

Evaluation of Asset Pricing Models in the South African Equities Market

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Commerce in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

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Abstract

Asset pricing models have been of interest since their origin in modern finance. The Capital Asset Pricing Model is a widely used tool and is one of the early developed asset pricing models in modern finance. There are continual improvements of this model with the evident multifactor models of [Fama and French \(2015\)](#), [Carhart \(1997\)](#) and the South African two – factor arbitrage pricing models of [Van Rensburg \(2002\)](#) and [Laird-Smith *et al.* \(2016\)](#). This research empirically investigates the performance of eight-different multi-factor asset pricing models in describing average portfolio returns in the South African Johannesburg Stock Exchange. We find that the [Carhart \(1997\)](#) four factor model comprising of the market factor, size factor, value factor and the momentum factor is the most parsimonious model and thus better explains the average portfolio returns in the South African JSE. This model is an improvement of the [Fama and French \(1992\)](#) three factor model. Additionally, we investigate the performance of the two factor Asset Pricing Theory (APT) model of [Laird-Smith *et al.* \(2016\)](#) and [Van Rensburg \(2002\)](#) that consists of the South African Financial Index (SAFI) and the South African Resources Index (SARI). We observe that the model performs better than the traditional CAPM that is widely used in industry. Adding the SAFI and the SARI to the six-factor model results in an eight-factor model that has a significant improvement in explaining average returns. The results indicate that the market factor, the South African Financial Index and the South African Resources Index (SARI) poorly explain each other but their linear combination improves the eight-factor asset pricing model in explaining average portfolio returns in the South African market. The eight – factor model comprises of the market, size, value, investment, profitability, momentum factors and the two South African indices namely, the South African Financials Index (SAFI) and the South African Resources Index (SARI).

Keywords: CAPM; JSE; APT; SAFI; SARI; Regression; Indices

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Chapter 1

Introduction

1.1 Introduction and Background

A fundamental principle of finance is determining the connection between risk and returns in financial assets. The capital asset pricing model (CAPM) provides the initial structure for quantifying risk- return relationship of of an investment. The model is considered to be the initial model of rationality since modern finance is derived from the proposition that markets are essentially rational. This method was initially developed by [Sharpe and Miller \(1964\)](#) and [Lintner \(1965\)](#) building on the model of portfolio choice developed by Markowitz (1952), leading to the birth of asset pricing theory. This field has attracted interest in the academic and practical finance environment with several improvements being developed in order to better cost the returns of an investment. The model assumes that investors are risk averse and that investors are concerned only about the mean and variance of their one-period investment returns when choosing among portfolios. This method further provides the ability to quantify risk and predict returns of financial assets.

There has been criticism of the CAPM model that has led researchers to develop improvements of this model postulating that market risk is itself made up of several separate systematic factors which should be taken into consideration when pricing an investment. It was argued that the market risk factor in the CAPM was not sufficient. Arbitrage Pricing Theory was then developed in order to fulfill this postulation coming up with several structured factors in the pricing model. The model built on this theory does not have a market risk's coefficient, the reason being that market risk is unspecified ([Fama and French, 2004](#)). Asset pricing theory (APT) does not need an unobservable market index to be identified hence concluded to be an improvement of the CAPM Model. The application of the CAPM and asset pricing theory (APT) continues to generate debate in the financial world.

Several studies show that a single factor linear relationship presented by the CAPM does not hold and that the market risk (represented by beta) alone does not explain relationship between risk and return. These studies include but are not limited to the findings obtained by [Basu \(1977\)](#), [Stattman \(1980\)](#), [Banz \(1981\)](#), [Rosenberg et al., \(1985\)](#) and [Bhandari \(1988\)](#). These findings indicate that non-market factors also contribute to asset risk-return relationships and hence portray that a single factor CAPM model does not hold. This led to further improvements of the CAPM model leading to development of multifactor models that take into consideration several factors that affect the risk-return relationship.

The South African market is an emerging market and is one of the best performing financial markets in Africa. It surely acts as a great market to test the applicability of various asset pricing models in the emerging markets in Africa. [Strydom and Charteris \(2013\)](#), performed an empirical test of the CAPM in the South African markets and concluded that the CAPM predicted returns in the South African financial market after eliminating the reliance on a risk-free proxy. Thus, supporting the applicability of the CAPM to forecast returns in the JSE. Furthermore, [Sacco \(2014\)](#) evaluated the accuracy of the CAPM, Fama and French 3 factor model and the Carhart model in calculating the cost of equity in the Johannesburg Stock Exchange over the period 2002 to 2012. The results obtained indicated that the CAPM is improved substantially by the 3-factor model and that the Carhart model performs the best in the South African context.

Concerns regarding the appropriate method for modelling the cross-section of returns still exist and have attracted great attention. [Van Rensburg \(2002\)](#) propelled for the two-factor Arbitrage Pricing Theory (APT) model to be employed to explain JSE share returns with the factors being the Financial Industrial Index and the Resources Index. These attributes were used as a representative of the dominant factors obtained from principal components analysis. [Laird-Smith et al. \(2016\)](#), carried out a research to examine the applicability of alternate beta estimates using the financial index and the resources index in the South African market as factors. They used factor analysis to identify the significant factors that better explain the South African financial market.

In this research we therefore seek to evaluate multi-factor asset pricing models in the South African context building on the research conducted by [Laird-Smith et al. \(2016\)](#) and [Fama and French \(2015\)](#). The study will integrate the South African Financial Index (SAFI) and the South African Resources Index (SARI) identified by

Laird-Smith *et al.* (2016) and Van Rensburg (2002) and the Fama and French factors. The Fama and French factors include the market, size, value, investment, profitability and momentum factors. We will further compare the applicability and performance of the CAPM, the two-factor APT model, the Fama-French three-factor model, the Carhart four-factor model, the Fama-French five-factor model, the six-factor model, the seven-factor model and the eight-factor model.

With limited studies conducted on the evaluation and testing of multi-factor asset pricing models in the South African context, there are important concerns on the models used to price an investment. It should be noted with caution that different markets are expected to possess varying and different characteristics and as such may provide different conclusions about the applicability of asset pricing models.

1.2 Research Problem

The recent developments in asset pricing models are generic, there has not been a concrete evaluation of which model better describes and predict the South African financial market. There is a possibility that the models currently in use perform poorly and thus can be improved in terms of predictive power and fit within the specific market of study. This arises from taking into consideration that the asset pricing models were developed during periods of different economic and stock market conditions. These were also implemented and tested in the American markets which intuitively behave differently from the emerging markets and the South African market in context. Since the markets in which these models were designed for are different from the South African market, we can immediately feel the gap and the risk of blindly using models that are broadly accepted without considering the nature and characteristics of African markets and thus may have different properties and mechanics to developed markets. Fama and French (1992), assert that a reliable and accurate model for the equities market is of relevance in practice and theory. This is where the research stems from, the lack of evaluation of these new developments in asset pricing within the South African equities atmosphere which leaves us exposed to use of models that can be significantly improved in predicting and describing expected asset returns in our markets. The Johannesburg Stock Exchange as an emerging market and one of the best performing markets in Africa, offers an ideal setting to investigate the asset pricing model(s) that better explain asset returns in emerging markets. The relative explanatory power of models is an obvious feature of interest when applying asset pricing models. This research follows the intuition that return on a assets is affected independently by several

systematic factors which could be of significance and relevance in explaining the returns. The CAPM is still widely used because it is the most parsimonious and hence making it essential for simple forecasting. However, this model has been argued to be less encompassing meaning that it does not accurately describe the average returns even though it is easier to use. Hence the search for a model that is both parsimonious and encompassing still continues with the aim to accurately explain the returns in a stock or portfolio investment.

1.3 Significance of the study

The asset pricing models are being improved more frequently and this area of study has attracted great attention from both the academic and practical world of finance. Most of asset pricing models were developed during periods of different economic activities and market behaviors', thus it is expected to obtain different outcomes when tested in different markets. There has been limited evidence of the applicability of asset pricing models in the South African markets. This indicates that there lacks a clear consensus of what model/(s) better describe stock returns in the South African market. Our research seeks to test the applicability of the CAPM, the two factor APT model of [Laird-Smith *et al.* \(2016\)](#), the Fama and French three-factor model, the Carhart four-factor model, the five-factor model, the six-factor model, the seven factor model and the eight-factor model in the South African financial market. This study provides a better understanding in model performance with the aim of better explaining and forecasting the returns on an investment. This study is of importance in the improvement and development of models in the finance environment and will be of great addition to the modern finance academic literature.

1.4 Aim of the Study

This study seeks to evaluate asset pricing models that better describe stock returns in the South African equities market.

1.5 Objectives

- To investigate the applicability of multi-factor asset pricing models in the Johannesburg Stock Exchange (JSE)

- To determine the parsimonious and most encompassing model in the South African market
- To evaluate the performance of the factor asset pricing models

1.6 Research Questions

- Which of these factor asset pricing models best fits the South African Market?
- How do the different factors affect the models in the South African context?
- Which model has a better explanatory performance?

Chapter 2

Literature Review

This section of the study outlines previous studies conducted by researchers and their findings. We further, indicate the contrast and motivation for the methods and aim of this study relative to the existing literature.

The merge between financial theory and practical investing was greatly observed in the 1950s and 1960s, with the formulation of portfolio theory by [Markowitz \(1952\)](#) and later on the Capital Asset Pricing Model of [Sharpe \(1964\)](#) and [Lintner \(1965\)](#) which was built on the basis portfolio theory. Current financial theory is based on three critical assumptions which are (i) market efficiency, (ii) there are arbitrage opportunities for investors and (iii) rationality of investors. The Markowitz theory of [Markowitz \(1952\)](#) is a theory that aims to optimize expected discounted returns in an investment at a minimum variance, thus achieving an efficient frontier for portfolio selection based on the “expected returns – variance of returns” rule. [Markowitz \(1952\)](#) further suggests that there is need for a probabilistic formulation of security analysis and outlines that better methods that account for more information can be found.

2.1 Capital Asset Pricing Models

The Capital Asset Pricing Model developed by [Sharpe \(1964\)](#) aims to explain the connection between average returns and risk in an investment. They extended the model of investor behavior to constructing symmetric asset prices in markets under risk conditions hence indicating the relationship between price and several constituents of its overall risk. They argued that in equilibrium there exists a consistency relationship between the expected return and systematic risk. Hence, motivating for the consideration of the model as a explaining of financial asset prices. This model measures the risk of an asset as the covariance of its returns with the

overall market returns thus quantifying risk. The CAPM was later modified by Black (1992) to a version known as zero-beta CAPM which adapts the model to a scenario where riskless investment option is not available. Mayers *et al.* (1972) shows that the zero-beta model remains identical to the CAPM structure when non-traded assets are included in the market proxy. This result attracts interest in examining how the recent asset pricing models are likely to behave when the portfolios include non-traded assets. Solnik (1974) and Black (1974) further extended the model to accommodate international investing. Scholes and Williams (1977) showed that the viability of the model is relatively vigorous if the homogenous expected returns assumption is dropped.

There exists a difficulty in implementing valid tests of CAPM, in which it posed a greater challenge in testing the early empirical CAPM in the 1960s due to lack of readily available databases of the capital markets, moreover, there was no computing power to process tremendous amounts of data. The test for applicability of the early empirical CAPM models is based on the assumption that high beta stocks have higher returns and the linear relationship between expected returns and beta. The Treynor-Sharpe-Litner-Mossin CAPM implies that the market risk factor is expected to be equivalent to the gradient of the line of best fit and the risk-free rate equivalent to the intercept. Whereas, in the zero-beta CAPM, the gradient should not be more than the market risk factor and the risk-free rate should be less than the intercept. These implications provided the ability to test the applicability of the CAPM.

Black *et al.* (1972) carried out a test of the CAPM obtaining a linear relationship between mean excess return and beta. They further observe that the gradient of the linear relationship was not constant for different periods and hence not tallying with the traditional CAPM. Most empirical tests employed the regression of security returns on their betas but this has been found to violate the assumptions of regression because beta is unknown and can be estimated with error. Black *et al.* (1972) and Fama (n.d.) attempted to solve this problem by constructing the “two-pass” methodology. Where, the first pass includes the time-series regression which gives estimates of the portfolio betas and the second pass; Black-Jensen-Scholes regress averages returns on the betas obtained from the first pass. This regression methodology is said to test the first principles CAPM and not the zero-beta CAPM. Fama (n.d.), overcame this problem by modifying the second pass, in which they perform cross-sectional regressions on monthly basis and then averaging the estimated risk premium. This allowed for the direct test of the validity of

the zero-beta CAPM. However, the two-pass method is said to fail to account for errors merely because the second pass beta estimates are obtained from the first pass which initially violated the assumptions of regression resulting in an estimated risk premium that is smaller than the true risk premium. [Black *et al.* \(1972\)](#) and [Fama \(n.d.\)](#) minimize this bias by forming sorted portfolios.

In light of the setback of the two-pass method, [Gibbons \(1982\)](#) proposed a methodology to directly test the limitation on returns due to the CAPM. This model utilizes maximum likelihood estimation hence estimating the beta and the risk premium at the same time. They conclude that the model solves for the error-in-variables problem and thus improving accuracy of estimates. It was obtained that this approach also rejects the CAPM hence raising further concerns on the viability of the CAPM.

[Roll and Solnik \(1977\)](#) challenged the definition of “market portfolio” in the CAPM asserting that the market index should be an index of wealth incorporating bonds, foreign assets, property, human capital and anything of value to the mankind. He further noted that if the representation for the market factor is mean-variance efficient, it will satisfy the equation of the security market line regardless of whether the CAPM holds or not. Hence concluding that the tests of the CAPM should be observed as tests for the mean-variance of the market portfolio proxy. However, [Shanken \(1987\)](#) and [Kandel and Stambaugh \(1987\)](#) argued that the stock market must be strongly related with the market even though it is not the true market portfolio. Even with this conclusion, their results indicate that the CAPM is still rejected.

Due to evidence of these tests rejecting the CAPM, there is growing evidence that there exist other risk factors that affect the returns in an equities market. Hence, forming the basis of motivation for the search of asset pricing models that significantly account for these factors in successfully explaining returns. [Basu \(1977\)](#) shows presence of the effect of price/earnings ratio on the returns of an asset, while [Banz \(1981\)](#) obtained that book-to-market equity is also a significant risk factor when pricing an asset. Further developments focused on extending the one-period assumption into a multi-period scenario since the CAPM is initially a static model. The static model was found to be unrealistic and hence the motivation of finding a model that would hold under a dynamic environment. In view of this insight, [Merton \(1973\)](#) improved the CAPM by developing the Intertemporal CAPM (ICAPM) that takes into account the assumption of continuity of time. This continuous-time assumption has been a major development in asset pricing, both in equilibrium and

derivative valuation. [Merton \(1973\)](#) concluded that the CAPM performs poorly under a dynamic environment but only improves under very special additional assumptions, moreover, the ICAPM caters for multi-factors in explaining expected returns in investment

2.2 Arbitrage Pricing Theory

Ross (1976) formulated the Arbitrage Pricing Theory (APT) Model with the objective to better explain expected returns using significant factors while satisfying the initial assumptions of the CAPM. This was a result of the observation that all risks in the market cannot be aggregated into a single risk factor as in the case of the CAPM. The APT allows for the inclusion of other significant sources of risk other than variances and covariances. In the APT model the assumption of linearity is extended to the linear relationship with the risk factors. The APT model and the ICAPM are similar, with the difference being that in the APT model the market risk is not included since it is assumed that the only sources of risk are common factors since perfect diversification is assumed. Hence the ICAPM becomes an APT if all portfolios are perfectly diversified. This raises questions as to whether perfect diversification is achievable. Previous studies show that the ICAPM and the APT are treated alike despite the differences in their assumptions.

Several CAPM modifications are variations of the APT model in which several factors are investigated in attempting to better explain the variation in the expected returns. Due to these variations, research in factors that significantly explain the expected returns has been widely done resulting in mixed conclusions. The APT allows an asset to be explained by many measures of systematic risk, with each measure capturing the response of the portfolio returns to the comparable factor of interest. Ross (1976) asserts that the APT is founded on the arbitrage relation rather than the equilibrium condition. There has been debate as to the choice of factors and acceptable number of factors to use in the APT model, hence; resulting in the existence of a wide variation of the APT models in Finance. [Roll and Ross \(1980\)](#) used factor analysis to investigate the statistically significant factors from the security prices data, in which their results indicated that there are four risk factors in the US. This approach has a setback in that the identified factors have no economic interpretation thus posing a challenge in finding significant market variable proxies. Use of quantifiable macroeconomic variables as risk factors has been employed as an alternative to factor analysis as portrayed by [Chen et al. \(1986\)](#). They argue that systematic factors that affect future dividends should be included as factors

of interest basing on the motion that stock prices are discounted expected future dividends. However, [Shanken \(1985\)](#) criticized the APT model, asserting that the approximation implied by the APT model is imprecise thus making it impossible to test for its validity.

2.2.1 Investigation of factors affecting stock returns and motivation for Multi-factor asset pricing models

[Stattman \(1980\)](#) and [Rosenberg \(1985\)](#) portray that high book-to-market (B/M) stocks have high expected returns and that the relationship is not accounted for by their betas. This results show evidence that US stock returns are positively correlated to book-to-market ratios. A positive correlation between expected common stock returns and ratio of debt equity while varying beta and firm size has been shown by [Bhandari \(1988\)](#). [Cha et al \(1991\)](#) in their research on Japanese stocks, identify a significant relationship between cross-sectional returns and four variables namely the earnings yield factor, size factor, value proxy and cash flow yield. In their results they indicate that the value proxy and cash flow yield have a strong positive relationship with expected returns.

[Fama and French \(1992\)](#) provide evidence on the significance of size and book-to-market factors in explaining the time series variation in portfolio returns. They indicate that small stocks with high returns and high book-to-market ratio (B/M) stocks show unspecified variables that result in uniform risks in returns that are not catered for by market returns. As a result, they indicate that their three-factor model that incorporates the market risk factor perform better than the CAPM and that the CAPM tests do not support the Sharpe-Litner-Black CAPM. However; there has been disapproval of the Fama-French three factor model by [Amihud et al. \(1992\)](#), stating that a better statistical approach estimates a significant positive correlation between beta and average returns. Furthermore, [Black \(1993\)](#) indicates that the size effect might be as a result of different market periods. Interestingly the size and price-to-book (P/B) ratio effect is observed to be as a result of investor panic instead of compensation for risk bearing as indicated by the results of [Lakonishok et al. \(1994\)](#).

Use of annual returns rather than monthly returns was observed to result in a stronger correlation between return and beta by [Kothari et al. \(1995\)](#) hence supporting the validity of the CAPM. They further claim that survivor bias in the sample may have exaggerated the relationship between P/B and returns observed by [Fama and French \(1992\)](#). [Kothari and Shanken \(1999\)](#) provide evidence that the price-to-

book ratio performs poorly in explaining the variation in expected returns of big firms and does not account for momentum and trading volume.

[Van Rensburg \(1999\)](#) identify that the gold price priced in South African Rand, the long bonds rates, the Dow Jones Industrial index and the level of gold, foreign exchange reserves and the All Gold residual market factors as significant factors within the structure of the APT over the period 1985 to 1995 in the South African financial market. The results indicate that the two-index model absorbs the effect of other macroeconomic variables. They achieved this using a non-linear seemingly unrelated regression approach of [Burmeister and McElroy \(1988\)](#) to do the cross-sectional analysis. They conclude that the influence of the macroeconomics variables on the Johannesburg Stock Exchange (JSE) is represented by the two factor APT model of [Van Rensburg and Slaney \(1997\)](#).

