

Simplified approaches for portfolio decision analysis

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Abstract

Traditional choice decisions involve selecting a single, best alternative from a larger set of potential options. In contrast, portfolio decisions involve selecting the best subset of alternatives — alternatives that together maximize some measure of value to the decision maker and are within their available resources to implement. Examples include capital investment, R&D project selection, and maintenance planning. Portfolio decisions involve a combinatorial aspect that makes them more theoretically and computationally challenging than choice problems, particularly when there are interactions between alternatives.

Several portfolio decision analysis methods have been developed over the years and an increasing interest has been noted in the field of portfolio decision analysis. These methods are typically called “exact” methods, but can also be called prescriptive methods. These are generally computationally-intensive algorithms that require substantial amounts of information from the decision maker, and in return yield portfolios that are provably optimal or optimal within certain bounds. These methods have proved popular for choice decisions — for example, those based on multiattribute value or utility theory. But whereas information and computational requirements for choice problems are probably manageable for the majority of diligent decision makers, it is much less clear that this is true of portfolio decisions. That is, for portfolio decisions it may be more common that decision makers do not have the time, expertise and ability to exert the effort to assess all the information required of an exact method.

Heuristics are simple, psychologically plausible rules for decision making that limit the amount of information required and the computation effort needed to turn this information into decisions. Previous work has shown that people often use heuristics when confronted with traditional choice problems in unfacilitated contexts, and that these can often return good results, in the sense of selecting alternatives that are also ranked highly by exact methods. This suggests that heuristics may also be useful for portfolio decisions. Moreover, while the lower information demands made by choice problems mean that heuristics have not generally been seen as prescriptive options, the more substantial demands made by portfolio decisions make *a priori* case for considering their use not just descriptively, but as tools for decision aid.

Very little work exists on the use of heuristics for portfolio decision making, the subject of this thesis. Durbach et al. (2020) proposed a family of portfolio selection

heuristics known collectively as *add-the-best*. These construct portfolios by adding, at every step, the alternative that is best in a greedy sense, with different definitions of what “best” is. This thesis extends knowledge on portfolio heuristics in three main respects.

Firstly, we show that people use certain of the *add-the-best* heuristics when selecting portfolios without facilitation, in a context where there are interactions between alternatives. We run an experiment involving actual portfolio decision making behaviour, administered to participants who had the opportunity to choose as many alternatives as they wanted, but under the constraint of a limited budget. This experiment, parts of which were reported in Durbach et al. (2020), provides the first demonstration of the use of heuristics in portfolio selections.

Secondly, we use a simulation experiment to test the performance of the heuristics in two novel environments: those involving multiple criteria, and those in which interactions between projects may be positive (the value of selecting two alternatives is more than the sum of their individual values) or negative (the opposite). This extends the results in Durbach et al. (2020), who considered only environments involving a single criterion and positive interactions between alternatives. In doing so we differentiate between heuristics that guide the selection of alternatives, called *selection heuristics*, and heuristics for aggregating performance across criteria, which we call *scoring heuristics*. We combine various selection and scoring heuristics and test their performance on a range of simulated decision problems. We found that certain portfolio heuristics continued to perform well in the presence of negative interactions and multiple criteria, and that performance depended more on the approach used to build portfolios (selection heuristics) than on the method of aggregation across criteria (scoring heuristics). We also found that in these extended conditions heuristics continued to provide outcomes that were competitive with optimal models, but that heuristics that ignored interactions led to potentially poor results.

Finally, we complement behavioral and simulation experimental studies with an application of both exact methods and portfolio heuristics in a real-world portfolio decision problem involving the selection of the best subset of research proposals out of a pool of proposals submitted by researchers applying for grants from a research institution. We provide a decision support system to this institution in the form of a web-based application to assist with portfolio decisions involving interactions. The decision support system implements exact methods, namely the linear-additive portfolio value model and the robust portfolio model, as well as two portfolio heuristics found to perform well in simulations.

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Chapter 1

Introduction

1.1 Background

People often have to deal with the problem of selecting a subset of alternatives to achieve a desired goal in the face of constraints such as money, space, time, etc. Similarly, many for-profit and not-for-profit organizations face the difficult task of allocating resources to various organizational units or projects, while in public administration, industrial and financial organizations there arises the need to balance benefits, costs and risks. The goal is therefore to select efficient portfolios. A portfolio is a set of projects or alternatives that a decision maker decides to implement based on some objective or subjective criteria.

Traditional choice problems involve selecting a single, best alternative from a larger set of potential options. These are in themselves complex, as the decision maker has to often deal with conflicting multiple criteria under a set of constraints, and is often faced with uncertainties regarding the performance of alternatives on some of the criteria. Trade-offs between the performance on different criteria needs to be carried out in the process and performance on relevant criteria needs to be aggregated to arrive at a final decision. The complexity is compounded when the decision maker is confronted with a portfolio decision problem and this can be thought of as a pooling of several choice problems into a more encompassing portfolio choice problem. Alternatives may interact with one another positively or negatively, in the sense that the value of selecting two alternatives may be more than the sum of their individual values (if the interaction is positive), or less (if the interaction is negative). As a result, individually optimal decisions – such as ranking alternatives independently and selecting the best few – may not be collectively optimal, and this leads to inefficient use of the total available resources. Portfolio decision analysis (PDA) often involves more alternatives than single choice problems and the stakes are higher than in choice problems considering the amount of resources committed and the economic, environmental and societal impacts of the consequences (Salo et al., 2011). This type of problem forms the subject of the thesis.

1.2 Statement of the problem

The methods for portfolio selection commonly found in the literature are generally computationally-intensive algorithms that require substantial amounts of information from the decision maker, and return portfolios that are provably optimal or optimal within certain bounds. These could be classified as prescriptive decision approaches. Several prescriptive portfolio decision analysis methods have been developed over the years and an increasing interest has been noted in this field. Salo et al. (2011) discuss several applications of decision portfolio analysis and the type of models used. Salo et al. (2016) present a general framework for portfolio decision analysis, discuss modeling challenges, present several application areas for these methods, and discuss how specific methods have been used to generate solutions.

Previous work has shown that people use heuristics when confronted with traditional choice problems (Tversky, 1972; Gigerenzer and Goldstein, 1996; Katsikopoulos, 2011). Heuristics are simple models for making decisions that rely on core psychological capacities. They do not necessarily use all available information, and they process the information by simple computations, and are easy to understand, apply and explain where time pressure, or the costs of gathering information, is high, and it might not be possible to employ the prescriptive methods required to find an optimal solution. This necessitates the use of heuristics, which can under certain conditions compete with optimal models (Katsikopoulos et al., 2018; Todd, 2007; R.M. Hogarth, 2005; Baucells et al., 2008a). Although prescriptive portfolio decision models are useful – and remain the preferred option where sufficient time and resources can be expended assessing their inputs and using these to guide the decision maker to a final decision – the increased demands made by portfolio decisions suggest that it is important to examine both prescriptive methods and heuristics for portfolio decision analysis when studying this topic.

Little work has been done on the use of heuristics in portfolio decision making. Keisler (1996, 2004, 2008) has looked at this issue but their focus was on collecting information about the values and cost of alternatives and interaction values but not on assessing heuristic performance. Durbach et al. (2020) proposed a family of portfolio selection heuristics known collectively as *add-the-best*. This set of heuristics constructs portfolios by adding, at every step, the alternative that is best in a greedy sense, with different definitions of what “best” is. They studied the performance of these heuristics in decision contexts involving a single-criterion and positive interactions only setting. The goal of this thesis is to extend knowledge on heuristics for portfolio decisions in three main respects.

Firstly, we aim to test whether participants’ choice behavior is consistent with the heuristics proposed in Durbach et al. (2020) in a purpose-designed experiment. A small number of past studies have examined behavioral portfolio decision making, but these have used heuristics that fall within so-called “consume-and-check” and “solve-and-schedule” approaches (Ball et al., 1998; Langholtz et al., 2002, 1993, 1997). Consume-and-check heuristics allocate sequentially resources to consume and check for resource constraint violation at each stage. Solve-and-schedule heuristics first specify a total objective function value and then distribute the resources among available alternatives so that the total value is achieved. These studies do not deal with interactions between alternatives and are based on relatively simple problem contexts where the decision maker can only employ a limited amount of resources, and is thus quite different from the portfolio problem we consider. The portfolio decision problem in our research work deals with the selection of a portfolio of alternatives under resource constraints and considers also the situation where there are interactions between alternatives.

Secondly, we extend the simulation-based test of heuristics for portfolio selections from the single-criterion, positive interaction context used by Durbach et al. (2020), to include multiple criteria and negative as well as positive interactions between alternatives. These conditions are common in practice and therefore it is important to understand the performance of proposed heuristics specifically designed for portfolio decisions in these contexts, which resemble what happens in real-world portfolio problems. Considering these heuristics in multi-criteria contexts requires separating out any heuristics that guide the selection of projects, called “selection heuristics”, from those used to aggregate performance across criteria, which we call “scoring heuristics”. It is reasonable to suggest that scoring heuristics may impact the performance of selection heuristics in this context. Our aim is, therefore, to examine the influence of scoring heuristics on the performance of selection heuristics.

Finally, we use a real-world application to demonstrate the use of both exact and heuristics methods to solve a small but realistic, and real-world, portfolio decision problem. The heuristics proposed in Durbach et al. (2020) have not so far been incorporated in a decision support system. The application involves selecting the best subset of research proposals from a pool of applicants. We construct a web-based decision support system implementing both exact and heuristics methods, and use this to facilitate decision making through a structured decision making process, allowing the decision maker to select a preferred set of proposals.

1.3 Methodology

To address our core objective of evaluating the use and effectiveness of simplified models for portfolio decision making, we make use of a three-pronged methodology, each with their own strengths and weaknesses:

Behavioral “laboratory” experiments Choice experiments conducted on student participants in controlled environments allow us to study how people make portfolio decisions without facilitation. Such a setting is required to help us understand how people choose portfolios in practice and which heuristics get used in the process. We set up an experiment in which participants selected a portfolio of alternatives under resource constraints. We then analyzed whether the selections made by participants were consistent with the set of selection heuristics under study. Behavioral experiments such as these allow for substantial sample sizes and for a degree of experimental control that is difficult or impossible to achieve in real-world settings. For this reason, the settings are inevitably somewhat artificial, often, as in our case, involving student participants and hypothetical decisions. There is no guarantee, therefore, that laboratory findings carry over to real-world decision making.

Simulation experiment Experiments conducted using computer simulation have been widely used to assess the quality or accuracy of different decision making strategies. These allow for much greater control over environmental and other conditions, and require no direct involvement from anyone other than the researcher. They require only for a benchmark or gold standard to be defined, relative to which other strategies can be evaluated. The linear-additive portfolio value model is the most widely-used prescriptive model of portfolio selection and, for this reason, is used as a benchmark in our simulations. However, while simulations are useful in judging model accuracy relative to a prescriptive benchmark, they cannot be used to investigate descriptive issues, and they also provide no information on decision making processes, such as the extent to which a model is easy to use, or no insight into the decision problem. Thus there are various issues that are central to decision aid that are simply not possible to investigate using simulation.

Real-world application Implementing a facilitated decision process for a real-world problem addresses the key weakness of both behavioral and simulation experiments: their artificiality. Using a real-world problem allows to assess how a decision maker would use the developed decision support system. The drawback, however, is that real-world applications are time-consuming and difficult to implement. Sample sizes are therefore typically very small – perhaps based on one or two applications – and so, although we gain much insight on these particular applications, general conclu-

sions cannot be drawn. Also, each application is typically a “once-off” decision, meaning that it is not possible to know what results would have been obtained with, for example, other kinds of decision support systems.

1.4 Research objectives

As discussed in Section 1.2, we want to test whether participants’ choice behavior is consistent with the heuristics for portfolio decision making proposed in Durbach et al. (2020), in a purpose-designed experiment. We have also identified the need to extend the simulation-based test of heuristics for portfolio selections to include multiple criteria and negative as well as positive interactions between alternatives. Beyond the need to examine the use and effectiveness of portfolio heuristics, we want to provide a decision support system for portfolio decisions.

In line with our discussion about previous work on portfolio decisions and research gaps identified in Section 1.2, we list more specifically our objectives as follows:

- To provide an overview of existing portfolio decision analysis methods and put them into a coherent framework.
- To study the use of heuristics in an experimental setting involving a set of tasks performed by participants in selecting portfolios of alternatives.
- To extend the proposed simulation framework in Durbach et al. (2020) based on a single criterion and positive interactions (between alternatives) to include multiple criteria and both positive and negative interactions to reevaluate the performance of heuristics for portfolio selections.
- To evaluate the aggregate performance across criteria of scoring heuristics and their influence on heuristics for portfolio selections.
- To apply both heuristics and prescriptive portfolio decision methods to a real-world portfolio decision problem and evaluate their effectiveness.
- To provide a decision support system to enable sound and effective portfolio decision making.

1.5 Overview of dissertation

This dissertation consists of six chapters including the current chapter (i.e. Introduction) which are outlined as follows:

Chapter 2 contains a literature review which is made of two parts. In the first part, we present and describe various prescriptive approaches to portfolio decision making.

We examine approaches under certainty and under uncertainty of outcomes. We review and compare the methods and highlight similarities and differences between them and how they complement each other. In the second part, we describe heuristics used for portfolio decision making. We cover selection heuristics and scoring heuristics and assess their performance in a decision-making context.

Chapter 3 describes an experiment on actual portfolio decision making behavior which was performed by 74 participants. One of the key aspects of subjective decision making is the use of heuristics by decision makers. For this reason we investigate the use of potential heuristics when participants make such decisions without facilitation and provide insights into decision making in a single criterion and positive interactions setting.

Chapter 4 presents a simulation experiment for the evaluation of heuristics for portfolio decisions in situations involving multiple criteria, and both positive and negative interactions between alternatives. We consider heuristics that guide the selection of alternatives and those that aggregate performance across criteria. We compare the results with those obtained in Durbach et al. (2020) where the same heuristics are examined in a single criterion and positive interactions setting.

Chapter 5 applied the two best-performing heuristics identified in the previous chapter and the linear-additive portfolio value model to a real-world problem. This application involves the selection of the best subset of research proposals out of a pool of proposals submitted by researchers who apply for grants to the research institution. We provided a decision support system to this institution in the form of a Web-based application to assist with portfolio decisions involving interactions and sensitivity analysis.

Chapter 6 provides a final summary of the main findings and lessons learned from previous chapters. We also include directions for future research and applications.

Chapter 2

Literature review

2.1 Outline of the chapter

The literature review chapter is made of two parts: the first part deals with methods of portfolio decision analysis that fall within the field of multi-criteria decision analysis and the second part deals with heuristics for portfolio decision analysis.

The chapter is organized as follows: Section 2.2 gives an overview on multi-criteria decision analysis; Section 2.3 draws comparisons between prescriptive portfolio decision methods and discusses how they could complement each other and Section 2.4 presents heuristics for portfolio decision analysis and explains them using a practical example. More details about portfolio methods and heuristics can be found in Appendix D.

2.2 Multi-criteria decision analysis

Multi-criteria decision analysis (MCDA) can be defined as a collection of formal approaches which seek to account for multiple criteria, often conflicting, in helping individual or groups explore decisions that matter (Belton and Stewart, 2002). The MCDA process begins with problem structuring i.e. helping decision makers identify fundamental objectives, alternatives, stakeholders, uncertainties and constraints. Fundamental objectives are also referred to as criteria and are the basis for evaluating alternatives. Criteria should satisfy the following properties: completeness, measurability, non-redundancy, operationally meaningful, well-defined and preference independence. Preference independence means that preferences with respect to a criterion or trade-offs between criteria do not depend on the level of another criterion. There are three main schools of thought within MCDA:

Value function methods These aim to provide an ordering of alternatives from the

most preferred to the least preferred by assigning each alternative a numerical score. Such scores are first assessed on individual criteria and are then aggregated into higher level preference together with inter-criteria information expressing the importance of the criteria i.e. criteria weights. The most popular and simplest value function is the additive model. The first step in defining a value function is to identify a measurable attribute or a qualitative scale which expresses the values of the decision maker. The weights of criteria can be assessed using the swing method or the trade-off method. In the swing method, weights are computed from point valuations of different alternatives whereas in the trade-off method, they are derived from the trade-off relationship between two criteria. (Keeney and Raiffa, 1976; Keeney, 1992; Belton and Stewart, 2002)

Goal and reference point methods These seek to identify the alternatives that are closest to desirable goals. This is why the first step is to set desirable levels of achievement for each criterion or aspiration levels. Each criterion needs to be defined on a measurable scale. An attribute can be defined in a maximizing sense where the goal is the minimum level of performance or in a minimizing sense where the goal is the maximum level of performance. The problem in this instance can be solved as a linear programming problem and is referred to as goal programming. A goal could also be a desirable level of performance which the decision maker seems to achieve as closely as possible. In this instance, the problem is referred to as a reference point approach and can be solved as a Tchebycheff goal programming problem (Charnes and Cooper, 1961; Wierzbicki, 1980, 1999).

Outranking methods These are based on a pairwise comparison of alternatives in terms of each criterion and then preferences are aggregated across criteria to establish the strength of preference of one alternative over another. Unlike the value function method, the output is not a value for each alternative but an outranking relationship defined on the set of alternatives. An alternative outranks another alternative if there is a strong argument that it is at least as good as the other alternative and there is no strong argument to the contrary. The weights of criteria are not swing weights as defined in the value function method but are importance weights that express the relative importance of a criterion and are understood as “voting power” i.e. the extent to which the majority is in favor of an outranking relationship. There are few outranking models such as ELECTRE, PROMETHEE and GAIA. They differ in terms of preference functions, that allow for grades of preferences and parameters that need to be specified (Roy and Bouyssou, 1993; Roy, 1996).

MCDA has traditionally been applied to ranking problems or choice problems, i.e. the selection of the best alternative out of a set of available alternatives, but was later

extended to portfolio selection problems. Portfolio decision analysis falls into methods used under certainty and those under uncertainty.

Methods of portfolio selection have been developed to deal with different contexts of decision making. Most of these methods deal with traditional choice problems but have been extended to deal with portfolio problems. We will examine portfolio decision making under certainty and uncertainty, as explained below:

- (a) Portfolio decision making under certainty: this is the context where decision outcomes are deterministic. If no external uncertainty is at play, there is no need to use probability-based portfolio methods. It is assumed that errors in the data are negligible and that accurate information regarding the performance of alternatives is available. Under this category, we reviewed the following methods: the resource constrained data envelopment analysis (DEA-RC) (Cook and Green, 2000), equity model (Bana e Costa, 2001; Bana e Costa and Lawrence, 2007; Keeney and Raiffa, 1976; Keeney, 1992), the analytic hierarchical process (AHP) (Saaty, 1977, 1994; Ishizaka and Labib, 2011; Subramanian and Ramanathan, 2012; Yang and Tzeng, 2011; Saaty, 2004; Sarkis, 1999; Cheng and Li, 2004; Jiang et al., 2011; Abdollahi et al., 2015), the linear-additive portfolio value model (LAPVM) (Pisinger, 1997, 1995; Jaszkiwicz, 2002; Martello et al., 2000; Stummer and Heidenberger, 2003), PROMETHEE V (de Almeida et al., 2014; de Almeida and Vetschera, 2012), ELECTRE-TRI (Mousseau et al., 2002, 2011) and the reference point method (Stewart, 2016; Liesiö, 2014). Details are found in Appendix D.
- (b) Portfolio decision making under uncertainty or risk: the decision maker is confronted with external uncertainty or risk. This type of uncertainty occurs when outcomes depend on external forces which are beyond the control of decision makers such as a recession, a storm, the rate of industry growth, *etc.* Under uncertainty, it may not be possible to obtain accurate information about the performance of alternatives with respect to given criteria. Under risk, one could account for possible turn of “events” or “states” for the purpose of decision making, and attach probabilities to these. One could also consider a number of plausible scenarios and detail events within each of the scenarios to account for how the future might turn out. We reviewed the following methods uncertainty or risk: robust portfolio modeling (RPM) (Liesiö et al., 2008; Salo and Punkka, 2005; Ollil, 2013; Mild et al., 2015; Lindstedt et al., 2008), data envelopment analysis and the balanced scorecard model for portfolio selection (DEA-BSC) (Eilat et al., 2006, 2008; Greenberg and Hegerich, 1970; Martello et al., 2000), the scenario-based portfolio selection with complete probability and utility information (Liesiö and Salo, 2012; Rockafellar and Uryasev, 2000; Dentcheva and

Ruszczynski, 2006; Liesiö et al., 2007; Villarreal and Karwan, 1981; Kiziltan and Yucaoglu, 1983) and the scenario-based portfolio selection with incomplete probability and utility information (Liesiö and Salo, 2012). Details are found in Appendix D.

In this chapter we review and compare existing portfolio selection methods and attempt to develop a general framework for combining the strengths of the various portfolio decision analysis methods.

It is essential for portfolio decision analysis methods to account for interactions between alternatives which either results in gains or losses. There are 3 types of interactions between alternatives: resource interactions, where different alternatives share common resources and save on the total amount required when they are run simultaneously; benefit interactions, where the impact of a portfolio of alternatives is more than the sum of the individual impact of alternatives; and outcome interactions, where the probability of success of an alternative depends on the success of another alternative.

Solutions are impacted by the value of parameters and inputs that get used in the model. A few approaches have been developed to evaluate the impact of baselines on portfolio solutions obtained by applying value-based method (Clemen and Smith, 2009; Golabi and Sicherman, 1981; Morton, 2015; Liesiö and Punkka, 2014; Clemen and Smith, 2009; Bana e Costa and Lawrence, 2007; Kirkwood, 1997). A baseline is defined in this context as the value for not selecting an alternative. We summarize these approaches in Appendix D.

2.3 Comparison of prescriptive portfolio decisions methods

2.3.1 Interaction between alternatives

We note that some of the methods presented in Section 2.2 allow for interactions between alternatives while others do not. Methods which capture interactions are: data envelopment analysis (DEA), the analytic hierarchical process (AHP), robust portfolio modeling (RPM), the linear-additive portfolio value model (LAPVM) and the reference point method. Methods which do not account for interactions are: equity model, ELECTRE-TRI, PROMETHEE V and the scenario-based portfolio selection with (in)complete probability and utility information.

Interactions are generally captured in four ways:

Predicted synergistic input/benefit increments

When no interactions take place, overall benefits/inputs are simply the sum of individual alternatives. In the event of interactions, resulting benefits represent either a gain or

loss compared to the sum of individual items. There may also be savings on the amount of resources to operate a combination of alternatives because of synergies. In addition, synergistic benefit increments can be weighted by their probability of occurrence, i.e. probability of success when interactions take place. Outcomes are given as the sum product of probability and benefit levels. This is the approach followed by DEA, RPM and LAPVM.

An interaction matrix in percentage points

Interactions can be displayed in the form of a matrix showing a percent increase/decrease of the performance of one alternative as a result of its interaction with another alternative (2-way interaction). The interaction matrix is made of elements ranging from -1 to 1, with negative values showing a conflict between alternatives and positive values showing the extent to which alternatives interact with each other. Diagonal elements are set to zero. This is the approach followed by AHP. The value of an alternative given interactions with other alternatives is the weighted sum of individual scores on criteria plus interaction contributions of other alternatives to the alternative under consideration. Interaction gains are given as a proportion of the weighted sum of individual scores on criteria for the alternative under consideration. Weights, in this context, are criterion importance.

Virtual alternatives or constraints

Interactions can be modelled in the form of virtual (hypothetical) alternatives or linear constraints to capture either resource savings or output benefits. Resource gains can be specified as alternatives with negative inputs but zero outputs, and output benefits as zero cost but positive outputs. Constraints can also be added to the algorithm to effect these virtual alternatives when synergistic alternatives are at play. This is the approach followed by RPM.

The additive multilinear value function

Interactions can be modelled using an additive multilinear value function. The weighting function can be derived by asking the decision maker to assess trade-offs between criteria considering scenarios where additional alternatives are added to a given portfolio. This will help specify the additive multilinear value function. Alternatively, the difference standard sequence technique can be used. For a given criterion, the decision maker is asked to define the number of alternatives k_1 such that a change from k_0 to k_1 alternatives is equivalent to a change from 0 to k_0 alternatives. The process can be repeated until

all the weighting functions are obtained and the additive multilinear value function is derived. Liesiö (2014) concedes that this approach needs to be tested and further developed in real-life case studies.

2.3.2 Robustness of alternatives/solution

Sensitivity analysis for individual alternatives and for portfolios differ in purpose. Sensitivity analysis for individual alternatives is very useful as it helps to show the effect of changing input parameters on the selection of the best alternative. The change is meant to reflect uncertainty in the mind of the decision maker. It is also meant to show how the performance of individual alternatives may change as a result of changing priorities and values. Out of two equally performing alternatives, decision makers may opt for the one whose performance is the most robust. The general approach to conducting this analysis is to vary weights, preference/indifference thresholds and other input parameters in order to measure the effect of this change on the performance of alternatives. This approach has been widely used in additive value function-based models, data envelopment analysis and outranking methods (Belton and Stewart, 2002; Thanassoulis, 2001).

For a portfolio, the aim of the sensitivity analysis is to see how changing values will affect the composition of non-dominated portfolios whose objective values are better than the corresponding objective values of other portfolios. The goal is also to assess how often an alternative belongs to non-dominated portfolios: whether an alternative is always or sometimes part of non-dominated portfolios or whether it is never part of any of them. In light of this, robustness portfolio modeling yields a robustness measure called core index (CI), which is the proportion of non-dominated portfolios that include a given alternative (Liesiö et al., 2007). Robustness portfolio modeling has been developed in the additive value function framework. Core alternatives are desirable as they are part of every non-dominated portfolio. Exterior alternatives are not viable as they belong to none of the non-dominated portfolios. A portfolio comprised mostly with core alternatives is said to be more robust than a portfolio comprised mostly of exterior alternatives.

2.3.3 Preference modeling

Different approaches to preference modeling have been implemented in portfolio decision making. Preference with respect to given objectives can be captured through the use of a value function. Value functions are essential in evaluating the performance of alternatives and portfolios.

There are different approaches to defining a value function. We have value functions assessing the performance of each alternative across a set of criteria. We also have value functions assessing the difference in performance between any two alternatives. Preference/indifference thresholds can also be embedded into value functions to capture further performance aspects. The shape of value functions vary widely; we can have a linear shape, U-shape, V-shape, etc. Aggregation across criteria (attributes) can be done via an additive or a multiplicative approach. Criteria can be measured on an ordinal, interval or ratio scale and there are implications for this and some of this is discussed under Appendix D.1.8.

We need to recognize that value functions for portfolio problems may require further assumptions apart from those specified for value functions for alternatives only. Liesiö (2014) discussed multiattribute functions for portfolio problems and we present this work in Appendix D.1.8. Researchers may need to explore this area further to improve assessment of portfolios.

2.3.4 Portfolio solutions

We consider features of portfolio solutions given by the methods we have surveyed in this chapter. We examine whether the methods yield one best portfolio or a set of “best” portfolios. We also discuss the concept of portfolio optimality.

Portfolio methods fall into two groups. In the first group, we have methods that yield a unique best portfolio solution and in the second group, methods that yield a group of “best” portfolio solutions or non-dominated portfolios. Most of the methods discussed in this chapter fall into the first group. For methods in the second group, the decision maker is supplied with an array of non-dominated portfolios instead of a single best portfolio and has to choose their most preferred portfolio from this collection of portfolios. This type of solutions is produced by RPM, reference point method and the scenario-based portfolio selection with (in)complete probability and utility information.

Portfolio optimality is defined differently by the different methods and the best portfolios are identified in terms of: highest benefit-to-cost ratio, highest value, desired category, expected utility and minimum risk (variance).

Solutions yielding highest benefit-to-cost ratio identify the best performing portfolios in terms of benefit-to-cost ratios or weighted output to weighted input ratios. The decision maker chooses the subset of efficient alternatives that is within their budget

or meets their strategic goals. As such, the efficient frontier is unique and therefore a single portfolio within budget or resource constraints space is selected. This is the type of solutions produced by equity model and DEA-RC (DEA without interactions). These methods can be implemented via a visual interface.

Highest yielding value solutions identify portfolios that return the highest weighted value sum. AHP and LAPVM select portfolios which maximize total benefit. PROMETHEE identifies portfolios yielding the highest net flow or dominance.

Solutions that assign alternatives to a pre-specified category do so based on performance levels. The set of alternatives is effectively subdivided into subsets, each belonging to a given category. The subsets which belong to preferred categories form the optimal portfolio. The solution may also require further filtering in the form of constraints and is implemented as a linear program problem. This is the type of solution generated by the ELECTRI-TRI method.

Lastly, optimality is defined in terms of minimum risk (variance) or expected utility. The portfolio with maximum expected value under a defined level of risk is identified as the best portfolio. Alternatively, the analyst is tasked with performing pairwise comparisons of expected utility of portfolios to arrive at a set of non-dominated portfolios. Scenario-based portfolio selection with (in)complete probability and utility information apply this approach.

2.3.5 Further discussions

In this section, we have reviewed various methods for selecting optimal portfolios of alternatives. We have compared different approaches and discussed similarities and differences. In this concluding section, we will also discuss ways in which the methods can complement each other and suggest a coherent approach for doing this. We summarize, below, our main findings:

Solution types and Similarities The methods we have discussed aim to help decision makers and stakeholders select optimal portfolios. As we have seen, the concept of optimality is differently defined by different methods and optimality measures are given as: ratio of output to input (equity model and DEA), maximum weighted value sum (AHP, RPM and LAPVM), desired category (ELECTRI-TRI), dominance (PROMETHEE) and maximum expected utility (the scenario-based portfolio selection model with (in)complete probability and utility information). The types of solution are also different: some produce a unique

portfolio (equity model, DEA-RC, LAPVM, AHP, PROMETHEE and ELECTRI-TRI) and some a non-dominated set of portfolios (RPM, reference point method, DEA-BSC and the scenario-based portfolio selection model with (in)complete probability and utility information).

Methods: Strengths & Weaknesses Portfolio decision analysis methods, which depend on a prior enumeration of potential portfolios are quite burdensome and may require a lot of time and computing resources, especially if there is a high number of alternatives. They are of great value if the number of alternatives under consideration is relatively small and all potential portfolios can be enumerated quite easily. This is the case for the analytic hierarchical process (AHP) for portfolio problems. The remaining methods do not depend on complete enumeration but identify solutions using algorithms. Portfolio Decision Analysis methods that identify the set of non-dominated portfolios are of great help, but they leave the decision maker with the task of sorting through the non-dominated set to arrive at a final solution. This is the case for RPM, reference point method and the scenario-based portfolio selection model with (in)complete probability and utility information. Core alternative Indicators (CI) can be used to alleviate this process. They are a great help in this regard but they do not guarantee the absence of ties among portfolios. Lastly, ELECTRE-TRI yields a desired category or a set of desired categories comprising multiple alternatives. Further alternatives may be added to the desired set of alternatives by applying additional constraints.

We further observe that DEA calculates an efficiency score for each alternative and derives it by determining the value of constant multipliers or weights, in order to show each alternative in the best light. Such weights do not capture the relative importance of criteria as assessed by decision makers. They are instead “objectively” determined from the data. “Virtual” criteria capture the relative importance of criteria and are calculated as the product of weights and performance levels. Virtual criteria do not necessarily reflect weight in a MCDA sense. As already discussed, swing weights in MCDA are both a scaling constant and importance measure. Later DEA models introduce further constraints in the specification of weights apart from non-negativity constraints. This is an attempt to regulate the assignment of values to weights. For example, such constraints preclude large differences in weight between criteria when there is reasonable ground to assume that they are of similar importance (Cooper et al., 2000).

Ease of applying methods Models based on pairwise comparisons of alternatives are relatively easy to implement provided there is a small number of alternatives and the process can easily understood by decision makers. Outranking and AHP models use this

type of approach. The other models rest on the use of more or less complex algorithms to derive solutions and the process may not be easily understood by the decision maker. Outranking and AHP models, however, have their fair share of issues. The drawback for outranking methods is that the user needs to supply rather non-intuitive input values for a number of parameters (thresholds and preference functions) and this may lead to counter-intuitive results. AHP has also suffered from the issue of rank reversal as a result of a copy (or a near identical copy) of an alternative being added or removed from the current set (Ishizaka and Labib, 2011; Belton and Stewart, 2002).

How the methods complement each other and a framework for application:

The choice of an appropriate portfolio decision analysis method depends on the decision context: Are we certain or uncertain about decision outcomes? Do we have accurate measures of performance or only ranges of outcomes?

We consider two distinct cases as follows:

1. Under certainty of outcomes, the following methods apply:
 - (a) Resource constrained data envelopment Analysis (DEA-RC): It should be used if the decision maker is dealing with alternatives which use input factors to generate output factors.
 - (b) Equity model: if the decision maker wants to select alternatives based on a portfolio risk-adjusted benefit to cost.
 - (c) The analytic hierarchical process (AHP): It can be applied if the decision maker feels they are more comfortable expressing preference about alternatives and criteria via pairwise comparisons of these and can elicit a decision tree from stakeholders and interactions are at play in that decision context.
 - (d) The linear-additive portfolio value model (LAPVM): the decision maker wants to select the best subset of alternatives in a context where each alternative has both a value and weight or cost attached and there are interactions between alternatives..
 - (e) Outranking portfolio methods (ELECTRE-TRI & PROMETHEE): These can be applied if the decision maker feels they are more comfortable expressing preference about alternatives via pairwise comparisons of these. ELECTRE-TRI can be used when the decision maker believes that performance on criteria are non-compensatory i.e. strengths in one criterion do not necessarily compensate for weaknesses in other criteria. Otherwise, they can use PROMETHEE to model preference.

- (f) Reference point method: It can be applied if the decision maker can specify levels that they desire to achieve on the criteria.
2. Under uncertainty of outcomes, the following methods apply:
- (a) Robust portfolio model (RPM): It can be applied if the decision maker is dealing with a situation where there is incomplete score and weight information and there is a need to identify robust portfolios in that context.
 - (b) Data envelopment analysis and balanced scorecard model for portfolio selection (DEA-BSC): It can be applied when there are interactions between alternatives but outcomes are uncertain and alternatives are such that they use input factors to generate output factors.
 - (c) Scenario-based portfolio selection model with complete probability and utility information: It can be applied when the decision maker is confronted with decisions under risk where the goal is to maximize the expected utility of alternatives and there is complete probability and utility information.
 - (d) Scenario-based portfolio selection model with incomplete probability and utility information: It deals with decisions under risk and there is incomplete probability and utility information.

If one wishes to combine the strengths of the various methods that we have discussed, we suggest the following approach for screening, conducting further analysis and making a final decision:

Screening of alternatives/portfolios: in order to identify promising portfolios, we can apply DEA, outranking portfolio methods (PROMETHEE V/ELECTRE-TRI) and LAPVM. This step requires minimal input from decision makers such as thresholds and resource constraints.

Further analysis: This step elicits further input from decision makers and helps them gain more insight about the alternatives being analyzed. Such inputs express the values held by decision makers i.e. criteria weights, comparison of alternatives, value tree, utility functions, probability information, etc. At this stage, the user can apply AHP, equity model or the scenario-based portfolio selection model with (in)complete probability and utility information.

Final decision: this step allows for further interaction with decision makers. One can first identify the set of non-dominated portfolios by applying RPM and later review the

set to arrive at a preferred portfolio through sensitivity analysis and decision rules such as maximin or minimax-regret. The reference point method can also be used at this stage to identify a preferred portfolio solution. The decision maker can determine their budget and account for this in their decision making. It is worth saying that sensitivity analysis can be applied in any of the methods we have reviewed; however, in our opinion, it is much more structured in the RPM framework.

Future research As we have seen, it is crucial for methods to account for interactions between alternatives. Methods that do not currently account for interactions between alternatives can learn from those that do so. Interactions can be included in three distinct ways:

- (i) Predicted synergistic input or benefit increment (DEA, LAPVM and RPM)
- (ii) Synergistic matrix in percentage points (AHP)
- (iii) Virtual alternatives and constraints (RPM)

In instances where performance levels remain more or less unchanged over time, predicted synergistic inputs or outputs or virtual alternatives ((i) and (iii)) can be a good option for incorporating interactions in the model. The interaction matrix (see Section D.1.4), on the other hand, has the advantage of being applicable to situations where performance levels may change but we can reasonably assume that interactions remain unchanged. As we have seen, DEA, AHP, LAPVM and RPM cater for interactions. In this regard, the rest of the methods can learn from these four and this is an area for future developments. DEA, LAPVM and AHP allow only for two-way interactions between alternatives, i.e. only two alternatives can interact or enhance each other at a time. In real-world situations, it is possible that two or more alternatives would interact at a time. This type of complex interactions can be implemented in RPM and LAPVM. Future work should be directed towards enhancing DEA and AHP to allow for multiple interactions at once; we term this multiple-way interactions. This level of complexity will go a long way towards creating more pragmatic solutions. To the best of our knowledge current methods have only looked at the impact of baselines on solutions when there is no interaction between alternatives. Future research should investigate the impact of baselines when interactions are at play. When panel data or time series are available, it is essential to develop and apply appropriate methods of portfolio selection. This has potential to account for fluctuations in how alternatives perform over time and improve decision making. The issue of biases in decision making has been addressed in MCDA. It would be important to assess how the same issue play out in a portfolio context and develop approaches to address this. Outranking approaches could also be extended to deal with uncertainty in a portfolio context. Some of the portfolio methods we have discussed have

a user interface such as equity model and RPM. As far as we know, this is not the case for all the methods. Future research should also look into developing user-friendly interfaces for methods that do not have one.

2.4 Heuristics for portfolio decision analysis

2.4.1 Definition

It is true that information and computational requirements for choice problems are manageable for the majority of diligent decision makers but this is not the case for portfolio decisions. It may be more common that decision makers do not have the time, expertise and ability to exert the effort to assess all the information required of an exact method. Portfolio decision problems are made even more complex by the existence of interactions between alternatives. The use of exact methods under such circumstances is beyond the reach of the majority of decision makers. Heuristics provide strategies that can perform well in these conditions because they rely on simple search rules, aspiration levels, lexicographic rules and other principles. We also know that prescriptive decision making are mainly designed to work in a context where there is perfect knowledge of all outcomes of each outcome. This is the pillar of profit maximization and value functions presented in the previous section. However, in most real-world problems, decision making happens under uncertainty i.e. in situations where the set of alternatives and their consequences are not known. In such a context, prescriptive methods cannot work. Although robust portfolio modeling and scenario-based portfolio selection model with incomplete probability and utility information have been designed to work under uncertainty, they are computationally challenging for the average decision maker, who may lack the time or resources to implement them. Such situations call for the use of simplified models for portfolio decision making, also called heuristics (Gigerenzer and Gaissmaier, 2011; Gigerenzer and Mousavi, 2014).

Heuristics can be defined as models of decision-making that are used to describe peoples' actual behavior. These can be based on expertise or on observation and analysis of human behavior and on how people make good or even optimal decisions with little data (Keller and Katsikopoulos, 2016; R.M. Hogarth, 2005). Heuristics can also be defined as efficient cognitive processes, conscious or unconscious, that ignore part of the information (Gigerenzer and Mousavi, 2014). In this section we describe the selection and scoring heuristics used in the behavioral and simulation experiments (Chapters 3 and 4). Since we make extensive use of these heuristics in later chapters, we illustrate them with an example (in Section D.4.2) adapted from Durbach et al. (2020).

