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TOWARDS IMPROVING THE STATSCAN™ X-RAY IMAGE QUALITY THROUGH SLIDING MODE CONTROL OF THE C-ARM



Prepared by:

Mohammed Esmail (ESMMOH001)

Department of Human Biology, Biomedical Engineering

University of Cape Town

Supervised by:

Mohohlo Samuel Tsoeu

Department of Electrical Engineering,

University of Cape Town

Dr Lester John

Department of Human Biology

University of Cape Town

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Declaration

I, Mohammed Esmail, hereby declare that this dissertation is my own unaided work. This dissertation is being submitted for a degree of Master of Science in Medicine in Biomedical Engineering at the University of Cape Town. Except as stated below, neither the substance nor any part of this dissertation has been submitted in the past for any degree or examination in any other university. I have used the IEEE convention for citation and referencing. Each contribution to, and quotation in, this dissertation report from the work(s) of other people has been attributed, and has been cited and referenced.

Signature of the author: _____

Date: _____

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Abstract

This dissertation investigates methods for improving the image quality of a digital radiography system. The Lodox Statscan™ X-ray system provides a full-body scanned image for initial diagnosis. The system is driven by a permanent magnet linear motor (PMLM) controlled by a cascaded proportional-proportional integral controller (P-PI). Transient errors in the trapezoidal motion profile of the scanning C-arm may cause mismatches between the detector and the collimated beams from the X-ray source. This results in a partial degradation of image quality.

The Statscan™ X-ray system was investigated and the following constraints were identified: The scanning time is limited to 13 seconds and the maximum scan length is limited to 1.8 m. Since it was not possible to obtain the Lodox Statscan™ dynamics model, because of the similarity, a characteristic model was then developed using a DC motor in order to investigate the control dynamics. It is not advisable for a designer to manipulate the controllers on commercial machines except for changing the parameters for tuning. Therefore, a P-PI controller and a proportional-integral sliding mode controller (P-ISMC) as well as a Boundary Layer variant (P-ISMC+BD) were designed for fair comparison purposes.

Root locus, Bode diagrams and integral sliding mode control methods, respectively, were used to design the P-PI and P-ISMC controller groups. Each controller consists of an inner loop and an outer position loop. Proportional integral (PI) and integral sliding mode controllers (ISMC) were used for the inner loop. The two inner loop controllers were tuned, and then tested, before cascading them with the outer position loop. The simulation and experiments were conducted to compare each controller's performance on step set-point tracking, trapezoidal motion profile tracking, the time transient's specifications and robustness against disturbances.

In order to test image quality, 27 distance profiles were generated from P-PI, P-ISMC and P-ISMC+BD. In addition, four images captured by the Statscan™ were also selected. A time delay integrator (TDI) simulator was used on the distance profiles and the four images to generate 108 distorted image profiles.

Four-three-way ANOVAs compared the effects of different factors (*Controller (P-ISMC, P-PI and P-ISMC+BD)*, *Speed (3mm/s, 6mm/s and 9mm/s)* and *Position (25mm, 50mm and 100mm)*) and their interactions on *Settling Time*, *Final Position Error*, *Overshoot* and *Image Blurring Index*.

With respect to Control Outcomes: For the *Final Position Error*, the Controller results varied quite a bit and even though they indicate that P-ISMC+BD was statistically significantly better at the highest speed tested, an inspection of the statistical graphs show that this behavior was not systematic with respect to speed compared to the two other controllers; therefore it is inconclusive which controller can be considered better with respect to FPE. For *Settling Time*, both P-ISMC and P-PI controllers were found to be statistically significant and better than the P-ISMC+BD controller. For the *Overshoot*, the P-ISMC controller was best controller and P-ISMC+BD was the worst

With respect to Radiological Image Outcomes: For the *Image Blurring Index* there were no statistically significant differences ($p > 0.05$) between all three controllers on the *Image Blurring Index* but there were marginal ($0.10 > p > 0.05$) and close to marginally significant differences. Since this result was obtained using four images the addition of more images for the *Image Blurring Index* test may increase the statistical power and could change the results. The interim conclusion is that, despite the differences in Control performance, there was no significant different in Radiological Image outcome, although this needs to be examined in greater detail before a final conclusion can be reached.

The P-ISMC controller in the current study was developed in continuous time domain. This implementation has a limitation on sampling frequency, sample/hold effects and discretization error, as well as chattering. Therefore, we recommend considering the discrete time domain implementation for future studies as this may improve image quality.

Glossary

A/D	Analogue to digital converter
AILC	A-type iterative learning controller
Amax	Maximum acceleration
ANOVA	Analysis of variances
AP	Anterior-posterior
b	PI controller bias
CCD	Charge-coupled devices
CE06250	SERVOSTAR® CD series by Kollmorgen
C_n	The number of controllers under comparison
D/A	Digital to analogue convertors made
Df	Degree of freedom
df1	First degree of freedom
df2	Second degree of freedom
DSP	Digital signal processing
Dt300	Data acquisition card from Data Translation
e_p	Position error
$f(t)$	Control input disturbance
$f_y(s)$	the system position output disturbance
F	Levene Statistic
FPE	Final Position Error
GM	gain margin
IL-240- 50 AI	Ironless linear motor manufactured by Kollmorgen
ISMIC	Integral sliding mode control
k(s)	Controller feedback loop
LS 603	Linear optical encoder
LSSM	Linear slit-scanning mode
LSSR	Linear slit-scanning radiography

M	Means
MIMO	Multi-input and multi-output systems
MRAC	Model reference adaptive control
MRDSMC	Model reference discrete sliding mode control
MS	Mean squares
M_h	Image height
M_L	The energy of the Laplacian of the image
M_p	Maximum overshoot
N	Image width
N_d	Number of distorted image for both controllers
N_{dp}	The number distance profiles required for each controller
N_s	The number of selected images from the Statscan™ system
P	Proportional controller
P_error	Final position error
PASW Statistics 18	statistical software by SPSS Inc
PC	Personal computer
PI	proportional integral Controller
PID	Proportional integral Controller and derivative controller
P-ISM	Proportional- integral sliding mode control
P-ISM+BD	Proportional- integral sliding mode control plus –boundary width and disturbances
PLC	Programmable logic controller
PMLM	Permanent Magnet Linear Motor
$P.O$	Maximum percentage overshoot
P-PI	proportional-proportional integral control
PSF	Point-spread function
PM	Phase margin
RHP	Right-half-plane
RK4	4th order Runge-Kutta

rms	Root mean square
S-curve	Smoothing motion profile
SD	Standard deviation
Sgn	Signum function
SISO	Single input single output system
SMC	Sliding mode control
ss	Switching surface
SS	Sum of squares partial squares
s_0	The dominant closed loop pole
Std.Erro	Standard deviation error
T-curve	Trapezoidal motion profile
T_s	Sampling time
TDI	Time delay integration
TDIS	Time delay integration simulator
ts	Settling time
tr	Rising time
t_p	Maximum overshoot time
tp	maximum overshoot time
Vmax	The maximum velocity
VSC	Variable structure control
VSCS	Variable structure control systems
ZOH	Zero-order holds
ζ	Damping ratio
wrt	With respect to

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1. Introduction

The Lodox Statscan™ X-ray machine is a low-dose full-body scanner that uses a linear cascaded controller and a SERVOSTAR® drive (CE06250) to perform point-to-point linear positioning. This dissertation investigates improving the performance characteristics of the Statscan™ X-ray machine using proportional-integral sliding mode control (P-ISMC) and proportional-proportional integral control (P-PI). The objective of the study was to identify the capability of P-ISMC to track various set-points based on a trapezoidal motion profile, and how this influences the image quality. The P-ISMC performance was compared with that of the P-PI controller.

This first chapter of the dissertation presents the problem statement; outlines the purpose of the study; provides the limitations of the study in the context of available solutions and finally outlines how the dissertation development is organized.

1.1. Background to the study

In the last decade, digital radiography has been steadily replacing screen-film in many radiography departments. Nowadays, digital imaging manufacturers supply different types of digital imaging solutions based on various detectors and read-out technologies. The major advantages of digital radiography include: higher dynamic range of digital detectors, improved dose efficiency, less radiation exposure to patients and reduced operating cost because of the absence of film and chemicals for processing [1]. Based on this digital imaging revolution, Lodox Systems, a South African company, has developed Statscan™, a full-body linear slit-scanning X-ray machine [2], [3]. The Statscan™ X-ray machine technology was originally developed for the diamond-mining industry to reduce the theft of diamonds from the mine premises. However, it was soon realized that this technology could be applied in the medical field, and it was introduced in areas where there is a high incidence of penetrating injury [2], [4]. The machine is designed in such a way that the needs of patients with multiple injuries are provided for during the initial diagnosis in emergency rooms [5].

The Statscan™ machine makes use of linear slit-scanning and allows images to be taken over an angular range of 90°, i.e., from anterior-posterior (AP) views to lateral views in 1° increments. The X-ray beams of the Statscan™ X-ray machine are tightly collimated by a narrow slit into a fan beam of X-rays, such that the scattered radiation is reduced. Its detector operates synchronously with the X-ray fan beam and is sensitive only to direct X-rays from the fan beam. The detector consists of 12 charge-coupled devices (CCD) and operates in time delay integration (TDI) mode. These factors facilitate

scanning using a low dose of X-ray radiation. In addition, these features allow the high quality radiographic image to be acquired line by line by scanning the X-ray fan beam across the patient at a constant speed.

1.2. The system description

The Statscan™ X-ray machine works in a linear slit-scanning mode (LSSM). Its C-arm is driven by an ironless linear motor (permanent magnet linear motor (PMLM)), model IL-240- 50 AI, manufactured by Kollmorgen, and is equipped with a linear optical encoder (LS 603) by Heidenhain as the measurement feedback component with a precision of approximately 20 μm . The linear motor is used to drive the C-arm along a linear length of 1800 mm. The detector and the tube are mounted on opposite sides of the C-arm, facing each other. In high-precision positioning applications, this type of movement is known as point-to-point motion control [6]. Its fundamental operation requires a maximum constant velocity and position destination to be determined, and then the system must be precisely moved to the determined position destination. This PMLM is driven by a SERVOSTAR® (CE06250) drive. The position and the different speed settings of the C-arm travel are controlled by a programmable logic controller (PLC) which receives instructions about the operator settings. The Statscan™ X-ray machine is shown in Figure 1.1. Its total mass is approximately 1220 kg, while the total moving mass of the C-arm (including the linear motor coil, the X-ray tube, heat exchanger and detector) is 515 kg. Currently, the linear slit-scanning radiography (LSSR) control system consists of a cascaded control structure including an inner-loop velocity and an outer-loop position (P-PI) controller. A detailed design formulation for linear motors using P-PI controllers is discussed by Ding and Wu [6]. The design of LSSM has attracted increased attention from the conventional digital radiography community because it uses the best properties of linear scanning. Therefore, the Statscan™ X-ray machine has been promoted as a low-dose alternative to conventional digital radiography system.



Figure1.1: Statscan™ X-ray machine.

1.3. Statscan™ Image Degradation

Previous research conducted on the Statscan™ X-ray machine has focused mostly on comparisons between the linear slit-scanning technique and other types of digital radiography systems in terms of scatter and dose reduction in trauma and emergency applications [2], [3], [7], evaluating image quality and diagnostic accuracy of the Statscan™ in both children and adult chest imaging applications [8]. Statscan™ offers significant improvements compared to full-field digital radiography with respect to image quality in most applications. Specifically, the LSSR configuration with TDI reduces the image degradation caused by scatter radiation, and enhances the image resolutions, as well as the signal-to-noise ratio [7].

Despite these many advantages, the Statscan™ X-ray images are associated with image degradation for a very small section at the beginning or at the end of the scan. In the literature, the LSSR configuration with TDI can cause image degradation due to the mechanical system vibrations [9], [10], [11]. Vibration affects cause a mismatch between the X-ray source scanning speed and the detector clocking speed. In other words, the image information is allocated to the incorrect pixel during the scanning process, causing image degradation. This leads to image blurring. Furthermore, the vibration can cause fish-tailing of the C-arm where the source moves back and forth across the detector, resulting in banding as shown in Figure 1.2(a). The banding is a result of over- and underexposure on the image. In addition, the image can also become degraded by the patient moving relative to the motion of the C-arm [8]. The abnormal movements of the patient cause not only blurring, but also distortion [12]. The image

can become distorted by the patient movements because a TDI imaging system requires a relatively long exposure time to capture the entire image.

One possible approach to reduce the effect of scanner vibration and relative movement would be to use the recorded position or velocity profile with image processing techniques designed to reduce blurring and distortion [12], [13]. However, this solution is not as suitable as it may appear to be, for three main reasons.

Firstly the image is acquired, one row at a time from the TDI scanner (see Appendix B). The TDI scanner, which was purchased from an external commercial supplier, uses time delay integration to perform multiple scans of each row that are then averaged in order to increase the signal-to-noise ratio. If a simple position tracking (assuming a noisy motion profile) correction method were adopted, without actually correcting the physical motion noise, then each row would not have been scanned the same number of times. Hence the SNR as well as the exposure time of each row may differ, and this would result in horizontal banding in the image. Note that an over-exposure may saturate a CCD. For an example of this horizontal banding, see Figure 1.2(a).

Secondly, the TDI scanner is implemented in an embedded algorithm that is part of the CCD detector circuitry [14]. The results are transferred to the Statscan™ software (PC-based) one integrated row at a time. Using image processing-based motion noise correction would require real-time bi-directional communication between the PC-based software and the commercial embedded TDI algorithm in order to adjust the TDI process at each discrete position (i.e., to adjust the integration procedure). Whilst this can be achieved by modeling row-by-row degradation of the TDI image, taking into account movement sensor data, and then using the restoration method to compensate for the degradation (which has been done theoretically and experimentally by Zheng, Li and Chen [12]), this is not a viable solution for a device that includes standardised commercial components from third-party manufacturers

Furthermore, both the scan length and scanning time are externally defined parameters that may not be adjusted. For example, the scanning time is limited to 13 seconds as the commercial X-ray tube [Varian, UT, USA] would overheat beyond this maximum. The scan length is also set to a maximum of 1.8 m because of standardised Lodox trolley sizes (see the Lodox product specifications described in Appendix B.5). Under these constraints, the scope of this dissertation was determined by the preferred approach to solving the image quality degradation issue, which was to investigate the possibility of using different control systems to allow the operating velocity to be achieved in as short a time as possible, as well as to provide a smoother motion profile [9].

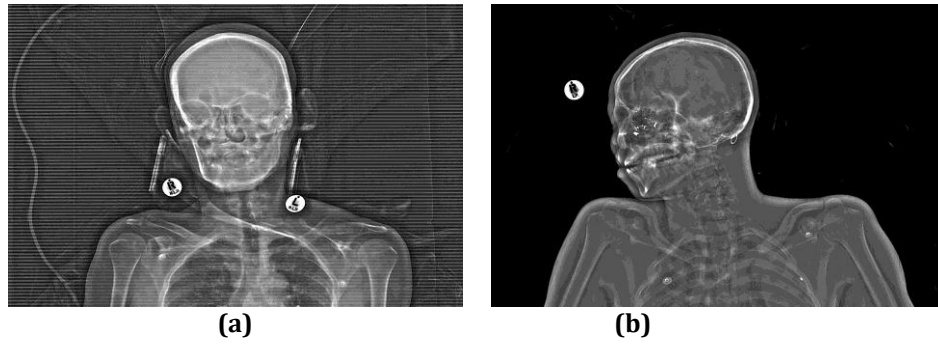


Figure 1.2: TDI image with banding strips (a) and normal TDI image (b).

A Trapezoidal motion profile (T-curve) and a smoothing motion profile (S-curve) are common motion profiles in industrial applications, (see Figure 2.1). They consist of an acceleration phase, a maximum velocity phase and a deceleration phase. Generally, the S- curve is better than the T-curve because it has a longer acceleration time and hence can achieve a smaller error. The T-curve is characterised by the sudden change during the acceleration phase and deceleration phase, thus causing a large vibration [9].

Currently, the Statscan™ X-ray machine implements a T-curve motion profile. Therefore it was decided to use this same profile for the initial simulations that were implemented as part of this dissertation. In addition, from the Statscan™'s documentation it was found that the Statscan™ X-ray machine starts scanning the patient, i.e., creating an image when the system steady state at constant speed of the motion profile has been reached. The Statscan X-ray machine is driven by a permanent magnet linear motor (PMLM) which is directly coupled to the load. It provides high accuracy position and velocity motions but these come at a cost of difficulties in controlling such systems. More specifically, the motor is sensitive to disturbances and parameter variations, frictional forces, cogging forces and electrostatic forces, as well as other external disturbances. These significantly degrade the controller's tracking performance [10].

In order to determine the extent of the image degradation when the system is at the beginning of the constant speed phase, a spatial resolution line-pair tool (Fluke Biomedical, OH, USA) was imaged. The beginning of the constant speed was chosen because this is where the change from acceleration to constant speed happened. A second change also happens at the end of the scan, going from constant speed to deceleration. As mentioned earlier, the system starts scanning the patient only when the system is at constant speed. The experimental results are shown in Figure 1.3. The image on the left was scanned at half speed (0.070 m/s) while that on the right was scanned at full speed (0.140 m/s). Both images were exposed at 50 kV, 125 mA, and the resulting digital images used 4x4 pixel binning. It can easily be observed that the image

degradation is greater at full speed than at half speed. This is due to higher vibrations induced during the higher acceleration and because of less X-ray flux during the exposure. Another reason for the image quality degradation may be due to a mismatch between the detector's clocking speed and the linear speed of the C-arm during the early part of the scan. We conclude that vibration forces have degraded the image quality by adding a partial image blurring. Hence, as mentioned, the trapezoidal velocity profile causes vibration to increase as the speed increases. Currently, the strategy implemented in the Statscan™ machine is to suppress the initial data that may be severely blurred. This lost data appears as a white area as shown at the top of the image in Figure 1.3(b)

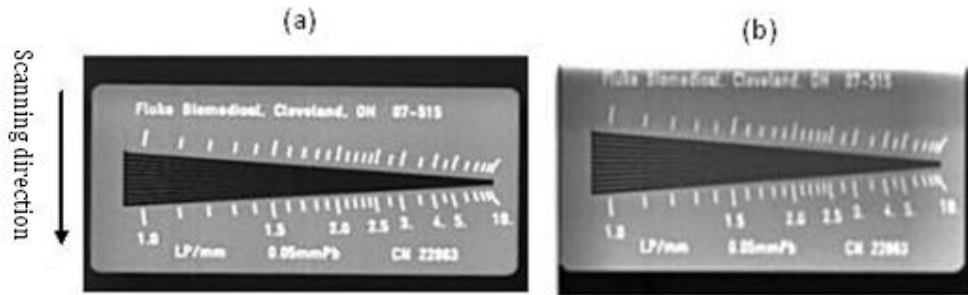


Figure 1.3: Statscan™ images of a line-pair tool acquired at half speed (a) and at full speed (b) with data loss.

In order to overcome the output transition problem and residual suppression, several studies have been conducted to improve high positioning accuracy systems. For example, a cascaded controller consisting of a proportional controller (P) in the outer loop, a proportional integral controller (PI) in the inner loop and an Anticipatory type iterative learning controller (A-ILC) were proposed to control a high positioning accuracy table driven by a linear motor (PMLM) [6]. The A-ILC is a repetitive learning controller which uses the previous cycle's information of the system's output to adjust the next cycle's control effort in order to adaptively improve the system performance. This approach is very good for systems that require repeated actions. Further, a novel motion control method has been designed to reduce the jerk and the resultant transient vibration in linear motion stages [9]. The velocity and the displacement S-profiles are calculated by integration and implemented on a digital signal processing (DSP) motion controller. In order to obtain a smoother movement without any sharp changes as stated earlier, the acceleration profile is designed based on a level-shifted cosine function. This method can be applied to a wide range of precision machines. Moreover, a sliding control combined with input shaping has also been proposed to reject flexible dynamics [15]. The command generation has proven beneficial for eliminating residual vibration from motion-induced dynamics. The first step of this method is generating an

input shaper (a sequence of pulses) from the desired input trajectory. The input shaper sequences are convolved into a shaped trajectory command. The shaped trajectory command is then used as a reference for the system. As an example, a demonstration of the input shaping method for an unshaped step input command is illustrated in Figure 1.4. Therefore, input shaping is one of several methods that could be used to reduce vibration. However, the downside is that it will increase the rise time [16]. Increasing the rise time is not desirable in the case of Statscan™ since it is essential for the system to settle as fast as possible because the scan time is limited to 13 seconds at full speed. The extent of this has not been quantified in this dissertation because this approach has not been considered as a method in this dissertation. The various proposed methods will be discussed in depth in Chapter 3, when the methodology is described.

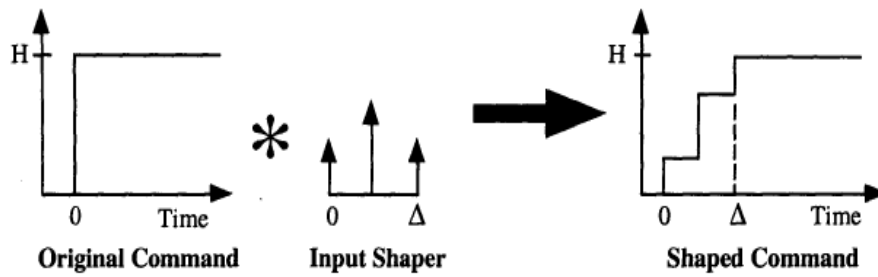


Figure 1.4: Input shaping step input [16].

1.4. Purpose of the study and limitations

X-ray image degradation results from a mismatch between the actual position of the X-ray source on the C-arm and the position assumed by the embedded TDI algorithm of the CCD detector. This mismatch is affected by controller tracking performance on the C-arm such as overshoot, settling time and the position vibration during the initial part of the scanning process, and the final position error.

The image degradation is in the form of image blurring and/or banding. Correcting for vibration by tracking the instantaneous position of the C-arm and using instantaneous image correction procedures is non-trivial because:

- 1) The embedded and commercial nature of the CCD module with the TDI algorithm complicates a row-by-row update and correction of the image. Such an update can only be implemented by a customised communications protocol that would require both hardware and software changes. This would not be industrially viable, particularly because of the extensive modifications to the embedded “off the shelf” TDI module that would be required.
- 2) Position vibration also results in over- and under-exposed image bands. Owing to the nature of X-ray sources, as well as the TDI method, these bands would also have different signal-to-noise ratios that cannot be corrected.

There are a number of additional constraints: For example, the maximum scan time is limited to 13 seconds (see Appendix B.6) by the specifications of the commercial X-ray source, i.e., to prevent over-heating, and the maximum scan length is limited by the standardised length of the patient trolley. Furthermore, the minimum scan speed is limited by the maximum X-ray dosage allowable (noting that the Statscan™ system was specifically designed and marketed as a low-dose X-ray system) and the maximum scan speed is limited by the minimum X-ray dosage requirement for an acceptable image.

The approach was therefore to investigate changing the control system in order to reduce settling time, overshoot, final position error and vibration. This was done in the laboratory and with computer-simulated environments. The current controller implemented in Statscan™ is the classical proportional-proportional integral cascaded controller (P-PI). In this dissertation the existing PI controller of the inner loop was compared to the new integral sliding mode control (ISMC) in order to improve image quality. The new controller in a cascaded structure will be a P-ISMC where P is the proportional controller of the outer loop. The P-ISMC controller was chosen because ISMC has many advantages, including insensitivity to parameter variations and model

uncertainties, external disturbance rejection, fast dynamic responses and good transient performance [15]. The one noted drawback of the sliding mode controller is the infinite-frequency magnitude oscillations around the sliding surface called chattering which can be reduced by boundary layer techniques.

The author hypothesises that use of the P-ISMC controller would result in improved image quality due to shorter settling time, reduced overshoot; lower final position error and less vibration than the P-PI controller, and this would translate into improved X-ray image quality as measured by the image blurring index. Therefore, the representative tracking tasks that will be used are the step set-point and trapezoidal motion profile, system transient specifications such as settling time, final position error and overshoot, as well as disturbances. In contrast, three-way ANOVA statistical tests will be carried out to examine both controllers tracking performance on transient specifications, as well as the image blurring index (see Chapter 5).

Although motion trajectory planning and system modelling in point-to-point control are essential steps before the design of a controller [6], it is good to point some limitations regarding the motion planning and the system modelling. One of the problems of the trapezoidal motion profile is the jerk force which causes the system to vibrate. However this is beyond the scope of this dissertation, which focuses instead on studying the tracking performance of the trapezoidal motion profile in order to test the image quality improvement. On the other hand, it has been difficult to generate a workable Statscan™ X-ray machine model. The linear motor (PMLM) can be modelled with a first-order or second-order model for velocity and position, respectively, as a DC motor model. Therefore, all the simulation and experimentation will be done on a DC motor.

Furthermore, although optimization can provide accurate comparisons between both controllers, both controllers' parameter tuning is based on a control engineering experience perspective. Thus, the ISMC tuning parameters were chosen by setting the rate of converging to the sliding surface (M or S_I), switching amplitude (k) and boundary layer width (δ) such that all ISMC parameters are positive, the system dynamics on the switching surface are stable, and the discontinuous controller is kept reasonable in order to avoid damaging the actuator (see section 3.3.5). On the other hand, the P-PI controller's parameter tuning was based on the root locus method for servo systems (see section 3.2.3).

1.5. Dissertation Plan Development

This dissertation is organised as follows:

Chapter 1

This chapter provides the problem statement and highlights the general overview of the entire dissertation.

Chapter 2

This chapter presents a literature review which provides the background theory on proportional-proportional integral control (P-PI), proportional-integral sliding mode control (P-ISMC) and DC motor modelling. A review of previous work on the subject of P-PI and P-ISMC, on point-to-point motion application, as well as on modelling and control of linear motors (PMLM) will be covered.

Chapter 3

This chapter covers the dissertation methodology. It provides the system model. This model is necessary for designing the controllers and initialising the simulation study. Validation tests are carried out to validate the model. Then the design of the PI, P-PI, ISMC and P-ISMC controllers for the DC motor are presented. The simulation and experimental setup will also be presented. Lastly, the image quality assessment methods will be presented.

Chapter 4

This chapter presents the PI, P-PI, ISMC and P-ISMC simulation results for velocity and position tracking tasks, step set-point, trapezoidal motion profile and the influence of disturbances on the system performance. The performance of the PI and ISMC are evaluated only by simulation, while the P-PI and P-ISMC are evaluated by means of both simulation and experiment.

Chapter 5

This chapter presents ANOVA test results for the controllers in terms of constant velocity overshoot, final position error, settling time and image blurring index.

Chapter 6

The findings are set out and the proportional-proportional integral (P-PI) controller's and the proportional-integral sliding mode controller's (P-ISMC) performance are discussed. In addition to the assessment of both controllers' performance, the discussion and findings on image quality will be presented.

Chapter 7

This chapter summarises the results and the conclusions of proportional–integral sliding mode control (P-ISMC) and its applicability to the system. Finally, recommendations and future work are discussed.

2. Literature Review

2.1. Introduction

The cascaded motion controller that is implemented on the Statscan™ X-ray machine (see Figure 1.1) comprises a linear cascaded controller and a SERVOSTAR® CD series CE06250 driver by Kollmorgen. It is generally not advisable for a user to manipulate the controllers on commercial machines except by changing the parameters for tuning. Therefore, it was necessary to design both a proportional-proportional integral controller (P-PI), as well as a proportional-integral sliding mode controller (P-ISMC) to allow for fair comparisons between the two.

This chapter starts with a literature review of cascaded linear position control. The general analysis and design of a P-PI controller for linear motion is then presented. In order to compensate for the effects of saturation, the integration stopping technique is used. In addition, a practical PI-controller implementation is also presented.

The second part of the literature review presents integral sliding mode control as an alternative solution. It starts with a brief review of the existing work on linear motion control along with some background theory concerning variable structure control (VSC) systems with integral sliding mode control. Finally, the third part of the literature review ends with a brief review of an image quality method that will be used to compare the performance of the proportional-integral sliding mode controller (P-ISMC) and the proportional-integral sliding mode controller-plus-boundary layer width and disturbances (P-ISMC+BD) with a P-PI controller.

In order to achieve fast motion, it is essential to control the acceleration or deceleration phases which cause high jerks that can lead to some residual vibration. Therefore, the trade-off between the speed of the fast motion and the vibration reduction has been one of the challenges faced in motion technology [9]. The literature on fast motion control highlights two directions of research, which have been conducted to date. One is to investigate how to plan the motion profile trajectory. The second direction is to investigate or use one of several advanced non-linear control methods. For example, output feedback adaptive robust control [10], non-linear sliding mode control [16], the loop shaping method [17] and discrete model reference sliding mode control [18] have been proposed in the literature for reducing tracking error when non-linearities (friction and dead band), external disturbances, plant uncertainties and parameters variations exist.

For example, in terms of motion planning, a practical efficient method for motion control based on asymmetric velocity profiles has been proposed by Rew, Ha and Kim [19]. This method allows the jerk magnitude to be manipulated during the deceleration phase by using a single parameter. In the same way, Zheng, Su and Müller have addressed an online smooth trajectory generation for industrial systems [20]. Two modified non-linear tracking differentiators were used to generate a smooth trajectory from ramp and step set-point references. The practical value of this method lies in the fact that it can be useful for implementation in most mechatronic systems without hardware alteration. This approach has also been shown to perform effectively for systems with dynamic friction. More examples for motion planning will be given in the literature review section.

However, in terms of practical implementation, the easiest way to control the motion is to use a trapezoidal motion profile. A review of the literature on control methods that are used to track trapezoidal motion profiles is presented in the following subsections. This dissertation implements a trapezoidal motion profile and will focus on investigating advanced control methods, specifically an integral sliding mode control (ISMC) tracking method.

2.2. Cascaded Linear Positioning Control Review

Linear feedback controllers such as PI and PID control are still the preferred controllers in modern industries, including motion control systems, owing to their simple structure and ease of implementation [21]. In principle, linear controllers usually compare the desired set-point signal with the measured output signal in order to generate the tracking error signal. Then, the control output signal $u(t)$ is generated based on the controlled error signal $e(t)$ [22]. Obviously, because of the parameter variations and the system non-linearities such as frictions and saturations that exist in the positioning systems, the PI/PID controller's performance sometimes cannot be consistent [21], [6]. In other words, the lack of control robustness against parameter variation and disturbance rejection will lead to inadequate control system transients and tracking performances.

Various approaches have been proposed in the literature for improving the PI/PID controller's performance by means of some form of robustness against the parameter variations and disturbance rejections [21], [22], [23], [24], [25]. For example, feed-forward is added to feedback linear controllers. This addition improves the tracking performance but requires plant model and input disturbance signals to be known [23]. In addition, in order to improve motion positioning performance and robustness against inertia variations, artificial neural network have been used by Ahamed and Htut

[24] to produce intelligent PID control. This method allows the PID controller to automatically learn, adapt and tune its parameters in case there is variation on the load during the control process. In the same way, a concurrent-relay PID control is proposed by Li and Tsang [21] for motor position servo systems using a PID controller and a dead band relay sub-controller. The PID controller is designed to regulate the system near steady-state while the dead band relay control is used to improve the transient performance. In contrast, in order to deal with the large tracking error and longer settling time, the dead band relay sub-controller design was based on variable structure theory. However, because of the high frequency oscillation and the stability of the closed-loop system problems that are found in the variable structure, an auxiliary limited integration and lead compensator have been added to the dead band relay controller. In both later methods the PID controller was designed separately by using standard methods such as root locus, Nyquist plots, pole-zero cancellation and so on, before combining to the artificial neural network and a dead band relay sub-controller. Further, in order to improve the tracking error with minimal overshoot and improve the settling time for the industry servo drive, an automatic mode switching of P/PI control tuning method has been proposed in the literature, where the PI and P speed controllers are switched between operating points based on torque command [25]. By using online spectrum analysis of the torque command in the frequency domain, the P/PI controller's operating points can be found automatically. The torque command was used as a tuning index because of the measurement noise that occurred with the speed sensor at low speeds.

In the literature, although these mentioned techniques perform well when compared to the classical PI/PID controllers, the P-PI cascaded linear control shown in Figure 2.2 still remains the dominant strategy for improving tracking errors, fixing the parameter variations and rejecting the disturbances for servo systems in industrial applications [6], [16], [26]. Therefore, in order to improve the P-PI controller performance, some modification to it has successfully been implemented in numerous high motion control applications [6], [26]. For instance, iterative learning controllers such as A-ILC, P-ILC and D-ILC can be used to improve the tracking performance of the cascaded P-PI controller [6], [26]. The cascaded controller is composed of a PI-controller in the inner loop and a P-controller in the outer loop. In this way two strategies have been used for good tracking performance in the case of disturbance and parameter variation. In the first strategy the A-type of iterative learning control scheme and a proportional control loop in the outer loop are designed to be variant during the motion process for compensating the system uncertainty as proposed by Ding and Wu [6]. In the second strategy the cascaded controller remains invariant during the motion process while the

A-ILC adjusts the reference trajectory at any cycle as proposed by Wu and Ding [26]. The desired reference trajectories that are tracked are the T-curve and S-curve motion profiles as shown in Figure 2.1, as well as the position motion profile. In this way, the reference trajectory can be adjusted cycle by cycle for improving the tracking performance.

The convergence analysis methods in the time domain and in the frequency domain have often been used to prove the advantage of the iterative learning control algorithms for improving the tracking performance. With such strategies it is common to generate the motion profile first and then to design a controller to track it. In experimental tests the proposed strategies can greatly improve the system performance when an A-ILC controller is used. In addition, an S-curve was found to be better than a T-curve. The important factor that has been noticed is that these strategies can be useful for commercial machines which allow the users to adjust only the reference command and don't allow the users to change the controllers or manipulate the control signals. In other words, although iterative learning control algorithms for improving the performance of the P-PI controller are beyond the scope of this dissertation, they may be investigated for the Statscan™ X-ray machine since the same Kollmorgen linear motor model (IL-240- 50 AI), Heidenhain linear optical encoder and SERVOSTAR® CD (CE06250) commercial servo drive, and cascade controller configuration have been used in these strategies as well as in the Statscan™ X-ray machine.

Motion profile planning has also been investigated as an alternative in the literature. As discussed in Chapter 1, a novel motion control method was designed to reduce the jerk and the resultant transient vibration in linear motion stages [9]. This study also proved that an S-curve is exhibited better performance in terms of speed and precision with a jerk profile when compared to a T-curve. The velocity and the displacement of the S-curve profiles were calculated by integration and then the S-curve motion profile was implemented on a DSP-based motion controller. In this way, smooth movement was achieved by reducing the settling time at lower speeds and with lower acceleration. However, the cost of this reduction required a lot of computation power and it was also more difficult to implement.

Figure 2.1(a) and Figure 2.1(b) show the motion profiles of a trapezoidal-curve (T-curve) and a smoothing-curve (S-curve), respectively. The V_{max} denotes the required maximum velocity, and A_{max} is the maximum acceleration. In Figure 2.1(a), section I denotes the constant velocity, and section II is the maximum acceleration. By varying the time of section II, we can reach different values of A_{max} , while variation of the length of section I leads to different end positions and different period of section II causes different accelerations [6],[9].

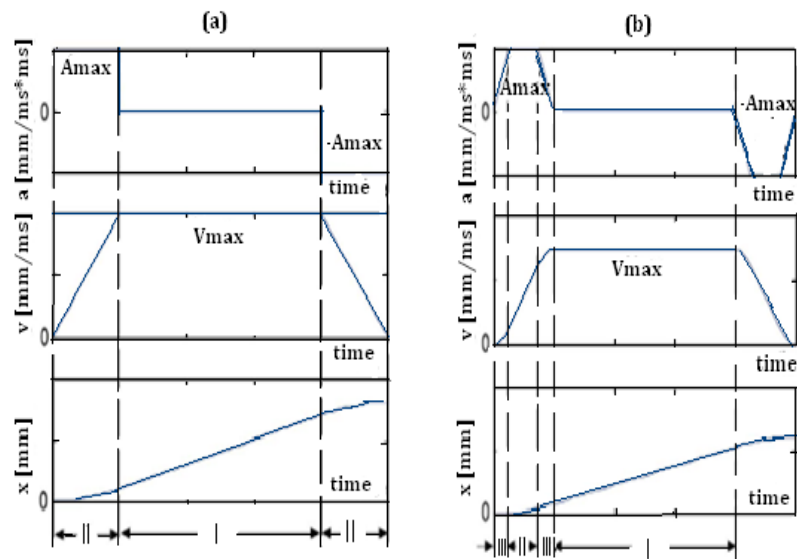


Figure 2.1: Trapezoidal-curve (a) and smoothing-curve (b) [6].

2.2.1. P-PI Control Structure and Design Analysis

A simplified structure of a cascaded linear controller consists of an inner speed loop and an outer position loop. Figure 2.2 illustrates the general form of a P-PI controller. This structure design is usually based on frequency domain techniques [16], [17], where $f(s)$, $f_y(s)$ and $n(s)$ are the system input disturbance, system output disturbance and the system input noise, respectively in volts. For example, the loop shaping frequency domain method is reported for optimal tracking performances [17], [27]. In these methods the P-PI controller can be tested and tuned from the inner loop to the outer loop. The tracking performance factors include closed-loop bandwidth, the damping ratio and the dynamic stiffness [27]. This tracking performance analysis is based on a second-order mathematical model of the system.

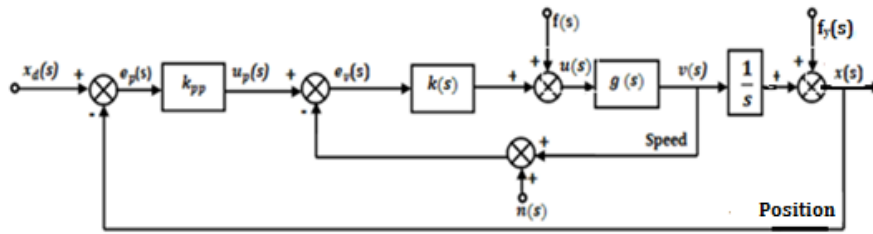


Figure 2.2: Simplified linear motor (PMLM) direct-driven axis with P-PI control structure.

The physical model of the PMLM (IL-240- 50 Al) that drives the Statscan™ X-ray machine (see Figure 1.1) can be described with the physical model shown in Figure 2.3. Its main components include a face-to-face permanent magnet assembly known as a stator, a coil assembly known as a translator or a mover, a magnet base and hall-effect sensors. A coil assembly is usually composed of a specific number of coils plus the hall-effect sensors. The main purpose of the hall-effect sensors is to detect the polarity of the stator for electronic commutation. Conceptually, the interaction between a permanent magnetic field and a travelling magnetic field will generate thrust force when the speed of the motor and the speed of the travelling magnetic field are the same and synchronized. For more information on the description of the physical model the reader is referred to work presented by Li, Chen and Zhou [28].

Let $x(t)$ be the position of the linear motor, then the mechanical dynamics of the motor can be described as:

$$M\ddot{x}(t) + D\dot{x}(t) + f(t) = F_m(t) \quad (2.1)$$

Where, M , D , $f(t)$ and $F_m(t)$ donate the mass, viscous friction coefficient, disturbance force and resultant force.

Let's K_f be the force constant of the motor and then the resultant forces can be given as

$$F_m(t) = K_f i(t) \quad (2.2)$$

where, $i(t)$ is the input current. For more details about these modeling the reader is referred to the work conducted by Cao and Soon Low [29].

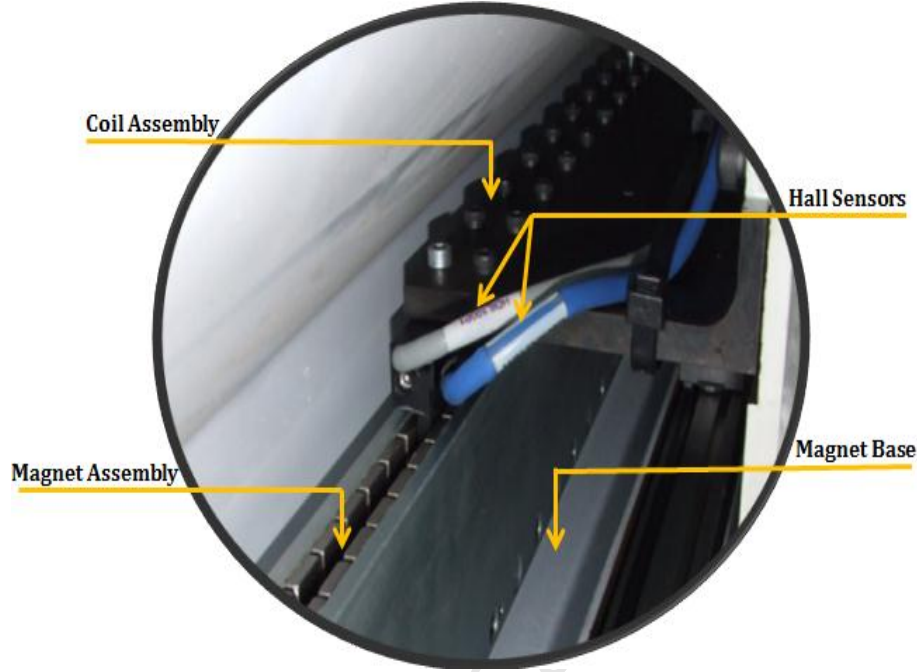


Figure 2.3: Ironless linear motor physical model.

