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Review Article

Application of AdS/CFT in Nuclear Physics

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We review some recent progress in studying the nuclear physics especially nucleon-nucleon (NN) force within the gauge-gravity duality, in context of noncritical string theory. Our main focus is on the holographic QCD model based on the AdS_6 background. We explain the noncritical holography model and obtain the vector-meson spectrum and pion decay constant. Also, we study the NN interaction in this frame and calculate the nucleonmeson coupling constants. A further topic covered is a toy model for calculating the light nuclei potential. In particular, we calculate the light nuclei binding energies and also excited energies of some available excited states. We compare our results with the results of other nuclear models and also with the experimental data. Moreover, we describe some other issues which are studied using the gauge-gravity duality.

1. Introduction

One of the fundamental ingredients of nuclear physics is the nuclear force with which point-like nucleons interact with each other. Since Yukawa, many potential models have been constructed which have been composed to fit the available NN scattering data. The newer potentials have only slightly improved with respect to the previous ones in describing the recent much more accurate data. As it is shown in [1], all of these potential models do not have good quality with respect to the pp scattering data below 350 MeV and just a few of them are of satisfactory quality. These models are the Reid soft-core potential Reid68 [2], the Nijmegen soft-core potential Nijm78 [3], the new Bonn pp potential Bonn89 [4], and also the parameterized Paris potential Paris80 [5]. These familiar one-boson-exchange potentials (OBEP) contain a relatively small number of free parameters (about 10 to 15 parameters) but do not have a reasonable description of the empirical scattering data. Also, most of these potentials, which have been fitted to the np scattering data, unfortunately do not automatically fit to the pp scattering data even by considering the correction term for the Coulomb interaction [1]. Of course, new versions of these potentials have been constructed such as Nijm I, Nijm II, Reid93 [6], CD-Bonn [7], and AV18 [8] which explain the empirical scattering data successfully. But they contain a large number of purely phenomenological parameters. For example, an updated (Nijm92pp [9]) version of the Nijm78 potential contains 39 free parameters.

On the other hand, there are many attempts to impose the symmetries of QCD using an effective Lagrangian of pions and nucleons [10, 11]. These models only capture the qualitative features of the nuclear interactions and could not compete with the much more successful potential models mentioned above.

Despite many efforts, no potential model has yet been constructed which gives a high-quality description of the empirical data, obeys the symmetries of QCD, and contains only a few number of free phenomenological parameters.

In recent years, holography or gauge-gravity duality gave us a new approach to hadronic physics [12] and make new progress in understanding the nuclear force.

Nuclear force, the force between nucleons, exhibits a repulsive core of nucleons at short distances. This repulsive core is quite important for large varieties of physics of nuclei and nuclear matter. For example, the well-known presence of nuclear saturation density is essentially due to this repulsive core. However, from the viewpoint of strongly coupled QCD, the physical origin of this repulsive core has not been well understood. Nuclear force especially the repulsive core has been studied using the AdS/CFT correspondence [13–16] and an explicit expression has been obtained for the repulsive core.

Also, there are many attempts to find a geometry dual to nucleus. Since nucleons are described by D-branes wrapping a sphere in curved geometry of holographic QCD, on a nucleus with mass number A there appears a $U(A)$ gauge theory. One can find the dual gravity by taking the large mass number limit $A \to \infty$ and obtained a near horizon geometry corresponding to the heavy nucleus. The corresponding supergravity solution has discrete fluctuation spectra comparable with nuclear experimental data [17]. As we know from the nuclear experiments, the nucleons of a heavy nuclei have coherent excitations which are called Giant resonances. These resonances exhibit harmonic behavior $E_n = nw(A)$ which is explained with phenomenological models such as the liquid drop model. The gauge-gravity duality can reproduce this behavior. Moreover, dependence to the mass number A is obtained by using the duality [17].

Among the holographic QCD models, the Sakai-Sugimoto (SS) [18, 19] and Klebanov-Strassler (KS) models [20] are the most interesting holographic models to study strong coupling regime of QCD. The SS model is based on ten-dimensional type-IIA string theory, with a background geometry given by N_c $D4$-branes. They fill four-dimensional Minkowski space-time and extend along a fifth extra dimension x_4 compactified on a circle whose circumference is parametrized by the Kaluza-Klein mass. Through this compactified dimension and antisymmetric boundary conditions for fermions supersymmetry is completely broken. Left- and right-handed chiral fermions are introduced by adding N_f $D8$- and N_f $D8$-branes which extend in all dimensions except for x_4. In this compact direction, they are separated by a distance $L \in [0, \pi/M_{\mathrm{KK}}]$. There are two possible background geometries called confined and deconfined phase. For more details about the setup of the model see the original papers by Sakai and Sugimoto [18, 19]. In this model, there is a nice topological interpretation of chiral symmetry breaking.

Chiral symmetry breaking is realized in the model as follows. A $U(N_f)$ gauge symmetry on the flavor branes corresponds to a global $U(N_f)$ at the boundary. Therefore, the bulk gauge symmetries on the $D8$- and $D8$-branes can be interpreted as left- and right-handed flavor symmetry groups in the dual field theory. The Chern-Simons term accounts for the axial anomaly of QCD, such that one is left with the chiral group $SU(N_f)_L \times SU(N_f)_R$ and the vector part $U(1)_V$. There is no explicit breaking of this group since the model only contains massless quarks. Spontaneous chiral symmetry breaking is realized when the $D8$- and $D8$-branes connect in the bulk. They always connect in the confined phase and whether they connect in the deconfined phase depends on the separation L of the $D8$- and $D8$-branes in the extra dimension x_4.

The Sakai-Sugimoto model is particularly suited for phenomenon related to chirality as chiral magnetic effect (CME) [21–25] since it has a well-defined concept for chirality and the chiral phase transition. It is straightforward to introduce right- and left-handed chemical potentials independently.

The chiral magnetic effect is a hypothetical phenomenon which states that, in the presence of a magnetic field B, a nonzero axial charge density will lead to an electric current along the direction of the B field [26–28]. Analysis of RHIC data appears to favor the presence of a CME in the quark-gluon plasma, although a better understanding of systematic errors and backgrounds is still needed. CME is studied in many holographic systems, following [29–34], including systems without confinement or chiral symmetry breaking in vacuum.

Also, predictions of the SS model are in good agreement with the lattice simulations such as the glueball spectrum of pure QCD [35, 36]. This model describes baryons and their interactions with mesons well [18, 19, 37–39]. It is shown that the baryons can be taken as point-like objects at distances larger than their sizes, so their interactions can be described by the exchange of light particles such as mesons. Therefore, one can find the baryon-baryon potential from the Feynman diagrams using the interaction vertices including baryon currents and light mesons [38]. But there are some inconsistencies. For example, the size of the baryon is proportional to $\lambda^{-1/2}$. Consequently in the large 't Hooft coupling (large λ), the size of the baryon becomes zero and the stringy corrections have to be taken into account. Another problem is that the scale of the system associated with the baryonic structure is roughly half the one needed to fit to the masonic data [40]. Also, the holographic models arising from the critical string theory encounter with the some Kaluza-Klein (KK) modes, with the mass scale of the same order as the masses of the hadronic modes. These unwanted modes are coupled with the hadronic modes and there is no mechanism to disentangle them from the hadronic modes yet. In order to overcome this problem, it is possible to consider the color brane configuration in noncritical string theory [41–44].

The noncritical string is not formulated with the critical dimension, but nonetheless has vanishing Weyl anomaly. A worldsheet theory with the correct central charge can be constructed by introducing a nontrivial target space, commonly by giving an expectation value to the dilaton which varies linearly along some spacetime direction. For this reason noncritical string theory is sometimes called the linear dilaton theory. Since the dilaton is related to the string coupling constant, this theory contains a region where the coupling is weak (and so perturbation theory is valid) and another region where the theory is strongly coupled [45, 46].

In such backgrounds the string coupling constant is proportional to $1/N_c$, so the large N_c limit corresponds to the small string coupling constant. However, contrary to the critical holographic models, in the large N_c limit, the 't Hooft coupling is of order one instead of infinity and the scalar curvature of the gravitational background is also of order one. So, it seems that the noncritical gauge-gravity correspondence is not very reliable. But studies show that the results of these models for some low energy QCD properties such as the meson mass spectrum, Wilson loop, and the mass spectrum of glueballs [45–47] are comparable with lattice computations. Therefore, noncritical holographic models still seem useful to study QCD.

One of the noncritical holographic models is composed of a $D4$ and anti $D4$ brane in six-dimensional non-critical string theory [43, 47]. The low energy effective theory on the intersecting brane configuration is a four-dimensional QCD-like effective theory with the global chiral symmetry $U(N_f)_L \times U(N_f)_R$. In this brane configuration, the six-dimensional gravity background is the near horizon geometry of the color $D4$-branes. This model is based on the compactified AdS_6 spacetime with constant dilaton. So the model does not suffer from large string coupling as the SS model. The meson spectrum [47] and the structure of thermal phase [48] are studied in this model. Some properties, like the dependence of the meson masses on the stringy mass of the quarks and the excitation number, are different from the critical holographic models such as the SS model.

We study the gauge field and its mode expansion in this noncritical holography model and obtain the effective pion action [49]. The model has a mass scale M_{KK} like the SS model in which we set its value by computing the pion decay constant. Then, we study the baryon [50] and obtain its size. We show that the size of the baryon is of order one with respect to the 't Hooft coupling, so the problem of the zero size of the baryon in the critical holography model is solved. But the size of the baryon is still smaller than the mass scale of holographic QCD, so we treat it as a point-like object and introduce an isospin $1/2$ Dirac field for the baryon [49]. We write a $5D$ effective action for the baryon field and reduce it to $4D$ using the mode expansion of gauge field and baryon field and obtain the NN potential in terms of the meson exchange interactions. We calculate the meson-nucleon couplings using the suitable overlapping wave function integrals and compare them with the results of SS model. Also, we compare the nucleon-meson couplings obtained from noncritical holographic model with the results of SS model and predictions of some phenomenological models. Our study shows that the noncritical results are in good agreement with the other available models.

On the other hand, one of the oldest problems of nuclear physics is the nuclear binding energies: the interactions between nucleons are very strong, while the nuclear matter is not relativistic. Nuclear binding energies are experimentally known with high accuracy while they are not predicted with sufficient accuracy using different theoretical models. Since, prediction of nuclear binding energy is a useful tool to test the goodness of a theoretical nucleon-nucleon (NN) interaction model, we use our NN holography potential to obtain the light nuclei binding energies. We construct a nuclear holographic model [50–53] in the noncritical base and calculate the nuclei potentials as the sum of their NN interactions. The minimum of the ground state potential is considered as the binding energy. Also, difference between this energy and the minimum of the excited state potential presents the excited energy for each state. In order to compute the potentials, we use the values of nucleon-meson coupling constants obtained from both the critical and noncritical holography models.

This paper is organized as follows. In Section 2, we briefly review the AdS/CFT correspondence. The noncritical holographic model is introduced in Section 3 and NN potential is constructed in this section. In Section 4, we construct a simple model to study light nuclei such as ^2D, ^3T, $^3\mathrm{He}$, and $^4\mathrm{He}$ and obtain their potential of ground and excited states and respective binding energies. Section 5 is devoted to a brief summary and conclusions. Also, some other topics, which are studied using the duality, are introduced in this section.

2. Review of AdS/CFT Correspondence

2.1. Historical Notes. Quantum chromodynamics (QCD) is the quantum field theory of the strong interactions which has two important properties, asymptotic freedom and confinement. Various analytical and numerical methods have been developed to study QCD. One example is perturbative QCD which works at small distances where the coupling is weak but fails to work at larger distances where the coupling becomes relatively strong in which case the problem is said to become nonperturbative. Examples of methods that study nonperturbative problems are effective field theories such as chiral perturbation theory, lattice QCD [64], Dyson-Schwinger equations (DSE) formalism [65], and gauge-gravity duality [12, 66, 67].

