THE ECONOMICS OF BUILDINGS

LIFE CYCLE COSTING

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I N D E X

INTRODUCTION

In today's economic climate where cost restraints are often introduced into building projects at the expense of quality, the following quotation attributed to J. Ruskin may be particularly apt:

"There is hardly anything in the world that some man cannot make a little worse and sell a little cheaper, and the people who consider price only are this man's lawful prey".

Any investment which requires large sums of money to be expended must be carefully appraised and evaluated beforehand in order to ensure that it fulfills the objectives of the investor. Real property investment is no exception and the associated problems are compounded by trying to strike a balance in the relationship between cost and quality. Life cycle costing is a method of appraising quality and total cost of buildings whereby the economic consequences of available alternatives are analysed to arrive at the optimum cost solution. This solution requires an investment which gives the greatest financial return for the least capital investment. That does not mean that the cheapest product is the best, but it does require that any analysis or appraisal must forecast and assess the economic consequences over the full life of the particular project.

In practice there are many constraints, such as town planning regulations, available finance, etc. which so limits the number of alternative solutions that a true optimum result cannot be achieved. Notwithstanding the above it is still essential to choose the best possible solution from the remaining available possibilities.

The motivations and objectives underlying each and every investment will be many and varied but the one characteristic common to most is "value-for-money". All real property investments have a value but measurement of this value is not always possible in monetary terms alone. Many of the benefits derived from ownership may be indirect or intangible with the resultant difficulty in assessing on purely economic grounds. Prestige value and contentment are cases in point. Since appraisal methods discussed in the following chapters are concerned solely with the economic aspects it must be clearly understood that they are only part of the total evaluation process which aids management in making responsible decisions. They are not substitutes for management's judgement. The final decision and responsibility still rests with the Client who must evaluate all aspects such as risks, costs, available finance, probabilites, etc. peculiar to his own particular situation in order that a total appraisal can be achieved to maximise the value of the investment.

The problems associated with real property investment are increased because of the magnitude and complexity of many modern buildings and the multiplicity of materials and forms of construction available. The high rate of inflation resulting in rapid escalation of building and maintenance costs is compounded by the cost of energy and has increased running expenses to unprecedented proportions. This has led to a change of investment strategy on the part of many informed investors who are now demanding a greater degree of expertise and sophistication from those involved in the economic appraisal of capital projects.

The professions associated with the building industry have long recognised that cost considerations have become a dominant feature of real property investments. In an attempt to achieve well formulated economic design solutions, within defined cost limits, cost planning and control is now fairly extensively used throughout this Country. However, maintenance and operating costs, although often recognised, are seldom taken into account. This was not particularly important in the late sixties and early seventies when energy costs, in particular, were relatively inexpensive and no energy shortage was envisaged. Rapid re-thinking has lately taken place and it is now realised that for cost planning and control to be truly effective the total cost throughout the life of the building must be considered.

If one is to arrive at a true reflection of the investment potential and total cost of a particular real property investment the appraisal method used must be capable of allowing for the following factors which, although by no means complete, incorporates those aspects which are required to give a valid and meaningful assessment of the investment.

1. Comparisons to be made with alternative investment opportunities

2. Limited life of certain components in a building which may have to be repaired or replaced

3. Comparisons to be made with alternative materials and/or forms of construction

4. The total costs and income generated throughout the effective life of the building including residual value at the expiry of the investment

5. The effect of the time value of money and the impact of inflation

6. Income tax implications

7. Adaptation to the individual characteristics of each specific project

Life cycle costing which is essentially similar in concept to costs-in-use or terotechnology, is a sophisticated method of appraisal capable of satisfying the abovementioned needs. It could be defined essentially as :-

> A FINANCIAL APPRAISAL TECHNIQUE WHICH ALLOWS VALID COMPARATIVE EVALUATION OF AVAILABLE ALTERNATIVE POSSIBILITIES BASED ON TIME-PHASED COSTS OVER A SPECIFIC INVESTMENT PERIOD IN ORDER TO ARRIVE AT THE OPTIMUM ECONOMIC SOLUTION.

The purpose of this thesis is to examine the economics of buildings with special reference to life cycle costing in order to illustrate how valid comparisons of alternative construction methods or material selection can be achieved with a view to optimizing total costs.

CHAPTER 1

THE TECHNIQUE OF LIFE CYCLE COSTING

THE TECHNIQUE DEFINED

Life cycle costing is the term applied to the measurement of total costs incurred over the full life of a building. These costs, resulting from a decision to build, may be divided into the following categories:

1. Capital costs - includes all initial costs related to the development of the new facility and is computed from inception of the project until the start of the accounting period.

2. Running or operating costs - embraces those costs required to operate the building and includes such items as energy costs, cleaning expenses, etc.

3. Maintenance costs - are periodic costs involving charges incurred in order to maintain the building in an acceptable standard of repair. Although periodic, these costs will be continuing and involve items such as redecorations.

4. Repair and replacement costs - includes expenditure on repair and replacement of components with limited lives.

5. Alteration and improvements costs - are costs of improving or adapting a facility to provide for a function not originally anticipated.

6. Residual or salvage costs - demolition or removal may constitute an expense whereas re-sale at the end of the investment period will contribute towards income and must be offset against expenses.

Division into the above categories is not absolute but is given to identify terminology used in this document. Interest, inflation, opportunity costs and income tax all have a bearing on the above and will be dealt with in subsequent chapters.

Income tax and inflation are two factors that increase the complexity and volume of life cycle cost study calculations. In order to provide a fairly simple and quick method of undertaking life cycle cost studies incorporating both income tax and inflation allowances I have developed a set of compound interest tables that accommodates increases in future costs arising from inflation and depreciation allowances based on the declining balance method. These tables can be found in the Appendix at the end of the thesis.

When predicting future recurring or replacement costs it will be necessary to take into account increases due to:-

1. Inflation
2. Difference in working conditions and scale of operations. Replacement will not necessarily be on the same scale as the original operation and will in all probability also be executed under more trying or difficult conditions.

3. The costs of demolishing and removing existing work as well as the protection of remaining structures and finishes while the work is in progress.

4. Costs of disturbances to and/or by the occupiers during building operations.

Life cycle cost analysis has already been defined as a financial appraisal technique allowing comparative evaluations of available alternative possibilities. It is essential that this is clearly understood since a life cycle cost expressed alone is almost meaningless. In essence, a life cycle cost provides a frame work for selection of alternative possibilities where there is more than one method of achieving certain defined objectives. These mutually exclusive possibilities may differ in both initial and running costs and this technique allows consideration of all relevant costs incurred by converting them to equal terms at common points in time.

Unless there are available alternatives there is no need for a decision to be taken and hence, no need for life cycle cost analysis. However, unless these alternatives are clearly defined,incorrect decisions may be made by failure to recognise possible substitutes. Even if several alternatives are considered, failure to formulate and define the objectives may still result in ineffective selections as a result of not considering the best solution. A poor alternative will obviously appear the best selection if compared with alternatives that are even worse.

Although many basic design decisions are subject to outside influences such as town planning regulations and, as such, are not candidates for life cycle costing, the majority have multiple options each with its own economic consequence. For life cycle costing to play a meaningful role in the investment appraisal process it must be introduced at the initial stage of the development, so that economic consequences of design and construction decisions can be assessed prior to implementation.

Financial assessments of set decisions, while obviously important, will not ensure that the best possible decision has been made.

Having defined the main alternatives which are to be analysed, the next step is to establish the life cycle of the investment. This life cycle should be set in consultation with the Client in order to ensure that it corresponds with his functional and / or economic objectives. This will also involve the establishment of start and end dates for the investment. The lives of building components or elements are not dependent upon policy decisions and estimating the anticipated useful life remains the responsibility of the design team. A further important aspect in this regard is that a life cycle cost analysis is concerned solely with the total cost over the determined life of the investment. Aspects such as obtaining increased rentals by the use of superior quality and more expensive finishes is regarded as a separate issue and has not been considered in this thesis.

The choice of interest rate is another extremely important aspect that requires Client participation. This is discussed in greater detail in later chapters but it is important to note that the interest rate is an individual matter that must be selected by the owner prior to implementation of life cycle cost analysis. A further important point is that all life cycle cost studies are assumed to apply to either 100 per cent borrowed or 100 per cent equity funded situations.

Although a tax rate of 50 per cent has generally been used in the examples contained in this thesis it must be remembered that this rate will be modified according to the tax position of each individual investor. Similarly the time lag factor between income accrued and actual payment of income tax together with the 'loan levy' portion has not been taken into account although both can be accommodated fairly easily.

Life cycle costing employed at the inception stage of a development can be used as a technique for determining whether or not to build.

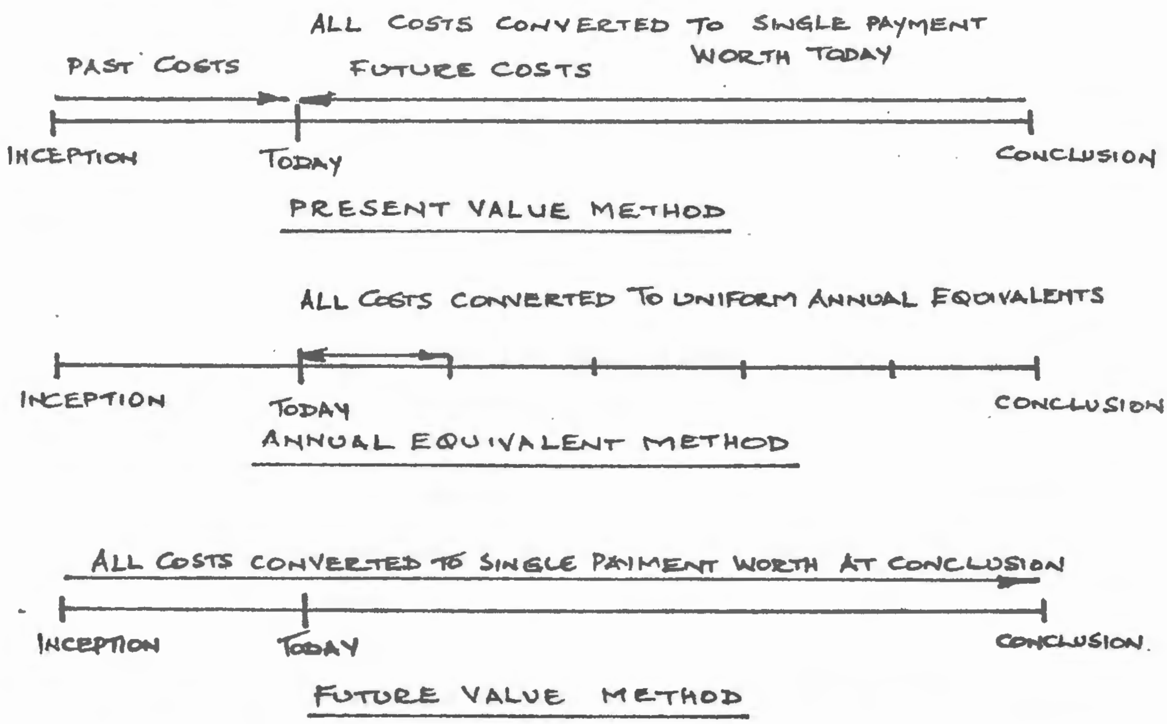
Once the decision to build has been taken the optimum basic design solution can be determined. During the initial stages it will be preferable to concentrate on those aspects which potentially have the largest impact on both capital and running costs. Thereafter, attention can be given to those items which are expected to be heavily maintenance intensive. This technique is ideally suited to assist in the selection of appropriate building components, and it is examples of this nature that have been used to illustrate the basic principles and techniques of life cycle costing. However it must be emphasized that the optimum solution representing the best value and the lowest total cost may not be capable of realisation due to shortage of capital on the part of the investor.

There are several methods which may be employed in order to give a fair evaluation of alternative proposals. The three procedures stated below, provide for all cash flows, regardless of when they are incurred, to be converted to equal terms at common points in time. The first two which are widely accepted and commonly used will be considered in detail. The third method has not found favour although it is suitable for use in life cycle cost studies.

1. Equivalent Present Value Sums - representing a total single payment, present worth.
2. Equivalent Annual Value Sums - representing uniform annual costs.
3. Equivalent Future Value Sums - representing a total single payment worth at the conclusion of the investment

This method is the inverse of the Present Value method but has not been considered in any detail.

These methods are clearly illustrated by the time line diagrams shown below:



TIME LINES ILLUSTRATING THE THREE METHODS EMPLOYED IN LIFE CYCLE COST TECHNIQUES

All these methods are fully interchangeable, the choice of method being dependent upon the user and his objectives. The equivalent annual value sum method is ideally suited to build or lease situations, but whichever technique is followed the concept of equivalence comparisons remains fundamental to life cycle costing.

EQUIVALENCE

For meaningful comparisons to be made of differing sums of money occurring at various points in time the concept that payments differing in quantity but made at different periods in time may be equivalent to one another is a fundamental supposition. The compounding and discounting formulae discussed hereafter enable us to compute values of differing sums occurring at various points in time, and so forms the basis for comparing alternative possibilities.

For example, using a present value of R1 000.00, an interest rate of 10 per cent and a time period of three years the following series of payments will be equivalent to one another. Inflation and income tax, which will modify these results, have been ignored.

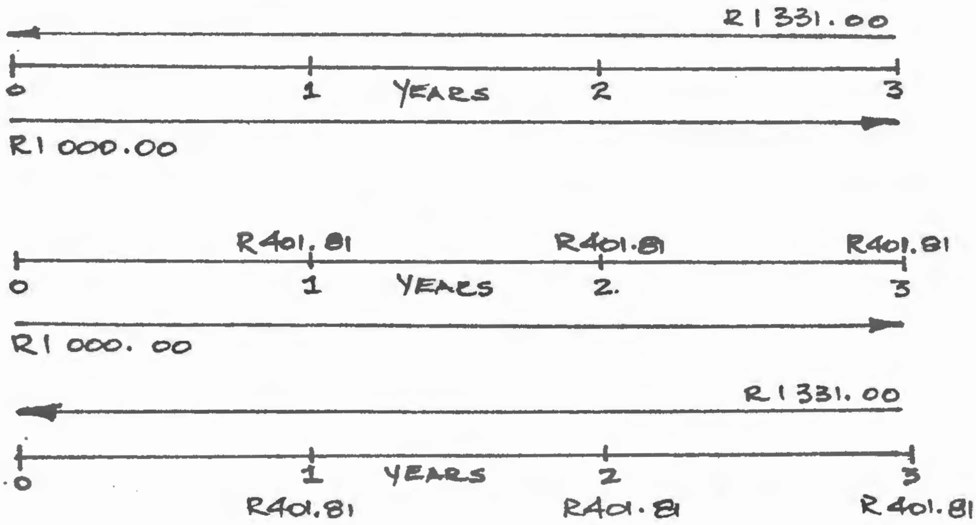
1. R1 000.00 today is equivalent to R1 331.00 receivable in three years time.

2. R1 331.00 receivable in three years time is equivalent to R1 000.00 today

3. R1 000.00 today is equivalent to R401.81 per annum for the next three years

4. R401.81 per annum for the next three years is equivalent to R1 331.00 receivable in three years time.

Any modification to the stipulated interest rate or time period will render different values and the payment series would not be equivalent.



TIME LINES ILLUSTRATING THE CONCEPT OF EQUIVALENCE

CHAPTER 2

COMPOUNDING AND DISCOUNTING FORMULAE

Since life cycle cost analysis deals with costs over the life of an investment, and involves both money received or expended today and money to be received or to be expended in the future, the effects of the time value of money will be fundamental to any analysis. Central to the 'time value of money' concept is the fact that money has the capacity to earn interest and, as such, the basic principles of compounding and discounting must be clearly understood. The derivations and explanations of the applicable mathematical formulae are available in numerous textbooks and will not be considered in this document. However, it is important that the functions performed by these formulae are clearly defined and understood so that they are capable of being used in their correct context.

SYMBOLS

The following symbols are employed in the mathematical formula in this document:

ATP designates the present value of money after payment of income tax

ATPLC designates the present value of expenditure after allowance for income tax on interest and tax relief on expenditure

ATPLCD designates the present value of depreciation charges after allowance for income tax on interest and tax relief which is to be deducted from expenditure

ATRS designates the real value of a sum of money, at a specified future point in time, after allowance for inflation and income tax

ATS designates a sum of money, at a specified future point in time, after payment of income tax

D designates the annual depreciation for income tax purposes

L designates the life of an asset

P designates the principal or present value of money

PMT designates a uniform series of payments which are paid at the end of each period

PMTD designates a uniform series of payments which are paid at the start of each period

R designates the residual value of an asset

RS designates the real value of a sum of money, at a specified future point in time, after allowance for inflation

S designates a sum of money at a specified future point in time b designates the rate of interest earned after payment of income tax

i designates the interest rate per period. In this document it shall also indicate the 'nominal' interest rate per annum and all given rates of interest shall, unless specifically stated otherwise, be deemed to mean interest compounded annually, i.e. an interest rate stated as 10% shall indicate that the interest rate is 10% compounded annually

m designates the inflation rate per annum

n designates the number of compounding periods

p designates the compounding period when they are shorter than one year

q designates the real interest rate after allowance for inflation

r designates the 'effective' interest rate when the interest periods are shorter than one year

t designates the percentage tax payable on accrued interest

COMPOUND INTEREST

For an investment or transaction continuing over a period of time, interest may be calculated in one of two ways:-

(i) At the end of each interest period the interest earned is paid to the investor. The capital remains constant and hence interest payments are unchanged throughout the investment life. This is termed simple interest.

(ii) At the end of each interest period the interest earned is added to the capital sum thus increasing the capital amount. This re-investment of interest at the end of each interest period is termed compounding and the total amount at the termination of the investment life is called the compounded amount. The difference between the compounded amount and the original capital investment is known as compound interest.

The investment appraisal techniques referred to in this document all involve the principle of compound interest. The operation of compound interest is fundamental to all formulae used in life cycle costing and reflects the 'time value of money' concept - namely that money has the capacity to 'multiply' with time. Simple interest, because interest does not earn interest, is unable to recognize this growth factor and as such is unsuitable for real property investment appraisal techniques.

In order to visualize the concept of compounding, the following simple example is given.

EXAMPLE 1

If R1 000.00 is invested for three years at 10 per cent interest what is the total sum at the end of that period?

Principal at start of year 1	1 000.00	
Interest for year 1	100.00	
Principal at start of year 2	1 100.00	
Interest for year 2	110.00	
Principal at start of year 3	1 210.00	
Interest for year 3	121.00	
TOTAL SUM		R1 331.00

The mathematical formula for compound interest of a single payment amount is:-

$$S = P(1 + i)^n$$

and the problem in example 1 is solved as follows:-

$$\begin{aligned} S &= P(1 + i)^n \\ &= 1000(1 + ,10)^3 \\ &= 1000 \times 1,331 \\ &= \underline{R1\ 331.00} \end{aligned}$$

It must be noted that this formula applies only to a single payment amount invested at the start of the accounting period. The time line below shows clearly the results of the annual compounding of R1 000.00 invested at 10 per cent per annum for three years.

INVESTMENT

R1 000.00	R1 100.00	R1 210.00	R1 331.00
Year 0	Year 1	Year 2	Year 3

Life cycle costing involves the economic appraisal of various alternative possibilities each having diverse life spans. For valid comparisons to be made it is generally convenient to calculate the present value instead of the future or compounded value. This process is known as discounting and is the reciprocal of the method for calculating a future sum. The formula for the present value of a future single payment is:-

$$P = S\left[\frac{1}{(1 + i)^n}\right] \quad \text{or} \quad P = \frac{S}{(1 + i)^n}$$

This function is described in the following example.

EXAMPLE 2

What is the present value of R1 331.00 receivable in three years time if money can be invested at 10 per cent interest?

$$\begin{aligned} P &= \frac{S}{(1 + i)^n} \\ &= \frac{1331}{(1 + ,10)^3} \\ &= \frac{1331}{1,331} \\ &= \underline{R1\ 000.00} \end{aligned}$$

To avoid unnecessary and laborious calculations, compound interest tables may be referred to for problems of this nature. These tables are readily available and the factors contained therein will be used in certain examples. This does not diminish the importance of a clear understanding of the basic principles and functions of the mathematical formulae. Lack of an absolute comprehension of these fundamentals will invariably result in incorrect usage of compound interest tables.

NOMINAL AND EFFECTIVE RATES OF INTEREST

Compound interest is generally expressed as an annual rate, although it may be paid over daily, monthly, quarterly or half-yearly periods. Since the interest rate is the 'interest rate per period' the effect of compounding periods shorter than one year must be recognised.

EXAMPLE 3

If R1 000.00 is invested for three years at 10 per cent interest, compounded quarterly, what is the total sum at the end of that period?

$$S = P\ (1 + i)^n$$

$$= 1000 \left[1 + \frac{,10}{4}\right]^{12}$$

$$= 1000\ (1 + 0,025)^{12}$$

$$= \underline{R1\ 344.89}$$

The stated annual interest rate for compound periods of less than one year is termed the 'nominal interest rate' whereas the actual rate of interest earned in one year is termed the 'effective interest rate'.

In the previous example the 10 per cent nominal interest rate results in an effective interest rate of 10,38 per cent when compounded quarterly. The mathematical formula for the calculation of the effective interest rate is:

$$r = \left[1 + \frac{i}{p}\right]^{p} - 1$$

so that

$$r = \left[1 + \frac{,10}{4}\right]^{4} - 1$$

$$= (1{,}025)^{4} - 1$$

$$= \underline{10{,}38\%}$$

It is important that the interest rate per period corresponds with the compounding period interval, and mixtures such as monthly interest rates with half-yearly periods are incorrect.

The interest rates and periods for a nominal interest rate of 12 per cent per annum compounded for 3 years is demonstrated below:-

Compounding time	Interest rate	No. of periods
Annually	0,12	3
Half yearly	0,06	6
Quarterly	0,03	12
Monthly	0,01	36

One interesting aspect concerning economic viability studies is that the effective interest rate is rarely utilised even though most payments and revenues are conducted on a monthly basis. The convention of compounding for a yearly period is generally adopted and the assumption is that both income and expenditure will be single lump sum payments occurring at the end of each year. Where investments involve large payments, appreciable differences to the cash flow could result and it is my contention that in studies of this nature serious consideration should be given to the use of effective interest rates.

For present value appraisal techniques involving compounding periods of greater frequency than one year, the effective interest rate must first be calculated before computing the present value.

EXAMPLE 4

What is the present value of R1 344.89 receivable in three years time if money can be invested at 10 per cent interest, compounded quarterly?

The procedure used in the previous example is again adopted to realise an effective interest rate of 10,38 per cent.

$$P = \frac{S}{(1 + i)^n}$$

$$= \frac{1344.89}{(1,1038)^3}$$

$$= R1\ 000.04$$

This discrepancy is a result of rounding off to four decimal places. Similar results, also as a result of rounding off, will be encountered in the following chapters.

ANNUITIES

An annuity involves a series of equal payments received or expended at regular periods of time and, as such, property investment appraisal techniques lend themselves ideally to common usage of these formulae. An 'ordinary annuity' is an annuity in which payments are made at the end of each period whereas an 'annuity due' is an annuity in which payments are made at the start of each period.

(i) Ordinary Annuity

The formula for the sum of an ordinary annuity is:-

$$S = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

which arises from the formula for compounding a single payment amount. The use of this formula is best explained by a practical example.

EXAMPLE 5

If R1 000.00 is invested at 10 per cent interest at the end of each year for a period of three years what is the total sum at the end of that period?

$$S = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

$$= 1000 \left[\frac{(1 + ,10)^3 - 1}{,10} \right]$$

$$= R3\ 310.00$$

The time line below illustrates the above example:-

0	End year 1	End year 2	End year 3
	Payment 1	Payment 2	Payment 3
	R1 000.00	R1 000.00	R1 000.00

$$S = 1000\ (1 + i)^2 + 1000\ (1 + i) + 1000$$

$$= 1210 + 1100 + 1000$$

$$= R3\ 310.00$$

(ii) Annuity Due

The mathematical formula for equal payments paid at the start of each period is:-

$$S = PMTD \left[\frac{(1 + i)^n - 1}{i} \right] (1 + i)$$

Considering the problem in example 5 but with payments made at the start of each period.

$$S = 1000 \left[\frac{(1 + ,10)^3 - 1}{i} \right] (1 + ,10)$$

$$= 1000 \times \frac{,331}{,10} \times 1,10$$

$$= R3\ 641.00$$

The difference between an 'ordinary annuity' and an 'annuity due' is clearly shown in the time line below:-

O	End year 1	End year 2	End year 3
Payment 1	Payment 2	Payment 3	No payment
R1 000.00	R1 000.00	R1 000.00	

$$\begin{aligned} S &= 1000\ (1 + i)^3 + 1000\ (1 + i)^2 + 1000\ (1 + i) \\ &= 1331 + 1210 + 1100 \\ &= \underline{R3\ 641.00} \end{aligned}$$

Unless expressly stated otherwise, annuity formulae used in the following chapters will be the ordinary annuity formula.

As in the case of the compound interest formulae it will generally be more convenient to express the annuities in terms of present value. The formula for the present value of an ordinary annuity is:-

$$P = PMT \left[\frac{(1 + i)^n - 1}{i}\right] \left[\frac{1}{(1 + i)^n}\right]$$

$$\therefore P = PMT \left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right]$$

EXAMPLE 6

What is the present value of R1 000.00 invested at 10 per cent interest at the end of each year for a period of three years?

$$\begin{aligned} P &= PMT \left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right] \\ &= 1000 \left[\frac{(1 + {,}10)^3 - 1}{{,}10(1{,}10)^3}\right] \\ &= \underline{R2\ 486.85} \end{aligned}$$

Therefore, if R2 486.85 is invested at 10 per cent interest for a period of three years the future value would be:-

$$S = P \quad (1 + i)^n$$
$$= 2\ 486.85\ (1 + ,10)^3$$
$$= \underline{R3\ 310.00} \quad \text{(see example 5)}$$

The mathematical formula for the present value of an annuity due is expressed as:-

$$P = PMTD \left[\frac{(1 + i)^n - 1}{i(1 + i)^n} \right] (1 + i)$$

EXAMPLE 7

What is the present value of R1 000.00 invested at 10 per cent interest at the start of each year for a period of three years?

$$P = 1\ 000 \left[\frac{(1 + ,10)^3 - 1}{,10(1 + ,10)^3} \right] (1 + ,10)$$

$$= 1\ 000 \left[\frac{,331}{,1331} \right] 1,10$$

$$= \underline{R2\ 735.54}$$

If this amount was invested at 10 per cent interest for a period of three years the future sum would amount to:-

$$S = P \quad (1 + i)^n$$
$$= 2735,54 \quad (1,10)^3$$
$$= \underline{R3\ 641.00}$$

SINKING FUND PAYMENTS

Sinking fund payments are annual payments invested to produce a required sum at the end of a specific period. The formula for an ordinary annuity is $S = PMT \left[\frac{(1 + i)^n - 1}{i}\right]$ where the future sum is calculated on the basis of annual payments, interest rate and duration of the investment. Therefore, based on this equation the formula for sinking fund payments will be:-

$$PMT = S \left[\frac{i}{(1 + i)^n - 1}\right]$$

In this equation it is once again assumed that the payments are made at the end of each period and that no interest will be applied to the final payment.

EXAMPLE 8

If R3 310.00 is required at the end of three years and money is invested at 10 per cent interest, what amount must be invested at the end of each year to obtain the required amount?

$$PMT = S \left[\frac{i}{(1+i)^n - 1}\right]$$

$$= 3310 \left[\frac{,10}{(1 + ,10)^3 - 1}\right]$$

$$= \underline{R1\ 000.00}$$

With a basic understanding of the formulae described in the preceding pages, life cycle costing of alternative possibilities can be undertaken. However, it must be emphasised that indiscriminate application of the formulae can lead to serious error and it is essential to ensure that the equation used is the correct one to perform the function intended.

CHAPTER 3

INCOME TAX AND INFLATION

Two factors which have significant cost implications are income tax and inflation. Each of these will be dealt with separately.

INCOME TAX IMPLICATIONS

Broadly speaking income tax is a charge levied on the gross income after deduction of permissable expenditure. The percentage tax payable depends upon the tax position of the investor, but since most property investors operate not as private individuals but as companies, the problems associated with income tax calculations are alleviated to a certain degree. In spite of the fact that the company tax rate is not determined by the income generated, constant changes in the tax structure make it extremely difficult to predict with any degree of certainty the effects of income tax over a prolonged period of time.

It is not the intention to describe in detail the tax system which is in itself a lengthy and complicated subject, but merely the general techniques involved in order to illustrate the effects of income tax on life cycle costing. The first consideration will be the effects of income tax on accrued interest.

Assuming that the percentage tax payable is constant over the period of the investment the formula for an after tax single payment compound interest amount is:-

$$ATS = P\left[1 + i\ (1 - t)\right]^n$$

The income tax implications of a single payment investment can be seen in the following example:-

EXAMPLE 9

If R1 000.00 is invested for three years at 10 per cent interest and the tax payable is 50 per cent, what is the total sum at the end of that period?

$$\begin{aligned} ATS &= P \quad \left[1 + i\ (1 - t)\right]^n \\ &= 1000 \quad \left[1 + ,10\ (1 - ,5)\right]^3 \\ &= 1000 \quad (1,05)^3 \\ &= \underline{R1\ 157.63} \end{aligned}$$

This can be verified by tracing the history of the investment as follows:-

Principal at start of year 1		1 000.00
Interest for year 1		100.00
		1 100.00
Tax payable on R100.00 interest	(50%)	50.00
Principal at start of year 2		1 050.00
Interest for year 2		105.00
		1 155.00
Tax payable on R105.00 interest	(50%)	52.50
Principal at start of year 3		1 102.50
Interest for year 3		110.25
		1 212.75
Tax payable on R110.25 interest	(50%)	55.12
Future value		R1 157.63

To calculate the rate of interest earned after the payment of income tax the following formula applies:-

$$b = i\ (1 - t)$$

Therefore, if income tax is 50 per cent, a 10 per cent interest rate would give an effective rate of 5 per cent after payment of tax.

$$b = i\ (1 - t)$$
$$= ,10\ (1 - ,5)$$
$$= \underline{,05 \text{ or } 5 \text{ per cent}}$$

To achieve a stipulated after tax interest rate the formula to be adopted is:-

$$b = i\ (1 - t)$$
$$\therefore\ i = \frac{b}{1 - t}$$

A required after tax interest rate of 10 per cent per annum with income tax deductions of 50 per cent would need an initial interest rate of:-

$$i = \frac{b}{1 - t}$$
$$= \frac{,10}{1 - ,5}$$
$$= \underline{,20 \text{ or } 20 \text{ per cent}}$$

For income tax allowances to be incorporated into an annuity formula the interest rate must be adjusted to allow for the income tax percentage.

EXAMPLE 10

If R1 000.00 is deposited at the end of each year for a period of three years at 10 per cent interest and the tax rate is 50 per cent the future value will be:-

$$b = i\ (1 - t)$$
$$= ,10\ (,5)$$
$$= \underline{,05}$$
$$ATS = PMT \left[\frac{(1 + b)^n - 1}{b}\right]$$
$$= 1000 \left[\frac{(1,05)^3 - 1}{,05}\right]$$
$$= \underline{R3\ 152.50}$$

The aforementioned is further illustrated by means of the following time line:-

0	End year 1	End year 2	End year 3
	Payment 1	Payment 2	Payment 3
	R1 000.00	R1 000.00	R1 000.00

$$
\begin{aligned}
S &= 1000\,(1 + b)^2 + 1000\,(1 + b) + 1000 \\
&= 1000\,(1 + ,05)^2 + 1000\,(1 + ,05) + 1000 \\
&= 1102{,}50 + 1050 + 1000 \\
&= \underline{R3\ 152.50}
\end{aligned}
$$

Similarly when considering after-tax present values the discount rate must be adjusted to allow for income tax. This formula will read:-

$$ATP = \frac{S}{[1 + i\,(1 - t)]^n} \quad \text{or} \quad ATP = \frac{S}{(1 + b)^n}$$

EXAMPLE 11

What is the present value of R1 157.63 receivable in three years time if interest is 10 per cent and the tax payable 50 per cent?

$$
\begin{aligned}
ATP &= \frac{S}{[1 + i\,(1 - t)]^n} \\
&= \frac{1157{,}63}{[1 + ,10\,(1 - ,5)]^3} \\
&= \underline{R1\ 000.00}
\end{aligned}
$$

The after-tax present value formula for an annuity is:-

$$ATP = PMT\left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right] \quad \text{or} \quad ATP = PMT\left[\frac{(1 + b)^n - 1}{b}\right]\frac{1}{(1 + b)^n}$$

EXAMPLE 12

What is the present value of R1 000.00 deposited at the end of each year for a period of three years at 10 per cent interest if the tax rate is 50 per cent?

$$ATP = PMT \left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right]$$

$$= 1000 \left[\frac{(1 + ,05)^3 - 1}{,05(1 + ,05)^3}\right]$$

$$= \underline{R2\ 723.25}$$

So far income tax payments and their effects on accrued interest have been investigated. A further consideration must be the effects of income tax on net income from the investment. Tax payments on income may be treated as negative cash flows. However, as far as life cycle costing is concerned the analysis represents only expenses incurred in the life of the investment. For tax implications to be accommodated the tax relief gained by expenses must be treated as a reduction of maintenance costs.

EXAMPLE 13

What is the present value of R1 157.63 expended in three years time if interest is 10 per cent and the tax payable 50 per cent?

$$ATPLC = \left[\frac{S}{(1 + b)^n}\right] (1 - t)$$

$$= \left[\frac{1157,63}{(1,05)^3}\right] X\ ,5$$

$$= \underline{R500.00}$$

The formula for an after tax present value of a series of uniform expenditures occurring at the end of each period would therefore read:-

$$ATPLC = PMT \left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right] (1 - t)$$

or

$$ATPLC = PMT \left[\frac{(1 + b)^n - 1}{b}\right]\left[\frac{1}{(1 + b)^n}\right](1 - t)$$

EXAMPLE 14

What is the present value of R1 000.00 expended at the end of each year for a period of three years at 10 per cent interest if the tax rate is 50 per cent?

$$ATPLC = 1000\left[\frac{(1 + ,05)^3 - 1}{,05(1 + ,05)^3}\right] ,5$$

$$= 1000 \times \frac{,157625}{,05788125} \times ,5$$

$$= R1\ 361.63$$

Thus income tax paid on accrued interest has the same effect as reducing the interest rate and increasing the present value. However, should future payments represent an expense which is tax deductible, the present value amount must then be reduced by the tax relief factor. This results in the unique position of having increased present values on the one hand being offset by reduced present values on the other, both of which are occasioned by the same factor, income tax.

