

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.



Theory-Consistent Formal Risk Measures: Using Financial Market Data from a Middle Income Context

Dissertation presented in partial fulfilment of the requirements for
the M. Com (Economics)

By: Neryvia Pillay
PLLNER002

Supervisor: Prof. Johannes Fedderke

Date: 4 July 2007

Word count: 5378 words

Declaration

1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretend that it is one's own.
2. I have used the Harvard convention for citation and referencing. Each contribution to, and quotation in, this dissertation from the work(s) of other people has been attributed, and has been cited and referenced.
3. This dissertation is my own work.
4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
5. I acknowledge that copying someone else's assignment or essay, or part of it, is wrong, and declare that this is my own work.

Signature

Signed by candidate

Abstract

While economic theory assumes that risk is of central importance in financial decision making, it is difficult to measure the uncertainty faced by investors. Commonly used proxies for risk (such as the moving standard deviation of an asset) are not firmly grounded in economic theory. Risk measures have been developed by other studies, but these are generally based on subjective weights attaching to a range of objective component indicators and are difficult to replicate.

The contribution of this paper is to develop a methodology to construct risk measures. It has the advantages of being explicitly grounded in economic theory and easily replicable. We illustrate this methodology by specific application to the South African context.

The time-varying risk measure developed in this paper is consistent with the expectations hypothesis and captures the asymmetric nature of shocks. This measure reflects investors' risk perceptions and accords with the literature on risk in South Africa.

Table of Contents

Abstract.....	i
Table of Contents.....	ii
List of Illustrations.....	iii
1. Introduction	1
2. The Expectations Hypothesis	4
3. Data	6
4. Evidence of a Time-Varying Risk Premium	8
4.1 Empirical Methodology for Testing the Expectations Hypothesis.....	8
4.2 Cointegration Analysis: Full Sample Period	10
4.3 Cointegration Analysis: Rolling Sample Period.....	12
4.4 Evidence of a Time-Varying Risk Premium	14
5. Constructing a Theoretically Consistent Risk Measure	18
6. Conclusion	25
References	26
Appendix A: Impulse Response Functions	30

List of Illustrations

Figures

1. Yields to Maturity (Levels)	6
2. Yields to Maturity (First Differences)	7
3. Number of Cointegrating Vectors	13
4. Tests that Homogeneity Restrictions Hold Jointly	13
5. Tests that Homogeneity Restrictions Hold for the Spread Between $R(36, t)$ and $R(3, t)$	13
6. Tests that Homogeneity Restrictions Hold for the Spread Between $R(120p, t)$ and $R(3, t)$	13
7. News Slope Curve for $y(36, t)$	17
8. News Slope Curve for $y(120p, t)$	17
9. Risk Measure Component from First CV	19
10. Risk Measure Component from Second CV	19
11. Final Risk Measure	20
12. Comparison of Risk Measures	22

Tables

1. Unit Root Tests for Yields	8
2. Test Statistics for Number of Cointegrating Vectors	10
3. Estimates of Restricted Cointegrating Relations	11
4. Tests that Homogeneity Restrictions Hold	11
5. EGARCH-M Models for $y(36, t)$ and $y(120p, t)$	15
6. Engle and Ng Test for $y(36, t)$ and $y(120p, t)$	16
7. Unit Root Tests for Risk Measure	23
8. Test Statistics for Number of Cointegrating Vectors	23
9. Tests that Homogeneity Restrictions Hold	24

1. Introduction

Risk is defined as the uncertainty of future rates of return, and is of central importance in financial decision making. It is generally assumed that investors are risk averse so that the riskier an asset, the greater must its return be to induce risk averse investors to hold the asset. The implication is that investors face a risk-return trade-off when allocating their funds.

While economic theory assumes that risk is important, unfortunately it is difficult to construct a measure that appropriately captures the risks faced by investors. The contribution of this paper is to develop a methodology to construct a theoretically grounded measure of risk. We illustrate this methodology by specific application to the South African context.¹

The methodology developed here constructs a risk measure that is consistent with the expectations hypothesis of the term structure of interest rates. The framework of the expectations hypothesis enables us to extract the risk implied by the returns on government bonds. Since government bonds are free of default risk, and their prices are determined in the bond market, yields on government bonds can be assumed to reflect the pure country risk perceived by investors.

The expectations hypothesis is the simplest and most commonly used theory for explaining the term structure of interest rates.² The expectations hypothesis holds that long-term rates are related to expectations of future short-term rates. Thus, changes in the shape of the term structure reflect a changed outlook for future interest rates relative to current rates.

¹ Fedderke et al. (2001) develop a risk measure based on political instability for South Africa, which is shown to have an impact on the real sector through the investment channel in Kularatne (2002), Mariotti (2002) and Fedderke (2004).

² There are two main alternative theories: the liquidity preference hypothesis developed by Hicks (1946) and the market segmentation hypothesis of Culbertson (1957). The liquidity preference hypothesis also involves the expected values of future spot rates, but emphasises the risk preferences of agents. The risk premium is assumed to increase with the time to maturity of the bond. Under the market segmentation hypothesis, agents have strong preferences for specific maturities. Thus, bond prices are determined in separate markets, and bonds of neighbouring maturities are not substitutes.

The expectations hypothesis has been widely studied for US data. One implication of the expectations hypothesis is that changes in the yield curve directly reflect changes in expected future rates. In general, the empirical evidence does not support this prediction and thus points to a rejection of the expectations hypothesis (see, for example, Shiller (1979), Shiller, Campbell and Schoenholtz (1983), Mankiw and Summers (1984), Mankiw (1986), and Campbell and Shiller (1991)).

In their seminal paper, Engle and Granger (1987) employ cointegration techniques to test the expectations hypothesis. The expectations hypothesis posits that yields of different maturities will move systematically together over time, which implies that they may be cointegrated.

Engle and Granger (1987) and Campbell and Shiller (1987) find that the one-month US Treasury bill rate is cointegrated with the yield to maturity on the 20-year US Treasury bond. These studies use a single equation Engle-Granger approach to cointegration. Hall et al. (1992) was the first study to suggest that the term structure might be well modelled as a cointegrating system. They employ twelve yield series and find that while some yields are cointegrated, other yields are not – a finding inconsistent with the expectations hypothesis. In addition, they find that the expectations hypothesis holds for some sub-periods of their sample, but not for others; a further contradiction of the expectations hypothesis.

The evidence on the expectations hypothesis is therefore mixed. One possibility for the failure of the expectations hypothesis is the presence of a time-varying risk premium. However, studies that have attempted to incorporate a time-varying risk premium using various proxies have also generated mixed results. Many of these studies still do not find support for the expectations hypothesis – for example Shiller, Campbell and Schoenholtz (1983), Froot (1989) and Simon (1989); while some of the studies do find support for the expectations hypothesis, such as Tzavalis and Wickens (1997).³ Harris (2001) uses panel data techniques with the risk premium

³ The variables used to capture the risk premium include a moving standard deviation of short term rates, a variable representing the relative volumes issued of different maturities (Shiller, Campbell and Schoenholtz, 1983), survey data on interest rate expectations (Froot, 1989), a risk premium that is specified to be proportional to the volatility of excess returns using instrumental variables (Simon,

captured by fixed effects. This approach has the advantage of allowing a risk premium that varies with maturity and over time without the need for a proxy that may not be based on economic theory, but the study still finds that the expectations hypothesis is rejected.

The development of fully specified general equilibrium models of the term structure, such as affine-yield models (which have the property that log bond yields are linear in the state variables), may be viewed as another response to the inadequacies of the expectations hypothesis.

In this study we allow for a time-varying risk premium that also captures the asymmetric nature of shocks. Our approach does not use a proxy for the time-varying risk premium, but instead derives a risk premium that is explicitly consistent with the expectations hypothesis. Since the risk premium is derived from the expectations hypothesis, it cannot be incorporated in a test of the expectations hypothesis. However, it does provide a clear indication of the structure the time-varying risk premium would have to exhibit for the expectations hypothesis to hold. This allows us to compare the structure of standard proxies of the time-varying risk premium to the structure that is required for the expectations hypothesis to hold, to gain an indication of how close standard proxies have been to the time-varying risk premium that is consistent with the expectations hypothesis. It also provides an explicit, theoretically grounded measure of risk.

The remainder of the paper is organised as follows. Section 2 outlines the theoretical framework associated with the expectations hypothesis. Section 3 examines the data used in the empirical analysis and establishes the univariate characteristics of the data. Section 4 examines the empirical evidence on the central predictions of the expectations hypothesis. The empirical analysis points to a rejection of the expectations hypothesis, which we show to be attributable to a time-varying risk premium. Section 5 then constructs a time-varying risk measure that is consistent with the expectations hypothesis (modified to allow a time-varying risk premium) and with the literature on risk in South Africa. Section 6 concludes.

1989), and the ex post excess holding period return of one maturity as a proxy for the term premium of another maturity (Tzavalis and Wickens, 1997).

2. The Expectations Hypothesis

Define $R(k, t)$ as the yield to maturity of a k period zero coupon bond at time t , and $F(1, t + j)$ as the forward rate contracted at time t to buy a one period zero coupon bond in $t + j$ which matures at the end of that period. Note that $R(1, t + j)$ denotes the actual rate or yield and it is not necessarily equal to the one period forward rate, $F(1, t + j)$ for $j > 0$. It must be the case that $F(1, t) = R(1, t)$.