2.4.2 Heuristics for portfolio selection

Simon (1955, 1956), the father of bounded rationality, published their work in which they argued that people rely on simple strategies to deal with situations of sparse resources. Simon (1979) contended that classical model of rationality require knowledge of all the relevant alternatives, their consequences and probabilities, and a predictable world without surprises. These conditions, however, are rarely met for the problems that individuals and organizations face. In this school of thought, heuristics are viewed as simple strategies for solving ill-structured problems. Another school of thought called the “heuristics and biases research” gained prominence in the 1970s. Tversky and Khaneman (1974), the founding fathers of this school of thought, defined heuristics as mental shortcuts resulting in judgments that often violate the laws of logic, probability, or other benchmarks of rational choice. Judgments deviating from principles of logic and statistics are explained by people’s reliance on heuristics that lead to systematic cognitive biases. The heuristics and biases approach view heuristics as inferior strategies for solving well-structured problems where optimal solutions can be obtained by logical or statistical principles (Artinger et al., 2015a). Building on the work of Simon (1955, 1990), Gigerenzer et al. (1999) undertook to determine the environments in which particular heuristics work well and why this is the case (Gigerenzer et al., 2012). Gigerenzer et al. (1999) developed formal models of heuristics that they argued can be faster, frugal and more accurate when compared to standard benchmark strategies. This approach has been termed fast and frugal heuristics and exploits the study of ecological rationality to determine in which environments heuristics can be more successful than other methods (Artinger et al., 2015a). Gigerenzer and Goldstein (1996) presented heuristics such as take-the-best, tallying, weighted tallying, unit-weight linear model, etc. The take-the-best heuristic assumes a subjective rank order of criteria (cues) according to their validities. The highest ranking criterion which discriminates between the two alternatives is the the best criterion and we choose the alternative with the positive criterion value. In the tallying heuristic, the number of positive criterion values for each alternative is tallied across all criteria and the alternative with the largest number of positive criterion values is chosen. Weighted tallying is identical with tallying, except that it weighs each criterion according to its ecological validity. The ecological validity of a criterion is the relative frequency with which the criterion correctly predicts the target variable. The unit-weight linear model is the same as for tallying, only the assignment of values is different when the criterion value is negative (-1 versus 0, respectively).

In spite of all these advances, little work has been done on the use of heuristics in portfolio decision making. Keisler (1996, 2004, 2008) has studied this issue but the focus was on collecting information about the values and cost of alternatives and interaction

values but not on assessing heuristic performance.

Durbach et al. (2020) proposed selection heuristics that use a single criterion or an overall score (across criteria) when adding alternatives to a portfolio. These were adapted from the heuristics presented in Gigerenzer et al. (1999) and Keisler (2009). The construction of a portfolio was done sequentially by considering, at each stage, alternatives that have not been added yet. A stopping rule was needed to terminate portfolio construction after a specified number of consecutive constraint violations. In this section, we present and discuss this family of heuristics called *Add-the-best* or *selection heuristics*.

Add-the-best At each stage, the heuristics add an alternative that has the highest value according to a well-defined criterion. In the event of ties, these will be broken randomly. Attempts to add an alternative are terminated after ν consecutive violations of the budget constraints or when no alternatives remain to be added. Heuristics in this family differ from one another based on the criterion used to evaluate alternatives.

We illustrate *add-the-best* heuristics by examples in Tables D.2 and D.3 and explain how they are calculated in the Appendix Section D. We provide here a brief description of the heuristics.

We first describe heuristics that ignore quantitative information about alternative interactions and then we describe those that use quantitative information about alternative interactions.

Ignoring quantitative information about alternative interactions

We consider five heuristics that do not use quantitative information about alternative interactions in the calculation of selection values: *Highest Value* (HV), *Lowest Cost* (LC), *Unit Value* (UV), *Pareto*, and *Unit Value with Synergy* (UVwS). The first three heuristics (HV, LC and UV) entail a sorting of alternatives based on values, cost or a combination of both. The fourth (*Pareto*) is based on a dominance relationship between alternatives, and the last one (UVwS) uses binary information about whether an alternative is involved in any interactions. These heuristics were proposed in Durbach et al. (2020) and are given as follows:

***Highest value* (HV)** adds alternatives in descending order of their values.

***Lowest cost* (LC)** adds alternatives in ascending order of their costs.

Unit value (UV) adds alternatives in descending order of their value-to-cost ratios. Values are based on marginal alternative values only.

Pareto adds a randomly chosen alternative provided it is within budget and is not dominated by any alternative not already in the portfolio. An alternative is non-dominated if there is no other alternative yielding a higher return at equal or lower cost.

Unit Value with Synergy adds alternatives from the set of alternatives that are involved in at least one positive interaction based on the *Unit Value* heuristic; i.e. alternatives are sorted by their marginal value-to-cost ratios and added in this order. Alternatives are added until the set is exhausted. Then, alternatives from outside the set are added again using *Unit Value*.

Using quantitative information about alternative interactions

Unlike the previous group of heuristics, this group accounts for quantitative information about alternative interactions in the selection of portfolios. In other words, they consider the benefits or losses resulting from certain alternatives being selected together, and fall under the “Added Value” class in the *add-the-best* family. Individual heuristics within this class differ from one another on whether they account for all interactions or a subset of them. Heuristics that consider only a subset of interactions are frugal, as they do not consider at all the available information.

Added Value adds the alternative whose inclusion would lead to the largest increase in overall portfolio value per unit cost. Gains include both marginal values of the alternative and interaction values resulting from a combination of alternatives being simultaneously included in the portfolio.

Added Value heuristics searches across the alternatives not yet added to see whether their inclusion would complete any interaction subset. It considers all interaction subsets. Variants of *Added Value* (*Added Value Most*, *Added Value Least* and *Added Value Random*) do not consider all interaction subsets, as we shall see below.

Variants of *Added Value* can be defined as follows:

Added Value Most considers only interaction subsets involving the alternative that contributes the *most* to portfolio value per unit cost, i.e. the alternative with the highest value-to-cost ratio.

Added Value Least considers only interaction subsets that involve the alternative that currently contributes the *least* to portfolio value per unit cost, i.e. the alternative with the lowest value-to-cost ratio.

Added Value Random randomly chooses one of the alternatives already in the portfolio and considers only the interaction subsets involving this alternative.

2.4.3 Heuristics for overall scoring of alternatives

In most situations, alternatives are evaluated based on multiple criteria. Each alternative scores a value on each criterion and these are aggregated into an overall value for the alternative based on some rule. We discuss here approaches for assigning an overall value to alternatives. The prescriptive approach for scoring alternatives in a multiple criteria problem is the weighted additive linear function (WADD). In this approach, each alternative gets assigned an overall value which is the weighted sum of individual values on criteria. WADD requires the assessment of several parameters, some of which are difficult to assess, and previous work in non-portfolio contexts has shown that good performance can be obtained with simpler heuristics (Katsikopoulos et al., 2018; Todd, 2007; R.M. Hogarth, 2005; Baucells et al., 2008a).

We look at a set of heuristics for aggregating the values of alternatives across criteria. These were presented in Payne et al. (1993). We call this group of heuristics *scoring heuristics* and they are given as follows:

Equal Weighting Function (EQW): The aggregate score of an alternative is given as the mean of individual scores on all criteria. This is a simplified version of WADD, which ignores information about the relative importance of each criterion.

Lexicographic (LEX): The aggregate score of an alternative is represented by its value on the most important criterion, i.e. the criterion with the highest weight value.

Tallying (TAL): The aggregate score of an alternative is given as the count of good features of the alternative, i.e. the number of criteria whose scores exceed some predefined cut-off. Other variants of the tallying heuristic can be applied, such as the count of bad features or a combination of both good and bad features. The tallying heuristic is also called “the frequency of good and bad features”.

Each alternative can thus be scored according to WADD, EQW, LEX and TAL. The WADD score is the ideal score, as it accounts for all available information. Therefore, it serves as a benchmark against which the scores from the other scoring heuristics can be judged.

We illustrate *add-the-best* heuristics by examples in Table D.4 and explain how they can be calculated in the Appendix Section D.

2.4.4 Discussion and future research

In this section, we have presented and explained selection heuristics proposed in Durbach et al. (2020). These simple models are designed for portfolio selection and do not rely on all information available. Heuristics that account for interaction quantitative values fall under the “Added Value” class in the *add-the-best* family. In the presence of a bigger set of alternatives, these heuristics can become quite overwhelming for the decision maker. Therefore, future work should find ways to simplify “Added Value” heuristics in this case. Although the *add-the-best* heuristics can be applied successfully in a portfolio context, it is unclear how they would work under risk. In this case, one could account for possible turn of “events” or “states” for the purpose of decision making, and attach probabilities to these. Further work should be done to extend the heuristics to address risk in decision making. We can also consider other forms of interactions apart from benefit interactions, which are already included in current heuristics, such as resource or cost interactions presented in Villarreal and Karwan (1981); Liesiö et al. (2007). This will, for example, extend the heuristics to account for resource gains. Lastly, current heuristics are in the *add-the-best* family. Other classes of heuristics are presented in the literature such as recognition heuristic, fluency heuristic, 1/N rule, imitate the majority, etc. (Gigerenzer and Mousavi, 2014). These classes could also be adapted to portfolio decision problems and this would provide a richer toolbox of heuristics to the decision maker and enable them to address a broader range of portfolio decision problems.

In this chapter, we reviewed both prescriptive portfolio decision methods and heuristics for portfolio decision analysis. In the next chapter, we will evaluate the performance of heuristics against optimal portfolios constructed from a linear-additive portfolio value function, the most used prescriptive portfolio decision method.

Chapter 3

A descriptive study of unfacilitated portfolio decision making

3.1 Introduction

In this chapter, we aim to understand how people go about selecting portfolios without facilitation. Within the field of decision making, we have both prescriptive and descriptive decision theories. Prescriptive decision theory aims to guide a decision maker towards an optimal alternative: it prescribes a solution thought to be the best under the circumstances. Descriptive decision theory, also referred to as Hypothesis decision theory aims to model intuitive decision making.

Most of the literature about portfolio decision analysis deals with prescriptive decision analysis. Over the years, several approaches have been developed to guide the selection of the best portfolios. Some of the most well-known methods for traditional one-out-of- n choice problems have been extended to deal with portfolio problems. In the previous chapter, we presented the following portfolio decisions methods: resource-constrained data envelopment analysis (Cook and Green, 2000; Eilat et al., 2006), equity model (Bana e Costa, 2001; Bana e Costa and Lawrence, 2007), the analytic hierarchical process (Jiang et al., 2011), outranking portfolio methods (Mousseau et al., 2011), robust portfolio modeling (Liesiö et al., 2007, 2008), reference point models (Stewart, 2016), scenario-based portfolio selection models (Rockafellar and Uryasev, 2000; Dentcheva and Ruszczyński, 2006). All these models have one feature in common: they rely on optimization algorithms to find the best solution.

Besides prescriptive approaches to decision making, there is a field of behavioral decision making that looks at the practice of decision making. One of the key aspects of behavioral decision making is the use of heuristics by decision makers. A heuristic can be defined as a method (a sequence of operations) for searching through the decision space (Payne et al., 1993). Heuristics are sometimes labelled “fast and frugal” as they utilize

a minimum of time, knowledge, and computation to make adaptive choices (Gigerenzer et al., 1999). A heuristic can be viewed as a strategy for decision making and well-known managerial heuristics have been employed by managers in making decisions such as satisficing, tallying and $1/N$, lexicographic strategies, recognition, etc. (Artinger et al., 2015b). Payne et al. (1993) suggest that people make trade-offs between cognitive effort and accuracy of strategy. Moreover, they show that heuristics are not all-purpose tools but strategies that can perform well in particular environments.

This chapter aims to shed light on how people go about selecting portfolios without facilitation. We aim to show, in this experiment, that people use the heuristics proposed in Durbach et al. (2020) when selecting portfolios without facilitation, in contexts where there are interactions between alternatives and shared implications for decision making. In order to investigate participants' behavior we ran an experiment where people were tasked with selecting the best portfolio out of a set of alternatives. We then examined the distribution of their portfolio selections as well as how the portfolios were constructed. The final selections made by participants are shown in the following GitHub repository: <https://github.com/DKantu01/Experimental-Data>.

The rest of this chapter is organized as follows: Section 3.2 explains the research hypotheses being analyzed; Section 3.3 presents the information and statistical methods used for this analysis; Section 3.4 presents the results; Section 3.5 highlights key findings; the final section discusses the implications of the findings .

3.2 Research hypotheses

Previous work has found that task complexity negatively affects the accuracy of responses (Kormos and Trebits, 2012; Robinson, 2001), i.e. more complex tasks get a lower response accuracy. Task complexity can be measured by the number of alternatives, number of criteria, interactions, time constraints, etc. Payne et al. (1993) argue that decision behavior is highly contingent on (task) factors that impact task complexity, and task complexity appears to affect the quality of decisions and methods of processing information. For this reason, we varied the degree of difficulty across tasks to study the impact of complexity of tasks on the responses. We investigated whether the finding in Payne et al. (1993) applies in the context of portfolio decisions by testing the following hypotheses:

Hypothesis (H1a): introducing interactions negatively affects the quality of portfolio selections (Task 2 vs Task 1).

Hypothesis (H1b): increasing the number of alternatives negatively affects the quality of portfolio selections (Task 3 vs Task 2).

Different decision strategies require different amounts of computational effort. It has been established, for example, that the number of alternatives to select from impacts the effort required to make a decision and that there is a positive correlation between task complexity and amount of effort exerted by participants, i.e. participants exert more effort when tasks become more difficult (Johnson and Payne, 1985). Again, we expect this to apply in the portfolio context, as stated in the following hypotheses:

Hypothesis (H2a): introducing interactions increases participant effort (Task 2 vs Task 1).

Hypothesis (H2b): increasing the number of alternatives increases participant effort (Task 3 vs Task 2).

Decision making is known to be influenced by the consideration of cognitive effort in traditional choice problems. It has been established that as decisions become more complex, people tend to use simpler heuristics (Payne et al., 1993). Portfolio decisions are complex by nature, and so we expect some degree of heuristic use. Our primary objective in this chapter is to test whether there is evidence for the use of any of the heuristics proposed in Durbach et al. (2020) in portfolio selections. Firstly, our hypothesis (H3) is that the proportion of selections associated with a given heuristic is not consistent with what might be expected if alternatives were added or removed at random. Secondly, our hypothesis (H4) is that selections are consistent with heuristics being studied in this research work.

It is commonly thought that people learn to make better decisions by trial and error. In the context of this experiment, if decision quality improves, we expect a consistent decline in the proportion of selected dominated alternatives (with respect to the baseline) as participants continue to make selections. The baseline can be measured as the number of dominated alternatives available at the moment of selection. Our hypothesis (H5) is that people are less likely to choose a dominated alternative later on in the selection process.

3.3 Materials and Methods

3.3.1 Survey participants

We presented participants with three tasks, each requiring the selection of a portfolio within a specific context. Participants were students from the African Institute of Mathematical Sciences (AIMS) and the Cape Peninsula University of Technology (CPUT), chosen randomly. In total, 74 students, aged between 20 and 30, took part in the survey. There were 43 female and 31 male participants in total. The students from AIMS were studying towards a Masters degree in mathematical sciences whereas students from

CPUT were studying towards a bachelors degree in environmental management. Tasks were performed individually on a computer in a number of hosted laboratory sessions of 15 to 20 participants per session. We gave a small monetary incentive to each participant.

3.3.2 Task design

The experiment was coded using the R Shiny framework, with C# and Microsoft SQL used to interface with and store data from the web-based front end. In each session the following instructions were given:

“In this experiment your aim is to get as many points as possible. You get points by selecting alternatives. You can choose as many alternatives as you want, but each alternative has a cost, and you have a limited budget. Your aim is to choose the combination of alternatives that gives you the maximum number of points, while not exceeding your budget.”

For tasks involving interaction between alternatives, additional information was given:

“Each alternative has its own points value, but some combinations of alternatives go well together. If you select two or more alternatives that go well together, you get extra points (on top of the alternative’s own points). In other words, the number of points you get for a alternative can depend on other alternatives you might have already chosen, so be careful when making your selections!”

Participants were shown a video repeating these instructions and providing a short walk-through illustration of how to perform the task.

The tasks were as follows:

Task 1 A small portfolio problem involving five alternatives with no interactions between alternatives. The total portfolio value is the sum of the values of the individual alternatives.

Task 2 A small portfolio problem involving five alternatives with interactions between alternatives. The value of two alternatives may be higher or lower than the sum of the values of each alternative depending on whether there are positive or negative interactions.

Task 3 A larger portfolio problem involving ten alternatives with interactions between alternatives.

All participants were given the three tasks above in random order. However, some participants did not attempt all of the tasks and hence there was a small discrepancy in sample sizes across tasks (Task 1: $n = 73$; Task 2: $n = 74$; Task 3: $n = 72$). A fixed budget

was available to each respondent and they were not allowed to submit a portfolio that exceeded the amount of funds available. We also displayed the overall points and cost of the currently-selected set of alternatives, and the amount of money they still had available to spend. In addition, we showed potential points (returns) and cost that would result from adding each of the alternatives not currently selected or from removing each of the alternatives from the set of currently-selected alternatives. The new total was computed taking into account the synergistic gains where interactions (between alternatives) were at play.

Alternatives with corresponding values (points), cost, available budget and interaction values are displayed in Tables 3.1 to 3.4. There are either two-way (between two alternatives at once) or three-way (between three alternatives at once) interactions and we show added returns that would result from two or three alternatives being selected in the same portfolio for Task 2 and Task 3 respectively (Tables 3.2 and 3.4).

	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Return	1	1	2	2	5
Cost	1	2	3	4	2
Budget: 7					

Table 3.1: Data for Task 1 and Task 2, comprising values and costs of alternatives, and available budget.

	Alt1,2 (A_1)	Alt1,3 (A_2)	Alt1,2,3 (A_3)	Alt2,3,4 (A_4)
Interactions	3	3	3	3

Table 3.2: Interaction values for Task 2. The values are the extra amount gained if all alternatives in the interaction subset $A_i, i = \{1, \dots, 4\}$ are included in the portfolio. The first interaction subset is made of alternatives 1 & 2 and earns an extra 3 points when both alternatives are selected.

	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10
Return	100	150	120	210	80	180	110	150	40	40
Cost	60	100	40	90	40	100	40	60	80	40
Budget: 280										

Table 3.3: Data for Task 3, comprising values and costs of alternatives, and available budget.

We explore the following set of heuristics that would provide a plausible explanation of selections made by participants:

Highest value (HV) adds alternatives in descending order of their values.

Lowest cost (LC) adds alternatives in ascending order of their costs.

	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10
Alt1	0	110	190	130	70	210	80	90	130	70
Alt2	110	0	70	170	80	190	85	130	240	80
Alt3	190	70	0	180	85	90	80	210	210	85
Alt4	130	170	180	0	70	170	75	110	130	70
Alt5	70	80	85	70	0	70	70	70	70	75
Alt6	210	190	90	170	70	0	80	110	130	70
Alt7	80	85	80	75	70	80	0	80	80	80
Alt8	90	130	210	110	70	110	80	0	85	75
Alt9	130	240	210	130	70	130	80	85	0	70
Alt10	70	80	85	70	75	70	80	75	70	0

Table 3.4: Interaction values for Task 3. The values show the extra amount gained if all alternatives in the interaction subset are included in the portfolio. The interaction subset made of alternatives 1 and 2, for example (second column, first row) yields a gain of 110 points when both alternatives are selected.

Unit value (UV) adds alternatives in descending order of their value-to-cost ratios. Values are based on marginal alternative values only.

Added Value (AV) adds the alternative whose inclusion would lead to the largest increase in overall portfolio value per unit cost.

Other the “Other” option points to selections that can’t be explained with known heuristics.

Tasks were hosted on shinyapps.io and can be accessed at:

Task 1: <https://iandurbach.shinyapps.io/simpleportfolioexp/>

Task 2: <https://iandurbach.shinyapps.io/simpleportfolioexpwithinteractions/>

Task 3: <https://iandurbach.shinyapps.io/portfolioexperiment/>

3.3.3 Statistical analysis

We assessed the decision quality of portfolio selections in two ways: the percentage of participants who chose the optimal portfolio, and the mean portfolio value for each task. In the second approach, we calculated the mean value across all portfolios selected in a given task. The value of a selected portfolio is given as a percentage of the optimal portfolio’s value. A portfolio is optimal if there is no other portfolio giving a higher return within the budget constraint. Given that the return of the optimal portfolio is 8 in Task 1 (Table 3.5), the value of a selected portfolio is 50% if its return is 4. We show optimal portfolios in Table 3.5.

The percentage of participants who chose the optimal portfolio is denoted PP_{opt} and

Cost and Return values of optimal portfolios			
Task	Optimal portfolio	Return	Cost
Task 1	{1, 3, 5}	8	6
Task 1	{2, 3, 5}	8	7
Task 1	{1, 4, 5}	8	7
Task 2	{1, 2, 3}	13	6
Task 3	{1, 3, 5, 7, 8, 10}	2010	280

Table 3.5: The table shows (the) optimal portfolio(s) for each task and corresponding earnings and costs. For Task 1, there are 3 optimal portfolios whereas there is only one optimal portfolio per task for the remaining tasks (Task 2 & Task 3).

is calculated as follows:

$$PP_{opt} = \frac{N_{part_{opt}}}{N_{part}} \quad (3.1)$$

where $N_{part_{opt}}$ is the number of participants choosing the optimal portfolio and N_{part} the total number of participants in a task.

The value of portfolio P can be found as follows (Stummer and Heidenberger, 2003):

$$V(P) = \sum_{j=1}^n x_j z_j + \sum_{i'=1}^I x'_{i'} g_{i'} \quad (3.2)$$

$$g_{i'} = \{0, 1\} \quad \forall i'$$

where P is a portfolio, z_j is a binary variable indicating whether alternative j is selected in a portfolio, $x_j = \sum_{i=1}^m \omega_i x_{ij}$, the weighted additive value across criteria of proposal j , $x'_{i'}$ is the incremental change in value if at least $m_{i'}$ alternatives in interaction subset $A_{i'}$ are included in the portfolio, and $g_{i'} = 1$ if at least $m_{i'}$ alternatives in interaction subset $A_{i'}$ are selected, and $g_{i'} = 0$ otherwise.

The mean portfolio value for a given task is given as:

$$\text{Mean Portfolio Value} = \frac{\sum_{P=1}^N V(P)/V(P_{opt})}{N} \quad (3.3)$$

where $V(P)$ is the value of portfolio P , $V(P_{opt})$ is the value of optimal portfolio and N is the number of portfolios selected in a task.

In order to test hypothesis H1a, we compared for Task 1 and Task 2 the percentage of participants who chose the optimal portfolio and the mean portfolio value. In order to test Hypothesis H1b, we compared the same metrics (percentage of participants choosing optimal portfolio and mean portfolio value) for Task 2 and Task 3.

Cognitive effort has been measured in various ways. Johnson and Payne (1985) used Elementary Information Processes (EIPs). In this approach, called “cost of thinking”,

heuristics are described as a sequence of mental events. Another approach suggests that cognitive effort is multidimensional consisting of time (latency), cognitive strain (self-reported subjective measure) and total cognitive effort (cost of thinking) (Gopher and Donchin, 1986). Eye-tracking studies use fixation count and pattern as measures of cognitive effort (Djamasbi et al., 2013). In addition, websites usage, gaming and eye tracking have been used traditionally and successfully as a technique for measuring cognitive load in reading, psycholinguistics, writing, and language acquisition (Rayner, 1998). It seems to us that cognitive effort should be measured appropriately given the context in which it is measured and the resources available to the researcher. In our experiment, we considered latencies, cost of thinking and usage (a frequency measure). However, we could not track successfully the time spent by participants on the tasks (latency) as most of them participated in the experiment remotely. Ties between heuristics made it difficult to use cost of thinking. For this reason, we opted for a frequency measure i.e. the mean number of selections and deselections made by participants for each task as a measure for the amount of effort applied by participants in carrying out a portfolio selection.

In order to test Hypothesis H2a, we compared the mean number of selections and deselections for Task 1 and Task 2. In order to test Hypothesis H2b we compared the same metrics for Task 2 and Task 3.

Heuristics vary in their level of complexity. There are more basic heuristics where participants focus on a single item of information such as cost or earnings and attempt to either minimize cost (e.g. *Lowest Cost*) or maximize return (e.g. *Highest Value*). There are more complex heuristics where participants consider multiple items of information at once, such as both cost and return or marginal changes in both cost and return, etc. and attempt to optimize these. Examples of this kind are ratio-based heuristics such as *Added Value* and *Unit Value*.

Assessing the extent to which people use any given heuristic is complicated by the fact that often (a) different heuristics lead to the selection of the same alternative (there are ties between heuristics), and (b) a single heuristic may lead to the selection of any of the sets of alternatives, where alternatives are tied on the criterion used by the heuristic. We can illustrate this by the example in Table 3.6. Suppose we have four alternatives, each associated with a return value and a cost value. People can select alternatives by any of the 4 given heuristics: *Lowest Cost (LC)*, *Highest Value (HV)*, *Unit Value (UV)* and *Other (Unknown)*.

If a participant's strategy is to maximize return, they will choose alternative 1 as it gives the highest return (3 value units). If their strategy is to minimize cost they will choose either of alternatives 1 and 3, as they are the least expensive (1 cost unit). If their strategy is to maximize return-to-cost ratio, they will choose alternative 1 as it has the highest return-to-cost ratio (3). If alternative 1 is selected, all 3

Alternatives	Return	Cost	Return-to-cost Ratio	LC	HV	UV	Other
1	3	1	3.0	1/3	1/3	1/3	
2	1	2	0.5				
3	2.5	1	2.5	1			
4	2	4	0.5				1

Table 3.6: Examples of heuristic ties. The table shows earnings, costs and return-to-cost ratios, and heuristics that were applied to the 4 alternatives. Heuristics *Lowest Cost*, *Highest Value*, and *Unit Value* account for the selection of alternative 1,, i.e. they all tie in this instance.

heuristics (*Highest Value*, *Lowest Cost* and *Unit Value*) account for this selection, i.e. they tie. So, we assigned the 3 heuristics to alternative 1 but shared the vote equally among heuristics,, i.e. each gets 1/3 vote. There is no obvious reason why someone would choose alternative 4. For this reason we assigned this selection to “*Other*” heuristic.

Another challenge in assessing whether participants used heuristics is that our experimental setup did not tell participants which alternatives had interactions. In the absence of this information, some selections and deselections (using rules similar to those in Table 3.6) would have been made to gather this information. Strategies for information gathering introduce noise to the selection process and thus to our assessment of heuristic use. However, the alternative option, i.e. providing the interaction information, would have biased their responses. For this reason, we chose to analyze both the order in which alternatives were added or removed before the final submission as well as the final submitted portfolios.

We analyzed how alternatives were added and removed by participants and linked each addition and removal to potential heuristics. For each addition or removal, we took into consideration alternatives already included in the portfolio. We did this for each respondent and looked at the following heuristics: *Lowest Cost*, *Highest Value*, *Added Value* and *Unit Value*. Selections that could not be accounted for by these heuristics are classified as “*Other*”. We did not include *Unit Value with Synergy* and *Pareto* in this analysis as the data collected does not allow us to infer whether participants’ selections were based on interacting alternatives or on non-dominated alternatives. In cases where the selected alternative corresponded to more than one heuristic, the vote was shared evenly between those heuristics, as in Table 3.6. In this way, we were able to compute across all participants weighted proportions of the use of each heuristic and test for significant differences between them to establish which heuristics were most used.

We tested whether the observed proportions are different from those that would arise

from a random selection of alternatives (Hypothesis H3). Under the null hypothesis, the proportion of selections is consistent with what might be expected if alternatives were added or removed at random. In order to test this, we simulated a distribution of proportions associated with each heuristic, under the null “random selection” model. We did this by creating a random sample of participants of the same size as the actual sample, with the same distribution of additions and removals as observed in the experiment. We added alternatives at random until the required number of removals was completed for each participant and removed any alternative causing a violation of budget constraint and an additional alternative selected at random and replaced them with new alternatives. We repeated this procedure 2000 times and calculated the proportions associated with each heuristic under the null “random selection” model. We tested observed proportions of heuristics against simulated proportions of heuristics under the “random selection” model for significant differences. A significant difference between these proportions is an indication that the observed proportions of heuristics are unlikely to arise from a random selection strategy.

We investigated whether selections were consistent with heuristics being studied in this research work (Hypothesis H4) . To reduce the influence of removals of alternatives, we also assessed the first few selections and analyzed how the addition of alternatives was consistent with the use of a particular heuristic. We also identified any participants that made consistent use of a single heuristic in the first three selections (Task 1 and Task 2) or six selections (Task 3). For each task, we analyzed the ordering of alternatives in the final submitted portfolios and compared this with the ordering dictated by the heuristics, i.e. the order in which each heuristic (*Lowest Cost*, *Highest Value*, *Added Value* and *Unit Value*) would have selected alternatives. We did this for each respondent and counted the number of times we could assign a given heuristic to the ordering of alternatives in the final submitted portfolios.

We tracked the number of selected dominated alternatives (denoted n_{sd}) against the number of available dominated alternatives (denoted n_{ad} , i.e. the baseline) and denoted the proportion of selected dominated alternatives with respect to the baseline as $P_d = n_{sd}/n_{ad}$. This can be interpreted as the proportion of all available dominated alternatives where the selected alternative is dominated. We analyzed the trend of the proportion of selected dominated alternatives by fitting a regression model of P_d against an index of selection to check whether there was a trend. This was done to test Hypothesis H5.

3.4 Results

In Table 3.7 we show the proportion of participants choosing the optimal portfolio and the corresponding mean portfolio value for each task.

Task	Percent. of Resp.	Mean Value
Task1	70%	89%
Task2	22%	77%
Task3	7%	69%

Table 3.7: Proportion of participants choosing the optimal portfolio and mean portfolio value given by task.

Participants were much more likely to select the optimal portfolio in Task 1 than in Task 2 (70% vs 22%, $z = 5.84$, $p < 0.001$, see Figure 3.1). The mean portfolio value was also significantly higher in Task 1 than Task 2 (89% vs 77%, $t = 2.523$, $DoF = 145$, $p = 0.013$, Table 3.7). Together these results provide strong support for Hypothesis 1a. Also, participants were much more likely to select the optimal portfolio in Task 2 than in Task 3 (22% vs 7%, $z = 2.566$, $p = 0.010$, see Figure 3.1). The mean portfolio value was also significantly higher in Task 2 than Task 3 (77% vs 69%, $t = 2.137$, $DoF = 143$, $p = 0.034$, Table 3.7). Together these results provide support for Hypothesis 1b. We also observed that, despite selecting the optimal portfolio less often, most participants in Task 2 still selected two out of the three alternatives making up the optimal portfolio (Figure 3.1).

Overall, we can conclude that when selection tasks become more complex with the introduction of interactions and a high number of alternatives, the quality of selections deteriorates, as fewer optimal solutions are selected for the most part and the overall mean portfolio return decreases.

We tested for a significant difference between the mean number of selections and deselections in Task 2 vs Task 1 to verify Hypothesis H2a. We used the mean number of selections and deselections as the measure of effort in each task as shown in Table 3.8. The mean number of selections and deselections was slightly higher (but not significantly higher) in Task 2 than in Task 1 (8 vs 6, $t = 1.461$, $DoF = 145$, $p = 0.073$, Table 3.8). Given this, we found only weak evidence that the amount of effort exerted in Task 2 was higher than the amount of effort exerted in Task 1. We also tested for significant difference between mean number of selections and deselections in Task 3 vs Task 2 to verify Hypothesis H2b. The mean number of selections and deselections was slightly higher (but not significantly higher) in Task 3 than in Task 2 (10 vs 8, $t = 1.428$, $DoF = 144$, $p = 0.078$, Table 3.8). Given this, we found only weak evidence that the amount of effort exerted in Task 3 was higher than the amount of effort exerted in Task 2.

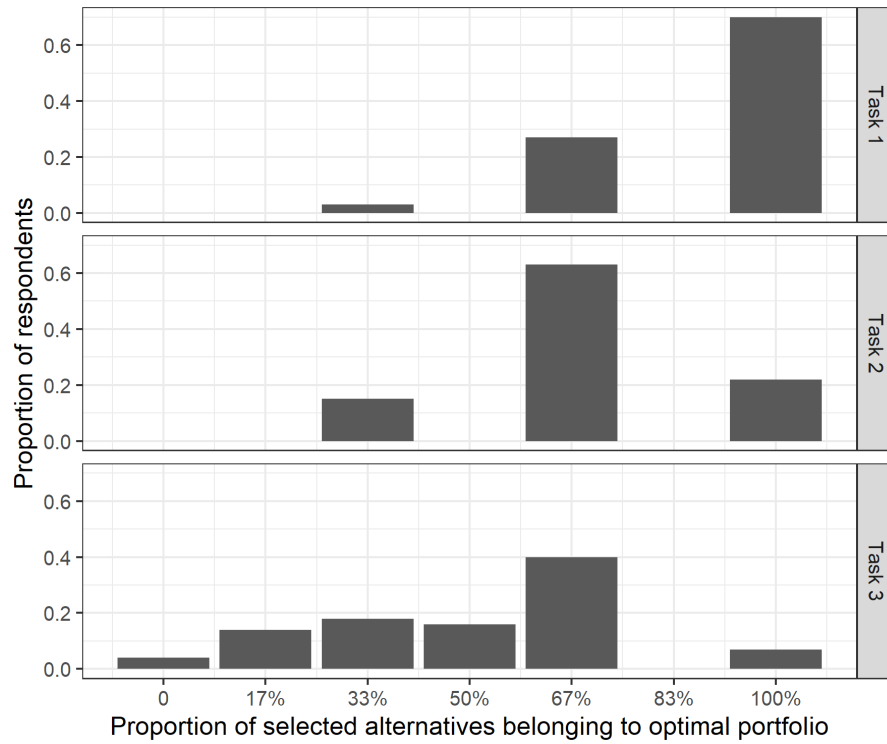


Figure 3.1: The proportion of participants (Y-axis) who selected portfolios of a given degree of optimality (X-axis) in all 3 tasks. The degree of optimality of a portfolio is given as the proportion of the alternatives (making up the portfolio), which belong to the optimal portfolio. A portfolio with two thirds of all (selected) alternatives belonging to the optimal portfolio is 67% optimum. For example, 70%, 22% and 7% of participants selected optimal portfolios (100% optimum) in tasks 1, 2 and 3 respectively.

Judging by the mean number of selections and deselections, it seems that participants applied more effort in Task 3, followed by Task 2 and Task 1 respectively. However, there was only weak evidence that the differences between tasks were significant. For this reason, we found weak support for hypotheses H2a and H2b.

	Task 1	Task 2	Task 3
Number of SD	428	586	731
Number of Participants	73	74	72
mean SD	6	8	10

Table 3.8: The table shows a measure of effort exerted by participants in doing the tasks. The metric used is the mean number of selections and deselections by task as recorded across the sample of participants. In Task 1, for example, participants make 6 selections and deselections on average.

We compared the observed proportions of heuristics to those measured under a random selection of alternatives. In Task 1, the proportions that were consistent with the *Highest Value* and *Unit Value* heuristics were unlikely to arise from a random selection strategy ($p = 1/2000$ and $p < 1/2000$ respectively, see Figure 3.2). In Task 2, the proportions that were consistent with the *Unit Value*, *Highest Value* and *Added Value* were unlikely to arise from a random selection strategy ($p = 3/2000$, $p = 113/2000$ and $p = 10/2000$ respectively, see Figure 3.2). In Task 3, the proportions that were consistent with the *Unit Value*, *Highest Value* and *Added Value* were unlikely to arise from a random selection strategy ($p < 1/2000$, $p < 1/2000$ and $p < 1/2000$ respectively, see Figure 3.2). The proportion of “*Other*” heuristics, i.e. selections that could not be explained by any of the heuristics, was much lower than the proportion that would be expected if selections were made randomly ($p < 1/2000$, see Figure 3.2). These findings confirm Hypothesis H3, i.e. the observed proportions of heuristics in all three tasks are different from those that would arise from a random selection of alternatives.

Having established that the observed proportions of heuristics in all three tasks are different from those that would arise from a random selection of alternatives, we investigated whether selections were consistent with the heuristics being studied in this research work. We looked at this from three angles in the below paragraphs: weighted proportions in the first two selections, the proportion of participants using a single heuristic, and heuristics consistent with the most popular portfolios (i.e. those selected by the most participants).

- (a) The weighted proportions of the first two selections consistent with each heuristic are shown in Table 3.9. In both Task 1 and Task 2, alternatives 5 and 1 were the most selected in the first selection. Alternative 5 was chosen by *Highest Value* and *Unit Value* heuristics, whereas alternative 1 was chosen by *Lowest Cost*. In

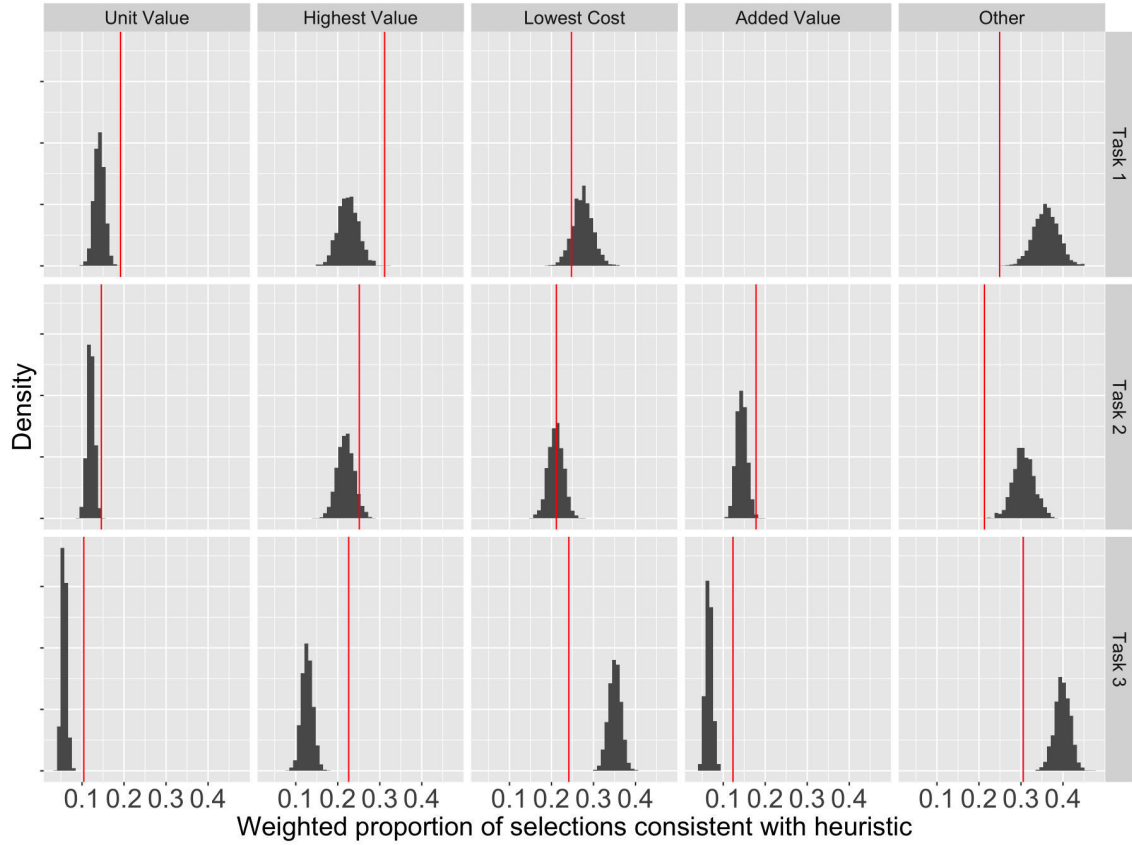


Figure 3.2: The proportion of selections of alternatives that were consistent with each heuristic (red vertical lines). As at any stage in the process different heuristics can select the same alternative, these proportions are of limited value on their own. We therefore compare each one against a distribution of proportions generated by a random selection heuristic (grey histograms; see text for details). In cases where the same alternative is selected by different heuristics, that selection “vote” is distributed evenly between those heuristics, and hence the proportion is a weighted one.

Task 3, alternatives 3 and 4 were the first most-selected alternatives. Alternative 3 was selected by *Lowest Cost*, *Added Value* and *Unit Value* whereas alternative 4 was selected by *Highest Value*. Regarding second selections, alternative 3 was the most selected in both Task 1 and Task 2 regardless of which alternative was already selected. In Task 1, alternative 3 was selected by *Highest Value* and *Unit Value*, whereas in Task 2 it was selected by *Added Value*. In Task 3, alternative 4 was selected by *Highest Value* if alternative 3 had already been selected. On the other hand, alternative 3 was selected by *Lowest Cost*, *Added Value* and *Unit Value* if alternative 4 had already been selected. Beyond the second selection, the distribution of selected alternatives became more even as it was dependent on which other alternatives had already been selected at this point.