It should be noted that it was not possible to obtain parameters values of the system model such as the system gain A_g and time constant T using a step test method on the physical system shown in Figure 2.3 because it is difficult to submit various step values through a SERVOSTAR® CD series CE06250 driver to the physical system. In addition, the system is in commercial use. Permission to manipulate its parameters was not given. Therefore, because of similarity, an alternative model was obtained by using step tests on a laboratory scale DC motor (see section 3.1, Chapter 3).

2.2.1.1. The System Model

The system's velocity is described by means of a first-order model with a moment of inertia and viscous friction [25]. If A_g and T are system gain and time constant, respectively, the first-order model of the system can be described as in Equation (2.3). The system model in Equation (2.3) will be derived by means of a graphical identification using a step test. The details of the step test will be given later in Chapter 3.

$$g(s) = \frac{A_g}{Ts + 1} \frac{V}{V} \quad (2.3)$$

If Equation (2.3) is taken through the integrator, the system position model can be found too, such that

$$g_p(s) = \frac{x(s)}{u(s)} = \frac{A_g}{s(Ts + 1)} \quad rad/V \quad (2.4)$$

Equation (2.4) represents the transfer function of the position from the control input signal $u(s)$ to the position output signal $x(s)$. Accordingly, the system position model has two poles $s = 0$ and $s = -1/T$. In Figure 2.2 the tracking error $e_p(t)$ is the difference between the desired position $x_d(s)$ and the measured position $x(s)$.

2.2.1.2. Proportional Integral Control Formula

A velocity control design criterion is fully described by Li and Tsang [21]. The PI controller transfer function parameters, namely proportional gain, integral gain and system response speed factors, are denoted as k_{vp} , k_{vi} and $b = \frac{1}{k_{vi}}$ respectively. For simplicity the PI-controller transfer function is represented as:

$$k(s) = k_{vp} \left(1 + \frac{b}{s} \right) \quad (2.5)$$

In the literature, the velocity loop controller is usually either a P- or a PI-controller. Sometimes even a PID-controller is used. The theoretical analysis discussions of these forms are reported in the literature [23], [16], [27]. However, in this dissertation only the analysis of the cascaded controller which consists of the PI controller for the velocity inner loop and the P-controller for the position outer loop will be considered. For more details about the use of the proportional controller in the outer position loop see sections 3.1.6 and 3.2.3.

2.2.1.3. The Analysis of P-PI Closed Loop Control

Consider a velocity controller loop with a proportional-integral (PI) controller and a position controller with proportional gain k_{pp} . If the PI-controller has a structure as given in Equation (2.5) and the position model is represented with Equation (2.4), the output position signal $x(s)$ in Equation (2.6) is related to the desired position reference $x_d(s)$, the input disturbance $f(s)$ in volts and the speed measurement noise $n(s)$. As seen on the block diagram in Figure 2.2, by applying the superposition concept, the relation transfer functions between the desired position reference $x_d(s)$, the input disturbance $f(s)$ and the speed measurement noise to the output position signal $x(s)$ are represented by Equations (2.7), (2.8) and (2.9), respectively.

$$x(s) = G_p(s) x_d(s) + G_d(s)f(s) + G_n(s)n(s) \quad (2.6)$$

with

$$G_p(s) = \frac{x(s)}{x_d(s)} = \frac{k_{pp} k(s) g(s)}{s(1 + k(s)g(s)) + k_{pp} k(s) g(s)} \quad (2.7)$$

$$G_d(s) = \frac{x(s)}{f(s)} = \frac{g(s)}{s(1 + k(s)g(s)) + k_{pp} k(s) g(s)} \quad (2.8)$$

$$G_n(s) = \frac{x(s)}{n(s)} = \frac{-k(s) g(s)}{s(1 + k(s)g(s)) + k_{pp} k(s) g(s)} \quad (2.9)$$

The transfer function from the input external disturbance $f(s)$ to the output position $x(s)$ (Equation (2.8)) is called the compliance transfer function of the tracking system. The inverse of the compliance transfer function is defined as the system's dynamic stiffness. Dynamic stiffness is an important characteristic of servo systems' control because it indicates the controller disturbance rejection capacity [16], [17]. The relation between the tracking error $e_p(s)$ and the desired reference position input $x_d(s)$, the input disturbance $f(s)$ and the external velocity noise $n(s)$ can be expressed as

$$\begin{aligned} e_p(s) &= x_d(s) - x(s) \\ e_p(s) &= G_{ep}(s) x_d(s) - G_{ed}(s)f(s) - G_{en}(s)n(s) \end{aligned} \quad (2.10)$$

where,

$$G_{ep}(s) = \frac{e_p(s)}{x_d(s)} = \frac{(1 + k(s)g(s))s}{(1 + k(s)g(s))s + k_{pp} k(s) g(s)} \quad (2.11.a)$$

$$G_{ed}(s) = \frac{e_p(s)}{f(s)} = \frac{g(s) (1 + k(s)g(s))}{(1 + k(s)g(s))s + k_{pp} k(s) g(s)} \quad (2.11.b)$$

$$G_{en}(s) = \frac{e_p(s)}{n(s)} = \frac{-k(s)g(s)(1 + k(s)g(s))}{(1 + k(s)g(s))s + k_{pp} k(s) g(s)} \quad (2.11.C)$$

In order to analyse the P-PI controller's tracking performance, the proportional controller in the position loop is modified such that a set-point signal with a constant velocity can be generated as

$$r(s) = \frac{v_{in}}{s^2} \quad (2.12)$$

where $r(s)$ is the ramp set-point and v_{in} is desired constant velocity. The steady state tracking error is derived by using the final value theorem, Equation (2.11), results in

$$e_p(\infty) = G_{ep}(\infty) + G_{ed}(\infty) + G_{en}(\infty) \quad (2.13)$$

Since the system closed-loop in Equation (2.6) will result in a 3rd order feedback system, the design procedure of the P-PI cascaded controller has become more complicated. One of the approaches to overcome this complication is to design the inner loop and the outer loop in Figure 2.2 separately so that the design specifications can be satisfied [6], [16], [27]. The most important design specification is the bandwidth which must be selected so that the bandwidth of the velocity loop is higher than that of the position outer loop. In this approach, the Standard control methods such root locus, Bode diagrams and input shaping have been extensively used in order to find the controller gains in Figure 2.2 so that the design specifications such as settling time or bandwidth and percentage of overshoot can be obtained. Furthermore, the controller must be tuned from the inner loop to the outer position loop. In this dissertation Bode diagrams and root locus were used for designing the two loops; see section 3.2 for more details.

2.2.2. Integrator windup

In order to ensure the best performance of the controller in a practical implementation, some non-linear effects must be considered. The main non-linear effect with a PI-controller is the phenomenon called integrator windup. The main cause of integrator windup is the interaction between the integral action and saturation which makes the system behave as an open loop configuration [27]. For this reason, the integral term becomes large (wind up) and the control action becomes higher than in the absence of saturation. The large integral term causes saturation of the control variable even after the process output reaches its reference value and a large overshoot occurs and an oscillatory response may result because of limited motor current and motor speed [30]. In order to avoid this problem four major techniques have been reported, namely conditional integration, limited integration, tracking anti-windup and modified tracking anti-windup. Some of these techniques are active only during saturation and can provide limited improvement in the performance. Others recover from saturation by extra feedback compensation with a tuning process [31].

Of late, an anti-windup algorithm for positioning systems can be used whereby priority is given to the proportional control output of the speed control PI. This method can provide a smooth response compared to the previous strategies [30]. In contrast, for this dissertation the integration stopping technique is used to recover the integration windup since the main project purpose is to compare the P-PI controller with P-ISMC.

2.2.2.1. PI Formula Implementation

The PI-controller, previously mentioned in Equation (2.5), is implemented by the following difference equations, where T_s is the sampling time, k_{vi} and k_{vp} are defined as in the earlier section 2.1.1.2, $e_v(k)$ is the error between the desired constant velocity and the output velocity and finally, $u(k)$ is the PI control action

$$e_v(k) = v_{in}(k) - v(k) \quad (2.14)$$

$$I(k+1) = I(k) + k_{vi} e_v(k) T_s \quad (2.15)$$

$$u(k) = I(k) + k_{vp} e_v(k) \quad (2.16)$$

2.2.2.2. Conditional Integrator (Integration Stopping)

This strategy is fully discussed under the name of conditional integration or sometimes integration stopping [30], [31], [32]. This technique is based on the idea that the integration process must be stopped when the integral value of the PI-controller reaches the actuator limit value. The integration stopping method is shown in Figure 2.4.

$$|I(k)| > |u(k)_{Limit}| \quad (2.17)$$

$$I(k+1) = I(k) \quad (2.18)$$

Note: in Figure 2.4 the term $I(k)$ represents the integral sample, $e_v(k)$ represents the velocity error and T_s represents the sampling time.

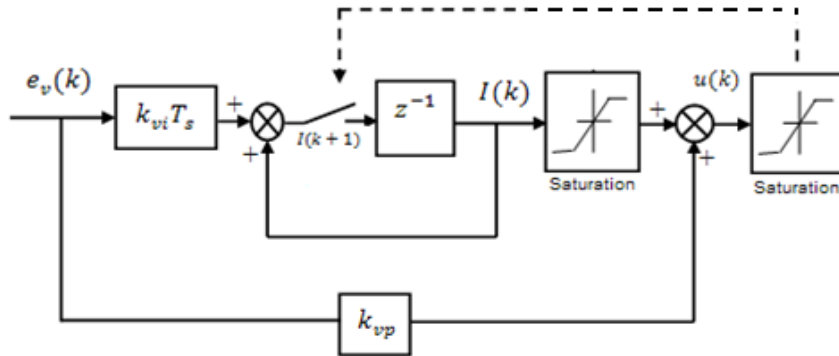


Figure 2.4: PI-controller using integrator with limiter.

2.3. Review of Sliding Mode Control for Motion Control Applications

In recent years direct transmission between the servo motors and the load has achieved much higher motion accuracy. In practice, however, it is difficult to control such systems. The linear servo motors (PMLM) are sensitive to disturbances and parameter variations, especially because the system's inertia influences the motion response performance [33]. As mentioned in the previous subsection, the classical cascaded controller (P-PI) is a well-known tracking controller used in most servo systems. However, it is not robust to parameter variations and disturbances rejection. As a result, various alternative methods have been suggested for replacing the PI-control type scheme performance [34]. Generally, SMC is known to yield high robustness properties to parameter variations and disturbance rejection and is one preferred approach to non-linear tracking control of motion servo systems. This design method is presented in more detail later in this chapter. However, some examples of previous work on this application are presented here.

A conventional SMC scheme can be used to improve tracking performance but the discontinuous controller exhibits chattering (see section 2.4.3) which is undesirable with a direct servo system.

A continuous, accurate and robust sliding mode tracking controller based on a disturbance observer on the direct servo system was proposed [35]. The effects of the disturbances and uncertainties are removed by using a continuous controller. The results showed that the proposed scheme improved the tracking performance and removed chattering through efficient compensation by the disturbance observer.

A model reference discrete sliding mode control (MRDSMC) on a high precision SISO servo system was studied [18]. This control configuration is shown in Figure 2.5, where the desired response $x_d(s)$ to a set-point reference signal $r(s)$ is determined by a reference model which provides the reference trajectories that can be tracked by the controller.

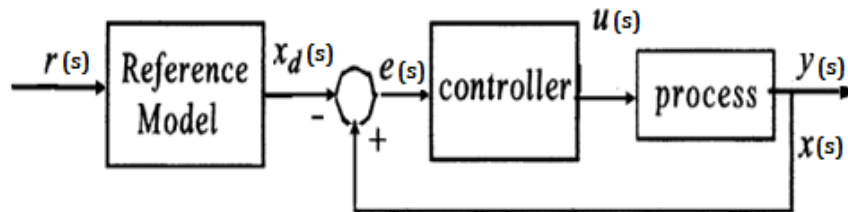


Figure 2.5: The model reference control system [18].

The servo system friction was compensated for by a one-step delayed approximation scheme. The model reference structure converted the servo problems into simple regulator problems and the controller becomes insensitive to parameter uncertainties, without the need for complicated friction identification methods. The results showed that an MRDSMC cannot eliminate the friction effects when reference trajectories crossed zero velocity. Thus, an additional integral action was needed to design the MRDSMC such that the tracking errors could be improved.

A novel sliding mode control scheme for tracking control of a permanent magnet linear motor (PMLM) with two modes of operation was proposed in the literature [34]. The speed mode profile is segmented into three phases, namely acceleration, constant speed and deceleration, while the position has one phase. The velocity controller tracks the speed that is specified with a trapezoidal motion profile. However, the multi-segment SMC controller is composed of a position control mode and a velocity control mode. At any velocity segment a new switching function is defined and a corresponding linear feedback sliding mode controller is designed. Moving from segment to segment the velocity controller suppresses any uncertainties that occur in the velocity segments. The position segment controller improves the system response and stops at the desired position.

The most interesting comparison study of sliding mode control and linear cascaded control was conducted by Jamaludin, Brussel and Swevers [16]. The two controllers were applied in simulation and practice to an x-y milling process. Both controllers' performance was assessed on reference set-point tracking and circular tests. In summary, the sliding mode controller worked well but required relatively high-bandwidth to ensure better stiffness, while the classical cascaded controller worked well but required reasonably low-bandwidth to ensure better stiffness. When both controllers were tested for tracking errors, the cascaded control worked better on tracking errors than the sliding mode control did.

In the same way an iterative learning variable structure controller for high-speed and high-precision point-to-point motion application was proposed by Wu and Ding [33]. In this method the motion is divided into two stages that do not depend on motion profiles, namely the high speed-stage and the high-accuracy position stage. Then the controllers are designed to track the two phases. By developing an iterative learning law, a position switch is determined. Then the position sub-stage is controlled with sliding mode control. The system performance was improved with no overshoot and a reduction of settling time. However, in order to achieve a zero overshoot and reduce settling time, discontinuous projection as a coordination mechanism is used. It modifies

the iterative learning law such that the control law becomes robust to the disturbance that destabilizes the learning process [33].

Alternatively, a sliding mode control scheme that is combined with input shaping is proposed to reject flexible dynamics [16]. Input shaping is implemented by convolving a sequence of impulses with a desired system command. The convolved signal is then used as the reference input to the controller. The input shaper is designed from a simple system model estimation of the natural frequencies and damping ratios. It is noted that motion profile controllers such as classical linear controllers and embedded controllers (DSPs) have some drawbacks, including the sudden change in the jerk profile over time in the case of classic controllers, and the difficulty of the implementation and the necessary computation power in the case of embedded controllers.

2.4. Fundamentals of Variable Structure with Sliding Mode Control

This section presents the fundamental design of sliding mode control in general and various design methods in the literature that will be used in the methodology (Chapter 3).

2.4.1. Background

Variable structure control (VSC) with sliding mode control is a concept that was first proposed in the Soviet Union. Since 1977, when Utkin published his survey paper of VSS with sliding modes, this concept has been attracting the attention of control engineering researchers worldwide. This method has many qualities, including insensitivity to parameter variations and model uncertainties, external disturbance rejection, fast dynamic responses and good transient performance [36], [37].

The research of VSC in the early days was focused on single-input and single-output systems. Since then, various new methodologies have been developed, such as eigenvalue assignment and the Filippov approach. In recent years, the research in sliding mode control has taken on a general design form and many applications are valid for linear and non-linear, multi-input and multi-output (MI/MO) systems [38].

2.4.2 Brief Fundamentals of Variable Structure Control Systems

The variable structure control systems (VSCS) are a class of systems whereby the control law is deliberately allowed to change its structure at any instant during the control process according to some defined rules which depend on the state of the system [36], [39]. The design concept in VSCS is, therefore, first to define a design rule called a switching surface, which decides how the control structures should change, and then to design a control law which changes its structure based on the defined

switching surface [40]. The key feature of VSCS has been explained in detail in the literature using single-input, single-output, and second-order systems [36], [39], [41]. In these studies, the systems that result from the combination of two variable structures show properties not seen in any separate system structure. For instance, the system trajectories in the phase plane were constructed from parts of trajectories of separate structures as shown in Figure 2.6. The system structures on the upper portion of Figure 2.6 are not stable when they are separate, but the motion presented in the lower section of Figure 2.6 shows that when the two structures are combined, the system trajectories move towards the switching surface. When the trajectories reach the switching surface they move asymptotically towards the equilibrium point. In this way stable motion is achieved.

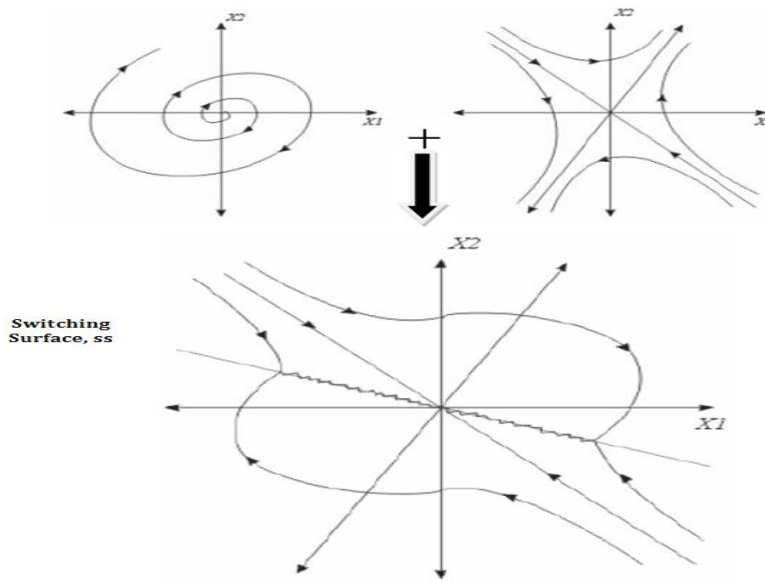


Figure 2.6: Asymptotically stable VSCS consisting of two unstable structures [39].

The general variable structure control design with sliding mode control is composed of two separate steps. The first is the selection of a switching surface in state space to satisfy certain design specifications, including linearization, reduction of order and asymptotic stability. The second is the design of the control structure to bring the trajectories of the switching function onto the sliding surface in a finite amount of time and keeping them there [42].

The dynamics of a process with discontinuous control and equivalent control can be divided into two phases-[38]:

- i. The reaching phase: When the distance to the switching surface is not equal to zero, the process is said to be in the reaching phase. The trajectories in the phase plane are brought onto the sliding surface from an initial location.

- ii. The sliding phase: This phase expresses the dynamic motion in which the trajectories are confined to the sliding surface and then slide towards the equilibrium point.

In the literature, the aforementioned motion is called ideal or regular sliding mode control (SMC) motion. However, the existence of ideal SMC does not guarantee that the trajectories will be confined to the sliding surface and that the system will be stable. Therefore, in order to ensure stability and to reject disturbances, Filipov's vector condition must be checked and Lyapunov's second method should be used. Although the equivalent control handles the matched uncertainties and disturbances on the switching surface, the discontinuous control and unmatched disturbances can affect the trajectories of the ideal sliding motion [36]. Owing to this, chattering will be experienced in practice and the induced high frequency dramatically decreases the process performance.

To illustrate some of the general ideas and properties of the VSCS with sliding mode control mentioned previously in this chapter, a simple example is presented where a second-order non-linear equation is used as a process for sliding mode control design. In addition, the process is assumed to be in a controllable canonical form called the regular form that is derived for systems undergoing sliding motions on switching surface; see section 3.3.1 for more details.

Consider a second-order system that is in the regular form such that:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= h(x) + g(x)u + \xi(t)\end{aligned}\tag{2.19}$$

where $h(x)$ and $g(x)$ are known functions and $\xi(t)$ represents the input disturbance. The general switching function $s(x)$ can then be defined as

$$s(x) = \left(\frac{d}{dt} + m\right)^{n-1}x = 0\tag{2.20}$$

where $x = [x \ \dot{x} \dots x^{(n-1)}]^T$ is the state vector, m is a positive constant, which determines the bandwidth of the desired control system and n is the state dimension [40].

For example, if $n = 2$.

$$s(x) = \frac{dx}{dt} + mx(t)\tag{2.21}$$

This can be rewritten,

$$s(x) = m x_1(t) + x_2(t)\tag{2.22}$$

where $x_1(t) = x(t)$ and $x_2(t)$ is the time derivative of $x_1(t)$.

This yields the following system behaviour once the system has reached the sliding surface, which guarantees that both $x_1(t)$ and $x_2(t)$ converge exponentially to zero.

$$x_2(t) = -mx_1(t) \implies \dot{x}_1(t) = x_2(t) = -mx_1(t) \quad (2.23)$$

The next step is to determine a control structure that ensures that the system trajectories will be on the sliding surface in a finite amount of time ($t \geq t_{sm}$) and that they will remain there indefinitely ($s(x) = \dot{s}(x) = 0$), where t_{sm} is the sliding mode time. The time derivative of the sliding surface is found to be

$$\dot{s}(x) = m \dot{x}_1(t) + \dot{x}_2(t) = mx_2(t) + h(x) + g(x) u(t) + \xi(t) \quad (2.24)$$

Suppose that $\xi(t)$ meets the requirement that the inequality $|\xi(t)| \leq D$ where D represents the external matched disturbance, then $u(t)$ should be selected such that a Lyapunov's function $V(s) = \frac{1}{2}s^2$ satisfies the Lyapunov stability criterion which states that the time derivative of Lyapunov's function must be a negative definite function in order for the system to be stable. In other words, the reaching condition $s\dot{s} < -\eta|s|$ is satisfied where η is a positive design scalar [36], [37], [38]. It should be noted that in some of these publications the reaching condition is represented by $s\dot{s} < 0$. Therefore, in this dissertation these are both considered as reaching conditions. Considering the time derivative of the switching surface given in Equation (2.24) it follows that

$$\begin{aligned} \dot{V}(s) &= s\dot{s} < -\eta|s| \\ &= s|m x_2(t) + h(x) + g(x) u(t) + \xi(t)| < -\eta|s| \end{aligned} \quad (2.25)$$

In order to ensure that the reaching condition given in Equation (2.25) and the sliding mode $s(x) = 0, \dot{s}(x) = 0$ are attained, the controller can be chosen as:

$$u(t) = u_{eq}(t) + u_d(t) \quad (2.26)$$

where the term $u_d(t)$ is the discontinuous control law designed to bring the system trajectories towards the sliding surface and $u_{eq}(t)$ is the equivalent control designed to cancel all matched uncertainties. In the literature the discontinuous control can be defined in several strategies. For example, it can be defined as a bang-bang controller [38] as shown Equation (2.27):

$$u_d(t) = -k \operatorname{sgn}(s) \quad (2.27)$$

where a scalar parameter k needs to be determined and $\operatorname{sgn}(s)$ is a signum-function which is defined as

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{if } s(x) > 0 \\ -1 & \text{if } s(x) < 0 \end{cases} \quad (2.28)$$

In sliding mode control design theory the value of k is chosen to be large enough such that the reaching condition in Equation (2.25) is satisfied, where $k = D + \eta$, and D represents a matched disturbances to the input of the system [40].

Assuming that in a finite time t_{sm} the system states hit the sliding surface such that the solution to Equation (2.24) satisfies $s(x) = 0$ and $\dot{s}(x)=0$ for all $\geq t_{sm}$, then an ideal sliding motion is said to have taken place [36]. If the disturbance to the process in Equation (2.24) is assumed to be zero, then the equivalent control can be expressed as:

$$u_{eq}(t) = \frac{1}{g(x)} (-mx_2(t) - h(x)) \quad (2.29)$$

Substituting into Equation (2.25) for the controller $u(t)$ in Equations (2.26), (2.27) and (2.29) gives:

$$\begin{aligned} s\dot{s} &= s(m x_2(t) + h(x) + g(x) u_{eq}(t) + g(x) u_d(t) + \xi(t)) \\ &= s(m x_2(t) + h(x) + g(x) \frac{1}{g(x)} (-mx_2(t) - h(x)) - \\ &\quad g(x) k \operatorname{sgn}(s) + \xi(t)) \\ &= -g(x) k |s| + s \xi(t) < |s|(\xi(t) - g(x) k) \end{aligned} \quad (2.30)$$

By choosing $k = \frac{\xi(t)}{g(x)} + \eta$ then the inequality in Equation (2.25) is satisfied. Note for simplicity the notation s in Equations (2.25) and (2.30) sometimes is used instead of $s(x)$ and $s(e)$ throughout this dissertation, where $s(e)$ is the switching surface that is defined in the case of the tracking error; see section 3.3.3.

In a practical situation, using the controller that is defined in Equation (3.26) will result in slower attainment of the switching surface [36], [40]. Therefore, an alternative control structure can be defined to improve the system transient response. In other words, the controller can be established such that the time taken to induce sliding motion should be minimized and the region in which sliding motion takes place maximized. Thus, in practice, the modification to Equation (2.26) is to add the term $-\phi s(x)$, where ϕ is a positive design scalar and $s(x)$ is a switching surface, hence,

$$u(t) = u_{eq}(t) + u_d(t) - \phi s(x) \quad (2.31)$$

Again, as argued earlier a new inequality condition must be satisfied in order to establish this new controller which can be described as

$$s\dot{s} \leq \phi s^2 - \eta |s| \quad (2.32)$$

where $s^2 \geq 0$, η means that the reaching condition has been ensured and sliding mode will be obtained. By ignoring the reaching condition term and with some manipulation, the rate of change in the switching surface can be obtained as follows:

$$\frac{d}{dt}|s(t)| \leq \phi |s(t)|. \text{ This implies } |s(t)| \leq |s(0)|e^{-\phi t} \quad (2.33)$$

where $|s(0)|$ presents the initial distance away from the switching surface and ϕ determines the time at which the sliding switching surface will be reached.

2.4.3. Chattering

The one noted drawback of the sliding mode controller is a phenomenon known as chattering, which is defined as finite-frequency magnitude oscillations around the sliding switching surface, which are due to the discontinuous nature of a control action that switches between two system structures.

The high-frequency switching can excite un-modelled dynamics to cause dynamic and static friction forces. Two mechanisms cause chattering [43]:

- i. The existence of parasitic dynamics in sensors and actuators that cause small-amplitude, high-frequency oscillation in the region around the switching surface.
- ii. The switching surface non-linearities required to implement the controller in Equation (2.27) imply an infinite-frequency oscillation.

2.4.3.1 Chattering Alleviation Review

Since chattering is the most significant disadvantage in practical implementations of sliding mode controllers, chattering alleviation has become an important issue in sliding mode research. A lot of research has been conducted to overcome chattering in real-time. The most common approach to reduce chattering is the boundary layer approach [40]. The basic idea is to replace the signum-function in Equation (2.28) of discontinuous control $u_d(t)$ with a saturation function to smooth the $sgn(s)$ term in the thin boundary layer about the sliding surface. This approach will be used for this dissertation and, therefore, will be discussed later in this chapter. In addition, the continuous approximation method has also been presented as an alternative method [16], [36]. Here, as described, the signum-function in Equation (2.28) is also replaced by a continuous approximation, which takes the following form:

$$v_\delta(s) = \frac{s}{|s| + \delta} \quad (2.34)$$

where, the positive constant δ represents the degree of the continuous approximation. A comparison between the discontinuous signum function and its continuous approximation is shown in Figure 2.7

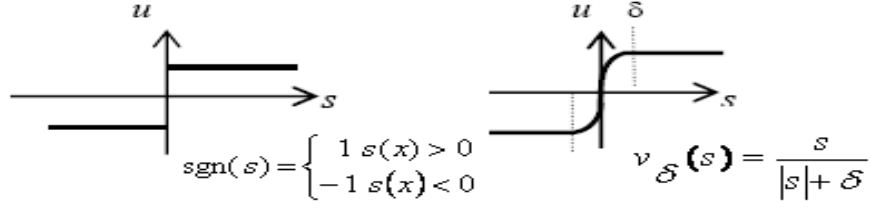


Figure 2.7: The smooth approximation of the signum function [16].

2.4.3.2 Boundary Layer Control

As previously described, to smooth the $sgn(s)$ term in the discontinuous control the thin boundary layer about the switching surface can be defined as

$$B(x) = \{x: |s(x)| \leq \delta\}, \quad \delta > 0 \quad (2.35)$$

where δ is the boundary layer width [40].

In real-time implementation, this is achieved by replacing the signum function in Equation (2.27) with a saturation function. This gives

$$u(t) = u_{eq}(t) - k \cdot sat(s) \quad (2.36)$$

and the saturation function [51] is determined as

$$\begin{aligned} sat(s) &= \{sgn(s) \quad \text{if } |s| > \delta \\ &= \left\{ \frac{s}{\delta} \right\} \quad \text{if } |s| \leq \delta \end{aligned} \quad (2.37)$$

2.4.3.3. Restriction of Boundary Layer Technique

Although a boundary layer is an attractive solution, some practical problems have been raised in the literature. For example, the small or large choice of the boundary layer width can more or less effectively alleviate the chattering, but on the other hand, this could affect the control robustness against parameter variations and disturbance rejection [44]. As a result, the system states are no longer driven to the origin, but rather to a region around the switching surface. Thus, the switching function is never reached. This compromises the robustness against parameter variations and disturbance rejection properties that are the biggest advantages of sliding mode controllers.

2.5. Controllers Comparative Performance Measures

In the reviewed literature, time responses and frequency responses are important performance measures as they show the behaviour of the system in the time domain and the frequency domain [2], [16], [27], [45]. These enable the control engineer to examine some significant control measures such as overshoot, settling time, steady state tracking error, phase margin, gain margin, oscillations on the output, and

saturation on the actuator. Prior to control design, time domain and frequency domain performance measure specifications must be determined [6], [10], [45]. For example, in high precision positioning applications, the performances indices about a position error include

- The maximum absolute value of positioning error as an index of transient positioning performance.
- Root mean square (rms) for average positioning performance.
- Index of positioning speed (how fast the system moves).

However, in this dissertation the basic design requirements include:

- 1) The maximum absolute value for the final position error of the motor should be less than 2 mm used as an index of transient positioning performance.
- 2) The maximum ramp's constant speed overshoot (M_p) should be less than 0.2 mm/sec.
- 3) The settling time (t_s) for reaching constant speed should be 4 seconds while the index of the position move should be fast from start of motion to end of motion; , the smaller the index positioning speed, the faster the point-to-point motion.

Figure 2.8 shows the time domain responses specifications such as rising time (t_r), settling time (t_s), maximum of overshoots (M_p), maximum overshoot time (t_p) and steady-state tracking error [46]. These time domain responses specifications performances are evaluated in Chapter 4 and Chapter 5. In addition, the frequency domain specifications will include phase margin, gain margin, and bandwidth for designing the linear P-PI controller (see section 3.2 in Chapter 3).

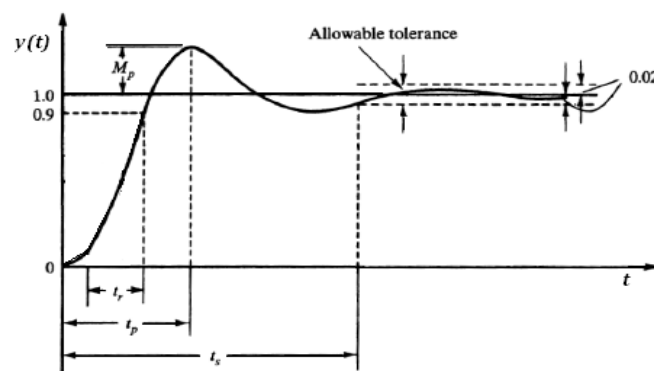


Figure 2.8: Some time responses specification measures where t_s = settling time, M_p = overshoot at time (t_p) and final position error within allowable tolerance [46].

2.6. Image Quality Measures

In the previous discussion (see Chapter 1), the linear slit scanning with TDI operation mode offered potential performance enhancements when compared to other digital radiography systems. It reduces the image degradation caused by scatter radiation, and enhances image quality, as well as the signal-to-noise ratio [7], [13]. These characteristics are useful in LSSR with TDI imaging but the cost of these is the need of a longer exposure which may result in image degradation. The effects of vibration, relative patient movement, measurement errors (control tracking errors) and the radiation dose on the TDI image quality are quantified as follows:

- A lack of synchronisation between the clocking speed of the CCD cameras in the detector and the linear scanning speed of the C-arm will cause image blurring to occur [12]. This occurs in the acceleration and deceleration phases during the scan. During these phases, the Statscan™ will suppress image acquisition, resulting in data loss (see Figure 1.3(b)). The scanning speed of the C-arm can become unsynchronised by the vibration force that arises from the mechanical systems. In addition, the image can also become blurred and distorted by abnormal patient movement relative to the motion of the platform [11]. Image degradation means that image information is being allocated to the incorrect pixel during the TDI process (see Appendix B). In addition, the vibration can cause fish-tailing of the C-arm where the source moves back and forth across the detector, resulting in banding. The banding is a result of over- and underexposure on the image (see Figure 1.2(a)).
- The image contrast resolution, and hence the image quality, can be enhanced by a higher X-ray dose. Conversely, the image quality might be reduced if the scan speed is too high because the dose to the area being scanned will be lower. However, on the Lodox Statscan™ the technique factors, including the given scan speed, are automatically selected from a database according to the scan procedure (body part that needs to be imaged). The dose for each technique is dependent on several technique factors, namely the kV and mA settings for the X-ray tube, as well as the scan speed. These have already been optimised on the Lodox Statscan™.
- From a positioning control system's perspective, the mechanical systems can experience vibration because of the control motion profile tracking errors during the acceleration and deceleration phases. Thus, the control tracking performance along the linear axis is very important, because any sudden change in the acceleration and deceleration phases causes vibrations and

longer settling times[9], [10]. Therefore, the control tracking error measurements also cause the TDI image to be degraded.

In order to quantify the image quality of the Statscan™ X-ray machine, one of several possible image blurring index quality measures was utilized. Image quality is based on the energy content in the higher frequencies in the spatial domain [47]. A focus measure is defined as a quantity which measures the degree of blurring of an image and is called the image blurring index. It has a maximum value for the best-focused image and decreases as image blurring increases [48]. A good focus value should satisfy some essential requirements such as being independent, monotonic, having good discrimination power and being able to combat noisy and low-contrast images conditions.

Various image focus measures are mentioned in the literature, the common ones being variance of image grey levels, norm of the image gradient, 2-norm of the image gradient, 1-norm of the second derivatives of the image, and energy of the Laplacian of the image [49]. Most are measures of the high spatial frequency components of an image. This is because the high spatial frequency components of an image are suppressed by blurring, regardless of the point-spread function (PSF).

In this dissertation, the energy of the Laplacian of the image will be used to compare the quality of Statscan™ images taken at various constant speeds. The main reason for this is because the energy of the Laplacian rated highly as image blurring index in a comparative study among various focus measures [50]. This blurring index measure analyses high spatial frequencies associated with image border sharpness using the Laplacian operator (see Equation (3.61)) which is quite important in the case of Statscan™ [51]. Although the difference between blurring index measures is not big, the energy of the Laplacian is preferred among the other measures because of its tolerance to additive noise [52]. Note that blurring indexes have not previously been applied to X-rays in the literature. However, the blurring of pixels and its quantification is applicable to all images. Hence, it is adopted here.

2.7. Summary

The main observations from the literature and theory survey cited here are that, in general, sliding mode control (SMC) is a good candidate for controlling high-speed and high-positioning servo motors. In order to use SMC in real-time, the boundary layer method must be used to alleviate chattering. Although the addition of a boundary layer can eliminate chattering, there is still a trade-off between a reduction in the tracking precision and control robustness against parameter variations and disturbance rejection.

3. Methodology

This chapter provides the simulation, experimental and image quality assessment and control verification methodology used for the analysis. The system model was defined using step tests. The design and stability of a proportional–proportional integral (P-PI) controller were checked using bode diagrams. The design and stability of a proportional–integral sliding mode controller (P-ISMC) were based on Lyapunov's stability criteria and integral sliding mode design methodology. The qualitative comparison of image quality assessment and controller performance verification is carried out using a three-way ANOVA method in SPSS. ANOVA was chosen because it is a widely used statistical method for comparing several groups means to assess a set of experimental conditions, called factors, which influence a measured output quantity (the dependent variable i.e., *Settling Time*) [53]. Moreover, in comparison with other tests such as a simple t-test, ANOVA has an advantage over them because it can detect an effect of interaction between independent variables (factors) by allowing for the study of two or more factors [54]. Such effects in the model include the main effects in each factor and the interaction between the factors. The main effect is defined to be a simple effect of a factor on a dependent variable while the interaction is the variation among the differences between means for different levels of the other factor [55]. The various tests of three-way ANOVA used in this dissertation are explained in Appendix A.

3.1. System Model Identification

System identification is necessary before a controller can be designed. For example, the model used for computer simulations should represent the behaviour of the real physical system. In this dissertation the model of the system is obtained using step tests because they are simple and have wide application in practice.

3.1.1. Step Tests

The step test is practical, simple and widely implemented for physical systems [56]. The step test data that were obtained relate the system input to the system output for an open-loop system. The physical model obtained was for a DC motor which can be described as a first-order system. The input-output relation is given as in Equation (2.2), where, the system gain (A_g) is given by: $A_g = \frac{\Delta y}{\Delta u}$ and T is the time-constant when the system reaches 63% of settled output.

Figure 3.1 shows the experimental set-up. It consists of an operation amplifier, an attenuator, a servo drive (pre-amplifier and servo amplifier), a DC motor, a tachometer unit, a low-pass filter for filtering the measurement noise, a PC and an 8-bit ADC/DAC (Dt300 DAQ card) from *Data Translation* for analogue and digital conversion. The whole measurement was done in open loop, where $u(k)$ is the sample control data, and $v(t)$ and $v(k)$ in volts are the continuous and sample angular velocities, respectively.

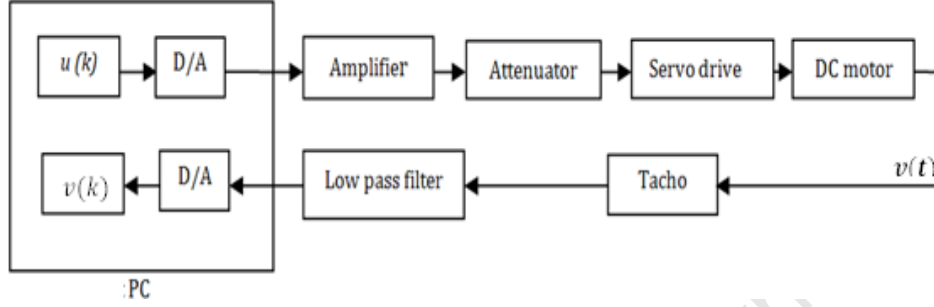


Figure 3.1: the model experimental setup.

Due to the influence of a dead band, the step input at 7 volts is taken as the initial step input. Thus, the following three transfer functions (G_1, G_2, G_3) in Equation (3.1) were obtained from 6.5 to 4.0 volts, 4.0 to 5.5 volts and 5.5 to 3.0 volts, as shown in Figure 3.2.

$$\begin{aligned}
 G_1(s) &= \frac{v_1(s)}{u_1(s)} = \frac{2.02}{1.72s + 1} \quad \frac{V}{V} \\
 G_2(s) &= \frac{v_2(s)}{u_2(s)} = \frac{2.27}{1.28s + 1} \quad \frac{V}{V} \\
 G_3(s) &= \frac{v_3(s)}{u_3(s)} = \frac{2.03}{1.48s + 1} \quad \frac{V}{V}
 \end{aligned} \tag{3.1}$$

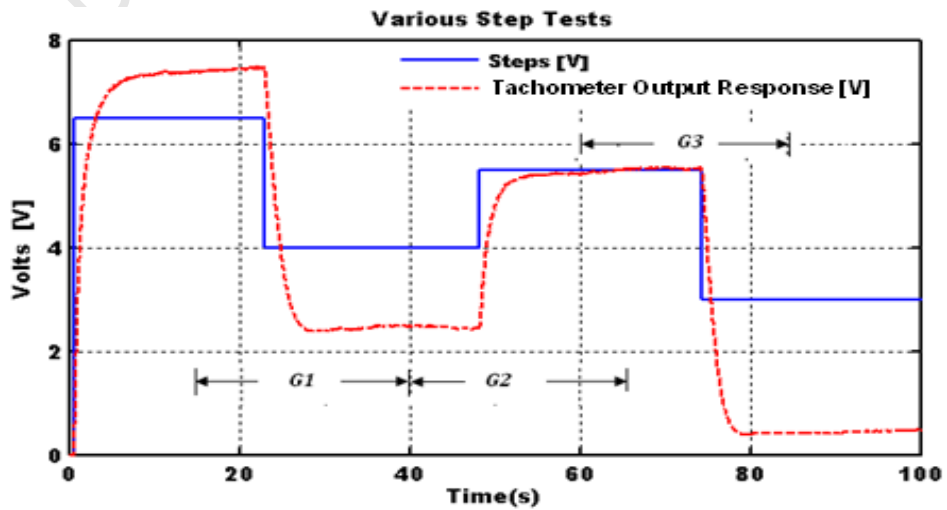


Figure 3.2: Various step tests.

It should be noted that the system output in Figure 3.2 has a slow mode in the first step and some overshoot in the second output because the system is exhibiting dead band. This is mainly caused by coulomb friction which works to oppose the direction of the system output [57]. Moreover, it is usually higher when the control input signal is around zero which can be clearly observed when the output response has dropped to around 2.5 volts – corresponding to 4 volts step input. Therefore, by deducting the system output response from the corresponding step input, the dead band size is around 1.5 volts.

