UNIVERSITY OF CAPE TOWN

FACULTY OF EDUCATION

An investigation into children's developing mathematical abilities.

A minor dissertation presented in partial fulfilment of the requirements for the Degree of

Master of Primary Education

by Jean Gurney

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DECLARATION

I hereby declare that the whole of this thesis, unless specifically indicated to the contrary in the text, is my own original work and that it has not been submitted for any degree in any other university.

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ABSTRACT

This study examines children's developing mathematical abilities during the first three years of their schooling. More particularly, children in grades one, two and three of three different primary schools, in two different regions, had their performances on eleven mathematics tasks monitored over the course of 1996 to examine their developing mathematical abilities. These abilities were investigated in terms of task-particular performances and the assumed competencies (internal mental processes) underlying these performances. The data was generated through the use of a repeated measures design. The theory of the methods used to gather the data and to analyse the results is rooted in Vygotsky's (1978) experimental-developmental approach to the study of higher mental functions. This method of observation proved to be successful to the degree that it allowed for the study of changes in children's performances over a seven month period. The overall findings of the study revealed that the subjects in the sample population had the developmental readiness with which to improve their mathematical abilities. However, when this developmental readiness had to be taken further through formal instruction, their performances were inadequate. The investigation exposed the complexity and importance of language in the successful development of mathematical concepts. The data indicated that the subjects' learning was neither in advance of their development nor was it indicative of the constructivist approach to the task of teaching. Furthermore, there existed a conflict between spontaneous and formal knowledge in engaging with school mathematics tasks.

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CHAPTER ONE

INTRODUCTION

In this project, I examine children's developing mathematical abilities during the first three years of schooling. More particularly, children in grades one, two and three of three different primary schools, in two regions, had their performances on eleven mathematics tasks monitored over the course of 1996 to examine their developing mathematical abilities. These abilities were investigated in terms of task particular performances and the assumed competencies (internal mental processes) underlying these performances.

The purpose of this study is to discuss the students' learning outcomes and ways in which mathematical performance and competence can be improved in their first three years of schooling. Schooling is commonly regarded as the process through which children are taught to develop their learning and thinking skills. The development of thinking skills in and through mathematics as a school subject is highly regarded in most education programmes. This is particularly evident at primary school level where the grounding for further mathematical development is set. Reasons for teaching mathematics are motivated in a junior primary mathematics course book (Flanagan, 1995:02) as follows:

> Interest in the subject comes from the public feeling that mathematics opens doors to the sciences and technology and the fact that the primary school is where the foundations of sound mathematical understanding takes place. In fact we know that you cannot train for some professions and careers unless you have mathematics.

The above quotation elaborates the importance of mathematics learning and teaching in terms of the social access it provides to the job market.

In South Africa in the past, social inequalities were perpetuated in schools and skilled work in the job market was reserved for whites. Chisholm (in Gordon, 1993:176) makes the following observation: "[o]ne of the most profoundly pernicious legacies of apartheid schooling has been the concentration of mathematical and scientific knowledge in the white community." The division of education in South Africa into

19 departments of education and "[t]he vast regional and racial disparities in the provision of resources for the different schooling systems..." (Flanagan, 1992:27) resulted in stark inequalities in terms of educational access. The resulting poor success rates that black students have had, particularly with the subject of mathematics is well documented (Gordon, 1992; Marsh, 1993; Muller and Taylor, 1995; Slammert, 1993) and is evident by the small number of "...matriculants interested in, and capable of engaging in mathematics at university level" (Craig and Winter, 1992/2:45). Marsh (1993:292) describes mathematics education in South Africa as being in "a crisis situation" and draws attention to Spira's finding of the fact "...that of every ten thousand Black children who enter the school system only one emerges eventually with matriculation exemptions in mathematics and [physical] science". These findings highlight the need for mathematical research into learning and teaching in South African primary schools if attempts to transform the legacy of apartheid education are to be successful. It is at primary school level that the seeds for later success with mathematics are sown, yet there "...is a dearth of research evidence - particularly that of primary education - to inform progressive educational decision making" (Flanagan, 1992:27). This investigation takes place against this background.

This project investigates the developing mathematical abilities of 27 children in Gauteng and the North West province. The sample includes Zulu, Setswana and North-Sotho speakers in their first three years at primary school. At the time of the study, the teachers were following a course that included a constructivist perspective but they themselves had not had any exposure to this perspective before. The constructivist approach to mathematics teaching is currently the most popular approach to the learning and teaching of mathematics in primary schools in South Africa. While this implementation of constructivist ideas is not new in South Africa, it has never been particularly widespread. The most common teaching approaches in classrooms in South Africa have traditionally been based on behaviourist models. However, the constructivist approach has been embraced by many educators as 'the answer' to children's difficulties and frustrations in classrooms (Von Glasersfeld, 1995). Constructivist principles are viewed by some as a progressive break away from the traditionally conservative methods of mathematics teaching of the past.

The changes in approaches to teaching and learning are reflected in the content and structure of the new curriculum, forms of pedagogy, methods of assessment, modes of organisation and of management, all of which are being touted as more meaningful and relevant to students in primary mathematics classes (cf. WCED Mathematics Syllabus, 1996). Similarly, constructivist principles to mathematics learning and teaching are included in the PREP Junior Primary Mathematics Course (Flanagan, 1995) on which the tasks for this study have been based. The tasks were designed to provide insight into the particular performances and assumed mathematical competence of students being taught from a constructivist perspective from an early age.

The overall findings of the study revealed that the subjects in the sample population had the developmental readiness with which to improve their mathematical abilities. However, low performance values were revealed on particular tasks requiring formal instruction and learning. The findings were disappointing given the large developmental gains that children make in their first three years of schooling (Slavin, 1988).

The outline of this dissertation is as follows. Chapter one describes the overall focus of the study within the South African context and highlights some of the more significant observations and results. Chapter two provides the theoretical background and the literature review which focuses on the learning and teaching of mathematics during junior primary schooling. The review centres around four related emphases: (i) the nature of mathematics; (ii) the relationship between development and learning; (iii) approaches to teaching and (iv) differences between informal and formal knowledge and learning. Chapter three explains the research design and the methodology used for the study. Chapter four is the results chapter where the subjects' responses to the tasks are recorded. Chapter five is a discussion chapter which highlights the most interesting results in terms of the four emphases reviewed in chapter two. Chapter six is the concluding chapter which summarises important findings and makes recommendations for further research.

CHAPTER TWO

THEORETICAL BACKGROUND

Introduction

The focus of this chapter is on theories that inform the investigation of children's developing mathematical abilities in terms of both their competence and performance. Competence refers to: the children's tacit and explicit knowledge (of mathematics in this case), the resultant mastery of the manipulation of the system of numbers (e.g. abstract concepts) and the ability to engage with mathematical tasks (appropriate for a given age) independently and successfully (cf. Chomsky, 1975). This is an idealised concept of the competence underlying mathematics performance, where performance refers to the children's observable actions on given tasks.

The theoretical review that follows emphasises the following aspects of mathematics learning and teaching:

- the **nature** of **mathematics** in learning and teaching
- the relationship between **development** and **learning**
- **approaches** to the teaching task
- the differences between **informal** (spontaneous) and **formal** (school) knowledge and learning

These emphases allow for an analysis of that which learners bring to the learning and teaching situation (e.g. developmental gains and spontaneous knowledge), and draws attention to that which the teaching task must confront (e.g. gaps between everyday and school tasks) if it is to assist children in becoming independent and successful learners, particularly at mathematics.

2.1 The nature of mathematics in learning and teaching

There are several special considerations that distinguish the learning and teaching of mathematics from other subjects. For the purposes of this study and review, the **nature** of mathematics in learning and teaching refers specifically to the nature of the

language in mathematics (or, mathematics as a non-natural language). This can be further elaborated as follows:

- abstract concepts

  e.g. the symbols of mathematical operations such as those for addition, subtraction, multiplication and division, fractions and time;

- specialised terminology where the words are specific to mathematics

  e.g. one thing is equal to another; two-thirds of a quantity; and

- mathematics register where words which have everyday meanings take on new mathematical meanings e.g. divide; share; less; faces.

These distinguishing features are characteristic of the language in mathematics learning and teaching. They are fundamental considerations for teaching children[1] to engage with mathematical tasks independently and successfully. The importance of language in mathematics classrooms is emphasised by Flanagan (1995:60) as follows:

> Schools don't usually recognise the importance of language for learning; schools usually put emphasis on facts and information, on content, and only see language as the medium through which we express what we know...Yet every teacher is a teacher of language because every subject or discipline has particular concepts which need to be understood in order to produce knowledge. And language is the tool we use to think about and produce knowledge. For example, in mathematics division, multiplication, the properties of a circle, sets and subsets, place value are all mathematical terms...so...teachers must be responsible for teaching the language of mathematics.

This quotation emphasises that the success of children's performances on mathematical tasks is largely dependent on the extent to which they master the required mathematical language. Conceptual understanding cannot develop without understanding the language of mathematics.

The language spoken in mathematics classrooms is particularly complex because of the combination of 'ordinary' and 'mathematical' language that is used. 'Ordinary' refers to everyday usages of words and 'mathematical' to the particular meanings appropriate to the language of mathematics (Pimm, 1987, emphasis in original). The latter is sometimes referred to as being part of the 'mathematical register', and is described as follows:

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[1] The words students, children, subjects and learners are used interchangeably in this project for interests sake.

> ...the meaning that belongs to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself)...We should not think of a mathematical register as consisting solely of terminology, or of the development of a register as simply a process of adding new words (Halliday in Pimm, 1987:76).

Some registers are used on certain occasions and not on others. Therefore part of learning mathematics involves learning the 'language game' that is specific to the subject of mathematics. Confusion commonly occurs because of the many specialised terms which do not have meaning in 'ordinary' English (e.g. quadrilateral, parallelogram, hypotenuse etc.) as well as the borrowed words from everyday language (e.g. divide, some, all, less, share, degrees, faces, etc.), that are part and parcel of mathematics teaching. Because of this mixture of what Pimm (1987:88) terms "ordinary" and "mathematical" English, students sometimes try to understand what they hear in terms of their everyday understandings of the meanings of words, when in fact something very different is intended. Pimm (1987:88) warns that a "...failure to distinguish between these two can result in incongruous errors and breakdowns in communication". He therefore recommends that children be made aware of the mathematics register that is used in class and that teachers help their students to understand how to keep different meanings and uses separate:

> Pupils at all levels must become aware that there are different registers and the grammar, the meanings and the uses of the same terms and expressions all vary within them and across them Without this awareness, little sense can be made of a non-trivial part of mathematics usage (Pimm, 1987:75, original emphasis).

Children need to know when to use one meaning or another and for this to happen, the teacher has to be aware of the different registers that are being used in the class. Children should therefore by given many opportunities in class to talk mathematically so that they can become knowledgeable about the mathematics register and the way in which it is used. Adler (1996:7&8) makes this point clearly:

> [l]earners can develop familiarity and confidence using new educated and educational discourses only by using them...they also need opportunities to practise being users of educated discourses. Often there is a mismatch between the educational discourse in play ... the ways with words being used

> in the classroom ... and the educated discourse they are meant to be entering. So, in relation to mathematical discourse, the teacher's role is to translate what is being said into academic discourse, to help frame discussion, pose questions, suggest real life connections, probe arguments and ask for evidence.

Anomalous responses and contradictions expressed by students are good indicators of differing meanings or interpretations which can be picked up by teachers. When this occurs, the different registers have to be made explicit to the class (Pimm, 1987:77&78). Pimm (1987:110) is clear on the importance of distinguishing between different registers when he says "...the greatest danger is that the unexplained extension of concepts can too often result in the destruction rather than the expansion of meaning" (original emphasis).

The implication of this for teachers is to make the differences between everyday and mathematical meanings explicit so that the confusion is minimal. However, this is not as easy as it may sound. In a research project conducted by Adler (1996:20), she noted that the teacher's "...focus on language obscured rather than enabled access to mathematical practice" and that the explicit focus on the form of words being explained served to "inadvertently obscure" rather than to aid mathematical understanding. These kinds of linguistic hurdles are evident in other subject areas too, but not necessarily to the same extent in which one encounters them in mathematics.

In fact one of the most pertinent problems in classroom practice of mathematics teaching does not concern the learners as much as it does the teachers. Often teachers are unable to recognise the everyday understandings that their students may have. Knowing how to bridge the gap between everyday and school knowledge requires specialised skills and training which has been acknowledged by many researchers. Nunes, Schliemann and Carraher (1993:150) indicate that the problem lies in the fact that the "...teachers themselves have grown up with a gap between informal and formal mathematics". Davis (1996:150) suggests that it is somewhat dependent on "the skill of the teacher to facilitate the student's production of specific inscriptions without having explicitly announced them or the means for their production". The

skill of the teacher has a definite influence on the way in which students develop their mathematical understandings.

The use of a combination of different languages becomes even more complicated in multilingual classrooms. Teachers shift between the first language of the students, the language of learning (English) and the 'language' of mathematics, i.e. three languages. Combining languages together in this way can easily lead to confusion and misunderstandings unless teachers are meticulous about the ways in which language is used in the classroom context (cf. Adler, 1996). Flanagan (1995:64) emphasises the crucial role that teachers play in this regard as follows:

> Teachers have to be especially careful with the ways in which they themselves use language and the ways in which they introduce children to the correct language. It is a heavy responsibility on the teacher but very rewarding if done properly because then the kids become good at maths.

The significance of the language used in mathematics learning and teaching cannot be overemphasised. As stated above, it is the responsibility of the teacher to ensure that the children are provided with the correct language to develop their conceptual understanding of abstract mathematical ideas. It is also her responsibility to differentiate between the ordinary and mathematical language used in the classroom context. If teachers clarify the meanings and importance of the language used by themselves and by their students, they will have made a major contribution to the development of the competence required by students to successfully engage with mathematical problems and tasks.

2.2 The relationship between development and learning

When analysing teaching and learning in school children, the relationship between development and learning has to be addressed. There are a variety of theoretical positions which view this relationship in different ways. Some view development and learning as being completely independent, others regard them as being inseparable while still others advocate a mutually dependent and interactive relationship between the two (cf. Vygotsky, 1978:79-81).

For the purposes of this study I draw on two main cognitive theorists, Piaget and Vygotsky. Piaget (1972) conceptualised development along different stages based on

biological adaptation to the environment. Each of his stages encapsulates a different (logical) level of knowledge that reflects the manner in which one thinks. Piaget's ideas about developing cognitive abilities emphasise "**...universal** capacities of the ability to know" (Craig, 1985:55). Piaget did not regard himself as an educationalist or even as a child psychologist and therefore did not directly theorise about the development of particular individuals or groups, nor did he advocate any particular approach to learning and teaching. Vygotsky (1978:132), on the other hand, recognised the relationship between biological bases of behaviour and social conditions as being a critical issue in the development of higher mental functions and he did make suggestions for learning and teaching. He examined people within particular contexts and argued that the way children think about mathematics in terms of how they solve problems and understand certain concepts is greatly influenced by sociohistorical conditions. Craig (1985:57) describes this sociohistorical position of Vygotsky when she says, "...knowledge is not "fixed" in logico mathematical structures but tied to sociohistorical processes and it is therefore necessary to ask how the mind develops within these particular constraints".

Vygotsky (1978) recognised the significance of a relationship between development (as biologically driven) and learning (as dependent on particular communicative relationships) and devised an experimental-developmental method for studying higher mental functions. This method provides the means by which to study the changing mathematical abilities of children through a reconstruction of each stage of the process of development (see chapter 3.2). The principle upon which the reconstruction of developmental stages is based as a means of explaining the dynamics of cognitive change is cited by Craig (1985) as follows:

> Piaget reconstructed the genesis of logico-mathematical thought with reference to both the history of the child and the growth of knowledge. In this sense, Piaget has created a theory of, and method for, the study of change. Despite the apparent differences with Vygotsky regarding the origin of higher mental functions in humans, Piaget may be regarded as the greatest exponent of the developmental method advocated by Vygotsky.

My research is a study of the change and development of children's mathematical abilities in a manner exemplified by Piaget and made explicit by Vygotsky in terms of

his experimental-developmental method. Piaget's theory of cognitive development is used to inform the discussion on development and Vygotsky's theory of mediated learning is used to inform learning and teaching.

Piaget's theory of cognitive development

At the turn of this century, children were still regarded by many people as miniature adults.

> Human development was assumed to follow a smooth progression from infancy to adulthood. By the age of six or seven children were believed to think in much the same way as adults, and it was assumed that all they lacked was experience and education (Slavin,1988:21).

