

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.

This thesis is dedicated to Erin

*Without your unwavering words of encouragement,
this dream would never have come true*

University of Cape Town



UNIVERSITY OF CAPE TOWN

UYUNIVESITHI YASEKAPA • UNIVERSITEIT VAN KAAPSTAD

BLAST IMPACT & SURVIVABILITY RESEARCH UNIT



Blast loading of sandwich panels with
thin-walled tube cores

Matthew David Theobald

Thesis Presented for the Degree of

DOCTOR OF PHILOSOPHY

in the Department of Mechanical Engineering

UNIVERSITY OF CAPE TOWN

February 2007

Declaration

I, Matthew Theobald, hereby:

- (a) Grant the University free license to reproduce this thesis, entitled "*Blast loading of sandwich panels with thin-walled tube cores*", in whole or in part, for the purpose of research.
- (b) Declare that:
 - (i) This thesis is my own unaided, both in conception and execution, and that apart from the normal guidance of my supervisor, I have received no assistance apart from that stated in the "Acknowledgements" section of the thesis.
 - (ii) This thesis or any part of this thesis has not been submitted in past, or is being, or is to be submitted in this or any other form for a degree at the University of Cape Town or any other University.
 - (iii) I am now presenting this thesis for examination for the degree of PhD in Mechanical Engineering.

Matthew Theobald
February 2007

Abstract

An investigation into the response of a novel sandwich panel under blast loading is presented. Thin-walled tubular members have been used extensively as energy absorbers under impact conditions. The progressive folding mechanism induced by axial crushing of the tubes allows for efficient dissipation of kinetic energy. The present study investigates the use of such tubes as a sandwich core material. Several thin-walled tubes are configured in parallel, between mild-steel face sheets. A programme of experimental, numerical, and analytical investigations are conducted to characterise the energy absorption properties of the sandwich structure under blast loading. Particular attention is given to the influence of geometric panel variables on the global response to uniform blast loading. It is anticipated that well designed panels will be able to efficiently dissipate large amounts of kinetic energy.

Three primary panel layouts are identified, consisting of panels with four, five and nine tubes within the core. The positioning of the tubes within the panel is defined by a single parameter. The remaining panel design variables are identified as the tube thickness, tube aspect ratio, and top plate thickness. To investigate the influence of material properties on the response of the panels, annealed mild steel and 6063-T6 aluminium alloy tubes are selected as the core material. A material characterisation study is conducted using quasi-static techniques as well as a Split-Hopkinson Pressure Bar (SHPB) setup to obtain material properties at high strain rates. The material properties obtained from the tests are input into numerical and analytical models of the panels.

A series of experimental blast tests are conducted on the proposed panels. Panels with four and five tubes of constant geometry in the core are constructed using the annealed mild steel and aluminium tubes. Hemispherical indentation triggers are created in the tube sidewalls at their midpoint. The panels are loaded using a square blast tube. A disc of PE4 plastic explosive is detonated at one end of the blast tube while the other end is exposed to the test panels. A ballistic pendulum is used to measure the impulse transferred to the panel as a result of the blast load. The panels are subjected to varied charge masses of explosive.

At smaller blast impulses, irregular buckling modes are observed for both the steel and

aluminium tube core panels primarily due to poor trigger performance. For blasts which utilise the full stroke of the tubes however, symmetric buckling modes are observed. In all cases panel crush distance increases with increasing impulse and decreases with an increasing number of tubes in the panel core. Further to this, at equal charge mass, the aluminium tube cores show significantly higher crush distance than identical steel tube panels.

Numerical analysis of the experiments, using the explicit-dynamic finite element code ABAQUS/Explicit, agrees reasonably well with global panel response. Energy absorption is primarily confined to the core tubes with only minimal plastic strains developing in the top plate. For both core materials, the mean crush force, and hence capacity of the panels is found to increase with increasing impulse.

A numerical parametric study is conducted to quantify the effect of geometric variables on the performance of the panels. Buckling stability of the panel cores is highly dependent on the positioning of the tubes within the panel, particularly when using a small number of tubes. Energy absorption within the core increases with decreasing tube thickness, provided core compaction does not occur. The aspect ratio of the tubes is observed to have little influence on panel performance, however stability can be improved in some cases with a small aspect ratio. Increased panel top plate thickness is found to minimise instabilities in the panel core and increase the blast absorption capacity of the panels. Further to this, the ability of the panels to dissipate energy from non-uniform blasts is increased when increasing the top plate thickness.

Simple analytical formulae are derived for the tube based panels, as well as aluminium foam based panels. The analytical model for the tube core panels agrees well with numerical and experimental results. In general, the tube core panels perform favourably with the foam core panels. Panels with higher density foam are found to be able to attenuate the same blasts as tube based panels but use a smaller thickness and slightly less mass. This is countered however, by significantly larger force levels transmitted to the structure the panels are protecting. Further investigations are required to determine the validity of foam core panels under blast loads.

The investigations presented in this thesis indicate that sandwich-type panels can be used as an energy absorbing structure under blast loading. The efficiency of the panels is highly dependent however, on the panel geometry and on the uniformity of the blast load itself.

Acknowledgements

The author wishes to express his gratitude to many of the staff of the Department of Mechanical Engineering, at the University of Cape Town. In particular, the significant time and effort put forth by the technical staff within the department. The author wishes to thank Mr. G. Newins, and Mr. H. Emrich for their assistance in preparation of the tubes and Mr. L. Watkins for creating the tensile test and Split-Hopkinson bar specimens. Special thanks goes to Mr. H. Tomlinson for the countless hours he spent in helping to construct the test panels. Thanks also go to Mrs. P. Park-Ross of the Centre for Materials Engineering for her assistance with the tensile testing rig.

The author also wishes to thank all members of the Blast Impact and Survivability Research Unit (BISRU) for all their assistance and support. Particular thanks go to Mr. D. Bonochris and Ms. A. van der Westhuizen for sharing their expertise in material modelling. Special thanks goes to Mr. T. Cloete for his assistance with many areas, particularly with theoretical modelling.

Financial support of this work was provided by the Armaments Corporation of South Africa (ARMSCOR) through direct funding, and through the University of Cape Town Postgraduate Scholarship and the University of Cape Town Entrance Merit Scholarship. The author wishes to thank these organisations for their continued support. Thanks also go to the Department of Mechanical Engineering and the Department of Mathematics for their financial support through lecturing and tutoring posts.

The author would like to thank Prof. G. Nurick, who supervised this thesis. Without his guidance and knowledge, this work would not have been possible. The author wishes to extend special thanks his parents David and Kathy, and his sister Amy for their support over the years. Lastly, but certainly not least, the author wishes to extend his warmest thanks to his other family, including Erin, Raymond, and Helen Goodchild. Their encouragement during the authors time in South Africa has been essential in its success.

Table of Contents

Declaration	i
Abstract	iii
Acknowledgements	v
Table of Contents	vi
List of Tables	xi
List of Figures	xiii
Nomenclature	xxi
1 Introduction	1
1.1 Background	1
1.2 Problem Definition	2
1.3 Thesis Outline	4
2 Literature Review	7
2.1 Crushing of Thin-Walled Tubes	7
2.1.1 Buckling Modes	8
2.1.2 Buckling Transition	11
2.1.3 Material Properties and Geometric Imperfections	13
2.1.4 Foam and Honeycomb Filled Sections	18
2.1.5 Blast Loading	23
2.1.6 Analytical Models	24
2.1.7 Optimization	29
2.2 Blast Loading of Plates and Sandwich Panels	32
2.2.1 Blast Loading	32
2.2.2 Blast Loading of Sandwich Structures	33
2.2.3 Alternative Core Structures	41
2.2.4 Optimisation of Sandwich Structures	46
3 Material Characterisation	47
3.1 Introduction	47
3.2 Quasi-Static Tensile Behaviour	49
3.2.1 Test Apparatus	49
3.2.2 Construction of True Stress vs. True Strain Curves	50
3.2.3 Quasi-Static Results	51

3.3	Tensile Test Simulations	53
3.3.1	Mass Scaling	56
3.3.2	Simulation Results	58
3.4	Dynamic Material Behaviour	61
3.4.1	Test Apparatus	61
3.4.2	Calibration of the SHPB	62
3.4.3	SHPB Data Analysis	65
3.4.4	Dynamic Test Results	68
3.5	Plasticity Model	70
3.5.1	The Johnson-Cook Model	70
3.5.2	The Cowper-Symonds Model	71
3.5.3	Alternative Constitutive Models	71
3.5.4	Modelling of Thermal Softening	72
3.5.5	Model Parameters	74
4	Panel Design and Finite Element Model	81
4.1	Introduction	81
4.2	Panel Design	82
4.2.1	Tube Design	82
4.3	Finite Element Model	84
4.3.1	Element Selection	84
4.3.2	Panel Model	84
4.4	Blast Loading Model	85
4.4.1	Uniform Loading	85
4.4.2	Non-Uniform Loading	86
4.5	Finite Element Convergence Study	88
4.6	Quantitative and Qualitative Analysis of Panel Behaviour	92
4.6.1	Tube Efficiency Measures	92
4.6.2	Panel Efficiency Measures	93
4.6.3	Qualitative Measures	96
5	Validation of the Numerical Model using Experimental Testing	99
5.1	Introduction	99
5.2	Test Programme	99
5.3	Panel Manufacture	101
5.4	Tube Preparation	103
5.5	Finite Element Model of Experimental Panels	105
5.6	Quasi-Static Experiments	108
5.7	Blast Loading Experimental Procedures	110
5.7.1	Panel Loading	110
5.7.2	The Ballistic Pendulum	112

5.8	Blast Test Results	116
5.8.1	Impulse Measurements	116
5.8.2	Post-Test Panel Measurements	117
5.8.3	Quantitative Results	118
5.8.4	Qualitative Results	123
5.9	Finite Element Analysis of Experimental Tests	129
5.9.1	Prediction of Panel Crush Distance	129
5.9.2	Prediction of Buckling Behaviour	130
5.9.3	Energy Absorption Properties of the Experimental Panels	131
5.9.4	Temperature Effects	138
6	Analytical Modelling	141
6.1	Introduction	141
6.2	Phase I: Application of Blast Load	141
6.2.1	Model Assumptions	141
6.2.2	Finite Element Model	143
6.2.3	Elastic-Plastic Wave Propagation	143
6.2.4	Prediction of Peak Top Plate Velocity	146
6.3	Phase II: Panel Response	148
6.3.1	Mean Crushing Force	150
6.3.2	Panel Capacity and Stroke Length	152
6.4	Model Verification	155
6.5	Panel Performance	158
6.6	Theoretical Comparisons with Foam Core Panels	163
6.6.1	Analytical Foam Model Under Blast Loading	163
6.6.2	Foam Core Response to Blast Loading	170
6.6.3	Mass and Thickness Comparison of Tube and Foam Based Cores	173
7	Design Studies	179
7.1	Parametric Study Outline	179
7.1.1	Design Variables	179
7.1.2	Finite Element Model	180
7.1.3	Parametric Study Formulation	181
7.2	Design Study Results	183
7.2.1	Panel Layout	183
7.2.2	Tube Geometry Effects	191
7.2.3	Top Plate Geometry Effects	198
7.3	Non-Uniform Loading of Optimal Panels	201

8	Conclusions	209
8.1	Material Characterisation of Tube Materials	210
8.2	Panel Design and Performance Measurement	210
8.3	Experimental Investigations	211
8.3.1	Blast Loading Rig	211
8.3.2	Experimental Results	211
8.4	Analytical Panel Modelling	213
8.4.1	Analytical Predictions of the Influence of Design Parameters on Panel Response	213
8.4.2	Foam Core Panel Response to Blast Loading	214
8.4.3	Comparison of Tube and Foam Core Panels	214
8.5	Numerical Modelling of Panel Response	216
8.5.1	Tube Layout	216
8.5.2	Tube Geometry Effects on Panel Performance	216
8.5.3	Top Plate Geometry Effects on Panel Performance	217
8.5.4	Panel Response to Non-Uniform Loading	218
8.6	Recommendations for Future Work	218
	References	220

List of Tables

3.1	Chemical Composition of the steels used in this study	47
3.2	Chemical composition of the 6063-T6 aluminium used in this study . . .	48
3.3	Tensile specimen dimensions.	50
3.4	Mass scaling schemes used in the convergence study of tensile simulations.	57
3.5	Calibration data and bar properties used in the SHPB experimental setup.	65
3.6	General material properties for all materials.	74
3.7	Strain-rate hardening and thermal softening parameters for the tube materials.	76
3.8	Johnson-Cook material constants for the top plate steel.	79
5.1	Summary of panel and tube geometries investigated experimentally. . . .	100
5.2	Loading magnitudes in terms of the mass of PE4 detonated for each of the core tube materials.	101
5.3	Comparison of quasi-static energy absorption parameters for experimental and finite element analysis.	108
5.4	Summary of experimental results and comparisons with finite element simulations.	121
5.5	Panel efficiencies obtained from the finite element simulations of experimental tests.	137
6.1	Validation of critical parameter formulas.	158
6.2	Foam core properties and blast load properties used in the validation test cases of the proposed aluminium foam model.	169
6.3	Critical response parameters for the panels and loading conditions given in Table 6.2.	170

7.1	Plate design variables used in the parametric study.	180
7.2	Tube design parameters corresponding to maximum, median, and minimum stability under variation of λ	184
7.3	Individual tube response for panel <i>A</i> as a function of λ	189
7.4	Individual tube response for panel <i>B</i> as a function of λ	189
7.5	Individual tube response for panel <i>C</i> as a function of λ	190
7.6	Tube wall thickness effects on panel performance for the optimal values of λ in panels <i>A</i> , <i>B</i> , and <i>C</i>	196
7.7	Tube aspect ratio effects on panel performance for the optimal values of λ in panels <i>A</i> , <i>B</i> , and <i>C</i>	197
7.8	Top plate thickness effects on panel performance for the optimal values of λ in panels <i>A</i> , <i>B</i> , and <i>C</i>	200
7.9	Individual tube response for the panels investigated under non-uniform blast loading.	202
8.1	Summary of performance measures for optimal values of λ in the parametric study on tube layout.	216
8.2	Performance measures for the highest performing panels, in terms of core efficiency, under simultaneous variation in tube geometric parameters. . .	217
8.3	Performance measures for the highest performing panels, in terms of core efficiency, under variation in top plate thickness.	217

List of Figures

1.1	Schematic of the proposed panel, illustrating the the energy absorbing mechanism initiated by the blast load.	3
2.1	The three primary modes of buckling.	8
2.2	Typical force-deformation curves under global and progressive buckling. .	9
2.3	Plastic wave speed effects on the stress state of circular and square tubes and the effects of strain hardening on the plastic wave speed.	11
2.4	Transition diagram from progressive to global bending under quasi-static and dynamic loading for square tubes.	12
2.5	Transition diagram for square tubes under different dynamic loading velocities.	13
2.6	Effects of annealing on the crushing behaviour of tubes under compression.	14
2.7	Influence of various triggers on the peak buckling load and buckling mode of square steel tubes.	15
2.8	Schematic of possible full and half dent triggers in square tubes.	17
2.9	Typical aluminium foam material properties and the interaction effect of foam filler on the crushing behaviour of square tubes.	18
2.10	Photographs and force-displacement curves of mono-tubal and bitubal square tubes with and without aluminium foam filler under progressive buckling.	20
2.11	Photographs of a square aluminium tube with an attached mass before and after blast.	23
2.12	Influence of explosive loading parameters on the duration of the initial compression phase of square tubes and the resulting correction to existing formulae for crush distance.	23

2.13	Two generic types of energy absorbing structures (type I and type II) and corresponding idealized model of a type II structure.	25
2.14	Schematic of the basic folding elements used in theoretical analysis of the crushing of square tubes.	26
2.15	Examples of response surfaces in the optimization of cylindrical tubes under dynamic loading.	30
2.16	Optimization of tapered cylindrical tubes.	30
2.17	A typical pressure-time history on a structure under blast loading.	33
2.18	Schematic of foam based protective cladding and its effect on the pressure distribution on the protected structure.	34
2.19	Blast wave interaction with rigid panels and deformable foam core panels.	35
2.20	Schematics of a double layered foam cladding.	35
2.21	Deformed shape of foam core sandwich panels under blast loading at various time instances.	36
2.22	Dependence of densification strain and pressure intensity on the transmitted stress using cellular material.	37
2.23	Photographs of honeycomb core after subjected to uniform and localised blast loading.	39
2.24	Plot of non-dimensional energy partitioning between the face sheets and the core for tetragonal truss and square honeycomb cores.	40
2.25	Schematic and deformation modes of a web type layered cladding under blast loading.	41
2.26	Photographs of layered web based cladding before and after blast loading.	42
2.27	Schematic of a mass-spring model for a unit cell of a web plating cladding structure.	42
2.28	Plot of force transfer and energy absorption for a web plate sacrificial cladding structure.	43
2.29	Periodic cellular lattice structures and their deformation under quasi-static loading.	44
2.30	Energy absorption per unit mass of various lattice structures.	45
2.31	Schematic of a corrugated sandwich panel under optimization and corresponding design variables.	46

3.1	Schematic of tensile specimens.	49
3.2	Engineering stress-strain curves of the materials used in this study. . . .	52
3.3	Idealised geometry of the fractured surface in the tensile specimen.	53
3.4	Finite element mesh of 1/8 th symmetry specimen model used in tensile simulations.	56
3.5	Plot of results of the mass scaling convergence study.	57
3.6	Simulated and experimental force-displacement curves for the initial and final approximations of the post-UTS tensile behaviour.	59
3.7	Equivalent plastic strain localisation.	60
3.8	Final true stress-strain curves for the materials used in this study.	60
3.9	Schematic of the SHPB test apparatus used in this work.	61
3.10	Voltage signals used in the calibration of the SHPB.	62
3.11	Schematic of the SHPB with stresses in the bar and specimen variables .	65
3.12	Dynamic stress-strain curves from SHPB tests.	69
3.13	Fitting of strain-rate sensitivity models to experimental data.	75
3.14	Comparison of material models with experimental data including temperature effects.	77
3.15	True stress-strain curves at various temperatures for the tube materials. .	78
4.1	Schematic of tube layouts used in the study. Dashed lines indicate where symmetry is used in the FEM model.	82
4.2	Schematic of the cross-section and longitudinal design of the tubes used in the sandwich construction.	83
4.3	Panel models implemented in ABAQUS with circular hole imperfections.	85
4.4	Contour plot of a non-uniform pressure distribution on the top plate. . .	88
4.5	Variation of peak pressure as function of decay constant and radius of uniform pressure.	89
4.6	Single tube finite element models for various mesh densities.	90
4.7	Convergence study results.	91

4.8	Graphical definitions of the individual tube performance measures used in this study.	94
4.9	Tube crushing modes observed in numerical simulations.	96
5.1	Schematic of the bonding methodology between the core tubes and the top and bottom plates.	102
5.2	Photograph of the bonding between the tubes and the top and bottom plate.	102
5.3	Photographs of typical aluminium tube panels after construction.	103
5.4	Schematic of the apparatus used to create the hemispherical indentations in the tube wall.	105
5.5	Indentation model implemented in ABAQUS.	106
5.6	Panel model with indentations implemented in ABAQUS.	107
5.7	Plot of experimental and numerical quasi-static force-displacement curves.	109
5.8	Schematic of the blast tube used to direct the pressure load onto the panels.	111
5.9	Photographs of explosive layout and the mounting of the blast tube on the ballistic pendulum.	112
5.10	Schematic of pendulum geometry.	114
5.11	Plot of impulse transferred to the panels as a function of explosive mass.	117
5.12	Mid-point crush distance as a function of applied impulse for all experimental tests cases.	119
5.13	Plot of experimental and finite element predictions of mid-point crush distance as a function of applied impulse.	120
5.14	Experimental and numerical deformation of the core tubes for panel <i>B</i> with steel tubes.	124
5.15	Experimental and numerical deformation of the core tubes for panel <i>C</i> with steel tubes.	125
5.16	Experimental and numerical deformation of the core tubes for panel <i>B</i> with aluminium tubes.	126
5.17	Experimental and numerical deformation of the core tubes for panel <i>C</i> with aluminium tubes.	127
5.18	Plot of equivalent plastic strain distribution along the midline of the central tube for selected tests.	128

5.19	Plot of force-displacement and absorbed energy in the core and top plate from finite element simulations of experiments on steel tube panels. . . .	132
5.20	Plot of force-displacement and absorbed energy in the core and top plate from finite element simulations of experiments on aluminium tube panels.	133
5.21	Numerically computed mean crush force as a function of axial displacement for experimental tests.	134
5.22	Temperature distribution and influence of thermal softening on the energy absorption properties of the proposed panels.	139
6.1	Idealised bi-linear material model used in the modeling of panel response during blast loading.	142
6.2	Idealised model of the proposed panels in terms of stress wave propagation.	143
6.3	Plot of yield behaviour and impulse transfer during application of the blast load.	146
6.4	Plot comparing the peak top plate velocity as a function of blast duration for panels <i>A</i> , <i>B</i> and <i>C</i>	148
6.5	Contour plot showing the variation in the energy equivalent yield stress as a function of tube geometry for the annealed mild steel used in this study.	151
6.6	Plot comparing analytically predicted crush distances with finite element analysis and experiments.	156
6.7	Plot comparing analytical and finite element predictions for the test panels used in the experimental tests.	157
6.8	Plots of impulse capacity as a function of tube aspect ratio and thickness for panels <i>A</i> , <i>B</i> , and <i>C</i>	159
6.9	Plots of impulse capacity as a function of tube thickness and top plate thickness for panels <i>A</i> , <i>B</i> , and <i>C</i>	160
6.10	Plots of specific impulse capacity as a function of tube thickness and top plate thickness for panels <i>A</i> , <i>B</i> , and <i>C</i>	161
6.11	Schematic of the foam core panel investigated in this study.	163
6.12	Plot of foam model verification.	169
6.13	Plot of impulse capacity and specific impulse capacity as a function of foam core panel design variables.	171

6.14	Plot of idealised reaction forces at the base of the tube and foam core panels for an applied impulse of 40 Ns.	172
6.15	Contour plots of low-density foam based panel thickness and mass required to have the same impulse capacity as the tube based panel with the corresponding design parameters.	174
6.16	Contour plots of high-density foam based panel thickness and mass required to have the same impulse capacity as the tube based panel with the corresponding design parameters.	175
6.17	Plot of required foam core panel thickness and mass to tube core panel thickness and mass to absorb the maximum tube core impulse.	176
6.18	Design space partitioning in terms of thickness and mass efficiency to the response of the maximum impulse of the tube core panels.	177
7.1	Flowchart outlining the parametric design study procedure.	182
7.2	Plots of variation in core efficiency and stability as a function of the tube place variable λ	183
7.3	Contour plots of the equivalent plastic strain fields in the quarter symmetry model of panel top plates, at different time increments, for optimal and sub-optimal tube placements as defined in Table 7.2.	185
7.4	Partitioning of dissipated kinetic energy between the core tubes and the top plate for different tube layouts.	187
7.5	Axial view of panel cores with λ values corresponding to the minimum, median, and maximum performance for each panel layout.	188
7.6	Plots of core efficiency and the stability parameter as a function of λ and tube thickness h_t for panel layouts <i>A</i> , <i>B</i> , and <i>C</i>	192
7.7	Plots of core efficiency and the stability parameter as a function of λ and tube aspect ratio <i>R</i> , for panel layouts <i>A</i> , <i>B</i> , and <i>C</i>	193
7.8	Contour plots of core efficiency as a function of tube thickness and aspect ratio.	195
7.9	Plots of core efficiency and the stability parameter as a function of λ and top plate thickness h_p for panel layouts <i>A</i> , <i>B</i> , and <i>C</i>	199
7.10	Plot of non-uniform pressure distribution corresponding to decay parameters $m = 10, 20, 30, 40$ using the relation given in Eq. 4.7.	203

7.11 Plots of variation in core efficiency and panel stability as a function of the non-uniform loading decay parameter m	204
7.12 Axial view of panel cores with 2.0 mm and 4.0 mm thick top plates subject to non-uniform loading.	205
7.13 Influence of top and bottom plate contact on the total reaction force at the panel base.	206

University of Cape Town

Nomenclature

Upper Case Symbols

A, B, n	quasi-static material parameters used with the Johnson-Cook Model
A_p	panel top plate area
C	dynamic material parameter used with the Johnson-Cook Model
C_0	speed of sound in the SHPB
C_e	elastic wave speed of the tubes
C_{pl}	plastic wave speed of the tubes
D, q	material parameters used with the Cowper-Symonds model
E	Young's modulus of a material
F	reaction force at the proximal end of the tube
F_m	mean crushing force
I	Impulse
I_e	impulse transfer due to elastic wave propagation
I_{pl}	impulse transfer due to elastic-plastic wave propagation
I_{max}	maximum impulse capacity a panel can be subjected to without compaction
I_{sp}	impulse capacity per unit mass of a panel
KE	kinetic energy in the panel at a given time instance
K_{mom}	momentum based calibration factor for the SHPB
K_s	stress calibration factor for the SHPB
K_{th}	theoretical calibration factor for the SHPB
M_0	plastic collapse moment of the square tubes
M_d	mass of the top plate and densified foam in the foam core panel
M_p	panel top plate mass
M_t	total mass of the tubes in the panel
P_0	peak pressure applied to the panel from blast loading
R	tube aspect ratio
St_e	stroke efficiency
T	instantaneous material temperature

T_{melt}	material melting temperature
T_{room}	temperature at which the material flow stress become temperature dependent
U_i	plastically dissipated energy in tube i
U_1	displacement of the foam core top plate after application of the blast load
U_{core}	plastically dissipated energy in the panel core
U_{plate}	plastically dissipated energy in the panel top plate
V_0	panel top plate velocity after application of the blast load
V_0^{crit}	maximum top plate velocity a panel can have without compaction
V_1	velocity of the foam core top plate after application of the blast load
V_{pl}	particle velocity in the tubes due to elastic-plastic wave propagation
V_y	particle velocity in the tubes at initial yield
W	external work applied to the panel through application of the blast load

Lower Case Symbols

c_{sp}	material specific heat
h_p	panel top plate thickness
h_t	tube wall thickness
l	length of tube in the panel
l_{min}	minimum foam core thickness required to absorb a given impulse without compaction
m	temperature material parameter used with the Johnson-Cook Model
n_t	number of tubes in the panel
t_0	pressure pulse duration from blast loading
t_m	analytically predicted time at which panel deformation is maximised
t_c	analytically predicted time at which panel compaction occurs
t_{c0}	time at which foam core panel compaction occurs during application of the blast load assuming $t_{c0} < t_0$
t_{c1}	time at which foam core panel compaction occurs occur after application of the blast load assuming $t_{c1} < t_m$
t_y	total time at initial yield
u_m	analytically predicted maximum panel crush distance assuming compaction does not occur
w_t	mean tube width

Greek Symbols

ΔL_{pl}	extensometer displacement in tensile tests
δ_i	crush distance for tube i
δ_{mean}	mean crush distance for the panel

$\bar{\epsilon}_{pl}$	equivalent plastic strain
$\dot{\bar{\epsilon}}_{pl}$	equivalent plastic strain-rate
$\dot{\epsilon}_0$	reference strain-rate
ϵ_f	tensile failure strain
ϵ_D	densification strain of aluminium foam
ϵ_{pl}	true plastic strain from tensile tests
η	inelastic heat fraction
λ	ratio of λ_1 to λ_2 as shown in Fig. 4.1
λ_1	distance between the panel centre and the axis of the tube as shown in Fig. 4.1
λ_2	distance between the panel centre and corner of the top plate as shown in Fig. 4.1
ν	poisson ratio of the material
ρ_f	aluminium foam density
ρ_{f0}	aluminium foam parent material density
ρ_p	panel top plate density
ρ_t	tube material density
σ	true stress in tensile tests
$\bar{\sigma}$	equivalent flow stress
σ_0	plateau stress of aluminium foam
σ_c	characteristic flow stress of the tube material
σ_d	dynamic flow stress
σ_s	static flow stress
σ_{eng}	engineering stress in tensile tests
σ_f	tensile failure stress
σ_t	instantaneous stress in the tubes
ξ_{core}	core efficiency of the panel
ξ_{crush}	crush force efficiency of the tube or panel
ξ_{def}	deformation efficiency of the tube or panel
ξ_{st}	stability parameter of the panel
ξ_{total}	total efficiency of the tube or panel

Chapter 1

Introduction

1.1 Background

The need to protect structures and vehicles from blasts is of increasing importance within the current political climate of terrorist activity, and internal and external armed conflicts. In conflicts throughout the world, lives are endangered by explosions due to such ordnances as Improvised Explosive Devices (IEDs) and landmines. Injuries and fatalities can be minimised if the personnel subjected to explosions are protected within their surrounding structure.

Of particular concern is the use of landmines within conflict scenarios. Landmines can be classified into two general groups [1]. The first being anti-personnel mines (APM) which are designed to be triggered by individuals. The results often kill or maim the victim. The second classification of landmine is the anti-tank or anti-vehicle landmine (ATM/AVM) which are designed to destroy or disable vehicles. The effect on the passengers of vehicles subjected to anti-vehicle mines varies depending on the severity of the blast, the location of the mine with respect to the vehicle, and of course on the protective measures used by the vehicle itself.

Landmines have been used extensively in warfare since WWI [2], however the use of such devices has continued into current conflicts. Most international treaties and agreements, such as the Ottawa Convention, focus on the banning and removal of anti-personnel mines due to the direct human cost from accidental tripping of the mines. The Landmine Monitor Report [3] provides a thorough annual report on the worldwide status of their use. Anti-tank mines are however, of considerable concern due to widespread use and superior destructive power. For example, the Coalition war against the Taliban in Afghanistan saw newly contaminated areas with anti-tank mines [4]. A cache of over 10,000 anti-tank mines was discovered and destroyed in July, 2004 [4]. Even before the Coalition invasion

of Afghanistan, it is reported that approximately 2 million anti-tank/anti-vehicle mines are scattered about the country [2]. On the African continent, the Ugandan government's conflict with the rebel group known as the Lord's Resistance Army has exemplified the dangers of anti-tank mines. A significant number of the mines are recovered each year by the Ugandan government, however many civilian and military deaths are reported due to vehicles detonating anti-tank mines [4]. In many areas, such as northern Uganda, the nature of the conflicts themselves makes determination of exact landmine contamination impossible, however it is clearly a difficult problem with a potential for significant human loss.

In an attempt to counter the destruction caused by anti-tank and anti-vehicle mines, advances have been made in the protection of vehicles which are subjected to mine blasts during standard operation. The pioneering blast protected vehicle was conceived in South Africa in the 1970's as a joint venture between Mechem and the CSIR, and is known as the Casspir [5], which is currently manufactured by BAE Systems. The innovative design of the Casspir uses a V-shaped hull to deflect the blast away from the vehicle midline, towards the outside of the vehicle. The design has been so successful, that even the most modern of blast resistant vehicles are still utilizing the V-shaped hull. The highly successful Buffalo and Cougar vehicles [5], created by Force Protection Inc. utilise the V-shaped hull and are widely used by American forces in Iraq. Despite the past and current success of V-shaped hulls for blast protection, the design does have significant drawbacks which are discussed in the next section. It is therefore necessary to continually improve upon existing designs to increase passenger safety and maximise operational performance of the vehicles which are exposed to blasts.

1.2 Problem Definition

Due to the limitations and difficulties posed by current landmine protection systems, new research is required to investigate possible alternatives. The commonly used V-shaped hull deflects the mine blast away of the vehicle cabin and hence the passengers. The amount of deflection capable using such a hull is increased when narrowing the shape of the V. This has the undesirable effect of reducing ground clearance and increasing the hull mass. Due to the varied terrain mine protected vehicles are exposed to, it is clear that such restrictions can significantly impede mobility. Studies have been conducted to improve the currently existing design [6, 7], however the work of this thesis is devoted to exploring an alternative protection system.

An alternative methodology for the protective structure of vehicle hulls exposed to landmine blasts is to use a flat, energy absorbing panel, with a minimum of thickness in order

to maximise ground clearance. In addition to ground clearance, a flat solution lowers the centre of gravity of the vehicle, reducing the chance of the vehicle toppling. The protective structure must however, have sufficient energy absorbing capacity to dissipate the kinetic energy induced by blast loads. If the panel is loaded beyond its capacity, localised protection of the vehicle may be compromised. This must also work in tandem with the requirement that the forces transmitted to the protected structure be controlled and do not exceed certain thresholds.

As will be discussed in detail in section 2.1, thin-walled square tubes can absorb significant energy when crushed in the axial direction. A recent study has extended this idea into blast loading of square tubes [8]. This thesis furthers this by proposing the idea of sandwiching multiple tubes between metal plates. Fig. 1.1 illustrates the philosophy of the proposed panels.

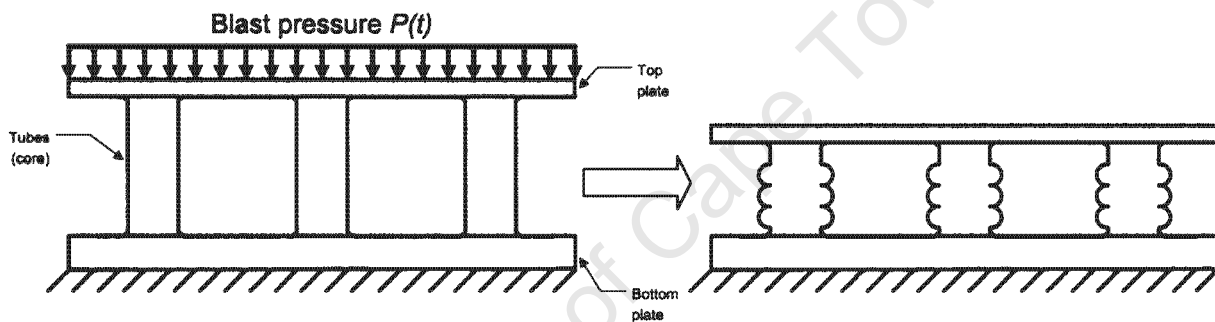


Figure 1.1: Schematic of the proposed panel, illustrating the the energy absorbing mechanism initiated by the blast load.

The research reported in this thesis investigates the following problems:

1. Characterization of small scale thin-walled tube materials at low and high rates of strain
2. Construction methodologies of tube core panels
3. Performance and stability of tube core panels subjected to actual blast loads
4. Modelling of the proposed panels and blast loads
5. Influence of tube and top plate geometry on the global response of the panels
6. Determination of accurate design formulae for the blast loading capacity of the proposed panels
7. Comparative analysis with existing sandwich panel materials

To address the problems described above, a programme of experimental, numerical and analytical investigations is undertaken. In an attempt to solve the problems identified with the proposed sandwich panels under blast loading, the following is included within the scope of the research:

- Material characterization of annealed mild steel and 6063-T6 aluminium alloy tubes, and hot rolled mild steel plates, using quasi-static and dynamic testing equipment, coupled with finite element analysis of tensile tests
- Blast testing on selected test panels, with top plate dimensions 150 mm $\times$ 150 mm, using the ballistic pendulum at the Blast Impact and Survivability Research Unit (BISRU), University of Cape Town
- Development of numerical models of the panels under blast loading using the ABAQUS/Explicit nonlinear finite element software
- Numerical parametric study
- Development of simple analytical formulae, for the proposed panels *and* for aluminium foam core panels to predict critical blast loads for a given panel configuration

It must be stressed that the research presented in this thesis is kept as general as possible. Although the work was initially commissioned with improvements to landmine protection technology as the primary focus, the research and subsequent results are applicable to the general field of cladding design under blast loading.

1.3 Thesis Outline

This thesis has the following plan of development:

- Literature Review
 - Review of the relevant literature on the crushing use square tubular elements.
 - Review of the relevant literature on energy absorbing sandwich structures, particularly under blast loading.
- Material Characterisation
 - Material testing of the panel materials at quasi-static and dynamic strain-rates.

- Identification of constitutive material parameters for use in finite element analysis.
- Numerical Modelling
 - Identification of panel design parameters.
 - Development of a finite element model of the panels.
 - Identification of measures to quantify the performance of the numerically modelled panels.
- Experimental Investigations
 - Outline of panel construction.
 - Development of a blast loading rig.
 - Details of blast test results and numerical modelling of the experiments.
- Analytical Modelling
 - Development of design formulas to predict the proposed panel response under blast loading.
 - Development of design formulas to predict aluminium foam core response under blast loading.
 - Comparative analysis of the proposed panels with aluminium foam core panels.
- Design Studies
 - Outline of a numerical parametric design study of panel response to variation of panel design variables.
 - Details of parametric study results and identification of panel design variables which influence the response.
- Conclusion and Recommendations
- References

Chapter 2

Literature Review

2.1 Crushing of Thin-Walled Tubes

The literature relating to the axial crushing of thin-walled tubular structures is extensive. Over the past several decades researchers have studied the energy absorption properties of circular and square tubes under quasi-static and dynamic loading conditions. These studies include experimental, theoretical, and computational investigations. This chapter attempts to give an overview of these studies and describe the state-of-the-art in the field of tube crushing as energy absorbers. Particular attention is given to dynamic loading of metallic square tubes.

The significant interest in the energy absorption properties of thin-walled stems primarily from the transportation industry. In crash and impact scenarios, kinetic energy must be dissipated to maintain structural integrity and hence ensure passenger safety. As will be discussed in this chapter, tubular structures are desirable for these purposes. An ideal energy absorber is one where the stroke length, in this case, the crush distance, is maximized while maintaining nearly constant force levels. Under most circumstances, thin walled metallic tubes display these qualities.

The pioneering work in this field is traced back to Pugsley and Macauley [9] and Alexander [10]. Since these works were published significant progress has been made in characterising the crushing process in thin walled tubes. Recent literature review papers on the behaviour of energy absorbing devices have been published by Alghamdi [11], and Jones [12].

2.1.1 Buckling Modes

Experiments conducted on thin-walled tubes have resulted in three primary buckling modes. The modes are:

1. Euler (global) buckling
2. Progressive buckling
3. Dynamic plastic buckling

The mode in which a tube buckles is an active area of research and depends on a number of factors such as cross-sectional geometry, tube length, material properties and loading.

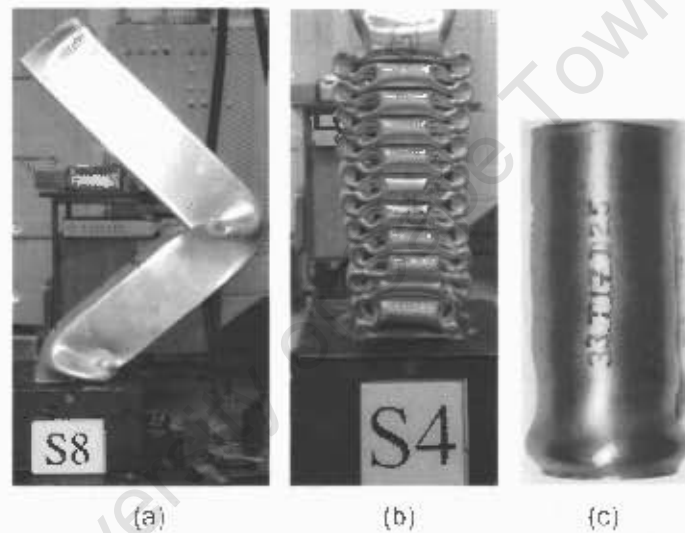


Figure 2.1: (a) Photograph of a square aluminium tube under Euler buckling. [13] (b) Photograph of a square aluminium tube under progressive buckling. [13] (c) Photograph of cylindrical aluminium tube under dynamic plastic buckling. [14]

2.1.1.1 Euler Buckling

Euler, or global buckling is characterized by the formation of a single plastic hinge in the tube. As the tube continues to collapse, the tube folds over on itself. Global buckling of a square aluminium tube is shown in the photograph of Fig. 2.1 [13]. The force-deflection curve of tube which buckles globally will have a form similar to that shown in Fig. 2.2a [13]. Since the dissipated energy is the area under this curve, this buckling mode is highly undesirable.

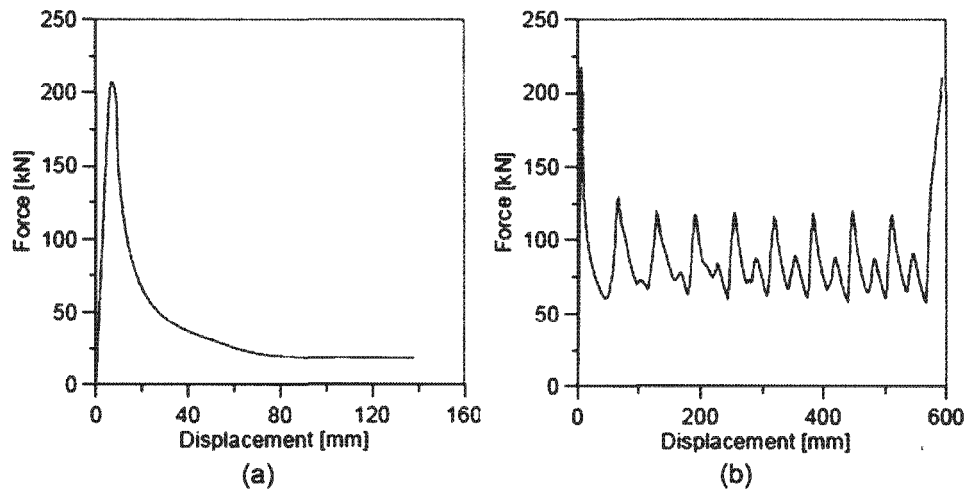


Figure 2.2: (a) Plot of a typical force-deflection curve for a tube under Euler buckling. [13]. (b) Plot of a typical force-deflection curve for a tube under progressive buckling. [13]

2.1.1.2 Progressive Buckling

Unlike global buckling, progressive buckling is characterised by a sequence of localised folding events. If a quasi-static or dynamic load is applied to a tube member such that the tube yields, its load carrying capacity decreases (Fig. 2.2a) [13]. Under progressive buckling, the load decreases to a local minimum and then begins to increase again until the creation of another fold. This process may repeat while the load oscillates about the mean buckling load as shown by the example in Fig. 2.2b [13]. An example of the resulting crushing pattern from such a load-deflection curve is given in Fig. 2.1b. Each pair of peaks and valleys of the force deflection curve is associated with a lobe in the crumpled tube. The size and number of lobes formed is primarily a function of the tube geometry. Comprehensive theoretical descriptions of the crumpling process have been published by Jones [14], Johnson *et al.* [15], and Wierzbicki and Abramowicz [16].

Lobe formation under progressive buckling can be classified into several modes. Jones [14] has identified two crush modes for circular tubes. The first is an axisymmetric, or concertina mode where ring shaped lobes form down the length of the tube. The second mode is the non-axisymmetric diamond mode. Theoretical methods have shown that thicker tubes ($r/h < 40 - 45$, where r and h are the tube radius and thickness respectively) deform axisymmetrically, while thinner tubes buckle in diamond mode.

Of more relevant interest to the present work is the deformation modes of square tubes. Abramowicz and Jones [17] theoretically predicted four crush modes and confirmed this behaviour experimentally. The modes are described as

1. **Symmetric mode.** Square tubes crushing in this mode deform in one of three ways within each layer and each layer deforms similarly along the length of the

tube. Firstly, two opposite lobes move outward, while the other two move inward. Secondly, three lobes can move inward while the other moves outward. Finally, all lobes within a layer can move inward.

2. **Asymmetric mixed collapse mode A.** In this crushing mode, the tube buckles with a combination of symmetric layers and layers with three lobes moving outwards and one inwards.
3. **Asymmetric mixed collapse mode B.** In this crushing mode, the tube buckles with a combination of symmetric layers and layers with two adjacent lobes deforming outwards and the other two lobes deforming inwards.
4. **Extensional mode.** In this mode, all lobes within a layer move outward.

2.1.1.3 Dynamic Plastic Buckling

When a tube is subject to quasi-static loading, longitudinal and radial inertia effects are insignificant in the buckling process. Under certain high-velocity loading conditions, dynamic plastic buckling may occur instead of dynamic progressive buckling. Dynamic plastic buckling is characterized by an initial wrinkling with sustained plastic flow along the tube length as shown in Fig. 2.1c [12]. Until recently, the perception among researchers was that dynamic plastic buckling is a phenomenon associated only with high velocity impacts. Karagiozova [18] has disproved this however, and found that sustained axial plastic flow can occur at lower velocities for high density, ductile materials.

Karagiozova and Jones [19] developed a discrete model to numerically investigate stress wave propagation effects on dynamic buckling in cylindrical tubes. These same authors, in another study [20] on the influence of stress waves found that cylindrical shells made of strain-rate sensitive material will undergo dynamic progressive buckling, due to initial strain localisation, under high velocity impact. For rate insensitive materials, the development of dynamic plastic buckling is governed by the inertia properties of the cylindrical shell. Karagiozova and Jones [21] and Karagiozova *et al.* [22] investigated further the dynamic effects on elastic-plastic buckling in cylindrical tubes and found the initiation of dynamic plastic buckling to be highly sensitive to loading parameters and material properties, particularly the hardening modulus. The influence of axial and lateral inertia effects in the dynamic buckling of square tubes was examined by Karagiozova [23], and Karagiozova and Jones [24]. The authors found that geometrically equivalent circular and square tubes under the same loading may show different buckling behaviour. Faster axial plastic stress wave propagation in the square tubes resulted in dynamic plastic buckling whereas the equivalent circular tube buckles progressively. The dependence of plastic wave speeds on the stress state for square and circular tubes is given in Figs. 2.3a

and 2.3b respectively. Figure 2.3c gives the variation in plastic wave speed for different hardening ratios. Other works of note are from Karagiozova and Jones [25] where the authors investigated the phenomenon using a multi-degree of freedom spring mass model. Although not directly related to dynamic buckling of tubes, Harrigan *et al.* [26] examined inertia effects on tube inversion and the dynamic crushing of aluminium honeycomb.

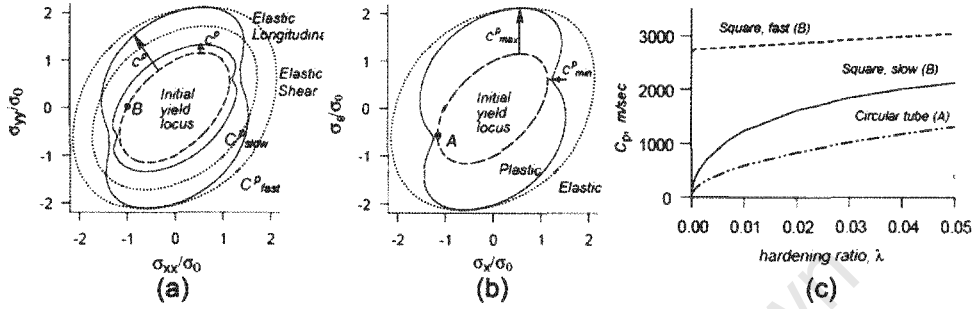


Figure 2.3: (a) Example of a stress state variation for a square tube. (b) Example of a stress state variation for a circular tube. (c) Plot of variation in plastic wave speed between square and circular tubes as a function of the hardening ratio. [24]

2.1.2 Buckling Transition

For any energy absorbing protective structure, ensuring the ideal collapse mode is important. In the context of tube crushing, this implies understanding why tubes buckle globally instead of progressively. The transition from global to progressive buckling mode under static and dynamic loading is an active area of research. The first dedicated study on this subject appears to be from Andrews *et al.* [27]. In this study, the authors constructed a classification chart, using experimental tests that describes the relationship between tube length and thickness to the buckling modes for cylindrical aluminium tubes described in the previous section. They found that Euler buckling was more likely to occur in thick tubes with large slenderness ratios. Korneck [28] investigated the effect of square tube dimensions on buckling mode and found similar results to Andrews *et al.* [27]. This transition was studied Abramowicz and Jones [29] for circular and square steel tubes with various cross sections and lengths. This study found significantly lower critical lengths for thick cylindrical tubes than the results of Andrews *et al.* [27]. This may be due however, to the difference in strain-hardening of the steel (as opposed to aluminium) used in Abramowicz and Jones [29]. The transition classification diagram for square tubes under quasi-static and dynamic loading, determined by this study is given in Figs. 2.4a and Figs. 2.4b respectively. In this figure, Eq. (29) refers to an empirically

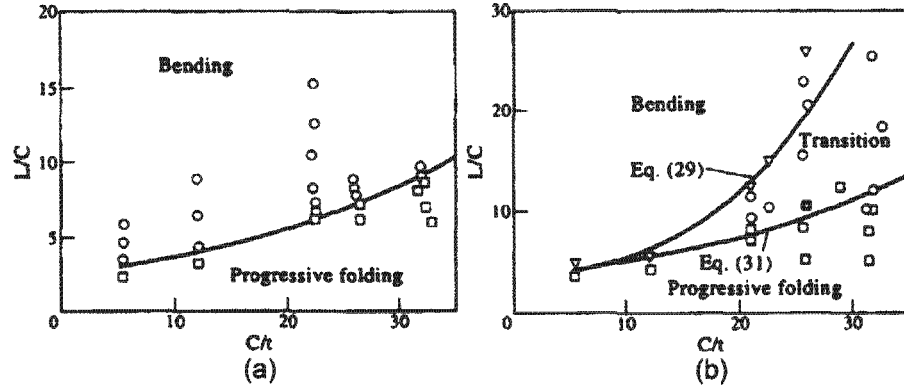


Figure 2.4: $\square$ progressive buckling; $\circ$ transition from progressive collapse to global bending; ∇ global bending. (a) Buckling mode maps under quasi-static loading. (b) Buckling mode maps under dynamic loading. [29]

derived formula for the critical length under dynamic loading and is given here by

$$\left(\frac{L}{C}\right)_{cr} = 2.435e^{(0.08\frac{C}{t})} \quad (2.1)$$

where L and C are the length and width of the tube respectively, and t is the wall thickness of the tube. In general the value of $\left(\frac{L}{C}\right)_{cr}$ which marks the transition from progressive to global buckling for a given thickness was found to be larger under dynamic loading than for quasi-static loading. Abramowicz and Jones [29] also found a third distinct mode under dynamic loading whereby the tube would begin to buckle progressively and then revert to global buckling or vice versa.

Jensen *et al.* [13, 30] studied buckling transition in square aluminium tubes for various cross sections and impact velocities. The collapse modes for various aspect ratios with impact velocities of 13 m/s and 20 m/s are given in Figs. 2.5a and 2.5b respectively. It must be noted that Jensen *et al.* [13, 30] define the tube width as b and the thickness as h which is not consistent with Abramowicz and Jones [29]. The results clearly show a large dependence on impact velocity. At quasi-static and impact velocities up to 13 m/s the critical global slenderness (L/b) was found to be an increasing function of the cross-sectional slenderness (b/h). The same function was found to be decreasing however for impact velocities of 20 m/s.

This behaviour was further confirmed experimentally and numerically by Karagiozova and Alves [31]. This study also highlighted the importance of material properties, particularly strain hardening, coupled with inertia effects when characterising buckling transition. The authors found that materials with high yield stress and low strain hardening perform better than materials with a low yield stress and high strain hardening. This also implies however that using a single, averaged yield stress in theoretical analysis may not be suitable. Karagiozova and Alves [32] also conducted a theoretical analysis of buckling

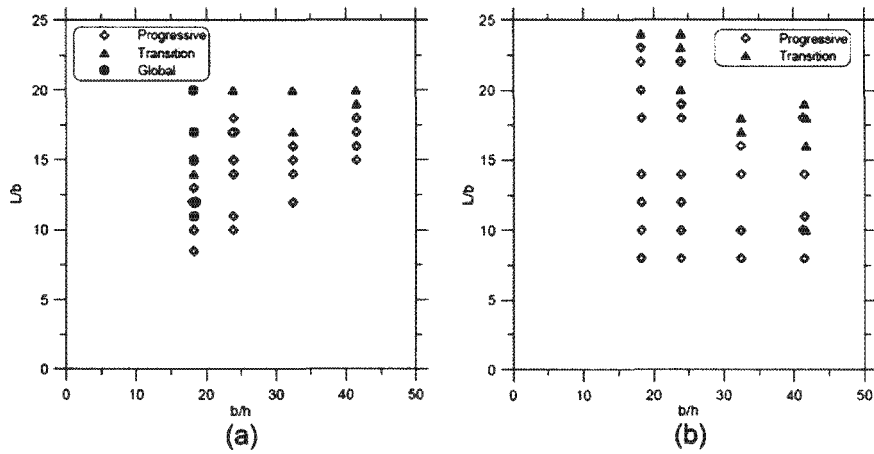


Figure 2.5: (a) Buckling classification diagram with an impact velocity of 13 m/s. (b) Buckling classification diagram with an impact velocity of 20 m/s. [13]

transition as a follow on study.

Kim *et al.* [33] examined the effects of varying the cross section for aluminium extruded tubes. Although the focus of the study was not the transition from progressive to global buckling, the authors found that smaller thickness to width ratios led to a progressive mode. The number of folds decreased with increased thickness which caused global buckling to occur. This study, along with Kim and Lee [34] also found that symmetric folds tended to occur in the thicker specimens as opposed to asymmetric folds in the thinner tubes. Guillow *et al.* [35] studied buckling transition for 6060 – T5 aluminium circular tubes including the construction of a mode classification diagram in terms of diameter to thickness ratio and length to diameter ratio. The diagram is not shown for the sake of brevity. Also of limited interest, is the study conducted by Teramota and Alves [36] in which the buckling transition of axially impacted open shells is investigated.

2.1.3 Material Properties and Geometric Imperfections

2.1.3.1 Annealing

As discussed in the previous sections, material properties such as yield and strain hardening can have significant influence on the dynamic buckling properties of metallic tubes. The process of annealing involves subjecting the metal to large temperatures for an specified period of time to increase ductility and remove residual stresses. The influence of this process has received limited attention in the literature. Gupta and Gupta [37] and Gupta [38, 39] reported effects of annealing for aluminium and mild steel circular tubes. The authors found that for aluminium tubes of diameter to thickness ratios (D/t) between 10 and 40, the work hardened as-received tubes folded in diamond mode whereas

the annealed tubes deformed axisymmetrically. Steel tubes, on the other hand deformed in an axisymmetric mode in their as-received state and in diamond mode when annealed as shown in Fig. 2.6b.

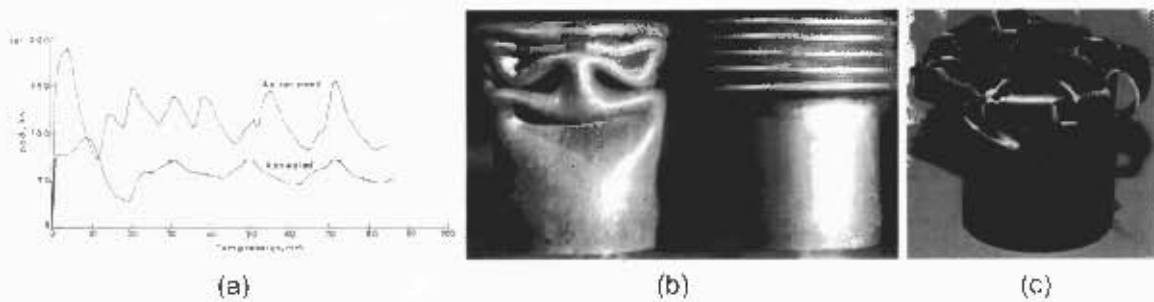


Figure 2.6: (a) Plot of force deformation curve for a circular steel tube, post and pre annealing. [37] (b) Collapse modes for a circular steel tube, post and pre (from left to right) annealing. [37] (c) Fragmentation mode of a brittle metal tube. [40]

The change in microstructure from the annealing process influences more than just the buckling shape. The force-displacement curve, and therefore the energy absorption in the tubes is highly modified as shown in Fig. 2.6a. Further to this, brittle metals may be subject to fracture and petalling under some loading conditions. Fig. 2.6c gives an example of a petaled tube made from a brittle metal. This tearing is of increasing concern if the tube is already damaged or has induced imperfections.

2.1.3.2 Geometric Imperfections

Examination of the force-displacement diagram for a tube under axial compression (e.g. Fig. 2.2b) clearly shows that the force required to create the first lobe is much higher than for subsequent lobes in most cases. If this force is too large in a practical protection system, the elastic limit of the protected structure may be compromised. Further to this, these large peaks also imply large decelerations which may enter unsafe limits. Ideally, the deceleration is a constant function of time throughout the stroke.

In order to overcome these large peak forces, the tubes have an induced imperfection, often called a trigger, built into the structure. These triggers are most often indentations, grooves, or circular cut-outs placed in the shell wall. Induced imperfections not only have the effect of lowering the initial peak force, but also influencing the buckling mode. Firstly, the formation of the the first lobe tends to follow the shape of initial indentations [41–43]. Circular cutouts allow for the adjustment of the effective dimensions of the tube [37, 41, 43, 44]. The investigations of these researchers have shown that the appropriate inclusion of imperfections can lead to progressive buckling where Euler buckling would occur in the non-triggered tube.

One method of introducing a geometric imperfection into a tube is to quasi-statically pre-buckle the tube. Langseth *et al.* [42] performed a series of experiments on pre-buckled tubes. In this study the author investigated square tubes of length 80 mm with wall thicknesses of 1.8 – 2.5 mm. As expected, the peak load was found to be reduced by including a pre-buckle with the amplitude of the pre-buckle controlling this reduction.

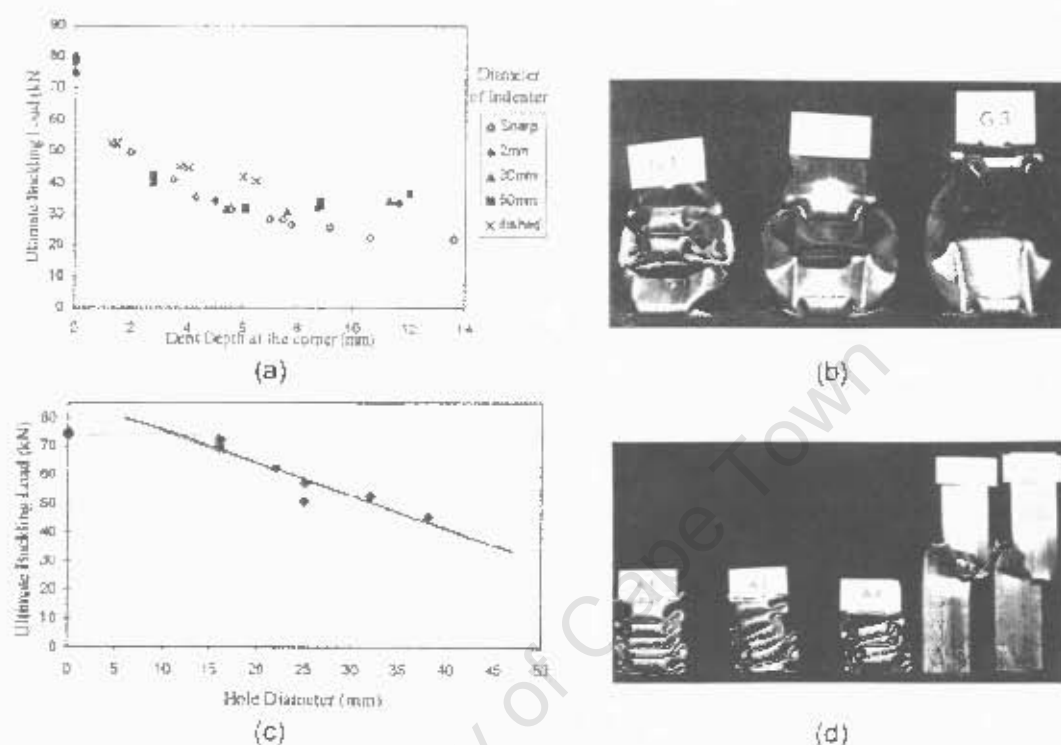


Figure 2.7: (a) Plot of peak buckling load as a function of dent depth for various cylindrical parallel indenters and for hemispherical indenters. (b) Photographs of buckling mode shapes resulting from hemispherical indentations of various depths. (c) Plot of peak buckling load as function of hole diameter for circular cutout triggers. (d) Photographs of buckling mode shapes resulting from circular cutout triggers. [41]

Similar to pre-buckling, the use of opposing indentations can reduce the peak load and improve buckling behaviour. Marshall and Nurick [41] and Marshall [45] investigated such imperfections under quasi-static loading. In Marshall and Nurick [41] opposing parallel indentations created using cylindrical indenters. The tubes examined in the study were 50.8 mm wide and 300 mm with a thickness of 1.2 mm. The cylindrical indenters had diameters ranging from 2 mm - 50 mm. The indentation depth ranged from 3 mm - 12 mm. The authors found that that diameter of the indenter did not significantly influence the ultimate buckling load. The ultimate buckling load did decrease with increasing indentation depth as shown in Fig. 2.7a. Further to this reduction in buckling load is the effect on lobe size. The size of the first lobe was found to increase with increasing indentation depth. This trend can be seen in Fig. 2.7b. Dished indentations created using a hemispherical indenter were also investigated in this study. The reduction in buckling

load was found to be less profound than for the parallel indentations. As with parallel indentations, the size of the first lobe increased with increasing dent depth, however the effect was more pronounced due to opposite sides of the tubes coming in contact. Fig. 2.7b shows the resulting buckling modes for tubes with hemispherical indenters at various indentation depth. It is clear from the figure that buckling stability is reduced with increasing indentation depth.

In the same study, the use of opposing holes was investigated. Several tests were performed with hole sizes ranging from 16 mm - 38 mm. Significant reduction in peak buckling load was observed with increasing hole size as shown in Fig. 2.7c. Linear regression revealed that for the given tubes, hole sizes less than 10.5 mm would not influence the peak buckling load. The size of the first lobe was found to decrease with increasing hole size. At larger hole sizes, the buckling became skew with tearing occurring in some specimens. Fig. 2.7b shows this result. Nannuci *et al.* [46] numerically modelled the tests of Marshall and Nurick [45] and obtained reasonable correlation with experiments. Gupta and Gupta [37] also investigated the influence of circular holes, however using circular tubes. In this study, single pairs of opposite holes are studied as well as two and three pairs of opposing holes. Further to this, the use of 3 and 4 opposing holes in a cross-sectional plane is investigated. In general, the authors found that deformation began at the holes and that the length where Euler buckling occurs is increased with the inclusion of holes.

Surko [43] studied the influence of circular holes on the buckling strength of box columns. The tubes under question had dimension 140 mm $\times$ 140 mm with a thickness of approximately 1.6 mm. This study, along with Meng *et al.* [47], utilized von Karman's postulate of effective plate width to calculate the buckling load of square tubes. The von Karmann effective width of structural plating is given by Surko [43] as

$$h_e = 1.9t \sqrt{\frac{E}{\sigma_y}} \quad (2.2)$$

where t is the plate thickness, E is the Young's modulus, an σ_y is the yield stress of plate material. This implies that if the circular hole diameter ϕ is smaller than

$$\phi \leq C - h_e \quad (2.3)$$

where C is the outer tube width, then the hole will have no effect on the buckling characteristics of the tube. Marshall and Nurick [41] gives a more general form of the

effective plate width as

$$h_e = \frac{2\pi t}{\sqrt{12(1 - \mu^2)}} \sqrt{\frac{E}{\sigma_y}} \quad (2.4)$$

where μ is the poisson ratio, E is the elastic modulus and σ_y is the initial yield stress. With a poisson ratio of $\mu = 0.3$, Eq. (2.4) reduces to Eq. (2.2). For the tube dimensions and material used in Marshall and Nurick [41], the effective tube width exceeds the actual tube width implying that a hole of any diameter will reduce the buckling load. The authors found however, experimentally, holes of diameter less than 10.5 mm showed no appreciable decrease in buckling load. The authors contribute this discrepancy to the extra loading bearing capacity of the tube edges.

Lee *et al.* [48] numerically and experimentally investigated the effect of trigger dents on the quasi-static energy absorption of square tubes. The authors examined various configurations of full-dent and half-dent triggers as shown in Fig. 2.8. The tubes are constructed of aluminium and have a length of 300 mm with a width of 50 mm and a thickness of 2 mm. Computer simulations and compressive tests correlated well. The results showed that dents located at the pre-determined folding sites, energy absorption was improved. Global bending occurred however if the triggers were placed independently of the folding location. The use of half-dent indenters resulted in the same number of folds as the full-dented tubes, however force levels were higher which increased the total energy absorption.

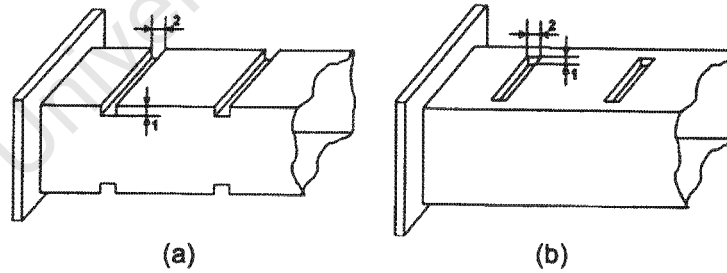


Figure 2.8: Schematics of full dent (a) and half dent triggers (b) in square tubes as investigated by Lee *et al.* [48].

2.1.4 Foam and Honeycomb Filled Sections

As will be shown in section 6.6, the energy absorption properties of aluminium foam are of significant interest in the present study. Aluminium foam is an attractive choice for an absorber due to the high mass efficiency and compression characteristics. Specifically, the foams show the characteristics of an ideal absorber in that the compressive force is nearly constant throughout the crushing process. A typical force-displacement curve for aluminium foam is given in Fig. 2.9a [49]. As can be seen from Fig. 2.9a, under uni-axial

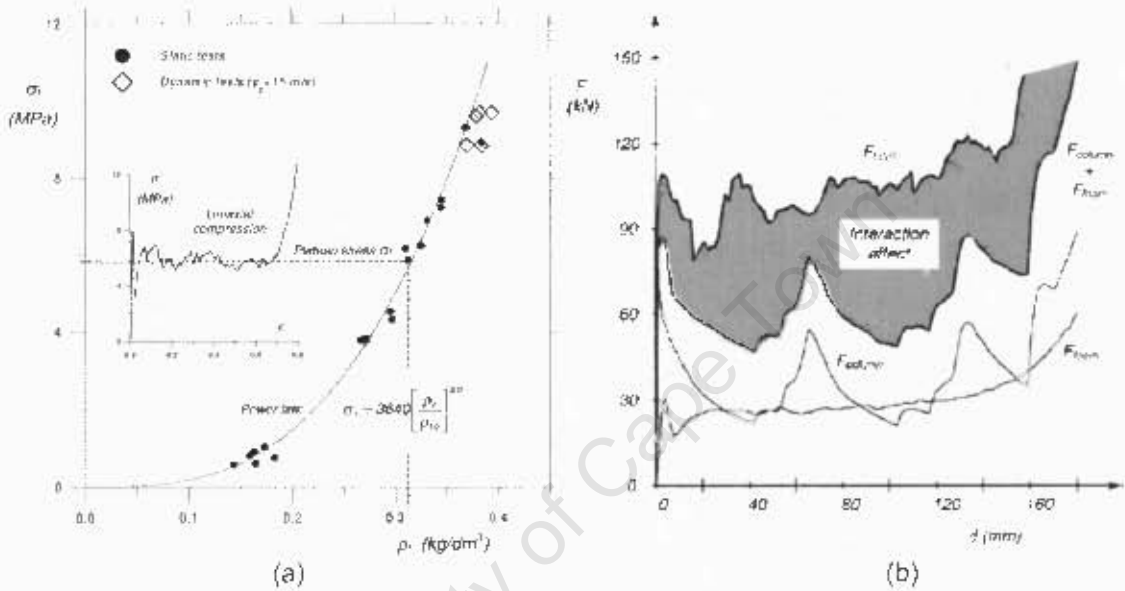


Figure 2.9: (a) Plot of typical aluminium foam material properties, [50] (b) Plot of typical interaction effects of aluminium foam filled tubes, [51]

compression, aluminium foams have a distinct plateau of nearly constant stress σ_f , up to the point of compaction. This plateau stress increases with increased foam density which is also shown in Fig. 2.9a. Further to this, the plateau stress increases with increasing impact velocity. This plateau stress typically occurs up to strain values of 70 – 80%.

Although the aluminium foam is desirable as a sandwich core material in plate structures, it can also be used as part of a metallic tube structure to improve energy absorption. This can be accomplished by filling a circular or square tube with the foam material. Further to this, bitubal elements may also be used which have outer and inner profiles with foam filler in between. The purpose of this is to provide an interaction effect whereby the tube and the foam are a more efficient absorber than each component alone. Fig. 2.9b [52] gives an example of such coupling between the tube and the foam. It is clear that the crush force, and hence energy absorption of the foam filled column is significantly higher throughout the stroke than either the tube or foam alone.

Seitzberger *et al.* [53] experimentally and numerically studied the axial compression of

square and circular steel tubes with aluminium foam filler. The foam densities varied between $\rho_f = 0.47 - 0.7 \text{ g/cm}^3$. Interaction effects between the foam and steel wall increased energy absorption in square tubes by up to 60 % provided progressive buckling occurred. High foam densities tended to initiate global modes, primarily in circular tubes. Seitzberger *et al.* [54] studied monotubal and bitubal steel tubes with various cross sections with aluminium foam filler. The authors found that interaction effects between the foam core and tube wall improved mass efficiency substantially. Specifically, the authors found that more than a third of mean force level was attributed to interaction effects. The total energy absorbing capacity of foam filled elements was found to not be increased so dramatically however. The inclusion of aluminium foam reduces the total crush distance and hence the stroke efficiency (ratio of crush distance to element length). The specific energy (the ratio of energy absorbed to absorber mass) was found to increase by up to 60 % over the tube structure alone. The crushing mode was found to vary considerably between foam filled and not foam filled sections. Figs. 2.10a-c [54] gives examples of square tubes with and without foam filler. The reduction in stroke efficiency when including foam is clear from this figure. The experimentally obtained force-displacement curves corresponding to empty, foam filled, and bitubal tubes are shown in Fig. 2.10d. In Fig. 2.10d, curves denoted by SE refer to empty columns, SM refers to monotubal foam filled tubes and SB refers to bitubal foam filled tubes. Larger crush forces are observed with foam filled tubes however the stroke of the absorber is reduced due to an increased distance between the lobes. Higher foam densities tended to result in increased global bending, as opposed to progressive folding with an optimal foam density of $0.3 - 0.6 \text{ g/cm}^3$. Bitubal (Fig. 2.10c) foam filled elements also appeared to provide increased specific energy over monotubal foam filled elements provided global bending is avoided.

Sanstosa and Wierzbicki [55] experimentally and numerically investigated the effect of including aluminium foam or honeycomb in prismatic columns on bending performance. The authors found bending resistance to be significantly increased by including the lightweight filler, particularly if the filler is properly bonded to the column wall. Further to this, bending resistance is improved using the lightweight filler as opposed to using a thicker tube wall. Hanssen *et al.* [56] experimentally and analytically investigated the bending performance of aluminium foam filled extrusions. They found that the foam filler increased the peak load and this peak load was limited by the tensile failure strain of the foam core. After failure in the core, the bending moment in the beam decreased but this decrease was independent of the aluminium foam density. Bending performance was also investigated by Shahbeyk *et al.* [57]. The authors also found improved energy absorption by including the foam filler and determined the foam to be more mass efficient than increasing the tube thickness.

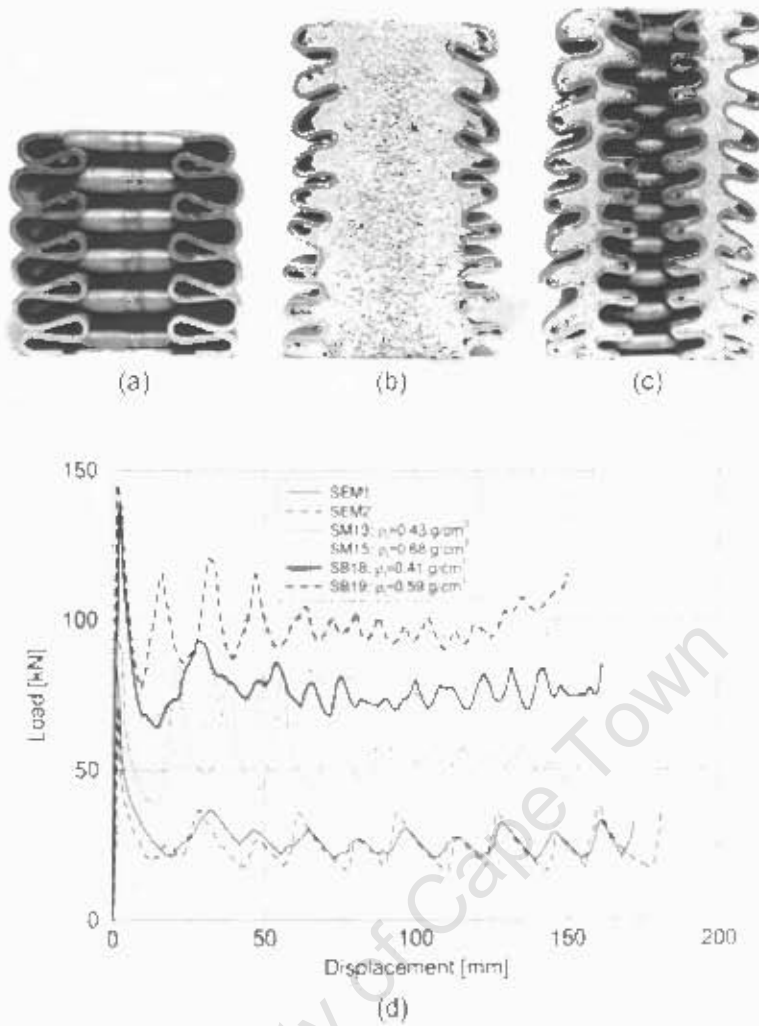


Figure 2.10: (a) Photograph of an empty progressively buckled square tube under quasi-static compression. (b) Photograph of a foam filled ($\rho_f = 0.43 \text{ g/cm}^3$) progressively buckled square tube under quasi-static compression. (c) Photograph of a bitubal foam filled ($\rho_f = 0.59 \text{ g/cm}^3$) progressively buckled square tube under quasi-static compression. (d) Plot of force-displacement curves for empty, and monotubal and bitubal foam filled tubes under quasi-static compression. [54]

Sautosa and Wierzbicki [58] numerically investigated the crash behaviour of foam filled column. The study found that aluminium honeycomb filling is more weight efficient than aluminium foam. This may however, be due to the unidirectional loading used in the study. The authors constructed simple formulae for the quasi-static mean crushing load P_m , of honeycomb and foam filled sections respectively as

$$F_{m,honeycomb} = 14\sigma_0 t^{5/3} b^{1/3} + b^2 \sigma_h \quad (2.5)$$

and

$$F_{m,foam} = 14\sigma_0 t^{5/3} b^{1/3} + 2b^2 \sigma_f \quad (2.6)$$

where σ_0 is the effective plastic flow stress, t is the tube wall thickness, b is the column width, σ_h is the crushing strength of the honeycomb and σ_f is the crush strength of the aluminium foam. The value of σ_h is given as

$$\sigma_h = 3.22\sigma_0(\rho_h/\rho_s)^{5/3} \quad (2.7)$$

where, in terms of Eq. (2.7), σ_0 is the plastic flow stress of honeycombs cell wall, ρ_h is the mass density of the honeycomb while ρ_s is the mass density of the solid material. The value of σ_f is

$$\sigma_f = \sigma_0(\rho_f/\rho_s)^{1.5} \quad (2.8)$$

Langseth *et al.* [59] and Hanssen *et al.* [49–52, 60] have conducted extensive numerical and experimental studies on aluminium extrusions filled with aluminium foam. Hanssen *et al.* [49] experimentally studied the static crushing of such tubes and developed simple formulae and design rules to describe the interaction effects between the tube and the foam. The formula for the mean crush force F_m developed in [49] as presented by Hanssen *et al.* [52] is

$$F_m = 13.06\sigma_0 b_m^{1/3} t^{5/3} + \sigma_f b_i^2 + C_{\text{avg}} \sqrt{\sigma_f \sigma_0} b_m t \quad (2.9)$$

where $b_m = b - t$, $b_i = b - 2t$, and C_{avg} is the interaction constant which is taken as 5.0 in Hanssen *et al.* [49]. The remaining variables are the same as those in Eq. (2.6). This additive formula is composed of three parts. The first term is the mean force from the tube only, the second term is for the foam only, and the final term models the interaction effect. The effects of dynamic loading on circular foam filled tubes with triggers is investigated by Hanssen *et al.* [50] where a design formula similar to Eq. (2.9) is proposed. Static and dynamic loading of square foam filled tubes with triggers was investigated experimentally by Hanssen *et al.* [51] with the primary purpose of developing design formulae under dynamic conditions. The authors found the crush force to be very insensitive to velocity. The aluminium foam is rate insensitive and thus increases in force levels are attributed to inertia effects in the tube walls. This is reflected in the resulting design formula under dynamic loading differing from Eq. (2.9) only in the first term to amplify the contribution of the tube wall in the total mean crush force.

A desirable characteristic of aluminium foam is that it maintains its absorption capabilities when loaded in various directions. The foam must be considered to be anisotropic however, due to the manufacturing process [61]. Despite the anisotropic behaviour, tube structures that have aluminium foam filler should clearly maintain energy absorbing properties when the loading is oblique. Borvik *et al.* [61] investigated circular foam filled tubes

under oblique quasi-static loading angles of 0, 4, 15 and 30 degrees. The authors found the influence of load angle to be significant. Specifically, large reductions in peak force and energy absorption were observed with increasing load angle with a more pronounced effect for foam filled tubes. Interestingly, the specific energy was found to be increased in the foam filled extrusions under axial loading but was lower than the empty sections under oblique loading. LS-DYNA simulations were carried out and showed good agreement with the experiments. Reyes *et al.* [62] conducted a similar study using square aluminium tubes with aluminium foam filler. The general observations were that under oblique loading, it is preferable to use an empty tube with a thicker wall than to have a thinner wall and use a foam filler, given approximately equal mass. This applies to most performance measures except for peak force which is lower when using a foam filler over a thicker tube. Also of note, Reyes *et al.* [63] studied the oblique loading of non-foam filled square aluminium columns. As expected the authors found a significant drop in performance even with a small loading angle.

In the studies by Borvik *et al.* [61] and Reyes *et al.* [62] acknowledge that the apparent inefficiencies could be overcome through optimization of parameters such as tube geometry and foam density could improve results. A mass optimization study was carried out by Hanssen *et al.* [52] to investigate if mass improvements can be expected using a foam filler. The authors used established design formulae to present a graphical optimization method based on cross-sections of the design space. The design variables were the foam plateau stress, column wall strength, column width, column thickness, and tube length. The study determined that mass savings could be obtained when using a foam filler subject to several prescribed constraints on the design variables and structural response. These results are highly dependent on the foam quality (the plateau stress as a function of density) which are likely to increase as the manufacturing process improves.

A recent study was conducted by Wang *et al.* [64] where top hat sections were filled and partially filled with aluminium foam. The authors found that using partially filled tubes showed the best performance in terms of peak load, specific energy and structural stability. Other recent work is by Shahbeyk *et al.* [65] in which a complete numerical study of foam filled columns under dynamic axial loading is given with an emphasis on the extension of the Desphande and Fleck [66] constitutive model for aluminium foam.

2.1.5 Blast Loading

Relatively little research has been done on the blast loading of tube structures. Karagiozova *et al.* [8] studied the behaviour of circular and square aluminium tubes subject to axial blast loads. The loads were transferred to the tubes via a small attached mass at one end of the tube. The impulse transferred to the tube was measured with the ballistic pendulum. Explosive charges were used which resulted in the applied impulse ranging from 6.68 – 12.46 Ns. Fig. 2.11a,b shows the experimental setup before blast and post blast respectively. The study included a finite element analysis which predicted

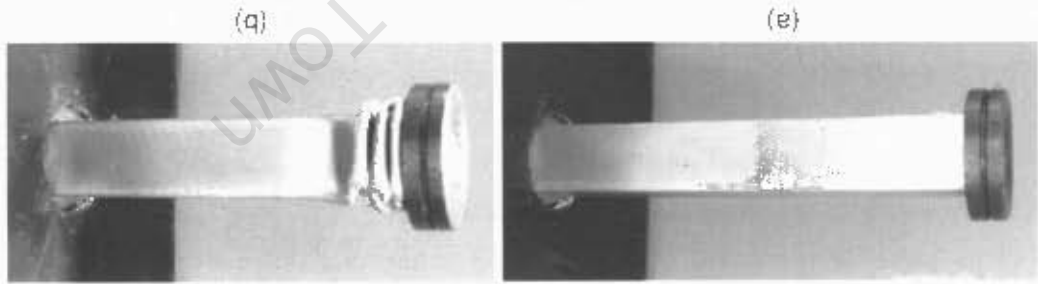


Figure 2.11: (a) Photograph of a square aluminium tube with attached mass before blast and (b) post-blast. [8]

the crush mode and deformed distance well. Existing theoretical formulae were found to significantly underestimate mean force levels. The experimental setup used in this study can be viewed as a high velocity-small mass impact which is therefore sensitive to load characteristics and material properties [8]. Under such loading the energy absorbed during the initial compressive phase becomes significant and is not accounted for in classical models.

