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UNIVERSITY OF CAPE TOWN

SCHOOL OF ECONOMICS

**The Black-Scholes Model and the Pricing of Stock Options
in South Africa**

Dissertation submitted in partial fulfilment of the requirements for the degree of
Masters in Economics (MCom) on the 6th of September 1999.

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1.Introduction

Option Pricing Theory (OPT), along with the Capital Asset Pricing Model, the Theory of Capital Structure, and the Efficient Markets Hypothesis, form one of the pillars of modern finance theory. Central to OPT is the Black-Scholes model, the first option-pricing model derived within a general equilibrium framework, and therefore consistent with all arbitrage conditions an asset pricing model must satisfy.

An attempt is made at explaining this model, and the first part of the paper is devoted to this objective. The appreciation of the theoretical elegance of the Black-Scholes model can be considerably enhanced through the understanding of the issues that made (and still make in the case of American put options) the derivation of an equilibrium model of option pricing such an immense task. With the intention of emphasising such issues, the first section of Part One covers the option pricing models that had been suggested before Black and Scholes (and Merton). This helps to put the Black-Scholes model in context, as well as facilitate an understanding of the approach Black and Scholes adopted in developing their model. Its derivation is the central focus of section 3, the second section of Part One.

The second part of the paper contains an attempt at testing the Black-Scholes model, first in its "pure form," and then adjusted to account for the possibility of early exercise. Simple regression tests are performed, where daily prices of a sample of stock options traded on the Johannesburg Stock Exchange are used as dependent variables in regression equations. Black-Scholes model prices are computed, and used as the explanatory variables in these equations. But before the tests could be conducted, the distributions of the underlying assets' returns had to be examined and due consideration had to be given to the estimation of the volatility parameters. Part Two starts with a very brief overview of the South African exchange-traded stock options market. This is followed by a description of the data used in the tests, and discussions on the statistical behaviour of the underlying assets. A discussion on volatility estimating follows, and the test results are then presented.

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Any errors in this paper are of my sole responsibility.

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PART ONE: OPTION PRICING THEORY

2.1. Basic Definitions and Notation

An *option* is a contract whose value, like that of any other derivative security, depends on the value of another asset or security, and time. There are two general types of options: 'European' and 'American'. A *European option* is a contract that gives its owner the right (but not the obligation) to buy or sell a certain quantity of an asset, physical or financial, at a specified price, and on a specified date. The specified date is the *maturity (or expiration) date*, and the specified price is known as the *exercise (or strike) price*. Holders of *American options* have the right to buy or sell the underlying asset (i.e. to exercise the option) on or before the maturity date. A *call* option gives its owner the right to buy and a *put* option the right to sell. The option pricing models reviewed in the next two sections concern the pricing of European call options.

The following notation will be used throughout the paper:

- t current date;
- T maturity date;
- C price of a call option at t;
- C_T price of a call option at T;
- r risk-free interest rate;
- S stock price at time t;
- X exercise (strike) price of the option;
- σ standard deviation.

2.2. Option Pricing Models Prior To Black-Scholes

2.2.1. Bachelier - 1900

The first attempt at creating a model for the pricing of options (that can be found in today's finance literature) was provided by Louis Bachelier, in 1900.

The central assumption in deriving a theoretical model for the pricing of options, concerns the statistical process which describes the behaviour of the underlying asset over time.¹ Bachelier believed that transactions in the stock market are the result of buyers and sellers having different expectations of the future stock price, and that it would be unreasonable to expect that, on average, either buyers or sellers could consistently make better predictions than the others. From these postulations, he conjectured that at any moment, the market as a whole cannot be expected to go either up or down. Positive and negative expectations cancel each other when one considers the aggregate of market participants at any moment in time. On average then, the expected price change per unit of time is assumed to be zero (Bernstein, 1992:20).²

He also noted that stock price volatility increases as the time horizon over which the volatility is measured is increased. This led him to investigate the mathematical properties of the behaviour of particles subject to random shocks as they move in

¹ The underlying asset in the models described in this section is a share of common stock.

² It should be clear that Bachelier did not ignore the fact that, what motivates investors towards buying stocks, is the expectation of gains from future appreciation in the price of the stocks. He simply states that the reason for people to be able to do so, is that others believe such appreciation will not materialise - otherwise prices would keep rising indefinitely. The net effect of buyers and sellers, at the same instant of time, having these different views of the likely future behaviour of prices is, according to Bachelier, that the expected change in the stock price will on average, be equal to zero. (Unless buyers and sellers simultaneously receive new information that causes their views on the future price to converge.)

space - known in physics as Brownian Motion³; Bachelier showed that the standard deviation of stock prices increases in proportion to the squared root of time.

He then assumed that the movement of stock prices over time follows arithmetic Brownian Motion⁴ without drift, and with variance of σ^2 per unit of time. (If the statistical process that describes the movement of the stock price over time has no drift, then on average the stock price will not tend to move either up or down.) This assumption implies that the stock price is a normally distributed random variable, generated by a statistical process comparable to that generated by successive flips of a fair coin - there is an equal probability of getting either outcome, and each outcome is independent of the previous outcome. But more specifically, if the stock price follows arithmetic Brownian motion without drift, the probability of the price rising or falling by one absolute unit (e.g. one Rand) are equal, irrespective of the price level (Smith, 1976:15).

Bachelier also assumed the expected return on a call option to be equal to the expected change on the value of the underlying asset, which, as stated previously, was assumed to be equal to zero.

The value of a call option was then expressed as the expected value of the option at maturity. Since a call option will only be exercised if the stock price is higher than the exercise price, the value of a call at maturity will be

$$C_T = \text{Max}(S_T - X ; 0).$$

The reason for this is that, if it is the case that on the date at which the option expires, the stock price is larger than the exercise price, the holder of the option can obtain the stock by exercising his/her right to buy it for $X (< S)$. The option will be worth the difference between the stock and exercise prices. Alternatively, if at expiration of the

³ The mathematical representation of Brownian Motion, for a long time credited to the physicist Albert Einstein, was apparently first discovered by Bachelier as he investigated the behaviour of stock prices (see for example Bernstein, 1992, pp.18-22 and Ingersoll, 1989, pp.202).

⁴ This process is commonly known today as the 'Random Walk'.

option contract the stock price is lower than the exercise price, the option will not be exercised. The right to buy the stock for X will be worthless. In sum, the value of a call option at maturity will always be the greater of, the difference between the stock and exercise prices, and zero.

Bachelier's pricing model can therefore be expressed as the pay-off to the option holder at maturity, integrated over the probability density function of the terminal stock price, for values of the stock price above the exercise price,

$$C = E(C_T) = E [\text{Max}(S_T - X; 0)] = \int_X^{\infty} (S_T - X) N'(S_T) dS_T$$

where $E(C_T)$ is the expected terminal value of the call option and $N'(S_T)$ is the normally distributed probability density function of S. Solving for this expected value (see Smith, 1976:48) gives us Bachelier's formula for the valuation of call options:

$$C = SN\left(\frac{S - X}{\sigma\sqrt{T - t}}\right) - XN\left(\frac{S - X}{\sigma\sqrt{T - t}}\right) + \sigma\sqrt{T - t} \cdot N'\left(\frac{X - S}{\sigma\sqrt{T - t}}\right)$$

where $N'(\)$ and $N(\)$ are the standard normal⁵ and cumulative standard normal⁶ density functions, respectively.

Although probably very advanced for its time, this model contains some substantial flaws. Arithmetic Brownian motion is clearly not an adequate representation of the

⁵ $N'(Z) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{1}{2}z^2}$. This function describes the values the standard variable may assume, and the probability of the variable assuming each of these values. For a normally distributed (standard) variable the density function is a bell-shaped curve with a zero-mean.

⁶ $N(z) = \int_{-\infty}^z \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{1}{2}z^2} dz$, where $z = (X - \mu)/\sigma$; and $X \sim N(\mu, \sigma^2)$. This function gives us the probability that the standard normal variable (Z) assumes a value between $-\infty$ and z (i.e.: probability that Z is less than z). It represents the area under the density function to the left of z.

statistical process followed by stock prices. First, investors are concerned with the proportional, not absolute change in price. Second, the assumption of arithmetic Brownian motion and normally distributed prices ignores the fact that stocks represent shares of the equity of public companies, the owners of which cannot be held liable for more than the value of their shareholdings. The value of such stocks cannot be negative. The distribution of stock prices cannot therefore be symmetrical - although in theory they can rise by any percentage, stock prices cannot fall by more than 100%. The assumption of arithmetic Brownian motion and normally distributed prices implies that stock prices may become negative.

Assuming a mean expected change in stock prices of zero implies that economic agents are risk-neutral, and ignores the time value for money (note that the expected terminal value of the option was not discounted to obtain the present value). If interest rates are positive and investors are risk-averse (i.e. they prefer a certain outcome P to an expected outcome P), stocks will be expected, at any moment and on average, to yield a return at least equal to the rate of interest on a bond with no probability of default (the risk-free rate). Also, differences in the variability of the price of an option and the underlying stock were not investigated.

Despite what is today acknowledged as an enormous contribution to the fields of financial economics and mathematical statistics, Bachelier's work went unnoticed for more than half-century until it was accidentally found in the University of Chicago's library and circulated to some famous economists. Bachelier had failed to obtain a "*mention tres honorable*" for his Doctoral thesis (where the work partly summarised here was developed), and as a result could only get an academic position in a small university in provincial France. He lived as an unknown and frustrated man (Bernstein, 1992:19).

2.2.2. Sprenkle - 1964

Clearly rooted in Bachelier's work, the next important option-pricing model formulated within a probabilistic framework is that of Sprenkle, published in 1964. Sprenkle notes that it is the percentage, and not absolute change in the stock price that matters. He assumes that stock prices follow geometric Brownian motion with positive drift⁷. This means that the probability of a one percent increase in the stock price equals the probability of a one percent decrease, irrespective of the stock price level (Smith, 1976:15). Positive drift means that the random walk followed by the stock price has an upward trend. The drift factor can be interpreted as the mean rate of return on the stock. The mean rate of return and the variance of the stock price are assumed to be constant. The model thus allows for the existence of positive interest rates and risk-averse behaviour.

The assumption of a stock price generated by geometric Brownian motion is consistent with a log-normal distribution of the possible stock price, at the end of a finite time interval – such as the life of an option (Sprenkle, 1964:428). It is the natural logarithm of the stock price, and not the stock price itself, which can be assumed to be normally distributed. The log-normal distribution, unlike the normal which is bell-shaped, is skewed to the right - that is, the right tail is fatter than the tail on the left-hand side of the distribution. It serves as a better approximation of the actual distribution of stock prices, reflecting the fact that while stock prices can increase by any amount, they cannot become negative (note that natural logs are always positive).

This seems to be about as far as Sprenkle's model differs from Bachelier's. He expressed the value of a call option as the expected terminal value of the option,

$$E(C_T) = \int_X^{\infty} (S_T - X)L(S_T)dS_T ,$$

⁷ Although the deeper technical aspects of this process are beyond the scope of this paper, geometric Brownian motion is looked at in more detail in section 3.

where $L(S_T)$ is the lognormal density function (this function is now used to calculate the expected value instead on the normal, because the terminal stock price is assumed to be log-normally distributed).