3.1.1.1. Model Parameters Uncertainties Capturing

Given three transfer functions (G_1, G_2, G_3) with time constants and system gains as in Equation (3.1), the system parameter variation or plant uncertainties can be captured as follows:

- 1) Obtain the average time constant and average system gain by taking the average of the system gain and the average of the system time constant from the three transfer functions (G_1, G_2, G_3).
- 2) Determine the minimum and maximum gain error by subtracting the minimum and the maximum gains of the three transfer functions in Equation (3.1) from the average gain.
- 3) Repeat 2 by subtracting the minimum and maximum time constants of the three transfer functions in Equation (3.1) from the average time constant.
- 4) Obtain the average error by taking the average of the minimum and maximum errors such that the minimum and maximum uncertainty error can be determined.

This concept for capturing the model parameters' uncertainties from Equation (3.1) is illustrated in Table 3.1, where the average gain error obtained was ± 0.13 and the average time constant error obtained was ± 0.22 .

Table 3.1. Model Parameters Uncertainties.

Parameter	Transfer functions			Average	Error		Average Error
	$G_1(s)$	$G_2(s)$	$G_3(s)$		Minimum	Maximum	
A_g	2.02	2.27	2.03	2.11	0.09	0.16	± 0.13
T	1.72	1.28	1.48	1.49	0.21	0.23	± 0.22

From Equation (3.1), the average system gain for the three transfer functions (G_1, G_2, G_3) was found to be 2.11 and the average system time constant was 1.49 seconds. Therefore the nominal first-order model with uncertainties is given as follows:

$$g(s) = \frac{v(s)}{u(s)} = \frac{2.11 \pm 0.13}{1.49 \pm 0.22s + 1} \frac{V}{V} \quad (3.2)$$

It should be noted that the state space model of the uncertainty transfer function $g_{un}(s)$ $= \pm \frac{0.13}{0.22s + 1}$ is given as state space model and accounted for in the design in section 3.3.1.

3.1.2. Validation of the System Model

In order to implement both control algorithms, a matched model is needed to meet the design specification. Therefore, it is important to ensure that the system model is valid. Figure 3.3 shows the model validation test graph between the model and the physical system. However, at zero volts initial condition the dead band is compensated for with 1 volt at the output of the pre-operation amplifier. Therefore, the system gain became 2.52. In addition, the sampling time is chosen to be 109 milliseconds based on the Nyquist sampling criteria to avoid aliasing [58]. Subsequently, the sampling frequency must be equal to or greater than twice the system closed-loop bandwidth. Owing to this the 109 milliseconds was found to be within the Nyquist criteria bandwidth choice. Thus the validated nominal model is given by:

$$g(s) = \frac{v(s)}{u(s)} = \frac{2.52}{1.36s + 1} \frac{V}{V} \quad (3.3)$$

and the position model is given by:

$$g_p(s) = \frac{x(s)}{u(s)} = \frac{2.52}{s(1.36s + 1)} \text{ rad/V} \quad (3.4)$$

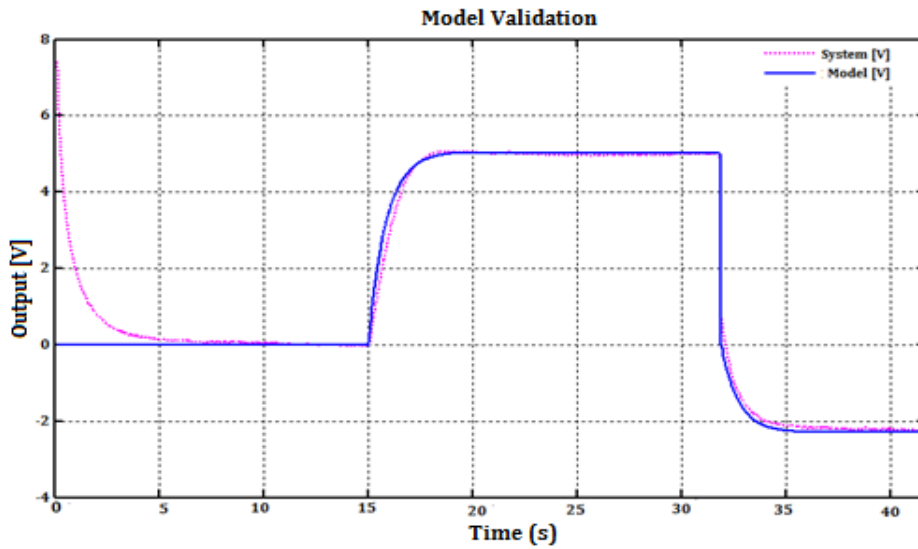


Figure 3.3: The system model validation of the step test data and the calculated model.

The setup of position transfer function in Equation (3.4) usually has two types of sensors: the potentiometer to measure angular position and the tachometer for angular velocity (see Figure 3.1.). In order to determine the relationship between the angular position (radian), the angular velocity (radian/sec) and the sensors outputs in volts, the voltage output of both sensors needs to be calibrated corresponding to actual angular position and angular velocity measurement. The conversion from voltage (volts) to angular position (radian) in Equation (3.4) is expressed as $\theta = \frac{V_p}{k_\theta} \text{ volts} / \left(\frac{\text{volts}}{\text{rad}}\right)$ (see Equation C.1 in Appendix C) where θ is angular position in radian, V_p is the potentiometer output voltage in volts and k_θ is the proportionality constant in $\frac{\text{volts}}{\text{rad}}$. More details can be found in Appendix C.

3.1.3. State Space Formulation

The phase plane is a popular way of analysing single-input single-output (SISO) systems [36]. However, in this dissertation the velocity and position transfer functions are presented in state space forms. The general linear process models of SISO can be seen in Equations (3.5).

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A} \mathbf{x}(t) + \mathbf{B} u(t) + d \\ y(t) &= \mathbf{C}^T \mathbf{x}(t)\end{aligned}\tag{3.5}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the state vector, $u(t)$ and $y(t)$ are the input and output variables, $\mathbf{B} \in \mathbf{R}^n$ is the input vector, $\mathbf{C} \in \mathbf{R}^n$ is the output vector, $\mathbf{A} \in \mathbf{R}^{n \times n}$ is the system matrix and d is the system disturbance.

3.1.3.1. The Velocity Continuous State Space Model

Using the velocity transfer function model in Equation (3.3), the continuous state space equations were obtained as shown in Equation (3.6).

$$\begin{aligned}\dot{x}(t) &= -0.74 x(t) + u(t) \\ y(t) &= 1.85 x(t)\end{aligned}\tag{3.6}$$

In the integral sliding control method, the idea is to improve the system tracking response in the presence of uncertainties. This is done by adding an extra integrator within the boundary layer so that the system will result in an extra state (see the integral sliding mode control simplified configuration of the inner loop in Figure 3.4).

The design analysis on the integral sliding mode control method can be found in the work discussed by Edwards and Spurgeon [36], as well as in the study that was conducted for non-linear systems by Zhao and Zhao [59].

Using the velocity transfer function in Equation (3.3) and an extra integrator, the continuous equation of the integral sliding mode control in the inner loop was obtained as shown in Equation (3.7). In a similar way the state space of system uncertainties that was captured in section 3.1.1.1 is given in section 3.3.1, Chapter 3.

$$\begin{aligned}\dot{x}(t) &= \begin{pmatrix} -0.74 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) \\ y(t) &= (0 \quad 1.85) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}\end{aligned}\tag{3.7}$$

3.1.3.2. Continuous Position State Space Model

The system position model given in Equation (3.4) for the P-controller in the outer position loop can be represented similarly for the inner loop state space models in Equation (3.7) that are used for the integral sliding mode controller.

An additional integrator was added to the position transfer function model in Equation (3.4). The simplified block diagram of this configuration is shown in Figure 3.3. As a result of the integral before the sliding mode controller (SMC) and the integral after the inner loop, the continuous state space equation for the outer position loop was obtained as follows:

$$\begin{aligned}\dot{x}(t) &= \begin{pmatrix} -0.74 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(t) \\ y(t) &= (0 \quad 0 \quad 1.85) x(t)\end{aligned}\tag{3.8}$$

It should be noted that the values of Equations (3.6), (3.7) and (3.8) were obtained using MATLAB. Likewise, in this dissertation, the position output is calculated by integrating the speed as shown in Equation (3.9).

$$y(t) = \int_0^t v(\tau) d\tau\tag{3.9}$$

where $v(\tau)$ is the velocity output of the inner loop and t is time of the simulation's length and $y(t)$ is the position output.

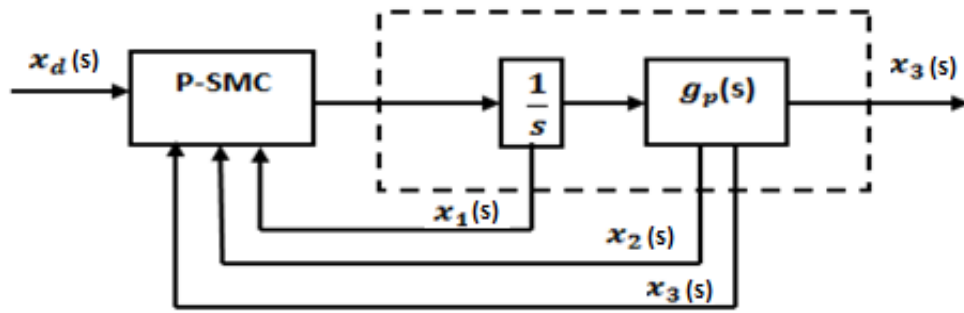


Figure 3.4: Block diagram of proportional-integral sliding-mode control (P-ISMC).

3.1.4. The System Stability

For any linear time-invariant system, the system stability must be determined. The stability of linear time-invariant systems is defined when there are no poles of the characteristic equation located in the right-half-plane (RHP) of the complex plane. A detailed stability analysis is discussed by Maciejowski [60]. The Equation (3.2) was found to be stable.

3.1.5. Controllability and Observability

Controllability and observability are important concepts in modern control theory. Specifically, the two concepts are important in designing non-linear integral sliding mode control, since all states must be controllable and observable to allow a full input control signal of the system's stability [36]. Controllability implies that all of the states in the state space model are influenced by the controller signal and observability implies that the system states can be estimated from the system output.

The velocity and position became second and third order owing to an additional integrator (see Figure 3.4). The controllability and observability tests can be determined if the rank of the controllability matrix and the observability matrix have the same rank as the order of the system. The system states in Equation (3.7) and Equation (3.8) have ranks of 2 and 3, respectively. Therefore, the state space representations of the velocity and position responses are controllable and observable, as the controllability matrix and the observability matrix have the same rank as the system.

3.1.6. Trapezoidal Motion Profile

Sliding mode and linear cascaded controllers will attempt to drive the system to velocity and position states described by the desired reference. In practice, this reference input needs to be generated before the controller tracks it automatically. In order to compare the tracking performance of both controllers, trapezoidal profiles are established from an outer position loop that generates the velocity set-point over time. The outer position controller will accept the error in the distance moved and the proportional controller is used to generate the outer position control action. If this control action is ramped to a certain maximum velocity, the trapezoidal motion profile is bounded and the acceleration, constant speed and deceleration are established. If

$x(t)$, $x_d(t)$, $e_p(t)$ and $u_p(t)$, are the output position, desired position, position set-point error and the outer position loop control action, respectively, the following equations illustrate the concepts of trapezoidal motion profile generation from the outer position loop.

$$e_p(t) = x_d(t) - x(t) \quad (3.10)$$

$$u_p(t) = k_{pp} e_p(t) \quad (3.11)$$

In most cases found in the literature, the PI controller usually is designed for the inner loop in order to effectively suppress the steady state speed errors. When it comes to the outer loop control design there are different controls types have been selected in order to improve the system performance. For instance, as seen in Chapter 2, an outer loop control with proportional control plus an A-type iterative learning controller was designed by Ding and Wu [6]. In another study, a traditional proportional controller only was designed for the outer loop by Jamaludin, Brussel, and Swevers [16]. In other studies even PI and PID controllers for the position loop design were also reported [23]. Thus the proportional controller only in Equation (3.11) was chosen for this dissertation because the proportional controller will be enough for the outer loop since there is integration in the position loop by nature [27].

The ramp set-point $r(s)$ in this dissertation is described by Equation (3.12), where u_p is made to be equal to the desired constant velocity v_{in} , see section 2.2.1.3

$$r(s) = \frac{u_p}{s^2} \quad (3.12)$$

3.2. Design and Stability of a P-PI controller on Bode Diagrams

This section discusses the design and stability of a P-PI controller as mentioned in Chapter 2, section 2.2.1.3. The parameters of the PI controller are obtained and tuned based on a gain margin and a phase margin, related to the open-loop transfer function of the inner loop, while the parameter of proportional controller is obtained based on a root locus method. The velocity and position models of the system are based on system models described in Equation (3.3) and Equation (3.4). First, the PI-controller parameters in the inner loop are analysed and then, the P-controller parameters in the outer loop are analysed.

3.2.1. Design and Analysis of the Inner Loop

PI-type control is the most common method in use today because of its simple structure, high gain margin at low frequency, high phase margin at high frequency and easy implementation in practice [61]. As mentioned in the reviewed literature, once the system modeling is obtained, frequency analysis approaches are used to fulfill the design and tuning of the PI-controller parameters from inner loop to outer position loop [16], [17], [27]. For PI-controller parameters, bode diagrams can be used to design and tune the controller parameters from a knowledge of the gain margin and the phase margin of the velocity open-loop frequency response function of the system. Hence, the phase margin and the gain margin can show the system stability margin and its transient response.

In most servo systems, usually the velocity output signal can be estimated or measured by differentiating the position signal through an optical encoder whilst a low-pass filter is used to suppress the noise [6]. Conversely, in this dissertation, the velocity feedback is assumed directly measurable through a velocity feedback sensor such as a tachometer, and the noise is considered to be zero. From Figure 2.2 given in section 2.2.1, Chapter 2, the transfer functions for open-loop and closed-loop of the inner velocity loop can be described as follows:

$$v_{ol}(s) = k(s) g(s) \quad (3.13)$$

$$v_{cl}(s) = \frac{v(s)}{r(s)} = \frac{k(s) g(s)}{1+k(s) g(s)} \quad (3.14)$$

where $k(s)$ is the PI-controller, $g(s)$ is the velocity open-loop transfer function, $v_{ol}(s)$ is the open-loop transfer function and $v_{cl}(s)$ is the velocity closed-loop transfer function.

The following outline is the design specifications for the inner loop:

- The PI controller gain should be chosen to be high to reduce the effects of the disturbances.

- The critical damping factor should be selected as in most mechanical systems $0.77 \leq \zeta \leq 1$
- The gain margin (GM) and phase margin (PM) should be selected to ensure stability and a good transient.
- The dynamics of the inner loop must be faster than those of the outer position loop, the settling time less than 1 second.
- The maximum percentage overshoot ($P.O.$) should be 0.1 %.
- Zero steady-state error.

In order to study the behaviour of the system, the bode diagram of the velocity modelled in Equation (3.3), is drawn in Figure 3.5.

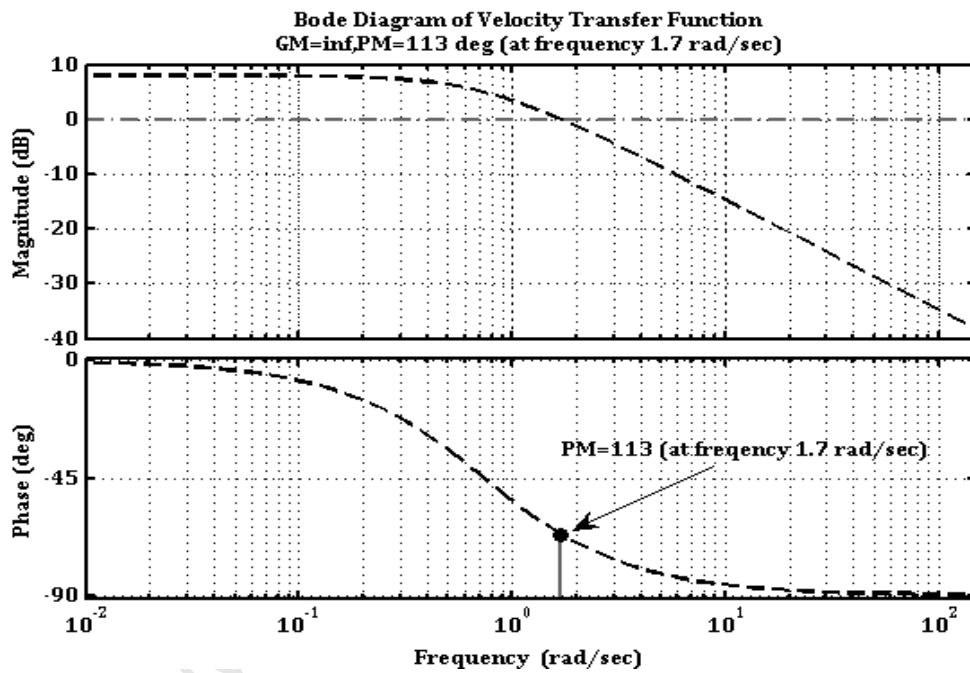


Figure 3.5: Bode diagram of velocity model transfer function.

By examining Figure 3.5, it can be seen that there is a gain margin of infinite magnitude, a phase margin of 113 degrees at 1.7 rad/sec and a bandwidth frequency of 1.52 rad/sec. From the relationship $\zeta = PM/100$, it follows that $\zeta = 1.13$. In order to fulfill this design specification, the inner velocity open- and closed-loops in Equation (3.13) and Equation (3.14) were examined.

The relation between the damping ratio (ζ) and percentage maximum overshoot (M_p) can be described as shown in Equation (3.15) [46].

$$M_p \% = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad (3.15.a)$$

$$\zeta^2 = \frac{\ln 2(M_p)}{\pi^2 + \ln 2(M_p)} \quad (3.15.b)$$

Using $M_p=0.001$ (design specification), for this reason $\zeta = 0.83$. Owing to this, about 83 degree is required in order to fulfill the design specifications. Since the current phase margin is 113 degrees, the phase margin needs to be dropped to 83 degrees. The system transfer function in Equation (3.3) is a type 0 system. Therefore, a type 1 controller is required to achieve zero steady-state error, $k(s) = \frac{k_{vp}}{s}$. Adding an integrator gives a denominator with a pole at $s=0$ which will reduce the current phase margin by 90 degrees, as shown in Figure 3.6 (note that this corresponds to the inner-loop system of Figure 2.2 with k_{pp} removed).

It can be noticed that PM dropped to about 20 degrees when an integral controller $k(s) = \frac{2.34}{s}$ was used. This will destabilize the system. It could be improved by just introducing a zero of the structure $s+0.8$.

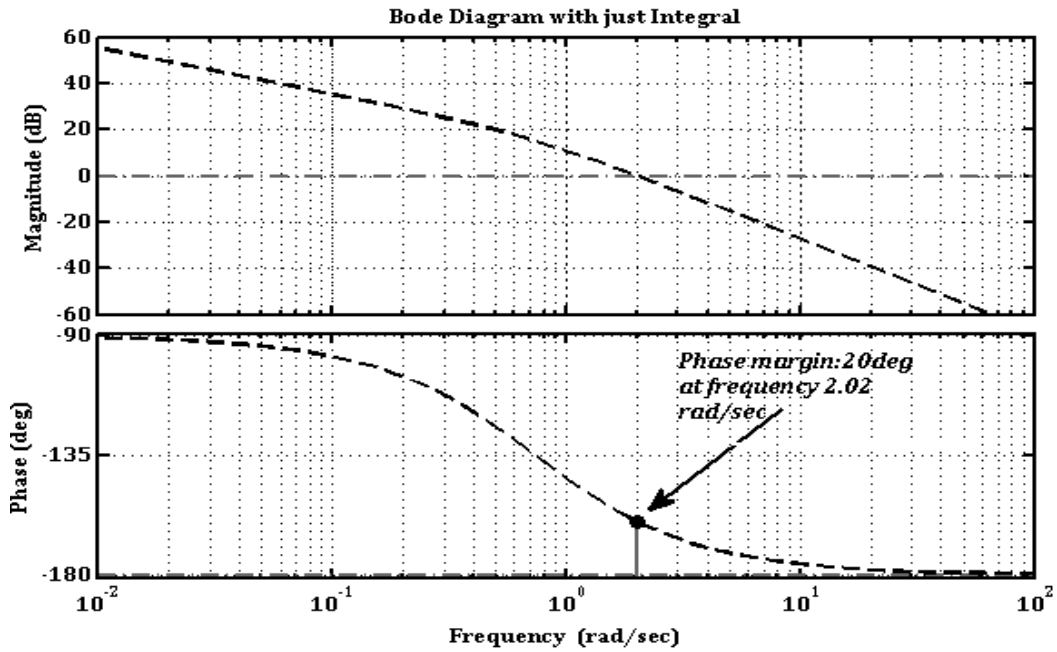


Figure 3.6: Bode plot after adding integral controller k_{vp}/s added to the velocity model.

Now if we combine this, the total controller is as in Equation (2.6), $k(s) = \frac{2.34(s+0.80)}{s}$, which is the PI controller, where $k_{vp} = 2.34$, $b = 1.87$ and $k_{vi} = 0.54$.

Figure 3.7 shows bode diagrams of the velocity open-loop from Equation (3.13) based on the chosen PI-controller's parameters. The PM was 89.3 degrees (at 4.34 rad/sec) and the GM was an infinite magnitude.

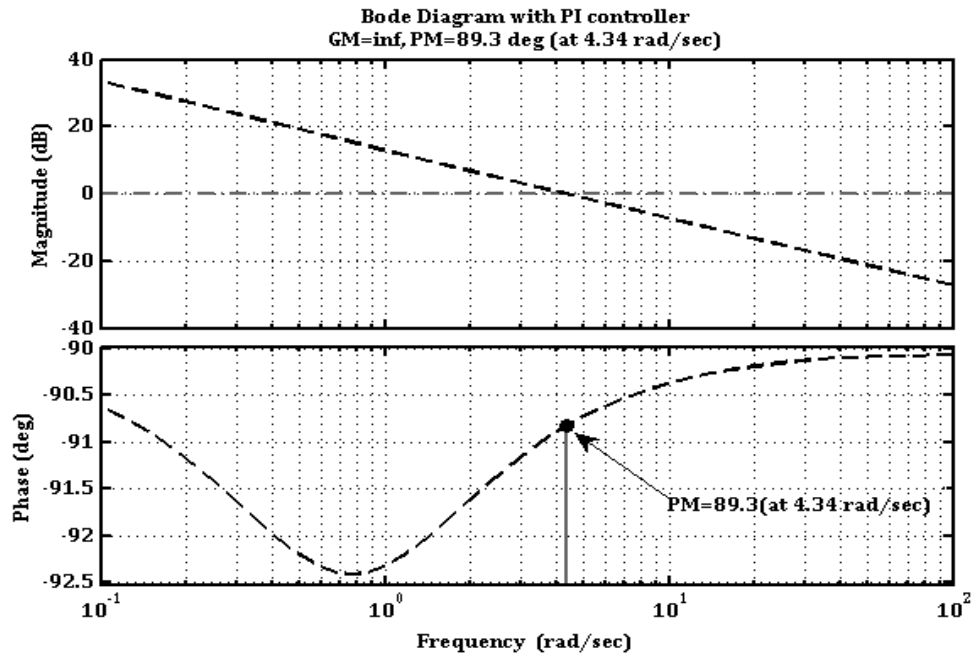


Figure 3.7: Bode plot for the velocity open-loop after the PI has been included.

In order to ensure that the stability and design specifications are met, a unit step-test was used to validate the velocity closed-loop performance as shown in Figure 3.8.

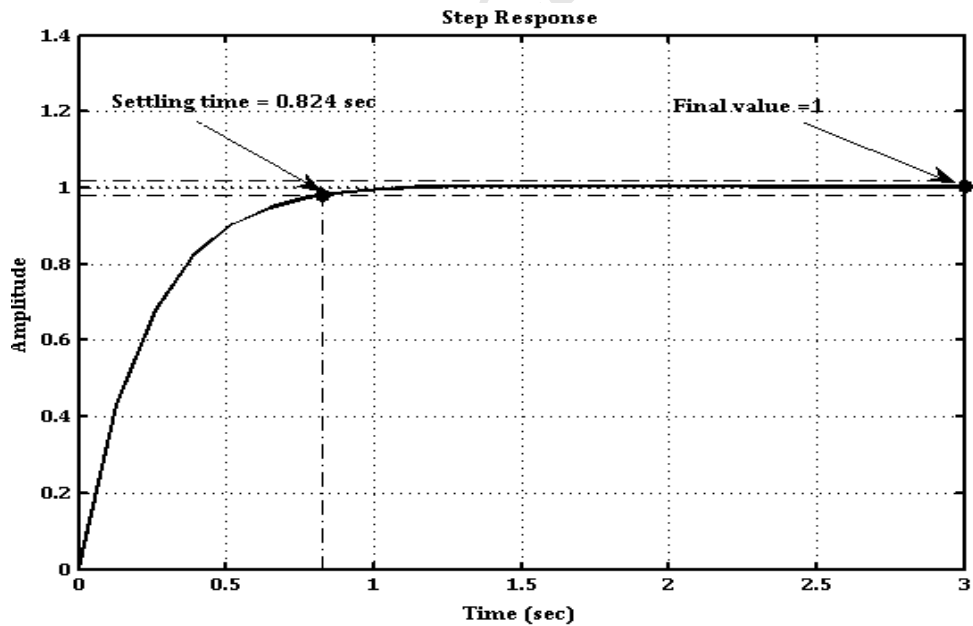


Figure 3.8: Velocity closed-loop unite step response with PI controller.

The inner closed-loop (velocity loop) step response satisfies the design specifications; the overshoot is less than 0.1%, there is no steady-state error and the settling time is less than 1 second.

3.2.2. Design and Analysis of the Outer Loop

The design of the outer position loop is usually done after the inner loop has been designed because the characteristics of the inner loop have an effect on the outer position loop [6], [16]. Owing to this, the PI-controller parameters that have been chosen for the velocity loop in the previous section are also used for the position open-loop and closed-loop analysis. The outer position loop controller design specification include the bandwidth of the inner velocity loop which is sufficiently greater than that of outer position loop, no overshoot and zero steady-state error.

From the previous section, the resulting transfer function of the inner closed-loop based on Equation (3.14) is

$$v_{cl}(s) = \frac{v(s)}{u_p(s)} = \frac{5.90 (s + 0.80)}{1.36 s^2 + 6.90 s + 4.72} \quad (3.16)$$

The open loop transfer function of the outer position loop $q_p(s)$ becomes

$$q_p(s) = \left(\frac{k_{pp} 5.90 (s + 0.80)}{1.36 s^2 + 6.90 s + 4.72} \right) \frac{1}{s} \quad (3.17)$$

The outer position transfer function in Equation (3.17) is a type 1 system and hence, there is no need to introduce an integrator. For this reason the outer position controller k_{pp} is chosen as the proportional control gain. In order to determine the outer loop control gain, the root locus method is applied. The outer position transfer function ($q_p(s)$) in Equation (3.17) can be rewritten as :

$$q_p(s) = \gamma \frac{N_{q_p}(s)}{D_{q_p}(s)} = 5.90 k_{pp} \frac{(s + 0.80)}{1.36 s^3 + 6.90 s^2 + 4.72 s} \quad (3.18)$$

where q_p is the open-loop transfer function of the outer position loop, γ is root locus constant gain, $N_{q_p}(s)$ and $D_{q_p}(s)$ are Laplace polynomials where the coefficients of their highest Laplace operator are unity (monic polynomials).

Now we can sketch the root locus for Equation (3.18) and select the closed-loop pole that satisfies the specifications. The three open- loop poles are placed at 0.0, -0.82 and -4.23, $n_\infty = 2$, asymptote angles = 90° , 270° , asymptote centre = -2.14 and break away = -2.14.

The dominant closed loop pole (s_0) is chosen to be $s_0 = -0.51$ since zero overshoot is required. At this pole, the root locus gain and the controller gain are computed.

$$\gamma = 5.9 k_{pp} = \left| \frac{D_{qp}(-0.51)}{N_{qp}((-0.51))} \right| = \left| \frac{(0.51 + 4.35)(0.51 + 0.80)}{(0.51 + 0.8)} \right| = 4.90$$

$$\therefore k_{pp} = 0.824$$
(3.19)

Figure 3.10 shows the root locus of the outer position open-loop and closed-loop poles with $k_{pp} = 0.82$. The position closed-loop poles (pink colour) are at -3.1 , -1.2 and -0.8 , yielding a stable system. At this controller gain ($k_{pp}=0.82$), the outer position closed-loop ($h_p(s)$) would then be:

$$h_p(s) = \frac{(4.90 s + 3.90)}{1.36 s^3 + 6.90 s^2 + 9.60 s + 3.90}$$
(3.20)

Whose unit step response is shown in Figure 3.10. From this figure, the outer position loop satisfies the design specification since the settling time of the outer position is about three times less than the inner loop settling time. Furthermore, Equation (3.20) has zero steady-state error for the step response as $\lim_{s \rightarrow 0} h_p(s) = 1$.

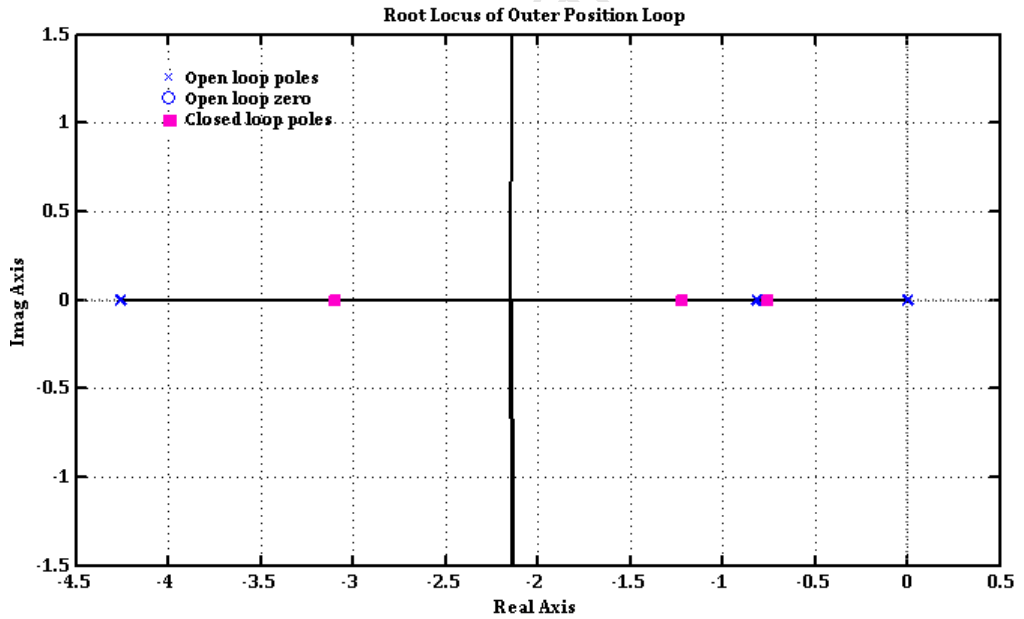


Figure 3.9: The root locus plot for the outer position loop.

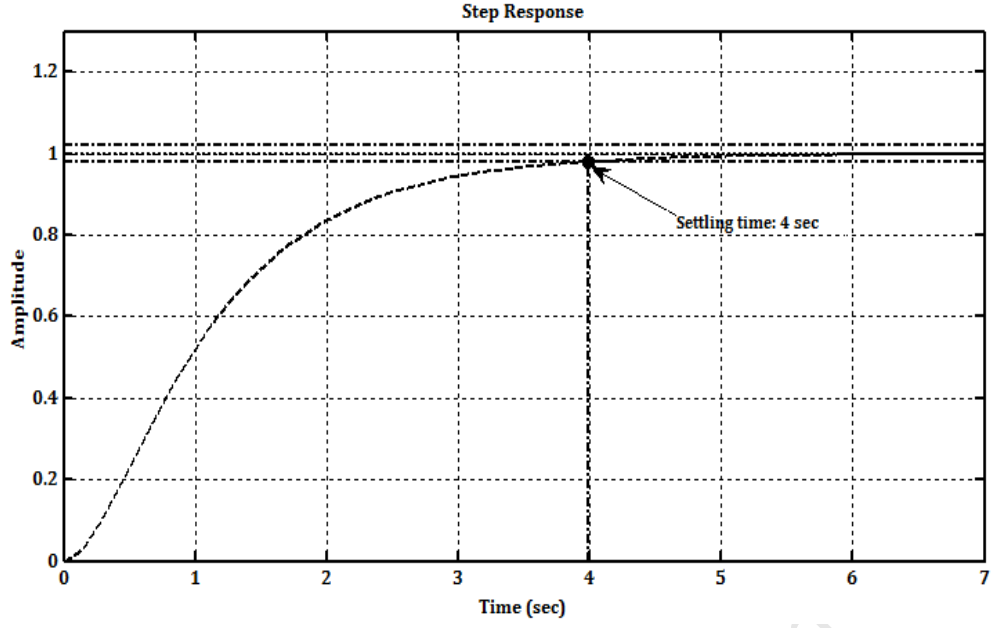


Figure 3.10: Position closed-loop unit step response.

3.2.3. P-PI Parameters Tuning

As described in section 3.2, the P-PI controller's three parameters to be tuned are: the outer loop position proportional gain, the inner loop proportional gain and the inverse of the integral gain of the PI-controller in the s-plane (k_{pp} , k_{vp} and b), respectively. The P-PI parameters were tuned using the root locus method for the continuous time and the discrete P-PI controller to achieve smooth, critically damped responses if the settling time and the system gain are given [62]. The detailed information on the root locus tuning method for the continuous time and discrete P-PI controller for electrical servos is discussed by Żabiński and Trybus [62]. This tuning method is applicable for discrete and continuous time domain systems. The P-PI controller tuning process is done accordingly to the following guidelines:

- 1) Since the feedback system in Figure 2.2 is of 3rd order, two of its three poles must be the same.
- 2) If the settling time and the system gain are given or known, the following tuning rules can be applied to obtain the P-PI parameters:

$$k_{pp} = \frac{4}{t_s}, \quad k_{vp} = \frac{27}{A_g t_s}, \quad b = \frac{108}{A_g t_s^2} \quad (3.21)$$

Where t_s is the required settling time and A_g is the system model gain.

Example 3.1

If the settling time is given, $t_s=4$ seconds and the system gain is given, $A_g = 2.52$ as in Equation (3.3), find the controller parameters k_{pp} , k_{vp} and b to get a smooth, damped response for the system configuration shown in Figure 2.2 and compared this with the parameters of the inner loop in section 3.2.1 and the outer loop in section 3.2.2.

Solution: Given the settling time to be 4 seconds and the system model gain to be 2.52, we can calculate the continuous P-PI controller parameters by using the tuning guide lines in Equation (3.21), where

$$k_{pp} = \frac{4}{t_s} = 4/4 = 1, \quad k_{vp} = \frac{27}{A_g t_s} = 27/10.08 = 2.679 \text{ and } b = \frac{108}{A_g t_s^2} = \frac{108}{40.32} = 2.68.$$

For this example, the result of the step response is shown in Figure 3.11. Accordingly, the overshoot is almost zero and the settling time is approximately 4 seconds, so the specification have been satisfied for a cascaded P-PI configuration design in this example.

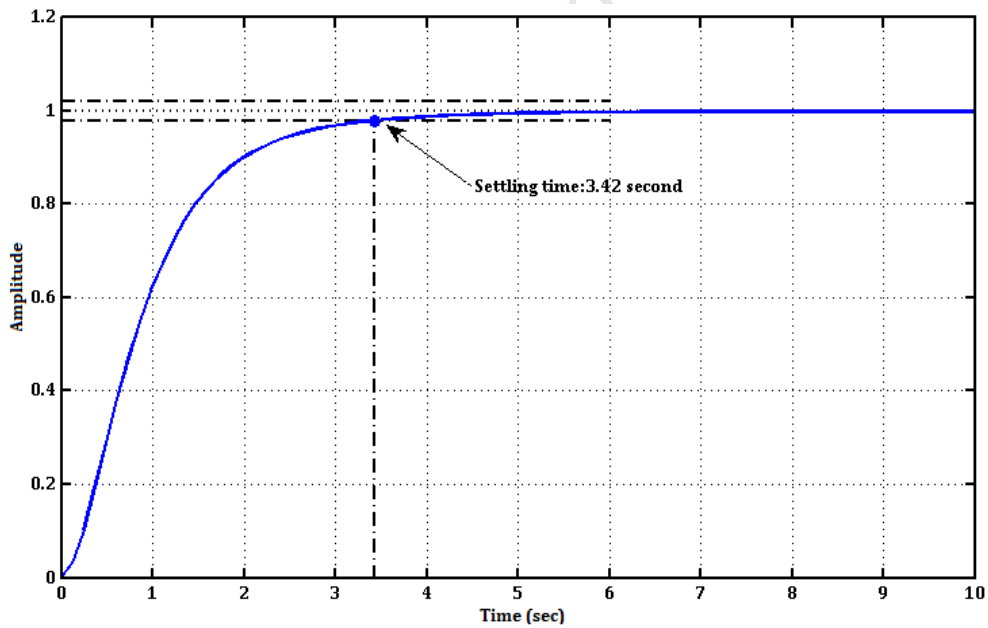


Figure 3.11: Tuned P-PI step response.

3.3. Design and Stability of P-ISMC Controller

The mathematical model of the system enables us to verify the properties and performance of the controller through a computer simulation. In this section the state space equations, namely Equation (3.7) and Equation (3.8) are used to design and check the stability of the ISMC and the P-ISMC controller (see Figure 3.3). Prior to the inner loop design, the system state space model in Equation (3.7) is transformed into an appropriate regular form for designing the switching surface and the integral sliding mode. Further, for good tracking performance, the reference tracking definition is required. Then, the design and analysis of the inner loop is presented and the system stability is checked in the sense of Lyapunov's stability criteria [37]. These three major design steps are presented and discussed in the following sections.

3.3.1. Regular Form Representation

In order to design the switching surface and a sliding mode controller, the state space model for the inner velocity loop in Equation (3.7) is simply transformed into the regular form. The regular form is the controllability canonical form. This is established by using an invertible transformation matrix that transforms the system state-space model into the regular form. This transformation can be established if the pair (A, B) is assumed controllable. Hence, $x(t) = T_r z(t)$ can be used to transform the system state space model into the new coordinate, where $T_r \in \mathbb{R}^{n \times n}$ is a square invertible transformation matrix. In-depth discussions about regular form transformation have been given by Edwards and Spurgeon [36], as well as by Pisano *et al.* [63]. The regular form representation yields into partitioned sub-systems as:

$$\dot{x}_1(t) = A_{11} x_1(t) + A_{12} x_2(t) \quad (3.22.a)$$

$$\dot{x}_2(t) = A_{21} x_1(t) + A_{22} x_2(t) + B_2 u(t) \quad (3.22.b)$$

Where, $u(t) \in \mathbb{R}^m$ represents the system input, $A_{11} \in \mathbb{R}^{(n-m) \times (n-m)}$, $A_{12} \in \mathbb{R}^{(n-m) \times m}$, $A_{21} \in \mathbb{R}^{m \times (n-m)}$, $A_{22} \in \mathbb{R}^{m \times m}$, $B_2 \in \mathbb{R}^{m \times m}$ is a square matrix that has an order equal to the number of the inputs. By using the T_r matrix the regular form matrices are obtained as follows:

$$A_r = T_r A T_r^{-1} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, B_r = T_r B = \begin{pmatrix} 0 \\ B_2 \end{pmatrix} \quad (3.23)$$

Thus, the system in the new coordinates can be represented as follows:

$$\dot{x}(t) = (A_r + A_r) x(t) + B_r u(t) \quad (3.24)$$

With $x(t)$ being the new state vector and the pair (A_r, B_r) is in the regular form.

Most importantly, the described regular form in Equations (3.23) and (3.24) is based on the work conducted by Edwards and Spurgeon [36]. In this process the transformation matrix T_r is obtained by performing a QR-decomposition technique on the input matrix B such that the decomposed matrix is decomposed into a unitary matrix Q ($QQ^T = I_n$) and a right triangular matrix R , where I_n is unity matrix. In this dissertation the MATLAB function “qr” which performs the QR-decomposition technique is used in order to obtain the transformation matrix T_r [36].

To consider the system parameter variations that are discussed in section 3.2.2.2, and the disturbances in our transformation, Equation (3.24) can be modified as follows [36], [37]:

$$\begin{aligned}\dot{x}(t) &= (A_r \pm \Delta A_r) x(t) + B_r (u(t) + \xi(t)) \\ y(t) &= (C + \Delta C) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}\end{aligned}\tag{3.25}$$

Where $\Delta A_r \in [\Delta A_{r_max}, A_{r_min}]$ represents the bounded parameters uncertainties matrix, $\xi(t)$ is the disturbance and $\Delta C \in [\Delta C_{min}, \Delta C_{max}]$, where the subscript *min* and *max* denote the minimum and maximum uncertainty values. This uncertainty matrix regular form as shown in Equation (3.26) is obtained from the system parameter variation that are captured in section 3.1.1.1 where ΔA uncertainty matrix before transformed to the regular form.

$$\Delta A_r = T_r \Delta A T_r^{-1} = \begin{pmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{pmatrix}\tag{3.26}$$

Illustrative Case Study 3.2

Consider the SISO system $\dot{z}(t) = (A + \Delta A) z(t) + B (u(t) + \xi(t, x))$ and $y(t) = C + \Delta C)z(t)$ from Equation (3.7) and the system uncertainties matrices given with

$A = \begin{pmatrix} -0.74 & 0.00 \\ 1.00 & 0.00 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\Delta A = \begin{pmatrix} -4.55 & 0.00 \\ 1.00 & 0.00 \end{pmatrix}$, $C = (0.00 \quad 1.85)$ and $\Delta C = (0 \quad 0.59)$. Convert the system and system parameter variations matrices into the regular form.

