

FISH STOCK ASSESSMENT BY A STATISTICAL ANALYSIS  
OF ECHO SOUNDER SIGNALS

BY

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SUBMITTED IN PARTIAL FULFILMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF  
MASTER OF SCIENCE IN ENGINEERING

UNIVERSITY OF CAPE TOWN

OCTOBER 1986

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## ABSTRACT

A means of assessing the quantity of exploitable fish in the sea is a requirement for effective management of the resource. Sonar is widely used in this regard, as it provides a rapid means of assessment. Two acoustic assessment techniques currently used are the echo counting and echo integration methods. The echo counting method requires that only single fish echoes are present in the backscatter from the shoal, while the echo integration technique requires an a-priori knowledge of the average target strength of the fish in the shoal. A novel method of assessment has been proposed. It relies on the relationship between the statistics of the backscatter from a volume distribution of scatterers and the number of scatterers contributing to the backscatter at any one time. The attraction of the method when applied to the estimation of number density of fish, is that estimates can be produced in the presence of overlapping echoes, and that knowledge of the target strength of the fish is unnecessary. The application of this method to acoustic fish stock assessment is investigated in this work. Current methods of assessment are reviewed and the theory of the statistical method is given. A computer simulation of the scattering problem gives a useful insight into the effects of sample size and density on the accuracy of the method. The method has been applied to the assessment of fish at sea, where it was run in tandem with an echo integrator. The results obtained with the two techniques are compared. Reasons for discrepancies are proposed and problems in the application of the method are identified.

## ACKNOWLEDGEMENTS

I would like to thank my supervisor, Professor P.N. Denbigh, for his help, insight and encouragement.

I am also grateful to the Sea Fisheries Research Institute, and in particular to Mr I. Hampton, for arranging my participation in the anchovy survey cruise. Mr Hampton also provided extensive advice during the course of the project.

Support for this work was provided by the Council for Scientific and Industrial Research Foundation for Research Development.

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## 1.0 INTRODUCTION

The collapse of the Southeast Atlantic pilchard industry in 1970 (Cram and Hampton, 1976) demonstrated the need for a rationalized program for the management of the fishing industry. Possible reasons for the collapse were increased pressure on the available stocks, and the increased efficiency of modern fishing equipment.

"It is evident that fisheries need closely controlled management if they are to survive the greatly increased efficiency of search and capture, brought about partly by acoustic methods of detection and aimed fishing" (Mitson, 1983).

Management of the fishing industry requires a reliable method for the estimation of the extent of the fish resource. In the past, these data were limited to an analysis of catch and fishing effort statistics, as reported by the industry (Mitson, 1983). Such reports afford at best only an indirect estimate of fish stocks and they may be unreliable since they require the cooperation of the fishing industry.

Sonar techniques provide a way of investigating the normally inaccessible undersea environment. An application of this technology is the acoustic survey of fish populations to provide a direct estimate of abundance. Acoustic surveys are rapidly and reliably conducted when and where required. The estimate of fish stock resulting from an acoustic survey is known as the acoustic biomass.

Acoustic techniques have been used by skippers in fish finding applications since 1933 (Balls, 1948) and in fisheries research since 1935 (Sund, 1935). Early attempts to estimate the abundance of fish stocks by acoustic means relied on the examination of traces on sounder charts (Cushing, 1952), or the counting of individual fish echoes, either using operators

(Richardson et al, 1959) or automatically (Mitson and Wood, 1961).

A technique known as echo squared integration (or simply echo integration) has largely replaced the echo counting technique as the means of estimating fish abundance at sea. This method relies on the linear relationship between the energy (or squared and integrated echo) backscattered from a fish shoal and the number of fish contributing to the backscatter. This is equivalent to a linear relationship between the mean number density of fish and the mean squared intensity of the backscatter. The major advantage over counting techniques is that it is unnecessary to be able to discern individual echoes in the backscatter. This means that the method produces useful estimates over a far greater range of densities than the echo counter. Careful calibration of the sonar used for the echo integrator is a requirement of the method, as is an a-priori knowledge of the average target strength of the fish in the shoal, so that the measure of mean square intensity may be converted to the number density. The sonar may be calibrated using conventional techniques such as the use of copper spheres as standard targets (Urick, 1975), but fish target strength is difficult to measure and data are not readily available for every species. The estimate of number density is inversely proportional to the average target strength of the individuals in the shoal, and so errors in the measurement of target strength will directly result in errors in the abundance estimate. The need for accurate a-priori knowledge of fish target strength is the major drawback of the echo integration method.

A novel method of determining the number density of random scatterers was described by Wilhelmij (1983) and Wilhelmij and Denbigh (1984), who based their work on a theory derived by Pusey et al (1974) for electromagnetic scattering. The method relies on the relationship between the statistics of the backscatter from random scatterers and the number of scatterers contributing to the backscatter. Wilhelmij and Denbigh described how the method may be applied to the scattering of acoustic waves and

performed experiments in a water tank, using as scatterers a randomized array of polystyrene spheres suspended on nylon line. Estimates of number density were compared with the true number density and good agreement was obtained.

Applications suggested for the technique were remote histology or ultrasonic tissue characterization, acoustic ocean bottom identification and acoustic fish stock assessment. The major advantage of the statistical method over the echo integrator for acoustic fish stock assessment is that the estimate of abundance is independent of the average target strength of the individual fish in the shoal while an order of magnitude increase in the useful range of densities over echo counting methods is possible. Also, calibration of the sonar is limited in principle to measurement of the transducer beamwidth and the transmit pulse length.

This work extends the application of the statistical method to fish stock assessment. Acoustic techniques in the assessment of fisheries, and the problems associated with them are reviewed. The statistical method is described and the statistical theory is extended so as to be more valid for scattering by live fish in a shoal. A computer simulation of the scattering problem is presented giving a useful insight into the random uncertainty associated with an estimate produced with a given amount of backscatter data. An attempt was made to verify the statistical method by conducting an experiment on live fish in an aquarium tank. No meaningful results were obtained as experimental difficulties were encountered, which could not readily be overcome. The results of sea trials conducted during a survey cruise of the Sea Fisheries Research Vessel "R.S. Africana", where the statistical method was used in tandem with an echo integrator, are presented. Difficulties experienced in applying the statistical method to real fish shoals are described. Estimates of fish density which were obtained using the statistical method are compared with estimates obtained by echo integration and possible reasons for differences in the estimates are discussed.

## 2.0 BACKGROUND

A discussion of fisheries management and assessment is given here. Current methods of fish stock estimation and shortcomings are described, with reference to the literature. Motivation for the novel approach described in this work can then be given.

### 2.1 Need for Assessment.

The increase in the world's population has resulted in a need to obtain more food from the sea. This, coupled with the greatly increased effectiveness of the world's fishing fleets, largely due to the use of sonar aids for detection and capture, has resulted in increased pressure on the fish resource (McElroy, 1977). The consequence of overexploitation is depletion of stock which is serious both from a biological and an economic point of view.

Control of the fishing industry is thus necessary for its survival and knowledge of present stock and the effects of exploitation is a requirement for effective management. In the words of Mitson (1983):

"The work of fisheries research aims to understand biological processes which underlie the response of fish stocks to exploitation, to make assessments of the stocks and to give appropriate and timely advice about how they should be managed"

### 2.2 Assessment Methods.

#### 2.2.1 Non Acoustic Methods.

The traditional method of assessing fish abundance relies on studies of commercial catch and effort data combined with

government surveys of fish and egg populations (McElroy, 1977). This provides an indirect estimate of resources; only relative changes in population are indicated. Relative changes are useful because the effect that a decrease in population has on the yield may be evaluated.

Non acoustic methods of direct assessment of fish density include fishing on a shoal using a trawl net with doors of known dimensions. The volume of water intersected by the doors of the trawl net in a sampling trawl may easily be evaluated and the catch may be counted or weighed. A direct estimate of number density or mass density results. Not all fish in the path of the net are caught; the efficiency of the trawl gear must be estimated. Factors affecting efficiency of the gear may be fish avoidance and gear selectivity (Bayona, 1984). The method is considered to be innacurate, and is slow since ship speed is limited while trawling. Only a single depth channel is sampled by the net in a particular trawl.

### 2.2.2 Acoustic Methods.

The acoustic methods of assessment are, in general, faster and more reliable than the non-acoustic methods and have become the standard means of survey.

#### 2.2.2.1 Historical Development.

Kimura (1929, cited by Johannesson and Mitson, 1983) first used an acoustic method to detect fish.

Experiments were conducted by sending 200kHz continuous waves across ponds containing goldfish, and echo signals were recorded on an oscillograph. It was noted that as the number of fish intercepted by the acoustic beam varied, so the signal recorded by the oscillogram varied.

The first application of fisheries acoustics at sea was the spotting of herring shoals using an "echometer" made by Marconi. The device was fitted to the fishing vessel of Ronnie Balls in 1933 and the resulting improvement in fishing efficiency was reported by him in 1948.

The use of an echo sounder in fisheries research was reported by Oscar Sund in "Nature" of June 1935. It is noted in this letter that new insights into the distribution of cod were obtained with the instrument. It was concluded that there is a relationship between water temperature, the concentration of oxygen and hydrogen ions in the water and concentrations of fish. No attempt at abundance estimation was made and Sund notes that:

"A true estimate of the quantity of fish represented by marks of different types can, however, only be gained by further study in connexion with the use of suitable fishing implements."

The first attempt at quantitative evaluation of echos related the thickness of traces on sounder charts to the number of soundings producing the line (Cushing, 1952). Charts were examined using a microscope and the thickness of compound soundings (traces made with the chart stationary for a number of pulses) measured. In this way a crude measurement of relative abundance could be made. Various tests of the reliability of the survey were devised including the comparison of measurements with commercial landings, comparison with egg distributions and repetition of measurements. Good agreement is reported.

A manual method of echo counting using operators counting signals from a cathode ray tube has been reported by Richardson et al (1959). The obvious disadvantages of a

manual counting method are that operators cannot accurately judge amplitudes and that they tire quickly. This is the motivation given by Mitson and Wood (1961) for the design of an automatic device for the counting of fish echoes. This echo counter was used in trawls for cod off the West coast of Spitzbergen. The device was evaluated in terms of the correlation between abundance estimates and trawl catches. An improvement in accuracy over the visual method of Richardson is recorded. The reliability of the counter was greatly affected in the presence of noise, the cause of which was often adverse weather conditions. The fundamental disadvantage of the echo counter, namely that it is only useful in the presence of non-overlapping echoes, is not pointed out.

The introduction of the echo integrator (Dragesund and Olsen, 1965 cited by Johannesson and Mitson, 1983) overcame this fundamental difficulty. The echo integrator relates abundance to the acoustic energy backscattered from a shoal and produces an estimate of fish density in the presence of overlapping echoes. Accurate calibration of the sonar used, as well as a-priori knowledge of the average target strength of the fish in the shoal is a requirement of the method and represents the major difficulty. This method is now the most widely used in marine assessment applications with the echo counter being used mostly in riverine applications and on tuna at sea (Yamanaka et al, 1977).

A method of estimating shoal number density by a statistical analysis of the acoustic signals backscattered from shoals has been suggested (Wilhelmij and Denbigh, 1984) and represents a novel approach to abundance estimation. The motivation for the method is that, unlike the echo counter, estimates in the presence of overlapping echoes are possible, and unlike the echo integrator, a-priori knowledge of fish target strength

and accurate calibration of the sonar used are not required. The method is the subject of this work and is discussed in detail in later chapters.

The recent work of Stanton (1985) also uses the statistical properties of echoes from fish shoals to derive estimates of number density. The probability density function (pdf) of the peak of the echo in a finite range gate, or echo peak pdf, is plotted. Note that this pdf is a function of the size of the time gate as well as of the backscatter statistics, there being a greater chance of a large peak in a wide gate than in a narrow gate. Extremal theory is used to derive the expected echo peak pdf's for overlapping and non-overlapping echoes, and for the transition between the two regimes, and these are compared with pdf's derived from data collected from biological sound scatterers. The critical density corresponding to the transition pdf is calculated. The fish density is then categorized as being greater than, less than or equal to this value depending on the shape of the echo peak pdf. This is a rough estimate, and in principle, the method proposed by Wilhelmij and Denbigh should provide a more exact estimate.

A more detailed description of echo counting and echo integration systems is now given.

### 2.2.2.2 Echo Counting.

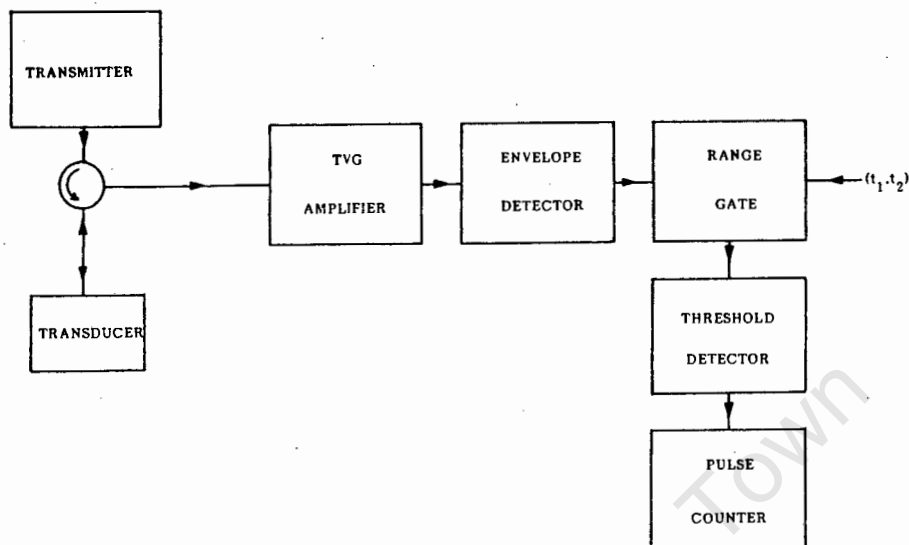


Fig 2.1 Block diagram of echo counter.

(From Ehrenberg and Lytle, 1972).

The operator selects a depth channel for the device excluding the ocean floor. Echos from the depth channel are gated out of the signal, and directed through a time varied gain (TVG) amplifier which removes the dependence of echo strength on range. An envelope detector follows, producing the envelope of the signal. The threshold device detects the presence of a fish when the detector output exceeds a threshold level, which is placed above the level of ambient noise, and sends a pulse to the counter. The count then is a measure of the number of fish detected. A correction must be made to the count due to the fact that the same fish may be detected on successive pings as the fish moves across the beam of the sonar. Such a correction is a function of the speed of the survey vessel, speed of the fish, transducer beam pattern and the pulse repetition frequency (PRF) of the sonar (Yamanaka et al, 1977).

As already mentioned, the principle drawback of the system is that when echoes from two or more fish overlap only a single count is recorded on the counter. This causes the system to underread. The echo counter is only a useful method of assessment at low number densities, when the acoustic echo from the fish shoal consists mostly of peaks originating from single targets. The maximum density is of course related to the resolution of the particular sonar used.

### 2.2.2.3 Echo Integration.

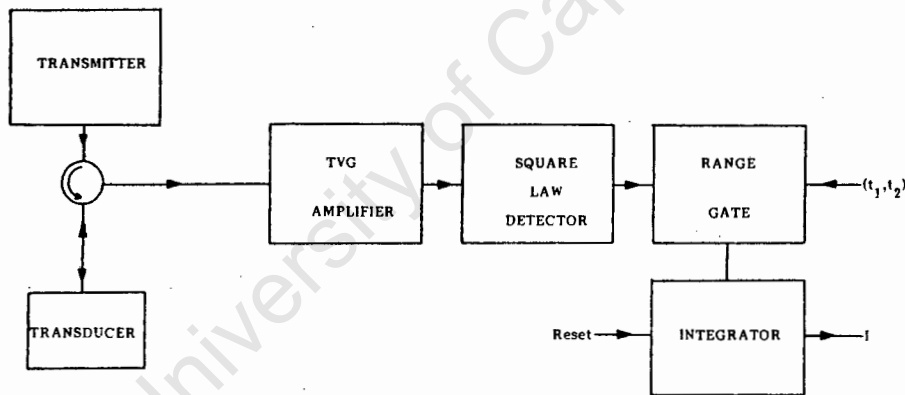


Fig 2.2 Block diagram of a simple echo integrator.  
(From Ehrenberg and Lytle, 1972).

The echo integrator depends on the principle that the energy backscattered from the insonified shoal is proportional to the number of fish causing the backscatter. More correctly called the echo squared integrator, the device measures the energy by integrating the backscattered intensity, proportional to the square

of the sonar receiver output, over a range gate corresponding to the depth channel of interest. The device is reset at the start of an integration interval corresponding to a segment of the survey line followed by a survey vessel, and the output at any time then represents the accumulated energy backscattered from targets insonified up until that time. By hypothesis this is proportional to the number of targets encountered.

The operator of the device must try to exclude echo energy originating from spurious sources, such as the ocean floor or scatter from bubbles caused by turbulence in rough seas, by careful setting of the integrated depth channel.

The hypothesis that the energy measured by the integrator is proportional to the number of fish insonified requires proof. The most convincing verification of the linearity of fisheries acoustics and hence the validity in principle of the echo integration method is given by the results of experiments conducted by Foote (1983). Strictly, linearity applies only in the absence of extinction (weakening of the incident pulse on fish deep in the shoal due to scattering by earlier targets). This criterion sets an upper bound on shoal densities to which the echo integrator may be applied without errors but this condition is far more relaxed than that applicable to the echo counter.

In order to relate the energy measured by the echo integrator to the number of fish, the average energy scattered by a single fish must be known. This is a function of the particular species being surveyed, the frequency of the sonar used, the depth of the shoal and the length and mass of the particular specimens. Variation with in target strength with fish behavior may

occur. There is therefore always some uncertainty associated with such data. Furthermore, estimates of mean target strength for many species are unavailable.

The quantity that is estimated by the simplified system described above is the total number of fish encountered over an integration interval which by hypothesis is directly proportional to the total echo energy measured over the interval. In practice, the quantity which is of interest is the mean density of the fish over the interval. (This may be expressed as a number density or a mass density.) A system to measure fish density will now be described.

The concept of volume backscattering strength is useful when describing a system to measure density. The volume backscattering strength, denoted  $s_v$  if expressed as a linear ratio or  $S_v$  if expressed in decibels, is defined as the ratio of the intensity scattered towards the transducer by the scatterers in a volume of  $1 \text{ m}^3$ , measured in the far field of the backscatter and referred to  $1 \text{ m}$  from the volume, to the intensity incident on the volume (Urick, 1975). Mathematically:

$$S_v = 10 \log \frac{I_{\text{scat}}}{I_{\text{inc}}} \quad (\text{dB}) \quad (2.1)$$

$I_{\text{scat}}$  = Intensity of the sound scattered towards the source.

$I_{\text{inc}}$  = Intensity of the incident plane wave.

The volume backscattering strength is a characteristic of a particular volume distribution of scatterers. (For a single target the analogous quantity would be the target strength.) The linearity hypothesis can then be applied

to suggest that the density of fish in a shoal is proportional to the volume backscattering strength of the shoal. The average density of fish encountered during a survey interval is then directly proportional to the mean of  $S_v$  over the survey line.

The basic theory needed for an understanding of the operation of an echo integrator based on these principles will now be derived. This theory is usually explained using decibel units. For clarity, linear units will be used in this explanation, and the corresponding decibel equations will be given where appropriate.

Consider a shoal of mean fish density  $\bar{\rho}$  fish/m<sup>3</sup>. Let the mean volume backscattering strength for the shoal be  $\bar{S}_v$  and the mean backscatter cross section of an individual fish be  $\bar{\sigma}_f$ .

The mean target strength of an individual fish is then  $\bar{\sigma}_f/4\pi$ .

The assumption of linearity may be expressed mathematically as

$$\bar{S}_v = \bar{\rho} \bar{\sigma}_f/4\pi \quad (2.2)$$

In decibel units with  $\bar{S}_v = 10 \log \bar{S}_v$

and  $\overline{TS} = 10 \log \frac{\bar{\sigma}_f}{4\pi}$  this may be written as

$$\bar{S}_v = 10 \log \bar{\rho} + \overline{TS}$$

or

$$\bar{\rho} = 10^{0.1\{\bar{S}_v - \overline{TS}\}} \quad (2.3)$$

The mean target strength of an individual fish is known a-priori so the measurement of mean number density is

reduced to a problem of measuring the mean volume backscattering strength of the shoal. Firstly, a sonar equation relating the instantaneous backscattered intensity of volume reverberation at the receive transducer to the volume backscattering strength of the fish in a shoal at a range  $R$  will be derived. This backscattered intensity can then be related to the squared voltage out of the sonar receiver. Integration over the survey track and depth annulus sampled then gives the relationship between mean squared received voltage and the mean volume backscattering strength.

The volume of scatterers contributing to the return signal at any one time is termed the reverberating volume,  $V$ . This is equivalent to the volume of the insonifying acoustic pulse. Normally,  $V$  is calculated on the basis of an equivalent idealized beampattern characterised by a beam solid angle  $\psi$ . Within the idealized beam the insonification is uniform, and equal in intensity to the on axis intensity of the real beam; outside the idealized beam the intensity is zero. The equivalent beam solid angle is calculated to give the same average returned intensity from a volume distribution of scatterers as the actual non-ideal beam. For a circular transducer of radius  $a$ , operating at a frequency such that the acoustic wavelength is  $\lambda$ , it may be shown that the equivalent beam solid angle is (Johannesson and Mitson, 1983):

$$\psi = 5.9 (\lambda/2\pi a)^2 \quad (2.4)$$

For a sonar with a square pulse of length  $\tau$ , operating in a medium with sound speed  $c$ ,

$$V = R^2 \psi (c\tau/2). \quad (2.5)$$

Let the transmitted on axis far field sound intensity referred to 1 metre from the transmit transducer be  $I_t$ .

The intensity at a range  $R$  is then  $\frac{I_t}{R^2} e^{-\alpha R}$

where  $\alpha$  is the sound absorption coefficient of seawater.

The intensity scattered towards the receiver referred to 1 m from the scatterers, by definition of the volume backscattering strength, is

$$\begin{aligned} I_{\text{scat}} &= \frac{I_t}{R^2} e^{-\alpha R} \cdot s_v \cdot V \\ &= \frac{I_t}{R^2} e^{-\alpha R} \cdot s_v \cdot R^2 \psi \cdot \frac{c\tau}{2} \\ &= I_t e^{-\alpha R} \cdot s_v \cdot \psi \cdot \frac{c\tau}{2} \end{aligned}$$

The intensity of the echo at the receiver is then

$$I_{\text{rec}} = \frac{I_t}{R^2} e^{-2\alpha R} \cdot s_v \cdot \psi \cdot \frac{c\tau}{2}$$

The squared voltage out of the sonar receiver is directly proportional to this intensity. If  $k$  is the constant of proportionality, having units of  $v^2/(Wm^{-2})$ , then

$$v^2 = k \frac{I_t}{R^2} e^{-2\alpha R} \cdot s_v \cdot \psi \cdot \frac{c\tau}{2}$$

Assuming an ideal tvg law of  $R^2 e^{2\alpha R}$ , with  $R$  in metres (in practice any static gain associated with the tvg amplifier may be absorbed into  $k$ ), then,

$$v^2 = kI_t s_v \Psi(ct/2)$$

For a survey along a track from  $x_1$  to  $x_2$ , sampling over a depth annulus from  $R_1$  to  $R_2$ ,

$$\begin{aligned} \overline{v^2} &= \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \frac{1}{R_2 - R_1} \int_{R_1}^{R_2} v^2 dR dx \\ &= kI_t \Psi(ct/2) \left\{ \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \frac{1}{R_2 - R_1} \int_{R_1}^{R_2} s_v dR dx \right\} \\ &= kI_t \overline{s_v} \Psi(ct/2) \end{aligned}$$

or,

$$\overline{s_v} = \frac{\overline{v^2}}{kI_t \Psi(ct/2)} \quad (2.6)$$

In logarithmic units, this is equivalent to

$$\overline{s_v} = 10 \log \overline{v^2} - \text{SRT} - G - \text{SL} - 10 \log \frac{ct}{2} - 10 \log \Psi \quad (2.7)$$

where,

SRT = transducer receiving sensitivity  
in (dB re 1 V/dB re 1  $\mu$ Pa),

G = the total static gain of the receiving equipment,

SL = the source level of the transmit transducer  
=  $10 \log I_t$ .

Note that in a digital system, the mean square voltage may be computed by summing samples of the squared received voltage and dividing by the number of samples; this is equivalent to the integration described above. The voltages may be represented by their equivalent digital values, and an appropriate conversion must then be included in equation 2.7.

All that remains is to calibrate the echo integrator. The objective is that, by using a standard target, the value of  $kI_t$ , or  $SRT + G + SL$  may be determined. It is unnecessary to determine these individually.

A standard target, such as a copper sphere, of cross section  $\sigma_s$ , is placed on the axis of the sonar beam at a range  $R_s$ .