Task 1	LC	HV	AV	UV	Other	Most selected
1st select	34%	20%	0%	20%	26%	Alt 1 & Alt 5
2nd select	21%	28%	7%	7%	38%	Alt 3
Task 2	MC	HV	AV	UV	Other	Most selected
1st select	26%	16%	16%	16%	27%	Alt 1 & Alt 5
2nd select	15%	34%	10%	6%	36%	Alt 3
Task 3	MC	HV	AV	UV	Other	Most selected
1st select	18%	21%	10%	10%	42%	Alt 3 & Alt 4
2nd select	25%	30%	7%	6%	32%	Alt 3 & Alt 4

Table 3.9: Proportion of selections (of alternatives) that were consistent with each heuristic for each task. This is given with the most selected alternatives in each task.

- (b) Table 3.10 shows the percentage of participants using a single heuristic in each of the first three (Task 1 and Task 2) or six (Task 3) selections. This strategy would have paid off if participants had applied the right heuristic consistently, they would have reached the optimal solution within the first three or six selections. For example, if participants had used *Unit Value* consistently in Task 1, they would have selected one of the optimal portfolios, namely alternatives $\{1, 3, 5\}$. On the other hand, if participants had applied *Lowest Cost* consistently in Task 2, they would have selected the optimal portfolio $\{1, 2, 3\}$. Similarly, if participants had used *Lowest Cost* consistently in Task 3, they would have ended up with the optimal portfolio, namely alternatives $\{10, 7, 5, 3, 1, 8\}$. However, we observed that a small proportion of participants used a single heuristic throughout tasks 1 and 3 (10.9% and 1.4% of participants in Task 1 and Task 3, respectively) but a third of participants did so in Task 2 (31.2% of participants in Task 2). So the strategy was not widespread, especially in Tasks 1 and 3.

Judging by the percentage of respondents who selected an optimal portfolio by consistently using a single heuristic across tasks, we think that this was not a well-thought-out approach in general, and it seems to be a response to the complexity in the selection tasks. As we have seen, most of the respondents using a single heuristic in both Task 1 and Task 2 (60% in Task 1 and 92% in Task 2) stuck to heuristics that led them to non-optimal portfolios. However, in Task 3 it was a very small number of respondents (1.4% of respondents) who consistently used a single heuristic, and all those respondents stuck to the heuristic (*Lowest Cost*) leading to the optimal solution. It seems that those who stuck to a single heuristic in Task 3 had thought out the problem well. The proportion of respondents using a single heuristic as a function of the degree of complexity of tasks can be depicted as an inverted U-shape function: as task complexity increases, usage of a single heuristic increases until it reaches a certain point beyond which respondents are deterred and begin to reduce single heuristic usage. It might be that they are deterred from using a single heuristic as this strategy becomes more difficult to implement when the level of complexity increases, with a high number of alternatives coupled with interactions among alternatives (Task 3). It will require further investigation to understand single heuristic patterns.

Task	Percent
Task 1	10.9%
Task 2	31.2%
Task 3	1.4%

Table 3.10: The proportion of participants who use a single heuristic for the first three selections (Task 1 and Task 2) or six selections (Task 3).

- (c) In both Task 1 and Task 2, the most popular portfolio was made of alternatives $\{1, 3, 5\}$ (Table 3.11). In Task 1, the most frequent ordering 1-3-5 and the second most frequent ordering 5-3-1 were consistent with a combination of the *Lowest Cost*, *Highest Value* and *Unit Value* heuristics as shown in Table 3.12, although the ordering 1-3-5 also involved unknown heuristics. In Task 2, the most frequent ordering 5-3-1 was added in the same order as the *Highest Value* heuristic as shown in Table 3.11, although it could also be constructed by combining the *Lowest Cost*, *Highest Value*, *Added Value* and *Unit Value* heuristics. Most participants that selected alternative 5 as their first selection stuck to it throughout the experiment, trying to build combinations from this initial choice. This led a significant proportion of participants (i.e. over 60%) to select suboptimal solutions in their final submissions (see Figure 3.1). This indicates a path dependency, i.e. the state

of the solution depends on the path taken to reach it (Hämäläinen and Lahtinen, 2016). In this case, the initial selection of alternative 5 dictated the final selections participants ended up with. As Task 3 had almost the same number of participants as the other tasks but more alternatives to choose from, it had a large number of possible combinations of alternatives. For this reason there was little likelihood of getting different orderings of the same portfolio in the selections. The most frequent ordering 3-4-7-8-5 is consistent with the *Unit Value* except for alternative 4 (Table 3.12).

Selection order for alternatives appearing in the most frequently chosen portfolios

Task 1: no interactions			Task 2: with interactions			Task 3: with interactions	
Portfolio	Order R-1	Order R-2	Portfolio	Order R-1	Order R-2	Portfolio	Order R
{1,3,5}	1-3-5 (11)	5-3-1 (10)	{1,3,5}	5-3-1 (13)	5-1-3 (10)	{3,4,5,7,8}	3-4-7-8-5 (4)
{2,3,5}	5-3-2 (7)	5-2-3 (3)	{1,2,3}	3-1-2 (5)	1-2-3 (4)	{4,6,8}	4-6-8 (3)
{1,4,5}	5-4-1 (2)	4-5-1 (2)	{2,3,5}	5-3-2 (5)	3-5-2 (1)	{1,3,5,7,8,10}	1-3-5-7-8-10 (2)
{1,2,4}	1-2-4 (3)	2-1-4 (1)	{3,4}	4-3 (3)	3-4 (1)	{3,4,8,9}	3-4-8-9 (2)
{1,2,5}	5-2-1 (1)	1-5-2 (1)	{1,2,5}	2-5-1 (1)	1-2-5 (1)	{3,4,7,8,10}	3-4-7-8-10 (1)

Table 3.11: For each portfolio we show the order in which the alternatives making up the portfolio were added. We show the two most popular orderings, which in most cases account for the majority of participants. The number of participants using each sequence is shown in parentheses

Task	Alternatives Selected	Returns	Cost	LC	HV	AV	UV	Other
Task 1	{1}	1	1	1				
	{1, 3}	3	4					1
	{1, 3, 5}	8	6	1	1	1	1	
Task 1	{5}	5	2		1	1	1	
	{5, 3}	7	5		1			
	{5,3,1}	8	6	1		1		
Task 2	{5}	5	2		1	1	1	
	{5,3}	7	5		1			
	{5,3,1}	11	6	1	1	1		
Task 3	{3}	120	40	1		1	1	
	{3,4}	510	130		1			
	{3,4,7}	775	170	1			1	
	{3,4,7,8}	1325	230		1	1	1	
	{3,4,7,8,5}	1700	270	1			1	

Table 3.12: Most popular sequence of selections (Task 1), i.e. the sequence that had the highest frequency of occurrence. This table shows how the most popular sequence was constructed. At each stage of the selection process, we display the overall earnings and costs and heuristics that were applied.

Based on the evidence presented in points (a), (b) and (c) above, we concluded that selections were consistent with the heuristics being studied in this research work (Hypothesis H4).

Finally, we tracked the proportion of selected dominated alternatives with respect to the baseline (the number of available dominated alternatives), denoted P_d . Figure 3.3 shows the relationship between P_d and selection order, with regression results shown in Table 3.13. Beyond the first few selections, we observed that the number of selected dominated alternatives became small. At selection 9 (with respect to the order of selection), for example, the number of selected dominated alternatives was zero in Task 1. Small numbers introduce noise in P_d due to fluctuations in small numbers and such fluctuations should not be overstated. For this reason, we limited the number of selections to 9 for this exercise.

Model results show that the coefficient in Task 1 is both negative and significant, giving a clear indication of a decline in the proportion of selected dominated alternatives. No significant relationship was found for both Task 2 and Task 3. The results confirm Hypothesis H5 in Task 1: people are less likely to choose a dominated alternative later on in their selections. However, Hypothesis H5 is not confirmed in both Task 2 and Task 3.

Task	Variables	Coefficients	St. Error	t Stat	P-value	Model Fit	
Task 1	Intercept	0.37	0.07	5.33	0.00	R Squared	0.63
	Time	-0.04	0.01	-3.49	0.01	Observations	9
Task 2	Intercept	0.28	0.10	2.76	0.03	R Squared	0.27
	Time	-0.03	0.02	-1.62	0.15	Observations	9
Task 3	Intercept	0.30	0.12	2.49	0.04	R Squared	0.30
	Time	-0.04	0.02	-1.73	0.13	Observations	9

Table 3.13: Model results for a fitted linear regression of the proportion of dominated alternatives selected (relative to the number of dominated alternatives available for selection) on selection order.

3.5 Discussion

We analyzed the quality of selections in the portfolio experiment. We observed that as selection tasks became more complex, the quality of selections deteriorates. Hence, 70% of participants selected optimal solutions (100 % optimum) in Task 1 against 22% in Task 2 against 7% in Task 3. This was also corroborated by the overall mean return value by task (Task 1: 89%; Task 2: 77%; Task 3: 69%). Therefore, we can conclude the quality of selections deteriorates with task complexity. We also found that the amount of effort made to perform a portfolio selection task, as measured by the mean number of selections and deselections, increased with task complexity, although the support for this finding was slightly weak. Finally, we investigated whether participants chose fewer dominated

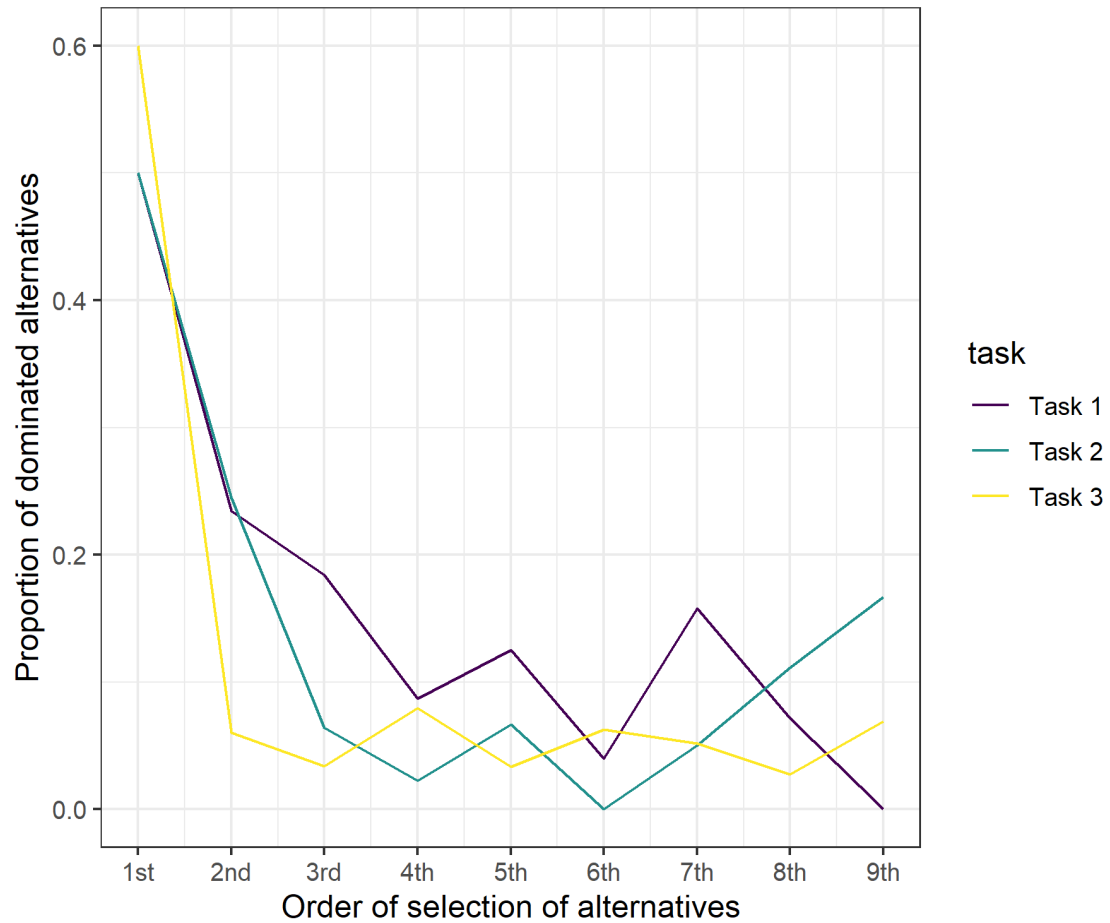


Figure 3.3: Trend analysis for all three tasks. The figure shows the ratio of the number of selected dominated alternatives relative to the number of all available selected dominated alternatives. A decreasing ratio means that fewer dominated alternatives are selected over time (time is equal to the order of selections) and this is an indication of improving decision making.

alternatives and were able to make better choices as they went along. We found evidence of a declining number of dominated alternatives in Task 1 but, in the more complex tasks, we did not observe a significant decline. The results pointed to better choices being made in the simple task over time. We believe that the number of dominated alternatives can be of limited use in the more complex tasks where there are interactions between alternatives because the addition of a dominated alternative to a portfolio can have a beneficial impact if it carries positive interactions.

Our results show that participants selected portfolios in a way that was not consistent with a random selection of alternatives. Overall, we observed a widespread use of heuristics across the sample of participants and that individuals used a variety of heuristics. More specifically, we saw that most participants used *Highest Value*, *Lowest Cost* and *Unit Value*, heuristics that ignore interactions, in the first few sequences and most popular sequences. In addition, participants added sequentially alternatives to their portfolios in an ordering that was mostly consistent with the *Highest Value* and *Unit Value* heuristics. Also, the most popular selection of alternatives were consistent with the *Unit Value* or *Highest Value* heuristics.

3.6 Conclusion

We set out to understand how participants use heuristics in a portfolio selection experiment without facilitation. Participants were presented with 3 tasks in varying degrees of complexity, involving either 5 (Task 1 and 2) or 10 (Task 3) alternatives, and without (Task 1) or with (Task 2 and 3) interactions between alternatives.

Research has shown that task complexity impacts decision quality in choice problems (Kormos and Trebits, 2012; Robinson, 2001; Payne et al., 1993), i.e. poor choices are made in more complex problems. Our results show that this is also true in portfolio decision problems: we found that the quality of selections deteriorates in the more complex tasks involving more alternatives and interaction effects.

Previous studies have shown that in choice problems participants exert more effort when tasks become more difficult (Johnson and Payne, 1985). We found no conclusive evidence that this is the case in portfolio problems. This is an area for future research.

As we have seen, research has established that in traditional choice problems people tend to use simple heuristics as decisions become more complex (Payne et al., 1993). Our results have established that the same finding applies in a portfolio decision context, and

have provided evidence that the selection of portfolios was consistent with the heuristics studied in this research work.

Chapter 4

A simulation-based assessment of simplified portfolio heuristics

In the previous chapter we presented evidence for the use of Add-the-best heuristics developed in Durbach et al. (2020) in unfacilitated portfolio selections. In this chapter, we turn our attention to the more prescriptive question of whether heuristics for portfolio decisions can return good decisions – in the sense of returning portfolios that are in some sense close to what might have been achieved using an exact method, if sufficient time and resources were available to do so.

Durbach et al. (2020) presented a number of heuristics for portfolio decision analysis (See Chapter 2) and compared their performance with that of so-called optimal portfolios constructed by optimizing a linear additive portfolio value function. They found that two heuristics, *Added Value* and *Unit Value with Synergy*, were competitive with theoretically optimal models under a fairly wide range of environmental conditions, and identified environmental conditions under which these heuristics performed best (i.e. when interactions between projects possessed a “nested” rather than random structure, and occurred between projects that were individually good). Their study, however, considered only single-criterion decision making and restricted all interactions between projects to be positive, i.e. beneficial. The purpose of this chapter is to relax these two restrictions, and extend the simulation framework used by Durbach et al. (2020) to cover situations involving multiple criteria, and both positive and negative interactions. We use this extended simulation framework to reevaluate the performance of portfolio selection heuristics in Durbach et al. (2020) under these expanded set of conditions, and in particular to assess whether the *Added Value* and *Unit Value with Synergy* heuristics continue to perform well in more challenging environments involving multiple objectives and both negative and positive project interactions. In considering a multi-criteria setting we need to consider not only heuristics that guide the selection of alternatives (which we call “selection heuristics”) but also heuristics that aggregate performance across criteria

(which we call “scoring heuristics”). The latter are available from research into traditional one-out-of-n choice problems (See Chapter 2).

The chapter is organized as follows: Section 4.1 describes the heuristics that are used in the experiment; Section 4.2 details how the experiment was designed and conducted; Section 4.3 presents and discusses the performance of heuristics under both positive and negative interactions; Section 4.4 presents and discusses the performance of heuristics under multiple objectives; a final section concludes with a summary of insights gained and suggestions for future research.

4.1 Heuristics

Durbach et al. (2020) proposed a family of heuristics, collectively called add-the-best, which we briefly describe here (See Chapter 2 for details). Add-the-best heuristics construct portfolios sequentially by considering, at each stage, alternatives that have not been added yet. The authors divided heuristics into two broad categories – those that use interaction information and those that do not.

Heuristics that do not use interaction information include:

Highest value (HV) adds alternatives in descending order of their values.

Lowest cost (LC) adds alternatives in ascending order of their costs.

Unit value (UV) adds alternatives in descending order of their value-to-cost ratios. Values are based on marginal alternative values only.

Heuristics that use interaction information include:

Unit Value with Synergy adds alternatives from the set of alternatives that are involved in at least one positive interaction based on the *Unit Value* heuristic.

Added Value adds the alternative whose inclusion would lead to the largest increase in overall portfolio value per unit cost. Variants of the *Added Value* heuristic consider only interaction subsets involving the alternative that contributes the *most* to portfolio value per unit cost (*Added Value Most*) or consider only the alternative that contributes the *least* to portfolio value per unit cost (*Added Value Least*) or consider only the interaction subsets involving an alternative chosen at random (*Added Value Random*).

We also look at a set of heuristics for aggregating the values of alternatives across criteria (Payne et al., 1993). We call the proposed group of heuristics *scoring heuristics*. The following scoring heuristics will be used in this simulation study:

Equal Weighting Function (EQW): The aggregate score of an alternative is given as the mean of individual scores on all criteria.

Lexicographic (LEX): The aggregate score of an alternative is represented by its value on the most important criterion.

Tallying (TAL): The aggregate score of an alternative is given as the count of good features of the alternative.

The normative scoring approach is the weighted additive method (WADD). We generated weights from a uniform distribution with lower and upper bounds given as zero and one respectively. The WADD score is ideal because it accounts for all available information. Therefore, it serves as a benchmark against which the scores from the scoring heuristics can be judged.

4.2 Simulation study design

4.2.1 Generating individual values and costs

We summarize the general structure of the simulation in Figure 4.1. The problem context is defined by the number of alternatives n , the number of criteria m , the individual values x_{ij} of alternative j measured on criterion i and costs c_j associated with each alternative j , and total budget R . We simulated problems involving $n = 50$ alternatives. Alternative values were taken from three different data distributions: uniform, positively skewed and negatively skewed, and each with six criteria. This gives a set of three problems in total. The experimental design was meant to help us to consider various conditions under which portfolio selections could occur and, in this way, mimic real world conditions. We opted for six criteria in order to describe a middle size problem.

We run two main experiments, one adds negative interactions to the simulation in Durbach et al. (2020), but remains in the single-objective context; the other uses multiple criteria with both positive and negative interactions. In each experiment, we generated 3000 datasets for each of three problems discussed above.

In the experiment to test both positive and negative interactions, the set-up is as follows: In the uniform case, we drew the values from the uniform distribution $U[0, 20]$.

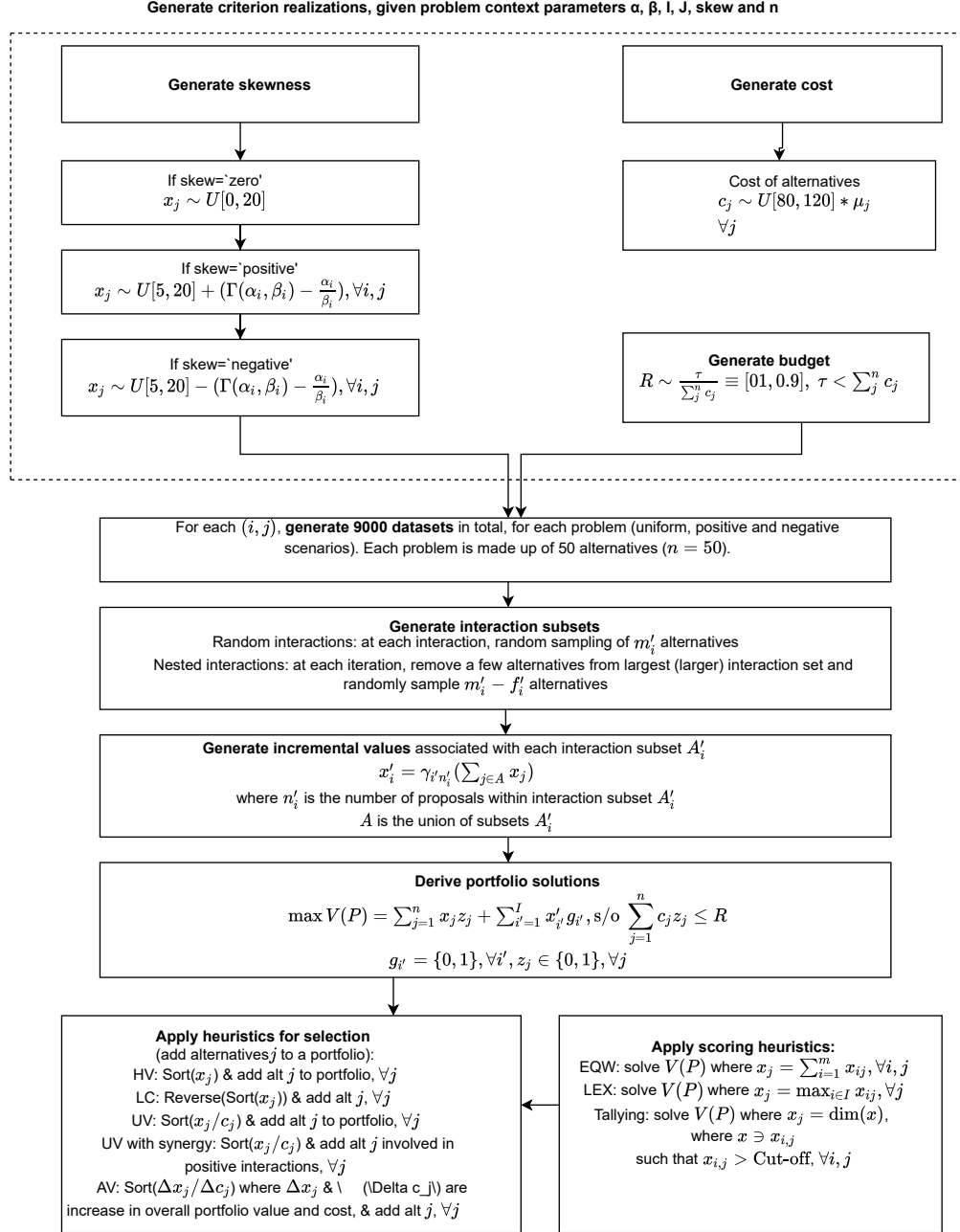


Figure 4.1: Outline of the simulation experiment

For the positively skewed data sets, we generated values from a gamma distribution with parameters $\alpha = 5$ and $\beta = 2$. For the negatively skewed condition, we used the same gamma distribution but transformed each value by subtracting it from the maximum value and adding the mean value.

In the multiple criteria experiment, the set-up is as follows: In the uniform case, we drew values of criteria from the uniform distribution $U[0, 20]$. For the positively skewed data sets, we generated values of criteria from a gamma distribution with parameters $\alpha_i = \{4 + i\}, i = \{1, \dots, 6\}$ and $\beta_i = \{1 + i\}, i = \{1, \dots, 6\}$. Negatively skewed values are generated from the same gamma distribution but each value is transformed by subtracting it from the maximum value and adding the mean value.

The cost of alternatives in both the single-criterion and multi-criteria cases were generated by sampling uniformly between 80 and 120 and multiplying by the alternative value. Generating values and costs in this way means that value per unit costs are, on average, uncorrelated with value and weakly negatively correlated with cost (uniform: -0.2; skewed: -0.1). We varied the available budget by choosing the proportion ζ to lie between 0.1 and 0.9 in increments of 0.1.

4.2.2 Running portfolio selection models

We first used heuristics for portfolio selection to construct portfolios. We implemented all nine variants of *add-the-best* heuristics: *Added Value*, added value with the most valuable alternative (*Added Value Most*), added value with the least valuable alternative (*Added Value Least*), added value with a random alternative (*Added Value Random*), *Unit Value*, *Unit Value with Synergy*, *Highest Value*, *Lowest Cost* and *Pareto*. We refer to the portfolios constructed using these heuristics as heuristic portfolios.

We also identified the optimal portfolio by solving the linear program given in Equation 4.1 in Section 4.2.5. The value of the optimal portfolio and heuristic portfolios will be the sum of scores of the alternatives making up these portfolios. The performance of the heuristics can be judged by comparing the values of heuristic portfolios with the value of the optimal portfolio.

4.2.3 Creating interactions between alternatives

In the following section, we describe two ways of constructing subsets of interacting alternatives, which we term *random* and *nested*. Informally, random interactions imply that there is no structure linking lower- and higher-order interaction subsets, beyond the fact that these are all drawn from a common pool of alternatives. For example, two-way

interaction subsets are generated independently of three-way interaction subsets: the set of alternatives involved in two-way interactions may be entirely different from the set involved in three-way interactions. With nested interactions, a low-order interaction subset (one containing relatively few alternatives) is generated by sampling the required number of alternatives from one of the already-generated higher-order interaction subsets.

Defining this formally requires some additional notation. Suppose we have a set $\mathcal{J}$ made up of J alternatives, e.g. for $n = 12$, we have alternatives P_1 to P_{12} . Let $\mathcal{J}'$ be the subset of $\mathcal{J}$ containing those n' alternatives from which interdependencies will be drawn. This set is indexed by $j \in \mathcal{J}'$ such that $j = \{1, 2, \dots, n'\}$.

Interaction subsets A_k containing interactions are indexed by k where $k = \{1, \dots, K\}$ and $\mathcal{K}$ is the set of all interaction subsets A_k .

Suppose we have four interaction subsets ($K = 4$) such that $A_1 = \{P_1, P_3, P_4\}$, $A_2 = \{P_3, P_4, P_6\}$, $A_3 = \{P_4, P_7\}$ and $A_4 = \{P_1, P_9\}$ with subscript $k = \{1, 2, 3, 4\}$. This means $\mathcal{K} = \{A_1, A_2, A_3, A_4\}$; $\mathcal{J}' = \{P_1, P_3, P_4, P_6, P_7, P_9\}$ and the size of $\mathcal{J}'$ is 6, i.e. $|\mathcal{J}'| = 6$.

Further, let $\{\mathcal{K}_{n''}\}_{n''=1}^{N''}$ be a partition of $\mathcal{K}$ according to the number of alternatives making up the interactions, with $\mathcal{K}_{n''} \subseteq \mathcal{K}$ denoting all interactions involving exactly n'' alternatives such that $n'' \in \mathcal{N} = \{1, \dots, N''\}$. Many of these sets may be empty including, by definition, $\mathcal{K}_1$ (since an interaction requires at least two alternatives). In the interaction above, we have $\{\mathcal{K}_{n''}\}_{n''=1}^3$ given as $\mathcal{K}_1 = \emptyset$, $\mathcal{K}_2 = \{A_3, A_4\}$ and $\mathcal{K}_3 = \{A_1, A_2\}$. The size of each $\{\mathcal{K}_{n''}\}$ is given by $\kappa_{n''} = \{|\mathcal{K}_{n''}|\}$ such that $\kappa_1 = \{|\mathcal{K}_1|\} = 0$; $\kappa_2 = \{|\mathcal{K}_2|\} = 2$; $\kappa_3 = \{|\mathcal{K}_3|\} = 2$.

Interactions between alternatives in $\mathcal{J}'$ are simulated by specifying (a) the size of the largest interaction subset N'' , and (b) $\kappa_{n''}$, the number of interactions of order n'' (n'' interacting alternatives) that should be generated, for each $n'' \in \mathcal{N}$. In the above example, N'' and $\kappa_{n''}$ are specified as follows: $n'' = 3$ and $\kappa = (0, 2, 2)$. This results in two two-way and two three-way interactions between a subset of six alternatives randomly selected from the full set ($|\mathcal{J}'| = 6$). We describe below the procedure for creating random and nested interactions:

Random interactions Each subset $A_k, k \in \mathcal{K}_{n''}$ is obtained by randomly sampling n'' alternatives from $\mathcal{J}'$.

Nested interactions We begin by constructing the largest desired interaction by randomly sampling $N_0'' = N''$ alternatives, repeating this as many times as desired, i.e. $\kappa_{N_0''}$ times. This gives the set of largest interactions $\mathcal{K}_{N_0''}$. We then construct the next-largest interactions, each comprising $N_1'' < N_0''$ alternatives, by randomly sampling one of the largest interactions $\mathcal{K}_{N_0''}$, and randomly removing $N_0'' - N_1''$ alternatives. We repeat this procedure – randomly sampling one larger interaction and randomly removing $N_0'' - N_1''$ of its alternatives – until we have constructed the required number $\kappa_{N_1''}$ of distinct interaction subsets. This procedure can be repeated until all interactions have been generated. At each step s , we compute the largest remaining interaction size N_s'' , and then randomly sample one element from $\mathcal{K}_{N_{s-1}''}$ and randomly remove $N_{s-1}'' - N_s''$ of its alternatives, continuing this until we have $\kappa_{N_s''}$ distinct interactions of order N_s'' .

Alternatives are selected with selection probabilities (a) equal across alternatives, (b) directly proportional to their value-to-cost ratio x_j/c_j , where $x_j = \sum_i w_i x_{ij}$ is the overall value of alternative j using WADD and c_j is the cost of alternative j , in which case alternatives that are individually better are more likely to be involved in positive interactions, or (c) inversely proportional to x_j/c_j , in which case worse alternatives are more likely to be involved in interactions. Conditions (b) and (c) are expected to help and hinder heuristics, respectively.

Our simulations generate interactions between a subset of $n' = 10$ alternatives. We allow for both positive and negative interactions, which serve to either increase or decrease portfolio value. We vary, as a parameter of the simulations, how the interacting alternatives (i.e. those included in $\mathcal{J}$) are selected: with probability proportional to their overall value-cost ratios x_j/c_j , in which case interactions involve relatively good alternatives; or inversely proportional to x_j/c_j , in which case relatively poor alternatives tend to be selected; or with equal selection probabilities. We generate interactions up to order 5, i.e. $N' = 5$, with $\kappa = (0, 10, 8, 6, 2)$. Interactions are generated either randomly or in nested fashion, as a parameter of the simulation.

4.2.4 Computing values and costs of interactions

Let x_j be the overall benefit value of alternative j obtained with WADD and c_j be the overall cost of alternative j .

We set the incremental benefit of completing interaction subset A_k to be a proportion γ of the sum of the values of alternatives in A_k , i.e. $x'_k = \gamma \sum_{j \in A_k} x_j$, with $\gamma \in \{0, |0.5|, |1|\}$, a parameter of the simulation. Positive or negative interactions mean that incremental

benefit x'_k are either added or subtracted, respectively, from the sum of initial overall benefit x_j . Higher-value alternatives thus resulted in interactions with higher absolute values, although as these alternatives also tend to cost more, lower-value alternatives may still be preferred per unit cost. We chose values of γ so that interactions contribute a substantial proportion of the overall value of the optimal portfolio (i.e. half or equal the overall value of the optimal portfolio), on a trial-and-error basis. Our motivation here was to avoid making overly favourable claims for those heuristics that ignore interactions between alternatives.

4.2.5 Computing values of portfolios

The optimal portfolio will maximize the overall value of benefits and incremental benefits due to interactions between alternatives, given a set of constraints. This is solved as a linear programming problem. If we apply budget constraints, the problem can be formulated as in Stummer and Heidenberger (2003):

$$\begin{aligned}
\text{maximize} \quad & V(P) = \sum_{j=1}^n x_j z_j + \sum_{k=1}^K x'_k g_k \\
\text{subject to} \quad & \sum_{j=1}^n c_j z_j \leq R \\
& g_k = \{0, 1\}, \quad \forall k
\end{aligned} \tag{4.1}$$

where $z_j \in \{0, 1\}$ is a variable such that $z_j = 1$ if alternative j is included in portfolio P ($z_j = 0$ otherwise) and $g_k \in \{0, 1\}$ such that $g_k = 1$ if all alternatives in interaction subset A_k are included in portfolio P ($g_k = 0$ otherwise); c_j is the cost of alternative j and R denotes the available budget.

If there are no interactions between alternatives x'_k is equal to zero.

4.2.6 Evaluating heuristics for portfolio decisions

Because heuristics have been used at two levels - first for aggregating values across criteria and second for constructing portfolios - we need to consider a combination of these two levels in order to comprehensively evaluate how heuristics perform under various conditions: no interactions, random vs nested interactions and varying budget levels. We first evaluated the performance of scoring heuristics and then evaluated the performance of selection heuristics both described in Chapter 2.

In order to assess the performance of scoring heuristics, we first assessed the overall value of each alternative using WADD as well as all scoring heuristics (EQW, LEX and TAL). We then applied the optimal portfolio selection approach only to select which alternatives go into the portfolio. We thus construct all portfolios using by solving the linear programming problem in equation (4.1), but use different approaches (WADD, heuristics) to compute the inputs to this problem. We denote the optimal portfolios generated in this way as WADD + optimal, EQW + optimal, LEX + optimal and TAL + optimal. WADD + optimal is the ideal portfolio and we used it as the benchmark for assessing the remaining optimal portfolios. The values of optimal portfolios will be the sum of the overall values of alternatives making up the portfolios plus interaction values.

In order to assess the performance of selection heuristics, we fixed WADD as the scoring approach for all alternatives. We then generated portfolios using selection heuristics and the optimal portfolio selection approach. Just as before, the values of portfolios are the sum of the overall scores of the alternatives making up each portfolio. We used the value of optimal portfolios (WADD + optimal) as the benchmark for assessing the performance of heuristic portfolios.

4.3 Heuristic performance under positive and negative interactions

In this study, we first ran portfolio selection simulation involving both positive and negative interactions between alternatives for single-criterion problems. The performance of heuristics is assessed by comparing the mean value of portfolios selected with the heuristics and the mean value of optimal portfolios. We looked at the performance of heuristics under different budget levels. This is important as budget levels will impact the composition of feasible portfolios and ultimately determine which portfolio is optimal. Our main findings are that results obtained previously for positive interactions only (Durbach et al., 2020) largely hold true for negative interactions as well. This becomes clear in the comparison of Figures 4.2 and 4.3.

We first describe a number of findings that are similar to those in (Durbach et al., 2020), and extend these results into the domain of negative interactions. Overall, we found that *Added Value* was the best-performing heuristic. However, there was still a noticeable gap between its mean value and optimal mean values at lower levels of budget (i.e. the value of *Added Value* portfolios is between 73% and 90% of the value of optimal portfolios) but the gap narrows at higher levels (beyond 50% of total costs),

i.e. the value of *Added Value* portfolios is almost equal to the value of optimal portfolios. *Unit Value with Synergy* was the second-best performing heuristic and was almost as good as the optimal portfolio approach for budget values between 20% and 30% of total costs, i.e. the value of *Unit Value with Synergy* portfolios is almost equal to the value of optimal portfolios, but worsens slightly beyond this interval. This result shows that information about which alternatives are involved in interactions is critical and that the performance of a heuristic that accounts for this information is greatly improved even if the actual interaction values are not accounted for (Figure 4.2a & Figure 4.2b). Heuristics that ignored interactions (*Lowest Cost*, *Highest Value*, *Unit Value* and *Pareto*) are the worst-performing - even being equivalent to a random selection of alternatives - because they ignore key information about alternatives (Figure 4.2b).

Median performance also tended to increase as budgets increased from low to moderate values. This increase happened earlier for *Added Value* and *Unit Value with Synergy* heuristics, and latest for heuristics that do not consider interactions. The initial decrease in performance at very low budget levels is probably an artefact in that at extreme budget proportions (0 or 1) all approaches select the same portfolio (an empty portfolio or a portfolio of all alternatives). The performance of heuristics that do not account for interaction values was more stable than the performance of heuristics that account for interaction values. The large amount of uncertainty, particularly for the heuristics that include interaction information, demonstrate that interaction information can sometimes be very useful (can return in excellent portfolios) but can also be highly misleading (can return poor portfolios). The latter will presumably occur when certain of the interaction subsets lead the decision maker to the selection of underperforming portfolios (Figure 4.4).

We also noted that in the absence of interactions, heuristics had similar relative performance i.e. performance relative to optimal portfolios. The size of interaction was inversely related to the performance of heuristics. As interactions increased in value, heuristic performance worsened. Because heuristics are frugal, they don't use all available information and ignore some of the interactions between alternatives. When values of interactions are large, ignoring them resulted in bigger penalties. *Added Value* and *Unit Value with Synergy* were less sensitive to the size of interactions i.e. they performed rather similarly regardless of the size of interactions (Figure 4.5).

Two results were different to those reported for positive-only interaction environments. Firstly, negative interactions resulted in a bigger gap between *Added Value* and its variants (*Added Value Most*, *Added Value Least* and *Added Value Random*) than positive interactions did. This happened because the variants consider only a subset of all

interactions, involving a random alternative or the alternative that currently contributes the most or the least to portfolio value per unit cost. The selected alternative (most or least valuable or random alternative), in a number of instances, interacted negatively with other alternatives and this resulted in a loss of value vis-à-vis *Added Value* (Figure 4.2a).

Secondly, negative interactions resulted in a similar performance of *Unit Value with Synergy* and *Pareto* at higher levels of budget. This was due to the fact that *Unit Value with Synergy* worsened slightly at higher levels when negative interactions were at play. However, the gap was substantial between the two heuristics when there were only positive interactions (Figure 4.2b).

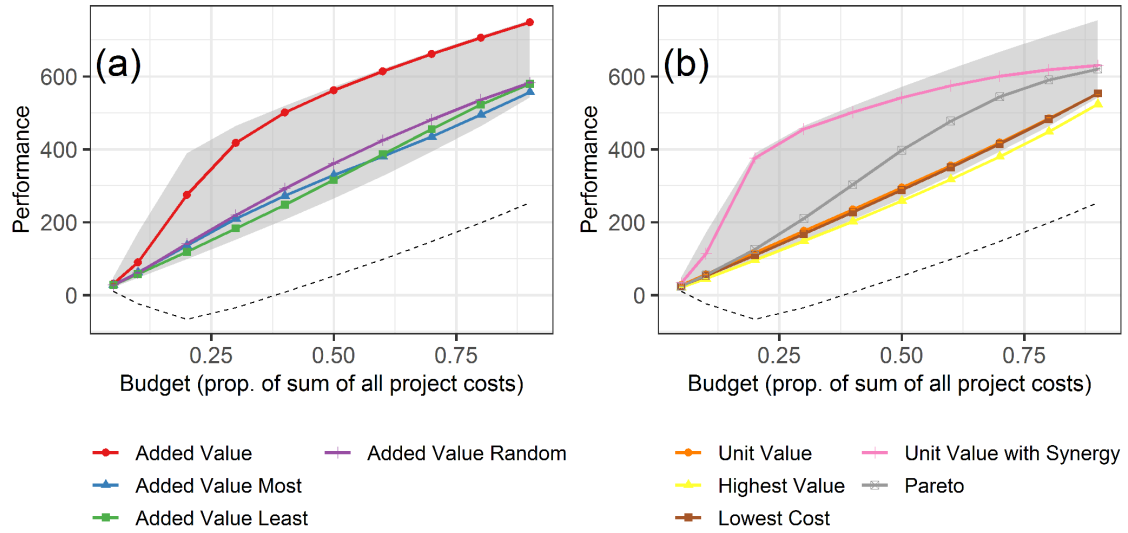


Figure 4.2: Mean values of portfolios selected by fast-and-frugal heuristics for portfolio decisions under different budget constraints for both positive and negative interactions. Panel (a) shows heuristics that consider quantitative alternative interactions; panel (b) shows heuristics that do not. Confidence intervals around these means are negligible (smaller than the symbols used to plot the means). The grey polygon plots the envelope between the value of the optimal portfolio (selected using the integer program in Section 4.2.5) and the mean value returned by a random selection of alternatives, which we consider to be a useful lower bound for benchmarking performance. The dashed line denotes the value of the nadir portfolio.

