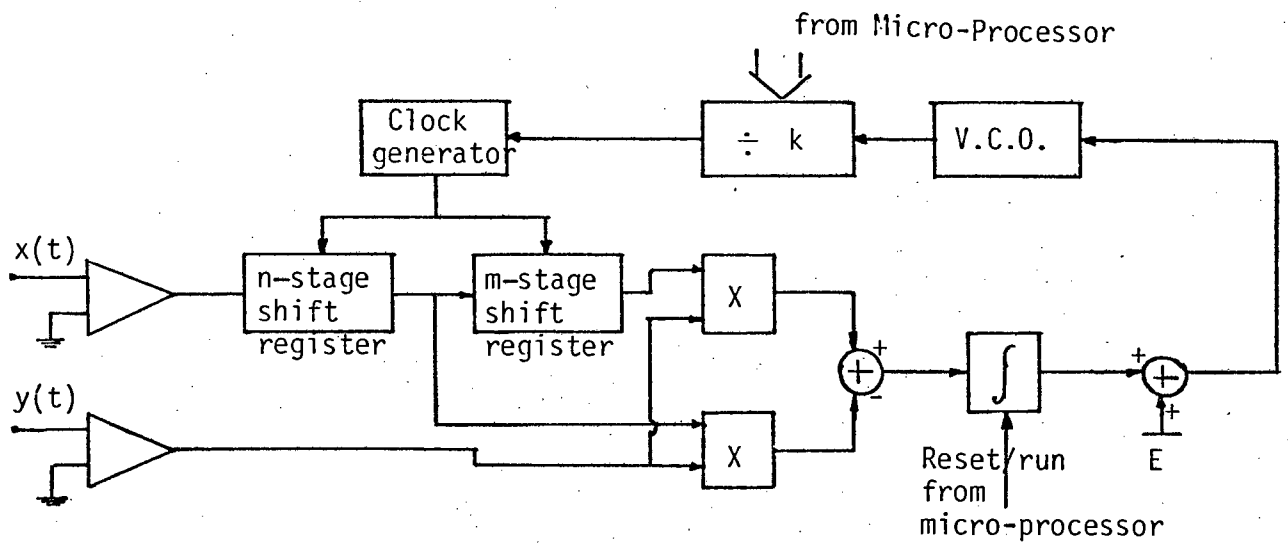


FIG. 7.9 Two-point difference correlator with interface

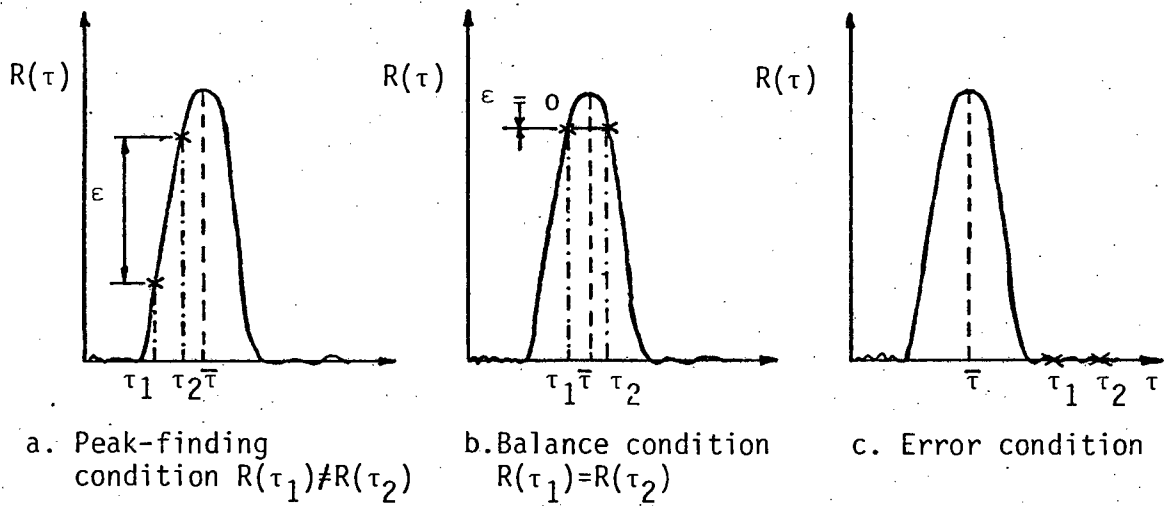


FIG. 7.10 Principle of operation of two-point difference correlator

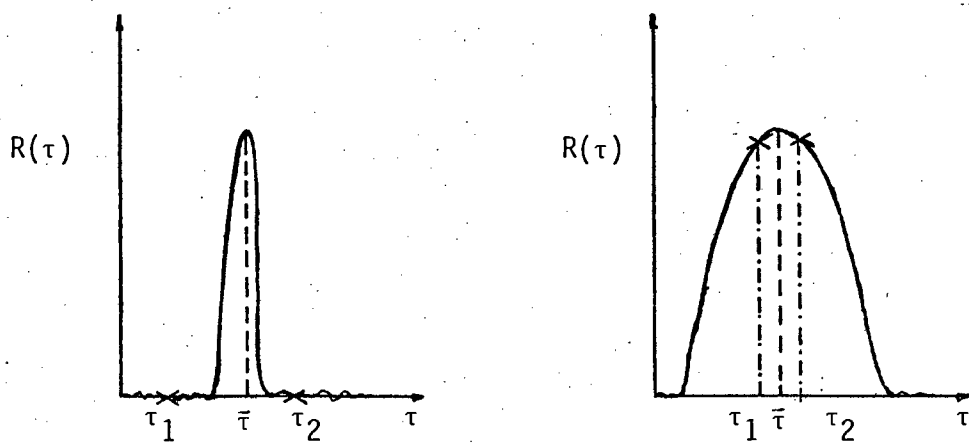


FIG. 7.11 Unwanted balance conditions

are unquantized. However, for simplicity, digital shift-registers will be used in this design. The two delay lines are cascaded and controlled by a common clock generator. During every clock cycle, the data contained within the delay lines is shifted by one position with a new sample entering the first stage and the data in the last stage being lost. Since the data is shifted through one stage per clock cycle, the actual amount by which a signal is delayed is determined by the number of delay line stages, k , and the clocking frequency, f , viz.,

$$\text{delay } \tau = k \cdot \Delta t = k/f$$

The two-point difference correlator calculates the correlation function at two delay values determined by the number of stages of the two delay lines and the clocking frequency as shown in Figure 7.10. If an n -stage delay line followed an m -stage is used, then the two delay times, τ_1 and τ_2 at which the correlation function is evaluated are given by:

$$\tau_1 = n/f \quad \text{and} \quad \tau_2 = (n + m)/f$$

By varying the clock rate, f , it is possible to vary the two delay times τ_1 and τ_2 . If the difference in the value of the correlation function at the two delay times, viz., $\epsilon = R_{xy}(\tau_1) - R_{xy}(\tau_2)$ is used to control a voltage controlled oscillator which in turn clocks the delay lines, it is possible to bring the two-point difference correlator into the balance position shown in Figure 7.10. Once locked onto a correlation peak, the correlator will track the peak since the error voltage applied to the V.C.O. is always of the correct polarity to alter the clock rate in a manner which ensures tracking, i.e., negative feedback is used to reduce the error voltage to near zero.

Assuming a symmetrical correlation peak, the actual peak position, β , is given by the arithmetic mean of τ_1 and τ_2 , i.e.,

$$\beta = \frac{\tau_1 + \tau_2}{2} = \frac{2n+m}{2f}$$

$$\text{i.e., } \beta \propto 1/f$$

The peak position is hence inversely proportional to the clocking frequency f . However, since in correlation flowmetering the flow velocity is given by:

$$\text{velocity, } v = \frac{\text{sensor spacing, } d}{\text{transit time, } \beta}$$

$$\text{i.e., } v = \frac{2d}{2n + m} \cdot f$$

$$\text{i.e., } v = Kf \quad \text{where } K = \frac{2d}{2n + m} = \text{constant}$$

the flow velocity is directly proportional to the clocking frequency.

Of primary importance is a suitable choice for the number of stages of each delay line (n and m). This choice depends on, *inter alia*, the delay range which must be covered and the bandwidth of the signals to be correlated. The latter factor is relevant since the signal bandwidth determines the width of the correlation peak. The difference in time delay between τ_1 and τ_2 , viz., $\Delta\tau = \tau_2 - \tau_1 = m/f$ must be such that the correlation peak is straddled by the two-point difference correlator. Too large a difference can result in the situation shown in Figure 7.11a in which the width of the peak is smaller than the time delay difference, $\Delta\tau$. On the other hand, too small a difference can cause the two-point difference correlator to be situated at the crest of the peak as shown in Figure 7.11b. Since the slope of the correlation function is very small in this region, the performance of the correlator is degraded.

4.2 Hardware Description

A complete circuit diagram of the microprocessor controlled two-point difference correlator is given in Appendix E. A 1024 bit MOS shift-register is used as the primary delay element, n . In order to vary the length of the second shift-register (m), a programmable 1-63 bit shift-register is used. The length of this shift-register is determined by the settings of a series of toggle switches.

For a 5 m sec to 500 m sec delay range, the range of the clock frequency is approximately 200 kHz to 2 kHz. This exceeds the Nyquist sampling rate of the flow signals under all conditions. Further design details are included in Appendix G.

5. INTERFACE

Of the two interfaces described in Section 7.2 for controlling the two-point difference correlator, that based on a frequency divider is most suitable. The advantages of this system are:

1. The V.C.O. used requires only a small, say 10:1, tuning range since all the other frequencies necessary to cover a 100:1 delay range are produced by frequency division.

2. Since the tuning range of the V.C.O. is small (10:1 instead of 100:1 as required by the other interface), greater linearity and sensitivity is possible.
3. A division routine for finding the approximate voltage that must be applied to the V.C.O. is not required by the microprocessor correlator since this division is done automatically by the frequency divider.

The programmable frequency divider required must be capable of accepting a binary word from the microprocessor correlator and divide its input frequency f by this factor k to give an output at a frequency f/k .

The circuit shown in Figure 7.12 achieves this programmable frequency division by using a down counter which is decremented by one starting from the specified division factor k whenever it receives an input pulse. This continues until an underflow occurs, the underflow pulse being used to reload the division factor k into the down counter by triggering a monostable which in turn enables the preset control of the counter. For every k input pulses where k can have any value from 1 to a value depending on the capacity of the down counter, a single underflow pulse is produced. These underflow pulses represent the output of the frequency divider.

Figure 7.9 shows the complete interface circuitry. In addition to the programmable frequency divider it consists of an integrator which is also controlled by the microprocessor correlator and a V.C.O. When the reed relay is enabled, the integrator is reset to a reference voltage V_0 . This voltage produces a reference frequency $f_0 = k V_0$ where k is the conversion factor of the V.C.O. which after division by n should result in a shift rate that places the difference correlator in the approximate position of the correlation peak. After resetting the integrator and outputting the division factor n , the microprocessor correlator disables the reed-relay thereby permitting the integrator to integrate the two-point difference correlator's error voltage applied to its input and so locking onto and tracking the correlation peak.

6. READ-OUT

As was shown earlier, the shift rate of the two-point difference correlator is directly proportional to the flow velocity since

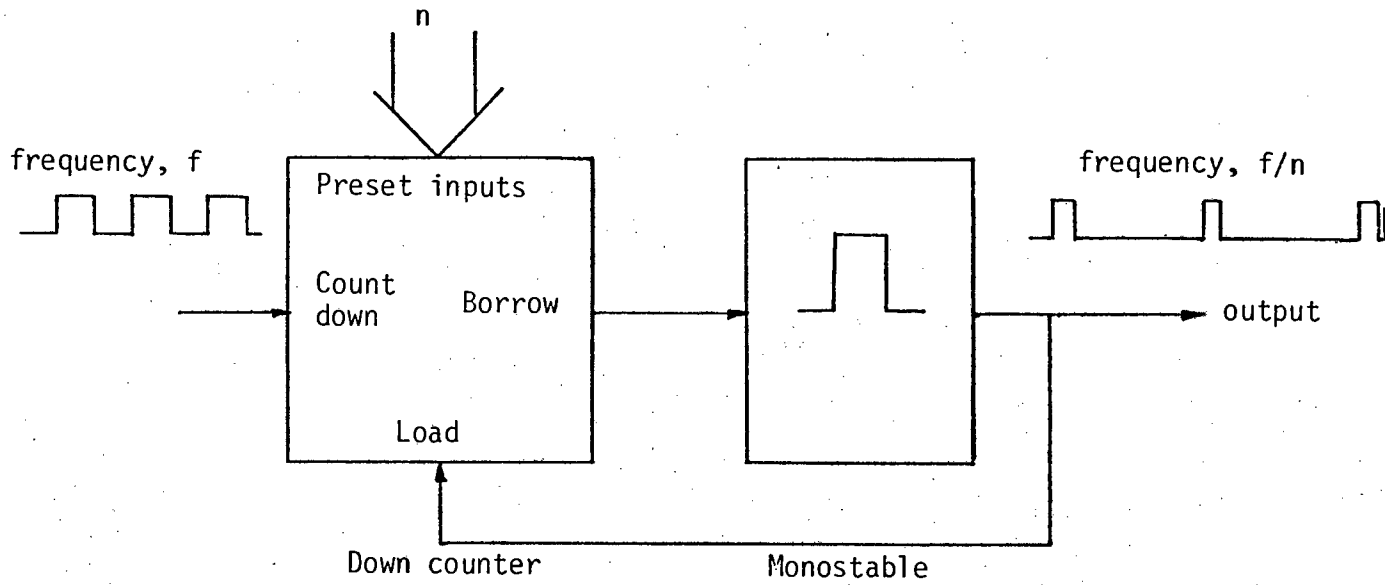


FIG. 7.12 Programmable frequency divider

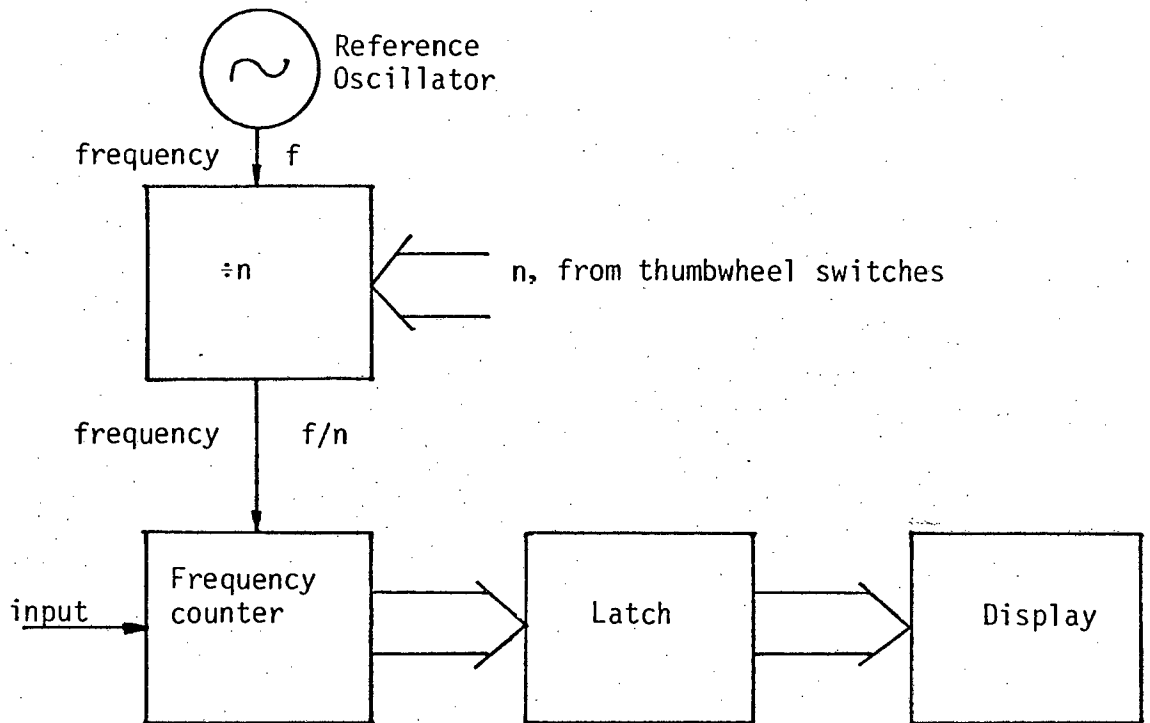


FIG. 7.13 Read-out

$$v = d/\beta = \frac{2d}{2n + m} \cdot f = K \cdot f$$

where $K = \frac{2d}{2n + m}$

A simple frequency counter could thus be used as the read-out for the flow meter. The constant of proportionality between the flowrate and shifting frequency, K , does, however, depend on the transducer spacing, d , which will vary from application to application. In order to be able to calibrate the flowmeter and to include the effects of variable transducer spacing, a frequency counter with variable gating interval could be used.

The block diagram of the read-out is shown in Figure 7.13. A stable reference oscillator is used as basis for generating a variable gating interval. This reference frequency is divided by a programmable frequency divider similar to that described in Section 7.4. The division factor n is set up manually via a series of thumb-wheel switches. The output of the frequency divider controls the time interval for which the unknown frequency f is applied to the frequency counter. At the end of this gating period, the frequency counter output is latched and displayed. The counter is then reset to zero and the process is repeated.

If an analogue signal proportional to flow velocity is required for control purposes, it is a simple matter to convert the clock frequency into an analogue signal using a frequency to voltage convertor. Furthermore, if the total volume of fluid flowing through the pipe is required, an accumulator can be used to count the total number of clock pulses over a period of time.

PART II : SERIAL PEAK-FINDING TWO-POINT DIFFERENCE CORRELATOR

7. BASIC SERIAL TWO-POINT DIFFERENCE CORRELATOR

The second peak-finding correlator to be described represents a different approach to the problem of directing a two-point difference correlator to the approximate position of a cross-correlation peak. The proposed correlator locates the approximate position of a correlation peak by evaluating the cross-correlation function at a time delay that lies midway between the two-point difference correlator time delays. This point is swept down from a maximum time delay, τ_{MAX} , towards a minimum time delay, τ_{MIN} . In the absence of any peak, the correlation coefficient is small (zero). As a correlation peak is

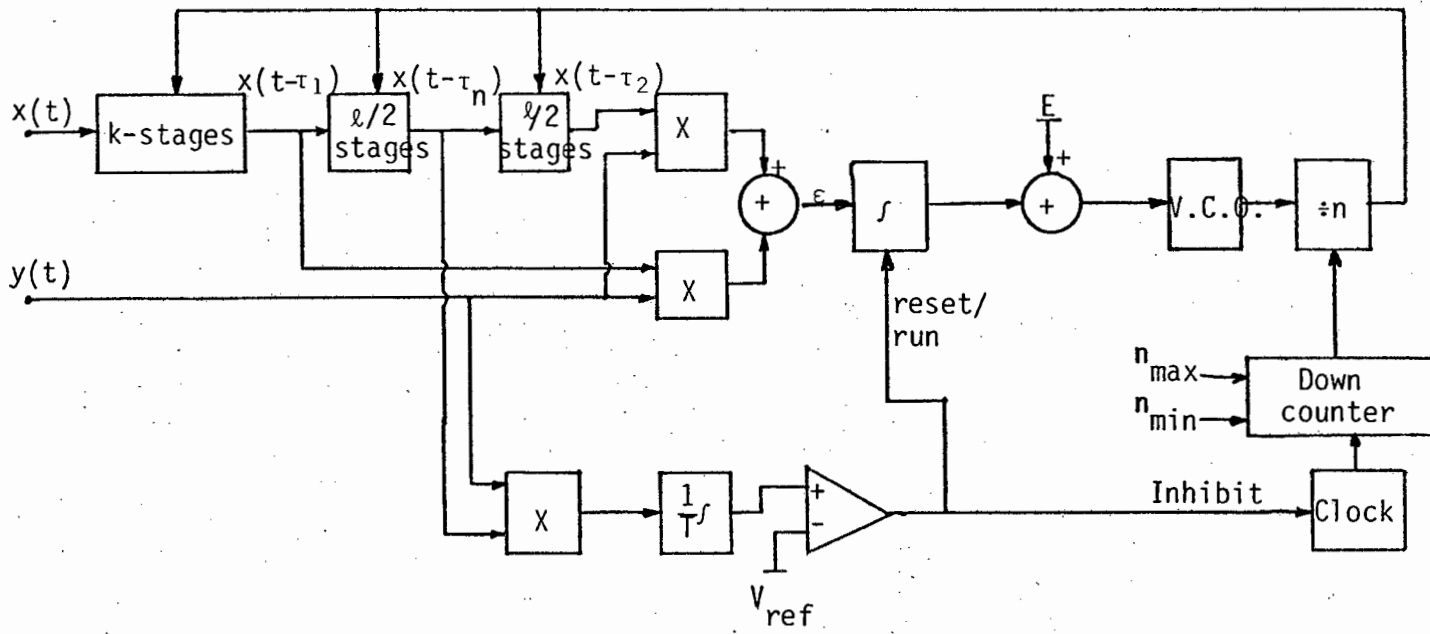


FIG. 7.14 Serial peak-finding two-point difference correlator

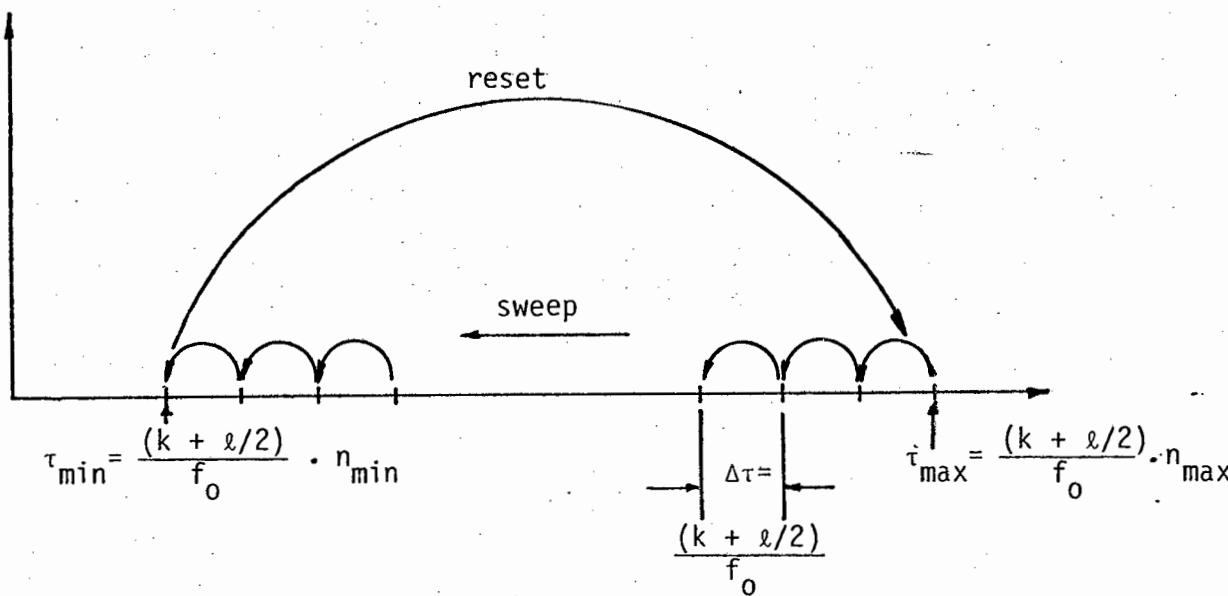


FIG. 7.15 Time-delays at which correlation function is evaluated during the sweep mode

approached, the value of the calculated correlation coefficient increases. When this value exceeds some externally set threshold, the downward time-delay sweep is inhibited and the two-point difference correlator, which has been disabled up to this stage, is enabled. The difference correlator now can accurately locate and track the peak.

A diagram of this correlator is shown in Figure 7.14. The downward sweep of the time-delay is achieved by applying a reference voltage to the V.C.O. with the control voltage equal to zero, hence keeping its frequency fixed at f_0 (say). A frequency divider then divides this base frequency by an integer, n , to yield a lower frequency $f = f_0/n$ that is applied to the shift-registers.

The divisor n is obtained from a counter that is decremented starting from some presettable maximum value n_{MAX} to a presettable minimum value n_{MIN} . The shift rate, f , hence, starts at $f_{min} = f_0/n_{MAX}$ and changes in steps to $f_0/n_{MAX}-1$, $f_0/n_{MAX}-2, \dots, f_0/n_{MIN} + 1, f_0/n_{MIN}$. The corresponding time delays at which the correlation function is evaluated are thus (see Figure 7.15):

$$\tau_{MAX} = \frac{k+l/2}{f_{MIN}} = \frac{k+l/2}{f_0} \cdot n_{MAX}, \frac{k+l/2}{f_0} (n_{MAX}-1) \dots, \frac{k+l/2}{f_0} \cdot n, \dots, \frac{k+l/2}{f_0} \cdot n_{MIN}$$

In general, the delay corresponding to a divisor n is given by

$$\tau_n = \frac{k+l/2}{f_0} \cdot n$$

The correlation function thus is evaluated sequentially at equal delay intervals of $\Delta\tau = (k+l/2)/f_0$ starting at $\tau_{MAX} = (k+l/2)/f_0 \cdot n_{MAX}$

and decreasing towards $\tau_{MIN} = (k+l/2)/f_0 \cdot n_{MIN}$

If the threshold is not exceeded during a sweep, the counter is reset to n_{MAX} and another sweep is initiated. If the threshold is exceeded for $n = n_p$ (say), the sweep is inhibited at this point. This sets the shift rate to $f_p = f_0/n_p$ and the corresponding time delay to

$$\tau_p = \frac{(k+l/2)}{f_0} \cdot n_p$$

The two-point difference correlator is now enabled and its error voltage controls the base frequency f_o of the V.C.O. to accurately locate and track the peak. When straddling the correlation peak, the shift-rate is $f_p' = f_o' / n_p$ where f_o' is the corrected V.C.O. frequency and the corresponding peak position for a symmetrical peak is:

$$\tau_p' = \frac{(k+l/2)}{f_o'} \cdot n_p = \frac{k+l/2}{f_p'}$$

If at any stage the peak is lost, i.e., the evaluated correlation coefficient drops below the threshold, the correlator is reset and a new sweep is started.

8. SERIAL TWO-POINT DIFFERENCE CORRELATOR WITH IMPROVED TRACKING

In the preceding section it was shown how a correlation peak is located and tracked. The following situation could arise - assume a peak has been located at $n = n_p$, i.e., at $\tau_p' = \frac{k+l/2}{f_o'} n_p$. The peak now changes position causing the error voltage applied to the V.C.O. to change in order to keep the difference correlator in the balance position. A stage will be reached when the V.C.O. runs out of range at which point the difference correlator will lose the peak and be reset.

In order to avoid this condition, additional logic is included as shown in Figure 7.16 which maintains the control voltage, ϵ , within a range $+E$ to $-E$ volts (say). This range is the useful operating range of the V.C.O. While the control voltage lies within this range, the divisor is set at $n = n_p$. As soon as the control voltage departs from this range, the divisor is either increased or decreased depending upon which limit is exceeded. This is achieved by incrementing or decrementing the counter from $n = n_p$ until the control voltage is zero.

This improved tracking correlator is thus able to track the correlation peak over the entire delay range τ_{MAX} to τ_{MIN} once the peak has been located.

9. ADAPTIVE SERIAL TWO-POINT DIFFERENCE CORRELATOR

For accurate peak location, it is important that the two time delay values (τ_1 and τ_2) of the two-point difference correlator lie on the slopes of the cross-correlation function. The situations shown in Figures 7.17(a & b) must be avoided. The optimum positions, in fact, of the two time delays, τ_1 and τ_2 are where the slopes of the correlation peak are a maximum, i.e., at the points of inflection.

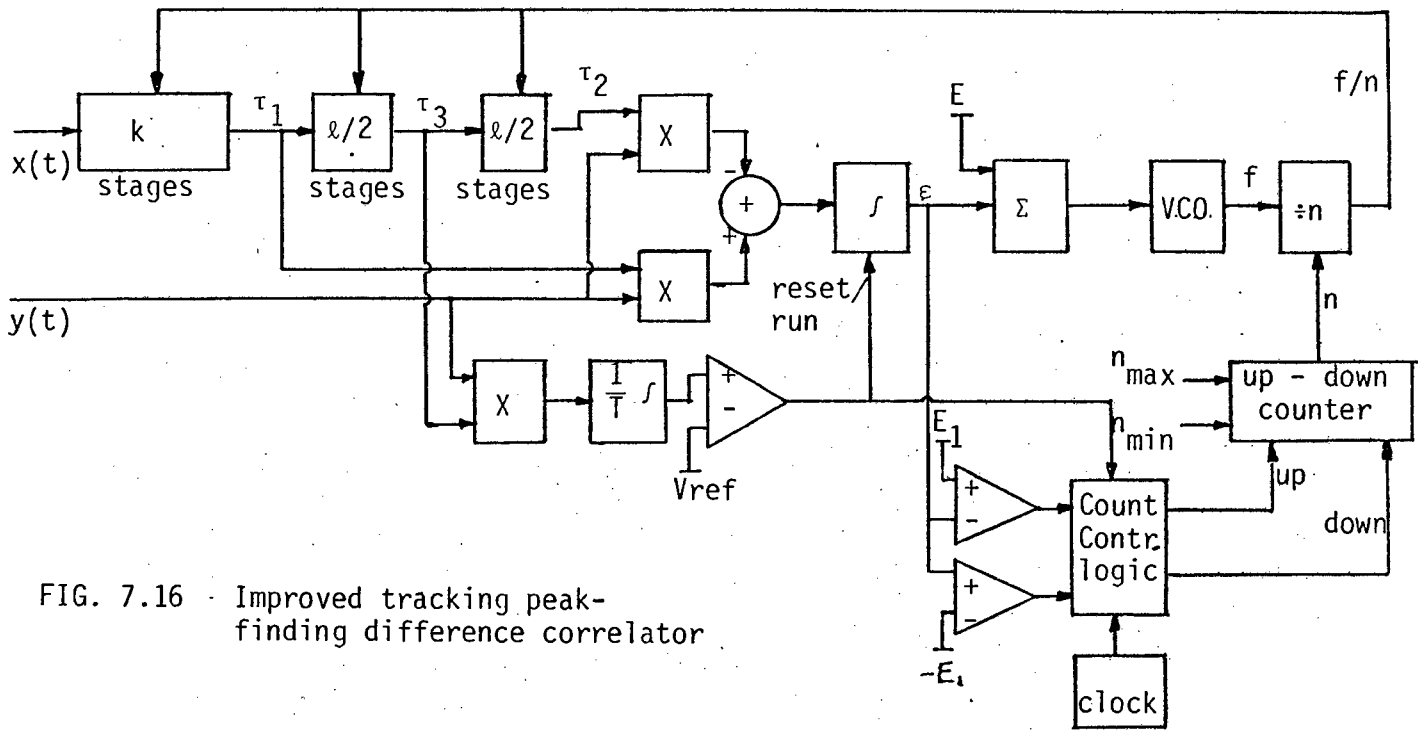


FIG. 7.16 Improved tracking peak-finding difference correlator

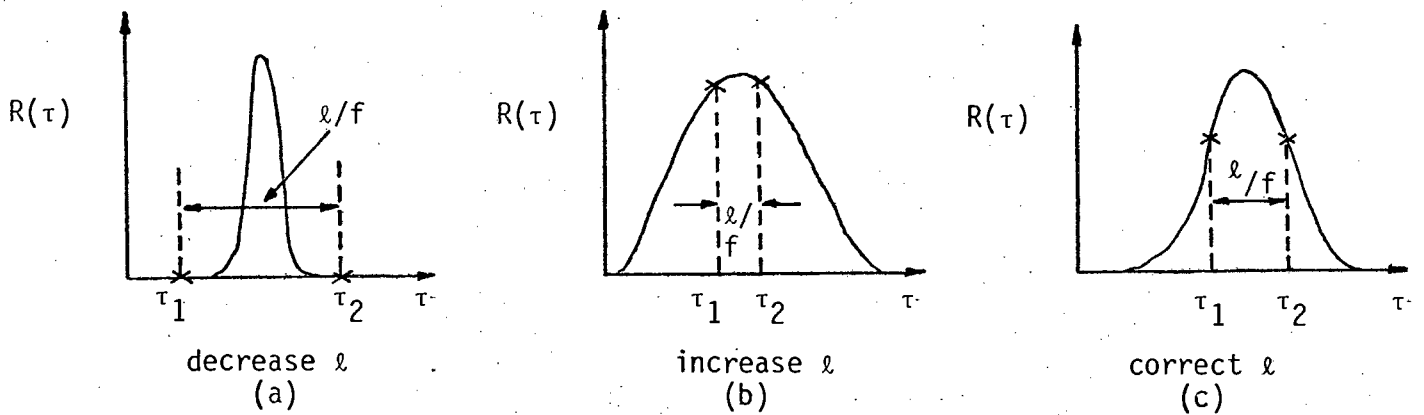


FIG. 7.17 Effect of correlation peak width on choice of l

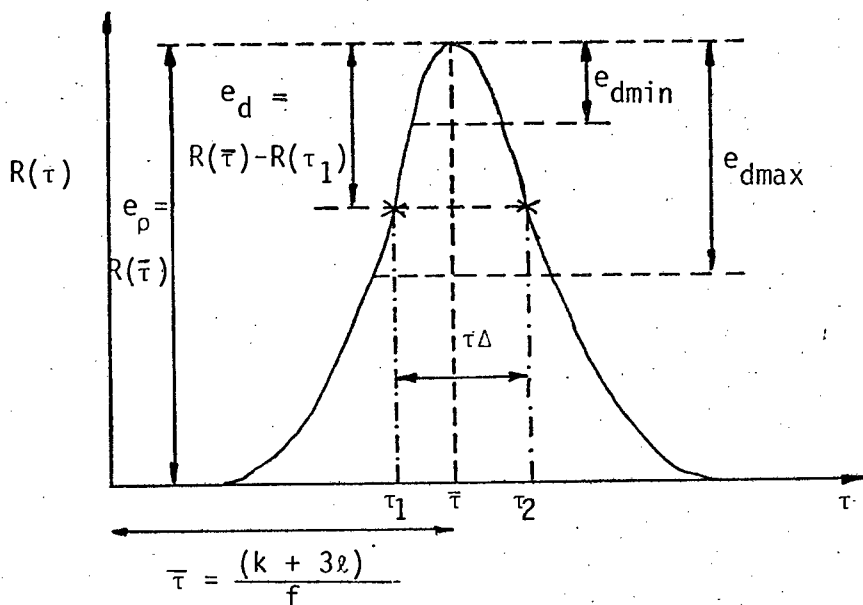


FIG. 7.19 Adaptive difference correlator attempts to adjust $\Delta\tau$ so that $e_{dmin} < e_d < e_{dmax}$

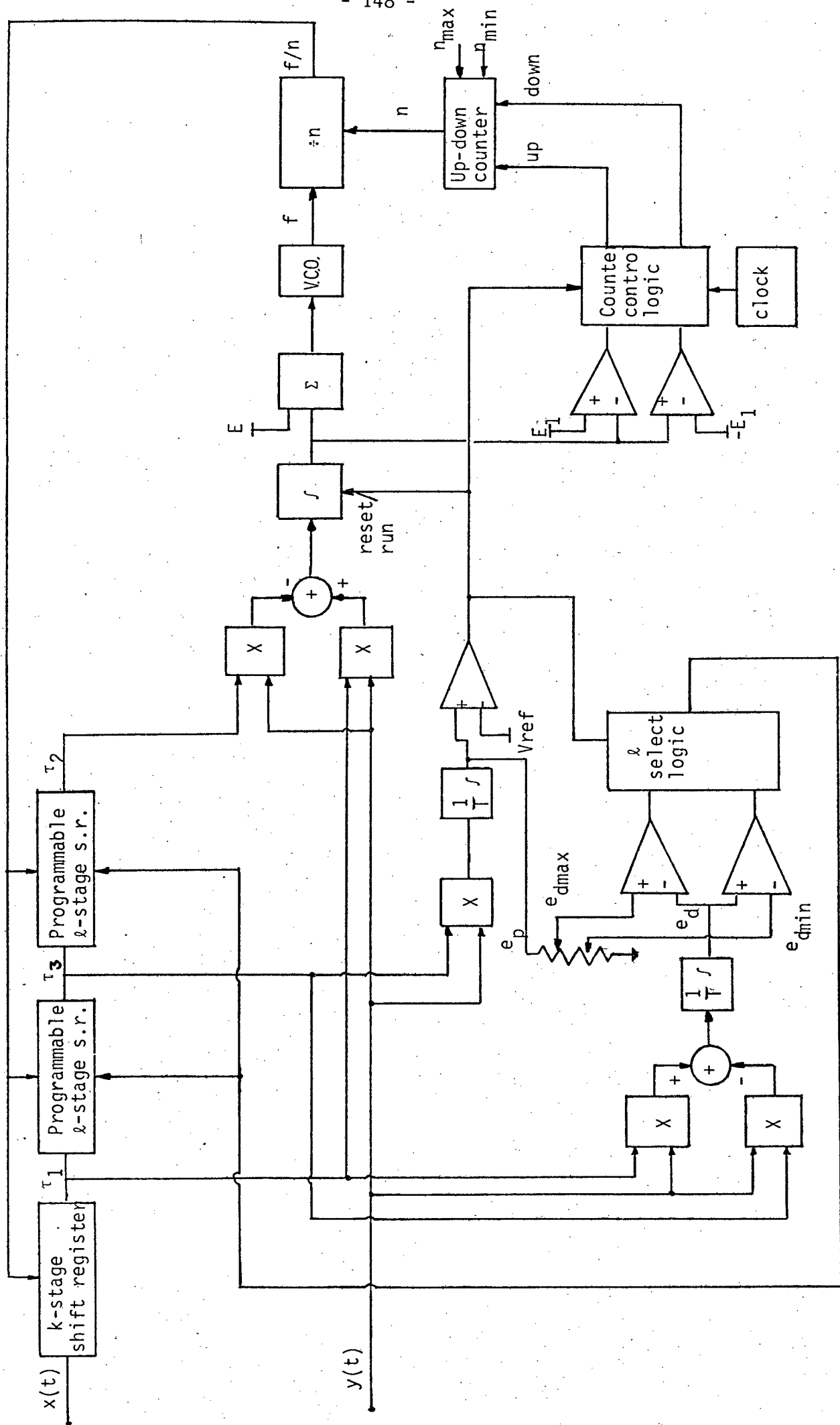


FIG. 7.18 Adaptive peak-finding two-point difference correlator

Assume that the length of the first shift-register is k and that a peak exists at $\tau = \tau_p$. The shift rate of the two-point difference correlator in the balance position is hence:

$$f_p = (k + \ell/2)/\tau_p$$

This shift rate now determines the time delay difference, $\Delta\tau$, between the two-point difference correlator delay values

$$\Delta\tau = \tau_2 - \tau_1 = \ell/f_p$$

The length of the shift-register, ℓ , must now be chosen so that τ_1 and τ_2 lie on the slopes of the correlation peak as shown in Figure 7.17c.

The diagram of such an adaptive difference correlator can be seen in Figure 7.18. This correlator attempts to maintain the difference, e_d , between the correlation function at the peak position, viz., $\tau_p = (k + \ell/2)/f_p$ and at one of the two-point difference time delays $\tau_2 = (k + \ell)/f_p$ within a specified range relative to the magnitude of the correlation function, e_p at the peak position $\tau = \tau_p$ as shown in Figure 7.19.

The correlator hence uses the value of the correlation function at the peak position as its reference and adjusts the length of the shift-register stages ℓ accordingly. Initially, during the peak finding sweep, a small value of ℓ is chosen to avoid the situation shown in Figure 7.17a when a peak is eventually located. Once a peak has been located, an up/down counter is used to adjust the lengths of three programmable shift-registers to suit the particular circumstances. If the difference $e_d = R_{xy}(\tau_p) - R_{xy}(\tau_2)$ is less than the minimum of the specified range e_{dMIN} , ℓ is increased so as to increase the difference between τ_p and τ_1 , thereby increasing $R_{xy}(\tau_p) - R_{xy}(\tau_2)$. Similarly if the difference e_d is greater than the maximum specified value e_{dMAX} , the shift-register lengths, ℓ , are decreased so as to reduce the difference between τ_p and τ_2 and thereby also reduce the difference e_d .

10. DISCUSSION

The serial peak-finding two-point difference correlator is a versatile instrument since:

- (i) The delay range τ_{MAX} to τ_{MIN} is determined by presenting n_{MAX} and n_{MIN} and can be made large.

- (ii) The sweep rate is determined by the rate at which the counter is decremented and can easily be varied.
- (iii) The time delay intervals between steps is $(k + \ell/2)/f_0$, i.e., it depends on the base frequency, f_0 , and the shift-register lengths k, ℓ . Both of these can be selected to give adequate resolution thus ensuring that a peak is not overlooked.
- (iv) The correlator adjusts the length of the shift-registers ℓ depending on the width of the correlation peak, i.e., the bandwidth of the flow signals.
- (v) The threshold at which a peak is located can be adjusted.
- (vi) An output that is directly related to the flow velocity is available.
- (vii) In addition, the correlator has all the advantages of a two-point difference correlator.

The main disadvantage of the correlator is that if a large delay range is to be covered, the initial period of time required to locate a correlation peak can be long depending on the peak position. However, once a peak has been located, the response time of the correlator is equal to that of the two-point difference correlator which is fast. The correlator, furthermore, cannot distinguish between multiple correlation peaks should more than one peak exist. It will lock onto the first peak it encounters which exceeds the set threshold. This is not necessarily a drawback since in most instances only a single peak will exist and otherwise the most significant peak, i.e., the peak giving the flow velocity, is situated at larger time delays than the other peaks. Since the correlator sweeps down from τ_{MAX} , this peak will be the first to be encountered.

A detailed circuit diagram of the serial two-point difference correlator including the improved tracking and adaptive features, can be found in Appendix F. A 1024 stage shift-register is used for the main delay line (k) and three programmable 1-63 stage shift-registers are used for the remaining shift-registers. This results in a total of 1088 stages to the mid-point of the two-point difference correlator delay values and a range of 2 to 126 stages between these two delay values. The read-out is identical to that described for the micro-processor correlator.

11. COMPARISON OF MICRO-PROCESSOR AND SERIAL PEAK-FINDING CORRELATORS

The operation and characteristics of both the microprocessor-controlled two-point difference correlator and the serial two-point difference correlator are identical once a peak has been located. The adaptive feature of the latter correlator can be included directly with the former correlator if required. Alternatively, the selection of the shift-register length, ℓ , could be made under micro-processor control.

The main difference between the two correlators is the manner in which the approximate peak position is found initially. The micro-processor correlator examines the entire delay range, τ_{MAX} to τ_{MIN} at any instant of time, whereas the serial correlator evaluates the correlation function at only a single time-delay value, which is swept down from τ_{MAX} toward τ_{MIN} . The microprocessor correlator can distinguish between multiple peaks and will direct the two-point difference correlator to the largest of these. On the other hand, the serial correlator will lock onto the first peak that exceeds the pre-set threshold. The latter correlator can, therefore, not distinguish between peaks.

This clearly defines the areas of application of the two methods, the use of the serial two-point difference correlator being limited to those applications in which a single peak is known to exist. This method, however, can also be applied if the correlation peak of interest lies at a larger time-delay value than the spurious peaks, since the correct peak will then be the first to be encountered on the downward time-delay sweep. It is envisaged that the microprocessor-based instrument would be more valuable in development situations in which the flow and fluid conditions are not wellknown. The serial two-point difference correlator, however, can be applied to simple, well-defined situations in which only a small time-delay range has to be covered.

The response time for the serial correlator under the worst conditions is longer than that of the microprocessor correlator. The maximum response time of the latter correlator is equal to one complete averaging cycle (5-20 seconds depending on the application). On the other hand, the maximum response time of the serial correlator occurs when the correlation peak is situated at the far end of the sweep range. If a large delay range and small step increments are used, the time taken to reach the peak can be fairly long, depending on the application.

The hardware costs of both correlators are low, of the order of a few hundred rand.

CHAPTER 8

TWO-POINT DIFFERENCE CORRELATOR ANALYSIS

1. INTRODUCTION

A two-point difference correlator is the essential element of both correlators described in the previous chapter. In order to optimize these correlators, it is necessary to analyze the response of a two-point difference correlator and understand its characteristics.

A two-point difference correlator forms a closed-loop system in which an error voltage controls the frequency of a V.C.O. so as to reduce the error to a minimum. The correlator attempts to adjust, by means of a variable delay line, the mean delay time of the upstream signal, to equal the unknown delay time of the downstream signal. The manner in which the correlator is able to locate and track the unknown time delay between upstream and downstream signals is of considerable interest.

A method frequently used to evaluate a system is to examine its response to a step input. In the case of a two-point difference correlator, an input in the conventional sense does not exist. The physical inputs to the correlator are two random signals that are correlated at some unknown delay time. A step input would correspond to a sudden change in the delay time between the two random signals. A simulation based on such inputs is difficult since the two-point difference correlator response depends on the shape of the cross-correlation function between the inputs. It is very difficult to specify explicitly two random signals which, when correlated, give rise to a desired cross-correlation function. A different approach is thus necessary.

Consider again the operation of the serial two-point difference correlator described previously. Initially, in the peak search mode, the two-point difference correlator is disabled and the time delay is swept downwards. Disabling the difference correlator is achieved by resetting the integrator and applying a fixed reference voltage to the V.C.O. The two-point difference correlator comes into operation once the calculated correlation coefficient exceeds the externally set threshold. This corresponds to the position shown in Figure 8.1. At this point, one or both of the two time-delay values, τ_1 and τ_2 , of the two-point difference correlator, are situated on the slope of the correlation function. The integrator is now switched from the reset to the run mode in which the error voltage applied to its input is integrated and changes the clock frequency so as to bring the difference correlator into the

balance position. This variation of clock frequency with time is the response of interest since the clock frequency is directly proportional to flow velocity.

The method of analysis hence assumes that some specified correlation function exists. The two-point difference correlator is initially situated on a slope of a correlation peak and is in the reset mode. At time $t = 0$, it is switched to the run mode and the variation of frequency with time is recorded. This variation will give the transient response of the correlator.

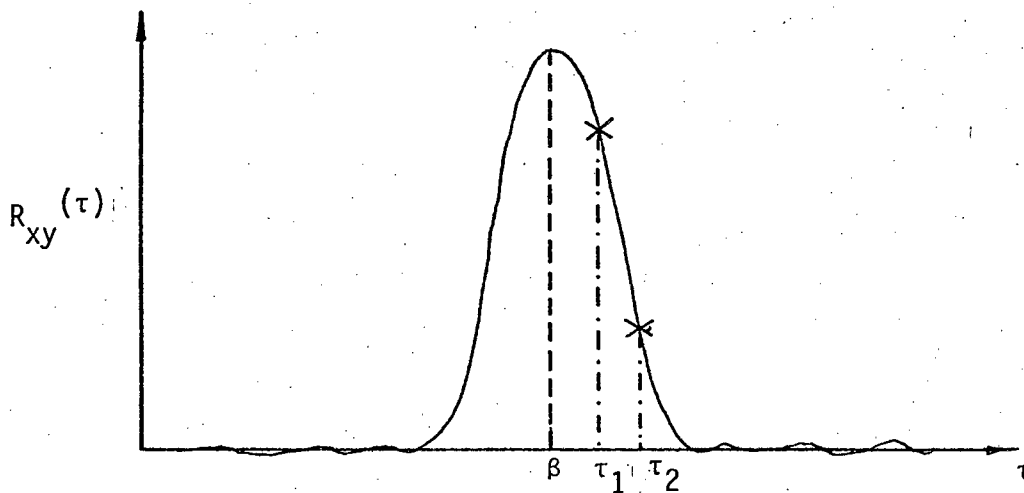


FIGURE 8.1 : Initial Two-point Difference Correlator Position

2. FIRST ORDER MODEL

The basic two-point difference correlator is again shown in Figure 8.2 and a block diagram of the first order model can be seen in Figure 8.3. A one-to-one correspondence between the parameters of the actual circuit and the model is evident. The main points to note are that the clocking frequency, $f(t)$, gives rise to the two time delay values $\tau_1(t) = n/f(t)$ and $\tau_2 = -(n+m)/f(t)$ and these in turn produce replicas of the signal, $x(t)$, delayed in time by $\tau_1(t)$ and $\tau_2(t)$. These are multiplied with the downstream signal, $y(t)$, to form two products the difference of which is the error signal applied to the integrator of the correlator.

The model can be simplified if the following assumptions are made. The product of upstream and downstream signals, $x(t-\tau(t)).y(t)$, is a random signal the mean value of which is equal to the cross-correlation coefficient between the signals for the particular delay $\tau(t)$. For example, if $x(t-\tau(t))$ and $y(t)$ are completely uncorrelated, then this mean value is zero; if the signals are identical, the mean value equals a maximum. It is thus possible to write this product in the form:

$$\begin{aligned} x(t-\tau(t)).y(t) &= \overline{x(t-\tau(t)).y(t)} + [x(t-\tau(t)).y(t)]' \\ &= R_{xy}(\tau(t)) + [x(t-\tau(t)).y(t)]' \end{aligned}$$

where the primed second term is a random waveform having zero mean value. The first term, $R_{xy}(\tau(t))$, is the main factor determining the response of the two-point difference correlator whereas the second term can be considered as a noise signal that can be neglected.

The model can now be redrawn as shown in Figure 8.4 in which the two time delays, $\tau_1(t)$ and $\tau_2(t)$, give rise to two correlation coefficients, $R_{xy}(\tau_1(t))$ and $R_{xy}(\tau_2(t))$. Note that these are all functions of time. The latter values depend on the shape of the cross-correlation function, $R_{xy}(\tau)$, which can be specified as desired. Initially a reference voltage, E , is applied to the V.C.O. to set its frequency to $f(0)=f_0$. The value of the voltage, E , is such that either one or both of the two delay values $\tau_1(0) = n/f(0)$ or $\tau_2(0) = (n+m)/f(0)$ lie on the slope of the correlation function. At time $t=0$, the switch, S , is closed and the correlator locks onto the correlation peak.

It is shown in Appendix G, that the equation for the clock rate is:

$$\frac{d}{dt}.f(t) = k.k_1\{R(n/f(t)) - R(n+m/f(t))\} \dots\dots\dots (8.1)$$

with initial condition $f(0) = f_0 = k.E$.

This equation shows that the rate of change of the clock rate is directly related to the difference in the magnitude of the correlation function at the two time delay values, τ_1 and τ_2 . This clearly indicates that the response depends on the correlation function. Equation (8.1) is a first order non-linear differential equation and can be solved numerically. However, it is first necessary to specify the shape of the correlation function $R(\tau)$. Let this function be Gaussian in shape (see Figure 8.5), that is:

$$R(\tau) = e^{-\left(\frac{\tau}{\sigma} - \frac{\bar{\tau}}{\sigma}\right)^2} \dots\dots\dots (8.2)$$

where $\bar{\tau}$ is the peak-position and σ determines the width of the peak. Equation (8.1) thus becomes:

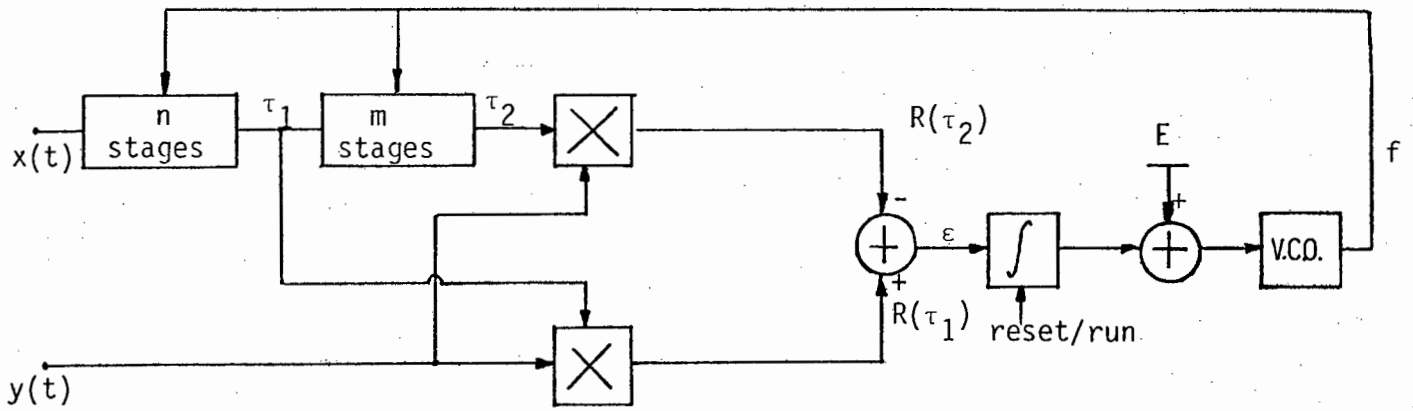


FIG. 8.2 Basic two-point difference correlator

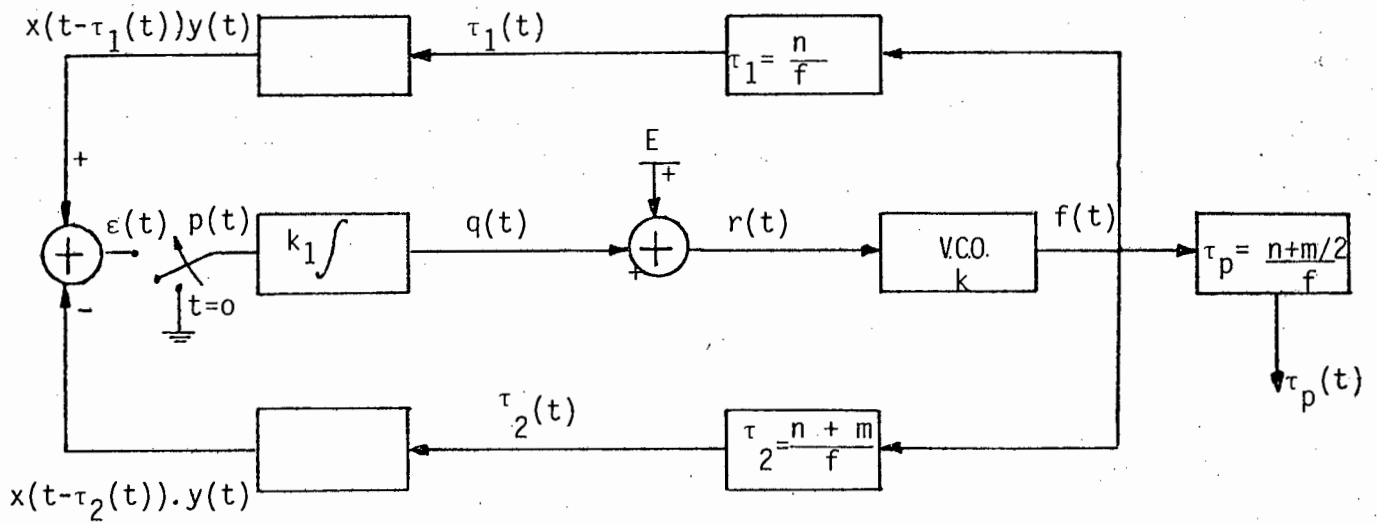


FIG. 8.3 Block diagram of first-order model

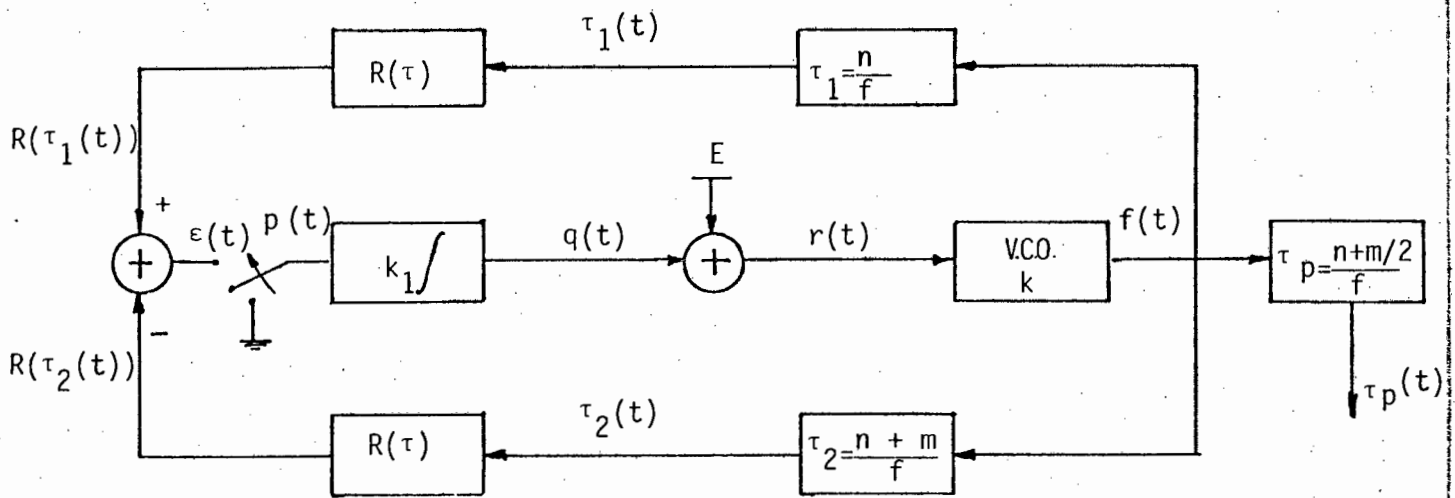


FIG. 8.4 Redrawn block diagram of first-order model

$$\frac{d}{dt} f(t) = k.k_1 \left[e^{-\left(\frac{n}{\sigma.f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma.f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} \right] \dots\dots\dots (8.3)$$

Solving this equation hopefully, will result in the time delay midway between τ_1 and τ_2 , viz., $\tau_m = (n+m/2)/f$ to tend towards the peak position $\bar{\tau}$ as time tends to infinity.

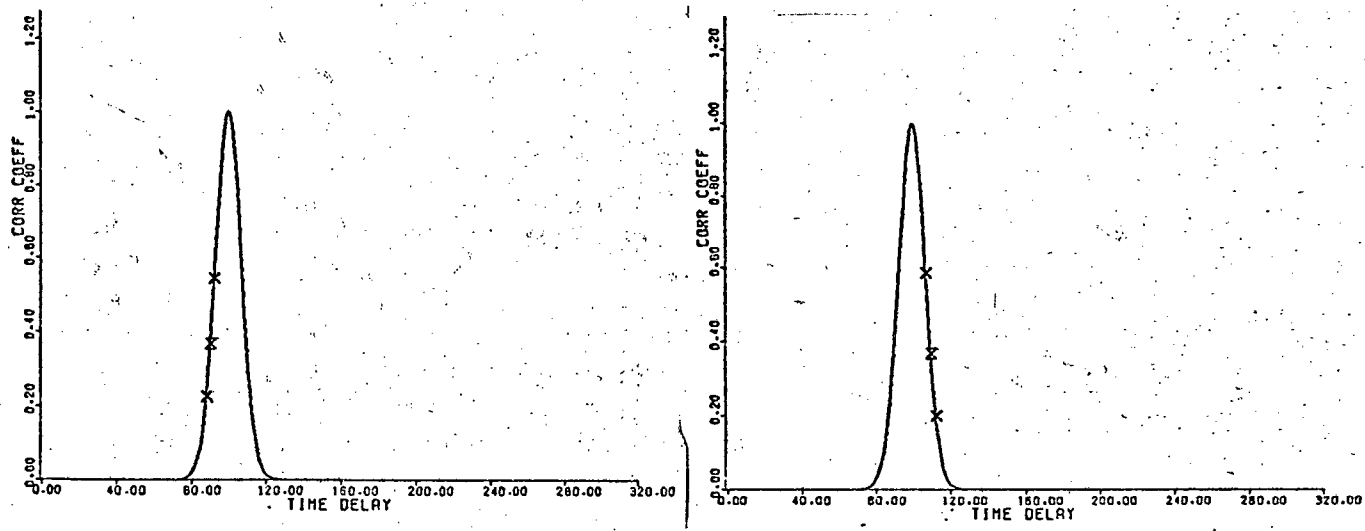


FIG. 8.5 - Gaussian correlation function with initial conditions shown.

A numerical solution of the above differential equation requires numerical values for the constants. These were chosen to correspond approximately with the actual values used in the hardware two-point difference correlator, viz.,

$$\begin{aligned} n &= 1000 \\ m &= 50 \\ k.k_1 &= 30000 \\ \bar{\tau} &= 0,10 \\ \sigma &= 0,01 \end{aligned}$$

The peak position is thus at a time delay of 100 m secs and has a standard deviation of 10 m secs as shown in Figure 8.5. The value of E is chosen such that:

- (i) $\tau_M(0) = 0,09 \text{ secs} = 90 \text{ m secs}$ and
- (ii) $\tau_M(0) = 0,11 \text{ secs} = 110 \text{ m secs}$

This corresponds to the two-point difference correlator being situated first on the left-hand slope and secondly on the right-hand slope of the correlation peak (Figure 8.5).

Since $\tau_M = (n + m/2)/f = 1025/f$, this results in initial clock rates of:

$$(i) \quad f_1(0) = 1025/0,09 = 11,388 \text{ kHz and}$$

$$(ii) \quad f_2(0) = 1025/0,11 = 9,318 \text{ kHz}$$

The two-point difference correlator delay values hence are initially located at:

$$(i) \quad \tau_1(0) = n/f_1(0) = 0,0878 \text{ secs}$$

$$\text{and } \tau_2(0) = (n+m)/f_1(0) = 0,0922 \text{ secs}$$

and

$$(ii) \quad \tau_1(0) = n/f_2(0) = 0,1071 \text{ secs}$$

$$\text{and } \tau_2(0) = (n+m)/f_2(0) = 0,1127 \text{ secs}$$

Typical results of the simulation are shown in Figures 8.6. In both cases, the clock frequency, mid-point delay value and error voltage have been plotted as functions of time. As can be seen, the two-point difference correlator locates the correlation peak with no overshoot in its response. This is to be expected since a first-order differential equation describes the model.

The time to reach the final value was determined for various gains and is given in Table 8.1.