For a transmit intensity of  $I_t$  at 1 m, the intensity at the standard target is

$$I_{inc} = \frac{I_t}{R_s^2} e^{-\alpha R_s}$$

The intensity scattered towards the receiver at 1 m from the target is

$$I_{scat} = \frac{I_t}{R_s^2} e^{-\alpha R_s} \cdot \sigma_s / 4\pi$$

And the intensity at the receiver

$$I_{rec} = \frac{I_t}{R_s^4} e^{-2\alpha R_s} \cdot \sigma_s / 4\pi$$

The mean square voltage out of the receiver is

$$\overline{v_s^2} = k \frac{I_t}{R_s^4} e^{-2\alpha R_s} \cdot \sigma_s / 4\pi$$

giving

$$kI_t = \frac{\overline{v_s^2} R_s^4 e^{2\alpha R_s}}{\sigma_s / 4\pi} \quad (2.8)$$

In dB's, the equivalent equation is

$$SRT + G + SL = 10 \log \overline{v_s^2} + 2TL - TS_s \quad (2.9)$$

with

$$TL = 20 \log R_s + \alpha R_s$$

$$TS_s = 10 \log \sigma_s / 4\pi$$

The result obtained in this way for  $SRT + G + SL$  is then used in equation 2.7 to determine  $\overline{S}_v$ .

The range to the standard target is measured and used in calculating the transmission loss. Time varied gain is normally not used in calibrations, since there is normally some inaccuracy associated with a tvg amplifier. Any static gain associated with the tvg amp used in surveys is not present in the calibration and should be accounted for by adding its value in dB's to the left hand side of equation 2.9.

The volume density of the fish shoal may be expressed as either as a number density or as mass density. In either case, equation 2.3 is used to convert the mean volume

backscattering strength to a density estimate. The value of mean target strength used may be given as the mean target strength of an individual, or is often given as the target strength in dB's of unit mass of fish. If given for 1 kg of fish this is denoted  $\overline{TS}_{kg}$ . Equation 2.3 will produce a number density or a mass density depending on the units of target strength used.

The abundance information gathered during a survey is often displayed on a map giving contours of effective shoal surface density. Surface density is a measure of the total number or mass of fish in the water column under unit area of sea surface. Like the volume density, surface density can be expressed as a number density or as a mass density. Surface density at a point is calculated by multiplying the mean volume density at that point by the water depth. In practice this is often achieved by adding the depth expressed in dB re 1m to the volume backscattering strength to produce the surface backscattering strength. This quantity may be averaged and converted to a surface density in a similar way as for volume density.

#### 2.2.2.4 Relative performance.

Ehrenberg and Lytle have evaluated the theoretical limits to the performance of the echo counter and integrator systems. A statistical model of a fish shoal, which assumes that the fish are Poisson distributed in volume, is constructed. The estimate of number of fish in the reverberating volume is termed  $N$ . The average number in that volume is the expected value of  $N$  or  $E\{N\}$ , which is a parameter of the Poisson distribution. (The notation  $\langle N \rangle$  used for the mean number per resolution cell volume later in this work may be identified with  $E\{N\}$ .) The performance of the integrator and counter systems is then

evaluated in terms of the mean squared error, defined as

$$\overline{e^2} = E\{(N - E\{N\})^2\} \quad (2.10)$$

The normalized mean squared error is defined as  $\overline{e^2}/(E\{N\})$ .

Theoretical expressions for the normalized mean squared error of the counter and integrator systems are derived. These are plotted as a function of average number density for a typical set of operating conditions. These are a circular transducer with a 20 degree beamwidth, 1ms pulse duration, 400 pulses with no beam overlap and a depth channel of 40 - 80 meters.

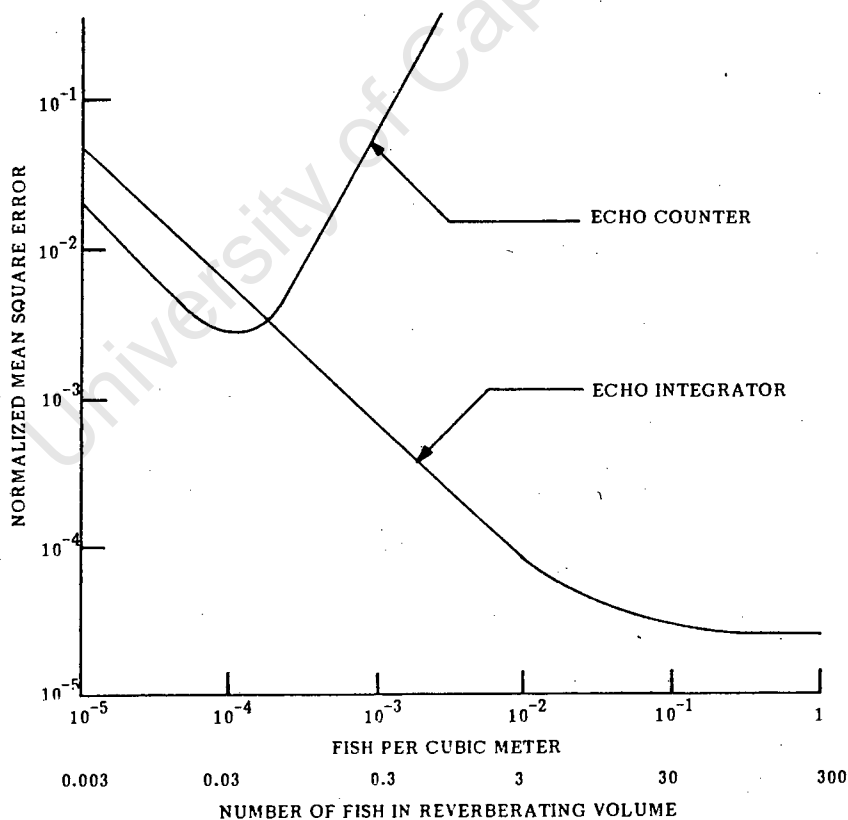


Fig 2.3 Errors of echo counter and echo integrator.  
(After Ehrenberg and Lytle, 1972)

Note that the horizontal axis has also been labeled with the number of fish in the reverberating volume at the mid-range of 60 meters. This scaling is considered to give a better insight than simply scaling with number density as in Ehrenberg and Lytle, as the error as a function of the number of scatterers in the reverberating volume is independent of the parameters of the particular sonar and of the range of the scatterers. The reverberating volume is calculated on the basis of a cone shaped beam of 20 degree beamwidth and a square 1 ms pulse length, using equation 2.5, and the number of scatterers in the reverberating volume is given by multiplying by the density of fish in the shoal.

At low number densities the error in both systems is mainly due to fluctuations of number density about the true mean and the finite sampling interval. In addition, statistical fluctuations in fish target strength cause an additional error in the integrator estimate; the counter is unaffected by such errors and is thus the better estimator in this region. As number density increases, echoes begin to overlap, and the performance of the echo counter deteriorates while the performance of the echo integrator improves since the fluctuations in fish target strength are less significant in this region. Note that systematic errors due to miscalibration of the sonar and echo integrator and inaccuracies in the estimates of mean fish target strength are not accounted for in this analysis.

### 2.3 Fish Acoustics.

It has already been pointed out that accurate knowledge of the average target strength of a particular species of fish is vital when using an echo integrator to provide an abundance estimate. This section outlines the problems associated with the measurement of  $\overline{TS}$ , and briefly reviews some measurement techniques and some published results.

#### 2.3.1 Fish as Acoustic Targets.

Fish are irregular structures with different parts having dissimilar densities. The power reflected by a fish back to a receiver thus depends on the orientation of the fish to the insonifying signal. The most significant reflector is the swim bladder, an air filled sac with used by the fish to maintain neutral bouyancy. Only certain species of fish have a swim bladder. Experiments have shown that for these fish the swim bladder is responsible for up to 90% of the echo signal (Foote, 1980).

In order to maintain neutral bouyancy, the volume of the swim bladder must change with changes in ambient sea pressure for example those accompanying migrations in the water column. The accommodation of the swim bladder to changes in pressure may lag the change in pressure. This results in a hysteresis effect; the TS of a fish which is stationary in the water column may change in time depending on previous vertical migrations. This introduces errors in tabulated values of fish target strength. The change in target strength due to depth adaptation may be of the order of 1dB (Mitson, 1983).

Variation in target strength with behaviour has also been observed. This may be caused by changes in the average aspect angle of the fish to the sonar, since the target strength of a fish has directional character. Diurnal

variation has been observed since behaviour patterns may change from day to night.

Possible variations of the sort described above should be kept in mind when interpreting published data.

### 2.3.2 Measurement of Fish Target Strength.

Measurement of fish target strength has been achieved by mounting dead or anesthetized specimens on a special jig and plotting polar diagrams of backscatter cross section as a function of pitch, roll and yaw (Haslett et al, 1973). The mean dorsal aspect target strength is required to calibrate an echo integrator, so these measurements must be averaged over angle in some way depending on the expected angle distribution. The acoustic properties of the specimens may be different to those of live or conscious fish. Techniques of measurement on live fish in tanks or cages have been developed, but there is evidence to suggest that the acoustic properties of artificially confined fish may be different to those of free swimming specimens (Ehrenberg, 1982). So called 'in situ' techniques where measurements are performed at sea on suitable low density aggregations are now the preferred method of measurement. The difficulty is that when a single fish echo is observed, its amplitude depends not only on the target strength of the fish, and the sonar parameters, but also on the position of the fish in the acoustic beam. Ways of removing this effect include the use of statistical methods to extract the average effect of the beam pattern from a collection of single fish measurements. Alternatively, the dual (or split) beam echo sounder (Ehrenberg, 1984, Foote et al, 1984) provides a simple way of performing in situ measurements, but requires expensive equipment.

The target strength of a particular species of fish depends on factors such as size of the specimen and the frequency of the insonifying signal. At a particular frequency, empirical formulae for mean target strength in terms of length have been produced by fits to measured data. A logarithmic dependence on length is expected of the form:

$$TS = m \log l + b \quad (\text{dB}) \quad (2.11)$$

It is common practice to derive an empirical formula of the same type giving the mean target strength of 1 kilogram of fish. As mentioned earlier, the notation for this is  $\overline{TS}_{\text{kg}}$ . A similar length dependence occurs in this formula.

Measurements of mean fish target strength as a function of mean length have been made by various workers on herring at 38 kHz. Regression is used to determine the constants  $m$  and  $b$ . Because they are used in calculating the echo integrator estimates in chapter 5, a comparison of the resulting empirical formulae is useful. They are:

Nakken and Olsen (1977) (in situ):

$$\overline{TS}_{\text{kg}} = -14.12 \log \bar{l} - 15.07 \quad (2.12)$$

Edwards and Armstrong (1982) (caged free swimming fish):

$$\overline{TS}_{\text{kg}} = -17.09 \log \bar{l} - 10.06 \quad (2.13)$$

These authors also report diurnal variations of 2 to 3 dB in measured target strength.

Halldorsson and Reynisson (1982) (in situ):

$$\overline{TS}_{\text{kg}} = -10.9 \log \bar{l} - 20.9 \quad (2.14)$$

Note that in all these formulae the subtraction of 30 dB converts the units of  $\overline{TS}_{\text{kg}}$  from dB re 1 kg to dB re 1 g.

Estimates of  $\overline{TS}_{kg}$  using these formulae are tabulated over a typical range of lengths to facilitate comparison.

l (cm)	8	9	10	11	12	13	14
N & O	-27.9	-28.5	-29.2	-29.8	-30.3	-30.8	-31.3
E & A	-25.5	-26.4	-27.1	-27.8	-28.5	-29.1	-29.6
H & R	-29.9	-30.5	-31.0	-31.4	-31.8	-32.2	-32.6

Table 2.1 Comparison of herring target strength per kilogram estimates of various authors.

It can be seen that discrepancies of up to 4 dB exist between the estimates of the various authors. Depending on which formulation is used, differences in the estimate of abundance by the echo integrator of a factor of up to 2.3 may result. It should be noted that the first and last of these formulae were derived from measurements conducted on fish at sea, while the second is based on data collected on caged specimens. The possibility exists that the caged fish behave differently to free swimming specimens causing an error in the target strength collected on these specimens. The geographic location of the experiments varied, and factors such as time of day and depth induced effects are unaccounted for. In the light of the discussion in section 2.3.1, this may partly explain discrepancies in the formulae. The usefulness of a system which can produce an estimate of fish density independent of target strength is apparent.

### 3.0 THEORY OF THE STATISTICAL METHOD

The statistical method of determining the number density of fish shoals relies on an analysis of the statistical character of sonar backscatter from a random volume distribution of fish. The statistical background needed for an understanding of this analysis is given in this chapter; then the theory relating backscatter statistics and number density of scatterers is reviewed.

#### 3.1 Statistical Background.

Random variables are variables which cannot be described absolutely in terms of some parameter, for example time. Such variables have to be described with the aid of statistics. It is only possible to give the probability that the variable will take on a value within a specified range.

For a random variable  $x$ , the probability that it will take on a value within a specified range is given by integrating the probability density function (pdf) for the variable over the range. The pdf may be denoted  $p(x)$  and has the following properties:

$$\int_{-\infty}^{\infty} p(x) dx = 1 \quad (3.1a)$$

$$\int_{x_1}^{x_2} p(x) dx = \text{probability that } x \text{ will take on a value} \quad (3.1b)$$

between  $x_1$  and  $x_2$ .

There are standard functional forms for those probability density functions which occur often. Three of these are referred to in this work and are given here:

The Gaussian pdf:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad (3.2a)$$

The Rayleigh pdf:

$$p(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad x \geq 0 \quad (3.2b)$$

The exponential pdf:

$$p(x) = \frac{1}{\langle x \rangle} e^{-\frac{x}{\langle x \rangle}} \quad x \geq 0 \quad (3.2c)$$

In these formulae,  $\langle x \rangle$  is the mean of the random variable, and  $\sigma$  the standard deviation.

The pdf's of random variables may be partially characterized by their moments. The moments may be computed exactly from their definition if the functional form of the pdf of the random variable is known.

The theoretical definition of the moment of a pdf is given in terms of the function  $p(x)$ . The  $n$ 'th moment of  $p(x)$  is defined as:

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n p(x) dx \quad (3.3)$$

The first moment, given by

$$\langle x \rangle = \int_{-\infty}^{\infty} x p(x) dx$$

is just the mean of  $p(x)$ .

The n'th normalized moment of  $p(x)$  is given by dividing the n'th moment by the mean raised to the power  $n$ :

$$\frac{\langle x^n \rangle}{\langle x \rangle^n} = \frac{\int_{-\infty}^{\infty} x^n p(x) dx}{\left\{ \int_{-\infty}^{\infty} x p(x) dx \right\}^n} \quad (3.4)$$

It may be shown that all the moments of order one to infinity uniquely specify  $p(x)$ . This suggests that a few of the moments of a pdf contain information about some of the important character of the pdf.

An estimate of the moments of a pdf may be computed from a finite number of samples of the random variable; these are then known as sample moments. The sample moments are useful in characterizing the pdf of a random variable when the functional form of the pdf is unknown.

The n'th sample moment can be determined from  $k$  samples  $x_i$  of a random variable  $x$ . The same notation used for the exact moment will be used to denote the sample moment.

$$\langle x^n \rangle = \frac{1}{k} \sum_{i=1}^k x_i^n \quad (3.5)$$

Notice that knowledge of the exact form of the pdf is unnecessary in calculating the n'th sample moment. This is often the case when the  $x_i$  are samples of random physical data.

The n'th normalized sample moment is then given by the n'th sample moment divided by the sample mean to the power  $n$ . The sample mean is just the first sample moment.

$$\begin{aligned}
 \frac{\langle x^n \rangle}{\langle x \rangle^n} &= \frac{\frac{1}{k} \sum_{i=1}^k x_i^n}{\left\{ \frac{1}{k} \sum_{i=1}^k x_i \right\}^n} \\
 &= \frac{k^{n-1} \sum_{i=1}^k x_i^n}{\left\{ \sum_{i=1}^k x_i \right\}^n} \tag{3.6}
 \end{aligned}$$

The n'th sample moment is an estimate of the n'th moment of the true pdf of the samples. As k becomes large, the estimate converges to the true moment.

To simplify the text, the sample moments will from now on simply be referred to as moments.

### 3.2 Application to Identical Random Scatterers.

Random scatterers are scatterers which are randomly distributed in volume. If such scatterers are irradiated or insonified, the resultant scattered field will have random amplitude and phase fluctuations. If the backscattered field is detected, for example by a sonar receiver transducer, the detected waveform at any time will be the sum of independent, randomly phased contributions from the scatterers which are insonified at that time. Since the scatterers are randomly distributed, the resultant waveform will have random amplitude and phase fluctuations. It is therefore necessary to describe the waveform statistically.

If a large number of scatterers contribute to the backscatter at any one time, the receive signal waveform will be the sum of the many independent randomly phased contributions due to each of the scatterers. The central limit theorem then predicts that the

statistics of the backscattered signal will be noiselike, and can be described by the Gaussian pdf. The envelope of the signal will then be Rayleigh distributed and the intensity, which is the square of the signal envelope, will have an exponential probability density function. If the number of scatterers is not large, however, then the statistical distribution of the backscatter is not readily determined. It is apparent that the statistics of the signal will deviate from Gaussian, those of the envelope will deviate from Rayleigh and those of the intensity from exponential. A measure of this statistical deviation may then be used to produce an estimate of the number of scatterers contributing to the backscatter.

It has been found that the normalized moments of the intensity pdf  $p(I)$  provide a useful measure of the statistics of the backscatter. It can easily be shown that if the intensity pdf is exponential of the form:

$$p(I) = \frac{1}{\langle I \rangle} e^{-I/\langle I \rangle} \quad (3.7)$$

then, from the definition, the  $n$ 'th normalized moment is

$$\frac{\langle I^n \rangle}{\langle I \rangle^n} = n! \quad (3.8)$$

Exponential statistics in the backscattered intensity corresponds to a large number of scatterers contributing to the backscatter. When the number of scatterers contributing to the backscatter is not large, however,  $p(I)$  will deviate from exponential, and this deviation will be reflected in the value of the  $n$ 'th normalized moment. This is the basis of a method proposed by Wilhelmij (1983), and Wilhelmij and Denbigh (1984) of estimating the number

density of random scatterers. The method makes use of formulae, originally derived in studies of electromagnetic scattering by Pusey, Schaefer and Koppel (1974). The formulae give the first three normalized moments as a function of  $\langle N \rangle$ , the mean number of scatterers contributing to the backscatter. They are:

$$\frac{\langle I^2 \rangle}{\langle I \rangle^2} = 2 + \frac{1}{\langle N \rangle} \quad (3.9a)$$

$$\frac{\langle I^3 \rangle}{\langle I \rangle^3} = 6 + \frac{9}{\langle N \rangle} + \frac{1}{\langle N \rangle^2} \quad (3.9b)$$

$$\frac{\langle I^4 \rangle}{\langle I \rangle^4} = 24 + \frac{72}{\langle N \rangle} + \frac{34}{\langle N \rangle^2} + \frac{1}{\langle N \rangle^3} \quad (3.9c)$$

It is interesting to note that, as  $\langle N \rangle$  tends to infinity, the normalized moments tend to  $n!$  in these three cases, as expected for an exponential pdf.

These formulae are derived under the assumption that all scatterers have the same scattering strength, and that they are Poisson distributed in volume. The Poisson volume distribution is formulated as

$$P_{\langle N \rangle}(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!} \quad (3.10)$$

and gives the probability that  $N$  scatterers contribute to the backscatter at any one time in a distribution where the mean

number of contributing scatterers is  $\langle N \rangle$ .  $\langle N \rangle$  in this formula is a parameter which is constant in time, which is equivalent to requiring that the statistics of the backscatter are stationary. In the sonar context,  $\langle N \rangle$  is the mean number of fish in the reverberating volume, or the sonar resolution cell.

In an error analysis (Wilhelmij and Denbigh, 1984) formulae are derived relating the expected random uncertainty in the estimates of  $\langle N \rangle$  to the true  $\langle N \rangle$  and the number of samples used to obtain an estimate. The sensitivities of the estimates of  $\langle N \rangle$  to errors in the normalized moments are derived by differentiation of equations 3.9. Manipulation gives the incremental error in an estimate of  $\langle N \rangle$  in terms of the incremental error in the normalized moments of intensity. The standard deviations in the normalized moments are derived assuming exponential intensity statistics. The error in the estimate of  $\langle N \rangle$  is obtained by substituting the standard deviation of the normalized moment for the incremental error. The resulting expression for the random uncertainty in an estimate of  $\langle N \rangle$  is not the standard deviation, merely the error in the estimate of  $\langle N \rangle$  when the error in the corresponding normalized moment of intensity is equal to its standard deviation. Moreover, the analysis is valid only for small errors and large  $\langle N \rangle$ . The analysis is useful as a comparative study of the relative errors which can be expected when using the various normalized moments to produce estimates of  $\langle N \rangle$ . It does indicate that the second normalized moment can be expected to be the best measure of number density. The errors in  $\langle N \rangle$  are shown to be inversely proportional to the square root of the number of samples used in producing the sample normalized moment. A way of estimating the standard deviation in an estimate of  $\langle N \rangle$ , produced using the second normalized moment, by simulation, is discussed in chapter 4.

### 3.3 Extension to Rayleigh Scatterers.

When applied to live fish, the assumption that the scatterers are identical, which was used in deriving the equations 3.9 above, is considered to be invalid. For an individual fish, time variation in fish target strength is observed as the fish flexes and changes its angle to the horizontal. The work of Clay and Heist (1984) suggests that the time variation in echo amplitude from an individual fish may be statistically described by the Ricean distribution. For fish large in terms of wavelengths this Ricean distribution reduces to the Rayleigh distribution. Clay and Heist consider that Rayleigh statistics may be used when the fish are more than 25 wavelengths long. Note that this criterion is somewhat arbitrary, and that the statistics of the echo amplitudes from fish do depend on how active the fish are. In Stanton and Clay (1986), the requirement for Rayleigh statistics is relaxed, for active fish, to requiring that the fish be more than 15 wavelengths long.

For a group of large fish, the echo amplitude will vary in time independently for each fish according to a Rayleigh pdf. Therefore, at a single instant in time, the echo amplitude will vary from fish to fish according to this distribution. The assumption of a Rayleigh distribution of echo amplitudes has been used by Denbigh (1984) to derive a revised formula for the second normalized moment of intensity. It is assumed in the derivation that the mean echo amplitude is the same for each fish. The formula is

$$\langle N \rangle = \frac{2}{\langle I^2 \rangle / \langle I \rangle^2 - 2} \quad (3.11)$$

The derivation of this formula has also been published in Denbigh and Weintraub (1986). A reprint of this paper is included in the appendix.

This can be compared with equation (3.9a) reproduced here with  $\langle N \rangle$  the subject.

$$\langle N \rangle = \frac{1}{\langle I^2 \rangle / \langle I \rangle^2 - 2} \quad (3.9a)$$

The formula derived assuming a Rayleigh distribution of echo amplitudes from individual scatterers therefore predicts exactly twice the number of scatterers compared to the formula which assumes identical amplitudes from each scatterer. It is important to note that equation 3.11 assumes a Rayleigh distribution of instantaneous echo amplitudes but it also assumes equal time averaged echo amplitudes. This means that the equation is only strictly valid for a shoal consisting of identical fish. It is the time variation of individual fish echoes due to movement and flexing of the fish that produces the Rayleigh distribution of amplitudes and not the distribution of fish sizes.

#### 3.4 Application to Sonar.

Equations 3.9a and 3.11 may be used to estimate  $\langle N \rangle$  from measurements of the first two moments of backscattered intensity. (Attention will hereafter be confined to the second normalized moment as Wilhelmij and Denbigh (1984) have established, both in the theory discussed in section 3.2, and experimentally that this is the best estimator of number density.) In the sonar context  $\langle N \rangle$  is the mean number of scatterers in the sonar resolution cell and the number density may be estimated by dividing the estimate of  $\langle N \rangle$  by the the volume of the resolution cell.

### 3.4.1 Ideal Case.

Consider a sonar emitting a pulse of frequency  $f_0$  or angular frequency  $\omega_0$ . Assume the pulse length of the sonar is  $\tau$  and that the pulse is perfectly square. The interpulse period is  $T$ .

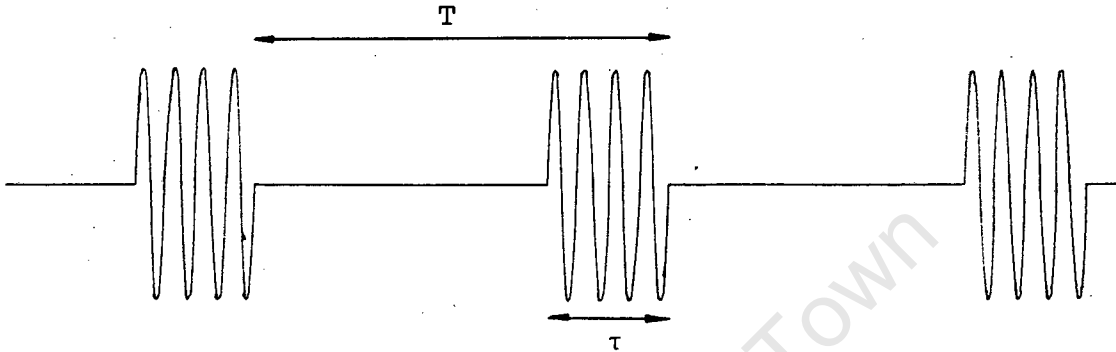


Fig 3.1 Pulse train of pulse length  $\tau$ , interpulse period  $T$ .