To solve for this expectation, the following theorem for solving integrals that include the lognormal density function will be used (from Smith, 1976:16):

Theorem: *If S is a variable whose value at the end of any finite time interval is log-normally distributed; $L(S_T)$ is a log-normal density function of S at T (the end of a finite time interval); and C is a function of S such that*

$$C_T = \lambda S_T - \gamma X, \text{ if } S_T - \psi X \geq 0 \text{ and}$$

$$C_T = 0 \quad \text{if } S_T - \psi X < 0,$$

Then $E(C_T)$, the expected terminal value of C , is given by the area under the part of the curve (meaning the curve that describes the log-normal distribution of S_T) for which $S_T \geq \psi X$. That is, we cut the probability density function of S_T at $S_T = \psi X$, and calculate the value of the area under the remainder of the curve. This value is given by

$$E(C_T) \equiv \int_{\psi X}^{\infty} (\lambda S_T - \gamma X) L(S_T) dS_T,$$

which has the following solution:

$$= e^{\rho(T-t)} \lambda SN \left(\frac{\ln\left(\frac{S}{X}\right) - \ln\psi + \left(\rho + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) - \gamma X N \left(\frac{\ln\left(\frac{S}{X}\right) - \ln\psi + \left(\rho - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right)$$

where λ , γ , and ψ are arbitrary constants, ρ is the expected growth rate in S [$e^{\rho(T-t)} \equiv E(S_T/S)$] and N , as before, is the cumulative standard normal density function. \square

This theorem can be used to solve for Sprenkle's expected terminal value of the option by noting that for this integral $\lambda = \gamma = \psi = 1$:

$$E(C_T) = e^{\rho(T-t)} SN \left(\frac{\text{Ln}\left(\frac{S}{X}\right) + \left(\rho + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) - XN \left(\frac{\text{Ln}\left(\frac{S}{X}\right) + \left(\rho - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right)$$

Sprenkle later included an adjustment for risk by multiplying X in the above formula by a parameter for the level of risk-aversion. He also recognised that an option is riskier than the underlying stock⁸ (see Sprenkle, 1964:416), but this observation was not built into the derivation of the model. Moreover, the expected value of the option at maturity is not discounted when calculating the current option price - the time value of money is ignored, despite the assumption of a positive drift in the stochastic process.

2.2.3. Boness - 1964

Boness's model is very similar to Sprenkle's. The crucial difference is that Boness does not ignore the time value of money. He also recognised the importance of risk, but assumed that all stocks on which options are traded have the same risk profile, and, for simplicity, that investors are risk-neutral. Accordingly, no distinction is made between the expected rates of return or risk levels of options and the underlying stocks. The expected return on the stock is used as the discount rate in calculating the present value of the expected terminal value of the option (see Ingersoll, 1989:203, and Smith, 1976:17).

Boness starts by expressing the expected value of a call option at maturity as the probability that the option will be exercised (which is the probability that the stock price at maturity will exceed the exercise price), multiplied by the difference between the conditional expected value of the stock price at maturity given that this price is larger than the exercise price, and the expected value of the strike price, given the same condition,

⁸ The reason for this difference is explained later in this paper.

$$E(C_T) = [E(S_T/S_T > X) - E(X/S_T > X)] \text{Prob}(S_T > X).$$

By definition, the probability that the stock price at maturity (which is log-normally distributed) will exceed the exercise price, is given by the area under the log-normal density function of S_T , for values of S_T larger than X ; formally,

$$\text{Prob}(S_T > X) = \int_X^\infty L(S_T) dS_T$$

The exercise price is a constant; therefore its expected value (conditional or unconditional) is the exercise price itself;

$$E(X/S_T > X) = E(X) = X.$$

Lastly, the conditional expected value of S_T given that S_T is larger than X , is the unconditional expectation of S if one considers only values of S_T higher than X , divided by the probability that S_T will be larger than X (which was defined above). Formally,

$$E(S_T/S_T > X) = \frac{\int_X^\infty S_T L(S_T) dS_T}{\int_X^\infty L(S_T) dS_T}.$$

Substituting these three definitions into Boness's expression for the expected value of the option on expiry, we get:

$$\begin{aligned} E(C_T) &= \left[\frac{\int_X^\infty S_T L(S_T) dS_T}{\int_X^\infty L(S_T) dS_T} - X \right] \int_X^\infty L(S_T) dS_T \\ &= \int_X^\infty S_T L(S_T) dS_T - X \int_X^\infty L(S_T) dS_T \end{aligned}$$

$$= \int_X^{\infty} (S_T - X)L(S_T)dS_T$$

Note that this does not differ in any way from the expression Sprengle used to express the expected terminal value of a call option. But this expectation is then discounted at ρ , the expected rate of return on the stock, to get the present value of the option

$$C = e^{-\rho(T-t)} E(C_T) = e^{-\rho(T-t)} \int_X^{\infty} (S_T - X)L(S_T)dS_T$$

Note that this integral is equal to the integral solved in the above theorem, with $\lambda = \gamma = e^{-\rho(T-t)}$ and $\psi = 1$. The theorem can therefore be used to obtain Boness's (call) option pricing formula:

$$C = e^{-\rho(T-t)} e^{\rho(T-t)} SN \left(\frac{\ln\left(\frac{S}{X}\right) - \ln\left(\rho + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) - e^{-\rho(T-t)} XN \left(\frac{\ln\left(\frac{S}{X}\right) - \ln\left(\rho - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right)$$

$$= SN \left(\frac{\ln\left(\frac{S}{X}\right) + \left(\rho + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) - e^{-\rho(T-t)} XN \left(\frac{\ln\left(\frac{S}{X}\right) + \left(\rho - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right)$$

It will be seen that the only difference between this formula and the Black-Scholes formula is the use of the expected rate of return on the stock as the discount rate, instead of the risk-free rate.

2.2.4. Samuelson - 1965

Samuelson was the first to suggest a model, which explicitly recognises that stocks and options have different risk characteristics. The higher volatility of option prices is assumed to imply a higher expected rate of return for options. He assumed, as

Sprenkle and Boness did, that the distribution of stock prices is lognormal, and that stock prices follow geometric Brownian motion with positive drift (Smith, 1976:18). The value of a call option is posited as being given by the expected value of the option at maturity, discounted by the expected rate of return on the option (κ),

$$C = e^{-\kappa(T-t)} E(C_T) = e^{-\kappa(T-t)} \int_X (S_T - X) L(S_T) dS_T .$$

This integral can also be solved through direct application of the theorem in page 6, by letting $\lambda = \gamma = e^{-\kappa(T-t)}$ and $\psi = 1$:

$$C = e^{(\rho-\kappa)(T-t)} SN \left(\frac{\text{Ln}\left(\frac{S}{X}\right) + \left(\rho + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) - e^{-\kappa(T-t)} XN \left(\frac{\text{Ln}\left(\frac{S}{X}\right) + \left(\rho - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right)$$

Samuelson does not suggest a procedure for estimating the expected rate of return on the option and admits this to be a weakness of the model (Smith, 1976:20).

Furthermore, discounting the expected terminal value of the option at the expected rate of return on the option implies that this rate can be assumed to be constant. To see why this is inadequate, consider first a situation where the option is "deeply in the money" - that is, the stock price is much larger than the exercise price. The probability that such an option will be exercised is very high, and an increase of one Rand in the stock price would be matched by an increase of approximately one Rand in the value of the option. If, for example, the term to maturity is one day, it is possible that both assets would appreciate by the same amount. But a call option allows its holder to take a position in the underlying stock with a much smaller cash disbursement. And a call is always worth less than the underlying stock (a rational investor would not pay more for the right to buy an asset than the asset is currently worth). So, even if the two assets move by the same absolute amount, the percentage change in the price of the option will be higher. The less in the money the option is, the larger will this difference be.

Now consider a scenario where the stock price is so much lower than the exercise price, (a call option is said to be "out of the money" if the stock price is lower than the strike price) that the probability of exercise is very low and the option is almost worthless. An increase in S of one Rand would mean a much smaller increase in the value of the option. But as a percentage of the option price, this increase would be much higher than that in S , and than the increase in the option price if it were in the money (Scholes, 1998:353).

It is clear that, not only is the expected rate of return on an option higher than the return expected on the underlying asset - as explicitly recognised in Samuelson's model - but also, the volatility (and therefore expected return if investors are risk-averse) of an option varies as the stock price changes, as we know it will if it follows a random-walk. Specifically, "*the higher the stock price relative to the exercise price, the safer the option, although the option is always riskier than the stock. The option's risk changes every time the stock price changes*" (Brealey and Myers, 1996:573).⁹

Note that accepting that the expected return (or riskiness) of an option is not constant puts a severe limitation on the theoretical validity of any model derived by simply estimating the expected value of the option at maturity (in itself not likely to be an easy task), and using 'the' expected return on the option to discount this expectation to the present. The procedure would only be satisfactory if it were possible to specify one expected rate of return on the option that could serve as an appropriate discounting factor - i.e.: if the option's risk could be assumed constant.

3. THE BLACK-SCHOLES MODEL - 1973

3.1. Behaviour of the Underlying Asset and Other Assumptions

⁹ It should be added that the option value also varies over time. The less the time to maturity the narrower the range within which the stock price is expected to be at maturity.

Black and Scholes assumed, as did Sprenkle, Boness, and Samuelson, that the distribution of stock prices is log-normal, and that stock prices follow geometric Brownian motion with a constant positive drift, and constant variance σ^2 . This process can be represented by the following stochastic differential equation:

$$\frac{dS}{S} = \mu dt + \sigma dz. \quad (3.1)$$

The parameter μ , the drift component of the process, is the stock's expected (or mean) rate of return. μdt is the value of this return over a small period dt . The parameter σ is the volatility of the stock price. The standard deviation of the rate of return on the stock is normally used as an approximation of this parameter. Both the drift and volatility parameters are assumed constant. dS is the change in the stock price over dt , and dS/S represents the rate of change or the proportional return on the stock over dt . z is a stochastic variable which follows a Weiner process, and dz the change in its value over dt . The term σdz represents the stochastic component of the stock's return. By definition, if z follows a Weiner process the values of dz over two different short intervals of time are independent and dz is defined as

$$dz = \varepsilon \sqrt{dt},$$

where ε is a random drawing from a standardised normal distribution (Hull, 1997:210).

For an intuitive understanding of equation 3.1 and the role of the stochastic component, consider what the path of the stock would be in the absence of randomness in the process. Imagine σ is zero. Then the stochastic term is cancelled and the equation is reduced to an ordinary differential equation

$$\frac{dS}{S} = \mu dt \quad \text{or} \quad \frac{1}{S} \frac{dS}{dt} = \mu,$$

which can be solved using simple classical calculus to obtain the path of S over time:

$$\int \frac{1}{S} \frac{dS}{dt} dt = \int \mu dt$$

$$\ln S = \mu t + c$$

$$S_t = S_0 e^{\mu t}$$

where S_0 is the value of S at time zero. This shows that with $\sigma = 0$, the asset price is non-stochastic and predictable. The Weiner process dz in equation 3.1 may be interpreted as a term to represent the variability that accompanies the path followed by the stock price if, as assumed, it follows geometric Brownian motion (Hull, 1997:212).

Despite some discrepancies, equation 3.1 has proved to be a very good model of the actual behaviour of stocks and stock-indices (Wilmott, Howison and Dewynne, 1995:22).

Other assumptions about conditions in the market for the option and the underlying stock, made in deriving the model, are the following:

- The short-term interest rate is known and constant through the life of the option;
- The stock (the underlying asset in the analysis) pays no dividends or other distributions;
- No transaction costs are incurred in buying or selling the option or the stock;
- Investors are able to borrow an amount equivalent to any fraction of the stock price at the risk-free rate, and it is possible to buy and sell any number (or fraction) of the stock;
- Short selling is permitted - that is, an investor is able to sell an asset he/she does not own by getting it from someone who does, and agreeing to buy it in the future to give it back to the person who 'lent' it.
- There are no arbitrage opportunities in asset markets, so that all securities with the same risk characteristics must be priced to earn the same return.

3.2. The Black-Scholes-Merton Analysis

The central idea in the Black-Scholes model, is the possibility of combining an option (or a number of options) and the underlying stock into a portfolio, in such a manner that the return on this portfolio is not affected by market fluctuations. That such a portfolio can be constructed is a result of the correlation between fluctuations in the prices of the option and the underlying stock. The price of a call (put) option is positively (negatively) correlated with the underlying stock price.

Consider a portfolio consisting of a short position on a call option, and some quantity of the underlying stock. The gain to the holder of this portfolio from a given increase in the stock price will be at least partly offset by a loss on the short position taken on the option. This is simply because the rise in the stock price causes the value of the call, which must be repaid, to increase as well. Thus, there must be some number of a given stock which, when combined with a short position on one option to buy that stock will result in a portfolio that, over a very small period of time, will be risk-less. For this portfolio, any gain (loss) on the long position will be completely offset by a loss (gain) on the short position such that the return on the portfolio, over this very small period of time, will be certain.

For the portfolio to remain risk-free over time, the number of stocks would have to be constantly adjusted. These adjustments would be necessary to reflect the fact that the rate of return on an option varies as the value of the underlying asset changes (as explained previously).