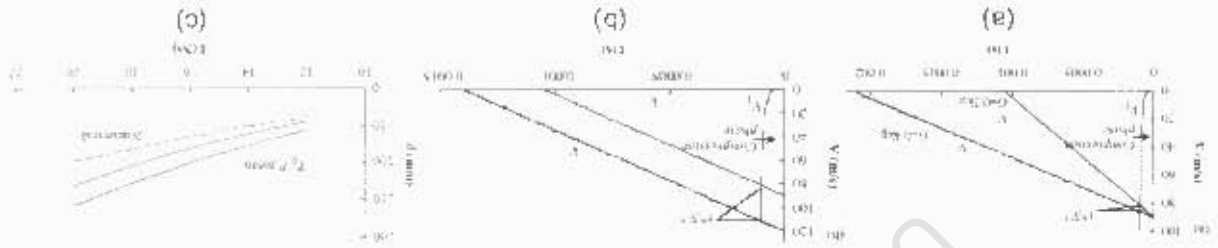


Figure 2.12: Influence of the explosive loading on the duration of the compression phase and proximal end velocities for circular tubes with (a) $I = 18$ Ns and $I = 36$ Ns (b) and proximal end velocities for circular tubes with (a) $I = 18$ Ns and $I = 36$ Ns (b) due to plastic hinge rotation. (c) Predicted crush distance using the traditional formula (T_0/P_{mean}) and using Eq. (2.12) and the comparison with numerical results for a 0.9 mm thick cylindrical tube with an attached mass of $G = 0.2$ kg. [8]

This shortcoming of the existing analytical formulae was theoretically investigated by

Karagiozova *et al.* [8]. An expression for the dissipated energy during the initial compression phase was developed for circular shells. Figs. 2.12a,b give the attached mass velocity characteristics for a circular tube [8]. The velocity at the proximal end of the tube as a function of time is given by

$$V = V_0 - \frac{A\sigma_0}{G}t \quad (2.10)$$

where G is the attached mass, A is the cross sectional tube area, $\sigma_0 = \sigma_y + E_h V_0 t$ (σ_y and E_h are the yield stress and hardening modulus respectively), and $V_0 = I/G$. The time $t = t^*$ is the time where the initial compression phase ends. This occurs at the intersection of the proximal end velocity curve due to plastic hinge rotation (V_1) with the proximal end velocity. The expression for V_1 is determined analytically but is not reported here. The energy at the end of the compression phase is therefore

$$T_c = G [V_0^2 - V(t^*)^2] / 2 \quad (2.11)$$

A relatively accurate crush distance formula is then given by the authors as

$$\delta = \frac{T_0}{(T_0 - T_c)} \frac{3^{1/4}}{2(\pi h)^{2/3} R^{1/2} \sigma_y [1 + (V_0/4RD)^{1/q}]} \quad (2.12)$$

Fig. 2.12c shows the analytically predicted crush distance using the classical models and using Eq. (2.12). The results show significant improvement when compared to numerical results using ABAQUS/Explicit. This result is significant for this thesis as it indicates that under blast loading, compression energy during the initial phase of buckling may be significant when analytically modelling the panels.

2.1.6 Analytical Models

Although numerical and experimental methods can provide good understanding of the crushing mechanism of circular and square tubes, theoretical studies can improve upon this understanding. Further to this, the development of closed form solutions can provide useful tools for the efficient calculation of such parameters as mean crush force and crush distance. This can also be invaluable if analytical optimization of the structure is required. This section discusses some of the analytical models and results regarding square and circular tubes conducted by researchers. Attention is primarily given to square tubes as this relates to the proposed study, however the axisymmetry of circular tubes makes theoretical analysis easier.

In the most general sense, there are two distinct types of plastically deforming structures. These were first identified by Calladine and English [67] and later expanded by Tam and

Calladine [68]. These structures are characterized by their load-deflection curves. Fig. 2.13 [69] gives examples of force-displacement curves for the two types which are denoted as type I and type II.

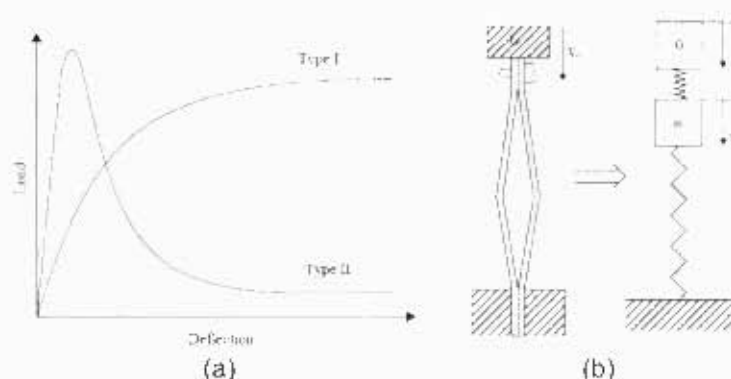


Figure 2.13: (a) Plot of load-deflection curves for the two types of generic structures. (b) A one-dimensional spring-mass model for a pair of pre-bent plates under impact loading. [69]

As can be seen, type I structures harden and exhibit a flat-topped load-deflection curve. Such structures include transversely loaded beams or radially loaded rings. Type II structures on the other hand, have an initial peak followed by structural softening as the deflection increases. Axial buckling columns and tubes exhibit such behaviour. Gao *et al.* [69, 70] theoretically and numerically investigate such structures.

2.1.6.1 Mean Static Crush Load

There has been a reasonable number of studies conducted on analytically characterising the crushing mechanisms and energy absorption. As the deformation in the folding of square tubes is a complex process, researchers have attempted to solve the tube crushing problem by modelling a series of basic elements. Once the absorption properties of the element are identified, the crushing behaviour of an entire tube can be approximated by combining these elements. Wierzbicki and Abramowicz [16] introduced the basic element shown in Fig. 2.14a [17] which models one quarter of a square tube. This is known as a type I collapse element and models symmetric collapse (section 2.1.1.2). A type II is shown in Fig. 2.14b [17] which models extensional collapse modes. The mixed modes described in section 2.1.1.2 can be modelled by using combinations of type I and type II elements to create a complete lobe. These elements contain fixed horizontal plastic hinges and inclined plastic hinges. Clearly, as the element deforms plastic deformation is confined to these hinges. This is not realistic however as the travelling hinges will propagate plastic deformations. A more realistic and kinematically admissible element

is shown in Fig. 2.14c [16]. With this element, plastic deformation occurs in the shaded regions. The flat trapezoidal regions are connected by a toroidal shell.

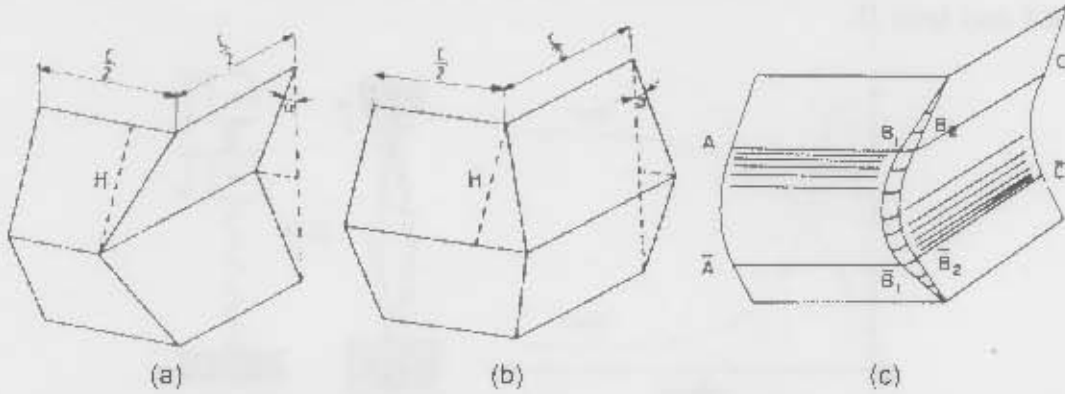


Figure 2.14: Schematic of the basic folding elements used in theoretical analysis of the crushing of square tubes. (a) Type I element, [17] (b) Type II element, [17] (c) Improved type I element with plastic deformation zones, [16]

The previously described folding elements allow for analytical studies on the energy absorbing properties of tubes under axial crushing while maintaining kinematic constraints. From this, design formulae can be obtained which approximate the true behaviour of square tubes. Of particular interest is the mean crushing force F_m . The internal energy absorption in each element can be calculated, including that of the toroidal shell element and equated to the external work to obtain an expression for the mean crush force. Wierzbicki and Abramowicz [16] developed a formula for a tube collapsing in a symmetric mode by combining four type I elements and is given by

$$\frac{F_m^2}{M_0} = 38.12 \left(\frac{C}{h} \right)^{\frac{1}{3}} \quad (2.13)$$

where

$$F_0 = \frac{1}{4} \sigma_0 h^2 \quad (2.14)$$

and σ_0 is the yield stress of the material, h is the tube wall thickness, and C is the tube width. Abramowicz and Jones [17] developed further formulae for mixed mode and extensional collapse. For asymmetric mixed mode A, two layers are used which are composed of six type I and two type II elements. Equating external work to internal work gives

$$\frac{P_m^A}{M_0} = 33.05 \left(\frac{C}{h} \right)^{\frac{1}{3}} - 2.44 \left(\frac{C}{h} \right)^{\frac{2}{3}} - \frac{\pi}{2} \quad (2.15)$$

Similarly, for asymmetric mixed mode B, seven type I and one type II elements are used

resulting in

$$\frac{P_m^B}{M_0} = 35.34 \left(\frac{C}{h} \right)^{\frac{1}{3}} + 1.354 \left(\frac{C}{h} \right)^{\frac{2}{3}} + \frac{\pi}{4} \quad (2.16)$$

And finally for extensional collapse mode

$$\frac{P_m^E}{M_0} = 8\pi \left(\frac{C}{h} \right)^{\frac{1}{2}} + 2\pi \quad (2.17)$$

The limitation of these formulae is that they assume that elements completely flatten during crushing which is clearly an idealization. Abramowicz and Jones [17] extended the previously described model by including the effective crushing distance. The authors expressed the effective crushing for symmetric crushing (δ_s) as

$$\frac{\delta_s}{2H} = 0.73 \quad (2.18)$$

where $2H$ is the height of the folding element. Similarly for both mixed mode and extensional modes the effective crush distance is

$$\frac{\delta_E}{2H} = 0.77 \quad (2.19)$$

Improved design formulae for the mean crush force which are denoted as $\bar{P}_m$ can be obtained by assuming $\bar{P}_m \delta = F_m 2H$ for each of the modes using Eqs. (2.13)-(2.17). This implies the external work is the same, however the internal dissipation occurs over a distance $\delta < 2H$ which implies a larger mean crush force. This leads to an updated expression for the mean crushing load under symmetric collapse mode as

$$\frac{\bar{F}_m^S}{M_0} = 52.22 \left(\frac{C}{h} \right)^{\frac{1}{3}} \quad (2.20)$$

Similarly expressions for the mean crush load under asymmetric mode A and B are given respectively as

$$\frac{\bar{F}_m^A}{M_0} = 42.92 \left(\frac{C}{h} \right)^{\frac{1}{3}} + 3.17 \left(\frac{C}{h} \right)^{\frac{2}{3}} + 2.04 \quad (2.21)$$

and

$$\frac{\bar{F}_m^B}{M_0} = 45.90 \left(\frac{C}{h} \right)^{\frac{1}{3}} + 1.75 \left(\frac{C}{h} \right)^{\frac{2}{3}} + 1.02 \quad (2.22)$$

and finally, under extensional mode the mean crush load is given as

$$\frac{\bar{F}_m^E}{M_0} = 32.64 \left(\frac{C}{h} \right)^{\frac{1}{2}} + 8.16 \quad (2.23)$$

2.1.6.2 Mean Dynamic Crush Load

Of primary interest in the present study is the how the mean static crushing load scales under dynamic loading. Strain rate sensitive material will show an increase in plastic flow stress at higher strain rates. This implies that the mean crushing load will increase under dynamic loading.

The most commonly used strain rate model was suggested by Cowper and Symonds and is given as

$$\frac{\sigma_d}{\sigma_s} = \left[1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{q}} \right] \quad (2.24)$$

where σ_d is the dynamic yield stress at the plastic strain rate $\dot{\epsilon}$, σ_s is the static yield stress and D and q are material parameters. Commonly used values for D and q are 40.4 s^{-1} and 5 respectively [14]. This model is discussed further in section 3.5. Under the assumption that the cross sectional area of the tube remains constant under progressive buckling, the dynamic crush load P_m^{dyn} to the static load P_m^{st} is simply

$$\frac{F_m^{\text{dyn}}}{P_m^{\text{st}}} = \left[1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{q}} \right] \quad (2.25)$$

In order to use this scaling factor, the mean strain rate $\dot{\epsilon}$ must be expressed in terms of the impact parameters, specifically the dimensions of the square tube and impact velocity. Abramowicz and Jones [17] determined that for symmetric buckling

$$\dot{\epsilon} = 0.33 \frac{V_0}{C} \quad (2.26)$$

where V_0 is the initial impact velocity. Therefore, for progressive symmetric buckling

$$\frac{F_m^{\text{dyn}}}{M_0} = 52.22 \left(\frac{C}{h} \right)^{\frac{1}{3}} \left[1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{q}} \right] \quad (2.27)$$

Abramowicz and Jones [17] also found similar expressions for the mean strain rate under asymmetric mixed modes A and B . It is then straight forward to modify Eqs. 2.21 and 2.22 to include strain rate effects.

2.1.7 Optimization

The proposed structure in this thesis is controlled by several design variables which affects the performance under blast loads. For this reason it is ideal for optimization of the design parameters. Structural optimization is well researched field however nearly all studies investigate the elastic response of structures under static loading. Most optimization methods require several finite element simulations which limits the practical use of structural optimization for non-linear responses such as those to blast loads. Recently however, researchers have conducted structural optimization of tube structures in order to improve their crashworthiness properties.

Yamazaki and Han [71] maximized the energy absorption of circular tubes under dynamic loading. The authors optimised the wall thickness and shell radius while keeping the mass of the tube fixed at 0.53 kg. The objective function was taken as the energy absorbed by the tube before bottoming out when impacting a rigid wall at 10 m/s. Constraints were placed on the design variables as well as on the mean crushing force.

In the study, the authors deal with the efficiency problems inherent to impact problems by using the Response Surface Methodology (RSM). In traditional optimization, the objective function, in this case energy absorption, must be evaluated by conducting a finite element analysis. Therefore simply evaluating this function may take several hours. Using the RSM, the objective function (and possibly constraints based on structural response) is approximated using some analytical function, normally a polynomial. The surface is created using selected number of finite element analysis. Once this approximate surface is created, the optimization algorithm can seek the optimum solution based on this surface. Once the optimum solution is found based on the surface, the accuracy of the solution at this point is compared to the finite element solution. If the accuracy is not within a predefined tolerance, the response surface is reconstructed and the process begins again. Forsberg and Nilsson [72] and Jansson *et al.* [73] provide overviews of the use of response surfaces in crashworthiness applications. Redhe *et al.* [74] compare the performance of response surface based optimization and stochastic optimization in crashworthiness applications.

In Yamazaki and Han [71], the response surface is constructed in an efficient way using the Design-of-Experiment technique. This is a technique based on statistical theory where the response of the structure can be determined extremely accurately without conducting a full factorial design. A factorial design is one where all combinations of design variables are examined and the resulting response is determined. For example, if the optimization study has four design variables which each have three examined values then it takes $3^4 = 81$ finite element analysis to get the complete response behaviour. Yamazaki and Han [71] use the Design-of-Experiment to construct orthogonal polynomials representing

the structural response. Examples of the response surfaces in the study are given in Fig. 2.15. The mathematical programming technique is then used to find the optimal solution. In the numerical example given, three design cycles were needed with the optimal solution at the allowable maximum limit of the mean force while buckling is progressive with the most number of axisymmetric folds created.

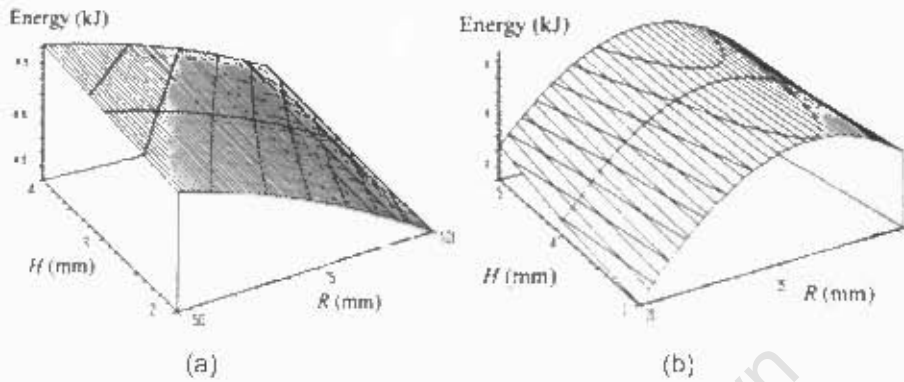


Figure 2.15: (a) Response surface for the first design cycle of the optimization study of Yamazaki and Ilan [71]. (b) Response surface for the second design design cycle of the optimization study of Yamazaki and Ilan [71].

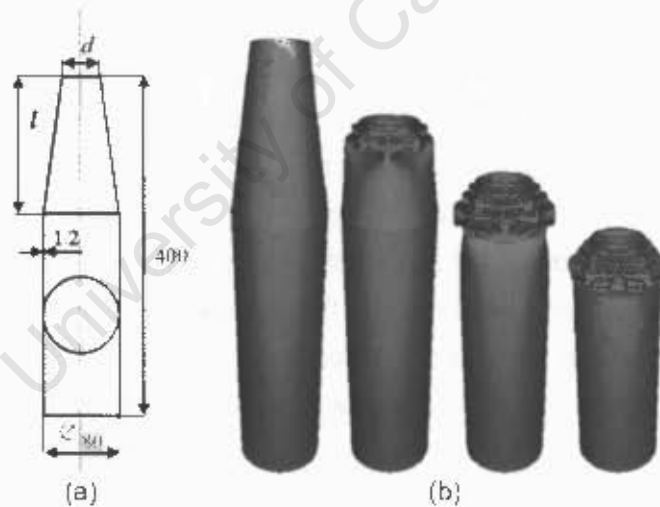


Figure 2.16: (a) Tapered tube layout and design variables used in the shape optimization by Chiandussi and Avallo [75]. (b) Crush behaviour for one of the optimum solutions. [75]

Chiandussi and Avallo [75] used shape optimization to maximize the performance of tapered tubular structures. Shape optimization is a less general form of optimization where parameters defining the overall shape of the structure are the design variables. In this study the shape parameters refer to the taper length and the diameter of the narrow end of the tube. This is pictured by the schematic in Fig. 2.16a. The objective function in the study is a load uniformity parameter which is not defined in the paper.

This parameter is minimized using a program called OPTISTAT which is implemented by the authors. This study also utilizes response surfaces to increase efficiency.

Two local minimums are found corresponding to two distinct tapered tube designs denoted solution A and solution B. Both solutions are examined with respect to small perturbations in wall thickness and Young's modulus. Solution A was found to be less sensitive to perturbations in these parameters however the value of the objective function is larger for this design. This is indicative of the difficulty with structural optimization is that small changes in the optimum structure can result in a large reduction in performance.

Hanssen *et al.* [52] conducted an optimization study on square aluminium columns with aluminium foam filler. This study was briefly discussed in section 2.1.4. This study differs significantly from the previously mentioned studies as the objective functions and constraints are analytically derived expressions as opposed to polynomials which are fit to numerical results. A simplified geometric approach is then taken to determine the optimum structure subject to a number of constraints.

2.2 Blast Loading of Plates and Sandwich Panels

A review of the relevant literature on blast loading is presented here. In general, the work published on blast and dynamic loading is extensive and mostly beyond the scope of this thesis. For this reason, the literature review primarily focuses on blast loading of sacrificial cladding, including foam and honeycomb based sandwich panels. Some attention is also given to research on sandwich type structures under quasi-static compression which may in the future, be solutions to the problems posed in this thesis. Special attention is given to aluminium foam cladding as it is considered in this thesis as a direct alternative to using thin-walled tubes.

2.2.1 Blast Loading

An explosion is a rapid release of stored energy characterised by a bright flash and an audible blast [76]. Energy is released in the form of heat, light, sound and a shock wave in the surrounding air. The shock wave consists of a region of highly compressed air moving at high velocities travelling outward from the point of detonation. As the shock wave it travelling at very high velocities, it induces impulsive or dynamic pressure loading on the structures in which it encounters. It should be noted that the Steel Construction Institute [77] defines impulsive loads as those in which the duration of loading is significantly shorter than the natural period of the structure experiencing the load. This implies that the structure cannot show any significant response during the loading application. This is in contrast to quasi-static loading in which the period of loading is much longer than the natural period of the structure. The load duration regime in between impulsive and quasi-static loading is referred to as dynamic. Impulsive blast loading of interest in the present work.

Sources of blast loading can vary significantly and include the intentional detonation of plastic explosives or the accidental bursting of pressure vessels containing detonable gases. Regardless of the source of the blast loading, the effect on the structures in encounters is generally characterised as a pressure pulse which a large peak pressure and short pulse duration. Such a pressure pulse is described in Fig. 2.17 [77]. Of note in the figure is the region of over-pressure defined by a large peak pressure that decays over time followed by a region of under-pressure where the pressure acts in a direction away from the face of the structure. The pressures in this region are typically small and are often omitted in the analysis of structures under blast loading.

As the pressure loading is impulsive the exact peak pressures and time duration are considered to be of little consequence provided the total impulse is correctly observed [77]. Assuming the pressure is distributed uniformly over the structure, the impulse is simply

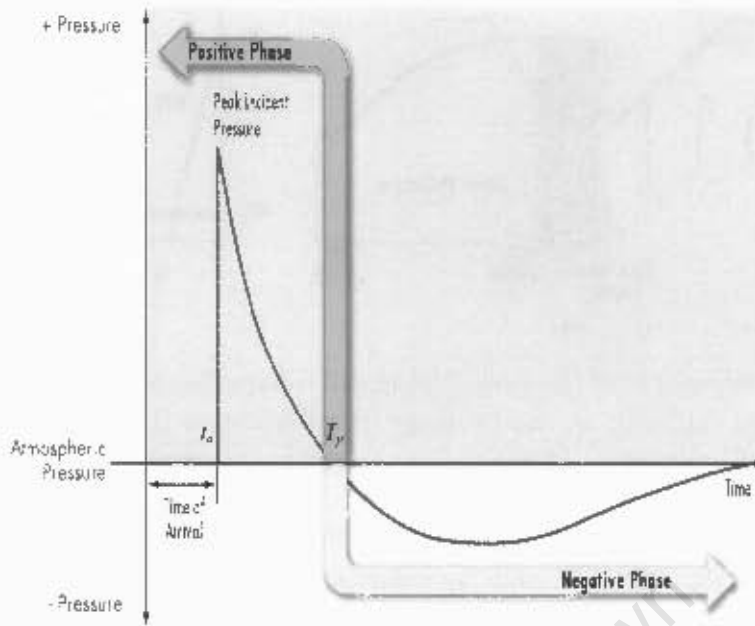


Figure 2.17: A typical pressure-time history on a structure under blast loading [77].

the area under the pressure-time curve. The modelling of the pressure pulse is discussed in more detail in section 4.4.

2.2.2 Blast Loading of Sandwich Structures

2.2.2.1 Aluminium Foam Cladding

The use of aluminium foam would appear to be an ideal structure for absorbing energy under blast loads. Aluminium foam used as the core material sandwiched between aluminium cover sheets was examined by Hanssen *et al.* [78]. The use of aluminium foam as a protective cladding is pictured in Fig. 2.18a [78]. Without the protective cladding extremely large pressures are generated by the blast which are experienced directly by the structure. With the use of foam cladding, the pressure experienced by the protective structure is dispersed over a longer duration with a smaller peak pressure. This is illustrated in Fig. 2.18b [78].

In the study by Hanssen *et al.* [78], full scale blast tests were performed using between 1.0 – 2.5 kg of PE4. The panels are attached to the bob of large pendulum and the rotation of the bob is used to determine the impulse transfer to the pendulum. The authors found that experimentally, the foam panels actually increased the energy and impulse transfer to the pendulum. An analytical solution is derived which shows that the stress level experienced by the protected structure is equal to the plateau stress of the foam. The total impulse transferred to the main structure is the same however with

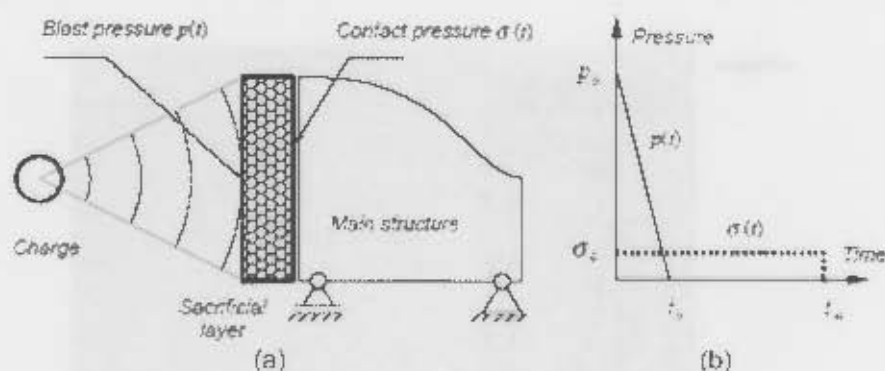


Figure 2.18: (a) Schematic of the potential use of a foam based cladding. (b) Plot of the influence of a foam cladding on the pressure distribution to the protected structure. [78]

and without the cladding. This implies that only local protection of the structure is attainable, however it is controllable by selecting foam with the desired plateau stress. The increased impulse transfer with inclusion of the foam panel is attributed to changes in the shape of the cover plate. As the foam is able to deform instantly, the shape of the cover plate attains a double curvature thereby forming a paraboloid type surface. This is illustrated in Fig. 2.19. The authors of Hanssen *et al.* [78] refer to a study by Liang *et al.* [79] that found that blast waves interacting with a paraboloid shaped surface resulted in a larger amplification of pressure than if the wave interacted with a surface of single curvature or a planar surface. The experimental observations of Hanssen *et al.* [78] concurred with this as larger dishing in the cover plate resulted in increased energy transfer to the pendulum.

A similar study was recently undertaken by Ma and Ye [80]. In this paper, the authors examine a double-layer aluminium foam cladding separated by steel cover plates. The study is analytical and numerical and compared the performance of four different claddings. Similar to the study by Hanssen *et al.* [78], the authors derived their solution using a rigid-perfectly plastic-locking material model which greatly simplifies the analysis while providing accurate results when compared to finite element simulations, particularly for low density foams. Single and dual layered claddings are analyzed with a low and high density foams. The low density foam has a density of 200 kg/m^3 and a plateau stress of 1.5 MPa while the high density foam has a density of 300 kg/m^3 and a plateau stress of 3.0 MPa . Under quasi-static loading, with identical low-density foam, a dual layer cladding will absorb twice as much energy as a single layered foam. Mixing high and low density foams increased the energy absorption by a further 42 %. Under blast loading, the dual layered foam increases the maximum allowable impulse by 76 %. Using high density foam on the rear layer (RL in Fig. 2.20) increases the capacity of the panels by a further 27 % however the stress transmitted to the protected structure is doubled. Using higher density foams on the front layer (FL) gave no significant increase in capacity

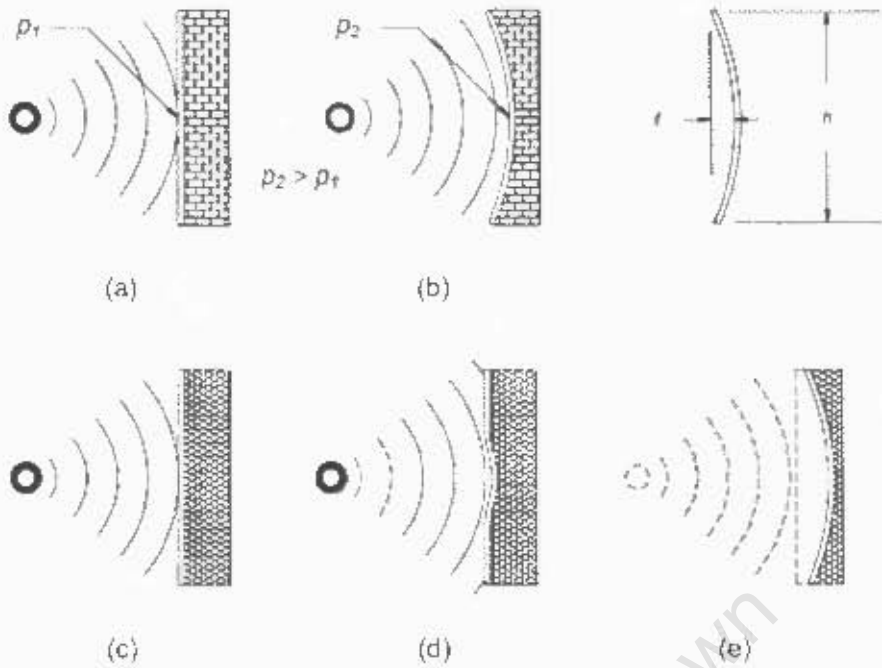


Figure 2.19: Blast wave interaction with rigid panels and deformable foam core panels. (a) planar rigid surface (b) curved rigid surface (c) cover plate on foam (d) plate and foam deforms and (e) final shape of component.

over the low density foam.

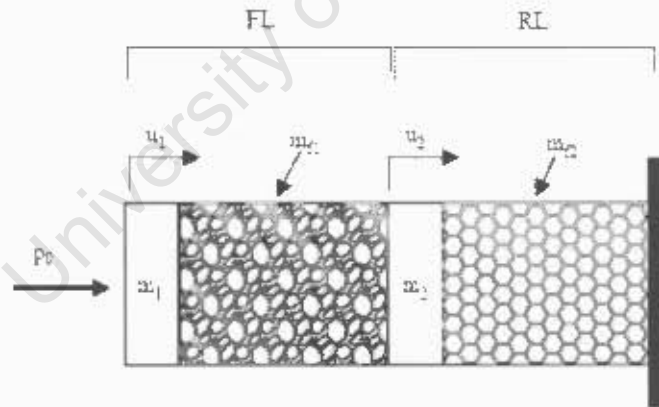


Figure 2.20: Schematic of a double layered foam cladding as proposed by Ma and Ye [80].

A recent study on blast loading of aluminium foam panels was conducted by Nemat-Nasser *et al.* [81]. This study differs significantly from the aforementioned studies in that the panels are clamped circular sandwich plates and hence do not have a rigid back plate. The authors modeled such panels using the dynamic finite element package LS-DYNA with the pressure distribution determined by the ConWep function. The blast loading is assumed to be generated from 2 g of TNT. It was concluded that due to high velocities at the panel centre ($V > 100$ m/s) that a shock front develops inducing much larger displacements in the front plate than in the back plate. This is illustrated by the finite

element simulations shown in Fig. 2.21 [81].

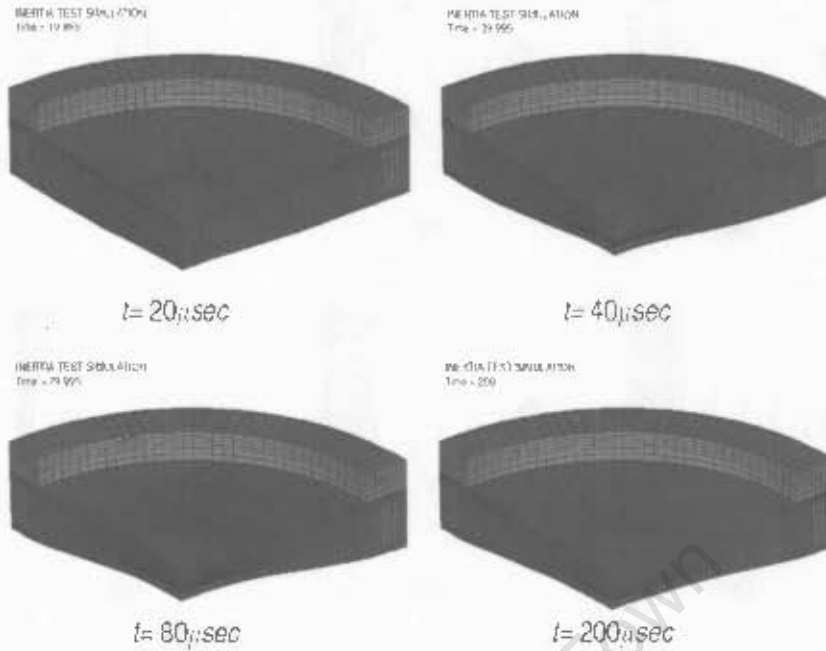


Figure 2.21: Deformed shape of foam core sandwich panels under blast loading at various time instances. [81]

When using foam cladding as a blast attenuator, the designer must be careful that the exact opposite behaviour does not occur. Li and Meng [82] determined that when a blast load is intensive, the transmitted pressure can actually be enhanced and exceed the incident peak pressure. The authors studied analytically, the parameters which determine whether attenuation or enhancement of the blast occurs when using a foam cladding against a rigid support. They found that the densification strain ε_1 and the pulse duration τ_d influenced the maximum pressure which is applied to the rigid surface. Fig. 2.22a shows the influence of ε_1 on the dimensionless transmitted stress to the protected structure. The loading corresponding to the data in this figure is a pressure pulse of maximum pressure to plateau stress ratio $p = P/\sigma_y = 3$ and a dimensionless pulse duration of $\tau_{test}/\tau_d = 1$ where τ_{test} is given as

$$\tau_d = \frac{T}{L/\sqrt{E/\rho}} \quad (2.28)$$

In Eq. (2.28) T is pulse duration, L is foam length, E is the elastic modulus and ρ is the density of the cellular material.

Clearly, for larger densification strains the transmitted pressure approaches the plateau stress of the foam. Fig. 2.22b shows the influence of the pulse intensity on the transmitted stress for various pulse durations. The densification strain is fixed at 68.9% in all curves. The straight line denoted as $R = 1$, corresponds to the transition point between

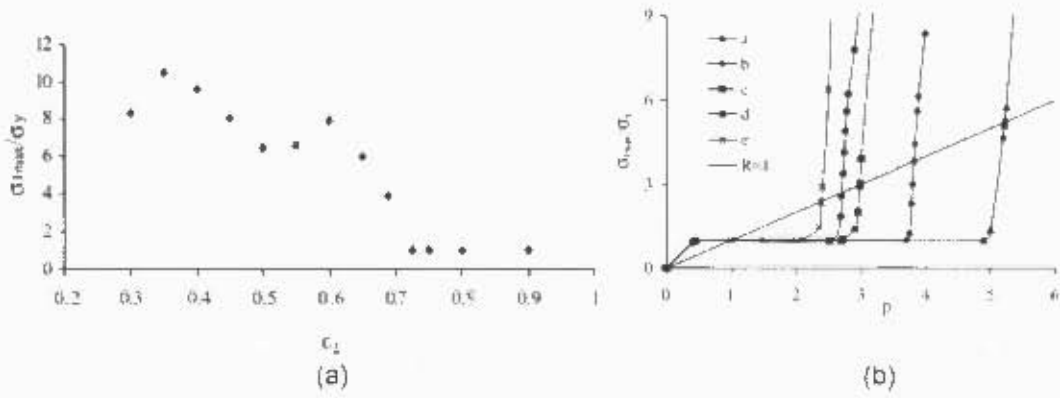


Figure 2.22: (a) Dependence of the dimensionless transmitted stress on densification strain ϵ_1 with $p = 3$ and $\tau_d = 1$. (b) Dependence of the dimensionless transmitted stress on the pressure intensity for pulse durations a: $\tau_d = 0.5$, b: $\tau_d = 0.75$, c: $\tau_d = 1.0$, d: $\tau_d = 1.25$, e: $\tau_d = 1.5$. [82]

attenuation and enhancement of the blast pressure. From Fig. 2.22 it is clear that as the pulse duration increases, the maximum input pressure that will result in the transmitted pressure being below the plateau stress of the material decreases.

Other studies on aluminium foam cladding for protection include Radford *et al.* [83] where clamped sandwich beams are investigated under shock loading. The shock loading is simulated by impacting the beams with a metallic foam striker which is shown to resemble underwater blasts [84]. The study analytically and experimentally investigated monolithic beams, and sandwich beams with stainless steel corrugated and pyramidal truss, and aluminium foam cores. The sandwich beams all performed better than the monolithic beam with the pyramidal core beams the weakest of the sandwich structures. Gama *et al.* [85] studied the use of aluminium foam as part of a multi-layer, multi-material armour protection system. Under ballistic loading the author found favorable results for such a configuration. Crupi and Montanini [86] experimentally investigated aluminium foam sandwich panels under static and dynamic three-point bending tests.

The material properties of aluminium foam under high strain rate has been studied in the literature and a brief selection are discussed here. Paul and Ramamurty [87] studied the strain rate sensitivity of closed cell aluminium foam Alporas for strain rate between $3.33 \times 10^{-5} \text{ s}^{-1}$ to $1.6 \times 10^{-1} \text{ s}^{-1}$. The plateau stress was found to increase bilinearly with a slow increase up to strain rates of $3.33 \times 10^{-3} \text{ s}^{-1}$ with a sharper increase after this rate. Dannemann and Lankford [88] also investigated Alporas foams but at strain rates of 400 s^{-1} to 2500 s^{-1} with a split Hopkinson pressure bar setup. Strain rate sensitivity was observed particularly for higher density foams. It is suggested by the authors that the cell shape is responsible for this increase at higher density. Desphande and Fleck [89] studied the strain rate sensitivity for the closed cell Alulight and open cell Duocel foams using a split Hopkinson pressure bar and direct impact tests at strain rates between 10^{-3} s^{-1}

to approximately $5 \times 10^3 \text{ s}^{-1}$. The authors found both foams to be strain rate insensitive in this range of strain rates. Dannemann and Lankford [88] attributed this discrepancy between the closed wall Alporas and Alulight to the manufacturing process (casting vs. powder processing). Ramachandra *et al.* [90] studied the impact behaviour of Alporas aluminium foam for impact velocities ranging from 3 – 30 m/s. The energy absorbed per unit volume was found to increase with increasing impact velocity, particularly for velocities greater than 5 m/s. Zhao *et al.* [91] also studied the rate sensitivity of metallic foams under impact. Two aluminium foams (IFAM, Cymat) are tested where IFAM is a similar foam to Alulight. The IFAM foam is characterised by cell wall buckling in the crush process while for the Cymat foam crushing is enabled by the fracture of cell walls. The authors found that only the IFAM foam was rate sensitive however, and conclude that this is likely due to microinertia effects presented by Zhao *et al.* [92]. Under dynamic loading, strains along the fold line in a progressively collapsing wall have higher strains due to inertia causing the strain hardening of the base material to come into effect.

2.2.2.2 Honeycomb Core Sandwich Cladding

Despite the wealth of literature regarding the response of honeycomb core sandwich panels to quasi-static and impact loading there has been relatively few studies conducted on blast loading of such structures. A recent study was conducted by Chi [93] however. In the study, the author investigated circular aluminium honeycomb core sandwich panels under blast loading. Specifically, the study compared the deformation of sandwich structures with steel face sheets and empty air core with that of a honeycomb core. The author found that for a given impulse, the maximum deformation of the panels is increased when using a honeycomb core as opposed to an empty core. Also investigated is the different response of the panels when subjected to uniform and localised blast loads. The author found that under uniform blasts, deformations are distributed over the entire exposed area and the resulting shape is similar to the dome shape expected when monolithic plates and blasted uniformly. Fig. 2.23a shows a typical honeycomb core after uniform blast loading. For localised loading, severe damage is found in the centre of the honeycomb core even for small impulses. This results in larger maximum panel deformation than for the uniformly loaded case. Fig. 2.23b illustrates this damage to the core with increasing impulse applied locally. Of note is the complete disintegration of the honeycomb material within the centre of the panel.

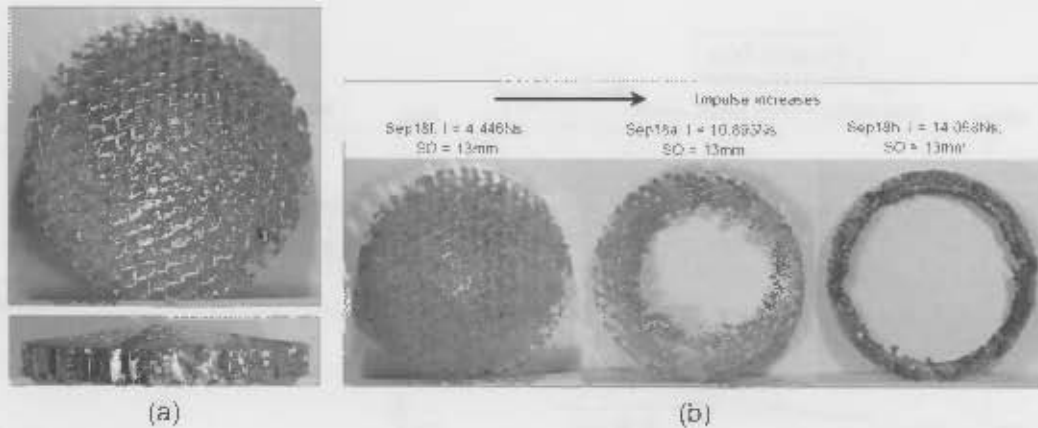


Figure 2.23: (a) Photograph of the deformed honeycomb core subjected to a uniform blast with an impulse of 30.293 Ns. (b) Photographs of resulting honeycomb core subject to localised blast loading. [93]

2.2.2.3 General Sandwich Cladding

The linear and non-linear response of generalized orthotropic weak core (transverse shear only) sandwich panels under blast loading is studied by Librescu *et al.* [94]. The core is sandwiched between laminated face sheets. The authors generated sets of governing equations under air-blast and underwater blast loading which were solved using methods based on the Extended Galerkin Method. Some general results were that deflections due to air-blast loadings were found to be much greater than for underwater blasts. The laminate orientation was also found to play a large role in structural dampening and total deflection of the panels. The ideal ply orientation depended heavily on the transverse shear moduli of the core. Hause and Librescu [95] conducted a similar study where closed form solutions are developed for the dynamic response of sandwich panels with orthotropic cores to a variety of blast pulses (rectangular, sinusoidal, sonic boom).

Studies conducted by Xue and Hutchinson [96,97] investigated the blast loading of sandwich structures from a theoretical and computational standpoint. The work presented in [96] is a preliminary study which compares the performance in terms of strength and energy absorption of monolithic steel plates and tetragonal truss sandwich panels. The panels are circular and have clamped boundary conditions thereby allowing deformations of both the top and bottom face sheets. The authors found that the sandwich panels can sustain substantially larger impulses than solid metal plates of the same mass. The authors conducted a follow-on comparative study in [97]. In the study, the performance of tetragonal truss, square honeycomb, and folded plate sandwich cores are compared to steel plates of equal mass using the ABAQUS finite element package. It was again found that sandwich structures outperform equal mass steel plates in terms of impulsive resistance. This was shown to be particularly true at smaller impulses. Further to this,

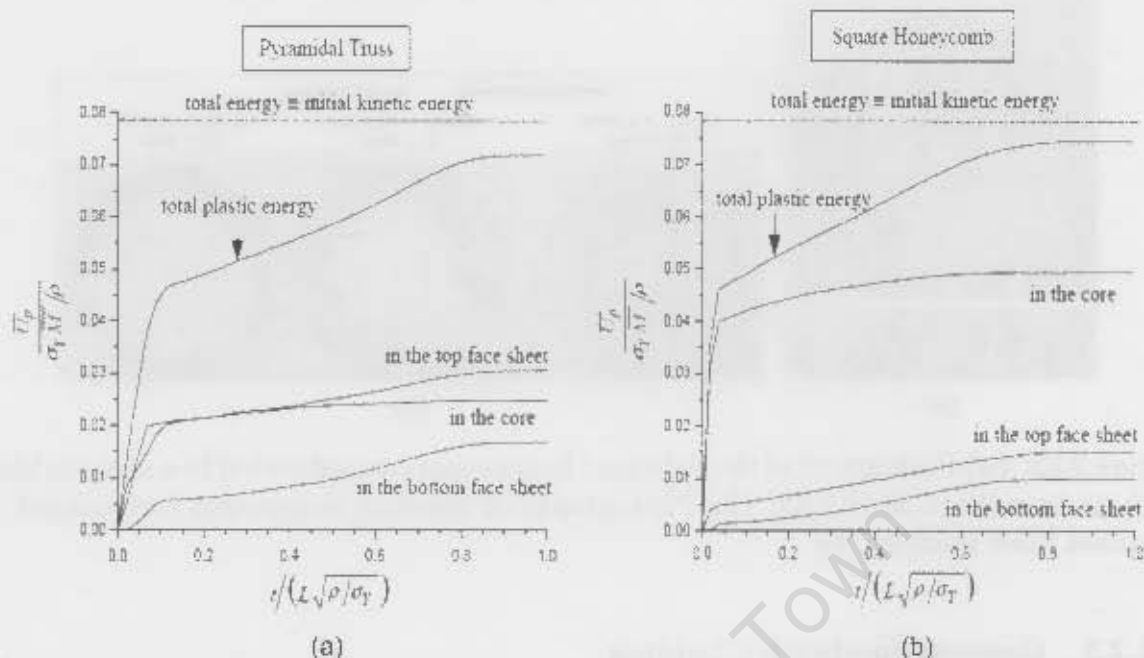


Figure 2.24: Plot of non-dimensional energy partitioning between the face sheets and the core for (a) Tetragonal truss core and (b) Square honeycomb core. The non-dimensional applied impulse applied to the panels is $1/(\bar{M}\sqrt{\sigma_y/\rho}) = 0.25$. [96]

the square honeycomb and folded plate cores significantly outperformed the tetragonal cores due to in-plane stretching. Although the study primarily focused on deflections of the top and bottom face sheets, the partitioning of dissipated plastic energy between the core and the face sheets is also examined. Fig. 2.24 shows the partitioning of the non-dimensional energy as a function of non dimensional time. It should be noted that in Fig. 2.24, $\bar{M}$ and L refers to the mass per unit area and length of the panel respectively. For the given example the non-dimensional applied impulse is $1/(\bar{M}\sqrt{\sigma_y/\rho}) = 0.25$. Note that a much larger portion of the initial kinetic energy is dissipated in the honeycomb core as opposed to the tetragonal truss core. The authors also showed that improvements gained using sandwich panels over solid plates are magnified when subjecting the panels to underwater blasts. It is noted however that the results presented in [97] may be overly optimistic as a complete model of the fluid structure is not used. The authors counter this limitation however by noting that the results presented for the sandwich panel are not fully optimised.

2.2.3 Alternative Core Structures

2.2.3.1 Blast Loading of Web Plate Claddings

Other forms of sacrificial cladding have been proposed in the literature. In terms of blast loading, Guruprasad and Mukherjee investigated analytically and numerically [98] and experimentally [99], a layered web cladding under blast loading. A schematic of a three-layered cladding structure is given in Fig. 2.25a. Each layer is composed of a base plate, a cover plate, and a web plate, all made of mild steel. The layers collapse successively with minimal force transfer to the protected structure.

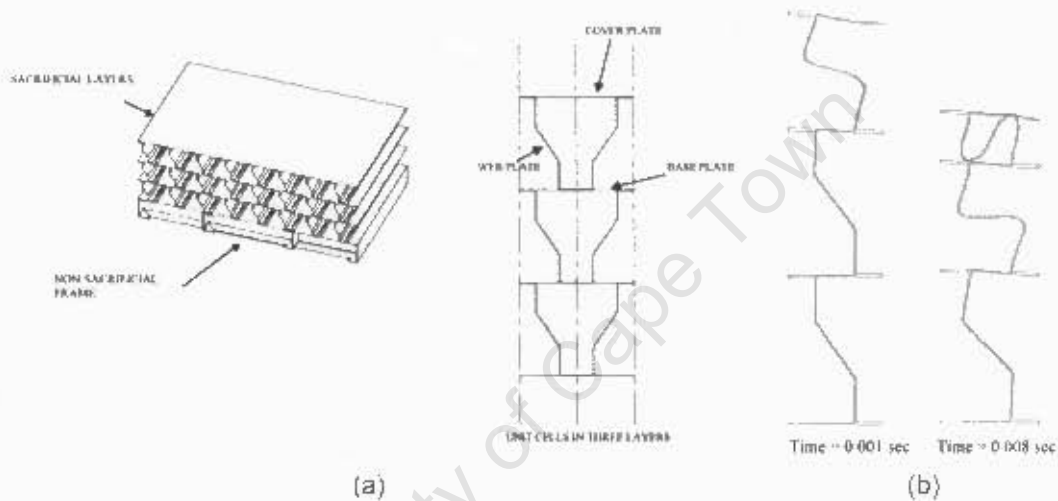


Figure 2.25: (a) Configuration of sacrificial cladding with a non-sacrificial frame. (b) Deformation modes at $t = 0.001$ s and $t = 0.008$ s for a three-layered cladding with a sheet thickness of 1.6 mm. [98]

This configuration has several advantages in terms of energy absorption under blast loads. Plastic hinge locations are well defined and hence the performance of each layer is predictable, consistent and tailorable to the expected blast loading. Strains are small and hence fracture within the web plates is unlikely. The layered cladding in the aforementioned study is also space efficient in that it absorbs a large amount of energy within a given thickness. In Guruprasad and Mukherjee [98] a plate strain model as well a simplified mass-spring model was developed and analyzed under a triangular pressure pulse to simulate the blast loading. The two models showed good correlation with each other. In terms of energy absorption, the cladding structure was found to be efficient and provided blast isolation for the protected structure.

This absorption behaviour was confirmed experimentally in Guruprasad and Mukherjee [99]. The experimental setup is pictured in Fig. 2.26a. The panels had dimensions of 1 m $\times$ 2 m with one to three layers. Each layer had a thickness of 100 mm. TNT explosives

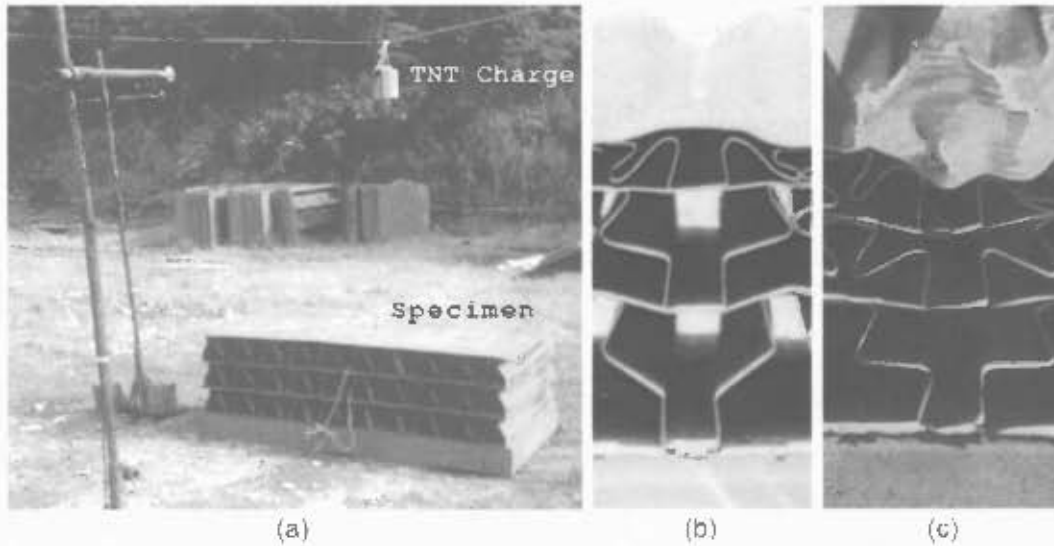


Figure 2.26: (a) Photographs of experimental setup using layered web cladding. (b) Photographs of deformed shape of a layered cladding subject to a 4.1 kg TNT charge positioned 2 m from the panel. The thickness of the web plate is 1.6 mm. (c) Photographs of deformed shape of a layered cladding subject to a 4.1 kg TNT charge positioned 2 m from the panel. The thickness of the web plate is 1.2 mm. [99]

ranging from 2.5 – 5.0 kg were hung above the cladding. Pressure gauges recorded the blast pressure and the deformation of each layer was also recorded. The analytical and experimental results compare reasonably well, however some over-prediction in the deformation was observed due to some modes of dissipation not being included in the simple model. Figs. 2.26b,c show the deformed cladding resulting from two 4.1 kg blasts.

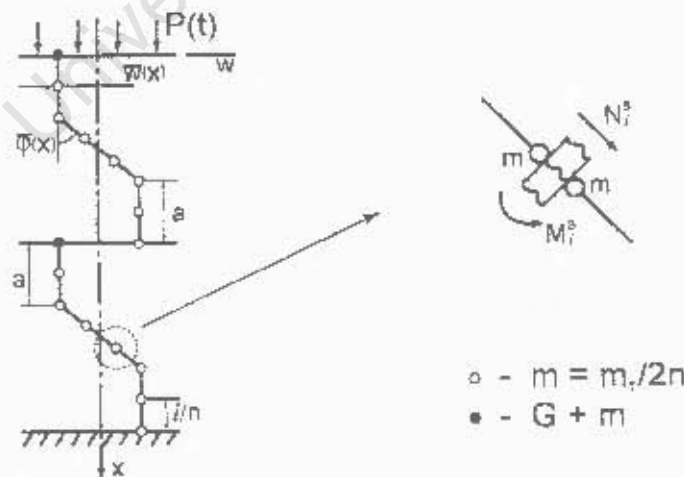


Figure 2.27: Schematic of a mass-spring model for a unit cell of a web plating cladding structure. [100]

Karagiozova and Jones [100] theoretically investigated a layered cladding structure similar to the previously described structure under a triangular blast pulse. The study examined

two layers of mild steel and aluminium web plates using a lumped mass model with links connected by springs as shown in Fig.2.27. Note that the angle of the web plates as shown in the figure is given by φ . In the study, two different values of φ were examined. A small value of φ corresponds to a nearly vertical web plate. The work by Karagiozova and Jones [100] focused on the deformation modes, force transfer between the layers as well as the influence of strain rate. The mild steel claddings showed higher energy absorbing capacity than the aluminium panels. Strain rate sensitivity showed little influence on the absorption properties of the cladding due to the velocity transfer properties of the pressure pulse. Specifically, that the velocity applied to the structure rises gradually as opposed to impact scenarios where the velocity rise is instantaneous. Force transfer for a unit cell with two different starting geometries and materials is given in Figs. 2.28a and 2.28b. The cladding response in Fig. 2.28a is for an aluminium cladding with a larger angle on the web plates. In Fig. 2.28b, the web plates are at a greater angle (i.e. nearly vertical) and made of mild steel. For two layered claddings it was found that stiffer web plates transferred larger forces to the subsequent layer. The stiffness of each layer is defined by the material used and initial angle of the web plates. Small angles can therefore transfer large loads to the subsequent layer and the main structure. Further to this, small angled web plates showed unpredictable deformation modes. Fig. 2.28c shows the energy absorption in each layer of an aluminium web plate with small web plate angle. It is clear from this figure that very little energy is absorbed in the second layer until a distinct point where the first layer collapses.

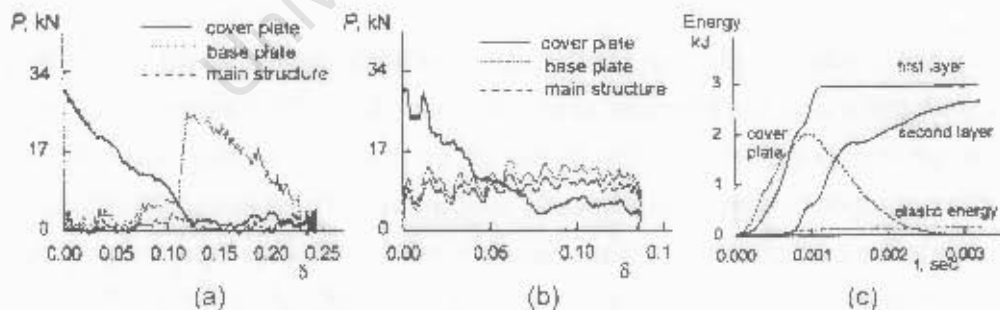


Figure 2.28: Force transfer to the base plate and main structure of a unit cell with the following web plate configurations: (a) Aluminium web plates with a large value of φ . (b) Steel web plates with a small value of φ . (c) Energy absorption within each component of an aluminium cladding with a small value of φ . [100]

2.29a), woven metallic textile, and aluminium honeycomb cores. The energy absorbed per unit mass ($W_m \rho_s$) for the various configurations is given in Fig. 2.30a. All results are normalized with respect to the yield stress of the solid material σ_y . Clearly at low peak strengths (σ_{pk}), honeycomb cores performance is highest. At higher peak strengths, tetrahedral truss structures outperform other structures indicating it may be more suitable for high intensity loading situations.

Queheillait *et al.* [103] investigated the compressive behaviour of metallic lattice hollow truss sandwich structures. Square and diamond structures as shown in Fig. 2.29a are studied. As the truss elements are hollow tubes, the square configuration highly resembles the structure posed in this thesis. Fig. 2.29b shows a square and a diamond shaped lattice under quasi-static compression at 0% and 40% strain. The peak stress σ_{pk} of both lattice structures is a function of the plastic buckling stress of a hollow cylinder and is given as

$$\sigma_{pk} = \pi^2 k^2 E_t \left(\frac{a_o^2 - a_i^2}{4l^2} \right) \sin^2 \omega \bar{\rho} \quad (2.29)$$

where k is a constant defining the rigidity of the joints, E_t is the tangent modulus of the parent material, a_o and a_i are the outside and inside radii of the truss respectively, l is the length of the column, ω is the angle the truss makes with the face sheet, and $\bar{\rho}$ is the relative density. Queheillait *et al.* [103] found that the hollow truss structures have the highest peak strength of all cellular metal topologies presented in the literature.

A comparison into the energy absorption characteristics, similar to that of Kooista *et al.* [102] is given for the diamond and square truss lattices and summarized in Fig. 2.30b. It is clear that such lattices compare favorably with pyramidal lattice truss and metal textiles.

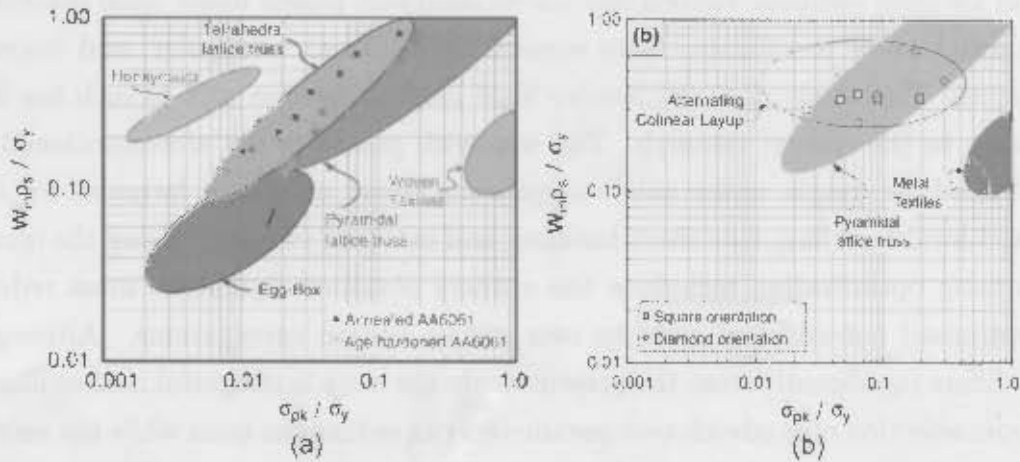


Figure 2.30: (a) Energy absorption per unit mass of various lattice structures. (b) Energy absorption per unit mass of square and diamond orientation lattice structures. [102]

2.2.4 Optimisation of Sandwich Structures

Liang *et al.* [104] conducted a mass optimisation study of corrugated sandwich structures under blast loads. The design variables used in the study are shown in Fig. 2.31. The mass of such a structure is optimised using a combined Feasible Direction Method combined with the Backtrack Programming Method. Axial compression, bending, buckling, and manufacturing considerations are used as constraints. A significant assumption is made that the blast load can be represented by a static pressure. The results show that the corrugation leg (L_c), corrugation angle (θ), core thickness (t_c) and corrugation pitch (S) are important design parameters for the core component with respect to buckling strength, and that the face sheet thickness (t_f) are important design parameters for the face sheet component with respect to axial stress.

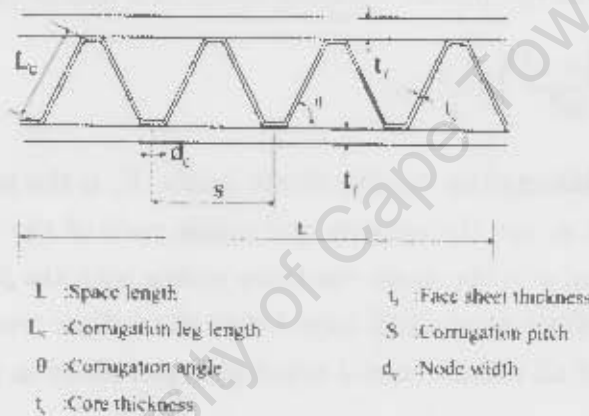


Figure 2.31: Schematic of corrugated sandwich panels with the design variables used in the optimization study by Liang *et al.* [104].

Tian and Lu [105] optimise various corrugated sandwich panels under axial compression. The sandwich cores investigated were square, hat-stiffened, triangular, and trapezoidal corrugations. This study does not involve blast loading however and as such has limited importance to the current research. The sandwich panels in the aforementioned study are optimised for weight under axial compression based on design formulae for critical global and local buckling, face sheet buckling and material yielding. Using the quadratic programming optimization technique the authors obtained significant mass reductions using optimised hat-stiffened sections over simple square corrugations. Although the loading differs significantly from the present study the work is insightful in that illustrates how proper selection of sandwich core parameters can reduce the mass while not sacrificing the structural properties of the panel.

Chapter 3

Material Characterisation

3.1 Introduction

To investigate the response of the proposed panels under blast loading conditions, rigorous material characterization needs to be undertaken. Although mild steel is a relatively well understood material, variation in the manufacturing process and composition of low-carbon steel occurs resulting in widely varying material characteristics. This is particularly true in the current application as the low-carbon steel used for the top plate and for the tubes undergoes a different rolling process (hot rolled and cold rolled respectively). This increases the work hardening in the tubes and must be accounted for when modelling the flow stress of the material.

It should be noted that the steel used for the top and bottom plates are obtained from Vulcan SteelTM while the steel tubes are supplied by TubeconTM. The aluminium tubes are supplied by Non-Ferrous MetalsTM. The chemical composition of the steels used in this study are given in Table 3.1 and the aluminium is given in Table 3.2. For both tube materials, the chemical composition is obtained directly from the supplier, however the chemical composition of the top plate is found in [106].

Table 3.1: Chemical Composition of the steels used in this study

Steel	C %	Mn %	P %	S %	Si %
tubes	0.10	0.60	0.025	0.030	0.30
top plate	0.15	1.00	0.035	0.035	0.30

The modelling of materials under blast loads requires quasi-static and dynamic constitutive parameters. The following sections describe the experimental and analysis proce-

Table 3.2: Chemical composition of the 6063-T6 aluminium used in this study

Cu %	Mg %	Si %	Fe %	Mn %	Zn %	Ti %	Cr %
0.10	0.45	0.2	0.35	0.1	0.1	0.1	0.1

dures used to obtain the constitutive behaviour of the materials used in the panels. To summarize, there are four materials of interest in this study

1. Mild steel cut from square tubes of dimension $25.4 \text{ mm} \times 25.4 \text{ mm} \times 2.04 \text{ mm}$. All material specimens are centred about the centreline of the tube.
2. Annealed mild steel cut from tubes described in the previous point. The tubes are annealed for 60 min at 900°C in an autoclave. The tubes are allowed to cool in the oven and are removed when the temperature in the oven is reasonably near room temperature.
3. 6063-T6 aluminium alloy tubes of dimension $25.4 \text{ mm} \times 25.4 \text{ mm} \times 2.04 \text{ mm}$. The tubes are analyzed only in their as-received state.
4. Mild steel cut from hot rolled 4.0 mm plates.

The as-received mild steel tubes are not subject to a full material characterization as the tubes are not used in either numerical simulations or in experiments. The numerical simulations and experimental work using the steel tubes utilizes only the annealed tubes. In terms of the numerical study conducted in chapter 7, tubes of various aspect ratios are studied. Due to the nature of cold-formed steel tubing, tubes with larger widths will have less work hardening along the longitudinal axis than narrower tubes. For this reason, modelling tubes with various widths using experimental quasi-static and dynamic tests from a single width tube is insufficient. The annealing process removes the work hardening effect on the flow stress in the tubes allowing for characterization of a wide range of cross sections.

3.2 Quasi-Static Tensile Behaviour

3.2.1 Test Apparatus

In order to obtain the quasi-static stress-strain parameters, several tensile tests are performed. All tests are conducted on a 200 kN Zwick universal testing machine at the Centre for Materials Engineering at the University of Cape Town. The tests are performed at a crosshead speed of 2 mm/min which corresponds to an approximate strain-rate of $\dot{\epsilon} = 8.33 \times 10^{-3} \text{ s}^{-1}$. The tensile forces are obtained via a load cell on the machine and transferred via a digital acquisition device to a PC. An extensometer is used and the force as a function of the extensometer clip displacement is recorded. For samples with no extensometer, the crosshead displacement is used.

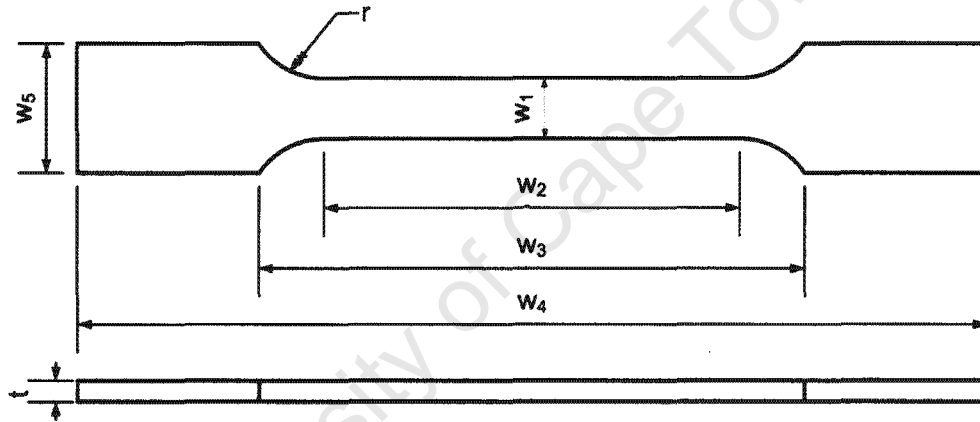


Figure 3.1: Schematic of flat dogbone tensile specimen. The values corresponding to the widths are found in Table 3.3

Tensile specimens are created using several methods. All specimens however use the flat dogbone shape as given in Fig. 3.1. The actual dimensions of the specimens for each of the cases described here is given in Table 3.3. The dimensions are based upon available source material and extensometer properties of the testing machine. Specimen dimensions are based on the British Standard Method for Tensile Testing of Metals [107]. The specimens diverge slightly from the recommendations due to technical requirements of using the extensometer. This does not effect the resulting material characterization in terms of the purpose of this work as the constitutive behaviour is required only for analytical and finite element material models.

For the top plate the specimens were cut from the top plates directly where all plates are cut from a single sheet. The dimensions of these specimens correspond to Specimen A in Table 3.3. To obtain material parameters from the tubes, specimens are obtained using two separate methods. For the mild steel tubes, specimens are cut from 25.4 mm wide

Table 3.3: Tensile specimen dimensions.

Specimen	w_1 (mm)	w_2 (mm)	w_3 (mm)	w_4 (mm)	w_5 (mm)	r (mm)	t (mm)
A	8.0	60.0.0	80.0	170.0	25.0	15.0	4.0
B	8.0	60.0.0	80.0	170.0	25.0	15.0	2.0

tubes with a thickness of 2.0 mm before and after annealing. The aluminium specimens are cut, parallel to the extrusion direction, from tubes with same dimensions as the steel tubes. These tube specimens have dimensions given as Specimen B in Table 3.3. The specimens cut from the annealed tubes have a slightly smaller thickness than nominal values given in Table 3.3 due to loss from scaling.

3.2.2 Construction of True Stress vs. True Strain Curves

This section describes the procedures used to construct the quasi-static true stress vs. true strain curves. The procedure varies somewhat depending on the nature of the material as special attention must be paid to materials that do not conform ideally to the Johnson-Cook model.

For all materials the process begins by determining the plastic displacement vs. load curves. As previously mentioned, a Zwick tensile machine is used to obtain the tensile behaviour in the specimen. The raw data from this process at small displacements is not the exact force behaviour in the specimen and must be accounted for. This occurs primarily for two reasons. Firstly, the deflection is not measured directly on the specimen, but instead is measured by the extensometer. Thus, the compliance of the deflection measuring device will be added to the compliance of the specimen. This is not a significant problem as the determination of Young's modulus is not of importance for the current application. Secondly, at the beginning of the test, the specimen must "bed in" where the grips of the specimen deform to the shape of the the jaws of the test machine.

To overcome the difficulties associated with the test machine, certain steps are taken. When using an extensometer, a linear curve is fit through the region of the load vs. displacement curve corresponding to the approximately linear section. The slope of this curve is denoted as the effective elastic force modulus F_{eff} . This is clearly related to the observed Elastic modulus of the material but has units of force. The curve is shifted so that the linear fit goes through the point $\Delta L = 0$. This removes the "bedding in" described in the previous paragraph. The actual plastic displacement ΔL_{pl} is then com-

puted using Eq. (3.1).

$$\Delta L_{pl} = \Delta L - \frac{F}{F_{eff}} \quad (3.1)$$

The engineering stress σ_e is then calculated as

$$\sigma_{eng} = \frac{F}{A_0} \quad (3.2)$$

where A_0 is the initial area of the specimen. Likewise, the plastic engineering strain is calculated as

$$\varepsilon_{eng}^{pl} = \frac{\Delta L_{pl}}{L_0} \quad (3.3)$$

where L_0 is the gauge length. For all tests this value is $L_0 = 40$ mm. The true logarithmic plastic strain is then calculated using Eq. (3.4) [108]

$$\varepsilon_{pl} = \int_{L_0}^L \frac{dL}{L} = \ln \left(\frac{L}{L_0} \right) = \ln \left(1 + \frac{\Delta L_{pl}}{L_0} \right) = \ln (1 + \varepsilon_{eng}^{pl}) \quad (3.4)$$

where L is the instantaneous gauge length. The true stress in the material is then calculated using Eq. (3.5) [108]

$$\sigma = \frac{F}{A} = \frac{F}{A_0(L_0/L)} = \sigma_{eng} \left(\frac{L}{L_0} \right) = \sigma_{eng} (1 + \varepsilon_{eng}^{pl}) = \sigma_{eng} e^{\varepsilon_{pl}} \quad (3.5)$$

where A is the instantaneous cross sectional area. Eqs. (3.4) and (3.5) are only valid up to the point of necking. That is, up to the point where the strain can be assumed uniform over the gauge length. At this point an instability develops and the strain localises to a point. This is known as the diffuse necking point and is characterised by an increase in strain resulting in a decrease in load. The point at which this occurs is at the Ultimate Tensile Strength (UTS). Figs. 3.12a-c gives the true stress vs. plastic strain up to the UTS for the materials used in this study. The curves shown correspond to the tensile test resulting in the largest engineering strain.

3.2.3 Quasi-Static Results

The plastic engineering stress-strain curves for the four materials used in this study are given in Fig. 3.2. Of particular note is the reduced flow stress and increased failure strain in the annealed steel tubes as opposed to the as-received state.

The purpose of Fig. 3.2 is to illustrate the repeatability of the tensile tests. For all materials, behaviour up to UTS shows good repeatability. For the annealed mild steel

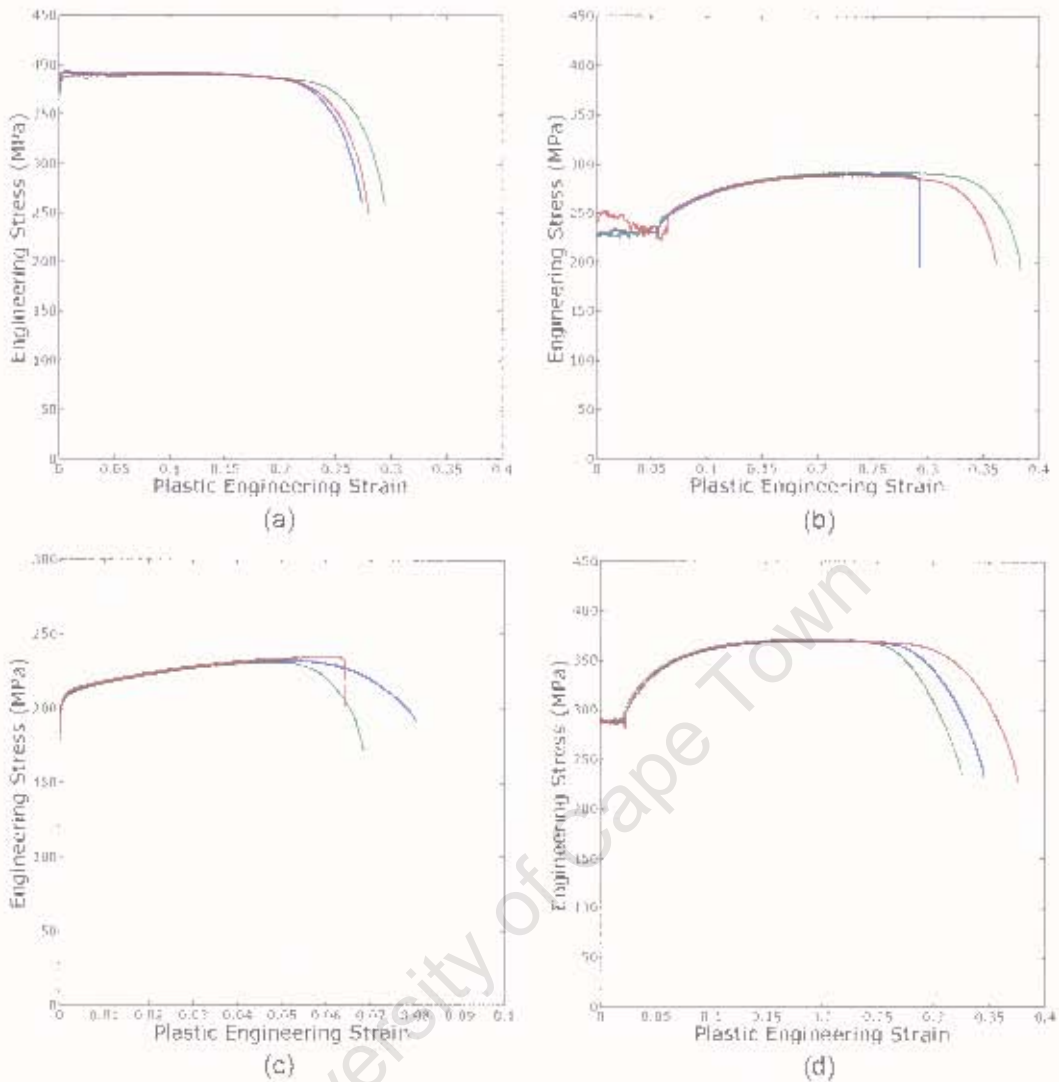


Figure 3.2: Engineering stress-strain curves for (a) As received mild steel tubes, (b) Annealed mild-steel tubes, (c) As received aluminium tubes, (d) Top plate steel.

tubes there is some discrepancy in the liders band but the behaviour after the band is the same for all tests. This is likely due to some slight differences in the annealing of the tubes. This is inconsequential however as the reference test used to compute the quasi-static behaviour is not the test showing the anomalous liders band behaviour.

Also of note is that for the annealed steel tubes and the aluminium tubes is that for each material a test resulted in a vertical stress-strain curve after the UTS. This is a result of necking occurring outside the extensometer where the measured extensometer displacement is zero while the force continues to drop. This is also irrelevant to the analysis as the material behaviour post-UTS is not taken from the force-deformation behaviour of these tensile tests but is instead obtained from tensile test simulations.

3.3 Tensile Test Simulations

The final strain for each of the materials given in Fig. 3.12 is not the true failure strain of the material. The failure strain ε_f (assuming the specimen is stretched to destruction) may be expressed as

$$\varepsilon_f = \ln \left(\frac{A_0}{A_f} \right) \quad (3.6)$$

where A_f is the final measured specimen area. The final area of each specimen is calculated based on the "skewed bowtie" configuration given in Fig. 3.3. Due to the small cross sectional area of the specimens cut from the tubes, accurate measurements of this areas are not possible and more advanced techniques are required. The specimens cut from the top plate are large enough for this type of analysis however more advanced techniques are used for the sake of accuracy.

Initially, the values of each dimension in the figure (th_m , th_l , th_r and w). are determined for both cross-sections of each top plate specimen and averaged. The failure stress is calculated simply as

$$\sigma_f = \frac{F_f}{A_f} \quad (3.7)$$

where F_f the tensile force at failure. Although Eq. (3.7) gives a reasonable approximation to the stress at fracture, it does not account for the triaxial stress state of the material at rupture. The classical method used to account for this is given by Bridgeman [109], however this only applies to cylindrical specimens. For the flat dogbone specimens used in this study, the MLR [110] scaling factor is used and is given by

$$S_{MLR} = 1 - 0.6058(\varepsilon_f - \varepsilon_{neck})^2 + 0.6317(\varepsilon_f - \varepsilon_{neck})^3 - 2.1070(\varepsilon_f - \varepsilon_{neck})^4 \quad (3.8)$$

where ε_{neck} is the true strain where diffuse necking begins. The failure stress is therefore multiplied by Eq. (3.8) to give the corrected failure stress σ_f^{corr} . The MLR method is still not ideal as it assumes a constant stress state across the face of the specimen. As such, more advanced techniques are employed.

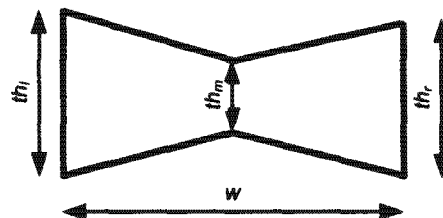


Figure 3.3: Idealised geometry of the fractured surface in the tensile specimen.

Although Eq. (3.8) can be used for a direct correction of tensile failure stress, the use of tensile simulations is much more accurate. This is particularly true in the case of the thin specimens cut from the thin tubes. In these cases predicting the failure stress and strain is very difficult without the use of simulation due to the very small area of fracture and unknown geometry of this area. Even with the larger top plate specimens the failure strain will not be accurate as the resulting value will be an average strain over the fractured specimen face. True failure strains can be approximated from simulation by examining strains at fracture in individual elements. Material failure is not required for the present study however such an analysis is included for a complete material characterization.

The simulation of quasi-static and dynamic tensile tests is a standard practice to verify and validate constitutive models [111–117]. Of particular note is Wierzbiki and Lee [116] where the authors utilized tensile test simulations to construct the post UTS stress strain curve behaviour of high strength steel for use in blast simulations.

With the use of tensile test simulation, a trial stress-strain curve is first determined which is simply a guess as to true constitutive behaviour of the material. The tensile tests discussed in section 3.2.1 can then be simulated using finite element analysis. The simulated specimen can be extended up to the point of extensometer extension in the experimental tensile test. The resulting force-displacement curve is then compared to the experimental curve. The trial stress-strain curve is then modified until the simulated and experimental force-displacement curves agree acceptably well. Due to the different damage mechanisms in the mild steel and aluminium specimens a different methodology is carried out to find the trial stress-strain curves.