Assume that the sonar transducer has an idealized beam pattern such that there is uniform insonification within the beam and zero insonification elsewhere. The beam may be characterized by the solid angle  $\Omega$  which it subtends. (In the discussion of the echo integrator in chapter 2, the symbol  $\Psi$  has been used for the solid angle subtended by that equivalent beam which returns the same energy as the real beam.  $\Omega$  is used for that solid angle which it is assumed is subtended by the real beam; this may be chosen on different criteria to  $\Psi$ , such as the nulls or 3 dB points of the real beam.) For a rectangular transducer, the beam has beamwidths in two dimensions of  $\beta$  and  $\phi$ . For a disk transducer, the beam may be approximated in the far field by a cone of apex angle  $\theta$ .

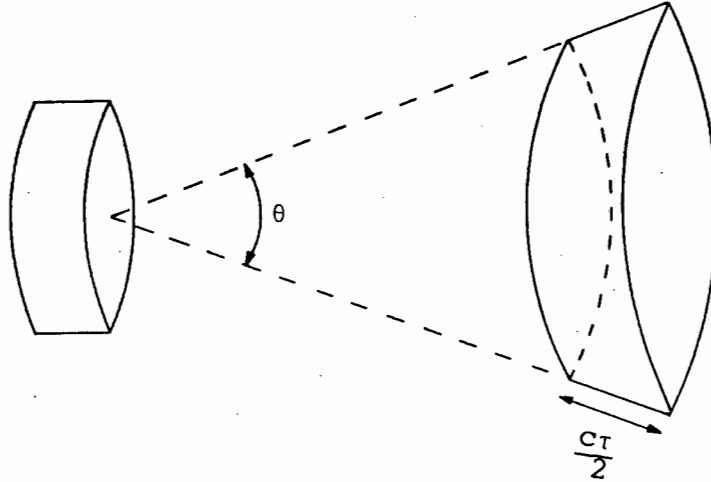


Fig 3.2 Far field beam pattern of idealized disk transducer, showing the reverberating volume.

(The assumed beam shape of the rectangular transducer, used in the experiments of Wilhelmij, is given in figure 4.2.)

The relationship between apex angle  $\theta$  and solid angle  $\Omega$  for the disk transducer is given by:

$$\Omega = 2\pi (1 - \cos \theta/2) \quad (3.12)$$

For a rectangular transducer the solid angle may be approximated for small angles by:

$$\Omega = \beta\phi \quad (\beta, \phi \text{ in radians}) \quad (3.13)$$

A time  $t$  after transmission the receiver will be receiving backscatter from a range  $R = ct/2$  where  $c$  is the speed of sound. The surface area of the pulse can be determined from the definition of the solid angle as:

$$S = \Omega R^2 = \frac{1}{4} c^2 t^2 \Omega \quad (3.14)$$

The range resolution of a pulse of pulse length  $\tau$  is given by  $c\tau/2$ . The sonar resolution cell is thus a conical segment of length equal to the range resolution with the area of the further face equal to  $S$ . For fairly large ranges and short pulses the beam divergence over a pulse length may be neglected and the pulse approximated by a cylinder of length equal to the range resolution and cross sectional area  $S$ . The resolution cell volume as a function of time is thus:

$$\begin{aligned} V(t) &= S \times \text{range resolution} \\ &= (c^3 \Omega \tau t^2) / 8 \end{aligned} \quad (3.15)$$

For scatterers with a constant number density,  $\rho$ , scatterers per cubic metre the number of scatterers in the resolution cell volume at any time  $t$  is given by:

$$\langle N \rangle(t) = \rho V(t) = 1/8 (\Omega \rho c^3 \tau t^2) \quad (3.16)$$

The time dependence of the resolution cell volume thus introduces a time dependence in  $\langle N \rangle$  even in a situation where the number density is constant. Because the normalized moments are related to  $\langle N \rangle$  through equations 3.9 or 3.11 the normalized moments will also have a time dependence, hence the intensity pdf will change with time. This is equivalent to saying that the intensity statistics of the backscatter are non-stationary. The equations 3.9 and 3.11 have been derived assuming stationary statistics; that is, it is assumed that the spatial distribution of scatterers may be described by a Poisson distribution with  $\langle N \rangle$  constant in time.

Wilhelmij and Denbigh (1984) proposed a scheme for dealing with non stationary statistics in the backscatter. Consider an ensemble of  $k$  independent backscatter envelope records. Each record is sampled  $m$  times from  $t_1$  to  $t_m$ . Each envelope record consists of independent samples due to a change in the scatterer distribution between pings or in the position of the transducer relative to the scatterers. It is thus possible to obtain the  $n$ th normalized sample moment using  $k$  samples, one from each record, taken at a single time  $t_i$ . If the density is constant for each record, the statistics of these samples will be stationary, since the resolution cell size corresponding to each will be equal. Each envelope record thus yields a sample at each time  $t_i$  corresponding to a particular value of  $N(t_i)$  and  $V(t_i)$ . One sample from each record may be used to calculate the normalized moments corresponding to a particular range. This results in an estimate of  $\langle N \rangle$  for this range.

There are now  $m$  estimates of the normalized moments and  $m$  corresponding estimates of  $\langle N \rangle$ . Two ways of averaging the estimates are apparent. Perhaps the most obvious approach is the following. For each estimate  $\langle N \rangle(t_i)$  the density estimate corresponding to that time may be calculated according to:

$$\rho_i = \frac{\langle N \rangle(t_i)}{V(t_i)}$$

These estimates can then be averaged to give an estimate of scatterer number density for the particular distribution. It was pointed out by Wilhelmij (1983) that this method of averaging is unsatisfactory. This is because the estimate of  $\langle N \rangle$  is the reciprocal of the difference between the normalized moment, which is greater than 2, and 2. A number of estimates of the normalized moment will be scattered about the true value. Symmetrical scatter in the estimate of the normalized moment will result in asymmetrical scatter in the individual estimates of  $\langle N \rangle$ . This is because a low value of

the moment results in a greater error in  $\langle N \rangle$  than a high value of the moment. The estimate of density obtained using the method of averaging described above will therefore probably result in an estimate of density which will have a high bias. Furthermore, if a particular estimate of the normalized moment turns out to be very nearly equal to 2, the corresponding value of number density may be enormously large and this may cause the averaged number density to be meaningless.

Another possibility is to average the normalized moments  $\langle I^2 \rangle / \langle I \rangle^2 (t_i)$  to obtain an averaged value  $\langle I^2 \rangle / \langle I \rangle_{av}^2$ .

A simple expression for the number density in terms of this averaged normalised moment in the case of Rayleigh scatterers is obtained by substituting equation 3.11 in equation 3.16, solving for the second normalized moment, and averaging over the time interval  $t_1$  to  $t_m$ :

$$\frac{\langle I^2 \rangle}{\langle I \rangle^2}(t) = 2 + \frac{2}{0.125\rho\tau c^3\Omega t^2}$$

hence

$$\frac{\langle I^2 \rangle}{\langle I \rangle_{av}^2} = \frac{1}{t_m - t_1} \int_{t_1}^{t_m} 2 + \frac{2}{0.125\rho\tau c^3\Omega t^2} dt$$

The estimate of mean number density is then given by integrating and solving for  $\rho$ . This gives:

$$\rho = \frac{2}{(\langle I^2 \rangle / \langle I \rangle_{av}^2 - 2)(0.125c^3\Omega\tau t_1 t_m)} \quad (3.17)$$

This formula is identical to that derived in Wilhelmij and Denbigh (1984) apart from the factor of 2 in the numerator

due to the assumption of a Rayleigh distribution of echo amplitudes.

A different form of equation 3.17 may be obtained by writing:

$$\langle N \rangle_{av} = \frac{2}{\langle I^2 \rangle / \langle I \rangle_{av}^2 - 2}$$

then

$$\rho = \frac{\langle N \rangle_{av}}{0.125c^3 \Omega \tau t_1 t_m} \quad (3.18)$$

### 3.4.2 Non Ideal Case.

A practical sonar cannot have all the ideal properties of the sonar described above. The practical constraints which are imposed must often be dealt with in an approximate way.

#### 3.4.2.1 Non Ideal Beam Pattern.

In practice the beam patterns of the transmit and receive transducers are non-uniform within the first nulls, and sidelobes exist outside these nulls. A polar diagram of the beam of a disk transducer is given in fig 3.1.

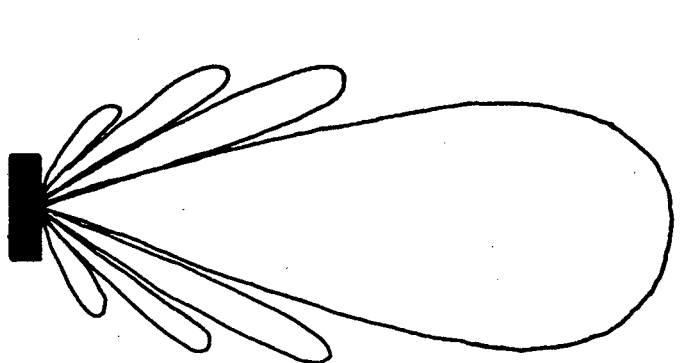


Fig 3.3 Beam pattern of non-ideal disk transducer.

The appearance of the beam in three dimensions is the solid of rotation of the beam pattern about the indicated axis of symmetry. The effect of this non-ideal beam is twofold. Firstly individual scatterers will have variation of their insonification depending on exactly where they are positioned in relation to the transmit transducer. The position of the scatterers in the beam of the receive transducer will affect the magnitude of the received echo. Secondly, scatterers in the sidelobes will also contribute to the backscatter, increasing the effective value of  $\langle N \rangle$ .

The problem of the amplitude variation of the scatterers due to their position in the beam is equivalent to the scatterers being non-identical. It will affect the pdf of individual echo amplitudes.

#### 3.4.2.2 Non-ideal Pulse Shape.

The ideal pulse was assumed to be perfectly rectangular with pulse length  $\tau$ . In fact, the limited bandwidth of the transducer will cause the pulse to have a shape that is not exactly square.

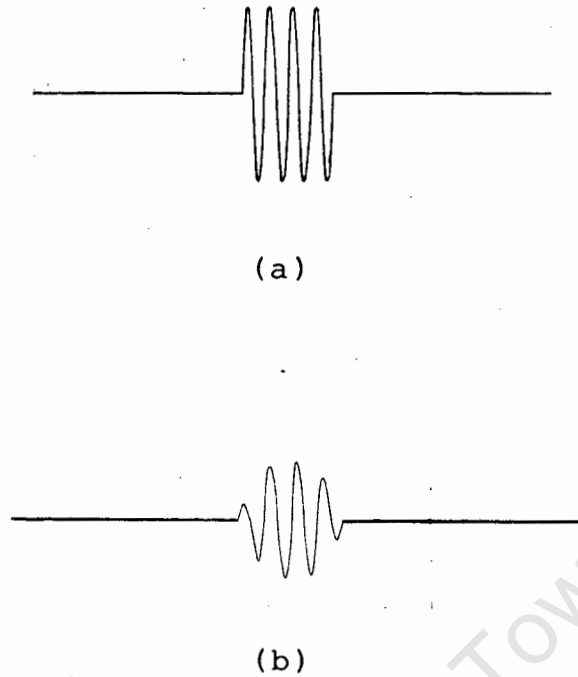


Fig 3.4 (a) Ideal pulse shape and (b) Bandlimited pulse shape.

Where the pulse is bandlimited, the choice of pulse length is to a certain extent arbitrary. For example,  $\tau$  may be chosen as the time from null to null, or the time between half power (3dB) points, or it may be chosen as the length of that square pulse which has the same energy as the bandlimited pulse. The pulse also introduces some shading of the individual echo amplitudes so that the assumption of uniform scatterers is further invalidated. The pdf of Rayleigh scatterers may also be affected.

## 4.0 EXPERIMENTAL WORK

The statistical method of determining number density for the case of identical random scatterers has already been experimentally investigated in a small tank experiment (Wilhelmij, 1983, Wilhelmij and Denbigh, 1984). The methods used, encouraging results obtained and conclusions of the experiment are briefly reviewed in this chapter. Further work is then described including a computer simulation of the scattering problem applied to an examination of the effect of sample size and shoal density on the random uncertainties in the estimates, and an attempt to apply the method to live fish in an aquarium tank. Experimental difficulties associated with the live fish experiment prevented conclusive results being obtained, but the equipment designed for the experiment is described for possible future use. The experiment was intended primarily to test the applicability of the assumptions made about the statistics of echo strengths from individual fish in the derivation of equation 3.11.

### 4.1 Previous Experimental Work on the Scatterer Model.

Wilhelmij (1983) performed experiments on a model fish shoal which consisted of expanded polystyrene spheres strung on monofilament nylon. The nylon was suspended from a series of fifteen frames which were then stacked so as to form a three dimensional scatterer distribution.

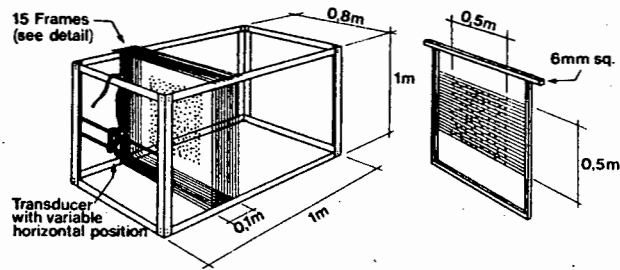


Fig 4.1 The random scatterer model (Wilhelmij and Denbigh, 1984).

The transducers used in the experimental sonar were rectangular ceramics of dimensions 24mm by 3mm. Their resonant frequency was 500kHz with a bandwidth of approximately 60kHz. The configuration was bistatic in that separate transmit and receive transducers were used. The nominal beamwidths of both transducers were quoted as 37 degrees and 4.6 degrees in the short and long dimensions respectively. These are 3dB values predicted from the theoretical directivity pattern of the transducer. The far field beam shape was then assumed to be a fan beam of uniform intensity within the 3 dB beam and zero intensity elsewhere.

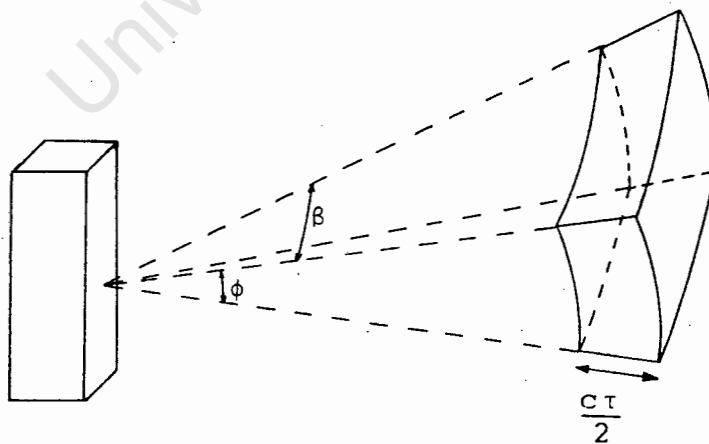


Fig 4.2 Diagram of ideal far field beam shape, for rectangular transducer, showing resolution cell.

A pulse of 12 microseconds was used and assumed to be rectangular. (The bandwidth of the transducers is quoted as 60kHz, so it is likely that the transmitted acoustic pulse was somewhat longer, and that it is not rectangular.)

The polystyrene spheres used as scatterers ranged in diameter from 3mm to 6mm. For the purposes of this experiment they were assumed as being equivalent to approximately identical scatterers in order for equations 3.9 to be valid. The densities investigated varied between 9 500 scatterers per cubic metre and 76 000 scatterers per cubic metre in steps of 9 500 scatterers per cubic metre. This corresponds to a  $\langle N \rangle_{av}$  ranging from 0.625 to 5.00 scatterers per resolution cell in steps of 0.625 scatterers per resolution cell. (The midrange resolution cell volume was used in evaluating  $\langle N \rangle_{av}$  from the scatterer number density.) Independent envelope records were obtained by moving the transducer position relative to the frames and by shuffling the order in which the frames were stacked.

The backscatterer from the volume distribution of scatterers was processed by passing the receive transducer output through a tuned receiver amplifier and an envelope detector. The envelope was digitized and loaded into an HP-85 microcomputer for processing. The envelope samples were squared by the computer to produce samples proportional to the intensity of the received backscatter and the sample normalized moments calculated. The averaging described in section 3.4.1 was performed in order to overcome the effect of non stationary statistics caused by the beam divergence. An equation similar to 3.17, but with a 1 in the numerator, derived on an assumption of identical scatterers was used to produce estimates of number density from the averaged normalized moments. These could then be compared with the true number densities. Estimates of  $\langle N \rangle_{av}$  using the third and fourth normalized moments were also produced.

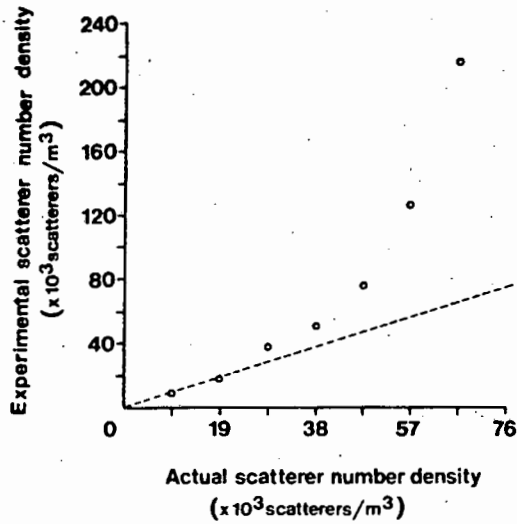


Fig 4.3 Graph of estimates of number density obtained from the second normalized moment against true number density (Wilhelmij and Denbigh, 1984).

At low number densities, there is good correspondence between the actual and experimentally estimated values of  $\rho$  with a rapidly increasing positive error in the estimate as the value of  $\rho$  gets larger. The increasing error in the estimates is attributed to two factors; the existence of multiple scattering, and background reverberation from the monofilament nylon used in the construction of the model.

Multiple scattering occurs when in addition to first order echoes from individual scatterers being added at the receiver, a component of scatter which is the result of the echo from one scatterer being intercepted and rescattered by another target. The statistics back at the transducer will then reflect more scatterers than are actually present in the resolution cell. The effect of multiple scattering naturally increases with number density and results in a positive error.

The nylon monofilaments themselves produce a significant amount of backscatter which adds to the backscatter from the polystyrene

$\langle N \rangle$ , a greater number of intensity samples is required for convergence of the higher order sample moments to their true values. This is consistent with the theoretical findings of Wilhelmi and Denbigh.

The conclusions of the experiment may be summarized as follows: The requirements for accurate prediction are that multiple scattering should not be appreciable, that a substantial number of independent envelopes are available for a sufficiently accurate statistical analysis, that the number of scatterers in the resolution cell is small and that the measurements of intensity should not be influenced significantly by noise or reverberation. Although it would appear that the higher order moments should provide a better estimate of number density due to their greater deviation at any particular value of  $\langle N \rangle$  from the Rayleigh moment of  $n!$ , the second normalized moment is in fact shown to be the most accurate estimator of number density since compared to higher order moments, fewer samples are needed for its convergence. This work concentrates on estimates by the second moment for this reason.

#### 4.2 Simulation.

The error analysis by Wilhelmi and Denbigh was intended to give an indication of the relative random errors in the estimates of number density by the statistical method using the different normalized moments. The sensitivities of estimates of  $\langle N \rangle$  to errors in the various normalized moments are obtained by differentiation of equations 3.9 with respect to the relevant normalized moment. Expressions for the standard deviations in the normalized moments are then obtained as a function of the number of samples used to produce them, assuming Rayleigh statistics in the amplitude samples. The use of differentiation means that the analysis is only valid for small errors, and the assumption of Rayleigh statistics implies that  $\langle N \rangle$  is fairly large. The statistical method depends on the presence of

non-Rayleigh statistics in the amplitude samples and is therefore most useful when  $\langle N \rangle$  is fairly small. The error analysis is therefore useful as a way of comparing the errors in estimates by the various moments under various conditions, but it does not provide an indication of the absolute error in a particular estimate. The expression for the standard deviation in the second normalized moment, derived under the above assumptions is reproduced here,

$$\text{s.d.} \left\{ \frac{\langle I^2 \rangle}{\langle I \rangle^2} \right\} = \frac{2}{\sqrt{M}} \quad (4.1)$$

where  $M$  = number of independent intensity samples.

and the error in  $\langle N \rangle$ , by substitution of this expression in the formula relating incremental errors in  $\langle N \rangle$  and  $\langle I^2 \rangle / \langle I \rangle^2$  is,

$$\left| \frac{\Delta \langle N \rangle}{\langle N \rangle} \right| = \frac{2}{\sqrt{M}} \langle N \rangle \quad (4.2)$$

This is the approximate error in  $\langle N \rangle$  when the error in the normalized moment is exactly the standard deviation. It is not the standard deviation of the estimate of  $\langle N \rangle$ .

Notice that the expression for standard deviation of  $\langle I^2 \rangle / \langle I \rangle^2$  is independent of the value of  $\langle I^2 \rangle / \langle I \rangle^2$ . This is a consequence of the assumption of Rayleigh statistics in the backscattered amplitude samples, implying that the second normalized moment is always exactly 2. Considering as an example, the case where  $\langle N \rangle = 0.5$ , then  $\langle I^2 \rangle / \langle I \rangle^2 = 4$ , and then one would not expect the standard deviation to be identical to the case where  $\langle N \rangle = 10$  and  $\langle I^2 \rangle / \langle I \rangle^2 = 2.1$ .

The simulation of the scattering model described here gives a useful indication of the absolute random errors which can be expected when using a certain number of samples of backscatter to

calculate the second normalized moment and hence to predict a particular value of  $\langle N \rangle$ .

#### 4.2.1 Simulation design.

In this simulation, a random scatterer distribution is modelled and the intensity envelope derived by vector addition of backscatter from individual scatterers at the transducer. The statistics of the intensity envelope can then be calculated.

The steps of the simulation are as follows. Firstly, a spatial distribution function is chosen. This gives the probability of a resolution cell having a particular number  $N_i$  of scatterers within. The true mean number of scatterers  $\langle N \rangle$  is a constant parameter of the spatial distribution. The Poisson volume distribution with constant  $\langle N \rangle$  is chosen initially, but the simulation may be later extended to other distributions. The formulation of the Poisson distribution is given in chapter 3 (equation 3.10). Using a Poisson distribution with constant  $\langle N \rangle$  is equivalent to having stationary statistics in the calculated backscatter samples. Under these circumstances, the fractional error in a particular estimate of density is identical to the fractional error in the corresponding estimate of  $\langle N \rangle$ .

In the simulation a large number of independent samples of intensity are to be calculated. Each intensity sample corresponds to scatterers in a single resolution cell. If there are  $M$  resolution cells, then there will result  $M$  independent samples of intensity. The spatial distribution is used to predict how many cells  $Q$  contain each particular number  $N_i$  of scatterers.

$$Q = C P_{\langle N \rangle} (N_i) \quad (4.3)$$

It is adequate to calculate Q for a limited number of values of  $N_i$  because a value of  $N_i$  is soon reached where  $P_{\langle N \rangle}(N_i)$  is negligible. For example, with  $\langle N \rangle = 6$ , and  $N_i = 30$ ,

$$P_6(30) = \frac{6^{30} e^{-6}}{30!} = 2.1 \times 10^{-12}$$

which is clearly a negligible probability. For the purposes of the simulation, a maximum value of  $N_i = 30$  is used.

Once the number of cells with each particular number  $N_i$  of scatterers has been calculated, the cells are analysed each yielding a random intensity sample. The random position of scatterers within each particular cell is equivalent to each contributing to the signal at the transducer with a random phase uniformly distributed between 0 and  $2\pi$ . This phase may be generated by multiplying a random number uniformly distributed between zero and one, by  $2\pi$ .

The amplitudes of the backscatter from the individual scatterers are assigned according to the assumed amplitude distribution. An identical amplitude may be assigned to each (amplitude arbitrarily equal to one), or the amplitudes may be generated randomly according to a Rayleigh distribution. The Rayleigh distributed random numbers are generated using a formula derived by Fox (1984):

$$y = \sqrt{2 \sigma^2 \ln 1/x} \quad (4.4)$$

where:

x = uniformly distributed random number in the range 0 to 1.

y = Rayleigh distributed random number with standard deviation  $\sigma$ .

For each cell the signals from each scatterer are summed vectorially. This is achieved by decomposing the amplitude of each scatterer into in-phase and quadrature components:

$$A_{I_i} = A_i \cos \phi_i \quad (4.5)$$

$$A_{Q_i} = A_i \sin \phi_i$$

These components are then summed and the resultant amplitude sample obtained.

$$A_i = \sqrt{(\sum A_{I_i})^2 + (\sum A_{Q_i})^2} \quad (4.6)$$

The intensity sample is then the square of this amplitude sample. Each cell thus produces a single intensity sample giving M samples in total. The sample normalized second moment may then be calculated, using equation 3.6, giving an estimate for  $\langle N \rangle$ .

Each time the simulation is performed, a single estimate of  $\langle N \rangle$  will result which will differ from the true value since only a finite number of samples are used to produce it. If a number of such simulations, or trials, are performed, these estimates may be analysed to produce the sample mean of the estimates and the sample standard deviation.

Programs to simulate scattering by identical and Rayleigh scatterers have been written. The programs were written in FORTRAN and run on the UNIVAC 1100 system of the University of Cape Town. The URAND library routine was used to generate uniformly distributed pseudo random numbers. Listings of the programs may be found in the Appendix.