Solution:

The system pair matrices $(A + \Delta A, B)$ are controllable and rank $(B) = 1$. Therefore, the QR decomposition is performed on matrix B to obtain the transformation matrix T_r as $T_r = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. By substituting $x(t) = T_r z(t)$, the regular form for the nominal system state space model is obtained as follows: :

$$\dot{\mathbf{x}}(t) = \begin{pmatrix} 0.00 & 1.00 \\ 0.00 & -0.74 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 0.00 \\ 1.00 \end{pmatrix} u(t) \quad (3.27.a)$$

$$y(t) = [(0.00 \quad 1.85)] \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

Then the general regular form that involves the system state space nominal model, the system uncertainties and the input disturbance is determined as follows

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \left[\begin{pmatrix} 0.00 & 1.00 \\ 0.00 & -0.74 \end{pmatrix} \pm \begin{pmatrix} 0.00 & 1.00 \\ 0.00 & -4.55 \end{pmatrix} \right] \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0.00 \\ 1.00 \end{pmatrix} [u(t) + \xi(t)] \\ y(t) &= [(0.00 \quad 1.85) \pm (0.00 \quad 0.59)] \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \end{aligned} \quad (3.27.b)$$

where,

$$\begin{aligned} A_r + \Delta A_r &= \left[\begin{pmatrix} 0.00 & 1.00 \\ 0.00 & -0.74 \end{pmatrix} \pm \begin{pmatrix} 0 & 1 \\ 0 & -4.55 \end{pmatrix} \right], B_r = \begin{pmatrix} 0.00 \\ 1.00 \end{pmatrix} \text{ and} \\ C + \Delta C &= [(0.00 \quad 1.85) + (0 \quad 0.59)]. \end{aligned}$$

3.3.2. Reference Tracking Model

The main objective of this dissertation is to design a tracking controller for the system model. The important pre-concept is to define a model of the tracking error before designing the sliding mode control based on an integral action method. A detailed design procedure using integral sliding mode method is discussed by Li and Wikander [18] as well as Slotine and Li [40]. If $\mathbf{x}_d$ is the reference set-point trajectory of the trapezoidal motion profile, then the reference tracking error for the inner loop will be given as :

$$\begin{aligned} \mathbf{x}_d(t) &= [x_d(t) \quad \dot{x}_d(t)] \\ \mathbf{e}(t) &= [\mathbf{x}(t) - \mathbf{x}_d(t)] = \begin{bmatrix} x_1 - x_d \\ x_2 - \dot{x}_d \end{bmatrix} \end{aligned} \quad (3.28)$$

Then, the continuous model of the tracking error is given as follows:

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= -0.74 e_2(t) + u(t) + \xi(t) \end{aligned} \quad (3.29)$$

3.3.3. Design and Analysis of the Inner Loop Control

The aim of the next step is to design a tracking controller to track the model of the tracking error such that the tracking error $e_p(t)$ is bounded and approaches zero. The design of the integral sliding mode controller has been based on the regular form model given in Equation (2.27). The model of the tracking error is described by Equations (3.28) and (3.29). However, the design procedures are the same as presented in Chapter 2. This section first considers the design of the switching surface and then looks at the design of the integral sliding mode control.

3.3.3.1. Switching Surface Design

The design of the switching surface is crucial because it allows the linearization, reduction of order and asymptotic stability of sliding mode dynamics [42], [64]. Suppose the switching surface $s(e) \in R^n$ is given as shown in Equation (3.30). Then the design problem will be to find the switching surface matrix $S \in R^{m \times n}$ which is defined as.

$$s(e) = S e(t) \quad (3.30)$$

where $S \in R^{m \times n}$ should be chosen so that the matrix SB is non-singular and $e(t) \in R^{n \times 1}$ represents the state tracking error vector.

To design the switching surface, the partitioned subsystems of Equation (3.22) can be rewritten as shown in Equation (3.31) with the assumption that the system is subjected to a disturbance $\xi(t) \in R^n$ and the uncertainties ΔA_r .

$$\dot{e}_1(t) = [A_{11} + \Delta A_{11}] e_1(t) + [A_{12} + \Delta A_{12}] e_2(t) \quad (3.31.a)$$

$$\dot{e}_2(t) = [A_{21} + \Delta A_{21}] e_1(t) + [A_{22} + \Delta A_{22}] e_2(t) + B_2 [u(t) + \xi(t)] \quad (3.31.b)$$

where $e_1(t) \in R^{n-m}$ and $e_2(t) \in R^m$ are the sub-system's state tracking errors.

In a similar manner to the regular form presented in section 3.3.1, the switching surface also can be partitioned in the new coordinates as

$$s(e) = (S_1 \ S_2) \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \quad (3.32)$$

where $S_1 \in R^{m(n-m)}$ and $S_2 \in R^{m \times m}$ are the required design parameters which should be chosen such that $s(e) = 0$.

When $s(e) = 0$ Equation (3.32) can be solved for $e_2(t)$ to obtain Equation (3.33) and substituted into the subsystem in Equation (3.31.a) to produce Equation (3.35).

$$e_2(t) = -M e_1(t) \quad (3.33)$$

With $M \in \mathbb{R}^{m \times (n-m)}$ being defined as

$$M = S_2^{-1} S_1 \quad (3.34)$$

$$\dot{e}_1 = (A_{11} - A_{12} M) e_1(t) \quad (3.35)$$

From Equation (3.35) the problem of the switching design has become a linear state feedback control problem, where the matrix $(A_{11} - A_{12} M)$ describes the dynamics of the system's closed-loop where A_{11} is the state matrix, A_{12} is the input matrix and M is the state feedback gain matrix that is described by Equation (3.34). This allows the design of the switching surface matrix M to be obtained using the traditional state feedback methods such as *LQR* or pole placement [36],[64]. However, this purpose can only be obtained if the pair matrices (A, B) , as well as the pair matrices (A_{11}, A_{12}) , are controllable. For fewer numerical errors, the matrix S_2 can be considered as a scaling factor and its value can be chosen to be the identity matrix such that the matrix S_1 can be made equal to the obtained feedback gain matrix M . Once the matrix M is obtained in the next step, the switching matrix S must be transformed into the original coordinates as follows:

$$S = (M \quad I_m) T_r^T \quad (3.36)$$

where I_m represents the identity matrix with order m .

In this dissertation, the pole placement method was used to obtain the matrix M by using the MATLAB "place" command on the closed-loop system shown in Equation (3.35). In order to ensure stability of the sliding mode dynamics, a set of $(n-m)$ negative eigenvalues of the matrix $(A_{11} - A_{12} M)$ must be assigned in the left half plane. Sometimes the determination of the feedback gain matrix M does not satisfy the design dynamics requirement along the switching surface. Thus, the design process must be repeated until satisfactory dynamics along the switching surface are achieved. For example, in order to obtain better position tracking, the value of the S_1 or M matrix can be obtained using the tuning guidelines discussed in the section 3.3.5 by increasing the non-zero values of S_1 until the desired response is obtained. The values of the matrix M or S_1 when the pole placement is used have been presented in sections 4.1.2, 4.2.2 and 4.3.2 of Chapter 4.

3.3.3.2. Velocity Controller Design and Stability Analysis

Once the switching surface is found, the next step is to determine the control structure $u(t)$ which forces the error $e(t)$ towards the switching surface and then converge along the surface to the origin. In other words, the control structure $u(t)$ must be selected so that the desired trajectories are followed by the closed-loop control system so that the process of sliding mode control is met, specifically sliding phase when $s(e) = 0$, $\dot{s}(e) = 0$ and the reaching phase when $s(e) \neq 0$ [37]. Once the sliding process has occurred, the system can be described by invariant properties called the invariant set which makes the sliding mode insensitive to disturbances and certain system parameter variations [34]. Conceptually, the control structure $u(t)$ can consist of equivalent control and discontinuous control. The system uncertainties and disturbances can be maintained and rejected by using Lyapunov's stability theory in the design of the sliding mode control [34], [37]. This theory states that the process can be stable if the time derivative of a given positive definite function $V(s)$ is the negative definite function, i.e., $\dot{V}(s) < 0$, which means that the reachability condition is satisfied. For purpose of design, the lyapunov function $V(s)$ can be defined as Equation (3.37) which satisfies the conditions given in Equation (3.38).

$$V(s) = \frac{1}{2} s^2 \quad (3.37)$$

- $V(s) > 0, \forall s \neq 0$
 - $V(s) = 0, s = 0$
 - $V(s) \rightarrow \infty, s \rightarrow \infty$
- (3.38)

In order to guarantee stability, the control structure $u(t)$ must be chosen such that the reachability condition described by Equation (3.39) is satisfied [37].

$$\dot{V}(s) = s \dot{s} < 0, \quad s \neq 0, \quad (3.39)$$

Using this reachability condition, the sliding mode control structure can be chosen similar to Equation (2.26), namely:

$$u(t) = u_{eq}(t) + u_d(t) \quad (3.40)$$

where the equivalent control $u_{eq}(t)$ and the discontinuous control $u_d(t)$ components can be obtained as follows:

Suppose the time derivative of the switching surface determined in the previous section is described by Equation (3.41) and the sub-systems in Equation (3.31) are substituted into Equation (3.41) when $\dot{s}(e) = 0$, then Equation (3.42) can be obtained.

$$\dot{s}(e) = S_1 \dot{e}_1(t) + S_2 \dot{e}_2(t) \quad (3.41)$$

$$\dot{s}(t) = S_1[(A_{11} + \Delta A_{11}) e_1(t) + (A_{12} + \Delta A_{12}) e_2(t)] + S_2 [(A_{21} + \Delta A_{21}) e_1(t) + (A_{22} + \Delta A_{22}) e_2(t) + B_2 (u(t) + \xi(t))] \quad (3.42)$$

Then at $\dot{s}(e) = 0$, and assuming that the system uncertainties matrix $\Delta A_r = 0$ and the disturbance $\xi(t) = 0$ the equivalent control $u_{eq}(t)$ can be obtained as,

$$u_{eq}(t) = -\frac{1}{S_2 B_2} [(S_1 A_{11} + S_2 A_{21}) e_1(t) + (S_1 A_{12} + S_2 A_{22}) e_2(t)] \quad (3.43)$$

Where the matrix $S_2 B_2$ is non-singular while S_1 and S_2 were determined in the previous section.

Different discontinuous control strategies have been used in order to deal with the system parameter variations and disturbances. For example, relay control, bang-bang, enforced control and others have been discussed in the literature [36], [38]. In this dissertation the bang-bang control strategy is used as given in Equation (3.44). The illustration of its derivation is shown below.

$$u_d(t) = -k \operatorname{sgn}(s) \quad (3.44)$$

When Equations (3.42), (3.40) and (3.43) are substituted, inequality in Equation (3.39) becomes,

$$\begin{aligned} \dot{V}(s) = s(e) \{ & S_1[(A_{11} + \Delta A_{11}) e_1(t) + (A_{12} + \Delta A_{12}) e_2(t)] + \\ & S_2 [(A_{21} + \Delta A_{21}) e_1(t) + (A_{22} + \Delta A_{22}) e_2(t) + \\ & B_2 u_d(t) + B_2 \xi(x, t) - \frac{1}{S_2} [(G_1 A_{11} + S_2 A_{21}) e_1(t) + \\ & (S_1 A_{12} + S_2 A_{22}) e_2(t)]] \} < 0 \end{aligned} \quad (3.45)$$

or,

$$\begin{aligned} \dot{V}(s) = s(e) \{ & S_1[\Delta A_{11} e_1(t) + \Delta A_{12} e_2(t)] + S_2 [\Delta A_{21} e_1(t) + \\ & \Delta A_{22} e_2(t) + B_2 u_d(t) B_2 \xi(x, t)] \} < 0 \\ & \leq |s(e)| \{ S_1[|\Delta A_{11}| |e_1(t)| + |\Delta A_{12}| |e_2(t)|] + \\ & S_2 [|\Delta A_{21}| |e_1(t)| + |\Delta A_{22}| |e_2(t)| + B_2 |\xi(t)| + B_2 u_d(t)] \} < 0 \end{aligned} \quad (3.46)$$

To satisfy the reaching condition in Equation (3.46), the discontinuous control can be chosen as,

$$\begin{aligned} u_d(t) = -\operatorname{sgn}(s) (B_2 S_2)^{-1} [& S_1 (|\Delta A_{11}| |e_1(t)| + |\Delta A_{12}| |e_2(t)|) \\ & + S_2 (|\Delta A_{21}| |e_1(t)| + |\Delta A_{22}| |e_2(t)| + B_2 |\xi(t)|)] \end{aligned} \quad (3.47)$$

where k can be given as,

$$\begin{aligned} k = (B_2 S_2)^{-1} [& S_1 (|\Delta A_{11}| |e_1(t)| + |\Delta A_{12}| |e_2(t)|) \\ & + S_2 (|\Delta A_{21}| |e_1(t)| + |\Delta A_{22}| |e_2(t)| + B_2 |\xi(t)|)] \end{aligned} \quad (3.48)$$

Illustrative example 3.2

Consider the problem of controlling the continuous model of the tracking error in Equation (3.27), with the system uncertainties ΔA_r and disturbance $\xi(t)$. Using the nominal model from Equation (3.27) and substituting in Equation (3.42), then Equation (3.42) during sliding mode can be given as follows:

$$\dot{s}(t) = S_1 e_2(t) + S_2 [-0.74 e_2(t) + u(t)] = 0 \quad (3.49)$$

From Equations (3.43) and (3.49) the $u_{eq}(t)$ of the control structure $u(t)$ is

$$u_{eq}(t) = -\frac{1}{S_2} [S_1 e_2(t) - 0.74 S_2 e_2(t)] \quad (3.50)$$

To ensure stability, the reaching condition from Equation (3.39) can be described as:

$$\dot{V}(s) = s\dot{s} < 0 \quad (3.51)$$

$$\dot{V}(s) = s[S_1 e_2(t) + S_2 (-0.74 e_2(t) + u(t))]$$

By choosing the controller to be

$$u(t) = u_{eq}(t) - k \operatorname{sgn}(s) \quad (3.52)$$

where $u_d(t) = -k \operatorname{sgn}(s)$ and $u_{eq}(t)$ is obtained as in Equation (3.50), by considering the system uncertainties and disturbance as in Equations (3.27), (3.42) and (3.45)), then the reaching condition in Equation (3.51) will result in

$$\begin{aligned} \dot{V}(s) &= s\dot{s} < 0 \\ &= s[S_1 e_2(t) + |\Delta A_{12}| S_1 e_2(t) - 0.74 S_2 e_2(t) + \Delta A_{22} S_2 e_2(t) + S_2 u_{eq}(t) \\ &\quad + S_2 u_d(t) + S_2 \xi(t)] \\ &= s[|\Delta A_{12}| S_1 e_2(t) + \Delta A_{22} S_2 e_2(t) +] + s[S_2 u_d(t) + S_2 \xi(t)] \\ &\leq |s| [|\Delta A_{12}| |S_1| |e_2(t)| + |\Delta A_{22}| |S_2| |e_2(t)|] + |s| [S_2 |u_d(t) + \xi(t)|] < 0 \end{aligned} \quad (3.53)$$

Where $\Delta A_{11} = 0.00$, $\Delta A_{12} = \pm 1.00$, $\Delta A_{21} = 0.00$, $\Delta A_{22} = \pm 4.55$, $B_2 = 1.00$, S_1 and $S_2 = 1.00$ are determined in section 3.3.3.1.

From Equation (3.52) the discontinuous control of the control structure $u(t)$ can be determined as shown in Equation (3.54)

$$u_d(t) = -\operatorname{sgn}(s) [S_1 |e_2(t)| + 4.55 |e_2(t)| + |\xi(t)|] \quad (3.54)$$

$$\text{where } k = S_1 |e_2(t)| + 4.55 |e_2(t)| + |\xi(t)|$$

In summary, the motion of the process now comprises a reaching phase and a sliding phase. In the reaching phase the trajectories starting outside $s(e)$ from any initial

condition converge to the switching surface in a finite time. Once the trajectory reaches the switching surface the process enters the sliding phase. This motion of the system is now confined to the switching surface and then converges towards the origin. Consequently, the dynamics of the system can be reduced to a lower-order linear model. The results for implementing the switching surface design and the controller structure $u(t)$ design with/without disturbances are discussed and described in Chapter 4.

3.3.4. Boundary Layer and the Continuous Approximation Control

As mentioned in Chapter 2, chattering is a challenge that will occur during the practical implementation of the sliding mode control. The chattering needs to be mitigated. In this section, boundary layer and continuous approximation controllers, respectively, are designed as follows:

$$u(t) = u_{eq}(t) - k \cdot \text{sat}\left(\frac{s}{\delta}\right) \quad (3.55)$$

$$u(t) = u_{eq}(t) - k \frac{s}{|s| + \delta} \quad (3.56)$$

where $u(t)$ is the total control action, $u_{eq}(t)$ is the equivalent control, s is the switching function and δ is the boundary layer width.

3.3.4.1. Alternative Equivalent Control

The common sliding mode control law, which is in use, comprises two components, namely equivalent control and discontinuous control. The equivalent control is the linear component part of the control input, and its goal is to stabilise the nominal linear system against some known matched uncertainties. Under assumptions that an ideal sliding mode motion took place ($\dot{s}(e) = 0$), the equivalent control is calculated as presented in Equation (3.50).

For motion within the boundary layer width, $s(e)$ can have a small value for the boundary layer width but the differential of $s(e)$ takes finite values and does not tend towards zero within the boundary layer width. As a result, it is difficult in practice to replace uncertainties by using action of equivalent control [65]. In order to overcome this, the equivalent control in Equation (3.50) can be modified within the boundary layer where the matrix (BS) is non-singular.

$$u_{eq}(t) = u_{eq}(t) + (BS)^{-1}s(e) \quad (3.57)$$

3.3.4.2. System Stability Analysis

Lyapunov's second method is a common method for determining the stability of the sliding mode surface [37], [64]. It states that if the states of the system trajectories on the sliding surface are stable, then the system is stable. Consider the Lyapunov's scalar function $V(s) = \frac{1}{2} s^2$ and that the time-derivative satisfies the inequality as in Equation (3.39). This equation states that the squared distances from the system's state to sliding surface must decrease along any of the system's trajectories. Therefore, satisfying this sliding condition and defining the input trajectories makes the sliding surface the solution to tracking the desired trajectory in an invariant set [10].

3.3.4.3. Additional Control Elements

In order to improve the response of the system, an additional control element $-\phi s(e)$ can be added to the control structure $u(t)$ [36] as follows:

$$u(t) = u_{eq}(t) + (BS)^{-1}s(e) + u_d(t) - \phi s(e) \quad (3.58)$$

3.3.4.4. The Observer Design

In control theory it is very important to estimate system states for better system performance. In sliding mode control theory, many sliding mode observer designs have been proposed. For example, an asymptotic observer to prevent chattering by generating an ideal sliding motion in a supplementary loop is proposed by Lee and Utkin [66]. Furthermore, the Luenberger observer, the Utkin observer and the Awalcott-Zak observer are discussed by Edwards and Spurgeon [36]. In all these approaches, the observer design should form part of the structure on control policy where estimated states will be utilised in a state feedback control law.

However, in this dissertation, the observer design is based on the work conducted by Drakunov and Utkin[67]. Compared with the Luenberger observer, the Utkin observer and the Awalcott-Zak observer approaches, such an observer has better robustness properties because a finite-time observer can be developed by means of equivalent control in order to estimate the system parameter variations. Moreover, the observer design can be based on a regular form given in section 3.3.1. The observer is designed using the well-known standard linear pole placement technique for linear and non-linear systems. The standard pole placement observer equation is given by

$$\dot{\hat{x}} = A\hat{x}(t) + B u(t) + L (y(t) - \hat{y}(t)) \quad (3.59)$$

Under the appropriate selection of observer gain L , the sliding motion occurs on the switching surface when $y(t) - \hat{y}(t) = 0$.

3.3.5. Outer Position Loop Analysis and Parameters Tuning for P-ISMC

The block diagram representation of a P-ISMC controller is illustrated in Figure 3.3. This control configuration consists of two loops, namely a velocity inner loop which is controlled by the ISMC, and an outer position loop which is controlled by a proportional controller ($u_p(t)$). The outer position value is calculated by using an integrator to integrate the output from the velocity inner loop (see the Equation (3.9)).

For robust performance against parameter variations and disturbances rejection, the ISMC parameters need to be tuned. The tuning parameters were chosen based on guidelines that were given by Kaya [68]. The controller designer had to set the switching function parameters (S_1, S_2), the discontinuous controller constant (k), the boundary layer width (δ) and the sliding mode attended rate parameter (ϕ). The guidelines stated that all ISMC parameters must be positive, that Equation (3.39) must satisfy the reaching condition in order for the system to be stable, and that the discontinuous controller must be kept reasonable in order to prevent damaging the actuator. This requires that the trade-off between the controller robustness against parameter variations and disturbances and sluggish performances have to be taken into consideration.

3.4. Simulation and Experimental Setup

This section presents the controllers simulation and experimental setup. First the PI, P-PI, ISMC and P-ISMC controllers' simulation implementation is presented. Then the comparison between P-PI controller and P-ISMC experimental data is presented. The simulation step size and experimental sampling time is chosen to be 0.109 seconds. The control input and system output disturbances are chosen to be 1.50 and 1.25, respectively. In order to compensate for the static friction as experienced during system modelling, a dead band of ± 2.5 was taken into consideration during these simulations. However, the comparison performance of controllers will be based on the settling time; final position error and velocity overshoot and control action. The PI and P-PI controller tuning guidelines are those presented in section 3.2.3, whereas the ISMC and P-ISMC controller stability and tuning were based on Lyapunov's second method and ISMC tuning guidelines (see sections 3.3.4 and 3.3.5).

It should be noted that although input shaping is important in vibration suppression in point-to-point applications, the possibility of input shaping was not considered in this dissertation because it beyond the scope of this dissertation.

3.4.1. Simulation of Controllers

The simulations for the PI, P-PI, ISMC and P-ISMC controllers were performed using program codes written in MATLAB. The great advantage of using MATLAB's simulation program codes is that they allow the same program codes to be converted into C++ for experimental tests.

The velocity state space models from Equations (3.6) and (3.7) were used to simulate step-tests at 5 mm/sec for the PI and ISMC controllers. On the other hand, to observe the performance of the P-PI and P-ISMC controllers, a simulation study was conducted for tracking the trapezoidal motion profile. The closed-loop position is a trapezoid too and can be obtained by integrating the velocity trapezoidal motion profile. Therefore, the position state space model from Equation (3.8) will be used to track the trapezoidal motion profile at a maximum velocity of 5 mm/sec and a desired position of 100 mm.

A 4th order Runge-Kutta (RK4) will be chosen as the differential equation solver for ISMC and P-ISMC simulations. The main aim of the RK4 is to calculate the plant's next state value based on the value of the state derivatives using initial conditions provided to the plant. This method is known for its simplicity, and also directly provides the value of the states [69]. The RK4 was first implemented in MATLAB and then in C++ because it is faster than MATLAB, and because C++ was a part of the skills-development component of the degree. This will allow the entire code to be established for simulating the closed-loop system and for estimating the system states during the experimental implementation. If h and t are the step size and time, respectively, the RK4 numerical algorithm has the following sequence of operation:

$$\begin{aligned}k_1 &= h * f(t, x) \\k_2 &= h * f\left(t + \frac{h}{2}, x + \frac{k_1}{2}\right) \\k_3 &= h * f\left(t + \frac{h}{2}, x + \frac{k_2}{2}\right) \\k_4 &= h * f(t + h, x + k_3) \\x(t + h) &= x(t) + (k_1 + k_2 + k_3 + k_4)/6 \quad [70]\end{aligned} \tag{3.60}$$

On other hand, it should be noted that the accuracy of the simulation depends on the step size. A better precision simulation can be obtained with a smaller step size but this will require more computational time [70]. For this reason, an appropriate step size can be chosen when the computational time is reasonable, and better precision simulation performance can be achieved. In this dissertation these have been obtained when the step size was chosen to be 0.109 seconds.

In summary, to assess the tracking performance of the PI, ISMC, P-PI and P-ISMC controllers effectively, several simulations were conducted with different desired inputs and their parameters. A step set-point and trapezoidal motion profile with and without disturbances were used to evaluate the speed and position tracking performance of the system transients. The step set-point simulation responses with/without disturbances were conducted for PI and ISMC controllers to evaluate the tracking performance of the system velocity overshoot and velocity settling time (see the simulation results in section 4.1). In addition, the trapezoidal simulation responses with and without disturbances were conducted for the P-PI and P-ISMC controllers to evaluate the tracking performance of the system velocity overshoot, velocity settling time, final position error and disturbance rejection (see the simulation results in section 4.2).

3.4.2. The Experimental (Physical model) Setup

The experimental setup, given in Figure 3.12, consists of a DC motor, its servo drive, a power supply, a preamplifier, an attenuator amplifier and a servo motor DAQ card with a personal computer (Intel(R) Celeron AT/AT compatible). For point-to-point applications, the experimental setup is based on a computer-controlled system. The computer will be connected to the DC motor by means of an analogue-to-digital converter (A/D) and digital-to-analogue convertors (D/A) made by *DATA TRANSLATION INC.* This type of implementation is discussed in detail by Gambier [71]. The velocity is directly measurable in radians per seconds through a tachometer. (In order to meet the linear velocity (mm/sec) and linear position (mm) characteristics, angular velocity and angular position are converted to linear velocity and linear position within the code written in the personal computer. Motion profiles with/without disturbances are implemented on the setup at a speed of 5 mm/sec and a desired position of 100 mm.

The C\C++ programs for both controllers' experimental implementation are run and compiled based on Newmat library version 11 and Borland compiler version 5.5. However, the P-ISMC controller's C\C++ code is coded such that the RK4 will act as an observer which estimates system states. The *DATA TRANSLATION DAQ* card (DT300) outputs voltages to the motor through an operation amplifier, attenuator, pre-amplifier and DC motor driver as shown in Figure 3.12. The motor tachometer voltage is input into the computer ADC (DT300). The components from the operation amplifier to the preamplifier were adjusted so that the minimum and maximum voltages could not exceed the DT300 card's range by attenuating the motor gain through a voltage divider built into the attenuator. The sampling frequency was chosen based on the Nyquist

sampling criteria. For this reason, in this experimental setup, the sampling time was chosen to be 0.109 seconds which equates to a sampling frequency of 9.174 Hz. The results of the experimental setup shown in Fig. 3.12 are presented in Chapter 4.

In summary, experiments were tested only for a trapezoidal motion profile with and without disturbances to evaluate the tracking performance of the system transients. These were conducted for the P-PI and P-ISMC controllers to evaluate the tracking performance of the system velocity overshoot, velocity settling time, final position error and disturbance rejection (see the simulation results in section 4.3).

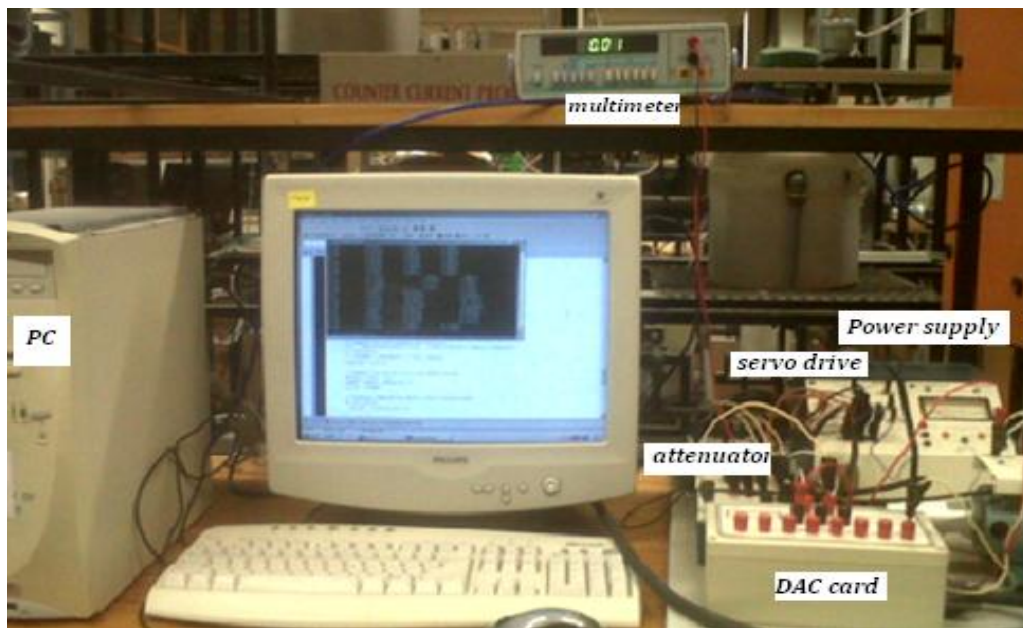


Figure 3.12: The experimental setup.

3.5. Image Quality Assessment and Control Verification Methods

Image quality assessment is an important process for determining the amount of image degradation that may be caused by a sudden change in the acceleration phase of the trapezoidal motion profile, the mechanical rigidity mismatch between the base and the C-arm and the challenges of optimally controlling the linear motor (PMLM). As mentioned in Chapter 1 and Chapter 2, these factors cause a mismatch between the scanning speed of the C-arm and the clocking speed of the detector while working in time delay integration mode (TDI) resulting in image blurring [11],[72],[73].

In order to determine the influence of the controller's tracking performance on image quality improvement; four X-ray images taken with the Statscan™ (see section 5.1) were distorted using the distance profiles that were generated by the controllers and a time delay integration simulator code(see Appendix B). Then, the image focus measure was used to assess the image quality at scanning speeds of 3 mm/s, 6 mm/s and 9 mm/s, using 25 mm, 50 mm and 100 mm position references. Finally, a three-way ANOVA statistical test was used to compare the performance of the controllers (see sections 5.2.1 - 5.2.4).

3.5.1. Position Distance Profile

The area under the curve of the trapezoidal motion profile (see Figure 2.1.a) is the total distance profile (see Figure 3.14). The total distance profile measurement is in millimeters from the simulation output of both controllers. It is used to scan the four X-ray images taken with the Statscan™ X-ray machine from the initial position to the final position. Simulation results of distance profiles are created using a proportional-integral sliding mode controller (P-ISMC) and a proportional-proportional integral cascaded controller. In order to explore the influence of boundary layer width and disturbance, additional simulation distance profiles are generated using a proportional-integral sliding mode controller (P-ISMC)-plus-boundary width and disturbances (BD). These have been called P-SMC+BD. The distance profiles are kept in a file to be loaded later for image scanning. The TDI software scanning algorithm converts the distance profile into pixels. Before scanning, the image size must be modified to add a blank area at the beginning and at the end. This helps the TDI software scanning algorithm to scan all selected test images without failing. In order to increase the amount of distorted image noise, some noise is added to return random noise. For more details see Appendix B.

3.5.2. TDI Software Algorithm

The Statscan™ detector consists of 12 charge-coupled devices (CCDs) operating in time delay integration (TDI) mode. The physical principle of CCD detectors and their use in digital radiography systems is discussed by Körner *et al.* [1]. In this mode, each row of pixels in the CCD is sequentially added to the next row and finally clocked out to memory. In order to obtain high-quality images, the linear motion of the radiation source and detector should be precisely synchronised to the clocking speed of the CCD or else blurring will occur (see Figure 1.3). The Statscan™ can operate at three different clocking speeds (see Chapter 2, section 1.3). At the beginning of a scan the C-arm has to be accelerated from standstill to one of those speeds, in as short a time as possible. Note that the speeds of the Statscan™ C-arm are not the same as those scanning speeds that were mentioned in section 3.5 for the image quality test. This is because these speeds are for a real Statscan™ X-ray machine while in section 3.5 the speeds are for an alternative DC motor model where the DAQ card limitation is taken into consideration.

The configuration of the Statscan™ detector is illustrated in Figures (3.13) and (3.14) [74]. The twelve CCD sensors used are CCD30-11 type. The CCD30-11 is a high-performance and high-speed binning operation sensor with a wide dynamic range. The size of each CCD sensor is 1024 width x 255 length in pixels. The pixel size of the CCDs is 26 μm . Fiber optic tapers increase the pixel size to 60 μm , whilst obviously still retaining the 1024x256 resolution. There are twelve of these CCD sensors arranged in a row of the Statscan™ making up the detector. The physical size of the 12 CCD detectors is 690 x 15 mm² but this also includes the spaces between the 12 CCDs where they are against each other. The data sheet for the CCD30-11 sensor can be found in Appendix B.4.

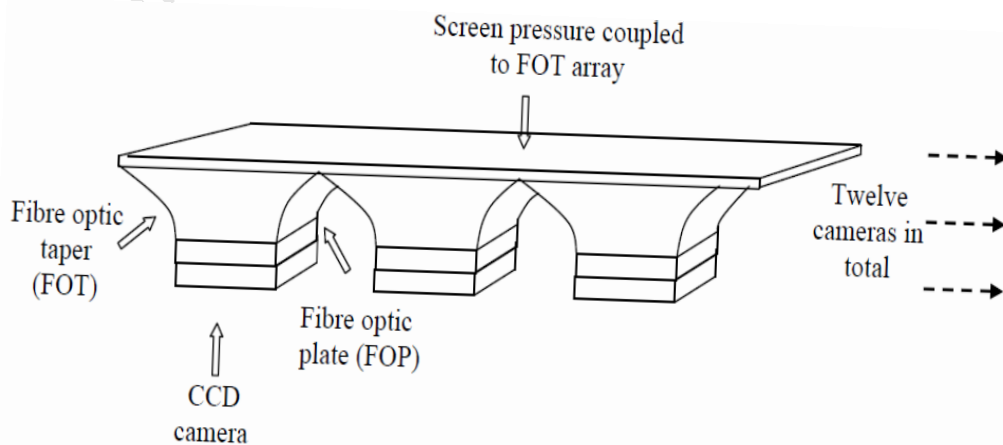


Figure 3.13: Schematic of Statscan™ detector.

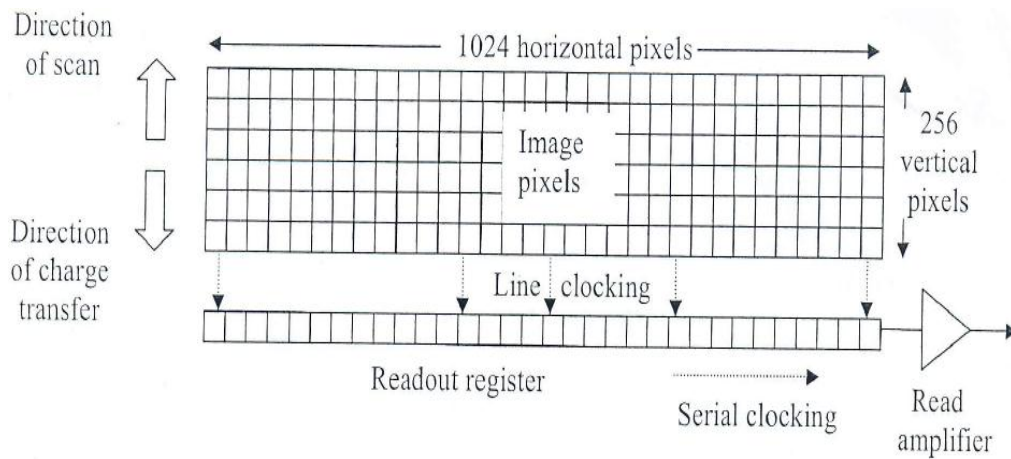


Figure 3.14: The illustration of image acquisition by TDI effect in the CCD.

In order to quantify the image quality using image focus measure, TDI software algorithms written by Dr Lester John from the University of Cape Town were used to simulate the time delay integration used in the Statscan™ X-ray scanner using simulated distance profiles. This software was written in MATLAB version 2007. The main aim of this algorithm is to scan the four images taken with the Statscan™ X-ray scanner. More details about the work of TDI software can be found in Appendix B.

The TDI algorithm operation mode starts by converting the Statscan™ image into a double-size image. Then the double double-size mage is loaded into the algorithm. In the next step the algorithm uses the distance profile from the file to scan the loaded image. This code assumes that the TDI algorithm always starts scanning at the top of the image. Figure 3.15 illustrates the conversion of the distance profile into pixels. This conversion is done by dividing the length and width of the detector by the detector's effective pixel size (0.06 mm). In this case the height is 250 pixels and the width is 11500 pixels.

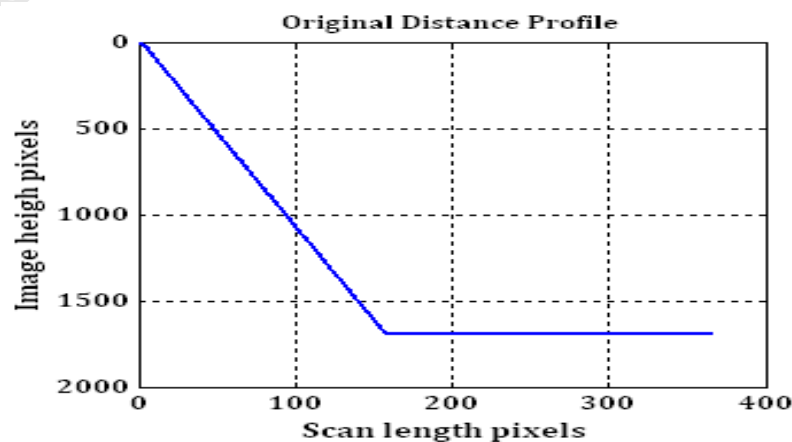


Figure 3.15: The 100 mm distance profile at 6 mm/sec converted into pixels using the detector width and height.

3.5.3. Measurement of the Image Blurring Index

Since there are a number of different focus measures, the candidate measure that will be used in this dissertation is the energy of the Laplacian of the image. It was used to compare the quality of the distorted Statscan™ images taken at various constant speeds. The main reason for using this metric is because the energy of the Laplacian of the image is a good focus measure in terms of tolerating the additive noise [52].

The image blurring index process is determined in three steps: First, convolve the image with a smoothing operator; second, measure the image property; and finally, measure the image focus by using the energy of the Laplacian of the image operator [50]. The images focus value for an image with height (M_h) and width (N) were calculated from the images distorted by the distance profile of both controllers and the TDI software algorithm at three scanning speeds. If the images focus values of the distance profiles using P-ISMC are greater than in the case of the P-PI controller, it will be an indication that P-ISMC provides superior image quality. The energy of the Laplacian of the image (M_L) [51] is given by:

$$M_L = \sum_i^{M_h} \sum_j^N (I_{ii} + I_{jj})^2 \quad (3.61)$$

where g is the image's grey level function at pixels (ij) , the M_h and N are the height and width of image, respectively, I_{ii} and I_{jj} are the second derivative of I with respect to i and j , respectively, and their summation can be described as

$$\begin{aligned} I_{ii} + I_{jj} = & -I(i-1, j-1) - 4 \times I(i-1, j) - I(i-1, j+1) \\ & -4 \times I(i, j-1) + 20 \times I(i, j) - 4 \times I(i, j+1) \\ & I(i+1, j-1) - 4 \times I(i+1, j) - I(i+1, j+1) \end{aligned} \quad (3.62)$$

3.5.4. ANOVA Statistical Test

Analysis of variance (ANOVA) is an established method for testing the equality of three or more means simultaneously by using variances. The explanation and details about the ANOVA tests can be found in Appendix A. In this dissertation a three-way ANOVA test is used to test the tracking performance of the P-ISMC, the P-ISMC+BD and the P-PI, using version PASW Statistics 18 statistical software by SPSS Inc.

The three-way ANOVA tests should be run for the final position tracking error (called *Final Position Error*), constant speed settling time (called *Settling Time or Settling*), constant speed overshoot (called *Overshoot*) and image blurring index (called *Image*

Blurring Index) for each single dependent variable one at a time using the linear model (univariate) of SPSS.

In summary, a three-way ANOVA test was used for the following factors and dependent variables:

i. Factors: (*Controller*Speed*Position*):

- *Controller*: P-PI, P-ISMC and P-ISMC+BD
- *Speed*: 3, 6 and 9 mm/s
- *Position*: 25, 50 and 100 mm

ii. Dependent Variables:

- *Final Position Error* (see Chapter 5.2.1)
- *Constant Speed Settling Time* (see Chapter 5.2.2)
- *Constant Speed Overshoot* (see Chapter 5.2.3)
- *Image Blurring Index* (see Chapter 5.2.4)

4. Simulation and Experiment Results

This chapter presents computer simulation and experimental (i.e., physical model) results. The simulations were done in MATLAB, while experimental implementations were coded in C++, linked with a Newmat matrix library and compiled on a Borland C++ compiler [75]. The step set-point simulation response with/without any control input and system output disturbances were simulated for the PI and ISMC controllers to evaluate the system velocity overshoot, velocity settling time, and the control input and the system output disturbance rejection (see section 4.1). On the other hand, trapezoidal motion profile simulations and experiments with and without disturbances were simulated and implemented for the P-PI and P-ISMC to evaluate the system velocity overshoot, velocity settling time, final position error and the control input and the system output disturbance rejection (see sections 4.2 and 4.3). In order to compensate for the integration windup and saturations, the PI and P-PI controllers were subjected to integration stopping in the simulation and the experiment (i.e., physical model). In addition, because the DAQ card that was used is limited to ± 10 volts, the minimum and maximum of the actuator in the simulation and the experimental setup is limited to the DAQ card's limit volts. It should be noted that simulation and experiment (i.e., physical model) were conducted on a DC motor whose model is different compared to the Statscan™, because the system dynamics' parameters are different. Therefore, the simulation time for the DC motor will also be different (i.e., longer than 13 seconds).