All tensile simulations use the experimentally obtained true stress-strain data up to UTS. For the mild steel (annealed tubes and top plate), the post UTS behaviour is approximated by projecting the tangent slope of the true stress strain curve up to a strain of 3.0. This strain is arbitrary provided it is larger than the true fracture strain of the material. The tangent slope σ'_0 is computed by fitting a straight line through the final ten points of the true stress strain curve up to the UTS. The slope of this line is taken as the tangent slope. This is done as a starting point as it results in the maximum allowable flow stress at any given strain after UTS. The slope of this line is then adjusted until the experimental force-displacement behaviour is obtained. Specifically, the value of μ in Eq. (3.9) is adjusted until convergence is obtained.

$$\sigma' = \mu \sigma'_0 \quad (3.9)$$

The stress-strain curves are input into ABAQUS/Explicit in tabular form. The spacing between all points up to a strain of 3.0 is based on using 100 points up to the UTS. This gives a uniform strain spacing for a given material, however the spacing will be slightly

different for each material. For all materials the experimental force-displacement curves are compared to the experimental curve resulting in maximum tensile deflection.

The post-UTS behaviour of the aluminium specimens is significantly different from the mild steel. Fracture occurs soon after UTS is reached and the slope is expected to approach zero. To simulate this an approach similar to Lee and Wierzbicki [116] is used. The true stress-strain curve from the experiments is used up to UTS and a power law of the form $A + B\varepsilon^n$ is used for the post UTS behaviour. The trial curve is obtained by using the stress at a strain of 0.5 as a control point. This stress is denoted as σ_t . The values of A , B and n are then computed such that the stress at UTS and the slope at UTS agree with the experimental curve and that the power law goes through the control point. The stress at the control point is iteratively modified until reasonable agreement between the experimental and computed force-displacement curves.

At this point it would be beneficial to discuss the finite element mesh used in the tensile simulations. Fig. 3.4 shows the finite element mesh for the annealed mild steel used in tensile simulations. The finite element model for all materials is essentially the same with only small variation in the parallel width and specimen thickness. Note the gradual increase in mesh density nearing the central point of the model. This is required as localised necking significantly elongates the elements in this region.

Due to the high computation cost of tensile simulations, symmetry in the tensile specimen is exploited to reduce the computational work at each time increment. It is clear from the Fig. 3.4 that symmetry boundary conditions are applied at the longitudinal and transverse midsurface. In terms of the degrees of freedom this implies that $u_2 = \theta_1 = \theta_2 = 0$ on the bottom surface (s_1) while $u_1 = \theta_2 = \theta_3 = 0$ on the front facing surface (s_2) in Fig. 3.4 where u_i refers to nodal displacements in the i direction and θ_i refers to nodal rotations about axis i . Symmetry is also applied on the midsurface in respect to the thickness of the specimen. This implies $u_3 = \theta_1 = \theta_2 = 0$ on the surface s_3 . Therefore only $1/8^{\text{th}}$ of the actual tensile specimen is modelled.

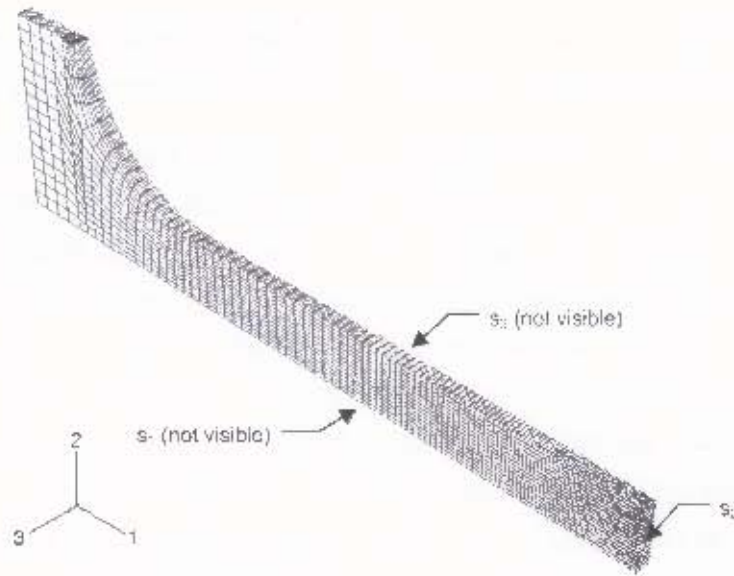


Figure 3.4: Finite element mesh of $1/8^{\text{th}}$ symmetry specimen model used in tensile simulations.

3.3.1 Mass Scaling

The difficulty in simulating tensile tests is that the tests have a long absolute time duration under quasi-static conditions. Using ABAQUS/Explicit with element-by-element stability the time step is computed based on Eq. (3.10).

$$\Delta t \leq \min \left(L_e \sqrt{\frac{\rho}{\hat{\lambda} + 2\hat{\mu}}} \right) \quad (3.10)$$

where L_e is the characteristic element length associated with an element. The variables $\hat{\lambda}$ and $\hat{\mu}$ are the effective Lamé's constants for the material. Both of these constants are proportional to the Elastic Modulus of the material.

From Eq. (3.10) it is clear that the critical time increment is based on the material density and hence the elastic wave speed of the material. To overcome this difficulty, the method of mass scaling is used where the density of the material is adjusted in order to obtain a particular time step. With an increased element mass, the time required for an elastic wave to propagate through an element is increased and hence a larger stable time step can be used. This methodology may only be used at low velocities where inertia effects are negligible. Artificially increasing the element mass influences the inertia properties just as using a higher rate of loading. Even at low velocities, care must be given to ensure that a convergent solution is obtained. If the mass scaling is too great in magnitude, a solution can be obtained that is inaccurate due to inertia effects. This is particularly difficult in the case of tensile test simulations as the mesh density changes significantly after localised necking.

To determine the ideal mass scaling a convergence study is undertaken. The ideal mass scaling parameters are sought which give suitable results while maintaining a tractable completion time on the computational facilities available. To determine the ideal mass scaling scheme, three scaling schemes are examined for the initial case where the slope of the true stress-strain curve equals the tangent of the curve up to UTS. The first scheme involves a target time increment of 5×10^{-7} s with the time step updated every 500 intervals. The second scheme involves a target time increment of 2×10^{-7} s with the time step updated every 100 intervals. Finally, a very small mass scaling scheme is used where the target time increment of 1×10^{-7} s with the time step updated every 100 intervals. Tests at this fidelity are extremely computationally expensive and hence if the stress-strain curve at less expensive models yields reasonable results, these mass scaling parameters should be used. These schemes are summarized in Table 3.4.

Table 3.4: Mass scaling schemes used in the convergence study of tensile simulations.

Scaling scheme	Target time increment (s)	Intervals between updates
1	5×10^{-7}	500
2	2×10^{-7}	100
3	1×10^{-7}	100

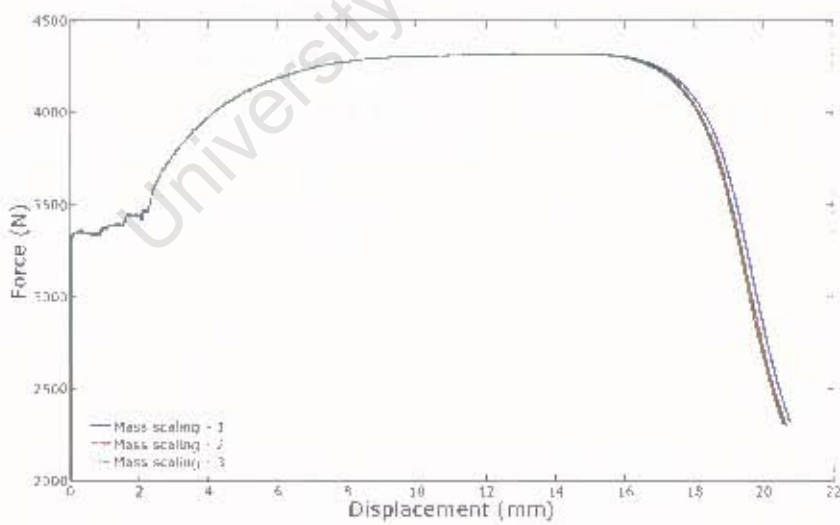


Figure 3.5: Plot of results of the mass scaling convergence study.

Simulations are conducted using each of these mass scaling schemes using a simulated crosshead speed of approximately 0.033 m/s. The material used for the simulations is the annealed mild steel. The densities of the top plate steel are the same and therefore a separate convergence study is not required. The density of the aluminium is less than that of the steel and therefore mass scaling scheme used for the steel will be conservative for

The stress at the transmitter bar/specimen interface is

$$\sigma_{s_2}(t) = \frac{\sigma_t(t)A_{\text{bar}}}{A_s(t)} \quad (3.42)$$

The use of Eq. (3.41) is termed a ‘2-wave analysis’ [120] as it uses information from two waves (σ_i, σ_r) while Eq. (3.42) implies a ‘1-wave analysis’. The 2-wave analysis is the stress at the front of the specimen and is inherently subject to large oscillations, primarily near the yield stress. Therefore, the stress vs. strain curve is then constructed by combining Eqs. (3.38) and (3.42) which removes time as a variable.

3.4.4 Dynamic Test Results

The dynamic stress-strain curves, including the quasi-static true stress-strain curves for each of the studied materials is given in Fig. 3.12. The curves are displayed such that the maximum dynamic or static (up to UTS) strain is given. Note that the dynamic compression tests for the aluminium specimens give much larger strains than the quasi-static tensile tests. On the other hand, the thick top plate specimens give smaller strains than the quasi-static tensile tests.

There is a large differential between the strain-rates tested, however all are of the order $\dot{\epsilon} \approx 10^3 \text{ s}^{-1}$. The variation is due the required specimens sizes and strength of the materials at high strain-rate. Tests are first done on the annealed mild steel specimens with impact velocities appropriate to the bar diameter and specimen size. strain-rates in the range of $\dot{\epsilon} \approx 1800 - 3400 \text{ s}^{-1}$ are tested. The range of strain-rates is obtained by varying the striker mass and gas gun pressure. The wider range of strain-rates tested for the annealed tubes over the other materials is due to the nature of the present application. Throughout this study, the tubes are subjected to a relatively large range of strain-rates and therefore it is advantageous to include more experimental data when determining rate dependent model parameters.

Strain-rates above and below the previously described range are not possible for the given specimen size and testing equipment. Larger strain-rates would require impact velocities that are too large therefore risking the bar becoming plastic and in addition, excessive compression of the specimens can give poor results. Similarly low strain-rates require a much larger striker than the current gas gun can utilize. To obtain strain-rates in the top plate specimens at the same magnitude as the thin annealed specimens would require impact velocities that could damage the bars and strain gauges. To obtain these strain-rates, thinner specimens would need to be machined. Testing the aluminium specimens presents further difficulty due to the small specimens and low strength even at high

the aluminium. The stress-strain curve used in the convergence study corresponds to the post-UTS behaviour obtained from using the tangent slope at UTS up to a strain of 3.0. Fig. 3.5 shows the results of the study. Of concern for the convergence study is of course, if the numerical force-deformation curves converge. As can be seen from the figure, mass scaling schemes 2 and 3 give very similar results clearly indicating convergence. As the mass scaling scheme 3 requires several hours more computational time than scheme 2, the time increment and increment updating scheme used for the simulations corresponds to scheme 2.

3.3.2 Simulation Results

Tensile test simulations are conducted on the three materials used in this study. The optimal curves are found by manual adjustment of the corresponding stress parameter described in the previous section. The search for the optimal value is concluded when the experimental and numerical curves agree visually. The true optimal parameters are beyond the scope of this study as this would require complete mathematical optimization. Fig. 3.6 shows the results of the tensile simulations. For each material, the initial approximation and optimal force-displacement curves are given up to the onset of fracture in the experimental curve. Exact curves are not attainable for any of the materials as damage is not included in the model. This is acceptable as the resulting stress-strain curves would not be significantly different and exact fracture strains are not required for this study. Examining Fig. 3.6 shows that the force-displacement behaviour at the onset of localised necking is captured well using the techniques presented here. For the aluminium however, the optimal curve parameter captures the onset of localised necking well but begins to diverge near the point of fracture. This is likely a result of not including damage in the material model.

Although not directly relevant to the current study the fracture strain of the material can be determined with reasonable accuracy using tensile test simulation. Eq. (3.6) determines the average strain over the fracture surface. In reality, the strain is not uniform across this surface when fracture initiates. Fig. 3.7 shows the plastic strain distribution at the final tensile force measured by experimentation. For both steels the fracture strain is approximately 200 % while for the aluminium the fracture strain is approximately 70 %. For all materials the strain attains its maximum value at the centroid of the specimen and decreases towards the outer surface of the specimen. The fracture strain has been shown to be mesh sensitive however and therefore using this technique for fracture applications requires the fracture strain to be presented as a function of element size.

Fig. 3.8 gives the final quasi-static true stress-strain curves obtained from experimentation and numerical simulation. It is clear from these plots that the post-UTS hardening

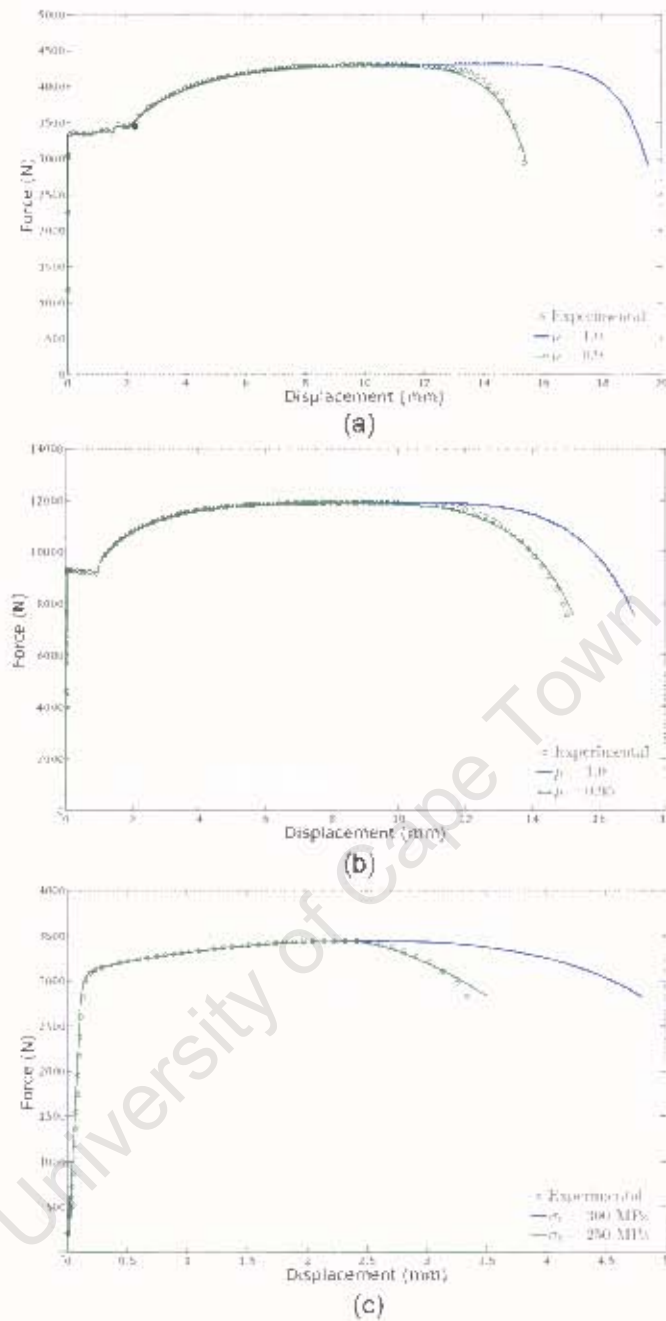


Figure 3.6: Simulated and experimental force-displacement curves in obtaining the post-UTS tensile behaviour of (a) Annealed steel tubes (b) Top plate steel and (c) Aluminium tubes.

behaviour of the steels corresponds almost to the tangent slope at UTS. It is also interesting to note that the hardening is quite similar between both steels however the yield stress of the annealed steel is much lower.

Of note is that for the aluminium, the post UTS flow stress shows only negligible strain hardening, particularly when compared to the steels. This is expected however as the material is a T6 temper aluminium alloy. For such alloys, researchers have assumed horizontal stress-stress strain curves beyond UTS [118]. For comparative purposes the

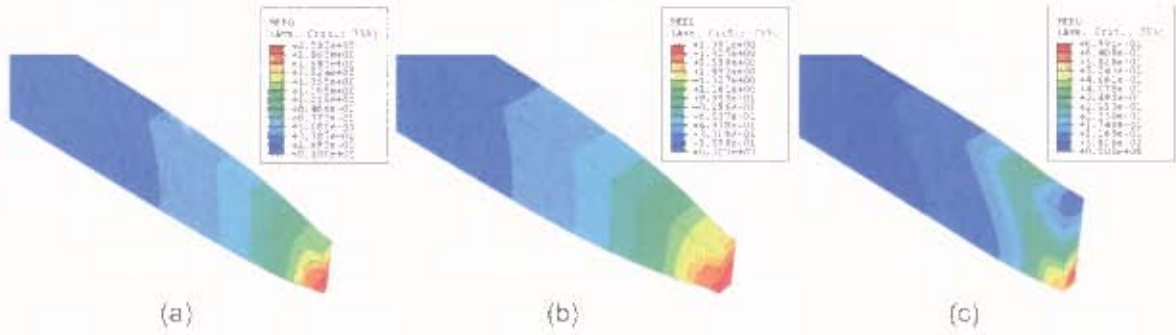


Figure 3.7: Contour plot of plastic strain localisation for (a) Annealed steel tubes (b) Top plate steel and (c) Aluminium tubes.

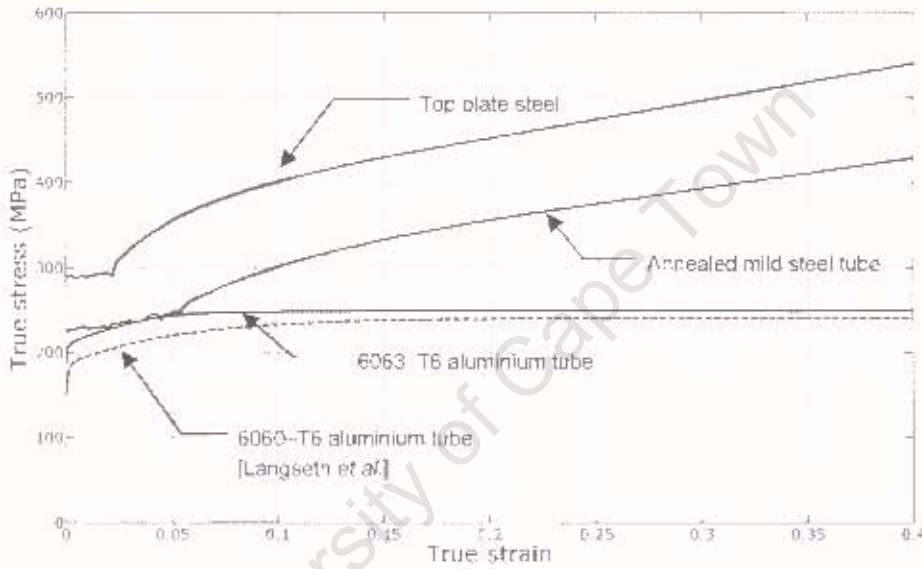


Figure 3.8: Final true stress-strain curves obtained from experimentation and simulation for the materials used in this study.

material model used by Langseth *et al.* [119] for 2.0 mm 6060-T6 aluminium alloy extrusions is presented in Fig. 3.8. The model used is a five parameter model where the equivalent plastic flow stress is obtained as

$$\bar{\sigma} = \sigma_0 + \alpha\beta + (1 - \alpha)\beta - \sigma_v \quad (3.11)$$

where

$$\beta = Q_1 [1 - \exp(-C_1 \bar{\epsilon}_p)] + Q_2 [1 - \exp(-C_2 \bar{\epsilon}_p)] \quad (3.12)$$

For the 6060-T6 alloy the material parameters are $\sigma_0 = 152$ MPa, $C_1 = 861.90$, $C_2 = 19.29$, $Q_1 = 35.10$ N/mm², $Q_2 = 54.00$ N/mm², $\sigma_v = 0$. The alloy used in [119] is a 6060 alloy and hence the initial yield stress is lower, however the hardening properties at large strains are similar.

3.4 Dynamic Material Behaviour

3.4.1 Test Apparatus

In order to characterise the dynamical material properties for the materials used in this study, dynamic material testing is undertaken using a Split Hopkinson Pressure Bar (SHPB). Detailed descriptions of the SHPB setup and theory are documented extensively in the literature. A good overview was developed by Gray [120] while details of the SHPB used for the current work can be found in Marais [121]. For the current work an existing SHPB system is used. The experimental schematic is given in Fig. 3.9.

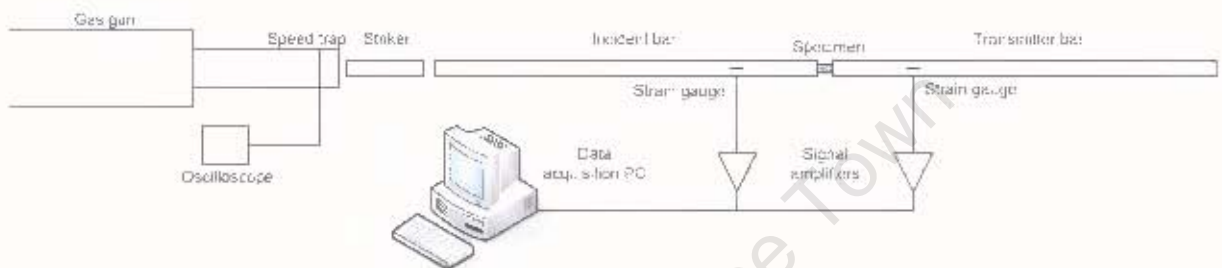


Figure 3.9: Schematic of the SHPB test apparatus used in this work.

The gas gun has an associated pressure valve that can launch the striker using the specified pressure. A speed trap is attached to the barrel of the gun which transmits a signal to an oscilloscope. As the distance between trap points is known (81 mm) the velocity of the projectile can be obtained. This is significant for calculating the calibration factor of the system.

The cross-sectional diameter of the striker is 19.03mm and for most tests has a mass of 546.4 g however some of the tests are conducted using a longer striker of mass 864.7 g. The incident and transmitter bars are identical and have a length of 2.0 m and a cross sectional diameter of 20.0 mm. The bars are constructed using silvered steel. Strain gauges are mounted longitudinally on the incident and transmitted bars. The strain gauges are positioned precisely 1.0 m from the face adjacent to the specimen. The bars are then mounted to the system using bearing and alignment fixtures that allow the bars to move freely while maintaining precise axial alignment. The voltages recorded at the strain gauges are transferred through an amplifier and recorded by the data acquisition PC.

Cylindrical specimens are cut from all materials. For specimens cut from the aluminium and annealed mild steel tubes the nominal specimen diameter is $d_0 = 4.0$ mm with nominal length $l_0 = 2.0$ mm. For the specimens cut from the top plate the dimensions are nominally $d_0 = 5.0$ mm and $l_0 = 4.0$ mm. The dimensions for the tube samples

correspond to optimal dimensions given by Gray [120] based on minimization of the error due to longitudinal and radial inertia. This requirement gives the relationship

$$\frac{l_0}{d_0} = \sqrt{\frac{3\nu}{4}} \tag{3.13}$$

where l_0 and d_0 are the initial specimen length and diameter respectively and ν is the poisson ratio of the test material. For material with $\nu = 0.33$ this gives an optimal ratio of $l_0/d_0 = 0.5$. Using this ratio for the top plate specimens however is not possible as the specimen to bar diameter becomes too large to ensure that bar will not become plastic during the test.

3.4.2 Calibration of the SHPB

In order to convert the voltage signal from the strain gauges, the calibration factor K of the system must be determined. Three methods are presented here for determining K . For each set of tests that are conducted, all three methods are used to determine K as a check to ensure there are no problems with the setup. For a calibration test, the SHPB is set up as in Fig. 3.9 except that there is no specimen and the two bars in perfect contact effectively resulting in one long bar. The projectile is fired at an arbitrary velocity and this velocity is recorded by the speed trap. The resulting signal in each bar is typical of that in Fig. 3.10a.

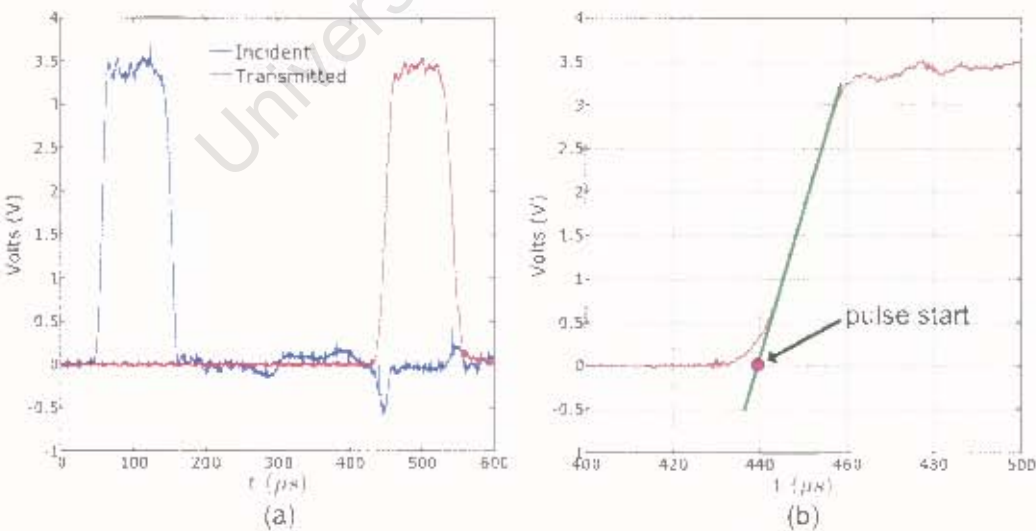


Figure 3.10: (a) Typical voltage signal used to calibrate the SHPB. (b) Pulse idealisation used in determining elastic wave speed.

Firstly, the theoretical calibration factor is presented. It can be shown that stress is

related to the voltage in the strain gauge by

$$\sigma(t) = \left[\frac{4E_{\text{bar}}}{G_{\text{amp}}K_{\text{gf}}NV_{\text{in}}} \right] V_{\text{read}}(t) \quad (3.14)$$

where E_{bar} is the Young's modulus of the bar. For the setup used in this study, the remaining constants are $K_{\text{gf}} = 2.11$, $N = 4$, $G_{\text{amp}} = 1000$. Therefore

$$\sigma(t) = \left[\frac{E_{\text{bar}}}{2110V_{\text{in}}} \right] V_{\text{read}}(t) = K_{\text{th}}V_{\text{read}}(t) \quad (3.15)$$

where

$$K_{\text{th}} = \frac{E_{\text{bar}}}{2110V_{\text{in}}} \quad (3.16)$$

Therefore the theoretical calibration factor is dependent only on the Young's modulus of the bar and value of $V_{\text{in}}(t)$ which is read directly from the test equipment. The advantage of this calibration factor is that it is not influenced by dispersion and requires no extra tests to be carried out. It is however highly dependent on the accurate calculation of E_{bar} . The determination of this value is discussed later in this section.

The second calibration factor used is the momentum balance calibration factor K_{mom} . This calibration factor is based on the momentum of the striker before and after impact with the incident bar. The rebound velocity of the striker is calculated as

$$v_{\text{reb}} = \left[\frac{\frac{A_s}{A_b} - 1}{1 + \frac{A_s}{A_b}} \right] v_0 \quad (3.17)$$

where A_s and A_b are the striker and bar cross sectional areas respectively and v_0 is the initial velocity of the striker as measured by the speed trap in Fig. 3.9. The total impulse transferred during impact is given by

$$I = M_{\text{striker}}(v_0 - v_{\text{reb}}) \quad (3.18)$$

where M_{striker} is the mass of the striker. It can then be shown that

$$I = \int_{t_1}^{t_2} A_b \sigma(t) dt = A_b K_{\text{mom}} \int_{t_1}^{t_2} V_{\text{read}}(t) dt \quad (3.19)$$

where t_1 and t_2 are the start and end times of the incident pulse. The exact start times of the pulse are difficult to determine. In order to obtain a consistent method, the incident pulse is integrated until the integral begins to decrease (loss of momentum). This may slightly underestimate the integral of Eq (3.19), however this amount is insignificant.

Therefore

$$K_{\text{mom}} = \frac{I}{A_b \int_{t_1}^{t_2} V_{\text{read}}(t) dt} \quad (3.20)$$

This calibration factor is strongly influenced by the calculation of the rebound velocity v_{reb} and on the dispersion present in the incident signal.

The final calibration factor is the stress calibration factor K_s . The stress in the bar at a given time is given as

$$\sigma_b = \rho C_0 v_b \quad (3.21)$$

where ρ is bar density, v_b is the particle velocity in the bar, and C_0 is the speed of sound in the bar given by

$$C_0 = \sqrt{\frac{E_{\text{bar}}}{\rho}} \quad (3.22)$$

The maximum stress in the bar is obtained when the particle velocity is $v_b = v_0 \frac{A_s}{A_b} / \left(1 + \frac{A_s}{A_b}\right)$. Therefore, the maximum stress in the bar is given by

$$\sigma_b^{\text{max}} = \frac{\rho C_0 v_0 \frac{A_s}{A_b}}{1 + \frac{A_s}{A_b}} \quad (3.23)$$

The average maximum voltage $V_{\text{read}}^{\text{max}}$ is found by integrating the voltage over the pulse duration and dividing by the pulse duration. Therefore the stress calibration factor is given by

$$K_s = \frac{\sigma_b^{\text{max}}}{V_{\text{read}}^{\text{max}}} \quad (3.24)$$

The final consideration is the calculation of the sound speed C_0 and hence the Young's modulus of the bars. An accurate method for doing this is to use the incident and transmitted pulses of Fig. 3.10. The distance between the strain gauges on the incident and the transmitter bars is 2.0 m. If the time between pulses is determined from a calibration test, the elastic wave speed is therefore known. Unlike the previously described impulse calculation, determining the exact start time of the pulse is essential. To do this, the rise component of the incident and transmitted pulse is fitted with a straight line as shown in Fig. 3.10b. The point of crossing with the time axis is taken as the start of the pulse. This will slightly overestimate the start time of each pulse however we are only interested in the time differential which is very accurate using this method. Once C_0 is determined Eq. (3.22) is then used to compute the Young's modulus.

Table 3.5: Calibration data and bar properties used in the SHPB experimental setup.

Test	$V_{in}(V)$	C_0 (m/s)	E_{bar} (GPa)	K_{th} (MPa/V)	K_{meas} (MPa/V)	K_s (MPa/V)
1	2.54	5132	206	38.44	36.92	37.83
2	2.53	5140	207	38.72	37.48	37.51
3	2.53	5117	205	38.37	37.82	37.69
Avg.	-	5130	206	-	37.41	37.68

Table 3.5 gives the calculated bar properties and calibration factors used in this work. The resulting values of C_0 and E_{bar} correspond well with published values of silvered steel. Also of note is the good correlation between the calibration factors. This provides a high level of confidence that the SHPB system is configured correctly. The theoretical calibration factors will vary depending on the amplification of the voltage through the strain gauges. It is therefore much more important that the calibration factors are similar within a single calibration test. The three calibration tests were conducted at different testing sessions. The second session involved the actual material test used in this study. The first is shown for repeatability. The momentum calibration factor is the constant used to compute the stress-strain curves in the following sections. This is because the momentum calibration factor most closely resembles the actual conditions within the bar as opposed to theoretical values associated with the the strain gauges. Any misalignment of the gauges will not be reflected using the theoretical value.

3.4.3 SHPB Data Analysis

This section describes the procedures used to analyze the data from the SHPB and hence construct dynamic stress-strain curves for the test materials. To begin, a brief derivation of the primary SHPB equations is given. The derivations are based on elementary one-dimensional wave propagation theory. For elastic bars such as those used here, the theory is very well understood and can be found easily in the literature [121] [122] .

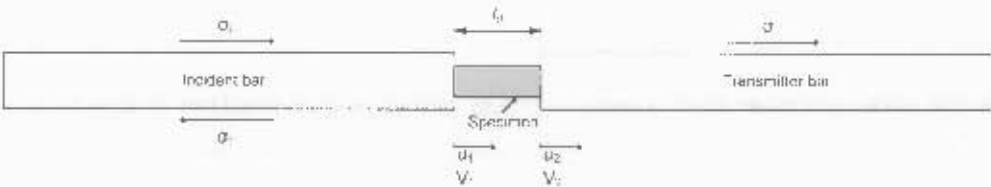


Figure 3.11: Schematic of the SHPB with stresses in the bar and specimen variables

From elementary wave theory, the one dimensional wave equation is given by the second

order partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{C_0^2} \frac{\partial^2 u}{\partial t^2} \quad (3.25)$$

Where C_0 is again the sound speed in the bar. The general solution to Eq. (3.25) is given as

$$u(x, t) = f(x - C_0 t) + g(x + C_0 t) = u_i + u_r \quad (3.26)$$

which reduces to

$$u(x, t) = f(x - C_0 t) \quad (3.27)$$

if considering wave motion only in the positive x direction as given in Fig. 3.11. The strain in the bar is therefore found by differentiating Eq. (3.27) to get

$$\varepsilon(x, t) = \frac{\partial u(x, t)}{\partial x} = f' \quad (3.28)$$

Similarly, the particle velocity in the bar is given as

$$v(x, t) = \frac{\partial u(x, t)}{\partial t} = -C_0 f' \quad (3.29)$$

where f' denotes the derivative of f with respect to $z = x - C_0 t$. Substituting Eq. (3.28) into Eq. (3.29) gives

$$\frac{\partial u}{\partial t} = -C_0 \frac{\partial u}{\partial x} \quad (3.30)$$

which clearly reduces to

$$v = -C_0 \varepsilon \quad (3.31)$$

Making the substitution $\sigma = E\varepsilon$ and $E = C_0^2 \rho$ reduces this equation to

$$\sigma = -\rho C_0 v \quad (3.32)$$

The negative sign indicates that a tensile pulse moves in the positive x direction, however the standard convention in wave theory is take compressive pulses as positive and hence the negative sign is removed. The particle velocities $\Delta v_i, \Delta v_r, \Delta v_t$ due to each of the pulses indicated in Fig. 3.11 can then be written as

$$\Delta v_i = \frac{\sigma_i}{\rho C_0}, \quad \Delta v_r = -\frac{\sigma_r}{\rho C_0}, \quad \Delta v_t = \frac{\sigma_t}{\rho C_0} \quad (3.33)$$

where each of the stresses is found by multiplying the voltage signals by the theoretical calibration factor K_{th} . The velocity of the specimen at the incident bar/specimen interface v_1 is simply the sum of the particle velocities induced by the incident and reflected waves. In terms of the stress signal this gives

$$v_1(t) = \frac{1}{\rho C_0} [\sigma_i(t) - \sigma_r(t)] \quad (3.34)$$

Integration of Eq. (3.34) gives the displacement at the incident bar/specimen interface. This gives

$$u_1(t) = \int_0^t v_1(t) dt = \frac{1}{\rho C_0} \int_0^t [\sigma_i(t) - \sigma_r(t)] dt \quad (3.35)$$

Similarly, the displacement at the transmitter bar/specimen interface is calculated as

$$u_2(t) = \frac{1}{\rho C_0} \int_0^t \sigma_t(t) dt \quad (3.36)$$

The instantaneous length of the specimen is therefore

$$l_s(t) = l_0 - u_1(t) + u_2(t) \quad (3.37)$$

The strain in the specimen can now be determined using Eq. (3.38)

$$\varepsilon(t) = \int_{l_s(t)}^{l_0} \frac{dl}{l} = \ln l_0 - \ln l_s(t) = \ln \left(\frac{l_0}{l_s(t)} \right) \quad (3.38)$$

The strain-rate is then calculated as

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d}{dt} \left[\ln \left(\frac{l_0}{l_s(t)} \right) \right] = \frac{u'_1(t) - u'_2(t)}{l_s(t)} = \frac{v_1(t) - v_2(t)}{l_s(t)} \quad (3.39)$$

Finally, to obtain the true stress in the specimen, incompressible plasticity is assumed therefore $A_0 l_0 = A_s(t) l_s(t)$ where A_0 is the initial cross-sectional area of the specimen and $A_s(t)$ is the instantaneous cross-sectional specimen area. Therefore the instantaneous area of the specimen is

$$A_s(t) = \frac{l_0 A_0}{l_s(t)} \quad (3.40)$$

The stress at the incident bar/specimen interface is then

$$\sigma_{s1}(t) = \frac{[\sigma_i(t) - \sigma_r(t)] A_{bar}}{A_s(t)} \quad (3.41)$$

The stress at the transmitter bar/specimen interface is

$$\sigma_{s2}(t) = \frac{\sigma_t(t)A_{\text{bar}}}{A_s(t)} \quad (3.42)$$

The use of Eq. (3.41) is termed a ‘2-wave analysis’ [120] as it uses information from two waves (σ_i, σ_r) while Eq. (3.42) implies a ‘1-wave analysis’. The 2-wave analysis is the stress at the front of the specimen and is inherently subject to large oscillations, primarily near the yield stress. Therefore, the stress vs. strain curve is then constructed by combining Eqs. (3.38) and (3.42) which removes time as a variable.

3.4.4 Dynamic Test Results

The dynamic stress-strain curves, including the quasi-static true stress-strain curves for each of the studied materials is given in Fig. 3.12. The curves are displayed such that the maximum dynamic or static (up to UTS) strain is given. Note that the dynamic compression tests for the aluminium specimens give much larger strains than the quasi-static tensile tests. On the other hand, the thick top plate specimens give smaller strains than the quasi-static tensile tests.

There is a large differential between the strain-rates tested, however all are of the order $\dot{\epsilon} \approx 10^3 \text{ s}^{-1}$. The variation is due the required specimens sizes and strength of the materials at high strain-rate. Tests are first done on the annealed mild steel specimens with impact velocities appropriate to the bar diameter and specimen size. strain-rates in the range of $\dot{\epsilon} \approx 1800 - 3400 \text{ s}^{-1}$ are tested. The range of strain-rates is obtained by varying the striker mass and gas gun pressure. The wider range of strain-rates tested for the annealed tubes over the other materials is due to the nature of the present application. Throughout this study, the tubes are subjected to a relatively large range of strain-rates and therefore it is advantageous to include more experimental data when determining rate dependent model parameters.

Strain-rates above and below the previously described range are not possible for the given specimen size and testing equipment. Larger strain-rates would require impact velocities that are too large therefore risking the bar becoming plastic and in addition, excessive compression of the specimens can give poor results. Similarly low strain-rates require a much larger striker than the current gas gun can utilize. To obtain strain-rates in the top plate specimens at the same magnitude as the thin annealed specimens would require impact velocities that could damage the bars and strain gauges. To obtain these strain-rates, thinner specimens would need to be machined. Testing the aluminium specimens presents further difficulty due to the small specimens and low strength even at high

3.5 Plasticity Model

As the quasi-static and dynamic behaviour cannot be measured at all strains and strain-rates, mathematical models must be employed to be used in computational tools. Once the parameters of these models are identified, continuous functions are obtained describing the constitutive behaviour of the materials. Various models are considered and described here.

3.5.1 The Johnson-Cook Model

The constitutive behaviour of the top plate steel is modelled in ABAQUS/Explicit using the the Johnson-Cook plasticity model [123]. The Johnson-Cook model is often used in modelling of large deformation and failure problems in isotropic plates and hence is suitable for the current application. A further advantage is that the Johnson Cook model is included with ABAQUS/Explicit and thus does not require a user created VUMAT. The material flow stress is described as function of equivalent plastic strain, plastic strain-rate, and temperature by

$$\bar{\sigma} = [A + B\bar{\epsilon}_{pl}^n] \left[1 + C \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right] [1 - T^{*m}] \quad (3.43)$$

where

$$T^* = \left[\left(\frac{T - T_{room}}{T_{melt} - T_{room}} \right) \right] \quad (3.44)$$

In the first term of Eq. (3.43), $\bar{\sigma}$ is the equivalent plastic flow stress as defined by the von Mises yield criterion and $\bar{\epsilon}_{pl}$ is the equivalent plastic strain. The parameters A , B , and n are material constants which are discussed extensively in the following sections. For the second term C is the strain-rate constant for the material at the reference strain-rate $\dot{\epsilon}_0$ and $\dot{\bar{\epsilon}}_{pl}$ is the instantaneous equivalent plastic strain-rate. The third term models the effect of temperature on the yield stress where T is material temperature, T_{melt} is the melting temperature of the material and m is the thermal softening fraction.

The parameters A , B and n in Eq. (3.43) describe the quasi-static constitutive behaviour at the reference strain-rate $\dot{\epsilon}_0$. The parameter A is the yield stress as defined by the Johnson-Cook model as it is the flow stress at zero plastic strain. This value does not necessarily need to correspond to the yield stress on the true stress-strain curve as a lower value may provide a more accurate curve over the entire plastic range. The values B and n represent the strain hardening parameters of the material.

strain-rates. For those tests a large striker at low velocity is used to ensure excessive deformation of the specimen does not occur.

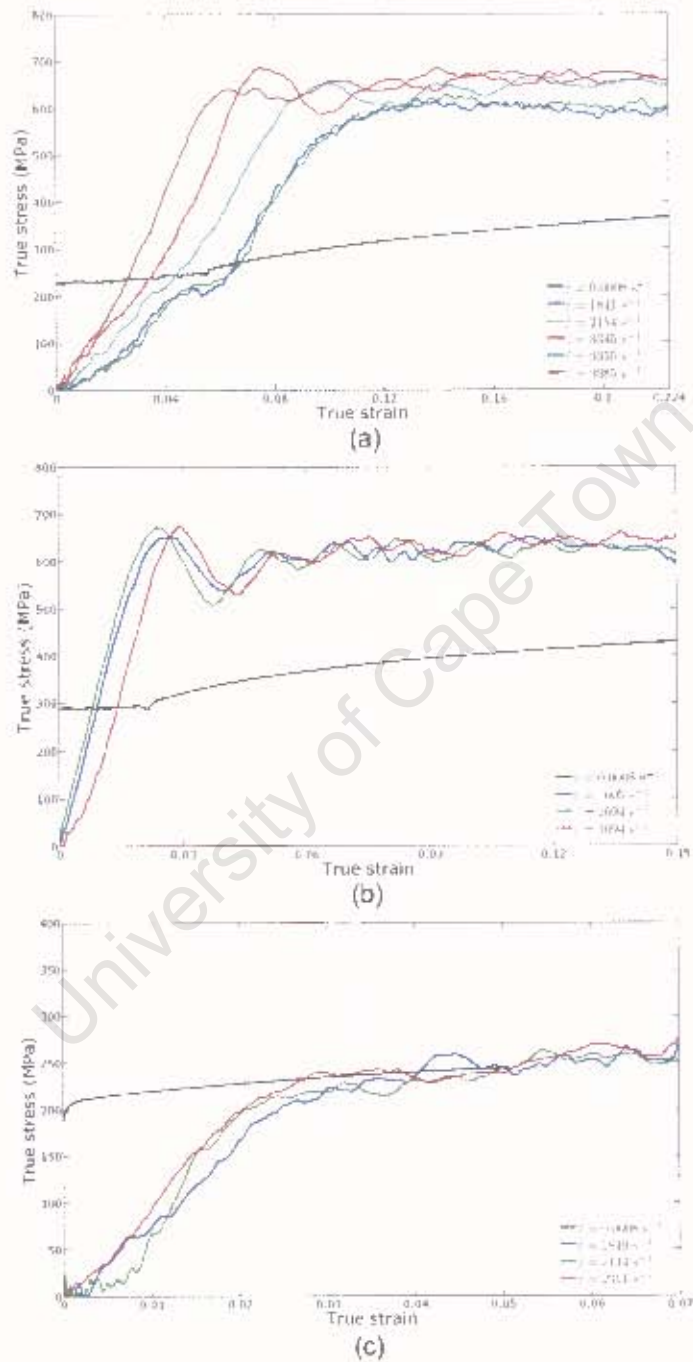


Figure 3.12: Dynamic stress-strain curves from SHPB tests for (a) Annealed steel tubes (b) Top plate steel and (c) Aluminium tubes.

3.5 Plasticity Model

As the quasi-static and dynamic behaviour cannot be measured at all strains and strain-rates, mathematical models must be employed to be used in computational tools. Once the parameters of these models are identified, continuous functions are obtained describing the constitutive behaviour of the materials. Various models are considered and described here.

3.5.1 The Johnson-Cook Model

The constitutive behaviour of the top plate steel is modelled in ABAQUS/Explicit using the the Johnson-Cook plasticity model [123]. The Johnson-Cook model is often used in modelling of large deformation and failure problems in isotropic plates and hence is suitable for the current application. A further advantage is that the Johnson Cook model is included with ABAQUS/Explicit and thus does not require a user created VUMAT. The material flow stress is described as function of equivalent plastic strain, plastic strain-rate, and temperature by

$$\bar{\sigma} = [A + B\bar{\epsilon}_{pl}^n] \left[1 + C \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right] [1 - T^{*m}] \quad (3.43)$$

where

$$T^* = \left[\left(\frac{T - T_{room}}{T_{melt} - T_{room}} \right) \right] \quad (3.44)$$

In the first term of Eq. (3.43), $\bar{\sigma}$ is the equivalent plastic flow stress as defined by the von Mises yield criterion and $\bar{\epsilon}_{pl}$ is the equivalent plastic strain. The parameters A , B , and n are material constants which are discussed extensively in the following sections. For the second term C is the strain-rate constant for the material at the reference strain-rate $\dot{\epsilon}_0$ and $\dot{\bar{\epsilon}}_{pl}$ is the instantaneous equivalent plastic strain-rate. The third term models the effect of temperature on the yield stress where T is material temperature, T_{melt} is the melting temperature of the material and m is the thermal softening fraction.

The parameters A , B and n in Eq. (3.43) describe the quasi-static constitutive behaviour at the reference strain-rate $\dot{\epsilon}_0$. The parameter A is the yield stress as defined by the Johnson-Cook model as it is the flow stress at zero plastic strain. This value does not necessarily need to correspond to the yield stress on the true stress-strain curve as a lower value may provide a more accurate curve over the entire plastic range. The values B and n represent the strain hardening parameters of the material.

3.5.2 The Cowper-Symonds Model

An alternative method to the Johnson-Cook model was used in the publications regarding the numerical simulations of the panels [124] [125]. In these papers the Cowper-Symonds relation is used which models the strain-rate sensitivity of the material. The ratio of dynamic to static flow stress is given by

$$\frac{\sigma_d}{\sigma_s} = \left[1 + \left(\frac{\dot{\epsilon}_{pl}}{D} \right)^{\frac{1}{q}} \right] \quad (3.45)$$

where σ_d is the dynamic flow stress at the uniaxial plastic strain-rate $\dot{\epsilon}_{pl}$, σ_s is the static flow stress and D and q are material parameters. This methodology for strain-rate dependence can be combined with an appropriate hardening model. In the aforementioned papers, Eq. (3.45) was combined with tabulated values from tensile tests for a similar steel. This was a reasonable approach for these studies as they are intended to compare the numerical behaviour of different panels and not to simulate experimental tests. The values of D and q in these papers is $D = 844 \text{ s}^{-1}$ and $q = 2.207$ which were found experimentally using locally obtained mild steel [122].

3.5.3 Alternative Constitutive Models

The models given in the current section are not used in the panel simulations but are presented for completeness and to illustrate the difficulties in the constitutive modelling of materials and possible solutions to these difficulties.

The use of the Johnson-Cook formulation given in Eq. (3.43) presents several challenges related to strain hardening and strain-rate hardening. Firstly, the Johnson Cook model is only considered valid at high strain-rates ($\dot{\epsilon} > 1.0 \text{ s}^{-1}$). An example of how to overcome this is given by Kang *et al.* [126] where a quadratic relation given by Eq. (3.46) is used.

$$\bar{\sigma} = [A + B\bar{\epsilon}_{pl}^n] \left[1 + C_1 \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) + C_2 \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right)^2 \right] \quad (3.46)$$

The current study induces large strain-rates ($\dot{\epsilon}_{pl} \approx 10^3 - 10^4 \text{ s}^{-1}$) and therefore the classic Johnson-Cook constitutive equation is applicable. The second difficulty is that the Johnson Cook model is only ideal for metals that exhibit divergent plastic flow curves. That is, materials which show increased strain-rate hardening with increased plastic strain. Xu *et al.* [127] proposed a solution to this using the modified Johnson Cook model given by

$$\bar{\sigma} = B\bar{\epsilon}_{pl}^n \left[1 + C\bar{\epsilon}_{pl}^n \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right] \quad (3.47)$$

Another model examined in this study to uncouple the strain-rate hardening and the strain hardening is given as

$$\bar{\sigma} = \left[A \left[1 + C \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right] + B \bar{\epsilon}_{pl}^n \right] \quad (3.48)$$

The final model which was considered in this study is given by

$$\bar{\sigma} = [A + B \bar{\epsilon}_{pl}^n] + \left[C \ln \left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right] \sigma_0 \quad (3.49)$$

where σ_0 is the flow stress at the reference strain-rate $\dot{\epsilon}_0$. As with Eq. (3.48), this constitutive equation uncouples the strain-rate and strain hardening but has practical advantages. Firstly, the C value calculated is the same as that in the classical Johnson-Cook model which allows for comparisons with published values. Secondly, the C value can be computed using only the raw experimental data. In using Eq. (3.48), the reference stress-strain curve must be expressed using the Johnson-Cook model before computing C . As the scaled test data is directly used in the computational model, fitting a curve through the data is unnecessary. It is clear that the models described by Eq. (3.48) and Eq. (3.49) best model materials where the dynamic stress-strain curve can be obtained from a translation of the quasi-static curve.

Söderberg and Sellgren [128] investigated several constitutive models, including some of the models presented here, applied to dynamic axial crushing of dual phase steels. They found that despite the large differences in the models at the continuum level, the strain hardening component of the models showed little effect on the energy absorption properties of the tubes when simulated using the FEM. They concluded that the ability of the model to capture the strain-rate effects of the material is of much more importance.

3.5.4 Modelling of Thermal Softening

The final consideration of the material model is temperature dependence. Large plastic deformations in the tube wall and top plate generate high temperatures which softens the material thereby decreasing the flow stress. The change in temperature from adiabatic heating in the material when increasing the equivalent plastic strain from $\bar{\epsilon}_{p0}$ to $\bar{\epsilon}_{p0} + \Delta \bar{\epsilon}_{pl}$ is given by Eq. (3.50).

$$\Delta T = \frac{\eta}{\rho c_{sp}} \int_{\bar{\epsilon}_{p0}}^{\bar{\epsilon}_{p0} + \Delta \bar{\epsilon}_{pl}} \bar{\sigma}(\bar{\epsilon}_{pl}) d\bar{\epsilon}_{pl} \quad (3.50)$$

The parameter η is the inelastic heat fraction which is taken to be 0.9 for this study. The parameters ρ and c_{sp} are the material density and specific heat respectively. The effects

of temperature on the flow stress can then be modelled using any constitutive model as

$$\bar{\sigma}_{\text{temp}} = \bar{\sigma} [1 - T^{*m}] \quad (3.51)$$

where $\bar{\sigma}_{\text{temp}}$ is the flow stress including thermal softening and T^* is given by Eq. (3.44) and the thermal softening fraction is given by m .

In Theobald and Nurick [124] [125] an alternative method for temperature dependence was used for the tubes and top plate. This method was proposed by Masui *et al.* [129]. The effect of temperature on Young's modulus is given by Eq. (3.52) and the effect on the equivalent flow stress is given by Eq. (3.53).

$$E = \begin{cases} 207 \times 10^9 - (58.34 \times 10^6)T & T \leq 600^\circ\text{C} \\ 3.1 \times 10^5(T - 1000) + 97 \times 10^9 & 600^\circ\text{C} < T \leq 1100^\circ\text{C} \end{cases} \quad (3.52)$$

$$\bar{\sigma}_{\text{temp}} = \begin{cases} \bar{\sigma} & T \leq 200^\circ\text{C} \\ \bar{\sigma}[1 - 0.00178(T - 200)] & 200^\circ\text{C} < T < 700^\circ\text{C} \\ \bar{\sigma}[0.133 - (T - 700)3.884 \times 10^{-4}] & 700^\circ\text{C} \leq T \leq 1000^\circ\text{C} \end{cases} \quad (3.53)$$

Eqs. (3.52) and (3.53) describe the temperature effects as piecewise polynomials that are linear in T . The coefficients were determined by Masui *et al.* [129] by fitting experimentally obtained data from hot rolled mild steel. In this model Young's modulus decreases linearly for all $T > 0$ however the flow stress is unaffected for all $T < 200^\circ\text{C}$. This temperature model is not used in the present work however, as the variation in the steel manufacturing needs to be accounted for.

3.5.5 Model Parameters

This section describes the methodology used in obtaining the parameters required for the previously described models. Table 3.6 gives the remaining material properties which do not relate directly to these constitutive models but are still required for analysis. Note that the yield stress σ_{yield} is included however the explicit value of this stress is not needed for computation but is included for sake of comparison.

Table 3.6: General material properties for all materials.

Material	ρ (kg/m ³)	ν	E (GPa)	σ_{yield} (MPa)	c_{sp} (J/kg °C)
annealed steel	7850	0.30	200	223	452
top plate steel	7850	0.30	200	287	452
6063-T6 aluminium	2710	0.33	67	188	920

3.5.5.1 Tube Materials

For the annealed mild steel the strain-rate is modelled using the Cowper-Symonds relation given by Eq. (3.45). The aluminium is assumed to be strain-rate insensitive. To obtain the constants the curve fitting toolbox in Matlab is used. Firstly, the stress as a function of strain-rate is determined at a single strain value. The stress values are obtained at a strain of $\epsilon = 0.12$. This corresponds to the smallest possible strain after ringing up of the specimen during dynamic tests. The smallest strain value is used in order to minimize temperature effects on the flow stress. To account for the oscillatory behaviour the curve is integrated over the range $\epsilon = 0.12 \pm 0.01$ to obtain the mean stress over this range. Stress values at low strain-rate ($\dot{\epsilon} = 8.33 \times 10^{-4} \text{ s}^{-1}$ and $\dot{\epsilon} = 8.33 \times 10^{-2} \text{ s}^{-1}$) are obtained from the tensile test results given in section 3.2.3. The higher strain-rate tensile test is required to create a control point between the quasi-static result and the high strain-rate results from the SHPB results. As discussed in section 3.4.4 there are two distinct strain-rate regimes resulting from the SHPB tests. The resulting points are shown on a log scale in Fig. 3.13a. Optimal values for D and q are determined using least-squares techniques. Specifically, the optimal parameters are found by finding the best fit D and q being sure to include the scaling of the quasi-static stress down to the static value. The resulting optimal values for D and q are determined to be $D = 2731 \text{ s}^{-1}$ and $q = 3.419$.

The resulting curve is plotted against the experimental data in Fig. 3.13a. From the results it is apparent why several interior data points are required to get an accurate strain-rate model. The higher strain-tensile test result provides the required "flatness" of the curve at low strain-rates. The two sets of dynamic test results ensure the slope of the

curve at higher strain-rates is as accurate as possible. Ideally, stress values at strain-rates of approximately $10^1 - 10^2 \text{ s}^{-1}$ would be obtained to control the behaviour of the curve at medium strain-rates. The experimental apparatus required for this is not available at the University of Cape Town. Although this technique is not employed here it is of note that Markiewicz *et al.* [130] used an inverse approach to obtain values of D and q for tube crushing applications. Specifically, the authors found optimal values of D and q based on matching numerical simulations of experimental tube crushing experiments. The authors validated, with success, this method using three point bending however they noted that this inverse approach is only applicable to cases where the deformation mechanism is similar to that of lobe formation during tube crushing.

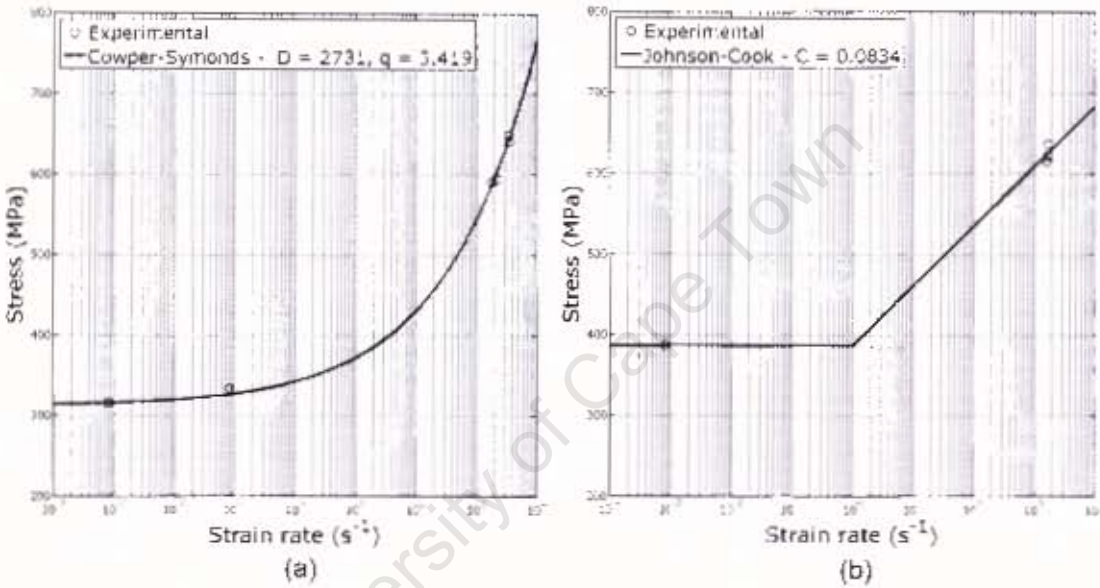


Figure 3.13: Fitting of strain-rate sensitivity models to experimental data for (a) Annealed steel tubes using Cowper-Symonds and (b) Top plate steel using Johnson Cook.

For both the annealed steel and aluminium the material softening due to temperature is modelled using Eq. (3.51). The melting temperature T_{melt} for the steel is taken to be 1527°C which corresponds to commonly accepted values in the literature [113,131]. For the aluminium the melting temperature is 600°C which is provided by the tube manufacturer.

For this model the only parameter to be obtained is m . Ideally, quasi-static isothermal tensile tests would be conducted to obtain this value however such tests are not possible at present using local facilities. For the aluminium, m is taken to be $m = 1.0$ which corresponds to a linear decrease in yield stress from room temperature to zero at the melting temperature. This value is commonly used for materials where temperature dependence is not known explicitly. A value of $m = 1.0$ was used by Park *et al.* [132] and Dabbousi and Nemes [133] however values such as $m = 0.859$ [134], $m = 0.703$ [135], and

$m = 1.131$ [136] have been reported.

To obtain an approximate m value for the annealed mild steel one can examine the original test data from the SHPB tests. Fig. 3.14a gives the stress-strain curve scaled using the Cowper-Symonds relation at a strain-rate of 3362 s^{-1} which corresponds to the average of the dynamic stress-strain curves shown in the graph. It is clear that the modelled curve begins to diverge at strains beyond 12%. If thermal softening is included however using an m value of 0.7 the resulting curve follows the experimental curve reasonably well which is also shown in Fig. 3.14a. For this curve, the stress-strain behaviour is created incrementally using Eq. (3.50) to determine the increase in temperature for a given increase in strain. The temperature dependent stress is then scaled according to this increase in temperature.

It should be noted however that this thermal softening value may be slightly artificial in that the material appears to show no increase in hardening with increased strain-rate. The smaller m value is therefore partially due to compensation for the limitation of the strain-rate model. Uncoupled models such as that given by 3.49, which result in a curve parallel to the quasi-static curve were considered. Although such uncoupled models give a more accurate fit to the experimental data, their implementation in ABAQUS requires the use of a VUMAT which significantly increases the computation time. In addition to this, without complete temperature dependent experiments determining the influence of thermal softening on the "flattening" of the dynamic curves is difficult. For these reasons, the uncoupled models are not used here. The value of $m = 0.7$ is however, well within the range of published values for mild steel however [113,131,137]. Of particular interest is the study by Campbell and Ferguson [138] where the authors study the temperature dependence of mild steel annealed under identical conditions to those of this study. Although the study predates the Johnson-Cook temperature model, the authors found a significant influence of temperature, even at temperatures below $T < 200 \text{ }^{\circ}\text{C}$. It also should be considered that the thermal softening parameter is obtained using data at a much higher strain-rate than is commonly used in the literature. Temperature sensitivity has been shown to increase at higher strain-rates [139]. The stress-strain curves for the annealed mild steel and aluminium at various temperatures according to Eq. (3.53) are given by Figs. 3.15a and 3.15b respectively.

Table 3.7: Strain-rate hardening and thermal softening parameters for the tube materials.

material	$D \text{ (s}^{-1}\text{)}$	q	m	$T_{\text{room}} \text{ (}^{\circ}\text{C)}$	$T_{\text{melt}} \text{ (}^{\circ}\text{C)}$
annealed steel	2731	3.419	0.7	23	1527
6063-T6 aluminium	—	—	1.0	23	600

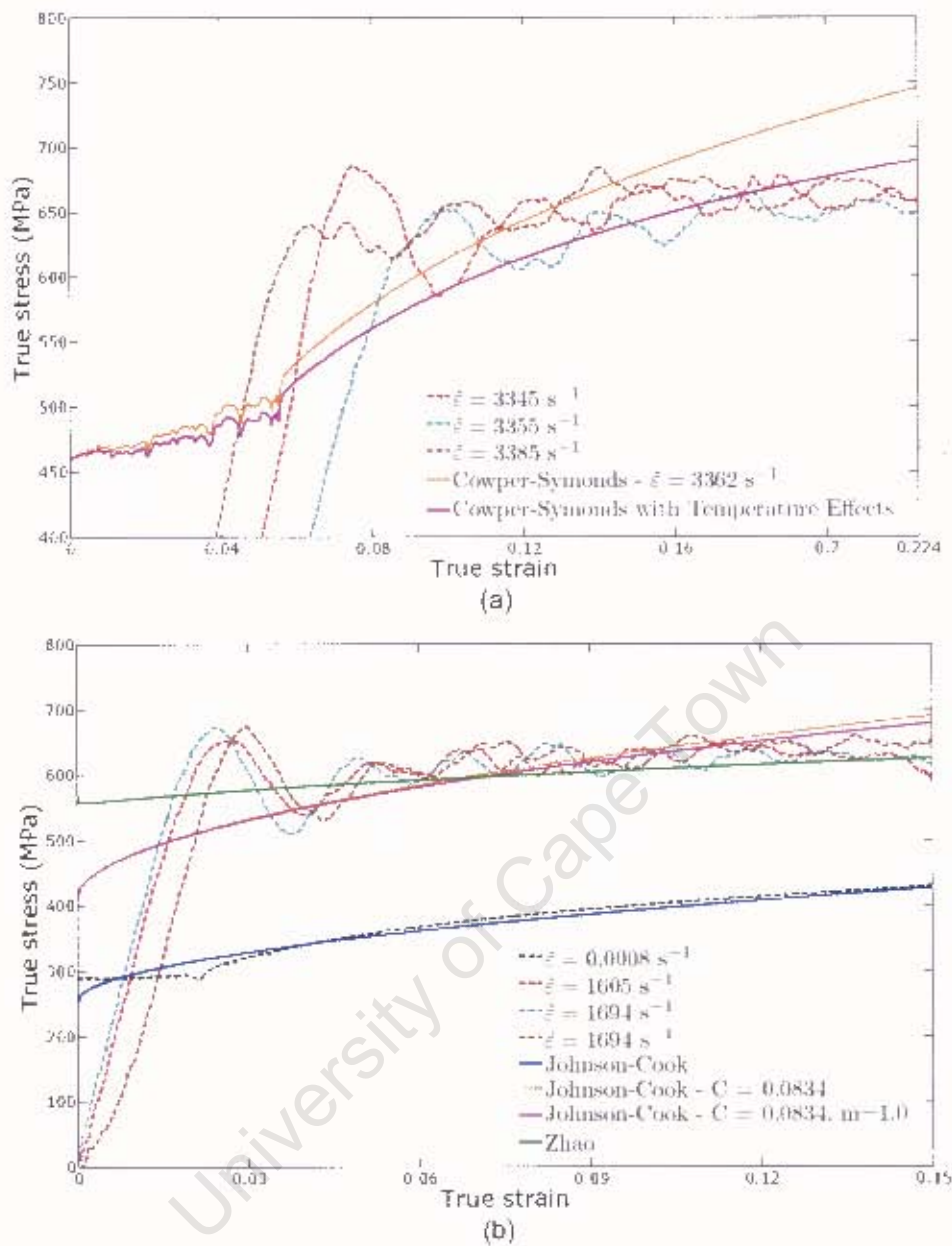


Figure 3.14: Comparison of material models with experimental data including temperature effects for (a) Annealed steel tubes and (b) Top plate steel. For both figures, models are scaled using a strain-rate corresponding to the mean strain-rate of the experimental curves for the material.

3.5.5.2 Top Plate Steel

For the top plate steel, the complete Johnson-Cook model is used. The values of A , B , and n are obtained by fitting the first component of Eq. (3.43), in the least squares sense, through the quasi-static results obtained from tensile tests and simulations. The model parameters are obtained by curve fitting up to 50 % strain using the curve fitting toolbox in Matlab. As the exact yield stress is not significant for the present analysis the fit is obtained without constraint of the value of A . This gives a more accurate fit over the

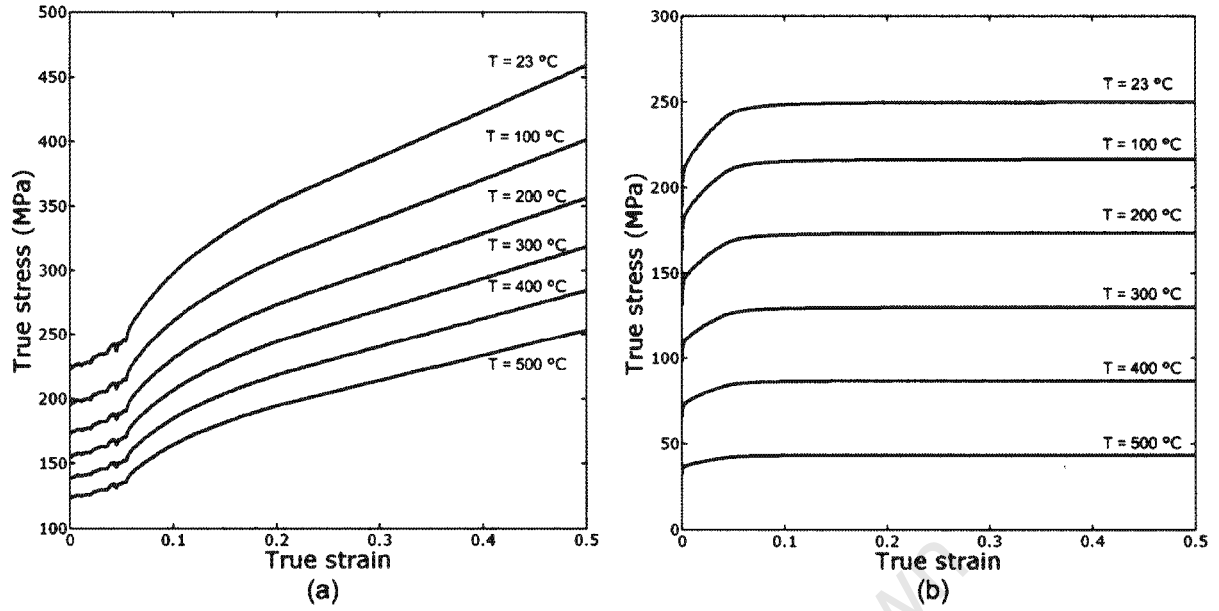


Figure 3.15: True stress-strain curves at various temperatures for (a) Annealed steel tubes and (b) Aluminium tubes.

complete curve but results in the A value having no physical meaning. The Johnson-Cook model fitted to the quasi-static data is given in Fig. 3.14b. The resulting Johnson-Cook constants are given in Table 3.8.

To compute the strain-rate constant C in the Johnson-Cook model, the flow stress is related to strain-rate as shown in Fig. 3.13b. Least-squares techniques are then used to find the optimal C value assuming a reference strain-rate of $\dot{\epsilon}_0 = 1.0 \text{ s}^{-1}$. If the reference strain-rate is taken as the quasi-static test strain-rate, the linear (in log space) behaviour will significantly overestimate the stress at low strain-rates while underestimating the stress at large strain-rates. This reference strain-rate is commonly used in studies that employ the Johnson-Cook model.

The thermal softening parameter m is taken to be 1.0 for the top plate steel. As previously mentioned, this gives a linear softening of the material with no load carrying capacity at the melting temperature. This m value is used by Park *et al.* [132] with similar values used in other studies as previously discussed. The quasi-static curve scaled to a strain-rate of 1664 s^{-1} and is plotted with and without temperature in Fig. 3.14b. The results show that the computed curve slightly overestimates the experimental curve indicating that a non-unity value of m may be preferred. As with the annealed steel, with discrepancy is likely due however, to the strain hardening not scaling with strain-rate.

For comparison with published models and constants the model proposed by Zhao [140]

is included in Fig. 3.14b. This model, excluding temperature effects is given by

$$\bar{\sigma} = A + B\bar{\epsilon}_{pl}^n + (C - D)\bar{\epsilon}_{pl}^m \log\left(\frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\bar{\epsilon}}_0}\right) + E\dot{\bar{\epsilon}}_{pl}^k \quad (3.54)$$

The parameters given in Zhao [140] for hot-rolled mild steel are $A = 145$ MPa, $B = 550$ MPa, $C = 35$ MPa, $D = 45$ MPa, $E = 18.5$ MPa, $n = 0.42$, $m = 0.3$, and $k = 0.3$. Examining the the plot of the model with these parameters and comparing to the experimentally obtained curves at high strain-rate it is clear that there is very good correlation. Note that these values are the values determined by the author for their steel and not parameters fit to the steel used in the current study. This gives a high degree of confidence that the experimental material behaviour determined in this study is accurate. Comparing the model proposed by Zhao clearly shows the difference in hardening between models that have uncoupled the strain-rate and strain hardening from those that couple the two phenomenon such as the Johnson-Cook model.

Table 3.8: Johnson-Cook material constants for the top plate steel.

A (MPa)	B (MPa)	n	C	m	T_{room} (°C)	T_{melt} (°C)
254.5	461.7	0.5201	0.0834	1.0	23	1527

Chapter 4

Panel Design and Finite Element Model

4.1 Introduction

In this chapter the conceptual and numerical modelling considerations of the panel are discussed. Specifically, the global panel design and corresponding design variables are presented. The finite element model is also presented including the blast loading model. All finite element analysis conducted in this thesis use the explicit finite element code ABAQUS/Explicit. Dynamic finite element codes can be grouped according to their time integration scheme, however all codes discretise the time duration of the structural response into discrete timesteps. Implicit methods assume nodal accelerations vary linearly over a timestep and have the advantage that relatively large timesteps can be used. Large scale matrix inversions are required using implicit techniques and hence are more suitable for relatively long duration dynamic events. Explicit finite element codes assume a constant acceleration over a given timestep resulting in trivial matrix inversions at the cost of requiring very small stepsizes. Such methods are more suitable for short duration responses such as blast loading of small scale structures. In explicit time integration methods, nodal velocities are updated at each time step using velocities and accelerations only from the previous time step using the central difference method.

All finite element modelling in this thesis is implemented by interfacing the *Python* scripting language with ABAQUS. Specifically, all models, including geometry, loading, materials, meshing and output extraction is done through the creation of Python scripts. As a result, implementing parametric design studies, and possibly complete optimisation studies requires significantly less work from the user.

4.2 Panel Design

The global design of the panels is discussed here. In the design of the proposed panels, several factors must be considered. At the most general level, the number of tubes within a panel and the placement of the tubes within a panel must be considered. Optimization of the number of tubes within a panel is a difficult problem within the field of integer programming and is beyond the scope of this thesis. For this reason, three sets of panels with four, five, and nine tubes are chosen for investigation. Fig. 4.1 shows the layouts used in the study. These numbers are based on several factors. Firstly, symmetry of

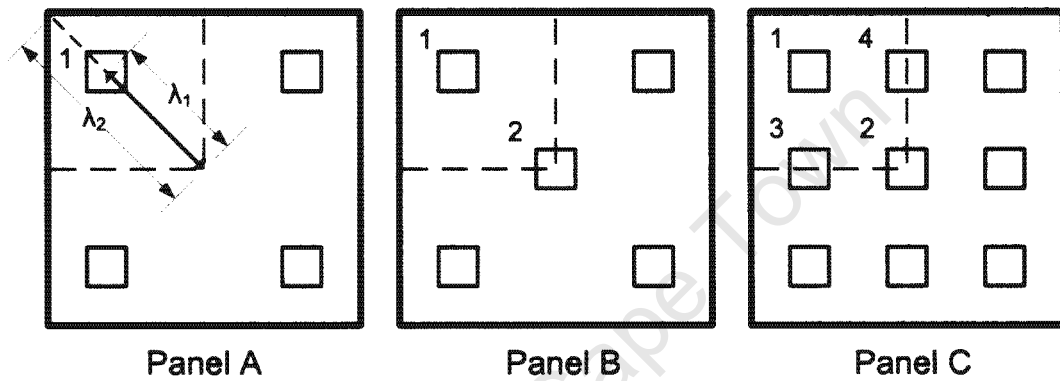


Figure 4.1: Schematic of tube layouts used in the study. Dashed lines indicate where symmetry is used in the FEM model.

the panel is essential to ensure equal distribution of the loading to each tube. Secondly, dimensional requirements of the tubular elements limits the number of tubes that can be included in a panel. Specifically, a large number of tubes in a panel of a given thickness would require slender tubes which are either unavailable or would require aspect ratios exceeding reasonable values to ensure global buckling does not occur.

Within each panel, the position of the tubes varies along a line 45° to the horizontal. The variable $\lambda = \lambda_1/\lambda_2$, where λ_1 and λ_2 are shown in Fig. 4.1, is introduced as the parameter defining tube position along this line. The term λ_1 is the distance from the panel centre to the centre of tube 1. The position of all tubes are therefore defined by this parameter. The tube numbering used throughout the thesis is given in Fig. 4.1.

4.2.1 Tube Design

The tube design and dimensions are given here. The tube length l for all panels is fixed at 75.0 mm. This length is chosen based on preliminary numerical investigations. The results obtained from these simulations are then compared to the limitations of the experimental blast facilities at the University of Cape Town. Ideally, the length used

will be small enough to reach the maximum stroke length for large impulses and/or weak tubes. Conversely, the tubes should be long enough to allow a large range of loadings and resulting tube buckling patterns.

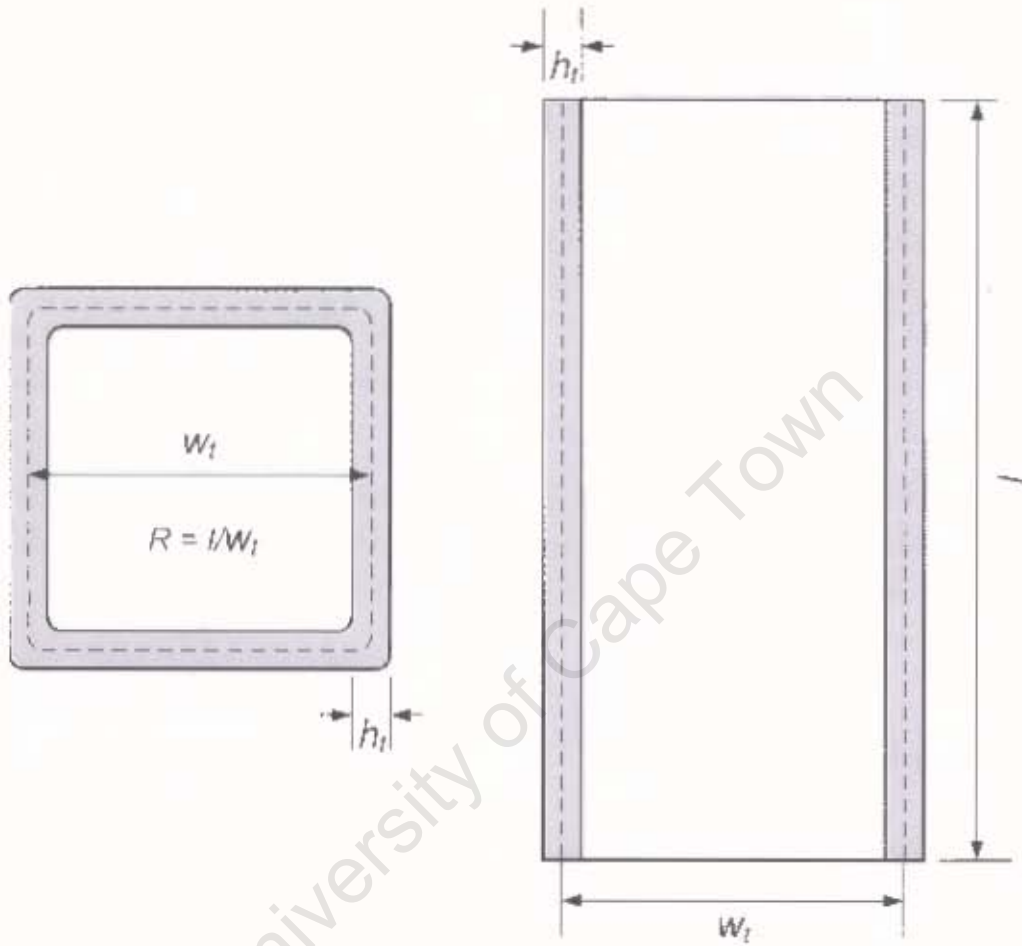


Figure 4.2: Schematic of the cross-section and longitudinal design of the tubes used in the sandwich construction.

The cross-sectional and longitudinal design of each tube is given in Figure 4.2. The width of each tube is defined as the mean width, that is, the distance between the midplanes of opposing tube walls. Using this definition of tube width allows the thickness of the tube walls to be modified without altering the mean aspect ratio. The aspect ratio of all tubes is defined as the length of the tube between the top and bottom plates divided by the mean width. That is, $R = l/w_t$. The design variable h_t denotes the wall thickness of the tubes. The values of h_t numerically and experimentally investigated are given in the respective sections.

4.3 Finite Element Model

4.3.1 Element Selection

The tube walls are modelled in ABAQUS/Explicit using the S4R shell elements. These elements are 4-node quadrilateral, doubly curved, thin or thick shell elements with reduced integration and hourglass control. The elements are formulated for finite membrane strains and are suitable for large strain behaviour. Integration through the shell thickness is through Gauss quadrature with five integration points. Gauss quadrature gives much more accurate solutions over Newton-Cotes methods such as Simpson's rule using the same number of integration points. This comes at the cost of not having the solution to state variables at the shell surfaces. For the present study this is not a necessary requirement and hence Gauss quadrature is used.

The top plate is modelled using 8-node linear brick continuum elements with reduced integration. These elements are labeled C3D8R in ABAQUS. For the shell and continuum elements each node has translational degrees of freedom denoted as u_x, u_y, u_z and rotational degrees of freedoms $\theta_x, \theta_y, \theta_z$.

4.3.2 Panel Model

The present study investigates two distinct panel models, differentiated only by the initial imperfections, or triggers. The first model uses circular holes as described in the previous sections. Specifically, all panels are designed such that the hole diameter is $0.25l/R$. This model is used extensively in chapter 7 and to a limited extent in chapter 6. The second model used in numerical simulations is specific to the experiments of chapter 5 and is discussed in that chapter.

Each tube in the panel has a rigid plate slightly larger than the tube base positioned at the distal end of the tube in order to record individual tube reaction forces. These forces are then added to obtain the total force transfer to the base structure. A final rigid surface is added to model the remainder of the base plate. The only purpose of this surface is to allow for modelling of the top plate coming in contact with the base structure as a result of large top plate deformation or excessive crushing of the core tubes. Fig. 4.3 shows the ABAQUS models for panels *A*, *B* and *C* with the rigid base structures removed for clarity. Note that quarter-symmetry is used in all models for computational efficiency. The finite element mesh is also not shown as the tube mesh used for all simulations has a high density as will be shown in the convergence study in section 4.5. It should be noted that four C3D8R continuum elements with a width of 1.0 mm are used through the thickness of the top plate for all analysis.