Before QCD, in the 1960's string theory was introduced as a model to describe the strong interactions [68]. It was able to explain the organization of hadrons in Regge trajectories, describing them as rotating strings. After the formulation of QCD, string theory took a different direction, becoming a possible candidate for a unified theory of all the forces. Nevertheless, some string interpretation of hadron spectra was not abandoned; for example, a meson is sometimes described as a quark and an antiquark connected by a tube of strong interaction flux [69, 70]. This picture establishes a link between QCD and string theory, which becomes even more evident in the limit of large number of colors N [71]. 't Hooft proposed that in this limit the gauge theory may have a description in terms of a tree level string theory; in particular, the leading Feynman diagrams in the $1/N$ expansion are planar and look like the worldsheet of a string theory. For example, a meson can be represented by two quark lines propagating in time connected by a dense sheet of gluons, reminding the worldsheet swept out by a string through time. In 1997, these studies found a possible new framework in the so-called AdS/CFT correspondence [12], a conjecture introduced by Maldacena relating a supergravity theory in ten dimensions to a supersymmetric gauge theory in four dimensions. This correspondence has been extended to a gauge theory as $SU(3)_c$, thus proving some link between QCD and a higher dimensional theory in a curved space-time.

2.2. D-Branes and AdS Space. The most important property of D-branes is that they contain gauge theories on their world volume. In particular, the massless spectrum of open strings living on a Dp-brane contains a (maximally supersymmetric)

$U(1)$ gauge theory in $p+1$ dimensions. Moreover, it appears that if we consider the stack of N coincident D-branes, then there are N^2 different species of open strings which can begin and end on any of the D-branes, allowing us to have (maximally supersymmetric) $U(N)$ gauge theory on the world-volume of these D-branes. Now, if N is sufficiently large, then this stack of D-branes is a heavy object embedded into a theory of closed strings that contains gravity. This heavy object curves the space which can then be described by some classical metric and other background fields.

Thus, we have two absolutely different descriptions of the stack of coincident Dp-branes. One description is in terms of the $U(N)$ supersymmetric gauge theory on the world volume of the Dp-branes, and the other is in terms of the classical theory in some gravitational background. It is this idea that lies at the basis of gauge-gravity duality.

One important example is $D3$-branes which can also be seen as solutions of ten-dimensional type IIB supergravity at low energies, with metric of the form [72]:

$$ds^2 = \left(1+\frac{L^4}{r^4}\right)^{-1/2}\left[-dt^2+d\vec{x}^2\right] + \left(1+\frac{L^4}{r^4}\right)^{1/2}\left[dr^2+r^2 d\Omega_5^2\right], \quad (1)$$

where

$$L^4 = 4\pi g_s N^2 \alpha'. \quad (2)$$

Here, g_s is the string coupling constant which is related to the constant dilaton as ($g_s = e^{\Phi}$). Also, there is N_c units of $F_{[5]}$ flux. L is the only length scale in the solution. This metric interpolates between a throat geometry and a ten-dimensional Minkowski region.

If we take the near horizon limit of the solution given in (1), $r \ll L$, and redefine $z = L^2/r$, we can completely decouple the Minkowski region and are left with a throat geometry which is given by

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2+d\vec{x}^2+dz^2\right] + L^2 d\Omega_5^2, \quad (3)$$

which is the Poincaré wedge of the direct product of five-dimensional anti-de-Sitter space and a five-sphere ($\mathrm{AdS}_5 \times S^5$). The isometry group of this space is given by $SO(4,2)\times SO(6)$, though if we include fermions, the full supersymmetric isometry group is $SU(2,2 \mid 4)$. Note that this is exactly the same as the full global symmetry group of the low energy limit of the open string sector (i.e., SYM theory).

We see that the radius L, of both the AdS throat and the S^5, in string units is given in terms of the gauge theory parameters as

$$L^4 = g_{\mathrm{YM}}^2 N_c {\alpha'}^2 = \lambda {\alpha'}^2. \quad (4)$$

Therefore, in order that the stringy modes be unimportant, $L \gg \sqrt{\alpha'}$, which translates into gauge theory language as $\lambda = g_{\mathrm{YM}}^2 N_c \gg 1$.

2.3. $\mathcal{N}=4$ Super Yang-Mills Theory. $\mathcal{N}=4$ $SU(N)$ supersymmetric Yang-Mills theory (SYM) in four dimensions (the dimensionality of the world volume of the $D3$-branes) has one vector field, A_μ, six scalars fields ϕ^I ($I = 1,\ldots,6$), and four fermions χ_α^i, $\chi_{\dot\alpha}^{\bar i}$ ($i, \bar i = 1,2,3,4$) which are in the $\mathbf{4}$ and $\bar{\mathbf{4}}$ representations of the $SU(4) = SO(6)$ R-symmetry group.

This theory naturally arises on the surface of a $D3$-brane in type IIB superstring theory. Open strings generate a massless gauge field in ten dimensions. When the open string ends are restricted to a $3+1$ dimensional subspace the ten components of the gauge field naturally break into a $3+1$ dimensional gauge field and 6 scalar fields. The fermionic superpartners naturally separate to complete the $3+1$ dimensional supermultiplets.

The beta function of $\mathcal{N}=4$ SYM theory vanishes to all orders in perturbation theory, $\beta = 0$. This implies that the theory is conformal with conformal symmetry group $SO(4,2)$ also at the quantum level. Moreover, this theory has a global $SU(4)R$ symmetry group. The complete superconformal group is $SU(2,2 \mid 4)$, of which both $SO(4,2)$ and $SU(4)$ are bosonic subgroups.

2.4. The AdS/CFT Correspondence. The AdS/CFT correspondence, which was first suggested by Maldacena [12] in 1997, states that type IIB string theory on $(\mathrm{AdS}_5 S^5)_N$ plus some appropriate boundary conditions (and possibly also some boundary degrees of freedom) is dual to $\mathcal{N}=4, d=3+1\, U(N)$ super Yang-Mills. There are three different versions of this conjecture [73], depending on the precise form of the limits taken. In the strong version, type IIB string theory on $\mathrm{AdS}_5 \times S^5$ is dual to $SU(N_c)$ SYM theory. The mild version relates classical type IIB strings on $\mathrm{AdS}_5 \times S^5$ to planar $SU(N_c)$ SYM theory. But the mostly adopted form of the conjecture is the weak regime (in the SUGRA limit) which specializes further to the case in which λ is large. In this limit, strongly coupled $\mathcal{N}=4$ $SU(N)$ Yang-Mills theory is mapped to supergravity on $\mathrm{AdS}_5 \times S^5$; the inverse string tension α' goes to zero.

A precise way in which the two theories can be mapped into each other was proposed independently by Gubser et al. [41] and by Witten [66]. Since the boundary of the AdS_5 space, namely, $S^3 \times R$, is equivalent to $R^{3,1}$, which is a copy of the Minkowski space, plus a point at infinity, the authors suggested a recipe to link the gravity theory in the bulk (AdS space) to the field theory on the boundary (Minkowski space). In this sense, the AdS/CFT correspondence can be considered as a holographic projection of the supergravity theory in the bulk to the field theory on the boundary.

Despite the fact that there is no proof of the AdS/CFT correspondence taking account of its string-theoretical origin yet, the huge amount of symmetry present almost guarantees that the AdS/CFT correspondence should hold. When proceeding to less symmetrical situations below, generalized gauge-gravity dualities remain a conjecture though.

2.5. QCD versus SYM. It would be useful if the four-dimensional theory on the boundary was QCD, since this would allow us to explore its nonperturbative regime by studying a perturbative dual theory. However, the field theory described by the correspondence is a supersymmetric theory with conformal invariance, while QCD has none of these features. The most important differences between the two theories are as follows [73].

(i) QCD confines while SYM is not confining.

(ii) QCD has a chiral condensate while SYM has no chiral condensate.

(iii) QCD has a discrete spectrum while that of SYM is continuous.

(iv) QCD has a running coupling while SYM has a tunable coupling and is conformal.

(v) QCD has quarks while SYM has adjoint matter.

(vi) QCD is not supersymmetric while SYM is maximally supersymmetric.

(vii) QCD has $N_c = 3$ in real life, while the AdS/CFT correspondence holds for large N_c.

However, the gauge-gravity duality can be expanded to more field theories by changing the supergravity theory. This gives a possibility to search for a field theory that is closer to QCD and has a gravity dual.

(i) For example, considering multiple $D3$-branes on curved backgrounds leads to an interesting family of $\mathcal{N} = 1$ superconformal field theories [74, 75] which contain adjoint matter fields. Also, one can introduce the confinement and break the conformal symmetry by deforming the background further. This leads to chiral symmetry breaking and a running coupling constant [20].

(ii) Also, theories looking like $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in the IR can be obtained by considering higher dimensional D-branes wrapped on certain submanifolds of the ten-dimensional geometry [76, 77].

(iii) Deformations of the geometry lead to nonsupersymmetric, nonconformal gauge theories which display confinement and chiral symmetry breaking [18, 19, 78–81].

(iv) Fundamental matter can be added to the gauge theory by introducing $D7$-branes [82]. In the quenched approximation, $N_f \ll N_c$, their effect on the background geometry is ignored. Also, dynamical quarks can be added to this geometry [82].

(v) Recently, some phenomenological models have been suggested which are motivated by the AdS/CFT but not within the full string theory framework. These models are known as AdS/QCD [83–86].

(vi) Also, an approach similar to AdS/QCD is introduced based on the noncritical string theory in $d \neq 10$ dimensions [42, 86, 87].

3. Holographic QCD from the Noncritical String Theory

The key idea of construction of holographic models with flavors was given by Karch and Katz [88]. In these models, two stacks of flavor branes, branes, and antibranes are added to the geometry as a probe, so that the back reaction of the flavor branes is negligible (probe approximation). This approximation is reliable when $N_f \ll N_c$, where N_c and N_f refer to the number of colors and flavors, respectively.

Of course, the brane/antibrane system is unstable, since the branes and antibranes will tend to annihilate. This is reflected in the presence of tachyons in the spectrum. But, it should make sense within the context of perturbation theory. The point where the tachyon field vanishes corresponds to a local maximum of the tachyon potential, and thus it is part of a classical solution. The one-loop effective action in an expansion around this solution should be well defined, even though the solution is unstable, and, in particular, it should have a well-defined phase. It was conjectured that at the minimum of the tachyon potential, the negative contribution to the energy density from the potential exactly cancels the sum of the tensions of the brane and the antibrane, thereby giving a configuration of zero energy density (and hence restoring spacetime supersymmetry). Therefore, the various gauge and gravitational anomalies, which arise as one-loop effects, cancel and as we expected theory is perturbatively well-behaved [72–92].

In this section, we study a model which is similar in many aspects to the SS model [18], a holographic model based on the critical string theory. But, we try to solve some inconsistencies of the SS model in describing the baryons via the noncritical AdS_6 model.

3.1. AdS_6 Model. In the presented noncritical model, the gravity background is generated by near-extremal $D4$-branes wrapped over a circle with the antiperiodic boundary conditions. Two stacks of flavor branes, namely, $D4$-branes and anti-$D4$-branes, are added to this geometry and are called flavor probe branes. The color branes extend along the directions t, x_1, x_2, x_3, and τ while the probe flavor branes fill the whole Minkowski space and stretch along the radius U which is extended to infinity. The strings attaching a color $D4$-brane to a flavor brane transform as quarks, while strings hanging between a color $D4$ and a flavor $\overline{D4}$ transform as antiquarks. The chiral symmetry breaking is achieved by a reconnection of the brane and antibrane pairs. Under the quenched approximation ($N_c \gg N_f$), the reactions of flavor branes and the color branes can be neglected. Just like the SS model, the τ coordinate is wrapped on a circle and the antiperiodic condition is considered for the fermions on the thermal circle. The final low energy effective theory on the background is a four-dimensional QCD-like effective theory with the global chiral symmetry $U(N_f)_L \times U(N_f)_R$.

