

Assessment of the South African sardine resource using data from 1984-2015: Results at the joint posterior mode for the two mixing-stock hypothesis

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The Operating Model (OM) for the South African sardine resource has been updated from that used to develop OMP-14 given four more years of data. The model has been altered from previous assessments to now model infection of sardine by a “tetracotyle”-type metacercarian parasite, and includes data on this parasite prevalence-by-length in the likelihood. A Hockey-Stick stock recruitment relationship is used for this OM, with a different median recruitment and higher variability estimated for the west stock during “peak years”. Time-invariant natural mortality is assumed to be 1.0 year⁻¹ for juveniles and 0.8 year⁻¹ for adults as before. The total resource abundance is estimated to be 688 thousand tons in November 2015, with the west stock consisting of 142 thousand tons and the south stock consisting of 546 thousand tons. These biomasses are well below the long term average of 1 039 and 492 thousand tons for the total resource and west stock respectively, and near the long term average of 547 thousand tons for the south stock. The west stock has experienced below average recruitment in 11 out of the 12 most recent years.

Introduction

The assessment of the South African sardine resource has been revised and updated using data available up to November 2015. Two primary hypotheses regarding the sardine stock structure have been agreed for investigation. The first considers sardine distributed off the west and south coasts of South Africa to form a single homogeneous “stock” (or “population”) (de Moor and Butterworth 2016). The second considers the sardine to consist of a western stock and southern stock with some mixing between the two. This document presents results at the joint posterior mode for the two mixing-stock hypothesis only.

The two mixing-stock hypothesis postulates a “west” stock distributed west of Cape Agulhas and a “south” stock distributed south-east of Cape Agulhas, with movement from the “west” to the “south” stock in November each year. de Moor and Butterworth (2015) considered the mixing to be only the movement of annually-varying west stock recruits to the south stock as they become 1 year old each November. In addition they assumed no mixing prior to 1994 when there are no recruit survey data for the south coast. This document extends that previous hypothesis by allowing for older west stock fish to move to the south stock and by estimating mixing for all the years considered in the assessment.

This assessment also includes new data on the proportion of sardine-by-length infected by a digenean ‘tetracotyle-type’ metacercarian endoparasite. The working hypothesis assumed in this document is that only sardine distributed to the west of Cape Agulhas, i.e. west stock sardine, are infected with this parasite, based on the assumption that the parasite host is only found west of Cape Agulhas (Weston 2013, van der Lingen et al. 2015). Under this hypothesis, infection of sardine on the south coast can result only from west-to-south movement.

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Population Dynamics Model

The same generalised operating model for the South African sardine resource is used for both the single and two mixing-stock hypotheses, and the data used in this assessment are listed in de Moor *et al.* (2016). The model is detailed in Appendix A and all the parameters are defined in Tables A1 and A2. The particular difference when fitting the two mixing-stock hypothesis to the data compared to the single-stock hypothesis is that both abundance indices and proportion-at-length data are disaggregated west and south of Cape Agulhas, and the negative log likelihood include terms for each of these spatially separate components. In addition, the parasite data are used only in the likelihood for the two mixing-stock hypothesis.

Key features of this model include:

- The model is age-structured with a plus group of age 5. A distribution of length-at-age is used to model the length-structure of the population at fixed times during the year, and the length-at-age-0 differs by year to allow for variations in the time of peak recruitment (thus being able to accommodate early/late recruitment).
- Recruitment to each stock is dependent on the spawner biomass of that stock only (though the equations are generalised to allow for alternative assumptions to be made in robustness testing).
- In the two mixing-stock hypothesis, permanent west-to-south movement is modelled, but no south-to-west movement is permitted.

Key differences in this model compared to that of de Moor and Butterworth (2015) include:

- Spawner biomass is calculated assuming a maturity-at-length ogive which changes over time, rather than assuming all sardine mature at age 2, and using weight-at-length rather than weight-at-age.
- The trawl survey selectivity-at-length is assumed to be logistic (hence allowing for some escapement of small fish); reduced availability (a decrease in selectivity) at larger lengths is no longer modelled.
- The estimated stock-specific commercial selectivity-at-length curve is described by a logistic distribution at greater lengths rather than an inverted lognormal distribution. Time-varying commercial selectivity is assumed, with selectivity varying by quarter and between four pre-specified periods (1984-1986, 1987-1997, 1998-2001, 2002-2015).
- Instead of assuming that the small (<14cm) sardine bycatch is measured without error, for numerical computation convenience a small error is allowed and a fishing mortality is estimated for this bycatch to assist with model convergence.
- The informative prior distribution for the bias associated with the acoustic survey has been recalculated assuming a lognormal rather than normal distribution.
- To aid stable parameter estimation, the stock-specific lengths at ages 1 and 3 are estimated instead of the von-Bertalanffy growth curve parameters themselves.
- To account for variation in the time of the recruitment peak each year, the annual length-at-age 0 is estimated to vary with additive normal error about a median value at 1st November.

- In the two mixing-stock hypothesis, movement of age 2+ sardine is assumed to be non-zero, and is a time-invariant proportion of the proportion of age-1 sardine moving each year.
- In the two mixing-stock hypothesis, infection of sardine by the parasite is modelled to occur in an annual pulse on 1st November and the proportion of sardine infected is age-invariant (but excludes recruits of age 0 on 1st November). Infection by the parasite is assumed to result in no change to growth, maturity, natural or fishing mortality.

In addition, a number of other prior distributions have been modified and/or parameters have been re-parameterised to assist with model convergence.

Natural mortality

A number of combinations of median juvenile and adult natural mortality values are examined, covering the range 0.6 to 1.2 year⁻¹, for the case where a Hockey Stick stock recruitment relationship is assumed. For realism, only combinations with $\bar{M}_j^S \geq \bar{M}_{ad}^S$ are considered.

Results and Discussion

Natural mortality

Table 2 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. Given the choice of prior distributions, the ratio k_r^S/k_N^S is by definition less than 1. Combinations of natural mortality which result in $k_r^S/k_N^S < 0.5$ are considered less plausible and are not considered further.

To maintain consistency with previous assessments, the base case hypothesis continues to assume $\bar{M}_j^S = 1.0$ and $\bar{M}_{ad}^S = 0.8$. This choice was made considering both the single and two-mixing stock hypotheses jointly. There is only one case where the fit to the data (in terms of the negative log likelihood) is better than $\bar{M}_j^S = 1.0$ and $\bar{M}_{ad}^S = 0.8$ at the joint posterior mode. However those improvements were not considered sufficient to warrant changing from the selection used in the most recent assessment, given also the major difficulties that a change would introduce for maintaining a comparable risk definition

Results at the posterior mode

The model fits to the time series of survey abundance indices of November biomass and May recruitment surveys are shown in Figures 1 and 2. In both cases the fits to the survey data are reasonably good. As for the single stock hypothesis (de Moor and Butterworth 2016), the model under-predicts the west stock recruitment in May 2010 as it is unable to reconcile the conflicting data of an above average recruitment estimate in May 2010, with almost no increase in the November biomass estimate from 2009 to 2010. The model is also not able to fully replicate the annual switches from relatively high to relatively low west stock recruitment between 1994 and 1997, given the constraint of also fitting the November biomass during this same period. The sudden and short-

lived increase in survey biomass for the south stock in November 2011 was also under-predicted given the low and absent survey estimates of west and south stock recruitment in May 2011, respectively.

The bias in the surveys is estimated to be 0.75 for the November acoustic survey and 0.51 for the May recruit survey, and the model assumes the coverage of the south stock recruits by the May survey is the same as that of the west stock recruits.

The estimated stock-recruitment relationships are shown in Figure 3. As for the single stock assessment (de Moor and Butterworth 2016), the median recruitment to the west stock is estimated to be much higher during peak years (2000-2004), with a much higher standard deviation about the curve during these years compared to “non-peak” years. The model estimated annual proportions of age 1 and age 2+ sardine moving from the west to south stock are shown in Figure 4. The proportion of 2+ sardine moving is estimated at the joint posterior mode to be 80%¹ of that of the 1 year olds. These results continue to show that the west stock is estimated to be substantially more productive than the south stock, with the observed peak in biomass in the south stock in the early 2000s estimated to have primarily resulted from movement of west stock fish to the south stock, than to have originated from south stock recruitment.

The model estimated survey trawl selectivity is shown in Figure 5 with the residuals from the fit to the November survey length frequency data given in Figure 6. de Moor et al. (In review) show that some improvement in the comparison of average model predicted proportions-at-length with those from the survey (Figure 7) is obtained when the parasite prevalence-at-length data are excluded from the model. This indicates there may be some conflict in these two sets of length-structured data. In addition, the remaining differences in the averages for the south stock may be indicative of the hypothesised south coast winter spawning, but further analyses to investigate this have yet to be carried out.

The model estimated commercial selectivity is shown in Figure 8. While the model is able to fit the data better by allowing commercial selectivity to differ by quarter and year (results not shown here), this variability does not remove all of the systematic residual patterns in the model fit to the commercial proportion-at-length data (Figures 9 and 10). The average (over all years and quarters) model predicted commercial proportions-at-length matches the general pattern of that observed, when considering that the variability in the normal distribution for small lengths is the same for all years (Figure 10). Further work could allow for time-varying changes in further commercial selectivity parameters or alternatively model some selectivity parameters with a random walk rather than with pre-specified time blocks to see if an improved fit would be warranted given the additional parameters estimated.

¹ Note this point estimate at the joint posterior mode may not be reflective of the marginal posterior distribution (e.g. see de Moor et. al. In Review).

A key factor in the model fits to the proportion-at-length data is the model estimated growth curve (Figure 11) and variability about this curve (Figure 12). The estimated annual residuals about an average age at which length is zero, chosen to mimic differences between early and late peak recruitment, allowed a better fit to the model.

As parasite prevalence-at-length data are only available from 2010 onwards, the infection rate is estimated from 2008 onwards and fixed at an arbitrary rate prior to 2008. The estimated rates of infection of west stock sardine by the parasite are estimated to vary substantially between years (Figure 13). The model is able to trace the observed prevalence-at-length from November surveys sufficiently well for the west stock, with some good fits (e.g. 2015) to the south stock data too (Figure 14). Figure 15 shows the model estimated annual proportions-by-length moving, which provides a visual link between the model estimated proportions-by-age which move (Figure 4) and the informative prevalence-by-length data. While the proportions are estimated to be large in some recent years, the numbers moving were more substantial in the early 2000s (Figure 15b).

Figure 16 shows the model estimated harvest rates and approximate instantaneous fishing mortality rates (see Appendix C). Table 2 gives the model estimated loss to predation compared to the loss to fishing mortality.

Summary

This document has detailed the results for the updated assessment of the South African sardine resource, assuming a two mixing-stock hypothesis. Further robustness testing needs to be carried out and will be reported in a subsequent document. This recommended base case hypothesis assumes a different constant median west stock recruitment over the peak years of 2000-2004, together with $\bar{M}_j^S = 1.0$ and $\bar{M}_{ad}^S = 0.8$. This is consistent with the base case hypothesis recommended for the single stock hypothesis (de Moor and Butterworth 2016).

These results have shown yet further progress towards a new two mixing-stock hypothesis for South African sardine. In particular, the direct fitting of the model to the parasite prevalence-at-length data from the November surveys from 2010 to 2015 has resulted in more precise estimates of movement in recent years (de Moor et al. submitted) and the ability to estimate a time and age-invariant proportion parameter allowing for age 2+ sardine to also move permanently from the west to the south stock.

The total resource biomass in November 2015 is estimated to be 688 thousand tons, with the west stock consisting of 142 thousand tons and the south stock consisting of 546 thousand tons. These biomasses are well below the long term average of 1 039 and 492 thousand tons for the total resource and west stock respectively, and near the long term average of 547 thousand tons for the south stock. The west stock has experienced below average recruitment in 11 out of the 12 most recent years. Harvest rates on the west stock have decreased since the peak in 2006 – 2008, but remain higher than that for the south stock in recent years.

References

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Table 1. The contributions to the objective function at the posterior mode for a range of combinations of juvenile, \bar{M}_j^S , and adult, \bar{M}_{ad}^S , natural mortality for models assuming the Hockey Stick stock recruitment relationship. The ratio of the multiplicative bias in the recruit survey to that in the November survey, k_r^S/k_N^S , is given for diagnostic purposes.

Shaded rows represent what are considered unrealistic values for this ratio.

\bar{M}_j^S	\bar{M}_{ad}^S	-ln(Posterior)	$\Delta \{-\ln(\text{Likelihood})\}$	-ln(Likelihood)					-ln(Prior)					k_N^S	k_r^S	k_r^S/k_N^S
				Nov	Rec	Com Prop-at-length	Survey Prop-at-length	Prevalence	k_{ac}^S	ε_y^S	$move_{1,y}$		ε'_y			
0.6	0.6	697.53*	7.8	61.7	64.6	-385.9	-360.9	1281.3	-1.33	25.94	-27.75	40.67	-0.84	0.77	0.77	1.00
0.8	0.6	697.07*	7.6	61.3	64.6	-385.9	-360.9	1281.4	-1.32	25.74	-27.75	40.67	-0.84	0.77	0.68	0.88
0.8	0.8	692.15	2.7	59.7	64.6	-384.8	-365.3	1281.5	-1.42	25.68	-27.73	40.64	-0.85	0.75	0.56	0.75
1	0.6	699.02	9.6	60.8	64.0	-385.7	-357.0	1280.4	-1.36	25.63	-27.85	40.80	-0.84	0.76	0.62	0.81
1	0.8	689.22	0.0	60.5	64.5	-387.3	-365.4	1280.6	-1.42	25.51	-27.69	40.64	-0.84	0.75	0.51	0.68
1	1	685.84	-3.1	58.5	63.9	-383.6	-370.2	1281.3	-1.44	25.14	-27.81	40.82	-0.83	0.74	0.45	0.61
1.2	0.6	702.07	12.4	61.5	65.2	-383.7	-358.9	1281.3	-1.41	25.25	-27.73	41.36	-0.85	0.75	0.58	0.78
1.2	0.8	695.04	5.7	58.6	64.4	-383.2	-364.4	1283.3	-1.44	25.00	-27.66	41.16	-0.85	0.74	0.47	0.64
1.2	1	694.80*	5.1	62.0	64.1	-384.5	-364.7	1281.2	-1.41	24.43	-27.60	42.09	-0.85	0.72	0.41	0.57
1.2	1.2	697.38	7.4	63.6	64.5	-382.8	-364.3	1279.4	-1.36	24.67	-27.59	41.94	-0.85	0.71	0.37	0.53
1.4	0.6	705.29	14.7	64.6	64.7	-382.6	-360.5	1281.5	-1.41	26.06	-27.68	41.39	-0.85	0.75	0.54	0.72
1.4	0.8	697.27	6.6	63.0	64.0	-383.8	-365.7	1282.0	-1.44	26.24	-27.62	41.16	-0.85	0.74	0.45	0.61
1.4	1	697.09	5.0	63.9	63.2	-386.0	-363.4	1280.2	-1.44	26.59	-27.65	42.37	-0.84	0.74	0.40	0.54
1.4	1.2	697.81	7.8	64.5	64.4	-382.8	-364.8	1279.4	-1.36	24.73	-27.57	41.93	-0.85	0.71	0.34	0.49
1.4	1.4	694.79	1.2	63.9	64.5	-382.8	-368.0	1276.5	-1.44	29.05	-27.59	41.31	-0.84	0.73	0.34	0.46

* Non positive definite Hessian

Table 2. The annual estimated sardine loss to predation (in '000t), P (Appendix C), compared to the annual sardine directed and total catch (in '000t).

