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TECHNIQUES FOR GENERAL MODELLING OF INNOVATION STRATEGIES IN
ENTRANT-INCUMBENT SCENARIOS

BY

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This thesis contributes to the literature in industrial organization economics on strategic behavior with respect to innovation. In particular, it seeks to extend the applicability of conclusions drawn from a Nash Equilibrium framework in response to the criticisms of Sutton (1991, 1998) through the use of sensitivity analysis. In addition, the focus of the investigation is the link between innovation and persistence of monopoly. The work of Gilbert & Newbery (1982) is used as a foundation for building a new analytical framework for application to innovation games in the traditional 'incumbent-entrant' scenario, but in the absence of the usual restrictive patenting assumption. The analysis of this framework is achieved by justifying an axiomatic definition of the payoffs to players in various games and then using a computer software program to generate the Nash Equilibrium solution for many hundreds of scenarios. The sensitivity of the N.E. solution to changes in model parameters can then be ascertained. The general conclusion drawn from this approach is that the findings of much of the literature hold only under certain highly restrictive conditions. The investigation shows that barriers to entry are of far more relevance to the identity of the innovator than the characteristics of competition in the market once entry is achieved. It is also demonstrated that the incumbent is more likely to innovate first in an industry that demonstrates a systemically high level of growth. Scenarios of particular unintended risk to social welfare through regulatory attempts at stimulating competition are also identified.

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University of Cape Town

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Introduction

Theoretical background and motivation

The body of literature concerning investigation into market structure hypotheses, both theoretical and empirical, supports the point raised by Sutton (1998) that the results derived from any model of product innovation must derive from, if not industry specific variables, then assumptions made as to the nature of the innovative process. Consensus on outcome is impossible when debate continues on the inputs that lead to that outcome. For example, Gilbert and Newbery (1982) used a deterministic model of product innovation to derive the result that an incumbent firm is more likely to innovate than a potential entrant. Reinganum (1983) then derived the opposite result using a stochastic model of innovation, namely that the entrant was more likely to innovate. Although debate continued between these authors as to whether their differences in result could be directly attributed to the particular assumptions made about the nature of the innovative process (Reinganum arguing in favour, Gilbert and Newbery noting other differences in assumptions) the point is made. Whilst debate continues on the way in which a game of product innovation is set up, consensus shall never be reached on a general result.

The models of Gilbert and Newbery (1982) and Reinganum (1983) focused on innovation occurring within an incumbent monopolist framework. This marked a departure from other models of product innovation being published at the time which were concerned with a single innovation being produced by a number of identical

agents (see Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980) and Reinganum (1982) for examples of such models). This again illustrates the problems associated with reaching a consensus on the issue. The analysis of a situation in which a monopolist is threatened by entry is likely to offer different insights into analysis of a patent race than between homogenous firms. The incumbent-entrant framework is one that has aroused great interest not least due to the diametrically opposed conclusions drawn by separate authors. It is this framework that is the subject of this thesis.

The framework is generally a simple one. An incumbent firm, operating in a particular product market, is normally involved in a patent race for a superior product with another firm from outside of the industry, the entrant. It is often the case that entry into the market, post innovation is free. The industry post-innovation can then be described with reference to a number of variables. The characteristics of competition in the secondary market are clearly important. The degree of cannibalisation of the original product by the innovation is also critical. Does the innovation render the original product obsolete? Finally the possible shelving of products is another way of structuring the post-innovation environment differently. By shelving, I mean the removal of a product from the industry. Generally this would be a choice by the incumbent to either keep the original product or the innovation in the market. Given all of these characteristics, the various models in the literature attempt to formulate an understanding of whether the entrant or incumbent has the greater incentive to innovate.

The idea that a potential entrant enjoys a greater incentive to innovate than the incumbent has been termed 'the Incumbent's Curse' by Chandy and Tellis (2000). One would expect industries in which it applies to be characterised by cycles of entrant innovation, subsequent leadership and incumbency, and finally usurpation by new innovating entrants. The concept of incumbent failure to innovate receives support in theoretical modelling (Ghemawat 1991, Reinganum 1983) but also faces opposition (Gilbert and Newbery 1982). Empirical case studies (Christensen 1993, Henderson and Clark 1990) can also offer support for the concept of incumbent failure but no inference could be drawn in any general way, owing to the criticism raised by Sutton (1998), that the results rest on industry specific variables. Henderson and Clark (1990), for instance, focus on photolithographic aligners, while Ghemawat (1991) analyses private branch exchanges. Even when these industries are accurately modelled, something itself always open to question, generalization from such restricted cases is perilous. While it is possible that a theoretical model designed for a specific industry may accurately reflect the nature of the product innovation game in that industry, it is also possible that any assumptions made in order to render the modelling possible may be corrupting the result.

Certainly any model that, in seeking to explain a result in a specific industry, generates a general result must come under some suspicion. Indeed, in this particular scenario, one would not expect a general result under any circumstances. The very fact that there is empirical support for both points of view suggests that the search for a general result is futile. Even though there are documented examples as mentioned of incumbency inertia, there are equally well-documented examples of incumbency innovation. For example, General Electric introduced fluorescent lamps and Philips

introduced compact disc players. Any model that contradicts either possibility in entirety must be suspect, to say the least.

This criticism applies to Ghemawat (1991), which, in endeavouring to explain the usurpation of an incumbent firm in the private branch exchange industry, deploys a model that generates a general result which predicts incumbency failure as always and inevitably occurring. If such a result were obtained by introducing limitations on the model that were motivated by attempting to accurately describe the industry in question, thereby limiting up front the scope of the result, then at least *this* criticism could be deflected. However, in the case of Ghemawat's model we find assumptions on the nature of price competition in the secondary product market and the relative size of the secondary product market to the original product market that are not empirically motivated, but are introduced for the sake of analytic tractability. I demonstrated in Wolf (2001) that these assumptions alone, if applied to a static game of product innovation with a non-conditional stochastic innovative process, could limit the game solution to a single Nash equilibrium, in which the entrant attempted to innovate and the incumbent did not. Therefore the conclusion drawn by the model, that we should expect incumbency failure, was driven entirely by the constraints of the empirically unmotivated assumptions made.

It is vulnerability to this sort of criticism that Sutton (1998) seeks to avoid in his investigation into the relationship between market structure and R&D. As in the majority of the innovation literature, Sutton is most concerned with the relationship between market structure and the societal level of R&D undertaken. The present project, however, takes a less exclusively welfarist approach. Whilst the optimal level

of R&D for society would be interesting to policy makers at the governmental level, it is also interesting to examine conditions governing the optimal level of R&D for a company. What this project seeks to achieve is an understanding of the incumbent – entrant scenario, independent of assumptions made as to the specific nature of any particular industry.

Sutton's argument is that owing to the definition of a Nash Equilibrium as an equilibrium in strategies, any solution must be determined by a formulation of the actions available to the players as parts of their game strategies. The resulting dependence of analytical solutions on precise initial restrictions on which actions are available to firms in innovation races is the source of much of the methodological criticism of the standard incumbent-entrant framework in the literature. High sensitivity of outcomes to very particular action-set specifications has been, as we shall document in detail later, the source of great difficulty in settling on stable, unequivocal results that hold across persuasive ranges of models. One can always argue, it seems, over whether the representation of a situation being modelled accurately captures real industrial situations of recurring relevance. Sutton (1991, 1998) seeks to address this concern by adopting what he calls a 'Bounds' approach. This method seeks to identify general principles underlying the competitive mechanisms of firms across wide spectra of industries and derive from these pre-strategic restrictions on the sets of possible outcomes. Such understanding amounts to determining 'boundaries' around outcomes within which any solution derived from strategic considerations should fall. It is crucial for the sake of the critical point that these boundaries be derived without recourse to specification of the structures of particular games.

For example, Sutton's (1991) investigation into the relationship between market size and market concentration shows that, whilst a statistical inverse relationship exists, there is a bound to the decline of market concentration as a market grows in size. Sutton identifies two forms of endogenous sunk costs, advertising and research and development (R&D), that are likely to lead to this bound on the level of concentration. This is because firms typically find it rational to raise their advertising and R&D expenditures as market size increases. This increase in sunk costs means that less competition is necessary to ensure firms are profitable. Sutton then is able to demonstrate that strategic factors influence the statistical relationship. The bounds approach predicts a lower bound to the decline of market concentration as a market grows in size, conditional on the potential for competitive escalation of sunk costs by firms. By focusing on a statistical relationship, albeit with a strategic element to the approach, Sutton is able to concentrate on the outcome of a game rather than an equilibrium specified with recourse to strategies. He is therefore able to avoid the problems discussed earlier that arise from the necessarily hyper-precise specification of strategies within a Nash Equilibrium framework.

A key aim of this investigation is to respect Sutton's philosophy, but without joining him in abandoning Nash equilibrium as the primary solution concept. Thus I shall instead attempt to extend the NE framework attempt to derive the most general result possible, but, as noted above, follow Sutton in not applying restrictive assumptions about the nature of the industry in which the analysis is taking place. It may thus seem that I am committed to denying an unavoidable trade-off, seeking to gain *greater* generality using *fewer* analytical resources. However, what will be novel in my

approach is use of sensitivity analysis to measure the relative robustness of various NE. I shall demonstrate that this methodology offers an excellent way of representing a 'population' of Nash Equilibria in much the same way that Sutton seeks to enclose all possible Nash solutions within his more logically demanding solution concept.

Here, then, is the agenda for this project. First we must examine the degree to which the assumptions made in the setting up of a model affect the result. This will involve re-working the model of Gilbert & Newbery (1982), dropping their assumption that patents are always available and achieved. This in itself will demonstrate the importance of industry-specific assumptions in driving putatively general results. I shall then subject the less restricted model to the full scope of the sensitivity analysis methodology. Following this I shall extend the model into two further generations and use sensitivity analysis to compare the results of these models with other results from the recent literature.

The methodology employed here is a crucial component of this thesis. Not only is it important from a purely practical perspective in generating the results of the investigation, but a central theme of the thesis is the failure of most of the literature to address Sutton's concerns as discussed above.. It is my belief that the methodology of sensitivity analysis is a vast improvement, permitting general analysis limited by far fewer restrictions.. Using techniques made available by newly available, but off-the-shelf, software, I am able to generate novel ways of representing the influences of market structure on innovation, yielding a picture both richer and more general than can be obtained from the standard algebraic representations traditional in the literature.

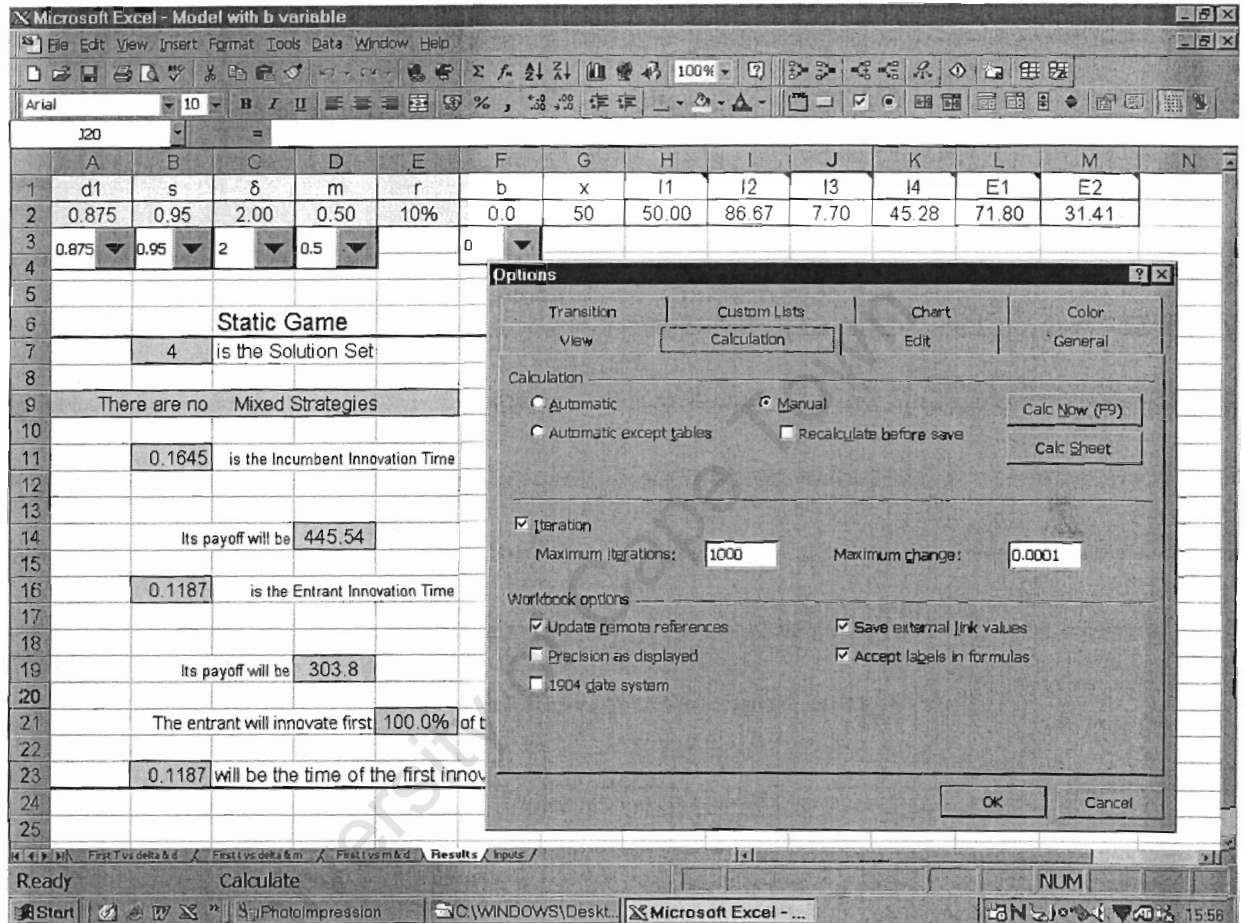
That is not to claim that sensitivity analysis is being offered as the basis for an ultimate solution, at least here.. First, we shall see that the effectiveness of the methodology declines as the structure of the game evolves into a more complicated form (though it still at least says *something* about games that are beyond the range of what the standard algebraic literature has even attempted to model). Second, whilst the methodology allows one to make far fewer assumptions about the nature of the industry in which the innovation game takes place, it is still necessary for some framework to be imposed in order for there to be something to analyse. In the static version of the game, this means one must continue to impose, following Gilbert & Newbery (1982), the idea of a strategy as being an investment level in R&D, given some relationship between that investment and the expected time of innovation. I reiterate, however, that this is a much more modest, and general, framework of assumptions than anything found in the relevant literature to date.

The analytic technique

This section demonstrates and explains how the results that are to be discussed are generated. Having built a game, I use the spreadsheet program Microsoft Excel™ to calculate its Nash Equilibria through a process of iteration. This is done because the equations derived for the NE are only obtainable using the Lambert W-function, a feature not contained on the program. However, this function is itself an iterative algorithm and so I can derive the results through my own iterative process. The software offers this capability as well as the flexibility to specify the number of

iterations required and the accuracy which is required. The screenshot below shows the relevant option window within the software..

Screenshot 1: MS Excel™ allows for solutions derived through iteration.



What is of primary importance is that through a series of logical formula, and sheer calculative power, we can use the program to derive the iterated NE solution for any specification of industry specific variables we choose. The spreadsheet representing the model simply takes the input variables that have been defined by the modeller, uses them to generate the payoff functions and then derives the NE from these. The process is no different from that which would be followed manually for a far simpler framework. It is the increased processing power of the computer that makes analysis of this nature possible.

In order to better illustrate how this works I shall work through an far simpler example than the models which shall form the bulk of this thesis. I shall adopt a basic Cournot framework in which one firm has a lower cost of production. The industry is described as one in which there are two firms. Total industry output is $Q = q_1 + q_2$ with an industry demand function of $P(Q) = a - Q$. The cost functions of the firms are $C_1(q_1) = cq_1$ and $C_2(q_2) = 2cq_2$.

We shall make the assumption that $c < \frac{a}{3}$.

The strategies of the firms are quantities drawn from $S_i = [0, \infty]$ or actually $S_i = [0, a]$ as $P(Q)=0$ for $Q>a$. The firms choose their strategies simultaneously.

The payoffs are profit, given as

$$\begin{aligned}\pi_1(q_1, q_2) &= q_1 [P(q_1 + q_2) - c] \\ \pi_2(q_1, q_2) &= q_2 [P(q_1 + q_2) - 2c]\end{aligned}$$

In N.E. a strategy must maximise utility for each player subject to the choice of the other player.

So the strategy pair (q_1^*, q_2^*) is an N.E. if, for each firm i , q_i^* solves the maximisation problem $\max_{0 \leq q_i < \infty} \pi_i(q_i, q_j^*)$.

The solution to this problem is

$$\{q_1^*, q_2^*\} = \left\{ \frac{a}{3}, \frac{a-3c}{3} \right\} \text{ with total industry output as } \frac{2a-3c}{3}.$$

This very basic example gives an expression for the solution with two inputs, namely a and c . We can then input this information into the software program to calculate numerical scenarios. Screenshot 2 below demonstrates how the equations may be

inputted into the spreadsheet. The blue lines merely demonstrate the path of the inputs to the output cells.

Screenshot 2: The basic Cournot example in a spreadsheet.

	A	B	C	D	E
1		a	c		
2					
3					
4	NE output for firm 1		=B2/3		
5	NE output for firm 2		=(B2-(3*C2))/3		
6	NE output for industry		=C4+C5		
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					
24					

Naturally it is simple for the software to calculate a numerical example. However, in the interests of testing the sensitivity of any solution to the inputs we must look at many scenarios. To do this, it is necessary to create a data table which calculates the desired output cell by altering the specified input cells according to the choice of the modeller. In screenshot 3 below, I demonstrate a data table which generates the output cell for the NE output for the industry (“C6”) for various vectors of the inputs a (“B2”) and c (“C2”).

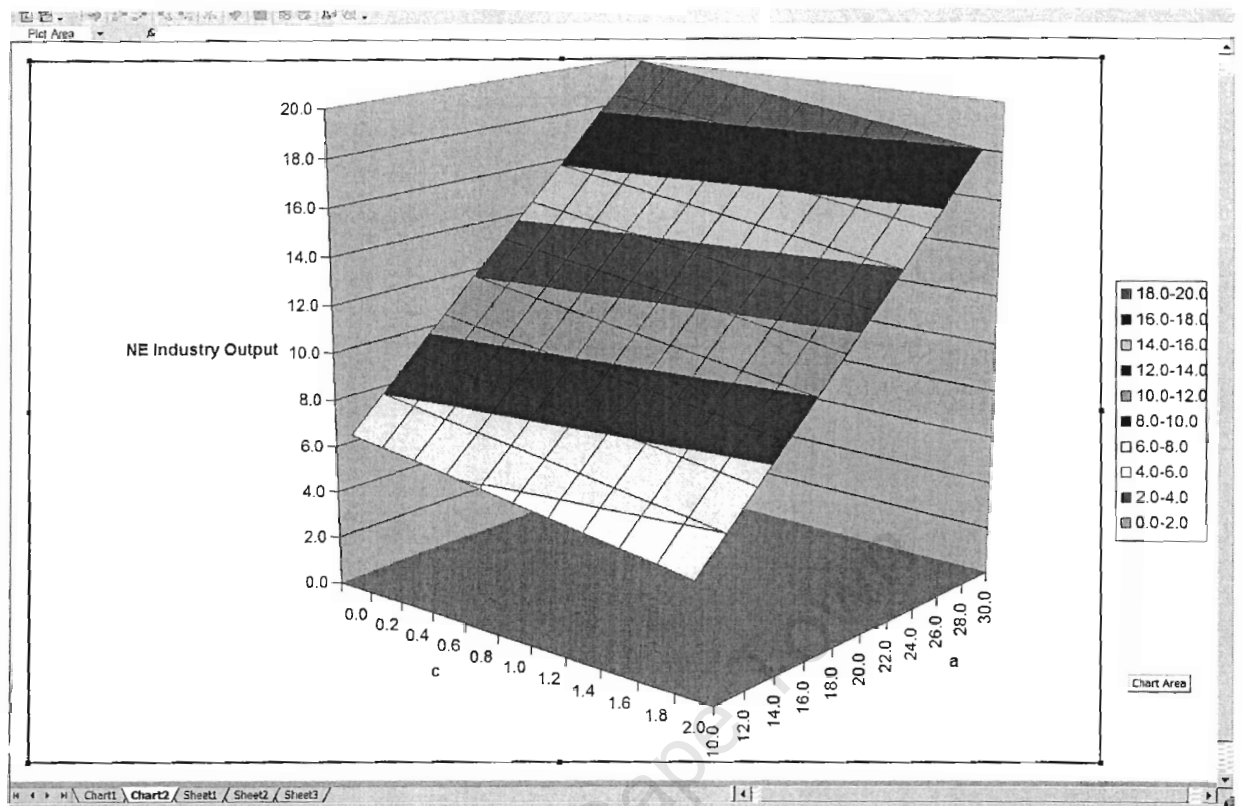
Screenshot 3: A data table derived from the Cournot example

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		a	c											
2		20	1											
3														
4	NE output for firm 1		6.67											
5	NE output for firm 2		5.67											
6	NE output for industry		12.3											
7														
8			c											
9			0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
10	a	10.0	6.7	6.5	6.3	6.1	5.9	5.7	5.5	5.3	5.1	4.9	4.7	
11		12.0	8.0	7.8	7.6	7.4	7.2	7.0	6.8	6.6	6.4	6.2	6.0	
12		14.0	9.3	9.1	8.9	8.7	8.5	8.3	8.1	7.9	7.7	7.5	7.3	
13		16.0	10.7	10.5	10.3	10.1	9.9	9.7	9.5	9.3	9.1	8.9	8.7	
14		18.0	12.0	11.8	11.6	11.4	11.2	11.0	10.8	10.6	10.4	10.2	10.0	
15		20.0	13.3	13.1	12.9	12.7	12.5	12.3	12.1	11.9	11.7	11.5	11.3	
16		22.0	14.7	14.5	14.3	14.1	13.9	13.7	13.5	13.3	13.1	12.9	12.7	
17		24.0	16.0	15.8	15.6	15.4	15.2	15.0	14.8	14.6	14.4	14.2	14.0	
18		26.0	17.3	17.1	16.9	16.7	16.5	16.3	16.1	15.9	15.7	15.5	15.3	
19		28.0	18.7	18.5	18.3	18.1	17.9	17.7	17.5	17.3	17.1	16.9	16.7	
20		30.0	20.0	19.8	19.6	19.4	19.2	19.0	18.8	18.6	18.4	18.2	18.0	
21														
22														
23														
24														

All of the cells in the table above represent the total industry output for the game under the inputs that correspond to that position in the table on the respective axes.

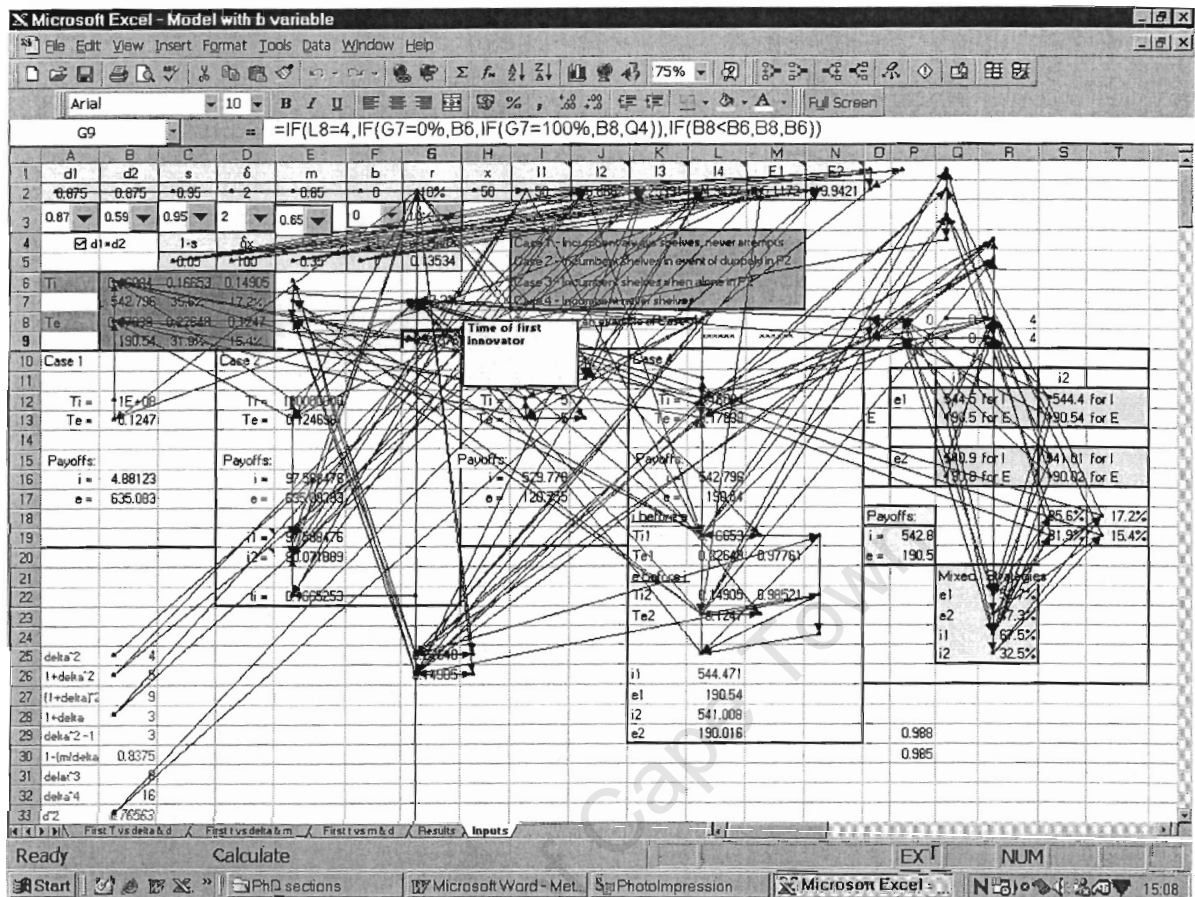
Much of the time a table of data is not a perspicacious way of representing the results drawn from the model, since general patterns are often buried from view in them as games become more complex than the present example. I will thus often represent the data in graphical form. For the above simple example this does not particularly aid the understanding as the relationships between the inputs and outputs are linear. As we shall see through this thesis, this is often not the case and the graphical representation of data is an invaluable way of examining results. Screenshot 4 below therefore shows the data from the above table in graphical form.

Screenshot 4: A graphical depiction of the data table



This process is in essence what I do for the far more complicated models that I shall derive in the main body of the thesis. At this stage, having not yet introduced the variables to be examined nor the structure in which I shall examine them, it would be premature to try to fully demonstrate the method by which the results will be obtained. But although the actual models will be more complicated, it will be just as straightforward to derive their results as it is for this very simple Cournot example. The screenshot below shows the links between the inputs and outputs in one of the models that I shall build. This looks somewhat more complicated than it actually is as, owing to the structure of the game, some of the results must be obtained through iteration and this causes a circular set of references in the model.

Screenshot 5: One of the models used in the thesis.



The logic of the representational schema remains the same as in the trivial Cournot example. There is a series of inputs from which the outputs are generated. However, this screenshot demonstrates the overall complexity of the model and explains why deriving a result in this way manually is not feasible. It is only with the power of machine processing that this methodology becomes an option. The potential for obtaining novel, interesting results is therefore unsurprising, as it was simply not possible to perform analyses of this nature until relatively recently.

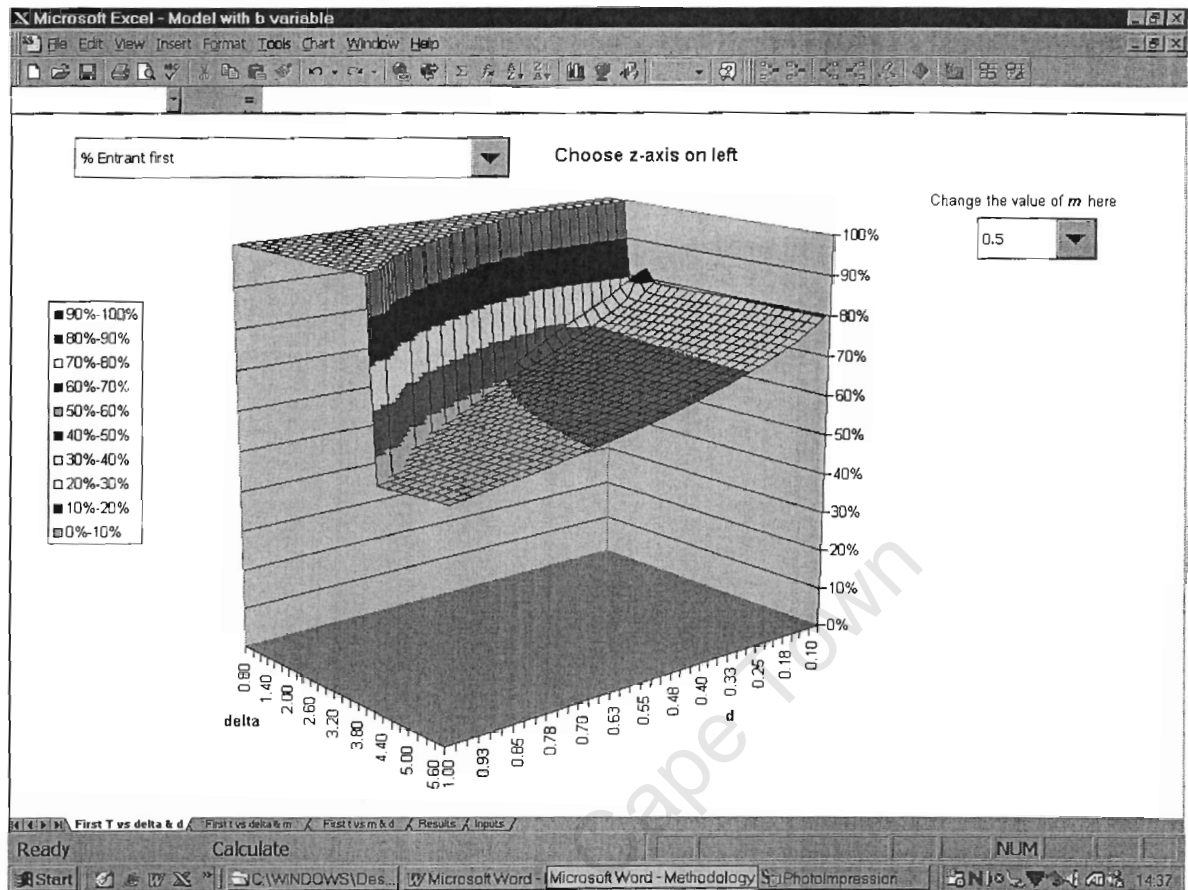
In the same way as in the Cournot example, we can generate data tables which will give us an entire population of outputs for various vectors of inputs.

Screenshot 6: An actual data table of results from the thesis.

The screenshot shows a Microsoft Excel spreadsheet titled "Model with b variable". The active cell is B5, containing the value "i60". The formula bar shows "=BY3". The spreadsheet contains a large table of numerical data. The columns are labeled BW, Bx, BY, BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL, CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ, DA, DB, DC, DD, DE. The rows are numbered 4 through 43. The data includes variables such as "Incumbent R&D investment", "Entrant R&D investment", "Total Industry R&D investment", "Time of first innovation", "% Entrant first", "Boone - who gets patent", and "Boone - incentive for lab". The values are generally small, ranging from 0.00 to 1.00, with some values like 0.025 and 0.005.

Once these data have been obtained, it is again a simple task to convert the table into a graphical form that assists analysis. Whilst the graph and data table will mean nothing to the reader yet, as the model has not yet been developed, this section should serve to explain how the graphs and data tables, in which form the results are presented throughout the thesis, are derived.

Screenshot 7: The data table converted into graphical form.



One thus obtains a highly readable and readily interpretable compression of a game much too complex to be fully shown algebraically, let alone on a tree.

The use of this representational system for displaying the results of complex games allows one to respect the philosophy of Sutton without abandoning Nash Equilibrium analysis. Exact specification of industry-specific variables, and the resultant dependence of the solution on this specification, is avoided. Instead a full spectrum of possibilities for these variables, and the full set of vectors of the combinations of these variables, may be analysed simultaneously leading to the generation and visualisation of a population of Nash Equilibria. This system allows one to observe a 'dynamic' Nash Equilibrium across the entire spectrum of possible industry-specific variables.

Gilbert & Newbery (1982) revisited with no patents – the four solution set model

In this section I shall outline in detail the relevant literature that forms the basic framework around which this investigation is built. I shall also discuss what additional modifications I shall make to this framework and then state this framework in an algebraic form. I shall also present the Nash Equilibrium solution to this model.

We first outline the original model due to Gilbert and Newbery (1982):

An established firm has a monopoly position in the sale or manufacture of a product (labelled product 1). Entry into the monopolized industry can only take place through the invention and patenting of a single patentable substitute for the monopolist's product. The cost of inventing the substitute (labelled product 2) depends only on the expected lag before a patentable design can be produced. The date of invention, T , is a deterministic function of the time path of expenditures. The present value of an optimal expenditure path defines a cost function $C(T)$, that is a decreasing function of the invention date. The cost function is the same for all firms engaged in research and development for the substitute product.

The strategy space for each firm is restricted to the research and development expenditure on product 2 and the price(s) the firm charges for the product(s) it sells. Before the innovation, the incumbent earns profits from his monopoly in the original product market. This is given as $\pi_i(P_i^1)$ where $P_x^y(x = i, e; y = 1, 2)$ denotes firm x 's maximising choice of price for product y . After patenting, profits to the incumbent if

if patents are given as $\pi_i(P_i^1, P_i^2)$. If the entrant patents then the incumbent earns $\pi_i(P_i^1, P_e^2)$ and the entrant earns $\pi_e(P_i^1, P_e^2)$.

Invention is treated as a deterministic function of expenditure. Therefore, the amount that the entrant will pay for the patent in a competitive bidding process with other entrants will be

$$(1) \quad C(T) = \int_T^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt$$

which are the returns to the entrant of holding the patent from time T , discounted to the present value at an interest rate r .

In this case the incumbent profits would consist of the monopoly in the 1st product market up until time T and a reduced profit thereafter according to the effect of the new product on the original product market. This is given as

$$(2) \quad V_{entry}^i = \int_0^T \pi_i(P_i^1) e^{-rt} dt + \int_T^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt$$

But what if the incumbent was to consider inventing before time T , i.e. $T-\varepsilon$ where ε is some small positive number? The cost of obtaining the patent would then be $C(T) + \delta(\varepsilon)$ where δ represents the extra amount invested to reduce innovation time by ε . The returns from doing so would be given as

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But what if the incumbent was to consider inventing before time T , i.e. $T-\varepsilon$ where ε is some small positive number? The cost of obtaining the patent would then be $C(T) + \delta(\varepsilon)$ where δ represents the extra amount invested to reduce innovation time by ε . The returns from doing so would be given as

$$(3) \quad V_{patent}^i = \int_0^{T-\varepsilon} \pi_i(P_i^1) e^{-rt} dt + \int_{T-\varepsilon}^{\infty} \pi_i(P_i^1, P_i^2) e^{-rt} dt - [C(T) + \delta]$$

In the limit, both ε and δ tend to zero. If we also substitute in equation (1) for $C(T)$ then an expression for the difference between profits with pre-emption and profits with entry is

$$(4) \quad \begin{aligned} V_{patent}^i - V_{entry}^i &= \int_T^{\infty} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_T^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt - \int_T^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt \\ &\equiv \int_T^{\infty} \left\{ \pi_i(P_i^1, P_i^2) - \left[\pi_i(P_i^1, P_e^2) + \pi_e(P_i^1, P_e^2) \right] \right\} e^{-rt} dt \end{aligned}$$

This gives the result that the incumbent profits from patenting exceed those from not patenting if:

$$(5) \quad \pi_i(P_i^1, P_i^2) > \pi_i(P_i^1, P_e^2) + \pi_e(P_i^1, P_e^2)$$

In other words, if there is any reduction in total industry profit as a result of patenting, then the incumbent has a greater incentive to patent the innovation than the entrant. This inequality will weakly hold in all scenarios as there are no situations in which two firms can outperform a monopoly as, if he wishes, the monopolist could always duplicate the actions of the two duopolists.

Gilbert & Newbery (1982) is an example of the type of Incumbent-Entrant model that I have described in the introduction.. The conclusion of their model is that under

certain conditions the incumbent will pre-emptively patent an innovation and thus monopoly will persist in an industry. The article generated great interest and was followed closely by Reinganum (1983), another very similar model that generates opposite results, which is also constantly referenced in the literature,. Reinganum opted to use a stochastic process of innovation, in contrast to the deterministic process adopted by Gilbert & Newbery. I shall now offer a brief summary of the Reinganum model.

Suppose an incumbent, firm 1, and an entrant, firm 2, can engage in R&D to reduce cost from c_h to c_l . The R&D process is a Poisson process with firm i 's success rate being $h(x_i)$, where x_i is firm i 's R&D expenditures at any moment and h is a concave and increasing function with $h'(0) = \infty$. Starting from time 0, the probability that firm i has not succeeded at time t is

$$(6) \quad 1 - \int_0^t h(x_i) e^{-h(x_i)s} ds = 1 - (1 - e^{-h(x_i)t}) = e^{-h(x_i)t}$$

The probability that at time t none of the firms has made the discovery is $e^{-[h(x_1)+h(x_2)]t}$.

Firm 1's discounted sum of expected profit, as a function of x_1 and x_2 , is

$$(7) \quad V_1 = \int_0^{\infty} e^{-rt} e^{-[h(x_1)+h(x_2)]t} \left[\pi^m(c_h) - x_1 + h(x_1) \frac{\pi^m(c_l)}{r} + h(x_2) \frac{\pi^d(c_h, c_l)}{r} \right] dt$$

$$= \frac{\pi^m(c_h) - x_1 + h(x_1) \frac{\pi^m(c_l)}{r} + h(x_2) \frac{\pi^d(c_h, c_l)}{r}}{r + h(x_1) + h(x_2)}$$

Firm 2's discounted sum of expected profit is

$$\begin{aligned}
 (8) \quad V_2 &= \int_0^{\infty} e^{-rt} e^{-[h(x_1)+h(x_2)]t} \left[h(x_2) \frac{\pi^d(c_h, c_l)}{r} - x_2 \right] dt \\
 &= \frac{h(x_2) \frac{\pi^d(c_h, c_l)}{r} - x_2}{r + h(x_1) + h(x_2)}
 \end{aligned}$$

The question of whether $x_1^* > x_2^*$ within the Nash Equilibrium pair (x_1^*, x_2^*) that results from this specification is determined by the trade-off between two effects: the replacement effect and the efficiency effect. The replacement effect says that since by innovation the incumbent replaces itself, the incumbent has less incentive than the entrant. The efficient effect says that innovation by the incumbent avoids the dissipation of rents through competition when the entrant innovates. Either effect may dominate. Reinganum demonstrates this by considering the case of drastic innovation. As under drastic innovation the efficient effect is zero, the replacement effect is dominant and the entrant is more likely to innovate first.