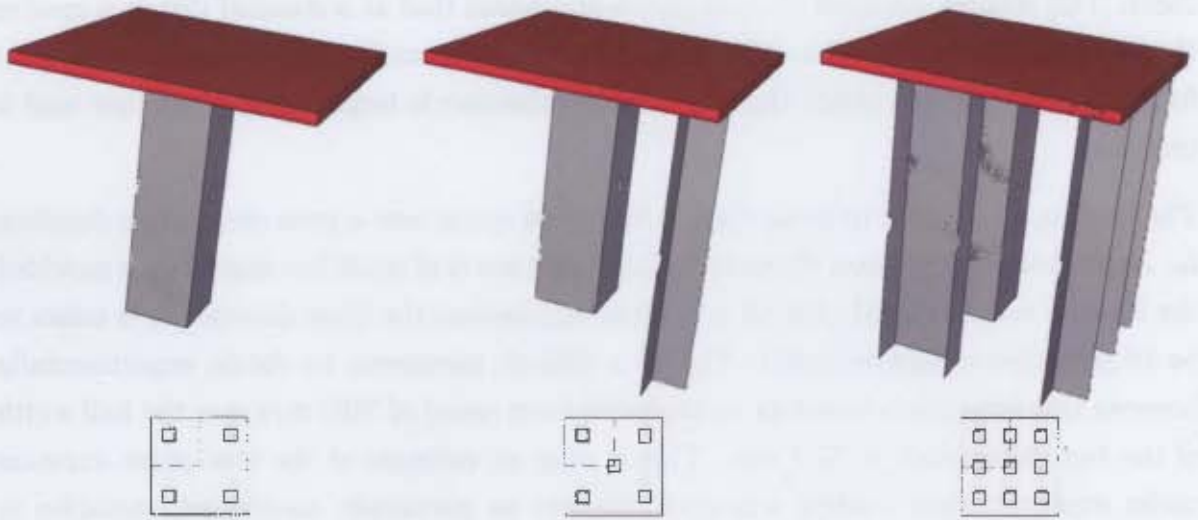


Figure 4.3: Panel models implemented in ABAQUS for panels *A*, *B*, and *C*. For all panels $\lambda = 0.600$, $R = 4$ and $h_p = 3.0$ mm.

4.4 Blast Loading Model

Thus far, only the geometric structure of the panels has been described. Before validating the complete numerical model, the method of loading the structure must be given. The accurate loading of a structure under blast loads is a complex process. The fluid-structure interaction is difficult to characterise exactly, however some simplifications can be made provided certain conditions are met. A common method for modelling the pressure distribution on the plate is to use the Jones-Wilkins-Lee (JWL) equation of state. Using such a methodology a Lagrangian mesh is used for the explosive which expands and interacts with the Lagrangian mesh used for the structure itself. Examples of work where such methodologies are employed can be found in Grobbelaar and Nurick [141] and Daykin [6]. Using this approach has several difficulties however, in that the mesh becomes severely distorted causing numerical instabilities. Further to this the momentum transferred from the explosive to the plate has been shown to be mesh sensitive. A possible solution to this, which was considered for this study is to use the hydro-dynamic code AUTODYN which allows for Eulerian and Lagrangian meshes. Using this methodology a pressure-time distribution on the top plate can be determined for a given explosive setup.

4.4.1 Uniform Loading

As blast loading is a complex problem it is often convenient to assume a simplified pressure distribution over the exposed surface. Marchand and Alfawakhiri [76] suggest that if the blast load initiates from a distance at least one half the maximum dimension of the exposed structure then the pressure can be assumed to be uniform over the structure.

Jacob [142] studied clamped circular plates and found that at a standoff distance greater than the diameter of the plate, the load appeared to be uniform in space. It is assumed for the majority of this study that the standoff distance is large enough that that load is uniform.

The loading is assumed to be uniform in time and space over a prescribed pulse duration t_0 . As the load is impulsive, the distribution over time is of much less importance provided the impulse remains fixed. For all numerical simulations the blast duration t_0 is taken to be $10 \mu\text{s}$ unless otherwise noted. This is a difficult parameter to obtain experimentally however this estimate is based on an explosive burn speed of 7500 m/s over the half width of the top plate which is 75.0 mm . This is only an estimate of the true pulse duration under expected blast loading scenarios however as previously mentioned, variation in pulse duration for a fixed impulse will have little effect on the response of the structure. A square pressure pulse is used which is often used in the modelling of plates under uniform blast loading [96, 143–146]. This implies a pressure-time relationship given by

$$P(t) = \begin{cases} P_0 & 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (4.1)$$

where the constant pressure P_0 is calculated as

$$P_0 = \frac{I}{A_p t_0} \quad (4.2)$$

The next section generalizes the loading such that the applied pressure pulse is not perfectly uniform over the exposed panel area.

4.4.2 Non-Uniform Loading

In reality the pressure distribution over the panel may not be perfectly uniform. Even in the case where suitable results can be obtained from a uniform pulse it is of interest to be able to predict the response of an energy absorbing structure in the case where the pressure is not uniform. The ability of the structure to attenuate the blast may be severely compromised under these conditions and as such, must be accounted for.

In general if the total impulse applied to the panel is given by I then the pressure distribution must satisfy

$$I = \int \int_{\Omega} \int_0^{t_0} P(\Omega, t) dt d\Omega \quad (4.3)$$

where Ω is an arbitrary region on the top plate of the panel and t_0 is the loading duration.

Since the loading is impulsive the time distribution of the load is of significantly less importance than the spatial distribution and therefore the load is assumed to be constant over the time duration t_0 . Integrating over time then gives

$$I = t_0 \int \int_{\Omega} P(\Omega) d\Omega \quad (4.4)$$

The non-uniform spatial pressure distribution investigated here is given by the a circular region centred at the plate midpoint with an exponentially decaying pressure outside this radius. Converting to polar co-ordinates, the pressure at any given location on the panel surface at time t is

$$P(r, t) = \begin{cases} P_0 & r \leq a, \quad 0 \leq t \leq t_0 \\ P_0 e^{-m(r-a)} & r > a, \quad 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (4.5)$$

where r is the radial distance from the centre of the panel, a is the radius of constant pressure, m is the pressure decay constant, P_0 is peak pressure. Clearly, as the decay constant m approaches 0, the pressure approaches uniformity over the panel. An example of this distribution is given Fig. 4.4.

The objective here is to obtain the peak pressure P_0 for a given impulse, constant radius a and decay constant m . This allows the distribution shape to vary for the impulse used in the parametric study. To begin, the applied impulse can be expressed in terms of the load described in Eq. (4.5) as

$$I = 2\pi t_0 \int_0^a P_0 r dr + 8t_0 \int_0^{\pi/4} \int_a^{w/\cos\theta} P_0 r e^{-m(r-a)} dr d\theta = \pi a^2 P_0 t_0 + I_2 \quad (4.6)$$

where w is the half-width of the panel top plate. In Eq. 4.6, the second integral I_2 utilizes 1/8th symmetry. The first integral is trivial and the internal integral of the double integral can be computed analytically. This integral can not be evaluated analytically, however it is simple to evaluate numerically. The difficulty occurs due to the square region of integration as opposed to standard localised loading problems [147] where the region of pressure distribution is circular. A Matlab code is developed that implements a gauss integration scheme using four gauss points. The peak pressure P_0 is therefore computed as

$$P_0 = \frac{I}{t_0 \left[\pi a^2 + 8 \int_0^{\pi/4} \left\{ \frac{-e^{\left(\frac{m(-w+a \cos \theta)}{\cos \theta} \right)} (mw - \cos \theta) + \cos \theta (1+ma)}{m^2 \cos \theta} \right\} d\theta \right]} \quad (4.7)$$

Fig. 4.5a gives the peak pressure P_0 as function of m and a while Fig. 4.5b gives P_0 as a function of m for a fixed a and shows the pressure obtained when using a uniform pressure distribution over the surface of the top panel. Clearly, from this figure, as the decay constant m tends to zero, the peak pressure approaches the pressure of the uniform distribution given by Eq. (4.2).

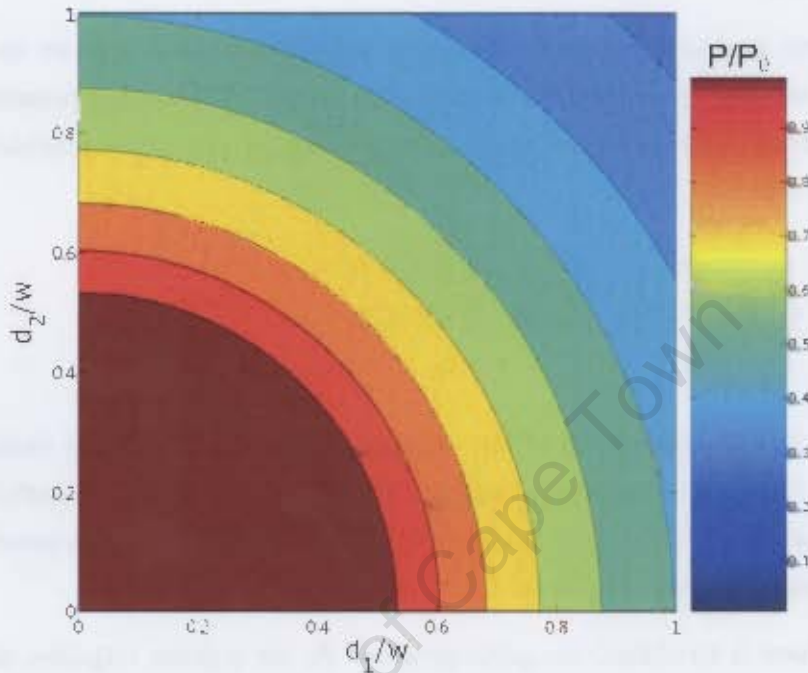


Figure 4.4: Normalized pressure contour plot for an impulse of 50 Ns. The panel is 150×150 mm with $m = 20$, $a = 40$ mm, $P_0 = 316.35$ MPa and a time duration of $10 \mu\text{s}$. The horizontal and vertical distances from the panel centre are represented by d_1 and d_2 respectively while w refers to the half width of the panel.

4.5 Finite Element Convergence Study

To determine the mesh size required for an accurate solution to the problem of blast loading of the panels, a convergence study is undertaken. If the chosen element size is too large, then the resulting output may not be accurate. Conversely if the element size is too small, the computational time required for a solution may be unreasonable. A convergence study allows for the determination of the minimum element size required for an accurate solution.

Instead of conducting the convergence study on the entire panel, only a single mild steel tube is modelled. Conducting the convergence study on a single tube allows mesh related difficulties to be isolated easily with a minimum of computational resources. Once the

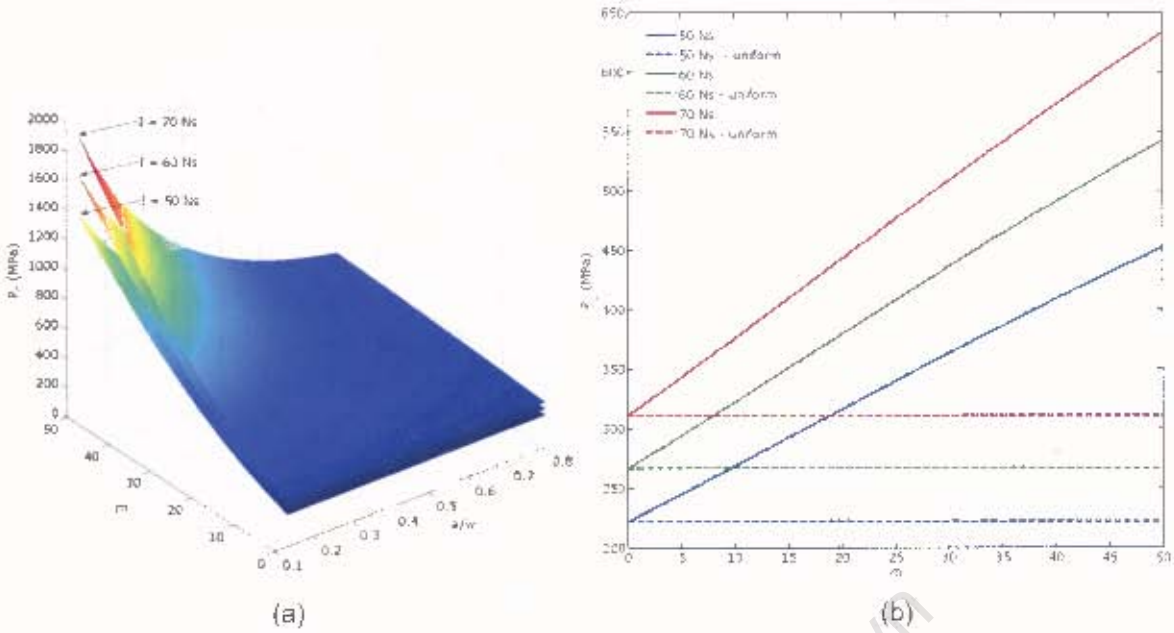


Figure 4.5: (a) Plot of peak pressure as a function of decay constant m and uniform pressure radius a for impulses of 50 Ns, 60 Ns, and 70 Ns. (b) Plot of peak pressure for uniform and non-uniform (with varying decay constant) pressure distributions for impulses of 50 Ns, 60 Ns, and 70 Ns with a pulse duration of $10\mu s$. For the non-uniform pressure distribution the radius of constant pressure is $a = 40$ mm.

optimal mesh is found for the single tube, the results can be applied to an entire panel. The material used in the convergence study is annealed mild steel.

For the convergence study, the tube has length $l = 75.0$ mm, thickness $h_t = 0.6$ mm and aspect ratio $R = 1$. The tube has an attached rigid mass of 0.5 kg and is subjected to an impulsive load of 10 Ns with a pulse duration of $2\mu s$. This mass and impulse results in similar initial velocities at the proximal end of the tube to those in the complete panel subjected to normal blast loading conditions. The pressure pulse is assumed to be rectangular and uniform over the surface of the attached mass. The uniform pressure is thus computed using Eq. (4.2) where the exposed area of attached mass is 552.25 mm^2 .

The single tube is modelled using quarter symmetry in ABAQUS. A small hole of width $0.25/R$ is inserted at the midpoint of the tube. This is to remove any mesh dependency on the position where the buckling initiates and to ensure symmetric lobe formation. The numerical models of chapter 7 use this type of trigger imperfection and hence they are ideal for the convergence study.

The reaction force at the base of the tube is recorded using a discrete rigid surface attached to the distal end of the tube using a tie constraint. The reference point for this surface has all translational and rotational degrees of freedom fixed which has the effect of fixing these degrees of freedom of the nodes at the base of the tube. The nodes at the proximal end of the tube have rotation degrees of freedom fixed to reasonably model the bonding

between the top plate and the tube. The displacement of the tube is recorded at the nodes at the proximal end of the tube however the reaction force is recorded at the distal end. Because of the short duration of the crushing event under blast loading there is a small time differential between when the force and the displacement at this force. This time differential, although small, should be accounted for. Without accounting for this there will be a region on force-displacement curve where there is displacement without any applied force. The time offset corresponds to the time required for an elastic stress wave to propagate the length of the tube, therefore the true force at the proximal end of the tube at time t is given by

$$F_{\text{proximal}}(t) = F_{\text{distal}}(t + l/C_e) \quad (4.8)$$

where C_e is the elastic wave speed of the material which is taken as 5130 ms^{-1} and l is the tube length.

The mesh size used in the convergence study is characterised by setting the number of elements along the length of the tube. In determining the number of elements along the width of the tube the element aspect ratio is kept as close to 1 : 1 as possible. A refined mesh is constructed in the immediate vicinity of the hole. This is done by creating a small partition around the hole. The size of the partition varies slightly to ensure the width is an integer multiple of the element width. This ensures all elements outside the partition have edges parallel or perpendicular to the tube edge and hence are free of distortion.

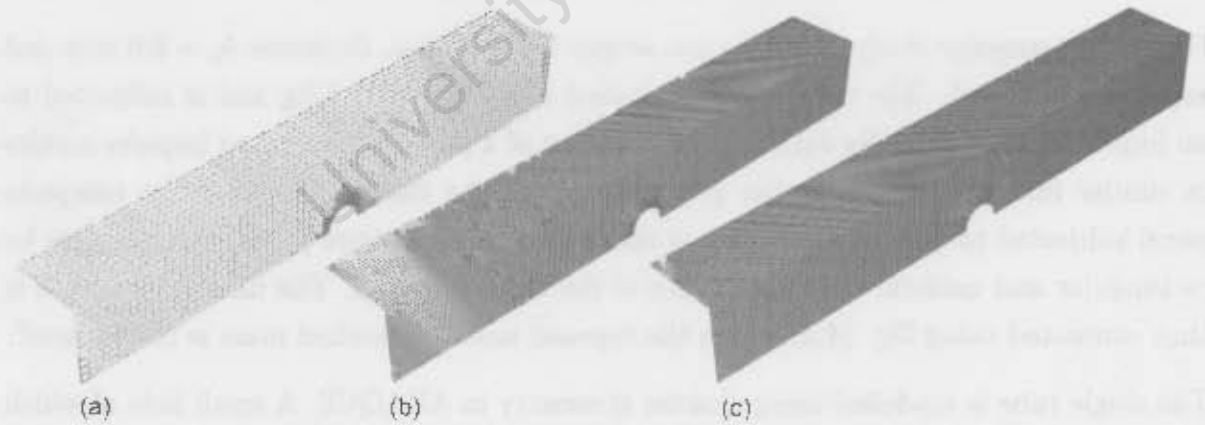


Figure 4.6: Single tube finite element models for mesh densities corresponding to (a) $N = 80$ (b) $N = 180$ (c) $N = 220$

Mesh sizes corresponding to $N = 80, 100, 150, 180, 200, 220$ are examined in ABAQUS/Explicit under the previously described loading conditions. The meshes corresponding to $N = 80, 180, 220$ are given in Fig. 4.6a, b, and c respectively. Convergence is determined based on maximum crushing distance as this is a critical parameter in this study.

The results of the convergence study are presented in Fig. 4.7. The crush distances

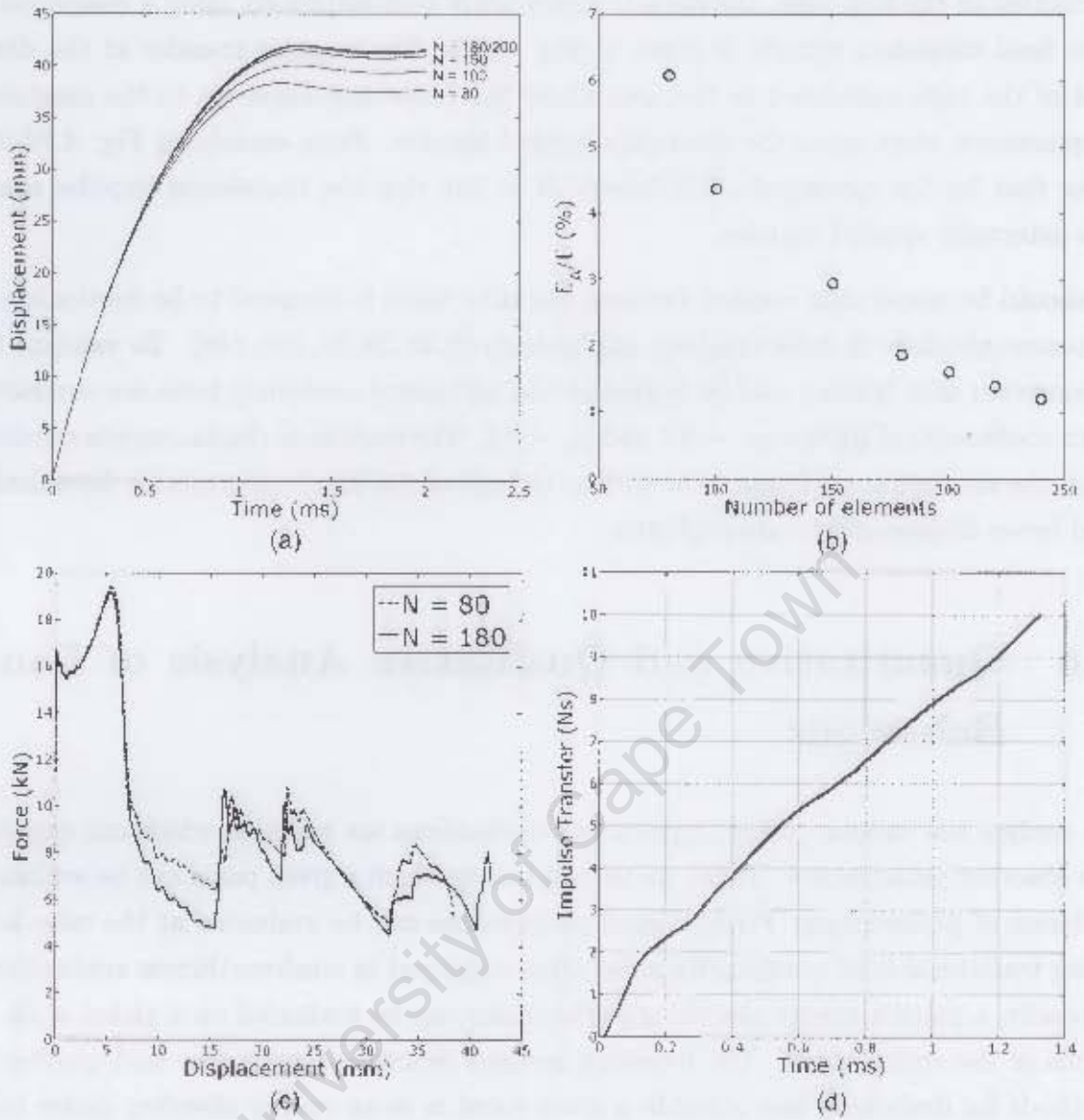


Figure 4.7: (a) Convergence of maximum crush distance for various mesh densities. (b) Ratio of artificial strain energy to total plastic work for various mesh densities. (c) Force-displacement curves for mesh densities $N = 80$ and $N = 180$. (d) Impulse transfer as function of time up to the point of maximum displacement for the $N = 180$ mesh.

as a function of time for mesh densities corresponding to $N = 80, 100, 150, 180, 200$ are given in Fig. 4.7a. It is clear that the displacements are nearly identical at $N = 180$ and $N = 200$ indicating that a mesh density corresponding to $N = 180$ is suitable under the present loading conditions. Fig. 4.7b gives the ratio of artificial strain energy to plastic work. Artificial strains are introduced by ABAQUS/Explicit to control hourglassing. The ABAQUS manual recommends this energy be kept below 5% of the plastic work. As can be seen from Fig. 4.7b, this guideline is satisfied for $N > 100$, therefore the converged density of $N = 180$ more than satisfies this requirement. Fig. 4.7c shows the resulting force-displacement curves for $N = 80$ and $N = 180$. It is clear that even during the

formation of the first lobe, the force is significantly over-estimated using a coarse mesh. The final validation criteria is given in Fig. 4.7d. The impulse transfer at the distal end of the tube calculated as the area under the force-time curve up to the maximum displacement must equal the externally applied impulse. From examining Fig. 4.7d it is clear that for the converged mesh density $N = 180$, that the transferred impulse equals the externally applied impulse.

It should be noted that contact between the tube walls is assumed to be frictionless as is commonly done in tube crushing applications [8, 21, 24, 31, 119, 148]. To validate the assumption that friction can be neglected two additional numerical tests are conducted with coefficients of friction $\mu_k = 0.1$ and $\mu_k = 0.3$. The maximum displacements resulting from the simulation are found to be within 0.02 mm of the frictionless contact formulation and hence friction effects are negligible.

4.6 Quantitative and Qualitative Analysis of Panel Behaviour

To analyze the various panels, appropriate mechanisms are required which can quantify the absorber performance. There are two aspects in which a given panel can be evaluated in terms of performance. Firstly, panel performance can be evaluated at the tube level using traditional tube crushing formulae often employed in crashworthiness applications. Secondly, a panel's energy absorbing performance can be evaluated on a global scale, in terms of the entire panel. The following sections describe quantitative and qualitative methods for describing how suitable a given panel is as an energy absorber under blast loading.

4.6.1 Tube Efficiency Measures

Ideally, an energy absorber should maximize the stroke length while maintaining a constant force level [41]. Throughout the analysis, i refers to the tube number as per Fig. 4.1. Ignoring elastic deformations, the plastic deformation energy in tube i is given by Eq. (4.9) where δ_i is the maximum axial tube displacement and $F_i(u)$ is the reaction force at the proximal end of tube i as a function of displacement (u). In cases where the tubes does not crush symmetrically, the value of δ_i is calculated as the mean crush distance at each of the four corners of the tube.

$$U_i(\delta_i) = \int_0^{\delta_i} F_i(u) du \quad (4.9)$$

The mean force (F_m) at the proximal end of the tube i is therefore described by

$$F_m^i(\delta_i) = \frac{U_i(\delta_i)}{\delta_i} \quad (4.10)$$

The relative deformation of tube i which quantifies the ratio of crush distance to the total length l of the absorber is given as

$$\xi_{\text{def}}^i = \frac{\delta_i}{l} \quad (4.11)$$

If the crush distance δ_i reaches the point where compaction occurs, force levels increase monotonically as deformations increases. The relative deformation where this occurs is known as the stroke efficiency and is found by Jones [14], for square tubes, to be given by Eq. (4.12)

$$St_e = \frac{\delta_{\text{max}}^i}{l} = 0.73 \quad (4.12)$$

The crush force efficiency given by Eq. (4.13) measures the consistency of the force levels throughout the stroke.

$$\xi_{\text{crush}}^i = \frac{U_i(\delta_i)}{F_{\text{max}}^i \cdot \delta_i} = \frac{F_m^i(\delta_i)}{F_{\text{max}}^i} \quad (4.13)$$

The final tube efficiency measure is given by Eq. (4.14). This performance measure combines Equations 4.11 and 4.13 to include the effective crush distance in the calculation of crush force efficiency.

$$\xi_{\text{total}}^i = \frac{U_i(\delta_i)}{F_{\text{max}}^i \cdot l} = \xi_{\text{crush}}^i \cdot \xi_{\text{def}}^i \quad (4.14)$$

This measure quantifies the performance of the absorber against an ideal absorber over the crush distance δ_i . The stroke, crushing, and total efficiencies are described in detail by Hanssen *et al.* [52] and Jones [14]. The performance measures here are depicted graphically in Fig. 4.8

4.6.2 Panel Efficiency Measures

The aforementioned performance measures apply only to single tubes and hence using these measures to quantify total panel behaviour is cumbersome. The difficulty is that describing the performance of a given design requires providing measures for each individual (independent) tube. To quantify the performance of the complete panels using the measures presented in section 4.6.1 the total panel crushing force is found by summing

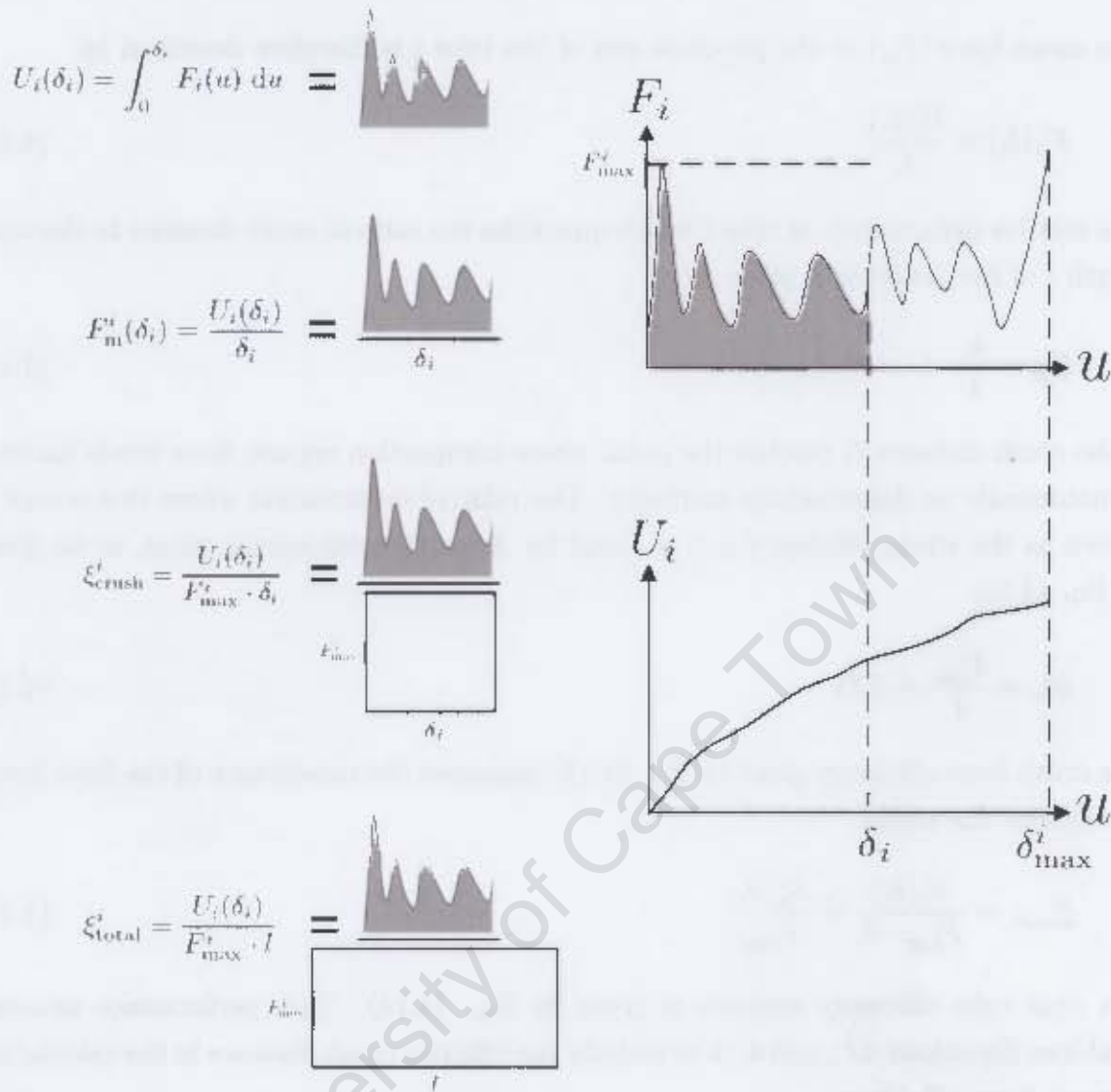


Figure 4.8: Graphical definitions of the individual tube performance measures used in this study.

the individual crushing forces of the core tubes. From this, the efficiency measures can be computed in a similar way to that of a single tube.

The classical efficiency measures described by Equations 4.11-4.14 are insufficient in quantifying the performance of a panel in that various geometric panel properties will inherently influence these properties. Specifically, panels using thick tubes, or a large number of tubes giving a large impulse capacity will display a small stroke efficiency under a single load. In an attempt to provide a meaningful measure of panel performance that is largely unaffected by loading magnitude a new efficiency parameter is introduced. If one ignores stored elastic strain energy, frictional dissipation, and artificial strain energy due to hourglass control, then at a given time instant

$$W = KE + U_{core} + U_{plate} \quad (4.15)$$

where W is the externally applied work from the blast load, KE is the kinetic energy remaining in the panel at a given time, and U_{core} and U_{plate} are the dissipated plastic energy in the tubes and top plate respectively. After the blast load has been absorbed $KE = 0$ and the dissipated energy has been partitioned between the core and the top plate. Ideally the tubes in the core will absorb as high a proportion of the input energy as possible since large plastic strains in the top plate likely indicate instability in the core tubes. To quantify this the core (tubes) efficiency ξ_{core} is given by Eq. (4.16). This measure is simply the ratio of energy absorbed by the core to the total input energy of the system.

$$\xi_{\text{core}} = \frac{U_{\text{core}}}{W} = \frac{1}{W} \sum_{i=1}^{n_t} \int_0^{\delta_i} F_i(u) du \approx \frac{2A_p \rho_p h_p}{I^2} \sum_{i=1}^{n_t} \int_0^{\delta_i} F_i(u) du \quad (4.16)$$

where A_p , ρ_p , and h_p are the top plate area, density and thickness respectively and $U_{\text{core}} = \sum_{i=1}^{n_t} U_i$. A value of ξ_{core} approaching 1 implies the desired result of nearly rigid body motion of the top plate.

Although the energy absorbing efficiency is of primary importance when determining if a given panel design is a suitable blast absorber, the buckling stability is also of significance. There is a clear correlation between buckling stability and energy absorption efficiency as tubes that undergo global buckling or partial global buckling will absorb less energy than a tube that buckles in a progressive symmetric manner. Further to this, instabilities in the buckling of the core tubes implies a larger amount of the energy absorbing responsibility is shifted to the top plate, reducing the total capacity of the panel. To quantify this, a stability parameter is introduced. The mean displacement of the corners of each tube in the panel is clearly

$$\delta_{\text{mean}} = \frac{1}{4n_t} \sum_{i=1}^{4n_t} \delta_i \quad (4.17)$$

Therefore, the stability parameter ξ_{st} is then simply defined as the sum of the absolute differences between each tube corner displacement and the mean displacement which is then divided by the mean displacement to normalize the result.

$$\xi_{\text{st}} = \frac{\sum_{i=1}^{4n_t} |\delta_i - \delta_{\text{mean}}|}{\delta_{\text{mean}}} \quad (4.18)$$

It must be noted that stability parameter comparisons are only valid within a specified tube layout (A , B , or C). The measure simply quantifies the differential between nodal crush distances as measured at each corner of the tubes in the panel. In panel A tests, for example, the stability parameter is limited by the width of the tube only, as there is only one unique tube in the panel. Panel B designs however have two (assuming quarter

symmetry is utilized) independent tubes which may have significantly different crush distances which will influence the resulting value of Eq. 4.18.

It should also be noted that momentum is conserved at the interface of the panel core and the structure at the bottom plate. This implies the total impulse transferred to the bottom plate is the same for all panels regardless of the performance in terms of the above measures. Thus, if t_i is the time required for tube i to reach its maximum deformation δ_i then

$$I = \sum_{i=1}^{n_t} \int_0^{t_i} F_i(t) dt \quad (4.19)$$

From a structural protection point of view, the panel will ideally ‘smooth’ out the applied impulse on the bottom plate. It is clear that the total force levels will be much higher as n increases, however the total time required to dissipate the kinetic energy is much lower. It is thus an engineering optimization problem to determine the ideal absorber based on the required absorption capacity of the panels and the allowable peak force levels on the bottom plate. For example, panel A will give the lowest total force levels on the bottom plate, however panel A has a much smaller absorption capacity than panels B and C.

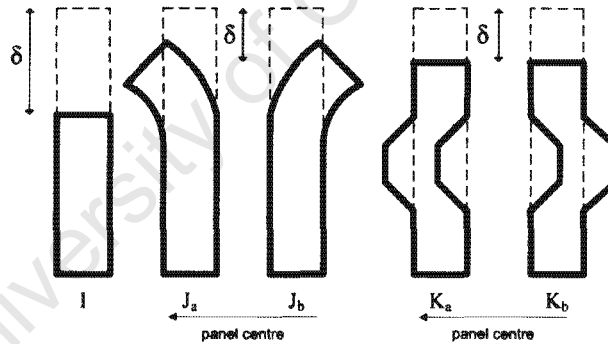


Figure 4.9: Tube crushing modes observed in numerical simulations.

4.6.3 Qualitative Measures

In addition to these quantitative absorber performance measures, qualitative measures are also used to analyse the numerical results. Controlling the formation of plastic lobes during the crushing process is important for ensuring efficient energy absorption. To maximize the stroke length, Euler buckling must be avoided. The overall shape of the crushed tubes in this study are classified into three modes as shown in Fig. 4.9. Mode I crush behaviour describes perfect dynamic plastic or progressive symmetric buckling. Mode J behaviour is identified with pure global buckling, or a combination of some progressive buckling with global buckling dominating the energy absorption process. Mode

K describes crush behaviour where large global, non-axial deformations occur primarily away from the proximal end of the tube. Tubes which buckle in this mode show little lateral displacements at the proximal end of the tube, unlike mode J global buckling. Modes J and K are further subdivided based upon the direction in which the non-axial global displacements occur.

University of Cape Town

Chapter 5

Validation of the Numerical Model using Experimental Testing

5.1 Introduction

In order to validate any numerical models, experimental tests are required to determine if the numerical model can capture the behaviour of the proposed panels under blast loading. If the models can reasonably simulate the true response of the panels, further numerical investigations can be accomplished with confidence that the results indicate panel response under real blast loading conditions. The methodology used to experimentally test the proposed panels is given here. The goal of this chapter is to discuss firstly, the panel construction and preparation. Secondly, to compare the quasi-static response of the tubes used in the panels to finite element simulations. Finally, this chapter describes the experimental blast loading methodology and results from tests on selected panels. It must be noted that, due to the difficulty of conducting experimental blast tests on the proposed panels, the experiments presented in this chapter are primarily for the purposes of validation of the numerical model.

5.2 Test Programme

In order to validate the numerical model and experimentally investigate the performance of the panels under study several blast tests of actual panels are undertaken. The manufacture of the panels is difficult and consumes significant resources therefore a careful selection of test panels is required. The methodology used in selecting the panels is described here.

The tube materials used in the study are the annealed mild steel and 6063-T6 aluminium alloy. Although it would be beneficial for showing the effects of annealing, panels of non-annealed steel are not tested due to manufacturing time constraints. The results of such tests would be highly dependent on the size of tube chosen due to variations in work hardening which would limit applicability to further research. The purpose of testing the annealed mild-steel and aluminium is to examine the response of materials with similar yield stress but much different strain-rate sensitivity.

Panels *B* and *C* as given in Fig. 4.1 are examined experimentally. Panel *A* is not examined as it is highly sensitive to any non-uniformity in the loading. The differences in response between panels *B* and *C* is expected to provide insight into impulse capacity as function of the number of tubes in the panel core.

The tubes forming the core of all panels have the approximate geometric parameters given in Table 5.1. The top plate thickness for all panels is 4.0 mm. This top plate thickness is chosen to ensure nearly rigid body displacements during the blast event. This has the purpose of isolating the energy absorption responsibility to the tubes in order to study the response of the tubes to blast loading. The tube positions, which are defined entirely by the value of λ , are selected based on the preliminary results found in [125]. It is expected that these λ values are near the optimal value due to these initial numerical investigations. It should however be noted that due to the thick top plate, the λ value used here is expected to have little effect on the overall panel response to blast loading. Using a thick top plate minimises top plate deformations thereby limiting interaction effects between the top plate and the tubes. These interaction effects were found to vary significantly with λ in [125] and are discussed thoroughly in chapter 7. Table 5.1 summarises the relevant panel and tube geometries investigated experimentally.

Table 5.1: Summary of panel and tube geometries investigated experimentally.

Material	Configuration	λ_B	λ_C	h_p (mm)	h_t (mm)	R
Steel	<i>B, C</i>	0.61	0.70	4.0	0.61	3.49
Aluminium	<i>B, C</i>	0.61	0.70	4.0	0.62	3.44

To summarize, two panels configurations are investigated for each material. For each configuration however, three loading magnitudes are applied as shown in Table 5.2. A small, medium, and large charge mass is selected for each panel configuration and core material with a single charge mass in common for both materials. For a given tube material, using a single set of charge masses allows for direct comparison between responses of panels *B* and *C*.

Table 5.2: Loading magnitudes in terms of the mass of PE4 detonated for each of the core tube materials.

Material	$M_{\min}$ (g)	M_{med} (g)	$M_{\max}$ (g)
Steel	28	33	38
Aluminium	13	20	28

5.3 Panel Manufacture

The construction of the panels is described here. The fastening of the tubes to the bottom plate and top plate is found to be a difficult task. Under blast loading large stresses may develop at the plate/tube interfaces and thus a strong bond is required. Unfortunately, welding techniques require too much heat to ensure structural integrity in the thin tubes during the welding process in addition to ensure material properties remain unchanged. This is particularly true when welding aluminium to steel. Further to this, warping in the top and bottom plates due to excessive heat renders the panels unusable. Several epoxy techniques are also tried, however all tests using such methods are found to be unsuccessful due to debonding of the tubes and plates.

A compromise to welding and epoxy is used in this study for bonding the tubes to the top and bottom plate. Soft solder is used for both interfaces and materials. For the steel to steel bonding soft solder is used and for the aluminium to steel bonding aluminium solder is used with a specialized flux for such applications. Silver solder was also used in preliminary tests however this bond was found to be excessive in terms of heat and therefore the soft solder is used in the final tests. Although a welding torch is still required, the applied heat is minimised.

Fig. 5.1 gives an idealized schematic of the bonding process. For the top plate square grooves are milled into the plates such that the inside width of the grooves is the same as the inside width of the tubes. The tubes are then inserted into the grooves. The purpose of this is to ensure that the tubes do not shear from the top plate. This is unlikely using the thick 4.0 mm top plates however this method is utilized as thinner plates may be used in future studies. The groove is cut such that there is a trough type structure after insertion of the tube. The trough is filled with solder as shown in Fig. 5.1. The encroachment of the solder on the tube is minimal due to the trough and thus can be neglected in numerical models.

In the bottom plate, the process differs significantly. Holes of approximately the same size as the tubes are laser-cut through the plates. A fillet of approximately 45° is then manually created using a file. The panel is turned upside down with 75 mm support pillar

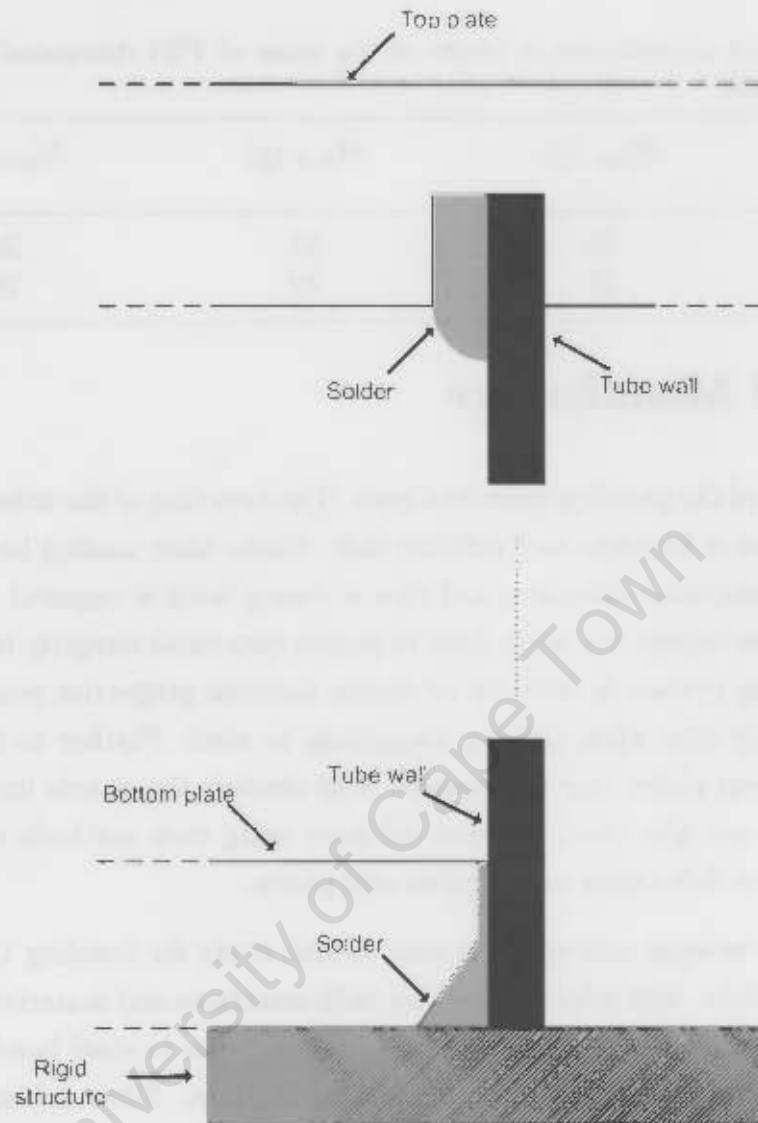


Figure 5.1: Schematic of the bonding methodology between the core tubes and the top and bottom plates.

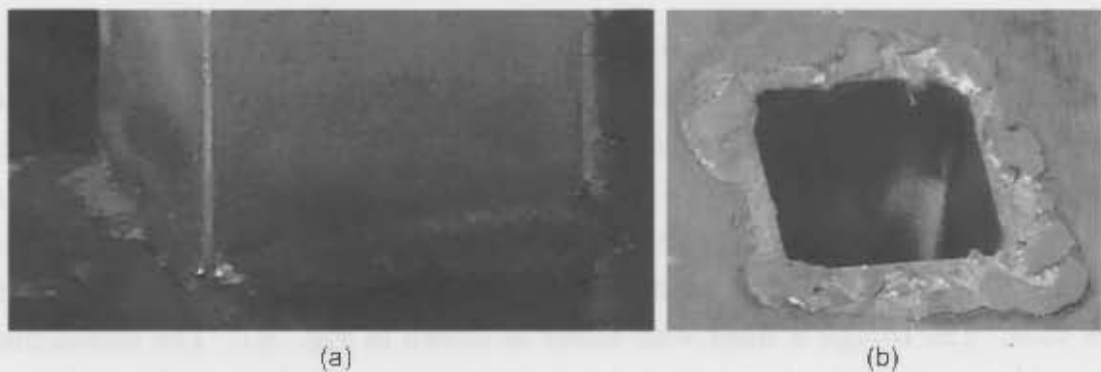


Figure 5.2: (a) Photograph of the bonding between the top plate and the proximal end of the tube. (b) Photograph of the underside of the bottom plate showing the bonding of the bottom plate with the distal end of the tube.

holding the bottom plate at the correct level. The volume of the bottom plate removed by creating the fillet is then filled with solder. After hardening, the resulting structure is then sanded flat leaving only solder remaining in the filleted region. The reason for this is that the panel must sit flush with a thick rigid plate which is connected to the ballistic pendulum as described in section 5.7.

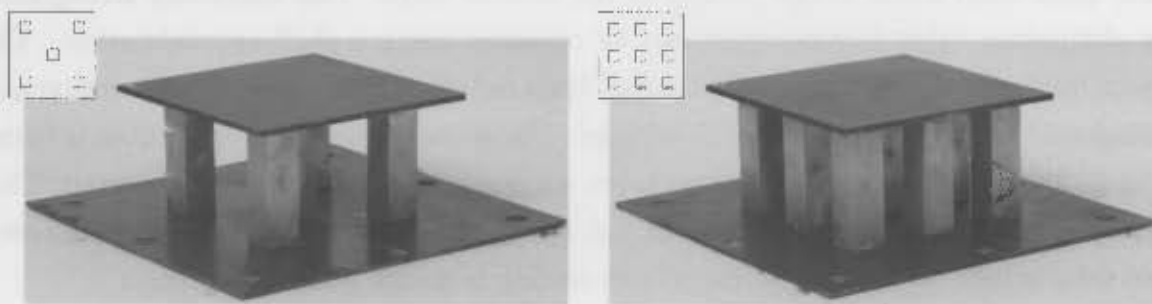


Figure 5.3: Photographs of typical aluminium tube panels after construction.

5.4 Tube Preparation

For all experimental investigations the tube thickness is set to a nominal value 0.6 mm. Commercially available tubes are not available in this thickness however, and as such the tubes must undergo processing, using a milling machine, to obtain the desired thickness. It should be noted that the steel tubes are annealed before shaving in order to minimize the effect of scaling on the final tube thickness. The initial lengths of tubes are cut into small pieces of length approximately 175 mm before shaving allowing for two tubes per section to be obtained. The tubes in their as-received state have a nominal thickness of 2.0 mm with an outside width of 25.4 mm. After shaving, a random selection of five steel and aluminium tubes are selected for dimensional testing of thickness reduction. This reduction in thickness is difficult to measure however due to the very small thicknesses in question. This is particularly true for the annealed steel tubes as the scaling from the annealing process on the interior walls causes difficulties in measurements. The process also results in slightly larger thicknesses near the midpoint of the tube walls in the annealed steel tubes. This result does not appear in the extruded aluminium tubes. Also of note, due to the extrusion process, the aluminium tubes contain filleted edges at the interface of the inside faces of the tube. This effectively thickens the tube at the edges.

To obtain the thickness of the steel tubes a vernier is used to measure the thickness at three approximately equidistant points on each of the four tube walls. Ideally a micrometer would be used however the slightly parabolic shape of the tube wall prevents the use of such a tool. The wall thickness of the aluminium tubes is measured at three

equidistant points on each tube wall using a micrometer. The thickness are then averaged to obtain the mean thickness used in all experimental simulations. The mean thickness of the steel tubes is found to be 0.61 mm and for the aluminium tubes a thickness of 0.62 mm is obtained. It should be noted however that these thicknesses are only an approximation as the steel tubes are found to be slightly thicker at the midpoint while the aluminium tubes have small fillets at the corners of the interior walls. This effectively strengthens the aluminium tubes beyond what can be obtained using a shell approximation. The aspect ratio of the shaved tubes is obtained from outside width of the sample section and is measured with a vernier and then averaged. The mean width of the steel tubes is found to be 21.49 mm and for the aluminium tubes a mean width of 21.83 mm is observed. This corresponds to aspect ratios of $R = 3.49$ and $R = 3.44$ respectively. The slightly narrower steel tube is likely due to small losses from scaling from the annealing process.

Once the tubes have been processed to the desired thickness, imperfections are created in the tube walls. The purpose of the imperfections is to induce buckling at a predetermined location along the tube length. If each tube in the panel buckles at the same location, it is expected that crushing stability will be maximised. In addition to this, creating a relatively large induced imperfection ensures any small imperfections in the tube wall introduced in the thickness reduction do not dominate the buckling process. It should be noted that initial experimental tests used tubes with circular cutouts of various sizes. Such imperfections created difficulties in that if the cutouts were too small, they did not influence the buckling mode while larger holes resulted in ductile fracture at various locations around the the hole perimeter.

To circumvent the problem of ductile fracture, small indentations are created in the opposing tubes wall at the midpoint in terms of length and width. The dent is created with a hemispherical steel indenter with a radius of 10.0 mm. The dent is created using the apparatus shown in Fig. 5.4. The tube is inserted into a plastic clamp which contains a hole for the denter to move though. The denter is placed in the hole and the rig is placed on a Zwick universal testing machine. The denter is then displaced a distance of 2.0 mm at a constant speed of 50 mm/min. A small pre-load of 100 N is applied to the denter to remove any compliance in the compression rig, thereby ensuring the measured crosshead displacement is the same as the denter displacement. The dent dept is measured for 20 samples for each of the steel and aluminium tubes. After elastic unloading, the final mean dent dept is found to be $\delta_s = 1.74$ mm and $\delta_a = 1.45$ mm for the steel and aluminium tubes respectively. The mean deviation is found to be 0.063 mm and 0.066 mm for the steel and aluminium tubes respectively indicating a reasonable level of repeatability.

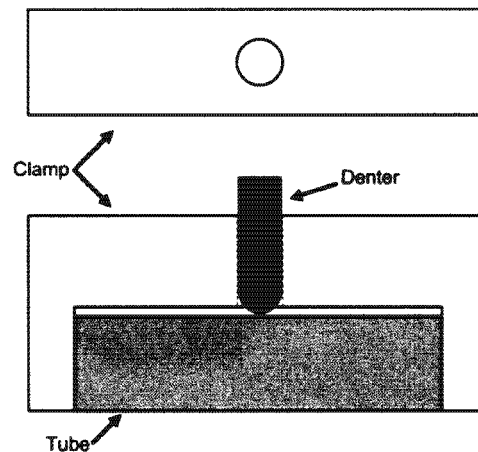


Figure 5.4: Schematic of the apparatus used to create the hemispherical indentations in the tube wall.

5.5 Finite Element Model of Experimental Panels

For all experimental tests presented in this chapter, the corresponding numerical predictions are given. The details of the numerical model used for comparison with experiments is given here.

In chapter 4 the finite element model of the general panel design is described. A slightly modified panel model is required for experimental validation due to the indentations in the tube walls. In these panels the initial imperfections are created using hemispherical indenters. The modelling of the indentation process described in section 5.4 is given here. The indenter is modelled as a discrete rigid surface in ABAQUS. For all analysis the indenter has a diameter of 10 mm. The geometric model of the indenter and quarter-symmetry tube is given in Fig. 5.5a. In the figure, the tube is composed of annealed mild steel and has a thickness of 0.63 mm and aspect ratio 3.44. These dimensions correspond to the quasi-static experimental tests done in the section 5.6. Contact between the tube and the indenter is limited to a small area of the tube slightly larger than the indenter.

Conducting simulations of the indentation at the velocities used in the actual experiments is not feasible and therefore a suitable numerical velocity must be chosen. Under quasi-static crushing conditions the indent is created by displacing the indenter, normal to the tube wall at a velocity of 0.5 m/s up to a maximum depth of 2.0 mm. This indentation velocity is chosen as it is the highest velocity observed where inertia effects in the tube wall are negligible. Difficulty occurs however, when extending the model to blast loading conditions. The time scales, and hence strain rates associated with the indentation process differ vastly from that of blast loading. In simulations involving the steel tubes, strain rate effects must be included in the material model. The difficulty with this is that the

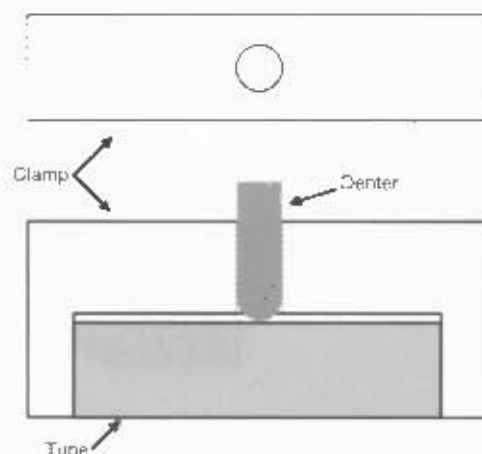


Figure 5.4: Schematic of the apparatus used to create the hemispherical indentations in the tube wall.

5.5 Finite Element Model of Experimental Panels

For all experimental tests presented in this chapter, the corresponding numerical predictions are given. The details of the numerical model used for comparison with experiments is given here.

In chapter 4 the finite element model of the general panel design is described. A slightly modified panel model is required for experimental validation due to the indentations in the tube walls. In these panels the initial imperfections are created using hemispherical indenters. The modelling of the indentation process described in section 5.4 is given here. The indenter is modelled as a discrete rigid surface in ABAQUS. For all analysis the indenter has a diameter of 10 mm. The geometric model of the indenter and quarter-symmetry tube is given in Fig. 5.5a. In the figure, the tube is composed of annealed mild steel and has a thickness of 0.63 mm and aspect ratio 3.44. These dimensions correspond to the quasi-static experimental tests done in the section 5.6. Contact between the tube and the indenter is limited to a small area of the tube slightly larger than the indenter.

Conducting simulations of the indentation at the velocities used in the actual experiments is not feasible and therefore a suitable numerical velocity must be chosen. Under quasi-static crushing conditions the indent is created by displacing the indenter, normal to the tube wall at a velocity of 0.5 m/s up to a maximum depth of 2.0 mm. This indentation velocity is chosen as it is the highest velocity observed where inertia effects in the tube wall are negligible. Difficulty occurs however, when extending the model to blast loading conditions. The time scales, and hence strain rates associated with the indentation process differ vastly from that of blast loading. In simulations involving the steel tubes, strain rate effects must be included in the material model. The difficulty with this is that the

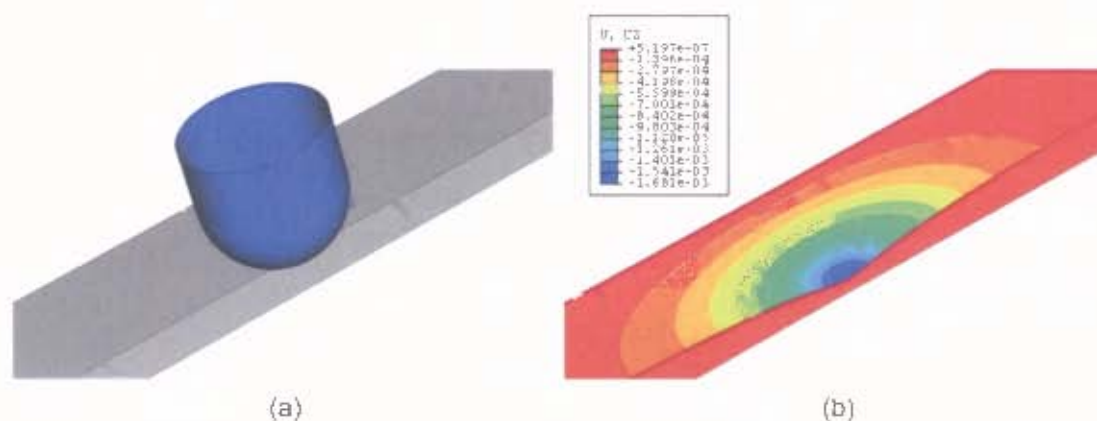


Figure 5.5: (a) Model of a single annealed steel tube of thickness 0.63 mm and aspect ratio 3.44 with indenter before indentation occurs, implemented in ABAQUS. (b) Indented tube after removal of the indenter with a contour plot showing deformation in the lateral direction.

simulated indentation velocity is not the true indentation velocity and hence strain rate hardening is introduced that does not occur experimentally. The experimental indentation process is slow enough that strain rate effects can be ignored. As ABAQUS/Explicit does not allow multiple material definitions at a particular point in the geometry some simplifications have to be made. Indentation simulations are conducted at a velocity of 0.5 m/s with strain rate effects removed from the material model. Lower indentation velocities are then simulated and compared to the strain rate independent case. It is found that the final indentation depth at a velocity of 0.2 m/s differs negligibly from the rate independent model and hence strain rate effects have little effect at this velocity. The aluminium material model is strain-rate insensitive and hence the higher indentation velocity of 0.5 m/s is sufficient.

Once the indenter has reached its 2.0 mm maximum displacement, the indenter is removed at the indentation velocity. During this phase, the indent unloads elastically. The slow unloading is done to ensure that the rebound velocity of the dent is minimized by keeping the dented tube in contact with the indenter. If the indented tube is allowed to freely unload, significant oscillations in the tube wall displacement are observed. Fig. 5.5b shows the deformed shape and transverse displacement field of the tube after indentation. Note that although the peak displacement at the dent midpoint is the prescribed 2.0 mm, the final displacement after elastic unloading for this example is approximately 1.68 mm.

For the mean tube geometries presented in Table 5.1 the simulated indentations depths are found to be $\delta_s = 1.65$ mm and $\delta_a = 1.20$ mm for the steel and aluminium tubes respectively. The indentation depth for the steel tubes is predicted reasonably well however for the aluminium tubes the simulated depth is significantly less than that of the experimental depths. This is likely due to the the filleted edges in the actual aluminium

tube walls acting as a stiffener during the elastic unloading.

The single tube indentation model is easily extended into a complete panel with indented tubes. Fig. 5.6 shows typical ABAQUS models for panels *B* and *C*. Each indentation is created as described for the single tube case. For the complete panel however the indenters are physically removed from the panel before the blast load is applied to the top plate. Note that the stress state in the tubes after indentation is maintained at the start of the blast event to effectively model the residual stresses in the tube from creating the imperfection. The alternative to this is to create the tube geometry directly from a separate denting analysis. This is clearly computationally more efficient as only a single denting analysis needs to be done for a given tube geometry and aspect ratio. The disadvantage is of course, that even for a small changes in the tube geometry and material that a new dent profile needs to be pre-determined. Further to this, the tubes in the panel will be free of residual stresses at the start of the blast event which is not indicative of the true stress state.



Figure 5.6: Panel model with indentations for panels *B* and *C* implemented in ABAQUS.

For both tube materials, the uniaxial stress-strain curve is implemented in tabular form, as a function of temperature for strains up to a value of 3.0. During the folding process, large localized strains develop in the corners of the fold. Element deletion is used if the equivalent plastic strain in an element exceeds 3.0. This is generally only required for excessively large blasts on aluminium tubes where the low melting point of the material can result in highly localised areas where an element has nearly no load carrying capacity. Such behaviour can effect stability of the analysis, while removal of the element has negligible influence on the structural stiffness of the tube. For the top plate the Johnson-Cook constitutive model is used with the model parameters given in section 3.5.5.2.

For all finite element simulations of experimental tests, the loading is assumed to be

applied as a uniform, square pressure pulse as described in section 4.4.1. The duration of the pulse is assumed to be $10 \mu\text{s}$ in all tests. The total impulse is measured experimentally using the ballistic pendulum as will be described in section 5.7.2. From these measurements, the constant pressure from the blast can be computed using Eq. (4.2).

5.6 Quasi-Static Experiments

Quasi-Static tests are conducted on single tubes of the same length and aspect ratio as are used in the complete panel blast tests. The purpose of this is to validate the material and finite element model by comparing experimental force levels with those obtained through finite element simulations. At present, reaction forces cannot be measured during blast tests and therefore quasi-static tests are required to compare reaction forces. Two steel tubes and two aluminium tubes are cut from a single section, after thickness reduction, to obtain a uniform thickness. The mean thickness of the steel tubes is found to be 0.63 mm while the aluminium tubes are found to have a mean thickness of 0.62 mm. The aspect ratios of the steel and aluminium tubes are found to be $R = 3.44$ and $R = 3.45$ respectively.

The tubes are crushed on a Zwick universal testing machine at a constant velocity of 3.75 mm/min. This crushing velocity is chosen as it gives a strain rate approximately equal to the quasi-static tensile tests conducted in section 3.2. This allows for simulations to be conducted at higher velocities than the actual tests by omitting strain-rate effects from the numerical model. The tubes are crushed a distance of 50.0 mm. Fig. 5.7a and Fig. 5.7b shows the resulting force-displacement curves for the steel and aluminium tubes respectively. Also included in the figures are the corresponding numerical simulations of the quasi-static crushing experiments. Table 5.3 gives the quantitative results of the experiments. In the table, $F_{\max}$ refers the initial peak buckling force, F_{high} refers to the peak forces at successive lobe formation and finally F_m refers to the average force over the stroke which is simply the total absorbed energy divided by the crush distance.

Table 5.3: Comparison of quasi-static energy absorption parameters for experimental and finite element analysis.

Material	$F_{\max}$ (kN) (EXP)	$F_{\max}$ (kN) (FEM)	F_{high} (kN) (EXP)	F_{high} (kN) (FEM)	F_m (kN) (EXP)	F_m (kN) (FEM)
Steel	9.53	10.19	5.84	6.26	4.02	3.66
Aluminium	7.86	8.32	5.67	5.37	3.42	2.94

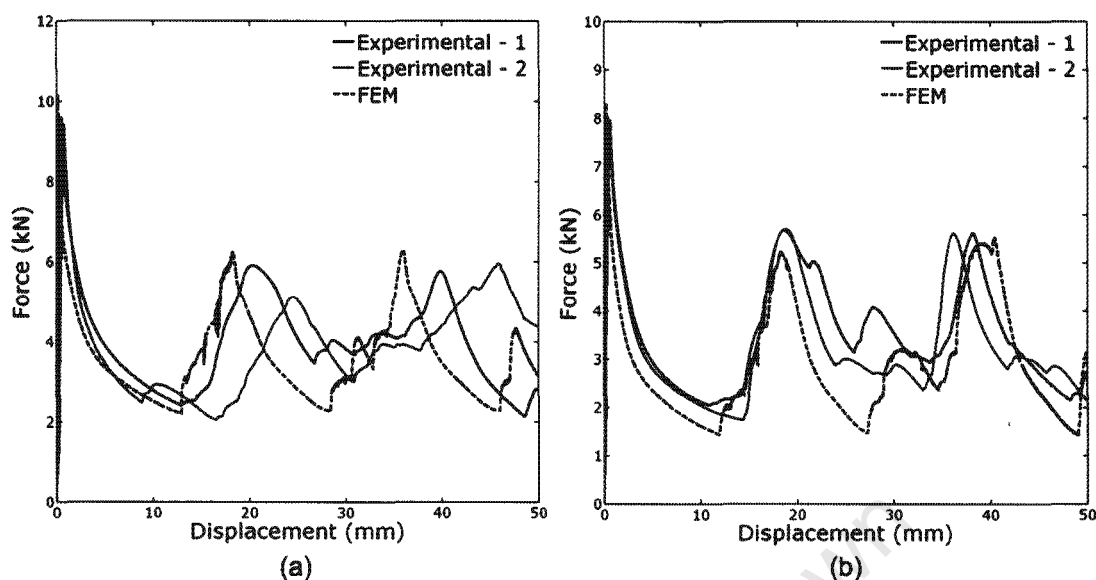


Figure 5.7: Plot of experimental and numerical quasi-static force-displacement curves for (a) Annealed mild steel tubes and (b) Aluminium tubes.

In terms of buckling initiation it is found that all tubes buckle at the location of the initial indentation as expected. During the second experiment using the steel tube, partial global buckling initiates and hence progressive, symmetric buckling ceases. For this reason experiment 2 in Fig. 5.7a is not included in any energy based calculations as it not suitable for comparison with a quarter-symmetry numerical model which must buckle progressively. The force-displacement curve is included however as the peak force and the force over the first lobe formation are valid. All other steel and aluminium tubes are found to buckle in a progressive symmetric manner, however some slight anti-symmetric lobe formation is observed in the first steel tube experiment. This behaviour is a temporary effect as the final buckling shape is reasonably symmetric. This is likely due to inconsistencies in the wall thickness which cause lobes to begin forming at a particular location on a tube wall before the corresponding locations on the other wall have buckled. This has the effect of initiating global buckling however the effect is quickly reversed as the other tube walls buckle to form a nearly symmetric lobe.

Reasonable agreement is found between the experimental and numerical force-displacement curves as shown in Fig. 5.7 and the corresponding energy absorption parameters shown in Table 5.3. For both materials the numerically obtained forces are found to be slightly lower throughout the stroke. This is particularly true for the aluminium tubes. This is expected however due to the relatively large filleted interfaces on the interior tube walls. This is not accounted for in the finite element model using shells. The increased stiffness at the tube corners increases the experimentally obtained crushing force. The underestimation of the force levels for the steel tubes is likely due to the slight parabolic shape in the tube wall as the tubes tend to be thicker at the midpoint.

For both materials the peak buckling load is found to be slightly higher in the numerical model than in the experiments. This is a result of the hemispherical indentation depth being under-predicted in the numerical model. The subsequent peak forces however, are predicted with a high degree of accuracy. In summary, the numerical predictions agree well enough with the experimental data to ensure a high level of confidence in the models ability to capture the force-displacement behaviour of the tubes used in this study.

5.7 Blast Loading Experimental Procedures

5.7.1 Panel Loading

The panels investigated experimentally are subject to uniform blast loads. Although such loads are trivial to generate numerically, experimentally uniform loading due to blast loading is a difficult problem. Initially, tests were conducted using the method presented in Chung Kim Yuen and Nurick [143] where concentric annuli of explosive are detonated at a very short standoff distance from the panel. Such tests did not appear to give a reasonably uniform load however and resulted in significant deformation near the point of detonation.

To circumvent this problem a testing rig similar to that of Jacob [142] is developed. In the aforementioned study the author generated reasonably uniform loads on clamped circular steel plates by exposing the plates only at the distal end of a thick circular blast tube while detonating a disc of explosive at the other end. As discussed in section 4.4.1, the author found the load to be uniform provided the standoff was at least the diameter of the plate. For the present study a similar shock tube is used however this tube is square due the geometry of the top plate. Fig. 5.8 shows a schematic of the testing rig used here. The panel is fixed to a thick rigid plate at the base of the panel, and the blast tube is adjusted such that the other end of the tube is flush against the top plate of the panel. Under this configuration only the base of the panel is constrained and the top plate is free to move as the core tubes buckle.

The blast tube is 190 mm long which exceeds the width of the plate and hence the pressure applied to the top plate is likely to be reasonably uniform. As previously mentioned however, this is a difficult parameter to verify experimentally and hence thick top plates for the panels are used to minimize the effect of non-uniformity in the loading. The inside area of the blast tube is 150 mm $\times$ 150 mm while the top plate of the panel is 156 mm $\times$ 156 mm. The reason the top plate is slightly larger than the exposed area is to minimize the escaping of any gases in a direction along the axis of the ballistic pendulum. If there

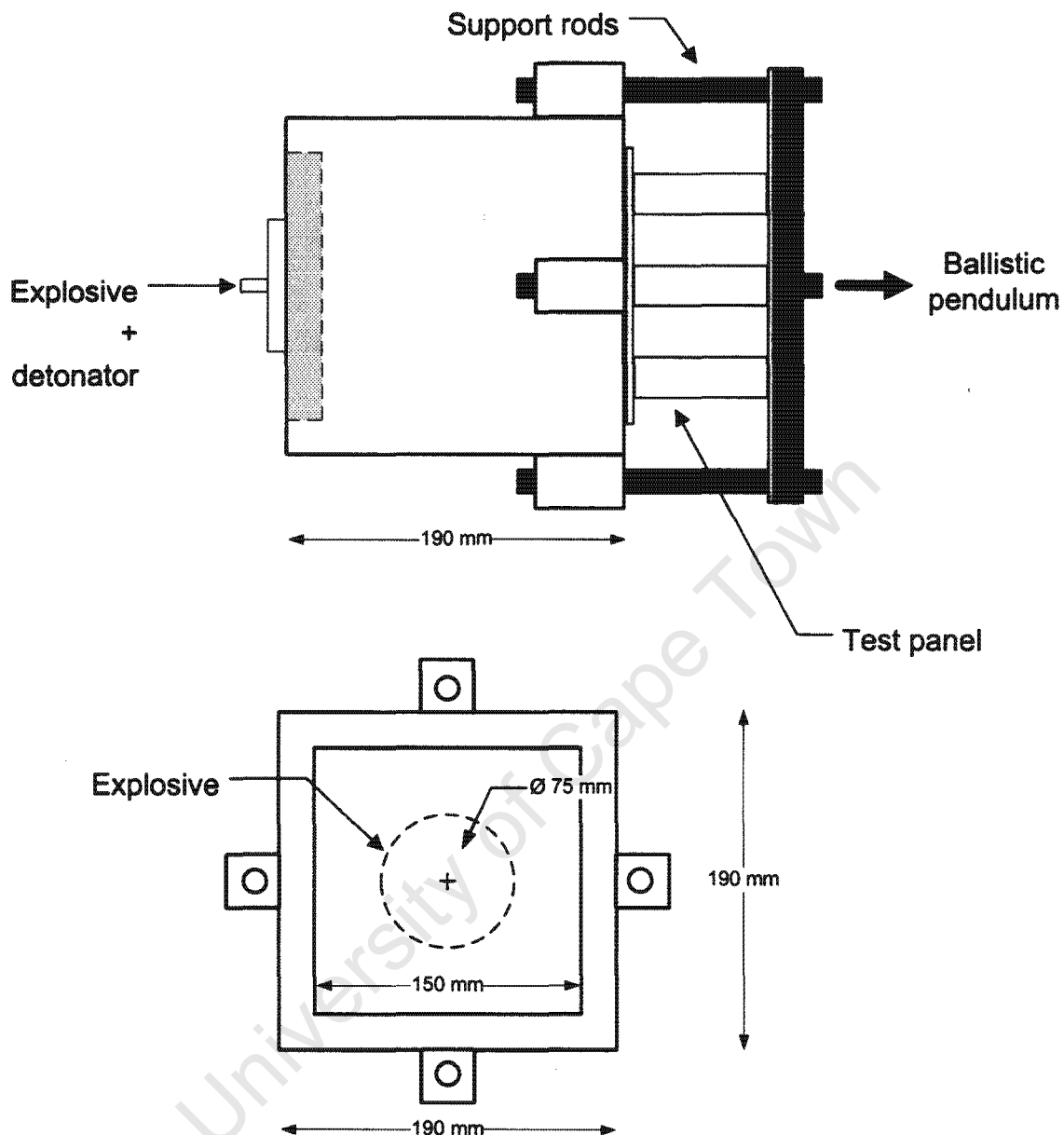


Figure 5.8: Schematic of the blast tube used to direct the pressure load onto the panels.

are gaps created during the blast event the ballistic pendulum may be subjected to loads that the panel is not, resulting in an over measurement of the applied impulse.

The pressure loading on the panel is generated by detonating a circular disc of PE4 explosive of diameter 75.0 mm. The explosive is placed on a 150 mm $\times$ 150 mm polystyrene pad of approximate density 18.15 kg/m³ as shown in Fig. 5.9a. The polystyrene is then placed at the end of the tube opposite to that of the location of the panel as shown in Fig. 5.9b. After detonation of the explosive, the polystyrene pad disintegrates and as such, has no apparent effect on the loading of the panel.

The technical data for PE4 explosive as reported by Wharton et al. [149] is briefly discussed here. The PE4 explosive used in this study is composed of 88 % RDX and 12 %

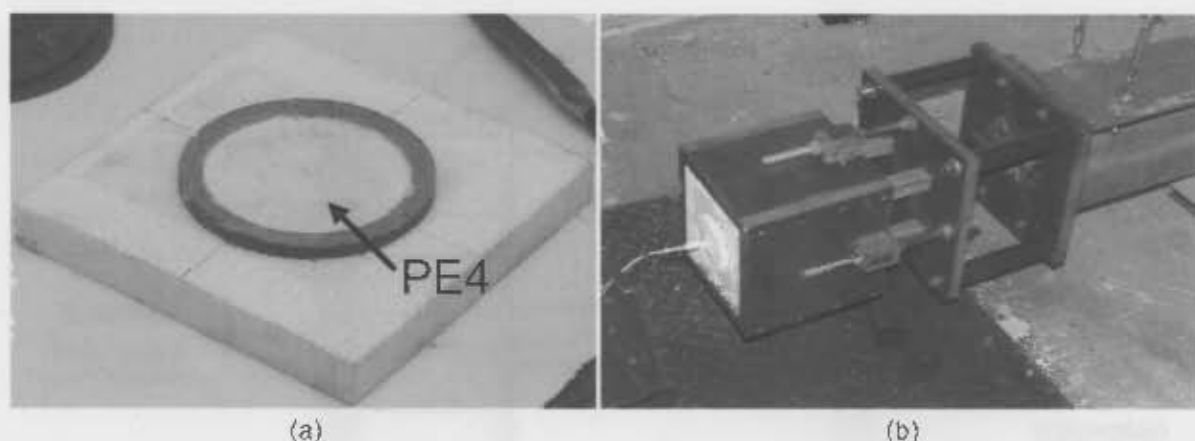


Figure 5.9: (a) Photograph of the layout of the PE4 explosive disc on the polystyrene pad. (b) Photograph of the front end of the ballistic pendulum with the attached shock tube. Also shown in the photograph is the explosive and detonator held in place by the polystyrene pad.

lithium grease. The density is 1.6 g/cm^3 and the strength in terms of TNT equivalence is 130 %. The detonation speed is approximately 8200 m/s however values as low as 7500 m/s have been used in blast loading analysis [143].

5.7.2 The Ballistic Pendulum

Although recording experimental measurements is difficult under blast loading conditions, the use of a ballistic pendulum allows for the recording of applied impulse measurements. Some studies that utilize the ballistic pendulum under blast loading include Lump [150], Nurick [151], and more recently, Hanssen *et al* [78] and Enstock and Smith [152]. Most of the literature cited uses the pendulum merely as tool to record applied impulse, however Enstock and Smith [152] investigate the validity of the pendulum itself. Specifically, the authors investigated the ability of the pendulum to discriminate between different formulations of PE4 explosive. The formulations included the use of barium sulphate and copper oxide which were both found to increase the impulse transfer to the pendulum.

5.7.2.1 Theoretical Impulse Calculations

Simple calculations can be performed following a blast test on the ballistic pendulum to determine the impulse transferred to the panel. Nurick [151] provides a good overview of the pendulum theory. The analysis is simplified by assuming that the pendulum maintains its orientation throughout the blast response. This is a valid assumption if the pendulum is properly balanced in all directions. This implies that the ballistic pendulum can be treated as a simple pendulum where rotational inertia of the pendulum and wires

are ignored. The linearized equation of motion of a simple pendulum is given as

$$\ddot{x} + 2\beta\dot{x} + \omega_n^2 x = 0 \quad (5.1)$$

where

$$\beta = C/2m_p \quad (5.2)$$

and

$$\omega_n = 2\pi/T \quad (5.3)$$

C is the damping coefficient, m_p is the total mass of pendulum including the specimen and balancing weights, and T is the natural period of motion of the pendulum. The solution of the second order differential equation given in Eq. (5.1) is given by

$$x(t) = \frac{(e^{-\beta t})\dot{x}_0 \sin \omega_d t}{\omega_d} \quad (5.4)$$

where $\dot{x}_0$ is the initial velocity of the pendulum and ω_d is given by

$$\omega_d = \sqrt{\omega_n^2 - \beta^2} \quad (5.5)$$

Let x_1 be the maximum forward horizontal displacement of the pendulum and x_2 be the maximum backward displacement of the pendulum. The distances x_1 and x_2 occur at $t = T/4$ and $t = 3T/4$ respectively. Substituting these times into Eq. (5.4) gives

$$x_1 = \frac{\dot{x}_0}{2\pi} e^{-\beta T/4} \quad (5.6)$$

and

$$x_2 = -\frac{\dot{x}_0}{2\pi} e^{-3\beta T/4} \quad (5.7)$$

solving for β in terms of x_1/x_2 gives

$$\beta = \frac{2}{T} \ln \frac{x_1}{x_2} \quad (5.8)$$

and therefore the initial velocity can be calculated as

$$\dot{x}_0 = \frac{2\pi}{T} x_1 e^{-\beta T/4} \quad (5.9)$$

Finally, the impulse can be calculated as

$$I = m\dot{x}_0 \quad (5.10)$$

The period T can be determined by simply allowing the pendulum to oscillate from a fixed position. The values of x_1 and x_2 must be calculated however. The difficulty in computing the impulse stems from difference in pen position from the horizontal displacement of the pendulum. This difference must be accounted for. Figure 5.10 gives the geometric description of the pendulum motion with associated geometric variables.

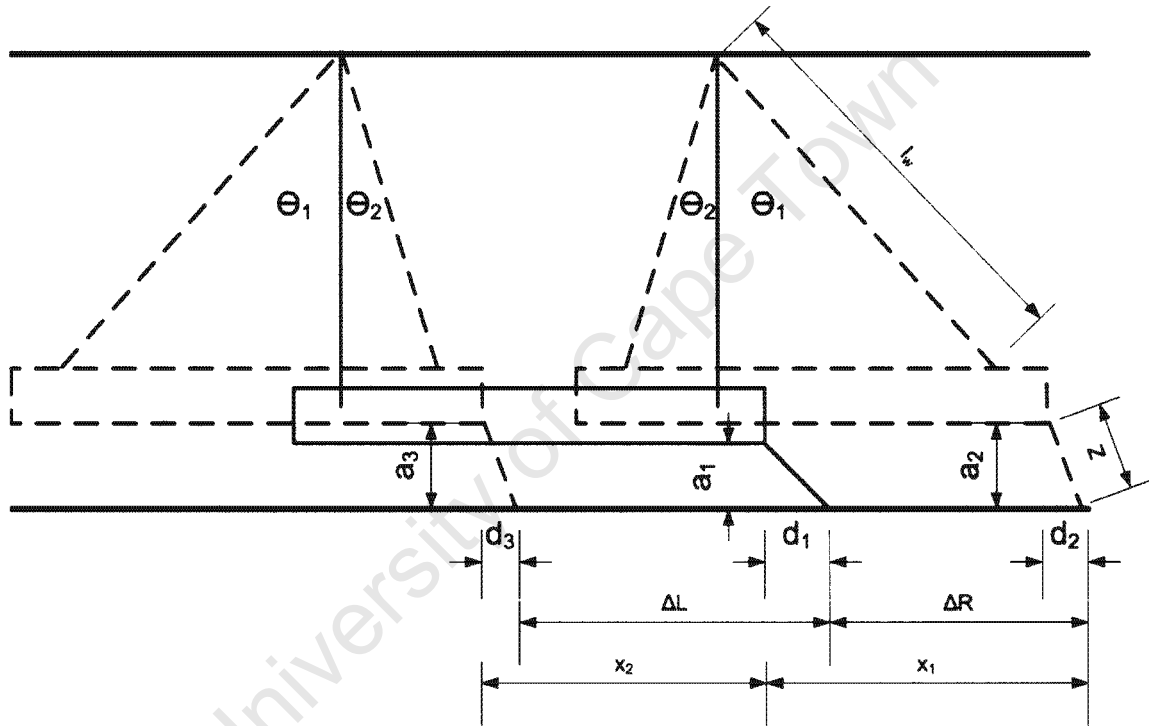


Figure 5.10: Schematic of pendulum geometry. Solid lines indicate initial pendulum position while dashed lines indicate the pendulum position and the maximum forward and backward positions.