In this model, the near horizon gravity background at low energy is [47]

$$ds^2 = \left(\frac{U}{R}\right)^2 \left(-dt^2 + dx_i dx_i + f(U)\, d\tau^2\right) + \left(\frac{R}{U}\right)^2 \frac{dU^2}{f(U)}, \quad (5)$$

where R is the radius of the AdS space. Also $f(U)$ and RR six-form field strength, $F_{(6)}$, are defined by the following relations:

$$F_{(6)} = Q_c \left(\frac{U}{R}\right)^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge d\tau,$$
$$f(U) = 1 - \left(\frac{U_{\mathrm{KK}}}{U}\right)^5. \quad (6)$$

In order to obtain solutions of near extremal flavored AdS_6, the values of dilaton and R_{AdS} are considered as

$$e^{\phi} = \frac{2}{3}\frac{Q_f}{Q_c^2}\left(\sqrt{1 + \frac{6Q_c^2}{Q_f^2}} - 1\right),$$
$$R_{\mathrm{AdS}}^2 = \frac{90}{12 + Q_f^2/Q_c^2 - \left(Q_f^2/Q_c^2\right)\sqrt{1 + 6Q_c^2/Q_f^2}}. \quad (7)$$

This relation indicates that the R_{AdS} and dilaton depend on the ratio of the number of colors ($\sim Q_c$) and flavors ($\sim Q_f$). Under the quenched approximation, the values of the dilaton and AdS radius can be rewritten as

$$R_{\mathrm{AdS}}^2 = \frac{15}{2}, \qquad e^{\phi} = \frac{2\sqrt{2}}{\sqrt{3}Q_c}, \quad (8)$$

where Q_c is proportional to the number of color branes, N_c.

To avoid singularity, the coordinate τ satisfies the following periodic condition:

$$\tau \sim \tau + \delta\tau, \quad \delta\tau = \frac{4\pi R^2}{5U_{\mathrm{KK}}}. \quad (9)$$

Also, the Kaluza-Klein mass scale of this compact dimension is

$$M_{\mathrm{KK}} = \frac{2\pi}{\delta\tau} = \frac{5}{2}\frac{U_{\mathrm{KK}}}{R^2}, \quad (10)$$

and dual gauge field theory for this background is nonsupersymmetric. Also, the Yang-Mills coupling constants can be defined as a function of string theory parameters using the DBI action as follows:

$$g_{\mathrm{YM}}^2 = \frac{g_s}{\mu_4 (2\pi\alpha')^2 \delta\tau}, \quad (11)$$

where $\alpha' = l_s^2$ is the Regge slope parameter and l_s is the string length. Also, the 't Hooft coupling is $\lambda = g_{\mathrm{YM}}^2 N_c$.

3.2. Meson Sector. In AdS/QCD, there is a gauge field living in the bulk AdS whose dynamics is dual to the meson sector of QCD such as pions and higher resonances. The gauge field on the $D4$-brane includes five components, A_μ ($\mu = 0, 1, 2, 3$) and A_U. The $D4$-brane action is given by Pahlavani et al. [49]:

$$S_{D4} = -\mu_4 \int d^5 x e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)} + S_{\mathrm{CS}}, \quad (12)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$, $(M, N = 0, 1, \ldots, 5)$ is the field strength tensor, and A_M is the $U(N_f)$ gauge field on the $D4$-brane. The second term in the above action is the Chern-Simons action and $\mu_4 = 2\pi/(2\pi ls)^5$. It is useful to define the new variable z as

$$U_z = \left(U_{\mathrm{KK}}^5 + U_{\mathrm{KK}}^3 z^2\right)^{1/5}. \quad (13)$$

Then by neglecting the higher order of F^2 in the expansion, the $D4$-brane action can be written as [49]

$$S_{D4} = -\tilde{\mu}_4 \left(2\pi\alpha'\right)^2 \int d^4 x\, dz \times \left[\frac{R^4}{4U_z^{5/2}} \eta^{\mu\nu}\eta^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} + \frac{25}{8}\frac{U_z^{9/2}}{U_{\mathrm{KK}}^3} \eta^{\mu\nu} F_{\mu z} F_{\nu z}\right] + \mathcal{O}\left(F^3\right), \quad (14)$$

where $\tilde{\mu}_4$ is

$$\tilde{\mu}_4 = \sqrt{\frac{3}{2}} \frac{N_c U_{\mathrm{KK}}^{3/2}}{5R^3} \mu_4. \quad (15)$$

The gauge fields A_μ ($\mu = 0, 1, 2, 3$) and A_z have a mode expansion in terms of complete sets $\{\psi_n(z)\}$ and $\{\phi_n(z)\}$ as

$$A_\mu\left(x^\mu, z\right) = \sum_n B_\mu^{(n)}\left(x^\mu\right) \psi_n(z),$$
$$A_z\left(x^\mu, z\right) = \sum_n \varphi^{(n)}\left(x^\mu\right) \phi_n(z). \quad (16)$$

After calculating the field strengths, the action (14) is rewritten as

$$S_{D4} = -\tilde{\mu}_4 \left(2\pi\alpha'\right)^2 \int d^4 x\, dz \times \sum_{m,n} \left[\frac{R^4}{4U_z^{5/2}} F_{\mu\nu}^{(m)} F^{\mu\nu(n)} \psi_m \psi_n + \frac{25}{8}\frac{U_z^{9/2}}{U_{\mathrm{KK}}^3} \left(\partial_\mu \varphi^{(m)} \partial^\mu \varphi^{(n)} \phi_m \phi_n + B_\mu^{(m)} B^{\mu(n)} \dot{\psi}_m \dot{\psi}_n - 2\partial_\mu \varphi^{(m)} B^{\mu(n)} \phi_m \dot{\psi}_n\right)\right], \quad (17)$$

where the over dot denotes the derivative respect to the z coordinate.

We introduce the following dimensionless parameters:

$$\tilde{z} \equiv \frac{z}{U_{\mathrm{KK}}}, \qquad K(\tilde{z}) \equiv 1 + \tilde{z}^2 = \left(\frac{U_z}{U_{\mathrm{KK}}}\right)^5, \tag{18}$$

and find that the functions ψ_n ($n \geq 1$) satisfy the normalization condition as

$$\tilde{\mu}_4 (2\pi\alpha')^2 \frac{R^4}{U_{\mathrm{KK}}^{3/2}} \int d\tilde{z} K^{-1/2} \, \psi_n \psi_m = \delta_{nm}. \tag{19}$$

Also, we suppose that the functions ψ_n ($n \geq 1$) satisfy the following condition:

$$\tilde{\mu}_4 (2\pi\alpha')^2 \frac{R^4}{U_{\mathrm{KK}}^{3/2}} \int d\tilde{z} K^{9/10} \partial_{\tilde{z}} \psi_m \partial_{\tilde{z}} \psi_n = \lambda_n \delta_{nm}. \tag{20}$$

Using (19) and (20), an eigenvalue equation is obtained for the functions ψ_n ($n \geq 1$) as

$$-K^{1/2} \partial_{\tilde{z}} \left(K^{9/10} \partial_{\tilde{z}} \psi_m \right) = \lambda_m \psi_m. \tag{21}$$

The orthonormal condition for ϕ_n are as follows:

$$(\phi_m, \phi_n) \equiv \frac{25}{4} \tilde{\mu}_4 (2\pi\alpha')^2 U_{\mathrm{KK}}^{5/2} \int d\tilde{z} K^{9/10} \phi_m \phi_n = \delta_{mn}. \tag{22}$$

We find that the functions $\phi_{(n)}$ and $\dot{\psi}_n$ are related together. In fact, we can consider $\phi_n = m_n^{-1} \dot{\psi}_n$ ($n \geq 1$). Also, there exists a function $\phi_0 = C/K^{9/10}$ which is orthogonal to $\dot{\psi}_n$ for all $n \geq 1$:

$$(\phi_0, \phi_n) \propto \int d\tilde{z} \partial_{\tilde{z}} \psi_n = 0, \quad (\text{for } n \geq 1). \tag{23}$$

We use the normalization condition $1 = (\phi_0, \phi_0)$ to obtain the normalization constant C. Finally by using an appropriate gauge transformation, the action (14) becomes

$$S_{D4} = -\int d^4x \left[\frac{1}{2} \partial_\mu \varphi^{(0)} \partial^\mu \varphi^{(0)} \times \sum_{n \geq 1} \left(\frac{1}{4} F_{\mu\nu}^{(n)} F^{\mu\nu(n)} + \frac{1}{2} m_n^2 B_\mu^{(n)} B^{\mu(n)} \right) \right], \tag{24}$$

where $B_\mu^{(n)}$ is a massive vector meson of mass $m_n \equiv \lambda_n^{1/2} M_{\mathrm{KK}}$ for all $n \geq 1$ and $\varphi^{(0)}$ is the pion field, which is the Nambu-Goldstone boson associated with the chiral symmetry breaking [49].

It is useful to make another gauge choice, namely, the $A_z = 0$ gauge. Actually, we can transform to the new gauge through a suitable gauge transformation and obtain the following new gauge fields:

$$\begin{aligned} A_z(x^\mu, z) &= 0, \\ A_\mu(x^\mu, z) &= -\partial_\mu \varphi^{(0)}(x^\mu) \psi_0(z) + \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z). \end{aligned} \tag{25}$$

Table 1: The ratio of the obtained eigenvalues of (21) compared with the results of the KS [54], DKS [55], and SS model [18] and the ratio of meson masses.

k	$\left(\frac{\lambda_{k+1}}{\lambda_k}\right)_{\mathrm{AdS}_6}$	$\left(\frac{\lambda_{k+1}}{\lambda_k}\right)_{\mathrm{DKS}}$	$\left(\frac{\lambda_{k+1}}{\lambda_k}\right)_{\mathrm{KS}}$	$\left(\frac{\lambda_{k+1}}{\lambda_k}\right)_{\mathrm{SS}}$	$\left(\frac{\lambda_{k+1}}{\lambda_k}\right)_{\mathrm{Exp}}$
1	2.76	1.97	2.68	2.34	2.51
2	5.58	3.56	5.63	4.92	3.65
3	9.55	5.49	8.88	6.97	4.45

Function $\psi_0(z)$ is calculated through

$$\psi_0(z) = \int_0^z dz' \phi_0(z') = C U_{\mathrm{KK}} \tilde{z} F_1 \left(0.5, 0.9, 1.5, -\tilde{z}^2\right), \tag{26}$$

where F_1 is well-known hypergeometric function. It should be noted that the massless pseudoscalar meson appears in the asymptotic behavior of A_μ, since we have

$$A_\mu(x^\mu, z) \longrightarrow \pm 1.8 C U_{\mathrm{KK}} \partial_\mu \varphi^{(0)}(x^\mu) \quad (\text{as } z \longrightarrow \pm\infty). \tag{27}$$

In order to calculate the meson spectrum, it is necessary to solve the (21) numerically by considering the normalization condition (19).

Since (21) is invariant under $\tilde{z} \rightarrow -\tilde{z}$, we can assume ψ_n to be an even or odd function. In fact, the $B_\mu^{(n)}$ is a four-dimensional vector and axial vector if ψ_n is an even or odd function, respectively. Equation (21) is solved numerically using the shooting method to obtain the mass of lightest mesons. Our results are compared with the results of the SS, KS, and DKS models and experimental data in Table 1. As is clear, our results are in good agreement with the experimental data [49].

3.3. Pion Effective Action. Now, we just consider the pion field in the gauge field expansion and use the non-Abelian generalization of the DBI action to find the effective pion action [49]:

$$S_{D4} = -\tilde{\mu}_4 (2\pi\alpha')^2 \int d^4x \, \mathrm{tr} \left(A \left(U^{-1} \partial_\mu U \right)^2 + B \left[U^{-1} \partial_\mu U, U^{-1} \partial_\nu U \right]^2 \right), \tag{28}$$

where the coefficients A and B are defined by the following relations [49]:

$$\begin{aligned} A &\equiv 2 \frac{25}{8} \frac{1}{U_{\mathrm{KK}}^3} \int d\tilde{z} U_z^{9/2} \left(\partial_{\tilde{z}} \hat{\psi}_0(\tilde{z}) \right)^2 = \frac{25}{4} \frac{U_{\mathrm{KK}}^{1/2}}{3.6}, \\ B &\equiv 2 \frac{R^4}{4} \int dz \frac{1}{U_z^{5/2}} \psi_+^2 (\psi_+ - 1)^2 = \frac{0.16 R^4}{2 U_{\mathrm{KK}}^{3/2}}. \end{aligned} \tag{29}$$

If we compare (33) with the familiar action of the Skyrme model [93]

$$S = \int d^4x \left(\frac{f_\pi^2}{4} \operatorname{tr}\left(U^{-1}\partial_\mu U\right)^2 + \frac{1}{32e^2} \operatorname{tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2 \right), \tag{30}$$

it is possible to calculate the pion decay constant f_π and dimensionless parameter e in terms of the noncritical model parameters [49]:

$$\begin{aligned} f_\pi^2 &= 4\tilde{\mu}_4\left(2\pi\alpha'\right)^2 A = \sqrt{\frac{3}{2}}\frac{45\mu_4\left(2\pi\alpha'\right)^2}{3.6R^3} N_c M_{\mathrm{KK}}^2, \\ \frac{1}{e^2} &= 32\tilde{\mu}_4\left(2\pi\alpha'\right)^2 B = \sqrt{\frac{3}{8}}\mu_4\left(2\pi\alpha'\right)^2 R N_c. \end{aligned} \tag{31}$$

It is clear from the above equations that the parameters f_π and e depend on N_c as $f_\pi \sim \mathcal{O}(\sqrt{N_c})$ and $e \sim \mathcal{O}(1/\sqrt{N_c})$, respectively. It is coincident with the result obtained from the SS model and also QCD in large N_c. We fix the M_{KK} such that the $f_\pi \sim 93$ MeV for $N_c = 3$. So, we obtain $M_{\mathrm{KK}} = 395$ MeV for our holographic model [49]. It should be noted that M_{KK} is the only mass scale of the noncritical model below which the theory is effectively pure Yang-Mills in four dimensions.