Her claim that it was the stochastic nature of the innovation technology that could explain the difference in the conclusions drawn is disputed by Gilbert & Newbery in a published comment. They claim that it is the difference in the order of play that primarily accounts for the discordant conclusions, Reinganum having opted for simultaneous moves whereas they had given the incumbent firm a first mover advantage. Reinganum's reply in the same edition of the journal rightly rejects this criticism, noting that not only is there 'no explicit discussion in G-N's 1982 article

regarding the order of play in their game', (p.243) but that owing to the structure of the game 'the simultaneous-move equilibrium is identical to the equilibrium specified by G-N.'(p.243). Whilst it could be argued that Gilbert & Newbery's conclusion that the incumbent would pre-emptively patent implies a first mover advantage, (for how else could pre-emption occur?), Reinganum is right that this is not explicitly stated. Indeed Gilbert & Newbery even state that a 'formal model of the patent competition is that of an auction market. Each firm enters a bid which is the maximum present-value amount that the firm will spend on research and development. (Firms can be thought of as bidding for R&D services).' (p.517). Given that one can derive the same result from both simultaneous move 'sealed-bid' auctions and sequential bid auctions, this comment can be seen as supporting Reinganum's point that the distinction is not relevant. Salant (1984) also suggests that the outcome could be altered if patents were considered to be transferable, a characteristic not incorporated by Gilbert & Newbery.

It is clear, then, that the results of Gilbert & Newbery have already been criticised for their sensitivity to certain highly particular assumptions made in the formulation of the game. What then, do I intend to add to the discussion? First, the assumption of the availability of patents has never been itself dropped. This, then, is one novelty I will introduce. Second, I intend to build further generations of a non-patent based innovation model within the incumbent-entrant framework. After demonstrating the impact of the removal of the patenting assumption I shall go on to demonstrate the further significance of removing other assumptions from the model *framework* shared by all of the authors just discussed. In doing so, I will demonstrate my contention that a basic algebraic formulation of results is not sufficient to convey understanding of

truly general and systematic industrial properties, but that an alternative approach has the power to do so.

This section then seeks to examine the significance of assuming the availability of patenting, in terms of its influence on the result of the general entrant-incumbent model. Following Gilbert and Newbery, a deterministic model demonstrating the persistence of monopoly may be constructed. Whereas Gilbert & Newbery allowed innovations to be patented, I shall reconstruct the model, not allowing patents. This is not an exercise in criticising the presence of the patenting assumption. Rather, it is an investigation into the significance of the assumption on the outcome of the model. Empirically, innovation competition often occurs in contexts not well, or at all, governed by readily obtainable and effective patent protection; so a truly general model should not assume it, but should have resources for introducing it as a variable where appropriate.

A model such as this is clearly highly simplified and the authors make no claims that it offers a full explanation of the innovative process. If we remove the assumption of a patentable innovation then the deterministic model of innovation changes considerably. I also make no claim that this offers a complete model. All I am attempting to do within this section is to show that an alternative result to Gilbert & Newbery's can be derived just by removing the patenting assumption. This is something we should expect for, as Harris & Vickers (1985) point out, in the Gilbert & Newbery model, the incumbent is only concerned that no rival firm wins the patent. By removing the ability to patent the innovation, we are removing the power of the

incumbent to prevent rival firms innovating. This alone alters the strategic element of the game significantly and we shall see how, under certain conditions, the result is indeed overturned.

The removal of the patenting assumption is a move for which there is some theoretical and empirical support. Greenstein & Ramey (1998) discuss in their model the fact that patent protection is not available to many innovators. Bessen & Maskin (2000) claim that the computer software industry has offered historically weak patent protection. Mansfield (1985) found that information concerning 'the detailed nature and operation of a new product or process generally leaks out within about a year. (p.217). This might seem to be support for the 'fast second' strategy propounded by Baldwin & Childs (1969) in which some companies may invest in an ability to copy an innovation quickly rather than in a R&D program. This would certainly alter the nature of the game. However, analysing the model without patents is a mere extension of this theory. By allowing both firms to enter the innovation product market, we can represent this idea of imitation but without placing any constraints on the lag-time that must follow a successful innovation. An investment in R&D by a firm can also be seen as an investment in imitating capability. If one firm innovates first, then the other firm can imitate a period of time later, that period of time being a function of how far it is has progressed along its own innovative process. Both firms make an investment in R&D that can result in either innovation, if the firm innovates first, or imitation of the innovation if it does not innovate first.

Imitation then would only have distinct economic significance if it was expected to be less costly than innovation itself. Naturally this can be incorporated into the model. However, I am attempting to isolate the direct influences on innovation within a specified, but still highly general, market structure, and derive an understanding of them. If the ability to 'free-ride' on the innovative process were introduced then this would dramatically alter the framework of the game and certainly negate the comparison between my results and those obtained by Gilbert & Newbery (1982). It would certainly cease to be an investigation into the direct effects of market structure on innovation, instead becoming an investigation into the effect of imitation on the incentive to innovate. This is an important issue, but not the topic of the present study.

We can specify the removal of the patenting assumption explicitly by alluding to the equivalence of a patent race to the situation of an independent R&D laboratory that auctions its innovation to the highest bidder, as in Gilbert & Newbery (p.517). Hence we can assert that the race is won by the highest bidder / largest investor with certainty. We can use the same argument to support the process in this model. Purchasing the rights to the innovation no longer guarantees exclusivity but the amount paid determines how soon the rights are granted. Investing the most also does not guarantee the first innovation but it does guarantee the expectation of the first innovation.

The structure of play is also important here. Gilbert & Newbery (1984), in a comment on Reinganum (1983), say that the fact that the incumbent has a first-mover advantage in Gilbert & Newbery (1982) is significant and an important determinant of the differing results obtained in the simultaneous move game of Reinganum (1983).

Although I earlier declared my support for the contention by Reinganum (1984) that this order of play was never explicitly stated in the Gilbert & Newbery model, I want here to clarify the fact that the whole argument is superfluous. In my framework, the distinction is irrelevant. The game is one of complete and perfect information and hence both firms would know the strategy choice of the other whether the game was simultaneous move, or sequential. Another difference between these authors is the use of a stochastic innovative process by Reinganum as opposed to the deterministic approach adopted by Gilbert & Newbery. This again is no cause for concern as the choice does not affect the results to be obtained within this model if we do not consider attitudes to risk. Were we to obtain the result that the entrant innovates first 100% of the time within our model, this may be interpreted in one of two ways. If the innovative process is deterministic, i.e. the identity of the first innovator is determined in an auction for the product, then this does indeed mean that the entrant innovates first with certainty. If, however, the innovative process is stochastic, i.e. each firm adopts its own R&D program with the uncertainty that this entails, then this result means that 100% of the time the entrant is merely expected to innovate first. This is naturally a very different claim than one that says that the entrant will always innovate first. The choice between a deterministic and stochastic process therefore affects the way in which we can interpret the results, not the way in which the game should be set up.

Other extensions to the framework, such as the idea that an innovation may have a different value to each firm, or that the firms are pursuing different innovations, shall be considered later. Also, to be introduced later is the ability of the firm to shelve its old product. Within Gilbert & Newbery, only the possibility of shelving the

innovation was considered. The option to shelve the original product to be replaced by the innovation (a more intuitive scenario) was not introduced. As this was not part of the original model we shall not introduce it until the next section. For now, we wish to see only the significance of the patenting assumption on the original model as a demonstration of the methodology we are employing.

I shall now re-state the model framework of Gilbert & Newbery with the patenting assumption removed.

The Innovation Sub-Game

Players and Timeline

Two firms – an incumbent monopolist (I) and a potential entrant (E) can potentially generate and commercialise an innovation. Both firms may commercialise the innovation subject only to the condition that they must have first generated it themselves, i.e. there can no imitation. The game is a continuous-time investment game in which each player, at the beginning of the game, chooses a strategy C_i from the strategy set $S_i = [0, \dots, \infty]$, a fact that is common knowledge before the game. I assume that the game carries on indefinitely and that there are no further entrants into the market. The strength of this assumption is reduced by the adoption of a discount rate of future payoffs, r .

We can derive expressions for the profit made in each market in the same way as Gilbert & Newbery. In the original product market, only the incumbent firm exists. The payoff here per time period for the incumbent is as defined earlier;

$$\pi_i(P_i^1)$$

In the secondary product market there are several possible set-ups in which one, both or neither of the two firms produce in this product market. If neither firm innovates and enters the market then the payoffs continue as (5) for the incumbent and zero for the entrant. When only one firm is in the secondary product market then the payoffs, per time period are

$$(9) \quad \pi_x(P_i^1, P_x^2) \quad (x = e, i)$$

When both firms successfully innovate and enter the secondary product market, then the payoffs per time period are

$$(10) \quad \pi_x(P_i^1, P_{ei}^2) \quad (x = e, i)$$

The payoffs in this game will alter according to which firm innovates first. We need to consider two possibilities. The first is when the entrant innovates before the incumbent and the second is when the incumbent innovates before the entrant. We must also consider the possibility of the incumbent firm shelving an innovation, or shelving the original product. We shall assume that the entrant, having incurred the

cost of innovation, shall never shelve the product. That is, there is always some non-negative profit to be made in the secondary product market. This may appear to be an unnecessary restriction. However in this section I shall also demonstrate the importance of the threat of entry on the innovative process. To introduce a cost of entering the market post-innovation would be to reduce this threat. Since we know that the influence of introducing this cost will be to increase persistence of leadership within the market, we can remove it from the present analysis. We are interested in the causes of incumbent persistence that are directly attributable to market structure; any in-built advantage to the incumbent would contaminate these results. Given this, I shall now specify a series of propositions which shall lead into the formal derivation of the game.

Proposition 1:

$$\pi_e(P_i^1, P_{ei}^2) \geq 0$$

Proposition 1 follows directly from my above argument, that there is always non-negative profit available to the entrant in the secondary product market. Proposition 1 sets the condition that the entrant will always commercialise an innovation when it has generated one.

I earlier referred to the possibility of the incumbent shelving the innovation as described by Gilbert & Newbery. We can easily determine when shelving occurs and can derive a series of algebraic conditions for the various configurations of possible shelving. The decision to shelve is made by the incumbent firm. There are four possible shelving frameworks in which the game can play out. That is to say, there

are four possible configurations that the secondary product market could take in the event of the incumbent innovating, each corresponding to a decision as to when and whether to shelve the innovation. We shall refer to each of these configurations as “Solution Sets” as the payoffs will be different in each case.

The first configuration is when the incumbent will always shelve his new product. That is to say, regardless of the presence or otherwise of the entrant in the secondary product market, the incumbent firm, having innovated would not introduce the product to the market. As a result of this the incumbent would not attempt innovation. No firm would incur a cost of innovation, however small, to develop a product for which there are no circumstances in which it would introduce the product. Given this, we can go on to define Proposition 2.

Proposition 2:

If $\pi_i(P_i^1) > \pi_i(P_i^1, P_i^2)$ and $\pi_i(P_i^1, P_e^2) > \pi_i(P_i^1, P_{ei}^2)$ then the incumbent firm will always shelve the innovation and hence, never attempt. This is defined as solution set 1.

Of course, there would be circumstances in which the incumbent would wish to introduce the innovation. This may be only when the incumbent has a monopoly in the secondary product market, only when there is a duopoly in the secondary product market, or both. These scenarios are specified by Propositions 3, 4 and 5.

Proposition 3:

If $\pi_i(P_i^1) < \pi_i(P_i^1, P_i^2)$ and $\pi_i(P_i^1, P_e^2) > \pi_i(P_i^1, P_{ei}^2)$ then the incumbent firm would shelve the innovation when the entrant has also innovated but not when the incumbent has innovated on its own. Such a scenario could occur if competition in the new product market was so severe that it significantly increased the cannibalisation of the old product market. This is defined as solution set 2.

Proposition 4:

If $\pi_i(P_i^1) > \pi_i(P_i^1, P_i^2)$ and $\pi_i(P_i^1, P_e^2) < \pi_i(P_i^1, P_{ei}^2)$ then the incumbent will shelve the innovation when it is the only innovator but not when the entrant firm has innovated. Such a scenario could occur when the new product market significantly cannibalises the old product market. This is defined as solution set 3.

Proposition 5:

If $\pi_i(P_i^1) < \pi_i(P_i^1, P_i^2)$ and $\pi_i(P_i^1, P_e^2) < \pi_i(P_i^1, P_{ei}^2)$ then the incumbent would never shelve an innovation. This is defined as solution set 4.

As we are presently considering a game of perfect information then players will know before the game what, if any, shelving will take place. Therefore we must consider all

Research Investment Technology

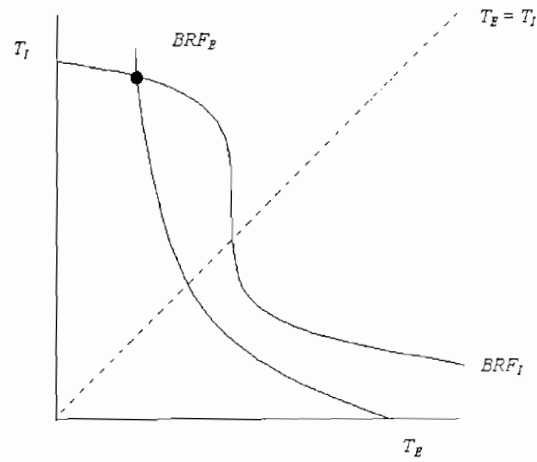
The research technology is a simple one. The choice of strategy C_i results in a time of innovation T_i where $T_i = \frac{1}{C_i}$ with probability 1. I have already discussed the arguments for maintaining the deterministic innovation process as followed by Gilbert & Newbery.

Derivation of Nash Equilibrium

The payoff to each firm will depend to a large extent on whether it is the first to innovate and thus faces a reduced degree of competition. We must consider as possibilities both the scenario in which the incumbent innovates first and that in which the entrant innovates first. For any time of innovation by the other firm, there will be a single best response choice of strategy for the other firm. The key aspect from our point of view is where the Nash Equilibria lie, or where these functions intersect. There are three possible classes of solutions to the game. First there are the two classes of games in which there is a single Nash Equilibrium in pure strategies, and one of the firms innovates before the other. The other class of game is when there are two Nash Equilibria in pure strategies, one in which Firm 1 innovates first and one in which Firm 2 innovates first. In this instance, I shall take as the equilibrium of the game the Nash Equilibrium in mixed strategies that results from the bi-matrix game obtained from the two pure strategies that form part of one of the Nash Equilibria in pure strategies.. Figure 1 below gives a graphical depiction of a scenario in which there is a single Nash Equilibrium in pure strategies, and the entrant innovates before the incumbent. The intersection of the best response functions occurs only once, and

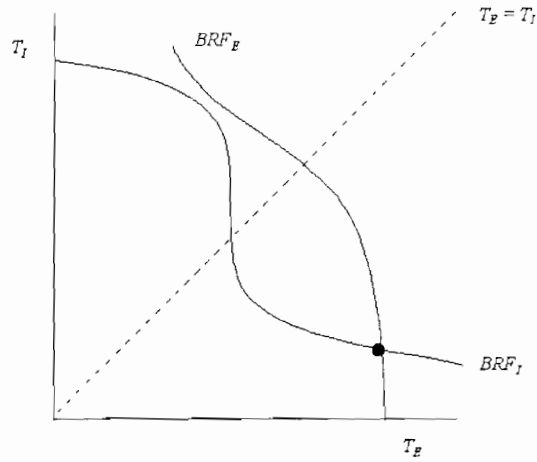
this is in the region where the time of innovation for the incumbent is greater than that of the entrant.

Figure 1: A single N.E. where the entrant innovates first



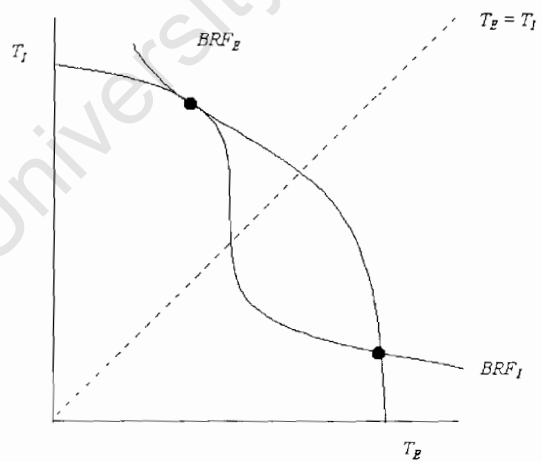
A similar scenario in which the incumbent innovates first could be represented as follows. Again we see a single intersection between the best response functions. Within scenarios of this nature the solution is relatively simple. Both firms will be able to calculate the single Nash Equilibrium solution and thus would expect the same outcome.

Figure 2: A single N.E. where the incumbent innovates first



There will of course be scenarios in which there exists more than one Nash Equilibrium in pure strategies. In this case the best response functions could look as follows.

Figure 3: Multiple N.E. - either firm may be the first to innovate



This would be a perfectly intuitive scenario. Neither firm would like to incur the cost of innovating early if they thought the other firm was going to innovate early as well. There are two stable solutions in pure strategies, one for each firm innovating first. Of course neither firm actually knows who will innovate first when making an investment decision. In the case where there is more than one Nash Equilibrium we shall use a mixed strategy solution. The firms will randomise between the two Nash Equilibrium pure strategies such that the other firm is indifferent between their own Nash equilibrium pure strategies. This will give us a third Nash Equilibrium in mixed strategies. This is the appropriate solution as we do not want to impose any restriction on which Nash Equilibrium in pure strategies the firms should expect.

We can now continue to analyse the Nash Equilibria within the solution sets we have defined through the shelving activity of the incumbent firm. We shall see that the class of solutions involving multiple Nash Equilibria in pure strategies, and necessitating the mixed strategy approach, only occur in solution set 4. This is because in all of the other solution sets there is shelving activity. Given this either the incumbent shelves in the event of duopoly, in which case there could not be a Nash equilibrium in which the incumbent innovates after the entrant, or the incumbent shelves if it is the first to innovate, in which case there could not be a Nash equilibrium in which the incumbent innovates before the entrant. Therefore, if there is any shelving then it follows that there is no Nash Equilibrium in mixed strategies.

Solution set 1 - The incumbent firm always shelves.

As the incumbent firm always shelves an innovation and therefore never attempts innovation, the entrant firm effectively acts in isolation. The payoff to the entrant firm is therefore

$$(11) \quad \int_{T_e}^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt - [C(T_e)] \\ = \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(12) \quad T_e = \sqrt{\frac{1}{\pi_e(P_i^1, P_e^2) e^{-rT_e}}}$$

which can be solved either through iteration or use of the Lambert w-function to give

$$(13) \quad T_e = -\frac{2}{r} W\left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}}\right)$$

The payoff to the incumbent would therefore be

$$(14) \quad \int_0^{T_e} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt \\ = \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r}$$

Solution Set 2 – The incumbent firm only shelves in the event of a duopoly in the secondary product market.

Once again the entrant can act as though in isolation. This is due to the fact that regardless of the order of innovation, the entrant will be the only firm in the secondary product market from the point at which it innovates. The payoff to the entrant firm is therefore again

$$(15) \quad \int_{T_e}^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt - [C(T_e)] \\ = \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(16) \quad T_e = \sqrt{\frac{1}{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \text{ or}$$

$$(17) \quad T_e = -\frac{2}{r} W\left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}}\right)$$

The incumbent must therefore choose whether to attempt innovation at all or to accept the payoff

$$(18) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_i}}{r}$$

If the incumbent were to attempt innovation then it would restrict its strategy space to those innovation dates which are earlier than the date of the entrant's innovation. This is because if it innovated after the entrant it would simply shelve the innovation. The payoff received by the incumbent in this case would be

$$(19) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt - [C(T_i)]$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^1, P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(20) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \text{ or}$$

$$(21) \quad T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^1, P_i^2)}} \right)$$

The decision of the incumbent will therefore be based on choosing the higher of the payoffs. The situations in which the incumbent shall attempt innovation within this solution set are comparatively specific. The level of competition must be sufficiently fierce, and the size of monopoly profits sufficiently big, that the incumbent firm has an incentive to cannibalise his own market before the entrant innovates. This occurs

because the incumbent, when it innovates first, does not face the price competition from the original product market that the entrant would. Therefore, only the incumbent can truly extract monopoly profit from the innovation.

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Solution Set 3 – The incumbent shelves when it is the only firm in the secondary product market but not when there is a duopoly.

In this scenario one would never expect a Nash equilibrium outcome in which the incumbent innovates before the entrant. This is because the incumbent would shelve its innovation if it innovated before the entrant. Therefore, for any date of innovation achieved by the entrant the best response for the incumbent could never be an innovation date earlier than this. As a result of this we only have to consider situations in which the incumbent firm innovates after the entrant if we are concerning ourselves with Nash Equilibria. Given this scenario we can specify the payoff to the entrant as

$$(22) \quad \int_{T_e}^{T_i} \pi_e(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_e(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_e)]$$

$$\frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_e(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\pi_e(P_i^1, P_{ei}^2) e^{-rT_i}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(23) \quad T_e = \left(\frac{1}{\sqrt{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \right) \text{ or}$$

$$(24) \quad T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right)$$

The incumbent will receive a payoff of

$$(25) \quad \int_0^{T_e} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{T_i} \pi_i(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_i(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_i)]$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1)}{r} e^{-rT_e} + \frac{\pi_i(P_i^1, P_e^2)}{r} e^{-rT_e} - \frac{\pi_i(P_i^1, P_e^2)}{r} e^{-rT_i} + \frac{\pi_i(P_i^1, P_{ei}^2)}{r} e^{-rT_i} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(26) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_{ei}^2) e^{-rT_i} - \pi_i(P_i^1, P_e^2) e^{-rT_i}}} \text{ or}$$

$$(27) \quad T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(p_i^1, p_{ei}^2)}} \right)$$

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Solution Set 4 – The incumbent does not shelve the innovation.

Consider first the entrant innovating before the incumbent who does not shelve his innovation. In this case, the game is identical to that played in Solution Set 3. If however the incumbent innovates before the entrant then a different set of payoffs is required. In this case the payoff to the entrant would be

$$(28) \quad \int_{T_e}^{\infty} \pi_e(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_e)] \\ = \frac{\pi_e(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(29) \quad T_e = \left(\frac{1}{\sqrt{\pi_{ei}(P_i^1, P_e^2)} e^{-rT_e}} \right) \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_{ei}(P_i^1, P_e^2)}} \right)$$

The incumbent will receive a payoff of

$$(30) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_i)] \\ = \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^1, P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(31) \quad T_i = \frac{1}{\sqrt{\pi_i(p_i^1, p_i^2)e^{-rT_i} - \pi_i(p_i^1)e^{-rT_i}}} \quad \text{or}$$

$$(32) \quad T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}}\right)$$

This is perhaps the most interesting solution set. The other three solution sets in which the incumbent shelves the innovation under various conditions may all be solved to give a unique Nash equilibrium. Multiple Nash Equilibria are possible, however, under conditions where the incumbent will never shelve the innovation.

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Payoff Structure

The game I have just described, and its solutions, offered a series of calculations that must be undertaken in order to derive the Nash Equilibria. First the market specific characteristics must be determined to specify the shelving activity of the incumbent in all scenarios. This gives a solution set into which the game falls. The solution set offers some clue as to whether there is a Nash Equilibrium in which the entrant innovates first, the incumbent innovates first, or whether there are multiple Nash Equilibria. The purpose of defining the solution sets was to simplify the search for the Nash Equilibrium of the game. However I must now define the market specific characteristics that characterize each solution set. The result of the game will depend on a number of variables that will be defined as those that affect the payoffs to the firms in the various market conditions. I need to determine and define those variables that affect all of the possible scenarios.

I shall define the following variables.

Definition 1:

Define the variable s to represent the degree of closeness of product 2, the new innovation, to product 1, the original product in n -dimensional product space. Let l represents the distance between the two products in n -dimensional product space then

$s = \frac{1}{1+l}$. The variable s is therefore non-negative, increasing in l and bounded by

$[0,1]$.

A value of $s = 1$ would represent the fact that the innovation and the original product are perfect substitutes. As s declines, so we should see the effect of other variables on the original product market decline. That is, the effects of price competition or branding will be at their most visible when the two goods are perfect substitutes. If the two goods are not related in any way, represented by a value for s of zero, then the introduction of the innovation should have no effect on the market of the original product.

Definition 2:

Let d represent the percentage of monopoly profit that can be obtained in a product market, as a result of competition. The variable d is bounded by $[0,1]$ where 0 represents extreme price competition and 1 represents perfect collusion.

This variable represents the ferocity of price competition, independent of all other variables. Where goods are perfect substitutes, one would expect the influence of price competition to be more marked. This is covered by the s variable. A value of $d = 0$ clearly represents pure Bertrand price competition in which prices are competed down to marginal cost and therefore profits are zero.

The Cournot case can also be easily represented. In the general case, where a market is defined by the inverse demand function $P(Q) = a - Q$ and costs of production are

$C_i(q_i) = cq_i$ with no fixed costs, monopoly profit is $\frac{(a-c)^2}{4}$. Under Cournot

competition, industry profit is $\frac{2(a-c)^2}{9}$. The Cournot game could therefore be

represented by a value for d of $\frac{\frac{2}{9}}{\frac{1}{4}} = \frac{8}{9} = 0.889$. The benefit of the present

methodology is that I do not have to make any assumption regarding the nature of price competition in a theoretical framework. I can test the outcome of the model for a series of different levels of price competition and see how sensitive the model is to any assumptions made about this variable.

Definition 3:

Let m represent the proportion of the original market secured by the incumbent firm in the event of there being an innovation. A value of 1 represents a situation such that, with no other influences, the incumbent would keep the entire market if the entrant innovated. A value of 0 implies that the incumbent has no residual consumer loyalty and thus the entirety of the original product market is potentially subject to cannibalisation. The variable m is bounded by $[0,1]$.

Clearly in many markets, the incumbent may retain some advantage in the secondary product market due to customer loyalty or branding, or have a locked in level of demand for the original product that affects the potential gains to innovation by an entrant. This variable is intended to capture all of these possibilities. This advantage may be because of simple branding or it may be because the original product complements the innovation in some way and this gives the incumbent, as the sole producer of the original product, an advantage in the competitive market for the innovation.

This variable also introduces the concept of strategic innovation. To a certain extent the incumbent may be able to invest in raising the level of this variable and lock-in its customers. As a function of the cost of this, an incumbent firm may prefer to invest in protection against any innovation than in the innovation itself. This is similar to the concept of pre-emptive patenting described by Gilbert & Newbery (1982).

Definition 4:

Let x be the monopoly profit available in the original product market. As all other variables are either shares, percentages or multiples of the profit in the original product market, we must have some reference point. The model I will develop generates results through iteration based on this number. As a pure baseline device its value can be set arbitrarily. The numerical value does not affect the model as we are interested in the direction of the influences for our analysis, not the actual numbers generated by the model. It is relative magnitudes in sensitivity that will interest us.

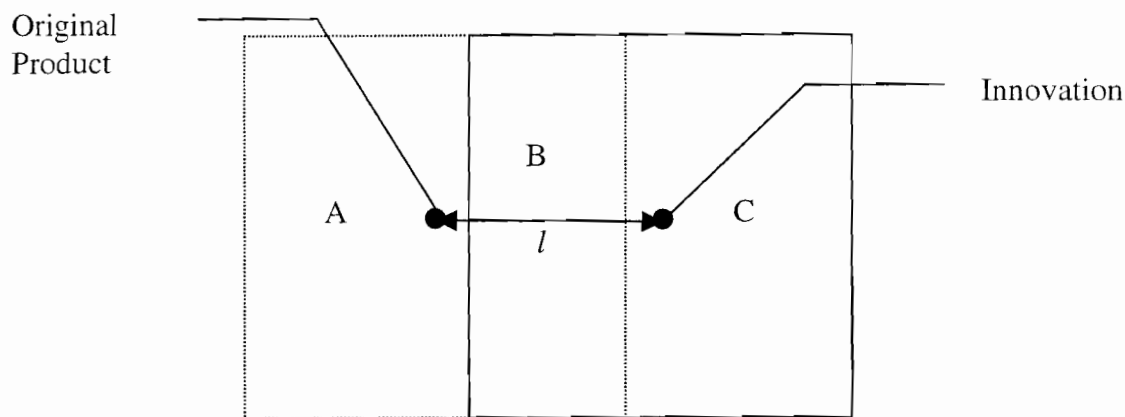
Definition 5:

Let δ represent the ratio of profitability from the innovation. That is to say, the factor by which the monopoly profit in product market 2 relates to the monopoly profit in product market 1. The monopoly profit in product market 2 therefore is defined as δx . The variable δ is non-negative and bounded by $[0, \infty]$.

i.e. the strategic interaction that occurs when the innovation game begins and this would occur under the conditions of whatever level of m is prevailing in the model at that time. The results obtained from this investigation regarding the sensitivity of the result to the level of m will offer an insight as to the effectiveness of the strategy of investing in m at an earlier stage of the game without having to model any explicit cost structure for doing so.

It is best to think of the interaction between the markets and the effects on profits in two stages. At the first stage we consider what percentages of the market are subject to the various influences of cannibalisation. Close substitutes will face a greater effect from a given level of price competition than products that are not close substitutes. Therefore one can say that the degree of substitutability between products defines the scope for cannibalisation of profit, with actual levels of cannibalisation then determined by other factors. The degree of substitutability between products has been defined by the variable s . If the innovation is close to the original product in product space then the level of s will be high. One can think of each market as squares of uniform area, as drawn below. The market comparison area of a given product x is represented by a Euclidean square of arbitrary internal metric with x at its geometric centre. Now we can compare levels of substitutability between two products x and y by examining the extent to which market comparison areas of various sizes around them intersect. The value of the variable s can be understood by reference to this measurement heuristic..

Figure 4:



$$A = C = (1 - s) \equiv \frac{l}{1 + l}$$

$$B = s \equiv \frac{1}{1 + l}$$

Therefore if s is equal to 1, the innovation is a perfect substitute for the original product. In this case, the two squares overlap exactly and the entire market for each product is a site for their competition with each other.

Axiom 1: The scope for the cannibalisation of profit by another product is defined by the degree of substitutability between the products.

Having determined what area of each market is subject to which cannibalising influences on profit, the second stage is to determine exactly how much cannibalisation can take place within this area. That has been defined by the variable m . A high degree of incumbent lock-in will prevent much market share being available for cannibalisation. In the area that is subject to competitive influences one must then ask what the allocation of profit to each firm is in each area.

In non-intersecting areas, where there is only one firm, the profit is the monopoly profit as it is isolated from the influence of the other product. Therefore this would be x for the original product market and δx for the innovation market. In the intersecting areas, the profit to each firm is determined by whether or not they are the innovator, and by d . But the effect that the latter variable has on the profit is determined by the value of δ . If δ is high, then one would not expect to see the effect of fierce competition taking profit away from an entrant which has produced a higher quality product. In other words, the variable d representing price competition is secondary. Its effect only occurs after the influence of δ and m have defined its scope.

This assumption is a core component of this investigation. Although all of the variables I have described are independent of each other, it is clear that their effect on profit in the relevant markets cannot be. The degree to which the influences on profit, such as the ferocity of competition, can affect profit will be related in part to the degree of substitutability. For any given ferocity of price competition, one would expect the effect to be greater the closer the products are in substitutability. Equally, the δ variable will affect the influence of the other variables. If δ captures quality dynamics or cost reductions, then profit from an innovation with a high degree of δ should be immune to some degree from the influences of price competition, that degree being increasing in δ . In fact I am implicitly defining innovation as a dampening or amplifying influence on the other variables that determine profits in 2-firm market scenarios with imperfect substitutes. If price competition were not affected by δ , then I would no longer be isolating innovation within the investigation.

Although, one can say that the d variable affects profit to the players in the form of a certain defined relationship, does it not undermine our investigation to define that relationship in this restricted way? The answer to this criticism is my standard one again: whichever way one defines the relationship, its significance will also be subject to the same scrutiny as all other assumptions that I am trying to test within this framework. I shall maintain consistency by stating that all the variables in the equations I am about to define, that are subject to dampening or amplifying through δ , shall be subject to an influence of δ . I have argued previously that δ , as a representation of quality or cost-structure, should control the influence of d . The form that this control should take is now being defined. However I have no empirical basis for selecting one relationship over another. There is no good reason for not choosing a cubic form of δ for example. But it is clear that by raising the exponential power of the δ variable, I would simply be increasing the sensitivity of the model to the δ variable by giving it a greater scope for defining outcomes. The effect would be to make the model less or more sensitive respectively to that variable. What will be interesting from this analysis will be the direction of the influences on the equilibrium outcomes, especially if this turns out to be a non-monotonic relationship. The exact nature of the relationship is not within the scope of a general analysis. It is the objective of a case-specific analysis to determine the exact structure, as opposed to the general direction, of any relationship between variables.

Axiom 2: Cannibalisation of a product shall be driven primarily by the δ variable, which can amplify or dampen the other cannibalising influences.

I shall specify the direction of the influences and whether they are to be amplified or dampened in δ . By maintaining the consistency in only subjecting the variables to a non-exponential form of the δ variable, I defend myself from the accusation that the sensitivities that will become apparent in the results were contrived from the outset. It is a fundamental assumption that there does exist a sensitivity to the δ variable, through the way in which innovation is being defined. One can always argue that the profit functions that will follow do not accurately describe a particular industry. They are not supposed to represent any specific industry. Rather, they are intended to define a highly general framework in which 'innovation' can be isolated from other aspects of competitive interaction, and which is empirically adequate in capturing directions and signs of factor influences. This then permits us to investigate the comparative sensitivities of particular models to changes in the variables.

Using these axioms, I can propose expressions for the payoffs per time period and the conditions under which shelving occurs. I can establish bounds on the profit available to each firm within the intersecting area and then define functions that determine the position within these bounds, as a function of our defined variables.

I shall now move on to defining the profits to each firm under each of the range of situations discussed earlier.

The case where the incumbent has innovated and the entrant has not.

This is a fairly straightforward case as the entrant simply receives a payoff of 0. As only the incumbent is in either market, the branding variable and ferocity of competition variable are discarded. The profit in the intersecting area will be a combination of δx and x . The variable δ also acts as a cannibalisation variable; this is the effect from innovation. If δ is greater than 1, I would expect the innovation to be either of a higher quality or lower cost than the original product. If this is the case then the level of demand will be higher for the innovation than for the original product. Therefore the profit available in the intersecting area shall be weighted towards either the innovation profit or the original product. I shall say that the profit in an intersecting area is given as

$$\frac{\delta}{1+\delta} \delta x + \frac{1}{1+\delta} x \equiv \frac{1+\delta^2}{1+\delta} x$$

All that remains is to determine the degree to which the new product market cannibalises the old market. This is a function of s . Hence,

$$(33) \quad \pi_i(p_i^1, p_i^2) = (1-s)x + (1-s)\delta x + s \left(\frac{1+\delta^2}{1+\delta} x \right) \text{ for all } \delta$$

One can easily determine from this equation the condition for the incumbent to shelve the innovation, in the case of it being the sole innovator. This is also the condition necessary for the incumbent to attempt innovation at all if it were a protected monopoly. In the preceding section I defined this condition as $\pi_i(p_i^1, p_i^2) < \pi_i(p_i^1)$.

From this I can say that the incumbent would shelve the innovation if it were the sole innovator where $\delta < 2s - 1$. Where this condition holds there will be no Nash Equilibria in which the incumbent innovates first.

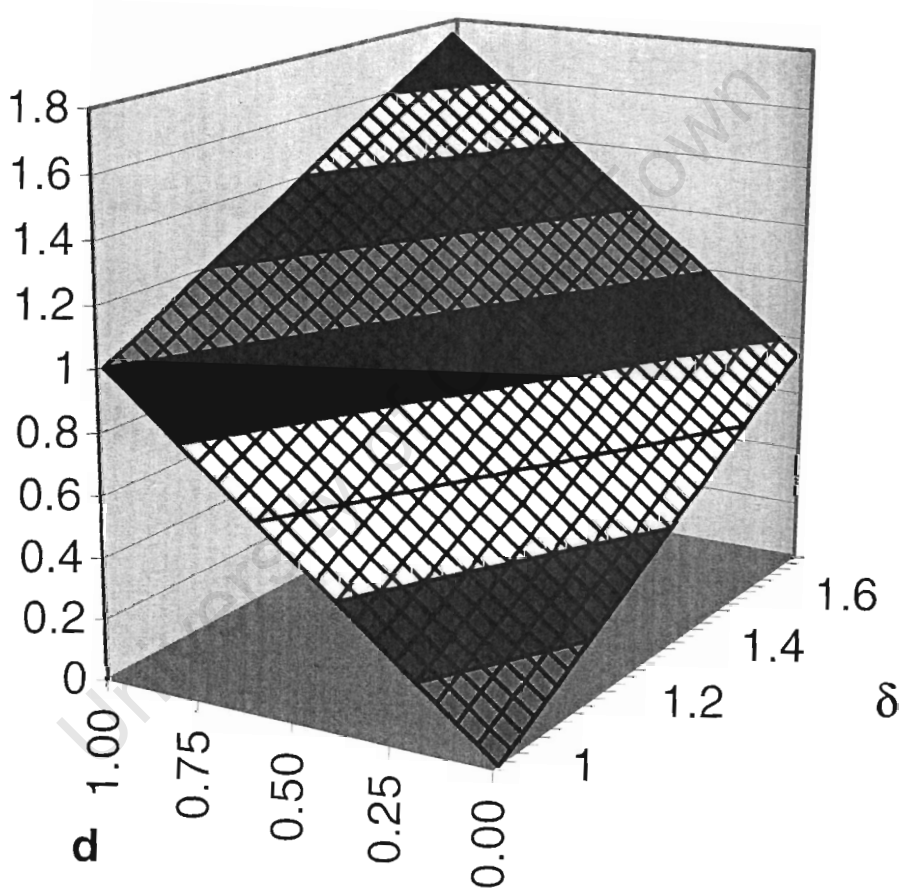
The case where the entrant has innovated and the incumbent has not.

It is important to ensure that for values of $\delta \neq 1$ the diminished or amplified effect of competition and branding variables is demonstrated. First we look at the case where $\delta > 1$. We know that the entrant in this case would receive a profit of $(1 - s)x$ plus a share of the profit available in the area of intersection. We know that the maximum profit available to the entrant in the intersecting area is δx . But a proportion m of this market is unavailable to the entrant through lock-in of the incumbent's market share. This means that the maximum profit available to the entrant is $(1 - m)\delta x$. However only a proportion, $\frac{1}{\delta}$, of this shall be affected by the d variable. This is to ensure that if there is a high level of δ , representing a better product, then fierce competition cannot destroy all the profit of the innovator. The effect of d must be diminishing in δ . As I stated earlier, I shall use δ as the amplifying variable. This gives us the profit equation, $\left(\frac{d}{\delta} + \frac{(\delta - 1)}{\delta}\right)(1 - m)\delta x$. We can see how this equation conforms to our logical reasoning from Graph 1. The profit becomes less sensitive to the d variable as δ increases. Equally, as $d \rightarrow 1$, the profit tends to $(1 - m)\delta x$.