An efficient market will arbitrage away any expected profit between the alternatives of either (1) buying a k period bond at time t , or (2) buying a sequence of one period bonds. This yields the following relationship:

$$R(k, t) = \frac{1}{k} \left[\sum_{i=1}^k F(1, t + (i-1)) \right] \quad (1)$$

Although the forward rate typically differs from the corresponding actual rate, the forward rate is likely to be related to the expected actual rate. In particular, we assume:

$$F(1, t + j) = E_t[R(1, t + j)] + \tau(1, t + j) \quad (2)$$

where E_t denotes expectations based on the information available at time t , and $\tau(1, t + j)$ denotes the risk or term premium. This premium could arise, for example, because there is uncertainty about future interest rates, generating risks on forward contracts. In the “pure expectations hypothesis” the term premia are zero, while in the “expectations hypothesis” they are nonzero but constant.

Substituting (2) into (1) yields:

$$R(k, t) = \frac{1}{k} \left[\sum_{i=1}^k E_t[R(1, t + (i-1))] \right] + L(k, t) \quad (3)$$

$$\text{where } L(k, t) = \frac{1}{k} \left[\sum_{i=1}^k \tau(1, t + i) \right] \quad (4)$$

Equation (3) is considered the fundamental relationship of the expectations hypothesis. It states that the yield on a k period bond is a weighted average, with equal weights summing to one, of the current and expected future yields on a

sequence of one period bonds. However, equation (3) cannot be directly estimated since none of the right-hand-side variables are directly observable.

Provided that yields are $I(1)$, it is straightforward to show that under the expectations hypothesis the yields are cointegrated. Subtracting $R(1, t)$ from both sides of equation (3) gives:

$$[R(k, t) - R(1, t)] = \sum_{i=1}^{k-1} \left(\frac{k-i}{k} \right) E_t [\Delta R(1, t+i)] + L(k, t) \quad (5)$$

$$\text{where } \Delta R(1, t+i) = R(1, t+i) - R(1, t+(i-1)) \quad (6)$$

Equation (5) implies the yields on a long- and short-maturity bond differ by the liquidity premium, and by expected changes in the short rate over the horizon of the bond.

If $R(1, t+i)$ is a $I(1)$ process, then $\Delta R(1, t+i)$ is stationary. Provided the premia $L(k, t)$ are stationary, the right hand side of equation (5) is stationary.

Recall that $L(k, t) = \frac{1}{k} \left[\sum_{i=1}^k \tau(1, t+i) \right]$, where $\tau(1, t+j)$ denotes the risk or term premium associated with the forward rate. Under the expectations hypothesis, these risk premia are assumed to be non-zero but constant, rendering the $L(k, t)$ stationary.

Hence, the left-hand-side of equation (5) must be stationary and, under the expectations hypothesis, each yield $R(k, t)$ is cointegrated with $R(1, t)$. We define the spread between the yields $R(k, t)$ and $R(1, t)$ as $S(k, 1, t) = R(k, t) - R(1, t)$. It follows that the spread, $S(k, 1, t)$, is the stationary linear combination that results from the cointegration of $R(k, t)$ and $R(1, t)$.

Note that the spread vector associated with any two yields (i.e. $S(k, j, t)$) is just a linear combination of two of the spread vectors defined using the one period yield (i.e. $S(k, 1, t)$ and $S(j, 1, t)$). Since linear combinations of stationary variables are also stationary it follows that any spread vector is cointegrated.

3. Data

The analysis is conducted on nominal yield data obtained from the South African Reserve Bank. We use the yields to maturity on: the three month Treasury bill, denoted $R(3, t)$; zero to three year government bonds, denoted $R(36, t)$; and ten year and over government bonds, denoted $R(120p, t)$.

The sample consists of 304 monthly observations from January 1981 to April 2006. Figure 1 plots the yield, and Figure 2 the differenced yield data.

Figure 1: Yields to Maturity (Levels)

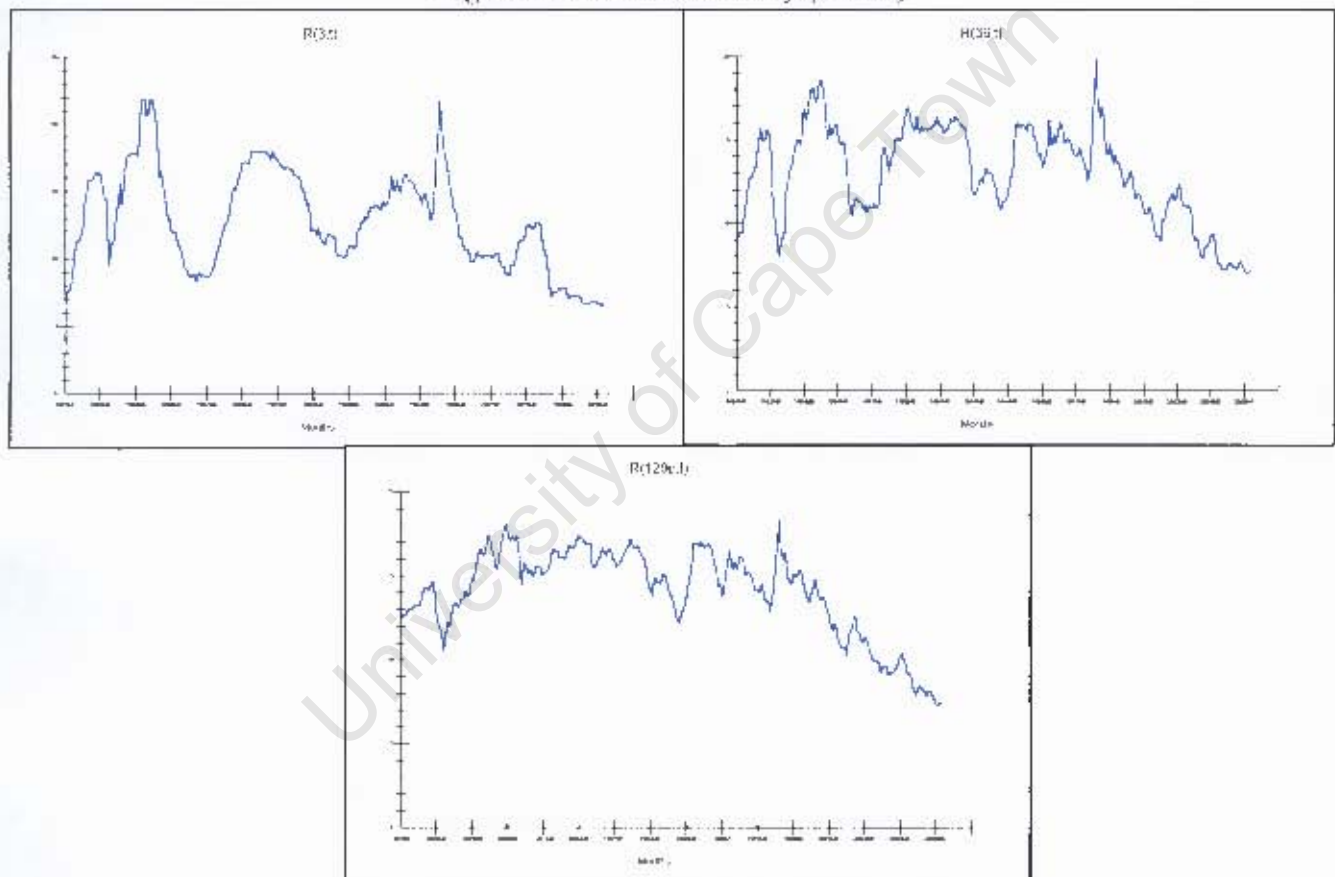


Figure 2: Yields to Maturity (First Differences)