4.4 Heuristic performance in the presence of multiple criteria

Results reported in the previous section were found to hold in nearly identical fashion when multiple criteria were present, and as a result we show results graphically in the

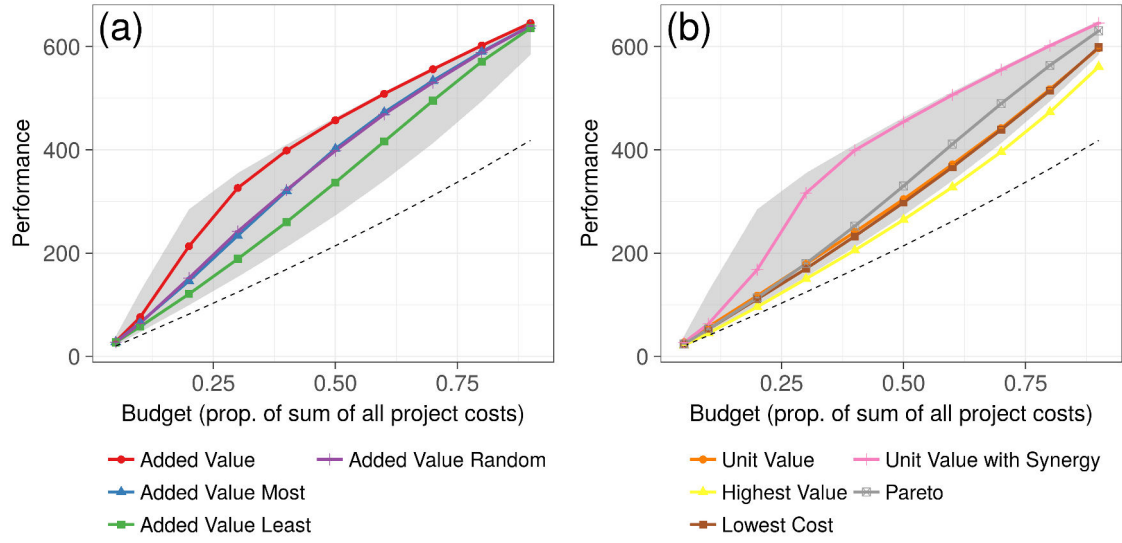


Figure 4.3: Mean values of portfolios selected by fast-and-frugal heuristics for portfolio decisions under different budget constraints for positive only. Panel (a) shows heuristics that consider quantitative alternative interactions; panel (b) shows heuristics that do not. Confidence intervals around these means are negligible (smaller than the symbols used to plot the means). The grey polygon plots the envelope between the value of the optimal portfolio (selected using the integer program in Section 4.2.5) and the mean value returned by a random selection of alternatives, which we consider to be a useful lower bound for benchmarking performance. The dashed line denotes the value of the nadir portfolio.

appendix Section B. We found that the addition of multiple criteria caused no change in the relative performance of heuristics used to select portfolios or on the effects of environmental variables such as budget and interaction type. We also found that the *Added Value* and *Unit Value with Synergy* heuristics continued to perform better than other heuristics, and often returned performance comparable with the optimal portfolio. Heuristic performance also worsened with larger interactions.

We also evaluated scoring heuristics i.e. heuristics that aggregate alternative values across criteria. We evaluated Equal Weighting (EQW), Lexicographic (LEX) and Tallying (TAL) scoring heuristics and compared them to WADD, the ideal scoring approach. Here are the main findings:

At lower budget levels, differences between mean values of portfolios selected by scoring heuristics and the mean value of those selected by WADD were rather small in absolute terms (Figure 4.6a), i.e differences were less than 5% of portfolio mean values but were larger in relative terms (Figure 4.6b). We found that overall heuristics ranked as follows: WADD > EQW > LEX > TAL (Figure 4.6b). EQW heuristic mean values were the closest to WADD mean values at higher levels of budget and consequently were

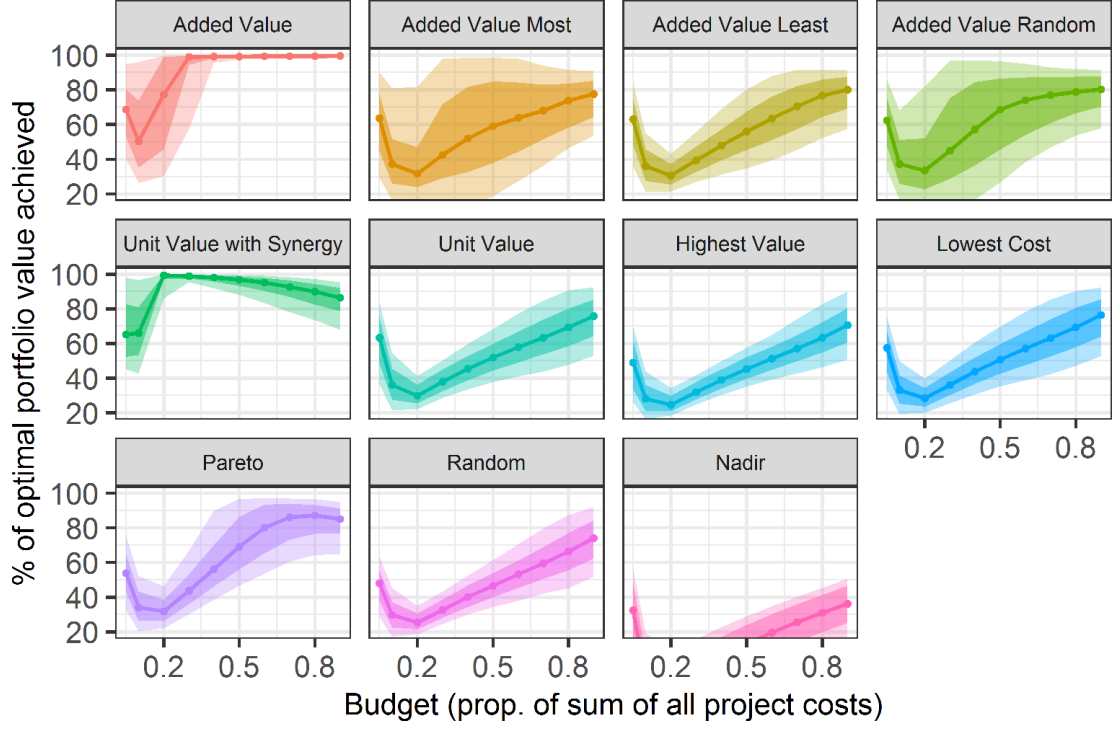


Figure 4.4: Quantiles of fast-and-frugal portfolio performance, where these have been normalized relative to the optimal portfolio (value = 100). Solid lines plot median performance, darker shaded areas plot the envelope between lower and upper quartiles, and lighter shaded areas the envelope between the 10% and 90% quantiles. This figure complements Figure 4.2 by showing relative performance as well as the highly variable performance of some heuristics.

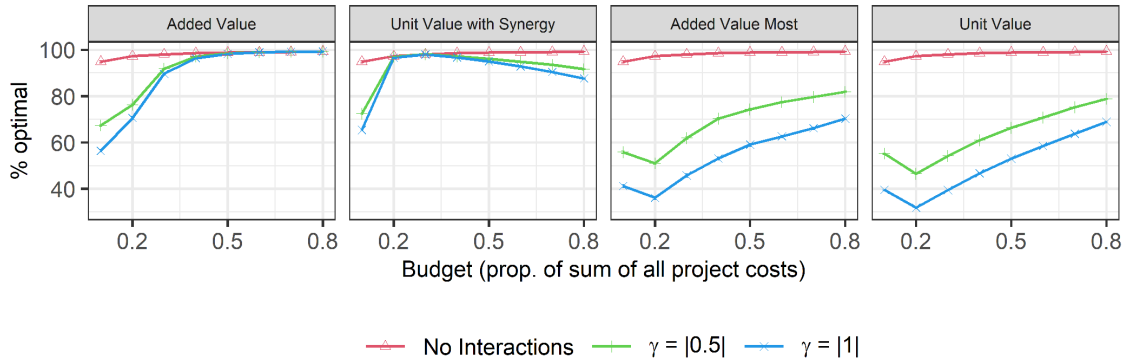


Figure 4.5: Relative performance of *add-the-best* variants vary greatly with the magnitude of alternative interactions. Alternatives making up an interaction subset each have a value, and hence a sum exists for the interaction subset. The γ parameter indicates the proportion of this sum that is awarded when the entire interaction subset is selected. Lines indicate mean relative performance for different values of γ , with error bars denote 95% confidence intervals around these means.

a good approximation for WADD at these levels. The EQW heuristic is in principle closer to WADD because it uses all available information (like WADD) except weight values. In practice, criteria are not always weighted equally - hence the loss in accuracy experienced by the EQW heuristic with respect to WADD optimal values. The LEX heuristic capitalises on the most important criterion and is likely to be more optimal when the weight of the most important criterion is substantially different from the remaining criteria. However, this is not always the case, hence the loss in accuracy with this approach. It is remarkable that the LEX heuristic outperforms the TAL heuristic in relative terms, although the latter requires more information. Arguably the LEX heuristic is the simplest approach in computational terms.

We also observed that the relative performance of scoring heuristics (performance relative to portfolios selected using WADD) was better when there were interactions between alternatives, across all budget levels but was low at lower and moderate levels when there were no interactions. This shows that accounting for interactions is essential to improving heuristic performance (Figure 4.7).

The results indicate that, in general, the structure of interactions – nested or random – did not influence to a great extent the performance of heuristics, except in the case of the TAL heuristic (Figure 4.8). We believe that the decline in the performance of the TAL heuristic in the nested structure happened when the initial interacting alternatives did not pass criterion thresholds or when those that did interacted negatively. As subsequent interaction subsets were drawn from existing interaction subsets in this case, they had the same pattern as the initial subsets.

4.5 Conclusion

Heuristics have mainly been used in settings where the decision maker has to choose the best option out of a set of alternatives. Although research has been conducted on the use of heuristics in portfolio decisions (Martignon et al., 2008), there remain a number of issues to explore, such as understanding how such heuristics are best used when there are interactions between alternatives, or how heuristics perform when interactions between alternatives turn out to have a negative impact.

As with every simulation study, it is evident that the results of the experiment are dependent on the values of the parameters used in the simulation, and this is a limitation of the study. It could also be that variants of the heuristics used in this study or even other heuristics not considered in the current study could yield new insights.

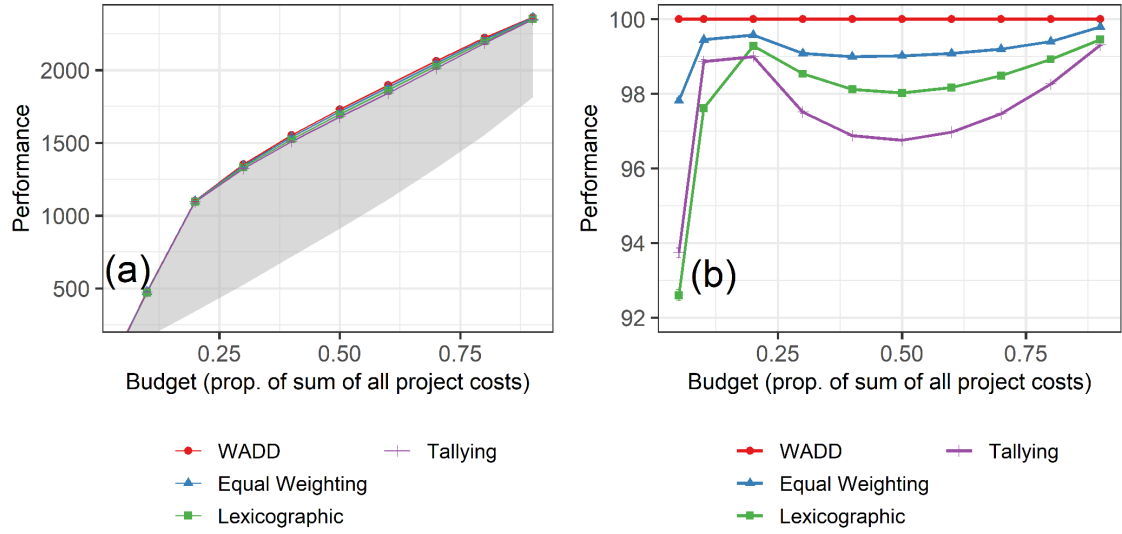


Figure 4.6: Mean values of portfolios selected by scoring portfolio heuristics under different budget constraints. Confidence intervals around these means are negligible (smaller than the symbols used to plot the means). Panel (a) shows the absolute performance of scoring heuristics when the budget ranges from 5% to 90% of all alternative costs. The grey polygon plots the envelope between the value of the optimal WADD portfolio and the mean value returned by a random selection of alternatives, which we consider to be a useful lower bound for benchmarking performance. Panel (b) shows the relative performance of scoring heuristics when the budget ranges from 5% to 90% of all alternative costs.

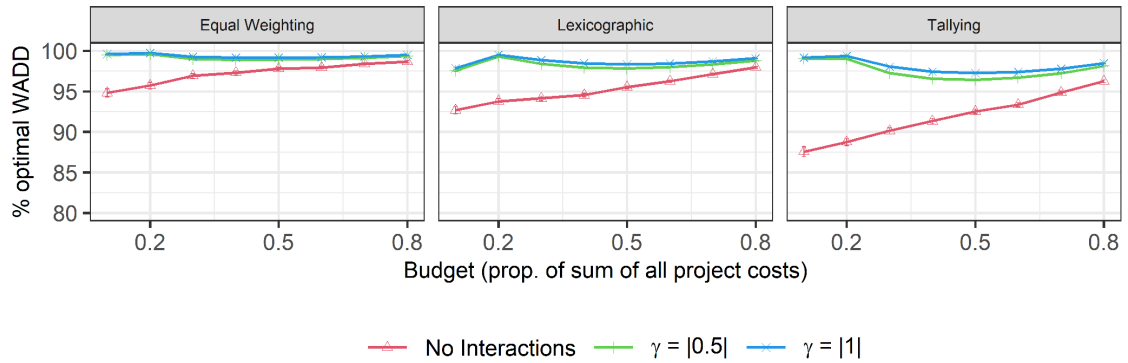


Figure 4.7: The relative performance of scoring heuristics varies greatly with the presence or absence of interactions. Alternatives making up an interaction subset each have individual alternative values, and hence a sum exists for the interaction subset. The γ parameter indicates the proportion of this sum that is awarded when the entire interaction subset is selected. Lines indicate mean relative performance for different values of γ , with error bars denoting 95% confidence intervals around these means.

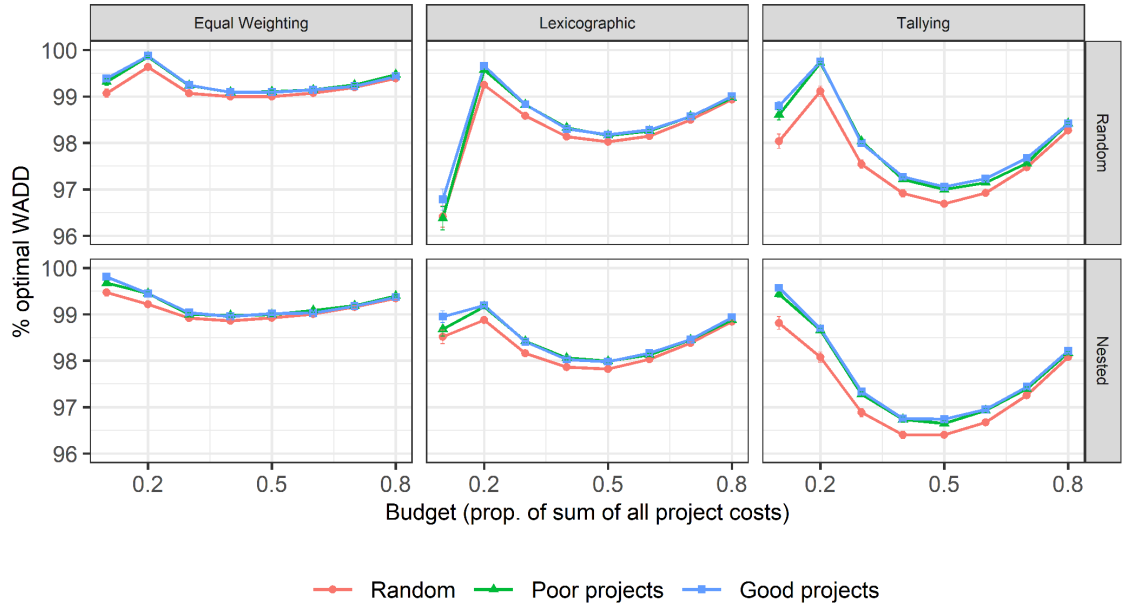


Figure 4.8: Mean relative portfolio value as a function of how alternatives interact with one another, for the EQW, LEX and TAL heuristics. The rows indicate whether higher-order interactions are nested within lower-order ones, or are random. Line colours denote whether alternatives involved in interactions have high value-to-cost ratios (“good” alternatives), low value-to-cost ratios (“poor” alternatives), or whether the selection is random. Vertical error bars denote the 95% confidence intervals of the means.

In this experiment, we have considered single-cue heuristics, i.e. heuristics used for selecting alternatives based on a single criterion, such as maximum profit, minimum cost, etc. We have looked at the “*add-the-best* family” of heuristics adapted from those found in Gigerenzer et al. (1999) and Keller and Katsikopoulos (2016). These include, on the one hand, heuristics that consider information about interactions between alternatives such as the *Added Value* heuristic with all its variants (*Added Value Most*, *Added Value Least* and *Added Value Random*) and *Unit Value with Synergy* and, on the other hand, heuristics that do not consider interactions, such as *Unit Value*, *Highest Value*, *Lowest Cost* and *Pareto*. All these heuristics fall under the umbrella of “construction” or “selection heuristics”, i.e. rules that guide the selection of a portfolio.

Durbach et al. (2020) investigated the above heuristics for portfolio decisions when there are positive interactions between alternatives. We have used the same framework as in Durbach et al. (2020) but have extended the study to look at the selection of portfolios when there are negative interactions, and also to consider multiple criteria in the selection of portfolios. Both conditions are realistic scenarios in the context of portfolio decision-making.

We found that results for cases where negative interactions occur are similar to the positive-only cases shown in the Appendix Section B. Adding negative interactions does not harm the performance of heuristics. This is reassuring, as many practical cases will have both.

Added Value and *Unit Value with Synergy* stood out from all the heuristics tested in this experiment and gave an indication that they can lead to better decisions than the other heuristics. *Unit Value with Synergy* was found to work exceptionally well at lower budget levels when there are negative interactions. Heuristics that ignore interactions were the worst-performing heuristics, as it turned out that they ignored vital information.

In the multiple-criteria experiment, each alternative scores a value on each criterion and these are aggregated into an overall score for the alternative based on some rule. We explored a few heuristics for assigning an overall score to alternatives with multiple criteria. We call this family of heuristics: “scoring heuristics”. The normative scoring approach is the weighted additive linear function (WADD). In this approach, each alternative gets assigned an overall score which is the weighted sum of the scores on the criteria. We note that WADD requires the assessment of several parameters (e.g. the weights of all criteria, all interaction values, etc.), some of which are difficult to assess. We looked at three well-known scoring heuristics: Equal Weighting (EQW), Lexicographic (LEX) and Tallying (TAL) scoring heuristics (Payne et al., 1993). We used the same selection heuristics as those used in the single-criterion case. Results for the multiple criteria case were also largely similar to the single-criterion case. Once again, we found that, overall, *Added Value* and *Unit Value with Synergy* performed better than the other heuristics. Heuristics that ignore interactions were again the worst-performing heuristics.

We evaluated the performance of scoring heuristics by comparing the values that they assign to alternatives with the value assigned by the WADD function to the same alternatives. We found that all scoring heuristics are a good approximation for WADD at lower levels of budget. EQW, in particular, is a good approximation for WADD at all levels of budget. However, the mean values of heuristics were not too different from the mean value produced by WADD in absolute values, i.e. differences were less than 5% of the mean values. Therefore we can conclude that the performance of a portfolio depends more on the approach used to build the portfolio (selection heuristics) than on the method of aggregation across criteria (scoring heuristics). In other words, the main result here is that the heuristic used to choose alternatives is much more important in determining final portfolio value than how performance is aggregated across criteria.

Chapter 5

A decision support system for selecting research proposals

5.1 Introduction

In this chapter, we apply portfolio decision analysis methods and heuristics that we presented in previous chapters to a practical problem of portfolio selection. Individuals and organizations are often faced with the problem of selecting the best portfolio and need guidance for better practice in terms of making those decisions. Our goal is to develop a decision support system for the selection of research proposals at a research institution. Research institutions typically allocate funding for prospective research proposals in a given research area. Usually, as in the case here, there may be a fixed number of research proposals that can be funded, with each proposal funded up to some maximum amount. Applicants submit research proposals and these, as well as the applicants making them, are assessed by the research institution. The decision problem here is to select, from a larger pool of research proposals, the best subset knowing that some combinations of proposals work better than others. Although this application deals with a specific portfolio decision problem, the principles we will learn from this application can be applied to other decision contexts.

Besides the fact that we wanted to provide a decision support system for portfolio selections, we wanted to test the hypothesis (H6) that the web-based application is an effective tool for sensitivity analysis and learning.

The chapter is organized as follows: Section 5.2 describes the structuring of the problem; Section 5.3 describes the web-based application built for the problem, including a brief summary of the portfolio decision methods employed; Section 5.4 presents the results; and Section 5.5 discusses the results and a final section concludes with guidance for practice and suggestions for future research.

5.2 Applying portfolio decision making to select funding applications

The first step in solving a portfolio decision problem is to define the decision context i.e. the set of preferences, constraints and policies that have an effect on the decision makers and stakeholders. When conducting multiple criteria decision analysis in a decision context, it is required to first lay a foundation by structuring the problem, then identifying alternatives and interactions between them and finally, build a model. We deal with these issues below.

5.2.1 Background of decision problem

In this research project, we provide support for a process of selecting research proposals submitted by researchers who apply to the research institution for grants. The management of this institution have opted to remain anonymous. The institution was founded as a centre of excellence for post-graduate training, research and educational outreach. It provides a unique space for international and local collaboration, knowledge exchange and joint research projects. Key stakeholders are donors (or their representatives) who provide grants to researchers and independent assessors who are tasked with scoring each research proposal against a set of criteria defined by the institution. The decision maker is the research manager, who is ultimately responsible for selecting the best set of research proposals based on the scores and associated information assigned to proposals by assessors. Alternatives are research proposals, hereafter referred to as “proposals”. In the most recent round of applications, ten assessors rated twenty-four proposals. For confidentiality and data privacy reasons, the institution did not want to use the data from an actual application round, and so we generated a second set of 10×24 scores with similar properties, showed these to the research manager, and led them through the process described in this chapter.

Proposals are at postgraduate, research masters, doctoral, and postdoctoral levels. In some funding streams, applicants are students; in other streams, they may be established researchers with their own independent research groups. Because funding, accommodation and other resources are rather limited, the research institution can only accommodate a limited number of researchers. The goal of the decision process is to select proposals from the most promising candidates, who have a good track record and whose research proposals align with thematic areas defined by the

research department, and that speak of excellence in research and implementation. There are interaction effects between proposals: some combinations of proposals would be preferred to others based on how well they, as a group, meet the goals set by the research institution. The research manager wants to select the best combination of proposals i.e. the subset of proposals that would yield the highest research contribution.

Up to this point, the selection process has been rather subjective and would take weeks to months complete, each year. A scientifically sound decision support system would streamline the selection process and make it more transparent and effective. We had a series of online meetings, one face-to-face meeting and email exchanges with the research manager to guide the decision process. Details of these can be found in Table 5.1. Face-to-face contact was limited due to the COVID-19 pandemic.

Meetings	Medium	Subject	Duration
First	Online meeting	Discussion of goals and objectives of research institution	80 minutes
Second	Online meeting	Discussion of criteria for selection of proposals	90 minutes
Third	Face-to-face	Elicitation of value tree, weights and interaction values	150 minutes
Fourth	A series of emails	Further elicitation of value tree, weights and interactions	
Fifth	A series of emails	Introduction of the web-based application	
Sixth	A series of emails	Feedback about the web-based application	
Seventh	A series of emails	Release of a new version of the web-based application	
Eighth	A series of emails	Feedback about the web-based application	
Ninth	A series of emails	Discussion of the results	
Tenth	Online meeting	Further Discussion of the results	120 minutes
Eleventh	A series of emails	Further Discussion of the results	
Twelfth	A series of emails	Feedback about overall decision process and conclusion	

Table 5.1: Details of meetings and discussions held with the research manager of the research institute that we have helped with a decision support system. We show the subject discussed at each meeting and the format of the meetings i.e. whether they were online or face-to-face or whether we held discussions via email. Also, we show the duration of the meetings where applicable.

5.2.2 Value Tree

An in-depth consideration of the problem setting allows for a broader perspective and more creativity in finding a solution to the problem. A key step in solving a portfolio decision problem is to define the structure of the decision problem. In order to do this, stakeholders and decision makers need to articulate their fundamental objectives and goals. These are also referred to as criteria and are the basis for evaluating alternatives. Belton and Stewart (2002) state that criteria should satisfy the following properties: completeness, measurability, non-redundancy, operationally meaningful and well-defined and preference independence. Preference independence means that criteria should be defined such that judgment can be made based on performance on a criterion without

referring to performance on other criteria. This is a key requirement for the additive linear value function. The decision maker needs also to specify how criteria will be measured. Generally, criteria can be organized into a value tree, i.e. an hierarchy of objectives. These can be broken down into lower-level criteria or even aggregated into higher-level criteria. Lower-level criteria are sub-objectives that describe the higher-level criteria and make the latter more measurable and concrete.

In the course of the discussion with the research manager, we were able to verify that they could always make preference judgments about proposals based on each criterion independently of other criteria, and that the criteria in this study were unambiguous and the research manager could also specify a scale for each one of them, thus making them measurable and operational. The research manager was satisfied that the criteria capture all important aspects of the decision problem.

We conducted a series of interviews with the research manager mostly online meetings because of the COVID-19 pandemic. The discussion was essential in order for us to develop a clear description of the decision problem and understand fundamental objectives. We uncovered a range of criteria and grouped into these four families: Rating of Applicant, Quality of proposal, Relevance of proposal and Equity (Figure 5.1).

Below we discuss the meaning of the criteria and briefly outline a measurable attribute or a qualitative description of each one of them. More information about measurable attributes or qualitative scales can be found in Tables C.3 and C.4 where we show the scale of the criteria. We also show the value tree in Figure 5.1. Criteria used are similar to those used in the literature in a similar context (Belton and Stewart, 2002; Joshi, 2014).

- Recognition of prior work: A grant applicant who has received a number of awards for their research work or for their academic achievements is highly recognized. Achievement on this criterion is measured by the number of awards received by the grant recipient.
- Quality of publications: the quality of the applicant's publications are rated based on the substance of their publication. Achievement on this criterion is measured by the number letters/communications, articles, supplementary articles, review articles or research notes published by the applicant. Each of the outputs denotes a given degree of contribution to the research field.
- Impact of prior work: the impact of prior research work in terms of the number of publications and the number of citations. A combination of both the number of

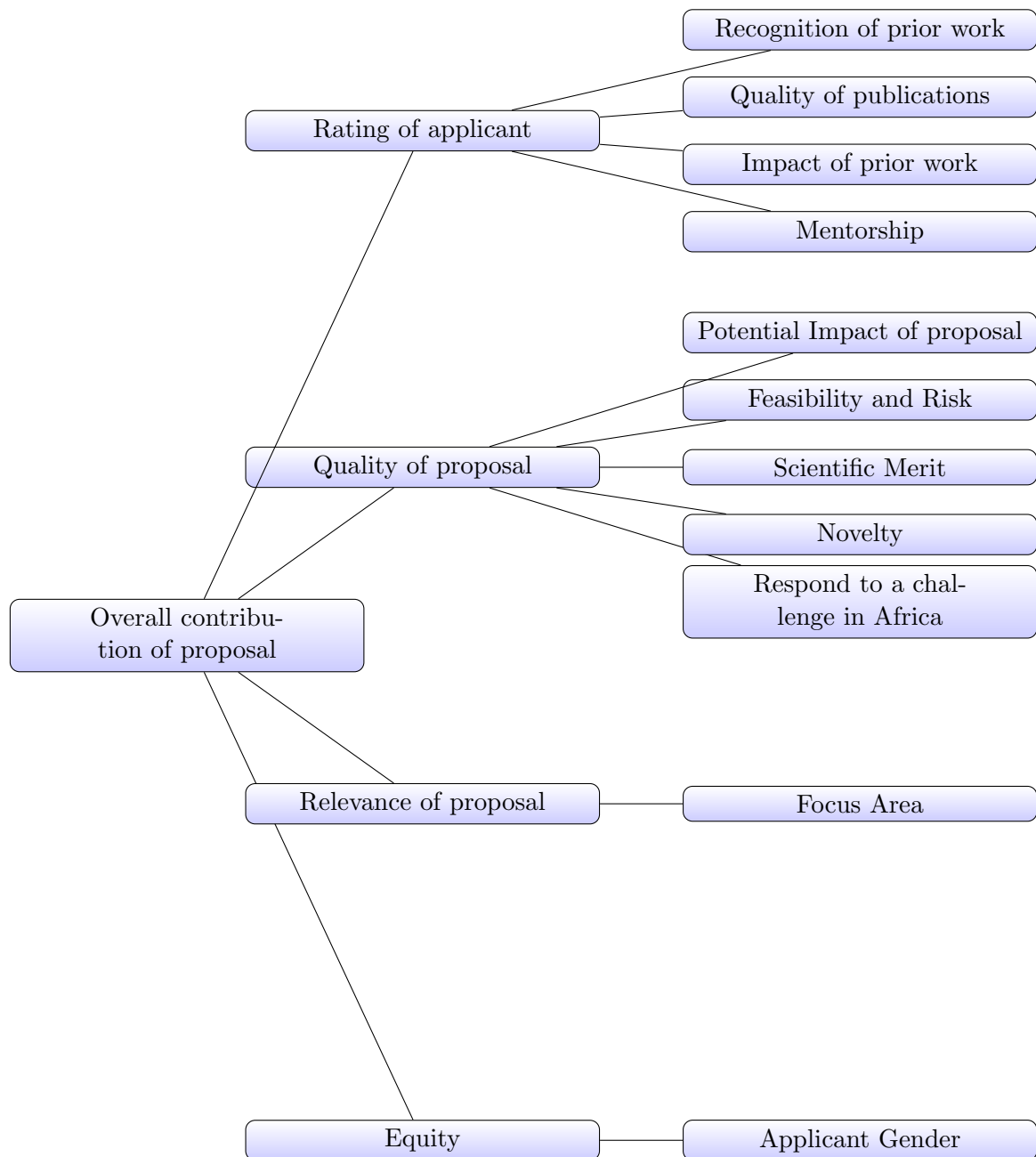


Figure 5.1: Value tree as assessed by the decision maker. The overall quality of a proposal is first broken down into mid-level objectives: the rating of the applicant, quality of the proposal, the alignment of the proposal with the goals of the institute, and whether the proposal furthers equity targets. Each of these is further broken down into lower-level criteria, to be measured with attributes.

publications and the number of citations provide a scale to measure achievement on this criterion.

- **Mentorship:** this criterion assesses whether the applicant has shown the ability to supervise or teach or tutor students. It is measured by the number of students being supervised, taught or tutored.
- **Potential Impact of proposal:** A research proposal can fall within either applied or pure research. If it is applied research, the aim may be to inform policies and seek to find a solution for an immediate problem facing society, or an industrial/business organization. The impact of the proposal, in this case, is to bring about socio-economical and environmental changes. If it is pure research, the aim is to bring a deeper understanding of the theoretical aspects of a phenomenon and formulate a theory. The impact of the proposal, in this case, is mainly knowledge creation. Achievement on this criterion is measured whether the project falls within applied or pure research and whether it has potential to be published.
- **Feasibility & Risk:** The feasibility and risk associated with a proposal is based on the applicant's academic strength and likelihood of completing the project. The selection panel would evaluate the methodology and decide whether the applicant could implement the project within the time frame. Achievement on this criterion is measured by the likelihood for the applicant to complete the project based on previous related work, clarity of definition of the project, etc.
- **Scientific merit:** A research proposal should represent good science. A good research proposal entails both clear research objectives and questions and pertinent hypotheses. Reputation is key for a science institution. They would not want to be associated with any project that would damage their brand if the methodology was found to be wrong or end results turned out to be scientifically incorrect. Achievement on this criteria is rated based on problem design, methodology, objectives and potential contribution to knowledge. A project that does well on each of these items is highly scored.
- **Novelty:** A proposal should generate knowledge and uncover hidden information. This would indicate the likelihood of the final results being published or contribute to solving a pressing problem. Achievement on this criterion is measured by the degree of novelty of a project: whether it is groundbreaking i.e. it opens up entire new areas of research or whether it repeats existing research or extends it.
- **Respond to a challenge in Africa:** Degree to which proposal directly addresses a challenge in Africa. The project receives a score based on whether it is purely theo-

retical or whether it is still theoretical but has a limited applied aspect or whether it is applied research and gives a solution to a practical problem.

- **Focus Areas:** Most funding calls are made with specific aims and objectives in mind, expressed as themes. Proposals should clearly specify how they align with these objectives. Achievement on this criterion is measured by the degree to which the proposal is aligned with the thematic areas outlined in the call for funding proposals: Science, Technology, Engineering and Mathematics.
- **Gender:** this criterion aims to redress the balance between men and women in the research field. Often women are at a distinct disadvantage in the African context, having less opportunities than men and facing more obstacles than men in their pursuit of a career, especially in Sciences. In the spirit of balancing equity and good research, the criterion has been assigned the lowest weight value. We also found the same criterion used in Belton and Stewart (2010).

5.2.3 Eliciting Weights

Weights of criteria in MCDA are a scaling constant reflecting both the scale of the criterion and its importance. There are a few approaches to assessing weights: the trade-off method, the direct-ratio method and the swing method. The trade-off method means that you ask for the exchange relationship of two criteria for which the decision maker is indifferent. This is generally done by comparing two alternatives that differ only in the values of the two criteria that are being considered. In the direct-ratio method, you first rank criteria in terms of their “importance”. The level of importance of criteria is established with no regard to the scale of criteria. After this, the decision maker estimates to what extent a given criterion is more important than another one and expresses this as a ratio of the one criterion to the other. Although the direct-ratio method is often used, it is logically flawed because a criterion is not important apart from the difference between the levels of performance on the criterion (Eisenfuhr et al., 2010).

In this application, we opted for the swing method as it is the most intuitive approach, in our view, and is easy to convey to the decision maker. To establish swing weights, criteria should first be ranked based on the importance assigned to improvement from the worst to the best level of achievement on each criterion and then improvement registered on the criteria should be compared against improvement registered on the most highly ranked criterion.

We established criterion weights, assessing weights within families of criteria (higher-level objectives), followed by a comparison of the most highly weighted criterion in each

family, and using worst-to-best swings as a basis for that comparison. Weights were reasonably equally distributed across criteria, with the most important criterion, impact of prior work, receiving a weight a little under three times as large as the least important criterion, gender ($w = 0.13$ vs $w = 0.05$, as seen in the *Cumulative Weights* column in Table 5.2). Out of the families of criteria, applicant rating was rated most important, followed by proposal quality, relevance of proposal, and gender equity. All criterion weights are shown in Table 5.2.

Family of Criteria	Criteria	Cumulative Weights	Relative Weights
Rating of Applicant	Recognition of prior work	0.08	0.19
	Quality of publications	0.11	0.28
	Impact of prior work	0.13	0.31
	Mentorship	0.09	0.22
Quality of proposal	Potential Impact of proposal	0.09	0.20
	Feasibility and Risk	0.11	0.25
	Scientific Merit	0.08	0.18
	Novelty	0.07	0.15
	Respond to a challenge in Africa	0.10	0.23
Relevance of proposal	Focus Area	0.10	0.10
Equity	Applicant Gender	0.05	0.05

Table 5.2: Weights of criteria are shown in the “Cumulative Weights” and “Relative Weights” columns respectively. Cumulative weights are weights standardized across all criteria to sum up to 1. For example, the cumulative weight of *Recognition of prior work* constitute 8% of total importance weight. Relative weights are weights standardize within families of criteria. For example, the relative weight of *Recognition of prior work* constitutes 19% of the importance weight of the *Rating of an Applicant*.

5.2.4 Rating of alternatives

Each proposal is scored across all criteria. Scores are related to the levels of performance along a criterion and can be specified using a value function. There are a few approaches for determining value functions: the direct-rating method, the difference standard sequence technique and the bisection method. Details of these approaches can be found in Eisenfuhr et al. (2010). In this application, assessors used the direct-rating method. Each alternative is rated directly following these steps: First, the best and worst values on each criterion are specified, and then alternatives are sorted based on the decision maker’s preference regarding their performance. Lastly, a direct evaluation of the alternatives is carried out. The best-performing alternative receives a score of 100 and the worst-performing alternative a score of 0 and the other alternatives receive a score between 0 and 100. We show in the appendix Section C the average scores achieved by the proposals in Table C.2. We also show in the appendix Section C the scales of the criteria in Table C.3.

5.2.5 Specifying interactions between alternatives

It is essential for portfolio decision analysis methods to account for interactions between alternatives as these could result either in gains or losses. Generally speaking, there are three types of interactions between alternatives, although they all don't apply to this application:

Resource interactions When there are resource interactions, different alternatives share common resources and save on the total amount required when they are run simultaneously.

Benefit interactions When there are benefit interactions, the impact of a portfolio of projects is either more or less than the sum of individual project impacts.

Outcome interactions When there are outcome interactions, the probability of success of a alternative depends on the success of another alternative.

In this context, the success of a research proposal is independent of the success of other research proposals and the combination of research proposals does not induce a saving of resources by the research institution. For this reason, resource and outcome interactions do not apply in this case. Benefit interactions are more appropriate for this analysis: each proposal has its own value, but some combinations of proposals go well together. If two or more proposals that go well together are selected, the total benefit gained is more than the sum of the proposals' benefits if they were selected in isolation. In other words, the value gained from a proposal depends on other proposals already selected.

The decision maker provided interaction factors in the same way they provided input judgments about value scales and weight values (Belton and Stewart, 2002). Based on their extensive experience in dealing with research proposal selections, the decision maker estimated that interaction effects existed between any proposals that have an excellent score on the following criteria: impact of prior work, quality of publications, feasibility and risk, respond to a challenge in Africa, focus area and recognition of prior work. They also estimates that proposals which have an excellent score on the remaining criteria do not earn extra benefits. Proposals were rated on each criterion with a score between 0 and 100. The decision maker reckons that an excellent score should be defined as 90 points and above on each criterion.

The interaction multiplier (γ) is the sum of the interaction values $\gamma_{i'n_{i'}}$. The interaction value $\gamma_{i'n_{i'}}$ increases with $n_{i'}$, the number of projects in the portfolio that have

an excellent score on a criterion i' and that carry interaction effects as a result of this. For example, $\gamma_{i'3}$ ($\gamma_{i'n_{i'}}$ for three good proposals) is more than $\gamma_{i'2}$ ($\gamma_{i'n_{i'}}$ for two good proposals). This is similar to the way Liesiö (2014) builds multiattribute value functions for portfolio decision analysis i.e. the interaction benefit increases with the number of good alternatives. The research manager specified $\gamma_{i'n_{i'}}$ values, as shown in Table 5.3. For instance, on the criterion “Impact of prior work”, $\gamma_{12} = 0.07$ for 2 proposals, $\gamma_{13} = 0.10$ for 3 proposals, $\gamma_{14} = 0.12$ for 4 proposals, etc.