Hence,

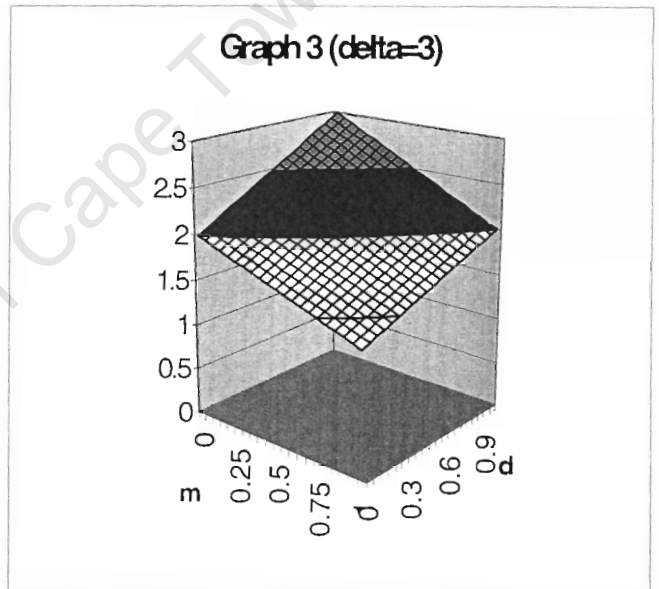
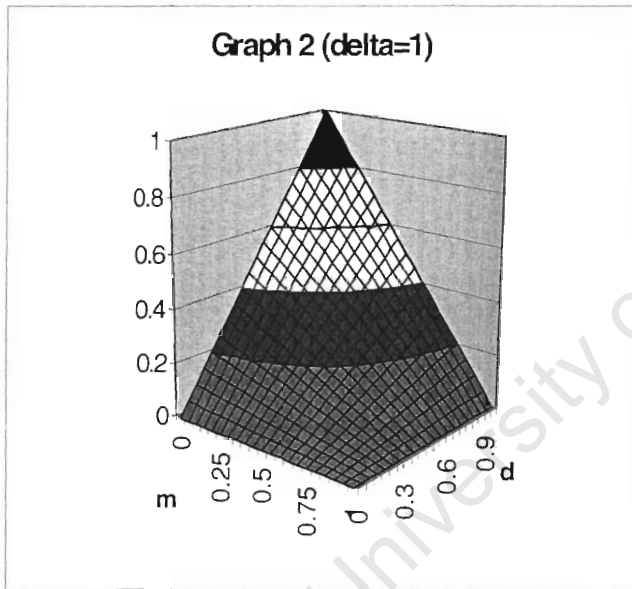
$$(34) \quad \pi_c(p_i^1, p_e^2) = \left[(1-s)\delta x + s(1-m) \left(\frac{d}{\delta} + \frac{(\delta-1)}{\delta} \right) \delta x \right] \text{ for } \delta \geq 1$$

Graph 1 – How a product with $m = 0$ and $x = 2$ is cannibalised by d for innovation with various levels of δ



Graphs 2 and 3 show how the influences of m and d change the profit to the innovator within the intersecting area for a low value of δ (1) and a high value of δ (2).

When δ is high, the ferocity of competition has a lesser effect and is unable to reduce the profit to the innovator within the intersecting area to zero. When δ is equal to one, representing no improvement on the original product, the d variable does affect the profit within the intersecting area to its full degree.

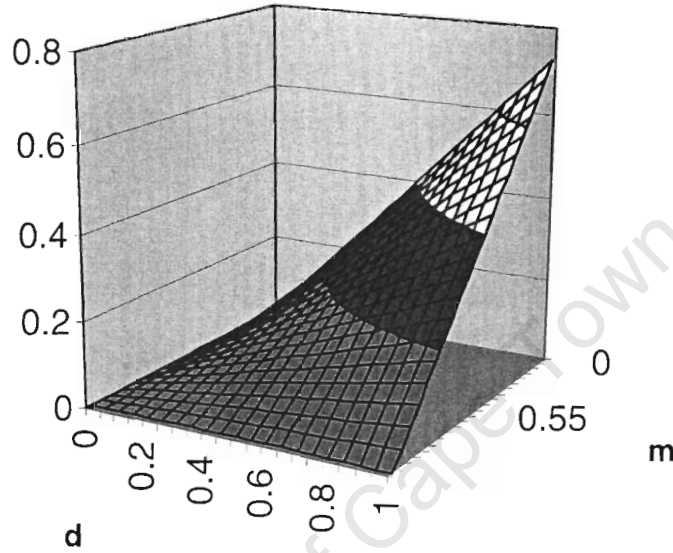


When $\delta < 1$, then the effect of the levels of d must be amplified rather than dampened. Therefore the maximum profit available in the intersecting area, $(1 - m)\delta x$, becomes subject to an amplified level of d , namely $d\delta$. This gives $d\delta(1 - m)\delta x$ as the profit in the intersecting area to the entrant. This relationship is depicted in Graph 4 for $\delta = 0.8$.

Hence,

$$(35) \quad \pi_e(p_i^1, p_e^2) = (1-s)\delta x + sd\delta^2(1-m)x \text{ for } \delta < 1.$$

Graph 4



For the incumbent firm, as non-innovator, a profit of $(1-s)x$ is assured. The share in the intersecting area in the event of $\delta > 1$ is determined as follows. The m variable determines the degree to which the remainder of the market is locked in by the incumbent. The profit available in the intersecting area to the incumbent as non-innovator, x , becomes subject to an amplified level of d , namely $\frac{d}{\delta}$. The profit therefore becomes $\left(\frac{d}{\delta}(1-m) + m\right)x$. It is clear that for values of δ greater than 1, the cannibalising effect of the d variable is increasing in δ .

Hence,

$$(36) \quad \pi_i(p_i^1, p_e^2) = \left[(1-s)x + s \left(\frac{d}{\delta} (1-m) + m \right) x \right] \text{ for } \delta \geq 1$$

For $\delta < 1$, the effect of d needs to be dampened. A proportion, δ shall be affected by the d variable. This will result in the profit in the intersecting area being worth $x(m + (1-m)(\delta d + 1 - \delta))$.

Hence,

$$(37) \quad \pi_i(p_i^1, p_e^2) = [(1-s)x + s(m + (1-m)(\delta d + 1 - \delta))x] \text{ for } \delta < 1$$

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The case where both firms have innovated

This situation becomes slightly more complicated in the sense that there are now three products to consider, the original product and a duopoly in the innovation product market. I shall begin by considering cases where $\delta > 1$.

The incumbent firm will receive an amount $(1-s)x$ from the original product market and an amount to be determined from the new product market. The share to the incumbent in the new product market should be expressed in terms of the m variable even though this refers to the degree to which the customers in the old product market are locked in. Here I am defining that section of the new product market which is independent of the original product market and thus should not be affected directly by the degree of lock-in. The sense in which I am using the m variable here is as a proxy to determine the market share to the incumbent of that independent section of the market. It is a reasonable assumption that if the incumbent brand is one that generates a high degree of customer loyalty then this would be a good proxy for capturing an assumption to the effect that the incumbent would gain a market share greater than 50% in a market which it entered simultaneously with another innovator. Equally if there is no degree of lock-in in the original product market then the incumbent should be considered as being on an equal footing, *ceteris paribus*, with the other innovator. Therefore I shall define the profit to the incumbent in the new product market, outside of the area intersecting with the original product, as $(1-s)\left(\frac{1+m}{2}\right)d\delta x$. This ensures that if there is no lock-in, the incumbent receives a 50% market share, increasing in m . It should be noted here that the m variable does not isolate the profit from the

cannibalising influence of the d variable. This is because the m variable in this instance is not actually isolating market share for the incumbent; it is simply being used as a proxy for any incumbency advantage in the innovation product market.

Within the intersecting area the maximum profit available to the incumbent is

$\frac{1+\delta^2}{1+\delta}x$, which we recall is the weighted average of the profit available in the

original product market and in the new product market. Of this, the proportion m is isolated from competition. The remainder is $(1-m)$, of which, in accordance with the

principles of the model, the proportion $\frac{1}{\delta}$ is subject to the influence of d^2 , the

remainder being subject to the influence of d . By this assumption the double effect of the price competition variable, representing the competition between the old product

market and the competitive new product market, is emphasized as δ tends to 1. I can

then define the profit in the intersecting area as $\frac{1+\delta^2}{1+\delta} \left(m + (1-m) \left(\frac{d^2}{\delta} + \frac{\delta-1}{\delta} d \right) \right) x$.

Hence,

$$(38) \quad \pi_i(p_i^1, p_{ei}^2) = \left((1-s) + (1-s) \left(\frac{1+m}{2} \right) d\delta + s \left(\frac{1+\delta^2}{1+\delta} \left(m + (1-m) \left(\frac{d^2}{\delta} + \frac{\delta-1}{\delta} d \right) \right) \right) \right) x$$

For $\delta \geq 1$.

Where $\delta < 1$, the influences on the profit within the intersecting area change. Within

the intersecting area the profit is $\frac{1+\delta^2}{1+\delta}x$ of which $(1-m)$ is subject to competition.

Again, in line with previous assumptions, of this the proportion δ is subject to the influence of d^2 . The remainder is not. This gives $\delta d^2 + (1 - \delta)$ as the function describing the effects of the competition variable on the profit for values of $\delta < 1$. As δ tends to 0, the innovation has a reduced competitive impact on the original product.

Hence,

$$(39) \quad \pi_i(p_i^1, p_{ei}^2) = \left((1-s) + (1-s)d \frac{1+m}{2} \delta + s \left(\frac{1+\delta^2}{1+\delta} (m + (1-m)(\delta d^2 + (1-\delta))) \right) \right) x$$

for $\delta < 1$

The profit for the entrant is thenceforth easy to calculate. It receives no profit from the original product market and the maximum profit which it can achieve in the intersecting area, δx , is subject to the same process of cannibalisation as I have just outlined in the case of the incumbent. The one exception is in the event where $\delta < 1$, in which case all of the profit will be subject to the d variable. In the case of the incumbent a proportion of the profit, decreasing in δ , was isolated from this cannibalising influence to represent the profit of the original product.

Hence,

$$(40) \quad \pi_e(p_i^1, p_{ei}^2) = \left((1-s)d \left(1 - \frac{1+m}{2} \right) \delta + s \left(\delta(1-m) \left(\frac{d^2}{\delta} + \frac{\delta-1}{\delta} d \right) \right) \right) x \text{ for } \delta \geq 1$$

$$(41) \quad \pi_e(p_i^1, p_{ei}^2) = \left((1-s)d \left(1 - \frac{1+m}{2} \right) \delta + s((1-m)(\delta(\delta d^2))) \right) x \text{ for } \delta < 1$$

The four solution set model – analysis

This model is only the first generation in the evolution of the model from the original Gilbert & Newbery framework. Naturally any results obtained from this analysis come with this caveat. It is hoped that by the end of the investigation, results will hardly need to come with any caveat at all. Of course, there must always be some restriction on the game in terms of structure. In the Introduction, I discussed the merits of the bounds approach adopted by Sutton (1998). This methodology cannot allow us to totally escape the need to make some specifications on the structure of the game. However, as the generations of the present model evolve we shall gradually introduce more and more flexibility into the structure.. This will allow us to describe a vast array of industry specifications and thus go a long way towards resolving the problems of traditional analysis outlined by Sutton.

We need to analyse the model to see how the variables defined earlier drive the solution set of the game. In general, we will be interested in configurations that display a high value of s . A low value of s would indicate that the products are not close substitutes in n -dimensional product space and this, in the limit, implies that we are no longer working within the incumbent-entrant framework at all.. A value of zero for s would imply that we were effectively considering an entrant – entrant framework instead.

As Beath, Katsoulacos & Ulph (1989) rightly point out, there are two forces that will drive R&D effort. They refer to them as a “profit incentive” and a “competitive

threat” (p.74). Common sense tells us that this is correct within this framework. A monopolist would only be incentivised to innovate if it were profitable to do so. Naturally, a valuable innovation will attract more R&D effort. Equally, a monopolist may be incentivised to innovate if, by doing so, it can protect its market position. Without patents, the incumbent firm can no longer secure a monopoly by innovating first but it may still have an incentive to innovate if an innovation would cannibalise the original product to such an extent that the incumbent must innovate to stay in the market.

All of the relevant forces can be modelled using the variables that have been defined here. The profit incentive would be captured through δ in the case where firms act in isolation. In a two firm scenario however, the variables that affect profit in a duopoly scenario, such as m and d , will also have an influence. All of the variables will contribute towards the competitive threat force. However, we might expect d to have the greater effect. This is because this variable represents the ferocity of competition that the incumbent would face in the original product market with the introduction of a new product. This would also lead to an indirect effect as a competitive new product market would lead to an increased degree of cannibalisation of the old product market.

For the analysis, we shall hold r to be fixed at a level of 10%. The reason for this is that the effects of changes in the level of r are obvious. A higher discount rate simply reduces the incentive to invest now in future profit streams and thus reduces the incentive of either firm to innovate. Given that we are interesting ourselves in what

drives innovation, not what drives non-innovation, we shall keep variations in the level of r out of the analysis.

One of the main advantages of this methodology is that it allows us to quickly generate outcomes from many variations of the same game. We can use this ability to examine what shelving of products occurs under what conditions. We specified that there were four solution sets within the model, each referring to a shelving rule for the incumbent. Table 1 shows, for a specified selection of variables, which of the four solution sets is yielded, and thus what shelving behaviour by the incumbent occurs for varying levels of δ and ferocity of price competition. Using the following key as an aid to understanding the table, it should be clear that the decision of the incumbent as to under what conditions to shelve the innovation, if at all, is sensitive to both of these variables.

Solution Set	<i>Shelving Behaviour</i>
1	Incumbent always shelves innovation
2	Incumbent only shelves in event of duopoly
3	Incumbent only shelves when sole innovator
4	Incumbent never shelves innovation

As one might expect, all of the solutions for levels of δ greater than 1 are in solution sets 2 and 4 where the incumbent never shelves the innovation when it is the sole innovator. We should expect this as there is no reason why a more profitable innovation should not be introduced into the market. Whether the innovation is

shelved by the incumbent when the entrant is also in the market depends on whether the extra profit derived from the new product is enough to outweigh the additional cannibalisation of the original product that occurs as a result of the competitive new product market.

Table 1 :Solution sets for levels of d and δ in the case where $s=0.95, m=0, r=10\%$.

δ																											
d		0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
0.000	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.025	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.050	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.075	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.100	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.125	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.150	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.175	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.200	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.225	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.250	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.275	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.300	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.325	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.350	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.375	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.400	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.425	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.450	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.475	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.500	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.525	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.550	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.575	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.600	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.625	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.650	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.675	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.700	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.725	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.750	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.775	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.800	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.825	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.850	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.875	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.900	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.925	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.950	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.975	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
1.000	3	3	3	3	4	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4

We earlier stated that we would focus on high levels of s for the analysis. Table 2 examines whether this will unduly affect the results generated by the model.

Table 2: Solution sets for levels of d and δ in the case where $s=0.99, m=0, r=10\%$.

		δ																											
		0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3		
d	0.000	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.025	3	3	3	3	3	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.050	3	3	3	3	3	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.075	3	3	3	3	3	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.100	3	3	3	3	3	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.125	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.150	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.175	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.200	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.225	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.250	3	3	3	3	3	2	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.275	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.300	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.325	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.350	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.375	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.400	3	3	3	3	3	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.425	3	3	3	3	3	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.450	3	3	3	3	3	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.475	3	3	3	3	3	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.500	3	3	3	3	3	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.525	3	3	3	3	3	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.550	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.575	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.600	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.625	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.650	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.675	3	3	3	3	3	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.700	3	3	3	3	3	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.725	3	3	3	3	3	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.750	3	3	3	3	3	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.775	3	3	3	3	3	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.800	3	3	3	3	3	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.825	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.850	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.875	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.900	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.925	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.950	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	0.975	3	3	3	3	3	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	1.000	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	

The small movement of the fulcrum levels demonstrates that excessive sensitivity at high levels of s is not a problem. This was an important check. Owing to the methodology used, the model generates the Nash Equilibria by calculating reaction functions through an iterative process. Often, in models of this nature, problems can arise with numbers close to 1 and zero. This is because the terms s and $(1-s)$ appear within the equations and thus a number very close to zero could result. There are also exponential and reciprocal functions within the model and a number very close to zero can produce an asymptotic result. That is the reason why the check for excessive sensitivity at high levels of s was necessary. Having determined that excessive sensitivity is not a problem, attention can refocus on the argument for having chosen a high value of s in the first place, rather than on any unusual results that are analytical artefacts of such a choice.

Table 3 shows an identical model as Table 1 but with a different value of s . By lowering the value of s to 0.75, the effect of changes in s now becomes clear. As s declines, the cannibalistic effect of competition in the secondary market diminishes and the incumbent is less likely to shelve any innovation. Therefore we see more situations in which the incumbent does not shelve. Situations in which the incumbent will shelve the innovation in the event of a duopoly in the new product market are generally insensitive to changes in the level of s . This supports the earlier observation that whether the innovation is shelved by the incumbent when the entrant is also in the market depends on whether the extra profit derived from the new product is enough to outweigh the additional cannibalisation of the original product that occurs as a result of the competitive new product market.

Table 3: Solution sets for levels of d and δ in the case where $s=0.75, m=0, r=10\%$.

δ	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	
0.000	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.025	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.050	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.075	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.100	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.125	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.150	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.175	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.200	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.225	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.250	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.275	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.300	4	4	4	4	4	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.325	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.350	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.375	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.400	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.425	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.450	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.475	4	4	4	4	4	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.500	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.525	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.550	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.575	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.600	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.625	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.650	4	4	4	4	4	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.675	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.700	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.725	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.750	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.775	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.800	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.825	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.850	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.875	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.900	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.925	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.950	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.975	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
1.000	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

The main concerns of this section were to first demonstrate the usefulness and capability of the methodology and also to examine the significance of the patenting assumption within the Gilbert & Newbery model. The model will allow us to calculate the percentage of cases in which the entrant will innovate before the incumbent. Gilbert & Newbery suggests that this should be 0%, Reinganum (1983) suggests that it is greater than 50%. We can easily use the methodology to not only see who, if anyone, is correct but also what is driving the respective results. I shall show that both of these conclusions hold only under specific conditions. The Gilbert & Newbery model offers only the single possible result of 0% because it is a patent-based model, with the innovation being awarded with certainty to the highest bidder, as though it was produced by an independent laboratory. Their result arises from the mathematical inequality that the profit to a monopolist must always be at least as great as the sum of the profit of two duopolists. The Reinganum model introduces a stochastic innovative process and therefore cannot yield such a definitive result. This model has no patenting and therefore is not driven by the simple analytic inequality that determines the Gilbert & Newbery version. The results of any specification of the model, however, can range across the entire spectrum of possibilities, demonstrating the importance of introducing industry-specific criteria. As empirical observations can be found to support both points of view, it is reassuring to note that the results of the present model do not contradict either; instead, they specify both earlier models as special cases.

Table 4: % of games in which entrant innovates first for levels of d and δ in the case where $s=0.95, m=0, r=10\%$.

		δ																											
		0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80		
d	0.000	100%	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		
	0.025	100%	100%	100%	100%	100%	82%	84%	84%	85%	85%	85%	86%	86%	86%	86%	86%	86%	86%	87%	87%	87%	87%	87%	87%	87%	87%	87%	
	0.050	100%	100%	100%	100%	100%	75%	77%	79%	79%	80%	81%	81%	81%	82%	82%	82%	82%	82%	83%	83%	83%	83%	83%	83%	83%	83%	83%	
	0.075	100%	100%	100%	100%	100%	68%	72%	74%	75%	76%	77%	78%	78%	78%	79%	79%	79%	80%	80%	80%	80%	80%	80%	80%	80%	81%	81%	
	0.100	100%	100%	100%	100%	46%	62%	67%	70%	72%	73%	74%	75%	75%	76%	76%	77%	77%	77%	77%	78%	78%	78%	78%	78%	78%	79%	79%	
	0.125	100%	100%	100%	100%	37%	56%	63%	66%	68%	70%	71%	72%	73%	74%	74%	75%	75%	75%	76%	76%	76%	76%	77%	77%	77%	77%	77%	
	0.150	100%	100%	100%	100%	27%	49%	58%	62%	65%	67%	69%	70%	71%	71%	72%	73%	73%	73%	74%	74%	74%	75%	75%	75%	75%	75%	76%	
	0.175	100%	100%	100%	100%	14%	42%	52%	58%	61%	64%	66%	67%	68%	69%	70%	71%	71%	72%	72%	73%	73%	73%	74%	74%	74%	74%	74%	
	0.200	100%	100%	100%	100%	32%	46%	53%	57%	61%	63%	65%	66%	67%	68%	69%	70%	70%	71%	71%	71%	71%	72%	72%	72%	73%	73%	73%	
	0.225	100%	100%	100%	100%	21%	38%	48%	53%	57%	60%	62%	64%	65%	66%	67%	68%	68%	69%	70%	70%	70%	70%	71%	71%	71%	71%	72%	
	0.250	100%	100%	100%	100%	100%	29%	41%	48%	53%	56%	59%	61%	62%	64%	65%	66%	67%	67%	68%	68%	69%	69%	69%	70%	70%	70%	70%	
	0.275	100%	100%	100%	100%	100%	100%	33%	42%	48%	52%	55%	58%	60%	61%	63%	64%	65%	66%	66%	67%	67%	68%	68%	69%	69%	69%	69%	
	0.300	100%	100%	100%	100%	100%	100%	100%	35%	42%	48%	52%	54%	57%	59%	60%	62%	63%	64%	65%	65%	66%	66%	67%	67%	68%	68%	68%	
	0.325	100%	100%	100%	100%	100%	100%	100%	100%	35%	42%	47%	51%	53%	56%	58%	59%	60%	62%	63%	63%	64%	65%	65%	66%	67%	67%	67%	
	0.350	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	35%	41%	46%	49%	52%	54%	56%	58%	59%	60%	61%	62%	63%	64%	64%	65%	65%	
	0.375	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	34%	40%	45%	48%	51%	53%	55%	57%	58%	59%	60%	61%	62%	63%	63%	63%	
	0.400	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	33%	39%	43%	46%	49%	51%	53%	55%	57%	58%	59%	60%	61%	62%	62%	
	0.425	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	31%	37%	41%	44%	47%	50%	52%	54%	55%	56%	58%	59%	59%	59%	
	0.450	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	34%	39%	42%	45%	48%	50%	52%	53%	55%	56%	57%	
	0.475	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	31%	36%	40%	43%	45%	48%	50%	51%	53%	54%
	0.500	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	32%	36%	40%	43%	45%	47%	49%	51%	
	0.525	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	33%	36%	39%	42%	44%	47%	47%	
	0.550	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	32%	36%	39%	41%	
	0.575	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	
	0.600	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	
0.625	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.650	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.675	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.700	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.725	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.750	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.775	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.800	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.825	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.850	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.875	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.900	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.925	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.950	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
0.975	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
1.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		

In Table 4, displaying these results ‘X’ denotes a case where each firm would never choose to innovate if it were to be the second innovator. This means that they adopt a mixed strategy in which one of the strategies is to innovate at ∞ , hence the average time of innovation is also ∞ and no result can be obtained on who innovates first.

Table 4 demonstrates limited support for the result of Reinganum's (1983) conclusion that the entrant shall always innovate first more often than the incumbent. So the result of Gilbert & Newbery (1982) is already overturned as a general principle. In the vast majority of scenarios the entrant innovates before the incumbent more than 50% of the time. This certainly refutes the results of Gilbert & Newbery as a general analysis and although not exactly replicating the result of Reinganum, is support for the notion that the entrant innovates before the incumbent. However, there is a key aspect of the analysis missing.

We were forgetting that a key variable, m , is held fixed at 0, representing no consumer lock-in by the incumbent. We should examine the effects of changes in m on the industries with different characteristics. Table 5 demonstrates how, with the presence of some lock-in to the incumbent, the result that the entrant is generally expected to innovate first is no longer valid. With a value of m of 0.30, for various configurations of δ and d , not only are there more results in which the entrant will be expected to innovate first less than 50% of the time, there are cases in which the incumbent always innovates before the entrant. The condition derived earlier still applies however, namely that the entrant always innovates first where $\delta \leq 2s - 1$. It is already clear that although we might be able to draw conclusions as to what industry characteristics lead to a particular result, it is not going to be the case that strong conclusions, applicable to all circumstances, will be reached. This conforms to a realistic view of the world and indeed, to evidence from this world. Rather than waste time in an attempt to generate a proof that is already disproved through observation, efforts can be directed towards attempting an understanding of the factors that determine these different results. Therefore, neither the Reinganum result nor the

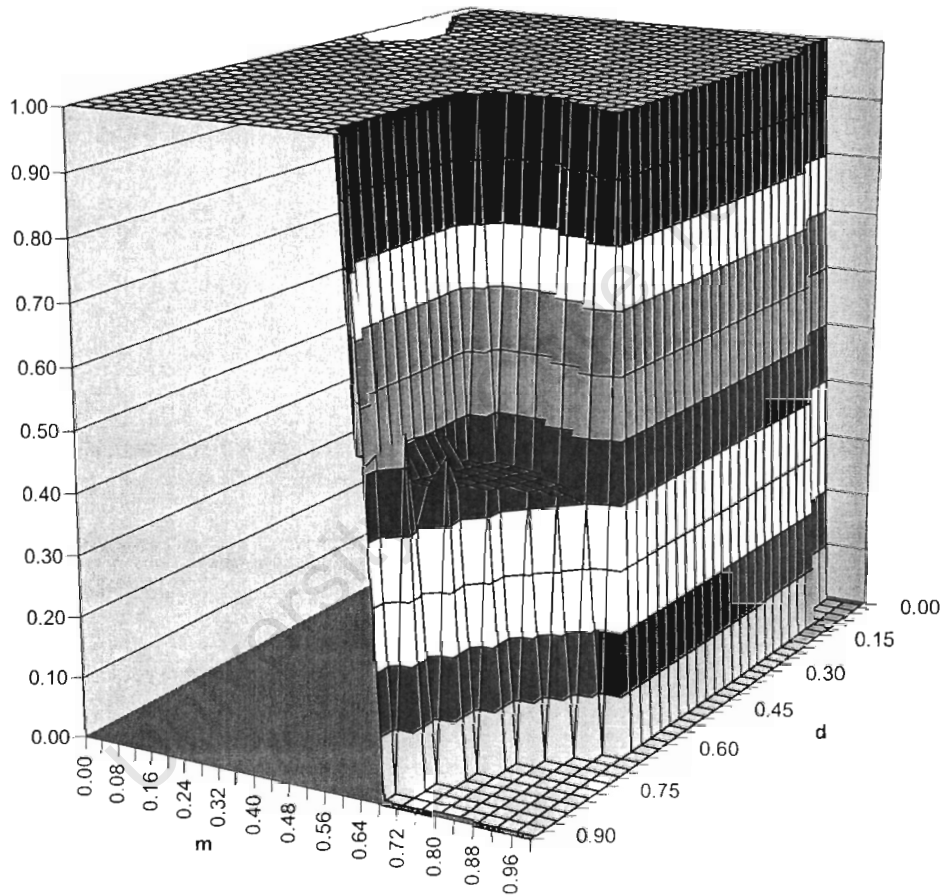
Gilbert & Newbery result stand up to this analysis. Clearly the question of who innovates first is sensitive to δ , m and d .

Table 5: % of games in which entrant innovates first for levels of d and δ in the case where $s=0.95$, $m=0$, $r=10\%$.

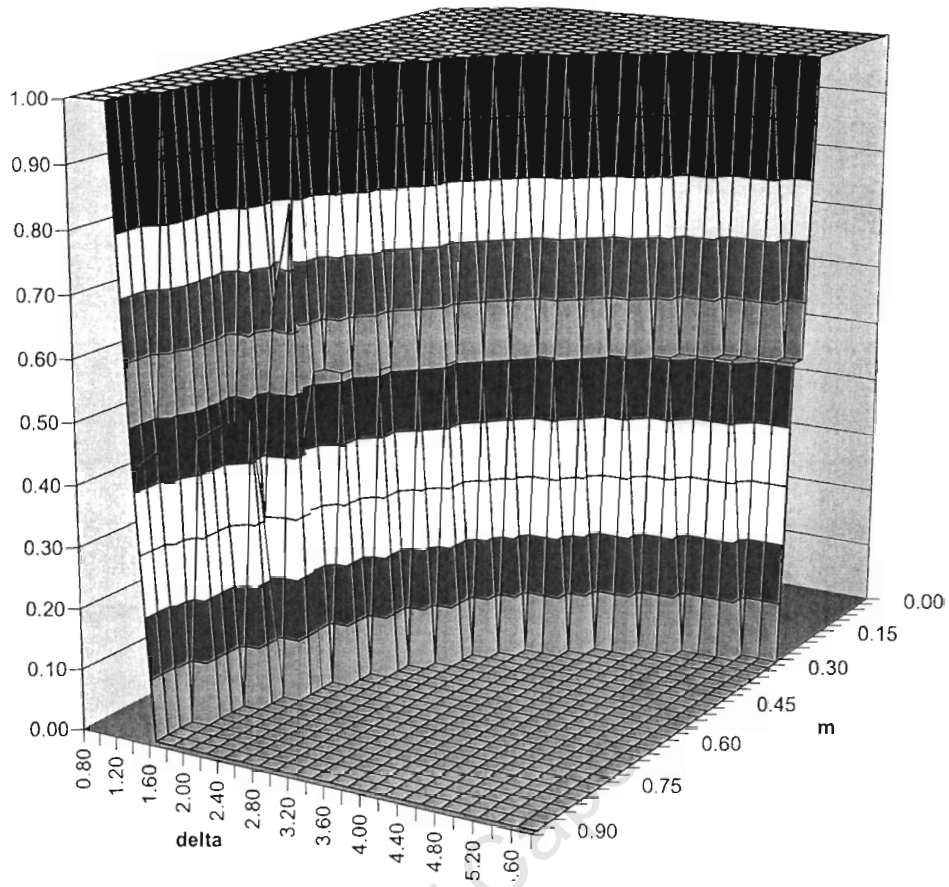
		δ																											
		0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80		
d	0.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%		
	0.025	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	
	0.050	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	
	0.075	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	21%	21%	
	0.100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	24%	24%	24%	24%	24%	24%	24%
	0.125	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	26%	26%	26%	26%	26%	26%	26%	26%	26%	26%	26%	26%
	0.150	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	27%	27%	27%	27%	27%	27%	27%	27%	27%	27%	27%	27%	27%	27%
	0.175	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%	29%
	0.200	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	47%	60%	65%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%	30%
	0.225	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	55%	62%	66%	32%	32%	32%	32%	32%	32%	32%	32%	32%	32%	32%	32%	32%	32%
	0.250	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	48%	57%	62%	66%	69%	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%
	0.275	100%	100%	100%	100%	100%	100%	100%	100%	100%	35%	50%	58%	63%	66%	69%	34%	34%	34%	34%	34%	34%	34%	34%	34%	34%	34%	34%	34%
	0.300	100%	100%	100%	100%	100%	100%	100%	100%	100%	38%	51%	58%	63%	66%	69%	35%	35%	35%	35%	35%	35%	35%	35%	35%	35%	35%	35%	35%
	0.325	100%	100%	100%	100%	100%	100%	100%	100%	100%	39%	51%	58%	62%	66%	68%	70%	36%	36%	36%	36%	36%	36%	36%	36%	36%	36%	36%	36%
	0.350	100%	100%	100%	100%	100%	100%	100%	100%	100%	39%	50%	57%	62%	66%	68%	70%	37%	37%	37%	37%	37%	37%	37%	37%	37%	37%	37%	26%
	0.375	100%	100%	100%	100%	100%	100%	100%	100%	100%	38%	49%	56%	61%	65%	68%	70%	72%	37%	37%	37%	37%	37%	37%	37%	37%	37%	26%	27%
	0.400	100%	100%	100%	100%	100%	100%	100%	100%	100%	36%	48%	56%	61%	65%	68%	70%	72%	38%	38%	38%	38%	38%	38%	38%	38%	26%	27%	27%
	0.425	100%	100%	100%	100%	100%	100%	100%	100%	100%	33%	46%	54%	60%	64%	67%	70%	71%	39%	39%	39%	39%	39%	39%	39%	26%	27%	28%	28%
	0.450	100%	100%	100%	100%	100%	100%	100%	100%	29%	44%	53%	59%	63%	67%	69%	71%	73%	40%	40%	40%	40%	40%	26%	27%	27%	28%	29%	29%
	0.475	100%	100%	100%	100%	100%	100%	100%	100%	24%	41%	51%	58%	63%	66%	69%	71%	73%	40%	40%	40%	40%	40%	26%	27%	28%	29%	30%	30%
	0.500	100%	100%	100%	100%	100%	100%	100%	100%	17%	37%	49%	56%	62%	65%	68%	71%	73%	74%	41%	41%	26%	27%	28%	29%	30%	31%	31%	32%
	0.525	100%	100%	100%	100%	100%	100%	100%	100%	100%	32%	46%	54%	60%	65%	68%	70%	72%	74%	41%	25%	26%	27%	29%	30%	31%	31%	32%	32%
	0.550	100%	100%	100%	100%	100%	100%	100%	100%	100%	26%	42%	52%	59%	64%	67%	70%	72%	74%	42%	26%	27%	28%	29%	30%	32%	32%	33%	33%
	0.575	100%	100%	100%	100%	100%	100%	100%	100%	100%	17%	37%	49%	57%	63%	67%	70%	72%	74%	57%	26%	27%	29%	30%	31%	33%	34%	34%	34%
	0.600	100%	100%	100%	100%	100%	100%	100%	100%	100%	5%	31%	46%	55%	61%	66%	69%	72%	74%	57%	26%	28%	29%	31%	32%	34%	35%	35%	35%
0.625	100%	100%	100%	100%	100%	100%	100%	100%	100%	23%	41%	52%	60%	65%	69%	72%	74%	56%	56%	56%	28%	30%	32%	33%	35%	37%	37%	37%	
0.650	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	35%	49%	58%	64%	68%	71%	73%	55%	55%	55%	29%	31%	33%	35%	36%	38%	38%	38%	
0.675	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	26%	44%	55%	62%	67%	71%	73%	55%	55%	55%	29%	31%	34%	36%	38%	40%	40%	40%	
0.700	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	37%	52%	60%	66%	71%	73%	54%	54%	54%	54%	54%	32%	35%	38%	40%	43%	43%	
0.725	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	47%	58%	65%	71%	73%	54%	54%	54%	54%	54%	54%	33%	36%	40%	43%	46%	46%	
0.750	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	40%	55%	64%	71%	73%	53%	53%	53%	53%	53%	53%	35%	38%	42%	46%	51%	51%	
0.775	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	50%	52%	62%	70%	73%	52%	52%	52%	52%	52%	53%	53%	41%	46%	51%	57%	57%	
0.800	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	52%	52%	52%	52%	52%	52%	52%	44%	50%	58%	66%	66%	
0.825	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	51%	51%	51%	51%	51%	52%	52%	48%	57%	0%	0%	0%	
0.850	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	51%	51%	51%	51%	51%	51%	51%	0%	0%	0%	0%	0%	
0.875	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	50%	51%	51%	51%	51%	51%	0%	0%	0%	0%	0%	0%	
0.900	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	50%	50%	50%	50%	50%	50%	0%	0%	0%	0%	0%	0%	
0.925	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	49%	50%	50%	50%	50%	50%	0%	0%	0%	0%	0%	0%	
0.950	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	49%	49%	49%	49%	49%	49%	0%	0%	0%	0%	0%	0%	
0.975	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	
1.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	

The best way in which to utilise the methodology is to generate a graphical representation of the result of who innovates first as a function of the three variables. This will give us a clear picture of the sensitivity of the result to the respective variables. As we can only depict two variables at a time, three three-dimensional graphs will be necessary.

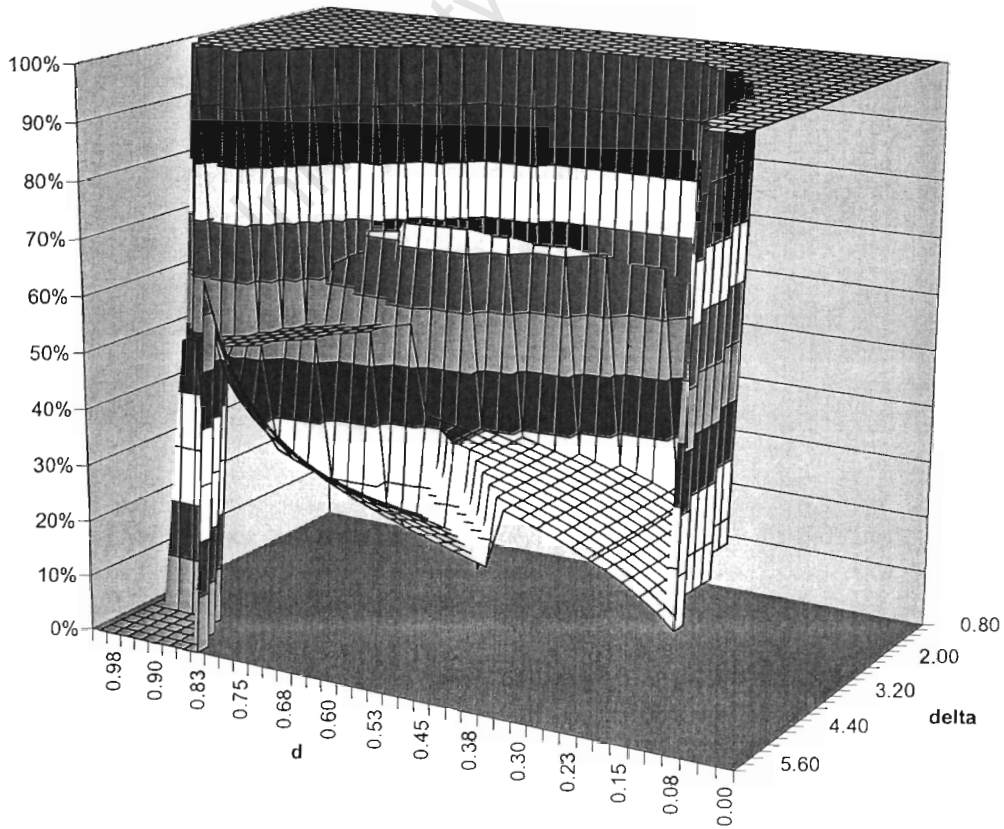
Graph 5: % of games in which the entrant innovates first for $\delta=2$



Graph 6: % of games in which the entrant innovates first for $d = 0.875$



Graph 7: % of games in which the entrant innovates first for $m = 0.3$



The above graphs dramatically demonstrate the power of this methodology. It is clear that the oft-discussed question of who innovates first in an incumbent – entrant framework relies crucially on the characteristics of the industry. This is not an unexpected result. The simple existence of empirical support for each side indicates that there must be a reason why some cases demonstrate persistent leadership and others don't. It is extremely unlikely that this is not due to some characteristic of the industry. Of course the uncertain nature of the innovative process could mean that the observed variations or consistencies in industry leadership could simply be the result of a random process. However this is statistically extremely improbable. The sensible hypothesis is that the observed contrasts between industries results from contrasts in the observed industries. The above results demonstrate visibly how this might be the case.

These results do not rely on the exact equations we derived. We made assumptions as to how sensitive profit was to the defined variables. However the direction of the influences is far less subjective. Altering the sensitivity will not alter the structure of the above graphs, it would simply speed up or slow down the reaction of the function to the respective variable. Therefore, these results rely only on the propositions made with regard to the direction of the effect of each variable on profit in each scenario.

We can also use the model to look at innovation from the societal aspect. A lot of the literature on R&D focuses not on the profit maximization of individual firms but on social welfare maximization through promotion of R&D. From the perspective of a policy-maker this is of more relevance. A policy maker would be indifferent as to which firm innovates first but would be concerned with the time of the first

innovation, with respect to growth, and with the total investment made in R&D, again for its contribution towards aggregate demand. We should therefore examine how the levels of δ and d affect the time of first innovation. We would expect the time to diminish as δ increases. This is what Beath, Katsoulacos & Ulph (1989) would refer to as the profit incentive driving the innovation. But profit also diminishes as d increases. This is an important observation. The Schumpeterian hypothesis is that competition, in the sense of an unconcentrated market structure, harms innovation, as the supernormal profits obtained under a concentrated market structure that allow R&D to take place are reduced. Here competition, in the sense of price competition, harms innovation within a specified market structure. That is, if we have a duopoly, to encourage innovation, we should allow collusion. Of course, policy makers would not only have a singular aim. The important point is that there is a trade-off for policy makers between innovation and competition in the product market.

We now seek to support and build on the results of Greenstein & Ramey (1998) which themselves are a reversal of the results of the conclusions of Arrow (1962) in the case of process innovations. They found that a blockaded monopoly and a competitive industry provide identical returns to innovation if the innovation is non-drastic. If the monopolist can be threatened with entry then it provides strictly greater incentives to innovate than a competitive industry. In this model, we maintain the monopolist incumbency framework but instead refer to competition within the secondary product market in the event of a duopoly. We shall find that the time of the first innovation is earlier for a protected monopoly, where $\delta > 2s - 1$, than in a competitive industry. Our results will demonstrate what was clear from the above analysis, that innovation happens soonest, as a direct result of the most investment in

R&D, in the situation where the monopolist is threatened by entry and there is perfect collusion in the duopoly scenario.