[Rensburg and Robertson \(2003\)](#) further carried out a research on whether factor loading based exposure to size and price earnings (PE) perform better in explaining returns on the JSE. They employed Daniel and Titman methodology to test for this relationship and their results indicated that the allocated values are able to distinguish the spread in future values rather than the factor loadings. This was mainly because of the fact that the information of the attributes is not lagging than the risk factor loadings. They then concluded that it is better to specify asset pricing models using attribute values rather than factor loadings and that there exists an inverse relationship between beta and returns in the JSE; thus, invalidating the CAPM. Moreover, [Rensburg and Robertson \(2003\)](#) also went on to use cross sectional regression to address concerns regarding the discerning of the identity of the style-based factors that explain JSE stock returns. Their results indicate that the size and price-to-earnings are explanatory variables and capture style effects.

[Van Rensburg \(2001\)](#) further extended their study in search of investigating the specifications of the risk factors that describe expected returns of stocks in the Johannesburg Stock Exchange. They achieved this by using cluster analysis to examine the interrelationships between these style factors. Student t-test was used to test for the significance of the differences in portfolio returns. Their results show that earnings-to-price (value factor), market cap (quality) and momentum form a parsimonious model on the JSE. They conclude that to better explain returns of industrial stocks, the two-factor APT model needs to be supplemented with quantifiable securities exposures to these risk factors. This forms the motivation for investigating the augmented two factor model in the Johannesburg Stock Exchange.

Strugnell *et al.* (2011), built their research on findings obtained by Rensburg and Robertson (2003) by validating the significance of size and value effects on the JSE. However, they used a different estimation approach called the Dimson Aggregated Coefficients method to combat the OLS weakness to thin trading. In their results they find that the relationship between beta and returns loses its statistical significance, hence contradicting the inverse relationship findings of Rensburg and Robertson (2003). The researchers highlight that the reliability of the data is subject to question. They propose the incorporation of transaction costs as an extension of the research and also the construction of a multifactor equilibrium pricing model.

Al-Ajimi (2015) used the methodology suggested by Fama (n.d.) for testing the CAPM based on the cross-section and also investigate anomalies presented by thin trading. They use rates of return for different intervals (daily, monthly, yearly) in the Bahrain Stock Exchange and their results show that the estimated betas are insensitive to the length of period used and the impact of thin trading on beta depends on the method used to account for thin trading.

Rahim and Nor (2006) carried a comparison of the Fama-French three-factor model and the liquidity-based three factor model in predicting portfolio returns. They aimed at evaluating the accuracy in forecasting of two liquidity based three-factor models which were formulated to improve the Fama-French three-factor model. They used mean absolute percentage errors and Theil's inequality coefficient to measure the forecast errors. They conclude that the three-factor models outperform the CAPM. However, they suggest that predicting stocks traded on the Bursa Malaysia is improved by catering for illiquidity risk in a three-factor model.

Bello (2008) statistically investigate the CAPM, the Fama-French three-factor model and Cahart's extension of the Fama-French three-factor model in which they use actively domestic equity mutual funds. They use Amemita's criterion to compare goodness of fit, the prediction of sum of squares statistics (PRESS) and Mallow's Cp statistic for prediction comparison. They conclude that the difference between the three models is insignificant with respect to goodness of fit but the Fama-French three-factor model is a significant improvement of the CAPM with respect to quality of prediction. However, they obtain that Cahart's model is an improvement of the Fama-French three-factor model. They also conclude that there is no harmful collinearity in their analyses.

2.2.2 Motivation for the development of the Fama and French five-factor model

A theoretical motivation for the CAPM anomalies is provided by [Fama and French \(2015\)](#) by use of the dividend discount model. This model shows that the present market value of a company should be equal to the discounted value of all expected future dividends. By this intuition the valuation formula shows that the market value is positively correlated to the book-to-market ratio (B/M). The formula further provides evidence for relationship of expected stock returns with earnings and change in dividends. However, [Fama and French \(2015\)](#) find that earnings and change in dividends seem to be weak proxies in forecasting future stock returns.

The earnings and change in dividends (investment) were further investigated by [Novy-Marx \(2013\)](#), resulting in evidence of a strong relationship between gross profitability and stock returns. They assert that gross profitability performs better as a proxy than current earnings because investments that are treated as expenses reduce current earnings. They note that expensed investments decrease earnings without increasing book value. In their results they find a negative relationship between gross profitability and book-to-market ratio, indicating that a combination of these factors significantly improves the asset pricing modeling.

[Aharoni et al. \(2013\)](#) further investigated the relationship between returns and the variables in the discount-dividend model. They obtain that at the per share level the investment factor is small and statistically not significant as indicated by [Fama and French \(2017\)](#). However, they find that the profitability factor calculated as income before expenses, the investment factor measured as annual asset growth and book-to-market ratio are all significantly correlated to stock returns at firm level. They argue that the anomalies found in [Fama and French \(2017\)](#) were related to their proxies of profitability and investment which were done at per share level. They assert that variation in the number of stocks is likely to decrease the correlation between returns and investment per share and hence the relationship is enhanced when the factors are investigated at firm level. In their results they further obtain that the investment factor is less vigorous when dealing with high book-to-market (B/M) firms. They conclude by raising concerns on the performance of an asset pricing model based on Miller and Modigliani (1996) in comparison to the Fama-French three-factor model.

A recent and important research conducted by [Fama and French \(2015\)](#) that improves their earlier three-factor model by taking into consideration profitability

and investment factors in the model, thus building on the evidence of [Novy-Marx \(2013\)](#) and [Aharoni et al. \(2013\)](#). This five-factor model aims to capture the significant factors that cannot be accounted for, by the three-factor model. These factors are motivated by the dividend-discount model. Operational profitability (OP) is used as a measure of a firm's profitability and the variability in total assets as a measure of investment. [Fama and French \(2017\)](#) employ time-series regression to test this proposed model and also the mean variance efficient tangency portfolio that takes into account the risk-free asset, the market portfolio and the factors of interest. Fama and French (2015, 2016) tested the five-factor model on U.S data and international data, their results show that the model performs better than both their earlier three-factor model and the CAPM in explaining returns on factor sorted portfolios.

In a more recent research, [Laird-Smith et al. \(2016\)](#) validate the results obtained by [Van Rensburg \(2002\)](#) that the All Share Index performs poorly in explaining the returns in the JSE and thus suggesting the two-factor model that replaces the market proxy with the South African Financial Index (SAFI) and the South African Resources Index (SARI), obtained through factor analysis. They employed symmetric regression outlined by [Draper and Yang \(1997\)](#), that assumes presence of error in asset returns instead of the Ordinary Least Squares (OLS) approach. In comparing the two methods, they indicate that symmetric regression produces more stable beta parameters than the OLS approach and also that there maybe possible relationships between some stocks/portfolios and factors that remains unexplored when using the OLS approach. They recommend future research to consider an improved measure of stability when comparing systematic risk measures and also investigation as to whether an inverse relationship in the CAPM will exist under alternative estimation methodologies. They question whether the model will behave and perform differently if a different estimation approach is used.

[Sundqvist et al. \(2017\)](#) carried out a research investigating if a Fama-French five-factor model can describe expected returns in the Nordic markets. They compare the five-factor model's performance to that of the Fama-French three-factor model and the CAPM model. In their results they find that the size-effect is minimal and small stocks in the Nordic markets show lower market betas when compared to big stocks. They use the GRS statistics and analysis of regression intercepts to test the applicability of the models. They find that the five-factor model provides a mean-variance efficient portfolio from its independent risk factors but however fails to improve the intercepts obtained in regressions on three factors. Their result fur-

ther indicates that the CAPM is rejected in GRS tests on Size-Investment portfolios. However, the GRS test rejected all models applied to size-profitability sorted portfolios.

Sacco (2014) evaluates the performance of the Fama and French three-factor model and the Cahart Model in calculating the cost of equity in the Johannesburg Stock exchange over the period 2002 to 2012. The results obtained show that the components of the three-factor model are able to add some robustness to the CAPM but they are highly volatile and therefore may lead to inconsistent results. They further suggest that the F-F three factor model and Cahart model present an improvement in constructing an asset pricing model that satisfy empirical evaluation.

[Karp and van Vuuren \(2017\)](#) further tested the validity and accuracy of the CAPM and Fama and French three-factor model in the Johannesburg Stock Exchange. They find that the models perform poorly due to inadequate market representation, market liquidity constraints, non-quantifiable risk factors and volatility in emerging markets. They also indicate that the value premium contributes a large portion to the changes in returns than the size factor and that it is more evident in higher book-to-market portfolios .

A more recent study was done to introduce human capital as the sixth factor in the Fama-French five-factor model by [Roy and Shijin \(2018\)](#). Concerns have been raised in previous research on whether human capital is a key factor in explaining the expected returns of a stock or portfolio. This factor might be motivated by the fact that returns are a product of a firm's workforce. The authors used the ordinary least squares (OLS) method and the generalized method of moments base robust instrumental variables technique (IVGMM) to estimate the parameters of the six-factor asset pricing model. Their results indicate that the IVGMM technique outperforms the OLS approach. They conclude that the human capital factor contributes equally as the Fama-French five factors in explaining the expected returns on portfolios.

2.3 Final Remarks

Finding an approach to better cost asset returns is a field that still continues to attract interest from the academics and professionals. The ICAPM has been likened to the APT model but we note that the ICAPM incorporates the market factor while the APT replaces the market factor with the significant factors. Hence, we ob-

serve that the Fama-French five-factor model fits the ICAPM definition and the two-factor APT model of [Laird-Smith *et al.* \(2016\)](#)) and [Van Rensburg \(2002\)](#) fit the definition of the APT model. Considering the prominent two-factor APT model for the South African equities market and the Fama-French five-factor model we therefore are interested on whether integrating these two models will improve the ability to explain the returns of a portfolio or stock in the Johannesburg Stock Exchange (JSE). Moreover, [Rensburg and Robertson \(2003\)](#) concluded that for the industrial shares, the two-factor APT model needs to be combined with style-risk factor exposures. Hence, this forms the motivation for investigating the augmented model (Fama-French five factor and two-factor APT model) in the Johannesburg Stock Exchange. Taking into account the rising interest of asset pricing models as in finance, we seek to build on historical research to investigate the performance of a multi-factor asset pricing model that integrates factors obtained by [Laird-Smith *et al.* \(2016\)](#)) and [Van Rensburg \(2002\)](#) through factor analysis and the [Fama and French \(2015\)](#) factors in the South African equities markets. This will be done by use of several statistical, mathematical and operations research methods to be explained in detail in chapter 3.

Chapter 3

Methodology

The methodologies we employ are heavily inclined to meeting the main objectives of this study. We seek to compare the performance of multifactor asset pricing models in explaining expected portfolio returns and this implies conducting empirical statistical analysis of time series data to reach solid conclusions. Time-series data has certain characteristics and assumptions that need to be satisfied before statistically using them. Tests and necessary transformations are conducted in order to carry out a significant statistical and empirical analysis of the time series data collected. It is therefore critical to test for these assumptions before carrying out statistical analysis on the collected data. If the model assumptions are not satisfied the results and conclusions will be inevitably wrong and very costly. Moreover, using a technique suited for some type of data on a different type may lead to inaccurate and skewed results. Thus, it is important to classify different types of data formally in order to use the appropriate models.

3.1 Data Selection

In order to conduct our study, monthly percentage returns (including dividends) and accounting data which includes total assets, total liabilities, shares outstanding and operating income for stocks traded in the Johannesburg Stock exchange (JSE) during the period March 2006 to June 2019, is retrieved from Bloomberg and IRESS. The chosen starting period was motivated by [Laird-Smith *et al.* \(2016\)](#); stating that it corresponds to the live date of a number of broad market indices such as the South African Resources Index. We use Python and Matlab programming languages to conduct empirical analysis of the data collected. These two languages are object-oriented languages and are widely used in the financial sector, they provide a great platform to analyse data efficiently. Analysis conducted in one platform is performed in the other one as a platform validation exercise.

Below is the financial data collected:

- Individual stock returns
- JIBAR rates
- All Share Index returns
- Market capitalization
- Book equity
- Market equity
- Earnings before taxes
- Total Assets
- South African Financial Index
- South African Resource Index

We use this data to construct the variables and portfolios required for our statistical analysis in order to meet our main objective of the study.

3.1.1 Data Filtering

In the quest to make the collected sample of data applicable, we introduced the filtering process which was suggested by [Ince and Porter \(2006\)](#). The initial aim was to incorporate the human capital factor and investigate its importance in explaining portfolio returns in the Johannesburg Stock Exchange (JSE) but due to data accessibility constraints we decided to exclude it from the study. We employ static screening to filter out dead stocks as they may lead to bias in analysis and conclusions. Stocks which were not updated from the start date are screened out of the sample. The following screening process was conducted:

- Filter for static variable instrument type
- Stocks with missing data
- Double-check for dead stocks
- Multiple instances for stocks

[Rensburg and Robertson \(2003\)](#) and [Ward and Muller \(2012\)](#) found that most of the companies that are not included in the All Share Index are too small and too illiquid for accurate evaluation and hence excluded these from their sample. We take into account all active stocks in our study in order to minimize sample bias problems.

3.1.2 Thin Trading in the JSE

Basiewicz and Auret (2009) assert that the Johannesburg Stock Exchange is an illiquid market and thin trading should be taken into consideration when carrying an analysis of the stocks traded. Rensburg and Robertson (2003) attempted to overcome the thin trading issue by including shares whose turnover ratio is greater than 0.01%. Moreover, Sacco (2014) indicate that the lack of a diverse set of investors trading small stocks will continue to result in mispriced share prices in the market. However, we exclude dead stocks and include all the active stocks.

3.2 Variable definitions

This section provides the definitions of the variables used in constructing the factors to be investigated.

3.2.1 Dependent Variables

In this study we seek to explain the variation of expected asset returns in the Johannesburg Stock Exchange (JSE). Therefore, we use expected portfolio returns as the dependent variable of interest denoted by y_t :

$$y_t = R_{it} - R_{ft} \quad (3.1)$$

where:

y_{it} - the risk-premium return of portfolio i at time t

R_{it} - the overall return of asset or portfolio i at time t

R_{ft} - the risk-free return obtained from the one month JIBAR rate at time t.

3.2.2 Explanatory Variables

Market capitalization

We use the market capitalization as a proxy for the size factor for each stock/company and this is calculated by multiplying the share price at the 31st of December of each year with the shares outstanding at the 31st of December for that year.

$$MarketCap_t = SharePrice_t \times SharesOutstanding_t \quad (3.2)$$

Book equity

This is the value of a company's shares, it can be viewed as the amount available for distribution to the shareholders. It is a key measure that can be used to gauge a stock's valuation. It is calculated as shown in equation 3.35:

$$Bookequity_t = (TotalAssets)_t - (TotalLiabilities)_t \quad (3.3)$$

Book-to-Market ratio (B/M)

This ratio is used to obtain the value of a company/stock. It is calculated by dividing the book equity variable by the market cap as shown in equation 3.4 below:

$$(B/M)_t = \frac{BookEquity_t}{MarketCap_t} \quad (3.4)$$

Operating Profit (OP)

This variable is used to represent the profitability of a company listed in the exchange before taking into account interest and tax. It is calculated by dividing operating income by book equity.

$$(OP)_t = \frac{OperatingIncome_t}{Bookequity_t} \quad (3.5)$$

Investment (Inv)

This variable is the measure of a company's investability. It is the change of total assets over time and is calculated using the formula shown in equation 3.6:

$$Inv_t = \frac{TotalAssets_{t-1} - TotalAssets_{t-2}}{TotalAssets_{t-2}} \quad (3.6)$$

Momentum (UMD)

Short term momentum effects of shares indicate shares with increasing value that can be predicted to continue to increase over a certain period of time. Academic research has proven to use the 12-2 month momentum which measures the total return of a stock over the past 12 months leaving out the previous month this is also evidenced by the French data library. It is categorized as a performance factor since it is expected for winners to continue performing well in the near future. We calculate momentum by finding the average return on the two high previous period return portfolios minus the average return on the two low previous period return portfolios as shown in equation 3.7:

$$UMD_t = \frac{1}{2}(SmallHigh + BigHigh) - \frac{1}{2}(SmallLow + BigLow) \quad (3.7)$$

3.3 Factor Construction

In order to formulate factors under consideration we define breakpoints for size, book-to-market, operating profitability, investment and momentum variables. We follow the same process used by [Fama and French \(2015\)](#) to formulate the factors for the Johannesburg Stock Exchange (JSE). We use the JSE median size to divide all stocks into two size groups, small (S) and big (B); we then divide the other risk factors into three segments; low (L), medium (M) and high (H).

Size Percintile Break Points	
Small (S)	<50%
Big (B)	>50%

Tab. 3.1: Size Percintile Break Points

The book-to-market ratio, the operational profitability factor, the investment factor and the momentum factor are sorted into into three segments; low (L), medium (M) and high (H) as shown in table 3.2:

Factor Percintile Break Points	
Low (L)	0% - 30%
Medium (M)	30% - 70%
High (H)	70% - 100%

Tab. 3.2: Factor Percintile Break Points

We further sort the factors into six portfolios in the same manner conducted by [Fama and French \(2015\)](#), with the aim to formulate the independent risk factors of interest. We obtain the portfolios by finding the intersection of the segments as indicated by table 3.3 below. Six portfolios are constructed, that is, Small/Low (SL), Small/Medium (SM), Small/High (SH), Big/Low (BL), Big/Medium (BM) and Big/High (BH).

Factor	Low (30%)	Medium (40%)	High (30%)
Small Size (50%)	SL	SM	SH
Big Size (50%)	BL	BM	BH

Tab. 3.3: Sorting on Size and Factor into six portfolios by finding the intersection of the two factors

SMB (Size Factor)

We represent the size risk premium by the SMB portfolio obtained by differencing monthly percentage returns for the three big portfolios (B/L, B/M, B/H) from the average percentage returns of the three small portfolios (S/L, S/M, S/H).

SMB_t - portfolio of small asset returns minus big stocks returns at time t.

$$SMB_{B/M} = \frac{1}{3}((SH + SM + SL) - (BH + BM + BL)) \quad (3.8)$$

$$SMB_{OP} = \frac{1}{3}((SR + SM + SW) - (BR + BN + BW)) \quad (3.9)$$

$$SMB_{INV} = \frac{1}{3}((SC + SM + SA) - (BC + BW + BA)) \quad (3.10)$$

Therefore the size premium is defined as:

$$SMB = \frac{1}{3}(SMB_{B/M} + SMB_{OP} + SMB_{INV}) \quad (3.11)$$

High Minus Low (HML) Factor

HML_t - factor obtained by finding the difference between high and low diversified portfolios of Book-to-Market (B/M) stocks at time t

$$HML = \frac{1}{2}((SH + BH) - (SL + BL)) \quad (3.12)$$

RMW Factor

RMW_t - it stands for Robust Minus Weakness; it is the difference between the returns on diversified portfolios of stock with robust and weak profitability at time t

$$RMW = \frac{1}{2}((SR + BR) - (SW + BW)) \quad (3.13)$$

Conservative Minus Aggressive(CMA) Factor

CMA_t - the difference between returns on diversified portfolios of the stocks of conservative and aggressive investment firms.

$$CMA = \frac{1}{2}((SC + BC) - (SA + BA)) \quad (3.14)$$

e_{it} - error term associated with the portfolio i at time t

Where:

SH - Small size stocks with High value

SM = Small size stocks with Medium value

SL - Small size stocks with Low value

SR = Small size stocks with robust profitability

SW - Small size stocks with weak profitability

SC - Small stocks with conservative investment

SA - Small stocks with Aggressive investment

BH - Big size stocks with High value

BM = Big size stocks with Medium value

BL - Big size stocks with Low value

BR = Big size stocks with robust profitability

BW - Big size stocks with weak profitability

BC - Big stocks with conservative investment

BA - Big stocks with Aggressive investment

The factor exposures for the factors of interest are (respectively): $\beta_i, s_i, h_i, r_i, c_i, h_i, u_i, l_i, f_i, k_i,$

3.3.1 Portfolio Construction

We test the applicability of asset pricing models on the portfolios constructed using the method used by [Fama and French \(2015\)](#) in their five-factor model. We use sets of factors to account for patterns observed in average returns. The portfolios are formed using portfolios constructed on size and book to market, size and operating profitability, size and investment and size and momentum.