Year	West Stock			South Stock		
	Directed Catch	Total Catch	Loss to Predation	Directed Catch	Total Catch	Loss to Predation
1984	27.178	27.178	32.098	0.000	0.000	1.353
1985	30.843	30.843	111.682	0.000	0.000	25.489
1986	30.639	30.639	171.597	0.000	0.000	66.544
1987	26.703	33.529	308.801	0.000	0.000	73.270
1988	28.338	34.527	366.227	0.000	0.000	55.783
1989	25.837	36.065	573.890	0.171	0.171	70.335
1990	48.832	56.416	666.829	0.453	0.453	81.044
1991	44.981	52.043	498.079	1.155	1.155	84.455
1992	38.551	51.680	486.552	2.509	2.509	108.424
1993	40.385	47.003	705.761	2.144	2.144	129.644
1994	76.688	91.902	723.216	4.361	4.361	195.607
1995	89.936	111.590	780.690	4.370	4.370	164.216
1996	85.232	93.843	888.730	6.480	6.480	112.592
1997	117.365	124.253	1216.093	6.315	6.315	229.251
1998	114.090	127.410	1344.137	4.095	4.095	158.315
1999	89.320	96.767	1073.429	5.561	5.561	373.445
2000	137.436	144.747	895.972	7.126	7.126	550.868
2001	129.380	143.240	1929.481	10.380	10.380	610.602
2002	198.608	211.185	2402.631	24.568	24.568	1592.368
2003	218.321	228.773	2140.505	57.372	57.372	1991.073
2004	287.500	293.872	875.087	79.532	79.532	1724.521
2005	154.285	159.964	301.384	142.373	142.373	1117.139
2006	71.563	80.816	336.954	136.873	136.873	502.729
2007	73.327	77.266	203.895	84.126	84.126	261.815
2008	34.802	38.730	231.170	46.036	46.036	172.753
2009	69.140	72.213	338.013	33.474	33.474	208.624
2010	58.928	76.908	815.362	27.444	27.444	136.035
2011	79.398	90.802	806.739	29.966	29.966	171.206
2012	69.973	77.650	587.995	27.147	27.147	185.136
2013	54.576	57.925	590.118	35.734	35.734	88.827
2014	55.416	61.505	394.059	34.666	34.666	282.056
2015	51.989	65.245	626.910	21.128	21.128	197.455

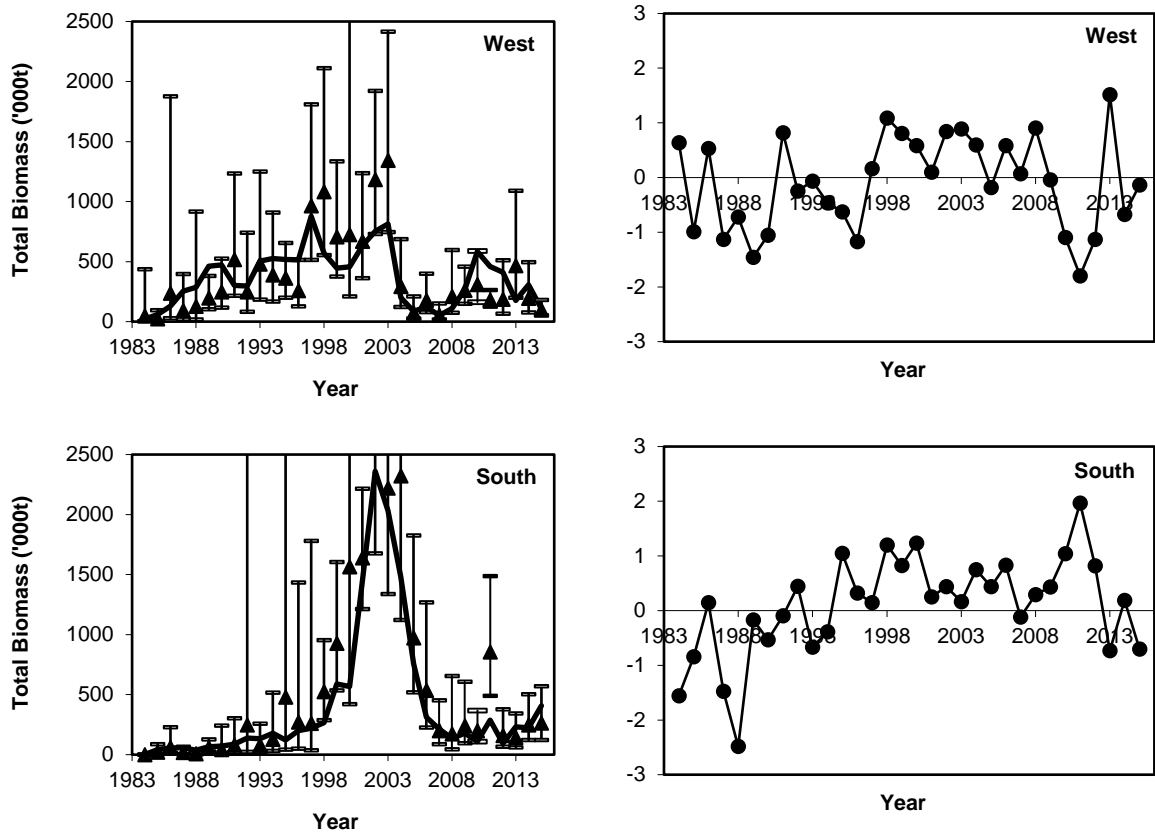


Figure 1. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2015. The observed indices are shown with 95% confidence intervals. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate) from the fits are given in the right hand plots.

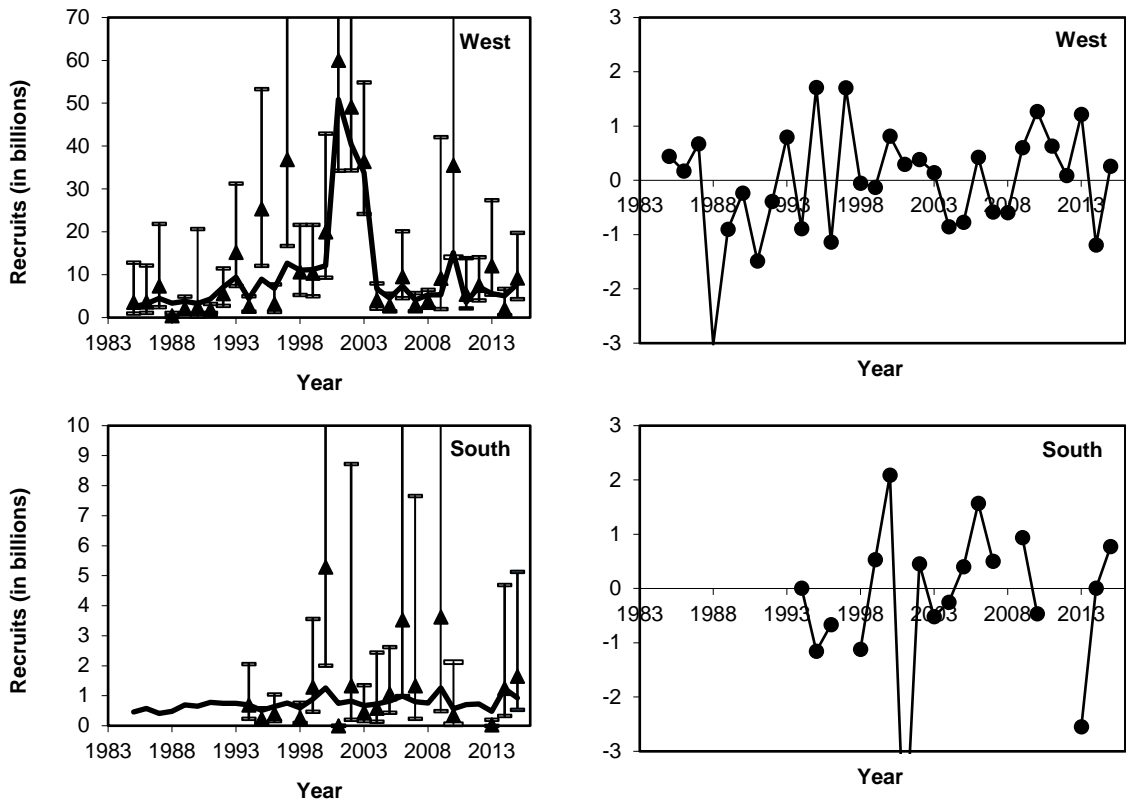


Figure 2. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2015. The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plots.

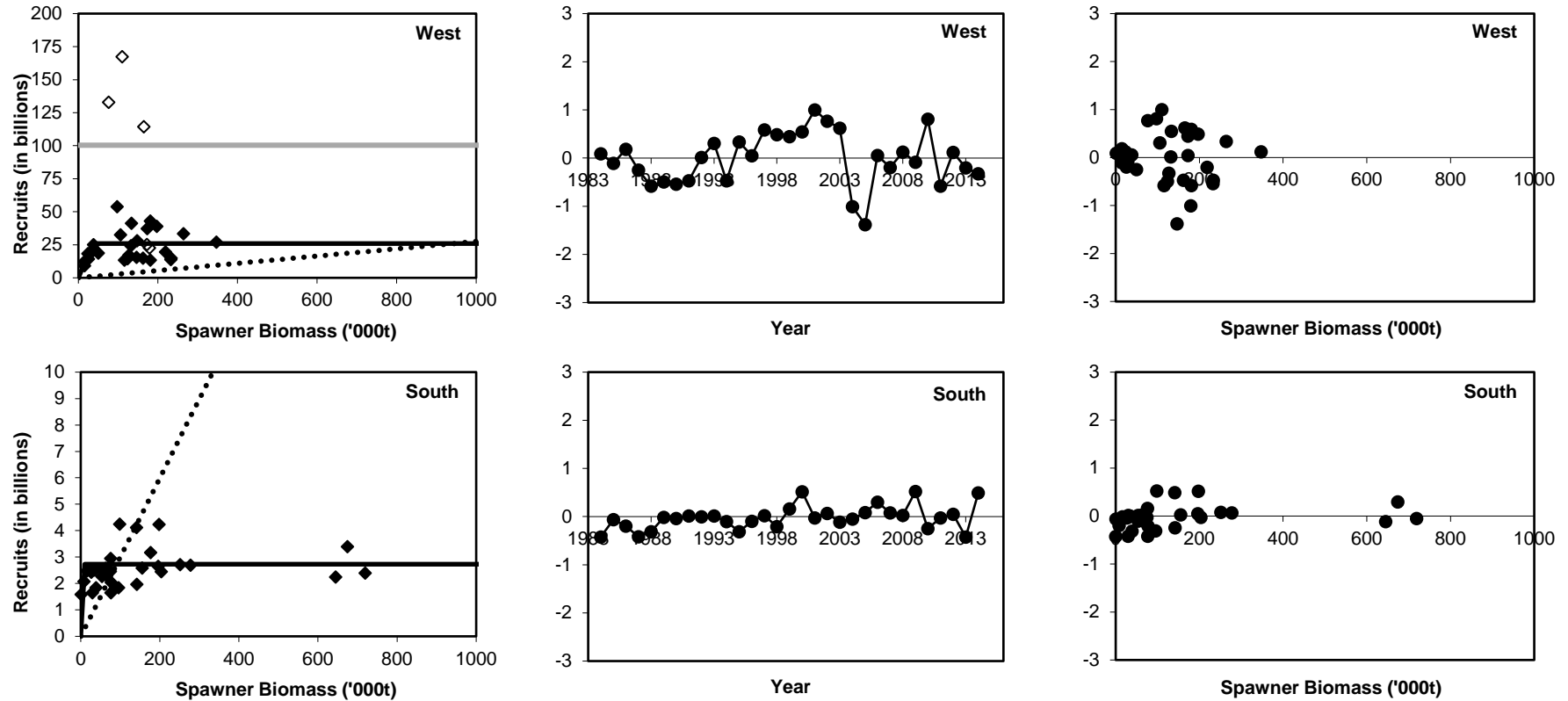


Figure 3. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2014 with the estimated Hockey stick stock recruitment relationships are shown in the left side plots. The grey line shows the median 2000-2004 west stock recruitment and the open diamonds correspond to these same ‘peak’ years. The dotted line indicates the replacement line. The standardised residuals for the fits are given in the centre and right side plots, against year and against spawner biomass respectively.

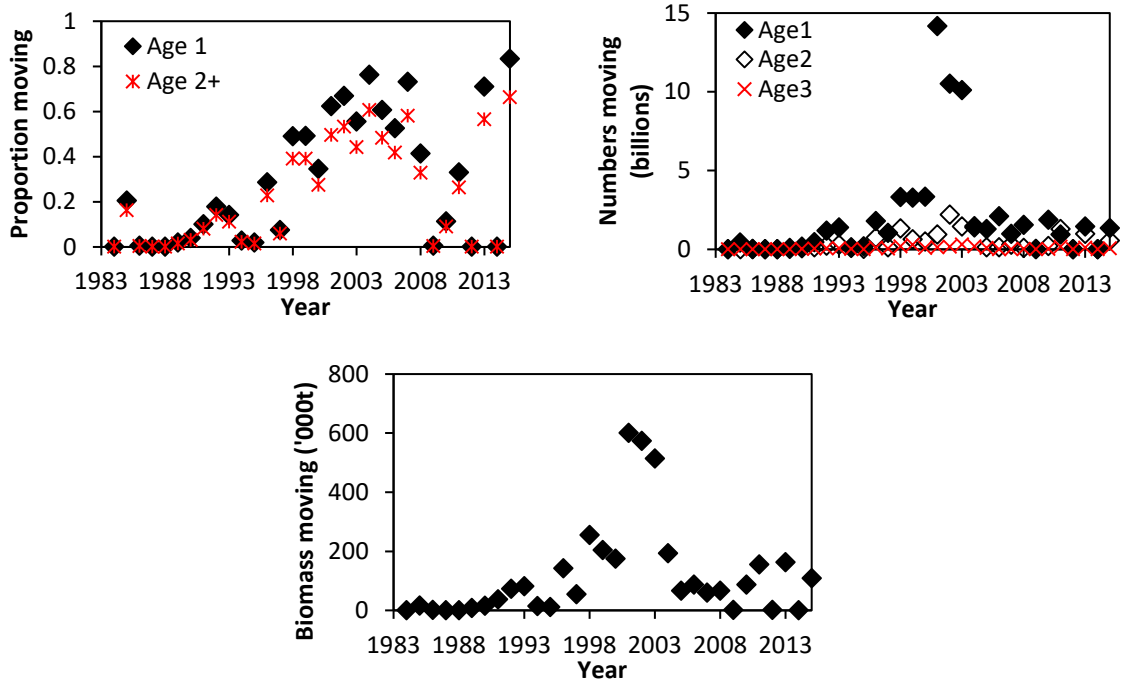


Figure 4. Model estimated proportion of 1-year-olds and 2+-year-olds which move from the “west” stock to the “south” stock in November. The right hand plot shows the numbers of 1-, 2- and 3-year olds moving while the lower plot shows rough² estimates of the annual biomass moving from the west to south stock.