Assuming that a building is to be repainted in three years time at a cost of R1 331.00 and the interest rate used is 10 per cent.

The present value of the future sum would, in this instance, be R1 000.00 If a tax payment of 50 per cent was introduced the effect on the accrued interest would be to increase the present value to R1 149.77. Commercial developments would be allowed to deduct painting as an expense against income and the applied tax relief factor results in a present value of R579.89

Another important aspect in respect of income tax implications is the factor of depreciation.

DEPRECIATION

In this document depreciation shall mean depreciation for tax purposes and is the amount or percentage, allowed by the Receiver of Revenue, to be deducted from income. Although the building itself cannot, in normal circumstances, be depreciated, certain components therein may be subject to depreciation and tax relief obtained therefrom. No fixed rates of depreciation or list of components are laid down since they are subject to agreement with the Receiver. In order to give a clearer indication of what allowances might be permissible the following is a list given by Mr. F Bihl (Project Management Seminar - UNISA October 1977) of allowances granted by the Receiver on an actual project. In this particular case the percentages were adjusted on a 'straight line' basis whereas the more common practice would be the use of the 'reducing balance' method.

Annual Depreciation (for income tax purposes)

Lifts (movable parts only)	5%
Escalators (movable parts only)	5%
Electric stoves	10%
Gas meters	10%
Loose standing partitions	7½%
Incinerators	7½%
Waste compactor	10%
Public address system	10%
Air compressors	10%
Water softener plant	10%
Water boilers	5%
Fire hose reels	5%
Fire extinguishers	5%
Booster pumps	5%
Sump pump	5%
Blinds and curtains	25%
Carpets	25%
Tea kitchen fittings	10%
Electric geysers	10%

Air conditioning equipment (movable parts only)	10%
Vacuum cleaners	20%
Lockers	10%

As far as depreciation is concerned the main problem arises with the salvage value, if any, of the component. Where the tax value of the asset is less than the salvage value then the difference must be treated as income for that tax year. However, where the tax value of the asset is more than the salvage value then the difference in this case may be treated as further depreciation and adjusted against income received during that particular tax year. For the purpose of examples it will be assumed that reasonably correct estimates have been made for the longevity and salvage values of given assets.

Straight line method of depreciation:

This is the simplest method which depreciates an asset by the same amount each year. It must be noted that this method can only be used with the prior consent of the Receiver. The formula for the straight line method of depreciation is:-

$$D = \frac{P - R}{L}$$

Where the tax value of an asset has been written down to nil, no further depreciation allowance may be claimed for that asset.

For example, consider a set of lockers with a first cost of R2 500.00, an estimated life of 10 years (which would be allowable according to the 10 per cent allowance supplied above), and an estimated residual value of R500.00 The annual depreciation charge would be computed as follows:-

$$D = \frac{P - R}{L}$$

$$= \frac{2500 - 500}{10}$$

$$= \frac{2000}{10}$$

$$= \underline{R200.00}$$

The annual depreciation charge represents a positive cash flow and as in the case of tax allowances for maintenance costs can conveniently be treated as a reduction of negative costs of a life cycle cost analysis. In the above example the R200.00 depreciation per annum must first be converted to present value and the tax saving thereon offset against incurred costs. If the interest rate is 10 per cent and the tax rate 50 per cent the amount to be offset against capital costs would be:-

Present value factor of R1.00 per annum over 10 years at 5 per cent (10 per cent less tax)
= 7,7217

Therefore present value of R200.00 per annum for 10 years at 5 per cent
= R200.00 X 7,7217
= R1 544.35 less 50 per cent tax
= R 772.17 which must be deducted from maintenance costs

OR

$$ATPLCD = PMT \left[\frac{(1 + b)^n - 1}{b(1 + b)^n} \right] (1 - t)$$

$$= 200 \left[\frac{(1 + ,05)^{10} - 1}{,05(1 + ,05)^{10}} \right] (1 - ,5)$$

$$= 200 \left[\frac{1.62889 - 1}{,05 \times 1.62889} \right] ,5$$

$$= \underline{R772.17}$$

Reducing Balance Method of Depreciation:

This method reduces the asset by the same percentage each year and gives high early depreciation but never reaches nil value. The use of this method means that the total annual costs of an asset are more likely to be evened out to uniform amounts since maintenance and repair costs are generally low in earlier years and tend to increase with the age of the asset.

There is no arithmetic formula for calculating the present value of a reducing balance method of depreciation so that the procedure to be adopted will, off necessity, be time-consuming and laborious.

EXAMPLE 15

What is the present value allowance to be made against expenditure for carpets, to an office building, costing R4 000.00 initially? Interest rate is 10 per cent and tax rate 50 per cent. The depreciation allowance is 25 per cent and the life of the component four years.

YEAR	VALUE AT BEGINNING	DEPRECIATION	VALUE AT END
1	R4 000.00	R1 000.00	R3 000.00
2	R3 000.00	R 750.00	R2 250.00
3	R2 250.00	R 562.50	R1 687.50
4	R1 687.50	R 421.88	R1 265.63

Since there is no salvage value in this case R1 265.63 must also be included in year four as a further depreciation allowance. The depreciation allowances are converted to present values using an interest rate of 5 per cent (10 per cent less tax rate).

YEAR	DEPRECIATION ALLOWANCE		P V FACTORS (5%)	P V
1	R1 000.00		,9524	R 952.40
2	R 750.00		,9070	R 680.25
3	R 562.50		,8638	R 485.89
4	R 421.88		,8227	R 347.08
	R1 265.63	(Non depreciated amount)	,8227	R1 041.23
				R3 506.85
		Less 50 per cent tax		R1 753.43
Allowance to be offset against expenditure				R1 753.42

Note: Should the asset realise a residual or salvage value greater than the tax value of the asset at the end period the difference between the tax asset and salvage value would constitute income and must be deducted from the depreciation allowance.

IMPACT OF INFLATION

Inflation is the declining value of the purchasing power of money which can be a critical factor in investment decisions, especially with today's high rate of inflation. Two problems associated with inflation which may require solving are:-

1. The anticipated effects on future costs
2. The real value of future costs (i.e. the value in today's purchasing power of money)

The anticipated future cost of a single payment amount is easily calculated by means of compound interest formulae.

EXAMPLE 16

What will be the cost of an article in three years time costing R1 000.00 today, if inflation is taken at 5 per cent?

$$S = P(1 + i)^n$$
$$= 1000(1 + ,05)^3$$
$$= R1\,157.63$$

Where the real value is to be calculated for a future sum of a single payment amount invested at compound interest, the effects of inflation can readily be included in the calculations.

The expression for a real interest rate after allowance for inflation will be:-

$$q = \left[\frac{1 + i}{1 + m}\right] - 1$$

Returning to example 1 on page 15. If R1 000.00 is invested for three years at 10 per cent interest, the future value amounts to R1 331.00 However, R1 331.00 receivable in three years time will not have the same purchasing power as R1 331.00 today. Price increases in the intervening years will have caused the erosion of the real value of money. Provided the inflation rate is known then the real purchasing power, in today's terms, can be calculated. This is illustrated by the following example:-

EXAMPLE 17

If R1 000.00 is invested for three years at 10 per cent interest, what is the real value at the end of that period if inflation is expected to be 5 per cent per annum?
For the real purchasing power to be calculated the 10 per cent interest rate must be adjusted to allow for inflation.

$$q = \left[\frac{1 + i}{1 + m}\right] - 1$$

$$= \left[\frac{1,10}{1,05}\right] - 1$$

$$= \underline{,04762} \quad \text{or } 4,762 \text{ per cent}$$

$$RS = P\ (1 + q)^n$$

$$= 1000\ (1 + ,04762)^3$$

$$= \underline{R1\ 149.77}$$

which is equivalent (in real terms) to R1 331.00 receivable in three years if inflation is 5 per cent per annum.

This can be proved as follows:-

Principal at start of year 1	1 149.77
Inflation for year 1	57.49
Principal at start of year 2	1 207.26
Inflation for year 2	60.36
Principal at start of year 3	1 267.62
Inflation for year 3	63.38
Principal at end of year 3	R1 331.00

For a real rate of interest to be achieved, after allowance for the reduced purchasing power of money, then the applicable rate for inflation must be incorporated in the overall interest rate. One method would be to add the required rate of interest and the annual inflation rate. In periods of low interest and inflation rates this would be fairly accurate; however, with the higher rates experienced in present times, the adjusted interest rate should be calculated as follows:-

$$\text{Adjusted interest rate} = (1 + i)\ (1 + m) - 1$$

Assuming that a real rate of interest of 10 per cent is required, after allowance for 5 per cent inflation per annum, on a single payment amount of R1 000.00 invested for three years then the adjusted interest rate will be:-

$$\text{Adjusted interest rate} = (1 + i)\ (1 + m) - 1$$
$$= 1{,}10 \times 1{,}05 - 1$$
$$= {,}155 \text{ or } 15{,}5 \text{ per cent}$$

$$\therefore\ S = P\ (1 + i)^n$$
$$= 1000\ (1 + {,}155)^3$$
$$= \text{R1 } 540.80$$

which is the amount receivable in three years time and is equivalent to R1 331.00 in today's purchasing power of money if inflation is 5 per cent per annum.

i.e. $P = \dfrac{S}{(1 + 1)^n}$

$= \dfrac{1540,80}{(1 + ,05)^3}$

$= \underline{R1\ 331.00}$

When computing the real value of a series of uniform payments at the end of each period, the simplest method is to calculate the future value of the series and then discount that amount by using the interest rate for inflation.

The formula for an ordinary annuity is:-

$$S = PMT \left[\frac{(1 + i)^n - 1}{i}\right]$$

which must then be adjusted by the present value formula:-

$$\frac{1}{(1 + m)^n}$$

$$\text{Thus}\quad RS = PMT \left[\frac{(1 + i)^n - 1}{i}\right] \left[\frac{1}{(1 + m)^n}\right]$$

$$= PMT \left[\frac{(1 + i)^n - 1}{i\,(1 + m)^n}\right]$$

<u>EXAMPLE 18</u>

If R1 000.00 is invested at 10 per cent interest at the end of each year for a period of three years what will be the real value of the total amount at the end of that period if inflation is 5 per cent per annum?

$$RS = PMT \left[\frac{(1 + i)^n - 1}{i(1 + m)^n}\right]$$

$$= 1000 \left[\frac{(1 + ,10)^3 - 1}{,10\ (1 + ,05)^3}\right]$$

$= \underline{R2\ 859.30}$

Considering the future costs of a series of uniform payments that will increase with inflation. We find that each payment in the series will suffer different degrees of inflation and each value will have to be calculated separately.

Example 19

If the yearly income is R1 000.00 and is expected to increase by 5 per cent per annum, what will the present value be at the end of three years if interest is taken at 10 per cent?

Year	Income		P V Factor (10%)	P V
1	1000 (1 + ,05)	= 1 050.00	,9091	954.56
2	1000 $(1 + ,05)^2$	= 1 102.50	,8264	911.11
3	1000 $(1 + ,05)^3$	= 1 157.63	,7513	869.73
				R2 735.40

Or if the future value is required:-

Year	Income		F V Factor (10%)	F V
1	1000 (1 + ,05)	= 1 050.00	1,21	1 270.50
2	1000 $(1 + ,05)^2$	= 1 102.50	1,10	1 212.75
3	1000 $(1 + ,05)^3$	= 1 157.63	1,00	1 157.63
				R3 640.88

The amount of R3 640.88 is equivalent to R2 735.40 if interest is taken at 10 per cent per annum.

Unfortunately both income tax and inflation are real problems associated with the majority of investment decisions and their cumulative effect must be assessed.

INFLATION AND INCOME TAX

For a single payment amount invested at compound interest for a stipulated number of years the mathematical formula is:-

$$S = P \ (1 + i)^n$$

which when tax is taken into account, changes to:-

$$ATS = P \ (1 + b)^n$$

and when inflation is considered will be:-

$$RS = P \ (1 + i)^n \left[\frac{1}{(1 + m)^n}\right]$$

The formula for allowing for income tax and inflation would therefore be:-

$$ATRS = P \ (1 + b)^n \left[\frac{1}{(1 + m)^n}\right]$$

EXAMPLE 20

If R1 000.00 is invested for three years at 10 per cent interest what is the real value at the end of that period if the tax rate is 50 per cent and inflation 3 per cent per annum?

$$ATRS = P \ (1 + b)^n \left[\frac{1}{(1 + m)^n}\right]$$

$$= P \ (1{,}05)^3 \left[\frac{1}{(1{,}03)^3}\right]$$

$$= \underline{R1\ 059.39}$$

This method can be simplified by the use of the formula:-

$$ATRS = P \left[\frac{1 + b}{1 + m}\right]^n$$

$$= 1000 \left[\frac{1{,}05}{1{,}03}\right]^3$$

$$= \underline{R1\ 059.39}$$

The technique for adjusting the interest rate to allow for inflation is:-

$$q = \left[\frac{1 + i}{1 + m}\right] - 1$$

If (1 + i) is substituted by (1 + b) then the formula would read:-

$$ATq = \left[\frac{1 + b}{1 + m}\right] - 1$$

$$= \left[\frac{1 + ,05}{1 + ,03}\right] - 1$$

$$= \underline{,019417} \text{ or an interest rate of } 1,9417 \text{ per cent.}$$

Therefore, employing the normal compound interest formula

$$ATRS = P \quad (1 + q)^n$$

$$= 1000 \quad (1 + ,019417)^3$$

$$= \underline{R1\ 059.39}$$

Similarly, the formula for an ordinary annuity may be adjusted to allow for both income tax and inflation.

$$RS = PMT \left[\frac{(1 + i)^n - 1}{i(1 + m)^n}\right]$$

Substituting 'b' for 'i' the formula now reads:-

$$ARTS = PMT \left[\frac{(1 + b)^n - 1}{b(1 + m)^n}\right]$$

EXAMPLE 21

If R1 000.00 is invested at the end of each year for three years at 10 per cent interest what would be the real value at the end of that period if inflation is 3 per cent per annum and income tax 50 per cent?

$$ARTS = PMT \left[\frac{(1 + b)^n - 1}{b(1 + m)^n}\right]$$

$$= 1000 \left[\frac{(1 + ,05)^3 - 1}{,05(1 + ,03)^3}\right]$$

$$= \underline{R2\ 884.98}$$

As life cycle costing is concerned primarily with the values of future cash flows, income tax and the effects of inflation on future costs will be of prime importance.

EXAMPLE 22

It is estimated that it will cost R1 000.00 to paint the external facade of a building at today's costs. What is the present value of the cost of painting in three years time if the interest rate is 10 per cent, inflation 3 per cent and income tax 50 per cent?

The steps to be taken would be as follows:-

(a) Calculation of the future cost of repainting

$$S = P\ (1 + m)^n$$

(b) Calculation of the present value after tax of the future cost

$$ATP = P\ (1 + m)^n \times \frac{1}{(1 + b)^n}$$

(c) Calculation and adjustment of the effects of income tax on the net income i.e. the tax relief factor

$$ATPLC = P\ (1 + m)^n \times \frac{1}{(1 + b)^n} \times (1 - t)$$

$$= P \left[\frac{1 + m}{1 + b}\right]^n \times (1 - t)$$

$$\therefore ATPLC = 1000 \left[\frac{1 + ,03}{1 + ,05}\right]^3 (1 - ,5)$$

$$= \underline{R471.97}$$

As described previously, each payment in a series of uniform payments will suffer different degrees of inflation and hence each payment will have to be calculated separately.

EXAMPLE 23

It is estimated that it will cost R1 000.00 to paint the external facade of a building at today's costs. What is the present value of the total cost of painting if the building

/is to be

is to be repainted at the end of each year for the next three years? Interest is 10 per cent, inflation 3 per cent and income tax 50 per cent.

Year	Income	P V Factor (5%) (after tax)	P V
1	1000 (1 + ,03) = 1 030.00	,9524	980.97
2	$1000\ (1 + ,03)^2$ = 1 060.90	,9070	962.24
3	$1000\ (1 + ,03)^3$ = 1 092.73	,8638	943.90
		Present value	2 887.11
		Less tax 50%	1 443.55
		ATPLC	R1 443.56

S U M M A R Y

Before considering further practical examples it might be expedient to briefly summarise the most important formulae discussed previously.

1. Future value of a single payment amount:-

$$S = P\ (1 + i)^n$$

2. Future value of a series of uniform payments made at the end of each period:-

$$S = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

3. Present value of a single payment amount:-

$$P = \frac{S}{(1 + i)^n}$$

4. Present value of a series of uniform payments made at the end of each period:-

$$P = PMT \left[\frac{(1 + i)^n - 1}{i(1 + i)^n} \right]$$

OR

4. $$P = PMT \left[\frac{(1 + i)^n - 1}{i}\right]\left[\frac{1}{(1 + i)^n}\right]$$

5. Annual sinking fund payments:-

$$PMT = S \left[\frac{i}{(1 + i)^n - 1}\right]$$

6. After tax future value of a single payment amount:-

$$S = P \quad (1 + b)^n$$

7. After tax future value of a series of uniform payments made at the end of each period:-

$$S = PMT \left[\frac{(1 + b)^n - 1}{b}\right]$$

8. After tax present value of a single payment amount:-

$$P = \frac{S}{(1 + b)^n}$$

9. After tax present value of a series of uniform payments made at the end of each period:-

$$P = PMT\left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right]$$

OR

$$P = PMT \left[\frac{(1 + b)^n - 1}{b}\right]\left[\frac{1}{(1 + b)^n}\right]$$

10. After tax present value of a single payment expenditure:-

$$ATPLC = \left[\frac{S}{(1 + b)^n}\right] (1 - t)$$

11. After tax present value of a series of uniform expenditures occurring at the end of each period:-

$$ATPLC = PMT\left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right] (1 - t)$$

OR

$$ATPLC = PMT \left[\frac{(1 + b)^n - 1}{b}\right]\left[\frac{1}{(1 + b)^n}\right] (1 - t)$$

12. Calculation of the real interest rate after allowance for inflation:-

$$q = \left[\frac{1 + i}{1 + m}\right] - 1$$

13. Present value of a single payment expenditure after allowance for income tax and inflation:-

$$ATPLC = P\left[\frac{1 + m}{1 + b}\right]^{n} (1 - t)$$

These developed formulae will form the basis for the life cycle cost analyses which are discussed in detail in the following chapters.

CHAPTER 4

THE PRESENT VALUE METHOD OF COMPARISON

GENERAL

The present value method of comparison is a well established method that comprises converting all costs in the analysis, using a pre-determined interest rate, to a single present day sum. It is important to remember that the most economic solution for a series of costs has the lowest present value, whereas for a series of incomes the best solution would have the highest present value.

This technique is relatively simple to execute and because of the method of presentation,offers a readily understandable approach. This is particularly advantageous when one considers that many Clients and Managers, for whom the appraisals are undertaken, have had no formal education in the field of economics or mathematics. Another distinct advantage of the present value system is that it is readily and easily converted to the annual method of comparison.

One of the problems associated with analyses of this type is the comparison of alternatives with different life spans. For valid comparisons to be made the alternatives must be compared over equal periods of time.

When comparisons are required between alternatives of this type, the problem can be solved by selecting or determining a time period equivalent to the shortest life cycle under consideration. This is known as the determined life cycle. At the chosen cut off point values can be assigned to all assets continuing into the future, i.e. residual or salvage values, so providing a true analysis for each of the alternatives over the same number of years.

As an illustration of the problem of unequal life spans, consider the following situation.

A machine to be used on a contract lasting three years has an initial cost of R20 000.00 and annual running costs of R20 000.00 Its life expectancy is only three years and will have no salvage value at the end of that period. An alternative machine is also available at an initial cost of R43 000.00, but has an estimated life of ten years. At the end of three years the predicted re-sale value is R19 000.00 Operating costs will be R12 000.00 per year. If the required rate of interest is 10 per cent, which machine would be the more viable proposition?

<u>Machine A</u>

Initial cost of machine	20 000.00
Operating costs:	
Present value of R20 000.00 for 3 years	
20 000 X 2,4869	49 738.00
Total present value	R69 738.00

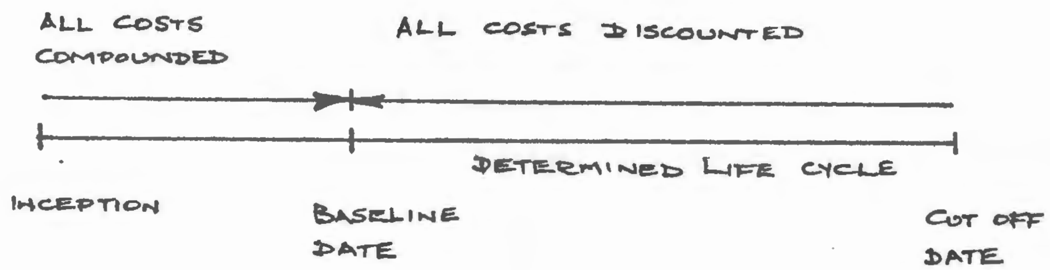
<u>Machine B</u>

Initial cost of machine	43 000.00
Operating costs:	
Present value of R12 000.00 for 3 years	
12 000 X 2,4869	44 764.00
Residual value of machine after 3 years	87 764.00
R19 000.00	
Present value of residual	
19 000.X 0,75131	14 275.00
Total present value	R73 489.00

Machine A offers the most economic solution.

With all the techniques under consideration cost will be the only method of comparison. When comparisons are made on this basis it must be assumed that all the alternatives are capable of fulfilling the required functional objectives. The design team must, therefore, ensure that the technical capabilities of each alternative is satisfactory. The only other criterian that may affect the decision is termed the irreducible factor. An irreducible factor is one that has no alternatives and, as such, may outweigh any cost considerations.

We have already stated that the present value method discounts all future costs to today's value. In order to maintain an equivalent economic relationship, costs incurred prior to that date will have to be compounded at the stipulated discount rate. It may not always be convenient or practical to use the present day as a baseline date, and in property investment appraisal techniques it is generally accepted that the start of the accounting period is the most suitable point in time. However, no matter what baseline is chosen, it is essential that all costs be time phased to that particular date. It is also obvious that costs occurring at baseline date need not be compounded or discounted.



TIME LINE ILLUSTRATING THE CONCEPT OF COMPOUNDING AND DISCOUNTING TO BASELINE DATE

THE PRESENT VALUE METHOD OF COMPARISON AND LIFE CYCLE COST ANALYSIS

For convenience, all future costs incurred shall be assumed to be annual costs occurring as single payment amounts at the end of each year. Before undertaking a practical example the six basic cost categories referred to in Chapter 1 will be examined and discussed in order to decide upon the method of compounding or discounting to be adopted.

1. Capital costs

   If these costs are incurred at baseline date no further consideration is necessary. Costs occurring prior to this time must be converted to the baseline date by means of compound interest formulae, using the stipulated interest rate. For single payment amounts the formula $P\ (1 + b)^n$ would apply, whereas for uniform payment amounts the formula $PMT \left[\frac{(1 + b)^n - 1}{b} \right]$ will be used.

   It should be noted that as these costs are of a capital nature there is no relief from income tax. The only adjustment for tax purposes is that derived from accrued interest or, in this instance, from the opportunity rate. Opportunity cost is the term employed for loss of income on money invested in an undeveloped site or uncompleted project. This loss is equivalent to the income that could have been earned had it been profitably invested elsewhere. In this case income earned from the investment would have been subject to income tax and deducted accordingly. Opportunity costs used will, therefore, be calculated net of tax.

   Many property developments have phased building operations, allowing Clients to lease parts of the scheme prior to contract completion date. In these developments income will be derived prior to baseline date.

This income, although reducing the capital cost, is subject to taxation and tax allowances must be made. At this point it will be as well to remember that many matters concerning property taxes are extremely complex and require the advice of an expert, specialising in such aspects, to evaluate each individual case. Tax aspects in these studies have been confined to the effects of taxation on life cycle costing and for this reason alone certain general assumptions have been made.

2. Running Costs

As these are in the form of annual costs, the formula as illustrated below will be used if no escalation is anticipated.

$$PMT \left[\frac{(1 + b)^n - 1}{b(1 + b)^n}\right]\left[1 - t\right]$$

Where increased costs due to inflation are considered the cumbersome and time consuming method of assessing each annual cost separately must be adopted.

3. Maintenance Costs

These are generally periodic costs and, as such, will have to be adjusted individually. The applicable formula in this case will be:-

$$P\left[\frac{1 + m}{1 + b}\right]\left[1 - t\right]$$

The tax depreciation allowance will apply to many components falling under this and the previous category, which, after the necessary tax relief factor, must be deducted from the present value sum.

4. Repair and Replacement Costs

Will also form single payment amounts which must be converted by the same formula applicable to 3 above.

Care should be taken to ensure that these costs are not of a capital nature since, if so, the tax relief factor will no longer apply. In that case the formula must read:-

$$P \left[\frac{1 + m}{1 + b}\right]^n$$

5. <u>Alteration and Improvement Costs</u>
The tax relief factor will obviously not be used in expenses of this nature as they definitely constitute capital costs, and the single payment amount formula must once again be exercised. Costs of this nature are rarely visualized at baseline date and so will not generally be introduced into the life cycle cost analysis. Exceptions may be realised in the case of office buildings, where some allowance may be made for changes in office layout, as a result of varying tenancies.

6. <u>Residual or Salvage Costs</u>
These amounts constitute income which are to be treated as negative factors to be deducted from the present value sum. It will again be necessary to decide which amounts are subject to tax or tax relief since appreciable differences could result as a consequence of incorrect decisions. Generally speaking, residual value will relate to the resale of the development and tax implications need not apply since there is, at present,in this Country no capital gains tax. Taxation from the point of view of salvage value will depend on the amount of depreciation previously allowed and each case should be evaluated individually.

The selection of the baseline date has already been considered, but this is not the complete picture since a cut off or expiry time must also be established in order to fix the determined life cycle. Traditionally one tends to think the useful life of a building lasting in the region of fifty to sixty years.

Clients, either through functional or economic objectives, might have different ideas and the determined life cycle of the particular investment under consideration should be mutually agreed upon. Other problems encountered in this regard will be the estimated lives of individual components. If there is any doubt as to the probable life of any particular component a sensitivity analysis should be introduced and 'maximum' and'minimum' lives could be used to determine what effects they will have on the analysis.

Implicit to all life cycle cost techniques is the assumption that equal interest rates will be used for all comparisons. In making these comparisons it may result that the solution representing the best value and lowest total cost may not be the one requiring the minimum capital expenditure but one, in fact, which the Client cannot afford. In this case all the facts should be presented to the Client who must make the final choice.

There is one school of thought that considers it unnecessary to take into account escalation of future costs in appraisals of this nature. Their argument is that price increases will affect all alternatives equally and thus make no difference to the final choice. This may be the case in certain instances, but where replacement lives are different for alternative components or materials, the cost consequences of inflation can be appreciable. For this reason, I am of the opinion that where price increases can be justifiably predicted they should be included in the appraisal process. Of importance is that where inflation is allowed in any part of the appraisal it must be continued throughout. There is no justification for applying price increases to one component and not another.

In order to show the differences that may be obtained by introducing the effects of inflation into life cycle cost studies, the following example will be calculated by:-

(a) Applying no inflation or tax factors

(b) Applying inflation effects only

(c) Applying tax implications only

(d) Applying both the effects of inflation and taxation implications

EXAMPLE 24

The economic consequences of two alternative floor coverings are to be evaluated. The floor area to be covered is 1 000 square metres and the life of the building is estimated to be forty years. The applicable interest rate for the investment is 10 per cent and the tax rate 50 cents in the Rand. Inflation over this period is expected to average 12 per cent per annum and will apply to both maintenance and replacement costs.

The two types of floor coverings under consideration are:-

(a) Vinyl flooring at an initial cost of R8.00 per square metre and an anticipated life of fifteen years. Maintenance costs are expected to be 90 cents per square metre per annum while a premium of 50 per cent will be placed on replacement costs for taking up existing flooring, disruption, etc.

(b) Carpeting at an initial cost of R15.00 per square metre and a life expectancy of nine years. Maintenance costs are calculated at R1.20 per square metre per annum with a 10 per cent premium on replacement costs.

VINYL FLOORING

Initial Costs - R8 000.00

Replacement Cost Premium - 50 per cent

A. NO ALLOWANCE FOR INCOME TAX OR INFLATION

Replacement costs every 15 years

Replacement cost premium = 50 per cent	
P V of R8 000.00 in year 0	8 000.00
	c/f8 000.00

b/f	8 000.00
P V of R8 000.00 (1 + 0,5) in year 15	
$= \frac{12\ 000}{(1 + 0{,}10)^{15}}$	2 872.70
P V of R8 000.00 (1 + 0,5) in year 30	
$= \frac{12\ 000}{(1 + 0{,}10)^{30}}$	687.70
Annual Maintenance Costs	
P V of R900.00 per annum for 40 years	
$= 900 \left[\frac{(1 + {,}10)^{40} - 1}{{,}10(1 + {,}10)^{40}}\right]$	8 801.15
Total Present Value	R20 361.55

B. ALLOWANCE FOR INFLATION ONLY

Replacement costs every 15 years

P V of R8 000.00 in year 0	8 000.00
P V of R8 000.00 (1 + ,05) in year 15	
$= 12\ 000 \left[\frac{1 + {,}12}{1 + {,}10}\right]^{15}$	15 723.94
P V of R8 000.00 (1 + ,05) in year 30	
$= 12\ 000 \left[\frac{1 + {,}12}{1 + {,}10}\right]^{30}$	20 603.52

Annual Maintenance Costs

$$A = 900 \left[\frac{1 + {,}12}{1 + {,}10}\right]$$

P V of R900.00 in year	1	$= A^1$	916.36
"	2	$= A^2$	933.02
"	3	$= A^3$	949.99
"	4	$= A^4$	967.26
"	5	$= A^5$	984.85
"	6	$= A^6$	1 002.75
"	7	$= A^7$	1 020.99
"	8	$= A^8$	1 039.55
		c/f	52 142.23

			b/f	52 142.23
P V of R900.00 in year 9	=	A^{9}		1 058.45
" 10	=	A^{10}		1 077.70
" 11	=	A^{11}		1 097.29
" 12	=	A^{12}		1 117.24
" 13	=	A^{13}		1 137.55
" 14	=	A^{14}		1 158.24
" 15	=	A^{15}		1 179.30
" 16	=	A^{16}		1 200.74
" 17	=	A^{17}		1 222.57
" 18	=	A^{18}		1 244.80
" 19	=	A^{19}		1 267.43
" 20	=	A^{20}		1 290.47
" 21	=	A^{21}		1 313.94
" 22	=	A^{22}		1 337.83
" 23	=	A^{23}		1 362.15
" 24	=	A^{24}		1 386.92
" 25	=	A^{25}		1 412.13
" 26	=	A^{26}		1 437.81
" 27	=	A^{27}		1 463.95
" 28	=	A^{28}		1 490.57
" 29	=	A^{29}		1 517.67
" 30	=	A^{30}		1 545.26
" 31	=	A^{31}		1 573.36
" 32	=	A^{32}		1 601.97
" 33	=	A^{33}		1 631.09
" 34	=	A^{34}		1 660.75
" 35	=	A^{35}		1 690.94
" 36	=	A^{36}		1 721.69
" 37	=	A^{37}		1 752.99
" 38	=	A^{38}		1 784.86
" 39	=	A^{39}		1 817.32
" 40	=	A^{40}		1 850.36
Total Present Value				R97 547.57

For the sake of clarity the mechanics of repetitive calculations, although obviously computed, have not been illustrated in the following examples. In such cases the calculations referring to the start and end periods only have been shown.

C. ALLOWANCE FOR INCOME TAX ONLY

Income tax rate 50 per cent

$$b = i\,(1 - t)$$
$$= ,10 \quad (1 - ,5)$$
$$= ,05$$

Replacement Costs every 15 years

For the purposes of this exercise replacement costs shall be deemed to be of a capital nature and not subject to tax allowances.