The simulation and experiment (i.e., physical model) that were conducted can be summarized as follows:

- 1) Step set-point simulations:
 - i. Factors: (*Controller*Speed*)
 - *Controller*: PI and ISMC
 - *Speed* : 5 mm/sec
 - ii. Outcome Dependent Variables:
 - Constant speed *Overshoot*
 - Constant speed *Settling Time*
- 2) Trapezoidal motion profile simulation and experimental (i.e., physical model)
 - i. Factors: (*Controller*Speed*Position*):
 - *Controller*: P-PI, P-ISMC
 - *Speed*: 5 mm/s

- *Position*: 100 mm
- ii. Outcome Dependent Variables:
- Constant speed *Overshoot*
 - Constant speed *Settling Time*
 - *Final Position Error*

4.1. PI and ISMC Controller's Simulation Results

This section shows the PI and ISMC simulation results for step set-point tracking designed for velocity inner loop. The PI controller was implemented digitally using the difference equation method based on Equation (2.16), Chapter 2. The PI controllers' parameters were obtained using bode plots discussed in section 3.2.3. On the other hand, the ISMC simulation was performed on the system's regular form in Equation (3.27) using the control algorithm in Equation (3.58) which consists of an ISMC controller and a continuous approximation boundary layer. In order to estimate the states, the observer mentioned in section 3.3.4.3 was used. The switching function and ISMC parameters and gains were obtained using the pole placement method in section 3.3.3.1 and sliding mode control tuning guidelines given by Kaya [68] that were presented in section 3.3.5. The PI controller's parameters gains and the ISMC controller's gains and parameters, including the observer gain L , observer poles and poles locations, can be seen in Table 4.1.

Table 4.1: The PI and ISMC controllers' parameters and gains.

Controller	Gains	Observer parameters
ISMC	$S_2=1.00$	$L=[0.77 \ -0.02]^T$
	$M=S_1=4.28$	Pole = -0.20
	$K=13.26$	Pole = -0.30
	$\Phi=0.05$	Pole location= -0.60
	$\delta =1.05$	Pole location= -0.90
PI	$k_{vp}=15.90$	
	$b=24.17$	
	$k_{vi}=0.04$	

4.1.1. PI Simulation Time Responses to Step Input

This section presents the PI simulation results. The simulation is performed on the state space model in Equation (3.6) using calculated values for k_{vp} and b gains as shown in Table 4.1. The step set-point is chosen to be 5 mm/sec with a sampling time

of 0.109 seconds. In addition, the simulations were run for the control input constant disturbance ($f(t) = 1.5$ volts) introduced at 1.9 seconds and system output constant disturbance ($f_v(t) = 1.25$ volts) added at time 5.5, as well as with no disturbances, where $f_v(t)$ is assumed to be the constant disturbance to the input of the velocity feedback system. For instance, the constant control input disturbance and constant system output disturbance were shown in Figure 2.2.

Step-test time responses for the PI controller without and with disturbances can be seen in Figure 4.1. Figures 4.1(a) and 4.1(b) show the velocity error (a), velocity output (b) and controller signal responses (c), respectively.

From these plots (left and right of Figure 4.1), it can be seen clearly that the system reaches a steady state after 1 second. There is a small overshoot on the velocity output response and the final steady state velocity error is 0.0940. Due to integrator windup, the PI controller seems to generate a relatively high action (10 volts) before returning to 3.5 volts at the system steady-state.

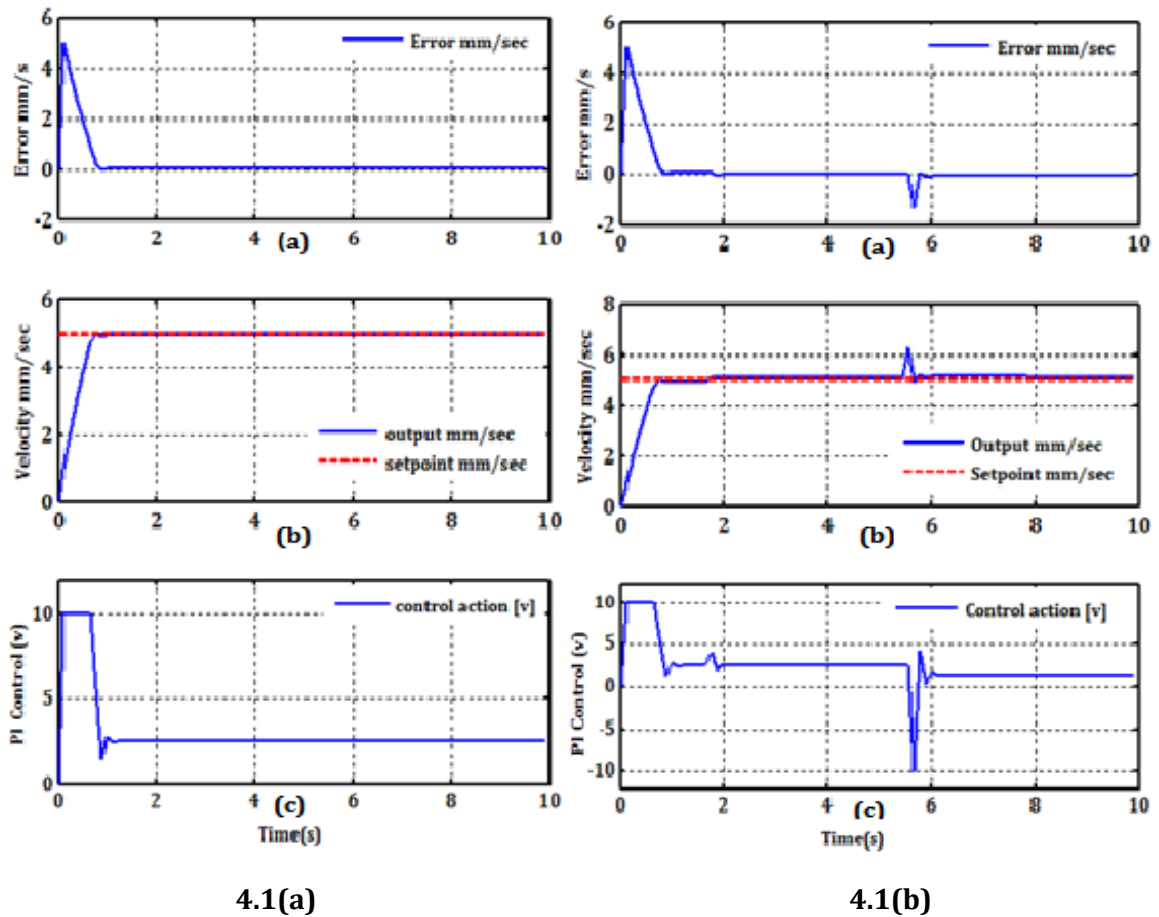


Figure 4.1: Simulated time responses of PI controller without disturbances 4.1(a) and with control input disturbance ($f(t) = 1.5$ volts at 1.9seconds) and system input disturbance ($f_v(t) = 1.25$ volts at 5.5 seconds) 4.1(b).

The system overshoots by 0.006 mm/sec when disturbances are added at simulations times 1.9 seconds and 5.5 seconds. This affects the robustness of the controller tracking performance because the limiter limits the PI controller signal at ± 10 volts but the integration windup phenomenon still degrades the reference tracking of the step set-point [30]. Evidence of this can be seen in Figure 4.1(b) where disturbances are added at 1.9 and 5.5 seconds. As a result of the type number correction due to the integrator part in the PI controller (type 1 controller), the PI controller seems to reject the control input disturbance more effectively than the system output disturbance.

To eliminate the effect of integration windup that was incorporated in Figure 4.1, the anti-integration-windup method that was discussed in section 2.2.2.2, Chapter 2, was used in the simulation. A step set-point of magnitude 10 was simulated with/without the anti-windup integration method. Figure 4.2 shows the velocity output responses and the PI controller action with/without the integration stopping method. It is clearly seen that the closed-loop system with saturation (with dashed lines) produces an excessive overshoot and a long settling time, whereas the closed-loop system with the integration stopping, anti-windup method (solid line) produces zero overshoot and a small settling time. In short, the anti-integration-windup method has improved the controller tracking performance by limiting and stopping the integrator value from growing excessively.

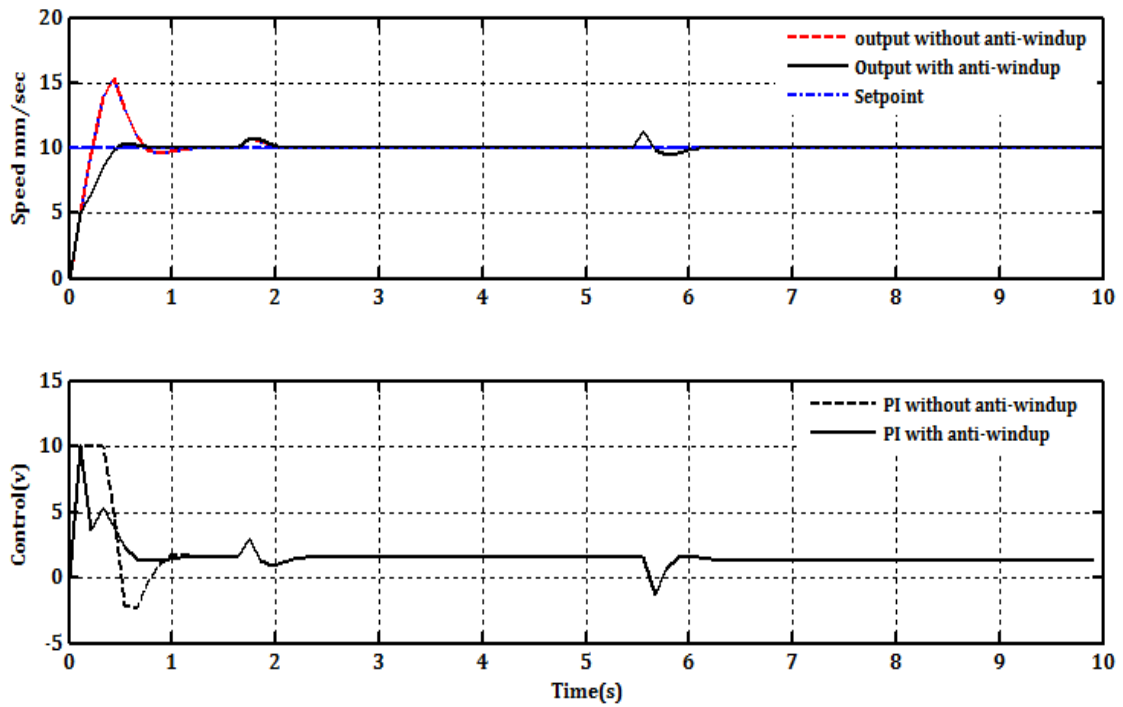


Figure 4.2: The closed-loop dynamic with PI controller with and without anti-windup technique applied to the system model in Equation (3.3).

4.1.2. ISMC Simulation Time Responses to Step Input

Similar to the PI, a step input (5 mm/sec) without and with control input and system output disturbances simulations added at simulation times 4 seconds and 9 seconds, respectively, was carried out with the ISMC. The switching surface matrix $\mathbf{S}$ parameters (S_2 and $M = S_1$) and the ISMC control gains used in the simulation are determined by the methods outlined in Chapter 3.3 using a regular form in section 3.3.1, the reference model in section 3.3.3.2, the pole placement method in section 3.3.3.1, the Lyapunov stability method in section 3.3.3.2 and ISMC tuning guidelines in section 3.3.5. The ISMC controller's parameter values for K , Φ , boundary layer width δ and switching matrix parameters values for $S_2 = 1$ and $M = S_1$ are tabulated in Table 4.1 with corresponding time responses shown in Figure 4.3. These switching matrix values are obtained as results of the following:

According to the case study illustrated in section 3.3.3, first the transformation matrix T_r was computed as, $T_r = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, which leads to decomposition results given in Equation (3.27), and therefore, the matrix sub-blocks for sliding mode control are obtained as

$A_{11}=0.00$, $A_{12}=1.00$, $A_{21}=0.00$ and $A_{22}=-0.74$, The matrix M that places the eigenvalues of the Matrix $A_{11} - MA_{12}$ in the location -4.2750 on the left side of the s-plane was computed by using the MATLAB "place" command which yields $M = 4.28$. Owing to this, the switching surface matrix is obtained as $S = [4.28 \ 1.00]$.

As stated in section 2.4.3, the development of high-frequency oscillations in the sliding mode controller generally are the main disadvantage to its implementation. To evaluate the effect of the chattering, first the regular form in Equation (3.27) is simulated without/with control input and system output disturbances and all ISMC controller parameters, except that the chattering alleviation method is not included. Chattering corresponding to time responses can be found and seen in Appendix D. Figures D.1.a and D.1.b are the simulation results of the ISMC controller without disturbances, and simulations results with disturbances, respectively. It can be seen that both corresponding responses are oscillatory. This is undesirable in a practical implementation since it may lead to wear and tear in the mechanical systems [36].

However, to overcome the chattering problem, the system was simulated with the obtained ISMC controller gains and switching surface parameters, while the $\text{sgn}(s)$ function is replaced by the boundary layer technique with width $\delta = 1.05$ to reduce the chattering influence, as well as to improve the tracking error performance.

The corresponding time responses with the boundary layer technique are shown in Figure 4.3. Figures 4.3(a) and 4.3(b) represent simulation responses for the velocity output and observer (a), the ISMC controller signal (b) and the switching surface responses (c), respectively. From these simulation results some observations can be recorded. These plots indicate that the ISMC has good step input tracking performance, but the corresponding responses overshoot by 0.0954. Moreover, the system reaches a steady state after 2 seconds. The boundary layer gives the ISMC more practical advantages since the steady-state error can be reduced. In addition, at corresponding simulation times 4.5 and 10.5 seconds, respectively, the ISMC controller provides good disturbance rejection. Nevertheless, this was obtained with relatively high control effort (10 volts) before the system reached the steady-state.

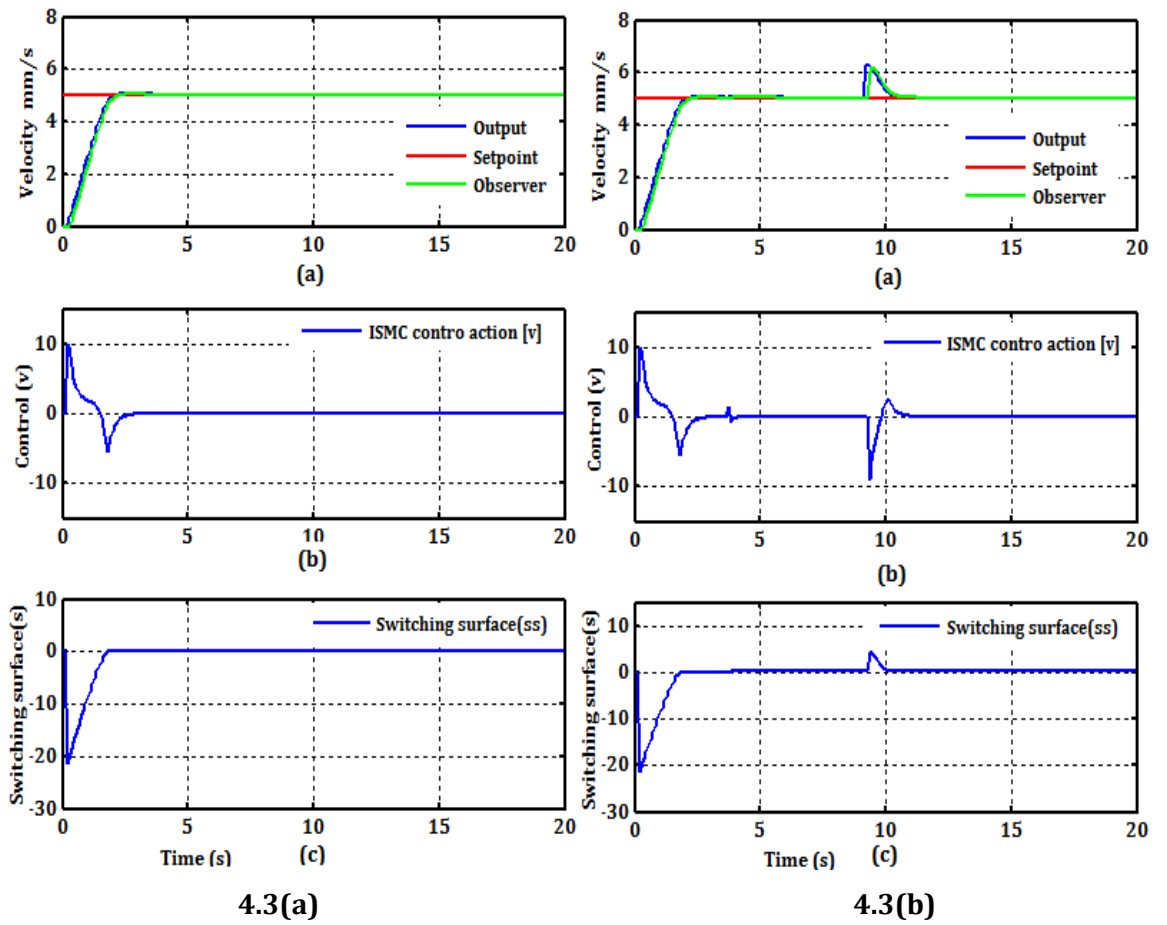


Figure 4.3: ISMC with boundary layer simulation time responses: 4.3(a). Simulation results without disturbances and 4.3(b). Simulation results with control input disturbance ($f(t) = 1.5$ volts at 4 seconds) and system output disturbance ($f_v(t) = 1.25$ volts at 9 seconds).

4.2. P-PI and P-ISMC Simulations Time Responses

This section presents the time responses of the P-PI and P-ISMC controllers' simulation. Their tracking performances were tested on a trapezoidal motion profile, control input and system output disturbances. The main objective of these tests is to compare the P-PI and P-ISMC for the velocity and position tracking performances and disturbance rejections. To reduce chattering in control, the discontinuous signum function is replaced by the boundary layer technique that is linear within a boundary layer width (see Equation (3.56) in section 3.3.4). The P-PI controller parameters and gains, and the P-ISMC controller's parameters and gains, including the observer gain L , observer poles and pole locations on the s -plane, are tabulated in Table 4.2.

Table 4.2: P-PI and P-ISMC controllers' simulation gains and parameters.

Controller	Gains	Others
P-ISMC	$M = S_1 = 3.50$	$L = [0.77 \ -0.02]^T$
	$S_2 = 1.00$	$Pole = -0.20$
	$K = 12.33$	$Pole = -0.30$
	$\Phi = 0.29$	$Pole\ location = -0.60$
	$\delta = 2.99$	$Poles\ location = -0.90$
	$k_{pp} = 1.42$	
P-PI	$k_{vp} = 1.90$	
	$b = 1.20$	
	$k_{vi} = 0.83$	
	$k_{pp} = 1.12$	

4.2.1. P-PI Time Responses to Trapezoidal Motion Profile

This section presents the P-PI simulation results. The simulation is performed on the state space model in Equation (3.7). The P-PI controller gains were chosen as in Table 4.2. The trapezoidal motion profile was set to 100 mm for position and the maximum speed was set at 5 mm/sec with a sampling time of 0.109 seconds. The tracking steady-state errors with and without disturbances are tabulated in Table 4.3.

Table 4.3: The final steady-state for the velocity and position errors.

Tracking error	$E_{v-\max} (mm/s)$		$E_{p-\max} (mm)$	
	No disturbances	Disturbances	No disturbances	Disturbances
P-PI Simulation	0.00	0.02	0.00	0.52

Figure 4.4(a) and (4.4(b) show the P-PI time responses with and without disturbances for position error (a), position output (b), trapezoidal response (c) and corresponding control signal (d). From Figure 4.4.a, which represents the simulation results without disturbances, it can be seen that the P-PI achieves zero overshoot response when it is well tuned. The system reaches a steady state after 4 seconds. Also, it can be seen that the P-PI controller control effort is 2.6 volts before the system reaches steady state (2 volts at steady-state).

To further assess the performance of the P-PI controller, the control input constant disturbance ($f(t) = 1.5$ volts) and system output constant disturbance ($f_y(t) = 1.25$ volts) are introduced into the feedback loop. The first disturbance (1.5 volts) was introduced to the input of the inner loop PI-controller when the simulation time was 4.36 seconds. The second disturbance (1.25 volts) was introduced to the position output when the simulation time equals 15.4 seconds.

The P-PI time responses with disturbances are shown in Figure 4.4.b. Looking at the trapezoidal motion tracking, the P-PI controller provides satisfactory control performance in spite of system output disturbance rejections. For a corresponding control input disturbance, an extra control effort was provided by the P-PI, and extra time (2.74 seconds) is needed for it to return to a steady state.

It should be noted that the position set-point that is shown in Figure 4.4 and in the other figures throughout this chapter represents the target position and not the position ramp set-point.

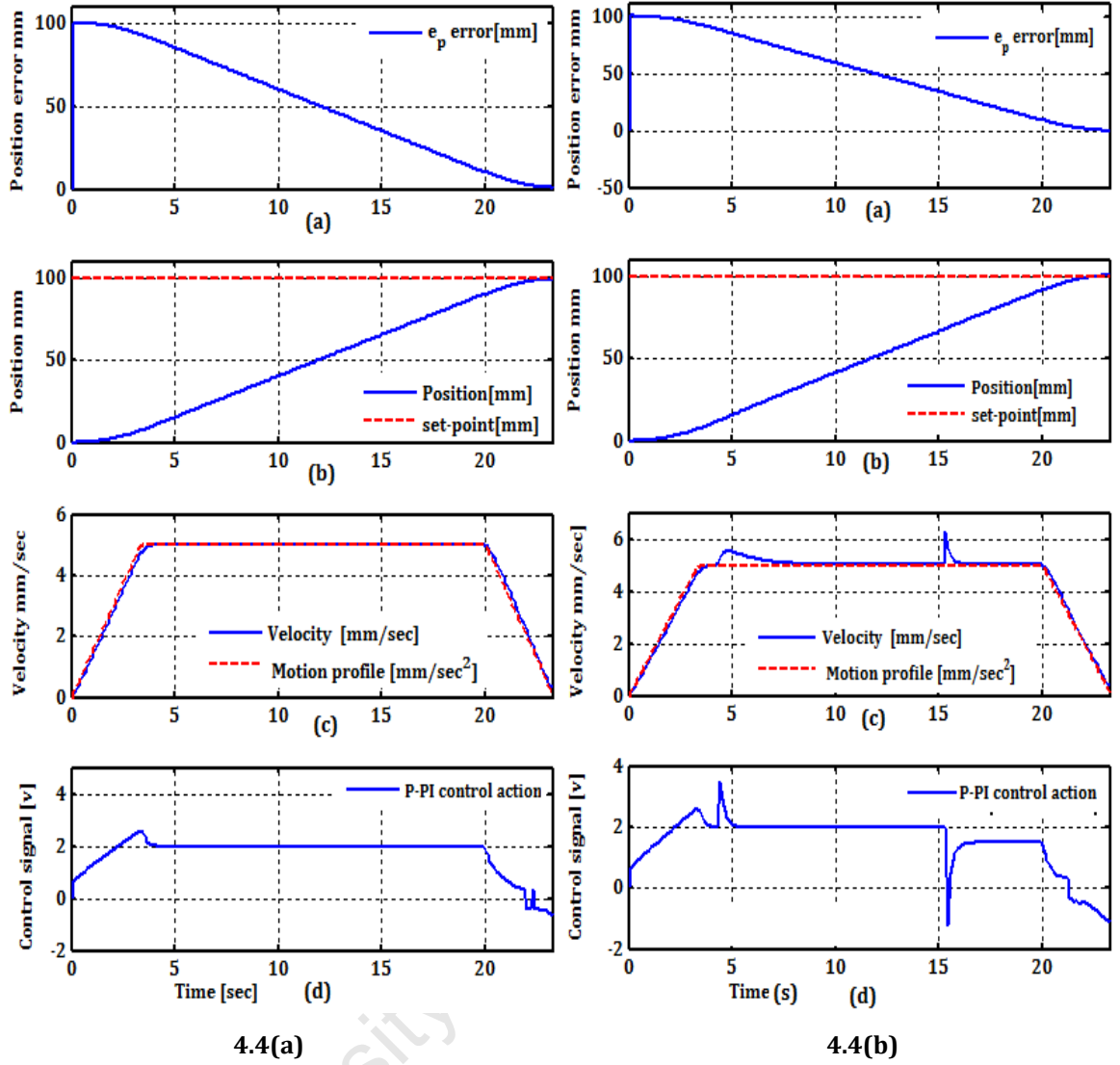


Figure 4.4: Simulation results for P-PI controller: 4.4(a). Simulation results without disturbances and 4.4(b). Simulation results with control input disturbance ($f(t) = 1.5$ volts) introduced at 4.36 seconds and system input disturbance ($f_y(t) = 1.25$ volts) introduced at 15.4 seconds.

4.2.2. P-ISMC Time Responses to Trapezoidal Motion Profile

This section presents the simulation results of the P-ISMC (see Figure 3.4). The simulation is similar to that of the P-PI from the previous section. In order to reduce the influence of the high-frequency oscillations, a boundary layer width was implemented. The P-ISMC parameter values for k_{pp} , K , Φ , boundary layer width δ , switching matrix parameter values for $S_2 = 1.00$ and $M = S_1$, observer gain L and pole locations are tabulated in Table 4.2. All the parameters in the P-ISMC controller and switching surface matrix were chosen to achieve the best transient responses and stability. All these parameters and gains used in the simulation are determined in a similar way to ISMC control that was discussed in section 4.1.2. It can be seen that the matrix $\mathbf{M}$ is

chosen to be 3.5 in order to place the eigenvalues of the matrix $A_{11} - MA_{12}$ in the location -3.5000 which makes the switching surface matrix to become $S = [3.50 \ 1.00]$.

Figure 4.5 shows the time responses for position error (4.5.a), position output (4.5.b), trapezoidal motion profile plus velocity responses (4.5.c), corresponding control signal (4.5.d) and sliding mode-switching surface (ss) (4.5.e). Figure 4.6 shows the time responses for corresponding variables discussed in Figure 4.5 plus the control input and the system model output disturbances. The simulation tracking error results without and with disturbances are tabulated in Table 4.4.

Table 4.4: The final steady state for the velocity and the position errors.

Tracking error	$E_{v-\max} \text{ (mm/s)}$		$E_{p-\max} \text{ (mm)}$	
	No disturbances	Disturbances	No disturbances	Disturbances
P-ISMC Simulation	0.00	0.04	0.00	0.72

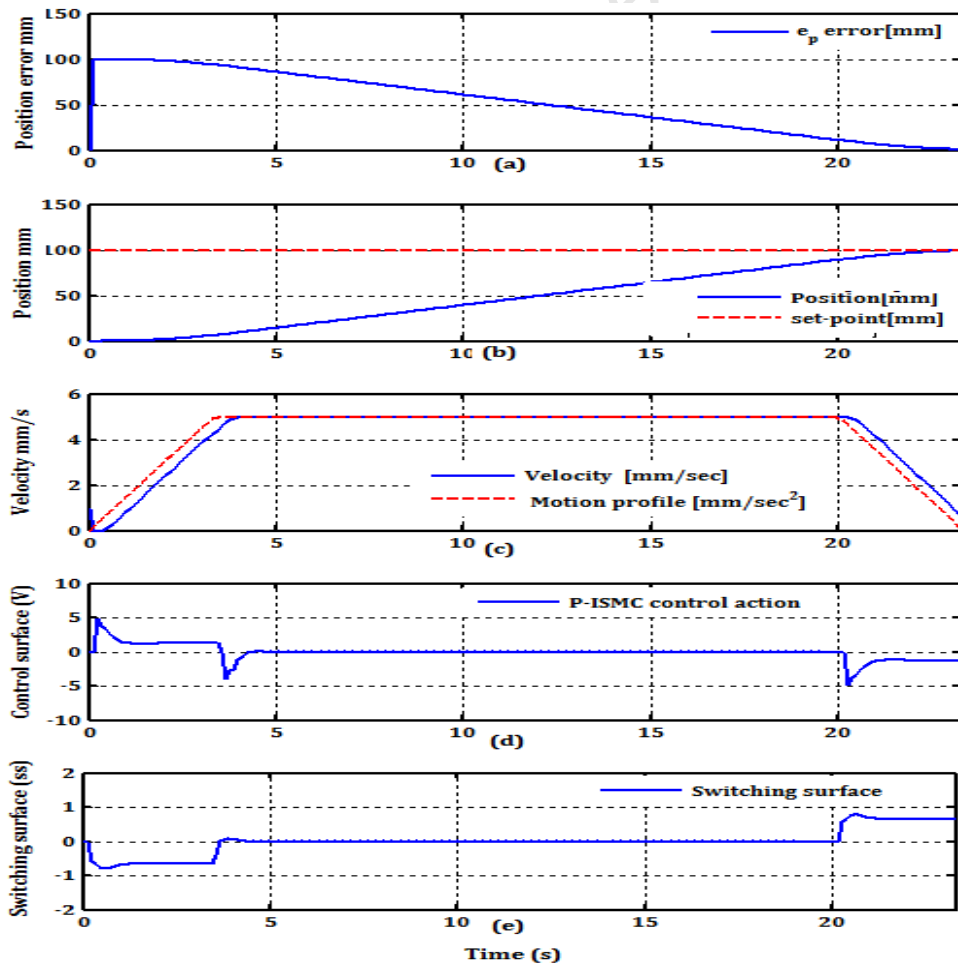


Figure 4.5: Simulations time responses for P-ISMC controller with boundary layer width ($\delta = 2.99$) and tracking performance without including disturbances.

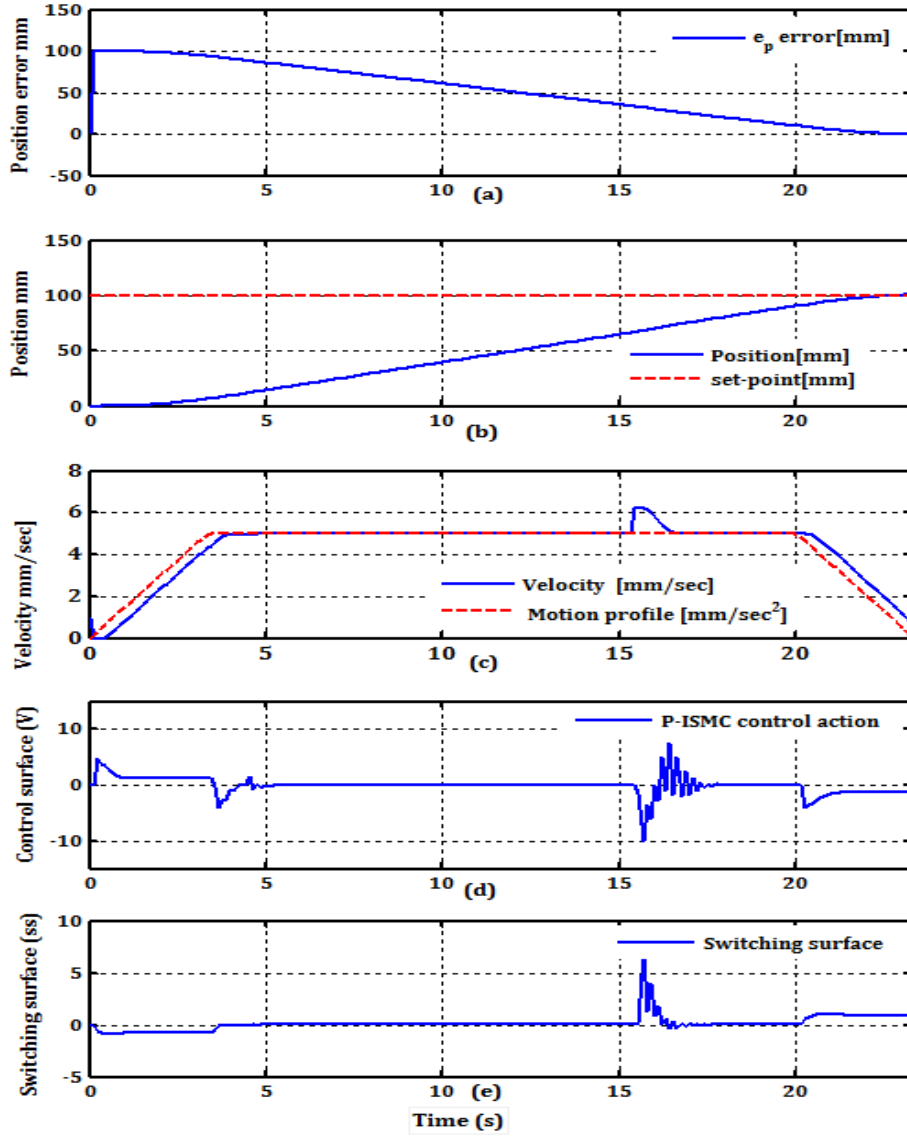


Figure 4.6: Simulation time responses for P-ISMC with boundary layer width ($\delta = 2.99$) and with control input disturbance ($f(t) = 1.5$ volts at 4.36 seconds) and system output disturbance ($f_y(t) = 1.25$ volts at 15.4 seconds).

Looking at the two plots of the switching surface simulations in Figures 4.5.e and 4.6.e, the system states converged to the origin, so system stability was achieved [36]. In addition, as can be seen in Figures 4.5.c and 4.6.c the tracking performance of the P-ISMC is not good during acceleration and deceleration phases when compared to the P-PI controller. This may be the result of the integrator in the P-PI controller which improves the tracking performance. Also, at constant velocity both controllers have good tracking performance since their tracking errors are zero when there are no disturbances. The corresponding controller effort is 1.5 volts before the system reaches a steady state. By looking at the steady-state tracking errors in Table 4.4, the P-ISMC gives higher final velocity error (0.04 mm/sec) and higher final position error (0.72 mm) when compared to the P-PI controller.

By observing Figure 4.6, it can be noted that the P-ISMC controller is not affected by a control input disturbance (at 4.5 seconds). Furthermore, when the simulation length is 15.4 seconds, the controller provides a better response in spite of system output disturbance rejections in comparison to the P-PI.

4.3. Experiment Time Responses

This section presents the experimental results of both controllers simulated in the previous section. It must be noted that the linear motors (PMLM) provide linear velocity (mm/sec) and linear position (mm) while the DC motors provide rotational motions (rad/sec). The simulation and experimental tests for the DC motor model are described in section 3.1 and the motor setup is shown in Figure 3.12, assembled and fitted such that there is a match between the linear velocity and linear position characteristics. In these experiments, the velocity reference was set to 5 mm/s and the desired position set-point was set to 100 mm. In this implementation, the effects of disturbances, settling time, final velocity and final position error are considered.

4.3.1. P-PI Experiment Time Response

In this section, the P-PI tracking performance is tested on the DC motor platform shown in Figure 3.12. The position is set to 100 mm and the trapezoidal motion profile with constant speed equal to 5 mm/sec and high acceleration/deceleration set to ± 1.5 mm/sec² were utilized. The trapezoidal motion profile duration of the acceleration and deceleration is 4 seconds. In order to accelerate to the constant velocity in 4 seconds one of the P-PI controller parameters that was obtained in the simulation (see Table 4.2) differed slightly in the experimental setting. This may be as a result of modelling uncertainties which affect the performance of the P-PI controller. Owing to this, the PI controller's gains that were used in the experiment were chosen as $b = 1.22$, $k_{vi} = 0.82$, $k_{vp} = 2.35$, and the proportional controller's as $k_{pp} = 1.12$.

The time responses without disturbances for the position error (a), position output (b), trapezoidal motion profile and velocity output(c) and control signal (d) are illustrated in Figure 4.7. The controller time responses with disturbances listed in Table 4.2 for the position error (a), position output (b), trapezoidal motion profile tracking(c) and control signal (d) are illustrated in Figure 4.8.

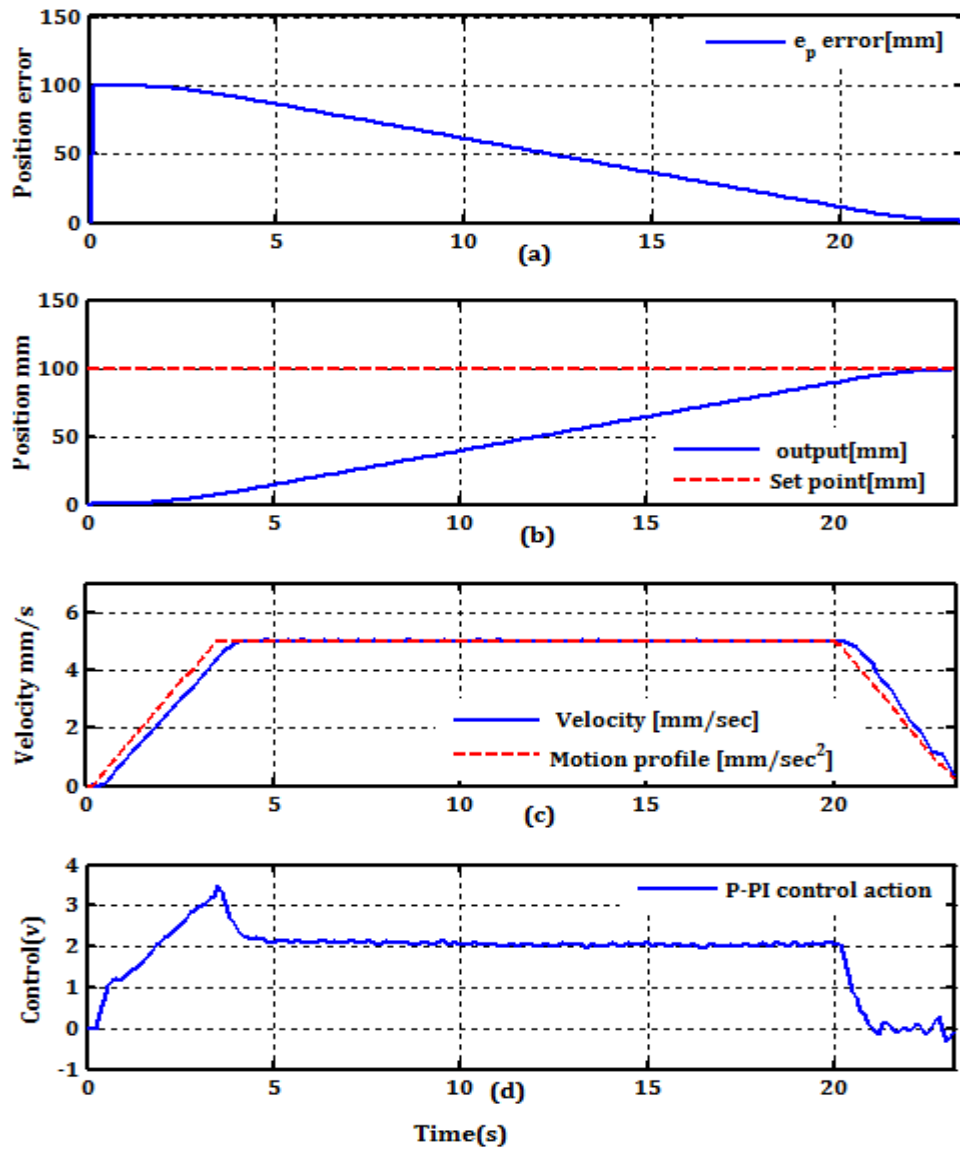


Figure 4.7: Experimental time responses of P-PI controller without disturbances.

As shown in Figure 4.8, during the acceleration and deceleration phases the P-PI controller requires 3.60 and -0.22 volts, respectively as control effort whilst an effort of 2.11 volt is necessary when the system reaches the steady state. By looking at the tracking errors, the P-PI's final velocity error is 0.02 while its final position error is 1.27 mm. The system reaches a velocity steady state after 4 seconds which equals the acceleration time and there is an overshoot of 0.01.

Figure 4.9 shows the P-PI controller's performance with the same tuning parameters when the control input disturbance was 1.5 volts and the system output distance was 1.25 volts, applied at 7 and 13.5 seconds, respectively. In the experiment, the high acceleration /deceleration are set to $\pm 1.5 \text{ mm/sec}^2$, the desired position is set to 100 mm and the maximum speed is set to 5 mm/sec.