$n_{i'}$	Imp. of work	Qual. of public.	Feasib. & Risk	Resp. to Chal.	Focus Area	Recognit.
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0.07	0.05	0.03	0.02	0.01	0.01
3	0.10	0.07	0.05	0.03	0.02	0.01
4	0.12	0.09	0.06	0.04	0.03	0.02
5	0.13	0.11	0.08	0.06	0.03	0.02
6	0.15	0.13	0.09	0.07	0.04	0.03

Table 5.3: Interaction between proposals generate value benefits, also called interaction benefits. An interaction benefit is a factor of the sum of weighted additive value of proposals involved in interactions. The interaction value ($\gamma_{i'n_{i'}}$) increases with the number of proposals ($n_{i'}$) in the portfolio that have an excellent score on a given criterion. The table shows a cross tabulation of $\gamma_{i'n_{i'}}$ for relevant criteria broken down by the number of proposals. The criteria are listed respectively as: impact of prior work, quality of publications, feasibility and risk, respond to a challenge in Africa, focus area and recognition of prior work.

The sets of proposals that have an excellent score on criteria i' for which interaction effects exist; S'_i , $i' = \{1, ..4\}$ are:

- $S_1 = \{15, 17, 19\}$ defined on the criterion “Impact of prior work” whose weight is 0.13.
- $S_2 = \{6, 20\}$ defined on the criterion “Quality of publications” whose weight is 0.11.
- $S_3 = \{9, 10, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24\}$ defined on the criterion “Feasibility and Risk” whose weight is 0.11.
- $S_4 = \{14, 17, 18, 22\}$ defined on the criterion “Respond to a challenge in Africa” whose weight is 0.10.

The interaction set A is the union of interaction subsets $A_{i'}$ drawn from the above 4 sets (S_1 to S_4) and represents all proposals involved in interaction. Interaction subsets $A_{i'}$ are formed whenever any two or more of the proposals in the four sets above are included in a portfolio.

5.3 Developing a web-based interface for decision support

We developed a web-based application to provide support to the decision maker in choosing the best set of proposals. The application presents a range of solutions to choose from: a linear-additive portfolio value model, robust portfolio modeling and simplified portfolio models using heuristics. These methods were described in detail in Chapter 2 but for completeness, and to keep this chapter self-contained, we briefly summarize them again here before describing the decision support system interface. We applied robust portfolio modeling as we assumed that the research proposal scores were uncertain, being observed between the best and worst scores given by the assessors. We have included heuristic solutions in the web-based application because heuristic portfolios (portfolios selected by heuristics) are useful as they can be selected under uncertainty or used if the decision maker does not have the resources to find the optimal solution for a very complex portfolio decision problem (e.g. a much higher number of alternatives or the presence of many more interactions between them).

5.3.1 Methods

Linear-additive portfolio value model

The linear-additive portfolio value model, also called the knapsack model, has been formulated in a context where one has to pack a subset of n alternatives into a knapsack of capacity c . Each of the n alternatives has value x_j and weight ω . The aim is to select the best subset, i.e. the subset whose total weight would not exceed c but would still yield maximum value (Pisinger, 1997, 1995). This problem can be solved as an integer linear programming problem with binary decision variable $z_j = \{0, 1\}$, which takes a unit value when alternative j is included in the subset and is otherwise set to zero.

The linear-additive portfolio value model can be specified as follows (Jaszkiewicz, 2002; Martello et al., 2000):

$$\begin{aligned} \max \quad & V(P) = \sum_{j=1}^n x_j z_j \\ \text{subject to} \quad & \sum_{j=1}^n c_j z_j \leq R \\ & z_j \in \{0, 1\}, \quad \forall j \end{aligned} \tag{5.1}$$

where P is a portfolio, coefficients x_j and c_j are value and weight (cost) for alternative j , respectively, R denotes a knapsack capacity and z_j is a binary variable indicating whether alternative j is selected in a portfolio. $z_j \in \{0, 1\}$ is a variable such that $z_j = 1$ if alternative j is included in portfolio P and $z_j = 0$ if alternative j is not included in

portfolio P . This is known as the integrality constraint.

Linear-additive portfolio value model with interactions So far the linear-additive portfolio value model we have discussed deals with the selection of a portfolio in the absence of interactions between alternatives. However, this model has been extended to tackle instances of interactions between alternatives. Suppose we also have $\hat{I}$ interaction subsets $A_{i'}^+$ ($i' = 1, \dots, \hat{I}$), defined on criteria that carry interaction effects denoted i' .

The optimal value of portfolio P can be found as follows (Stummer and Heidenberger, 2003):

$$\begin{aligned} \max V(P) &= \sum_{j=1}^n x_j z_j + \sum_{i'=1}^I x'_{i'} g_{i'} \\ g_{i'} &= \{0, 1\} \quad \forall i' \end{aligned} \tag{5.2}$$

where $x_j = \sum_{i=1}^m \omega_i x_{ij}$ the weighted additive value across criteria of proposal j and $x'_{i'}$ is the incremental change in value if at least $m_{i'}$ alternatives in interaction subset $A_{i'}$ are included in the portfolio. $g_{i'} \in \{0, 1\}$ is a variable such that $g_{i'} = 1$ if at least $m_{i'}$ alternatives in interaction subset $A_{i'}$ are selected and $g_{i'} = 0$ otherwise.

We formulate incremental values $x'_{i'}$ as follows: $x'_{i'} = \gamma_{i' n_{i'}} (\sum_{j \in A} x_j)$ where $n_{i'}$ is the number of proposals within interaction subset $A_{i'}$, A is the union of interaction subsets $A_{i'}$. As explained in Section 5.2.5, $\gamma_{i' n_{i'}}$ increases with the number of projects $n_{i'}$ in the portfolio that have an excellent score on a criterion i' and carry interaction effects as a result of this.

We can now render equation (5.2) as follows:

$$\begin{aligned} \max V(P) &= \sum_{j=1}^n x_j z_j + \sum_{i'=1}^I (\gamma_{i' n_{i'}} (\sum_{j \in A} x_j)) g_{i'} \\ &= \sum_{j=1}^n x_j z_j + \sum_{i'=1}^I (\gamma_{i' n_{i'}} g_{i'}) (\sum_{j \in A} x_j) \\ &= \sum_{j=1}^n x_j z_j + \gamma \sum_{j \in A} x_j \\ g_{i'} &= \{0, 1\} \quad \forall i' \end{aligned} \tag{5.3}$$

where I is the number of interaction subsets $A_{i'}$, i' being a criterion that carry interaction effects and $g_{i'} = 1$ if $n_{i'}$ proposals in $A_{i'}$ are selected ($g_{i'} = 0$ otherwise). The interaction multiplier (γ) is the sum of the interaction values $\gamma_{i' n_{i'}}$.

For example, if the decision maker selects a portfolio made up of proposals $\{3, 17, 19, 23\}$ whose values are shown in Table 5.4, we can see that proposals $\{17, 19, 23\}$ are involved in interactions (See proposals with values highlighted in yellow). Therefore, we will have 2 ($K = 2$) interaction subsets $A_1 = \{17, 19\}$ ($n_1 = 2$ proposals) drawn from S_1 with $\gamma_{12} = 0.07$ and $A_3 = \{17, 19, 23\}$ ($n_3 = 3$ proposals) drawn from S_3 with $\gamma_{33} = 0.05$; $A = \{17, 19, 23\}$. The overall value of portfolio $P = \{3, 17, 19, 23\}$ can be calculated as explained in Table 5.5. The S sets (S_1 and S_3) are defined in Section 5.2.5 as the sets of proposals that have an excellent score on the criteria i' for which interaction effects exist.

Proposal	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	Weighted Sum
3	70	60	70	70	75	80	70	70	50	45	0	63.6
17	90	70	90	80	80	90	80	60	40	50	0	71.6
19	90	70	90	70	80	80	70	70	70	50	100	76.8
23	80	67	90	70	80	70	80	75	70	50	0	70.6
Weight	0.13	0.11	0.11	0.10	0.10	0.09	0.09	0.08	0.08	0.07	0.05	

Table 5.4: An example of a selected portfolio. Suppose the decision maker has selected proposals $\{3, 17, 19, 23\}$ as their preferred portfolio. The values highlighted in yellow are excellent scores i.e. values of proposals that equal or exceed 90 points on each criterion. Criterion values are given on a scale from 0 to 100 points. Criteria C_1 to C_{11} are given respectively as follows : impact of prior work, quality of publications, feasibility and risk, respond to a challenge in Africa, focus areas, potential impact of proposal, mentorship, scientific merit, recognition of previous work, novelty and gender. Proposals 17, 19 and 23 have excellent scores on criteria “impact of previous work” and “feasibility and risk”. The table also shows weights of criteria and weighted additive values across criteria for each proposal.

Portfolio	Equation	Value
Weighted Sum	$\sum_{j=1}^n x_j z_j$	63.6+71.6+76.9+70.6 282.5
Interaction Benefit	$\gamma \sum_{j \in A} x_j$	(0.07+0.05)*(71.6+76.9+70.6) 25.5
Overall	$\sum_{j=1}^n x_j z_j + \gamma \sum_{j \in A} x_j$	308.1

Table 5.5: The overall value of a selected portfolio is the sum of weighted additive value of all proposals plus the interaction benefit. The interaction benefit is a multiple of the sum of weighted additive values of proposals involved in interactions.

Robust Portfolio Modeling (RPM)

In order to select the optimal portfolio, we need to solve a linear programming problem. At times there are organizational and environmental constraints that have to be taken into account, such as budgetary constraints, logical constraints (mutually exclusive alternatives), positioning constraints (minimum number of projects in an area) and threshold

constraints (minimum acceptable level). If there is complete information about weights and scores of alternative j on criteria i , the optimal portfolio P can be given as follows:

$$\begin{aligned} \max V(P) &= \sum_{j=1}^n z_j \sum_{i=1}^m \omega_i x_{ij} \\ \text{Subject to } \sum_{j=1}^n z_j c_j &\leq R \\ z_j &\in \{0, 1\} \quad \forall j \end{aligned} \tag{5.4}$$

where x_{ij} is the performance of alternative j on criterion i , ω_i is the weight of criterion i such that $\{\sum_{i=1}^m \omega_i = 1, \omega_i \geq 0\}$, c_j is the cost of alternative j , R is the amount of budget available and z_j defined as in equation (5.1).

RPM models interactions between alternatives as in Stummer and Heidenberger (2003). Following equation (5.2), RPM incorporates constraints that specify interactions when at least a given number of synergistic alternatives are included in the solution. Similarly, further interactions are specified when, at most, a given number of synergistic alternatives are included in the solution. Details of this are shown in Liesiö et al. (2008).

The RPM model presented by Liesiö et al. (2008) models portfolio selection in the presence of incomplete performance score and weight information. The set of feasible scores is given as $S_x = \{X \in \mathbb{R}^{m \times n} \mid \underline{X} \leq X \leq \bar{X}\}$ where $\underline{X}$ and $\bar{X}$ are the lower and upper bounds of the score interval, respectively. In the same way, the set of feasible weights S_w is spanned by t extreme vectors, denoted $W_{ext} = [\omega^1, \dots, \omega^r, \dots, \omega^t] \in \mathbb{R}^{m \times t}$ (Salo and Punkka, 2005). Non-dominated portfolios are identified by performing pairwise comparisons of portfolios (P and P') for all extreme weight vectors ω^r in W_{ext} .

$P > P'$ i.e. Portfolio P dominates portfolio P' if and only the value of portfolio P for the lowest performance scores $\underline{X}$ is higher than the value of portfolio P' for the highest performance scores $\bar{X}$, for every extreme weight vectors ω^r . This is given as follows:

$$\begin{aligned} \sum_{j=1}^n z_j \sum_{i=1}^m \omega_i \underline{x}_{ij} &\geq \sum_{j'=1}^n z_{j'} \sum_{i=1}^m \omega_i \bar{x}_{ij'} \\ \sum_{j=1}^n z_j \sum_{i=1}^m \omega_i \underline{x}_{ij} &\neq \sum_{j'=1}^n z_{j'} \sum_{i=1}^m \omega_i \bar{x}_{ij'} \end{aligned} \tag{5.5}$$

where $j \in \{P \setminus P'\}$ are projects in portfolio P that are not included in portfolio P' and $j' \in \{P' \setminus P\}$ are projects in portfolio P' that are not included in portfolio P , ω_i is the weight of criterion i such that $\omega_i \in \omega^r$, and $\underline{x}_{ij}$ and $\bar{x}_{ij'}$ are elements of $\underline{X}$ and $\bar{X}$

which are the lower and upper bounds of performance scores interval, respectively. This problem is equivalent to a multi-objective zero-one linear programming problem. The set of non-dominated portfolios is called Π_N .

Liesiö et al. (2008) present an algorithm in which pairwise comparisons are conducted systematically as per equation (5.5) and this results in the construction of the set of the non-dominated portfolios. A partial enumeration of portfolios is made possible by the elimination of portfolios that cannot become non-dominated. In this application, we use a similar algorithm. However, we are able to enumerate all portfolios because of the reasonable size of the portfolio selection problem. We identify portfolios that are non-dominated across all extreme weights and these are the RPM portfolios.

In order to assess the quality of portfolios, RPM supplies a number called the core index. It is computed for individual alternatives and is calculated as the share of non-dominated portfolios which include the given alternatives. For Project j , the core index CI is given as $CI(x_j, S) = |\{P \in \Pi_N \mid x_j \in P\}|/|\{\Pi_N\}|$. Based on this index, alternatives can be classified into: core alternatives (they are part of every non-dominated portfolio i.e. $CI = 1$), borderline alternatives (they belong to some non-dominated portfolios but not to all i.e. $0 < CI < 1$) and exterior alternatives (they belong to none of the non-dominated portfolios i.e. $CI = 0$). A portfolio made up of core alternatives is thought to be more robust than other portfolios. The core index can also be defined as the relative frequency at which each proposal appears in non-dominated portfolios, with a score of 1 indicating that the proposal is found in every non-dominated portfolio and a score of 0 indicating that none of the robust portfolios contains the proposal. One can look at the core index to decide on the value of each proposal.

Simplified portfolio models using heuristics

In the previous chapter, we presented heuristics commonly used for portfolio selection. These heuristics consider benefits or losses resulting from certain alternatives being selected together. We conducted a simulation study to assess the performance of these heuristics and found that *Unit Value with Synergy* and *Added Value* were the best performing heuristics. For this reason, we use them in this chapter for portfolio selection alongside the Robust Portfolio Modeling approach.

We briefly describe here the two heuristics:

Unit Value with Synergy (UVwS) Adds alternatives according to the *Unit Value* heuristic from the set of alternatives that are involved in at least one positive inter-

action i.e. alternatives are sorted by their marginal value-to-cost ratios and added in this order. Alternatives are added until the set is exhausted. Then, it adds alternatives from outside the set, again using *Unit Value*.

Added Value (AV) Adds the alternative whose inclusion would lead to the largest increase in overall portfolio value per unit cost. Gains include both marginal values of the alternative and interaction values resulting from a combination of alternatives being simultaneously part of the portfolio. *Added Value* searches across alternatives not added yet to see whether their inclusion would complete any interaction subset.

5.3.2 Overview of decision support system

The application provides the following features and solutions:

- “Instructions” tab: it gives a general description of the web-based application and available features (See Figure 5.2).

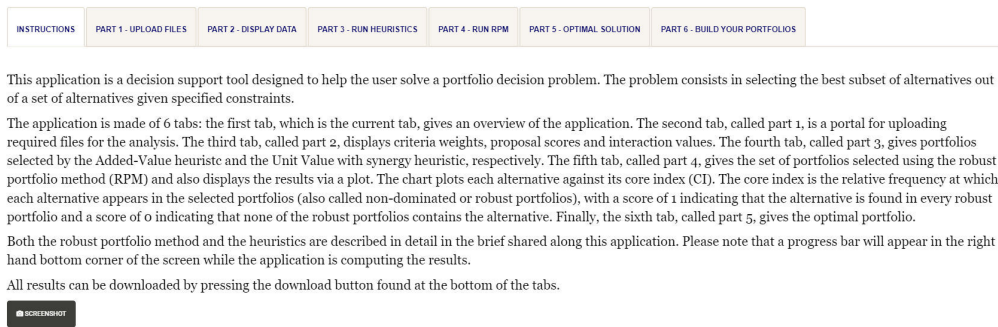


Figure 5.2: The Instructions tab gives a general description of the application and the remaining tabs. It also lists the portfolio selection methods used by the application and briefly describes the results and finally includes instructions for users.

- “Upload files” tab: the user must upload an XLSX file containing individual proposal scores, interaction values and labels, in different sheets. The weights of criteria are set using a series of input sliders on the same tab. Weights are automatically rescaled to sum to one (See Figure 5.3).
- “Display data” tab: this tab provides the user with a visual summary of the input data, prior to optimization. Slider bars provide the user with control over (a) which criterion is shown and (b) which proposals are shown. Weights, scores, and interaction values are all shown via charts or tables. The user can adjust the weights in the previous tab, and view the updated values on this tab. A “Reset weights” button allows the user to return to the initial set of weights (See Figure 5.4).

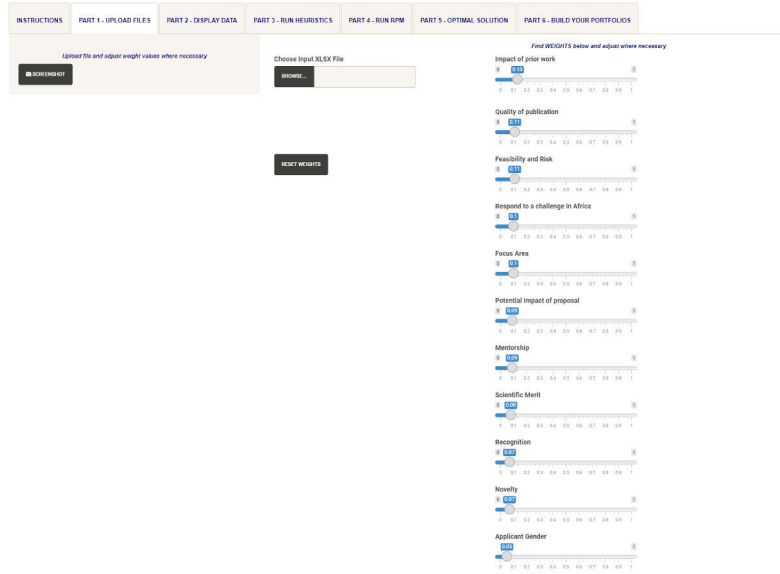


Figure 5.3: The upload tab allows the user to upload the Xlsx input file containing the scores of proposals, interaction values and labels via a file-select field called “Browse”. It also allows the user to view and adjust weight values, that are displayed as input sliders, and reset the weights to default values by pressing the “RESET WEIGHTS” button. For example, the default weight value for the “Impact of prior work” criterion is 0.13.

- “Run Heuristics”, “Run RPM” and “Optimal solution” tabs: these three tabs provide the user with the portfolios selected by heuristics (*Added Value* and *Unit Value with Synergy*), RPM, and the linear-additive portfolio value model respectively. These methods are only run once the user clicks the “Run” button. The sheets show the proposals selected, and information about each of the proposals and the overall portfolio value (See Figure 5.5).
- “Build your portfolio” tab: this tab allows the user to construct their own portfolios through the use of checkboxes, again displaying the proposals selected, and information about each of the proposals and the overall portfolio value. This allows the user to experiment with any selected portfolios, and conduct sensitivity analysis (See Figure 5.6).

The “Build your portfolio” tab displays the portfolios selected by the user and resulting overall values as they apply any heuristic they deem appropriate. Proposals are shown as checkboxes on the left sidebar panel and the user can select multiple proposals at once. The tab also provides functionality to track the portfolios that the user selected previously.

- From “Display data” to “Optimal solution” tabs, the user can change the portfolio size by moving the “portfolio size” slider to the desired value (Step 1 on the tabs).

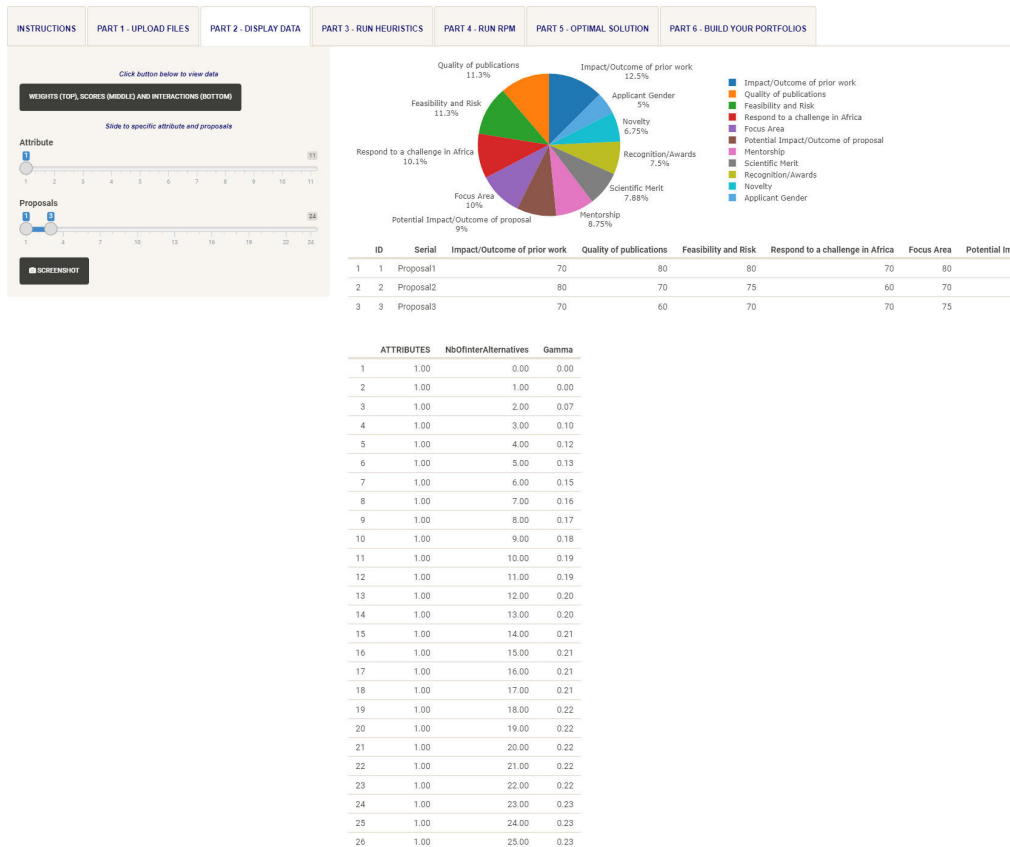


Figure 5.4: The “Display Data” tab displays weights, the scores of proposals and interaction values. The user has the ability to display the scores for selected proposals and interaction values for selected criteria using the “Proposals” and “Attribute” sliders, respectively.

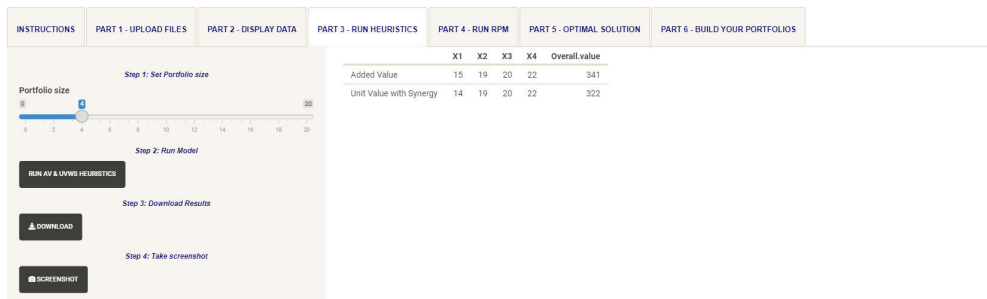


Figure 5.5: The “Run Heuristics” tab displays the portfolios that are selected using the *Added Value* and *Unit Value with Synergy* heuristics. The “RPM” and “Optimal Solution” tabs have a layout similar to the layout “Run Heuristics” tab, except that they display the RPM and the linear-additive portfolio value model respectively.

INSTRUCTIONS

PART 1 - UPLOAD FILES

PART 2 - DISPLAY DATA

PART 3 - RUN HEURISTICS

PART 4 - RUN RPM

PART 5 - OPTIMAL SOLUTION

PART 6 - BUILD YOUR PORTFOLIOS

Portfolio size

0

4

20

Step 1: Set Portfolio size

Step 2: Select proposals

From list below

☐ p1

☐ p2

☐ p3

☐ p4

☐ p5

☒ p6

☐ p7

☐ p8

☐ p9

☐ p10

☐ p11

☒ p12

☐ p13

☐ p14

☒ p15

☐ p16

☒ p17

☐ p18

☐ p19

☐ p20

☐ p21

☐ p22

☐ p23

☐ p24

DISPLAY YOUR PORTFOLIO

DOWNLOADED

SCREENSHOT

Step 3: Download Results

Step 4: Take screenshot

Display your own Portfolio

	X1	X2	X3	X4	Overall value
Selected portfolio	4	8	15	17	310
Selected portfolio1	6	8	15	17	310
Selected portfolio2	6	12	15	17	304

Figure 5.6: The “Build your Portfolios” tab displays the portfolios selected by the user using their own heuristics and overall values of these portfolios and tracks also previous selections made by the user.

The slider is set to size 6 by default. Each model can be run by pressing the “Run” button (or “Find Optimal Solution” or “Display your portfolio” buttons) (Step 2 on the tabs) and the results get displayed on the page i.e. the proposals that are part of the portfolio solutions and their overall values get displayed. A small progress window shows at the right bottom of the tabs while the solutions are being computed. Finally, the user can download the results by pressing the download button (Step 3 on the tabs).

This application is available online at:

https://sciencedata.shinyapps.io/portfolio_application/

5.4 Results

5.4.1 Heuristics, RPM and optimal solutions

In typical application mode, the research institution decides beforehand how many proposals can be funded with a given budget. In some cases there is a small amount of flexibility i.e. additional funding can be found, if there are a number of exceptional proposals. In this application, we examined solutions selecting between four and six proposals (Table 5.6).

We assessed the quality of portfolios produced by heuristic methods and RPM against optimal portfolios. Optimal portfolios are those selected using the linear-additive portfolio value model. In this section we show that both the overall portfolio value (the first approach) and percentage of proposals in common with the optimal portfolio (the second approach) can guide portfolio decision making.

In the first approach, we compared standardized overall values achieved by portfolios. We standardized overall values such as the optimal overall value is 100 points. The standardized overall values are shown in Table 5.6. For example, the *Added Value* heuristic is made up of proposals {15, 19, 20, 22} and yields an overall value of 99.

We found that on average RPM solutions, also known as non-dominated solutions, performed better than heuristic solutions. The latter scored an average of 95 whereas the former scored an average of 99. For each problem size, we observed that the optimal portfolio was among RPM portfolios but not among heuristic solutions.

In the second approach, the measure of quality of a portfolio is the percentage of its proposals that are part of the optimal portfolio. If a portfolio had all the proposals found

in the optimal portfolio, it got a score of 1 and if none of its proposals were in the optimal portfolio, it got a score of 0. The scores are presented in Table 5.7. For example, for the four proposal-solutions, three quarters of proposals in the *Added Value* portfolio are found in the optimal portfolio (See Table 5.6). We found that on average RPM solutions performed better than heuristic solutions: the latter scored an average of 0.60 whereas the former scored an average of 0.89. This means RPM solutions have more in common with optimal solutions in terms of portfolio composition than heuristic solutions.

For problems involving five proposals, we observed that the RPM portfolio is the same as the optimal portfolio.

4 Proposal-Solutions					5 Proposal-Solutions					6 Proposal-Solutions								
Heuristic Solutions																		
	P1	P2	P3	P4	Overall Value	P1	P2	P3	P4	P5	Overall Value	P1	P2	P3	P4	P5	P6	Overall Value
Added Value	15	19	20	22	99	14	15	19	20	22	97	1	14	15	19	20	22	95
	14	19	20	22	93	1	14	19	20	22	90	1	14	15	19	20	22	95
RPM Solutions																		
	P1	P2	P3	P4	Overall Value	P1	P2	P3	P4	P5	Overall Value	P1	P2	P3	P4	P5	P6	Overall Value
Portfolio1	9	15	17	19	96	6	15	17	19	20	100	6	9	15	17	19	20	97
Portfolio2	14	15	17	19	99							6	14	15	17	19	20	99
Portfolio3	15	17	19	20	100							6	15	17	19	20	22	100
Portfolio4	15	17	19	22	100							6	15	17	19	20	23	99
Portfolio5	15	17	19	23	98							6	15	17	19	20	24	98
Portfolio6	15	17	19	24	97													
Optimal Solution																		
	P1	P2	P3	P4	Overall Value	P1	P2	P3	P4	P5	Overall Value	P1	P2	P3	P4	P5	P6	Overall Value
Portfolio1	15	17	19	22	100	6	15	17	19	20	100	6	15	17	19	20	22	100

Table 5.6: 4-6 Proposal-Solutions: Portfolios selected using linear-additive portfolio value model, robust portfolio modeling (RPM) and simplified portfolio models in the form of heuristics (*Added Value* (AV) and *Unity Value With Synergy* (UVwS)) and their corresponding overall values. For example, for the 4 proposal-solutions, the *Added Value* heuristic is made up of proposals {15, 19, 20, 22}. Overall values are standardized such that optimal portfolios have a value 100.

	4 Proposals	5 Proposals	6 Proposals
Heuristic Solutions			
Added Value	0.75	0.60	0.67
UV with Synergy	0.50	0.40	0.67
RPM Solutions			
Portfolio1	0.75	1.00	0.83
Portfolio2	0.75		0.83
Portfolio3	0.75		1.00
Portfolio4	1.00		0.83
Portfolio5	0.75		0.83
Portfolio6	0.75		

Table 5.7: The quality of portfolios selected by *Added Value* and *UV with Synergy* heuristics and RPM, also called heuristic and RPM portfolios, is assessed as a proportion of optimal portfolio proposals found in the heuristic and RPM portfolios. Columns show portfolio quality for 4 to 6 proposal-solutions. Optimal portfolios were already shown in Table 5.6.

Core indices are shown in Table 5.8. We observed that proposals {15,17,19} had a mean score of 1, being part of every RPM portfolio and were therefore strongly recommended by RPM. We observed that these proposals were also part of every optimal portfolio (Table 5.6).

It is worth noting that proposals which are part of optimal portfolios, i.e. proposals {6,15,17,19,20,22}, stood out from the rest. They scored 80 or 90 points on two or three of the most important criteria: recognition of previous work, quality of publications and impact of prior work. When they performed below 80 points on any of the three criteria, they compensated this by performing exceptionally well (90 points) on the other two criteria (See Table C.2).

	PROPOSALS									
	<i>P6</i>	<i>P9</i>	<i>P14</i>	<i>P15</i>	<i>P17</i>	<i>P19</i>	<i>P20</i>	<i>P22</i>	<i>P23</i>	<i>P24</i>
4 Proposal-Solutions	0	0.17	0.17	1	1	1	0.17	0.17	0.17	0.17
5 Proposal-Solutions	1	0	0	1	1	1	1	0	0	0
6 Proposal-Solutions	1	0.2	0.2	1	1	1	1	0.2	0.2	0.2
Average	0.67	0.12	0.12	1.00	1.00	1.00	0.72	0.12	0.12	0.12

Table 5.8: Core Index of proposals defined as the relative frequency at which each proposal appears across RPM portfolios, also known as non-dominated portfolios. A score of 1 indicates that the proposal is found in every non-dominated portfolio and a score of 0 indicates that the proposal is not found in any of the non-dominated portfolios.

5.4.2 Comparisons with unfacilitated proposal selections

We conducted an experiment with the decision maker where they selected portfolios without facilitation, i.e. without looking at the portfolios suggested by the application, by only considering the average profile of the proposals and the weight attached to the criteria and interaction values as displayed by the application. The decision maker later explained that they computed the weighted average of the proposals and sorted them in descending order. They chose proposals from the list until they reached the total number of proposals relative to the portfolio size they were interested in i.e. portfolio sizes four to six. We show the results in Table 5.9. The approach described by the decision maker is equivalent to the *Unit Value* heuristic.

	P1	P2	P3	P4	P5	P6	Overall Value	Optimality
4 Proposal-Solution	14	19	20	22			93	0.50
5 Proposal-Solution	1	14	19	20	22		90	0.40
6 Proposal-Solution	1	14	15	19	20	22	95	0.67

Table 5.9: Portfolios selected by the decision maker without facilitation i.e. without the help of the web-based application. For each portfolio size, we show respectively the proposals selected in the middle column, corresponding overall values and the percentage of its proposals that are part of the optimal portfolio (Optimality).

We have noted that in our case study *Unit Value* portfolios match those selected using the *Unit Value with Synergy* for portfolios size four to six (See Table 5.6) because proposals scoring higher on the most important criteria were involved in interactions. This is due to the setup of the current case study but the outcome could be different in a different context.

5.4.3 Sensitivity Analysis

The first aspect of the sensitivity analysis was done by changing the value of weights and observing new results. The research manager wanted to investigate whether the optimal portfolios were robust solutions i.e. portfolios would still be optimal even if his or her preferences were to change. They looked at the following issues in order to check whether optimal portfolios were robust solutions:

- Do optimal portfolios (e.g. {6, 15, 17, 19, 20, 22}), given in Table 5.6, remain optimal even if the preferences of the research manager were to change?
- Would a plausible change of weights lead to different portfolios?

The research manager looked at five scenarios translating the weight changes they wished to make. Initial and subsequent weights are shown in Table 5.10. Initial weights (weight set 1) lead to the following optimal portfolios for sizes four to six, respectively: $\{15, 17, 19, 22\}$, $\{6, 15, 17, 19, 20\}$ and $\{6, 15, 17, 19, 20, 22\}$. We subsequently computed the optimal portfolio using the other weights (weight sets 2 to 6) and observed resulting portfolios as displayed by the application. In weight set 2, for example, the weight of the most rated criterion (“Impact of prior work”) was increased from 0.13 to 0.2 and this was compensated by a decrease in the four least rated criteria to a value of 0.05. A similar approach was applied in the next weight sets. In each instance (weight sets 2 to 6), the resulting portfolio was the same as the initial one for five and six proposal-solutions. For 4 proposal-solutions, we got a different solution in weight set 5: proposal 22 got dropped and was replaced by proposal 20. Based on this, the decision maker was satisfied that portfolio $\{6, 15, 17, 19, 20\}$ and $\{6, 15, 17, 19, 20, 22\}$ (5 and 6 proposal-solutions) were robust choices. Portfolio $\{15, 17, 19, 22\}$ (4 proposal-solution) was almost robust and only ceased to be the optimal solution when the last two sets of weights were applied. We argue that given that the weights in step 5 and 6 are quite extreme, e.g. the first criterion gets a weight worth 40% to 50% of total impact of weights, this portfolio should still receive favorable consideration. It is also interesting to note proposal 20 had a higher mean core index than proposal 22 (Table 5.8). It makes, therefore, sense that proposal 20 replaced 22, as it had a stronger robustness profile, yielding the following portfolio: $\{15, 17, 19, 20\}$.

Weight sets	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
1	0.13	0.11	0.11	0.1	0.1	0.09	0.09	0.08	0.07	0.07	0.05
2	0.2	0.11	0.11	0.1	0.1	0.09	0.09	0.05	0.05	0.05	0.05
3	0.2	0.13	0.13	0.12	0.1	0.07	0.07	0.05	0.05	0.05	0.03
4	0.3	0.2	0.1	0.09	0.08	0.07	0.06	0.04	0.03	0.02	0.01
5	0.4	0.3	0.2	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
6	0.5	0.3	0.07	0.05	0.02	0.01	0.01	0.01	0.01	0.01	0.01

Table 5.10: The research manager conducted a sensitivity analysis by changing the weights of criteria. We term this analysis: weight sensitivity analysis. The research manager specified 11 criteria and the list of criteria C_1 to C_{11} is given respectively as follows : recognition of previous work, quality of publications, impact of prior work, mentorship, potential impact of proposal, feasibility and risk, scientific merit, novelty, respond to a challenge in Africa, focus areas and gender. The research manager wanted to investigate how the change of weights would impact the composition of optimal portfolios. Initial weights are shown in the first row. Subsequent weights are shown in rows 2 to 6.

The second aspect of the sensitivity analysis was done by the selection and deselection of proposals in order to improve the initial portfolio selected by the decision maker without facilitation. The research manager had the following question in mind: Can I

improve on my initial portfolio by adding and dropping proposals?

The research manager used the selection/deselection feature found on the web-based application to conduct the sensitivity analysis. At each step, they either added and removed proposals. Resulting portfolios are shown in Table C.1 in the appendix Section C. The research manager explained that they followed a process of trial and error to improve their portfolio but they were guided by the overall values of portfolios as displayed by the web-based application. We should note that overall values of portfolios are inclusive of interaction values. The research manager was able to generate many portfolios, among which was the optimal portfolio shown as portfolio 25 in Table C.1.

For each project selection, we identified which heuristics were consistent with that selection (Table C.1). We found that the decision maker used the *Added Value Random* heuristic for the most part, followed by the *Lowest Cost* heuristic. As already mentioned, the optimal portfolio was one of the portfolios selected in the course of the sensitivity analysis. The decision maker retained the optimal portfolio as their best portfolio because it had the maximum overall value among the generated portfolios. The value of this exercise is to show how the decision maker can discover which combinations of alternatives work well together and appreciate the incremental value resulting from adding or removing individual alternatives. This finding confirms the hypothesis (H6) that the web-based application is an effective tool for sensitivity analysis and learning.

5.5 Discussion

5.5.1 Possible extensions

In this chapter, we have analyzed heuristic portfolios among various types of portfolios. We suggest that this analysis be extended to include combinations of current heuristics as well as other heuristics as this will give a better understanding of heuristic portfolios. We only included *Added Value* and *Unit Value with Synergy* heuristics in our case study because simulation studies in the previous chapter established these heuristics as the best performing ones among the set of heuristics that we tested. In addition, both heuristics performed almost as well as optimal solutions under conditions highlighted in Section 4.3. However, these heuristics may not always be the best approaches in a different context, and it may also be advantageous to show portfolios constructed using other heuristics to the decision maker in an effort to stimulate their thinking about alternative portfolios and the reasons why one portfolio would be better than another one. Given that it is computationally simple to construct portfolios using heuristics, future work should

provide the decision maker with portfolios from a greater range of heuristics.

5.5.2 Feedback from the decision maker

The research manager found the web-based application intuitive and easy to use. They also found the visualization of proposal data and portfolio solutions in both graphical and tabular format useful. In their view, the application makes it easy to assess interaction benefits and identify optimal portfolios. They found the ability to select proposals at will and determine the size of portfolios to be useful features. They observed that the web-based application reduces processing time for the selection of proposals, and removes a lot of manual processes used in the past and speeds up the process of identifying the best proposals among the set of available proposals. Finally, based on their previous experience in selecting portfolios of proposals, they concluded that the web-based application reduces the associated biases in the unfacilitated selection of proposals.

5.6 Conclusions

This chapter describes the construction of a real-world decision support system to assist in the funding of grant proposals at a research institution. We have built a web-based application as an integral part of the decision support system. We applied portfolio decision analysis methods and heuristics to the selection of the best subset of proposals in the presence of possible interactions between proposals. We specifically applied the linear-additive portfolio value model, robust portfolio modeling and simple portfolio models using *Added Value* and *Unit Value With Synergy* heuristics to the selection of proposals and these were automated via the web-based application. For each possible portfolio size, we show all the portfolios selected using these models and provide insights about the robustness of portfolio solutions. We should point out that if it is possible to generate the optimal portfolio, then this is obviously the preferred approach. The other methods (RPM and heuristics) are more for providing information about other potentially good portfolios, which can either be compared the optimal portfolio, if the optimal portfolio is available (so, as extra information), or as a way of generating portfolios if it is not possible to compute the optimal portfolio. The application comes with built-in capability for running sensitivity analysis. Results are downloadable and selections made by the user are traceable, making the tool more effective. The decision support system that we have provided still requires an analyst to help the decision maker provide preferences in terms of individual criteria, the aggregation model allowing trade-offs between criteria and interaction values. Based on all this information, the web-based application provides an overall evaluation of alternatives in the form of an

overall value for each alternative.