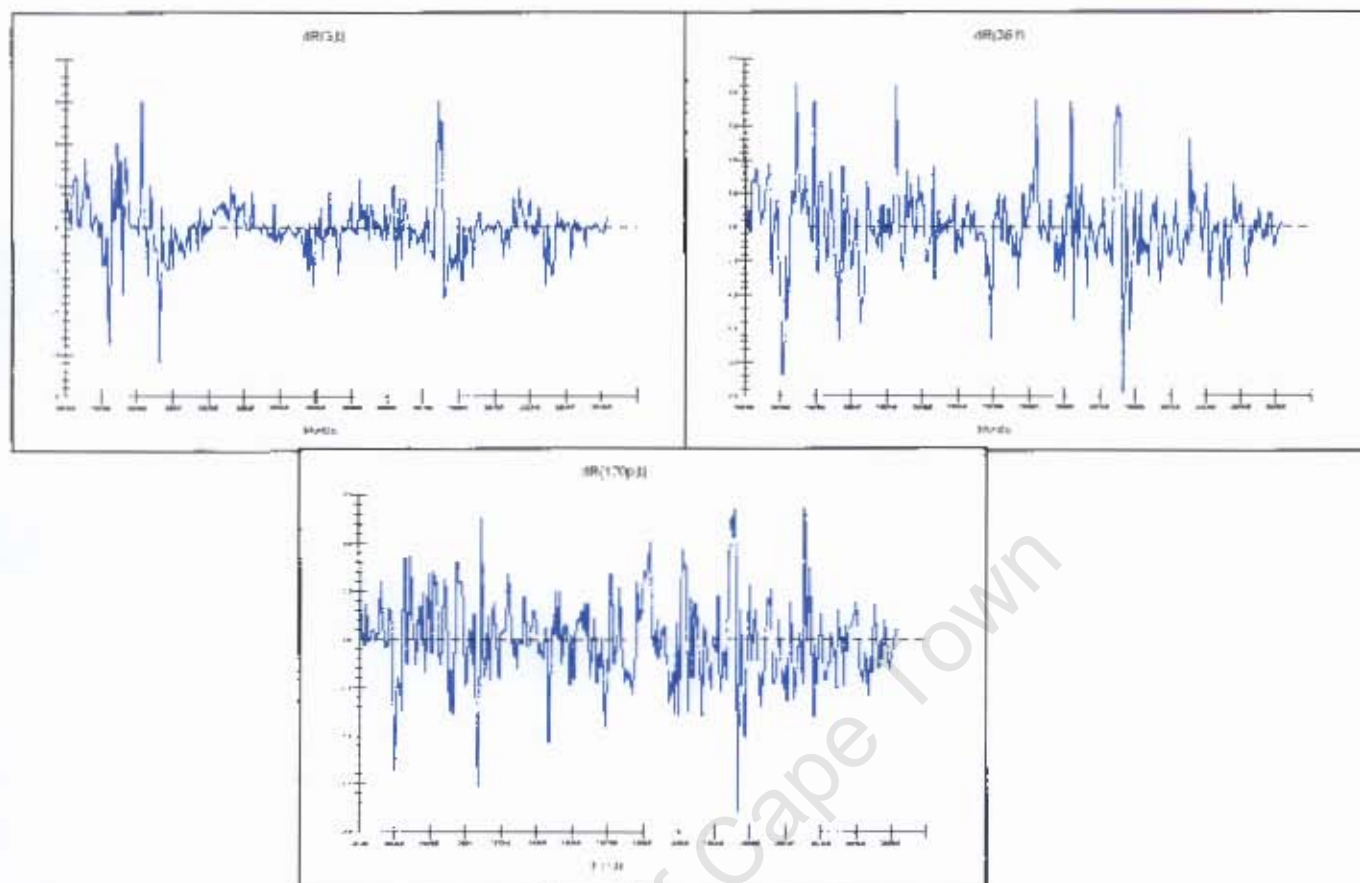


Table 1 reports the relevant ADF tests for stationarity.⁴ Results confirm that the $R(3, t)$, $R(36, t)$ and $R(120p, t)$ series are $I(1)$ processes, with an absence of trend in all series, and drift present only in the $R(120p, t)$ series.

⁴ See Dickey and Fuller (1979, 1981).

Table 1: Unit Root Tests for Yields

Test statistics for 1 unit root						
Variable	Model	τ_τ	Φ_3	τ_μ	Φ_1	τ
$R(3, t)$	ADF(2)	-3.323	6.078	-2.482	3.081	-0.715
$R(36, t)$	ADF(1)	-3.338	6.139	-2.551	3.265	-0.675
$R(120p, t)$	ADF(1)	-2.658	4.483	-1.456	6.085*	-0.655
Test statistics for 2 unit roots						
Variable	Model	τ_τ	Φ_3	τ_μ	Φ_1	τ
$R(3, t)$	ADF(1)	-9.204*	42.360*	-9.144*	41.810*	-9.160*
$R(36, t)$	ADF(1)	-10.572*	55.899*	-10.505*	55.178*	-10.521*
$R(120p, t)$	ADF(1)	-11.232*	63.085*	-11.103*	61.636*	-11.109*
5% Critical values for T=300						
		τ_τ	Φ_3	τ_μ	Φ_1	τ
		-3.4267	6.340	-2.8716	4.630	-1.945

Note: * significant at 5% level,

τ : no constant or trend, τ_μ : constant, τ_τ constant and trend in regression.

4. Evidence of a Time-Varying Risk Premium

The empirical evidence of this section explores the results that emerge from the cointegration tests outlined in the theory section above. The evidence points to a rejection of the expectations hypothesis. In addition, a rolling sample period analysis is conducted to determine whether the expectations hypothesis breaks down over certain sub-periods of the sample. GARCH-M modelling is used to determine whether the failure of the expectations hypothesis is attributable to a time-varying risk premium.

4.1 Empirical Methodology for Testing the Expectations Hypothesis

We employ the standard Johansen technique for multivariate cointegration.⁵ Consider the general k -dimensional VAR specification given by:

$$Z_t = A_1 Z_{t-1} + \dots + A_m Z_{t-m} + \delta + u_t \quad (7)$$

where m is the lag length, δ deterministic terms and u_t a Gaussian error term.

⁵ See Johansen (1988) and Johansen and Juselius (1990).

Reparametrization provides the general VECM specification:

$$\Delta Z_t = \sum_{i=1}^{j-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-j+1} + u_t \quad (8)$$

The existence of r cointegrating relationships amounts to the hypothesis that:

$$H_1(r) : \Pi = \alpha\beta' \quad (9)$$

where Π is $p \times p$ and α, β are $p \times r$ matrices of full rank. Therefore, $H_1(r)$ is the hypothesis of reduced rank of Π . Where $r > 1$ issues of identification arise,⁶ resolved by means of restrictions on the loading matrix (α), the matrix representing the short-run dynamics (Γ), or on the cointegrating space (β).⁷

The expectations hypothesis predicts that any spread vector is cointegrated. If we considered a set of n yields, then we would expect $(n - 1)$ cointegrating relationships since the spread vectors are linearly independent.⁸ Since for the present study we are using three yields we expect to find two cointegrating vectors, i.e. we expect the Π matrix to have a rank of two.

The previous discussion suggests very specific cointegrating relationships (where the spreads are defined in terms of $R(3, t)$):

$$\Pi Z_{t-j+1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \beta' \begin{pmatrix} R(3, t-j+1) \\ R(36, t-j+1) \\ R(120p, t-j+1) \end{pmatrix} \quad (10)$$

$$\text{where } \beta' = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad (11)$$

The above specification is overidentified, since we require only four restrictions to exactly identify the model.

⁶ See Wickens (1996), Johansen and Juselius (1990, 1992), Pesaran and Shin (1995a, 1995b), and Pesaran, Shin and Smith (1996).

⁷ See the discussion in Greenslade et al. (1999).

⁸ While it may be relevant to perform unit root tests on the spreads to test the expectations hypothesis, Hall et al. (1992) note that a unit root test on a spread tests the null hypothesis that the spread vector is not cointegrating instead of the required null that the spread vector is cointegrating. In addition, if we have more than two yields, there will be more than one spread vector. Unit root tests are performed on individual spread vectors and do not test the joint hypothesis that all the spread vectors are cointegrating.

Should we not find the required number of cointegrating vectors or the cointegrating relationships predicted by the theory, this indicates a rejection of the expectations hypothesis. The failure of the expectations hypothesis may suggest that an alternative theory is more appropriate for the term structure (see the discussion in the introduction). Another possibility is the presence of a time-varying risk premium, which is the one explored in this paper.

4.2 Cointegration Analysis: Full Sample Period

Estimation proceeds by means of the VECM technique with a VAR of order two.⁹ Based on the results from Table 1, we proceed with unrestricted intercepts and no trends in the VAR. The λ_{max} and λ_{trace} test statistics for the number of cointegrating vectors (r) are reported in Table 2.

Table 2: Test Statistics for Number of Cointegrating Vectors

Cointegration with unrestricted intercepts and no trends in the VAR		
302 observations from 1981M3 to 2006M4 . Order of VAR = 2.		
List of eigenvalues: .095127 .029317 .0042314		
Null	λ_{max}	λ_{trace}
$r = 0$	30.1881*	40.4547*
$r \leq 1$	8.9860	10.2665
$r \leq 2$	1.2806	1.2806

Note: * significant at 5% level

The λ_{max} and λ_{trace} test statistics indicate that we can reject the null hypothesis that $r = 0$ at the 5% level. However, we cannot reject the null hypothesis that $r \leq 1$ with either of the test statistics. These tests indicate that we have one cointegrating vector in the data, contradicting the first theoretical prediction of the expectations hypothesis theory of the term structure.

However, given the small sample properties of the cointegration tests, and the strong theoretical prior, we proceed with VECM estimation on the assumption of two cointegrating vectors.

⁹ Tested for VAR lag structure and test statistics indicate VAR of order two.

We impose the following exactly identifying restrictions on the cointegrating vectors: the coefficients on $R(3, t)$ are restricted to one, and the coefficients on $R(120p, t)$ and $R(36, t)$ are zero restricted in the first and second vectors respectively. The estimates of these restricted cointegrating relations are reported in Table 3.¹⁰

Table 3: Estimates of Restricted Cointegrating Relations

Cointegration with unrestricted intercepts and no trends in the VAR		
302 observations from 1981M3 to 2006M4 . Order of VAR = 2, r = 2.		
	Vector 1	Vector 2
$R(3, t)$	-1.0000 (*none*)	-1.0000 (*none*)
$R(36, t)$	0.8653 (0.31876)	0.0000 (*none*)
$R(120p, t)$	0.0000 (*none*)	0.7012 (0.40581)

Note: standard errors in parentheses

Results for the tests on the overidentifying restrictions of the expectations hypothesis are reported in Table 4.