TABLE 8.1

$G = k.k_1$	$f(0) = 11,388$ kHz	$f(0) = 9,318$ kHz
3 000	3,25 secs	2,95 secs
30 000	0,325 secs	0,295 secs
300 000	0,0325 secs	0,0295 secs

The response time is thus clearly inversely proportional to the gain. This is also evident from equation 8.1 since the rate of change of the clock frequency is directly proportional to the gain, $k.k_1$. A further point to note is that the response time is shorter when the two-point difference correlator initially is located on the right-hand slope of the correlation peak (i.e., when

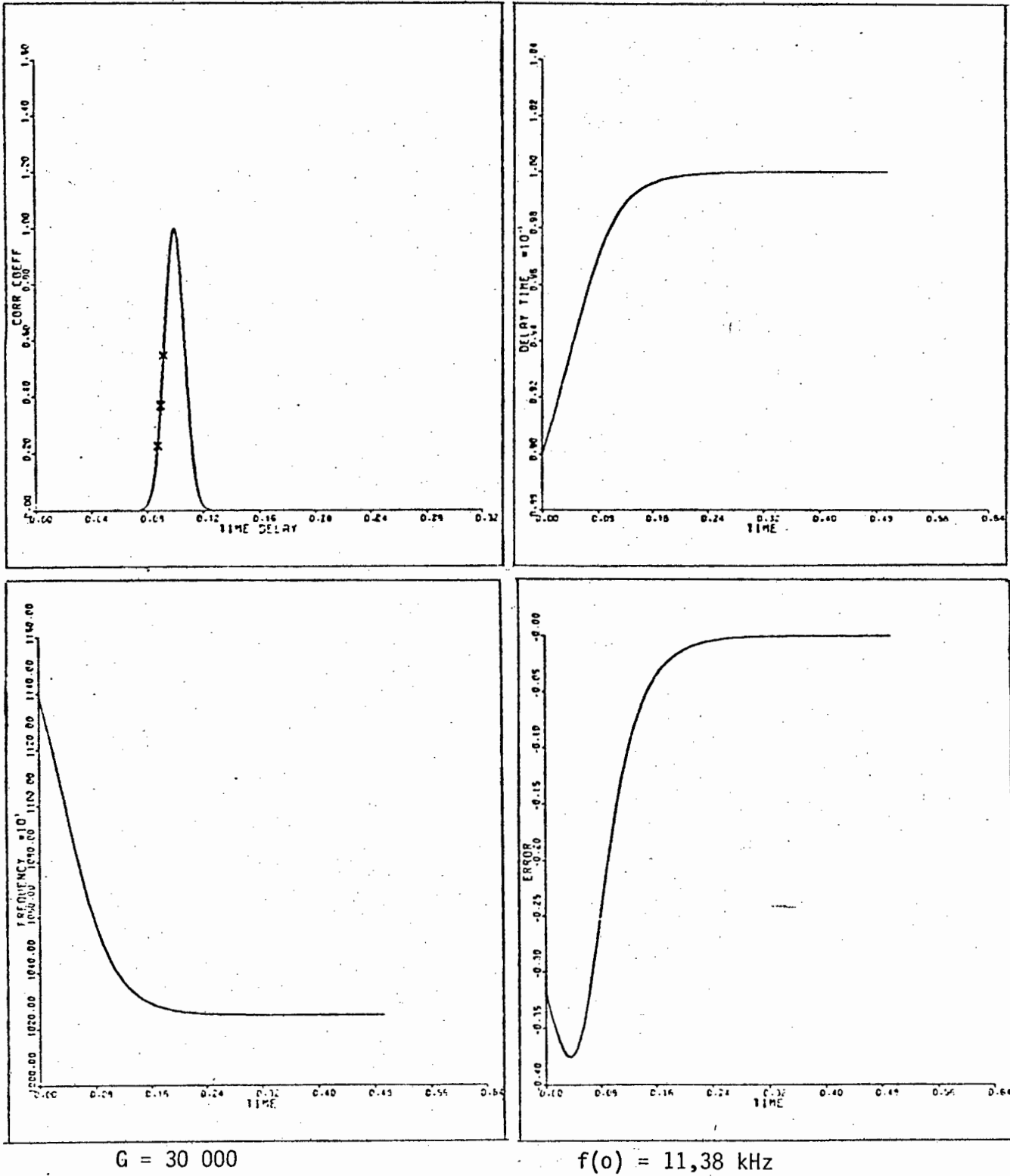
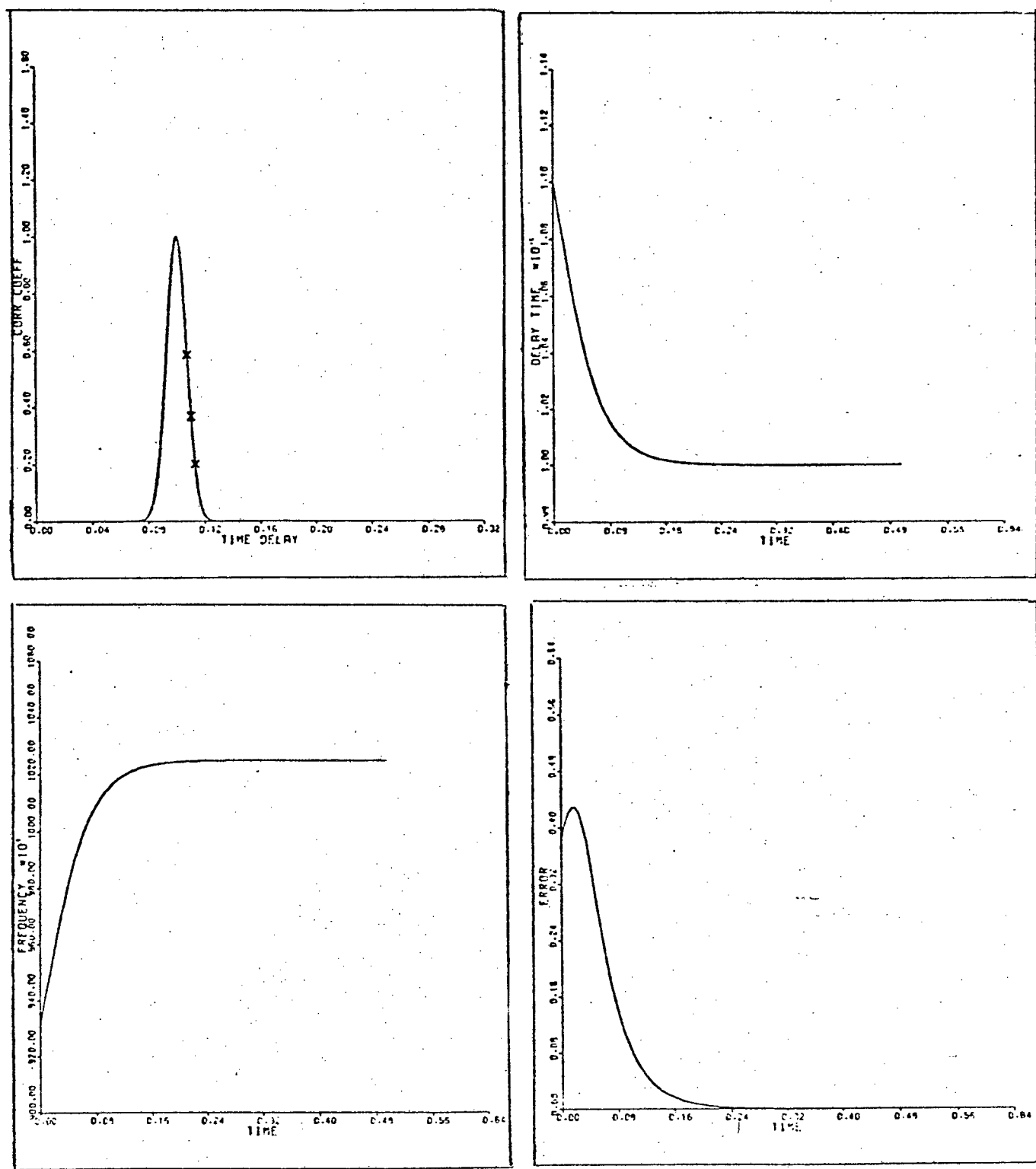


FIG. 8.6a - Typical response curves of first-order two-point difference correlator model.



$G = 30\,000$

$f(o) = 9,32\text{ kHz}$

FIG. 8.6b - Typical response curves of first-order two-point difference correlator model.

$f(0) = 9,318 \text{ kHz}$). The reason for this is that at this lower clocking frequency, the time-delay difference $\Delta\tau = \tau_2 - \tau_1 = m/f$ is larger, resulting in a larger error voltage and hence a faster response.

3. SECOND ORDER SIMULATION

In the two-point difference correlator of Figure 8.2, the actual error voltage applied to the integrator consists of a random pulse train having one of three possible levels $(-V, 0, +V)$ at any instant. For large integrator gains, i.e., fast response, this produces considerable random fluctuations at the integrator output. To improve the performance of the two-point difference correlator, a low-pass filter with impulse response, $h(t)$, is used to smooth the signal applied to the integrator. This hopefully will reduce the integrator output fluctuations even for large integrator gains. (See Fig.8.7)

The differential equation for the model has been derived in APPENDIX G assuming an R-C averager for the low-pass filter $h(t)$, and a Gaussian correlation peak (EQU 8.2). It is

$$\frac{d^2f}{dt^2} + \frac{1}{RC} \cdot \frac{df}{dt} + \frac{k \cdot k_1}{RC} \left[e^{-\left(\frac{n+m}{\sigma \cdot f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n}{\sigma \cdot f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} \right] = 0$$

with initial conditions

$$f(0) = k \cdot E = f_0$$

$$\frac{df(0)}{dt} = k \cdot k_1 \left[e^{-\left(\frac{n}{\sigma \cdot f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma \cdot f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2} \right]$$

The model is hence described by a second-order non-linear differential equation. Using the same numerical values for the constants as in Section 8.2, the equation has been solved via a fourth-order Runge-Kutta method for various combinations of RC , k and k_1 . Plots of clock frequency, mid-point time delay and error voltage versus time are shown in Figures 8.8 with the same initial values used previously, viz.,

(i) $f_1(0) = 11,388 \text{ kHz}$ and

(ii) $f_2(0) = 9,318 \text{ kHz}$.

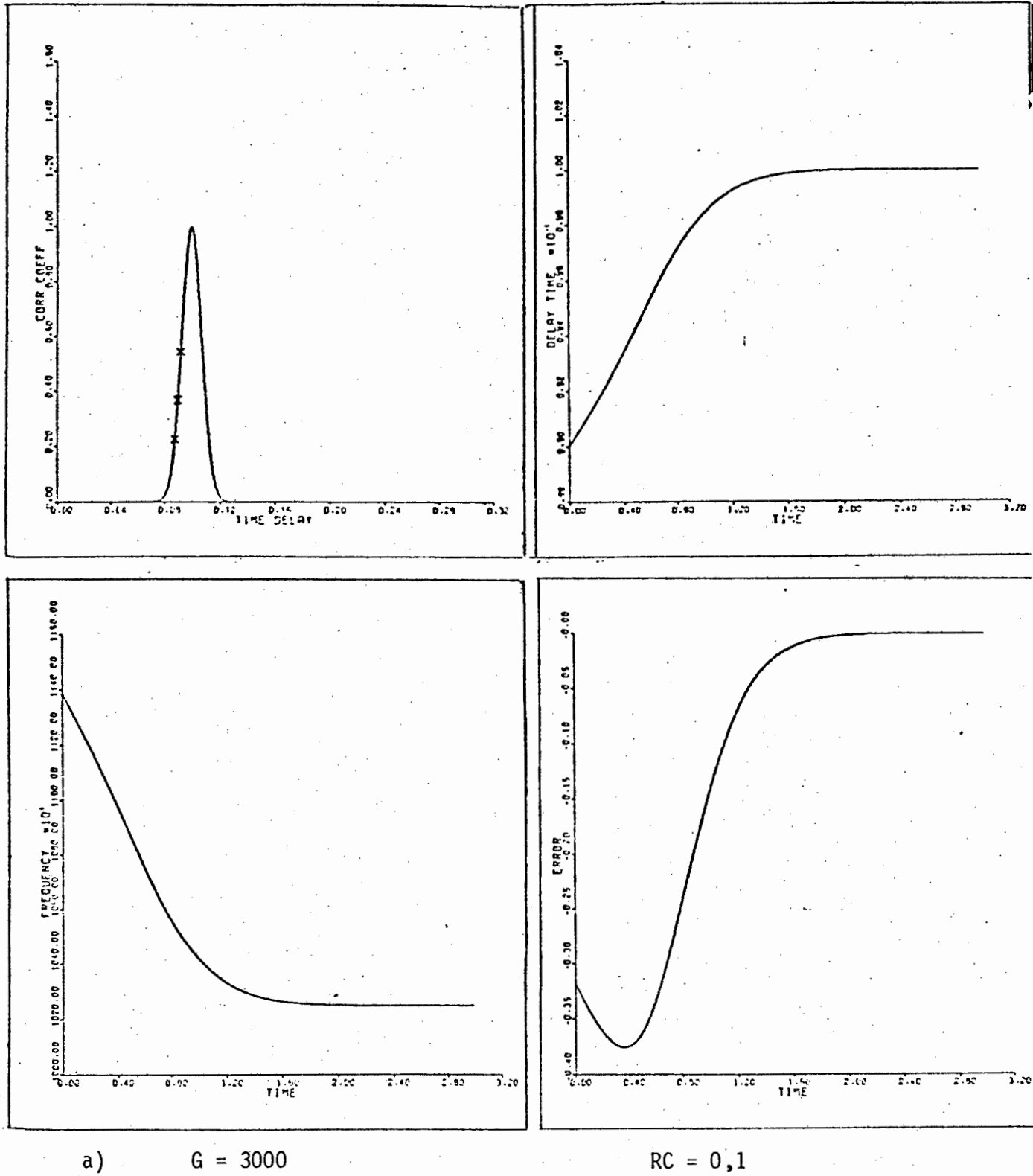
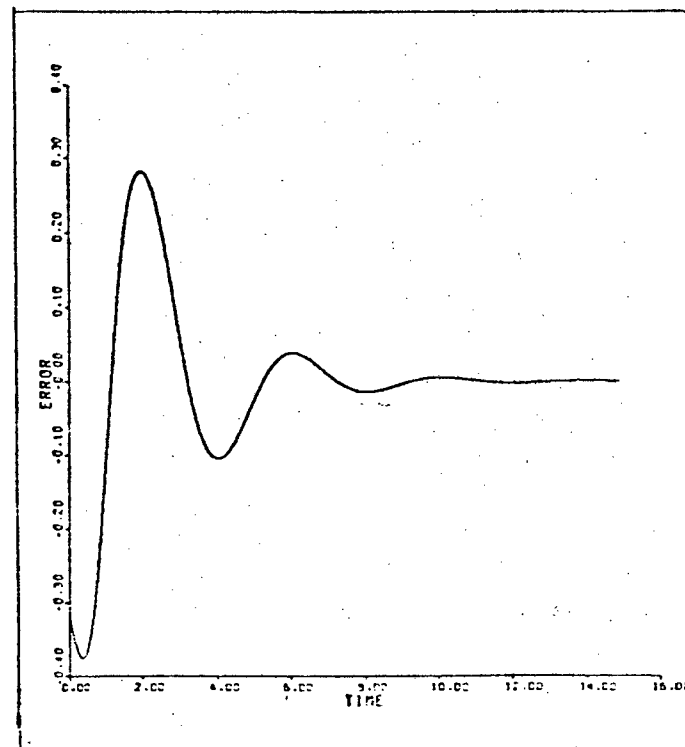
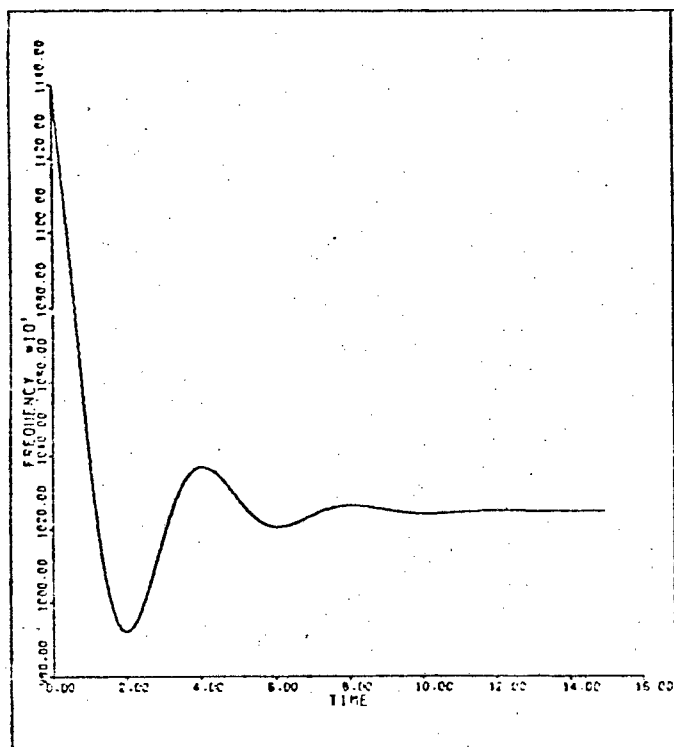
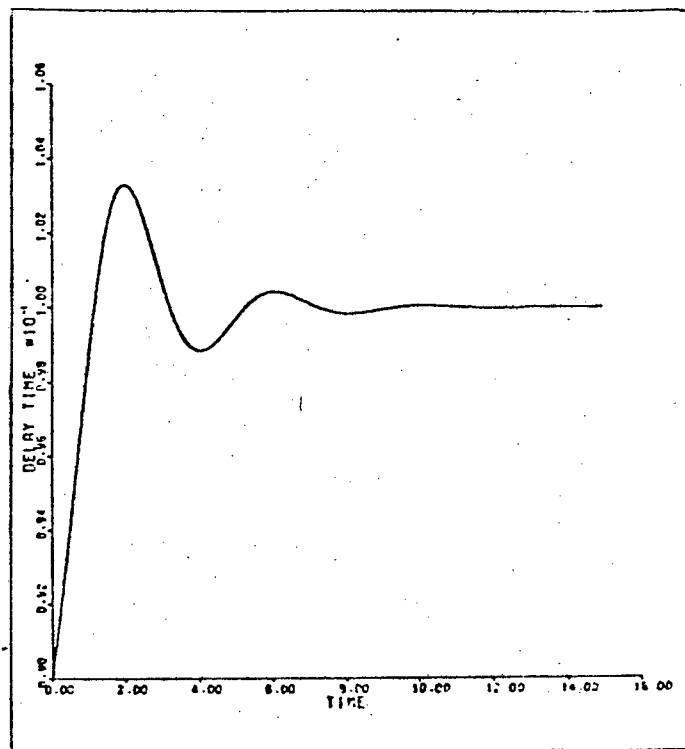
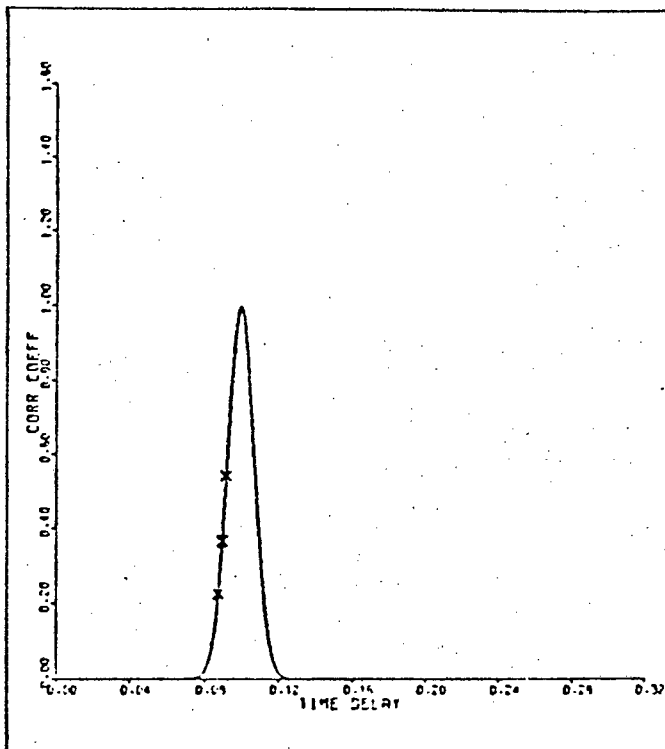
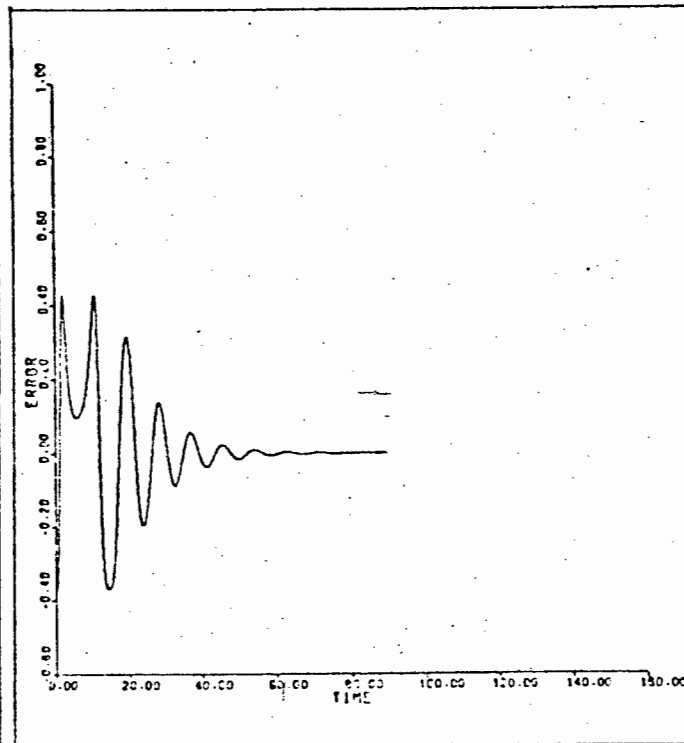
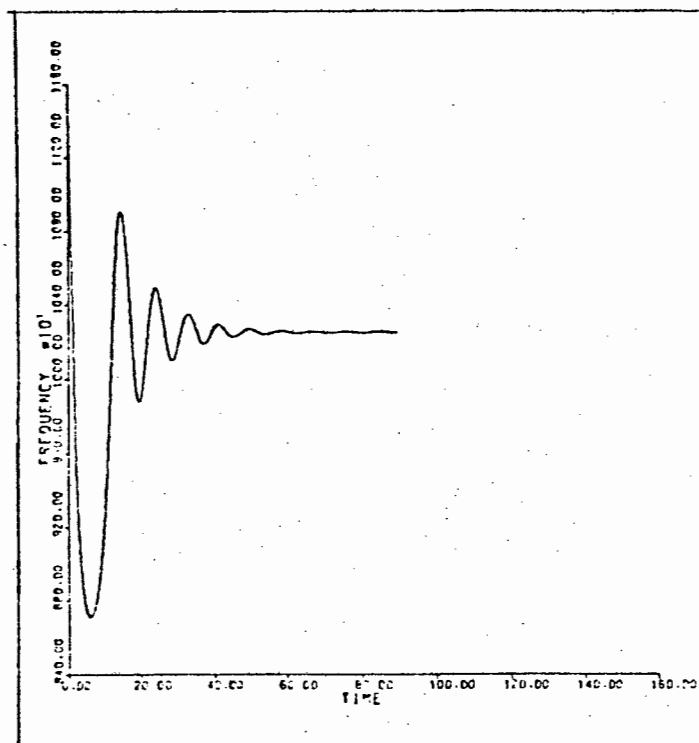
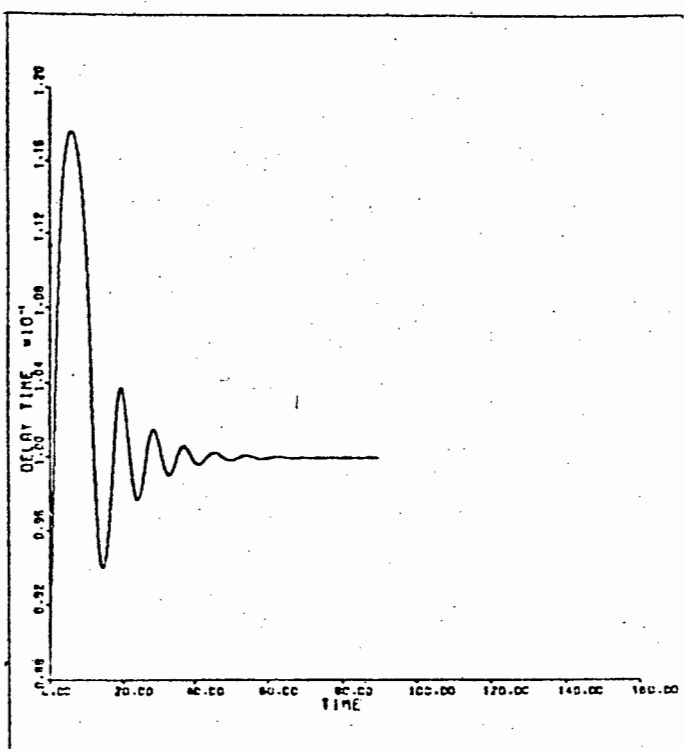
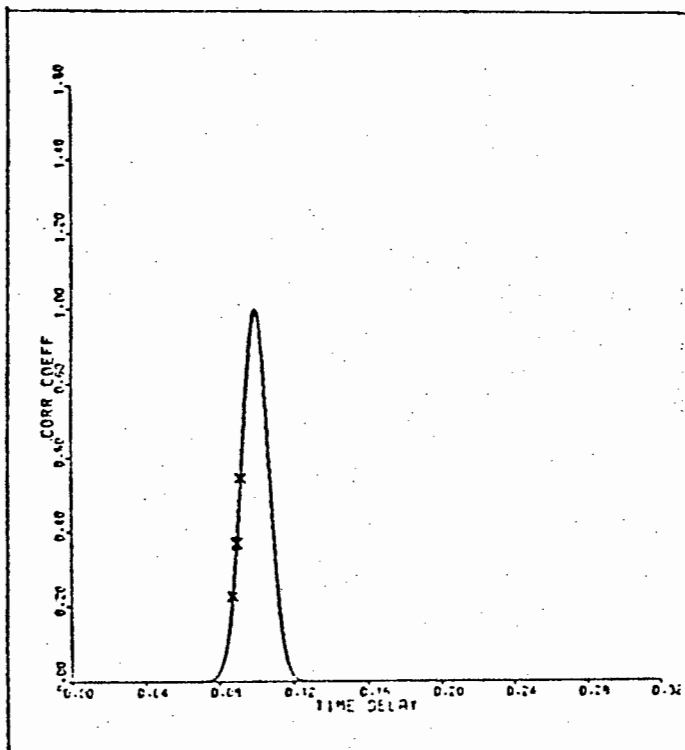


FIG. 8.8 - Typical response curves of second-order two-point difference correlator model



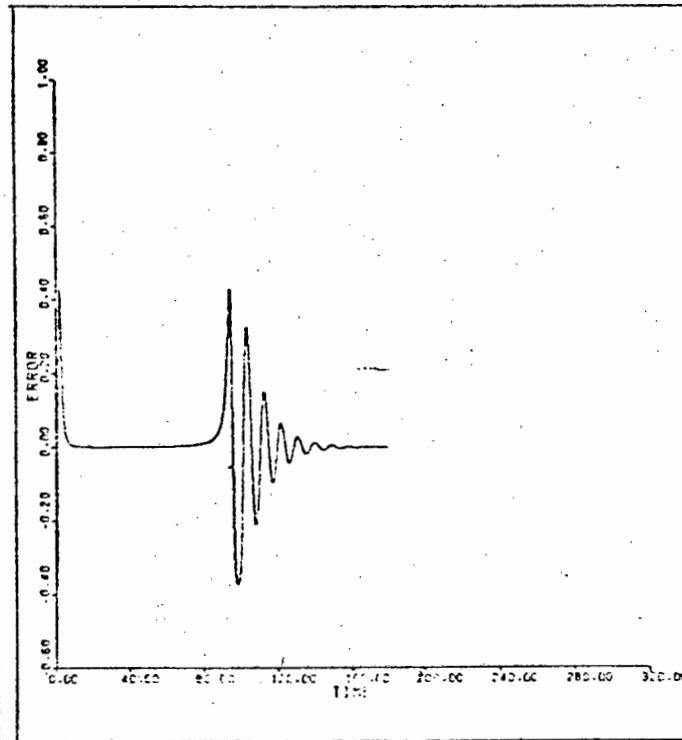
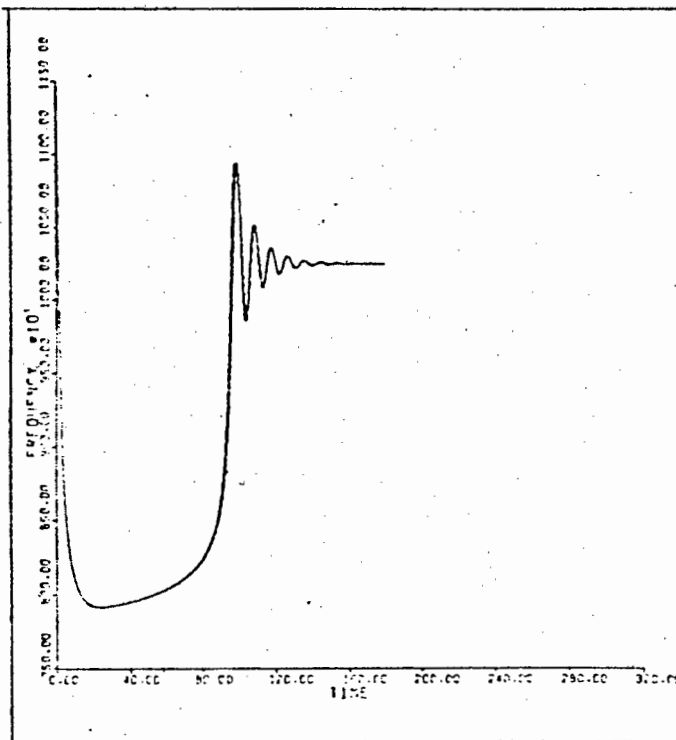
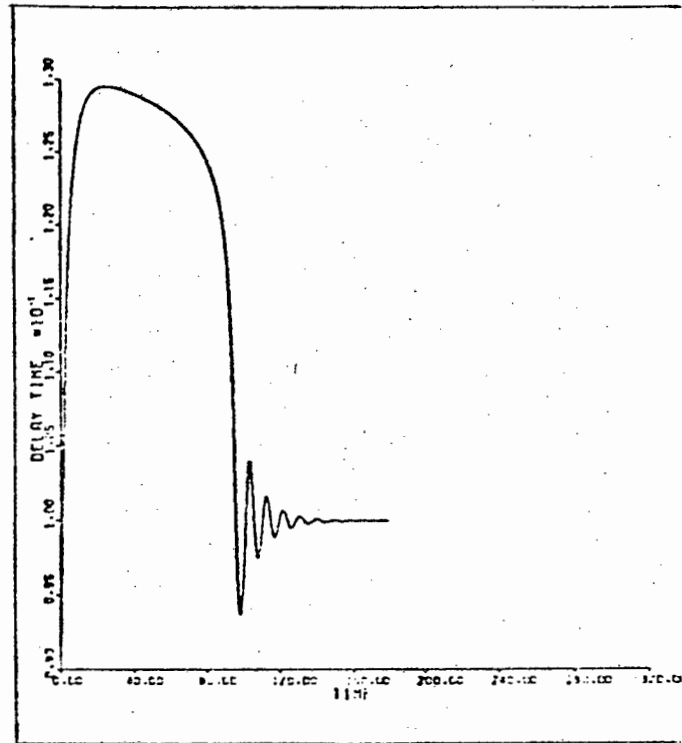
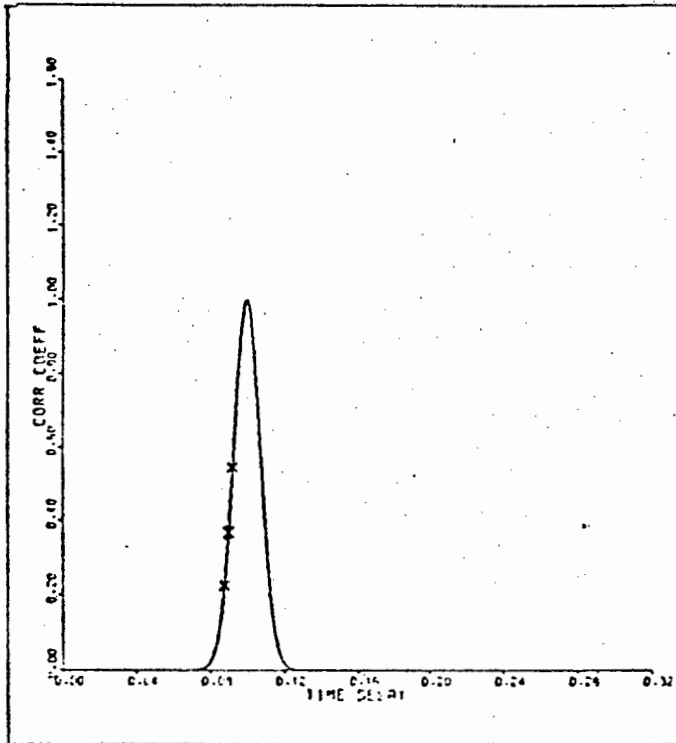
b) $G = 3000$

$RC = 1.0$



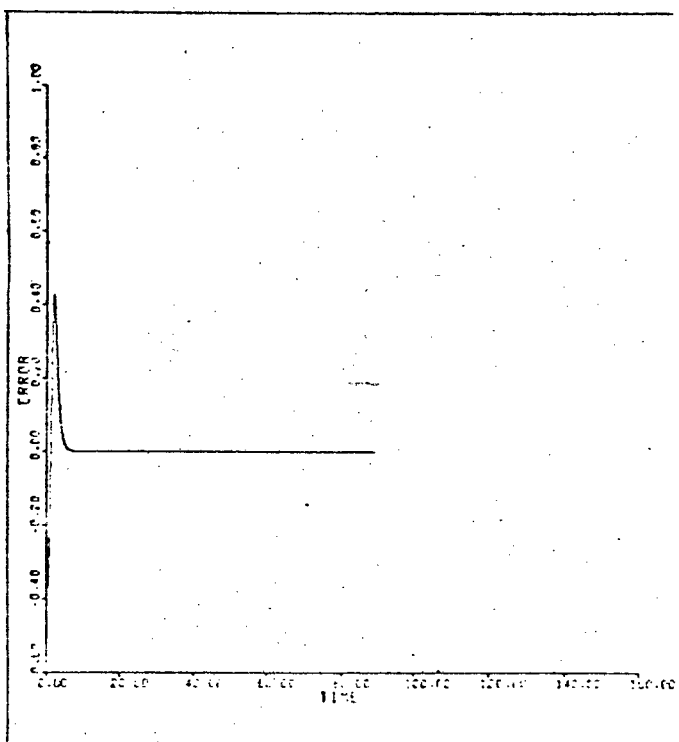
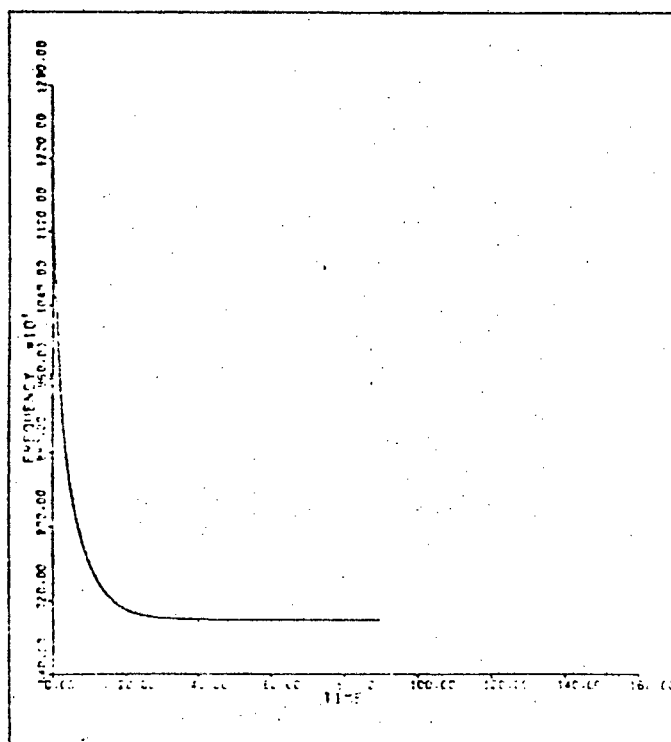
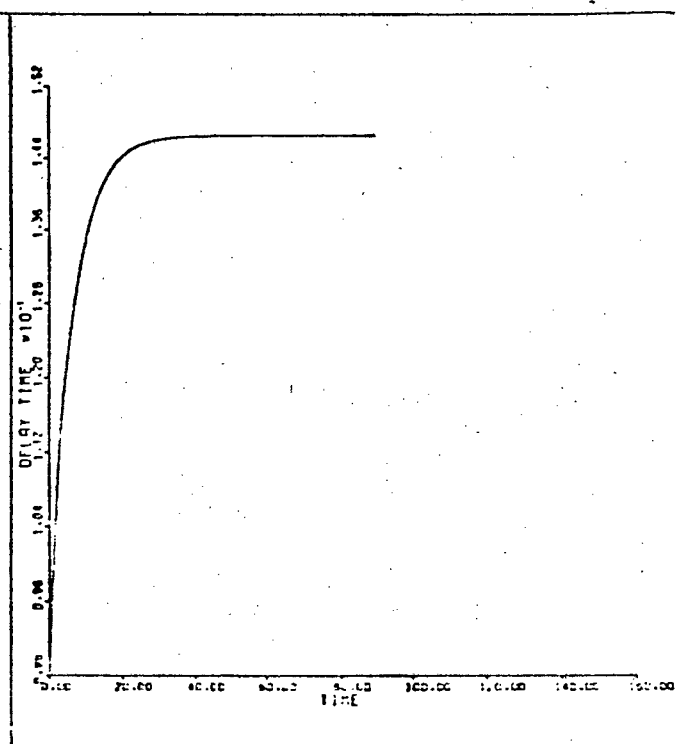
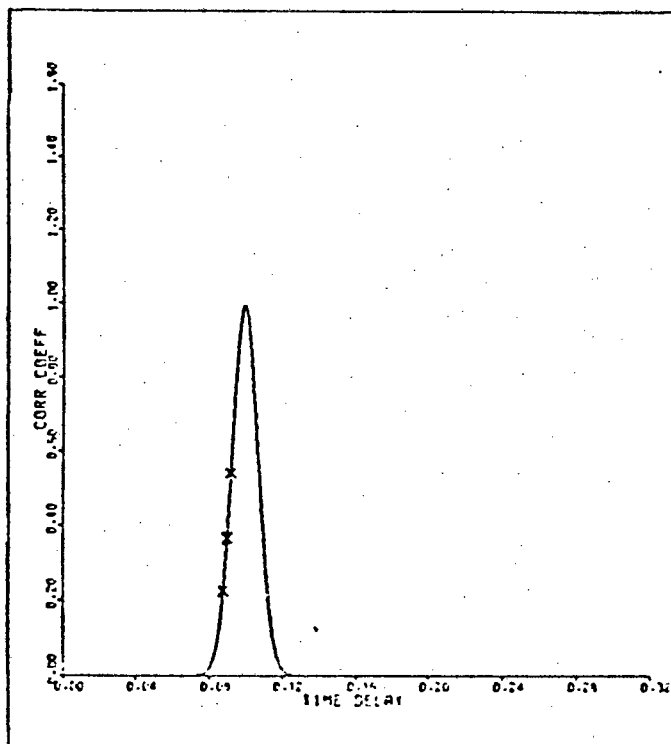
c) $G = 3000$

$RC = 5,0$



d) $G = 3000$

$RC = 5,5$

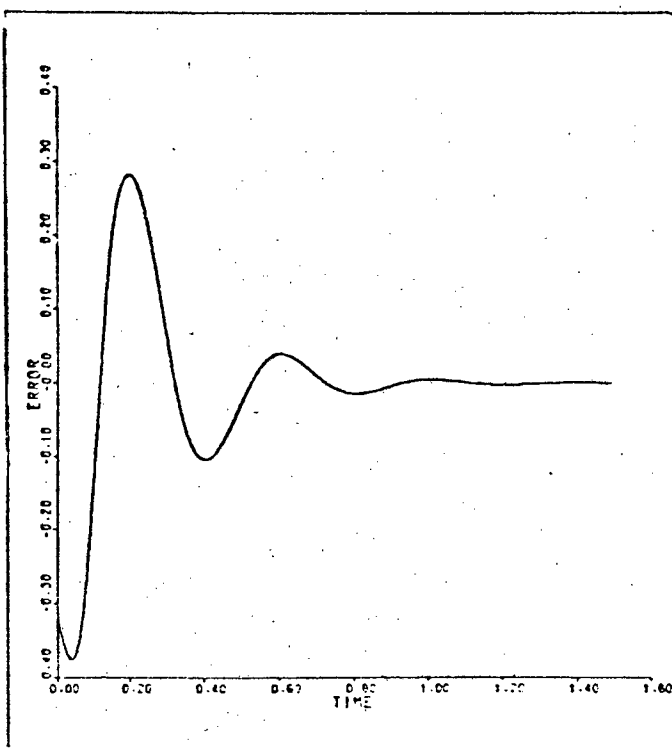
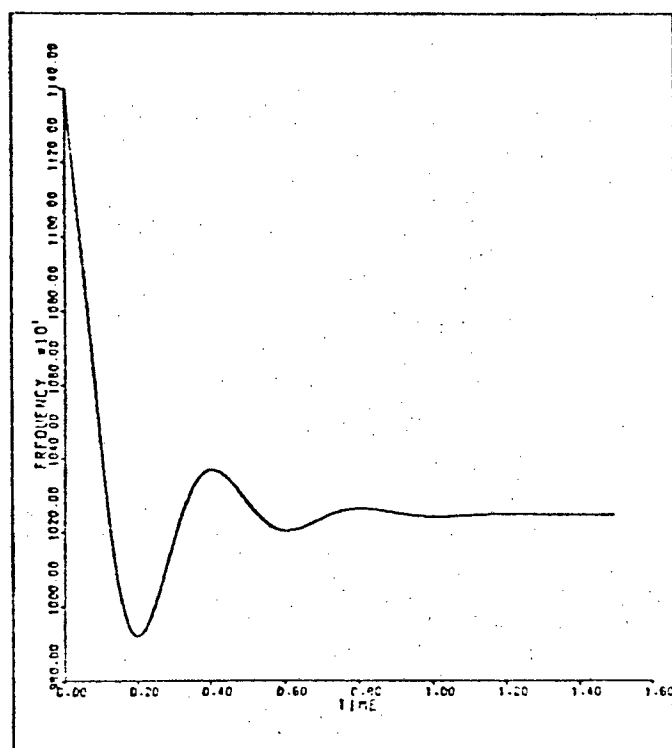
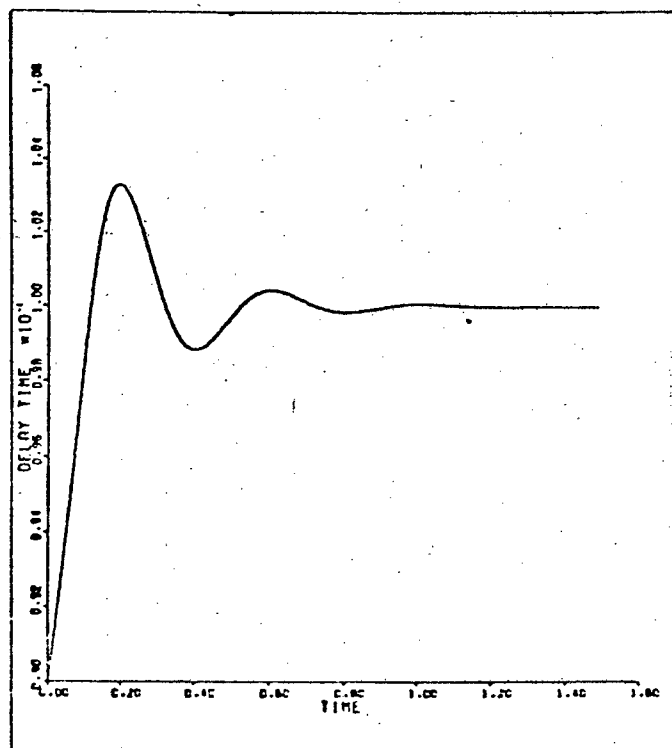
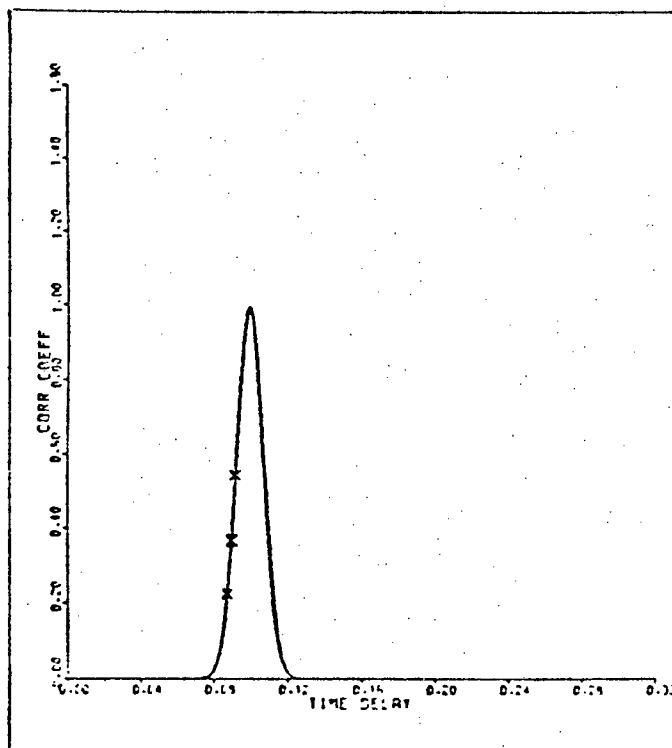


Peak not located

e)

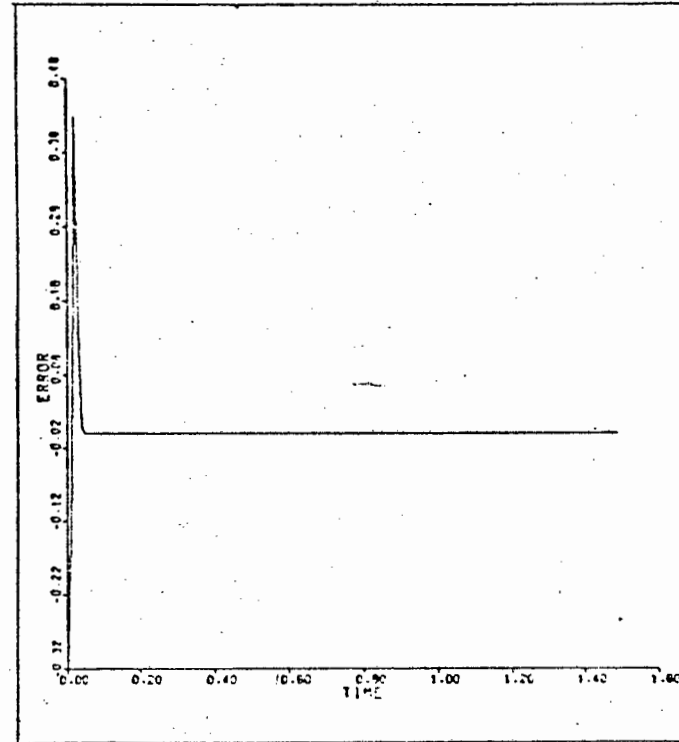
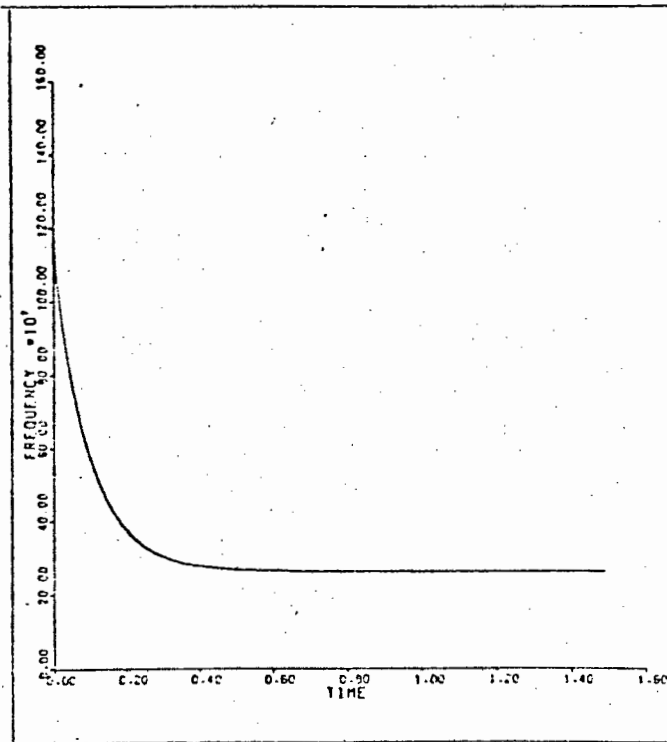
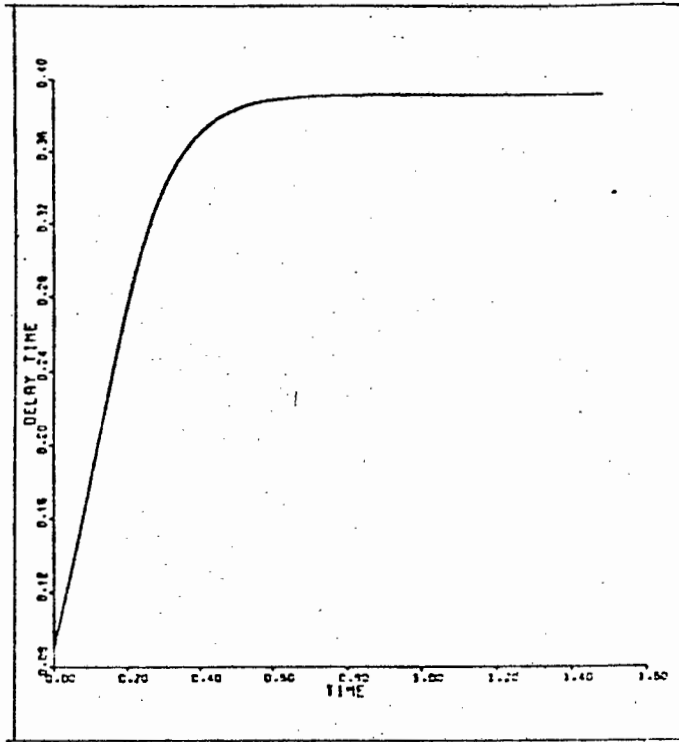
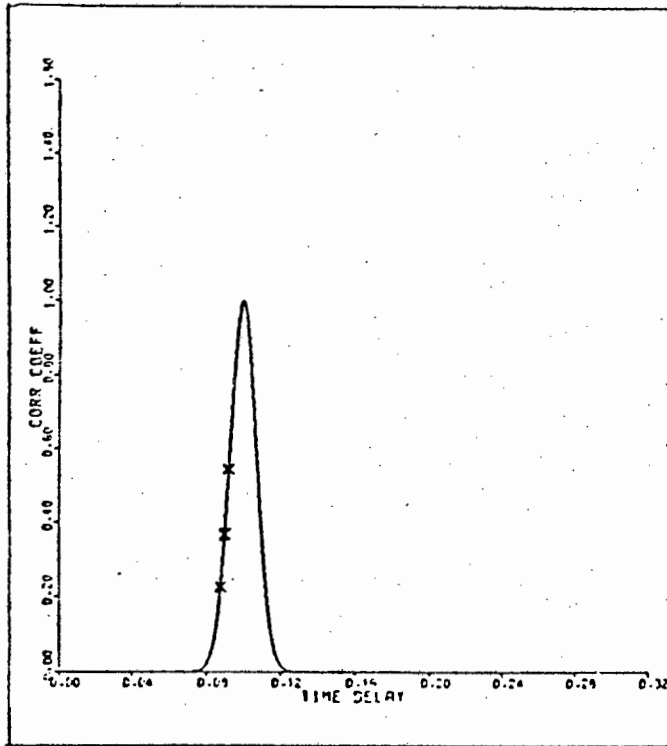
G = 3000

RC = 6.0



f) $G = 30\ 000$

$RC = 0,1$



Peak not located

g) $G = 30\,000$

$RC = 0,1$

It is evident from the plots obtained that both underdamped and overdamped responses are possible. This is typical of second order systems. For a fixed value of gain $G = k.k_1$, the system shows no overshoot for small values of time constant, RC (Figure 8.8a). As RC increases (i.e., longer averaging times), overshoot commences, (Figure 8.8b,c and d), and increases until eventually the correlation peak is no longer located (Figure 8.8e). Furthermore, for a fixed time-constant, RC , the response changes from no overshoot to overshoot until eventually the peak is no longer located as the gain is increased (Figure 8.8f and g).

4. ACTUAL RESPONSE OF TWO-POINT DIFFERENCE CORRELATOR

In order to verify experimentally the results obtained in the previous simulations, the response of a hardware two-point difference correlator was recorded. The set-up used is shown in Figure 8.9. A random band-limited signal is applied to a variable delay analogue delay-line. The delayed and undelayed signals form the inputs to a two-point difference correlator and are also applied to a HONEYWELL SAICOR proprietary correlator in order to examine the shape of the cross-correlation function. The peak position is determined by the delay set on the delay line and the width of the peak is controlled by the bandwidth of the random signals. Both are chosen to correspond approximately with the values used in the simulation, viz., peak position at 100 m sec and peak width about 30 m secs. The actual correlation peak used is shown in Figure 8.10. It differs considerably from the smooth Gaussian correlation peak assumed in the simulation.

The main difference is that the Gaussian peak tends to zero uniformly on either side of the peak position, whereas the peak shown in Figure 8.10 exhibits random fluctuations on either side of the peak position. This will prevent the limiting results (Figure 8.8b) obtained in the simulation to be verified experimentally.

A two-point difference correlator with $n = 1024$ and $m = 54$ was constructed with an external reset/run control on the integrator. With the integrator in the reset mode, the DC voltage applied to the V.C.O. was adjusted so that either one or both of the two-point difference correlator delay values was located on the slopes of the correlation peak. The integrator was then switched to its run mode and the voltage applied to the V.C.O. was recorded. Since $f(t) = k.r(t)$, this voltage is directly proportional to the clocking frequency.

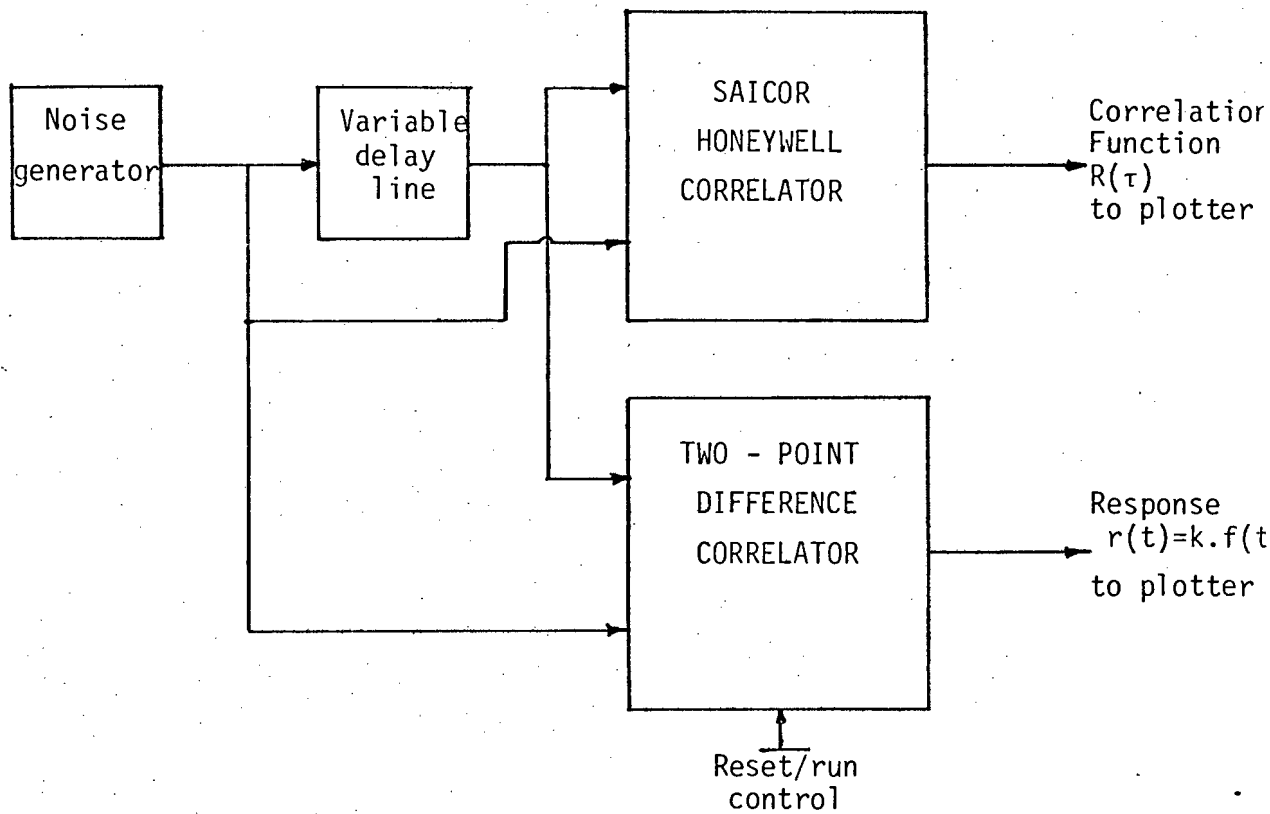


FIG. 8.9 Set-up for obtaining two-point difference correlator response

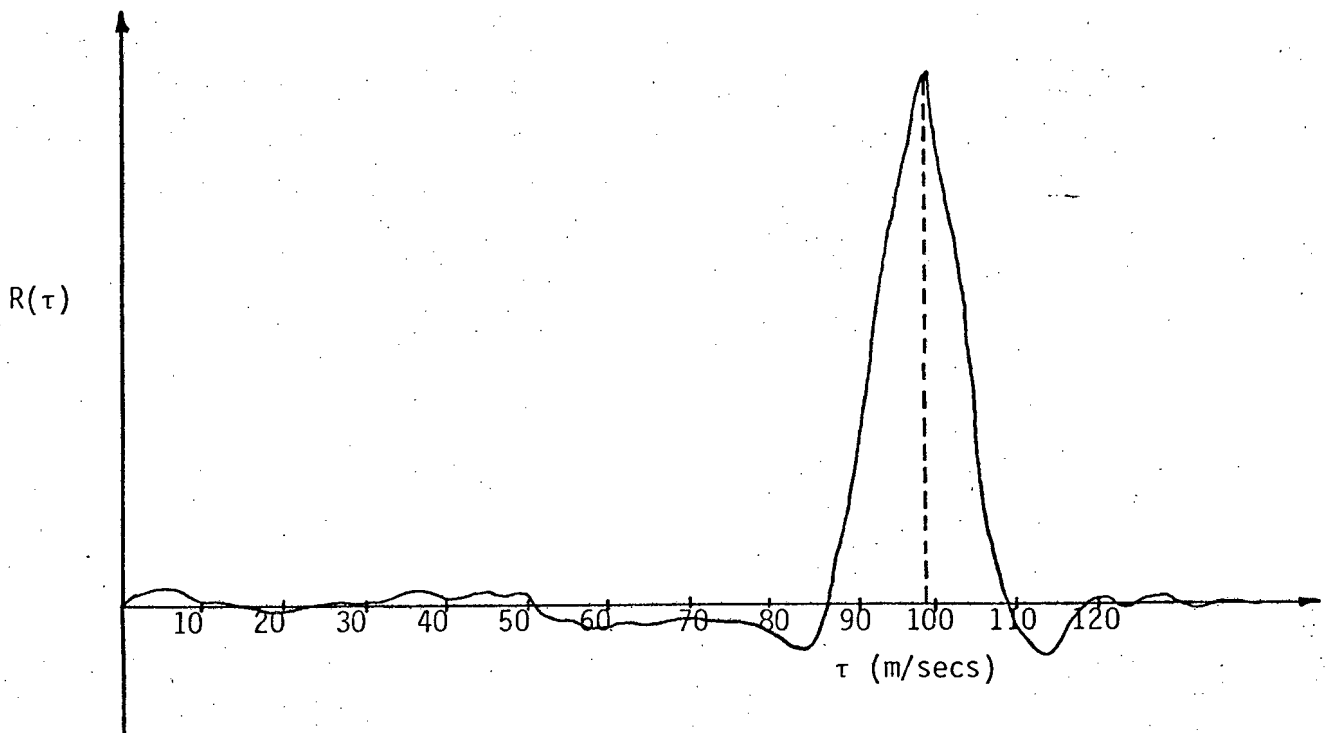


FIG. 8.10 - Correlation function used for testing two-point difference correlator response

The response of the basic two-point difference correlator without the additional RC averager is shown for three values of gain in Figure 8.11. The results agree with the first order simulation curves and confirm that the basic correlator is a first order system. The response shows no overshoot and as concluded previously, the response time decreases with increasing gain. The fluctuations about the final value, however, increase with an increase in gain and this sets an upper limit on the highest gain that can be used if a fast response is desired. The cause of these random fluctuations will be examined later.

An RC averager was added, hereafter, to the two-point difference correlator with a buffer placed between the averager and the integrator (Figure 8.12). This is necessary so as to isolate these two elements since this was assumed to be the case for the model .

Plots of modified two-point difference correlator response for various values of loop gain, G , and averager time constant, RC , can be seen in Figures 8.13 and 8.14. The values of G and RC have been chosen to correspond approximately with the values used in the simulation. Figure 8.13(a,b,c) show the response for fixed gain settings, G , but varying RC time constant. For low gain (Figure 8.13a) a smooth response is obtained that exhibits no overshoot for small time constants. As the time constant is increased, overshoot commences but for further increases the correlation peak is no longer located. A large gain (Figure 8.13b) results in a faster response time and overshoot becomes evident at a smaller time constant than previously. A further increase in gain (Figure 8.13c) produces a very fast response but unfortunately the sensitivity of the correlator is too high, resulting in excessive output fluctuations.

Figure 8.14 shows response curves for fixed averager time-constant but varying gain. It again is apparent that for a specific time constant, the system is overdamped for small gains and progressively becomes more underdamped as the gain is increased.