### 4.2.2 Simulation results and discussion.

As a test of the program a run consisting of 100 trials for the case  $\langle N \rangle = 10$  has been performed. 3000 independent samples of intensity were used to produce each estimate of  $\langle N \rangle$ . Each amplitude was identical. In this case the relatively high value of  $\langle N \rangle$  should make the assumption of a Rayleigh distribution in the backscattered amplitude a reasonable approximation so that equation 4.1 above should be valid.

The results of the run are:

Mean normalized moment = 2.0888, s.d. = 0.0376

Mean estimate of  $\langle N \rangle = 19.4$ , s.d. = 44.8

Note that a second identical run, consisting of 100 trials, and with a different seed for the random number generator, produced a similar mean moment and standard deviation, indicating that 100 trials is a sufficient number for the sample values of the average moment and its standard deviation to be an accurate reflection of their true values. Wild fluctuations in the estimates of  $\langle N \rangle$  occur in this region, because the difference between the normalized moment and 2 is so small; the mean estimate of  $\langle N \rangle$  and its sample standard deviation have not converged as a result.

The predicted standard deviation of the normalized moment by equation 4.1 is 0.0365 which ties up well with the sample standard deviation above. The high positive error of the estimate of  $\langle N \rangle$  and the very high value of the associated standard deviation are the result of the very high sensitivity of the estimate of  $\langle N \rangle$  to negative errors in the normalized moment when it is very close to two. This causes some of the trials to produce, by chance, estimates of  $\langle N \rangle$  with very large positive errors. For example, one of the trials in this run produced a normalized moment of 2.0023, resulting in a single estimate of  $\langle N \rangle = 434!$

A similar run, but with  $\langle N \rangle = 4$  was performed. This time, the sample standard deviation of the normalized moment was 0.534. Equation 4.1 predicts that this standard deviation should again be 0.0365, independent of  $\langle N \rangle$ . The discrepancy indicates that the assumption of Rayleigh statistics in the backscatter amplitudes used in deriving equation 4.1 is invalid in this case. The statistical method is most useful when  $\langle N \rangle$  is of this order, so the simulation should be useful in this range for producing estimates of error in  $\langle N \rangle$  for a given sample size.

Runs have been performed for 500 and 1000 independent samples (cells) per trial for identical and Rayleigh scatterers, and various values of  $\langle N \rangle$ . Results are presented in the following tables.

True $\langle N \rangle$	Mean $\langle I^2 \rangle / \langle I \rangle^2$	Mean est. of $\langle N \rangle$	S.d. in est. of $\langle N \rangle$
0.10	11.81	0.10	0.007
0.50	3.915	0.53	0.071
1.00	2.944	1.10	0.205
2.00	2.439	2.69	1.51
4.00	2.235	-1.31	58.0

Table 4.1 Simulation results for identical scatterers. 100 trials were performed at each density using 500 samples per trial.

True $\langle N \rangle$	Mean $\langle I^2 \rangle / \langle I \rangle^2$	Mean est. of $\langle N \rangle$	S.d. in est. of $\langle N \rangle$
0.10	11.80	0.10	0.006
0.50	3.931	0.52	0.053
1.00	2.962	1.06	0.157
2.00	2.473	2.22	0.507
4.00	2.225	4.72	5.49

Table 4.2 Simulation results for identical scatterers. 100 trials were performed at each density using 1000 samples per trial.

True <N>	Mean $\langle I^2 \rangle / \langle I \rangle^2$	Mean est. of <N>	S.d. in est. of <N>
0.10	20.76	0.11	0.014
0.50	5.914	0.52	0.078
1.00	3.926	1.08	0.197
2.00	2.982	2.19	0.616
4.00	2.486	4.80	2.04

Table 4.3 Simulation results for Rayleigh scatterers. 100 trials were performed at each density using 500 samples per trial.

True <N>	Mean $\langle I^2 \rangle / \langle I \rangle^2$	Mean est. of <N>	S.d. in est. of <N>
0.10	21.43	0.10	0.011
0.50	5.952	0.51	0.068
1.00	3.948	1.05	0.150
2.00	2.975	2.11	0.369
4.00	2.507	4.23	1.14

Table 4.4 Simulation results for Rayleigh scatterers. 100 trials were performed at each density using 1000 samples per trial.

An interesting point to notice is that there is a positive bias in the estimates of  $\langle N \rangle$ . The reasons for this are outlined in section 3.4.1 and earlier in this section. The nonsensical estimate in the last line of table 4.1 is caused by a negative error in the normalized moment in some of the trials causing the moment to go slightly below 2; the resulting estimate of  $\langle N \rangle$  for that trial will be very large negative and it causes a bias in the mean. Notice that a sensible value for the estimate of  $\langle N \rangle$  can still be obtained by substitution of the mean normalized moment in equation 3.9. This is the reason why the normalized moments are averaged rather than the individual estimates of density in the scheme outlined for handling non-stationary statistics. It may be concluded that, with 500 samples of intensity

backscatter from identical scatterers, estimates when  $\langle N \rangle \geq 4$  are unreliable.

A second point to note is that the higher the number of samples used in producing estimates of  $\langle N \rangle$  result in a decrease both in the bias and in the standard deviation. In the case of Rayleigh scatterers, it can be seen that, if 500 samples are available for averaging, an estimate with a random uncertainty of 25% or less corresponds to a value of  $\langle N \rangle$  less than about 2. If 1000 samples are available, the value of  $\langle N \rangle$  should be less than about 4.

It is also interesting to note that in general the Rayleigh formula produces estimates with a smaller standard deviation. This is due to the fact that at any particular value of  $\langle N \rangle$  the difference between the normalized moment and 2 is twice as great in the Rayleigh case, and, as a result, the sensitivity of the estimates to negative errors in the normalized moments is not as great.

An examination of the effect of different numbers of samples on the mean and standard deviation for the particular case of  $\langle N \rangle = 2$  has been conducted. Rayleigh scatterers were assumed. The results are presented in table 4.5.

M	Mean est of $\langle N \rangle$	S.d. in est. of $\langle N \rangle$	$\frac{\text{S.d.} \times \sqrt{M}}{\langle N \rangle^2}$
100	3.10	1.85	4.62
200	2.59	0.90	3.18
400	2.36	0.63	3.15
800	2.20	0.46	3.25
1000	2.11	0.41	3.24
1400	2.08	0.31	2.90
2000	2.03	0.28	3.13
2500	2.09	0.21	2.62
3000	2.05	0.19	2.60

Table 4.5 Estimates of  $\langle N \rangle$  and standard deviations for different numbers of intensity samples. Actual  $\langle N \rangle = 2$ .

The standard deviation and bias are seen to decrease with increasing number of samples. The last column in the table shows that, as predicted by equation 4.2, the standard deviation of the estimate of  $\langle N \rangle$  is approximately inversely proportional to the square root of the number of intensity samples available for averaging. However, it is clear that the equation does not predict the actual standard deviation. In the case of  $\langle N \rangle = 100$ , the error in  $\langle N \rangle$  is large, so that the differentiation used in deriving equation 4.2 is invalid, and this proportionality does not hold.

### 4.3 Experiments on Caged Live Fish.

Experiments were conducted on live fish in tanks. These were situated at the premises of the Sea Fisheries Research Institute in Sea Point, Cape Town. The objective of the experiment was to confirm the applicability of equation 3.11 to live fish large in terms of wavelengths. A number of problems were encountered which resulted in the work being inconclusive. The equipment devised and suggested procedure will be described for possible future use.

#### 4.3.1 Experimental Arrangement.

The fish, which were anchovy, were confined in a cylindrical tank of 3 metre diameter with suitable water aeration equipment. They were fed three times a day. The exact number of fish varied from day to day as individuals died and the tank was periodically restocked. The size distribution of fish also changed as the population grew, and new members were introduced. The tank could support a maximum of approximately 150 individuals.

One of the requirements of the experiment was to insonify the fish from the same aspect as would the depth sounder in a survey vessel. This required that the sonar transducer be positioned above the fish pinging downwards thus insonifying the fish from the dorsal aspect.

#### 4.3.2 The Experimental Sonar.

The sonars used in acoustic surveys of fish are usually depth sounders with disk transducers. In order to model the practical fisheries problem the transducers used in the experimental sonar are disk ceramics of effective diameter 14mm and resonant frequency 615kHz. This frequency corresponds to a wavelength of 2.4mm and gives an approximate 3dB beamwidth angle of 0.17 rad or 10 degrees. This is suitable for use in a small tank without unacceptable reverberation from the sides of the tank. The typical length of fish in the tank was approximately 12cm, which is equivalent to 50 wavelengths. This should make Rayleigh statistics an acceptable model of the fish as acoustic scatterers if the spread in size of the individual specimens is not too large.

In a shipborne echo sounder, frequencies are generally limited to 120kHz so that propagation losses due to sound

absorption and dispersion are acceptable. This corresponds to a wavelength of 12.5mm so that the fish are then 10 wavelengths long. This is the major difference between the tank model and the situation at sea, since the relatively long wavelength makes Rayleigh statistics only a rough description of individual echo amplitudes.

The far field distance of the experimental transducer is given by the square of the diameter divided by the wavelength and is equal to 81mm. It is important that the experiments are conducted with the scatterers in the far field of the transducer.

The sonar required in addition to the transducer a transmitter, a tuned receiver amplifier with a front end matched to the transducer, an envelope detector, and digital sampler (or digital storage oscilloscope) suitably interfaced to a microcomputer. A block diagram of the experimental given is given here. Included in the block diagram is a circuit which enables computer control of the transmitter pulse length. Reasons for the inclusion of this are given later. Circuit details are in the appendix.

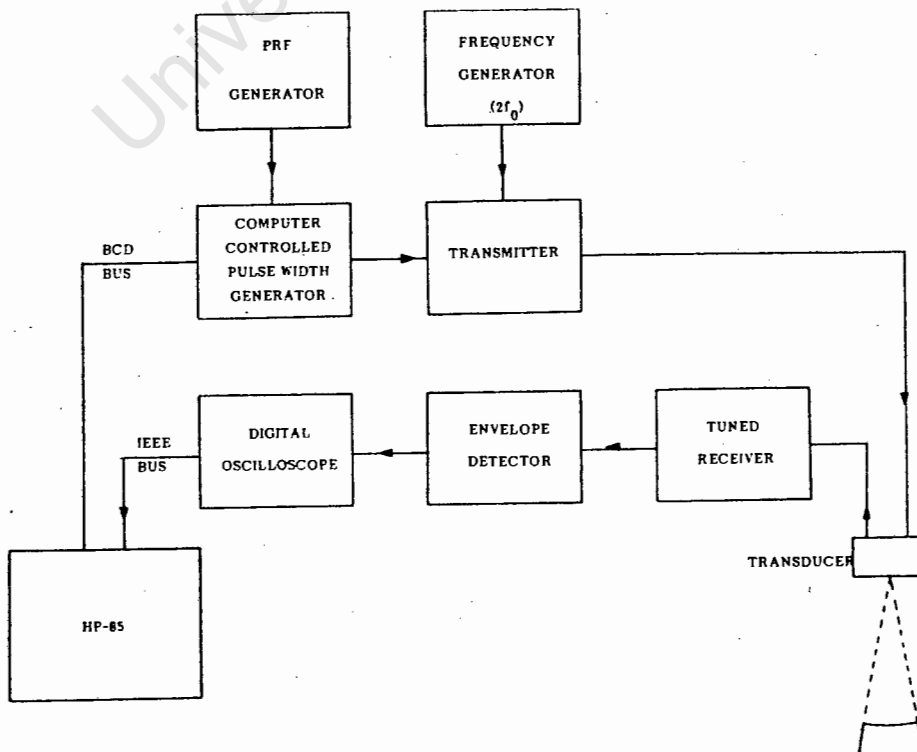


Fig 4.5 Block diagram of experimental sonar.

### 4.3.3 Experimental Difficulties.

Two major difficulties were encountered in the performance of this experiment. The first of these concerned fish behaviour, and the second involved the problem of experimental control.

The experiment requires that the fish maintain a constant number density, constant range, and remain reasonably consistently within the beam of the transducer. When the fish swim unrestricted in the tank, they spend very little time in the sonar beam. An attempt was made to confine the fish to the perimeter of the tank by building a cylindrical cage which would exclude them from the centre of the tank. The cage is 2.5 metres in diameter which leaves a 25cm channel on either side of the tank.

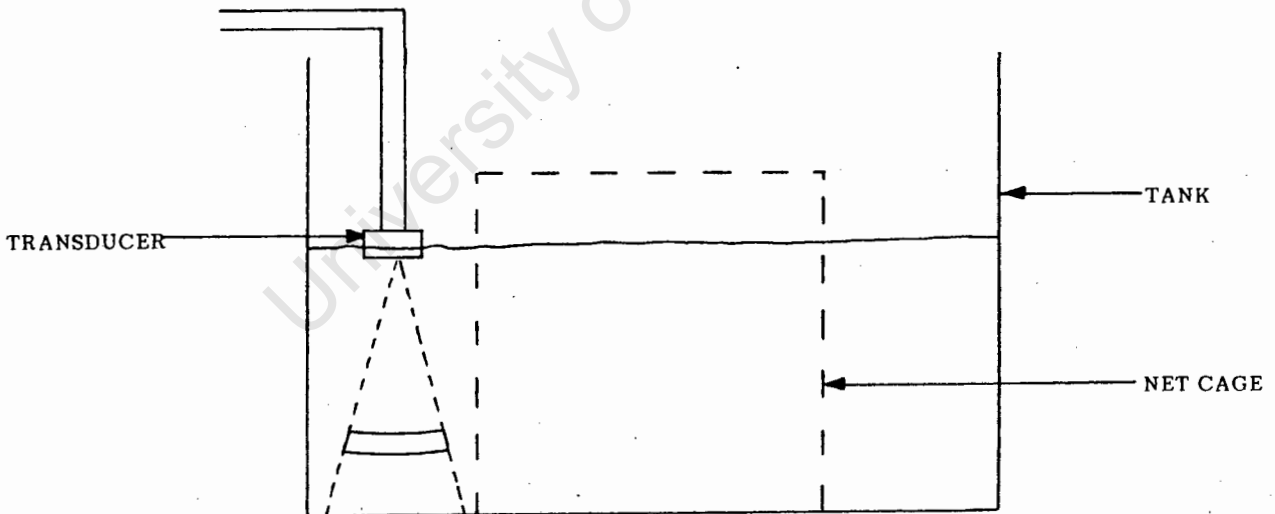


Fig 4.6 Sketch of experimental arrangement.

With the central cage included, the fish swam in circles around the edge of the tank. However the fish still did not maintain a constant range or constant density.

The second problem is one of experimental control. The statistical method produces an estimate of number density

and this estimate must then be compared with the actual number density. There must be some way of determining this actual density. One possibility which was considered is to photograph the shoal under the transducer at the time of each ping. This is an impractical idea, however, since to record the number density of the shoal photographically requires two photographs for each ping. Only a few independent samples are available from each ping; the one hundred or so pings required for an experiment required impractically many photographs.

A method of experimental control has been proposed which does not require photographs. This uses the facility designed into the experimental sonar which enables the pulse length to be adjusted under computer control. Each ping is paired with an adjacent ping; the first has a long pulse length, and overlapping targets are present in the resulting envelope record. The second ping has a much shorter pulse length such that mostly single targets are present in the envelope record; echo counting may be used to determine the number density in this case. The idea is that software be used to sort the data on the basis of the echo counting method into pings with constant average number density. The statistical method can then be used with the corresponding records containing overlapping targets to produce an estimate of number density for each group of envelope records. This estimate can then be compared with one obtained using the echo counting method.

Unfortunately to the difficulties associated with controlling the behaviour of the fish prevented the extraction of any useful results from these experiments in the time available.

## 5.0 SEA TRIALS

This chapter describes testing of the statistical method of fish stock assessment on real fish shoals at sea. The tests were performed during ten days of a routine survey cruise of the Sea Fisheries Research Institute Vessel "R.S. Africana". The aims of the cruise are discussed and the acoustic survey equipment on the ship is described. Tests in which the statistical method was used in tandem with the echo integration system, used routinely on the ship for acoustic surveys of fish, are described.

Difficulties inherent in the application of the statistical method to real fish became apparent during the tests. These both limited the application of the method to suitable groupings of fish, and caused errors in the estimates. Estimates of number density by the statistical method are presented and compared with those produced for the same fish by the echo integrator. Reasons for discrepancies are discussed.

### 5.1 The Cruise.

The 1985 Anchovy Recruitment Survey cruise was conducted by the Sea Fisheries Research Institute (SFRI) from 20th May to 10th June 1985. The primary objective of the cruise was to estimate the biomass of anchovy recruits on the South and West coasts of South Africa, by acoustic survey. Anchovy recruits are young fish, spawned in the previous spawning season (October 1984) from which the major portion of the exploitable resource is drawn. Secondary objectives included the elucidation of recruitment and migration patterns between SWA and the Cape East coast, investigation of other fish of commercial importance, and investigation of the relationship between anchovy distribution and the hydro-biological environment (Hampton, 1985).

The cruise consisted of two legs. Leg 1 was conducted along the South coast between Cape Town and Mossel Bay (20th to 30th May). The second leg surveyed the West coast between Cape Town and

Luderitz. The author participated in the first leg of the cruise.

The first leg consisted of two phases. Phase 1 followed a zig-zag grid within 30 nautical miles of the coast starting in False Bay and stretching eastwards to the Gouritz river mouth, near Mossel Bay. The purpose of this was to determine the approximate eastward limit of the anchovy. Phase 2 was the return trip, consisting of an intensive rectangular survey grid, set up on the basis of the observations made during phase 1. Figs 5.1 (a) and (b) are maps of the survey pattern for leg 1 (Hampton, 1986).

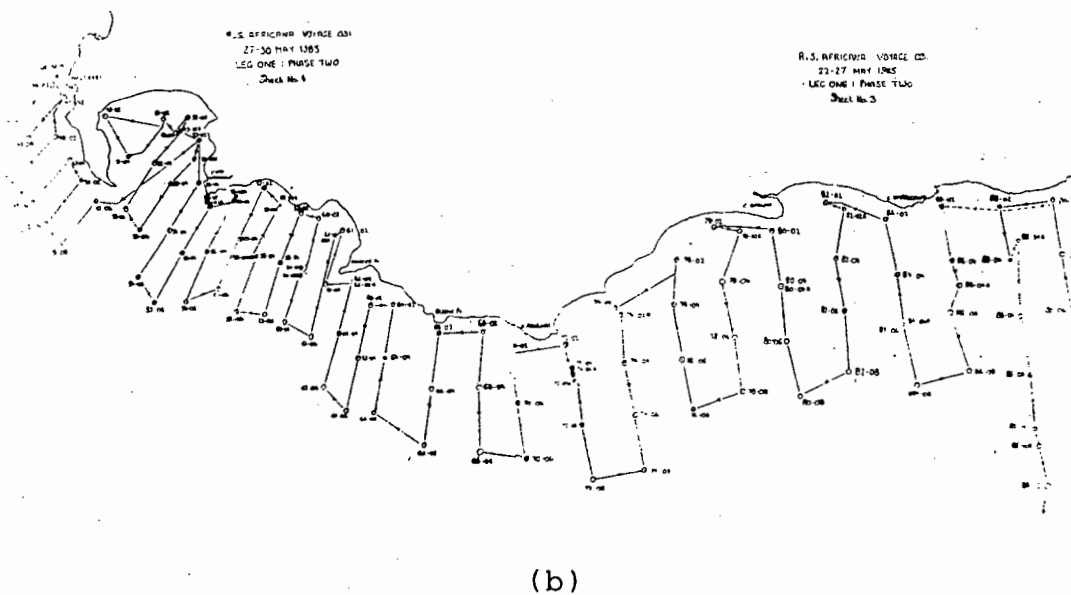
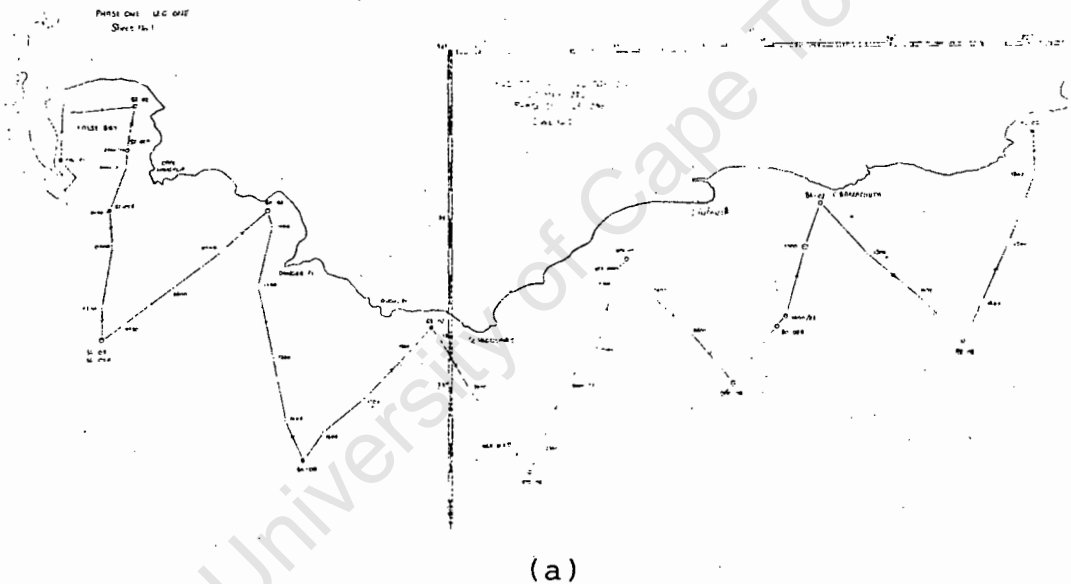


Fig 5.1 Survey pattern for leg 1 (a) phase 1, and (b) phase 2.

Stations are positioned every 10 nautical miles on the survey grid and are numbered with a four figure code such as 52-02. The ship steamed at a constant survey speed of approximately twelve knots between stations. At each station the ship stopped and various tests such as temperature and salinity profile measurements and plankton sampling were performed. The times between stations are referred to as intervals.

Species identification was by means of midwater trawls conducted on suitable shoals which had been detected acoustically. Measurements of fish length and mass distribution were also made on the trawl samples. Shoals often consisted of a mixture of species; the species composition of the shoal could then be estimated from the trawl sample, and this used in calculating the biomass of each species alone.

## 5.2 The Acoustic Systems of the R.S. Africana.

The method used for acoustic survey of pelagic fish by SFRI is echo integration. The ship is equipped with a custom built digital echo integrator and data logging system (Scientific Computers, 1985). Two commercial Simrad echo sounders, the EK-S 38 and the EK-S 120, operating at 38kHz and 120kHz respectively are the sonars used in the system. The sounders have conventional electrostatic paper recorders. Analog time-varied gain amplifiers are provided in the sounder receivers to correct for the dependence of echo strength on range. The TVG correction may be set to a  $40 \log R + 2\alpha R$  characteristic when using the sounder in a single target situation, or to  $20 \log R + 2\alpha R$  when it is applied to a volume distribution of scatterers. This latter law is the appropriate setting when using the sounder output as the input to an echo integrator (see chapter 2). The outputs from the echo sounders are fed into the two channels of the custom data processing and logging system.

Each channel of the data logging system has several components. A high quality linear envelope detector takes as its input the output one of the echo sounders and gives its envelope as input to the sampling analog to digital converter. The envelope is sampled at times corresponding to the depth channel of interest. This channel is set by the operator to exclude spurious signals such as the bottom echo. (The bottom echo can also be excluded automatically, since it is far greater in amplitude than the sound scatter from biological targets. If the signal exceeds a certain threshold, indicating the bottom echo, a pulse is sent to the data logger instructing it to stop recording that ping. Part of the signal sent to the paper recorder at this time is blanked, resulting in a distinct white line on the paper record; the sounder is said to be in white line mode under these circumstances. The second time around bottom echo from the previous ping remains a problem and may have to be excluded by the operator). To save recording magnetic tape and to prevent the quantity of data collected becoming unmanageable, only pings containing sound scattering layers of interest are logged. This is achieved by "vetting" the signal. Vetting is achieved by the equipment having an operator selectable "threshold and window". The signal must exceed the threshold voltage for a length of time at least as long as the window for it to be recorded.

The digitized signal is recorded under the control of a microcomputer. Two modes of operation are provided; integrator mode and fast logging mode. In the fast logging mode, the complete sampled signal envelope (providing it has satisfied the vetting criteria) is recorded on tape for later analysis. In integrator mode each ping is squared and integrated over 1m depth channels and only the results of this integration is logged. Other data which may be useful, such as details of sounder settings, are stored as a header for each integration interval.