We formulate the value weight portfolios sorting using size and profitability, size and investment and size and momentum. We use the JSE median size to divide all

Size Percintile Break Points	
Small (S)	<50%
Big (B)	>50%

Tab. 3.4: Size Break points

stocks into two size groups, small (S) and big (B); we then devide the other factors into quantiles i.e Low, 2, 3, 4 and High, with each quantile divided into equal 20% segments.

Factor Percintile Break Points	
Low (L)	0% - 20%
2	20% - 40%
3	40% - 60%
4	60% - 80%
High (H)	80% - 100%

Tab. 3.5: Factor Percintile Break Points

3.4 Correlation tests

It is important to investigate how the risk factors used in the models are related to each other in the hope of providing in-depth analysis of the models and the characteristics of the variables used to construct it. We therefore, use the Pearson correlation test to investigate how the factors are related to each other.

3.5 Generalized Linear Models

We use multiple-linear Regression models to analyze the relationship between expected portfolio returns and the risk factors under consideration in our study. These models provide an overview insight of how the asset pricing models explain the variation in expected returns and also how they perform relative to each other. It is important to note that in order to use these models certain statistical assumptions need to be satisfied to avoid spurious regression. A linear regression will yield biased coefficients if it omits a variable that is important in explaining the dependent

variable and correlated with the other variables in the regression, hence yielding unreliable inference.

3.5.1 Assumptions of the Generalized Linear Models

We take into consideration assumptions of general linear models before carrying statistical modelling of portfolio returns.

1. The model is linear in parameters
2. There is no perfect multi-collinearity in the explanatory variables
3. Explanatory variables are strictly exogenous with respect to the population error
4. The population error is homoscedastic

3.6 Time Series data and Model Assumptions

It is of great importance to identify the type of data being used when carrying out statistical tests in order to prevent the risk of misusing methods and models that may lead to misleading and costly conclusions. In our study, we deal with time series data and hence this type of data has certain assumptions that it needs to satisfy before being statistically analyzed. Moreover, an acceptable time series model must encompass the true data-generating process meaning that it must capture all of the time series properties of the process of interest. The residuals from an encompassing model must be completely lacking in systematic information. Furthermore, a good model must be parsimonious meaning that it should be as small as possible only possessing variables that are significant.

3.6.1 Stationarity Tests

We use the useful explanatory data analysis techniques related to the stationarity of a process. These are the auto-correlation and partial auto-correlation functions for a process followed by formal stationarity tests. The model parameters are estimated using regression modelling and hence a test for presence of autocorrelation is conducted to investigate if the data satisfies the necessary assumption of regression models.

Auto-correlation Function (ACF)

This is also known as serial correlation; it reveals the correlation between points separated by various time lags. We define the auto-correlation function rho at lags using the standard definition of correlation as follows:

$$\rho_s = \frac{Cov(y_t, y_{t-s})}{\sqrt{Var(y_t)} \times \sqrt{Var(y_{t-s})}} \quad (3.15)$$

If the process is stationary the equation reduces to:

$$\rho_s = \frac{Cov(y_t, y_{t-s})}{Var(y_t)} \quad (3.16)$$

Where:

y_t - returns of a stock or portfolio at time t

y_{t-s} - returns of a stock or portfolio at time s ; given that $t < s$

This is calculated for a number of lags and we study the pattern to get information about the time series properties such as persistence and oscillation.

Partial Auto-correlation Function

This function considers the correlation of different lags of the process conditional on the mutual correlation with intervening lags. This is calculated for a number of lags and we study the pattern to get information about the time series data and compare to the patterns observed in the ACF.

Let y_t be an arbitrary process. De-meaning the process:

$$y_t^* = y_t - E(y_t) : \quad (3.17)$$

$$y_t^* = \phi_{11}y_{t-1}^* + u_t \quad (3.18)$$

$$y_t^* = \phi_{21}y_{t-1}^* + \phi_{22}y_{t-2}^* + u_t : \quad (3.19)$$

$$y_t^* = \phi_{31}y_{t-1}^* + \phi_{32}y_{t-2}^* + \phi_{33}y_{t-3}^* + u_t \quad (3.20)$$

3.6.2 Formal Stationarity tests

Studies indicate that standard statistical results do not hold for non-stationary data, therefore, different tests have been developed with different assumptions, implementations and null hypotheses. The most widely used tests are the Augmented Dickey Fuller (ADF) and Phillips-Peron tests which tests the null of a root against

an alternative of stationarity; and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, which test the null of stationarity against an alternative of an unspecified type of non-stationarity. In this study we employ the Augmented Dickey Fuller (ADF) test.

Augmented Dickey Fuller

We use the Augmented Dickey Fuller in this study to formally test for stationarity and hence validating the results obtained from the ACF and PACF. The ADF tests for the presence of a unit root in the data. If the unit root exists we transform the data using an appropriate transformation like log normal transformations.

The Augmented Dicky Fuller (ADF) Test procedure

$X_t \sim AR(p)$ model with mean μ given by;

$$X_t - \mu = \phi_1(X_{t-1} - \mu) + \phi_2(X_{t-2} - \mu) + \dots + \phi_p(X_{t-p} - \mu) + Z_t \quad (3.21)$$

where $Z_t \sim N(0, \sigma^2)$

Rewriting the model as;

$$Y_t = \nabla X_t = \phi_1^* + \phi_2^* X_{t-1} + \dots + \phi_p^* X_{t-p+1} + Z_t \quad (3.22)$$

where;

$$\phi_0 = \mu(1 - \phi_1 - \phi_2 - \dots - \phi_p)$$

$$\phi_1^* = \sum_{i=1}^p \phi_i - 1$$

$$\phi_j^* = \sum_{i=j}^p \phi_i; j = 2, \dots, p$$

Test results for the ADF test are obtained by the following hypothesis;

H_0 : the data is not stationary

H_a : the data is stationary

Rejection criterion for the ADF test: Reject H_0 if $p < \alpha$

3.7 Asset Pricing Models

In this study, we test eight asset pricing models namely; the Capital Asset Pricing Model (CAPM), 2-factor APT of [Laird-Smith *et al.* \(2016\)](#) and [Van Rensburg \(2002\)](#), Fama and French 3 factor model, the Fama and French 5 factor model, the six factor model, the seven factor model and the eight factor model that integrates the 2 factor APT model with the 6 factor model. Unlike most extended variants of multifactor models, the eight factor model includes the two South African indices, namely the South African Financial Index and the South African Resources Index.

3.7.1 Multiple-linear Ordinary Least Squares Regression

We use this method to regress the proposed factors against the market returns in order to understand how well the risk factors explain the dependent variable (returns). Regression is done several times for different portfolios constructed specifically for this research. Regression enables us to evaluate how much variation in the dependent variable do the exogenous variables explain by the use of the R-squared statistic ranging from 0 to 1.

3.7.2 South African Indices

[Laird-Smith *et al.* \(2016\)](#) and [Van Rensburg \(2002\)](#) assert that the South African equities market is better explained by two factors namely; the South African Resources Index (SARI) and the South African Financial index (SAFI) which they obtained as proxies for the factors identified using factor analysis. Therefore, in our study we employ these findings in order to investigate whether integrating the two factor model comprising of these factors and the Fama-French risk factors will lead to a model that better explains the returns of equities in the Johannesburg Stock exchange (JSE).

The terms used in the asset pricing models are as follow:

$SARI_t$ - South African Resource Index at time t

$SAFI_t$ - South African Financial Index at time t

R_{it} - the overall return of asset or portfolio i at time t

R_{ft} - the risk-free return proxied by the one-month JIBAR rate at time t.

R_{mt} - the overall market return at time t.

α_{ji} - intercept for model j

β_i - this is the degree of change in the dependent variable for every 1-unit of change in the explanatory variable in the asset or portfolio i

3.7.3 Capital Asset Pricing Model

This model forms the basis for asset pricing and is still widely used in the global financial markets. It is defined as follows:

$$R_{it} - R_{ft} = \alpha_{1i} + \beta_i(R_{mt} - R_{ft}) + e_{it} \quad (3.23)$$

3.7.4 Two-factor APT Model

Laird-Smith *et al.* (2016) and Van Rensburg (2002) found evidence that the factor model comprising of the South African Resource index and the South African Financial Index (SAFI) as the explanatory factors performs better in the South African equities market as compared to the CAPM. We therefore, use this model to compare it to other multifactor models of interest in this study. The model is defined as:

$$R_i = \alpha_{2i} + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + e_i \quad (3.24)$$

3.7.5 Fama & French Three-Factor Model

Fama and French concluded that the traditional CAPM lacks the factors that explain the variation of the returns and hence proposed the 3-factor model that takes it account the size factor and the value factor. In this study, we investigate the performance of this model in comparison to the other multifactor asset pricing models of interest. The model is defined as:

$$R_{it} - R_{ft} = \alpha_{3i} + \beta_i(R_{mt} - R_{ft}) + s_iSMB_t + h_iHML_t + e_{it} \quad (3.25)$$

3.7.6 Cahart's Four-Factor Model

The Cahart's four factor model adds the momentum factor to the Fama-French three-factor asset pricing model. The model is normally used as a fund evaluation model for active investments.

$$R_{it} - R_{ft} = \alpha_{4i} + \beta_i(R_{mt} - R_{ft}) + s_iSMB_t + h_iHML_t + u_iUMD_t + e_{it} \quad (3.26)$$

3.7.7 Fama & French Five-Factor Model

Fama and French (2014) further developed a five-factor model as an improvement to their three-factor model. This model aims to take into account the profitability and investment factors in explaining variation stock returns. It is our interest to investigate how this model explains returns in the South African equities market as compared to the other models:

$$R_{it} - R_{ft} = \alpha_{5i} + \beta_i(R_{mt} - R_{ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (3.27)$$

3.7.8 Six-Factor Model

This model is an improvement of the Fama-French five-factor model that adds the momentum factor to the five factor model. We therefore, test this model in our research and compare its performance to the other multifactor asset pricing models under consideration.

$$R_{it} - R_{ft} = \alpha_6 + \beta_i(R_{mt} - R_{ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.28)$$

3.7.9 Seven-Factor Model

Motivated by the findings of [Laird-Smith *et al.* \(2016\)](#), [Van Rensburg \(2002\)](#) and [Fama and French \(2015\)](#), we therefore seek to integrate the two - factor APT model and the five-factor model. However, this model replaces the market with the South African Financial Index and the South African Resources Index. The model is defined as follows:

$$R_{it} - R_{ft} = \alpha_7 + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.29)$$

3.7.10 Eight -Factor Model

We further extend the seven factor model to an eight-factor model that comprises of the market factor, the South African Financial Index and the South African Resources index jointly. We seek to investigate the performance of this model in explaining the expected returns of the cross sectional portfolios formed.

$$R_{it} - R_{ft} = \alpha_8 + \beta_i(R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.30)$$

Individual explanatory variables

A pressing issue in a model is whether we need all the factors to adequately explain the asset/portfolio returns. We therefore, carry out statistical tests to investigate the significance of the factors.

Hypothesis Tests for explanatory variables

In order to identify the significance of the independent variables we carry out the hypothesis tests. We therefore also seek to test if the independent variables have no partial effect on the returns of stock/portfolio. Hence, the null hypothesis is a joint test of the independent variables $H_0 - \beta_i = \beta_j = 0$

$$H_a - \beta_i \neq \beta_j \neq 0$$

Test statistic: F ratio

$$\frac{RSS_R - RSS_U)/q}{RSS_U/(n - k)} \quad (3.31)$$

Where:

RSS_R - Residual Sum of Squares of the restricted model

RSS_U - Residual Sum of Squares of the unrestricted model

q - Is the number of restrictions or the number of equalities in the null hypothesis.

The decision rule is:

If $F < F_{q,n-k}^\alpha$ reject H_0

If $F > F_{q,n-k}^\alpha$ fail to reject H_0

With this test alone, we are not able to determine which variables have a partial effect on the returns of a stock or portfolio, all or one may affect the returns. The F statistic is used to test the exclusion of a group of variables when the variables in the group are highly correlated. Moreover, the p-value maybe used to make a decision instead of the F statistic

The p-value is defines as:

$$p = Pr(F > F|H_0)$$

Where:

F - is the actual value of the test statistic

F' - is the Snedecor's F random variable with (q,n-k) degrees of freedom

The decision rule then becomes:

Reject H_0 if $\alpha > p$

Fail to reject H_0 if $\alpha < p$

We are also interested in testing for the significance of each of the independent variables in explaining returns.

$$H_0: \beta_j = \beta_0$$

$$H_a: \beta_j < \beta_0$$

The t-statistic is given by:

$$t = \frac{\beta_{OLS} - \beta_0}{s.e(\beta_{OLS})} \quad (3.32)$$

Rejection Criteria: Reject the null hypothesis if:

$$t < t_{crit}(\alpha)$$

3.8 Factor Spanning Tests

We use factor spanning tests in order to test if a risk factor can be explained by the combination of the other risk factors. These tests help determine if the factors contain unique information about the average returns and hence testing for redundancy. We perform these tests by regressing average returns of each risk factor against the returns of all other risk factors to investigate which risk factor is redundant and which one is not. Therefore, eight regressions will be performed to carry out the spanning tests as follows:

Market factor as the dependent factor

$$(R_{mt} - R_{ft}) = \alpha + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.33)$$

The South African Financial Index as the dependent factor

$$R_{SAFI} = \alpha + (R_{mt} - R_{ft}) + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.34)$$

The South African Resources Index as the dependent factor

$$R_{SARI} = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.35)$$

The Size(SMB) Factor as the dependent factor

$$SMB_t = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + h_iHML_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.36)$$

The Value (HML) Factor as the dependent factor

$$HML_t = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + r_iRMW_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.37)$$

The Profitability Factor (RMW) Factor as the dependent factor

$$RMW_t = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + c_iCMA_t + u_iUMD_t + e_{it} \quad (3.38)$$

The Investment (CMA) Factor as the dependent factor

$$CMA_t = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + u_iUMD_t + e_{it} \quad (3.39)$$

The Momentum (UMD) Factor as the dependent factor

$$UMD_t = \alpha + (R_{mt} - R_{ft}) + \beta_{Fi}R_{SAFI} + \beta_{Ri}R_{SARI} + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (3.40)$$

3.9 Asset Pricing Model Performance Evaluation

In this section we seek to answer the main objectives of this research by evaluating the performance of each of the asset pricing models relative to each other. We use the ordinary least squares $R^2_{adjusted}$ statistic to determine the model that better explain the variation of the expected returns of the portfolios constructed. Furthermore, we investigate the intercepts obtained when applying the regression models. When evaluating asset pricing models, it is said that if the intercept is zero it indicates that the model fully captures the returns of the portfolio or asset under consideration. We therefore conduct this test using the GRS test statistic discussed in the next sections.

3.9.1 Adjusted R-Squared Statistic

In order to measure the variation in the dependent variable we use the adjusted R-squared which corrects the issue of over-fitting presented by the R-squared statistic. A good model will have a relatively high R-squared statistic. However, it is of best practice to combine this test with other statistical tools to determine a good model.

$$R^2_{adjusted} = 1 - \frac{SS_{res}/df_e}{SS_{tot}/df_t} \quad (3.41)$$

The adjusted R-squared value can also be calculated using the R-squared value and the degrees of freedom as follows:

$$R^2_{adjusted} = 1 - \frac{(1 - R^2)(N - 1)}{N - p - 1} \quad (3.42)$$

Where

SS_{res} = the sum of squares of residuals

SS_{tot} = total sum of squares

df_t = degrees of freedom $n-1$ of the population variance of the dependent variable

df_e = degrees of freedom $n - p - 1$ of the estimate of the underlying population error

variance

R^2 = sample R-squared

N = total sample size

p = number of predictors

3.10 GRS Tests

We use the GRS test as a test of mean variance efficiency between the formulated portfolios and the risk factors. This test was developed by [Gibbons *et al.* \(1989\)](#). The test determines whether the intercept values from individual model regressions are jointly non-significant and thus investigating if a model fully explains portfolio returns. In simple terms, the method tests if the intercept is significantly different from zero. In asset pricing tests, a model is said to fully capture the returns of a portfolio if its intercept is not significantly different from zero.

$$fGRS = \frac{T}{N} \times \frac{T - N - L}{T - L - 1} \times \frac{\hat{\alpha}^T \times \Sigma^{-1} \times \hat{\alpha}}{1 + \bar{\mu}^T \times \hat{\Omega}^{-1} \times \bar{\mu}} \sim F(N, T - N - L) \quad (3.43)$$

Where:

$\hat{\alpha}$ is an $N \times 1$ vector of estimated intercepts

$\hat{\Sigma}$ is an unbiased estimate of the residual covariance matrix

$\bar{\mu}$ is a $L \times 1$ vector of the factor portfolios' sample means

$\hat{\Omega}$ is an unbiased estimate of the factor portfolios' covariance matrix

Test results for the GRS test are obtained by the following hypothesis;

$$H_0: \alpha = 0$$

$$H_a: \alpha \neq 0$$

Level of significance: 5%

Test statistic: fGRS

Decision criteria for the fGRS test: Reject H_0 if $p < \alpha$

3.11 Conclusion

In this section we have identified quantitative techniques to estimate the parameters of the asset pricing models to be used in this study. We further elaborate on tools to be used to evaluate the estimates and the performance of the model. The models are evaluated using the adjusted R-squared statistic and the error metric tools. A good model shall be determined by a relatively high adjusted R-squared value and a very small error in the forecasting performance.

Chapter 4

Data Analysis and Results

This chapter presents the analysis and results of asset pricing models together with the portfolios used to test the models. The goal of this research is to investigate the performance of asset pricing models in explaining average excess returns on value-weight portfolios sort in constituents of size, B/M, profitability, investment and momentum. The value-weight portfolios are constructed using the risk factors under consideration. We begin with statistical tests to investigate the properties of the collected data and we proceed to constructing the test portfolios using value-weight portfolios. In constructing the portfolios, we follow the methodology applied in prior literature by sorting stocks using quantiles and halves of factors as discussed in Chapter 3. First, the average excess return patterns in these portfolios are examined and discussed. Then as conducted in prior literature, we investigate the characteristics of the returns of these portfolios as we seek to test the asset pricing models on them.