3.4. Baryon in AdS_6. In this section we aim to introduce baryon configuration in the noncritical holographic model. As is known, in the SS model the baryon vertex is a $D4$-brane wrapped on a S^4 cycle. Here in six-dimensional configuration, there is no compact S^4 sphere. So, we introduce an unwrapped $D0$-brane as a baryon vertex instead [94]. In analogy with the SS model, there is a Chern-Simons term on the vertex world volume as

$$S_{\mathrm{CS}} \propto \int dt A_0(t), \tag{32}$$

which induces N_c units of electric charge on the unwrapped $D0$-brane. In accordance with the Gauss constraint, the net charge should be zero. So, one needs to attach N_c fundamental strings to the $D0$-brane. In turn, the other side of the strings should end up on the probe $D4$-branes. The baryon vertex looks like an object with N_c electric charge with respect to the gauge field on the $D4$-brane whose charge is the baryon number. This $D0$-brane dissolves into the $D4$-brane and becomes an instanton solution [94]. It is important to know the size of the instanton in our model. In the SS model, it is shown that the size of an instantonic baryon goes to zero at large 't Hooft coupling limit which is one of the problems of the SS model in describing the baryons [37].

Let us consider the DBI action in the Yang-Mills approximation for the $D4$-brane:

$$S_{\mathrm{YM}} = -\frac{1}{4}\mu_4\left(2\pi\alpha'\right)^2 \int e^{-\phi}\sqrt{-g_{4+1}} \operatorname{tr} F_{mn}F^{mn}. \tag{33}$$

The induced metric on the $D4$-brane is

$$g_{4+1} = \left(\frac{U}{R}\right)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{R}{U}\right)^4 \frac{dU^2}{f(U)}\right). \tag{34}$$

It is useful to define the new coordinate w

$$dw = \frac{R^2 U^{1/2} dU}{\sqrt{U^5 - U_{\mathrm{KK}}^5}}. \tag{35}$$

Using this coordinate, the metric (34) transforms to a conformally flat metric:

$$g_{4+1} = H(w)\left(dw^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \quad H(w) = \left(\frac{U}{R}\right)^2. \tag{36}$$

Also, the w coordinate can be rewritten in terms of the z coordinate introduced in (13) as

$$dw = \frac{2}{5}\frac{R^2 U_{\mathrm{KK}}^3 dz}{\left(U_{\mathrm{KK}}^5 - U_{\mathrm{KK}}^3 z^2\right)^{7/10}}. \tag{37}$$

Note that in the new conformally flat metric, the fifth direction is a finite interval $[-w_{\max}, w_{\max}]$ because

$$w_{\max} = \int_0^\infty \frac{R^2 U^{1/2} dU}{\sqrt{U^5 - U_{\mathrm{KK}}^5}} \simeq \frac{R^2}{U_{\mathrm{KK}}} 1.25 < \infty. \tag{38}$$

We can approximate w near the origin $w \simeq 0$ as

$$w \simeq \frac{2}{5}\left(\frac{R}{U_{\mathrm{KK}}}\right)^2 z, \tag{39}$$

and using relation (10), we obtain

$$w \simeq \frac{z}{M_{\mathrm{KK}} U_{\mathrm{KK}}} \quad \text{or} \quad M_{\mathrm{KK}} w \simeq \frac{z}{U_{\mathrm{KK}}}, \tag{40}$$

or, equivalently,

$$U^5 \simeq U_{\mathrm{KK}}^5 \left(1 + M_{\mathrm{KK}}^2 w^2\right). \tag{41}$$

In analogy with the SS model, this relation implies that M_{KK} is the only mass scale that dictated the deviation of the metric from the flat configuration and it is the only mass scale of the theory in the low energy limit. (It should be noted that the $D4$-branes come with two asymptotic regions at $w \to \pm w_{\max}$ corresponding to the ultraviolet and infrared region near the $w \simeq 0$).

Equation (33) is rewritten in the conformally flat metric (36) as

$$\begin{aligned} S_{\mathrm{YM}}^{D4} &= -\frac{1}{4}\mu_4\left(2\pi\alpha'\right)^2 \int d^4x dw e^{-\phi}\left(\frac{U(w)}{R}\right) \operatorname{tr} F_{mn}F^{mn} \\ &= -\int dx^4 dw \frac{1}{4e^2(w)} \operatorname{tr} F_{mn}F^{mn}. \end{aligned} \tag{42}$$

Thus, the position dependent electric coupling $e(w)$ of this five-dimensional Yang-Mills is equal to [30]

$$\frac{1}{e^2(w)} \equiv \frac{\sqrt{3/2}\mu_4\left(2\pi\alpha'\right)^2 R N_c}{5} M_{\mathrm{KK}}\left(\frac{U}{U_{\mathrm{KK}}}\right). \tag{43}$$

Also, for a unit instanton we have

$$\frac{1}{8\pi^2}\int \mathrm{tr}\ F\wedge F = \frac{1}{16\pi^2}\int \mathrm{tr}\ F_{mn}F^{mn} = 1. \tag{44}$$

Inserting the above relations in (42), we obtain the energy of a point-like instanton localized at $w = 0$ as

$$m_B^{(0)} = \frac{\sqrt{3/2}\,4\pi^2\mu_4\left(2\pi\alpha'\right)^2 R}{5} N_c M_{\mathrm{KK}}. \tag{45}$$

By increasing the size of the instanton, more energy is needed because $1/e^2(w)$ is an increasing function of $|w|$. So the instanton tends to collapse to a point-like object. On the other hand, N_c fundamental strings attached to the $D4$-branes behave as N_c units of electric charge on the brane. The Coulomb repulsions among them prefer a finite size for the instanton. Therefore, there is a competition between the mass of the instanton and Coulomb energy of fundamental strings. For a small instanton of size ρ with the density $D(x^i, w) \sim \rho^4/(r^2 + w^2 + \rho^2)^4$, the Yang-Mills energy is approximated as

$$\sim \frac{1}{6} m_B^{(0)} M_{\mathrm{KK}}^2 \rho^2, \tag{46}$$

and the five-dimensional Coulomb energy is

$$\sim \frac{1}{2} \times \frac{e(0)^2 N_c^2}{10\pi^2\rho^2}. \tag{47}$$

The size of a stable instanton is obtained by minimizing the total energy [49]:

$$\rho_{\mathrm{baryon}}^2 \simeq \frac{1}{\sqrt{3/2}\,2\pi^2\mu_4(2\pi\alpha')^2} \frac{1}{M_{\mathrm{KK}}^2}. \tag{48}$$

As it is stated in the previous section, in the SS model (the critical version of dual QCD) the size of the instanton goes to zero because of the large 't Hooft coupling limit. However in the noncritical string theory, the 't Hooft coupling is of order one. So, the size of the instanton is also of order 1 but it is still smaller than the effective length of the fifth direction $\sim 1/M_{\mathrm{KK}}$ of the dual QCD.

3.5. Nucleon-Nucleon Potential. In the previous section, we demonstrated that the size of the baryon in the noncritical holographic model is smaller than the scale of the dual QCD and we can assume that the baryon is a point-like object in five dimensions. Thus as a leading approximation, we can treat it as a point-like quantum field in five dimensions. In the rest of this paper, we will restrict ourselves to fermionic baryons because we intend to study the nucleons. So, we consider odd N_c to study a fermionic spin 1/2 baryon. We choose $N_c = 3$ in our numerical calculations for realistic QCD. Also, we will assume $N_F = 2$ and consider the lowest baryons which form the proton-neutron doublet under $SU(N_F = 2)$. All of these assumptions lead us to introduce an isospin 1/2 Dirac field, $\mathcal{N}$ for the five-dimensional baryon.

The leading $5D$ kinetic term for $\mathcal{N}$ is the standard Dirac action in the curved background along with a position dependent mass term for the baryon. Moreover, there is a coupling between the baryon field and the gauge filed living on the flavor branes that should be considered. Therefore, a complete action for the baryon reads as [49]

$$\begin{aligned}\int d^4x dw \Bigg[&-i\overline{\mathcal{N}}\gamma^m D_m \mathcal{N} - i m_b(w)\overline{\mathcal{N}}\mathcal{N} \\ &+ g_5(w)\frac{\rho_{\mathrm{baryon}}^2}{e^2(w)}\overline{\mathcal{N}}\gamma^{mn}F_{mn}\mathcal{N}\Bigg] \\ &- \int d^4x dw \frac{1}{4e^2(w)}\ \mathrm{tr}\ F_{mn}F^{mn},\end{aligned} \tag{49}$$

where D_m is a covariant derivative, ρ_{baryon} is the size of the stable instanton, and $g_5(w)$ is an unknown function with a value as $w = 0$ of $2\pi^2/3$ [38]. γ^m are the standard γ matrices in the flat space and $\gamma^{mn} = 1/2[\gamma^m, \gamma^n]$.

The factor $\rho_{\mathrm{baryon}}^2/e^2(w)$ is used for convenience. Usually, the first two terms in the action are called the minimal coupling and the last term in the first integral refers to the magnetic coupling.

A four-dimensional nucleon is the localized mode at $w \simeq 0$ which is the lowest eigenmode of a five dimensional baryon along the w direction. So, the action of the five-dimensional baryon must be reduced to four dimensions. In order to do this, one should perform the KK mode expansion for the baryon field $\mathcal{N}(x_\mu, w)$ and the gauge field $A(x_\mu, w)$. The gauge field has a KK mode expansion presented in (16). The baryon field also can be expanded as

$$\mathcal{N}_{L,R}(x^\mu, w) = N_{L,R}(x^\mu) f_{L,R}(w), \tag{50}$$

where $N_{L,R}(x^\mu)$ is the chiral component of the four-dimensional nucleon field. Also the profile functions, $f_{L,R}(w)$, satisfy the following conditions:

$$\begin{aligned}\partial_w f_L(w) + m_b(w) f_L(w) &= m_B f_R(w), \\ -\partial_w f_R(w) + m_b(w) f_R(w) &= m_B f_L(w),\end{aligned} \tag{51}$$

in the range $w \in [-w_{\max}, w_{\max}]$, and the eigenvalue m_B is the mass of the nucleon mode, $N(x)$. Moreover, the eigenfunctions $f_{L,R}(w)$ obey the following normalization condition:

$$\int_{-w_{\max}}^{w_{\max}} dw\left|f_L(w)\right|^2 = \int_{-w_{\max}}^{w_{\max}} dw\left|f_R(w)\right|^2 = 1. \tag{52}$$

It is more useful to consider the following second-order differential equations for $f_{L,R}(w)$:

$$\begin{aligned}\left[-\partial_w^2 - \partial_w m_b(w) + (m_b(w))^2\right] f_L(w) &= m_B^2 f_L(w), \\ \left[-\partial_w^2 + \partial_w m_b(w) + (m_b(w))^2\right] f_R(w) &= m_B^2 f_R(w).\end{aligned} \tag{53}$$

As we approach $w \to \pm w_{\max}$, $m_b(w)$ diverges as $m_b(w) \sim 1/(w \mp w_{\max})^2$ and the above equations have normalizable eigenfunctions with a discrete spectrum of m_B. Note that the term $-\partial_w m_b(w)$ is asymmetric under $w \to -w$. It causes that $f_L(w)$ tends to shift to the positive side of w and the opposite behavior happens for $f_R(w)$. It is important in the axial coupling of the nucleon to the pions.