It has only been in relatively recent times that children's cognitive, personal and social developmental needs have come to be regarded as being important and different to those of the adult population. Instead of advocating continuous theories of gradual development, various theorists proposed that "...children do not develop gradually, but rather go through a series of stages of development. The abilities that children gain in each subsequent stage are not simply "more of the same"; at each stage children develop qualitatively different understandings, abilities and beliefs" (Slavin, 1988:21).

Piaget describes four distinct stages of cognitive development. These stages range from the most basic sensorimotor stage to the final formal operational stage and are summarised by Slavin (1988) as follows. The sensorimotor stage (from birth to about two years) is the stage in which children explore the world using their senses and motor skills. The second stage is the preoperational stage (from about two years to about seven years) during which time children start to use symbols to mentally represent objects. The third stage is the concrete operational stage (from about seven years to about eleven years) during which children can perform operations such as reciprocity and inversion and have gained the ability to solve tasks where conservation is required (the concept that certain properties of an object remain the same regardless of changes in other properties). The final stage is the formal operational stage (from about eleven years to adulthood) during which time the ability to deal with hypothetical situations and to monitor one's own thinking is developed.

Most of the children in my study fall into the concrete operational stage. Once again Slavin's work (1988:30-31) is drawn upon for a broad overview of the kinds of cognitive abilities expected from children in this stage. Children in the concrete operational stage no longer experience difficulties with conservation problems because they acquire the concepts of reversibility (the ability to perform a mental operation and then reverse one's thinking to return to the starting point). They are able to respond to inferred reality which means that they are able to see the same things in terms of other meanings. They also are able to recognise relationships between things, are less egocentric than in the other two stages and are able to see things from other people's perspectives.

Another important task that children learn during the concrete operational stage is to order things according to a particular attribute. When ordering objects in a particular way they have to be able to compare separate but related bits of information along a scale. Once they are able to order objects in this manner, they can then master a related skill known as "transitivity" which "requires the mental arrangement and comparison of objects" (Slavin, 1988:31).

There are other operations which develop during this stage which are important to mathematics learning. These include: invariance of properties such as length, inclusion, invariance of shape under geometrical transformations, the use of place value, use of algebraic symbolism to refer to an identifiable object or length and so on (Brown in Floyd, 1979:364&366). However, it should be borne in mind that the stage generally spans five years although the ages at which children progress from stage to stage has been shown to cover a wide range. In particular, the onset of formal operations seems "...likely to occur on average much later than 11 years" (Brown in Floyd, 1979:361).

Slavin goes on to say that although children at this developmental level acquire a number of new skills and concepts during this stage of their development, they still have difficulty with abstract thought. Flavell, in Slavin,

points out that a concrete operational child still has "an earthbound, concrete, practical minded sort of problem-solving approach, one that persistently fixates on the perceptible and inferable reality right there in front of him". In other words the child can "form concepts, see relationships, and solve problems, but only so long as they involve objects and situations that are familiar" (Slavin, 1988:31).

According to Piaget (1972), it is impossible to skip any of the developmental stages, but it is likely that a child will exhibit behaviours that are characteristic of more than one stage at any given time. The tasks used in my study (see Appendix A) do not require performances beyond the scope of the ability of a concrete operational child and are therefore appropriate for the children in the sample population.

While Piaget did not directly theorise about learning and teaching, many other researchers have used his principles to make recommendations for classroom practice (see, for example, Bruner, 1966). One principle deduced from Piaget's stage theory of development, that has been used extensively is the idea that **development precedes learning.** This principle arises out of his finding that developmental stages are largely fixed and that certain concepts can not be taught at earlier developmental stages. Recently, however, several researchers have established cases in which Piagetian tasks can be taught at earlier developmental stages. Piaget's response to these findings has been to argue that the children must have been on the verge of the next developmental stage, but as Slavin (1988:35) points out "...some...Piagetian tasks can be taught to children well below the age at which they usually appear without instruction" (cf. Bruner, 1966; Brown in Floyd, 1979).

There are many critics of Piaget who have conducted research to argue that Piaget underestimated children's abilities. Recent findings suggest that children can "...succeed on simpler forms of Piaget's tasks that require the same skills" (Slavin, 1988:35). By using less abstract language in the instructions that are given to the children and by using simpler forms of Piaget's tasks, children's abilities have been shown to be more competent than Piaget originally thought (Black, 1981; Boden, 1980; Donaldson, 1978; Gelman, 1979).

Despite the criticisms, Piaget's cognitive theories have still had far reaching implications in terms of classroom practice. The most notable of these implications has been the idea that teaching must be adjusted **to the level of the child**. The thinking behind this recommendation arises out of the Piagetian principle of fixed developmental levels and the notion that children cannot learn certain concepts before they have reached a particular stage. This principle has resulted in the formulation of a variety of pre-programmed teaching materials which have step-by-step instructions to get learners from one level to another in particular hierarchical (developmental) sequences.

'Discovery learning' is one of the ways in which Piaget's ideas have been put into classroom practice (Bruner, 1966). Discovery learning is a problem-solving, learner-centred approach which emphasises the acquisition of skills at an individual pace and where thinking is central to the learning process. Students are meant to discover things for themselves within this approach and the teacher's role is seen to be that of a 'facilitator' whose primary responsibility is to provide materials and to set up situations in which appropriate problem-solving skills and concepts can be 'discovered'. For more details of classroom implications of Piaget's work, refer to chapter 2.3.

Vygotsky's theory of mediated learning

Unlike Piaget, Vygotsky does theorise about how learning and teaching happens and his notion of a zone of proximal development has been especially influential in educational circles. Traditionally, children's abilities have been assessed by means of formal tests that are administered to determine the level of each child's independent understanding. Then Vygotsky (1978) presented the alien notion that a child could be given a test and receive assistance with it. He proved that "...the capability of children with equal levels of mental development to learn under a teacher's guidance varied to a high degree" (Vygotsky, 1978:86). Hence came the formulation of what is now well known as the zone of proximal development (zpd), defined as:

> ...the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978:86).

The implications of the zpd on learning and development are enormous in that a skilled teacher is a crucial factor in the child's learning and development. The development of cognitive growth, according to Vygotsky (1978), occurs within the learner's zone of proximal development through spoken language. The teacher pushes the learner's thinking beyond the understanding that she would have accomplished without guidance.

This emphasis on the zpd goes **against** the view of learners constructing their own knowledge. The zpd emphasises the role of the teacher as the 'constructor' or mediator of new knowledge to the learners. She is the one who provides the link between the known and the unknown through her teaching. Vygotsky (1978:86) explains this further by likening "...those functions that have not yet matured but are in the process of maturation" to "buds" or "flowers" of development, rather than to "fruits" of development. This example demonstrates the difference between what is and what can or will be, i.e. the actual and potential for mental development.

The zpd provides a model for explaining the development of internal higher mental functions on the level of individual action through a mediated account of instruction and learning. This is done by considering the dynamic developmental state of the child in addition to the development that has already occurred. The implication of this for teachers is revolutionary and conflicts with the Piagetian principle that teaching should be directed at the level of development that children have already reached. Instead, Vygotsky (1978) proposes that teaching should be directed at a level that is currently in **advance** of the child's development so as to encourage the "bud" to blossom.

Craig (1996:49&50) discusses the zpd in terms of the teaching task and suggests that there is no point in directing teaching at the child's level of development, because then there is no gap between the known and the unknown, and there is no

motivation for learning. This leads her to conclude that unless there is conflict between what a learner knows and can do and the demands or constraints of the task, no new knowledge or thinking will be gained. Thus it is essential for teachers to know how to present tasks to pupils in ways that are unfamiliar in terms of content or form. They need to know how to bridge between content and form for learning to take place.

The teacher's approach in the classroom influences the manner in which the environment is arranged. A child should be able to reach a point at which she is able to reflect and to become more conscious of her learning. Bruner (in Wertsch, 1985:24-25) states this succinctly:

> If the child is enabled to advance by being under the tutelage of an adult or a more competent peer, then the tutor or the aiding peer serves the learner as a vicarious form of consciousness until such a time as the learner is able to master his own action through his own consciousness and control. When the child achieves that conscious control over a new function or conceptual system, it is then that he is able to use it as a tool. Up to that point the tutor in effect performs the critical function of "scaffolding" the learning task to make it possible for the child, in Vygotsky's words, to internalise external knowledge and convert it into a tool for conscious control.

It is the teacher's job then to provide the 'scaffolding' with which her learners will be able to bridge the gap between the known and the unknown and that which they can do with support and that which they can do independently.

The tasks (see Appendix A) formulated for this research project were designed to discover the learners' limits by eliciting actions that indicate what the subjects can and can not do. When the subjects made errors, their limits were exposed and their lack of understanding became apparent. By analysing observational data regarding variations in the ways in which subjects tackled tasks (particularly the errors that they made) recommendations could be made for teachers as to the types of 'scaffolding' that could be used in classrooms in ways that would raise their learners to new heights of understanding. Towards this aim, different approaches to teaching are now considered.

2.3 Approaches to the task of teaching

In discussions that have to do with mathematics education, two basic contrasting views can be identified. The classical view is that mathematics is a codified body of knowledge which emphasises what children should know. The other perspective is

the view that mathematical knowledge is the result of a learner's activity of problem-solving through the construction of relationships and patterns.

This research project investigates the development of children's mathematical abilities in junior primary classrooms in African schools. In the past, the classical view of teaching and learning mathematics has been practised in these classrooms. Recently, some teachers have shifted towards applying constructivist principles to their teaching. The reasons for this shift and the understandings upon which these changes have been based warrants an overview of both the classical and the constructivist perspectives.

Behaviourism

The classical view of knowledge is rooted in the pedagogical theories of behaviourism. Behavioural learning theories tend to emphasise observable behaviours such as skills or knowledge that can be demonstrated. This view of knowledge is the traditional authoritarian approach to learning and teaching that has dominated classrooms all over the world and particularly within South African schools.

Behaviourists believe that knowledge is something on the outside of learners. For them, learning involves the acquisition of abilities that are not innate. Learning is dependent on experience. The origin of knowledge is seen to be in the environment. Senses are used to detect the stimuli in the external world and the mind then detects patterns in these stimuli.

It has only been since the late nineteenth century that people have begun to study learning in a scientific manner, by conducting experiments to understand how people and animals learn. Important researchers within the behaviourist approach to learning include scientists such as Pavlov, Thorndike and Skinner.

The importance of Pavlov's work is as much in the methods he used for his experiments as in the results. Pavlov emphasised observable and careful measurement and the systematic exploration of various aspects of learning that form the

cornerstones of scientific research methods today. His research helped advance the scientific study of learning but his findings have few applications to the learning that takes place in schools (from Slavin, 1988:111).

Thorndike viewed most behaviour as a response to stimuli in the environment and he linked behaviour to physical reflexes. As a result of his experiments he formulated Thorndike's Law of Effect which stated that an act followed by a satisfying change in the environment is more likely to be repeated in similar situations and that an act followed by an unsatisfactory change in the environment is less likely to be repeated in similar situations (from Slavin, 1988:112). This law suggests that the consequences of one's present behaviour would greatly affect one's future behaviour. Skinner's work (1953) elaborated on the relationship between behaviour and its consequences. His research focused on observing the changes in subjects' behaviour that resulted from changing the consequences of their behaviour.

There are many other important researchers who advocate a behavioural model of teaching and many different interpretations of the tasks of teaching within behaviourist classrooms. Some of these implications are discussed below.

The implications of behavioural studies for the classroom have been enormous. Within this view, content is seen to be central to the learning process and is taught through pre-determined sequences of small steps in order to take the learners to the desired behaviours. Little consideration is given to the prior knowledge of individual learners who are generally taught at a common pace. Different levels of observable behaviours are taught, the principles of which can be found in works such as Bloom's *Taxonomy of educational objectives* (Bloom, 1956).

Meaning is understood to be given to the learners by the teacher who is considered to be the expert and this is accomplished through the use of repetition, drills and memorisation. The popularised term 'Talk and Chalk" is indicative of the kind of method most commonly employed by teachers utilising this approach. Positive feedback and the reinforcement of desirable behaviours are used by teachers to help

them formulate procedures for discipline and classroom management as well as to help students master particular kinds of knowledge and skills. Assessment is usually accomplished by means of psychometric and other educational tests which are specifically designed to test knowledge that is measurable in terms of specific objectives.

Based on the behaviourist view of learning, the teaching of mathematics emphasises an accumulation of skills in a set sequence which is systematically and hierarchically ordered. The order in which numbers are learned is controlled, starting with low numbers and working one's way up. Tables are taught and reinforced through drills and repetition and mathematical calculations are taught through the application of mechanical procedures and set methods which emphasise accuracy over understanding about how numbers work.

Slavin (1988) describes the strengths of the basic principles of behavioural learning theories as "firmly established" and states that they "...have been demonstrated under many different conditions". He goes on to say that "[t]hese principles are useful for explaining much of human behaviour, and they are even more useful in changing behavior" (1988:139). But there are also extensive limitations to behavioural theories. By focusing exclusively on observable behaviour, behaviourists neglect processes that are less visible and which are central to school learning. The kinds of processes that are difficult to observe directly include concept formation, problem-solving and thinking and learning from text-processes that form an integral part of one's education.

There are many problems that arise out of the behaviourist approach to learning and teaching. Some of these problems are particularly evident when students try to transfer their school learning to more practical everyday situations. They commonly find that the knowledge that they have acquired in school is of absolutely no use out of the context of the classroom. The issue of learning transfer is one of the problems of a behaviourist approach to learning and teaching to which constructivists believe they have solutions.

Constructivism

The constructivists regard mathematical knowledge as a learner's activity of problem-solving through the construction of relationships and patterns. From the constructivist's perspective cognisance of the real world is achieved through mental actions, where actions are understood as "...all activity by which we bring about a change in the world around us or by which we change our situation in relation to the world" (Sinclair in Steffe & Wood, 1990:01).

This is an authoritative view of learning and teaching that has become firmly established in education debates world-wide. Its dominance is apparent if one surveys the current literature in the field of mathematics education. Constructivists believe that mathematics education is controlled by the learner and constructed 'inside' the learner.

Constructivist theory is not new. Even Socrates with his use of questioning as a method of teaching showed that children should be allowed to construct their own knowledge (Sehlare, 1993:185). However, in the 1930s, Piaget, who is perceived by many as this century's pioneer of the constructivist approach to cognition, was still considered as unconventional (Von Glasersfeld, 1995:54). According to Von Glasersfeld, "[i]t was not until the 1960s that the decay in faith of the existing scientific objective knowledge made itself particularly felt" (in Sehlare, 1993:185). It was only then that the emphasis in education in the western world shifted to "...**how** children learn rather than **what** teachers should teach" (Sehlare, 1993:185, my emphasis).

Constructivism is relevant to my research project because the course work on which the study is based is rooted in a constructivist understanding of knowledge. The PREP Junior Primary Mathematics Course notes that traditional teaching that has been based on behaviourist methods has been discredited and advocates a constructivist approach in which "...most mathematical knowledge has to be

constructed by the children themselves with the help of the teacher" (Flanagan, 1995:09).

Since this project investigates children's developing mathematical abilities it is important to understand the theory on which the classroom practice has supposedly been based so as to gain some insight into the kind of learning outcomes that can be expected. Therefore, a thorough grasp of the constructivist debate serves to usefully inform the findings of the research.

According to Von Glasersfeld (1995) constructivism is the most important theoretical perspective to have emerged in mathematics education in the past few decades. However, it is also a term that has become popularised, so there is a great deal of diversity in terms of how it is understood and implemented in classrooms. Gordon (1993) and Phillips (1995) both point out that constructivism has a wide range of goals and intentions and that it is by no means a unified perspective. It lacks clear vision and agreed upon teaching methods. Gordon (1993:175) argues that the differences between various constructivist viewpoints "...are rooted in the way in which programmes conceive of the 'social' as embodied in philosophical and psychological notions of constructivism". These are important considerations to bear in mind when discussing constructivism in mathematics education. Paul Ernest (1993:168) identifies three constructivist models which he distinguishes from each other by unpacking their underlying metaphors. These models are information processing, radical constructivism and social constructivism. In order to give a complete overview of the constructivist perspective, these three models are described below.

Information processing constructivism

Ernest (1993) describes this model as being based on the metaphor of the mind as a computer or machine and as operating from the absolutist assumption that mathematics knowledge is infallible. From this point of view the mind is viewed as actively processing information, memorising and retrieving data. Information processing "...recognises that knowing involves actively processing, that it is

individual and personal, and that it is based on previously acquired knowledge" (Ernest, 1993:169). It represents a significant change from the traditional transmission view of teaching in that children are encouraged to reflect on error patterns and to look for alternatives in their work.