Were the incumbent were to act in isolation, protected from entry, its payoff would be

$$(42) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{\infty} \pi_i(P_i^1, P_i^2) e^{-rt} dt - [C(T_i)]$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(43) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \text{ or}$$

$$(44) \quad T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^1, P_i^2)}} \right)$$

From Proposition 7 we know that the only variables that affect profit to the incumbent when it holds a monopoly in both markets are s and δ . As we have been holding s fixed at a level of 0.95, our attention should focus on the effect of changes in δ on the decision to innovate by the incumbent when it is not facing a threat of entry. We saw earlier that it was a necessary condition for the incumbent innovating first that $\delta > 2s - 1$. This was because the force of profit incentive would never be enough to make the incumbent invest in innovating a less profitable innovation. The entrant does have a profit incentive, though, for these levels of δ and therefore will always innovate first. This condition now becomes a condition for the incumbent attempting innovation at all when the monopoly is protected. The table below shows a

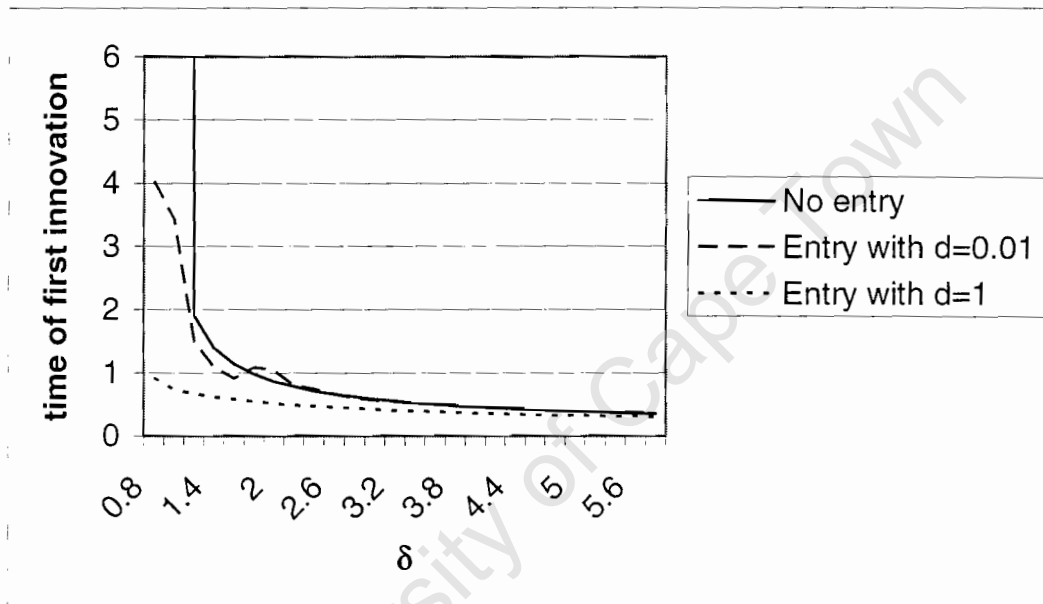
comparison of the time innovation occurs with threat of entry against the time innovation occurs when an incumbent acts in isolation.

Table 6: Comparisons of time of innovation for levels of δ in the case where $s=0.95$, $m = 0.5$, $r = 10\%$ and $x = 2$ for a non-drastic innovation.

δ

	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80
No entry	Never	Never	1.92	1.40	1.14	0.98	0.86	0.78	0.71	0.66	0.62	0.58	0.55	0.52	0.50	0.48	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.36	0.35
Entry with $d=0.01$	4.03	3.44	1.50	1.10	0.91	1.08	1.04	0.80	0.74	0.66	0.60	0.56	0.53	0.51	0.49	0.47	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.37	0.36
Entry with $d=1$	0.92	0.73	0.67	0.62	0.58	0.54	0.51	0.49	0.47	0.45	0.43	0.42	0.40	0.39	0.38	0.37	0.36	0.35	0.34	0.34	0.33	0.32	0.32	0.31	0.30	0.30

Graph 8: Graph of Table 6



We must look at different levels for d since, although d does not affect the time of innovation when entry is blockaded, it does when entry is possible. We use a value of 0.01 for d to represent fierce competition because with a value of 0 analysis is confounded by the aforementioned fact that the average time of innovation is ∞ when this is allowed as one of the components of a vector of mixed strategies.

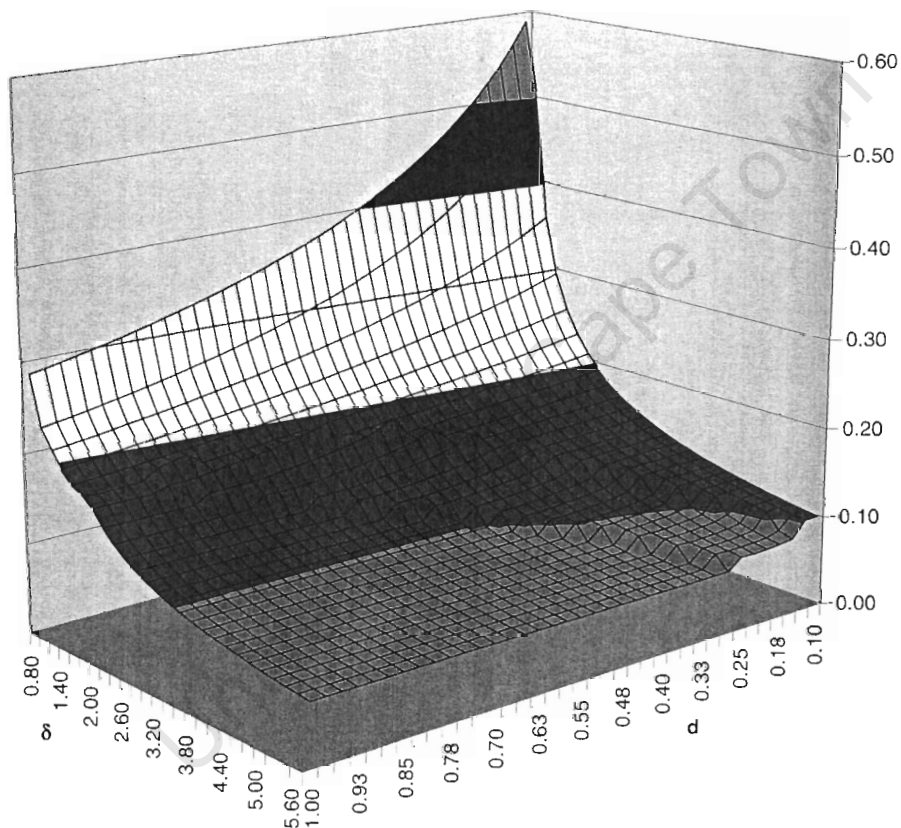
We have already seen mathematically that when the incumbent is not threatened by entry, innovation only occurs when the condition $\delta > 2s - 1$ is fulfilled. An incumbent will only cannibalise its own market if it is being replaced with something better. We can also see from the data that, when this condition is fulfilled, the time of innovation is almost the same when entry is blockaded as when entry is allowed but competition is fierce - in fact it is sometimes earlier. Essentially this is saying that competition, by reducing profits, discourages innovation. When competition is so fierce that it reduces profits to almost zero, as in the case when d is very small, then the threat of entry has no impact on the time of innovation. These results therefore unequivocally support the insight of Beath, Katsoulacos & Ulph (1989), that the forces of 'profit incentive' and 'competitive threat' drive innovative activity. These forces are maximised when there are no barriers to entry and perfect collusion in the duopoly market, precisely the circumstances in which, within our model, investment in R&D is maximised. It also builds on the results of Greenstein & Ramey (1998) in that it demonstrates the same conclusions with competition in the secondary product market that they show with competition in the primary product market.

Purely from the aspect of encouraging innovation, it appears that the best framework in which to encourage R&D expenditure, and thereby reduce innovation time, is to ensure that entry is possible and then allow the firms to collude. It is not that greater concentration must always lead to greater innovation. Rather, the point is that reduced barriers to entry allied with the possibility of collusion in a duopoly together cause a reduction in the time to innovation. This may be a worrying conclusion for makers of competition policy. The ideal for consumers is for there to be both

innovation and competition in the market for that innovation. The results shown above demonstrate that the two aims can oppose one another or at least imply trade-offs.

However, all is not lost for consumers and policy makers. The result described above is itself highly sensitive to the level of δ . The graph below demonstrates this.

Graph 9: Time of first innovation for neutral branding influences



At a low level of d , representing a high degree of competition, the time to the first innovation is higher. Also the time to innovation is more sensitive to changes in δ at low levels of d . This has implications for competition policy. If δ is sufficiently high, the disadvantage of a later innovation time resulting from the prevention of collusion, is reduced. For a given level of δ , increased competition increases the time of a successful innovation. In terms of looking purely at the time of a successful

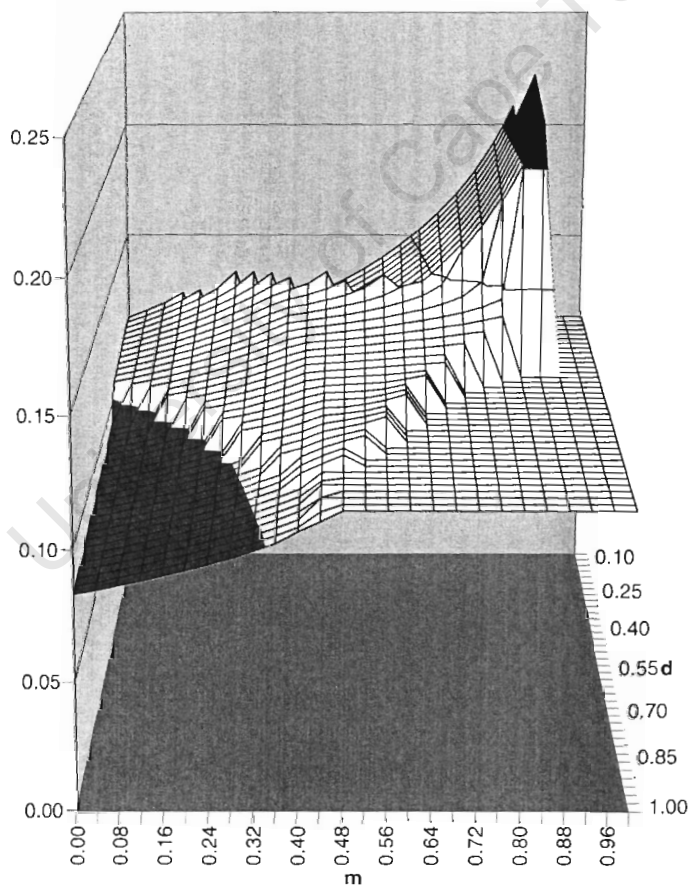
innovation, the worst-case scenario for any level of δ is either an incumbent in a blockaded industry or an industry facing the threat of entry but with perfect Bertrand competition in the event of entry. The best-case scenario is an industry facing the threat of entry but with a perfectly collusive duopoly if innovation occurs. For competition policy makers, this represents a difficult balancing act. Monopoly is clearly bad for innovation and consumers. If we allow entry and enforce strict competition, this will not improve the situation. To allow a perfectly collusive duopoly is better than staying with the original duopoly as, although consumers are still paying monopoly prices, at least innovation has occurred. There will have to be a trade-off made in which innovation is delayed as a result of the restriction of collusion.

The important caveat to this result for policy makers is the sensitivity of the relationship between innovation time and competition to δ . When δ is high then the sensitivity of the innovation time to the level of competition is reduced. Therefore at high levels of δ , a high level of competition will have a far smaller negative impact in the time of innovation than it would at a lower level of δ . The implication of this is that policy makers should tailor their policing of industries according to industry levels of δ . Where δ is high, they can fiercely police a sector to protect consumers without worrying about any significant impact on innovation. Where δ is low, if they wish to encourage innovation, they should consider backing off to allow firms to earn the supernormal profits required for innovation.

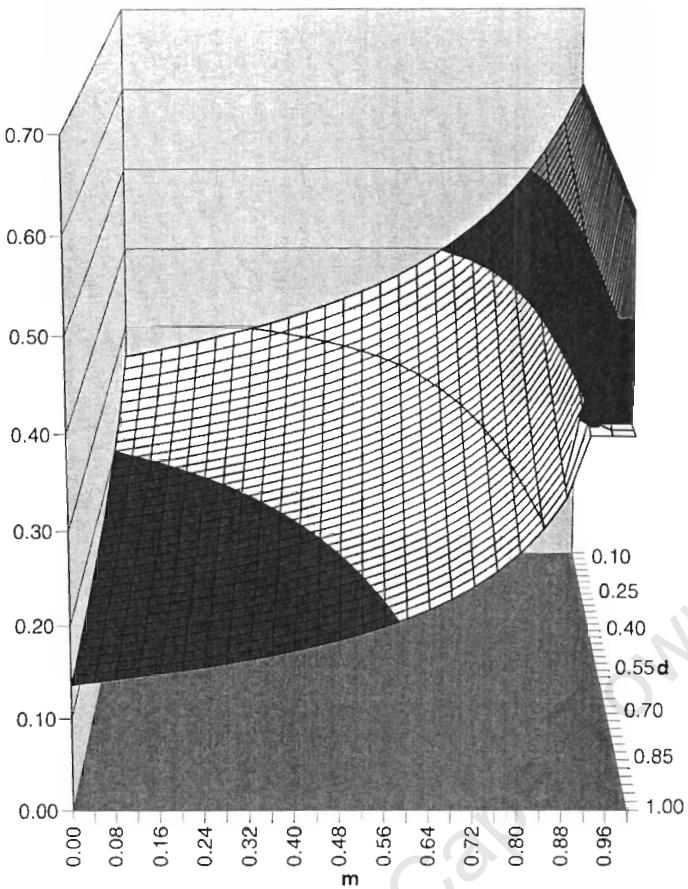
It could well be that industries are characterised by a systemic level of δ . Some industry sectors may exhibit strong, innovation-driven growth over a prolonged

period. In this case, policy makers should define the industry as a ‘high- δ ’ industry and use this as their basis for making a decision on how best to regulate the industry. The incumbent in such an industry may or may not hold an advantage if an entrant comes into the market. Therefore a policy maker would be interested in observing how the time of first innovation changes for various levels of m and d , d being the variable under the control of the policy maker and m being observed. It is likely that the policy maker would have to make an estimate of the range into which m falls. It is for these types of scenarios that the methodology used becomes increasingly useful.

Graph 10: Time of first innovation for a high $\delta(3)$ industry



Graph 11: Time of first innovation for a low d (1.15) industry



We can see from the above graphs that the time of first innovation is potentially more sensitive to changes in m and d within industries that exhibit a high level of δ . This means that a policy maker might have to be extremely careful in deciding how strictly it wishes to enforce competition in this industry. We can see from the graph for the high δ industry that, as the advantage to the incumbent increases, i.e. m diverges away from 0, the policy maker can enforce an increasing amount of competition onto the industry without unduly affecting the time of the first innovation. However, there is a fulcrum level of d for which the time of first innovation is extremely sensitive. Increasing d above this level and reducing the speed of innovation may outweigh the gains to the policy maker from increased consumer surplus. The beliefs of the policy maker as to the level of m are therefore significant in determining the ‘safe’ level of price competition that the policy maker can enforce without harming innovation.

The low δ industry exhibits a much smoother relationship between the time of first innovation, m and d . As a result the policy maker can be more confident that a small change in the decision it makes will not give a dramatically different result. Policy makers may well take the view that high δ industries generate growth and thus should be left alone in order to help drive the innovation and avoid the possibility of the problem outlined above. Low δ industries may be monitored closely for the benefit of consumers as the overall effect on growth will be far less.

It is clear then that the most simple of changes to the basic model of Gilbert & Newbery (1982), namely the removal of the patenting assumption has led to a complete reversal of the conclusions drawn. This justifies the arguments set out at the beginning of this investigation that the majority of models derived in the literature are not sufficiently robust for their equilibrium solutions to survive the testing of the assumptions made during their construction. As well as demonstrating the significance of the patenting assumption in the determination of the Nash Equilibrium, I have also been able to show how, with the use of sensitivity analysis methodology, important policy implications may be drawn from the observed impact of industry-specific variables. In this case I was able to demonstrate that the consequences of attempts by policy makers to promote competitive pricing on investment in innovation have a far less uniform profile for high growth industries.

Increasing the strategic options for the incumbent – the nine solution set model

The previous section demonstrated the significant changes that can occur in the solutions to the game when a seemingly small change is made to the structure. This section shall serve two purposes. The first is that it will offer another illustration of the sensitivity of the game theoretic result to small changes in the framework. The second is that it will integrate in to the framework an important addition that was not included by Gilbert & Newbery (1982), that is, the opportunity for the incumbent firm to shelve the original product after it has achieved innovation. In the previous section I explained, whilst outlining the model of Gilbert & Newbery, that only the opportunity to shelve the innovation was built in to the framework. This is no doubt due to the desire on the part of the authors to demonstrate their conclusion more forcefully, that the incumbent would win the auction for the patent solely to pre-empt the entrant, not for the innovation itself. Intuitively however, it is more satisfactory to allow the incumbent this option within our model.

The shelving of products can only occur when the incumbent has innovated. There are two scenarios in which this may have occurred. Either the incumbent has innovated and is the only firm in the product market or it has innovated and is now part of a duopoly in the new product market. The incumbent could choose to shelve the original product, the new innovation or neither. The choice of these three options may be determined by a simple analysis of which is preferable. The incumbent would never shelve both products at the same time as that would leave it with nothing. For the same reason it would not shelve the original product before it has innovated. Given this, we can determine that there are nine possible shelving configurations.

There can be described as solution sets that correspond to the shelving that occurs in each scenario.

Table 7: Configuration of solution sets in the nine solution set model

Solution Set	Shelve when only Incumbent has innovated	Shelve when both have innovated
1	New product	New product
2	New product	Old product
3	New product	Neither product
4	Old product	New product
5	Old product	Old product
6	Old product	Neither product
7	Neither product	New product
8	Neither product	Old product
9	Neither product	Neither product

This is not to suggest that a Nash Equilibrium solution could exist within each of these solution sets. We are merely stating here that the actions of the incumbent with regard to the shelving of products will change the framework within which the game is being played. The identification of Nash Equilibria within these frameworks is the next challenge.

The best way to examine the solution sets is to state that there are two possible classes into which the Nash Equilibria can be grouped. There are those in which the

incumbent firm innovates before the entrant, and there are those in which the entrant innovates before the incumbent.

Nash Equilibria in which $T_i > T_e$

In this type of Nash equilibrium the shelving decision of the incumbent firm in the event of innovating first is irrelevant as it always innovates after the entrant. Therefore the Nash Equilibrium strategy for the incumbent is determined by maximising the payoff condition

$$\begin{aligned}
 (45) \quad & \int_0^{T_e} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{T_i} \pi_i(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} e^{-rt} dt - [C(T_i)] \\
 & = \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_i(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} e^{-rT_i}}{r} - \frac{1}{T_i}
 \end{aligned}$$

If $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2)$ then the optimum strategy for the incumbent must be

to not innovate. Any strategy in which the incumbent innovates at a time, T_i , where $T_e < T_i < \infty$ cannot be optimal as the incumbent would shelve the new product at the time of innovation. No firm would incur a cost of innovation, however small, to develop a product for which there are no circumstances in which it would introduce the product. In this scenario the entrant may choose its strategy in the knowledge that

the incumbent will never innovate, thereby essentially knowing that it will be the only firm in the new product market.

The payoff to the entrant firm is therefore

$$(46) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt - [C(T_e)] \\ &= \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(47) \quad T_e = \sqrt{\frac{1}{\pi_e(P_i^1, P_e^2) e^{-rT_e}}}$$

which can be solved either through iteration or use of the Lambert w-function to give

$$(48) \quad T_e = -\frac{2}{r} W\left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}}\right)$$

The payoff to the incumbent would therefore be

$$(49) \quad \begin{aligned} & \int_0^{T_e} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt \\ &= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} \end{aligned}$$

Proposition 6:

In the set of Nash Equilibria in which $T_i > T_e$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2)$ then

the Nash Equilibrium Solution is $\{T_i, T_e\} = \left\{ 0, -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}} \right) \right\}$. This will

be true for solution sets 1, 4 and 7.

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If $\max \begin{bmatrix} \pi_i(P_i^1, P_e^2) \\ \pi_i(P_e^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_e^2)$ then the incumbent firm will shelve the old product in

the event of a duopoly. Given this scenario we can specify the payoff to the entrant as

$$(50) \quad \int_{T_e}^{T_i} \pi_e(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_e(P_e^2) e^{-rt} dt - [C(T_e)] \\ = \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_e(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\pi_e(P_e^2) e^{-rT_i}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(51) \quad T_e = \left(\frac{1}{\sqrt{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \right) \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right)$$

The incumbent will receive a payoff of

$$(52) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{T_i} \pi_i(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_i(P_e^2) e^{-rt} dt - [C(T_i)] \\ = \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_i(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\pi_i(P_e^2) e^{-rT_i}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(53) \quad T_i = \frac{1}{\sqrt{\pi_i(P_e^2) e^{-rT_i} - \pi_i(P_i^1, P_e^2) e^{-rT_i}}} \text{ or}$$

$$(54) \quad T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(p_{ei}^2)}}\right)$$

Proposition 7:

In the set of Nash Equilibria in which $T_i > T_e$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_{ei}^2)$ then the

Nash Equilibrium Solution is

$$\{T_i, T_e\} = \left\{ -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(p_{ei}^2)}}\right), -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}}\right) \right\}.$$

This will be true for solution sets 2, 5 and 8.

If $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ then the incumbent firm will not shelve either

product in the event of a duopoly. Given this scenario we can specify the payoff to the entrant as

$$(55) \quad \frac{\int_{T_e}^{T_i} \pi_e(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_e(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_e)]}{\frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_e(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\pi_e(P_i^1, P_{ei}^2) e^{-rT_i}}{r} - \frac{1}{T_e}}$$

Maximising this with respect to T_e gives

$$(56) \quad T_e = \left(\frac{1}{\sqrt{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \right) \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right)$$

The incumbent will receive a payoff of

$$(57) \quad \frac{\int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_e}^{T_i} \pi_i(P_i^1, P_e^2) e^{-rt} dt + \int_{T_i}^{\infty} \pi_i(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_i)]}{\frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{\pi_i(P_i^1, P_e^2) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_{ei}^2) e^{-rT_i}}{r} - \frac{1}{T_i}}$$

Maximising this with respect to T_i gives

$$(58) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_{ei}^2)e^{-rT_i} - \pi_i(P_i^1, P_e^2)e^{-rT_i}}} \text{ or}$$

$$(59) \quad T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(P_i^1, P_{ei}^2)}}\right)$$

Proposition 8:

In the set of Nash Equilibria in which $T_i > T_e$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_e^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ then

the Nash Equilibrium Solution is

$$\{T_i, T_e\} = \left\{ -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(P_i^1, P_{ei}^2)}}\right), -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{r^2}{\pi_e(p_i^1, P_e^2)}}\right) \right\}.$$

This will be true for solution sets 3, 6 and 9.

Nash Equilibria in which $T_i < T_e$

In this set of Nash equilibria, the payoff to the incumbent shall be of the form

$$\int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} e^{-rt} dt + \int_{T_e}^{\infty} \max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} e^{-rt} dt - [C(T_i)]$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} e^{-rT_i}}{r} - \frac{\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} e^{-rT_e}}{r} + \frac{\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} e^{-rT_e}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(60) \quad T_i = \frac{1}{\sqrt{\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \quad \text{or}$$

$$T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(P_i^1) - \max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix}}} \right)$$

We already know that if $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1)$ then no Nash Equilibria exist in

which the inequality $T_i < T_e$ is satisfied. This is because the term inside the square root in the above equation for the optimum strategy would be zero, thus suggesting

that the optimum time for the incumbent to innovate would be ∞ or never. As the Nash Equilibria within this set are constrained to satisfy the inequality $T_i < T_e$, then this could not be the case.

We are therefore stating that no Nash Equilibrium can exist within solution sets 1, 2 and 3 in which the incumbent firm innovates before the entrant. This is because for any innovation time by the entrant, the best response to this by the incumbent could only be, at best, the same time of innovation as the entrant. It could never be a best response to innovate earlier than the entrant in order to shelve the product. Within these solution sets the incumbent would always regret innovating before the entrant as it could have paid less for the innovation to occur later at no detriment to itself.

Proposition 9:

If $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1)$ then no Nash Equilibria exist in which the inequality

$T_i < T_e$ is satisfied. This is true for solution sets 1, 2 and 3.

In the event that $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2)$ then we may find Nash Equilibria in

which the incumbent innovates before the entrant. Where

$\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ then this is a structure in which the incumbent firm

shall never shelve either product.

In this case the payoff to the entrant would be

$$(61) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_e)] \\ & = \frac{\pi_e(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(62) \quad T_e = \left(\frac{1}{\sqrt{\pi_e(P_i^1, P_{ei}^2) e^{-rT_e}}} \right) \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_{ei}^2)}} \right)$$

The incumbent will receive a payoff of

$$(63) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_i)]$$

$$= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^1, P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(64) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \quad \text{or}$$

$$T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}} \right)$$

Proposition 10 :

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ and the

inequality $T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$$\left\{ -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}} \right), -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_{ei}^2)}} \right) \right\}$$

This is true for solution set 9.

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_{ei}^2)$ the incumbent

firm will shelve the new product only in the event of a duopoly in the new product market. The payoff to the entrant firm is therefore

$$(65) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_{ei}^2) e^{-rt} dt - [C(T_e)] \\ &= \frac{\pi_e(P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(66) \quad T_e = \sqrt{\frac{1}{\pi_e(P_{ei}^2) e^{-rT_e}}} \text{ or } T_e = -\frac{2}{r} W\left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}}\right)$$

The payoff received by the incumbent in this case would be

$$(67) \quad \begin{aligned} & \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_{ei}^2) e^{-rt} dt - [C(T_i)] \\ &= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^1, P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_i} \end{aligned}$$

Maximising this with respect to T_i gives

$$(68) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \text{ or}$$

$$(69) \quad T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}}\right)$$

Proposition 11:

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_{ei}^2)$ and the inequality

$T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$\left\{ -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}}\right), -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}}\right) \right\}$. This is true for solution set

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When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_e^2) \\ \pi_i(P_e^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2)$ the incumbent

firm will shelve the new product only in the event of a duopoly in the new product market. The payoff to the entrant firm is therefore

$$(70) \quad \int_{T_e}^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt - [C(T_e)] \\ = \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_e}$$

Maximising this with respect to T_e gives

$$(71) \quad T_e = \sqrt{\frac{1}{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right)$$

The payoff received by the incumbent in this case would be

$$(72) \quad \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^1, P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt - [C(T_i)] \\ = \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^1, P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^1, P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_i}$$

Maximising this with respect to T_i gives

$$(73) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^1, P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \text{ or}$$

$$(74) \quad T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}} \right)$$

Proposition 12:

$$\text{When } \max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^1, P_i^2) \quad \text{and} \quad \max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2) \quad \text{and the}$$

inequality $T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$$\left\{ -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^1, p_i^2)}} \right), -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}} \right) \right\}.$$

This is true for solution set 7.

If $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ then this is a structure

in which the incumbent firm shall only shelve the old product when it is the only firm to have innovated.

In this case the payoff to the entrant would be

$$(75) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_e)] \\ &= \frac{\pi_e(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(76) \quad T_e = \left(\frac{1}{\sqrt{\pi_e(P_i^1, P_{ei}^2) e^{-rT_e}}} \right) \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_{ei}^2)}} \right)$$

The incumbent will receive a payoff of

$$(77) \quad \begin{aligned} & \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_{ei}^2) e^{-rt} dt - [C(T_i)] \\ &= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_i} \end{aligned}$$

Maximising this with respect to T_i gives

$$(78) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^2)e^{-rT_i} - \pi_i(P_i^1)e^{-rT_i}}} \quad \text{or}$$

$$T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^2)}}\right)$$

Proposition 13 :

When $\max\begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max\begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_{ei}^2)$ and the inequality

$T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$$\left\{ -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^2)}}\right), -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{r^2}{\pi_e(P_i^1, P_{ei}^2)}}\right) \right\}.$$

This is true for solution set 6.

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_{ei}^2)$ the incumbent firm

will shelve the new product only in the event of a duopoly in the new product market.

The payoff to the entrant firm is therefore

$$(79) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_{ei}^2) e^{-rt} dt - [C(T_e)] \\ &= \frac{\pi_e(P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(80) \quad T_e = \sqrt{\frac{1}{\pi_e(P_{ei}^2) e^{-rT_e}}} \quad \text{or} \quad T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}} \right)$$

The payoff received by the incumbent in this case would be

$$(81) \quad \begin{aligned} & \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_{ei}^2) e^{-rt} dt - [C(T_i)] \\ &= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_{ei}^2) e^{-rT_e}}{r} - \frac{1}{T_i} \end{aligned}$$

Maximising this with respect to T_i gives

$$(82) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \quad \text{or}$$

$$(83) \quad T_i = -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(P_i^2)}}\right)$$

Proposition 14:

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_{ei}^2)$ and the inequality

$T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$$\left\{ -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(P_i^2)}}\right), -\frac{2}{r}W\left(-\frac{1}{2}\sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}}\right) \right\}.$$

This is true for solution set 5.

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_e^2) \\ \pi_i(P_e^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2)$ the incumbent

firm will shelve the new product only in the event of a duopoly in the new product market. The payoff to the entrant firm is therefore

$$(84) \quad \begin{aligned} & \int_{T_e}^{\infty} \pi_e(P_i^1, P_e^2) e^{-rt} dt - [C(T_e)] \\ &= \frac{\pi_e(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_e} \end{aligned}$$

Maximising this with respect to T_e gives

$$(85) \quad T_e = \sqrt{\frac{1}{\pi_e(P_i^1, P_e^2) e^{-rT_e}}} \text{ or } T_e = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right)$$

The payoff received by the incumbent in this case would be

$$(86) \quad \begin{aligned} & \int_0^{T_i} \pi_i(P_i^1) e^{-rt} dt + \int_{T_i}^{T_e} \pi_i(P_i^2) e^{-rt} dt + \int_{T_e}^{\infty} \pi_i(P_i^1, P_e^2) e^{-rt} dt - [C(T_i)] \\ &= \frac{\pi_i(P_i^1)}{r} - \frac{\pi_i(P_i^1) e^{-rT_i}}{r} + \frac{\pi_i(P_i^2) e^{-rT_i}}{r} - \frac{\pi_i(P_i^2) e^{-rT_e}}{r} + \frac{\pi_i(P_i^1, P_e^2) e^{-rT_e}}{r} - \frac{1}{T_i} \end{aligned}$$

Maximising this with respect to T_i gives

$$(87) \quad T_i = \frac{1}{\sqrt{\pi_i(P_i^2) e^{-rT_i} - \pi_i(P_i^1) e^{-rT_i}}} \text{ or } T_i = -\frac{2}{r} W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^2)}} \right)$$

Proposition 15:

When $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} = \pi_i(P_i^2)$ and $\max \begin{bmatrix} \pi_i(P_i^1, P_e^2) \\ \pi_i(P_e^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} = \pi_i(P_i^1, P_e^2)$ and the inequality

$T_i < T_e$ is satisfied, the Nash Equilibrium Solution is given as

$$\left\{ -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(P_i^1) - \pi_i(P_i^2)}} \right), -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_i^1, P_e^2)}} \right) \right\}.$$

This is true for solution set 4.

I shall summarise the above solutions in matrix form below. The first solution matrix displays Nash Equilibrium solutions where the incumbent innovates before the entrant. The second solution matrix displays the Nash Equilibrium solutions where the entrant innovates before the incumbent.

Solution Matrix 1: Incumbent innovating first

$\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} =$			
$\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} =$	$\pi_i(P_i^1, P_i^2)$	$\pi_i(P_i^1)$	
$\pi_i(P_i^1, P_{ei}^2)$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, P_{ei}^2)}} \right) \end{array} \right\}$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(P_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, P_{ei}^2)}} \right) \end{array} \right\}$	<p>No N.E. exist</p>
$\pi_i(P_{ei}^2)$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}} \right) \end{array} \right\}$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(P_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(P_{ei}^2)}} \right) \end{array} \right\}$	<p>No N.E. exist</p>
$\pi_i(P_i^1, P_e^2)$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(p_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, P_e^2)}} \right) \end{array} \right\}$	$\left\{ \begin{array}{l} -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1) - \pi_i(P_i^2)}} \right), \\ -\frac{2W}{r} \left(\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, P_e^2)}} \right) \end{array} \right\}$	<p>No N.E. exist</p>

Solution Matrix 2: Entrant innovating first

$T_i > T_e$		Nash Equilibrium Solution Sets
$\max \begin{bmatrix} \pi_i(P_i^1, P_{ei}^2) \\ \pi_i(P_{ei}^2) \\ \pi_i(P_i^1, P_e^2) \end{bmatrix} =$	$\pi_i(P_i^1, P_{ei}^2)$	$\left\{ -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(P_i^1, P_{ei}^2)}} \right), -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}} \right) \right\}$
	$\pi_i(P_{ei}^2)$	$\left\{ -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{-r^2}{\pi_i(p_i^1, p_e^2) - \pi_i(P_{ei}^2)}} \right), -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}} \right) \right\}$
	$\pi_i(P_i^1, P_e^2)$	$\left\{ \infty, -\frac{2}{r}W \left(-\frac{1}{2} \sqrt{\frac{r^2}{\pi_e(p_i^1, p_e^2)}} \right) \right\}$

The addition of the option for the incumbent to shelve the original product may greatly increase the complexity of the algebraic formulation of the model but it does not necessitate a similar change to the specification of the profit functions.

In this event that the incumbent may wish to shelve the original product when it is the only innovator, the payoff would be

$$(88) \quad \pi_i(p_i^2) = \delta x \text{ for all } \delta$$

We must also consider the scenario where the incumbent would choose to shelve the original product after both firms have innovated. In this case the payoffs would be

$$(89) \quad \begin{aligned} \pi_i(p_{ei}^2) &= m\delta x + (1-m)\frac{d\delta x}{2} \\ \pi_e(p_{ei}^2) &= (1-m)\frac{d\delta x}{2} \end{aligned} \text{ for all } \delta$$

In this section, I have simply increased the action set of the incumbent to include a more intuitive possibility. The incumbent is now allowed, within the model, to shelve the old product after it has innovated. This was not an option originally included by Gilbert & Newbery, although the reason for this was that it allowed them a more forceful demonstration of their conclusion. A feature of their model was that the incumbent desired the patent solely as a barrier to entry. My expansion of strategic options has led to the increase in the combinations of shelving behaviour to nine scenarios. Having derived the NE for these scenarios and the additional profit functions we are now positioned to derive results from the model.

Analysis of static model with nine solution sets

This extended version of the model gives the ability to the incumbent of shelving the original product after innovation. We have seen whilst setting out the framework of this model in the preceding section that this changes some of the conditions for Nash Equilibria. What remains to be determined is whether this extension to the framework alters in any significant way the results obtained under the framework of the four solution set model.

One would not expect to see the incumbent firm innovating less often as a result of this change, rather the reverse. This is because the previous model, in not allowing the incumbent to shelve the original product, was placing a constraint on the actions of the incumbent and, potentially, reducing the returns from innovation. As a result, the incumbent would invest a smaller amount in R&D when faced with this constraint than when not. The incumbent could never gain a higher payoff in the earlier model than in the more sophisticated one, as any vector of product placement in the previous model can be replicated by the incumbent in the new one.

Although we know that the incumbent receives a weakly dominant payoff in this model, it remains to be seen whether this will dramatically alter the result of the preceding section. Perhaps the first analysis of this model should seek to see if the introduction of the opportunity to shelve the original product is taken up.

Table 7 :Solution sets for levels of d and δ in the case where $s=0.95, m=0, r=10\%$.

δ	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
0.000	3	3	3	3	3	7	7	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.025	3	3	3	3	3	7	7	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.050	3	3	3	3	3	7	7	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.075	3	3	3	3	3	7	7	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.100	3	3	3	3	3	7	7	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.125	3	3	3	3	3	7	7	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.150	3	3	3	3	3	7	7	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.175	3	3	3	3	3	7	7	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.200	3	3	3	3	3	7	7	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.225	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.250	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.275	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.300	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.325	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.350	3	3	3	3	3	7	7	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.375	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.400	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.425	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.450	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.475	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.500	3	3	3	3	3	7	7	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.525	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.550	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.575	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.600	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.625	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.650	3	3	3	3	3	7	7	4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.675	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.700	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.725	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.750	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.775	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.800	3	3	3	3	3	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.825	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.850	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.875	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.900	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.925	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.950	3	3	3	3	3	7	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
0.975	3	3	3	3	3	9	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
1.000	3	3	3	3	3	9	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Shelving of the old product takes place in solution sets 2, 4, 5 and 6. It is clear from the above table that, at high levels of δ , these solution sets dominate the analysis. The significance of introducing this choice for the incumbent becomes clearer when we remind ourselves that the only solution sets within this model that were possible

within the previous model were 1, 3, 7 and 9. The limited cases in which these solution sets appear demonstrate the extent to which the analysis was limited by the failure to allow this option for the incumbent.

Whereas in the previous version of the model, cases with a high value of δ led to a situation of no shelving, the opportunity to shelve the old product is readily taken with a high level of δ . Solution sets 5 and 6 both imply that the original product will be shelved when only the incumbent has innovated. The original product will only be shelved in the event of a duopoly in the secondary product market in solution set 5, solution set 6 having no shelving in this event.

A note is required here on solution set 6 as it may seem somewhat strange. It requires the incumbent to shelve the original product when it has a monopoly in the innovation product market but to re-introduce the original product when the entrant innovates. We should state here that we have not introduced any costs involved in removing and introducing the products into the market so any results here discussed come with this caveat. This is admittedly unrealistic as the reintroduction of a product that has previously been shelved could incur some potentially significant costs, such as re-branding or the re-establishment of supply lines and networks. It would be relatively simple to introduce into the model the necessary conditions to prevent a product being reintroduced after it had been shelved. However, it is no more acceptable to impose an assumption that this is not possible than it is to assume that it is possible. The purpose of this extension to the model was to examine whether the restriction on the actions of the incumbent was potentially significant in the determination of the results obtained. To then impose additional restrictions on the incumbent's action on no

empirical basis seems perverse. We shall instead simply acknowledge the fact that reintroduction may be uneconomical and issue this caveat with our results.

We can see from the table that the move from solution set 5 to 6, and hence the decision to re-introduce the original product is sensitive to both d and δ . The incumbent will reintroduce the product at extreme levels of d , both high and low, and decreasingly often as δ increases. The first condition for the game being in solution sets 5 and 6 is that the original product is shelved when the incumbent is the first innovator. This can be given as $\pi_i(p_i^2) > \pi_i(p_i^1, p_i^2)$.