Table 4: Tests that Homogeneity Restrictions Hold

302 observations from 1981M3 to 2006M4 . Order of VAR = 2, r = 2.		
Null Hypothesis	Restriction	Test statistic
Homogeneity restrictions on $S(36,3,t)$ and $S(120p,3,t)$ hold jointly	$\beta_{21} = \beta_{32} = 1$	2.3149
Homogeneity restrictions on $S(36,3,t)$ hold	$\beta_{21} = 1$	0.2190
Homogeneity restrictions on $S(120p,3,t)$ hold	$\beta_{32} = 1$	0.5567

Note: * significant at 5% level

The first hypothesis test in Table 4 examines whether the homogeneity restrictions hold jointly. The test statistic indicates that we cannot reject the overidentifying restrictions imposed by the expectations hypothesis. The remaining two tests are conducted to determine whether the homogeneity restrictions hold separately for each cointegrating vector. Again the test statistics for the tests that $\beta_{21} = 1$ and $\beta_{32} = 1$

¹⁰Impulse response functions (see Appendix A) confirm stability of the cointegrating vectors.

indicate that we cannot reject the null hypotheses that the overidentifying restrictions hold separately.

These results indicate mixed evidence for the expectations hypothesis. The first central prediction of the expectations hypothesis is that there will be two cointegrating vectors present in the data. However, this prediction is not supported by the λ_{max} and λ_{trace} test statistics. If, in accordance with the theory, we impose two cointegrating vectors then we cannot reject the overidentifying restrictions implied by the expectations hypothesis, providing empirical support for the second central prediction of the expectations hypothesis.

We find empirical support for only one of the two central predictions of the expectations hypothesis. Consequently, we cannot conclude that the expectations hypothesis holds for South Africa over our sample period.

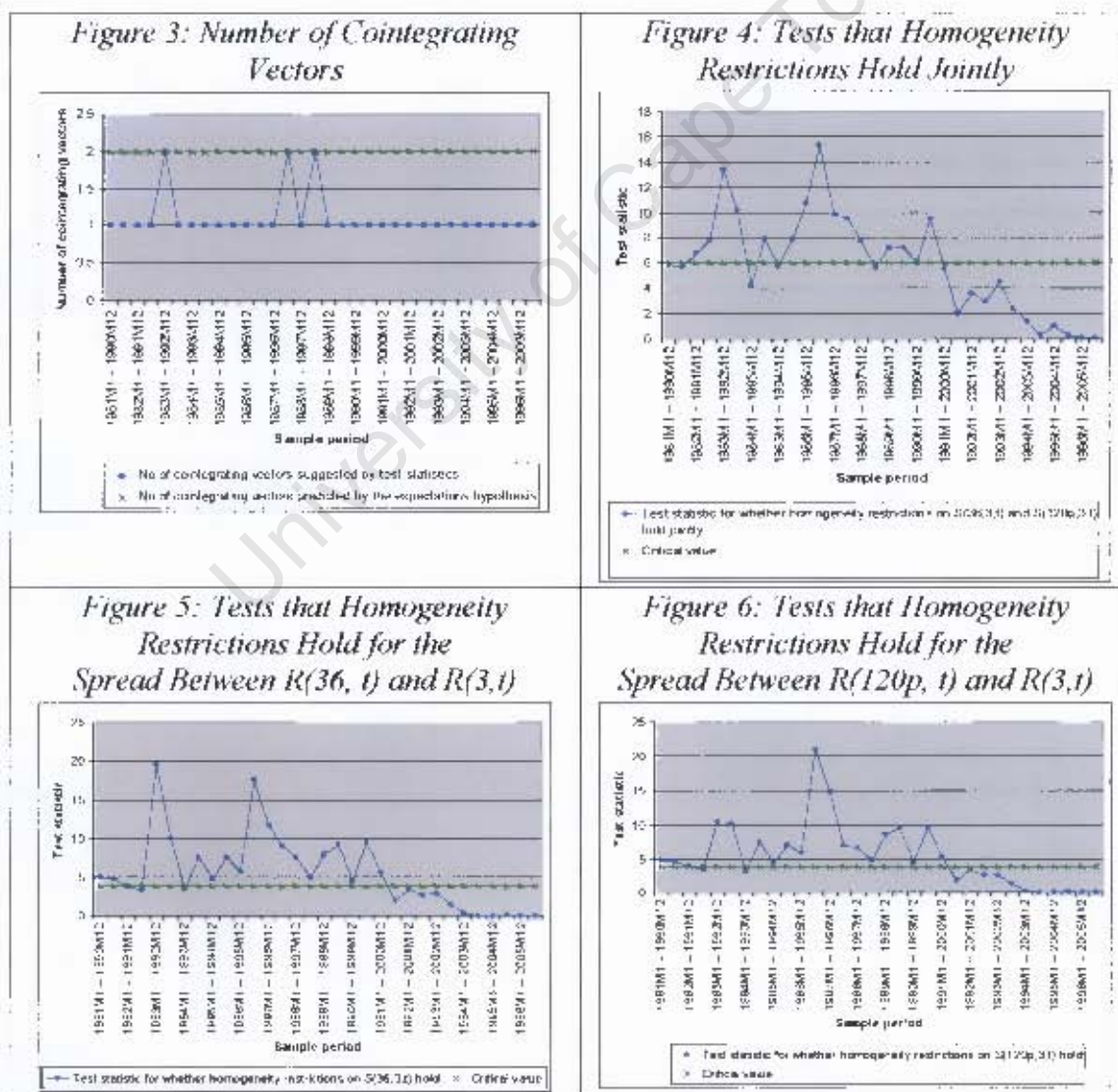
4.3 Cointegration Analysis: Rolling Sample Period

One possibility is that the expectations hypothesis may have broken down over some of the sub-periods covered by our sample. The relatively high political instability faced by South Africa during the 1980s (see Fedderke et al. (2001)) renders this possibility plausible. Consequently, we now investigate whether the expectations hypothesis holds for sub-periods of our full sample period. As before, estimation proceeds by means of the VECM technique with a VAR of order two with unrestricted intercepts and no trends. We roll a ten-year sample period forward by six months at a time.

Figure 3 shows the results from the λ_{max} and λ_{trace} test statistics for the number of cointegrating vectors. As can be seen, in general, we still find only one cointegrating vector, and not the two predicted by the expectations hypothesis, for most sub-periods of the study. Specifically, we find only three sub-periods (1983M1 – 1992M12, 1987M7 – 1997M6 and 1988M7 – 1998M6) with the required two cointegrating vectors.

Figure 4 plots the test statistics of the test for whether the homogeneity restrictions hold jointly (conditional on there being two cointegrating vectors). Figure 5 plots the test statistics to determine whether the homogeneity restrictions hold for the spread between $R(36, t)$ and $R(3, t)$, and Figure 6 plots the test statistics to determine whether the homogeneity restrictions hold for the spread between $R(120p, t)$ and $R(3, t)$.

In these three figures, the critical value is indicated by the crossed line, while the test statistic is indicated by the dotted line. Where the test statistic is greater than the critical value we can reject the null that the homogeneity restrictions hold. The figures show that we can reject (both jointly and for each cointegrating vector) the overidentifying restrictions imposed by the expectations hypothesis over most of the eighties while we cannot reject them for most of the nineties.



These results also fail to support the first central prediction of the expectations hypothesis, with the tests generally indicating that fewer than the required number of cointegrating vectors is present in the data. Conditional on there being two cointegrating vectors, the homogeneity restrictions implied by the expectations hypothesis are now also rejected over the eighties, though they continue to hold over the nineties. In effect, support for the second central prediction of the expectations hypothesis is weakened relative to the full sample tests, since we find that the homogeneity restrictions do not hold for the first ten years of the sample period whereas previously full sample tests confirmed the homogeneity restrictions.

4.4 Evidence of a Time-Varying Risk Premium

A second possibility for the failure of the expectations hypothesis is that the risk premium is time-varying. The expectations hypothesis assumes that the risk premia are non-zero but constant, which yields the prediction that the spread between any two yields must be cointegrated (see equation (5)). The theoretical assumption that the risk premium is constant renders it stationary, which is crucial to the cointegration testing procedure carried out above. If the risk premium is time-varying, the testing procedure may become invalid and we cannot draw inferences about the expectations hypothesis from the results.

Since our expectation is that the excess holding yield is a positive function of its own risk, proxied here by its conditional variance, the appropriate model is GARCH-M. Given the presence of asymmetries in the GARCH-M estimation, we proceed by the EGARCH-M formulation of the estimation procedure (Nelson, 1991).