The echo squaring and integration processes are performed numerically on the sampled signal if it satisfies the signal vetting criteria. Scaling of the A/D output and inclusion of the

sonar calibration parameters are also performed by the computer. The output from the echo integrator is in the form of a printout giving the mean volume backscattering strength from various depth channels selectable within the sampled range by the operator. The mean volume backscattering strength is denoted MVBS on the printout, is given in dB's, and is identical to the quantity  $\overline{S}_V$  discussed in chapter 2. The integrator is reset and started by the operator at the start of the integration interval. At the end of the interval, the integrator is stopped, and a printout of mean volume backscattering strength from the various depth channels obtained. The grand mean volume backscattering strength (GMVBS) is the mean volume backscattering strength for the interval, integrated and averaged over the entire water column. The integration may be performed as many times as required with different start and stop depths and channel widths, but when the integrator is restarted again for the next interval a reset occurs and all the signal information about the previous interval is lost. An example of the output from the integrator is given in Fig 5.2.

```

SYSTEM A
  DATE 1985 5 27 22: 2
  POSITION 34 35.33 S 19 15.60 E

A. INTEGRATION START DEPTH : 17
B.                   STOP DEPTH : 21
C.                   CHANNEL WIDTH : 1

Min. Start Depth : 5 (m)
Max. Stop Depth  : 30 (m)
Grand VBS        :- 46.5 dB
Grand MVBS       :- 46.6 dB

Depth Range (m)      VBS (dB)      MVBS (dB)
  17 - 18            - 47.1        - 47.2
  18 - 19            - 46.8        - 46.9
  19 - 20            - 46.3        - 46.4
  -20 - 21          - 46.1        - 46.2

```

Fig 5.2 Output of echo integrator.

The difference between volume backscattering strength and mean volume backscattering strength is that the VBS is calculated on the basis of pings which satisfy the vetting threshold and window and are hence logged. The MVBS is the VBS scaled by the factor (pings logged/total pings sampled). It is assumed that pings which are not logged contain no energy and hence that this scaling produces the MVBS averaged over all pings in the interval. This processing assumes that ship speed and the sounder pulse repetition frequency are constant over the interval.

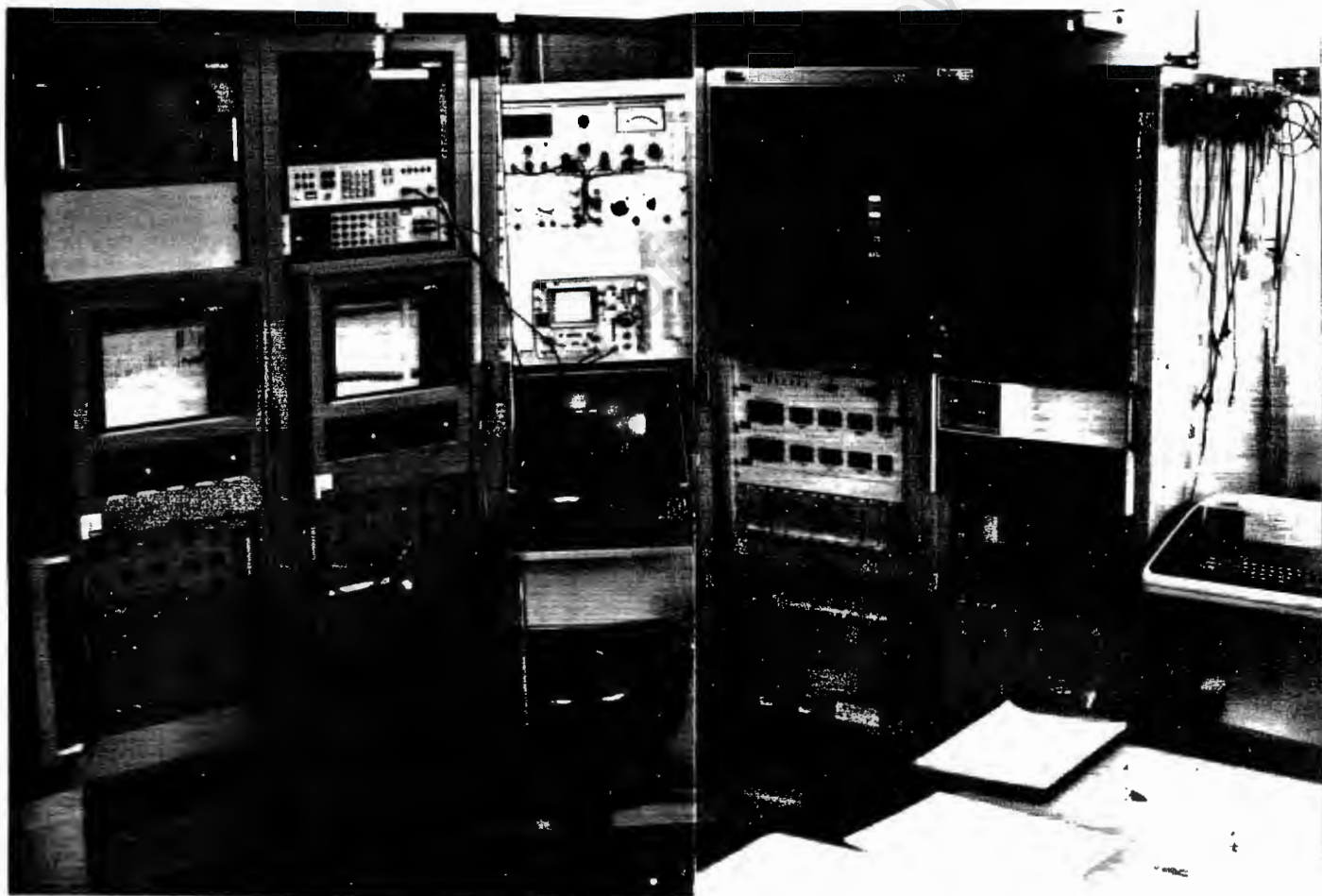
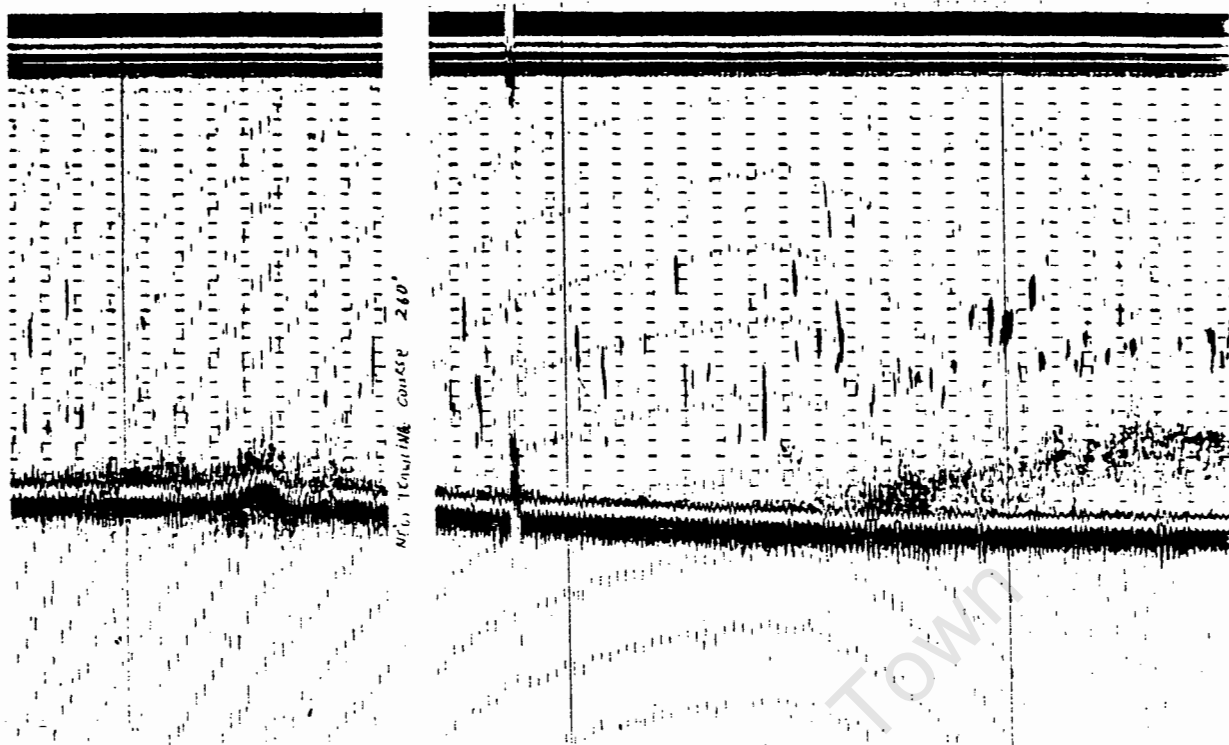


Fig 5.3 Acoustics laboratory on Africana, showing, from left, depth sounders and electrostatic recorders, operators console, custom digitizing and logging equipment, control computer and printer.

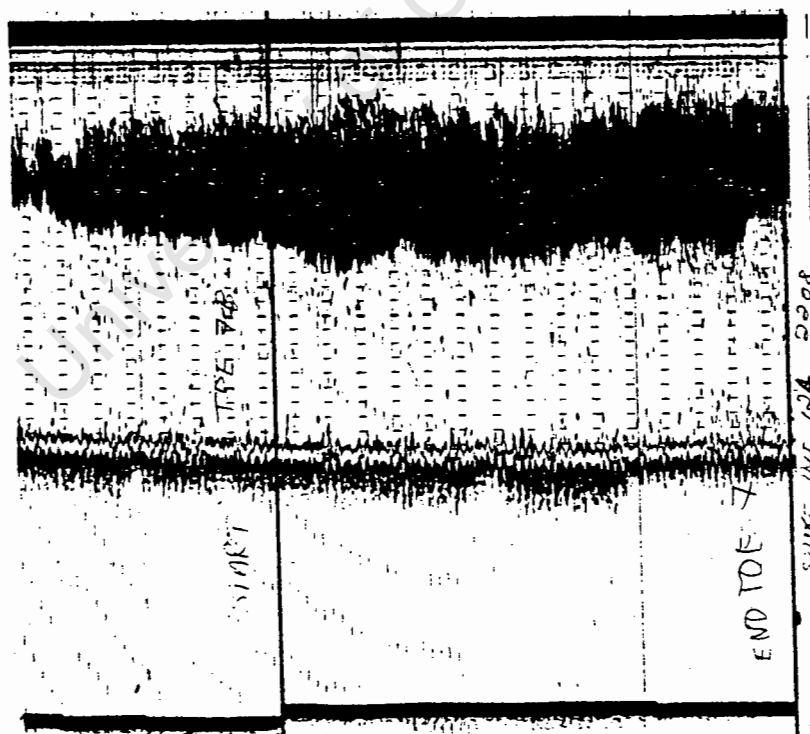
### 5.3 Initial Observations.

There is a great difference in the behaviour of anchovy between day and night. During the day, the fish concentrate into extremely dense shoals, which at a survey speed of 12 knots are insonified typically for 10 pings before the ship passes over them. The number of independent envelope samples obtained from such a short insonification is inadequate for working out a statistically significant normalized moment of intensity. At night, however, the shoals spread out into a more diffuse sound scattering layer which may be several miles in horizontal extent and several tens of metres deep. Such a distribution of fish is termed an aggregation (McElroy, 1977). Examples of sounder charts of shoals during the day and aggregations at night are given here (Figs 5.4 (a) and (b)):

It became clear from these observations that experiments using the statistical method on anchovy could only be conducted at night, when suitable aggregations were present.



(a)



(b)

Fig 5.4 Sounder charts of (a) typical daytime anchovy shoals and (b) typical nighttime aggregation.

## 5.4 Experimental Equipment and Procedure.

### 5.4.1 Additional Hardware.

The lack of suitable software for reading data logged in fast logging mode from the tape drives of the custom acoustic logging system, coupled with the unfamiliarity of the computer system aboard the Africana, encouraged the design of a microcomputer system for logging acoustic envelope records. For this purpose, a HP-85 microcomputer system, floppy disk drives and a digital oscilloscope with a built in IEEE-488 interface (the Philips PM3305) were taken on board for the duration of the cruise.

The signals from the echo sounders can be extracted after tvg and detection. They thus represent the signal envelopes. They are used as the input to the digital oscilloscope. Pings are then digitized on the oscilloscope and downloaded to microcomputer via the IEEE-488 interface. The computer then stores the envelope records on floppy disk. The major drawback of this system is its speed limitation. Because of the time taken to transfer a recorded ping from oscilloscope to computer, every ping can not be logged. In addition it is necessary to store a group of records to disc periodically when the internal memory of the computer fills. The disk access time for this operation is about a minute.

### 5.4.2 Software.

A requirement of the record logging software is flexibility of range and record length. The depth and thickness of the aggregation can be observed and entered before the commencement of an experiment. It is a requirement of the statistical method that samples only be taken while the pulse is completely contained in the volume distribution of scatterers. It is therefore necessary to examine the

aggregation on the sounder record, decide on the depth range of interest, and take samples only from this depth range. This is illustrated in figure 5.5.

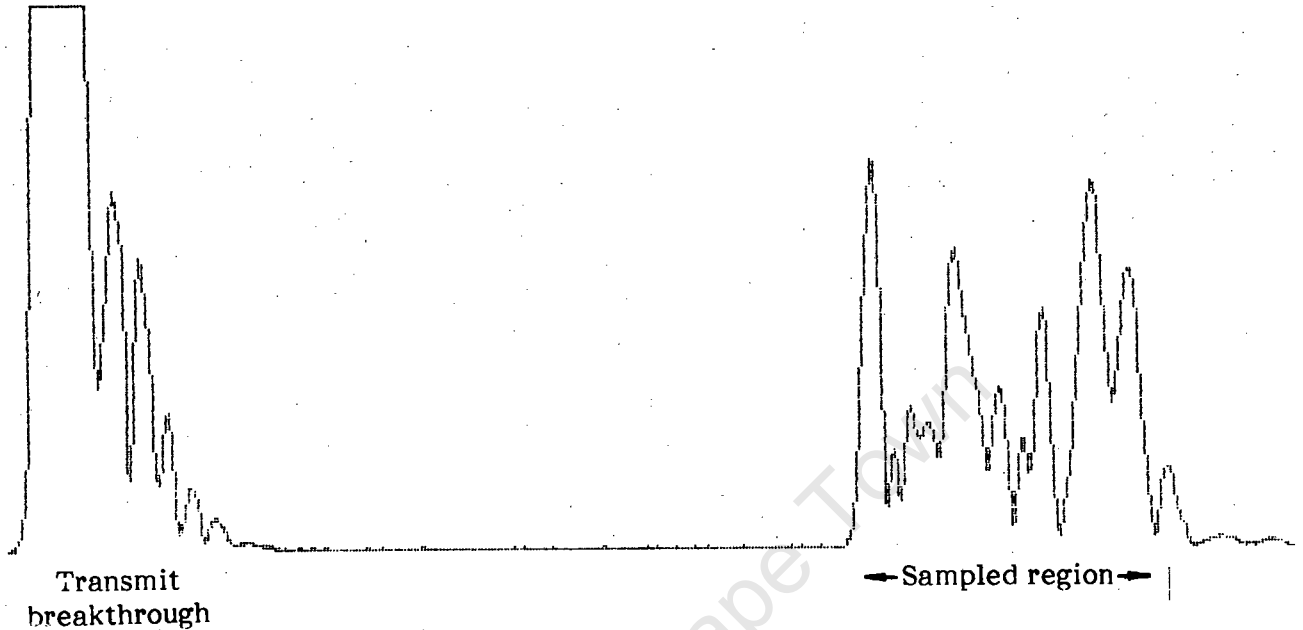


Fig 5.5 A typical envelope record, showing sampled region.

The program therefore runs under operator control. The sample range of interest is entered by the operator. This depends on the depth range of interest, and the time base setting of the oscilloscope. A listing of the sampling program is given in the appendix.

Figure 5.6 is a block diagram of the experimental configuration.

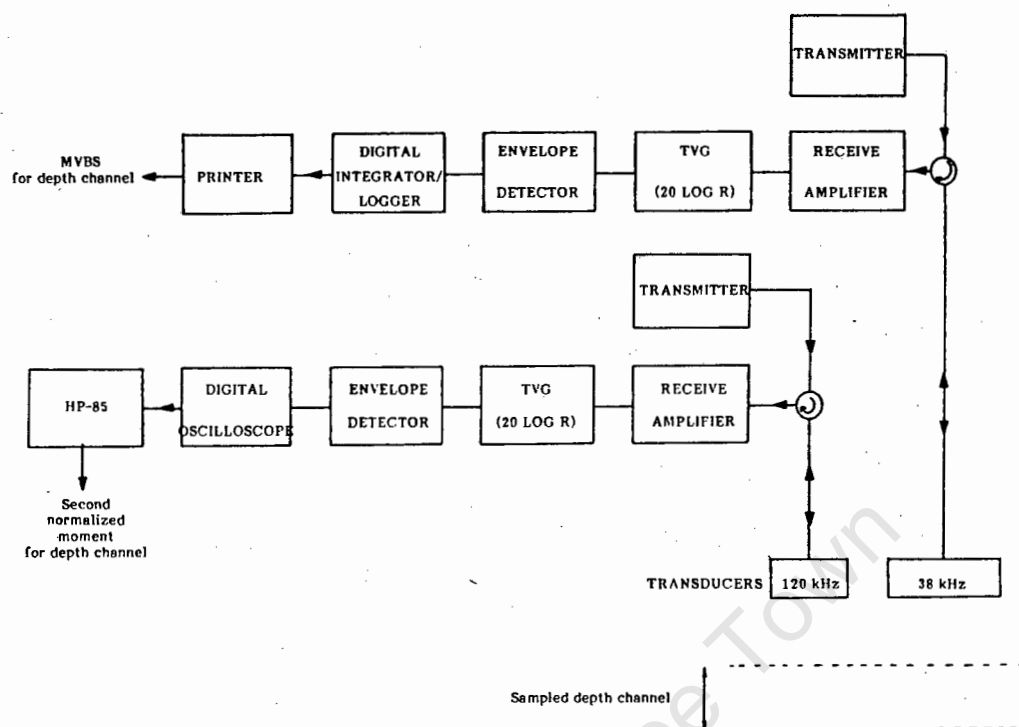


Fig 5.6 Block diagram of the experimental setup on the Africana, showing the digital oscilloscope and HP-85 computer connected to the 120 kHz sounder and the echo integrator running in tandem off the 38 kHz equipment.

### 5.5 Target Density Experiments.

Over the course of the cruise, experiments were conducted on six suitable aggregations. (Experiments were started on other sound scattering layers, but the ship passed over them before sufficient samples could be taken for a density estimate to be realistically calculated.) Both the EK-S 120 and the EK-S 38 sounders were used. The echo integrator was run concurrently with the first five experiments which were conducted on nighttime aggregations which consisted predominantly of anchovy, although other species such as redeye were present. Experiment 6 was conducted on a species called lightfish, which is present in extensive distributed aggregations off the West coast. They were encountered on the last day of the cruise off Hout Bay. The lightfish target does have disadvantages; it is not a very strong

target, and no target strength data are available for it so that echo integrator estimates cannot be calculated. A sounder chart of the lightfish aggregation is of interest, and is given here (Fig 5.7).

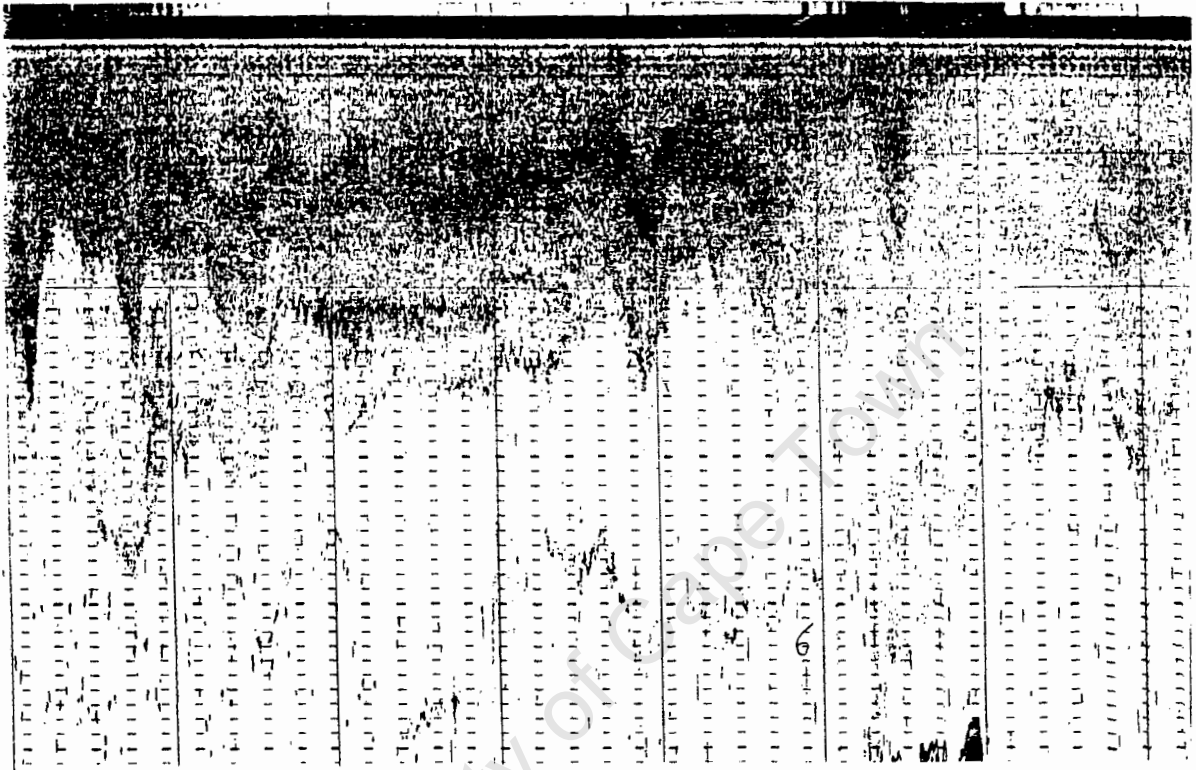


Fig 5.7 Sounder chart of lightfish sound scattering layer. The vertical and horizontal extent of the aggregation may make it a suitable target for assessment by the statistical method. The limited dynamic range of the electrostatic paper recorder may give the impression that this is a fairly strong target; examination of the echoes on an oscilloscope shows that they are fairly weak.

The conditions under which the experiments were conducted and other relevant data such as sounder settings and fish mass and length statistics have been summarized in table 5.1.

Expt number	Date Time	Sounder	Pulse length (ms)	Number of envelope records	Indep. samples per rec.	Sampled depth channel (m)	Trawl data length (cm)	mass (g)	Comments
1	22-05 03h30	EK-120	0.14	80	18	22.5-24.4	-	-	No trawl conducted
2	25-05 21h30	EK-120	0.31	30	16	30-33.75	11.6	14.7	Sampled channel changed after ping 30, because aggregation depth had changed. Pulse length increased at the same time.
			0.68	70	7	33.75-37.5			
3	27-05 18h20	EK-120	0.68	50	7	15-18.75	11.5	15.0	Groups of 10 pings at different pulse lengths interleaved.
			0.86	50	6				
4	27-05	EK-38	0.80	100	6	16.9-20.6	10.7	12.0	
5	28-05 18h20	EK-38	0.80	50	12	15.0-22.5	12.5 10.0	15.1	Aggregation density decreased after ping 50. (Observed on sounder chart.) Bimodal length in trawl sample.
6	30-05 04h10	EK-120	0.68	50	37	19.0-38.0	-	-	Lightfish target-no trawl. No means of estimating target strength. Groups of 10 pings at different pulse lengths interleaved.
			0.86	50	29				

Table 5.1 Summary of experimental conditions.

As many samples as the extent of each aggregation would allow were taken. In experiments 2, 3 and 6 two different pulse lengths were used to establish the effect of an increasing resolution cell volume on the estimate of number density. The estimates should be consistent despite the change in the average number of insonified scatterers, and consequent change in the intensity statistics caused by an increased  $V$ . In experiments 3 and 6, groups of ten pings at the two pulse lengths were interleaved, in an attempt to minimize the effect of possible long term differences in average shoal density. In some cases the ship passed over the aggregation before the maximum of 100 envelope records could be collected.

It is desirable to have as many independent samples per ping as possible. This indicates that a short pulse length should be

used which is also favourable insofar as the statistical method produces its most accurate estimates at low values of  $\langle N \rangle$ . However with weak targets too short a pulse length results in an unacceptable degradation in the signal to noise ratio of the returned signal. The target was examined in each case and a decision as to what pulse length to use was made on the basis of the nature of the target.

### 5.5.1 Analysis of Data, and Results.

The averaged second normalized sample moment has been calculated in each experiment. The averaging scheme, described in section 3.4.1 in which the range slice sampled in each experiment is divided up into range bins and the sample moments calculated for each is used. The sample moments are then averaged giving  $\langle I^2 \rangle / \langle I \rangle_{av}^2$ . Equation 3.17 is then used to produce an estimate of number density for the sampled range slice

The trawl data are needed in the calculation of the echo integrator estimates and to facilitate their comparison with the estimates from the statistical method. The mean length is used in the calculation of the mean target strength of the fish which is used in obtaining the echo integrator estimate. The target strength is estimated by the target strength to length relationship (Hampton, 1985; Halldorsen and Reynisson, 1982):

$$TS_g = -(10.9 \log \bar{l} + 50.9) \quad \text{dB re 1 g}$$

$\bar{l}$  = mean length of fish in the relevant trawl sample in cm. In practice, the distribution of fish lengths in the trawl is measured, and the peak of the distribution or the modal length is taken as the mean.

It should be appreciated that the use of this formula, which is critical to the accuracy of the estimate of mass density by the echo integrator method, is an interim measure. The measurements of target strength used in producing this fit were performed in situ on Icelandic herring. Data on fish target strength for the South African anchovy are simply unavailable. Justification for the use of this fit is that herring are considered to be physiologically comparable to anchovy and the experiments were performed at 38 kHz, which is the echo integrator frequency used on the survey. Further, the estimate of total acoustic biomass produced on recent surveys ties up well with independent estimates produced from biological surveys (Hampton, 1986).