We have learnt the following lessons from this research:

The decision maker failed to integrate interactions between proposals when making portfolio selections without facilitation: We observed that for each problem, the decision maker selected the *Unit Value* portfolios instead of optimal portfolios when selecting without facilitation. This suggests that the decision maker was not able to account for interactions into their selections. This highlights the value of the decision support we provided to them including the web-based application, which provides optimal solutions and other solutions such RPM and *Added Value* heuristic, that explicitly include interaction values.

Both overall portfolio value and percentage of proposals in common with the optimal portfolio can guide decision making: We know that a portfolio with a higher overall value is more desirable. It should be noted that a portfolio that has more proposals in common with the optimal portfolio may not have a higher overall value than the portfolio that has less proposals in common with the optimal portfolio. However, the latter approach helps to understand how portfolios are different from each other. Because both approaches look at different aspects of the performance of portfolios, we believe that they should be used in conjunction with each other for a better understanding of the performance of portfolios.

RPM method can help with sensitivity analysis (Hypothesis H6): If the decision maker does not know the weights of criteria but is interested in selecting a portfolio that is robust against this uncertainty, they could look at the (mean) core index of each of the proposals in the portfolio being considered (See Table 5.8) and select proposals with a higher core index. In the 4 proposal-solutions (Table 5.6), for instance, RPM portfolios $\{15, 17, 19, 20\}$ and $\{15, 17, 19, 22\}$ differ by one proposal: 20 vs 22, respectively. It turns out that proposal 20 has a higher core index than proposal 22 rendering the former portfolio more robust than the latter. As seen in Section 5.4.1, portfolio $\{15, 17, 19, 22\}$ is the optimal portfolio for portfolios of size 4 but portfolio $\{15, 17, 19, 20\}$ has effectively the same value as the optimal portfolio i.e. 99.63, that was rounded to 100. This shows us that there are a few proposals that are key to a good portfolio (proposals 15, 17 and 19 in this instance), but then a few more that can be substituted by different proposals that would still offer a similar contribution to the overall portfolio, e.g. proposal 22 in portfolio $\{15, 17, 19, 22\}$ was substituted by proposal 20, in the example above, to form portfolio $\{15, 17, 19, 20\}$. This means

that we can choose a different set of proposals to the optimal portfolio, but still come up with a portfolio that is nearly as good as the optimal portfolio.

The web-based application is a useful tool for learning about interactions and sensitivity analysis (Hypothesis H6): The web-based application computes heuristic, RPM and optimal portfolios by factoring in interaction values. The decision maker used the application to conduct a sensitivity analysis by changing weights of criteria to investigate the robustness of portfolios, on the one hand, and then by adding and removing proposals to build their preferred portfolio, on the other hand. In this way, they could work out which proposals work well together and interact better and were even able to find the optimal portfolio using this method.

We observed that the decision maker selected portfolios using the *Unit Value* heuristic when selecting without facilitation. This observation is in line with the one made in Section 3.5: a lot of participants used *Highest Value* and *Unit Value*, heuristics that ignore interactions. We also found that *Unit Value with Synergy* and *Added Value* still performed well against optimal solutions and produced portfolios with standardized overall values ranging between 90 and 99 (the optimal value being standardized to 100 points) and this is in line with the results in the previous chapter. Finally, another contribution of this work is the automation of heuristics for portfolio selections (*Highest Value* and *Unit Value*), which is a unique feature of our decision support system as the heuristics themselves are a novel contribution.

Chapter 6

Conclusions and Future Research

6.1 Discussion

We know that computational requirements for choice problems are manageable in the majority of situations but this is not true of portfolio decisions. It is also true that often decision makers do not have the time, expertise and ability to exert the necessary effort to assess all the information required of an exact method. The difficulty is further compounded by the existence of interactions between alternatives making computation of portfolio solutions by exact methods even more challenging. Heuristics provide strategies that can perform well under such circumstances due to the fact that they rely on simple search rules, aspiration levels, lexicographic rules and other principles. In addition, situations where there is perfect knowledge of all outcomes call for the use of prescriptive decision making such as profit maximization and value functions. In most real-world problems, decision making happens under uncertainty, i.e. where the set of alternatives and their consequences are not known. Such situations call for the use of simplified models for portfolio decision making (heuristics) as prescriptive methods cannot work in those instances (Gigerenzer and Gaissmaier, 2011; Gigerenzer and Mousavi, 2014).

Heuristics have been studied mainly for choice problems. Although some work has been conducted on heuristics in a portfolio context, the research has been limited in scope and no attempt was made at assessing heuristic performance. Our aim was to verify in Chapter 3 the use of heuristics proposed in Durbach et al. (2020) in portfolio selections, in contexts where there are interactions between alternatives. Our results established that participants selected portfolios in a way that was not consistent with a random selection of alternatives. We verified that most participants made selections consistent with heuristics studied in this research work, especially those that ignore interactions. The finding confirms the use of the proposed heuristics but also indicate that, overall, people fail to account for interaction values in their portfolio selections. Research has established that in traditional choice problems people tend to use simple heuristics as

decisions become more complex (Payne et al., 1993). Our results also confirm that people tend to use simple heuristics in portfolio selection problems.

Having established the use of heuristics in portfolio selections, we set out to evaluate the performance of heuristics in Chapter 4. We compared the performance of heuristics to optimal models under a considerable range of conditions. We looked at two classes of heuristics: heuristics for the selection of portfolios (selection heuristics) and heuristics for aggregating the performance of alternatives across criteria (scoring heuristics). The first iteration of our simulation experiment included both positive and negative interactions. The second iteration involved multiple criteria. We found that having negative interactions does not harm the performance of heuristics. We found that certain portfolio heuristics (*Added Value* and *Unit Value with Synergy*) performed well in the presence of negative interactions and multiple criteria and can provide outcomes that are competitive with optimal models. This is reassuring, as many practical problems will have both negative and positive interactions. These results are an extension to the work presented in Durbach et al. (2020) which only looked at the performance of heuristics in environments involving single criteria and positive interactions between alternatives. We also verified heuristics that ignore interactions lead to less optimal results. We observed that the performance of a portfolio depends more on the approach used to build the portfolio (selection heuristics) than on the method of aggregation across criteria (scoring heuristics). To our knowledge, this something that had not been investigated yet in the literature.

Lastly, we wanted to provide a decision support system for the selection of portfolios in Chapter 5 in the form of a web-based application. This is to assist decision makers to apply heuristics that we have presented in this research work and compare the results thereof with those from prescriptive portfolio decision analysis methods. Both heuristics and exact portfolio decision analysis methods are explained in Chapter 2. The practical application was concerned with the selection of the best subsets of research proposals at a research institution. The main findings were firstly that the web-based application is a useful tool for learning about interactions and for conducting sensitivity analysis. The decision maker used the application to conduct a sensitivity analysis of portfolios by changing weights of criteria to investigate the robustness of portfolios and then by adding and removing proposals to build their preferred portfolios. The decision support system provided optimal portfolios along with a set of portfolios, also known as heuristic portfolios, that are selected by portfolio heuristics. Heuristic portfolios are useful as they could be selected under uncertainty or used if the decision maker does not have the resources to find the optimal solution for a very complex portfolio decision problem (e.g.

a much higher number of alternatives or the presence of many interactions between them).

In a recent literature review spanning the years 2006 to 2019, Liesiö et al. (2021) found that roughly half of the decision support systems use exact algorithms, leading to (globally) optimal solutions, whereas the other half use approximate algorithms, leading to reasonably good solutions without guarantees of optimality. The vast majority of software deal with MCDA problems involving the selection of a single, best alternative from a larger set of potential options. There are only a handful of software that deal with portfolio decision making, some of which involve interactions and sensitivity analysis: HiPriority, PROBE, Expert Choice, DPL 8, etc. Weistroffer and Li (2016). Likewise, our software deals with portfolio decision making involving interactions and also helps with sensitivity analysis. The merit of our decision support system is to support the implementation of heuristics for portfolio selections. To our knowledge, this is a novel feature, understandably enough, as these heuristics have only been proposed recently in Durbach et al. (2020). As was shown in our simulation study in Chapter 4, these heuristics can perform as well as prescriptive portfolio decision analysis methods in certain conditions and are required when we do not have perfect knowledge of all outcomes and probabilities of each outcome.

As with all experiments, there are limitations to the approach followed in the thesis, that need to be considered in interpreting the results. Participants in the behavioral experiment were all university students and generally more adept at doing complex calculations and were better at handling portfolio decision making than the average decision maker. We suggest that future behavioral experiments use samples that are more representative of the average decision maker. Also, our simulation experiment results are dependent on the parameters that we have used in the simulation. The results could be different if different parameter values were used. For example, we do not claim that *Added Value* and *Unit Value with Synergy* will perform well under any circumstances. Instead, we report the finding that in our simulation experiment both heuristics were the best performers under the conditions highlighted in Chapter 4.

In the next section, we explain the main findings in detail.

6.2 Summary and Conclusion

We set out to examine the use and effectiveness of simplified models for portfolio decision making and provide a decision support system for portfolio decisions. Making use of heuristics proposed in (Durbach et al., 2020), i.e. the Add-the-best family of heuristics,

we aimed to use a behavioral experiment to assess portfolio heuristic use in unfacilitated decision making, extend the simulation framework to assess the performance of these heuristics in a new environment involving multiple criteria and both positive and negative interactions, and to apply both exact and heuristic methods in an applied setting. In this final chapter, we summarize the main findings and discuss implications for portfolio decision making. Our main findings can be listed as follows:

Finding 1: We found evidence that people use heuristics in actual portfolio decision making.

There is evidence that people use heuristics (Tversky, 1972; Gigerenzer and Goldstein, 1996; Katsikopoulos, 2011) when confronted with traditional (one-out-of-n) choice problems. It has been found that heuristics can perform as well as theoretically optimal models under reasonably well-known conditions (Katsikopoulos et al., 2018; Todd, 2007; R.M. Hogarth, 2005; Baucells et al., 2008b; Buckmann and Şimşek, 2017), some of these conditions often occur in real-world contexts (Şimşek, 2013) and decision makers use heuristics, particularly when time pressure or the cost of gathering information is high. However, the literature did not provide evidence that heuristics are used in portfolio selections (Bröder and Newel, 2008; Ford et al., 1989). This is what we sought to investigate in our research work. In the experiment that we conducted, we gave participants a set of tasks where they had an opportunity to select portfolios without facilitation. The tasks varied in their levels of complexity going from a simple task (task 1) of selecting alternatives with no interactions between them to more complex tasks involving either interactions (task 2) or more alternatives (task 3). We found a widespread use of heuristics by the participants and that individuals used various heuristics. Our results show that participants selected portfolios in a way that was not consistent with a random selection of alternatives but was instead consistent with the *Highest Value* and *Unit Value* heuristics. Also, participants added sequentially alternatives to their portfolios in an ordering that was mostly consistent with the *Highest Value* and *Unit Value* heuristics. Finally, the most popular selection of alternatives were consistent with the *Unit Value* or *Highest Value* heuristics.

Finding 2: Adding negative interactions does not harm the performance of heuristics. This is reassuring, as many practical problems will have both negative and positive interactions.

Added Value and *Unit Value with Synergy* stood out from all the heuristics tested in this experiment giving an indication that they can lead to better decisions than the other heuristics. The variant of *Added Value* came second, although the gap between *Added Value* and its variants (*Added Value Most*, *Added Value Least* and *Added Value Random*)

was larger than under positive interactions only. The heuristics that ignore interactions (*Lowest Cost*, *Highest Value*, *Unit Value* and *Pareto*) came in the last position. The performance of heuristics was assessed based on the mean value of portfolios selected by the heuristics relative the mean value of optimal portfolios. The *Pareto* heuristic performed similarly to the *Unit Value with Synergy* at higher levels of budget i.e. between 80% and 100% of total costs. The continued good performance of the *Unit Value with Synergy* is particularly noteworthy. Because this heuristic starts by excluding all alternatives not involved in positive interactions, it may have been expected that the heuristic would perform well in the positive-only context studied in Durbach et al. (2020). However, it is much less obvious that the heuristic would perform well when alternatives are also involved in negative interactions.

Negative heuristics have been discussed in a portfolio context (Stummer and Heidenberger, 2003; Litvinchev et al., 2013) but we have not found research specifically dealing with how negative interactions affect the performance of heuristics. Our research deals with this issue and shows that adding negative interactions does not harm the performance of heuristics.

Finding 3: Heuristics can provide outcomes that are competitive with optimal models under a considerable range of conditions.

In our simulation experiment, we used the linear-additive portfolio value model to identify optimal portfolios. we found that portfolios selected with *Added Value* and *Unit Value with Synergy* were closer to the optimal portfolio. This is very significant given the fact that these heuristics consider only a fraction of required information by optimal methods, hence they are frugal. The benefit is noticeable when one considers in particular the *Unit Value with Synergy* heuristic. It only requires information about which alternatives are involved in any positive interactions, with no regard to the magnitude of interaction values and then sorts alternatives based on their value-to-cost ratios. We found that the *Unit Value with Synergy* was almost as good as the optimal portfolio approach for budget values between 20% and 30% of total costs. This heuristic illustrates a good balance between effort and accuracy that is often desired in decision making. As seen, prescriptive models can be quite complex and require information that may not at times be available or the cost of obtaining such information can be exorbitant. It is very comforting to know that under these conditions heuristics are a viable option. This finding is in line with Keisler (2008) who showed that the strategy that applies equal weights to all criteria achieved values closer to optimal portfolio values. Similar results are found in Winterfeldt and Edwards (1986), showing that inexact weights work rather well.

In addition, the *Added Value* and *Unit Value with Synergy* were found to perform best when interactions were nested rather being random and took place between individually good alternatives (Durbach et al., 2020).

Finding 4: Heuristics that ignore interactions lead to less optimal results.

Heuristics that ignored interactions (*Lowest Cost*, *Highest Value*, *Unit Value* and *Pareto*) are the worst-performing — even being equivalent to a random selection of alternatives at times — because they ignore key information about alternatives. We observed a substantial gap between their performance and optimal methods. This is in line with Keisler (2005) who showed that identifying interactions added substantial value to portfolios compared to the baseline, i.e. the strategy that ignores interactions. In fact, the optimal portfolio value was on average worth almost as twice as the baseline. Similar results are also found in Clemen and Kwit (2001); Rzaza et al. (1990).

Finding 5: The performance of a portfolio depends more on the approach used to build portfolios (selection heuristics) than on the method of aggregation across criteria (scoring heuristics).

Durbach et al. (2020) proposed heuristics for the selection of portfolios (selection heuristics). In addition to these, we looked at heuristics for aggregating the performance of alternatives across criteria. Because we deal with a multi-criteria decision context, we aggregated the performance of alternatives on individual criteria using scoring heuristics to derive an overall score for each alternative across criteria. The following scoring heuristics were used: the equal weighting function (EQW), lexicographic (LEX) and tallying (TAL) heuristics and the prescriptive scoring approach (the weighted additive method (WADD)).

We evaluated the performance of scoring heuristics by comparing the values that they assign to alternatives with the value assigned by the WADD function to the same alternatives. We found that all scoring heuristics are a good approximation for WADD at lower levels of budget. EQW, in particular, is a good approximation for WADD at all levels of budget. However, the mean values of heuristics were not too different from the mean value produced by WADD in absolute values, i.e. differences were less than 5% of the mean values. This shows that the heuristics used to choose alternatives (*Highest value*, *Lowest cost*, *Unit value*, *Unit Value with Synergy* and *Added Value*) are much more important in determining final portfolio value than how performance is aggregated across criteria (EQW, LEX and TAL). Results for the multiple criteria case under the WADD scoring were also largely similar to the single-criterion case. Once again, we found that, overall, *Added Value* and *Unit Value with Synergy* performed better than the other heuristics. Heuristics that ignore interactions were again the worst-performing heuristics.

We have not found research specifically comparing selection heuristics and scoring heuristics. However, the successful use of selection heuristics has been documented in several studies in choice problems. Albar and Jetter (2013) showed that the tallying heuristic can be an effective strategy for evaluating new product ideas and performed as well as a regression model with almost an eighty percent success rate. Similarly, the take-the-best heuristic was the best-performing strategy in non-compensatory environments (no trade-offs among criteria) (Hogarth and Karelaia, 2007) and had a higher predictive accuracy with an increased sample size (Katsikopoulos et al., 2010). Gigerenzer and Goldstein (2011) also show that the recognition heuristic can be a very effective decision strategy across a wide range of tasks. For this reason, it is not be surprising that our research indicates that the portfolio selection aspect is more important than the scoring one.

Finding 6: Overall, we found that decision makers failed to integrate interactions when making portfolio selections without facilitation.

We assisted a research institution dealing with an actual portfolio decision problem. The management is faced with the selection of the best subset of research proposals out of a pool of proposals submitted by researchers who apply for grants to the research institution. We observed in section 3.4 that a lot of participants used *Highest Value* and *Unit Value*, heuristics that ignore interactions. The same observation was made in this practical problem as the research manager selected the *Unit Value* portfolios instead of optimal portfolios when selecting without facilitation and thus ignored interactions although they were aware of interaction values.

Klein et al. (1986) found that fire ground commanders, who operate under uncertainty and time pressure, that prohibit the generation and comparison of options, rely on the recognition of an object to make a successful prediction of the target variable. This approach has also been applied in many other decision-making contexts such as offshore oil installations and system designs (Artinger et al., 2015a). Dhimi (2003) observed that over 85 percent of decisions in magistrate courts were correctly predicted by a fast and frugal tree of heuristics. Achleitner et al. (2012) showed that people relied on one of two cues, voluntary accountability efforts and reputation, to invest in social ventures. These studies point to the fact that decision makers, when faced with complex situations, ignore part of available information. Therefore, the finding that decision makers failed to integrate interactions when making portfolio selections without facilitation is consistent with previous findings.

We provided a decision support in the form of a Web-based application to assist with portfolio decisions involving interactions and sensitivity analysis. The web-based application computes portfolios selected by heuristics, robust portfolio model and linear-additive portfolio value model (optimal portfolio) by factoring in interaction values. We implemented the *Added Value* and *Unit Value with Synergy*, which were the two best-performing heuristics in the simulation experiment. The decision maker used the application to conduct a sensitivity analysis by changing weights of criteria to investigate the robustness of portfolios and by adding and removing proposals to build their preferred portfolio and compared this to the optimal portfolio. They were able to identify which proposals work well together and interact better and were even able to work out the optimal portfolio using this method. An interesting finding was that they could choose a different set of proposals to the optimal portfolio, but still come up with a portfolio that is nearly as good as the optimal portfolio.

Our decision support system was designed to help a research institution in selecting the best portfolio of research proposals and was tested by its research manager only. We acknowledge that the observations made about the performance of the Web-based application and the feedback received need to be understood in this context.

6.3 Questions for future research

We have gained a fresh understanding in the field of behavioral decision making and have been able to harness the benefits of both prescriptive methods and heuristics in portfolio decision making. In spite of our findings, there remains a number of issues that could be addressed in future research. Here are specific questions that could be answered:

Question 1: How do people select portfolios under risk without facilitation?

We suggest that future research should be done on how people select portfolios under risk without facilitation. In such cases, an individual would have to select the best portfolio of alternatives with uncertain outcomes. It is necessary in such instances to specify the probability distribution of outcomes. It will be interesting to see whether we will see the same patterns that were observed in the deterministic case, i.e. under certainty of outcomes. It will be intriguing to see what new insights are gained in the new experiment. We presented in Chapter 2 the scenario-based portfolio selection with complete probability and utility information, a prescriptive methods under risk (Liesiö and Salo, 2012; Rockafellar and Uryasev, 2000; Dentcheva and Ruszczyński, 2006; Liesiö et al., 2007; Villarreal and Karwan, 1981; Kiziltan and Yucaoglu, 1983). In this new experiment, we can compare

portfolios selected by participants with optimal portfolios derived from the prescriptive method under risk.

Question 2: How do people select portfolios based on multiple objectives?

In the thesis, we looked at portfolio decision problems with one objective in the behavioral experiment, i.e. the amount of profit realized by each alternative. This was also done in Durbach et al. (2020). Researchers could consider more than one objective in future work. This will add to the complexity of the problem. As such, portfolio decision analysis even with one objective can be quite complex if you include more alternatives and allow for interactions between them. However, a portfolio decision problem with multiple objectives is even more complex but is closer to real-life portfolio decision problems as it is generally the case that portfolio selection happens in this context. The proposed research work will help confirm whether we have a behavioral pattern in the multiple objectives experiment different from the one observed in the single objective experiment.

Question 3: How do we select portfolios in multiple categories at once?

In our experiment, we focused on the selection of the best subset of alternatives, i.e. we identified alternatives in the “category” referred to as the best subset (optimal solution). However, it is possible that decision makers are interested in a few categories rather than a single one: this is similar to classification problems discussed in Mousseau et al. (2002, 2011) where students are assigned to different streams. This would extend the decision problem we have dealt with. It would, therefore, be interesting to see how the simulation experiment could be extended to deal with this type of problems.

Question 4: How do heuristics perform under risk?

Our simulation experiment was based on the assumption that the values of alternatives on relevant criteria and interactions between them were well known. Future research can look at the performance of heuristics under risk and make recommendations on how to best apply heuristics in such instances. Prescriptive methods such as the scenario-based portfolio selection with complete probability and utility information (Liesiö and Salo, 2012; Rockafellar and Uryasev, 2000) are designed for this type of situations and could serve as a benchmark against which the performance of heuristics can be measured. Although selection heuristics in Durbach et al. (2020) can be applied successfully in a portfolio context, it is unclear how they would work under risk. In this case, one could account for possible turn of “events” or “states” for the purpose of decision making, and attach probabilities to these. Further work should be done to extend selection heuristics to this case. Also, a non-linear model could be used as a benchmark against which the performance of

heuristics and it would be interesting to see whether outcomes will be different from those obtained in this research work.

Question 5: How do heuristics perform when we account for other forms of interactions ?

The methods used in this research work account for value interactions between alternatives i.e. the added value resulting from the inclusion of alternatives that work well together. For further research, it would be interesting to look at other types of interactions. Future work could extend our research by modeling cost interactions as well. Cost interactions are also included in the robust portfolio modeling (RPM) (Villarreal and Karwan, 1981; Liesiö et al., 2007) and this could inform this extension. In addition, we could model outcome interactions as those presented in DEA-BSC (Eilat et al., 2006, 2008; Greenberg and Hegerich, 1970; Martello et al., 2000). Outcome interactions are such that the probability of success of an alternative depends on the success of another alternative. Including new types of interactions will give the audience a broader understanding of the impact of interactions on portfolio selections. This would also broaden the range of applications in portfolio decision analysis.

Question 6: How do we refine heuristics that account for interaction values?

We have presented selection heuristics proposed in Durbach et al. (2020) and explained them. These models are designed for portfolio selections and do not rely on all information available. Heuristics that account for interaction values fall under the “Added Value” class in the *add-the-best* family. In the presence of a bigger set of alternatives, these heuristics can become quite overwhelming for the decision maker. Therefore, future work should find ways to simplify “Added Value” heuristics in this case.

Question 7: How do we create new heuristics for portfolio decision problems?

Heuristics presented in the thesis are in the *add-the-best* family. Other classes of heuristics are presented in the literature such as recognition heuristic, fluency heuristic, 1/N rule, imitate the majority, etc. (Gigerenzer and Mousavi, 2014). These classes could also be adapted to portfolio decision problems and this would provide a richer toolbox of heuristics to the decision maker and enable them to address a broader range of portfolio decision problems.

Question 8: How to investigate learning by decision makers in portfolio decision problems involving interactions?

We have acknowledged the limitations of the number of dominated alternatives as an in-

indicator of learning in the tasks where there were interactions between alternatives. It is possible that when people switch heuristics, it could be an indication that they are learning to make better choices. However, the act of switching heuristics is not in itself enough to infer learning but needs to be associated with a positive outcome such as the selection of a better portfolio. This could be explored in future work to investigate learning by decision makers in portfolio decision problems involving interactions.

Question 9: How to investigate the effort made by decision makers in portfolio decision problems by tracking choices made by the same individuals across tasks?

In analyzing the effort made by participants across tasks, we found that there was only weak evidence that differences between tasks were significant. Given this finding, there is need to conduct further investigation on this subject. In this dissertation, we did not track choices made by the same individuals across tasks. This approach could help us better understand effort made by participants in carrying out portfolio selection tasks. As the experiments in our research work were not designed to track the same individuals across tasks, we believe that this approach should be explored in future research.

Appendix A

Notations used

The following symbols and abbreviations are used in this thesis:

- Alternatives j, k , etc.
- x_{ij} : performance level of alternative j on criterion (output) i
- r_{sj} : performance level of alternative j on criterion (input) s
- ω_i / v_s : (global) weight of criterion i / s
- ω_{ij} : (local) weight of criterion i for alternative j
- z_j : binary variable $\in \{0, 1\}$ for alternative j . If j is selected, $z_j = 1$
- Pr : probability
- p / x : alternative $p /$ value of alternative p
- P : portfolio P
- Π : Portfolio set
- Π_N : non-dominated portfolio set
- Π_F : feasible Portfolio set
- s_{jk} : benefit interactions between DMU j & DMU k
- V/U : value / utility function
- DMU(s): Decision Making Unit(s)
- s_q : scenario q
- X : alternative scores/performance levels matrix

Appendix B

Simulation results for multi-criteria setting

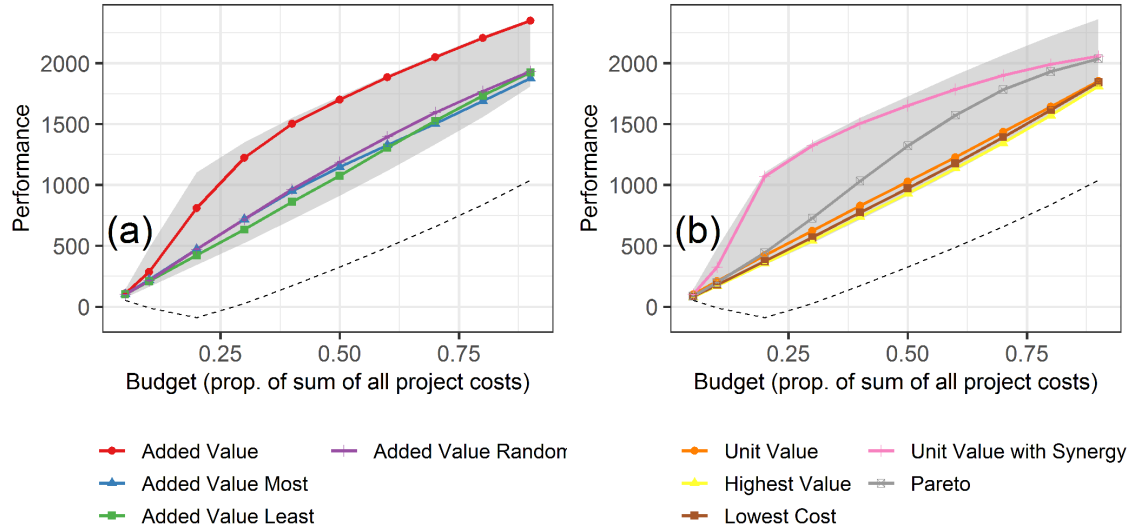


Figure B.1: Mean values of portfolios selected by fast-and-frugal portfolio heuristics under different budget constraints. Panel (a) shows heuristics that consider quantitative alternative interactions; panel (b) shows heuristics that do not. Confidence intervals around these means are negligible (smaller than symbols used to plot the means). The grey polygon plots the envelope between the value of the optimal portfolio (selected using the integer program in Chapter 3) and the mean value returned by a random selection of alternatives, which we consider a useful lower bound for benchmarking performance. The dashed denotes the value of the nadir portfolio.

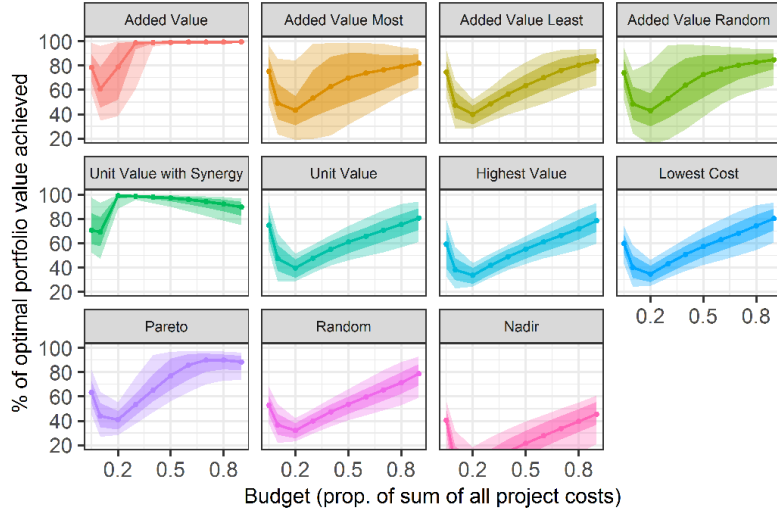


Figure B.2: Quantiles of fast-and-frugal portfolio performance, where these have been normalized relative to the optimal portfolio (value = 100). Solid lines plot median performance, darker shaded areas plot the envelope between lower and upper quartiles, and lighter shaded areas the envelope between 10% and 90% quantiles. This figure complements Figure B.1 by showing relative performance as well as the highly variable performance of some heuristics.

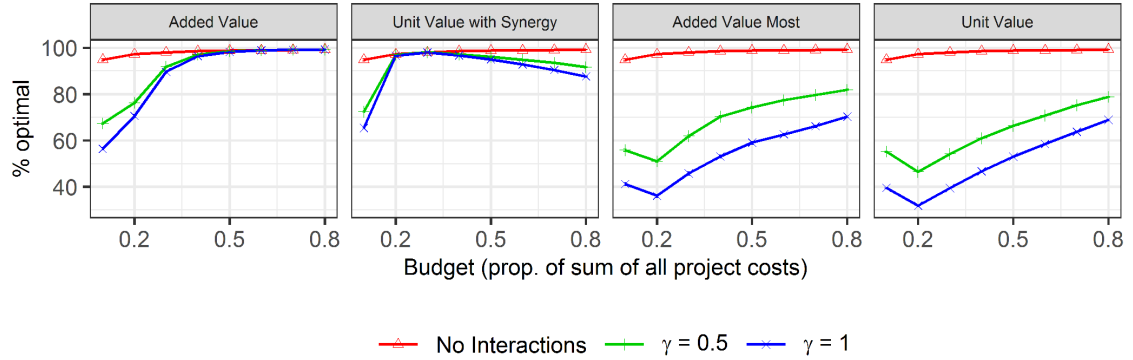


Figure B.3: Relative performance of *add-the-best* variants vary greatly with the magnitude of alternative interactions. alternatives making up an interaction subset each have individual alternative values, and hence a sum exists for the interaction subset. The γ parameter indicates the proportion of this sum that is awarded when the entire interaction subset is selected. Lines indicate mean relative performance for different values of γ , with error bars denote 95% confidence intervals around these means.

Appendix C

Additional data for application

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>	<i>P5</i>	<i>P6</i>	<i>Overall Value</i>	<i>Heuristic</i>
Portfolio1	1	14	15	19	20	22	512	Unit Value with Synergy
Portfolio2	2	14	15	19	20	22	507	Added Value Random
Portfolio3	3	14	15	19	20	22	501	Added Value Random
Portfolio4	4	14	15	19	20	22	508	Added Value Random
Portfolio5	5	14	15	19	20	22	504	Added Value Random
Portfolio6	6	14	15	19	20	22	528	Added Value Random
Portfolio7	7	14	15	19	20	22	507	Added Value Random
Portfolio8	8	14	15	19	20	22	513	Added Value Random
Portfolio9	9	14	15	19	20	22	507	Added Value Random
Portfolio10	10	14	15	19	20	22	513	Added Value Random
Portfolio11	11	14	15	19	20	22	507	Added Value Random
Portfolio12	12	14	15	19	20	22	501	Added Value Random
Portfolio13	13	14	15	19	20	22	510	Added Value Random
Portfolio14	14	15	16	19	20	22	508	Added Value Random
Portfolio15	14	15	17	19	20	22	531	Lowest Cost
Portfolio16	14	15	18	19	20	22	508	Added Value Random
Portfolio17	14	15	19	20	21	22	519	Added Value Random
Portfolio18	14	15	19	20	22	23	515	Added Value Random
Portfolio19	14	15	19	20	22	24	512	Lowest Cost
Portfolio20	1	15	17	19	20	22	525	All heuristics except Lowest Cost
Portfolio21	2	15	17	19	20	22	520	Added Value Random
Portfolio22	3	15	17	19	20	22	514	Added Value Random
Portfolio23	4	15	17	19	20	22	521	Added Value Random
Portfolio24	5	15	17	19	20	22	517	Added Value Random
Portfolio25	6	15	17	19	20	22	541	Added Value Random
Portfolio26	7	15	17	19	20	22	519	Added Value Random
Portfolio27	8	15	17	19	20	22	526	Added Value Random
Portfolio28	9	15	17	19	20	22	520	Added Value Random
Portfolio29	10	15	17	19	20	22	525	Added Value Random
Portfolio30	11	15	17	19	20	22	520	Added Value Random
Portfolio31	12	15	17	19	20	22	513	Added Value Random
Portfolio32	13	15	17	19	20	22	523	Added Value Random
Portfolio33	15	16	17	19	20	22	521	Added Value Random

Table C.1: The research manager conducted a sensitivity analysis by removing and adding proposals to their initially selected portfolio. The initial portfolio $\{1, 14, 15, 19, 20, 22\}$ was selected using the Unit Value with Synergy heuristic. The research manager wanted to see whether they could improve on their initial portfolio by adding or removing proposals. The sensitivity analysis was conducted via the web-based application that we developed to assist the research institute in the selection of the best portfolios of proposals. The analysis was made possible by a feature that was added to the web-based application, allowing the user to either select or deselect proposals at will. We show at each step proposals selected, overall value and heuristic applied.

	Imp. of work	Qual. of public.	Feasib. Risk	Resp. to Chal.	Focus Area	Imp. of prop.	Mentorship	Scient. Merit	Recog. of work	Novelty	Gender
Proposal1	70	80	80	70	80	70	80	70	60	40	100
Proposal2	80	70	75	60	70	60	70	55	60	60	100
Proposal3	70	60	70	70	75	80	70	70	50	45	0
Proposal4	70	70	70	65	70	80	90	60	55	40	100
Proposal5	70	70	80	60	70	70	90	65	50	70	0
Proposal6	80	90	80	70	70	80	90	60	40	50	0
Proposal7	70	70	65	50	70	70	80	70	53	70	100
Proposal8	80	70	60	70	90	70	80	70	70	60	0
Proposal9	80	60	90	65	70	70	60	70	47	40	0
Proposal10	70	70	90	70	65	65	90	75	60	60	0
Proposal11	70	70	80	55	70	65	80	70	45	60	100
Proposal12	60	60	70	70	62	70	80	75	70	50	0
Proposal13	70	60	90	70	70	57	90	55	20	40	100
Proposal14	80	70	90	70	70	90	80	60	40	50	100
Proposal15	90	70	90	80	80	60	80	70	60	60	0
Proposal16	70	60	70	80	70	75	80	50	65	60	100
Proposal17	90	70	90	80	80	90	80	60	40	50	0
Proposal18	80	80	80	90	57	90	80	53	60	40	0
Proposal19	90	70	90	70	80	80	70	70	70	50	100
Proposal20	80	90	90	70	70	70	70	70	60	60	100
Proposal21	70	50	90	60	90	80	90	60	80	50	0
Proposal22	80	80	90	60	90	90	80	70	50	50	100
Proposal23	80	67	90	70	80	70	80	75	70	50	0
Proposal24	80	80	90	60	70	70	75	60	70	45	0

Table C.2: The research institute awards funding to the best portfolio of research proposals. We could not show the real scores assigned to proposals by independent assessors because of confidentiality issues. We generated scores with similar properties as the real ones. The table shows the average scores generated for the research proposals of twenty-four applicants. Scores are assigned on a scale of 0-100 for each criterion. In this problem, the decision maker specified 11 criteria and their names are shown in the top row.

The scale of the criteria is defined in the tables below:

Recognition of prior work	
Awards/Recognition	Rating
0	0
1	30
2	70
≥ 3	100

Quality of publications				
Published/Peer Reviewed	Letter/Communication	Article	Supplemental Article	Review Article/Research note
1	50	10	5	1
5	80	50	10	5
10	100	80	50	10
15		100	80	50
20			100	80
≥ 25				100

Impact of prior work					
Number of Publications					
Number of Citations	1	5	10	15	≥ 20
0	10	20	30	40	50
1	20	30	40	50	60
5	30	40	50	60	70
10	40	50	60	70	80
15	50	60	70	80	90
≥ 20	60	70	80	90	100

Table C.3: The table shows the measurement scales used for the assessment of value functions for the following criteria: recognition of prior work, quality of publications and impact of prior work.

Mentorship			
Number of students	Supervision	Teaching	Tutoring
0	0	0	0
1	40	50	10
5	60	70	20
10	70	80	30
> 10	90	100	50

Potential Impact of proposal	
	Applied
Publishable	100
Unpublishable	50

Feasibility & Risk			
Feasibility & Risk	High	Medium	Low
Total score (from table below)	0-64	65-84	85-100

	Score(Max)	Actual score
Candidate matched funding	20	X
Previous related work	10	X
Clear and well defined proposal	40	X
Time frame	15	X
Location (Africa)	15	X
Total	100	X

Scientific Merit			
High {1,2,3,4}	Medium High {1,2,3}	Medium Low {1,2}	Low {1}
76-100	51-75	26-50	1-25
Where 1=Problem design;	2=Methodology;	3=Objectives;	4=Knowledge contrib.

Novelty			
	High {1,2}	Medium {3}	Low {4}
Originality	61-100	31-60	1-30
Where 1=Groundbreaking research;	2=Major extension of research;	3=Minor extension of research;	4=Repeats research

Respond to a challenge in Africa			
Research Problem	Pure & Theoretical	Theoretical & Applied (Non-industrial)	Applied (Real life)
Score	1-60	61-80	81-100

Focus Area	
STEM	100
Non-STEM	0
STEM means: Science, Technology, Engineering & Mathematics	

Applicant Gender	
Male	0
Female	100

Table C.4: The table shows the measurement scales used for the assessment of value functions for the following criteria: mentorship, potential impact of proposal, feasibility & risk, scientific merit, novelty, respond to a challenge in Africa, focus area and applicant Gender.

Appendix D

Portfolio decision analysis methods

D.1 Portfolio decision analysis under certainty

In this section, we present relevant basic concepts and methods that deal with portfolio decisions under certainty: data envelopment analysis (DEA) portfolio approach without interactions, equity model, the analytic hierarchical process (AHP), linear-additive portfolio value model (LAPVM), outranking methods and reference point method. The portfolio models developed from these methods operate on the assumption that we have precise performance metrics for the alternatives being considered. We also present multiattribute value theory for portfolios and discuss the impact of baselines on portfolio solutions.

D.1.1 Basic models

Prior to presenting the methods that we have just mentioned, we briefly describe a few basic portfolio models that we have found in the literature, namely the scoring, checklist and Q-Sort models. These methods are either of a qualitative nature or are less rigorous than those we will present in the remainder of the section.

Scoring Model

The scoring model is also called weighted factor scoring (Archer and Ghasemzadeh, 1999; Rengarajan and Jagannathan, 1997). In this model, alternatives are ranked based on the weighted score assigned to each alternative. Criteria are chosen such that they represent the values and needs of the organization that conducts the assessment of alternatives. Alternatives are scored in terms of their contribution to each criterion. The score can be captured with different scales. Some have used a 3-point scale (High=3,

Medium=2, Low=1), while others have used a 5, 7 or 10-point scale. Weights are elicited for each criterion. Respondents or stakeholders assign a weight representing the perceived importance they attach to a given criterion. Weights are assessed on a continuous scale from 1 to 10. A zero weight is not allowed since all criteria are judged important. Criterion scores are multiplied by their corresponding weights and then the products are added together to yield a total weighted score for each alternative. The total weighted score corresponds to an overall benefit measure for each alternative. In the final step, alternatives are ranked based on this measure. A subset of the top alternatives is chosen as the optimal portfolio. It is worth noting that in the scoring model weights are not defined as in Multiple Criteria Decision Analysis (MCDA) where they are scaling constants representing the correspondence between value units on one criterion compared to another.