The general trend of the experimental results agrees with the results obtained in the simulation. Differences exist in the precise gains and time constants at which certain characteristics (e.g., overshoot) appear but this is to be expected since firstly, the correlation curves used are not identical and

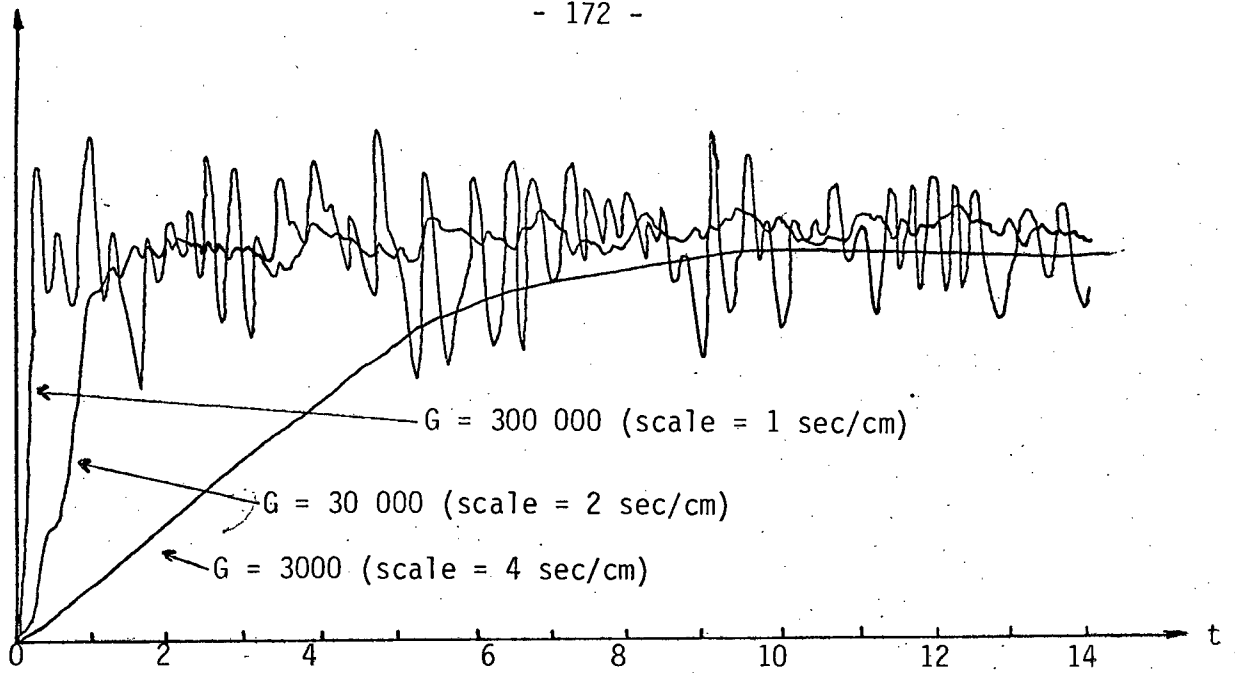


FIG. 8.11 Basic two-point difference correlator response for various gains

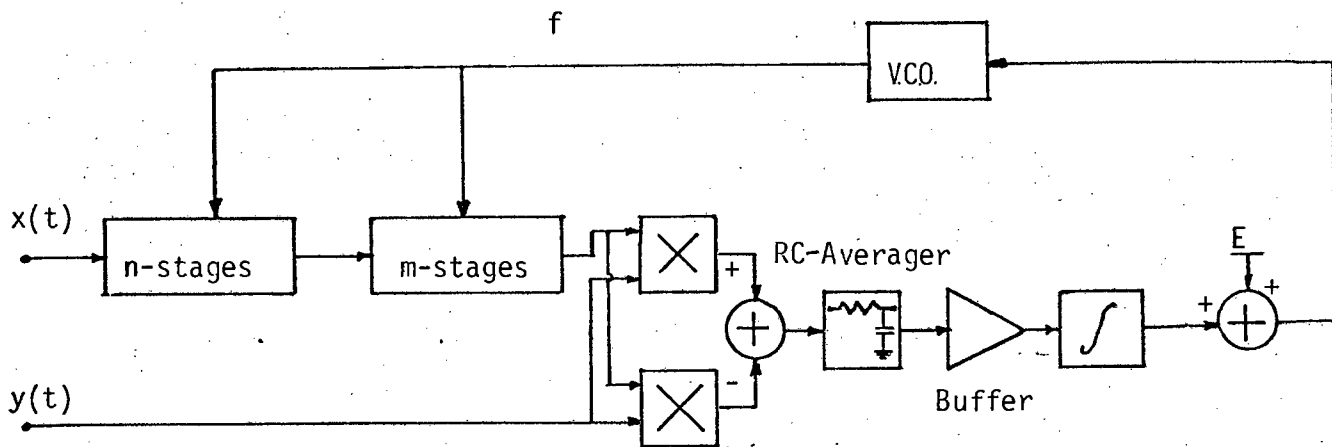


FIG. 8.12 Modified two-point difference correlator

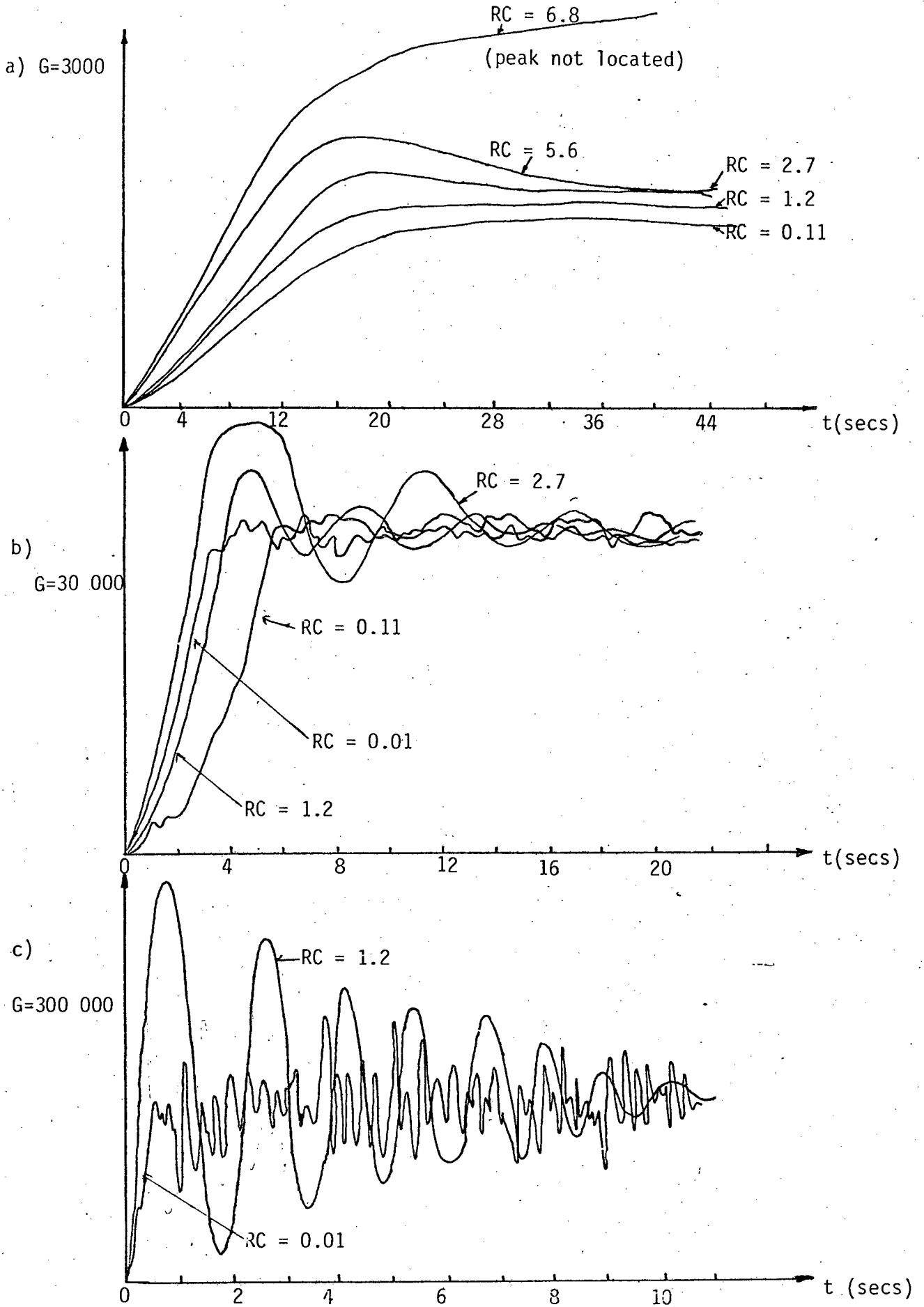


FIG. 8.13 Response of modified two-point difference correlator for different averager time constants at fixed gains

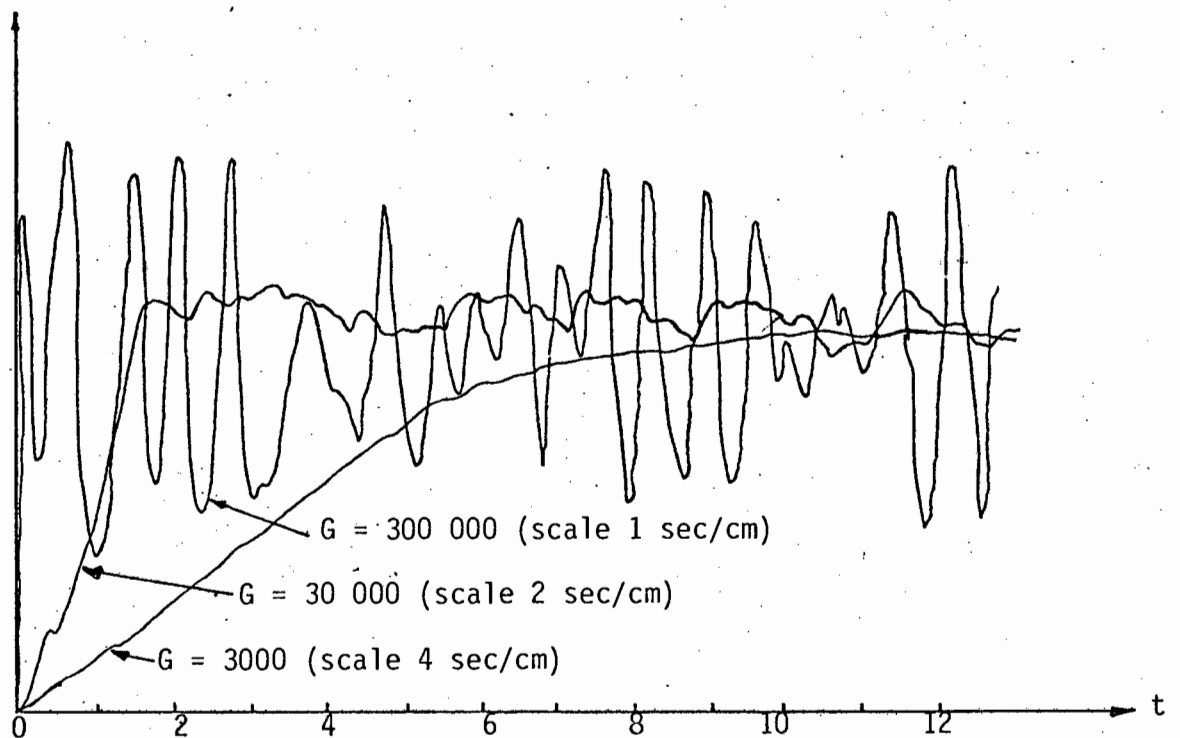
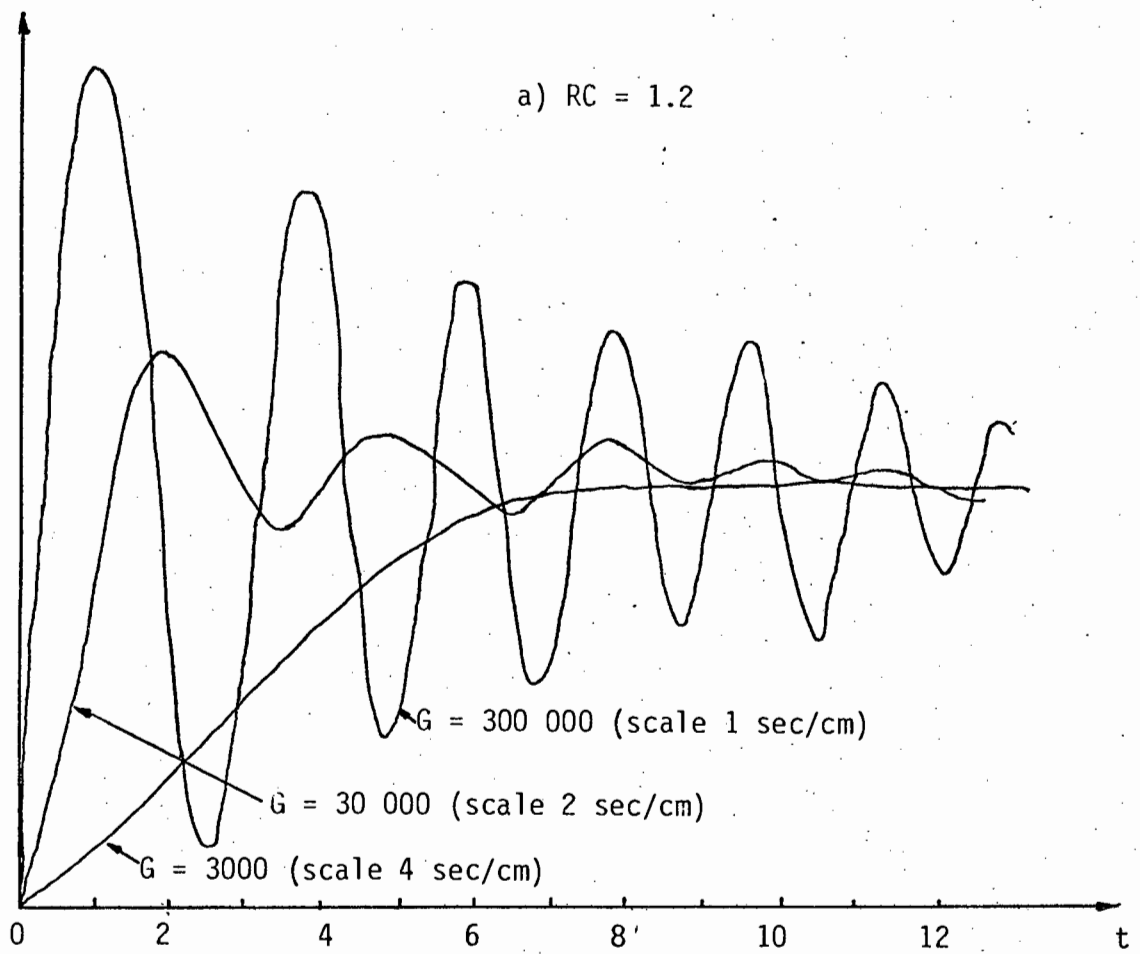


FIG. 8.14 Response of modified two-point difference correlator for different gains at fixed averager time constants

secondly, the parameters of the correlator used in the model are not exactly equal to those of the hardware correlator.

The random fluctuations in the experimental responses can be explained if the assumption made in Section 8.2 is examined. There it was assumed that the product of up and downstream signals $x(t-\tau).y(t)$ can be expressed as

$$x(t - \tau).y(t) = \overline{x(t-\tau).y(t)} + [x(t-\tau).y(t)]'$$

where the second term has a zero mean value. The primary factor determining the response of the correlator is the first term

$$\overline{x(t-\tau).y(t)} = R_{xy}(\tau)$$

The primed term was neglected in the simulation and this resulted in the smooth theoretical response curves. In practice this term is present and it can be considered to be a noise signal with zero mean. It also is applied to the integrator of the two-point difference correlator in addition to the DC term $R_{xy}(\tau)$. The integrator output thus is:

$$q(t) = k_1 \cdot \int R_{xy}(\tau) dt + k_1 \cdot \int [x(t-\tau).y(t)]' . dt$$

The second term will also have a zero mean value and it gives rise to the random fluctuations observed in the experimental results.

The main aim, namely to see if the proposed models are valid and can be used to predict the performance of the two-point difference correlator, has been confirmed. Since the correlation function and correlator parameters used in the simulation can be defined arbitrarily, the models can easily be modified if greater agreement between experimental and theoretical results is required.

5. OPTIMIZATION OF TWO-POINT DIFFERENCE CORRELATOR

The ideal two-point difference correlator must have a fast response, good sensitivity and exhibit only small fluctuations about the steady-state value. As is evident in the previous section, these unfortunately are conflicting requirements and a compromise must be found.

The basic two-point difference correlator is a first order system and its response depends on both the gain, $k.k_1$, and the difference in the value of the correlation function at its two delay times, $\Delta R = R(\tau_1) - R(\tau_2)$ (Equation 8.1). By increasing either of these, the response time can be reduced. The effect of varying the gain for a fixed correlation peak has been shown previously in Figure 8.11. Figure 8.15 shows the response for correlation peaks having the same shapes but different peak positions. Clearly the response time is longer for peaks located at small time delays since at such delays, the clock rate, $f(t)$, is high and hence $\Delta\tau = \frac{m}{f}$ is small resulting in a small, ΔR . In correlation flowmetering, however, the correlation peaks fortunately become narrower as the flow velocity increases (Chapter 9). This will compensate for the decrease in $\Delta\tau$ and tend to speed up the response. Similarly, for a fixed correlation peak position, the response time becomes shorter as the correlation peak is made narrower by increasing the bandwidth of the flow signals (Figure 8.16).

The modified two-point difference correlator is a second order system. As was shown previously, it exhibits both under and overdamped responses and sometimes does not locate the correlation peak at all! Its response depends on the gain, $k.k_1$, the averager time constant, RC , and the difference, ΔR . The selection of component values to optimize the response under all conditions is difficult. In Figure 8.17 are shown response curves with a particular set of component values for identical correlation peaks situated at different time delays. This shows that if the correlator is optimized for a particular correlation peak, it will not necessarily be optimized for other correlation peaks. In some cases, a very unstable oscillating response is obtained whereas for other peaks, the correlator is unable to locate the peak at all.

Since the first order correlator locates a correlation peak irrespective of the shape or position of the peak, it is concluded that such a correlator is most suitable for general purpose applications. However, an averager with a small time constant can be included to reduce the steady-state fluctuations slightly without introducing the problems discussed above. The response of such a correlator for various peak positions and shapes is shown in Figure 8.18. As the bandwidth of the flow signals increases, so the output fluctuations decrease. The response for $B = 500$ Hz in Figure 8.18b is exceptionally good.

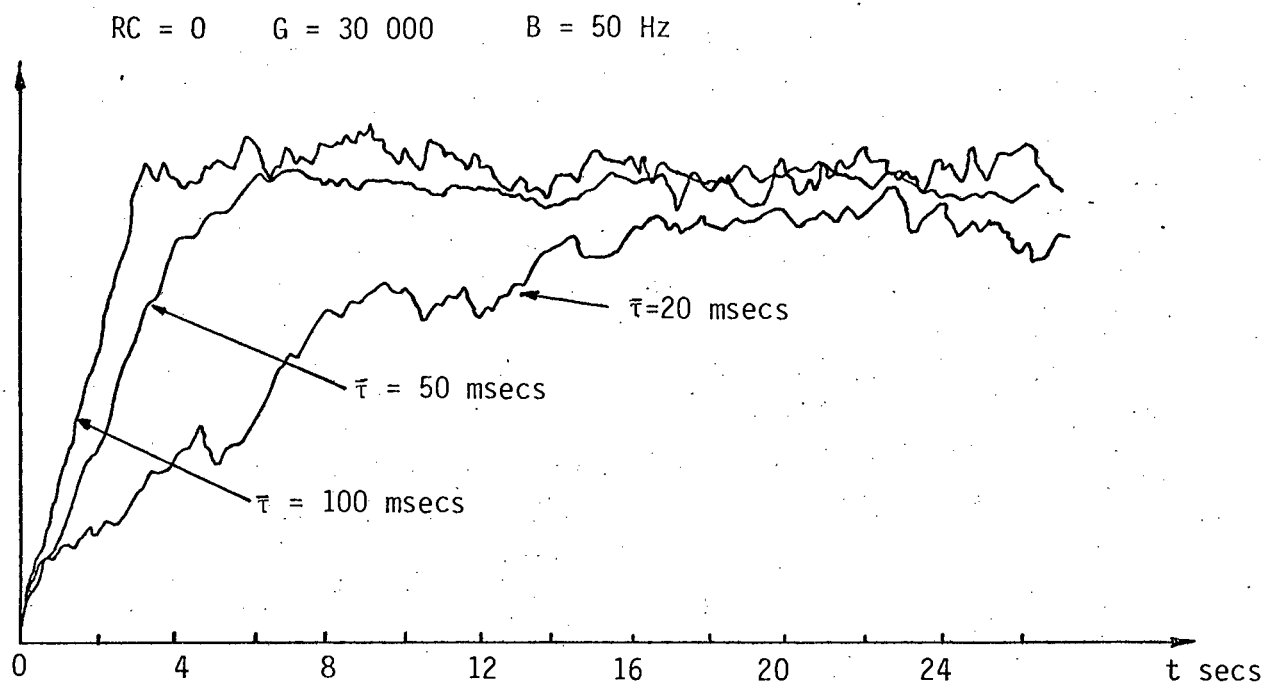


FIG. 8.15 Effect of peak-position on basic two-point difference correlator response (fixed correlation function shape)

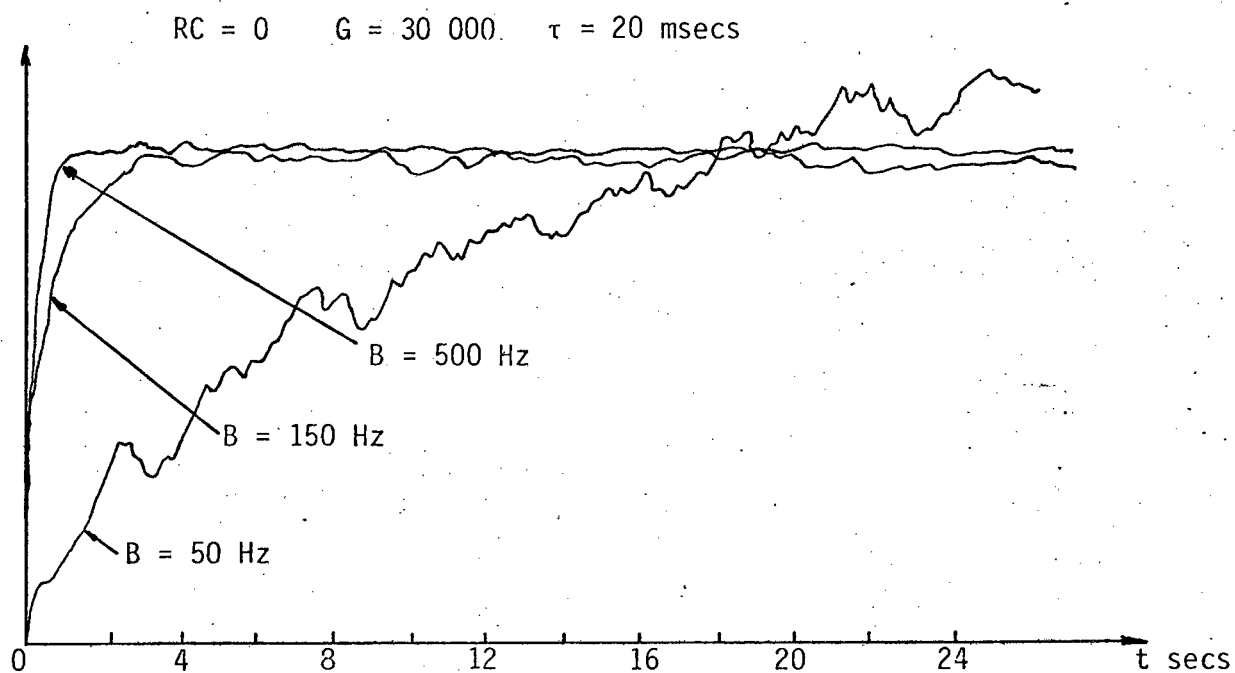


FIG. 8.16 Effect of correlation function shape on basic two-point difference correlator response (fixed peak position)

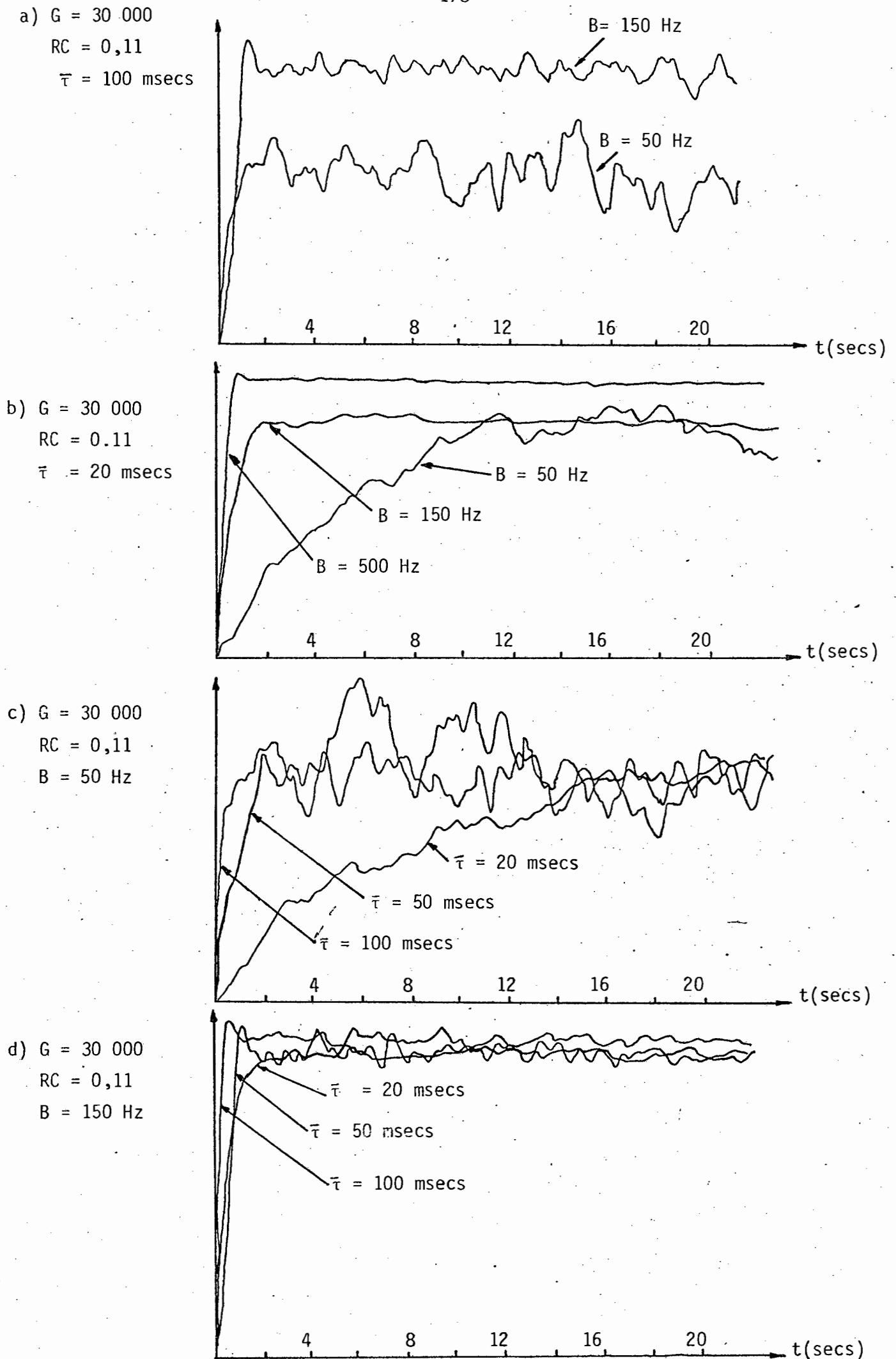
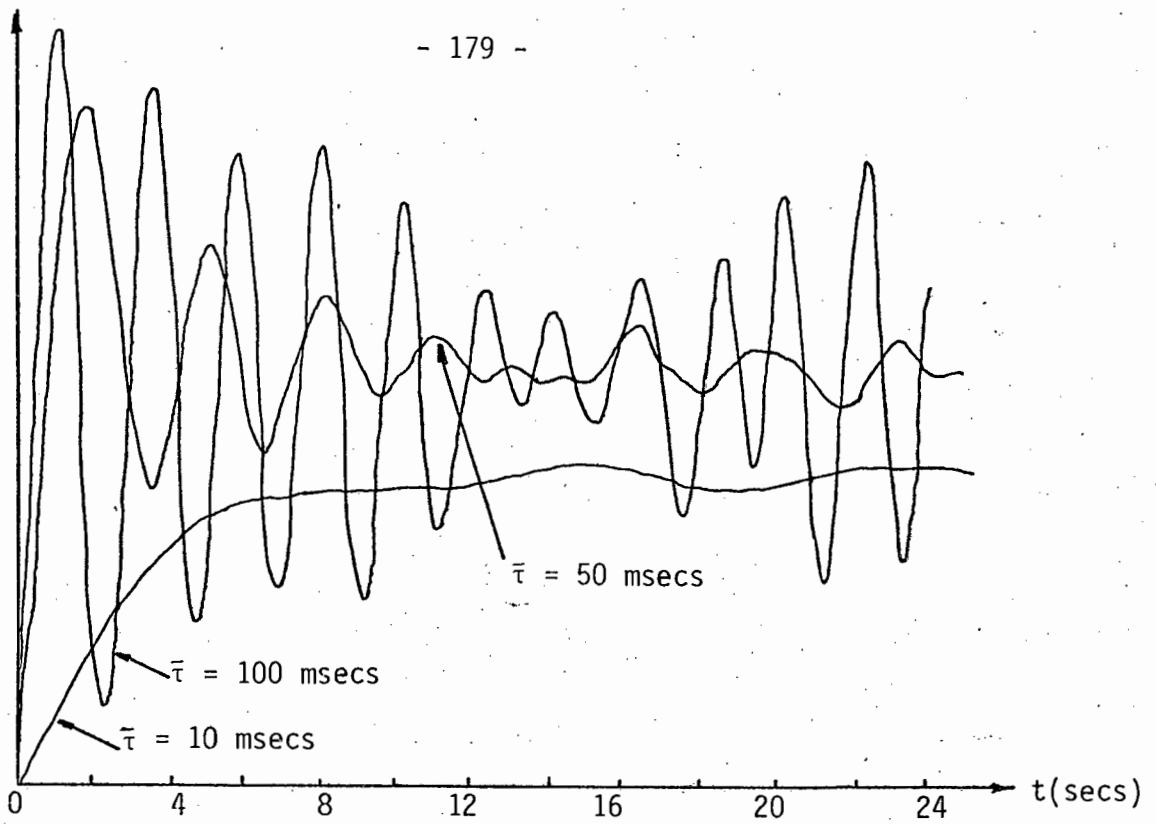


FIG. 8.18 Response of 'optimized' two-point difference correlator for various correlation function shapes and peak positions

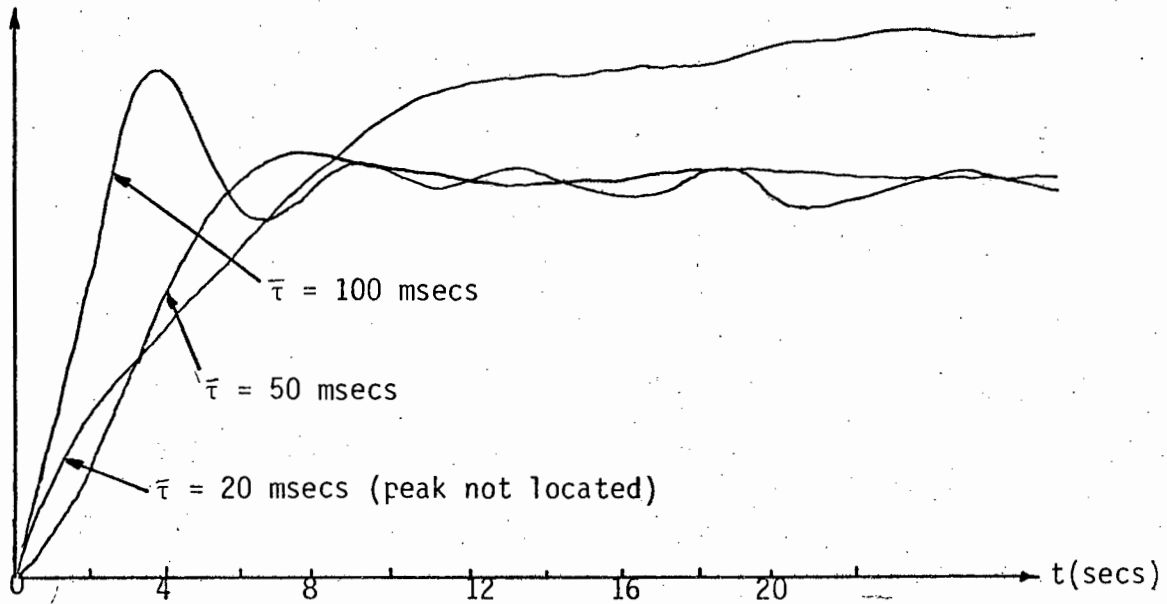
a) $G=300\ 000$

$RC=1,2$



b) $G=30\ 000$

$RC=1,2$



c) $G=30\ 000$

$RC=2,7$

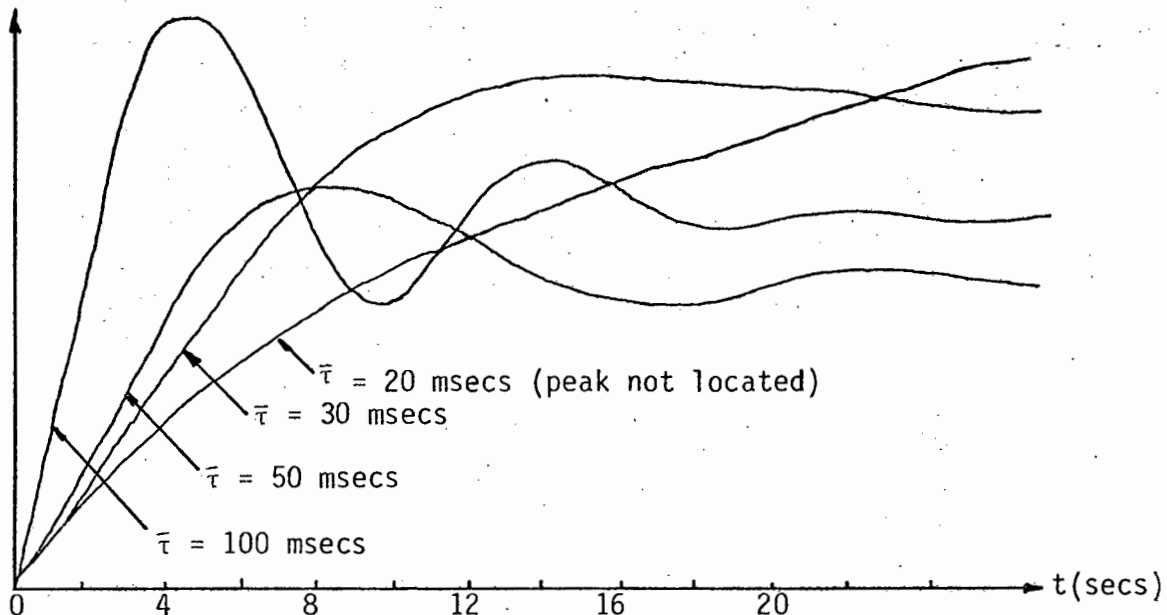


FIG. 8.17 Effect of correlation peak position on modified two-point difference correlator response (fixed correlation function shape)

CHAPTER 9

RESULTS

1. INTRODUCTION

This chapter is divided into two sections. Section A contains an analysis of the flow signals in the frequency, amplitude and time domains whereas Section B examines the performance of the correlation flowmeters that have been developed. The results were obtained using a test-rig in the Department of Civil Engineering at the University of Cape Town. The rig is shown in Figure 9.1 and is of the gravimetric flying-start- and flying-finish-type with static weighing (Reference 7). The heart of the system is a flow diverter which causes the flow through the test section to bypass the weight tank during the periods when the flow is accelerating up to its steady value and decelerating to standstill at the end of a test, but diverts the steady flow into the weight tank for a measured time interval. The diverter is operated pneumatically and is used to start and stop an electronic counter as it moves from the sump to the weight tank and back again. The scale of the weight tank has a capacity of 800 kg with a 1 kg resolution. The fluid is circulated by a rubberlined centrifugal pump with a maximum flow rate of about $150 \text{ m}^3/\text{hr}$. A pinch valve is used to control the flow rate.

The main disadvantage of the system is that the flow range is limited to about 6:1. The reason for this is that the pinch valve cannot be closed too much as excessive back pressure may cause pipe failure.

Initially water was circulated in the system but later solids were added. For the former case, the ultrasonic flow sensors were mounted on a horizontal section of 10 cm i.d. P.V.C. pipe. This is acceptable since radial symmetry of the velocity profile will exist for water flow. Due to the difference in density between the water and solids settling will take place. This results in one of several flow regimes (Reference 1).

- (i) Pseudo-homogeneous flow in which the particles are uniformly distributed over the cross-section of a pipe. This condition exists at high flow rates and results in a symmetrical velocity profile.
- (ii) Heterogeneous suspension in which a definite particle concentration gradient exists.
- (iii) Sliding bed in which the fluid velocity is just sufficient to move particles that lie on the floor of the pipe.
- (iv) Saltation and stationary bed in which particles are repeatedly picked up by the liquid and deposited further along the pipe.

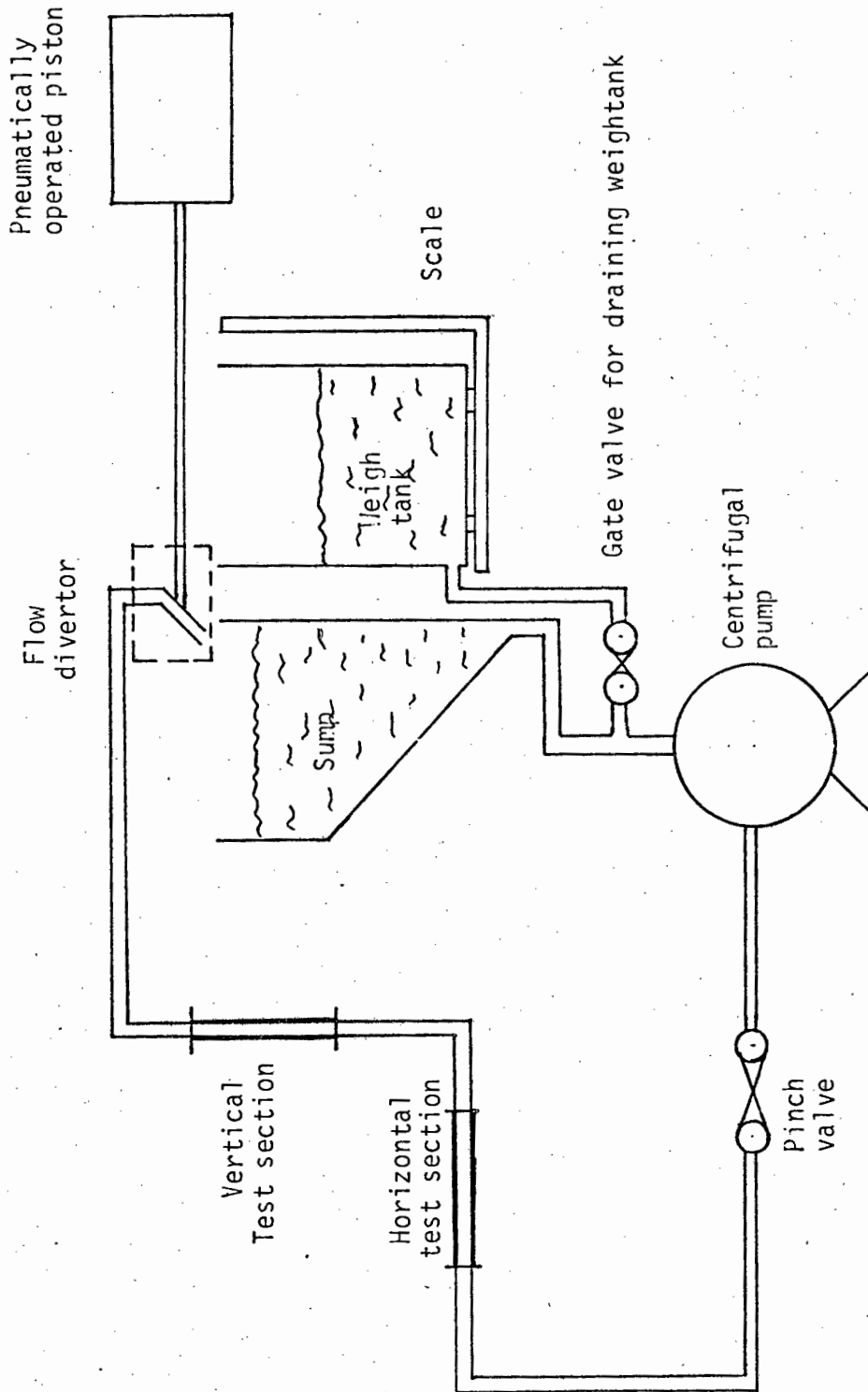


FIG. 9.1 Test rig for flow measurements

Which of these flows is present depends on the mean flow velocity in the pipe as well as on fluid and particle properties such as relative density, viscosity, particle size distribution and particle concentration. The latter three flow regimes produce an asymmetrical velocity profile in a horizontal section of a pipe. It is thus necessary to mount the flow sensors on a vertical section of pipe since the particles will be uniformly dispersed and the velocity profile symmetrical for such a section.

SECTION A : ANALYSIS OF FLOW SIGNALS

2. FREQUENCY DOMAIN ANALYSIS

It is important to know the frequency content of the flow signals since their bandwidth determines the bandwidth of the receiver circuitry. The frequency spectra were obtained at various flow velocities using a SPECTRAL DYNAMICS real-time frequency analyzer and are shown in Figure 9.2. The maximum energy is located in the low frequency components with a gradual roll-off towards higher frequency components. This implies that the low frequency turbulent eddies (i.e., the spatially large eddies) are the main energy bearing components.

As the flow velocity increases, so the bandwidth of the flow signals increases. This effect has been plotted in Figure 9.3 in which an approximately linear relationship between flow velocity and bandwidth is evident. The increase in bandwidth can be ascribed to two factors. Firstly, as the flowrate increases, so the level of turbulence increases resulting in eddies of smaller spatial dimension containing a larger portion of the turbulent energy. Secondly, any eddy or discontinuity is swept passed the flow sensors in a shorter period of time.

Both factors produce an increase in flow signal bandwidth but it is not possible to specify which factor dominates.

A further factor that influences the bandwidths of the flow signals is the size of the flow sensors in the direction of flow. This effect is discussed in Chapter 10 where it is shown that the sensors average the flow disturbances within their fields of view, thus acting as low-pass filters.

The frequency spectra of the flow signals with sediment in the water show similar characteristics as in Figure 9.2. However, for the same flowrate, the bandwidth with sediment is reduced (Figure 9.4). The reason for this is

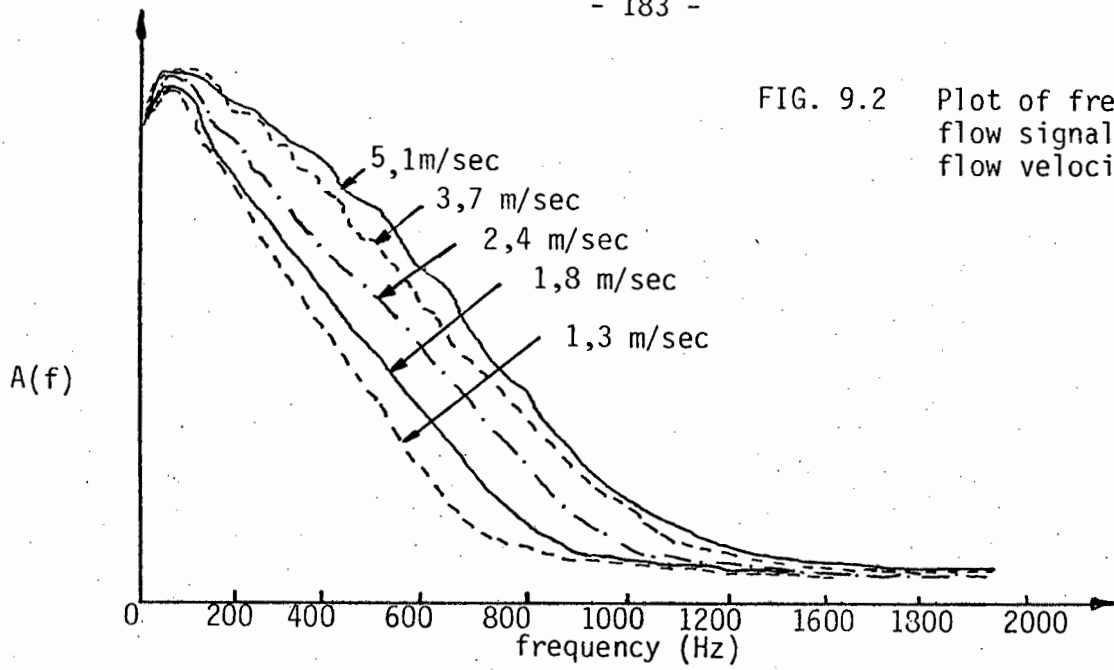


FIG. 9.2 Plot of frequency spectra of flow signals for different flow velocities

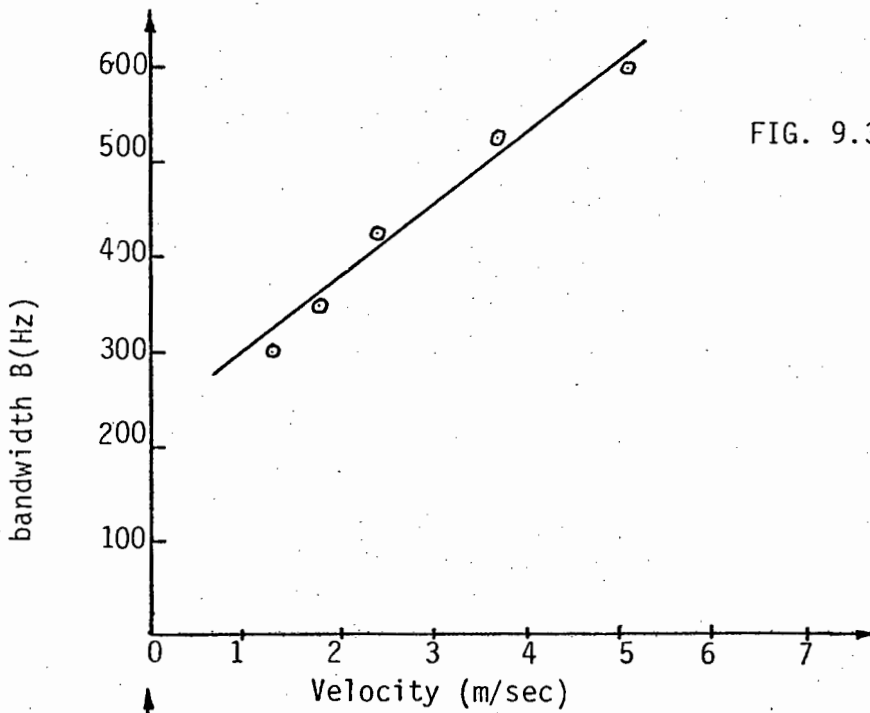


FIG. 9.3 Plot of flow signal bandwidth vs flow velocity

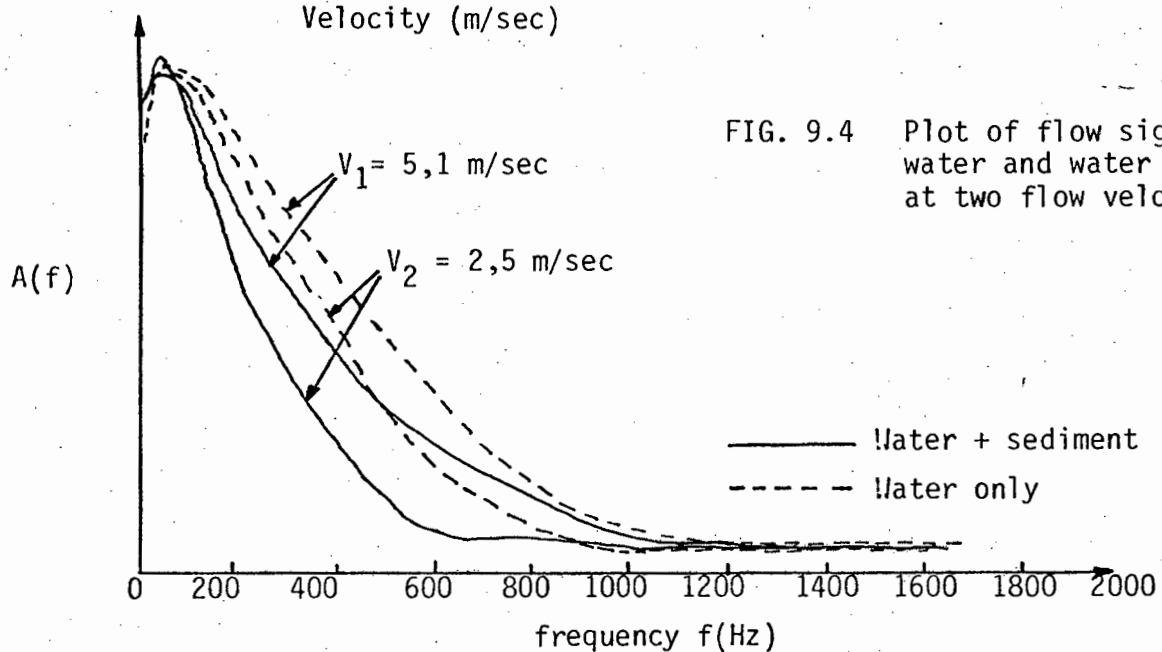


FIG. 9.4 Plot of flow signal spectra for water and water plus sediment at two flow velocities

that due to their inertia, larger particles are mainly borne by the low frequency eddies whereas smaller particles can follow the higher frequency turbulence components. Hence the particle size distribution influences the frequency distribution of the main energy bearing eddies. Clearly in the case of Figure 9.4, the particles are predominantly large and thus accentuate the low frequency components. It has been suggested that the frequency spectra of flow signals can be used to determine the particle size distribution of slurries or sediments (References 85 and 103).

3. AMPLITUDE DOMAIN ANALYSIS

A random signal is described in the amplitude domain by its probability density function $w_x(x)$ or its cumulative probability distribution function, $W_x(x)$. These contain the same information about a signal and are related simply by an integral, i.e.,

$$W_x(x) = \int_{-\infty}^x w_x(x) dx$$

It is important to know the probability density function of flow signals, since if it is Gaussian, the results of Chapter 5 are valid, that is, the relationship between the correlation functions of signals amplitude quantized to two levels and the original signals is:

$$\rho_{x'y'} = \frac{2}{\pi} \cdot \sin^{-1} \rho_{xy} \quad \text{where} \quad \rho_{xy} = \frac{\overline{x \cdot y}}{\sigma_x \cdot \sigma_y}$$

Plots of the probability density function and cumulative distribution function for a typical flow signal are shown in Figure 9.5. From these it appears that the process generating the flow signals is Gaussian.

A chi-squared goodness of fit test for a normal distribution was performed on the measured probability density function and no significant difference between the experimental and ideal functions was found. There thus is no reason to suspect the measured data to be non-Gaussian.

4. TIME DOMAIN ANALYSIS

4.1 Auto-correlation Functions

The auto-correlation function and the power spectral density function of a signal form a Fourier transform pair. They are two different ways of representing the same information. The auto-correlation function of a typical

FIG. 9.5a Probability density function of flow signals

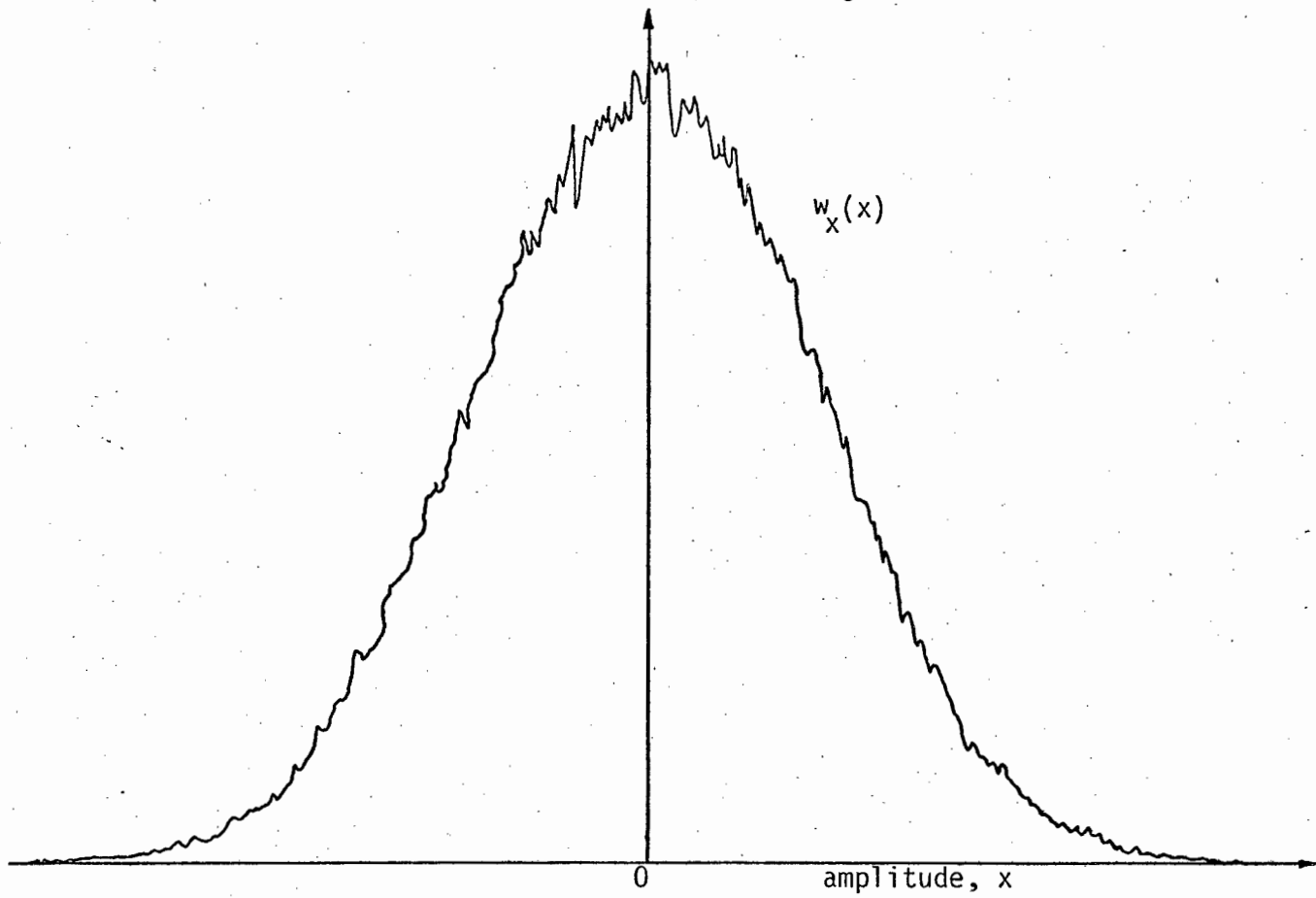
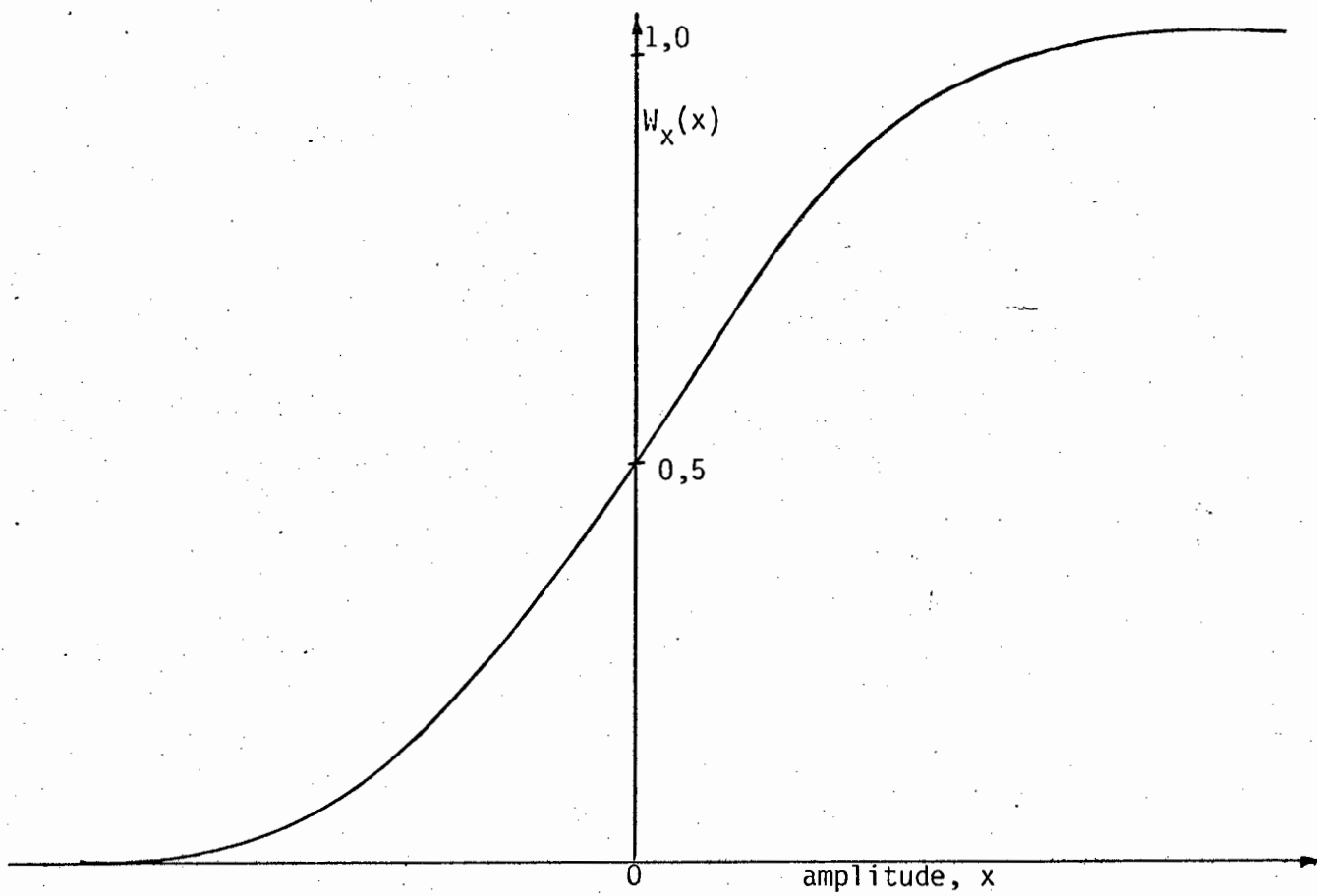


FIG. 9.5b Cumulative probability distribution function of flow signals



flow signal is plotted in Figure 9.6 for various flow velocities. As the flow-rate is reduced, so the width of the auto-correlation function is increased. This can be explained in terms of the inverse spreading action of a Fourier transform pair. As the flowrate decreases so the bandwidth of the flow signals decreases and as a result the auto-correlation function becomes wider. Consider for example, a signal having a flat power spectrum of the form:

$$G_X(f) = \begin{cases} k & f \leq B \\ 0 & f > B \end{cases}$$

The auto-correlation of this signal is of the form:

$$R_{XX}(\tau) = \frac{\sin(2\pi B\tau)}{2\pi B\tau}$$

The first zero of $R_{XX}(\tau)$ occurs at $\tau=1/2B$ and the width of the auto-correlation function is thus inversely proportional to the signal bandwidth. In Figure 9.7, the reciprocal of auto-correlation peak width, $1/\tau_w \propto B$, is plotted as a function of flow velocity. As was evident in Figure 9.3, a linear relationship again appears to exist between signal bandwidth and flow velocity.

4.2 Cross-correlation Functions for Water

(a) Effect of varying sensor spacing at fixed flow velocity

It was shown in Chapter 2 that the flow of fluid in a pipe has a non-frozen pattern of turbulence. The flow signals measured at the upstream and downstream sensors will not be identical and as a result the normalized cross-correlation function does not have a maximum value of unity. As the sensor spacing is increased, the flow signals become less similar and the cross-correlation function peak value decreases. To illustrate this effect, a series of cross-correlation functions at different sensor spacings but constant flow rate are shown in Figure 9.8a. Due to the physical size of the mounts, the transducers could not be separated by less than 5.1 cm.

In Figure 9.8b, the cross-correlation peak positions are plotted against sensor spacing for three flow velocities. A linear relationship that passes through the origin exists. This is to be expected since $d = v \cdot \tau_p$ and for fixed velocity, the spacing, d , is directly proportional to the peak position, τ_p .

The envelopes of the cross-correlation functions are shown in Figure 9.8c for two flow rates. These envelopes are the moving frame auto-correlation functions, i.e., the auto-correlation functions that would be measured in a

FIG. 9.7a Plot of auto-correlation functions for different flow rates

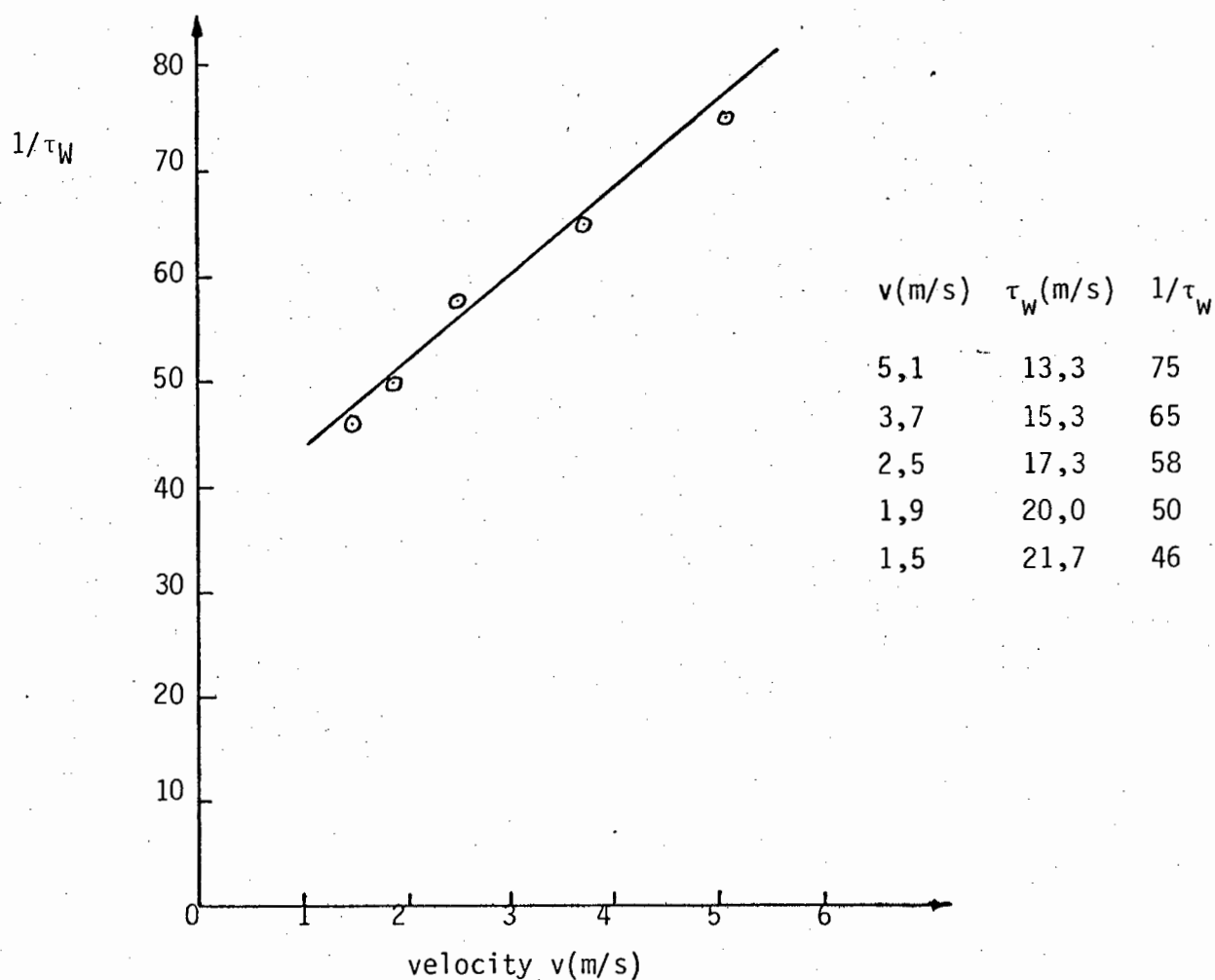
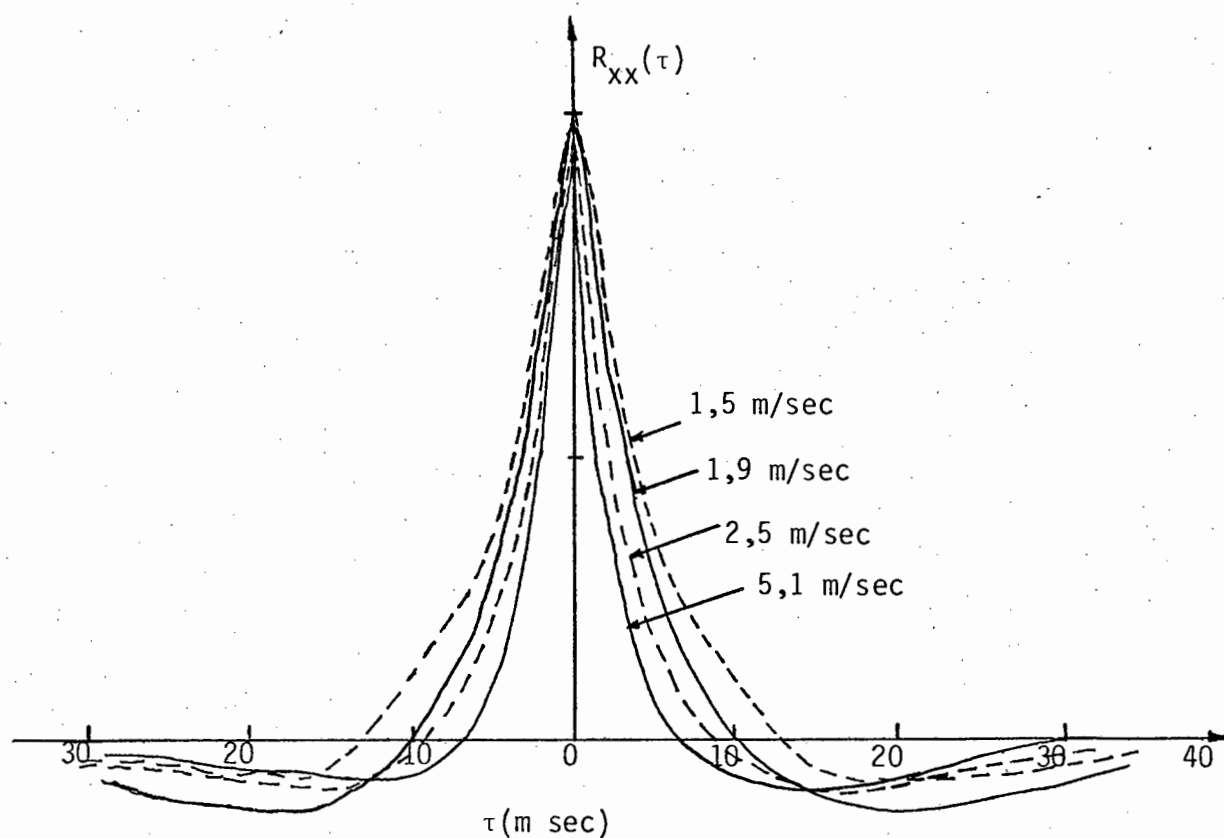


FIG. 9.7b Plot of reciprocal of auto-correlation peak width vs flow velocity.