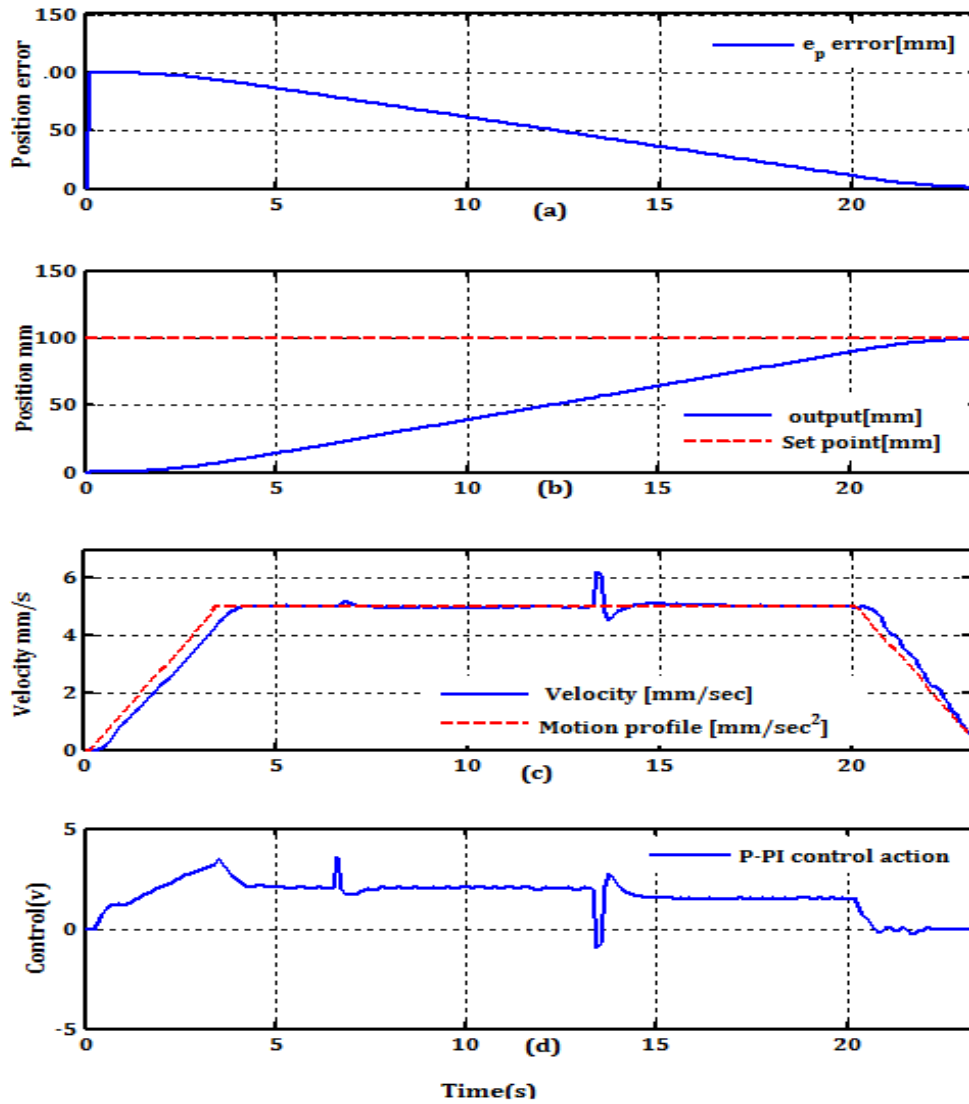


Figure 4.8: Experimental time responses of P-PI controller tracking performance with control input disturbance ($f(t) = 1.5$ volts at 7 seconds) and system output disturbance ($f_y(t) = 1.25$ volts at 13.5 seconds).

Figure 4.8 it can be seen that the P-PI controller performance seems to be influenced by disturbances. First, it produces the same magnitude of control signal when the system in a steady state, but within the acceleration and deceleration phases the control magnitude was produced (3.6 volts and -0.22 volts, respectively). Secondly, there is variation in the final velocity error (0.17 mm/sec) and the final position error (1.72

mm) compared to the case without disturbances. In addition, the system reaches a velocity steady state after 4 seconds; there is an overshoot of 0.09.

Looking more closely where the control input constant disturbance at the times of 7 seconds, and the system output constant disturbance at 13 seconds, are added during the experiment, the proposed controller shows satisfactory disturbance rejection against the control input constant disturbance, whereas its performance against the system output constant disturbance made the system response overshoot and then undershoot for approximately 2 seconds before return to steady state. However, these latter results were obtained at a cost of a longer time taken in order for the control system to reject the system output disturbance when compared to its performance on the control input disturbance. This may be a result of the PI controller in the inner loop which well-known for its disturbance rejection [6], [16] while the outer position loop is used as a proportional controller.

4.3.2. P-ISMC Experiment Time Responses

In this section, the P-ISMC tracking performance is tested on the DC motor platform shown in Figure 3.12. In this experiment, the same position set-point and same trapezoidal with the same constant speed, acceleration/deceleration and time duration as in the P-PI experiment are used. The P-ISMC gains and parameters were selected as follows: $k_{pp}=1.12$, $k = 6.50$, $M = S_1 = 3.99$, $\phi = 0.09$, $\delta = 1.55$, and the observer gain $L = [0.67 \ -0.02]^T$.

The time responses without disturbances for the position output (a), trapezoidal motion profile and velocity output (b), switching surface (c) and control signal (d) are illustrated in Figure 4.9.

The controller's time responses with disturbances listed in Table 4.2 for the position output (a), trapezoidal motion profile and velocity output (b), switching surface and control signal (d) response are illustrated in Figure 4.10.

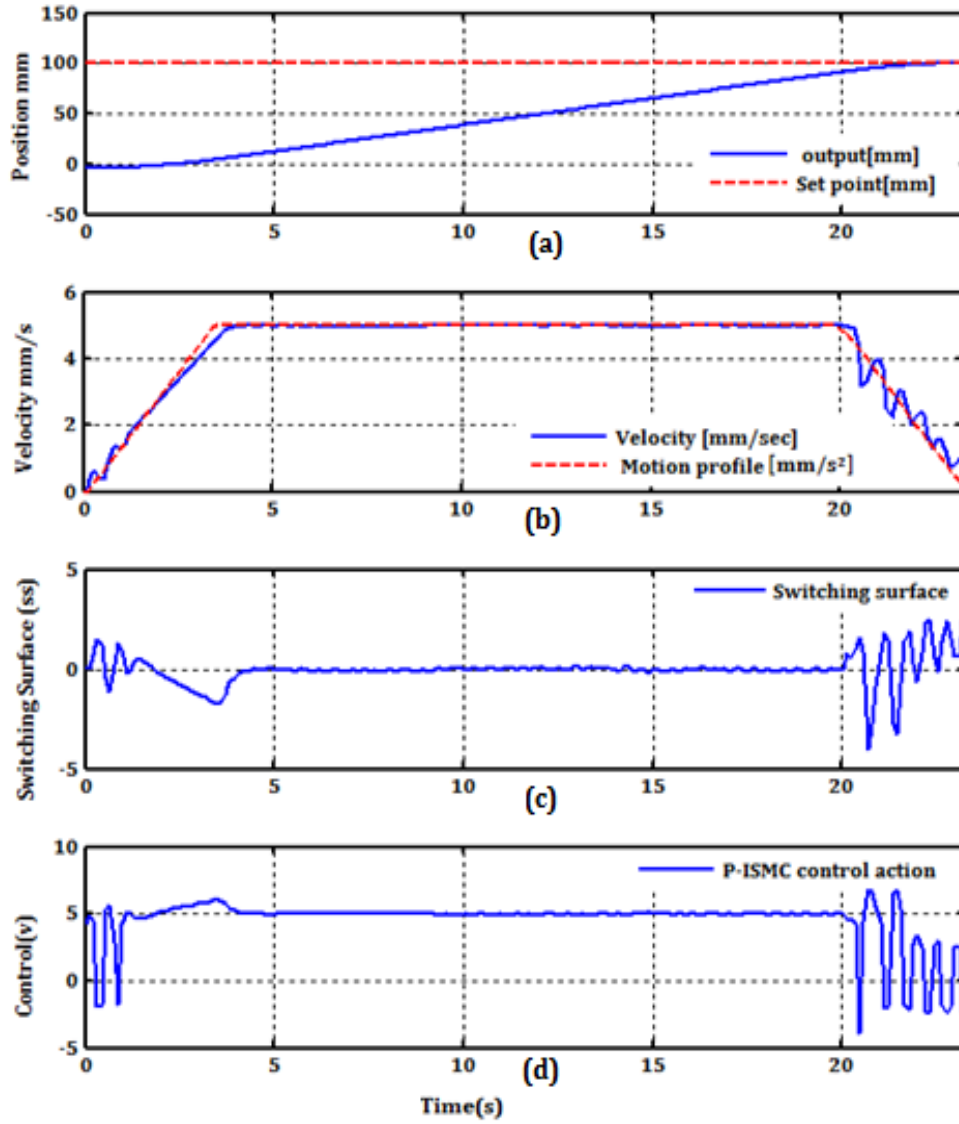


Figure 4.9: Experimental time responses for P-ISMC controller with boundary layer width ($\delta = 1.55$) and tracking performance without disturbances.

From Figures 4.9 and 4.10 it can be seen that the P-ISMC controller produces an oscillatory 4.8 volts magnitude during the acceleration and deceleration phases, whilst at steady state it gives 5 volts. The switching surface time responses (see Figure 4.9.c) converge to zero and there is no overshoot. The system reaches a velocity steady state after 4 seconds. The final velocity error remains the same (0.07 mm/sec) for both tests but there is variation in the final position error (0.66 –1.96 mm) as a result of the disturbances.

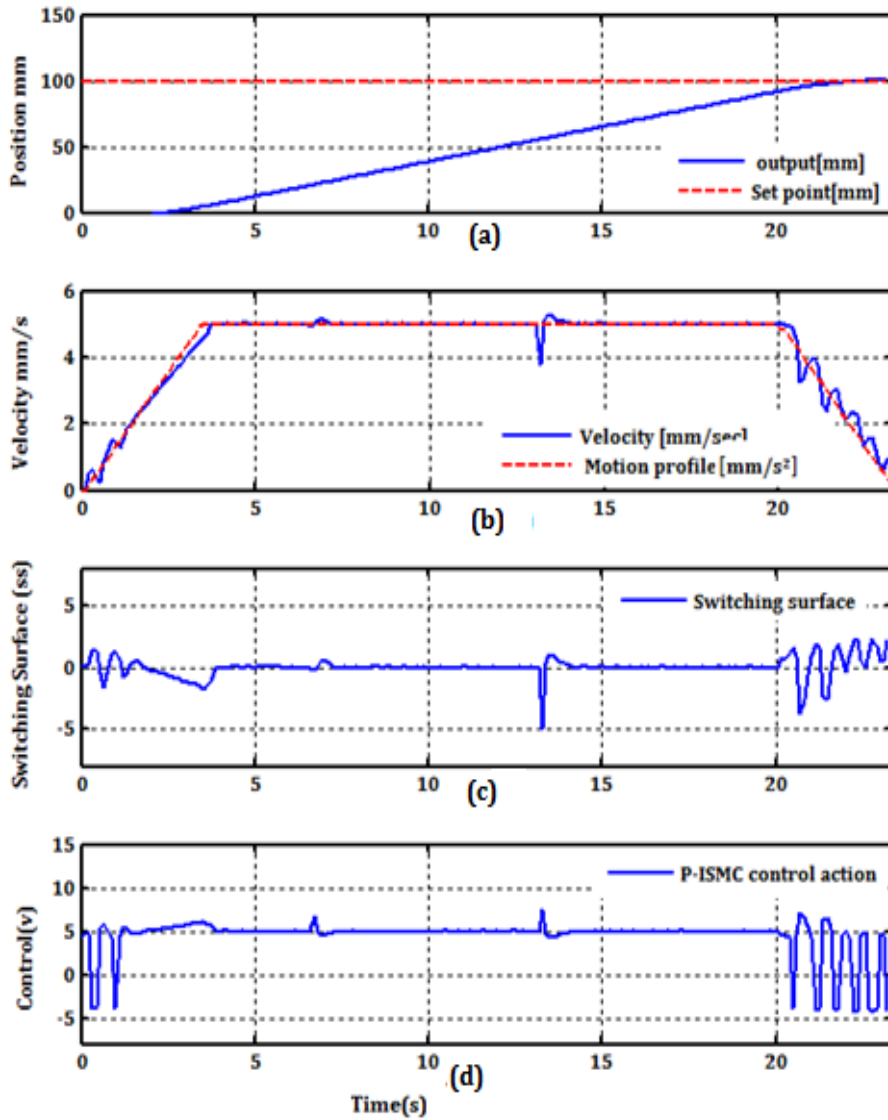


Figure 4.10: Experimental time responses for P-ISM controller with boundary layer width ($\delta = 1.55$) and tracking performance with control input disturbance ($f(t) = 1.5$ volts at 7 seconds) and system output disturbance ($f_y(t) = 1.25$ volts at 13.5 seconds).

Looking more closely at the simulation times of 7 and 13 seconds when the two disturbances were added (see Figure 4.10), the P-ISM controller is not influenced by the controller input disturbance (1.5 volts). On the other hand, although the system response overshoots and undershoots, the P-ISM provides a very good system output disturbance rejection (1.25 volts). It took only 1 second (compared to 2 seconds for the P-PI) before returning to steady state. Furthermore, during the acceleration phase and at the end of the trapezoidal motion profile, the P-ISM oscillates around the final position error. This oscillation may be because the final position error is larger.

4.4. Summary of Simulation and Experimental Results

For a better comparison, the controllers' simulation and experimental response time specifications are summarised and tabulated in Table 4.5 and Table 4.6. These tables show that both controllers are capable of providing satisfactory performance of the system with small differences between them.

Table 4.5: Summary of simulation performances between P-PI and P-ISMC.

Time Response Specifications	P-PI		P-ISMC	
	No disturbances	With disturbances	No disturbances	With disturbances
Settling time	4.0 sec	4.0 sec	4.0 sec	4.0 sec
Velocity final steady state error	0.00	0.02	0.00	0.03
Position final steady state error	0.00	0.38	0.00	0.44
Velocity max overshoot	0.00	0.00	0.00	0.00
Control effort during acceleration	2.60 volts	2.60 volts	1.50 volts	1.50 volts
Control effort during deceleration	-1.00 volts	-1.00 volts	- 1.50 volts	-1.50 volts
Control effort at steady-state	2.00 volts	2.00 volts	0.00 volts	0.00 volts

Table 4.6: Summary of experimental performances between P-PI and P-ISMC.

Time response Specifications	P-PI		P-ISMC	
	No disturbances	With disturbances	No disturbances	With disturbances
Settling time	4.0 sec	4.0 sec	4.0 sec	4.0 sec
Velocity steady state error	0.02	0.17	0.07	0.07
Position steady state error	1.27	1.72	0.66	1.96
Velocity max overshoot	0.01	0.09	0.00	0.00
Control effort during acceleration	3.60 volts	3.60 volts	+5.00 to -2.00 volts	±4.80 volts
Control effort during deceleration	0.00	0.00	±4.80 volts	±4.80 volts
Control effort at steady-state	2.10 volts	2.10 volts	5.00 volts	5.00 volts

5. Verification and Image Quality Results

The results obtained from the controllers outlined in the previous chapter were used to generate two comparison data sets. One set comprises data required for image blurring index measuring and the second comprises the parameters required for both controllers' performance verification. The data generated are for the distorted images with the following dependent variables: *Final Position Error*, *Settling Time*, *Overshoot*, (see Figure 2.7) and *Image Blurring Index* (see section 2.6 and section 3.5). In terms of the controllers, the P-ISMC and P-ISMC+BD are almost the same type of controller, even though in the case of the P-ISMC+BD a maximum boundary layer width was used compared to the P-ISMC. Therefore, for ANOVA tests the controllers P-ISMC, P-ISMC+BD and P-PI will be compared. In order to determine the performance, these controller-generated data sets are subjected to ANOVA as described in the previous chapter. Ultimately, the results of the ANOVA will be utilized as a basis for comparison for the controllers' performances.

5.1. Distorted Test Images

In order to prepare the distorted images for image quality assessing, four images that were scanned using the Statscan™ X-ray system were provided by Lodox Systems, namely images of the hand, chest, skull and knee. In addition, the 27 distance profiles were obtained based on both controllers' gains tuning parameters as discussed in section 4.1. Out of 27 distance profiles, 18 distance profiles belong to the P-ISMC and the P-ISMC+BD. In terms of control, the P-ISMC and the P-ISMC+BD are almost the same type of controller except that for the P-ISMC+BD a maximum boundary layer width was used compared to the P-ISMC. In order to have nine distance profiles for each controller, the simulation test was run at speeds of 3 mm/s, 6mm/s and 9 mm/s with 25 mm, 50 mm and 100 mm positions. The three positions were repeated for every constant velocity set-point. Then the distorted test images were obtained using the TDI simulator discussed in Chapter 3 by selecting one distance profile and an image at a time. If N_s is the number of selected images from the Statscan™ X-ray machine, N_{dp} is the number of distance profiles required and C_n is the number of controllers under comparison, then the number of generated distorted images N_d is given by Equation (5.1). In this project, the total number of distorted images was 108 and the number of index measure values was also 108. Out of the 108 distorted images, 72 were generated by the P-ISMC and the P-ISMC+BD.

$$N_d = N_s \times N_{dp} \times C_n \quad (5.1)$$

Example 5.1 demonstrates the number of distorted images using the chosen images from the Statscan™ X-ray system and the controllers' distance profiles.

Example 5.1

Determine the number of distorted images of a control system whose four images from the Statscan™ X-ray system are subjected to three controllers (P-PI, P-ISMC and P-ISMC+BD) when nine distance profiles for each controller are selected.

Solution:

$N_s = 4, N_{dp} = 9$ and $C_n = 3$. Then:

$N_d = 3 \times 9 \times 4 = 108$ (Distorted images)

5.1.1. Distorted Images of P-PI Controller

The remaining 36 P-PI's distorted images were obtained by processing the four Statscan™ X-ray machine images and nine distance profiles from the P-PI with the TDI simulator. An explanation of the TDI simulator can be found in Appendix B. Four output distorted images from the TDI simulator out of 36 distorted images are shown in Figure 5.1. The four images on the left side are the original Statscan™ X-ray machine images. The four images on the right side represent the distorted images after using the P-PI controller's distance profile and the TDI simulator. These four distorted images on the right were obtained when the distance profile was chosen to be 100 mm and the speed was 3 mm/s.

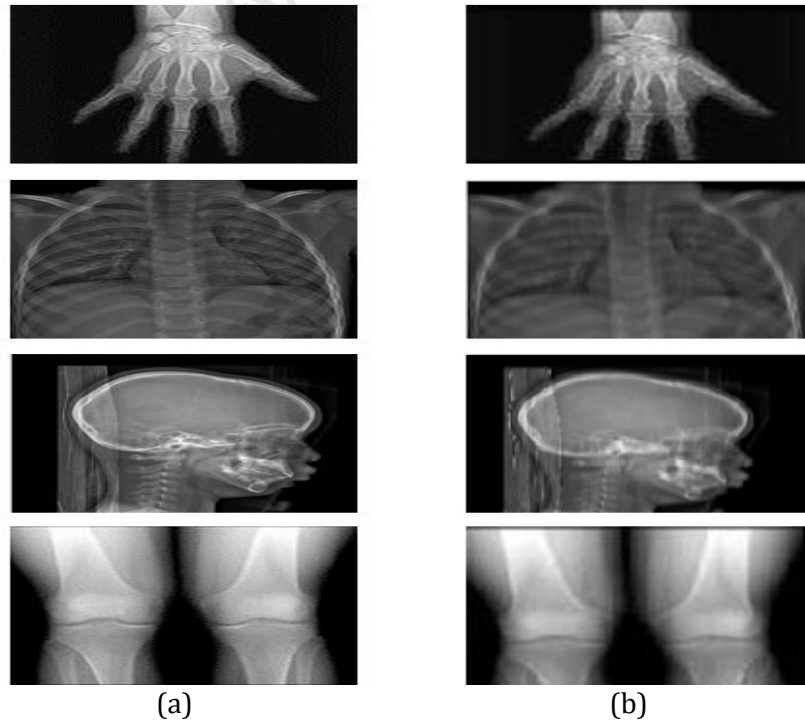


Figure 5.1: Four test images (a) and corresponding P-PI controller distorted images(b) using a distance of 100 mm and velocity of 3 mm/s.

5.1.2 Distorted Images of P-ISMC and P-ISMC+BD Controllers

This section presents the distorted images of P-ISMC and P-ISMC+B.

5.1.2.1. Distorted Images of P-ISMC Controller

In a similar way, the P-ISMC's 36 distorted images were obtained by processing the four Statscan™ X-ray machine images and the nine distance profiles from P-ISMC with the TDI simulator (see Appendix B). The output distorted images from the TDI simulator for four out of 36 distorted images are shown in Figure 5.2. In the same way as in the previous section, the four images on the left side are original Statscan™ X-ray machine images and the four images on the right side represent the distorted images. These four distorted images were obtained when the distance was 100 mm and speed was 3 mm/s.

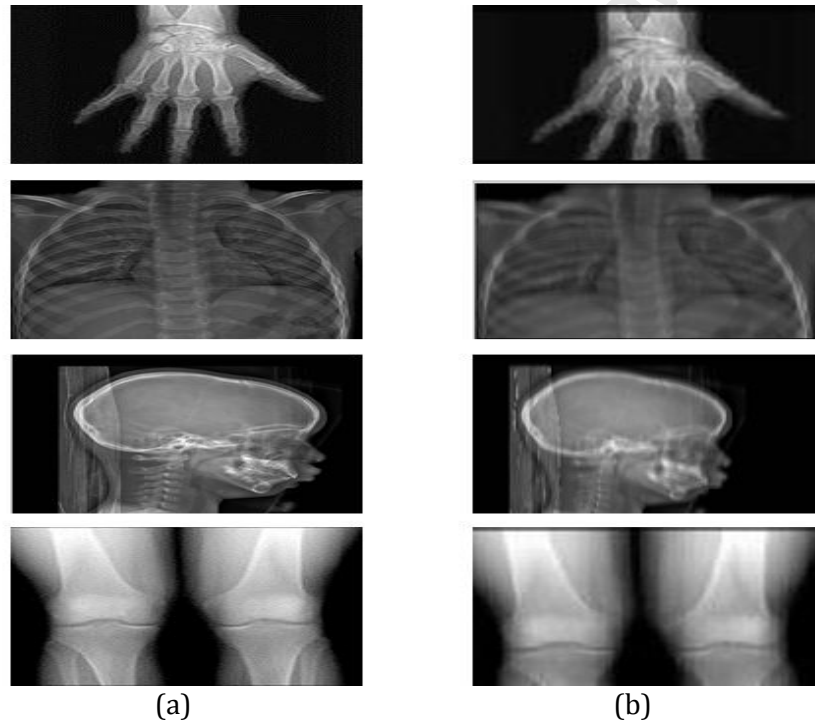


Figure 5.2: Four test images (a) and corresponding P-ISMC controller distorted images (b) using a distance of 100 mm and velocity of 3 mm/s.

5.1.2.2. Distorted Images of P-ISMC+BD Controller

The P-ISMC+BD's 36 distorted images were obtained by processing the four Statscan™ X-ray machine images and nine distance profiles from the P-ISMC+BD controller with the TDI simulator (see Appendix B). The output distorted images from the TDI simulator for four out of 36 distorted images are shown in Figure 5.3. The four images on the left are original Statscan™ X-ray machine images and the four images in the

right represent the distorted images. These four distorted images were obtained when the distance profile was 100 mm and the speed was 3 mm/s.

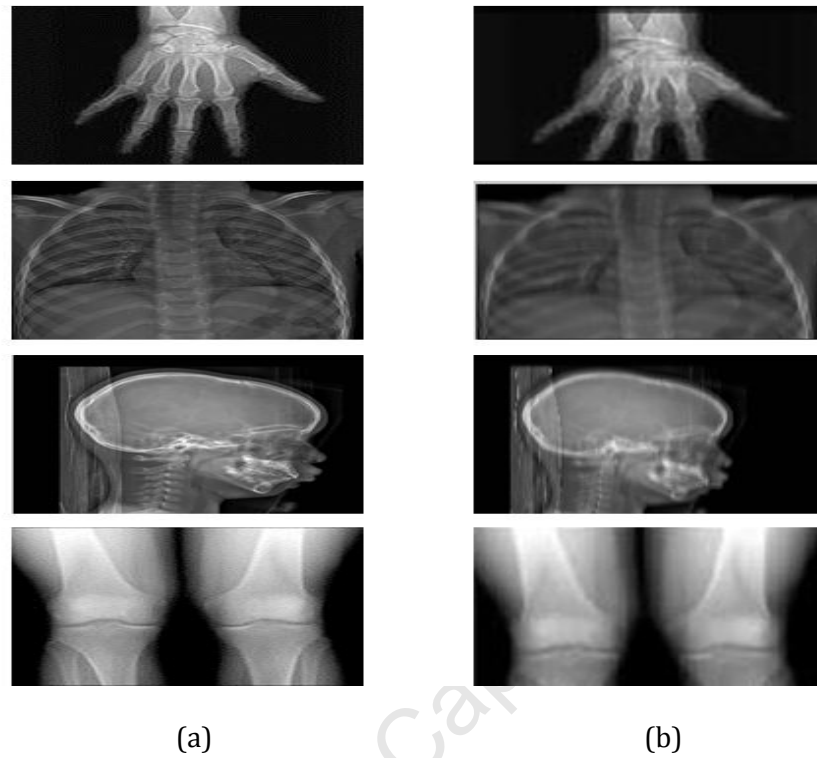


Figure 5.3: Four test images (a) and corresponding P-ISMC+BD controlle distorted images(b)using a distance of 100 mm and velocity of 3 mm/s.

5.2. Linear Model Analysis

This section presents the three-way ANOVA results for the P-PI, P-ISMC and P-ISMC+BD controllers. It further outlines the use of the general linear model (univariate model) performance measures to compare the overshoot of the constant speed settling time response, final position error and image blurring index of distorted images. The overshoot of constant velocity, settling time and final position errors data set is based on simulations of controllers that were tested in Chapter 4. The image quality measure (image blurring index) is based on the output of distorted images from the TDI simulator by using the energy of the Laplacian focus measure [43.] As previously experimentally tested by the author as shown in Figure 1.3, the amount of data lost varied between the half speed and full speed. However, if the difference between speeds in each category is very small, then the amount of data lost for those speeds will be similar. For example, at speeds of 2.8 mm/s, 2.9 mm/s, 3 mm/s, 3.10 mm/s and 3.20 mm/s a similar amount of data is lost. Therefore, all these speeds can be treated as 3 mm/s. Similarly, using the same assumption, the speed categories of 6 mm/s and 9 mm/s were considered and evaluated. At each velocity set-point, the simulation

population size was five. However, for each position set-point, the population size was 15 runs, which means five runs for each velocity set-point. As a result, the population size of three position set-points was 45 runs. Therefore, the total simulation population for all three controllers together was 135 and, as discussed in section 5.1, the population size of the image quality test was 108 simulation runs for the three controllers. These sizes resulted from a random selection of four images for the image quality runs and distance profiles.

As discussed in Chapter 3, ANOVA was used to test and verify each controller's performance on image quality and mentioned parameters, although, as stated earlier in section 5.1, P-ISMC and P-ISMC+BD are similar (except that the P-ISMC+BD controller has a higher boundary layer width ($\delta = 3.77$)). This was done for further analysis and for exploring the boundary layer width's influence on the integral sliding mode control robustness against the parameter variations and disturbances.

In a three-way ANOVA with three factors (with multiple levels) there are the main effects, "2-way interactions" between two of the three factors, and "3-way interactions" between all three factors. It should be noted that the asterisk "*" is the 2-way and 3-way interaction sign that is used in the SPSS software package [SPSS Inc., Chicago, IL, USA], see sections 5.2.1-5.2.4. In the three-way ANOVA analysis the controller factor will be classified in three levels, namely P-PI, P-ISMC and P-ISMC+BD, with the position and the *Speed* factor also in three levels. The dependent variables data for the three-way ANOVA are given in Appendix E (*Overshoot*, *Settling Time* and *Final Position Error*) and the dependent variable *Image Blurring Index* data in Appendix F. These dependent variables data were generated using simulations for three velocities and three positions for each controller. However, the ANOVA methodology makes use of a number of assumptions that have to be checked before the results may be accepted, specifically, the assumptions that the residual values are both normally distributed and homogenous (have equal variance).

a) Test of Normality:

The assumption of normality applies only to the residual values, not to the raw data. However, since the residuals are a function of the raw data, its normality increases the likelihood of normality of the residuals.

Normality can be visually explored by examining a histogram of the data or by looking at the skewness and kurtosis statistics (assumed to equal 3 and 0 respectively for a normally distributed variable)[76]. Formal normality tests may also be used, such as the Kolmogorov-Smirnov and Shapiro-Wilk tests. These tests are supported by the SPSS software package [76]. These formal tests compare the distribution of the variable to a

normally distributed set of data with the same mean and standard deviation. The null hypothesis of these tests is that the data are normally distributed, and hence, this hypothesis is rejected if $p < 0.05$. However, it has been shown by Field [76] that these tests are very dependent on the sample size, and as such, more emphasis is placed on an examination of histograms and a general assessment. Additionally, it should be noted that the F-test (used in ANOVA) is reasonably robust to small amounts of skewness [77].

General descriptive statistics, the results of formal tests of normality and histograms of the raw data with the normal distribution overlaid are shown in Tables 5.1 and 5.2 and Figure 5.4, for the dependent variables: *Overshoot*, *Final Position Error*, *Settling Time* and *Image Blurring Index*.

The formal test results indicate that none of the four dependent variables are normally distributed ($P \leq 0.001$). However, the degree of skewness of *Final Position Error* is small (-0.095), and the mean and median values appear to be quite similar, see Table 5. The use of ANOVA for this dependent variable may thus be acceptable, given the robustness of the F-test [76],[77], although residual analysis will still be required.

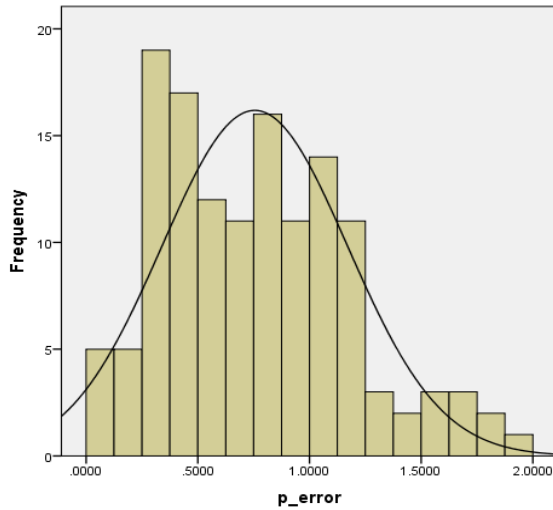
The histograms of *Overshoot* and the *Image Blurring Index* are incredibly skewed however, and that of *Settling* indicates a triple peak (which was determined to be a result of the varying *Speed* factor levels on further investigation). Severe non-normality of the data can be corrected by the removal of outliers or by using one of various transformation methods. The data can be transformed by using log transformation, square root transformation, reciprocal transformation and reverse transformation. In this dissertation, log transformation (log base e) was employed, together with the removal of outliers where required. Specifically, the dependent variables *Overshoot* and *Image Blurring Index* were log transformed, and outliers were removed for both the *Image Blurring Index* and *Settling Time* analyses. In all instances, residual analysis was performed, and the normality (and homogeneity) of the residuals was examined prior to the acceptance of any results.

Table 5.1: Descriptive statistics for raw data for the various dependent variables.

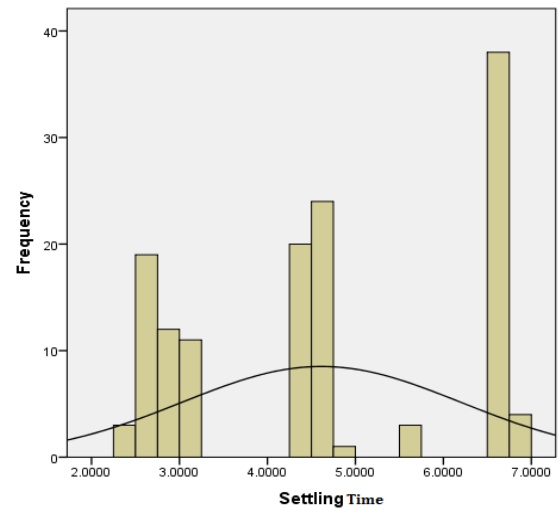
Variable	N	Mean	Median	Std.	Percentile		Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	25 th	75 th	Statisti	Std.	Statisti	Std.
<i>Overshoot</i>	135	0.0736	0.015	0.149	0.002	0.041	2.783	0.209	7.366	0.414
<i>p_error</i>	135	0.755	0.704	0.416	0.407	1.008	0.584	0.209	-0.095	0.414
<i>Settling</i>	135	4.613	4.500	1.582	2.979	6.590	0.117	0.209	-1.438	0.414
<i>Image Blurring Index</i>	108	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	5.614	0.233	45.107	0.461

Table 5.2: Test of normality for various dependent variables.

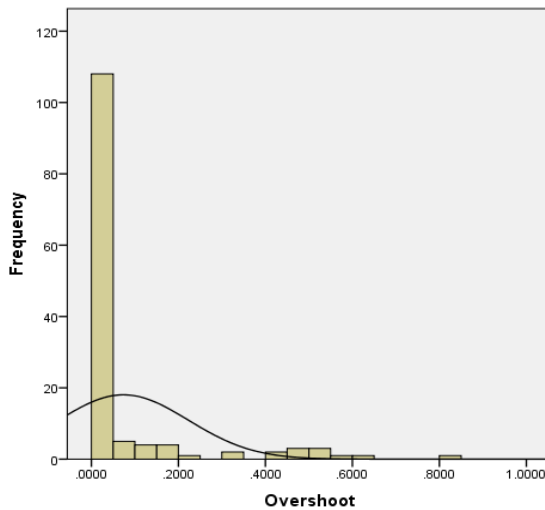
Dependent Variable	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	p	Statistic	df	p
<i>p_error</i>	0.079	135	0.037	0.964	135	0.001
<i>Overshoot</i>	0.385	135	< 0.001	0.0528	135	< 0.001
<i>Settling</i>	0.197	135	< 0.001	0.857	135	< 0.001
<i>Image Blurring Index</i>	0.217	108	< 0.001	0.553	108	< 0.001



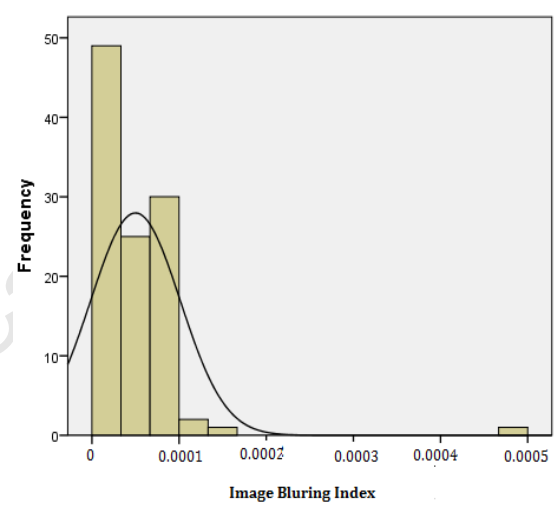
(a)



(b)



(c)



(d)

Figure 5.4: Frequency histograms of the dependent variables: *Final Position Error* (abbreviated to *p_error*) (a), *Settling Time* (b), *Overshoot* (c), and *Image Blurring Index* (d).

b) Test of Homogeneity (Equal Variance Assumption):

In addition to the assumptions of normality and homogeneity for the residuals, the ANOVA methodology also assumes that each data group that is compared comes from a population with equivalent spread. Either the Bartlett's or the Levene's test may be used to assess this assumption. The results of the Levene's test for the raw data of the dependent variables: *Final Position Error*, *Settling time*, *Overshoot* and *Image Blurring Index* are shown in Table5.3. From these results it can be seen that assumption of equal variance is not violated ($p > 0.05$).

Table 5.3: Test of Homogeneity of Variance for the various dependent variables.

		Levene Statistic	df1	df2	p
<i>Overshoot</i>	Based on Mean	1.178	2	132	0.311
	Based on Median	0.319	2	132	0.727
	Based on Median and with adjusted df	0.319	2	118.377	0.727
	Based on trimmed mean	0.849	2	132	0.430
<i>p_error</i>	Based on Mean	2.949	2	132	0.056
	Based on Median	2.847	2	132	0.062
	Based on Median and with adjusted df	2.847	2	125.811	0.062
	Based on trimmed mean	2.959	2	132	0.055
<i>Settling</i>	Based on Mean	0.140	2	132	0.869
	Based on Median	0.223	2	132	0.801
	Based on Median and with adjusted df	0.223	2	131.269	0.801
	Based on trimmed mean	0.139	2	132	0.870
<i>Image Blurring Index</i>	Based on Mean	1.240	2	105	0.293
	Based on Median	.636	2	105	0.531
	Based on Median and with adjusted df	.636	2	45.711	0.534
	Based on trimmed mean	.690	2	105	0.504

The ANOVA test was run for three fixed factors (*Controller*, *Position* and *Speed*) and one dependent variable from *Overshoot*, *Settling Time*, *Final Position Error* and *Image Blurring Index* at a time. However, in addition to the above assumptions, the following points have to be addressed to explore the ANOVA results:

- The hypotheses of three-way ANOVA (see Equation (A.2) in Appendix A) should be tested as to whether or not they are statistically significant for the factor, e.g., *Controller* or a combination of two factors or more, e.g., *Controller* and *Position* or *Controller*, *Speed* and *Position*. A factor or combination of two factors or more is statistically significant when there is a direct influence of the factor or their combination on the dependent variable, e.g., *Overshoot* ($p < 0.05$); otherwise, the three-way ANOVA hypotheses should be rejected and there is no statistically significant difference.
- Because ANOVA compares many means, when statistically significant effects are detected, it is often followed by pair-wise tests, for example, the Tukey's HSD test, applied to the significant direct and interaction effects (see Equation (A.5) in Appendix A). Where there is no significant effect, then applying Tukey's test

following ANOVA is unnecessary. More details on the three-way ANOVA procedure and analysis can be found in the literature [53], [54], [55], [78]. See Appendix A for a further explanation on the ANOVA method.

The factors (*Controller*, *Speed* and *Position*) and above statistical procedures were applied to the dependent variables: *Final Position Error* (section 5.2.1), *Settling Time* (section 5.2.2), *Overshoot* (section 5.2.3) and *Image Blurring Index* (section 5.2.4):

5.2.1. ANOVA Analysis of Final Position Error

Table 5.4 shows the results of a three-way (*Controller*, *Speed*, *Position*) ANOVA that was performed on the dependent variable *Final Position Error*.

Of the main effects, only *Speed* and *Position* were statistically significantly different ($F(2,108) = 54.880$, $p < 0.001$ and $F(2,108) = 6.888$, $p = 0.002$ respectively), indicating that there was no difference in the mean value of *Final Position Error* between the various *Controller* categories.

All 2-way and 3-way interactions were statistically significantly different ($p < 0.001$ for all). This indicates that the various effects of *Controller*, *Speed* and *Position* on the dependent variable were not independent of each other, i.e. that the *Speed* effect was altered by the *Controller* and *Position*, that the *Position* effect was altered by the *Speed* and *Controller*, and that the *Controller* effect was altered by the *Speed* and *Position*.

Table 5.4: Three-way ANOVA interactions for dependent variable *Final Position Error*.

Source	SS	df	MS	F	p
Controller	0.039	2	0.019	0.320	0.727
<i>Speed</i>	6.647	2	3.323	54.880	< 0.001
<i>Position</i>	.834	2	0.417	6.888	0.002
<i>Controller * Speed</i>	2.322	4	0.581	9.587	< 0.001
<i>Controller * Position</i>	1.388	4	0.347	5.729	< 0.001
<i>Speed * Position</i>	1.772	4	0.443	7.313	< 0.001
<i>Controller * Speed * Position</i>	3.646	8	0.456	7.527	< 0.001
Error	6.540	108	0.061		
Total	100.096	135			

Figure 5.5 shows the histogram of the standardized residuals from this analysis. Although slightly peaked, the residuals appear to be acceptably symmetrically distributed. The scatter plot of the standardized residuals against the predicted values from the linear model shown in Figure 5.6 does indicate a slight increasing pattern

however, which infers a decrease in the precision of the results for larger values of *Final Position Error*.

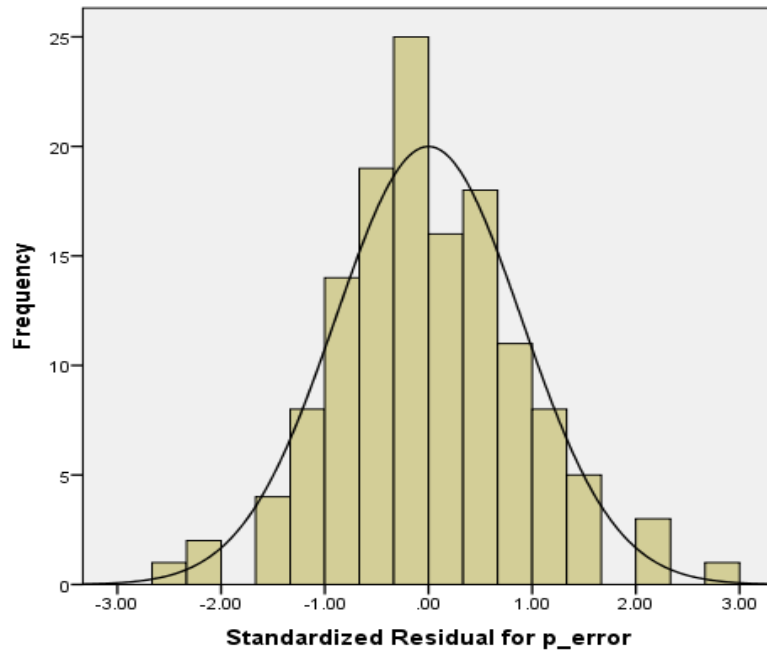


Figure 5.5: Residual Analysis plot for the *Final Position Error*.