The echo integrator biomass estimate is given by a modified form of equation 2.8:

$$\overline{\rho}_{\text{int}} = f \cdot 10^{0.1(\text{MVBS} - \overline{\text{TS}}_g + \Delta\text{TVG})} \quad \text{g/m}^3 \quad (5.1)$$

This equation has been modified to account for two of the imperfections of the echo integration equipment used. The factor  $\Delta\text{TVG}$  (dB) is included to account for imperfections in the sonar TVG amplifier which does not exactly follow the ideal  $20 \log R + 2\alpha R$  curve. This factor is extracted from a table giving the TVG error appropriate to a particular range slice. (Hampton, 1985). The factor  $f$  corrects for the miscalibration of the integrator during certain parts of the cruise. The use of  $\overline{\text{TS}}_g$  in this formula indicates that the resulting biomass estimate is in units of grams per cubic metre. As such this estimate is unsuitable for direct comparison with an estimate of number density. The mean mass of the trawl sample is used to convert the biomass estimate to units of number of fish per cubic metre. This is done by simply dividing the mass density estimate by the mean mass of

the individuals in the trawl sample. If  $\bar{w}$  is the mean mass of the individuals in the sample in grams:

$$\bar{\rho}(\text{no}/\text{m}^3) = \frac{\bar{\rho}(\text{g}/\text{m}^3)}{\bar{w}} \quad (5.2)$$

The integration is performed over the same range slice sampled by the statistical method giving an estimate appropriate to that range slice. Experiment 1 corresponds to an aggregation for which no trawl data are available. The calculation of an integrator estimate has been accomplished by assuming average values for  $\bar{w}$  and  $\bar{l}$ . No means of estimating the target strength of the lightfish target is available and an echo integrator estimate can therefore not be produced.

Table 5.2 compares the estimates of number density produced by the two methods.

Experiment number	Pulse duration (ms)	Statistical estimate of (number/m <sup>3</sup> )	Integrator estimate of (number/m <sup>3</sup> )	<u>Integ. est.</u> Stat. est
1	0.14	0.55	1.57	2.9
2	0.31	0.076	0.34	4.5
3	0.68	0.24	0.67	2.8
4	0.80	0.43	1.19	2.8
5	0.80	0.26	0.68	2.6
6	0.68	0.52	-	-

Table 5.2 Comparison of estimates of number density by statistical and echo integration methods.

A linear regression of the statistical estimates onto the integrator estimates gives:

$$\rho_{\text{stat}} = 0.373\rho_{\text{int}} - 0.021 \quad (5.3)$$

This line is plotted together with the data points in fig 5.8.

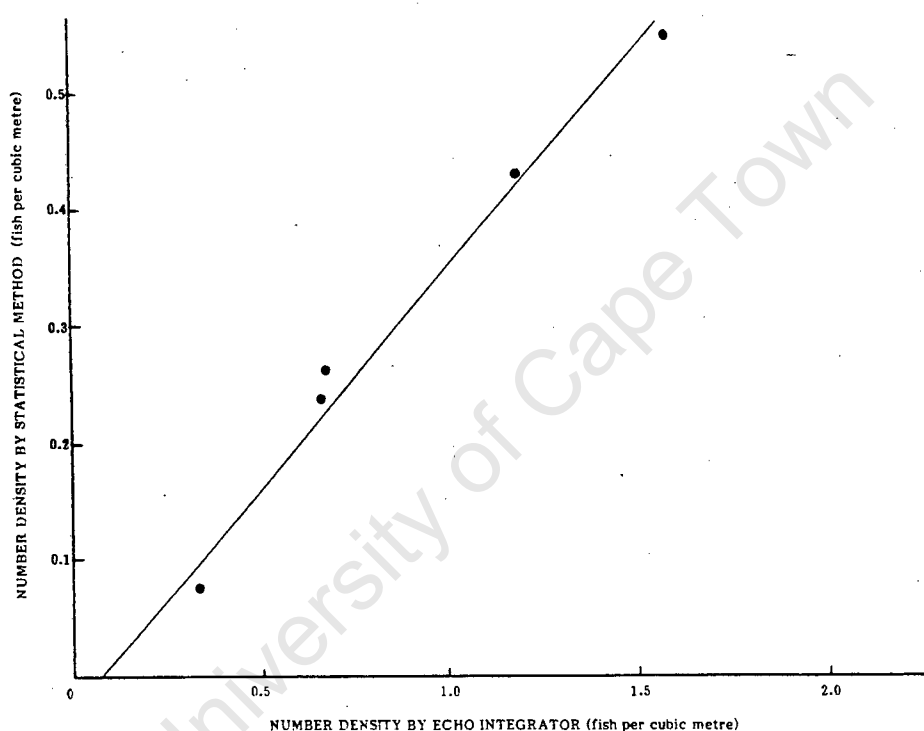


Fig 5.8 Graph of estimates of number density by statistical method against estimates by echo integration.

The integrator estimates are seen to be on average about 3 times larger than the statistical estimates. The correlation coefficient  $r = 0.98$  indicating a high degree of linearity between the estimates by the echo integrator and statistical methods. This value of  $r$  is based on only a few data pairs, however. It is possible therefore that it may be a spuriously high sample value even though there is no linear relationship between the two variables. A test of

significance, which is a test of the hypothesis that the true correlation coefficient is zero, may be performed. It can be shown that the statistic

$$z = \frac{\sqrt{(n-3)}}{2} \ln \left\{ \frac{1+r}{1-r} \right\} \quad (5.4)$$

has a sampling distribution which is approximately the standard normal distribution (Miller and Freund, 1965).

Substituting in this formula with  $n = 5$  and  $r = 0.98$ , a value of  $z = 3.25$  is obtained. From a table of the normal distribution it is seen that this correlation coefficient indicates that there is less than a 0.01% chance that the data are not linearly correlated

Table 5.3 shows that the statistical estimates of number density are relatively unchanged when the resolution cell volume is increased.

Experiment number	Pulse duration (ms)	Statistical estimate (number/m <sup>3</sup> )
2	0.68	0.07
3	0.86	0.28
6	0.86	0.75

Table 5.3 Statistical estimates of number density for increased pulse lengths.

### 5.5.2 Discussion.

Although the estimates produced by the two methods show little absolute agreement, an encouraging linearity is observed in the data; increasing statistical estimates are generally accompanied a proportional increase in integrator estimates.

The estimates for the first five experiments are calculated on the basis of 300-1440 independent samples (depending on shoal extent and the pulse length chosen). A random uncertainty will be associated with each estimate. An idea of the extent of this random uncertainty may be obtained by extracting a standard deviation from the simulation results. For the anchovy,  $\langle N \rangle_{av}$  varied between 0.6 and 1.7 for the five experiments. As a representative example of random uncertainty, from table 4.3 for  $\langle N \rangle = 2$  and 500 independent samples, the standard deviation of the estimate is found to be 0.6, or about 30% of  $\langle N \rangle$ . This random error cannot explain the large error in these results, especially as most of the results correspond to smaller  $\langle N \rangle$  and more independent samples in which case a smaller random uncertainty is expected. Also, such random errors cannot explain why the statistical estimates are consistently too low.

Four possible explanations for the discrepancy are proposed. The first one is a possible error in the echo integrator estimate, and the others describe reasons for errors in the estimates by the statistical method.

Firstly, as already discussed, there is limited confidence in the integrator result due to uncertainty in the measure of anchovy target strength. If the target strength data used are consistently a factor of two too low, the integrator estimate will be consistently a factor of two too high. The size of this error depends firstly on how much uncertainty there is in the the empirical formula relating target strength to length, and secondly on how applicable a formula based on herring target strength data is to anchovy. It is difficult precisely to quantify this error, but the discussion in section 2.3.1 shows that an error of a factor of 2 is possible.

The use of equation 3.17 to produce the statistical estimates implies that a Rayleigh distribution of individual echo

amplitudes is assumed. Clay and Heist (1984) specify that for Ricean statistics to be approximated by Rayleigh statistics, the fish should be more than  $25\lambda$  long. In Stanton and Clay (1986), the criterion for Rayleigh statistics if the fish are 'active' is that they are more than  $15\lambda$  long. Typically the anchovy were  $10\lambda$  long at 120 kHz so that the assumption of Rayleigh statistics in their backscattered amplitudes is not strictly valid. Secondly, the derivation of this equation also assumes that the time average of the echo amplitudes from each fish are equal requiring that the fish should all be of equal length. Although the fish do tend to collect in constant age groupings, there is always some spread in lengths and this assumption is also not strictly valid. The variation in acoustic intensity across the beam of the echo sounder will also affect the statistics of the individual echoes.

Another source of error is the somewhat arbitrary choice of beam solid angle as that corresponding to the 3 dB beamwidth of the transducer when used as a transmitter only. A rough calculation using the fact that the theoretical one way beamfunction is proportional to  $2J_1(k\text{asin}\theta)/k\text{asin}\theta$  shows that if the beam solid angle  $\Omega$  had been calculated on the basis of the two way beampattern, it would have been less by a factor of 1.9, and the resulting density estimates would be greater by this factor.

The most likely explanation for the discrepancies is thought to be the effect that inhomogeneities in the aggregation density have on the statistical method estimate. The integrator is linear insofar as it will produce an estimate of mean biomass for an inhomogeneous aggregation. The statistical method does not have this fortunate property. Consider as a simple example that half of the samples correspond to a region where  $\langle N \rangle = 2$  and half to where  $\langle N \rangle = 4$ .

aggregation corresponds to a value of  $\langle N \rangle = 4$  and the other half corresponds to a value of  $\langle N \rangle = 8$ . A similar calculation to the one above shows that an estimate of  $\langle N \rangle = 3.6$  results.

This simple example indicates that macroscopic variations in aggregation density will result in the statistical estimate having a negative bias. It is therefore probable that at least part of the discrepancy between the statistical estimates and the integrator estimates can be attributed to this factor. The extent of the error can not be estimated without knowledge of the nature of the density distribution of each particular aggregation.

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## 6.0 CONCLUSIONS

This thesis has extended the theoretical and experimental work of Wilhelmi and Denbigh on the relationship between the statistical character of backscatter from random scatterers and the scatterer number density. Of the applications suggested for the work, attention has been confined to acoustic fish stock assessment. A discussion of current assessment techniques provides background and motivation for the use of this method.

The relationship between the normalized second moment of intensity and the number of scatterers contributing to the backscatter has been extended to situations where the scatterers are not identical. The statistical distribution of the amplitudes of backscatter from individual scatterers chosen is the Rayleigh distribution. Justification for this choice in terms of the application of the method to real fish is given.

The statistical method has produced encouraging results in the small tank experiment reported by Wilhelmi and Denbigh, when the number of scatterers in the resolution cell is small. These authors also derived theoretical expressions, under certain simplifying assumptions, for the expected random uncertainty in an estimate of number density produced with a given number of independent samples of intensity. A discussion of these formulae is given in this work, showing that the simplifications may not be valid in cases of interest. A computer simulation of scattering by identical scatterers, and scatterers with a Rayleigh distribution of scattering strengths, has been implemented and has been found useful in producing a better indication of the expected random uncertainty in an estimate.

Estimates of number density of fish aggregations at sea have been produced. In contrast to the echo counter, these estimates can be produced in the presence of overlapping echos. In contrast to the echo integrator, knowledge of the average target strength of

individuals in the shoal and accurate calibration of the sonar used are not required. In order to obtain sufficient samples to produce an estimate, shoals or aggregations extending over a large area are required. When used for the survey of anchovy, this means that assessment with the statistical method can only be achieved at night. This may not be the case with other species.

The estimates of number density produced in the sea trials have shown large discrepancies when compared with estimates produced by an echo integrator. The estimates are seen to be considerably and consistently too low. In part, such errors can be attributed to uncertainty in the echo integrator estimates, uncertainty in the calculation of resolution cell volumes, and random errors in the estimates due to the averaging of relatively few samples. However, the greatest source of error is considered to be caused by the sensitivity of the method to macroscopic variations in average number density. There is no reason to suppose that shoals or aggregations of fish should be homogeneous in density. The effect of variations in shoal density is to cause a low bias in the estimates produced by the statistical method. This is consistent with what has been observed in the sea trial experiments. The echo integrator is not susceptible to this error.

A method of overcoming this source of error has been suggested. It should be possible to divide the shoal into regions of constant average density based on measurements of received energy. An estimate of number density without bias may be produced for each region. Averaging over the whole shoal will then produce the final estimate.

Apart from the errors in the estimate, the major drawback of the statistical method is the requirement that many samples are needed to produce an estimate. This places requirements on the type of shoal or aggregation surveyed. The echo integrator is not limited in this way. For this reason, it may be useful to apply the statistical method in tandem with an echo integrator

when suitable targets are encountered. The resulting independent estimate of number density can be used to check the calibration of the echo integrator. If no calibration for the integrator is available due to lack of target strength data for the particular species being surveyed, an in situ calibration may be produced which can be used for the rest of the survey.

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## APPENDICES

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```

10 C*****
12 C
14 C
16 C
18 C For a distribution of scatterers, randomly distributed in volume
20 C according to a Poisson distribution with parameter <N>, this program
22 C calculates the expected distribution of values of N. For each of these
24 C 'cells' containing a particular number, N, scatterers, a phasor
26 C summation gives the resultant amplitude for that cell. Each of the N
28 C phasors is assigned an identical amplitude, arbitrarily equal to 1,
30 C and a uniformly distributed random phase. The calculation is performed
32 C for many cells giving the resultant amplitude for each. The square
34 C of this amplitude is then the intensity sample for that cell. The
36 C second normalized moment is then calculated and used to estimate <N>
38 C using equation 3.9a. The simulation is performed many times and the
40 C sample means of the normalized moment and <N> and their standard
42 C deviations are calculated.
44 C*****
46 C
48 C
90     DOUBLEPRECISION FACT,Q,AVERAG
95 C Arrays for storing amplitude, intensity and squared intensity samples
96 C and estimates of <N> and <I**2>/<I>**2 for each trial.
100     REAL A(4000),I(4000),I2(4000),MOM(100),EST(100)
105 C For averaging moments.
110     AVMOM=0
111 C For averaging <N>.
112     AVEN=0
115     PI=3.14159265
120 * Enter number of trials and <N>.
130     READ (5,*)ITOTAL,AVERAG,ISEED
131     WRITE(*,51)
135 51     FORMAT('IDENTICAL SCATTERERS')
136     PRINT*,'True <N> = ',AVERAG
137     WRITE(*,50)
140 50     FORMAT('    TRIAL    <I>        <I**2>        <I**2>/<I>**2    <N
150     &>')
160 *
170 * Trial loop.
180     DO 100 ITRIAL=1,ITOTAL
190         ENUM=0
200 * Initialize arrays.
210         DO 101 M=1,4000
220             A(M)=0.
230             I(M)=0.
240             I2(M)=0.
250 101     CONTINUE
260 * Total number of cells in CELLS.
270     CELLS=1000.
280 * Determine number of cells with zero fish.
290     Q=CELLS*EXP(-AVERAG)
300     IZERO=INT(Q)
310 * I(M) is the intensity from the M'th cell.
315 C IZERO cells will have zero intensity.
320     DO 102 M=1,IZERO
330         I(M)=0.
340 102     CONTINUE
350 *
360 * Determine how many cells IC contain IFISH fish.
370 * Assume a negligible number of cells contain more than 60 fish.

```

## IDENTICAL SCATTERER SIMULATION

```

380          DO 103 IFISH=1,60
390 * Calculate factorial of IFISH.
395          FACT=1.
400          DO 104 K=IFISH,1,-1
410              FACT=FACT*K
420 104          CONTINUE
425 C Apply Poisson law to work out number of cells with IFISH
426 C fish.
430          IC=INT(Q*AVERAG**FLOAT(IFISH)/FACT)
435          IF((IC+IZERO).GT.3000)GO TO 2000
440 * For the cells containing this number of fish, sum IFISH phasors
450 * of identical amplitude and uniformly distributed random phase.
460          DO 105 N=IZERO+1,IZERO+IC
470              F=0.
480              G=0.
485 C This loop sums the phasors for 1 cell.
490              DO 106 K=1,IFISH
492 C Work out random phase for single fish echo.
495                  RANDOM=URAND(ISEED)
500                  PHAS=RANDOM*2.*PI
505                  RANDOM=URAND(ISEED)
506 C Assign identical amplitude to each cell.
510                  AMP=1.0
512 C Decompose into in-phase and quadrature components and sum.
520                  F=F+AMP*COS(PHAS)
530                  G=G+AMP*SIN(PHAS)
540 106          CONTINUE
545
550 * Calculate the amplitude, intensity, and squared intensity
560 * from the N'th range cell.,
570          A(N)=(F*F+G*G)**0.5
580          I(N)=A(N)**2.
590          I2(N)=I(N)**2.
600 105          CONTINUE
610          IZERO=IZERO+IC
620 103          CONTINUE
630 2000         SUM1=0.
640          SUM2=0.
650          SUM3=0.
655 C Sum amplitudes, intensities and squared intensities.
660          DO 107 N=1,IZERO
670              SUM1=SUM1+A(N)
680              SUM2=SUM2+I(N)
690              SUM3=SUM3+I2(N)
700 107          CONTINUE
705 C Work out means.
710          AV1=SUM1/FLOAT(IZERO)
720          AV2=SUM2/FLOAT(IZERO)
730          AV3=SUM3/FLOAT(IZERO)
731 C Work out normalized moments.
732          OMENT=AV3/AV2**2
734          UMBER=1./(OMENT-2.)
740          WRITE(*,10)ITRIAL,AV2,AV3,OMENT,UMBER
750 10          FORMAT(6X,I2,2X,F8.3,4X,F10.2,9X,F7.4,6X,F7.3)
760          MOM(ITRIAL)=OMENT
765          EST(ITRIAL)=UMBER
770          AVMOM=AVMOM+OMENT
775          AVEN=AVEN+UMBER
776 C Back for next trial.
780 100          CONTINUE
785 C Work out averages.
790          AVMOM=AVMOM/FLOAT(ITOTAL)

```

## IDENTICAL SCATTERER SIMULATION

```
795     AVEN=AVEN/FLOAT(ITOTAL)
800 * Calculate standard deviation of normalized moment
810     SUMDEV=0.
815     DEV2=0.
816 C Calculate standard deviation of norm. mom. and <N>.
820     DO 108 J=1,ITOTAL
830         SUMDEV=SUMDEV+(MOM(J)-AVMOM)**2
835         DEV2=DEV2+(EST(J)-AVEN)**2
840 108 CONTINUE
845 C Print averages and standard deviations.
850     WRITE(*,20)AVMOM,SQRT(SUMDEV/FLOAT((ITOTAL-1))),AVEN,SQRT(DEV2/FLO
851     &AT(ITOTAL-1)),1./(AVMOM-2.)
860 20  FORMAT(' Mean moment = ',F10.4,/' Standard deviation = ',F8.5
861     &,/' Mean estimate of <N> = ',F10.4,/' Standard deviation = ',F10.5,
862     &,/' Average <N> = ',F10.4)
870     STOP
875     INCLUDE UCT*ASCII.URAND
880     END
```

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```

6 C
10 C *****
15 C
20 C          RAYSIM
21 C
22 C For a distribution of scatterers, randomly distributed in volume
24 C according to a Poisson distribution with parameter <N>, this program
26 C calculates the expected distribution of values of N. For each of these
28 C 'cells' containing a particular number, N, scatterers, a phasor
30 C summation gives the resultant amplitude for that cell. Each of the N
32 C phasors is assigned a Rayleigh distributed random amplitude, and a
34 C uniformly distributed random phase. The calculation is done for many
36 C cells, giving a resultant amplitude for each. The intensities for each
38 C cell are obtained by squaring the corresponding amplitude sample. The
40 C normalized second moment may then be calculated and used to estimate <N>.
42 C The simulation is repeated many times and the sample standard deviations
44 C in the normalized moments and in <N> are calculated and printed.
46 C *****
48 C
50 C
90          DOUBLEPRECISION FACT,Q,AVERAG
95 C Arrays for storing amplitude, intensity, and squared intensity
96 C samples. Maximum number of cells = 4000. Last 2 arrays are
97 C for storing estimates of the moments and <N>.
100         REAL A(4000),I(4000),I2(4000),MOM(100),EST(100)
105 C For averaging moments.
110         AVMOM=0
111 C For averaging <N>.
112         AVEN=0
115         PI=3.14159265
120 * Enter number of trials and <N>, random number seed and number of cells.
130         READ (5,*)ITOTAL,AVERAG,ISEED,CELLS
131         WRITE(*,51)
135 51      FORMAT('RAYLEIGH SCATTERERS')
136         PRINT*,'True <N> = ',AVERAG,'Cells = ',CELLS
137         WRITE(*,50)
140 50      FORMAT('      TRIAL      <I>          <I**2>          <I**2>/<I>**2      <N>
150          &>')
160 *
170 * Trial loop.
180         DO 100 ITRIAL=1,ITOTAL
190             ENUM=0
200 * Initialize arrays.
210             DO 101 M=1,4000
220                 A(M)=0.
230                 I(M)=0.
240                 I2(M)=0.
250 101         CONTINUE
260 * Total number of cells in CELLS.
280 * Determine number of cells with zero fish from Poisson volume distribution
290             Q=CELLS*EXP(-AVERAG)
300             IZERO=INT(Q)
310 * I(M) is the intensity from the M'th cell.
315 C IZERO cells will have zero intensity.
320             DO 102 M=1,IZERO
330                 I(M)=0.
340 102         CONTINUE
350 *
360 * Determine how many cells IC contain IFISH fish.
370 * Assume a negligible number of cells contain more than 60 fish.
380             DO 103 IFISH=1,60

```

## RAYLEIGH SCATTERER SIMULATION

```

390 * Calculate factorial of IFISH.
395     FACT=1.
400     DO 104 K=IFISH,1,-1
410         FACT=FACT*K
420 104     CONTINUE
425 C Apply Poisson law to work out number of cells IC with IFISH fish.
430     IC=INT(Q*AVERAG**FLOAT(IFISH)/FACT)
435     IF((IC+IZERO).GT.4000)GO TO 2000
440 * For the cells containing this number of fish, sum IFISH phasors
450 * of Rayleigh amplitude and uniformly distributed random phase.
460     DO 105 N=IZERO+1,IZERO+IC
470         F=0.
480         G=0.
485 C This loop sums the phasors for 1 cell.
490         DO 106 K=1,IFISH
495             RANDOM=URAND(ISEED)
500             PHAS=RANDOM*2.*PI
502 C Work out Rayleigh amplitude for single fish echo.
505             RANDOM=URAND(ISEED)
510             AMP=SQRT(2.*ALOG((1/RANDOM)))
515 C Decompose into in-phase and quadrature components, and sum.
520             F=F+AMP*COS(PHAS)
530             G=G+AMP*SIN(PHAS)
540 106     CONTINUE
550 * Calculate the amplitude, intensity, and squared intensity
560 * from the N'th range cell.,
570         A(N)=(F*F+G*G)**0.5
580         I(N)=A(N)**2.
590         I2(N)=I(N)**2.
600 105     CONTINUE
605 C Work out number of cells containing next number IFISH of fish.
610         IZERO=IZERO+IC
620 103     CONTINUE
630 2000     SUM1=0.
640         SUM2=0.
650         SUM3=0.
655 C Sum amplitudes, intensities and squared intensities.
660         DO 107 N=1,IZERO
670             SUM1=SUM1+A(N)
680             SUM2=SUM2+I(N)
690             SUM3=SUM3+I2(N)
700 107     CONTINUE
705 C Work out normalized moment and estimate of <N> and print.
710         AV1=SUM1/FLOAT(IZERO)
720         AV2=SUM2/FLOAT(IZERO)
730         AV3=SUM3/FLOAT(IZERO)
732         OMENT=AV3/AV2**2
734         UMBER=2./(OMENT-2.)
740         WRITE(*,10)ITRIAL,AV2,AV3,OMENT,UMBER
750 10     FORMAT(6X,I2,2X,F8.3,4X,F10.2,9X,F7.4,6X,F7.3)
760         MOM(ITRIAL)=OMENT
765         EST(ITRIAL)=UMBER
766 C Sum for averaging.
770         AVMOM=AVMOM+OMENT
775         AVEN=AVEN+UMBER
776 C Back for next trial.
780 100     CONTINUE
785 C Work out averages.
790         AVMOM=AVMOM/FLOAT(ITOTAL)
795         AVEN=AVEN/FLOAT(ITOTAL)
800 * Calculate standard deviation of normalized moment and of individual estimate
810         SUMDEV=0.