FIG. 9.8a Plot of cross-correlation functions as function of sensor spacing for fixed flow velocity (water; 10 cm PVC pipe)

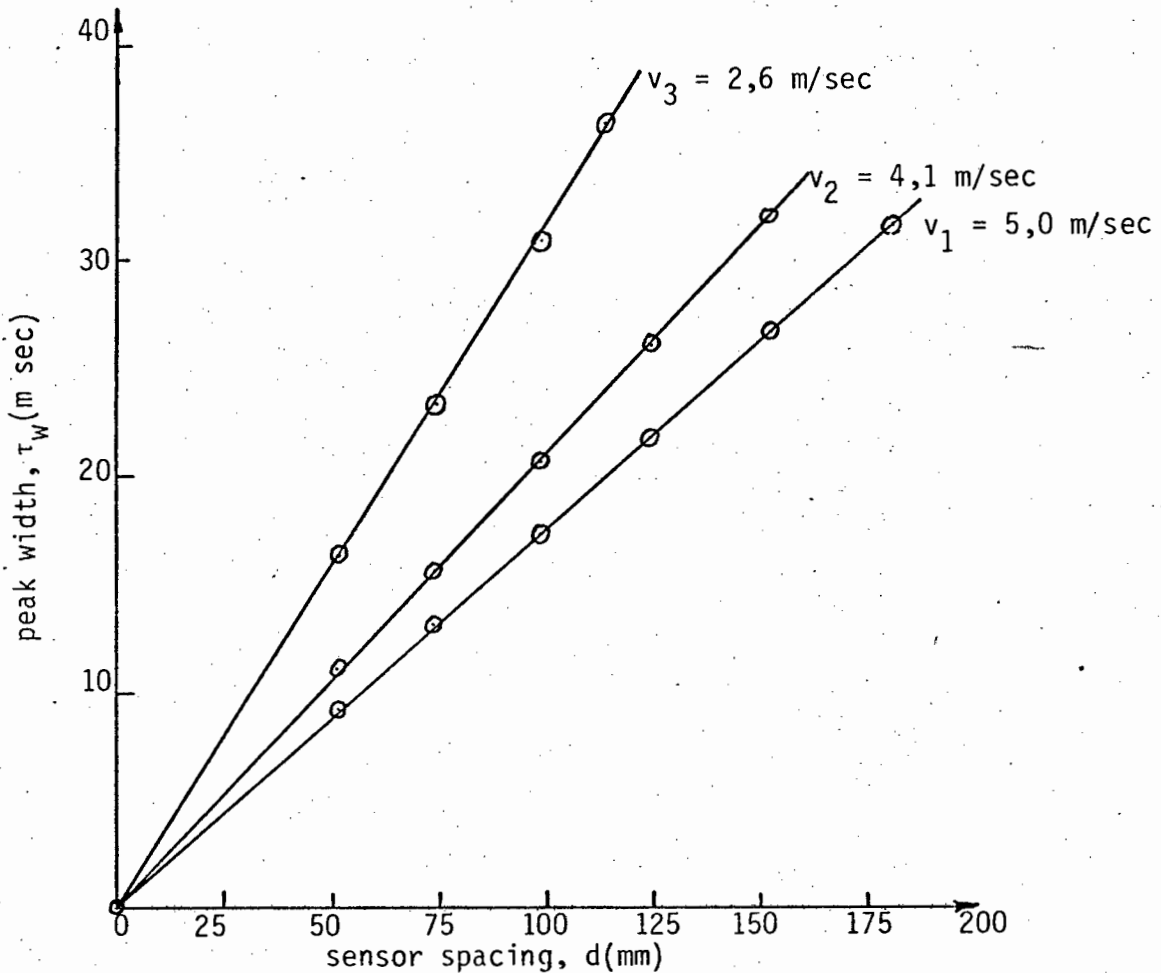
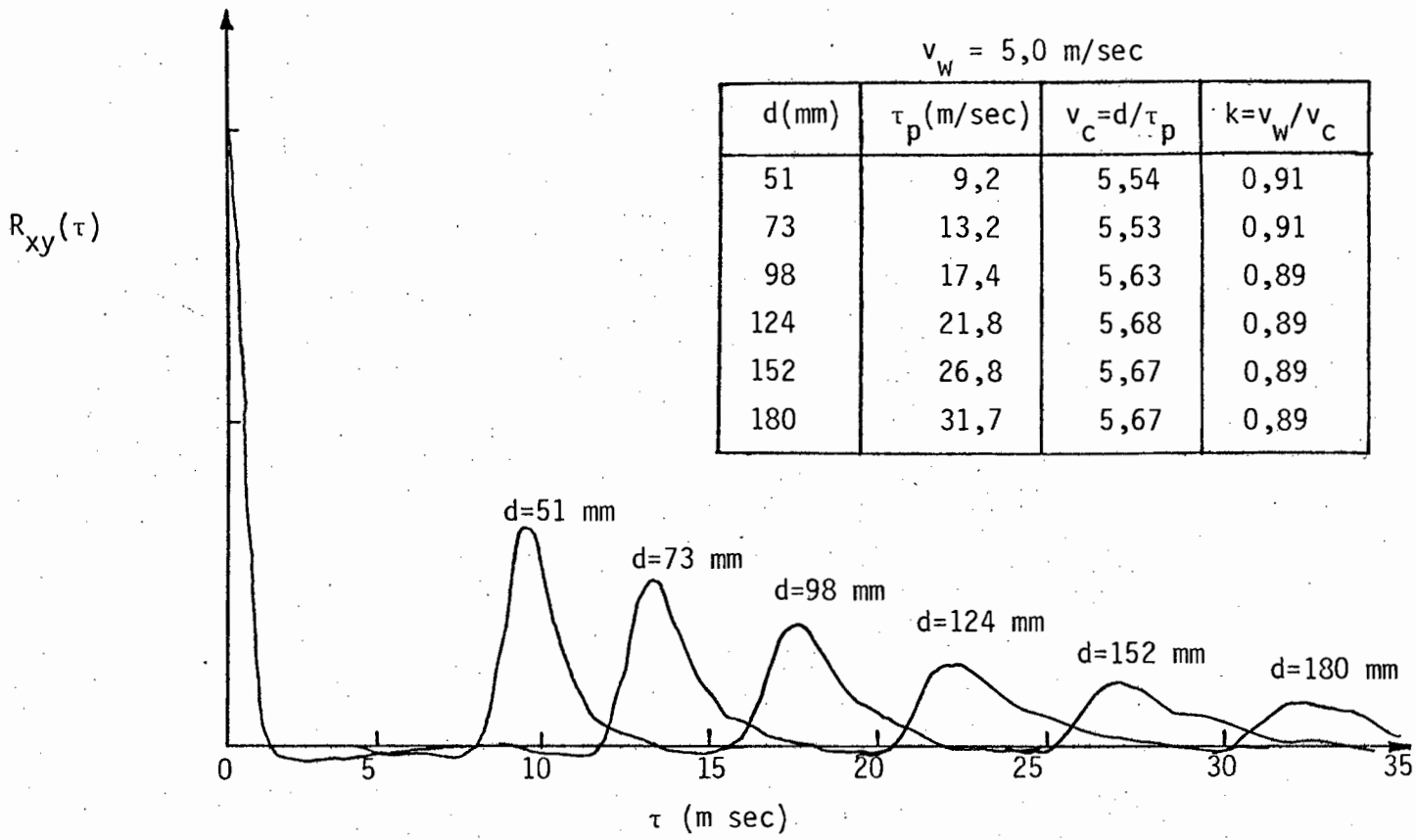
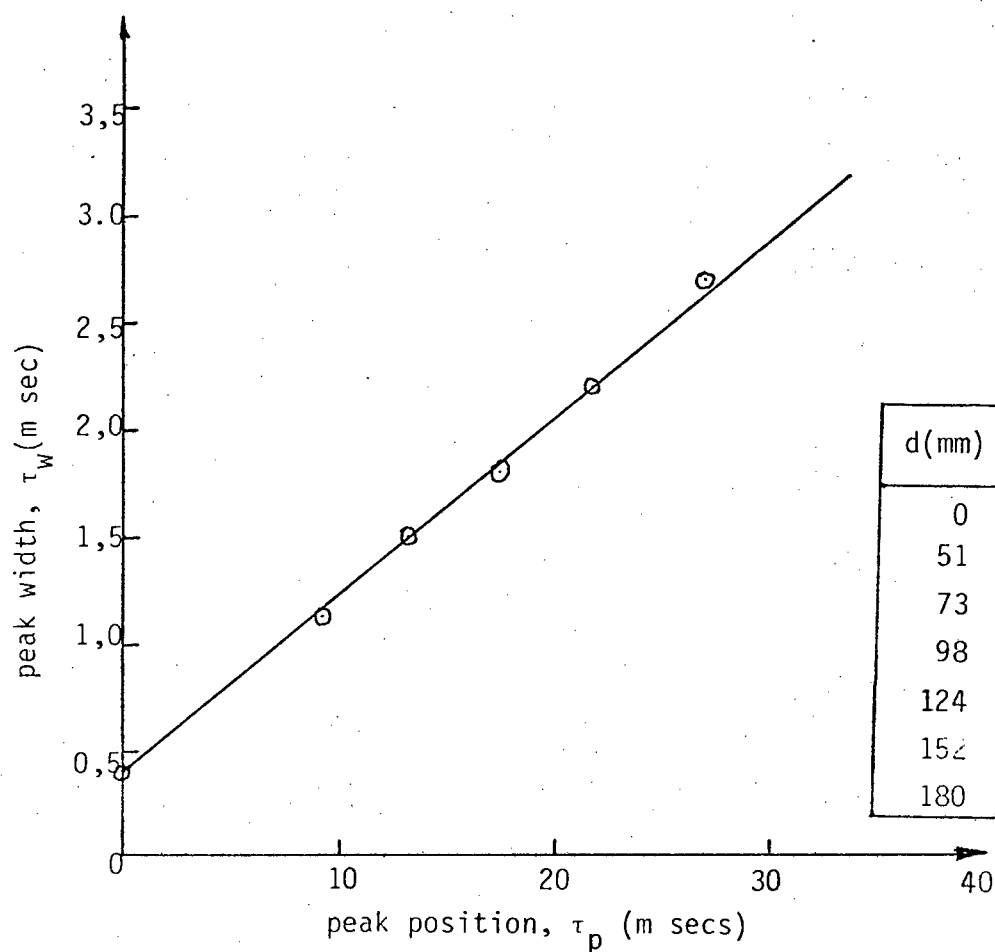
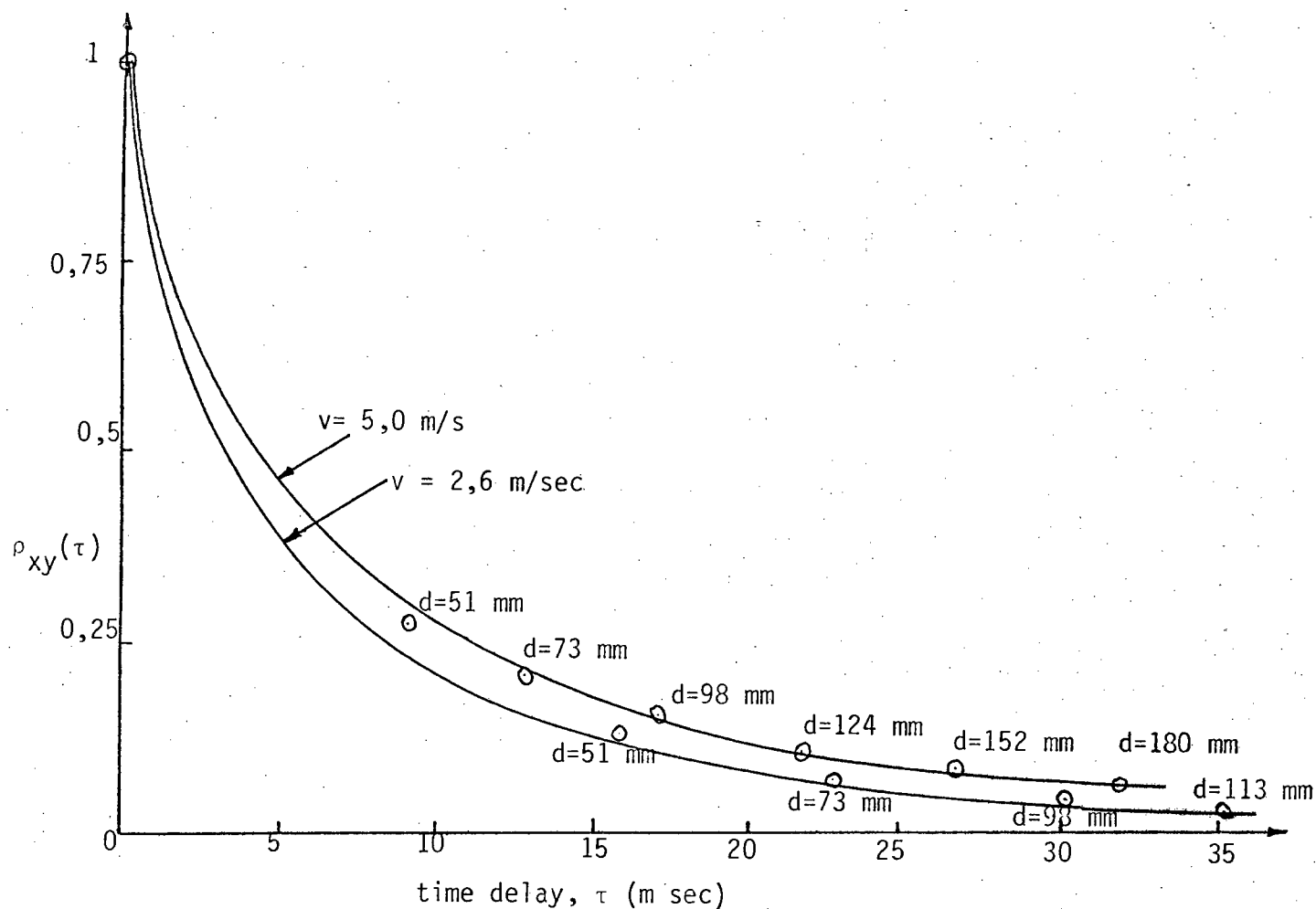


FIG. 9.8b Plot of peak position vs sensor spacing for fixed flow velocity

FIG. 9.8c Plot of moving frame auto-correlation function for two flow velocities



d (mm)	τ_p (m sec)	τ_w at $\frac{2}{3} R_{XY}(\tau_p)$
0	0	0,4
51	9,2	1,2
73	13,2	1,5
98	17,4	1,8
124	21,8	2,2
152	26,8	2,7
180	31,7	3,6

FIG. 9.8d Plot of cross-correlation peak width vs peak position at constant flow rate

frame of reference moving at the convection velocity of the turbulent eddies. In this frame of reference, the decay of the eddies with time is a minimum, thus yielding the widest possible auto-correlation function and is due only to the change in the structure of the turbulence due to shear stresses.

In a stationary frame of reference on the other hand, the eddies are convected passed the observer and this leads to a much narrower auto-correlation function. The time-delay at which the moving frame auto-correlation function decays to $1/e$ of its initial value gives the eddy life-time. From Figure 9.8c it appears that the eddy lifetime is longer at high flow velocities than at lower flow velocities.

The widths of the cross-correlation functions at a height of $1/3$ the peak value of the particular correlation function are plotted against peak position in Figure 9.8d. As the sensor spacing increases (i.e., increased time delay), the correlation peaks become wider and a linear relationship appears to exist between peak width and peak position at fixed flowrate. This increase in peak width has been discussed in Chapter 2 and is due to velocity profile effects. In addition, the cross-correlation peaks are not symmetrical and exhibit a tail on their right-hand sides. This again is due to velocity profile effects since the transit-time of fluid between upstream and downstream sensors varies depending on the radial position of the fluid.

(b) Effect of varying flow velocity at fixed sensor spacing

In the following set of results, the sensor spacing is fixed and the flowrate is varied. This is the mode in which an actual correlation flow-meter would operate. Typical cross-correlation curves at a particular sensor spacing are shown in Figure 9.9a. Again, due to the non-frozen nature of the turbulence, the peak values decrease with reduced flow velocity. As the transit time of the flow patterns between sensors increases, the flow patterns undergo greater change resulting in a smaller degree of correlation.

The correlation flow velocity, $v_c = d/\tau_p$, is plotted against the weightank flow velocity, v_w , in Figure 9.9b. As expected a linear relationship passing through the origin is seen to exist. Ideally the slope of this line should be 45° , that is, the ratio $k = v_w/v_c = 1$. However, due to velocity profile and linear averaging effects (see Chapter 10), a ratio of $v_w/v_c = 0.89$ is obtained. The result of Figure 9.9b confirms that it is possible to measure flowrate accurately via the correlation technique.

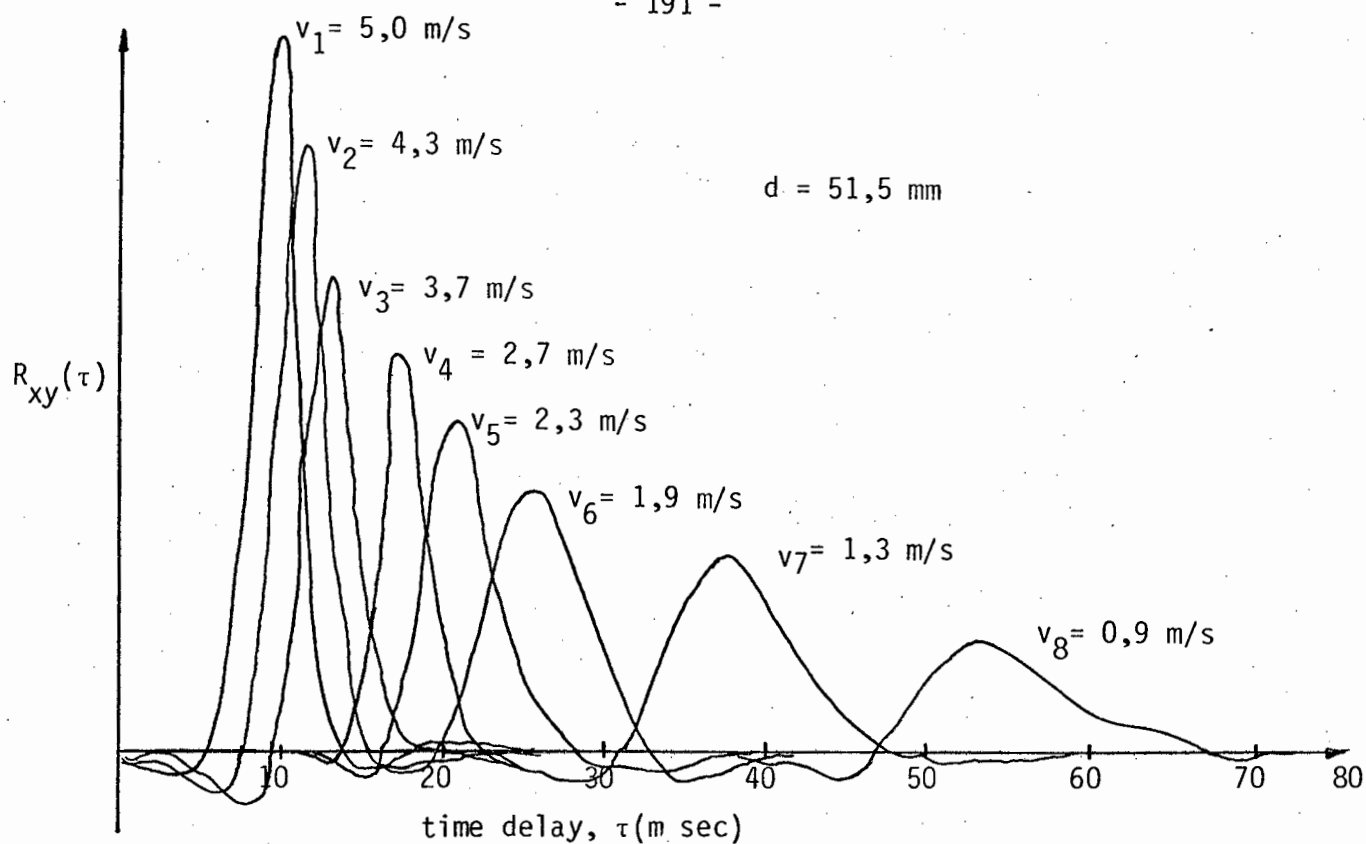


FIG. 9.9a Plot of cross-correlation functions vs flow velocity for fixed sensor spacing

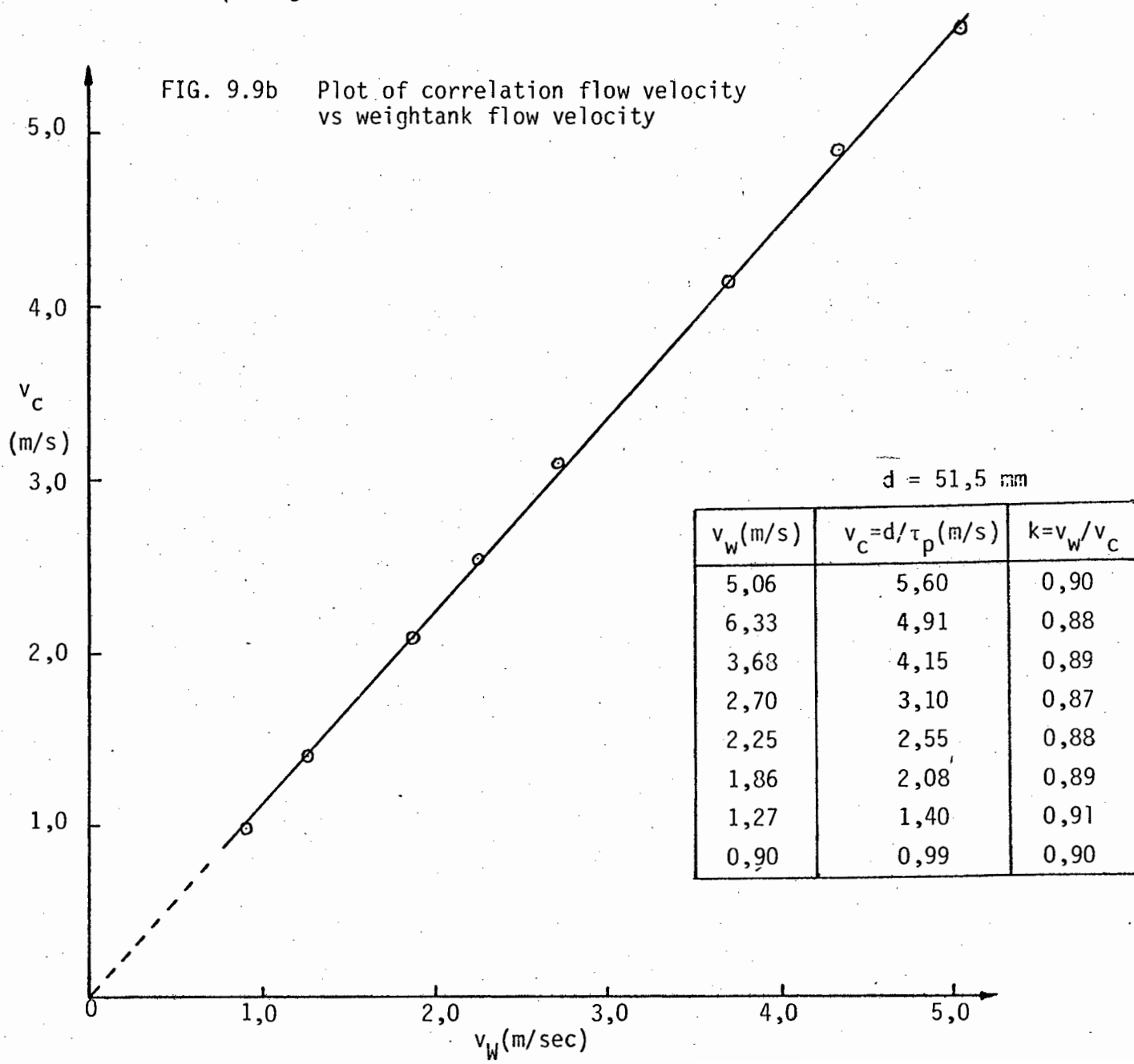


Figure 9.9c shows a number of cross-correlation peaks shifted to zero delay. The spreading (i.e., widening) of the peaks with decreasing flow velocity is clearly evident and an approximately linear relationship between peak width and flowrate exists (Figure 9.9d).

(c) Effect of varying signal bandwidth

The effect of flow signal bandwidth on cross-correlation functions was investigated next. The reason for this is that in non-frozen turbulent flow, the various turbulent components do not all necessarily travel at the same velocity.

In Chapter 2 it was shown that the Fourier transform of a cross-correlation function, $R_{xy}(\tau)$, gives the cross-spectral density function, $G_{xy}(f)$. This is a complex function and can be expressed as:

$$G_{xy}(f) = C_{xy}(f) - j.Q_{xy}(f)$$

where $C_{cy}(f)$ is the co-spectrum and

$Q_{xy}(f)$ the quad-spectrum (see Reference 245)

Rewriting in Polar form gives:

$$G_{xy}(f) = |G_{xy}(f)| \cdot e^{-j \theta_{xy}(f)}$$

$$\text{where } |G_{xy}(f)| = \sqrt{C_{xy}^2(f) + Q_{xy}^2(f)}$$

$$\text{and } \theta_{xy}(f) = \arctan [Q_{xy}(f)/C_{xy}(f)]$$

It is now possible to define a coherence function

$$\gamma_{xy}^2(f) = |G_{xy}(f)|^2 / G_x(f) \cdot G_y(f)$$

where $G_x(f)$ and $G_y(f)$ are the power spectral density functions of the signals $x(t)$ and $y(t)$ respectively. The coherence function is always less than or equal to one and gives the degree of coherence (c.f. correlation) between the components of the signals $x(t)$ and $y(t)$ at the frequency, f .

The function of interest in this section is the phase term, $\theta_{xy}(f)$. This phase is the phase lead of signal $x(t)$ relative to $y(t)$. The time delay between these signals can now be written as a function of frequency:

FIG. 9.9c Plot of shifted cross-correlation functions for various flow velocities

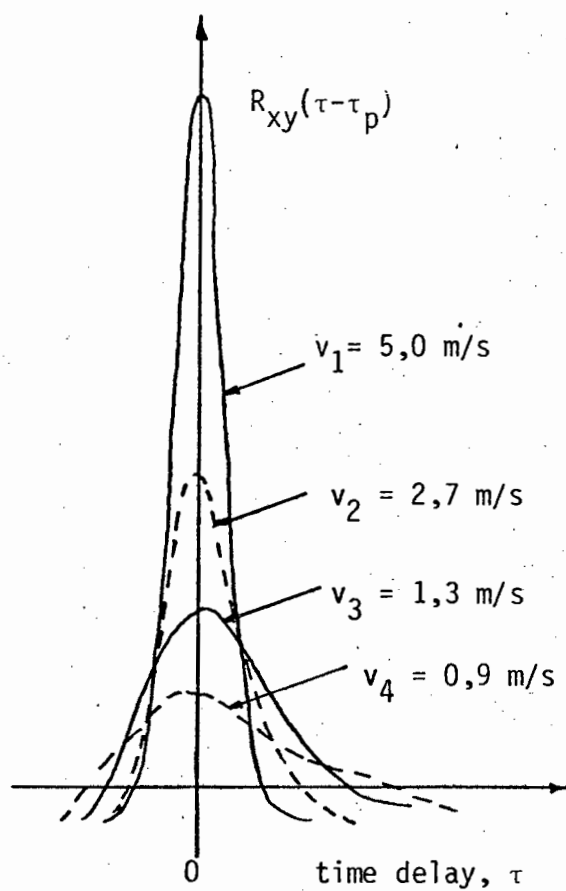
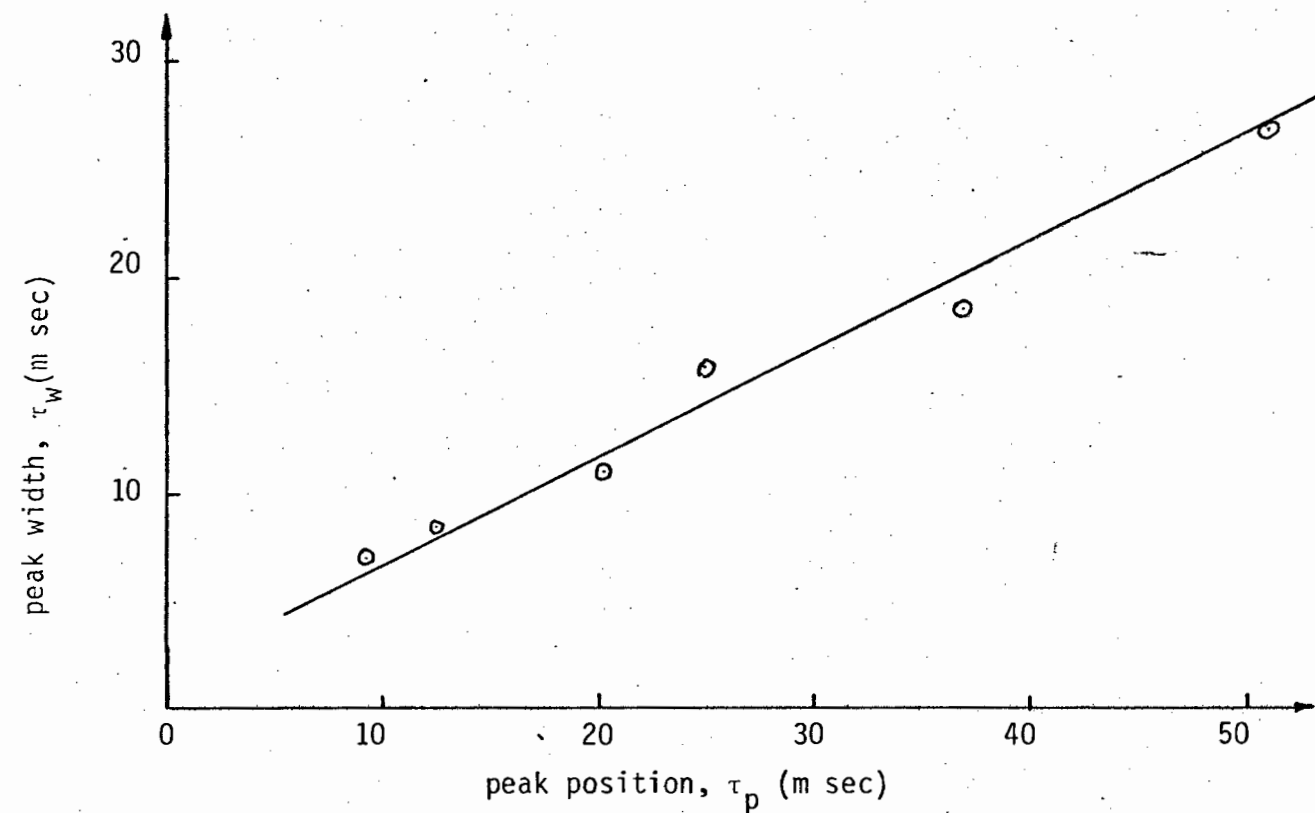


FIG.9.9d Plot of cross-correlation functions peak width vs peak position for fixed sensor spacing



$$\tau(f) = \theta_{xy}(f)/2\pi f$$

and the convection velocity of the turbulent components is:

$$v(f) = d/\tau(f) = 2\pi df/\theta_{xy}(f)$$

If the phase shift $\theta_{xy}(f)$ varies linearly with frequency, i.e., $\theta_{xy}(f) = k.f$ as is the case for a pure time delay, then $v(f) = \frac{2\pi d}{k} = V_0$ which is constant and independent of frequency. This corresponds to frozen-pattern turbulent flow. For non-frozen turbulent flow, such a linear relationship does not exist and different turbulent components travel at different velocities.

To investigate the effect of receiver bandwidth on cross-correlation functions, the flow signals were limited in frequency to approximately 30 Hz, 60 Hz, 200 Hz and 500 Hz. Figure 9.10 shows the auto-correlation functions of the filtered signals all taken at the same flow rate. The width of the autocorrelation curves, clearly is related inversely to the signal bandwidth.

In Figure 9.11a-d, cross-correlation curves at fixed flowrate but varying sensor spacing are shown for four different signal bandwidths. Using these curves, the cross-correlation peak positions have been plotted against sensor spacing in Figure 9.12a. In all cases, a linear relationship that passes through the origin exists. The slopes of the lines, however, vary and are larger for smaller bandwidths. The mean calibration factors have been calculated and are also shown in Figure 9.12a. From these results it can be concluded that the low frequency turbulent components (small wave-number) travel slower than the higher frequency turbulent components since as the bandwidth is increased so the calibration factor becomes smaller. This result is in agreement with that of FISHER and DAVIES (Reference 147). Instead of low-pass filtering they band-pass filtered the flow signals and their results indicated an approximately linear rise in convection velocity with frequency.

The decay of the flow patterns was determined, hereafter, by plotting the moving-frame auto-correlation functions. Figure 9.12b shows that the lower frequency components have a longer life-time, i.e., decay more slowly, than the higher frequency turbulence components. This is because the shear stresses are larger for the high frequency components (large wave-number), resulting in greater distortion.

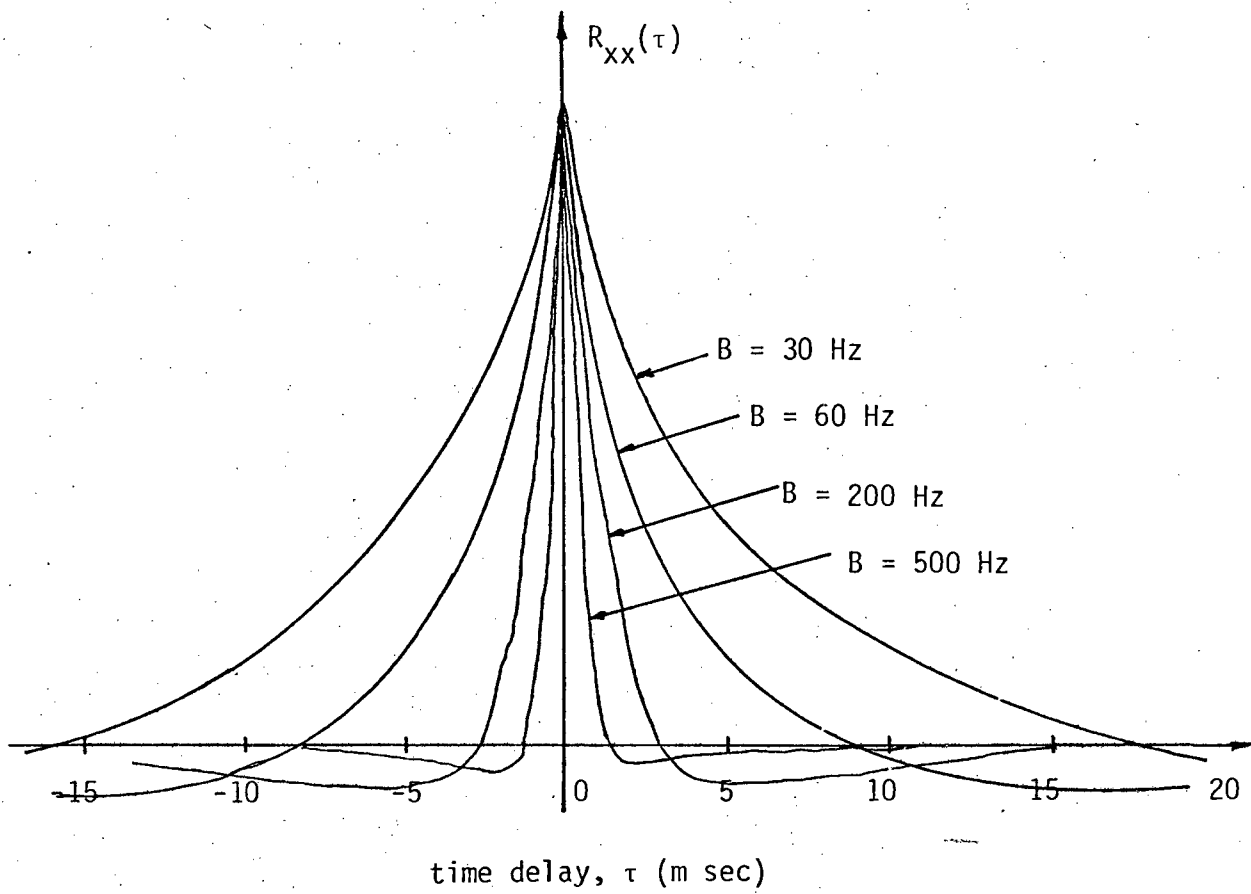
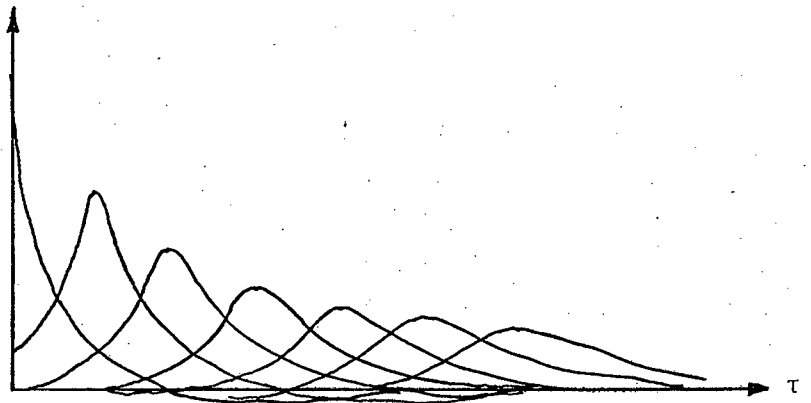


FIG. 9.10 Plot of auto-correlation functions of flow signals at fixed flow velocity but different amplifier bandwidths

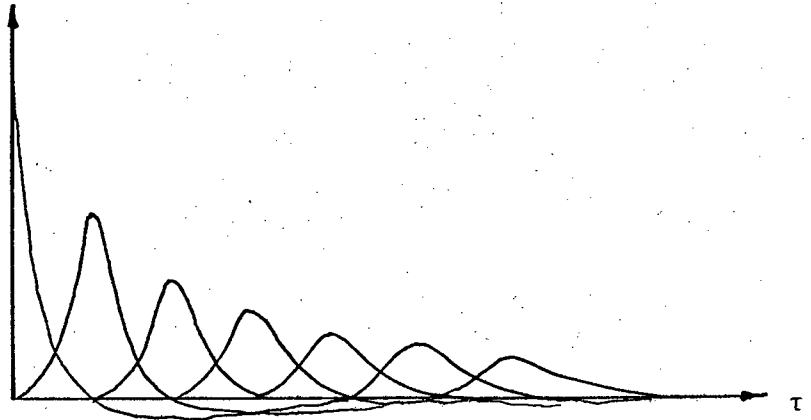
a) $B=30$ Hz

$R_{xy}(\tau)$



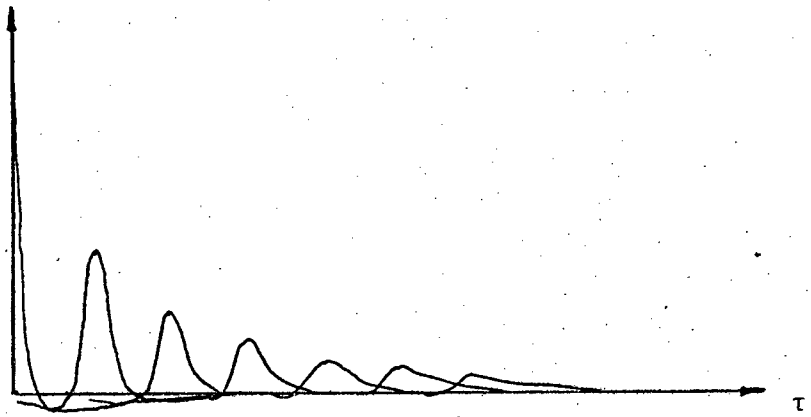
b) $B = 60$ Hz

$R_{xy}(\tau)$



c) $B = 200$ Hz

$R_{xy}(\tau)$



d) $B = 500$ Hz

$R_{xy}(\tau)$

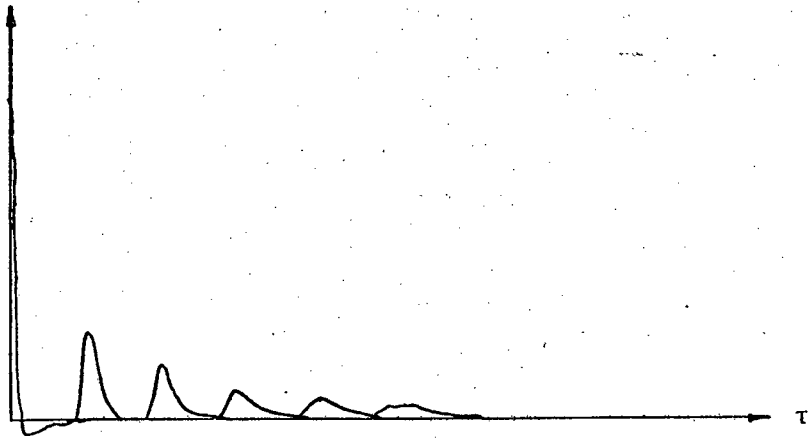


FIG. 9.11 Plot of cross-correlation functions for different sensor spacing at fixed flow velocities

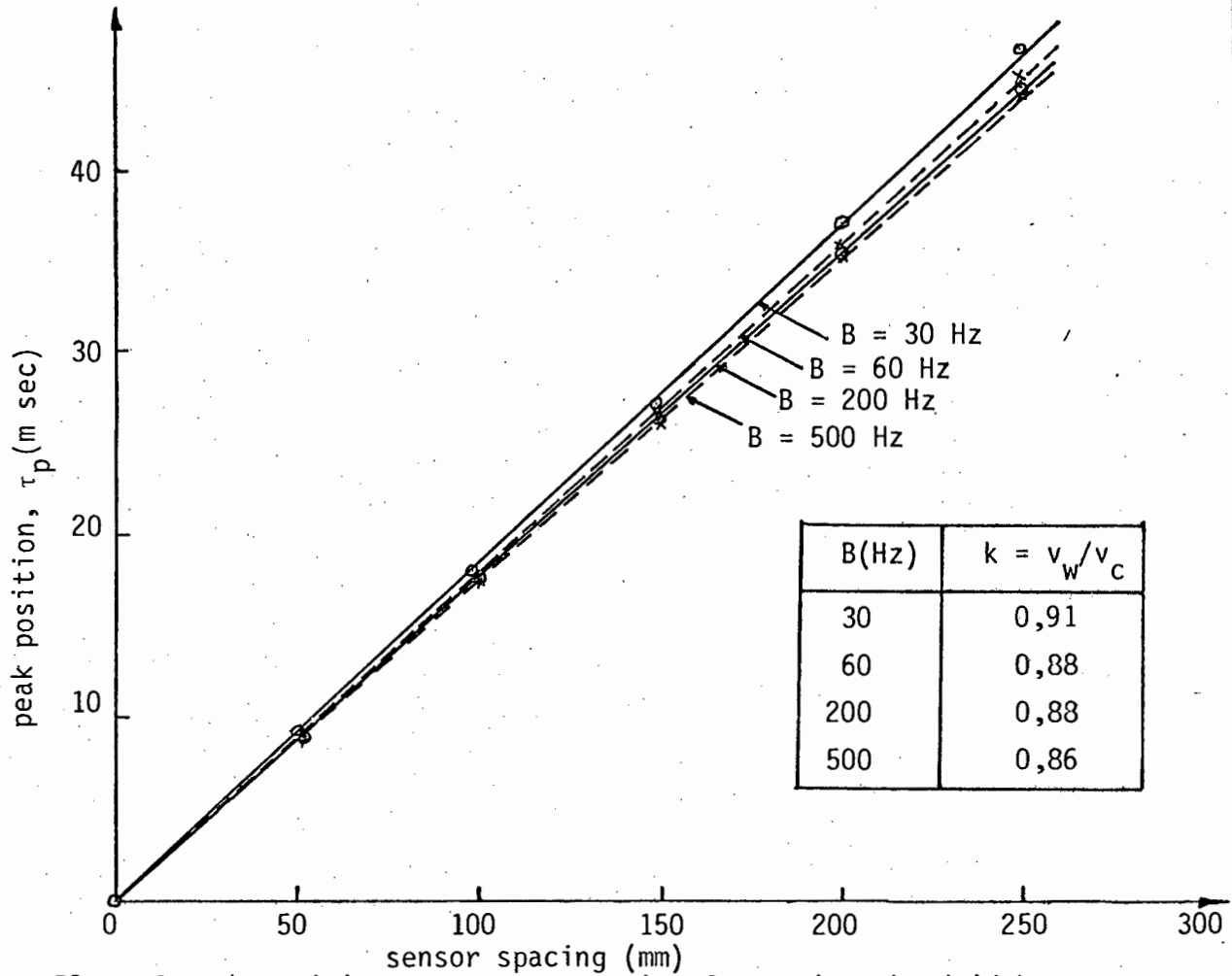


FIG. 9.12a Plot of peak position vs sensor spacing for various bandwidths

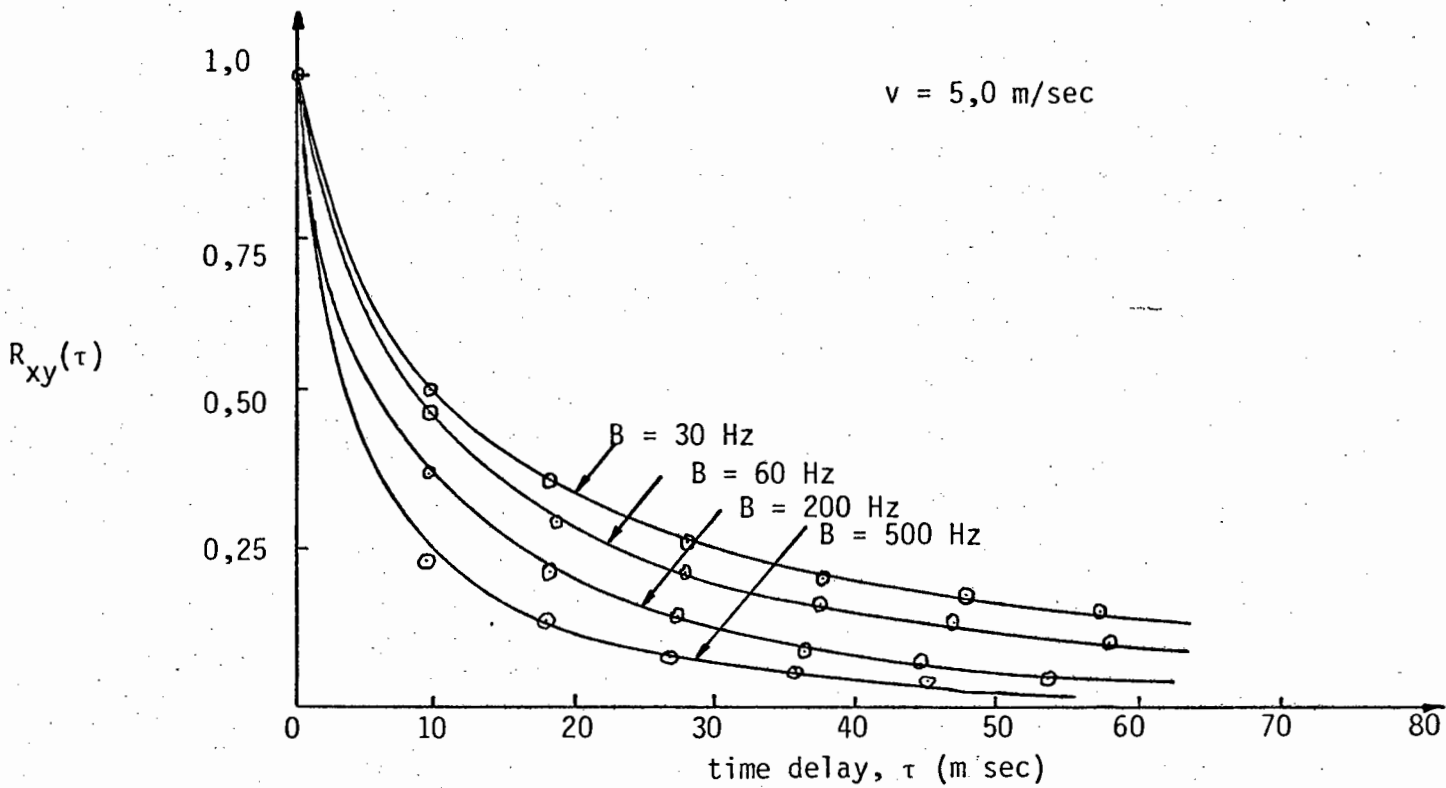


FIG. 9.12b Plot of moving frame auto-correlation functions for different bandwidths at fixed flow velocity

In Figure 9.13a, cross-correlation functions measured at the same sensor spacing and velocity have been superimposed for various signal bandwidths. These curves illustrate clearly the manner in which a decrease in bandwidth produces both an increase in cross-correlation peak height and width. The peak widths have been plotted again as a function of sensor spacing (Figure 9.13b) and indicate a linear relationship.

4.3 Cross-correlation functions with sediment

The previous Section has shown that good results can be obtained for the flow of pure water. Since the main aim is to measure slurry flows, sand was added to the water. Ideally mining slurry should have been used but as large quantities were not available, sand was an adequate substitute. The particle-size distribution of the sediment is 80%, <1mm and concentrations of about 10 per cent by weight were used.

Figure 9.14a-c, show typical cross-correlation curves for different sensor spacing at fixed flow velocities. The peak positions are plotted against sensor spacing in Figure 9.15a and the moving-frame auto-correlation functions in Figure 9.15b. The results exhibit the same characteristics as for the flow of water. A comparison between the moving frame auto-correlation functions for water and water plus sediment at the same flow rate is shown in Figure 9.16. The main feature to note is that the decay rate of the flow patterns is much smaller with sediment included than it is for pure water. Since in the former case, not only the turbulent eddies but also the sediment particles produce flow signals, a more distinct flow pattern is available.

The effect of varying the flow velocity for a fixed sensor spacing is shown in Figure 9.17a and a plot of correlation flow velocity versus weightank velocity is given in Figure 9.17b. The correlation peaks again are more distinct and decay at a slower rate than is the case for water flow.

In conclusion it is clear that better results will be obtained for two or three phase flows than for single phase flows.

FIG. 9.13a Diagram showing effect of bandwidth on shape of cross-correlation functions

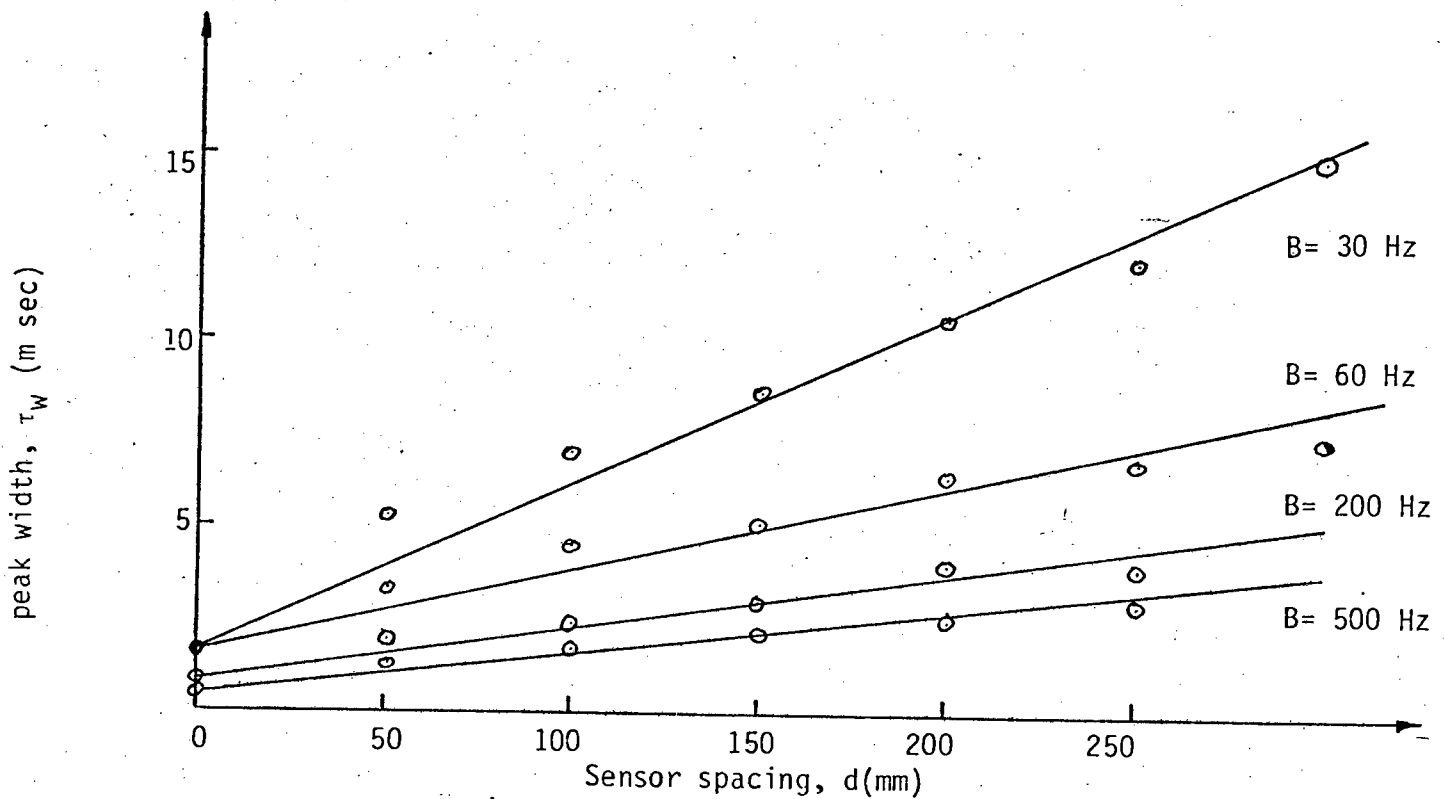
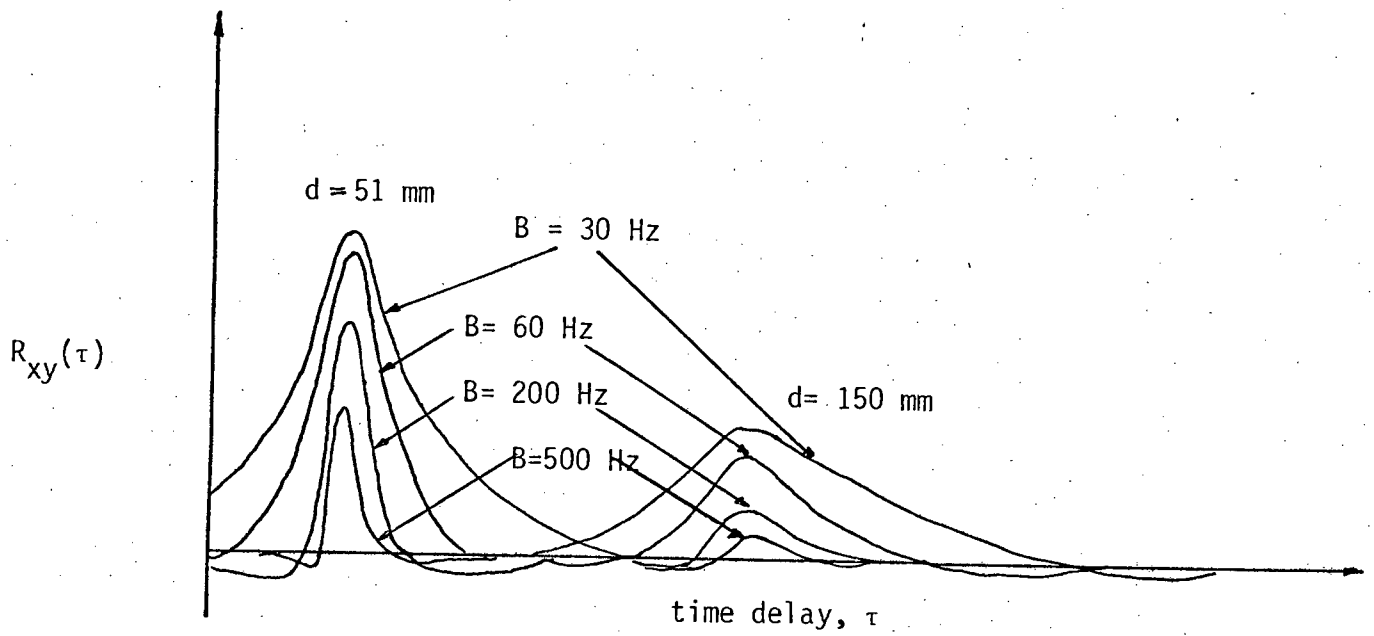
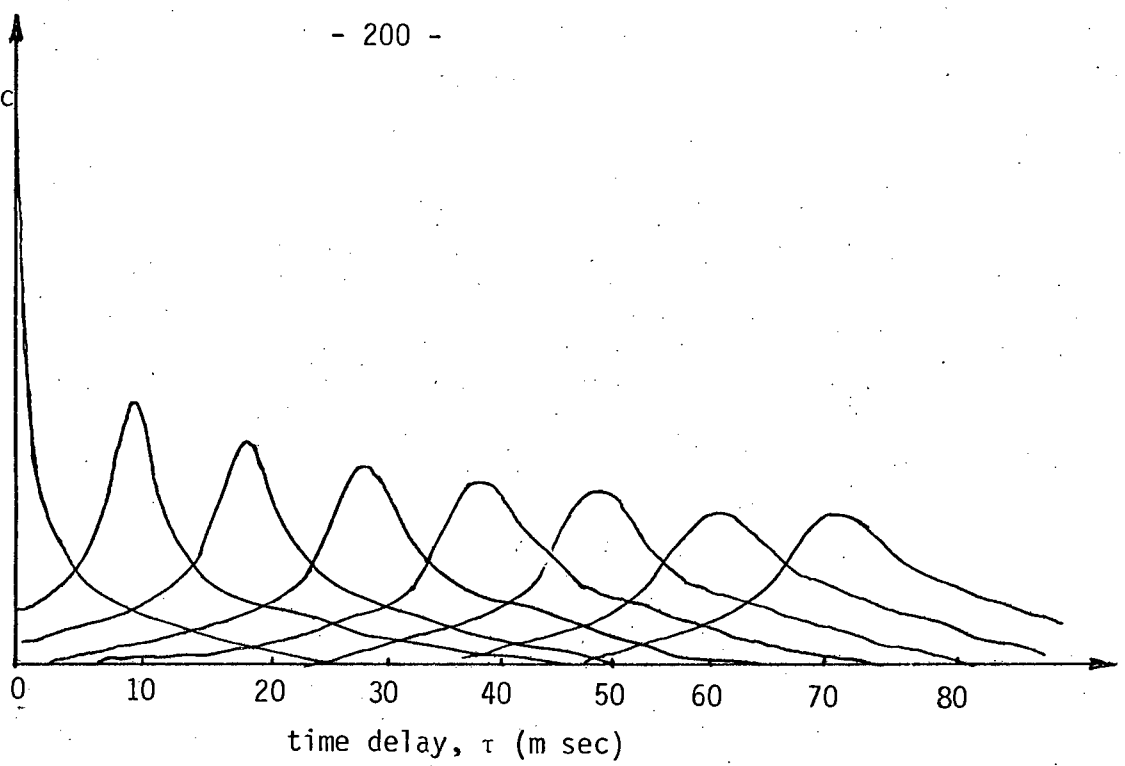


FIG. 9.13b Plot of cross-correlation function peak width vs sensor spacing for various bandwidths

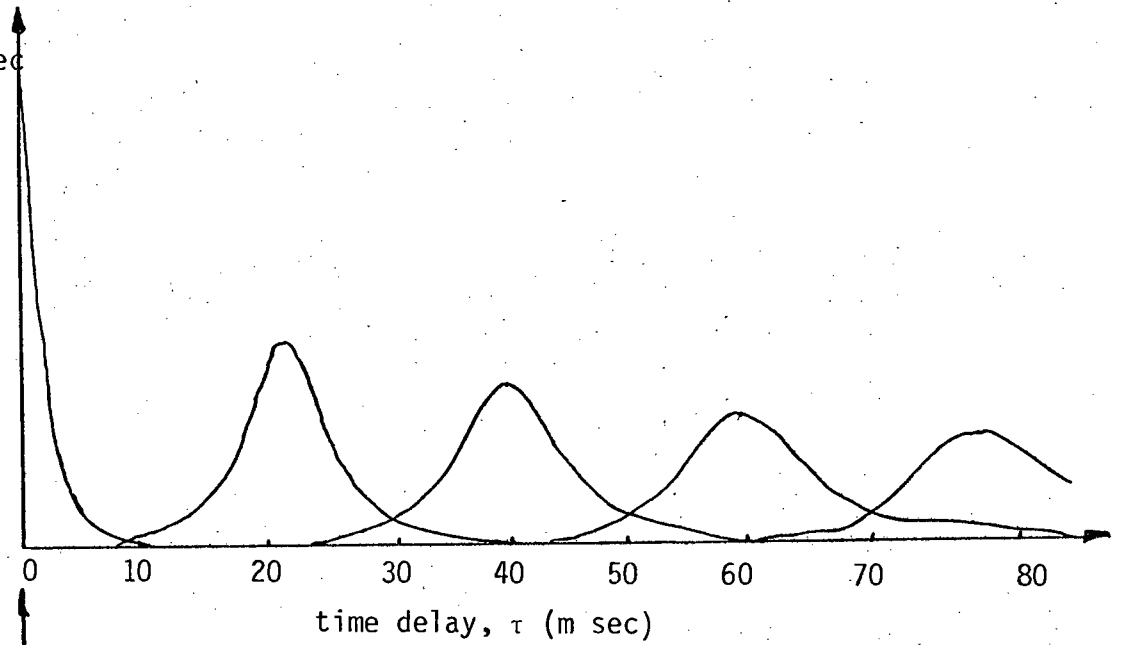
a) $v_1 = 5,2 \text{ m/sec}$

$R_{xy}(\tau)$



b) $v_2 = 2,6 \text{ m/sec}$

$R_{xy}(\tau)$



c) $v_3 = 1,4 \text{ m/sec}$

$R_{xy}(\tau)$

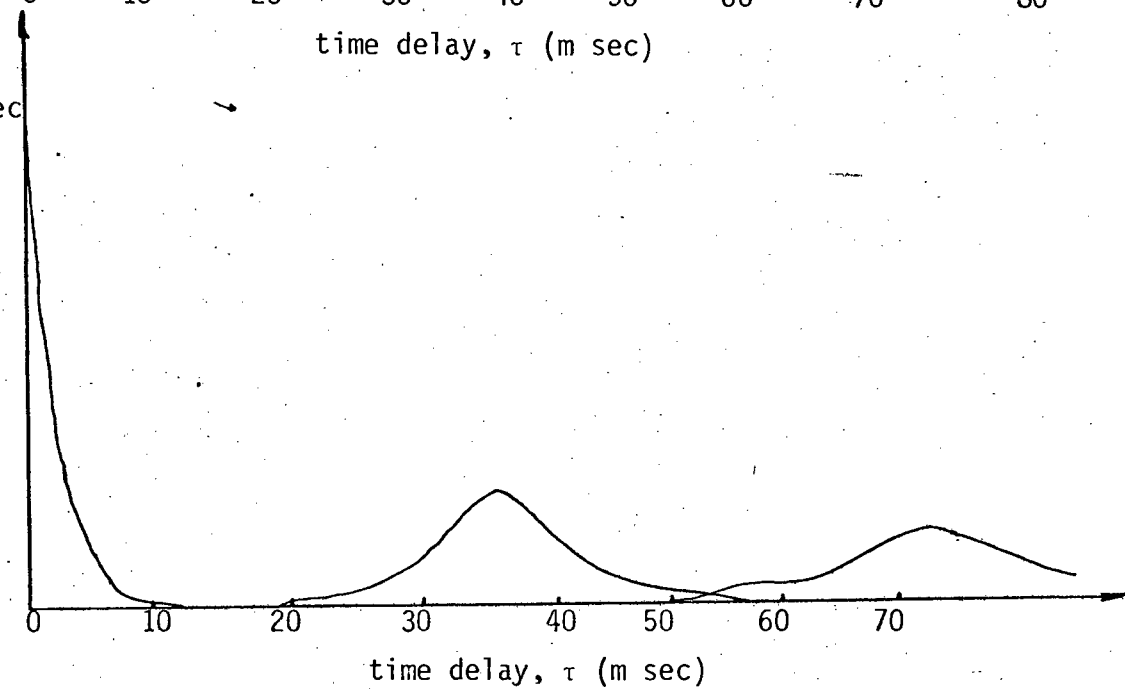


FIG. 9.14 Plot of cross-correlation functions for different sensor spacings at fixed flowrate for water + sediment

FIG. 9.15a Plot of correlation peak position vs sensor spacing (water plus sediment)

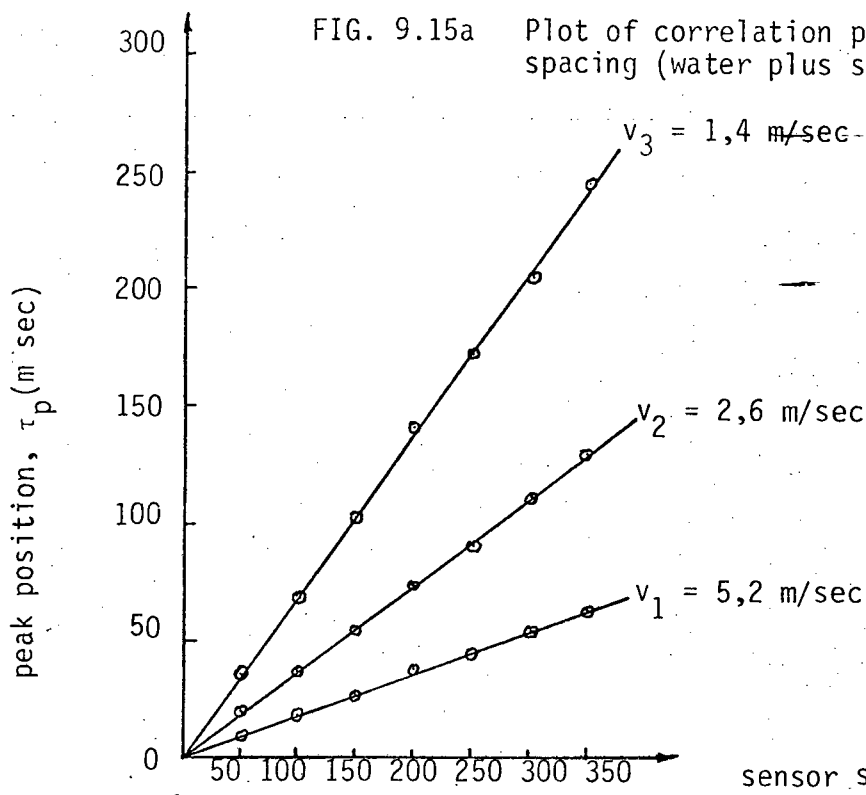


FIG. 9.15b Moving frame auto-correlation functions for different flow velocities (water plus sediment)