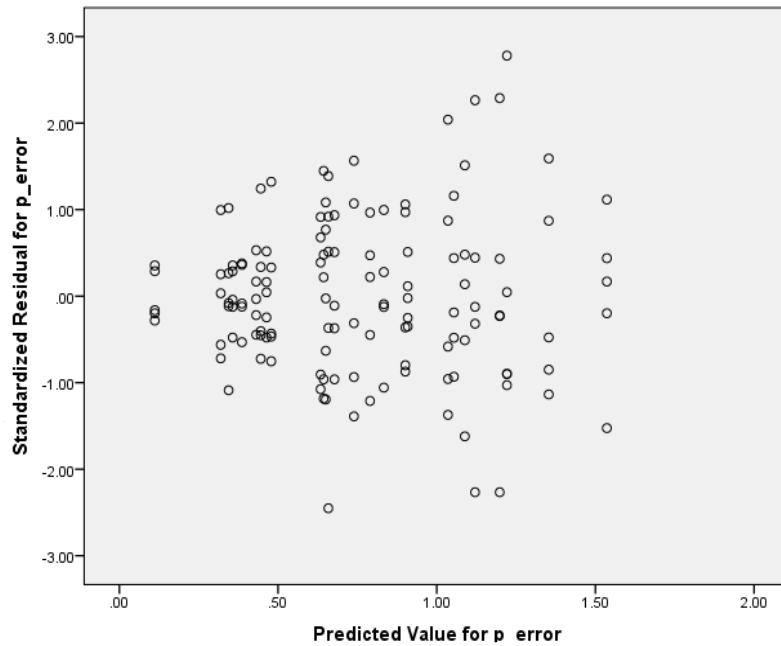


Figure 5.6: Scatter plot for the standardized *Final Position Error*.

Figures 5.7-10 show the *Controller*Speed*, *Controller*Position*, and *Speed*Position* 2-way interaction plots and the *Controller*Speed*Position* 3-way interaction plot for the dependent variable *Final Position Error*. The corresponding pair-wise Tukey tables are in the Appendix A.3-A.6. Direct effects and interactions not involving the Controller factor are not discussed further here, since the focus of this dissertation is to

investigate the effect of the Controller. A summary of results is presented at the end of this section 5.2.1

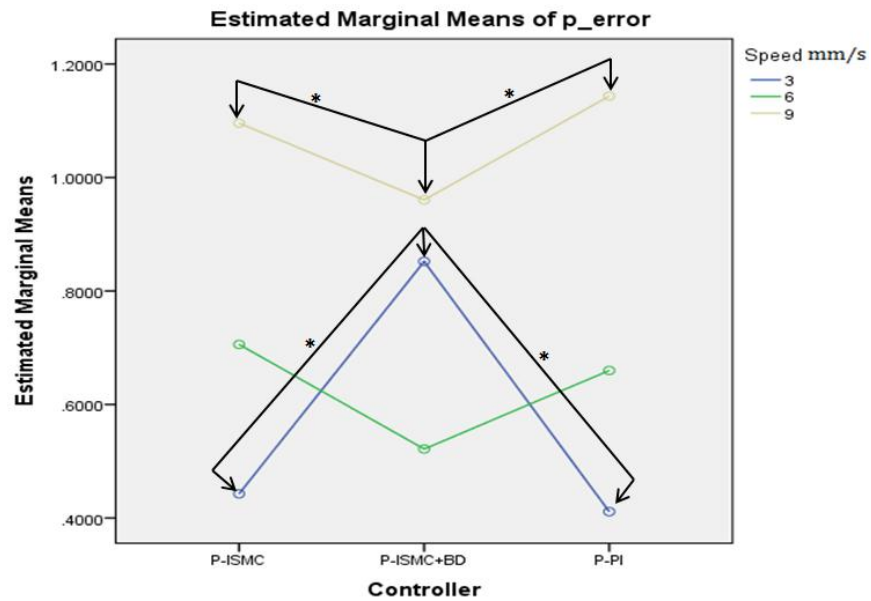


Figure 5.7: Graph of the interaction of *Controller* and *Speed* in the *Final Position Error*, where * = Significant difference at level of 5%; between controllers represented by the solid black line. See Table A3.

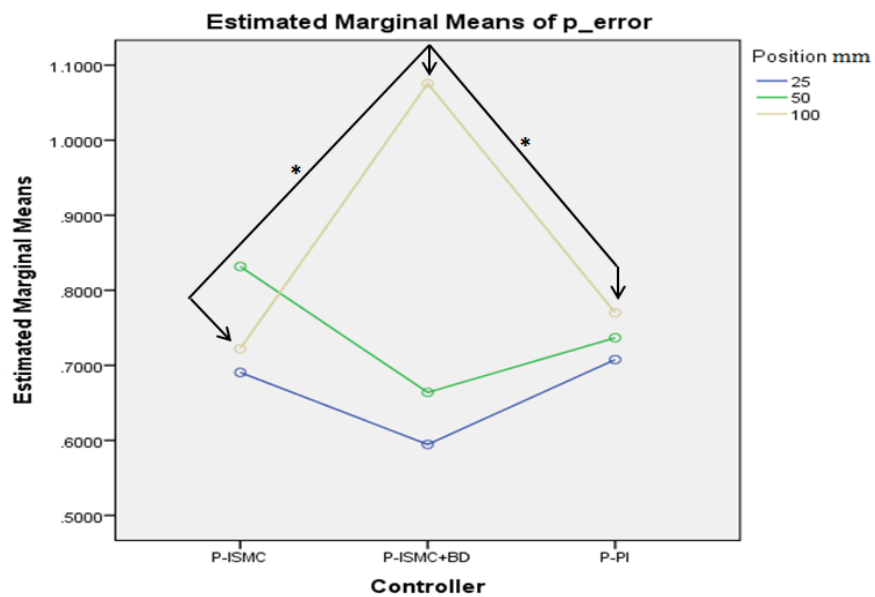


Figure 5.8: Graph of the interaction of *Controller* and *Position* in the *Final Position Error*; where * = Significant difference at level of 5%; between controllers represented by the solid black line. See Table A.4.

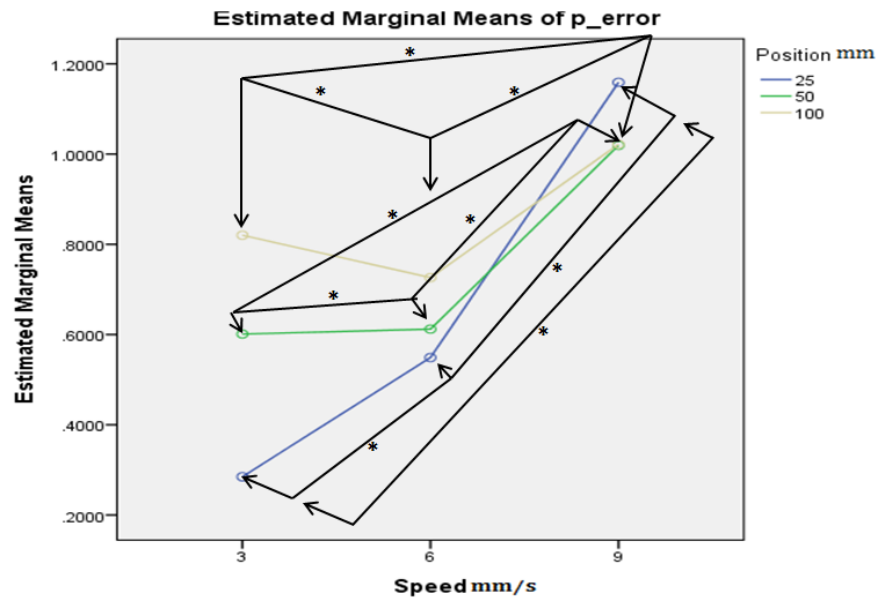


Figure 5.9: Graph of the interaction of *Speed* and *Position* in the *Final Position Error*; where * = Significant difference at level of 5%; between speed is represented by the solid black line. See Table A.5

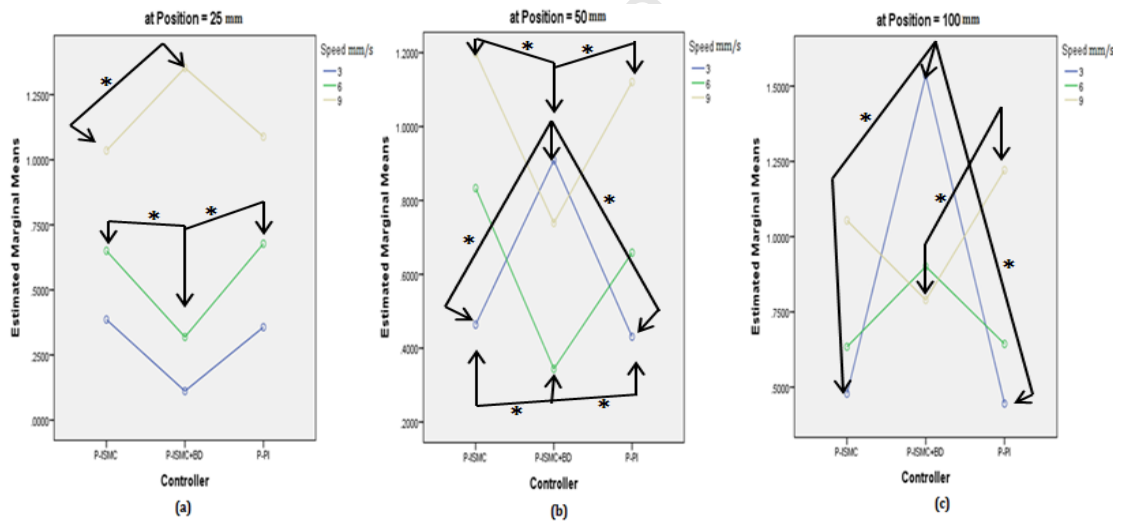


Figure 5.10: *Controller*Speed*Position* interaction estimated marginal means of *Final Position Error* at *Position* = 25 mm (a), *Position* = 50 mm (b) and *Position* = 100 mm (c); where * = Significant difference at level of 5%; between controllers represented by the solid black line. See Table A.6

Table 5.5: A multi comparisons of *Final Position Error* for the *Speed* using Tukey HSD.

(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
3	6	-0.060462	0.0518798	0.476	-0.183752	0.062828
	9	-0.498018*	0.0518798	< 0.001	-.0621308	-.374727
6	3	0.060462	0.0518798	0.476	-0.062828	0.183752
	9	-0.437555*	0.0518798	< 0.001	-0.560845	-0.314265
9	3	0.498018*	0.0518798	< 0.001	0.374727	0.621308
	6	0.437555*	0.0518798	< 0.001	0.314265	0.560845

Table 5.6: A multi comparisons of *Final Position Error* for the *Position* using Tukey HSD.

(I) Position	(J) Position	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
25	50	-0.079871	0.051879	0.277	-0.203161	0.043419
	100	-0.191676*	0.051879	0.001	-0.314966	-0.068386
50	25	0.079871	.0518798	0.277	-.0043419	0.203161
	100	-0.111804	0.051879	0.084	-0.235095	0.011486
100	25	0.191676*	0.051879	0.001	0.068386	0.314966
	50	0.111804	0.051879	0.084	-0.011486	0.235095

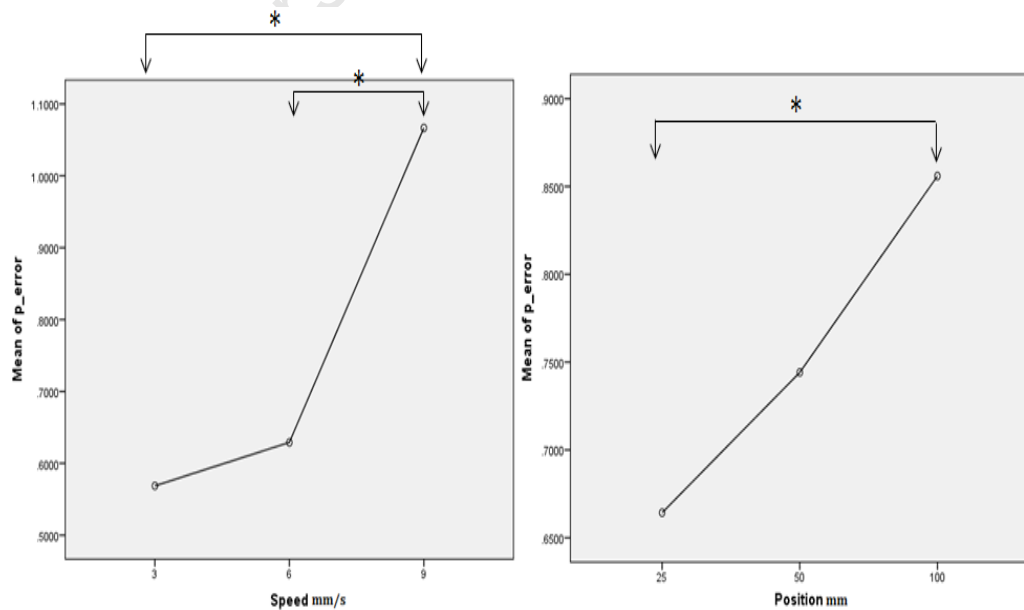


Figure 5.11: The estimated marginal means plots of *p_error* for various levels of the *Speed* and *Position*, where * = Significant difference at level of 5% between *Speed* categories and *Position* categories represented by the solid black line.

Summary 5.2.1 of ANOVA Analysis of Final Position Error (FPE):

1. Only the statistically significant ($p < 0.05$) main effects and interactions that involve the *Controller* factor are summarized here, since they are relevant to the focus of this dissertation.
2. There was no main effect for the *Controller*, however there were interactions for the Controller factor.
3. There was a ***Controller*Speed*** interaction (Table 5.4, Table A.3, Figure 5.7) such that at the highest speed (9mm/s) P_ISMC+BD had the lowest (i.e. best) FPE, but at the lowest speed P_ISMC+BD had the highest (i.e. worst) FPE.
4. There was a ***Controller*Position*** interaction (Table 5.4, Table A.4, Figure 5.8) such that at *Position* = 100mm P_ISMC+BD had the highest (i.e. worst) FPE.
5. There was a ***Controller * Speed*Position*** interaction (Table 5.4, Table A.5, Figure 5.10) such that:
 - a. At *Position* = 25mm: At *Speed* = 6 mm/s P-ISM+BD has the lowest (i.e. best FPE), but is higher (i.e. worse) than P-ISM at *Speed* = 9 mm/s.
 - b. At *Position* = 50mm: At *Speed* = 6 mm/s and 9mm/s, P-ISM+BD has the lowest (i.e. best), but is higher (i.e. worse) at 3mm/s.
 - c. At *Position* = 100mm: At *Speed* = 9mm/s, P-ISM+BD is lower (i.e. better) than P-P, but at 3mm/s it is the highest (i.e. worst).
6. The *Controller* results are quite variable since for some combinations of *Speed* * *Position*, the P-ISM+BD is the best controller, but for others it is the worst controller. This controller generally appears to perform better (wrt FPE) at higher speeds.

5.2.2. ANOVA Analysis of Settling Time

In Figure 5.4.b, section 5.2, the histogram of settling time indicates triple peaks in the distribution of *Settling Time*. Further investigation indicated that each peak was related to a particular *Speed* category (which factor is included in the ANOVA). Table 5.7 shows the results of a three-way (*Controller*, *Speed*, *Position*) ANOVA that was performed on the raw data of the dependent variable *Settling Time*, which yielded significant results for the *Controller* ($F(2,108) = 22.336$, $p < 0.001$) and *Speed* ($F(2,108) = 4875.608$, $p < 0.001$) main effects, and *Controller*Speed* ($F(4,108) = 4.432$, $p = 0.002$) 2-way interaction. The *Position* effect was only marginally significant ($F(2,108) = 2.688$, $p = 0.073$), and the remaining 2- and 3-way interactions were not statistically significant. The lack of significant interactions between *Controller* and *Position* and between *Speed* and *Position*, and finally between *Speed*, *Controller* and *Position*, indicate that the effect of *Controller* on *Settling Time* is not influenced by *Position*.

Table 5.7: Three-way ANOVA interactions for dependent variable *Settling Time*.

Source	SS	df	MS	F	p
<i>Controller</i>	1.506	2	0.753	22.336	< 0.001
<i>Speed</i>	328.762	2	164.381	4875.608	< 0.001
<i>Position</i>	0.181	2	0.091	2.688	0.073
<i>Controller * Speed</i>	0.598	4	0.149	4.432	0.002
<i>Controller * Position</i>	0.070	4	0.018	0.520	0.721
<i>Speed * Position</i>	0.269	4	0.067	1.994	0.101
<i>Controller * Speed * Position</i>	0.129	8	0.016	0.480	0.868
Error	3.641	108	0.034		
Total	335.156	134			

Figure 5.12 shows the residual analysis for the raw data of the *Settling Time*. In this plot, the residuals are not normal, there are outliers present in the data and the data are too peaked. These same outliers are observed in the scatter plot of the Predicted values versus standardized residuals shown in Figure 5.13 and in the scatter plot of Cook's Distance versus standardized residuals shown in Figure 5.14. This latter graph indicates that the outliers are influential observations. These observations (detailed in Table 5.8) were thus removed, and the ANOVA analysis was rerun.

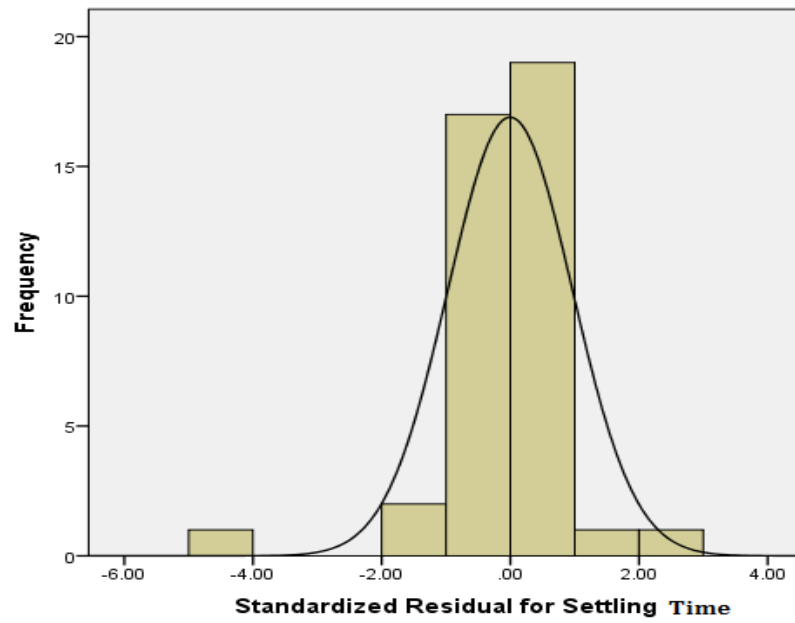


Figure 5.12: Residual Analysis plot for the *Settling Time*.

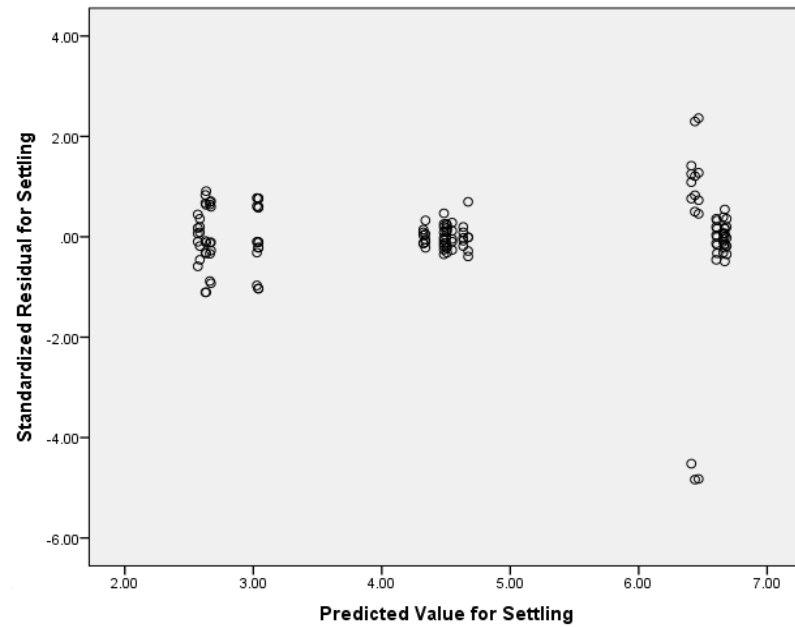


Figure 5.13: Scatter plot for the standardized *Settling Time*.

Table 5.8: Outlier observations at 32, 77 and 82.

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>	<i>Controller</i>	<i>Residual</i>	<i>N</i>	<i>Cooksd</i>
P-PI	9	25	0.2325	0.9632	5.55	P-PI	-4.84	32	0.27
P-ISMC	9	25	0.3199	0.6976	5.58	P-ISMC	-4.83	77	0.27
P-ISMC	9	50	0.001	1.7616	5.58	P-ISMC	-5.05384	82	0.236494

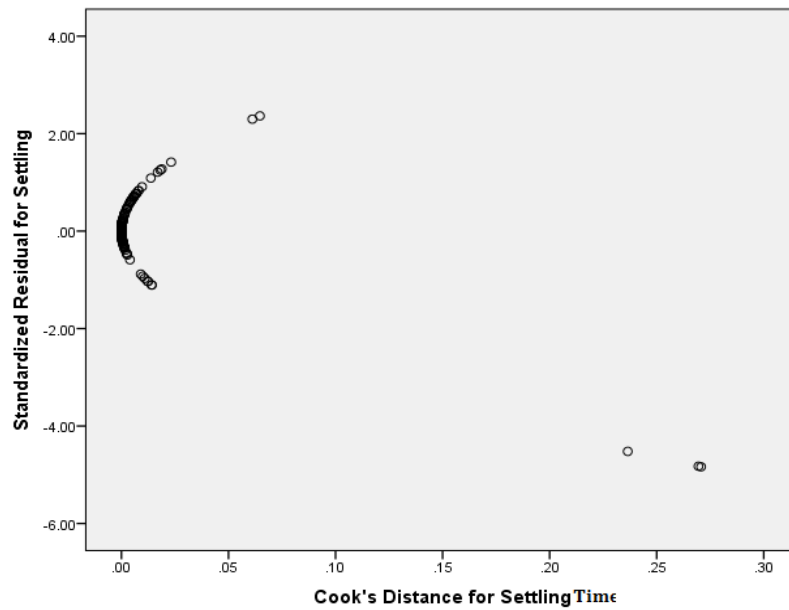


Figure 5.14: Outliers influential observations plot.

The results of the three-way ANOVA that were obtained after the influential observations were removed are shown in Table 5.9. The *Position* main effect and *Speed*Position* 2-way interaction are now significant, although the *Controller*Position* and *Controller*Speed*Position* interactions remain non-significant. Figure 5.15 shows the histogram of the residual values. Although not perfectly symmetric, there are no longer any outliers, and the skewness (as reported in Table 5.10) seems to be acceptable. The scatter plot of the predicted values versus the standardized residuals shown in Figure 5.16 also appears to be somewhat improved, although the middle band does appear to have slightly less variability than those to either side.

Table 5.9: Three-way ANOVA results for *Settling Time* without outliers.

Source	SS	df	MS	F	p
<i>Controller</i>	1.031	2	0.516	66.566	0.001
<i>Speed</i>	327.308	2	163.654	21131.822	0.001
<i>Position</i>	0.051	2	0.025	3.278	0.042
<i>Controller * Speed</i>	0.862	4	0.216	27.835	0.001
<i>Controller * Position</i>	0.017	4	0.004	0.556	0.695
<i>Speed * Position</i>	0.289	4	0.072	9.344	0.000
<i>Controller * Speed * Position</i>	0.023	8	0.003	0.378	0.930
Error	0.813	105	0.008		
Total	3114.998	132			

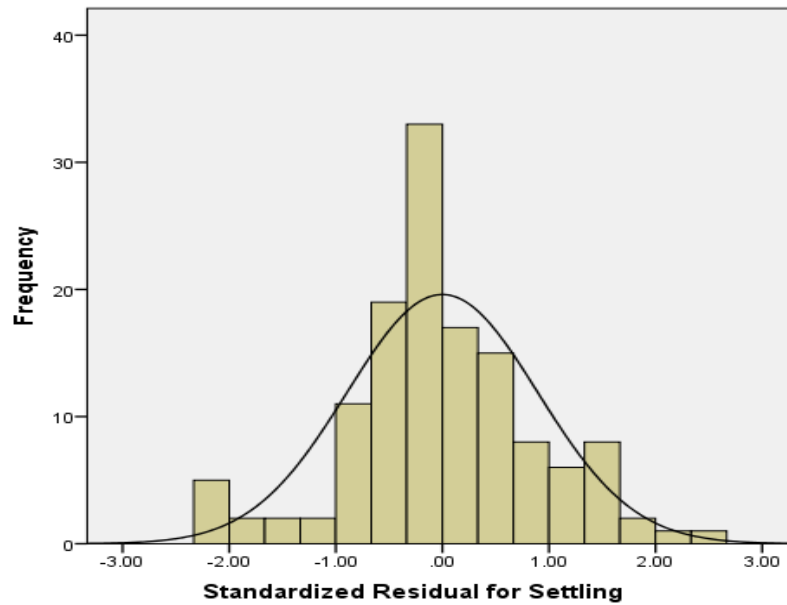


Figure 5.15: Residual Analysis plot for the Settling Time without the outlier.

Table 5.10: Descriptive statistics (Mean, Median, Skewness and Kurtosis for Residuals *Settling Time*)

Variable	N	Mean	Median	Std. Deviation	Skewness		Kurtosis		Percentile	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error	25%	75%
Settling	132	0.000	0.046	0.895	-0.034	0.211	0.717	0.419	-0.455	-0.409

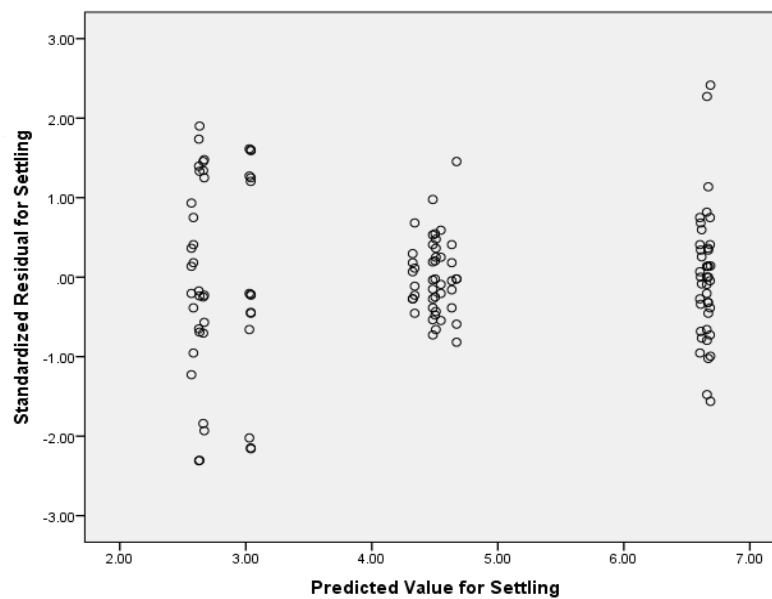


Figure 5.16: Residual scatter analysis plot for the *Settling Time* without outliers.

Figures 5.17 - 5.19 shows the *Controller*Speed*, *Speed*Position*, 2-way interaction plots and the direct effects (*Controller*, *Speed*, *Position*) for the dependent variable *Settling Time*. The corresponding pair-wise Tukey tables are in Tables 5.11- 5.13 and the Appendix A.7-A.8. Direct effects and interactions not involving the Controller factor are not discussed further here, since the focus of this dissertation is to investigate the effect of the Controller. A summary of results is presented at the end of this section 5.2.2

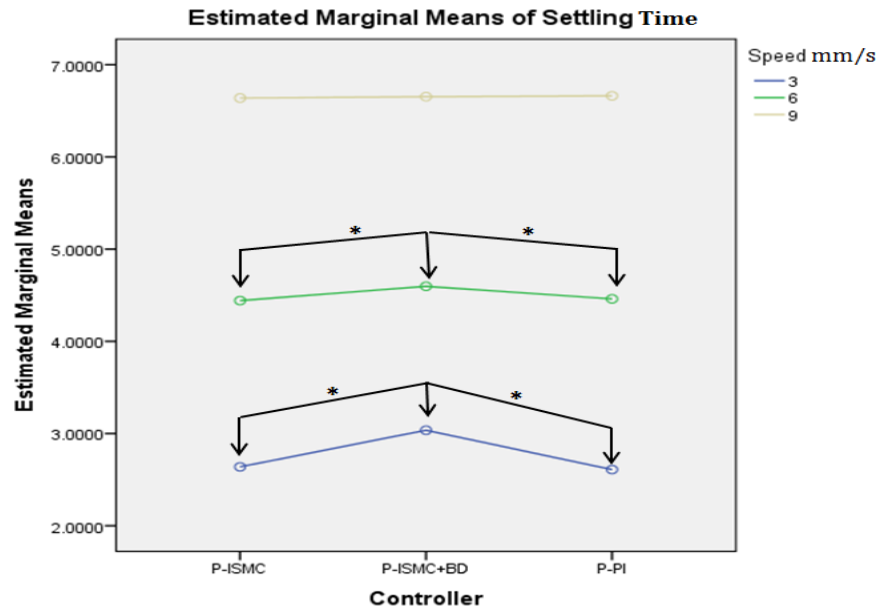


Figure 5.17: Graph of the interaction of *Controller* and *Speed* in the *Settling Time* Significant difference at level of 5%; between controllers is represented by the solid black line,. See Table A.7.

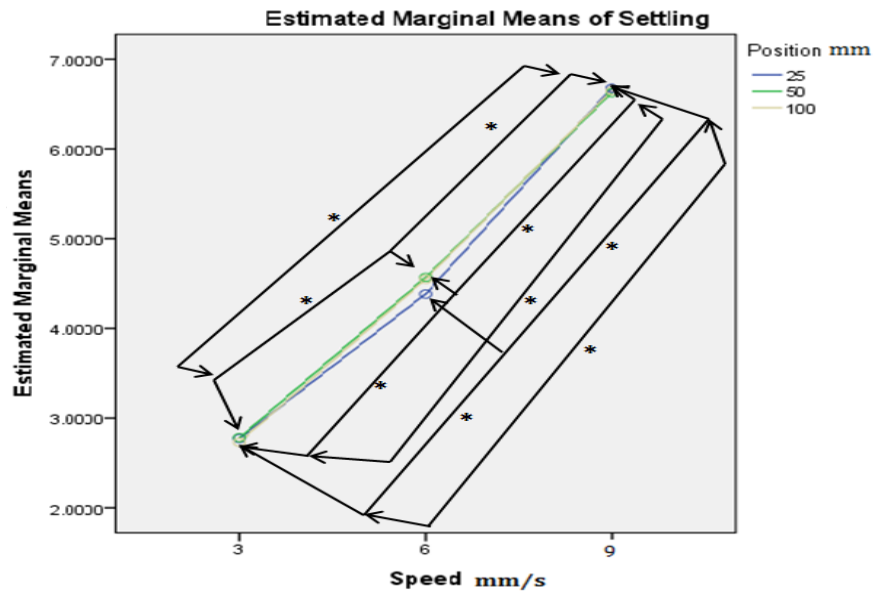


Figure 5.18: Graph of the interaction of *Position* and *Speed* in the *Settling Time*, significant difference at level of 5%; between speed is represented by the solid black line. See Table A.8.

Table 5.11: A multi comparison of *Settling Time* for controllers using Tukey *HSD*.

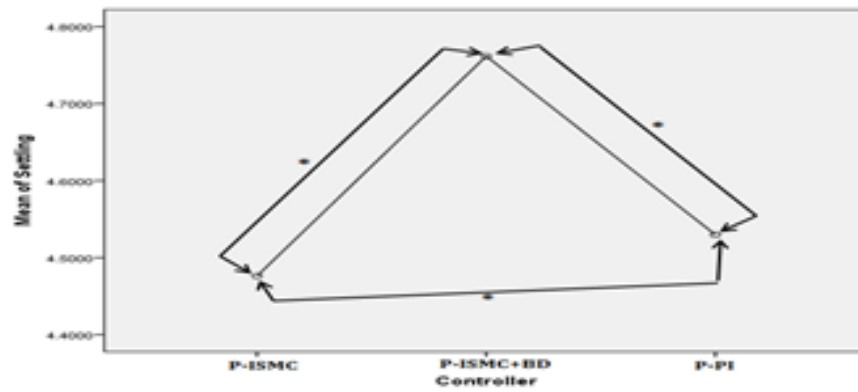
(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
P-ISM C	P-ISM C+BD	-0.285481*	.0018767	< 0.001	-0.330097	-0.240864
	P-PI	-0.053587*	0.018871	0.015	-0.098451	-0.008723
P-ISM C+BD	P-ISM C	0.285481*	0.018767	< 0.001	0.240864	0.330097
	P-PI	0.231894*	0.018658	< 0.001	0.187537	0.276251
P-PI	P-ISM C	0.053587*	0.018871	0.015	0.008723	0.098451
	P-ISM C+BD	-0.231894*	0.018658	< 0.001	-0.276251	-0.187537

Table 5.12: A multi comparison of *Settling Time* for the speeds using Tukey *HSD*.

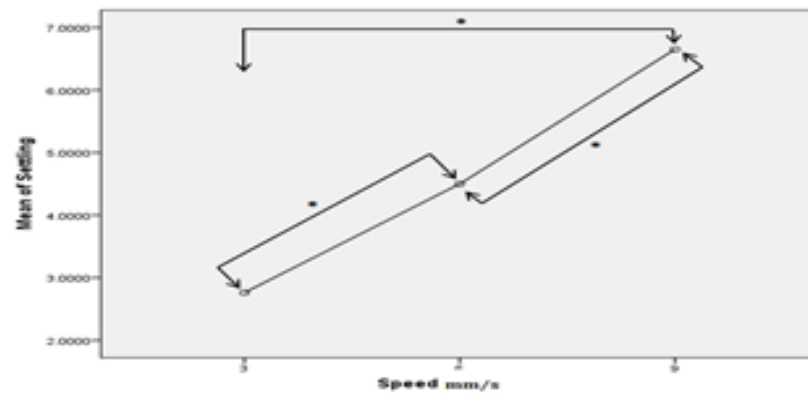
(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
3	6	-1.738244*	0.0185525	< 0.001	-1.782351	-1.694138
	9	-3.889492*	0.0188809	< 0.001	-3.934380	-3.844604
6	3	1.738244*	0.0185525	< 0.001	1.694138	1.782351
	9	-2.151248*	0.0188809	< 0.001	-2.196135	-2.106360
9	3	3.889492*	0.0188809	< 0.001	3.844604	3.934380
	6	2.151248*	0.0188809	< 0.001	2.106360	2.196135

Table 5.13: A multi comparison of *Settling Time* for the positions using Tukey *HSD*.

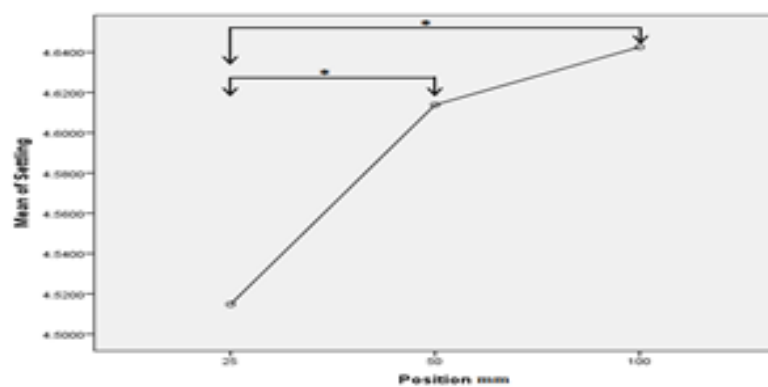
(I) Position	(J) Position	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
25	50	-0.09907*	0.018871	< 0.001	-0.143937	0.054209
	100	-0.127765*	0.018767	< 0.001	-0.172382	-0.083148
50	25	0.099073*	0.018871	< 0.001	0.054209	0.143937
	100	-0.028692	0.018658	0.278	-0.073049	0.015665
100	25	0.127765*	0.018767	< 0.001	0.083148	0.172382
	50	0.028692	0.018658	0.278	-0.015665	0.073049



(a)



(b)



(c)

Figure 5.19: The estimated marginal means plots of *Settling Time* for the main effect of *Controller* (a), *Speed* (b) and *Position*(c). Where * = significant difference at level of 5% is represented by the solid black line, See Tables 5.11 - 5.13.

Summary 5.2.2 of ANOVA Analysis of Settling Time

1. Only the statistically significant ($p < 0.05$) main effects and interactions that involve the *Controller* factor are summarized here, since they are relevant to the focus of this dissertation.
2. There was a direct effect for the *Controller*, such that P-ISMC has the lowest (i.e. best) Settling Time and P-PI is lower (i.e. better) than P-ISMC+BD. (See Figure 5.19)
3. There was a ***Controller*Speed*** interaction (Table A.7, Figure 5.17) such that P-ISMC+BD at Speed= 3mm/s and 6mm had highest (i.e. worst) *Settling Time* and the remaining controllers are not different at low to mid speeds. There is no difference at high speeds (9mm/s).
4. The *Controller*speed* interaction results indicates that P-ISMC and P-PI controllers are generally equivalent and the best controllers for lower Settling times and that P-ISMC+BD is the worst controller, except at high (9mm/s) speeds where all controllers are equivalent.

5.2.3. ANOVA Analysis of Overshoot

Table 5.14 shows the results of a three-way (*Controller*, *Speed*, *Position*) ANOVA that was performed on the raw data of the dependent variable *Overshoot* (no transformation), which yielded non- statistically significant results for the *Controller*Position* ($F(4,108) = 0.744$, $p = 0.564$) and the *Controller*Speed*Position* ($F(8,108) = 1.907$, $p = 0.066$) interactions. The remaining main effects and interactions are statistically significantly different since $p < 0.05$. However, the histogram in Figure 5.20 shows that the *Overshoot* residuals have a distinctly non-normal distribution. In addition, the scatter plot (predicted values versus residual for *Overshoot* shown in Figure 5.21.) indicates that the scatter is increasing rather than random. This means that the ANOVA analysis shown in Table 5.14 is unacceptable. This was corrected by using the log transformation (base e) on the raw data of the dependent variable *Overshoot*.

Table 5.14: Three-way ANOVA interactions for dependent variable *Overshoot*.

Source	SS	df	MS	F	p
<i>Controller</i>	0.084	2	0.042	11.869	<0.001
<i>Speed</i>	0.555	2	0.278	78.176	<0.001
<i>Position</i>	0.795	2	0.397	111.872	<0.001
<i>Controller * Speed</i>	0.45	4	0.011	3.139	0.017
<i>Controller * Position</i>	0.011	4	0.003	0.744	0.564
<i>Speed * Position</i>	1.057	4	0.264	74.382	<0.001
<i>Controller * Speed * Position</i>	0.054	8	0.007	1.907	0.066
Error	0.384	108	0.004		
Total	2.989	134			

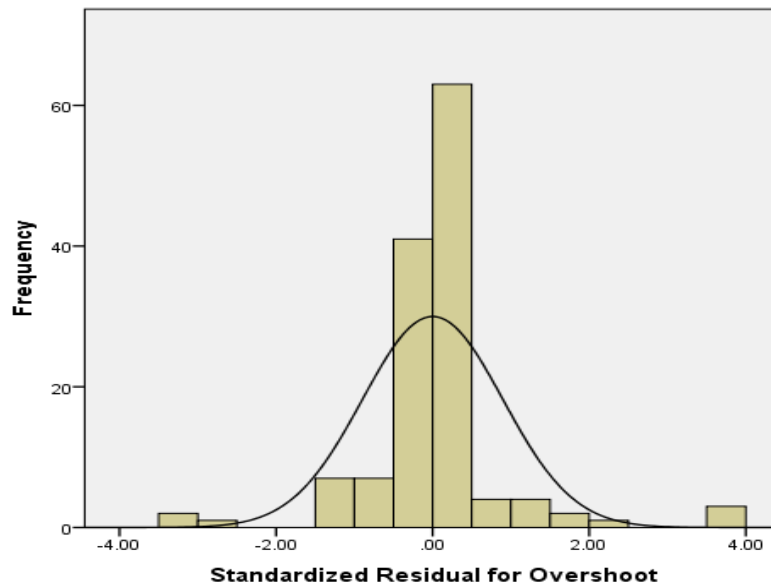


Figure 5.20: Standardized residuals for the *Overshoot* before log transformation.