Following Engle, Lilien and Robins (1987) we define the one quarter excess holding yield on the longer bonds relative to the three month T-bill as:

$$y(i, t) = R(i, t) - R(3, t) - 1 + \frac{R(i, t)}{R(i, t+1)} \quad (12)$$

where $R(i, t)$ is the quarterly yield to maturity on the longer term bond (i.e. the zero to three year and the ten year and over bonds).

We hypothesise that the excess holding yield is a function of its own risk, measured by the conditional variance (h^2). The riskless return is given by μ . The conditional variance is a function of both the magnitude and sign of shocks, yielding the following model (Nelson, 1991):

$$y_t = \mu + \gamma h_t^2 + \varepsilon_t \quad (13)$$

$$\log h_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \left(\frac{\varepsilon_{t-i}}{h_{t-i}} \right) + \sum_{i=1}^q \alpha_i^* \left(\left| \frac{\varepsilon_{t-i}}{h_{t-i}} \right| - E \left(\left| \frac{\varepsilon_{t-i}}{h_{t-i}} \right| \right) \right) + \sum_{i=1}^p \delta_i \log h_{t-i}^2 \quad (14)$$

Table 5 gives the results from the EGARCH-M specifications of the excess returns of bonds with zero to three years to maturity ($y(36,t)$), and of the excess returns of bonds with ten years and over to maturity ($y(120p,t)$).

Table 5: EGARCH-M Models for $y(36,t)$ and $y(120p,t)$

100 observations used for estimation from 1981Q1 to 2005Q4		
	Dependent variable is $y(36,t)$	Dependent variable is $y(120p,t)$
	EGARCH-M (3,1)	EGARCH-M (3,1)
<i>Regressor</i>		
constant	-0.53365*** (0.14464)	-0.63568*** (0.20002)
h^2	15.8379*** (5.6972)	10.9492*** (2.8840)
<i>Conditional variance h^2</i>		
constant	-2.5541*** (0.44510)	-1.9790*** (0.26824)
$(\varepsilon/h)_{t-1}$	0.23444** (0.10726)	0.25980*** (0.072083)
$ \varepsilon/h _{t-1} - E(\varepsilon/h _{t-1})$	-0.17751 (0.12599)	-0.074395 (0.049332)
$\log h_{t-3}^2$	0.23851* (0.12217)	0.25129*** (0.094573)

Note: standard errors in parentheses,

*** significant at 1% level, ** significant at 5% level, * significant at 10% level

Given the significance of the h^2 terms in the mean equation, results confirm the presence of a time-varying risk premium. While the insignificance of the $|\varepsilon/h|_{t-1} - E(|\varepsilon/h|_{t-1})$ terms suggests that the magnitude of the shock does not

impact the conditional variances; the positive and statistically significant coefficients on $(\varepsilon/h)_{t-1}$ indicate that volatility increases (decreases) when shocks are positive (negative), suggesting an asymmetrical effect of excess return shocks.

To confirm the asymmetrical effect of shocks we run the the Engle and Ng (1993) test for asymmetry, with results reported in Table 6. The S variable is a binary variable for negative shocks. The ε^+ and ε^- variables control for positive and negative shocks respectively.

Table 6: Engle and Ng Test for $y(36,t)$ and $y(120p,t)$

<i>Regressor</i>	Dependent variable is ε^2 from $y(36,t)$	Dependent variable is ε^2 from $y(120p,t)$
constant	-0.044287*** (0.010039)	-0.10403*** (0.011423)
S	-0.028491** (0.013621)	0.0074888 (0.018192)
ε^+	-0.69647*** (0.036871)	0.84123*** (0.028389)
ε^-	0.52842*** (0.040187)	-0.73764*** (0.037033)

Note: standard errors in parentheses,

*** significant at 1% level, ** significant at 5% level, * significant at 10% level

The significance of the ε^+ and ε^- terms indicate that both positive and negative shocks affect the conditional variances of both $y(36,t)$ and $y(120p,t)$. However, the S term is significant only for $y(36,t)$, suggesting that the sign of shocks impacts the conditional variance of $y(36,t)$ only. These results confirm the asymmetrical effect of $y(36,t)$ shocks, while evidence for the asymmetrical effect of $y(120p,t)$ shocks is weaker. This distinction is further confirmed by the news curves provided by Figures 7 and 8.

Figure 7: News Slope Curve for $y(36,t)$

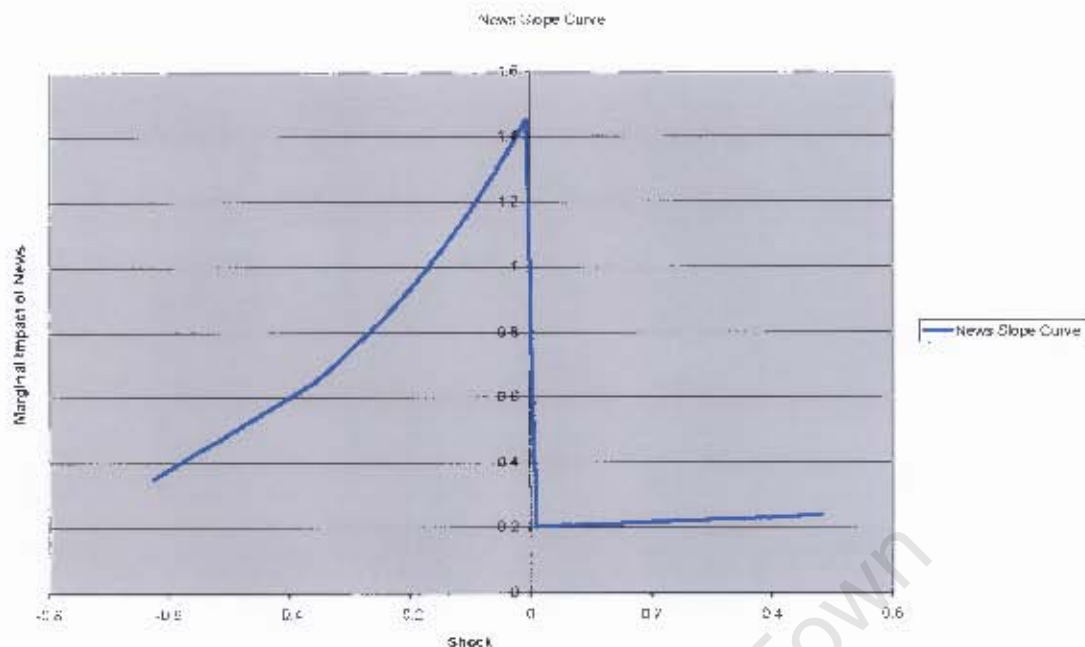
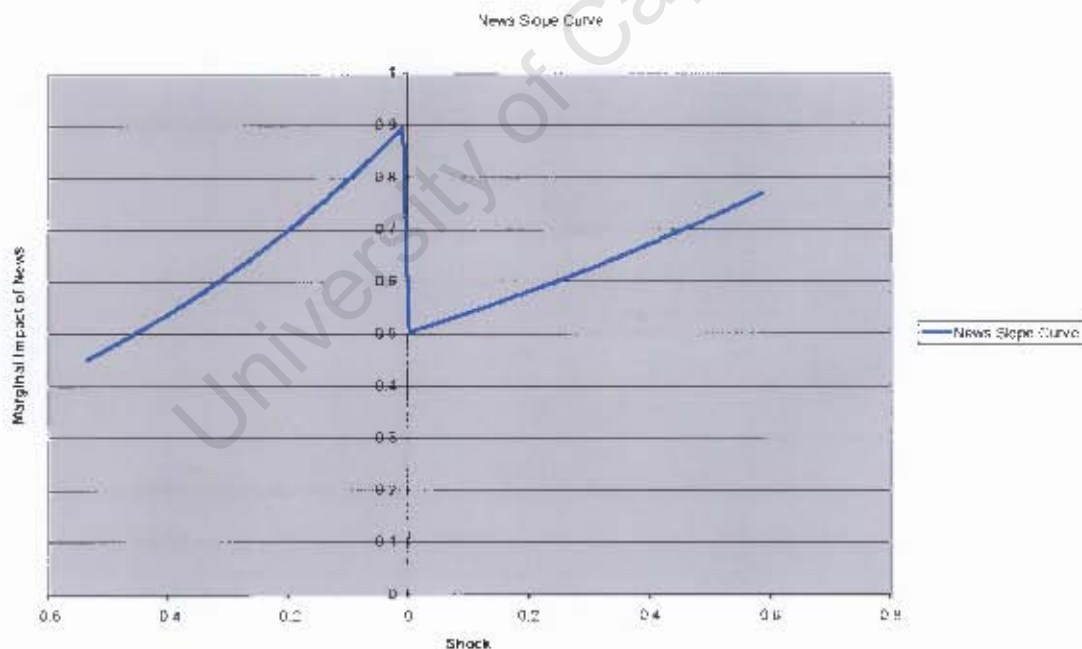


Figure 8: News Slope Curve for $y(120p,t)$



In summary, there is evidence to suggest that both excess returns $y(36,t)$ and $y(120p,t)$ depend on their conditional variances, consistent with a time-varying risk premium. While we find clear evidence of an asymmetric impact of shocks for $y(36,t)$, evidence for asymmetries with respect to $y(120p,t)$ is weaker. This suggests risk premia that

are time-varying, asymmetric and distinct between the two spreads examined in this paper. This violates the constant risk premia assumption of the expectations hypothesis, and potentially renders the standard cointegration tests inappropriate.