The measurable variables are the wire length l_w , the initial vertical displacement from the pen position a_1 , the pen length Z , and the maximum forward and backward translations of the pen tip ΔR and ΔL respectively. Simple trigonometry gives the following relationships for the geometric variables given in Figure 5.10. The differences between the initial position of the pendulum and the tip of the pen d_1 is

$$d_1 = \sqrt{Z^2 - a_1^2} \quad (5.11)$$

The differences between the forward swing of the pendulum and the forward movement

of the pen d_2 is given by

$$d_2 = \sqrt{Z^2 - a_2^2} \quad (5.12)$$

where

$$a_2 = l_w(1 - \cos \theta_1) + a_1 \quad (5.13)$$

The differences between the backward swing of the pendulum and the backward movement of the pen d_3 is given by

$$d_3 = \sqrt{Z^2 - a_3^2} \quad (5.14)$$

where

$$a_3 = l_w(1 - \cos \theta_2) + a_1 \quad (5.15)$$

therefore x_1 can be described as

$$x_1 = l_w \sin \theta_1 \quad (5.16)$$

and x_2 is given as

$$x_2 = l_w \sin \theta_2 \quad (5.17)$$

The measurable quantities ΔR and ΔL are therefore defined as

$$\Delta R = x_1 - d_1 + d_2 = l_w \sin \theta_1 - \sqrt{Z^2 - a_1^2} + \sqrt{Z^2 - (l_w(1 - \cos \theta_1) + a_1)^2} \quad (5.18)$$

and

$$\Delta L = x_2 + d_1 - d_3 = l_w \sin \theta_2 + \sqrt{Z^2 - a_1^2} - \sqrt{Z^2 - (l_w(1 - \cos \theta_2) + a_1)^2} \quad (5.19)$$

Newton's method is implemented in Matlab to solve for θ_1 and θ_2 in Eqns. 5.18 and 5.19. The values of x_1 and x_2 can then be computed using Eqns. 5.16 and 5.17 respectively. The damping value β is then computed along with the initial velocity $\dot{x}_0$. Finally, the impulse is determined using Eq. (5.10).

5.8 Blast Test Results

The results of the experimental blast tests are presented here. This section is structured in the following manner. Firstly, the impulse measurements are discussed. The post-experiment measurement methodology is then briefly described. The crushed panels after blast loading are then illustrated along with the corresponding finite element simulations. The qualitative and quantitative experimental results are given along with the corresponding analysis. This analysis is strictly limited to experimental observations however as the finite element analysis is described in section 5.9. Finite element results are presented in figures, tables and graphs in this section, however this is simply for sake of comparison.

5.8.1 Impulse Measurements

Fig. 5.11 shows the impulse transfer to the ballistic pendulum as a function of charge mass. The mass of the 1 g detonator is included in the total charge mass. It must be noted that the measured impulse is independent of the response of the panels as a result of conservation of momentum. The natural period of the pendulum greatly exceeds the duration of response of the panels and thus the swing of the pendulum is independent of the energy absorbed by the panels. This implies that charges of equal mass will result in nearly equal impulse provided the testing rig does not change. The results of Fig. 5.11 show this to be the case for the tests conducted. There are small variations in impulse at equal charge mass however this is likely due to small changes in the detonation process.

Two curves corresponding to linear least squares fits of the data is also included. The solid line is the general linear fit while the dashed is the best fit obtained when forcing the curve to go through the origin. Note that the general least squares fit of the data is significantly higher than zero at zero charge mass. This is not unexpected however as there are relatively few data points, most of which are at large charge masses and therefore the general linear relationship given can only be assumed to be valid within the given range of charge masses. For comparison purposes, Jacob [142] conducted blast tests using a cylindrical blast tube at standoff distances of 15 – 300 mm with charge masses of 5 – 15 g. The author obtained a charge mass (m) to impulse relationship of $I = 1.960m + 1.640$ with 90 % confidence lines corresponding to ± 1.31 Ns and 99 % confidence lines corresponding to ± 2.69 Ns. The author of [142] also observed that if limiting the analysis to 300 mm standoff distance the relationship between charge mass and impulse is $I = 2.03m + 2.86$. The slopes of the linear curve fits agree closely with the results of the present study. It should be noted that this relationship is not required in any theoretical or computational analysis and is included merely for completeness.

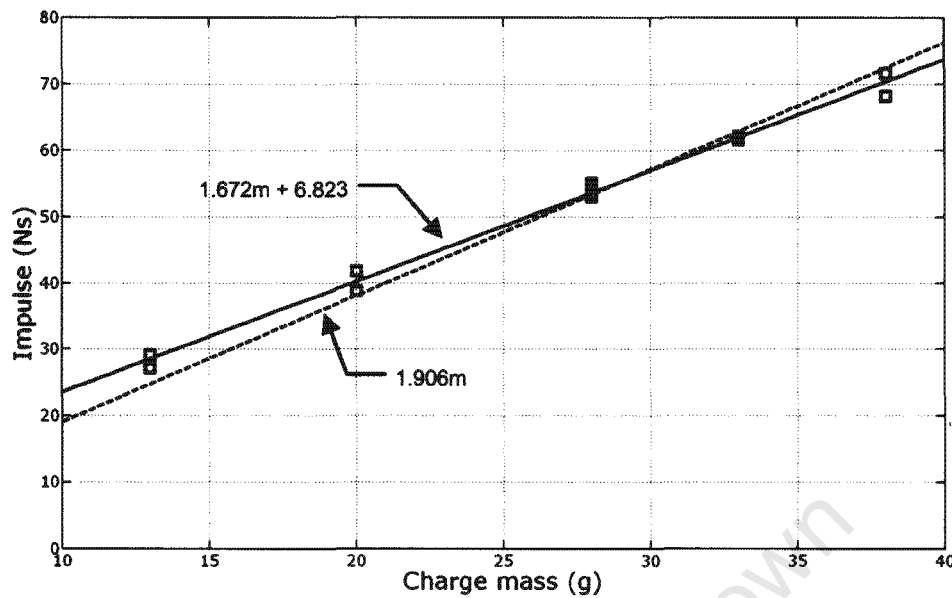


Figure 5.11: Plot of impulse transferred to the panels as a function of explosive mass. For the linear curve fits, m refers to the total mass of the explosive, including the detonator.

5.8.2 Post-Test Panel Measurements

The methodology used to measure the final crush distance is briefly discussed here. For several tests the soldering process used to bond the base of the tubes to the bottom plate is found to be insufficient. In these test cases the core tubes appear to remain in the bottom plate cutouts and hence there is no global lateral tube movement. The solder does fail however, and thus there is some local deformation at the tube locations that are initially fixed to the bottom plate. For these cases, it is assumed the core tubes become physically removed from the bottom plate after the maximum crush distance (i.e. during rebounding) is obtained and therefore this debonding should have minimal effects on any post-test measurements.

Crush distances are measured using a vertical height gauge. Before measurements are taken bottom plates are removed from all panels in which the plate did not become debonded during testing. This is accomplished by re-heating the solder and carefully removing the tube and top plate from the bottom plate. No deformation in the tubes is observed during this process. The crush distance for each tube is measured by positioning the panel with the top plate facing downwards and then placing a small plate over the exposed cross-section of the tube. The small plate has approximately the same cross-sectional dimensions as the tubes in the panel. The distance crushed by a given tube can then be measured at the midpoint of the small plate. The deformations in the top plate are negligible and therefore the top plate sits nearly flush with the measuring table. For panel *B* tests, all tube crush distances are measured and for panel *C* tests, the centre and outermost tubes are measured (tubes 1 and 2 in Fig. 4.2).

The experimentally obtained mid-point crush distances for all tests is presented in Fig. 5.12. A summary of experimental and finite element results is presented in Table 5.4. Fig. 5.13 illustrates the experimentally obtained crush distances, for each core material, including information on the skewness of the top plate after blasting. In Fig. 5.13 the error bars indicate the minimum and maximum observed crush distances as measured at the tubes in the panel. Figs. 5.14 - 5.17 show the crushed tubes and top plate for the tests conducted in this study. For all photographs, the bottom plate is removed for clarity and the panel positioned such that the top plate is facing downwards. In all figures and tables the naming convention used for the panels is

$$\text{material} - \text{panel} - \text{charge mass}$$

where for the *material* field *S* indicates steel is the tube material and *A* indicates aluminium. The *panel* field corresponds to panel *B* or *C* as per Fig. 4.1.

5.8.3 Quantitative Results

5.8.3.1 Steel Tube Cores

The results of Table 5.4 and Figs. 5.12 - 5.13 show a significant increase in core crush distance with increasing impulse. In terms of the panel *B* designs, a large increase in crush is observed when moving from 53.05 Ns blast to the 62.13 Ns blast however only a small increase is found when examining the 68.24 Ns. This is clearly due to panels reaching its maximum impulse capacity at approximately 62.13 Ns. For test *S - B - 38* it can then be assumed that deformations near the the end of the crushing process are limited to localised squashing of the tube beyond the maximum stroke length which induces large forces to the base structure.

It is also of note that for test *S - B - 28* the difference between the largest and smallest measured tube crush distance is much larger than for other tests. This would appear to indicate that for blasts that do not fully utilize the capacity of the panel, global instabilities can develop. If the blast is large enough to require the full panel stroke however, the final crush distances for each tube in the panel is nearly the same indicating a stable response to the blast. Without the time history of the tests it is difficult to determine if initial panel skewness develops, resulting in some of the panel tubes compacting while others still are undergoing progressive folding. It must be noted that for all tests, for both materials, that the differentials in observed crush distance are not symmetric. Specifically, even for large crush distance differentials, the top plate remains flat. As a result of this, the tube within a given panel with the largest crush distance will be opposite along a diagonal line of action from the tube with the smallest crush distance. This is

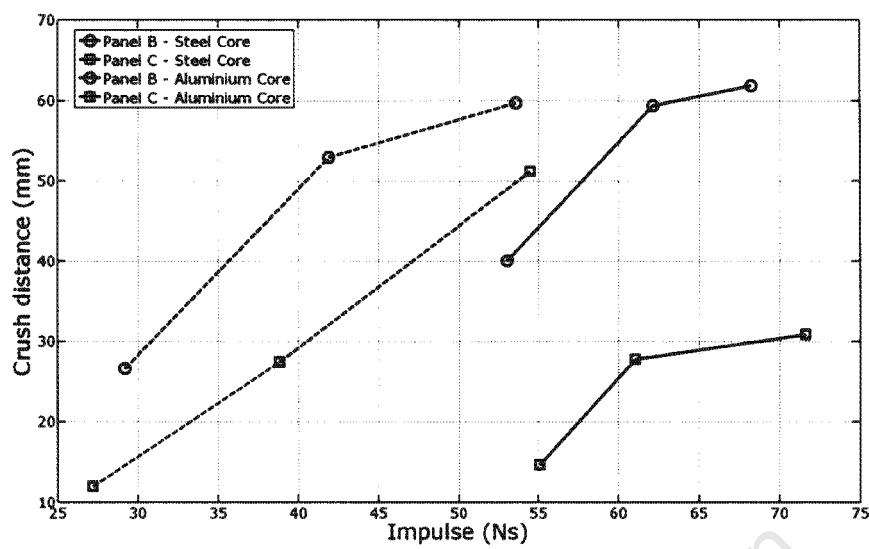


Figure 5.12: Mid-point crush distance as a function of applied impulse for all experimental tests cases.

an interesting result when considering the loading rig. Although the uniformity of the pressure pulse is questionable, the loading is expected to be symmetric about the primary axis of the top plate. The observed skewness is therefore likely a result of uneven detonation in the disk of PE4 explosive. It is also observed however that in panel tests with large top plate skewness, the tube with the largest crush distance appears to have buckling initiated at the trigger. Conversely, in these panels, the tube with the smallest crush distance buckles far from the trigger. This indicates that buckling initiation may have significant influence on the global stability of the proposed panels.

For the panels *C* tests, a significant increase in crush distance is observed for the two smaller blasts. The larger blast corresponding to test *S – C – 38* shows an unexpectedly small crush distance in comparison to test *S – C – 33*. Despite the relatively large increase in impulse (10.6 Ns) only a small increase in crush distance is found. A possible explanation for this can be found by examining the distal end of the tubes in test *S – C – 38*. Due to apparently early debonding of the tubes and the bottom plate, significant, highly localised deformations occur in the tube at the position where the tubes are normally bonded to the bottom plate. Plastic hinges are formed with a large angles of rotation at these hinges. A relatively significant amount of energy can be absorbed through such deformations while adding comparatively little to the total crush distance.

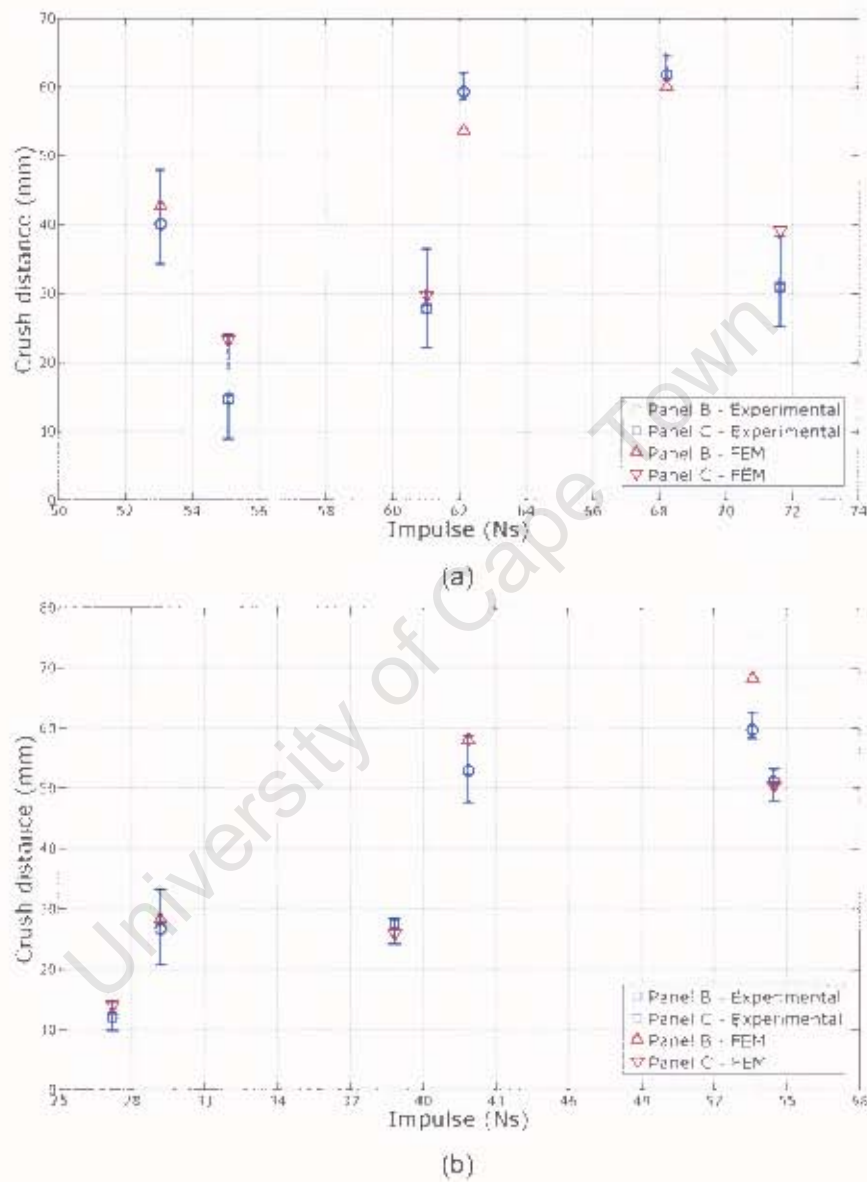


Figure 5.13: Plot of experimental and finite element predictions of mid-point crush distance as a function of applied impulse for (a) Steel tube cores and (b) Aluminium tube cores. Error bars indicate the minimum and maximum crush distances observed for the panel.

Table 5.4: Summary of experimental results and comparisons with finite element simulations.

Test	Material	n_t	PE4 mass (g)	I (Ns)	δ_{mid} (mm) (EXP)	δ_{mid} (mm) (FEM)	Rel. error (%)
$S - B - 28$	Steel	5	28	53.05	40.05	42.63	6.44
$S - B - 33$	Steel	5	33	62.13	59.36	53.64	9.64
$S - B - 38$	Steel	5	38	68.24	61.85	60.14	2.76
$S - C - 28$	Steel	9	28	55.09	14.69	23.26	58.34
$S - C - 33$	Steel	9	33	61.63	27.78	29.65	6.73
$S - C - 38$	Steel	9	38	71.63	30.90	39.24	26.99
$A - B - 13$	Aluminium	5	13	29.17	26.65	28.35	6.38
$A - B - 20$	Aluminium	5	20	41.87	52.93	58.04	9.65
$A - B - 28$	Aluminium	5	28	53.59	59.73	68.34	14.41
$A - C - 13$	Aluminium	9	13	27.21	11.99	13.97	16.51
$A - C - 20$	Aluminium	9	20	38.83	27.46	25.89	5.72
$A - C - 28$	Aluminium	9	28	54.49	51.17	50.36	1.58

Test	Energy abs. (kJ) (FEM)	F_{max} (kN) (FEM)	F_m (kN) (FEM)	Compaction (EXP)	Comments (EXP)
$S - B - 28$	1.743	94.36 (106.58)	40.25	N	tearing, partial bottom plate debonding
$S - B - 33$	2.399	95.54 (106.54)	44.34	Y	tube 1 debonding from top plate
$S - B - 38$	2.891	96.11 (106.58)	47.49	Y	tearing, bottom plate debonding
$S - C - 28$	1.881	169.88 (191.61)	80.54	N	bottom plate debonding
$S - C - 33$	2.359	171.27 (191.63)	80.28	N	-
$S - C - 38$	3.189	173.13 (191.82)	81.65	N	bottom plate debonding
$A - B - 13$	0.537	61.65	18.74	N	-
$A - B - 20$	1.109	61.67	19.02	N	-
$A - B - 28$	1.759	61.68	24.84	Y	bottom plate debonding
$A - C - 13$	0.462	110.88	32.77	N	-
$A - C - 20$	0.946	110.97	36.55	N	-
$A - C - 28$	1.864	111.06	36.80	N	-

5.8.3.2 Aluminium Tube Cores

The aluminium tube core panels also show significant increase in midpoint crush distance with increasing impulse, as given in Figs. 5.12 - 5.13. Only a small increase is observed however, in test $A - B - 28$ over test $A - B - 20$ indicating that compaction has occurred and that the maximum impulse of the panel B designs with the given geometry is approximately 41.87 Ns. As is the case for the steel tube panels, the panel B tests in which compaction does not occur results in relatively large top plate skewness. This skewness, characterized by large differences in tube crush distances within a given panel, does not appear to be as severe for the aluminium tube panels as for the steel tube panels. For the panel C tests the observed crush distance forms a strong linear trend as none of the tests fully compact the core tubes. Visual examination of test $A - C - 28$ however, shows that panel is near compaction and hence the impulse capacity of the panel C designs is approximately 54.49 Ns. It is also worthy of note that the crush distances at the common charge mass of 28g between the steel and aluminium cores are significantly higher in the aluminium core panels. For the panel B tests the aluminium core panel shows a crush distance approximately 49 % higher than the steel tube panel subjected to the same charge mass. For the panel C test, the aluminium tube panel shows a crush distance 248 % larger than the steel tube panel. This large percentage increase is partially due to the unusually small crush distance found in test $S - B - 28$. These large increases in crush distances when using aluminium tubes illustrates the influence that strain hardening, and in particular, strain rate hardening has on the response of the panels under blast loading conditions.

5.8.4 Qualitative Results

5.8.4.1 Steel Tube Cores

Figs. 5.14 and Figs. 5.15 show the steel tube test results for panels *B* and *C* respectively. For the panel *B* design under the 28 g blast the buckling mode is found to be irregular with each tube in the panel having lobe formations at different locations. In each of the tubes except for the central tube, buckling appears to initiate at the indentation trigger. In the central tube, lobe formation occurs at the proximal end of the tube, ignoring the trigger completely. Also of note for this test is that tearing occurs along the edge of one of the outermost tubes in the panel. For the larger blasts corresponding to 33 g and 38 g of explosive, the buckling modes are observed to be much different. In both cases, the resulting tubes show a nearly symmetric crushing pattern. This is particularly true for the test *S-B-33*. This result, when coupled with the resulting crushing pattern of test *S-B-28* indicates that the core tubes may be able to "recover" from initial instabilities in the buckling process. It is however possible that the symmetric buckling patterns observed are due to buckling initiation occurring at the triggers in tests *S-B-33* and *S-B-38*. For tubes that undergo significant crushing, the lobe formations are of the symmetric type. Further to this, for panels that crush to compaction, it is found that two to three lobes are formed in each tube where, in general, two of the lobes are large with the third having a very small lobe size.

The results of the panel *C* tests show similar buckling behaviour to that of test *S-B-28*. Specifically, the lobe formations are highly irregular, particularly so for tests *S-C-28* and *S-C-33*. Buckling does appear to initiate at the trigger in several of the tubes in these panels however. In test *S-C-28*, buckling initiates at the trigger only in the outermost tubes, with the largest crush distance. Buckling initiates at the trigger in all tubes in test *S-C-33* on the side of the panel corresponding to maximum crush distance. For the largest blast test, *S-C-38*, most of the tubes show buckling initiating at the trigger with tubes with small crush distance buckling at the proximal ends of the tubes. It should also be noted that the progressive buckling of tube which buckle at the trigger appears to be confined to the upper half of the tube, closest to the top plate. Further to this, in cases where the lobe formation are not degenerate, the lobe formations appear to be of the symmetric type.

5.8.4.2 Aluminium Tube Cores

Figs. 5.16 and Figs. 5.17 show the steel tube test results for panels *B* and *C* respectively. Similarly to the panel *B*, steel tube tests with small crush distance, the aluminium tube panel subject to the 13 g blast shows highly irregular lobe formations. It is also found

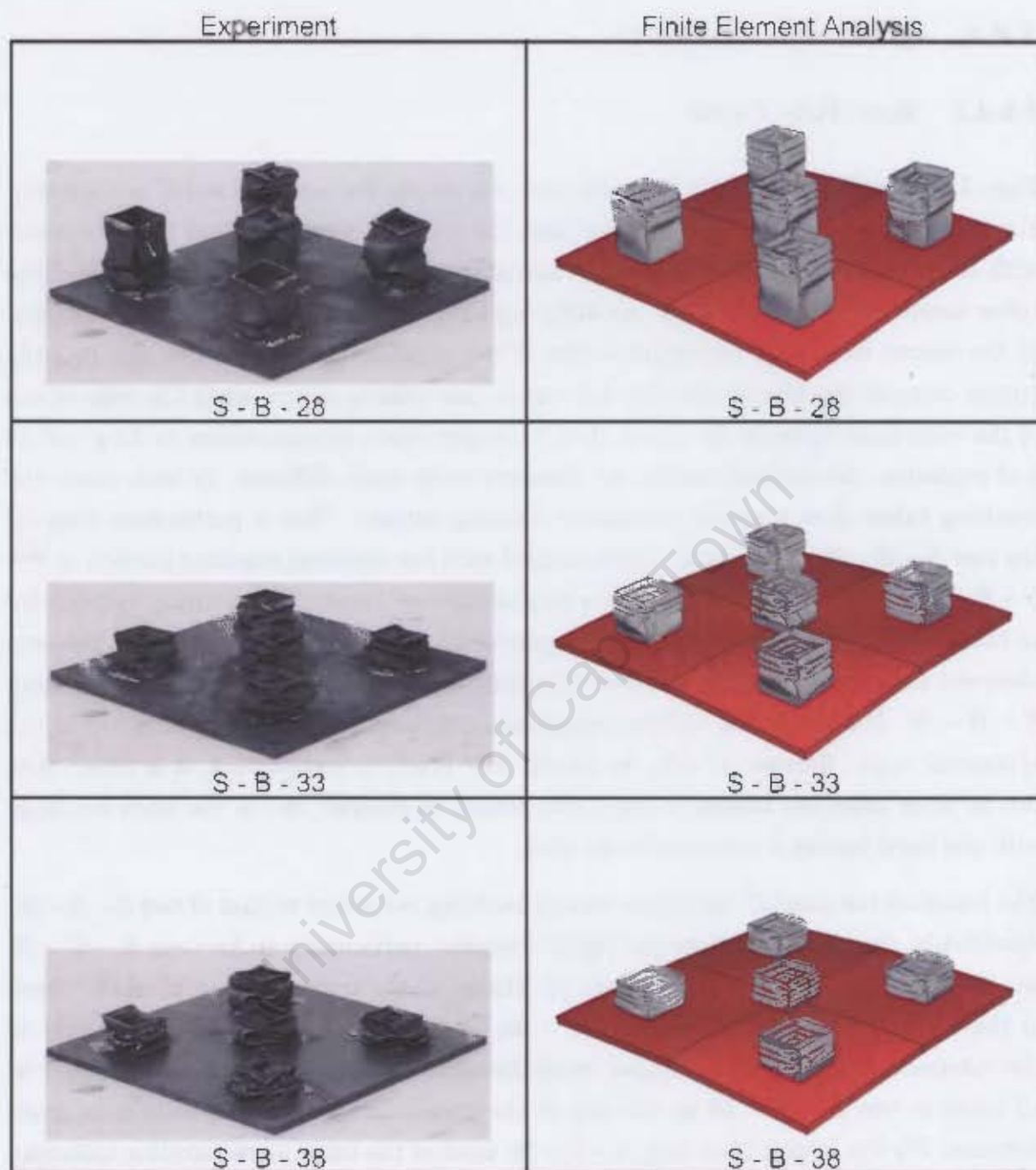


Figure 5.14: Experimental and numerical deformation of the core tubes for panel *B* with steel tubes.

that only three of the five tubes buckle at the trigger in this test. For the tube with the smallest crush distance, extensional modes develop at the proximal and distal ends of the tube which does not occur in any of the steel tube tests. For the larger blast tests *A - B - 20*, and *A - B - 28*, buckling is found to generally be of the progressive symmetric type with some irregularity found in the test panel *A - B - 28*. In this test the lobes are found to be much larger and highly non-uniform in thickness across the tube width.

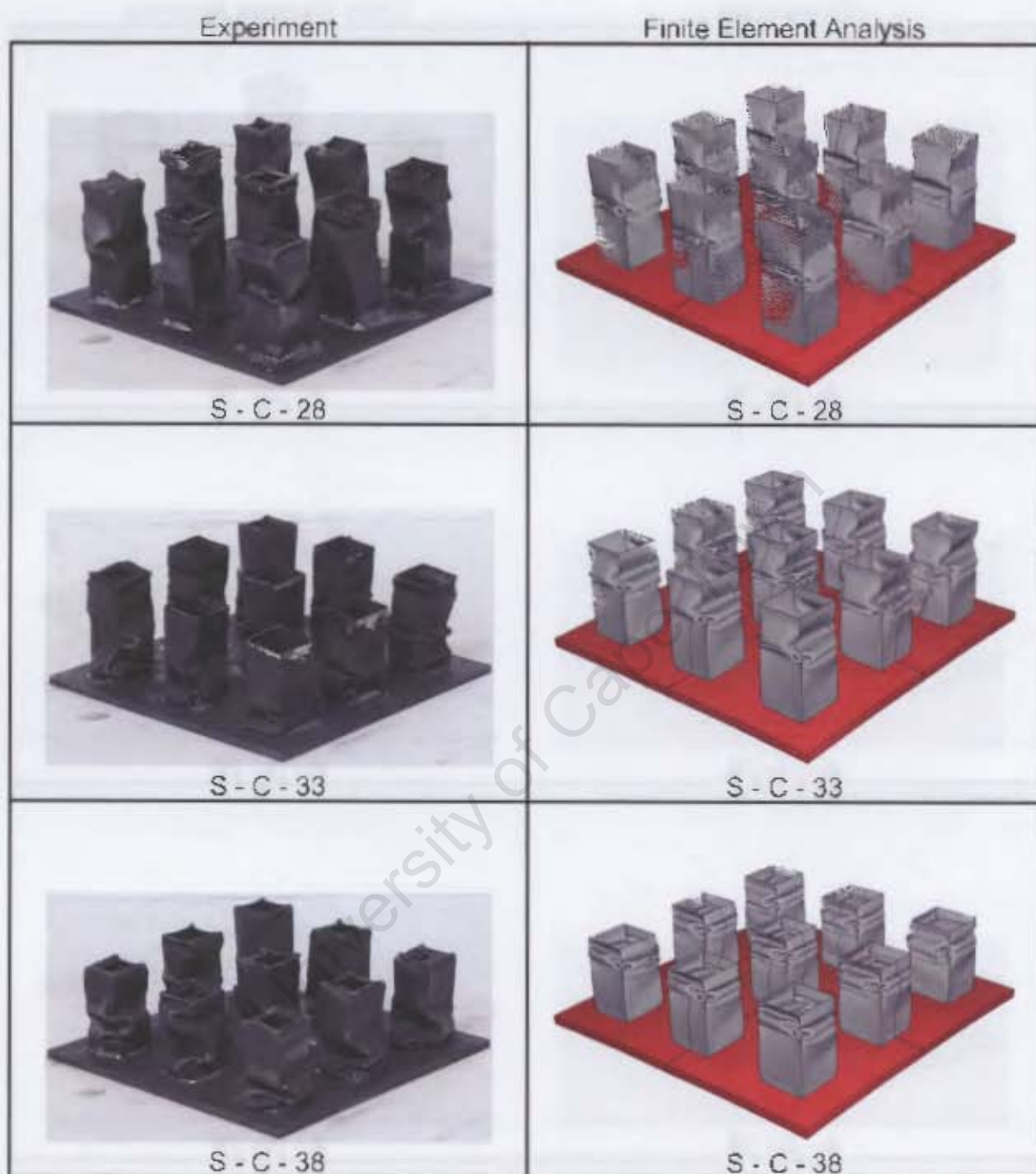


Figure 5.15: Experimental and numerical deformation of the core tubes for panel *C* with steel tubes.

For tests *A - C - 13* and *A - C - 20*, the trigger is ignored in almost all tubes. In these cases where the trigger is ignored, a mixture of extensional and symmetric lobes are formed at the distal and proximal end of the tubes. This buckling pattern is consistent with the experimental and numerical study by Karagiozova *et al.* [8] in which square aluminium tubes are subject to blast loads. The authors found that for a rate insensitive material, rapid localisation of plastic strain occurs at both ends of the tube due to slow plastic wave speeds. A flat wrinkle is found to be formed at the proximal end while the

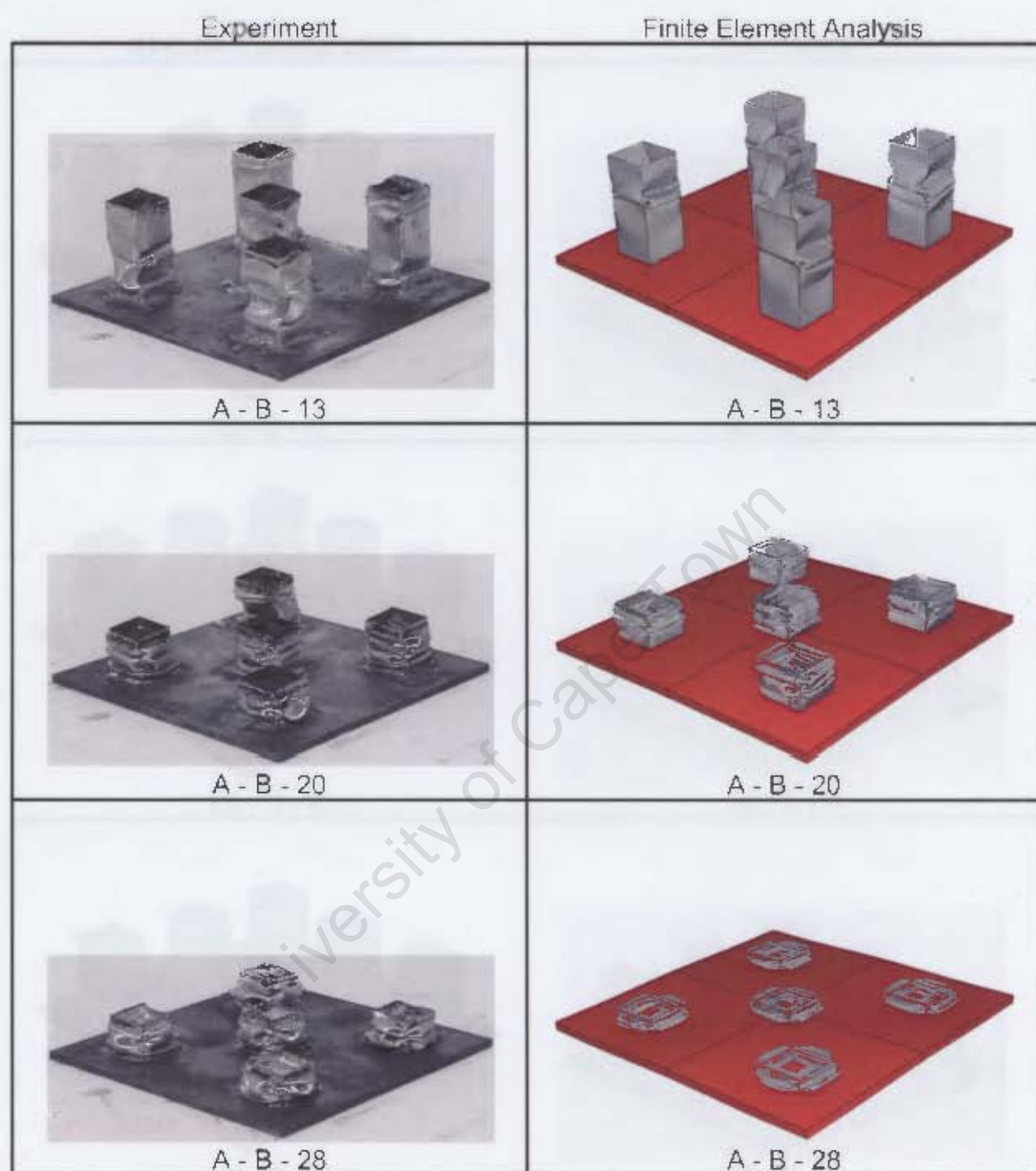


Figure 5.16: Experimental and numerical deformation of the core tubes for panel *B* with aluminium tubes.

reflection of the plastic stress wave forms a wrinkle at the distal end. The authors note that for a rate sensitive material, plastic strain are distributed more evenly along the shell length. Numerically, it is observed that symmetric folds are developed in the rate sensitive material and not in the rate insensitive material. Such behaviour is consistent with the experimental observations of the current study when comparing the response of the steel and aluminium tubes. The expected localisation of plastic strain at the distal and proximal end of the shell effectively eliminates the effectiveness of the trigger in most

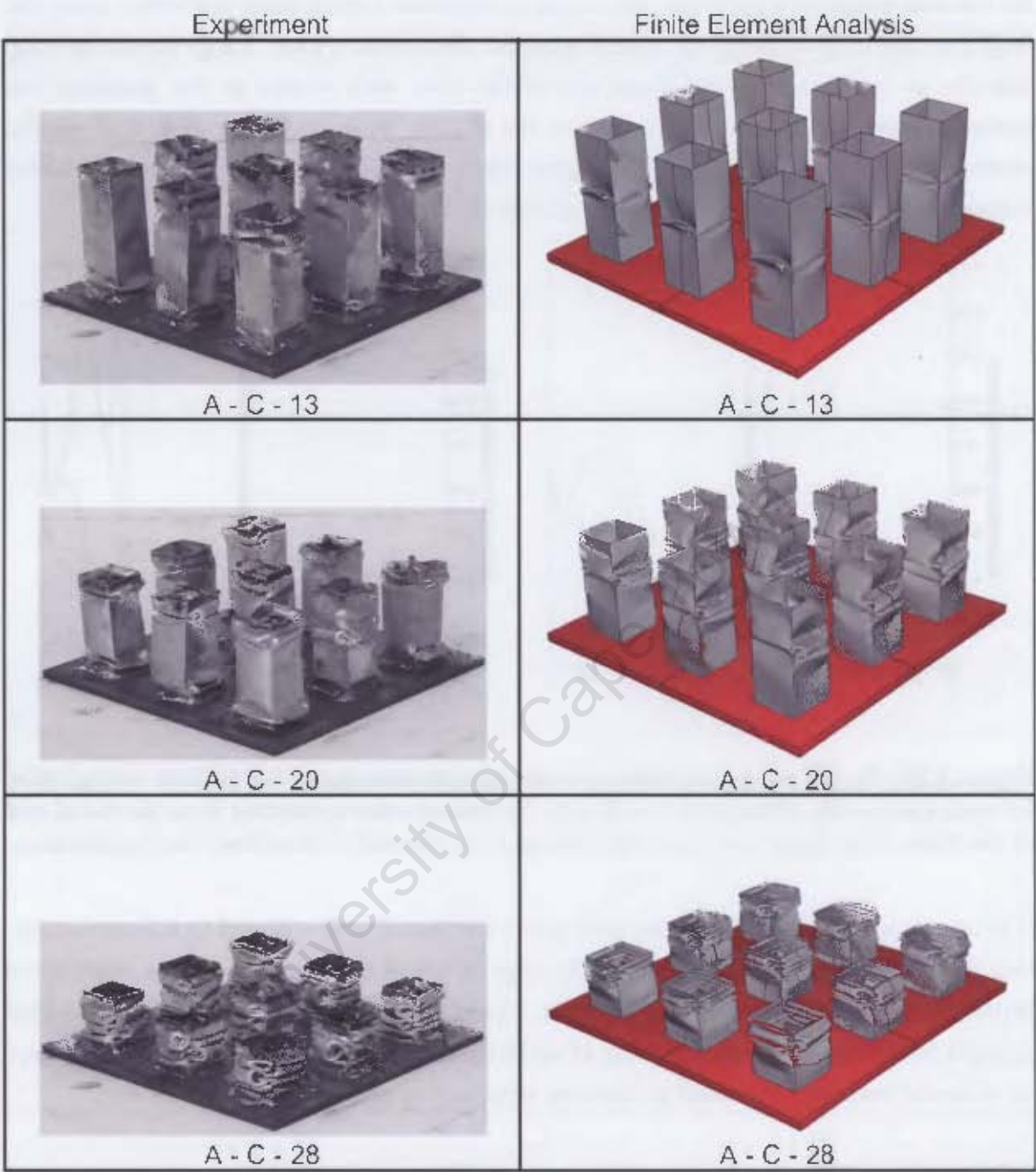


Figure 5.17: Experimental and numerical deformation of the core tubes for panel *C* with aluminium tubes.

cases.

Although the majority of the finite element results are discussed in the following sections it is beneficial here to investigate numerically, the strain localisation observed experimentally. Fig. 5.18 shows the numerically determined equivalent plastic strain distribution, at several time intervals along the midline of the central tube, for tests *S* – *B* – 28 and *A* – *B* – 28. The times are given with respect to the end of the blast load. It is clear that

for the rate-sensitive steel tubes, the strain is distributed much more uniformly along the length of the tube than for the rate insensitive aluminium tubes. Large strains develop initially at the proximal and distal end of the tube with strains at the proximal end having comparable magnitude to that at the trigger. From this it is clear that careful consideration must be given to the trigger depth in tubes composed of rate insensitive material to ensure buckling initiates at the desired location.

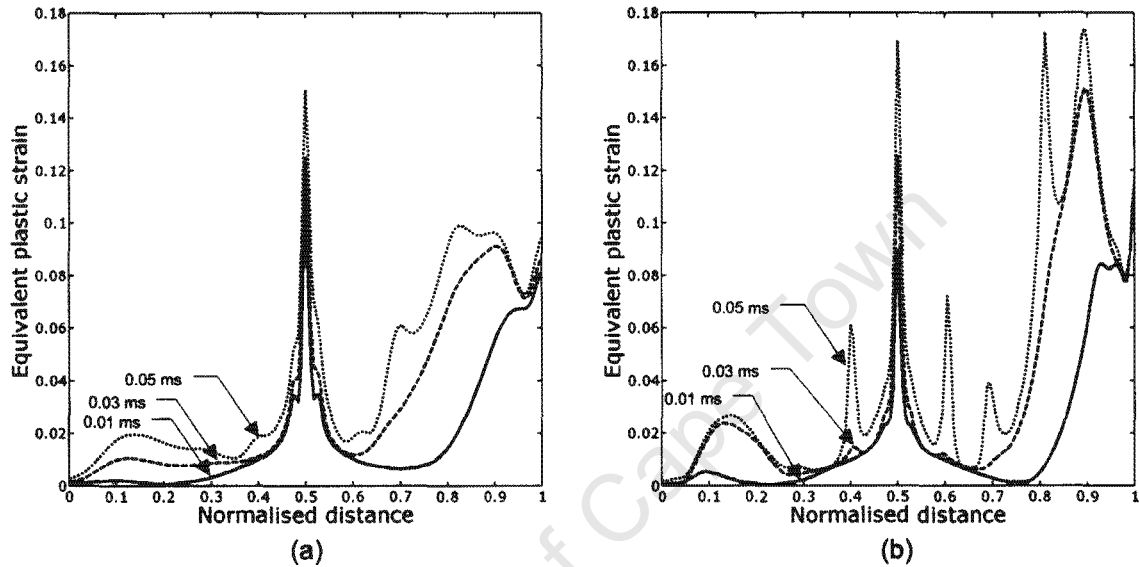


Figure 5.18: Plot of equivalent plastic strain distribution along the midline central tube for tests (a) $S-B-28$ and (b) $A-B-28$. Strains are shown starting from the distal end of the tube. The times are given with reference to the end of the blast load application.

It is interesting to note that in the steel tube case, test $S-B-28$ and to a lesser extent, test $S-B-38$ showed tearing along the edge of one of the core tubes. The much more brittle 6063-T6 aluminium tubes however, showed no tearing in the tube walls. This appears to indicate that the ductility of material under uni-axial tension has little effect on material tearing when used as the core structure of the proposed panels.

5.9 Finite Element Analysis of Experimental Tests

5.9.1 Prediction of Panel Crush Distance

The plot shown in Fig. 5.13 gives the finite element predictions of the final crush distances computed using ABAQUS/Explicit. The absolute numerical values of the crush distances are given in Table 5.4. All crush distances are measured at the panel midpoint on the top plate. In the finite element simulations the panels crush to a maximum distance and then undergo a slight rebounding, resulting in small amplitude oscillations in the top plate displacement. Crush distance values are estimated as the mid-point of the maximum and minimum crush distances within the period of oscillation.

For the steel tube results shown in Fig. 5.13a, the finite element analysis shows reasonable agreement with the experimental tests. Correlation is found to be particularly good for the panel *B* designs. For the panel *C* designs, the finite element predictions are not as accurate, except for test *S* – *C* – 33 in which an accurate prediction is obtained. It was also noted in section 5.8.3 that the experimental crush distance for test *S* – *C* – 38 is remarkably low and reasons were presented explaining this behaviour. For these reasons it is expected that the finite element simulations will over-predict for this test. For test *S* – *C* – 28, similar reasons can be used to partially explain the over-prediction. Examining the photograph of this test in Fig. 5.15 shows that for the central tube there are very localised deformations at the distal end due to solder failure early in the test. As this is the primary point of measurement this artificially reduces the total crush distance a small amount. It should also be noted that test *S* – *C* – 28 shows the largest relative top plate skewness of all the tests.

The results of the finite element simulations from the aluminium tube panels are given in Fig. 5.13b. For the panel *B* designs, the finite element model tends to over-predict the crush distance, particularly in cases where the panel reaches its compacted crush distance. The crush distance is predicted well for test case *A* – *B* – 13 where the panel does not undergo compaction. As the finite element model over-predicts in the test that undergo compaction, it is clear that the finite element model does not properly simulate the tube deformation mechanisms in the region at, or beyond the maximum stroke length. The over-predicted crush distances are likely due to the differences in lobe formation between the finite element model and experiments. This is discussed further in the section 5.9.2.

The finite element predictions of panel *C* crush distances, with aluminium tubes agree extremely well with experimental results in all test cases. This further emphasizes the result that the response of the aluminium tube core panels can be predicted well provided that the panel does not undergo compaction. This is a desirable result as it is

important to be able to numerically predict the panel response to determine if a given panel *will* compact under specified blast loading conditions.

5.9.2 Prediction of Buckling Behaviour

From a qualitative standpoint, the buckled shape of the tubes, is predicted poorly in nearly all test cases. This can be seen by comparing the experimental tests to the finite element results in Figs. 5.14 - 5.17. The primary cause of the discrepancy is poor performance of the hemispherical indentation trigger at the midpoint of each tube. In experimental test cases using aluminium tubes, where the trigger is ignored, lobe formation tends to begin at the proximal and distal ends of the tubes. Numerically, buckling begins at the trigger in all cases. This behaviour may be partially due to the numerically predicted indentation depth exceeding the experimental one. It was noted in section 5.5 that the discrepancy in indentation depth is largest in the aluminium tubes due to the filleted interface between the walls. This is not accounted for in the numerical model. It is clear that a larger indentation depth should be used in the experimental panels to obtain the same location of the initial lobe in the numerical simulations.

In all numerical simulations, buckling initiates at the trigger and progresses along the lower half of the tube. This also appears to be contrary to the experimental behaviour, particularly in the steel tube panels. In these panels, the experiments show buckling to be confined to the proximal end, close to the top plate, even for tubes that buckle at the trigger. It is unclear how successive lobes form in the experimental aluminium tube panels as the given tests that are not at or near compaction only have lobe formations at the distal and proximal ends of the tube. For tests which use the full stroke of the tube it is inconclusive as to how the lobes form with respect to time. It should be noted that in the photographs of the experimental tests, the section of the tube that is initially constrained by the bottom plate is included.

The size, and hence the number of lobes is determined reasonably well in the numerical simulations of the steel tube panels. For the steel tubes, the large strain hardening and strain rate hardening leads to increased lobe size in comparison with the aluminium tubes. In the fully crushed panels, the number of lobes in the numerical simulations is found to be between three and four which compares favorably with the experiments. Of interest however is that in the finite element simulations, due to the progressive buckling being confined to the lower half of the tubes, localised compaction occurs. This is a process where lobes which have already been formed become compressed further, despite the upper half of the tube remaining unbuckled. This is found to be a temporary phenomenon as folds begin to form in the upper half of the tube. This is discussed further in the following section. In terms of buckling mode however, this process has the effect of

reducing the lobe size in the lobes closest to the base of the tubes. This can be seen in Fig. 5.14.

The size of the lobes in simulations of the aluminium tube panels is found to be smaller than that of the experimental tests. This behaviour is found to be particularly true for the simulation of test case $A - C - 28$. This is again partially due to the localisation of buckling to a single end of the tube where lobes become crushed despite the full stroke of the tubes not being utilised. A further contribution to the underestimation of lobe size may also be from underestimation of the hardening properties of the 6063-T6 aluminium. It was determined in section 3.3.2, using tensile test simulation that the strain hardening beyond the UTS of the material to be negligible which may not be entirely accurate. This is further supported by examining the quasi-static force-displacement curves shown in Fig. 5.7. An underestimation of the strain hardening may explain why the numerically computed reaction force has a lower minimum value than the experimental curves. The material is also assumed to be completely rate insensitive however it is possible that slight rate sensitivity exists in the material which cannot be measured using the SHPB setup utilised in section 3.4.

Despite the poor prediction of the local folding behaviour of the tubes within the panel the crush distance is predicted well in nearly all test cases. This indicates that global response of the panels can be predicted accurately, however local behaviour of the panel core requires further research. Specifically, the implementation of the trigger requires further investigation into the effect of the trigger on localised deformation patterns.

5.9.3 Energy Absorption Properties of the Experimental Panels

As it has been determined that the global response of the proposed panels is predicted reasonably well it is interesting to examine further, the finite element simulations of the experiments. Specifically, the force transfer and energy absorption properties of the panels is numerically investigated. Figs. 5.19 and 5.20 give plots of the force-displacement curves and energy absorption partitioning for the steel and aluminium core panels respectively. The forces shown in the figures are obtained as the summation of the reaction forces at the distal end of each tube in the panel. As the top and bottom plates do not come into contact during the simulations, these forces represent the total force transfer to the rigid body at the base of the panels. For all force and energy plots, the curves are shown up to maximum of the average tube displacement, measured at the corners of the tubes.

The peak and mean forces as measured up to maximum displacement are shown in Table 5.4. The mean force however, is a function of displacement and hence comparing the mean force at maximum crush distance is not necessarily appropriate for an application

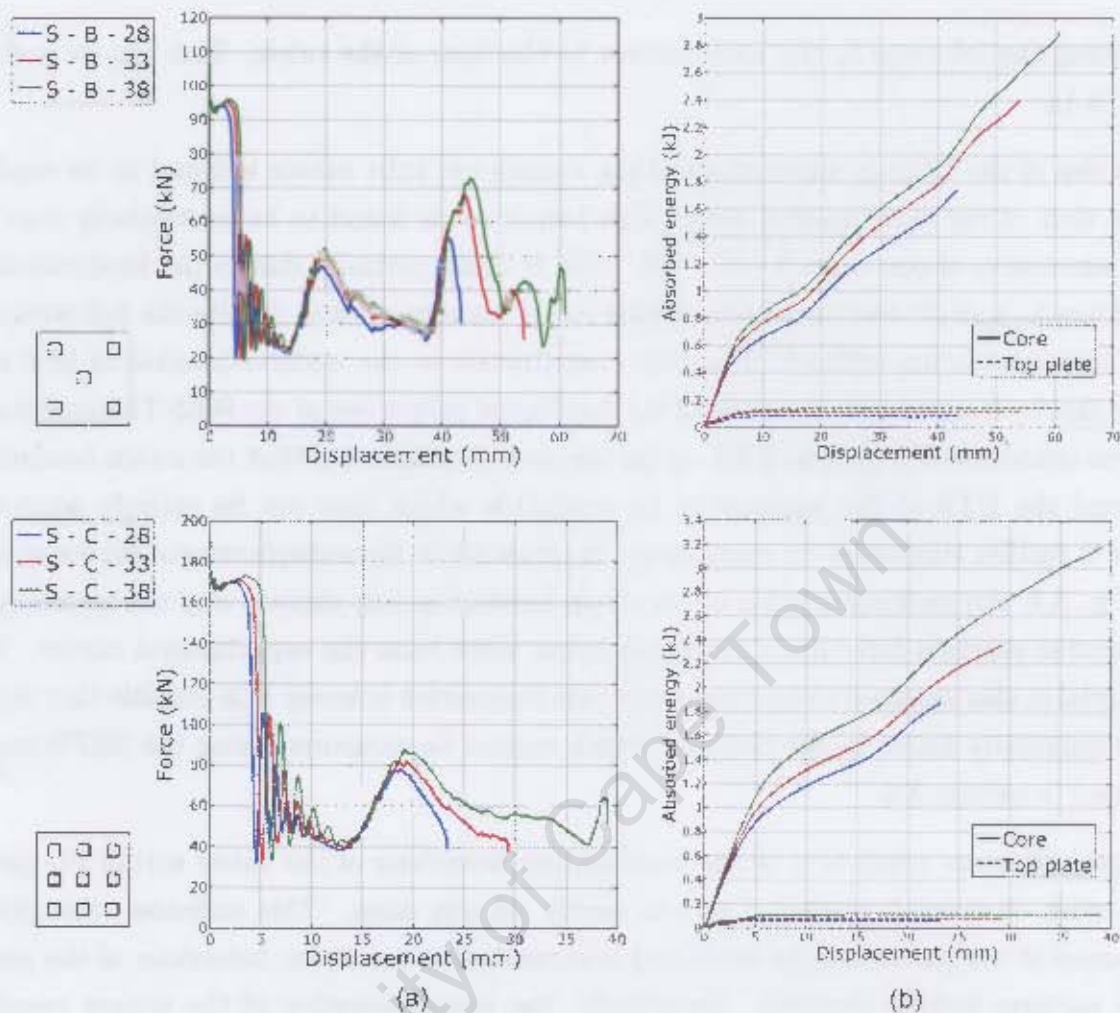


Figure 5.19: (a) Plot of force-displacement curves for the finite element analysis of experiments on steel tube panels. (b) Plot of dissipated plastic energy partitioning between the core and top plate for the finite element analysis of experiments on steel tube panels.

such as this where the crush distance is not the same for all tests. The mean crush force as function of displacement for the steel and aluminium tube panels is given in Figs. 5.21a and 5.21b respectively. It is clear from these plots that the mean force peaks at the onset of local buckling and then decreases and eventually asymptotes to a nearly constant level provided the tubes crush a sufficient distance. It is clear, primarily in the panel C tests that even if a panel has higher mean force levels than another test throughout their common crush distance that the panel that crushes further may still have a nearly identical, or possible lower mean force at the maximum crush distance. This implies that when comparing mean force levels of various panels to obtain energy absorption properties one must consider the mean force over the full crush distance and not the value at maximum crush distance.

For the steel tube panels, there is a clear increase in absorbed energy at a given displacement when increasing the applied impulse. As the annealed mild steel is rate sensitive

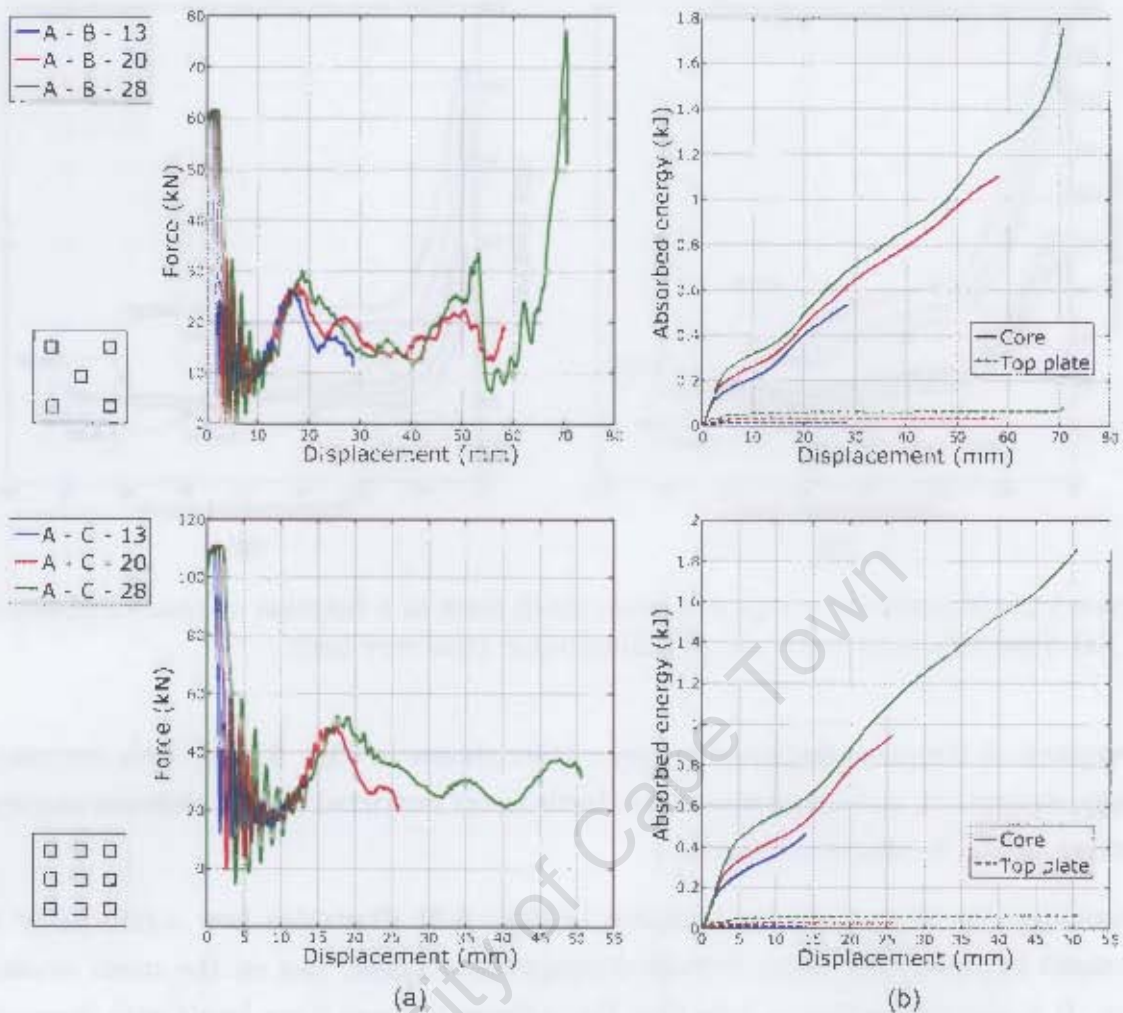


Figure 5.20: (a) Plot of force-displacement curves for the finite element analysis of experiments on aluminium tube panels. (b) Plot of dissipated plastic energy partitioning between the core and top plate for the finite element analysis of experiments on aluminium tube panels.

this result is not unexpected. This rate sensitivity also results in a slight increase in peak force levels with increasing impulse. Of note for the steel tube tests is that the peak reaction force does not correspond to the dynamic buckling load of the tubes. Examination of Fig. 5.19 shows an instantaneous spike in the reaction force due to the inertia of the tube itself. The magnitude of this peak reaction force is independent of the loading velocity. This spike in the reaction force is common in the dynamic crushing of tubes. For the current application, the spike in reaction force actually exceeds the critical buckling force. For this reason, it must be noted that for all calculations requiring the peak force, the true critical buckling force is used.

The duration of the initial compressive phase is also increased significantly at higher impulses due to increased velocity in the top plate causing more shortening in the tube at the onset of buckling. This elongated compressive phase of buckling results in the

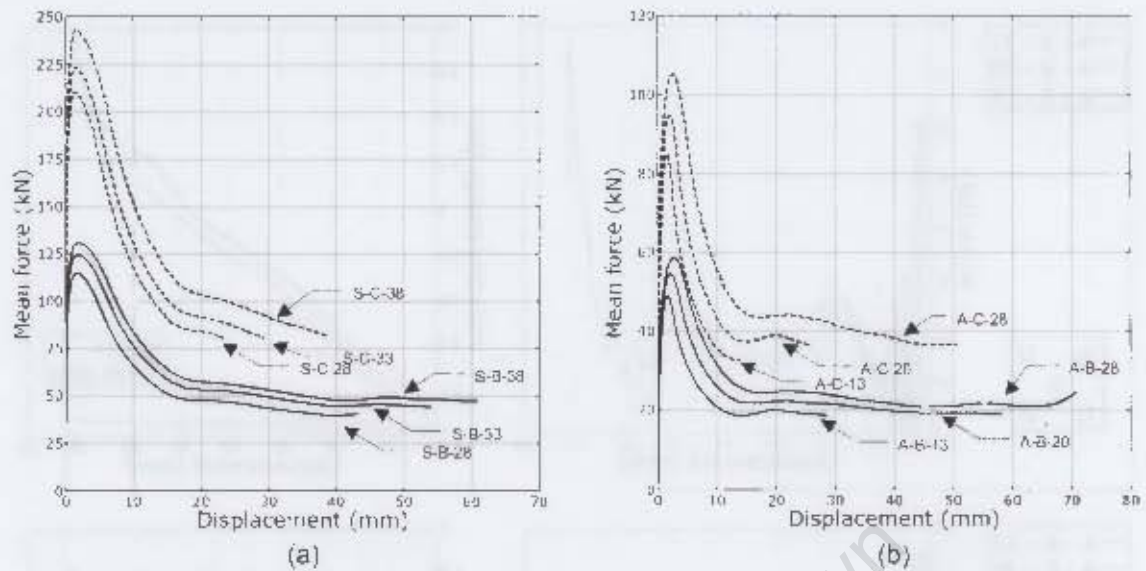


Figure 5.21: Numerically computed mean crush force as a function of axial displacement for (a) Steel tube core tests and (b) Aluminium tube core tests.

divergence of the core dissipated energy curves shown in Fig. 5.19b. This increase in energy absorption, and hence mean force levels is not restricted to rate sensitive materials however as will be discussed shortly.

Examining the mean force levels shown in Fig. 5.21 illustrates how significantly the increased impulse, and hence increased compressive phase, has on the mean crushing force. It is also interesting to note that the increase in mean force levels with increasing impulse appears to be primarily confined to the initial phase of buckling. This would indicate that strain rate effects play a comparatively small role in the increased energy absorption when moving to higher impulses. It is however clear, from examining the quasi-static mean crush forces presented in Table 5.3, accounting for the number of tubes in the panel, and comparing to the mean crush forces under blast loading, that strain rate effects have significant influence on the mean crush force. It is also interesting to note a small increase in mean force level in tests $S - B - 33$ and $S - B - 38$ at approximately 45 mm displacement. This is due to localized compaction of the tubes as a result of progressive folding being confined to the bottom half of the tube. The force-displacement curves from the tests shown in Fig. 5.19a show a monotonic increase in force levels well beyond the previous peak load which is usually indicative of crushing beyond the capacity of the tube. This is a result in increased squashing of the lobes which have already been completely formed. Folds are however then formed in the top half of the tubes and progressive, symmetric buckling then continues until the input energy has been dissipated.

As is the the case for the steel tube cores, the aluminium tube panels show a clear increase

in mean force levels when increasing the blast impulse, and hence the velocity of the top plate. This is an interesting result as the material model for the aluminium assumes strain rate insensitivity. In studies by Langseth *et al.* [118, 119] the authors observed similar behaviour in square aluminium tubes, including a T6 temper alloy similar to that studied here. The authors found significant increases in the mean dynamic load over the quasi-static load despite the rate insensitivity of the material. The phenomenon is attributed to lateral inertia effects in the side walls associated with a high velocity impact. It was also determined by Langseth *et al.* that the increased mean load is specifically related to impact velocity and not impact energy as no discernible increase in mean load was found when increasing the impact mass while maintaining a constant impact velocity.

In the present study, it is clear from Fig. 5.20 that the primary differences between the tests at increasing impulse is during the initial compression phase. This phase is characterised in the force-deformation curve by the region of deformation up to the onset of buckling. This onset of buckling clearly occurs at larger axial displacements with increasing impulse, however the force at which buckling occurs is unchanged. Thus, the time and the force at which buckling occurs is invariant however, the higher velocity top plate results in more compression during this time therefore increasing the dissipated plastic energy at the onset of buckling. This is in partial contrast to the findings of Langseth *et al.* [119]. They found that the lateral inertia effects stiffened the tube, delaying the onset of buckling, thereby increasing the peak buckling load. The study by Langseth *et al.* [119] used only imperfection-type triggers however, as opposed to the relatively large indentations used here. For the present study, it is clear that the initial compression phase is increased in terms of energy absorption however the peak load is controlled entirely by the indentation depth and not by lateral inertia effects. This is significant in terms of the experimental tests in that several of the tubes do not buckle at the trigger but instead buckle near the top or bottom of the tube. The peak buckling load in these tubes must be assumed significantly higher than those that buckle at the trigger. This is consistent with the experimental observations that the crush distance is higher in the panel locations where the tubes appear to initiate buckling at the trigger. The delay in localised buckling in the tubes which do not buckle at the trigger appears to have significant influence on the global stability of the panels.

As with the steel core panels, it is interesting to analyze the mean crush force levels as a function of crush distance for the aluminium tube panels. As with the steel tubes panels, the increase in mean force appears to be dominated by the initial compression phase of buckling. This phase is also shown to be considerably longer, in terms of displacement at the onset of buckling, in the panel *B* simulations. This would indicate that for the rate-insensitive aluminium tubes, the energy absorbed during the initial compression phase is highly sensitive to the velocity of the top plate which is increased in the panel *B* tests due

to lower core resistance during the application of the load. This effect is so pronounced that the absorbed energy of the smallest blast in a panel C test ($A - C - 13$) is nearly identical to that of the largest blast panel B test ($A - B - 28$) at a crush distance of approximately 4.8 mm.

5.9.3.1 Panel Efficiency Measures

A final note is made regarding some of the efficiency parameters described in section 4.6.1. The results for selected efficiency measures along with the final mean crush force for the numerical simulations of the experimental tests are shown in Table 5.5.

The deformation efficiency ξ_{def} is clearly higher in panel B tests and increases with increasing impulse. This is largely irrelevant however, as it is directly a function of the applied impulse. It does provide a direct measure of comparison for how much of the panel stroke is utilised for a given load. The same trends and limitations also apply to the total efficiency ξ_{total} .

The crush force efficiency is generally considered to be of critical importance when evaluating the performance of an absorber. For the steel tube, panel B simulations, the crush force efficiency increases with increasing impulse. This is expected as the peak force shows only small increases with increasing impulse however energy absorption before the onset of buckling is significant. For the panel C designs however the crush force efficiency is found to be nearly constant and of approximately the same magnitude as test $S - B - 33$. This is however, due to low crush distances not allowing for the mean crush force to approach it's asymptotic level. This further emphasis the need to evaluate the performance of an absorber over the full range of deformation, as opposed to single parameters obtained at the final crush distance.

The aluminium tube panels show a lower crush force efficiency than the steel tube panels with identical geometric characteristics. As opposed to the panel B tests with steel tubes, the value of ξ_{crush} is nearly identical between tests $A - B - 13$ and $A - B - 20$. This is expected however, as the asymptotic mean crush force increases only slightly with increasing impulse, despite the large increase in mean force observed at smaller crush distances. As the material is assumed to be rate insensitive, the increased mean load is primarily confined to the compressive phase of buckling and becomes of less importance at larger crush distances. The crush force efficiency is artificially high in test $A - B - 28$ due to a large mean force resulting from panel compaction. As is the case with the steel tube panels, the crush force efficiency for the panel C tests is found to be similar to that of the the panel B tests with a slight increase in efficiency with increasing impulse.

The core efficiency parameter ξ_{core} simply measures the partitioning of the plastic dissi-

pation between the tubes and the top plate. This is generally an important measure as large plastic deformations in the top plate may lead to global buckling of the core tubes and hence failure of the panel in terms of structural protection. Because of the thick top plate used in the experiments, only small deformations are observed, experimentally and numerically. This is quantitatively observed by examining the core efficiencies presented in Table 5.5. In all test cases the panel core absorbs nearly all of the required kinetic energy. For all tests using steel tubes, the ratio of energy absorbed in the tube to the energy absorbed in the top plate is nearly constant with a higher core efficiency found in the panel *C* simulations. Increased core efficiency with increasing number of tubes in the panel is expected as the tubes effectively constrain the motion of the top plate. Similar core efficiencies are found in the aluminium tube panels with only test *A – B – 20* having a significantly smaller core efficiency. This is due to the core tubes compacting while significant kinetic energy remains to be dissipated. The top plate therefore undergoes plastic deformation, primarily at the corners of the plate. This increase in plastic deformation in the top plate, near the end of the dissipation process, can clearly be seen in Fig. 5.20.

Table 5.5: Panel efficiencies obtained from the finite element simulations of experimental tests.

Test	F_m (kN)	ξ_{def}	ξ_{crush}	ξ_{total}	ξ_{core}
<i>S – B – 28</i>	40.25	0.578	0.427	0.246	0.925
<i>S – B – 33</i>	44.34	0.722	0.464	0.335	0.927
<i>S – B – 38</i>	47.49	0.812	0.494	0.401	0.925
<i>S – C – 28</i>	80.54	0.311	0.474	0.148	0.942
<i>S – C – 33</i>	80.28	0.392	0.469	0.183	0.942
<i>S – C – 38</i>	81.65	0.521	0.472	0.246	0.940
<i>A – B – 13</i>	18.74	0.382	0.304	0.116	0.932
<i>A – B – 20</i>	19.02	0.778	0.309	0.240	0.935
<i>A – B – 28</i>	24.84	0.944	0.403	0.380	0.904
<i>A – C – 13</i>	32.77	0.188	0.296	0.056	0.929
<i>A – C – 20</i>	36.55	0.345	0.329	0.114	0.935
<i>A – C – 38</i>	36.80	0.675	0.331	0.224	0.933

5.9.4 Temperature Effects

Thermal softening of the material due to heat generated through plastic work, contrasts the strain-rate hardening in the high velocity dynamic crushing of tube structures. It is of interest to determine what temperatures are expected and the influence that these changes in temperature have on the response of the panels to blast loading. To compare and understand the increase in temperature due to the creation of plastic folds, a steel tube and an aluminium tube panel are examined, both of which show large crush distances. Specifically, simulations of tests $A-B-20$ and $S-B-38$ are investigated. Temperatures are observed at the nodes along the midline of the tube. Temperature distribution along this path at the final crush distance is shown in Fig. 5.22a. The distance along the tube is normalized with zero referring to the distal end of the tube.

For both materials, the increase in temperature along the axis is not substantial, however the increase is significant enough to warrant temperature inclusion in the material model. This can be seen by examining the temperature effects on the stress-strain curves shown in Fig. 3.15. It must be noted that the temperatures along the middle of tube, as shown here will be lower than at the corners of the plastic folds created during the crushing process. In these locations, large localised plastic strains, and hence large temperatures develop. It is more important however to examine the temperatures at the middle of the lobe. The peak temperatures in Fig. 5.22a are located at the centre of the plastic hinges created by the folding process. Of note is the highly localised temperatures in the aluminium tubes, as opposed to the steel tubes. This is a result of the much smaller lobe sizes in the aluminium tubes. In the steel tube panel, the decay in temperature is much more gradual. High temperatures are also found at the constrained boundaries at the proximal and distal end of the tubes for both materials.

To understand the significance, if any, of thermal softening on the response of the panels, finite element simulations are conducted without adiabatic temperature effects. The force-displacement behaviour of tests $A-B-20$ and $S-B-33$, with and without temperature effects is shown in Fig. 5.22. Slightly larger force levels are observed when neglecting temperature although the difference is negligible in the aluminium tube case. The maximum crush distance is slightly shortened as a result of this. It is clear that temperature effects, although not significant should be included in the finite element model of the panels.

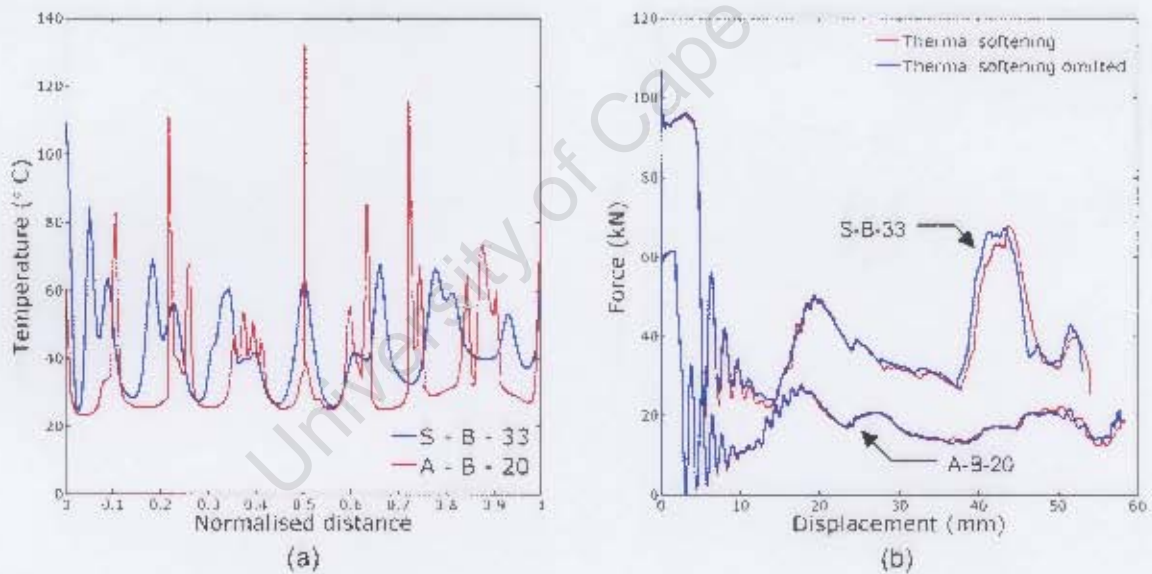


Figure 5.22: (a) Plot of typical temperature distribution along the axis of the central tube in steel and aluminium tube cores. The plot shows the temperature distribution from the distal to the proximal end of the tube. (b) Influence of thermal softening on the force-displacement behaviour of steel and aluminium tube cores.

Chapter 6

Analytical Modelling

6.1 Introduction

In this chapter, simple analytical results are presented. The purpose of this is to provide elementary, closed form design formulae⁴ that can be used to estimate the response of the panels investigated in this study under blast loading. This is accomplished by partitioning the response into two phases, the first being the blast itself and the second being the crushing of the core tubes. The first phase is analyzed from the viewpoint of elementary stress wave transfer. The purpose of this is to determine the kinetic energy remaining to be dissipated by the panels after the blast event. Once this has been determined, well established results for the crushing of thin-walled square tubes can be used to obtain the global response of the panels to a given blast load. Further to this, critical design parameters can be obtained that provide further understanding of the behaviour of the panels under blast loads. A final analytical investigation is undertaken to compare the response of the proposed panels to those using lightweight aluminium foam core panels. Such a study is important as it allows for a comparison of the thin-walled tube core paradigm with another type of core as opposed to simply comparing different tube based cores.

6.2 Phase I: Application of Blast Load

6.2.1 Model Assumptions

A simple analysis of the impulse transfer during the application of pressure pulse is given here. This analysis is done to obtain reasonable estimations of the top plate velocity and impulse transferred during application of the load. A complete understanding of the

stress state behaviour within the tubes during the first phase of panel response aids in the prediction in post-buckling deformation.

First, the following assumptions are made to simplify the analysis. The top plate is considered to be rigid. This is necessary as the analysis is based on the top plate being a non-deformable mass which transfers the impulse to the panel core. The assumption is justified as deformations are minimal, provided the pulse duration is short. The second assumption is that the material can be characterised with a bi-linear stress-strain curve. This implies that that state of stress is defined as

$$\sigma = \begin{cases} E\varepsilon & 0 \leq \varepsilon \leq \varepsilon_y \\ E\varepsilon_y + G(\varepsilon - \varepsilon_y) & \varepsilon > \varepsilon_y \end{cases} \quad (6.1)$$

where E is the elastic modulus of the material, G is the hardening modulus and ε_y is the strain at yield. Fig. 6.1 illustrates such constitutive behaviour. Also given in Figure 6.1 is the plastically dissipated energy and stored elastic energy for an arbitrary stress-strain pair (σ, ε) . For all calculations done in section 6.2 a yield stress $\sigma_y = 225 \text{ MPa}$ and a hardening modulus of $G = 700 \text{ MPa}$ is assumed. These values correspond closely to the material properties obtained in chapter 3. It should be noted that strain-rate sensitivity of the material is not included in the analysis of phase I.

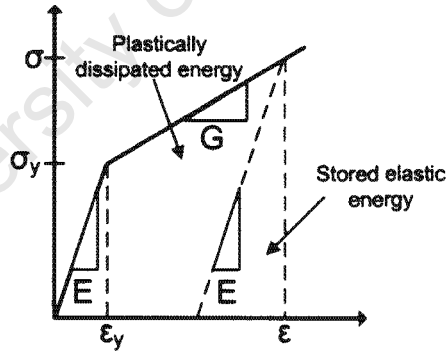


Figure 6.1: Idealised bi-linear material model used in the modeling of panel response during blast loading.

In addition to the the simplified material model during phase I, in terms of modelling the blast load, the analysis assumes a pressure pulse of the form

$$P(t) = \begin{cases} P_0 & 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (6.2)$$

where P_0 is a constant pressure and t_0 is the pulse duration.

6.2.2 Finite Element Model

It should be noted that for all finite element comparisons in section 6.2 the panels have geometric parameters $h_t = 0.6$ mm, $h_p = 3.0$ mm, and $R = 4$. As the top plate is considered rigid for the analysis the value of λ is of little importance however values of $\lambda = 0.550, 0.650$, and 0.750 are used in modelling panels A, B, and C respectively. Although the analysis in section 6.2 is formulated for an arbitrary linear elastic material with linear hardening properties, finite element comparisons during application of the blast load are made using a material similar to that of the annealed mild steel. This is for the sake of brevity and is reasonable as strain rate effects are not included in the analytical model during the blast load application. Models are implemented in ABAQUS/Explicit as described in chapter 4, however rigid elements are used in modelling the top plate. The simple bi-linear material model described in this section is implemented using the Johnson-Cook material model with $A = 225$ MPa, $B = 700$ MPa and $n = 1$.

6.2.3 Elastic-Plastic Wave Propagation

The initial transfer of impulse to the tubes is analyzed using simple one dimensional wave propagation. Figure 6.2a illustrates the simple model used here with the elastic-plastic wave propagation illustrated for a single tube. Initial imperfections in the tube geometry are ignored for simplicity. Figure 6.2b shows the free-body diagram of the top plate

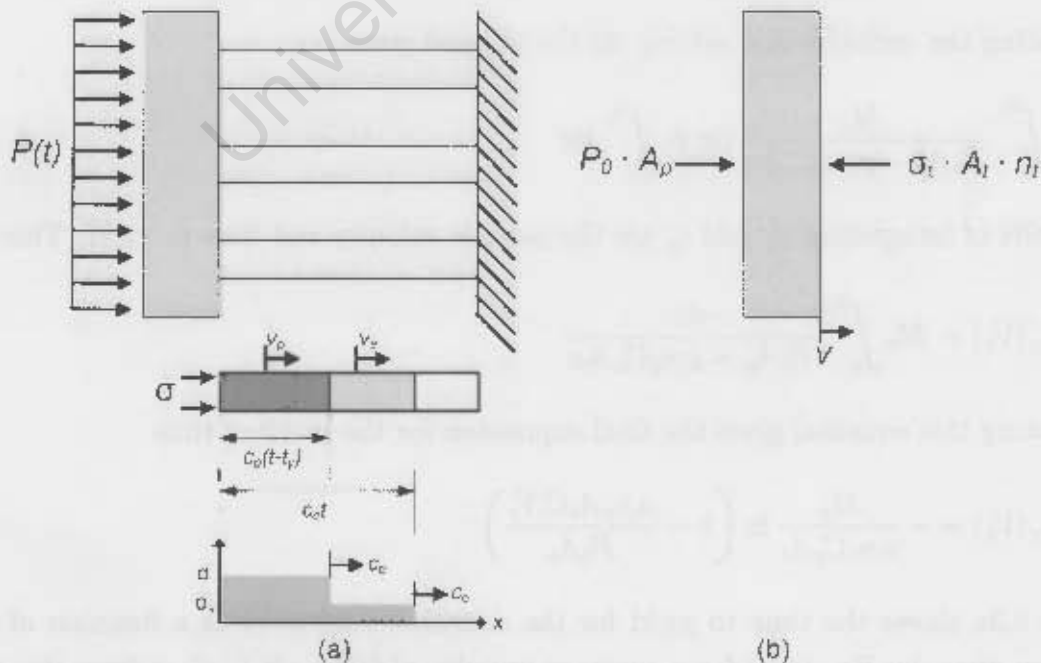


Figure 6.2: (a) Stress wave model of the proposed panels. (b) Free body diagram of the top plate.

subject to a constant pressure P_0 . The term A_t refers to the area of a single tube while n_t is the number of tubes in the panel. The stress in the tubes σ_t is defined as

$$\sigma_t = \rho_t C_e v \quad (6.3)$$

where ρ_t is the material density, v is the particle velocity, and C_e is the elastic wave speed given as

$$C_e = \sqrt{\frac{E}{\rho_t}} \quad (6.4)$$

The first thing to consider is when yielding of the tube material occurs, and how that is related to the blast duration. From this, the momentum transfer due to elastic wave propagation can be obtained. Balancing the forces in Figure 6.2 gives

$$P_0 A_p - n_t \sigma_t A_t = M_p \frac{dv}{dt} \quad (6.5)$$

where A_p and M_p are the plate area and mass respectively and v is the instantaneous plate velocity and since the plate is rigid, the particle velocity at the interface of the tubes and the plate are equal. Eq. (6.5) gives a simple separable differential equation. Firstly, The particle velocity at yield can be calculated as

$$V_y = \frac{\sigma_y}{\rho_t C_e} \quad (6.6)$$

Separating the variables and setting up the integral gives

$$\int_0^{V_y} \frac{M_p}{P_0 A_p - \rho_t n_t C_e A_t v} dv = \int_0^{t_y} dt \quad (6.7)$$

the limits of integration V_y and t_y are the particle velocity and time at yield. Therefore

$$t_y(V_y) = M_p \int_0^{V_y} \frac{dv}{P_0 A_p - \rho_t n_t C_e A_t v} \quad (6.8)$$

integrating this equation gives the final expression for the yielding time

$$t_y(V_y) = -\frac{M_p}{\rho_t n_t C_e A_t} \ln \left(1 - \frac{\rho_t n_t A_t C_e V_y}{P_0 A_p} \right) \quad (6.9)$$

Figure 6.3a shows the time to yield for the annealed mild steel as a function of pulse duration given by Eq. (6.9) for a constant impulse of 50 Ns. It is clear from this figure that the core tubes yield significantly before the end of the pulse. Although not figured, identical aluminium tubes give a blast duration to yield ratio of approximately 6 – 7.