P V of R8000.00 in year 0	8 000.00
P V of R8 000.00 (1 + 0,5) in year 15	
$= \dfrac{12\ 000}{(1 + ,05)^{15}}$	5 772.21
P V of R8 000.00 (1 + 0,5) in year 30	
$= \dfrac{12\ 000}{(1 + ,05)^{30}}$	2 776.53

Annual Maintenance Costs

P V of R900.00 per annum for 40 years	
$= 900 \left[\dfrac{(1 + 0,05)^{40} - 1}{,05(1 + ,05)^{40}}\right](1 - ,5)$	7 721.59
Total Present Value	**R24 270.33**

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Replacement Costs every 15 years

P V of R8 000.00 in year 0	8 000.00
P V of R8 000.00 (1 + 0,5) in year 15	
$= 12\ 000 \left[\dfrac{1 + 12}{1 + ,05}\right]^{15}$	31 594.54
P V of R8 000.00 (1 + 0,5) in year 30	
$= 12\ 000 \left[\dfrac{1 + ,12}{1 + ,05}\right]^{30}$	83 184.60

Annual Maintenance Costs

$$C = 900 \left[\dfrac{1 + ,12}{1 + ,05}\right](1 - ,5)$$

c/f 122 779.14

		b/f	122 779.14
P V of R900.00 in year 1	=	C^1	480.00
" 2	=	C^2	512.00
" 3	=	C^3	546.13
" 4	=	C^4	582.54
Years 5 to 38 not illustrated			-
P V of R900.00 in year 39	=	C^{39}	5 576.13
" 40	=	C^{40}	5 947.87
Total Present Value			R210 745.02

CARPETING

Initial Costs - R15 000.00

Replacement Cost Premium - 10 per cent

A. NO ALLOWANCE FOR INCOME TAX OR INFLATION

Replacement Costs every 9 years

P V of R15 000.00 in year 0	15 000.00
P V of R15 000.00 (1 + 0,10) in year 9 $= \frac{16\ 500}{(1 + ,10)^9}$	6 997.61
P V of R15 000.00 (1 + 0,10) in year 18 $= \frac{16\ 500}{(1 + ,10)^{18}}$	2 967.67
P V of R15 000.00 (1 + 0,10) in year 27 $= \frac{16\ 500}{(1 + ,10)^{27}}$	1 258.58
P V of R15 000.00 (1 + 0,10) in year 36 $= \frac{16\ 500}{(1 + ,10)^{36}}$	533.76

Annual Maintenance Costs

P V of R1 200.00 per annum for 40 years $= 1200 \left[\frac{(1 + ,10)^{40} - 1}{,10(1 + ,10)^{40}}\right]$	11 734.86
Total Present Value	R38 492.48

B. ALLOWANCE FOR INFLATION ONLY

Replacement Costs every 9 years

P V of R15 000.00 in year 0	15 000.00
P V of R15 000.00 (1 + 0,10) in year 9 $= 16\,500 \left[\frac{1 + ,12}{1 + ,10}\right]^{9}$	19 404.93
P V of R15 000.00 (1 + 0,10) in year 18 $= 16\,500 \left[\frac{1 + ,12}{1 + ,10}\right]^{18}$	22 821.28
P V of R15 000.00 (1 + 0,10) in year 27 $= 16\,500 \left[\frac{1 + ,12}{1 + ,10}\right]^{27}$	26 839.11
P V of R15 000.00 (1 + 0,10) in year 36 $= 16\,500 \left[\frac{1 + ,12}{1 + ,10}\right]^{36}$	31 564.30

Annual Maintenance Costs

$$B = 1\,200 \left[\frac{1 + ,12}{1 + ,10}\right]$$

P V of R1 200.00 in year 1 = B^1	1 221.82
" 2 = B^2	1 244.03
" 3 = B^3	1 266.65
" 4 = B^4	1 289.68
Years 5 to 38 not illustrated	-
P V of R1 200.00 in year 39 = B^{39}	2 423.09
" 40 = B^{40}	2 467.15
Total Present Value	**R186 589.76**

C. ALLOWANCE FOR INCOME TAX ONLY

Replacement Costs every 9 years

P V of R15 000.00 in year 0	15 000.00
P V of R15 000.00 (1 + 0,10) in year 9 $= \frac{16\,500}{(1 + ,05)^{9}}$	10 636.05
c/f	25 636.05

b/f	25 636.05
P V of R15 000.00 (1 + 0,10) in year 18 $= \frac{16\ 500}{(1 + ,05)^{18}}$	6 856.09
P V of R15 000.00 (1 + 0,10) in year 27 $= \frac{16\ 500}{(1 + ,05)^{27}}$	4 419.50
P V of R15 000.00 (1 + 0,10) in year 36 $= \frac{16\ 500}{(1 + ,05)^{36}}$	2 848.85

Annual Maintenance Costs

P V of R1 200.00 per annum for 40 years

$$= 1\ 200 \left[\frac{(1 + ,05)^{40} - 1}{,05(1 + ,05)^{40}}\right] (1 - 0,5) \qquad 10\ 295.45$$

Total Present Value	R50 055.94

The above makes no allowance for depreciation that may be granted by the Receiver on the initial cost of carpeting. Tax relief has been calculated on the straight line method in order to prevent laborious arithmetic calculations, but the principles and procedures to be adopted would also apply to the declining balance method.

Tax allowance for depreciation

Initial cost of carpeting - R15 000.00

Depreciation per annum for first 9 year period — R1 666.67

P V of R1 666.67 per annum for 9 years

$$= 1\ 666.67 \left[\frac{(1 + ,05)^{9} - 1}{,05(1 + ,05)^{9}}\right] ,5 \qquad 5\ 923.20$$

Cost of carpeting after 9 years - R16 500.00

Depreciation per annum for second 9 year period

- R 1 833.33

c/f	R5 923.20

	b/f	R5 923.20
P V of R1 833.33 per annum for 9 years after year 9		
$= 1\ 833.33 \left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^9}\right]$	,5	4 199.94
P V of R1 833.33 per annum for 9 years after year 18		
$= 1\ 833.33 \left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^{18}}\right]$	,5	2 707.32
P V of R1 833.33 per annum for 9 years after year 27		
$= 1\ 833.33 \left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^{27}}\right]$	,5	1 745.16
P V of R1 833.33 per annum for 4 years after year 36		
$= 1\ 833.33 \left[\frac{(1+,05)^4-1}{,05(1+,05)^4}\right]\left[\frac{1}{(1+,05)^{36}}\right]$	,5	561.21
Total depreciation		R15 136.83

Note: Although the carpeting was not fully depreciated by year 40, it is assumed that no further tax allowance will be granted by the Receiver.

Present value of carpeting with no depreciation allowance	50 055.94
Tax depreciation allowance	15 136.83
Present value of carpeting after tax depreciation	R34 919.11

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Replacement Costs every 9 years

P V of R15 000.00 in year 0	15 000.00
P V of R15 000.00 (1 + 0,10) in year 9	
$= 16\ 500 \left[\frac{1+,12}{1+,05}\right]^9$	29 494.60
c/f	44 494.60

b/f	44 494.60
P V of R15 000.00 (1 + 0,10) in year 18	
$= 16\ 500 \left[\frac{1 + ,12}{1 + ,05}\right]^{18}$	52 723.10
P V of R15 000.00 (1 + 0,10) in year 27	
$= 16\ 500 \left[\frac{1 + ,12}{1 + ,05}\right]^{27}$	94 245.25
P V of R15 000.00 (1 + 0,10) in year 36	
$= 16\ 500 \left[\frac{1 + ,12}{1 + ,05}\right]^{36}$	168 468.22

Annual Maintenance Costs

$$D = 1\ 200 \left[\frac{1 + ,12}{1 + ,05}\right] (1 - 0,5)$$

P V of R1 200.00 in year 1	= D^1	640.00
" 2	= D^2	682.67
" 3	= D^3	728.18
" 4	= D^4	776.72
Years 5 to 38 not illustrated		-
P V of R1 200.00 in year 39	= D^{39}	7 434.84
" 40	= D^{40}	7 930.49
Total Present Value		**R477 219.05**

Tax allowance for depreciation

Initial cost of carpeting - R15 000.00

Depreciation per annum for first 9 year period

- R 1 666.67

P V of R1 666.67 for 9 years

$$= 1\ 666.67 \left[\frac{(1 + ,05)^9 - 1}{,05(1 + ,05)^9}\right](1 - 0,5) \qquad 5\ 923.20$$

Cost of carpeting after 9 years

$1\ 650\ (1 + ,12)^9 =$ R45 755.80

Depreciation per annum for second 9 year period

= R 5 083.98

c/f 5 923.20

	b/f	5 923.20

P V of R5 083.98 per annum for 9 years after year 9

$$= 5083.98 \left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^9}\right] \quad ,5 \qquad 11\ 646.80$$

Cost of carpeting after 18 years

16 500 $(1 + ,12)^{18}$ - R126 884.44

Depreciation per annum for third nine year period

- R 14 098.27

P V of R14 098.27 per annum for 9 years after year 18

$$= 14\ 098.27\left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^{18}}\right] \quad ,5 \qquad 20\ 819.24$$

Cost of carpeting after 27 years

16 500 $(1 + ,12)^{27}$ - R351 860.53

Depreciation per annum for fourth 9 year period

- R 39 095.61

P V of R39 095.61 per annum for 9 years after year 27

$$= 39\ 095.61\left[\frac{(1+,05)^9-1}{,05(1+,05)^9}\right]\left[\frac{1}{(1+,05)^{27}}\right] \quad ,5 \qquad 37\ 215.46$$

Cost of carpeting after 36 years

16 500 $(1 + ,12)^{36}$ - R975 736.97

Depreciation per annum for fifth 9 year period

- R108 415.22

P V of R108 415.22 per annum for 4 years after year 36

$$= 108\ 415.22\left[\frac{(1+,05)^4-1}{,05(1+,05)^4}\right]\left[\frac{1}{(1+,05)^{36}}\right] \quad ,5 \qquad 33\ 187.78$$

Total depreciation	R108 792.48
Present value of carpeting with no depreciation allowance	477 219.04
Tax depreciation allowance	108 792.38
Present value of carpeting after tax depreciation	R368 426.66

S U M M A R Y

		Vinyl Tiles	Carpeting
1.	Present values with no allowance for income tax or inflation	R 20 361.55	R 38 492.48
2.	Present values with allowance for inflation only	R 97 547.57	R186 589.76
3.	Present values with allowance for income tax only	R 24 270.33	R 50 055.94
4.	Ditto with tax allowance for depreciation	R 24 270.33	R 34 919.11
5.	Present values with allowances for income tax and inflation	R210 745.02	R477 219.05
6.	Ditto with tax allowance for depreciation	R210 745.02	R368 426.66

In all the methods under consideration vinyl tiles appear to be the most economic choice and would seem to confirm the belief of those who assert that it is unnecessary to take taxation and inflation into account in economic studies of this nature. However, let us examine the results when 'Econo' carpeting is used costing R10.00 per square metre initially and having a life expectancy of ten instead of nine years.

In order to simplify the calculations it is assumed that although a cheaper product is used the maintenance costs for 'Econo' carpeting will be equivalent to the maintenance costs for the more expensive carpeting.

'ECONO' CARPETING

A. NO ALLOWANCE FOR INCOME TAX OR INFLATION

Replacement Costs every 10 years

P V of R10 000.00 in year 0	10 000.00
P V of R10 000.00 (1 + 0,10) in year 10 $= \frac{11\ 000}{(1 + ,10)^{10}}$	4 240.98
P V of R10 000.00 (1 + 0,10) in year 20 $= \frac{11\ 000}{(1 + ,10)^{20}}$	1 635.08
P V of R10 000.00 (1 + 0,10) in year 30 $= \frac{11\ 000}{(1 + ,10)^{30}}$	630.39

Annual Maintenance Costs

As previously calculated	11 734.86
Total present value	R28 241.31

B. ALLOWANCE FOR INFLATION ONLY

Replacement Costs every 10 years

P V of R10 000.00 in year 0	10 000.00
P V of R10 000.00 (1 + 0,10) in year 10 $= 11\ 000 \left[\frac{1 + ,12}{1 + ,10}\right]^{10}$	13 171.83
P V of R10 000.00 (1 + 0,10) in year 20 $= 11\ 000 \left[\frac{1 + 12}{1 + ,10}\right]^{20}$	15 772.46
c/f	38 944.29

b/f	38 944.29
P V of R10 000.00 (1 + 0,10) in year 30 $= 11\,000 \left[\frac{1 + ,12}{1 + ,10}\right]^{30}$	18 886.56
<u>Annual Maintenance Costs</u>	
As previously calculated	70 960.14
Total present value	**R128 790.99**

C. <u>ALLOWANCE FOR INCOME TAX ONLY</u>

<u>Replacement Costs every 10 years</u>

P V of R10 000.00 in year 0	10 000.00
P V of R10 000.00 (1 + 0,10) in year 10 $= \frac{11\,000}{(1 + ,05)^{10}}$	6 753.05
P V of R10 000.00 (1 + 0,10) in year 20 $= \frac{11\,000}{(1 + ,05)^{20}}$	4 145.78
P V of R10 000.00 (1 + 0,10) in year 30 $= \frac{11\,000}{(1 + ,05)^{30}}$	2 545.15
<u>Annual Maintenance Costs</u>	
As previously calculated	10 295.45
Total present value	**R33 739.43**

<u>Tax allowance for depreciation</u>

Initial cost of carpeting - R10 000.00

Depreciation per annum for first 10 year period - R 1 000.00

P V of R1 000.00 per annum for 10 years $= 1\,000 \left[\frac{(1 + ,05)^{10} - 1}{,05(1 + ,05)^{10}}\right](1 - 0,5)$	3 860.87
c/f	3 860.87

b/f 3 860.87

Cost of carpeting after 10 years - R11 000.00

Depreciation per annum for second 10 year period - R 1 100.00

P V of R1 100.00 per annum for 10 years after year 10

$$= R1\ 100.00 \left[\frac{(1+,05)^{10}-1}{,05(1+,05)^{10}}\right]\left[\frac{1}{(1+,05)^{10}}\right](1-,5)$$ 2 607.26

P V of R1 100.00 per annum for 10 years after year 20

$$= R1\ 100.00 \left[\frac{(1+,05)^{10}-1}{,05(1+,05)^{10}}\right]\left[\frac{1}{(1+,05)^{20}}\right](1-,5)$$ 1 600.63

P V of R1 100.00 per annum for 10 years after year 30

$$= R1\ 100.00 \left[\frac{(1+,05)^{10}-1}{,05(1+,05)^{10}}\right]\left[\frac{1}{(1+,05)^{30}}\right](1-,5)$$ 982.65

Total depreciation	R9 051.41
Present value of carpeting with no depreciation allowance	R33 739.43
Tax depreciation allowance	9 051.41
Present value of carpeting after tax depreciation	R24 688.02

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Replacement Costs every 10 years

P V of R10 000.00 in year 0 10 000.00

P V of R10 000.00 (1 + 0,10) in year 10

$$= 11\ 000 \left[\frac{1+,12}{1+,05}\right]^{10}$$ 20 973.94

P V of R10 000.00 (1 + 0,10) in year 20

$$= 11\ 000 \left[\frac{1+,12}{1+,05}\right]^{20}$$ 39 991.45

c/f 70 965.39

b/f	70 965.39
P V of R10 000.00 (1 + 0,10) in year 30 $= 11\,000 \left[\frac{1 + ,12}{1 + ,05}\right]^{30}$	76 252.55
Annual Maintenance Costs	
As previously calculated	117 287.88
Total present value	R264 505.82

<u>Tax allowance for depreciation</u>

Initial cost of carpeting - R10 000.00

Depreciation per annum for first 10 year period - R 1 000.00

P V of R1 000.00 per annum for 10 years $= 11\,000 \left[\frac{(1 + ,05)^{10}-1}{,05(1 + ,05)^{10}}\right] ,5$	3 860.87

Cost of carpeting after 10 years

$11\,000\ (1 + ,12)^{10}$ - R34 164.33

Depreciation per annum for second 10 year period - R 3 416.43

P V of R3 416.43 per annum for 10 years after year 10 $= R3\,416.43 \left[\frac{(1 + ,05)^{10}-1}{,05(1 + ,05)^{10}}\right]\left[\frac{1}{(1 + ,05)^{10}}\right] ,5$	8 097.76

Cost of carpeting after 20 years

$11\,000\ (1 + ,12)^{20}$ - R106 109.22

Depreciation per annum for third 10 year period - R10 610.92

P V of R10 610.92 per annum for 10 years after year 20 $= R10\,610.92 \left[\frac{(1 + ,05)^{10}-1}{,05(1 + ,05)^{10}}\right]\left[\frac{1}{(1 + ,05)^{20}}\right] ,5$	15 440.17
c/f	R27 398.80

b/f R27 398.80

Cost of carpeting after 30 years

$11\ 000\ (1 + ,12)^{30}$ - R329 559.14

Depreciation per annum for fourth 10 year period

- R 32 955.91

P V of R32 955.91 per annum for 10 years after year 30

$$= R32\ 955.91 \left[\frac{(1 + ,05)^{10} - 1}{,05(1 + ,05)^{10}}\right]\left[\frac{1}{(1 + ,05)^{30}}\right] ,5 \quad 29\ 440.10$$

Total depreciation R56 838.90

Present value of carpeting with no depreciation allowance R264 505.82

Tax depreciation allowance 56 838.90

Present value of carpeting after tax depreciation R207 666.92

S U M M A R Y

	Vinyl Tiles	Carpeting	Econo Carpeting
1. Present values with no allowance for income tax or inflation	R 20 361.55	R 38 492.48	R 28 241.31
2. Present values with allowance for inflation only	R 97 547.57	R186 589.76	R128 790.99
3. Present values with allowance for income tax only	R 24 270.33	R 50 055.94	R 33 739.43
4. Ditto with tax allowance for depreciation	R 24 270.33	R 34 919.11	R 24 688.02

	Vinyl Tiles	Carpeting	Econo Carpeting
5. Present values with allowances for income tax and inflation	R210 745.02	R477 219.05	R264 505.82
6. Ditto with tax allowance for depreciation	R210 745.02	R368 426.66	R207 666.92

Although, under all conditions, carpeting has a total cost far in excess of vinyl tiles, 'Econo' carpeting appears the most economic solution when the effects of income tax and inflation are considered. Apart from demonstrating the necessity for using accurate and realistic cost data and interest rates, it also confirms that unless the user is positive that future price changes and income tax will affect all transactions equally and as such make no difference to the choice of alternatives, then they must be included in life cycle cost studies. This is again demonstrated in the following example.

EXAMPLE 25

A residential building has an external facade area of 150 square metres. The Client requires a life cycle cost analysis over the estimated life of forty years at a discount rate of 12 per cent. The two materials under consideration are facings at an initial cost of R5.00 per square metre and plaster and paint costing R3.30 per square metre. The estimated maintenance expenses, at today's rates, are repointing facings every ten years at a cost of R2.50 per square metre, and painting and patching plaster every three years at a cost of R1.25. Escalation of costs is assumed to average 10 per cent per annum and income tax 50 per cent. Which is the most economic material to use considering that the property is to be built for letting purposes and as such company tax procedures will apply?

FACINGS

A. NO ALLOWANCE FOR INFLATION OR INCOME TAX

Initial cost

150 square metres at R5.00		750.00

Maintenance costs

Costs at today's rates - 150 square metres at R2.50 - R375.00

P V of R375.00 in year 10	$\frac{375}{(1 + ,12)^{10}}$	120.74
P V of R375.00 in year 20	$\frac{375}{(1 + ,12)^{20}}$	38.88
P V of R375.00 in year 30	$\frac{375}{(1 + ,12)^{30}}$	12.52
Total Present Value		R922.14

B. ALLOWANCE FOR INFLATION ONLY

Initial cost

150 square metres at R5.00		750.00

Maintenance costs

P V of R375.00 in year 10 = 375	$\left[\frac{1 + ,10}{1 + ,12}\right]^{10}$	313.17
P V of R375.00 in year 20 = 375	$\left[\frac{1 + ,10}{1 + ,12}\right]^{20}$	261.53
P V of R375.00 in year 30 = 375	$\left[\frac{1 + ,10}{1 + ,12}\right]^{30}$	218.41
Total Present Value		R1 543.11

C. ALLOWANCE FOR INCOME TAX ONLY

Initial cost

150 Square metres at R5.00		750.00
Maintenance costs		
P V of R375.00 in year 10	$\left[\frac{375}{(1+,06)^{10}}\right] ,5$	104.70
P V of R375.00 in year 20	$\left[\frac{375}{(1+,06)^{20}}\right] ,5$	58.46
P V of R375.00 in year 30	$\left[\frac{375}{(1+,06)^{30}}\right] ,5$	32.65
Total Present Value		**R945.81**

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Initial cost

150 Square metres at 5.00		750.00
Maintenance costs		
P V of R375.00 in year 10	$= 375 \left[\frac{1+,10}{1+,06}\right]^{10} ,5$	271.56
P V of R375.00 in year 20	$= 375 \left[\frac{1+,10}{1+,06}\right]^{20} ,5$	393.31
P V of R375.00 in year 30	$= 375 \left[\frac{1+,10}{1+,06}\right]^{30} ,5$	569.65
Total Present Value		**R1 984.52**

PLASTER AND PAINT

A. NO ALLOWANCE FOR INCOME TAX OR INFLATION

Initial cost

150 Square metres at R3.00		450.00

Maintenance costs

Costs at today's rates - 150 Square metres at R1.25 - R187.50

P V of R187.50 in year 3	$\frac{187.50}{(1+,12)^{3}}$	133.46
P V of R187.50 in year 6	$\frac{187.50}{(1+,12)^{6}}$	94.99
P V of R187.50 in year 9	$\frac{187.50}{(1+,12)^{9}}$	67.61
P V of R187.50 in year 12	$\frac{187.50}{(1+,12)^{12}}$	48.13
P V of R187.50 in year 15	$\frac{187.50}{(1+,12)^{15}}$	34.26
P V of R187.50 in year 18	$\frac{187.50}{(1+,12)^{18}}$	24.38
P V of R187.50 in year 21	$\frac{187.50}{(1+,12)^{21}}$	17.35
P V of R187.50 in year 24	$\frac{187.50}{(1+,12)^{24}}$	12.35
P V of R187.50 in year 27	$\frac{187.50}{(1+,12)^{27}}$	8.79
P V of R187.50 in year 30	$\frac{187.50}{(1+,12)^{30}}$	6.26
P V of R187.50 in year 33	$\frac{187.50}{(1+,12)^{33}}$	4.45
P V of R187.50 in year 36	$\frac{187.50}{(1+,12)^{36}}$	3.17
P V of R187.50 in year 39	$\frac{187.50}{(1+,12)^{39}}$	2.26
Total Present Value		R907.46

B. ALLOWANCE FOR INFLATION ONLY

Initial cost

150 Square metres at R3.00	450.00

Maintenance costs

$$B = 187.50 \left[\frac{1 + ,10}{1 + ,12}\right]$$

P V of R187.50 in year 3	$= B^3$	177.63
" 6	$= B^6$	168.29
" 9	$= B^9$	159.43
Years 12 to 33 not illustrated		-
P V of R187.50 in year 36	$= B^{36}$	98.01
" 39	$= B^{39}$	92.86
Total Present Value		R2 153.94

C. ALLOWANCE FOR INCOME TAX ONLY

Initial cost

150 Square metres at R3.00	450.00

Maintenance costs

P V of R187.50 in year 3	$= \left[\frac{187.50}{(1 + ,06)^3}\right] ,5$	78.71
" 6	$= \left[\frac{187.50}{(1 + ,06)^6}\right] ,5$	66.09
" 9	$= \left[\frac{187.50}{(1 + ,06)^9}\right] ,5$	55.49
Years 12 to 33 not illustrated		-
P V of R187.50 in year 36	$= \left[\frac{187.50}{(1 + ,06)^{36}}\right] ,5$	11.51
" 39	$= \left[\frac{187.50}{(1 + ,06)^{39}}\right] ,5$	9.66
Total Present Value		R890.20

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Initial cost

150 Square metres at R3.00	450.00

Maintenance costs

$$C = 187.50 \left[\frac{1 + ,10}{1 + ,06} \right]$$

P V of R187.50 in year 3	=	C^3 (1 - 0,5)	104.77
" 6	=	C^6 (1 - 0,5)	117.08
" 9	=	C^9 (1 - 0,5)	130.84
Years 12 to 33 not illustrated			-
P V of R187.50 in year 36	=	C^{36} (1 - 0,5)	355.71
" 39	=	C^{39} (1 - 0,5)	397.52
Total Present Value			**R3 338.29**

S U M M A R Y

		Present values	
		Facings	Plaster and Paint
1.	No allowance for inflation or income tax	R 922.14	R 907.46
2.	Allowance for inflation only	R1 543.11	R2 153.94
3.	Allowance for income tax only	R 945.81	R 890.20
4.	Allowance for income tax and inflation	R1 984.52	R3 338.29

Considering the same problem but substituting 10 per cent for the interest rate and 12 per cent for the inflation rate the effects on the results would not be inconsiderable.

FACINGS

A. NO ALLOWANCE FOR INFLATION OR INCOME TAX

Initial cost

150 Square metres at R5.00		750.00
Maintenance Costs		
P V of R375.00 in year 10	$= \dfrac{375}{(1 + ,10)^{10}}$	144.58
P V of R375.00 in year 20	$= \dfrac{375}{(1 + ,10)^{20}}$	55.74
P V of R375.00 in year 30	$= \dfrac{375}{(1 + ,10)^{30}}$	21.49
Total Present Value		R971.81

B. ALLOWANCE FOR INFLATION ONLY

Initial cost

150 Square metres at R5.00		750.00
Maintenance costs		
P V of R375.00 in year 10	$= 375 \left[\dfrac{1 + ,12}{1 + ,10}\right]^{10}$	449.04
P V of R375.00 in year 20	$= 375 \left[\dfrac{1 + ,12}{1 + ,10}\right]^{20}$	537.70
P V of R375.00 in year 30	$= 375 \left[\dfrac{1 + ,12}{1 + ,10}\right]^{30}$	643.86
Total Present Value		R2 380.60

C. ALLOWANCE FOR INCOME TAX ONLY

Initial costs

150 Square metres at R5.00		750.00
	c/f	750.00

	b/f	750.00

<u>Maintenance costs</u>

P V of R375.00 in year 10	$\left[\frac{375}{(1+,05)^{10}}\right](1-,5)$	115.11
P V of R375.00 in year 20	$\left[\frac{375}{(1+,05)^{20}}\right](1-,5)$	70.67
P V of R375.00 in year 30	$\left[\frac{375}{(1+,05)^{30}}\right](1-,5)$	43.38
Total Present Value		**R979.16**

D. ALLOWANCE FOR INCOME TAX AND INFLATION

<u>Initial cost</u>

150 Square metres at R5.00	750.00

<u>Maintenance costs</u>

P V of R375.00 in year 10		
= 375	$\left[\frac{1+,12}{1+,05}\right]^{10}(1-,5)$	357.51
P V of R375.00 in year 20		
= 375	$\left[\frac{1+,12}{1+,05}\right]^{20}(1-,5)$	681.67
P V of R375.00 in year 30		
= 375	$\left[\frac{1+,12}{1+,05}\right]^{30}(1-,5)$	1 299.76
Total Present Value		**R3 088.94**

PLASTER AND PAINT

A. NO ALLOWANCE FOR INCOME TAX OR INFLATION

<u>Initial cost</u>

150 Square metres at R3.00		450.00
	c/f	450.00

	b/f	450.00
Maintenance costs		
P V of R187.50 in year 3	$\frac{187.50}{(1 + ,10)^3}$	140.87
P V of R187.50 in year 6	$\frac{187.50}{(1 + ,10)^6}$	105.84
P V of R187.50 in year 9	$\frac{187.50}{(1 + ,10)^9}$	79.52
Years 12 to 33 not illustrated		-
P V of R187.50 in year 36	$\frac{187.50}{(1 + ,10)^{36}}$	6.07
P V of R187.50 in year 39	$\frac{187.50}{(1 + ,10)^{39}}$	4.56
Total Present Value		R1 002.71

B. ALLOWANCE FOR INFLATION ONLY

Initial cost

150 Square metres at R3.00	450.00

Maintenance costs

$$D = 187.50 \left[\frac{1 + ,12}{1 + ,10}\right]$$

P V of R187.50 in year 3	=	D^3	197.91
" 6	=	D^6	208.91
" 9	=	D^9	220.51
Years 12 to 33 not illustrated			-
P V of R187.50 in year 36	=	D^{36}	358.69
" 39	=	D^{39}	378.61
Total Present Value			R4 081.82

C. ALLOWANCE FOR INCOME TAX ONLY

Initial cost

150 Square metres at R3.00	450.00

Maintenance costs

P V of R187.50 in year 3	$\left[\frac{187.50}{(1 + ,05)^3}\right](1-,5)$	80.98
	c/f	530.98

	b/f	530.98
P V of R187.50 in year 6	$\left[\frac{187.50}{(1+,05)^6}\right](1-,5)$	69.96
" 9	$\left[\frac{187.50}{(1+,05)^9}\right](1-,5)$	60.43
Years 12 to 33 not illustrated		-
P V of R187.50 in year 36	$\left[\frac{187.50}{(1+,05)^{36}}\right](1-,5)$	16.19
" 39	$\left[\frac{187.50}{(1+,05)^{39}}\right](1-,5)$	13.98
Total Present Value		**R956.06**

D. ALLOWANCE FOR INCOME TAX AND INFLATION

Initial costs

150 Square metres at R3.00 — 450.00

Maintenance costs $E = 187.50 \left[\frac{1+,12}{1+,05}\right]$

P V of R187.50 in year 3	=	E^3	(1-,5)	113.78
" 6	=	E^6	(1-,5)	138.08
" 9	=	E^9	(1-,5)	167.58
Years 12 to 33 not illustrated				-
P V of R187.50 in year 36	=	E^{36}	(1-,5)	957.21
" 39	=	E^{39}	(1-,5)	1 161.69
Total Present Value				**R6 516.97**

SUMMARY

	Present Values: Interest rate 12 %, Inflation rate 10%		Interest rate 10 %, Inflation rate 12%	
	Facings	Plaster & Paint	Facings	Plaster & Paint
1. No allowance for inflation or income tax	R 922.14	R 907.46	R 971.81	R1 002.71

	Present Values			
	Interest rate 12% Inflation rate 10%		Interest rate 10% Inflation rate 12%	
	Facings	Plaster & Paint	Facings	Plaster & Paint
2. Allowance for inflation only	R1 543.11	R2 153.94	R2 380.60	R4 081.82
3. Allowance for income tax only	R 945.81	R 890.20	R 979.16	R 956.06
4. Allowance for income tax and inflation	R1 984.52	R3 338.29	R3 088.94	R6 516.97

By increasing the inflation rate and decreasing the interest rate the future costs assume greater significance by virtue of the fact that their present values are increased. In the above example, plaster and paint have greater maintenance costs and, hence, a higher proportion of future costs will be severely affected by the high inflation rate. The effects of interest rates on life cycle cost studies and on building economics generally will be discussed in greater depth in following chapters.

CHAPTER 5

THE ANNUAL EQUIVALENT METHOD OF COMPARISON

GENERAL

The basis of the annual equivalent method of comparison is to convert all costs into a series of uniform cash flows in order that comparisons with alternative possibilities can be achieved. There is no implication that the payments will be made in such a manner but merely the provision of a method of measurement of each alternative. The alternative with the lowest annual equivalent cost is obviously the more economic solution.

The annual equivalent method comprises two elements, viz:

1. Interest on capital invested, and
2. Recovery of capital on investments with limited lives, i.e. the sinking fund concept.

For instance, the annual equivalent cost of a machine costing R10 000.00 and lasting five years, if interest is taken to be 10 per cent, will be:-

<u>Interest on Capital Investment</u>

R10 000.00 at 10 per cent	-	R1 000.00

<u>Annual Sinking Fund to realise R10 000.00 after 5 years if invested at 10 per cent</u>

$$PMT = S\left[\frac{i}{(1+i)^{n}-1}\right]$$

$$= 10\ 000\left[\frac{,10}{(1+,10)^{5}-1}\right] \qquad R1\ 637.97$$

Annual Equivalent Cost	R2 637.97

We have already seen that $S = P(1+i)^n$ and so substituting $P(1+i)^n$ for S the above formula can be stated as:-

$$PMT = P(1+i)^{n}\left[\frac{i}{(1+i)^{n}-1}\right]$$

$$\therefore PMT = P\left[\frac{i(1+i)^{n}}{(1+i)^{n}-1}\right]$$

which will give the annual interest on the principal plus the yearly amount needed to invest in a sinking fund. The formula for the present value of a series of uniform payments is:-

$$P = PMT\left[\frac{(1+i)^{n}-1}{i(1+i)^{n}}\right]$$

$$\therefore PMT = P\left[\frac{i(1+i)^{n}}{(1+i)^{n}-1}\right]$$

which represents the formula for the annual equivalent of a present value. Converting from one method to another is thus an easily achieved task. Both methods, therefore, allow for the recovery of capital.

Another method of computing the annual equivalent cost is by the 'Capital Recovery with a Rate-of-Return Method'. In this concept interest is earned only on the declining balance, i.e. similar to the payment of instalments on a house mortgage. As such the annual payments of R2 637.97 on an initial cost of R10 000.00 would be expressed as follows:-

Capital Investment	10 000.00
Interest at 10% for year 1	1 000.00
Balance at end of year 1	11 000.00
Repayment at end of year 1	2 637.97
Balance at start of year 2	8 362.03
Interest at 10% for year 2	836.20
Balance at end of year 2	9 198.23
Repayment at end of year 2	2 637.97
Balance at start of year 3	6 560.26
Interest at 10% for year 3	656.02
Balance at end of year 3	7 216.28
Repayment at end of year 3	2 637.97
Balance at start of year 4	4 578.31
Interest at 10% for year 4	457.83
Balance at end of year 4	5 036.14
Repayment at end of year 4	2 637.97
Balance at start of year 5	2 398.17
Interest at 10% for year 5	239.81
Balance at end of year 5	2 637.98
Repayment at end of year 5	2 637.97
Balance at start of year 6	R Nil

The sinking fund method concept is the method which shall be discussed in this document.

Since the annual equivalent method allows for the recovery of capital, adjustments have to be made to the calculation where there is no expiry of the capital sum, i.e. the asset will have a residual value. If the machine, in the above example, had a residual value of R2 000.00 at the end of the five year life then a sinking fund need not be set up for the full amount.

The investment would be considered in two parts, the first part being the R8 000.00 which requires annual interest plus an amount to amortise the depreciating value of the asset. The second part amounts to R2 000.00 and, because there is no capital sum disappearance, need be calculated on an interest only basis.

The formula for the annual equivalent of a present value amount is:-

$$PMT = \left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$$

so that if R = Residual value, the formula can be altered to read:-

$$PMT = (P - R)\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] + Ri$$

which will calculate interest and redemption on the depreciating element of the investment and interest only on the portion that requires no capital recovery.

The annual equivalent cost of the machine in the example referred to earlier amounts to R2 637.97. This was calculated on the basis of the investment realising no residual value at the expiry of the five year term. Assuming a residual value of R2 000.00 then the problem could be solved in the following manner:-

<u>Interest of Capital Investment</u>

R10 000.00 at 10%	R1 000.00
<u>Annual Sinking Fund</u> to realise R8 000.00	
(R10 000.00 less R2 000.00) after 5 years if invested at 10%	
$PMT = 8\ 000 \left[\frac{,10}{(1 + ,10)^5 - 1}\right]$	R1 310.38
Annual Equivalent Cost	R2 310.38

This could be calculated in one stage, using the formula developed previously.

$$PMT = (P - R)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] + Ri$$

$$= 10\ 000 - 2\ 000\left[\frac{,10(1+,10)^5}{(1+,10)^5 - 1}\right] + (2\ 000 \times ,10)$$

$$= \underline{R2\ 310.38}$$

As in the case of the present value method if valid comparisons are to be made the life cycle must first be determined.