Moreover, given the differential structure of time varying conditional variances across the term structure, the error structures are potentially complex, differing across the various excess returns. This suggests that if we are to construct a single risk measure that is consistent with all the spread vectors of the expectations hypothesis, the implication is that we may have to extract the risk implied in each of the error structures individually.

5. Constructing a Theoretically Consistent Risk Measure

One of the crucial assumptions of the expectations hypothesis is that the risk premium is constant, which yields the result that any spread vector must be cointegrated (see Section 2). However, the evidence presented in Section 4 rejects this prediction of the expectations hypothesis. Furthermore, this rejection is shown to be the result of a time-varying risk premium.

Since the estimated cointegrating vectors are the spread vectors, they do not allow for a time-varying risk premium. It follows that this time-varying risk premium must be contained in the error processes of the cointegrating vectors. Therefore, we construct a risk measure based on the errors from the estimated single-equation cointegrating vectors. A Hodrick-Prescott filter is run through each error process, and the resulting series are combined to generate the risk measure that is consistent with the expectations hypothesis.¹¹

Figures 9 and 10 illustrate the errors from the individual single-equation cointegrating vectors, and the smoothed errors that form the components of the risk measure. These figures suggest that the first error structure contributes the overall trend to the final

¹¹ We deviate from the optimal choice of the smoothing coefficient for the Hodrick-Prescott filter suggested by Harvey and Jaeger (1993). We experimented with various smoothing coefficients to generate a risk measure that is consistent with the expectations hypothesis.

risk measure, while the second contributes the fluctuations. The final risk measure is an equally weighted combination of the smoothed errors from the individual single-equation cointegrating vectors and is illustrated in Figure 11.

Figure 9: Risk Measure Component from First CV

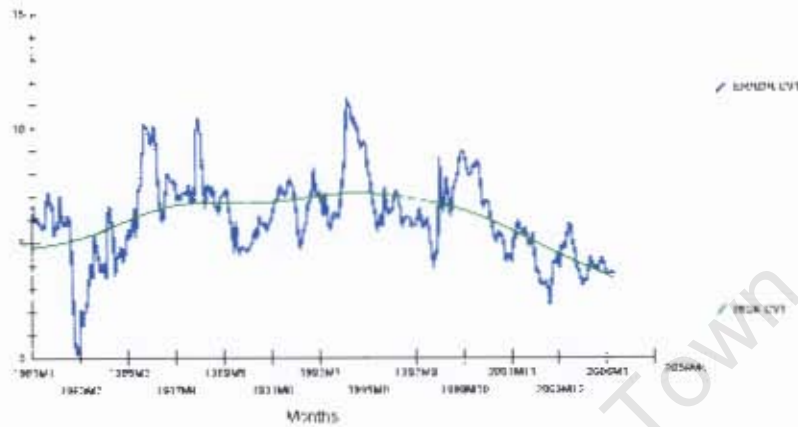


Figure 10: Risk Measure Component from Second CV

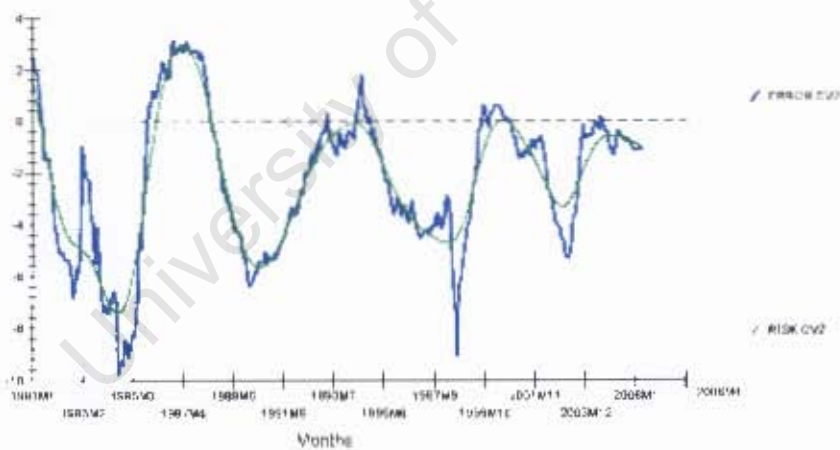


Figure 11: Final Risk Measure

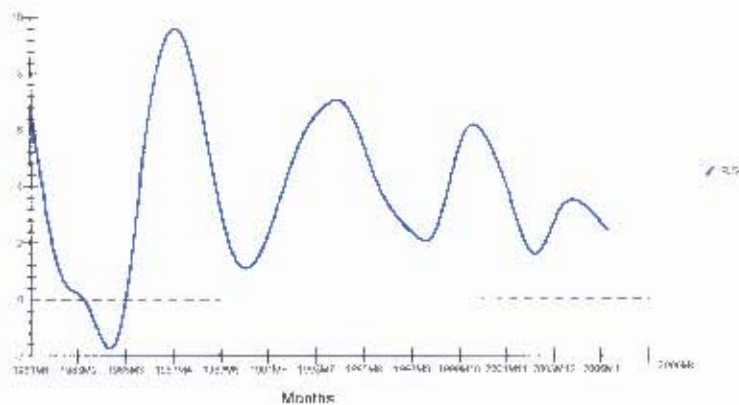


Figure 11 shows that this constructed risk measure is time-varying, and is broadly consistent with the literature on risk in South Africa.

Risk spiked in 1986 and 1994 (though at a lower level than in 1986) as a result of the increased political risk of those years. The year 1986 was a time of peak political instability (Fedderke et al., 2001) characterised by widespread sabotage activities on the part of the African National Congress (ANC) in resistance to apartheid, a second state of emergency with associated extra-judicial forms of repressive state activity, and growing international pressure to dismantle the apartheid system (SAIRR, 1987, 1988). The first democratic elections were held in 1994, and in the lead-up to the elections it was unclear whether the elections would take place peacefully or be derailed by violent resistance. This period saw high racial tensions, sabotage attempts by white right-wing groups, and increased political fatalities (SAIRR, 1995).

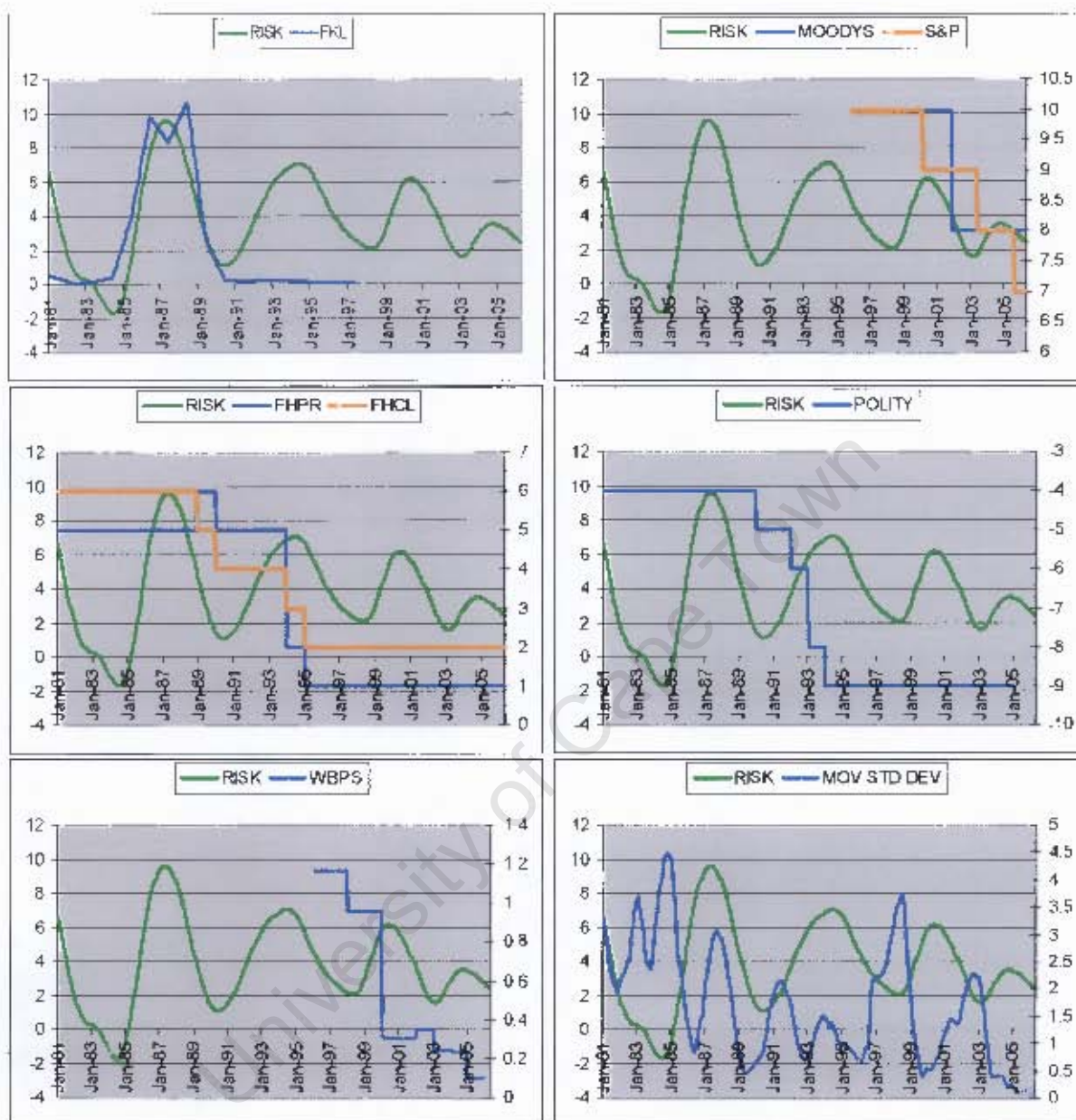
The spikes in 2001 and 2004 are largely associated with increased exchange rate risk in those years. The currency crisis of 2001 was caused by various factors, including a spillover effect of deteriorating economic conditions in Argentina, the political turmoil in neighbouring Zimbabwe, and the tightening of the enforcement of exchange controls (see Bhundia and Ricci, 2006). There was also a global stock market and economic downturn and the beginning of a US technical recession in 2001 (Aron and Muellbauer, 2006), which may have increased the risks associated with emerging market economies. In 2004, the currency strongly appreciated (SARB,