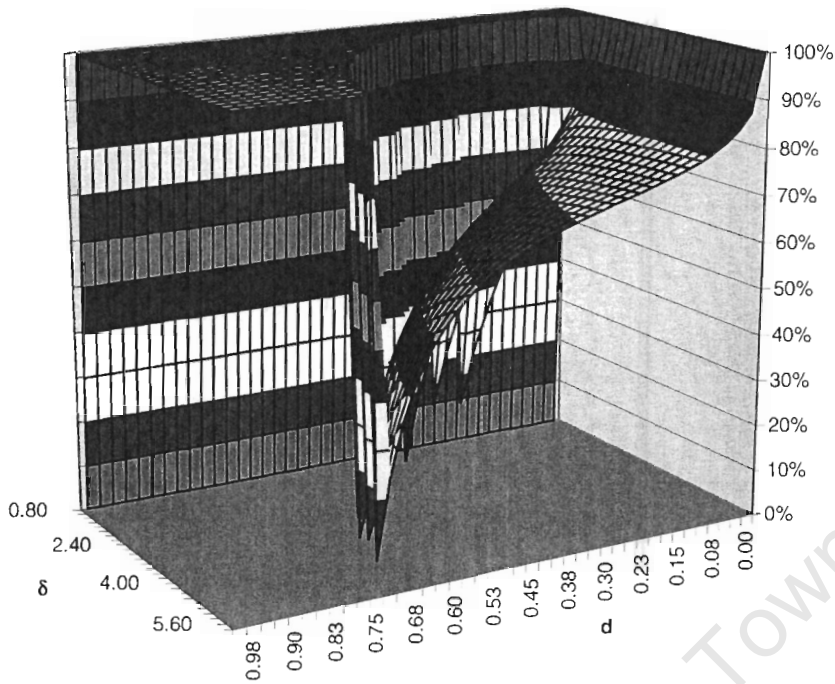
Lemma: The incumbent, when first innovator never shelves the original product for values of s less than or equal to 0.5. If s is greater than 0.5, the incumbent shelves the original product if $\delta > \frac{1}{2s-1}$.

Proof: The incumbent, when shelving the original product is forgoing the payoff $(1-s)x$. The extra payoff received is $s\left(\delta - \frac{1+\delta^2}{1+\delta}\right)$ which is constrained by the condition $s\left(\delta - \frac{1+\delta^2}{1+\delta}\right) \leq sx$. Hence, it cannot be profitable to shelve the original product and receive $(1-s)x$ for an amount which is less than sx for values of s below 0.5. Where shelving of the original product is feasible, the condition for doing so is $\delta x > (1-s)x + (1-s)\delta x + s\left(\frac{1+\delta^2}{1+\delta}x\right)$. This simplifies to $\delta > \frac{1}{2s-1}$.

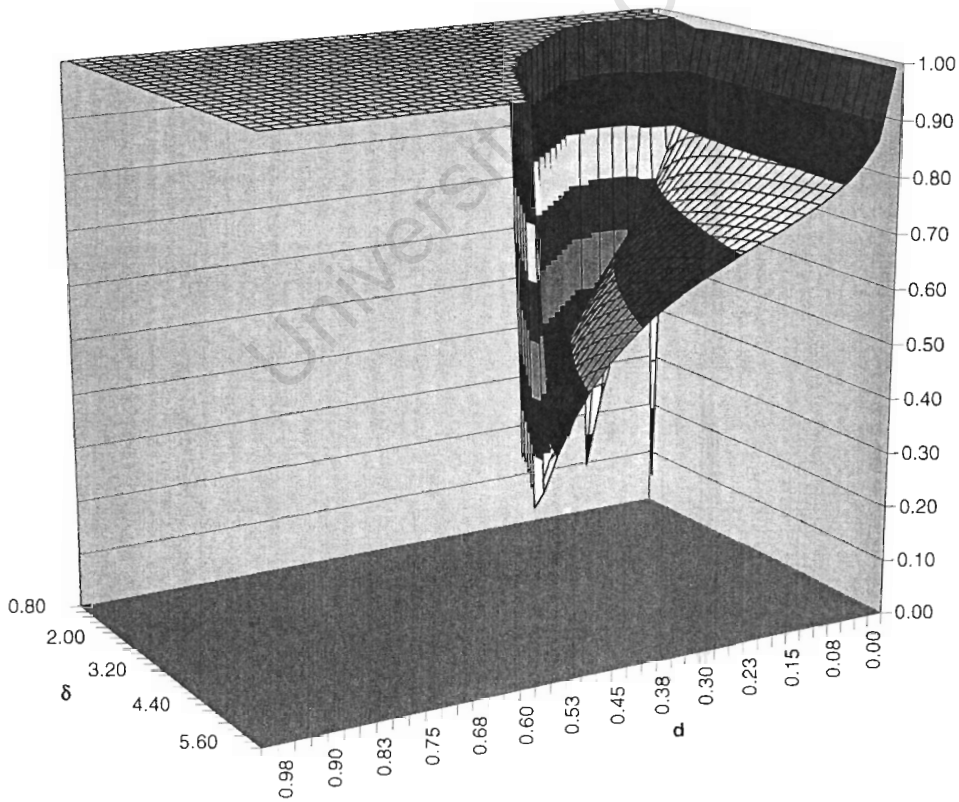
Whether the product is reintroduced after the entrant innovates is determined by whether the incumbent receives a higher payoff from the original product than it does in the secondary market given the degree of cannibalisation that occurs there. We should remember that the products are vertically differentiated in the sense that the presence of the original product constrains the profit that may be obtained in the secondary product market. Whether or not the incumbent reintroduces the product is a function of the forces of cannibalisation, that we know to be m , d and δ , only the latter two being represented in the above table. At extreme levels of d , different forces have preponderance. With fierce competition, the profit in the secondary product market is already heavily cannibalised, and the reintroduction of the original product is therefore not costly to the incumbent. Equally, with low levels of competition, the reintroduction of the original product will not unduly raise the cannibalisation of the innovation profit stream. The sensitivity of the solution set to δ is explained by the fact that the effect of the d variable is itself a function of δ .

A natural question is to ask whether the results of the four solution set model hold up within the new framework of the model. Graph 12 demonstrates the sensitivity of the % of games in which the entrant innovates first to d and δ for a non-drastic innovation with a value of m of 0. This graph is followed by the graph earlier depicted in the analysis of the four solution set model for an identical set-up, which is here replicated for ease of comparison.

Graph 12 : % of games in which entrant innovates first for $s = 0.95$, $m = 0$ in the nine solution set model



Graph 13 : % of games in which entrant innovates first for $s = 0.95$, $m = 0$ in the four solution set model



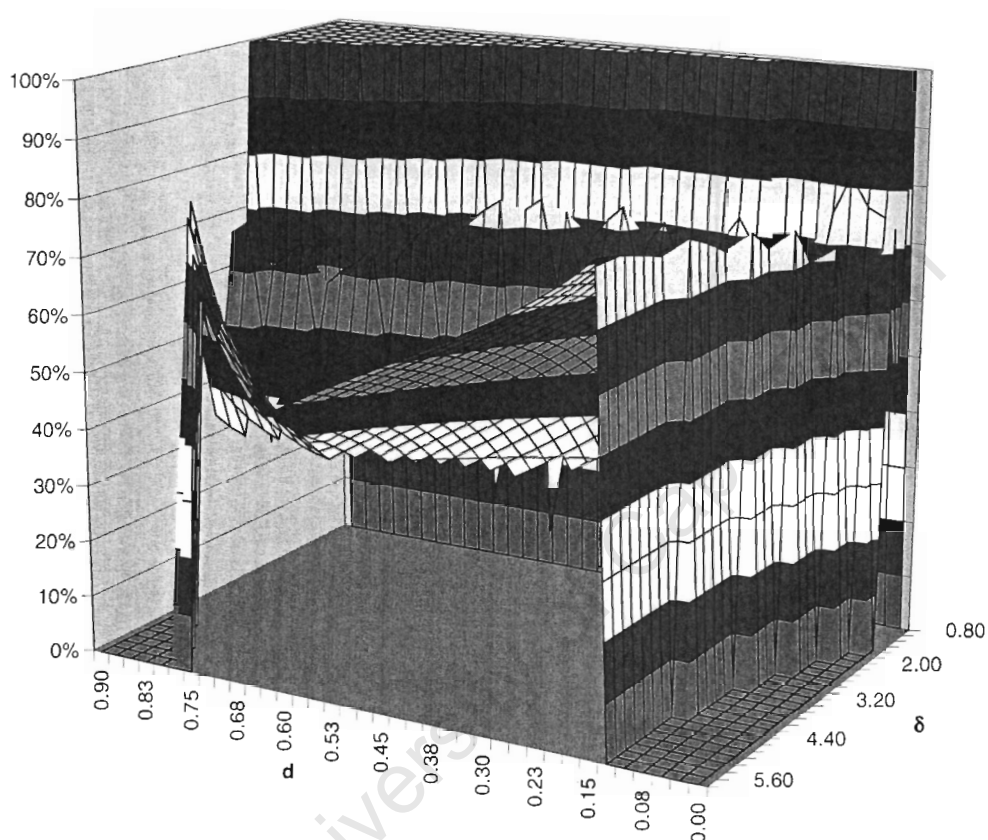
A simple visual check shows that the results have changed with respect to the effect of changes in the value of d , representing price competition. At some levels the

entrant innovates first significantly less often within the nine solution set model than in the four solution set model. As discussed this is not altogether surprising as allowing the incumbent greater freedom of choice was unlikely to have reduced its incentive to innovate. Nowhere do we see the entrant innovating first more often in the nine solution set model than in the previous version. Hence we see the introduction of the new solution sets altering the results in favour of the incumbent innovating. We could go on to test this new model against further criteria. Testing different values of m , comparing drastic with less drastic innovations, and analysing the time of the first innovation spring to mind as in the previous section. It may be more interesting, however first to examine how this model supports or refutes, or indeed extends results obtained in the extensive body of literature. Naturally we are constrained somewhat to looking at games of a similar structure, or at least a structure that we can replicate relatively painlessly within the confines of this flexible methodology. Examples of such papers and where this result is of relevance are Vickers (1986) and more recently Boone (2001).

Vickers (1986) showed that an incumbent / low-cost firm innovates more under strict price competition (Bertrand) and an entrant / high-cost firm innovates more under Cournot conditions. Clearly this result is at odds with our analysis. Graph 13 clearly shows that the entrant always innovates first when price competition has a low degree of ferocity, i.e. d is high. This result reverses dramatically at a critical level of d , that level being a function of δ . After this critical level, the result is increasing in d . Therefore, although there are degrees of price competition in which the incumbent is more likely to innovate first, this is not the case under any extreme level of price

competition whether that be fierce or benign. This discrepancy may in part be removed by removing the assumption that the level of m is equal to 0. If we set the value of m to 0.3 then we obtain an altogether different result.

Graph 14 : % of games in which entrant innovates first for $s = 0.95$, $m = 0.3$ in the nine solution set model



With the existence of lock-in the result changes considerably. With intense price competition, the incumbent does generally innovate more often, in line with the result of Vickers. At low levels of price competition however, the result of Vickers holds only for low values of δ .

We must remember that Vickers (1986) is a model of a patent race, and so only one firm can have the innovation. We shall have to replicate this framework within the

analysis for a fair comparison of results. This does add more weight to the evidence behind the importance of the patenting assumption. But we must also recognise the fact that both of these authors acknowledge that winning the patent does not secure a monopoly forever. Rather, the possibility must be considered that other innovations will be introduced in the future and the present innovation will be cannibalised to some extent. In this case, the non-patenting assumption to some extent replicates the possibility that another firm can enter the market later with another innovation. Within this model, that is an identical innovation, not a realistic assumption. However, in the next section we will introduce the possibility of each firm pursuing a different innovation. In that case, the non-patenting assumption should be a proxy for patents with a sequence of innovations. If a patent only protects an innovation until the other firm innovates, then that is no different from a game in which two firms each pursue different innovations in the absence of patenting.

We can easily re-introduce the idea of a patent being sold by an independent R&D laboratory into the model. Rather than be drawn into an investigation into auction theory, we shall follow the result from a second-price sealed bid auction framework. That is to say that the firm that places the highest valuation on the innovation purchases it, and the price paid is the valuation placed on the innovation by the loser of the auction. Normally to determine who has the highest valuation we would use the following method. For the incumbent, we must consider what it would receive in

the event of purchasing the innovation, namely $\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix}$, against what it would receive if it did not purchase the innovation, $\pi_i(P_i^1, P_e^2)$. The difference

between the two values represents the incentive to the incumbent. The entrant will receive zero if it loses the auction and therefore its incentive is the payoff in the event of winning the auction, $\pi_e(P_i^1, P_e^2)$.

If $\left[\max \begin{bmatrix} \pi_i(P_i^1, P_i^2) \\ \pi_i(P_i^2) \\ \pi_i(P_i^1) \end{bmatrix} - \pi_i(P_i^1, P_e^2) > \pi_e(P_i^1, P_e^2) \right]$ then the incumbent will win the

auction with probability one. If the inequality is reversed then the entrant wins with certainty and in the event of both firms having an equal incentive, they will each obtain the innovation 50% of the time. In this case however, we must consider that the award of the patent does not guarantee a monopoly forever and we must take that into account. The way in which we shall determine the proxy for the winner of the auction will be to use the results of the investment by each firm in R&D. These investments were made within a framework in which a successful innovation could be followed later by an innovation from the other player. Given this, the amount invested within this game by each player should serve as a proxy for the amount each firm should be prepared to bid in an auction for the patent.

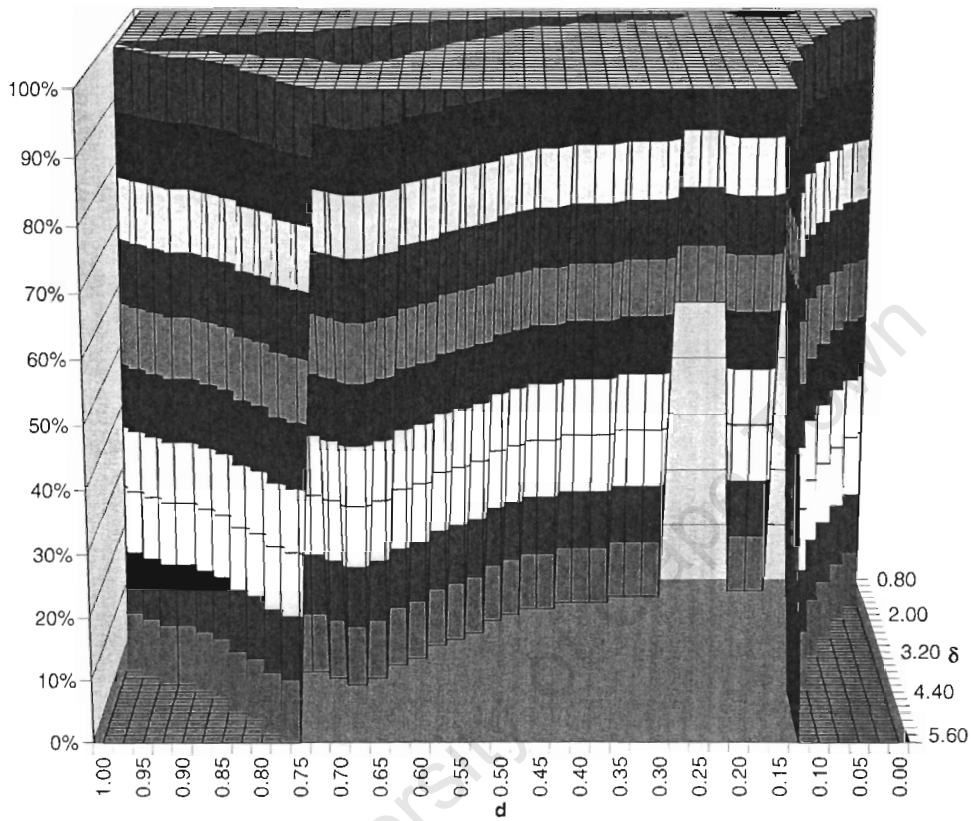
Boone (2001) generalises the result of Vickers (1986) in that in industries characterised by a high degree of competition rather than simply Bertrand conditions, the low cost firm (incumbent) buys the innovation and leadership persists. Equally as competition decreases, the high-cost firm buys the innovation and leap-frogging occurs. Both of the above cited papers are analysing a cost-reducing innovation. This is important as it means it is appropriate in this context to maintain a level of δ that is the same to both firms. The δ variable defines profit in the innovation market in the case of a monopoly. In this case, the profit levels would not necessarily be different

for one firm or the other, if they held a monopoly. Of course, if the entrant won the auction, it still would not hold a monopoly as, owing to the fact that the products are vertically differentiated, the profit to the entrant would still face some constraint from the original product. Therefore, we might expect to see sensitivity of this result to all the three main variables, m , d , and δ .

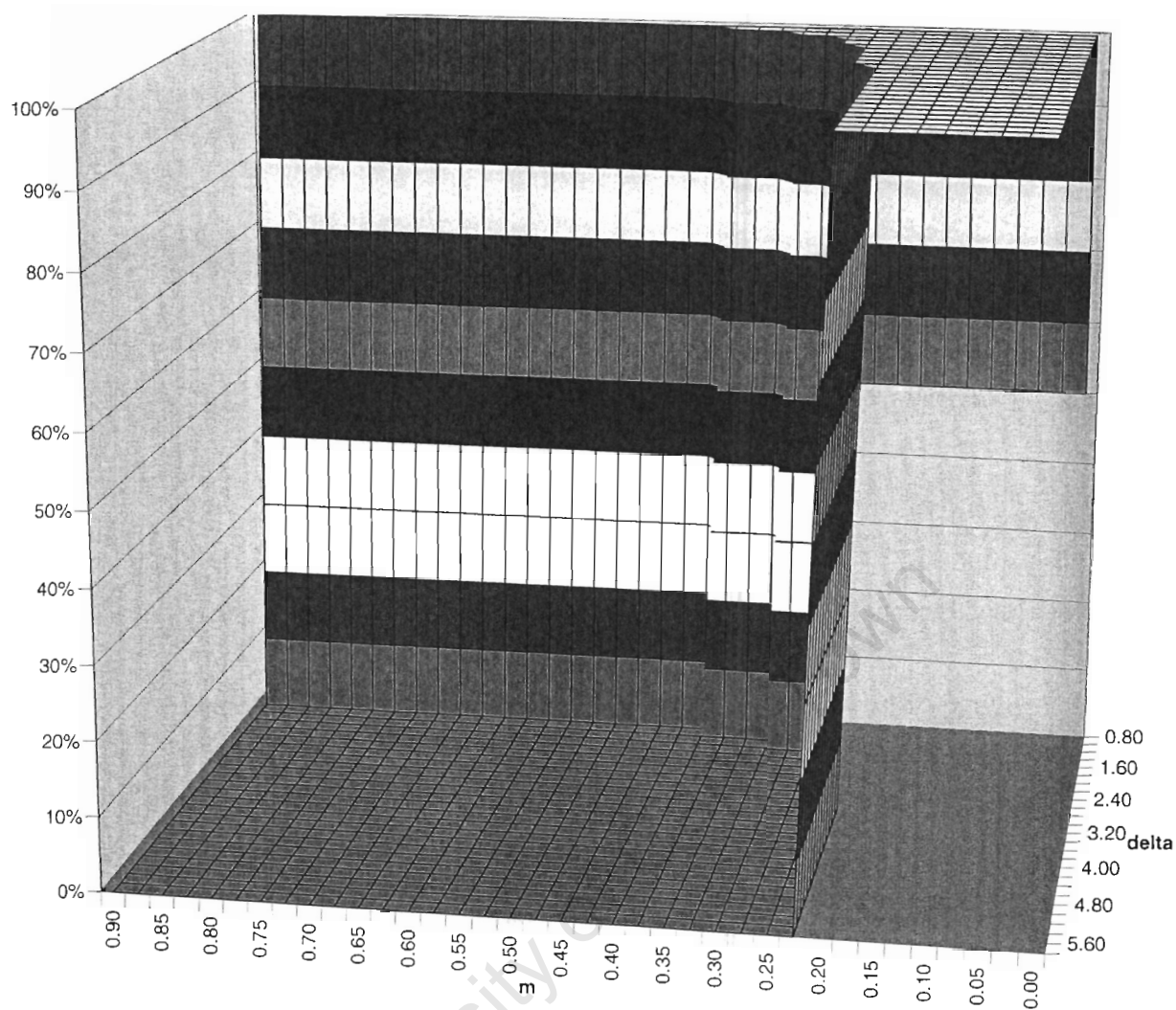
Graphs 15 and 16 below demonstrate that this is emphatically the case. We should not be alarmed at the apparent severity of the sensitivity of the result to the variables. First, within the patenting scenario, the result with respect to how often the entrant buys the patent can only ever be 100, 0% or in rare cases 50%. This exaggerates the sensitivity of the result when viewed in this graphical form. More importantly, we should remember that the sensitivities of all of the results to the variables were defined into the model by the specification of the equations. The point was made then that we were not seeking to define exactly how quickly profit responds to each variable. Instead we were stating the direction of that influence, a far less controversial exercise. The speed at which the result heads in that defined direction is naturally something that differs from industry to industry. It is precisely for this reason that we did not seek to specify anything but the most simplistic structure on the profit functions. It is the job of empirical researchers to determine the exact nature of these functions. What is important, then, in viewing these results is to acknowledge that their significance does not lie in precisely *where* the fulcrum points exist but in whether they exist at all. Equally, we are not interested in the precise levels of the variables that deliver a result but in how the result changes as the variables move. The nature of this relationship is the only thing of interest as we are seeking to

determine what are these industry-specific variables that are driving the observed differences in empirical results.

Graph 15 : % of games in which the entrant buys the patent for $m = 0.3$



Graph 16 : % of games in which the entrant buys the patent for $d = 0$

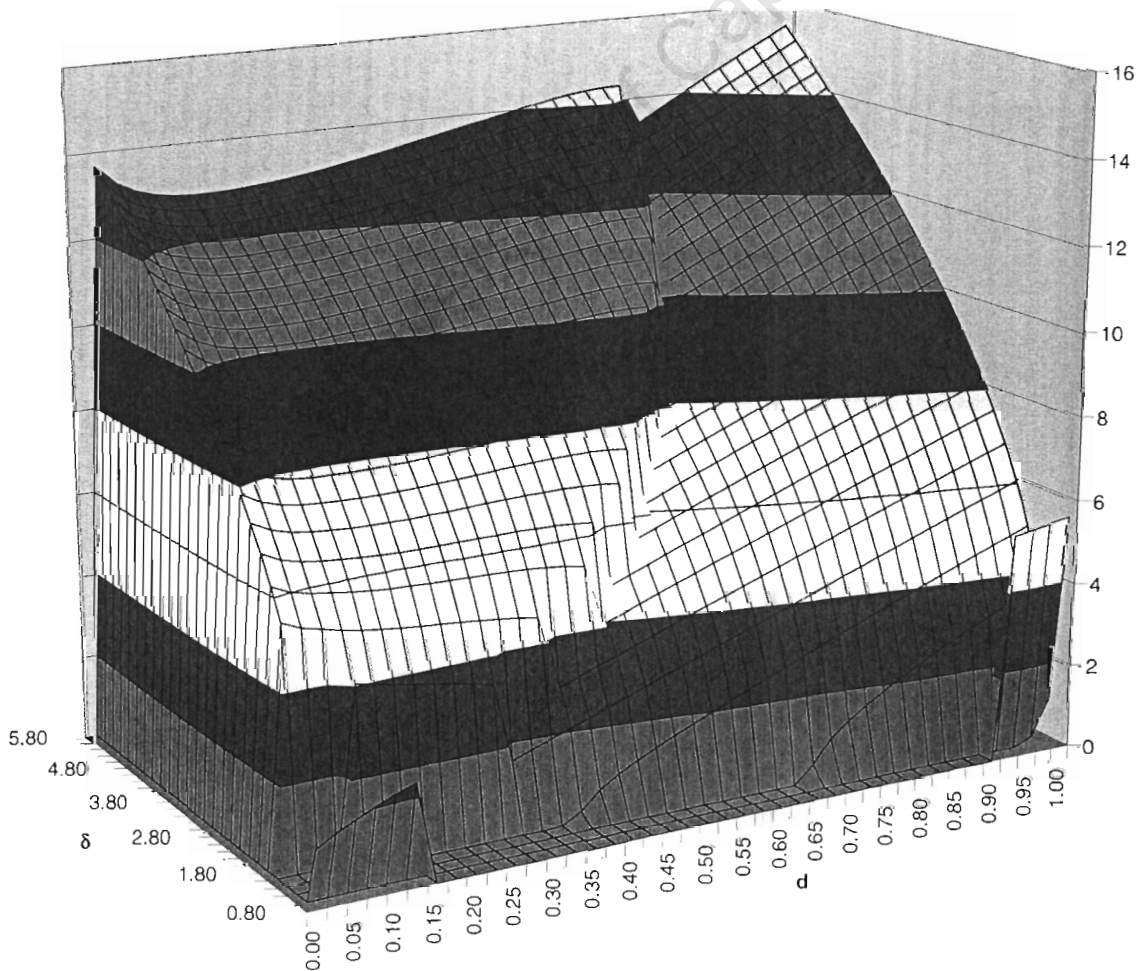


Graphs 15 and 16 offer a qualification of the result that under conditions of fierce competition, the incumbent will buy the patent. Both graphs demonstrate that the result is sensitive to both δ and m . This is not precisely in accord with the results of Vickers (1986) and Boone (2001). We can re-analyse this question within the next generation model.

Boone (2001) extended the analysis of Vickers (1986) and found a result that is close to the heart of this investigation. Boone found that the relationship between ferocity of competition and incentives to innovate were non-monotonic and therefore industry-

specific. This is exactly the type of result that this project expected to find from the start. In the same way that a patenting assumption may only be suitable for a specific industry, so must a generalist result that is only backed up by a portion of the empirical data rest on sample-specific characteristics within the data set. The following graph demonstrates support from this model for Boone's contention that the incentive to innovate is non-monotonic with respect to price competition. For the reasons outlined above, we are not fully replicating Boone's model but rather generating the result of the lowest firm investment in R&D. This is a proxy for the incentive to an independent R&D laboratory, given the second price sealed-bid auction framework.

Graph 17 : Incentive to an independent R&D laboratory to pursue patent for $m = 0$



Graph 17 confirms that the relationship is indeed non-monotonic in relation to price competition. This result is crucial to the heart of this investigation. It offers visual confirmation of the notion that conclusions drawn from analysis are generally a function of the characteristics of the industry under investigation.

This section has advanced the analysis considerably. I simply introduced an extra strategic option for the incumbent, one that should be considered entirely reasonable, that if the incumbent should be allowed to choose to not introduce the innovation to the market, it should also have the option to remove the original product from the innovation from the market in favour of the innovation. I then analysed the result from this new framework and found that not only did it alter the results of the four solution set model, it also showed that other results from the literature were themselves sensitive to the variables I have defined.

Differentiated innovations – firm-specific δ variables

In this section I seek to address a possible criticism of the four and nine solution set models. The motivation for both of these models was the removal of the patenting assumption, while the distinction between them was the strategic opportunities available to the incumbent. However, in removing the patenting assumption, I have also removed any notion that this should be considered a patent race game. Given that no patent is available, the analogy drawn by Gilbert & Newbery, and myself, to the idea of an independent R&D laboratory auctioning the innovation to the players does not hold as well as it did. It is perfectly reasonable to argue that the innovations sought by the respective firms are not identical, though equally this must remain a possibility. I shall therefore introduce the possibility of both firms pursuing different innovations. We must still assume, however, that the products should retain *some* non-trivial degree of similarity; otherwise, as noted earlier, we are no longer modelling innovation as *strategic* behaviour.

Thus far we have assumed that the innovation being targeted by the respective firms lies on the same point in product space. In reality, these innovations may well differ. This would also make more sense of the removal of the patenting assumption. Now the lack of patent protection can be explained not simply as an industry characteristic but as a result of the products occupying different points in product space. There could very well be three separate product markets all interacting with each other. The degree to which this interaction occurs will still be a function of the distance between the products in product space.

s_1 measures the degree of overlap between the original product and the innovation of the Incumbent.

s_2 measures the degree of overlap between the original product and the innovation of the Entrant.

s_3 measures the degree of overlap between the innovations of the Incumbent and the entrant.

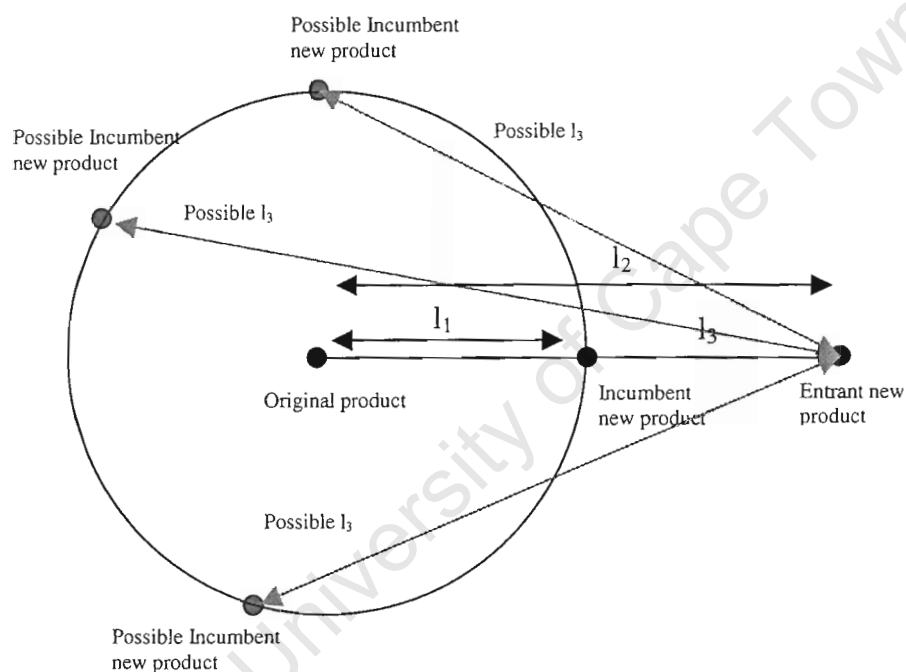
$s_i = f(l_i) \forall i, i = 1, 2, 3$ where l_i is the distance between two points in product space.

The relationship is defined as $s_i = \frac{1}{1+l_i}$.

The level of l_3 is constrained by the levels of l_1 and l_2 . If the incumbent's innovation is very similar to the original product, i.e. l_1 is low, then it should not be possible for there to be a simultaneously high level of l_3 with a low level of l_2 . The levels of l_1 and l_2 will determine a possible range in which l_3 can lie as it is a function of the two variables. The levels of l_1 and l_2 represent the relationship between the new innovations and the original product. Within a 2-dimensional product space this cannot entirely define the length of l_3 . The position within the range of the level of l_3 , defined by the levels of l_1 and l_2 , can be biased towards either a high or low level. This bias shall be defined by the variable v . How the level of this variable is determined shall be discussed later. A value of 0.5 for v represents no bias, a value of 1 a maximum bias towards a low level of l_3 , a value of 0 a maximum bias towards a high level of l_3 . The minimum level of l_3 can be given as the absolute value of the difference between the levels of l_1 and l_2 . The closest that the two innovations could

lie to each other in product space is if they were on the same trajectory from the original product. The minimum level of l_3 can represent this case. Figure 5 shows how the innovation of the incumbent can maintain a constant level of proximity to the original product (the radius of the circle) whilst having a variable level of proximity to the innovation of the entrant.

Figure 5: Proximity of the two innovations in product space – there is a range of possibilities



The radius of the circle in Figure 5 shows the range of positions that the innovation of the incumbent can be in product space whilst maintaining a constant degree of substitutability from the original product. For any given two innovations there is therefore a range of values for the possible degree of substitutability between them.

Only when the two innovations are the same degree of substitutability from the original product can the level of l_3 be zero. The maximum level of l_3 can be given as the sum value of the levels of l_1 and l_2 . The minimum level, as I have stated, is the absolute difference between the two values.

Therefore the range of values that l_3 can take is $\{|l_1 - l_2|, (l_1 + l_2)\}$. Where within this range the value lies shall be determined by the level of v which shall be a fraction between zero and 1. Therefore the actual level of l_3 will be $l_3 = |l_1 - l_2| + v((l_1 + l_2) - |l_1 - l_2|)$ which gives us the solution

$$(90) \quad s_3 = \frac{1}{1 + l_3} \equiv \frac{1}{1 + |l_1 - l_2| + v((l_1 + l_2) - |l_1 - l_2|)}$$

If we wish to know s_3 as a function of s_1 and s_2 then we can say

$$(91) \quad s_3 = \frac{1}{1 + \left| \frac{1 - s_1}{s_1} - \frac{1 - s_2}{s_2} \right| + v \left(\left(\frac{1 - s_1}{s_1} + \frac{1 - s_2}{s_2} \right) - \left| \frac{1 - s_1}{s_1} - \frac{1 - s_2}{s_2} \right| \right)}$$

Clearly the level of v is important in determining the level of s_3 . The level of s_3 measures the degree to which the companies are working on the same innovation. In the signalling game to come later, this will be important. As companies do not know what research the other firm is undertaking, any signal of R&D will only be a conscious signal of the level of the closeness of their new product to the original product. It will also define a range of values for the level of s_3 . However, to know the actual level of v and therefore s_3 , some form of spying would have to take place.

This should not change the earlier model significantly in the cases where there are only two products in the market. However, as the innovations are no longer the same then we must introduce separate δ variables for each innovation, namely δ_I for the innovation of the incumbent and δ_E for the innovation of the entrant. One assumption that it shall be necessary to make is that δ_I is always greater than or equal to 1. This is not a particularly restrictive assumption; it merely states that an incumbent would not be investing money in producing a less profitable product than he has already. We have seen in previous generations of the model that the incumbent does not attempt innovation for values of δ less than 1, and it is with some confidence that we introduce this restriction into this generation of the model.

We shall now define the profit equations for each firm under each combination of products. Within this section we shall necessarily reduce the time spent explaining the construction of the equations where the logic followed, and assumptions made, are identical to the procedures outlined fully in the previous section. There will of course be some aspects of the equations that are new as a result of the introduction of greater flexibility. These aspects will be explained in detail.

The case in which the incumbent has innovated and the entrant has not.

Once again there will be an area of each product market, both initial and secondary, which will not be cannibalised by each other. The profit in the intersecting area shall again be the weighted average of the profits available in each market, namely $\frac{1 + \delta_I^2}{1 + \delta_I}$.

This gives,

$$(92) \quad \pi_i(p_i^1, p_i^2) = (1 - s_1)x + (1 - s_1)\delta_I x + s_1 \frac{1 + \delta_I^2}{1 + \delta_I} x$$

The incumbent may wish to shelve the original product in the event of innovation. In this case the payoff would be

$$(93) \quad \pi_i(p_i^2) = \delta_I x$$

The entrant would of course receive no payoff in both of these scenarios.

The case in which the entrant has innovated and the incumbent has not.

When the entrant is the sole innovator, profit is determined by the same procedure as in the previous model. However, we are now specifying a specific value of δ for the innovation of the entrant, δ_E . For values less than 1, we see the maximum profit to the entrant increasingly affected by the price competition variable as δ_E declines. The effect of the price competition amplifies as δ_E moves away from 1 by a factor of δ_E as before. The effect of the incumbent advantage is also amplified as δ_E declines. For values of δ_E greater than 1, we see the opposite effect, namely a dampened effect of the cannibalisation variables as the level of δ_E increases. For the incumbent as non-innovator, the maximum profit available in the intersecting area is amplified as δ_E increases and cannibalisation for the incumbent is increasing in δ_E . We can therefore define the profit to each player in this scenario as,

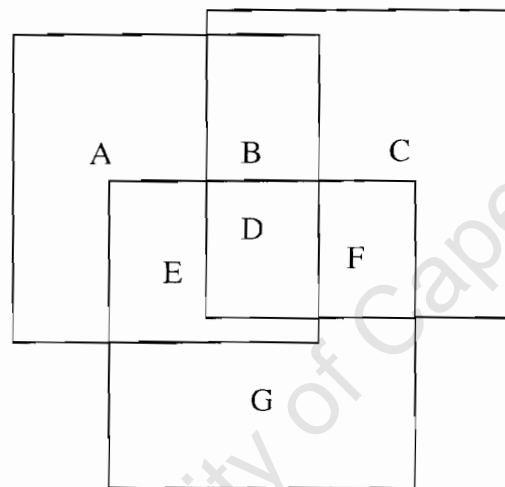
$$(94) \quad \pi_e(p_i^1, p_e^2) = \begin{cases} [(1-s_2)\delta_E x + s_2(1-m)(d\delta_E)\delta_E x] \text{ for } \delta_E < 1 \\ \left[(1-s_2)\delta_E x + s_2(1-m) \left(\frac{d}{\delta_E} + \frac{(\delta_E-1)}{\delta_E} \right) \delta_E x \right] \text{ for } \delta_E \geq 1 \end{cases}$$

$$(95) \quad \pi_i(p_i^1, p_e^2) = \begin{cases} [(1-s_2)x + s_2(m + (1-m)(\delta_E d + 1 - \delta_E))x] \text{ for } \delta_E < 1 \\ \left[(1-s_2)x + s_2 \left(m + \frac{(1-m)d}{\delta_E} \right) x \right] \text{ for } \delta_E \geq 1 \end{cases}$$

The case where both firms have innovated.

In the case where there are three products, the situation becomes vastly more complicated as there are now a possible nine, rather than three, market segments. The three levels of s_1 , s_2 and s_3 will determine the midpoints of the market areas and therefore the degrees of overlap of the areas. As the points in product space represent the midpoints of the area of market influence, high levels of s will lead to a high degree of overlap between markets. This will therefore mean a greater percentage of the market is subject to the influences that result from competition.

Figure 6: Interaction of the three products in product space



If all the points in product space were to lie on each other, the total area would be equivalent to one full area, or monopoly market. This entire area would be labelled 'D' in the above diagram. This case occurs where $s_1=s_2=s_3=1$. Whenever one of the s variables is equal to zero, representing a total absence of substitutability, the area D must also be equal to zero. We shall therefore define the area D as the product of the three s variables. We should note that we are not actually interested in the actual area

of D. This is simply a tool to allow us to make a statement about which cannibalisation forces are at work on what proportion of each market.

$$(96) \quad D = s_1 s_2 s_3$$

Following this, the area B+D must be equal to the level of s_2 . To recall, the degree of overlap of the two products in product space is defined by the level of s_2 . This area is also equal to B+D. As $B = (B + D) - D$ it follows that

$$\begin{aligned} E = s_1 - s_1 s_2 s_3 & \quad A = 1 - B - D - E \\ F = s_3 - s_1 s_2 s_3 & \quad \text{and } C = 1 - B - D - F \\ G = 1 - E - D - F & \end{aligned}$$

Given this and, assuming the area of each square to be 1, we can say that the total area of the three intersecting areas is

$$(97) \quad 3 - s_1 - s_2 - s_3 + s_1 s_2 s_3$$

From this we know the relative size of each segment of the market and can easily define the influences on profit within the sections.

The majority of the segments have already been analysed in previous sections, namely those that represent an interaction between a single innovation and the original product (B, E), and those that show a product isolated from the cannibalisation influences (A, C and G). The explanation that follows shall therefore focus on the other areas.

For the incumbent, a profit of x is obtained in the area A representing the original product. A profit of δx is obtained in the area C representing the innovation of the incumbent. A profit of $\left(\frac{1+\delta_l^2}{1+\delta_l}x\right)$ is obtained in the area B, representing the weighted average of the profit available from the intersecting area between the original product and the innovation of the incumbent.

Within area D, the maximum profit available to the incumbent is the weighted average of the profit available to each of the two products, namely $\frac{1+\delta_l^2}{1+\delta_l}x$. As $\delta_E <$

1, then as δ_l rises or δ_E falls, the effect of the ferocity of competition shall diminish. Therefore, we can, in accordance with the modelling principles we have often

repeated, state that the proportion, $\frac{\delta_E}{\delta_l}$ shall be subject to the influence of a squared d

variable. The remaining proportion shall be more heavily weighted towards the influence of d than no price competition if the ratio of δ_E to 1 is small. Therefore the

expression, $\left(\left(1-\frac{\delta_E}{\delta_l}\right)+d\delta_E\left(1-\frac{\delta_E}{\delta_l}\right)+d^2\left(\frac{\delta_E}{\delta_l}\right)\right)$ shall represent the extent to which

the profit in this area is reduced by the ferocity of competition. If we take into account the degree to which the market has been locked-in by the incumbent then we can define the profit available to the incumbent in this area as

$$\left(m+(1-m)\left(\left(1-\frac{\delta_E}{\delta_l}\right)+d\delta_E\left(1-\frac{\delta_E}{\delta_l}\right)+d^2\left(\frac{\delta_E}{\delta_l}\right)\right)\right)\delta_E x.$$

Within the area E, the profit is determined by the interaction between the original product and the innovation of the entrant. This has previously been described. The area F describes the interaction between the two innovations. The maximum profit available to the incumbent here is $\delta_I x$. The effects of the price competition and incumbency advantage variables are now determined by the ratio of profit available to each innovation. As δ_I rises, the price competition variable should be dampened and the incumbency advantage variable amplified. The reverse occurs if δ_E rises, although we should remember that this is a value below 1.