4.1 Statistical Analysis of the Risk Factors

4.1.1 Correlation Analysis

We begin by anticipating the possibility of multicollinearity issues arising in the formulation of asset pricing models as expected with the application of general linear models. A correlation analysis of the risk factors of interest is conducted for the period March 2006 - June 2019. This test does not test for causation but it acts as an early indicator of possible associations in the risk factors under consideration. Table 1 presents the findings of the correlation analysis.

The size, value and investment factors are negatively correlated with the market factor. In the Instabul market, Eraslan (2008) finds a similar relationship with the size and value excess returns. The negative correlation of the size and market factor is observed in studies conducted in the Pakistan market by [Ali *et al.* \(2018\)](#).

	Market	Size	Value	Profitability	Investment	Momentum	SARI	SAFI
Market	1.00							
Size	-0.25	1.00						
Value	-0.22	0.44	1.00					
Profitability	-0.12	0.10	-0.10	1.00				
Investment	0.00	0.04	0.05	-0.09	1.00			
Momentum	0.01	-0.21	-0.46	0.12	-0.02	1.00		
SARI	0.01	0.05	-0.10	-0.08	0.05	-0.10	1.00	
SAFI	0.13	-0.06	-0.03	-0.12	0.17	-0.02	0.18	1.00

Tab. 4.1: Correlation matrix for risk factors

However, in the developed markets [Fama and French \(2015\)](#) found a positive correlation between the size factor and the market factor stating that small stocks tend to have higher betas than big stocks. The positive correlation between size and value merits comment; this positive correlation might be due to the proportion of large market capitalization stocks in our sample. A negative correlation of -0.36 is observed between the size factor and excess market factor, a similar relationship is observed between investment and profitability. We further investigate autocorrelations and test for stationarity of the risk factors in the next section.

4.2 Stationarity tests

Using non-stationary data in statistical models results in spurious outcomes and thus biased conclusions. Hence, we test for stationarity in the data collected in order to avoid poor conclusions resulting from spurious regressions. We therefore begin by conducting the autocorrelation and partial autocorrelation tests.

4.2.1 Autocorrelation

We conduct autocorrelation tests to investigate the degree to which each of the series (variable) moves relative its own lagged values over time.

Figure 4.1 shows the autocorrelation plots for each independent factor. The height of the spikes indicates the value of the autocorrelation function for the lag. The autocorrelation plots show little or no statistical evidences of autocorrelation in all the factors, meaning that there is low likelihood of autocorrelation in the data. This indicates that each factor is not highly autocorrelated.

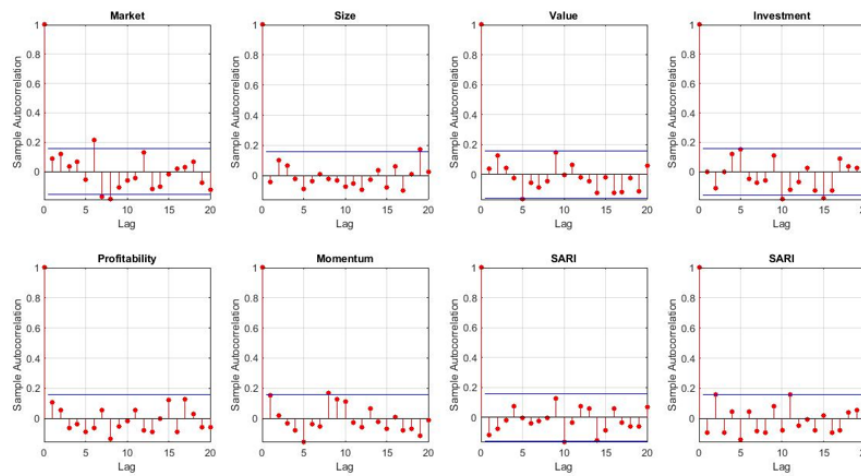


Fig. 4.1: Factor Autocorrelations

4.2.2 Partial Autocorrelation

If there are first order autocorrelations present, the second-order coefficient is also most likely to be statistically significant. Thus, we seek to validate the autocorrelation results and further identify the effects of lower-order coefficients using partial autocorrelation. A partial autocorrelation coefficient analyzes correlation with lower-order coefficients removed.

Figure 4.2 shows autocorrelation plots of all the factors being investigated in this

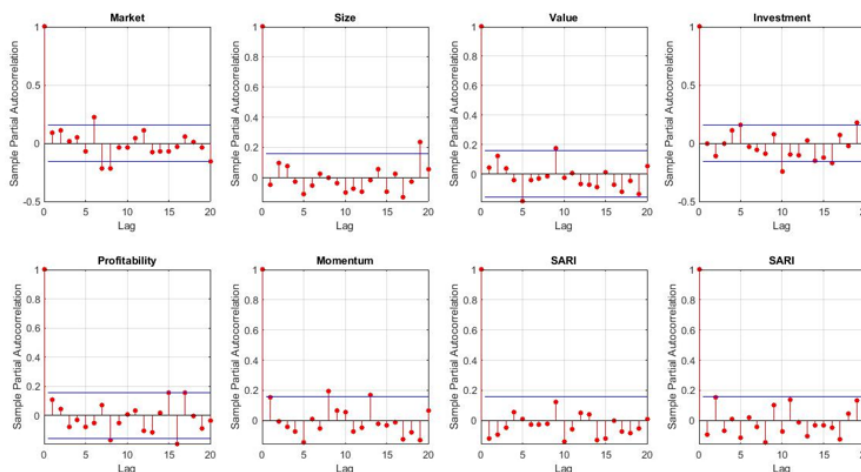


Fig. 4.2: Factor Partial Autocorrelations

study. There are significant autocorrelations at lags 6, 7 and 8 for the market factor, at lag 19 for the size factor, at lag 5 for the value factor, at lag 10 for the investment factor, at lags 8 and 16 for the profitability factor and at lag 8 for the momentum factor. Most of the lags for the factors are not significantly correlated and it is observed that the significant correlations exist in the mid lags. We therefore proceed with the Augmented Dickey Fuller (ADF) test to further statistically investigate the stationarity of the risk factors.

4.2.3 Augmented Dickey Fuller (ADF) Test

We carry out the ADF test to examine if our data is stationary. If the data is non-stationary, it is transformed by first differencing and then tested for stationarity once again to investigate if it has been transformed. The ADF test results in table 4.2 show that factor returns are stationary and hence satisfy the ordinary least squares assumptions.

H_0 : There is evidence of a unit root in the risk factor

H_1 : There is no evidence a unit root in the risk factor

Level of Significance: 5%

Test statistic: p-values

Decision criteria: For all the $p < 0.05$ we reject the null hypothesis, otherwise for $p > 0.05$ we fail to reject the null hypothesis. We therefore reject H_0 for all the risk factors that we study on the South African JSE and conclude that there are no unit roots in the independent factors. Hence, the independent risk factors are stationary at the five percent significance level.

Conclusion: Since we rejected H_0 for all the risk factors, we conclude that there is no presence of unit roots in the independent risk factors of the South African JSE.

P-Values for all the risk factor are represented in Figure 4.3.

Logical 0 = There are unit roots in the data

Logical 1 = There are no unit roots in the data

Factor	P-Value	stat	logical
Market	3.44E-03	-11.46	1
Size	1.00E-03	-12.98	1
Value	1.28E-03	-11.95	1
Profitability	1.89E-03	-11.25	1
Investment	1.55E-03	-12.57	1
Momentum	1.69E-03	-10.16	1
SAFI	7.62E-03	-12.34	1
SARI	6.98E-03	-13.27	1

Tab. 4.2: Risk Factor Returns ADF Test

From this hypothesis we are then guided accordingly to formulate asset pricing models without statistically violating model assumptions.

4.2.4 Normality Tests

In this section we check for the validity of the normality assumption in our factors. Figure 4.3 presents the Q-Q plots for the market factor, size factor, value factor, investment factor, profitability factor, momentum factor, South African Financial Index and the South African Resources Index (SARI). The market factor and the SARI factor show existence of outliers in their tails as compared to the rest of the factors; this is observed in the lower points that deviate from the line. However, the rest of the factors have values that lie close to the line of best fit and hence their distributions have the similar shape with the theoretical normal distribution.

4.2.5 Independent Factor Distributions

Figure 4.4 shows the histograms of all the independent factors being investigated. The size factor shows a wider spread than the rest of the factors and the peaks of the factors range within the same value showing consistency in the returns of the South African equities market.

4.2.6 Anderson - Darling Test

We further carry out the Anderson - Darling test to validate previous observations. This test investigates if our data is indeed normally distributed and hence providing guidance on whether the normality assumption is met so that we may proceed with the modelling techniques.

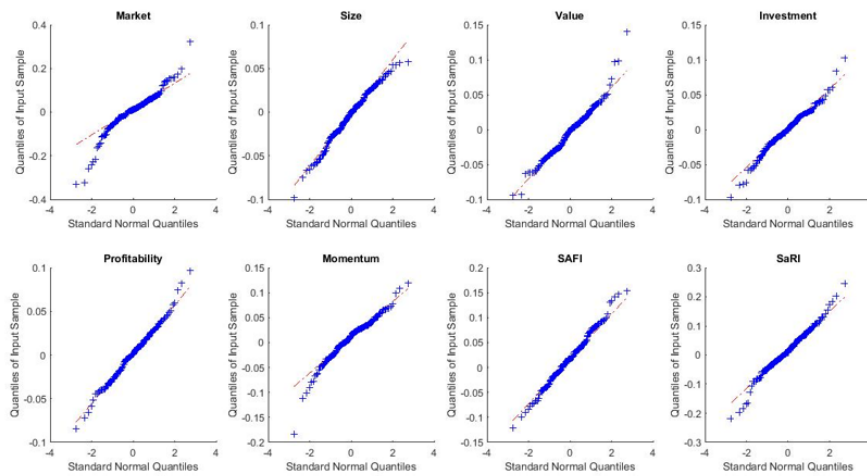


Fig. 4.3: Q-Q plots for Risk Premiums

H_0 : The risk factors are normally distributed

H_1 : The risk factors are normally distributed

Level of Significance: 5%

Test statistic: p-values

P-Values for all the risk factors are represented in Table 4.3.

Decision criteria: since the p-value of the market factor and the momentum factor are below the value of 0.05 and hence resulting with a logical result of 1, we reject the null hypothesis and conclude that the two factors are not normally distributed at 5% significance level. The rest of the factors have a p-value greater 0.05 and with a logical result of 0, thus we fail to reject the null hypothesis and conclude that the remaining five factors are normally distributed. We therefore carryout necessary transformations to normalize the Market factor and the Momentum factor so as to avoid spurious results in our models. The Market factor and the Momentum factor were normalized using first differencing.

Logical 0 = There are unit roots in the data

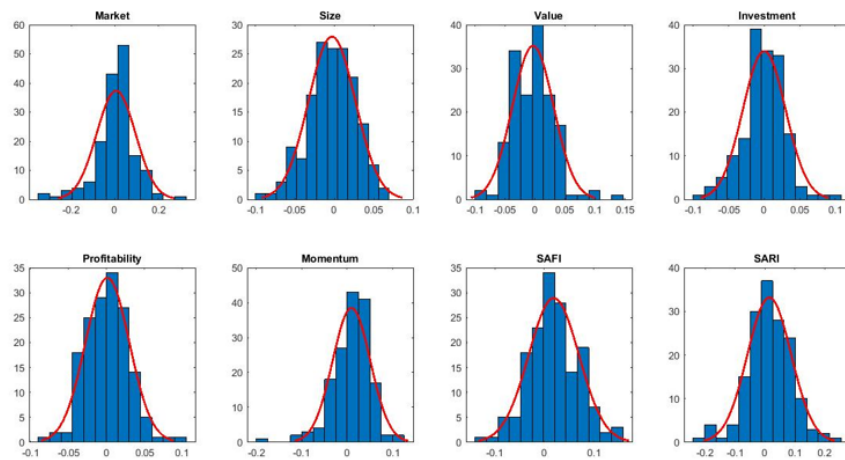


Fig. 4.4: Distrintuion of the independent Risk Factors

Factor	P-Value	adtest	logical
Market	5.00E-04	3.1561	1
Size	0.2746	0.4506	0
Value	0.0917	0.6429	0
Profitability	0.6326	0.29	0
Investability	0.0441	0.7702	0
Momentum	7.93E-04	1.4759	1
SAFI	0.6201	0.2944	0
SARI	0.1209	0.5948	0

Tab. 4.3: Risk Factor Returns Anderson Darling Test

Logical 1 = There are not unit roots in the data

4.2.7 The Independent Risk Factors

Figure 4.5 presents plots of the independent factors after testing regression assumptions and conducting necessary transformations. The plot shows monthly returns for the market factor, size factor, value factor, investment factor, profitability factor, momentum factor, the indices South African Resource Index (SARI) and the South African Financial Index (SAFI). The time-series plot shows that all the risk factors are stationary about some constant mean.

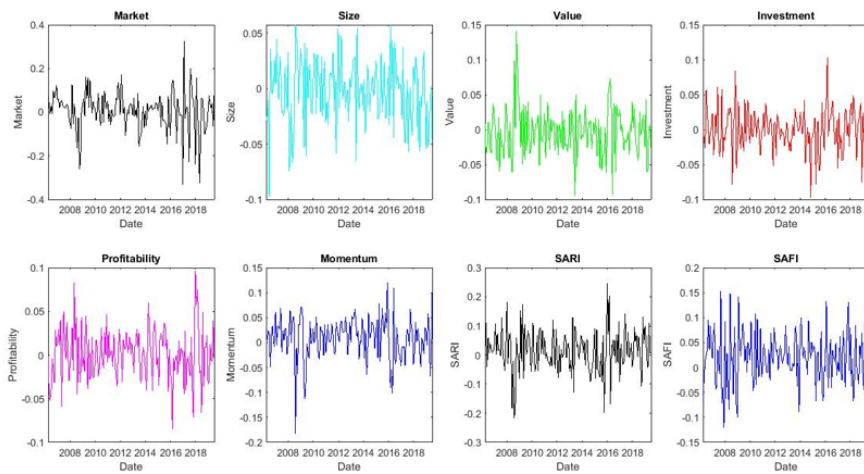


Fig. 4.5: Risk Factor Premium Returns

Size - Factor Portfolios (2 x 2 sorts)

The initial stage is to investigate patterns associated with portfolios formed on size, value (B/M), profitability, investment and momentum factors in average returns we seek to explain in this study. This section consists of the analysis conducted on the average monthly excess returns for four portfolios formed independently. These portfolios are formed on stocks sorted into two size groups (small and big) and two groups (high and low) for each factor. It is important to analyze the factor-sorted portfolios when evaluating asset-pricing model as this takes into consideration the different aggregational levels of stock returns.

Size - B/M (Value) (2 x 2) Portfolios

Table 4.4 shows average returns of the four value-weight (VW) portfolios formed from sorting independent stocks into two size groups (small and big) and two B/M groups (high and low). The breakpoints of the size and B/M are obtained by using the market capitalization breakpoints. Stocks are assigned independently to each of the portfolios formed using these break points.

In each B/M column there is a decrease in the average returns from small stocks to big stocks (the size effect). [Fama and French \(2015\)](#) obtain a similar finding in the US market using portfolios sorted using the 5 x 5 sorting approach. This effect is further investigated on portfolios formed on 2 x 5 Size-B/M portfolios. The

Size - Value Portfolios		
	Low	High
Small	1.67%	-0.66%
Big	1.56%	0.43%

Tab. 4.4: Size - Value (2 x 2) Returns

average returns drop from low B/M to high B/M for each size row. The returns of the small stocks drop from 1.67% per month to an average of -0.66% per month whereas for the big stocks the average returns drop from 1.56% to 0.43%. This observation weakly validates that the value effect is stronger among small stocks; the value effect is the relation between B/M and average return.

Size - Inv (2 x 2) Portfolios

Table 4.5 shows the mean excess returns of the four value-weight portfolios formed on two size groups and two investment groups. [Fama and French \(2015\)](#) define the investment independent factor as the growth of total assets for the fiscal year ending t-1 divided by total assets at the end of period t-2.

An increase in the returns is observed across the size rows, suggesting that high

Size - Inv Portfolios		
	Low	High
Small	0.16%	0.47%
Big	1.19%	1.28%

Tab. 4.5: Size - Investment (2 x 2) Returns

investment stocks have a higher return as compared to the low investment stocks. The small stocks show a higher change in returns across the row size from small size - low investment portfolio (0.16%) to small size - high investment portfolio (0.47%) as compared to the change observed in the big portfolio stocks. [Fama and French \(2015\)](#) observed the size effect in these portfolios, concluding that small stock portfolios portray higher expected returns than big stocks. However, in our study we observe that small stocks have lower average returns than big stocks, which is in contradiction to the finding obtained in the developed market.

Size - Profitability (2 x 2) Portfolios

Table 4.6 shows average excess returns for four portfolios formed on two size groups and two profitability (OP) groups.

Higher average excess returns are observed in the high profitability quantile port-

Size - OP Portfolios		
	Low	High
Small	-0.05%	1.11%
Big	1.09%	1.31%

Tab. 4.6: Size - Profitability (2 x 2) Returns

folio than the low profitability quantile portfolios. This observation is known as the profitability effect and is evident in previous literature such the findings of [Chen et al. \(2011\)](#) and [Fama and French \(2015\)](#). The relation between profitability and average returns is relatively small for the small size - low profitability portfolio.

Size - Momentum (2 x 2) Portfolios

Table 4.7 portrays average excess returns for four portfolios formed on two size groups (small and big) and two momentum groups (low and high).

Similar average returns are observed in the Small Size – High Momentum port-

Size - Mom Portfolios		
	Low	High
Small	-0.12%	1.03%
Big	1.03%	0.95%

Tab. 4.7: Size - Momentum (2 x 2) Returns

folio and the Big Size – Low Momentum. This weakly indicates that small stocks with high momentum behave relatively the same as big stocks with low momentum. This is an interesting observation and merits comment. A low average return of -0.12% is observed in the portfolio of small stocks with low momentum as compared to other portfolios. This is probably due to the fact that small companies with low momentum are expected to continue to portray low returns in the periods under study. Hence, we will name this effect the momentum effect in addition to the size and value effects observed in the previous analysis.

Value Weighted Portfolios Sorts (2 x 5)

We further search for clarified patterns in the above sorts using 2 x 5 classifications. This clarifies the characteristics of the various effects observed in 2 x 2 classifications and more so, identify possible effects that exist as we broaden the classification approach from a 2 x 2 to a 2 x 5 approach.

4.2.8 Size – Value Portfolios

Table 4.8 below shows average excess returns for 10 portfolios formed on two size groups (small and big) and five value (B/M) groups.