One of the implications for classroom practice within this model is that is makes no allowance for teachers who do not agree with the understanding that reasoning and mathematical knowledge is unproblematic or ultimately knowable (Ernest, 1993:169). It is also unable to provide answers to the problems that arise when the building of knowledge is idiosyncratic or is found to have faulty foundations. This model is primarily a theoretical one and I am not aware of any leading educationists who advocate the direct implementation of this model in classrooms.

Radical constructivism

This is a sceptical position in theories of knowledge. The central metaphor for the mind is of an evolutionary organism operating on the central concept of the notion of 'the survival of the fittest' which is evident in Piaget's notions of adaptation to the environment (Ernest, 1993:168). Not only is knowledge built up by the cognising subject, but according to Von Glasersfeld, "...the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality" (in Ernest, 1993:169). The environment is seen to be 'experienceable' but, unlike information processing constructivism, it is not seen to be ultimately 'knowable'. The onus for the discovery of mathematics rests solely on the learner who operates within the ordinary life-world of everyday experience.

This stance operates from a 'fallibilist' view of mathematics. It emphasises the essential subjective nature of all experiences including the idea that experiences and language can never be assumed to be the same from one individual to the next. Von Glasersfeld (1995:02) explains that "...the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. What we make of experience constitutes the only world we consciously live in".

The complications and the implications for classroom practice within this perspective are highlighted by critics of this view who point out that it provides the scope for individuals to be seen to be able to construct whatever reality they like, while disregarding the role of society and the social interaction in the development of knowledge. Ernest (1993:171) explains this further:

> ...the cognisant subject appears to be near-hermetically sealed in a privately constructed world of its own. Its representations of the world and indeed of other human beings are personal and idiosyncratic. Indeed, the construe of other persons is driven by whatever representations best fit the cognising subject's needs and purposes.

The teaching process also gets called into question by the apparently common belief that this position implies:

> ...that mathematical learning would be a process of spontaneous, unguided, independent invention...Constructivist theory is then interpreted to imply that students' learning should be natural and that teachers should not tell them anything as they attempt to make sense of their worlds (Cobb, Yackel and Wood, 1992:27).

Since many critics do not believe that mathematical knowledge is independent of social and cultural influences, suggestions that teachers should not attempt to influence students' constructive efforts becomes nothing short of indefensible and is what I believe to be one of the major shortcomings of the approach. As a result of the individual nature of this approach, "...there is never only *one* right way of teaching" (Von Glasersfeld, 1995:176, original emphasis). Therefore while radical constructivism informs an understanding of the ways in which children construct mathematical concepts, teaching remains a conjectural affair with a wide variety of understandings.

Social constructivism

This position is the newest form of constructivism and has Vygotskian roots. The metaphor upon which it is based is one of conversation where thought is seen to be based on internalised conversation and meaning is seen to be socially constructed. For Ernest (1993:170), "[t]he social constructivist model of the world is that of social reality, the socially constructed world which creates (and is constrained by) the shared

experience of the underlying physical and social worlds". Ernst (1991:42) elaborates this point by describing mathematics as a social construction because "...language, rules and agreement play a key role in establishing and justifying the truths of mathematics".

Vygotsky (1978:132) recognises the relation between the biological bases of behaviour and social conditions as being a critical issue in the development of higher mental functions. He states that the way that children think about mathematics in terms of how they solve problems and understand certain concepts is greatly influenced by sociohistorical conditions. Children's understandings of mathematics, according to Vygotsky (1978), is impacted upon by the social conditions in which they live and learn. Teaching is understood to be a "...social activity in which meaning is mediated to and negotiated with the learner" (Moll, 1989:715). Conversational methods are used by the teacher to provide the learners with the new conceptual language which they will internalise and which will then become the basis of higher forms of cognitive activity (Moll, 1989).

The importance of language is stressed in the PREP Junior Primary Mathematics course book (Flanagan, 1995:60) in a way which favours sociohistorical theory over behaviourism:

> Schools don't usually recognise the importance of language for learning; schools usually put emphasis on facts and information, on content, and only see language as the medium through which we express what we know....Yet every teacher is a teacher of language because every subject or discipline has particular concepts which need to be understood in order to produce knowledge. And language is the tool we use to think about and produce knowledge.

The study of mathematics from within this realm of constructivism cannot be seen to be independent of the learner. The source of the changes in the knower-known relationship is found in the biological process of adaptation whereby external experiences are changed to fit into systems of cognition (Piaget, 1972). Since these external experiences are essentially social, the very objectivity of mathematical knowledge is seen to be social, as it is based on the acceptance of linguistic rules

which are necessary for communication and which are socially constructed. Mathematical knowledge then, can be seen to be the means by which relationships are constructed and generated (Ernst, 1991). This is the Vygotskian response to the radical constructivist conundrum.

Of the three models presented herein, social constructivism seems to be the most reasonable. It not only highlights the problem areas of the other two models, but also acknowledges the extreme complexities between learning and development. The relationship between development, which is basically biologically driven, and learning, which is dependent upon particular communicative relationships, is also recognised as being further complicated by the demands of formal schooling. Whatever approach to the teaching task is adopted, central to the learning-teaching situation is the intervention and even conflict between everyday and school knowledge.

2.4 Everyday and school knowledge

The sociohistorical developmental theories of Vygotsky can be used to provide insight into some of the discontinuities between everyday (spontaneous) and school (scientific) mathematics. For Vygotsky (1978), the two are completely separate in that "...development of 'scientific' thinking resides only in the context of schooling" (Adler, 1996:10). In other words, everyday knowledge and school knowledge are completely separate issues in terms of how they are learned and how they are taught. The Vygotskian perspective is summed up by Adler (1996:08), as follows:

> For Vygotsky, schooling and formalised instruction lead specifically to the development of metacognitive awareness on the one hand and to the development of what he called 'scientific concepts' on the other. The learning of new word meanings in school is not through direct experience with things or phenomena: rather, it is through a system of concepts. Vygotsky distinguished 'scientific' concepts from 'spontaneous' concepts, those concepts that are formed in our everyday activity. For Vygotsky, scientific concepts are unsystematised and saturated with experience. Nonetheless, scientific and spontaneous concepts, while distinct, interact with and influence each other.

This is of great significance because it means that Vygotsky in effect acknowledges the importance of prior knowledge and of gaining insight into the learner's everyday

understandings. 'Spontaneous' concepts, while regarded as being separate and different from 'scientific' concepts are recognised as having a strong influence on school learning. The ways in which 'spontaneous' and 'scientific' concepts influence and interact with one another interrelate with problems associated with 'learning transfer' and are the source of much research. There are wide-ranging opinions in the debate about school and everyday knowledge and the issues surrounding the roles that they play in teaching and learning (Cazden, 1988; Cole, 1985; Dowling, 1993; Gerdes, 1989; Moll, 1989; Nunes et al., 1993).

Some researchers argue **for** recontextualising everyday knowledge into the curriculum (Nunes et al., 1993) while others warn **against** the introduction of everyday knowledge into the classroom context (Dowling, 1993; Walkerdine, 1988). The findings of a research project carried out in Brazil by Nunes et al. (1993) conflicted with the implicit pedagogical assumption of teaching formal mathematical operations before applying them to verbal and real-life situations. The results showed that "...there are informal ways of mathematical calculations which have little to do with the procedures taught in school" (Nunes et al., 1985:21) and the researchers noted that:

> ...it seems quite possible that children might have difficulty with routines learned at school and yet at the same time be able to solve the mathematical problems for which these routines were devised in other more effective ways.

The findings from their research show conclusively that context-embedded problems are much more easily solved than ones without a context (Nunes et al., 1985:24). They concluded that mathematical operations should be introduced in contexts that are relevant to children's daily lives and exposed doubt "...about the pedagogical practice of teaching mathematical operations in a disembodied form before applying them to word problems" (Nunes et al., 1985:27). This recommendation is further emphasised by their suggestion that:

> ...educators should question the practice of treating mathematical systems as formal subjects from the outset and should instead seek ways of introducing these systems in contexts which allow them to be sustained by human daily sense (Nunes et al., 1985:28).

Kuzolin (in Gordon, 1992:05) is also critical of the disjuncture between school and everyday knowledge. For him, formal knowledge runs the risk of being "...applicable to a rather narrow range of topics learned in school" and he points out that "...school practice is full of situations in which a child becomes helpless when required to apply the concepts learned in the classroom to phenomena outside the school curriculum". Muller and Taylor (1995:268) support Kuzolin's view and state that it "...serves to emphasise the disjuncture between school knowledge and everyday life". Dowling's (1993) research has shown that the different British mathematics textbooks enforce an effective class discrimination in exactly this manner. Research conducted by Scribner and Cole (1973:553) led them to conclude that:

> ...school represents a specialised set of educational experiences which are discontinuous from those encountered in everyday life and that it requires and promotes ways of learning and thinking which often run counter to those nurtured in practical daily activities.

They accentuate the contrasting features of school learning and everyday learning while stressing that the two are constantly intermingled.

Nunes et al. (1994:148) also discuss the importance of preserving meaning during the mathematising of situations - a concept which they say has been greatly explored under realistic mathematics education. They describe it as follows: "...Realistic mathematics education involves posing to pupils in the classroom problems that require the consideration of empirical constraints as well as social and logical rules that apply outside school".

They go on to explain that this approach emphasises problem-solving in imagined situations and that procedural applications are not part of this understanding. The decisions that have to be made are intended to give sense to the problem and to provide children with examples of mathematics that they would be able to use outside of the classroom (Nunes et al., 1993:148). It is upon this model of mathematics that Nunes et al. base their conclusions about the implications for teachers' classroom practices. They suggest that the "models built in the classroom about problem situations arise from problem-solving activities" and that "[f]or this reason, the

models can work as a bridge between street and school mathematics" (1993:149). These researchers propose that students start from their own understandings and representations of situations and that they are then provided with the mathematical tools that help them connect various aspects of their knowledge.

Ethnomathematicians also focus on the ways in which "...cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics" (Barton, 1996:214). The ethnomathematicians stress that the school curriculum represents only one way of systematising and expressing mathematical activities, and that different local and cultural forms of mathematics practised in the communities are equally valid and worthy of recognition (Gerdes, 1989). However, Gerdes prefers neither to include nor exclude everyday notions of mathematics in the school curriculum. Taylor (1993:132) elaborates on this:

> Gerdes advocated the use of local knowledge as a bridge into formal mathematics. The assumption underlying this recommendation is that the knowledge canonised in the official curriculum is the desired outcome of mathematics education and, as such, represents the highest form of mathematical knowledge.

Walkerdine (1988) explores the precise nature of the bridge between local knowledge and formal mathematics and has made significant contributions to increasing the understanding of the learning and teaching of mathematics. Her research into the nature of recontextualisation is particularly interesting. She theorises about what happens when a set of signs that make sense in one discursive domain are transferred to a different set of meanings in another. She also points to the discontinuities between everyday and formal tasks. She advocates teaching practices in which teachers link the everyday 'non-mathematical' discourses of their students with formal school mathematics through a series of transformations. This is done "...by the formation of complex signifying chains, which facilitate the move into new relations of signification which operate with written symbols in which the referential content of the discourse is suppressed" (Walkerdine, 1988:128). In other words, Walkerdine advocates the use of everyday knowledge as a starting point to school mathematics.

However, knowing how to use everyday knowledge in ways that are effective and that do not serve to reinforce the differences between 'spontaneous' and 'scientific' concepts (Vygotskian terms, 1978) is no easy task. Teachers not only need to have mastered the language and mathematical concepts themselves, but they also have to think of ways of eliciting the children's everyday understandings from them so as to bridge the gap between the known and the unknown. Suitable problems and learning opportunities need to be presented to the students in ways that are interesting and useful in developing mathematical competence. Generating suitable problems in which the constraints of life are taken into account, is also no easy task. Textbooks are riddled with ridiculous so-called 'real-life' problems, which completely ignore the practicalities of daily life (Davis, 1996) and serve only to compound the complexities of dealing with the relationship between school and everyday mathematics. These are the kinds of problems that teachers have to confront on a daily basis if they want to be effective as mathematics teachers.

In this review I have discussed the nature of language in mathematics learning and teaching and have drawn attention to the need for teachers to master the mathematical terms and concepts themselves before they can effectively teach these ideas to their students. The importance of differentiating between ordinary and mathematical words has also been emphasised. The importance of directing teaching at levels that are in advance of students' development (see Vygotsky on mediated learning, chapter 2.2) has been stressed under discussions of the relationship between learning and development. This is so that children can advance from that which they already know to that which they still have to learn. Mediated learning requires special teaching skills and the importance of the spoken word is reiterated. Different approaches to mathematics teaching have been highlighted and the constructivist approach has received a particularly strong emphasis. Methods of teaching and learning outcomes which typify student learning within this approach are explained. Finally, the effectiveness of using children's spontaneous knowledge as a starting point for teaching scientific concepts has been discussed. The theories and ideas presented in

this review are drawn upon in chapter five by way of offering explanations for certain of the study's results.

CHAPTER THREE

RESEARCH DESIGN

3.1 Methodology

This project uses a **repeated measures design** as the means by which data are generated. The theory of the methods used to gather data and analyse results is rooted in Vygotsky's (1978) experimental-developmental approach to the study of higher mental functions.

This approach has been selected because it recognises the relationship between external and internal activity as a developmental relationship "...in which the major issue is how external processes are *transformed* to *create* internal processes" (Vygotsky in Wertsch, 1985:163, original emphasis). Leont'ev takes this point further by explaining that, "...the process of internalisation is not the *transferral* of an external activity to a pre-existing, internal 'plane of consciousness' : it is the process in which this plane is *formed*" (in Wertsch, 1985:163, original emphasis).

That is to say that the thinking process is not about a conscious transferral of learning activities into the mind or vice versa. Learning activities and how one thinks about the experiences are inextricably linked together. The relevance for this study is that the observable actions (learning activities) of the subjects provide the evidence of the internal processes (thinking) that are at work.

The Vygotskian experimental developmental approach sets itself apart from most other approaches of studying higher mental functions because it rejects "...both the assumption that the structures of external and internal activity are identical and the assumption that they are unrelated" (Wertsch and Stone in Wertsch, 1985:163). The assumptions to which they refer relate to studies which either disregard social contexts and physical influences on individuals' mental processes or regard external behaviour as the sole object of study by ignoring the complexities of internal psychological processes (Wertsch & Stone, 1985:162). Wertsch and Stone (1985:162) further stress

that "...[t]he tendency to focus solely on one or the other pole of this dichotomy has not been productive because it has consistently produced incompatible theories".

Vygotsky argues that the first position, i.e. that external and internal processes are the same, "makes the very notion of internalisation uninteresting or trivial" whereas the second position, i.e. that the external and internal processes are unrelated, "makes it unresolvable" (in Wertsch, 1985:163). Instead, Vygotsky calls for a "developmental analysis that returns to the source and reconstructs all the points in the development of a given structure" (in Wertsch, 1985:164).

According to Vygotsky (1978), the source of any mental function can be found in an external stage of its development because it is initially a social function before it becomes internalised. He considers all higher mental functions as internalised social relationships observable through the reconstruction of external actions. In my study the learning/development of the subjects was traced by studying the changes that occurred in their observable actions on tasks at repeated intervals.

Vygotsky's developmental methodology was chosen because it relies on different moments in the process of change and different vantage points are given from which to capture the development that takes place. There are three basic principles which form the basis of the Vygotskian approach to the analysis of higher mental functions. These include:

- analysing process not product, e.g. the administration of not one test, but a series of the same test given to subjects over a period of seven months with a view to explaining the conceptual changes that occur;
- explaining instead of merely describing, e.g. based on observations of the subjects' test performances, conclusions can be drawn as to the probable development of internalised higher mental functions;
- dealing with the problem of fossilised behaviour, i.e. recognising that some thinking processes are not evident by the external appearances of the actions that are being observed.

According to Vygotsky's first principle, **processes** must not be viewed as fixed products with forms that can be broken down into components. Instead, the process of development requires "a dynamic display of the main points making up the processes' history" (Vygotsky, 1978:61). Vygotsky's use of 'history' refers to changes that have taken place in the development up to that point. Instead of relying on one set of tests which would simply show the actual level of ability that the subjects have already accomplished, the data are used to establish learning trends and to give some indication of the learning outcomes of the subjects. Each stage in the process of development can be reconstructed through qualitative and quantitative analysis of the tasks and repeated measures over time.

In order to achieve this, the developmental process was traced over a period of seven months with a view to explaining conceptual changes that occur. The data are used both to describe the relationships among the findings in a given situation as well as to account for the changes that occur in these relationships as a function of time. The impact of social and cultural influences has also been highlighted for selected tasks.