This therefore means that we are in a position to define,

$$(98) \quad \pi_i(p_i^1, p_{ei}^2) = \left[\begin{aligned} & A(x) + B \left[\left(\frac{1 + \delta_I^2}{1 + \delta_I} \right) x \right] + C(\delta_I, x) \\ & + D \left[\left(\frac{1 + \delta_I^2}{1 + \delta_I} \right) \left(m + (1 - m) \left(\left(1 - \frac{\delta_E}{\delta_I} \right) + d \delta_E \left(1 - \frac{\delta_E}{\delta_I} \right) + d^2 \left(\frac{\delta_E}{\delta_I} \right) \right) \right) x \right] \\ & + E \left[\left((m + (1 - m)(\delta_E d + 1 - \delta_E)) x \right) \right] \\ & + F \left[\left(\left(m + (1 - m) \left(\frac{\delta_E}{\delta_I} d + \frac{\delta_I - \delta_E}{\delta_I} \right) \right) \delta_I x \right) \right] \end{aligned} \right] \text{ for } \delta_E < 1$$

In the case where $\delta_E \geq 1$, the situation becomes again more complicated. The areas A, B and C are the same as in the previous case. Within area D, the profit initially available is again the weighted average of the profit available to each of the three products, namely $\frac{1 + \delta_I^2}{1 + \delta_I}$. The relationship between δ_E and δ_I is now of crucial

importance. If δ_I is greater than δ_E then we would expect the cannibalisation of the profit due to d to decrease as the gap between the two δ variables increases. We are unable to state unequivocally which of the two variables is higher and therefore must specify a function for each case.

In the case where $(\delta_I - \delta_E) \leq 0$, the whole portion of the profit is subject to cannibalisation. The proportion that is subject to the influence of d^2 is decreasing in both δ_E and δ_I . This is denoted by $\frac{d^2}{\delta_I \delta_E}$. The remainder, $1 - \frac{1}{\delta_I \delta_E}$, is subject to d .

However, the extent to which the d variable is either amplified or dampened is a function of the relationship between the respective levels of δ . Therefore the

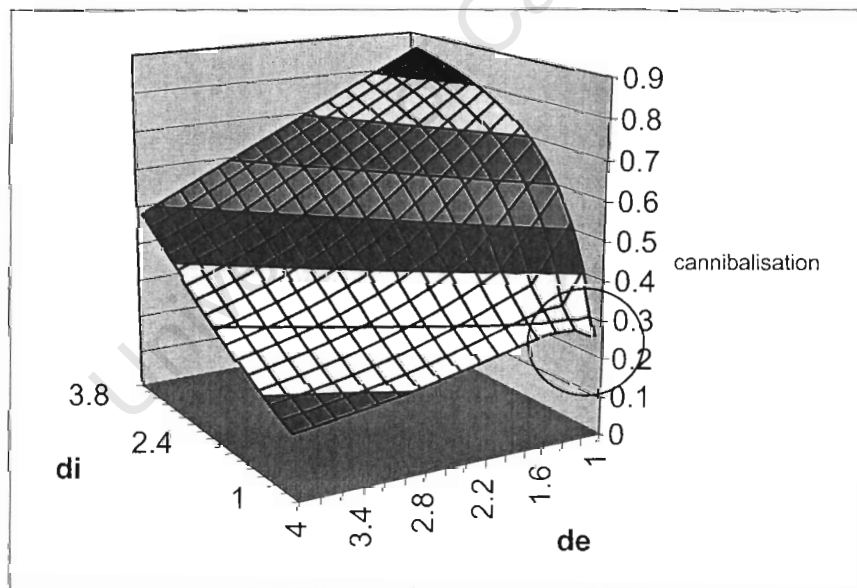
expression, $\left(\frac{d^2}{\delta_I \delta_E} + \left(1 - \frac{1}{\delta_I \delta_E} \right) \left(d \frac{\delta_I}{\delta_E} \right) \right)$ represents the extent to which the profit in

this area is reduced by the ferocity of competition. This may seem an unnecessarily complicated function. We must remember that it is not supposed to exactly replicate particular market dynamics, rather to describe axiomatically how the strength of price competition changes as the value of the respective innovations occur. A brief test reveals this to be the case. In the event that both δ_I and δ_E are equal to 1, we would expect the entire profit to be subject to the influence of the squared d variable. This function fulfils that criteria. As δ_E rises, with δ_I remaining at 1, cannibalisation increases. When both δ_I and δ_E are above 1, we see cannibalisation tend to the level of d when they are close, the cannibalisation being amplified or dampened appropriately.

In the case where $(\delta_I - \delta_E) > 0$, not all of the profit will be subject to cannibalisation. This will be a function of the ratio between the two δ variables. We shall state that the proportion $1 - \frac{\delta_E}{\delta_I}$ is not subject to any cannibalisation as it is increasing in δ_I but decreasing in δ_E . The proportion that is influenced by the d variable follows the same function as described above. We can therefore define the profit to the incumbent as a function of d as follows,

$$(99) \quad \begin{aligned} & \left(1 - \frac{\delta_E}{\delta_I}\right) + \left(\frac{\delta_E}{\delta_I}\right) \left(\frac{d^2}{\delta_I \delta_E} + d \left(1 - \frac{1}{\delta_I \delta_E}\right)\right) \text{ for } (\delta_I - \delta_E) > 0 \\ & \left(\frac{d^2}{\delta_I \delta_E} + \left(1 - \frac{1}{\delta_I \delta_E}\right) \left(d \frac{\delta_I}{\delta_E}\right)\right) \text{ for } (\delta_I - \delta_E) \leq 0 \end{aligned}$$

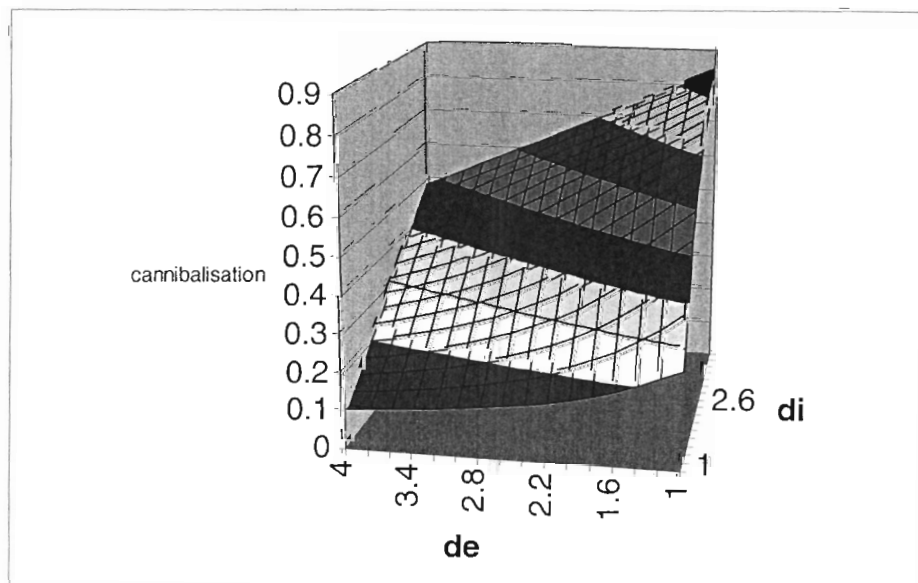
Graph 18: Non-monotonicity of the function



We can display this relationship graphically. The kink circled in the plane above demonstrates the dual nature of the function for the case of $d = 0.5$. As δ_I increases, the cannibalisation decreases. However, the relationship to δ_E is non-monotonic. We

can see that when δ_I is equal to one, cannibalisation is decreasing in δ_E for a short period. This problem arises as a result of there being two distinct equations within this function. In the move from one equation to the other, some of the profit changes from being cannibalised by a factor of d^2 to being cannibalised by a factor of $d \frac{\delta_I}{\delta_E}$ which is itself amplified or dampened by the respective δ variables. The relationship between the δ variables under these constraints is monotonic in relation to an exponential level of 1, but non-monotonic in relation to a level of 2. As δ_E increases there is a range in which the profit ceases to be cannibalised by d^2 and instead by $d \frac{\delta_I}{\delta_E}$. This is also the reason why this problem does not occur in any other generations of the model; it is specific to this version with individual δ variables. We can avoid this problem if, instead of using d^2 as a proxy for competition, we use d^{2n} where n is $\max\left(\frac{\delta_E}{\delta_I}, \frac{\delta_I}{\delta_E}\right)$. The graph below demonstrates that the new function conforms to the necessary criterion, namely monotonicity.

Graph 19: Monotonicity of the function



The function of the cannibalisation of the incumbent's payoffs due to price competition now becomes

$$(100) \quad \left(1 - \frac{\delta_E}{\delta_I}\right) + \left(\frac{\delta_E}{\delta_I}\right) \left(\frac{d^{\frac{2\delta_I}{\delta_E}}}{\delta_I \delta_E} + d \left(1 - \frac{1}{\delta_I \delta_E}\right) \right) \text{for } (\delta_I - \delta_E) > 0$$

$$\left(\frac{d^{\frac{2\delta_E}{\delta_I}}}{\delta_I \delta_E} + \left(1 - \frac{1}{\delta_I \delta_E}\right) \left(d \frac{\delta_I}{\delta_E} \right) \right) \text{for } (\delta_I - \delta_E) \leq 0$$

The profit available to the incumbent in area E is the same as previously described, being the intersecting area between the original product of the incumbent and the innovation of the entrant. The profit to the incumbent is constrained by a maximum of x and is cannibalised by the variable d which is itself amplified by a function of δ_E .

Within area F, if the incumbent has the highest value of δ , the effects of the cannibalising variable is as described for the case where δ_E is below 1. Where the incumbent has a lower value of δ , the cannibalisation of the incumbent profit in this area will be all be subject to the d variable, increasing in δ_E , and decreasing in δ_I .

We can therefore define the profit to the incumbent under this scenario as follows:

$$(101) \quad \pi_i(p_i^1, p_{ei}^2) = \left[\begin{aligned} & A(x) + B \left(\frac{1 + \delta_I^2}{1 + \delta_I} x \right) + C(\delta_I x) \\ & + D \left(\frac{1 + \delta_I^2}{1 + \delta_I} \right) \left(m + (1 - m) \left(\left(1 - \frac{\delta_E}{\delta_I} \right) + \left(\frac{\delta_E}{\delta_I} \right) \left(\frac{d^{\frac{2\delta_I}{\delta_E}}}{\delta_I \delta_E} + d \left(1 - \frac{1}{\delta_I \delta_E} \right) \right) \right) \right) x \\ & + E \left[\left((m + (1 - m)) (\delta_E d + 1 - \delta_E) \right) x \right] \\ & + F \left[\left(\left(m + (1 - m) \left(\frac{\delta_E}{\delta_I} d + \frac{\delta_I - \delta_E}{\delta_I} \right) \right) \delta_I x \right) \right] \end{aligned} \right] \text{ for } \delta_I > \delta_E \geq 1$$

$$(102) \quad \pi_i(p_i^1, p_{ei}^2) = \left[\begin{aligned} & A(x) + B \left(\frac{1 + \delta_I^2}{1 + \delta_I} x \right) + C(\delta_I x) \\ & + D \left(\frac{1 + \delta_I^2}{1 + \delta_I} \right) \left(\left(m + (1 - m) \left(\frac{d^2}{\delta_I \delta_E} + \left(1 - \frac{1}{\delta_I \delta_E} \right) \left(d \frac{\delta_I}{\delta_E} \right) \right) \right) \right) x \\ & + E \left(m + (1 - m) \left(\frac{d}{\delta_E} \right) \right) x \\ & + F \left(\left(\delta_I \left(m + (1 - m) d^{\frac{\delta_E}{\delta_I}} \right) \right) \right) x \end{aligned} \right] \text{ for } \delta_E \geq \delta_I$$

The conditions for the entrant profit payoffs are derived using a similar logic to that which generated the incumbent payoffs. We first analyse the cases where δ_E is less than 1. Within the area D, the maximum profit to the entrant is δ_E . The entrant faces price competition from the original product and also from the incumbent's innovation. It is therefore all subject to the influence of d^2 , and the cannibalisation is increasing in δ_I and decreasing in δ_E . This can be described as $\left(\frac{d^2 \delta_E}{\delta_I} \right)$.

Within the area E, the entrant can again obtain a maximum profit of δ_E . This is cannibalised by an amplified level of d . These functions are the same as those

specified for the entrant in the previous scenario for consistency. The area F, representing the interaction between the two innovations, responds in an identical manner but this time the strength of the influences is determined by the ratio of the respective δ variables. The payoff equation can thus be defined as

$$(103) \quad \pi_e(p_i^1, p_{ei}^2) = \left[\begin{array}{l} D \left(\delta_E (1-m) \left(\frac{d^2 \delta_E}{\delta_I} \right) x \right) \\ + E \left((\delta_E^2 d) (1-m) x \right) \\ + F \left(\left(\delta_E (1-m) \left(\frac{\delta_E}{\delta_I} d \right) \right) x \right) + G(\delta_E x) \end{array} \right] \text{ for } \delta_E < 1$$

In the case where δ_E is greater than 1, we again have the problem within the area D of whether it is greater than δ_I . We must therefore specify a separate function for each instance. In the event that $\delta_E < \delta_I$, all of the profit will be subject to the d variable. Having learned from the derivation of the profit function for the incumbent, we can specify a similar function for the entrant, reversing the respective δ variables. This would lead to the following functions.

$$(104) \quad \left(1 - \frac{\delta_I}{\delta_E} \right) + \left(\frac{\delta_I}{\delta_E} \right) \left(\frac{d^{\frac{2\delta_I}{\delta_E}}}{\delta_I \delta_E} + d \left(1 - \frac{1}{\delta_I \delta_E} \right) \right) \text{ for } (\delta_I - \delta_E) \leq 0$$

$$\left(\frac{d^{\frac{2\delta_I}{\delta_E}}}{\delta_I \delta_E} + \left(1 - \frac{1}{\delta_I \delta_E} \right) \left(d \frac{\delta_E}{\delta_I} \right) \right) \text{ for } (\delta_I - \delta_E) > 0$$

The function displays monotonicity and for the cases when the δ variables are equal, it returns a cannibalisation result that is d^2 when the δ variables are equal to 1 but tending to d as the δ variables increase.

Within the area E, the interaction is between the innovation of the entrant and the incumbent's original product. As δ_E is greater than 1, only the proportion $\frac{1}{\delta_E}$ is subject to the influence of d .

The area F, like the area D, depends on the relative size of the respective δ variables. If the entrant has the higher value of δ then a proportion, as a function of the ratio between the δ variables, is immune from the d variable. Dampening and amplifying of cannibalisation influences occurs in the same manner as previously described.

$$(105) \quad \pi_e(p_i^1, p_{ei}^2) = \left[\begin{array}{l} D \left(\delta_E (1-m) \left(1 - \frac{\delta_I}{\delta_E} \right) + \left(\frac{\delta_I}{\delta_e} \right) \left(\frac{d \frac{\delta_e}{\delta_I}}{\delta_I \delta_E} + d \left(1 - \frac{1}{\delta_I \delta_E} \right) \right) \right) x \\ + E \left((1-m) \left(\frac{d}{\delta_E} + \frac{(\delta_E - 1)}{\delta_E} \right) \delta_E x \right) \\ + F \left(\left(\delta_E (1-m) \left(\frac{\delta_I}{\delta_E} d + \frac{\delta_E - \delta_I}{\delta_E} \right) \right) x \right) + G(\delta_E x) \end{array} \right] \text{ for } \delta_E \geq \delta_I$$

$$(106) \quad \pi_e(p_i^1, p_{ei}^2) = \left[\begin{array}{l} D \left[\delta_E (1-m) \left(\frac{d^{\frac{2\delta_i}{\delta_e}}}{\delta_i \delta_E} + \left(1 - \frac{1}{\delta_i \delta_E} \right) \left(d \frac{\delta_e}{\delta_i} \right) \right) x \right] \\ + E \left((1-m) \left(\frac{d}{\delta_E} + \frac{(\delta_E - 1)}{\delta_E} \right) \delta_E x \right) \\ + F \left(\delta_E \left(d^{\frac{\delta_E}{\delta_i}} \right) (1-m)x \right) + G(\delta_E x) \end{array} \right] \text{ for } \delta_i > \delta_E \geq 1$$

In the event that the incumbent may wish to shelve his old product after innovation the following payoffs would occur. These relate to the areas F and G in the 3 products market and the profit functions therefore correspond to those stated for these areas.

$$(107) \quad \pi_i(p_{ei}^2) = \left[(1-s_3)(\delta_i x) + s_3 \left[m + (1-m) \left(\left(\frac{\delta_E}{\delta_i} d + \frac{\delta_i - \delta_E}{\delta_i} \right) \right) \delta_i x \right] \right] \text{ for } \delta_E < \delta_i$$

$$(108) \quad \pi_i(p_{ei}^2) = \left[(1-s_3)(\delta_i x) + s_3 \left[m + (1-m) \left(\left(d^{\frac{\delta_E}{\delta_i}} \right) \right) \right] \delta_i x \right] \text{ for } \delta_E \geq \delta_i$$

The entrant would receive the following payoffs as a result of this decision by the incumbent.

$$(109) \quad \pi_e(p_{ei}^2) = \left[(1-s_3)(\delta_E x) + s_3 \left[(1-m) \left(\delta_E \left(\frac{\delta_E}{\delta_i} d \right) \right) x \right] \right] \text{ for } \delta_E < \delta_i$$

$$(110) \quad \pi_e(p_{ei}^2) = \left[(1-s_3)(\delta_E x) + s_3 \left((1-m)\delta_E \left(\frac{\delta_I}{\delta_E} d + \frac{\delta_E - \delta_I}{\delta_E} \right) \right) x \right] \text{ for } \delta_E \geq \delta_I$$

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Analysis of dual innovation model

The introduction of the firm-specific δ variables completes the framework of the static model. We have already seen the implications of this for the representation of the innovations in product space. However, the crucial benefit of this additional parameter is that the individual δ variables can be seen in two ways. In the four solution set and nine solution set models we worked within the framework of a race to achieve the innovation of a specific product. This was a legacy of the initial analysis of the models of a patent race by Gilbert & Newbery (1982) and Reinganum (1983). In this situation, the two firms are competing to secure the patent on a particular product. It makes sense in that framework to suppose that one firm's successful innovation prevents the other firm from achieving it. When the patenting assumption is removed, however, then there is no longer any structural reason for imposing the restriction that the firms are necessarily attempting exactly the same innovation. They may be pursuing the innovation of similar and competing, but distinct, products, which would rationalize the absence of patent protection.

The introduction of distinct innovation products for each firm in turn led to the specification of a distinct δ variable for each product. This can be interpreted as representing two separate effects. It can be taken to mean that each firm is pursuing different innovations, each with an individual value. For example, the incumbent firm may be pursuing the innovation of a product that is inherently more valuable than the innovation being pursued by the entrant. But the value of each product is

assumed to be the same to each firm. That is, in this interpretation the products are different but the profitability of each product to its respective innovator is the same. This was the interpretation of the δ variable in the previous generations of the model. The other interpretation would be to view the respective δ variables as the firm-specific *value* of a particular innovation. In this way, two firms could race to produce the same innovation in the same way as we have represented in the previous models. However, now we have the ability to capture other possibilities, such as the ability of an incumbent firm to extract a greater degree of profit from an innovation. This could be for a number of reasons. It could be because the incumbent has a distribution network already in place and is therefore able to reach a larger market at a lower cost. These are not the same forces that are picked up by the m variable. That variable describes an advantage to the incumbent in the event of a duopoly for the same product, for such reasons as branding or indeed, the aforementioned distribution network. What we are describing here is the structural ability of an incumbent to extract greater profit from an innovation. Essentially, we are considering the possibility that the profit extracted from monopoly by an incumbent is greater than that extracted from monopoly by an entrant for an identical product. The difference between the δ variables for each firm can represent either one or a combination of the two effects described above.

The models of the previous section served in part to demonstrate the effectiveness of the sensitivity analysis methodology. The model in the present section is being put forward as the main thrust of this thesis and the tool with which questions relating to the relationship between market structure and innovation will be answered. The

analysis will take the form of three distinct sections. The first will ask the question that is at the heart of much of the literature in this field, namely who innovates first, or invests more, in the incumbent – entrant framework? The second will look at innovation from the social welfare perspective; that is from a perspective indifferent to the identity of the first innovator, but rather interested in the time of the first innovation. Finally we shall re-examine our results from the previous section, drawing comparison with the work of Vickers (1986) and Boone (2001).

The algebraic framework of the model has not altered from the nine solution set model. The only addition here is the introduction of the firm-specific δ variables. These affect the calculation of the profit functions in each given scenario but do not affect the structure of the game. As a result we can immediately move on to the analysis.

Who innovates first?

We shall assume a situation in which all of the respective s variables are equal to 0.95. This is in line with our earlier assumptions made in analysis of other models. As discussed earlier, the justification for this was that the innovations should be close substitutes, lest the significance and meaning of 'incumbency' become logically unclear. If the innovations had no relationship with the original product, then we are effectively looking at an entrant – entrant framework, which is not the topic of this project. Equally, to assume that innovations are perfect substitutes of the original products with which they compete tends to generate excessively sensitive results of

questionable empirical applicability. The result of these considerations was that we should consider the products to be close but not perfect substitutes, hence the adoption of the value of 0.95 for the s variables.

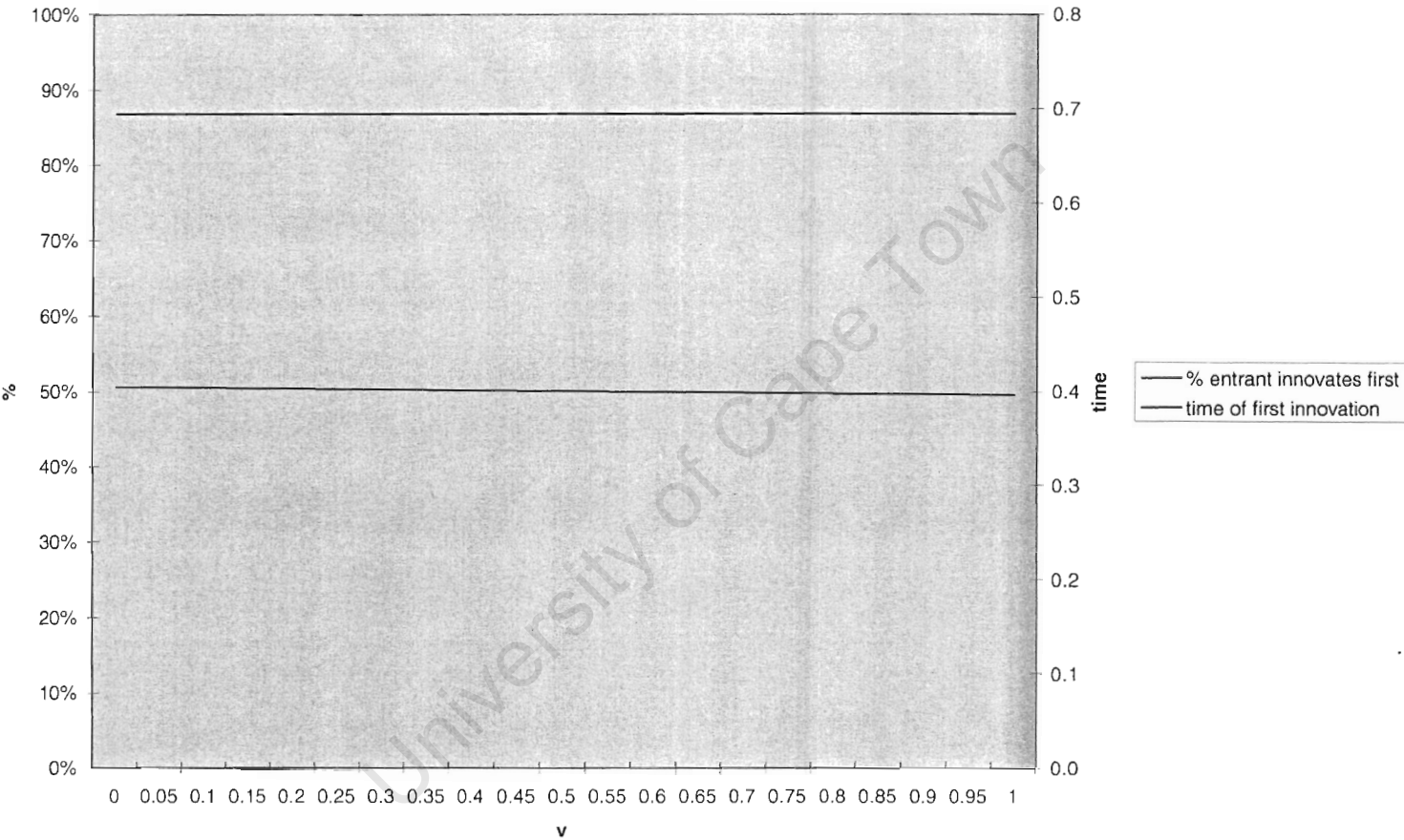
There is an additional assumption being made in this generation of the model that was not required earlier, namely, one concerning degree of substitutability between the respective innovations. In both the four and nine solution set models the firms were constrained to pursue identical innovations. Where this constraint is no longer to be imposed, but where we nevertheless must ensure that we still somehow talking about product competition, some looser but still definite restriction on how the two innovations relate to each other in product space must be used. If the degree of substitutability of each innovation is given, then this must define a range of possibilities for the distance between the two innovations in product space. That range was earlier defined as $\{|l_1 - l_2|, (l_1 + l_2)\}$. The exact number in the range is defined by a separate variable v earlier defined as v . For the levels of s_1 and s_2 assumed in the present model the degree of substitutability between the innovations ranges over $\{0.9, 1\}$. Recall that the relationship amongst the s variables is defined

by $s_3 = \frac{1}{1 + \left| \frac{1-s_1}{s_1} - \frac{1-s_2}{s_2} \right| + v \left(\left(\frac{1-s_1}{s_1} + \frac{1-s_2}{s_2} \right) - \left| \frac{1-s_1}{s_1} - \frac{1-s_2}{s_2} \right| \right)}$. This permits us to

calculate that a value for v of 0.5 then results in s_3 being equal to 0.95. Is it justifiable to suppose that the assumed value should be in the middle of the range? If the two firms are pursuing similar innovations then it may be more likely that the two innovations are closer to each other in product space than the original product. When the products are this close however, it is unlikely that the assumed value of v would have a significant impact on the result. Graph 20 below, which represents the results

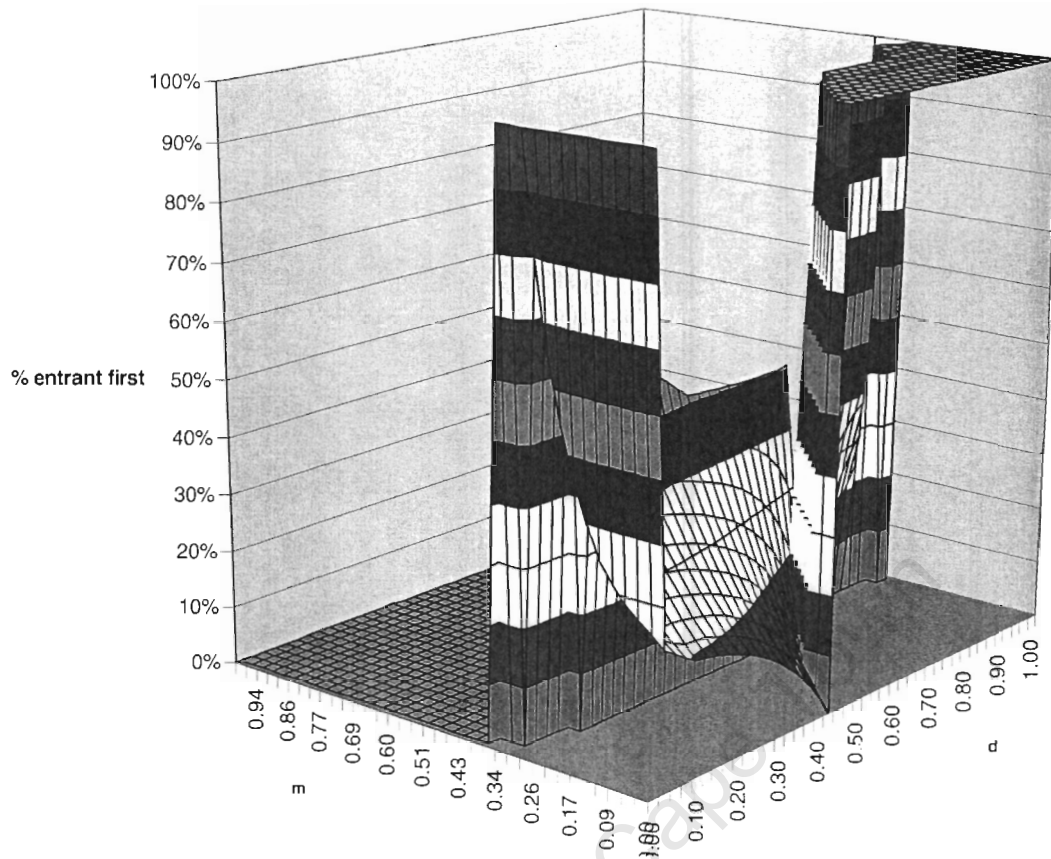
of who innovates first and the time of the first innovation for the full range of values of ν demonstrates that the level of ν is not critical. Hence for symmetry, we shall assume that level of ν that delivers the level of 0.95 for all s variables, namely 0.5.

Graph 20 : Demonstration of lack of sensitivity of results to changes in the ν variable

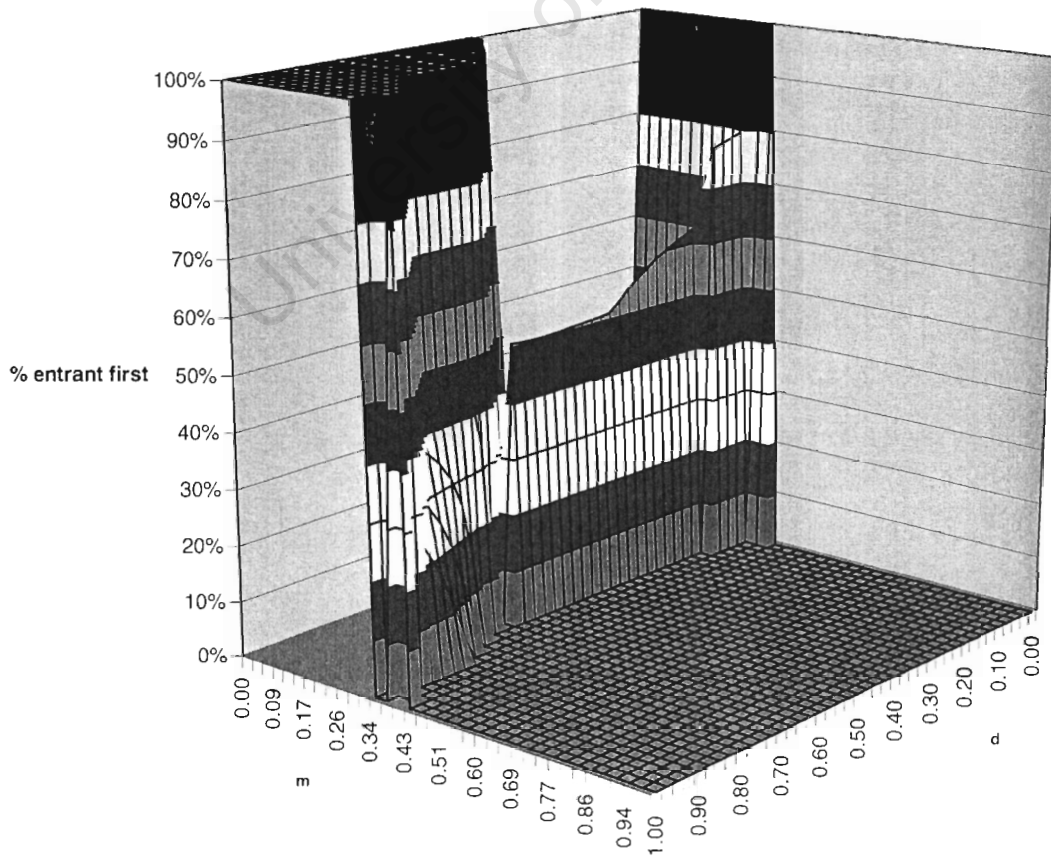


First we shall generate a result where both firms have an identical level of δ , in order to gain a reference point. In the analysis of the nine solution set model we saw that this result was not particularly sensitive to the level of δ , for values of δ greater than 1. As in that section of the analysis, we shall choose a level for the δ variables of 2 to facilitate the analysis of the influence of the d and m variables. To aid understanding the graph has been shown from two vantage points.

Graph 21: % of games in which the entrant innovates first for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=2$



Graph 21 v.2: % of games in which the entrant innovates first for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=2$

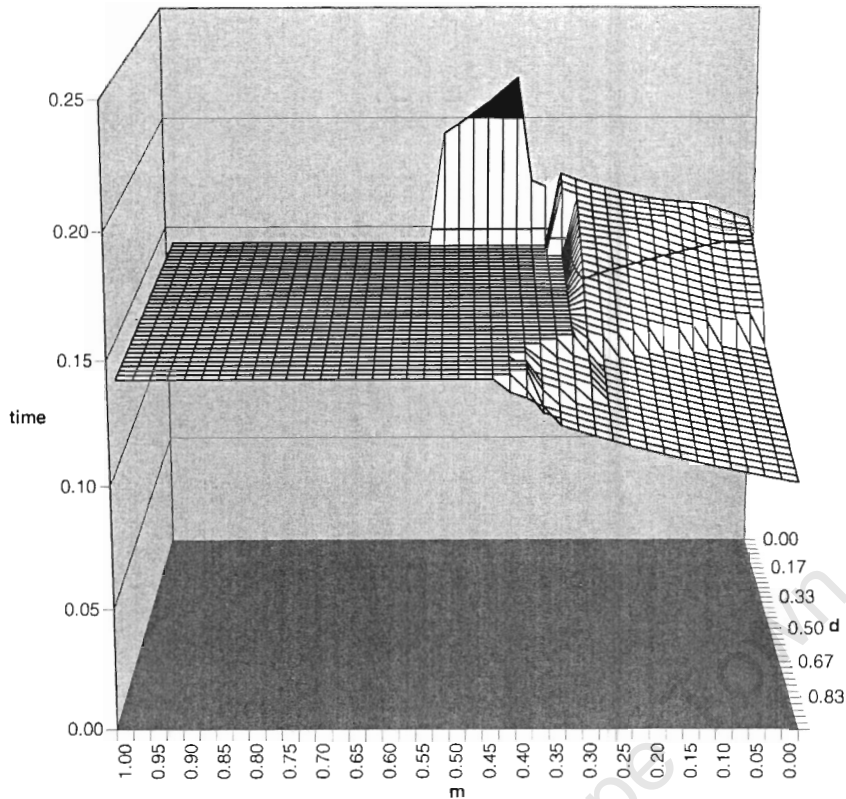


The result here is familiar from earlier versions of the model. It is clear that at strict levels of price competition this result becomes extremely sensitive to small changes in d . As the variable d approaches zero, representing Bertrand price competition, the result increases at some levels of m but when $d = 0$ the result jumps dramatically to 100%. This again illustrates the potential perils of making an assumption of strict Bertrand price competition rather than mere fierce competition. Looking at the same graph from the reverse direction illustrates the importance of the level of the m variable. We have previously discussed how the m variable may be considered to represent a barrier to entry in the market as it segregates a proportion of the original market from competition in the event of entry. While it will come as no surprise that as these barriers increase the entrant is less likely to innovate first, what is of particular interest is the seeming existence of some critical level of m that beyond which the entrant will never innovate first. Naturally this would imply that any model of product innovation that incorporates an assumption of even partial barriers to entry should be subjected to the present form of analysis to ensure that sensitivity to this assumption is not the primary driver of results.

Policy makers should also be concerned by this effect of the m variable. The analysis shows that the barriers to entry do not have to be complete to have a significant impact on the strategic outcome of the game. This highlights the important distinction between a monopoly that is blockaded and one that is not. The argument that an ability to make monopoly profits is necessary to encourage innovation dates back to Schumpeter (1942). In the earlier analysis of the four solution set model, I demonstrated, in agreement with the work of Beath, Katsoulacos & Ulph (1989), that the forces of 'profit incentive' and 'competitive threat' jointly drive innovative

activity. I showed in an analysis of the four solution set model that the time of innovation is almost the same when entry is blockaded as when entry is allowed but competition is fierce - in fact it is sometimes earlier. This implied that preventing supernormal profit in the post-innovation market when entry can occur is as harmful for encouraging innovation as allowing entry to be blockaded. In this more advanced model, it is now the m variable that represents a barrier to entry. As a result, the analytical framework no longer faces a binary choice between modelling blockaded and un-blockaded scenarios; by manipulating the m variable we can *select* the degree to which the market is protected against entry. In this context, the analysis just concluded shows that the market does not have to be totally blockaded for protective behaviour to have a significant impact on the strategic element, and hence the 'competitive threat', of the game. Graph 22 shows the time of the first innovation for the same scenario as described above, namely, where we set equivalent δ variables for both firms.

Graph 22: Time of first innovation for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=2$



Graph 22 clearly demonstrates that after a critical level of m , the sensitivity of the time of first innovation to changes in the ferocity of price competition is extremely low. This highlights the importance for policy makers of ensuring that barriers to entry in a market are minimized. The increased supernormal profit to which this leads does not encourage innovation. The profit that results from collusion in a market that may be entered freely does encourage innovation. Hence we can reaffirm our conclusion that the twin forces of ‘profit incentive’ and ‘competitive threat’ have a positive effect on innovation when acting in concert but not necessarily when acting independently.