Let Π denote the value of this portfolio (henceforth the 'hedge portfolio'). If the portfolio consists of a short position on one call option and a long position on some quantity Δ of the underlying stock, its value at any instant will be

$$\Pi = -C + \Delta S. \quad (3.2)$$

The change in its value over a small period of time dt is

$$d\Pi = -dC + \Delta dS. \quad (3.3)$$

dS is given by 3.1 which represents the behaviour the stock price is assumed to exhibit over time. But no assumption has been made with respect to the behaviour of the option over time. Fortunately, no assumption is needed. To determine dC and thereby specify the statistical process for the option, Black and Scholes proceeded by investigating the relation between the return of an option, and the stochastic process assumed to reflect the behaviour of stock price returns over time.

Given the assumptions mentioned in the previous sub-section of this paper, an option's value depends on the price of underlying stock, time, and on variables (volatility, the exercise price, and the rate of interest) which are assumed constant. The value of an option can therefore be expressed as $C(S,t)$, a function only of the underlying asset (the stock) and time. Ito's lemma can then be used to obtain an expression for dC . This lemma shows us how to relate a small change in C , a continuous function of S , to changes in S itself – when S follows a stochastic process.

To derive Ito's lemma (in the context of the specific stochastic process assumed here to govern S), note that an approximation of dC can be obtained by taking the total differential of $C(S,t)$:

$$dC = \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt$$

This expression gives the approximate effect of a simultaneous small change in S (dS) and t (dt) on C . $\partial C/\partial S$ and $\partial C/\partial t$ are the partial derivatives of C with respect to S and t , respectively. But a more precise expression for this effect is given by the Taylor series expansion of dC which, if we let dC and dt be infinitesimally small changes in S and t (ie: let the value of these changes tend toward zero) is

$$dC = \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} dS^2 + \dots$$

Normally, when we consider the limiting case where changes in the independent variables (S and t) are assumed to be infinitesimally small, all terms involving dS and

dt raised to a power equal or greater than two are ignored (because they become insignificant). But here, dS is

$$dS = \mu S dt + \sigma S dz = \mu S dt + \sigma S \varepsilon \sqrt{dt},$$

thus,

$$dS^2 = \mu^2 S^2 dt^2 + \sigma^2 S^2 \varepsilon^2 dt.$$

The first term in the right hand-side of this equation can be ignored since $dt^2 \approx 0$, but not the second term, thus the inclusion of the third term of the expansion. Then dS^2 can be re-written as

$$dS^2 = \sigma^2 S^2 \varepsilon^2 dt.$$

Furthermore, it has been shown (see references in Wilmott et al.,1995) that as $dt \rightarrow 0$, the square of dz, the Weiner process in 3.1, tends toward dt, ie: $\varepsilon^2 dt \rightarrow dt$. It follows that in the limiting case

$$dS^2 = \sigma^2 S^2 dt$$

The Taylor series expansion of dC when dS and dt are infinitesimally small can therefore be expressed as

$$dC = \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\sigma^2 S^2 dt)$$

Use 3.1 to substitute for dS, re-arrange, and the result is ¹⁰

¹⁰ In the more general case where C is a continuous function of S and t, and S follows a stochastic process such that over time

$$dS = a dt + b dz,$$

where a and b are functions of S and t, and dz is a stochastic term, Ito's lemma can be derived using the same procedure (expanding C around S and t, and noting that in the limiting case where $dt \rightarrow 0$, $dz^2 \rightarrow dt$). The lemma states that if the evolution of S over time can be described by the differential equation above, then the evolution of C will be:

$$dC = \left(\frac{\partial C}{\partial S} \mu S + \frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right) dt + \left(\frac{\partial C}{\partial S} \sigma S \right) dz \quad (3.4)$$

This is the random walk followed by the price of the call option (Wilmott et al., 1995:42), or any other derivative (or variable) whose value is a function only of S and t, and S follows the stochastic process described in 3.1. The term in (the first) brackets represents its drift rate, and the variance rate is $(\partial C/\partial S)^2 \sigma^2 S^2$, the square of the term inside the second brackets. All parameters are as previously defined. Most importantly, dz, the Weiner process in the equation for the option, is the same as the Weiner process responsible for the randomness in stock prices. This results from both C and S having volatility in S as the source of randomness in their behaviour over time (Hull, 1997:223).

Now that it has been shown how equation 3.1 can be used to derive an expression for the change in the value of a derivative on S, 3.1 and 3.4 can be substituted back into 3.3 to get a full description of how the value of the hedge portfolio changes over an infinitesimally small period of time. The change in the value of the portfolio over dt is

$$d\Pi = \left(-\frac{\partial C}{\partial S} \mu S - \frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \mu S \Delta \right) dt + \left(-\frac{\partial C}{\partial S} + \Delta \right) \sigma S dz. \quad (3.5)$$

It becomes clear that the stochastic term can be eliminated from the process by setting $(-\partial C/\partial S + \Delta)$ equal to zero. The quantity of stocks (Δ) that must be held to neutralise the effect on the portfolio of a change in the value of the short position, over dt, is therefore obtained as

$$-\frac{\partial C}{\partial S} + \Delta = 0, \text{ or } \Delta = \frac{\partial C}{\partial S}.$$

$$dC = \left(\frac{\partial C}{\partial S} a + \frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} b^2 \right) dt + \left(\frac{\partial C}{\partial S} b \right) dz.$$

If Δ is substituted by $\partial C/\partial S$ in 3.5, the stochastic term is cancelled and $d\Pi$ becomes

$$d\Pi = \left(-\frac{\partial C}{\partial S} \mu S - \frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \mu S \frac{\partial C}{\partial S} \right) dt + 0$$

$$= \left(-\frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right) dt. \quad (3.6)$$

This equation shows that if Δ is equal to $\partial C/\partial S$, which is a measure of the responsiveness of the option's value to an infinitesimally small change in the value of the stock, Π does not follow a random walk. And $d\Pi$, the change in the value of the portfolio over dt , becomes non-stochastic.¹¹ If there are no arbitrage opportunities in the market, the value of the option must be such that the rate of return on this portfolio over dt is equal to the risk-free rate r . If it were higher than the risk-free rate, investors would borrow an amount Π at r , buy (or construct) the portfolio, and earn a riskless profit since it is certain that at the end of dt the portfolio's value will be higher than the repayment on the loan ($\Pi + r\Pi dt$). The resulting demand for the portfolio would put upward pressure on its value until its rate of return equals r .

The opposite would happen if the return on the portfolio were less than r . Investors would sell the portfolio short and use the proceeds to buy default-free government bonds which are guaranteed to yield a higher rate of return, thereby locking a riskless profit. In this case arbitrage would put downward pressure on the value of the portfolio until its rate of return and the risk-free rate are equal (approximately equal in the presence of low transaction costs). In sum, the existence of arbitrageurs implies that in equilibrium,

¹¹ Setting $\Delta = \partial C/\partial S$ neutralises the effect of changes in the stock price on the option price, but the latter is also a function of time. For this reason the change in the value of the risk-free portfolio over time is not equal to zero.

$$\frac{d\Pi}{\Pi} = rdt, \text{ or } d\Pi = r\Pi dt$$

Substituting equations 3.2 (with $\Delta = \partial C/\partial S$) and 3.6 into this condition we get

$$\left(-\frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right) dt = r \left(-C + \frac{\partial C}{\partial S} S \right) dt. \quad (3.7)$$

Dividing both sides by dt results in

$$\left(-\frac{\partial C}{\partial t} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right) = -rC + \frac{\partial C}{\partial S} rS, \text{ or}$$

$$\frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial S} rS = rC, \quad (3.8)$$

which is the Black-Scholes (partial) differential equation.

Note that under the assumptions made in deriving the Black-Scholes equation and as a result of the generality of Ito's lemma, this equation would not differ if the derivative being considered were a put-option, or any other asset whose value depends solely on the stock price and time. A value for a derivative that violates equation 3.8 would only be possible in the presence of arbitrage opportunities.

3.3. The Black-Scholes (European) Call Option Pricing Formula

Deriving a formula for the price of a European option requires that equation 3.8 be solved. But obtaining a solution to equation 3.8 which is unique to a specific type of derivative requires the imposition of conditions that specify values of the required solution for possible values of the stock price and time. Three conditions must be specified to obtain a formula for the pricing of an option: the value of the option at maturity; the value of the option when the underlying asset becomes worthless; and the value of the option as the value of the underlying asset increases infinitely (Wilmott, Howison, and Dewynne, 1995:46). To obtain a formula for the value of a

call option, Black and Scholes impose the appropriate boundary conditions on 3.8, transform it into the heat exchange equation (used in physics to describe the transfer of heat under certain conditions), and solve it.¹²

However, there is a simpler and more intuitive approach. This is based on a very important property of the Black-Scholes equation: it does not include the expected rate of return on the underlying asset, and therefore the value of an option, whatever the solution to the equation turns out to be, will not be affected by it. μ , the term that represents this rate, is cancelled when a risk-free portfolio is constructed by setting Δ equal to $\partial C/\partial S$ (see the steps between equations 3.5 and 3.6). The intuitive explanation for this is that an investor who expects the price of a certain stock to go up (down) will consider both the stock and the call option on that stock to be underpriced (overpriced), so that the value of the call expressed in terms of the stock price will not be affected (Black, 1975:36).

Yet, μ is the only term in the analysis that incorporates investors' risk preferences - the more risk-averse an investor is, the higher will be the return he/she expects from a given stock. So, the value of the option is not affected by investors' attitudes towards risk.¹³ Investors may have different levels of risk aversion and expectations about the return on the underlying asset but value the option equally. This implies that the solution to the Black-Scholes equation that would be obtained assuming risk-aversion

¹² This method of solving the Black-Scholes equation is described in Wilmott, Howison, and Dewynne, 1995 (Chapters 4 and 5). Those unfamiliar with the methods used to solve second-order parabolic (partial differential) equations may find it refreshing to know that neither did Black, nor Scholes, when they derived the equation. In fact, they got the first glimpse of the Black-Scholes formula (the solution to this equation) by taking Sprenkle's formula, substituting the rate of return by the interest rate, and then checking that it is a solution to the differential equation. (See Black, 1989:5-6.)

¹³ This may seem counter-intuitive to those who read Black and Scholes' 1973 paper (without reading Merton (1973)), since they derive the option pricing model within the framework of the Capital Asset Pricing Model - a model which has as its very foundation the trade-off between risk and return. Merton (1973) showed that the CAPM does not have to be assumed to hold, in order to derive an equilibrium option-pricing model.

or risk-neutrality would be the same. But in a risk-neutral environment investors do not require compensation for additional risk in the form of increased returns, and as a result all securities are expected to earn the same rate of return which will be the risk-free rate r (Hull, 1997:239).

This insight considerably simplifies the procedure to find the formula for the option price. If all securities earn the risk-free rate, a value for the option which is consistent with equation 3.8 can be obtained by discounting, at the risk-free rate, the expected value of the payoff to the option holder at maturity:

$$C = e^{-r(T-t)} E(C_T) = e^{-r(T-t)} \int_X (S_T - X) L(S_T) dS_T. \quad (3.9)$$

In this case discounting the expected value of the option at maturity is valid because the risk-free rate is assumed to be constant. Again, this equation can be solved through direct application of the theorem described in page 6, with $\lambda = \gamma = e^{-r(T-t)}$ and $\psi = 1$, and substituting ρ by r . The result is the **Black-Scholes formula** for the price of a European call option:

$$C = SN \left(\frac{\text{Ln} \left(\frac{S}{X} \right) + \left(r + \frac{\sigma^2}{2} \right) (T-t)}{\sigma \sqrt{T-t}} \right) - e^{-r(T-t)} XN \left(\frac{\text{Ln} \left(\frac{S}{X} \right) + \left(r - \frac{\sigma^2}{2} \right) (T-t)}{\sigma \sqrt{T-t}} \right), \quad (3.10)$$

or, as more commonly expressed,

$$C = S N(d_1) - e^{-r(T-t)} X N(d_2),$$

where

$$d_1 = \left(\frac{\text{Ln} \left(\frac{S}{X} \right) + \left(r + \frac{\sigma^2}{2} \right) (T-t)}{\sigma \sqrt{T-t}} \right), \text{ and}$$

$$d_2 = \left(\frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \right) = d_1 - \sigma\sqrt{T-t}$$

The formula expresses the call at time t as equal to S , the stock price at time t , minus the exercise price discounted at the risk free rate, with each of these two terms multiplied by a probability. S is multiplied by the number of stocks that must be held against a short position on a call on a share of the same stock, since $\partial C/\partial S$, the partial derivative of C with respect to S , is equal to $N(d_1)$. $N(d_2)$, the term multiplied by the present value of the strike price, represents the probability that at maturity the stock price will exceed the strike and therefore the option will be exercised (Copeland and Weston, 1988:276).