The gauge field can be expanded in $A_z = 0$ gauge as follows [49]:

$$A_\mu(x,w) = i\alpha_\mu(x)\psi_0(w) + i\beta_\mu(x) + \sum_n B_\mu^{(n)}(x)\psi_{(n)}(w), \tag{54}$$

where α_μ and β_μ are related to the pion field $U(x) = e^{2i\pi(x)/f_\pi}$ by the following relations:

$$\begin{aligned}\alpha_\mu(x) &\equiv \left\{U^{-1/2}, \partial_\mu U^{1/2}\right\},\\ \beta_\mu(x) &\equiv \frac{1}{2}\left[U^{-1/2}, \partial_\mu U^{1/2}\right].\end{aligned} \tag{55}$$

Here, we use the above expansion along with the properties of $f_L(w) = \pm f_R(-w)$ and ψ_0 and $\psi_{(n)}$ under the $w \to -w$ transformation to calculate the four-dimensional action. It is worthwhile to note that again $\psi_{(2k+1)}(w)$ is even, while $\psi_{(2k)}(w)$ is odd under $w \to -w$, corresponding to vector $B_\mu^{(2k+1)}(x^\mu)$ and axial-vector mesons $B_\mu^{(2k)}(x^\mu)$, respectively. For simplicity, we neglect the Chern-Simons term in the baryon action (49).

By inserting the mode expansion of the nucleon field and gauge field into the baryon action [49],

$$\mathscr{L}_{\text{Nucleon}} = -i\overline{N}\gamma^\mu\partial_\mu N - im_B\overline{N}N + \mathscr{L}_{\text{vector}} + \mathscr{L}_{\text{axial}}, \tag{56}$$

where

$$\begin{aligned}\mathscr{L}_{\text{vector}} &= -i\overline{N}\gamma^\mu\beta_\mu N - \sum_{k\geq 0} g_V^{(k)}\overline{N}\gamma^\mu B_\mu^{(2k+1)}N\\ &\quad + \sum_{k\geq 0} g_{dV}^{(k)}\overline{N}\gamma^{\mu\nu}\partial_\mu B_\nu^{(2k+1)}N,\\ \mathscr{L}_{\text{axial}} &= -\frac{ig_A}{2}\overline{N}\gamma^\mu\gamma^5\alpha_\mu N - \sum_{k\geq 1} g_A^{(k)}\overline{N}\gamma^\mu\gamma^5 B_\mu^{(2k)}N\\ &\quad + \sum_{k\geq 0} g_{dA}^{(k)}\overline{N}\gamma^{\mu\nu}\gamma^5\partial_\mu B_\nu^{(2k)}N.\end{aligned} \tag{57}$$

Also, $g = g_{\min} + g_{\text{mag}}$ stands for all the couplings. We neglect the derivative couplings in the following calculations as a leading approximation. The various minimal couplings constants $g_{V,\min}^{(k)}$ and $g_{A,\min}^{(k)}$ as well as the pion-nucleon axial coupling $g_{A,\min}$ are calculated by the following suitable overlap integrals of wave functions:

$$\begin{aligned}g_{V,\min}^{(k)} &= \int_{-w_{\max}}^{w_{\max}} dw\left|f_L(w)\right|^2\psi_{(2k+1)}(w),\\ g_{A,\min}^{(k)} &= \int_{-w_{\max}}^{w_{\max}} dw\left|f_L(w)\right|^2\psi_{(2k)}(w),\\ g_{A,\min} &= 2\int_{-w_{\max}}^{w_{\max}} dw\left|f_L(w)\right|^2\psi_0(w).\end{aligned} \tag{58}$$

Also, we can compute the magnetic couplings using the following integrals [49]:

$$\begin{aligned}g_{V,\text{mag}}^{(k)} &= 2C_{\text{mag}}\int_{-w_{\max}}^{w_{\max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{\text{KK}}}\right)\\ &\quad\times\left|f_L(w)\right|^2\partial_w\psi_{(2k+1)}(w),\\ g_{A,\text{mag}}^{(k)} &= 2C_{\text{mag}}\int_{-w_{\max}}^{w_{\max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{\text{KK}}}\right)\\ &\quad\times\left|f_L(w)\right|^2\partial_w\psi_{(2k)}(w),\\ g_{A,\text{mag}} &= 4C_{\text{mag}}\int_{-w_{\max}}^{w_{\max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{\text{KK}}}\right)\\ &\quad\times\left|f_L(w)\right|^2\partial_w\psi_0(w),\end{aligned} \tag{59}$$

where we define C_{mag} as

$$C_{\text{mag}} = \frac{\sqrt{3/2}\mu_4\left(2\pi\alpha'\right)^2}{5}RN_c g_5(0)M_{\text{KK}}\rho_{\text{baryon}}^2. \tag{60}$$

Since the instanton carries only the non-Abelian field strength, the isoscalar mesons couple to the nucleon in a different formalism than the isovector mesons. Therefore, for the isoscalar mesons, such as the $\omega^{(k)}$ meson, only the minimal couplings contribute to

$$\begin{aligned}g_A^{\text{iso-scalar}} &= g_{A,\min},\\ g_A^{(k),\text{iso-scalar}} &= g_{A,\min}^{(k)},\\ g_V^{(k),\text{iso-scalar}} &= g_{V,\min}^{(k)}.\end{aligned} \tag{61}$$

However, the isovector mesons couple to the nucleon from both the minimal and magnetic channels. Thus, isovector meson couplings are [49]

$$\begin{aligned}g_A^{\text{iso-vector}} &= g_{A,\min} + g_{A,\text{mag}},\\ g_A^{(k),\text{iso-vector}} &= g_{A,\min}^{(k)} + g_{A,\text{mag}}^{(k)},\\ g_V^{(k),\text{iso-vector}} &= g_{V,\min}^{(k)} + g_{V,\text{mag}}^{(k)}.\end{aligned} \tag{62}$$

The is-scalar and isovector mesons have the same origin in the five-dimensional dynamics of the gauge field. In fact, if we write the gauge field in the fundamental representation, we could decompose the massive vector mesons as

$$B_\mu^{(2k+1)} = \begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix}\omega_\mu^{(k)} + \rho_\mu^{(k)}, \tag{63}$$

where $\omega_\mu^{(k)}$ and $\rho_\mu^{(k)}$ are the isoscalar and the isovector parts of a vector meson, respectively. Since the baryon is made out of N_c product quark doublets, the above composition for nucleon should be written as

$$B_\mu^{(2k+1)} = \begin{pmatrix}\frac{N_c}{2} & 0\\ 0 & \frac{N_c}{2}\end{pmatrix}\omega_\mu^{(k)} + \rho_\mu^{(k)}. \tag{64}$$

Therefore, there is an overall factor N_c between the isoscalar, $\omega_\mu^{(k)}$ and isovector, $\rho_\mu^{(k)}$ mesons. Indeed, there is a universal relation between the Yukawa couplings involving the isoscalar and isovector mesons:

$$\left|g_{\omega^{(k)}\mathrm{NN}}\right| \simeq N_c \times \left|g_{\rho^{(k)}\mathrm{NN}}\right|. \tag{65}$$

We solve the eigenvalue (51) numerically using the shooting method to obtain the wave function, $f_{L,R}$, and the mass, m_B, of the nucleon. In order to do the numerical calculation, we assume $N_c = 3$ for realistic QCD. Also as was mentioned in the previous section, we choose the value of $M_{\mathrm{KK}} = 0.395$ GeV to have the pion decay constant $f_\pi = 0.093$ GeV. We obtain the various couplings by evaluating integrals (58) and (59) and compare some of our results with the results of the SS model [37] in Table 2.

Also, using this noncritical model, the axial couplings are obtained as

$$g_{A,\mathrm{mag}} = 1.582, \qquad g_{A,\mathrm{min}} \simeq 0, \tag{66}$$

while in the previous analysis [18] using the SS model, these couplings are reported as

$$g_{A,\mathrm{mag}} = 0.7\frac{N_c}{3}, \qquad g_{A,\mathrm{min}} \simeq 0.13. \tag{67}$$

If we choose $N_c = 3$, then the SS model predicts $g_{A,\mathrm{mag}} = 0.7$ and $g_A = 0.83$. It should be noted that the higher order of $1/N_c$ corrections can be used to improve this result but the lattice calculations indicate that higher order of $1/N_c$ corrections are suppressed. Our results are a good approximation of the experimental data at leading order $g_A^{\mathrm{exp}} = 1.2670 \pm 0.0035$.

3.5.1. Nucleon-Meson Couplings. Our holographic NN potential contains just the vector, axial-vector, and pseudoscalar meson exchange potentials which have the isospin dependent and isospin independent components. The vector meson $(\omega^{(k)}, \rho^{(k)})$, axial-vector meson $(f^{(k)}, a^{(k)})$, and pseudoscalar meson $(\pi^{(k)}, \eta'^{(k)})$ couplings are related to the minimal and magnetic couplings as follows:

$$\begin{aligned}
g_{\omega^{(k)}\mathrm{NN}} &\equiv \frac{N_c g_V^{(k),\text{iso-scalar}}}{2} = \frac{N_c g_{V,\mathrm{min}}^{(k)}}{2},\\
g_{\rho^{(k)}\mathrm{NN}} &\equiv \frac{g_V^{(k),\text{iso-vector}}}{2} = \frac{g_{V,\mathrm{min}}^{(k)} + g_{V,\mathrm{mag}}^{(k)}}{2},\\
g_{f^{(k)}\mathrm{NN}} &\equiv \frac{N_c g_A^{(k),\text{iso-scalar}}}{2} = \frac{N_c g_{A,\mathrm{min}}^{(k)}}{2},\\
g_{a^{(k)}\mathrm{NN}} &\equiv \frac{g_A^{(k),\text{iso-vector}}}{2} = \frac{g_{A,\mathrm{min}}^{(k)} + g_{A,\mathrm{mag}}^{(k)}}{2},\\
\frac{g_{\pi^{(k)}\mathrm{NN}}}{2m_N} M_{\mathrm{KK}} &\equiv \frac{g_A^{\text{iso-vector}}}{2f_\pi} M_{\mathrm{KK}} = \frac{g_{A,\mathrm{min}} + g_{A,\mathrm{mag}}}{2f_\pi} M_{\mathrm{KK}},\\
\frac{g_{\eta'^{(k)}\mathrm{NN}}}{2m_N} M_{\mathrm{KK}} &\equiv \frac{N_c g_A^{\text{iso-scalar}}}{2f_\pi} M_{\mathrm{KK}} = \frac{N_c g_{A,\mathrm{min}}}{2f_\pi} M_{\mathrm{KK}}.
\end{aligned} \tag{68}$$

Table 2: Numerical results for axial and vector meson couplings in the noncritical holographic model of QCD. The values of vector couplings are compared with the SS model results [37].

k	$g_{A,\mathrm{min}}^{(k)}$	$g_{A,\mathrm{mag}}^{(k)}$	$g_{V,\mathrm{min}}^{(k),\mathrm{a}}$	$g_{V,\mathrm{min}}^{(k),\mathrm{b}}$	$g_{V,\mathrm{mag}}^{(k),\mathrm{a}}$	$g_{V,\mathrm{mag}}^{(k),\mathrm{b}}$
0	1.16	1.86	8.30	5.933	−1.988	−0.816
1	1.07	1.44	1.6488	3.224	−6.83	−1.988
2	0.96	0.862	1.9	1.261	−7.44	−1.932
3	0.67	0.14	0.688	0.311	−4.60	−0.969

[a]presented model results; [b]SS model results.