Checklist model

The checklist model is a simple model for screening and selecting alternatives. Checklists tend to be used as Go/Kill decision tools: alternatives are either selected or discarded. Unlike the scoring model, checklists are not used for ranking alternatives. They focus on individual alternatives rather than comparing alternatives (Archer and Ghasemzadeh, 1999; Rengarajan and Jagannathan, 1997).

First, key selection criteria are identified. If cost is, for example, one of the criteria, alternatives with a low cost will be preferred. Performance on each criterion can then be rated 'high', 'medium' or 'low'. Alternatively, alternatives can be evaluated on a set of No/Yes questions. The overall number of positive checks on criteria forms the basis for keeping or discarding an alternative. Alternatives that achieve all "yes" answers, or a specified number of "yes" answers, would qualify. One of the key limitations of this model is that it equally weighs criteria.

Q-Sort model

The Q-sort method is aimed at ensuring consensus within a group of participants about the prioritization of alternatives (Stenner and Marshall, 1996). It was designed to study the correlation between views expressed by different subjects. Each member of the group is either a decision maker or a stakeholder. Each alternative is represented by a card and each member of the group orders the cards according to their value. A facilitator coordinates discussions among group members about the alternatives to bring about a better understanding of the problem, and also summarizes all the results on a chart ensuring anonymity. This is meant to help participants engage more freely. Each

time results are presented and discussed, the participants go through another round of “sorting” or ordering alternatives according to their perceived importance, hence the name Q-sort. The group goes through several similar rounds, and each time an attempt is made at reaching consensus about the ranking of alternatives. The advantage of this method is its simplicity; however, the rationale behind the results may not always be easy to convey to outsiders and may all seem at times rather political.

D.1.2 The resource constrained data envelopment analysis (DEA-RC)

The resource constrained (DEA-RC) approach extends the standard DEA problem to portfolio analysis (Cook and Green, 2000). DEA is a method for assessing the relative efficiency of alternatives called homogeneous decision making units (DMUs) such as bank branches, university faculties, hospitals, *etc.* DEA evaluates an alternative by assigning input and output weights that maximize its relative efficiency compared to other alternatives. The standard CCR formulation for output efficiency evaluation for the 0th alternative, denoted alternative 0 is given as follows (Cook and Green, 2000):

$$\begin{aligned}
& \text{maximize} && V_0 = \sum_{i=1}^m \omega_{i0} x_{i0} \\
& \text{subject to} && \sum_{s=1}^q v_{s0} r_{s0} = 1 \\
& && \sum_{i=1}^m \omega_{i0} x_{ij} - \sum_{s=1}^q v_{s0} r_{sj} \leq 0, \quad j = 1, \dots, n \\
& && \omega_{i0} \geq 0, \quad i = 1, \dots, m \\
& && v_{s0} \geq 0, \quad s = 1, \dots, q
\end{aligned} \tag{D.1}$$

where x_{ij} denotes the value of output i for alternative j , r_{sj} denotes the value of input s for alternative j , and v_{sj} and ω_{ij} are weights of input s and output i for alternative j . The values V_0 are called efficiency scores and are used to rank alternatives. The scores are in the range $[0, 1]$ and efficient alternatives have a score of one.

In a portfolio decision making context, we define a portfolio as a composite alternative P made up of a single or multiple alternatives. It is formed by the combination or summation of its constituent alternative’s inputs and outputs. The problem can now be translated as follows (Cook and Green, 2000):

$$\begin{aligned}
& \text{maximize} && V(P) = \sum_{i=1}^m \omega_{iP} x_{iP} \\
& \text{subject to} && \sum_{s=1}^q v_{sP} r_{sP} = 1 \\
& && \sum_{i=1}^m \omega_{iP} x_{ij} - \sum_{s=1}^q v_{sP} r_{sj} \leq 0, \quad j \in J \\
& && \omega_{iP} \geq 0, \quad i = 1, \dots, m \\
& && v_{sP} \geq 0, \quad s = 1, \dots, q
\end{aligned} \tag{D.2}$$

where J is the set of all alternatives, x_{ij} denotes the value of output i for alternative j , r_{sj} denotes the value of input s for alternative j , x_{iP} denotes the value of output i for portfolio P , r_{sP} denotes the value of input s for portfolio P , and v_{sP} and ω_{iP} are weights of input s and output i for portfolio P . Note that as each portfolio P is a composite alternative, its output and input values can be calculated by, respectively, summing the output and input of the alternatives making up the portfolio.

We should note that Π , the set of all possible portfolios, is made of 2^n portfolios or alternatives, where n is the number of alternatives. Because it may be impossible to explicitly enumerate all the portfolios and solve optimization problem (D.2), an alternative formulation builds portfolios sequentially, ensuring that constraints on all resources are adhered to i.e. $\forall s \in S, \sum_{j \in J} r_{sj} \leq u_s$. In each iteration, we test whether a new alternative would satisfy the constraint and reject the alternative if it does not. Details of this algorithm are found in Cook and Green (2000).

The method was applied to the selection of projects in the Turkish iron and steel industry (Cook and Green, 2000). A total of thirty-seven projects was assessed against five criteria: direct and indirect economic contributions and technological, scientific and social contributions. A limited amount of resources was available and it turned out that a portfolio of sixteen projects was retained as the most optimal. Sensitivity analysis based on weight restrictions was conducted and it identified thirteen of the portfolio projects as “robust” or “core” projects, i.e. they got selected in all non-dominated portfolios.

D.1.3 Equity model

Decision makers are tempted at times to select portfolios based on benefit measures only. They sort alternatives by benefit values and select only those that their budgets can fund. In this way, alternatives are prioritized on the basis of benefits only. Bana e Costa (2001); Bana e Costa and Lawrence (2007) show that this approach does not give optimal results. Instead, they suggest “the value for money” approach: a portfolio gets selected based on the ratio of risk-adjusted benefit to cost.

In value theory, the simplest and most important form of the value function is the additive model. In this model (Bana e Costa and Lawrence, 2007) , the overall value of alternative j is given as $V_j = \sum_{i=1}^m \omega_i v_{ij}$ where ω_i is the weight of criterion i and v_{ij} is the value score assigned to the consequence of alternative j on criterion i . The value of an alternative, also called the risk-adjusted benefit, is the weighted sum of its values across criteria. In other words, the value of an alternative score on each criterion (benefit) is first multiplied by the corresponding criterion weight and then weighted values are summed across criteria to derive the overall risk-adjusted benefit. The optimal portfolio is formed based on the “value for money” principle given as the ratio of risk-adjusted benefit to cost: $r_j = V_j/C_j$ where C_j is the cost of implementing alternative j . The equity model identifies organizational functions, also called areas. Each alternative is appraised against relevant criteria in each area and the results are aggregated across areas as benefit-to-cost ratio.

Weights can be determined either by the trade-off method or the swing method. In the trade-off method, the decision maker specifies trade-offs within pairs of criteria and, based on this, weights are derived. In the swing method, artificial alternatives are simulated from the worst performing alternative (the alternative with the worst value on each criterion) by varying, each time, performance on each criterion to its maximum value. Then the decision maker ranks the artificial alternatives (criterion changes) and ranks are converted into weights (Eisenfuhr et al., 2010). The following conditions must be met in order to warrant the use of an additive model: simple preferential independence, mutual preferential independence and difference independence (Keeney and Raiffa, 1976; Keeney, 1992).

The equity model is mostly applied using software that provides a visual interface. For a given set of alternatives, the software displays an efficient frontier which is made up of all the alternatives that maximize the value for money. The efficient frontier is obtained by sorting alternatives on their value for money in descending order. The user specifies their budget beforehand and identifies the section on the efficient frontier that meets their budget by selecting alternatives until their budget is exhausted. The area under the efficient frontier represents all possible combinations of alternatives that are outperformed by alternatives on the efficient frontier. A given alternative can be evaluated by comparing it to the corresponding alternative on the efficient frontier. The interface provides a path for improving the alternative under consideration by projecting it to the efficient frontier. An alternative can be compared to a similar alternative that yields almost the same amount of benefit at a lower cost or costing almost the same but yielding a higher return. Several applications of this model have been made. It has been

used, for instance, in the selection of R&D projects for Allergan pharmaceutical company (Bana e Costa and Lawrence, 2007). Following discussions with stakeholders, key areas of performance were defined and a value tree was developed. The equity model was applied and subsequently recommendations regarding projects prioritisation were made. They were gradually well received by the management. As a result, the company experienced remarkable growth, and confidence in the model grew further. The method was also applied to the selection of capabilities for a new warship for the Royal Navy. The decision involved multiple stakeholders and conflicting objectives and a limited budget. The portfolio approach enable a solution within fifteen months and proved to be an effective procurement strategy (Phillips, 2011).

D.1.4 Analytic hierarchical process (AHP)

The analytic hierarchical process (AHP) is used for ranking alternatives in the presence of multiple conflicting and subjective criteria (Saaty, 1977, 1994). AHP is built on pairwise comparisons of alternatives. It is based on the premise that it is easy to compare two items (alternatives or criteria) at a time than all of them simultaneously. It develops a value tree akin to the one used in the value function method: a hierarchy of criteria that provides a structure of criteria and sub-criteria; hence, the name “analytic hierarchical process” (Ishizaka and Labib, 2011). Based on a value tree, AHP derives global (composite) priorities (weights) of alternatives, across criteria, from local priorities assessed on each criterion. A different structure, e.g.. value tree, may yield different results.

AHP is implemented in the following four steps: structuring of the decision problem, compilation of a preference matrix by making pairwise comparisons, computation of local weights and a consistency index and finally, an aggregation of local weights into global weights (Subramanian and Ramanathan, 2012). The analytical network process (ANP) (Saaty, 2004; Sarkis, 1999; Cheng and Li, 2004) is the general form of AHP. The later requires a unidirectional hierarchical structure while the former allows for interactions between criteria, i.e. criteria are connected to each other at each level (or within each cluster of criteria) and there is bidirectional dependence between levels. For this reason, ANP is called a system-with-feedback approach. ANP begins with pairwise comparisons performed as in AHP, then conducts additional pairwise comparisons by controlling for one criterion at a time. (Relative) weights (eigen vectors) of these additional comparison matrices are added to the initial (relative) weights to form a supermatrix. Following this step, the supermatrix is made column stochastic (each column sums to one) and is adjusted to account for interactions by raising it to a sufficiently large power until it

converges, i.e. weights become stable. The performance of each alternative is obtained as a sum of weighted criterion scores. Although ANP allows for interactions between criteria, the direction of interactions is not specified as such. Yang and Tzeng (2011) present an approach called decision making trial and evaluation laboratory (DEMATEL) that models a causal map of criteria and determines interconnections between them. DAMATEL can be used in conjunction with ANP to derive weights that reflect the influence level of criteria as shown by the structure of the causal map.

Portfolio AHP

The analytic hierarchical process (AHP) has been extended to deal specifically with portfolio selection. Jiang et al. (2011) present a multiple criteria decision making (MCDM) portfolio model using the AHP weighting method. This AHP-based MCDM approach allows for synergies between alternatives. Synergies can be specified such that the inclusion of an alternative could either be beneficial or detrimental to another alternative within a portfolio. In other words, the impact of an alternative depends on which other alternatives have already been included in the portfolio. Benefit interactions can be represented by a synergy matrix $S = |s_{jk}|$ where s_{jk} is the extent to which alternative j contributes to alternative k , and ranges from -1 to 1. Quantities s_{jk} are percent increase/decrease to alternative j value as a result of its interaction with alternative k . Negative values are an indication of conflicts between two alternatives. Quantities s_{jk} are summed up to compute overall synergistic benefits for alternative j as follows: $S_j = V_j \sum_{k=1}^n z_k s_{jk}$ where z_k is a binary variable indicating whether alternative k is selected in a portfolio and v_j is the value of alternative j obtained as the sum of its weighted performance across criteria.

The total value of a portfolio is obtained as the sum of the value of its constituent alternatives and their respective synergistic benefits :

$$V(P) = \sum_{j=1}^n z_j (V_j + S_j) \quad \forall j \quad (D.3)$$

where z_j is a binary variable indicating whether alternative j is selected in a given portfolio, V_j is the value of alternative j and S_j denotes overall synergistic benefits for alternative j .

The portfolio that gets selected is the combination of alternatives that yields the maximum value as defined in (D.3). An application of this method was made in the selection of re-manufacturing technology alternatives (Jiang et al., 2011). Four technologies were available, with synergies among them. Given a limited budget, the model selected a portfolio of three alternatives that returns maximum synergistic benefit. The results

were discussed with management and were validated.

While both ANP and the AHP-based MCDM portfolio model allow for interactions within AHP, they follow different approaches for doing this. ANP focuses on interactions between criteria, whereas AHP-based MCDM focuses on interactions between alternatives. Abdollahi et al. (2015) report an application of both ANP and DEA to a portfolio selection problem. ANP was used to determine the weights of criteria, for each supplier, and DEA was used to evaluate the efficiency of suppliers. Criteria fell into two classes: leanness (cost, quality and delivery) and agility (human, technological, managerial and cultural capabilities). Based on performance, suppliers were categorized into four groups: low leanness and low agility (LL); low leanness and high agility (LH); high leanness and low agility (HL) and high leanness and high agility (HH). Suppliers in the LL group were discarded but those in the HH group were commended as the best performing suppliers. LH and HL suppliers were selected as candidates for development programs. The goal was to help them become more efficient or sustainable.

D.1.5 Linear-additive portfolio value model (LAPVM)

The linear-additive portfolio value model, also called the knapsack model, has been formulated in a context where one has to pack a subset of n alternatives into a knapsack of capacity c . Each of the n alternatives has value x_j and weight ω_j . The aim is to select the best subset, i.e. the subset whose total weight would not exceed c but would still yield maximum value (Pisinger, 1997, 1995). This problem can be solved as an integer linear programming problem.

The linear-additive portfolio value model can be specified as follows (Jaszkiewicz, 2002; Martello et al., 2000):

$$\begin{aligned}
& \text{maximize} && V(P) = \sum_{j=1}^n x_j z_j \\
& \text{subject to} && \sum_{j=1}^n \omega_j z_j \leq R \\
& && z_j \in \{0, 1\}, \quad \forall j
\end{aligned} \tag{D.4}$$

where P is a portfolio, coefficients x_j and ω_j are value and weight for alternative j , respectively, R denotes a knapsack capacity and z_j is a binary variable indicating whether alternative j is selected in a portfolio. $z_j \in \{0, 1\}$ is a variable such that $z_j = 1$ if alternative j is included in portfolio P ($z_j = 0$ otherwise). This is known as the integrality

constraint.

Linear-additive portfolio value model with interactions So far the linear-additive portfolio value model we have discussed deal with the selection of a portfolio in the absence of interactions between alternatives. However, this model has been extended to tackle instances of interactions between alternatives. Suppose we also have $\hat{K}$ interaction sets A_k^+ ($k = 1, \dots, \hat{K}$). The optimal value of portfolio P can be found as follows (Stummer and Heidenberger, 2003):

$$\begin{aligned} \text{maximize} \quad V(P) &= \sum_{j=1}^n x_j z_j + \sum_{k=1}^K x'_k g_k + \sum_{k=K+1}^{\hat{K}} x''_k h_k \\ g_k &= \{0, 1\}, \quad h_k = \{0, 1\}, \quad \forall k \end{aligned} \quad (\text{D.5})$$

where x'_k is the incremental change in value if at least m_k alternatives in interaction subset A_k are included in the portfolio, and $g_k = 1$ if at least m_k alternatives in interaction subset A_k are selected and $g_k = 0$ otherwise. Similarly, x''_k is the incremental change in value if at most m_k alternatives in interaction subset A_k are included in the portfolio, and $h_k = 1$ if at most m_k alternatives in interaction subset A_k are selected and $h_k = 0$ otherwise.

D.1.6 Outranking methods

Several models have been developed within the outranking approach: ELECTRE, PROMETHEE and GAIA. Often multiple criteria use additive aggregation which is a compensatory method. The Outranking method falls within the class of non-compensatory preference structures where strengths in one criterion do not necessarily compensate for weaknesses in other criteria. Within the outranking class, PROMETHEE V and ELECTRE-TRI have been specifically developed to handle portfolio selection.

PROMETHEE V

In PROMETHEE, the performance level of alternative j on criterion i is given by x_{ij} . Given two alternatives, j and k , the difference in criterion performance between the two alternatives is given as $d_i(j, k) = x_{ij} - x_{ik}$. A preference function $F_i(j, k)$ is defined as a function of $d_i(j, k)$. Six types of preference function are available: 0-1 values, U-shape, V-shape, V-shape with indifference threshold, multilevel and Gaussian. These functions are defined with either an indifference threshold q or a preference threshold p or both. In addition, an intermediate threshold between q and p could also be defined. An outranking measure is determined for alternative j relative to k as follows:

$$\pi(j, k) = \sum_{i=1}^n \omega_i F_i(j, k) \quad (\sum_{i=1}^n \omega_i = 1) \quad (\text{D.6})$$

where ω_i are normalized weights for relevant criteria and $F_i(j, k)$ is a preference function.

The dominance (leaving) flow of alternative j is the extent to which it outranks the other alternatives:

$$\theta^+(j) = \sum_{k \in J} \pi(j, k) \quad (\text{D.7})$$

where J is the set of all alternatives and $\pi(j, k)$ is the outranking measure defined in (D.6). Likewise, the dominating (entering) flow of alternative j is the extent to which it is outranked by the other alternatives:

$$\theta^-(j) = \sum_{k \in J} \pi(k, j) \quad (\text{D.8})$$

where J is the set of all alternatives and $\pi(j, k)$ is an outranking measure defined as in equation (D.6).

A complete ranking of alternatives is achieved by defining a net flow: $\theta(j) = \theta^+(j) - \theta^-(j)$. The ordering of alternatives is conducted as follows: if $\theta(j) > \theta(k)$, then j is preferred to k and if $\theta(j) = \theta(k)$, then j is equally preferred to k .

PROMETHEE V has been specifically applied to portfolio selection. It uses the net flow defined above. The optimal portfolio is determined by solving a binary optimization program that maximizes the overall net flow of alternatives:

$$\begin{aligned} & \text{maximize} \quad \sum_j \theta_j z_j \\ & \text{subject to} \quad \sum_j c_{ij} z_j \leq R_i \quad \forall i \\ & \quad \quad \quad z_j \in \{0, 1\} \end{aligned} \quad (\text{D.9})$$

where $\theta(j)$ denotes the net flow for alternative j , c_{ij} is a cost element i for alternative j , R_i is budgetary constraint and $z = \{0, 1\}$ is a binary variable such that $z_j = 1$ if alternative j is included in portfolio P ($z_j = 0$ otherwise).

The problem with the solution of the model above is that it excludes alternatives with a negative net flow with respect to alternatives that are already included in the portfolio. de Almeida et al. (2014) point out that the alternatives that are excluded might actually improve the portfolio. They argue that these alternatives are not negative per se, but

are only outranked by all other alternatives. In effect, the inclusion of alternatives stops before a boundary solution is reached.

In order to correct for the deficiency, de Almeida and Vetschera (2012) propose the following formulation:

$$\begin{aligned}
& \text{maximize} && \sum_j \theta_j z_j \\
& \text{subject to} && \sum_j c_{ij} z_j \leq R_i \quad \forall i \\
& && \sum_j z_j = n \\
& && z_j \in \{0, 1\}
\end{aligned} \tag{D.10}$$

where $\theta(j)$ denotes the net flow for alternative j , c_{ij} is a cost element for alternative j , R_i is budgetary constraint, n is the maximum number of alternatives that can be allowed into a solution and $z = \{0, 1\}$ is a binary variable such that $z_j = 1$ if alternative j is included in portfolio P ($z_j = 0$ otherwise).

The new formulation forces the portfolio to include alternatives that have a negative net flow, especially when n exceeds the number of alternatives with a positive net flow. The algorithm takes a few iterations to reach a boundary solution, as n is incremented each time by one unit. Solutions from this linear programming problem are called c-optimal solutions (c stands for a constant and in this case $c = n$; n is the maximum number of alternatives). A boundary solution is reached at n^* . It is possible that a better solution exists for $n > n^*$; therefore, worth exploring solutions for different values of n until the maximum value is reached (n_{max}).

A procedure to implement c-optimal solutions is developed in (de Almeida et al., 2014). The authors apply PROMETHEE V with the c-optimal concept to select a portfolio of information system projects. This approach integrates both strategic and organizational goals in the evaluation of portfolios. The classical PROMETHEE V selects a portfolio of fourteen projects out of a total of twenty-five projects. The c-optimal portfolio is made of fifteen projects and proves to be a better solution in terms of the goals set by the decision maker, i.e. the c-optimal solution outperforms the PROMETHEE V solution on most of the criteria (de Almeida et al., 2014).

ELECTRE-TRI

ELECTRE-TRI assigns each alternative to a specific category. A set of categories must be first defined. Each alternative is considered independently of the other alternatives and the decision is made either to assign it or not to assign it to a specific category.

Reference levels or profiles are defined for each category and each alternative is compared to these in terms of its performance levels (Mousseau et al., 2002).

ELECTRE-TRI has been extended to deal with portfolio selection and this is solved either as a one stage or two stage-problem. In the one stage-problem, ELECTRE-TRI is applied as usual but portfolio constraints are also included. There could be a specification of a limit on the size of each category, for example. Or the decision maker could wish to set an upper limit on the cost of alternatives selected. Hence the need to achieve a compromise between ELECTRE-TRI classification and portfolio constraints. The problem is formulated as a mixed integer Program and can be solved in a few iterations by re-running the model and including a new set of constraints each time.

In the two-stage problem, the first stage is a multiple criteria sorting problem solved using ELECTRE-TRI. At this stage, alternatives are classified based on their individual performance levels. It could be that some alternatives are already admitted to the portfolio (most preferred category) based on the first stage results but the remaining alternatives (less preferred category) still need to be filtered. They are put on the waiting list, so to speak. The second stage is a linear programming problem in which the results of the first step are refined to fit portfolio selection constraints (Mousseau et al., 2011). Alternatives may have different features and fall into different groups such as gender, stream, career path, etc. In this case, the decision maker may want to select a balanced subset and therefore impose a minimum on the number of elements from each group that should be selected. Alternatives that were put on the waiting list can be added to the portfolio solution at this stage provided that requirements (specified by the decision maker) are met.

The algorithm in the second stage can be formulated as follows:

$$\begin{aligned}
& \text{minimize} && \sum_{l=1}^h d_l \\
& \text{subject to} && \sum_{j=1}^n \alpha_{jq} z_j \leq \beta_q + M d_l, \quad \forall q
\end{aligned} \tag{D.11}$$

where z_j is binary variable for each alternative that is still to be further assessed, α_{jq} is a coefficient guiding the choice of alternatives, β_q denotes group q size, M is an arbitrary large positive value and h is the total number of constraints. The binary variable d_l is added to each constraint and when $d_l = 1$, constraint l is disabled.

ELECTRE-TRI has been applied to support the selection of students for admission into a study program taking into account various institutional requirements (Mousseau

et al., 2011). The goal was to select at most 50 students from 76 applications. Students are assigned into 4 pre-specified categories, ranging from C_1 (least preferred) to C_4 (most preferred). In the first stage, students in C_1 are rejected and those in C_2 are judged insufficient performers. 29 students are assigned to C_4 and get automatically selected. The remainder gets selected from C_3 (27 students) in the second stage, with a requirement that a set number from each stream, professional track and gender gets included. The selection was made successfully and a subset of 50 students were admitted.

D.1.7 Reference point method

Stewart (2016) presents a multiple objective project portfolio selection based on reference points. Portfolios are assessed based on organizational objectives which can be grouped in 3 classes: directly quantifiable benefits, qualitative criteria and balance or distributional criteria. The goal for the first two classes is to maximize return or performance. The goal of the third class is to minimize imbalance with respect to each criterion, i.e. the discrepancy between the actual and the desired proportions. The notion of balance requires, for instance, that no client group is unfairly favored over the other groups or that long term goals are balanced with short and medium term goals. A chi-squared distance is used to measure imbalance.

A reference point method is used identify optimal solutions (portfolios) with respect to quantitative, qualitative and balance or distributional objectives. The user or analyst first need to identify an ideal point and a reference point. An ideal point (I_i) can be defined as the best value achievable among feasible portfolios on objective i , $i = 1, \dots, m$. For quantifiable benefits and qualitative criteria, ideal levels can be estimated by sorting alternatives on their values and selecting alternatives until resource constraints are violated. In the final step, the values of alternatives are summed up. Aspiration levels, making up a reference point, can be determined as a proportion of ideal point levels. For balance or distribution criteria, ideal values are set to zero as we would ideally like to reach the desired proportion of activity. There are also other ways of generating ideal and reference points.

In order to identify the optimal solution, we need to minimize a function given as:

$$S(P) = \sum_{i=1}^m \left[\frac{I_i - f_i(P)}{I_i - g_i} \right]^4 \quad (\text{D.12})$$

where I_i denotes an ideal point on criterion i , g_i denotes an aspiration level on criterion i , $f_i(P)$ is the value of objective function for Portfolio P on criterion i such as $f_i(P) < I_i$

and $g_i < I_i$ for maximizing objectives (quantitative and qualitative objectives) and vice versa for minimizing objectives (balance and distributional objectives). Objective function value $f_i(P) = \sum_{j=1}^n z_j x_{ij}$, $j \in P$ where z_j is a binary variable indicating whether alternative j is selected in portfolio P and x_{ij} is the performance of alternative j on criterion i .

The non-linear problem in (D.12) can be solved using a genetic algorithm. Stewart (2016) discusses ways the user or decision maker can modify reference points and update ideal points to explore the efficient frontier until they are satisfied with the solution. Details of this are included in Stewart (2016).

D.1.8 Value functions for portfolio problems and the impact of baseline values in portfolio selection

We discuss here multiattribute value functions for portfolios and the impact of baselines on solutions.

Multiattribute value functions for portfolio decision analysis

The primary function used in the literature to evaluate portfolio performance is the additive-linear value function. We will discuss the assumptions which underpin this model. It is possible to relax these assumptions and specify additive-multilinear value functions, which account for interactions between alternatives (Liesiö, 2014).

Suppose, we have X_0 defined as the matrix of criterion values or performance of alternatives on the set of criteria. Let us define the following:

- x_{ij} is the performance level of alternative j on criterion i .
- x_{ij}^* is the most preferred performance level of alternative j on criterion i .
- x_{ij}^0 is the least preferred performance level of alternative j on criterion i .
- I is the criteria set; J is the set of alternatives; J' is a subset of alternatives and $\bar{J}'$ is the complement of J' . Portfolio P is equivalent to subset J' .

Let us also define the following: x_{Ij} is the i -th row of X_0 and is also a set of criteria measuring the performance of an alternative; $x_{iJ'}$ is the J' -th column of X_0 and is also a portfolio criterion-specific performance. Assumptions for the additive-linear value function are then:

1. Preferences are not tied to specific alternatives but to levels of performance. Such preferences are alternative symmetric i.e. portfolios with equal performance levels are equally preferred even if these performance levels are yielded by different alternatives.
2. Each criterion measuring either the performance x_{ij} of an alternative or portfolio criterion-specific performance $x_{iJ'}$ is weak difference indifferent (WDI). The columns of X_0 given as $x_{iJ'}$ are WDI. In other words, the portfolio performance on a specific criterion is independent of the portfolio performance with respect to other criteria and the preference order of the changes in the levels of each criterion is independent of the preference levels of other criteria. The implication of this assumption is that the most (or least) preferred performance levels on a criterion is the same across alternatives i.e. $x_{ij}^* = x_i^*$ and $x_{ij}^0 = x_i^0$.
3. Each set of criteria measuring performance is difference independent. This means (each) row x_{Ij} is difference independent (DI). In other words, any change in the performance levels of a single alternative remains equally preferred even if performances of other alternatives are varied.

The implication of these assumptions is that the performance of an alternative on a particular criterion is independent of the performance of this alternative on other criteria and the performance of other alternatives. Based on these assumptions, we can formulate the additive-linear value function for portfolios as follows (Liesiö, 2014):

$$V(P) = \sum_{i=1}^m V_i(x_{iJ'}), \quad (D.13)$$

$$\text{where } V_i(x_{iJ'}) = \omega_i(1) \sum_{j=1}^n v_i(x_{ij}), \quad i = 1, \dots, m$$

where functions $V_i : x_{iJ'} \rightarrow [0, 1]$ and $v_i : x_i \rightarrow [0, 1]$ are respectively criterion-specific portfolio functions and value functions of an alternative defined as $V_i(x_{iJ'}) = V_i(x_{iJ'}, x_{iJ'}^0)$ and $v_i(x_{ij}) = V(x_{ij}, x_{ij}^0)/V(x_{ij}^*, x_{ij}^0)$ where $V(x_{iJ'}, x_{iJ'}^0)$ is the value function for the least preferred performance levels, for portfolio $P \equiv J'$, given on all criteria except for criterion i and $V(x_{ij}, x_{ij}^0)$ has all performance levels at their worst levels for all alternatives except for x_{ij} (performance of alternative j on criterion i). The weighting factor ω_i is the swing weight of criterion i as defined in the usual framework of MCDA.

Interactions between alternatives can be introduced by dropping the assumption that each set of criteria measuring the performance of an alternative is DI. The resulting value function is additive-multilinear. The DI assumption is relaxed such that each criterion measuring an alternative x_{ij} becomes conditionally DI of other criteria measuring the performance of the same alternative x_{ij} , given a fixed level of the other criteria $x_{I\bar{j}}$

(performance of the other alternatives on the criteria set). In other words, there are dependencies among alternatives i.e. a change in the performance level of an alternative on a particular criterion is dependent on the total level achieved on this criterion by the portfolio. As a result, the value of adding an alternative to a portfolio is no longer constant but is a function of the performances of the alternatives already included in the portfolio.

The additive-multilinear value function is formulated as follows (Liesiö, 2014):

$$V(P) = \sum_{i=1}^m V_i(x_{iJ'}), \quad (D.14)$$

$$\text{where } V_i(x_{iJ'}) = \sum_{J'' \subseteq J'} \omega_i(|J''|) \prod_{j \in J''} v_i(x_{ij}) \prod_{j \notin J''} v_i(1 - x_{ij})$$

where V_i and v_i are respectively criterion-specific portfolio functions and value functions of an alternative, weights $\omega_i(k) = V_i(x_{iJ''}^*, x_{i\overline{J''}}^0) \quad \forall J'' \subseteq J', \quad |J''| = k$ where $\omega_i(k)$ denotes criterion-specific value of portfolio $P \equiv J'$, having the most preferred levels x^* on criterion i for k alternatives but the least preferred levels on the remaining alternatives. The weighting function $\omega_i(k)$ is dependent only on the size of the set J'' (subset of J' having k alternatives) because of the symmetry property discussed earlier, i.e. $\omega_i(k)$ is dependent on the number of alternatives included in the portfolio rather than the specific alternatives included. The weighting function is made of n parameters defined on criterion i given as $\omega_i = \omega_i(1), \dots, \omega_i(n)$. Therefore, the additive-multilinear value function accounts for interactions between alternatives.

Liesiö also discusses ways of eliciting criterion-specific portfolio value functions and optimization models for maximizing portfolio value. For problems with a small number of alternatives, it is possible to enumerate all potential portfolios and identify the optimal solution. The weighting function $\omega_i(k)$ can be derived by asking the decision maker to assess trade-offs between criteria considering scenarios where additional alternatives are added to a given portfolio. This will help specify the additive multilinear value function.

For bigger problems, the difference standard sequence technique can be used. For a given criterion, the decision maker is asked to define the number of alternatives k_1 such that a change from k_0 to k_1 alternatives is equivalent to a change from 0 to k_0 alternatives. The process can be repeated until all the weighting functions are obtained and the additive multilinear value function is derived. Details can be found in (Liesiö, 2014). This approach suffers from a limitation: the procedure to estimate the value function becomes very complex when we have a high number of attributes as the number of parameters linearly increases in the number of attributes. Liesiö (2014) admits that

preference elicitation should be tested and further developed in real-life case studies.

The impact of baseline values in portfolio selection

We think of a baseline as the value of not implementing or selecting an alternative. The choice of a baseline score leads to different prioritization of alternatives. Traditionally, there is an implicit assumption in the multiattribute value function that the value of not implementing or selecting an alternative gives a value score of zero i.e. the worst possible outcome on each criterion. Such an assumption may not always be justified or even accurate. For instance, the assumption that the cost of not selecting an alternative leads to a negative financial impact is not always true (Clemen and Smith, 2009; Golabi and Sicherman, 1981; Morton, 2015). This can be true if one or more alternatives have negative financial contributions on the one criterion while having positive values on other criteria. The fact is that, in some instances, not doing a project may still yield a positive outcome.

A number of studies have looked at the impact of baselines on the selection of optimal portfolios. The baseline is typically obtained by asking the respondent or decision maker whether he/she is indifferent between selecting or not selecting a given alternative. If he/she is, the alternative is retained as the baseline (Golabi and Sicherman, 1981). If none of the existing alternatives qualify as a baseline, the decision maker is required to specify a hypothetical alternative such as they are indifferent between selecting and not selecting it. This can be quite burdensome for the decision maker. There are also situations where all the alternatives have been pre-selected and can therefore be implemented and these types of methods are not suited to these situations.

The value of a portfolio allowing for non-zero value scores (non-zero baseline) when an alternative is not selected or implemented is given as (Clemen and Smith, 2009):

$$V(P) = \sum_{j=1}^n [z_j V(x_j) + (1 - z_j) V_B] = \sum_{j=1}^n z_j [V(x_j) - V_B] + n V_B \quad (\text{D.15})$$

where $V(x_j) = \sum_{i=1}^m \omega_i V_i(x_{ij})$ is the value for alternative j across m criteria, V_B is the value of baseline B such that $V_B = \sum_{i=1}^m \omega_i V_i(x_{iB})$ and z_j is a binary variable indicating whether alternative j is selected in this portfolio.

The optimal portfolio is determined by solving the following integer linear program:

$$\begin{aligned} \text{maximize } nV_B + \sum_{j=1}^n z_j[V(x_j) - V_B] \\ \sum_{j=1}^n c_j z_j \leq R \end{aligned} \quad (\text{D.16})$$

where V_B is the baseline value, R denotes the available budget and c_j is the cost per alternative and z_j is a binary variable indicating whether alternative j is selected in this portfolio. This formulation of the portfolio value indicates that we are trying to maximize the incremental value associated with selecting alternatives against not selecting them.

We present two recent methods of determining the value of the baseline and identifying optimal portfolios:

First method (Optimality range): The starting point is to determine whether the decision maker would like to implement x_j^0 , the worst performing alternative. If they are willing to implement x_j^0 , the baseline has a negative value. In order to identify the baseline in this case, a method involving a comparison of multiple (k') worst alternatives x_j^0 , on the one hand, and multiple ($k' - 1$) best alternatives (x_l^*), on the other hand is used. The aim is to identify the value ($k' + 1$) where preference switches towards the latter portfolio (portfolio with x_l^*) (Morton, 2015). If we substitute x_j^0 and x_l^* in (D.16), we get the following equations:

$$k'V(x_j^0) + (n - k')V_B \geq (k' - 1)V(x_l^*) + (n - (k' - 1))V_B \quad (\text{D.17})$$

$$(k' + 1)V(x_j^0) + (n - (k' + 1))V_B \leq k'V(x_l^*) + (n - k')V_B \quad (\text{D.18})$$

Solving (D.17) and (D.18) yield V_B in the interval $[-k', -k' + 1]$

Second method (VCR approach): A benefit-to-cost ratio approach (Liesiö and Punkka, 2014; Clemen and Smith, 2009) can be applied to studying the impact of changing baseline values on the prioritization of alternatives. Priority can be assigned to alternatives based on the value-to-cost ratio (VCR) and alternatives with higher VCR are preferred (Bana e Costa and Lawrence, 2007; Kirkwood, 1997). VCR can be specified by taking into account the baseline V_B as follows: $VCR_j(V_B) = \max\{\frac{V(x_j) - V_B}{c_j}, 0\}$. Alternatives are sorted in a descending order. Each alternative will assume a rank within a certain interval.

It is worth mentioning that the methods presented here can be used in conjunction with one another. The starting point could be the optimality range (or VCR) approach and decision makers can then indicate whether they prefer one of the potentially optimal alternatives (solution corresponding to a given baseline interval bound). The choice of

a preferred alternative determines the baseline optimality range. Note that Clemen and Smith (2009) advocate baselines that are specific to the set of alternatives considered, i.e. baselines are defined as the value of “not selecting a given alternative” (D.16). Morton (2015) points out a few issues with this baseline formulation. Morton shows that the formulation is fine for the regular MCDA problem of selecting the best alternative out of a set of alternatives; however, the formulation suffers at times from a rank reversal issue when applied to portfolio problems. They highlight that the problem stems from using a value function measured on certain scale types (e.g. ordinal scales). An interval scale can be safely used in this case. The original formulation (i.e. formulation without baselines) suffers also from rank reversals because value functions are generally taken on interval scales. This can be remedied by using a ratio scale instead. Morton (2015) advocates the definition of baselines that are not specific to the set of alternatives considered but rather uses the concept of “not selecting any alternatives from the choice set at all”. It is, therefore, advisable to choose a baseline which is independent of the set of alternatives and does not change when new alternatives are added. Morton also points out that interval scales can still be used in the original formulation depending on the type of portfolio model used to combine value scores. For example, the maximum function can be safely used with interval scales.

D.2 Portfolio decision analysis under uncertainty or risk

In this section, we present four approaches for dealing with portfolio decisions under uncertainty or risk: robust portfolio modeling (RPM), data envelopment analysis and balanced scorecard model for portfolio selection (DEA-BSC) and the scenario-based portfolio selection with incomplete/complete probability and utility information.

D.2.1 Robust portfolio modeling (RPM)

The work presented by Liesiö et al. (2008) models portfolio selection in the presence of incomplete score and weight information. Suppose we have incomplete score information. The set of feasible performance scores is given as $S_x = \{X \in \mathbb{R}^{m \times n} \mid \underline{X} \leq X \leq \bar{X}\}$ where $\underline{X}$ and $\bar{X}$ are the lower and upper bounds of the score interval, respectively. In the same way, the set of feasible weights S_w is spanned by t extreme vectors, denoted $W_{ext} = [\omega^1, \dots, \omega^t] \in \mathbb{R}^{m \times t}$ (Salo and Punkka, 2005). Non-dominated portfolios are identified by performing pairwise comparisons of portfolios (P and P') for all extreme weight vectors ω^r in W_{ext} .

$P > P'$ i.e. Portfolio P dominates portfolio P' if and only the value of portfolio P for the lowest performance scores $\underline{X}$ is higher than the value of portfolio P' for the highest performance scores $\bar{X}$, for every extreme weight vectors ω^r . This is given as follows:

$$\begin{aligned} \sum_{j=1}^n z_j \sum_{i=1}^m \omega_i \underline{x}_{ij} &\geq \sum_{j'=1}^n z_{j'} \sum_{i=1}^m \omega_i \bar{x}_{ij'} \\ \sum_{j=1}^n z_j \sum_{i=1}^m \omega_i \underline{x}_{ij} &\neq \sum_{j'=1}^n z_{j'} \sum_{i=1}^m \omega_i \bar{x}_{ij'} \end{aligned} \quad (\text{D.19})$$

where $j \in \{P \setminus P'\}$ are alternatives in portfolio P that are not included in portfolio P' and $j' \in \{P' \setminus P\}$ are alternatives in portfolio P' that are not included in portfolio P , ω_i is the weight of criterion such that $\omega_i \in \omega^r$, and $\underline{x}_{ij}$ and $\bar{x}_{ij'}$ are elements of $\underline{X}$ and $\bar{X}$ which are the lower and upper bounds of performance scores interval, respectively. This problem is equivalent to a multi-objective zero-one linear programming problem. The set of non-dominated portfolios is called Π_N . In cases where there interactions between alternatives, the formulation in (D.19) can be expanded to account for this as shown in the linear-additive portfolio value model in (D.5).