All the variables required to use the Black-Scholes formula to calculate the option price, except the stock's volatility, are directly observable. The volatility can be estimated by calculating the standard deviation of returns, based on past stock price data. Thus, as noted by Merton (1989:216), the valuation of an option is not dependent on its existence. If options do exist however, the prices at which they are traded and the Black-Scholes formula can be used to compute the volatility implied in option prices.¹⁴

Finally, note that the only difference between the Black-Scholes formula and the formula derived by Boness (page 9), is the use of r , the risk-free rate, as the rate of return on the option instead of ρ , the rate of return on the underlying asset. "*If Boness had carried his assumption that investors are indifferent to risk to its logical conclusion [that the expected rate of return on the stock is equal to the risk-free rate], he would have derived the Black-Scholes equation*" (Ingersoll, 1989:204). However, it is clear that the derivation of the two models differs substantially. Boness did not formulate his model within the context of capital market equilibrium. He failed to demonstrate that a risk-free portfolio could be constructed, which would result in risk

¹⁴ Implied volatility is discussed in some detail in Part Two.

neutrality being acceptable as a basis to calculate the price of an option. Furthermore, although the Black-Scholes formula was obtained within a risk neutral environment, it is never assumed that investors are risk neutral. The formula is valid irrespective of risk preferences.

The next part of this paper contains an attempt at looking at the empirical validity of this model, in the context of the South African stock options market.

PART TWO: The Pricing of South African (Equity) Stock Options

4.1. The South African Exchange-Traded Equity Options Market – A Brief Overview

An attempt at creating an exchange-traded equity options market in South Africa was made during the early 1990's. However, no institution assumed responsibility for ensuring liquidity in the market. Despite a relatively active stock market, the options market was highly illiquid and this attempt was unsuccessful (Deutsche Morgan Grenfell Research, 1999a).

The options used in this study are traded on the equity options market, which was launched in October 1997, when Deutsche Bank issued options on shares listed on the Johannesburg Stock Exchange (JSE). The options, known somewhat confusingly as warrants, are also listed on the JSE.¹ The rules and costs which apply to trade in equity options are similar to those that apply to stock trading (Deutsche Morgan Grenfell Research, 1999b).

Standard Bank of South Africa, United Bank of Switzerland, and ING-Barings later joined the market. The issuers are responsible for delivery of, or payment for, the underlying asset upon exercise of the option contract. They also act as market makers to promote liquidity in the market (except for ING Barings, which guarantees delivery but does not act as market maker). There are currently 50 equity options being traded

¹ In the financial economics literature, the term '**warrants**' in the context of derivatives markets, refers to options to buy stock which are issued by the firm which also issues the underlying stock. Upon exercise of a warrant, the issuing firm issues the shares for delivery in accordance with the option contract, resulting in a dilution of the value of the underlying asset. The effect of this dilution is an important factor in the valuation of the warrant contract.

But the warrants traded in the South African warrants market are in fact issued by a third-party, with no involvement of the company which issues the underlying stocks. These securities are commonly referred to in the finance literature simply as stock options. (See, for example, Black and Scholes (1973), Brealey and Myers (1996), Copeland and Weston (1988), Cox and Rubinstein (1985), Hull (1997), or Merton (1973).) The only aspect about the 'warrants' traded in South Africa that is more common to warrants than plain stock options, is their longer term to maturity.

on the JSE, and 12 new listings have been announced (Deutsche Morgan Grenfell Research, 1999c). Deutsche Bank and Standard Bank are the main issuers. The options issued by the former and marketed by Deutsche Morgan Grenfell, its subsidiary, seem to be the most liquid in the market. Most listed options are American calls. American puts, as well as European calls and (mainly) puts are also traded.

4.2. Data

The data used in this study consist of daily observations of call option prices, prices of the underlying stocks, and the rate of interest on South African Government Bonds. These data, as well as data on trade volumes of the options and on dividend payments to holders of the underlying assets, were extracted from the I-NET Database, accessible from the computer laboratories in the Commerce Faculty of the University of Cape Town. Information on the terms of the option contracts, including exercise provisions, number of options convertible into one share of stock, and expiry dates, were obtained from the web-pages of DMG-South Africa and Standard Bank, as well as from private communication with DMG-South Africa.

The study covers the twelvemonth period between the 1st of June 1998 and 31st of May 1999, representing 251 trading days. In order to ensure that a reasonably long period is covered, only options for which a series of not less than twelve months was available by the end of this period were considered.

Of these, options that had any of the exercise provisions in the option contract altered were excluded.² In addition, one option was excluded due to very thin trading.³ These

² These were the Deutsche Bank option on South African Breweries stock, and the Standard Bank option on Goldfields Limited stock. The exercise price in these options has changed since they were issued.

³ This was the ING-Barings call on NEDCOR stock. The trading volume for this option is zero for a total of no less than 101 trading days during the sample period. (This is equivalent to approximately 4.8 calendar months) Also, the option is not traded for up to 5 days in a row. The liquidity of the five options remaining after these exclusions is considerably higher. One of them is not traded during a total of 34 days

exclusions were made in order to minimise violations of the assumptions made in deriving the model to be tested. (Recall that the exercise price was assumed constant, and investors are assumed to be able to continuously adjust their positions in the hedge portfolio.) The resulting sample of option price data consisted of daily closing price observations for five American call options. In total, 1255 daily price observations from these five options were used.

The underlying asset for each option is a share (or fraction thereof) of the common stock of a South African company whose shares are listed on the JSE. The daily closing price was used as the price of the underlying stock for each day in the sample.

Table 1 shows the JSE code for each of the options used; the option's maturity date; the first exercise date (the options' terms are such that they can be exercised any day between this date and the expiry date); and the name of the underlying stock.

<i>Option</i>	<i>Maturity Date</i>	<i>Exercisable From:</i>	<i>Underlying Stock</i>
1AMS	15.06.2000	01.02.1998	Anglo American Platinum Corporation
1CPX	16.03.2001	01.06.1998	Comparex
1DDT	15.06.2001	01.09.1998	Dimension Data Holdings
1ISC	17.03.2000	01.02.1998	ISCOR Limited
1SOL	16.06.2000	01.02.1998	SASOL Limited

Sources: Deutsche Morgan Grenfell and I-Net.

Merton (1973) has shown how the Black-Scholes model can be derived without assuming a constant interest rate during the life of the option. His derivation involves the construction of a hedge portfolio, consisting of an option, the underlying common stock, and risk free bonds, where the maturity date of the bonds coincides with the expiration date of the option (Merton 1973:164). It has since become common in empirical studies of option pricing models, to use the yield on a risk free bond whose maturity date is near the maturity date of the option, as a proxy for the interest rate parameter (see for example Whaley, 1982:711, or Lauterbach and Schultz, 1990:1188). This was the approach employed in this study, where the daily yield on

(ISCOR), another during 13 days (SASOL), and the others during 2 to 4 days throughout the sample – not ideal but hopefully reasonable enough to test the model.

SA Government bonds with maturity dates nearest the expiry dates of the option contracts were used.

Table 2 shows the code for each option; the abbreviated name of the SA Government bond whose annual yield was used as the proxy for the risk free rate; and the maturity date of each bond.

<i>Option</i>	<i>Bond</i>	<i>Bond's Maturity Date</i>
1AMS	RSA 148	20.05.2000
1CPX	RSA 088	15.04.2001
1DDT	RSA 174	15.07.2001
1ISC	RSA 083	15.04.2000
1SOL	RSA 148	20.05.2000

Source: I-Net.

4.3. Statistical Distribution of the Underlying Assets

Once the options to be included in the study had been selected, the statistical behaviour of the underlying asset for each of these options was examined. Figures one to five below, show the histograms and relevant statistics, for the distribution of the daily rate of return (the natural growth rate) for each underlying stock.⁴

⁴ The daily **rate of return** for each underlying stock was computed as

$$R_t = \text{Ln} \left(\frac{S_t}{S_{t-1}} \right),$$

the natural logarithm of the closing stock price at day t, divided by the closing stock price on the previous day. This is equivalent to the difference between the log of the closing stock price on day t, and the log of the closing price on the previous day. It represents the continuously compounded rate of return.

Figure 1: Distribution of the Daily Rate of Return on Amplats⁵ Stock.

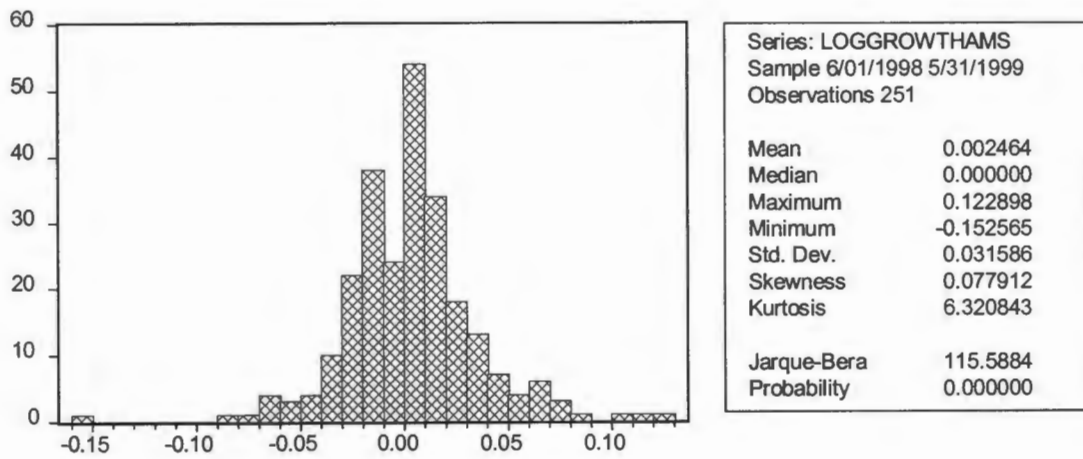
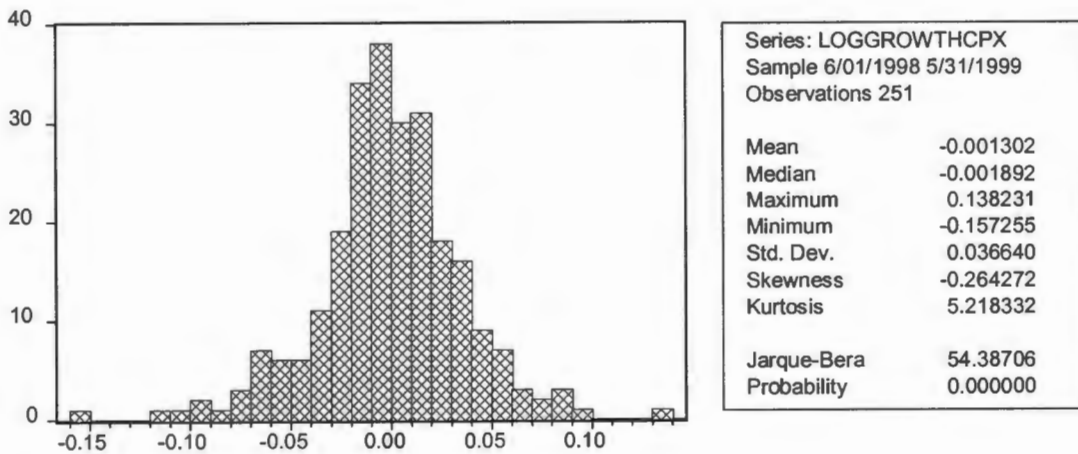


Figure 2: Distribution of the Daily Rate of Return on Comparex Stock.



⁵ Anglo American Platinum Corporation.

Figure 3: Distribution of the Daily Rate of Return on Digital Data Stock.