THE ANNUAL EQUIVALENT METHOD OF COMPARISON AND LIFE CYCLE COST ANALYSIS

The six basic cost categories in chapter 1 are referred to again. Unless otherwise stated, the method used shall include both income tax and inflation implications.

1. Capital Costs

Costs occurring prior to baseline date are capitalised to that point in time. As far as capital costs are concerned, the residual value, if any, must be evaluated at this stage since meaningful differences could result. Two formulae will apply. For those costs having no residual value the formula :-

$$PMT = P\left[\frac{b(1+b)^n}{(1+b)^n - 1}\right]$$

will be used, whereas for those investments of a capital nature with salvage value upon expiry, the relevant formula is :-

$$PMT = (P - R)\left[\frac{b(1+b)^n}{(1+b)^n - 1}\right] + Rb$$

2. Running Costs

As these are already in the form of annual costs they require no conversion other than tax relief adjustments, provided inflation is not taken into account. Where increased running expenses due to inflation are taken into account in the calculations, each annual cost must be separately assessed by calculating the increases envisaged, converting them back to a present value and then equating them to annual equivalent values. This is a cumbersome procedure as can be seen from the present value examples in the previous chapter.

The resultant formulae, with PMT taken to mean the annual payment for one year only, is as follows:-

$$AE = PMT \left[\frac{1 + m}{1 + b}\right]^{n^{(1)}} \left[1 - t\right] \left[\frac{b(1 + b)^{n^{(2)}}}{(1 + b)^{n} - 1}\right]$$

Where AE = Annual Equivalent.

(1) Is the number of years after which payment is to be made

(2) Is the determined life cycle of the investment

3. Maintenance Costs

As these are periodic costs which have to be individually adjusted the formula above will apply. The tax depreciation allowance, if applicable, can be adjusted by one of two methods. After offsetting the tax relief factor, the computed amounts can be deducted from either the present values or the annual equivalent values. Whichever method is used it will in all probability still require conversion to present values except, in the unlikely event of depreciation being assessed on a straight line basis and coinciding with the determined life cycle of the analysis.

4. Repair and Replacement Costs

In the event of costs of this nature constituting capital costs, the tax relief factor will fall away. The applicable formula for costs of this nature will therefore be:-

$$AE = P \overset{(1)}{\left[\frac{1 + m}{1 + b}\right]^n} \overset{(2)}{\left[\frac{b(1 + b)^n}{(1 + b)^n - 1}\right]}$$

5. Alteration and Improvement Costs

The formula used in 4 above will apply.

6. Residual or Salvage Costs

These costs or values can either be dealt with separately or incorporated into the formula as previously described. The results in both cases will be the same.

Returning to the previous example involving a machine with an initial cost of R10 000.00, a salvage value of R2 000.00 and a life expectancy of five years, at an interest rate of 10 per cent the annual equivalent cost amounted to R2 310.38 when applying the formula:-

$$PMT = (P - R) \left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] + Ri$$

Supposing the analysis is solved on the basis of the machine having no residual value accounted for in the initial operation, adjustment only being made at a later stage in the calculations, then the Annual Equivalent Value is computed as follows:-

$P \left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$		2 637.97
Residual Value after 5 years	= R2 000.00	
P V of R2 000.00 in year 5	= R1 241.84	
Annual Equivalent of R1 241.84 for 5 years at 10 per cent		327.59
Net Annual Equivalent Value		R2 310.38

One aspect that needs to be considered in ranking exercises of this nature is the question of maintenance and capital costs. Generally speaking high initial costs, based on wise material selection or design, will reduce maintenance costs. Limited financial resources may, however, preclude high initial costs even though a life cycle cost exercise favours this as the more economic total cost solution. Limited or short life buildings may also favour cheaper initial cost although this factor should be highlighted during the appraisal process.

EXAMPLE 26

The problem used in example 25 will be used to illustrate the annual equivalent method of comparison. Interest rate will be 12 per cent, inflation rate 10 per cent and income tax 50 per cent.

(a) Facings (Allowance for income tax and inflation)

Initial Costs

150 Square metres at R5.00 = R750.00

A E of R750.00 over 40 years at 12%

$$750\left[\frac{,06(1+,06)^{40}}{(1+,06)^{40}-1}\right] \qquad 49.85$$

Maintenance Costs

A E of R375.00 in year 10

$$= 375\left[\frac{1+,10}{1+,06}\right]^{10}(1-0,5)\left[\frac{,06(1+,06)^{40}}{(1+,06)^{40}-1}\right] \qquad 18.05$$

A E of R375.00 in year 20

$$= 375\left[\frac{1+,10}{1+,06}\right]^{20}(1-0,5)\left[\frac{,06(1+,06)^{40}}{(1+,06)^{40}-1}\right] \qquad 26.14$$

A E of R375.00 in year 30

$$= 375\left[\frac{1+,10}{1+,06}\right]^{30}(1-0,5)\left[\frac{,06(1+,06)^{40}}{(1+,06)^{40}-1}\right] \qquad 37.86$$

Total Annual Equivalent Costs R131.90

This method can be shortened considerably by converting the maintenance costs to present value, summing them and then converting to annual equivalent costs.

(b) Plaster and Paint

Initial Costs

150 Square metres at R3.00 = R450.00

A E of R450.00 over 40 years at 12%

$$450 \left[\frac{,06(1 + ,06)^{40}}{(1 + ,06)^{40} - 1}\right] \qquad 29.91$$

Maintenance costs

$$C = 187.50 \left[\frac{1 + ,10}{1 + ,06}\right]$$

P V of R187.50 in year	3	=	C^3 (1-0,5)	104.77
"	6	=	C^6 (1-0,5)	117.08
"	9	=	C^9 (1-0,5)	130.84
"	12	=	C^{12}(1-0,5)	146.22
"	15	=	C^{15}(1-0,5)	163.41
"	18	=	C^{18}(1-0,5)	182.61
"	21	=	C^{21}(1-0,5)	204.08
"	24	=	C^{24}(1-0,5)	228.06
"	27	=	C^{27}(1-0,5)	254.87
"	30	=	C^{30}(1-0,5)	284.82
"	33	=	C^{33}(1-0,5)	318.30
"	36	=	C^{36}(1-0,5)	355.71
"	39	=	C^{39}(1-0,5)	397.52
				R2 888.29

A E of R2 888.29 over 40 years at 12%

$$= 2\,888.29 \left[\frac{,06(1 + ,06)^{40}}{(1 + ,06)^{40} - 1}\right] \qquad 191.96$$

Total Annual Equivalent Costs — **R221.87**

To compare with the present value method these annual equivalent costs can be converted to present values.

A. Facings

P V of R131.90 for 40 years at 12%

$$= R131.90 \left[\frac{(1 + ,06)^{40} - 1}{,06(1 + ,06)^{40}} \right] \qquad \underline{R1\ 984.61}$$

B. Plaster and Paint

P V of R221.87 for 40 years at 12%

$$= R221.87 \left[\frac{(1 + ,06)^{40} - 1}{,06\ (1 + ,06)^{40}} \right] \qquad \underline{R3\ 338.32}$$

Which is equivalent to, within a few cents, the answers obtained in the previous chapter.

CHAPTER 6

EQUITY AND BORROWED CAPITAL

Inherent in both the Present Value Method and the Annual Equivalent Method of Comparison is the premise that capital may either be borrowed or equity financed, or a combination of both provided the interest rates correspond. Different rates of interest for equity funds and borrowed capital will render diverse results and as such life cycle cost studies must comprise 100 per cent financed or 100 per cent equity funded situations. A further element taken into account in these methods is the recovery of invested capital less residual value, if any.

As there seems to be some confusion between equity financed and borrowed money situations, especially with regard to income tax implications, this aspect will be dealt with in some depth. Before proceeding further, clarity must be reached on one point, namely that actual 'interest received' situations and 'opportunity cost' situations will be identical as far as economic consequences are concerned.

If capital is 100 per cent equity financed there is an 'opportunity cost' involved on that amount. For instance, if R1 000.00 was invested in a machine with an economic life of three years, with no salvage value at the end of that period, the annual cost to the investor, if interest is calculated at 10 per cent, would be computed as follows:-

Initial cost of investment -	R1 000.00	
Annual interest/opportunity rate at 10%		100.00
Annual investment into a sinking fund to recoup wasting asset of R1 000.00 over 3 years at 10%		302.11
Total Annual Cost		**R402.11**

If the capital amount had to be borrowed on an interest and redemption basis at 10 per cent interest then the annual costs would remain the same i.e. R402.11

Initial amount borrowed		1 000.00
Interest for year 1 on capital amount	100.00	
Capital redemption for year 1		302.11
Balance at start of year 2		697.89
Interest on balance for year 2	69.79	
Capital redemption for year 2		332.32
Balance at start of year 3		365.57
Interest on balance for year 3	36.56	
Capital redemption for year 3		365.55
Remaining balance		R .02

With money borrowed on an interest only basis and capital redemption at the end of the investment period the results would again be unchanged. Interest rates again calculated at 10 per cent.

Annual interest on borrowed capital	100.00
Annual investment into a sinking fund to repay R1 000.00 capital at end of 3 years	302.11
Total Annual Costs	R402.11

If income tax implications are introduced then once again all situations will be affected. Assuming the same conditions apply, but with a tax rate of 50 per cent incorporated into the calculations, the examples will again be computed in order to ascertain whether the changes will be equal for all studies.

The first example comprises a 100 per cent financed situation.

Annual interest after payment of tax at 50 per cent = R100.00 less 50%	50.00
Annual investment into a sinking fund to recoup wasting asset of R1 000.00 over 3 years at an after tax rate of 5%	317.21
Total Annual Cost	R367.21

The second example is a 100 per cent borrowed money situation on an interest and redemption basis. Income tax affects the calculations in the form of tax relief obtained on the interest portion of the repayments.

Year	Interest	Tax Relief	Cost	Redemption	Total
1	R100.00	R50.00	R50.00	R302.11	R352.11
2	69.79	34.89	34.90	332.32	367.22
3	36.56	18.28	18.28	365.55	383.83

This results in a different cash flow for each year and in its present form cannot be compared with the annual costs of the example above. This can be accommodated by converting to a present value sum and then re-converting to annual equivalent amounts. (5 per cent after tax rate must be used).

Present value of R352.11 in year 1	=	R335.35
Present value of R367.22 in year 2	=	R333.08
Present value of R383.83 in year 3	=	R331.57
Total present value		R1 000.00

which at 5 per cent interest, after tax, gives an annual equivalent value of R367.21.

If the interest only is paid on an annual basis and capital redeemed at the end of the investment period, the results obtained are identical to the 100 per cent financed example.

Annual interest on borrowed capital	100.00
Tax relief based on 50% tax rate	50.00
Net annual interest cost	50.00
Annual investment into a sinking fund to repay R1 000.00 capital at end of three years at an after tax rate of 5 per cent	317.21
Total annual cost	R367.21

Because one of the fundamental elements of discounted cash flows is the recoupment of initial investment capital, composite rates cannot be applied when derived from combinations of equity and borrowed capital at differing interest rates. Recoupment of capital implies the use of a sinking fund. With compound interest formulae, interest rates cannot be averaged and the same result still obtained. An extreme case is illustrated in order to clarify this important aspect.

R100 000.00 is a capital investment for a period of ten years with an interest rate of 10 per cent. The annual equivalent cost for this investment will be R16 274.54. If R50 000.00 of this investment is financed by owner's equity which he calculates at 4 per cent and the remainder is borrowed at 16 per cent, then the composite rate can be assumed to be 10 per cent on an interest only basis.

i.e. R50 000.00 at 4% = R2 000.00
R50 000.00 at 16% = R8 000.00

R10 000.00

which equals an average of 10 per cent.

However, by computing the two elements separately the results do not correspond when capital redemption is taken into account.

Annual equivalent of R50 000.00 for 10 years at 4 per cent	6 164.55
Annual equivalent of R50 000.00 for 10 years at 16 per cent	10 345.05
Total Annual Equivalent Cost	R16 509.60

The above emphasizes the necessity for assuming 100 per cent equity or borrowed money situation and using only one interest rate for all calculations.

CHAPTER 7

THE CHOICE OF INTEREST RATE AND ITS EFFECT ON LIFE CYCLE COST ANALYSIS

The choice of interest rate features prominently in all life cycle cost techniques. The applied rate should be dictated by the Client who must be consulted and made aware of the consequences of his decision. Since the rate of interest will materially affect the results, establishment of the correct interest rate will be critical to most appraisals and care should be taken to ensure that a realistic and accurate rate is selected. Major planning errors and uneconomic use of available resources can often be attributed to incorrect decisions based on inadequate knowledge or misunderstanding of the effects of interest rates and investment lives on property economics.

INTEREST RATES AND PRESENT VALUES

In order to demonstrate the effects of the choice of interest rate on the present value of a series of payments, consider the following:-

EXAMPLE 27

Which of the following sequences of cash outlays will be the most economic at interest rates of 3 per cent, 6 per cent and 9 per cent?

Project A - R1 000.00 today followed by R200.00 per annum for the next three years

Project B - R1 000.00 today followed by R100.00 for the next two years and R418.36 for the third year

P R O J E C T 'A'

TIME	AMOUNT	INTEREST RATE 3%	P V	INTEREST RATE 6%	P V	INTEREST RATE 9%	P V
0	R1 000.00	1,000	R1 000.00	1,000	R1 000.00	1,000	R1 000.00
1	200.00	,9709	194.18	,9434	188.68	,9174	183.48
2	200.00	,9426	188.52	,8900	178.00	,8417	168.34
3	200.00	,9151	183.02	,8396	167.92	,7722	154.44
	R1 600.00		R1 565.72		R1 534.60		R1 506.26

P R O J E C T 'B'

TIME	AMOUNT	INTEREST RATE 3%	P V	INTEREST RATE 6%	P V	INTEREST RATE 9%	P V
0	R1 000.00	1,000	R1 000.00	1,000	R1 000.00	1,000	R1 000.00
1	100.00	,9709	97.09	,9434	94.34	,9174	91.74
2	100.00	,9426	94.26	,8900	89.00	,9417	84.17
3	418.36	,9151	382.84	,8396	351.26	,7722	323.06
	R1 618.36		R1 574.19		R1 534.60		R1 498.97

From the above calculations the following can be deduced:-

1. When no interest is taken into account Project A will be the more economic

2. At a 3 per cent interest rate Project A will be the more economic

3. At 6 per cent interest rate both projects will cost the same

4. At a 9 per cent interest rate Project A will be the more expensive

The above provides a clear indication of the possible effects of the choice of an incorrect interest rate. In practice this problem is alleviated to some extent by the use of sensitivity analysis which will be considered in detail.

The higher the interest rate the less the present value of future amounts. Since the rate of interest depends, amongst other factors, upon the current rate of inflation interest rates tend to increase as inflation increases,and vice versa. Thus for those who advocate the application of life cycle costing excluding allowances for inflation absurd results can be obtained. This is particularly relevant in today's economic climate where interest rates of 14 per cent are not uncommon.

SENSITIVITY ANALYSIS

As forecasting is basic to life cycle cost analysis, assumptions have to be made when considering future costs, interest rates, inflation, etc. It is impossible to anticipate, with any degree of certainty, what future changes may occur as a result of economic or technological developments. Interest rates, for instance, will fluctuate constantly with changing economic conditions.

This uncertainty is in itself no reason to forego intelligent forecasts of future changes, since sensible predictions will be better than making no allowance whatsoever. However, since these estimates are subject to future changes it will be prudent to discover whether the analysis is sensitive to variations and, if so, whether it will radically affect the results. Hence the sensitivity analysis, which is an evaluation of various alternatives taking into account the 'upper' and 'lower' limits applicable to each of the variables constituting the analysis. These variables are the rate of interest, the duration or life of the investment or component and the estimate of initial capital cost. If the use of the 'minimum' and 'maximum' figures does not change the result of the study then it can be assumed that the decision will be the correct one. However, if changes to a variable significantly alters the result, the element under consideration must be viewed with suspicion and should be carefully assessed and investigated further before any final decision is taken.

In the previous example it was shown how, by changing the one variable, namely the rate of interest, it was possible to radically alter the results of the analysis.

In the case of example 27, the investments are extremely sensitive to fluctuations in the rate of interest and, as such, particular circumspection must be used in arriving at a decision. This example also serves to illustrate a further aspect, namely the concept of the 'break-even' point. By altering the value of one of the variables in the analysis it is possible to find a value that equates two alternatives, which in the example under consideration occurs at a 6 per cent rate of interest. This is particularly relevant where the choice between two alternatives will vary under different sets of conditions. If, for example, time is a variable it may well be of benefit to solve the 'break-even' point where one of the alternatives becomes economically more attractive than the other.

EXAMPLE 28

Assume the owner has the choice of two machines for use in his factory. Machine A has an initial cost of R50 000.00 and annual operating costs of R12 000.00, whereas machine B has an initial cost of R70 000.00 and annual operating costs of R5 000.00. If interest is calculated at 10 per cent and the time factor variable, at what point in time, if any, will it be more economic to use the initially more expensive machine?

	Machine A	Machine B
P V of capital costs at year 0	50 000.00	70 000.00
P V of running costs at end of year 1	10 909.09	4 545.45
Total Present Value	R60 909.09	R74 545.45
P V of running costs at end of year 2	9 917.36	4 132.23
Total Present Value	R70 826.45	R78 677.68
P V of running costs at end of year 3	9 015.78	3 756.57
Total Present Value	R79 842.23	R82 434.25
P V of running costs at end of year 4	8 196.16	3 415.07
Total Present Value	R88 038.39	R85 849.32

The above suggests that for an investment period of just under four years, Machine A would be the more economic. For any investment period of longer life cycle, Machine B becomes the more favourable selection.

INTEREST RATES AND ANNUAL EQUIVALENT COSTS

The annual equivalent cost is also dependant upon the three variables mentioned earlier, viz:-

(a) The rate of interest

(b) The duration or life of the investment

(c) The capital cost of the investment

Changes to any of the above will have significant economic implications to an appraisal. An increase in the interest rate or capital cost will produce a higher annual cost. Similarly, a shortening of the investment life will also result in increased annual costs. This is given greater clarity by table A which gives the annual equivalent costs per R1 000.00 at variable interest rates and time periods.

T A B L E 'A'

ANNUAL EQUIVALENT COSTS PER R1 000.00

TERM (Years)	INTEREST RATE (%)				
	8	9	10	11	12
5	250,46	257,09	263,80	270,57	277,41
10	149,03	155,82	162,75	169,80	176,98
15	116,83	124,06	131,47	139,07	146,82
20	101,85	109,55	117,46	125,58	133,88
25	93,68	101,81	110,17	118,74	127,50
30	88,83	97,34	106,08	115,02	124,14
35	85,80	94,64	103,69	112,93	122,32
40	83,86	92,96	102,26	111.72	121,30
45	82,59	91,90	101,39	111,01	120,74
50	81,74	91,23	100,86	110,60	120,42
55	81,18	90,79	100,53	110,35	120,24
60	80,80	90,51	100,33	110,21	120,13

Changes in interest rates have the same effect as relevant changes in capital costs considered over the same time period. For an investment life of 40 years a change in interest rate from 8 per cent to 10 per cent represents a capital cost increase of 21,94 per cent. A further rate increase to 12 per cent is equivalent to an additional 18,62 per cent capital investment.

A reduction in investment life also results in increased annual equivalent costs which, in turn represent higher capital investment equivalents. For an economic life of 15 years a change in investment rate from 8 per cent to 10 per cent is equivalent to a capital cost increase of 12,53 per cent, whereas an 11,68 per cent increase is reflected for an interest rate change from 10 per cent to 12 per cent.

Thus the shorter the investment period or life cycle the less the effect of any interest rate adjustment and vice versa. As the life of the investment increases so changes to interest rate take on greater significance. Also, the longer the life of the investment the lower the annual equivalent costs. This rate of fall declines dramatically in the earlier years until eventually relative stability is reached. The higher the rate of interest the earlier annual equivalent costs tend to stabilise, and at that point in time these costs are closely approaching equality with the annual interest rates.

Table B gives the annual equivalent costs over 20 and 50 years for capital investments ranging from R1 000.00 to R1 500.00

TABLE 'B'

CAPITAL COSTS AND ANNUAL EQUIVALENT COSTS

(a) Annual Equivalent Cost over 20 years:

Capital Cost (Rand)	INTEREST RATE (%)						
	6	7	8	8	10	11	12
1 000.00	87,18	94,39	101,85	109,55	117,46	125,28	133,88
1 100.00	95,90	103,83	112,04	120,50	129,21	138,13	147,27
1 200.00	104,62	113,27	122,22	131,46	140,95	150,69	160,65
1 300.00	113,34	122,71	132,41	142,41	152,70	163,25	174,04
1 400.00	122,06	132,15	142,59	153,37	164,44	175,81	187,43
1 500.00	130,78	141,59	152,78	164,32	176,19	188,36	200,82

(a) Annual Equivalent Cost over 50 years:

Capital Cost (Rand)	INTEREST RATE (%) 6	7	8	9	10	11	12
1 000.00	63.44	72,46	81,74	91,23	100,86	110,60	120,42
1 100.00	69,79	79,71	89,92	100,35	110,95	121,66	132,46
1 200.00	76,13	86,95	98,09	109,47	121,03	132,72	144,50
1 300.00	82,48	94,20	106,27	118,59	131,12	143,78	156,54
1 400.00	88,82	101,44	114,44	127,72	141,20	154,84	168,58
1 500.00	95,17	108,69	122,61	136,84	151,29	165,90	180,62

Thus at a 6 per cent interest rate the annual equivalent cost over 20 years for a capital cost of R1 100.00 is roughly equal to a capital cost of R1 500.00 for a 50 year period.

For an interest rate of 11 per cent the annual equivalent cost of R1 100.00 capital is approximately the same as the annual equivalent cost of R1 250.00 capital over a 50 year time span. The longer investment period is therefore more sensitive to interest rate changes than the shorter investment period. In practical terms these characteristics of interest rates will affect investment and design decisions whether the investor is consciously aware of the effects or not. Consider the problem of an office building with an estimated net annual income of R80 000.00, based on a gross area of 4 000 square metres yielding 3 000 square metres of lettable area at R3.00 per square metre per month. Annual expenses are assumed at R28.000.00 per annum. If a 10 per cent return is required the capitalised value of the scheme for a 50 year life would be calculated as follows:-

Net income per annum R80 000.00	
Capitalised value at 10% over 50 years	793 185.00
Land costs and all other costs associated with capital costs	143 185.00
Balance for building	R650 000.00

Which is equivalent to R162.50 per square metre building costs.

A reduction in interest rate of 2 per cent results in an allowable building cost of R208.87 per square metre - an increase of 28,5 per cent.

If the life of the building is estimated at 20 years the allowable building cost per square metre is reduced to R134.48 if interest is taken at 10 per cent, whereas at 8 per cent interest the square metre rate is increased to R160.57. This extra allowance represents 19,4 per cent, once again illustrating that short investment periods are less susceptible to interest rate changes than long periods.

It will be obvious that when interest rates are low it is sound practice to construct good quality buildings which, although having higher initial costs, should have longer lives and more economic running expenses. Conversely, in times of high interest rates there will be a tendency to concentrate on reductions in capital costs at the expense of running costs. For a building with an estimated economic life of 50 years the annual equivalent cost per R1 000.00 capital costs is approximately R100.00 for an interest rate of 10 per cent. The ratio between initial and running costs is ten to one, i.e. for every R10.00 capital cost the annual equivalent cost is R1.00, so that for every R1.00 annual expenses that can be saved an extra R10.00 in capital costs can be spent. Therefore, if one considers a small block of flats that cost R100 000.00 initially, with an investment life of 50 years and a return of 10 per cent the initial cost/annual equivalent cost ratio is ten to one. Based on these figures it would be beneficial to spend up to R1 000.00 extra on capital costs for every R100.00 saving on annual running expenses. Conversely, it would be economically profitable to spend up to R100.00 extra on annual running expenses for every R1000.00 reduction on capital expenditure.

If the interest rate is 13 per cent then the capital cost/annual equivalent cost ratio drops to 7,7 to 1 so that for every R1 000.00 increase in capital costs an equivalent of R130.00 saving must be made to the running expenses. Similar results are obtained for shorter life cycles which is the

/reason why

reason why short investment periods and higher interest rates encourage capital expenditure reduction and running cost increases.

If the predicted life of the building is assessed as having too short a life cycle it will have the same effect as reducing the capital cost/annual equivalent cost ratio, and tend to increase the probability of constructing a building having low initial costs coupled with higher running costs.

CHAPTER 8

THE EFFECTS OF FORECASTING ERRORS ON LIFE CYCLE COST ANALYSIS

In order to undertake a life cycle cost analysis certain assumptions have to be made and future occurrences must be predicted. Should these assumptions and predictions be incorrect they will obviously affect the results of the appraisal. What must be determined is whether decisions arising out of incorrect assumptions or forecasts will be critical to the project under consideration.

ERRORS IN DETERMINED LIFE CYCLES

The predictions of investment lives of components and elements is a basic requirement of life cycle cost studies. However, as can be seen in Table C on page 105 incorrect assumptions need not necessarily be critical to the economic appraisal, especially if only five or ten years separates actual from estimated lives. The higher the rate of interest used the less will be the effect of any error in a predicted life cycle. In any event the shorter the investment life the greater the effect of misjudgments in determining life cycles. It is also of less economic consequence to err by predicting too long a life than too short a life.

If problems are encountered in determining the life cycle of a particular component, sensitivity analysis can be used to examine the extent to which changing life cycles will affect the appraisal.

ERRORS IN INTEREST RATES

Table D on page 107 illustrates that incorrect interest rates will have far greater economic consequences than errors in predicted life cycles. The longer the investment life the greater will be the effect of any error in choice of rate of interest. This is as a result of the compounding effects of interest rates. The longer the life cycle the greater the compound interest and the greater the effect of any change or error in interest rate. The interest rate once again proves a critical factor that must be resolved with the Client before undertaking life cycle cost studies.

T A B L E 'C'

PERCENTAGE ERRORS FROM ACTUAL LIVES OF INCORRECTLY ESTIMATED LIVES - (ASSUMING EQUAL PRESENT VALUES).

Estimated Life (Years)	Actual Life (Years)						
	20	30	40	50	60	70	80
20	0	+23,4	+37,7	+46,4	+51,9	+55,2	+57,2
30	-18,9	0	+11,6	+18,8	+23,1	+25,8	+27,5
40	-27,4	-10,4	0	+ 8,2	+10,3	+12,7	+14,2
50	-31,7	-15,8	- 8,7	0	+ 3,7	+ 6,0	+ 7,4
60	-34,2	-18,8	- 9,4	- 3,6	0	+ 2,2	+ 3,5
70	-35,6	-20,5	-11,3	- 5,6	- 2,1	0	+ 1,3
80	-36,4	-21,6	-12,4	- 6,9	- 3,4	- 1,3	0

5% Interest Rate

T A B L E 'C'

PERCENTAGE ERRORS FROM ACTUAL LIVES OF INCORRECTLY ESTIMATED LIVES - (ASSUMING EQUAL PRESENT VALUES).

Estimated Life (Years)	Actual Life (Years)						
	20	30	40	50	60	70	80
20	0	+10,7	+14,9	+16,5	+17,1	+17,3	+17,4
30	- 9,7	0	+3,7	+5,1	+ 5,7	+ 5,9	+ 6,0
40	-12,9	- 3,6	0	+1,4	+ 1,9	+ 2,1	+ 2,2
50	-14,1	- 4,9	-1,4	0	+ 0,5	+ 0,7	+ 0,8
60	-14,6	- 5,4	-1,9	-0,5	0	+ 0,2	+ 0,3
70	-14,8	- 5,6	-2,1	-0,7	- 0,2	0	+ 0,1
80	-14,8	- 5,6	-2,2	-0,8	- 0,3	- 0,1	0

10% Interest Rate

T A B L E 'C'
(Continued).

	20	30	40	50	60	70	80
20	0	+4,9	+6,1	+6,4	+6,5	+6,5	+6,5
30	-4,7	0	+1,2	+1,4	+1,5	+1,5	+1,5
40	-5,8	-1,1	0	+0,3	+0,4	+0,4	+0,4
50	-6,0	-1,4	-0,3	0	+0,1	+0,1	+0,1
60	-6,1	-1,5	-0,4	-0,1	0	0	0
70	-6,1	-1,5	-0,4	-0,1	0	0	0
80	-6,1	-1,5	-0,4	-0,1	0	0	0

15% Interest Rate

TABLE 'D'

PERCENTAGE ERRORS FROM ACTUAL INTEREST RATES OF INCORRECTLY ESTIMATED INTEREST RATES - (ASSUMING EQUAL PRESENT VALUES).

Estimated Interest Rate	Actual Interest Rate						
	5	6	7	8	9	10	11
5	0	- 6,4	-12,2	-17,5	-22,3	-26,7	-30,7
6	+ 6,9	0	- 6,2	-11,9	-17,0	-21,7	-26,0
7	+14,0	+ 6,6	0	- 6,0	-11,5	-16,5	-21,1
8	+21,3	+13,5	+ 6,4	0	- 5,8	-11,1	-16,0
9	+28,8	+20,5	+13,0	+ 6,2	0	- 5,6	-10,8
10	+36,5	+27,7	+19,8	+12,5	+ 6,0	0	- 5,5
11	+44,4	+35,1	+26,7	+19,0	+12,1	- 5,8	0

15 Year Life Cycle

Estimated Interest Rate	Actual Interest Rate						
	5	6	7	8	9	10	11
5	0	-11,5	-20,9	-28,8	-35,5	-41,1	-45,7
6	+12,9	0	-10,7	-19,6	-27,1	-33,5	-38,9
7	+26,5	+12,0	0	-10,0	-18,4	-25,5	-31,6
8	+40,5	+24,4	+11,1	0	- 9,3	-17,3	-24,0
9	+55,0	+37,2	+22,5	+10,3	0	- 8,7	-16,2
10	+69,8	+50,3	+34,3	+20,9	+ 9,6	0	- 8,2
11	+84,4	+63,7	+46,2	+31,6	+19,3	+ 8,9	0

35 Year Life Cycle

TABLE 'D'
(Continued)

Estimated Interest Rate	Actual Interest Rate						
	5	6	7	8	9	10	11
5	0	-14,6	-25,8	-34,6	-41,3	-47,3	-52,1
6	+17,1	0	-13,1	-23,4	-31,6	-38,3	-43,9
7	+34,8	+15,1	0	-11,8	-21,3	-29,0	-35,4
8	+52,9	+30,6	+13,4	0	-10,7	-19,5	-26,7
9	+71,3	+46,3	+27,1	+12,0	0	- 9,8	-17,9
10	+89,9	+62,1	+40,9	+24,2	+10,8	0	- 9,0
11	+108,6	+78,1	+54,7	+36,4	+21,8	+ 9,8	0

60 Year Life Cycle

ERRORS IN INITIAL COSTS

Errors in estimating initial costs will obviously affect the complete appraisal process. However, the extent to which the accuracy of the initial estimate will be of vital significance in the total cost concept will also depend on the ratio of initial to running costs. Errors of this nature will inevitably give rise to concern since, as previously indicated, the financial resources of the Client may be limited and cash flow problems may be encountered if initial costs are substantially higher than anticipated.

It should be remembered that the basis of life cycle costing is to provide a framework on which decisions can be formulated and, as such, need not necessarily reflect actual cash flows provided that ranking orders remain unaffected. Similarly, errors reflected in common elements of the alternatives under consideration will not normally be of significant consequence since the alternatives will, in the majority of cases, be equally affected.

Only the effects of errors and changes in interest rates on capital costs have been considered. Since total cost is fundamental to life cycle costing, the ratio of initial costs to running costs will be a decisive factor and must be taken into account when considering the effects of prediction errors.

EXAMPLE 29

In order to visualise the effects of incorrect assumptions and predictions five alternate designs will be appraised, each having an initial cost of R100 000.00 but with differing running costs, thus producing unequal initial to running cost ratios. Changes will be introduced to interest rates and predicted lives in order to ascertain their effects on a comparative analysis. The 'norm' for each alternative scheme will be a 50 year life cycle and a 10 per cent interest rate.

Scheme	Annual Running Costs	Initial to Running Cost Ratio
A	R 1 000.00	100 to 1
B	R 2 000.00	50 to 1
C	R10 000.00	10 to 1
D	R20 000.00	5 to 1
E	R50 000.00	2 to 1

All results will be computed on the basis of present values and the results can be seen in Table 'E' on page 110 .

T A B L E 'E'

The effects of changes in interest rates and predicted life cycles on present values of combined initial and running costs expenses.

Life Cycle Years	Interest Rate %	Tax Rate %	SCHEMES A (100:1)	B (50:1)	C (10:1)	D (5:1)	E (2:1)
100	10	-	100,08	100,14	100,42	100,57	100,71
50	10	-	100,00	100,00	100,00	100,00	100,00
25	10	-	99,24	98,6	95,79	94,38	92,97
15	10	-	97,90	96,15	88,41	84,52	80,62
100	5	-	109,04	116,58	149,88	163,25	183,37
50	5	-	107,59	113,92	141,88	155,93	170,01
25	5	-	103,80	106,97	120,99	128,02	135,08
15	5	-	100,42	100,77	102,33	103,12	103,90
100	10	50	100,01	100,02	100,05	100,06	100,08
50	10	50	99,28	98,69	96,05	94,72	93,40
25	10	50	97,39	95,21	85,60	80,77	75,93
15	10	50	95,70	92,46	76,27	68,32	60,35
100	5	50	107,63	114,44	142,14	156,27	170,43
50	5	50	103,88	107,52	121,42	128,60	135,81
25	5	50	99,36	98,83	96,47	95,29	94,10
15	5	50	96,61	93,78	81,30	75,03	68,75
100	10	10	100,08	100,14	100,42	100,56	100,70
50	10	10	99,96	99,92	99,75	99,67	99,59
25	10	10	99,02	98,20	94,61	92,80	90,99
15	10	10	97,58	95,56	86,64	82,17	77,68
100	5	10	108,95	116,42	149,41	165,98	182,59
50	5	10	107,16	113,14	139,52	152,77	166,06
25	5	10	103,12	105,72	117,22	123,00	128,79
15	5	10	99,77	99,58	98,75	98,33	97,91

Table E shows that changes in predicted lives will have greater impact for lower rates of interest than higher rates of interest. As income tax has the same effect as reducing the interest rate,costs will be more sensitive to investment changes where tax rates are high. The ratio of initial to running costs is also significant when comparing the results of future changes. Where the ratio of initial to running costs is 100 to 1 and the interest rate 10 per cent a change in predicted life cycle from 15 to 100 years results in a difference in present value of only approximately two per cent. If the interest rate is changed to five per cent the difference is increased to nine per cent. As the ratio of initial to running costs increases so does the effect of any changes in predicted life cycles so that at a ratio of 2 to 1 at five per cent a change in predicted life cycle from 15 to 100 years results in a difference in present value of approximately 80 per cent.