² Calculated using the average of “west” and “south” stock weights-at-age.

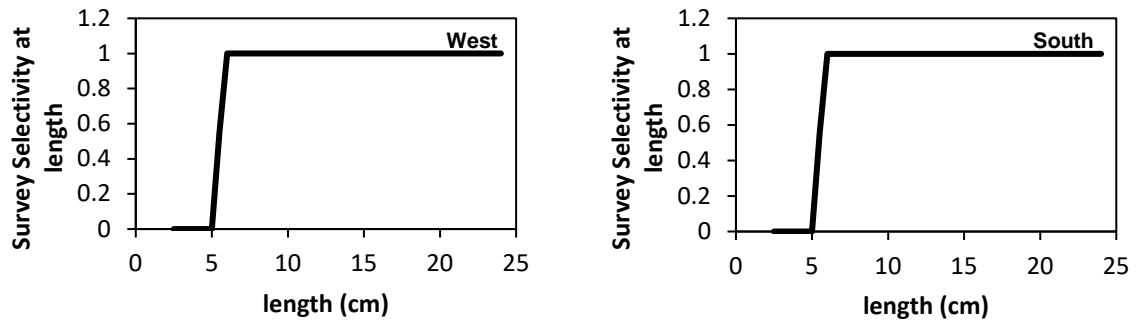


Figure 5. The model estimated November survey selectivity at length.

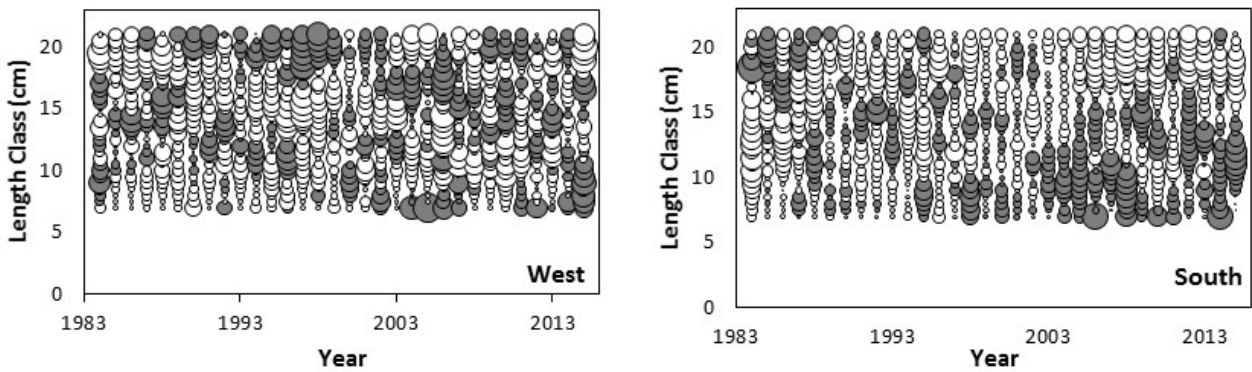


Figure 6. Residuals from the fit of the model predicted proportions-at-length in the November survey to the hydroacoustic survey estimated proportions. The left panels show the residuals for the minus length class (9cm) and the right panels show the residuals for the remaining length classes.

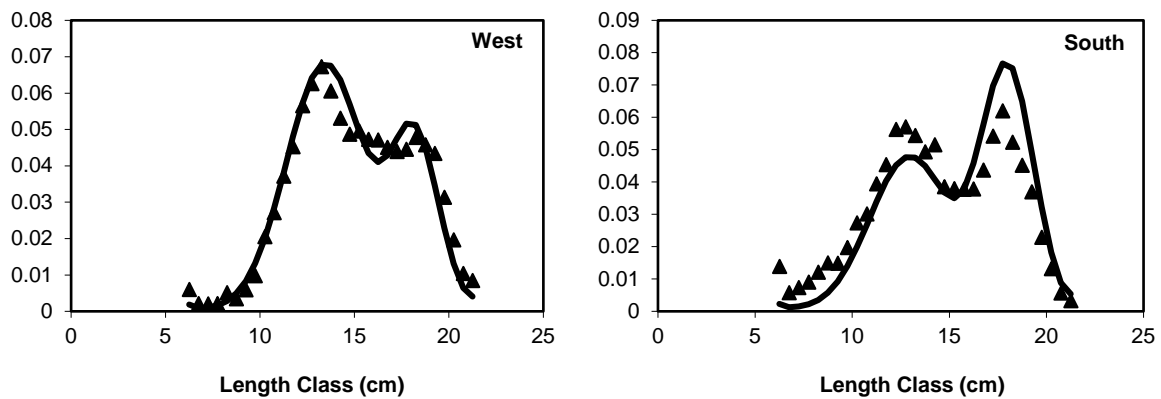


Figure 7. Average (over all years) model predicted and observed proportion-at-length in the November survey.

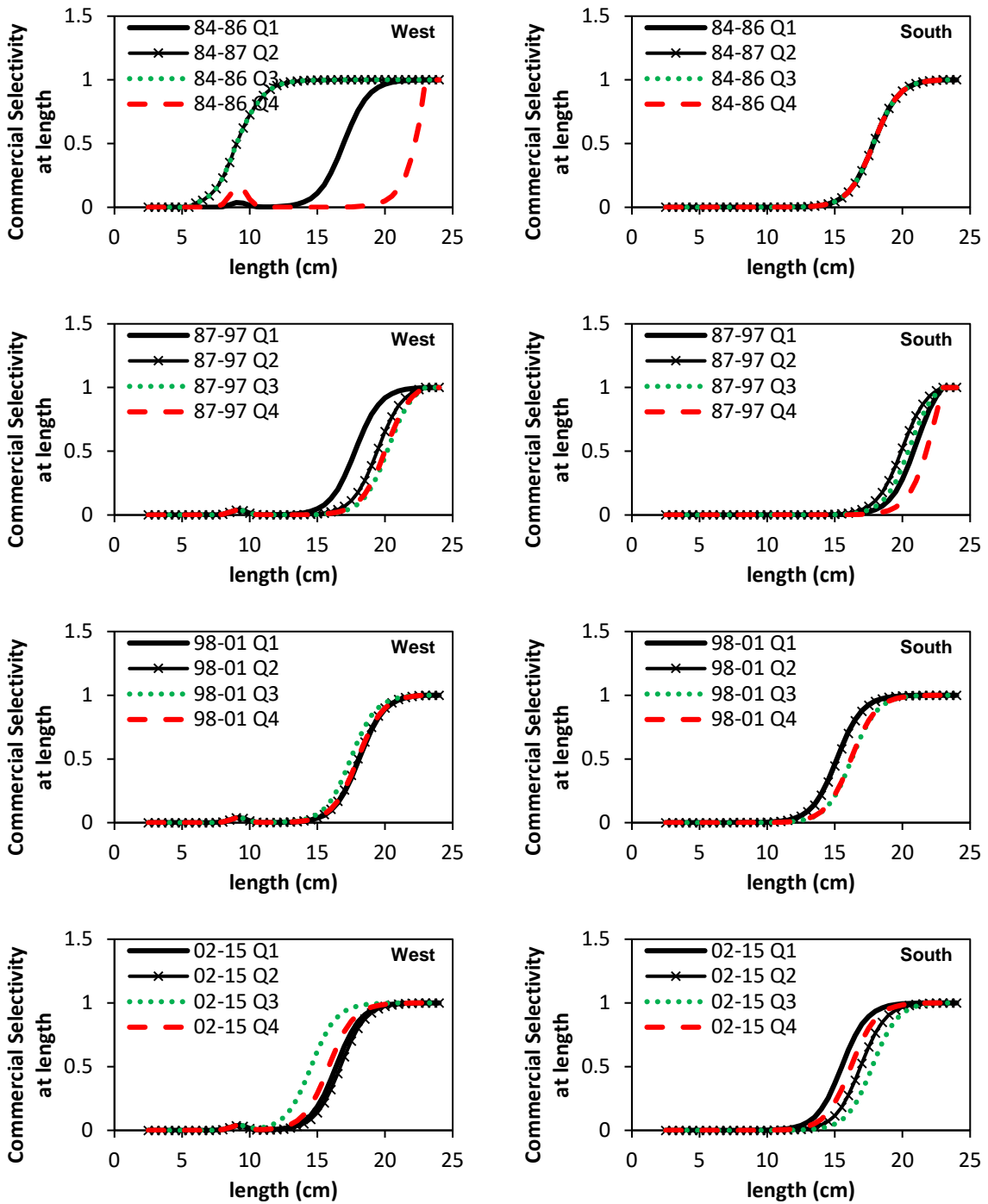


Figure 8. The model estimated commercial selectivity at length, which differs between four pre-specified time periods (the four rows) and quarters.

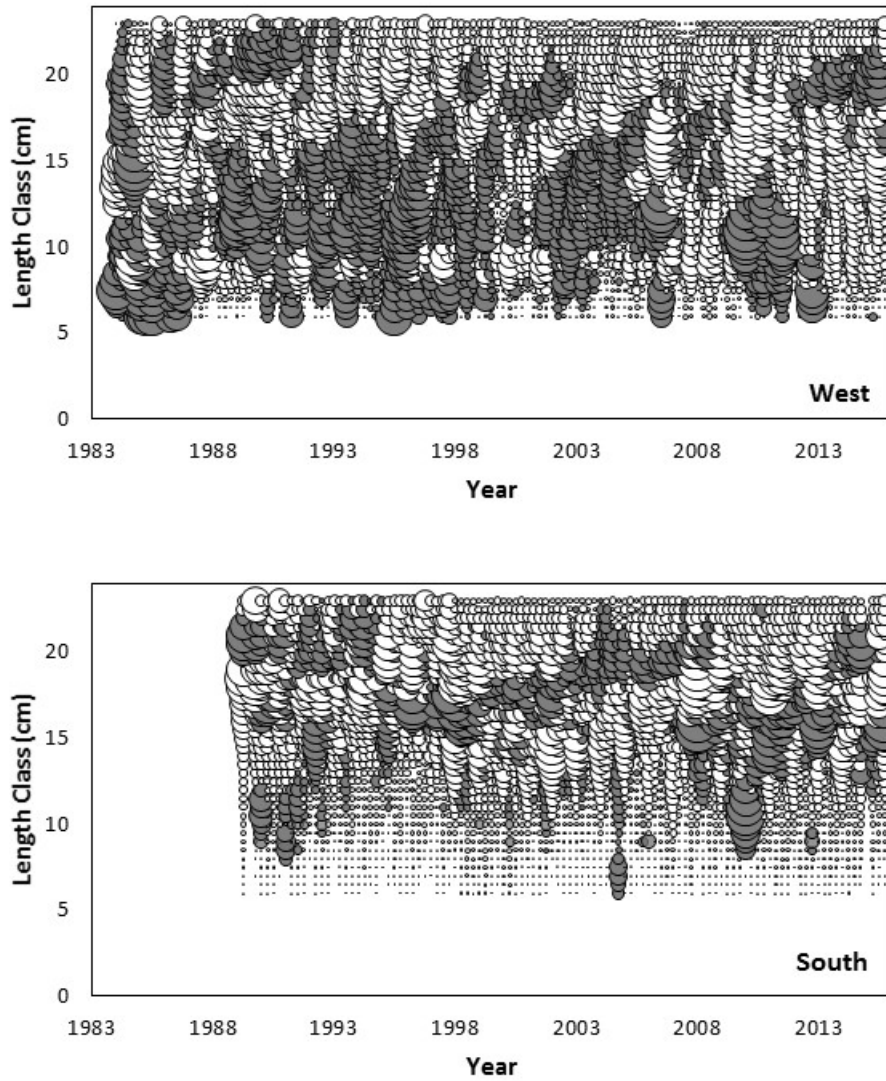


Figure 9. Residuals from the fit of the model predicted proportions-at-length in the quarterly commercial catch to the observed proportions.

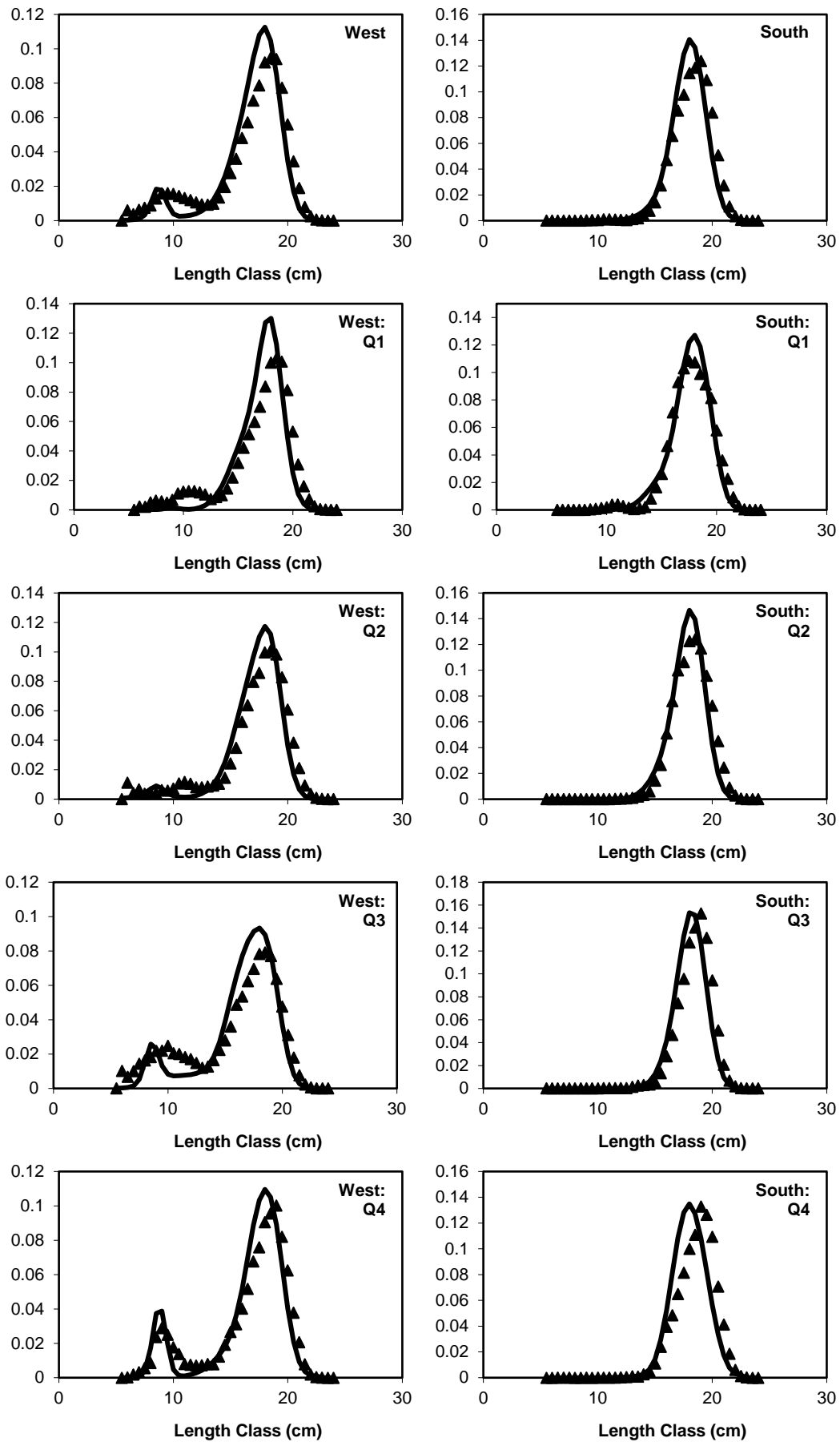


Figure 10. Average (over all quarters and years) model predicted and observed proportion-at-length in the commercial catch (top row), and average (over all years) quarterly model predicted and observed proportions-at-length in the commercial catch (subsequent rows).