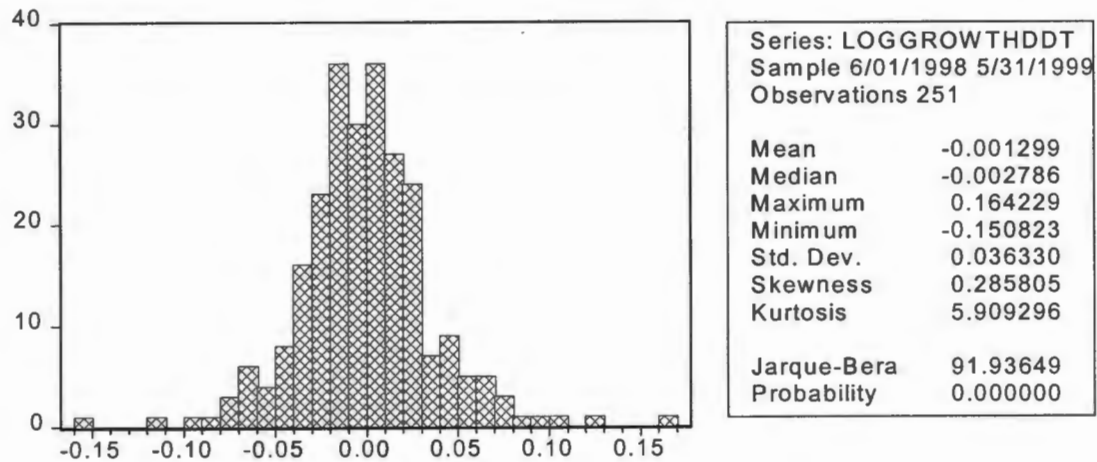


Figure 4: Distribution of the Daily Rate of Return on ISCOR Stock.

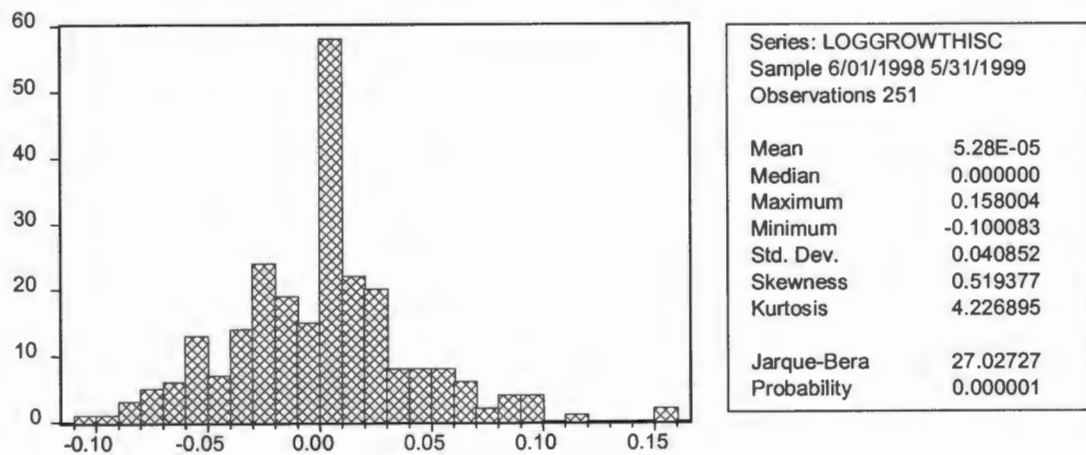
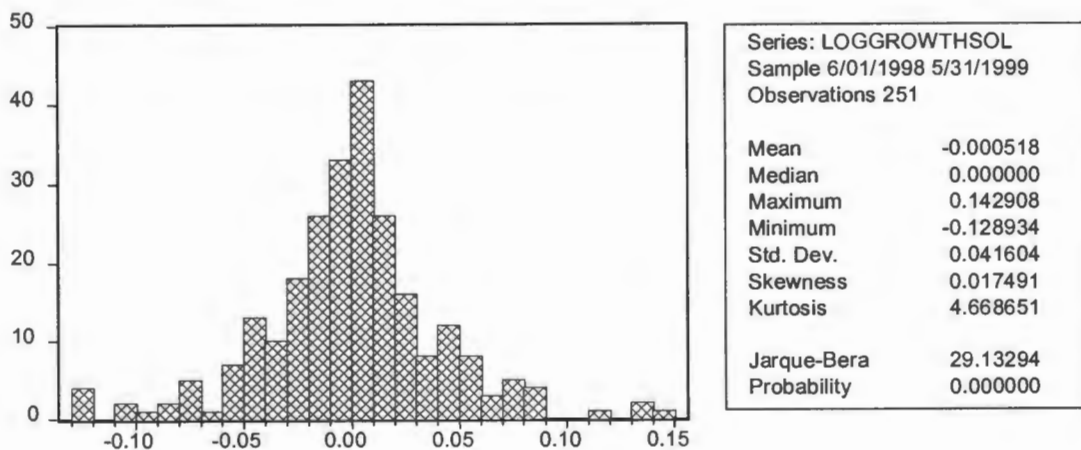


Figure 5: Distribution of the Daily Rate of Return on SASOL Stock.



A graphical analysis of these histograms shows that the distribution of the rate of return on these stocks is quite similar to the normal distribution. This suggests a strong possibility that the actual distribution of the stock prices is lognormal. The skewness and kurtosis parameters can be used to evaluate this possibility, and thereby determine the adequacy of assuming that the distribution of the rate of return on these stocks is normal.

The skewness of a symmetric distribution, such as the normal distribution, is equal to zero. The kurtosis of a normally distributed variable is equal to three.⁶ Thus, the extent to which it can be assumed that the distributions in figures one to five are normal depends on the proximity of the computed skewness and kurtosis estimates, to zero and three, respectively.

It has been shown that, in large samples of normally distributed data, the skewness estimate is normally distributed with mean zero and variance $6/N$; and the kurtosis estimate is normally distributed with mean three and variance $24/N$, where N is the sample size (Campbell, Lo, and MacKinlay, 1997:17).

Under the null hypothesis that the distributions are in fact normal, and given that N is equal to 251 for the samples shown here, the standard error for the skewness of these distributions is equal to the square root of $6/251$, while the standard error for the kurtosis is equal to the square root of $24/251$. This means that, at the 95% level, the confidence interval for the skewness is between -0.303 and 0.303 , while the confidence interval for the kurtosis is between 2.393 and 3.606 . At the 90% level, we are confident that if the sample distributions are normal, the skewness will be between -0.408 and 0.408 , while the kurtosis will be between 1.399 and 4.600 .

From the distribution statistics shown for each stock return sample, it can be seen that the range for the skewness estimates goes from -0.26 to 0.51 . Except for the ISCOR stock (figure 4), sample skewness estimates are not significantly different from zero. However, the sample kurtosis estimates range from 4.22 to 6.32 , and none of the estimates fits within the 95% confidence interval. The excess kurtosis present in the

sample stock return distributions is indicative of tails (of the distributions) that contain more mass than would be predicted by a normal distribution – i.e. the tails of the distributions are fatter than expected.

To the extent that significant excess kurtosis is present in the sample distributions, these results are consistent with what has been observed in the US stock market. However, the magnitude of the excess kurtosis observed for the five return distributions shown in this paper, are considerably smaller than what has been found for some samples (albeit much larger than those used here) of daily returns on US stocks.⁷ It must be noted that it is not being suggested that this serves as evidence that return distributions of (some) South African stocks are closer to the normal distribution than most US stocks. But it seems clear that the lognormal distribution serves as a reasonable approximation of the actual distribution of the stock prices for the five stocks used in this study. And the fact that some successful tests of stock option pricing models (which assume stock returns to be normally distributed) have been conducted using US stock data, can be used to support this conclusion.⁸

4.4. Volatility

4.4.1. Historical and Implied Volatility Estimates

As mentioned previously, the volatility parameter is the only input to the Black-Scholes formula that is not directly observable. It represents the diffusion rate of the Brownian motion, assumed to govern the evolution of the underlying stock over time. The simplest method to obtain an estimate for this parameter is to calculate the conventional standard deviation of the rate of return on the underlying stock, using

⁶ To see the formulas for calculating these parameters see, for example, Campbell, Lo, and MacKinlay (1997:17).

⁷ Campbell, Lo, and MacKinlay (1997:19-21) show the distribution statistics for the daily returns of ten representative stocks, listed on the New York Stock Exchange and the American Stock Exchange, for the 1962 to 1994 period ($T = 8179$). The sample excess kurtosis (difference between the kurtosis estimate and three) for the stock returns range from 3.35 to 59.49.

⁸ See, for example, Whaley (1982).

historical stock price data. This was the method used by Black and Scholes (1972) in an empirical study of their model,⁹ as well as by other early empirical studies of option pricing models.¹⁰

However, an historical measure of volatility may fail to reflect the market's view, at a point in time, of the future volatility of the stock. We can expect this to be the case whenever a shock hits the stock market. Since the market's assessment of future volatility will determine the volatility input when options are valued, a model that uses only past data to estimate this parameter will fail to price them correctly.

The most commonly used alternative to the estimation of volatility from past stock price data, is the use of the implied volatility. This measure is obtained by observing the market price of the option and feeding all other parameters into the option pricing formula. Then, through a numerical procedure (analytically inverting the Black-Scholes formula to obtain an expression for volatility is normally not possible), it is possible to find the value for the volatility parameter that is implicit in the formula. Given all other parameters, this is the estimate of volatility, which allows the model to perfectly explain the option price.

This measure has been used extensively in the literature. If various options on a given stock are available, the volatility implied by each of them is computed and the average of the individual volatilities, often weighted by the sensitivity of each option to the volatility parameter, is used as the estimate of volatility. A computationally simpler approach would be to use the volatility implied by the option that is nearest to being at-the-money, the only instance in which it is possible to invert the Black-Scholes formula (Brenner and Subrahmanyam, 1988:81). Empirically, this estimate performs as well as any weighted-average (Mayhew, 1995:10).

⁹ Note that Black and Scholes' seminal paper was published in 1973. Their test of the model was published before the model itself. The original theoretical paper had however been submitted for publication before the empirical study – but it was not accepted and had to be re-submitted, after the interference of more well known economists. (See Bernstein, 1992.)

Early empirical studies on the relative performance of historical and implied volatility estimates in predicting future volatility, showed evidence that implied measures are superior. More recent studies have shown that certain measures of historical volatility, some computed by using more than just the closing stock prices and others based on Auto-Regressive Conditional Heteroskedasticity models, perform much better than the conventional measure. The evidence on whether implied estimates perform significantly better than the non-conventional historical estimates is somewhat mixed (Mayhew, 1995:11).

To the author's knowledge, this issue has never been addressed in the South African context. But if research on this matter were to be conducted, and implied volatility found to be a better predictor of the future volatility of returns on South African stock, it would not follow that this measure should be preferred when computing the volatility input to an option pricing model. (Especially if the validity of the model is being tested.)

The reason for this is simple. We can only tell that the volatility measure we obtain by inverting the Black-Scholes formula represents the market's estimate of the underlying stock's volatility, *if the model is the correct model to value the option.* "*If the Black-Scholes formula does not hold, then the implied volatility is difficult to interpret since it is obtained by inverting the Black-Scholes formula.*" (Campbell, Lo, and MacKinlay, 1997:378.)

Even if the Black-Scholes model were the correct model to value stock options in South Africa (and it has never been shown that it is), the use of implied volatility estimates in this study would face another problem. It has been shown that the model performs better for options that are not deeply in or out of the money.¹¹ The correct procedure to estimate implied volatility would therefore be to invert the formula using the market price, over (for example) the last three months before 01-06-1998, of an option that is not too deeply in or out of the money. This estimate would then be used

¹⁰ See, for example, the study by Galai (1977).

¹¹ See Black and Scholes (1972), Black (1975), MacBeth and Merville (1979), and Brenner and Subrahmanyam (1988).

in valuing another option on the same stock. (If the Black-Scholes model holds, prices of different options on the same underlying asset should imply the same volatility.)

However, only one option on ISCOR stock exists, and it is considerably out of the money since March 1998. There are at least two options on the other stocks, but they are all very illiquid assets. And of these, a European call on the Digital Data stock (its JSE code is 5DDT), is the only option that is not considerably out of the money throughout the three months that precede the sample period. The use of a historical estimate of volatility was therefore chosen.