We further represent the findings in a graphical form in Figure 4.6 for ease of

		Size - Value				
		Low Value	2	3	4	High Value
Small		1.36%	1.41%	1.12%	0.46%	-1.30%
Big		1.68%	1.86%	1.48%	1.00%	-0.06%

Tab. 4.8: Size - Value (2 x 5) Returns

analysis and comparison:

Stocks are allocated independently to each of the portfolios formed. In each B/M

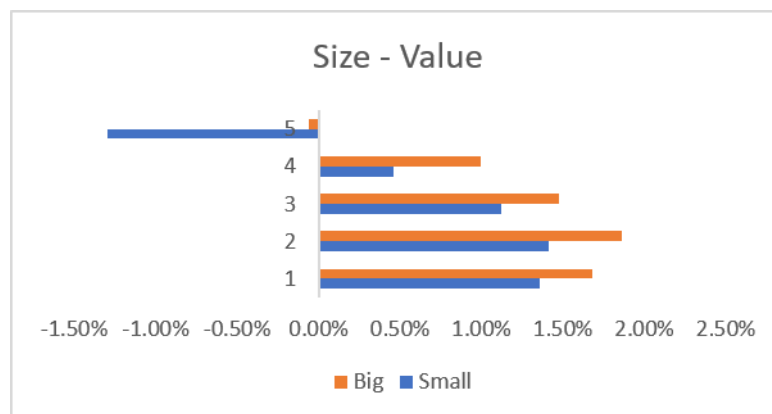


Fig. 4.6: Size - Value (2 x 5) Returns

column there is an increase in the average returns from small stocks to big stocks which is in contrast to what [Fama and French \(2015\)](#) observed in the US market, in which they find that the average returns small stocks are greater than big stocks portfolio returns. Our observation indicates the existence of the size effect observed by [Fama and French \(2015\)](#) but with an opposite effect. The average returns show

a general drop from low value stocks to high value stocks for each size row, with the second value quantile being an exception. The relationship between the value portfolios and average returns is stronger among small stocks as seen in the drop from 1.36% per month to an average of -1.30% per month for small stocks whereas for the big stocks the average returns drop from 1.68% to -0.06%. This observation is also contrary to the results obtained by [Fama and French \(2015\)](#), in which they conclude that for each size row there is an increase in average returns with value (B/M). Moreover, the highest value stocks indicate negative excess average returns in each size row and thus, this observation might be due to the assumption that high value stocks are at a risk of obtaining significant negative returns. High growth stocks have higher returns than high value stocks and the same observation is observed in the South African market by [Karp and van Vuuren \(2017\)](#).

4.2.9 Size – Investment Portfolios

We also sort stocks by size and investment factors independently using the 2 x 5 sorting approach and the resulting average excess returns of the obtained portfolios are shown in Table 4.9:

The investment factor is the variation in total assets for the fiscal year ending

Size - Investment					
	Low Inv	2	3	4	High Inv
Small	0.08%	-0.08%	-0.12%	-0.16%	0.31%
Big	1.62%	1.52%	1.26%	1.28%	1.79%

Tab. 4.9: Size - Investment (2 x 5) Returns

in $t-1$ divided by total assets at the end of year $t-2$ as defined by [Fama and French \(2015\)](#). Negative excess returns are observed in the small size - mid-quantile investment portfolios (2, 3, 4), this observation is worth taking note of. Each size row indicates a decreasing trend in the average returns with the highest investment portfolios being an exception. The same result was obtained in the US market conducted by [Fama and French \(2015\)](#). The extreme growth counterparts with the highest investment indicate the highest returns of 1.79% as compared to the rest of the portfolios. The big stock portfolios show significantly higher returns as compared to the small stock portfolios. This effect is worth taking into consideration as this shows the existence of the size effect across investment portfolios.

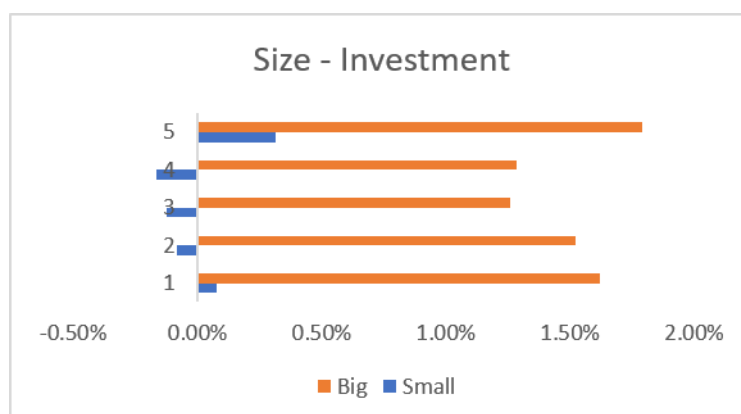


Fig. 4.7: Size - Investment (2 x 5) Returns

Small stocks show a smaller change in returns across the row size from small size - low investment portfolio to small size - high investment portfolio with a change from -0.12% to 0.31% as compared to the change observed in the big portfolio stocks from 1.26% to 1.79%. Fama and French (2015) observe a size effect in these portfolios, concluding that portfolios of smaller stocks have a higher average return than big stocks.

4.2.10 Size – Profitability Portfolios

Portfolios are also sort using the size and profitability factors in the same manner as the previous sorts, resulting in 10 value-weight portfolios. We therefore, seek to understand the patterns and characteristics within these portfolios before using them as test portfolios in evaluating the models. Table 4.10 shows the excess average returns of the ten value-weight portfolios:

There is no obvious relationship between size and profitability in the small size

	Size - Profitability				
	Low OP	2	3	4	High OP
Small	-0.84%	0.14%	-0.16%	0.82%	0.88%
Big	1.07%	1.33%	1.66%	1.61%	1.72%

Tab. 4.10: Size - Profitability (2 x 5) Returns

portfolios. A size effect is observed in each investment column as the big size - investment portfolios show higher returns than the small size – investment port-

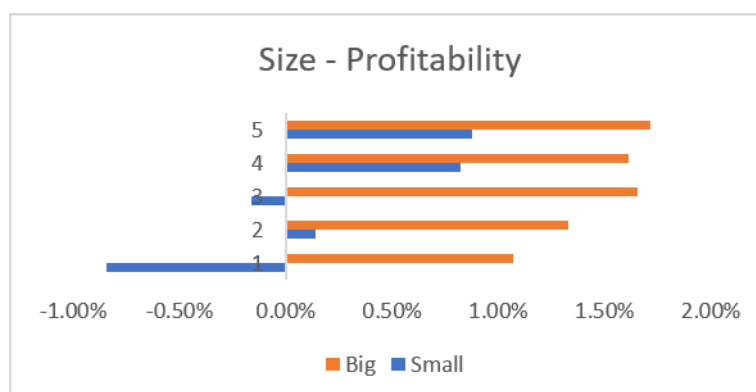


Fig. 4.8: Size - Profitability (2 x 5) Returns

folios. The highest return of 1.72% is observed in the big size – high investment portfolios, whereas the lowest returns of -0.84% are observed in the small size – low investment portfolios. The profitability effect identified by [Novy-Marx \(2013\)](#), [Fama and French \(2015\)](#) and others is present in our study. High profitability stocks are associated with higher returns than low profitability stocks.

4.2.11 Size – Momentum Portfolios

Table 4.11 shows average excess returns for 10 value-weight portfolios from independent sorts of stocks into momentum quantiles and two size portfolios. Momentum represents the prior twelve-month total returns minus the prior month's returns.

There are no obvious relationships between size and momentum and hence, we

Size - Momentum					
	Low Mom	2	3	4	High Mom
Small	-0.78%	0.49%	0.22%	0.54%	0.51%
Big	0.50%	0.15%	0.46%	0.37%	0.95%

Tab. 4.11: Size - Momentum (2 x 5) Returns

can conclude that that there is no significant size bias in the momentum portfolios.

However, high momentum portfolios show much higher returns than low momentum portfolios. We will define this relationship as the momentum effect in the

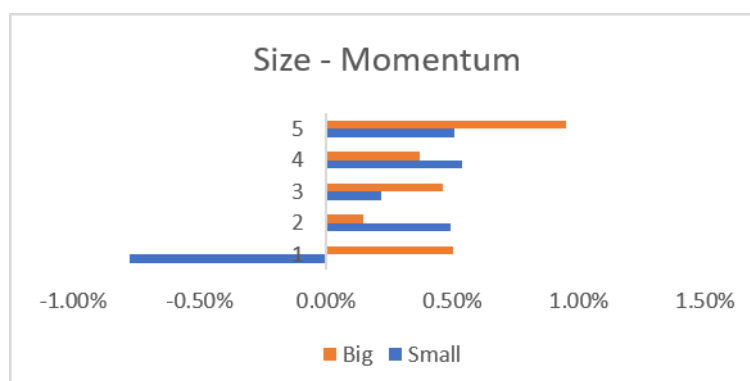


Fig. 4.9: Size - Momentum (2 x 5) Returns

portfolios. The highest return is observed in the big size – high momentum portfolios whereas the lowest return is observed in the small size – low momentum portfolio.

4.2.12 Value – Weight Portfolios (2 x 5 x 5)

We further attempt to disentangle the value, profitability, investment and momentum effects by sorting using three factors with the size factor held constant.

Portfolios formed on Size, Value and Profitability

Table 4.12 shows average returns for 50 Size – B/M – OP portfolios sorted on small and big stocks, quantile value stocks and quantile profitability stocks. In the small size portfolio, there is a significant decrease in average excess returns from the low value and low profitability portfolio to the low value and high profitability portfolio. Negative returns are observed in the small size - extreme high profitability stocks for each level of value. Low profitability portfolios portray much higher returns than high profitability portfolios when controlling for value.

Portfolios controlled on big size show higher excess average returns than the port-

Size - Value (B/M)- Profitability (OP)											
Small	Low Value	2	3	4	High Value	Big	Low Value	2	3	4	High Value
Low OP	1.60%	-1.37%	-1.47%	1.71%	-1.73%	Low OP	1.35%	1.19%	2.23%	0.53%	0.16%
2	0.86%	0.23%	2.45%	0.70%	-0.44%	2	1.24%	1.99%	2.33%	1.37%	0.49%
3	0.83%	1.29%	1.46%	0.43%	-1.95%	3	1.53%	2.55%	1.57%	0.99%	1.40%
4	1.08%	1.61%	1.45%	0.46%	-1.80%	4	2.41%	1.91%	0.95%	0.44%	1.25%
High OP	1.30%	1.62%	0.93%	0.46%	-1.83%	High OP	1.90%	1.33%	1.76%	0.55%	-0.27%

Tab. 4.12: Size - Value - Profitability Returns

folios controlled for small size. The portfolio of stocks in the highest value and profitability quantiles has a low average return of -1.83% per month indicating a negative exposure.

Portfolios formed on Size, Value (B/M) and Investment

Table 4.13 shows average excess returns for the 50 Size – Value – Investment portfolios. For small stocks, high value portfolios for each level of investment show negative returns and hence the lowest returns in all the portfolios.

The highest return of 2.50% is observed in the big stocks second value quantile

Size - Value (B/M) - Investment (Inv)											
Small	Low B/M	2	3	4	High B/M	Big	Low B/M	2	3	4	High B/M
Low Inv	2.22%	0.68%	1.91%	0.10%	-0.94%	Low Inv	1.83%	1.61%	1.61%	2.15%	-0.95%
2	-1.17%	1.81%	0.89%	0.21%	-1.48%	2	1.59%	2.13%	1.37%	1.42%	-0.53%
3	1.14%	0.61%	0.82%	0.93%	-1.75%	3	1.39%	1.10%	1.59%	0.51%	1.28%
4	1.07%	1.73%	1.10%	-0.19%	-2.27%	4	1.34%	1.59%	1.35%	0.30%	0.87%
High Inv	0.24%	1.65%	1.34%	0.43%	-1.16%	High Inv	1.62%	2.50%	1.80%	0.92%	1.42%

Tab. 4.13: Size - Value - Investment Returns

and second investment quantile portfolio followed by the small size – low value – low investment portfolio which is an interesting observation. The patterns in average returns in the small stocks are like those of the big stocks.

Portfolios formed on Size, Value (B/M) and Momentum

Table 4.14 shows average excess returns for 50 Size – Value – Momentum portfolios. Small stocks with high value for each level of investment show negative returns and hence the lowest returns in all the portfolios. In the big stocks, the stocks with extreme low momentum and extreme high value show the lowest and negative return of -1.30%. For the small portfolio, the low value portfolio and high momentum portfolio show a negative return of -2.09% which is the lowest in the entire portfolio sorts.

Size - Value (B/M) - Momentum (Mom)											
Small	Low B/M	2	3	4	High B/M	Big	Low B/M	2	3	4	High B/M
Low Mom	1.87%	0.21%	1.15%	0.38%	-1.93%	Low Mom	2.49%	1.16%	1.52%	1.31%	-1.30%
2	0.50%	1.76%	1.02%	0.66%	-1.03%	2	1.40%	0.69%	1.90%	0.72%	0.55%
3	1.52%	1.57%	0.32%	0.66%	-0.41%	3	1.80%	1.55%	0.85%	0.92%	1.11%
4	1.64%	0.78%	1.21%	0.35%	-0.83%	4	1.80%	2.57%	1.16%	1.26%	1.98%
High Mom	-2.09%	1.98%	2.00%	-0.54%	-0.56%	High Mom	1.91%	2.50%	1.57%	0.85%	0.61%

Tab. 4.14: Size - Value - Momentum Returns

Portfolios formed on Size, Profitability and Investment

Table 4.15 shows average excess returns for the 50 Size – Profitability – Investment portfolios. For small stocks with low profitability, there are negative returns associated with each level of investment. This evidence is quite interesting as this represents the type of counterparts that are low profitable but still invest a lot. High profitability stocks are associated with high average excess returns and this is known as the profitability effect identified by [Novy-Marx \(2013\)](#) and [Fama and French \(2015\)](#). The size effect is also evident in this table, big stocks are associated with higher returns as compared to small stocks.

Size - Profitability (OP) - Investment (Inv)											
Small	Low OP	2	3	4	High OP	Big	Low OP	2	3	4	High OP
Low Inv	-1.08%	-0.03%	0.13%	1.01%	1.39%	Low Inv	0.47%	2.05%	1.97%	1.40%	1.90%
2	-1.16%	0.06%	-0.73%	0.30%	1.26%	2	0.88%	1.08%	1.03%	1.66%	2.33%
3	-2.02%	-0.12%	0.26%	0.17%	0.35%	3	1.54%	0.95%	1.73%	1.03%	0.82%
4	-2.34%	-0.10%	-0.93%	0.76%	1.60%	4	0.85%	1.69%	1.16%	1.89%	0.89%
High Inv	-1.13%	0.81%	-0.44%	1.40%	0.76%	High Inv	2.70%	1.29%	1.44%	2.09%	1.82%

Tab. 4.15: Size - Profitability - Investment Returns

Portfolios formed on Size, Profitability and Momentum

Table 4.16 shows average excess returns for the 50 Size – Profitability – Momentum portfolios. Small stocks with low profitability still behave the same way as the in the previous portfolios. They show negative returns; this is expected at all momentum levels since these stocks are expected to perform in the same manner as they performed in their previous period.

Size - Value (B/M) - Momentum (Mom)											
Small	Low B/M	2	3	4	High B/M	Big	Low B/M	2	3	4	High B/M
Low Mom	1.87%	0.21%	1.15%	0.38%	-1.93%	Low Mom	2.49%	1.16%	1.52%	1.31%	-1.30%
2	0.50%	1.76%	1.02%	0.66%	-1.03%	2	1.40%	0.69%	1.90%	0.72%	0.55%
3	1.52%	1.57%	0.32%	0.66%	-0.41%	3	1.80%	1.55%	0.85%	0.92%	1.11%
4	1.64%	0.78%	1.21%	0.35%	-0.83%	4	1.80%	2.57%	1.16%	1.26%	1.98%
High Mom	-2.09%	1.98%	2.00%	-0.54%	-0.56%	High Mom	1.91%	2.50%	1.57%	0.85%	0.61%

Tab. 4.16: Size - Profitability - Momentum Returns

Portfolios formed on Size, investment and Momentum

Table 4.17 shows average excess returns for the 50 Size – Investment – Momentum portfolios. Small stocks with low investment are associated with negative returns across all levels of momentum with the high momentum levels having the lowest

returns. The highest returns of 2.24% and 2.11% are observed in the big stocks with high investment and high momentum.

Size - Investment (Inv) - Momentum (Mom)											
Small	Low Inv	2	3	4	High Inv	Big	Low Inv	2	3	4	High Inv
Low Inv	-0.50%	-0.73%	-0.07%	-1.24%	-1.22%	Low Inv	0.75%	0.57%	1.47%	0.49%	1.78%
2	-0.32%	1.25%	-0.07%	0.50%	0.02%	2	0.47%	1.46%	1.48%	0.85%	1.57%
3	1.16%	0.17%	-0.41%	-0.72%	0.81%	3	2.05%	0.81%	1.50%	1.50%	1.02%
4	1.07%	0.15%	0.96%	0.49%	0.57%	4	2.05%	1.74%	0.74%	1.26%	1.89%
High Inv	-0.43%	0.63%	1.65%	1.30%	0.83%	High Inv	1.67%	1.86%	1.37%	2.24%	2.11%

Tab. 4.17: Size - investment - Momentum Returns

4.3 Factor Spanning Test

In this section we investigate how the factors explain each other. We regress each factor against the rest of the factors. This method seeks to statistically test whether adding a set of factors can improve the performance of an asset pricing model. In other words, it seeks to test if each factor improves the performance of the model in describing average portfolio returns. We use seven factors in regression to explain the average returns on the eighth factor. For consistency and clarity we follow the notations and approach in [Fama and French \(2015\)](#). The 8 factors are regressed as shown in Table 4.18.

$R_M - R_F$ is the value-weight return on the market portfolio of all the sample stocks less the one-month JIBAR rate; SAFI (South African Financial Index) is the financial index in the South African JSE; SARI (South African Resources Index) is the resources index in the South African JSE; SMB (Small Minus Big) is the size factor; HML(High Minus Low B/M) is the value factor; CMA (Conservative Minus Aggressive investment) is the investment factor; RMW (Robust Minus Weak profitability) is the profitability factor and Mom (Momentum) is the momentum factor.

The factor spanning tests suggest that SAFI and the momentum are statistically significant with t-values of 5.17 and 3.47, however dropping the remaining factors does not improve the model in explaining the average portfolio returns. The intercepts indicate that the SMB, CMA, RMW and HML factors are better explained by the remaining factors as portrayed by the intercept values that tend to zero. Whereas the R-squared values show that the HML, $R_M - R_F$, SMB and momentum factors are better explained by the remaining factors as compared to other factors. Combining these findings suggest that adding the HML, SMB and the CMA factor

	Intercept	Rm-Rf	SAFI	SARI	SMB	HML	CMA	RMW	Mom	R-squared
Rm - Rf										
Coef	0.93%		-4.84%	12.57%	-25.94%	-9.64%	-60.66%	0.45%	-8.08%	27.69%
t-stat	1.18		-0.44	1.52	-1.40	3.36*	-2.98*	0.02	-0.39	
SAFI										
Coef	2.38%	-3.42%		14.23%	23.69%	-45.65%	9.63%	-19.98%	-33.99%	7.06%
t-stat	5.17*	-0.44		1.42	1.29	-2.35	0.56	-1.11	-3.32*	
SARI										
Coef	0.58%	15.97%	25.58%		-26.37%	9.72%	36.26%	-6.88%	9.18%	3.50%
t-stat	0.66	1.38	1.44		-1.03	0.46	1.58	-0.33	0.37	
SMB										
Coef	-0.11%	-5.15%	6.66%	-4.13%		35.93%	-8.93%	11.45%	4.33%	18.40%
t-stat	-0.37	-1.35	1.24	-1.01		3.49*	-0.77	1.06	0.67	
HML										
Coef	0.57%	-18.05%	-12.10%	1.43%	33.86%		-6.70%	-47.58%	-24.77%	41.77%
t-stat	1.95	-2.66*	-2.60*	0.46	3.54		-0.71	-1.59	-4.76	
CMA										
Coef	-0.19%	-12.01%	2.70%	5.65%	-8.89%	-7.08%		-3.84%	8.96%	4.93%
t-stat	-0.60	-3.02*	0.57	1.44	-0.81	-0.68		-0.40	0.98	
RMW										
Coef	0.26%	0.08%	-5.26%	-1.01%	10.72%	-14.64%	-3.61%		6.02%	-0.60%
t-stat	0.78	0.02	-1.09	-0.33	1.16	-1.61	-0.41		0.67	
Mom										
Coef	1.38%	-2.89%	-17.18%	2.58%	7.78%	-47.25%	16.17%	11.55%		14.38%
t-stat	3.47*	-0.39	-2.66*	0.36	0.64	-3.82	0.91	0.67		

Tab. 4.18: Factor Spanning

might not improve the asset pricing model. However, in the next sections we observe that the combined effect of the factors actually improves the performance of the eight-factor model when compared to the seven, six and five factor models.