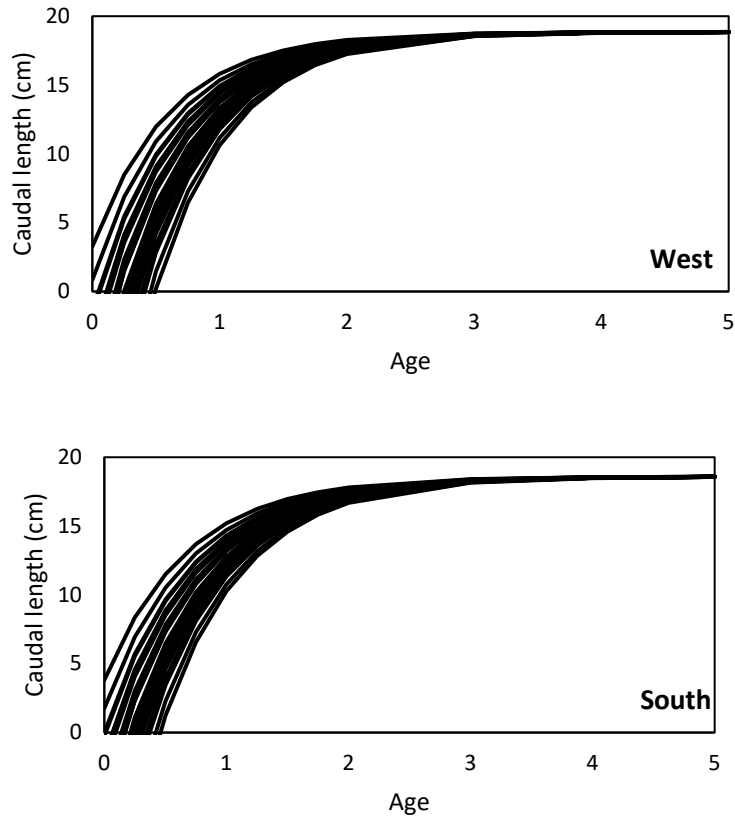


Figure 11. The annual von Bertalanffy growth curves estimated by allowing for auto-correlated residuals for the variation about the age at which length is zero.

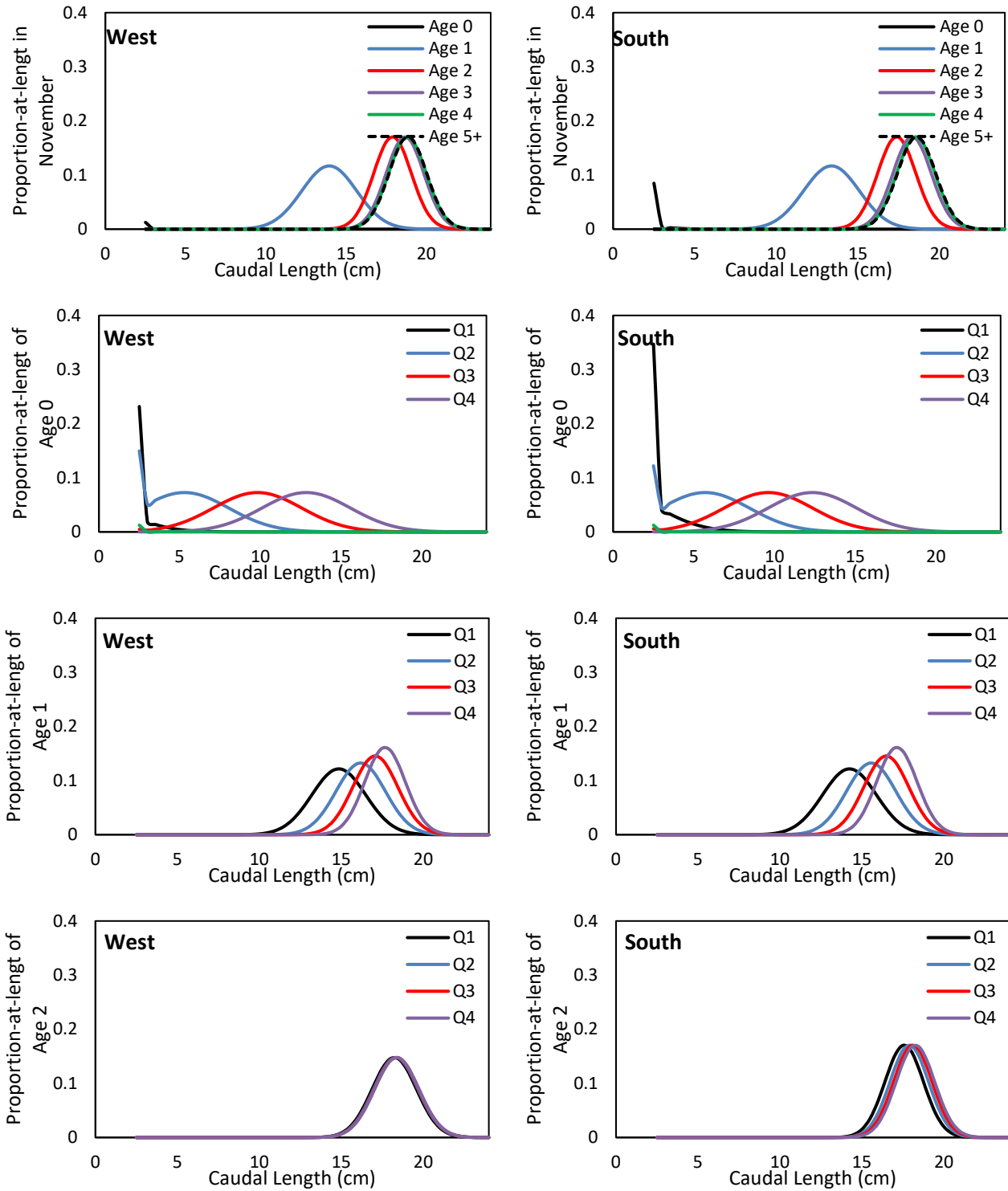


Figure 12. The model estimated distributions of proportions-at-length for each age, given at the time of the biomass survey (1 November, top row), and middle of each quarter of the year (corresponding to the times commercial catch is modelled to be taken) for age 0, 1 and 2 (subsequent rows).

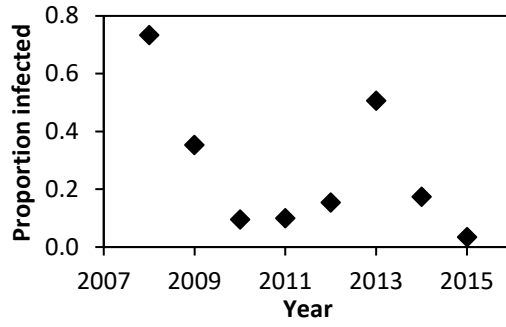


Figure 13. The model estimated proportion of west stock sardine infected with the parasite between 2008 and 2015. (Annual infection rate is arbitrarily assumed to be 0.2 prior to 2008.)

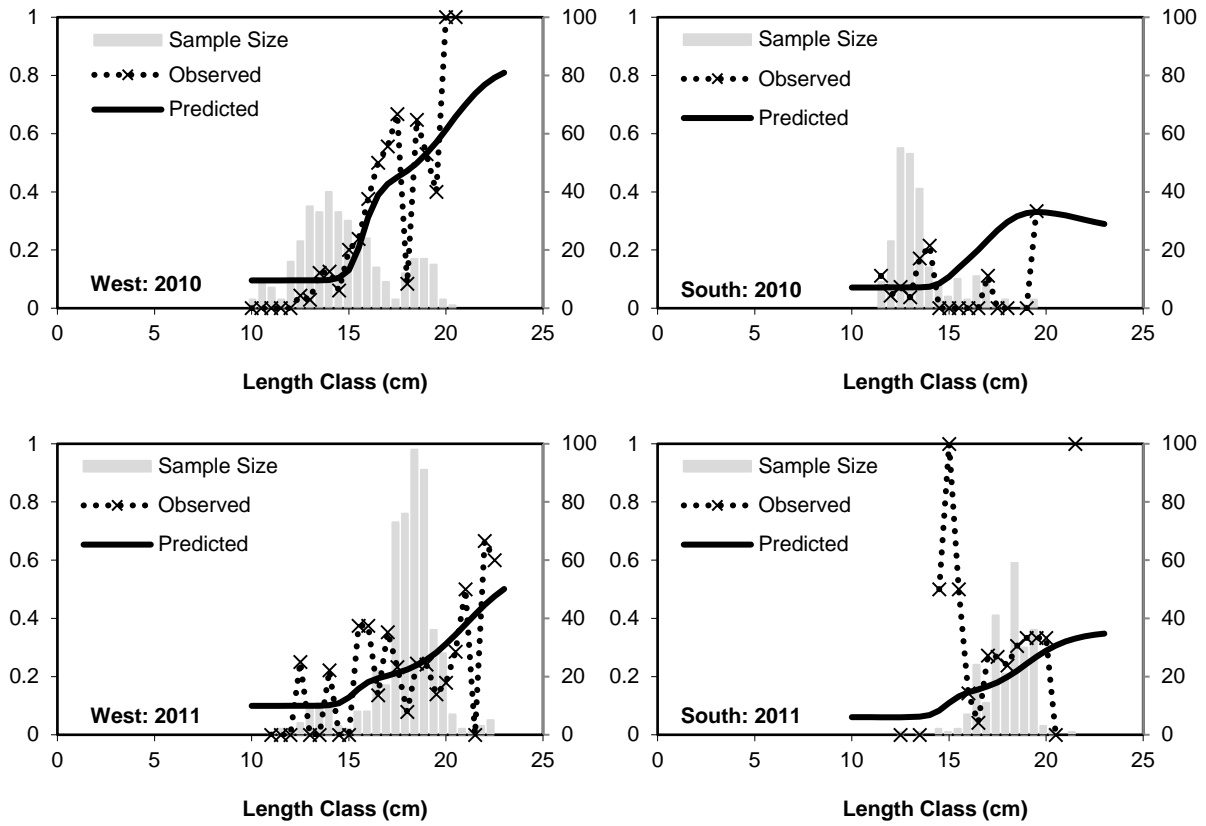


Figure 14. The model estimated proportions-at-length of west and south stock sardine infected with the parasite (i.e. parasite prevalence-by-length) between 2010 and 2015 together with the observed proportions-at-length. The sample size for each length class is given by the grey bars, plotted against the right vertical axis.

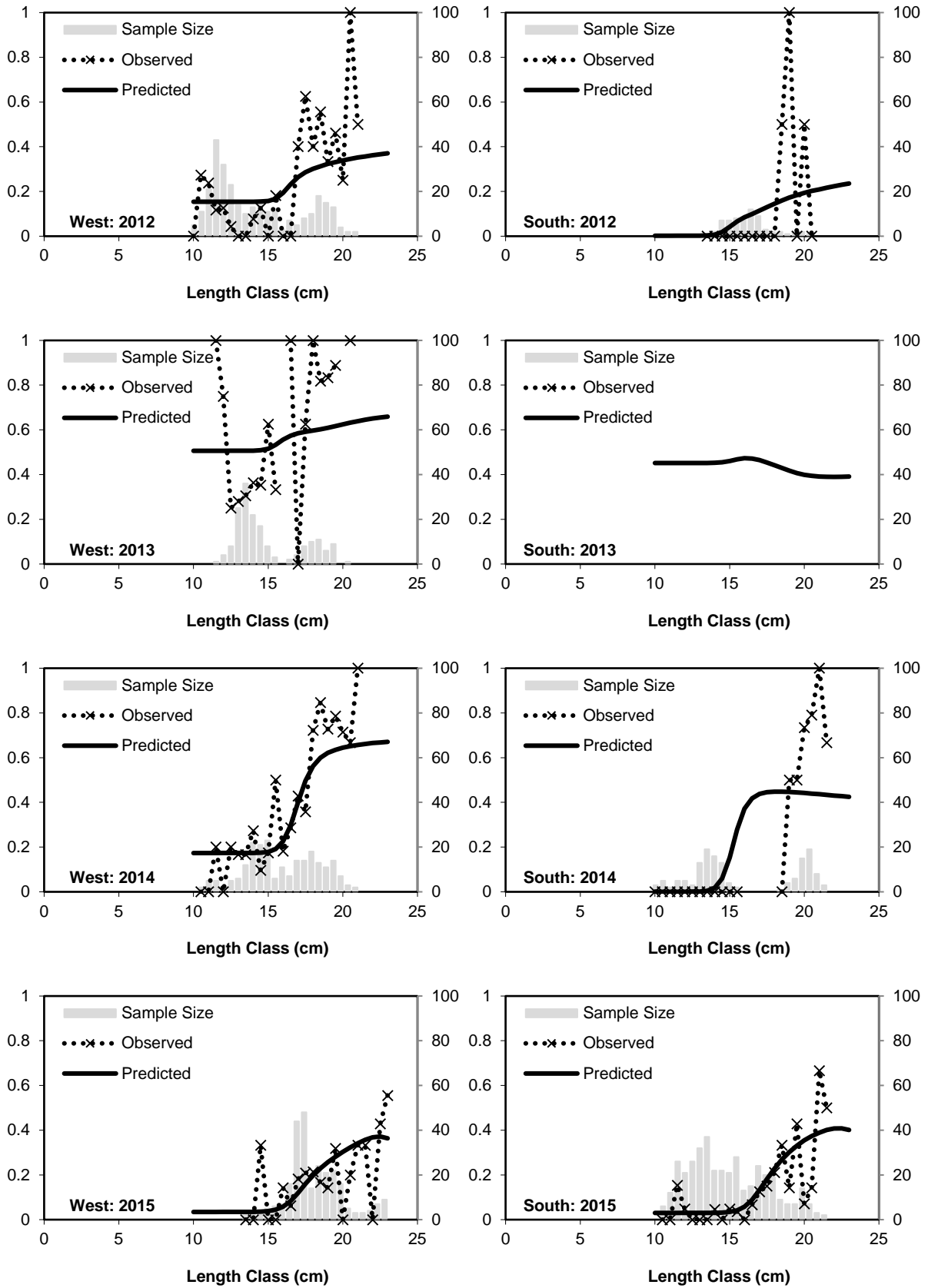


Figure 14 (continued).