Table 3: The values of different effective meson-nucleon couplings in the phenomenological interaction models [56], SS model [18], and our model.

g	AV18	CD-Bonn	Nijm (93)	Paris	SS model	Our model
g_{a^0}	9.0	9.0	9.0	10.4	—	—
g_σ	9.0	11.2	9.8	7.6	—	—
g_π	13.4	13.0	12.7	13.2	16.48	15.7
g_η	8.7	0.0	1.8	11.7	16.13	0.0
g_ω	12.2	13.5	11.7	12.7	12.6	11.57
g_ρ	—	3.19	2.97	—	3.6	3.15
g_{a^1}	—	—	—	—	3.94	1.51
g_{f^1}	—	—	—	—		1.74

All of the leading order meson-nucleon couplings are calculated numerically and compared with the predictions of the four modern phenomenological NN interaction models such as the AV 18 [8], CD-Bonn [7], Nijmegen (93) [6], and Paris [5] potentials in Table 3. Also, results of the SS model are presented in this table. It is necessary to mention here that the components of the phenomenological models are very different in strength, and if parameterized in terms of single meson exchange give rise to effective meson-nucleon coupling strengths, which also are similar. We explain different components of the NN potential below in detail.

The isospin dependent component of the vector potential which arises from a ρ meson exchange is roughly three times weaker than the isospin independent component. In a chiral quark model, it is expected to have $g_\omega = 3g_\rho$, but the value of the $\mathscr{R} = g_\omega/3g_\rho$ differs from the one in the above phenomenological interaction models. It is 1.66 for the CD-Bonn, 1.5 for the Nijmegen, and 0.77 in the Paris model. This ratio is about 1.2 in the SS model and equal to $\mathscr{R} = 1.33$ in our model. Actually, the NN phase shifts uniformly require a larger $\mathscr{R}$ than the chiral quark model prediction which is a mystery. However, in the resultant potential of the holographic QCD model, it can be explained by the contribution of the magnetic coupling in the vector channel.

4. Holographic Light Nuclei

In the holographic models, baryon is introduced as a D-brane wrapped on a higher dimensional sphere in the curved spacetime [17]. According to the fact that each nucleus is a set of A nucleons, so the collection of the A baryon D-branes can describe a nucleus with the mass number A. Then the dual gravity for the nucleus can be obtained by applying the AdS/CFT correspondence. The $U(A)$ gauge theory living in the gravity dual of QCD is difficult to treat; hence, the large A limit is considered for this dual geometry which corresponds to the heavy nuclei [95]. On the other hand, it is necessary to use the nucleon-nucleon potential to study the properties of light nuclei. In this section, we aim to study the holographic light nuclei such as ${}^2D$, ${}^3T$, ^{3}He, and ^{4}He. For this purpose, we consider a set of A instantonic baryons as a nucleus. It is known that the nucleons are stabilized at a certain distance in nuclei because of a binding force and a strong repulsive force due to the light meson exchanges. We assume that the nucleons have a uniform distribution in nucleus. Therefore, we consider a homogeneous distribution of D-branes in the R^3 space. In order to study the potential of nucleus, we should regard the interaction between these D-branes. It was shown that the size of baryon (instanton) is small and the interaction between two instantons can be explained by the OBEP potential [49]. In this section, we use this nucleon-nucleon potential to obtain the potentials of light nuclei. Also we calculate the binding energy of these nuclei. Then we impose different conditions on nucleon spins in order to obtain some excited states of the ^{4}He nucleus. Finally, we calculate the energy of these excited states and estimate their excited energy.

4.1. Nucleon-Nucleon Holography Potential. Two particle scattering Phase shift in different partial waves as well as the bound state properties of deuteron are experimental data for a two-nucleon system which identify the main properties of nucleon-nucleon interaction. But the potentials attained phenomenologically have many free parameters which are determined by fitting to the experimental data. Various mesons and their resonances play a special role in producing the nucleon-nucleon potential with the following rules.

(i) The long range part of the NN potential ($r > 3\,fm$) is mostly due to the one pion exchange mechanism.

(ii) Isoscalar mesons are responsible for the attractive interaction in the intermediate range of the potential ($0.7 < r < 2\,fm$).

(iii) Exchanging the vector meson ρ can explain the small attractive behavior of the odd-triplet state.

(iv) Vector mesons produce the strong short range repulsion.

Then by considering these facts the general one boson exchange nucleon-nucleon potential is written as [39]

$$V_{\mathrm{NN}} = V_\pi + V_{\eta'} + \sum_{k=1}^{\infty} V_{\rho^{(k)}} + \sum_{k=1}^{\infty} V_{\omega^{(k)}} + \sum_{k=1}^{\infty} V_{a^{(k)}} + \sum_{k=1}^{\infty} V_{f^{(k)}}, \tag{69}$$

which contains the pseudoscalar (π, η'), vector $(\rho^{(k)}, \omega^{(k)})$, and axial vector $(a^{(k)}, f^{(k)})$ meson exchange potentials, respectively. It should be noted that despite of the phenomenological NN interaction model, here we compute all of the nucleon-meson couplings contributing to the above potential using the noncritical holography model.

In our calculations, the leading parts of the potential come from the pseudoscalar meson π, isoscalar vector meson $\omega^{(k)}$, isovector vector meson $\rho^{(k)}$, and isovector axial vector meson $a^{(k)}$ exchange interactions:

$$\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}} \sim g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \sim \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}} \sim g_{a^{(k)}\mathcal{N}\mathcal{N}}. \tag{70}$$

One pion exchange potential (OPEP) has the following form:

$$V_\pi = \frac{1}{4\pi}\left(\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}}\right)^2 \frac{1}{M_{\mathrm{KK}}^2 r^3} S_{12}\vec{\tau}_1 \cdot \vec{\tau}_2. \tag{71}$$

Also, the holographic potentials for isospin singlet vector mesons $\omega^{(k)}$, isospin triplet vector mesons $\rho^{(k)}$, and the triplet axial-vector mesons $a^{(k)}$ are

$$\begin{aligned} V_{\omega^{(k)}} &= \frac{1}{4\pi}\left(g_{\omega^{(k)}\mathcal{N}\mathcal{N}}\right)^2 m_{\omega^{(k)}} y_0\left(m_{\omega^{(k)}} r\right), \\ V_{\rho^{(k)}} &\simeq \frac{1}{4\pi}\left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}}\right)^2 \frac{m_{\rho^{(k)}}^3}{3M_{\mathrm{KK}}^2} \\ &\quad \times \left[2y_0\left(m_{\rho^{(k)}} r\right)\vec{\sigma}_1\cdot\vec{\sigma}_2 - y_2(m_{\rho^{(k)}} r) S_{12}(\hat{r})\right]\vec{\tau}_1\cdot\vec{\tau}_2, \\ V_{a^{(k)}} &\simeq \frac{1}{4\pi}\left(g_{a^{(k)}\mathcal{N}\mathcal{N}}\right)^2 \frac{m_{a^{(k)}}}{3} \\ &\quad \times \left[-2y_0\left(m_{a^{(k)}} r\right)\vec{\sigma}_1\cdot\vec{\sigma}_2 + y_2\left(m_{a^{(k)}} r\right) S_{12}(\hat{r})\right]\vec{\tau}_1\cdot\vec{\tau}_2. \end{aligned} \tag{72}$$

In the above equations we have

$$\begin{gathered} S_{12} = 3\left(\vec{\sigma}_1\cdot\hat{r}\right)\left(\sigma_2\cdot\hat{r}\right) - \vec{\sigma}_1\cdot\vec{\sigma}_2, \\ y_0(x) = \frac{e^{-x}}{x}, \qquad y_2(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\frac{e^{-x}}{x}. \end{gathered} \tag{73}$$

The masses of all mesons are of the order M_{KK} and $m_{\rho^{(k)}} = m_{\omega^{(k)}} < m_{a^{(k)}}$. Also, the mass of pion in the holographic model is zero and its coupling constant to the nucleon in our approach is 15.7.

Finally, the holographic nucleon-nucleon potential becomes [51–53]

$$V_{\mathrm{NN}}^{\mathrm{holography}} = V_C(r) + \left(V_T^\sigma(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^S(r)\,S_{12}\right)\vec{\tau}_1\cdot\vec{\tau}_2, \tag{74}$$

where

$$V_C(r) = \sum_{k=1}^{P} \frac{1}{4\pi} \left(g_{\omega^{(k)}\mathcal{N}\mathcal{N}}\right)^2 m_{\omega^{(k)}} y_0\left(m_{\omega^{(k)}} r\right) m, \tag{75}$$

$$\begin{aligned} V_T^{\sigma}(r) = &\sum_{k=1}^{P} \frac{1}{4\pi} \left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}}\right)^2 \frac{m_{\rho^{(k)}}^3}{3M_{\mathrm{KK}}^2} \left[2y_0\left(m_{\rho^{(k)}} r\right)\right] \\ &+ \sum_{k=1}^{P} \frac{1}{4\pi} \left(g_{a^{(k)}\mathcal{N}\mathcal{N}}\right)^2 \frac{m_{a^{(k)}}}{3} \left[-2y_0\left(m_{a^{(k)}} r\right)\right], \end{aligned} \tag{76}$$

$$\begin{aligned} V_T^{S}(r) = &\frac{1}{4\pi} \left(\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}}\right)^2 \frac{1}{M_{\mathrm{KK}}^2 r^3} \\ &+ \sum_{k=1}^{P} \frac{1}{4\pi} \left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{\mathrm{KK}}}{2m_{\mathcal{N}}}\right)^2 \frac{m_{\rho^{(k)}}^3}{3M_{\mathrm{KK}}^2} \left[-y_2\left(m_{\rho^{(k)}} r\right)\right] \\ &+ \sum_{k=1}^{P} \frac{1}{4\pi} \left(g_{a^{(k)}\mathcal{N}\mathcal{N}}\right)^2 \frac{m_{a^{(k)}}}{3} \left[y_2\left(m_{a^{(k)}} r\right)\right]. \end{aligned} \tag{77}$$

It is shown that in the SS model, at the large enough distances, $p \simeq \sqrt{\lambda/10}$ is an acceptable value for these potentials. We consider the ten first terms of the above potentials in our numerical calculations both in SS and AdS_6 models.

In order to calculate the NN potential, the nucleon-meson coupling constants are needed. These couplings are calculated using the SS model at the large λN_c limit and presented in Table 4.

Also, we calculate the coupling values in the noncritical AdS_6 background. The obtained results are presented in Tables 5 and 6. In the following, we calculate the light nuclei potentials using the NN holography potentials coming from both SS and AdS_6 models.

4.2. Holographic Deuteron. Deuteron is the only bound state of two-nucleon system with the isospin $T = 0$, total spin $S = 1$, spin parity 1^+, and binding energy $E_B = 2.225$ MeV. In our holographic model, we suppose that deuteron is made of two instantonic baryons with $N_f = 2$ and $N_c = 3$ which are located at relative distance r in the R^3 space and consider the following potential for the deuteron:

$$V_{\text{deuteron}}^{\text{holography}} = V_C + \left(V_T^{\sigma}\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{S} S_{12}\right)\vec{\tau}_1 \cdot \vec{\tau}_2, \tag{78}$$

where $V_C(r)$, $V_T^{\sigma}(r)$, and $V_T^{S}(r)$ are presented in (75), (76), and (77), respectively. The super selection rules propose that

$$S_{12} = 2, \qquad \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, \qquad \vec{\tau}_1 \cdot \vec{\tau}_2 = -3. \tag{79}$$

The deuteron potential is calculated numerically using the results of both SS model and AdS_6 model. The minimum of this potential is considered as the deuteron binding energy. We choose the $N_c = 3$, $\lambda = 400$, and $m_N = 550$ MeV in the SS model.