4.4.2. Conventional and Extreme Value Estimates of Historical Volatility

The “classical” procedure for estimating volatility from historical stock price data, is to compute the mean rate of return on the closing stock price over some period, and calculate the sum of squared deviations of the observed returns from this mean, divided by the number of observations minus one. This gives us σ_c^2 , henceforth the conventional variance of returns,

$$\sigma_c^2 = \frac{1}{n-1} \sum_{t=1}^n (R_t - \mu)^2. \quad (4.1)$$

Where μ is the mean rate of return (the drift rate of the process that generates the stock price),

$$\mu = \frac{1}{n} \sum_{t=1}^n R_t$$

And n represents the number of observations in the sample used to calculate the volatility estimates. The conventional standard deviation, σ_c , is equal to the square root of the variance.

It should be noted that a measure of the stock’s past rate of return does appear in the formula after all. What matters is that the irrelevance of estimating the expected rate of return when valuing the option is maintained. Because the square of the logarithmic

daily change is being used (the same would apply if weekly data were used), the volatility estimates are insensitive to the estimate of the mean return (See Cox and Rubinstein, 1985:260).

Parkinson (1980) suggested that historical volatility measures could be improved by using the range between high and low prices, instead of the closing price. If this method is used, Parkinson (1980:63) suggests an unbiased extreme value estimator of σ^2 , the true variance of the process, which can be expressed as

$$\sigma_p^2 = \frac{1}{(4Ln2)n} \sum_{i=1}^n (Ln(h_i) - Ln(l_i))^2, \quad (4.2)$$

where h and l stand for the day's high and low prices, respectively.¹² He shows that the efficiency of this estimator, relative to the conventional "close-to-close" estimate, is 5.2. This means that the variance of the Parkinson (variance) estimator is, in theory, approximately one-fifth of the variance of the conventional (variance) estimator. Parkinson's deviation, σ_p , can be obtained by taking the squared root of the variance.

Garman and Klass (1980) suggest other extreme value estimators, using high, low, open, and close prices. They show that their estimators (with relative efficiency factors around 8) are more efficient than Parkinson's.

Beckers (1983:98) notes that the Garman and Klass "practical" estimator is essentially a weighted average of Parkinson's and the conventional estimator. He also notes that both, the estimators of Parkinson and those of Garman and Klass, will be biased downward, especially if trade is not very frequent, because "*high and low prices will respectively understate and overstate the 'true' values*" (Beckers, 1983:98). It follows

¹² Note that if daily data is used to calculate volatilities, these must be *annualised* before being used as inputs to the Black-Scholes formula. Given that the stock price is assumed to follow geometric Brownian motion, the annual deviation is obtained by multiplying the daily deviation by the square root of the number of trading days in the year, which is typically assumed to be equal to 252. The annual variance can be obtained by multiplying the daily variance by the number of trading days in the year (or simply by squaring the annual deviation).

that the theoretically higher efficiency of extreme value estimators does not imply they should be preferred over the conventional estimators.

To decide which historical estimator should be used, I followed Beckers' approach and concentrated on the conventional and Parkinson's estimators. He suggests a comparison of the predictive ability of each estimator, using simple regression equations. The test consists of determining which variance estimator is both the best predictor of itself, and of the alternative estimator.¹³

Accordingly, the daily high, low, and closing stock prices were used to compute the variance estimators, over the sample period. The value of each estimator on a given day in the sample period was obtained using the previous three months' prices. (Effectively, prices for the period between 01 March 1998 and 30 May 1999 were used.) Using these sample estimators, four simple regression equations were estimated for each of the five underlying stocks. The summarised results are shown below.

The first two equations show the predictive power of Parkinson's variance estimator, by regressing the conventional and Parkinson variances on the lagged (by one day) Parkinson variance. The last two equations do likewise for the conventional variance estimator: in the third equation the lagged (by one day) conventional variance is used to explain changes in the Parkinson estimator, and the same explanatory variable is used in the last equation to explain changes in the conventional variance. α is the coefficient of the intercept term, β the coefficient of the explanatory variable, and ε is a random error term. The t-statistics are shown in brackets. (The coefficient of the explanatory variable in each equation is highly significant.)

¹³ Beckers (1983:112) shows a proof that this will be the most accurate estimator of the true volatility.

Equation 1: $\sigma_c^2(t) = \alpha + \beta\sigma_p^2(t-1) + \varepsilon$

STOCK	α	β	R^2
Amplats	-0.061 (-6.15)	1.683 (34.98)	0.831
Comparex	0.053 (11.94)	0.259 (77.38)	0.960
Digital Data	0.092 (16.81)	0.747 (54.03)	0.921
ISCOR	0.132 (11.05)	0.728 (22.88)	0.678
SASOL	-0.005 (-1.078)	0.430 (40.87)	0.870

Equation 2: $\sigma_p^2(t) = \alpha + \beta\sigma_p^2(t-1) + \varepsilon$

STOCK	α	β	R^2
Amplats	0.001 (0.81)	0.991 (112.81)	0.980
Comparex	0.003 (0.65)	0.997 (226.16)	0.995
Digital Data	0.001 (0.52)	0.996 (190.96)	0.993
ISCOR	0.002 (0.87)	0.996 (145,76)	0.988
SASOL	0.001 (0.52)	0.996 (192.46)	0.993

Equation 3: $\sigma_p^2(t) = \alpha + \beta\sigma_c^2(t-1) + \varepsilon$

STOCK	α	β	R^2
Amplats	0.061 (15.11)	0.499 (37.12)	0.847
Comparex	-0.131 (-7.33)	3.237 (71.74)	0.954

Digital Data	-0.084 (-8.681)	1.216 (47.40)	0.900
ISCOR	-0.009 (-0.53)	0.941 (22.43)	0.669
SASOL	0.061 (6.644)	2.057 (46.96)	0.898

Equation 4: $\sigma_c^2(t) = \alpha + \beta\sigma_c^2(t-1) + \varepsilon$

STOCK	α	β	R^2
Amplats	0.000 (0.51)	0.995 (166.66)	0.991
Comparex	0.001 (0.58)	0.996 (174.59)	0.991
Digital Data	0.002 (0.92)	0.994 (170.40)	0.991
ISCOR	0.000 (0.09)	1.001 (98.22)	0.975
SASOL	0.000 (0.18)	0.998 (287.70)	0.997

The R^2 statistic shows the explanatory power of the independent variable for each equation estimated. It can be seen from a comparison of equations one and three, that the conventional variance estimator is a better predictor of the Parkinson variance (than the latter is of the former) for three of the stocks, namely Amplats, ISCOR and SASOL. The opposite is true for the other two stocks. That is, for Comparex and Digital Data (DiData), the Parkinson estimator predicts changes in the conventional variance with better accuracy than the conventional estimator predicts changes in the Parkinson variance.

The situation is reversed when we look at the estimators' power at predicting changes in their own values. Equations 2 and four show that for three stocks, (Comparex,

DiData, and ISCOR) the Parkinson estimator is a better predictor of itself. The opposite is true for the other two stocks.

This balance in predictive power, between the two estimators, is reinforced by the fact that the differences in predictive power are marginal. This may suggest that investors in the JSE pay as much attention to the range between high and low prices within trading days, as they do to close-to-close variance. (But this statement can only be confirmed with a study covering a larger sample – including more stocks and over a longer period.) In the light of these results, and given the downward bias that is characteristic of extreme value estimators, it was decided that the use of the conventional method to estimate the volatility parameters would be adequate for the purposes of this study. Accordingly, equation (4.2) was used to estimate the variance and standard deviation of stock returns, proxies for the volatility parameters in the option pricing formulas.

4.5. Tests of Option Valuation

4.5.1. Test One: the Unadjusted Black-Scholes Model

In an attempt at testing the validity of the Black-Scholes model to the pricing of call options on South African stock, an approach similar to that followed by Whaley (1982) was adopted. The fact that the options used in this study (like most options traded on the JSE) are of the American type was initially ignored. No adjustment was made for dividend payments, and the possibility of early exercise was not considered.

Black-Scholes model prices were computed for each of the five options, using the conventional volatility estimates described in the previous section. To examine the extent, to which changes in model prices can explain changes in the options' market prices, the following regression equation was estimated for each option:

$$C_a = \alpha + \beta C_m + \varepsilon.$$

C_a denotes the actual market price of the option, and C_m , the explanatory variable in the equation, is the model price. α , β and ϵ represent, respectively, the intercept term, the coefficient of the explanatory variable, and a random error term (as in the regressions in the previous section).

The summarised regression results are shown below (full regression outputs are shown in Appendix 1).

Test One: Summarised Regression Results (Black-Scholes Model)

Option	α	β	R^2
1AMS (Amplats)	-1.110 (-0.23)	0.792 (46.05)	0.89
1CPX (Comparex)	45.621 (20.03)	0.855 (84.18)	0.96
1DDT (DiData)	15.712 (16.39)	1.756 (60.97)	0.93
1ISC (ISCOR)	2.642 (4.43)	0.673 (4.43)	0.84
1SOL (SASOL)	7.148 (7.63)	0.606 (26.80)	0.74

In terms of explanatory power the model seems to perform reasonably well, with an average R^2 of 0.87. The model's performance in the case of the SASOL option is considerably poorer. The average R^2 if this option is excluded is 0.91. Despite the strong explanatory power of model prices for the DiData option, it must be noted that the model systematically (and grossly) underestimates the market value of this option. (See relevant graph in Appendix Three, p.60.) This result is consistent with those found by Black and Scholes (1972). They found that options deeply out of the money were found to be underestimated by the model (or that the market overprices these options, if the model is valid). The DiData option is deeply out of the money throughout the entire sample period. There is no marked systematic bias with respect to the other options.

However, Whaley (1982:43) notes that in the “ideal” situation where the regression of market price on model value yields a “line of perfect forecast” (perfect prediction of market prices by the model), α and β should be indistinguishable from zero and one, respectively. This is clearly not the case for any of the regressions estimated.

But most importantly, with respect to the explanatory power of the equations, the Durbin-Watson statistics (see Appendix 1) are all very close to zero. With a sample of over 200 observations and one independent variable, the lower bound for the Durbin-Watson statistic is 1.758, at the 5% significance level (1.664 at the 1% level). Since the Durbin-Watson for all regressions is far below the lower bound, it is clear that positive serial correlation (positive correlation of the error terms, which are assumed random in the “Ordinary Least Squares” (OLS) procedure used to generate the equation parameters) is present. The OLS procedure is therefore likely to have overestimated the R^2 statistics.

Given the proximity of the computed Durbin-Watson statistics to zero, it seems reasonable to assume that the (first-order) coefficients of correlation of the error terms are “close enough” to one - in which case perfect positive serial correlation is present.¹⁴ If the Durbin-Watson statistics are equal or very close to zero (and the coefficients of correlation close to one), a simple method to adjust the OLS estimation procedure is available. This is the first-difference method, which consists of converting the regression equation into a first-difference equation. The change (first difference) in the option price was used as the explanatory variable, and the change (first difference) in the theoretical value was used as the explanatory variable. The intercept (a constant) vanishes, and the following regression equation was estimated for each option:

$$\Delta C_a = \beta \Delta C_m + \epsilon,$$

where $\Delta C_a = C_a(t) - C_a(t-1)$, and $\Delta C_m = C_m(t) - C_m(t-1)$.

¹⁴ Estimates of the first-order coefficients of correlation (of the error terms) can be calculated, for large samples, as $\{1 - d/2\}$, where d represents the Durbin-Watson statistic. (Gujarati, 1995:430.)

Serial-correlation was eliminated. But the test results, summarised in the next table (full results in Appendix 2), were not impressive.

Test One: Summarised Regression Results (Black-Scholes Model) – *First-difference regression equations.*

Option	β	R^2
1AMS (Amplats)	0.557 (20.54)	0.62
1CPX (Comparex)	0.642 (18.27)	0.57
1DDT (DiData)	0.869 (13.91)	0.43
1ISC (ISCOR)	0.525 (17.98)	0.56
1SOL (SASOL)	0.326 (11.96)	0.36

4.5.2. Test Two: Black's Approximation (The possibility of early exercise)

In the absence of dividend payments, or other distributions to shareholders that the option holder is not entitled to, the absence of arbitrage opportunities is sufficient to ensure that it is never optimal for the holder of an American call option to exercise it before maturity. Ingersoll (1989:202) shows how this statement can be proved, by considering two portfolios. One consists of a share of stock. The other consists of a call option on the stock and a (zero-coupon) bond with a face value of X (the exercise price on the option). When the option expires, the first portfolio is worth S_T , the value of the stock at T. The value of the second portfolio has X as its lower bound, but if at T the stock is worth more than X, the portfolio is worth S – that is, at maturity the second portfolio is worth $\text{Max}(S_T, X)$. It follows that the value of the first portfolio can never be greater than the value of the second, thus

$$C(S, T-t) + Xe^{-r(T-t)} \geq S, \text{ and}$$

$$C(S, T-t) \geq S - Xe^{-r(T-t)} > S - X,$$

which shows that the pay-off from exercising before maturity (s-x) is always worth less than keeping the option "alive."