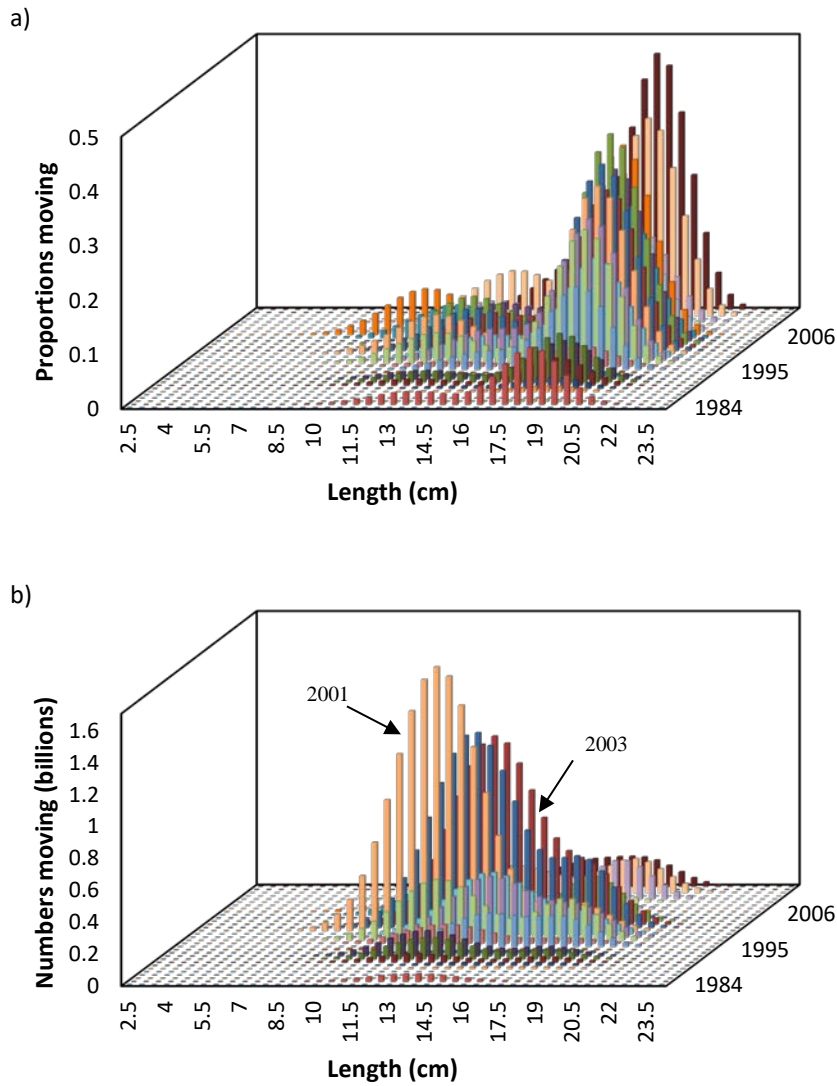


Figure 15. Model estimated annual a) proportion-by-length and b) numbers-by-length of sardine which move from the “west” stock to the “south” stock in November.

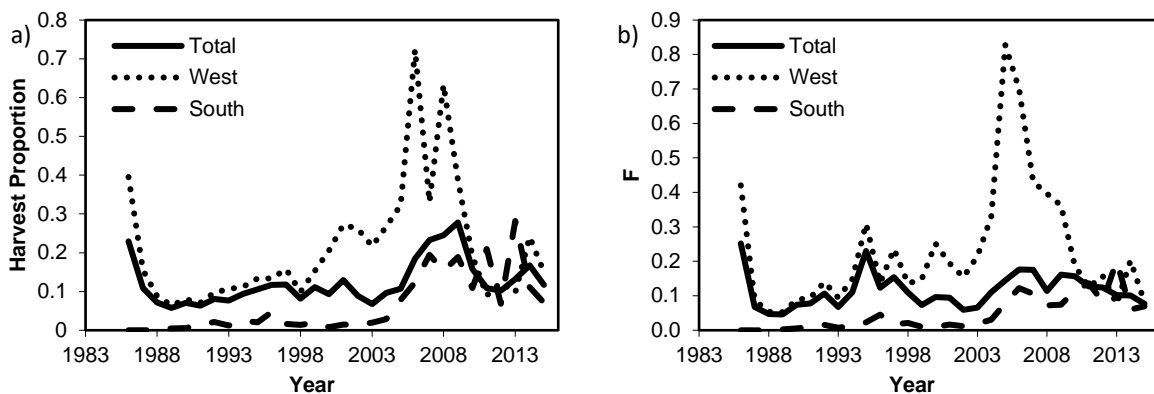


Figure 16. The harvest proportion (simply calculated as the observed annual (Nov-Oct) catch tonnage as a proportion of the model predicted total biomass), and the estimated fishing mortality on a fully selected length class of the “west” and “south” stocks and the total population.

Appendix A: Bayesian assessment model for the South African sardine resource

The assessment is run from November $y_1 = 1984$ to November $y_n = 2014$, with the following subscript notation:

- quarters $q=1$ denoting November $y-1$ to January y , $q=2$ denoting February to April y , $q=3$ denoting May to July y and $q=4$ denoting August to October y ;
- ages $a=0$ to a plus group of $a=5^+$;
- lengths from a minus group of $l=2.5^- \text{ cm}$ to a plus group of $l=24^+ \text{ cm}$;
- stocks $j=W$ or $j=S$ denote the west and south stocks, respectively, where only the west stock equations are used in the single stock hypothesis;
- infection $p=NI$ or $p=I$ denote the sardine uninfected and infected with the digenean ‘tetracotyle-type’ metacercarian endoparasite, respectively.

All parameters are defined in Tables S1 and S2.

Population Dynamics

Numbers-at-age at 1 November before movement or infection

$$N_{j,p,y,a}^{S*} = \left(\left(\left(N_{j,p,y-1,a-1}^S e^{-M_{y,a-1}^S/8} - C_{j,p,y,1,a-1}^S \right) e^{-M_{y,a-1}^S/4} - C_{j,p,y,2,a-1}^S \right) e^{-M_{y,a-1}^S/4} - C_{j,p,y,3,a-1}^S \right) e^{-M_{y,a-1}^S/4} - C_{j,p,y,4,a-1}^S \right) e^{-M_{y,a-1}^S/8}$$

$$p = I, NI, y_1 \leq y \leq y_n, 1 \leq a \leq 4$$

$$N_{j,p,y,a=5^+}^{S*} = \left(\left(\left(N_{j,p,y-1,4}^S e^{-M_{y,4}^S/8} - C_{j,p,y,1,4}^S \right) e^{-M_{y,4}^S/4} - C_{j,p,y,2,4}^S \right) e^{-M_{y,4}^S/4} - C_{j,p,y,3,4}^S \right) e^{-M_{y,4}^S/4} - C_{j,p,y,4,4}^S \right) e^{-M_{y,4}^S/8}$$

$$+ \left(\left(\left(N_{j,p,y-1,5^+}^S e^{-M_{y,5^+}^S/8} - C_{j,p,y,1,5^+}^S \right) e^{-M_{y,5^+}^S/4} - C_{j,p,y,2,5^+}^S \right) e^{-M_{y,5^+}^S/4} - C_{j,p,y,3,5^+}^S \right) e^{-M_{y,5^+}^S/4} - C_{j,p,y,4,5^+}^S \right) e^{-M_{y,5^+}^S/8}$$

$$p = I, NI, y_1 \leq y \leq y_n \quad (A1)$$

Infection of west stock sardine in the two mixing-stock hypothesis; in the single stock hypothesis $I_y = 0$ as the parasite data have no influence so that they are not included in the likelihood

$$N_{W,NI,y,a}^{S**} = (1 - I_y) N_{W,NI,y,a}^{S*} \quad y_1 \leq y \leq y_n, 1 \leq a \leq 5^+$$

$$N_{W,I,y,a}^{S**} = N_{W,I,y,a}^{S*} + I_y N_{W,NI,y,a}^{S*} \quad y_1 \leq y \leq y_n, 1 \leq a \leq 5^+$$

$$N_{S,p,y,a}^{S**} = N_{S,p,y,a}^{S*} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq a \leq 5^+ \quad (A2)$$

Movement of west stock ($j=W$) sardine to the south stock ($j=S$) in the two mixing-stock hypothesis; in the single stock hypothesis $move_{y,a} = 0$

$$N_{W,p,y,a}^S = (1 - move_{y,a}) N_{W,p,y,a}^{S*} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq a \leq 5^+$$

$$N_{S,p,y,a}^S = N_{S,p,y,a}^{S*} + move_{y,a} N_{W,p,y,a}^{S*} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq a \leq 5^+ \quad (A3)$$

Numbers-at-age mid-way through each quarter (for use in catch equations)

$$N_{j,p,y,1,a}^S = N_{j,p,y-1,a}^S e^{-M_{y,a}^S/8} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq a \leq 5^+$$

$$N_{j,p,y,q,a}^S = (N_{j,p,y,q-1,a}^S - C_{j,p,y,q-1,a}^S) e^{-M_{y,a}^S/4} \quad p = I, NI, y_1 \leq y \leq y_n, 2 \leq q \leq 4, 1 \leq a \leq 5^+ \quad (A4)$$

Numbers-at-length at 1 November (after infection and movement)

The model estimated numbers-at-length range from a 2.5cm minus group to a 24cm plus group, denoted 2.5⁻ and 24⁺, respectively, in the remaining text.

$$N_{j,p,y,l}^S = \sum_{a=0}^{5^+} A_{j,y,a,l}^{sur} N_{j,p,y,a}^S \quad p = I, NI, y_1 \leq y \leq y_n, 2.5^- \text{ cm} \leq l \leq 24^+ \text{ cm} \quad (A5)$$

The model predicted numbers-at-length of ages 1+ only are given by:

$$N_{j,p,y,l}^{S,1+} = \sum_{a=1}^{5^+} A_{j,y,a,l}^{sur} N_{j,p,y,a}^S \quad p = I, NI, y_1 \leq y \leq y_n, 2.5^- \text{ cm} \leq l \leq 24^+ \text{ cm} \quad (A6)$$

The proportion of sardine of age a in stock j that fall in length group l at 1 November, $A_{j,y,a,l}^{sur}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{j,y,a,l}^{sur} \sim N\left(L_{j,\infty}\left(1 - e^{-K_j(a-t_{0,j,y})}\right), g_{j,a}^2\right) \quad y_1 \leq y \leq y_n, 0 \leq a \leq 5^+, 2.5^- \text{ cm} \leq l \leq 24^+ \text{ cm} \quad (A7)$$

with

$$t_{0,j,y} = \begin{cases} t_{0,j} + \varepsilon_y^t & y = y_1 \\ t_{0,j} + \rho^t \varepsilon_{y-1}^t + \sqrt{1 - (\rho^t)^2} \varepsilon_y^t & y_1 < y \leq y_n \end{cases} \quad (A8)$$

Natural mortality

Natural mortality is modelled to vary annually in an autocorrelated manner around a median as follows (although the baseline assumes no such correlation – see Table A.1):

$$M_{y,a=0}^S = \bar{M}_j^S e^{\varepsilon_y^j} \quad \text{with } \varepsilon_{1984}^j = \eta_{1984}^j \quad \text{and } \varepsilon_y^j = \rho \varepsilon_{y-1}^j + \sqrt{1 - \rho^2} \eta_y^j, \quad y > y_1 \quad (A9)$$

$$M_{y,a=1+}^S = \bar{M}_{ad}^S e^{\varepsilon_y^{ad}} \quad \text{with } \varepsilon_{1984}^{ad} = \eta_{1984}^{ad} \quad \text{and } \varepsilon_y^{ad} = \rho \varepsilon_{y-1}^{ad} + \sqrt{1 - \rho^2} \eta_y^{ad}, \quad y > y_1 \quad (A10)$$

³ Given the allowance for early/late recruitment in varying $t_{0,y}$ estimates annually, there may be some proportion of this distribution below a length of zero (due to late recruitment). In these cases, this proportion is removed from the proportion-at-length of the minus length class.

⁴ Additive error allows for early or late recruitment. While the timing of recruitment may vary between stocks due to differing environmental conditions on the west and south coasts, the same autocorrelation parameters are assumed here for simplicity reasons.

Spawning biomass and biomass associated with the November survey

$$SSB_{j,y}^S = \sum_p \sum_{l=2.5^-}^{24^+} f_{j,y,l}^S N_{j,p,y,l}^{S,1+} w_{j,y,l}^S \quad y_1 \leq y \leq y_n \quad (\text{A11})$$

$$SSB_{j=W,y}^{eff,S} = \chi_W SSB_{W,y}^S + (1 - \chi_S) SSB_{S,y}^S \quad y_1 \leq y \leq y_n$$

$$SSB_{j=S,y}^{eff,S} = (1 - \chi_W) SSB_{W,y}^S + \chi_S SSB_{S,y}^S \quad y_1 \leq y \leq y_n \quad (\text{A12})$$

$$B_{j,y}^S = k_{j,N}^S \sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S w_{j,y,l}^S \quad y_1 \leq y \leq y_n \quad (\text{A13})$$

$$\text{where } w_{j,y,l}^S = w_{j,l}^S \times \frac{\tilde{w}_{j,y}}{\left(\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S w_{j,l}^S \right) / \left(\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S \right)} \quad y_1 \leq y \leq y_n, 2.5^- \text{ cm} \leq l \leq 24^+ \text{ cm} \quad (\text{A14})$$

Commercial selectivity

$$S_{j,y,q,l} = \begin{cases} 0 & l \leq 5.5 \text{ cm} \\ \chi_j \exp \left\{ -\frac{(l + 0.25 - \bar{l}_{1,j})^2}{(\sigma_1^{sel})^2} \right\} + \frac{1}{1 + \exp \left\{ -\frac{l - \bar{l}_{2,j,y,q}}{\sigma_{2,y}^{sel}} \right\}} & 6 \text{ cm} \leq l \leq l_{\max} = 23 \text{ cm} \\ S_{j,y,l} \max & l > l_{\max} \end{cases} \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4 \quad (\text{A15})$$

$$S_{j,y,q,a} = \sum_{l=2.5^-}^{23.5^+} A_{j,y,q,a,l}^{com} S_{j,y,q,l} \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4, 0 \leq a \leq 5^+ \quad (\text{A16})$$

$$\text{where } A_{j,y,q,a,l}^{com} \sim N \left(L_{j,\infty} \left(1 - e^{-\kappa_j (a + (2q-1)/8 - l_{0,y})} \right), \mathcal{G}_{j,a}^2 \right) \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4, 0 \leq a \leq 5^+, 2.5^- \text{ cm} \leq l \leq 24^+ \text{ cm} \quad (\text{A17})$$

Bycatch in the anchovy directed fishery

$$C_{j,p,y,q,a}^{bycatch} = \begin{cases} N_{j,p,y,q,a}^S F_{j,y,q,a}^{By} & 0 \leq a \leq 1 \\ 0 & 2 \leq a \leq 5^+ \end{cases} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq q \leq 4 \quad (\text{A18})$$

Catch in the directed sardine and round herring bycatch fisheries

$$C_{j,p,y,q,a}^{dir} = \left(N_{j,p,y,q,a}^S - C_{j,p,y,q,a}^{bycatch} \right) \mathcal{F}_{j,y,q,a} F_{j,y,q} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq q \leq 4, 0 \leq a \leq 5^+ \quad (\text{A19})$$

Total catch

$$C_{j,p,y,q,a}^S = C_{j,p,y,q,a}^{bycatch} + C_{j,p,y,q,a}^{dir} \quad p = I, NI, y_1 \leq y \leq y_n, 1 \leq q \leq 4, 0 \leq a \leq 5^+ \quad (\text{A20})$$

⁵ The biomass in $y_n = 2015$ excludes age 0 fish, although the contribution of age 0 fish to the total biomass should be minor.