Table 4: The values of meson-nucleon couplings and mass of mesons in the SS model. The values of $N_c = 3$, $\lambda = 400$, and $m_N = 550$ MeV are supposed in calculations.

k	m_{ω^k}	m_{a^k}	g_{ω^k}	g_{ρ^k}	g_{a^k}
0	0.818	1.25	2.1165	0.7055	0.8140
1	1.69	2.13	1.9312	0.6437	1.4202
2	2.57	3.00	1.8888	0.6296	2.0178
3	3.44	3.87	1.8740	0.6246	2.6067
4	4.30	4.73	1.8680	0.6226	3.1956
5	5.17	5.59	1.8636	0.6212	3.7931
6	6.03	6.46	1.8619	0.6206	4.3734
7	6.89	7.32	1.8602	0.6200	4.9623
8	7.75	8.19	1.8602	0.6200	5.5512
9	8.62	9.05	1.8593	0.6197	6.1401

Table 5: Numerical results of vector meson couplings to the nucleon for the ten lowest mesons using the AdS_6 model. Meson masses are in the M_{KK} unit.

k	$g^k_{V,\mathrm{mag}}$	$g^k_{V,\mathrm{min}}$	g_{ω^k}	g_{ρ^k}	m^2_{2k+1}
0	−1.9889	7.7251	11.5727	2.8630	0.5516
1	−6.8384	7.3315	10.9974	0.24	3.0593
2	−7.4493	7.2420	10.863	0.1036	7.6012
3	−4.6067	7.2211	10.8317	1.3072	14.1905
4	−4.4327	7.2147	10.8222	1.3910	22.8274
5	−6.6083	7.2133	0.8200	0.3024	33.5191
6	−6.1778	7.2137	10.8206	0.5179	46.2717
7	−4.0509	7.1740	10.7611	1.5616	60.3053
8	−4.4701	7.1725	10.7589	1.3512	76.8821
9	−6.5703	7.1714	10.7572	0.3005	95.4673

Table 6: Numerical results of axial-vector meson couplings to the nucleon for the ten lowest mesons using the AdS_6 model. Meson masses are in the M_{KK} unit.

k	$g^k_{A,\mathrm{mag}}$	$g^k_{A,\mathrm{min}}$	g_{a^k}	g_{f^k}	m^2_{2k}
0	4.2648	1.1659	2.7154	1.7489	1.5389
1	5.3813	1.0718	3.2301	1.6189	5.0877
2	7.8574	0.9692	4.4133	1.4539	10.6404
3	10.3344	0.6713	5.5028	1.0069	18.2525
4	12.8068	0.4188	6.6128	0.6282	27.9160
5	15.2780	0.3020	7.7900	0.4531	39.6300
6	17.7493	0.2743	9.0118	0.4115	53.4224
7	20.0849	0.2620	10.1734	0.3930	68.3462
8	22.528	0.2359	11.3820	0.3539	85.9293
9	24.9705	0.2061	12.5885	0.3092	105.5220

As we know, the t' Hooft parameter is of order one in noncritical holographic models. So, we choose the $N_c = 3$, $\lambda = 1$ values in our calculations in the AdS_6 model. Also, we use the obtained value for the nucleon mass $m_N = 920$ MeV in this model which is very close to the real value of nucleon mass. Numerical results are shown in Table 7.

Table 7: The obtained binding energy of ^{2}D, ^{3}T, ^{3}He, and ^{4}He nuclei with $N_c = 3$ and $m_N = 0.92$ GeV. The results have a good consistency with the experimental nuclear data. All energies are in MeV.

Nuclei	M_{KK}	$E_B^{\text{NC-H}}$	$E_B^{\text{C-H}}$ [51, 52]	E_{Exp} [57–60]
^2D	372	2.22	2.20	2.17 ± 0.0
^3T	600	8.432	1.03	8.48
^{3}He	372	7.8680	7.41	7.71
^{4}He	533	28.3527	28.58	28.30

4.3. Holographic Tritium. The next nucleus we considered here is tritium which is composed of three nucleons, two neutrons, and one proton. We propose an equilateral triangular configuration for the tritium nucleus in which the distance between each two nucleons is r. We suppose that the total potential of the nucleus is the sum of the all nucleon-nucleon interaction potentials which are parameterized in terms of a single parameter r. In fact, the radius of nucleus can be expresses in terms of parameter r. Finally, we write the following potential for the tritium:

$$\begin{aligned} V_{\text{Tritium}}^{\text{holography}} &= V_{12} + V_{13} + V_{23} \\ &= 3V_C(r) + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{S}(r)\,S_{12}\right)\vec{\tau}_1\cdot\vec{\tau}_2 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_3 + V_T^{S}(r)\,S_{13}\right)\vec{\tau}_1\cdot\vec{\tau}_3 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_2\cdot\vec{\sigma}_3 + V_T^{S}(r)\,S_{23}\right)\vec{\tau}_2\cdot\vec{\tau}_3. \end{aligned} \tag{80}$$

The super selection rules for this three-nucleon systems imply that

$$\begin{aligned} &S_{12} = 2, \quad \vec{\sigma}_1\cdot\vec{\sigma}_2 = 1, \quad \vec{\tau}_1\cdot\vec{\tau}_2 = -3, \\ &S_{13} = 0, \quad \vec{\sigma}_1\cdot\vec{\sigma}_3 = -3, \quad \vec{\tau}_1\cdot\vec{\tau}_3 = -3, \\ &S_{23} = 0, \quad \vec{\sigma}_2\cdot\vec{\sigma}_3 = -3, \quad \vec{\tau}_2\cdot\vec{\tau}_3 = 1. \end{aligned} \tag{81}$$

4.4. Holographic ^{3}He. In order to study the ^{3}He nucleus, it is necessary to add the repulsive Coulomb energy to the potential. So, we consider the following potential for the ^{3}He nucleus:

$$\begin{aligned} V_{^3\text{He}}^{\text{holography}} &= V_{12} + V_{13} + V_{23} \\ &= 3V_C(r) + E_c(r) \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{S}(r)\,S_{12}\right)\vec{\tau}_1\cdot\vec{\tau}_2 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_3 + V_T^{S}(r)\,S_{13}\right)\vec{\tau}_1\cdot\vec{\tau}_3 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_2\cdot\vec{\sigma}_3 + V_T^{S}(r)\,S_{23}\right)\vec{\tau}_2\cdot\vec{\tau}_3, \end{aligned} \tag{82}$$

where $E_c(r)$ is the Coulomb repulsion between two instantons carrying N_c unit of electric charge [14]. The protons of ^{3}He in the ground state have the opposite spin directions, so the spin parity of ^{3}He nucleus in the ground state is $(1/2)^+$. On the other hand, we should have $L + S + T = 1$ for a system of two nucleons. It is well known that the nucleons in the ground state of the ^{3}He are in $L = 0$ state. So, by using the super selection rules we obtain

$$\begin{aligned} &S_{12} = 0, \quad \vec{\sigma}_1\cdot\vec{\sigma}_2 = -3, \quad \vec{\tau}_1\cdot\vec{\tau}_2 = 1, \\ &S_{13} = 2, \quad \vec{\sigma}_1\cdot\vec{\sigma}_3 = 1, \quad \vec{\tau}_1\cdot\vec{\tau}_3 = -3, \\ &S_{23} = 0, \quad \vec{\sigma}_2\cdot\vec{\sigma}_3 = -3, \quad \vec{\tau}_2\cdot\vec{\tau}_3 = 1. \end{aligned} \tag{83}$$

If we consider another set of nucleons in ^{3}He such that the spin of protons is in a parallel direction, the spin parity of ^{3}He nucleus should be equal to $(3/2)^+$. By super selection rules, we have

$$\begin{aligned} &S_{12} = 2, \quad \vec{\sigma}_1\cdot\vec{\sigma}_2 = 1, \quad \vec{\tau}_1\cdot\vec{\tau}_2 = 1, \\ &S_{13} = 2, \quad \vec{\sigma}_1\cdot\vec{\sigma}_3 = 1, \quad \vec{\tau}_1\cdot\vec{\tau}_3 = -3, \\ &S_{23} = 2, \quad \vec{\sigma}_2\cdot\vec{\sigma}_3 = 1, \quad \vec{\tau}_2\cdot\vec{\tau}_3 = -3. \end{aligned} \tag{84}$$

We found that there is no bound state in this case both in SS and AdS_6 models. Thus, we conclude that there is no excited state for the ^{3}He nucleus.

4.5. Holographic ^{4}He. There is more than one possible configuration for a system with four nucleons. The most symmetric configurations are tetrahedron, diamond, and square configurations. If we suppose that the nucleons are located in the corners of a tetrahedron configuration which is made of four equilateral triangles, the distance between any two nucleons is similar. So, the total potential is sum of the 6 nucleon-nucleon interactions with the same relative distance. But, we know that the Coulomb interaction between protons prefers a larger proton-proton distance than neutron-neutron or neutron-proton distances. If two protons sit on the contrary corners of a square, then the proton-proton distance is larger than the neutron-proton distance. So, we consider the square configuration for the ^{4}He nucleus and write the potential of ^{4}He nucleus as the following form:

$$\begin{aligned} V_{^4\text{He}}^{\text{holography}} &= V_{12} + V_{13} + V_{14} + V_{23} + V_{24} + V_{34} \\ &= 4V_C(r) + 2V_C\left(\sqrt{3}r\right) + E_c\left(\sqrt{2}r\right) \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{S}(r)\,S_{12}\right)\vec{\tau}_1\cdot\vec{\tau}_2 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_3 + V_T^{S}(r)\,S_{13}\right)\vec{\tau}_1\cdot\vec{\tau}_3 \\ &\quad + \left(V_T^{\sigma}\left(\sqrt{2}r\right)\vec{\sigma}_1\cdot\vec{\sigma}_4 + V_T^{S}\left(\sqrt{2}r\right)S_{14}\right)\vec{\tau}_1\cdot\vec{\tau}_4 \\ &\quad + \left(V_T^{\sigma}\left(\sqrt{2}r\right)\vec{\sigma}_2\cdot\vec{\sigma}_3 + V_T^{S}\left(\sqrt{2}r\right)S_{23}\right)\vec{\tau}_2\cdot\vec{\tau}_3 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_2\cdot\vec{\sigma}_4 + V_T^{S}(r)\,S_{24}\right)\vec{\tau}_2\cdot\vec{\tau}_4 \\ &\quad + \left(V_T^{\sigma}(r)\,\vec{\sigma}_3\cdot\vec{\sigma}_4 + V_T^{S}(r)\,S_{34}\right)\vec{\tau}_3\cdot\vec{\tau}_4. \end{aligned} \tag{85}$$

Table 8: The obtained excited energy of ^{3}He and ^{4}He nuclei with $N_c = 3$ and $m_N = 0.92$ GeV. The results have a good agreement with the experimental nuclear data [57, 58]. All energies are in MeV.

Nuclei	J^P	M_{KK}	$E^{\text{NC-H}}_{\text{Ex}}$	$E^{\text{C-H}}_{\text{Ex}}$ [18, 19]	$E^{\text{Exp}}_{\text{Ex}}$ [57, 58]
^{3}He	$\frac{3}{2}^+$	—	—	—	No state
^{4}He	$2^-, T = 0$	395	21.8237	22.00	21.840
^{4}He	$2^-, T = 1$	395	25.1001	—	23.330
^{4}He	$1^-, T = 1$	305	23.658	23.17	23.640

4.5.1. Ground State. It is well known from the Pauli exclusion rule that the spins of two protons (neutrons) have opposite directions and the ^{4}He nucleus in the ground state has the spin parity 0^+. The super selection rules for this structure imply that

$$\begin{aligned}
&S_{12} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_2 = 1,\\
&S_{13} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_3 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_3 = -3,\\
&S_{14} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_4 = 1,\\
&S_{23} = 0, && \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_2 \cdot \vec{\tau}_3 = 1,\\
&S_{24} = 2, && \vec{\sigma}_2 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_2 \cdot \vec{\tau}_4 = -3,\\
&S_{34} = 0, && \vec{\sigma}_3 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_3 \cdot \vec{\tau}_4 = 1.
\end{aligned} \tag{86}$$

4.5.2. Excited States. Also, the potential of ^{4}He is obtained for its excited states with $(2^-, T = 1)$, $(2^-, T = 0)$, and $(1^-, T = 1)$ by considering various structures for the spin parity of nucleons. The holographic potential for each excited state has a minimum. The excited energies of these states can be regarded as the difference between the minimum point of potential in each state and the binding energy of nucleus.

If two nucleons (two protons or neutron) have the same spin directions and occupy the level $L = 1$, we find the excited level with 2^-, $T = 1$, and excited energy $E_{\text{ex}} = 23.330$ MeV. Super selection rules for this state lead to

$$\begin{aligned}
&S_{12} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_2 = 1,\\
&S_{13} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_3 = -3,\\
&S_{14} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_4 = 1,\\
&S_{23} = 0, && \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_2 \cdot \vec{\tau}_3 = 1,\\
&S_{24} = 2, && \vec{\sigma}_2 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_2 \cdot \vec{\tau}_4 = 1,\\
&S_{34} = 0, && \vec{\sigma}_3 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_3 \cdot \vec{\tau}_4 = -3.
\end{aligned} \tag{87}$$

Numerical values for the potential of this excited state are shown in Table 8. For this state we obtain $E_{\text{excited}} = 25.1005$ MeV using the value $M_{KK} = 395$ MeV. While such excited state is not predicted by the SS model [52].