This arbitrage condition is the conceptual justification for using the Black-Scholes model, which was derived assuming the option can only be exercised at maturity, to value American options when either the underlying stock pays no dividends or the dividends per share are too small.

If significant dividends are paid on the underlying stock of an American option, the above arbitrage condition may no longer hold, and the Black-Scholes model needs to be adjusted. The reason is that the option holder is not entitled to the dividend by virtue of owning the option. Yet, the payment of dividends by the firm implies less potential stock price appreciation than would be possible if the cash was reinvested instead. It may then become optimal for the option holder to exercise just before the dividend payment date.

An analytic formula for the valuation of American call options on dividend paying stock has been proposed and successfully tested (see Whaley, 1982). But it applies only to short term options, when no more than one dividend payment will be made until the option expires. (And the study by Whaley (1982) shows little difference between the performance of this model and an adjustment to Black-Scholes explained next.) But the options tested here have very long terms to maturity, and more than one dividend payment is expected before expiry.

Thus, the adjustment for the possibility of early exercise was made by use of an approximation suggested by Black (1975:41). This approximation is based on the observation that early exercise of a call option can only be optimal if done just before the dividend payment date.

It works as follows. Instead of computing a unique series for the model price, with the maturity date as the only date when the option can be exercised, various series are computed. First, the Black-Scholes values are re-computed, still assuming exercise at expiry, but this time the daily stock price is reduced by the present value of all

dividends expected to be paid before the option expires. Next, the same procedure is used to calculate a second series, but now it is assumed the option will be exercised just before the last dividend payment before the option's expiry date. (That is, a shorter term to maturity is assumed, and the daily stock price is reduced by the present value of all dividends to be paid until the final maturity, except the last one) Another series is computed using the same procedure, assuming the option will be exercised just before the next-to-last dividend payment, and so on, depending on the number of dividend payments to be made before the option expires. The value of the option for each day is the highest of the computed values.

However, Merton (1973:151) has proved that, except for the last dividend payment before the option's expiry, it will only pay to exercise before the dividend payment date, if the annual dividend divided by the exercise price exceeds the interest rate (otherwise the above arbitrage condition holds). Black (1975:41) notes that this condition is rarely satisfied.¹⁵ And as a result, that only two possible exercises dates need to be considered: the option's actual maturity date and the date just before the last dividend payment before the option's expiry. Thus, the first two series described in the previous paragraph are the only ones that have to be computed, and the value of the option is given by the highest of these.

This procedure was used to calculate the second series of model values for four of the five options that comprise this study's sample. (DiData has not paid any dividends for many years, and it is assumed no dividends will be paid during the life of the option, so that early exercise of this option is not optimal.)

The volatility measure used was the same as the one used to compute the first set of Black-Scholes model prices.¹⁶ It was assumed that dividends paid within the sample period had been perfectly anticipated by investors. The dividends expected to be paid

¹⁵ Indeed, over the twelve month period covered in my study, dividends are quite low (dividend yields around 3 to 4%), while interest rates fluctuate around 14%.

¹⁶ Hull (1997:249) notes that although in theory the volatility parameters should be re-computed using an adjusted stock price, in practice the volatility of the full stock price is commonly used. Given the small size of dividend payments made to holders of the

subsequently, including the last dividend before expiry, were assumed to be equal to the dividend paid in 1998 (or 1999 in the case of Amplats where semi-annual dividends are paid before the end of May). No extrapolation of past growth rates or any other procedure to estimate future dividends was used. (This is relatively common and acceptable if the dividends are small).

Future dividends were discounted using the yield on a bond that matures near the relevant option's expiry date (same as the yields used for test one). So, for any of the options, only one rate of discount is used in valuing the option in a given day, and the same rate is used to calculate the present value of the strike price and future dividends. (Again, this should be fine if the dividends per share are small, and the term of the option is very long.)

The regression equation used in Test One was estimated, this time with the model values obtained by Black's approximation, as C_m , the explanatory variable. The summarised results for each of the dividend paying stocks are shown below. (Full regression outputs in Appendix 3). *Graphs* showing the actual and model values are shown in Appendix 5.

Test Two: Summarised Regression Results (Black's Approximation)

Option	α	β	R^2
1AMS (Amplats)	45.427 (11.93)	0.764 (45.79)	0.89
1CPX (Comparex)	47.751 (21.27)	0.864 (84.50)	0.96
1ISC (ISCOR)	5.456 (8.57)	0.708 (29.73)	0.78
1SOL (SASOL)	8.246 (9.57)	0.619 (28.29)	0.76

four stocks, the choice of stock price used to compute the volatility measure can be expected to be irrelevant.

Again, strong positive serial correlation is present (the Durbin-Watson statistics are very close to zero). Transforming the series into first-differences and re-estimating the equations eliminated the correlation between adjacent error terms (full outputs in Appendix 4). Summarised results are shown in the next table.

Test Two: Summarised Regression Results (Black's Approximation) – First-difference regression equations.

Option	β	R^2
1AMS (Amplats)	0.607 (21.00)	0.63
1CPX (Comparex)	0.646 (18.25)	0.57
1ISC (ISCOR)	0.571 (18.46)	0.57
1SOL (SASOL)	0.323 (11.61)	0.34

As in Test One, adjusting for first-order serial correlation results in very large losses in explanatory power. For both tests, the R-squared statistics for the first-difference equations are generally very low.

The differences between results for tests One and Two are almost insignificant. The marginal improvements (except for SASOL) were a result of adjusting the Black-Scholes values by the present value of future dividends, since the values computed assuming early exercise were always lower than the value obtained by assuming the option is held until maturity and subtracting the present value of dividends from the stock price. In other words, the differences between the results for tests one and two, are mainly the result of adjusting the Black-Scholes values for the payment of dividends. Allowing for the possibility of early exercise had virtually no impact on the theoretical value of the options.

Two reasons for these observations are suggested. First, concerning the relative irrelevance of the possibility of early exercise, the crisis in international financial markets that started in Asia in the end of 1997 and the consequent withdrawal by

international investors from emerging markets affected the JSE quite substantially. In September 1998, the JSE lost approximately 35% of its value. Since the options included in the study were written before the magnitude of the crisis, and especially its contagion effect on South African markets, could have reasonably been foreseen, the strike prices in the contracts reflect earlier conditions. The result is that all options used here are considerably out of the money throughout most of the sample period. (The Digital Data option, not included in test two, is deeply out of the money throughout the entire period) And when they become in the money, the stock price exceeds the strike by small amounts. This makes early exercise unattractive, and it is likely that, given the very long term of the options, investors prefer to keep them alive in the expectation that general market conditions improve before the options expire.

Second, with respect to the fact that the improvements in explanatory power were marginal, the dividend payments are small. And adjusting option values for the payment of dividends, rather than the possibility of early exercise, as argued before, explains the (very small) increases in explanatory power. But dividend yields are on average between 2 and 5%. Since the options are out of the money through most of the sample, the ratio of annual dividends to the exercise price is smaller than the dividend yield. Yet interest rates are on average close to 14% throughout the sample. The impact of dividends is therefore small in the valuation of the options, and this may, at least partly, explain why the improvements in explanatory power were virtually negligible.

5. Reflections (on the Test Results) and Conclusion

The evidence presented suggests that the Black-Scholes model performs rather poorly in explaining actual market prices for options on the five stocks of common equity analysed in this study. On average, changes in model prices capture about half of variation on actual prices. In this respect, it is interesting to note that simple first-order Auto-Regressive models of the option market prices perform better, in terms of explanatory power, than regressions with theoretical values as the independent variable – once the series are converted to first-differences. The explanatory power of

AR(1) models, which are equivalent to estimating regressions where the lagged (by one period) value of the dependent variable is used as the independent variable, was over 70% for the five options (these results are not shown here.)

There are solid reasons for, a priori, expecting the Black-Scholes model to perform poorly over the sample period. It would certainly come as no surprise that volatility was not constant during the period. In fact, observed volatilities between September 1998 and January 1999 were (except for ISCOR's stock, the volatility of which increased substantially but only towards the end of the sample period) extremely high. (See Appendix 6, p.63.)

Hull (1997:501) notes that the pricing impact of stochastic volatility "*becomes progressively larger as the life of the option increases.*" (Hull, 1997:501) He also notes that this impact, in percentage terms, is potentially very large for options that are deeply out of the money. The options studied here have very long terms to maturity and are out-of-the-money, some deeply, throughout most of the sample period. These factors make their values particularly sensitive to the volatility parameter. It is therefore very likely that a stochastic volatility model would perform better in explaining variation in market prices.

The fact that the stock market suffered such severe losses during a considerable part of the sample period is likely to have negatively affected the performance of the model. What normally happens during market crashes is that it becomes difficult to adjust portfolio positions since most market participants want to sell and few want to buy new securities. Although volatility increases substantially during such periods, liquidity falls.

Yet, the assumption that investors are able to freely buy and sell (and short) stocks and options (as well as lend and borrow at the risk-free rate) to maintain a risk free portfolio, is central to the derivation of the model. In fact, this is perhaps the single most important assumption, since the theoretical strength of the model relies so heavily on the suggestion that, if the other assumptions hold, a price for an option that differs from the Black-Scholes value results in an arbitrage opportunity. What is probably a reflection of the possible violation of this assumption is the fact that the

model's performance was clearly poorer for the less liquid stocks – ISCOR and SASOL.

To the extent that the observed poor fits were a result of lack of liquidity in the market, it is unlikely that risk-free profit opportunities are available in the options market. But only limited inference can be made from these results to the market since the sample included a very small number of options. Rigorous tests of market efficiency are not possible yet, as most options in the market were issued very recently, and are very illiquid.

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Appendix 1: Full Regression Results for Test One

Estimated Equation: $C_a = \alpha + \beta C_m + \varepsilon$, where C_m is the value given by the **Black-Scholes Model**.

Option 1: Amplats

Dependent Variable: AMSCL (C_a)

Independent Variable: AMSBS11 (C_m)

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.110721	4.753378	-0.233670	0.8154
AMSBS11	0.792886	0.017217	46.05334	0.0000
R-squared	0.894933	Mean dependent var		210.5737
Adjusted R-squared	0.894511	S.D. dependent var		59.07573
S.E. of regression	19.18727	Akaike info criterion		8.754307
Sum squared resid	91669.67	Schwarz criterion		8.782398
Log likelihood	-1096.666	F-statistic		2120.910
Durbin-Watson stat	0.249183	Prob(F-statistic)		0.000000

Option 2: Comparex

Dependent Variable: CPXCL (C_a)

Independent Variable: CPXBS11 (C_m)

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	45.62142	2.276659	20.03876	0.0000
CPXBS11	0.855038	0.010156	84.18978	0.0000
R-squared	0.966062	Mean dependent var		224.8845
Adjusted R-squared	0.965926	S.D. dependent var		69.16235
S.E. of regression	12.76682	Akaike info criterion		7.939513
Sum squared resid	40584.96	Schwarz criterion		7.967604
Log likelihood	-994.4089	F-statistic		7087.919
Durbin-Watson stat	0.509153	Prob(F-statistic)		0.000000

Option 3: DiData

Dependent Variable: DDTCL (C_a)

Independent Variable: DDTBS11 (C_m)

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	15.71250	0.958357	16.39525	0.0000
DDTBS11	1.756466	0.028806	60.97606	0.0000
R-squared	0.937233	Mean dependent var		60.58566
Adjusted R-squared	0.936981	S.D. dependent var		38.74382
S.E. of regression	9.726060	Akaike info criterion		7.395431
Sum squared resid	23554.47	Schwarz criterion		7.423522
Log likelihood	-926.1266	F-statistic		3718.080
Durbin-Watson stat	0.211757	Prob(F-statistic)		0.000000

Option 4: ISCOR

Dependent Variable: ISCL (C_a)
 Independent Variable: ISCBS11 (C_m)
 Method: Least Squares
 Sample(adjusted): 6/01/1998 5/31/1999
 Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.642818	0.596405	4.431250	0.0000
ISCBS11	0.673667	0.018592	36.23378	0.0000
R-squared	0.840577	Mean dependent var		22.62550
Adjusted R-squared	0.839937	S.D. dependent var		8.991284
S.E. of regression	3.597220	Akaike info criterion		5.406136
Sum squared resid	3222.057	Schwarz criterion		5.434227
Log likelihood	-676.4700	F-statistic		1312.887
Durbin-Watson stat	0.229481	Prob(F-statistic)		0.000000

Option 5: SASOL

Dependent Variable: SOLCL (C_a)
 Independent Variable: SOLBS11 (C_m)
 Method: Least Squares
 Sample(adjusted): 6/01/1998 5/31/1999
 Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.148548	0.936099	7.636527	0.0000
SOLBS11	0.606385	0.022625	26.80193	0.0000
R-squared	0.742594	Mean dependent var		27.07570
Adjusted R-squared	0.741560	S.D. dependent var		17.72519
S.E. of regression	9.010947	Akaike info criterion		7.242694
Sum squared resid	20218.10	Schwarz criterion		7.270785
Log likelihood	-906.9581	F-statistic		718.3436
Durbin-Watson stat	0.064751	Prob(F-statistic)		0.000000

Appendix 2: Full Regression Results for Test One – First Difference Equations.