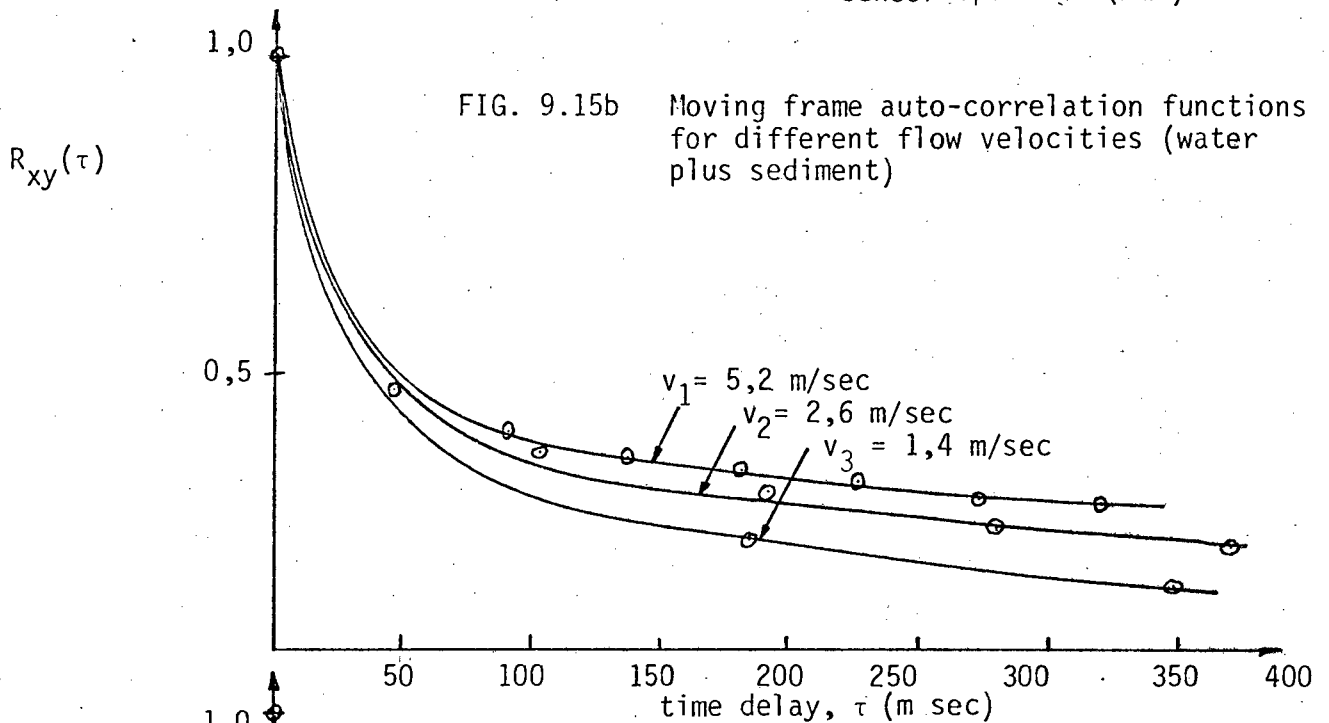
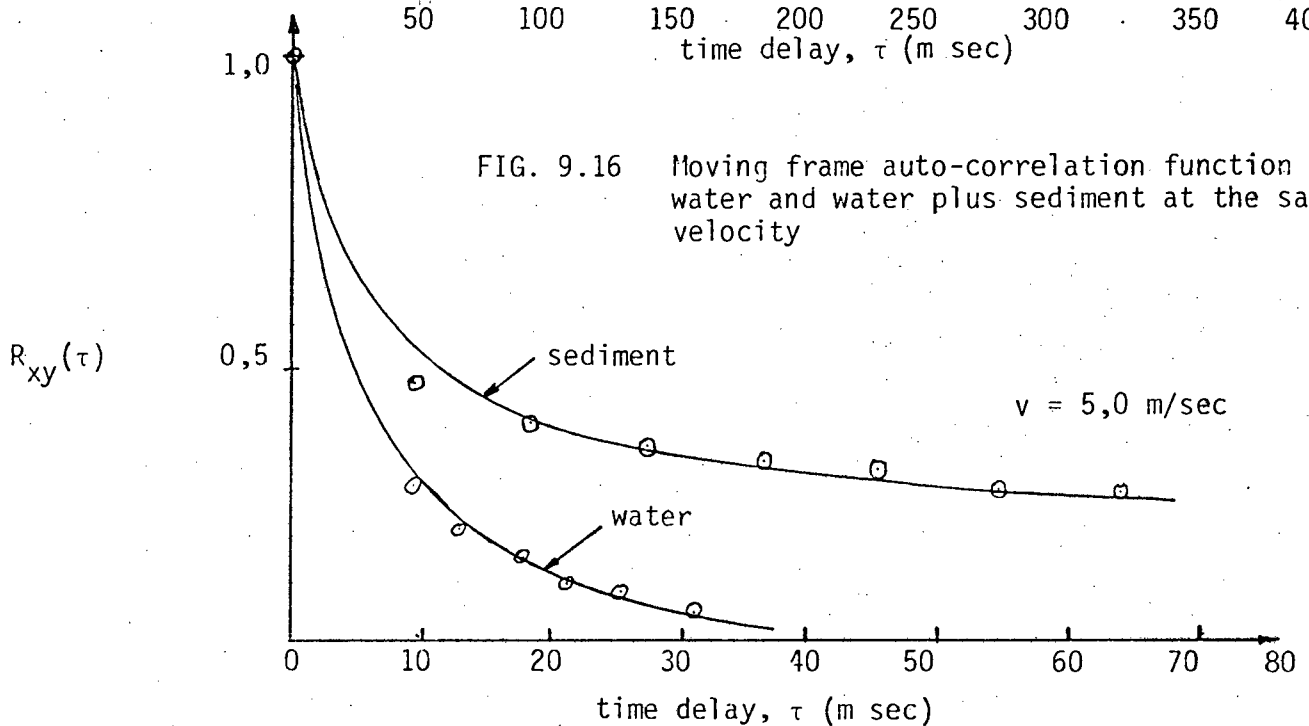


FIG. 9.16 Moving frame auto-correlation function for water and water plus sediment at the same flow velocity



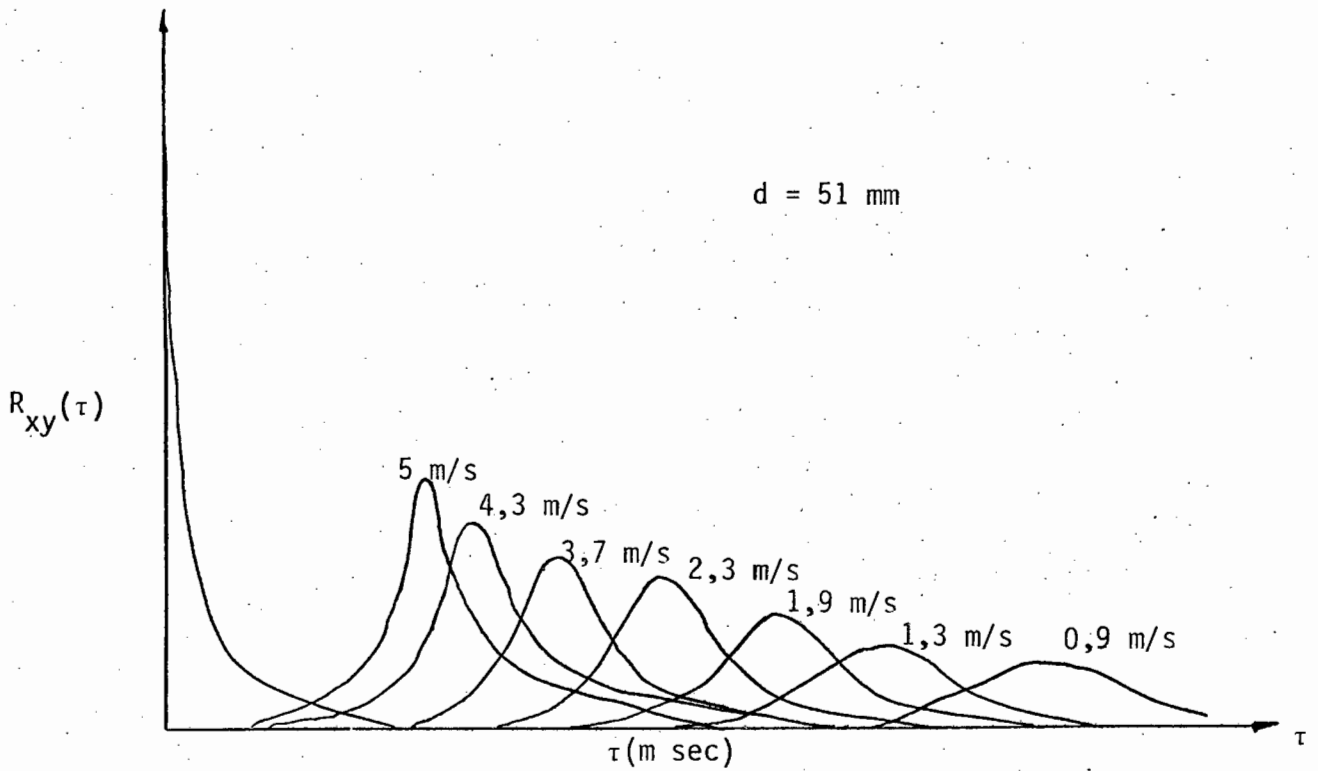


FIG.9.17a Effect of varying flow velocity for fixed sensor spacing (water plus sediment)

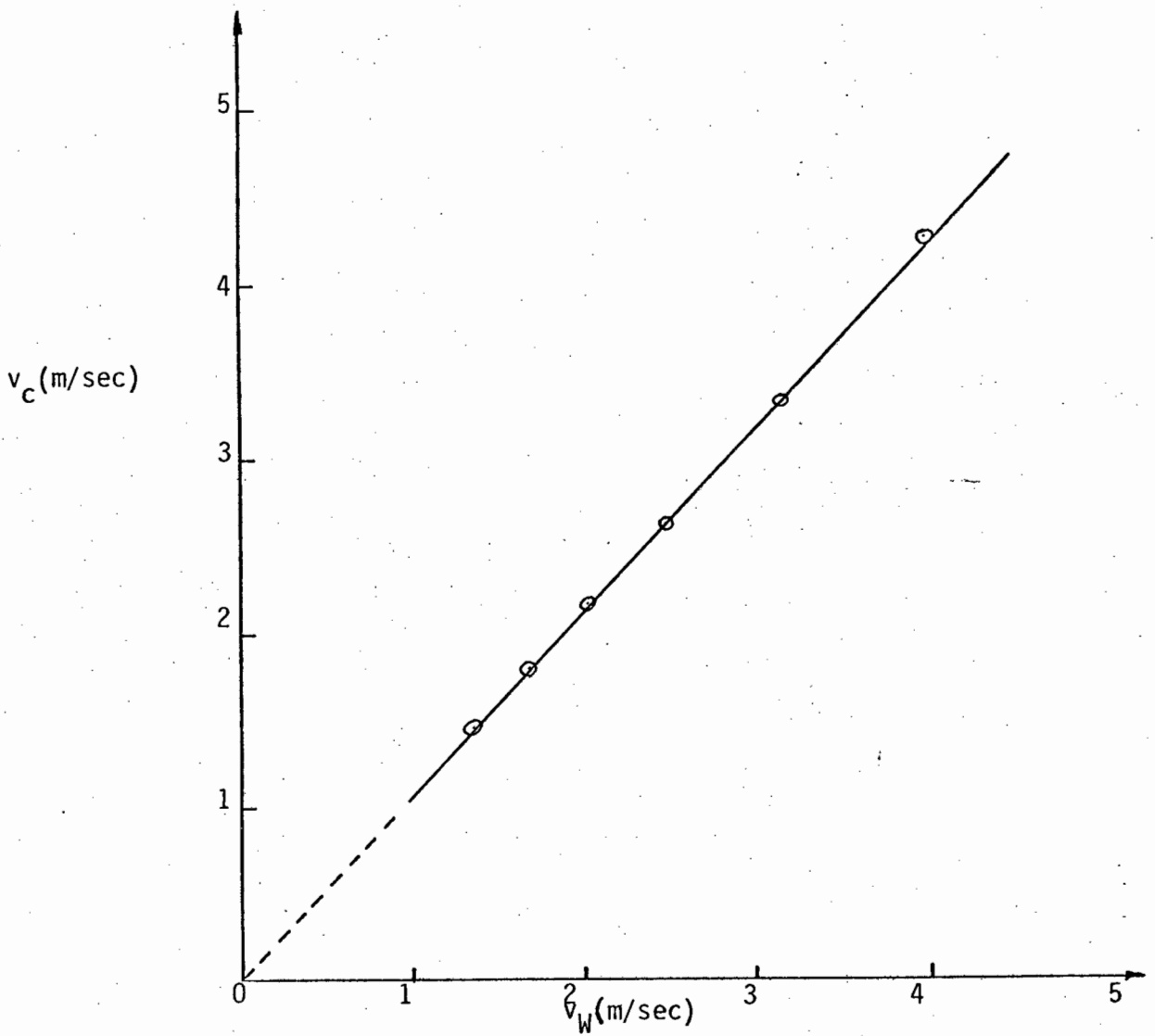


FIG. 9.17b plot of correlation flow velocity vs weightank flow velocity (water plus sediment)

SECTION B : EVALUATION OF DEDICATED CORRELATION FLOWMETERS

4. CALIBRATION OF CORRELATORS USING DELAY LINE

The correlation flowmeters that have been developed consist of two components, viz., flow sensors and a peak-seeking correlator. The performance of the correlators will firstly be evaluated using known input signals. Hereafter the characteristics of the correlators together with the flow sensors will be examined as a flowmeter.

To calibrate the peak-finding correlators a random signal was passed through an analogue delay line. The input and output of the delay line formed the two signals applied to the correlators as shown in Figure 9.18. The delay line time-delay could be varied from 10 m sec to 100 m sec in 10 m sec increments. It was found that the actual delay produced by the delay line did not equal exact the delay indicated by the device. A HONEYWELL SAICOR proprietary correlator was thus used to display the cross-correlation function and the peak position was obtained from this display. Since the SAICOR correlator is a digital instrument, the accuracy of the peak position is $\pm \frac{1}{2} \Delta t$ where Δt is the sampling interval used.

The output of both the micro-processor controlled and serial two-point difference correlators is a frequency that is inversely proportional to the correlation peak position. At each time delay, a number of output readings was taken using both a 1-sec and 10-sec counting interval on a frequency counter. The mean values and standard deviations were calculated for each input and the former are plotted against the reciprocal of peak position in Figure 9.19. The best fit line was calculated and is also shown in Figure 9.19. No non-linearity is evident in the calibration plot, any deviation from the straight line being due to scatter. This is to be expected since the correlator inherently is a linear device.

The correlation coefficient between input and output was calculated and yielded a value of $r = 0,9999$. This again confirms the good linearity of the correlators.

The precision or repeatability defined as $RP = 2 \sigma / \bar{x} \cdot 100\%$ is plotted against the output for both 1-sec and 10-sec counting times in Figure 9.20. The indicated percentages can be interpreted to mean that 95% (2σ) of the output readings lie within $RP1$ ($RP10$)% of the mean value for a counting interval of 1(10) secs. For a 1 sec count interval, the worst case repeatability is about $\pm 0,25\%$, whereas for a 10-sec interval this is reduced to $\pm 0,08\%$.

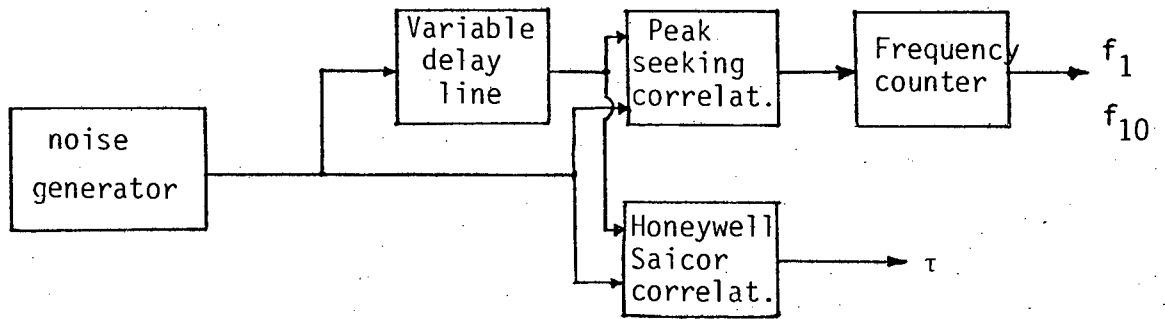


FIG. 9.18 Set-up for calibrating the peak-seeking correlators

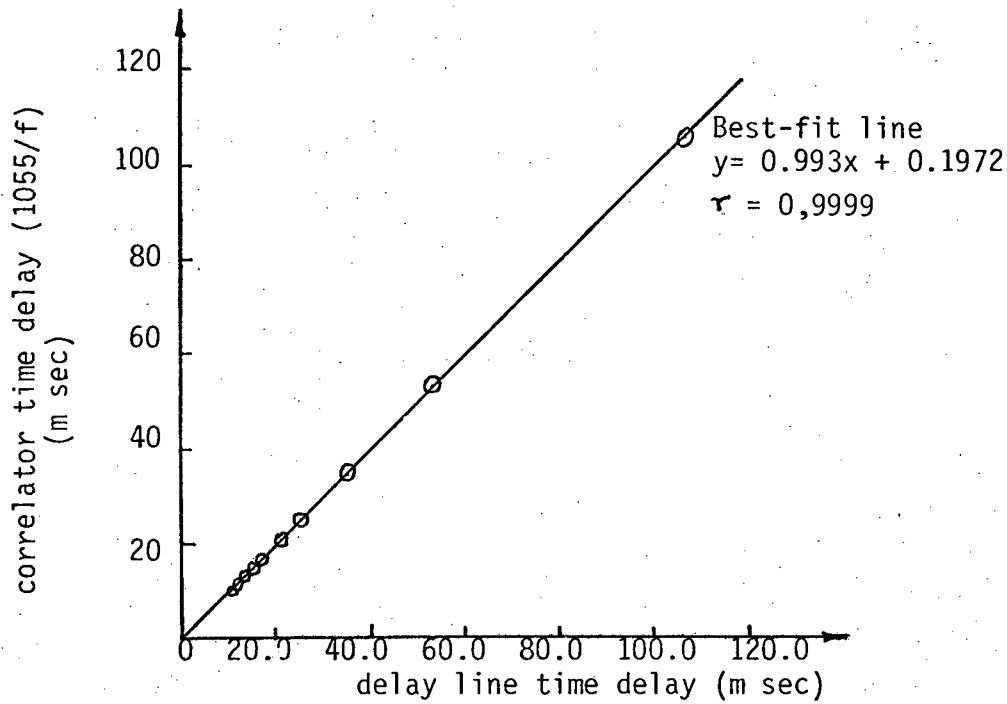


FIG. 9.19 Calibration of micro-processor based correlator using delay line

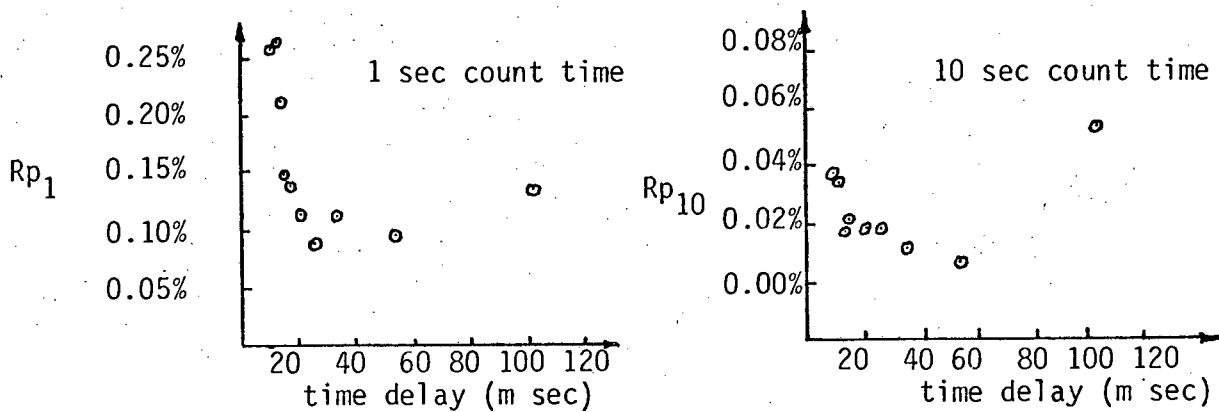


FIG. 9.20 Repeatability of peak-seeking correlator

The calibration curves of both the micro-processor controlled and serial two-point difference correlators are identical since they are both based on a two-point difference correlator. This correlator determines the accuracy of the devices, the difference between them merely being the manner in which the approximate peak position is located.

5. CALIBRATION OF CORRELATION FLOWMETERS WITH TEST-RIG

The correlation flowmeters that have been developed were calibrated using the same test-rig described in Section 9.1. For the flow of water the sensors were mounted on a horizontal test section but with sediment included, they were clamped to a vertical section of pipe.

For a fixed counting interval, the reciprocal of the flowmeter output is plotted as a function of sensor spacing in Figure 9.21 for three flow rates. Due to the construction of the test-rig only a small 6:1 flow range is possible. However, by varying both the sensor spacing and flow velocity, a correlation peak position range from 260 m secs ($v=1,35$ m/sec at $d = 350$ mm to 10 m sec $v=5,2$ m/sec at $d = 51$ mm), i.e., a 26:1 range was covered. The correlators satisfactorily located the correlation peaks over this range and will, in fact, operate over a much wider (e.g., 100:1) range.

Hereafter the correlation flowmeter output was plotted against the flow velocity as measured by a weightank at a fixed sensor spacing for both water (Figure 9.22) and sediment (Figure 9.23) flows. These figures again demonstrate the linear characteristic of the flowmeter. The repeatability of the output readings is plotted against flow velocity in Figure 9.24. It is not certain whether small flow variations existed since no reference flowmeter was available. In spite of this, repeatabilities of the order of $\pm 1,5\%$ were observed.

6. CONCLUSION

The absolute accuracy of a correlation flowmeter depends on both the sensor spacing and the correlation peak position. Depending on the application the sensor spacing will vary. A calibration factor has thus been included in the design of the flowmeters to adjust for such variations. As a result, the bias of the correlator (i.e., the difference between the mean output value

FIG.9.21 Plot of reciprocal of peak-seeking correlator output vs sensor spacing at fixed flow velocity

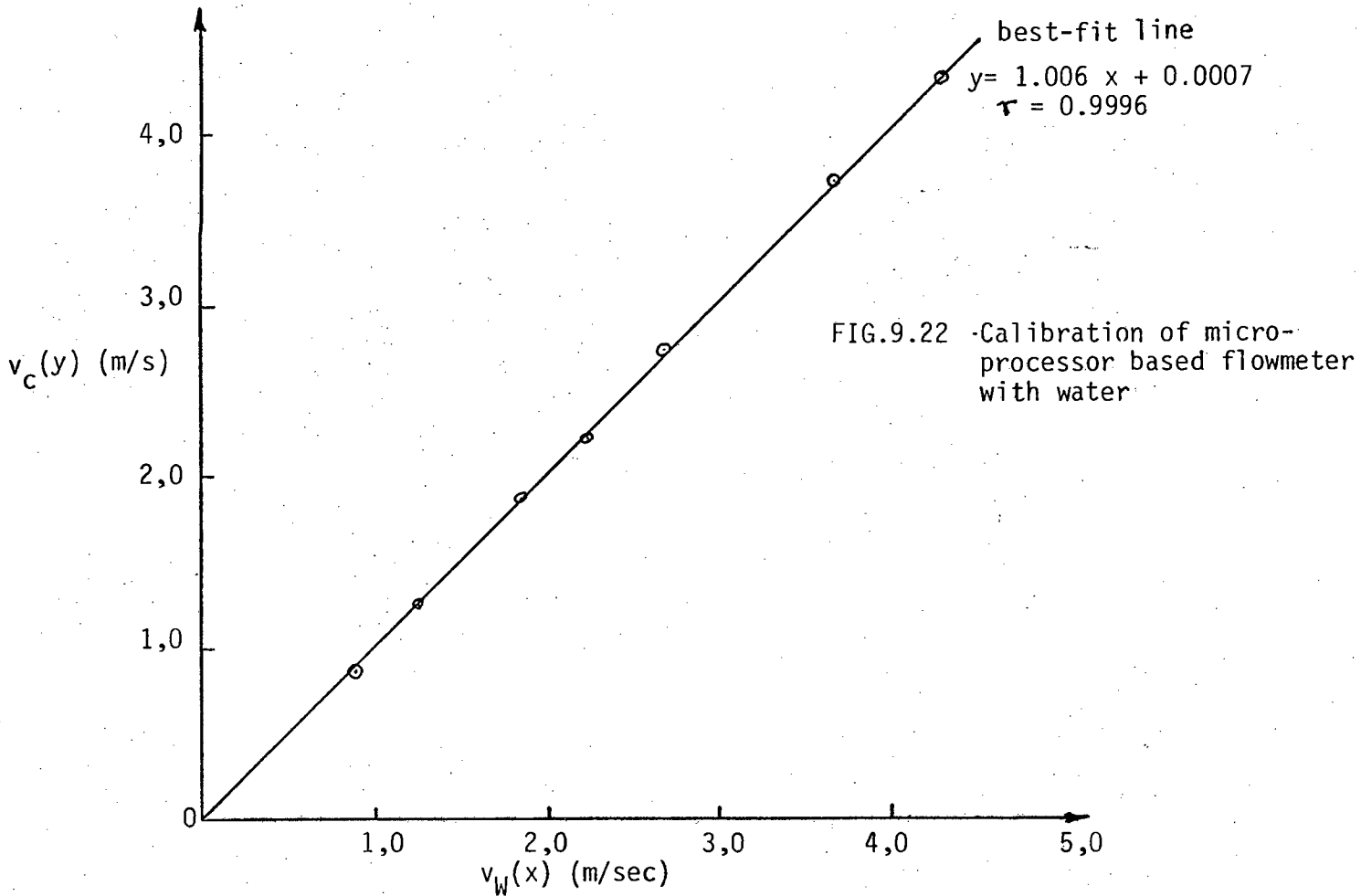
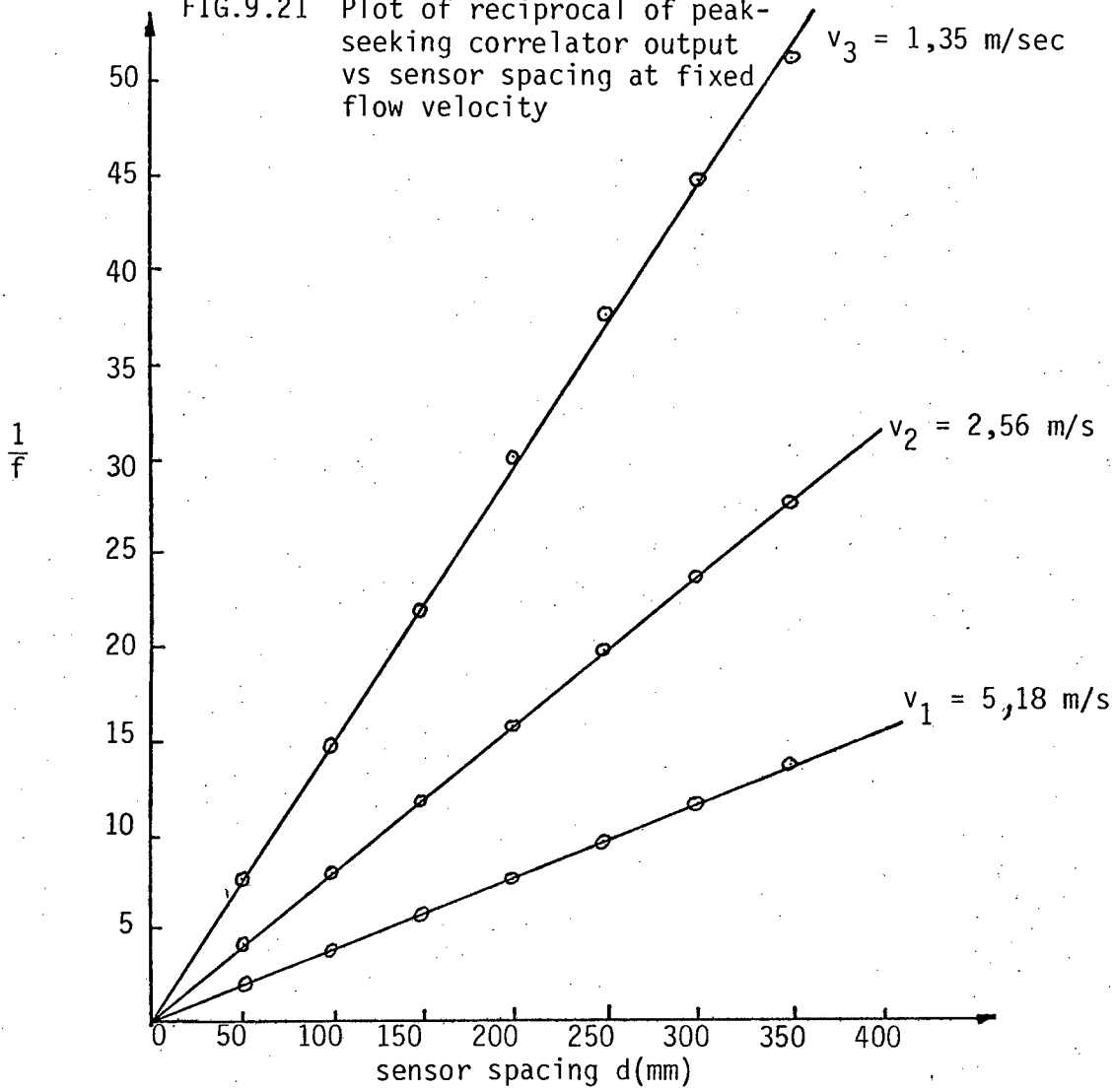


FIG. 9.23 Plot of flowmeter output vs weightank flow velocity at fixed sensor spacing for sediment

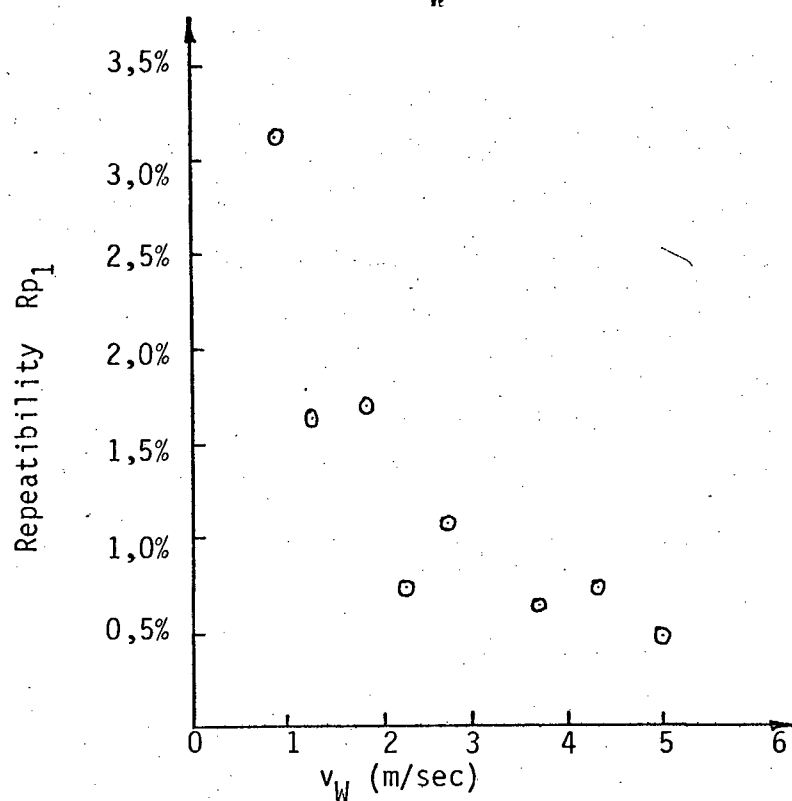
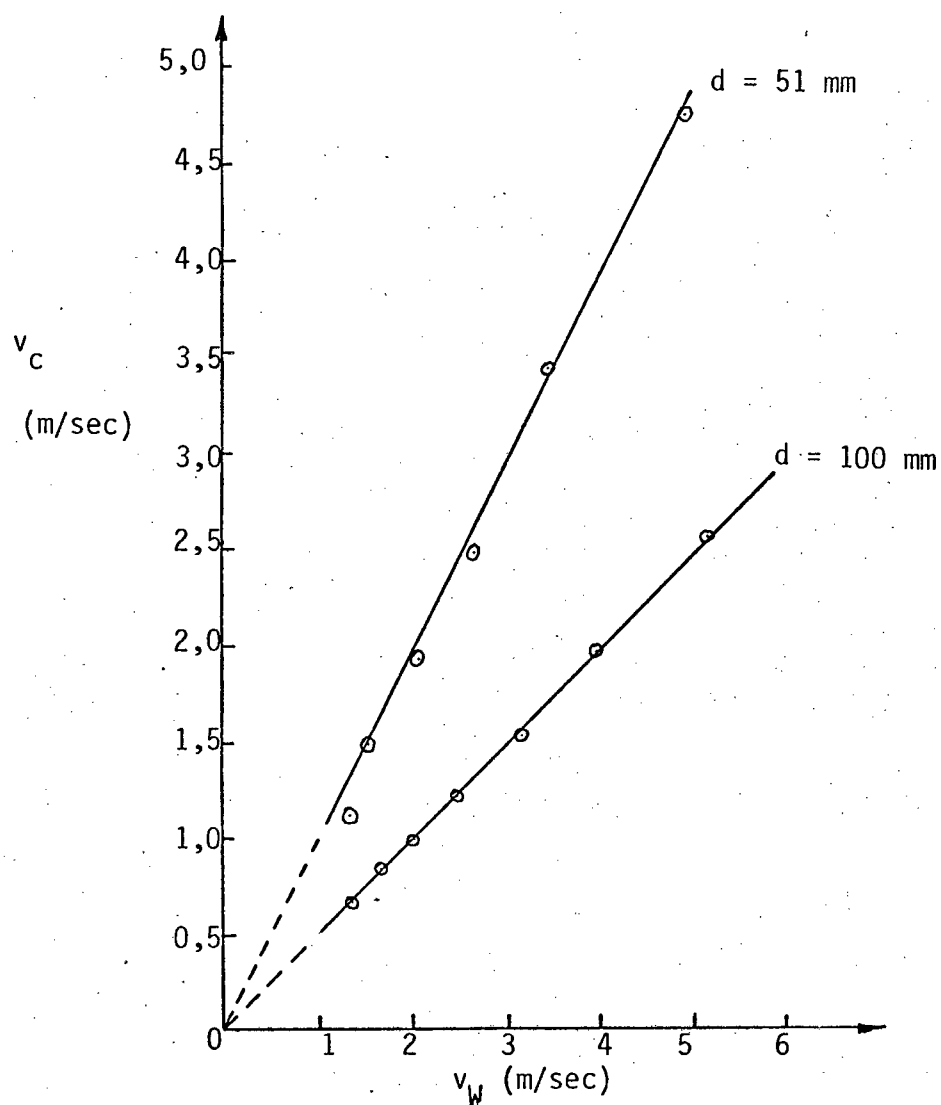


FIG. 9.24 Repeatability as function of flow velocity

and the true flow velocity) can be reduced to zero. The precision (or repeatability) of the correlators, that is their ability to give the same output for repeated applications of the input, is about $\pm 0,08\%$ with a probability of 95% for a 10-sec count interval. The precision when used as a flowmeter has not as yet been ascertained since no reference is available. The resolution of the correlators is determined by the number of digits of the frequency counter used and can be made very good.

CHAPTER 10

SOURCES OF ERROR

1. INTRODUCTION

In the previous chapters the operation of correlation flowmeters and the results that can be expected were examined. This chapter investigates the precautions which need to be taken to get reliable results when using such a flowmeter. Some of these precautions are common to all flowmeters whereas others apply specifically to correlation type flowmeters. Any instrument has its limitations and if not used properly, its full potential is wasted. A thorough understanding of the possible sources of error of a correlation flowmeter is thus necessary.

2. VELOCITY PROFILE EFFECTS AND FLOWMETER FACTOR

The parameter of interest in most flow applications is the mean flowrate, $\bar{v}$, or the total volumetric flowrate, Q . These two factors are related by the cross-sectional area of the pipe, i.e.,

$$Q = \bar{v}.A$$

If all fluid elements were to travel at the mean flow velocity, $\bar{v}$, a flat velocity profile (Figure 10.1a) would exist and a correlation flowmeter would measure the mean flowrate. Unfortunately, in pipe flow, not all fluid elements travel at the mean flow velocity. Due to, *inter alia*, viscous drag effects at the pipe walls, the flow velocity is zero at the walls (no-slip condition) and rises to a maximum at the centre of the pipe. A non-uniform velocity profile thus exists within the pipe, the shape of which depends on the Reynolds number of the flow. For low Re numbers ($Re < 1000$), the velocity profile is given by (Reference 58).

$$v(r) = \bar{v} \cdot \frac{n+2}{n} \left[1 - \left(\frac{r}{a} \right)^n \right] \dots\dots\dots (10.1)$$

where $\bar{v}$ is the true mean flow velocity, r the distance from the pipe axis and a the radius of the pipe.

In general, $n = 2$ and this leads to the wellknown parabolic velocity profile (Figure 10.1b) for laminar flow (Reference 183,243) viz.,

$$u(r) = \frac{4}{3} \bar{u} \left[1 - \left(\frac{r}{a} \right)^2 \right]$$

$$= u_{MAX} \left[1 - \left(\frac{r}{a} \right)^2 \right]$$

where $u_{MAX} = \frac{4}{3} \bar{u}$ is the velocity at the pipe centre.



For turbulent flow, the velocity profile can be described quite accurately by the following empirical equation (Reference 183) (see Figure 10.1c).

$$u(r) = \bar{u} \frac{(1+2m)(1+m)}{2m^2} \left[1 - \frac{r}{a} \right]^{1/n} \dots\dots\dots (10.2)$$

where m depends on the Re. number as follows:

Re	m
$10^3 - 10^5$	6 - 7
10^6	8,8
3×10^6	10

The turbulent velocity profile clearly is a lot flatter than the laminar profile but different fluid elements still travel at different velocities.

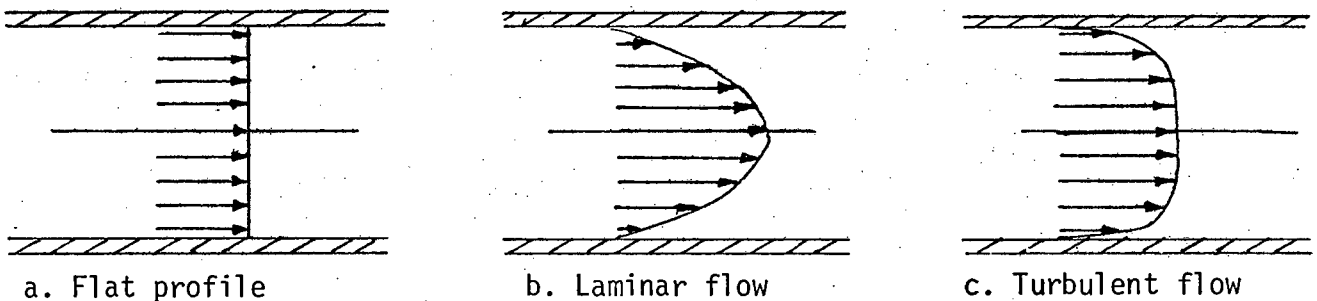


FIGURE 10.1 : Velocity Profiles in Pipes

A correlation flowmeter measures a velocity, u_c , that does not equal the mean flow velocity. In order to obtain the latter from the correlation velocity, a calibration factor, k , is required. This factor is defined as the ratio of the mean flow velocity to the correlation flow velocity, i.e.,

$$k = \bar{u} / u_c \quad \text{or} \quad \bar{u} = k u_c$$

The mean flow velocity is obtained by averaging the velocity over the cross-section area of a pipe with a weighting that increases with radial position, r , i.e.,

$$\bar{v} = \frac{1}{\pi a^2} \int_0^a 2\pi r v(r) dr$$

A correlation flowmeter with ultrasonic sensors, on the other hand, measures a velocity that is the linear average along a pipe diameter, i.e., it gives equal weighting to all radial positions.

$$v_c = \frac{1}{a} \int_0^a v(r) dr$$

This velocity clearly does not give sufficient weighting to the fluid near the perimeter of the pipe which is travelling slower than the mean flow velocity. A correlation flowmeter will thus over-estimate the mean flow velocity. The flowmeter calibration factor, k , can now be written as:

$$k = \frac{\frac{1}{\pi a^2} \int_0^a 2\pi r v(r) dr}{\frac{1}{a} \int_0^a v(r) dr} \dots\dots\dots (10.3)$$

Substituting equation (10.1) into equation (10.3) gives for laminar flow a calibration factor of

$$k = \frac{n+1}{n+2} = 0,75 \text{ for } n = 2$$

The correlation flow velocity thus exceeds the mean velocity by 33,3%.

Repeating the calculation for turbulent flow, (EQU 10.2) gives

$$k = \frac{2m}{2m+1}$$

$$= \begin{cases} 0,923 & \text{for } m = 6 \\ 0,937 & \text{for } m = 7 \\ 0,941 & \text{for } m = 8 \end{cases}$$

The correlation flow velocity thus again exceeds the mean flow velocity but only be about 5% to 7%.

In the transitional region between laminar and turbulent flows (Re no. from 2000 to 10 000 approximately), the profile correction factor, k , is not constant and lies between 0,75 and 0,93 as shown in Figure 10.2.

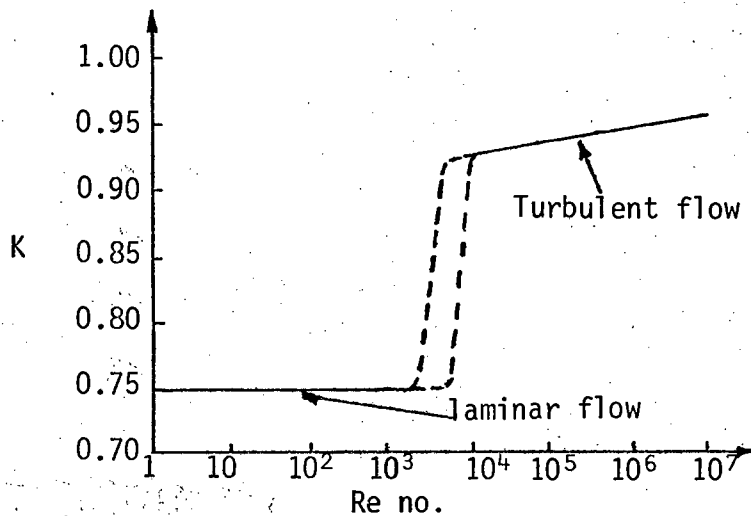


FIGURE 10.2 : Profile correction factor vs Reynolds number

Reference 51 proposes methods for reducing velocity profile effects by sampling a greater fraction of the cross-sectional flow area.

The velocity measured by a correlation flowmeter with ultrasonic sensors requires further clarification. Each flow signal represents the instantaneous spatial average of flow disturbances within its respective ultrasonic beam. These signals subsequently are cross-correlated and the position of the correlation peak is found. This peak is produced by the dominant disturbances or those undergoing the least change between upstream and downstream sensors. These disturbances are usually situated at the centre of the pipe and hence travel at a greater velocity than the mean flow velocity. This leads to a correlation flowmeter further overestimating the mean flow velocity.

3. ENTRY AND EXIT LENGTHS

A correlation flowmeter relies on the flow signals recorded at its two sensors to be similar. The flowmeter must thus be located at a point on the pipe at which the velocity profile is fully developed. For consider the case in which the flowmeter is situated in a region where the velocity profile is changing. The profile at the downstream sensor will then not be the same as at the upstream sensor. This results in poor correlation between the flow signals and a degradation in performance.

The length of straight pipe required to precede a correlation flowmeter is thus the minimum length for the velocity profile to become fully developed after it has been disturbed. Factors affecting the profile include bends, inlets and outlets. This minimum length again depends on the Re number of the flow. For laminar flow the entry length is given by $0,057(Re)$ pipe diameters according to Reference 243. So for $Re = 2000$, the highest value at which laminar flow can be counted on, the entry length is about 110 pipe diameters. For turbulent flow, the steady velocity profile is reached sooner and Reference 244 claims that the entry length can be expressed by :

$1.41 (Re)^{0.25}$ pipe diameters. For Re numbers from 2500 to 10^5 , entry lengths of the order of 10 to 30 pipe diameters are thus predicted. Since all slurry flows are turbulent, an entry length of, say, 30 pipe diameters should be sufficient for a correlation flowmeter.

The exit length, that is, the length of straight pipe downstream of the flowmeter is much shorter; the requirement again being that the effect of any downstream velocity profile disturbance does not extend sufficiently far upstream to unsettle the profile at the flowmeter.

4. SPATIAL AVERAGING OF SENSORS

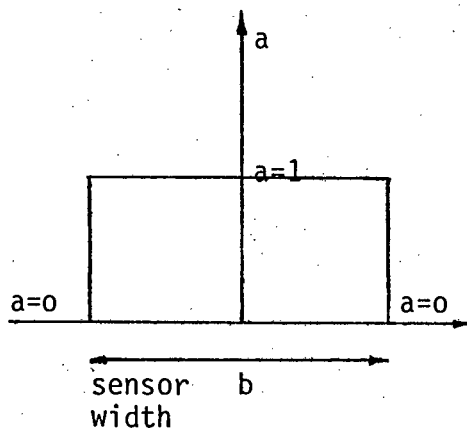
The effect of sensor geometry on the bandwidth of the flow signals has been examined by MESCH et al, (Reference 71, 105). The ultrasonic sensors must transform the moving spatial flow patterns into electrical signals. Due to their finite extent in the flow direction, a certain averaging occurs resulting in bandwidth reduction. This can be described for an idealized sensor by an aperture function, a , being 1 within the sensor field (i.e., within the ultrasonic beam) and zero outside (Figure 10.3a). After Fourier transforming, this corresponds to a spatial frequency response given by (Figure 10.3b)

$$A(m) = k \cdot \frac{\sin(\pi bm)}{(\pi bm)}$$

where b is the width of the sensor in the direction of flow, m the spatial frequency (cycles/metre) and k a constant.

It is shown in Reference (105) that the expression for the power spectral density of the detected signals contains the factor $|A(m)|^2$ where $m = f/v$ (v = velocity of fluid). Thus the aperture function of the sensors acts as a low-pass filter, its upper frequency cut-off, defined at the first zero of the $(\frac{\sin \pi bm}{\pi bm})$ function, is limited to $f_{MAX} = v/b$. The bandwidth of the flow signals can thus be increased by reducing the sensor width.

a) Ideal aperture function



b) Spatial frequency response

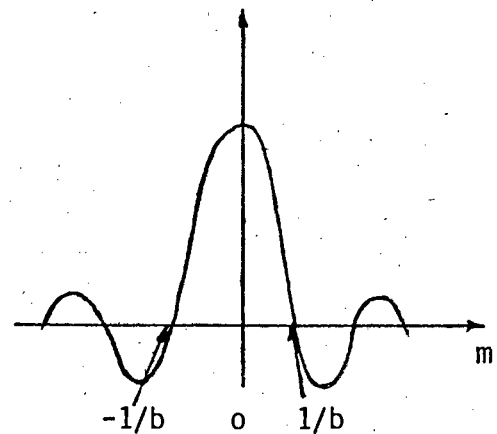


FIGURE 10.3 : Effect of Sensor Geometry

5. SENSOR SPACING

Two main factors determine the choice of sensor spacing. Ideally a large spacing is required for good accuracy since for a given absolute error, the relative error is thereby reduced. However, due to the non-frozen nature of the flow pattern, a large sensor spacing results in reduced correlation between the flow signals. A compromise between these conflicting requirements must thus be found.

CALVERT et al, (Reference 117) suggests that that the sensor spacing that maximizes the product of sensor spacing and normalized moving-frame auto-correlation function be used (see Figure 10.4). This yields an optimal spacing of between 1 to 2 pipe diameters. ONG (125) shows, using results for water flow in a 1" pipe, that a sensor spacing of between 3 and 4 pipe, diameters minimizes the standard error in the velocity measurement.

MESCH et al, (Reference 75) uses some simplifying assumptions and calculates the optimal spacing to be:

$$L_{opt} = 0,88 \cdot \frac{v^2}{\sigma \cdot \omega_0}$$

where v - flow velocity

σ - standard deviation of the velocity fluctuations about

ω_0 - cut-off frequency of power density spectrum of the flow signals

This spacing minimizes the relative standard deviation of the output clock frequency of the optimal tracking correlator.

From the above results it is evident that no exact solution exists to the problem of specifying the optimum sensor spacing for any application. Spacings ranging from 1 to 4 pipe diameters have been suggested. It is concluded that the effect of sensor spacing on measurement accuracy is not critical provided the spacing lies within the range given above. From the experience gained during the course of this project, a spacing of about 2 pipe diameters is recommended as a starting point.

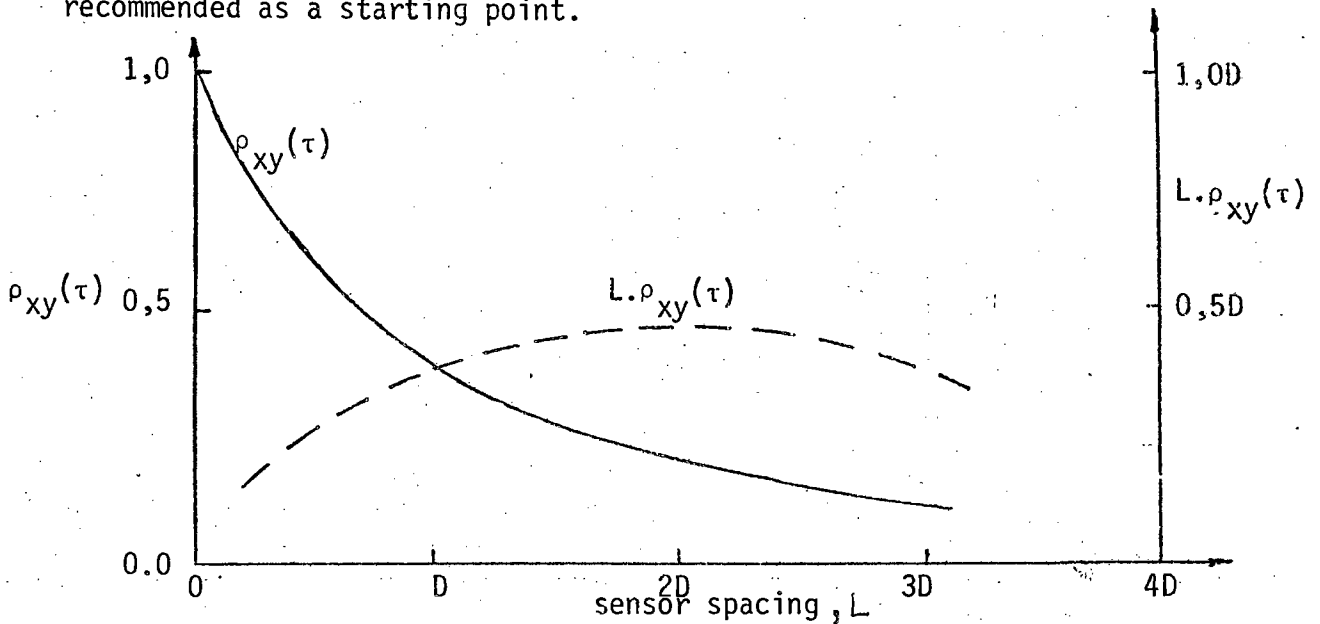


FIGURE 10.4 : Effect of Sensor Spacing

6. CROSS-COUPLING OF ULTRASONIC BEAMS

To investigate the effect of cross-coupling between up and downstream flow sensors, consider a disturbance situated at the centre of a pipe as shown in Figure 10.5. At time t_0 this disturbance will produce a variation at the upstream sensor. At time $t_0 + d/2v$ fluctuations will be produced at both up and downstream sensors due to cross-coupling of the ultrasonic beams. Finally at a time $t_0 + d/v$ the downstream sensor detects the disturbance. Cross-correlating the two received signals will hence produce peaks at time delays equal to $d/2v$ and d/v assuming that disturbances are located only at the centre of the pipe (Figure 10.6a). Cross-coupling has thus caused a second peak midway between the main peak and zero delay.

Correlatable disturbances, in general, are not only located at the centre of the pipe but may exist anywhere on a cross-section. Assuming a uniform distribution of disturbances over the pipe cross-section, all moving at the flow velocity v (i.e., a flat velocity profile), then the two received signals will be correlated for all delays from 0 to d/v due to cross-coupling of the ultrasonic beams. A correlation plot as shown in Figure 10.6b should thus be obtained. A main peak is still produced by the two direct beams.

In practice it is reasonable to assume that most of the dominant disturbances are situated near the centre of the pipe. If this is the case, cross-coupling again results in a minor peak at approximately $d/2v$ (Figure 10.6c).

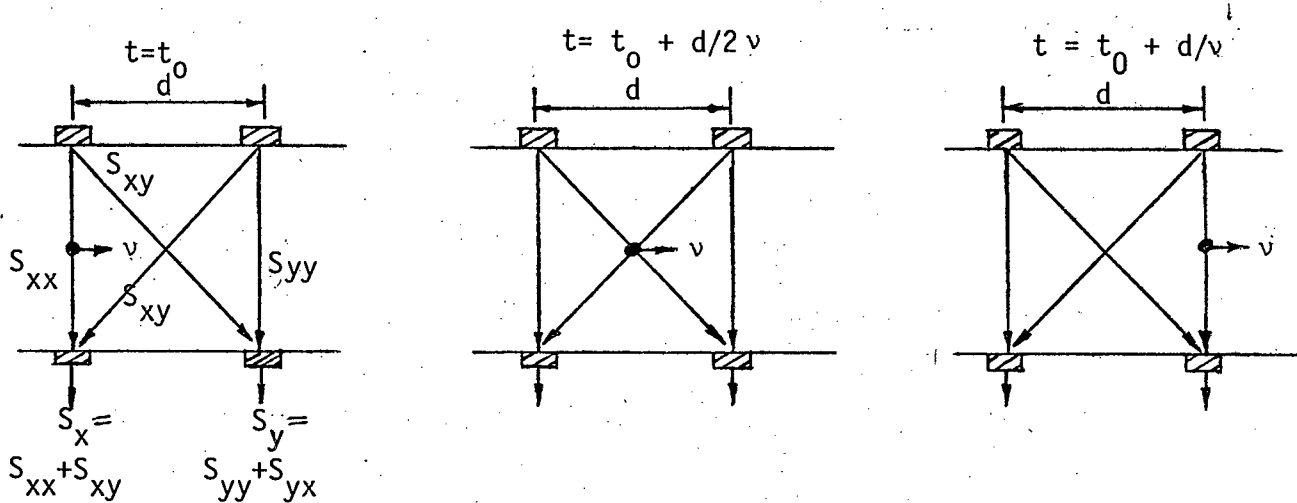


FIGURE 10.5 : Transducers with Cross-coupling

7. NON-PARALLEL ULTRA-SONIC BEAMS

It is important to ensure that the two ultrasonic beams are parallel to each other and in the same place. If they are not parallel, the transit time of a flow disturbance depends on its radial position and this leads to a broadening of the correlation peak as well as a shift of the peak position. This effect was investigated using optical sensors and the results obtained are shown in Figure 10.7.

If the beams are not in the same plane, disturbances incident on the upstream ultrasonic beam will not necessarily pass through the downstream beam. This leads to a poor if not to the absence of a cross-correlation peak.

By taking precautions during the mounting of the sensors, these problems can be avoided.

8. ADDITIONAL CORRELATION PEAKS

Various factors can cause minor correlation peaks to be present in addition to the desired peak. These include pipe vibrations and acoustic noise that is conveyed either along the pipe-wall or through the fluid. If these factors do not modulate the received high-frequency signal, they can be removed by a suitable band-pass filter. Otherwise they may produce a peak at

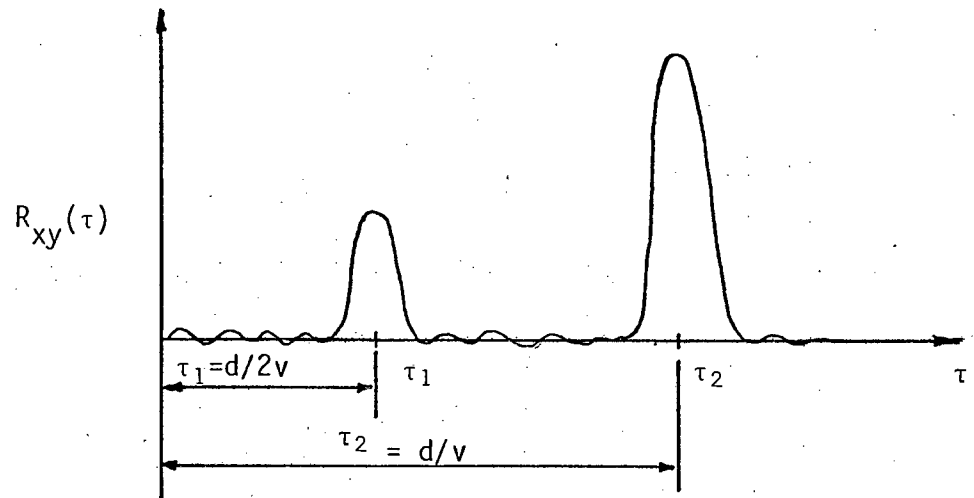


FIG. 10.6a Correlation plot with cross-coupling and disturbances only at centre of pipe

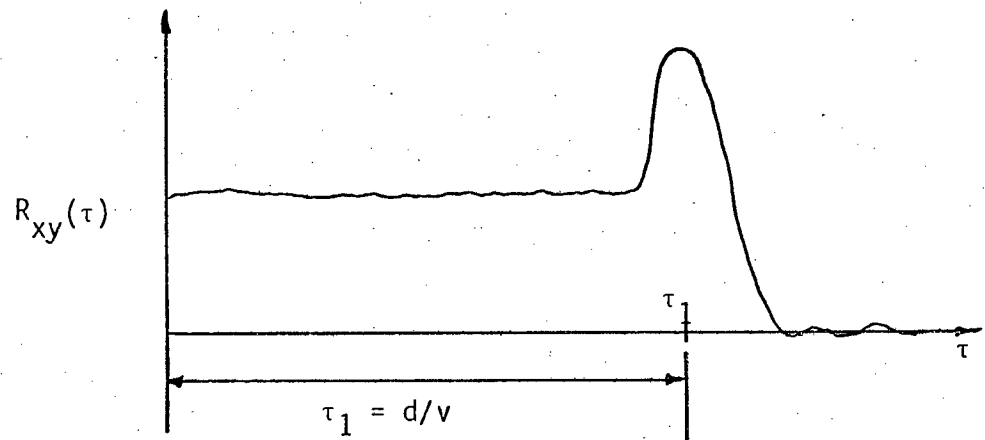


FIG. 10.6b Correlation plot with cross-coupling and uniform distribution of disturbances

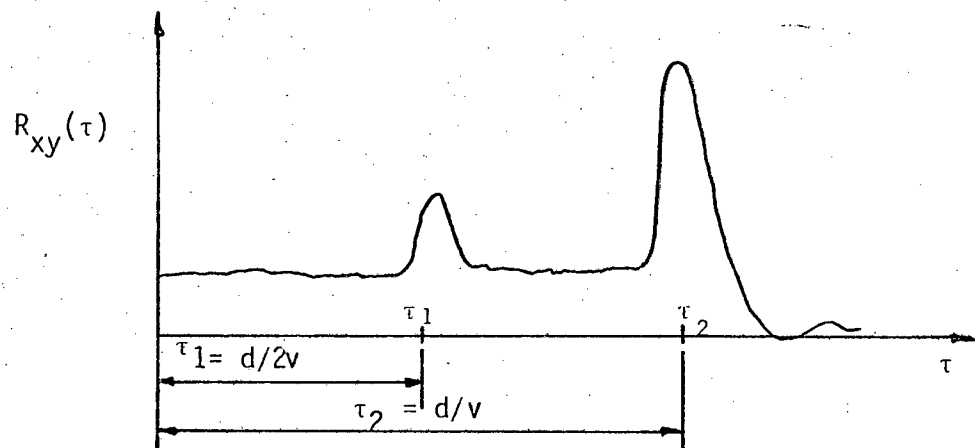


FIG. 10.6c Correlation plot with cross-coupling and non-uniform distribution of disturbances

a time delay equal to the transit time of sound between the sensors. This transit time is very short in comparison with flow velocities encountered in practice. As a result, the correlation flowmeter can be arranged to ignore peaks at very short time delays.

9. PHASE SHIFT ERRORS

It is important to ensure that the phase response of both up and downstream sensors including demodulators, amplifiers, etc., are identical. If the phase responses are linear with frequency but unequal, an additional, constant time delay will be produced between the flow signals. This causes a fixed shift in the position of the correlation peak and can be compensated for (References 105 and 139). If, however, the phases responses are non-linear and unequal, further peak position errors can be produced.

The effects of non-linear phase shifts, are shown in Figure 10.8. If all the frequency components of one signal are shifted by 90° (e.g., by differentiation or integration), the resulting correlation function is equal to the first derivative of the original correlation function, i.e.,

$$R_{xy}(\tau) = \frac{\partial}{\partial \tau} R_{xy}(\tau)$$

or

$$R_{xy}(\tau) = - \frac{\partial}{\partial \tau} R_{xy}(\tau) \quad (\text{Figure 10.8a})$$

In practice the phase responses do not differ by 90° but are much smaller. Typical correlation peaks for various phase responses are shown in Figure 10.9. Shifts in peak position still appear and will produce an error in correlation flowmetering.

Fortunately, phase differences between the two receivers can be made small by using identical circuits for both.

ONG(125) further shows that differences in acoustic path lengths of the two ultrasonic beams also can lead to phase shift errors. However, by careful mounting of the sensors, these errors again can be neglected.