Vygotsky's (1978) second principle recognises the limited usefulness of mere descriptions of particular objects of knowledge. He explains this in terms of Lewin's contention that:

> ...two phenotypically identical or similar processes may be radically different from each other in their causal-dynamic aspects and vice versa; two processes that are very close in the causal dynamic nature may be very different phenotypically (in Vygotsky, 1978:62).

In other words, an external description of the children's performances on the mathematical tasks in this research would be of little use in understanding their thinking. It is necessary to gain insight into their internal, mental differences (competencies) in order to understand their mathematical development. This is done by studying the subjects' actions and the changes in the ways they engage with the tasks. Based on observations of subjects' performances, conclusions can be drawn as to the probable development of internalised higher mental functions.

In order to observe subjects' performances in this way, tasks had to be devised to elicit responses that would give insight into the children's thinking. The tasks required the children to demonstrate their mathematical understandings using a variety of concepts and objects and to explain their actions in such a way that their understanding was made more apparent to the observer. The tasks were devised so that the subjects were not always able to do them correctly. This meant that their actions provided the external evidence required to show the internal intellectual processes that were at work.

The analysis of developmental research can be both quantitative and qualitative as it sets out to describe and interpret the processes that are developing. The quantitative analysis of this repeated measures study has been done in the form of statistical comparisons of the empirical data collected over a given time period. The data have been analysed in this way so as to identify patterns and themes from which conclusions can be drawn. The qualitative data recorded at each interview has been used to enrich these findings.

The advantages of using a repeated measures design for analysing human growth and development is described by Cohen and Manion (1994:69) as being:

> ...uniquely able to identify a typical pattern of development and to reveal factors operating on those samples which elude other research designs. They permit researchers to examine individual variations in characteristics or traits, and to produce individual growth curves...[and] are particularly appropriate when investigators attempt to establish causal relationships, for this task involves identifying changes in certain characteristics that result in changes in others.

When identifying patterns of development and drawing conclusions about developing internal mental processes, Vygotsky (1978) warns researchers of the problem of fossilised behaviour, his third principle requiring consideration when analysing higher mental functions.

Vygotsky (1978) describes fossilised behaviour as a series of processes that have already died or become "fossilized". By "fossilized" he means that the processes have

lost their original appearances and their external appearances do not tell anything about the processes involved. This tells us that when subjects know how to do mathematics, there is no manifest performance from which to gain insight into the internal processes (competencies) involved.

Bearing the notion of fossilised behaviour in mind, the tasks that form the focus of this study (see Appendix A), have been designed in such a way to elicit some 'work' before successful execution and some 'room' for discussion about their reasons for engaging with the tasks in the way that they do. Therefore, their manipulations of the objects which form part of the tasks provides the external evidence required to show the internal processes that are at work. These actions provide insight into the mental processes.

3.2 Method

The repeated measures design study using Vygotsky's experimental developmental method "artificially provokes or creates a process of psychological development" which enables researchers to gain insight into the learning/development of children (Vygotsky, 1978:61). Towards this aim, 11 tasks were created as follows:

- The INSET course aimed at teachers teaching mathematics to junior primary children (Flanagan, 1995) was used as a data basis;
- The anticipated learning outcomes were designed into the 11 tasks contained in Appendix A (see Craig, 1997);
- The data from this study were analysed in terms of likely learning outcomes if and when teaching was successful.

The set of 11 tasks was administered to the subjects on four different occasions so that the repeated measures over time could show the **changes** that take place over the course of a year. Instead of accounting for that which has already occurred, this experimental developmental method attempts to describe and interpret those higher mental functions that are in the process of developing.

3.3 Sample

The units of analysis comprise 27 children. The subjects were selected from three schools in Gauteng and North West provinces (see Appendix B). These schools all had teachers currently registered as PREP students in their second year of course work. The subjects were taken from PREP teachers' classes.

The first languages spoken at the three schools are Setswana, North-Sotho and Zulu. Three children from each of the first, second and third grade classes at each school made up the research sample (nine subjects from each school). A translator was used to explain each task in each subject's first language. Student selection was based on a complete range of abilities within each class, i.e. top, middle and bottom, as determined by the teacher.

The 27 subjects would have generated 108 data points (27 children x 4 visits) for the entire project, had there not been any absentees. However, on both the second and fourth visits one child was absent from Grade 1. The sample for these two visits was only 26 and therefore a total of 106 data points were generated for the entire project (27 children x 4 visits - 2 absentees = 106).

3.4 Procedure

Data was collected over a period of seven months at two month intervals, four visits in total. The first visit was in April 1996 and the last visit was in October 1996. Each student interview was observed and all responses given by the candidates on the Observation Schedule were recorded (see Appendix A). The Tasks were explained orally to each subject through the use of a translator who spoke to the children in their first language. The same translator was used for each child on the last three visits. The assumption was that the translator was fluent in Setswana, North-Sotho and Zulu. Unfortunately different translators had to be used on the first visit. The use of translators is recognised as a significant factor in that information obtained was not obtained directly from the primary source. However, most of the tasks required the subjects to demonstrate their answers using a variety of objects. Thus results were not

totally dependent on language. Furthermore, interviews were tape-recorded. I am confident that all information obtained both directly and indirectly from the children is reasonably accurate.

Table 3.1: Data collection schedule

Date	Description
April 1996	Observation and Data Collection, No. 1
June 1996	Observation and Data Collection, No. 2
August 1996	Observation and Data Collection, No. 3
October 1996	Observation and Data Collection, No. 4

CHAPTER FOUR

RESULTS

DESCRIPTIVE ANALYSIS OF TASK ENGAGEMENT

The results presented below reveal the trends in the data of a repeated measures study of children's developing mathematical abilities. The study is based on various tasks in the form of an Observation Schedule (see Appendix A). These same tasks were administered to the subjects on four occasions, at two-month intervals during the course of 1996.

Descriptions of the tasks and data presented in tables allow for the analysis and comparison of scores during the course of the study. Criteria used to determine the subjects' success on each task are explained and potentially significant patterns and trends are highlighted. All data are presented as percentages. The number of subjects (n) used in each test must be taken into account at all times. Percentages given per grade generally represent nine subjects, whereas percentages given per visit for the entire sample generally represent all 27 subjects. Consequently an apparently large deviation of as much as 11% could in fact only be indicative of a change in one subject's level of success.

TASK 1: When do you use numbers?

Task 1 is a straightforward task that asks the subjects to give descriptions of situations in which they use numbers. Any instance that they named was regarded as acceptable and was recorded by the observer.

Table 4.1a represents the **total sample's** ability to do the task in terms of its most basic requirements. Here, 'basic requirements' means that if a child was able to name any instance in which s/he used numbers, s/he was considered to have been successful at the task..

Table 4.1a[2]: The use of numbers

Visit	1	2	3	4
Task success rate in %	93	100	100	100
n	27	26	27	26

This table indicates that from the second visit onwards all 27 subjects were able to give examples of situations in which they use numbers. The score of 93% on the first visit reflects that two of the first graders said that they "never ever" used numbers. The other subjects had no difficulty in giving examples of the ways in which they use numbers in their daily lives, thus capturing the simplicity of this basic task.

The responses that the children gave for Task 1 were quite varied. However, for the purpose of analysis, answers have been grouped into six categories which are as follows:

A. numbers on doors, including numbers on houses in the street, numbers on house gates and numbers on classroom doors;
B. home appliances, including television channels, telephones, radio controls and dials and calendars;
C. watches and clocks;
D. prices on items in shops, money and shopping ;
E. mathematics at school including number charts, mathematical games, counting and all mathematical operations;
F. any other uses of numbers which do not fall into the above categories such as shoe sizes, car number plates, measuring tapes and scales used at the hospital.

The numbers given in Table 4.1b represent the average percentage of the subjects in each grade whose responses fell within the categories established over all four visits. The total shows the average score for the whole sample within the given categories.

[2] The number 1 refers to the task (see Appendix A) and the lower case letter refers to the different analyses undertaken per task.

Table 4.1b: The use of numbers

Category	A	B	C	D	E	F
Grade 1	50	26	18	18	56	12
Grade 2	53	31	25	17	92	53
Grade 3	33	53	39	8	92	36
n	27	27	27	27	27	27
Total	**45**	**37**	**27**	**14**	**80**	**34**

Table 4.1b shows that the category with the highest average percentage across all three grades is category E which represents the use of numbers at school. The emphasis on the use of numbers at school is very noticeable in the second and third grade classes. On average, only 56% of the first graders cited school as a situation in which they used numbers, whereas the averages in Grades 2 and 3 reached 92%. The emphasis on school mathematics is shown to increase with the number of years that the children have been at school. At the time of the final visit, only 50% of the subjects in Grade 1 mentioned school as a place where they use numbers, whilst 100% of the subjects in Grades 2 and 3 gave it as an example. This shows that the second and third graders primarily associated the use of numbers with school mathematics.

The second highest average percentages are found in Category A (numbers designating various locations) for the first and second graders (averages of 50% and 53% respectively) and Category B (home appliances) for the third graders (53%).

Numbers on watches (category C) are given as an example of using numbers by subjects in all three grades (see Task 8, Table 4.8a). Table 4.1b shows that the number of responses that fall within this category increased substantially over the course of the four visits for the entire sample. This finding is not particularly surprising since learning to tell time is part of the school syllabus for all three grades.

The infrequent citation of money as an example of using numbers seems surprising as Task 2 shows that the students have a certain expertise with money and its value and use.

Having described the data in terms of specific categories that emerged from the subjects' answers, analysis suggested a further grouping in terms of 'everyday' and 'school' examples. Responses are categorised as 'school', 'everyday' and 'both' and are defined as follows:

- school uses include counting, mathematical games and charts and all mathematical operations;
- 'everyday' uses include all five of the other categories in Table 4.1b (numbers designating locations, home appliances, watches, prices, money, etc.);
- 'both' means that the subject cited at least one 'everyday' example and one 'school' example.

This grouping highlights the apparent influence of these factors and emphasises their importance in terms of how the subjects think they use numbers. Table 4.1c is a summary of the total percentages of subjects who gave everyday examples of using number only, those who gave examples of school uses and those who gave examples of both on each respective visit (1-4).

Table 4.1c: The use of numbers

Visit	1	2	3	4	1	2	3	4	1	2	3	4
	Everyday Uses				School Uses				Both			
Grade 1	22	63	33	50	44	0	33	0	11	50	33	50
Grade 2	11	11	11	0	22	22	0	0	67	67	89	100
Grade 3	0	33	0	0	22	0	0	0	78	67	100	100
n	27	26	27	26	27	26	27	26	27	26	27	26
Total	**11**	**35**	**15**	**16**	**30**	**7**	**11**	**0**	**52**	**62**	**74**	**88**

Table 4.1c indicates that the more schooling the subjects have, the more aware they are of numbers both at school and in their daily lives. By the time of the last visit, all the second and third graders gave examples of school and everyday uses of numbers whilst only 50% of the first graders responded similarly. The other 50% of the first graders still related to their everyday usage of numbers, which suggests that the 'everyday' category is the most significant in their lives. An evaluation of the entire population indicates that 52% of the children gave examples of both everyday and school uses at the start of the year but responses increased to 88% by the time of the

fourth visit, showing an overall improvement in terms of awareness of, or familiarity with, numbers in both everyday and school situations.

The findings from Task 1 indicate that the subjects are very familiar with the use of numbers. They are aware of the importance of numbers both in their everyday lives as well as at school.

TASK 2: Do you get pocket money? If so, how much?

Task 2 focuses on how one uses numbers when dealing with money. For this task, the subjects were asked to demonstrate their understanding of the value of money by indicating what items they would choose to buy with the coins in front of them (R2, R1, 50c, 20c, 10c, 5c, 2c, 1c). They were also asked to give prices for the items that they wished to buy.

The subjects were considered to have a sound understanding of money if they were able to name items that they would like to buy, give reasonable prices for each item and identify the coins that they would use to pay for them. Table 2a represents the total sample's ability to use money in this way.

Table 4.2a: The use of money

Visit	1	2	3	4
% who understand money	93	96	100	100
% who receive pocket money	44	49	78	76
n	27	26	27	26

The table indicates that from the third visit onwards, all the children in the sample were able to demonstrate a sound understanding of the value of money. On the first visit, two first graders (neither of whom received pocket money) were only occasionally able to display the correct use of money. On the second visit, only one child was still struggling to show the cost of the items chosen and by the third visit, all the children were convincingly able to demonstrate efficient money usage.

A general familiarity with both the value and use of money was apparent during the administration of Task 2. This is reflected in the high percentages of subjects who showed expertise in this task. However, although the entire sample of subjects was adept with money, only five students referred to money as an example of 'using numbers' during the course of the study in Task 1 (see Table 4.1b, column D).

Table 4.2a also shows that the number of children who stated they received pocket money increased during the course of the study from 44% to 76%. This may be an effect of being asked whether or not they received money.

The overwhelming majority of the items selected for purchase by the children were food items. Table 4.2b is a summary of the average percentages of all items chosen by the children in each grade over the course of the four visits. The responses have been grouped into seven categories as follows:

A. bread;
B. chips, most often referred to as 'Simba';
C. drinks including juice, 'cool-aid', cooldrink and milk;
D. sweets, including sherbet, chocolates, chewing gum;
E. ice-creams, ice-blocks and ice lollies;
F. biscuits including cookies and cakes;
G. all other items not mentioned above including: books, rulers, popcorn, fruit, yoghurt, polony, pies, taxi fares and photographs.

Table 4.2b: The use of money

Category	A	B	C	D	E	F	G
Grade 1	62	53	41	64	20	5	18
Grade 2	92	50	56	56	14	33	22
Grade 3	64	44	47	44	33	5	33
n	27	27	27	27	27	27	27
Total	**73**	**49**	**48**	**55**	**23**	**15**	**25**

Category A (bread) at 73% has the highest average across the entire sample population. This demonstrates the importance of bread in the daily lives of the subjects, a fact that is further elaborated upon in Task 10. Category D (sweets and

chocolates) has the second highest average at 55%, followed closely by categories B (chips) and C (drinks) with 49% and 48% respectively.

Table 4.2 established that these subjects have a very sound understanding of money, its value and use, and the importance of bread as a daily food substance in their lives was emphasised.

TASK 3: Dividing objects into equal groups:

For this task, the subjects were asked to sort 21 objects (small cubes) into equal groups of 3, 2 and 4 respectively. Table 4.3a shows the subjects' ability to complete each of these three tasks correctly. Here 'correctly' means that the child used all 21 objects and was able to divide them into 2, 3 or 4 equal groups. Answers that were considered to be correct for each task are as follows:

- 2 equal groups: 2 groups of 10 and 1 remainder or 2 groups of 10 and 1 remainder which is cut in half;
- 3 equal groups: 3 groups of 7;
- 4 equal groups: 4 groups of 5 and 1 remainder or 4 groups of 5 and 1 remainder cut into quarters.

Table 4.3a: Dividing objects into equal groups

Visit	1	2	3	4
Division by 2				
Grade 1	11	0	0	13
Grade 2	0	33	33	56
Grade 3	22	56	56	44
Total average %	**11**	**31**	**30**	**38**
Division by 3				
Grade 1	0	0	0	0
Grade 2	0	11	44	55
Grade 3	0	22	44	44
Total average %	**0**	**12**	**30**	**35**
Division by 4				
Grade 1	0	25	0	0
Grade 2	11	33	44	55
Grade 3	11	44	44	33
Total average %	7	35	30	31
n	27	26	27	26
Overall average % of students able to do all 3 tasks correctly on each visit	**0**	**4**	**15**	**31**

Table 4.3a reveals that dividing objects into equal groups was difficult for these subjects. This is especially true for the first graders, a tiny minority of whom were occasionally able to do the task. A comparison of the second and third graders who were able to do the tasks shows very little difference between the two groups. This is somewhat surprising as one would expect third graders to be more competent than second graders given their assumed extra year of schooling. However, an analysis of the average ages and average number of years of schooling of the subjects in Grades 2 and 3 reveals little difference between the two groups (see Appendix B).

Table 4.3a also indicates steady improvement amongst the second and third graders on all three tasks when comparing their performances from the first to final visits. This success rate never exceeded 56% for any one grade and the total average percentage of the whole sample never exceeded 38%, a relatively low performance value.

On the last visit the task with the greatest success rate was the one requiring the subjects to divide by 2, followed closely by dividing by 3 and then dividing by 4. However, the difference between the number of subjects who were able to do the task is really quite minimal (10, 9 and 8 respectively). This suggests that the task of

division, i.e. the cognitive operations required, whether it be by 2, 3 or 4, does not appear to be increasingly difficult for these subjects. This means that subjects who understand the concept of division and who are able to divide by 2 are generally also able to divide by 3 and by 4.