This is particularly relevant to the type of life cycle cost analysis that is to be undertaken. For life cycle cost studies of alternative materials the annual running costs will generally be low in relation to initial costs and as such errors in predicted lives will not normally be of great consequence. However, when studies are concerned with mechanical installations or total project costs the running costs will take on greater significance and special care must be exercised when considering estimated life cycles. It should also be noted that the length of life cycle is more critical for short life investments than for long life investments.

CHAPTER 9

PROBLEMS ASSOCIATED WITH LIFE CYCLE COSTING

The practice of life cycle costing involves the assessment of total costs over a determined investment life and the choice of the optimum economic solution from among alternative designs, procedures, etc. In addition to those who promote the concept of life cycle costing there are many who are sceptical of the merits of its application in property development appraisal studies. In order to appreciate the realities that must be faced when applying life cycle cost studies, the associated problems together with the criticisms promoted by its detractors will be discussed. It will be seen that some of the criticisms, while superficially appearing valid, are in fact advantages of the technique.

The interaction between capital and running costs:

The interaction between capital and running costs is not always readily understood, and lower running costs will not always be achieved by increasing capital expenditure. The objectives of the investor will also be of paramount importance when considering the relationship between initial and running costs. For a project developed with the object of resale after completion, maintenance charges will be significant to the purchaser but not to the seller whose

/prime concern

prime concern will be the economies of initial costs. This brings us to one of the advantages of the technique of life cycle costing, namely that it forces the user to define his investment objectives, a factor that is often forgotten in appraisals of this nature. The long term investor is thus more concerned with total costs than the short term speculator or developer, but this does give rise to the very real problem of obsolescence.

Obsolescence and Durability:

Life cycle cost studies depend upon the predictions of anticipated lives of complete projects or components thereof. While errors in forecasting are not always critical, care must still be taken to ensure that unrealistic forecasts are not introduced into the appraisal process. It is very often difficult to assess the physical life of certain components, but functional or economic obsolescence is very often of greater importance. It must be remembered that once money has actually been spent it is very difficult to amend the decision in the light of changed future developments. Since most buildings are functionally obsolete prior to physical obsolescence alteration and renewal costs will be pertinent to life cycle cost studies. The associated problem in this connection is the virtual impossibility of forecasting with any degree of accuracy future costs of this nature.

Uncertainty in Forecasting:

While initial costs can be estimated with a certain degree of accuracy, future maintenance and running costs are extremely difficult to assess. This technique does, however ensure that proper estimates of future costs are considered, which in my opinion, is far better than no consideration whatsoever. As previously discussed, one of the methods of alleviating this problem is by undertaking sensitivity analysis to assess the results when the predictions are varied.

One of the main criticisms in this regard is the fact that interest rates, which have a definite impact on present values, tend to fluctuate over the years and cannot be forecast with any certainty. While appreciating the fact that the use of an incorrect interest rate may lead to wrong results, the effect of changes in interest rates, dictated to a large extent by the rate of inflation, is not as serious as it first seems. The interest rate used is generally made up of two components, namely, the interest required from the investment and the inflation rate applicable at the time. Thus during periods of high inflation interest rates will tend to be high and vice versa.

In order to demonstrate the effect of fluctuations in interest and inflation rates the following example is given:-

EXAMPLE 30

It is estimated that it will cost R1 000.00 to paint the external facade of a building at today's costs. What is the present value of the cost of painting if the building is to be repainted at the end of each year for the next five years? Interest is 8 per cent and inflation 5 per cent.

Year	Expenditure	P V
1	$1000\ (1 + ,05) = 1\ 050.00 \left[\frac{1}{(1 + ,08)}\right] =$	972.22
2	$1000\ (1 + ,05)^2 = 1\ 102.50 \left[\frac{1}{(1 + ,08)^2}\right] =$	945.22
3.	$1000\ (1 + ,05)^3 = 1\ 157.60 \left[\frac{1}{(1 + ,08)^3}\right] =$	918.96
4.	$1000\ (1 + ,05)^4 = 1\ 215.50 \left[\frac{1}{(1 + ,08)^4}\right] =$	893.44
5.	$1000\ (1 + ,05)^5 = 1\ 276.30 \left[\frac{1}{(1 + ,08)^5}\right] =$	868.62
		R4 598.42

If the inflation rate is omitted from the interest rate, the real rate of interest would be in the region of three per cent. The calculation could then be changed to rzad as follows:-

Year	Expenditure		P V
1.	$1\ 000 \left[\dfrac{1}{(1 + ,03)}\right]$	=	970.87
2.	$1\ 000 \left[\dfrac{1}{(1 + ,03)^2}\right]$	=	942.60
3.	$1\ 000 \left[\dfrac{1}{(1 + ,03)^3}\right]$	=	915.14
4.	$1\ 000 \left[\dfrac{1}{(1 + ,03)^4}\right]$	=	888.49
5.	$1\ 000 \left[\dfrac{1}{(1 + ,03)^5}\right]$	=	862.61
			R4 579.71

This is much the same result as achieved when separate allowances were made for both inflation and interest rates. Therefore, provided the real rate of interest remains unchanged fluctuations in inflation and thus overall interest rates should not materially affect results of the calculations.

Acquisition of Capital

The realities of acquiring the necessary capital to implement the optimum economic solution is one of the problems associated with life cycle costing. Apart from the fact that the funds might not be available it is often difficult to convince Clients of the long term advantages that can result from increasing capital budgets.

Salvage and Residual Values

In many studies the problem of residual values and salvage costs must be faced, which represent a further forecasting problem. These values are also invariably associated with the additional problems of depreciation and tax relief factors. Such problems are more likely to be encountered when short term life cycles are evaluated, as in the case of developments for the purpose of resale, and not long term investments.

Disturbance Costs

Difficulties may arise in accurately forecasting future replacement costs of certain components due to uncertainty as to the degree of disturbance that may be suffered during building operations. This problem is generally accounted for by introducing a percentage premium to allow for increased costs. This is obviously not an exact method and where feasible, estimates of additional expenses should be undertaken.

Indirect or Intangible Costs

Life cycle costing is an economic appraisal process capable of reaching conclusions based on cost considerations alone. Not all cost consequences can be costed directly since their effects on total costs are difficult to measure. For example, an increase in initial costs to enhance the appearance of a building may in the long term result in a high occupancy rate, or greater contentment of the inhabitants. Costs and returns of this nature are extremely difficult to measure in monetary terms alone. In addition there are aspects such as prestige that are impossible to quantify and as such cannot form part of a life cycle cost analysis.

Cost Data

A major problem in South Africa is the virtual absence of suitable reliable cost data in the right form and detail to undertake studies of the nature envisaged.

For life cycle costing to be introduced on a national scale concerted efforts will have to be made for the collection of the required cost information on a continuing basis which must be presented and structured in a suitable form so that it can be recaptured for re-use in future studies.

CHAPTER 10

THE USE OF COMPOUND INTEREST TABLES IN LIFE CYCLE COST STUDIES

Provided both income tax and inflation implications are omitted from the calculations, life cycle cost studies are straightforward and quick to implement. The relatively inexpensive sophisticated calculators that are readily available today and the numerous books containing standard compound interest tables can greatly simplify problems associated with the application of life cycle costing.

However, once income tax and inflation implications are introduced into the calculations the problems assume different proportions. There are, to my knowledge, no standard compound interest tables that allow for the effects of inflation nor are the calculators, referred to above, capable of accommodating inflation implications in a quick and easy method. Thus, without the use of a computer, a life cycle cost analysis can become a time consuming and cumbersome operation.

In order to provide a method of preparing life cycle cost studies that includes for the effects of inflation, I have developed a set of compound interest tables that accommodates increases in future costs arising from inflation.

In addition, separate tables to incorporate the depreciation allowance, based on the declining balance method, which also involves time consuming and laborious calculations, have been developed. These tables have been prepared to facilitate an easy and quick application of the appraisal process and are included in the Appendix on page 130 .

A standard format for calculation of life cycle cost studies is suggested and illustrated on page 121. While it will obviously be modified to suit the needs of different users it could serve as a basis for future development.

In order to demonstrate the application of the format and developed compound interest tables, part of the problem in example 24 will be used.

<u>EXAMPLE 31</u>

The total cost of a vinyl tile floor covering is to be evaluated. The area to be covered is 1 000 square metres and the life of the building is 40 years. The applicable interest rate for the investment is 10 per cent, the inflation rate 12 per cent and the tax rate 50 per cent. The initial cost of the covering is R8.00 per square metre with an anticipated life of 15 years. Maintenance costs are expected to be 90 cents per square metre per annum while a premium of 50 per cent will be placed on replacement costs. The answer to this problem is shown on page 122 and the results can be compared with the answer in example 24. This example makes no allowance for tax depreciation. In order to examine the effects of these allowances consider the following example.

<u>EXAMPLE 32</u>

The problem used in example 31 will form the basis for this example except that it will be assumed that a tax depreciation allowance of 10 per cent is allowed by the Receiver.

This calculation is slightly more complicated. The future values of the vinyl flooring at end of years 15 and 30 must first be calculated. Table P.V.D.I.R can then be referred to but the three present values obtained will be for years 0, 16 and 30 respectively. The latter two must therefore be converted to year 0 by use of standard discount tables. The solution to example 32 is shown on page 123.

It will therefore, be seen that the use of these tables enables life cycle costing to be undertaken in a practical and simple manner that will be easily understandable and can be used as a quick method of relating initial to user costs. The need for a fairly speedy, accurate method of determining total costs is, to my mind, an important factor if life cycle costing is to be accepted for general use.

LIFE CYCLE COST ANALYSIS SHEET DATE: ____________

PROJECT: ____________ COMPONENT: ____________

Determined Life Cycle ___ Years Interest Rate ___ % Inflation Rate ___ %
Tax Rate ___ % Depreciation Rate ___ % After Tax Interest Rate ___ %

1. CAPITAL COSTS

	P V Total	Present Value
a.		
b.		
c.		
Total R		R ______

2. ANNUAL RUNNING COSTS

	Years	Amount		P.V.U.S.F.		P.V.		T.R.F		P V Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
								Total	R	

Present Value: R ______

3. MAINTENANCE COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
e.			X		=		X		=	
f.			X		=		X		=	
								Total	R	

Present Value: R ______

4. REPAIR, REPLACEMENT, ALTERATION COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F.		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
								Total	R	

Present Value: R ______

5. RESIDUAL/SALVAGE VALUE/COSTS

	P.V.		T.R.F.		P.V. Total
a.		X		=	
b.		X		=	
Add/Omit Total				R	

Present Value: R ______

6. DEPRECIATION ALLOWANCE

	Year	Amount		P.V.D.I.R.		P.V.		P.V.F.		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
						Omit Total			R	
						X T.R.F.				

Present Value: R ______

TOTAL LIFE CYCLE COST R

STANDARD FORMAT

LIFE CYCLE COST ANALYSIS SHEET — DATE: 18 : 08 : 78

PROJECT: QUANTBUILD EXAMPLE 31. — COMPONENT: VINYL FLOORING.

Determined Life Cycle 40/15 Years — Interest Rate 10 % — Inflation Rate 12 %

Tax Rate 50 % — Depreciation Rate – % — After Tax Interest Rate 5 %

1. CAPITAL COSTS

	P V Total	Present Value
a.	8000.00	
b.		
c.		
Total R	8000.00	R 8 000.00

2. ANNUAL RUNNING COSTS

	Years	Amount		P.V.U.S.F.		P.V.		T.R.F		P V Total
a.	40	R900.00	X	195,47976	=	175931.78	X	0,5	=	87965.89
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	

Total R 87965.89 — R 87 965.89

3. MAINTENANCE COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
e.			X		=		X		=	
f.			X		=		X		=	

Total R — R

4. REPAIR, REPLACEMENT, ALTERATION COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F.		P.V. Total
a.	15	R12000.00	X	2,63288	=	31 594.56	X	–	=	31 594.56
b.	30	R12000.00	X	6,93205	=	83 184.60	X	–	=	83 184.60
c.			X		=		X		=	

Total R 114 779.16 — R 114 779.16

5. RESIDUAL/SALVAGE VALUE/COSTS

	P.V.		T.R.F.		P.V. Total
a.		X		=	
b.		X		=	

Add/Omit Total R — R

6. DEPRECIATION ALLOWANCE

	Year	Amount		P.V.D.I.R.		P.V.		P.V.F.		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	

Omit Total R

X T.R.F. — R

TOTAL LIFE CYCLE COST R 210 745.05

LIFE CYCLE COST ANALYSIS SHEET — DATE: 18 : 08 : 78

PROJECT: QUANTBUILD EXAMPLE 32. — COMPONENT: VINYL FLOORING.

Determined Life Cycle 40/15 Years   Interest Rate 10 %   Inflation Rate 12 %
Tax Rate 50 %   Depreciation Rate 10 %   After Tax Interest Rate 5 %

1. CAPITAL COSTS

	P V Total
a.	8000.00
b.	
c.	
Total R	8000.00

Present Value: R 8 000.00

2. ANNUAL RUNNING COSTS

	Years	Amount		P.V.U.S.F.		P.V.		T.R.F		P V Total
a.	40	R900.00	X	195,47976	=	175931.78	X	0,5	=	87965.89
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
								Total	R	87965.89

Present Value: R 87965.89

3. MAINTENANCE COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F		P.V. Total
a.			X		=		X		=	
b.			X		=		X		=	
c.			X		=		X		=	
d.			X		=		X		=	
e.			X		=		X		=	
f.			X		=		X		=	
								Total	R	

Present Value: R

4. REPAIR, REPLACEMENT, ALTERATION COSTS

	Year	Amount		P.V.S.P.F.		P.V.		T.R.F.		P.V. Total
a.	15	R12000.00	X	2,63288	=	31 594.56	X	–	=	31 594.56
b.	30	R12000.00	X	6,93205	=	83 184.60	X	–	=	83 184.60
c.			X		=		X		=	
								Total	R	114 779.16

Present Value: R 114 779.16

5. RESIDUAL/SALVAGE VALUE/COSTS

	P.V.		T.R.F.		P.V. Total
a.		X		=	
b.		X		=	
			Add/Omit Total	R	

Present Value: R

6. DEPRECIATION ALLOWANCE

	Year	Amount		P.V.D.I.R.		P.V.		P.V.F.		P.V. Total
a.	0-15	R8000.00	X	0,69968	=	5597.44	X	–	=	5597.44
b.	16-30	R65683.00	X	0,69968	=	45957.09	X	0,48101	=	22 105.82
c.	31-40	R359519.00	X	0,73802	=	265332.21	X	0,23137	=	61389.91
d.			X		=		X		=	
								Omit Total	R	89093.17
								X T.R.F.		0,5

Present Value: R 44546.59

TOTAL LIFE CYCLE COST R 166 198.46

CONCLUSION

There is no doubt that the measurement and control of total building cost is becoming increasingly necessary and important, although progressively more difficult, as rapid technological developments result in larger and more complex projects. If cost planning is to play a meaningful role in the property development process then the total cost philosophy must be fully appreciated and the appraisal technique capable of implementing this concept. An additional requirement is flexibility in order to adapt to the individual characteristics of each specific project.

Arguments may be raised to the effect that sophisticated mathematical appraisal techniques are not warranted when used in investment situations involving uncertain future predictions. I believe this to be incorrect since criticisms of this nature tend to perpetuate situations in which decisions are based on implicit assumptions, the implications of which are not always fully understood or even considered. One of the advantages of the life cycle cost technique is that it forces these implied assumptions to be defined and, as such, they are easily recognisable and can be appraised and evaluated accordingly.

The benefits derived from recording the economic consequences of alternative design solutions are important, not only to the building owner but also to the design team who should continually be made aware of the cost implications of their design. The cost planning process is thus carried to its logical conclusion and the designer will at all times have a cost framework within which to work.

These cost parameters are not based upon the narrow confines of initial costs alone, but represent the broad total cost concept so necessary to establish the optimum cost solution.

A further, and important, advantage of the life cycle costing process is that it requires greater Client co-operation and participation. It is of fundamental importance that policy matters related to the investment are established at an early stage of the development. For instance, the interest rate is an individual matter that must be determined by the building owner. Although the most important duty of the Client if usually the formation of the brief, the continuation of dialogue between Client and Consultants can only be of benefit to the progress and success of the project.

The importance of any analysis being presented in a straightforward and understandable format cannot be over-emphasised. It is also realised that for any technique to be truly acceptable it must be capable of accurate and speedy calculation. Despite the apparent complexity of some of the derived equations and the laborious calculations required for those situations reflecting the effects of inflation, the compound interest tables developed in chapter 10 can result in complicated problems being solved relatively easily and quickly.

The problem area still remains the collection of data on a continuing basis and in a suitable form that allows re-capture for use in future analyses. This very real problem while obviously adding uncertainty, should not deter the use of the technique. In the United States of America there is wide acceptance of life cycle costing as an important and sophisticated appraisal technique for use in the property industry.

With the energy crisis well advanced it is becoming increasingly evident that new approaches and techniques must be developed and introduced into the property investment appraisal process in South Africa. I believe that life cycle costing is the one method that satisfies the investor's requirements. It provides a clear presentation of the total time-phased costs over the full life of a building thereby allowing decisions to be taken with confidence so that optimum economic use can be made of all resources.

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A P P E N D I X

COMPOUND INTEREST TABLES

INCORPORATING INFLATION AND DEPRECIATION ALLOWANCES

TABLE PVUSF

(Present Value Uniform Series Factor)

This table performs the following functions:

1. Increases the values of a series of uniform payment amounts by the stipulated inflation rates

2. Converts the future inflated value at the end of each year to present value at the stipulated interest rate

3. Provides the total present value of the series of payments

TABLE : PVUSF
****

INFLATION RATE: 8 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.02846	1.02557	1.01887	1.00935	1.00000	.99083
2	2.11686	2.08653	2.05696	2.02813	2.00000	1.97257
3	3.23674	3.17472	3.11464	3.05643	3.00000	2.94530
4	4.39969	4.29400	4.19227	4.09434	4.00000	3.90910
5	5.60737	5.44526	5.29024	5.14195	5.00000	4.86406
6	6.86150	6.62941	6.40892	6.19935	6.00000	5.81026
7	8.16387	7.84739	7.54671	7.26663	7.00000	6.74778
8	9.51633	9.10017	8.71001	8.34389	8.00000	7.67670
9	10.92080	10.38875	9.89322	9.43122	9.00000	8.59710
10	12.37929	11.71414	11.09875	10.52871	10.00000	9.50905
11	13.89398	13.07740	12.32703	11.63645	11.00000	10.41264
12	15.46672	14.47961	13.57848	12.75455	12.00000	11.30794
13	17.10006	15.92188	14.85355	13.88310	13.00000	12.19502
14	18.79622	17.40536	16.15267	15.02219	14.00000	13.07396
15	20.55761	18.93123	17.47631	16.17193	15.00000	13.94484
16	22.38675	20.50069	18.62492	17.33242	16.00000	14.80773
17	24.23624	22.11500	20.19896	18.50375	17.00000	15.66271
18	26.25879	23.77543	21.59696	19.68603	18.00000	16.50984
19	28.30720	25.48330	23.02536	20.87936	19.00000	17.34920
20	30.43440	27.23997	24.47867	22.08384	20.00000	18.18086
21	32.64342	29.04683	25.95940	23.29958	21.00000	19.00489
22	34.93740	30.90531	27.46807	24.52668	22.00000	19.82136
23	37.31961	32.81689	29.00520	25.76525	23.00000	20.63034
24	39.79344	34.78309	30.57134	27.01539	24.00000	21.43190
25	42.36242	36.80546	32.16703	28.27722	25.00000	22.22610
26	45.03021	38.88562	33.79282	29.55084	26.00000	23.01302
27	47.80060	41.02521	35.44929	30.83636	27.00000	23.79272
28	50.67755	43.22593	37.13701	32.13390	28.00000	24.56526
29	53.66515	45.48953	38.85658	33.44356	29.00000	25.33072
30	56.76766	47.81780	40.60859	34.76546	30.00000	26.08915
31	59.98949	50.21259	42.39366	36.09972	31.00000	26.84063
32	63.33524	52.67581	44.21241	37.44645	32.00000	27.58521
33	66.80967	55.20940	46.06547	38.80576	33.00000	28.32296
34	70.41773	57.81538	47.95350	40.17778	34.00000	29.05394
35	74.16457	60.49582	49.87715	41.56262	35.00000	29.77822
36	78.05551	63.25284	51.83710	42.96040	36.00000	30.49585
37	82.09611	66.08864	53.83403	44.37124	37.00000	31.20690
38	86.29210	69.00546	55.86863	45.79527	38.00000	31.91142
39	90.64949	72.00561	57.94162	47.23261	39.00000	32.60948
40	95.17447	75.09148	60.05372	48.68338	40.00000	33.30114
41	99.87349	78.26552	62.20568	50.14771	41.00000	33.98645
42	104.75324	81.53025	64.39824	51.62573	42.00000	34.66547
43	109.82067	84.88826	66.63217	53.11756	43.00000	35.33826
44	115.08300	88.34221	68.90625	54.62332	44.00000	36.00488
45	120.54773	91.89484	71.22727	56.14316	45.00000	36.66539

TABLE : PVUSF
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INFLATION RATE: 8 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.03349	1.02370	1.01406	1.00465	.99539	.98630
2	2.10160	2.07165	2.04245	2.01397	1.98619	1.95909
3	3.20548	3.14444	3.08530	3.02799	2.97243	2.91855
4	4.34633	4.24265	4.14284	4.04672	3.95412	3.86487
5	5.52539	5.36688	5.21527	5.07019	4.93129	4.79823
6	6.74394	6.51775	6.30281	6.09842	5.90396	5.71880
7	8.00331	7.69590	7.40567	7.13149	6.87214	6.62676
8	9.30496	8.90196	8.52406	8.16926	7.83586	7.52228
9	10.65000	10.13660	9.65820	9.21191	8.79514	8.40554
10	12.04019	11.40050	10.80832	10.25941	9.75000	9.27670
11	13.47694	12.69435	11.97463	11.31178	10.70046	10.13592
12	14.96181	14.01886	13.15737	12.36904	11.64654	10.98337
13	16.49642	15.37476	14.35677	13.43122	12.58826	11.81921
14	18.08243	16.76279	15.57306	14.49834	13.52564	12.64360
15	19.72155	18.18371	16.80648	15.57043	14.45870	13.45670
16	21.41557	19.63830	18.05728	16.64750	15.38746	14.25866
17	23.16633	21.12736	19.32569	17.72958	16.31194	15.04964
18	24.97573	22.65171	20.61197	18.81669	17.23216	15.82978
19	26.84573	24.21218	21.91636	19.90886	18.14814	16.59923
20	28.77836	25.80963	23.23913	21.00611	19.05990	17.35814
21	30.77572	27.44493	24.58053	22.10846	19.96746	18.10666
22	32.83998	29.11898	25.94082	23.21594	20.87084	18.84492
23	34.97338	30.83270	27.32027	24.32857	21.77005	19.57307
24	37.17823	32.58703	28.71915	25.44638	22.66512	20.29125
25	39.45693	34.38293	30.13773	26.56939	23.55606	20.99959
26	41.81195	36.22139	31.57629	27.69762	24.44290	21.69823
27	44.24584	38.10341	33.03511	28.83110	25.32565	22.38729
28	46.76125	40.03003	34.51448	29.96985	26.20433	23.06692
29	49.36091	42.00231	36.01468	31.11390	27.07896	23.73724
30	52.04764	44.02132	37.53601	32.26327	27.94956	24.39837
31	54.82436	46.08818	39.07877	33.41799	28.81615	25.05045
32	57.69408	48.20401	40.64326	34.57806	29.67875	25.69359
33	60.65991	50.36998	42.22978	35.74354	30.53737	26.32792
34	63.72508	52.58728	43.83865	36.91444	31.39204	26.95356
35	66.89291	54.85712	45.47018	38.09079	32.24277	27.57063
36	70.16684	57.18075	47.12469	39.27261	33.08958	28.17925
37	73.55042	59.55944	48.80250	40.45992	33.93249	28.77953
38	77.04732	61.99450	50.50394	41.65276	34.77151	29.37159
39	80.66135	64.48725	52.22935	42.85115	35.60666	29.95554
40	84.39642	67.03908	53.97905	44.05511	36.43797	30.53149
41	88.25659	69.65138	55.75340	45.26467	37.26544	31.09955
42	92.24605	72.32558	57.55274	46.47985	38.08910	31.65983
43	96.36912	75.06315	59.37743	47.70069	38.90897	32.21243
44	100.63029	77.86560	61.22782	48.92721	39.72506	32.75746
45	105.03418	80.73445	63.10427	50.15942	40.53739	33.29503

TABLE : PVUSF
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INFLATION RATE: 9 %
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INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.04808	1.03810	1.07830	1.01869	1.00926	1.00000
2	2.14655	2.11574	2.08570	2.05642	2.02786	2.00000
3	3.29783	3.23443	3.17303	3.11355	3.05590	3.00000
4	4.50446	4.39574	4.29113	4.19044	4.09345	4.00000
5	5.76910	5.60129	5.44088	5.28746	5.14061	5.00000
6	7.09454	6.85277	6.62317	6.40498	6.19747	6.00000
7	8.48370	8.15192	7.83892	7.54339	7.26411	7.00000
8	9.93965	9.50056	9.08908	8.70308	8.34063	8.00000
9	11.46559	10.90058	10.37462	9.88445	9.42712	9.00000
10	13.06490	12.35394	11.69654	11.08790	10.52367	10.00000
11	14.74110	13.86266	13.05588	12.31384	11.63037	11.00000
12	16.49788	15.42886	14.45369	13.56270	12.74732	12.00000
13	18.33912	17.05472	15.89106	14.83490	13.87461	13.00000
14	20.26688	18.74252	17.36911	16.13088	15.01234	14.00000
15	22.29142	20.49462	18.88899	17.45108	16.16060	15.00000
16	24.41120	22.31346	20.45189	18.79596	17.31949	16.00000
17	26.63289	24.20159	22.05902	20.16598	18.48911	17.00000
18	28.96139	26.16165	23.71163	21.56161	19.66956	18.00000
19	31.40184	28.19638	25.41102	22.98332	20.86094	19.00000
20	33.95962	30.30862	27.15850	24.43161	22.06336	20.00000
21	36.64037	32.50133	28.95544	25.90697	23.27691	21.00000
22	39.45000	34.77757	30.80324	27.40990	24.50170	22.00000
23	42.39471	37.14053	32.70333	28.94093	25.73783	23.00000
24	45.48099	39.59350	34.65720	30.50057	26.98540	24.00000
25	48.71565	42.13992	36.66637	32.08937	28.24452	25.00000
26	52.10582	44.78335	38.73240	33.70786	29.51530	26.00000
27	55.65898	47.52748	40.85690	35.35660	30.79785	27.00000
28	59.38297	50.37615	43.04153	37.03616	32.09227	28.00000
29	63.28600	53.33334	45.28799	38.74712	33.39868	29.00000
30	67.37667	56.40318	47.59803	40.49006	34.71719	30.00000
31	71.66401	59.58997	49.97345	42.26557	36.04790	31.00000
32	76.15747	62.89816	52.41609	44.07427	37.39094	32.00000
33	80.86696	66.33237	54.92787	45.91678	38.74641	33.00000
34	85.80286	69.89741	57.51073	47.79373	40.11443	34.00000
35	90.97607	73.59825	60.16669	49.70576	41.49512	35.00000
36	96.39800	77.44009	62.89782	51.65353	42.88859	36.00000
37	102.08060	81.42828	65.70624	53.63771	44.29497	37.00000
38	108.03640	85.56940	68.59415	55.65898	45.71437	38.00000
39	114.27653	89.86624	71.56380	57.71803	47.14691	39.00000
40	120.82077	94.32781	74.61749	59.81556	48.59271	40.00000
41	127.67754	98.95934	77.75761	61.95230	50.05190	41.00000
42	134.86396	103.76731	80.98660	64.12897	51.52459	42.00000
43	142.39588	108.75845	84.30698	66.34633	53.01093	43.00000
44	150.28991	113.93972	87.72133	68.60514	54.51103	44.00000
45	158.56346	119.31838	91.23231	70.90617	56.02502	45.00000

TABLE : PVUSF
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INFLATION RATE: 9 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.04306	1.03318	1.02347	1.01395	1.00461	.99543
2	2.13104	2.10063	2.07097	2.04205	2.01385	1.98632
3	3.26587	3.20349	3.14306	3.08450	3.02774	2.97268
4	4.44957	4.34294	4.24031	4.14149	4.04630	3.95454
5	5.68424	5.52019	5.36332	5.21323	5.06955	4.93192
6	6.97208	6.73650	6.51269	6.29993	6.09752	5.90483
7	8.31538	7.99316	7.68904	7.40179	7.13023	6.87330
8	9.71652	9.29151	8.89301	8.51902	8.16770	7.83735
9	11.17800	10.63293	10.12524	9.65184	9.20995	8.79700
10	12.70241	12.01886	11.38640	10.80047	10.25700	9.75226
11	14.29247	13.45077	12.67716	11.96513	11.30988	10.70316
12	15.95100	14.93018	13.99822	13.14604	12.36560	11.64972
13	17.68095	16.45867	15.35029	14.34343	13.42719	12.59196
14	19.48539	18.03787	16.73410	15.55752	14.49367	13.52990
15	21.36754	19.66946	18.15039	16.78856	15.56507	14.46355
16	23.33073	21.35518	19.59993	18.03677	16.64141	15.39294
17	25.37846	23.09682	21.08350	19.30240	17.72271	16.31809
18	27.51437	24.89624	22.60189	20.58569	18.80899	17.23901
19	29.74226	26.75536	24.15593	21.88689	19.90028	18.15573
20	32.06609	28.67615	25.74644	23.20624	20.99659	19.06826
21	34.48999	30.66067	27.37429	24.54400	22.09796	19.97662
22	37.01827	32.71102	29.04035	25.90043	23.20440	20.88084
23	39.65542	34.82939	30.74552	27.27578	24.31594	21.78093
24	42.40613	37.01804	32.49072	28.67033	25.43260	22.67691
25	45.27529	39.27930	34.27689	30.08433	26.55441	23.56880
26	48.26801	41.61558	36.10499	31.51806	27.68139	24.45661
27	51.38960	44.02937	37.97600	32.97180	28.81356	25.34037
28	54.64561	46.52324	39.89093	34.44583	29.95095	26.22009
29	58.04183	49.09984	41.85081	35.94042	31.09358	27.09580
30	61.58430	51.76192	43.85670	37.45587	32.24148	27.96751
31	65.27932	54.51232	45.90967	38.99246	33.39467	28.83524
32	69.13345	57.35396	48.01084	40.55049	34.55317	29.69901
33	73.15354	60.28987	50.16133	42.13026	35.71701	30.55883
34	77.34675	63.32318	52.36230	43.73208	36.88621	31.41473
35	81.72053	66.45712	54.61494	45.35625	38.06080	32.26672
36	86.28266	69.69502	56.92045	47.00308	39.24080	33.11482
37	91.04124	73.04035	59.28009	48.67289	40.42624	33.95904
38	96.00474	76.49667	61.69512	50.36600	41.61714	34.79941
39	101.18198	80.06765	64.16683	52.08274	42.81353	35.63594
40	106.58216	83.75710	66.69657	53.82342	44.01544	36.46865
41	112.21488	87.56895	69.28569	55.58840	45.22288	37.29756
42	118.09016	91.50726	71.93559	57.37801	46.43588	38.12268
43	124.21844	95.57622	74.64769	59.19259	47.65448	38.94404
44	130.61062	99.78017	77.42346	61.03249	48.87869	39.76165
45	137.27806	104.12359	80.26439	62.89806	50.10855	40.57552