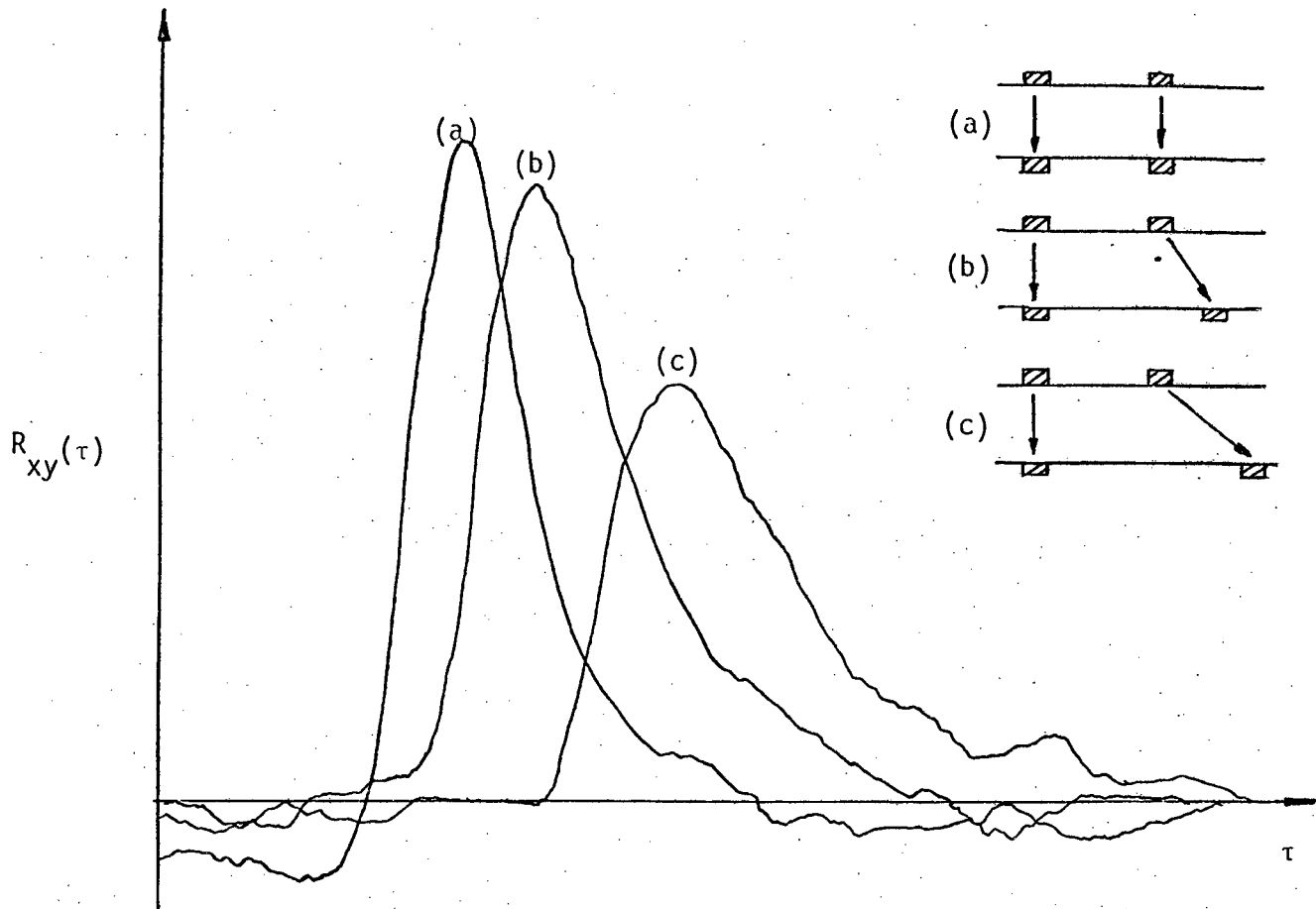


FIG. 10.7 Diagram showing effect of non-parallel beams on shape of correlation function

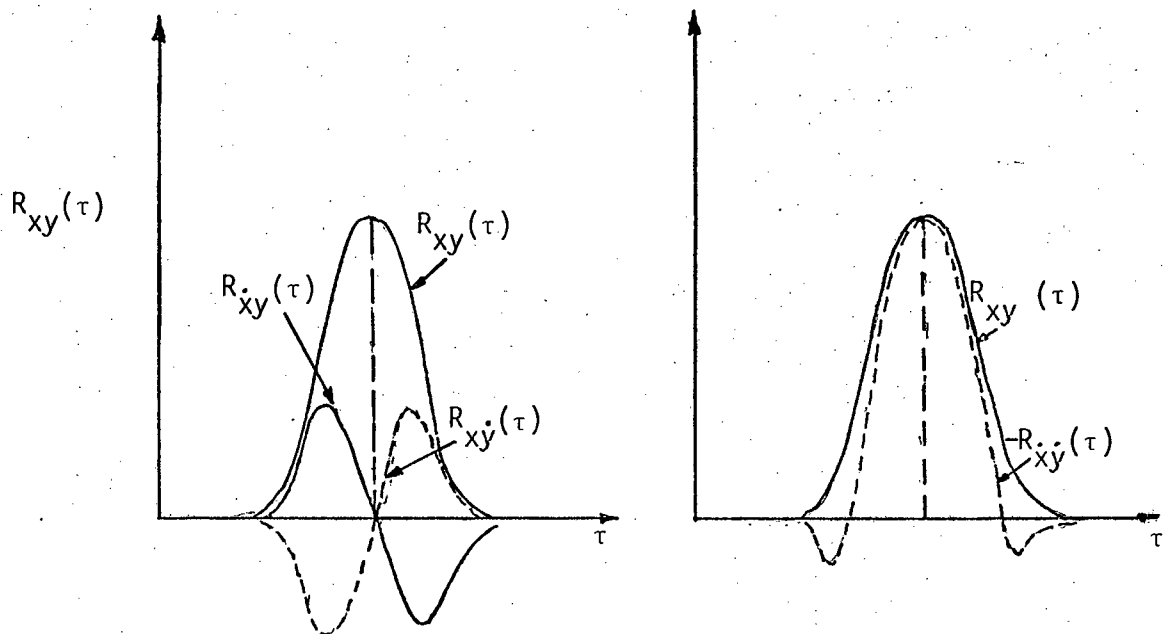


FIG. 10.8 Effect of 90° phase shifts on correlation functions

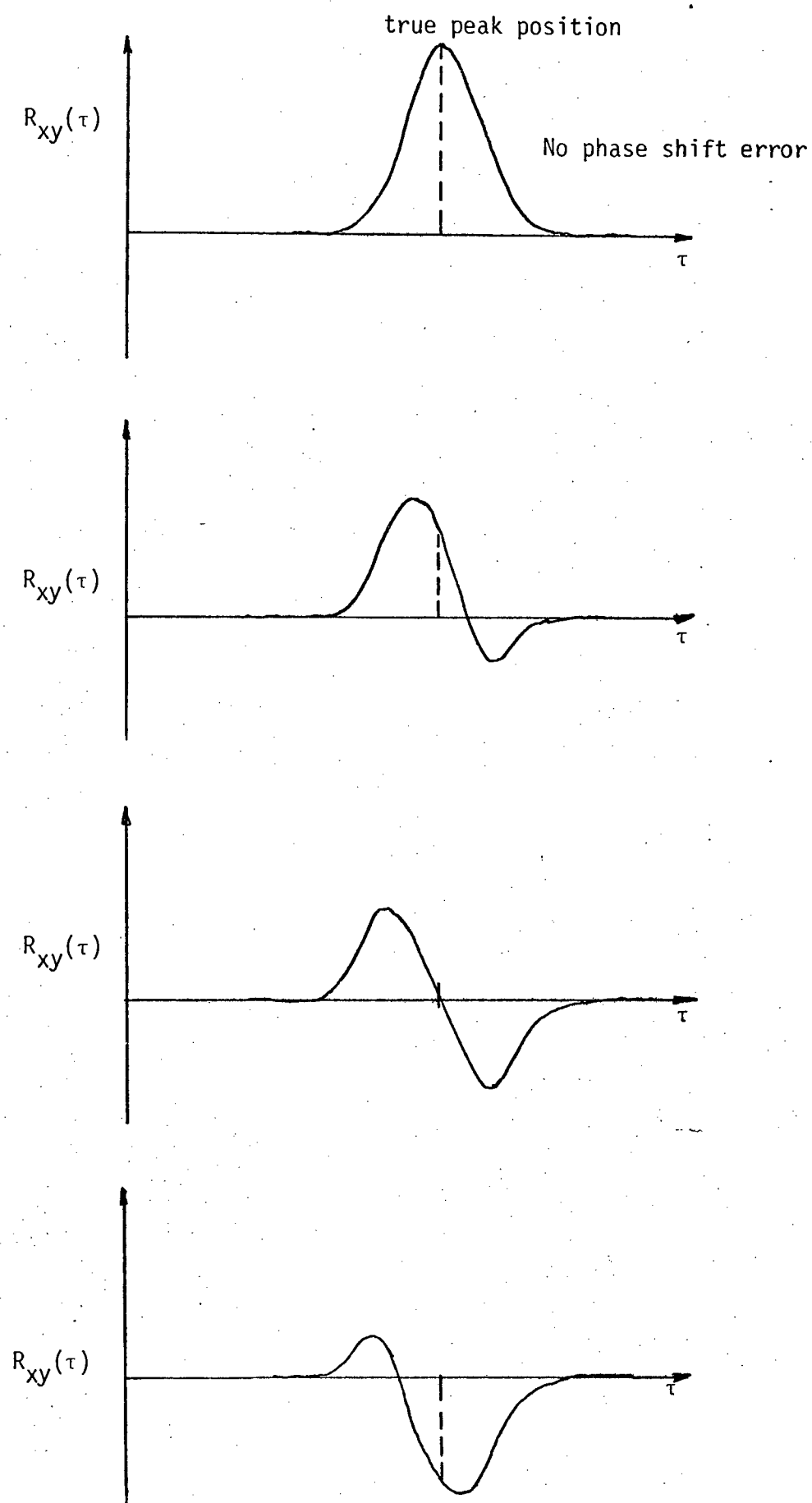


FIG. 10.9 Typical correlation plots for unequal phase shifts

10. CONCLUSION

This chapter has investigated some potential error sources and has suggested ways of avoiding or eliminating them. Clearly, the list is not complete and others will become apparent during the industrial operation of the correlation flowmeter. However, by using identical pairs of transducers at both the upstream and downstream measuring locations and by employing the same mounting technique, many problems can be circumvented.

CHAPTER 11

DISCUSSION, RECOMMENDATIONS AND CONCLUSIONS

1. GENERAL DISCUSSION

Slurry flowmetering is still one of the most difficult problems in the mining industry. Due to the tighter control of process efficiency, its accurate measurement has assumed greater importance. Current slurry flowmeters are either very expensive or require regular maintenance and careful operation precautions for satisfactory performance. The need for a cheap, robust flowmeter requiring a minimum of maintenance is thus clearly evident. A flowmeter based on correlation techniques with clamp-on ultra-sonic sensors, appears to satisfy all the requirements of a slurry flowmeter.

The principle of operation is based on measuring the transit time of naturally occurring flow discontinuities between two points a known distance apart in the direction of flow. By determining the position of the cross-correlation peak between the random flow signals generated at the two measuring points, this transit time can be obtained. Non-intrusive ultra-sonic sensors that are clamped onto the outside of a pipe do not affect leakage integrity and ensure easy and rapid flowmeter installation. Furthermore, since the sensors are not in contact with the process fluid, abrasion, corrosion and clogging problems are avoided. In addition, no pressure drop is produced nor are flow disturbances created.

In broad terms, the aims of this thesis were to investigate correlation flowmetering and develop a flowmeter suitable for industrial application. The thesis can be divided conceptually into two parts. The first consists of a review of all the aspects involved in correlation flowmetering. This includes an examination of correlation flowmeter models, sensors for generating flow signals, correlation techniques for simplifying correlation function evaluation, hardware correlators for determining correlation peak positions and sources of error and performance criteria. Since no complete review on the subject is available, this is considered to be a valuable contribution in itself.

The second objective was to examine each of the above aspects in depth in order to improve existing techniques as well as suggesting new ones. This lead, for example, to an attempt of using delta-modulation to simplify the problem of determining correlation functions. Unfortunately, no major improvement over Polar correlation was observed.

Simple, robust ultra-sonic sensors that easily can be installed and

removed were developed and shown to yield adequate results. The lack of a cheap dedicated peak-seeking correlator was recognized to be a major factor in preventing correlation flowmetering from being widely used in industry. A two-point difference correlator was found to satisfy the requirements of resolution, accuracy and response time but its inability to initially locate a correlation peak was a major disadvantage. As a result a new micro-processor controlled two-point difference correlator was developed and shown to satisfy all the requirements of a wide-range dedicated peak-seeking correlator. In addition, a cheaper serial two-point difference correlator was also developed for simple well-defined applications. Adaptive features that compensate for changes in cross-correlation function shape were included in the design.

In order to optimize the operation of a two-point difference correlator, a theoretical model was proposed. Experimental results confirmed the validity of the model. Typical flow signals were analyzed in the frequency, amplitude and time domains to determine their dependence on flow parameters. The correlators were firstly calibrated using known test signals and gave excellent repeatability and linearity. Thereafter tests were carried out on a test rig and promising results were obtained. Flowmeter linearity was verified over a 10:1 range and good repeatability was recorded under the circumstances.

2. SUMMARY OF RESULTS

With reference to the aim specified in Chapter 1, the following results were attained:

- (i) A comprehensive review of correlation flowmetering and all its associated aspects was undertaken. For the first time, a unified report on the subject is available.
- (ii) The characteristics of a correlation flowmeter were examined and factors affecting the shape of a correlation function were determined. The concept of a transit-time distribution function was shown to be extremely valuable in the analysis. The result, that the cross-correlation function is equal to the convolution of the transit time distribution function and the auto-correlation function of one of the flow signals was of specific interest. Asymmetrical cross-correlation peaks were shown to be caused mainly by velocity profile effects.
- (iii) Ultra-sonic transducers were shown to be most suitable for slurry flow applications. Their characteristics were examined and a clamp-on design was developed. The ultra-sonic sensors are located in shielded, protective cases that can be attached via clamp-on mounts

to any pipe over a specific size. Installation requirements are simple, maintenance is low and relocation of the sensors can readily be accomplished.

- (iv) Methods for simplifying correlation function evaluation were reviewed. Various degrees of amplitude quantization were investigated and its effect on correlation functions was determined. A novel technique based on delta-modulating the signals before they are correlated was evaluated, but shown to possess no advantages over Polar correlation for flowmetering applications. It was thus decided to design a correlator using the Polar technique.
- (v) Two new correlators were designed, constructed and evaluated. Both are based on a two-point difference correlator that operates by calculating the correlation function at two time delays and attempts to adjust these time delays in a closed loop manner, so that they straddle the correlation peak. This correlator has infinite resolution, a fast response, good accuracy, is simple and cheap and gives an output that is directly proportional to flow velocity. It, however, cannot locate a correlation peak if situated far from such a peak and thus has limited applicability. To overcome this deficiency, two devices were developed and locate the approximate position of a correlation peak and direct the two-point difference correlator to this position. The first device is micro-processor based and uses a serial-parallel mode of operation to locate the main correlation peak over a large (100:1) range. The second instrument sweeps the two-point difference correlator over the delay range of interest, the sweep being terminated when a peak is located.
The operation of both correlators is identical once a peak has been located since it then only depends on the operation of the two-point difference correlator. The main difference between the two correlators is the manner in which a peak is located initially. The micro-processor correlator examines the entire delay range at any instant and can thus distinguish between multiple peaks and select the largest. The serial device cannot discriminate between peaks. It will lock onto the first peak it encounters during its downward time delay sweep.
- (vi) In order to optimize the performance of a two-point difference correlator, its response was analysed. A first order non-linear differential equation was shown to describe the response of the basic correlator. It depends on the shape of the cross-correlation function as well as on correlator parameters. A modified version of the basic two-point difference correlator led to a second order non-linear

system. Both under and overdamped responses were obtained theoretically and verified experimentally. It was concluded that for general applications in which the shape of the correlation function is not known exactly, the basic two-point difference correlator is better than the modified version since the possibility of severe oscillations and instability do not arise.

- (vii) Typical flow signals, derived from the flow of both pure water and water plus sediment, were analyzed in the amplitude, time and frequency domains. It was confirmed that they possess Gaussian characteristics and that their frequency spectrum increases with an increase in flow velocity. The convection velocity as a function of frequency is not constant but is higher for the higher frequency components. The eddy life-time, however, decreases with increasing frequency. It can thus be concluded that the low frequency turbulent components decay more slowly than the higher frequency components.
- (viii) The performance of the two-peak-finding correlators was examined using known test signals. Excellent linearity was observed and repeatibilities of better than $\pm 0,08$ per cent were recorded. This confirms the potential of the correlators.
- (ix) The correlators were also evaluated on a test rig using the ultrasonic transducers. Good linearity was again evident, with repeatibilities of the order ± 1 per cent. Since no absolute standard was available, precise calibration was not possible. However, the results again confirmed the potential of the flowmeters as accurate devices.

3. FUTURE WORK AND RECOMMENDATIONS

(i) The correlation flowmeters developed during the course of this project appear to satisfy all the requirements for slurry flow-metering. However, in order to test their industrial performance with a view to commercial implementation, extensive trials need to be carried out on plant under various conditions of operation. Such tests will indicate if any aspects of the flowmeters need to be improved or modified. Specifically the effect of pipe size, pipe material and slurry concentration on the received ultra-sonic signals needs examination. The fact that most slurry pipe-lines are rubber lined may produce further practical problems. It is important to bear in mind that these are all specific problems that depend on the particular application. They, however, will not affect the principle of operation of the correlation flowmeters.

(ii) Further theoretical and experimental analysis of the characteristics of correlation flowmetering should be undertaken. This includes the determination of the transit-time distribution function and hence the velocity profile from the auto and cross-correlation functions of the up and downstream flow signals. Since these signals represent the linear average of discontinuities over the velocity profile if ultra-sonic sensors are used, the interpretation of the measured transit-time distribution functions requires further investigation in order to predict the velocity profile.

A calibration factor could be calculated from the measured velocity profile and used to correct the flowmeter reading in situ. Changes in the velocity profile could thus be compensated for automatically thereby further improving the potential of the correlation flowmeters. Once the feasibility of the above concepts has been established, the practical implementation with, for example, the micro-processor correlator needs to be examined.

(iii) Further adaptive features can be incorporated with the correlation. By examining the shape of the cross-correlation function, it may be possible to adjust the parameters of the two-point difference correlator for optimum performance. One such feature is the shift-register length adjustment of the serial two-point difference correlator. The same feature could be included with the micro-processor correlator. In addition, if the correlation function is assymetrical, further adjustments could be made.

- (iv) In conjunction with the previous paragraph, the theoretical performance of a two-point difference correlator can be further examined. The models that have been developed can be used to predict the response of the correlator under various conditions. Again, depending on the application, this allows the flowmeter parameters to be optimized.

4. CONCLUSION

This thesis has led to the development of a flowmeter possessing the following features:

- a) Cheap
- b) Robust
- c) Non-intrusive
- d) Clamp-on
- e) Accurate
- f) Wide-range
- g) Infinite resolution
- h) Linear
- i) Maintenance free
- j) Easy to install and remove
- k) No pressure drop
- l) Not prone to abrasion, corrosion clogging.

As a result, it is ideally suited for most slurry flowrate measurements and it may replace the electro-magnetic flowmeter in many applications.

LIST OF SYMBOLS

Re	Reynolds number
ρ	fluid density
v, v, u	fluid velocity
D	pipe diameter
η	fluid viscosity
c	velocity of sound
θ	angle
f	frequency
ϕ	phase angle
L	path length
d, x, L	sensor spacing
β, T_0, τ, τ_p	correlation peak position; transit time
R_{xx}	auto-correlation function
R_{xy}, C_{xy}	cross-correlation function
τ	time delay
$x(t), y(t), n(t)$	signals
T	averaging time
μ_x	ensemble average
N	sample size
$\sigma, \sigma_x, \sigma_y$	root mean square value; standard deviation
ρ_{xx}, ρ_{xy}	normalized correlation coefficients
S_{xx}, G_{xx}	power spectral density function
S_{xy}, G_{xy}	cross power spectral density function
C_{xy}	co-incidence spectrum
Q_{xy}	quadrature spectrum
$g(t)$	impulse response
$G(s)$	transfer function
$\delta(t)$	Dirac-delta impulse
$w_x(x)$	probability density function
$W_x(x)$	cumulative distribution function
$w(T, L)$	transit time distribution function
x, y, z	Cartesian co-ordinates
I, I_0, i	intensity
K, k	extinction coefficients
V	volume
A	area
R, r, a	radius

λ	wavelength
U_p	peak velocity
Z	impedance
r	reflection coefficient
Q	quality factor
L	inductance
C	capacitance
R	resistance
ω, ω	angular frequency
n_p, n_s	number of turns
t	transducer thickness
v, V	voltage
ϵ	permittivity
$W_X(u)$	characteristic function
$w_{xy}(x,y)$	joint probability density function
$W_{xy}(u, v)$	joint characteristic function
q	quantization interval
Δt	sampling interval
n, m, k, l	shift register lengths
ϵ	error
K	constant
G	gain
B	bandwidth
RC	time constant
v_c	correlation flow velocity
v_w	weightank flow velocity
k	calibration factor
τ_w	correlation function width
γ_{xy}	coherence function
r	correlation coefficient
RP	repeatability
Q	volume flowrate
$S_{xx}, S_{xy}, S_{yx}, S_{yy}$	direct and cross-coupled signals
m	spatial frequency

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REFERENCES

1. VOCADLO, J.J., and SAGOO, M.S. Slurry flows in pipes and pumps. *Trans. A.S.M.E., Journal of Engineering for Industry*. Feb. 1973. pp. 65-71.
2. BEHREND, G.M. Where we stand in slurry flow measurement. *The Canadian Mining and Metallurgical (C.I.M.) Bulletin; Computer Applications and Process Control*. Sep. 1973. pp. 70-75.
3. DALSTAD, J.I. Slurry pump selection and application. *Chem. Engng.* Apr. 1977. pp. 101-106.
4. PAULIS, N.J., and SILVERMETZ, D. Instrumentation for slurry systems. *Chem. Engng.* Apr. 1977. pp. 107-110.
5. KINGHORN, F.C., and MCHUGH, A. An international comparison of integration techniques for traverse methods in flow measurement. *La Houille Blanche no. 1*. 1977. pp. 49-58.
6. LIPTAK, B.G. Flowmetering accuracy. *Instrumentation Technology*. Jul. 1971. pp. 35-38.
7. HAYWARD, A.T.J. Methods of calibrating flowmeters with liquids - a comparative survey. *Measurement and Control*, vol. 10. Mar. 1977. pp. 106-116.
8. LIPTAK, B.G. Instruments to measure and control slurries and viscous materials. *Chem. Engng.* Feb. 1967. pp. 151-158.
9. CAVASENO, V. Flowmeter choices widen. *Chem. Engng.* Jan. 1978, pp. 55-57.
10. LOMAS, D.J. Selecting the right flowmeter. Part I : The six favorites. *Instrumentation Technology*. May 1977. pp. 55-62.
11. LOMAS, D.J. Selecting the right flowmeter. Part II : Comparing candidates. *Instrumentation Technology*. Jun. 1977. pp. 71-77.
12. TAYLOR, A. Flow measurement as an afterthought. *Meas. and Control*, vol. 9. Jun. 1976. pp. 207-208.
13. BAILEY, S.J. Flowmeters '75 : Many for on-stream use, some for effluents. *Control Engineering*. Nov. 1975. pp. 34-39.

14. BAILEY, S.J. Flowmeasurement '74. *Control Engineering*. May, 1974. pp. 36-40.
15. WILSON, M.P. A survey of mass flowmeters. *Instrumentation Technology*. Sep. 1971. pp. 63-68.
16. HAYWARD, A.T.J. Flowmeter choice. *Engineering*, Apr. 1977. pp. 307-310.
17. WEBB, A.S. Electro-magnetic flowmetering ... basics, products and applications. *Instrumentation Technology*. Mar. 1974. pp. 29-36.
18. BAKER, R.C. Some recent developments in ultra-sonic and electro-magnetic flow measurements at Imperial College. *IMEKO VII B.P.L.* 246. pp. 1-10.
19. MARKS, B. The evolution of the magnetic flowmeter. *Pulse*. Nov. 1977. pp. 14-19.
20. BLOCH, F. Nuclear induction. *Physical Review*, vol. 70, nos 7 and 8. Oct. 1946.
21. BATTOCLETTI, J.H., *et al.* Analysis of the n.m.r. blood flowmeter for pulsating flow. *IEEE Trans on Biomedical Engineering*, vol. BME19, no. 6. Nov. 1972. pp. 403-407.
22. BATTOCLETTI, J.H., *et al.* N.m.r. detection of low magnetization levels in flowing fluids. *IEEE Trans. on Magnetics*, vol. MAG-9, no. 3. Sep. 1973. pp. 451-456.
23. GENTHE, W.K., *et al.* N.m.r. applied to flow measurement. *Instrumentation Technology*. Nov. 1968. pp. 53-58.
24. VAN DER HEYDEN, W.H., and TOSCHIK, J.F. A n.m.r. flowmeter, a new simplified design. *Paper no. 2-7-180, ISA Flow Symposium, Pittsburg, Pennsylvania*. May, 1971.
25. MORSE, O.C., and SINGER, J.R. Blood velocity measurements in intact subjects. *Science*, vol. 17. Oct. 1970. pp. 440-441.

26. ROUGHTON, T.E., and CROSSE, P.A.E. A 'thermal transient' instrument for the measurement of flow in pipes by external means. *Journal of Physics E : Scientific Instruments*, vol. 7. 1974. pp. 621-626.
27. BENSON, J.M. Thermal flow sensors. *Instrumentation Technology*. Jul. 1971. pp. 39-43.
28. TARONI, A., and ZANARINI, G. Dynamic behaviour of thermistor flowmeters. *IEEE Trans. on Industrial Elec. and Control Instrum.*, vol. IECI-22, no. 3. Aug. 1975. pp. 391-396.
29. SADEH, W.Z., MAEDER, P.F., and SUTERA, S.P. A hot-wire method for low velocity with large fluctuations. *Review of Scientific Instruments*, vol. 41, no. 9. Sep. 1970. pp. 1295-1298.
30. RHODES, D.F. Measuring flow with radiotracers. *Instrumentation Technology*. Oct. 1975. pp. 43-48.
31. CLAYTON, C.G. Precise tracer measurements of liquid and gas flows *Nucleonics*. Jul. 1960. pp. 96-100.
32. WATKIN, T.D. Tracer techniques used in flow measurements. *A.S.M.E. Flow. Meas. Symposium, Pittsburgh, U.S.A.* Sep. 1966. pp. 139-148.
33. MANN, U., and CROSBY, E.J. Flow measurement of coarse particles in pneumatic conveyors. *Ind. Eng. Chem. Process Des. Dev.*, vol. 16, no. 1. 1977. pp. 9-13.
34. DECKER, M.M. The gyroscopic mass flowmeter. *The Engineer's Digest*, vol. 21, no. 7. Jul. 1960. pp. 92-93.
35. PAPADATOS, K., and SURCEK, W.Y. Low range flowmeter for pulverized solids. *Instrumentation Technology*. Aug. 1974. pp. 38-40.
36. TOMITA, Y., OZAWA, H., and HORIUCHI, T. Measurement of flow rate of solid particles by flow noise. *IMEKO VII, BFL/244*. pp. 1-10.
37. GERRARD, D. Ultrasonic flowmetering - the final challenge to the orifice plate. *Control and Instrum.* Feb. 1978. pp. 25-27.

38. McSHANE, J.L. Ultra-sonic flowmeter basics. *Instrumentation Technology*. Jul. 1971. pp. 44-47.
39. TORRANCE, J.W. Confusion and delusion in sonic flowmetering. *Pulse*. Mar. 1976. pp. 30-33.
40. LIPTAK, B.G., and KAMINSKI, R.K. Ultra-sonic instruments for level and flow. *Instrumentation Technology*. Sep. 1974. pp. 49-59.
41. LYNNWORTH, L.C. Industrial applications of ultrasound - a review. *IEEE Trans. on Sonics and Ultrasonics*, vol. SU-22, no. 2. Mar. 1975. pp. 71-100.
42. BARONE, A., and PAPPALARDO, M. Method for measuring the velocity of a fluid flow by means of electro-acoustic feedback. *Acustica*, vol. 24. 1971. pp. 211-215.
43. BROWN, A.E. Dual path ultra-sonic measurement of fluid flow. *Review of Scientific Instruments*, vol. 27, no. 9. Sep. 1966. pp. 1181-1186.
44. MCCARTHY, K., and WOODCOCK, J.P. Frequency modulated ultra-sonic doppler flowmeter. *Medical and Biological Engineering*. Jan. 1975. pp. 59-64.
45. BENDICK, P., and NEWHOUSE, V.L. Ultra-sonic random signal flow measurement system. *J.A.S.A.*, vol. 56, no. 3. Sep. 1974. pp. 860-865.
46. BESTOBELL INFORMATION NOTE. Ultra-sonic flow measurement of liquids or slurries in pipes.
47. KATO, K., KAKUSHO, O., and IZUMI, T. A new ultra-sonic doppler flowmeter that can detect flow direction. *The 6th I.C.A., Tokyo, Japan*. Aug. 21-28, 1968. M-1-5.
48. NIPPA, J., *et al.* Phase rotation for separating forward and reverse blood velocity signals. *IEEE Trans. Sonics and Ultra-sonics*, vol. SU-22, no. 5. Sep. 1975. pp. 340-346.

49. DE JONG, D.A., *et al.* A directional quantifying doppler system for measurement of transport velocity of blood. *Ultrasonics*. May 1978. pp. 138-141.
50. SAINZ, A., ROBERTS, V.C., PINARDI, G. Phase-locked loop techniques applied to ultra-sonic doppler signal processing. *Ultrasonics*. May, 1976. pp. 128-132.
51. LYNWORTH, L.C. Clamp-on ultra-sonic flowmeters ... limitations and remedies. *Instrumentation Technology*. Sep. 1975. pp. 37-44.
52. BAUMOEL, J. A comparative analysis of CONTROLOTRON clamp-on ultra-sonic flowmeter performance. *Pulse*. Jun. 1977. pp. 13-17.
53. ALBRIGHT, R. Relationship of doppler ultra-sonic scattered signal characteristics to flow and beam parameters. *J.A.S.A.*, vol. 59, no. 4. Apr. 1976. pp. 786-789.
54. MCCARTNEY, M.A. Phase-locked loops in ultra-sonic anemometry. *Acustica*, vol. 33. 1975. pp. 352-354.
55. OKUJIMA, M., and OHTSUKI, S. M-sequence modulated ultra-sonic doppler velocity meter. *7th I.C.A., Budapest*. 1971. 20-U-7. pp. 457-461.
56. ASSENZA, D., and PAPPALARDO, M. One path ultra-sonic flowmeter using electro-acoustic feedback. *7th I.C.A., Budapest*. 1971. 21-U-12. pp. 521-523.
57. GREENSPAN, M., and TSCHIEGG, C.E. Sing-around ultra-sonic Velocimeter for liquids. *Rev. Sci. Instrum.*, vol. 28, no. 11. Nov. 1957. pp. 897-901.
58. GESSNER, U. The performance of the ultra-sonic flowmeter in complex velocity profiles. *IEEE Trans. Bio-med. Eng.*, vol. BME-16, no. 2. Apr. 1969. pp. 139-142.
59. SCOTT, M.J. Doppler ultra-sonic flowmeters. *Electronics and Instrumentation*. Feb. 1979. pp. 46-50.

60. ATKINSON, P. An ultra-sonic fluctuation velocimeter. *Ultrasonics*. Nov. 1975. pp. 275-278.
61. PEDERSEN, N.E. Gas industry fluids absolute phase key to better flowmeter. *Ultrasonics*. Jul. 1977. pp. 147-148.
62. LOMAS, D.J. Vortex shedding flowmeter senses pressure fluctuations. *Control Engineering*. Mar. 1975. pp. 40-41.
63. CAMPBELL, R.J. New vortex-shedding flowmeter uses ultra-sonics. *Control Engineering*. Jul. 1973. p. 57.
64. ANON. Vortex shedding applied to flow metering. *Meas. and Control*, vol. 6. May, 1973. p. 179.
65. ANDREIEV, N. Fluidic oscillator measures flow. *Control Eng.* Sep. 1973. pp. 56-57.
66. HAYWARD, A.T.J. A linear orifice-plate flowmeter. *Journal of Physics E : Sci. Instr.*, vol. 9. 1976. pp. 440-442.
67. BEALE, R.B., and LAWLER, M.T. Development of a wall-attachment fluidic oscillator applied to volume flow metering. *ISA Flow Symposium, Pittsburg, Pennsylvania*. May 1971.
68. LORENZI, A. Evaluating the design of a prototype linear resistance flowmeter. *Control and Instrum.* Oct. 1975. pp. 24-27.
69. FISHER AND PORTER; MAGX Magnetic flow meter.
70. ANGLO AMERICAN ANNUAL REPORT, 1978.
71. SPILLER, R.L. The art and practice of orifice plate flowmetering. *Instrumentation Tech.* Jul. 1971. pp. 52-56.
72. FRITSCH, R., and MESCH, F. Non-contact speed measurement - a comparison of optical signals. *Meas. and Control*, vol. 6. Jul. 1973. pp. 293-300.
73. HILL, R.D., and MCGUNIGLE. Quasi-correlation circuit for thermal pulse, time of flight flowmeter. *I.S.A. Flow Symposium; Pittsburg, U.S.A.* May 1971.

74. HENRY, R.M., and BECK, M.S. Minimum cost conveying of solids in pipes using cross-correlation measurement systems with adaptive control. *IMEKO VII BFL/250* 1-8.
75. MESCH, F., and KIPPHAN, H. Solids flow measurement by correlation methods. *OPTO-Electronics*, vol. 4. 1972. pp. 451-462.
76. BECK, M.S., HOBSON, J.H., and MENDIES, P.J. Measurement of mass and volume flows of slurries and gaseous suspensions. *Meas. and Control*, 1971. pp. 49-54.
77. TOMPKINS, W.R., MONTI, R., and INTAGLIETTA, M. Velocity measurement by self-tracking correlator. *Rev. Sci. Instrum.*, vol. 45, no. 5. May, 1974. pp. 647-649.
78. KIPPHAN, H. Bestimmung des Feststoff-Volumenstromes einer zwei-phasen Strömung mit Hilfe der Korrelations-technik. *Chemie-Ing. Techn.*, vol. 47, no. 3. 1975. p. 105.
79. BECK, M.S., and PLASKOWSKI, A. Inferential measurement of fluid flow from the cross-correlation of temperature fluctuations in the system. *Electronic Letters*, vol. 3, no. 4. Apr. 1967. pp. 169-171.
80. WERTHER, J., and MOLERUS, O. Autokorrelation und Kreuzkorrelation zur Messung lokaler Blasengröße und Aufstiegsgeschwindigkeit. *Chemie-Ing. Technik*, vol. 43, no. 5. 1971. pp. 271-273.
81. FLEMONS, R.S. A new non-intrusive flowmeter. *N.B.S. Special Publ. 484; Proc. of Symp. on Flow in Open Channels and Closed Conduits*, Gaithersburg, M.D., Feb. 1977. pp. 319-333.
82. MATTHES, W., RIEBOLD, W., and DE COOMAN, E. Measurement of the velocity of gas bubbles in water by a correlation method. *Rev. Sci. Instr.*, vol. 41, no. 6. Jun. 1970. pp. 843-845.

83. ABEYSEKERA, S.A., and BECK, M.S. Cross-correlation techniques applied to pulsating flow measurement. *Meas. and Control*, vol. 3. Jun. 1970. pp. T109-T112.
84. BECK, M.S., and WAINWRIGHT, N. Flow-failure detector for powdered and granular material. *Control*. Jan. 1969. pp. 52-56.
85. BECK, M.S., and ONG, K.H. Slurry flow velocity, concentration and particle size measurement using flow noise and correlation techniques. *Meas. and Control*, vol. 8. Nov. 1975. pp. 453-463.
86. MESCH, F., KIPPHAN, H., and FRITSCH, R. Transit-time correlation - a survey on its application to measuring transport phenomena. *Trans. A.S.M.E., Journal of Dynamic Systems, Meas. and Control*. Dec. 1974. pp. 414-420.
87. ASHTON, M.W., and BENTLEY, P.G. Design study for on-line flow measurement by transit time analysis of temperature fluctuations. *Source unknown*.
88. PIPLIES, L. Feststoffgeschwindigkeit bei Gas-Feststoff-strömungen in vertikalen Rohren. *Chem.-Ing. Technik*, vol. 44, no. 6. 1972. pp. 394-399.
89. BECK, M.S., CALVERT, G., HOBSON, J.H., LEE, K.T., and MENDIES, P.J. Total volume and component flow measurement in industrial slurries and suspensions using correlation techniques. *Meas. and Control*, vol. 4. Aug. 1971. pp. 133-138.
90. ABEYSEKERA, S.A., and BECK, M.S. Liquid flow measurement by cross-correlation of temperature fluctuations. *Meas. Control*, vol. 5. Nov. 1972. pp. 436-439.
91. COULTHARD, J. The principle of ultra-sonic cross-correlation flowmetering. *Meas. and Control*, vol. 8. Feb. 1975. pp. 65-70.

92. COULTHARD, J. Ultra-sonic cross-correlation flowmeters. *Ultrasonics*. Mar. 1973. pp. 83-88.
93. OCHS, G.R., and MILLER, G.F. Pattern velocity computers : Two types developed for wind velocity measurement by optical means. *Rev. Sci. Instr.*, vol. 43, no. 6. 1972. pp. 879-882.
94. TERMAAT, K.P. Fluid flow measurement inside the reactor vessel of the 50 MW Dodeward nuclear power plant by cross-correlation of thermo-couple signals. *Journal of Physics E. Sci. Instr.* vol. 3. 1970. pp. 589-593.
95. TOMPKINS, W.R., and INTAGLIETTA, M. Circuit for approximate computation of reciprocal of time delay to maximum cross-correlation. *Rev. Sci. Instr.*, vol. 42, no. 5. May 1971. pp. 616-618.
96. BECK, M.S., COULTHARD, J., HEWITT, P.J., and SYKES, D. Flow velocity and mass flow measurement using natural turbulence signals. *Int. Conf. on Modern Devel. in Flow Meas.* Sep. 1971.
97. ANON. Water Research Instrumentation: 2 *Meas. and Control*, vol. 10. Oct. 1977. pp. 364-365.
98. FLOW MEASUREMENT REPORT. New solutions to flow measurement problems. *Chem. Processing*, vol. 15. Nov. 1972. pp. 75-76.
99. ARUNDEL, P.A. Measurement of local solids velocity in a pneumatic conveyor. *Chem. Processing*, vol. 15. Nov. 1972. pp. 77-80.
100. ANON. Flow Measurement Report : New slurry flowmeter using cross-correlation. *Chem. Processing*, vol. 15. Nov. 1972. pp. 83-85.
101. KWAN, H.K., and BECK, M.S. Mass flow measurement of solids in gravity conveyors using capacitance and ultrasonic transducers. *IMEKO VII BFL/251/1-7*,
102. BECK, M.S. DRANE, J., PLASKOWSKI, A., and WAINWRIGHT, N. A new method of measuring the mass flow of powder in a pneumatic conveyor using an on-line computer. *IEE Conf. Publ.*, no. 43. Jun. 1968. pp. 133-144.

103. BECK, M.S., LEE, K.T., and STANLEY-WOOD, N.G. A new method of evaluating the size of solid particles flowing in a turbulent fluid. *Powder Techn.*, vol. 8. 1973. pp. 85-90.
104. CORTI, M., DE AGOSTINI, A., and DE GIORGIO, U. Fast digital correlator for weak optical signals. *Rev. Sci. Instr.*, vol. 45, no. 7. Jul. 1974. pp. 888-893.
105. MESCH, F., DAUCHER, H-H, and FRITSCH, R. Geschwindigkeitsmessung mit Korrelationsverfahren Teil I & II. *Messtechnik*, vol. 7. 1971. pp. 152-168.
106. KIPPHAN, H., and MESCH, F. Methods from system theory applied to measuring stochastic transport processes. *Regelungstechnik*, vol. 9. 1975. pp. 299-304.
107. LESLIE, C.B., and JOHNSTON, T.F. Relative performance of three types of correlators. *6th I.C.A., Tokyo*. 1968. P-0-24.
108. BYWATER, R.E.H., MATLEY, W., and BROCK, D. Design of flexible phase reversal modulation correlator. *The Radio and Elec. Eng.*, vol. 46, no. 3. 1976. pp. 129-135.
109. JORDAN, J.R., and KELLY, R.G. Integrated circuit correlator for flow measurement. *Meas. and Control.*, vol. 9. Jul. 1976. pp. 267-270.
110. JORDAN, J.R., and BECK, M.S. Correlation function display and peak detection. *Electronic Letters*, vol. 8, no. 24. 1972. pp. 602-604.
111. TAI, I., HASEGAWA, K., and SEKIGUCHI, A. A real-time correlator with peak detector. *Journal of Physics E : Sci. Inst.*, vol. 8. 1975. pp. 206-208.
112. HAYES, A.M., and MUSGRAVE, G. Correlator design for flow measurement. *The Radio and Elec. Eng.*, vol. 43, no. 6. 1973. pp. 363-368.

113. IMAI, S. On a simplified hybrid analogue-digital correlator for sound measurements. *6th I.C.A. Tokyo* 1968, E-1-4.
114. FANNIN, P.C. Low frequency correlator using bucket-brigate analogue delay line. *Proc. IEE*, vol. 122, no. 12. 1975. pp. 1363-1366.
115. FANNIN, P.C. Extending the application potential of low-frequency correlators. *Proc. IEE*, vol. 123, no. 8. 1976. pp. 734-736.
116. HOLLIN, K.A., JONES, M.H., and FOWWEATHER, F. Construction of an analogue correlator for acoustical measurement. *Acustica*, vol. 34. 1976. pp. 323-328.
117. CALVERT, G., LEE, K.T., and ONG, K.H. Experiments in correlation flowmetering. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn. Bradford*. Jan. 1973. pp. 1-10.
118. MYLROI, M.G. Cross-correlation flow measurement systems - a review of their development, their versatility and their future. *Inst. Meas. and Control, FLOW-CON. 77 Proc. of Symp. Application of Flow Measuring Techniques, Sussex, U.K.* Apr. 1977. pp. 17-28.
119. GOH, C.K., and BENTLEY, J.P. Investigation of cross-correlation techniques to measure low velocity flue gases. *Inst. Meas. and Control, FLOW-CON. 77 Proc. of Symp., Application of Flow Measuring Techniques, Sussex, U.K.* Apr. 1977. pp. 409-432.
120. COULTHARD, J., and BRADLEY, D. Ultra-sonic correlation flowmetering applied to single phase liquids and gases. *Inst. Meas. and Control, FLOW-CON. 77 Proc. of Symp., Application of Flow Measuring Techniques, Sussex, U.K.* Apr. 1977. pp. 433-450.
121. BULLOCK, L.E. Flow measurement using correlation analysis with pseudo-random tag injection. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn, Bradford*. Jan. 1973. pp. 19-26.

122. DAVIES, P.O.A.L., and MERCER, C.A. Phase velocity measurement using cross-power spectra. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn, Bradford*. Jan. 1973. pp. 37-46.
123. JOHNSON, P.A. Ultra-sonic transducer systems for flow measurement by cross-correlation. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn, Bradford*. Jan. 1973. pp. 73-78.
124. HEWITT, P.J., and KING, P.W. Electrostatic charge generation in flowing liquids and solids. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn, Bradford*. Jan. 1973. pp. 91-102.
125. ONG, K. Hydraulic flow measurement using ultra-sonic transducers and correlation techniques. Ph.D. thesis, Bradford, 1975.
126. HENRY, R.M., and BECK, M.S. Incipient fault detection in pipe-lines transporting solid material. *Source unknown*.
127. MESCH, F. Systematische Übersicht zur Anwendung von Korrelationsverfahren in der Messtechnik. *Symp. "Korrelationsmethoden in der Messtechnik" VDI/VDE Gesellschaft Mess. & Regelungstechnik, Frankfurt am Main*, 1976.
128. LEE, K.T., BECK, M.S., and McKEOWN, K.J. An on-line instrument for measuring small quantities of dispersed non-conducting liquid in a conducting liquid. *Ins. Meas. and Control*, vol. 7. Sep. 1974. pp. 341-345.
129. REED, E.W., ALI, M.A., and GILL, K.F. The measurement of pulsating gas flow velocities using pseudo-random pressure pulses. *Trans. Ins. Meas. and Control*, vol. 7. Sep. 1974. pp. 346-348.
130. REED, E.W., MOBBS, F.R., and GILL, K.F. Measurement by correlation of small amplitude pressure wave propagation velocity in gas/liquid flow. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn, Bradford*, Jan. 1973. pp.

131. JORDAN, J.R. A correlation function and time-delay measuring system designed for large scale circuit integration. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn.*, Bradford. Jan. 1973. pp. 251-257.
132. INTAGLIETTA, M., and TOMPKINS, W.R. System for the measurement of velocity of microscopic particles in liquids. *IEEE Trans. Bio.-Med. Eng.*, vol. BME-18. Sep. 1971. pp. 376-377.
133. BUTTERFIELD, M.H., BRYANT, G.F., and DOWSING, J. A new method of strip-speed measurement using random waveform correlation. *Trans. Soc. Inst. Tech.*, vol. 13. Jun. 1961. pp. 111-123.
134. BECK, M.S., DRANE, J., PLASKOWSKI, A., and WAINWRIGHT, N. Particle velocity and mass flow measurement in pneumatic conveyors. *Power Technology*, vol. 2. 1968/69. pp. 269-277.
135. VOM STEIN, H.D., PFEIFER, H.J., and KOCH, B. Korrelationsmessungen an Gasströmungen mit Hilfe von Laserstrahlung. *Zeitschrift Für Angewandte Physik*, vol. 30, no. 2/3. 1970.
136. BENTLEY, P.G., and DAWSON, D.G. Fluid flow measurement by transit time analysis of temperature fluctuations. *Trans. Soc. Instrum. Tech.* Sep. 1966. pp. 183-193.
137. KASHIWAGI, H. Fluid velocity measurement by use of correlation technique. *Memoirs of Faculty of Engineering, Kumamoto University*, vol. XIX, no. 1. Feb. 1973. pp. 1-26.
138. LEITNER, J.R., and NAUDE, D.E.H. Slurry flowrate measurement using correlation. *S.A.I. Meas. and Control, Symposium on Metallurgical process Instrumentation, Randburg, South Africa.* Sep. 1978.
139. BOONSTOPPEL, F., VELTMAN, B., and VERGOUWEN, F. The measurement of flow by cross-correlation techniques. *IEEE Conf. Publ. no. 43. "Industrial Meas. Techn. for On-line Computers"*. Jun. 1968.

140. ROBERTS, P.A., and WEBB, P.J. Preliminary investigation into flow measurement of high temperature dust laden gases using correlation techniques. *Inst. Meas. and Control, Int. Symp. Meas. and Proc. Ident. by Correl. and Spectral Techn., Bradford.* Jan. 1973. pp. 27-36.
141. CIMORELLI, L., and EVANGELISTI, R. Experimental determination of the "slip-ratio" in a vertical boiling channel, under adiabatic conditions at atmospheric pressure. *Int. J. Heat Mass Transfer*, vol. 12. pp. 713-726.
142. BOURKE, P.J., DRAIN, L.E., and MOSS, B.C. Measurement of spatial and temporal correlation in turbulence in water by laser anemometry; *DISA Information no. 12.* 1971. pp. 17-20.
143. FISHER, M.J., and KRAUSE, F.R. The crossed-beam correlation technique. *J. Fluid Mech.*, vol. 28, Part 4. 1967. pp. 705-717.
144. FISHER, M.J., and KRAUSE, F.R. Remote sensing of local flow instabilities and turbulence. *A.S.M.E. Flow Meas. Symp. Pittsburgh, Pennsylvania.* Sep. 1966. pp. 171-193.
145. STEGEN, G.R., and VAN ATTA, C.W. A technique for phase speed measurements in turbulent flow. *J. Fluid Mech.*, vol. 42, Part 4. 1970. pp. 689-699.
146. KRAUSE, F.R., and WILSON, L.N. Optical crossed-beam investigation of local sound generation in jets. *NASA S.P.-189*, 1968. pp. 335-357.
147. FISHER, M.J., and DAVIES, P.D.A.L. Correlation measurements in a non-frozen pattern of turbulence. *J. Fluid Mech.*, vol. 18, no. 7. 1963. pp. 97-116.
148. KNAPP, C.H., and CARTER, G.C. The generalized correlation method for estimation of time delay. *IEEE Trans. Acoustics, Speech and Sig. Proc.*, vol. ASSP-24, no. 4. Aug. 1976. pp. 320-327.

149. WATTS, D.G. A general theory of amplitude quantization with applications to correlation determination. *IEE Monograph*, no. 481M. Nov. 1961. pp. 209-218.
150. WIDROW, B. A study of rough amplitude quantization by means of Nyquist sampling theory. *I.R.E. Trans. on CCT - Theory*, vol. CT-3, Dec. 1959. pp. 266-276.
151. WIDROW, B. Statistical analysis of amplitude quantized sampled-data systems. *A.I.E.E. Trans. Applic. and Ind.*, vol. 79, Part 2. Jan. 1961. pp. 555-568.
152. CHANG, K., and MOORE, D. Modified digital correlator and its estimation error. *IEEE Trans. Info. Theory*, vol. IT-16, no. 6. Nov. 1970. pp. 699-706.
153. JESPER, P., CHU, P.T., and FETTWEIS, A. A new method to compute correlation functions. *Int. Symp. Info. Theory, Brussels*. Sep. 1962. pp. 106-107.
154. MERCER, C.A. Note on digital estimation of correlation functions. *J. of Sound and Vib.*, vol. 27, no. 2. 1973. pp. 262-265.
155. VEIT, I. Anwendung der Korrelationstechnik in der Akustik and Schwingungstechnik. *Acustica*, vol. 35, no. 4. 1976. pp. 219-231.
156. OJALA, L., and RAUTANEN, E.T. On incremental algorithms for averaging and correlation computation. *IEEE Trans. Inst. and Meas.*, vol. IM-23, no. 1. Mar. 1974. pp. 90-94.
157. OWEN, E.W. Measurement of speed by cross-correlation using pulse frequency signals. *IEEE Trans. Ind. Elec. and Con. Instr.*, vol. IECI-25, no. 2. May 1978. pp. 167-172.
158. KIRLIN, R.L. Correlation and Fourier analysis in real-time, inexpensively. *Proc. IEEE*. Sep. 1976. pp. 1440-1442.

159. BROCH, J. On the measurement and interpretation of cross-power spectra. *6th I.C.A., Tokyo*. Aug. 1968. D-1-2.
160. LUBOW, B. Correlation entering new fields with real-time signal analysis. *Electronics*. Oct. 1966. pp. 75-81.
161. McDONNEL, A. and FORRESTER, J. Polarity coincidence technique for correlation function measurement and system response evaluation. *The Radio and Elec. Eng.*, vol. 40, no. 4. Oct. 1970. pp. 165-172.
162. KRAUSE, D.J., STEADMAN, J.W., and WILLIAMS, T.W. Effect of record length on noise induced error in the cross-correlation function. *IEEE Trans. Systems, Man and Cybernetics*, vol. SMC-2, no. 2. Apr. 1972. pp. 255-261.
163. KNOWLES, J., and TSUI, H.T. Correlating devices and their estimation errors. *Journal of Applied Physics*, vol. 38, no. 2. Feb. 1967. pp. 607-612.
164. SRIPAD, A.B., and SNYDER, D.L. A necessary and sufficient condition for quantization errors to be uniform and white. *IEEE Trans. on Acoustics, Speech, and Sign. Proc.*, vol. ASSP-25, no. 5. Oct. 1977. pp. 442-448.
165. VELTMAN, B.P., and KWAKERNAAK, T. Theorie und Technik der Polaritätskorrelation für die dynamische Analyse niederfrequenter Signale und Systeme. *Regelungstechnik*, vol. 9. 1961. pp. 357-364.
166. TUMFART, S. New instruments use probabilistic principles. *Electronics*. Jul. 1975. pp. 86-91.
167. TUMFART, S. Basic principles of stochastic-ergodic measuring techniques. *Norma Messtechnik Report*. May 1973.
168. SPILKER, J.J., and MAGILL, D.T. The delay-lock discriminator - an optimum tracking device. *Proc. I.R.E.* Sep. 1961. pp. 1403-1416.

169. NAKAMURA, S. A digital correlator using delta modulation. *IEEE Trans. Acoustics, Speech and Sig. Proc.*, vol. ASSP-24, no. 3. Jun. 1976. pp. 238-243.
170. VAN VLECK, J.H., and MIDDLETON, D. The spectrum of clipped noise. *Proc. IEEE*, vol. 54. 1966. pp. 2-19.
171. RUCHKIN, D.S. Error of correlation coefficient estimates from polarity coincidences. *IEEE Trans. Info. Theory*, vol. IT-11, Apr. 1965. pp. 296-297.
172. BERNDT, H. Correlation function estimation by a polarity method using stochastic reference signals. *IEEE Trans. Info. Theory*, vol. IT-14, no. 6. Nov. 1968. pp. 796-801.
173. HARTEBRODT, H. Schwingungen in extremalen Laufzeitfolgesystem mit Polaritätskorrelation. *Messen, Steuern, Regeln*, vol. 12, no. 6. Jun. 1969. pp. 245-250.
174. JACOBSON, S.A. The design of a stochastic-ergodic A-D convertor and its application to correlation measurement. B.Sc. (Eng) Thesis No. 43, Dept. of Elec. Eng. U.C.T. Nov. 1977.
175. NORMA SEM-ELECTRONIC CORRELATOR. PM4707E. Application note.
176. MACNAMARA, D.A. Digital delta-modulator correlator. B.Sc.(Eng) Thesis No. 23, Dept. of Elec. Eng. U.C.T. Nov. 1976.
177. WEINREB, S. Digital radiometer. *Proc. I.R.E.*, vol. 49, no. 9. Jun. 1961. pp. 1099-
178. ORSTEN, G.S.F. An analogue shift register. *Rev. Sci. Inst.*, vol. 41, no. 7. Jul. 1970. pp. 957-959.
179. New time-delay unit completes DISA time correlation analyzer. *DISA Info. no. 9*. Feb. 1970. pp. 37-40.
180. ANDERSON, G.E., and PERRY, M.A. A calibrated real-time correlator/averager/probability analyzer. *Hewlett-Packard Journal*, vol. 21. Nov. 1969. pp. 9-16.

181. HONEYWELL SAICOR CORRELATOR INSTRUCTION MANUAL.
182. HURLBUT, C.S. DANA'S MANUAL OF MINERLOGY, John Wiley and Sons, Inc.
183. SCHLICHTING, H. BOUNDARY LAYER THEORY. McGraw Hill Book Company, 1975.
184. WITTEK, G. Amplitudenmodulation von Ultraschallwellen durch turbulente Wasserströmungen. *Acustica*, vol. 27. 1972. pp. 7-14.
185. WITTEK, G. Modulation von Ultraschallwellen durch turbulente Wasserströmungen. *7th I.C.A. 20-U-3, Budapest*. 1971.
186. Lighthill, J. The Fourth Annual Fairy Lecture : The Propagation of Sound through Moving Fluids. *Journal of Sound and Vibration*, vol. 24, no. 4. 1972. pp. 471-492.
187. Lighthill, J. The Bakerian Lecture 1961. Sound generated aerodynamically.
188. Lighthill, J. On sound generated aerodynamically. I. General Theory. *Proc. Roy. Soc. London. A211*. 1952. pp. 564-587.
189. Lighthill, J. On sound generated aerodynamically. II. Turbulence as a source of sound. *Proc. Roy. Soc. London. A222*. 1954. pp. 1-32.
190. POWELL, A. Flow noise : A perspective on some aspects of flow noise and of jet noise in particular. Part I. *Noise Control Eng.*, vol. 8, no. 2. 1977. pp. 70-80.
191. POWELL, A. Flow noise : A perspective on some aspects of flow noise and of jet noise in particular. Part II. *Noise Control Eng.*, vol. 8, no. 3. 1977. pp. 109-119.
192. SIDDON, T.E. New correlation method for study of flow noise. *7th I.C.A., 25-N-12, Budapest*. 1971. pp. 533-537.
193. SCHMIDT, D.W., and TILMANN, P.M. Experimental study of sound-wave phase fluctuations caused by turbulent wakes. *J.A.S.A.*, vol. 47, no. 5. Part 2. 1970. pp. 1310-1324.
194. MIKHAILOV, I.G., and KOLTSOVA, I.S. Ultra-sonic scattering in heterogeneous systems. *7th I.C.A., 19-U-5, Budapest*. 1971. pp. 425-429.

195. ISHIMARU, A. Theory and application of wave propagation and scattering in random media. *Proc. IEEE*, vol. 65, no. 7. Jul. 1977. pp. 1030-1061.
196. FAVRE, A.J. Review on space-time correlations in turbulent fluids. *Trans. A.S.M.E. Journal of Applied Mechanics*. Jun. 1965. pp. 241-257.
197. WENZEL, A.R., and KELLER, J.B. Propagation of acoustic waves in a turbulent medium. *J.A.S.A.*, vol. 50, no. 3, Part 2. 1971. pp. 911-920.
198. DUNN, D.J. Turbulence and its effect upon the transmission of sound in water. *J. Sound and Vibration*, vol. 2, no. 3. 1965. pp. 307-327.
199. DAVIS, M.C. Coal slurry diagnostics by ultrasound transmission. *J.A.S.A.*, vol. 64, no. 2. Aug. 1978. pp. 409-410.
200. BULL, M.K. Wall-pressure fluctuations associated with subsonic turbulent boundary layer flow. *J. Fluid. Mech.*, vol. 28, Part 4. 1967. pp. 719-754.
201. ALLEGRA, J.R., and HAWLEY, S.A. Attenuation of sound in suspensions and emulsions : Theory and experiments. *J.A.S.A.*, vol. 51, no. 5. Part 2. 1972. pp. 1545-1564.
202. KRÄMER W., and DINKELACHER, A. Ultra-sonic study of turbulent water flow *6th I.C.A.*, K-5-1, Tokyo, 1968. pp. K41-K44.
203. SCHMIDT, D.W., and TILMANN, P.M. Über die Zirkulationsentwicklung in Nachläufen von Rundstäben. *Acustica*, vol. 27. 1972. pp. 14-22.
204. LIGHTHILL, J. On the energy scattered from the interaction of turbulence with sound or shock waves. *Proc. Camb. Phil. Soc.*, vol. 49. 1953. pp. 531-
205. CHOW, J.C.F. Dispersion and attenuation of sound in a highly concentrated suspension. *6th I.C.A.*, J-5-14. Tokyo. 1968. pp. J81-J84.
206. TALBOT, J.H. Method based on ultra-sonic attenuation. *J.S.A.I. Min. and Met.* Mar. 1973. pp. 265-268.

207. URICK, R.J. The absorption of sound in suspensions of irregular particles. *J.A.S.A.*, vol. 20, no. 3. May, 1948. pp. 283-289.
208. TAYLOR, G.I. The spectrum of turbulence. *Proc. Roy. Soc. London, Ser. A.*, vol. 164. 1938. pp. 476-490.
209. TAYLOR, G.I. Diffusion by continuous movement. *Proc. Lon. Math. Soc.*, vol. 20. 1920. pp. 196-212.
210. FAVRE, A.J., GAVIGLIO, J.J., and DUMAS, R. Space-time double correlations and spectra in a turbulent boundary layer. *J. Fluid. Mech.*, vol. 2. 1957. pp. 313-342.
211. TOWNSEND, A.A. The measurement of double and triple correlation derivatives in isotropic turbulence. *Proc. Camb. Phil. Soc.*, vol. 43. 1947. pp. 560-
212. HO, C., and KOVASZNY, L.S.G. Propagation of a coherent acoustic wave through a turbulent shear flow. *J.A.S.A.*, vol. 60, no. 1. Jul. 1976. pp. 40-45.
213. DINKELACHER, A., and STIEWITT, H. Modulation von Ultraschallwellen durch turbulente Wasserströmungen. *7th I.C.A., 20-U-4, Budapest.* 1971. pp. 449-452.
214. DEVIN, C. Survey of thermal, radiation and viscous damping of pulsating air bubbles in water. *J.A.S.A.*, vol. 31, no. 12. Dec. 1956. pp. 1654-1667.
215. BLUE, J.E., and McLEROY, E.G. Attenuation of sound in suspensions and gels. *J.A.S.A.*, vol. 44, no. 4. 1968. pp. 1145-1148.
216. CHOW, J.C.F. Attenuation of acoustic waves in dilute emulsions and suspensions. *J.A.S.A.*, vol. 36, no. 12. 1964. pp. 2395-2401.
217. DUYKERS, L.R.B. Sound attenuation in liquid/solid mixtures. *J.A.S.A.*, vol. 41, no. 5. 1967. pp. 1330-1335.
218. HAMPTON, L.D. Acoustic properties of sediments. *J.A.S.A.*, vol. 42, no. 4. 1967. pp. 882-890.

219. NISHI, R.Y. The scattering and absorption of sound waves by a gas bubble in a viscous liquid. *Acustica*, vol. 33, no. 2. 1975. pp. 65-74.
220. KAZYS, R., and LUKOSEVICIUS, A. Optimization of the piezo-electric transducer response by means of electrical correcting circuits. *Ultrasonics*. May, 1977. pp. 111-116.
221. FOSTER, F.S., and HUNT, J.W. The design and characterization of short pulse ultrasound transducers. *Ultrasonics*. May, 1978. pp. 116-122.
222. GOLL, J.H., AULD, B.A. Multilayer impedance matching schemes for broadbanding of water loaded piezo-electric transducers and high-Q electric resonators. *IEEE Trans. Sonics and Ultrasonics*, vol. SU-22, no. 1. Jan. 1975. pp. 52-53.
223. LAURA, P.A. Directional characteristics of vibrating circular plates and membranes. *J.A.S.A.*, vol. 40, no. 5. 1966. pp. 1031-1033.
224. PARROT, J. and STEPHENS, R.W.B. Ultra-sonic reflection at water-solid interfaces. *6th I.C.A., L-1-1, Tokyo*. Aug. 1968.
225. SCHNABEL, P. Dispersion of thickness vibrations of piezo-ceramic disc resonators. *IEEE Trans. Sonics and Ultrasonics*, vol. SU-25, no. 1. Jan. 1978. pp. 16-
226. FRY, W.J., and DUNN, F. Ultrasonic intensity gain by composite transducers. *J.A.S.A.*, vol. 34, no. 2. Feb. 1962. pp. 188-192.
227. RANZ-GUERRA, C., and RUIZ-AGUIRRE, R.D. Composite sandwich transducers with quarter-wavelength radiating layers. *J.A.S.A.*, vol. 58, no. 2. Aug. 1975. pp. 494-498.
228. REDWOOD, M. Experiments with the electrical analog of a Piezoelectric transducer. *J.A.S.A.*, vol. 36, no. 10. Oct. 1964. pp. 1872-1880.
229. KOSSOFF, G. The effects of backing and matching on the performance of Piezoelectric ceramic transducers. *IEEE Trans. on Sonics and Ultrasonics*, vol. SU-13. no. 1. Mar. 1966. pp. 20-30.

230. DEKKER, D.L., PIZIALI, R.L., and DOWG, E. Effect of boundary conditions on the u-s beam characteristics of circular discs. *J.A.S.A.*, vol. 56, no. 1. Jul. 1974. pp. 87-93.
231. SHAW, E.A.G., and SUJIR, R.J. Vibration patterns of loaded barium-titanate and quartz disks. *J.A.S.A.*, vol. 32, no. 11. Nov. 1960. pp. 1463-1467.
232. ZEMANEK, J. Beam behaviour within the nearfield of a vibrating piston. *J.A.S.A.*, vol. 49, no. 1, Part 2. 1971. pp. 181-191.
233. MEEKER, T.R. Thickness mode piezoelectric transducers. *Ultrasonics*. Jan. 1972. pp. 26-36.
234. GERBER, E.A., and SYKES, R.A. State of the art-quartz crystal units and oscillators. *Proc. IEEE.*, vol. 54, no. 2. Feb. 1966. pp. 103-
235. MINER, G.W., and LAURA, P.A. Calculation of the nearfield pressure induced by vibrating circular plates. *J.A.S.A.*, vol. 42, no. 5. 1967. pp. 1025-1030.
236. FOLDS, D.L. Speed of sound and transmission loss in silicone rubbers at ultra-sonic frequencies. *J.A.S.A.*, vol. 56, no. 4. Oct. 1974. pp. 1295-1296.
237. KIKUCHI, Y. Ultrasonic transducers. *7th I.C.A. 23-G-1. Budapest.* 1971.
238. HAFNER, E. The piezo-electric crystal unit - definitions and methods of measurement. *Proc. IEEE*, vol. 57, no. 2. Feb. 1969. pp. 179-201.
239. KINSLER, L.E., and FRY, A.R. FUNDAMENTALS OF ACOUSTICS. John Wiley and sons Inc.
240. BLITZ, J. FUNDAMENTALS OF ULTRASONICS. Butterworths, London 1963.
241. BLITZ, J. ULTRASONIC METHODS AND APPLICATIONS. Butterworths, London, 1971.
242. STEELE, R. DELTA-MODULATION SYSTEMS. Pentech Press, London, 1975.

243. MASSEY, B.S. MECHANICS OF FLUIDS, Van Nostrand Reinhard Company. 1975.
244. DAVIES, J.T. TURBULENCE PHENOMENA. Academic Press. 1972.
245. BENDAT J.S. and PIERSON, A.G. RANDOM DATA ANALYSIS AND MEASUREMENT PROCEDURES; John Wiley and Sons. Inc. 1971.
246. LANGE, F.H. CORRELATION TECHNIQUES. London Iliffe Books Ltd. 1967.
247. LEE, Y.W. STATISTICAL THEORY OF COMMUNICATION. John Wiley and Sons, New York. 1960.
248. PAPOULIS, A. PROBABILITY, RANDOM VARIABLES AND STOCHASTIC PROCESSES McGraw-Hill Book Co., New York. 1965.
249. KING, A. GOLD METALLURGY ON THE WITWATERSRAND. Transvaal Chamber of Mines, 1949.

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APPENDIX A

DERIVATION OF EXPRESSION FOR RECEIVED ULTRA-SONIC SIGNAL

In order to calculate the intensity of a received ultra-sonic beam, consider the system shown in Figure A.1 in which a collimated beam of radiation is assumed to propagate through a random medium in the x - direction (Reference 143). Assume that all the radiation that is emitted from the surface of a cross-section of the beam at point x , say, into the succeeding beam, arrives unchanged at the detector, that is, all radiation inputs between the cross-section and the detector are removed. Then

$$I(x, \dots) = I(x=x_{\text{receiver}}, \dots)$$

where

$I(x, \dots)$ radiative power emitted from beam cross-section at x

$I(x=x_{\text{receiver}}, \dots)$ - radiative power measured at the receiver

In this idealized case the surface $\Delta y \Delta z$, can be considered as the surface of a source emitting all the radiative power that was put into the receivers field of view before the cross-section at x .

The local change of radiative power inside the volume element adjacent to x can be obtained by repeating the measurement for a different cross-section at $x+\Delta x$ and by subtracting the result.

$$\Delta I = I(x+\Delta x, \dots) - I(x, \dots) = \frac{\partial I}{\partial x} \Delta x \dots \quad (\text{A.1})$$

This change in power is caused by acoustic absorption, scattering and refraction within the volume element ΔV . This loss of radiative power is proportional to the product of the incident radiative power $I(x, \dots)$ and the length Δx of the element i.e.,

$$\Delta I = - K(x, \dots) I(x, \dots) \Delta x$$

where $K(x, \dots)$ is the constant of proportionality that is called the extinction coefficient. Substituting into equation A.1 gives:

$$\frac{1}{I} \frac{\partial I}{\partial x} = - K(x, \dots)$$

The detected radiative power at position x is thus given by the radiation I_0

at the transmitter and the integral of the extinction coefficient of the medium in between:

$$I(x) = I_0 \text{EXP} \left[- \int_{x_{\text{transmitter}}}^x K(x, \dots) dx \right]$$

Similarly the radiative power at the receiver is given by:

$$I_{\text{receiver}} = I_0 \text{EXP} \left[- \int_{x_{\text{transmitter}}}^{x_{\text{receiver}}} K(x, \dots) dx \right]$$

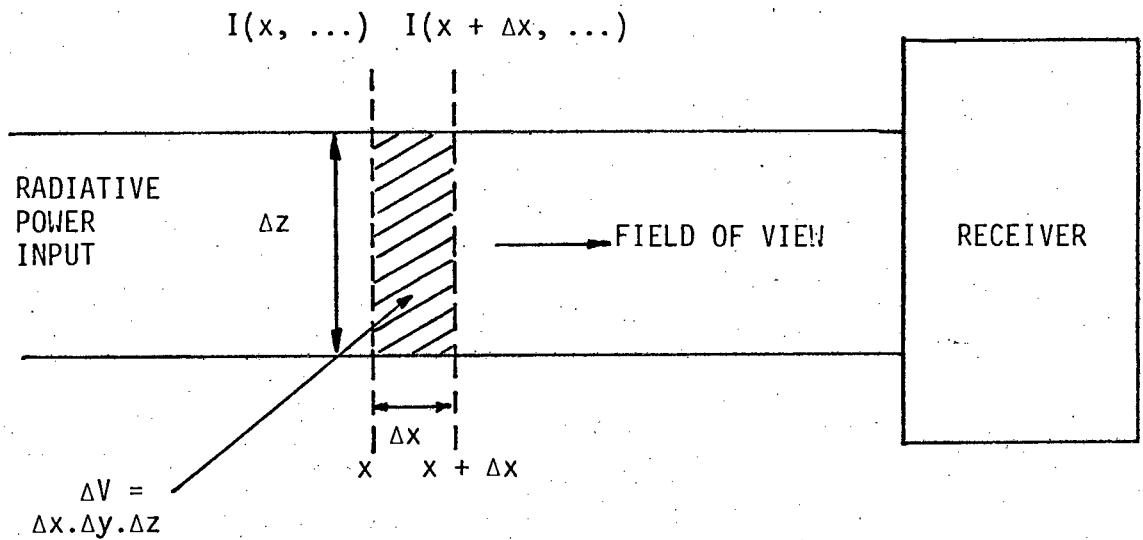


FIG. A1 DIAGRAM FOR DETERMINING RECEIVED ULTRA-SONIC RADIATION

APPENDIX B

PROPERTIES OF SONOX^(R) P5

SONOX P5

SONOX P5

SONOX® P5 ist eigens für Kleinleistungswandler geeignet und erlaubt mit seiner niedrigen Schwinggüte (Q_m ca. 90) einen breitbandigen Betrieb. Seine hohen Koppelfaktoren und Piezokonstanten eröffnen gleichzeitig eine vielseitige Anwendung für Körperschallsysteme und Hydroakustik, Luftschall und elektro-mechanische Wandler.