Another interesting fact revealed in the analysis of this task is the high percentage of subjects who made the same errors in their attempts to do the task. These findings are presented in Table 4.3b which includes the following categories:

A. The percentage of children who solved the problem of dividing the 21 objects into 2 equal groups separating out 10 equal groups of 2 and one remainder;
B. The percentage of children who gave any other incorrect responses to dividing the objects into 2 equal groups;
C. The percentage of children who solved the problem of dividing the objects into 3 equal groups by separating 7 groups of 3;
D. The percentage of children who gave any other incorrect responses to dividing the objects into 3 equal groups;
E. The percentage of children who solved the problem of dividing the objects into 4 equal groups by separating 5 groups of 4 and one remainder;
F. The percentage of children who gave any other incorrect responses to dividing the objects into 4 equal groups.

Table 4.3b: Typical errors made when dividing objects into equal groups

Category	A	B	C	D	E	F
Grade 1	44	50	41	56	44	50
Grade 2	53	17	61	11	55	17
Grade 3	50	5	47	11	36	19
n	27	27	27	27	27	27
Total % of whole sample	**51**	**24**	**50**	**25**	**44**	**28**

Note: Each of these percentages has been calculated for total observation of the four visits

Table 4.3b indicates quite clearly that the Grade 1 students had a far greater number of 'other' ways of dividing objects into equal groups (50%, 56% and 50% respectively) than the second and third graders whose 'other' average percentages never exceeded

19%. This finding suggests that the more schooling the children have had, the more similar their errors become.

The findings from Task 3 show clearly that these subjects have particular difficulties dividing objects into equal groups. Different divisors do not seem to alter this finding significantly. Another interesting observation arising from this task is that the percentage of subjects who make similar mistakes is directly linked to the number of years of schooling that they have had.

TASK 4: Matching objects in pairs:

Task 4 tests the subjects' abilities to match objects in pairs. This task could be considered as a test of school or everyday knowledge since either understanding would enable one to complete the task successfully. For this task the children were provided with seven round objects and seven long objects and asked to match these objects in pairs. They were also asked to give examples of pairs from their everyday experiences.

Many of the children did not understand the word 'pairs'. They devoted a great deal of time to just looking at the objects. On the first two visits they were strongly encouraged to make some attempt at the task, but on the last two visits, if they said that they didn't know what pairs were, they did not have to attempt the task. This information helps to explain the peculiar trends in Table 4.4a. Interestingly, the data reflect the children's unwillingness to try to do a task when given the option not to.

Table 4.4a represents the students' abilities to demonstrate a basic understanding of pairs. Here, 'basic understanding' is taken as some attempt to group the objects into twos of some sort, i.e. not necessarily the same object or shape but with some sort of a 'partner'.

Table 4.4a: Matching objects in pairs

Visit	1	2	3	4
Grade 1	22	67	11	11
Grade 2	55	55	44	55
Grade 3	55	67	44	22
n	27	26	27	26
Total average %	**44**	**63**	**31**	**31**

The average percentage of all the students able to do this task over the course of all four visits is 42% and the second graders prove to be the most consistently accomplished group.

The minimal success rate of Task 4 clearly reveals an inability to group objects together in pairs. The concept of 'pairs' was not adequately understood by the majority of the subjects.

An analysis of the responses given by the students when asked to describe what makes a pair revealed interesting findings. Most students were unable to answer the question, while those who were able to answer either described pairs as being "two-two" or as being "the same". Table 4.4b reflects the percentages of students who were able to offer any definition of a pair as well as those students able to name any correct example of a pair from their daily lives.

Table 4.4b: Defining and giving examples of pairs:

Visit	1	2	3	4
% of students able to define "pair"				
Grade 1	22	11	11	11
Grade 2	11	33	44	67
Grade 3	11	22	22	22
Total average %	11	22	26	30
% of students able to give examples of "pairs"				
Grade 1	11	11	11	11
Grade 2	44	33	67	78
Grade 3	33	44	44	33
n	*27*	*26*	*27*	*26*
Total average %	**37**	**22**	**41**	**41**

Table 4.4b indicates that more children were able to give examples of pairs from their daily lives than were able to describe pairs in any way. However, the performance values of both tasks were low - the average percentage never rose above 41%. An interesting finding, not presented in the tables, was that the way the students described pairs was reflected in the way that they engaged with the task. Those who described pairs as being in twos grouped the objects accordingly, while those who described the objects as being the same, placed all the pens together and all the nuts together.

TASK 5: Ordering objects in terms of length, size, weight and value:

Task 5, like Task 4, also tests abilities which could be attributed to a combination of 'everyday' and 'school' experiences. The ability to arrange objects in terms of a variety of characteristics clearly includes both of these influences. For this task, the subjects were asked to order objects as follows:

A. 3 different lengths of string from the shortest to the longest;
B. 3 differently sized buttons from the smallest to the largest;
C. 3 salt cellars of different weights from the heaviest to the lightest (one was full, one half-full and one was empty);
D. 3 coins of different sizes and values from the greatest to the least amount (these included a 50c coin, a 20c coin and a 5c coin).

Table 4.5 presents the percentages of subjects in each grade who were able to complete each task successfully. 'Successfully' is defined for each set of objects as follows:

String — the subject was able to identify and arrange the three different lengths of string in the correct order from the shortest to the longest

Buttons — the subject was able to identify and arrange the buttons from the smallest to the largest

Salt cellars — the subject was able to identify and arrange the cellars from the heaviest to the lightest

Coins — the subject was able to identify and arrange the coins from the greatest to the least value, i.e., 50c; 20c; 5c

Table 4.5: Ordering objects in terms of length, size, weight and value

Visit	1	2	3	4
A: String				
Grade 1	78	88	89	100
Grade 2	89	100	100	100
Grade 3	100	100	100	100
Total average %	**89**	**96**	**96**	**100**
B: Buttons				
Grade 1	100	88	89	100
Grade 2	100	100	100	100
Grade 3	100	100	100	100
Total average %	**70**	**88**	**96**	**100**
C: Salt cellars				
Grade 1	44	50	89	100
Grade 2	77	100	100	100
Grade 3	89	100	100	100
Total average %	**70**	**88**	**96**	**100**
D: Money				
Grade 1	44	38	67	100
Grade 2	77	89	89	100
Grade 3	67	100	100	100
Total average %	**65**	**77**	**85**	**100**
n	27	26	27	26

This table indicates that all the subjects had a relatively high success rate with all four of the tasks throughout the course of the four visits. The lowest recorded percentage is by Grade 1 students on the first visit (44%) for ordering objects by weight (salt cellars - heaviest to lightest). By the time of the fourth visit all subjects in all grades demonstrated that they were able to order each set of objects correctly. Improvement across the grades was most noticeable in Grade 1. The second and third graders generally did not seem to have much difficulty with any of the tasks right from the beginning of the year (the lowest average percentage for any one task on any one visit is 67% for the third graders for ranking money in terms of its value). Their success rates on all other tasks on all other visits never fell below an average of 77%.

The ranking of buttons (smallest to largest) was the easiest of the four tasks for the students and only two children were unable to do the task successfully during the course of the four visits. In these two cases, the children were able to identify the buttons as small, bigger and biggest but they were unable to rank them as such by placing them in the correct order.

The ordering of the string (shortest to longest) was also an easy task for the children. Once again, most of the problems or difficulties associated with the task were amongst the first graders.

Ordering the salt cellars by weight and the money by value generally presented the subjects with more difficulty than the other two tasks. The ranking of money was the hardest task for the children as is indicated by the comparatively low total average performance percentages for the first two visits (65% and 77% respectively). This is surprising given their obvious expertise with the value of money (see Task 2). The most common error was in ordering the 5c, 20c and 50c coins. Many of the children ordered the coins in terms of size, i.e., 50c-5c-20c (new coins were used) and not in terms of the actual value of the coins. The children were specifically instructed to order the coins from the greatest to the least value and they received further elaboration on the task by being told to order the coins from the one that could buy the

most to the one that could buy the least. By the time of the last visit in October, all students in the sample population were able to do the task successfully.

Results of Task 5 indicate that the subjects have little difficulty ordering objects in terms of length, size, weight and value. Ordering objects in terms of length and size was particularly easy.

TASK 6: Grouping objects in terms of colour, shape and function:

Task 6 entails testing the subjects' abilities to group a variety of objects according to specific attribute. Once again, like Task 5, this task can be seen to reflect a combination of everyday and school experiences.

The children were presented with nine objects which could be divided into three sets according to colour, shape and function. The objects that were used were as follows: 3 salt cellars, 3 pencils and 3 nuts. The salt cellars looked exactly the same but were different in weight; the pencils were all long, had sharp points and were red and black; and the nuts were still in their shells and were almost exactly the same size. The subjects were asked to sort the objects into groups that appeared to be most appropriate to them. They were then asked to explain their decisions. The subjects were then questioned on the characteristics the objects in each group shared and on the reasons for their association as well as their functions.

The most interesting finding on this task was the consistency in task performance, i.e. there was very little variation in terms of how the subjects chose to group the objects. Most of the object groupings were 'functional' in that the object's function was central to the grouping. This manner of grouping was described by the subjects as "same-same" whereby they placed the salt cellars in one group, the pencils in another and the nuts in another. The reasoning given for this was that the groups were made up of things that were the "same". This functional approach to grouping was noticeable across all three grades but particularly amongst the second and third graders who demonstrated little creativity in terms of how they did this task

throughout the course of the four visits. There was virtually no variation in their task performance. However, the Grade 1 subjects demonstrated considerable variety in their task performance. Several students grouped the objects in three groups with one pencil, one salt cellar and one nut in each group. Their explanation for this was also that each group was the "same". In both of the aforementioned cases, the apparent 'sameness' was taken for granted by the subjects concerned.

Table 4.6 reflects the percentages of subjects in each standard on each visit who grouped their objects as follows:

A. 3 pencils, 3 nuts and 3 salt cellars

B. any other configurations, e.g., 1 pencil, 1 nut and 1 salt cellar in each group

Table 4.6: Grouping objects in terms of colour, shape and function

Visit	A				B			
	1	2	3	4	1	2	3	4
Grade 1	78	87	55	75	22	12	44	25
Grade 2	100	78	89	89	0	22	11	11
Grade 3	100	100	100	100	0	0	0	0
n	27	26	27	26	27	26	27	26
Total average %	**93**	**89**	**81**	**89**	**7**	**8**	**19**	**12**

The percentages of subjects who gave exactly the same answers are high whereas the percentages showing those subjects who have 'other' ideas about how to tackle the task are much lower. This 'conformity' increases from Grade 1 to Grade 2 to Grade 3. In fact all the subjects in Grade 3 grouped the objects in this task in precisely the same way.

Results of Task 6 indicate that these subjects generally choose to group objects in a functional manner and that the number of children who deviate from this option, decreases with the number of years of schooling that they have had. The number of similar answers given are shown to increase with the number of years of schooling (see also Task 3, Table 4.3b).

TASK 7: Concepts of 'more' and 'less':

The children were each given a handful of small objects and they were asked to estimate (or guess) the number of objects in their hands. They then had to count the objects and say whether their estimation was more or less than the actual number.

Table 4.7 presents the percentage of subjects in each grade who were able to perform the task successfully. They were considered successful if they could establish whether their estimation was more or less than the actual number of objects.

Table 4.7: Identification of 'more' and 'less'

Visit	1	2	3	4
Grade 1	44	88	67	100
Grade 2	78	78	100	100
Grade 3	89	100	100	100
n	27	26	27	26
Total average %	**71**	**89**	**89**	**100**

Table 4.7 indicates quite clearly that the subjects had little difficulty with this task. The second and third graders were particularly adept, the third graders demonstrating the best performance. The subjects in Grade 1 exhibited the best improvement over the course of the four visits.

TASK 8: Telling the time and identifying a variety of concepts and objects:

Task 8 tests a variety of skills and concepts which reflect everyday and school learning processes. This is a five part task. The children were first asked whether they knew how to tell time on an analogue clock and were then asked to demonstrate this by reading the time (i). Next, they were presented with a box of objects and asked to pick out objects of different colours (ii), objects of different shapes (iii), the heaviest object in the box (iv) and the longest object in the box (v).

Table 4.8a reflects subjects' abilities to tell time. The percentages shown express the subjects' success as follows:

- Yes: These subjects were able to tell the time correctly. They were given at least five different opportunities to read a variety of 'times' off the clock. These times included a time on the hour, on the half hour, at a quarter to or quarter past the hour and two more 'difficult' times. Each time had to be read correctly.
- No: These subjects were unable to tell the time at all.
- Sometimes: These subjects were sometimes able to tell the time. They were usually able to tell the time on the hour or on the half- hour. If a subject was able tell the time correctly, even once, s/he was included in this category.

Table 4.8a: Telling the time

Visit	1	2	3	4
Grade 1				
Yes	11	0	0	0
No	89	67	67	55
Sometimes	0	22	33	33
Grade 2				
Yes	22	22	33	33
No	44	33	11	11
Sometimes	33	44	55	55
Grade 3				
Yes	11	22	44	55
No	44	11	33	11
Sometimes	44	67	22	33
Total average % able to do task across all 3 grades				
Yes	15	15	26	31
No	60	38	37	27
Sometimes	26	46	37	42
n	27	26	27	26

There was no improvement amongst the Grade 1 subjects in terms of telling the time correctly. However, the number of subjects who were sometimes able to tell the time increased from 0% on the first visit to 33%. This demonstrates an increased willingness to engage in the task itself.

The Grade 2 subjects also showed minimal improvement in terms of those who were able to do the task correctly (only one child more from the second to the fourth visits). Once again the number of subjects who were able to sometimes tell the time increased

from 33% on the first visit to 55 % on the fourth visit (an increase of two children). The third graders had the highest number of subjects able to tell the time correctly (55%) and only one subject was never able to do the task.

Considering the entire student sample population, the average percentage of subjects able to tell the time correctly increased from 15% to 31% and the average percentage of children sometimes able to tell the time increased from 26% to 42%. This suggests that a certain amount of learning was achieved during the course of the four visits, but the percentages of subjects able to do the task successfully was still relatively low. This is not an easy task because the concept of time is abstract and a sound understanding of numbers is required to master this skill.

The other activities for Task 8 proved to be less difficult for the subjects than telling the time. These activities are categorised below and results are reflected in Table 4.8b:

A. The percentage of subjects who were able to pick objects of different colours (at least six) out of a box;
B. The percentage of subjects who were able to pick objects of different shapes (at least four) out of a box;
C. The percentage of subjects able to identify the heaviest object in a box correctly. (the heaviest object, by a considerable amount, was a full salt cellar);
D. The percentage of subjects able to identify the longest object in a box correctly (a piece of string was by far the longest object in the box).

Table 4.8b: The identification of a variety of concepts and objects

	A				B				C				D			
Visit	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Gr. 1	89	100	100	100	22	11	44	22	33	55	33	67	100	100	89	100
Gr. 2	100	100	100	100	55	66	89	100	33	78	78	100	100	100	100	100
Gr. 3	100	100	100	100	67	89	78	89	55	89	67	89	100	89	100	100
n	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26
Total average %	**96**	**100**	**100**	**100**	**48**	**58**	**73**	**70**	**41**	**77**	**59**	**88**	**100**	**96**	**96**	**100**

Table 4.8b demonstrates that the easiest task in this group was the selection of objects of different colours (see column A). From the second visit onwards all the children were successfully able to pick out objects of a variety of colours without any difficulty.

A high success rate was also achieved on the task where the subjects' abilities at identifying the heaviest object in the box was tested (see column D). The heaviest item was the salt cellar which had been encountered previously in Tasks 4,5 & 6. Only two students in the entire sample population made mistakes on this task over the entire period of the four visits.

The shape-sorting task was difficult for some of the children, particularly for those in Grade 1 (Table 4.8b, column B). This finding is surprising in that many of these children were able to identify shapes in two dimensions (i.e. drawings of circles, triangles and the like) but they did not seem to transfer this knowledge to the three dimensional objects in the box. The second and third graders showed good improvement on this task during the course of the visits (55% to 100% and 67% to 89% respectively).

Column C presents the results of the task requiring identification of the longest item in the box. A surprisingly large number of subjects had difficulty with this task. The longest object in the box was a very long piece of string (previously encountered in Task 5). The most common error was the selection of a pen or a pencil. There were many pens and pencils of all different types and sizes in the box, yet the subjects rarely measured one against the other. There seemed to be a reluctance by the subjects to engage in what one may call 'investigative behaviour', i.e. a general unwillingness to search for an answer through trial and error. The overall percentage of children able to do this task increased from 41% to 88% over the course of all four visits.

The findings from Task 8 reveal a general improvement with time for all the tasks. The subjects who were best able to tell time correctly were in Grade 3 but even these

had a relatively low performance (55%). The other four tasks presented the subjects with relatively little difficulty. The ability to identify colours was mastered by all subjects from the time of the second visit and identification of the heaviest object also was relatively simple. Shape-sorting was hard for the first graders but not for the others and identification of the longest object was also not difficult.