TABLE : PVUSF

INFLATION RATE: 10 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.05769	1.04762	1.03774	1.02804	1.01852	1.00917
2	2.17640	2.14513	2.11464	2.08490	2.05590	2.02760
3	3.35965	3.29490	3.23217	3.17139	3.11249	3.05538
4	4.61117	4.49942	4.39187	4.28834	4.18865	4.09259
5	5.93489	5.76130	5.59534	5.43661	5.28474	5.13931
6	7.33498	7.08327	6.84422	6.61708	6.40112	6.19563
7	8.81584	8.46819	8.14023	7.83064	7.53818	7.26165
8	10.38214	9.91906	9.48514	9.07823	8.69629	8.33744
9	12.03880	11.43901	10.88081	10.36080	9.87585	9.42310
10	13.79154	13.03134	12.32914	11.67933	11.07725	10.51872
11	15.64437	14.69950	13.83213	13.03483	12.30090	11.62440
12	17.60462	16.44709	15.39183	14.42833	13.54721	12.74022
13	19.67796	18.27790	17.01039	15.86090	14.81660	13.86628
14	21.87092	20.19589	18.69003	17.33364	16.10950	15.00267
15	24.19040	22.20527	20.43305	18.84767	17.42634	16.14948
16	26.64369	24.31023	22.24194	20.40415	18.76757	17.30681
17	29.23852	26.51548	24.11899	22.00427	20.13364	18.47476
18	31.98305	28.82574	26.06677	23.64925	21.52500	19.65343
19	34.88592	31.24601	28.08816	25.34035	22.94213	20.84291
20	37.95626	33.78153	30.18583	27.07886	24.38550	22.04330
21	41.20374	36.43779	32.36265	28.86612	25.85560	23.25471
22	44.63857	39.22054	34.62162	30.70349	27.35293	24.47723
23	48.27157	42.13580	36.96583	32.59237	28.87798	25.71097
24	52.11416	45.18989	39.39850	34.53421	30.43128	26.95603
25	56.17844	48.36941	41.92297	36.53050	32.01334	28.21251
26	60.47720	51.74129	44.54271	38.58276	33.62470	29.48051
27	65.02396	55.25278	47.26130	40.69256	35.26590	30.76015
28	69.83304	58.93148	50.08248	42.86151	36.93749	32.05153
29	74.91956	62.78536	53.01012	45.09127	38.64004	33.35476
30	80.29953	66.82276	56.04824	47.38355	40.37411	34.66994
31	85.98989	71.05240	59.20100	49.74010	42.14030	35.99719
32	92.00854	75.46347	62.47274	52.16272	43.93919	37.33661
33	98.37442	80.12554	65.86794	54.65326	45.77140	38.68832
34	105.10756	84.98866	69.39125	57.21363	47.63754	40.05243
35	112.22915	90.08336	73.04752	59.84579	49.53824	41.42906
36	119.76160	95.42066	76.84177	62.55175	51.47413	42.81832
37	127.72862	101.01212	80.77920	65.33357	53.44587	44.22032
38	136.15527	106.86984	84.86521	68.19338	55.45412	45.63519
39	145.06808	113.00650	89.10541	71.13338	57.49957	47.06304
40	154.49509	119.43538	93.50562	74.15581	59.58290	48.50399
41	164.46596	126.17040	98.07187	77.26298	61.70481	49.95816
42	175.01208	133.22613	102.81043	80.45727	63.86601	51.42567
43	186.16661	140.61785	107.72781	83.74112	66.06723	52.90664
44	197.96469	148.36156	112.83075	87.11704	68.30922	54.40120
45	210.44342	156.47401	116.12625	90.58761	70.59272	55.90947

TABLE : FVUSF
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INFLATION RATE: 10 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.05263	1.04265	1.03286	1.02326	1.01382	1.00457
2	2.16066	2.12978	2.09967	2.07031	2.04166	2.01372
3	3.32701	3.26328	3.20154	3.14171	3.08371	3.02748
4	4.55475	4.44513	4.33962	4.23803	4.14017	4.04587
5	5.84711	5.67739	5.51510	5.35984	5.21123	5.06891
6	7.20748	6.96221	6.72921	6.50774	6.29710	6.09662
7	8.63945	8.30163	7.98322	7.68234	7.39798	7.12902
8	10.14679	9.69859	9.27844	8.88426	8.51408	8.16614
9	11.73346	11.15493	10.61623	10.11413	9.64561	9.20799
10	13.40364	12.67339	11.99798	11.37260	10.79278	10.25460
11	15.16173	14.25661	13.42514	12.66033	11.95581	11.30599
12	17.01235	15.90737	14.89921	13.97801	13.13492	12.36218
13	18.96037	17.62854	16.42172	15.32634	14.33033	13.42319
14	21.01092	19.42312	17.99427	16.70602	15.54227	14.48905
15	23.16939	21.29425	19.61849	18.11779	16.77097	15.55978
16	25.44146	23.24519	21.29609	19.56239	18.01665	16.63540
17	27.83312	25.27934	23.02883	21.04059	19.27955	17.71593
18	30.35065	27.40026	24.81851	22.55316	20.55991	18.80139
19	33.00068	29.61164	26.66701	24.10091	21.85797	19.89181
20	35.79019	31.91735	28.57625	25.68465	23.17398	20.98721
21	38.72652	34.32141	30.54824	27.30522	24.50818	22.08761
22	41.81739	36.82801	32.58504	28.96348	25.86083	23.19303
23	45.07094	39.44153	34.68877	30.66031	27.23218	24.30350
24	48.49573	42.16652	36.86164	32.39660	28.62249	25.41904
25	52.10077	45.00775	39.10592	34.17327	30.03202	26.53967
26	55.89555	47.97016	41.42396	35.99125	31.46103	27.66542
27	59.89005	51.05893	43.81817	37.85151	32.90980	28.79631
28	64.09479	54.27945	46.29107	39.75503	34.37860	29.93237
29	68.52083	57.63734	48.84524	41.70282	35.86771	31.07361
30	73.17982	61.13846	51.48335	43.69591	37.37740	32.22006
31	78.08402	64.78892	54.20815	45.73535	38.90796	33.37175
32	83.24634	68.59508	57.02250	47.82222	40.45968	34.52870
33	88.68035	72.56359	59.92934	49.95762	42.03286	35.69093
34	94.40037	76.70137	62.93171	52.14268	43.62778	36.85847
35	100.42144	81.01564	66.03274	54.37856	45.24475	38.03134
36	106.75941	85.51394	69.23569	56.66643	46.88408	39.20957
37	113.43096	90.20411	72.54391	59.00751	48.54607	40.39318
38	120.45364	95.09433	75.96085	61.40303	50.23104	41.58219
39	127.84594	100.19314	79.49008	63.85425	51.93929	42.77663
40	135.62731	105.50943	83.13529	66.36249	53.67117	43.97652
41	143.81822	111.05249	86.90030	68.92906	55.42699	45.18189
42	152.44023	116.83198	90.78904	71.55532	57.20709	46.39277
43	161.51603	122.85799	94.80558	74.24265	59.01180	47.60917
44	171.06951	129.14103	98.95412	76.99248	60.84146	48.83113
45	181.12580	135.69207	103.23900	79.80626	62.69641	50.05867

TABLE : PVUSF
****

INFLATION RATE: 11 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.06731	1.05714	1.04717	1.03738	1.02778	1.01835
2	2.20646	2.17469	2.14373	2.11354	2.08411	2.05538
3	3.42228	3.35610	3.29202	3.22993	3.16978	3.11144
4	4.71993	4.60507	4.49447	4.38806	4.28561	4.18688
5	6.10492	5.92531	5.75364	5.58948	5.43243	5.28205
6	7.58314	7.32104	7.07221	6.83582	6.61111	6.39732
7	9.16085	8.79653	8.45298	8.12875	7.82253	7.53305
8	10.84475	10.35633	9.89888	9.47001	9.06760	8.68962
9	12.64199	12.00526	11.41298	10.86141	10.34726	9.86741
10	14.56020	13.74842	12.99850	12.30483	11.66246	11.06681
11	16.60752	15.59119	14.65881	13.80221	13.01419	12.28922
12	18.79264	17.53926	16.39743	15.35556	14.40347	13.53204
13	21.12484	19.59865	18.21806	16.96698	15.83134	14.79868
14	23.61401	21.77572	20.12457	18.63864	17.29888	16.08856
15	26.27072	24.07719	22.12101	20.37280	18.80718	17.40211
16	29.10625	26.51017	24.21162	22.17178	20.35738	18.73976
17	32.13263	29.06218	26.40085	24.03802	21.95064	20.10196
18	35.36271	31.80116	28.69334	25.97402	23.58816	21.48915
19	38.81020	34.67551	31.09397	27.98240	25.27116	22.90180
20	42.48973	37.71411	33.60784	30.06585	27.00091	24.34037
21	46.41693	40.92634	36.24029	32.22719	28.77671	25.80533
22	50.60846	44.32213	38.99691	34.46933	30.60590	27.29717
23	55.08211	47.91197	41.88356	36.79529	32.48384	28.81638
24	59.85687	51.70694	44.90637	39.20820	34.41395	30.36347
25	64.95301	55.71877	48.07177	41.71131	36.39767	31.93895
26	70.39216	59.95964	51.38648	44.30800	38.43649	33.54334
27	76.19740	64.44326	54.85754	47.00176	40.53195	35.17716
28	82.39338	69.18287	58.49233	49.79622	42.68562	36.84096
29	89.00640	74.19332	62.29857	52.69514	44.89911	38.53529
30	96.06452	79.49008	66.28435	55.70244	47.17409	40.26071
31	103.59770	85.08950	70.45814	58.82216	49.51226	42.01779
32	111.63793	91.00890	74.82881	62.05850	51.91538	43.80711
33	120.21933	97.26655	79.40564	65.41583	54.38525	45.62926
34	129.37832	103.88178	84.19836	68.89867	56.92373	47.48484
35	139.15379	110.87503	89.21714	72.51170	59.53272	49.37447
36	149.58722	118.26789	94.47267	76.25979	62.21418	51.29877
37	160.72290	126.08320	99.97610	80.14801	64.97012	53.25838
38	172.60840	134.34510	105.73913	84.18158	67.80262	55.25395
39	185.29326	143.07911	111.77400	88.36594	70.71380	57.28613
40	198.83223	152.31220	118.09353	92.70672	73.70585	59.35560
41	213.28248	162.07290	124.71115	97.20978	76.78101	61.46304
42	228.70534	172.39135	131.64092	101.88117	79.94159	63.60915
43	245.16678	183.29943	138.89757	106.72720	83.18997	65.79463
44	262.73516	194.83082	146.49651	111.75439	86.52858	68.02022
45	281.48657	207.02115	154.45390	116.96951	89.95993	70.28665

TABLE : PVUSF

INFLATION RATE: 11 %

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.06220	1.05213	1.04225	1.03256	1.02304	1.01370
2	2.19047	2.15911	2.12854	2.09874	2.06965	2.04128
3	3.38892	3.32380	3.26073	3.19963	3.14038	3.08294
4	4.66192	4.54921	4.44076	4.33636	4.23578	4.13887
5	6.01410	5.83851	5.67065	5.51010	5.35642	5.20927
6	7.45038	7.19502	6.95251	6.72206	6.50288	6.29433
7	8.97600	8.62225	8.28853	7.97348	7.67576	7.39425
8	10.59652	10.12388	9.68100	9.26564	8.87566	8.50924
9	12.31783	11.70380	11.13231	10.59987	10.10321	9.63950
10	14.14621	13.36608	12.64494	11.97754	11.35904	10.78525
11	16.09832	15.11502	14.22149	13.40006	12.64381	11.94669
12	18.15123	16.95514	15.86465	14.86890	13.95818	13.12404
13	20.34246	18.89119	17.57724	16.38556	15.30284	14.31752
14	22.66998	20.92817	19.36219	17.95160	16.67848	15.52735
15	25.14228	23.07135	21.22257	19.56863	18.08582	16.75375
16	27.76835	25.32626	23.16155	21.23831	19.52559	17.99695
17	30.55777	27.69972	25.18246	22.96235	20.99853	19.25718
18	33.52069	30.19486	27.28876	24.74252	22.50541	20.53468
19	36.66791	32.82113	29.48406	26.58065	24.04701	21.82968
20	40.01089	35.58432	31.77212	28.47862	25.62413	23.14242
21	43.56181	38.49156	34.15686	30.43839	27.23759	24.47314
22	47.33360	41.55036	36.64236	32.46197	28.88823	25.82209
23	51.34000	44.76863	39.23288	34.55143	30.57690	27.18952
24	55.59560	48.15467	41.93286	36.70892	32.30448	28.57568
25	60.11590	51.71724	44.74693	38.93665	34.07186	29.98083
26	64.91737	55.46553	47.67990	41.23691	35.87997	31.40522
27	70.01749	59.40923	50.73680	43.61207	37.72974	32.84913
28	75.43485	63.55853	53.92286	46.06456	39.62213	34.31282
29	81.18916	67.92414	57.24355	48.59689	41.55812	35.79656
30	87.30141	72.51734	60.70455	51.21167	43.53872	37.30062
31	93.79384	77.35000	64.31179	53.91159	45.56496	38.82529
32	100.69011	82.43460	68.07144	56.69940	47.63789	40.37084
33	108.01533	87.78427	71.98994	59.57799	49.75858	41.93756
34	115.79619	93.41283	76.07402	62.55030	51.92813	43.52575
35	124.06103	99.33483	80.33067	65.61938	54.14767	45.13569
36	132.83995	105.56556	84.76718	68.78838	56.41835	46.76769
37	142.16493	112.12111	89.39115	72.06056	58.74135	48.42204
38	152.06993	119.01842	94.21050	75.43927	61.11788	50.09905
39	162.59103	126.27531	99.23348	78.92799	63.54917	51.79904
40	173.76655	133.91052	104.46870	82.53030	66.03647	53.52231
41	185.63720	141.94377	109.92513	86.24989	68.58109	55.26918
42	198.24621	150.39582	115.61211	90.09058	71.18434	57.03999
43	211.63951	159.26850	121.53938	94.05632	73.84757	58.83506
44	225.86589	168.64478	127.71710	98.15118	76.57217	60.65472
45	240.97717	178.48882	134.15585	102.37936	79.35955	62.49931

TABLE : PVWSF

INFLATION RATE: 12 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.07692	1.06667	1.05660	1.04673	1.03704	1.02752
2	2.23668	2.20445	2.17301	2.14237	2.11249	2.08332
3	3.49566	3.41808	3.35261	3.28921	3.22777	3.16818
4	4.83071	4.71262	4.59898	4.48964	4.38435	4.28290
5	6.27923	6.09346	5.91590	5.74617	5.58377	5.42830
6	7.83917	7.56636	7.30737	7.06141	6.82761	6.60523
7	9.51911	9.13745	8.77760	8.43811	8.11752	7.81455
8	11.32827	10.81328	10.33105	9.87914	9.45521	9.05715
9	13.27660	12.60083	11.97243	11.38751	10.84244	10.33395
10	15.37480	14.50755	13.70672	12.96637	12.28105	11.64589
11	17.63440	16.54139	15.53918	14.61900	13.77294	12.99394
12	20.06781	18.71081	17.47536	16.34886	15.32009	14.37909
13	22.68841	21.02486	19.52114	18.15955	16.92454	15.80237
14	25.51060	23.49318	21.68271	20.05486	18.59841	17.26482
15	28.54988	26.12606	23.96664	22.03873	20.31391	18.76752
16	31.82295	28.93446	26.37985	24.11531	22.10331	20.31159
17	35.34779	31.93009	28.92965	26.28892	23.95899	21.89914
18	39.14377	35.12543	31.62378	28.56410	25.88340	23.52836
19	43.23175	38.53379	34.47041	30.94560	27.87908	25.20345
20	47.63419	42.16939	37.47817	33.43839	29.94867	26.92465
21	52.37528	46.04734	40.65618	36.04766	32.09492	28.69322
22	57.48107	50.18383	44.01408	38.77886	34.37066	30.51047
23	62.97961	54.59608	47.56205	41.63769	36.62883	32.37773
24	68.90112	59.30248	51.31085	44.63011	39.02249	34.29638
25	75.27813	64.32265	55.27194	47.76236	41.50480	36.26784
26	82.14568	69.67749	59.45704	51.04097	44.07905	38.29356
27	89.54150	75.38932	63.87914	54.47279	46.74864	40.37504
28	97.50623	81.48194	68.55155	58.06498	49.51711	42.51380
29	106.08362	87.98073	73.48843	61.82503	52.38811	44.71143
30	115.32082	94.91278	78.70476	65.76078	55.36545	46.96954
31	125.26858	102.30697	84.21634	69.88044	58.45306	49.28980
32	135.98155	110.19410	90.03991	74.19260	61.65502	51.67392
33	147.51859	118.60704	96.19311	78.70627	64.97558	54.12366
34	159.94310	127.58084	102.69461	83.43086	68.41912	56.64083
35	173.32334	137.15290	109.56412	88.37623	71.99019	59.22728
36	187.73283	147.36309	116.82247	93.55269	75.69353	61.88491
37	203.25074	158.25396	124.49167	98.97104	79.53403	64.61569
38	219.96234	169.87089	132.59497	104.64258	83.51677	67.42162
39	237.95944	182.26728	141.15695	110.57915	87.64702	70.30479
40	257.34092	195.47976	150.20357	116.79313	91.93024	73.26731
41	278.21330	209.57841	159.76226	123.29748	96.37210	76.31137
42	300.69125	224.61697	169.86201	130.10577	100.97847	79.43921
43	324.89827	240.65809	180.53344	137.23221	105.75545	82.65313
44	350.96737	257.76863	191.80892	144.69166	110.70935	85.95551
45	379.04178	276.01987	203.72264	152.49968	115.84673	89.34878

TABLE : FVUSF
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INFLATION RATE: 12 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.07177	1.06161	1.05164	1.04186	1.03226	1.02283
2	2.22046	2.18863	2.15759	2.12733	2.09782	2.06901
3	3.45159	3.38509	3.32066	3.25824	3.19775	3.13908
4	4.77108	4.65526	4.54379	4.43649	4.33316	4.23358
5	6.18527	6.00369	5.83009	5.66406	5.50520	5.35307
6	7.70096	7.43520	7.18282	6.94302	6.71504	6.49812
7	9.32543	8.95490	8.60541	8.27552	7.96391	7.66931
8	11.06649	10.56823	10.10146	9.66360	9.25307	8.86724
9	12.93251	12.26096	11.67478	11.11019	10.58381	10.09252
10	14.93245	14.09922	13.32935	12.61713	11.95748	11.34577
11	17.07593	16.02950	15.06936	14.18715	13.37546	12.62764
12	19.37325	18.07871	16.89923	15.82289	14.83918	13.93877
13	21.83545	20.25418	18.82360	17.52710	16.35012	15.27984
14	24.47436	22.56368	20.64735	19.30265	17.90980	16.65153
15	27.30266	25.01547	22.97562	21.15253	19.51979	18.05453
16	30.33395	27.61832	25.21380	23.07985	21.18172	19.48956
17	33.58280	30.38153	27.56756	25.08784	22.89726	20.95736
18	37.06482	33.31499	30.04288	27.17989	24.66814	22.45867
19	40.79675	36.42918	32.64603	29.35951	26.49614	23.99426
20	44.79652	39.73524	35.38362	31.63037	28.38311	25.56490
21	49.08335	43.24500	38.26259	33.99629	30.33095	27.17140
22	53.67785	46.97100	41.29024	36.46125	32.34163	28.81458
23	58.60210	50.92656	44.47424	39.02940	34.41717	30.49528
24	63.87976	55.12583	47.82268	41.70505	36.55966	32.21435
25	69.53620	59.56382	51.34404	44.49270	38.77126	33.97267
26	75.59861	64.31647	55.04725	47.39705	41.05420	35.77113
27	82.09612	69.34071	58.94171	50.42297	43.41079	37.61065
28	89.05976	74.67450	63.03729	53.57556	45.84340	39.49217
29	96.52359	80.33691	67.34438	56.86012	48.35448	41.41665
30	104.52288	86.34819	71.87390	60.28217	50.94656	43.38507
31	113.09629	92.72982	76.63733	63.84747	53.62226	45.39843
32	122.28502	99.50464	81.64677	67.56202	56.38427	47.45775
33	132.13323	106.69687	86.91491	71.43206	59.23538	49.56409
34	142.68825	114.33222	92.45512	75.46410	62.17846	51.71852
35	154.00080	122.43800	98.28144	79.66492	65.21647	53.92214
36	166.12526	131.04319	104.40865	84.04159	68.35249	56.17607
37	179.11990	140.17855	110.85229	88.60146	71.58966	58.48146
38	193.04717	149.87676	117.62870	93.35222	74.93126	60.83948
39	207.97400	160.17249	124.75507	98.30185	78.38066	63.25134
40	223.97213	171.10255	132.24946	103.45867	81.94133	65.71827
41	241.11846	182.70603	140.13089	108.83136	85.61686	68.24152
42	259.49532	195.02440	148.41934	114.42895	89.41095	70.82237
43	279.19122	208.10173	157.13583	120.26086	93.32743	73.46215
44	300.30064	221.98478	166.30247	126.33690	97.37025	76.16220
45	322.92508	236.72318	175.94251	132.66729	101.54348	78.92389

T A B L E   P V S P F

(Present Value Single Payment Factor)

This table performs the following functions:

1. Increases the value of a single payment amount by the stipulated inflation rate

2. Converts the future inflated value to present value at the stipulated interest rate

TABLE : PVSPF

INFLATION RATE: 8 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.03846	1.02857	1.01887	1.00935	1.00000	.99083
2	1.07840	1.05796	1.03809	1.01878	1.00000	.98174
3	1.11988	1.08819	1.05768	1.02830	1.00000	.97273
4	1.16295	1.11928	1.07763	1.03791	1.00000	.96380
5	1.20768	1.15126	1.09797	1.04761	1.00000	.95496
6	1.25413	1.18415	1.11868	1.05740	1.00000	.94620
7	1.30237	1.21798	1.13979	1.06728	1.00000	.93752
8	1.35246	1.25278	1.16130	1.07726	1.00000	.92892
9	1.40447	1.28858	1.18321	1.08733	1.00000	.92040
10	1.45849	1.32539	1.20553	1.09749	1.00000	.91195
11	1.51459	1.36326	1.22828	1.10774	1.00000	.90359
12	1.57284	1.40221	1.25145	1.11810	1.00000	.89530
13	1.63334	1.44227	1.27507	1.12855	1.00000	.88708
14	1.69616	1.48348	1.29912	1.13909	1.00000	.87894
15	1.76139	1.52587	1.32364	1.14974	1.00000	.87088
16	1.82914	1.56946	1.34861	1.16049	1.00000	.86289
17	1.89949	1.61431	1.37406	1.17133	1.00000	.85498
18	1.97255	1.66043	1.39998	1.18228	1.00000	.84713
19	2.04841	1.70787	1.42640	1.19333	1.00000	.83936
20	2.12720	1.75667	1.45331	1.20448	1.00000	.83166
21	2.20902	1.80686	1.48073	1.21574	1.00000	.82403
22	2.29398	1.85848	1.50867	1.22710	1.00000	.81647
23	2.38221	1.91158	1.53713	1.23857	1.00000	.80898
24	2.47383	1.96620	1.56614	1.25014	1.00000	.80156
25	2.56898	2.02237	1.59569	1.26183	1.00000	.79420
26	2.66779	2.08016	1.62579	1.27362	1.00000	.78692
27	2.77039	2.13959	1.65647	1.28552	1.00000	.77970
28	2.87695	2.20072	1.68772	1.29754	1.00000	.77254
29	2.98760	2.26360	1.71957	1.30966	1.00000	.76546
30	3.10251	2.32827	1.75201	1.32190	1.00000	.75843
31	3.22183	2.39479	1.78507	1.33426	1.00000	.75148
32	3.34575	2.46322	1.81875	1.34673	1.00000	.74458
33	3.47443	2.53359	1.85306	1.35931	1.00000	.73775
34	3.60806	2.60598	1.88803	1.37202	1.00000	.73098
35	3.74684	2.68044	1.92365	1.38484	1.00000	.72428
36	3.89094	2.75702	1.95995	1.39778	1.00000	.71763
37	4.04060	2.83580	1.99693	1.41084	1.00000	.71105
38	4.19600	2.91682	2.03460	1.42403	1.00000	.70452
39	4.35739	3.00016	2.07299	1.43734	1.00000	.69806
40	4.52498	3.08587	2.11211	1.45077	1.00000	.69166
41	4.69902	3.17404	2.15196	1.46433	1.00000	.68531
42	4.87975	3.26473	2.19256	1.47802	1.00000	.67902
43	5.06743	3.35801	2.23393	1.49183	1.00000	.67279
44	5.26233	3.45395	2.27608	1.50577	1.00000	.66662
45	5.46473	3.55263	2.31902	1.51984	1.00000	.66051

TABLE : PVSPF
****

INFLATION RATE: 8 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.03349	1.02370	1.01408	1.00465	.99539	.98630
2	1.06811	1.04795	1.02837	1.00932	.99080	.97279
3	1.10388	1.07279	1.04285	1.01402	.98624	.95946
4	1.14085	1.09821	1.05754	1.01873	.98169	.94637
5	1.17906	1.12423	1.07243	1.02347	.97717	.93336
6	1.21855	1.15087	1.08754	1.02823	.97267	.92057
7	1.25937	1.17815	1.10286	1.03302	.96818	.90796
8	1.30155	1.20606	1.11839	1.03782	.96372	.89552
9	1.34514	1.23464	1.13414	1.04265	.95928	.88326
10	1.39019	1.26390	1.15012	1.04750	.95486	.87116
11	1.43675	1.29385	1.16631	1.05237	.95046	.85922
12	1.48487	1.32451	1.18274	1.05726	.94608	.84745
13	1.53461	1.35590	1.19940	1.06218	.94172	.83584
14	1.58601	1.38803	1.21629	1.06712	.93738	.82439
15	1.63912	1.42092	1.23342	1.07209	.93306	.81310
16	1.69402	1.45459	1.25080	1.07707	.92876	.80196
17	1.75076	1.48906	1.26841	1.08208	.92448	.79098
18	1.80940	1.52435	1.28628	1.08711	.92022	.78014
19	1.87000	1.56047	1.30439	1.09217	.91598	.76945
20	1.93263	1.59745	1.32277	1.09725	.91176	.75891
21	1.99736	1.63530	1.34140	1.10235	.90756	.74852
22	2.06426	1.67405	1.36029	1.10748	.90338	.73826
23	2.13340	1.71372	1.37945	1.11263	.89921	.72815
24	2.20495	1.75433	1.39688	1.11781	.89507	.71818
25	2.27870	1.79590	1.41858	1.12301	.89094	.70834
26	2.35502	1.83846	1.43856	1.12823	.88684	.69864
27	2.43389	1.88202	1.45882	1.13348	.88275	.68906
28	2.51541	1.92662	1.47937	1.13875	.87868	.67963
29	2.59966	1.97228	1.50020	1.14405	.87463	.67032
30	2.68673	2.01901	1.52133	1.14937	.87060	.66113
31	2.77672	2.06686	1.54276	1.15471	.86659	.65208
32	2.86972	2.11583	1.56449	1.16008	.86260	.64314
33	2.96583	2.16597	1.58652	1.16548	.85862	.63433
34	3.06517	2.21730	1.60897	1.17090	.85467	.62564
35	3.16783	2.26954	1.63153	1.17635	.85073	.61707
36	3.27393	2.32363	1.65451	1.18182	.84681	.60862
37	3.38358	2.37869	1.67781	1.18731	.84291	.60028
38	3.49691	2.43506	1.70144	1.19284	.83902	.59206
39	3.61403	2.49276	1.72541	1.19839	.83515	.58395
40	3.73507	2.55183	1.74971	1.20396	.83131	.57595
41	3.86017	2.61230	1.77435	1.20956	.82747	.56806
42	3.98946	2.67420	1.79934	1.21518	.82366	.56028
43	4.12307	2.73757	1.82469	1.22084	.81987	.55260
44	4.27117	2.80245	1.85039	1.22652	.81609	.54503
45	4.40389	2.86885	1.87645	1.23222	.81233	.53757

TABLE : PVBPF
****

INFLATION RATE: 9 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.04808	1.03810	1.02830	1.01869	1.00926	1.00000
2	1.09847	1.07764	1.05740	1.03773	1.01860	1.00000
3	1.15128	1.11869	1.08733	1.05713	1.02804	1.00000
4	1.20663	1.16131	1.11810	1.07689	1.03755	1.00000
5	1.26464	1.20555	1.14975	1.09702	1.04716	1.00000
6	1.32544	1.25148	1.18229	1.11752	1.05686	1.00000
7	1.38916	1.29915	1.21575	1.13841	1.06664	1.00000
8	1.45595	1.34864	1.25016	1.15969	1.07652	1.00000
9	1.52594	1.40002	1.28554	1.18137	1.08649	1.00000
10	1.59931	1.45336	1.32192	1.20345	1.09655	1.00000
11	1.67620	1.50872	1.35934	1.22594	1.10670	1.00000
12	1.75678	1.56620	1.39781	1.24886	1.11695	1.00000
13	1.84124	1.62586	1.43737	1.27220	1.12729	1.00000
14	1.92976	1.68780	1.47805	1.29598	1.13773	1.00000
15	2.02254	1.75210	1.51988	1.32020	1.14826	1.00000
16	2.11976	1.81884	1.56290	1.34488	1.15889	1.00000
17	2.22169	1.88813	1.60713	1.37002	1.16962	1.00000
18	2.32850	1.96006	1.65261	1.39563	1.18045	1.00000
19	2.44045	2.03473	1.69939	1.42171	1.19138	1.00000
20	2.55778	2.11224	1.74748	1.44829	1.20242	1.00000
21	2.68075	2.19271	1.79694	1.47536	1.21355	1.00000
22	2.80963	2.27624	1.84780	1.50293	1.22479	1.00000
23	2.94471	2.36296	1.90009	1.53103	1.23613	1.00000
24	3.08628	2.45297	1.95387	1.55964	1.24757	1.00000
25	3.23466	2.54642	2.00917	1.58880	1.25912	1.00000
26	3.39017	2.64343	2.06603	1.61849	1.27078	1.00000
27	3.55316	2.74413	2.12450	1.64874	1.28255	1.00000
28	3.72399	2.84867	2.18463	1.67956	1.29442	1.00000
29	3.90303	2.95719	2.24646	1.71096	1.30641	1.00000
30	4.09067	3.06984	2.31004	1.74294	1.31851	1.00000
31	4.28734	3.18679	2.37542	1.77551	1.33071	1.00000
32	4.49346	3.30819	2.44264	1.80870	1.34304	1.00000
33	4.70949	3.43421	2.51178	1.84251	1.35547	1.00000
34	4.93591	3.56504	2.58286	1.87695	1.36802	1.00000
35	5.17321	3.70085	2.65596	1.91203	1.38069	1.00000
36	5.42193	3.84184	2.73113	1.94777	1.39347	1.00000
37	5.68260	3.98819	2.80843	1.98418	1.40638	1.00000
38	5.95580	4.14012	2.88791	2.02127	1.41940	1.00000
39	6.24213	4.29784	2.96965	2.05905	1.43254	1.00000
40	6.54224	4.46157	3.05369	2.09753	1.44580	1.00000
41	6.85677	4.63153	3.14012	2.13674	1.45919	1.00000
42	7.18642	4.80797	3.22899	2.17668	1.47270	1.00000
43	7.53192	4.99114	3.32038	2.21736	1.48634	1.00000
44	7.89403	5.18127	3.41435	2.25881	1.50010	1.00000
45	8.27355	5.37866	3.51098	2.30103	1.51399	1.00000

TABLE : PVSPF
****

INFLATION RATE: 9 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.04306	1.03318	1.02347	1.01395	1.00461	.99543
2	1.08798	1.06745	1.04750	1.02810	1.00924	.99089
3	1.13483	1.10286	1.07209	1.04245	1.01389	.98636
4	1.19370	1.13945	1.09725	1.05699	1.01856	.98186
5	1.23467	1.17725	1.12301	1.07174	1.02325	.97738
6	1.28784	1.21631	1.14937	1.08670	1.02797	.97291
7	1.34330	1.25666	1.17635	1.10186	1.03271	.96847
8	1.40114	1.29835	1.20397	1.11723	1.03747	.96405
9	1.46148	1.34142	1.23223	1.13282	1.04225	.95965
10	1.52441	1.38593	1.26116	1.14863	1.04705	.95526
11	1.59006	1.43191	1.29076	1.16466	1.05188	.95090
12	1.65853	1.47941	1.32106	1.18091	1.05672	.94656
13	1.72995	1.52849	1.35207	1.19739	1.06159	.94224
14	1.80444	1.57920	1.38381	1.21409	1.06648	.93794
15	1.88215	1.63159	1.41629	1.23104	1.07140	.93365
16	1.96319	1.68572	1.44954	1.24821	1.07634	.92939
17	2.04773	1.74164	1.48357	1.26563	1.08130	.92515
18	2.13591	1.79942	1.51839	1.28329	1.08628	.92092
19	2.22789	1.85912	1.55404	1.30120	1.09129	.91672
20	2.32383	1.92079	1.59051	1.31935	1.09631	.91253
21	2.42390	1.98452	1.62785	1.33776	1.10137	.90836
22	2.52828	2.05035	1.66606	1.35643	1.10644	.90422
23	2.63715	2.11837	1.70517	1.37535	1.11154	.90009
24	2.75071	2.18865	1.74520	1.39455	1.11666	.89598
25	2.86916	2.26126	1.78617	1.41400	1.12181	.89189
26	2.99272	2.33628	1.82810	1.43373	1.12698	.88781
27	3.12159	2.41379	1.87101	1.45374	1.13217	.88376
28	3.25601	2.49387	1.91493	1.47403	1.13739	.87972
29	3.39622	2.57660	1.95988	1.49459	1.14263	.87571
30	3.54247	2.66208	2.00589	1.51545	1.14790	.87171
31	3.69502	2.75040	2.05297	1.53659	1.15319	.86773
32	3.85413	2.84164	2.10117	1.55803	1.15850	.86377
33	4.02010	2.93591	2.15049	1.57977	1.16384	.85982
34	4.19321	3.03331	2.20097	1.60182	1.16920	.85590
35	4.37378	3.13394	2.25264	1.62417	1.17459	.85199
36	4.56213	3.23791	2.30551	1.64683	1.18000	.84810
37	4.75858	3.34533	2.35964	1.66981	1.18544	.84422
38	4.96350	3.45632	2.41503	1.69311	1.19090	.84037
39	5.17724	3.57098	2.47172	1.71674	1.19639	.83653
40	5.40018	3.68945	2.52974	1.74069	1.20191	.83271
41	5.63272	3.81185	2.58912	1.76498	1.20744	.82891
42	5.87528	3.93831	2.64990	1.78961	1.21301	.82512
43	6.12828	4.06896	2.71210	1.81458	1.21860	.82136
44	6.39218	4.20395	2.77577	1.83990	1.22421	.81761
45	6.66744	4.34342	2.84093	1.86557	1.22986	.81387