In another structure, we suppose that the spins of two protons (or neutrons) have the same directions and one of them occupies the $L = 1$ level. In this case, the spin parity of the state is 2^-. It may be treated as excited state of ^{4}He nucleus with spin parity and isospin 2^-, $T = 0$, and the excited energy $E_{\text{ex}} = 21.840$ MeV. In order to calculate its holographic potential, the following values which are obtained from the super selection rules have been used:

$$\begin{aligned}
&S_{12} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_2 = -3,\\
&S_{13} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_3 = 1,\\
&S_{14} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_4 = 1,\\
&S_{23} = 0, && \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_2 \cdot \vec{\tau}_3 = 1,\\
&S_{24} = 2, && \vec{\sigma}_2 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_2 \cdot \vec{\tau}_4 = 1,\\
&S_{34} = 0, && \vec{\sigma}_3 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_3 \cdot \vec{\tau}_4 = -3.
\end{aligned} \tag{88}$$

The exited energy for this state is obtained about $E_{\text{excited}} = 21.8237$ MeV using the value $M_{KK} = 395$ MeV.

If the spin of proton (neutron) in the $L = 1$ level couples with the spin of the proton (neutron) in the $L = 0$ state, we find another excited state with the 1^-, $T = 1$, and the measured excited energy $E_{\text{ex}} = 23.640$ MeV. In this case, we have

$$\begin{aligned}
&S_{12} = 2, && \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, && \vec{\tau}_1 \cdot \vec{\tau}_2 = 1,\\
&S_{13} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_3 = -3,\\
&S_{14} = 0, && \vec{\sigma}_1 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_1 \cdot \vec{\tau}_4 = -3,\\
&S_{23} = 0, && \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, && \vec{\tau}_2 \cdot \vec{\tau}_3 = 1,\\
&S_{24} = 0, && \vec{\sigma}_2 \cdot \vec{\sigma}_4 = -3, && \vec{\tau}_2 \cdot \vec{\tau}_4 = 1,\\
&S_{34} = 2, && \vec{\sigma}_3 \cdot \vec{\sigma}_4 = 1, && \vec{\tau}_3 \cdot \vec{\tau}_4 = -3.
\end{aligned} \tag{89}$$

In this case, we obtain $E_{\text{excited}} = 23.658$ MeV by choosing the value $M_{KK} = 305$ MeV.

4.6. Numerical Results. In general, the considered potential in this model tends to zero at $r \to \infty$ and becomes infinity at small distances which is well established for nuclear knowledge. The minimum of the potential in the ground state is considered as the binding energy of nucleus. Moreover, the difference between the minimum of the excited state potential and the nucleus binding energy is considered as the excited energy of the corresponding state. We apply our method for the deuteron, ^2D, tritium, ^3T, and two isotopes of helium, namely, ^{3}He and ^{4}He nuclei.

To obtain the numerical results, $N_c = 3$ have been chosen for the realistic QCD. Also, we obtain the value of nucleon mass about $m_N = 0.92$ GeV which is very close to the experimental nucleon mass. In our numerical calculations there is only one free parameter M_{KK}. The results of binding energy and excited energies are compared with results of SS model and experiments in Tables 7 and 8. As it is indicated from the tables, our results are in good agreement with the experimental nuclear data. Moreover, our potential has only one free parameter which allows us to fit our results with the experimental data.

In Table 9, we compare our numerical results for the light nuclei binding energies with the predictions of the modern phenomenological NN potential models [61]. It

Table 9: 3N and 4N binding energies for various NN potentials [61] compared with our holographic model results and experimental values. C-H and NC-H refer to the critical holographic [39] and noncritical holographic potential [49] models, respectively. All energies are in MeV.

Potential	$E_B(T)$	$E_B(^3\mathrm{He})$	$E_B(^4\mathrm{He})$
CD Bonn	−8.012	−7.272	−26.26
AV18	−7.623	−6.924	−24.28
Nijm I	−7.736	−7.085	−24.98
Nijm II	−7.654	−7.012	−24.56
C-H	−1.03	−7.41	−28.58
NC-H	−8.4320	−7.8680	−28.3527
Exp.	−8.48	−7.72	−28.30

Table 10: Comparison of the ^{4}He binding energy obtained from our model with the results of some other theoretical models based on chiral low-momentum interactions [62, 63].

Method	$E_B(^4\mathrm{He})$ [MeV]
Faddeev-Yakubovsky (FY)	−28.65 (5)
Hyperspherical harmonics (HH)	−28.65 (2)
CCSD (CC with singles and doubles)	−28.44
Λ-CCSD (T) (CC with triples corrections)	−28.63
Critical holography model (SS model)	−28.58
Noncritical holography model (AdS_6 model)	−28.3527

is obvious that our results obtained using the noncritical holographic NN potential have a significant agreement with the experimental data. It should be noted that we calculated all of the parameters of noncritical holographic NN potential [49] and also our toy model for calculating the binding energy has just one free parameter which is the mass scale of the model, M_{KK}.

Also, we compare our results for the ^{4}He binding energy with the results obtained from other methods [62, 63] such as Faddeev-Yakubovsky (FY), Hyperspherical harmonics (HH), CCSD (CC with singles and doubles), and Λ-CCSD(T) (CC with triples corrections) in Table 10. It is necessary to mention that our model depends on just one parameter which is M_{KK}, whereas the other theoretical models in nuclear literatures have more than one parameter.

5. Conclusion

One of the applications of AdS/CFT correspondence is holography QCD and introduced to solve the strong coupling QCD such as the low-energy dynamics of hadrons in particular baryons. A lot of holography models are introduced to reproduce the QCD. Among them the SS model is one of the most successful models due to its accurate results. But, as we mentioned, the model encountered some inconsistencies in describing the baryons especially nucleons. For example, the mass scale of the model to describe the nucleons are the half of the one needs to describe the meson sector. Also, the size of baryon in the large t' Hooft limit goes to zero. On the other hand, all holographic QCD models based on the critical string theory suffer from the unwanted KK modes.

In order to investigate these issues, we employ the noncritical AdS_6 background and its field theory dual. We study the mesons and nucleons in this background and compute some of their features such as the vector-meson spectrum, pion decay constant, baryon binding energy, thermodynamic properties of baryonic matter, size of baryon, nucleon-nucleon interaction, and nucleon-meson coupling constants.

We review some obtained results below which show that our results not only are in a good agreement with the nuclear data but also are better than the SS model results.

(1) Just like the SS model, there exist some KK modes which come from the antiperiodic boundary conditions over the circle S^1. These modes have the masses of the same order of magnitude as the lightest glueballs of the four-dimensional YM theory. Critical holographic models, such as the SS model, have some extra KK modes too which do not belong to the spectrum of pure YM theory. These undesired KK modes come from the extra internal space over which ten-dimensional string theory is compactified, for example, the S^4 sphere in the SS model. In the noncritical holographic model, which we used here, there is no additional compactified sphere, so there are no such extra KK modes and the QCD spectrum is clear from them. Thus, it seems that our model based on the noncritical holography is much more reliable.

(2) We studied the dynamics of gauge field living on the flavor probe brane and obtained the spectrum of vector mesons. Our results were compared with the results of other holographic models and the experimental data. Also, we calculated the pion decay constant in terms of model parameter. We found the values of mass scale $M_{\mathrm{KK}} = 395\,\mathrm{MeV}$ to have pion decay constant $f_\pi = 92\,\mathrm{MeV}$.

(3) In order to study the nuclear physics in the holography frame, we investigated baryons which are defined by a vertex with N_c fundamental strings attached to the flavor brane. We obtained the binding energy of baryon in the noncritical AdS_6 model [31]. Baryon in holography is replaced by a solitonic instanton such that the instantonic number shows the baryon number. We used this definition of baryon in the AdS_6 model and calculated its size. We demonstrated that the size of baryon is of order one; therefore, the zero size of baryon in the holography SS model was solved here [49].

(4) Holographic models have a mass scale which is the low-energy scale of the model. In the SS model, the value of M_{KK} to describe the baryon should be half of the one to describe the mesons. The nucleon mass was obtained roughly 920 MeV using $M_{\mathrm{KK}} = 395\,\mathrm{MeV}$. So, our model could well describe the mesons and nucleons with the same mass scale.

(5) We employed the noncritical AdS_6 model to study the NN potential and nucleon-meson coupling constants. We derived the Yukawa coupling constants by exploring the dynamics of nucleon in the holography frame. We compared our results with the predictions of four modern phenomenological NN potential models. The remarkable point is that all nucleon-meson coupling constants have been calculated in the holography model, whereas these parameters were obtained by fitting to the empirical NN scattering data in the

phenomenological potentials. Our holography NN potential can be more accurate by considering the derivative couplings in the magnetic channels. In addition, the holography NN potential obtained using the AdS_6 model can be used widely in describing the nuclear structure and multinucleon systems such as the nuclear binding energy and NN scattering.

(6) The small value of nuclear binding energy is one of the interesting issues in nuclear physics. Despite of the power of strong interaction, the NN force is small: binding energy is only a few percent of the mass of the nucleons. In the holographic models, the exchange of heavy mesons is suppressed in the large N_c limit. As a result, the interaction of two nucleons is explained via the exchange of light mesons such as pion and ω-meson. The exchange of pion is responsible for the attractive long-range nuclear force, whereas, the exchange of ω-meson produce mainly medium-range repulsive force. If we suppose that the repulsion starts at distance $|x| \sim m_{\omega}^{-1}$, then the nuclear binding energy is of order $E_{\text{binding}} \sim (1/g_s)m_{\omega}$ which is much smaller than the nucleon mass. The above analysis motivated us to introduce a simple toy model to estimate the binding energy of multinucleons systems. We explained the model in the previous section in details. In general, the obtained nuclear potential has the familiar behavior in nuclear physics. In addition, despite of the small number of free parameters in our holography model, the obtained results have significant agreement with the experimental data.

(7) In our holography model for the light nuclei, we assumed that the setting of a small number of instantonic D-brane on the background does not change the background. In fact, we ignored the back reaction of baryon vertices and background geometry. It is clear that this assumption is correct just for the light nuclei. In fact, one can find a gravity dual for heavy nuclei by implying the AdS/CFT correspondence again. In this holographic description, the gauge theory on the nuclei with mass number A is $U(A)$. Study of the $U(A)$ gauge theory is hard, but the theory becomes more simple by taking the large A limit. In this limit, one can find the near horizon geometry dual to the gauge theory. The supergravity solution has a discrete spectrum which is the excited spectrum of heavy nuclei with mass A [17]. The result is in agreement with nuclear data manifestly. As we know from the nuclear experiments, the nucleons of a heavy nuclei have coherent excitations which are called Giant resonances. These resonances exhibit harmonic behavior $E_n = nw(A)$ which is explained with phenomenological models such as the liquid drop model. The gauge-gravity duality can reproduce this behavior. Also, dependence to the mass number A is obtained by using the duality [17].

In this regard, several issues can be studied. For example, if we put a probe brane as an external nucleon near the near-horizon geometry of A D-brane and consider the probe dynamics, the shell model potential of nuclear physics may be obtained.

On the other hand, since black holes are described by fluid dynamics holographically, one can speculate that the liquid drop model of heavy nuclei may be related to dual geometries through the holographic hydrodynamics. In fact, dissipation of excitations on a nucleus is a target of research for many decades.

(8) The repulsive core potential is one of the critical issues of nuclear physics that its origin is still not well understood. Nuclear force has been studied using the AdS/CFT correspondence [13–16] and an explicit expression has been obtained for the nuclear force which contains the repulsive core too. This potential behaves as r^{-2} in small distances. However, there are yet a lot of unanswered questions about the nuclear repulsive and attractive force.

Finally, it seems that the AdS/CFT correspondence is a new tool to solve the unanswered questions in nuclear physics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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