TASK 9: Identification of mathematical symbols and ability to do mathematical operations:

Task 9 involves school oriented activities which require formal instruction and school learning for success. For this task the children were asked to identify various mathematical symbols. They were given a hand full of objects and were asked to use them to demonstrate the meaning of each symbol (+, -, x, ÷, =). By doing so, they were showing their understanding by manipulating the objects in such a way that their grasp of the symbols was demonstrated.

Answers that were counted as correct for each symbol are as follows:

+	*** ***	(objects put out by the child)
	3 plus 3 is 6	(what the child says)
-	**/*	(3 objects put out by the child and then 1 taken away)
	3 minus 1 is 2	(what the child says)
x	** *	(objects put out by the child)
	2 times 1 is 2	(what the child says)
÷	**/**	(objects put out by the child and then split into 2 groups)
	4 divided by 2 is 2	(what the child says)
=	* *** or *** ***	(objects put out by the child)
	1+3=4 or 3=3	(what the child says)

Table 4.9a below reflects the subjects' abilities at identifying the various mathematical symbols and reveals the extent to which school learning has taken place. The latter is evident in that children do not generally encounter these sorts of mathematical symbols in everyday life.

Table 4.9a: Identification of mathematical symbols:

Visit:	+ 1	+ 2	+ 3	+ 4	- 1	- 2	- 3	- 4	× 1	× 2	× 3	× 4	÷ 1	÷ 2	÷ 3	÷ 4	= 1	= 2	= 3	= 4
Gr. 1	44	89	100	100	44	55	89	78	0	12	11	33	0	0	0	33	22	67	78	89
Gr.	100	100	100	100	67	100	100	100	67	67	67	89	22	78	55	55	44	89	89	89
Gr. 3	89	100	100	100	67	100	100	100	44	89	67	78	33	78	55	67	67	67	67	67
n	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26
Total average %	**78**	**96**	**100**	**100**	**70**	**88**	**96**	**96**	**37**	**58**	**48**	**69**	**19**	**54**	**37**	**52**	**44**	**77**	**78**	**85**

Significant improvements were achieved in all categories during the course of the study. A noticeable finding is that the subjects' abilities to identify mathematical symbols is highest for addition, followed by subtraction, multiplication and division. Significantly, this is the order in which the operations are taught at school. The first graders were the least competent at identifying the symbols, whereas there was little difference between the second and third graders.

Table 4.9b presents the subjects' abilities to demonstrate their understandings of the various mathematical operations.

Table 4.9b: Demonstration of mathematical operations

Visit:	+ 1	+ 2	+ 3	+ 4	- 1	- 2	- 3	- 4	× 1	× 2	× 3	× 4	÷ 1	÷ 2	÷ 3	÷ 4	= 1	= 2	= 3	= 4
Gr. 1	33	88	100	100	33	44	67	33	0	0	0	22	0	0	0	0	0	67	78	89
Gr. 2	100	100	100	100	100	100	100	100	67	67	67	78	33	11	22	33	55	89	89	89
Gr. 3	78	100	100	89	55	89	89	89	33	44	33	44	0	22	11	11	67	55	67	67
n	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26	27	26
Total average %	**70**	**96**	**100**	**96**	**63**	**81**	**85**	**77**	**33**	**38**	**33**	**48**	**11**	**22**	**11**	**15**	**41**	**73**	**78**	**85**

The subjects demonstrated that they were most capable of adding, followed by decreasing capabilities in subtracting, multiplying and dividing. The overall average performance percentages for division were very low, only 15% of the total sample population being able to demonstrate how to divide by the time of the fourth visit. The overall average percentage of students able to multiply correctly also was relatively low at 48%. By the time of the last visit, the second graders were by far the most competent at multiplication. They were superior to the third graders whose competence level was only at 44% by the time of the fourth visit. Neither adding nor subtracting seemed to present these children with many difficulties.

Results from Task 9 reveal that children learn to identify and demonstrate the use of symbols in the order in which they are taught in class. The concepts of addition, subtraction, multiplication and division were found to be increasingly difficult for the test subjects. By the end of the fourth visit relatively few children were able to demonstrate how to multiply or divide correctly (48% and 15% respectively).

TASK 10: Fractions

In Task 10, subjects were tested on their knowledge of fractions by means of a word problem involving sharing loaves of bread with friends. This task combines school learning and everyday knowledge together in a manner that generated interesting answers and revealed individual thought patterns, which previously had not been encountered in this study. For this task the children were asked to show how they would divide a loaf of bread if they had to share it with one friend. They were then asked to show how they would share it with two and three friends respectively. They were provided with rectangular shaped pieces of paper for their demonstrations. Scissors, rulers, pencils and pens were also available. Once they had completed the division of each loaf, they were then asked to name its different parts.

Correct responses to this task included any attempt to divide the 'loaves' into 2, 3 or 4 equal parts. (Most of the subjects had little regard for precision in this task and so some of the answers that were regarded as correct were very rough divisions).

Table 4.10a(i) presents the percentages of students in each grade who performed Task 10 'correctly' by attempting to divide a loaf of bread into 2, 3 and 4 equal parts respectively.

Table 4.10a(i): Dividing into fractions (bread)

Visit	1	2	3	4	1	2	3	4	1	2	3	4
	÷2	÷2	÷2	÷2	÷3	÷3	÷3	÷3	÷4	÷4	÷4	÷4
Gr. 1	78	75	78	67	11	38	11	25	44	55	67	44
Gr. 2	100	55	89	89	44	33	67	55	44	89	77	67
Gr. 3	89	89	78	67	33	55	33	55	78	67	78	67
n	*27*	*26*	*27*	*26*	*27*	*26*	*27*	*26*	*27*	*26*	*27*	*26*
Total average %	***89***	***73***	***81***	***77***	***30***	***42***	***37***	***46***	***56***	***73***	***74***	***62***

By the time of the fourth visit 77% of the subjects had attempted to divide the bread into 2 equal parts, 62% divided the bread into 4 equal parts, whilst only 46% of the students divided the bread into 3 equal parts. This finding is quite interesting because the subjects obviously found it easier to divide by 2 than 4 and easier to divide by 4 than by 3. This is different from the finding in Task 3 where it was equally difficult for the subjects to divide 21 objects into groups of 2, 3 or 4.

There was a variety of responses to this task. The number of individual and creative responses contrast dramatically with the uniform manner in which the previous tasks were performed. It was expected that the subjects would divide the loaves into halves, thirds and quarters, but this was not the case. They had highly individual ideas and personal concerns which were apparent from their answers. Responses include the following:

- cutting one or two slices for each friend;
- sharing some of the loaf and saving the rest for later;
- cutting extra pieces for brothers or sisters or grandmothers at home.

These responses display what one might call daily 'taught' behaviours which are the ways in which they have been shown to share bread. A careful analysis of the subjects' responses revealed that the task was not really testing the children's ability to do fractions, but that it was testing their ability to share. 'Sharing' and 'dividing' are of course two completely different concepts. 'Sharing' implies that one must give part of something away and that the way in which this is done is the decision of the person to whom the item belongs. 'Dividing' on the other hand, has implicit mathematical connotations which imply an equivalency of parts. The importance that

the subjects attached to bread as well as their familiarity with sharing bread in their daily lives probably skewed these results (see Task 2).

Consequently, an extra task was added for the third and fourth visits. The children were asked to divide a piece of paper into 2, 3 and 4 equal pieces. The intention was to eliminate the notion of 'sharing with friends' and to obtain a better indication of their understanding of fractions, by discounting the 'everyday' connotations of the task.

Table 4.10a(ii) presents the data collected on the last two visits when the subjects were simply asked to divide pieces of paper into 2, 3 and 4 equal parts respectively.

Table 4.10a(ii): Dividing into fractions (paper)

Visit	3	4	3	4	3	4
	÷2	÷2	÷3	÷3	÷4	÷4
Grade 1	100	88	22	33	67	33
Grade 2	100	100	78	78	100	89
Grade 3	100	100	67	78	100	89
n	27	26	27	26	27	26
Total average %	**100**	**96**	**56**	**65**	**89**	**73**

Results of this task are similar to those presented in Table 4.10a(i) in which it is apparent that the subjects were most comfortable dividing by 2, followed by dividing by 4 and then by 3. However, the percentages of children who were able to perform each task with paper was significantly higher than for the task dealing with bread. This indicates that the children's everyday understandings of sharing had a definite effect on the answers they gave to the original question (to show how they would divide a loaf of bread if they had to share it with friends, Task 10) and that the number of correct responses given can be improved by dissociating 'everyday' knowledge from the formal requirements of the task. The average scores for the entire sample population are presented in Table 4.10a(iii).

Table 4.10a(iii): Comparison of Tables 4.10a(i) and 4.10a(ii)

Visit	3 ÷2	4 ÷2	3 ÷3	4 ÷3	3 ÷4	4 ÷4
Total average % for bread problem	81	77	37	46	74	62
Total average % for paper problem	100	96	56	65	89	73
n	27	26	27	26	27	26

This table shows quite clearly that the percentage of subjects able to do the paper problem correctly exceeds the number who were able to do the bread problem correctly by between 11 and 19 percent for each task on both visits.

Results of Task 10 indicate that it was easiest for the subjects to divide by 2, then by 4 and then by 3 when working with fractions. The wording of the task requirements combines everyday knowledge with formal schooling in such a way as to evoke a wide variety of responses from the subjects. These responses reflect the confusion that can occur when differences between everyday and school knowledge are ignored.

Another notable finding that emerged from Task 10 concerned the naming of the fraction parts. I do not believe that the task involving the bread was truly indicative of the children's abilities to name fraction parts because the majority of their answers included parts, pieces and slices. Table 4.10b reflects the percentages of students who were able to name the fraction parts correctly on visits three and four on the task involving the division of paper into equal parts of 2, 3 and 4.

Table 4.10b: Naming fraction parts:

Visits % of students able to name fraction parts correctly	3 halves	3 thirds	3 quarters	4 halves	4 thirds	4 quarters
Grade 1	44	0	0	33	0	0
Grade 2	77	0	33	67	0	22
Grade 3	44	0	33	55	0	33
n	27	27	27	26	26	26
Total average %	**55**	**0**	**22**	**51**	**0**	**18**

Table 4.10b reflects low performance values and suggests that the children have not been taught the correct terminology for fraction parts.

TASK 11: Identifying ½ and the denominator and the numerator:

Task 11, like Task 9, is indicative of school learning in that subjects are required to identify certain mathematical symbols and terms. This is a very straightforward task which specifically tests school learning. Table 4.11 indicates the percentage of subjects in each grade who were able to perform the task correctly on each visit.

Table 4.11: The identification of ½ and of numerators and denominators

Visit	1	2	3	4
Ability to identify ½				
Grade 1	0	0	11	0
Grade 2	33	67	89	89
Grade 3	22	22	67	67
Total average %	**19**	**31**	**56**	**52**
Ability to identify numerators and denominators				
Grade 1	0	0	0	0
Grade 2	0	0	11	11
Grade 3	0	0	0	0
n	27	26	27	26
Total average %	**0**	**0**	**3**	**3**

Table 4.11 indicates that only one first grader was able to identify ½ correctly over the course of the four visits. However, the second and third graders improved progressively over the course of the four visits, from 33% to 89% and from 22% to 67% respectively. No significant improvement in identifying numerators or denominators was observed. Only two second graders were ever able to make correct identifications. Table 11a shows clearly that school learning occurred during the course of the year in respect of the identification of ½ (in Grades 2 and 3) but the teaching of numerators and denominators either did not occur or attempts to teach them were not successful.

CHAPTER FIVE

DISCUSSION

The data yielded a number of unexpected results that are cause for concern given the developmental competence that one expects children to display between the ages of six and eleven years old (e.g. ordering; inclusion; invariance of properties such as length and shape - all of which are included in the maths syllabus in Grades 1 & 2). The children's overall performance on the 11 tasks included in this project (see Appendix A) yielded positive[3] results on tasks that tested pre-number concepts and abilities (e.g. classifying, finding the relation between sets and ordering) which underlie more abstract mathematical operations. Their performances on tasks that required formal mathematical instruction (e.g. dividing objects into groups, multiplying and working with fractions) were generally low[4] (see Appendix C). The low performance on formal mathematical tasks probably warrants a thorough evaluation of the methods utilised in teaching mathematics at this level to the sample population used in this study. The overall trends are presented in the discussion that follows.

The results of the subjects' performances on the 11 tasks allow for the following analytic moves in terms of this discussion. Variable performances on tasks indicate different levels of competence, i.e. that the students have the tacit and explicit knowledge to master the tasks in question. Overall performance may or may not imply that their competence is not only task particular but may suggest that they have mastered mathematics, i.e. the manipulation of abstract concepts, familiarity with certain operations and a control over the system of numbers that is appropriate for their ages. When their performances on tasks are inadequate given the particular task demands, or inappropriate given their age level competencies, one may attribute this failure to teaching that does not utilise the children's developmental readiness or

[3] Positive results means that more than 70% of the subjects were able to do the tasks correctly by the time of the last visit (see Appendix C).

[4] Low performance means that less than 40% of the subjects were able to do the tasks correctly by the time of the last visit (see Appendix C).

spontaneous everyday familiarity with numbers. In addition, one would have to consider factors such as socio-economic conditions, student-teacher ratios, accessibility to teaching resources and mental development of 'slow' learners.

Of the 11 tasks, those which generated the high performance values (an average success rate of 70% or more at the time of the last visit, see Appendix C) were Tasks 1, 2, 5, 7, 8b, 9a, 10a(i) and 10a(ii). These findings indicate that the subjects have mastered the mathematical abilities listed below:

They are able to:

- classify objects according to a specific attribute (colour) (Task 8);
- classify objects according to a general attribute (shape) (Task 8);
- group objects functionally and name specific attributes e.g. colour, shape, size and function (Task 8);
- match objects on a one-to-one basis (Task 4);
- order sets of objects on the basis of a more-than and less-than relation in terms of size, shape weight and value (Task 5);
- name instances in which they use numbers (Task 1);
- use money and understand its value (Task 2);
- count and determine whether estimations are greater or less than another group of objects (Task 7);
- divide an object into halves and quarters (Task 10).

The high performance values (>70%) on these particular tasks suggest that the children are able to comprehend abstract concepts (e.g. time and use of money and numbers) and to perform mathematical operations (e.g. addition, subtraction, multiplication and division) successfully. However, low performance values (an average success rate of <40% at the time of the last visit, see Appendix C) on Tasks 3, 4, 8a, 9b, 10b, 11a and 11b require an explanation as to why these values did not improve significantly over the course of the study (see Appendix D).

These low performance values (<40% on Tasks 3, 4, 8a, 9b, 10b, 11a and 11b) indicate that the students are unable to grasp the task demands. They were unable to perform the following operations adequately:

- divide objects into equal groups (Task 3);
- match objects into pairs (Task 6);
- tell time correctly (Task 8);
- demonstrate the mathematical operations of multiplication and division (Task 9);
- identify certain mathematical symbols (Task 9);
- name fraction parts (Task 10b);
- identify the symbol ½ (Task 11a);
- identify numerators or denominators (Task 11b).

The differences between the tasks which revealed high performance values and those that revealed low performance values can be linked to the complexity of the task. High performance values were evident on tasks which could be described as 'easy' because not only do they fall within the scope of abilities appropriate for children of these ages, but they are also indicative of learning that can occur spontaneously in the course of everyday activities (Tasks 1, 2, 5, 8b, 9 and 10a). Low performance values on 'complex' tasks may be attributable to the need for formal instruction on these tasks (Tasks 3, 4, 8a, 10b, 11a and 11b).

It is unlikely that children would be able to recognise or demonstrate the mathematical symbols and operations of addition, subtraction, multiplication and division without formal instruction, unless these operations were needed as part of their daily activities. Even though this may seem to be the most parsimonious explanatory move, it does not rule out other explanations, e.g. that the children lack the cognitive abilities for the tasks, that the tasks do not elicit appropriate performances, and so forth.

If my distinction between learning that can occur **spontaneously** and learning which necessitates **formal instruction** is accepted, then one has to look to the task of teaching mathematics for possible explanations for the poor performances described above.

In what follows, I analyse the results presented in chapter four in terms of the following themes:

- the nature of mathematics in learning and teaching;
- the relationship between learning and development;
- teaching approaches;
- differences between spontaneous and formal knowledge and learning.

5.1 The nature of mathematics in learning and teaching

For the purpose of this study the nature of mathematics is primarily defined in terms of the **language** for mathematics; the latter in turn is defined in chapter two and includes abstract concepts, specialised terminology and the mathematics register.