SONOX® P5 is especially suitable for low-power transducers and its low quality factor (Q_m approx. 90) permits a wide band application. Its high coupling factors and piezo-coefficients provide a versatile number of uses e. g. for solid-conducted sound systems, hydroacoustics, airborne sound and electro-mechanical transducers.

Eigenschaften:

Characteristics:

Betriebstemperatur 150° C max.

Working temperature 150° C max.

Alterungsrate in %
pro Zeitdekade

Ageing rate in %
per time decade

Frequenz $\Delta f_s / f_s$ 0,3

Frequency $\Delta f_s / f_s$ 0,3

Diel.-Zahl $\Delta \epsilon_r / \epsilon_r$ -1,5

Permittivity $\Delta \epsilon_r / \epsilon_r$ -1,5

Weitere Daten siehe Tabelle

For further data see table

»Physikalische Eigenschaften der piezokeramischen Werkstoffe SONOX®«

"Physical characteristics of SONOX® piezoceramic materials"

SONOX P5

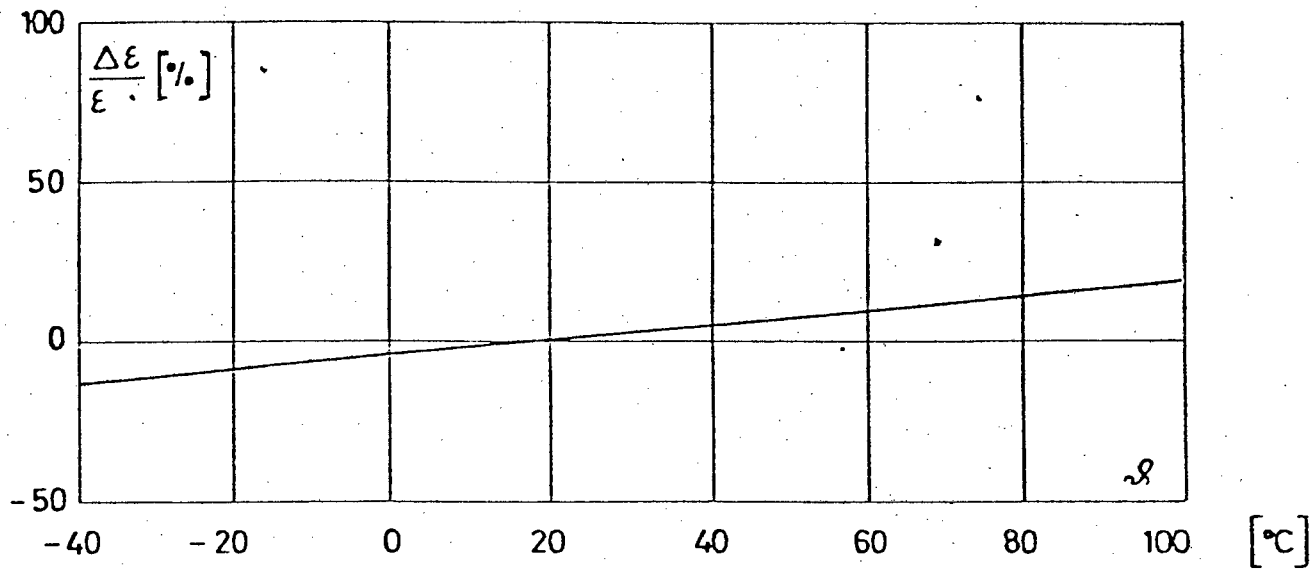
Temperaturabhängigkeit
an Planarschwingern:

SONOX P5

Temperature dependence
in planar vibrators:

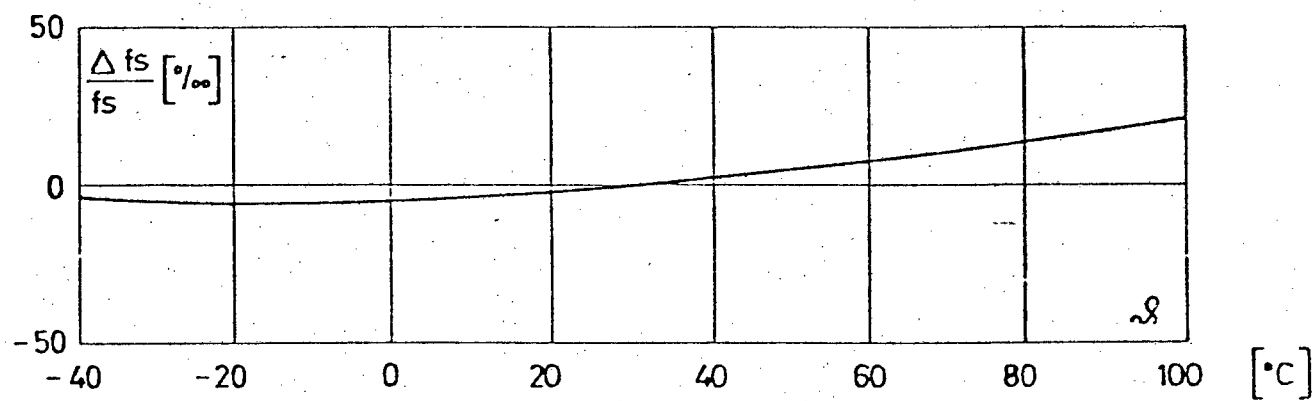
Dielektrizitätszahl $\epsilon_{33}^T/\epsilon_0$

Permittivity $\epsilon_{33}^T/\epsilon_0$



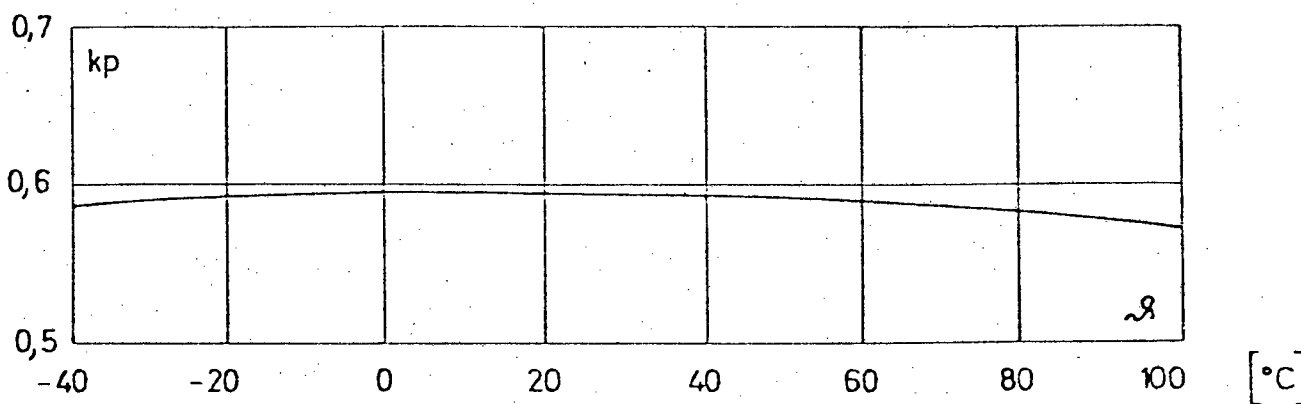
Serienresonanzfrequenz f_s

Series resonance frequency f_s



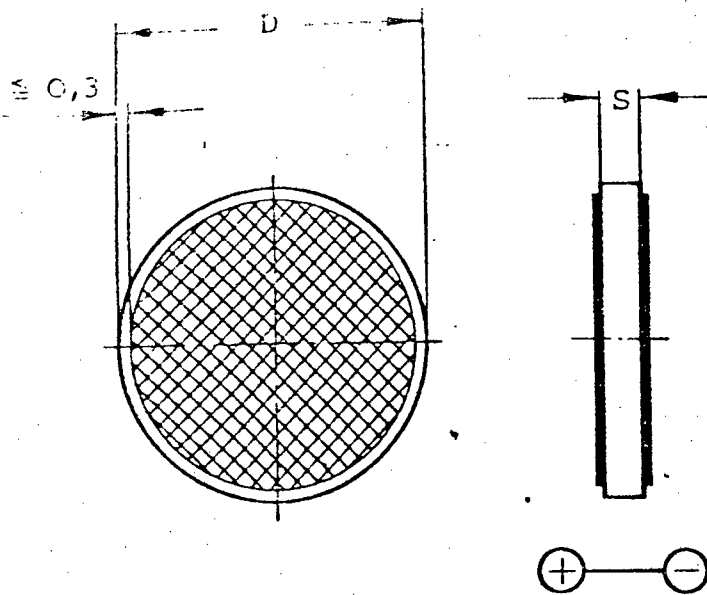
Koppelfaktor k_p

Coupling factor k_p



Anwendung	Beispiele für Werkstoffe SONOX®	P1	P2	P3	P4	P5	P51	P6	P61	P8
Ultraschall-Leistung	Reinigen, Entfetten Kunststoffschweißen Werkstoffbearbeitung Flüssigkeitszerstäubung			•	•					• •
Körperschall Hydroakustik	Materialprüfung Unterwasser-Meßsysteme Medizinische Diagnostik	• •		•	• • •	• • •	•			
Luftschall Elektroakustik	Mikrofone, Lautsprecher Ultraschall-Fernsteuerung Tonabnehmer				•	• •	• • •			
Elektronik	Bildabnehmer Akustische Verzögerungsleit. Filter Frequenzbest. Resonatoren		• •	• •		•		• •	•	
Hochspannungs- erzeugung	Zündung von Gasen Zündung von Feststoffen Piezotransformator			•	• •	•				•
Mechanisch-elektrische Wandler und Sondergebiete	Kraft- und Beschleunigungs- messer Mechanische Sensoren Tastaturen Drucker Piezorelais			•	• • • •	• • • •	• • • •			

Application	Examples for SONOX® materials	P1	P2	P3	P4	P5	P51	P6	P61	P8
High power ultrasonics	Degreasing, Cleaning Welding of Plastics Materials processing Atomisation of liquids			•	•					• •
Solid-conducted sound Hydroacoustics	Materials testing Underwater echo sounding Medical diagnosis	• •		•	• • •	• • •	•			
Airborne sound Electroacoustics	Microphones, Loudspeakers Ultrasonic remote control Pick-up cartridges				•	• •	• • •			
Electronics	Videovision cartridges Acoustic delay lines Filters Frequency controlling resonators		• •	• •		•		• •	•	
High-voltage generators	Gas ignition Solids ignition Piezoelectric transformers			•	• •	•				•
Mechanic-electrical transducers and Special fields	Force and acceleration meters Mechanical sensors Keyboards Printers Piezorelays			•	• • • •	• • • •	• • • •			



Scheiben
Dickenschwinger
Radialschwinger

Werkstoffe:

SONOX® P1, P2, P3,
P4, P5, P51, P6, P61, P8

Toleranzen:

Durchmesser D

Normalwerte:

$\pm 0,05$ mm bei $D \leq 20$ mm
 ± 2 % bei $D > 20$ mm

Bei Radialschwingern wird
die Toleranz von D durch die
Frequenztoleranz bestimmt.

Wanddicke S

$\pm 0,05$ mm

ohne Elektroden

Bei Dickenschwingern wird
die Toleranz von S durch
die Frequenztoleranz
bestimmt.

Frequenz f_s

± 10 % oder ± 20 %

Engere Toleranzen nach
Vereinbarung.

Für Dicken- und Radial-
schwinger.

Elektroden:

Silber eingebrannt.

Andere Elektrodenmetalle
auf Anfrage.

Wir bitten um Angabe, ob
gelötet wird.

Auf Wunsch liefern wir mit
Anschlußdrähten.

Sonderausführung:

Ab 8 mm Durchmesser
liefern wir halbierte
Scheiben mit 1 mm Trenn-
fuge.

Weitere Hinweise sind aus
dem technischen Teil des
Kataloges ersichtlich.

Bestellbeispiel:

500 Scheiben aus
SONOX® P4 ϕ 20 mm,
 $f_s = 2$ MHz ± 10 % lötlbar
versilbert.

Discs

Thickness Vibrators
Radial Vibrators

Materials:

SONOX® P1, P2, P3,
P4, P5, P51, P6, P61, P8

Tolerances:

Diameter D

Standard:

$\pm 0,05$ mm with $D \leq 20$ mm
 ± 2 % with $D > 20$ mm

With radial vibrators the
tolerance of D is determined
by the frequency tolerance.

Wall thickness S

$\pm 0,05$ mm

without electrodes

With thickness vibrators the
tolerance of S is determined
by the frequency tolerance.

Frequency f_s

± 10 % or ± 20 %

Closer tolerances on
request.

For thickness and radial
vibrators.

Electrodes:

Fired-on silver.

Other electrode metals on
request.

Please inform us, if parts
will be soldered.

On request we supply with
contact wires.

Special:

From 8 mm diameter we
supply half discs with 1 mm
parting line.

For further information see
technical section of the
catalogue.

Order example:

500 discs of SONOX® P4
 ϕ 20 mm,
 $f_s = 2$ MHz ± 10 %
solderable silver plated.

Werkstoffe:

SONOX® P1, P2, P3, P4,
P5, P6, P8.
Übersicht der lieferbaren
Durchmesser für die
gewünschten Frequenzen f_s
der Dickenresonanz bei
verschiedenen Werkstoffen.

Materials:

SONOX® P1, P2, P3, P4,
P5, P6, P8.
Table of diameters
available for desired
frequencies f_s of the
thickness resonance with
various materials.

Werkstoffe für Dicken-Resonanzfrequenzen

Materials for thickness resonance frequencies

D [mm]	f_s (kHz)								f_s (MHz)											
	150	200	250	300	400	500	600	800	1	2	4	5	6	8	10	12				
5	2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1	1	4 5	1 2 4 5	1 2	2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5				
7	1 4	1 4	1 4	1 4	4	4	1 4	1	1 4	1 4	1 4	1 4	1 4	1 4	1 4	1 4				
8	1 2 4	1 2 4	1	1	2 4	1 2 4	1 2	1	2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4				
10	1 2 4 5 8	1	1	2 4 5 8	1 2 4 5 8	1	1	2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8						
12	1 2	1 5	4 5	2 4 5	1 2	1	4 5	2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5						
15	1	4 5	2 4 5	1 2 4	1	4 5	2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5							
18		2 2	1 2	1		2 2	1 2	1 2	1 2	1 2	1 2									
20	4 5 8	2 4 5 8	1	1	2 4 5 8	2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8	1 2 4 5 8								
24	2 4 5	1 2	1	4 5	2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5	1 2 4 5								
25	2 4 8	1 2	1	4 8	2 4 8	1 2 4 8	1 2 4 8	1 2 4 8	1 2 4 8	1 2 4 8	1 2 4 8	1 2 4 8								
30	1 2 4	1 4	2 4	2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4										
35	1	1	4	1 4	1 4	1 4	1 4	1 4	1 4	1 4										
40	1	4 5	4 5	1 4 5	1 4 5	1 4 5	1 4 5	1 4 5	1 4 5	1 4 5										
50	4	4	1 4	1 4	1 4	1 4	1 4	1 4	1 4	1 4										
54		1	1	1	1	1	1	1	1											
60	2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2											
65		1	1	1	1	1	1	1	1											
68	1 4	1 4	1 4	1 4	1 4	1 4	1 4	1 4	1 4											
80	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4														

Die Ziffern 1 bis 8 stehen für:

1 SONOX® P1
2 SONOX® P2 und P6
4 SONOX® P3 und P4
5 SONOX® P5
8 SONOX® P8

The numbers stand for:

1 SONOX® P1
2 SONOX® P2 and P6
4 SONOX® P3 and P4
5 SONOX® P5
8 SONOX® P8

Andere Abmessungen und Frequenzen auf Anfrage.

Other sizes and frequencies on request.

	Symbol	Einheit Dimension	SONOX ² P1	SONOX	
Elektrische Eigenschaften Electrical Properties	1. Dielektrizitätszahl Permittivity	$\frac{T}{\epsilon_{33}/\epsilon_0}$	1	960	4
		$\frac{S}{\epsilon_{33}/\epsilon_0}$	1	790	2
		$\frac{T}{\epsilon_{11}/\epsilon_0}$	1	1050	8
		$\frac{S}{\epsilon_{11}/\epsilon_0}$	1	860	
2. Diel. Verlustfaktor bei 1 kHz Diel. dissipation factor at 1 kHz	$\tan \delta$	10^{-3}	7		
3. Spez. Durchg. Widerst. bei 25° C bei 200° C Volume resistivity at 25° C at 200° C	ρ_{25}	$\Omega \cdot \text{cm}$	10^{12}	11	
	ρ_{200}	$\Omega \cdot \text{cm}$	—	2.1	
4. Curie-Temperatur Curie temperature	T_c	°C	120	3	
Elektromechanische Eigenschaften Electro Mechanical Properties	5. Koppelfaktoren Coupling factors	K_p	1	—0,25	—0,
		K_{31}	1	—0,15	—0,
		K_{33}	1	0,43	0,
		K_{15}	1	0,42	0,
		K_t	1	0,33	
	6. Piezoelektr. Ladungskonstanten Piezoelectric charge coefficients	d_{31}	10^{-12}C/N	—40	—
		d_{33}	10^{-12}C/N	125	1
		d_{15}	10^{-12}C/N	205	
	7. Piezoelektr. Spannungskonstanten Piezoelectric voltage coefficients	g_{31}	10^{-3}Vm/N	—5	—
		g_{33}	10^{-3}Vm/N	14	
		g_{15}	10^{-3}Vm/N	22	
	8. Frequenzkonstanten Frequency constants	N_p	kHz · mm	3200	24
		N_t	kHz · mm	2600	21
		N_s	kHz · mm	—	9
		N_1	kHz · mm	2300	18
		N_3	kHz · mm	2300	16
Mechanische Eigenschaften Mechanical Properties	9. Dichte Density	γ	kg/dm ³	5,3	7
	10. Elastizitätskonstanten Elastic compliances	s_{11}^E	$10^{-12} \text{m}^2/\text{N}$	9	
		s_{33}^E	$10^{-12} \text{m}^2/\text{N}$	9	
		s_{55}^D	$10^{-12} \text{m}^2/\text{N}$	26	
		s_{11}^E	$10^{-12} \text{m}^2/\text{N}$	—	
		s_{33}^D	$10^{-12} \text{m}^2/\text{N}$	8	
		s_{55}^D	$10^{-12} \text{m}^2/\text{N}$	21	
	11. Poissonsche Querz. Poisson's ratio	σ		~0,3	~0
	12. Mech. Schwinggüte rad. Mech. quality factor rad.	Q_m	1	350	4

SONOX® P3	SONOX® P4	SONOX® P5	SONOX® P51	SONOX® P6	SONOX® P61	SONOX® P8
1100	1300	1600	3000	570	760	1000
580	680	800	1420	490	—	540
1210	1480	—	—	—	—	1160
580	870	—	—	—	— ^S	880
6	5	20	10	4	8	2
$4 \cdot 10^{12}$	10^{12}	10^{12}	$5 \cdot 10^{11}$	$3 \cdot 10^{12}$	10^{12}	10^{12}
$2 \cdot 10^8$	—	10^{10}	10^{10}	$7 \cdot 10^8$	10^9	—
345	325	340	200	430	—	290
—0,47	—0,55	—0,57	—0,60	—0,34	—0,43	—0,50
—0,30	—0,31	—0,33	—0,33	—0,19	—0,25	—0,30
0,60	0,68	0,70	0,71	—	—	0,60
0,60	0,64	0,67	—	0,39	—	0,50
—	0,50	0,47	—	0,38	—	—
—100	—130	—160	—210	—40	—	—95
240	280	340	510	—	—	215
305	450	—	—	—	—	290
—10	—11	—11	—8	—8	—	—11
24	26	24	19	—	—	25
37	34	—	—	—	—	28
2300	2210	2030	2000	2580	2400	2280
2000	1950	1900	1930	2200	—	2020
940	—	—	—	1180	—	—
1640	1600	1400	1490	1860	1690	1680
1530	1570	1500	1400	1880	—	1600
7,8	7,8	7,7	7,4	7,7	7,8	7,7
12	13	16	15	9	—	11
17	17	19	19	—	—	14
36	37	—	—	—	—	33
11	—	14	14	9	—	10
9	8	10	10	—	—	9
21	22	—	—	—	—	25
~0,3	~0,3	~0,3	~0,3	~0,3	~0,3	~0,3
310	400	90	100	1100	600	1000

APPENDIX C

THEORY OF AMPLITUDE QUANTIZATION

C.1 THE GENERAL QUANTIZER

A quantizer is a non-linear operator having the general input-output relation shown in Figure C.1a. Figure C.1b shows the special case of a unit quantizer from which the general quantizer can be derived by including gain factors and level shifts. The mathematical expression for the relationship between the input x and output x' of the general quantizer is a conditional one such that for:

$$aq + (n - \frac{1}{2})q < x \leq aq + (n + \frac{1}{2})q$$

$$x' = (n + c) r \quad n = 0, \pm 1, \pm 2, \dots$$

The output x' is thus a single-valued function of the input x . An input lying somewhere within a quantization 'box' of width q will yield an output corresponding to the centre of that box i.e. the input is rounded off to the centre of the box. The input to the quantizer can be either a continuous signal or a sampled signal.

C.2 RELATIONSHIP BETWEEN INPUT AND OUTPUT P.d.f.

The analysis that follows will be developed for sampled signals because high-order random processes are more easily described in sampled form. If the samples of a continuous variable x are all independent of each other, a first order probability density function (p.d.f.), $w_x(x)$, completely describes the process.

Quantization is like sampling in amplitude (Reference 150 and 151). It is a sampling process that acts not upon the function itself but upon its p.d.f. For consider a function having the p.d.f. shown in Figure C.2 applied to a quantizer. The input variable may take on a continuum of amplitudes, while the output assumes only discrete amplitudes. The p.d.f. of the output, $w_{x'}(x')$ consists of a series of impulses that are uniformly spaced along the amplitude axis, each one centered in a quantization box as shown in Figure C.2. Any input event (signal amplitude) occurring within a given quantization box is 'reported' as being at the centre of that box. Each impulse of the quantized p.d.f. must have an area (weight) equal to the area under the input p.d.f., $w_x(x)$ within the bounds of the corresponding quantization box. The probability of the quantized variable being at a certain level is equal to the total probability of the input lying within the corresponding quantization box. In other words, the p.d.f. of the quantizer output, $w_{x'}(x')$, consists of area samples of the input p.d.f. $w_x(x)$. The quantizer may be thought of as an area sampler acting upon the 'signal'

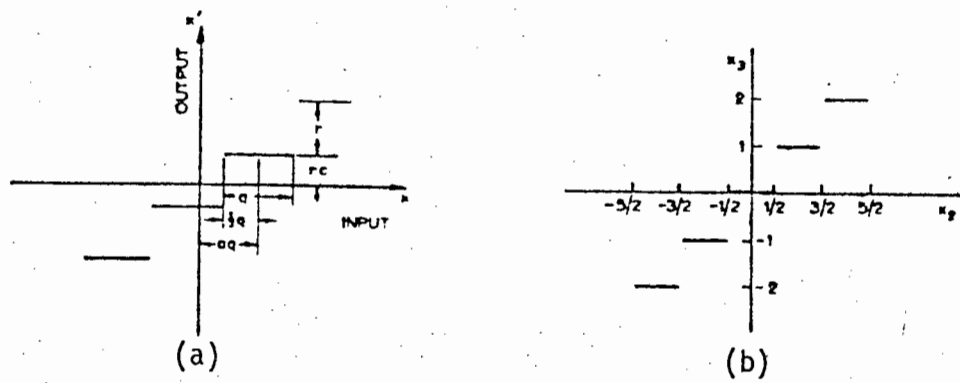


FIG. C.1 Quantizer input/output characteristics
a) General quantizer
b) Unit quantizer

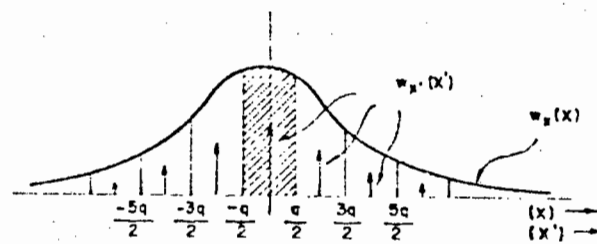


FIG. C.2 Area sampling of input p.d.f.

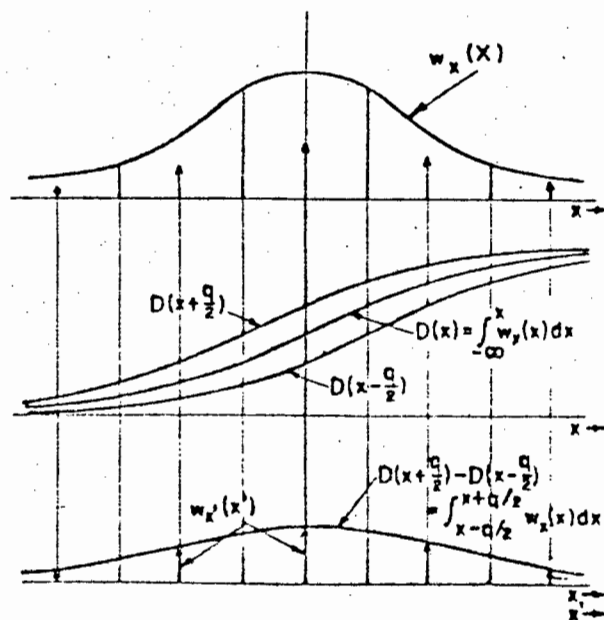


FIG. C.3 Construction of area samples

the input p.d.f.

Figure C.3 shows how $w_{x'}(x')$ may be constructed by sampling the difference function

$$h(x) = d(x + q/2) - d(x - q/2)$$

where $d(x)$ is the probability density distribution, the integral of the p.d.f. The difference function $h(x)$ may be written as follows:

$$h(x) = d(x + q/2) - d(x - q/2)$$

$$= \int_{x - q/2}^{x + q/2} w_x(x) \cdot dx = \int_{-\infty}^{\infty} w_x(x_1) \cdot f(x - x_1) dx_1$$

$$= w_x(x) \otimes f(x)$$

where $f(x)$ is a window function given by

$$f(x) = \begin{cases} 1 & \text{for } -q/2 < x < q/2 \\ 0 & \text{elsewhere} \end{cases}$$

and $\otimes$ denotes the convolution integral. The difference function $h(x)$, is thus obtained by convolving the input p.d.f. with a window function, $f(x)$ of the form given above. The p.d.f. of the quantizer output $w_{x'}(x')$ is finally obtained by impulse modulating the difference function with equally spaced unit impulses, i.e.

$$w_{x'}(x') = \sum_{n=-\infty}^{\infty} \left[\int_{-\infty}^{\infty} w_x(x) \cdot f(x' - x) \cdot dx \right] \cdot \delta(x' - n)$$

where $\sum_{n=-\infty}^{\infty} \delta(x' - n)$ is the modulating function consisting of equally spaced unit Dirac-delta impulses. The above expression for the output p.d.f. applies to the unit quantizer. The expression for the output p.d.f. of the general quantizer is derived in Reference 62 and is as follows:

$$w_{x'}(x') = \frac{1}{r} \sum_{n=-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} w_x(x) \cdot f \left[\frac{x'}{r} - c - \left(\frac{x}{q} - a \right) \right] dx \right\} \delta \left(\frac{x'}{r} - c - n \right)$$

As before, the output p.d.f. $w_{x'}(x')$ is seen to be the input p.d.f. $w_x(x)$ convolved with the window function $f(x)$ and the resulting convolution function impulse modulated with uniform spacing.

C.3 RELATIONSHIP BETWEEN INPUT AND OUTPUT CHARACTERISTIC FUNCTION

Further insight into quantization can be obtained by examining its effect in the characteristic function (c.f.) domain. By definition the characteristic function $W_X(u)$ of a variable x with p.d.f. $w_X(x)$ is the Fourier transform of its p.d.f., i.e.,

$$W_X(u) = \int_{-\infty}^{\infty} w_X(x) \cdot e^{-j \cdot x \cdot u} dx$$

Straight forward application of the formula to the expression for the p.d.f. of the quantizer output gives (Ref.149):

$$W_{X'}(u) = \sum_{k=-\infty}^{\infty} e^{-ju(c-a)} e^{-j2\pi ka} W_X\left(\frac{ur}{q} - \frac{2\pi k}{q}\right) \frac{\sin \frac{1}{2} (ur - 2\pi k)}{\frac{1}{2} (ur - 2\pi k)}$$

For the unit quantizer this becomes:

$$W_{X'}(u) = \sum_{k=-\infty}^{\infty} W_X(u - 2\pi k) \frac{\sin \frac{1}{2} (u - 2\pi k)}{\frac{1}{2} (u - 2\pi k)}$$

where $W_X(u)$ is the c.f. of the input p.d.f. and $\frac{\sin \frac{1}{2} u}{\frac{1}{2} u}$ is the Fourier transform of the window function $f(x)$.

The c.f. of the output is thus a repeated version of the c.f. of the input, modified by a factor of the form $\sin u/u$ as shown in Figure C.4. This is similar to the time-sampling case in which the spectrum of the output of the sampler is a repeated version of the spectrum of the input.

When the quantization frequency or fineness $\phi = 2\pi/q$ is twice as high as the highest frequency component contained in the shape of $w_X(x)$, the periodic sections of $W_{X'}(u)$ do not overlap and it is possible to recover $w_X(x)$ from the quantized distribution $w_{X'}(x')$.

This result is called the quantizing theorem, c.f. Nyquist theorem in time sampling case.

Consider a typical section of the quantized c.f. which is the product of two factors:

$$W_X(u) \cdot \frac{\sin (\pi u / \phi)}{(\pi u / \phi)}$$

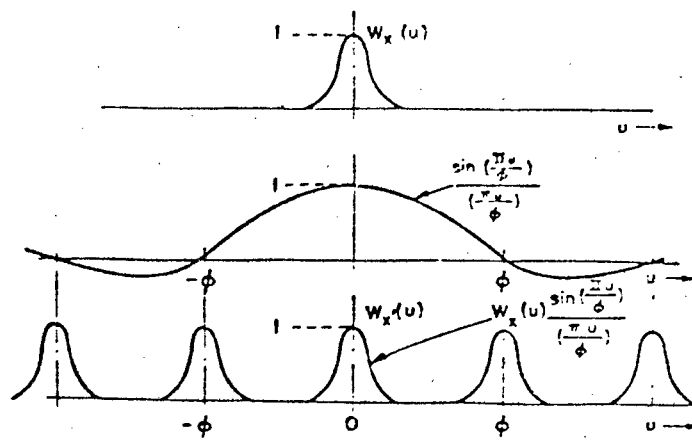


FIG. C.4 Formation of quantized characteristic function

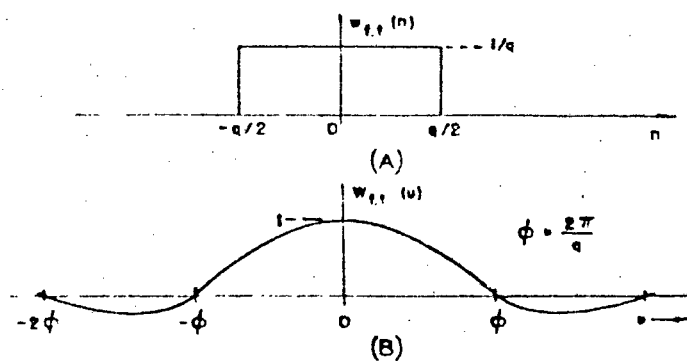


FIG. C.5 P.d.f. and c.f. of quantization noise

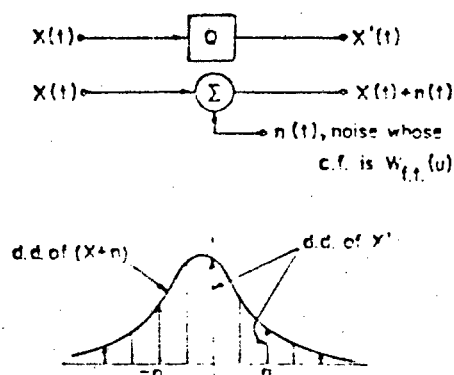


FIG. C.6 Comparison of quantization with noise addition

It is a well known fact that the C.F. of the sum of two statistically independent variables is the product of the individual C.F.'s. The typical section is therefore identical to the C.F. of the sum of the quantizer input and a statistically independent noise where C.F. is:

$$W_N(u) = \frac{\sin(qu/2)}{(qu/2)} = \frac{\sin(\pi u/\phi)}{(\pi u/\phi)} ; \phi = \frac{2\pi}{q}$$

This noise will ultimately be linked to quantization noise which has the flat topped p.d.f. as shown in Figure C.5.

It is interesting to compare quantization with the addition of an independent random noise whose p.d.f. is $w_N(x)$. This comparison is indicated in Figure C.6. The p.d.f. of the output is a series of impulses whose spacing is q . The impulse p.d.f. consists of impulse samples of the continuous p.d.f. under all conditions. The continuous p.d.f. can be uniquely derived from the impulse p.d.f. only when the quantizing theorem is satisfied.

It is a well known fact that the moments of a random variable are given by the derivatives of its C.F. at the origin ($u = 0$). For example, the k - th moment of x or the average of x^k is given by:

$$\overline{x^k} = (j)^{-k} \frac{d^k W_x(u)}{du^k} \bigg|_{u=0}$$

If the quantizing theorem is satisfied, the periodic sections of the quantized C.F. do not overlap and the derivatives of the quantized c.f. at the origin are the same as those of the typical section. Thus the moments of the two output signals in Figure C.6 would be the same.

Satisfaction of the quantizing theorem ensures that the moments of a quantized signal are the same as those of the sum of the unquantized signal and a statistically independent noise uniformly distributed between plus and minus half a quantization box.

C.4 P.d.f. OF QUANTIZATION NOISE

The investigation of the behaviour of the quantizer would be complete at this point (knowledge of the o/p C.F. that results for a given i/p C.F. would be sufficient to determine the noise C.F.) if it were true that quantization noise were statistically independent of the quantizer input.

The quantization noise signal is actually casually related to the i/p signal. Since the o/p of a quantizer is a single-valued function of the i/p, a given i/p yields a definite o/p and consequently a definite noise. The determination of the noise p.d.f. and C.F. is a separate problem.

Quantization noise may be regarded as the difference between an i/p variable and the value at the centre of the quantization box into which it falls. It is thus bounded and must under all conditions be distributed in some fashion between plus and minus half a box.

The distribution of quantization noises resulting from an i/p within the zeroth box may be constructed by plotting $w_x(x)$ between $-q/2 < x \leq q/2$. The noise distribution resulting from inputs within the first box may be obtained by considering $w_x(x)$ for values $q/2 < x < 3q/2$ recentered at the origin. Events taking place in the various boxes are exclusive to each other. The probability of a given noise magnitude arising is, therefore the sum of the probabilities of that noise arising from each box. Figure C.7 shows how the p.d.f. of the quantization noise may be constructed graphically from a plot of $w_x(x)$. This technique leads to a simple analytical derivation of the quantization noise C.F.

The summing process illustrated in Figure C.7 would be achieved in the following way. The p.d.f. $w_x(x)$ would be added to itself, shifted a distance q to the right, shifted $2q$ to the right etc., and shifted q to the left, $2q$ to the left etc. This infinite sum could then be multiplied by a window function which has the value unity over the range $-q/2 < x \leq q/2$ and zero elsewhere. The sum of the densities can be expressed as:

$$\sum_{m=-\infty}^{\infty} w_x(x + mq)$$

This expression can also be written as a convolution between $w_x(x)$ and a comb of unit impulses $\sum_{m=-\infty}^{\infty} \delta(x + m.q)$ spaced by a distance q from one another viz.

$$\begin{aligned} \sum_{m=-\infty}^{\infty} w_x(x + m.q) &= \int_{-\infty}^{\infty} w_x(x_1) \cdot \sum_{m=-\infty}^{\infty} \delta(x - x_1 + m.q) dx_1 \\ &= w_x(x) \otimes \sum_{m=-\infty}^{\infty} \delta(x + m.q) \end{aligned}$$

The transform of this sum is equal to ϕ times the impulse samples of $w_x(u)$ and is given by:

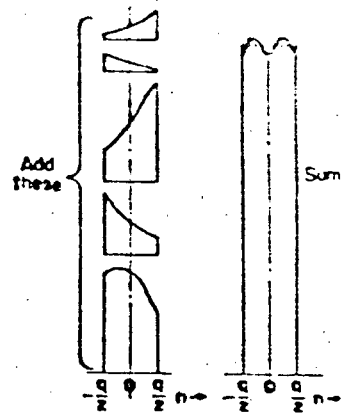
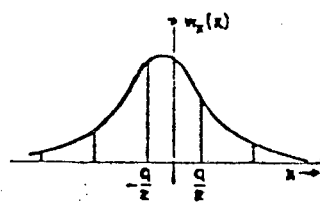


FIG. C.7 Construction of p.d.f. of quantization noise

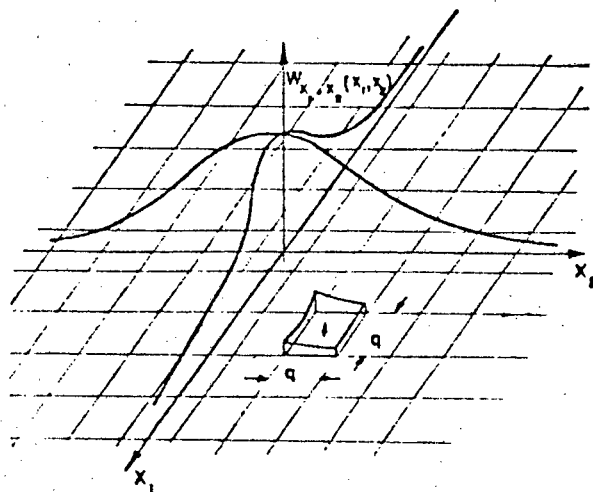


FIG. C.8 Volume sampling

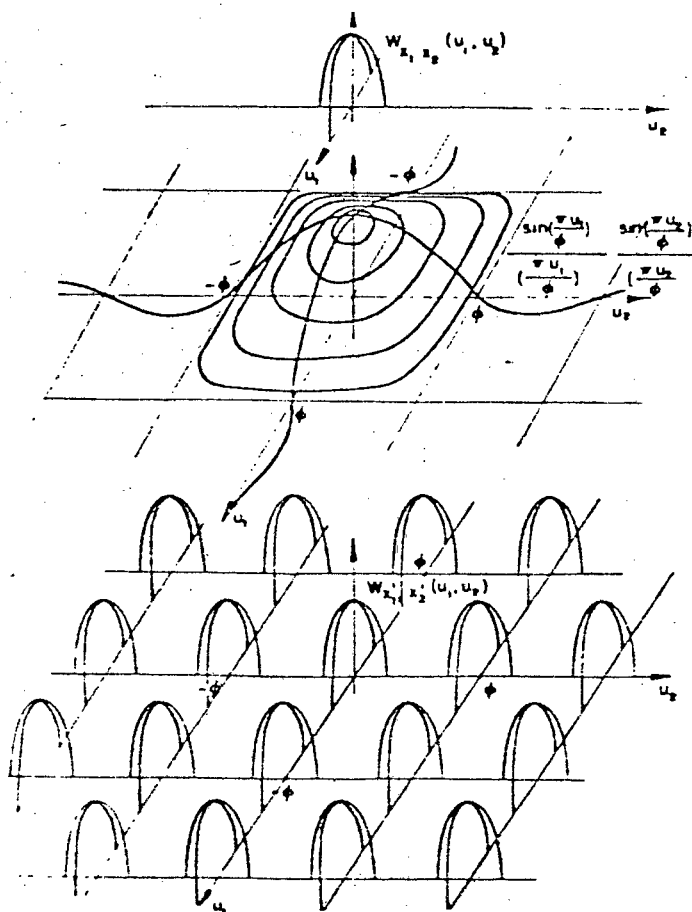


FIG. C.9 Formation of quantized two-dimensional characteristic function

$$\sum_{k=-\infty}^{\infty} \phi \cdot W_x(k \cdot \phi) \cdot \delta(u - k \cdot \phi)$$

Multiplication in the p.d.f. domain by the window function, $f(x)$, corresponds in the C.F. domain to convolution with

$$\frac{1}{\phi} \cdot \frac{\sin(\pi u / \phi)}{(\pi u / \phi)}$$

It follows, therefore that the C.F. of the quantization noise is:

$$W_n(u) = \sum_{k=-\infty}^{\infty} W_n(k \cdot \phi) \frac{\sin \pi(u/\phi - k)}{\pi(u/\phi - k)}$$

Now when the quantization theorem is satisfied the above sum has only one non-zero term which is

$$\frac{\sin(u\pi/\phi)}{(u\pi/\phi)}$$

This means that when the quantizing theorem is satisfied, the quantization noise has a $\sin u/u$ C.F. and a p.d.f. which is uniformly distributed between $\pm q/2$. A closer examination shows that the quantization noise will be precisely flat-topped even when the quantization theorem is only half satisfied, i.e., when the i/p C.F. $W_x(u)$ is zero for $|u| > \phi$ rather than $|u| > \phi/2$. It is interesting to note that satisfaction of the quantization theorem allows complete recovery of an original p.d.f. given the quantized p.d.f. Half-satisfaction of the quantization theorem allows only moments to be recovered. At the same time half-satisfaction of the quantization theorem ensure a flat topped quantization noise p.d.f.

C.5 QUANTIZATION OF BIVARIATE AND 2ND ORDER PROCESSES

A first order process as assumed in the previous analysis can be completely described by its first order p.d.f. For such a process all samples are completely independent. If the sampled variable is a second order process, a joint p.d.f. $w_{x_1 x_2}(x_1, x_2)$ is required to describe its statistics where x_1 and x_2 are respectively the first and second samples of an adjacent sample pair. Similarly the statistics of two statistically dependent processes x and y can be described with a joint p.d.f. $w_{xy}(x, y)$.

In order to examine the effects of amplitude quantization on correlation function determination, it is necessary to derive an expression for the joint p.d.f. and c.f. of the quantized data. Quantization of two variables x and y with joint p.d.f. $w_{xy}(x, y)$ can be described by a volume sampling process

on the input p.d.f. $w_{xy}(x,y)$ - see Figure C.8. The p.d.f. of the quantizer output $w_{x',y'}(x',y')$ consists of a set of impulses at the centres of each rectangle whose amplitudes (volumes, weights) equal the volume under the input p.d.f. $w_{xy}(x,y)$ within the bounds of their respective rectangles.

Associated with a joint p.d.f. is its c.f. $W_{xy}(u,v)$ given by the equation:

$$W_{xy}(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_{xy}(x,y) \cdot e^{-j(u \cdot x + v \cdot y)} dx \cdot dy$$

It can be shown (Reference 149) that for the bivariate case in which the variables x and y are quantized by general quantizers with parameters (a_1, q_1, c_1, r_1) and (a_2, q_2, c_2, r_2) respectively, the resulting output c.f. is given by equation 5.3 of Chapter 5.

The output c.f. again consists of repeated versions of the input c.f. each section modified by a term of the form $\frac{\sin u}{u} \frac{\sin v}{v}$ as shown in Figure C.9. Using the same results as in section C.4 it can be seen that a typical section of $W_{xy}(u,v)$ is the same as the c.f. of the sum of the quantizer input signal and statistically independent noises having c.f.'s

$$\frac{\sin \frac{1}{2} ur_1}{\frac{1}{2} ur_1} \quad \text{and} \quad \frac{\sin \frac{1}{2} vr_2}{\frac{1}{2} vr_2} \quad \text{respectively.}$$

If the c.f. of the quantizer input is bandlimited in 2-dimensions and if the quantization intervals q_1 , and q_2 are small enough, i.e., if

$$W_{xy}(u,v) = 0 \text{ for } |u| > \pi/q_1, \text{ and } |v| > \pi/q_2$$

then the repeated sections of $W_{x',y'}(u,v)$ do not overlap. If this 2-dimensional quantizing theorem is satisfied, the joint p.d.f. can be recovered from the joint output impulse p.d.f. Also if this theorem is only half-satisfied i.e.,

$$W_{x',y'}(u,v) = 0 \text{ for } |u| > \frac{2\pi}{q_1} \text{ and } |v| > \frac{2\pi}{q_2}$$

then the moments of x and y and all joint moments can be derived from the quantized variables. Under these conditions, all first and second order moments of the quantizer output are the same as if the quantizer were a source of additive, independent, uncorrelated, uniformly distributed noise. However, although quantization noise is, in many respects, like independent noise, it is casually connected to the input signal. Its p.d.f. will next be shown to be independent of the signals p.d.f. if the

quantitizing theorem is only half-satisfied.

C.6 P.d.f. OF QUANTIZATION NOISE FOR BIVARIATE PROCESS

A similar procedure as described in section C.4 is used to derive an expression for the c.f. of the quantization noise for a bivariate process. With reference to Figure C.8, the joint input p.d.f. is sliced along the rectangles of the quantization grid. The rectangles are then stacked and summed to give the 2-dimensional noise p.d.f. Joint input events give joint noise events whose probability are computed by summing their probabilities of occurrence within each box. This leads to the following expression for the c.f. of the quantization noise (151).

$$W_{N_x N_y}(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} W_{xy}(k\phi_1, l\phi_2) \frac{\sin\pi(\frac{u}{\phi_1} - k)}{\pi(\frac{u}{\phi_1} - k)} \cdot \frac{\sin\pi(\frac{v}{\phi_2} - l)}{\pi(\frac{v}{\phi_2} - l)}$$

The sum consists only of the following term if the quantizing theorem is half satisfied.

$$W_{xy}(0,0) \cdot \frac{\sin\pi u/\phi_1}{\pi u/\phi_1} \cdot \frac{\sin\pi v/\phi_2}{\pi v/\phi_2}$$

It follows that under these conditions, although x and y are statistically connected, their corresponding quantization noises are statistically independent and uniformly distributed between plus and minus one half a quantization box.

C.7 OUTPUT OF RELAY CORRELATOR WITH GAUSSIAN INPUT

For the relay correlator, the quantizer characteristic have the following form (assuming y to be quantized).

$$\begin{aligned} a_1 &= c_1 = 0 & ; & & q_1 &= r_1 & ; & & q_1 \rightarrow \infty \\ a_2 &= c_2 = 0 & ; & & r_2 &= 2 & ; & & q_2 \rightarrow 0 \end{aligned}$$

Substituting these values into equation 5.4 of Chapter 5 and assuming Gaussian input signals with characteristic function

$$W_{xy}(u, v) = \text{EXP} \left[-\frac{1}{2}(u^2 \cdot \sigma_x^2 + 2\rho \cdot \sigma_x \cdot \sigma_y \cdot v \cdot u + v^2 \cdot \sigma_y^2) \right]$$

the output of the relay correlator is:

$$\overline{x(\text{sgny})} = \lim_{q_2 \rightarrow \infty} \frac{2\rho\sigma_x\sigma_y}{q_2} \sum_{l=-\infty}^{\infty} \text{EXP} \left[-\frac{1}{2} \left(\frac{2\pi l\sigma_y}{q_2} \right)^2 \right]$$

Putting $y = l \cdot 2\pi\sigma_y/q_2$, $dy = 2\pi\sigma_y/q_2$ as $q_2 \rightarrow \infty$, the sum becomes an integral and

$$\begin{aligned} \overline{x(\text{sgny})} &= \rho \frac{\sigma_x}{\pi} \int_{-\infty}^{\infty} \text{EXP} \left[-\frac{1}{2} y^2 \right] dy \\ &= \sqrt{\frac{2}{\pi}} \cdot \rho \cdot \sigma_x. \end{aligned}$$

The output of a relay correlator for Gaussian inputs is thus directly proportional to the direct correlation coefficient, ρ .

C.8 OUTPUT OF POLAR CORRELATOR FOR GAUSSIAN INPUTS

For the polar correlator,

$$a_1 = c_1 = \frac{1}{2} \quad ; \quad r_1 = 2 \quad ; \quad q_1 \rightarrow \infty$$

$$a_2 = c_2 = \frac{1}{2} \quad ; \quad r_2 = 2 \quad ; \quad q_2 \rightarrow \infty$$

The output of this correlator for Gaussian input signals can be obtained via equation 5.4; however this leads to extensive arithmetic. (See Reference 149) A simpler method is to calculate the first joint moment of the signs of the two signals in the amplitude domain, i.e.,

$$\begin{aligned} \overline{(\text{sgnx}) \cdot (\text{sgny})} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\text{sgnx}) \cdot (\text{sgny}) w_{xy}(x,y) dx dy \\ &= \int_0^{\infty} \int_0^{\infty} w_{xy}(x,y) dx dy + \int_{-\infty}^0 \int_0^{\infty} w_{xy}(x,y) dx dy - \\ &\quad \int_0^{\infty} \int_{-\infty}^0 w_{xy}(x,y) dx dy - \int_{-\infty}^0 \int_{-\infty}^0 w_{xy}(x,y) dx dy \end{aligned}$$

where

$$w_{xy}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \text{EXP} \left[-\frac{1}{2} \frac{\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}}{1-\rho^2} \right]$$

Evaluating these integrals leads to the results

$$\overline{(\operatorname{sgn} x) \cdot (\operatorname{sgn} y)} = \frac{2}{\pi} \cdot \sin^{-1} \rho$$

C.9 OUTPUT OF MODIFIED CORRELATOR

The relationship between the joint characteristic function of quantized and unquantized signals is given by equation 5.3. In the case to be considered in this section the unquantized signals equal the input signal plus independent noise. The characteristic function of the unquantized signals is thus:

$$\begin{aligned} W_{x_1 y_1}(u, v) &= E \left[\exp \left\{ ju(x + a) + jv(y + b) \right\} \right] \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ju(x + a) + jv(y + b) \right] w(x, y, a, b) dx dy da db \end{aligned}$$

If $a(t)$ and $b(t)$ are independent of $x(t)$ and $y(t)$ and of each other, the joint probability density function can be written as:

$$w(x, y, a, b) = w_{xy}(x, y) \cdot w_a(a) \cdot w_b(b)$$

Therefore

$$\begin{aligned} W_{x_1 y_1}(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(jux + jvy) \cdot w_{xy}(x, y) dx dy \cdot \\ &\quad \int_{-\infty}^{\infty} \exp(jua) \cdot w_a(a) da \cdot \int_{-\infty}^{\infty} \exp(jvb) w_b(b) db \\ &= W_{xy}(x, v) \cdot W_a(u) \cdot W_b(v) \end{aligned}$$

where W_{xy} , W_a and W_b are the characteristic functions of x and y , a and b respectively.

Substituting the above characteristic function into equation 5.3 and then differentiating with respect to u and v at the origin as in equation 5.2, gives the output of the modified correlator. For quantizers with unity gain:

$$\begin{aligned} q_1 &= r_1 & ; & & a_1 &= c_1 \\ q_2 &= r_2 & ; & & a_2 &= c_2 \end{aligned}$$

the output equals (Reference 152)

$$\overline{x_1^1 \cdot y_1^1} = \overline{x \cdot y} + \sum_{k=-\infty}^{\infty} \text{EXP}(j2\pi k a_1) \cdot \frac{q_1}{2\pi k} \cdot (-1)^k \cdot W_a\left(\frac{-2\pi k}{q_1}\right).$$

$$\left. \frac{\partial}{\partial v} \left[W_{xy}\left(\frac{-2\pi k}{q_1}, v\right) \right] \right|_{v=0} + \sum_{l=-\infty}^{\infty} \text{EXP}(j2\pi l a_2) \cdot \frac{q_2}{2\pi l} \cdot (-1)^l \cdot$$

$$W_b\left(\frac{-2\pi l}{q_2}\right) \cdot \frac{\partial}{\partial u} \left[W_{xy}\left(u, \frac{-2\pi l}{q_2}\right) \right]_{u=0} -$$

$$\sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \text{EXP} \left[j2\pi (k a_1 + l a_2) \right] \cdot \frac{q_1 q_2}{4\pi^2 k \cdot l} \cdot (-1)^{k+l} \cdot W_a\left(\frac{-2\pi k}{q_1}\right).$$

$$W_b\left(\frac{-2\pi l}{q_2}\right) \cdot W_{xy}\left(\frac{-2\pi k}{q_1}, \frac{-2\pi l}{q_2}\right)$$

where the prime on the summation means 'omitting zero'. It is apparent from the above equation that the correlation function of the quantized signals x_1^1 and y_1^1 is equal to the true correlation function for arbitrary inputs if and only if the auxiliary noises $a(t)$ and $b(t)$ are such that:

$$W_a\left(\frac{-2\pi k}{q_1}\right) = 0 \quad \text{for } k \neq 0$$

$$W_b\left(\frac{-2\pi l}{q_2}\right) = 0 \quad \text{for } l \neq 0$$



Microcomputer Systems

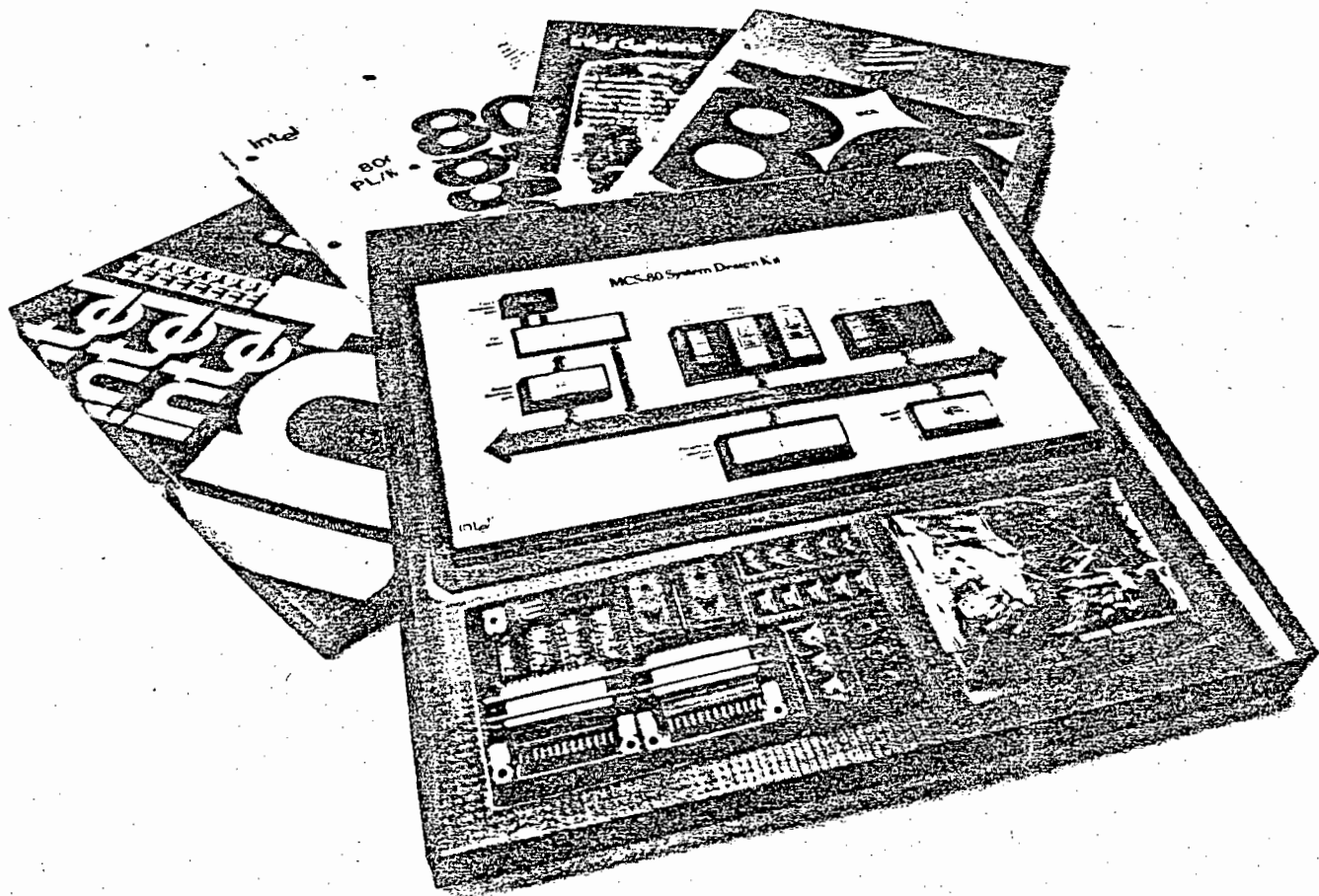
8080 SYSTEM DESIGN KIT (SDK-80)

- Complete Single Board Microcomputer System Including CPU, Memory and I/O
- Easy to Assemble Kit-Form
- High-Performance (2 μ s Instruction Cycle)
- Interfaces Directly with most Terminals (75-4800 Baud)
- Large Wire-Wrap area for Custom Interfaces
- Extensive System Monitor Software in ROM
- PC Board Format and Power, Compatible with INTELLEC[®] MDS
- Complete MCS-80[™] User's Library

The 8080 System Design Kit (SDK-80) is a complete, single board, microcomputer system in kit form. It contains all necessary components, including resistors, caps, crystal and miscellaneous hardware to complete construction. Included is a pre-programmed ROM that contains the system monitor for general software utilities and system diagnostics.

All that is required for operation are power supplies and a suitable terminal; TTY, CRT, etc., (level conversions and baud rate generation included on board).

The SDK-80 is an inexpensive, high-performance prototype system that has designed-in flexibility for simple interface to the users application.



SDK-80

SDK-80 SPECIFICATIONS

Central Processor

CPU: 8080A
Instruction Cycle: 1.95 microsecond
Tcy: 488 ns

Memory

ROM: 2K bytes (expandable to 4K bytes)
8708/8308
RAM: 256 bytes (expandable to 1K bytes) 8111
Addressing:
ROM 0000-0FFF
RAM 1000-13FF

Input/Output

Parallel: One 8255 for 24 lines (expandable to 48 lines).
Serial: One 8251 USART.
On-board baud rate generator (jumper selectable).
Baud Rates: 75 1200
 110 2400
 300 4800
 600

Interfaces

Bus: All signals TTL compatible.
Parallel I/O: All signals TTL compatible.
Serial I/O: RS232C/EIA
20 mil A. Current loop TTY
TTL (one TTL load)

Interrupts

Single level: Generates RST7 vector TTL compatible input.

DMA

Hold Request: Jumper selectable.

Software

System Monitor: Pre-programmed 8708 or 8308 ROM Address; 0000-03FF.

Features:

Display Memory Contents	(D)
Move blocks of memory	(M)
Substitute memory locations	(S)
Insert hex code	(I)
Examine Registers	(X)
Program Control	(G)
Power-up start or system reset start.	

I/O: Console Device (serial I/O)

Literature

Design Library:

8080 Users Manual
8080 Assembly Language Manual
PL/M™ Programming Manual
MDS Brochure
Reference Card (Programmers)
SDK-80 User's Guide

Connectors

I/O: 25 pin female (RS232C)
PCB: MDS format

Physical Characteristics (MDS Mechanical format)

Width: 12.0 in.
Height: 6.75 in.
Depth: 0.50 in.
Weight: approx. 12 oz.

Electrical Characteristics (DC Power)

V _{CC}	5V ±5%	1.3 Amps
V _{DD}	12V ±5%	.35 Amps
V _{BB}	-10V ±5%	.20 Amps
or	-12V ±5%	

Environmental

Operating Temperature: 0-70°C

CHAPTER 3

THEORY OF OPERATION

GENERAL

Now that you have assembled the structure of the SDK-80 it is time to discuss the internal composition of the design. We will do this by presenting the functional organization of the SDK-80 logic and, in the process, bring in the decisions that you, as the user, must make before completing the kit.

Figure 3-1 is a functional block diagram of the

SDK-80. It has been purposely drawn as simple as possible in order to give a basis for discussion. You will note that this figure shows only the major signals in the unit. For this reason, some occasional reference to the SDK-80 schematics (Appendix B) will be in order.

The text to follow describes each of the elements in the block diagram.

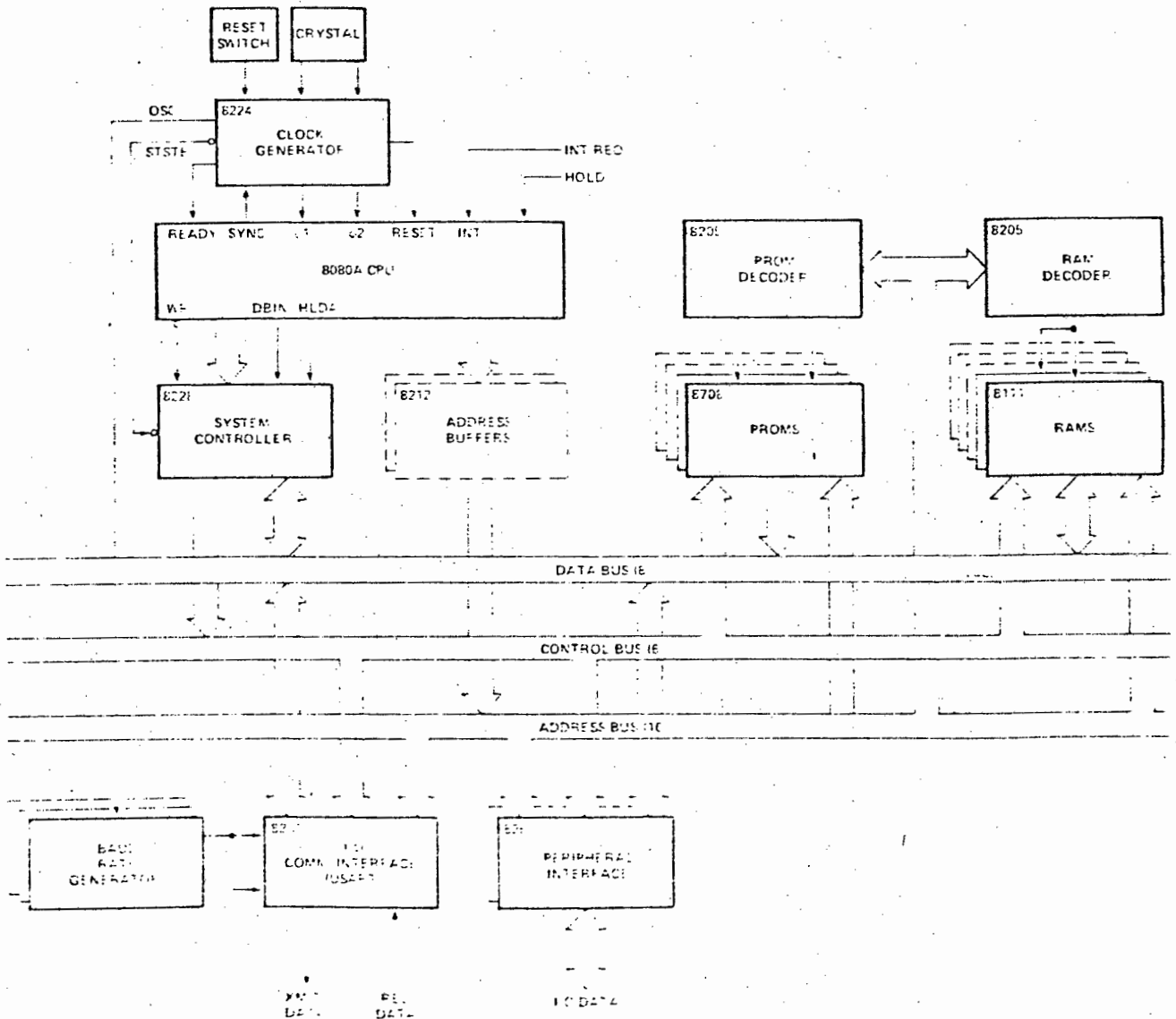


Figure 3-1. SDK-80 Functional Block Diagram.

CHAPTER 1

INTRODUCTION

GENERAL

The 8080 System Design Kit (SDK-80) is a complete microcomputer system in kit form. It is simple to assemble (construction time is 6 hours) and provides an excellent training/prototype vehicle for evaluation of the 8080 microcomputer system (MCS-80™).

The SDK-80 is an extremely flexible design and allows easy interface to an existing application or custom interface development.

An extensive system monitor is included in a pre-programmed ROM for general software utilities and system diagnostics.

The System Design Kit User's Guide will instruct the user how to assemble his kit and configure it to match the selected terminal and peripheral devices. It is suggested that the User's Guide be followed in the exact sequence that it is written to assure successful completion of the system.