Estimated Equation: $\Delta C_a = \beta \Delta C_m + \varepsilon$

Option: Amplats

Dependent Variable: AMSCLD (ΔC_a)

Independent Variable: AMSBSD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AMSBSD	0.557482	0.027135	20.54438	0.0000
R-squared	0.627312	Mean dependent var		0.912000
Adjusted R-squared	0.627312	S.D. dependent var		13.74847
S.E. of regression	8.393183	Akaike info criterion		7.096709
Sum squared resid	17540.94	Schwarz criterion		7.110795
Log likelihood	-886.0886	Durbin-Watson stat		1.933540

Option: Comparex

Dependent Variable: CPXCLD (ΔC_a)

Independent Variable: CPXBSD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CPXBSD	0.642971	0.035175	18.27904	0.0000
R-squared	0.572346	Mean dependent var		-0.504000
Adjusted R-squared	0.572346	S.D. dependent var		13.01289
S.E. of regression	8.509813	Akaike info criterion		7.124309
Sum squared resid	18031.81	Schwarz criterion		7.138395
Log likelihood	-889.5386	Durbin-Watson stat		2.603733

Option: DiData

Dependent Variable: DDTCLD (ΔC_a)

Independent Variable: DDTBSD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DDTBSD	0.869721	0.062481	13.91985	0.0000
R-squared	0.430123	Mean dependent var		-0.508000
Adjusted R-squared	0.430123	S.D. dependent var		4.408136
S.E. of regression	3.327711	Akaike info criterion		5.246438
Sum squared resid	2757.342	Schwarz criterion		5.260524
Log likelihood	-654.8048	Durbin-Watson stat		2.273600

Option: ISCOR

Dependent Variable: ISCLD (ΔC_a)

Independent Variable: ISCBSD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999
 Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ISCBSD	0.525988	0.029246	17.98496	0.0000
R-squared	0.565030	Mean dependent var	-0.008000	
Adjusted R-squared	0.565030	S.D. dependent var	2.488515	
S.E. of regression	1.641232	Akaike info criterion	3.832764	
Sum squared resid	670.7173	Schwarz criterion	3.846850	
Log likelihood	-478.0955	Durbin-Watson stat	2.285663	

Option: SASOL

Dependent Variable: SOLCLD (ΔC_a)
 Independent Variable: SOLBSD (ΔC_m)
 Method: Least Squares
 Sample(adjusted): 6/02/1998 5/31/1999
 Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SOLBSD	0.326249	0.027270	11.96380	0.0000
R-squared	0.360595	Mean dependent var	-0.200000	
Adjusted R-squared	0.360595	S.D. dependent var	2.403144	
S.E. of regression	1.921622	Akaike info criterion	4.148208	
Sum squared resid	919.4647	Schwarz criterion	4.162294	
Log likelihood	-517.5260	Durbin-Watson stat	2.467226	

Appendix 3: Full Regression Results for Test Two

Estimated Equation: $C_a = \alpha + \beta C_m + \varepsilon$, where C_m is the value given by **Black's Approximation**.

Option 1: Amplats

Dependent Variable: AMSCL

Independent Variable:

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	45.42708	3.806236	11.93491	0.0000
AMSBA	0.764520	0.016695	45.79329	0.0000
R-squared	0.893863	Mean dependent var		210.5737
Adjusted R-squared	0.893437	S.D. dependent var		59.07573
S.E. of regression	19.28469	Akaike info criterion		8.764437
Sum squared resid	92602.95	Schwarz criterion		8.792528
Log likelihood	-1097.937	F-statistic		2097.025
Durbin-Watson stat	0.206036	Prob(F-statistic)		0.000000

Option 2: Comparex

Dependent Variable: CPXCL

Independent Variable: CPXBA

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	47.75189	2.244680	21.27336	0.0000
CPXBA	0.864590	0.010231	84.50352	0.0000
R-squared	0.966305	Mean dependent var		224.8845
Adjusted R-squared	0.966170	S.D. dependent var		69.16235
S.E. of regression	12.72102	Akaike info criterion		7.932325
Sum squared resid	40294.29	Schwarz criterion		7.960417
Log likelihood	-993.5068	F-statistic		7140.845
Durbin-Watson stat	0.516558	Prob(F-statistic)		0.000000

Option 3: ISCOR

Dependent Variable: ISCCL

Independent Variable: ISCBA

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.456530	0.636011	8.579301	0.0000
ISCBA	0.708921	0.023843	29.73263	0.0000
R-squared	0.780235	Mean dependent var		22.62550
Adjusted R-squared	0.779353	S.D. dependent var		8.991284
S.E. of regression	4.223487	Akaike info criterion		5.727135
Sum squared resid	4441.622	Schwarz criterion		5.755226
Log likelihood	-716.7555	F-statistic		884.0294
Durbin-Watson stat	0.161505	Prob(F-statistic)		0.000000

Option 4: SASOL

Dependent Variable: SOLCL

Independent Variable: SOLBA

Method: Least Squares

Sample(adjusted): 6/01/1998 5/31/1999

Included observations: 251 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8.246477	0.860893	9.578980	0.0000
SOLBA	0.619232	0.021887	28.29167	0.0000
R-squared	0.762726	Mean dependent var		27.07570
Adjusted R-squared	0.761773	S.D. dependent var		17.72519
S.E. of regression	8.651401	Akaike info criterion		7.161256
Sum squared resid	18636.84	Schwarz criterion		7.189347
Log likelihood	-896.7376	F-statistic		800.4185
Durbin-Watson stat	0.073150	Prob(F-statistic)		0.000000

Appendix 4: Full Regression Results for Test Two – First Difference Equations.

Estimated Equation: $\Delta C_a = \beta \Delta C_m + \varepsilon$

Option: Amplats

Dependent Variable: AMSCLD (ΔC_a)

Independent Variable: AMSBAD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AMSBAD	0.607261	0.028908	21.00670	0.0000
R-squared	0.637684	Mean dependent var		0.912000
Adjusted R-squared	0.637684	S.D. dependent var		13.74847
S.E. of regression	8.275574	Akaike info criterion		7.068486
Sum squared resid	17052.80	Schwarz criterion		7.082571
Log likelihood	-882.5607	Durbin-Watson stat		1.978812

Option: Comparex

Dependent Variable: CPXCLD (ΔC_a)

Independent Variable: CPXBAD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CPXBAD	0.646314	0.035410	18.25217	0.0000
R-squared	0.571625	Mean dependent var		-0.504000
Adjusted R-squared	0.571625	S.D. dependent var		13.01289
S.E. of regression	8.516987	Akaike info criterion		7.125994
Sum squared resid	18062.23	Schwarz criterion		7.140080
Log likelihood	-889.7493	Durbin-Watson stat		2.604467

Option: ISCOR

Dependent Variable: ISCLD (ΔC_a)

Independent Variable: ISCBAD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ISCBAD	0.571134	0.030939	18.46008	0.0000
R-squared	0.577800	Mean dependent var		-0.008000
Adjusted R-squared	0.577800	S.D. dependent var		2.488515
S.E. of regression	1.616960	Akaike info criterion		3.802964
Sum squared resid	651.0252	Schwarz criterion		3.817050
Log likelihood	-474.3706	Durbin-Watson stat		2.282045

Option: SASOL

Dependent Variable: SOLCLD (ΔC_a)

Independent Variable: SOLBAD (ΔC_m)

Method: Least Squares

Sample(adjusted): 6/02/1998 5/31/1999

Included observations: 250 after adjusting endpoints

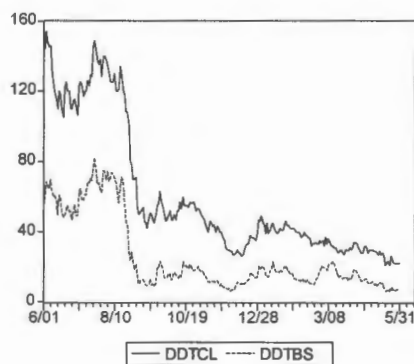
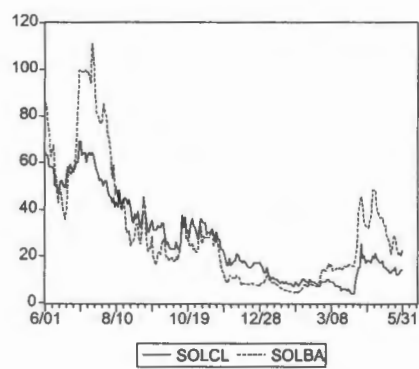
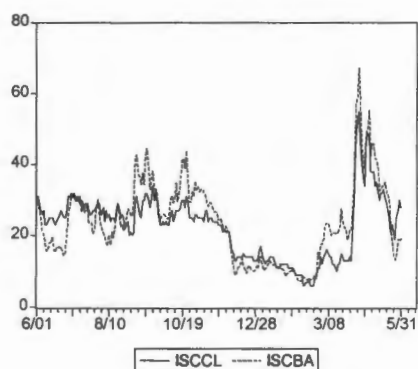
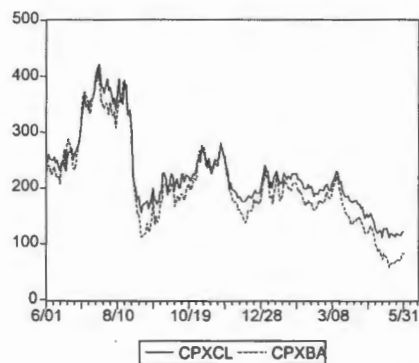
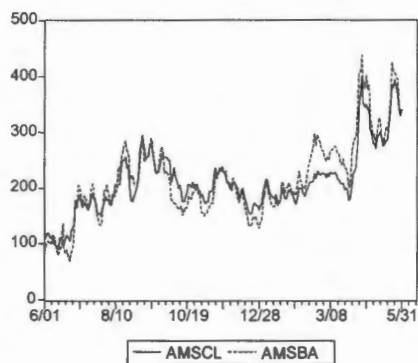
Variable	Coefficient	Std. Error	t-Statistic	Prob.
SOLBAD	0.323705	0.027874	11.61328	0.0000
R-squared	0.346829	Mean dependent var	-0.200000	
Adjusted R-squared	0.346829	S.D. dependent var	2.403144	
S.E. of regression	1.942196	Akaike info criterion	4.169508	
Sum squared resid	939.2594	Schwarz criterion	4.183594	
Log likelihood	-520.1885	Durbin-Watson stat	2.468566	

Appendix 5: Actual and Model Prices

CL: Option's Closing Price.

BA: Black's Approximation Value.

BS: Black-Scholes Value.



Note: No Black's approximation values were computed for the option on Digital Data (DDT). Thus the graph for this option compares actual values with unadjusted Black-Scholes values. The other graphs (relating to the four other options in the sample) compare actual values with the values obtained by applying Black's approximation.

Appendix 6: Volatility of Underlying Stocks during Sample Period (Conventional Standard Deviation)