The intercepts obtained from the regressions indicate presence of causation effects, moreover the R-squared value of the value factor is higher than that of the other factors. Fama and French (2015) concluded that the value factor in their research is redundant. They find evidence that adding the value factor does not improve the mean-variance efficient tangency portfolio. This is in line with our observations as shown by the R-squared of 41.77% as compared to the other factors. Furthermore, the test suggests that SAFI and Momentum are significant factors. The spanning tests suggests that the SMB factor and the CMA factor are explained by the other factors and hence add little to the description of expected returns of the South African equities market. The SMB intercept has an intercept of 0.11 ($t = 0.37$) which indicates that the factor might be redundant. However, the factor spanning inferences are said to be sample definitive. Further characteristics and significance of these factors will be investigated in the asset pricing tests section.

4.4 Asset Pricing Regression Details

We seek to discuss findings obtained from regressions applied to sorted portfolios. This section provides the path to comparison of the asset pricing models. We

examine regression details, more importantly the intercepts. In evaluating model performance, we pay attention to the exposures of the factor sorted portfolios to the factors under consideration. We seek to understand how the average return explanations change from the CAPM to the Eight – Factor model. In assessing and evaluating asset pricing models, if the intercept is indistinguishable from zero in the regression on any portfolio's excess returns then it is said to fully explain average portfolio returns. Hence, the closer the intercept to the origin (0.00%) the better the performance of the model in explaining the returns of the portfolio.

4.4.1 CAPM Regression Results

16 Value Weight Portfolios (2 x 2 sort)

CAPM regressions are run for the period 2006 – 2019 for 16 (2 x 2) portfolios and 40 portfolios sort on (2 x 5) sorts. We regress each value-weight portfolio using the market factor as the only explanatory variable.

Table 4.19 shows 2 x 2 sorted value weight portfolios regressed against the market

CAPM Intercepts					
a			t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.45%	-0.81%	Small Size	3.03*	-1.90
Big Size	1.35%	0.20%	Big Size	3.43*	0.41
<hr/>					
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.02%	0.27%	Small Size	0.04	0.60
Big Size	0.94%	1.08%	Big Size	2.17*	2.59*
<hr/>					
OP	Low	High	OP	Low	High
Small Size	-0.24%	0.96%	Small Size	-0.54	2.28*
Big Size	0.87%	1.12%	Big Size	2.03*	2.68*
<hr/>					
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.28%	0.83%	Small Size	-0.58	1.92
Big Size	0.86%	0.72%	Big Size	1.59	1.69

Tab. 4.19: CAPM Intercepts (2 x 2)

factor. We observe an intercept closest to zero in small stocks with low investments with a value of 0.02% (t = 0.04). Negative intercepts are observed in small stocks with high B/M (-0.81%), small stocks with low OP (-0.24) and in small stocks with low momentum (-0.28%). The CAPM intercepts are statistically significant in 6 out of 16 portfolios.

40 Value – Weight portfolios (2 x 5 sort)

CAPM Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.08%	1.17%	0.93%	0.20%	-1.53%	Small Size	1.83	2.45*	2.05*	0.45	-3.20*
Big Size	1.39%	1.62%	1.23%	0.77%	-0.53%	Big Size	3.42*	3.71*	2.74*	1.63	-0.82
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	-0.14%	-0.32%	-0.33%	-0.38%	0.01%	Small Size	-0.30	-0.75	-0.65	-0.80	0.03
Big Size	1.28%	1.29%	1.00%	1.01%	1.59%	Big Size	2.88*	2.84*	2.65*	2.32*	3.45*
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.07%	-0.11%	-0.37%	0.57%	0.62%	Small Size	-2.15*	-0.25	-0.77	1.24	1.43
Big Size	0.81%	1.07%	1.39%	1.35%	1.43%	Big Size	1.88	2.55*	3.12*	3.13*	3.40*
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-0.97%	0.26%	-0.03%	0.30%	0.22%	Small Size	-1.79	0.63	-0.08	0.72	0.46
Big Size	0.77%	0.88%	1.12%	1.45%	1.61%	Big Size	1.27	1.74	2.64*	3.49*	3.63*

Tab. 4.20: CAPM Intercepts (2 x 5)

Intercepts that tend to 0.00% are observed in small stocks with high investment (2 x 5) sorts with a value of (0.01%) and in small stocks in the third momentum quantile (2 x 5) sorts with a value of (-0.03). However, these intercepts are statistically insignificant in the model as portrayed by the t -static values that fall within the critical region. The CAPM intercepts are statistically significant in 18 out of 40 portfolios sorted using (2 x5) sorts. For detailed explanations, we describe our findings according to the specific portfolios sorted using each of the factors.

Size – B/M portfolios

The intercepts obtained using CAPM on the portfolios sorted using Size and B/M portfolios indicate intercepts ranging from 0.20 to 1.45 on all portfolios sorted using both 2 x 2 and 2 x 5 sorts. Small stocks with high value indicate a high negative intercept of -0.81%. The CAPM intercepts are negative for high value stocks and positive for the rest of the portfolios. The results suggest that the CAPM better explains the portfolio formed on big stocks with high value for both the 2 x 2 sort and 2 x 5 sort.

Size – Investment portfolios

Small stocks with high investment portray an intercept of 0.01 which is close to zero but observed not to be statistically significant in the model. This low intercept indicates that the CAPM performs better when applied to the portfolio of small stocks

with high investment. Furthermore, the CAPM performs better in small stocks than in big stocks when sorted for size and investment, as observed by the small stock intercepts that are closer to 0.00% as compared to the big stock intercepts for both (2 x 2) sorts and (2 x 5) sorts.

Size – Profitability portfolios

The intercepts for portfolios sort on size and operational profitability show small intercept values for small stocks on the 2nd and 3rd quantile of OP, indicating that the CAPM performs well in explaining the returns of these portfolios than the rest of the portfolios. The model does not perform well in explaining the returns of the portfolio of big stocks with high investment. Moreover, the CAPM explains returns of small stock portfolios better than the big stock portfolios excluding the small size - low investment portfolio.

Size – Momentum portfolios

Small stocks on the 3rd quantile of momentum have an intercept value that is closest to zero as compared to the rest of the portfolios. This portrays that the CAPM better explains the returns for the portfolio of smalls stocks that lie on the 3rd quantile of the momentum factor as compared to the rest of the portfolios. Big stocks of high momentum are poorly explained by the CAPM, this portfolio indicates the farthest intercept value (1.61%) from 0.00%.

4.4.2 Two Factor Asset Pricing Theory (APT) Regression Results

We run regressions using the two-factor model consisting of the South African Financial Index (SAFI) and the South African Resources Index (SARI) in a similar manner conducted by [Laird-Smith *et al.* \(2016\)](#), in which they investigate the total beta as the symmetric and stable risk measure in the Johannesburg Stock Exchange (JSE). They conclude that the total beta estimated from the two factor model is a more stable estimator for risk and return in the JSE. Their study emanates from findings obtained by [Van Rensburg \(2002\)](#) that shows evidence that the two factor Asset Pricing Theory model constituting of the JSE Financial-Industrial (CI21) and Resources (CI11) indices explains returns better than the CAPM. They further assert that the All Share Index (market proxy) is not mean variance efficient in the South African market thus implying that the CAPM does not hold in the JSE. We conduct a different test of the two factor model by testing the performance of the two factor model using portfolios sort on size and the other risk factors of interest

in our research. We therefore, first test the model on portfolios sorted using the 2 x 2 sorts and then proceed to more distributed portfolios sort using the 2 x 5 approach.

16 Value Weight Portfolios (2 x 2 sort)

The two factor APT model intercepts for most of the portfolios perform similarly to those obtained using the Capital Asset Pricing Model (CAPM) in the previous section.

The intercepts obtained from the 2 x 2 sorts regressions range from -0.75% to 1.44%,

Two - Factor Intercepts					
	a			t(a)	
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.44%	-0.75%	Small Size	1.86	-1.05
Big Size	1.42%	0.39%	Big Size	2.05*	0.47
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.06%	0.39%	Small Size	0.09	0.51
Big Size	1.01%	1.10%	Big Size	1.42	1.50
OP	Low	High	OP	Low	High
Small Size	-0.25%	1.06%	Small Size	-0.35	1.53
Big Size	1.01%	1.18%	Big Size	1.33	1.62
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.30%	0.86%	Small Size	-0.39	1.16
Big Size	1.01%	0.76%	Big Size	1.20	1.10

Tab. 4.21: Two - Factor Model Intercepts (2 x 2)

with small stocks showing intercepts that are much closer to zero. The intercept associated with big stocks of low value is the only one that is statistically significant out of all the portfolios. We further observe that the two factor model performs better in explaining returns of the portfolios sort using size and investment risk factors. As observed in the correlations matrix in the previous sections, the South African Resources Index and the South African Financial Index are not correlated to each other suggesting that they independently explain returns of stocks when applied to asset pricing models.

40 Value – Weight portfolios (2 x 5 sort)

We further test the two-factor APT model of [Laird-Smith et al. \(2016\)](#) to portfolios sort using the 2 x 5 sorts approach. We observe that the regression intercepts

range from -1.47% to 1.78%. There are only seven statistically significant intercepts obtained from regressing the two factors against the 40 portfolios. Small stocks indicate negative intercepts, and these are mostly evident in small stocks sorted with respect to investment. The two factor model improves the intercepts of portfolios of small stocks sorted on investment quantiles and this supports the findings we observe when the model is applied to the 2 x 2 sorted portfolios.

Two-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.05%	1.16%	0.95%	0.12%	-1.47%	Small Size	1.22	1.62	1.30	0.17	-1.90
Big Size	1.42%	1.78%	1.22%	0.71%	0.28%	Big Size	2.01*	2.45*	1.69	0.98	0.30
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	-0.17%	-0.20%	-0.33%	-0.37%	0.18%	Small Size	-0.21	-0.29	-0.44	-0.51	0.22
Big Size	1.53%	1.22%	1.06%	1.02%	1.48%	Big Size	1.96	1.78	1.50	1.38	2.02*
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.27%	0.07%	-0.36%	0.55%	0.84%	Small Size	-1.67	0.09	-0.48	0.75	1.17
Big Size	1.00%	1.03%	1.45%	1.47%	1.49%	Big Size	1.35	1.52	2.08*	1.96	2.01*
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-1.00%	0.32%	0.06%	0.39%	0.17%	Small Size	-1.22	0.48	0.09	0.54	0.22
Big Size	0.96%	0.80%	1.10%	1.58%	1.66%	Big Size	1.14	1.02	1.58	2.24*	2.27*

Tab. 4.22: Two - Factor Model Intercepts (2 x 5)

Size – B/M portfolios

The intercepts obtained using the Fama and French three factor model on the portfolios sorted using size and B/M portfolios show intercepts ranging from -1.47% to 1.78% on all portfolios sorted using both 2 x 2 and 2 x 5 sorts. The intercepts closest to the value 0.00 are observed in small stocks on the fourth quantile of the value factor, thus indicating that the two factor APT model performs better in this portfolio as compared to the other portfolios sorted on size and value factors. Small stocks with high value have a strong negative intercept of -1.47%. The two factor performs better in big stocks with high value as compared to the rest of the big stock portfolios and some of the small stock portfolios.

Size – Investment portfolios

Small stocks in the first four quantiles of the investment factor show negative intercepts which are also closer to 0.00%. The model explains the returns of portfolios of small stocks better than big stocks when sorting using the size and investment factors. The two-factor model performs better when applied to a portfolio of small

stocks with low investment and high investment. This could be as a result of firms that have a small market capitalization but invest a lot in the South African market.

Size – OP portfolios

The intercepts for portfolios sort on size and profitability show intercept values that are converging to 0.00% for small stocks on the 2nd quantile of the profitability factor (OP). A statistically significant intercept that strongly converges to zero is associated with big stocks that have low profitability. The model produces intercepts that are statistically insignificant for small stock portfolios, which could be a result of the returns inequality in the South African equities market. The two factor model performs better in the portfolio of small stocks that lie on the 2nd quantile of the profitability factor as shown by the intercept value of 0.07%. in smalls stocks and big stocks in the JSE.

Size – Momentum portfolios

The two-factor model performs better when applied to the portfolio of stocks formed on small stocks that lie on the third quantile of the momentum factor as shown by an intercept value of 0.06% that is closer to 0.00%. The portfolio of small stocks with low momentum has a negative intercept value of -1.00% which is the furthest from the origin among the small stock portfolios. Only two portfolios have statistically significant intercepts in the size - momentum portfolios.

4.4.3 Fama and French 3 Factor Regression Results

The intercepts for the Fama and French three factor asset pricing model are shown in tables 4.24 and 4.25 for both 2 x 2 sorts and 2 x 5 sorts respectively. Intercepts closest to zero by observation are associated with the portfolios containing small stocks with low profitability, small stocks with low momentum and big stocks with high value for which we conclude that the three factor model explains the returns of these portfolios better than the rest of the portfolios under consideration.

16 Value Weight Portfolios (2 x 2 sort)

There are ten statistically significant intercepts in the regressions conducted for each of the portfolios sort using the 2 x 2 sort. The three factor model better ex-

Three - Factor Intercepts					
	a		t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.64%	-0.68%	Small Size	3.67*	-1.63
Big Size	1.48%	0.36%	Big Size	3.95*	0.77
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.18%	0.40%	Small Size	0.44	0.89
Big Size	1.07%	1.18%	Big Size	2.59*	2.94*
OP	Low	High	OP	Low	High
Small Size	-0.08%	1.08%	Small Size	-0.20	2.59*
Big Size	1.02%	1.23%	Big Size	2.59*	3.03*
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.13%	0.96%	Small Size	-0.29	2.28*
Big Size	1.06%	0.86%	Big Size	2.13*	2.11

Tab. 4.23: Fama and French Three Factor Model Intercepts (2 x 2)

plains the returns of the portfolio of small stocks with low profitability as indicated by the intercept value of -0.08%.

40 Value – Weight portfolios (2 x 5 sort)

Small stocks through-out all the portfolios have on average, intercepts that converge to zero as compared to big stocks but however, appear to be statistically insignificant in most cases. This observation indicates that the Fama and French three factor model better explains the returns of small stocks than those of portfolios of big stocks. There are 20 statistically significant intercepts in the three-factor model applied to 40 portfolios sorted using the 2 x 5 sorting approach.

Size – B/M portfolios

Intercepts obtained using the Fama and French three factor model on the portfolios sorted using size and B/M portfolios indicate intercepts ranging from -1.40% to 1.70% on all portfolios sorted using both 2 x 2 and 2 x 5 sorts. The intercept closest to the origin (0.00%) is observed in the portfolio of small stocks on the fourth quantile of the value factor. Hence, suggesting that the three factor model explains the returns of this portfolio better than the other portfolios sort on size and value. The model performs better in the portfolio of big stocks with high value when compared to the rest of the big stocks portfolio formed using the remaining quantiles and factors.

Three-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.28%	1.34%	1.01%	0.32%	-1.40%	Small Size	2.32*	2.93*	2.23*	0.73	-2.99*
Big Size	1.51%	1.70%	1.35%	0.89%	-0.41%	Big Size	3.95*	3.92*	3.15*	1.94	-0.67
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	0.03%	-0.22%	-0.17%	-0.25%	0.10%	Small Size	0.07	-0.53	-0.33	-0.54	0.21
Big Size	1.39%	1.44%	1.09%	1.12%	1.70%	Big Size	3.14*	3.31*	2.66*	2.67*	3.78*
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-0.90%	0.02%	-0.25%	0.67%	0.75%	Small Size	-1.94	0.05	-0.53	1.48	1.72
Big Size	0.96%	1.22%	1.50%	1.45%	1.53%	Big Size	2.35*	3.12*	3.47*	3.45*	3.67*
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-0.85%	0.42%	0.08%	0.45%	0.32%	Small Size	-1.60	1.05	0.19	1.12	0.67
Big Size	0.99%	0.98%	1.24%	1.59%	1.68%	Big Size	1.77	1.98	3.03*	4.04*	3.88*

Tab. 4.24: Fama and French Three Factor Model Intercepts (2 x 5)

Size – Investment portfolios

The portfolio of small stocks with low investment portray an intercept of 0.03% ($t = 0.07$) which is close to zero, suggesting that the three factor model performs better when applied to this portfolio. The three-factor model has an intercept that is statistically significant and closer to 0.00% when applied to big stocks that lie on the third quantile of the investment factor. The model performs poorly on big stocks with high investment.

Size – OP portfolios

The intercepts for portfolios sort on size and operational profitability (OP) show intercept values that are converging to 0.00% for small stocks on the 2nd and 3rd quantile of OP. A statistically significant intercept that strongly converges to zero is associated with big stocks that have low profitability. The model produces intercepts that are statistically insignificant for small stock portfolios, which could be a result of a returns inequality in smalls stocks and big stocks in the JSE.

Size – Momentum portfolios

Small stocks in the 3rd quantile of momentum have an intercept value that is closest to zero as compared to the rest of the portfolios. The portfolio of big stocks with high momentum have the farthest intercept value from zero, thus portraying that the three factor model performs poorly in explaining the returns of this portfolio.

4.4.4 Carhart 4-Factor Regression Results

We further extend Fama and French's three factor model to a four-factor model applied by Carhart (1997). This four-factor model adds the momentum factor to the Fama and French three factors. We first apply the model to portfolios sorted using 2 x 2 sorts and then extend it to factors sorted using 2 x 5 sorts.

16 Value Weight Portfolios (2 x 2 sort)

The four-factor model produces regression intercepts that are statistically significant in most of the portfolios as compared to the previous three asset pricing models, suggesting that the model improves the estimated intercepts when applied to the portfolios sorted on size and the other risk factors. The model statistically performs well in small stocks with high momentum, this portfolio has a statistically significant intercept value of 0.84% (t = 2.11).

Four - Factor Intercepts					
a			t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.50%	-0.87%	Small Size	3.51*	-2.17*
Big Size	1.31%	0.20%	Big Size	3.53*	0.43
<hr/>					
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.06%	0.22%	Small Size	0.16	0.51
Big Size	0.85%	1.04%	Big Size	2.11*	2.73*
<hr/>					
OP	Low	High	OP	Low	High
Small Size	-0.28%	0.90%	Small Size	-0.72	2.26*
Big Size	0.93%	1.01%	Big Size	2.34*	2.67*
<hr/>					
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.33%	0.84%	Small Size	-0.72	2.11
Big Size	0.90%	0.63%	Big Size	1.77	1.62

Tab. 4.25: Carhart Four Factor Model Intercepts (2 x 2)

40 Value – Weight portfolios (2 x 5 sort)

Table 4.26 shows estimated intercepts for the portfolios from the 2 x 5 Size – Factor sorts. The portfolio that is better explained by the four factor model is the portfolio of small stocks on the third quantile of the momentum factor with a value of -0.09%. There are 19 portfolios that have statistically significant regression intercepts out of 40 portfolios.