SUGGESTIONS

The 8080 Microcomputer Systems User's Manual is included with the SDK-80 and it would be extremely beneficial to the user that he read and understand the operation of the 8080A and associated peripheral components prior to beginning the assembly of the SDK-80.

Every effort has been made to allow the SDK-80 to interface directly with most common terminals but with the wide array of display terminals available it is not possible to perfectly interface each one with the SDK-80 hardware and software. The user should carefully examine the requirements of his particular terminal interface and adapt the SDK-80 accordingly.

FUNCTIONAL DEFINITION

The SDK-80 is shipped with a complement of parts that allows the user to construct an operating small system with the following features:

- CPU: 8080A (see 8080 User's Manual for details)
1.95 μ s Instruction Cycle
- RAM: 8111 (static 256 x 4) 2 included for 256 byte storage
- ROM: 8708/8308 (1K x 8)
1 Pre-programmed system monitor
1 User-programmed (erasable 8708)
- I/O: 8251 (Programmable Communication Interface)
1 Serial communication with terminal
8255 (Programmable Peripheral Interface)
1 General user I/O, 24 lines
- Serial Interface: TTL
20mA current loop (TTY)
RS-232 (EIA)
- Baud Rate: User-selected by jumper or switch
75, 110, 300
600, 1200, 2400, 4800
- Interrupt: Single level, vectored (RST-7)

The SDK-80 has many designed-in features for expandability without the necessity of cutting PC runs or adding extra logic. The maximum configuration of the SDK-80 is as follows:

- RAM: 8111 (static 256 x 4)
Up to 8 for 1K x 8 storage
- ROM: 8708/8308 (1K x 8)
Up to 4 for 4K x 8 storage
- I/O: 8255 (Programmable Peripheral Interface)
2 General user I/O, 48 lines

Expanding the SDK-80 to the maximum configuration is a simple matter of purchasing the extra memory and I/O components and installing them on the board.

SYSTEM BUSESSES

The SDK-80 logic is built around three system busses: the data bus, the address bus and the control bus. All of the MCS-80™ components communicate via these three busses.

The system busses can be selectively enabled/disabled from the user system if the board is jumpered for that capability. Bus enable jumpering is described in the System Bus Enable section of Chapter 4.

Each bus is more fully described in the 8080 Microcomputer Systems User's Manual.

RESET SWITCH

The Reset Switch gives you the capability of forcing a reset to the SDK-80 logic at any time. When the switch is pressed, the Clock Generator will send a RESET signal throughout the system. The Reset Switch should be pressed each time you power-up the system.

CLOCK GENERATOR AND CLOCK CRYSTAL

The 8224 Clock Generator provides the primary timing to the system. It generates the high-level clocks necessary to drive the 8080A CPU, synchronizes the READY signal into the CPU, and transmits the power-up (and Reset Switch) reset signal.

Ø1 and Ø2 Clocks

Ø1 and Ø2 are 2.048 MHz clocks for the CPU. They are derived from OSC using an internal divide-by-nine function.

RESET Signal

RESET is the primary reset signal to the system logic. It is asserted both at power-up and when the Reset Switch is pressed. RESET clears the CPU, disables the RAM Decoder, and resets the USART. RESET is available to the user system at pad V.

READY Signal

READY can provide a synchronized READY to the CPU, derived from an external asynchronous RDYIN signal (pad P).

OSC Signal

OSC provides an 18.432 MHz input to the Baud Rate Generator. This 18.432 MHz rate was chosen for two reasons. First, it permits the 8080A CPU to run at very close to its maximum

speed. Second, it is a convenient rate to use in designing a simple, but highly stable, Baud Rate Generator.

STSTB (Status Strobe) Signal

At the beginning of each machine cycle, the CPU issues status information on its data bus. STSTB causes the 8228 System Controller to store this information into its status latch. STSTB is available to the user system as STATUS STROBE at pad J.

8080A CPU

The 8080A CPU is thoroughly described in the Intel® 8080 Microcomputer Systems User's Manual and need not be repeated here.

The CPU clocks, Ø1 and Ø2, will be supplied (at 2.048 MHz) by the Clock Generator.

The data bus will interface directly to the System Controller and the address bus will enter the system through the Address Buffers, if applicable.

There are two separate jumper-wire options with the CPU. The first option allows an external HOLD signal to be presented to the CPU via pad R. The second option allows an external READY signal to force a Wait state in the CPU. It should be pointed out, however, that the 8080A and SDK-80 memory chips have been designed to operate without Wait states. The option permits you to force a Wait if desired, though. Both of these jumper options are described in the Hold And Wait Options section of Chapter 4.

SYSTEM CONTROLLER

The 8228 System Controller generates the control bus signals that provide read and write functions for I/O and memory.

They are available to the user system as shown below:

- $\overline{I/O\ W}$ is at pad E
- $\overline{I/O\ R}$ is at pad L
- $\overline{MEMW}$ is at pad U
- $\overline{MEMR}$ is at pad T

The System Controller also buffers the data bus.

Interrupt

A single-level interrupt structure is provided such that whenever pad H ($\overline{INT\ REQ}$) is

grounded. the System Controller causes a Restart instruction (RST 7) to be inserted into the CPU. This feature provides a single interrupt vector without using additional components, such as an interrupt instruction port. Multiple level interrupts will require additional chips to be installed into the wire-wrap area.

ADDRESS BUFFERS (OPTIONAL)

The 8212 Address Buffers are not included in the System Design Kit, but must be added if more than a nominal amount of memory (more than 1024 bytes of RAM and more than 4K bytes of ROM) is used. The Address Buffers are tri-state TTL buffers that provide 15mA drive.

The address bus level can be forced to the high-impedance state by inputting a high level on pad S (SYSTEM BUS ENABLE), if the board is jumpered for this capability.

SDK-80 MEMORY

The SDK-80 has two types of memory. Its ROM Memory can accommodate from 1K to 4K bytes, where the lower 1K bytes are dedicated to the system monitor. Its RAM Memory can accommodate from 256 to 1K bytes, in which all but the uppermost 30 bytes (addresses 13E2-13FF) are useable by your system. Figure 3-2 is a map of SDK-80 memory.

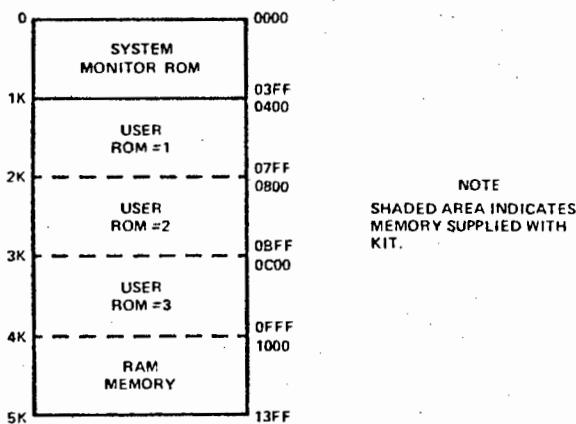


Figure 3-2. SDK-80 Memory Map.

ROM Decoder and ROM Memory

The SDK-80 can accommodate up to four 1024x8 8708/8308 Read Only Memory (ROM) chips.

The ROM that installs into board location A14 has been pre-programmed with the SDK-80 system monitor.

The 8708/8308 that installs into board locations A15, A16, and A17 can be used to hold a program that you have developed and checked out in RAM.

The 8205 ROM Decoder selects the ROM chip being addressed. Figure 3-3 shows the ROM address format.

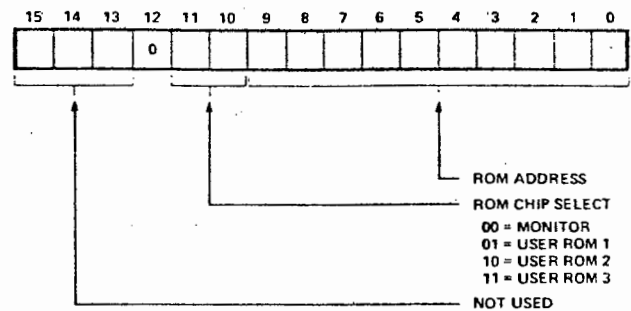


Figure 3-3. ROM Address Format.

RAM Decoder and RAM Memory

In the standard configuration, the SDK-80 can accommodate up to eight 256x4 Static MOS Random Access Memory (RAM) chips. Two of these chips are supplied in the System Design Kit, so users requiring only 256 bytes of memory need not install additional RAM chips.

The 8205 RAM Decoder selects the RAM chip pair being addressed. Figure 3-4 shows the RAM address format.

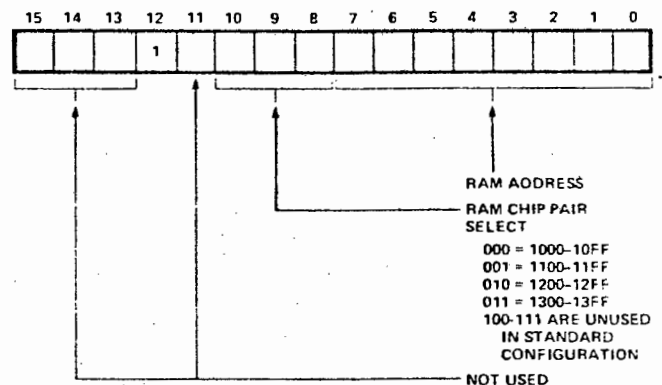


Figure 3-4. RAM Address Format.

RAM access is disabled whenever the RESET signal from the Clock Generator is asserted.

BAUD RATE GENERATOR

The Baud Rate Generator circuit supplies the transmitter and receiver clocks to the I/O Communication Interface. This circuit is made up of three IC chips: one 93S16 and two 74161s.

The Baud Rate Generator takes the 18.432 MHz OSC signal from the Clock Generator and, by internal division, generates a series of signals which represent baud rates between 75 and 4800. The baud rate that will be presented to the I/O Communication Interface is determined by jumper-wiring or a rotary switch. This selection will be discussed in the Baud Rate Selection section of Chapter 4.

I/O COMMUNICATION INTERFACE

The 8251 I/O Communication Interface is a Universal Synchronous/Asynchronous Receiver/Transmitter (USART) chip that accommodates any data communications required by the SDK-80 system. The I/O Communication Interface can accept parallel data from the data bus and send it serially to an external device. It can also accept serial data from an external device and put it onto the data bus in parallel form when eight bits have been collected. Figure 3-5 shows the address format for communications.

The baud rate at which the I/O Communication Interface will transmit and receive data is governed by the Baud Rate Generator.

The I/O Communication Interface circuit on the board also includes some jumpers that select the communication input/output level. Any of three levels may be selected.

- RS-232 level, which is typically used for CRT applications
- Current-loop level, for TTY applications
- TTL level.

The input/output level jumpering is discussed in the Communication Level Selection section of Chapter 4.

PERIPHERAL INTERFACES

The 8255 Programmable Peripheral Interfaces provide the user's primary access point to the SDK-80 data bus. One 8255 chip is supplied in the System Design Kit.

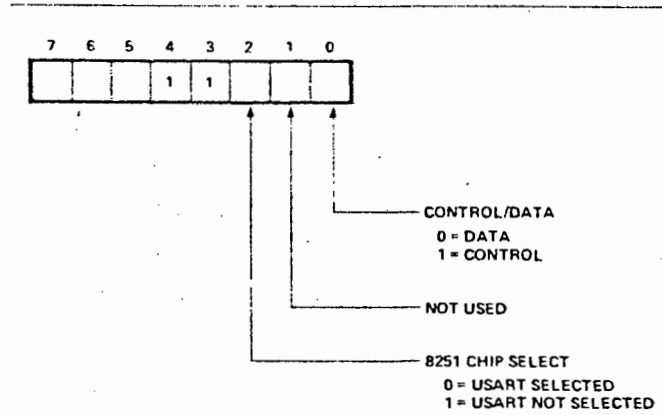


Figure 3-5. I/O Communication Interface Address Format.

Each Peripheral Interface chip provides three 8-bit parallel I/O ports, each of which is independently addressable. Figure 3-6 shows the address format for I/O port selection.

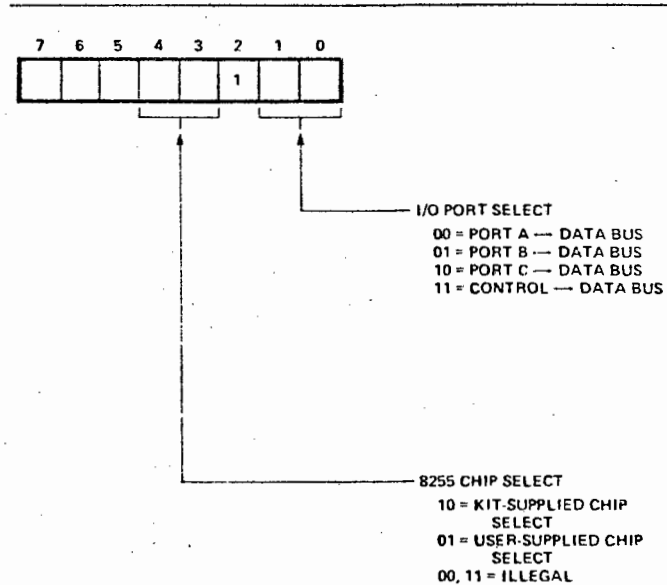


Figure 3-6. Peripheral Interface Address Format.

The output pins of the Peripheral Interfaces are totally uncommitted and may be jumper-wired to best suit your particular application. For example, they might be wired directly to the interface plugs or, alternately, they might be wired to standard TTL buffers in the wire-wrap area before coming back to the plugs. This wiring is further discussed in the Output Wiring section of Chapter 4.

APPENDIX E

MICRO-PROCESSOR CONTROLLED TWO-POINT DIFFERENCE CORRELATOR CIRCUIT DIAGRAMS

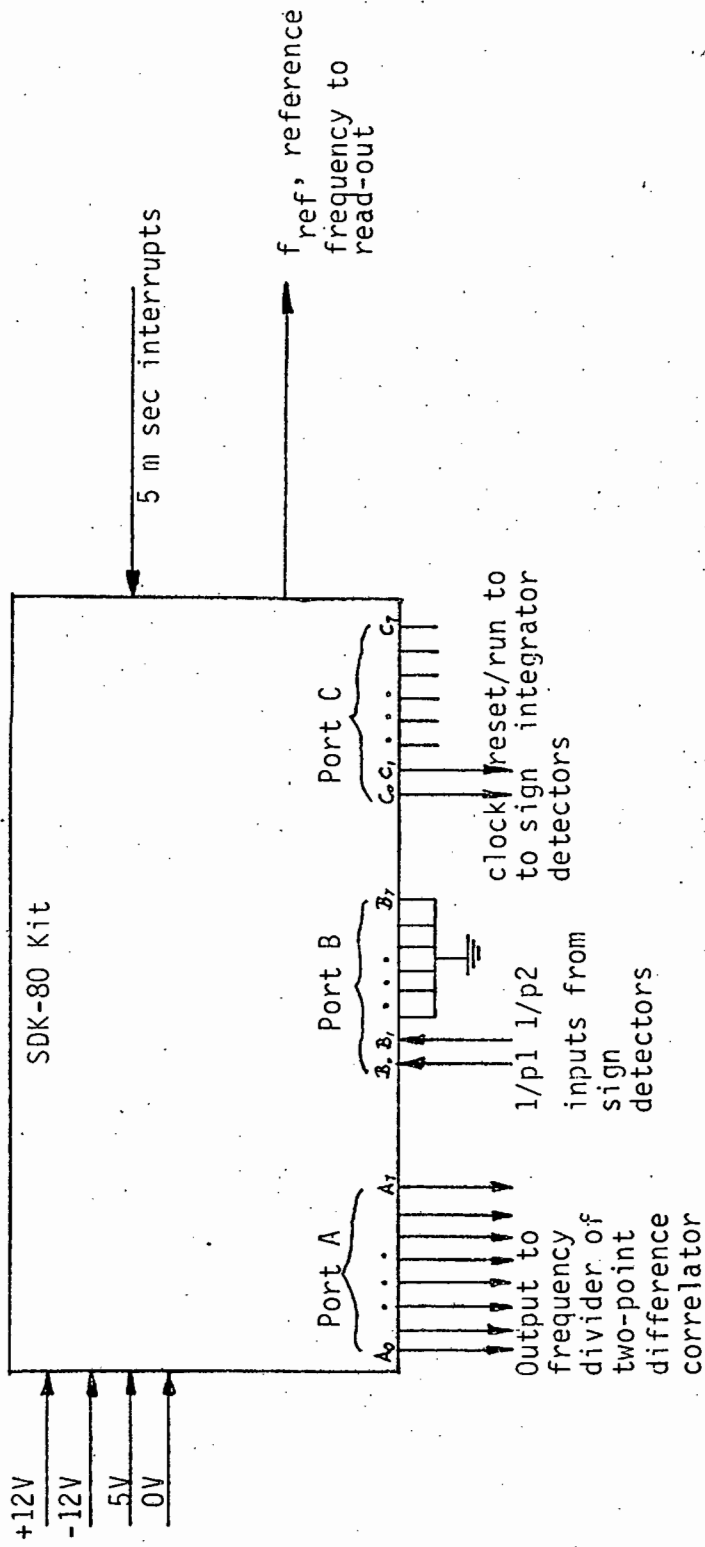


FIG. E.a Micro-processor correlator

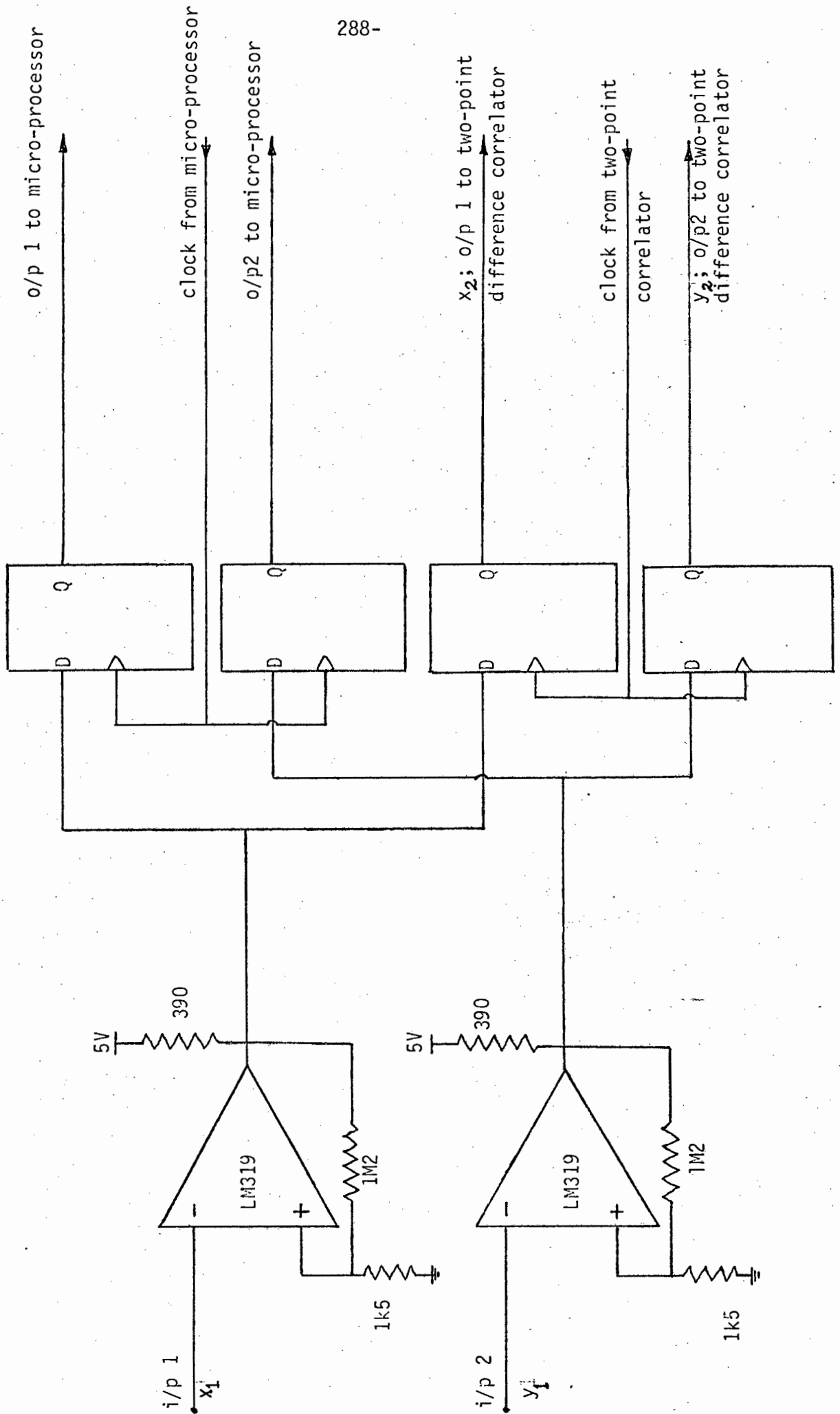


FIG. E.b Sign detectors

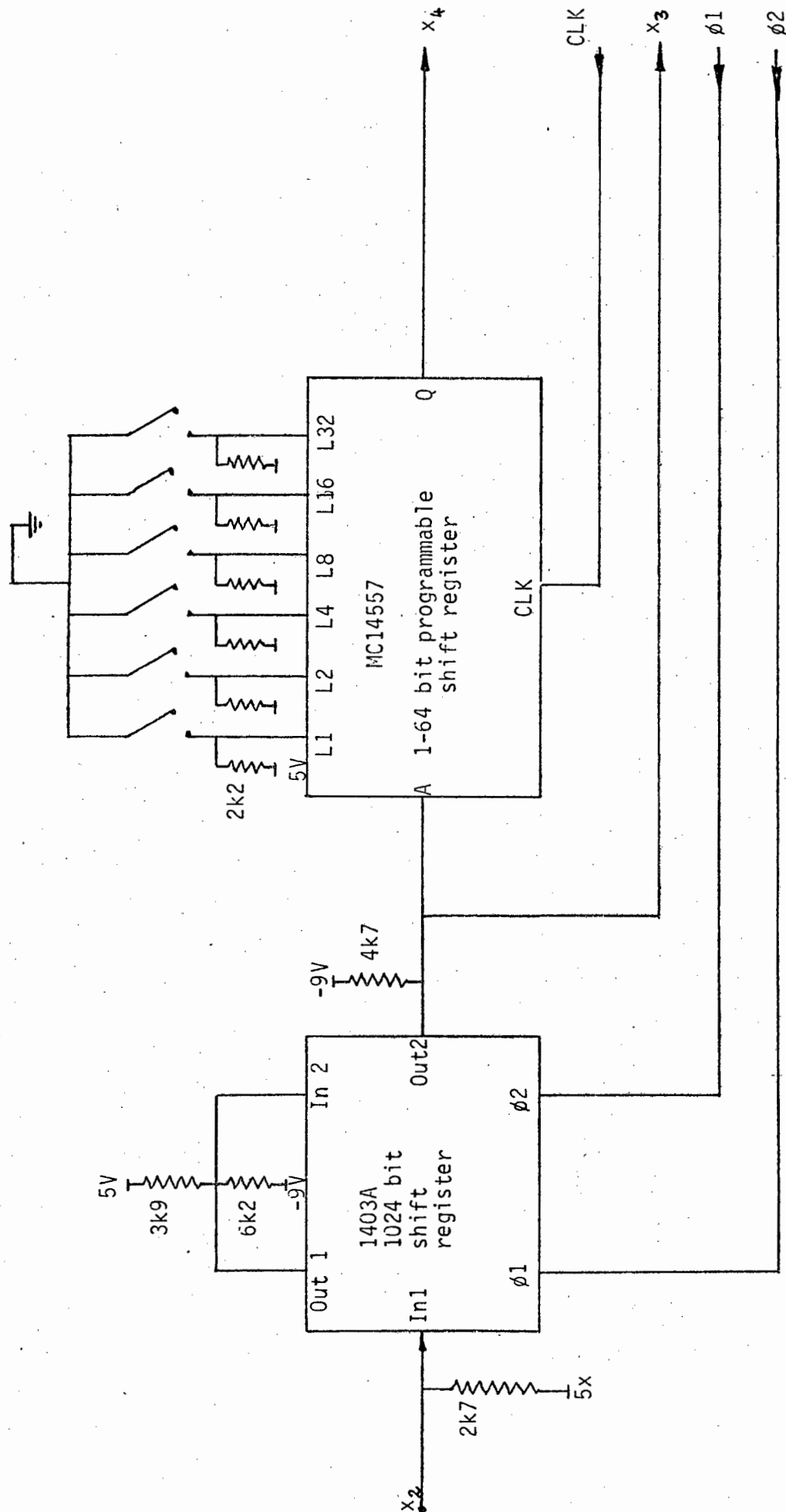


FIG. E.c Shift registers

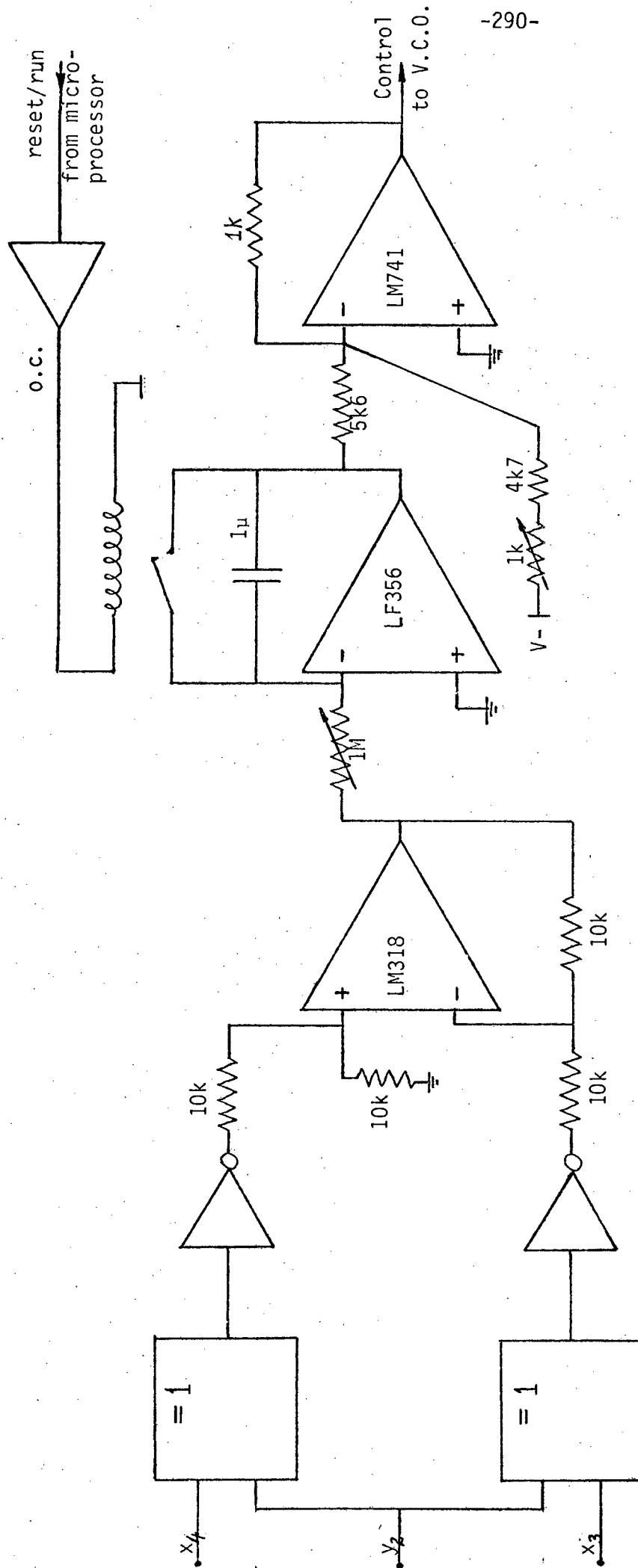


FIG. E.d Multiplier, integrator, level-shifter

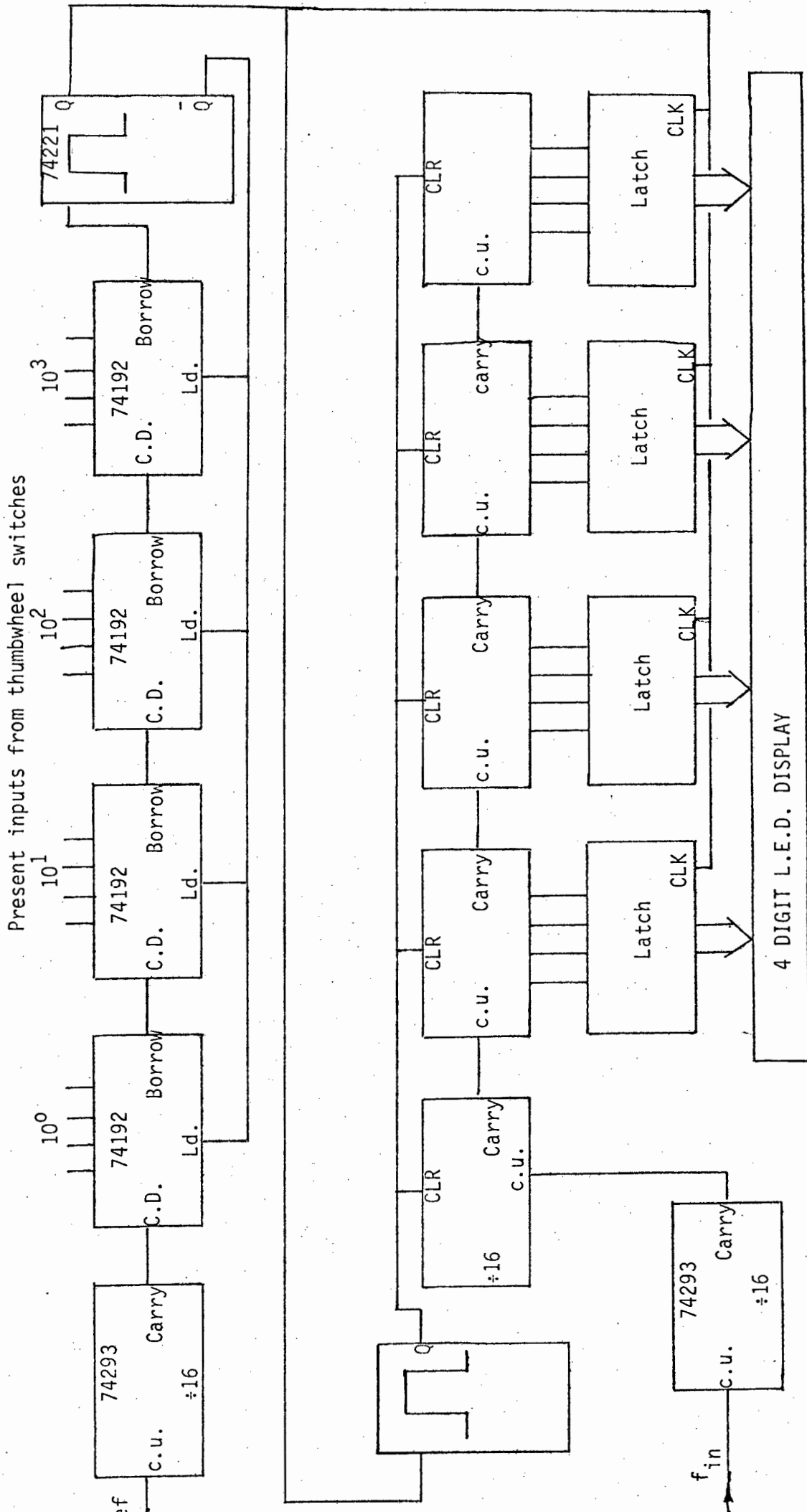


FIG. E.f Read-out

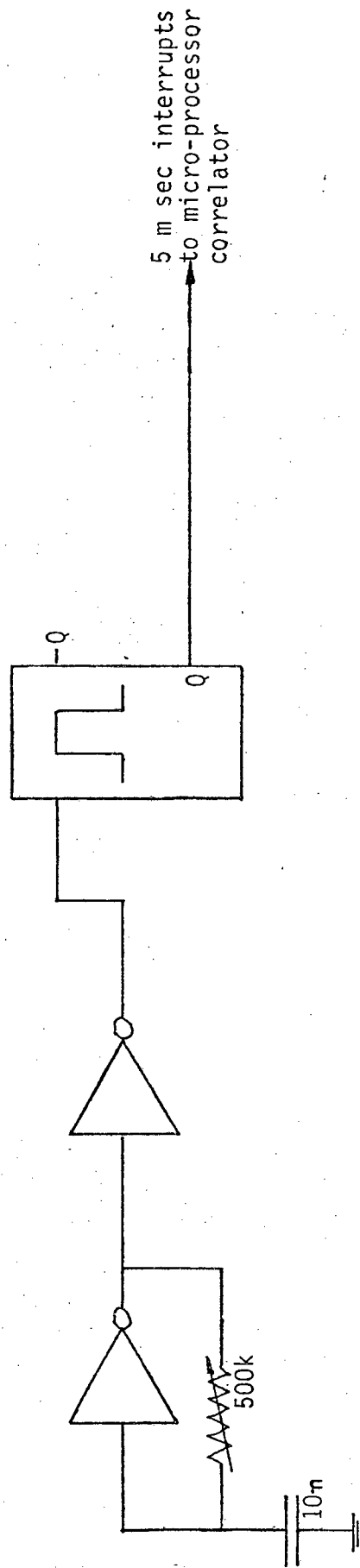


FIG. E.g Interrupt generator for micro-processor correlator

APPENDIX F

SERIAL TWO-POINT DIFFERENCE CORRELATOR

CIRCUIT DIAGRAMS

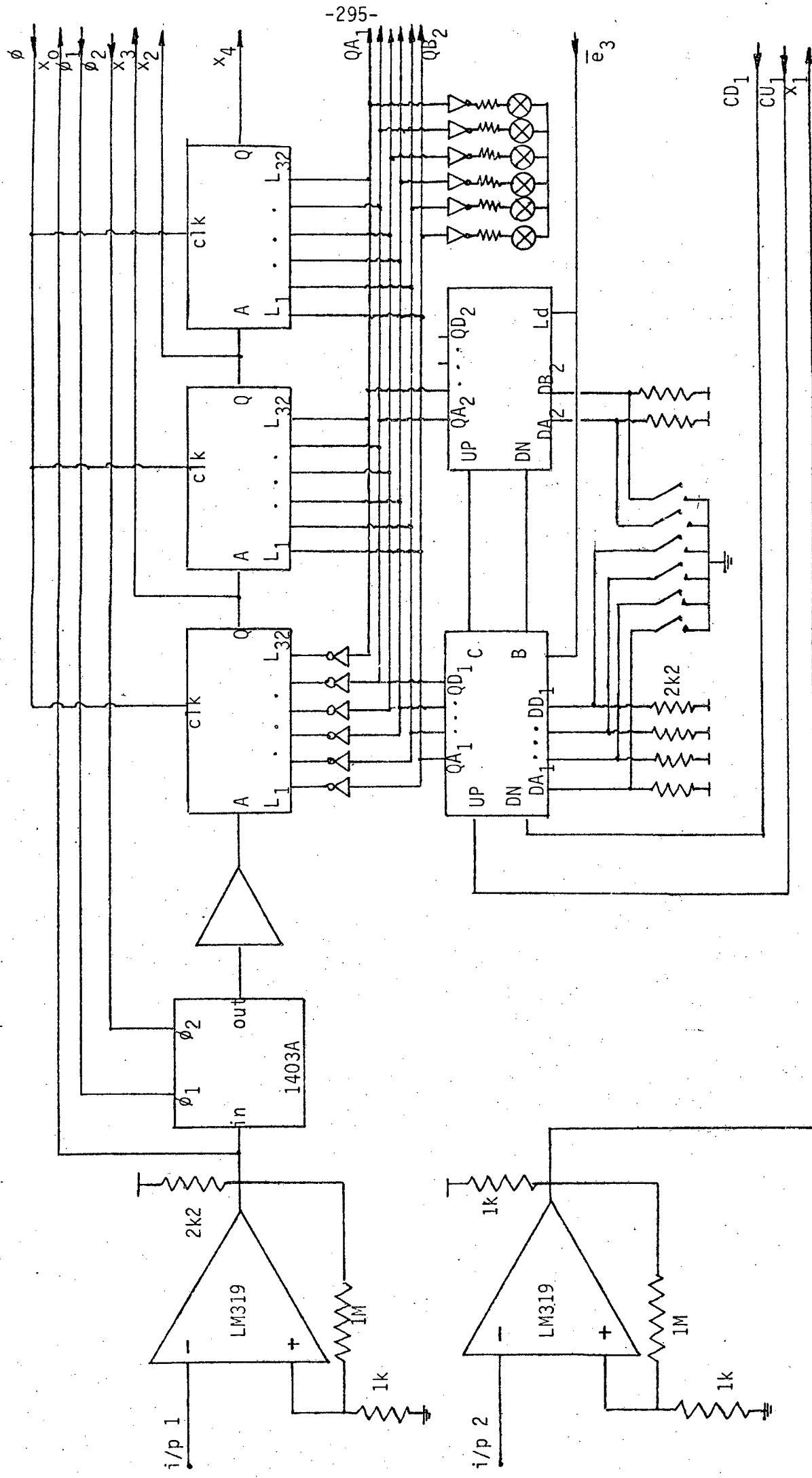


FIG. F.a Comparators and shift registers

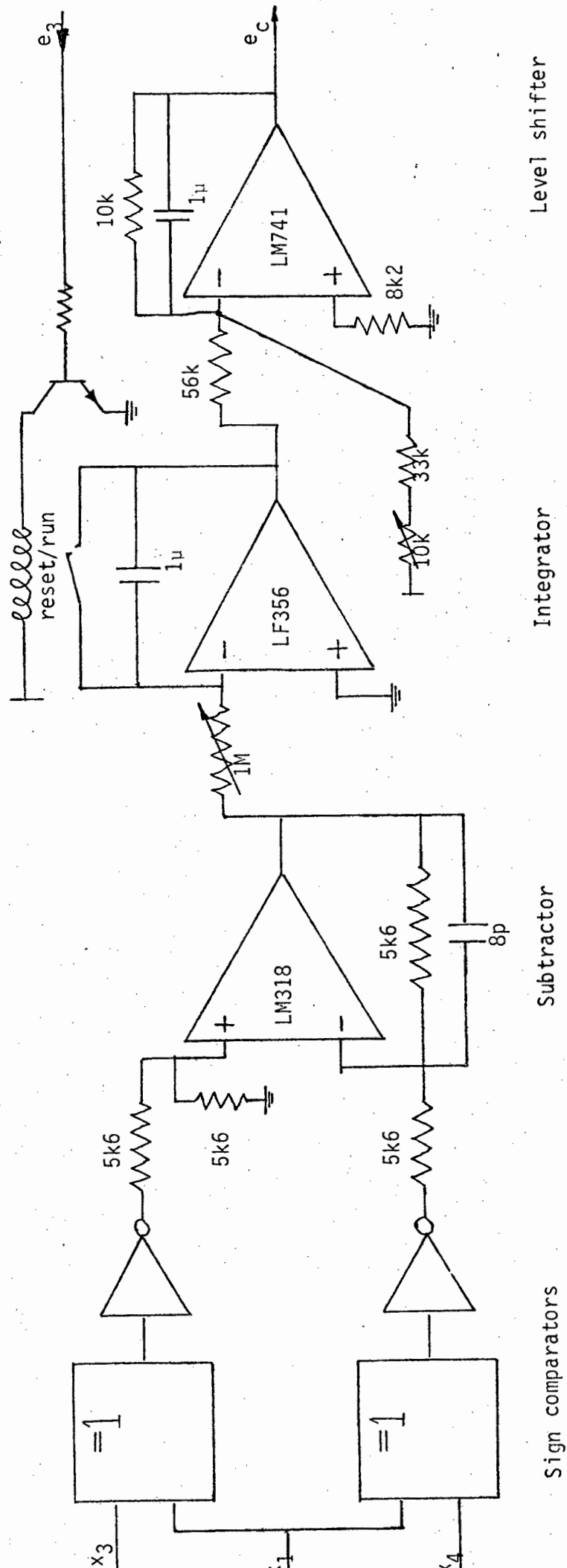


FIG. F.b - Two-point difference correlator

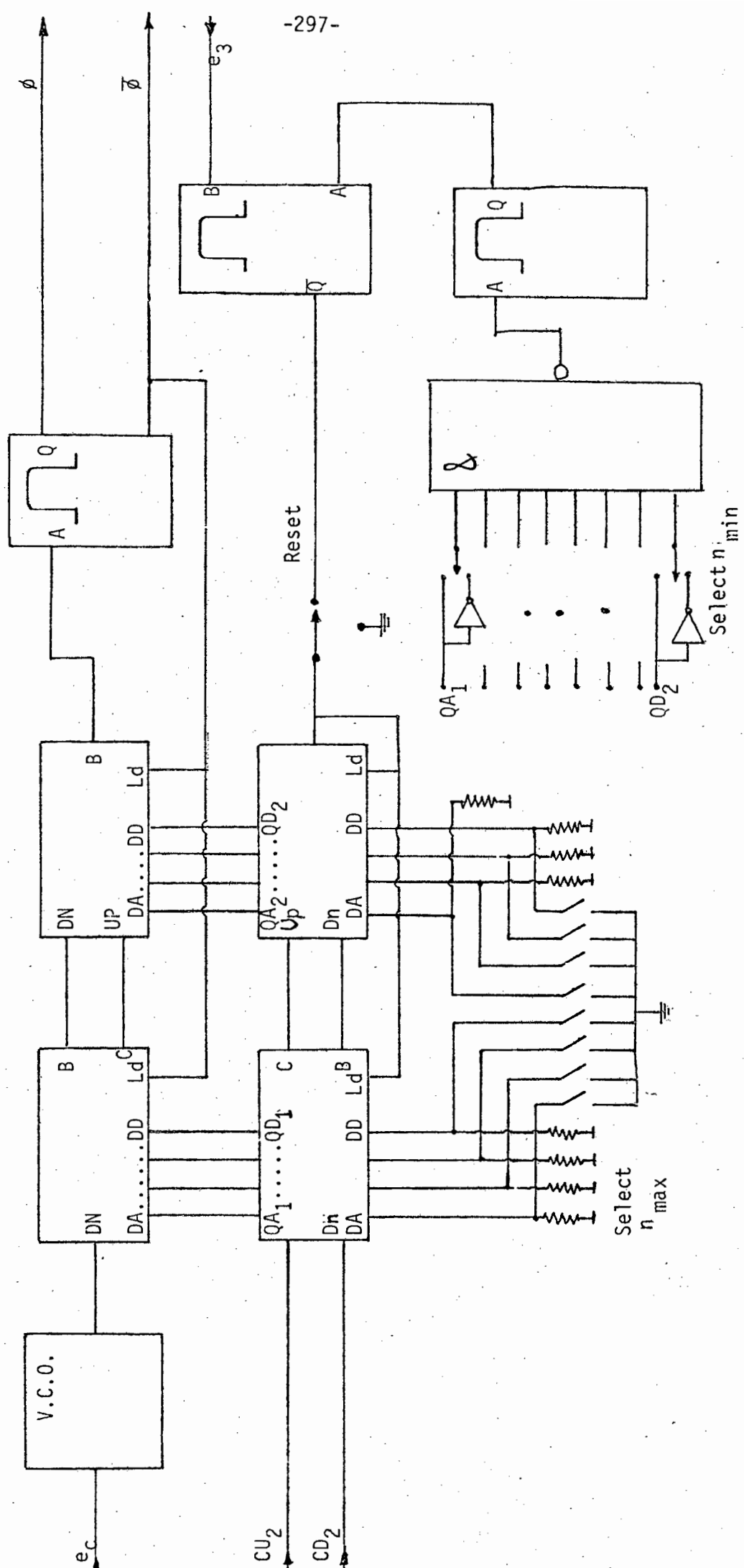


FIG. F.c - Programmable frequency divider and its control circuitry

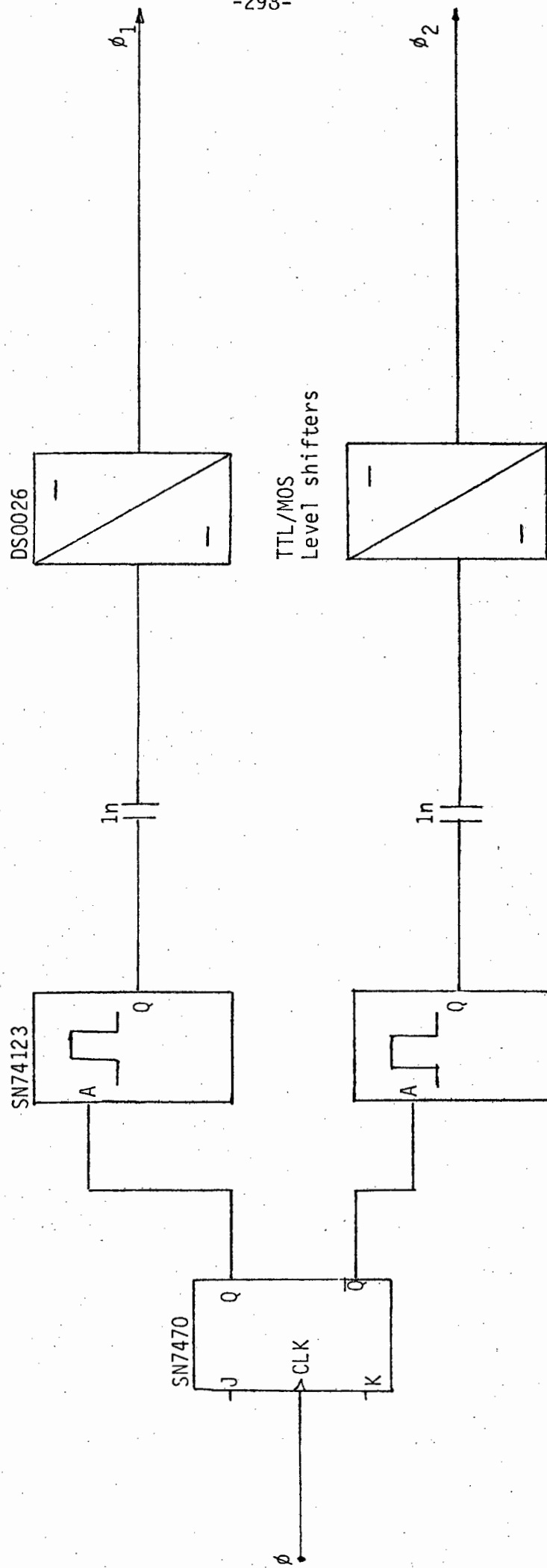


FIG. F.d - Two-phase MOS clock generator

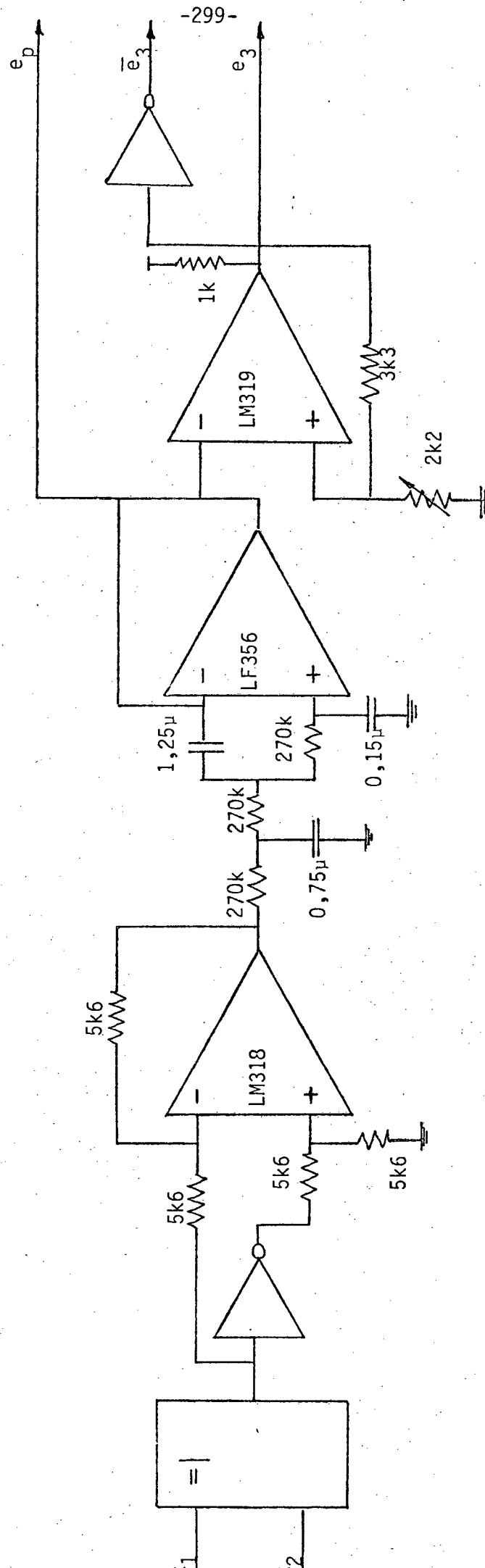


FIG. F.e - Correlation peak detector

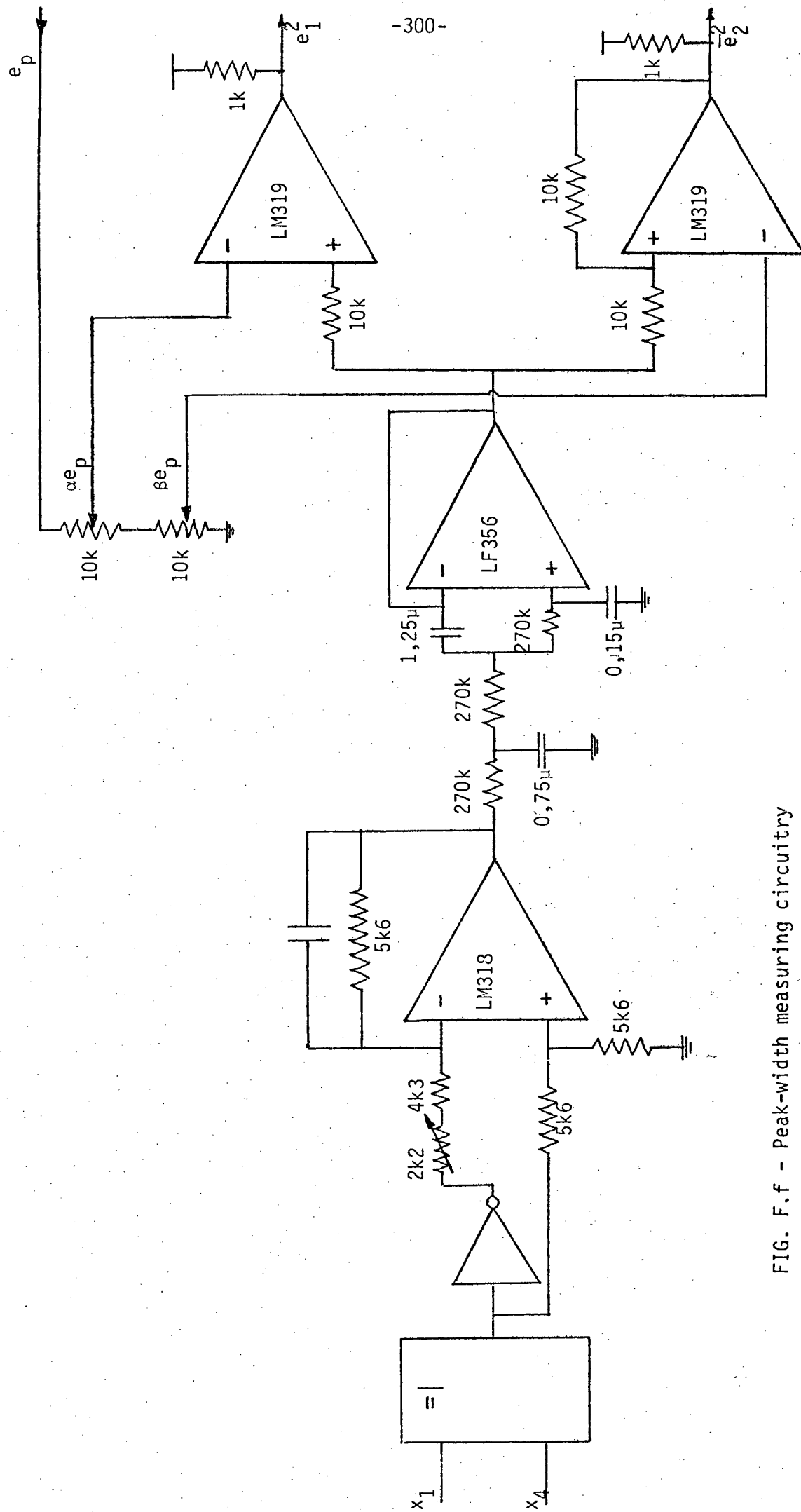


FIG. F.f - Peak-width measuring circuitry

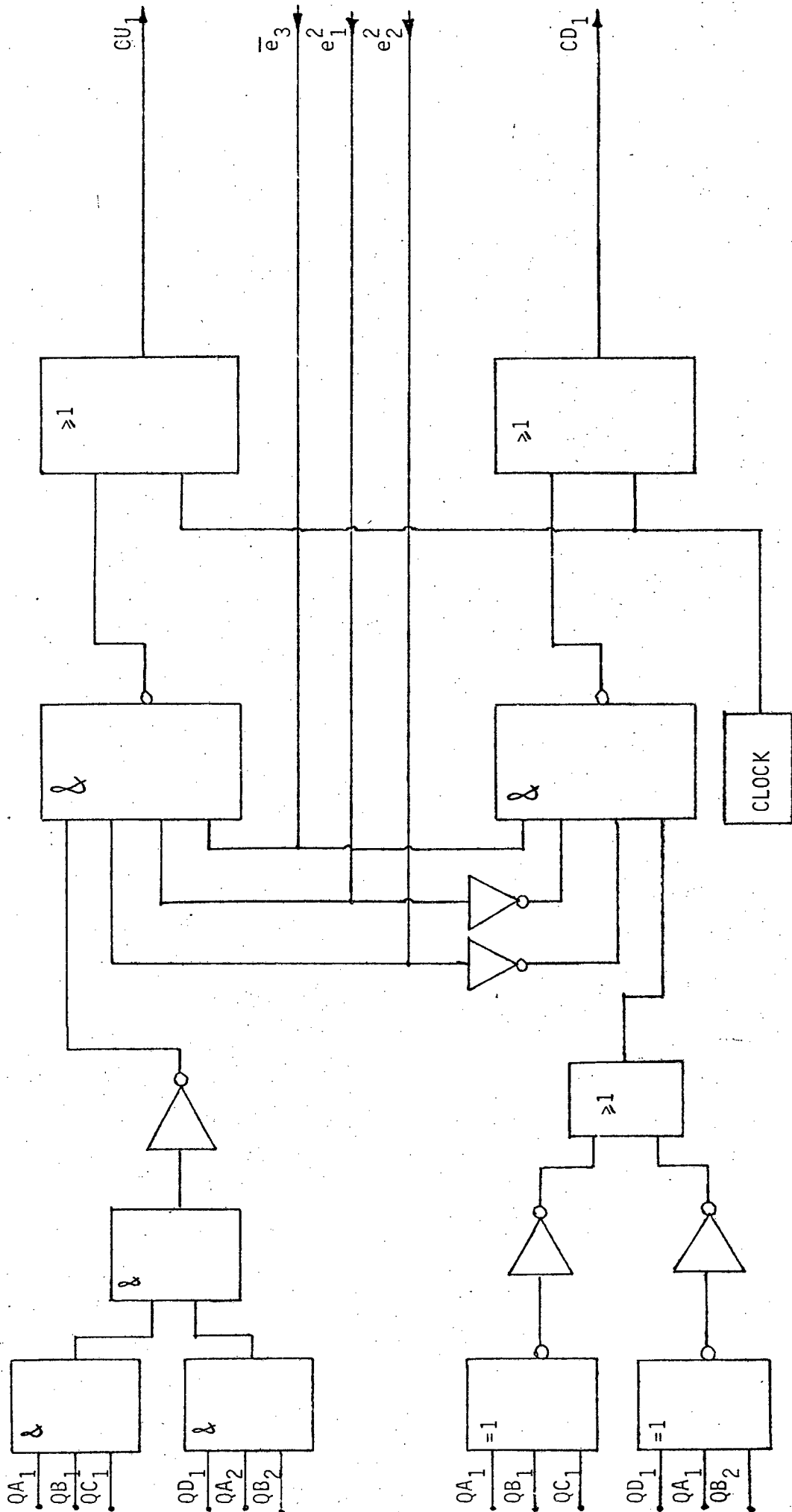


FIG. F.g - Shift register length control circuitry

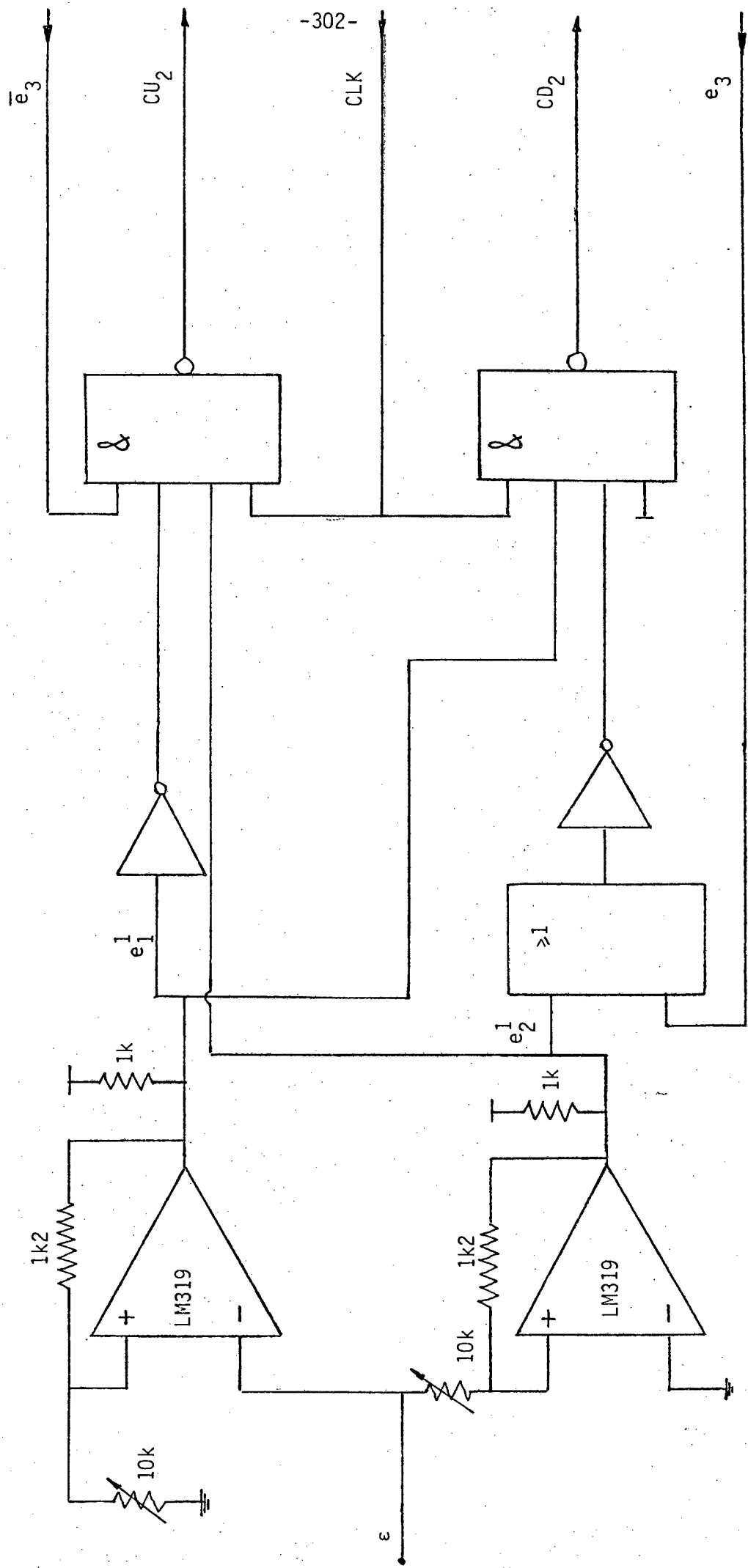


FIG. F.h - Error voltage limit control

APPENDIX G

TWO-POINT DIFFERENCE CORRELATOR SIMULATION

I.1 FIRST-ORDER MODEL

With reference to Figure 8.4 (Chapter 8), the following relations between the variables of the first-order model of the two-point difference correlator exist.

$$p(t) = \begin{cases} \epsilon(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$q(t) = k_1 \int_{-\infty}^t p(t') dt' = k_1 \int_0^t \epsilon(t') dt'$$

$$r(t) = E + q(t)$$

$$\begin{aligned} f(t) &= kr(t) = kE + kk_1 \int_0^t \epsilon(t') dt' \\ &= f_0 + kk_1 \int_0^t \epsilon(t') dt' \quad \text{where } f_0 = f(0) = kE \end{aligned}$$

$$\tau_1(t) = n/f(t) \quad ; \quad \tau_2(t) = (n+m)/f(t)$$

$$R_1(t) = R(\tau_1(t)) \quad ; \quad R_2(t) = R(\tau_2(t))$$

$$\epsilon(t) = R_1(t) - R_2(t)$$

An expression for the clock-rate, $f(t)$, can now be derived from the above equations, viz,

$$f(t) = kE + kk_1 \int_0^t \left\{ R(n/f(t')) - R(n+m/f(t')) \right\} dt'$$

Differentiating both sides gives:

$$\frac{d}{dt} f(t) = kk_1 \left\{ R(n/f(t)) - R(n+m/f(t)) \right\}$$

with initial condition:

$$f(0) = f_0 = kE$$

The basic two-point difference correlator thus is described by a first-order non-linear differential equation.

I.2 SECOND-ORDER MODEL

With reference to Fig. 8.7b (Chapter 8), the following relations exist between the variables of the modified two-point difference correlator model.

$$p(t) = \begin{cases} \varepsilon(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$q(t) = k_1 \int_{-\infty}^t p(t) dt = k_1 \int_0^t \varepsilon(t) dt$$

$$\tau(t) = E + q(t)$$

$$f(t) = k r(t) = kE + k k_1 \int_0^t \varepsilon(t) dt$$

$$\tau_1(t) = n/f(t) \quad ; \quad \tau_2(t) = (n + m)/f(t)$$

$$R'_1(t) = R(\tau_1(t)) \quad ; \quad R'_2(t) = R(\tau_2(t))$$

$$\varepsilon'(t) = R'_1(t) - R'_2(t)$$

$$\varepsilon(t) = h(t) \otimes \varepsilon'(t)$$

Using the above equations, an expression for the clock rate can be derived.

$$f(t) = kE + \int_0^t \varepsilon(t) dt$$

$$\text{where } \varepsilon(t) = h(t) \otimes \varepsilon'(t)$$

$$= \int_{-\infty}^{\infty} h(t - t') \varepsilon'(t') dt'$$

$$\text{and } \varepsilon'(t') = R(\tau_1(t')) - R(\tau_2(t'))$$

$$= R(n/f(t')) - R(n + m/f(t'))$$

Combining the above gives:

$$f(t) = kE + kk_1 \int_0^t \int_{-\infty}^{\infty} h(t - t') \left\{ R(n/f(t')) - R(n + m/f(t')) \right\} dt' dt \quad \dots I.1$$

Equation I.1 describes the clock rate of the modified two-point difference correlator as a function of time for any cross-correlation function, $R(\tau)$, and averager $h(t)$.

Assuming that an RC averager with impulse response

$$h(t) = \frac{1}{RC} e^{-t/RC}$$

is used and that the cross-correlation function is Gaussian in shape, i.e.,

$$R(\tau) = e^{-\left(\frac{\tau}{\sigma} - \frac{\bar{\tau}}{\sigma}\right)^2}$$

equation I.1 becomes:

$$f(t) = kE + kk_1 A \int_0^t e^{-t/RC} dt + \frac{kk_1}{RC} \int_0^t \int_0^t e^{-(t-t')/RC} \left\{ e^{-\left(\frac{n}{\sigma f(t')} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma f(t')} - \frac{\bar{\tau}}{\sigma}\right)^2} \right\} dt' dt$$

$$\text{where } A = e^{-\left(\frac{n}{\sigma f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2}$$

Differentiating gives:

$$\frac{d}{dt} f(t) = kk_1 A e^{-t/RC} + \frac{kk_1}{RC} \int_0^t e^{-(t-t')/RC} \left\{ e^{-\left(\frac{n}{\sigma f(t')} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma f(t')} - \frac{\bar{\tau}}{\sigma}\right)^2} \right\} dt'$$

with initial condition: $f(0) = kE$

Differentiating again and applying Leibnitz's rule for differentiation of integrals gives:

$$\frac{d^2}{dt^2} f(t) = -\frac{1}{RC} \frac{d}{dt} f(t) + \frac{kk_1}{RC} \left\{ e^{-\left(\frac{n}{\sigma f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma f(t)} - \frac{\bar{\tau}}{\sigma}\right)^2} \right\}$$

with initial conditions

$$f(0) = kE$$

$$f'(0) = kk_1 \left\{ e^{-\left(\frac{n}{\sigma f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2} - e^{-\left(\frac{n+m}{\sigma f(0)} - \frac{\bar{\tau}}{\sigma}\right)^2} \right\}$$

The modified two-point difference correlator can thus be described by a second-order non-linear differential equation.