Liesiö et al. (2008) present an algorithm to conduct these comparisons systematically and build the set of the non-dominated portfolios. Different rules can be used to choose the best solution among non-dominated portfolios, e.g. maxmin and minimax-regret. Given the information set, the maxmin picks the portfolio with the highest minimum overall value among non-dominated portfolios as the optimal solution. On the other hand, the minimax-regret recommends the portfolio that minimizes the worst overall gap in value among non-dominated portfolios.

In order to assess the quality of portfolios/solutions, the RPM supplies a number called the core index. It is computed for individual alternatives and is calculated as the share of non-dominated portfolios which include the given alternative. For alternative j the core index CI is given as $CI(x_j, S) = |\{P \in \Pi_N(S) \mid x_j \in P\}| / |\{\Pi_N(S)\}|$. Based on this index, alternatives can be classified into: core alternatives (they are part of every non-dominated portfolio i.e. $CI = 1$), borderline alternatives (they belong to some non-dominated portfolios but not to all i.e. $0 < CI < 1$) and exterior alternatives (they belong to none of the non-dominated portfolios i.e. $CI = 0$). A portfolio made up of core alternatives is thought to be more robust than other portfolios.

We will mention briefly a few reported applications of RPM. It has been used in environmental problems. One such example is concerned with the extraction of peat in Central Finland. Any approach to extraction has to be balanced with the risk to

water systems and threatened habitats. A number of possible actions (e.g. whether to extract peat from a predefined set of bogs or not) were considered and the goal was to select a portfolio of actions which would provide most peat while reducing damage to the environment. With the help of RPM, viable solutions were identified and proposed to stakeholders (Ollil, 2013). RPM has also been successfully applied at the Finish transport agency for bridge maintenance (Mild et al., 2015) to prioritize maintenance of bridges. Core index values were used to guide the selection process under incomplete information about weights and performance levels of alternatives. Further, RPM has been used to develop a portfolio of products at a telecommunications company (Lindstedt et al., 2008). Each product was defined as “a technology in a given customer segment” and was evaluated against three criteria: profit, risk and requirements. A few technologies were identified as winners across all segments and surprisingly some of the technologies previously thought to be self-evident did not score well. The results were presented to the management and adopted as key inputs in other strategic planning processes.

D.2.2 Data envelopment analysis and balanced scorecard model for portfolio selection (DEA-BSC)

This approach was developed to identify efficient, effective and balanced portfolios using the DEA methodology (Eilat et al., 2006). Unlike the traditional DEA method, it takes into account interactions between portfolios.

The data envelopment analysis and balanced Scorecard model for portfolio selection (DEA-BSC) assesses the performance of alternatives referred to as Decision Making Units (DMUs), given balance restrictions, i.e. constraints on the importance attached by an alternative to a particular set of outputs or “cards”. Each card measures a specific aspect of DMUs’ performance and is assessed by a given set of relevant outputs. Examples of cards could be internal growth, innovation, financial return, etc. The model is formulated as the basic DEA model (D.1) (also called CCR model) with additional constraints called balance restrictions. This approach ensures a balanced representation of the performance of an alternative with respect to key performance factors. Balance restrictions are formulated as virtual weight restrictions. A virtual weight is the product of the weight attached to an output (or input) and the performance value on the given output (or input).

For card c_l , restrictions are given as $l_{C_l} \leq \frac{\sum_{i \in C_l} \omega_i x_{i0}}{\sum_{i \in C_0} \omega_i x_{i0}} \leq u_{C_l}$, for $l = 1, \dots, h$. Cards $C_1, \dots, C_l, \dots, C_h$ are a partition of the set C_0 which includes all output measures, and l_{C_l} and u_{C_l} are lower and upper bounds on the importance of outputs in class C_l , respectively. The model yields a score for each alternative called DEA-BSC score

(Eilat et al., 2006). It also defines the following performance metrics: efficiency, being assessed as the CCR model efficiency score; risk as the product of the overall budget of the alternative and the probability that the alternative will not succeed and finally, the balance index given as the ratio of the DEA-BSC score to the CCR score.

A procedure is developed in (Eilat et al., 2008) to generate portfolios and we briefly explain it in this paragraph. First, the DEA-BSC model, discussed above, is used to evaluate the performance of individual alternatives with respect to efficiency, risk and balance. Expert opinion can help set minimum thresholds for both efficiency and balance and maximum thresholds for risk. Based on their performance, alternatives are either discarded or selected in the pool of alternatives from which portfolios will be drawn. Selected alternatives are ranked from highest scoring (lowest index) to lowest scoring (highest index). Portfolios of alternatives are then formed using a branch-and-bound algorithm (Greenberg and Hegerich, 1970; Martello et al., 2000). The initial portfolio is an empty set. The procedure begins with each alternative being added to the empty set to form branches (portfolios). In the next step, an alternative with a lower efficiency score gets added to each branch (portfolio). In each iteration, an alternative gets added to each portfolio until either the alternative with the lowest score is included or resource constraints are exceeded.

Once portfolios are generated, they are assessed in the same way individual alternatives are assessed in the DEA-BSC model but their aggregate input and output values are determined using certain accumulation functions. When there are no interactions, the accumulation function is simply the sum of individual alternative input and output values making up the portfolio.

When there are interactions between alternatives, output (or input) values of portfolio P are the sum of individual alternative output (or input) values plus predicted gain or losses due to interactions. The set of expected outputs (benefits) for portfolio P is given as $x_{iP} = \sum_{j=1}^n z_j Pr_j (x_{ij} + \sum_{k=1}^{j-1} z_k x'_{ijk})$ where Pr_j is the probability that alternative j will succeed, x_{ij} the value of alternative j on output i , x'_{ijk} are interactions between alternatives j and k for output i and $z = \{0, 1\}$ is a binary variable indicating whether alternative j is included in the portfolio. The set of inputs for portfolio P is defined similarly.

When there are interactions between alternatives but outcomes are uncertain, stakeholders can estimate probability Pr of success of an alternative given that another alternative has been selected. In this instance, output (or input) values of portfolio P are the sum of individual DMU output (or input) values plus predicted gain or losses due to interactions weighted by probability Pr . We can express the expected value of outputs

for portfolio P as: $x_{iP} = \sum_{j=1}^n z_j Pr_j (x_{ij} + \sum_{k=1}^{j-1} z_k Pr_k x'_{ijk})$ where $Pr_k = \sum_{j=1}^n z_j Pr_{jk}$ is the marginal change in the probability that alternative j will succeed is alternative k is undertaken. The matrix of probabilities Pr_{jk} is not necessarily symmetrical as alternative j may have greater impact on alternative k than the other way around. The DEA-BSC model has been applied to the selection of portfolios of technological projects for a large governmental agency whose task is to select and support such projects (Eilat et al., 2006). It was found that five portfolios achieved a raking of one. The model did not suggest a single best portfolio but allowed the decision maker to decide between a reduced number of alternative best choices.

D.2.3 Scenario-based portfolio selection with complete probability and utility information

We present in this section an approach for portfolio selection that account for decision makers' risk attitude. We assume that the decision maker has been able to provide relevant information about the likelihood of outcomes and risk preferences. We consider a number of plausible scenarios, each one with corresponding probability of occurrence.

Let us introduce some notations here. The set of alternatives is denoted $X = \{x_1, \dots, x_n\}$. The set of r disjoint scenarios is given as $\Omega = \{s_1, \dots, s_q, \dots, s_r\}$. Scenario probabilities belong to the set $Pr^0 = \{Pr \in \Re | Pr_q \geq 0, \sum_{q=1}^r Pr_q = 1\}$ and the set of utility functions is $U^0 = \{U : \Re \rightarrow [0, 1] | u(x) \geq u(x') \quad \forall x \geq x'\}$.

Risk neutral decision makers In this instance, the decision maker select the best portfolio as the one that maximizes the expected value. This is a linear programming problem given as follows (Liesiö and Salo, 2012):

$$\begin{aligned} \text{maximize} \quad & V(P) = \sum_{j=1}^n z_j \sum_{q=1}^r Pr_q x_{jq} \\ \text{subject to} \quad & \sum_{j=1}^n c_{jq} z_j \leq R_q \quad \forall q \\ & z_j \in \{0, 1\} \quad \forall j \end{aligned} \tag{D.20}$$

where x_{jq} the performance level for alternative j in scenario q , Pr_q denotes probability of scenario q , c_{jq} is a cost element for j in scenario q , and R_q is the budget in scenario q .

Risk taking decision makers For decisions under risk, the goal is to maximize the expected utility of alternatives. A utility function can be used to capture the decision

maker's risk attitude. The problem is now expressed as a non-linear programming problem:

$$\begin{aligned}
& \text{maximize} && V(P) = \sum_{q=1}^r Pr_q u\left(\sum_{j=1}^n z_j x_{jq}\right) \\
& \text{subject to} && \sum_{j=1}^n c_{jq} z_j \leq R_q \quad \forall q \\
& && z_j \in \{0, 1\} \quad \forall j
\end{aligned} \tag{D.21}$$

where u is the utility function and all other elements are defined as in (D.20).

Alternatively, risk measures and constraints can be used in the formulation of the problem and this will be solved as a linear program. For a risk level $\alpha \in (0, 1]$, we can specify the risk-adjusted probability set defined as $D_{Pr}^\alpha = \{d \in Pr^0 | d \leq \frac{Pr}{\alpha}\}$. We note that $\alpha = 1$ is the least risky level. The new solution is the maximum expected value obtained at extreme point of D_{Pr}^α provided that the value at risk exceeds a given risk threshold γ (Rockafellar and Uryasev, 2000; Dentcheva and Ruszczyński, 2006):

$$\begin{aligned}
& \text{maximize} && V(P) = \sum_{j=1}^n z_j \sum_{q=1}^r Pr_q x_{jq} \\
& \text{subject to} && \sum_{j=1}^n c_{jq} z_j \leq R_q \quad \forall q \\
& && \sum_{j=1}^n z_j \sum_{q=1}^r d_q x_{jq} \geq \gamma, \quad \forall d \in \text{ext}(D_{Pr}^\alpha)
\end{aligned} \tag{D.22}$$

where $\text{ext}(D_{Pr}^\alpha)$ are extreme points of the probability set D_{Pr}^α , γ is a risk threshold, and all other elements are defined as in (D.20). This is basically the same problem as in the case of a risk neutral decision maker but it is now complemented by a new set of constraints. Algorithms have been developed in Liesiö et al. (2007); Villarreal and Karwan (1981) and Kiziltan and Yucaoglu (1983) to solve the problem (D.20).

D.2.4 Scenario-based portfolio selection with incomplete probability and utility information

In this section, we present the scenario-based portfolio selection model with incomplete probability and utility information (Liesiö and Salo, 2012). We may have incomplete information when it is difficult to elicit probabilities, particularly in group settings where there may be conflicting views about the likelihood of events occurring. It may also be difficult to derive a unique utility function to describe a decision maker's risk attitude. It is in this context that we have to deal with imprecise parameter information. It should be noted that in this case, expected values are not crisp numbers, hence the need to

Extreme Points		Scenarios	Portfolios	
Pr^1	Pr^2		P	P'
0.3	0.7	S1	7	2
0.7	0.3	S2	4	7

Table D.1: Data for illustration of scenario-based portfolio selection with incomplete information

formulate an appropriate approach.

In order to identify non-dominated portfolios we need to look at expected utility of portfolios. If the expected utility of portfolio P is consistently strictly greater than or equal to portfolio P' across all scenarios, over the entire probability range, portfolio P dominates portfolio P' . The probability range is explored by considering its extreme points. We will include an example below to illustrate this.

Under risk, decision makers identify non-dominated portfolios by maximizing expected utility. In order to achieve this, we need to compare pairs of portfolios (P and P') based on their expected utility values. Considering plausible scenarios and feasible utilities, the expected utility difference is formulated as follows (Liesiö and Salo, 2012):

$$V(P) - V(P') = \sum_{q=1}^r Pr_q u\left(\sum_{j=1}^n z_j x_{jq}\right) - \sum_{q=1}^r Pr_q u\left(\sum_{j'=1}^n z'_{j'} x_{j'q}\right) \quad (\text{D.23})$$

where u is the utility function and x_{jq} the performance level for alternative j in scenario q , Pr_q denotes probability of scenario q . Because we do not have complete information about the utility function, we can compare the minimum and maximum values of expected utility differences at each extreme probability point to determine whether portfolio P dominates portfolio P' . P dominates P' i.e. $P \succ P'$ if and only if the minimum of the above problem is non-negative for all extreme points of the probability set and the maximum is strictly positive for some points of the probability set.

As a worked example to illustrate this approach, we consider a simple case where portfolios P and P' are made of one criterion (see Table ??). For example, the probability of scenario 1 is $Pr_1 \in [0.3, 0.7]$.

We sort the values of portfolios P and P' to obtain the set $\nu = \{2, 4, 7\}$. We then first need to work out the expected utility difference between portfolios P and P' at each extreme point of the probability set.

- At extreme point Pr^1 , the expected utility difference between P and P' is given

as $E_{Pr}[u(P)] - E_{Pr}[u(P')] = 0.3u(7) + 0.7u(4) - 0.3u(2) - 0.7u(7) = -0.3u(2) + 0.7u(4) - 0.4u(7)$

- At extreme point Pr^2 , the expected utility difference between P and P' is given as $E_{Pr}[u(P)] - E_{Pr}[u(P')] = 0.7u(7) + 0.3u(4) - 0.7u(2) - 0.3u(7) = -0.7u(2) + 0.3u(4) + 0.4u(7)$

If we consider an increasing utility function U^0 , the min value at Pr^1 is reached for $u(\nu) = (0, 0, 1) = -4$ and because it is negative, portfolio P cannot dominate P' .

Constraints can be introduced to define a risk averse utility function. The exact shape of this utility function U is not known. However, it can be specified through a set of constraints. The combination of probability and utility values can be defined as the unconstrained set $S^0 = Pr^0 \times U^0$. With the introduction of constraints, the set will be restricted as $S' = Pr' \times U'$ such that $Pr' \times U' \subseteq Pr^0 \times U^0$. Liesiö and Salo (2012) applied the model to explore the future operating environment (scenarios) of various industries over a period of twenty years. The data and results for the forest industry were presented in the paper. Scientists and experts advised on possible actions that could be taken in order to deal with different scenarios. In the light of incomplete probability and utility information, ranking of probability scenarios was elicited from experts. They also estimated the values of each action in each scenario. Liesiö and Salo experimented with different types of utility functions. The goal of this analysis was not to produce a single optimal portfolio but instead a list of actions to be analyzed further. The core index, already discussed in the RPM section (Section D.2.1), was also used as an indicator of the viability of each action. As a result of this work, five key actions out of a total of twenty-four were identified as the most promising, i.e. they were included in all dominated portfolios. The reduced list of actions was presented to stakeholders for further consideration (Liesiö and Salo, 2012).

D.3 Comparison of portfolio decisions methods

D.3.1 Interaction between alternatives

We note that some of the methods we have presented allow for interactions between alternatives while others do not. Methods which capture interactions are: data envelopment analysis (DEA), the analytic hierarchical process (AHP), the robust portfolio modeling (RPM) and the linear-additive portfolio value model (LAPVM). Methods which do not account for interactions are: equity model, ELECTRE-TRI, PROMETHEE V and the scenario-based portfolio selection with (in)complete probability and utility information.

Stewart (2016) discusses the potential of including interaction effects in the reference point method. Interactions are generally captured in four ways:

Predicted synergistic input/benefit increments

When no interactions take place, overall benefits/inputs are simply the sum of individual alternatives. In the event of interactions, resulting benefits represent either a gain or loss compared to the sum of individual items. There may also be savings on the amount of resources to operate a combination of alternatives because of synergies. In addition, synergistic benefit increments can be weighted by their probability of occurrence, i.e. probability of success when interactions take place. Outcomes are given as the sum product of probability and benefit levels. This is the approach followed by DEA, RPM and LAPVM.

An interaction matrix in percentage points

Interactions can be displayed in the form of a matrix showing a percent increase/decrease of the performance of one alternative as a result of its interaction with another alternative (2-way interaction). The interaction matrix is made of elements ranging from -1 to 1, with negative values showing a conflict between alternatives and positive values showing the extent to which alternatives act in synergy. Diagonal elements are set to zero. This is the approach followed by AHP. The value of an alternative given interactions with other alternatives is the weighted sum of individual scores on criteria plus interaction contributions of other alternatives to the alternative under consideration. Interaction gains are given as a proportion of the weighted sum of individual scores on criteria for the alternative under consideration. Weights, in this context, are criterion importance.

Virtual alternatives or constraints

Interactions can be modelled in the form of virtual (hypothetical) alternatives or linear constraints to capture either resource savings or output benefits. Resource gains can be specified as alternatives with negative inputs but zero outputs, and outputs benefits as zero cost but positive outputs. Constraints can also added to the algorithm to effect these virtual alternatives when synergistic alternatives are at play. This is the approach followed by RPM.

The additive multilinear value function

Interactions can be modelled using an additive multilinear value function. The weighting function can be derived by asking the decision maker to assess trade-offs between criteria considering scenarios where additional alternatives are added to a given portfolio. This will help specify the additive multilinear value function. Alternatively, the difference standard sequence technique can be used. For a given criterion, the decision maker is asked to define the number of alternatives k_1 such that a change from k_0 to k_1 alternatives is equivalent to a change from 0 to k_0 alternatives. The process can be repeated until all the weighting functions are obtained and the additive multilinear value function is derived. Liesiö (2014) concedes that this approach needs to be tested and further developed in real-life case studies.

D.3.2 Robustness of alternatives/solution

Sensitivity analysis for individual alternatives and sensitivity analysis for portfolios differ in purpose. Sensitivity analysis for individual alternatives is very useful as it helps to show the effect of changing input parameters on the selection of the best alternative. The change is meant to reflect uncertainty in the mind of the decision maker. It is also meant to show how the performance of individual alternatives may change as a result of changing priorities and values. Out of two equally performing alternatives, decision makers may opt for the one whose performance is the most robust. The general approach to conducting this analysis is to vary weights, preference/indifference thresholds and other input parameters in order to measure the effect of this change on the performance of alternatives. This approach has been widely used in additive value function-based models, data envelopment analysis and outranking methods (Thanassoulis, 2001; Belton and Stewart, 2002).

For a portfolio, the aim of the sensitivity analysis is to see how changing values will affect the composition of non-dominated portfolios. The goal is also to assess how often an alternative belongs to non-dominated portfolios: whether an alternative is always or sometimes part of non-dominated portfolios or whether it is never part of any of them. In light of this, the robustness portfolio model yields a robustness measure called core index (CI), which is the proportion of non-dominated portfolios that include a given alternative. The robustness portfolio model has been developed in the additive value theory framework. Core alternatives are desirable as they are part of every non-dominated portfolio. Exterior alternatives are not viable as they belong to none of the non-dominated portfolios. A portfolio comprised mostly with core alternatives is said to be more robust than a portfolio comprised mostly of exterior alternatives.

D.3.3 Preference modeling

Different approaches to preference modeling have been implemented in portfolio decision making. Preference with respect to given objectives can be captured through the use of a value function. Value functions are essential in evaluating the performance of alternatives and portfolios.

There are different approaches to defining a value function. We have value functions assessing the performance of each alternative across a set of criteria. We also have value functions assessing the difference in performance between any two alternatives. Preference/indifference thresholds can also be embedded into value functions to capture further performance aspects. The shape of value functions vary widely; we can have a linear shape, U-shape, V-shape, etc. Aggregation across criteria (attributes) can be done via an additive or a multiplicative approach. Criteria can be measured on an ordinal, interval or ratio scale and there are implications for this and some of this is discussed under Section D.1.8.

We need to recognize that value functions for portfolio problems may require further assumptions apart from those specified for value functions for alternatives only. Liesiö (2014) has done a good job of discussing multiattribute functions for portfolio problems and we present this work in Section D.1.8. Researchers may need to explore this area further to improve assessment of portfolios.

D.3.4 Features of portfolio solutions

We consider features of portfolio solutions given by the methods we have surveyed in this chapter. We examine whether the methods yield one best portfolio or a set of “best” portfolios. We also discuss the concept of portfolio optimality.

Portfolio methods fall into two groups. In the first group, we have methods that yield a unique best portfolio solution and in the second group, methods that yield a group of “best” portfolio solutions or non-dominated portfolios. Most of the methods discussed in this chapter fall into the first group. For methods in the second group, the decision maker is supplied with an array of non-dominated portfolios instead of a single best portfolio and has to choose their most preferred portfolio from this collection of portfolios. This type of solutions is produced by RPM, reference point method and the scenario-based portfolio selection with (in)complete probability and utility information.

Portfolio optimality is defined differently by the different methods and the best portfolios are identified in terms of: highest benefit-to-cost ratio, highest value, desired category, expected utility and minimum risk (variance).

Solutions yielding highest benefit-to-cost ratio identify the best performing portfolios in terms of benefit-to-cost ratios or weighted output to weighted input ratios. The decision maker chooses the subset of efficient alternatives that is within their budget or meets their strategic goals. As such, the efficient frontier is unique and therefore a single portfolio within budget or resource constraints space is selected. This is the type of solutions produced by equity model and DEA-RC (DEA without interactions). These methods can be implemented via a visual interface.

Highest yielding value solutions identify portfolios that return the highest weighted value sum. AHP and LAPVM select portfolios which maximize total benefit. PROMETHEE identifies portfolios yielding the highest net flow or dominance.

Solutions that assign alternatives to a pre-specified category do so based on performance levels. The set of alternatives is effectively subdivided into subsets, each belonging to a given category. The subsets which belong to preferred categories form the optimal portfolio. The solution may also require further filtering in the form of constraints and is implemented as a linear program problem. This is the type of solution generated by the ELECTRE-TRI method.

Lastly, optimality is defined in terms of minimum risk (variance) or expected utility. The portfolio with maximum expected value under a defined level of risk is identified as the best portfolio. Alternatively, the analyst is tasked with performing pairwise comparisons of expected utility of portfolios to arrive at a set of non-dominated portfolios. Scenario-based portfolio selection with (in)complete probability and utility information apply this approach.

D.4 Heuristics

D.4.1 Introduction

Heuristics can be described as models of decision-making that are used to describe peoples' actual behavior. These can be based on expertise or on observation and analysis of human behavior and how people make good (close to optimal) decisions with little data (Keller

and Katsikopoulos, 2016). Good decisions are in the sense of returning portfolios that are in some sense close to what might have been achieved using an exact method. In this section we describe the selection heuristics and scoring heuristics used in this experiment. Since we make extensive use of these heuristics in later chapters, we illustrate them with an example adapted from the one found in Durbach et al. (2020).

D.4.2 Heuristics for portfolio selection

Durbach et al. (2020) proposed selection heuristics that use a single criterion or an overall score (across criteria) when adding alternatives to a portfolio. These are adapted from the heuristics presented in Gigerenzer et al. (1999) and Keisler (2009). The construction of a portfolio is done sequentially by considering, at each stage, alternatives that have not been added yet. A stopping rule is needed to terminate portfolio construction after a specified number of consecutive constraint violations. We will discuss a family of heuristics called *Add-the-best* or *selection heuristics*.

Add-the-best At each stage, the heuristics add an alternative that has the highest value according to a well-defined criterion. In the event of ties, these will be broken randomly. Attempts to add an alternative are terminated after ν consecutive violations of the budget constraints or when no alternatives remain to be added. Heuristics in this family differ from one another based on the criterion used to evaluate alternatives.

We illustrate *add-the-best* by examples in Tables D.2 and D.3. Suppose that a decision maker must select a portfolio from five alternatives P_1 – P_5 with values x_j and costs c_j , given in Table D.2, where x_j is the overall score of alternative P_j obtained using one of the scoring heuristics we discuss in Section D.4.3; $j = \{1, \dots, 5\}$. Suppose that positive interactions exist between alternatives in the following subsets of alternatives: $\{P_2, P_3, P_4\}$ (interaction subset A_1); $\{P_2, P_3\}$ (interaction subset A_2); $\{P_2, P_4\}$ (interaction subset A_3); $\{P_3, P_5\}$ (interaction subset A_4) and $\{P_4, P_5\}$ (interaction subset A_5). If all of the alternatives in any of these interaction subsets are selected, an additional value of $B = 2$ is added to the value of the portfolio. The decision maker has a budget of $\zeta = 6$. The optimal solution is the set $\{P_1, P_2, P_3\}$, which returns a portfolio value of 10 at a cost of 6.

Ignoring quantitative information about alternative interactions

In this section, we consider five heuristics that do not use quantitative information about alternative interactions in the calculation of selection values: *Highest Value* (HV), *Lowest*

			<i>Unit Value</i>				<i>Highest Value</i>				<i>Lowest Cost</i>			
			Criterion value at stage				Criterion value at stage				Criterion value at stage			
x_j	c_j		0	1	2	3	0	1	2	3	0	1	2	3
P_1	3	1	3/1	—	—	—	3	3	—	—	1	—	—	—
P_2	5	4	5/4	5/4	5/4	5/4*	5	—	—	—	4	4	4	4*
P_3	1	1	1/1	1/1	1/1	—	1	1	1	—	1	1	—	—
P_4	4	3	4/3	4/3	—	—	4	4	4	4*	3	3	3	3*
P_5	1	2	1/2	1/2	1/2	1/2*	1	1	1	1*	2	2	2	—
Selection			P_1	P_4	P_3	—	P_2	P_1	P_3	—	P_1	P_3	P_5	—

Table D.2: A numerical illustration of fast-and-frugal portfolio heuristics ignoring quantitative interaction information. Columns show the information required by each heuristic at each iteration, i.e. as alternatives are sequentially added to the portfolio based on alternative values, costs, and value-to-cost ratios for *Unit Value*, *Highest Value* and *Lowest Cost*, respectively. Alternatives that cannot be added due to budget constraints are indicated with an asterisk.

Cost (LC), *Unit Value* (UV), *Pareto*, and *Unit Value with Synergy* (UVwS). The first three heuristics (HV, LC and UV) entail a sorting of alternatives based on values, cost or a combination of both. The fourth (*Pareto*) is based on a dominance relationship between alternatives, and the last one (UVwS) uses binary information about whether an alternative is involved in any interactions. These heuristics were also used in Durbach et al. (2020).

Let us begin with the first three heuristics. They are given as follows:

Highest value (HV) adds alternatives in descending order of their values.

Lowest cost (LC) adds alternatives in ascending order of their costs.

Unit value (UV) adds alternatives in descending order of their value-to-cost ratios. Values are based on marginal alternative values only.

Let us trace the selections made by the above heuristics (UV, HV and LC) in Table D.2.

The *Unit Value* heuristic selects alternatives in decreasing order of marginal value-to-cost ratio and sequentially adds alternatives P_1 , P_4 , and P_3 (see last row under “Unit Value”), after which the cost of adding either of the remaining alternatives exceeds the available budget. The selected portfolio has a total value of 8 and a cost of 5.

The *Highest Value* heuristic selects alternatives in decreasing order of value. In our example, P_2 , the alternative with highest value, is added first (stage 0), and then a selection is made from among the remaining alternatives. P_4 is next in line but can’t be selected as the total cost of $\{P_4, P_2\}$ would exceed the available budget at this stage i.e.

budget of 2. P_1 is the next candidate and can be chosen (stage 1) without exceeding the budget. At stage 2, there remains only one unit of budget available and either P_3 or P_5 can be selected as they have the same value. However, P_5 gets disqualified as it costs 2 units and therefore P_3 is selected. Thus *Highest Value* selects the portfolio $\{P_2, P_1, P_3\}$ that has value 11 (9 plus 2 given respectively as the sum of marginal values of alternatives and the value of interaction set A_2) and costs 6.

The *Lowest Cost* heuristic starts by selecting the cheapest alternatives, P_1 and P_3 , both having a cost of one unit. They can be added in any order (stages 0 and 1). The next cheapest alternative is P_5 (stage 2). Adding any other alternative after this would exceed the budget. Therefore, the final selection can be either $\{P_1, P_3, P_5\}$ or $\{P_3, P_1, P_5\}$, both having a value of 7 and a cost of 4.

We now turn to the fourth heuristic called *Pareto*. It makes use of dominance relationships. The “best” alternative is the one that is not dominated by any alternative that remains outside the portfolio.

Pareto adds a randomly chosen alternative provided it is within budget and is not dominated by any alternative not already in the portfolio. An alternative is non-dominated if there is no other alternative yielding a higher return at equal or lower cost.

In this instance, dominance relationships are based on marginal values and cost. We test for dominance among alternatives not yet included in the portfolio. While non-selected alternatives may be dominated by selected alternatives, the inclusion of the former may have the effect of improving the overall return of the portfolio. Because this heuristic involves a random selection from the set of non-dominated alternatives at each step, this may result in different solutions (sets of selected alternatives) if the heuristic were applied repeatedly.

The following alternatives are initially non-dominated: P_1 , P_2 and P_4 . Suppose that P_1 is picked first (stage 0). At the next stage (stage 1), P_3 joins the set of both non-dominated and not-yet selected alternatives $\{P_2, P_4\}$. Although P_3 is dominated by P_1 , the latter is already selected. Suppose P_2 is selected at stage 1. At the next stage, P_3 and P_4 are potential candidates. However, P_4 cannot be added because it would result in the cost of the resulting portfolio exceeding available budget. P_3 can be safely added at stage 2 and the resulting portfolio, $\{P_1, P_2, P_3\}$, has a value of 11 at a cost of 6. After this, no other alternatives can be added because of budget constraints.

Unit Value with Synergy adds alternatives from the set of alternatives that are involved in at least one positive interaction based on the *Unit Value* heuristic; i.e. alternatives are sorted by their marginal value-to-cost ratios and added in this order. Alternatives are added until the set is exhausted. Then, alternatives from outside the set are added again using *Unit Value*.

In the example in Table D.2, *Unit Value with Synergy* first establishes that the set of interacting alternatives is made up of all alternatives except for P_1 . It then adds alternatives from this set using the *Unit Value* heuristic, and thus adds P_4 , P_3 , and P_5 . It cannot add P_2 as this would exceed the available budget.

The *Unit Value with Synergy* stands out from the previous four heuristics and provides a bridge between this section and the next. While previous heuristics completely ignore interactions, *Unit Value with Synergy* accounts for the presence of interactions even though it does not incorporate their sizes and values.

Using quantitative information about alternative interactions

Unlike previous heuristics, this group of heuristics accounts for quantitative information about alternative interactions in the selection of portfolios. In other words, they consider the benefits or losses resulting from certain alternatives being selected together, and fall under the “Added Value” class in the *add-the-best* family. Individual heuristics within this class differ from one another on whether they account for all interactions or a subset of them. Heuristics that consider only a subset of interactions are frugal, as they do not consider at all the available information.

Added Value adds the alternative whose inclusion would lead to the largest increase in overall portfolio value per unit cost. Gains include both marginal values of the alternative and interaction values resulting from a combination of alternatives being simultaneously included in the portfolio.

Added Value searches across the alternatives not yet added to see whether their inclusion would complete any interaction subset. It considers all interaction subsets. Variants of the *Added Value* (*Added Value Most*, *Added Value Least* and *Added Value Random*) do not consider all interaction subsets, as we shall see below.

In the example in Table D.3, *Added Value* first adds P_1 (stage 0) and then P_4 (stage 1), as they yield the biggest and second biggest gains in portfolio value per unit cost. No interaction subsets have been completed at this stage. At the next stage (stage 2), we consider interaction subsets that may be completed with the addition of a new alternative: $A_3 = \{P_2, P_4\}$ should P_2 be added, and $A_5 = \{P_4, P_5\}$ should P_5 be added. The inclusion of P_2 would increase portfolio value by 7 at a cost of 4, while the inclusion of P_5 would increase portfolio value by 3 at a cost of 2. The inclusion of P_2 would violate the budget constraint. Thus P_5 is selected (stage 2). At this point, it is no longer possible to add any other alternative as this would drive the cost beyond the budget of 6 units. So, the final selection is $\{P_1, P_4, P_5\}$, yielding a value of 10 at a cost of 6.

Proj	x_j	c_j	<i>Added Value</i>				<i>Added Value Most</i>				<i>Added Value Least</i>			
			Criterion value at stage				Criterion value at stage				Criterion value at stage			
			0	1	2	3	0	1	2	3	0	1	2	3
P_1	3	1	3/1	–	–	–	3/1	–	–	–	3/1	–	–	–
P_2	5	4	5/4	5/4	7/4*	7/4*	5/4	5/4	5/4	5/4*	5/4	5/4	7/4*	7/4*
P_3	1	1	1/1	1/1	1/1	1/1*	1/1	1/1	1/1	–	1/1	1/1	1/1	1/1*
P_4	4	3	4/3	4/3	–	–	4/3	4/3	–	–	4/3	4/3	–	–
P_5	1	2	1/2	1/2	3/2	–	1/2	1/2	1/2	1/2*	1/2	1/2	3/2	–
Selection			P_1	P_4	P_5	–	P_1	P_4	P_3	–	P_1	P_4	P_5	–

Table D.3: A numerical illustration of fast-and-frugal portfolio heuristics making use of quantitative interaction information. The table shows, at each decision stage, the criterion value assigned by each heuristic to each of the eligible alternatives (i.e. the estimated increase in portfolio value per unit cost as alternatives are sequentially added to the portfolio). alternatives that cannot be added due to budget constraints are indicated with a superscripted asterisk.

Added Value Most considers only interaction subsets involving the alternative that contributes the *most* to portfolio value per unit cost, i.e. the alternative with the highest value-to-cost ratio.

Added Value Least considers only interaction subsets that involve the alternative that currently contributes the *least* to portfolio value per unit cost, i.e. the alternative with the lowest value-to-cost ratio.

Added Value Random randomly chooses one of the alternatives already in the portfolio and considers only the interaction subsets involving this alternative.

Let us consider how *Added Value* and its variants (*Added Value Least*, *Added Value Most* and *Added Value Random*) construct portfolios in Table D.3. All three heuristics begin with the selection of P_1 (stage 0), the alternative with the highest value-to-cost ratio. Then all three heuristics add P_4 (stage 1), the alternative with the second highest

value-to-cost ratio. No alternative gains additional value at this stage (stage 1) because there is no synergy between P_1 and P_4 . After this point, each heuristic takes a different path. *Added Value Most* now considers only interactions involving P_1 , the alternative in the portfolio with the highest value-to-cost ratio. No interaction subset will be activated at this stage (stage 2), as P_1 is not involved in any interactions. *Added Value Most* selects P_3 next (stage 2), as it returns the largest value gain per unit cost. No further selections can be made as this would cost more than the available budget. The final portfolio $\{P_1, P_4, P_3\}$ has a value of 8 and a cost of 5. As we have seen, *Added Value Least* has already selected P_1 and P_4 (stages 0 and 1). It now considers only the interactions involving P_4 , the alternative in the portfolio with the lowest value-to-cost ratio: $A_3 = \{P_2, P_4\}$ should P_2 be added, and $A_5 = \{P_4, P_5\}$ should P_5 be added. The selection of P_2 would cause the cost of the resulting portfolio to exceed available budget. Alternative P_5 is selected next (stage 2), as it returns the largest value gain per unit cost after P_2 . The final selection is $\{P_1, P_4, P_5\}$, giving a value of 10 at a cost of 6.

As we have already seen, *Added Value Random* has already selected P_1 and P_4 (stages 0 and 1). *Added Value Random* then randomly selects either P_1 and P_4 , and then only considers interactions involving these alternatives in the next step. Suppose P_1 is chosen. No interaction subsets are completed and the heuristic selects P_3 next (stage 2), as it returns the largest value gain per unit cost. At this point, any further selections would exceed the budget, and the selected portfolio is $\{P_1, P_4, P_3\}$.

D.4.3 Heuristics for overall scoring of alternatives

In most situations, alternatives are evaluated based on multiple criteria. Each alternative scores a value on each criterion and these are aggregated into an overall value for the alternative based on some rule. We discuss here approaches for assigning an overall value to alternatives. The normative approach for scoring alternatives in a multiple criteria problem is the weighted additive linear function (WADD). In this approach, each alternative gets assigned an overall value which is the weighted sum of individual attribute values on criteria. WADD requires the assessment of several parameters, some of which are difficult to assess, and previous work in non-portfolio contexts has shown that good performance can be obtained with simpler heuristics (Katsikopoulos et al., 2018; Todd, 2007; R.M. Hogarth, 2005; Baucells et al., 2008a).

We will look at a selection of heuristics for aggregating the values of alternatives across criteria. These were presented in Payne et al. (1993). We call the group of heuristics *scoring heuristics* and they are given as follows:

Equal Weighting Function (EQW): The aggregate score of an alternative is given as the mean of individual scores on all criteria. This is a simplified version of WADD, which ignores information about the relative importance of each criterion.

Lexicographic (LEX): The aggregate score of an alternative is represented by its value on the most important criterion, i.e. the criterion with the highest weight value.

Tallying (TAL): The aggregate score of an alternative is given as the count of good features of the alternative, i.e. the number of criteria whose scores exceed some predefined cut-off. Other variants of the tallying heuristic can be applied, such as the count of bad features or a combination of both good and bad features. The tallying heuristic is also called “the frequency of good and bad features”.

Each alternative can thus be scored according to WADD, EQW, LEX and TAL. The WADD score is the ideal score, as it accounts for all available information. Therefore, it serves as a benchmark against which the scores from the other scoring heuristics can be judged.

We illustrate scoring heuristics with the following small numerical example. Suppose that a decision maker constructs a portfolio from five alternatives P_1 – P_5 with values x_{ij} and costs c_j given in Table D.4, where x_{ij} is the value of alternative P_j on criterion i ; $i = \{1, 2, 3\}$ and $j = \{1, \dots, 5\}$. Suppose the values of the alternatives are drawn from $U[0.5, 5]$, a uniform distribution. We show in Table D.4 the overall scores obtained using WADD and other scoring heuristics. In addition, we show the ranks of alternatives based on their overall scores such that the alternative with the highest overall score gets a rank of 1.

	x_{1j}	x_{2j}	x_{3j}	c_j	Overall Score (x_j)				Ranks			
	$w_1=0.4$	$w_2=0.1$	$w_3=0.5$		WADD	EQW	LEX	TAL	WADD	EQW	LEX	TAL
P_1	1	3	0.6	1	1	4.6	0.6	1	4.5	4	5	2
P_2	0.5	3	1	2	1	4.5	1	1	4.5	5	4	2
P_3	2	2	2	3	2	6	2	0	2.5	2	2	4.5
P_4	2.25	1	2	4	2	5.25	2	0	2.5	3	2	4.5
P_5	5	2	2	2	3.2	9	2	1	1	1	2	2

Table D.4: A numerical illustration of WADD and heuristics for overall scoring of alternatives. Rows and columns show the relevant information for each alternative, such as the alternative value on each criterion (x_{ij}), costs (c_j), weights (w_i), and overall values and descending ranks for WADD, EQW, LEX and TAL. Ties are resolved by assigning the mean rank to the alternatives involved.

Let us look at how the overall scores are calculated for each alternative via WADD and scoring heuristics.

For WADD, we multiply the value of each alternative on a specific criterion by its corresponding weight to derive an criterion-weighted score and add these up to derive the overall score of the alternative. For example, the overall WADD score of P_1 is obtained as follows:

$$(x_{11} * w_1 + x_{21} * w_2 + x_{31} * w_3) = (1 * 0.4 + 3 * 0.1 + 0.6 * 0.5) = 1$$

For EQW, we add up the values of each alternative on individual criteria to derive the overall score of the alternatives. For example, the overall EQW score for P_1 is obtained as follows:

$$(x_{11} + x_{21} + x_{31}) = (1 + 3 + 0.6) = 4.6$$

For LEX, the overall score of each alternative is given by its value on the criterion with highest weight value i.e x_{3j} . For example, the overall LEX score for P_1 is 0.6.

For TAL, we count the values of each alternative above a threshold - 2.75 in this instance - to derive the overall score of the alternative. For example, the overall TAL score for P_1 is 1.

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