Once t_y has been determined the progress of the elastic stress wave is simply

$$d_e = C_e t_y \quad (6.10)$$

The impulse transferred to the tubes due to the elastic wave at yield and at the end of the blast event are calculated respectively as

$$I_y = V_y d_e \rho_t n_t A_t \quad (6.11)$$

and

$$I_e = V_y C_e t_0 \rho_t n_t A_t \quad (6.12)$$

The slower plastic wave speed is governed primarily by the hardening modulus of the material and is given by

$$C_{pl} = \sqrt{\frac{G}{\rho_t}} \quad (6.13)$$

The particle velocity within the plastic wave due to the elastic and plastic stress waves is then expressed as

$$V_{pl} = V_e + \frac{\sigma - \sigma_y}{\rho_t C_{pl}} \quad (6.14)$$

The impulse transferred to the tubes within the region of the plastic wave at the end of the blast event is therefore

$$I_{pl} = V_{pl} C_{pl} (t_0 - t_y) \rho_t n_t A_t \quad (6.15)$$

This implies that the total impulse applied at the end of the blast event is dependent on V_{pl} and hence, the total strain in the tubes at $t = t_0$.

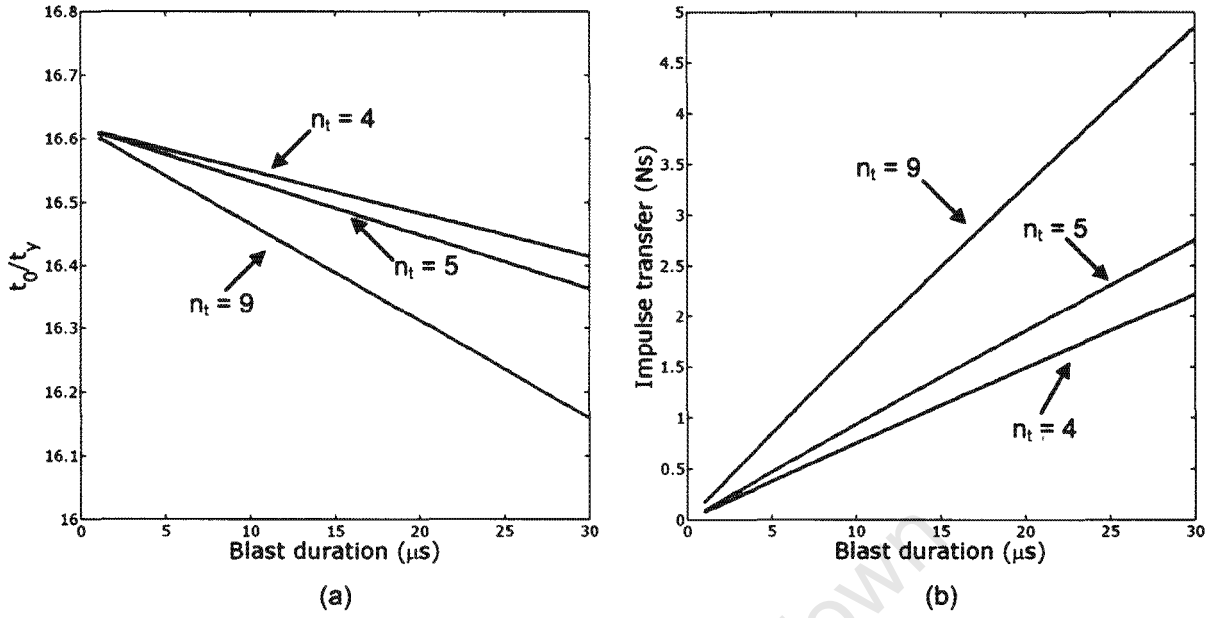


Figure 6.3: (a) Plot of the ratio of blast duration t_0 to yield time as a function of blast duration. (b) Plot of the total impulse transfer using application of the blast load as a function of the blast duration. For both plots the panel investigated is the annealed mild steel and has geometric parameters $h_{tectt} = 0.6$ mm, $h_p = 3.0$ mm, and $R = 4$.

6.2.4 Prediction of Peak Top Plate Velocity

Perhaps the most important prediction is that of determining the particle velocity at the tube-top plate interface, and hence the velocity of the top plate at the end of the blast event, denoted V_0 . This is important as it allows the engineer to predict the amount of energy that needs to be absorbed by the crushing process. A simple approximation can be used as given in Eq. (6.16)

$$V_0 \approx \frac{I}{A_p \rho_p h_p} \quad (6.16)$$

Where I is the prescribed blast impulse, ρ_p and h_p are the top plate density and thickness respectively. Note that from this point onwards the plate mass is expressed as $M_p = A_p \rho_p h_p$ in order to have the design equations expressed explicitly in terms of the top plate thickness. Eq. (6.16) assumes that the impulse applied from the blast causes the plate to instantaneously attain the velocity V_0 . This is only true if the top plate is detached from the tubes and allowed to attain the maximum velocity. The top plate velocity will approach the value as the pulse duration approaches zero. Specifically,

$$\lim_{t_0 \rightarrow 0} V_0 = \frac{I}{A_p \rho_p h_p} \quad (6.17)$$

A more accurate expression for initial top velocity can be obtained by assuming that the particle velocity in the tubes over the entire plastic wave is at the top plate velocity at $t = t_0$. Under this assumption, the total applied impulse can be partitioned as the momentum in the plate after the blast event plus the impulse transfer within the elastic and plastic stress waves. Keeping in mind that a portion of the particle velocity in plastic portion of the stress wave is due to elastic wave propagation, the impulse partitioning can be described as

$$I = I_{\text{plate}} + I_{\text{pl}} + I_{\text{e}} - I_{\text{pl, e}} \quad (6.18)$$

where $I_{\text{pl, e}}$ is the impulse transfer within the plastic stress wave due to elastic wave propagation. If using the results in the previous section, the impulse partitioning can be expressed as

$$I = V_0 A_p \rho_p h_p + V_0 C_{\text{pl}} (t_0 - t_y) \rho_t n_t A_t + V_y \{C_e t_0 - C_p (t_0 - t_y)\} \rho_t n_t A_t \quad (6.19)$$

therefore, the peak velocity of the top plate, in terms of the panel design variables can be approximated as

$$V_0 = \frac{I - 4V_y \{C_e t_0 - C_p (t_0 - t_y)\} \rho_t n_t h_t l / R}{A_p \rho_p h_p + 4C_{\text{pl}} (t_0 - t_y) n_t \rho_t h_t l / R} \quad (6.20)$$

From this expression it can clearly be seen that as the pulse duration t_0 (and yield time t_y) approaches zero, the initial top plate velocity approaches the velocity obtained by Eq. (6.16). Under the assumption that the entire plastic wave is travelling at this velocity, the impulse transferred to the tubes due to the elastic and plastic stress waves is given in Fig. 6.3b for a 50 Ns blast.

The peak velocity of the top plate as a function of pulse duration for a fixed impulse of 50 Ns is given in Fig 6.4 using Eq. (6.20). The analytical predictions for the top plate velocity agree well with the finite element results for small pulse durations however for longer pulses, the analytical predictions underestimate the peak velocity slightly. This is expected however, when examining the assumed stress state in the tubes as shown in Fig 6.2. The assumed constant levels of stress are not instantly obtained when subjected to the elastic and then plastic stress wave and in actuality will ramp up to the assumed stress level. This results in a slight overestimation of impulse transfer during the compressive phase of blast transfer, and hence a lower peak velocity.

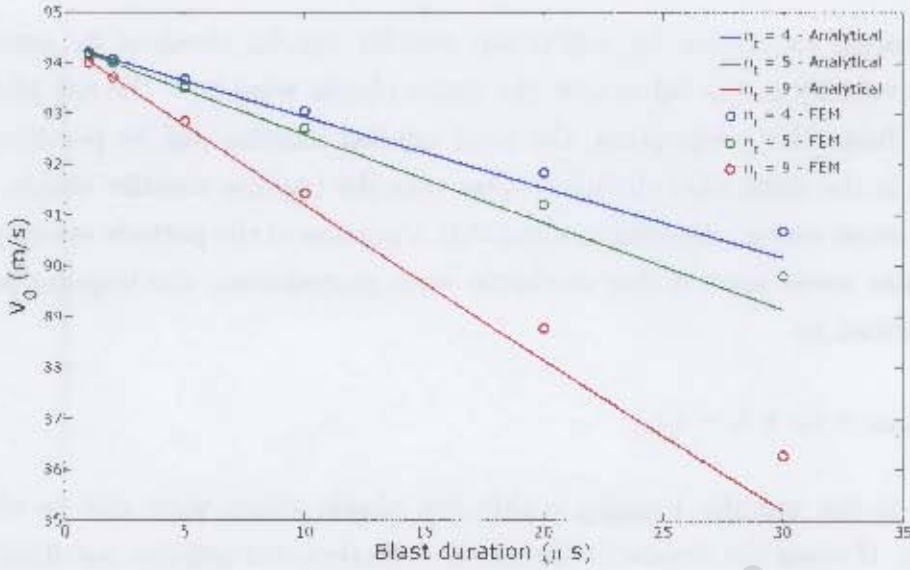


Figure 6.4: Plot comparing the peak top plate velocity as a function of blast duration for panels A, B and C. The geometric parameters of the panels are $h_t = 0.6 \text{ mm}$, $h_p = 3.0 \text{ mm}$, and $R = 4$.

6.3 Phase II: Panel Response

A simple theoretical analysis is given to predict the energy absorption properties of various panels. The basis of the simple analytical predictions is to reformulate the problem using well established theory of a tube structure impacted by a given mass at velocity V_0 . Under normal impact conditions, the proximal end of the tube attains the impact velocity instantaneously however, as described in the previous section this is not the case for blast loading. The energy absorbed by the core tubes during this initial compression phase is not accounted for in the analytical models presented in the literature. For this reason, the theory presented here formulates the problem in terms of dissipating the energy in the system after application of the blast load. In other words, the theory models the remaining impulse transfer to the base of the structure after the initial compression of the tubes during the blast load. This simplification is clearly only valid for short pressure pulses where the total kinetic energy to be dissipated by the folding process is nearly equal to the external work applied the panels.

The method used here is a similar approach to that of Karagiozova *et al.* [8] where the author determined the the velocity at the proximal end of the tube, and hence the energy remaining to be absorbed, at the end of the initial compression phase for a tube with an attached mass subject to blast loading. These parameters are then used to modify existing formulae for mean crushing forces. In the aforementioned study however, the attached mass attains its peak velocity instantaneously due to the short pulse duration and single tube configuration. The analysis of Karagiozova *et al.* focuses on the analytical modelling

of the compression phase of the *tube crushing process* and energy absorbed during this process. The present study cannot assume this instantaneous attainment of velocity and instead determines the energy dissipated during the blast event while the tubes are in compression. This is an important distinction however in the study by Karagiozova *et al.* and the present study the objective is to obtain a more realistic expression for the input energy required in existing analytical formulae.

For the proceeding discussion, the following assumptions are made in the analytical model

- The blast duration is short compared to the duration of crushing.
- The material is perfectly plastic.
- The shorting of the tube during the application of the blast load is negligible.
- The tubes in the panel core are long enough that simple elastic-plastic wave propagation theory can be used to model the stress state during the blast load.
- The energy absorbed by the top plate is negligible.

The first assumption is required due to the difficulty in modelling the tube behaviour in the case where simple propagation of a stress wave along the length of a square tube cannot be assumed. The result of Fig. 6.4 also shows that predicting the peak velocity of the top plate is difficult for longer pulses. The assumption of perfectly plastic material behaviour is common in analytical modelling and is required for several formulae. The accurate determination of this characteristic stress is discussed in the next section. The assumption of no tube shortening during the blast event is acceptable as the shorting is very small relative to the length of the tube and this shortening only influences the tube dimensions and stroke capacity in a negligible way. The fourth assumption implies that the tubes must be long enough such that stress wave reflections do not need to be accounted for in the model. The reflection will cause errors in the model if the pulse duration $t_0 > 2l/C_e$. In other words, the pulse duration must not be long enough such that the elastic stress wave can travel the length of the tube and back to the top plate. As the tube thickness is fixed at 75.0 mm for the study this corresponds to an approximate maximum pulse duration of 30 μ s. The final assumption is required as the deformation process within the top plate is very complex in general due to the boundary conditions, and is beyond the scope of this study. For relatively thick top plates under uniform loading however, this assumption is acceptable.

6.3.1 Mean Crushing Force

To obtain reasonably accurate results under dynamic loading, strain rate effects must be accounted for. Although the plastic strain rate is difficult to measure in a tube under progressive folding, Abramowicz and Jones [17] estimate that it can be expressed as

$$\dot{\epsilon}_p = 0.33V_0R/l \quad (6.21)$$

where V_0 is the initial plate velocity, R is the aspect ratio and l is the tube length. It should be noted that the tube aspect ratio refers to the mean aspect ratio of the tube and not the outer width. This is in keeping with standard practice in the analysis of shell structures [118]. Furthermore, direct comparisons with finite element shell solutions are possible. Eq. (6.21) can then be combined with the Cowper-Symonds relation

$$\frac{\sigma_d}{\sigma_s} = \left[1 + \left(\frac{\dot{\epsilon}_p}{D} \right)^{\frac{1}{q}} \right] \quad (6.22)$$

which was discussed in detail in section 3.5.2. Abramowicz and Jones [17] presented a simple formula for the mean dynamic crushing load of a square tube under dynamic loading. The formula scales the static crushing load F_m^{st} as

$$\frac{F_m^{\text{st}}}{M_0} = 52.22 \left(\frac{l}{h_t R} \right)^{1/3} \quad (6.23)$$

where M_0 is the plastic collapse moment of the square cross section given by $M_0 = \sigma_c h_t^2/4$. Note that Eq. (6.23) assumes a symmetric buckling mode. Some of the panels investigated here may not buckle symmetrically and could show asymmetric mixed mode B collapse for example. In this case, the static mean collapse mode is then expressed as [17]

$$\frac{F_m^{\text{st}}}{M_0} = 45.90 \left(\frac{l}{h_t R} \right)^{1/3} + 1.75 \left(\frac{l}{h_t R} \right)^{2/3} + 1.02 \quad (6.24)$$

The remainder of the analysis assumes a symmetric collapse mode for brevity. A large range of tube thicknesses are investigated analytically, many of which would be considered relatively thick and hence would buckle in a mixed mode as opposed to symmetric mode. It is assumed in the analysis that a trigger, such as a circular cut-out, or indentation is used in the panel which induces symmetric buckling.

In the above equations, the term σ_c is the characteristic stress which idealises the plastic flow of the material as perfectly plastic. There are various methods to account for strain hardening in the calculation of σ_c . A commonly used equation [118, 153] for determining

the energy equivalent flow stress is

$$\sigma_e = (\sigma_e^1 \sigma_e^2 \sigma_e^3)^{1/3} \quad (6.25)$$

where

$$\sigma_e^i = \frac{2}{(\varepsilon_e^i)^2} \int_0^{\varepsilon_e^i} \sigma(\varepsilon) \varepsilon \, d\varepsilon \quad i = 1, 2, 3 \quad (6.26)$$

and the plastic strain values are given as

$$\varepsilon_e^1 = 0.93(z)^{2/3}, \quad \varepsilon_e^2 = 0.69(z)^{1/3}, \quad \varepsilon_e^3 = 1.3(z)^{1/3} \quad (6.27)$$

where $z = h_t R/l$. This methodology of using a geometric dependent characteristic flow stress is particularly appropriate for the present study where a large range of tube geometries are examined. Fig. 6.5 gives a contour plot of the variation in the characteristic stress as a function of tube geometry for the annealed mild steel. The equivalent plot of aluminum is not shown as the material is nearly perfectly plastic for larger strains and hence the characteristic stress is almost equivalent to this plateau stress for all tube geometries. It should be noted that the simplified bi-linear material model of section 6.2 is not used here but instead the true-stress strain curves given in section 3.3.2 are used. The hardening modulus G is still required however for determination of V_0 and is computed using the slope of the linear fit of the true stress-strain curve up to the UTS of the material.

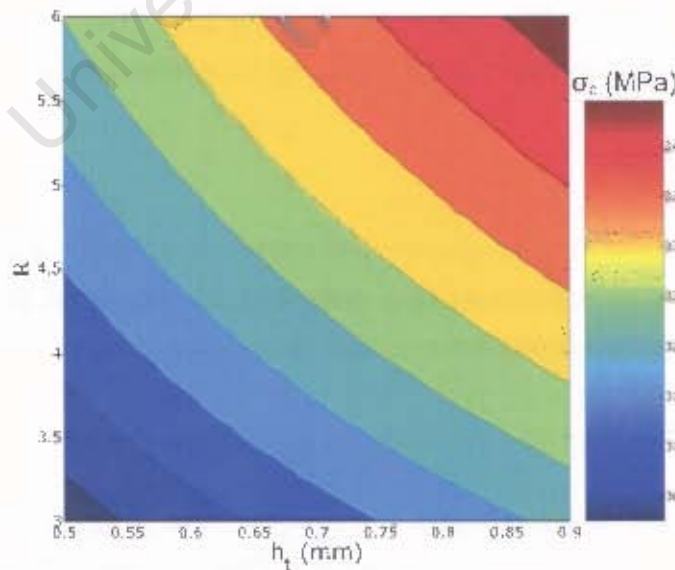


Figure 6.5: Contour plot showing the variation in the energy equivalent yield stress as a function of tube geometry for the annealed mild steel used in this study.

Once the characteristic stress is determined the mean force can be computed. Eq. (6.22)

is used to scale the mean load as

$$\frac{F_m^{\text{dyn}}}{F_m^{\text{st}}} = 1 + \left(\frac{0.33V_0 R}{lD} \right)^{\frac{1}{q}} \quad (6.28)$$

Therefore scaling the expression in Eq. (6.23) and substituting for M_0 gives

$$F_m = 13.05\sigma_c h_t^{\frac{5}{3}} \left(\frac{l}{R} \right)^{\frac{1}{3}} \left\{ 1 + \left(\frac{0.33V_0 R}{lD} \right)^{\frac{1}{q}} \right\} \quad (6.29)$$

This expression for the mean force has been shown to underestimate the true mean load [8]. Eq. (6.29) is derived based on the bending energy of the folding mechanism and assumes that the initial compression phase is a negligible portion of the absorbed energy. This is only true however if the impacting mass is much larger than the mass of the tube. Experiments reported by Karagiozova and Jones [24] and Karagiozova *et al.* [8] found that a tube subjected to a high velocity, small mass impact will have a considerably shorter deformed length δ than what is predicted by Eq. (6.29). High velocity impacts allow for lateral inertia effects to become predominant which results in the tube supporting the unbuckled shape for a longer period of time. As a result, the energy absorbing capacity of a tube is increased with increasing striker velocity.

6.3.2 Panel Capacity and Stroke Length

Some simple expressions are given here to predict the energy absorption capacity and crush distance of the proposed panels under blast loading. Firstly, Jones [14] determined that the ratio of maximum crush distance δ_{max} to tube length to be

$$St_c = \frac{\delta_{\text{max}}}{l} = 0.73 \quad (6.30)$$

for square tubes buckling in a progressive symmetric mode. For a panel with n_t tubes of length l in the core, it is assumed that each tube has the same mean crush force F_m therefore the total energy absorption capacity of the core is

$$E_{\text{max}} = n_t F_m St_c l \quad (6.31)$$

Inasmuch as Eq. (6.29) underestimates the mean load in a single tube, Eq. (6.31) will underestimate the capacity of the panel n_t -fold. This is in addition to the underestimation from the energy dissipated due to plastic deformation in the top plate. Using Eq. (6.31) as a simple design formula may over-design the panel at higher impulses but will provide a good factor of safety.

To examine the response of the panels further, some simple results from Jones [14] are presented. It is clear from a simple free body diagram that

$$n_t F_m = -A_p \rho_p h_p \frac{dv}{dt} \quad (6.32)$$

where $v(0) = V_0$ and $v(t_{\max}) = 0$ where $t_{\max}$ is the response duration. That is, at $t = t_{\max}$ all the kinetic energy from the blast is assumed to have been dissipated by the tubes. Therefore solving for dv/dt , integrating with respect to time and using the initial condition $v(0) = V_0$ gives

$$\int \frac{dv}{dt} dt = - \int \frac{n_t F_m}{A_p \rho_p h_p} dt \Rightarrow v(t) = V_0 - \frac{n_t F_m t}{A_p \rho_p h_p} \quad (6.33)$$

integrating Eq. (6.33) further with respect to time and applying the initial condition $u(0) = 0$ gives the deformation-time history

$$u(t) = \int \left\{ V_0 - \frac{n_t F_m t}{A_p \rho_p h_p} \right\} dt = V_0 t - \frac{n_t F_m t^2}{2 A_p \rho_p h_p} \quad (6.34)$$

To find the total panel response duration t_m , Eq. (6.33) is solved for t with $v = 0$ as this is when motion ceases. This gives

$$t_m = \frac{A_p \rho_p h_p V_0}{n_t F_m} \quad (6.35)$$

Alternatively, solving $\partial u / \partial t = 0$ for t will give the same result. The maximum crush distance, assuming compaction does not occur, is then simply $u(t_m)$ which is expressed as

$$u_m = \frac{A_p \rho_p h_p V_0^2}{2 n_t F_m} \quad (6.36)$$

If the energy absorption capacity of the core tubes is insufficient to absorb the blast then the deformation may not reach $u = u_m$ and the core compacts. This is an undesirable result as the forces at the base of the panel may exceed the peak buckling load of the tubes by several orders of magnitude. To determine the time when this occurs, the deformation-time relationship given in Eq. (6.34) is compared to the maximum allowable deformation $St_e l$. This relation is then solved for t to give the time at which compaction occurs as

$$t_c = \frac{V_0 A_p \rho_p h_p - \sqrt{A_p \rho_p h_p (V_0^2 A_p \rho_p h_p - 2 n_t F_m St_e l)}}{n_t F_m} \quad (6.37)$$

Therefore if $t_c \geq t_m$ then the panel will be able to absorb the prescribed blast load. The

complete solution to the equation of motion is given by

$$u_t(t) = \begin{cases} 0 & t \leq 0 \\ V_0 t - \frac{n_t F_m t^2}{2A_p \rho_p h_p} & 0 < t \leq t_m \text{ and } t < t_c \\ \frac{A_p \rho_p h_p V_0^2}{2n_t F_m} & t \geq t_m \text{ and } t < t_c \\ St_e l & t \geq t_c \end{cases} \quad (6.38)$$

It is of interest to analytically be able to predict the critical parameters which influence a panel's ability to absorb blast loading. A panel utilizes its full energy absorbing capacity if $u_m = St_e l$. In other words, the blast is dissipated completely when the tube has crushed a distance exactly equal to its maximum stroke length. This relation, expanded in terms of the tube parameters is

$$\frac{\kappa A_p \rho_p h_p V_0^2}{n_t \sigma_0 h_t^{\frac{5}{3}} \left(\frac{l}{R}\right)^{\frac{1}{3}} \left\{1 + \left(\frac{0.33 V_0 R}{l D}\right)^{\frac{1}{q}}\right\}} = St_e l \quad (6.39)$$

where $\kappa = 0.038314$. For a fixed panel design, the impulse capacity of the panel is found by first solving Eq. (6.39) for V_0 using an appropriate root finding algorithm (i.e. Newton's method) in Matlab. The solution to the problem is denoted V_0^{crit} . The maximum impulse the panel can sustain is therefore

$$I_{\max} = V_0^{\text{crit}} \{A_p \rho_p h_p + 4C_{pl} (t_0 - t_y) n_t \rho_t h_t l / R\} + 4V_y \{C_e t_0 - C_p (t_0 - t_y)\} \rho_t n_t h_t l / R \quad (6.40)$$

Alternatively, if the applied impulse is known for a particular application, the minimum thickness $l_{\min}$ required to absorb the load can be computed by solving Eq. (6.39) for l keeping in mind that the initial plate velocity is a function of the tube length using a fixed aspect ratio R . Finding the critical length is essential for blast attenuation application where thickness of the panel is more important than the mass. When examining panels with small thickness one must however consider the model assumptions, particularly those regarding the reflection of stress waves during application of the blast load.

6.4 Model Verification

The simple analytical model is compared to finite element analysis in ABAQUS/Explicit and experimental tests to determine the accuracy of the formulae under blast loading. The panels used for comparison are described in section 5.2. Although the number of panels used in the experimental analysis are limited, comparison of experimental and analytical crush distance is important in determining the validity of the given formulae for panel design.

The resulting crush distances for the steel tube and aluminium core are given in Figs. 6.6a and 6.6b respectively. It is clear that for the given panel geometries, the analytical and finite element models agree well provided the panel is not crushed beyond the theoretical maximum distance. Particularly good agreement is found for panel *C* tests with both core materials. For a number of the test cases, the analytical prediction is found to be slightly closer than the numerical prediction to the experimentally obtained crush distance. It is also clear from Fig. 6.6 that in test cases where the numerical result differs greatly from the experimental crush distance, that the analytical prediction does as well.

Despite the encouraging results of the test cases presented here, consideration must be given to the limitations of the model. Karagiozova *et al.* [8] found that when tubular structures are impacted by a small mass at a high velocity, the mean load computed using Eq. (6.29) underestimates the true mean load. This is due to the fact that the analytical prediction assumes that energy absorption due to initial compression is negligible which is not the case under the high velocity loading conditions investigated here. As the top plate used in the experiments, and hence the validation, is relatively thick, the influence of the mass of the "impactor" is a large portion of the total energy required to be dissipated. This effectively minimizes the influence of the initial compression phase of buckling on the total mean force. It is therefore expected that for a given impulse, a reduction in the top plate thickness will possibly lead to large underestimations in the mean crush force. For the test cases studied here, such behaviour can be seen by comparing the numerical and analytical mean crush forces as shown in Fig. 6.7. The figure indicates a relatively good agreement in mean crush force levels in nearly all test cases. For both the steel and aluminium core panels the numerically obtained mean crush force increases with increasing impulse. The reasons for this were discussed in section 5.9.3. A large contribution to this is from the increased energy absorption during the initial compression phase. For the steel tube core, the analytical mean crush does scale with increasing impulse, entirely due to strain-rate hardening however. For the aluminium tubes, the mean crush force is dependent only on geometry and hence is invariant to the applied impulse. It is clear that as the top plate velocity increases, this disparity will have an adverse effect on the predicted crush distance. It must be further noted that

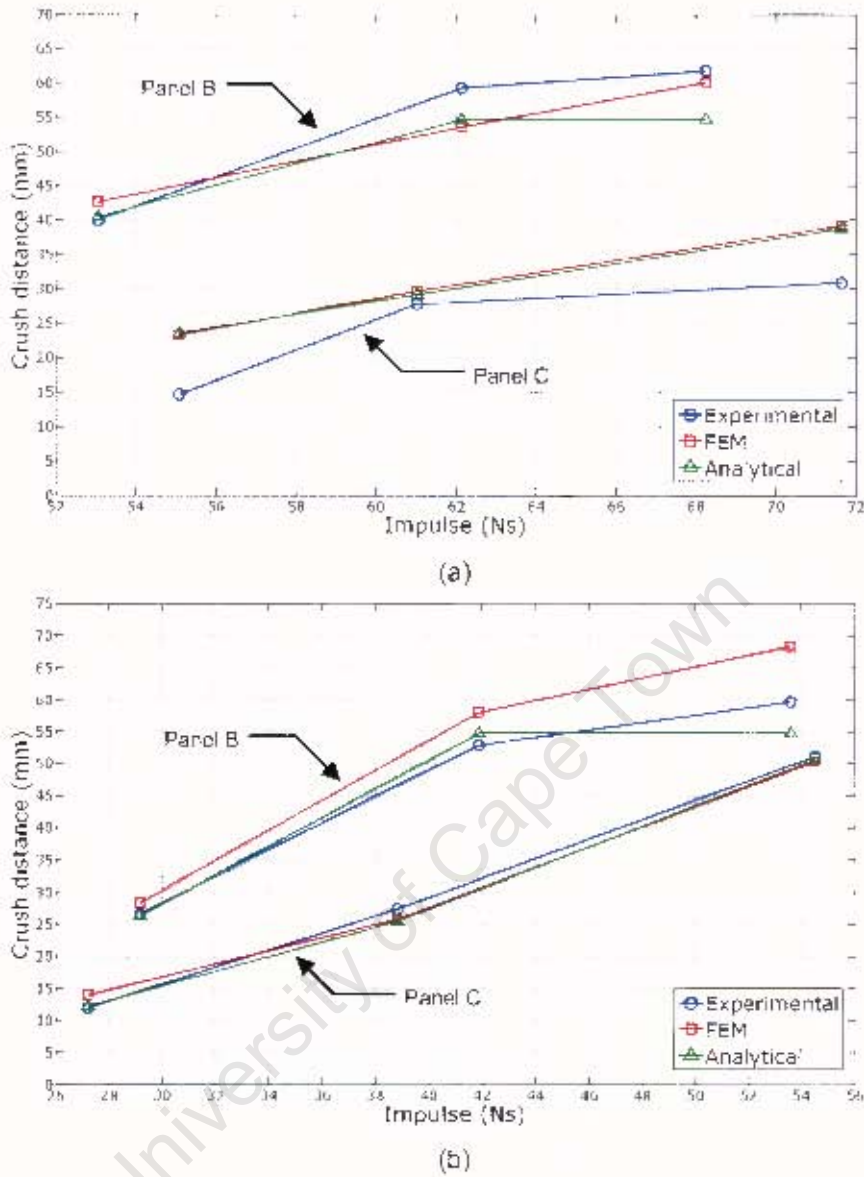


Figure 6.6: Plot comparing analytically predicted crush distances with finite element analysis and experiments with (a) Annealed mild steel tube core (b) Aluminium tube core.

as shown in section 6.2.4, the peak top plate velocity is slightly underestimated. This underestimation lowers the total energy absorption requirements in the analytical model effectively reducing the predicted crush distance. This simplification works in contrast to the omission of energy absorption during the initial compressive phase of buckling, thereby improving the accuracy of the prediction. It should also be noted that strain-rate effects will reduce the peak top plate velocity for the steel tube panels, which is not accounted for in the analytical model.

The correctness of the formulae given for calculation of the critical parameters of Eqs. 6.35, 6.36, 6.37, 6.39, and 6.40 are presented here. Note that it is the mathematical correctness of the formulae that is verified and not correlation with finite element results

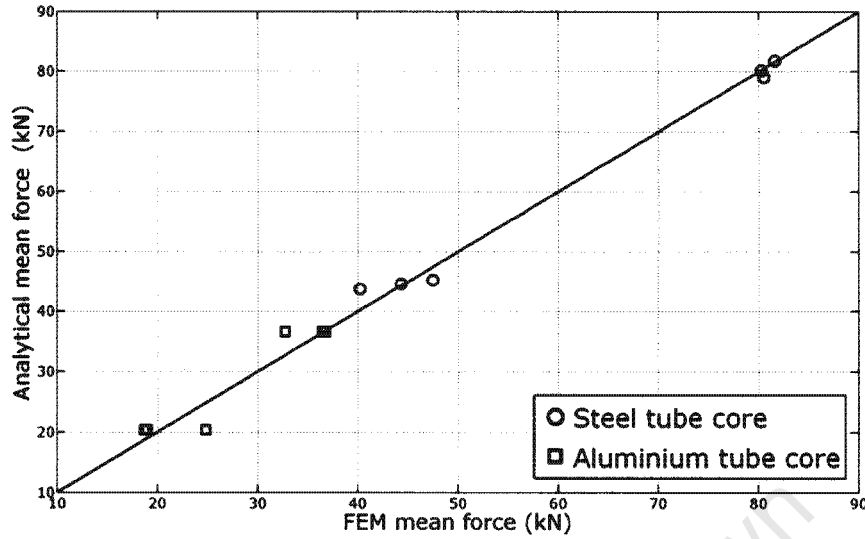


Figure 6.7: Plot comparing analytical and finite element predictions for the test panels used in the experimental tests.

as the latter are highly difficult to determine. Table 6.1 presents the test cases used for verification. Only panels *B* and *C* are verified as examining panel *A* provides no additional information. Test cases 1 and 3 are the standard case where the panel does not compact. Test cases 2 and 5 correspond to the case where the applied impulse exactly equals the impulse capacity of the respective panels. Finally, test cases 3 and 6 examine the case where the applied impulse is greater than the capacity of the panel. For test cases 1 and 3 the calculation of t_c results in a complex value indicating that the panels dissipates the required energy before compaction as desired. For both of the panel configurations the maximum allowable impulse is determined using Eq. (6.40) and these impulses are applied in test cases 2 and 5. The purpose of this to ensure that under these blast magnitudes that $u_m/l = St_e$ and that $t_c/t_m = l_{\min}/l = I_{\max}/I = 1$. The calculations presented in Table 6.1 indicate this to be true in test cases 2 and 5. Note that some of the parameters are slightly greater than one. This is a result of using a slightly higher critical impulse due to rounding the value to two digits after the decimal. The results of test cases 3 and 6 give the desired result of all critical parameters being then that unity except for $l_{\min}/l$ which must be greater than one indicating the minimum tube length is greater than that of the test case. It can therefore be concluded that the critical parameter formulae are correct.

Table 6.1: Validation of critical parameter formulas. In all test cases the material is annealed mild steel with $h_t = 0.6$ mm, $h_p = 3.0$ mm, and $R = 4$.

Test	Panel	I (Ns)	t_c/t_m (Eq. 6.35, 6.37)	u_m/l (Eq. 6.36)	$l_{\min}/l$ (Eq. 6.39)	$I_{\max}/I$ (Eq. 6.40)
1	<i>B</i>	40.00	—	0.46	0.58	1.28
2	<i>B</i>	51.39	1.00	0.73	1.01	1.00
3	<i>B</i>	53.00	0.76	0.77	1.06	0.97
4	<i>C</i>	60.00	—	0.53	0.78	1.19
5	<i>C</i>	71.23	1.00	0.73	1.02	1.00
6	<i>C</i>	74.00	0.74	0.78	1.08	0.96

6.5 Panel Performance

The performance of the panels according to the analytical model is presented here. Primary attention is given to the impulse capacity of a given panel and the impulse capacity per unit mass. The latter measure is referred to as the specific impulse capacity and is given by

$$I_{sp} = I_{\max}/(M_p + M_{\text{core}}) \quad (6.41)$$

where M_p and M_{core} are the top plate and core masses respectively. This expression is similar to the notion of specific energy but has more relevance to the present study on impulsive loading. Note that the specific impulse is defined in terms of the entire panel and not just the mass of the core tubes. The reason for this is that it is clear that under this definition the specific impulse would be maximized for the four tube panels of design A. For example, doubling the number of tubes in the core will double the core mass and approximately double the energy absorbing capacity however the critical velocity and hence impulse capacity is not doubled.

Firstly, the influence of the tube aspect ratio R is examined. Figs. 6.8a and 6.8b show the relationship between R and h_p and h_t respectively, in terms of impulse capacity. As can be clearly seen from the figures, the aspect ratio has little influence on the impulse capacity of the tubes. This is shown in chapter 7 to be consistent with the numerical results. This result is expected as when examining Eq. (6.29), the mean force is influenced only by the cube root of l/R . In other words, large increases in the tube width (reductions in R) only increase the mean force slightly. The energy equivalent flow stress is slightly reduced however with increased tube width. It should be noted however that the tube aspect ratio will have more influence in actual blast scenarios as the top plate cannot be considered rigid. In this case, significant plate deformation can initiate global buckling modes in the

tubes resulting in a much lower mean force in the core tubes. For this reason, only results at thicker top plate thicknesses can be considered accurate under true blast loading.

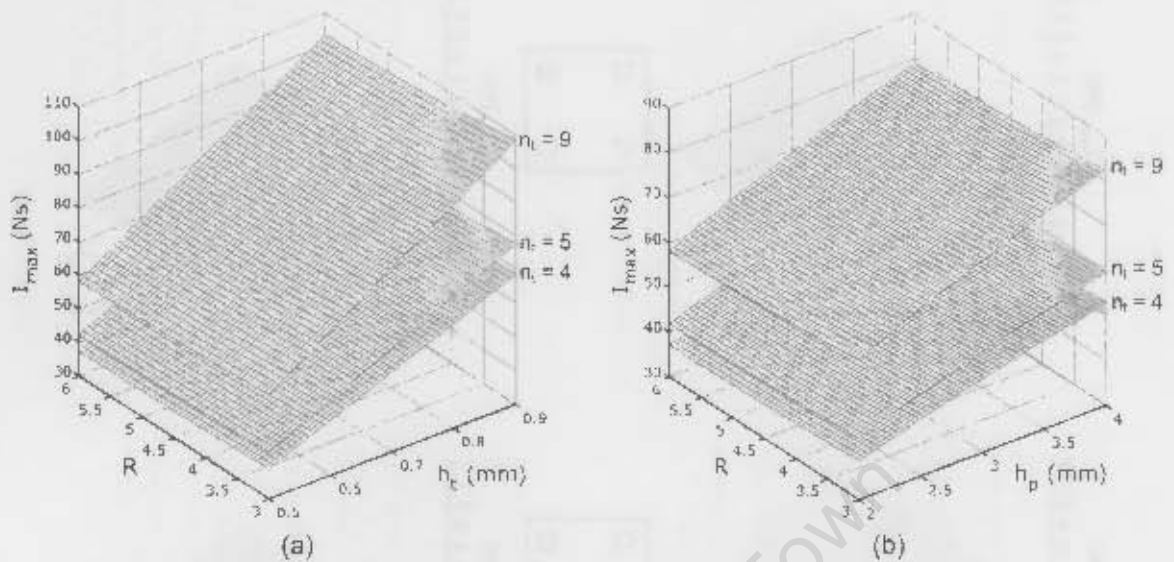


Figure 6.8: (a) Variation in impulse capacity as a function of R and h_t with $h_p = 3.0$ mm. (b) Variation in impulse capacity as a function of R and h_p with $h_t = 0.6$ mm.

The impulse capacities of steel and aluminium core panels are given in Fig. 6.9. For both steel and aluminium core panels the panel capacity is maximized at the highest thickness of tube and the thickest top plate. The impulse capacity is also shown to be highly sensitive to changes in tube thickness. It is also clear from the figure that significant increases in impulse capacity are obtained when increasing the number of tubes in the core. Further to this, the expected result of higher increase capacity in panels using the annealed mild steel. It is interesting to note that the steel tube show a larger sensitivity to changes in the top plate thickness. This is due to top plate thickness having a large influence on the initial velocity of the top plate and hence influence the strain rate hardening in the tubes.

Although it is clear that the panel absorption capacity can be increased using the thickest tubes and top plate the question still remains as to how to maximize the impulse capacity for a panel of a given mass. Fig. 6.10 shows the specific impulse of steel and aluminium tube panels as a function of tube thickness and top plate thickness. For both tube material the specific impulse is maximized for panels with a thin top plate and thick tubes. This is not unexpected however as the top plate constitutes a significant portion of the total panel mass. Consideration must be given to this result however as the assumption of an undeformable top plate becomes less valid as the top plate becomes thinner.

The specific impulse is similar for panels *A*, *B* and *C* with a small increase observed as a function of n_t . This is in contrast to the response in terms of absolute impulse capacity where there is a significant improvement in panel *C* over panels *A* and *B*. The annealed

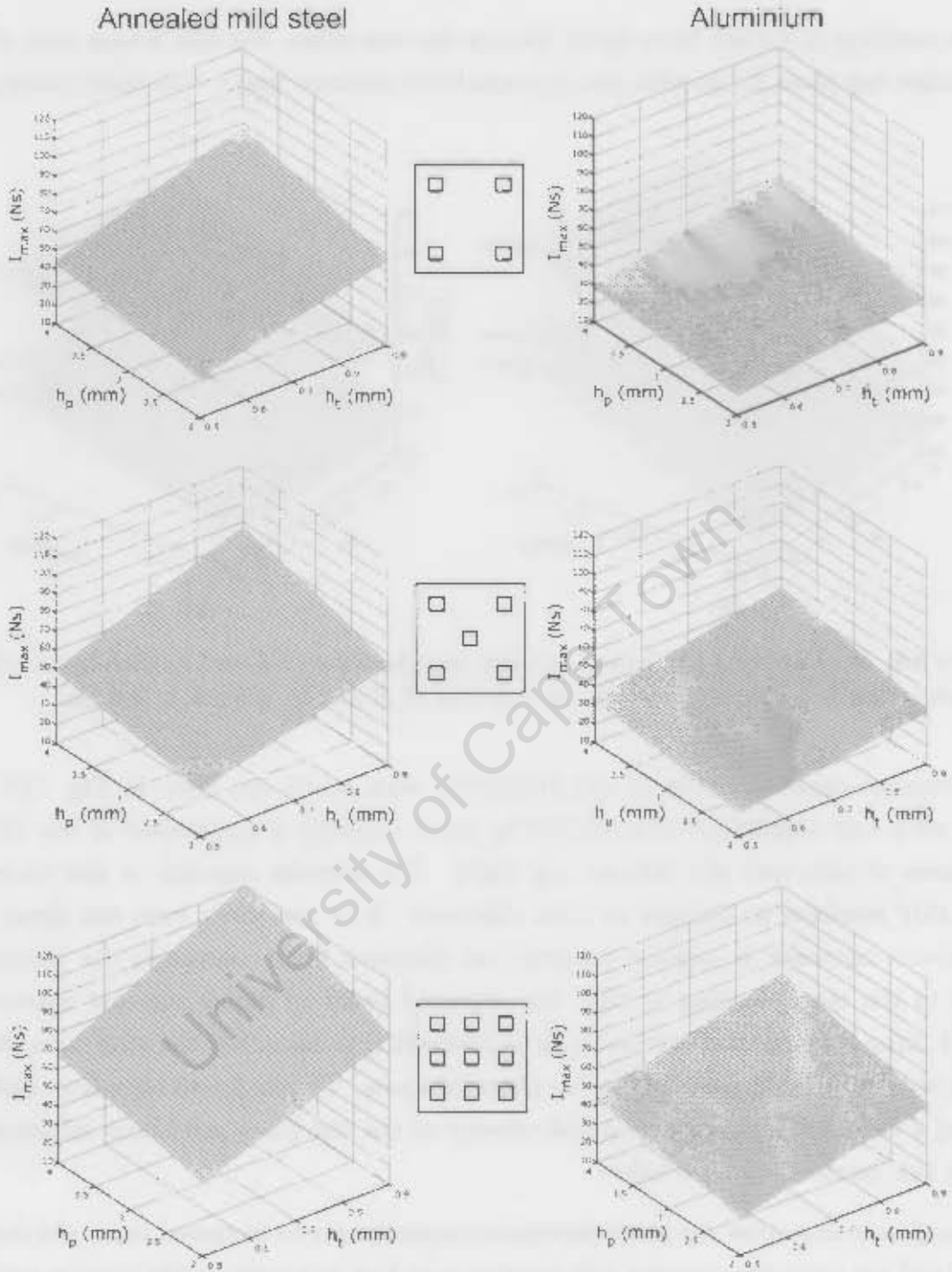


Figure 6.9: Plots of impulse capacity as a function of tube thickness and top plate thickness for panels A, B, and C.

steel tubes also have a higher specific impulse than the aluminium core tubes although the difference is not significant.

The analytical predictions of the specific impulse capacity provide an interesting challenge in terms of optimal panel design. In applications where mass is of critical importance the results imply the use of a panel C tube layout with thick steel tubes and small plate

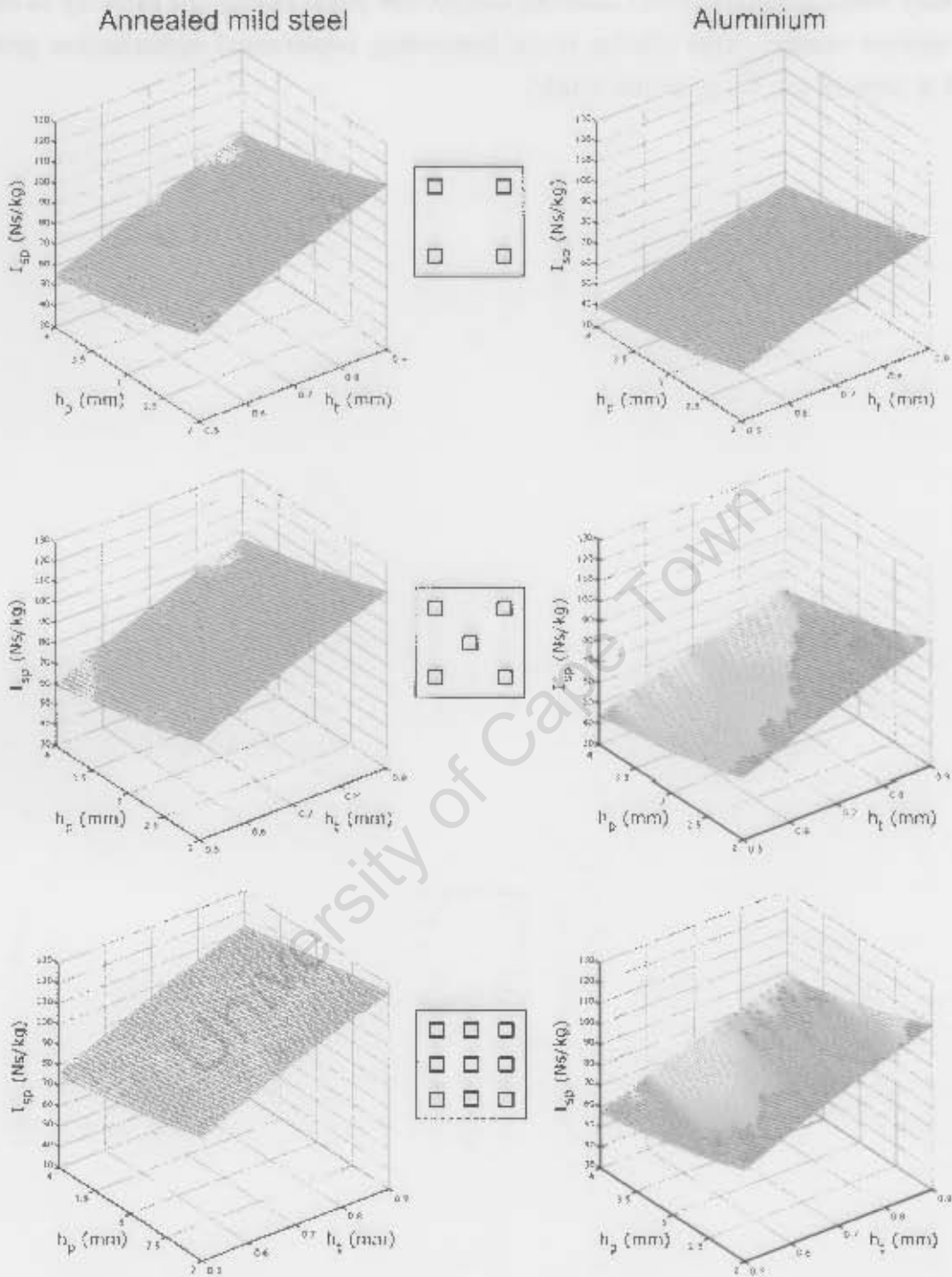


Figure 6.10: Plots of specific impulse capacity as a function of tube thickness and top plate thickness for panels A, B, and C.

thickness. The actual design variables can be chosen based on the expected blast loading and hence impulse capacity requirements of the panel. Although the specific impulse capacity favours a large number of thick tubes, such a design significantly increases the peak and mean force at the base of the panel. If the panels are used in a sacrificial cladding scenario, excessive reaction force magnitudes may compromise the protected structure.

This may result in failure of the absorber despite the panel having the capacity to absorb the required energy. This alludes to an interesting constrained optimization problem which is beyond the scope of this study.

University of Cape Town

6.6 Theoretical Comparisons with Foam Core Panels

As described in section 2.2.2.1, the use of metallic foam core sandwich panels is arguably the most suitable alternative to the structure proposed in this thesis. The relatively constant crush force and high configurability of the mass and absorption capacity of metallic foams makes them highly desirable for absorbing blast loads. This section theoretically compares the response and performance of aluminium foam panels and the proposed tube based core structure.

The response of aluminium foam under blast loading has been given little attention in the literature thus far. This is particularly true for comparisons of experimental tests and analytical models. It is therefore difficult to ensure that current models completely capture the true response of the foam panels. The purpose of the present analysis is give a global comparison of tube core and foam core panels under blast loading. The majority of this thesis focuses on comparisons among different tube core panels, in terms of various performance measures. It is of interest to compare the tube core panels to another energy absorbing structure. Such an analysis can determine if the panels presented in this thesis compare, at a general level, to existing sandwich configurations.

6.6.1 Analytical Foam Model Under Blast Loading

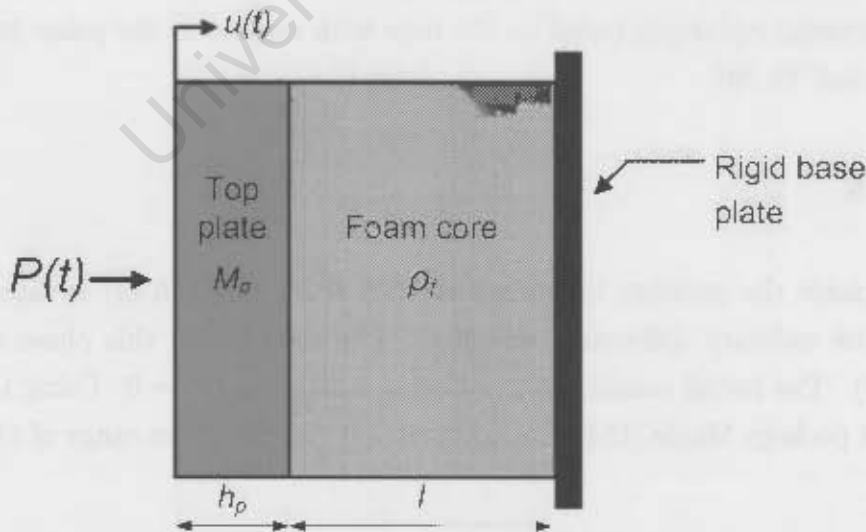


Figure 6.11: Schematic of the foam core panel investigated in this study.

Hanssen *et al.* [78] and Ma and Ye [80] used a one-dimensional shock propagation model to develop differential equations describing the behaviour of aluminium foam panels under impulsive loading. As the problem geometry is the same as presented in Hanssen *et al.* [78], the derivation of the differential equation of motion is not included here. For the

model, the loading is assumed to be given by

$$P(t) = \begin{cases} P_0 & 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (6.42)$$

where P_0 is the constant blast pressure and t_0 is the pulse duration. This is a rectangular pressure pulse as opposed to a triangular pulse used by the previously mentioned authors [78, 80]. It should be noted that the following analysis was reproduced using a triangular pulse for the purposes of validation and to compare the response differences between the pulse shapes. It was found that the rectangular and triangular pulses produced nearly identical results. The rectangular pulse is chosen to maintain consistency with the remainder of this work. The motion of the foam panel $u(t)$ is governed by the differential equation of Eq. (6.43).

$$\left[1 + \frac{\rho_f A_p}{M_p \varepsilon_D} u \right] \ddot{u} + \frac{\rho_f A_p}{M_0 \varepsilon_D} \dot{u}^2 + [\sigma_0 - p(t)] \frac{A_p}{M_0} = 0 \quad (6.43)$$

In Eq. (6.43), M_p is the mass of the cover plate, A_p is the cross sectional area of the panel, ρ_f is the foam density, ε_D is the densification strain of the foam and σ_0 is the crushing strength of the foam. Fig. 6.11 illustrates the foam core sandwich construction.

The differential equation given by Eq. (6.43) is a second order non-linear ordinary differential equation with piecewise coefficients. This implies careful consideration when determining the solution. The methodology used here is to break the equation into two separate differential equations based on the time with respect to the pulse duration t_0 as given by Ma and Ye [80].

Phase I: $t \leq t_0$

During this phase the pressure is applied as $P(t) = P_0$. Eq. (6.43) is thus reduced to a non-piecewise ordinary differential equation. The solution for this phase of motion is given by $u_0(t)$. The initial conditions are clearly $u_0(0) = \dot{u}_0(0) = 0$. Using the symbolic mathematical package Maple [154], the solution $u_0(t)$ for the given range of t is computed as

$$u_0(t) = \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t^2 (\sigma_0 - P_0))}}{\rho_f A_p} \quad (6.44)$$

From this solution, it is clear that $u_0(t) \leq 0$ if $P_0 \leq \sigma_0$. This implies that no deformation takes place if the peak pressure of the blast is less than the crushing strength of the foam. The variable $U_1 = u_0(t_0)$ is introduced here which represents the displacement of the

panel top plate at the end of the blast event and is given as

$$U_1 = u_0(t_0) = \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t_0^2 (\sigma_0 - P_0))}}{\rho_f A_p} \quad (6.45)$$

The velocity of the cover panel at the end of the blast event V_1 is given as

$$V_1 = \dot{u}_0(t_0) = \frac{\varepsilon_D A_p t_0 (P_0 - \sigma_0)}{\sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t_0^2 (\sigma_0 - P_0))}} \quad (6.46)$$

Furthermore,

$$\dot{u}_0(t) = 0 \Rightarrow \sigma_0 = P_0 \quad (6.47)$$

therefore if $\sigma_0 < P_0$ then deformation of the panel will continue to at least $t = t_0$ provided the panel does not compact. In other words, under a rectangular pulse the foam will not deform at all or will deform at least until the end of the pulse duration, or it will compact. Under a triangular pulse, if $P_0 < 2\sigma_0$ then deformation will cease before the end of the blast.

It is possible for the panel to cease motion during the blast event due to compaction of the foam. That is, the foam has crushed to its maximum distance $u = l\varepsilon_D$. This case can be ignored for the current application as the blast event is much shorter than the expected duration of response. Despite this, a short analysis is given for completeness. If the foam compacts before $t = t_0$, then $u_0(t_{c0}) = l\varepsilon_D$ where t_{c0} is the time at compaction and $t_{c0} < t_0$. Using Eq. (6.44) to solve $u_0(t_{c0}) = l\varepsilon_D$ gives

$$t_{c0} = \sqrt{\frac{\varepsilon_D l (2M_p + A_p l \rho_f)}{A_p (P_0 - \sigma_0)}} \quad (6.48)$$

Since Eq. (6.44) is monotonically increasing, and we know $P_0 > \sigma_0$, Eq. (6.48) will always have a real solution. The solution only makes physical sense however if $t_{c0} < t_0$. Therefore if $t_0 < t_{c0}$ then the panel will not compact before the end of the blast. Alternatively, for a given blast pulse the minimum thickness l of the panel required to ensure compaction does not occur during the blast loading is found by simply solving $u(t_0) = l\varepsilon_D$ which implies that l must satisfy

$$l \geq \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t_0^2 (\sigma_0 - P_0))}}{\rho_f A_p \varepsilon_D} \quad (6.49)$$

Assuming that the blast has not been attenuated and that the foam has not fully com-

pacted, then at the end of the blast the new effective cover plate mass M_d , is the original cover plate mass plus the mass of the densified foam which is considered rigid in the analysis. This mass is given by

$$M_d = M_p + \frac{U_1}{\varepsilon_D} \rho_f A_p = \sqrt{(M_p^2 \varepsilon_D - \rho_f A_p^2 t_0^2 (\sigma_0 - P_0)) / \varepsilon_D} \quad (6.50)$$

Phase II: $t > t_0$

If the peak pressure satisfies $P_0 > \sigma_0$ then deformation will continue past the duration of the blast t_0 if the blast is not dissipated. The solution for this phase of motion is denoted by $u_1(t)$. The motion of the cover panel is described by the following differential equation:

$$\left[1 + \frac{\rho_f A_p}{M_d \varepsilon_D} u_1' \right] \ddot{u}_1' + \frac{\rho_f A_p}{M_d \varepsilon_D} u_1'^2 + \sigma_0 \frac{A_p}{M_d} = 0 \quad (6.51)$$

This is a differential equation in terms of the cover plate displacement after the pulse duration given as $u_1' = u - U_1$. The initial conditions are $u_1'(0) = 0$ and $\dot{u}_1'(0) = V_1$ where V_1 is given by Eq. (6.46). The solution to the ordinary differential equation given by Eq. (6.51) is

$$u_1(t) = U_1 + \frac{-M_d \varepsilon_D + \sqrt{M_d^2 \varepsilon_D^2 - \rho_f A_p t' \varepsilon_D (A_p t' \sigma_0 - 2V_1 M_d)}}{\rho_f A_p} \quad (6.52)$$

where t' is the instantaneous time after the blast event given by $t' = t - t_0$. Using this substitution along with Eqs. (6.45), (6.46), and (6.50), the solution for this phase of motion is

$$u_1(t) = \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 (t_0^2 P_0 + \sigma_0 t^2 - 2t P_0 t_0))}}{\rho_f A_p} \quad (6.53)$$

Solving for $du/dt = 0$ gives the time when motion ceases as

$$t_m = t_0 P_0 / \sigma_0 \quad (6.54)$$

Therefore if the foam does not compact for some $t < t_m$ then at $t = t_m$ the blast has been fully attenuated and $u_1(t) = u_1(t_m)$ for all $t \geq t_m$.

To determine the time when compaction will occur, if at all, requires solving $u_1(t) = l \varepsilon_D$. Unlike u_0 , u_1 is not monotonically increasing and thus the solution to $u_1(t) = l \varepsilon_D$ will either have two real solutions or a complex conjugate pair. The latter set implies that

compaction does not occur and the displacement reaches its peak value before intersecting the line $u_1(t) = l\varepsilon_D$. For the case of real solutions, the smaller solution is clearly the time at compaction. This is given by

$$t_{c1} = \frac{A_p t_0 P_0 - \sqrt{A_p^2 t_0^2 P_0 (P_0 - \sigma_0) - A_p \sigma_0 l \varepsilon_D (2M_p + A_p l \rho_f)}}{A_p \sigma_0} \quad (6.55)$$

The complete solution to Eq. (6.43) is presented in Eq. (6.56). Comparisons involving t_{c1} are only valid if $t_{c1} \in \mathbb{R}$. If not then it is assumed $t_{c1} > t_m$.

$$u_f(t) = \begin{cases} 0 & t \leq 0 \text{ or } P_0 \leq \sigma_0 \\ \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t^2 (\sigma_0 - P_0))}}{\rho_f A_p} & 0 < t \leq t_0 \text{ and } t < t_{c0} \\ \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 (t_0^2 P_0 + \sigma_0 t^2 - 2t P_0 t_0))}}{\rho_f A_p} & t_0 < t < t_m \text{ and } t < t_{c1} \\ \frac{-M_p \varepsilon_D + \sqrt{\varepsilon_D (M_p^2 \varepsilon_D - \rho_f A_p^2 t_0^2 P_0 (1 - P_0 / \sigma_0))}}{\rho_f A_p} & t \geq t_m \text{ and } t < t_{c1} \\ \varepsilon_D l & (t < t_0 \text{ and } t \geq t_{c0}) \text{ or} \\ & (t \geq t_0 \text{ and } t \geq t_{c1}) \end{cases} \quad (6.56)$$

Now that the time when compaction occurs is determined the minimum panel thickness l required to absorb a given blast load can be determined. This is clearly a useful tool when designing such structures. The existence of a real value for $l_{\min}$, implies that the expression inside the square root of Eq. (6.55) must be positive. This expression is a quadratic in l which has the positive root given by

$$l_{\min} = \frac{-\sigma_0 M_p \varepsilon_D + \sqrt{\sigma_0^2 M_p^2 \varepsilon_D^2 + A_p^2 t_0^2 P_0 \rho_f \varepsilon_D (P_0 - \sigma_0)}}{\rho_f A_p \varepsilon_D \sigma_0} \quad (6.57)$$

therefore if $l \geq l_{\min}$ then the panel will absorb given blast without becoming fully compacted. If $l < l_{\min}$, then the remaining load is transferred directly to the protected structure resulting in stress levels much higher than σ_0 .

The final expression of interest is the the impulse capacity of a given panel. In other words, for a fixed panel configuration, the task is to determine what is the largest possible impulsive load that can be applied assuming a fixed pulse duration. If a given panel has used its entire energy absorbing capacity and completely attenuated the blast then $t_m = t_{c1}$. In other words, the time required to absorb the blast exactly equals the time when compaction occurs. This relationship can then be solved for P_0 using Eqs. 6.54 and

6.55 which results in a quadratic expression for P_0 . The positive root of this equation gives the expression for maximum allowable pressure $P_0^{\max}$. Multiplying this pressure value by $t_0 A$ gives the maximum allowable impulse $I_{\max}$ as

$$I_{\max} = \left\{ A_p t_0 \sigma_0 + \sqrt{A_p^2 t_0^2 \sigma_0^2 + 8 M_p A_p \sigma_0 l \varepsilon_D + 4 A_p^2 \rho_f \sigma_0 l^2 \varepsilon_D} \right\} / 2 \quad (6.58)$$

The solution given by Eq. (6.56) is compared with published models. Eq. (6.56) is implemented in Matlab along with the solutions given by Hanssen *et al.* [78] and Ma and Ye [80]. The test cases used are shown in Table 6.2. For all panels given in Table 6.2, the top plate has a thickness of $h_p = 3.0$ mm with a cross sectional area of 150×150 mm and a density of 7850 kg/m^3 . This is the typical mild steel top plate used in the numerical and analytical studies of the tube core panels.

To determine suitable foam properties design formulae from the literature are used. It is assumed for the purposes of this analytical investigation that the foam is similar to the Hydro Aluminium foam often studied in the literature. The plateau stress σ_0 is usually expressed in terms of the foam density using a power law of the form

$$\sigma_0 = C \left(\frac{\rho_f}{\rho_{f0}} \right)^n \quad (6.59)$$

where the values of C and n are 800 and 2.38 respectively [61,62]. The values also compare well with the reported density-stress curves in Reyes *et al.* [155]. The parameter ρ_{f0} is the density of the parent material in the foam cell wall which is taken as 2700 kg/m^3 . The densification strain ε_D is also a function of foam density and can be expressed as

$$\varepsilon_D = 1 - \alpha \left(\frac{\rho_f}{\rho_{f0}} \right) \quad (6.60)$$

The value of α is taken as $\alpha = 1.5$ as reported by Hanssen *et al.* [51,52] however values as low as $\alpha = 1.4$ [55,58,87] and as high as $\alpha = 2.0$ [80] have been used in similar studies. The value of $\alpha = 1.5$ is chosen as it is used in the analysis of the Hydro Aluminium foam. This α value appears to give rather high densification strains compared to other types of aluminium foams however this will provide an upper bound on the energy absorption capacity of the foams.

Table 6.2 outlines foam and blast properties of several test cases used to validate the proposed model. Test cases 1 – 3 give the response of a low density foam with large thickness. The tests represent blast attenuation, attenuation at the point of compaction, and compaction respectively. Test case 4 is included to show that at small pulse durations, combined with low resistance foam, that the square pulses used here converge to the triangular pulses used by Hanssen *et al.* [78] and Ma and Ye [80]. Test cases 5 – 7 are

Table 6.2: Foam core properties and blast load properties used in the validation test cases of the proposed aluminium foam model.

Test Case	Foam properties				Blast properties	
	ρ_f (kg/m ³)	σ_0 (MPa)	ε_D	l (mm)	I (Ns)	t_0 (μ s)
1	200.0	1.63	0.89	40.0	30.00	50.0
2	200.0	1.63	0.89	40.0	41.17	50.0
3	200.0	1.63	0.89	40.0	48.00	50.0
4	200.0	1.63	0.89	40.0	30.00	20.0
5	400.0	8.50	0.78	15.0	40.00	20.0
6	400.0	8.50	0.78	15.0	53.57	20.0
7	400.0	8.50	0.78	15.0	58.00	20.0

analogous to test cases 1 – 3 but use a higher density foam with a smaller thickness.

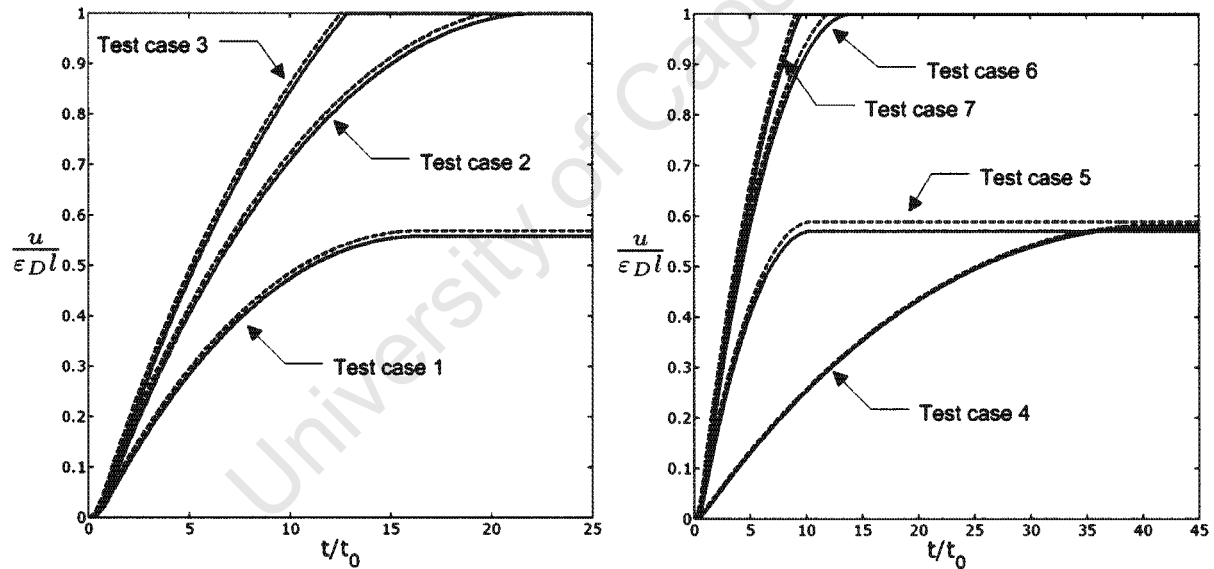


Figure 6.12: - - - Triangular pulse solution given by Hanssen *et al.* [78] and Ma and Ye [80]. — Proposed solution using a rectangular pulse. Plot of foam model verification for test cases (a) 1 – 3 and (b) 4 – 7.

It should be noted that the piecewise function given by Eq. (6.56) is implemented in Matlab based explicitly on the critical time calculations given by Eqs. (6.48), (6.54), and (6.55). Fig. 6.12 gives the displacement of the front panel as a function of time for each of the seven test cases. The solution presented here is compared to the triangular pulse results given by Hanssen *et al.* [78] and Ma and Ye [80]. For all test cases the front panel displacement modelled here is shown to agree very closely with the results from the literature. The shape of the pulse clearly has little effect on the resulting panel behaviour, particularly if the pulse duration is small and the foam has low strength. This is shown

by examining the results of test case 4 in Fig. 6.12b where the square and triangular pulses give nearly identical responses due to the small pulse duration.

Table 6.3 presents panel response parameters of interest for the test cases. Of note is the compaction time t_{c1}/t_0 is real only for test cases 3 and 7 even though compaction appears to occur at $t = t_m$ in cases 2 and 6. In the latter cases the applied impulse is just slightly lower than the maximum impulse and hence t_{c1}/t_0 in these cases has a small imaginary component and is not given. It is clear however from Fig. 6.12 that compaction does occur at $t = t_m$.

Table 6.3: Critical response parameters for the panels and loading conditions given in Table 6.2.

Test Case	$V_1(\text{ms}^{-1})$	t_{c0}/t_0	t_{c1}/t_0	t_m/t_0	$l_{\min}/l$	$I_{\max}/I$
1	52.49	5.60	—	16.33	0.56	1.37
2	72.95	4.73	—	22.41	1.00	1.00
3	85.36	4.37	12.84	26.13	1.31	0.86
4	54.94	8.68	—	40.83	0.58	1.35
5	67.27	4.39	—	10.46	0.57	1.34
6	92.01	3.74	—	14.01	1.00	1.00
7	100.03	3.59	9.46	15.17	1.16	0.92

6.6.2 Foam Core Response to Blast Loading

Now that a simple model has been developed for foam core panels under blast loads, the critical parameter formulae can be used to describe the panel's energy absorption capacity in similar terms to that of the tube based cores. Fig. 6.13 gives the impulse capacity and specific impulse capacity of a 75.0 mm thick aluminium foam core panels as a function of foam density and top plate thickness. As expected the impulse capacity is found to be maximized with a high foam density and a thick top plate. This is analogous to the tube based panel where thick tubes can be used in the core to maximize impulse capacity, however this increases the peak and mean force levels transmitted at the base of the panel which may give undesirable results.

As with the tube based cores the specific impulse capacity is found to be maximized using a thin top plate and high density foam. As is the case with the tube based cores, the top plate constitutes a significant portion of the panel mass while having a comparatively smaller effect on the amount of energy to be absorbed. The top plate influence on the specific impulse capacity of the panel is much less pronounced however with almost no reduction in specific impulse capacity when reducing the thickness of the top plate. It is

Table 6.2: Foam core properties and blast load properties used in the validation test cases of the proposed aluminium foam model.

Test Case	Foam properties				Blast properties	
	ρ_f (kg/m ³)	σ_0 (MPa)	ε_D	l (mm)	I (Ns)	t_0 (μ s)
1	200.0	1.63	0.89	40.0	30.00	50.0
2	200.0	1.63	0.89	40.0	41.17	50.0
3	200.0	1.63	0.89	40.0	48.00	50.0
4	200.0	1.63	0.89	40.0	30.00	20.0
5	400.0	8.50	0.78	15.0	40.00	20.0
6	400.0	8.50	0.78	15.0	53.57	20.0
7	400.0	8.50	0.78	15.0	58.00	20.0

analogous to test cases 1 – 3 but use a higher density foam with a smaller thickness.

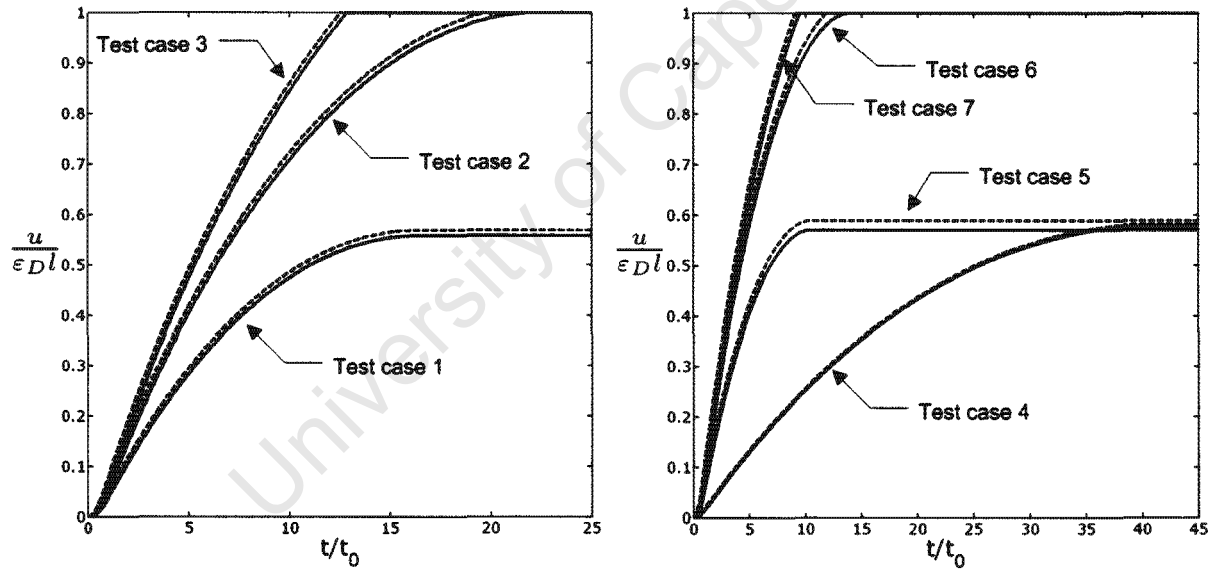


Figure 6.12: - - - Triangular pulse solution given by Hanssen *et al.* [78] and Ma and Ye [80]. — Proposed solution using a rectangular pulse. Plot of foam model verification for test cases (a) 1 – 3 and (b) 4 – 7.

It should be noted that the piecewise function given by Eq. (6.56) is implemented in Matlab based explicitly on the critical time calculations given by Eqs. (6.48), (6.54), and (6.55). Fig. 6.12 gives the displacement of the front panel as a function of time for each of the seven test cases. The solution presented here is compared to the triangular pulse results given by Hanssen *et al.* [78] and Ma and Ye [80]. For all test cases the front panel displacement modelled here is shown to agree very closely with the results from the literature. The shape of the pulse clearly has little effect on the resulting panel behaviour, particularly if the pulse duration is small and the foam has low strength. This is shown

by examining the results of test case 4 in Fig. 6.12b where the square and triangular pulses give nearly identical responses due to the small pulse duration.

Table 6.3 presents panel response parameters of interest for the test cases. Of note is the compaction time t_{c1}/t_0 is real only for test cases 3 and 7 even though compaction appears to occur at $t = t_m$ in cases 2 and 6. In the latter cases the applied impulse is just slightly lower than the maximum impulse and hence t_{c1}/t_0 in these cases has a small imaginary component and is not given. It is clear however from Fig. 6.12 that compaction does occur at $t = t_m$.

Table 6.3: Critical response parameters for the panels and loading conditions given in Table 6.2.

Test Case	$V_1(\text{ms}^{-1})$	t_{c0}/t_0	t_{c1}/t_0	t_m/t_0	$l_{\min}/l$	$I_{\max}/I$
1	52.49	5.60	—	16.33	0.56	1.37
2	72.95	4.73	—	22.41	1.00	1.00
3	85.36	4.37	12.84	26.13	1.31	0.86
4	54.94	8.68	—	40.83	0.58	1.35
5	67.27	4.39	—	10.46	0.57	1.34
6	92.01	3.74	—	14.01	1.00	1.00
7	100.03	3.59	9.46	15.17	1.16	0.92

6.6.2 Foam Core Response to Blast Loading

Now that a simple model has been developed for foam core panels under blast loads, the critical parameter formulae can be used to describe the panel's energy absorption capacity in similar terms to that of the tube based cores. Fig. 6.13 gives the impulse capacity and specific impulse capacity of a 75.0 mm thick aluminium foam core panels as a function of foam density and top plate thickness. As expected the impulse capacity is found to be maximized with a high foam density and a thick top plate. This is analogous to the tube based panel where thick tubes can be used in the core to maximize impulse capacity, however this increases the peak and mean force levels transmitted at the base of the panel which may give undesirable results.

As with the tube based cores the specific impulse capacity is found to be maximized using a thin top plate and high density foam. As is the case with the tube based cores, the top plate constitutes a significant portion of the panel mass while having a comparatively smaller effect on the amount of energy to be absorbed. The top plate influence on the specific impulse capacity of the panel is much less pronounced however with almost no reduction in specific impulse capacity when reducing the thickness of the top plate. It is

also important to note that unlike the tube based core, the analytical response of the foam based cores with a thinner top plate does not begin to violate any model assumptions. Specifically, significant top plate deformation is unlikely with the foam core which is not true for the tube based core.

It is of note that for the design variable values examined in this study, the foam core panels have a higher maximum impulse capacity than the tube based cores. This is somewhat arbitrary however as tubes with larger thickness can be investigated which clearly increase the capacity. The foam densities examined here correspond to approximately the maximum and minimum densities reported in the literature. Comparisons between these absolute impulse capacities therefore do not describe the performance of the panels and further investigations are required. This can be seen by examining the specific impulse capacities in Fig. 6.13b which shows the foam panels having similar specific impulse capacities despite the large difference in total impulse capacity.

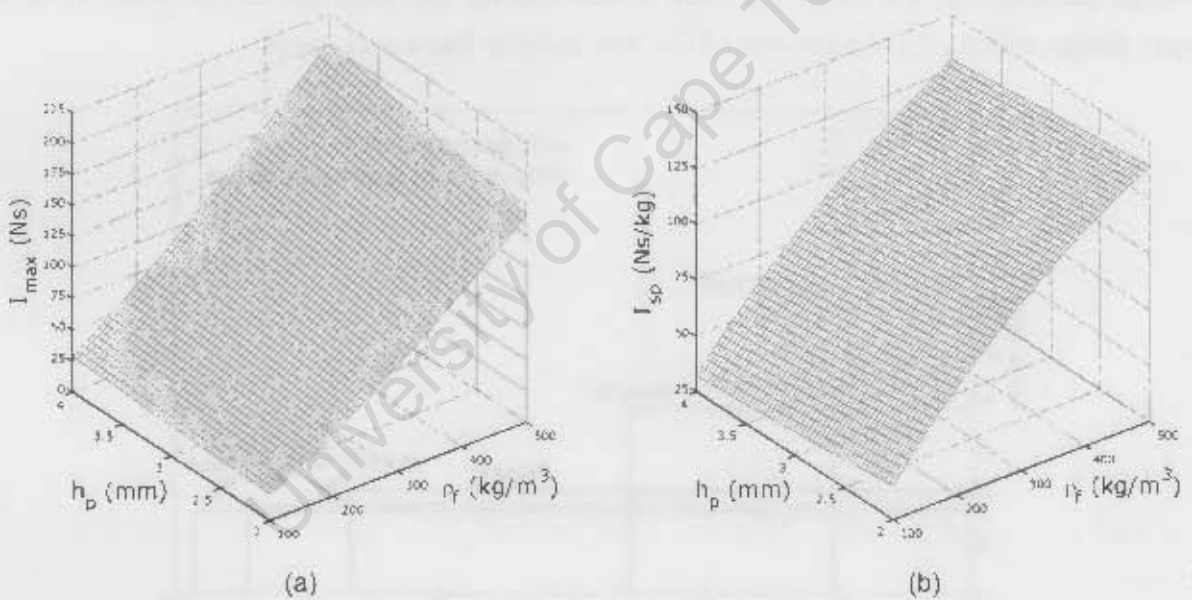


Figure 6.13: Plot of (a) Impulse capacity and (b) Specific impulse capacity as a function of the foam density ρ_f and top plate thickness h_p . The core thickness of the panel is 75.0 mm.

It is interesting to be able to not only predict and compare the impulse capacity of tube and foam based cores but to also compare the force transfer behaviour at the base of the panel. As previously mentioned, even if an energy absorbing device can attenuate a given blast, localized protection of the structure may be compromised. It is therefore the responsibility of the absorber to "smooth" out the load experienced by the structure at the base of the panel. It is important to note that the total impulse applied by the blast load must equal the integral of the force-time curve at the rigid base of the panel. This implies that momentum is conserved and the total impulse the protected structure experiences is not modified, however the force levels it is subjected to expected to be

much lower. Since the stress applied as the base of the foam core panels is the plateau stress up to densification we have the relation

$$t_m^f \sigma_0 A = t_m^t n_t F_m = I \quad (6.61)$$

where the superscripts f and t refer to the foam and tube based cores respectively. Note that the relation $t_m^t n_t F_m = I$ is not explicitly true as the formulation of F_m is based on the the impulse remaining to be transferred after application of the load as discussed in section 6.3. Fig. 6.14 shows typical idealised force transfer behaviour for steel tube and foam based panels under an applied impulse of 40 Ns with a $10 \mu\text{s}$ pulse duration. For both panel types, the core thickness is 75.0 mm with a top plate thickness of 3.0 mm. The tubes have a thickness of 0.6 mm with aspect ratio $R = 4$. It is clear from Fig. 6.14 that for the geometric and materials parameters examined in this study that the force levels and dissipation times are within the same range and depend strongly on the panel design parameters. Of particular note is how closely the response of the panel A tube core design mimics the behaviour of the low density foam core panel.