⁶ "Selectivity" is incorporated in $F_{j,y,q,a}^{By}$, as the sardine bycaught is typically independent of sardine abundance, but rather correlated with anchovy recruitment which varies from year to year.

Fished proportion of the available biomass from the bycatch in the anchovy directed fishery

$$\begin{aligned}
 F_{j,y,q=1,a=0}^{By} &= \frac{\sum_{m=11}^{12} \sum_{l < \text{cut}_{y,m}} C_{j,y-1,m,l}^{RLF, fleet=3} + \sum_{l < \text{cut}_{y,m}} C_{j,y,1,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=1,a=0}^S} & F_{j,y,q=1,a=1}^{By} &= \frac{\sum_{m=11}^{12} \sum_{l \geq \text{cut}_{y,m}} C_{j,y-1,m,l}^{RLF, fleet=3} + \sum_{l \geq \text{cut}_{y,m}} C_{j,y,1,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=1,a=1}^S} \\
 F_{j,y,q=2,a=0}^{By} &= \frac{\sum_{m=2}^4 \sum_{l < \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=2,a=0}^S} & F_{j,y,q=2,a=1}^{By} &= \frac{\sum_{m=2}^4 \sum_{l \geq \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=2,a=1}^S} \\
 F_{j,y,q=3,a=0}^{By} &= \frac{\sum_{m=5}^7 \sum_{l < \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=3,a=0}^S} & F_{j,y,q=3,a=1}^{By} &= \frac{\sum_{m=5}^7 \sum_{l \geq \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,q=3,a=1}^S} \\
 F_{j,y,q=4,a=0}^{By} &= \frac{\sum_{m=8}^{10} \sum_{l < \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,4,0}^S} & F_{j,y,q=4,a=1}^{By} &= \frac{\sum_{m=8}^{10} \sum_{l \geq \text{cut}_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}}{\sum_p N_{j,p,y,4,1}^S} \tag{A21}
 \end{aligned}$$

A penalty is imposed within the model to ensure that $F_{j,y,q,a}^{By} < 0.95$.

Fished proportion of the available biomass from the directed catch and round herring bycatch fisheries

$$\begin{aligned}
 F_{j,y,q=1} &= \frac{\sum_{fleet=1}^2 \sum_{m=11}^{12} \sum_{l \geq 6cm} C_{j,y-1,m,l}^{RFL, fleet} + \sum_{fleet=1}^2 \sum_{l \geq 6cm} C_{j,y,1,l}^{RFL, fleet}}{\sum_p \sum_{a=0}^{5+} (N_{j,p,y,1,a}^S - C_{j,p,y,1,a}^{bycatch}) \mathcal{S}_{j,y,1,a}} \\
 F_{j,y,q=2} &= \frac{\sum_{fleet=1}^2 \sum_{m=2}^4 \sum_{l \geq 6cm} C_{j,y,m,l}^{RFL, fleet}}{\sum_p \sum_{a=0}^{5+} (N_{j,p,y,2,a}^S - C_{j,y,2,a}^{bycatch}) \mathcal{S}_{j,y,2,a}} \\
 F_{j,y,q=3} &= \frac{\sum_{fleet=1}^2 \sum_{m=5}^7 \sum_{l \geq 6cm} C_{j,y,m,l}^{RFL, fleet}}{\sum_p \sum_{a=0}^{5+} (N_{j,p,y,3,a}^S - C_{j,y,3,a}^{bycatch}) \mathcal{S}_{j,y,3,a}} \\
 F_{j,y,q=4} &= \frac{\sum_{fleet=1}^2 \sum_{m=8}^{10} \sum_{l \geq 6cm} C_{j,y,m,l}^{RFL, fleet}}{\sum_p \sum_{a=0}^{5+} (N_{j,p,y,4,a}^S - C_{j,y,4,a}^{bycatch}) \mathcal{S}_{j,y,4,a}} \tag{A22}
 \end{aligned}$$

A penalty is imposed within the model to ensure that $S_{j,y,q,l} F_{j,y,q} < 0.95$. Fish $< 6\text{cm}$ were seldom⁷ caught and were thus not used in fitting this model. Commercial selectivity-at-length is fixed to zero for length classes $< 6\text{cm}$ (equation S12).

⁷ Less than 6% of the quarters west of Cape Agulhas, less than 2% of the quarters south-east of Cape Agulhas and less than 4% of the quarters for the whole coast.

Recruitment

For the west/single stock only, if $2000 \leq y \leq 2004$:

$$N_{j,p,y,a=0}^S = \begin{cases} c_j^S e^{\varepsilon_{j,y}^S - 0.5(\sigma_{j,r,peak}^S)^2} & \text{if } p = I \\ 0 & \text{if } p = NI \end{cases}$$

else:

$$N_{j,p,y,a=0}^S = \begin{cases} a_j^S e^{\varepsilon_{j,y}^S - 0.5(\sigma_{j,r}^S)^2} & \text{if } p = I \text{ and } SSB_{j,y}^S \geq b_j^S \\ \frac{a_j^S}{b_j^S} SSB_{j,y}^S e^{\varepsilon_{j,y}^S - 0.5(\sigma_{j,r}^S)^2} & \text{if } p = I \text{ and } SSB_{j,y}^S < b_j^S \\ 0 & \text{if } p = NI \end{cases} \quad y_1 \leq y \leq y_n \quad (\text{A23})$$

Carrying Capacity

$$K_j^S = a_j^S \sum_{a=1}^4 \bar{w}_{j,a}^S e^{-M_j^S - (a-1)\bar{M}_{ad}^S} + \bar{w}_{j,5+} e^{-M_j^S - 4\bar{M}_{ad}^S} \frac{1}{1 - e^{-\bar{M}_{ad}^S}}$$

$$K_{peak}^S = c_j^S \sum_{a=1}^4 \bar{w}_{j,a}^S e^{-M_j^S - (a-1)\bar{M}_{ad}^S} + \bar{w}_{j,5+} e^{-M_j^S - 4\bar{M}_{ad}^S} \frac{1}{1 - e^{-\bar{M}_{ad}^S}} \quad (\text{A24})$$

Number of recruits associated with the recruit survey

$$N_{j,y,r}^S = k_{j,r}^S \left(N_{j,NI,y,2,0}^S - C_{j,NI,y,2,0}^S \right) e^{-\left(1/8 + 0.5t_y^S/12\right)M_{y,0}^S} - \tilde{C}_{j,y,0bs}^S e^{-0.5t_y^S \times M_{y,0}^S/12} \quad y_1 \leq y \leq y_n \quad (\text{A25})$$

Multiplicative survey bias

$$k_{j,N}^S = k_{ac}^S \quad (\text{A26})$$

$$k_{j=W,r}^S = k_{cov}^S \times k_{ac}^S \quad (\text{A27})$$

$$k_{j=S,r}^S = k_{covS}^S \times k_{cov}^S \times k_{ac}^S \quad (\text{for the two mixing-stock hypothesis only}) \quad (\text{A28})$$

Survey trawl selectivity

$$S_{j,l}^{survey} = \begin{cases} 0 & l = 2.5^- \text{ cm} \\ \left[1 + \exp\left\{-\left(l + 0.25 - S_{50}\right)/\delta\right\}\right]^{-1} & 3\text{cm} \leq l \leq 24^+ \text{ cm} \end{cases} \quad y_1 \leq y \leq y_n \quad (\text{A29})$$

Proportion-at-length associated with the November survey

$$P_{j,y,l}^S = \begin{cases} \frac{\sum_p \sum_{l \leq 6cm} N_{j,p,y,l}^S S_{j,l}^{survey}}{\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S S_{j,l}^{survey}} & l = 6^- cm \\ \frac{\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S S_{j,l}^{survey}}{\sum_p N_{j,p,y,l}^S S_{j,l}^{survey}} & 6.5cm \leq l \leq 20.5cm \\ \frac{\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S S_{j,l}^{survey}}{\sum_p \sum_{l=21}^{23.5} N_{j,p,y,l}^S S_{j,l}^{survey}} & l = 21 - 23.5cm \\ \frac{\sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S S_{j,l}^{survey}}{\sum_p N_{j,p,y,24^+}^S S_{j,24^+}^{survey}} & l = 24^+ cm \\ \sum_p \sum_{l=2.5^-}^{24^+} N_{j,p,y,l}^S S_{j,l}^{survey} & \end{cases} \quad 8 \quad y_1 \leq y \leq y_n \quad (A30)$$

Proportion-at-length of fish infected with the parasite in November

$$P_{j,y,l}^S = \frac{N_{j,l,y,l}^S}{\sum_p N_{j,p,y,l}^S} \quad y_1 \leq y \leq y_n, 10cm \leq l \leq 23cm \quad (A31)$$

Catch-at-length from the directed and round herring bycatch fisheries

$$C_{j,p,y,q,l}^{dir} = \sum_{a=0}^{5^+} \left(N_{j,p,y,q,a}^S - C_{j,p,y,q,a}^{bycatch} \right) A_{j,q,a,l}^{com} S_{j,y,q,l} F_{j,y,q} \quad 9$$

$$p = I, NI, y_1 \leq y \leq y_n, 1 \leq q \leq 4, 2.5^- cm \leq l \leq 24^+ cm \quad (A32)$$

Proportion-at-length associated with the directed catch and round herring bycatch

$$P_{j,y,q,l}^{coml,S} = \begin{cases} \frac{\sum_p C_{j,p,y,q,l}^{dir}}{p} & 6cm \leq l \leq 22.5cm \\ \frac{\sum_p \sum_{l=6}^{24^+} C_{j,p,y,q,l}^{dir}}{\sum_p \sum_{l=23}^{24^+} C_{j,p,y,q,l}^{dir}} & l = 23^+ cm \\ \sum_p \sum_{l=6}^{24^+} C_{j,p,y,q,l}^{dir} & \end{cases} \quad 10 \quad y_1 \leq y \leq y_n, 1 \leq q \leq 4 \quad (A33)$$

Fitting the Model to Observed Data (Likelihood)

$$-\ln L = -\ln L^{Nov} - \ln L^{rec} - \ln L^{sur\ prop} - \ln L^{com\ prop} - \ln L^{prev} \quad (A34)$$

where

⁸ The inclusion of model predicted proportion-at-length 24⁺cm is deliberate to take into account the zero samples of 24⁺cm sardine in the survey.

⁹ Note the model predicted commercial catch of lengths <6cm is zero, from a zero commercial selectivity in equation A.13. This is consistent with the range of length classes in the observed commercial proportions-at-lengths.

¹⁰ Note the model predicted commercial catch of lengths <6cm is zero, from a zero commercial selectivity in equation A.13. This is consistent with the range of length classes in the observed commercial proportions-at-lengths.

$$-\ln L^{Nov} = \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \left\{ \frac{\left[5^5 \frac{\left| \ln(\hat{B}_{j,y}^S) - \ln(B_{j,y}^S) \right|}{\sqrt{(\sigma_{j,y,Nov}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,N}^S)^2}} \right]^5}{5^5 + \frac{\left| \ln(\hat{B}_{j,y}^S) - \ln(B_{j,y}^S) \right|}{\sqrt{(\sigma_{j,y,Nov}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,N}^S)^2}}} \right]^{2/5} + \ln \left[2\pi \left((\sigma_{j,y,Nov}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,N}^S)^2 \right) \right] \right\} \quad (A35)$$

$$-\ln L^{rec} = \frac{1}{2} \sum_j \sum_{y=y1+1}^{yn} \left\{ \frac{\left[5^5 \frac{\left| \ln(\hat{N}_{j,y,r}^S) - \ln(N_{j,y,r}^S) \right|}{\sqrt{(\sigma_{j,y,rec}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,r}^S)^2}} \right]^5}{5^5 + \frac{\left| \ln(\hat{N}_{j,y,r}^S) - \ln(N_{j,y,r}^S) \right|}{\sqrt{(\sigma_{j,y,rec}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,r}^S)^2}}} \right]^{2/5} + \ln \left[2\pi \left((\sigma_{j,y,rec}^S)^2 + (\phi_{ac}^S)^2 + (\lambda_{j,r}^S)^2 \right) \right] \right\} \quad (A36)$$

$$-\ln L^{sur\ prop1} = w_{prop1}^{sur} \sum_j \sum_{y=y1}^{yn} \left\{ \sum_{l=6}^{21^+} \left\{ \frac{\left(\sqrt{\hat{P}_{j,y,l}^S} - \sqrt{P_{j,y,l}^S} \right)^2}{2(\sigma_{j,ur}^S)^2} + \ln(\sigma_{j,sur}^S) \right\} + \frac{\left(0 - \sqrt{P_{j,y,24^+}^S} \right)^2}{2(\sigma_{j,ur}^S)^2} + \ln(\sigma_{j,sur}^S) \right\} \quad 11 \quad (A37)$$

$$-\ln L^{com\ prop1} = w_{prop1}^{com} \sum_j \sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=6}^{23^+} \left\{ \frac{\left(\sqrt{\hat{P}_{j,y,q,l}^S} - \sqrt{P_{j,y,q,l}^S} \right)^2}{2(\sigma_{j,com}^S)^2} + \ln(\sigma_{j,com}^S) \right\} \quad (A38)$$

$$-\ln L^{prev} = \sum_j \sum_{y=2010}^{2014} \sum_{l=5cm}^{23cm} \left\{ -n_{j,y,l}^{prev} \ln(P_{j,y,l}^S) - (N_{j,y,l}^{prev} - n_{j,y,l}^{prev}) \ln(1 - P_{j,y,l}^S) \right\} \quad (A39)$$

A “robustified likelihood” is used for the contributions from the hydro-acoustic surveys to ensure no undue influence from any extreme (outlying) values for residuals. The functional form chosen to robustify makes negligible difference for standardised residuals of magnitude three or less, but essentially treats large standardised residuals as if they do not exceed five in magnitude.

¹¹ The 21⁺ group in this equation consists of the length classes 21cm, 21.5cm, 22cm, 22.5cm, 23cm and 23.5cm.

Table A1. Assessment model parameters and variables with associated fixed values or prior distributions and, for derived variables, associated equation numbers. As the majority of prior distributions are uninformative, notes are provided only for informative priors and/or bounds.