TABLE : FVSPF

INFLATION RATE: 10 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.05769	1.04762	1.03774	1.02804	1.01852	1.00917
2	1.11871	1.09751	1.07690	1.05686	1.03738	1.01843
3	1.18325	1.14977	1.11753	1.08649	1.05659	1.02778
4	1.25152	1.20452	1.15970	1.11695	1.07616	1.03721
5	1.32372	1.26188	1.20347	1.14827	1.09609	1.04672
6	1.40009	1.32197	1.24888	1.18047	1.11638	1.05632
7	1.48086	1.38492	1.29601	1.21356	1.13706	1.06602
8	1.56630	1.45087	1.34491	1.24759	1.15811	1.07579
9	1.65666	1.51995	1.39567	1.28257	1.17956	1.08566
10	1.75224	1.59233	1.44833	1.31853	1.20140	1.09562
11	1.85333	1.66816	1.50299	1.35550	1.22365	1.10568
12	1.96025	1.74759	1.55970	1.39350	1.24631	1.11582
13	2.07334	1.83081	1.61856	1.43257	1.26939	1.12606
14	2.19296	1.91799	1.67964	1.47274	1.29290	1.13639
15	2.31948	2.00933	1.74302	1.51403	1.31684	1.14681
16	2.45329	2.10501	1.80879	1.55648	1.34123	1.15733
17	2.59483	2.20525	1.87705	1.60012	1.36607	1.16795
18	2.74453	2.31026	1.94788	1.64498	1.39136	1.17867
19	2.90286	2.42027	2.02139	1.69110	1.41713	1.18948
20	3.07034	2.53552	2.09767	1.73851	1.44337	1.20039
21	3.24748	2.65626	2.17682	1.78726	1.47010	1.21141
22	3.43483	2.78275	2.25897	1.83737	1.49733	1.22252
23	3.63300	2.91526	2.34421	1.88888	1.52505	1.23374
24	3.84259	3.05409	2.43267	1.94184	1.55330	1.24506
25	4.06428	3.19952	2.52447	1.99629	1.58206	1.25649
26	4.29876	3.35188	2.61974	2.05226	1.61136	1.26800
27	4.54676	3.51149	2.71859	2.10980	1.64120	1.27964
28	4.80908	3.67870	2.82118	2.16895	1.67159	1.29138
29	5.08652	3.85388	2.92764	2.22976	1.70255	1.30323
30	5.37998	4.03740	3.03812	2.29228	1.73407	1.31518
31	5.69036	4.22965	3.15276	2.35655	1.76619	1.32725
32	6.01865	4.43107	3.27174	2.42262	1.79889	1.33942
33	6.36588	4.64207	3.39520	2.49054	1.83221	1.35171
34	6.73314	4.86312	3.52332	2.56037	1.86614	1.36411
35	7.12159	5.09470	3.65627	2.63216	1.90070	1.37663
36	7.53245	5.33730	3.79425	2.70596	1.93589	1.38926
37	7.96702	5.59146	3.93743	2.78182	1.97174	1.40200
38	8.42665	5.85772	4.08601	2.85982	2.00826	1.41487
39	8.91281	6.13666	4.24020	2.94000	2.04545	1.42785
40	9.42701	6.42888	4.40021	3.02243	2.08333	1.44095
41	9.97087	6.73502	4.56625	3.10717	2.12191	1.45417
42	10.54612	7.05573	4.73856	3.19429	2.16120	1.46751
43	11.15454	7.39172	4.91738	3.28385	2.20122	1.48097
44	11.79808	7.74371	5.10294	3.37592	2.24199	1.49456
45	12.47873	8.11245	5.29550	3.47057	2.28350	1.50827

TABLE : PVSPF

INFLATION RATE: 10 %

YEAR	INTEREST RATE 4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.05263	1.04265	1.03286	1.02326	1.01382	1.00457
2	1.10803	1.08713	1.06681	1.04705	1.02784	1.00915
3	1.16635	1.13350	1.10187	1.07140	1.04205	1.01376
4	1.22774	1.18185	1.13808	1.09632	1.05646	1.01839
5	1.29236	1.23226	1.17548	1.12181	1.07106	1.02304
6	1.36037	1.28482	1.21411	1.14790	1.08587	1.02771
7	1.43197	1.33962	1.25401	1.17460	1.10088	1.03240
8	1.50734	1.39676	1.29522	1.20192	1.11610	1.03712
9	1.58667	1.45634	1.33779	1.22987	1.13153	1.04185
10	1.67018	1.51846	1.38175	1.25847	1.14717	1.04661
11	1.75809	1.58322	1.42716	1.28773	1.16303	1.05139
12	1.85062	1.65076	1.47407	1.31768	1.17911	1.05619
13	1.94802	1.72117	1.52251	1.34833	1.19541	1.06101
14	2.05055	1.79458	1.57255	1.37968	1.21194	1.06586
15	2.15847	1.87113	1.62422	1.41177	1.22870	1.07073
16	2.27207	1.95094	1.67760	1.44460	1.24568	1.07562
17	2.39166	2.03415	1.73274	1.47820	1.26290	1.08053
18	2.51753	2.12092	1.78968	1.51257	1.28036	1.08546
19	2.65003	2.21138	1.84850	1.54775	1.29806	1.09042
20	2.78951	2.30571	1.90924	1.58374	1.31601	1.09540
21	2.93633	2.40406	1.97199	1.62057	1.33420	1.10040
22	3.09087	2.50660	2.03680	1.65826	1.35265	1.10542
23	3.25355	2.61352	2.10373	1.69683	1.37135	1.11047
24	3.42479	2.72499	2.17287	1.73629	1.39031	1.11554
25	3.60504	2.84123	2.24428	1.77667	1.40953	1.12063
26	3.79478	2.96241	2.31804	1.81798	1.42901	1.12575
27	3.99450	3.08877	2.39421	1.86026	1.44877	1.13089
28	4.20474	3.22052	2.47290	1.90352	1.46880	1.13606
29	4.42604	3.35789	2.55417	1.94779	1.48911	1.14124
30	4.65899	3.50112	2.63811	1.99309	1.50969	1.14645
31	4.90420	3.65046	2.72480	2.03944	1.53056	1.15169
32	5.16232	3.80616	2.81435	2.08687	1.55172	1.15695
33	5.43402	3.96851	2.90684	2.13540	1.57318	1.16223
34	5.72002	4.13778	3.00237	2.18506	1.59492	1.16754
35	6.02107	4.31428	3.10104	2.23588	1.61697	1.17287
36	6.33797	4.49830	3.20295	2.28787	1.63933	1.17823
37	6.67155	4.69017	3.30822	2.34108	1.66199	1.18361
38	7.02268	4.89022	3.41694	2.39552	1.68497	1.18901
39	7.39230	5.09881	3.52923	2.45123	1.70826	1.19444
40	7.78137	5.31629	3.64521	2.50824	1.73188	1.19989
41	8.19091	5.54306	3.76501	2.56657	1.75582	1.20537
42	8.62201	5.77949	3.88874	2.62626	1.78010	1.21088
43	9.07580	6.02601	4.01654	2.68733	1.80471	1.21641
44	9.55348	6.28304	4.14854	2.74983	1.82966	1.22196
45	10.05629	6.55104	4.28488	2.81378	1.85495	1.22754

TABLE : FV5PF
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INFLATION RATE: 11 %
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INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	1.06731	1.05714	1.04717	1.03738	1.02778	1.01835
2	1.13915	1.11755	1.09656	1.07616	1.05633	1.03703
3	1.21582	1.18141	1.14829	1.11639	1.08567	1.05606
4	1.29765	1.24892	1.20245	1.15813	1.11583	1.07544
5	1.38499	1.32029	1.25917	1.20142	1.14682	1.09517
6	1.47822	1.39573	1.31857	1.24634	1.17868	1.11527
7	1.57771	1.47549	1.38077	1.29293	1.21142	1.13573
8	1.68390	1.55980	1.44590	1.34126	1.24507	1.15657
9	1.79724	1.64893	1.51410	1.39140	1.27966	1.17779
10	1.91821	1.74316	1.58552	1.44342	1.31520	1.19940
11	2.04732	1.84277	1.66031	1.49738	1.35173	1.22141
12	2.18512	1.94807	1.73862	1.55335	1.38928	1.24382
13	2.33220	2.05939	1.82063	1.61142	1.42787	1.26664
14	2.49917	2.17707	1.90651	1.67166	1.46754	1.28989
15	2.65671	2.30147	1.99644	1.73416	1.50830	1.31355
16	2.83553	2.43298	2.09061	1.79898	1.55020	1.33765
17	3.02638	2.57201	2.18923	1.86624	1.59326	1.36220
18	3.23008	2.71898	2.29249	1.93600	1.63752	1.38719
19	3.44749	2.87435	2.40063	2.00838	1.68300	1.41265
20	3.67953	3.03860	2.51387	2.08345	1.72975	1.43857
21	3.92720	3.21223	2.63245	2.16134	1.77780	1.46496
22	4.19153	3.39579	2.75662	2.24214	1.82719	1.49184
23	4.47365	3.58984	2.88665	2.32596	1.87794	1.51921
24	4.77476	3.79497	3.02281	2.41291	1.93011	1.54709
25	5.09614	4.01183	3.16540	2.50311	1.98372	1.57548
26	5.43915	4.24107	3.31471	2.59669	2.03882	1.60439
27	5.80524	4.48342	3.47106	2.69376	2.09546	1.63382
28	6.19598	4.73961	3.63479	2.79446	2.15367	1.66380
29	6.61302	5.01045	3.80624	2.89892	2.21349	1.69433
30	7.05812	5.29676	3.98578	3.00730	2.27498	1.72542
31	7.53319	5.59943	4.17379	3.11972	2.33817	1.75708
32	8.04023	5.91940	4.37067	3.23634	2.40312	1.78932
33	8.58140	6.25765	4.57683	3.35733	2.46987	1.82215
34	9.15899	6.61523	4.79272	3.48284	2.53348	1.85558
35	9.77547	6.99325	5.01879	3.61303	2.60899	1.88963
36	10.43343	7.39286	5.25553	3.74810	2.68146	1.92430
37	11.13568	7.81531	5.50343	3.88822	2.75595	1.95961
38	11.88520	8.26190	5.76303	4.03357	2.83250	1.99557
39	12.68516	8.73401	6.03487	4.18436	2.91118	2.03218
40	13.53897	9.23309	6.31953	4.34078	2.99205	2.06947
41	14.45025	9.76070	6.61762	4.50306	3.07516	2.10744
42	15.42286	10.31845	6.92977	4.67139	3.16058	2.14611
43	16.46094	10.90808	7.25665	4.84603	3.24838	2.18549
44	17.56889	11.53140	7.59894	5.02719	3.33861	2.22559
45	18.75141	12.19033	7.95739	5.21512	3.43135	2.26643

TABLE : PVSPF
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INFLATION RATE: 11 %
****

YEAR	INTEREST RATE 4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.06220	1.05213	1.04225	1.03256	1.02304	1.01370
2	1.12627	1.10698	1.08629	1.06618	1.04661	1.02758
3	1.19845	1.16469	1.13219	1.10089	1.07073	1.04166
4	1.27300	1.22541	1.18003	1.13673	1.09540	1.05593
5	1.35216	1.26930	1.22989	1.17374	1.12064	1.07040
6	1.43622	1.35651	1.28186	1.21196	1.14646	1.08506
7	1.52562	1.42723	1.33602	1.25142	1.17288	1.09992
8	1.62052	1.50163	1.39247	1.29216	1.19990	1.11499
9	1.72131	1.57992	1.45131	1.33423	1.22755	1.13026
10	1.82838	1.66228	1.51263	1.37767	1.25583	1.14575
11	1.94211	1.74894	1.57655	1.42252	1.28477	1.16144
12	2.06291	1.84012	1.64316	1.46884	1.31437	1.17735
13	2.19123	1.93605	1.71259	1.51666	1.34466	1.19348
14	2.32752	2.03698	1.78495	1.56604	1.37564	1.20963
15	2.47230	2.14318	1.86038	1.61703	1.40734	1.22640
16	2.62607	2.25491	1.93898	1.66968	1.43977	1.24320
17	2.78942	2.37246	2.02091	1.72404	1.47294	1.26023
18	2.96292	2.49614	2.10630	1.78017	1.50688	1.27750
19	3.14722	2.62627	2.19530	1.83813	1.54160	1.29500
20	3.34298	2.76319	2.28806	1.89797	1.57712	1.31274
21	3.55092	2.90724	2.38474	1.95977	1.61346	1.33072
22	3.77179	3.05880	2.48550	2.02359	1.65064	1.34895
23	4.00640	3.21827	2.59052	2.08946	1.68867	1.36743
24	4.25560	3.36604	2.69998	2.15749	1.72758	1.38616
25	4.52030	3.56257	2.81407	2.22773	1.76738	1.40515
26	4.80147	3.74829	2.93297	2.30026	1.80811	1.42439
27	5.10012	3.94370	3.05690	2.37516	1.84977	1.44391
28	5.41736	4.14930	3.18606	2.45249	1.89239	1.46369
29	5.75432	4.36561	3.32069	2.53233	1.93599	1.48374
30	6.11225	4.59320	3.46100	2.61478	1.98060	1.50406
31	6.49243	4.83266	3.60724	2.69991	2.02624	1.52467
32	6.89627	5.08460	3.75965	2.78782	2.07293	1.54555
33	7.32522	5.34967	3.91851	2.87859	2.12069	1.56672
34	7.78086	5.62857	4.08408	2.97231	2.16955	1.58819
35	8.26494	5.92200	4.25665	3.06908	2.21954	1.60994
36	8.77892	6.23073	4.43651	3.16900	2.27068	1.63200
37	9.32498	6.55555	4.62397	3.27218	2.32300	1.65435
38	9.90500	6.89731	4.81935	3.37872	2.37653	1.67701
39	10.52110	7.25689	5.02298	3.48872	2.43129	1.69999
40	11.17552	7.63521	5.23522	3.60231	2.48731	1.72327
41	11.87065	8.03325	5.45643	3.71959	2.54462	1.74688
42	12.60901	8.45205	5.68698	3.84069	2.60325	1.77081
43	13.39331	8.89268	5.92727	3.96574	2.66323	1.79507
44	14.22638	9.35628	6.17772	4.09486	2.72460	1.81966
45	15.11128	9.84404	6.43875	4.22818	2.78738	1.84459

TABLE : PVSPF
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INFLATION RATE: 12 %
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	INTEREST RATE					
YEAR	4%	5%	6%	7%	8%	9%
1	1.07692	1.06667	1.05660	1.04673	1.03704	1.02752
2	1.15976	1.13778	1.11641	1.09564	1.07545	1.05580
3	1.24698	1.21363	1.17960	1.14684	1.11528	1.08486
4	1.34505	1.29454	1.24637	1.20043	1.15658	1.11472
5	1.44852	1.38084	1.31692	1.25653	1.19942	1.14540
6	1.55994	1.47290	1.39147	1.31524	1.24384	1.17693
7	1.67994	1.57109	1.47023	1.37670	1.28991	1.20932
8	1.80916	1.67583	1.55345	1.44103	1.33769	1.24260
9	1.94833	1.78755	1.64138	1.50037	1.38723	1.27680
10	2.09820	1.90672	1.73429	1.57886	1.43861	1.31194
11	2.25960	2.03384	1.83246	1.65263	1.49189	1.34805
12	2.43341	2.16942	1.93618	1.72986	1.54715	1.38515
13	2.62060	2.31405	2.04578	1.81069	1.60445	1.42328
14	2.82219	2.46832	2.16157	1.89531	1.66387	1.46245
15	3.03928	2.63288	2.28393	1.98387	1.72550	1.50270
16	3.27307	2.80840	2.41321	2.07658	1.78940	1.54406
17	3.52484	2.99563	2.54980	2.17361	1.85568	1.58656
18	3.79598	3.19534	2.69413	2.27518	1.92441	1.63022
19	4.08798	3.40836	2.84663	2.38150	1.99568	1.67509
20	4.40244	3.63559	3.00776	2.49279	2.06959	1.72120
21	4.74109	3.87796	3.17801	2.60927	2.14625	1.76857
22	5.10579	4.13649	3.35790	2.73120	2.22574	1.81725
23	5.49854	4.41225	3.54797	2.85883	2.30817	1.86726
24	5.92151	4.70640	3.74880	2.99242	2.39366	1.91865
25	6.37701	5.02017	3.96099	3.13225	2.48231	1.97146
26	6.86755	5.35484	4.18520	3.27861	2.57425	2.02572
27	7.39582	5.71183	4.42210	3.43182	2.66959	2.08148
28	7.96473	6.09262	4.67241	3.59219	2.76847	2.13876
29	8.57740	6.49880	4.93688	3.76005	2.87100	2.19763
30	9.23720	6.93205	5.21633	3.93575	2.97734	2.25811
31	9.94776	7.39419	5.51159	4.11966	3.08761	2.32026
32	10.71297	7.88713	5.82357	4.31217	3.20196	2.38412
33	11.53704	8.41294	6.15320	4.51367	3.32056	2.44974
34	12.42451	8.97380	6.50150	4.72459	3.44354	2.51717
35	13.38024	9.57206	6.86951	4.94537	3.57108	2.58645
36	14.40949	10.21019	7.25835	5.17646	3.70334	2.65763
37	15.51791	10.89087	7.66920	5.41835	3.84050	2.73078
38	16.71160	11.61693	8.10330	5.67154	3.98274	2.80594
39	17.99710	12.39139	8.56198	5.93657	4.13025	2.88317
40	19.38149	13.21748	9.04662	6.21398	4.28322	2.96252
41	20.87238	14.09865	9.55869	6.50435	4.44186	3.04406
42	22.47795	15.03856	10.09975	6.80829	4.60637	3.12784
43	24.20702	16.04113	10.67144	7.12644	4.77698	3.21392
44	26.06910	17.11054	11.27548	7.45945	4.95390	3.30238
45	28.07441	18.25124	11.91372	7.80802	5.13736	3.39327

TABLE : FVSPF
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INFLATION RATE: 12 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	1.07177	1.06161	1.05164	1.04186	1.03276	1.02283
2	1.14869	1.12702	1.10595	1.08547	1.06556	1.04618
3	1.23113	1.19646	1.16307	1.13091	1.09993	1.07007
4	1.31949	1.27017	1.22313	1.17825	1.13541	1.09450
5	1.41419	1.34843	1.28630	1.22757	1.17204	1.11949
6	1.51569	1.43151	1.35273	1.27896	1.20984	1.14505
7	1.62447	1.51970	1.42259	1.33250	1.24887	1.17119
8	1.74106	1.61333	1.49605	1.38828	1.28916	1.19793
9	1.86602	1.71273	1.57332	1.44639	1.33074	1.22528
10	1.99994	1.81826	1.65457	1.50694	1.37367	1.25325
11	2.14348	1.93028	1.74001	1.57002	1.41798	1.28187
12	2.29732	2.04921	1.82987	1.63574	1.46372	1.31113
13	2.46220	2.17547	1.92437	1.70421	1.51094	1.34107
14	2.63891	2.30950	2.02375	1.77555	1.55968	1.37169
15	2.82830	2.45179	2.12827	1.84988	1.60999	1.40300
16	3.03129	2.60285	2.23818	1.92732	1.66193	1.43503
17	3.24885	2.76321	2.35376	2.00799	1.71554	1.46780
18	3.48202	2.93346	2.47532	2.09205	1.77088	1.50131
19	3.73193	3.11419	2.60315	2.17962	1.82800	1.53559
20	3.99977	3.30606	2.73759	2.27086	1.88697	1.57064
21	4.28683	3.50976	2.87897	2.36592	1.94784	1.60650
22	4.59450	3.72600	3.02765	2.46496	2.01068	1.64318
23	4.92425	3.95556	3.18400	2.56815	2.07554	1.68070
24	5.27766	4.19927	3.34844	2.67565	2.14249	1.71907
25	5.65644	4.45799	3.52136	2.78765	2.21160	1.75832
26	6.06241	4.73265	3.70321	2.90435	2.28294	1.79846
27	6.49751	5.02424	3.89446	3.02592	2.35659	1.83952
28	6.96384	5.33379	4.09558	3.15259	2.43261	1.88152
29	7.46363	5.66241	4.30709	3.28456	2.51108	1.92448
30	7.99930	6.01128	4.52952	3.42205	2.59208	1.96842
31	8.57341	6.38164	4.76344	3.56530	2.67570	2.01336
32	9.18873	6.77482	5.00944	3.71455	2.76201	2.05932
33	9.84821	7.19223	5.26814	3.87004	2.85111	2.10634
34	10.55502	7.63535	5.54021	4.03204	2.94308	2.15443
35	11.31255	8.10578	5.82632	4.20082	3.03801	2.20362
36	12.12446	8.60519	6.12721	4.37667	3.13602	2.25393
37	12.99464	9.13536	6.44364	4.55988	3.23718	2.30539
38	13.92727	9.69821	6.77641	4.75076	3.34160	2.35802
39	14.92683	10.29573	7.12637	4.94963	3.44940	2.41186
40	15.99814	10.93006	7.49439	5.15682	3.56067	2.46693
41	17.14633	11.60348	7.88143	5.37269	3.67553	2.52325
42	18.37692	12.31838	8.28845	5.59759	3.79409	2.58086
43	19.69584	13.07733	8.71649	5.83191	3.91648	2.63978
44	21.10942	13.88305	9.16664	6.07604	4.04282	2.70005
45	22.62445	14.73840	9.64004	6.33039	4.17323	2.76169

T A B L E   P V D I R

(Present Value Depreciation Including Residual)

This table performs the following functions:

1. Calculates the annual depreciation on the declining balance method at the stipulated depreciation rate

2. Calculates the value of the non depreciated amount at the end of the investment period

3. Converts all values at the end of each year to present value at the stipulated interest rate

4. Provides the total present value of the depreciation allowance

TABLE : PVDIR
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DEPRECIATION RATE: 2.5 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.96154	.95238	.94339	.93457	.92593	.91744
2	.92549	.90816	.89132	.87496	.85906	.84358
3	.89168	.86711	.84343	.82064	.79869	.77752
4	.86000	.82898	.79938	.77115	.74419	.71843
5	.83029	.79357	.75886	.72605	.69499	.66556
6	.80243	.76070	.72159	.68495	.65057	.61827
7	.77632	.73017	.68731	.64750	.61047	.57598
8	.75184	.70182	.65578	.61338	.57426	.53815
9	.72886	.67550	.62677	.58229	.54158	.50431
10	.70737	.65106	.60010	.55396	.51207	.47404
11	.68720	.62837	.57556	.52814	.48543	.44696
12	.66829	.60730	.55298	.50461	.46138	.42274
13	.65056	.58772	.53222	.48318	.43967	.40108
14	.63394	.56956	.51313	.46365	.42006	.38170
15	.61836	.55269	.49557	.44585	.40237	.36437
16	.60375	.53701	.47941	.42963	.38640	.34887
17	.59006	.52246	.46455	.41485	.37198	.33499
18	.57722	.50895	.45086	.40139	.35897	.32259
19	.56518	.49640	.43831	.38911	.34721	.31149
20	.55389	.48475	.42675	.37793	.33661	.30157
21	.54331	.47393	.41611	.36774	.32702	.29269
22	.53339	.46389	.40633	.35846	.31838	.28475
23	.52409	.45456	.39733	.35000	.31057	.27765
24	.51538	.44590	.38905	.34228	.30352	.27130
25	.50721	.43785	.38144	.33526	.29716	.26561
26	.49955	.43038	.37444	.32886	.29141	.26052
27	.49237	.42345	.36799	.32303	.28623	.25597
28	.48563	.41701	.36207	.31772	.28154	.25190
29	.47932	.41104	.35662	.31287	.27731	.24826
30	.47341	.40549	.35160	.30846	.27350	.24500

TABLE : PVDIR
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DEPRECIATION RATE: 2.5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95693	.94787	.93896	.93024	.92166	.91324
2	.91676	.89969	.88309	.86696	.85126	.83599
3	.87927	.85517	.83193	.80957	.78801	.76721
4	.84430	.81401	.78510	.75751	.73116	.70596
5	.81167	.77599	.74222	.71030	.68007	.65143
6	.78121	.74085	.70298	.66748	.63416	.60287
7	.75291	.70836	.66705	.62865	.59291	.55964
8	.72630	.67834	.63415	.59343	.55584	.52114
9	.70157	.65060	.60403	.56148	.52253	.48686
10	.67850	.62496	.57645	.53251	.49259	.45633
11	.65697	.60126	.55121	.50623	.46569	.42915
12	.63689	.57936	.52811	.48239	.44152	.40495
13	.61815	.55913	.50695	.46078	.41980	.36340
14	.60066	.54043	.48759	.44118	.40028	.36422
15	.58435	.52315	.46986	.42340	.38274	.34714
16	.56913	.50717	.45362	.40727	.36698	.33193
17	.55493	.49241	.43876	.39265	.35282	.31838
18	.54168	.47877	.42516	.37938	.34009	.30631
19	.52932	.46616	.41270	.36734	.32865	.29558
20	.51779	.45451	.40130	.35643	.31837	.28602
21	.50703	.44374	.39097	.34653	.30913	.27751
22	.49699	.43378	.38131	.33755	.30083	.26997
23	.48762	.42458	.37257	.32941	.29337	.26318
24	.47896	.41608	.36456	.32202	.28667	.25716
25	.47073	.40822	.35723	.31532	.28064	.25181
26	.46312	.40096	.35051	.30925	.27523	.24704
27	.45602	.39425	.34436	.30374	.27037	.24281
28	.44940	.38806	.33873	.29875	.26600	.23902
29	.44321	.38232	.33358	.29421	.26208	.23566
30	.43745	.37703	.32886	.29010	.25855	.23267

TABLE : PVDIR
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DEPRECIATION RATE: 5 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.96154	.95238	.94340	.93458	.92593	.91743
2	.92641	.90929	.89266	.87650	.86077	.84547
3	.89432	.87031	.84720	.82493	.80345	.78274
4	.86500	.83505	.80646	.77914	.75304	.72808
5	.83822	.80314	.76993	.73850	.70869	.68043
6	.81376	.77427	.73720	.70240	.66968	.63891
7	.79142	.74815	.70796	.67036	.63537	.60272
8	.77100	.72451	.68158	.64191	.60519	.57118
9	.75236	.70312	.65801	.61664	.57863	.54368
10	.73534	.68378	.63690	.59422	.55528	.51972
11	.71978	.66628	.61798	.57430	.53474	.49884
12	.70557	.65044	.60101	.55663	.51666	.48064
13	.69259	.63612	.58581	.54093	.50076	.46477
14	.68073	.62315	.57219	.52699	.48678	.45095
15	.66990	.61142	.55998	.51462	.47449	.43989
16	.66001	.60081	.54903	.50364	.46367	.42838
17	.65097	.59120	.53923	.49389	.45415	.41924
18	.64272	.58252	.53043	.48523	.44578	.41126
19	.63518	.57466	.52255	.47754	.43842	.40430
20	.62829	.56755	.51549	.47072	.43194	.39824
21	.62200	.56111	.50916	.46466	.42624	.39296
22	.61626	.55529	.50349	.45928	.42123	.38836
23	.61100	.55003	.49842	.45450	.41683	.38435
24	.60621	.54527	.49387	.45025	.41295	.38085
25	.60183	.54095	.48978	.44649	.40953	.37780
26	.59783	.53705	.48612	.44315	.40654	.37515
27	.59417	.53352	.48284	.44018	.40390	.37284
28	.59083	.53033	.47991	.43754	.40158	.37082
29	.58778	.52744	.47727	.43520	.39954	.36906
30	.58499	.52482	.47491	.43312	.39774	.36753

TABLE : PVDIR
****

DEPRECIATION RATE: 5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95694	.94786	.93897	.93023	.92166	.91324
2	.91780	.90092	.88453	.86857	.85306	.83797
3	.88220	.85865	.83597	.81409	.79301	.77267
4	.84985	.82058	.79264	.76593	.74042	.71602
5	.82044	.78631	.75400	.72338	.69437	.66687
6	.79370	.75544	.71952	.68578	.65405	.62422
7	.76940	.72765	.68677	.65255	.61875	.58722
8	.74730	.70262	.66135	.62318	.58794	.55512
9	.72721	.68009	.63688	.59723	.56078	.52727
10	.70894	.65980	.61506	.57429	.53709	.50312
11	.69234	.64152	.59559	.55402	.51634	.48216
12	.67725	.62507	.57823	.53611	.49817	.46397
13	.66353	.61025	.56274	.52028	.48227	.44819
14	.65106	.59691	.54893	.50630	.46835	.43450
15	.63972	.58489	.53660	.49394	.45615	.42262
16	.62941	.57408	.52561	.48301	.44548	.41232
17	.62003	.56434	.51580	.47336	.43613	.40337
18	.61152	.55556	.50705	.46463	.42794	.39562
19	.60378	.54766	.49924	.45730	.47078	.38889
20	.59673	.54055	.49228	.45063	.41451	.38305
21	.59033	.53415	.48607	.44475	.40901	.37799
22	.58452	.52837	.48053	.43955	.40420	.37359
23	.57923	.52318	.47559	.43495	.39999	.36979
24	.57442	.51850	.47118	.43089	.39630	.36648
25	.57005	.51429	.46725	.42730	.39308	.36361
26	.56606	.51050	.46374	.42413	.39025	.36112
27	.56246	.50708	.46062	.42132	.38778	.35897
28	.55917	.50401	.45783	.41884	.38560	.35710
29	.55619	.50125	.45534	.41665	.38371	.35547
30	.55348	.49876	.45312	.41472	.38205	.35406

TABLE : PVDIR
****

DEPRECIATION RATE: 7.5 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.96154	.95238	.94339	.93458	.92592	.91743
2	.92733	.91044	.89399	.87802	.86248	.84736
3	.89691	.87348	.85089	.82912	.80814	.78790
4	.86985	.84092	.81328	.78685	.76160	.73743
5	.84578	.81223	.78045	.75032	.72174	.69462
6	.82437	.78697	.75181	.71873	.68761	.65827
7	.80533	.76471	.72681	.69142	.65836	.62743
8	.78839	.74510	.70500	.66781	.63332	.60126
9	.77333	.72732	.68596	.64740	.61188	.57905
10	.75993	.71261	.66935	.62976	.59350	.56021
11	.74801	.69920	.65486	.61451	.57777	.54422
12	.73742	.68740	.64221	.60133	.56429	.53064
13	.72799	.67700	.63117	.58993	.55274	.51913
14	.71960	.66783	.62154	.58008	.54286	.50935
15	.71215	.65976	.61314	.57157	.53440	.50106
16	.70552	.65264	.60581	.56420	.52715	.49402
17	.69962	.64638	.59941	.55784	.52093	.48805
18	.69438	.64086	.59382	.55234	.51562	.48297
19	.68971	.63599	.58894	.54758	.51106	.47868
20	.68556	.63171	.58469	.54347	.50716	.47502
21	.68186	.62793	.58098	.53992	.50381	.47192
22	.67859	.62461	.57774	.53684	.50095	.46929
23	.67567	.62168	.57491	.53419	.49850	.46706
24	.67307	.61910	.57244	.53189	.49640	.46517
25	.67076	.61682	.57029	.52991	.49460	.46357
26	.66870	.61482	.56842	.52819	.49306	.46221
27	.66688	.61306	.56678	.52671	.49174	.46104
28	.66526	.61150	.56535	.52542	.49061	.46006
29	.66381	.61013	.56410	.52431	.48965	.45923
30	.66252	.60892	.56301	.52336	.48882	.45852

TABLE : FVDIR
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DEPRECIATION RATE: 7.5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95694	.94787	.93896	.93024	.92165	.91324
2	.91882	.90216	.88596	.87020	.85487	.83995
3	.88508	.86208	.83991	.81855	.79793	.77804
4	.85522	.82695	.79992	.77410	.74938	.72575
5	.82878	.79614	.76520	.73586	.70800	.68157
6	.80538	.76912	.73503	.70295	.67272	.64424
7	.78466	.74544	.70883	.67464	.64264	.61272
8	.76633	.72468	.68607	.65028	.61700	.58609
9	.75010	.70648	.66631	.62931	.59514	.56359
10	.73573	.69052	.64915	.61127	.57650	.54458
11	.72301	.67653	.63423	.59574	.56061	.52853
12	.71176	.66425	.62128	.58239	.54707	.51497
13	.70180	.65349	.61004	.57089	.53552	.50351
14	.69298	.64405	.60026	.56100	.52567	.49383
15	.68518	.63578	.59178	.55249	.51728	.48565
16	.67827	.62853	.58441	.54517	.51012	.47874
17	.67215	.62217	.57801	.53887	.50402	.47291
18	.66673	.61660	.57244	.53345	.49882	.46798
19	.66194	.61171	.56761	.52879	.49438	.46382
20	.65770	.60743	.56342	.52477	.49061	.46031
21	.65395	.60366	.55978	.52131	.48738	.45735
22	.65062	.60037	.55661	.51833	.48463	.45463
23	.64768	.59748	.55396	.51578	.48229	.45271
24	.64507	.59494	.55147	.51358	.48029	.45092
25	.64276	.59272	.54940	.51168	.47859	.44940
26	.64072	.59077	.54760	.51005	.47713	.44812
27	.63892	.58907	.54603	.50865	.47590	.44704
28	.63731	.58757	.54468	.50745	.47484	.44613
29	.63590	.58626	.54350	.50641	.47394	.44536
30	.63465	.58511	.54247	.50552	.47317	.44471

TABLE : PVCIR
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DEPRECIATION RATE: 10 %
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INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.96153	.95238	.94340	.93458	.92592	.91743
2	.92825	.91156	.89534	.87956	.86419	.84925
3	.89945	.87658	.85453	.83327	.81275	.79296
4	.87453	.84660	.81988	.79435	.76986	.74648
5	.85296	.82089	.79047	.76160	.73416	.70810
6	.83430	.79886	.76549	.73406	.70439	.67641
7	.81815	.77998	.74428	.71090	.67958	.65024
8	.80417	.76379	.72626	.69142	.65891	.62864
9	.79207	.74991	.71099	.67502	.64169	.61080
10	.78159	.73802	.69801	.66123	.62733	.59608
11	.77254	.72783	.68699	.64964	.61536	.58391
12	.76469	.71909	.67764	.63988	.60539	.57387
13	.75791	.71160	.66969	.63168	.59707	.56559
14	.75204	.70518	.66294	.62478	.59015	.55874
15	.74695	.69968	.65722	.61897	.58438	.55308
16	.74255	.69496	.65235	.61409	.57957	.54842
17	.73875	.69091	.64822	.60999	.57556	.54457
18	.73545	.68745	.64471	.60653	.57222	.54139
19	.73260	.68448	.64174	.60362	.56944	.53876
20	.73014	.68193	.63921	.60118	.56712	.53659
21	.72801	.67975	.63707	.59913	.56520	.53480
22	.72616	.67787	.63525	.59740	.56358	.53332
23	.72457	.67628	.63370	.59595	.56224	.53210
24	.72319	.67490	.63239	.59473	.56113	.53109
25	.72199	.67373	.63128	.59370	.56019	.53027
26	.72095	.67272	.63033	.59284	.55942	.52957
27	.72006	.67186	.62953	.59211	.55877	.52901
28	.71928	.67111	.62885	.59149	.55823	.52854
29	.71861	.67047	.62827	.59098	.55778	.52815
30	.71803	.66993	.62778	.59055	.55740	.52784