```

## RAYLEIGH SCATTERER SIMULATION

```
815     DEV2=0.
820     DO 108 J=1,ITOTAL
830         SUMDEV=SUMDEV+(MOM(J)-AVMOM)**2
835     DEV2=DEV2+(EST(J)-AVEN)**2
840 108 CONTINUE
845 C Print averages and standard deviations.
850     WRITE(*,20)AVMOM,SQRT(SUMDEV/FLOAT((ITOTAL-1))),AVEN,SQRT(DEV2/FLO
851     &AT(ITOTAL-1)),2./(AVMOM-2.)
860 20  FORMAT(' Mean moment = ',F10.4,/' Standard deviation = ',F8.5
861     &,/' Mean estimate of <N> = ',F10.4,/' Standard deviation = ',F10.5,
862     &,/' Average <N> = ',F10.4)
870     STOP
875     INCLUDE UCT*ASCII.URAND
880     END
```

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Sample output from Rayleigh scatterer simulation program

## RAYLEIGH SCATTERERS

True &lt;N&gt; = 4.0000000000000000

Cells = 1500.0000

TRIAL	<I>	<I**2>	<I**2>/<I>**2	<N>
1	7.481	139.42	2.4913	4.071
2	8.006	152.06	2.3722	5.374
3	7.898	152.68	2.4476	4.469
4	8.056	190.08	2.9287	2.154
5	8.257	185.12	2.7151	2.797
6	7.757	143.26	2.3805	5.256
7	8.074	164.97	2.5310	3.767
8	7.973	160.12	2.5188	3.855
9	8.231	168.26	2.4835	4.136
10	8.432	189.47	2.6652	3.006
11	8.086	159.71	2.4426	4.519
12	7.779	158.81	2.6244	3.203
13	7.919	156.12	2.4894	4.087
14	7.920	149.85	2.3891	5.140
15	7.830	151.84	2.4766	4.196
16	7.777	154.76	2.5585	3.581
17	8.383	192.76	2.7427	2.693
18	7.704	146.38	2.4665	4.287
19	8.305	176.26	2.5557	3.599
20	8.402	175.03	2.4793	4.173
21	7.414	130.49	2.3741	5.347
22	7.858	155.02	2.5107	3.917
23	8.578	177.55	2.4128	4.845
24	8.213	154.49	2.2904	6.886
25	8.045	159.63	2.4664	4.288
26	8.017	162.92	2.5346	3.741
27	8.382	215.92	3.0731	1.864
28	8.256	169.51	2.4868	4.109
29	7.806	151.01	2.4783	4.181
30	7.968	158.67	2.4994	4.005
31	8.226	176.40	2.6067	3.297
32	7.844	154.16	2.5057	3.955
33	7.741	157.46	2.6278	3.186
34	8.055	159.35	2.4557	4.389
35	7.732	146.34	2.4475	4.469
36	7.863	146.26	2.3654	5.474
37	7.976	160.97	2.5306	3.769
38	8.100	160.94	2.4530	4.415
39	7.944	161.20	2.5543	3.608
40	8.378	180.91	2.5772	3.465
41	8.156	162.27	2.4392	4.554
42	7.822	154.05	2.5176	3.864
43	7.873	155.40	2.5073	3.943
44	7.784	155.49	2.5661	3.533
45	8.188	193.75	2.8900	2.247
46	8.443	189.12	2.6528	3.064
47	8.103	159.85	2.4343	4.605
48	7.810	149.97	2.4584	4.363
49	7.843	150.96	2.4540	4.405
50	8.297	173.56	2.5212	3.837
51	8.095	171.45	2.6166	3.244
52	8.052	167.12	2.5778	3.461
53	8.359	179.96	2.5755	3.475
54	7.874	171.16	2.7607	2.629
55	7.796	148.88	2.4499	4.445
56	8.389	179.81	2.5550	3.603
57	8.291	169.45	2.4648	4.303
58	8.200	166.10	2.4700	4.256
59	7.751	136.22	2.2673	7.482

60	7.590	153.27	2.6606	3.028
61	8.132	155.77	2.3558	5.621
62	8.049	169.93	2.6229	3.211
63	7.515	150.27	2.6605	3.028
64	7.653	138.07	2.3577	5.591
65	8.336	173.52	2.4969	4.025
66	8.093	165.20	2.5220	3.831
67	8.083	160.31	2.4534	4.411
68	7.809	141.28	2.3166	6.316
69	8.088	163.85	2.5047	3.963
70	8.009	161.72	2.5215	3.835
71	8.040	150.91	2.3346	5.977
72	7.968	156.39	2.4630	4.320
73	8.587	190.82	2.5875	3.404
74	7.965	152.85	2.4096	4.883
75	8.395	176.22	2.5002	3.999
76	7.961	144.23	2.2758	7.251
77	7.826	141.45	2.3093	6.466
78	7.756	147.12	2.4457	4.488
79	8.007	152.95	2.3857	5.185
80	7.829	152.23	2.4838	4.134
81	7.987	151.96	2.3823	5.232
82	7.780	148.75	2.4573	4.373
83	8.136	165.28	2.4970	4.024
84	8.387	177.08	2.5175	3.864
85	7.854	160.99	2.6102	3.278
86	8.102	164.65	2.5083	3.935
87	7.891	156.37	2.5115	3.910
88	7.937	153.02	2.4292	4.660
89	7.815	148.35	2.4288	4.664
90	7.883	151.04	2.4305	4.646
91	8.169	167.65	2.5122	3.905
92	7.975	151.90	2.3881	5.153
93	8.129	166.64	2.5219	3.832
94	8.155	170.50	2.5638	3.547
95	7.904	155.28	2.4852	4.122
96	8.326	174.25	2.5137	3.894
97	7.922	152.45	2.4293	4.659
98	8.236	165.04	2.4330	4.619
99	7.650	146.48	2.5026	3.979
**	7.673	138.22	2.3474	5.757

Mean moment = 2.5032  
Standard deviation = .12652  
Mean estimate of <N> = 4.1991  
Standard deviation = .98796  
Average <N> = 3.9742  
END PROGRAM EXECUTION

@RESUME,P HIQUAL

## APPENDIX B

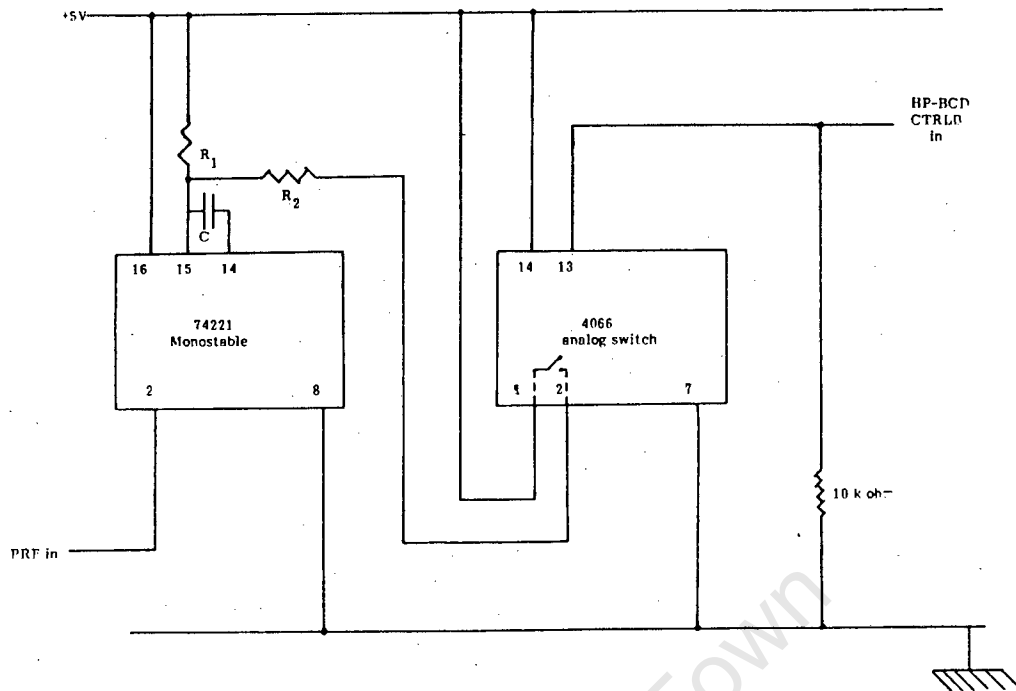


Fig B.1 Computer controlled pulse width generating circuit for gating transmitter. Requires PRF at TTL levels from pulse generator. Pulse width set by  $\tau = 0.7 RC$ .  
 $R = R_1$  if 4066 switch off or  $R_1$  in parallel if switch on.

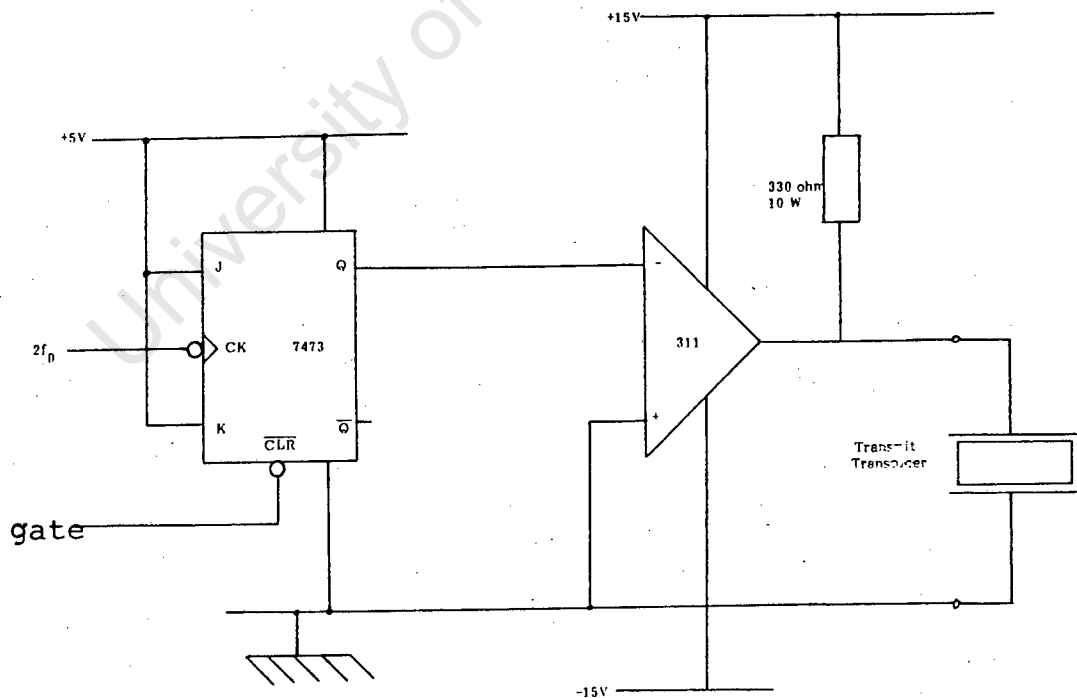


Fig B.2 Transmitter circuit. Requires  $2f_0$  from signal generator at TTL levels and gating pulse from previous circuit. Flip-flop divides frequency down. Transmit breakthrough is at twice the signal frequency. Output is 30V p-p gated square wave.

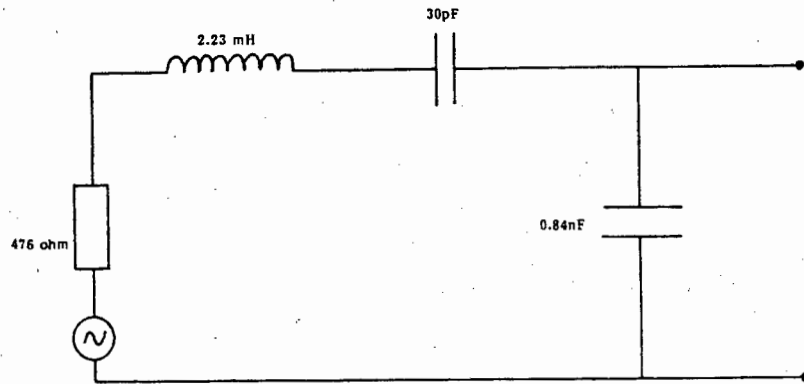


Fig B.3 Equivalent circuit of 615 kHz disk ceramic transducer.

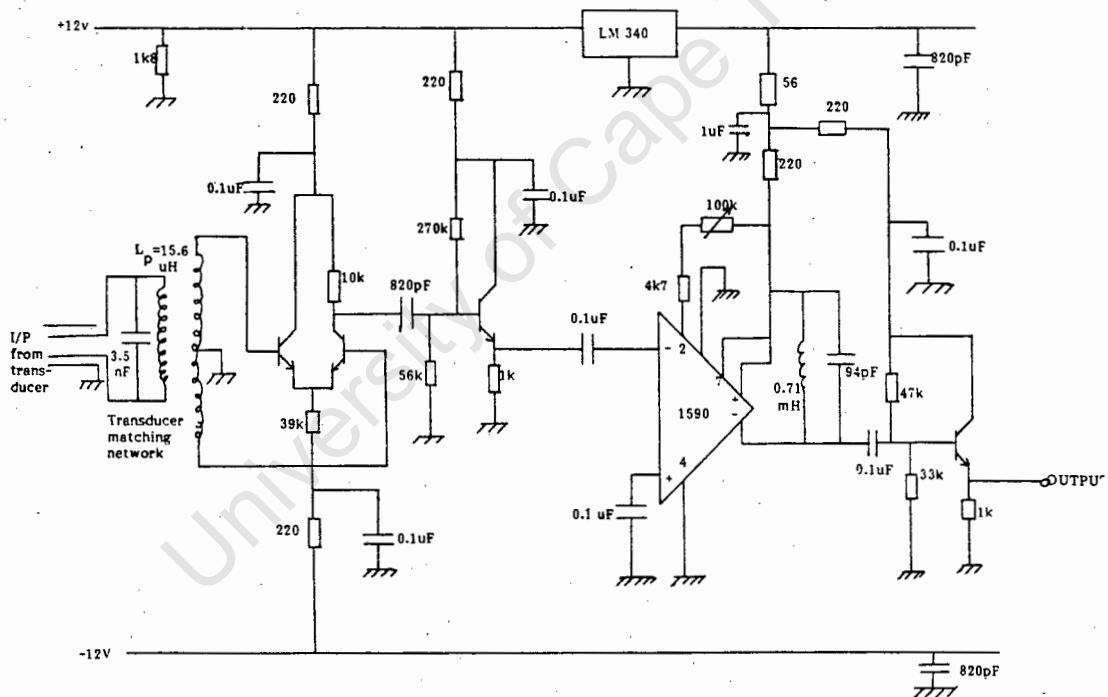


Fig B.4 Circuit diagram of matched receiver amplifier. The circuit is tuned to 615kHz and matched to the transducer equivalent circuit above. Input is dual balanced and output is earth referenced. The circuit was designed by Runciman (1986).

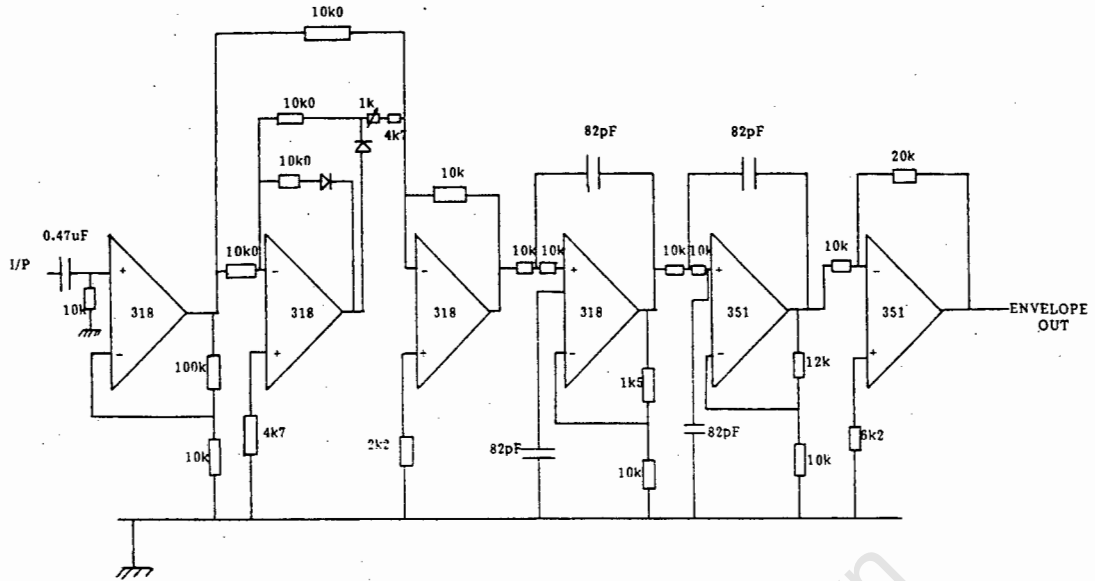


Fig B.5 Precision envelope detector circuit. Consists of a precision full wave rectifier circuit followed by a 4 pole Butterworth low pass filter. The design for the filter is in Horowitz and Hill (1980) p 158.

## APPENDIX C

```

0 !           FISH10
10 !          =====
10 ! Program for recording records of backscatter from anchovy shoals.
10 ! Used on an anchovy recruitment survey on the RS AFRICANA.
10 ! Time of survey: 20-30 May 1985
10 !
10 !
10 OPTION BASE 1
10 ! I is a buffer which stores 2000 samples before writing them to disc.
10 ! E$ stores a short descriptive title.
10 DIM I(10,200),E$[200]
20 ! "LIMITS" sets sample limits.
30 ON KEY# 1,"LIMITS" GOSUB 930
40 ! "RECORD" records a ping.
50 ON KEY# 2,"RECORD" GOTO 580
60 ON KEY# 3,"ABORT" GOTO 730 ! ABORT aborts program.
70 CLEAR @ DISP "Title of experiment";@ INPUT E$
80 ! Obtain filename for storing data from operator.
90 DISP "Date";@ INPUT D$
100 DISP "Data file name";@ INPUT F$
10 ! Create file for storing data. Maximum of 100 pings and 200 samples/ping
20 CREATE F#&":D701",10,16000
30 ASSIGN# 1 TO F#&":D701"
40 ! Get required timebase and required sample range from operator.
50 DISP "Required time base (ms/div)";@ INPUT T$
60 DISP "BGN, END sample numbers";@ INPUT B1$,E1$
70 CLEAR 7
80 ! Setup scope interface.
90 CONTROL 7,16 ; 1,10
100 OUTPUT 708 USING "K" ; "SPR"
110 OUTPUT 708 USING "K" ; "USP /"
20 ! Obtain equivalent digital value corresponding to ground.
30 ! Used for referring loaded samples to ground.
40 CLEAR @ DISP "Ground scope and press <CONT>." @ BEEP 1000,100 @ PAUSE
50 T=0
60 N1=VAL (E1$)-VAL (B1$)-1 ! Number of samples.
70 ! Set time gate.
80 OUTPUT 708 USING "K" ; "TIM "&T$&"E-03"
90 OUTPUT 708 USING "K" ; "BGN "&B1$&"/END "&E1$&"/CNT 1/DAT ?"
100 ! This loop samples ground values over the gate and sums them.
110 FOR I=1 TO N1
120 ENTER 708 ; T2
130 T=T+T2
140 NEXT I
150 ! Obtain averaged ground value.
160 A=T/N1
170 CLEAR
180 ! For each iteration of this outer loop, 10 pings are sampled and
190 ! stored in I(X,_). The entire array is then stored.
200 FOR H=1 TO 10
210 ! For each iteration of this loop one ping is sampled.
220 FOR X=1 TO 10
230 KEY LABEL
240 DISP "Record no.:";H;X
250 ! Interrupt signifies that a ping is ready for storing.
260 GOTO 560 ! Wait for interrupt.
270 ! Sets up required sample range to be downloaded from scope.
280 OUTPUT 708 USING "K" ; "TIM "&T$&"E-03"
290 OUTPUT 708 USING "K" ; "BGN "&B1$&"/END "&E1$&"/CNT 1/DAT ?"
300 ! This loop samples the ping.

```

```
610 FOR I=1 TO NI
620 ENTER 708 ; I(X,I)
630 NEXT I
640 NEXT X
650 CLEAR
660 DISP "Writing to disc...."
670 ! Refer ensemble of ten pings to ground...
680 MAT I=( -A)+I
690 ! ... and store on disc.
700 PRINT# 1,H ; I(,)
710 CLEAR
720 NEXT H
730 ASSIGN# 1 TO *
740 ! Reset IEEE interface.
750 RESET 7
760 ! Pertinent parameters are requested for the printing of an experimental
770 ! note.
780 DISP "System bandwidth";@ INPUT B$
790 DISP "Nominal pulse length";@ INPUT P$
800 INPUT V$
810 ! Print an experimental note for keeping with data disc.
820 PRINT E$
830 PRINT "Date: ";D$
840 PRINT "Filename: ";F$
850 PRINT "Time base: ";T$;"ms/div"
860 PRINT "Vertical gain: ";V$
870 PRINT "BGN and END sample numbers: ";B1$;E1$
880 PRINT "System bandwidth: ";B$
890 PRINT "Pulse length: ";P$
900 CLEAR @ DISP "*****RUN COMPLETE*****"
910 END
920 ! This subroutine used for changing sampled range if shoal range changes.
930 DISP "New BGN,END sample nos";
940 INPUT B1$,E1$
950 PRINT "BGN,END changed to: ";B1$,E1$
960 RETURN
```

```

10 !           MOMENT4
20 !           =====
30 ! Program for working out
40 ! normalized moments for each
50 ! range cell, and calculating
60 ! densities for each. The
70 ! mean moment and density is
80 ! also calculated.
90 !
100 ! Variable List.
110 ! =====
120 ! S(,) : 1 record of samples.
130 ! S1() :  $\sim I^2$  for each cell.
140 ! S2() :  $\sim I$  for each cell.
150 ! D#   : Data file name.
160 ! A,B  : Start, stop recs.
170 ! C    : Record step.
180 ! T    : Txducer beamwidth.
190 ! W    : Pulse length.
200 ! R4,R5: Range limits.
210 ! M4,M2:  $\langle I^2 \rangle, \langle I \rangle$ 
220 ! N    :  $\langle N \rangle$ 
230 ! V    : Resln cell vol.
240 ! I    : Cell number.
250 ! N1   : Norm. mom. for cell.
260 ! N5   : For summing norm.
270 !     : moments & averaging.
280 OPTION BASE 1
290 ! For storing envelope records and estimates for each cell.
300 CLEAR
310 DIM S(10,200),S1(20),S2(20)
320 CLEAR @ DISP "Data file name";
330 INPUT D#
340 ASSIGN# 2 TO D#&":D701"
350 MAT S1=( 0)@ MAT S2=( 0)
360 DISP "Start, stop records & step";
370 INPUT A,B,C
380 DISP "Beamwidth. (rad)";@ INPUT T
390 DISP "Pulse length (mS)";@ INPUT W
400 DISP "Range limits";@ INPUT R4,R5
410 R6=750*W*.001
420 R7=(R5-R4)/20
430 !
440 ! Record loop.
450 FOR R=A TO B STEP C
460 PRINT "Record number ";R
470 READ# 2,R ; S(,)
480 FOR R2=1 TO 10 ! Ping loop.
490 FOR I=0 TO 19 ! Cell loop.
500 FOR I2=I*10+1 TO I*10+10 ! Sample loop.
510 ! Sum the intensity and intensity squared.
520 ! The data stored in S are amplitude samples.
530 S1(I+1)=S1(I+1)+S(R2,I2)^4
540 S2(I+1)=S2(I+1)+S(R2,I2)^2
550 NEXT I2
560 NEXT I
570 NEXT R

```

```

580 NEXT R
590 ! Print a header for table.
600 PRINT " #====#"
610 PRINT " # Cell Mom. <N> Vres Roe #"
620 N5=0
630 FOR I=1 TO 20
640 !
650 ! Calculate <I^2>, <I>^2.
660 M4=S1(I)/((B-A+1)*100/C)
670 M2=S2(I)/((B-A+1)*100/C)
680 ! Calculate norm. mom.
690 N1=M4/M2^2
700 N5=N5+N1
710 N=2/(N1-2)
720 ! Resolution cell volume.
730 V=.2618*R6*T*T*((R4+(I-1)*R7)^2+(R4+(I-1)*R7)*(R4+I*R7)+(R4+I*R7)^2)
740 PRINT USING 750 ; I,N1,N,V,N/V
750 IMAGE " # ",DD,4X,D.DD,1X,DD.D,2X,DD.D,1X,D.DD," #"
760 NEXT I
770 PRINT " #====#"
780 N5=N5/20
790 PRINT "Data file:";D$
800 PRINT "Pulse length =";W
810 PRINT "Range limits";R4;R5
820 PRINT "Beamwidth =";T;"rad"
830 PRINT "<I^2>/<I>^2av = ";N5
840 PRINT "<N>av = ";2/(N5-2)
850 ASSIGN# 2 TO *
860 END

```

## APPENDIX D

## FISH STOCK ASSESSMENT BY A STATISTICAL ANALYSIS OF ECHO SOUNDER SIGNALS

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## ABSTRACT

In 1984 Wilhelmij and Denbigh<sup>1</sup> described a technique for determining the number density of random scatterers based upon a statistical analysis of acoustic backscattered signals. This paper examines the application of the method to the specific problem of estimating the number density of fish within a shoal using the signals obtained from an echo sounder.

## INTRODUCTION

If the return signal from a fish shoal at some given instant is caused by overlapping echoes from a very large number of fish, the central-limit theorem predicts that the amplitude of the echo waveform should have Gaussian statistics. This is equivalent to having an envelope which has Rayleigh statistics or an intensity which has exponential statistics, the intensity being the square of the envelope. If the average number of overlapping echoes contributing to the signal at each instant decreases to about ten or less, the central-limit theorem becomes non-applicable and there is a significant deviation of the statistics from those just mentioned. A suitable measure of this deviation can be used to predict the average number of overlapping echoes, i.e. the average number of fish in the resolution cell of the echo sounder. A division of this number by the volume of the resolution cell gives the number of fish per cubic meter, or the number density.