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
Annual numbers and biomass	$N_{j,p,y,a}^S$	Model predicted numbers-at-age a at the beginning of November in year y of stock j that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A1 - A3, A23	
	$N_{j,p,y,q,a}^S$	Model predicted numbers-at-age a mid-way through quarter q of year y of stock j that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A4	
	$M_{y,a}^S$	Rate of natural mortality of age a in year y	Year ⁻¹		$M_{y,0}^S = 1.0$ $M_{y,1+}^S = 0.1$	de Moor and Butterworth (2015)
	I_y	Proportion of uninfected west stock sardine that are infected with the endoparasite in year y (two mixing-stock hypothesis only)		$I_y \sim U(0,1)$ $2008 \leq y \leq y_n$ $=0, y_1 \leq y \leq 2007$		
	$move_{y,a}$	Proportion of west stock sardine of age a which move to the south stock at the beginning of November of year y (two mixing-stock hypothesis only)	-	$move_{y,1} \sim Beta(1.05,1.05)$ $, move_{y,2+} = \phi \times move_{y,1},$ $\phi \sim U(0,1)$		
	$SSB_{j,y}^S$	Model predicted spawning biomass of stock j at the beginning of November in year y	Thousand tons		A11	
	$SSB_{j,y}^{eff,S}$	Model predicted effective spawning biomass of stock j at the beginning of November in year y				
	$B_{j,y}^S$	Model predicted total biomass of stock j at the beginning of November in year y , associated with the November survey	Thousand tons		A13	

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Annual numbers and biomass	$f_{j,y,l}^S$	Proportion of stock j sardine that are mature in length class l in year y	-	$[1 + \exp\{-(l - 17.2)/1.17\}]^{-1}$ $1984 \leq y \leq 1987$ $[1 + \exp\{-(l - 18.6)/1.26\}]^{-1}$ $1988 \leq y \leq 1995$ $[1 + \exp\{-(l - 19.4)/1.40\}]^{-1}$ $1996 \leq y \leq 2003$ $[1 + \exp\{-(l - 17.4)/0.95\}]^{-1}$ $2004 \leq y \leq 2014$	Refit from data used by van der Lingen et al. (2006) using midpoints of length classes. Assuming maturity post-2003 reflects that of 1965-1975 as maturity is hypothesized to be density dependent (van der Lingen et al. 2006) and both these periods correspond to low biomass following a peak in abundance
	χ_j	Proportion of j -stock spawner biomass that contributes to the effective spawning biomass on the same coast		1.0	Alternative values will be tested in robustness tests
	$w_{j,l}^S$	Mean mass of sardine of stock j in length class l	Grams	$1.1639 \times 10^{-5} \times l^{3.03155}$	van der Lingen et al. (2006)
	$w_{j,y,l}^S$	Mean mass of sardine of stock j in length class l at the beginning of November in year y	Grams		A14
	$\tilde{w}_{j,y}$	Mean mass of sardine sampled from stock j during the November survey of year y	Grams		$\frac{\sum_p \sum_{l=3}^{23.5} N_{j,p,y,l}^S w_{j,l}^S}{\sum_p \sum_{l=3}^{23.5} N_{j,p,y,l}^S}$
	$\overline{w}_{j,a}^S$	Mean mass of age a from stock j sampled during each November survey, averaged over all years	Grams		$\sum_{l=2.5^-}^{24^+} A_{j,a,l}^{sur} w_{j,l}^S$

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$M_{y,a}^S$	Rate of natural mortality of age a in year y	Year ⁻¹		A9 and A10	Selected based on maximized joint posterior, and subject to a compelling reason to modify from previous assessment	
Natural Mortality	\bar{M}_j^S	Median juvenile rate of natural mortality	Year ⁻¹	1.0		
	\bar{M}_{ad}^S	Median rate of natural mortality for 1+ sardine	Year ⁻¹	0.8		
	ε_y^j	Annual residuals about juvenile natural mortality rate	-		A9	
	ε_y^{ad}	Annual residuals about natural mortality rate for 1+ sardine	-		A10	
	η_y^j	Normally distributed error in calculating ε_y^j	-	$N(0, \sigma_j^2)$		
	η_y^{ad}	Normally distributed error in calculating ε_y^{ad}	-	$N(0, \sigma_{ad}^2)$		
	σ_j	Standard deviation in the annual residuals about juvenile natural mortality	-	0		See robustness tests
	σ_{ad}	Standard deviation in the annual residuals about natural mortality for ages 1+	-	0		See robustness tests
ρ	Annual autocorrelation coefficient	-	0		See robustness tests	

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$N_{j,p,y,l}^S$	Model predicted numbers-at-length l at the beginning of November in year y of stock j that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A5		
$p_{j,y,l}^S$	Model predicted proportion-at-length l of stock j associated with the November survey in year y	-		A30		
$A_{j,y,a,l}^{sur}$	Proportion of age a of stock j sardine that falls in the length group l in November of year y	-		A7		
Proportions-at-length and growth curve	K_j	Somatic growth rate parameter for stock j	Year ⁻¹	$U(0,3)$		
	$L_{j,\infty}$	Maximum length (in expectation) of stock j	Cm	$L_{j,a=1} \sim U(5,25)$ $L_{j,a=3} \sim U(5,25)$	$L_{j,\infty}$ and $t_{0,j}$ derived from estimated length at ages 1 and 3	
	$t_{0,j,y}$	Age at which the length (in expectation) is zero in year y	Year		A8	
	$t_{0,j}$	Average age at which the length (in expectation) is zero		$\frac{1}{\kappa_j} \ln \left\{ \frac{e^{\kappa_j} (L_{j,a=1} - L_{j,a=3})}{L_{j,a=1} e^{-2\kappa_j} - L_{j,a=3}} \right\}$		
	ε_y^t	Annual residuals about the age at which the length is zero		$N(0,2)$		
	ρ^t	Autocorrelation coefficient in these residuals		$U(-1,1)$		
	$\mathcal{G}_{j,a}$	Standard deviation of the distribution about the mean length for age a of stock j	-	$U(0.01, 3), a = 0,1,2+$		Upper bound chosen to preclude unrealistically large lengths for very young fish
$p_{j,y,q,l}^{coml,S}$	Model predicted proportion-at-length l of stock j in the directed catch and round herring bycatch during quarter q of year y	-		A33		
$A_{j,y,q,a,l}^{com}$	Proportion of age a of stock j sardine that falls in the length group l mid-way through quarter q of year y	-		A17		

$P_{j,y,l}^S$

Model predicted proportion-at-length l of stock j that are infected with the endoparasite, at the time of the November survey in year y

A31

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$S_{j,l}^{survey}$	Survey selectivity-at-length l in the November survey for stock j	-		A29	Some smaller fish escape through the trawl net	
S_{50}	Length at which survey selectivity is 50%	Cm	$U(2.5,20)$			
δ	Inverse of slope of survey selectivity-at-length ogive when selectivity is 50%	-	$U(0.05,5)$			
$S_{j,y,q,l}$	Commercial selectivity-at-length l during quarter q of year y of stock j	-		A15		
$S_{j,y,q,a}$	Commercial selectivity-at-age a during quarter q of year y of stock j	-		A16		
Selectivity	χ_j	Height of the Gaussian component for stock j relative to the height of the logistic component	-	$U(0,1)$		
	$\bar{l}_{1,j}$	Mean of the Gaussian distribution for stock j	Cm	$U(5,15)$		
	$\bar{l}_{2,j,y,q}$	Length at 50% selectivity in the logistic component for stock j in quarter q of year y	Cm	$U(0,25)$		Estimated for four time periods 84-86, 87-97, 98-01, 02-15
	$(\sigma_1^{sel})^2$	Variance parameter of the Gaussian distribution	Cm	$U(2,7)$		
	$(\sigma_{2,y}^{sel})^2$	Variance parameter of the logistic distribution	Cm	$U(0,10)$		
Multiplicative bias	$k_{j,N}^S$	Multiplicative bias associated with the November survey of stock j	-		A26	
	$k_{j,r}^S$	Multiplicative bias associated with the recruit survey of stock j	-		A27 – A28	
	k_{ac}^S	Multiplicative bias associated with the hydro-acoustic survey	-	$\ln(k_{ac}^S) \sim N(-0.310, 0.094^2)$		Appendix B
	k_{cov}^S	Multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey	-	$\sim U(0.3,1)$		Lower bound selected in discussions with scientists on these surveys and their field experience
	k_{cov}^S	Multiplicative bias associated with the coverage of the south stock recruits by the recruit survey in comparison to the west stock recruits during the same survey	-	$\sim U(0,1)$		

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$C_{j,p,y,q,a}^S$	Model predicted number of age a fish of stock j caught during quarter q of year y that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A20		
$lcut_{y,m}$	Cut off length for recruits in month m of year y	Cm	de Moor et al. 2016		Differ by month and year as informed by the recruit surveys	
$C_{j,p,y,q,a}^{bycatch}$	Number of age a fish of stock j bycaught in the anchovy-directed fishery in quarter q of year y that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A18		
Catch	$C_{j,p,y,q,a}^{dir}$	Number of age a fish of stock j caught in the sardine-directed and round herring bycatch fisheries in quarter q of year y that are uninfected ($p = NI$) or infected ($p = I$) with the endoparasite	Billions		A19	
	$C_{j,p,y,q,l}^{dir}$	Number of length l fish of stock j caught in the sardine-directed and round herring bycatch fisheries in quarter q of year y	Billions		A32	
	$F_{j,y,q,a}^{By}$	Fished proportion in quarter q of year y for age class a of stock j , of bycatch in the anchovy-directed fishery	-		A21	
	$F_{j,y,q}$	Fished proportion in quarter q of year y for a fully selected age class a of stock j , by the directed and round herring bycatch fisheries	-		A22	
Initial Values	$N_{j,p,1983,a}^S$	Initial numbers-at-age a in stock j	Billions	$N_{j,NI,1983,a=1}^S \sim U(0,50)$	Alternatives considered for robustness tests	
				$N_{j,NI,1983,a}^S = 0, a \geq 2$		
				$N_{j,I,1983,a}^S = 0, 0 \leq a \leq 5^+$		

Table A1 (Continued).

Parameter/ Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
a_j^S	Maximum recruitment of stock j in the hockey stick model	Billions	$\ln(a_j^S) \sim U(0,5.5)$		Uninformative on log-scale as scale is not known <i>a priori</i> , with the maximum corresponding to about 10 million tons for K_j^S
c_j^S	Median recruitment of stock j in the hockey stick model during peak years	Billions	$\ln(c_j^S) \sim U(0,5.5)$		
b_j^S	Spawner biomass below which the expectation for recruitment is reduced below the maximum for stock j	Thousand tons	$b_{j=w}^S / K_{j=w}^S \sim \text{Beta}(1.05,1.05)$ $b_{j=S}^S / K_{j=S}^S = 0.001$		Insufficient information in the data to estimate the inflection point for the south stock
Recruitment	K_j^S	Carrying capacity for stock j	Thousand tons		A24
	K_{peak}^S	Carrying capacity during “peak” years (single stock hypothesis only)	Thousand tons		A24
$\varepsilon_{j,y}^S$	Lognormal deviation of recruitment of stock j in year y	-	$\varepsilon_{j,y}^S \sim N(0, (\sigma_{j,r}^S)^2)$ $\varepsilon_{j,y}^S \sim N(0, (\sigma_{peak,r}^S)^2)$ ¹²		Reflects the assumption of a different distribution applying over the peak period
$(\sigma_{j,r}^S)^2$	Variance in the residuals (lognormal deviation) about the stock recruitment curve of stock j	-	$\sim U(0.16,10)$		Lower bound chosen to restrict the influence of the stock recruitment curve on the assessment results
$(\sigma_{r,peak}^S)^2$	Variance in the residuals (lognormal deviation) about the west/total stock recruitment curve during “peak” years	-	$\sim U(0.16,10)$		
$N_{j,y,r}^S$	Model predicted number of juveniles of stock j at the time of the recruit survey in year y	Billions			A25

¹² During “peak” 2000-2004 years for the single stock hypothesis and for the west stock in the two mixing-stock hypothesis.

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes	
$-\ln L^{Nov}$	Contribution to the negative log likelihood from the model fit to the November 1+ survey biomass data	-		A35		
$-\ln L^{rec}$	Contribution to the negative log likelihood from the model fit to the recruit survey data	-		A36		
$-\ln L^{sur\ prop}$	Contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data	-		A37		
$-\ln L^{com\ prop}$	Contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data	-		A38		
$-\ln L^{sur\ prev}$	Contribution to the negative log likelihood from the model fit to the November parasite prevalence-at-length data	-		A39		
Likelihood	ϕ_{ac}^S	CV associated with factors which cause bias in the acoustic survey estimates and which vary inter-annually rather than remain fixed over time	-	$= 0.227$		Appendix B
	$(\lambda_{j,N/r}^S)^2$	Additional variance (over and above $(\sigma_{j,y,Nov/rec}^S)^2$ and $(\phi_{ac}^S)^2$) associated with the November/recruit surveys of stock j	-	$\sim U(0,10)$		
	W_{prop}^{sur}	Weighting applied to the remaining survey proportion-at-length data	-	$= 0.5 \times 0.167$		To allow for autocorrelation ¹³
	$\sigma_{j,sur}^S$	Standard deviation associated with the survey proportion-at-length data of stock j	-	$\sqrt{\sum_{y=y1}^{yn} \sum_{l=6^-}^{21^+} (\sqrt{\hat{p}_{j,y,l}^S} - \sqrt{p_{j,y,l}^S})^2} / \sum_{y=y1}^{yn} \sum_{l=6^-}^{21} 1$ 14		Closed form solution
	W_{prop}^{com}	Weighting applied to the commercial proportion-at-length data	-	$= 0.5 \times 0.04$		To allow for autocorrelation ¹⁵

¹³ Based upon data being available ~6 times more frequently than annual age data which contain maximum information content on this. Despite this downweighting of the size-structure information, the model struggled to reproduce the peak in the abundance data at the turn of the century. To provide a better fit to indices of abundance, a further 50% downweighting was applied to the size-structured data.

¹⁴ The 21⁺ group in this equation consists of the length classes 21cm, 21.5cm, 22cm, 22.5cm, 23cm and 23.5cm.

¹⁵ Based upon data being available ~4x6 times more frequently than annual age data which contain maximum information content on this. Despite this downweighting of the size-structure information, the model struggled to reproduce the peak in the abundance data at the turn of the century. To provide a better fit to indices of abundance, a further 50% downweighting was applied to the size-structured data.

Table A1 (Continued).

Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Likelihood $\sigma_{j,com}^S$	Standard deviation associated with the commercial proportion-at-length data of stock j	-		$\sqrt{\frac{\sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=6}^{23^+} \left(\sqrt{\hat{P}_{1,y,q,l}^{comlS}} - \sqrt{P_{1,y,q,l}^{comlS}} \right)^2}{\sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=6}^{23^+} 1}}$ $\sqrt{\frac{\sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=13}^{23^+} \left(\sqrt{\hat{P}_{2,y,q,l}^{comlS}} - \sqrt{P_{2,y,q,l}^{comlS}} \right)^2}{\sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=13}^{23^+} 1}}$	Closed form solution ¹⁶

¹⁶ A shorter range of lengths is used given the near absence of data outside this range, resulting in small/zero residuals, which would negatively bias this estimate.

Table A2. Assessment model data, detailed in de Moor et al. (2016).