Repeated evidence generated over the course of this study indicated low performance values on tasks that require the use of specialised terminology and mathematical language. The findings reflected minimal changes in terms of task performances over the course of the study which could be attributed to inadequate teaching of vocabulary and inappropriate use of language.

The first task in which difficulties with language was apparent was Task 4, where subjects were asked to put a group of 14 items (seven pens and seven nuts) into pairs. By the time of the last visit in October, more than half of the children still had no idea of what a pair was. The other 41% of the respondents were able to give at least one correct example of a pair from their everyday lives. They described pairs as when you group things together "in twos" or when you group "same things" together.

The way in which the children described the concept of a pair (i.e. the language used) was reflected in the manner in which they grouped the objects together. Those who described pairs as being "in twos" grouped the objects together in pairs while those who described pairs as being "the same" placed all the pens together and all the nuts together. These findings are significant for three reasons. Firstly, there is the indication that the children have not been taught the concept of pairs. This is evident

by their reluctance to engage with the task and their inability to give any examples of pairs from their daily lives. Secondly, being able to give an example of a pair from one's daily life is shown to be an inadequate factor in terms of determining one's ability to group objects together in pairs. The children who were able to give examples of pairs were not all able to distinguish between groups of objects that were "the same" and a pairs of objects grouped "in twos". Children who were able to give a correct example of a pair were not necessarily able to group the objects "in twos". Although they had some notion of what a pair was, they were unable to transfer this knowledge to the task at hand. Thirdly, the language used to describe a pair was shown to be an important factor in determining success on the task because the students grouped the objects in precisely the way that they defined a pair, i.e. if they knew that pairs were made up of "twos" they were successful on the task. The abilities of the children able to define pairs correctly and to engage successfully with the task, highlights the mediating function of language in the development of abstract concepts such as pairs (cf. Vygotsky, 1978).

More evidence of the inappropriate use of language was forthcoming from Task 10b where students were asked to divide pieces of paper into two, three and four equal parts respectively and they were then asked to name the parts. By the time of the last visit, just over half the students were able to name and recognise halves and a meagre percentage were able to name and recognise thirds and quarters. The children appeared not to have been provided with the correct terminology with which to understand fractions. Misunderstandings were further evident by the fact that thirds and quarters were commonly referred to as halves by the same students who had identified halves correctly. Other names given for fractions included "pieces" and "parts". 'Pieces' and 'parts' are examples of ordinary language from which mathematical concepts like fractions can be developed but once again there are important differences that need to be clarified and made explicit. One of the most important differences between 'pieces'/'parts' and fractions is that fractions are equal parts of a whole. In addition to not knowing the correct terminology for particular fractions, the students demonstrated a lack of precision when asked to divide the paper into fractions and so the parts were seldom divided equally. This lack of attention to

exactitude is significant because it further demonstrates that the very criteria which characterise fractions have not been successfully explained to the students. Unless one understands that fractions are equal parts of wholes, one cannot claim to have understood the concept mathematically. It appears that the children in this study have not been afforded such opportunities and that teachers have not emphasised the necessary language sufficiently because of the low performance values on Task 10b.

In some of the tasks confusion could be linked to the intermingling of ordinary and mathematical language. This was most evident in Task 10a where children were required to share loaves of bread with 1, 2 and 3 friends respectively. Task 10a demonstrated clearly how children's everyday knowledge can interfere with what is considered to be 'correct' mathematics, e.g. many students brought their personal circumstances to bear on the answers they gave.

There are several plausible reasons for the answers given by the children and for the 'problems' one perceives in their answers if one looks at the wording of the task. The wording is as follows:

- If you have to share a loaf of bread with one friend, how will you divide it?
- And with 2 friends?
- And with 3 friends?

The first word to consider is 'share' because different people have different ordinary conceptions of what it means 'to share' something. A variety of different conceptions were evident in answers given by the children where bread was cut into slices and again when pieces of bread were "saved" for siblings and other family members. At no stage were the students instructed to share the bread equally and so their everyday notions of 'sharing' came into play. This is further evidenced by the reasons given when the children were asked to elaborate on why it was that they received a larger portion of the loaf. The responses given were as follows:

- "It's my bread";
- "I'm the hungriest";
- "I paid for it".

Asking the children to divide a loaf of bread equally between themselves and one, two and three friends respectively would perhaps have avoided some of the confusion caused by the original wording.

However, even the use of the word 'friend' is not without complication because it implies that the person is someone special - someone whom it would be easy to favour in a distribution problem. The very fact that the children were asked to share with their friends greatly affected some of the children's responses. In one case where a child gave her friend more bread that she apportioned to herself, she explained it as follows:

- "I like my friend".

Another child responded similarly saying:

- "My friend is hungrier".

The above comments illustrate that some of the children solved the problem by apportioning different sized pieces of bread to their friends on the basis of their personal knowledge of the needs of the people involved. They did not separate their everyday interpretation of the task from solving the problem mathematically and could therefore not be regarded as having been successful on the task.

There were other words that the children found confusing and had difficulty comprehending which affected their task performances. One such word was 'one half'. Only about half of the subjects were able to recognise the symbol for a half at the time of the last visit (Task 9) and their low performances on Task 10b indicated inadequate conceptual mastery of the task. When asked to name the fraction parts, i.e. halves, thirds and quarters, they were commonly named "half-half"; "half-half-half" and "half-half-half-half" respectively. 'Half' to these students appears to be a name for a word like 'piece'/'part' instead of denoting the specific mathematical attributes that one would expect.

The findings which reveal that students' misconceptions on formal tasks are caused by the interference of their everyday interpretations of language (e.g. 'share') are in

agreement with the findings of researchers such as Walkerdine (1988) and Cazden (1988). They conducted similar studies that showed the dual meanings of words like 'share' in everyday existence and in formal mathematics. Another similar finding is noted by Gordon (1992:09) in a transcript she quotes from a study of mathematics learning in a school in Soweto. A child who appeared not to understand remainders had a perfectly valid reason based on life-experiences for the answer he had given. This study also supports the Vygotskian (1978) notion of the importance of spoken language in the development of cognitive growth and the data reveals that the mediation of the mathematical language needed by the children has been ineffective. This ineffective mediation has resulted in poor performance values on tasks requiring the understanding of specialised mathematical terms and concepts (see Vygotsky on mediated learning, chapter 2.2).

5.2 The relationship between learning and development

The relationship between learning and development is complex and this complexity is exacerbated by the fact that internal mental processes are not directly observable or measurable. However, the tasks used in this study were designed in such a way as to elicit observable actions which could be used to provide insight into the internal processes at work. The tasks were designed to test abilities that fall within the scope of a child at a concrete operational level and one would therefore anticipate the children's performance values to be high, if not perfect across tasks by the last visit.

The subjects' developmental readiness is evident in their performances on Tasks 1, 2, 5, 7, 8b (Appendix C). These abilities are all indicative of children who are within Piaget's concrete operational level (cf. Slavin, 1988) and suggest that the subjects have the competence with which to perform successfully on the tasks in the schedule (Appendix A). As such, one could suggest that their developmental gains are sufficient for learning to occur, yet their performance values on Tasks 3, 8a, 9b, 10b, 11a and 11b are low. The question which needs answering has to do with why there is this apparent discrepancy between developmental readiness and task performance.

The difference between the tasks which revealed low performance values and the tasks where performance values were high has been attributed to the kind of knowledge and learning required to master the task. The tasks which revealed high performance values tested abilities which I have described as easy because they can be learned spontaneously and do not necessarily require formal instruction. Many of the tasks with high performance values at the time of the last visit also had high performance values at the time of the first visit and are therefore not indicative of learning during the course of the study.

The tasks which revealed low performance values have been described as complex and requiring formal instruction. Low performance values on Tasks 3, 4, 8a, 9b, 10b, 11a and 11b at the time of the first visit did not improve significantly over the course of the four visits, and performance values were still low at the time of the last visit (Appendix D). The lack of improvement and the minimal changes in the data over the course of the study indicates that learning was inadequate. Given the children's apparent developmental readiness, their lack of improvement is disappointing and suggests that learning has not kept pace with their development. This requires an explanation.

The absence of learning was most striking on Task 9b where the changes in students' performances on the mathematical operations of addition, subtraction, multiplication and division were negligible. The only significant changes were exhibited by the first-graders in their abilities to 'add'. Overall performances on the mathematical operations of multiplication and division were particularly poor. The data revealed that only four students improved their grasp of multiplication during the study and that only one student in Grade 3 made advances in terms of division. This indicates that the learning that one would expect from children of these ages has not occurred.

Further shortcomings in terms of the kind of learning that one would expect from children at this level is evident in Tasks 3, 9 and 10 and centres on the concept of division. The aforementioned tasks (Tasks 3, 9 and 10) all test division skills in different ways.. In Tasks 3 and 10, the children demonstrated that they were able to

divide, but their responses were not regarded as mathematically correct. In Task 9 the number of subjects who were able to demonstrate the concept of division never exceeded two on any one visit.

In Task 3, instead of dividing objects into 2, 3 and 4 **equal** groups, subjects counted the objects into groups with 2, 3 and 4 objects in each. They used their knowledge of counting and ability to count, rather than dividing objects in the manner required by the task. This finding suggests that although the children have some idea of what it means to divide, they have not acquired the mathematical wherewithal with which to interpret the problem of division. Instead, they hear certain numbers and simply count, i.e. they use their familiarity with counting and try to transfer it to another situation where it simply does not work.

The findings on Task 9 emphasise the apparent difficulties that these children have with division by the extremely low performances at all three grade levels. It should be noted here that only one single example of division was required for the children to be considered successful on this task. While I am not suggesting that all children should have mastered the concept of division within their first three years of schooling, the low performance values do seem to point to inadequate teaching by the fact that so few students are able to recognise the symbol for division or to demonstrate how one divides in any way whatsoever. This is particularly unusual given the fact that many of them understand the mathematical operations of addition and subtraction more than adequately. This suggests that they have the prior conceptual knowledge on which to base the learning of the operations of multiplication and division.

In Task 10, when asked how they would divide a loaf if they had to "share a loaf of bread with one, two or three friends", the children demonstrated that they know how to divide but they paid little attention to the mathematical requirements of the task. A pre-requisite for division is that the parts must be exactly equal. This basic grasp of the task was barely evident in subjects' responses.

The above examples of how the children went about dividing their loaves of bread illustrate gaps between what the children are able to do and the mathematical requirements of the tasks. There has been a breakdown in the learning that should have occurred. This breakdown could be attributed to the ways in which the concepts are being taught to the students. The mediation of the concepts required by the students is the responsibility of the teacher and requires a great deal of skill (cf. Vygotsky, 1978). Teachers themselves must be in control of the relevant mathematical concepts and skills that the students are supposed to learn (cf. Nunes et al., 1993 and Davis, 1996). In order for learning to be in advance of development (instead of lagging behind), teachers need to mediate within the learners' zone of proximal development and to provide the necessary scaffolding with which to bridge the gap between the known and the unknown (cf. Bruner, 1966). If children's mathematical performances are to improve they need to acquire more than task particular skills - they need to acquire mathematical competence. For this to happen they need to be given a variety of learning opportunities so that their learning can be transferred from one particular situation to a more general one (Bruner, 1966). These sorts of teaching and learning opportunities appear to be missing in the performances of the children.

5.3 Approaches to the task of teaching

As described in chapter two, the subjects in this study are supposedly taught from a (sociohistorical) constructivist perspective. Children who are taught from within this perspective are expected to display certain characteristic task performance. Some of the findings from this study conflict with the learning outcomes that one would expect from students who have been instructed in this way. These contradictions are evident in many of the task performances, and are described below.

Two instances in which the data suggests that a constructivist approach was not being effectively implemented were evident by the children's unwillingness to tackle a task when not forced to do so (Task 4) and the poor investigative abilities displayed by the children (Task 8). Children taught within the constructivist model are encouraged to

solve problems through trial and error and should be more willing to attempt new tasks than these data reveal.

The manner in which students (exposed to a constructivist approach to teaching) solve problems should demonstrate a confidence in trying various solutions to the task at hand. The data in this study do not show this. Creative and individual responses to problems are encouraged by constructivists and children are taught to explore a variety of possible solutions to tasks. However, the findings from Tasks 3 and 6 suggest that these children's responses became more typical in proportion to the number of years of schooling that they had. This was evident by the fact that the first graders showed far greater variety and creativity in the kinds of responses that they gave (Tasks 3 & 6) and in the kinds of errors that they made (Task 3) than the second and third graders.

The fact that whatever teaching had occurred was not learned by the majority of the sample population may point to the teacher as the source of the problem, given the similarity of the errors that were being made: it would be highly unlikely for students to have all had similar misconceptions without the teaching task being somewhat to blame. The similar errors made by the students suggests they were taught the wrong thing. In essence, the teaching that was practised was 'successful' but in some way it failed to be effective. In other words the students had learned well how not to divide or multiply (cf. Flanagan, 1992).

5.4 Differences between spontaneous and formal knowledge

The differences between spontaneous and formal knowledge were a particular feature of this study. At the start of this chapter, attention was drawn to the fact that the students generally performed well on tasks which could be learned spontaneously and that did not necessarily require formal instruction, while low performance values were generated on tasks which required formal teaching. In many instances throughout the study the students appeared to rely solely on their spontaneous knowledge. This resulted in high performance values on certain tasks but was counterproductive on others.

The use of spontaneous knowledge on Tasks 1, 2, 5 and 6 appeared to assist in successful performances. This was first evident in Task 1, where the children were able to give a wide variety of examples in which they used numbers, both at school and in their everyday lives. In Task 2, the subjects all demonstrated a certain expertise and general familiarity with money and its value and use. Their performances of the operations of addition and subtraction while indicating the cost of various items surpassed the high values attained when demonstrating these operations in Task 9b both in terms of performance values and in terms of the complexities of the examples that were given. Their good performances suggest that the high success rate was assisted by the students' spontaneous knowledge on this task. The relative ease with which they were able to order objects according to certain attributes and to form groups on a functional basis (Tasks 5 & 6 respectively) are also indicative of spontaneous learning that has contributed to high performance values (refer to Piaget on ordering in the concrete operational stage, chapter 2.2).

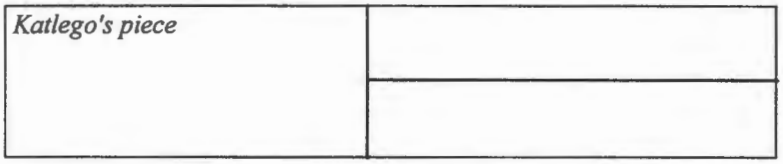
However, there were also times when the students' spontaneous understandings were shown to be very different from the formal requirements of the task and their use of this knowledge prevented them from attaining high performance values. The differences between spontaneous and formal knowledge were particularly apparent in the children's performances on Task 10 and it is on this task that the remainder of this discussion is focused.

In chapter two some of the complexities of incorporating the 'everyday' or spontaneous concepts with formal teaching were discussed. Task 10 highlights some of the potential confusions in using 'informal' knowledge to solve formal problems. This task required children to demonstrate how they would divide a loaf of bread if they had to share it with 1, 2 or 3 friends. The responses to this task incorporated a variety of 'everyday' beliefs and conceptions as to how bread should be shared. The diagrams below illustrate some of the solutions most commonly given by the children.

Bright's (Grade 2) answer to sharing his loaf with one friend:

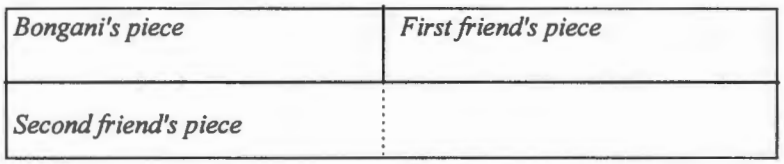
His reasoning for taking the largest piece is that the bread belongs to him. He names the parts "half-half".

Katlego's (Gr 2) answer to sharing her bread with two friends:



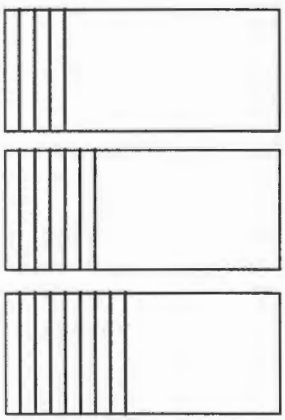
She says that if the others are still hungry, she'll give them some of hers.