The original paper by Wilhelmij and Denbigh was based upon the work of Pusey et al<sup>2</sup> for electromagnetic scattering and used the second normalised moment of backscattered intensity as a pertinent measure of the statistical properties of the waveform. The second normalized moment of intensity is the second moment of intensity divided by the square of the first moment, or  $\langle I^2 \rangle / \langle I \rangle^2$ . For the case of identical scatterers having a constant scattering strength and a Poisson volume distribution, Pusey et al showed that if the statistics are stationary this second normalized moment is related to the average number of scatterers by the formula

$$\langle N \rangle = \frac{1}{\langle I^2 \rangle / \langle I \rangle^2 - 2} \quad (1)$$

If the number of scatterers is large enough for the central-limit theorem to apply, we would find  $\langle I^2 \rangle / \langle I \rangle^2 = 2$ , giving  $\langle N \rangle = \infty$  from the above formula. Table 1 gives a few examples showing the relationship between  $\langle N \rangle$  and the second normalized moment based upon a rearrangement of this formula.

Table 1. The second normalized moment of intensity as a function of  $\langle N \rangle$ .

$\langle N \rangle$	$\langle I^2 \rangle / \langle I \rangle^2 = 2 + 1/\langle N \rangle$
64	2.016
32	2.031
16	2.063
8	2.125
4	2.25
2	2.5
1	3
0.5	4

It becomes clear from Table 1 that a very high precision is needed in the value of the second normalized moment if large values of  $\langle N \rangle$  are to be deduced from it with any accuracy.

The paper by Wilhelmij and Denbigh served three main purposes. It verified the applicability of the technique using experimental backscattered ultrasonic signals from a random matrix of polystyrene beads in a water tank. It showed how the problem of non-stationary statistics due to beam divergence may be overcome by a suitable averaging procedure. It also predicted theoretically, for different values of  $\langle N \rangle$ , how many independent measurements of backscattered intensity are needed in order to obtain estimates of  $\langle N \rangle$  with given accuracies.

This present paper has four main objectives. Firstly, it presents evidence to suggest that a Rayleigh distribution of echo amplitudes is more appropriate to the scattering from fish in a fish shoal than the assumption of identical echo amplitudes. Secondly, a theoretical derivation is given relating  $\langle N \rangle$  to the second normalized moment when the scatterers have a Rayleigh distribution of target strengths. Thirdly, it presents computer simulations to determine the error in estimates as a function of the true value of  $\langle N \rangle$  and of the number of independent measurements. Fourthly, it presents results obtained by applying the method to echo sounder signals obtained from real fish shoals at sea and compares them with number density estimates based upon echo integration. Much of the material has been described by the authors at conferences<sup>3,4</sup>. Reference 4 regrettably contains some mistakes in the calculations of the estimates by statistical analysis.

#### THE PDF OF ECHO ENVELOPES FROM INDIVIDUAL FISH

Considering an individual fish as a flexible line array of many small scattering points, its echo is determined by the sum of contributions from each scattering point. The result of interference between these point scatterers will vary with time and will be different for each fish. If the contributions are of similar amplitude but random phase, the central-limit theorem may be applied once again to suggest that the echo envelope will vary from fish to fish in accordance with a Rayleigh distribution. A recent paper by Clay and Heist gives evidence that the Rician PDF is a more accurate description of acoustic scattering by individual live fish. The physi-

cal explanation is that the backscattered signal may be considered to have two components, one which varies in a noise like manner with fish orientation and flexing, and one which does not vary. The noise like component arises from interference between the distributed scattering points along the body of the fish. The constant component arises from the swim bladder when it is small enough in wavelengths to produce constant conditions for interference irrespective of orientation and flexing. The envelope of the backscattered signal can then be thought of as arising from the envelope of two superposed components, a sinewave and noise, thus leading to the classical Rician distribution.

It is proposed that the sonar used for the assessment of fish number density by the statistical technique would have a narrow beam and a good range resolution in order to achieve a small average number of scatterers in the resolution cell. It follows that high operating frequencies are needed, and that the swim bladder is likely therefore to have an appreciable dimension in wavelengths. In these circumstances, and in accordance with the findings of Clay and Heist for fish more than  $25\lambda$  long, the constant amplitude component will be small and a close approximation to a Rayleigh PDF may be expected. This Rayleigh distribution of echo amplitudes is the case considered in the following analysis.

## THEORY

The measures of envelope statistics used in the technique are the first and second moments of intensity, where the intensity is the square of the envelope. Consider initially that there are a fixed number of fish  $N$  contributing to the return signal at any instant. As an example let  $N = 4$  so that there are four overlapping echoes  $a \cos(\omega t + \alpha)$ ,  $b \cos(\omega t + \beta)$ ,  $c \cos(\omega t + \gamma)$  and  $d \cos(\omega t + \delta)$  where the amplitudes  $a$ ,  $b$ ,  $c$  and  $d$  may be considered random variables with a Rayleigh PDF, and where the phases  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  may be considered random variables with a uniform distribution between  $0$  and  $2\pi$ . The resultant intensity, which is the square of the envelope, will be termed  $I_4$ , where the subscript  $4$  denotes that it results from the sum of  $4$  echoes. By adding the in-phase components of the four terms, and then the quadrature components, it follows that

$$I_4 = (a \cos \alpha + b \cos \beta + c \cos \gamma + d \cos \delta)^2 + (a \sin \alpha + b \sin \beta + c \sin \gamma + d \sin \delta)^2$$

Making use of the relationships  $\cos^2 x + \sin^2 x = 1$ , and  $\cos x \cdot \cos y + \sin x \cdot \sin y = \cos(x-y)$ , this can be expanded to become

$$I_4 = (a^2 + b^2 + c^2 + d^2) + 2ab \cos(\alpha-\beta) + 2ac \cos(\alpha-\gamma) + 2ad \cos(\alpha-\delta) + 2bc \cos(\beta-\gamma) + 2bd \cos(\beta-\delta) + 2cd \cos(\gamma-\delta)$$

$$\langle I_4 \rangle = \langle a^2 \rangle + \langle b^2 \rangle + \langle c^2 \rangle + \langle d^2 \rangle = 4\langle a^2 \rangle$$

assuming that each amplitude has the same mean value.

Squaring the expression for  $I_4$  and making use of the fact that all cross product terms such as  $2ab \cos(\alpha-\beta)$ ,  $2ac \cos(\alpha-\gamma)$  have zero mean, we obtain

$$\langle I_4^2 \rangle = \langle a^4 \rangle + \langle b^4 \rangle + \langle c^4 \rangle + \langle d^4 \rangle + \langle 4a^2b^2 \rangle + \langle 4a^2c^2 \rangle + \langle 4a^2d^2 \rangle + \langle 4b^2c^2 \rangle + \langle 4b^2d^2 \rangle + \langle 4c^2d^2 \rangle$$

The format of this expression is chosen to show how the terms originate and to enable a generalized prediction of  $\langle I_N^2 \rangle$  for values of  $N$  other than 4.

In the above example and noting that  $\langle a^2 b^2 \rangle = \langle a^2 \rangle^2$  if the amplitudes are independent random variables, we obtain

$$\langle I_4^2 \rangle = 4 \langle a^4 \rangle + 4(3+2+1) \langle a^2 \rangle^2$$

A similar derivation for  $N$  overlapping echoes gives

$$\langle I_N \rangle = N \langle a^2 \rangle$$

$$\langle I_N^2 \rangle = N \langle a^4 \rangle + 2N(N-1) \langle a^2 \rangle^2$$

which makes use of the simplification that  $\{(N-1) + (N-2) + \dots + 2 + 1\}$  equals  $N(N-1)/2$ .

For the Rayleigh distribution  $p(a) = a/\sigma^2 e^{-a^2/2\sigma^2}$

it is readily shown that  $\langle a^2 \rangle = 2\sigma^2$  and  $\langle a^4 \rangle = 8\sigma^4$ . Arbitrarily putting  $\sigma = 1/\sqrt{2}$  it follows that

$$\langle I_N \rangle = N \tag{2}$$

$$\langle I_N^2 \rangle = 2N + 2N(N-1) \tag{3}$$

The next step is to proceed from the situation where the the number of fish in each resolution cell is fixed at  $N$ , and to consider instead fish which are Poisson distributed in volume. The Poisson distribution predicts that the probability that there are  $N$  fish in a resolution cell and is given by

$$P_{\langle N \rangle}(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!} \tag{4}$$

where  $\langle N \rangle$  is the average number of fish in the resolution cell. We can now take the moments  $\langle I_N \rangle$  and  $\langle I_N^2 \rangle$  corresponding to a fixed number of fish  $N$  in the resolution cell, and multiply them by the probability  $P_{\langle N \rangle}(N)$  that there are  $N$  fish in the resolution cell. We do this for every value of  $N$  for which  $P_{\langle N \rangle}(N)$  is significant and then perform summations of these weighted first and second moments. This gives us averaged intensity moments  $\langle I \rangle$  and  $\langle I^2 \rangle$  which are appropriate to fish that have a Poisson distribution in volume and have a Rayleigh distribution of amplitudes.

Using the expression for  $\langle I_N \rangle$  given in Eq.2 we obtain

$$\langle I \rangle = \sum_{N=1}^{\infty} \langle I_N \rangle P_{\langle N \rangle}(N) = \sum_{N=1}^{\infty} N P_{\langle N \rangle}(N)$$

=  $\langle N \rangle$ , by the definition of a mean value.

Doing the same operation for the value of  $\langle I_N^2 \rangle$  given in Eq.3 we obtain

$$\begin{aligned} \langle I^2 \rangle &= \sum_{N=1}^{\infty} \langle I_N^2 \rangle P_{\langle N \rangle}(N) \\ &= \sum_{N=1}^{\infty} 2N P_{\langle N \rangle}(N) + \sum_{N=1}^{\infty} 2N(N-1) \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!} \\ &= 2\langle N \rangle + 2e^{-\langle N \rangle} \sum_{N=2}^{\infty} \frac{\langle N \rangle^N}{(N-2)!} \end{aligned}$$

$$= 2\langle N \rangle + 2e^{-\langle N \rangle} \left\{ \frac{\langle N \rangle^2}{0!} + \frac{\langle N \rangle^3}{1!} + \frac{\langle N \rangle^4}{2!} + \dots \right\}$$

Making use of the expansion  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

this simplifies to

$$\langle I^2 \rangle = 2\langle N \rangle + 2\langle N \rangle^2$$

$$\langle I^2 \rangle / \langle I \rangle^2 = 2 / \langle N \rangle + 2$$

$$\text{or } \langle N \rangle = \frac{2}{\langle I^2 \rangle / \langle I \rangle^2 - 2} \quad (5)$$

The relationship previously used was given by Eq.1. This assumed scatterers which were Poisson distributed in volume but which had identical backscattering cross-sections. Experimental estimates of the density of a randomized volume of polystyrene spheres were obtained using this formula and have shown good agreement with the true densities. The new equation gives number densities which are exactly twice as great. It is believed that the Rayleigh distribution of echo amplitudes assumed in this new equation is likely to be more appropriate to fish in a shoal. This applies even if the fish do not vary in size. When the fish length drops below 25 wavelengths, it is expected that the amplitude statistics will become non-Rayleigh and that the correct constant in the numerator of Eq.5 will now lie between one and two. An extension to the preceding analysis can determine this constant if the amplitude statistics of the individual fish are known.

#### COMPUTER SIMULATIONS AND ERRORS IN ESTIMATES

Using either Eq.1 or Eq.5 it is clear that an error in the second normalized moment, arising from the use of too few samples, will cause an error in the estimate of the mean number of scatterers in the resolution cell. The original error analysis by Wilhelmij and Denbigh was valid only for very small errors and should therefore be applied with caution. Although less general than this original analysis a useful insight into errors has been obtained by a computer simulation of a scattering model.

In the simulation it is assumed that there are M independent measurements of intensity, each arising from interference between the scatterers from M different resolution cells. The Poisson distribution of Eq.4 tells us what number Q out of these M measurements arises from cells containing N scatterers, i.e.

$$Q = M P_{\langle N \rangle}(N) \quad (6)$$

where  $\langle N \rangle$  is the average number of scatterers. For each cell containing N scatterers, an intensity may be computed by performing a vector addition of N complex amplitudes, each having a random phase and a Rayleigh distributed envelope. The M intensities computed in this way may be used to determine a value for the second normalized moment and hence to predict a mean number of scatterers by using Eq.5. One such run must be expected to give an answer which is different from the true value of  $\langle N \rangle$ , due to the use of too few intensity values in calculating the second normalized moment of intensity. However, very many such simulations may be performed and an analysis of the results produces a sample mean of the estimates and the standard deviation of the estimates. Table 2 is derived from many simulations each based on 500 independent measurements of intensity. The number of simulations was adequate for the results to have converged to be closely equal to the values shown.

Table 2. Simulation results for 500 samples, showing bias and errors in the estimates of  $\langle N \rangle$ .

$\langle N \rangle$	mean of the estimate of $\langle N \rangle$	standard deviation of the estimate of $\langle N \rangle$
0.1	0.11	0.013
0.5	0.52	0.079
1.0	1.1	0.20
2.0	2.2	0.62
4.0	4.8	2.0

An interesting point to note is that there is a bias in the estimate of  $\langle N \rangle$ . This arises because the value of second normalized moment in Eq.5 is always greater than 2 and the subtraction of 2 in the denominator causes a greater error if the error in second normalized moment is too low by some amount than if it is too high by the same amount. Concerning the standard deviation, it is relevant to note that a technique capable of producing an estimate of fish number density to an accuracy of 30% would be considered a useful technique. If 500 independent measurements were available an interpolation of the results of Table 2 shows that, neglecting the bias error, this accuracy is achieved if the number of scatterers in the resolution cell is 2.1 or less.

The corresponding table for 1000 independent measurements is given in Table 3. It is seen that, as is to be expected, the bias and the standard deviation are both reduced. An accuracy of 30% corresponds now to a value of  $\langle N \rangle$  equal to about 4.5.

Table 3. Simulation results for 1000 samples, showing bias and errors in the estimates of  $\langle N \rangle$ .

$\langle N \rangle$	mean of the estimate of $\langle N \rangle$	standard deviation of the estimate of $\langle N \rangle$
0.1	0.1	0.011
0.5	0.52	0.068
1.0	1.04	0.15
2.0	2.11	0.37
4.0	4.23	1.14

#### SEA TRIALS

In 1985 an Anchovy Recruitment Survey was conducted by the Sea Fisheries Research Institute aboard the R.S. Africana. The main objective of the cruise was to obtain an estimate by acoustic survey of the young anchovy biomass on the South and West coasts of South Africa. The method used for the survey was echo integration using the ship's SIMRAD EK-38 and EK-120 scientific echo sounders. One of the authors participated in the first leg of the cruise and the outputs of the echo sounder were recorded and used to calculate fish density estimates using the statistical technique. These were compared with the estimates made by the Sea Fisheries Research Institute using the echo integrator. In the light of the experience gained during the first few days of the cruise a few observations can be made regarding the suitability of the statistical method for acoustic survey of anchovy. During the day, the fish concentrate into extremely dense shoals which, at a survey speed of 12 knots are insonified typically for 10 pings before the ship has passed over them. The number of independent envelope samples

obtained from such a short insonification was found to be inadequate for working out a statistically significant estimate of the normalized moment of intensity. At night, however, the fish spread out into a more diffuse sound scattering layer, which may be several miles in horizontal extent, and several tens of metres deep. Examples of sounder charts of shoals during the day and at night are given in Figure 1.

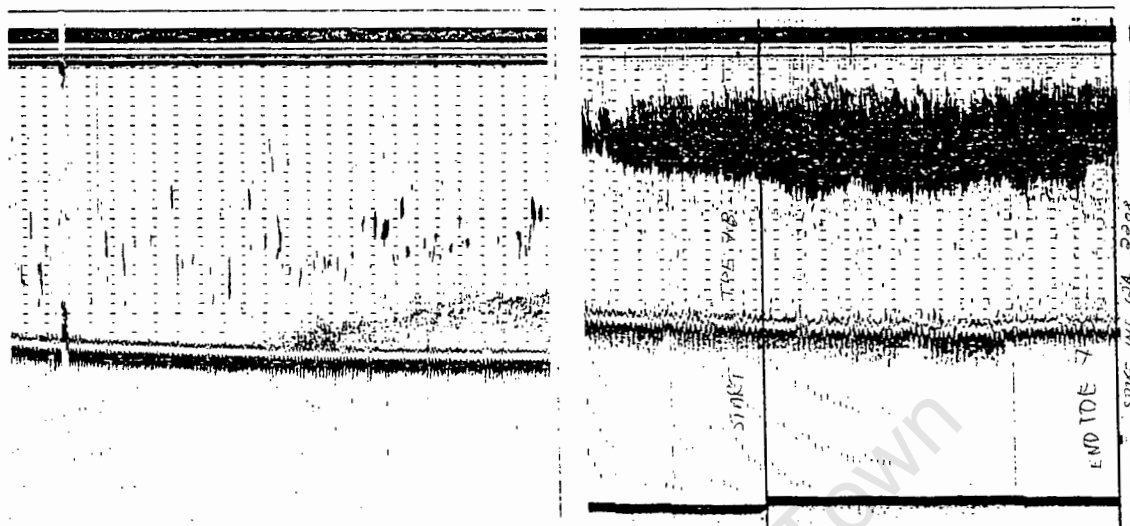


Figure 1. Day and night time echo soundings of anchovy.

Experiments were conducted on suitable night-time aggregations of anchovy. It was arranged that the echo integrator should run concurrently with the statistical method. This achieved a standard against which the estimates using the statistical technique could be compared. It should be noted, however, that echo integration results are generally recognized to be prone to considerable error. A second way of verifying the new technique was to examine the effect of increasing pulse length. An increase in pulse length should not change the estimates of density.

The experiments were conducted when suitable aggregations were present. As many samples as the extent of the shoal would allow were taken. Results are presented in Table 4. The statistical estimates were obtained using the formula

$$\rho = \frac{2}{(\langle I^2 \rangle / \langle I \rangle^2)_{av} - 2} \times (0.125c^3 \Omega \tau t_1 t_m) \quad (7)$$

This is an extension of Eq.5 which takes into account the volume of the resolution cell, including its change with range due to beam divergence. Apart from the modifying factor of 2 already discussed, the derivation of this is as given by Wilhelmij and Denbigh. In this formula  $c$  is the velocity of sound;  $\Omega$  is the beam solid angle taken somewhat arbitrarily to be that corresponding to the 3dB beamwidth of the transducer when used solely as a transmitter;  $\tau$  is the pulse duration;  $t_1$  and  $t_m$  are the time limits of the intensity data that are used;  $(\langle I^2 \rangle / \langle I \rangle^2)_{av}$  is obtained by calculating  $\langle I^2 \rangle / \langle I \rangle^2$  for each range bin doing the averaging over all pings, and then averaging these second normalised moments over the range bins. In a typical experiment there were 50 pings, and 12 range bins from each ping which gave statistically independent measurements of intensity.

Table 4. Estimates of number density by statistical and echo integration methods.

Experiment number	Pulse duration (ms)	Statistical estimate of $\rho$ (number/m <sup>3</sup> )	Integrator estimate of $\rho$ (number/m <sup>3</sup> ) *	Integ. est. / Stat. est.
1	0.14	0.55	2.1	3.8
2	0.31	0.076	0.39	5.1
3	0.68	0.24	0.78	3.2
4	0.80	0.43	1.42	3.3
5	0.80	0.26	0.81	3.1
6	0.68	0.52	-	-

Experiments 1 - 5 were conducted on different night-time anchovy aggregations. Experiment 6 was conducted on a species of small pelagic fish called lightfish, which is present in extremely extensive distributed aggregations on the Atlantic coast. The second column of the table gives the pulse duration. An estimate of number density based on Eq.7 is shown in Column 3. The estimate of number density from the echo integrator is given in Column 4, and the ratio of echo integrator to statistical method estimates is in Column 5: It will be noted that this ratio is large. No integrator results are available for Expt. 6 due to a lack of target strength data for the lightfish. It may be interesting to note that, for the anchovy, the number of fish in the mid-range resolution cell varied between 0.6 and 1.7 for the five experiments.

Table 5 shows the effect of increased pulse lengths for experiments 2, 3 and 6. It is seen that the density estimates are very similar to those of Table 4.

Table 5. Statistical estimates of number density for increased pulse lengths.

Experiment number	Pulse duration (ms)	Statistical estimate (number/m <sup>3</sup> )
2	0.68	0.070
3	0.86	0.28
6	0.86	0.75

## DISCUSSION

Estimates of fish number density have been obtained using the echo integrator method and using the new method based upon the statistical properties of the echo waveform. The echo integrator results must be regarded as approximate as they rely on assumptions made of fish target strength. However, errors using echo integrator methods are likely to be insufficient to account for the large discrepancies between the two methods. It appears from column 5 of Table 4 as though the statistical technique is not yet producing useful measures of fish number density. Work is proceeding to explain sources of error in the statistical technique and the most plausible so far, not yet proven, is that they arise because of variations in the density of the fish shoal. There is a non linearity in the method which gives rise to errors. Suppose, for example, that half of the echo sounder measurement correspond to a region where  $\langle N \rangle = 2$  and half to where  $\langle N \rangle = 4$ .

\* Note that these estimates were calculated on the basis of a slightly incorrect target strength-length relationship, and that they therefore differ from the estimates of table 5.2 in the text.

The linearity of the echo integration technique is such that the estimate would be the mean value, or  $\langle N \rangle = 3$ . This is not the case with the statistical technique as can be seen by the following argument. Eq. 5 gives

$$\langle I^2 \rangle / \langle I \rangle^2 = 2 + 2/\langle N \rangle$$

For those measurements corresponding to  $\langle N \rangle = 2$ , this gives  $\langle I^2 \rangle / \langle I \rangle^2 = 3$ . Arbitrarily, putting  $\langle I \rangle = 1$  this signifies  $\langle I^2 \rangle = 3$ . For the measurements corresponding to  $\langle N \rangle = 4$ , we would expect twice the echo intensity or  $\langle I \rangle = 2$ . Therefore, for this region  $\langle I^2 \rangle = 2^2(2+2/4) = 10$ . Considering all the measurements together we obtain  $\langle I \rangle = 1/2.(1+2) = 1.5$  and  $\langle I^2 \rangle = 1/2.(3+10) = 6.5$ . It follows that the estimate of  $\langle N \rangle$  using all the measurements together would be

$$\langle N \rangle = \frac{2}{6.5/1.5^2 - 2} = 2.25$$

This estimate is not the mean value for the two regions and the dangers of a non-uniform density are clearly revealed. In this example, as always, the effect of a non-uniform density is to produce an estimate which is less than the true average density. This bias is in agreement with the discrepancy between the experimental estimates using the statistical technique and those using the echo integrator. Work is at present under way to overcome this cause of error by dividing the fish shoals into regions of constant density, based upon measurements of returned energy, and to then apply the statistical technique to each of the regions separately. The density estimates for each region will then be combined to obtain an estimate of overall average density.

Another source of error in the estimates is believed to lie in the assumption of a Rayleigh distribution of echo amplitudes. Although almost certainly better than the assumption of constant amplitudes there are several reasons for doubting its strict validity. Firstly the anchovy are typically  $10\lambda$  long which is less than the  $25\lambda$  specified by Clay and Heist for the Rician statistics to be approximated by Rayleigh statistics. Secondly the assumption of constant fish size is not strictly accurate in practice. Thirdly there is no account made of the variation of acoustic intensity across the beam of the echo sounder. Further theoretical work needs to be done but it seems likely that these three effects largely compensate one another and that the assumption of a Rayleigh distribution is not grossly in error and not sufficient to account for the major discrepancies described earlier.

A third source of error may lie in the somewhat arbitrary choice of taking the beam solid angle to correspond to the 3 dB beamwidth of the transducer when used as a transmitter. This was done largely to be in keeping with the notation of Ref.1. It is interesting to note that, if the 3 dB beamwidth corresponding to the combined transmit/receive response is used, the beam solid angle is less by a factor of 1.9 and the density estimates are increased by this same large factor to give very much better agreement with the echo integrator estimates. A value of beam solid angle which is perhaps easiest to justify is  $4\pi$  divided by the transmitter/receiver directivity factor. A calculation on the basis of this increases the estimates by a factor of 1.3. Clearly there is scope for further theoretical work to examine the effects of beam shape.

## CONCLUSIONS

It is possible to obtain an estimate of fish number density by a statistical analysis of echo soundings. In contrast to the echo integrator, a knowledge of fish target strength is not needed. Experimental estimates of number density using a tank model of a fish shoal have produced good agreement with the true density. Experimental estimates of the number density of a real fish shoal have, however, shown large errors. The estimates have been considerably and consistently too low. One of causes appears to be a somewhat arbitrary choice of beam solid angle used in the calculation. Another appears to be a non-uniformity in the fish shoal. There is reason to hope that a technique for overcoming both these sources of errors may be found and that useful estimates of fish number density may become feasible.

## ACKNOWLEDGEMENTS

The authors would like to thank Mr Ian Hampton of the Sea Fisheries Institute for making possible our participation in the Anchovy Recruitment Survey, for providing the echo integration estimates, and for extensive advice generally.

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