Four-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.16%	1.14%	0.84%	0.17%	-1.60%	Small Size	2.15*	2.57*	1.94	0.41	-3.47*
Big Size	1.41%	1.54%	1.09%	0.64%	-0.69%	Big Size	3.61*	3.51*	2.69*	1.47	-1.08
<hr/>											
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	0.01%	-0.40%	-0.44%	-0.49%	-0.06%	Small Size	0.01	-0.97	-0.93	-1.09	-0.13
Big Size	1.29%	1.24%	0.84%	0.90%	1.55%	Big Size	2.74*	2.89*	2.02*	2.28*	3.60*
<hr/>											
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.08%	-0.18%	-0.47%	0.56%	0.57%	Small Size	-2.36*	-0.42	-1.02	1.26	1.32
Big Size	0.85%	1.06%	1.25%	1.28%	1.31%	Big Size	1.98	2.63*	3.01*	3.19*	3.27*
<hr/>											
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-1.04%	0.18%	-0.09%	0.34%	0.16%	Small Size	-1.97	0.47	-0.22	0.87	0.35
Big Size	0.75%	0.74%	1.00%	1.42%	1.58%	Big Size	1.26	1.56	2.61*	3.46*	3.56*

Tab. 4.26: Carhart Four Factor Model Intercepts (2 x 5)

Size – B/M portfolios

The intercepts obtained using the Fama and French four factor model on the portfolios sorted using Size and B/M portfolios indicate intercepts ranging from -1.60% to 1.54% on all portfolios sorted using both 2 x 2 and 2 x 5 sorts. The four-factor model improves the intercept convergence to zero when compared to the other models applied on the same portfolios. The intercepts statistically closest to the value of 0.00 are observed in big stocks on the third quantile of the value factor. There are 6 intercepts that are statistically significant and these are evenly spread between the big stocks and smalls stocks.

Size – Investment portfolios

Small stocks with extreme low investment portray a lower intercept value of 0.01% ($t = 0.01$) which is very close to zero, but the intercept is not statistically significant. We observe that the four-factor model draws the intercept closer to 0.00 on this portfolio as compared to the other models despite the intercepts being statistically rejected. The four-factor model has an intercept that is statistically significant and closer to 0.00% when applied to big stocks that lie on the third quantile of the investment factor, the model can be assumed to capture returns of this portfolio better than the rest of the portfolios. The regression intercepts are only statistically significant for big stock portfolios, therefore suspecting the investment effect being at play.

Size – OP portfolios

There are four statistically significant intercepts associated with the profitability portfolios formed using 2 x 5 sorts. The intercepts for portfolios sort on size and operational profitability show an intercept value that is converging to 0.00% for big stocks with low profitability. This observation shows that the four factor model explains this portfolio better among the big stock portfolios.

Size – Momentum portfolios

There are only two portfolios with statistically significant intercepts in the size – momentum portfolios and these are portfolios of big stocks with high momentum. The four factor model performs well when applied to the portfolio of small stocks that lie on the 3rd quantile of momentum as shown by an intercept value of -0.09% which is the closest to the origin when compared to the rest of the portfolios.

4.4.5 Fama and French 5 Factor Regression Results

16 Value Weight Portfolios (2 x 2 sort)

Table 4.27 shows five factor intercepts obtained from regressing the [Fama and French \(2015\)](#) factors against the sixteen value-weight portfolios. The intercepts range from -0.69% to 1.65%, this range is much closer to zero as compared to the other models applied to the same portfolios. Suggesting that the model improves the intercepts.

Five - Factor Intercepts					
a			t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.65%	-0.69%	Small Size	3.68*	-1.64
Big Size	1.48%	0.35%	Big Size	3.92*	0.75
CMA (Investment)			CMA (Investment)		
Small Size	0.19%	0.39%	Small Size	0.46	0.87
Big Size	1.07%	1.17%	Big Size	2.56*	2.89*
OP			OP		
Small Size	-0.08%	1.07%	Small Size	-0.19	2.55*
Big Size	1.01%	1.22%	Big Size	2.55*	3.02*
Mom (Momentum)			Mom (Momentum)		
Small Size	-0.13%	0.95%	Small Size	-0.28	2.26*
Big Size	1.06%	0.85%	Big Size	2.11*	2.09*

Tab. 4.27: Fama and French Five Factor Model Intercepts (2 x 2)

40 Value – Weight portfolios (2 x 5 sort)

Having run the five-factor model on sixteen portfolios sorted using 2 x 2 sorts, we further apply the model to forty portfolios sorted using 2 x 5 sorts in order to better assess the performance of the model.

The five-factor model significantly captures portfolio returns associated with small

Five-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.30%	1.35%	0.99%	0.31%	-1.40%	Small Size	2.35*	2.96*	2.19*	0.72	-3.00*
Big Size	1.51%	1.68%	1.33%	0.88%	-0.40%	Big Size	3.93*	3.90*	3.11*	1.90	-0.67
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	0.04%	-0.22%	-0.17%	-0.26%	0.09%	Small Size	0.09	-0.53	-0.34	-0.57	0.18
Big Size	1.37%	1.43%	1.09%	1.10%	1.70%	Big Size	3.12*	3.29*	2.65*	2.67*	3.77*
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-0.89%	0.03%	-0.27%	0.67%	0.74%	Small Size	-1.91	0.06	-0.58	1.46	1.70
Big Size	0.94%	1.22%	1.49%	1.44%	1.51%	Big Size	2.32*	3.10*	3.43*	3.43*	3.65*
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-0.85%	0.41%	0.07%	0.46%	0.30%	Small Size	-1.59	1.02	0.16	1.15	0.64
Big Size	0.99%	0.97%	1.24%	1.61%	1.65%	Big Size	1.74	1.97	3.01*	4.11*	3.79*

Tab. 4.28: Fama and French Five Factor Model Intercepts (2 x 5)

stocks that lie on the second quantile of the profitability factor, this portfolio shows an intercept value of 0.03%. There are 19 out of 40 portfolios that have statistically significant intercepts. There are much less portfolios with negative intercepts as compared to the models discussed in the previous sections.

Size – B/M portfolios

The intercepts obtained using the Fama and French five factor model on the portfolios sorted using size and B/M portfolios indicate intercepts ranging from -1.40% to 1.69% on all portfolios sorted using both 2 x 2 and 2 x 5 sorts. The five-factor model improves the intercepts as portrayed by intercepts converging to the origin. Six intercepts are statistically significant in this classification. Small stocks lying on the third quantile of the value factor show a statistically significant intercept value of 0.99% that is much closer to 0.00. Extreme value stocks portray negative intercepts.

Size – Investment portfolios

Small stocks with low investment portray a lower intercept value of 0.04% ($t = 0.09$) which is close to zero, indicating that the five factor model performs well when ap-

plied to this portfolio. We observe that the five-factor model draws the intercept closer to 0.00 on the portfolios of both small and high investment (0.09%). The five-factor model has an intercept that is statistically significant and closer to 0.00% when applied to big stocks with low profitability. The regression intercepts are only statistically significant for all the big stock portfolios, therefore suspecting that the model captures the investment effect.

Size – OP portfolios

All big stocks sorted on profitability have intercepts that are statistically significant. The intercepts for portfolios sort on size and operational profitability show an intercept value of 0.03% for small stocks lying on the second quantile of the profitability factor. This is the closest intercept to the origin (0.00%), hence, indicating that the five factor model better explains the returns associated with this portfolio.

Size – Momentum portfolios

Small stocks on the 3rd quantile of momentum have an intercept value that is closest to zero as compared to the rest of the portfolios. This observation suggest that the model performs well in explaining the returns of the portfolio associated with the intercept closest to the origin. There are only three portfolios with statistically significant intercepts in the size – momentum portfolios and these portfolios only consist of the big stocks.

4.4.6 Six – Factor Regressions

16 Value Weight Portfolios (2 x 2 sort)

The intercepts obtained from running the six-factor model on the portfolios sorted on 2 x 2 sorts are shown in Table 4.29. Small stocks with low investment portray an intercept value of 0.07%, which is the closest to 0.00% as compared to the rest of the portfolios. However, this intercept rejects the hypothesis that the intercept is statistically significant in the six-factor model. The six factor model portrays intercepts that are closer to the origin when applied to portfolios formed on size and investment.

Six - Factor Intercepts					
a			t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.51%	-0.87%	Small Size	3.51*	-2.16*
Big Size	1.32%	0.20%	Big Size	3.49*	0.42
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CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.07%	0.22%	Small Size	0.17	0.50
Big Size	0.86%	1.04%	Big Size	2.10*	2.68
<hr/>					
OP	Low	High	OP	Low	High
Small Size	-0.27%	0.89%	Small Size	-0.70	2.23*
Big Size	0.93%	1.00%	Big Size	2.32*	2.65*
<hr/>					
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.32%	0.84%	Small Size	-0.70	2.11*
Big Size	0.90%	0.63%	Big Size	1.75	1.61

Tab. 4.29: Six Factor Model Intercepts (2 x 2)

40 Value – Weight portfolios (2 x 5 sort)

The six-factor model is further applied to forty portfolios sorted using 2 x 5 sorts. Portfolios of small stocks sorted using the investment factor portray intercepts that are closest to zero, thus tallying with results obtained on the sixteen value weight portfolios of similar sort. An intercept value of 0.01% is observed in smalls stocks with low investment, suggesting that the model explains the returns of this portfolio better than the other portfolios. Comparing the six factor intercepts and the four factor intercepts, the two models explain the returns of all the portfolios with a similar performance as shown by intercepts that are precisely similar.

Six-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.17%	1.15%	0.83%	0.17%	-1.60%	Small Size	2.18*	2.62*	1.92	0.41	-3.47*
Big Size	1.42%	1.53%	1.08%	0.64%	-0.68%	Big Size	3.58*	3.50*	2.63*	1.44	-1.09
<hr/>											
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	0.01%	-0.40%	-0.44%	-0.49%	-0.07%	Small Size	0.03	-0.96	-0.93	-1.12	-0.15
Big Size	1.28%	1.25%	0.84%	0.89%	1.55%	Big Size	2.72*	2.81*	2.01*	2.26*	3.57*
<hr/>											
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.07%	-0.18%	-0.48%	0.55%	0.57%	Small Size	-2.33*	-0.41	-1.05	1.25	1.31
Big Size	0.84%	1.06%	1.25%	1.28%	1.30%	Big Size	1.96	2.62*	2.97*	3.17*	3.19*
<hr/>											
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-1.04%	0.17%	-0.10%	0.35%	0.15%	Small Size	-1.95	0.46	-0.24	0.90	0.33
Big Size	0.74%	0.73%	1.00%	1.43%	1.56%	Big Size	1.25	1.53	2.59*	3.55*	3.46*

Tab. 4.30: Six Factor Model Intercepts (2 x 5)

Size – B/M portfolios

The six-factor model produces statistically significant intercepts for small stocks with both low value and high value. High value stocks show negative intercepts indicating that the six-factor model largely captures the value effect.

Size – Investment portfolios

All small stocks sorted using investment show intercepts that are closest to zero with the low investment and high investment portfolios being the closest to zero. This observation suggests that the model performs better when applied to small stocks of various investment levels.

Size – OP portfolios

There are four statistically significant intercepts associated with the profitability portfolios. The intercepts for portfolios sort on size and operational profitability show intercept values that are converging to 0.00% for the portfolio consisting of big stocks lying on the second quantile of the profitability factor. The portfolio of small stocks with low profitability portfolio is the only portfolio portraying statistically significant low intercepts among the small stocks.

Size – Momentum portfolios

There are only three portfolios with statistically significant intercepts in the size – momentum portfolios. Big stocks in the 3rd quantile of momentum have an intercept value that is closest to zero as compared to the rest of the portfolios, thus, portraying that the model better explains this portfolio.

4.4.7 Seven Factor Regression Details

We run the seven-factor asset pricing model against the portfolios sorted using the 2 x 2 sorts and the 2 x 5 sorts. The results are shown in tables 4.31 and 4.32.

16 Value Weight Portfolios (2 x 2 sort)

There are no statistically significant regression intercepts in this model. The seven-factor intercept closest to zero is observed in small stocks with low investment with

an intercept value of 0.13%. ($t = 0.18$).

Seven - Factor Intercepts					
a			t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	1.55%	-0.81%	Small Size	1.95	-1.09
Big Size	1.40%	0.36%	Big Size	1.93	0.39
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.13%	0.34%	Small Size	0.18	0.44
Big Size	0.91%	1.06%	Big Size	1.18	1.42
OP	Low	High	OP	Low	High
Small Size	-0.27%	0.98%	Small Size	-0.36	1.39
Big Size	1.08%	1.07%	Big Size	1.34	1.42
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.33%	0.90%	Small Size	-0.43	1.22
Big Size	1.04%	0.65%	Big Size	1.13	0.89

Tab. 4.31: Seven Factor Model Intercepts (2 x 2)

40 Value – Weight portfolios (2 x 5 sort)

The seven-factor intercept for small stocks contained in the fourth quantile of the value factor is closer to the origin than the other intercepts; suggesting that the model performs well in explaining returns of this portfolio. Only 4 portfolios out of 40 portfolios have statistically significant intercepts and this is quite low when compared to the findings obtained for the other models.

Size – B/M portfolios

The seven-factor intercept value of -1.51% ($t = -1.87$) associated with the portfolio of small stocks with high value indicates that the model risk factors have a negative linear relationship with the portfolio returns. The observation portrays that returns of the portfolio will be negative, when assuming that all other factors are set to zero. The most improved intercept is observed in small stocks.

Seven-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	1.25%	1.09%	0.83%	0.06%	-1.51%	Small Size	1.35	1.49	1.13	0.08	-1.87
Big Size	1.49%	1.70%	1.06%	0.54%	0.22%	Big Size	2.00*	2.30*	1.40	0.70	0.23
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CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	0.03%	-0.25%	-0.44%	-0.56%	0.12%	Small Size	0.03	-0.35	-0.57	-0.76	0.15
Big Size	1.57%	1.17%	0.87%	0.91%	1.45%	Big Size	1.88	1.62	1.20	1.19	1.93
<hr/>											
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.23%	0.02%	-0.51%	0.53%	0.79%	Small Size	-1.51	0.03	-0.65	0.68	1.09
Big Size	1.06%	1.03%	1.27%	1.41%	1.36%	Big Size	1.33	1.45	1.73	1.84	1.80
<hr/>											
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-1.06%	0.22%	-0.03%	0.46%	0.10%	Small Size	-1.26	0.31	-0.04	0.61	0.12
Big Size	0.93%	0.64%	0.94%	1.59%	1.66%	Big Size	1.01	0.80	1.28	2.19*	2.24*

Tab. 4.32: Seven Factor Model Intercepts (2 x 5)

Size – Investment portfolios

All small stocks sorted using investment show intercepts that are closest to zero with the low investment and high investment portfolios being the closest to zero, portraying that the seven - factor model performs well when applied to these portfolios.

Size – Profitability (OP) portfolios

The portfolio with intercept closest to the origin is observed in the portfolio formed on smalls stocks that lie on the second quantile, indicating that the seven - factor model performs well in explaining returns of this portfolio. The intercepts for portfolios sort on size and operational profitability show intercept values that are converging to 0.00% for small stocks as compared to the big stocks.

Size – Momentum portfolios

There are two portfolios with statistically significant intercepts in the size – momentum portfolios. Big stocks in the 3rd quantile of momentum have an intercept value that is closest to zero as compared to the rest of the portfolios, suggesting a better performance of the model in explaining the returns of this portfolio.

Eight Factor Regression Details

The eight-factor model which consists of the Fama and French factors and the two South African indices, the South African Financial Index (SAFI) and the South African Resources Index (SARI) is regressed against the portfolios sort using (2 x 2) sorts and (2 x 5) sorts. The eight-factor model improves most of the intercepts by drawing them closer to zero in comparison to the rest of the multi-factor models under investigation in this study.

16 Value Weight Portfolios (2 x 2 sort)

Table 4.33 shows intercepts obtained from running the eight – factor model against the value weight portfolios sorted using the 2 x 2 sorts methodology. In these portfolios there is only one intercept that is statistically significant. Regression intercepts range from -0.92% to 1.28%, this range shows an improvement by the eight-factor model in converging the intercepts to 0.00%.

Eight - Factor Intercepts					
	a		t(a)		
B/M (Value)	Low	High	B/M (Value)	Low	High
Small Size	-0.92%	-0.92%	Small Size	-1.39	1.53
Big Size	1.28%	-0.25%	Big Size	2.26*	-0.41
CMA (Investment)	Low	High	CMA (Investment)	Low	High
Small Size	0.10%	0.22%	Small Size	0.16	0.32
Big Size	0.51%	1.03%	Big Size	0.95	1.81
OP	Low	High	OP	Low	High
Small Size	-0.42%	0.85%	Small Size	-0.68	1.42
Big Size	0.72%	0.90%	Big Size	1.30	1.51
Mom (Momentum)	Low	High	Mom (Momentum)	Low	High
Small Size	-0.36%	0.90%	Small Size	-0.53	1.44
Big Size	0.41%	0.58%	Big Size	0.65	1.03

Tab. 4.33: Eight - Factor Model Intercepts (2 x 2)

40 Value – Weight portfolios (2 x 5 sort)

The intercepts range from -1.59% to 1.89%, with intercepts associated with small stocks and low investment being the closest to the origin (0.00), noting that the closer the intercept is to the origin the better the model is in explaining the returns of the portfolio. This observation portrays that the model performs better

in explaining the returns of this portfolio. The improvement of the intercepts in converging to the origin show evidence that the linear combination of the market factor, the South African Financial Index (SAFI) and the South African Resources Index (SARI) improve the model performance in describing portfolio than when applied individually.

Eight-Factor Intercepts											
a						t(a)					
B/M (Value)	Low	2	3	4	High	B/M (Value)	Low	2	3	4	High
Small Size	0.77%	1.03%	0.60%	-0.12%	-1.59%	Small Size	0.81	1.53	0.94	-0.19	-2.17*
Big Size	1.30%	1.65%	1.15%	0.07%	-0.75%	Big Size	2.30*	2.44*	2.16*	0.11	-0.92
CMA (Investment)	Low	2	3	4	High	CMA (Investment)	Low	2	3	4	High
Small Size	-0.02%	-0.52%	-0.04%	-0.72%	-0.13%	Small Size	-0.03	-0.79	-0.06	-1.17	-0.17
Big Size	0.96%	1.07%	0.71%	0.77%	1.79%	Big Size	1.32	1.75	1.20	1.30	3.02*
OP (Profitability)	Low	2	3	4	High	OP (Profitability)	Low	2	3	4	High
Small Size	-1.48%	-0.24%	-0.49%	0.56%	0.36%	Small Size	-2.09*	-0.33	-0.72	0.78	0.62
Big Size	0.57%	0.79%	1.30%	1.37%	1.17%	Big Size	0.87	1.33	2.29*	2.12*	1.80
Mom (Momentum)	Low	2	3	4	High	Mom (Momentum)	Low	2	3	4	High
Small Size	-1.03%	0.22%	-0.46%	0.22%	0.09%	Small Size	-1.26	0.39	-0.76	0.39	0.16
Big Size	0.29%	0.38%	0.86%	1.10%	1.89%	Big Size	0.43	0.49	1.65	1.83	2.86*

Tab. 4.34: Eight - Factor Model Intercepts (2 x 5)

Size – B/M portfolios

The low intercept value of 0.07% ($t = 0.11$) for the portfolio consisting of big stocks on the fourth quantile of the value factor is improved by the eight-factor model. Extreme high value stocks show negative intercepts thus indicating that the value effect is absorbed by the eight-factor model.