TABLE : PVDIR
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DEPRECIATION RATE: 10 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95693	.94787	.93897	.93023	.92166	.91324
2	.91985	.90340	.88739	.87192	.85668	.84193
3	.88791	.86546	.84381	.82292	.80278	.78332
4	.66040	.33309	.60698	.78198	.75806	.73515
5	.83671	.80548	.77586	.74770	.72097	.69556
6	.81630	.78194	.74955	.71900	.69020	.66302
7	.79873	.76184	.72733	.69498	.66468	.63628
8	.78359	.74470	.70854	.67486	.64351	.61429
9	.77056	.73008	.69266	.65802	.67596	.59622
10	.75933	.71761	.67925	.64393	.61140	.58137
11	.74967	.70697	.66791	.63213	.59931	.56916
12	.74134	.69789	.65833	.62225	.58929	.55913
13	.73417	.69015	.65024	.61398	.58098	.55088
14	.72800	.66354	.64340	.60704	.57408	.54410
15	.72268	.67791	.63763	.60124	.56836	.53852
16	.71810	.67310	.63274	.59639	.56361	.53395
17	.71415	.66900	.62861	.59232	.55968	.53018
18	.71075	.66550	.62512	.58892	.55641	.52709
19	.70782	.66251	.62218	.58608	.55371	.52455
20	.70530	.65997	.61968	.58369	.55146	.52247
21	.70313	.65780	.61758	.56169	.54960	.52075
22	.70125	.65594	.61580	.58002	.54805	.51934
23	.69964	.65436	.61429	.57863	.54677	.51818
24	.69826	.65301	.61303	.57745	.54571	.51722
25	.69706	.65186	.61195	.57647	.54483	.51645
26	.69603	.65057	.61105	.57566	.54410	.51580
27	.69515	.65003	.61028	.57497	.54349	.51528
28	.69439	.64932	.60963	.57440	.54298	.51484
29	.69373	.64871	.60908	.57391	.54256	.51449
30	.69317	.64820	.60862	.57351	.54222	.51420

TABLE : PVDIR

DEPRECIATION RATE: 12.5 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.96154	.95238	.94339	.93459	.92593	.91743
2	.92917	.91270	.89666	.88108	.86591	.85115
3	.90195	.87963	.85809	.83733	.81729	.79794
4	.87904	.85207	.82625	.80156	.77789	.75523
5	.85976	.82911	.79997	.77230	.74598	.72094
6	.84355	.80997	.77827	.74837	.72012	.69341
7	.82991	.79402	.76037	.72881	.69916	.67132
8	.81843	.78073	.74558	.71281	.68219	.65359
9	.80878	.76966	.73338	.69973	.66844	.63935
10	.80065	.76043	.72331	.68902	.65730	.62793
11	.79382	.75274	.71499	.68027	.64828	.61875
12	.78806	.74633	.70813	.67312	.64097	.61138
13	.78323	.74098	.70246	.66727	.63504	.60547
14	.77913	.73654	.69778	.66249	.63024	.60072
15	.77572	.73263	.69391	.65858	.62636	.59690
16	.77285	.72974	.69073	.65537	.62320	.59385
17	.77043	.72716	.68810	.65275	.62065	.59139
18	.76838	.72502	.68592	.65061	.61858	.58942
19	.76666	.72323	.68412	.64986	.61691	.58784
20	.76572	.72174	.68264	.64742	.61555	.58657
21	.76400	.72051	.68141	.64626	.61445	.58555
22	.76298	.71947	.68040	.64530	.61356	.58474
23	.76212	.71861	.67957	.64452	.61284	.58408
24	.76140	.71790	.67888	.64388	.61225	.58355
25	.76079	.71730	.67831	.64335	.61177	.58313
26	.76027	.71681	.67785	.64292	.61139	.58278
27	.75985	.71639	.67747	.64256	.61108	.58251
28	.75945	.71605	.67714	.64228	.61083	.58229
29	.75917	.71576	.67688	.64204	.61062	.58211
30	.75891	.71551	.67666	.64185	.61046	.58197

TABLE : PVDIF
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DEPRECIATION RATE: 12.5 %
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INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95694	.94786	.93897	.93023	.92166	.91325
2	.92089	.90463	.88882	.87345	.85848	.84392
3	.89069	.86876	.84762	.82723	.80754	.78852
4	.86541	.83903	.81377	.78960	.76644	.74425
5	.84424	.81435	.78596	.75898	.73331	.70887
6	.82652	.79390	.76311	.73405	.70659	.68060
7	.81167	.77692	.74434	.71376	.68503	.65802
8	.79925	.76286	.72892	.69724	.66766	.63997
9	.78864	.75119	.71625	.68380	.65364	.62555
10	.78013	.74150	.70584	.67285	.64233	.61401
11	.77283	.73348	.69728	.66395	.63321	.60481
12	.76673	.72681	.69025	.65670	.62586	.59744
13	.76162	.72129	.68447	.65079	.61994	.59157
14	.75734	.71670	.67973	.64599	.61515	.58686
15	.75375	.71290	.67584	.64207	.61130	.58311
16	.75075	.70975	.67264	.63889	.60819	.58011
17	.74823	.70714	.67001	.63630	.60568	.57770
18	.74613	.70497	.66785	.63419	.60366	.57579
19	.74437	.70318	.66608	.63248	.60203	.57425
20	.74290	.70169	.66462	.63108	.60071	.57303
21	.74166	.70045	.66343	.62994	.59965	.57205
22	.74062	.69943	.66244	.62901	.59879	.57128
23	.73976	.69857	.66163	.62827	.59810	.57065
24	.73904	.69786	.66097	.62765	.59755	.57015
25	.73843	.69728	.66042	.62715	.59710	.56975
26	.73792	.69679	.65997	.62675	.59673	.56943
27	.73749	.69638	.65960	.62642	.59644	.56917
28	.73713	.69605	.65930	.62615	.59621	.56897
29	.73684	.69577	.65905	.62592	.59602	.56881
30	.73658	.69554	.65884	.62575	.59588	.56868

TABLE : PVDIR

DEPRECIATION RATE: 15 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.95694	.94787	.93897	.93023	.92166	.91325
2	.92191	.90586	.89026	.87506	.86029	.84590
3	.89343	.87202	.85138	.83145	.81221	.79362
4	.87025	.84475	.82036	.79696	.77455	.75304
5	.85140	.82278	.79559	.76969	.74503	.72154
6	.83607	.80509	.77582	.74813	.72192	.69709
7	.82360	.79083	.76004	.73108	.70381	.67811
8	.81345	.77933	.74746	.71760	.68963	.66338
9	.80520	.77007	.73741	.70694	.67851	.65194
10	.79846	.76262	.72939	.69851	.66980	.64306
11	.79303	.75661	.72299	.69185	.66299	.63617
12	.78858	.75177	.71788	.68658	.65764	.63082
13	.78497	.74787	.71390	.68241	.65346	.62667
14	.78203	.74473	.71055	.67912	.65018	.62344
15	.77965	.74220	.70795	.67651	.64760	.62094
16	.77770	.74016	.70587	.67445	.64559	.61900
17	.77612	.73851	.70422	.67283	.64401	.61749
18	.77484	.73718	.70290	.67154	.64277	.61632
19	.77380	.73612	.70184	.67053	.64181	.61541
20	.77295	.73525	.70100	.66972	.64105	.61470
21	.77226	.73456	.70033	.66908	.64046	.61415
22	.77170	.73400	.69980	.66858	.63999	.61372
23	.77125	.73356	.69937	.66819	.63963	.61339
24	.77087	.73320	.69903	.66788	.63934	.61313
25	.77057	.73291	.69876	.66763	.63911	.61293
26	.77032	.73267	.69854	.66743	.63893	.61277
27	.77013	.73249	.69837	.66727	.63879	.61265
28	.76996	.73234	.69823	.66715	.63869	.61256
29	.76982	.73222	.69813	.66705	.63860	.61249
30	.76972	.73212	.69803	.66697	.63854	.61243

TABLE : PVDIR

DEPRECIATION RATE: 15 %

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.95694	.94787	.93897	.93023	.92166	.91325
2	.92191	.90586	.89026	.87506	.86029	.84590
3	.89343	.87202	.85138	.83145	.81221	.79362
4	.87025	.84475	.82036	.79696	.77455	.75304
5	.85140	.82278	.79559	.76969	.74503	.72154
6	.83607	.80509	.77582	.74813	.72192	.69709
7	.82360	.79083	.76004	.73108	.70381	.67811
8	.81345	.77933	.74746	.71760	.68963	.66338
9	.80520	.77007	.73741	.70694	.67851	.65194
10	.79848	.76262	.72939	.69851	.66980	.64306
11	.79303	.75661	.72299	.69185	.66299	.63617
12	.78858	.75177	.71788	.68658	.65764	.63082
13	.78497	.74787	.71380	.68241	.65346	.62667
14	.78203	.74473	.71055	.67912	.65018	.62344
15	.77965	.74220	.70795	.67651	.64760	.62094
16	.77770	.74016	.70587	.67445	.64559	.61900
17	.77612	.73851	.70422	.67283	.64401	.61749
18	.77484	.73718	.70290	.67154	.64277	.61632
19	.77380	.73612	.70184	.67053	.64181	.61541
20	.77295	.73525	.70100	.66972	.64105	.61470
21	.77226	.73456	.70033	.66908	.64046	.61415
22	.77170	.73400	.69980	.66858	.63999	.61372
23	.77125	.73356	.69937	.66819	.63963	.61339
24	.77087	.73320	.69903	.66788	.63934	.61313
25	.77057	.73291	.69876	.66763	.63911	.61293
26	.77032	.73267	.69854	.66743	.63893	.61277
27	.77013	.73249	.69837	.66727	.63879	.61265
28	.76996	.73234	.69823	.66715	.63869	.61256
29	.76982	.73222	.69813	.66705	.63860	.61249
30	.76972	.73212	.69803	.66697	.63854	.61243

T A B L E P V D E R

(Present Value Depreciation Excluding Residual)

This table performs the following functions:

1. Calculates the annual depreciation on the declining balance method at the stipulated depreciation rate

2. Excludes the value of the non depreciated amount at the end of the investment period

3. Converts all values at the end of each year to present value at the stipulated interest rate

4. Provides the total present value of the depreciation allowance excluding residual

TABLE : PVDFR

DEPRECIATION RATE: 2.5 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.02404	.02381	.02358	.02336	.02315	.02294
2	.04658	.04592	.04527	.04465	.04405	.04346
3	.06771	.06645	.06522	.06405	.06292	.06161
4	.08752	.08551	.08357	.08173	.07995	.07823
5	.10609	.10321	.10045	.09784	.09533	.09291
6	.12350	.11965	.11598	.11252	.10921	.10604
7	.13982	.13491	.13026	.12589	.12174	.11779
8	.15512	.14908	.14340	.13808	.13305	.12830
9	.16946	.16224	.15548	.14919	.14326	.13770
10	.18291	.17446	.16660	.15931	.15248	.14611
11	.19552	.18581	.17682	.16853	.16080	.15363
12	.20734	.19635	.18622	.17693	.16831	.16036
13	.21842	.20613	.19487	.18459	.17509	.16639
14	.22881	.21522	.20283	.19157	.18121	.17176
15	.23855	.22366	.21015	.19793	.18674	.17658
16	.24768	.23149	.21688	.20372	.19173	.18089
17	.25624	.23876	.22307	.20900	.19624	.18474
18	.26426	.24551	.22877	.21381	.20031	.18819
19	.27178	.25178	.23401	.21819	.20398	.19127
20	.27883	.25760	.23883	.22218	.20730	.19403
21	.28544	.26301	.24326	.22582	.21029	.19650
22	.29164	.26803	.24734	.22914	.21299	.19871
23	.29745	.27269	.25109	.23216	.21543	.20068
24	.30290	.27702	.25454	.23491	.21763	.20245
25	.30801	.28104	.25771	.23742	.21962	.20403
26	.31280	.28477	.26063	.23971	.22141	.20544
27	.31729	.28824	.26331	.24179	.22303	.20670
28	.32150	.29146	.26578	.24369	.22449	.20783
29	.32545	.29445	.26805	.24542	.22581	.20884
30	.32915	.29723	.27014	.24700	.22700	.20974

TABLE : PVDER
****

DEPRECIATION RATE: 2.5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.02392	.02370	.02347	.02326	.02304	.02283
2	.04624	.04560	.04496	.04435	.04375	.04316
3	.06707	.06584	.06463	.06348	.06236	.06126
4	.08650	.08454	.08264	.08083	.07908	.07738
5	.10463	.10183	.09913	.09657	.09410	.09173
6	.12154	.11781	.11423	.11084	.10760	.10451
7	.13732	.13257	.12805	.12379	.11973	.11589
8	.15204	.14621	.14070	.13553	.13063	.12602
9	.16578	.15862	.15228	.14618	.14043	.13504
10	.17860	.17047	.16288	.15584	.14923	.14307
11	.19056	.18124	.17259	.16460	.15714	.15022
12	.20172	.19119	.18148	.17254	.16425	.15659
13	.21213	.20039	.18962	.17975	.17064	.16226
14	.22184	.20889	.19707	.18629	.17638	.16731
15	.23090	.21675	.20389	.19222	.18154	.17181
16	.23936	.22401	.21013	.19760	.18618	.17581
17	.24725	.23072	.21585	.20248	.19035	.17937
18	.25461	.23692	.22108	.20690	.19409	.18254
19	.26148	.24265	.22587	.21091	.19745	.18537
20	.26789	.24795	.23026	.21455	.20047	.18789
21	.27387	.25284	.23428	.21785	.20319	.19013
22	.27945	.25736	.23796	.22084	.20563	.19212
23	.28465	.26154	.24133	.22355	.20782	.19390
24	.28951	.26540	.24441	.22601	.20979	.19548
25	.29404	.26897	.24723	.22824	.21156	.19689
26	.29827	.27227	.24981	.23027	.21315	.19814
27	.30221	.27532	.25217	.23211	.21458	.19926
28	.30589	.27814	.25433	.23379	.21587	.20025
29	.30932	.28074	.25631	.23529	.21703	.20114
30	.31252	.28315	.25812	.23666	.21807	.20193

TABLE : PVDER

DEPRECIATION RATE: 5 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.04808	.04762	.04717	.04673	.04630	.04587
2	.09200	.09070	.08944	.08822	.08702	.08585
3	.13212	.12968	.12733	.12506	.12284	.12069
4	.16876	.16495	.16129	.15776	.15435	.15106
5	.20223	.19686	.19172	.18680	.18207	.17753
6	.23281	.22573	.21899	.21258	.20645	.22071
7	.26074	.25185	.24343	.23547	.22790	.23923
8	.28625	.27548	.26534	.25579	.24676	.25350
9	.30956	.29666	.28497	.27383	.26335	.26681
10	.33085	.31621	.30257	.28985	.27795	.27841
11	.35030	.33371	.31834	.30407	.29079	.28852
12	.36806	.34955	.33247	.31670	.30208	.29733
13	.38429	.36388	.34514	.32791	.31201	.30501
14	.39911	.37684	.35649	.33786	.32075	.31170
15	.41265	.38857	.36666	.34670	.32844	.31753
16	.42502	.39918	.37578	.35455	.33520	.32262
17	.43632	.40878	.38395	.36152	.34115	.32705
18	.44664	.41747	.39127	.36771	.34638	.33091
19	.45607	.42533	.39783	.37320	.35098	.33428
20	.46466	.43244	.40371	.37808	.35503	.33721
21	.47255	.43887	.40898	.38241	.35859	.33977
22	.47974	.44469	.41371	.38625	.36172	.34200
23	.48630	.44996	.41795	.38966	.36448	.34394
24	.49230	.45473	.42175	.39269	.36690	.34563
25	.49778	.45904	.42515	.39538	.36903	.34711
26	.50278	.46294	.42820	.39777	.37091	.34840
27	.50735	.46647	.43093	.39989	.37256	.34952
28	.51152	.46966	.43336	.40177	.37401	.35050
29	.51533	.47255	.43557	.40344	.37529	.35135
30	.51881	.47516	.43754	.40492	.37641	

TABLE : PVDER
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DEPRECIATION RATE: 5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.04795	.04739	.04695	.04651	.04608	.04566
2	.09135	.09007	.08883	.08761	.08643	.08528
3	.13089	.12850	.12619	.12393	.12176	.11965
4	.16694	.16310	.15951	.15603	.15269	.14947
5	.19952	.19426	.18923	.18440	.17977	.17534
6	.22923	.22232	.21574	.20947	.20348	.19778
7	.25624	.24759	.23939	.23162	.22424	.21725
8	.28079	.27034	.26049	.25120	.24242	.23414
9	.30311	.29083	.27931	.26850	.25834	.24880
10	.32340	.30928	.29610	.28379	.27228	.26152
11	.34185	.32569	.31107	.29730	.28448	.27255
12	.35662	.34085	.32443	.30924	.29516	.28212
13	.37387	.35432	.33635	.31979	.30452	.29042
14	.38773	.36645	.34698	.32912	.31271	.29762
15	.40033	.37737	.35646	.33736	.31988	.30387
16	.41178	.38721	.36492	.34464	.32616	.30929
17	.42219	.39607	.37246	.35108	.33166	.31399
18	.43166	.40404	.37919	.35677	.33647	.31807
19	.44027	.41122	.38519	.36180	.34069	.32161
20	.44509	.41769	.39054	.36624	.34438	.32468
21	.45570	.42351	.39532	.37017	.34761	.32735
22	.46167	.42875	.39958	.37364	.35044	.32966
23	.46755	.43347	.40336	.37671	.35292	.33167
24	.47289	.43772	.40677	.37942	.35509	.33341
25	.47775	.44155	.40979	.38181	.35699	.33492
26	.48217	.44500	.41249	.38393	.35865	.33623
27	.48618	.44810	.41490	.38580	.36011	.33737
28	.48983	.45090	.41705	.38745	.36138	.33836
29	.49315	.45342	.41896	.38891	.36250	.33922
30	.49617	.45569	.42067	.39020	.36348	.33996

TABLE : FVDER
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DEPRECIATION RATE: 7.5 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.07212	.07143	.07075	.07009	.06944	.06881
2	.13626	.13436	.13249	.13068	.12892	.12720
3	.19331	.18979	.18637	.18306	.17986	.17675
4	.24405	.23862	.23339	.22834	.22349	.21880
5	.28918	.28164	.27442	.26749	.26086	.25449
6	.32932	.31954	.31022	.30133	.29287	.28477
7	.36502	.35293	.34146	.33059	.32028	.31047
8	.39677	.38234	.36873	.35588	.34376	.33228
9	.42501	.40825	.39252	.37774	.36387	.35079
10	.45013	.43108	.41328	.39664	.38109	.36650
11	.47247	.45119	.43140	.41298	.39584	.37983
12	.49234	.46891	.44721	.42711	.40647	.39114
13	.51001	.48452	.46101	.43932	.41929	.40074
14	.52573	.49827	.47305	.44989	.42856	.40889
15	.53971	.51038	.48356	.45901	.43650	.41580
16	.55215	.52105	.49273	.46690	.44330	.42167
17	.56321	.53045	.50073	.47372	.44912	.42665
18	.57305	.53873	.50771	.47962	.45411	.43087
19	.58180	.54602	.51380	.48472	.45838	.43446
20	.59958	.55245	.51912	.48913	.46204	.43750
21	.59650	.55811	.52376	.49294	.46517	.44008
22	.60266	.56310	.52781	.49623	.46785	.44227
23	.60814	.56749	.53134	.49908	.47015	.44413
24	.61301	.57136	.53442	.50154	.47212	.44571
25	.61734	.57477	.53711	.50367	.47381	.44705
26	.62119	.57777	.53946	.50551	.47525	.44819
27	.62462	.58042	.54151	.50710	.47649	.44915
28	.62767	.58275	.54330	.50847	.47755	.44997
29	.63038	.58480	.54486	.50966	.47846	.45066
30	.63279	.58661	.54622	.51069	.47924	.45125

TABLE : PVDER
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DEPRECIATION RATE: 7.5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.07177	.07109	.07042	.06977	.06912	.06849
2	.13530	.13342	.13159	.12980	.12805	.12635
3	.19153	.18807	.18471	.18146	.17829	.17523
4	.24131	.23599	.23085	.22591	.22112	.21652
5	.28537	.27800	.27093	.26416	.25764	.25140
6	.32437	.31483	.30574	.29707	.28877	.28086
7	.35889	.34713	.33597	.32539	.31531	.30575
8	.38945	.37545	.36223	.34976	.33794	.32678
9	.41650	.40028	.38504	.37073	.35723	.34454
10	.44044	.42205	.40485	.38877	.37368	.35954
11	.46163	.44114	.42205	.40429	.38770	.37221
12	.48039	.45787	.43699	.41765	.39965	.38292
13	.49700	.47254	.44997	.42914	.40984	.39196
14	.51170	.48540	.46124	.43903	.41853	.39960
15	.52471	.49668	.47103	.44754	.42594	.40605
16	.53623	.50657	.47953	.45486	.43225	.41150
17	.54642	.51524	.48692	.46116	.43763	.41611
18	.55544	.52284	.49333	.46658	.44222	.42000
19	.56343	.52951	.49890	.47125	.44613	.42329
20	.57050	.53535	.50374	.47526	.44947	.42607
21	.57676	.54047	.50794	.47871	.45231	.42842
22	.58230	.54496	.51159	.48168	.45473	.43040
23	.58720	.54890	.51476	.48424	.45680	.43207
24	.59154	.55235	.51751	.48644	.45856	.43348
25	.59538	.55538	.51990	.48833	.46006	.43467
26	.59678	.55803	.52198	.48996	.46134	.43568
27	.60179	.56036	.52378	.49136	.46243	.43653
28	.60445	.56240	.52535	.49257	.46336	.43725
29	.60681	.56419	.52671	.49361	.46415	.43786
30	.60890	.56576	.52789	.49450	.46483	.43837

TABLE : PVDER

DEPRECIATION RATE: 10 %

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.09615	.09524	.09434	.09346	.09259	.09174
2	.17936	.17687	.17444	.17207	.16975	.16749
3	.25137	.24684	.24245	.23819	.23405	.23004
4	.31369	.30682	.30019	.29381	.28763	.28168
5	.36762	.35823	.34922	.34059	.33228	.32432
6	.41429	.40229	.39085	.37994	.36949	.35953
7	.45468	.44006	.42619	.41304	.40050	.38860
8	.49963	.47243	.45620	.44088	.42634	.41260
9	.51987	.50018	.48168	.46429	.44787	.43242
10	.54604	.52396	.50331	.48398	.46582	.44879
11	.56669	.54435	.52168	.50055	.48077	.46230
12	.58829	.56182	.53728	.51448	.49323	.47346
13	.60525	.57660	.55052	.52620	.50361	.48267
14	.61993	.58964	.56176	.53606	.51226	.49028
15	.63263	.60064	.57131	.54435	.51947	.49656
16	.64362	.61007	.57941	.55132	.52548	.50175
17	.65313	.61815	.58629	.55719	.53049	.50603
18	.66136	.62508	.59213	.56212	.53466	.50957
19	.66848	.63102	.59709	.56627	.53814	.51249
20	.67465	.63611	.60130	.56976	.54104	.51490
21	.67999	.64047	.60488	.57270	.54346	.51689
22	.68461	.64421	.60792	.57517	.54547	.51853
23	.68861	.64742	.61050	.57725	.54715	.51989
24	.69207	.65017	.61269	.57900	.54855	.52101
25	.69506	.65253	.61455	.58047	.54971	.52194
26	.69765	.65455	.61613	.58171	.55068	.52270
27	.69989	.65628	.61747	.58275	.55149	.52333
28	.70183	.65776	.61861	.58362	.55216	.52385
29	.70351	.65903	.61958	.58436	.55272	.52428
30	.70496	.66012	.62040	.58498	.55319	.52464

TABLE : PVDER
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DEPRECIATION RATE: 10 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.09569	.09479	.09390	.09302	.09217	.09132
2	.17811	.17565	.17325	.17090	.16862	.16638
3	.24909	.24463	.24031	.23610	.23204	.22807
4	.31022	.30348	.29698	.29069	.28464	.27878
5	.36287	.35368	.34487	.33639	.32827	.32046
6	.40821	.39651	.38534	.37465	.36446	.35472
7	.44726	.43304	.41954	.40668	.39448	.38288
8	.48089	.46421	.44844	.43350	.41938	.40602
9	.50986	.49080	.47286	.45595	.44004	.42504
10	.53481	.51348	.49350	.47475	.45718	.44067
11	.55630	.53283	.51094	.49049	.47139	.45352
12	.57480	.54934	.52568	.50367	.48318	.46408
13	.59074	.56342	.53814	.51470	.49296	.47276
14	.60447	.57543	.54867	.52393	.50107	.47989
15	.61629	.58568	.55757	.53166	.50780	.48575
16	.62647	.59442	.56509	.53813	.51338	.49057
17	.63524	.60188	.57144	.54355	.51801	.49453
18	.64279	.60824	.57681	.54809	.52185	.49779
19	.64929	.61367	.58135	.55189	.52504	.50047
20	.65489	.61830	.58518	.55507	.52768	.50267
21	.65971	.62225	.58842	.55773	.52987	.50449
22	.66386	.62562	.59116	.55996	.53169	.50597
23	.66744	.62849	.59347	.56183	.53320	.50719
24	.67052	.63094	.59543	.56339	.53445	.50819
25	.67317	.63303	.59708	.56470	.53549	.50902
26	.67546	.63481	.59848	.56580	.53635	.50970
27	.67743	.63633	.59966	.56672	.53706	.51026
28	.67913	.63763	.60066	.56749	.53765	.51072
29	.68059	.63874	.60150	.56813	.53814	.51110
30	.69185	.63969	.60221	.56867	.53855	.51141

TABLE : PVDER
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DEPRECIATION RATE: 12.5 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.12019	.11905	.11792	.11682	.11574	.11468
2	.22131	.21826	.21526	.21235	.20951	.20674
3	.30639	.30093	.29561	.29047	.28548	.28064
4	.37797	.36982	.36194	.35436	.34703	.33996
5	.43819	.42723	.41669	.40660	.39690	.38758
6	.49886	.47507	.46189	.44932	.43730	.42581
7	.53149	.51494	.49920	.48426	.47003	.45650
8	.56736	.54816	.53000	.51283	.49655	.48114
9	.59754	.57585	.55542	.53619	.51804	.50092
10	.62293	.59892	.57641	.55529	.53545	.51680
11	.64429	.61815	.59373	.57091	.54955	.52954
12	.66226	.63417	.60803	.58369	.56098	.53977
13	.67738	.64752	.61983	.59414	.57024	.54798
14	.69010	.65865	.62957	.60268	.57774	.55457
15	.70080	.66792	.63761	.60967	.58382	.55986
16	.70981	.67565	.64425	.61538	.58874	.56411
17	.71739	.68209	.64973	.62005	.59273	.56752
18	.72376	.68746	.65425	.62387	.59596	.57026
19	.72912	.69193	.65798	.62699	.59858	.57246
20	.73363	.69566	.66106	.62954	.60070	.57422
21	.73743	.69877	.66360	.63163	.60242	.57564
22	.74062	.70136	.66570	.63334	.60381	.57678
23	.74331	.70352	.66743	.63474	.60494	.57769
24	.74557	.70532	.66886	.63588	.60585	.57842
25	.74747	.70682	.67004	.63681	.60659	.57901
26	.74907	.70807	.67102	.63757	.60719	.57948
27	.75042	.70911	.67183	.63819	.60768	.57986
28	.75155	.70998	.67249	.63870	.60807	.58016
29	.75250	.71070	.67304	.63912	.60839	.58040
30	.75330	.71130	.67349	.63946	.60865	.58060

TABLE : PVCEP
****

DEPRECIATION RATE: 12.5 %
****

INTEREST RATE

YEAR	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.11962	.11848	.11737	.11628	.11521	.11416
2	.21978	.21675	.21380	.21093	.20812	.20538
3	.30364	.29825	.29303	.28797	.28305	.27827
4	.37386	.36565	.35812	.35067	.34347	.33652
5	.43266	.42191	.41160	.40171	.39220	.38306
6	.46129	.46841	.45554	.44325	.43150	.42025
7	.52311	.50697	.49164	.47706	.46319	.44997
8	.55763	.53896	.52130	.50458	.48675	.47372
9	.58653	.56549	.54567	.52698	.50936	.49270
10	.61073	.58749	.56569	.54521	.52598	.50786
11	.63099	.60574	.58214	.56005	.53938	.51998
12	.64796	.62087	.59565	.57213	.55019	.52966
13	.66217	.63342	.60675	.58196	.55891	.53740
14	.67407	.64363	.61587	.58996	.56594	.54358
15	.68403	.65246	.62337	.59647	.57161	.54852
16	.69237	.65962	.62953	.60177	.57618	.55247
17	.69935	.66556	.63459	.60609	.57987	.55567
18	.70520	.67049	.63875	.60960	.58284	.55814
19	.71010	.67458	.64217	.61246	.58574	.56015
20	.71420	.67797	.64498	.61479	.58717	.56176
21	.71763	.68078	.64729	.61668	.58873	.56305
22	.72050	.68311	.64918	.61822	.58999	.56408
23	.72291	.68504	.65074	.61948	.59100	.56490
24	.72493	.68664	.65202	.62050	.59182	.56556
25	.72662	.68797	.65307	.62133	.59248	.56608
26	.72803	.68907	.65393	.62201	.59301	.56650
27	.72921	.68998	.65464	.62256	.59344	.56683
28	.73020	.69074	.65522	.62301	.59379	.56710
29	.73103	.69137	.65570	.62337	.59407	.56731
30	.73172	.69189	.65609	.62367	.59430	.56748

TABLE : PVDER
****

DEPRECIATION RATE: 15 %
****

INTEREST RATE

YEAR	4%	5%	6%	7%	8%	9%
1	.14354	.14218	.14085	.13953	.13825	.13699
2	.26030	.25673	.25326	.24986	.24656	.24333
3	.35527	.34902	.34298	.33710	.33141	.32587
4	.43252	.42339	.41459	.40608	.39768	.38995
5	.49535	.48329	.47174	.46062	.44995	.43969
6	.54646	.53156	.51735	.50375	.49075	.47830
7	.58803	.57045	.55375	.53785	.52271	.50827
8	.62184	.60178	.58281	.56481	.54775	.53154
9	.64934	.62702	.60600	.58613	.56736	.54960
10	.67171	.64736	.62451	.60299	.58273	.56362
11	.68991	.66375	.63928	.61632	.59477	.57450
12	.70471	.67695	.65107	.62686	.60420	.58295
13	.71675	.68759	.66048	.63519	.61159	.58951
14	.72654	.69616	.66799	.64178	.61738	.59460
15	.73451	.70307	.67398	.64699	.62191	.59855
16	.74099	.70863	.67876	.65111	.62546	.60162
17	.74626	.71311	.68258	.65437	.62824	.60400
18	.75055	.71672	.68563	.65695	.63042	.60585
19	.75404	.71963	.68806	.65899	.63213	.60728
20	.75688	.72197	.69000	.66060	.63347	.60839
21	.75919	.72386	.69155	.66187	.63452	.60925
22	.76107	.72538	.69279	.66288	.63534	.60992
23	.76260	.72661	.69378	.66368	.63598	.61044
24	.76384	.72760	.69457	.66431	.63648	.61084
25	.76485	.72840	.69520	.66481	.63687	.61115
26	.76567	.72904	.69570	.66520	.63718	.61139
27	.76634	.72956	.69610	.66551	.63742	.61158
28	.76688	.72998	.69642	.66576	.63761	.61173
29	.76732	.73032	.69668	.66595	.63776	.61184
30	.76768	.73059	.69688	.66610	.63788	.61193

TABLE : PVDFR

DEPRECIATION RATE: 15 %

INTEREST RATE

YEAS	4.5%	5.5%	6.5%	7.5%	8.5%	9.5%
1	.14354	.14218	.14085	.13953	.13825	.13699
2	.26030	.25673	.25326	.24986	.24656	.24333
3	.35527	.34902	.34298	.33710	.33141	.32587
4	.43252	.42338	.41459	.40608	.39788	.38995
5	.49535	.48329	.47174	.46062	.44995	.43969
6	.54646	.53156	.51735	.50375	.49075	.47830
7	.58803	.57045	.55375	.53785	.52271	.50827
8	.62184	.60178	.58281	.56481	.54775	.53154
9	.64934	.62702	.60600	.58613	.56736	.54960
10	.67171	.64736	.62451	.60299	.58273	.56362
11	.68991	.66375	.63928	.61632	.59477	.57450
12	.70471	.67695	.65107	.62686	.60420	.58295
13	.71675	.68759	.66048	.63519	.61159	.58951
14	.72654	.69616	.66799	.64178	.61738	.59460
15	.73451	.70307	.67398	.64699	.62191	.59855
16	.74099	.70863	.67876	.65111	.62546	.60162
17	.74626	.71311	.68258	.65437	.62824	.60400
18	.75055	.71672	.68563	.65695	.63042	.60585
19	.75404	.71963	.68806	.65899	.63213	.60728
20	.75688	.72197	.69000	.66060	.63347	.60839
21	.75919	.72386	.69155	.66187	.63452	.60925
22	.76107	.72538	.69279	.66288	.63534	.60992
23	.76260	.72661	.69378	.66368	.63598	.61044
24	.76384	.72760	.69457	.66431	.63648	.61084
25	.76485	.72840	.69520	.66481	.63687	.61115
26	.76567	.72904	.69570	.66520	.63718	.61139
27	.76634	.72956	.69610	.66551	.63742	.61158
28	.76688	.72998	.69642	.66576	.63761	.61173
29	.76732	.73032	.69668	.66595	.63776	.61184
30	.76768	.73059	.69688	.66610	.63788	.61193