The results obtained in the analysis of the nine solution model were in general agreement with the conclusions drawn by Boone (2001), that the incumbent is more

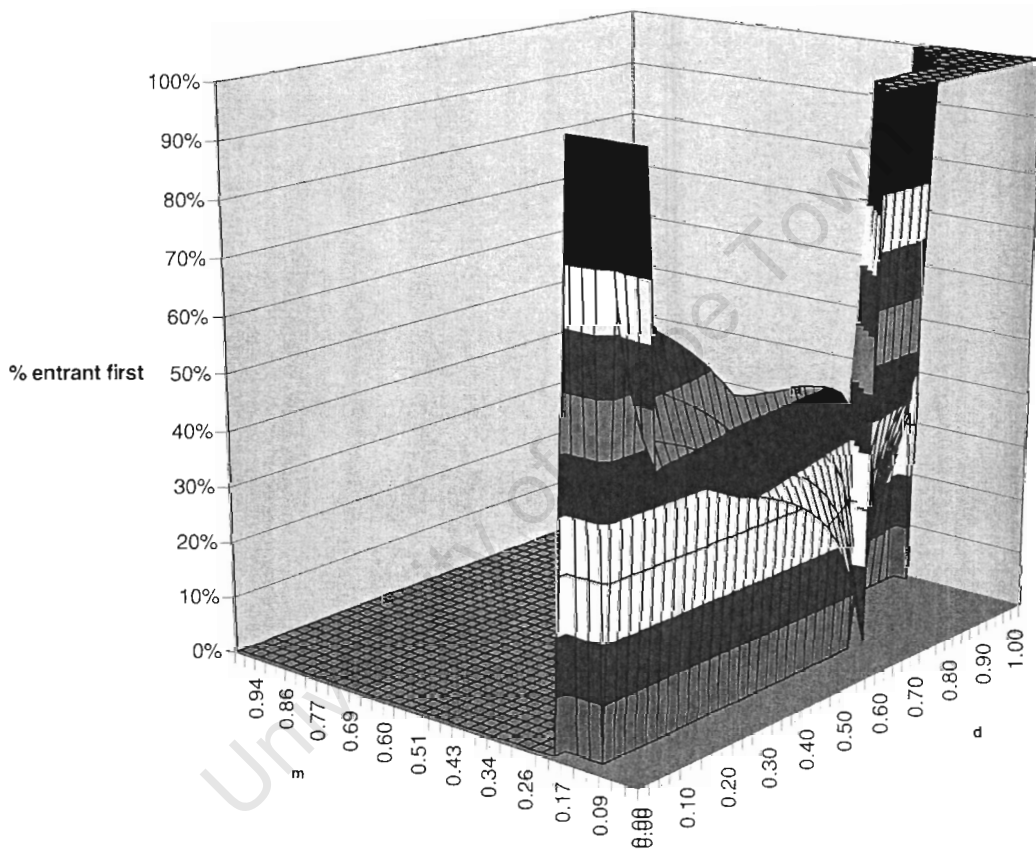
likely to buy the patent (or innovate first) under conditions of fierce price competition whilst the entrant is more likely to buy the patent under conditions of benign price competition. Vickers (1986) reached the same conclusion, but characterised fierce competition as the Bertrand case and benign competition as the Cournot case. The caveat to this result found here was that it is sensitive to the levels of both δ and m . I shall now subject this to re-analysis in the context of the dual innovation model, again identifying points of change in the identity of the first innovator where δ for each firm is 2, followed examination of a scenario in which the incumbent has an advantage.

Where there are no barriers to entry, i.e., when m is zero, the result shown on Graph 21 is broadly in line with the conclusion of Vickers and Boone in that at high levels of price competition (low d), the incumbent tends to innovate first, whereas the entrant tends to innovate first when there is less competition. This relationship clearly breaks down where d is zero but this has more to do with an analytic artefact identified earlier, namely, extreme sensitivity at levels close to zero when we are multiplying or dividing by terms that may themselves be zero. Again, the importance of barriers to entry affecting the strategic outcome of the game is demonstrated.

What will be now of interest are the results obtained when the firms have different levels of δ . Intuitions might expect that the incumbent will show a higher δ variable for the reasons outlined at the beginning of this section, that is, because of its intrinsic (i.e., *ceteris paribus*) ability to extract more profit from an innovation than the entrant. If we alter the levels of the δ variables to reflect a value of 2 for the incumbent and

1.8 for the entrant, a 10% advantage to the incumbent, we can illustrate the significance of having introduced this flexibility into the model. This scenario is depicted graphically in Graph 23.

Graph 23: % of games in which the entrant innovates first for $s_1=s_2=s_3=0.95$ and $\delta_I = 2$, $\delta_E = 1.8$



The difference in the results is significant. The ability of the incumbent to exploit an innovation to a greater degree than the entrant certainly alters the innovation game. In fact we can see graphically that for a greater range of levels of m , the incumbent will always be expected to innovate first regardless of the degree of competition. Given that these values of m represent an advantage to the incumbent in the event of a duopoly we might expect there to be a positive correlation between these values of m

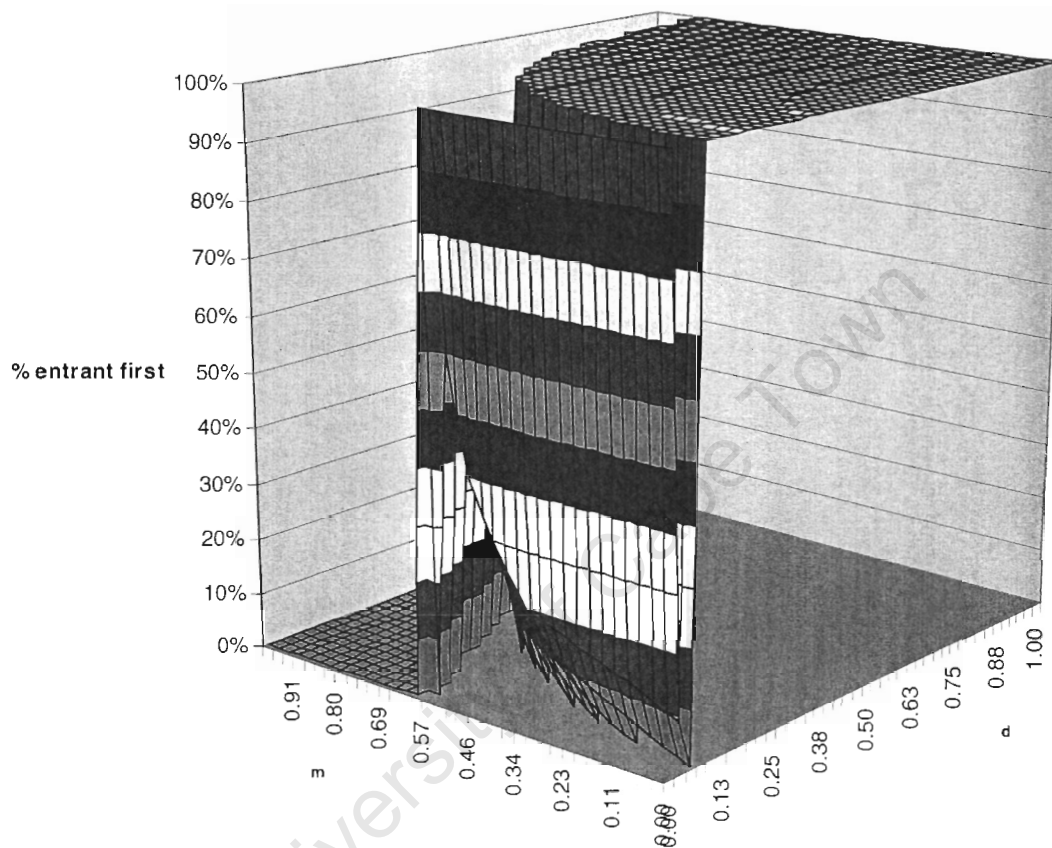
and the scenario in which the incumbent can extract a greater profit from the innovation than the entrant. To some extent we could consider the difference in the respective δ variables to be a substitute for variations in m . That is to say, the ability of the incumbent to gain incremental benefit from an innovation relative to an entrant, *ceteris paribus*, can be just as much considered a barrier to entry as the notion that some of the original market was protected from competition, represented by m .

As this is the final generation of the static model we must ensure that we conduct a full and thorough analysis. The above graphs represent the sensitivity of the result to the changes in m and d . However we have only seen the effect of one change in the δ variables. To fully understand the effects of changes in the δ variables we must perform sensitivity analysis using a different combination of these variables. We have seen one case that confirms what logic would lead us to expect, that if one firm has a greater incentive to innovate a particular product then this will increase the number of scenarios in which the firm holding this advantage innovates first. One possibility that we have not yet investigated, however, is the concept of an industry characterised by some *general and systematic* level of δ . Endogenously high growth industries, for example, could be characterised by systematically high levels of δ . The question is whether the dynamic between the incumbent and entrant alters between systemically high and systemically low δ industries.

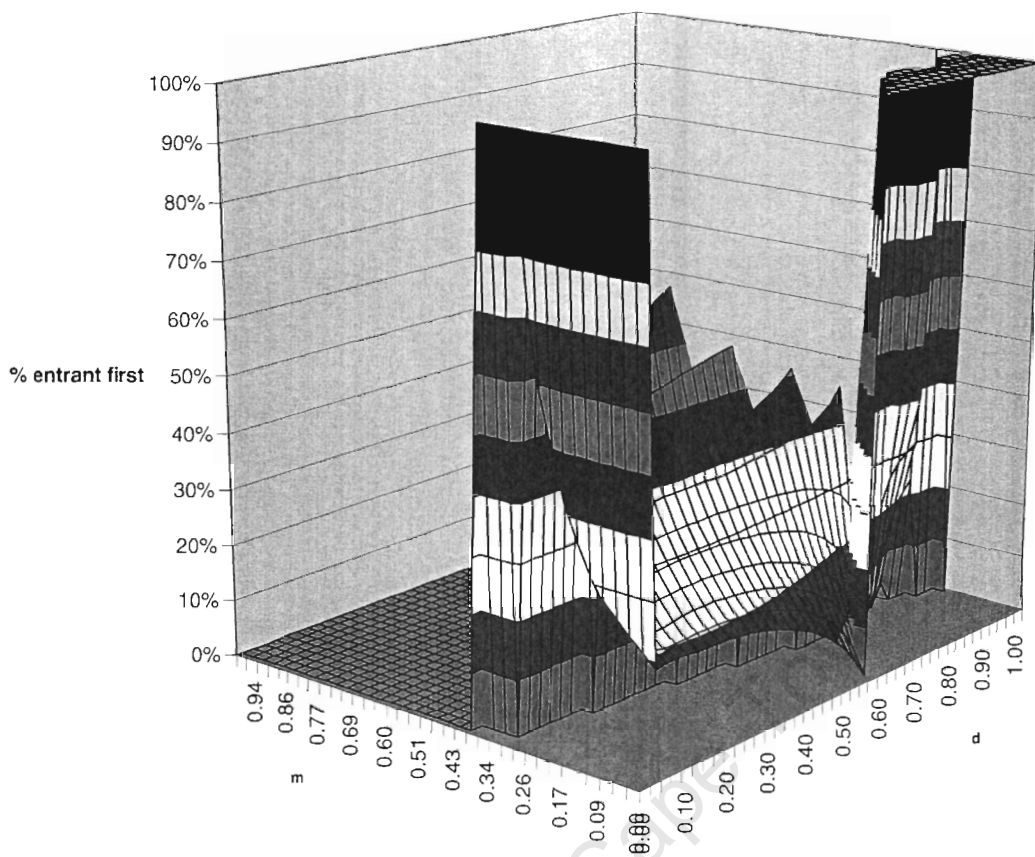
We shall examine two examples of industries. One of these shall be considered low growth and each firm shall have a δ variable equal to 1.15; the other will be high growth, with δ variables equal to 3. This means that we are not looking to offer any advantage to one firm over the other. Each firm has the same profit opportunities from

the innovation. The question is whether the dynamic is different in industries with different systemic δ .

Graph 24: % of games in which the entrant innovates first for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=1.15$ (low growth)



Graph 25: % of games in which the entrant innovates first for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=3$ (high growth)

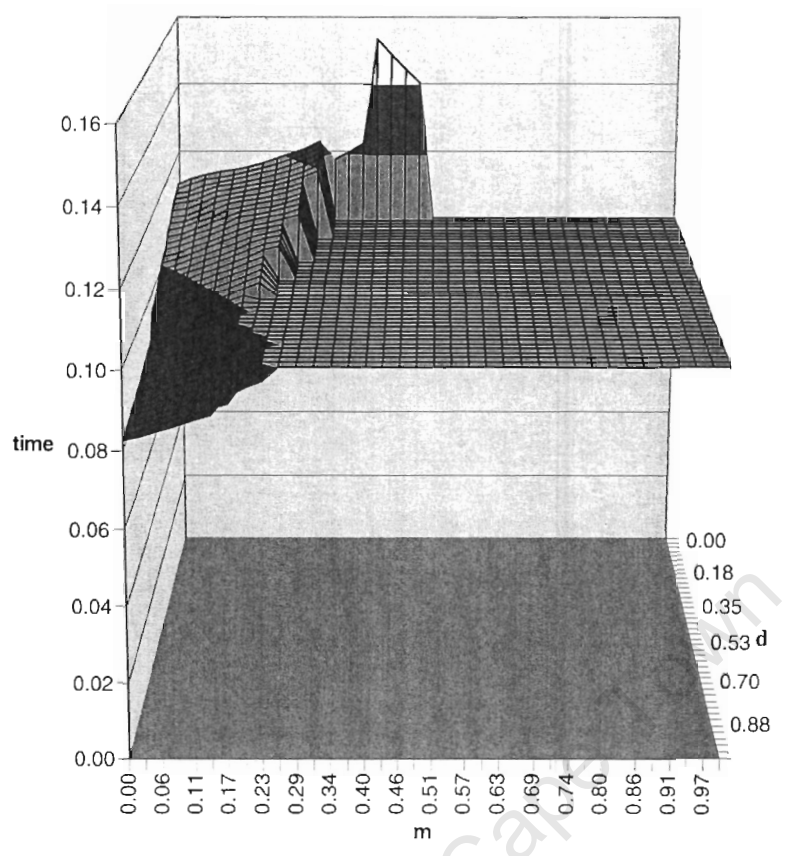


The competitive dynamic is dramatically altered as we move from a low growth to a high growth industry. The incumbent in the high growth case innovates first under a far greater range of industry characteristics than in the low growth case. We can rationalise this with reference to the identified twin forces that drive innovative activity. The profit incentive in this case is the same for both firms, neither having an ability to extract more value from the innovation than the other. However the incumbent must consider the increased cannibalisation, or indeed earlier cannibalisation, of the original product that may occur through its innovative activity. This effect clearly declines as the value of the innovation relative to that of the original product becomes greater. Also, in a high growth industry there is a greater competitive threat to the incumbent, as the original product will be cannibalised more effectively. To recall, I defined the degree of cannibalisation as an increasing

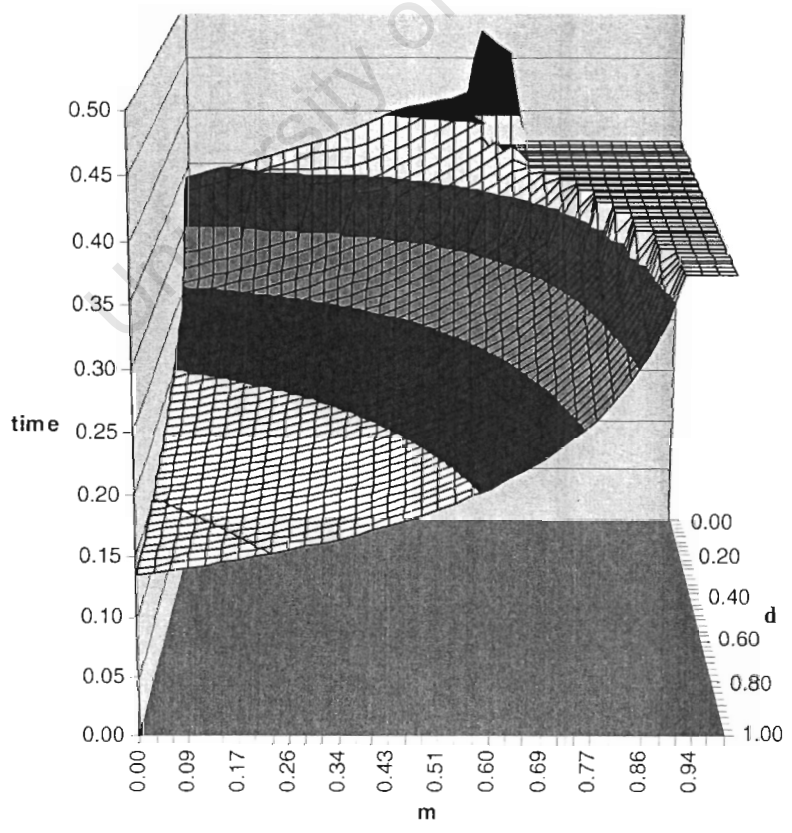
function of the δ variable. The entrant clearly has no competitive threat to encourage innovation. Hence, the proportion of games in which the incumbent will innovate first will increase as δ increases.

Given that the systemic level of δ in an industry clearly affects the strategic element of the game, this may suggest that the ability of a policy-maker to influence the time of innovation might also be affected when growth is high. When we allow the possibility that a policy-maker can, in some way, determine the level of d in which the game is played, our analysis of the four solution set model demonstrated that there was often a trade-off between allowing super-normal profit in the industry and encouraging investment in the innovation. If the strategic element of the game is reduced by a high systemic level of growth, is this trade-off also reduced?

Graph 28: Time of first innovation for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=3$ (high growth)



Graph 29: Time of first innovation for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=1.15$ (low growth)



Graphs 28 and 29 demonstrate a striking disparity between the reactions of high and low growth industries to the influence of d . It appears that in a low growth industry, there is a far wider range of possible values of m for which increased price competition will delay the innovation. This can be explained by the fact that the result in the high growth industry, at high levels of m , is driven primarily by the innovation of the incumbent, the incumbent innovating first more often. There is sufficient profit incentive for the incumbent to pursue the innovation even if in isolation and the entrant is partially locked out of the market. This reduces the impact of price competition, as the entrant does not enter the market until later. This conclusion concurs with that drawn in the analysis of the four solution set model, namely that in industries characterised by a high level of δ , the time of first innovation is less affected by any attempt of policy-makers to restrict super-normal profit in the post-entry market.

In the low growth industry, the entrant is more often the first innovator and thus the impact of price competition is more pronounced. When the incumbent is the first innovator there is no competition whereas when the entrant innovates first there is immediate competition. This result may then imply that policy-makers should be more ready to force competition on high-growth markets as in these circumstances it has less effect on the time of innovation. However we should consider that often the lack of sensitivity of the time of first innovation is partially due to the fact that the entrant is locked out of the market. For high values of m , the incumbent has a partially locked-in market and thus the time of first innovation, when the incumbent generally innovates first, reflects the decision of the incumbent. This decision is

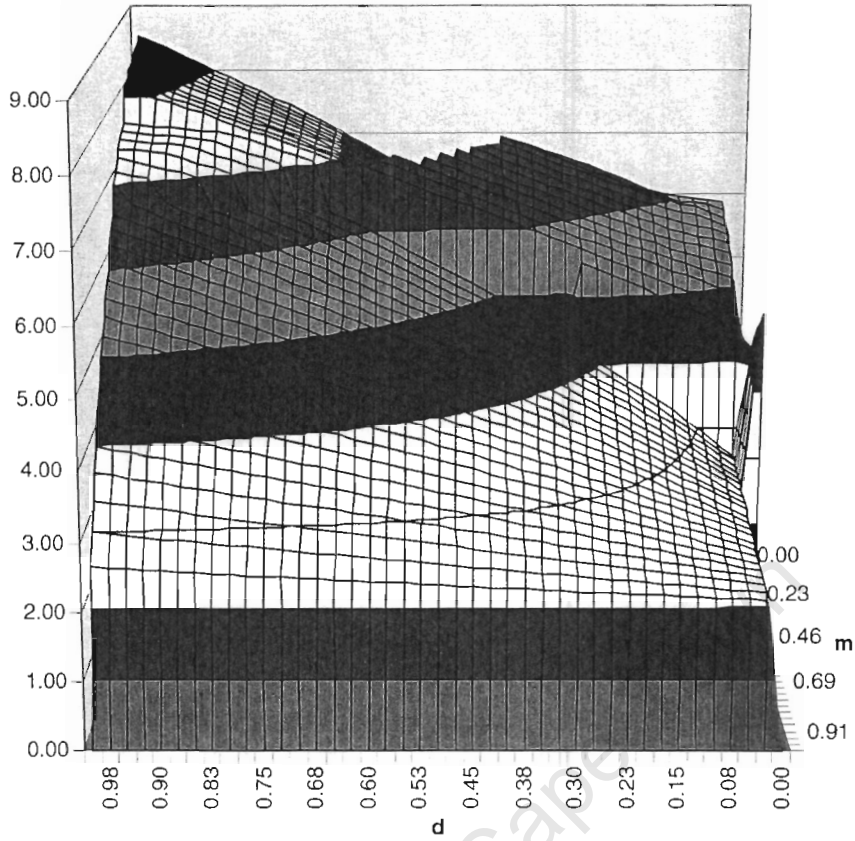
driven mainly by the profit available to the incumbent in the portion of the market segregated by the m variable from competitive forces and hence the result is not particularly sensitive to the value of d . In these conditions, the policy of requiring strict price competition in the event of a duopoly in the post-innovation market actually acts more as a disincentive to the entrant as the profit available in that market is reduced. Therefore the incumbent's monopoly position is protected in both markets. The existence of the original barrier to entry, denoted by the level of m , ensures that the incumbent innovates first. The threat by the policy-maker to enforce a strict level of price competition in the case of duopoly reduces any incentive for the entrant to innovate after the incumbent. Therefore the policy-maker, in attempting to ensure competition is actually increasing barriers to entry.

In this sort of circumstance, the attentions of the regulator might then be better directed at ensuring that barriers to entry do not exist in markets, i.e. that m be manipulated, if possible, to be low. Where m is reduced there is not, we have seen, such a dramatic difference between high and low growth industries in their sensitivity to price competition. When m is low in a high growth industry, the influence of d on the time of first innovation remains significant. The conclusion to draw from this is that the effect of barriers to entry in reducing the strategic element of the game is felt more keenly in high growth markets.

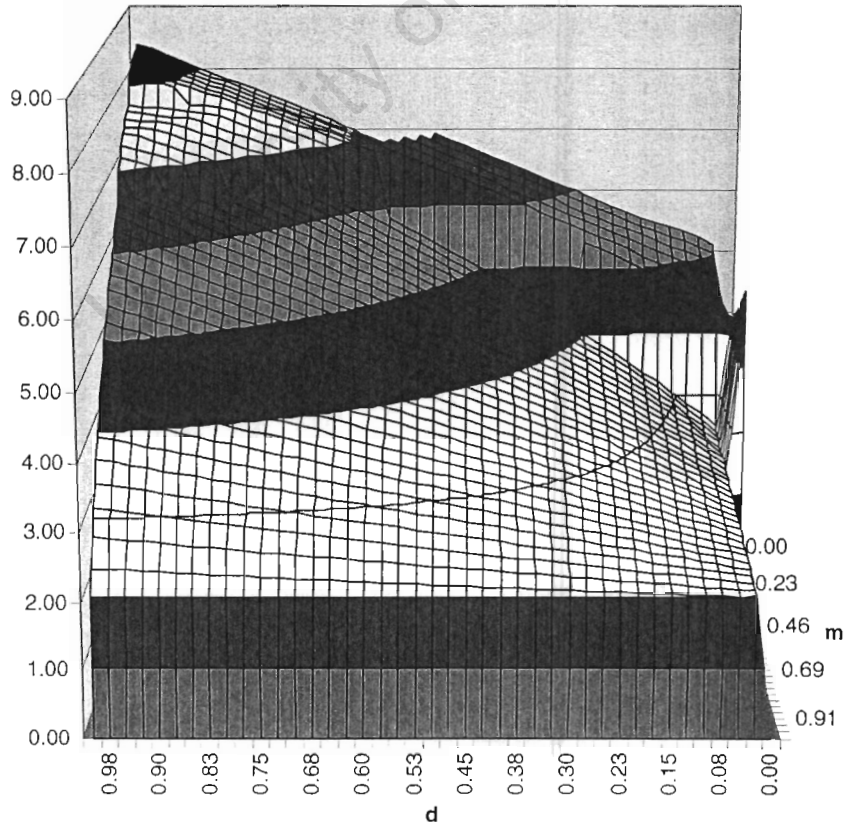
Comparison with Boone (2001)

If we recall, a key result from Boone, for which we were able to offer support in the analysis of the nine solution set model, was that the incentive to an independent R&D laboratory is non-monotonically related to price competition. In this section I aim to reassess these results in the context of the dual innovation model. In the analysis of the nine solution set model we obtained results on the minimum level of investment made in R&D by the incumbent or entrant. This was a proxy for the incentive for the independent R&D laboratory. We choose this rather than the straight cost of buying the patent, as used by Vickers (1986) and Boone (2001), because the models of these authors retain a patenting assumption but also take into account the possibility of another innovation coming into the market at a later time. Purchasing the patent does not secure a monopoly for infinity in their analysis. This characteristic is also seen in my model as there is no patent, hence both firms can enter the market. Thus, the incentives of the firms to innovate remain broadly similar in my model to what governs firm behaviour in the models of Vickers and Boone. Innovating first does secure a monopoly in the market for the innovation but the length of time that this monopoly lasts is uncertain *ex ante*. Within the models of Vickers and Boone, the incentive to an independent R&D laboratory to innovate was given as the price paid for the innovation in a second-price, sealed-bid auction framework. Given that the ability of both firms to innovate in my model corresponds to the idea that a patent does not offer a monopoly for infinity, I approximate within my model the amount paid to an independent R&D laboratory in a second-price sealed-bid auction framework by using the minimum individual firm investment that occurs in a scenario in which another can follow an innovation.

Graph 30: Incentive for independent R&D laboratory for $s_1=s_2=s_3=0.95$ and $\delta_I=\delta_E=2$



Graph 31: Incentive for independent R&D laboratory for $s_1=s_2=s_3=0.95$ and $\delta_I = 2, \delta_E = 1.8$



From the above graphs we can see that, when m is low, there is indeed non-monotonicity in the relationship between the incentive to the independent R&D laboratory and d . It is also clear that the non-monotonicity declines, though does not disappear, when the incumbent has an advantage. The reason for this observation is that, as Boone acknowledges, the non-monotonicity results from the fact that the identity of the purchaser of the patent changes. As we have already seen in this section, the changes in the δ variables result in the incumbent innovating first more often. This would therefore correspond to the declining perceivable non-monotonicity. Equally, a high level of m ensures that the incumbent innovates first more often and therefore if this is true for all levels of d then the relationship would be monotonic as there would be no change in the purchaser of the patent.

The results here therefore fully correspond to the conclusions drawn by Boone (2001). The significance of this is that it highlights the theme referred to continually throughout this thesis, that broad generalisations drawn from examining the effects of tightly restricted variables on the strategic outcome of a game is often open to criticism as depending with high sensitivity on the restrictions.. In this section I was able to demonstrate that while there are indeed scenarios demonstrable in my model in which the results of Vickers and Boone hold, we can also observe ranges of industry parameterizations in which they do not. The additional variables that I included in my model that are not present in those of Vickers and Boone represent the concepts denoted by δ and m . Neither of these concepts is intuitively exotic, and yet their impact on results is significant. This supports Sutton's criticism of the restrictions of the Nash Equilibrium framework as an analytical tool but also, I

believe, justifies the use of the sensitivity analysis methodology as a way of meeting it without leaving the domain of conventional game theory..

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Conclusion

This project began with a rather simple goal: to investigate the significance of the patenting assumption on the conclusion drawn from the model of Gilbert & Newbery (1982). It is fair to say that in the literature on innovation within the incumbent-entrant framework, progress has been slow. Only more recently have models been developed in which consideration has been given to the fact that results may only hold under certain conditions, Vickers (1986) and Boone (2001) being notable examples. Equally, it is only recently that an examination has been undertaken as to whether the methodology itself is to blame for the difficulty that has been encountered in producing stable and univocal results.. I refer here to Sutton (1991, 1998) in which the very concept of Nash Equilibrium as a suitable vehicle for analysis is called into question. The argument here is that a Nash Equilibrium is a vector of strategies and that to have such a vector one must have strategies. A strategy must strictly define an action set for a player, and this in turn leads to difficulties in extending results to general conditions..

This investigation agrees with Sutton in that many of the conclusions drawn can be demonstrated to rely so heavily on certain assumptions made in developing models as to render them virtually useless for guiding policy, either by individual firms or by regulators. However, I have sought to remain within the framework of Nash Equilibria for the sake of its rigor and clarity, and to look for a way of extending the methodology to avoid the narrow scope problems I have emphasized in criticism of others. This has been partially successful. I say partially as the point made by Sutton about the shortcomings of the Nash Equilibrium approach are perfectly valid and, as

such, cannot be completely overcome. However, I believe that through the methodology of sensitivity analysis I have been able to extend the conclusions drawn from a typical game theoretic model.

First, I took the original Gilbert & Newbery model and simply altered the framework by removing the option to patent the innovation. Then, through an axiomatic definition of the payoffs to the players under different scenarios I was able to use my distinctive methodology to generate Nash Equilibrium outcomes for hundreds of scenarios. The relationship between the outcome of the game and the levels of various input variables could then be displayed in graphical form. Whereas previously these inputs were defined as fixed values at the beginning of the game, the sensitivity analysis methodology defers any need for restrictive assumptions, allowing a fuller picture of the competitive dynamic to be modelled..

It should be noted that this is not a criticism of the authors of twenty years ago who derived the original results. This methodology is only possible because of the advances of technology and the improvement in microprocessors. It is my contention that even ten years ago this methodology would have been impractical for all but those with access to enormous number-crunching capacity. Economics is not the only area of academia that has changed considerably with the advent of the microcomputer and it should come as no surprise that we should see an updating of much of the work that was done previously, using the tools that are now available to us.

The result of my first model did indeed reach a significantly different conclusion, and showed the Gilbert and Newbury result to be highly sensitive to very specific market

specific conditions. The first aim of this project was therefore achieved rather easily. A further assumption that I then removed was the idea that the incumbent could only shelve the innovation, rather than the old product.

This assumption arose due to the removal of the first assumption. In the original model, there could never be a scenario in which the incumbent faced with entry could chose to shelve the innovation as there could only ever be one firm with the innovation. Entry implied that the incumbent did not have the innovation. Without patenting there could be an option to shelve the innovation by the incumbent when faced with entry. I also increased the strategic options available to the incumbent, allowing the possibility of shelving the original product in favour of the innovation. This section was an interesting exercise, examining what may have seemed a relatively minor change to the framework of the model and observing significant differences in the results. It served to again illustrate how placing a seemingly small constraint on the actions of a player within the game can significantly alter the results drawn from the model. This is precisely the type of problem that Sutton refers to in his project and why he looked to an alternative method of analysis.

The final model was an attempt to push the Nash Equilibrium approach to its boundaries and to introduce as much flexibility to the analysis as was feasible without removing all structure from the game. This required maintaining the original algebraic framework of the model but introducing a far greater variety of possible situations within this framework. It gave us the flexibility to examine an advantage to the incumbent in the secondary product market, a feature that proved highly influential on results.

The results obtained from the thesis were plentiful. First, with respect to the methodology employed in Industrial Organisation and Game Theory, I believe this thesis has demonstrated that there is scope for sensitivity analysis. Rather than search for *an* answer, this thesis sought to understand what made the answers come out as they have. To aim for a particular target, by definition, restricts the route one can take to that target. This in turn discredits the target obtained. The advantage of sensitivity analysis is that it goes some way towards addressing this criticism. It actually allows the model to generate all the possible answers and then further examination of the population of answers leads to understanding of the drivers of the solution. This was certainly true with all of the models constructed in this thesis.

In the first model, the four solution set model, I was able to show immediately that which firm innovates first is dependent on the industry characteristics in the context of which the game is played. I went on to demonstrate the result that pursuit of a policy of simultaneously reducing barriers to entry and also enforcing strict price competition can be harmful to the expected social rate of innovation. In fact, in some cases, innovation would happen faster in a blockaded monopoly scenario than in a scenario with free entry and strict price competition. In order to maximise the amount of innovation a policy maker should adopt the seemingly perverse policy of removing barriers to entry and then allowing collusion to take place. This is in agreement with the work of Beath, Katsoulacos & Ulph (1989), according to which the forces of 'profit incentive' and 'competitive threat' drive innovative activity. The profit incentive comes from the possibility of collusion and the competitive threat comes from the removal of barriers to entry. Given that the best way to obtain perfect

collusion is a merger, this result suggests that in industries that exhibit a high degree of innovation, the policy maker should, at least under some circumstances, allow consolidation of the industry post-innovation.

The final two models derive their frame of reference primarily from the contributions of Vickers (1986) and Boone (2001). According to these analyses, in conditions of strict price competition the incumbent is more likely to innovate first, whereas under Cournot conditions, or indeed collusion, the entrant is more likely to innovate first. I was able to offer qualified support for this result but with an important caveat. First, my model contained an important extra variable, namely the variable m , representing the degree of lock-in of customer base that the incumbent was able to carry forward into the secondary product market. It was demonstrated that this was an important constraint on the result.

Of more significance was the sensitivity of the result to the level of δ in the industry. I was able to show that the incumbent is far more likely to innovate first in a high growth industry than in a low growth industry, *ceteris paribus*. The rationale for this is that as the new product market becomes increasingly valuable, the ratio of future profit streams from innovation to profit streams from the original product increases and at the limit, as the value of the innovation tends to ∞ , the relative value of the original product tends to zero.

I was also able to demonstrate that a policy maker has far greater capacity for adversely affecting the time of innovation in a low growth industry than in a high growth industry. The importance of this result is not to suggest that a policy maker

can pressure a high growth industry with impunity, but to identify the basis for the result in the fact that if the reward is great enough for the incumbent, the potential price competition does not affect the innovation decision for the incumbent. The crucial point here is that what is driving this result is the fact that any degree of lock-in for the incumbent can have the result of reducing the effectiveness of policy, and that this relationship is increasing with the growth rate in the industry. This implies that, in high growth industries, policy makers should focus more on reducing barriers to entry than on price competition, as the latter is less effective where the former is maintained..

In conclusion, I believe that this thesis has gone some way to addressing the concerns of Sutton (1998) in building a new methodology around the relatively sparse literature in technical microeconomics on the process, so crucial to economic growth and change, of innovation races.

Bibliography

- Beath, J., Katsoulacos, Y. & Ulph, D. 1995, 'Game-Theoretic Approaches to the Modelling of Technological Change', in *Handbook of the Economics of Innovation and Technological Change*, ed. P. Stoneman, Blackwell, Oxford, pp. 132-181.
- Bessen, J. & Maskin, E., 'Sequential Innovation, Patents, and Imitation', *Massachusetts Institute of Technology Department of Economics Working Paper 00/01 January 2000*
- Binmore, K. 1992, *Fun and Games: A text on game theory*, D.C. Heath and Company, Lexington, Massachusetts.
- Boone, J. 2001, 'Intensity of Competition and the Incentive to Innovate', *International Journal of Industrial Organizations* 19(5): 705-726.
- Chandy, R. & Tellis, G. 2000, 'The Incumbent's Curse? Incumbency, Size and Radical Product Innovation', *Journal of Marketing*, vol. 64, pp. 1-18.
- Christensen, C. 1993, 'The Rigid Disk Drive Industry: A History of Commercial and Technological Turbulence', *Business History Review*, vol. 67, pp. 531-588.
- Cohen, W.M. 1995, 'Empirical Studies of Innovative Activity', in *Handbook of the Economics of Innovation and Technological Change*, ed. P. Stoneman, Blackwell, Oxford, pp. 182-264.
- Robert M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W Function", *Advances in Computational Mathematics*, volume 5, 1996, pp. 329--359.
- Dasgupta, P. & Stiglitz, J. 1980, 'Uncertainty, Industrial Structure, and the Speed of R&D', *Bell Journal of Economics*, vol. 11, pp. 1-28.
- Dosi, G. 1988, 'Sources, procedures and microeconomic effects of innovation', *Journal of Economic Literature*, vol. 36, pp. 1126-1171.
- Foster, R. 1986, *Innovation: The Attacker's Advantage*, Summit Books, New York.
- Ghemawat, P. 1991, 'Market Incumbency and Technological Inertia', *Marketing Science*, vol. 10 (Spring), pp. 161-171.
- Ghemawat, P. 1997, *Games Businesses Play: Cases and Models*, The MIT Press, Cambridge, Massachusetts.
- Gilbert, R. & Newbery, D. 1982, 'Preemptive Patenting and the Persistence of Monopoly', *American Economic Review*, vol. 72, pp. 514-526.

- Gilbert, R. & Newbery, D. 1984, 'Uncertain Innovation and the Persistence of Monopoly: Comment', *American Economic Review*, vol. 74, pp. 238-242.
- Greenstein, S. & Ramey, G. 1998, 'Market Structure, Innovation and Vertical Product Innovation', *International Journal of Industrial Organization*, vol. 16, pp. 285-311
- Grossman, G. & Shapiro, C. 1986, 'Optimal Dynamic R&D Programs', *Rand Journal of Economics*, vol. 17, pp. 581-593.
- Grossman, G. & Shapiro, C. 1987, 'Dynamic R&D Competition', *Economic Journal*, vol. 97, pp. 372-387.
- Henderson, R. & Clark, K.B. 1990, 'Architectural Innovation: The Reconfiguration of Existing Product Technologies and the Failure of Established Firms', *Administrative Science Quarterly*, vol. 35, pp. 9-30.
- Henderson, R. 1993, 'Underinvestment and Incompetence as Responses to Radical Innovation: Evidence from the Photolithographic Alignment Equipment Industry', *RAND Journal of Economics*, vol. 24, pp. 248-271.
- Kamien, M.I. & Schwartz, N.L. 1982, *Market Structure and Innovation*, Cambridge University Press, Cambridge.
- Lee T. & Wilde, L. 1980, 'Market Structure and Innovation: A Reformulation', *Quarterly Journal of Economics*, vol. 94, pp. 429-436.
- Loury, G.C. 1979, 'Market Structure and Innovation', *Quarterly Journal of Economics*, vol. 93, pp. 395-410.
- Nelson, R.R. & Winter, S.G. 1982, *An Evolutionary Theory of Economic Change*, Harvard University Press, Cambridge and London.
- Reinganum, J. 1982, 'A Dynamic Game of R&D: Patent Protection and Competitive Behaviour', *Econometrica*, vol.50, pp. 671-688.
- Reinganum, J. 1983, 'Uncertain Innovation and the Persistence of Monopoly', *The American Economic Review*, vol.73, pp. 739-748.
- Reinganum, J. 1984, 'Uncertain Innovation and the Persistence of Monopoly: Reply', *The American Economic Review*, vol.74, pp. 243-246.
- Reinganum, J. 1985, 'Innovation and Industry Evolution', *Quarterly Journal of Economics*, vol. 100, pp. 81-100.
- Rosenbloom, R. & Christensen, C. 1994, 'Technological Discontinuities, Organizational Capabilities, and Strategic Commitments', *Industrial and Corporate Change*, vol. 3, pp. 655-685

- Salant, S. 1984, 'Comment on Gilbert and Newbery's 'Preemptive Patenting and the Persistence of Monopoly,' *American Economic Review*, vol. 74, pp. 247-50.
- Schumpeter, J.A. 1942, *Capitalism, Socialism and Democracy*, Harper, New York.
- Sutton, J. 1991, *Sunk Costs and Market Structure*, The MIT Press, Cambridge, Massachusetts.
- Sutton, J. 1998, *Technology and Market Structure*, The MIT Press, Cambridge, Massachusetts.
- Tirole, J. 1988, *The Theory of Industrial Organization*, The MIT Press, Cambridge, Massachusetts.
- Utterback, J.M. 1994, *Mastering the Dynamics of Innovation*, Harvard Business School Press, Boston, Massachusetts.
- Vickers J. 1986, 'The Evolution of Market Structure when there is a Sequence of Innovations', *Journal of Industrial Economics*, 1986, 35, 1-12.
- Von Hippel, E. 1988, *The Sources of Innovation*, Oxford University Press, Oxford.
- Wolf, G. 2000, 'Product innovation as a static game of incomplete information in a non-Bayesian environment', *Unpublished Masters Thesis*, University of Cape Town.

Appendix

The appendix contains the script for the computer models discussed in the Introduction. This script, within Microsoft Excel™, generates the graphs used in this thesis to analyse the models. They now follow in the order derived in the thesis; the four solution set model; the nine solution set model; and the dual innovation model.