Size – Investment portfolios

Small stocks indicate negative values but that are much closer to zero than portfolios sorted on factors in a similar fashion. There is only one intercept that is found to be statistically significant among these portfolios and it is the regression intercept for big stocks with extreme high investment, 1.79% ($t = 3.02$).

Size – OP portfolios

Small stocks in the first three quantiles of OP have negative intercepts and the extreme low OP small stocks being the only portfolio with a statistically significant

intercept among these portfolios.

Size – Momentum portfolios

The portfolio consisting of small stocks with high momentum portray an intercept value of 0.09%, which is the closest to zero among the momentum sorted portfolios. There is only one statistically significant intercept among these portfolios, and it is the portfolio of big stocks with extremely high momentum.

4.4.8 Model Performance Evaluation

GRS Test

In assessing and evaluating asset pricing models, if the intercept is indistinguishable from zero in the regression on any asset/portfolio's excess returns then it is said to fully explain the expected returns. We therefore use the GRS statistic of [Gibbons *et al.* \(1989\)](#) in a similar manner as [Fama and French \(2015\)](#) to test the hypothesis of the asset pricing models. The main objective is to compare the performance of the CAPM, the two-factor APT model of [Laird-Smith *et al.* \(2016\)](#), the Fama and French three-factor model, the Carhart model, the Fama and French five – factor model, the six – factor model, the seven – factor model and the eight – factor model. Table 4.34 shows statistics obtained from regressions and GRS tests.

Table 4.35 show results for tests of the ability of the eight factor models to explain monthly average returns on portfolios in the JSE. The GRS test statistic, the p-value given by $p(\text{GRS})$, the absolute value of the intercept given by $A|a|$ and the adjusted R-squared given by AR-squared are key statistics in conducting the test. The GRS test rejects all the asset pricing models under investigation, this was also the case for [Fama and French \(2015\)](#) in the US market. This is because asset pricing models are basic suggestions about expected returns hence bound to be rejected in tests of power. We therefore, use relative performance in comparing the models using the GRS statistic, intercepts and the adjusted R-squared.

Asset Pricing Test

An asset pricing model is said to explain the average returns of a stock/portfolio if its regression intercept is not significantly different from zero. This is drawn

Model	GRS	p(GRS)	A a	AR-squared	Model	GRS	p(GRS)	A a	AR-squared
<i>Panel A: 10 Size - B/M portfolios</i>					<i>Panel A: 10 Size - OP portfolios</i>				
CAPM	2.25	0.00	0.63%	32.12%	CAPM	2.44	0.00	0.57%	33.67%
Two Factor Model	2.69	0.00	0.72%	22.11%	Two Factor Model	2.12	0.00	0.63%	21.79%
FF 3 Factor	2.78	0.00	0.76%	34.10%	FF 3 Factor	2.35	0.01	0.69%	35.24%
Carhart's 4 Factor	1.88	0.01	0.57%	34.57%	Carhart's 4 Factor	1.78	0.01	0.51%	35.91%
FF 5 Factor	2.03	0.00	0.75%	33.10%	FF 5 Factor	2.91	0.00	0.69%	34.48%
6 - Factor	1.98	0.00	0.57%	33.58%	6 - Factor	2.39	0.00	0.51%	35.18%
7 - Factor	0.92	0.00	0.67%	26.94%	7 - Factor	2.26	0.00	0.57%	26.87%
8 - Factor	2.10	0.01	0.41%	34.65%	8 - Factor	2.03	0.01	0.39%	35.90%
<i>Panel A: 10 Size - Investment portfolios</i>					<i>Panel A: 10 Size - Mom portfolios</i>				
CAPM	2.88	0.00	0.50%	32.39%	CAPM	2.10	0.00	0.56%	32.45%
Two Factor Model	2.59	0.00	0.54%	21.75%	Two Factor Model	2.70	0.00	0.60%	22.25%
FF 3 Factor	2.66	0.00	0.62%	33.78%	FF 3 Factor	1.52	0.00	0.69%	34.41%
Carhart's 4 Factor	2.33	0.00	0.44%	34.40%	Carhart's 4 Factor	2.01	0.01	0.50%	35.20%
FF 5 Factor	2.27	0.00	0.62%	32.83%	FF 5 Factor	1.57	0.00	0.68%	33.72%
6 - Factor	2.48	0.00	0.44%	33.44%	6 - Factor	1.90	0.02	0.50%	34.51%
7 - Factor	1.86	0.00	0.49%	26.15%	7 - Factor	2.86	0.00	0.54%	26.94%
8 - Factor	2.12	0.01	0.39%	34.16%	8 - Factor	2.72	0.01	0.36%	35.71%

Tab. 4.35: GRS Test

from the statistical principle that alphas/intercepts are error terms emanating in the cross-sectional linear relationship between expected returns and factor betas. Hence, if these intercepts are zero, the model is concluded to explain the expected returns of the portfolio better.

Regression intercepts

In seeking to achieve the goal of evaluating asset pricing models in the South African JSE, we assess the characteristics of the overall regression intercepts for each of the models applied to 56 portfolios used as the dependent variables.

Figure 4.10 shows the graph of aggregate intercepts for the models under study. We observe that the regression intercepts closer to zero are intercepts for the eight-factor model, the four – factor model and the six – factor model. The model with the aggregate closest to 0.00% is deemed to perform better. Thus, with regards to intercepts analysis, the four-factor model, the six-factor model and the eight-factor model describe average portfolio returns better than the rest of the models. The seven factor performs worse than the six factor model because it does not consist of the market factor which improves model performance when combined with the other factors. This observation is further shown by the improvement of the model

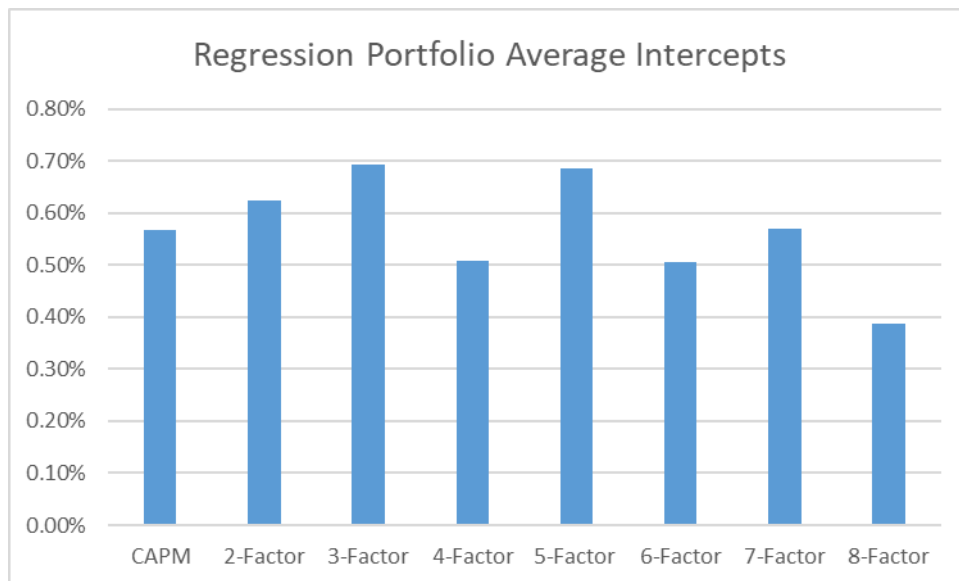


Fig. 4.10: Regression Intercepts

after adding the market factor to the seven factor model. The four factor model performs better than the five factor and six factor model because the momentum factor has been found to be significant in the South African context, while the investment factor performs poorly. Thus, leading to an improvement in the performance of the four-factor asset pricing model. Furthermore, we observe that the cumulative log-performance of the momentum factor to be significantly larger than any of the other factors in the JSE.

In Figure 4.11 we observe the general distribution of the regression intercepts for each of the asset pricing models. It is evident that the eight – factor model intercepts are much smaller and closer to zero. The rest of the models are distributed in similar manner, we therefore, conclude that adding the indices to the six – factor model improves the model intercepts.

4.4.9 Final Remarks

We obtain evidence that the market factor, the South African Financial Index and the South African Resources Index perform better jointly than independently in explaining the returns of portfolios as shown by the eight - factor model that improves the regression intercepts by drawing them closer to the origin. The analysis of regression intercepts portrays that the eight – factor model, the four – factor



Fig. 4.11: Regression Intercepts

model and the six-factor model significantly improves regression intercepts, drawing them closer to the origin, that is, the value of 0.00%. The results thus suggest that these models better explain the expected returns of the portfolios as compared to the rest of the models. The Carhart four factor model appears to describe expected returns in the South African equities markets than the other multifactor models excluding the eight-factor model, this result indicates that the model is parsimonious. For the portfolios sorted on size and investment factors the Carhart four - factor model has the highest adjusted R – squared value of 34.40% despite having few factors. A similar observation is observed throughout the rest of the portfolios, with the eight -factor model being slightly higher than the Carhart four – factor model.

Chapter 5

Conclusion

In this section we seek to conclude our study and provide suggestions for future research areas in the asset pricing field.

When stocks are sorted using the size and value factors, we observe that the size effect is present in these portfolios as shown by the increase in the average returns from small stocks to big stocks. A strong relationship between value portfolios and average returns is more evident in small stocks. This is observed in the drop in average returns from 1.36% to -1.30% in small stocks and from 1.68% to -0.07% for big stocks. The change is much smaller in big stocks than in small stocks. For each size quantile there is a decrease in average returns with value. Moreover, a negative excess return is observed in extreme high values. Portfolios sort on size and investment factors portray big stocks that have significantly higher expected returns than small stocks. We further observe that small stocks with extreme low profitability show extremely negative returns as compared to the rest of the portfolios. Portfolios formed on size and momentum show extremely negative returns on the portfolio of small stocks with extremely low momentum.

Factor spanning tests conducted on the risk factors indicate lack of presence of multicollinearity in independent risk factors. The R-squared value (41.77%) for the value factor indicates existence of causation effects in explaining the rest of the other factors. This observation agrees with the findings of Fama and French (2015) showing that the value factor is redundant in explaining expected returns. In their results they show that the four-factor model without the value factor and the five-factor model have statistically indifferent explanatory powers. Our results show that the size factor and the investment factor add little to the description of expected returns. Furthermore, we obtain evidence that the market factor, the South African Financial Index and the South African Resources index jointly improve the explanatory power of asset pricing models but do not explain each other indepen-

dently. We therefore conclude that the market factor, the South African Financial Index and the South African Resources Index are jointly significant in explaining equity returns in the South African context.

Our main objective is to test the applicability of popular asset pricing models and evaluate their performance within the South African equities market. The models investigated include the Capital Asset Pricing Model (CAPM), the two-factor model initially investigated by Van Ransburg (2002) and further investigated by Laird-Smith (2016), the Fama and French three factor model, the Carhart four-factor model, the Fama and French five factor model, the six factor model and the seven – factor model that includes the factors and the two South African indices.

5.0.1 The Capital Asset Pricing Model

The CAPM is applied to the portfolios sort on the independent factors. When applied to the portfolios sort on size and value we observe that there are only six out of sixteen statistically significant portfolios in portfolios sort using 2 x 2 sorts. This low number of statistically significant portfolios is expected since asset pricing models perform poorly in power tests. Applying the CAPM to the portfolios sort using the 2 x 2 methodology produces intercepts that range from -0.81% to 1.45% for the Size – Value portfolios, 0.02% to 1.08% for the size – investment portfolios, -0.24% to 1.12% for size – profitability portfolios and -0.28% to 0.86% for the portfolios sort on size – momentum. Model intercepts are improved when applied to portfolios sort on size and momentum factors, with the intercepts range being much closer to the origin (0.00%) as compared to the other portfolios. Intercepts obtained when the CAPM is applied to portfolios sort on 2 x 5 sorts range from -1.53% to 1.62% for the portfolios sort on Size – Value portfolios, -0.33% to 1.59%, from -1.07% to 1.43% for the Size – Profitability portfolios, from -0.97% to 1.61% for portfolios sort on Size and Momentum. The above results show that the CAPM performs better on portfolios sorted using the 2 x 2 methodology.

5.0.2 The two – factor APT model (South African Financial Index and South African Resources Index)

In the portfolios sort using the 2 x 2 methodology we observe that the model produces regression intercepts that range from -0.75% to 1.44% for portfolios formed using size and value factors, 0.06% to 1.10% for portfolios sort using size and investment factors, from -0.25% to 1.18% for portfolios sort using size and profitabil-

ity and from -0.30% to 1.01% for portfolios sort using size and momentum. The two-factor model improves regression intercepts for most of the portfolios when applied to portfolios sort using the 2 x 2 methodology. When applied to portfolios sorted using the 2 x 5 methodology, we see that regression intercepts range between 0.12% and 1.78% for the size – value portfolios, -0.37% and 1.53% for stocks sort on size and investment, -1.27% and 1.49% for stocks sort on size and profitability. -1.00% and 1.66% for stocks sorted on size and momentum. The two factor model slightly performs better than the CAPM when assessed on the values of the regression intercepts and adjusted R-squared values.

5.0.3 The Fama and French Three Factor Model

The Fama and French three factor model improves the regression intercepts associated with smalls stocks of low profitability (-0.08%) despite the intercept being rejected by tests for significance. Portfolios sort on size and value show regression intercepts that range from -0.68 to 1.64%, the size – investment portfolios show regression intercepts that range from 0.18% to 1.18% , the intercepts obtained from portfolios sort on size and profitability have regression intercepts that range from -0.08% to 1.23% and the portfolios sort on size – momentum portray intercepts that range from -0.13% to 1.06%. These obtained intercepts converge to zero when compared to CPM intercepts for the same portfolios.

5.0.4 The Carhart Four Factor Model

Regression intercepts for portfolios formed using size and value factors range from -0.87% to 1.50% when the Carhart four factor model is used. The size and investment portfolios have regression intercepts ranging from 0.06% to 1.04%, which converges much closer to zero than the intercepts obtained using the CAPM and the two-factor model. The size and profitability portfolios have intercepts that range from -0.28% to 1.01% thus showing an improvement in the intercepts. The size- momentum portfolios have regression intercepts ranging from -0.33% to 0.90%. The Carhart four factor model has a better performance in explaining average returns of portfolios as compared to the rest of the models. Hence, this provides evidence that this model is both parsimonious and applicable in the South African context.

5.0.5 The Fama and French Five factor model

The Fama and French five factor model was also tested on factor portfolios sorted using 2 x 2 sorts and 2 x 5 sorts. The regression intercepts range from -0.69% to

1.65% for all the portfolios, this indicates that the model improves the intercepts as compared to the other model. The portfolios sort on Size and Value have regression intercepts ranging from -0.69 to 1.65% which is slightly similar to the intercepts produced by the Fama and French three factor model. The size and investment factors have regression intercepts of the range 0.19% to 1.17% which is very similar to the intercepts obtained using the Fama and French three factor model. The Size and profitability portfolios show intercepts that range from -0.08% to 1.22%. and the size – momentum portfolios have regression intercepts ranging from -0.13% to 1.06%. One could reach a weak conclusion that the FF3 model and the FF5 model perform comparably in the same manner, thus bringing the argument that the other two factors may be redundant in the South African markets.

5.0.6 The Six factor model

The six-factor model is a model that adds the momentum factor to the five-factor model. It is statistically expected for an ordinary least squares model to improve as the number of independent variables are increased. We therefore seek to assess the performance of this model in comparison to other multifactor models under study. We observe that the regression intercepts obtained from this model range from -0.87% to 1.51% for all the portfolios. The size and value portfolios produce intercepts ranging from -0.87% to 1.51%, a similar result was also observed in the same portfolios from the other models. Portfolios sort on size and investment show regression intercepts ranging from 0.07% to 1.04%, while the size and profitability portfolios have regression intercepts ranging from -0.27% to 1.00%. The size – momentum portfolios portray regression intercepts that range from -0.32% to 0.90%. These intercept values converge even much closer to the origin (0.00%) and hence one can conclude that the model improves the description of expected returns in the portfolios sort on the Fama and French factors.

5.0.7 The Seven-factor model

Further to the well-known multifactor models, we attempted to assess the performance of the hybrid asset pricing model that combines the independent factors excluding the market factor and the two South African sector indices namely the South African Financial Index (SAFI) and the South African Resources Index (SARI). Applying the model to the factor sorted portfolios we obtain regression intercepts that range from -0.81% to 1.55%. Furthermore, all the portfolios produce intercepts that are statistically insignificant in the model. This observation is worth

noting and should be taken into consideration for future research as different modelling methodologies may result in different outcomes. Applying the model to the forty portfolios sorted using 2 x 5 sorts yields only four statistically significant regression intercepts.

5.0.8 The eight-factor asset pricing model

Following the observations of the seven-factor model, we further extended our investigations to assessing the performance of the eight-factor model which is an extension of the seven-factor model but with the market factor added to the factors in this case. The model is also applied to the factor sorted portfolios and we obtain that in overall the regression intercepts range from -0.25% to 1.39%. Small stocks with low value still exhibit the furthest intercept from the origin as was the case with all the other models. The model intercepts generally converge to the origin (0.00%), hence, indicating that the model produces better intercepts and thus performing better in describing expected returns.

5.0.9 Asset Pricing Model Performance

In conducting the GRS test, all the models are rejected and this is expected since previous studies show that asset pricing models perform poorly in power tests. However, Fama and French (2015) assert that what is important is the ability of the models to describe expected returns and not the statistical significance of the model. The absolute intercepts show that the eight-factor model performs better than the rest of the models, this is portrayed by the absolute intercepts that converge to zero. Size – Value (B/M) portfolios have an average absolute intercept of 0.41% which is the closest to the origin, whereas the Carhart four factor model and the six-factor model have the same average absolute intercept of 0.57%. The adjusted r-squared values for the portfolios sort on Size – Value (B/M) show that the eight-factor model has the highest value of 34.65%, however, the Carhart four factor model show a similarly high value of 34.57% despite having half the number of independent factors.

The Carhart four factor and the eight-factor model have consistently higher adjusted r-squared values and intercepts that converge to the origin. The Carhart four factor model and the six-factor model have similar average intercept values,

this is worth noting. The Fama and French three factor and five factor models have average intercept values that are the furthest from the origin (0.00%) when compared to the other models. However, the two models with the South African indices and that exclude the market factor, namely the two-factor model and the seven-factor model, have the lowest adjusted r-squared values when compared to the other multifactor models.

The Carhart four factor model, the six-factor model and the eight-factor model explain the expected returns of factor portfolios in this study. Based on the results obtained, we conclude that the Carhart four factor model is the most parsimonious model since it has the highest adjusted R-squared values and intercepts that diverge to zero the most. We therefore, conclude that the Carhart four factor model performs the best in explaining average portfolio returns in the Johannesburg Stock Exchange.

5.0.10 Future Research

In this study, we assumed a linear relationship between the risk factors (independent variables) and the portfolios (dependent variables). This study could be extended by relaxing the linear relationship assumption between variables and incorporate other predictive variables such as proxy for economic conditions. Advances in machine learning methods provides opportunity for further research in determining the structure of these relationships in asset pricing models.

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