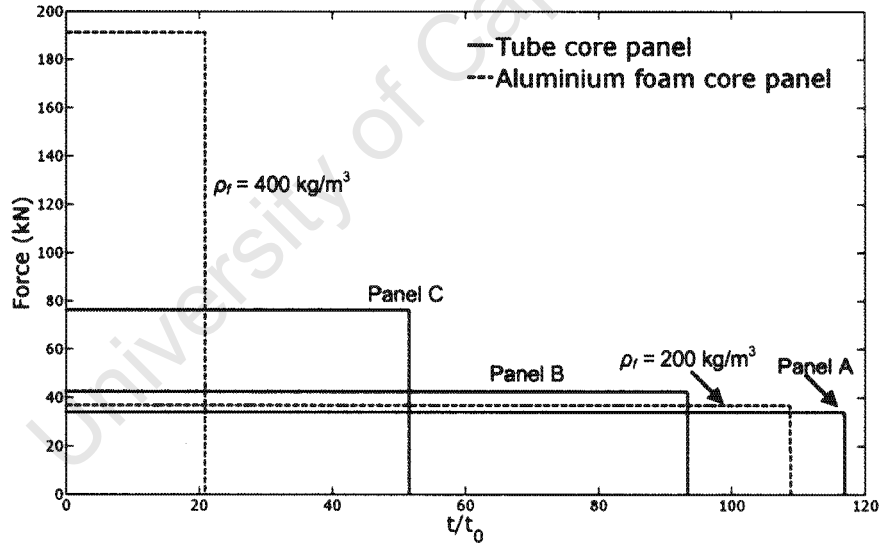


Figure 6.14: Plot of idealised reaction forces at the base of the tube and foam core panels for an applied impulse of 40 Ns with a $10 \mu\text{s}$ pulse duration. For both panel types, the core thickness is 75.0 mm with a top plate thickness of 3.0 mm. The tubes have a thickness of 0.6 mm with aspect ratio $R = 4$.

Simple comparisons of tube and foam based cores show that performance in terms of impulse capacity and force transfer to the protected structure are similar and depends strongly on the design parameters. The next section investigates this further by expanding on the criteria of comparison.

6.6.3 Mass and Thickness Comparison of Tube and Foam Based Cores

To study further, the energy absorbing capabilities of the foam core panels in terms of the tube based panels, several investigations are undertaken. Although fixing the foam core thickness to the thickness of the tube based panels and comparing the capacities and corresponding mass is useful, it does not give a complete characterisation of the panels. It should be kept in mind that under the proposed industrial application of the panels, that not only energy absorption, and mass efficiency are of importance but also the overall panel thickness. An energy absorbing panel may be of small mass and absorb the required energy, however if it requires an excessive thickness it may become unsuitable for a given application where space is limited.

To compare the thickness based performance of the tube and foam based cores, a simple numerical investigation is performed. Firstly, for the steel and aluminium tube panels, the maximum impulse as a function of tube thickness and top plate thickness is computed. For each of these tube based designs, the minimum thickness foam based panel required to absorb the corresponding impulse is found using Eq. (6.57). This procedure is done for a low density 200 kg/m^3 and a high density 400 kg/m^3 foam. The top plate thickness is set to the maximum value of 4.0 mm in order to give a lower bound on the minimum thickness. Fig. 6.15a and Fig. 6.15b show contour plots of results corresponding to the steel and the aluminium core tubes respectively for a low density foam (200 kg/m^3). Fig. 6.16a and Fig. 6.16b provide the results using a higher density foam (400 kg/m^3). Specifically, the total foam panel to tube core panel thickness ratio is given along with the corresponding mass ratio. The areas outlined in red correspond to tube based panels designs which require a larger thickness/mass than the foam based panel.

For the low density foam it is observed that the majority of steel tube core design space results in a smaller required thickness than the foam core panel. For the steel tube core, only for relatively small tube thickness and top plate thickness can the low density foam panel absorb the maximum impulse using a smaller thickness. This region is significantly expanded when comparing the aluminium tube core to the foam core due to the smaller impulse capacities at identical tube core panel thickness. For all but the largest tube thickness and top plate thickness does the tube core panel result in a smaller required thickness. In terms of mass ratios for the steel tube based panels, all tube core designs have a smaller mass than the foam core design that with the same impulse capacity. This is also observed for the aluminium core tubes as all low density foam core panels required a higher mass than the tube based core. This is slightly misleading however as the foam core panels have a fixed top plate thickness of 4.0 mm which contributes a significant portion of the overall panel mass. The large top plates do allow for much smaller core

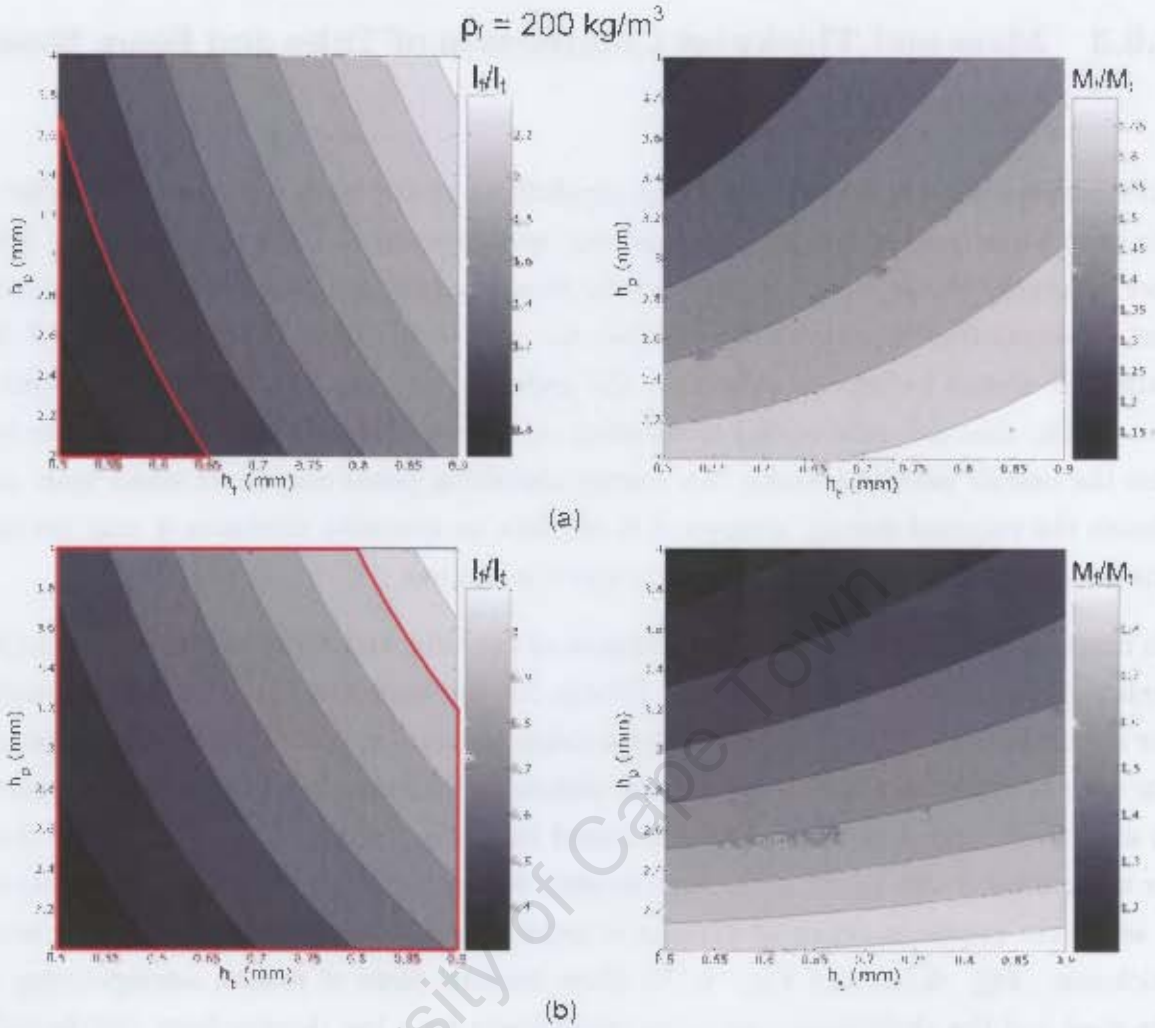


Figure 6.15: Contour plots showing the ratio of foam panel to tube based panel thickness and mass required to have the same impulse capacity as the panel *C* design with (a) annealed mild steel tube based core and (b) aluminium tube based core. The foam panels have a top plate thickness of 4.0 mm and foam density 200 kg/m^3 . Areas outlined in red indicate regions in design space where the thickness/mass required of the foam panels is less than that of the corresponding tube based panel.

thicknesses however which is the purpose of this investigation.

The results for the high density foam are significantly different than for the low density foam. For the steel and aluminium tube panels, the high density foam core is able to dissipate the required impulse using a much smaller thickness than the tube based panels. In terms of mass however, only the steel tube based panels with large tube thickness and top plate thickness require a larger mass than the foam core panel that can dissipate the same impulse. For the aluminium tubes, the larger mass of the high density foam results in all tube based designs requiring a smaller mass than the foam panel with equivalent impulse capacity despite the significantly smaller thickness.

The results presented in Fig. 6.15 and Fig. 6.16 indicate that in terms of minimizing

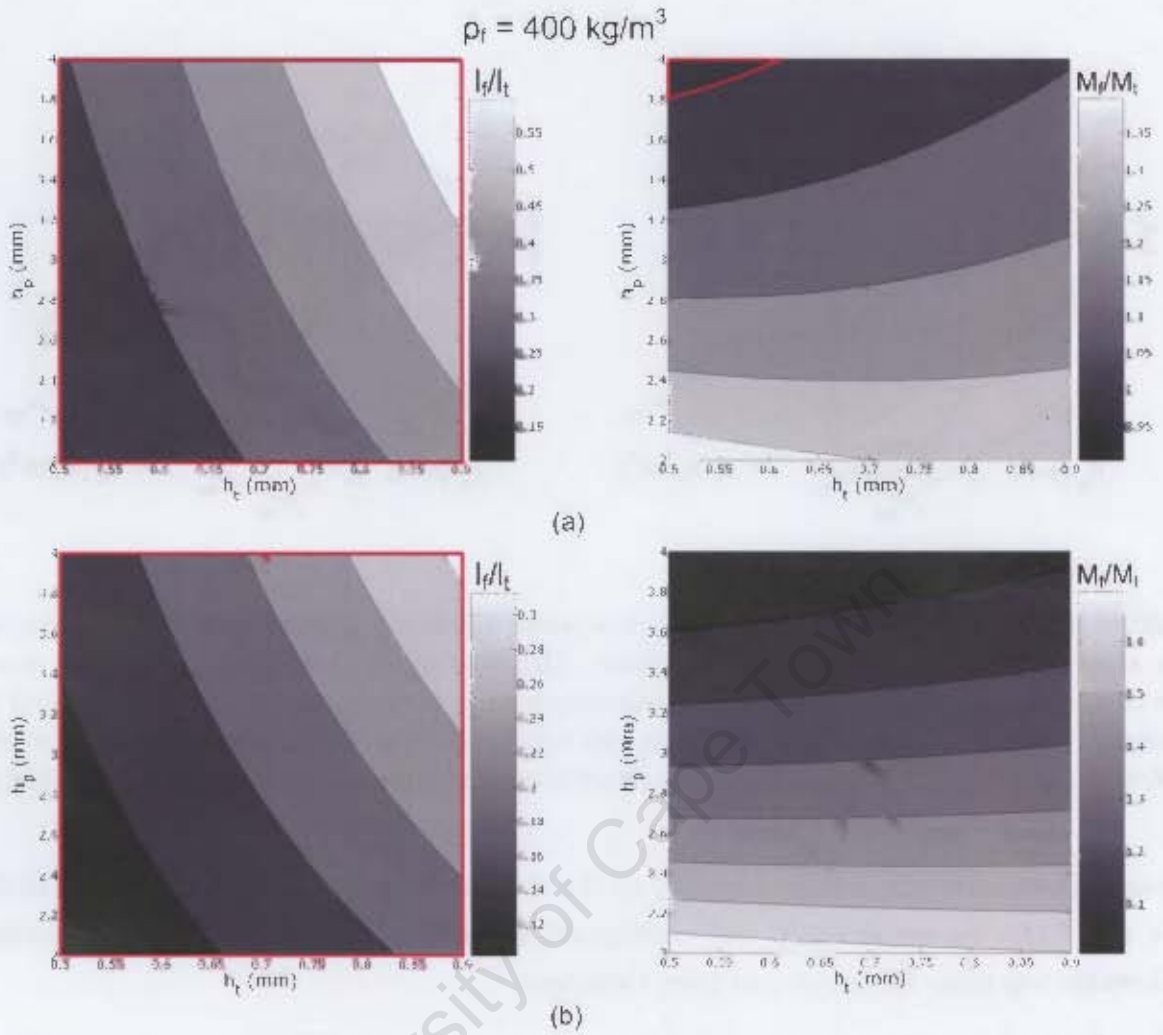


Figure 6.16: Contour plots showing the ratio of foam panel to tube based panel thickness and mass required to have the same impulse capacity as the panel *C* design with (a) annealed mild steel tube based core and (b) aluminium tube based core. The foam panels have a top plate thickness of 4.0 mm and foam density 400 kg/m^3 . Areas outlined in red indicate regions in design space where the thickness/mass required of the foam panels is less than that of the corresponding tube based panel.

the overall thickness of the energy absorbing panel that an aluminium foam core panel with high density may be preferable to a tube based core. In terms of mass however the improvements in thickness minimization can result in increased mass due to the higher density core.

Further insight into impulse dissipating capabilities of the tube and foam based cores can be obtained by examining the required foam core panel thickness and mass to have the same impulse capacity as a given tube core panel. These critical thickness and masses can then be directly compared to the thickness and mass of the steel or aluminium tube based panel with the target impulse capacity. Of particular interest is the steel and aluminium core panel designs that result in the highest impulse capacity within the given design

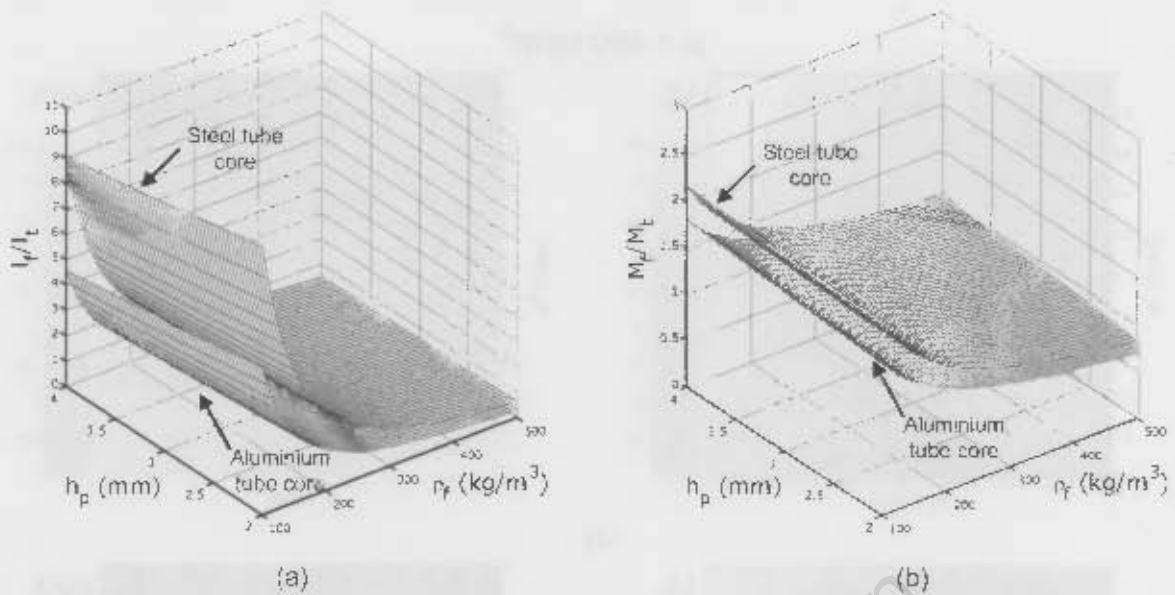


Figure 6.17: (a) Plot of required foam core panel thickness to tube core panel thickness to absorb the maximum tube core impulse. (b) Plot of required foam core panel mass to tube core panel mass to absorb the maximum tube core impulse. In the plots l_f and l_t refers to the total foam panel thickness and total tube core thickness respectively while M_f and M_t refers to the total foam core panel mass and tube core panel mass respectively.

space. The maximum impulse capacity of the steel and aluminium core panels is 119.89 Ns and 72.66 Ns respectively and correspond to panel C' designs with the maximum allowable top plate thickness and tube thickness.

Fig. 6.17a shows the total foam panel to tube core panel thickness ratio required to dissipate the maximum impulse capacity of the tube core panels as a function of the foam core density and top plate thickness. Fig. 6.17b provides the corresponding mass ratios of the panel in Fig. 6.17a. In the calculation of mass ratios, the steel tube results are normalized by the mass 1.0642 kg and the aluminium tube results are normalized by 0.8300 kg. These are the masses of the panels resulting in the maximum impulse capacity. Of particular note in Fig. 6.17a is the highly non-linear relationship between the required panel thickness and foam density where for a low density foam and thin top plate the panel thickness can be more than 10 times that of the steel tube core panel.

The critical thickness and mass information provided in Fig. 6.17 can be further utilized to discern interesting properties of the tube and foam based cores. The aim is to be able to answer the question of, given a particular foam density and top plate thickness, will the minimum thickness and mass of the foam core panel required to absorb the maximum tube core impulse be larger than the tube core panel? In other words, the set of points in foam design space requiring in a thicker/heavier panel in order to absorb the required impulse needs to be determined. To determine the foam designs that require the exact same thickness as the tube based core, the plane $l_f/l_t = 1$ is intersected with the surfaces

given in Fig. 6.17a. This curve of intersection is then projected onto the foam design space. The same method is applied to the mass ratios given in Fig. 6.17b.

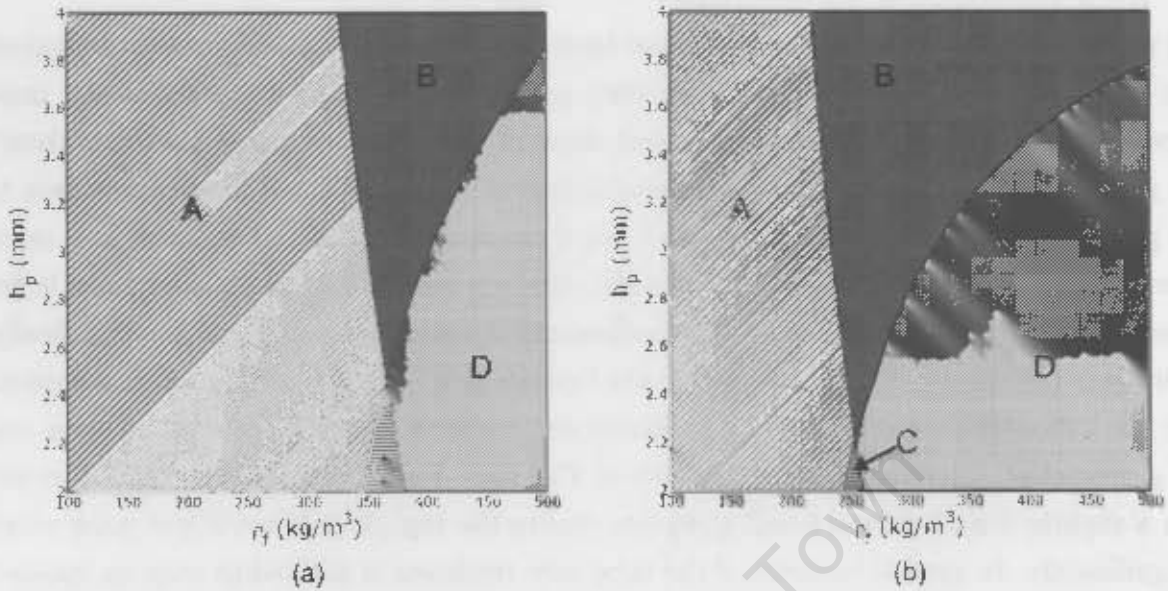


Figure 6.18: Design space partitioning in terms of thickness and mass efficiency to the response of the maximum impulse for (a) Steel tube cores and (b) Aluminium tube cores.

For each tube core material, two curves are generated which can then be plotted in the foam design space. Fig. 6.18 shows the results of this procedure. The design space is subdivided by forming areas bounded by the previously defined iso-thickness and iso-mass curves and the bounding area of the design space. The resulting areas are labelled A-D and the behaviour of the panels in each region can be described as the following

- A: The foam core panel required has a larger thickness and mass than the tube core panel with maximum impulse capacity.
- B: The foam core panel required has a smaller thickness but larger mass than the tube core panel with maximum impulse capacity.
- C: The foam core panel required has a larger thickness but smaller mass than the tube core panel with maximum impulse capacity.
- D: The foam core panel required has a smaller thickness and smaller mass than the tube core panel with maximum impulse capacity.

Therefore, for example from Fig. 6.18a, any foam core panel with foam density and top plate thickness in region A requires a thickness and mass greater than the thickness and mass of the highest capacity steel tube panel. If a foam design is chosen from region C however, the required thickness of the panel is greater than that of the steel tube panel

however the mass of the resulting panel is lower than that of the steel tube panel. For all foam designs in region D, the foam panel thickness and mass required are less than the thickness and mass of the highest capacity tube core panel.

It should be noted that analysis presented here does not, in general, determine the regions in foam design space where the foam core panel has a minimum thickness and mass greater than the *minimum* thickness and mass of the tube core panel that can absorb a given impulse. The design space partitioning of Fig. 6.18 is a direct comparison to a particular tube core panel design and not a particular impulse. The latter is a much more difficult problem as for a given impulse, the minimum thickness or mass panel is not easily attainable and requires a more mathematical optimization technique. Specifically, for every critical impulse value I within the bounds given in Fig 6.9 there exists a contour in the tube design space resulting in panels designs with I as the impulse capacity and a corresponding minimum core thickness of 75.0 mm. Each point on the contour results in a slightly different total panel thickness due to the top plate however the mass varies significantly. In general however, if the tube core thickness is allowed to vary, as opposed to being fixed at 75.0 mm then for each tube thickness and top plate thickness there exists a different minimum panel thickness and corresponding mass for a given impulse. The design parameters corresponding to the minimum panel thickness and mass would need to be obtained using optimization techniques. The analysis presented above however is in terms the maximum allowable impulse the of the tube core panels which is at the edge of the design space and hence the panel with this impulse capacity is unique. This panel is therefore implicitly the minimum mass panel over the given range of design values and assuming a fixed tube core thickness.

Chapter 7

Design Studies

7.1 Parametric Study Outline

In this section a parametric design study is presented. The purpose of the study is to determine the influence of various design parameters on the response of the proposed panels. As the construction of the panels is laborious and expensive, conducting a numerical study using the finite element method is a desirable alternative. Since the components of the model are validated experimentally in chapter 5, the numerically predicted response should give a high level of confidence that the experimental behaviour is similar. Although a formal optimisation study is not presented, design level observations can be made from a parametric study to determine the optimal panel design variables. Performance of the panels investigated here is evaluated using the measures given in section 4.6. This provides valuable insight into future mathematical optimisation studies.

7.1.1 Design Variables

The design parameters and variables which are used in the parametric study are presented here. It is useful at this point to define a naming convention for the panels in terms of the design variables described here. The naming scheme is given as

$$panel - \lambda - h_t - h_p - R$$

Where *panel* refers to the panel layout(*A*, *B*, *C*) as given in Fig. 4.1. The parameter λ is also defined in Fig. 4.1. The parameters h_t and h_p are the tube and top plate thickness respectively, both given in mm. The tube aspect ratio is given by *R*. Table 7.1 summarises the values of the design variables examined in this study.

Table 7.1: Plate design variables used in the parametric study.

Design variable	Values used in numerical study
Panel layout	A, B, C (Fig. 4.1)
λ	0.450, 0.475, 0.500, 0.525, 0.550, 0.575 0.600, 0.625, 0.650, 0.675, 0.700
Tube thickness (h_t)	0.50 mm, 0.55 mm, 0.60 mm, 0.65 mm, 0.70 mm
Top plate thickness (h_p)	2.0 mm, 2.50 mm, 3.00mm, 3.50 mm, 4.00 mm
Tube aspect ratio (R)	3, 3.5, 4, 4.5, 5, 5.5, 6

7.1.2 Finite Element Model

The finite element model of the panels used in this parametric study is presented in section 4.3. Of particular note is the use of circular cut-out imperfections as shown in Fig. 4.3, as opposed to the hemispherical indentation triggers used in the experimental investigations. As a result of the large number of panels investigated in this numerical study, the creation of indentation triggers is not practical due to the computational time required in creating the dents. The cut-out triggers require no additional computational time while still having the desired effect of symmetric buckling initiation, at the same location, within each tube of a panel.

For the design study presented in this chapter, the tube material is assumed to be the annealed mild steel discussed in chapter 3. Although conducting a similar parametric study using aluminium tubes would be beneficial, the focus of the study is to gain an understanding of the *geometric* panel variables and their influence on absorption performance. This can be accomplished to a reasonable degree using only a single material model. The annealed mild steel is ideal for these purposes as it has the highest impulse capacity. Further to this, results, if any, relating to rate sensitivity can be obtained using the mild steel. As the mild steel is annealed according to the procedure described in section 3.1, the variation in work hardening resulting from variation in tube aspect ratio can also be neglected. This is clearly desirable when examining a large range of tube geometries.

The blast load is assumed to be a 50 Ns square pulse with a $10 \mu s$ duration. The limitation of examining only a single load is a requirement of the computational complexity of the problem. The load is carefully chosen according to the certain requirements. The load must be large enough to induce compaction for panel designs which are expected to have a small impulse capacity. This is a requirement as it is of interest to examine the effect panel compaction has on various panel efficiency measures. The load must not be too large however, so as to excessively deform the weak panels to the point where numerical instabilities develop. The impulse must also be large enough to deform panels with large

impulse capacity to the point where meaningful quantitative measures can be extracted.

7.1.3 Parametric Study Formulation

The purpose of this study is to investigate the influence of various geometric panel design variables on the absorption performance of the panels. Ideally, a complete factorial design study that simultaneously varies all the design parameters described in section 7.1.1 is carried out. The proposed problem is highly non-linear and thus requires significant computational time, and hence a full factorial study is not practical. In order to obtain an accurate characterization of panel performance in design space, a simplified method is used.

Firstly, the panel layout, in terms of tube position, is varied according to the values of λ in Table 7.1. For these simulations, the plate thickness is fixed at 3.0 mm while the tube thickness is fixed at 0.6 mm. The constant top plate thickness is chosen as it expected to be thick enough to not deform excessively under standard loading conditions, however it is not so thick that it removes any interaction effects with the crushing modes of the core tubes. The constant tube thickness is chosen as it is the mean wall thickness and corresponds to the nominal tube thickness used in the experiments of chapter 5. The tube aspect ratio is also fixed at $R = 4$ as this corresponds to a slightly more slender tube than the tubes investigated experimentally.

Once this phase is completed, the value of ξ_{st} is computed for each panel design. The values of λ corresponding to the maximum, median and minimum value of ξ_{st} are identified for each panel layout (A, B , and C). Note that the design with minimum ξ_{st} is considered to be the most stable. Panels, with tube positions corresponding to each of these λ values, in addition to values lying on the edge of the design boundary are then subjected to variation in tube geometry and top plate geometry. The purpose of this is to determine if the panel design variables can improve/reduce stability and performance in panels with poorly positioned tubes.

In the design study, the tube thickness (h_t) is varied while tube aspect ratio is fixed at $R = 4$ and the top plate thickness is fixed at $h_p = 3.0$ mm. Tube aspect ratio is then varied with the remaining design variables fixed at $h_t = 0.6$ mm and $h_p = 3.0$ mm. Finally, h_p is varied while tube geometric parameters are fixed at $h_t = 0.6$ mm and $R = 4$. The procedure is outlined in Fig. 7.1. Using this methodology, combined with appropriate interpolation provides the performance of the panels at ‘slices’ of the design space.

In addition to variation of panel design parameters at different tube positions, a further study on the influence of tube geometry is conducted. For this set of numerical investi-

gations, only the optimal tube layout is examined, while tube thickness and aspect ratio are varied. This is an interesting study as it allows for a clear determination of how geometric tube parameters influence panel performance assuming the tubes are placed in an optimal or near-optimal position.

The parametric study presented here conducts numerical simulations of the response of 273 unique panel geometries under a fixed blast loading. Although this is not as robust of a methodology as using a mathematical optimisation algorithm, it is deemed to be a sufficient quantity of numerical tests to observe trends in the response and efficiency of the panels under the given blast load.

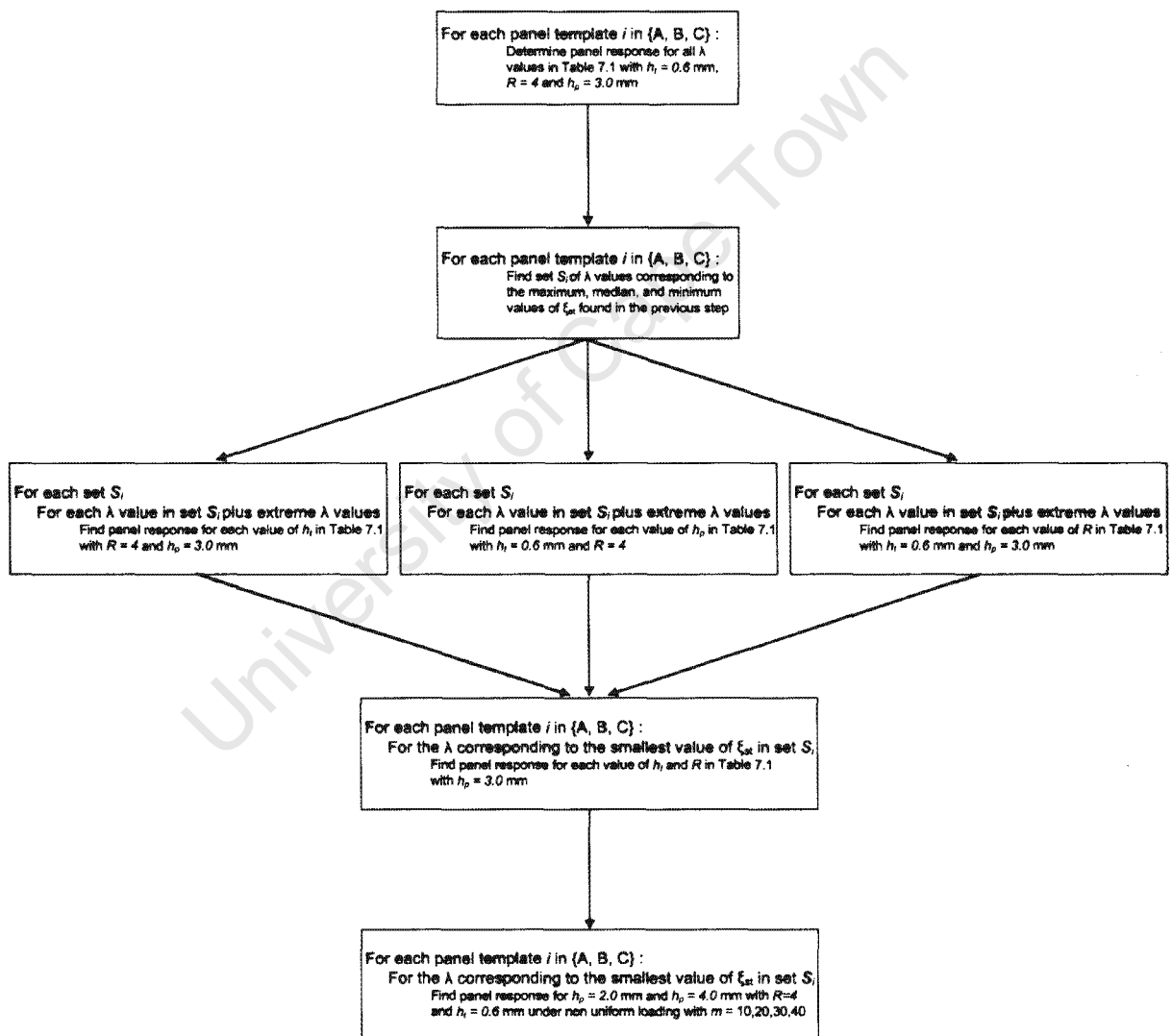


Figure 7.1: Flowchart outlining the parametric design study procedure.

7.2 Design Study Results

7.2.1 Panel Layout

The positions of the tubes within a panel are varied according to the λ values presented in Table. 7.1. The variation in core efficiency ξ_{core} and the stability parameter ξ_{st} as a function of λ is given in Figs. 7.2a and 7.2b respectively. Tables 7.3-7.5 outline the quantitative measures for each tube obtained from variation in tube placement.

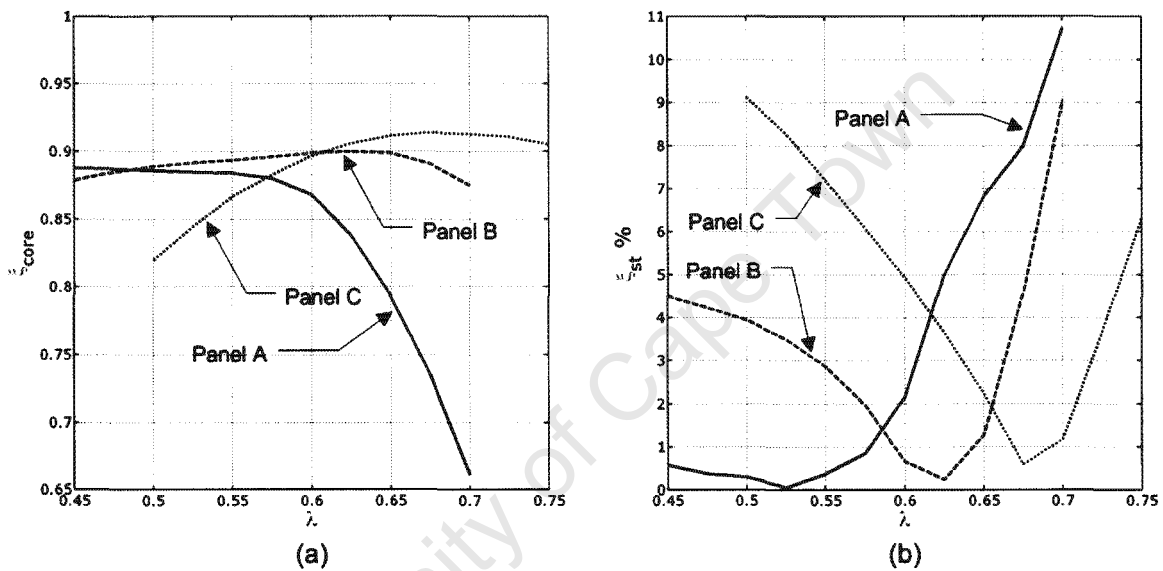


Figure 7.2: (a) Plot of variation in core efficiency as a function of λ for panels A, B, and C. (b) Plot of variation in the stability parameter as a function of λ for panels A, B, and C.

It is clear from Fig. 7.2a that placement of the tubes has a strong influence on the ratio of kinetic energy absorbed in the core to that of the top plate. This is shown to be particularly true for panel A designs. For panel A layouts, the core efficiency is found to plateau for small values of λ and then drop significantly when the tubes become further from the panel centre. This is a result of large dishing at the panel centre inducing severe mode J_a buckling in the panel tubes. For large values of λ , the position of the tube simulates a clamped boundary, effectively increasing the exposed area of the top plate. This results in larger deformations at the panel centre. From a tube stability perspective, the top plate would ideally be rigid, resulting in zero bending applied at the top of the tube, producing symmetric progressive buckling. Under actual blast conditions the top plate deforms and applies a bending moment to the proximal end of the tubes. These plate interaction effects implicitly define an oblique loading problem as studied by Borvik *et al.* [61] and Reyes *et al.* [63] where the authors found that with even a small load angle,

global buckling occurs. This trend concurs with the buckling sensitivities found in this study.

Fig. 7.3 illustrates the influence of tube placement on the development of strains in the top plate. It is clear that for optimal values of λ , the equivalent plastic strain state of the top plate is largely unchanged between $t = 0.2$ ms and $t = 1.0$ ms for all panel layouts. For unstable tube layouts, large strains develop in the top plate throughout the absorption process.

At smaller values of λ , using a panel *A* layout, the relatively thick top plate minimizes deformations near the plate corners resulting in no apparent loss of efficiency. Fig. 7.2b indicates a clear relationship between core efficiency and panel stability in that as the stability parameter increases (i.e. stability decreases), the core efficiency decreases. Stability is found to be maximised using a panel *A* layout with $\lambda = 0.525$. The values of λ corresponding to maximum, median, and minimum stability are given in Table 7.2 for panels *A*, *B*, and *C*.

Table 7.2: Tube design parameters corresponding to maximum, median, and minimum stability under variation of λ .

Panel	Design _{max}	Design _{med}	Design _{min}
<i>A</i>	<i>A</i> - 0.525 - 0.6 - 3.0 - 4.0	<i>A</i> - 0.575 - 0.6 - 3.0 - 4.0	<i>A</i> - 0.700 - 0.6 - 3.0 - 4.0
<i>B</i>	<i>B</i> - 0.625 - 0.6 - 3.0 - 4.0	<i>B</i> - 0.525 - 0.6 - 3.0 - 4.0	<i>B</i> - 0.700 - 0.6 - 3.0 - 4.0
<i>C</i>	<i>C</i> - 0.675 - 0.6 - 3.0 - 4.0	<i>C</i> - 0.600 - 0.6 - 3.0 - 4.0	<i>C</i> - 0.500 - 0.6 - 3.0 - 4.0

Similar behaviour is observed for panel *B*. Stability and core efficiency improves up to the optimal tube position with the corresponding value of $\lambda = 0.625$. Beyond this point, both measures decrease as larger plate deformations occur near the centre of the panel. The loss of efficiency is clearly much less severe than in the panel *A* case. It is interesting to examine the quantitative measures presented in Table 7.4, particularly the relationship between core efficiency and the crush distances of the individual tubes in the panel. At small values of λ , the crush distance of the outermost tube is somewhat larger than in the centre tube. As λ increases to the optimal value, the crush distance ratio approaches unity. Beyond the λ value corresponding to optimal core efficiency, the crush distance at the centre tube exceeds that of the outer tube. This indicates that maximising the panel efficiency requires nearly equal distribution of the energy absorbing responsibility to the tubes composing the panel core.

Variation in λ for panel *C* shows a similar pattern to that of panel *B*. For small values of λ , the large stiffness near the central area of the panel results in large deformations initiating at the edges of the top plate, before the onset of buckling in the core. As λ increases, top plate deformation decreases to higher rigidity at the outer edges of the top plate. The optimal value of λ , in terms of core efficiency and stability is found to occur

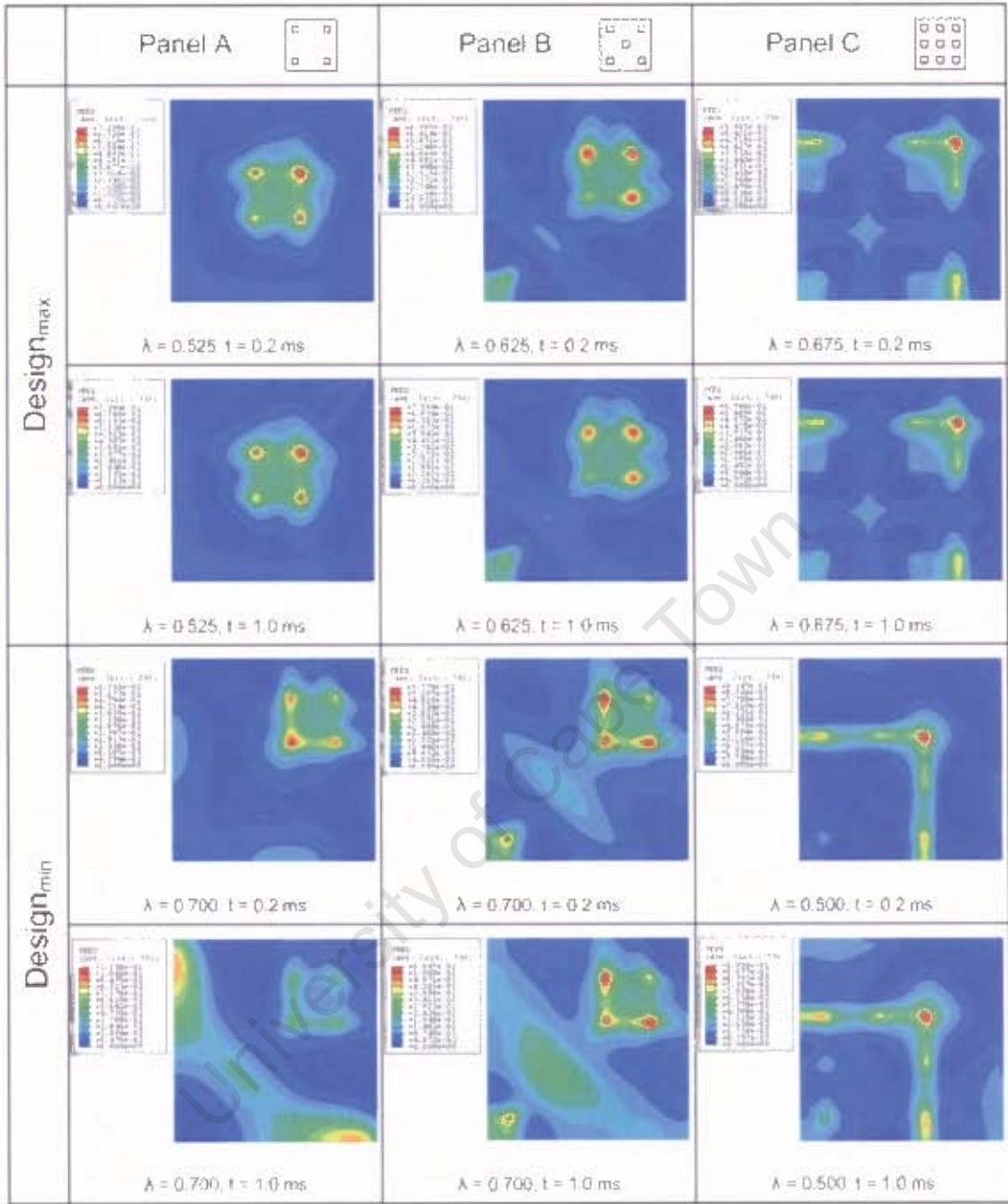


Figure 7.3: Contour plots of the equivalent plastic strain fields in the quarter symmetry model of the panel top plates, at different time increments, for optimal and sub-optimal tube placements as defined in Table 7.2. The strain fields are plotted on the undeformed shape of the top plate.

at $\lambda = 0.675$. Examination of the crush distances in Table 7.5 again indicate that core energy absorption efficiency is maximised when each tube in the core has nearly equal crush distance.

The parametric study presented here allows for further understanding of the energy absorption properties of the proposed panels, beyond that of the experimental investigations in chapter 5. Figs. 7.4a-c gives the partitioning of the dissipated kinetic energy between

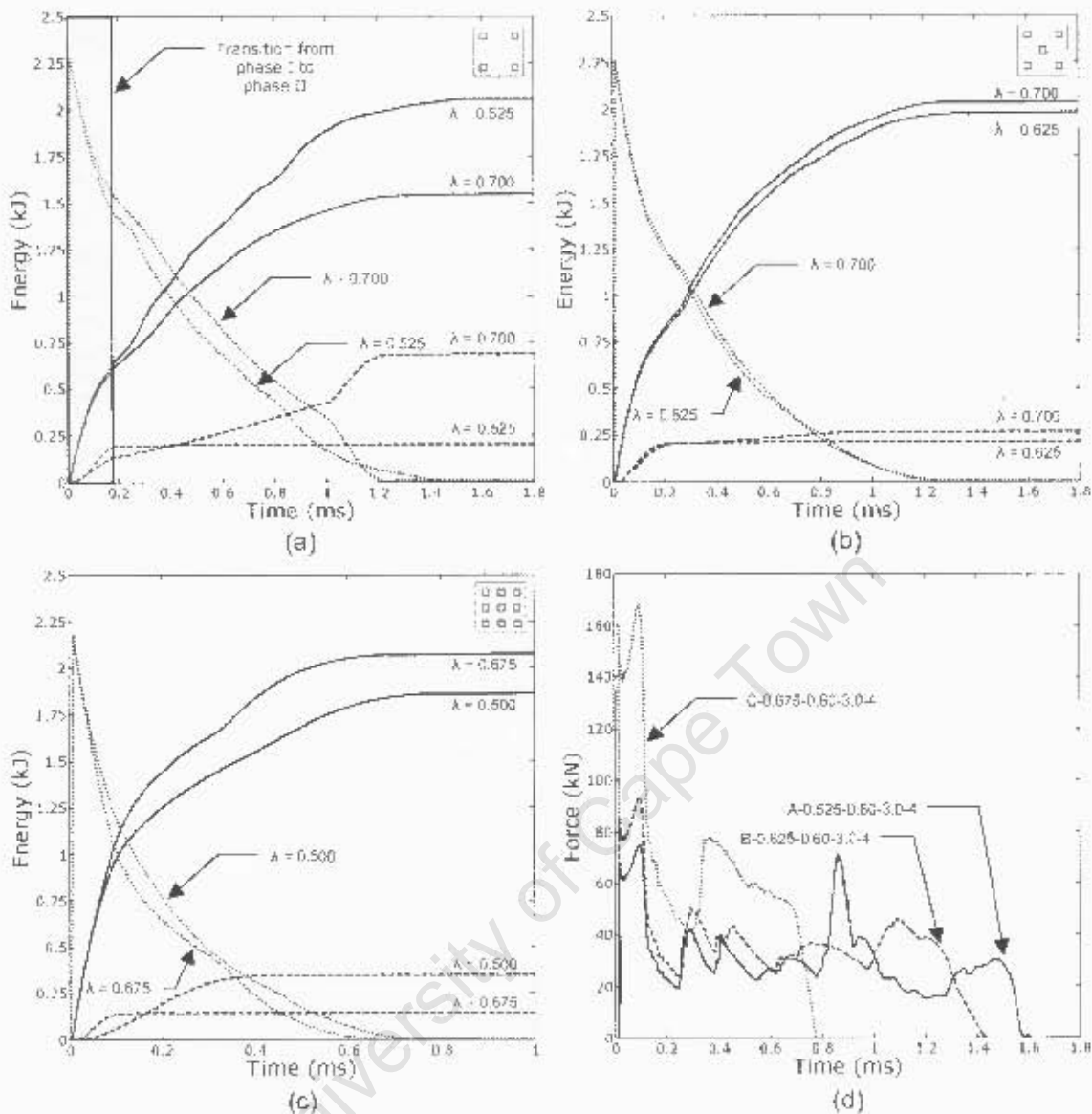


Figure 7.4: Partitioning of dissipated kinetic energy between the core tubes and the top plate for (a) Panel A (b) Panel B and (c) Panel C. In Figs. (a) - (c) — core energy absorption --- top plate energy absorption ... kinetic energy remaining to be dissipated. (d) Plots of force-time behaviour for the optimal tube positions for panels A, B, and C. The transition from phase I to II is given in (a).

It is clear from Figs. 7.4a-c that the deformations, and hence the plastic energy dissipation behaviour in the top plate at the end of phase I significantly influences the overall panel performance. Specifically, if the top plate deformation exceeds a certain threshold, symmetric progressive tube crushing cannot initiate and overall panel performance is reduced significantly, in terms of core energy dissipation. To illustrate the effect of panel stability on the resulting core crushing mode, axial depictions of the panels cores after dissipation of the blast load are shown in Fig. 7.5. It is clear from this figure that panel A designs are most sensitive to top plate interaction effects with the core tubes. For panels

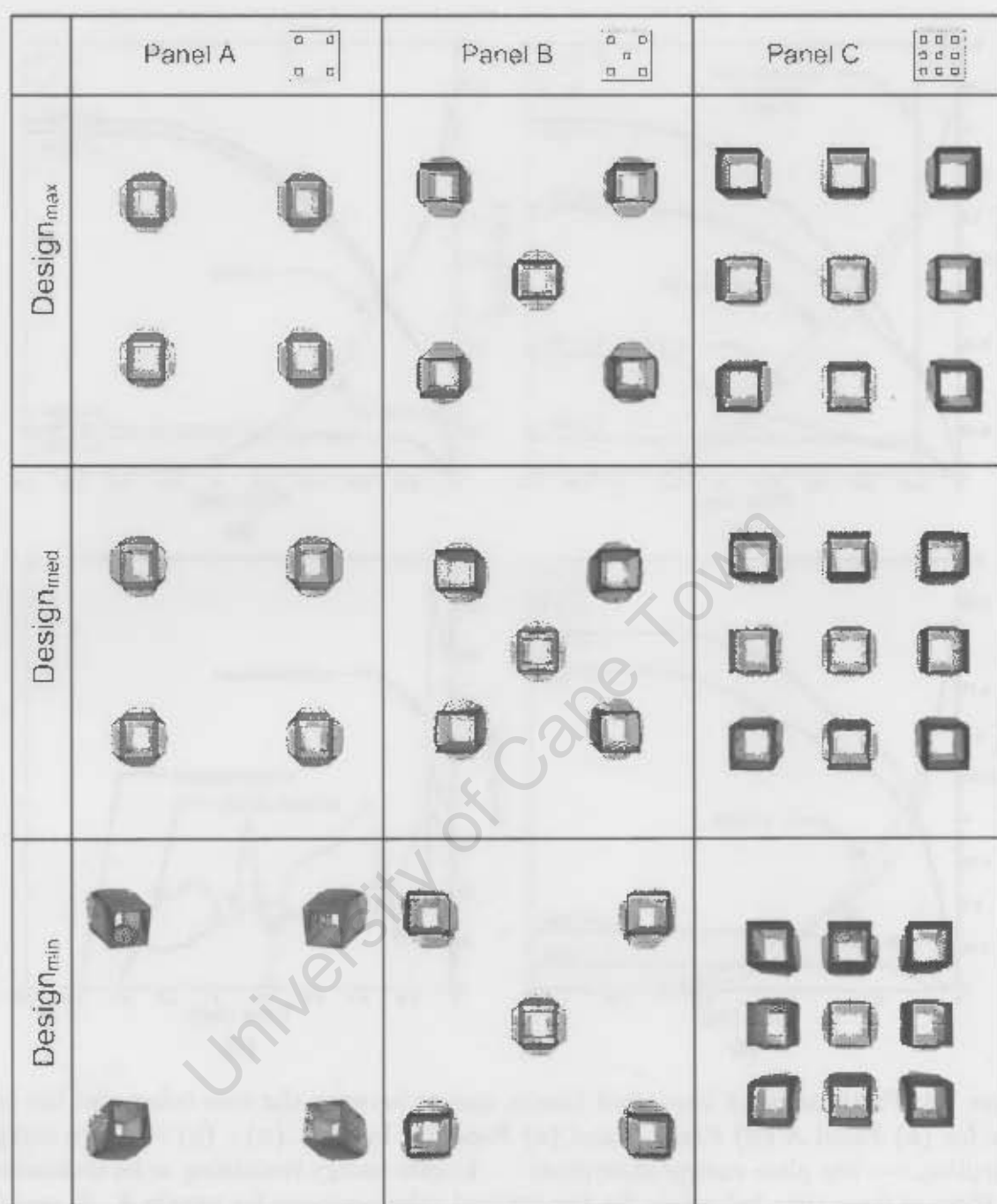


Figure 7.5: Axial view of panel cores with λ values corresponding to the minimum, median, and maximum performance for each panel layout.

B and *C*, even the designs corresponding to the most inefficient tube placements show relatively little loss of stability while the tube in panel *A* undergo severe global (mode J) buckling.

Table 7.3: Individual tube response, in terms of the measures presented in section 4.6, for panel *A* as a function of λ . For all designs, $h_t = 0.60$ mm, $R = 4.0$, and $h_p = 3.0$ mm. Mean forces are presented in terms of individual tubes and hence must be scaled to obtain the total mean force for the panel.

<i>Panel</i> – λ	δ (mm)	F_m (kN)	ξ_{total}	Crush mode	ξ_{core}
<i>A</i> – 0.450	57.28	9.00	0.366	I	0.888
<i>A</i> – 0.475	57.52	8.96	0.366	I	0.887
<i>A</i> – 0.500	57.70	8.92	0.366	I	0.886
<i>A</i> – 0.525	57.65	8.91	0.366	I	0.885
<i>A</i> – 0.550	57.70	8.89	0.366	I	0.884
<i>A</i> – 0.575	57.31	8.91	0.364	I	0.880
<i>A</i> – 0.600	56.61	8.90	0.360	I/J _a	0.868
<i>A</i> – 0.625	55.41	8.78	0.347	I/J _a	0.838
<i>A</i> – 0.650	53.73	8.56	0.329	J _a	0.793
<i>A</i> – 0.675	50.89	8.39	0.305	J _a	0.736
<i>A</i> – 0.700	46.09	8.33	0.275	J _a	0.661

Table 7.4: Individual tube response, in terms of the measures presented in section 4.6, for panel *B* as a function of λ . For all designs, $h_t = 0.60$ mm, $R = 4.0$, and $h_p = 3.0$ mm. Mean forces are presented in terms of individual tubes and hence must be scaled to obtain the total mean force for the panel.

<i>Panel</i> – λ	δ (mm)		F_m (kN)		ξ_{total}		Crush mode		ξ_{core}
tube	1	2	1	2	1	2	1	2	
<i>B</i> – 0.450	45.73	41.13	8.85	8.94	0.287	0.260	J _b /K _a	I	0.860
<i>B</i> – 0.475	46.03	41.52	8.85	8.95	0.289	0.263	J _a /K _a	I	0.866
<i>B</i> – 0.500	46.17	41.83	8.85	8.96	0.290	0.265	J _a /K _a	I	0.869
<i>B</i> – 0.525	46.18	42.22	8.86	8.97	0.291	0.268	K _a	I	0.872
<i>B</i> – 0.550	46.18	42.82	8.85	8.97	0.291	0.272	I/K _a	I	0.874
<i>B</i> – 0.575	46.16	43.77	8.84	8.98	0.291	0.278	I/K _a	I	0.876
<i>B</i> – 0.600	46.18	45.29	8.81	8.94	0.290	0.287	I	I	0.879
<i>B</i> – 0.625	46.18	46.13	8.78	8.94	0.289	0.292	I	I	0.880
<i>B</i> – 0.650	45.81	48.16	8.76	9.03	0.287	0.302	I/K _b	I	0.879
<i>B</i> – 0.675	44.64	47.21	8.72	9.14	0.278	0.324	I/K _b	I	0.871
<i>B</i> – 0.700	42.74	49.97	8.68	9.21	0.265	0.350	J _a /K _a	I	0.855

Table 7.5: Individual tube response, in terms of the measures presented in section 4.6, for panel C as a function of λ . For all designs, $h_t = 0.60$ mm, $R = 4.0$, and $h_p = 3.0$ mm. Mean forces are presented in terms of individual tubes and hence must be scaled to obtain the total mean force for the panel.

$Panel - \lambda$	δ (mm)		F_m (kN)		ξ_{total}		Crush mode		ξ_{core}
tube	1	2	1	2	1	2	1	2	
$C - 0.500$	22.61	17.45	9.79	10.14	0.120	0.128	J_b	I	0.820
$C - 0.525$	22.84	17.88	9.87	10.43	0.127	0.133	J_b	I	0.845
$C - 0.550$	23.03	18.43	9.95	10.63	0.134	0.139	J_b/K_a	I	0.867
$C - 0.575$	23.05	19.00	10.03	10.76	0.140	0.145	J_b	I	0.883
$C - 0.600$	22.96	19.61	10.13	10.81	0.145	0.150	J_b/K_a	I	0.897
$C - 0.625$	22.77	20.29	10.22	10.80	0.148	0.155	I/K_a	I	0.906
$C - 0.650$	22.49	21.14	10.30	10.72	0.148	0.160	I	I	0.912
$C - 0.675$	22.26	22.01	10.27	10.62	0.142	0.165	I	I	0.914
$C - 0.700$	21.91	22.76	10.28	10.55	0.142	0.170	I	I	0.913
$C - 0.725$	21.23	23.84	10.33	10.44	0.142	0.176	I	I	0.911
$C - 0.750$	20.70	25.19	10.17	10.37	0.142	0.185	I/K_b	I	0.905

$Panel - \lambda$	δ (mm)		F_m (kN)		ξ_{total}		Crush mode		ξ_{core}
tube	3	4	3	4	3	4	3	4	
$C - 0.500$	20.66	20.75	9.73	9.53	0.145	0.143	J_b	J_b	0.820
$C - 0.525$	20.93	20.98	9.97	9.77	0.150	0.147	J_b	J_b	0.845
$C - 0.550$	21.19	21.20	10.11	9.99	0.153	0.151	J_b/K_a	J_b/K_a	0.867
$C - 0.575$	21.47	21.39	10.20	10.18	0.156	0.155	J_b	J_b	0.883
$C - 0.600$	21.67	21.47	10.30	10.33	0.159	0.158	J_b/K_a	J_b/K_a	0.897
$C - 0.625$	21.82	21.54	10.39	10.45	0.161	0.160	I/K_a	I/K_a	0.906
$C - 0.650$	22.03	21.67	10.43	10.53	0.163	0.162	I	I	0.912
$C - 0.675$	22.24	22.03	10.41	10.46	0.164	0.164	I	I	0.914
$C - 0.700$	22.33	22.40	10.40	10.39	0.165	0.165	I	I	0.913
$C - 0.725$	22.61	22.64	10.35	10.41	0.166	0.168	I	I	0.911
$C - 0.750$	23.27	22.93	10.21	10.32	0.169	0.169	I/K_b	I/K_b	0.905

7.2.2 Tube Geometry Effects

As described in section 7.1.3, two distinct parametric studies are conducted regarding the influence of tube geometry on panel performance. Firstly, for each panel layout (*A*, *B*, and *C*), and for each value of λ given in Table 7.2, plus the extreme values of λ (0.450, 0.700), the response of the panel is determined with varying tube thickness and then with varying aspect ratio. A second parametric study is then conducted where the tube thickness and aspect ratio are simultaneously varied, however the value of λ is fixed at the optimal value for each panel layout.

Response surfaces plots for core efficiency and panel stability under variation in tube thickness and placement are given in Fig. 7.6. For all response surfaces presented in this chapter, the surfaces are created using cubic spline interpolation in Matlab. To supplement the response surfaces, numerical values of energy absorption and efficiency parameters, as a function of tube thickness, for the optimal tube positions are given in Table 7.6.

For panel *A* designs, core efficiency and panel stability is found to decrease with increasing tube thickness for all tube placements. This is found to be particularly true for the unstable positioning corresponding to $\lambda = 0.700$. As the tube thickness increases, the initiation of tube core buckling is delayed which allows for larger top plate deformation during phase I of panel motion. For tube placements which are unstable at the mean thickness $h_t = 0.6$, the instability is found to be increased when increasing the tube thickness. This is an important consideration in the design of the proposed panels as theoretical panel capacity is clearly maximised with thick tubes, however the global stability of the panels may be jeopardised as a result. The influence of tube thickness on top plate interaction effects was found to be more significant in the earlier parametric study given in Theobald and Nurick [125]. In the aforementioned study however, the material model used for the tubes corresponded to a steel with higher yield stress and larger strain-rate sensitivity. The top plate used in the study was also slightly thinner ($h_p = 2.5$ mm). Contrary to this trend is the reduced core efficiency for very thin tubes with panel *A*, even at the most stable tube placement. This is a result of tube core compaction necessitating the top plate to deform within phase II of panel motion, to dissipate the kinetic energy remaining in the panel at the time of compaction.

In panels *B* and *C*, core efficiency and panel stability are found to increase with decreasing tube thickness. This decrease in core efficiency is found to be most significant for panel *C* designs with λ values corresponding to tube positions close to the centre of the panel. The excessively large stiffness near the centre of the panel allows the free edges of the top plate to undergo large deformations before the onset of buckling. Also of note is that for all panel *B* and *C* designs, core efficiency is a strictly increasing function of tube

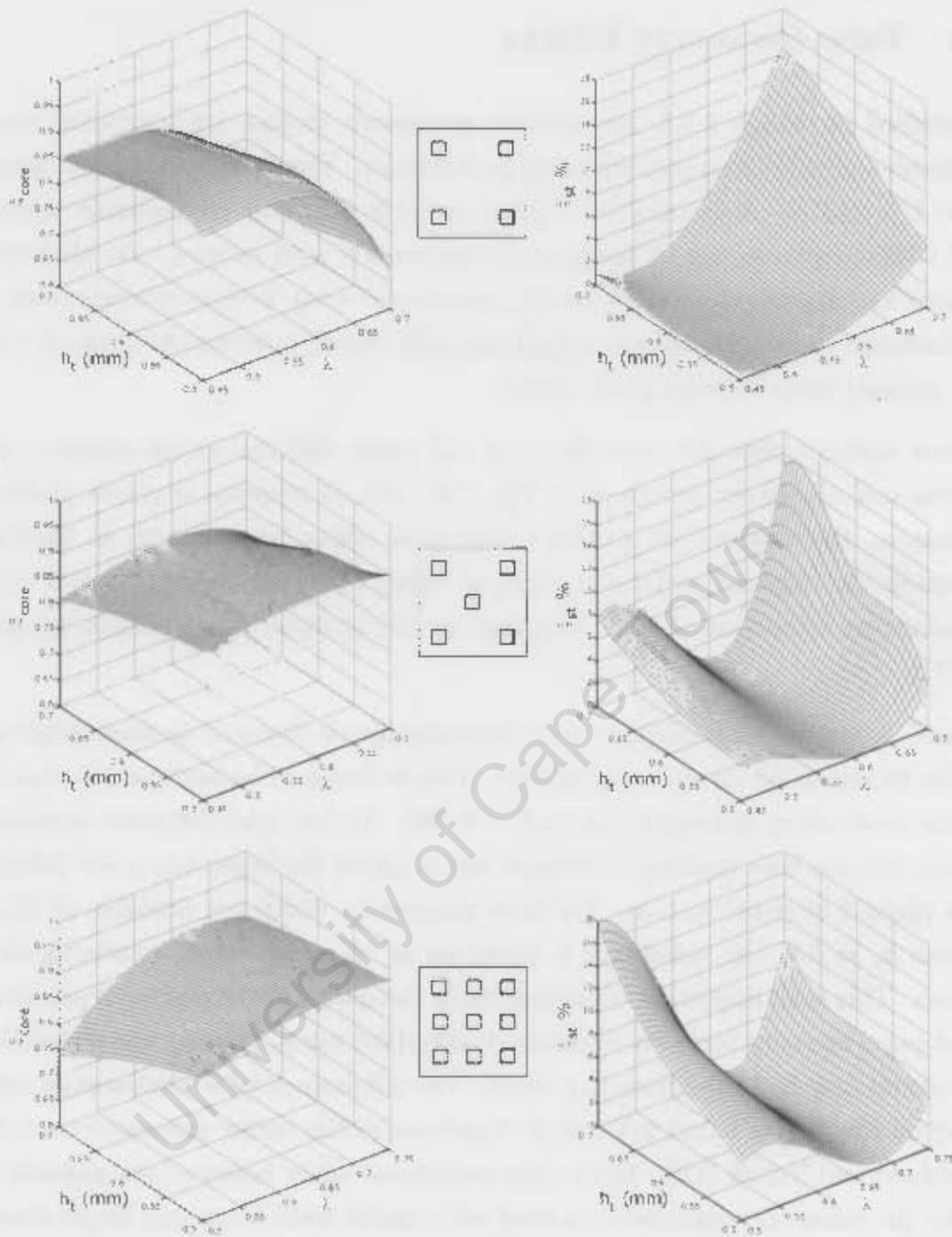


Figure 7.6: Plots of core efficiency and the stability parameter as a function of λ and tube thickness h_t for panel layouts A, B, and C.

thickness as compaction does not occur in any of the designs.

Examination of Table 7.6 shows that for all three panel layouts (A, B, C) that the peak and mean crushing force of the panel increases significantly with increasing tube thickness. For panel A, a general increase in crush force efficiency is observed with increasing tube thickness. The crush force efficiency is artificially high in test A – 0.525 – 0.6 – 3.0 – 4 however, due to core compaction increasing the mean force. The increase in ξ_{crush} with increasing tube thickness is found to be more significant in panels B and C.

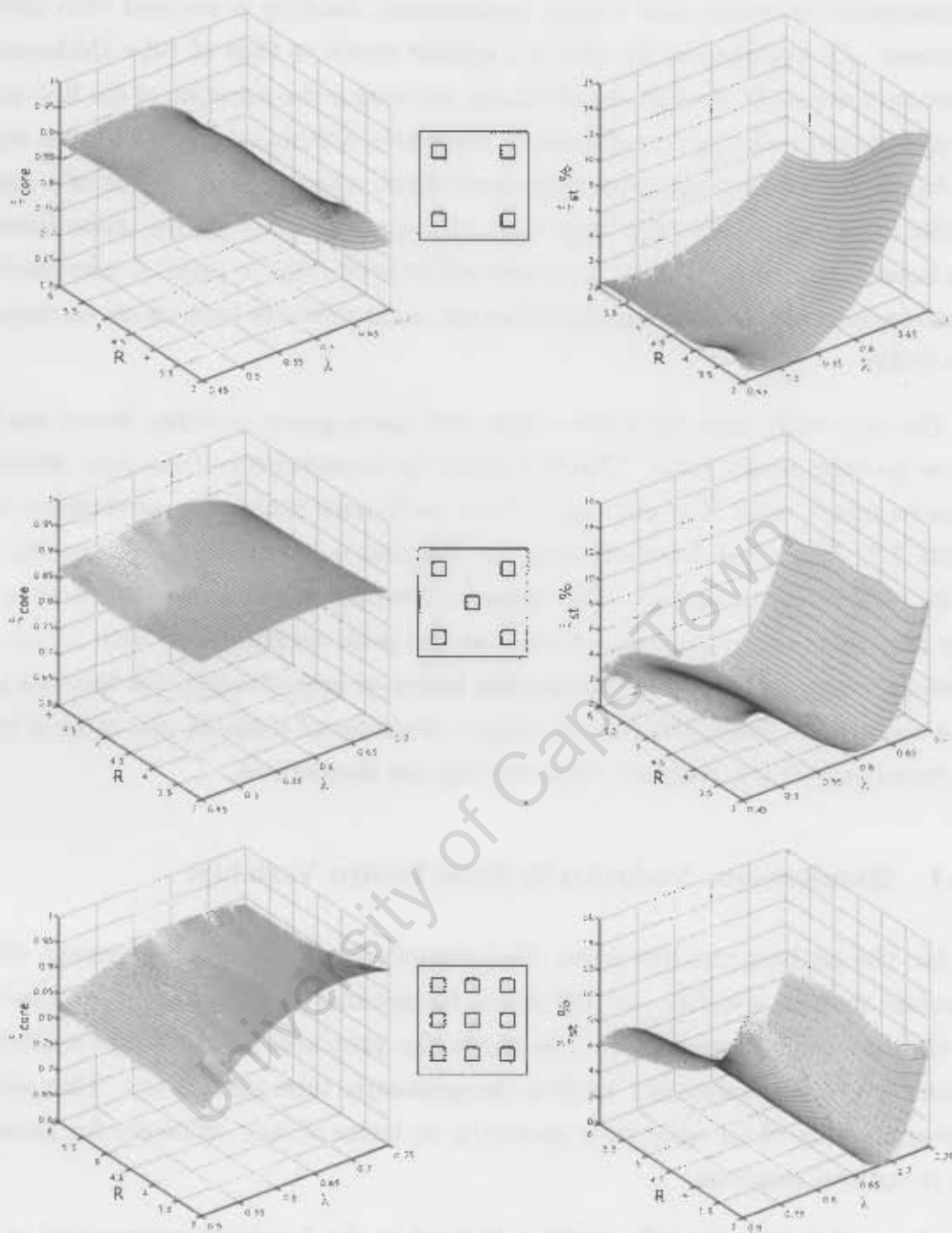


Figure 7.7: Plots of core efficiency and the stability parameter as a function of λ and tube aspect ratio R , for panel layouts A, B, and C.

For panel A, B, and C designs, core efficiency is found to be nearly invariant with respect to changes in the tube aspect ratio R . As the aspect ratio increases, the mean crushing force is found to decrease, however the total crush distance increases resulting in nearly identical core energy absorption in all test cases. For stable tube layouts the stability parameter is actually found to slightly increase with decreasing tube aspect ratio in panel A. This however, appears to be simply a result of the increased initial node spacing at the tube corners. This increase in stability with increasing aspect ratio is somewhat

counterintuitive as under most impact applications, stability is reduced with increasing slenderness. An explanation for this is a similar result to that of tube thickness. The core becomes over-stiff through the thickness, increasing the duration of the first phase of panel motion and reducing core efficiency. Therefore, although individual tubes are more prone to global buckling with increased slenderness, when applied as part of a sandwich core, the increased aspect ratio may have the opposite effect as top plate interaction effects become more pronounced. It is observed however, that in panel *A*, core efficiency is reduced significantly with increased slenderness at the unstable tube layout corresponding to $\lambda = 0.700$.

As is the case with tube thickness, peak and mean panel crushing forces are highly sensitive to tube aspect ratio. This is despite the insensitivity of the core efficiency to changes in aspect ratio. The increase in force levels with decreasing aspect ratio is given in Table 7.7. The crush force efficiency is observed to increase with increasing aspect ratio for all three panel layouts. This measure however, is sensitive to the scaling of the trigger dimensions with tube aspect ratio as the peak buckling load with increase with less slender tubes, however if the trigger size increases appropriately, the increase in force can be controlled. Using the current trigger diameter of $0.25l/R$, the ratio of peak to mean crush force is not constant when varying the slenderness.

7.2.2.1 Simultaneous Variation in Tube Design Variables

Thus far, the analysis and discussion has primarily focused on tube geometry effects as they relate to tube stability and efficiency, for optimal and poor tube positions. It is of further interest to numerically investigate the proposed panels at the optimal tube positions, while simultaneously varying the geometric tube parameters. This allows for the determination of the ideal tube geometry, in terms of core efficiency, for panels with tubes in optimal positions.

For each panel layout, the tube position is fixed at the λ value corresponding to design $\text{Design}_{\max}$ indicated in Table 7.2. For each of the tube thicknesses in Table 7.1, the tube aspect ratio is varied according to the values given in Table 7.1. Contour plots of the result for panels *A*, *B*, and *C* are given in Figs. 7.8a-c respectively.

For panel *A* designs, the response in tube designs space can be partitioned into three approximate regions. The boundaries of these regions are indicated in Fig. 7.8a by the dashed lines. The area between the dashed lines indicates tube dimensions which result in a near plateau of highly efficient response. Tube dimensions resulting from a combination of low wall thickness and large aspect ratio result in a core that is unable to dissipate the energy introduced by the 50 Ns blast. This is indicated in Fig. 7.8a

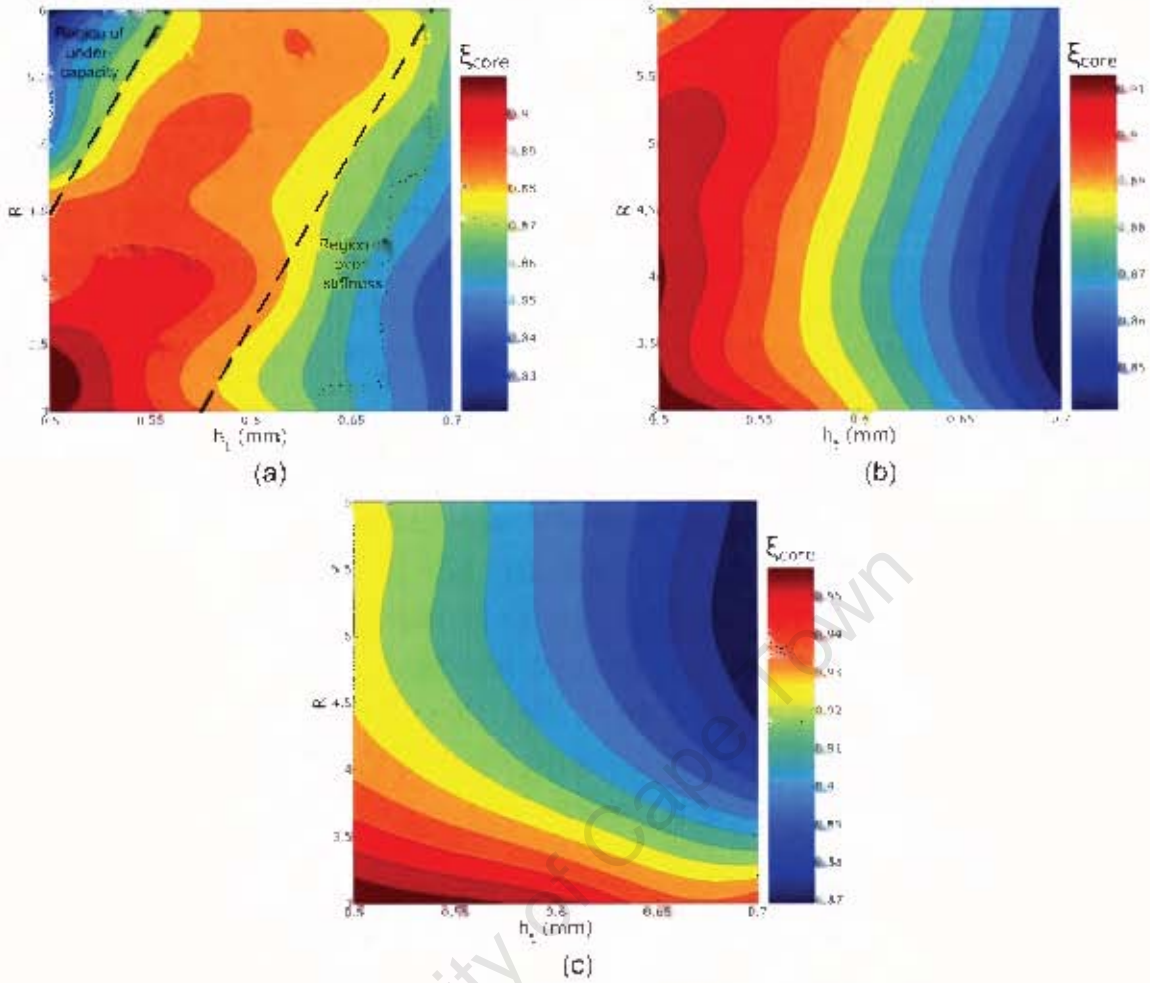


Figure 7.8: Contour plots of core efficiency as a function of tube thickness and aspect ratio for (a) Panel A ($\lambda = 0.525$) (b) Panel B ($\lambda = 0.625$) and (c) Panel C ($\lambda = 0.675$). In all numerical models, the top plate thickness is fixed at $h_p = 3.0$ mm.

as the "region of under-capacity". As previously described, core efficiency is reduced as core compaction requires the top plate to dissipate the kinetic energy remaining in the panel at compaction. Performance is found to be maximised when the crush distance is very near the point of compaction. In particular, core energy absorption is found to peak with tube geometry parameters $R = 3.0$, $h_t = 0.5$ mm. With this tube geometry the core efficiency is found to be $\xi_{core} = 0.912$.

Although energy absorption is maximised when the core is relatively weak, tube geometries which result in large peak and mean force levels can reduce core efficiency. Such behaviour is observed within the region indicated by "region of over-stiffness" in Fig. 7.8a. For tube geometries within this region of design space, buckling loads increase to the point where top plate deformation in the top plate during phase 1 of panel motion becomes significant. Although the ratio of core to top plate energy absorption is reduced in this region, the capacity of the panels is greatly increased. This however must be

balanced with the limitations on the allowable reaction force levels at the base of the panel.

Panel *B* shows similar global behaviour to panel *A*, however the region in tube design space resulting in core compaction is significantly reduced. Specifically, only for combinations of the lowest tube thickness and largest aspect ratio are compaction effects on the core efficiency observed. The plateau of nearly equal core absorption found in panel *A* is not observed in panel *B*. The core efficiency is found to reduce significantly in panel designs not near the compaction boundary. Similarly to panel *A*, the optimal panel *B* tube geometry occurs at $R = 3.0$, $h_t = 0.5$ mm with a core efficiency of 0.919.

Compaction does not occur for any tube geometry in panel *C*. As a result of this, core efficiency is found to be a decreasing function of aspect ratio for any tube thickness. Core energy absorption is again maximised at $R = 3.0$, $h_t = 0.5$ mm with a core efficiency of 0.965. This indicates that core efficiency reaches its highest value using a panel *C* tube layout. With the small aspect ratio ($R = 3.0$) the top plate is sufficiently constrained during phase I of panel motion, resulting in very little plate deformation. The thin tube wall thickness causes buckling to initiate early, allowing the progressive crushing of the tubes to dominate the energy absorption process.

Table 7.6: Tube wall thickness effects on panel performance for the optimal values of λ in panels *A*, *B*, and *C*. For all designs, $R = 4.0$, and $h_p = 3.0$ mm.

Panel – λ – h_t	δ_m (mm)	$F_{\max}$ (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{score}
<i>A</i> – 0.525 – 0.50	66.44	60.95	31.26	0.513	0.454	0.892
<i>A</i> – 0.525 – 0.55	63.95	67.99	32.75	0.482	0.411	0.900
<i>A</i> – 0.525 – 0.60	57.65	74.89	35.66	0.476	0.366	0.885
<i>A</i> – 0.525 – 0.65	49.62	81.54	40.45	0.496	0.328	0.865
<i>A</i> – 0.525 – 0.70	43.31	87.90	45.11	0.513	0.296	0.844
<i>B</i> – 0.625 – 0.50	61.11	76.13	34.68	0.456	0.371	0.914
<i>B</i> – 0.625 – 0.55	52.78	84.91	39.45	0.465	0.327	0.899
<i>B</i> – 0.625 – 0.60	46.17	93.51	44.04	0.471	0.290	0.880
<i>B</i> – 0.625 – 0.65	39.01	101.79	50.93	0.500	0.260	0.861
<i>B</i> – 0.625 – 0.70	32.82	109.71	58.98	0.538	0.235	0.841
<i>C</i> – 0.675 – 0.50	32.48	137.15	65.79	0.480	0.208	0.936
<i>C</i> – 0.675 – 0.55	26.07	153.04	80.77	0.528	0.183	0.926
<i>C</i> – 0.675 – 0.60	22.18	168.62	93.41	0.554	0.164	0.914
<i>C</i> – 0.675 – 0.65	19.19	183.58	106.00	0.577	0.148	0.900
<i>C</i> – 0.675 – 0.70	16.48	197.84	121.00	0.612	0.134	0.885

Table 7.7: Tube aspect ratio effects on panel performance for the optimal values of λ in panels A , B , and C . For all designs, $h_t = 0.6$ mm, and $h_p = 3.0$ mm.

$Panel - \lambda - R$	δ_m (mm)	F_{max} (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{score}
$A - 0.525 - 3.0$	46.78	98.62	43.47	0.441	0.275	0.881
$A - 0.525 - 3.5$	52.09	85.74	39.18	0.457	0.317	0.881
$A - 0.525 - 4.0$	57.65	74.89	35.66	0.476	0.366	0.885
$A - 0.525 - 4.5$	59.94	66.05	34.58	0.524	0.418	0.891
$A - 0.525 - 5.0$	61.34	59.20	33.78	0.571	0.467	0.889
$A - 0.525 - 5.5$	62.60	54.00	33.17	0.614	0.513	0.889
$A - 0.525 - 6.0$	63.35	49.99	32.79	0.656	0.554	0.889
$B - 0.625 - 3.0$	35.82	123.36	56.89	0.461	0.220	0.889
$B - 0.625 - 3.5$	41.49	107.14	48.96	0.457	0.253	0.882
$B - 0.625 - 4.0$	46.17	93.51	44.04	0.471	0.290	0.880
$B - 0.625 - 4.5$	48.44	82.41	42.31	0.513	0.332	0.884
$B - 0.625 - 5.0$	51.89	73.86	39.72	0.538	0.372	0.888
$B - 0.625 - 5.5$	53.76	67.35	38.55	0.572	0.410	0.891
$B - 0.625 - 6.0$	55.81	62.35	37.44	0.600	0.447	0.897
$C - 0.675 - 3.0$	17.27	222.08	123.41	0.556	0.128	0.954
$C - 0.675 - 3.5$	19.83	193.19	105.75	0.547	0.145	0.931
$C - 0.675 - 4.0$	22.18	168.62	93.41	0.572	0.164	0.914
$C - 0.675 - 4.5$	24.27	148.58	84.94	0.577	0.185	0.905
$C - 0.675 - 5.0$	25.96	133.14	79.37	0.596	0.206	0.901
$C - 0.675 - 5.5$	28.09	121.39	73.55	0.606	0.227	0.900
$C - 0.675 - 6.0$	29.92	112.35	69.22	0.616	0.246	0.900

7.2.3 Top Plate Geometry Effects

A final geometric parameter investigation is performed into the influence of top plate thickness on the buckling and energy absorption properties of each panel. The panels with design parameters given in Table 7.2 are examined for the plate thicknesses given in Table 7.1. The thickness of the top plate is an important design variable as it greatly influences the mass of the panel as well as the total kinetic energy the panel needs to absorb. Fig. 7.9 gives the response surfaces in terms of core efficiency and panel stability as a function of λ and h_p , for each of the panel layouts. Table 7.8 gives numerical parameters relating to energy absorption for stable tube placements.