Quantity	Description	Units / Scale	Shown in Figure
t_y^S	Time lapsed between 1 May and the start of the recruit survey in year y	Months	
$\tilde{C}_{j,y,obs}^S$	Number of juveniles of stock j caught between 1 May and the day before the start of the recruit survey in year y	Billions	
$C_{j,y,m,l}^{RFL,fleet}$	Number of fish in length class l landed by $fleet$ in month m of year y of stock j . $fleet = 1$ denotes the sardine directed fishery, $fleet = 2$ denotes the sardine bycatch with round herring (1984-2011) or ≥ 14 cm sardine bycatch (2012-14) and $fleet = 3$ denotes the juvenile sardine bycatch with anchovy (1984-2011) or < 14 cm sardine bycatch (2012-15)	Billions	
$\hat{B}_{j,y}^S$	Acoustic survey estimate of biomass of stock j from the November survey in year y	Thousand tons	Fig. 1
$\sigma_{j,y,Nov}^S$	Survey sampling CV associated with $\hat{B}_{j,y}^S$ that reflects survey inter-transect variance	-	Fig. 1
$\hat{N}_{j,y,r}^S$	Acoustic survey estimate of recruitment of stock j from the recruit survey in year y	Billions	Fig. 2
$\sigma_{j,y,rec}^S$	Survey sampling CV associated with $\hat{N}_{j,y,r}^S$ that reflects survey inter-transect variance	-	Fig. 2
$\hat{P}_{j,y,l}^S$	Observed proportion (by number) of stock j in length group l in the November survey of year y	-	Fig. 7
$\hat{P}_{j,y,q,l}^{S,coml}$	Observed proportion (by number) of the directed catch and round herring bycatch of fish of stock j and length group l during quarter q of year y	-	Fig. 10
$n_{j,y,l}^{prev}$	Number of sardine of stock j in length class l sampled from the November survey in year y that were tested and found to be infected with the endoparasite	Numbers	Fig. 14
$N_{j,y,l}^{prev}$	Number of sardine of stock j in length class l sampled from the November survey in year y that were tested for infection with the endoparasite	Numbers	Fig. 14

Appendix B. Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

The probability density functions (pdfs) for the bias in the May and November acoustic survey that relate directly to the acoustic survey (rather than, for example the coverage of the stock), k_{ac}^S , and the CV associated with variable error factors which cause bias in the acoustic survey estimates, ϕ_{ac}^S , have been updated from that calculated by de Moor and Butterworth (2015).

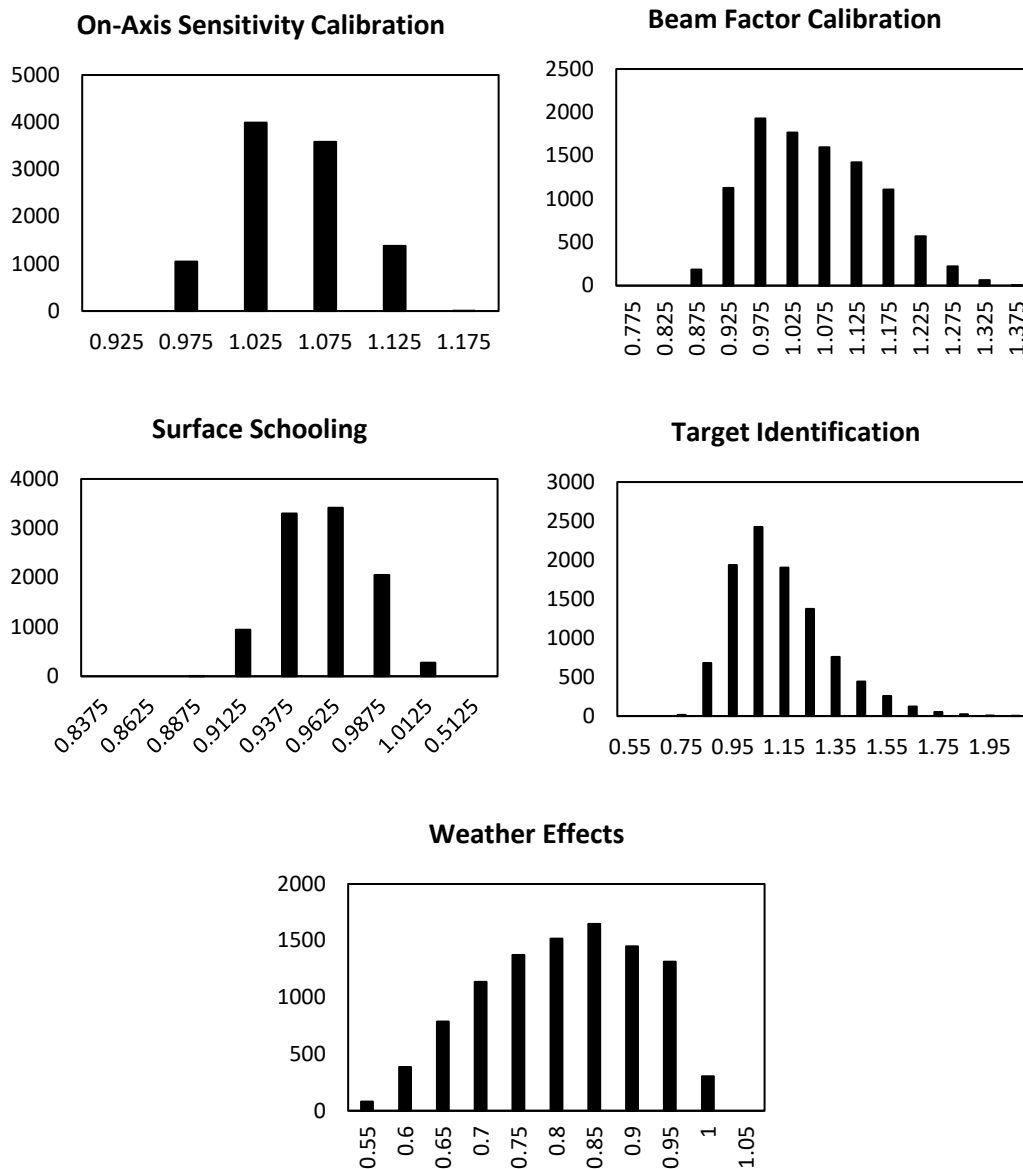
Anon. (2000) distinguished two different types of acoustic error factors (Table B1). Constant error relates to a factor whose value is not known exactly, but whatever it is, it is the same for each year. In contrast variable errors relate to a factor whose true value will change from one year to the next.

Ten thousand samples were drawn from the individual pdfs for each source of error. These sampled numbers were then all inverted so as to correspond directly to k_{ac}^S which applies to the model biomass rather than observed biomass. The inverted sample of constant errors is denoted as C_j , $j = 1, \dots, 10000$ for the only constant error factor (calibration – beam factor), and the inverted sample of variable errors – or errors that vary inter-annually – is denoted as V_j^k , $j = 1, \dots, 10000$ for error factor k . Histograms of the samples are given in Figure B1.

As the survey biomass estimates are considered in log-space in the likelihood (equations S32-S33), the distributions of k_{ac}^S and ϕ_{ac}^S are similarly considered in log-space. Histograms of the $\ln(C_j)$ and $\ln(V_j^k)$ samples are given in Figure S10. The median of $\ln(k_{ac}^S)$ is subsequently calculated as the median of the sample: $\ln(C_j) + \sum_k \ln(V_j^k)$, $j = 1, \dots, 10000$, which is -0.310. The standard deviation of $\ln(k_{ac}^S)$ is based only on the log of the constant factor, and thus it is calculated as the standard deviation of the sample: $\ln(C_j)$, $j = 1, \dots, 10000$, which is 0.094. The prior distribution for $\ln(k_{ac}^S)$ is taken to be normally distributed, i.e. $\ln(k_{ac}^S) \sim N(-0.310, 0.094^2)$ (Figure B2). The standard deviation of the log of the variable factors is considered similar to additional variance in the likelihood calculation, and is calculated as the standard deviation of the sample: $\sum_k \ln(V_j^k)$, $j = 1, \dots, 10000$, giving $\phi_{ac}^S = 0.227$. There may, however, still be systematic errors relating to the target strength that have not been taken into account in these pdfs. These could be taken into account through sensitivity tests by using alternative priors for k_{ac}^S .

Table B1. Individual error factors for hydro-acoustic surveys of sardine biomass, where the values define trapezium form pdfs (Anon. 2000). Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias factor k_{ac}^S in the model.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Variable ¹⁸
(Beam factor)	0.75 ¹⁷	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Variable ⁶
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable



¹⁷ This was originally reported as 0.8 in Anon 2000, but subsequently corrected (I. Hampton pers. Comm.).

¹⁸ This was recorded in Anon. (2000) as random error denoting that it would be positive or negative rather than purely positive or negative.

Figure B1. The histograms of 10 000 samples of the individual error factors C_j and V_j^k .

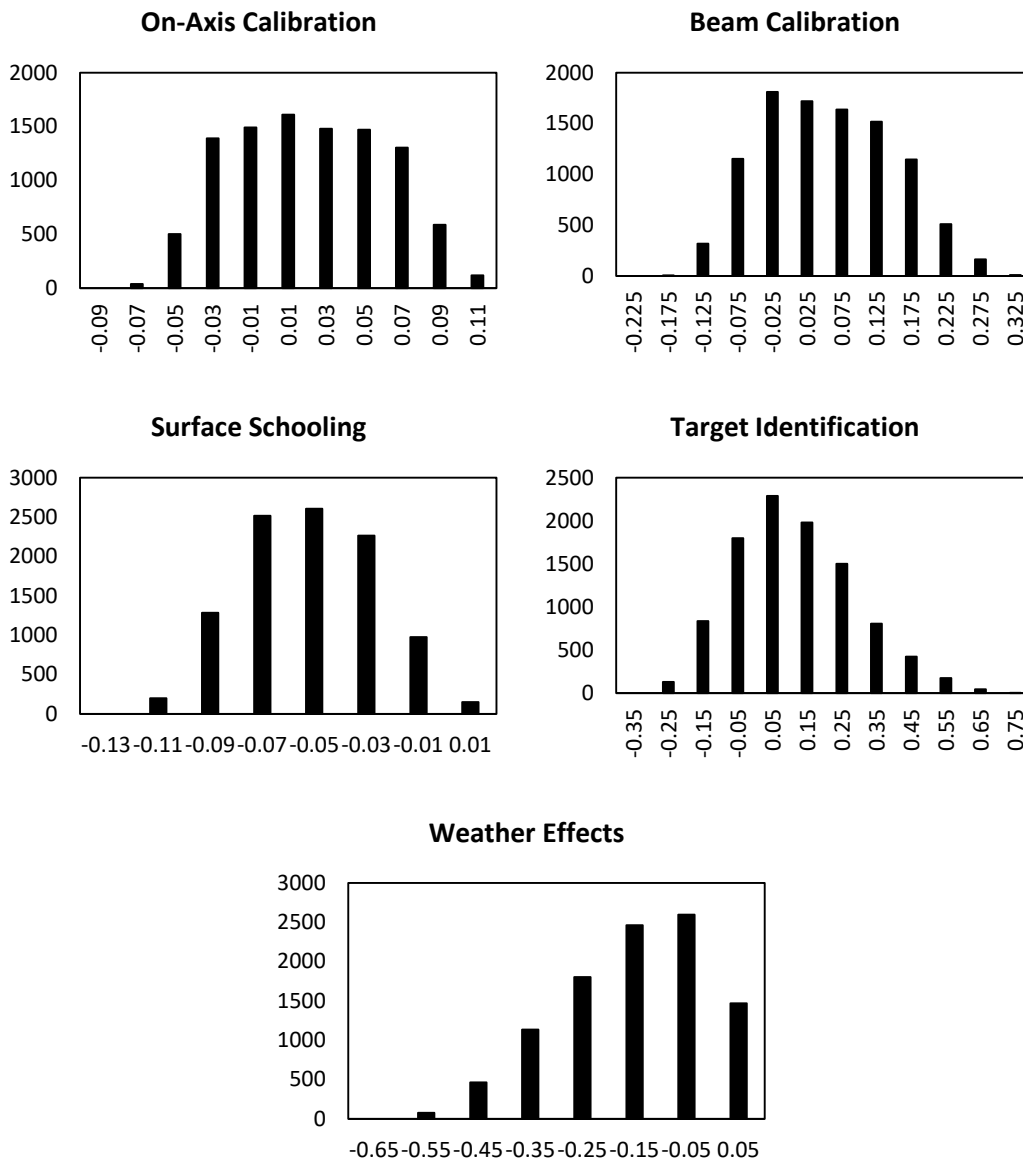


Figure B2. The histograms of 10 000 samples of the individual error factors $\ln(C_j)$ and $\ln(V_j^k)$.

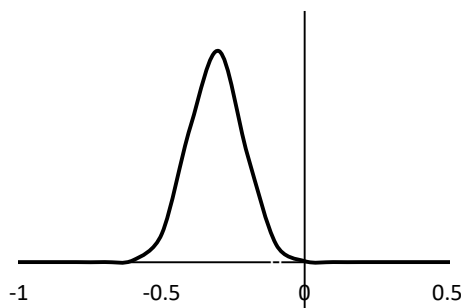


Figure B3. The resultant assumed prior distribution for $\ln(k_{ac}^s)$.

Appendix C. Calculating Instantaneous Fishing Mortality and the Loss to Predation

Considering only 1+ fish which primarily contribute to the directed sardine catch, and assuming that natural mortality, M , and commercial selectivity is constant over all ages of age 1 and above, the numbers of 2+ fish available at the end of the year is given by:

$$N_{j=W,y,2+}^S = N_{j=W,y-1,1+}^S e^{-(F_{j=W,y}^S + M)} (1 - move_{y,2+})$$

$$N_{j=S,y,2+}^S = N_{j=S,y-1,1+}^S e^{-(F_{j=S,y}^S + M)} + move_{y,2+} N_{j=W,y-q,1+}^S e^{-(F_{j=W,y}^S + M)} \quad (C.1)$$

where $N_{j,y-1,1+}^S$ denotes the total (over all ages 1+ or 2+) number of sardine of stock j in November $y-1$ (i.e. at the beginning of the year), and $F_{j,y}^S$ denotes instantaneous annual fishing mortality of sardine stock j in year y .

The biomass of sardine annually lost to predation, $P_{j,y}^S$, is calculated assuming for simplicity that catch is taken half way through the year

$$P_{j,y}^S = \sum_{a=1}^5 \left\{ N_{j,y-1,a-1}^S \left(1 - e^{-0.5M_{y,a-1}} \right) + \left(N_{j,y-1,a-1}^S e^{-0.5M_{y,a-1}} - \sum_q \left(C_{j,y,q,a-1}^{By} + C_{j,y,q,a-1}^{Dir} \right) \right) \left(1 - e^{-0.5M_{y,a-1}} \right) \right\} \times \frac{1}{2} \left(w_{j,y-1,a-1} + w_{j,y,a} \right)$$

$$+ \left\{ N_{j,y-1,5+}^S \left(1 - e^{-0.5M_{y,5+}} \right) + \left(N_{j,y-1,5+}^S e^{-0.5M_{y,5+}} - \sum_q \left(C_{j,y,q,5+}^{By} + C_{j,y,q,5+}^{Dir} \right) \right) \left(1 - e^{-0.5M_{y,5+}} \right) \right\} \times \frac{1}{2} \left(w_{j,y-1,5+} + w_{j,y,5+} \right) \quad (C.2)$$