2004) which may have resulted in exchange rate risk if investors perceived the exchange rate to be overvalued.

The peaks of the risk measure have been declining over time, suggesting that the uncertainty surrounding the South African investment climate is improving. In Figure 12 we compare the risk measure constructed here (*RISK*) with some commonly used risk measures: the political instability measure developed by Fedderke, de Kadt and Luiz (2001), which we denote *FKL*; Moodys' and Standard and Poors' ratings on domestic currency long-term government bonds; Freedom House indexes of political rights (*FHPR*) and civil liberties (*FHCL*); the Polity index; the World Bank political stability index (*WBPS*); and the moving standard deviation of the short rate (widely used in the international literature as a proxy for time-varying risk in tests of the expectations hypothesis).

The *FKL* measure was only constructed on a yearly basis until 1996, nevertheless a high correlation emerges between the two risk measures over the 1980s. This suggests that our constructed risk measure captures the political risk faced by investors over this period. The new measure also suggests that the *FKL* measure underestimates risk subsequent to the 1980s. The present measure is theory-consistent, while the *FKL* measure is a subjective measure and was constructed by means of the Delphi technique. Given that the *FKL* measure was constructed in the mid-1990s, it is not surprising that there may have been a framing effect of the reduced levels of political tension of the 1990s, relative to the 1980s.

Figure 12: Comparison of Risk Measures



There is a low correlation of our risk measure with all other risk measures, with our measure giving a more nuanced representation of risk in South Africa. Moreover, our risk measure captures the total risk (including political, economic and exchange rate risk) of investing in the country whereas the other measures capture mainly political risk.

We note specifically that the moving standard deviation of the short rate, which is widely used in the literature as a proxy for time-varying risk, does not match our risk

measure well. This suggests that tests using the moving standard deviation as a proxy for the time-varying risk premium are unlikely to generate empirical support for the expectations hypothesis,¹² given that our risk measure is explicitly derived from the error structure of the expectations hypothesis test, to render the latter confirmed.

Table 7 reports relevant ADF unit root tests.¹³ Results confirm that the risk measure is an $I(0)$ process.

Table 7: Unit Root Tests for Risk Measure

Model	τ_τ	τ_μ
ADF(4)	-3.837*	-3.807*
5% Critical values for T=300		
	τ_τ	τ_μ
	-3.4267	-2.8716

Note: * significant at 5% level,
 τ_μ : constant, τ_τ : constant and trend in regression.

We now show that this risk measure is consistent with the expectations hypothesis (modified to allow a time-varying risk premium). Since the risk measure is $I(0)$, it will not enter into the long-run equilibrium relationships, but instead into the dynamics of the system. Estimation proceeds, as before, with a VAR of order two (as suggested by the Schwarz Bayesian Criterion) with unrestricted intercepts and no trends.

Table 8: Test Statistics for Number of Cointegrating Vectors

Cointegration with unrestricted intercepts and no trends in the VAR		
302 observations from 1981M3 to 2006M4 . Order of VAR = 2.		
List of eigenvalues: 0.15252 0.093748 0.0028164		
Null	λ_{max}	λ_{trace}
$r = 0$	49.9758*	80.5558*
$r \leq 1$	29.7283*	30.5800*
$r \leq 2$	0.85175	0.85175

Note: * significant at 5% level

¹² We tested the expectations hypothesis in the presence of the moving standard deviation risk measure. Results continue to reject the expectations hypothesis. Results are available from the authors on request.

¹³ See Dickey and Fuller (1979, 1981).

The λ_{max} and λ_{trace} test statistics now indicate that we can reject the null hypotheses that $r = 0$ and that $r \leq 1$ at the 5% level. Thus, we find two cointegrating vectors, consistent with the expectations hypothesis.¹⁴ Table 9 reports the tests of the overidentifying restrictions implied by the expectations hypothesis.

Table 9: Tests that Homogeneity Restrictions Hold

302 observations from 1981M3 to 2006M4 . Order of VAR = 2, r = 2.		
Null Hypothesis	Restriction	Test statistic
Homogeneity restrictions on $S(36,3,t)$ and $S(120p,3,t)$ hold jointly	$\beta_{21} = \beta_{32} = 1$	3.5337
Homogeneity restrictions on $S(36,3,t)$ hold	$\beta_{21} = 1$	3.5337
Homogeneity restrictions on $S(120p,3,t)$ hold	$\beta_{32} = 1$	0.84477

Note: * significant at 5% level

The first hypothesis test in Table 9 examines whether the homogeneity restrictions hold jointly. The remaining two tests are conducted to determine whether the homogeneity restrictions hold separately for each cointegrating vector. We cannot reject the null hypothesis for any of these tests, indicating that the homogeneity restrictions imposed by the expectations hypothesis hold.

Thus, our evidence shows that our time-varying risk premium is consistent with the expectations hypothesis.

The inference is that we have constructed a theory-consistent measure of risk in South Africa that reflects the risk perceived by investors.

¹⁴ The emergence of two cointegrating vectors on inclusion of an $I(0)$ variable is explained by the fact that the ADF statistics on the risk measure are borderline stationary. Explicitly controlling for the near non-stationary risk measure is thus likely to generate more stationary linear combinations of the variables in the expectations hypothesis test – which is the result we find.

6. Conclusion

While economic theory assumes that risk is of central importance in financial decision making, it is difficult to measure the uncertainty faced by investors. Commonly used proxies for risk (such as the moving standard deviation of an asset) are not firmly grounded in economic theory. Risk measures have been developed by other studies (such as Fedderke et al., 2001), but these are generally based on subjective weights attaching to a range of objective component indicators and are difficult to replicate.

The contribution of this paper is to develop a methodology to construct risk measures. It has the advantages of being explicitly grounded in economic theory and easily replicable. We illustrate this methodology by specific application to the South African context.

The methodology developed here constructs a risk measure that is consistent with the expectations hypothesis of the term structure of interest rates. One of the crucial assumptions of the expectations hypothesis is that the risk premium is constant, which yields the result that any spread vector must be cointegrated. However, the evidence presented in this paper rejects the central predictions of the expectations hypothesis. This rejection is shown to be the result of a time-varying risk premium.

The evidence suggests risk premia that are time-varying, asymmetric and distinct between the two spreads examined in this paper. Given the differential structure of time varying conditional variances across the term structure, the error structures are potentially complex, differing across the various excess returns.

On this basis, we construct a time-varying risk measure that is consistent with the expectations hypothesis and captures the asymmetric nature of shocks. This measure reflects investors' risk perceptions and accords with the literature on risk in South Africa.

References

- Aron, J. and Muellbauer, J. 2006. "Review of Monetary Policy in South Africa since 1994", *CSAE WPS/2006-0*, Centre for the Study of African Economies, Oxford University.
- Bhundia, A.J. and Ricci, L.A. 2006. "The Rand Crises of 1998 and 2001: What Have We Learned?", in *Post-Apartheid South Africa: The First Ten Years*, Nowak, M. and Ricci, L.A. (eds). IMF.
- Campbell, J.Y. and Shiller, R.J. 1987. "Cointegration and Tests of Present Value Models", *The Journal of Political Economy*, 95 (5), 1062-1088.
- Culbertson, J.M. 1957. "The Term Structure of Interest Rates", *Quarterly Journal of Economics*, 71, 485-517.
- Dickey, D.A. and Fuller, W.A. 1979. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, 74, 427-431.
- Dickey, D.A. and Fuller, W.A. 1981. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", *Econometrica*, 49 (4), 1057-1072.
- Engle, R.F. and Granger, C.W.J. 1987. "Co-Integration and Error Correction: Representation, Estimation, and Testing", *Econometrica*, 55 (2), 251-276.
- Engle, R.F., Lilien, D.M. and Robins, R.P. 1987. "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model", *Econometrica*, 55 (2), 391-407.
- Engle, R.F. and Ng, V.K. 1993. "Measuring and Testing the Impact of News on Volatility", *The Journal of Finance*, 48 (5), 1749-1778.