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	A	B	C
1	d1	d2	s
2	=A143	=IF(A5=TRUE,D_one,B203)	0.95 =C203
3	0.875 <input checked="" type="checkbox"/> d1=d2	0.59	
4			1-s
5	TRUE		=1-S
6	Ti	=IF(L8=1,B12,IF(L8=2,E12,IF(L8=3,I12,L12)))	=IF(L8=4,L19,"")
7		=IF(L8=1,B16,IF(L8=2,E16,IF(L8=3,I16,L16)))	=IF(L8=4,S18,"")
8	Te	=IF(L8=1,B13,IF(L8=2,E13,IF(L8=3,I13,L13)))	=IF(L8=4,G25,"")
9	=IF(L8=1,"*****","")	=IF(L8=1,B17,IF(L8=2,E17,IF(L8=3,I17,L17)))	=IF(L8=4,S19,"")
10	Case 1		
11			

	D	E	F
1	δ	m	b
2	=D200	=E203	=F113
3	2	0.5	0
4	δx	1-m	1-b
5	=delta*X	=1-M	=1-F2
6	=IF(L8=4,G26,"")	=IF(G25<G26,1,0)	
7	=IF(L8=4,T18,"")	=IF(L23<L19,1,0)	
8	=IF(L8=4,L23,"")	=IF(E6*E7=1,1,IF(G25<G26,2,IF(L23<L19,3,4)))	
9	=IF(L8=4,T19,"")	=IF(L8=2,"*****", "")	
10	Case 2		
11			

	G	H	I	J
1	r	x	I1	I2
2	=G200	50	=X	=one_minus_S*X+one_minus_S*delta_X+(S*(F5*(B26/B28)*X+(F2*delta_X)))
3	0.1			
4	e^-delta		<p>Case 1 - Incumbent always shelves, never attempts</p> <p>Case 2 - Incumbent shelves in event of duopoly in P2</p> <p>Case 3 - Incumbent shelves when alone in P2</p> <p>Case 4 - Incumbent never shelves</p>	
5	=EXP(-delta)			
6	=IF(O2=4,IF(R4<R5,"I","E"),"E")			
7	=IF(L8=4,IF(L25=1,IF(O8=1,IF(O9=1,0%,IF(O9=4,100%,IF(O9=2,(IF(G25<G26,100%,0%)),IF(L23<L19,100%,0%))),IF(E8=1,T18+T19+S19,IF(E8=2,T19+T18,IF(E8=3,T19+S19,T19))),IF(L11=1,0%,100%)),IF(L8=3,IF(I12<I13,0%,100%),IF(L8=2,IF(E12<E13,0%,100%),100%)))			
8	=O8		This game is an example of Case	
9	=IF(L8=4,IF(G7=0%,B6,IF(G7=100%,B8,Q4)),IF(B8<B6,B8,B6))		=IF(L8=3,"*****",")")	
10		Case 3	=EXP(-interest_rate*J11)	
11			=IF(L8=3,IF(I_four-I_three>0.2,((I_four*J10)-(I_three*J10))^0.5,100000000),5)	

	K	L	M
1	I3	I4	E1
2	=IF(delta>=1,(one_minus_S*X)+(S*F5*M*D_one*X/B25),(one_minus_S*X)+(S*F5*((delta*D_one+1-delta)*(M+(one_minus_M*(1-delta))))))	=IF(delta>=1,X*(one_minus_S+(one_minus_S*D_one*M*delta)+S*((F5*((B26/B28)*((B33/delta)+((D_one*(delta-1)/delta)))*(M+M*((one_minus_M/(1+(delta/2)))))))+(F2*M*D_one*delta)),X*(one_minus_S+(one_minus_S*D_one*M*delta)+S*((F5*((B26/B28)*((delta*B33)+delta_X*(1-(delta*(1-(M^0.585)))))))+(F2*M*delta*D_one))))	=IF(delta<1,(one_minus_S*delta_X)+(S*((F5*((delta^3)*D_one*one_minus_M*X)))+(F2*delta_X)),(one_minus_S*delta_X)+(S*((F5*((1-(M/delta))*((D_one/delta)+((delta-1)/delta)*delta_X)))+(F2*delta_X))))
3			
4			
5			
6			
7			
8		=O2	
9		=IF(L8=4,"*****", "")	=IF(L8=4,"*****", "")
10	Case 4		
11		=IF(L12<L13,1,0)	

	N	O	P
1	E2	=IF(J2<I2,1,0)	=IF(L2<K2,1,0)
2	=IF(delta<1,X*((one_minus_S*one_minus_M*delta*D_one)+S*((F5*(delta*((delta*B33)+1-delta))*(delta*(1-(M^0.585))))+(F2*one_minus_M*D_one*delta))))),X*((one_minus_S*one_minus_M*delta*D_one)+S*((F5*(delta*((B33/delta)+(D_one*(delta-1)/delta))*(1-(M+M*((1-M)/(1+(delta/2)))))))+(F2*one_minus_M*D_one*delta))))))	=IF(O1=1,IF(P1=1,1,3),IF(P1=1,2,4))	
3			
4			
5	=IF(O2=4,IF(N24=1,IF(O9=0,1,0),0),0)		
6			
7			
8		=IF(P8=1,1,IF(P9=1,1,0))	=IF(Q12>S12,IF(Q15>S15,1,0),IF(Q15<S15,1,0))
9		=IF(O8=1,IF(P8=1,R8,R9),0)	=IF(Q13>Q16,IF(S13>S16,1,0),IF(S13<S16,1,0))
10			
11			

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	Q	R	S	T
1	=IF(L19<L23,(S19*L19),(S19*L23))			=(i_two/interest_rate)*(EXP(-interest_rate*B8))
2				
3	=IF(G26<G25,(T18*G26),(T18*G25))			=IF((S1-B9)>B7,1,0)
4	=S18*L19+T19*L23			
5				
6				
7				
8	=IF(P8=1,IF(Q12>S12,"i1","i2"),0)			=IF(Q8="i1",IF(Q13>Q16,1,3),IF(Q8="i2",IF(S13>S16,2,4),R9))
9	=IF(P9=1,IF((Q13-Q16>0),1,2),0)			=IF(Q9=1,IF(Q12>S12,1,2),IF(Q9=2,IF(Q15>S15,3,4),R8))
10	I			
11	i1		i2	

	A	B	C
12	Ti =	100000000	
13	Te =	$=((E_one*(EXP(-interest_rate*B13)))^{-0.5})$	$=EXP(-interest_rate*B13)$
14	Payoffs:		
15	i =	4.88122764021196	
16	e =	$=((E_one*(EXP(-interest_rate*B13)/interest_rate)) - (1/B13))$	
17			
18			
19			
20			
21			

	D	E	F
12	Ti =	=IF(E19>E20,100000000,E22)	
13	Te =	=(E_one*(EXP(-interest_rate*E13)))^-0.5)	
14	Payoffs:		
15	i =	=MAX(E19:E20)	
16	e =	=(E_one*((EXP(-interest_rate*E13))/interest_rate)-(1/E13))	
17	i1 =	=(1/interest_rate)*(I_one+(EXP(-interest_rate*E13)*(I_three-I_one))))	
18	i2 =	=(1/interest_rate)*(I_one+(EXP(-interest_rate*E22))*(I_two-I_one))+((EXP(-interest_rate*E13)*(I_three-I_two)))-1/E22	
20			
21			

	G	H	I	J
12		Ti =	=IF(J11<I13,I13,J11)	=EXP(-interest_rate*I12)
13		Te =	=IF(L8=3,IF(E_one>0,((E_one *J13))^-0.5,100000000),5)	=EXP(-interest_rate*I13)
14		Payoffs:		
15		i =	$=(1/\text{interest_rate}) * (I_one - (I_one * J13) + (I_three * J13) - (I_three * J12) + (I_four * J12)) - (1/I12)$	
16		e =	$=(1/\text{interest_rate}) * ((E_one * J13) - (E_one * J12) + (E_two * J12)) - (1/I13)$	
17				
18				
19				
20				
21				

Four Solution Set IV

	K	L	M
12	Ti =	=IF(L8=4,IF(L25=4,R4,IF(L25=3,G26,IF(L25=2,L19,IF(O9=0,R4,IF(R8=1,L19,G26))))), "")	
13	Te =	=IF(L8=4,IF(L25=4,R5,IF(L25=3,L23,IF(L25=2,G25,IF(O9=0,R5,IF(R8=1,G25,L23))))), "")	
14	Payoffs:		
15	i =	=IF(L8=4,IF(L25=4,P19,IF(L25=3,L28,IF(L25=2,L26,P19))), "")	
16	e =	=IF(L8=4,IF(L25=4,P20,IF(L25=3,L29,IF(L25=2,L27,P20))), "")	
17	i before e		
18	Ti1	=IF(L8=4,IF(I_two>I_one,(-(I_one*M19)+(I_two*M19))^-0.5,10000000),5)	
19		=EXP(-interest_rate*L19)	
20	Te1	=IF(G25<L19,L19,G25)	=IF(G25<L19,M19,H25)
21	e before i		

	N	O	P
12			e1
13		E	
14			
15			e2
16			
17			
18		Payoffs:	
19	=IF(L19<G25,1,0)	i =	=IF(O8=0,S18*Q12+T18*S12+Q15*S19+T19*S15,IF(O9=1,Q12,IF(O9=2,S12,IF(O9=3,Q15,S15))))
20		e =	=IF(O8=0,S18*Q13+T18*S13+Q16*S19+T19*S16,IF(O9=1,Q13,IF(O9=2,S13,IF(O9=3,Q16,S16))))
21			

Four Solution Set Model

	Q	R	S	T
12	=L26	for I	=IF(L8=4,IF(G26>G25,((1/interest_rate)*(I_one-(I_one*H25)+(I_three*H25)-(I_three*H26)+(I_four*H26))-1/G26),((1/interest_rate)*(I_one-(I_one*H26)+(I_two*H26)-(I_two*H25)+(I_four*H25))-(1/G26))), "") for I	
13	=L27	for E	=IF(L8=4,IF(G26>G25,((1/interest_rate)*(E_one*H25)-(E_one*H26)+(E_two*H26))-1/G25),(E_two*H25/interest_rate)-1/G25), "") for E	
14				
15		for I	=L28	for I
16		for E	=L29	for E
17				
18			=R22*R24	=R25*R22
19			=R24*R23	=R25*R23
20				
21	Mixed	Strategies		

	A	B	C
22			
23			
24			
25	delta^2	$=D2^2$	
26	$1+\text{delta}^2$	$=1+B25$	
27	$(1+\text{delta})^2$	$=B28^2$	
28	$1+\text{delta}$	$=1+D2$	
29	$\text{delta}^2 - 1$	$=B25-1$	
30	$1-(m/\text{delta}^2)$	$=1-(M/B25)$	
31	delat^3	$=D2^3$	
32	delta^4	$=D2^4$	
33	d^2	$=D_one^2$	

	D	E	F
22	ti =	=IF(I_two-I_one>0.1,((I_two*G22)-(I_one*G22))^0.5,100000000)	
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			

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	G	H	I	J
22	=IF(E22>100,2,EXP(- interest_rate*E22))			
23				
24				
25	=IF(L8=4,IF(E_two>0,((E_two*H25))^-0.5,100000000),5)		=IF(G25>100,0.02,EXP(- interest_rate*G25))	
26	=IF(L8=4,IF(I_four>I_three,((I_four- I_three)*H26)^-0.5,100000000),5)		=IF(G26>100,0.02,EXP(- interest_rate*G26))	
27				
28				
29				
30				
31				
32				
33				

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	K	L	M
22	Ti2	=IF(\$G\$26<\$L\$23,L23,G26)	=IF(\$G\$26<\$L\$23,M23,H26)
23	Te2	=IF(L8=4,IF(E_one>0,((E_one*M23)^-0.5),100000000),5)	=IF(L23>100,2,EXP(-interest_rate*L23))
24			
25		=IF(N24=1,1,IF(N19=0,IF(N22=0,4,3),2))	
26	i1	=(1/interest_rate)*(l_one-(l_one*M19)+(l_two*M19)-(l_two*H25)+(l_four*H25))-(1/L19)	=IF(L26>L28,1,2)
27	e1	=E_two*H25/interest_rate-(1/G25)	=IF(L27>L29,1,0)
28	i2	=(1/interest_rate)*(l_one-(l_one*M23)+(l_three*M23)-(l_three*H26)+(l_four*H26))-1/G26	
29	e2	=(1/interest_rate)*((E_one*M23)-(E_one*H26)+(E_two*H26))-1/L23	
30			
31			
32			
33			

	N	O	P
	=IF(L23<G26,1,0)		
22			
23			
24	=N22*N19		
25			
26			
27			
28			
29			=M23
30			=H26
31			
32			
33			

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	Q	R	S	T
22	e1	$=(S15-Q15)/(Q12-Q15-S12+S15)$		
23	e2	$=1-R22$		
24	i1	$=(S16-S13)/(Q13-S13-Q16+S16)$		
25	i2	$=1-R24$		
26				
27				
28				
29				
30				
31				
32				
33				

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Nine Solution Set Model

	A	B	C	D	E
1	d1	d2	s	δ	m
2	=A143	=IF(A5=TRUE,D_ one,B203)	0.95 =C203	=D200	=E203
3	1 <input checked="" type="checkbox"/> d1=d2	0.59		2	0.5
4			1-s	δx	1-m
5	TRUE		=1-S	=delta*X	=1-M
6	Ti	=IF(L8=1,B12,IF(L8=2,E12,IF(L8=3 ,I12,L12)))	=IF(L8=4,L19,"")	=IF(L8=4,G26,"")	=IF(G25<G26,1,0)
7		=IF(L8=1,B16,IF(L8=2,E16,IF(L8=3 ,I16,L16)))	=IF(L8=4,S18,"")	=IF(L8=4,T18,"")	=IF(L23<L19,1,0)
8	Te	=IF(L8=1,B13,IF(L8=2,E13,IF(L8=3 ,I13,L13)))	=IF(L8=4,G25,"")	=IF(L8=4,L23,"")	=IF(E6*E7=1,1,IF(G25 <G26,2,IF(L23<L19,3, 4)))
9	=IF(L8=1,"*****", "")	=IF(L8=1,B17,IF(L8=2,E17,IF(L8=3 ,I17,L17)))	=IF(L8=4,S19,"")	=IF(L8=4,T19,"")	=IF(L8=2,"*****", "")
10					

	A	B	C	D	E
11					
12					
13					
14					
15					
16					
17					
18					
19					

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	A	B	C	D	E
20					
21					
22					
23					
24					
25	delta^2		$=D^2$		
26	$1+\text{delta}^2$		$=1+B25$		
27	$(1+\text{delta})^2$		$=B28^2$		
28	$1+\text{delta}$		$=1+D2$		
29	$\text{delta}^2 - 1$		$=B25 - 1$		
30	$1 - (m/\text{delta}^2)$		$=1 - (M/B25)$		
31	delta^3		$=D^3$		
32	delta^4		$=D^4$		
33	d^2		$=D_one^2$		

Nine Solution Set Model

	F	G	H	I
1	b	r	x	l1
2	=F113	=G200	50	=X
3	0.9	0.1		
4	1-b	e ^{-delta}		
5	=1-F2	=EXP(-delta)		
6		=IF(O2=4,IF(R4<R5,"I","E"),"E")		
7		=IF(L8=4,IF(L25=1,IF(O8=1,IF(O9=1,0%,IF(O9=4,100%,IF(O9=2,(IF(G25<G26,100%,0%),IF(L23<L19,100%,0%))),IF(L23<L19,100%,0%))),IF(E8=1,T18+T19+S19,IF(E8=2,T19+T18,IF(E8=3,T19+S19,T19))),IF(L11=1,0%,100%),IF(L8=3,IF(I12<I13,0%,100%),IF(L8=2,IF(E12<E13,0%,100%),100%)))		
8		=O8		This game is an example of Case
9		=IF(L8=4,IF(G7=0%,B6,IF(G7=100%,B8,Q4)),IF(B8<B6,B8,B6))		=IF(L8=3,"*****",")
10				

	F	G	H	I
11				
12				
13				
14				
15				
16				
17				
18				
19				

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	F	G	H	I
20				
21				
22				
23				
24				
25		=IF(L3>3,IF(OR(M=1,D_one=0),100000000,((E_two*H25)^0.5)))	=IF(G25>100,0.02,EXP(-interest_rate*G25))	
26		=IF(L8=4,IF(I_four>I_three,((I_four-I_three)*H26)^0.5,100000000),5)	=IF(G26>100,0.02,EXP(-interest_rate*G26))	
27				
28				
29				
30				
31				
32				
33				

Nine Solution Set Model

	J	K	L
1	I2	I3	I4
2	$=MAX(one_minus_S*X+one_minus_S*delta_X+(S*(F5*(B26/B28)*X+(F2*delta_X))),X,delta_X)$	$=IF(delta>=1,(one_minus_S*X)+(S*F5*M*D_one*X/B25),(one_minus_S*X)+(S*F5*((delta*D_one+1-delta)*(M+(one_minus_M*(1-delta))))))$	$=MAX(IF(delta>=1,X*(one_minus_S+(one_minus_S*D_one*M*delta)+S*((F5*((B26/B28)*((B33/delta)+(D_one*(delta-1)/delta)))*(M+M*((one_minus_M/(1+(delta/2)))))))+(F2*M*D_one*delta)),X*(one_minus_S+(one_minus_S*D_one*M*delta)+S*((F5*((B26/B28)*((delta*B33)+delta_X)*(1-(delta*(1-(M^0.585)))))))+(F2*M*delta*D_one)),I_three,X*delta*M*D_one)$
3	$=IF(I_two=I_one,"new",IF(I_two=delta_X,"old","none"))$	$=IF(I_four=I_three,"new",IF(I_four=(delta_X*M*D_one),"old","none"))$	$=IF(J3="new",IF(K3="new",1,IF(K3="old",2,3)),IF(J3="old",IF(K3="new",4,IF(K3="old",5,6)),IF(K3="new",7,IF(K3="old",8,9))))$
4			
5			
6			
7			
8			4
9			$=IF(L8=4,"*****","")$
10	$=EXP(-interest_rate*J11)$	Case 4	

	J	K	L
11	=IF(L8=3,IF(L_four-l_three>0.2,((L_four*J10)-(L_three*J10))^-0.5,100000000),5)		=IF(L12<L13,1,0)
12	=EXP(-interest_rate*I12)	Ti =	=IF(L3<4,L22,IF(L25=4,R4,IF(L25=3,G26,IF(L25=2,L19,IF(O9=0,R4,IF(R8=1,L19,G26))))))
13	=EXP(-interest_rate*I13)	Te =	=IF(L3<4,L23,IF(L25=4,R5,IF(L25=3,L23,IF(L25=2,G25,IF(O9=0,R5,IF(R8=1,G25,L23))))))
14			
15		Payoffs:	
16		i =	=IF(L3<4,L28,IF(L25=4,P19,IF(L25=3,L28,IF(L25=2,L26,P19))))
17		e =	=IF(L3<4,L29,IF(L25=4,P20,IF(L25=3,L29,IF(L25=2,L27,P20))))
18		<u>i before e</u>	
19		Ti1	=IF(L3>3,(-(I_one*M19)+(I_two*M19))^-0.5,5)

	J	K	L
20		Te1	=IF(G25<L19,L19,G25)
21		e before i	
22		Ti2	=IF(\$G\$26<\$L\$23,L23,G26)
23		Te2	=IF(L8=4,IF(E_one>0,((E_one*M23)^-0.5),100000000),5)
24			
25			=IF(N24=1,1,IF(N19=0,IF(N22=0,4,3),2))
26		i1	=(1/interest_rate)*(l_one-(l_one*M19)+(l_two*M19)-(l_two*H25)+(l_four*H25))-(1/L19)
27		e1	=E_two*H25/interest_rate-(1/G25)
28		i2	=(1/interest_rate)*(l_one-(l_one*M23)+(l_three*M23)-(l_three*H26)+(l_four*H26))-1/G26)
29		e2	=(1/interest_rate)*((E_one*M23)-(E_one*H26)+(E_two*H26))-1/L23
30			
31			
32			
33			

Nine Solution Set Model

	M	N	O
1	E1	E2	=IF(J2<I2,1,0)
2	=IF(delta<1,(one_minus_S*delta_X)+S*((F5*((delta^3)*D_one*one_minus_M*X)+(F2*delta_X)),(one_minus_S*delta_X)+S*((F5*((1-(M/delta))*((D_one/delta)+((delta-1)/delta))*delta_X)+(F2*delta_X))))	=IF(K3="none",IF(delta<1,X*((one_minus_S*one_minus_M*delta*D_one)+S*((F5*(delta*((delta*B33)+1-delta))*delta*(1-(M^0.585))))+(F2*one_minus_M*D_one*delta))),X*((one_minus_S*one_minus_M*delta*D_one)+S*((F5*(delta*((B33/delta)+(D_one*(delta-1)/delta))*1-(M+M*((1-M)/(1+(delta/2)))))+(F2*one_minus_M*D_one*delta))),IF(K3="old",X*delta*one_minus_M*D_one,E_one))	=IF(O1=1,IF(P1=1,1,3),IF(P1=1,2,4))
3			
4			
5		=IF(O2=4,IF(N24=1,IF(O9=0,1,0),0),0)	
6			
7			
8			=IF(P8=1,1,IF(P9=1,1,0))
9	=IF(L8=4,"*****",")		=IF(O8=1,IF(P8=1,R8,R9),0)
10			

	M	N	O
11			
12			
13			E
14			
15			
16			
17			
18			Payoffs:
19	=EXP(-interest_rate*L19)	=IF(L19<G25,1,0)	i =

	M	N	O
	=IF(G25<L19,M19,H25)		e =
20			
21			
22	=IF(\$G\$26<\$L\$23,M23,H26)	=IF(L23<G26,1,0)	
23	=IF(L23>100,2,EXP(- interest_rate*L23))		
24		=N22*N19	
25			
26	=IF(L26>L28,1,2)		
27	=IF(L27>L29,1,0)		
28			
29			
30			
31			
32			
33			

	P	Q
1	=IF(L2<K2,1,0)	=IF(L19<L23,(S19*L19),(S19*L23))
2		=IF(G26<G25,(T18*G26),(T18*G25))
3		=(S18*L19)+(T19*L23)
4		=Q1+Q2+Q3
5		
6		
7		
8	=IF(Q12>S12,IF(Q15>S15,1,0),IF(Q15<S15,1,0))	=IF(P8=1,IF(Q12>S12,"i1","i2"),0)
9	=IF(Q13>Q16,IF(S13>S16,1,0),IF(S13<S16,1,0))	=IF(P9=1,IF((Q13-Q16>0),1,2),0)
10		i

	P	Q
11		i1
12	e1	=L26
13		=L27
14		
15	e2	=IF(L8=4,IF(L19>L23,((1/interest_rate)* (l_one-(l_one*M23)+(l_three*M23)- (l_three*M19)+(l_four*M19))- 1/L19),((1/interest_rate)*(l_one- (l_one*M19)+(l_two*M19)- (l_two*M23)+(l_four*M23))-(1/L19))),"
16		=IF(L8=4,IF(L19>L23,((1/interest_rate)* (E_one*M23)- (E_one*M19)+(E_two*M19))- 1/L23),(E_two*M23/interest_rate- (1/L23))),"
17		
18		
19	=IF(O8=0,S18*Q12+T18*S12+Q15*S19+T19 *S15,IF(O9=1,Q12,IF(O9=2,S12,IF(O9=3,Q1 5,S15))))	

	P	Q
20	=IF(O8=0,S18*Q13+T18*S13+Q16*S19+T19*S16,IF(O9=1,Q13,IF(O9=2,S13,IF(O9=3,Q16,S16))))	
21		Mixed
22		e1
23		e2
24		i1
25		i2
26		
27		
28		
29	=M23	
30	=H26	
31		
32		
33		

	R	S	T
1		$=(l_two/interest_rate)*(EXP(-interest_rate*B8))$	
2		$=IF((S1-B9)>B7,1,0)$	
3			
4	$=IF(O9=0,(L19*R24)+(G26*R25),IF(R8=1,L19,G26))$		
5	$=IF(O9=0,(G25*R22)+(L23*R23),IF(R9=1,G25,L23))$		
6			
7			
8	$=IF(Q8="i1",IF(Q13>Q16,1,3),IF(Q8="i2",IF(S13>S16,2,4),R9))$		
9	$=IF(Q9=1,IF(Q12>S12,1,2),IF(Q9=2,IF(Q15>S15,3,4),R8))$		
10			

Nine Solution Set Model

	R	S	T
11		i2	
12	for I	=IF(L8=4,IF(G26>G25,((1/interest_rate)*(I_one-(I_one*H25)+(I_three*H25)-(I_three*H26)+(I_four*H26))-1/G26),((1/interest_rate)*(I_one-(I_one*H26)+(I_two*H26)-(I_two*H25)+(I_four*H25))-(1/G26))), "") for I	
13	for E	=IF(L8=4,IF(G26>G25,((1/interest_rate)*((E_one*H25)-(E_one*H26)+(E_two*H26))-1/G25),(E_two*H25/interest_rate)-1/G25), "") for E	
14			
15	for I	=L28	for I
16	for E	=L29	for E
17			
18		=R22*R24	=R25*R22
19		=R24*R23	=R25*R23

	R	S	T
20			
21	Strategies		
22	$=(S15-Q15)/(Q12-Q15-S12+S15)$		
	=1-R22		
23			
24	$=(S16-S13)/(Q13-S13-Q16+S16)$		
	=1-R24		
25			
26			
27			
28			
29			
30			
31			
32			
33			

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	A	B	C	D	E	F	G
1	v	s3	s2	δi	d	s1	δe
2	=A143	=1/(1+B6+(A2*(B4+B5-B6)))	=C200	=D200	=E143	=G200	=H200
3	0.5	=1-B2	0.95	2	0.05	0.95	2
4			=1-C2				
5		=(1-S)/S		=D2*X			1-s
6		=(1-C2)/C2					=1-S
7		=ABS(B4-B5)			TRUE		
8				Ti	=O12	=IF(R9=0,O19,"")	
					=O16+M60	=IF(R9=0,U24,"")	
				Te	=O13	=IF(R9=0,O20,"")	

	H	I	J	K	L	M
1	m	r	x	I1	I2	I3
2	0.5 =I200	0.1 =J200	2	=X	=(one_minus_S*X)+(one_minus_S*D4)+(S*X*D26/D28)	=D4
3	=IF(MAX(I_one:I_three)=I_three,I_two,1,IF(MAX(I_one:I_three)=I_three,I_three,2,3))					=IF(MAX(I_four:I_six)=I_five,1,IF(MAX(I_four:I_six)=I_six,2,3))
4	δex	1-m	e^-delta			
5	=delta*X	=1-M	=EXP(-delta)			
6	=IF(R9=0,O22,"")	=IF(O20<O22,1,0)	=IF(E_one=4,IF(#REF!<U5,"I","E"),"E")		I on own	
7	=IF(R9=0,U25,"")	=IF(O23<O19,1,0)	=IF(O25=1,IF(R8=1,IF(R9=1,0%,IF(R9=4,100%,IF(R9=2,(IF(O20<O22,100%,0%))),IF(O23<O19,100%,0%))),IF(I8=1,W18+W19+V19,IF(I8=2,W19+W18,IF(I8=3,W19+V19,W19))),IF(O11=1,0%,100%))		I shelves	=IF(L3=1,"None",IF(L3=2,"Old","New"))
8	=IF(R9=0,O23,"")	=IF(I6*I7=1,1,IF(O20<O22,2,IF(O23<O19,3,4)))				

	N	O
1	14	15
2	$=IF(\text{delta}<1,(C3*X)+(C2*X*((1-\text{delta}+(\text{delta}*D_one))*M+(one_minus_M*(1-\text{delta})))),(C3*X)+(C2*X*M*D_one)/(F25)))$	$=IF(\text{delta}<1,((F11*X)+(F12*X*D26/D28)+(F13*D4)+(F14*X*(D26/D28))*((1-(\text{delta}/D2))+D_one*\text{delta}*(1-(\text{delta}/D2)))+(D33*\text{delta}/D2))*((1-(\text{delta}/D2)*(1-M^0.585))))+(F15*X*((D_one*\text{delta})+1-\text{delta})*(M+one_minus_M*(1-\text{delta})))+(F16*D2*X*((D_one*\text{delta}/D2)+((D2-\text{delta})/D2))*(M+one_minus_M*(1-(\text{delta}/D2))))),IF(\text{delta}<D2,(F11*X)+(F12*X*D26/D28)+(F13*D4)+(F14*X*(D26/D28))*((1-(\text{delta}/D2))+((\text{delta}/D2)*((D_one^(2*D2/\text{delta}))/D2*\text{delta}))+((1-(1/(D2*\text{delta}))*D_one))))*(M+((1-M)*(1-(\text{delta}/D2)+((1/6)^(D2*\text{delta})/(\text{delta}+D2)))))))+(F15*X*((D_one/F25)*(M)))+(F16*D2*X*(D_one*(\text{delta}/D2)+((D2-\text{delta})/\text{delta})*(M+(one_minus_M*(1-(\text{delta}/D2))))),((F11*X)+(F12*X*D26/D28)+(F13*D4)+(F14*X*(D26/D28))*((D_one^(2*D2/\text{delta}))/D2*\text{delta}))+((1-(1/(D2*\text{delta}))*D_one*(D2/\text{delta}))*M*(D2/\text{delta}))+((1/6)^\text{delta})))+(F15*X*((D_one/F25)*(M)))+(F16*X*D2*(D_one^(D2/D2))*(M*D2/\text{delta})))$
3	$=IF(L3=1,IF(M3=1,9,IF(M3=2,8,7)),IF(L3=2,IF(M3=1,6,IF(M3=2,5,4)),IF(M3=1,3,IF(M3=2,2,1))))$	
4		
5		
6	Both	
7	$=IF(M3=1,"None",IF(M3=2,"Old","New"))$	
8	This game is Solution Set	=N3

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	P	Q	R	S
1	16	E1	E2	E3
	$=IF(\delta < D2, (B3 * D4) + (B2 * X * D2 * ((D_one * \delta / D2) + ((D2 - \delta) / D2)) * (M + one_minus_M * (1 - (\delta / D2))))), (B3 * D4) + (B2 * X * (D2 * (D_one^\delta / D2)) * (M * D2 / \delta)))$	$=IF(\delta < 1, ((C3 * \delta / D2) + (C2 * D_one * X * (\delta^3 * one_minus_M))), ((C3 * \delta / D2) + (C2 * \delta * X * (1 - (M / \delta)) * ((D_one / \delta) + ((\delta - 1) / \delta))))))$	$=IF(\delta < 1, (F14 * X * \delta * ((D33 * \delta) / D2) * ((\delta / D2) * (1 - (M^{0.585})))) + (F15 * X * (\delta^3 * D_one * one_minus_M) + (F16 * X * \delta * (D_one * \delta / D2) * (1 - (M + ((1 - M) * (1 - (\delta / D2)))))) + (F17 * \delta * X), IF(\delta < D2, (F14 * X * \delta * (((D_one^\delta * D2 / \delta) / (D2 * \delta)) + ((1 - (1 / (D2 * \delta)))) * (D_one * \delta / D2))) * (M + ((1 - M) * (1 - (\delta / D2) + ((1 / 6) * ((\delta * D2) / (\delta + D2)))))) + (F15 * X * \delta * (1 - (M / \delta)) * ((D_one / \delta) + ((\delta - 1) / \delta))) + (F16 * \delta * X * (D_one^\delta / D2)) * (1 - (M + (one_minus_M * (1 - (\delta / D2)))))) + (F17 * \delta * X), (F14 * X * \delta * ((1 - (D2 / \delta) + (D2 / \delta) * ((D_one^\delta * D2 / D2) / (D2 * \delta)) + (D_one * (1 - (1 / (D2 * \delta)))) * (1 - (M * D2 / \delta) + ((1 / 6)^\delta))) + (F15 * X * \delta * (1 - (M / \delta)) * ((D_one / \delta) + ((\delta - 1) / \delta))) + (F16 * \delta * X * (D_one^\delta / D2)) * (1 - (M + (one_minus_M * (1 - (\delta / D2)))))) + (F17 * \delta * X))$	$=IF(G2 < D2, (B3 * G2 * J2) + (B2 * G2 * X * (E2^\delta / D2)) * (1 - (H2 + (I5 * (1 - (G2 / D2))))), ((B3 * G2 * J2) + (B2 * G2 * X * (E2 * ((D2 / G2) + ((G2 - D2) / G2)) * (1 - (H2 * ((D2 / G2)))))))$
2				
3				
4				
5		=IF(R9=0,1,0)		
6				
7				
8			=IF(S8=1,1,IF(S9=1,1,0))	=IF(T12>V12,IF(T15>V15,1,0),IF(T15<V15,1,0))

	T	U	V	W
1			$=(l_two/interest_rate)*(EXP(-interest_rate*F8))$	
2			$=IF((V1-F9)>F7,1,0)$	
3		$=(V18*O19)+(W19*O23)$		
4	$=(V18*MIN(O19,O20))+(V19*MIN(O19,O23))+(W18*MIN(O22,O20))+(W19*MIN(O20,O23))$	$=IF(R9=0,(O19*U24)+(O22*U25),IF(T8="i1",O19,O22))$		
5		$=IF(R9=0,(O20*U22)+(O23*U23),IF(T9=1,O20,O23))$		
6				
7				
8	$=IF(S8=1,IF(T12>V12,"i1","i2"),0)$	$=IF(T8="i1",IF(T13>T16,1,3),IF(T8="i2",IF(V13>V16,2,4),U9))$		

	A	B	C	D	E	F	G
9						=O17+M65	=IF(R9=0,U22,"")
10							
11				=1-D12-D14 D15	A		=D11/\$DS18
12				=C^2-D14	B		=D12/\$DS18
13				=1-D12-D14 D16	C		=D13/\$DS18
14				=B^2-C^2*S	D		=D14/\$DS18
15				=S^2-D14	E		=D15/\$DS18
16				=B^2-D14	F		=D16/\$DS18
17				=1-D15-D14 D16	G		=D17/\$DS18
18				=SUM(D11: D17)			
19							

	H	I	J	K	L	M
9	=IF(R9=0,U23,"")		=IF(J7=0,F6,IF(J7=100%,F8,T4))			
10						
11						
12				=OR(O8=7,O8=8,O8=9)		
13				=OR(O8=6,O8=5,O8=4)	=IF(K15="A",I_two,IF(K15="B",I_three,0))	
14				=OR(O8=1,O8=2,O8=3)	=IF(K23="A",E_two,IF(K23="B",E_three,E_one))	
15	I must pay			=IF(K12=TRUE,"A",IF(K13=TRUE,"B","C"))		
16						
17	to buy out E			=IF(L3=1,I_two,IF(L3=2,I_three,I_one))		
18				=IF(M3=1,I_five,IF(M3=2,I_six,I_four))	=IF(M3=1,E_two,IF(M3=2,E_three,E_one))	
19	Max signal					

	N	O
9		
10		
11		=IF(O12<O13,1,0)
Ti =		=IF(O25=4," ",IF(O25=3,O22,IF(O25=2,O19,U4)))
12		
Te =		=IF(O25=4," ",IF(O25=3,O23,IF(O25=2,O20,U5)))
13		
14		
Payoffs:		
15		
i =		=IF(O25=4," ",IF(O25=3,O28,IF(O25=2,O26,S19)))
16		
e =		=IF(O25=4," ",IF(O25=3,O29,IF(O25=2,O27,S20)))
17		
<u>i before e</u>		
18		
Ti1		=IF(K15="C",100000000,IF(L13>I_one,(-(I_one*P19)+(L13*P19))^-0.5,100000000))
19		

	P	Q	R	S
9	=IF(R8=1,IF(S8=1,U8,U9),0)			=IF(T13>T16,IF(V13>V16,1,0),IF(V13<V16,1,0))
10	E			
11				
12				e1
13				
14				
15				
16				
17				
18	Payoffs:			
19	=EXP(-interest_rate*O19	=IF(O19<O20,1,0)	i =	=IF(R8=0,V18*T12+W18*V12+T15*V19+W19*V15,IF(R9=1,T12,IF(R9=2,V12,IF(R9=3,T15,V15))))

	T	U	V	W
9	=IF(S9=1,IF((T13-T16>0),1,2),0)			
10	=IF(T9=1,IF(T12>V12,1,2),IF(T9=2,IF(T15>V15,3,4),U8))			
11	i1		i2	
12	=O26	for I		
13	=O27	for E		
14				
15	=IF(O19>O23,((1/interest_rate)*(l_one-(l_one*P23)+(l_four*P23)-(l_four*P19)+(K18*P19))-1/O19),((1/interest_rate)*(l_one-(l_one*P19)+(K17*P19)-(K17*P23)+(K18*P23))-(1/O19)))	for I	=O28	for I
16	=IF(O19>O23,((1/interest_rate)*(E_one*P23)-(E_one*P19)+(L18*P19))-1/O23,(L18*P23/interest_rate-(1/O23)))	for E	=O29	for E
17				
18			=IF(R9=0,U22*U24,"")	=IF(R9=0,U25*U22,"")
19			=IF(R9=0,U24*U23,"")	=IF(R9=0,U25*U23,"")

Dual innovation model

	A	B	C	D	E	F	G
20							
21							
22							
23							
24			delta i		delta e		
25			delta^2	=D2^2	delta^2		=delta^2
26			1+delta^2	=1+D25	1+delta^2		=1+F25
27			(1+delta)^2	=(1+D2)^2	(1+delta)^2		=F28^2
28			1+delta	=1+D2	1+delta		=1+delta
29			delta^2 - 1	=D2^2-1	delta^2 - 1		=F25-1
30			1-(m/delta^2)	=1-(M/D25)	1-(m/delta^2)		=1-(M/delta^2)
31			delta^3	=D2^3	delta^3		=delta^3
32			delta^4	=D2^4	delta^4		=delta^4
33			d^2	=D_one^2	d^2		=D_one^2
34			m^0.585	=M^0.585	m^0.585		=M^0.585
35			1-m^0.585	=1-D34	1-m^0.585		=1-F34
36			0.5*delta^2	=0.5*D25	0.5*delta^2		=0.5*F25

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	N	O
20	Te1	=IF(K15="C","",IF(L14=0,100000000,((L14*P20)^-0.5)))
21	e before i	
22	Ti2	=IF(I_four<L21,((-I_four+L21)*P22)^-0.5,10000000)
23	Te2	=((E_one*P23)^-0.5)
24		
25		=IF(Q24=1,1,IF(Q19=0,IF(Q22=0,4,3),2))
26	i1	=((1/interest_rate)*(I_one-(I_one*P19)+(K17*P19)-(K17*P20)+(K18*P20))- (1/O19))
27	e1	=L18*P20/interest_rate-(1/O20)
28	i2	=((1/interest_rate)*(I_one-(I_one*P23)+(I_four*P23)- (I_four*P22)+(K18*P22))-1/O22)
29	e2	=(1/interest_rate)*((E_one*P23)-(E_one*P22)+(L18*P22))-1/O23
30		
31		
32		
33		
34		
35		
36		

	P	Q	R	S
20	=EXP(- interest_rate*O20)		e =	=IF(R8=0,V18*T13+W18 *V13+T16*V19+W19*V1 6,IF(R9=1,T13,IF(R9=2, V13,IF(R9=3,T16,V16))))
21	=EXP(- interest_rate*O22	=IF(O23<O22,1,0)		
22	=EXP(- interest_rate*O23)			
23				
24		=Q22*Q19		
25				
26	=IF(O26>O28,1,2)			
27	=IF(O27>O29,1,0)			
28				
29				
30				=O25
31				
32				
33				
34				
35				
36				

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	T	U	V	W
20				
21	Mixed	Strategies		
22	e1		=IF(R9=0,(V15-T15)/(T12-T15-V12+V15),"")	
23	e2		=IF(R9=0,1-U22,"")	
24	i1		=IF(R9=0,(V16-V13)/(T13-V13)	
25	i2		=IF(R9=0,1-U24,"")	
26				
27				
28				
29				
30				
31				
32				
33				
34				
35				
36				

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