Bongani's (Gr 1) answer for sharing his bread with 2 friends:



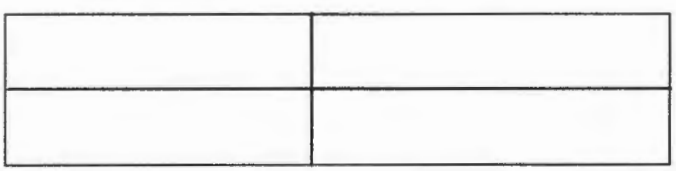
He says that he will divide the bread in half with his one friend, and that they will then each give the other friend half of theirs. He names all the pieces "half".

Jabulani's (Gr 2) responses to sharing bread with one, two and three friends



He names the parts as "2 slice - 2 slice - 2 slice" and says that he will save the rest of the bread for later.

Khamo's (Gr 3) answer to how she will share her loaf with two friends:



The extra piece is for her grandmother at home. She names the parts "half, half, half, half".

While one can accept that children will use their spontaneous understandings to help them solve mathematical problems (Walkerdine, 1988; Scribner & Cole, 1973), the teacher's task is to move the learners from that which they already know towards that which they need to know in order to perform successfully on the task (Vygotsky, 1978).

On the third and fourth visits a further task was introduced which eliminated 'bread' and the notion of 'sharing with friends' from the original problem. This generated higher performance values which gives added emphasis to the importance of language as a tool for the successful mediation of scientific/formal knowledge. That the sharing of bread, for example, is a very real part of the children's daily routine in the playground at school simply underlines the Vygotskian position on sociohistorical influences on learning. Spontaneous knowledge does not just turn into formal knowledge without a purposeful act of teaching (Walkerdine, 1988). Established traditions and ideas about day-to-day dealings with bread will remain precisely that unless intervention at the scientific level of concepts occurs. The relationship between spontaneous and formal knowledge is very complex and it is the task of the teacher to bridge the spontaneous learning in which the children's understanding is grounded to the formal requirements of the subject of mathematics (Vygotsky, 1978). The fact that these children performed better on the task requiring division of paper rather than bread gives meaning to the importance of the formal teaching of mathematics as a means by which to avoid the confusions that can occur when the differences between spontaneous and formal knowledge are not made explicit.

CHAPTER SIX

CONCLUSION

In this investigation of children's developing mathematical abilities, the following trends were apparent:

- the complexity and importance of language in the successful development of mathematical concepts;
- learning that was not in advance of development;
- learning that was not indicative of the constuctivist approach;
- a conflict between spontaneous and formal knowledge and learning in engaging with school mathematics tasks.

Each of these is concluded below.

The distinguishing features characteristic of the language used in mathematics learning are fundamental considerations for teaching children to engage with mathematical tasks successfully and independently. Without the appropriate language, children will not be able to develop their conceptual understandings adequately. In addition, teaching mathematics within the South African context requires extra special consideration of the issues surrounding language because of the multilingual composition of most classes, i.e. at least three 'languages' are commonly involved: the first language of the learners, the language of instruction (English) and the 'language' of mathematics. The use of all these different languages within the classroom context accentuates the difficulties and complexities of teaching mathematics. The results in this study underscore the importance of language in mathematics learning and teaching.

A further aspect of the complexities of teaching mathematics can be found in the relationship between ordinary and formal language used. There were numerous examples in this study in which ordinary language prevented the appropriate understanding of the formal language required by the task and this resulted in poor task performances. The importance of language and the skill of the teacher as significant factors in mathematics learning has been emphasised by a large number of

researchers. Although there is not always agreement on how one should deal with the complex nature of the concepts and skills required, researchers are united in their recognition of the differences between the 'languages' used and the need to make these differences apparent to the learners (cf. Adler, 1996; Davis, 1996; Scribner & Cole, 1973; Vygotsky, 1978).

The relationship between learning and development was another interesting pattern which emerged from the study. Generally, children's performances seemed to indicate a developmental readiness for mathematics learning and teaching. However, when this developmental readiness had to be taken further through formal instruction, their performances were inadequate. The children's low performances on the tasks which required formal instruction therefore lead one to conclude that, in general, their learning lags behind their development.

Possible reasons for learning not keeping pace with the developmental gains made by the children were attributed to poor mediation and inadequate learning opportunities, including teaching. There were indications that teaching had not been directed at levels that were in advance of the children's development and the mediation skills of the teachers were once again called into question. This finding was not completely unexpected given the history of inadequate and ineffective teaching in the schools under investigation (cf. Chisholm, 1993; Gordon, 1992; Marsh, 1993; Muller & Taylor, 1995; Slammert, 1993). The development of mathematical abilities is impeded by teaching that does not take cognisance of the developmental readiness of the children.

In respect of the teaching approach used in this study, teachers were supposedly teaching from a constructivist perspective. My research design does not allow for comment on teacher performance as such, but the trends in the data lead to some conjecture as to the origins of some of the learning outcomes and it would be amiss not to take the approaches to teaching into account. Most noteworthy were those occasions during the course of the study in which the children's performances conflicted with what one would expect from those subjected to a constructivist

approach to teaching (cf. Ernest, 1993; Von Glasersfeld, 1995). The students' unwillingness to tackle certain tasks, their poor investigative behaviours and their general inability to provide any sort of disparity in terms of the solutions that they offered to problems were not indicative of children who have benefitted from the basic principles of constructivism. This may mean that the students have not been adequately taught from a constructivist perspective or it may be indicative of weaknesses of a contructivist approach to teaching mathematics.

Another significant pattern in the data relates to the differences between spontaneous and formal learning. While the learners' spontaneous knowledge contributed to the students' successful performance on some of the tasks (e.g. the ability to demonstrate a sound understanding of the value and use of money), it severely hampered their success on others (e.g. the inability to demonstrate adequate understanding of fractions). While the students demonstrated that they had the necessary spontaneous knowledge on which good teaching and mediation could be based (to meet the formal task requirement), their performances did not illustrate such learning gains. Findings which reflect discontinuities between spontaneous and formal knowledge find resonance in a number of studies in other parts of the world (Dowling, 1993; Nunes et al., 1993; Scribner & Cole, 1973; Walkerdine, 1988) as reviewed in chapter two.

The trends and patterns that are highlighted above are based on the data obtained through a repeated measures design and the use of a schedule which tested children's performances on 11 mathematical tasks. The tasks were designed in such a way as to elicit observable actions from the subjects. These actions were then used to provide insight into the internal mental processes at work. This method of observation proved to be successful to the degree that it allowed for the study of changes in children's performances over a seven month period. The absence of an assessment of the statistical significance of changes in performance across the four visits limits the power of the conclusions drawn.

The repeated measures design was particularly appropriate for this investigation given the absence of baseline data and the dearth of research in mathematics learning and

teaching at primary school level in South Africa. In addition, this method allowed for the consideration of the social context of the study and of the influences on learners.

However, as with all (empirical) research projects there are also limitations to the conclusions that are reached which need to be considered. The use of a translator meant that the data was not obtained from the primary source and although the results appear to be truly representative of the children's abilities, the assumption that the tasks were clearly understood does underlie the findings. In addition, the two absentees during the course of the study decreased the already small sample size and thus the evidential basis for the conclusions that have been drawn. At best the trends that have been highlighted are indicative of possible hypotheses for further exploration.

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APPENDIX A

MATHEMATICS JUNIOR PRIMARY:
learning outcomes

Researcher: _____

Note: In the space provided, make detailed notes about what the child does as well as questions s/he may ask you, and problems s/he experiences.

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 1:	**When do you use numbers?** (Obtain from child a description of the situation(s) plus how s/he uses numbers in it).

OBSERVATION SCHEDULE

Struggles to understand
(Researcher must repeat question in different ways to child)

Yes	Sometimes	No

Engages task immediately

Yes	Sometimes	No

Completes task successfully

Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 2: **Do you get pocket money? If so, how much?**

(Get the child to show you using a set of the following coins: R2; R1; 50c; 2 x 20c; 2 x 10c; 2 x 5c; and old 2c and a small 1c coin, how much the things s/he buys from the shop cost.)

OBSERVATION SCHEDULE

Struggles to understand
(Researcher must repeat question in different ways to child)

Yes	Sometimes	No

Engages task immediately

Yes	Sometimes	No

Completes task successfully

Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 3: **Ask the child to sort 21 objects into equal groups. Firstly 3 equal groups; then 2 and then 4.**

(Observe what s/he does when there are objects left over. Question her/him about the meaning of 'divide equally' and ask her/him to check each group by counting the number of objects in it. Ask her/him how s/he knows that the groups are equal.)

OBSERVATION SCHEDULE

Struggles to understand (Researcher must repeat question in different ways to child)		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 4:	**Present the child with 7 round objects and 7 long objects. Ask her/him to match these in pairs; then ask the child to give you more examples in her/his ordinary, everyday experiences of 'pairs'.** (Question her/him on that which makes the things a pair.)

OBSERVATION SCHEDULE		
Struggles to understand **(Researcher must repeat question in different ways to child)**		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No
Other comments:		

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 5: **Present the child with the following sets:**

- three different length strings
- a small, slightly bigger and still bigger button
- three objects of slightly different weight
- a 5 cent coin, a 20 cent coin and a 50 cent coin

In each case ask her/him to sort/order these as follows:

- from the shortest to the longest
- from the smallest to the biggest
- from the least to the greatest amount

When the groups are sorted, ask her/him to explain in each case how s/he did the sorting.

OBSERVATION SCHEDULE

Struggles to understand (Researcher must repeat question in different ways to child)		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 6: **Present the child with nine objects which could be divided according to colour, shape and function into 3 sets of 3 objects each. Ask her/him to sort the objects into a group to which they belong and to explain her/his reasons for the way in which s/he did it.**

Question the child then, on:

- the characteristics they share
- why these objects are associated
- their functions

OBSERVATION SCHEDULE

Struggles to understand (Researcher must repeat question in different ways to child)		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 7: **Give each child a handful of small objects. Ask each child to *estimate* the number of counters/objects in her/his hands. Then ask each one to *count* the objects and to say whether her/his estimation was:**

- more or less than the correct number
- correct
- to decide whose estimation was closest to the correct number

OBSERVATION SCHEDULE

Struggles to understand (Researcher must repeat question in different ways to child)		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 8: **Ask the children whether anyone can read the time on (an analog) watch. Ask her/him to demonstrate this by reading the time. Present them furthermore with a collection of objects and ask each child to:**

- pick out the objects of different colours
- pick out the objects of different shapes
- show you the heaviest thing
- show you the longest thing

OBSERVATION SCHEDULE

Struggles to understand
(Researcher must repeat question in different ways to child)

Yes	Sometimes	No

Engages task immediately

Yes	Sometimes	No

Completes task successfully

Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 9: **Give the child a handful of objects and ask her/him to show you (or to demonstrate) what the following signs/symbols tell one to *do* with the objects in hand:**

- + (plus)
- - (minus)
- x (multiply)
- ÷ (divide)
- = equals

OBSERVATION SCHEDULE

Struggles to understand
(Researcher must repeat question in different ways to child)

Yes	Sometimes	No

Engages task immediately

Yes	Sometimes	No

Completes task successfully

Yes	Sometimes	No

Other comments:

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 10: **Ask the child to give you examples of 'fraction talk' which s/he uses every day. Then ask her/him the following questions:**

- If you have to share a loaf of bread with one friend, how will you divide it?
- And with 2 friends?
- And with 3 friends?

Note her/his demonstration of division plus her/his naming of the parts.

OBSERVATION SCHEDULE

Struggles to understand (Researcher must repeat question in different ways to child)		
Yes	Sometimes	No
Engages task immediately		
Yes	Sometimes	No
Completes task successfully		
Yes	Sometimes	No
Other comments:		

DATE: _____ SCHOOL: _____

PUPIL: _____ CLASS: _____

TASK 11	**Write ½ on a piece of paper and then ask the child what it means, and after s/he has answered to show you the *numerator* and the *denominator.***

OBSERVATION SCHEDULE

Struggles to understand
(Researcher must repeat question in different ways to child)

Yes	Sometimes	No

Engages task immediately

Yes	Sometimes	No

Completes task successfully

Yes	Sometimes	No

Other comments:

APPENDIX B

SAMPLE STUDENTS: NAME, SEX, GRADE, AGE AND YEARS IN SCHOOL.

NUMBER OF CHILDREN	NAME	BOY/GIRL	GRADE	AGE IN YEARS	YEARS IN SCHOOL
1	RUBEN	BOY	1	9	2
2	MMAPHEKWANE	GIRL	1	6	1
3	THEMBISO	BOY	1	6	1
4	BRIGHT	BOY	2	10	3
5	KATLEGO	GIRL	2	7	2
6	PALESA	GIRL	2	8	3
7	GABRIEL	BOY	3	11	3
8	LEBOGANG	GIRL	3	9	4
9	ORAPOLENG	BOY	3	9	3
10	JABULANI	BOY	1	7	1
11	BONGANI	BOY	1	9	3
12	LETSOABA	GIRL	1	6	1
13	SEBUSISO	BOY	2	9	2
14	VANESSA	GIRL	2	7	2
15	UNATHI	GIRL	2	8	2
16	SIFISA	BOY	3	8	3
17	MXOLISI	BOY	3	9	3
18	THEMBI	GIRL	3	7	3
19	LEBOGANG	BOY	1	6	1
20	SIZWE	BOY	1	6	1
21	NTHABISENG	GIRL	1	7	2
22	WILLIAM	BOY	2	9	3
23	SHARON	GIRL	2	8	2
24	MILLICENT	GIRL	2	8	2
25	KHAMO	GIRL	3	8	3
26	CHARMAINE	GIRL	3	8	3
27	CHRISTOPHER	BOY	3	10	4
TOTAL		13 GIRLS 14 BOYS		215	63
AVERAGE				8	2,3

APPENDIX C

PERFORMANCE CHART SUMMARISING THE TOTAL SAMPLE'S AVERAGE PERFORMANCE VALUES ON EACH TASK AT THE TIME OF THE LAST VISIT.

TASK NUMBER	TASK DESCRIPTION	LOW PERFORMANCE VALUES 1-39%	MIDDLE RANGE PERFORMANCE VALUES 40-69%	HIGH PERFORMANCE VALUES 70-100%
1	The use of numbers			✓
2	The use of money			✓
3	Dividing objects into equal groups	✓		
4	Matching objects in pairs	✓		
5	Ordering objects in terms of length, size, weight and value			✓
6	Grouping objects in terms of colour, shape and function	no correct answer stipulated	no correct answer stipulated	no correct answer stipulated
7	Identification of 'more' and 'less'			✓
8a	Telling time	✓		
8b	Identification of objects in terms of colour, shape, weight and length			✓
9a	Identification of mathematical symbols		✓(×, ÷)	✓(+, -, =)
9b	Demonstration of mathematical operations	✓(÷)	✓(×)	✓(+, -, =)
10a (i)	Dividing into fractions (bread)		✓(÷3, ÷4)	✓(÷2)
10a (ii)	Dividing into fractions (paper)		✓(÷3)	✓(÷2, ÷4)
10b	Naming fraction parts	✓(÷3, ÷4)	✓(÷2)	
11a	Identification of ½	✓		
11b	Identification of numerators and denominators	✓		

APPENDIX D

CHART SUMMARISING THE TOTAL SAMPLE'S AVERAGE IMPROVEMENT ON EACH TASK OVER THE COURSE OF THE STUDY

☞ ***GIVEN THE SMALL SAMPLE SIZE, AN APPARENTLY LARGE IMPROVEMENT OF 9-11% IS ONLY INDICATIVE OF A CHANGE IN 3 STUDENTS' PERFORMANCES.***

TASK NUMBER AND DESCRIPTION		FIRST VISIT	FOURTH VISIT	% IMPROVEMENT
1.	Use of numbers	93	100	7
2.	Use of money	93	100	7
3.	Dividing objects into equal groups	0	31	31
4a.	Matching objects in pairs	44	31	-14
4b.	Able to define a pair	11	30	19
4c.	Able to give examples of pairs	11	30	19
5.	Ordering objects in terms of length, size, weight and value	65	100	35
6.	Grouping objects in terms of colour, shape and function	no correct answer stipulated	no correct answer stipulated	
7.	Identification of 'more' and 'less'	71	100	39
8a.	Telling time	15	31	16
8b.	Identification of objects in terms of :			
	colour	96	100	4
	shape	48	70	22
	heaviest objects	41	88	47
	longest object	100	100	0
9a.	Identification of mathematical symbols:			
	+	78	100	22
	-	70	96	26
	×	37	69	32
	÷	19	52	33
	=	44	85	41
9b.	Demonstration of mathematical operations:			
	+	70	96	26
	-	63	77	14
	×	33	48	15
	÷	11	15	4
	=	41	85	44
10.	Fractions:			
	÷2	89	77	-29
	÷ 3	30	46	16
	÷ 4	56	62	6
11a.	Identification of ½	19	52	33
11b.	Identification of numerators and denominators	0	3	3