- Fedderke, J.W. 2004. "Investment in Fixed Capital Stock: Testing for the Impact of Sectoral and Systemic Uncertainty", *Oxford Bulletin of Economics and Statistics*, 66(2), 165-187.
- Fedderke, J.W., de Kadt, R.H.J. and Luiz, J.M. 2001. "Indicators of Political Liberty, Property Rights and Political Instability in South Africa: 1935-97", *International Review of Law and Economics*, 21, 103-134.
- Froot, K.A. 1989. "New Hope for the Expectations Hypothesis of the Term Structure of Interest Rates", *The Journal of Finance*, 44 (2), 283-305.
- Greenslade, J., Hall, S.G. and Henry, S.G.B. 1999. "On the Practical Identification of Cointegrated Systems with an Application to UK Wage and Price Setting", Paper presented at the African Econometrics Society Conference 1999.
- Hall, A.D., Anderson, H.M. and Granger, C.W.J. 1992. "A Cointegration Analysis of Treasury Bill Yields", *The Review of Economics and Statistics*, 74(1), 116-126.
- Harris, R.D.F. 2001. "The Expectations Hypothesis of the Term Structure and Time-Varying Risk Premia: A Panel Data Approach", *Oxford Bulletin of Economics and Statistics*, 63 (2), 233-245.
- Harvey, A.C. and Jaeger, A. 1993. "Detrending, Stylized Facts and the Business Cycle", *Journal of Applied Econometrics*, 8 (3), 231-247.
- Hicks, J.R. 1946. *Value and Capital*. London: Oxford University Press.
- Johansen, S. 1988. "Statistical Analysis of Cointegration Vectors", *Journal of Economic Dynamics and Control*, 12, 231-254.
- Johansen, S. and Juselius, K. 1990. "Maximum Likelihood Estimation and Inference on Cointegration – with Applications to the Demand for Money", *Oxford Bulletin of Economics and Statistics*, 52, 162-210.

Johansen, S. and Juselius, K. 1992. "Testing Structural Hypotheses in a Multivariate Cointegrating Analysis of the PPP and the UIP for UK", *Journal of Econometrics*, 53, 211-244.

Kularatne, C. 2002. "An Examination of the Impact of Financial Deepening on Long-Run Economic Growth: An Application of a VECM Structure to a Middle-Income Country Context", *South African Journal of Economics*, 70(4), 647-687.

Mankiw, N.G., 1986. "The Term Structure of Interest Rates Revisited", *Brookings Papers on Economic Activity*, 1986 (1), 61-96.

Mankiw, N.G. and Summers, L. 1984. "Do Long-Term Interest Rates Overreact to Short-Term Interest Rates?", *Brookings Papers on Economic Activity*, 1984, (1), 223-242.

Mariotti, M. 2002. "An Examination of the Impact of Economic Policy on Long-Run Economic Growth: An Application of a VECM Structure to a Middle-Income Context", *South African Journal of Economics*, 70(4), 688-724.

Nelson, D.B. 1991. "Conditional Heteroskedasticity in Asset Returns: A New Approach", *Econometrica*, 59(2), 347-370.

Pesaran, M.H. and Shin, Y. 1995a. "Long run Structural Modelling", Unpublished manuscript, University of Cambridge.

Pesaran, M.H. and Shin, Y. 1995b. "An Autoregressive Distributed Lag Modelling Approach to Cointegration Analysis", *DAE Working Paper 9514*, Department of Applied Economics, University of Cambridge.

Pesaran, M.H., Shin, Y. and Smith, R. 1996. "Testing for the Existence of a Long run Relationship", Unpublished manuscript, University of Cambridge.

SAIRR. 1987. *Race Relations Survey 1986 Part 1*. Johannesburg: South African Institute of Race Relations.

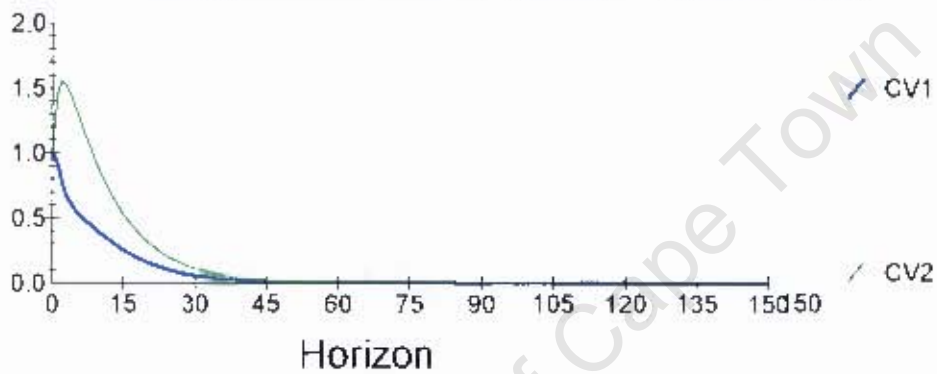
- SAIRR. 1988. *Race Relations Survey 1986 Part 2*. Johannesburg: South African Institute of Race Relations.
- SAIRR. 1995. *Race Relations Survey 1994/95*. Johannesburg: South African Institute of Race Relations.
- SARB. 2004. *Quarterly Bulletin December 2004*. Pretoria: South African Reserve Bank.
- Shiller, R.J. 1979. "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure", *The Journal of Political Economy*, 87 (6), 1190-1219.
- Shiller, R.J., Campbell, J.Y. and Schoenholtz, K.L. 1983. "Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates", *Brookings Papers on Economic Activity*, 1983 (1), 173-223.
- Simon, D.P. 1989. "Expectations and Risk in the Treasury Bill Market: An Instrumental Variables Approach", *The Journal of Financial and Quantitative Analysis*, 24 (3), 357-365.
- Tzavalis, E. and Wickens, M.R. 1997. "Explaining the Failures of the Term Spread Models of the Rational Expectations Hypothesis of the Term Structure", *Journal of Money, Credit and Banking*, 29 (3), 364-380.
- Wickens, M.R. 1996. 'Interpreting Cointegrating Vectors and Common Stochastic Trends', *Journal of Econometrics*, 74, 255-271.

Appendix A: Impulse Response Functions

These are based on the estimates of the restricted cointegrating relationships given in Table 3 in the paper.

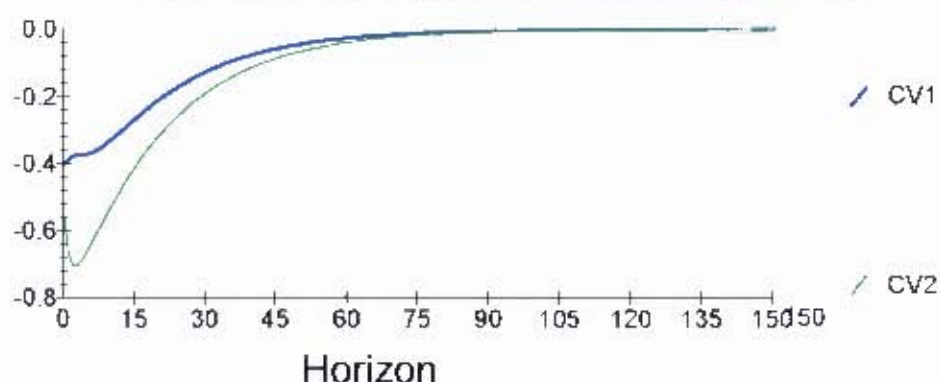
A.1 Persistence Profile of Cointegrating Vectors to System Wide Shocks

Persistence Profile of the effect of a system-wide shock to CV'(s)

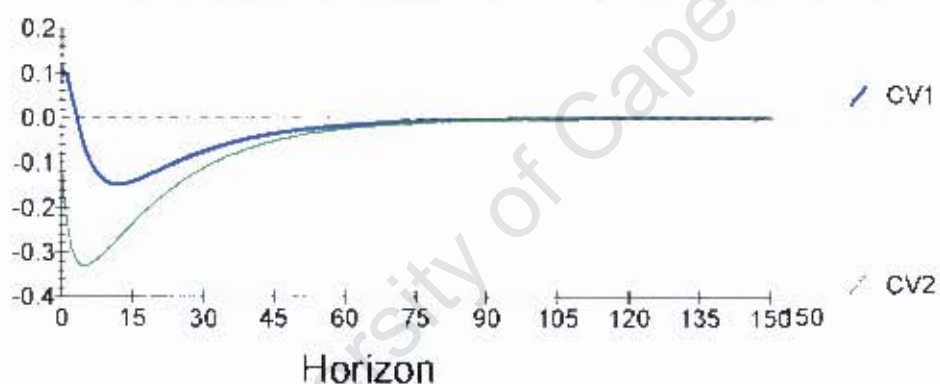


A.2 Impulse Responses of Cointegrating Vectors to Shocks in Equations

Generalized Impulse Response(s) to one S.E. shock in the equation for R3



Generalized Impulse Response(s) to one S.E. shock in the equation for R36



Generalized Impulse Response(s) to one S.E. shock in the equation for R120P