For panel *A* tube layouts, core efficiency is found to improve with increasing top plate thickness, for stable tube positions. This is an expected result as the thicker top plate will not undergo significant deformation during the onset of buckling in the core tubes. For unstable tube positions, the increase in core efficiency, and stability is found to be much more significant. This is exemplified by examining the response at the various λ values with $h_p = 4.0$ mm. For this top plate geometry, the difference between the maximum and minimum core efficiencies is found to be only 5.9 %. Further to this, the difference in maximum and minimum stability parameters is 3.5 %. This is a strong indication that a thick top plate may reduce interaction effects with the core tubes and initiates a symmetric, progressive buckling modes, whereas a thinner top plate induces global, mode J buckling for a given tube positioning. Conversely, thin top plates appear to enhance instability in the tubes in designs with λ values that result in instabilities at the nominal top plate thickness of 3.0 mm. This can be seen by examining the steep drop in core efficiency for thin top plates at values of λ near 0.700. It is also worthy of note that the crush force efficiency results in an opposing trend to the core efficiency as the increased velocities resulting from the thinner top plate significantly increase the mean force levels due to strain-rate effects, while having only a marginal effect on the peak crushing force of the panels.

Panels *B* and *C* show similar trends to that of panel *A*. Slight increases in core efficiency are observed with increasing top plate thickness, while panel designs which are found to be unstable in section 7.2.1, are highly sensitive to top plate thickness. This is shown to be particularly true in panel *B*, where significant reductions in core efficiency, coupled with large increases in the stability parameter, are observed for $\lambda \approx 0.500$ and $\lambda \approx 0.750$.

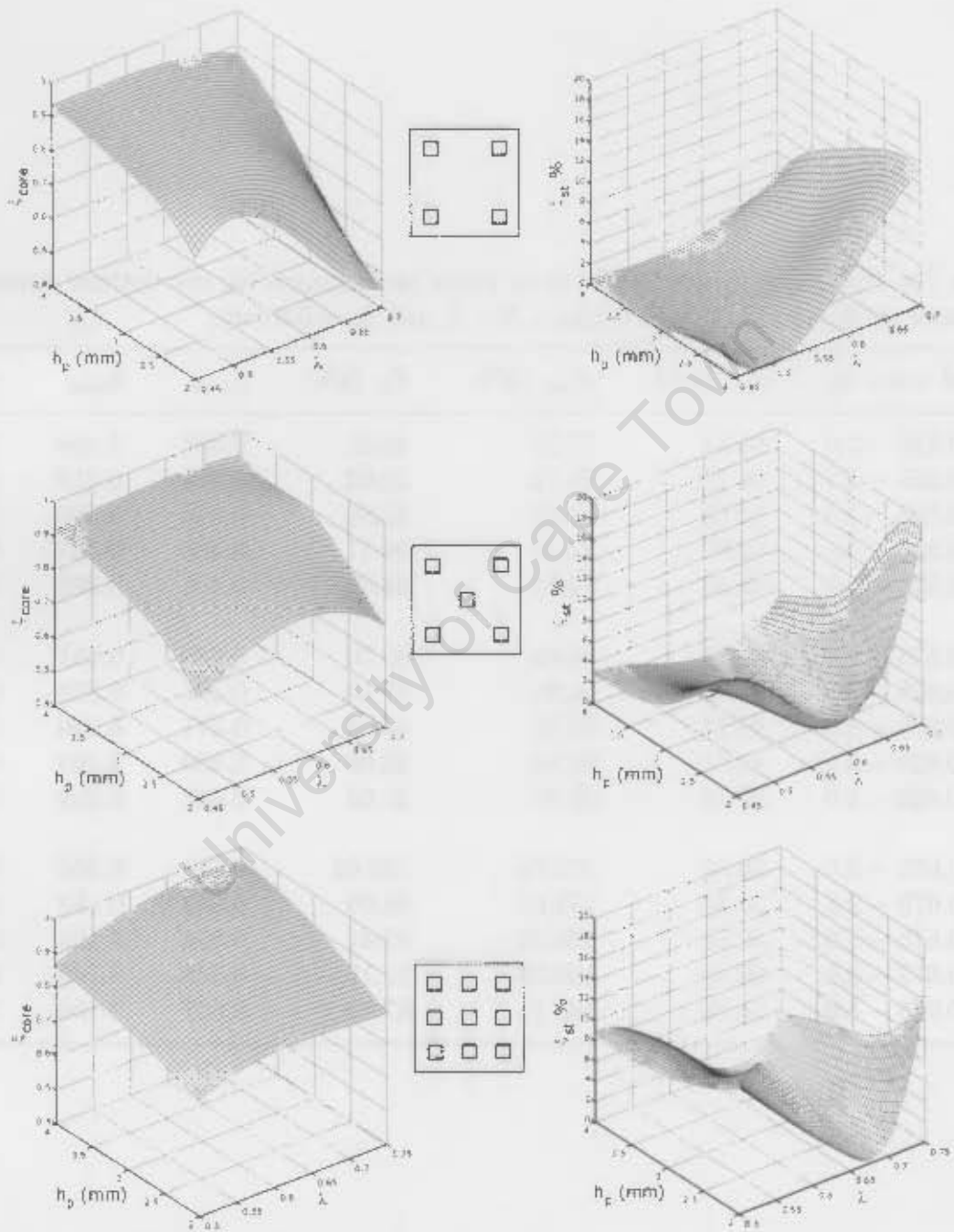


Figure 7.9: Plots of core efficiency and the stability parameter as a function of λ and top plate thickness h_p for panel layouts A, B, and C.

Table 7.8: Top plate thickness effects on panel performance for the optimal values of λ in panels A , B , and C . For all designs, $R = 4$, and $h_t = 0.6$ mm.

$Panel - \lambda - h_p$	δ_m (mm)	$F_{\max}$ (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{score}
$A - 0.525 - 2.0$	64.44	77.50	44.01	0.568	0.488	0.818
$A - 0.525 - 2.5$	61.29	76.12	39.03	0.513	0.419	0.860
$A - 0.525 - 3.0$	57.65	74.89	35.66	0.476	0.366	0.885
$A - 0.525 - 3.5$	53.07	73.70	34.11	0.463	0.327	0.908
$A - 0.525 - 4.0$	49.32	72.64	33.06	0.455	0.299	0.933
$B - 0.625 - 2.0$	54.52	96.65	50.70	0.525	0.381	0.802
$B - 0.625 - 2.5$	49.65	94.93	47.10	0.496	0.328	0.846
$B - 0.625 - 3.0$	46.17	93.51	44.04	0.471	0.290	0.880
$B - 0.625 - 3.5$	42.53	92.16	42.68	0.463	0.263	0.915
$B - 0.625 - 4.0$	39.16	90.91	41.68	0.458	0.239	0.938
$C - 0.675 - 2.0$	26.65	173.74	100.92	0.581	0.206	0.798
$C - 0.675 - 2.5$	24.23	171.01	96.09	0.562	0.182	0.860
$C - 0.675 - 3.0$	22.18	168.62	93.41	0.554	0.164	0.914
$C - 0.675 - 3.5$	20.32	166.29	91.15	0.548	0.148	0.950
$C - 0.675 - 4.0$	18.78	163.74	87.88	0.537	0.134	0.965

7.3 Non-Uniform Loading of Optimal Panels

Throughout this thesis, the significant assumption of uniform loading is assumed. Specifically, that the applied pressure pulse is uniform over the top plate and constant with time over the pulse duration t_0 . It is shown, that for the experimental investigations of chapter 5, this assumption is reasonable, however the loading rig is designed specifically to generate nearly uniform loads. The analysis of this chapter has focused on how the response of the proposed panels changes when the variables defining the structure of the panels is varied.

If the proposed panels are to be utilised in actual blast conditions, where the application of pressure loading is not controlled, it is of interest to understand how variations in loading uniformity influences panel response. The mathematical model for the non-uniform loading of the panels is presented in section 4.4.2. Under non-uniform loading, the blast load is defined by the total impulse I , the radius of uniform pressure a , the pulse duration t_0 , and the decay constant m . Once these parameters have been defined, the peak pressure P_0 is determined using Eq. 4.7.

For the analysis of this section, the impulse is fixed at $I = 50$ Ns. This is same impulse used throughout the parametric study, and hence allows for an isolation of the effects of non-uniformity as the total momentum transfer to the panels is same throughout. The radius of uniform pressure is fixed at $a = 40$ mm. As with all numerical simulations in this chapter, the pulse duration is $t_0 = 10$ μ s. The parameter which is allowed to vary is the decay constant m . The values used in this study are $m = 10, 20, 30, 40$. As the value of m increases, the pressure P_0 within the radius of constant pressure increases. As the impulse is identical for all values of m , this effectively shifts the pressure away from the top plate boundaries, to the centre of the panel. Fig. 7.10 gives the resulting pressure distribution for the m values investigated here. The pressure plot utilizes quarter-symmetry of the top plate.

Panel layouts A , B , and C are studied under non-uniform loading. The λ values used correspond to the optimal values given in Table 7.2. The goal of the investigation is to determine the degree to which efficiency and stability is reduced when increasing the non-uniformity of the loading on panels which have stable cores under uniform loading. As it is expected that the top plate thickness will have significant influence on the response to non-uniform loading, a thin and a thick top plate of thickness $h_p = 2.0$ mm and $h_p = 4.0$ mm respectively are used in the analysis. Finite element analysis are conducted for each layout (A , B , C), and for each top plate thickness, with loading decay parameters of $m = 10, 20, 30, 40$.

Fig. 7.11a gives the resulting variations in core efficiency as a function of the decay

Table 7.9: Individual tube response for the panels investigated under non-uniform loading. Note that in the table, W refers refers to the external work done on the entire panel by the non-uniform blast load.

$Panel - \lambda - h_p - m$	δ (mm)		F_m (kN)		V_0 (m/s)		Crush mode		W (kJ)
tube	1	2	1	2	1	2	1	2	
$A - 0.525 - 2.0 - 10$	57.19	—	9.23	—	110.71	—	I/ J_a	—	3.56
$A - 0.525 - 2.0 - 20$	54.32	—	9.73	—	111.35	—	J_a	—	3.81
$A - 0.525 - 2.0 - 30$	53.75	—	8.59	—	109.99	—	J_a	—	4.19
$A - 0.525 - 2.0 - 40$	52.92	—	8.25	—	112.06	—	J_a	—	4.67
$A - 0.525 - 4.0 - 10$	49.22	—	8.40	—	65.93	—	I	—	1.79
$A - 0.525 - 4.0 - 20$	49.88	—	8.41	—	66.34	—	I/ J_a	—	1.92
$A - 0.525 - 4.0 - 30$	52.07	—	8.07	—	65.71	—	I/ J_a	—	2.11
$A - 0.525 - 4.0 - 40$	53.35	—	7.87	—	64.44	—	J_a/K_b	—	2.34
$B - 0.625 - 2.0 - 10$	48.63	67.42	8.75	13.50	102.88	134.85	J_a	I	3.54
$B - 0.625 - 2.0 - 20$	48.44	68.64	7.25	14.57	93.44	158.75	J_a	I	3.80
$B - 0.625 - 2.0 - 30$	49.47	68.38	6.35	15.21	83.31	182.25	J_a	I	4.18
$B - 0.625 - 2.0 - 40$	49.92	69.07	5.96	16.60	77.87	204.68	J_a	I	4.65
$B - 0.625 - 4.0 - 10$	38.86	42.10	8.14	9.16	59.23	77.82	I/ J_a	I	1.79
$B - 0.625 - 4.0 - 20$	38.38	46.42	7.85	9.95	54.06	91.93	K_b/J_a	I	1.92
$B - 0.625 - 4.0 - 30$	36.09	51.02	7.48	10.92	48.64	106.00	J_a	I	2.11
$B - 0.625 - 4.0 - 40$	33.70	52.63	7.55	11.74	43.97	119.57	J_a	I	2.34
$C - 0.675 - 2.0 - 10$	24.27	33.07	10.33	12.46	95.30	134.94	J_a/K_b	I	3.46
$C - 0.675 - 2.0 - 20$	22.19	40.84	9.31	13.41	82.13	158.84	J_a	I	3.71
$C - 0.675 - 2.0 - 30$	21.92	49.61	8.00	13.96	69.42	182.38	J_a	I	4.09
$C - 0.675 - 2.0 - 40$	23.59	56.29	6.69	15.18	57.62	204.89	J_a	I	4.55
$C - 0.675 - 4.0 - 10$	18.57	20.13	9.06	11.18	56.28	77.83	I	I	1.76
$C - 0.675 - 4.0 - 20$	17.28	23.15	8.59	12.07	48.75	92.07	I/ J_a	I	1.88
$C - 0.675 - 4.0 - 30$	14.09	24.89	9.71	13.20	41.63	106.79	I/ J_a	I	2.07
$C - 0.675 - 4.0 - 40$	13.01	27.22	9.65	14.00	35.14	120.35	I/ J_a	I	2.30

$Panel - \lambda - h_p - m$	δ (mm)		F_m (kN)		V_0 (m/s)		Crush mode		W (kJ)
tube	3	4	3	4	3	4	3	4	
$C - 0.675 - 2.0 - 10$	28.84	29.39	11.74	11.68	118.92	118.91	J_a/K_b	J_a/K_b	3.46
$C - 0.675 - 2.0 - 20$	32.35	31.93	11.82	11.68	126.39	126.40	J_a/K_b	J_a/K_b	3.71
$C - 0.675 - 2.0 - 30$	35.17	35.41	11.63	11.27	131.94	131.95	J_a	J_a	4.09
$C - 0.675 - 2.0 - 40$	37.41	37.72	11.32	10.85	135.67	135.66	J_a	J_a	4.55
$C - 0.675 - 4.0 - 10$	19.38	19.31	10.14	10.07	69.03	69.03	I	I	1.76
$C - 0.675 - 4.0 - 20$	20.51	20.07	10.32	10.25	73.04	73.04	I/ J_a	I/ J_a	1.88
$C - 0.675 - 4.0 - 30$	19.99	20.08	10.79	10.50	76.26	76.24	I/ J_a	I/ J_a	2.07
$C - 0.675 - 4.0 - 40$	20.44	20.69	11.05	10.59	78.46	78.46	I/ J_a	I/ J_a	2.30

parameter m . Table 7.9 gives the individual tube responses for each of the panels investigated. In the table, V_0 refers to the average peak velocity at the proximal end of each tube, as measured at the tube corners. The expression W refers to the total external work done on the entire panel by the blast load. Of particular interest in the data of Table 7.9 is the significant increase in initial velocity, crush distance, and mean force levels in tube 2 of panels B , and C , with increasing value of the decay constant m . This is a result of

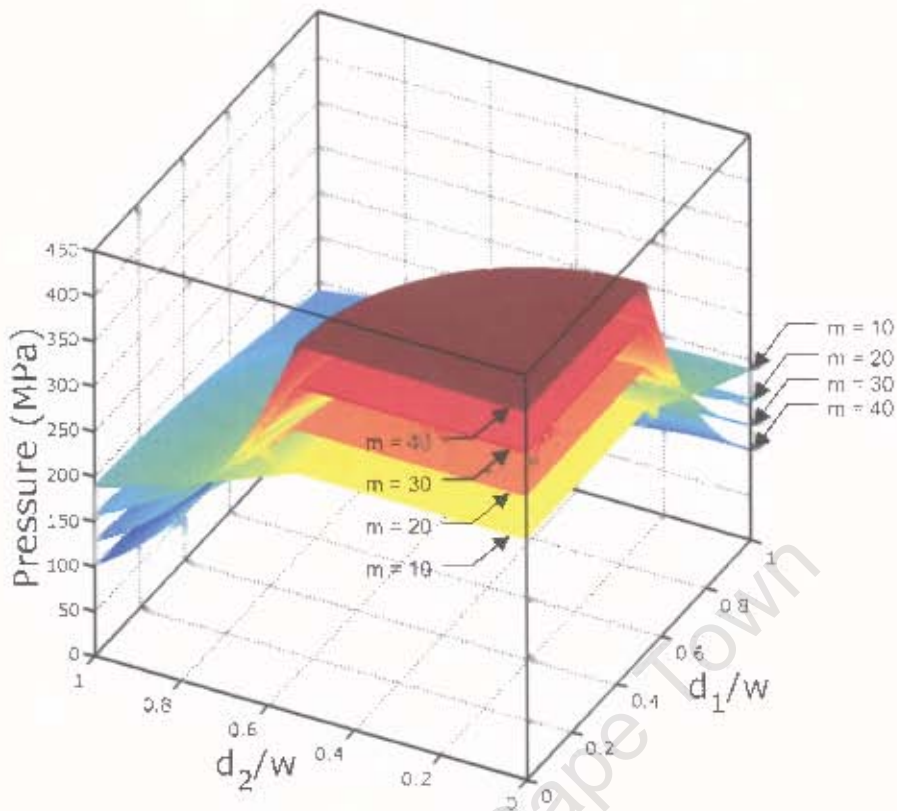


Figure 7.10: Plot of non-uniform pressure distribution corresponding to decay parameters $m = 10, 20, 30, 40$ using the relation given in Eq. 4.7. For all pressure distributions the impulse is fixed at $I = 50 \text{ Ns}$, the radius of constant pressure is $a = 40 \text{ mm}$ and the pulse duration is $t_0 = 10 \mu\text{s}$. The variables d_1 and d_2 refer to the distance along the primary axis of the top plate while w refers to the half width of the top plate as has the value $w = 75.0 \text{ mm}$.

increasing pressure at the panel centre, which increases the particle velocity of the top plate in the area adjacent to tube 2. This increases the energy absorption responsibility of the tube, however strain-rate hardening is also increased, thereby increasing the capacity of the tube.

From Fig. 7.11a, it is clear that when loading the panels with the 2.0 mm top plate, core energy absorption is found to be reduced significantly when increasing the decay constant. The reduction in efficiency is most pronounced with panel A layouts with panel C showing only a small reduction in core efficiency. For uniform loading ($m = 0$), all three panel layouts have nearly identical core efficiency however, due to the lack of the central tube, panel A shows a large reduction in efficiency for even a small amount of non-uniformity in the load. Then central supports provided by the extra tubes in panels B and C results in a relatively small reduction in efficiency for small changes in the decay constant. This is shown to be particularly true for panel C where only a negligible reduction in efficiency is observed at $m = 10$.

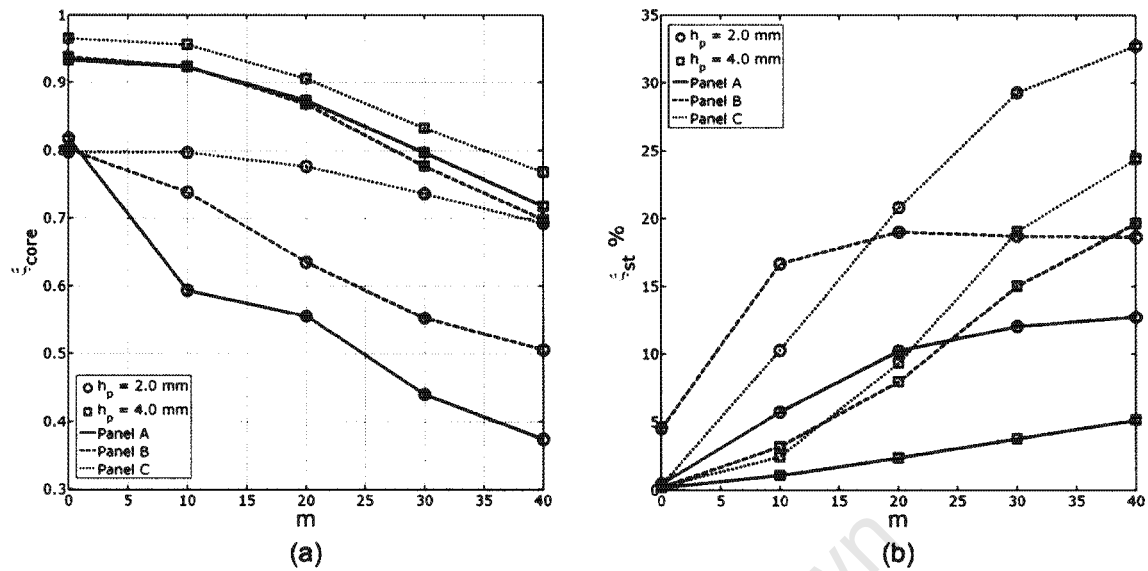


Figure 7.11: (a) Plot of variation in core efficiency as a function of the the decay parameter m for optimal tube positions. (b) Plot of variation in the stability parameter as a function of the the decay parameter m for optimal tube positions.

Numerical tests using the 4.0 mm top plate also show a substantial reduction in core efficiency at large values of m , however the reductions are less than that observed using the 2.0 mm plate. It is interesting to observe that panels A and B show nearly the same core efficiency for all values of m . For $m > 10$ the core efficiency is actually marginally higher in panel A than in panel B. This indicates that when using a thicker top plate, the addition of the central tube has little effect on the resulting core efficiency when loading the panels using a non-uniform pressure distribution.

The stability parameter is also analyzed as a function of the decay parameter m . It must again be noted that the comparison of stability parameter between different panel layouts does not provide any relevant information. Under uniform loading, large variations in the stability parameter generally correspond to instabilities developing in the core cubes which induce mode J and mode K buckling shapes. Under non-uniform loading, it is expected that, due to the pressure being concentrated at the panel centre, that core efficiency and stability parameter will show a correspondence, however changes in the buckling mode as per Fig. 4.9 will be different to that of uniform loading.

Fig. 7.11b gives the stability parameter as a function of m for the panels tested in this study. Under uniform loading ($m = 0$), the value of ξ_{st} is close to zero in all test cases. As expected, the value of ξ_{st} is found to increase when increasing the decay parameter m . For each panel layout, the stability parameter is significantly higher when loading the panels with a 2.0 mm top plate indicating a much larger differential in the crush distance at the tube corners within the panels. For higher decay constants As is the case with uniform loading, increases in the stability parameter correspond to decreases

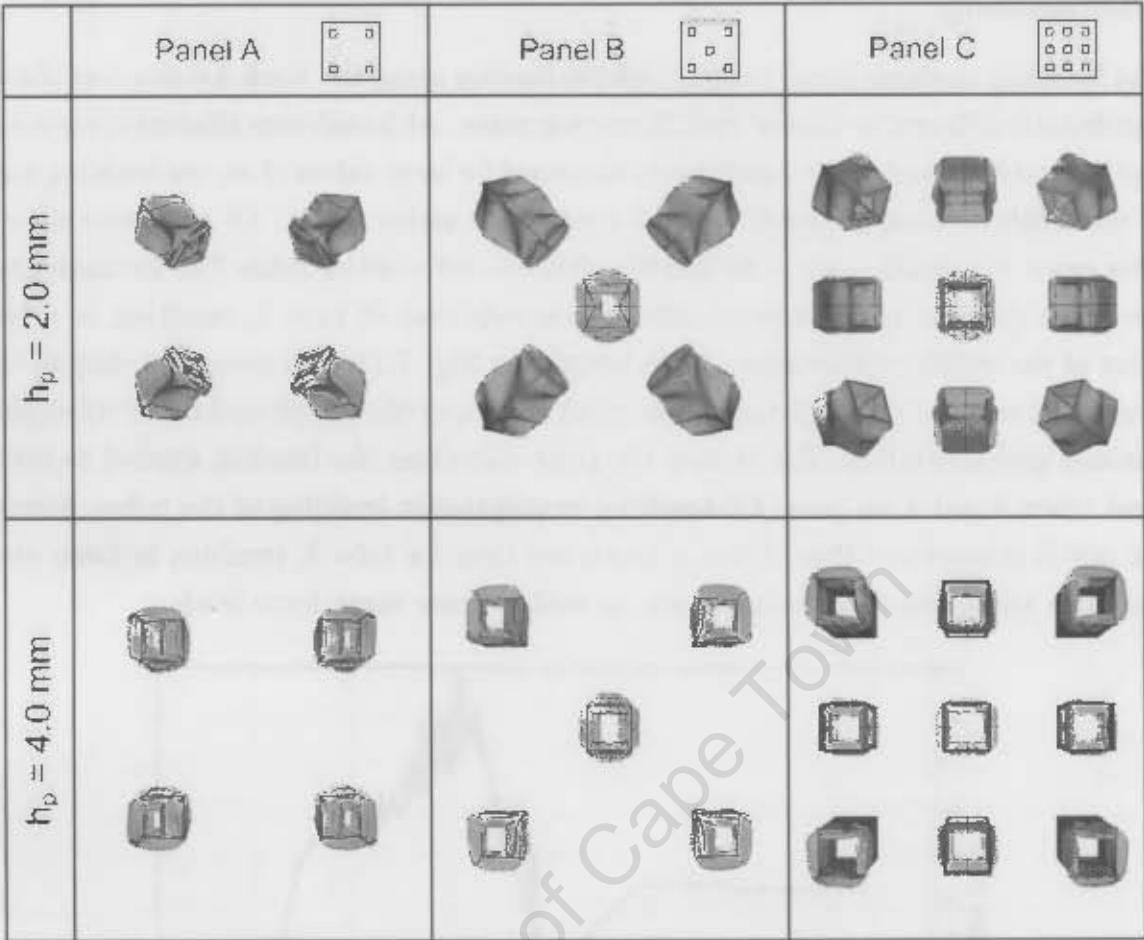


Figure 7.12: Axial view of panel cores with 2.0 mm and 4.0 mm thick top plates subject to non-uniform loading with decay parameter $m = 40$.

core efficiency as differential in crush distance of the core tube requires the top plate to deform, therefore reducing the total energy absorbed by the core tubes.

It is of interest to compare the resulting core deformation modes with the stability parameters presented in Fig. 7.11b. The axial view of the crushed tubes with $m = 40$ for the two plate thicknesses examined in the parametric loading study are given in Fig. 7.13. It is clear that for all panel layouts with a 2.0 mm top plate, the large non-uniformity of the load induces severe mode J buckling behaviour in the non-central tubes. Under uniform loading panel C layouts result in stable crushing patterns even for tubes in sub-optimal positions however, under non-uniform loading, tubes 1, 3, and 4 undergo significant global buckling. The individual tube responses presented in Table 7.9 quantify the effect of the global bending of the core tubes on energy absorption. For numerical tests which result in such buckling behaviour such as $B = 0.625 - 2.0 - 40$ and $C = 0.675 - 2.0 - 40$, tube 1 is found to have very low mean force levels compared to tube 2, which must crush in a symmetric mode due to model symmetry. For test $B = 0.625 - 2.0 - 40$, the increased loading on the tube 2 results in compaction of the tube while tube 2 still has significant

stroke remaining.

The buckling mode response to non-uniform loading using the thick 4.0 mm top plate is significantly different to that of the 2.0 mm top plate. Although core efficiency is reduced, and the stability parameter significantly increased for large values of m , the buckling mode of the crushed tubes is found to be of a relatively stable mode. Of particular note, is tests cases $B = 0.625 - 4.0 - 40$ and $C = 0.675 - 4.0 - 40$ in Table 7.9. In these cases, the crush distance of the central tube far exceeds that of tube 1, resulting in a large value of the stability parameter. From examining Fig. 7.12, it is clear that despite that large differences in crush distance, the crush modes of the centre and outer tubes show minimal global bending. The thicker top plate minimises the bending applied to tube 1 (and tubes 3 and 4 for panel C) resulting in progressive buckling of the tubes, however the initial velocities of these tubes is much less than for tube 2, resulting in lower crush distances and hence energy absorption, as well as lower mean force levels.

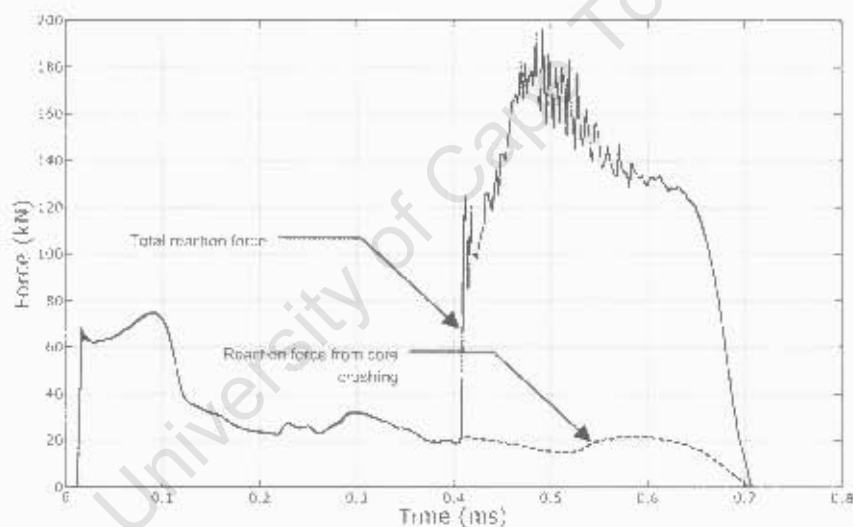


Figure 7.13: Influence of top and bottom plate contact on the total reaction force at the panel base for panel $A = 0.525 - 0.6 - 2.0 - 4$ with $m = 40$.

A final consideration of non-uniform loading is of the case where top plate dishing is significant enough that the top and bottom plate come into contact with one another. This is clearly an undesirable mode of deformation as the impact of the top plate with the base plate will increase force transfer to the protected structure, possibly loading the structure beyond safe levels. It is shown in section 7.2.1 that top plate impact is possible for uniform loading as well, if the tubes are positioned in a sub-optimal manner.

An example of the effect of top plate impact with the bottom plate is given in Fig. 7.13. The panel A design $A = 0.525 - 0.6 - 2.0 - 4$ is subject to the 50 Ns with the decay constant $m = 40$. The large dishing in the centre of the top plate induces severe mode I buckling in the core tubes as shown in Fig. 7.12. The resulting force-time curve for the

the tube is typical of square tubular member under global bending and is indicated in Fig. 7.13. The total panel reaction force due to the crushing of the tubes is given by the solid line up to the point where the dashed line begins and then by the dashed line. The total reaction force, including the impact of the top plate on the rigid bottom plate is given by the solid line. It is clear that, under the given loading scheme and panel geometry, the top plate impact increases force transfer to the protected structure significantly beyond the peak buckling load of the core tubes.

University of Cape Town

Chapter 8

Conclusions

Sandwich panels which use thin-walled square tubes as a core material are investigated under blast loading. The response of the novel panel structure is determined using analytical, experimental, and numerical techniques. The analysis presented in this thesis primarily focuses on the global response and efficiency of the panels in terms of energy absorption. The result of this is to determine the suitability of using the panels as a protective structure under blast loading. This work has

- Characterised the tube materials at quasi-static and high strain-rates.
- Identified three baseline layouts of the tubes in the panel core and has developed quantitative and qualitative measures to measure panel performance under blast loading.
- Conducted experimental tests to validate a numerical model.
- Created simple analytical design formulae to model the response of the proposed panels to blast loading. Further to this, analytical design formulae for similar panels using aluminium foam are developed and used for comparison with the proposed panels.
- Conducted a parametric study using ABAQUS/Explicit to determine the influence of several design variables on the performance of the panels.

This chapter describes the primary conclusions of the work described above and gives recommendations for future work involving the proposed panels.

8.1 Material Characterisation of Tube Materials

A comprehensive material characterisation study is conducted on the tube materials. Mild steel tubes, 6063-T6 aluminium alloy extrusions, and 4.0 mm hot rolled mild steel are characterised at strain-rates as low as $\dot{\epsilon} = 8.33 \times 10^{-3} \text{ s}^{-1}$ using a Zwick universal testing machine and at strain-rates as high as $\dot{\epsilon} = 3385 \text{ s}^{-1}$ using a Split Hopkinson Pressure Bar. To remove work hardening and improve ductility, the mild steel tubes are annealed at 900 °C for 1 hour. For the tube materials, the yield stress is found to be slightly higher in the annealed mild steel tubes and the strain hardening is found to be significantly higher. The ductility of the aluminium alloy is found to be much lower than for the mild steel tubes. The top plate steel and annealed mild steel tubes are observed to be highly strain-rate sensitive while the aluminium extrusions are found to be strain-rate insensitive.

8.2 Panel Design and Performance Measurement

In order to reduce the complexity of the panel layout problem, three general layouts are chosen. The three layouts have four, five and nine tubes within the panel respectively. These layouts are found to be ideal in terms of the present investigation as they allow for a large range of blast absorption capacities as well as inducing several different modes of panel deformation.

In order to make quantitative comparisons between panels, simple formulae are devised, in addition to the classical tube crushing performance measures such as crush force and total efficiency. The primary measure is the core efficiency which is simply the ratio of the energy absorbed in the core to the total external work done on the blast load. This measure is found to correspond closely to the stability parameter which measures the relative difference in the tube crush distances within the panel core. These measures are found to sufficiently characterise the global response of the proposed panels under blast loading.

Qualitative analysis of finite element results shows that the crushing modes of the core tube can be classified into three primary modes:

1. Mode I: The tube buckles in a progressive, symmetric mode with little or no global bending.
2. Mode J: The tube buckles primarily under global bending.

3. Mode K: The tube buckles in a combination of axial crushing with global lateral displacements away from the proximal end of the tube.

8.3 Experimental Investigations

8.3.1 Blast Loading Rig

To generate reasonably uniform blast loads on the proposed panels a new loading rig is utilised for experimental tests. The loading rig is used in conjunction with a standard ballistic pendulum used in blast experiments. The blast load is generated by detonating a disc of PE4 explosive at the end of a square blast tube. The other end of the blast tube is exposed only to the panel top plates allowing for increases explosive stand-off distance while ensuring the ballistic pendulum is loaded only by the panels under test. Negligible top plate deformation is observed indicating the blast is uniform to an acceptable level.

8.3.2 Experimental Results

8.3.2.1 Steel Tube Cores

Quasi-static tests conducted on the tubes used in the complete panels is conducted using a Zwick universal testing machine. Peak and mean force levels are found to be higher in the steel tubes than in the aluminium tubes. Reasonable correlation with numerical results from ABAQUS/Explicit are obtained in terms of peak and mean force levels. Buckling is observed to initiate at the hemispherical indentation trigger at the tube midpoint.

Under blast loading, panel crush distances are found to increase with increasing charge mass and impulse. Further to this, the panels *C* layouts are observed to have significantly higher impulse capacities than panel *B* designs. The steel tube core panels are found to buckle in an irregular pattern for lower impulses which do not crush the tubes to the point of compaction. The initiation of similar progressive symmetric buckling among each of the tubes is found to be controlled by the trigger. The depth of the hemispherical indentation is found to be insufficient to ensure that lobe formation is initiated at the desired location along the tube length. Despite the irregular buckling patterns at small impulses, the panels which crush to compaction show a symmetric lobe formation pattern indicating that the panels may be able to "recover" from initial instabilities and use the full stroke of all tubes to absorb the blast load.

8.3.2.2 Aluminium Tube Cores

Just as with the steel tubes, quasi-static crushing of the aluminum tubes used in the blast tests is conducted. Similar to the steel tube case, buckling is observed to initiate at the trigger. Numerically simulations are found to under-estimate the mean load. It is concluded that this is due to the filleted interior edges of the aluminium extrusions strengthening the tubes, which is not accounted for in the numerical model. As the tubes investigated here are of a smaller scale than those usually reported in the literature, this difficulty is unique to the present problem and should be investigated in further studies.

Trends in the response of the aluminium core panels to changes in impulse and panel layout are found to be similar to that of the steel tube panels. At identical charge mass, the aluminium tube cores show much larger crush distances in both panel *B* and *C* layouts. The trigger design is found to be ineffective in almost all tubes. Lobes are formed near the proximal and distal ends of the tubes. Numerical analysis of the blast tests indicate that the rate insensitivity of the material, combined with the high velocity loading, contributes to the observed mode of buckling. In future implementations of panels using the aluminium tubes, proper trigger design must be investigated to account for the buckling behaviour found in the experimental tests.

8.3.2.3 Numerical Modelling of Experimental Blast Tests

Finite element models implemented in ABAQUS/Explicit are found to reasonably predict the crush distance of experimental tests. Numerical predictions are found to be particularly accurate for panel *C* layouts using aluminium tube cores. The numerically predicted buckling pattern is found to correspond poorly with experimental results. This appears to be a result of the poor performance of the trigger in the experimental tests however, and as such, should improve with improvements in the trigger design. It can be concluded that the global response of the panels is predicted well, however modelling of local deformation modes requires further improvement.

Numerical analysis of the energy absorption properties of the panels indicates that for the panel geometries investigated experimentally, nearly all of the energy from the blast load is dissipated by the core tubes. For the steel tubes panels, peak forces increase marginally with increasing impulse due to strain-rate hardening. For both steel and aluminium tube panels, mean crush forces also increase with increasing impulse. Increased energy absorption occurs during the initial compression phase of core buckling, with increased impulse, for both the rate sensitive and rate insensitive materials. The indentation triggers appear to control the critical buckling load of the panels, as the peak force levels are found to be invariant to changes in top plate velocity for the rate insensitive aluminium.

Adiabatic temperature effects due to large plastic deformations, have very little influence on the response of the proposed panels. Exclusion of temperature effects from the material model shortens the resulting crush distance marginally. Despite the limited effect of thermal softening on the panel response, temperature should be included in the material model for the panels to obtain maximum accuracy.

8.4 Analytical Panel Modelling

8.4.1 Analytical Predictions of the Influence of Design Parameters on Panel Response

Simple analytical design formulae are developed to predict the response of the proposed panels to a uniform blast load. The model, which describes the transient response of the panel top plate, is used to determine critical initial top plate velocities, and hence impulses. Good correlation with experimental and numerical results are obtained.

It is found that unlike impact problems involving the crushing of thin-walled tubes, the "impacting mass" which for the present problem is the top plate, cannot be assumed to apply an instantaneous velocity to the tubes within the panel core. The velocity of the top plate is found to increase gradually as the blast load is applied and obtains the peak velocity at the end of the blast event. Simple theory of elastic and plastic wave propagation are used to show that the impulse transfer during the application of the blast is not negligible and accounting for this allows for relatively accurate approximations of the peak top plate velocity and hence the energy absorption requirements of the folding process in the core tubes.

The simple analytical model confirms the experimental results in that impulse capacity is significantly increased when increasing the number of tubes in the panel. Increased flow stress and strain-rate hardening in the steel tubes also results in significantly increased blast absorbing capacity. Panel impulse capacity for both tube materials is maximised when using thick top plate with thick tubes. The specific impulse capacity however, defined as the impulse capacity per unit of panel mass is maximised when using thick tubes with a thin top plate. Specific impulse capacities are found to increase with increasing number of tubes in the panel and are noticeably higher in panels using steel tube cores. The aspect ratio of the tubes is found to have minimal influence on the capacity of the panels however it is noted that under actual blast scenarios, a large aspect ratio may lead to significant reduction in panel stability which is not captured in the analytical model.

8.4.2 Foam Core Panel Response to Blast Loading

A simple analytical model is developed to obtain the transient response of the panel top plate, however using an aluminium foam core. The panel thickness is fixed at the same value as that of the tube core panels. The foam properties examined are typical of those reported in the literature.

The impulse capacity of the foam core panels is observed to be highly dependent on the foam density. At higher foam densities, the plateau stress of the material increases significantly while reducing the densification strain, and hence the allowable crush distance before compaction. Impulse capacity is maximised at the highest foam density and thickest top plate however specific impulse capacity is found to be maximised at the highest density foam with the thinnest top plate.

8.4.3 Comparison of Tube and Foam Core Panels

For the range of tube and foam design variables examined in this thesis, the panels with the highest blast absorption capacities are obtained from foam core panels with high foam density and a thick top plate. Due to the significant mass of such panels however, the maximum specific impulse capacity is nearly identical for the tube and foam core panels. For low density foam cores however, the impulse capacity and particularly the specific impulse capacity is found to be much lower than low capacity panel *C*, tube core panels. This is an important consideration in mass critical applications. The density of foam which is available, may result in a panel with much lower specific impulse capacity than what can be obtained using a tube based panel, thereby increasing the mass of the protective structure beyond the optimal level.

A comparison of the space efficiency of the panels is also conducted as possible use a protective vehicle structure requires that the total panel thickness be kept to a minimum. For steel tube, panel *C* designs, nearly all panel geometries result in a thinner panel than that of a low density (200 kg/m^3) foam core panel, using the thickest top plate, with the same impulse capacity as the corresponding tube based panel. The mass of the resulting panels are significantly larger than that of the tube based panel. When comparing aluminium tube panels, most panels designs result in a low density foam core panel that is thinner than the tube core panel with the same impulse capacity. The resulting foam core panels are of larger mass however than the aluminium tube core panels with the same impulse capacity. Using a high density (400 kg/m^3) foam core panel, for all tube based panel designs, the foam core panel with the same impulse capacity is significantly thinner than the tube core panel. Further to this, only steel tube panels with very thin tubes result in a lower mass panel.

A final comparison between the two cladding structures is made by determining the foam panel design parameters which require a larger thickness/mass panel in order to have the same capacity as the largest capacity tube core panel. It is found that there exists combinations of foam density and top plate thickness which, for each tube material,

- Requires a panel of larger thickness and mass than the highest capacity tube core panel.
- Requires a panel of smaller thickness but has a larger mass than the highest capacity tube core panel.
- Requires a panel of larger thickness but has a smaller mass than the highest capacity tube core panel.
- Requires a panel of smaller thickness and mass than the highest capacity tube core panel.

It is however observed that the crush force levels are generally much higher with the higher density foam in comparison to panel *C* tube based cores. The allowable force levels will depend on the particular application. This must be balanced with the impulse absorbing requirements. It can therefore be concluded that the performance of the proposed panels when compared to foam core panels depends strongly on the foam density. High density foams, combined with the appropriate top plate, can generally absorb a given impulse using a thinner and lighter panel than what can be constructed with a tube core panel. Further experimental and numerical investigation must be conducted on foam core panels however, to determine the validity of such a structure as a protective cladding under blast loads.

8.5 Numerical Modelling of Panel Response

A numerical parametric study is conducted in ABAQUS/Explicit for the purpose of determining the influence of tube and top plate geometry on the efficiency and stability of the panels. A large range of steel tube panel geometries are investigated under a common impulse and the results are compared. The conclusions draw from the study are briefly described here.

8.5.1 Tube Layout

The stability and energy absorption properties of the proposed panels are strongly influenced by positioning of the tubes within the panel. For each panel layout *A*, *B*, and *C* an optimal radial distance from the panel centre is found where energy absorption in the core tubes is maximised and crush distances of each core tube is nearly identical. For tube placements which are far from optimal, large top plate interaction effects induce global bending modes in the core tubes, reducing the energy absorption capacity of the panel core. Panel *A* layouts are found to be particularly sensitive to sub-optimal tube placements and hence are not recommended for use in actual blast scenarios. Table 8.1 summarises the results of the parametric study on tube layout.

Table 8.1: Panel performance measures for the optimal values of λ , in terms of core stability, in panels *A*, *B*, and *C*, obtained from the parametric study on tube layout. For all designs, $R = 4$, $h_t = 0.6$ mm, and $h_p = 3.0$ mm.

<i>Panel</i> – λ	δ_m (mm)	$F_{\max}$ (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{core}
<i>A</i> – 0.525	57.65	74.89	35.66	0.476	0.366	0.885
<i>B</i> – 0.625	46.17	93.51	44.04	0.471	0.290	0.880
<i>C</i> – 0.675	22.18	168.62	93.41	0.554	0.164	0.914

8.5.2 Tube Geometry Effects on Panel Performance

As is observed analytically, the aspect ratio of the tubes has minimal effect on panel response. Small aspect ratios appear to increase stability in panel *C* layouts however, due to increases support of the top plate prior to core buckling. Tube thickness is found to have significant influence on panel response. Small changes in tube thickness results in large changes in crush distance. For stable tube placements, large tube thicknesses are found to slightly decrease core efficiency due to delayed buckling in the core tubes allowing larger initial displacements in the top plate. For unstable tube layouts, large

tube thicknesses are found to decrease panel stability. Core efficiency is found to be maximised with tube thicknesses resulting in crush distances near the point of compaction. This implies that to maximise the energy absorption in the core, the tube thickness should be chosen based on the maximum expected blast load. This is impractical however as any underestimation of the expected loading will result in compaction, which will significantly increase force transfer to the protected structure. Table 8.2 summarises the optimal panel designs obtained from simultaneous variation in tube core geometric properties.

Table 8.2: Performance measures for the highest performing panels, in terms of core efficiency in panels *A*, *B*, and *C*, under simultaneous variation in tube geometric parameters. For all designs, $h_p = 3.0$ mm.

<i>Panel</i> – λ – h_t – R	δ_m (mm)	$F_{\max}$ (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{core}
<i>A</i> – 0.525 – 0.5 – 3.0	62.84	79.03	33.61	0.425	0.356	0.912
<i>B</i> – 0.625 – 0.5 – 3.0	50.92	98.78	41.60	0.421	0.286	0.919
<i>C</i> – 0.675 – 0.5 – 3.0	23.50	177.81	92.55	0.521	0.163	0.965

8.5.3 Top Plate Geometry Effects on Panel Performance

The numerical investigations clearly indicate a significant increase in panel performance with increasing top plate thickness. Thick top plates are found to minimise top plate interaction effects which induce global buckling modes in the panel core. The initial kinetic energy in the panels decreases with increasing top plate thickness, significantly increasing the impulse capacity of the panels. The top plate constitutes a significant portion of the total panel mass however, and as such the performance benefits gained from maximising the top plate thickness must be coupled with the mass efficiency requirements of the application. Table 8.3 summarises the optimal panel designs obtained from the parametric study on top plate thickness.

Table 8.3: Performance measures for the highest performing panels, in terms of core efficiency in panels *A*, *B*, and *C*, under variation in top plate thickness. For all designs, $h_t = 0.6$ mm and $R = 4.0$.

<i>Panel</i> – λ – h_p	δ_m (mm)	$F_{\max}$ (kN)	F_m (kN)	ξ_{crush}	ξ_{total}	ξ_{core}
<i>A</i> – 0.525 – 4.0	49.32	72.64	33.06	0.455	0.299	0.933
<i>B</i> – 0.625 – 4.0	39.16	90.91	41.68	0.458	0.239	0.938
<i>C</i> – 0.675 – 4.0	18.78	163.74	87.88	0.537	0.134	0.965

8.5.4 Panel Response to Non-Uniform Loading

The influence of load non-uniformity is investigated numerically. For all panel layouts, thin top plates induce instabilities in the panel core, even for small non-uniformities in the pressure loading. Panel A layouts are found to be most sensitive to loading non-uniformity. Using a thick top plate is found to minimise the reduction in stability and efficiency when loading the proposed panels under non-uniform blast loading. This further emphasises the importance of using a sufficiently thick top plate under actual blast conditions.

8.6 Recommendations for Future Work

The work presented in this thesis presents the groundwork for the use of the thin-walled tubes as sandwich material in the construction on protective panels under blast loads. Significant work remains in the study of the proposed panels in order to understand and improve performance under blast loading. The following recommendations are made regarding future work on the proposed panels:

1. The variation in panel designs used in experimental tests should be greatly increased. This includes variation in tube thickness and aspect ratio and also in the thickness of the top plate. The experiments presented in this thesis are not extensive enough to ascertain complete experimental trends and are primarily for validation of finite element models. Experimental tests on different panel geometries will provide further insight into panel behaviour and possibly allow for further improvements on existing analytical and numerical models.
2. Larger scale tests should be conducted which utilise several of the proposed panels subjected to a much larger charge mass. The current study focuses on a small structure which may compose a single cell of a complete protective cladding. If the proposed panel design is to be incorporated into actual protective cladding, it would be of interest to study the behaviour of the proposed structure configured in a cellular layout. From this, it can be determined if interaction effects from the individual panels have an effect on the energy absorbing properties of the complete structure. It will also be of interest to determine how the panels respond to the non-uniform blast loading associated with such an application.
3. The ballistic pendulum should be modified to include a Hopkinson bar setup which can be used to record the reaction forces at the panel base. Although a difficult problem, the use of Hopkinson bars to record the force history at the base of the

tubes would provide valuable information into how the proposed panels transfer impulse to the protected structure. The reaction force history would also provide a further means of validating the numerical models.

4. Further investigation should be conducted into a proper trigger mechanism to ensure similar buckling characteristics among the core tubes. It is shown in the present work that the initiation of buckling at the indentation trigger has significant influence on resulting crush mode of the panel core. Further studies using indentation triggers of differing radii and depth as well as different trigger mechanisms could improve the performance of the proposed panels.
5. The response of the panels to localised loading should be investigated experimentally. The current study focuses primarily on uniform loading with some numerical studies using non-uniform blast loads. It is of interest to determine experimentally, the response of the panels to localised blast loads to determine if role of the panel as an energy absorbing device is compromised when loaded primarily near the panel centre.
6. Numerical investigation using the hydrocode AUTODYN should be conducted on the loading rig used here to ensure uniformity of the specific impulse applied to the panel top plate. The experimental testing rig developed for this study appears to generate reasonably uniform blast loads based on the resulting structural response. It would be beneficial for future studies to be able to simulate the pressure history over the face of the panels. Such an exercise may allow for optimisation of the blast tube to generate the desired loading. In addition, the numerically predicted pressure loading can be used to parameterise the pulse shape used in the numerical model of the panels themselves.
7. Mathematical optimization, using analytical results and possibly the Response Surface Methodology, should be conducted to find the optimal panel response subject to constraints on the design variables and on peak force levels. In the present study, a parametric study is conducted to determine how the geometric design variables influence panel response. A true optimal panel cannot be obtained from such a methodology as all design variables are not varied independently. Further to this, in practice, the optimal configuration would depend heavily on mass and peak force constraints which must be incorporated into a complete optimisation study.

References

- [1] Landmine and explosive remnants of war safety handbook. Technical report, United Nations Mines Action Service, 2005.
- [2] Landmine monitor report 2000. Technical report, International Campaign to Ban Landmines, 2000.
- [3] Landmine monitor report 2006. Technical report, International Campaign to Ban Landmines, 2006.
- [4] Explosive remnants of war and mines. Technical report, Mines Action Canada, Actiongroup Landmine.de, Landmine Action (UK), 2005.
- [5] Geneva International Centre for Humanitarian Demining. *Mechanical Demining Equipment Catalogue*, chapter 7, pages 207–227. 2006.
- [6] L.A. Daykin. Modelling the effects of explosives on v-plates. Master's thesis, University of Cape Town, 2005.
- [7] K. Genson. Vehicle shaping for mine blast damage reduction. Master's thesis, University of Maryland, 2006.
- [8] D. Karagiozova, G.N. Nurick, and S. Chung Kim Yuen. Energy absorption of aluminium alloy circular and square tubes under an axial explosive load. *Thin Walled Structures*, 43(6):956–982, 2005.
- [9] A.G. Pugsley and M. Macaulay. The large scale crumpling of thin cylindrical columns. *Quarterly Journal of Mechanics and Applied Mathematics*, 13(1):1–9, 1960.
- [10] J.M. Alexander. An approximate analysis of the collapse of thin cylindrical columns. *Quarterly Journal of Mechanics and Applied Mathematics*, 13:10–15, 1960.
- [11] A. Alghamdi. Collapsible impact energy absorbers: An overview. *Thin Walled Structures*, 39(2):189–213, 2001.

- [12] N. Jones. Several phenomenon in structural impact and structural crashworthiness. *European Journal of Mechanics and Solids*, 22(5):693–707, 2003.
- [13] Ø. Jensen, M. Langseth, and O.S. Hopperstad. Experimental investigations on the behaviour of short to long square aluminium tubes subjected to axial loading. *International Journal of Impact Engineering*, 30(8-9):973–1003, 2004.
- [14] N. Jones. *Structural Impact*. Cambridge University Press, Cambridge, 1989.
- [15] W. Johnson and P.D. Soden. Inextensial collapse of thin-walled tubes under axial compression. *Journal of Strain Analysis*, 12(4):317–330, 1977.
- [16] T. Wierzbicki and W. Abramowicz. On the crushing mechanics of thin-walled structures. *Journal of Applied Mechanics*, 50(4a):727–734, 1983.
- [17] W. Abramowicz and N. Jones. Dynamic axial crushing of square tubes. *International Journal of Impact Engineering*, 2(2):179–208, 1984.
- [18] D. Karagiozova. Transition criteria between buckling modes of circular shells under axial impact. In M. Alves and N. Jones, editors, *Impact Loading of Lightweight Structures*, Florianopolis, Brazil, 2005. WIT Press.
- [19] D. Karagiozova and N. Jones. Dynamic elastic-plastic buckling of circular cylindrical shells under axial impact. *International Journal of Solids and Structures*, 37(14):2005–2034, 2000.
- [20] D. Karagiozova and N. Jones. Influence of stress waves on the dynamic progressive and dynamic plastic buckling of cylindrical shells. *International Journal of Solids and Structures*, 38(38-39):6723–6749, 2001.
- [21] D. Karagiozova and N. Jones. On dynamic buckling phenomena in axially loaded elastic-plastic cylindrical shells. *International Journal of Non-Linear Mechanics*, 37(7):1223–1238, 2002.
- [22] D. Karagiozova, M. Alves, and N. Jones. Inertia effects in axisymmetrically deformed cylindrical shells under axial impact. *International Journal of Impact Engineering*, 24(10):1083–1115, 2000.
- [23] D. Karagiozova. Dynamic buckling of elastic-plastic square tubes under axial impact - I: stress wave propagation phenomenon. *International Journal of Impact Engineering*, 30(2):143–166, 2004.
- [24] D. Karagiozova and N. Jones. Dynamic buckling of elastic-plastic square tubes under axial impact - II: structural response. *International Journal of Impact Engineering*, 30(2):167–192, 2004.

- [25] D. Karagiozova and N. Jones. Multi-degrees of freedom model for dynamic buckling of an elastic-plastic structure. *International Journal of Solids and Structures*, 33(23):3377–3398, 1996.
- [26] J.J. Harrigan, S.R. Reid, and C. Peng. Inertia effects in impact energy absorbing materials and structures. *International Journal of Impact Engineering*, 22(9-10):955–979, 1999.
- [27] K.R.F. Andrews, G.L. England, and E. Ghani. Classification of the axial collapse of cylindrical tubes under quasi-static loading. *International Journal of Mechanical Sciences*, 25(9-10):687–696, 1983.
- [28] H. Korneck. An investigation into the response of square box columns to axial loading. *Project No. 91, BSc. thesis, University of Cape Town*, 1992.
- [29] W. Abramowicz and N. Jones. Transition from initial global bending to progressive buckling of tubes loaded statically and dynamically. *International Journal of Impact Engineering*, 19(5-6):415–437, 1997.
- [30] Ø. Jensen, M. Langseth, and O.S. Hopperstad. Transition from local to global buckling: Quasi-static and dynamic experimental results. In *Proceedings of the 8th International Symposium on Plasticity and Impact Mechanics, IMPLAST 2003*, pages 118–125, New Delhi, India, 2003. Phoenix Publishing House.
- [31] D. Karagiozova and M. Alves. Transition from progressive buckling to global bending of circular shells under axial impact - part I: Experimental and numerical observations. *International Journal of Solids and Structures*, 41(5-6):1565–1580, 2004.
- [32] D. Karagiozova and M. Alves. Transition from progressive buckling to global bending of circular shells under axial impact - part II: Theoretical analysis. *International Journal of Solids and Structures*, 41(5-6):1581–1604, 2004.
- [33] D.-K. Kim, S. Lee, and M. Rhee. Dynamic crashing and impact energy absorption of extruded aluminium square tubes. *Materials & Design*, 19(4):179–185, 1998.
- [34] D.-K. Kim and S. Lee. Impact energy absorption of 6061 aluminium extruded tubes with different cross sectional shapes. *Materials & Design*, 20(1):41–49, 1999.
- [35] S.R. Guillow, G. Lu, and R.H. Grzebieta. Quasi-static axial compression of thin-walled circular aluminium tubes. *International Journal of Mechanical Sciences*, 43(9):2103–2123, 2001.
- [36] S.S. Teramoto and M. Alves. Buckling transition of axially impacted open shells. In *Proceedings of the 8th International Symposium on Plasticity and Impact Mechanics, IMPLAST 2003*, pages 126–133, New Delhi, India, 2003. Phoenix Publishing House.

- [37] N.K. Gupta and S.K. Gupta. Effect of annealing, size and cut-outs on axial collapse behaviour of circular tubes. *International Journal of Mechanical Sciences*, 35(7):597–613, 1991.
- [38] N.K. Gupta. Some aspects of axial collapse of cylindrical thin-walled tubes. *Thin-Walled Structures*, 32(1-3):111–126, 1998.
- [39] N.K. Gupta. IMPLAST symposia and large deformations - a perspective. *International Journal of Impact Engineering*, 27(9):901–918, 2002.
- [40] H.-W. Song, Z.-M. Wan, Z.-M. Xie, and X.-W. Du. Axial impact behaviour and energy absorption efficiency of composite wrapped metal tubes. *International Journal of Impact Engineering*, 24(4):385–401, 2000.
- [41] N.S. Marshall and G. N. Nurick. The effect of induced deformations on the formation of the first lobe of symmetric progressive buckling of thin walled square tubes. In N. Jones, D.G. Talaslidis, C.A. Brebbia, and G.D. Manolis, editors, *Proceedings of the 5th International Conference on Structures Under Shock and Impact, (SUSI V)*, pages 155–168, Thessaloniki, Greece, 1998.
- [42] M. Langseth, T. Berstad, O.S. Hopperstad, and A.H. Clausen. Energy absorption in axially loaded square thin-walled aluminum extrusions. In *Proceedings of the 3th International Conference on Structures Under Shock and Impact, (SUSI III)*, pages 401–410, Southampton, UK, 1994.
- [43] S.W. Surko. Crushing strength of box columns with partially-damaged plating. *International Journal of Mechanical Sciences*, 33(12):1017–1028, 1991.
- [44] L. Abah, A. Limam, and M. Dejeammes. Effects of cutouts on static and dynamic behaviour of square aluminium extrusions. In *Proceedings of the 5th International Conference on Structures Under Shock and Impact, (SUSI V)*, pages 133–142, Southampton, UK, 1998. Computational Mechanics Publications.
- [45] N. Marshall. Active control of passive safety in passenger motor vehicles. Master's thesis, University of Cape Town, 1995.
- [46] P. Nannucci, N.S. Marshall, and G.N. Nurick. A computational investigation of the progressive buckling of square tubes with geometric imperfections. In T.S. Lok and C.H. Lim, editors, *Proceedings of the 3rd Asia-Pacific Conference on Shock and Impact Loads on Structures*, pages 335–342, Singapore, 1999.
- [47] Q. Meng, S.T.S. Al-Hassani, and P.D. Soden. Axial crushing of square tubes. *International Journal of Mechanical Sciences*, 25(9-10):747–773, 1983.

- [48] S. Lee, C. Hahn, M. Rhee, and J. Oh. Effect of triggering on the energy absorption capacity of axially compressed aluminium tubes. *Materials & Design*, 20(1):31–40, 1999.
- [49] A.G. Hanssen, M. Langseth, and O.S. Hopperstad. Static crushing of square aluminium extrusions with aluminium foam filler. *International Journal of Mechanical Sciences*, 41(8):967–993, 1999.
- [50] A.G. Hanssen, M. Langseth, and O.S. Hopperstad. Static and dynamic crushing of circular aluminium extrusions with aluminium foam filler. *International Journal of Impact Engineering*, 24(5):475–507, 2000.
- [51] A.G. Hanssen, M. Langseth, and O.S. Hopperstad. Static and dynamic crushing of square aluminium extrusions with aluminium foam filler. *International Journal of Impact Engineering*, 24(4):347–383, 2000.
- [52] A.G. Hanssen, M. Langseth, and O.S. Hopperstad. Optimum design for energy absorption of square aluminium columns with aluminium foam filler. *International Journal of Mechanical Sciences*, 43(1):153–176, 2001.
- [53] M. Seitzberger, R.F. Rammerstorfer, H.P. Degischer, and R. Gradinger. Crushing of axially compressed steel tubes filled with aluminium foam. *Acta Mechanica*, 125(1-4):93–105, 1997.
- [54] M. Seitzberger, R.F. Rammerstorfer, H.P. Degischer, R. Gradinger, M. Blaimschein, and C. Walch. Experimental studies on the quasi-static axial crushing of steel columns filled with aluminium. *International Journal of Solids and Structures*, 37(30):4125–4147, 2000.
- [55] S. Santosa and T. Wierzbicki. Crush behaviour of box columns filled with aluminium honeycomb or foam. *Computers and Structures*, 68(4):343–367, 1998.
- [56] A.G. Hanssen, O.S. Hopperstad, and M. Langseth. Bending of square aluminium extrusions with aluminium foam filler. *Acta Mechanica*, 142(1-4):13–31, 2000.
- [57] S. Shahbeyk, N. Petrinic, and A. Vafaia. Numerical modelling of dynamically loaded metal foam-filled square columns. *International Journal of Impact Engineering*, 34(3):573–586, 2007.
- [58] S.P. Santosa, T. Wierzbicki, A.G. Hanssen, and M. Langseth. Experimental and numerical studies of foam filled sections. *International Journal of Impact Engineering*, 24(5):509–534, 2000.

- [59] M. Langseth, O.S. Hopperstad, and A.G. Hanssen. Crash behaviour of thin-walled aluminium members. *Thin-Walled Structures*, 32(1-3):127–150, 1998.
- [60] A.G. Hanssen, M. Langseth, and O.S. Hopperstad. Axial crushing of aluminium columns with aluminium foam filler. In *Proceedings of the 7th International Symposium on Structural Failure and Plasticity, IMPLAST 2000*, pages 401–407, Melbourne, Australia, 2000.
- [61] T. Børvik, O.S. Hopperstad, A. Reyes, M. Langseth, G. Solomos, and T. Dyngeland. Empty and foam-filled circular aluminium tubes subjected to axial and oblique quasi-static loading. *International Journal of Crashworthiness*, 8(5):481–494, 2003.
- [62] A. Reyes, O.S. Hopperstad, and M. Langseth. Aluminium foam-filled extrusions subjected to oblique loading: experimental and numerical study. *International Journal of Solids and Structures*, 41(5-6):1645–1675, 2004.
- [63] A. Reyes, M. Langseth, and O.S. Hopperstad. Square aluminium tubes subjected to oblique loading. *International Journal of Impact Engineering*, 28(10):1077–1106, 2003.
- [64] Q. Wang, Z. Fan, H. Song, and L. Gui. Experimental and numerical analysis of the axial crushing behaviour of hat sections partially filled with aluminium foam. *International Journal of Crashworthiness*, 10(5):535–543, 2005.
- [65] S. Shahbeyk, A. Vafai, and H.E. Estekanchi. A parametric study of the bending crash performance of empty and metal foam-filled box-beams. *International Journal of Crashworthiness*, 9(6):643–652, 2004.
- [66] V.S. Deshpande and N.A. Fleck. Isotropic models for metallic foams. *Journal of the Mechanics and Physics of Solids*, 48(6-7):1253–1283, 2000.
- [67] C.R. Calladine and R.W. English. Strain-rate and inertia effects in the collapse of two types of energy-absorbing structure. *International Journal of Mechanical Sciences*, 26(11-12):689–701, 1984.
- [68] L.L. Tam and C.R. Calladine. Inertia and strain-rate effects in a simple plate-structure under impact loading. *International Journal of Impact Engineering*, 11(3):349–377, 1991.
- [69] Z.Y. Gao, T.X. Yu, and G. Lu. A study on type II structures. part I: a modified one-dimensional mass-spring model. *International Journal of Impact Engineering*, 31(7):895–910, 2004.

- [70] Z.Y. Gao, T.X. Yu, and G. Lu. A study on type II structures. part II: dynamic behavior of a chain of pre-bent plates. *International Journal of Impact Engineering*, 2005(7):911–926, 2005.
- [71] K. Yamazaki and J. Han. Maximization of the crushing absorption of cylindrical shells. *Advances in Engineering Software*, 31(6):425–434, 2000.
- [72] J. Forsberg and L. Nilsson. Evaluation of response surface methodologies used in crashworthiness optimization. *International Journal of Impact Engineering*, 32(5):759–777, 2006.
- [73] T. Jansson, L. Nilsson, and M. Redhe. Using surrogate models and response surfaces in structural optimization with application to crashworthiness design and sheet metal forming. *Structural and Multidisciplinary Optimization*, 25(2):129–140, 2003.
- [74] M. Redhe, M. Giger, and L. Nilsson. An investigation of structural optimization in crashworthiness design using a stochastic approach. *Structural and Multidisciplinary Optimization*, 27(6):446–459, 2004.
- [75] G. Chiandussi and M. Avalle. Maximisation of the crushing performance of a tubular device by shape optimisation. *Computers and Structures*, 80(27-30):2425–2432, 2002.
- [76] K.A. Marchand and F. Alfawakhiri. Blast and progressive collapse, facts for steel buildings. Technical Report 2, American Institute of Steel Construction Inc., 2004.
- [77] The Steel Construction Institute. The effects of simplification of the explosion pressure-time history. Technical report, British Gas Research and Technology, 1992.
- [78] A.G. Hanssen, L. Enstock, and M. Langseth. Close-range blast loading of aluminium foam panels. *International Journal of Impact Engineering*, 27(6):593–618, 2002.
- [79] S.M. Liang, L.N. Wu, and R.L. Hsu. Numerical investigation of axisymmetric shock wave focusing over paraboloid reflectors. *Shock Waves*, 9(6):367–379, 1999.
- [80] G.W. Ma and Z.Q. Ye. Energy absorption of double-layer foam cladding for blast alleviation. *International Journal of Impact Engineering*, 34(2):329–347, 2007.
- [81] S. Nemat-Nasser, W.J. Kang, J.D. McGee, W.-G. Guo, and J.B. Isaacs. Experimental investigation of energy-absorption characteristics of components of sandwich structures. *International Journal of Impact Engineering*, 34(6):1119–1146, 2007.

- [82] Q.M. Li and H. Meng. Attenuation or enhancement - a one-dimensional analysis on shock transmission in the solid phase of a cellular material. *International Journal of Impact Engineering*, 27(10):1049–1065, 2002.
- [83] D.D. Radford, N.A. Fleck, and V.S. Deshpande. The response of clamped sandwich beams subjected to shock loading. *International Journal of Impact Engineering*, 32(6):968–987, 2006.
- [84] D.D. Radford, V.S. Deshpande, and N.A. Fleck. The use of metal foam projectiles to simulate shock loading on a structure. *International Journal of Impact Engineering*, 31(9):1152–1171, 2005.
- [85] B.A. Gama, T.A. Bogetti, B.K. Fink, C.-J. Yu, T. D. Claar, H.H. Eifert, and J.W. Gillespie. Aluminium foam integral armor: a new dimension in armor design. *Composite Structures*, 52(3-4):381–395, 2001.
- [86] V. Crupi and R. Montanini. Aluminium foam sandwiches collapse modes under static and dynamic three-point bending. *International Journal of Impact Engineering*, 34(3):509–521, 2007.
- [87] A. Paul and U. Ramamurty. Strain rate sensitivity of a closed cell aluminium foam. *Materials Science and Engineering A*, 281(1-2):1–7, 2000.
- [88] K.A. Dannemann and J. Lankford Jr. High strain rate compression of closed-cell aluminium foams. *Materials Science and Engineering A*, 293(1-2):157–164, 2000.
- [89] V.S. Deshpande and N.A. Fleck. High strain-rate compressive behaviour of aluminium alloy foams. *International Journal of Impact Engineering*, 24(3):277–298, 2000.
- [90] S. Ramachandra, R. Sudheer Kumar, and U. Ramamurty. Energy absorption characteristics of closed-cell aluminium foam under low velocity impact. In *Proceedings of the 8th International Symposium on Plasticity and Impact Mechanics, IMPLAST 2003*, pages 421–431, Phoenix, New Delhi, 2003.
- [91] H. Zhao, I. Elnasri, and S. Abdennadher. An experimental study on the behaviour under impact loading of metallic cellular materials. *International Journal of Mechanical Sciences*, 47(4-5):757–774, 2005.
- [92] H. Zhao and S. Abdennadher. On the strength enhancement under impact loading of square tubes made from rate insensitive metals. *International Journal of Solids and Structures*, 41(24-25):6677–6697, 2004.

- [93] Y.-C. Chi. Investigate the blast load response of honeycomb sandwich panels. *Project No. 35, BSc. thesis, University of Cape Town*, 2005.
- [94] L. Librescu, S.-Y. Oh, and J. Hohe. Linear and non-linear dynamic response of sandwich panels to blast loading. *Composites Part B: engineering*, 35(6-8):673–683, 2004.
- [95] T. Hause and L. Librescu. Dynamic response of anisotropic sandwich flat panels to explosive pressure pulses. *International Journal of Impact Engineering*, 31(5):607–628, 2005.
- [96] Z. Xue and J.W. Hutchinson. Preliminary assessment of sandwich plates subject to blast loads. *International Journal of Mechanical Sciences*, 45(4):687–705, 2003.
- [97] Z. Xue and J.W. Hutchinson. A comparative study of impulse-resistant metal sandwich plates. *International Journal of Impact Engineering*, 30(10):1283–1305, 2004.
- [98] S. Guruprasad and A. Mukherjee. Layered sacrificial claddings under blast loading part I - analytical studies. *International Journal of Impact Engineering*, 24(9):957–973, 2000.
- [99] S. Guruprasad and A. Mukherjee. Layered sacrificial claddings under blast loading part II - experimental studies. *International Journal of Impact Engineering*, 24(9):975–984, 2000.
- [100] D. Karagiozova and N. Jones. Energy absorption of a layered cladding under blast loading. In N. Jones and C.A. Brebbia, editors, *Proceedings of the 6th International Conference on Structures Under Shock and Impact, (SUSI VI)*, pages 447–56, Southampton, 2000. WIT Press.
- [101] H.N.G. Wadley, N.A. Fleck, and A.G. Evans. Fabrication and structural performance of periodic cellular metal sandwich structures. *Composites Science and Technology*, 63(16):2331–2343, 2003.
- [102] G.W. Kooistra, V.S. Deshpande, and H.N.G. Wadley. Compressive behavior of age hardenable tetrahedral lattice truss structures made from aluminium. *Acta Materiala*, 52(14):4229–4237, 2004.
- [103] D.T. Queheillalt and H.N.G. Wadley. Cellular metal lattices with hollow trusses. *Acta Materiala*, 53(2):303–313, 2005.
- [104] C.-C. Liang, M.-F. Yang, and P.-W. Wu. Optimum design of metallic corrugated core sandwich panels subjected to blast loads. *Ocean Engineering*, 28(7):825–861, 2001.

- [105] Y.S. Tian and T.J. Lu. Optimal design of compression corrugated panels. *Thin-Walled Structures*, 43(3):477–498, 2005.
- [106] Iscor data sheet. General purpose and commercial steels: hot rolled steel plate and strip for general applications. technical report file reference a2.1. Technical report, Iscor-flat steel, 2004.
- [107] British standard method for tensile testing of metals. Technical report, British Standards Institution, 1987.
- [108] J.E. Shigley and C.R. Mischke. *Mechanical Engineering Design*, 6th Metric Edition. McGraw Hill, 2003.
- [109] P.W. Bridgman. *Studies in Large Flow and Fracture*. McGraw Hill, New York, 1952.
- [110] G. Mirone. A new model for the elastoplastic characterization and the stress-strain determination on the necking section of a tensile specimen. *International Journal of Solids and Structures*, 41(13):3545–3564, 2004.
- [111] Z.L. Zhang, M. Hauge, J. Odegard, and C. Thaulow. Determining material true stress-strain curve from tensile specimens with rectangular cross-section. *International Journal of Solids and Structures*, 36(23):3497–3516, 1999.
- [112] K.S. Zhang and Z.H. Li. Numerical analysis of the stress-strain curve and fracture initiation for ductile material. *Engineering Fracture Mechanics*, 49(2):235–241, 1994.
- [113] T. Børvik, O.S. Hopperstad, S. Dey, E.V. Pizzinato, M. Langseth, and C. Albertini. Strength and ductility of weldox 460 E steel at high strain rates, elevated temperatures and various stress triaxialities. *Engineering Fracture Mechanics*, 72(7):1071–1087, 2005.
- [114] S. Okazawa, T. Usami, H. Noguchi, and F. Fujii. Three-dimensional necking bifurcation in tensile steel specimens. *Journal of Engineering Mechanics*, 128(4):479–486, 2002.
- [115] P. Matic, G.C. Kirby III, and M.I. Jolles. The relation of tensile specimen size and geometry effects to unique constitutive parameters for ductile materials. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 417(1853):309–333, 1988.
- [116] Wierzbicki and Lee. Fracture prediction of thin plates under localized impulsive loading. part II: discing and petalling. *International Journal of Impact Engineering*, 31(10):1277–1308, 2005.

- [117] D. MacDougall and N. Petrinic. A local optimisation method for obtaining material model parameters. *Engineering Computations*, 17(2):175–188, 2000.
- [118] M. Langseth and O.S Hopperstad. Static and dynamic axial crushing of square thin-walled aluminium extrusions. *International Journal of Impact Engineering*, 18(7):949–968, 1996.
- [119] M. Langseth, O.S. Hopperstad, and T. Berstad. Crashworthiness of aluminium extrusions: validation of numerical simulation, effect of mass ratio and impact velocity. *International Journal of Impact Engineering*, 22(9-10):829–854, 1999.
- [120] G. T. Gray III. Classic split-hopkinson pressure bar technique chapter 6a. Technical report, Los Alamos National Laboratory, 1999.
- [121] S.T. Marais. Development and proving of a split hopkinson pressure bar used for high strain rate materials testing. Master’s thesis, University of Cape Town, 2001.
- [122] S.T. Marais, R.B. Tait, T.J. Cloete, and G.N. Nurick. Material testing at high strain using the split hopkinson pressure bar. *Latin American Journal of Solids and Structures*, 1(3):319–339, 2004.
- [123] G.R. Johnson and W.H. Cook. A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures. In *Proceedings of the 5th International Ballistics Symposium*, pages 541–547, The Hague, 1983.
- [124] M.D. Theobald and G.N. Nurick. Numerical investigation of the response of sandwich panels subject to blast loads. In M. Alves and N. Jones, editors, *Impact Loading of Lightweight Structures*, pages 521–534, Florianopolis, Brazil, 2005. WIT Press.
- [125] M.D. Theobald and G.N. Nurick. Numerical investigation of the response of sandwich-type panels using thin-walled tubes subject to blast loads. *International Journal of Impact Engineering*, 34(1):134–156, 2007.
- [126] W.J. Kang, S.S. Cho, H. Huh, and D.T. Chung. Modified Johnson-Cook model for vehicle body crashworthiness simulations. *International Journal of Vehicle Design*, 21(4-5):424–435, 1999.
- [127] K. Xu, C. Wong, B. Yan, and H. Zhu. A high strain-rate constitutive model for high strength steel. Technical report, SEA paper no. 2003-01-0260, 2003.
- [128] A. Söderberg and U. Sellgren. Modelling of strain hardening and strain rate hardening of dual phase steels in finite element analysis of energy-absorbing components. Technical report, KTH - Machine Design, Royal Institute of Technology (KTH), Sweden.

- [129] T. Masui, T. Nunokawa, and T. Hiramatsu. Shape correction of hot rolled steel using an on line leveller. *Journal of Japan Society for Technology of Plasticity*, 28(312):81–87, 1987-1999.
- [130] E. Markiewicz, P. Ducrocq, and P. Drazetic. An inverse approach to determine the constitutive model parameters from axial crushing of thin-walled square tubes. *International Journal of Impact Engineering*, 21(6):433–449, 1998.
- [131] G.R. Johnson and W.H. Cook. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. *Engineering Fracture Mechanics*, 21(1):31–48, 1985.
- [132] M. Park, J. Yoo, and D. Chung. An optimization of a multi-layered plate under ballistic impact. *International Journal of Solids and Structures*, 42(1):123–137, 2005.
- [133] W. Dabboussi and J.A. Nemes. Modeling of ductile fracture using a dynamic punch test. *International Journal of Mechanical Sciences*, 47(8):1282–1299, 2005.
- [134] N.K. Gupta, M.A. Iqbal, and G.S. Sekhon. Experimental and numerical studies on the behavior of thin aluminium plates subjected to impact by blunt- and hemispherical-nosed projectiles. *International Journal of Impact Engineering*, 32(12):1921–1944, 2006.
- [135] T. Børvik, A.H. Clausen, M. Eriksson, T. Berstad, O.S. Hopperstad, and M. Langseth. Experimental and numerical study on the perforation of AA6005-T6 panels. *International Journal of Impact Engineering*, 32(1-4):35–64, 2005.
- [136] W. K. Rule and S.E. Jones. A revised form for the Johnson-Cook strength model. *International Journal of Impact Engineering*, 21(8):609–624, 1998.
- [137] V. Balden and G.N. Nurick. Numerical simulation of the post-failure motion of steel plates subjected to blast loading. *International Journal of Impact Engineering*, 32(1-4):14–34, 2005.
- [138] J.D. Campbell and W.G. Ferguson. The temperature and strain-rate dependence of the shear strength of mild steel. *Philosophical Magazine*, 21(169):63–82, 1970.
- [139] W.S. Lee and C.Y. Liu. Comparison of dynamic and compressive flow behavior of mild and medium steels over wide temperature range. *Metallurgical and Materials Transactions A*, 36A(11):3175–3186, 2005.
- [140] Han Zhao. A constitutive model for metals over a large range of strain rates. identification for mild-steel and aluminium sheets. *Materials Science and Engineering A*, A230(1):95–99, 1997.

- [141] W.P. Grobbelaar and G.N. Nurick. An investigation of structures subjected to blast loads incorporating an equation of state to model the material behaviour of the explosive. In *Proceedings of the 7th International Symposium on Structural Failure and Plasticity (IMPLAST 2000)*, pages 185–194, Melbourne, Australia, 2000.
- [142] N. Jacob. The effect of stand-off distance on the failure of thin plates subjected to blast loads. Master's thesis, University of Cape Town, 2005.
- [143] S. Chung Kim Yuen and G.N. Nurick. Experimental and numerical studies on the response of quadrangular stiffened plates. part I: Subjected to uniform blast loads. *International Journal of Impact Engineering*, 31(1):55–83, 2005.
- [144] M.D. Olson, J.R. Fagnan, and G.N. Nurick. Deformation and rupture of blast loaded square plates - predictions and experiments. *International Journal of Impact Engineering*, 13(2):279–291, 1993.
- [145] G.H. Farrow, G.N. Nurick, and G.P. Mitchell. Modelling of impulsively loaded circular plates using the abaqus/explicit finite element code. In *Proceedings of the 13th Symposium on Finite Element methods in South Africa*, Stellenbosch, South Africa, 8-20 January 1995.
- [146] R.J. Bimha, G.N. Nurick, and G.P. Mitchell. Modelling the deformation of blast-loaded stiffened square plates. In *Proceedings of the First South African Conference on Applied Mechanics (SACAM)*, Midrand, South Africa, 1-5 July 1996.
- [147] G.S. Langdon, S. Chung Kim Yuen, and G.N. Nurick. Experimental and numerical studies on the response of quadrangular stiffened plates. part II: localised blast loading. *International Journal of Impact Engineering*, 31(1):85–111, 2005.
- [148] G.M. Nagel and D.P. Thambiratnum. A numerical study on the impact response and energy absorption of tapered thin-walled tubes. *International Journal of Mechanical Sciences*, 46(2):201–216, 2004.
- [149] R.K. Wharton, S.A. Fromby, and R. Merrifield. Airblast tnt equivalence for a range of commercial blasting explosives. *Journal of Hazardous Materials*, 79(1):31–39, 2000.
- [150] D.M. Lumpp. An experimental investigation of circular plates, beams and stiffened circular plates subjected to impulsive loading. Master's thesis, University of Cape Town, 1990.
- [151] G.N. Nurick. *Large deformations of thin plates subjected to impulsive loading*. PhD thesis, University of Cape Town, 1987.

- [152] L.K. Enstock and P.D. Smith. Measurement of impulse from the close-in explosion of doped charges using a pendulum. *International Journal of Impact Engineering*, 34(3):487–494, 2007.
- [153] S. Santosa and T. Wierzbicki. Effect of an ultralight metal filler on the bending collapse behavior of thin-walled prismatic columns. *International Journal of Mechanical Sciences*, 41(8):995–1019, 1999.
- [154] Waterloo Maple Inc. Maple 8, April 2002.
- [155] A. Reyes, O.S. Hopperstad, A.G. Hanssen, and M. Langseth. Modeling of material failure in foam-bases components. *International Journal of Impact Engineering*, 30(7):805–834, 2004.