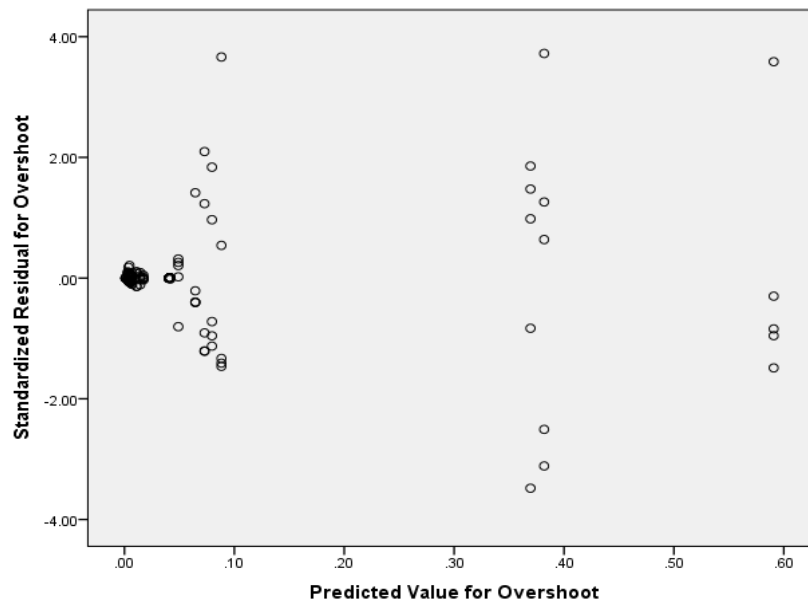


Figure 5.21: Residuals scatter graph before the log transformation.

The histogram and the normality curve results of the transformed data and raw data are shown in Figure 5.22. Figure 5.22(a) presents non-transformation raw data results while Figure 5.22.b presents the transformed data results. It can be seen that the transformed data are still not normal and have several peaks. However, they are no longer skewed and are an improvement on the data without transformation, (Figure 5.22.a). Further, the residuals of the transformed data are no longer skewed although their distribution is overly peaked (not normal based on kurtosis test) see Figure 5.23. The residuals scatter plot (predicated against standardized residual for $\ln(\text{Overshoot})$) shown in Figure 5.24) is much improved. Thus, the log transformation results are more acceptable for the ANOVA analysis.

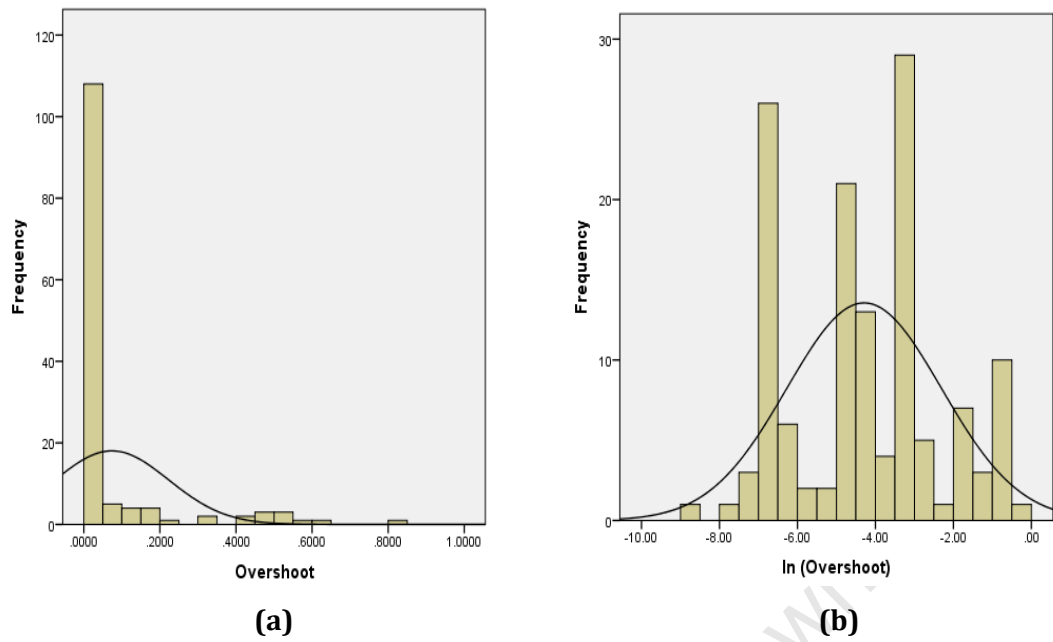


Figure 5.22: Histogram and normality curve for the raw data (a) and transformed data (b) for *Overshoot*.

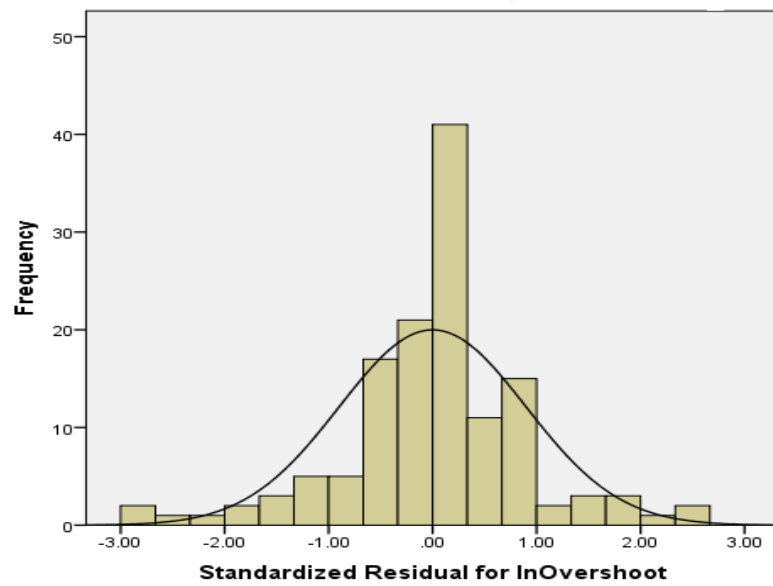


Figure 5.23: Histogram and normality curve for transformed data for $\ln(\text{Overshoot})$.

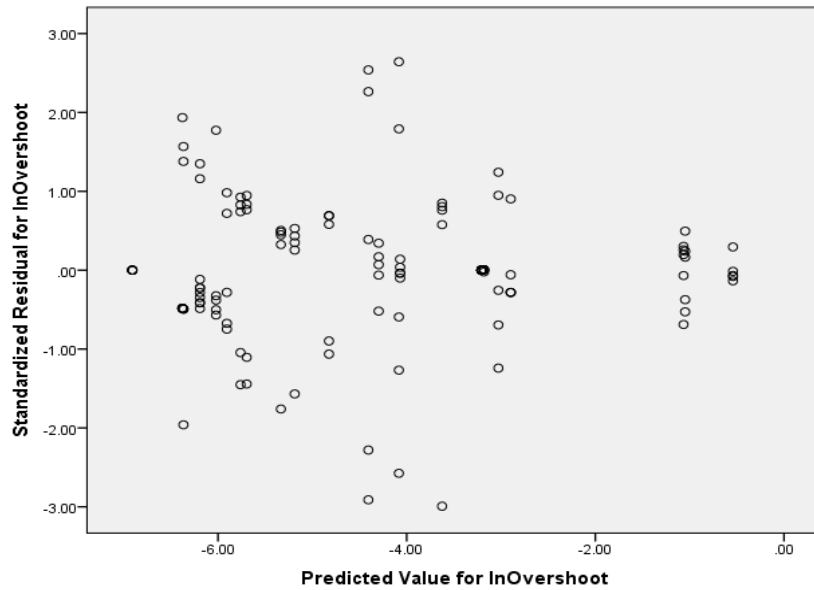


Figure 5.24: Residuals scatter plot for transformed data for $\ln(\text{Overshoot})$.

Table 5.15 shows the ANOVA results for the log transformation data, with all the main effects and interactions seen to be statistically significantly different.

Table 5.15: ANOVA results for dependent Variable $\ln(\text{Overshoot})$.

Source	SS	df	MS	F	p
Controller	110.798	2	55.399	46.018	< 0.001
Speed	26.825	2	13.412	11.141	< 0.001
Position	87.027	2	43.514	36.145	< 0.001
Controller * Speed	24.207	4	6.052	5.027	0.001
Controller * Position	22.463	4	5.616	4.665	0.002
Speed * Position	102.781	4	25.695	21.344	< 0.001
Controller * Speed * Position	24.136	8	3.017	2.506	0.015
Error	130.017	108	1.204		
Total	3012.696	135			

Figures 5.25-28 show the *Controller*Speed*, *Controller*Position*, and *Speed*Position* 2-way interaction plots and the *Controller*Speed*Position* 3-way interaction plot for the dependent variable $\ln(\text{Overshoot})$. The corresponding pair-wise Tukey tables are in Tables 5.16-5.18 and the Appendix Tables A.9-A.12. Direct effects and interactions not involving the Controller factor are not discussed further here, since the focus of this

dissertation is to investigate the effect of the Controller. A summary of results is presented at the end of this section 5.2.3

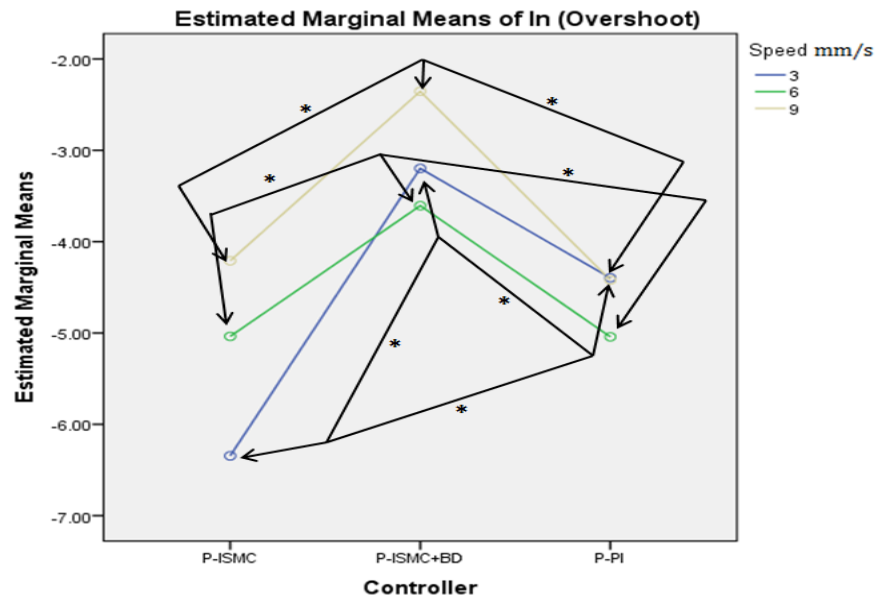


Figure 5.25: Graph of the interaction of *Controller* and *Speed* in the $\ln(\text{Overshoot})$. See Table A.9

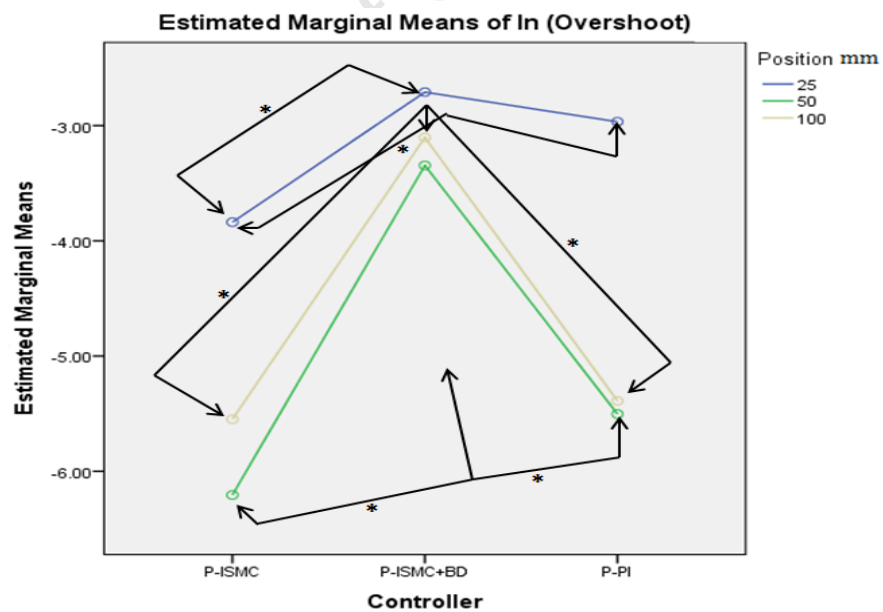


Figure 5.26: Graph of the interaction of *Controller* and *Position* in the $\ln(\text{Overshoot})$. See Table A.10; significant difference at level of 5% between controllers is represented by the solid black line

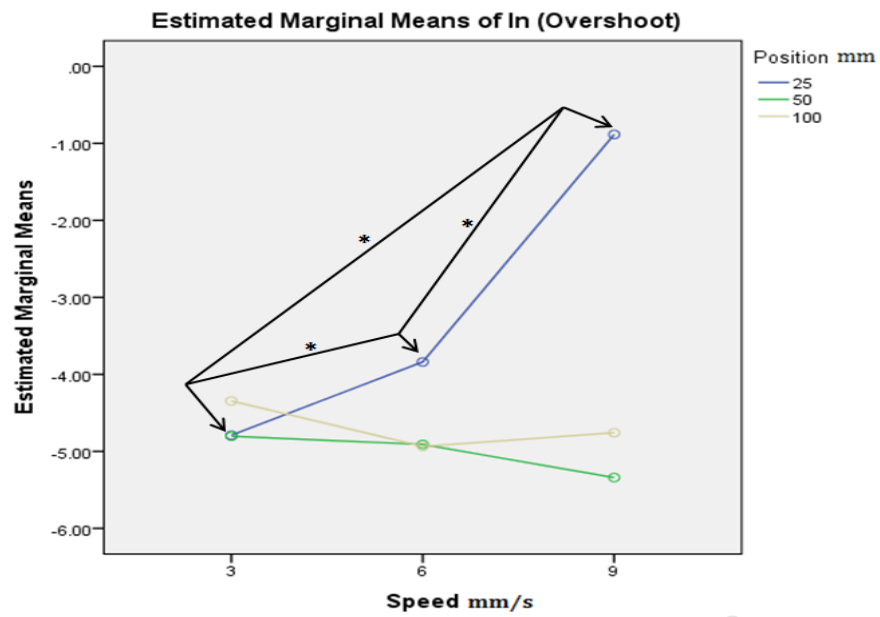


Figure 5.27: Graph of the interaction of Speed and Position in the $\ln(\text{Overshoot})$. See Table A.11, Where significant difference at level of 5%; between speeds is represented by the solid black line.

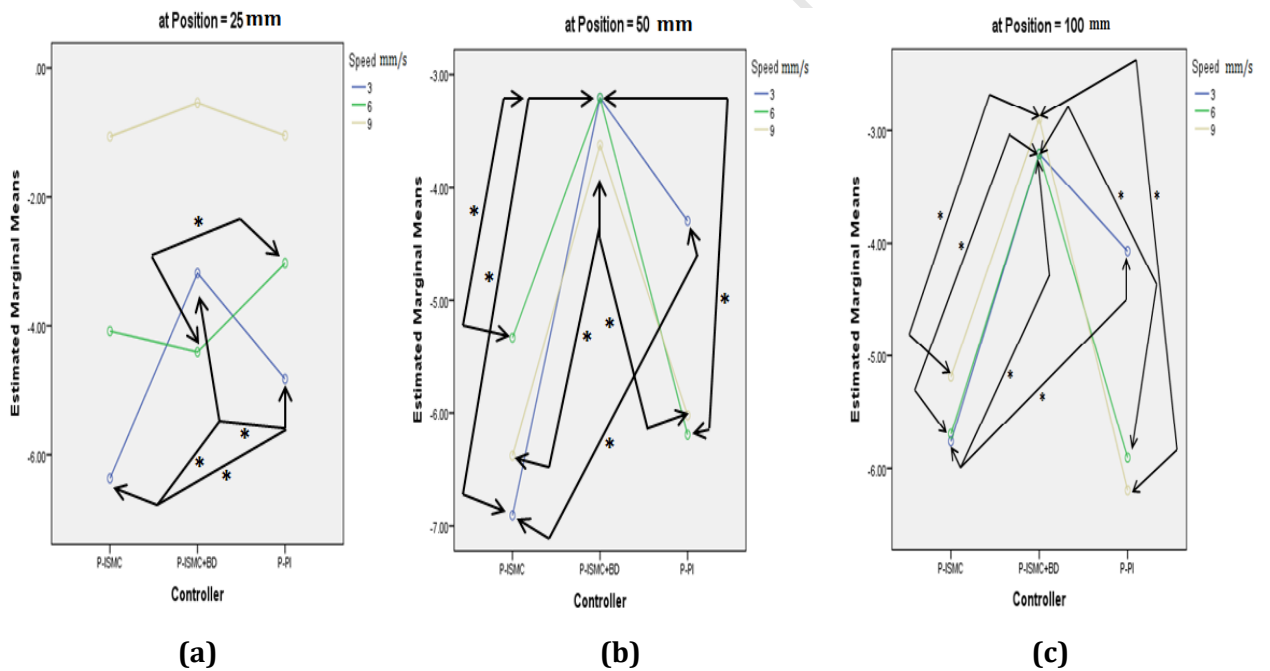


Figure 5.28: *Controller*Speed*Position* interaction of $\ln(\text{Overshoot})$ Position = 25 mm (a), Position = 50 mm (b) and Position = 100 mm (c). See Table A.12. where the significant difference at level of 5%; between controllers is represented by the solid black line.

Table 5.16: Multiple comparisons of $\ln(\text{Overshoot})$ for controllers using Tukey HSD.

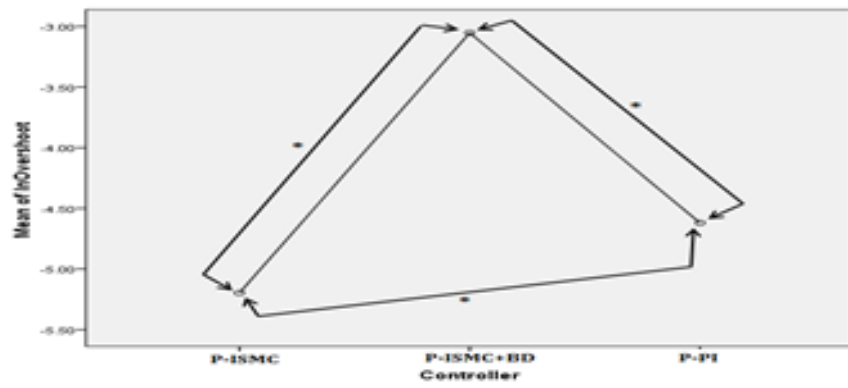
(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
P-ISMIC	P-ISMIC+BD	-2.1443*	.23131	< 0.001	-2.6940	-1.5946
	P-PI	-0.5776*	.23131	0.037	-1.1273	-0.0279
P-ISMIC+BD	P-ISMIC	2.1443*	.23131	< 0.001	1.5946	2.6940
	P-PI	1.5667*	.23131	< 0.001	1.0170	2.1164
P-PI	P-ISMIC	0.5776*	.23131	0.037	0.0279	1.1273
	P-ISMIC+BD	-1.5667*	.23131	< 0.001	-2.1164	-1.0170

Table 5.17: Multi comparisons of $\ln(\text{Overshoot})$ for the speeds using Tukey HSD.

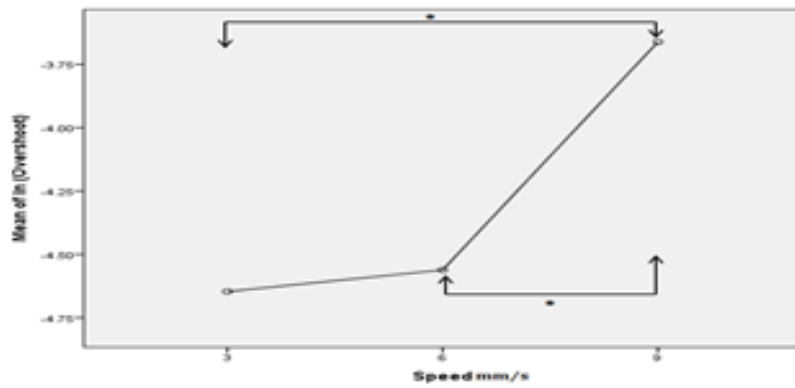
(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
3	6	-0.0850	0.23131	0.928	-0.6347	0.4647
	9	-0.9852*	0.23131	0.001	-1.5349	-0.4355
6	3	0.0850	0.23131	0.928	-0.4647	0.6347
	9	-0.9002*	0.23131	0.001	-1.4499	-0.3505
9	3	0.9852*	0.23131	< 0.001	0.4355	1.5349
	6	0.9002*	0.23131	< 0.001	0.3505	1.4499

Table 5.18: Multi comparisons of $\ln(\text{Overshoot})$ for the positions using Tukey HSD.

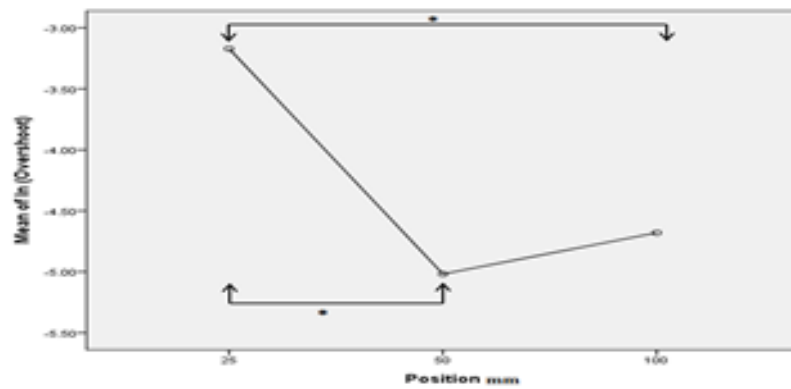
(I) Position	(J) Position	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
25	50	1.8470*	0.23131	< 0.001	1.2973	2.3967
	100	1.5087*	0.23131	< 0.001	0.9590	2.0584
50	25	-1.8470*	0.23131	< 0.001	-2.3967	-1.2973
	100	-0.3383	0.23131	0.313	-0.8880	0.2114
100	25	-1.5087*	0.23131	< 0.001	-2.0584	-0.9590
	50	0.3383	0.23131	0.313	-0.2114	0.8880



(a)



(b)



(c)

Figure 5.29: The estimated marginal means of $\ln(\text{Overshoot})$ at three levels of the *Speed*, *Position* and *Controller* factors, where * = statistically significant Difference at level of 5% is represented by the solid black line. See Tables 5.16-5.18.

Summary 5.2.3 of ANOVA Analysis of Overshoot

1. Only the statistically significant ($p < 0.05$) direct effects and interactions that involve the *Controller* factor are summarized here, since they are relevant to the focus of this dissertation.
2. There was a main effect for the *Controller*, as well as all 2-way interactions involving the *Controller* (*Controller*Speed*, *Controller*Position*) as well as the 3-way *Controller*Speed*Position* interaction.
3. There was a ***Controller*Speed*** interaction (Table A.9, Figure 5.25) such that P-ISM+BD had highest (i.e. worst) Overshoot at all speeds and P-ISM had the lowest (i.e. best) overshoot at low (3mm/s) speeds; but at higher speeds (6 and 9 mm/s) there is no difference between P-ISM and P-PI.
4. There was a ***Controller*Position*** interaction (Table A.10, Figure 5.26) such that P-ISM+BD had the highest (i.e. worst) overshoot at 50 and 100 mm positions and is also the highest at 25mm position but here it is not different to P-PI.
5. There was a ***Controller*Speed*Position*** interaction (Table A.12, Figure 5.28) such that:
 - a. At Position = 25mm: At Speed = 6 mm/s P-ISM+BD has a lower (i.e. better) overshoot than P-PI, but at Speed = 3 mm/s P-ISM+BD has the highest (i.e. worst) overshoot and P-ISM has the lowest (i.e. best). The controllers are not different at high speed (9mm/s) at this position.
 - b. At Position = 50mm: At Speeds = 6 and 9mm/s, P-ISM+BD has the highest (i.e. worst) overshoot, and at 3mm/s P-ISM has the lowest (best) overshoot.
 - c. At Position = 100mm: At Speed = 6 mm/s and 9mm/s, P-ISM+BD has the lowest (i.e. best). At 3mm/s P-ISM is the best.
6. There was a Controller main effect (Tables 5.16-5.18, Figure 5.29) such that P-ISM has the lowest (i.e. best) overshoot and P-ISM+BC is the worst.
7. In summary, in terms of overshoot, P-ISM is the best controller and P-ISM+BD is the worst.

5.2.4. Image Blurring Index

In Figure 5.4.d, section 5.2, the histogram of the *Image Blurring Index* indicates that the data are not normally distributed since the histogram is so skewed. This can be seen clearly in descriptive statistics and normality test results shown in Tables 5.1 and 5.2 respectively for the raw data of the *Image Blurring Index*. Table 5.19 shows the results of a three-way (*Controller*, *Speed*, *Position*) ANOVA that was performed on the raw data of the dependent variable *Image Blurring Index*, which yielded no significant results for either main effects or interactions. Residual analysis indicated a skewed distribution and violation of the ANOVA assumptions see Figure 5.30.

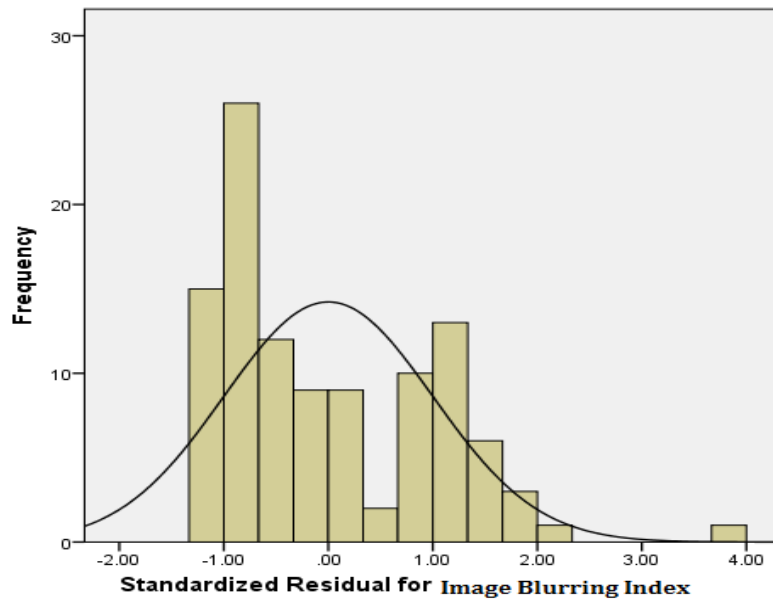


Figure 5.30: Standardized Residual for the *Image Blurring Index*.

A log transformation (base e) was used to correct the non-normality of the raw data of the *Image Blurring Index*.

Table 519: Three-way ANOVA interactions for dependent variable *Image Blurring Index*.

Source	SS	df	MS	F	p
<i>Controller</i>	3.962E-009	2	1.981E-009	0.699	0.500
<i>Speed</i>	6.190E-009	2	3.095E-009	1.092	0.341
<i>Position</i>	1.880E-009	2	9.400E-010	0.332	0.719
<i>Controller * Speed</i>	2.749E-009	4	6.874E-010	0.242	0.913
<i>Controller * Position</i>	5.510E-009	4	1.378E-009	0.486	0.746
<i>Speed * Position</i>	2.027E-008	4	5.069E-009	1.788	0.139
<i>Controller * Speed * Position</i>	1.177E-008	8	1.472E-009	0.519	0.839
Error	2.297E-007	81	2.835E-009		
Total	5.507E-007	108			

Figure 5.31 shows the frequency histogram plot for the log transformed data (base e). It can clearly be seen that the transformed data are still not normally distributed, although they may be more useable than the data without transformation.

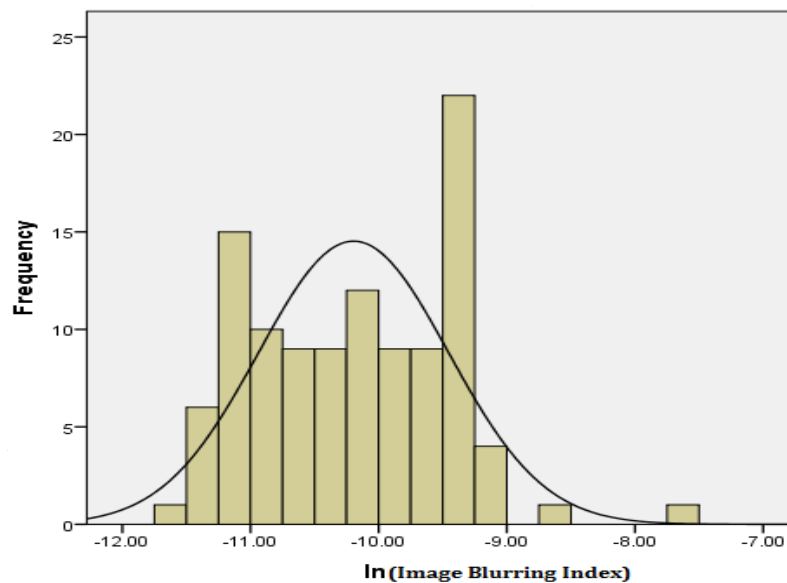


Figure 5.31: Histogram and normal analysis plots for the transformed data.

The below results (Table 5.20) are from the ANOVA performed on the log transformed data. Similarly to before, neither the main effects nor the interaction terms were significant, despite slight variations in the previous p-values.

Table 5.20: Three-way ANOVA analysis for the log transformed data.

Source	SS	df	MS	F	p
<i>controller</i>	2.413	2	1.206	2.027	0.138
<i>Speed</i>	1.392	2	0.696	1.169	0.316
<i>position</i>	0.605	2	0.302	0.508	0.604
<i>controller * Speed</i>	0.256	4	0.064	0.108	0.980
<i>controller * position</i>	0.566	4	0.141	0.238	0.916
<i>Speed * position</i>	3.760	4	0.940	1.580	0.188
<i>controller * Speed * position</i>	1.620	8	0.203	0.340	0.948
Error	48.205	81	0.595		
Total	11286.395	108			

Residual analysis for the transformed data shown in Figure 5.32 indicated the presence of an outlier, although the distribution of the residuals was found to be approximately normal (formally tested using Kolmogorov-Smirnov and Shapiro-Wilk tests of normality in Table 5.21). The same outlier is observed in the scatter plot of the predicted values versus standardized residuals shown in Figure 5.33 and in the scatter plot of the Cook's Distance versus standardized residuals shown in Figure 5.34. In the first graph the scatter is reasonably random with the exception of the outlier. However, the latter graph indicates that the outlier is an influential observation. This observation (detailed in Table 5.22) was therefore removed and the ANOVA analysis was rerun.

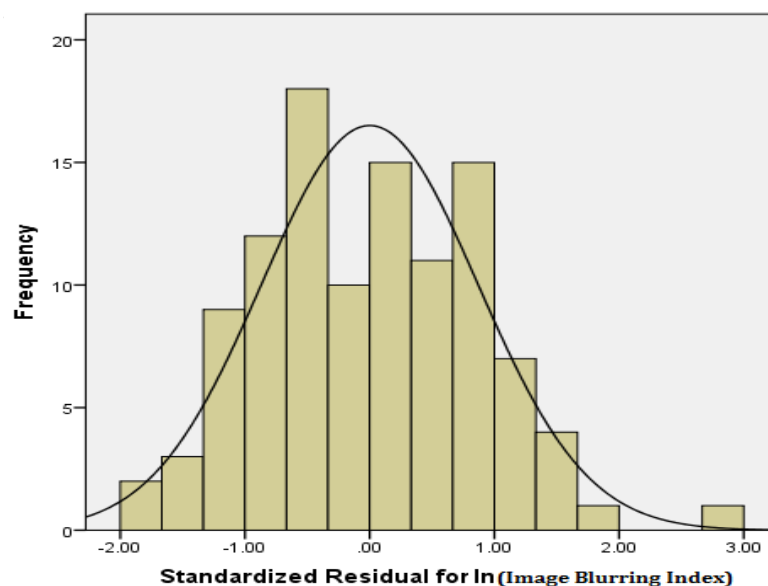


Figure 5.32: Residuals for $\ln(\text{Image Blurring Index})$ after the transformation.

Table 5.21: Residuals tests of normality for the transformed data.

Dependent variable	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	p	Statistic	df	p
Standardized Residual for $\ln(\text{Image Blurring Index})$	0.081	107	0.078	0.977	107	0.065

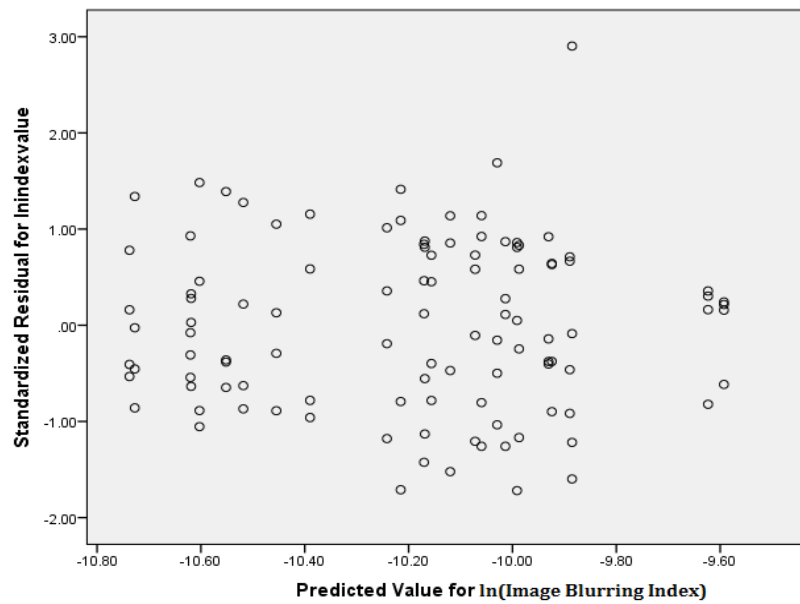


Figure 5.33: Standardized Residuals plot for $\ln(\text{Image Blurring Index})$.

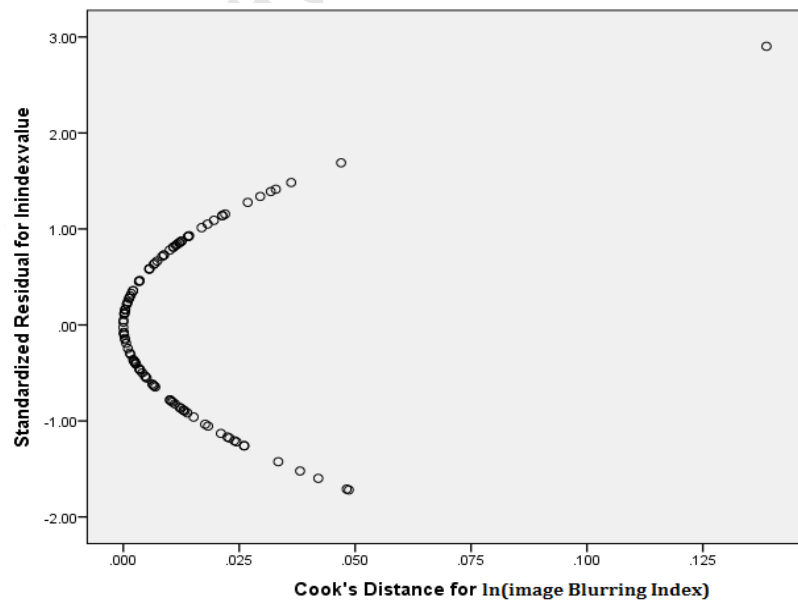


Figure 5.34: Outliers influential observation scatter plots for $\ln(\text{Image Blurring Index})$.

Table 5.22: Outlier observation at 106.

controller	Speed mm/ s	Position mm	image	N	Image Blurring index
P-ISMC+BD	9	100	skull	106	0.000478

The results of the three-way ANOVA that were obtained after the influential observations were removed are shown in Table 5.23.

In comparison to previous results, the main control effect is now marginally significantly different at the 10% level. Multiple comparison tests (Tukey's HSD *post-hoc* test, Table 5.24) are also close to the marginal significant level of 0.10. Therefore these tests are inconclusive here, as it is possible that greater statistical power (i.e. tests with more images and hence more statistical samples) may be necessary in order to track down differences due to controllers on the *Image Blurring Index*. The effect may be very subtle, but it is possible that it may be important to the expert Radiologists.

Table 5.23: Three-way ANOVA analysis for $\ln(\text{Image Blurring Index})$ without outlier.

Source	SS	df	MS	F	p
<i>controller</i>	2.989	2	1.494	2.879	0.062
<i>Speed</i>	1.146	2	0.573	1.104	0.336
<i>position</i>	0.926	2	0.463	0.892	0.414
<i>controller * Speed</i>	0.908	4	0.227	0.437	0.781
<i>controller * position</i>	0.699	4	0.175	0.337	0.853
<i>Speed * position</i>	2.046	4	0.512	0.986	0.420
<i>controller * Speed * position</i>	2.010	8	0.251	0.484	0.864
Error	41.518	80	0.519		
Total	11227.935	107			
Corrected Total	52.253	106			

Table 5.24: Multi comparisons of $\ln$ (*Image Blurring Index*) for controllers using Tuckey HSD.

(I) <i>Controller</i>	(J) <i>Controller</i>	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval	
					Lower Bound	Upper Bound
P-ISMIC	P-ISMIC+BD	0.3555	0.17101	0.101	-0.0529	0.7639
	P-PI	0.3440	0.16980	0.113	-0.0615	0.7495
P-ISMIC+BD	P-ISMIC	-0.3555	0.17101	0.101	-0.7639	0.0529
	P-PI	-0.0115	0.17101	0.997	-0.4199	0.3969
P-PI	P-ISMIC	-0.3440	0.16980	0.113	-0.7495	0.0615
	P-ISMIC+BD	0.0115	0.17101	0.997	-0.3969	0.4199

Summary 5.2.4 of ANOVA Analysis of *Image Blurring Index*

1. Only the statistically significant ($p < 0.05$) direct effects and interactions that involve the *Controller* factor are summarized here, since they are relevant to the focus of this dissertation.
2. There was a marginal main effect for the *Controller* ($p = 0.062$), see Table 5.23.
3. There were multiple comparison results tests (Tukey's HSD *post-hoc* test, Table 5.24) close to the marginal significant level of 0.10 for the P-ISMIC, P-PI and P-ISMIC+BD controllers.
4. Whilst the *ANOVA* results indicate that there is no statistically significant difference between the P-ISMIC, P-PI and P-ISMIC+BD controllers in terms of the *Image Blurring Index* and hence the image quality, the marginal and close to marginal significance values suggest that higher power (i.e. using more images) statistical tests need to be carried out before it can be concluded that there is no difference, since the marginal significance could be due to lower statistical power as a result of too few samples.

6. Discussion and Conclusions

In this chapter, the analysis and discussion of computer simulations and experimental results of the controller presented in Chapter 4 and the statistical results presented in Chapter 5 are discussed. This discussion first touches on the control perspective view and secondly, on the results obtained from the ANOVA statistical method that were carried out for *Final Position Error*, constant speed *Settling Time*, constant speed *Overshoot*, and *Image Blurring Index*, as presented in Chapter 5.

6.1. Performances Study between P-PI and P-ISMC Controllers

From the results presented in Chapter 4, it may be concluded that both P-PI and P-ISMC controllers provide comparable performance for the physical system (Figure 3.12). The three cases under which the simulation tests were carried out were step set-point (for PI and ISMC), tracking trapezoidal motion profile and robustness tests against controller input and system output disturbances. However, experimental tests were carried out only trapezoidal motion profile tracking and robustness tests against controller input and system output disturbances rejection.

From Chapter 4, comparisons between the simulation and experimental results with PI, ISMC, P-PI and P-ISMC controllers are discussed.

6.1.1. Step Time Responses

The time responses for the system in Figure 3.12 with the PI controller are shown in Figure 4.1 and those with ISMC are shown in Figure 4.3 and Appendix D. From these figures the step tracking response shows that differences in control performances obtained by PI and ISMC may be varying. Compared to the PI controller, the step tracking error performance of the ISMC looks better. On the other hand, compared to the ISMC controller, the settling time response of the PI controller is lower. Figures 4.1b and 4.4b show the time responses during the control input and system output disturbances, respectively. From these figures, it is clear that the ISMC controller's velocity response is robust against the disturbances rejection, whereas the PI controller is affected by the disturbances.

6.1.2. Tracking of Velocity and Position in Trapezoidal Motion Profile

In section 4.4, Table 4.5 and Table 4.6 show the summary of the simulation time response performances and the experimental time performances between the P-ISMC and P-PI controllers quantitatively. The first observation which is seen in the data tabulated in Table 4.5 is the similarity in the performance between the P-ISMC controller and P-PI controller for the dependent variables *Settling Time*, *Position Final Error* and velocity maximum *Overshoot* for cases with and without disturbances. This observation is based on the controllers' comparative performance measures that are presented in section 2.5. For instance, these results meet the comparative performance measures to be satisfied where the final position error for both simulation and experiment is less than 2 mm and the speed's maximum overshoot is less than 0.2 mm/sec, and the settling time to reach the constant speed is 4 seconds.

In term of control effort, both controllers produced low control action in order to track the trapezoidal motion profile. However, there were variations in the control effort for the P-PI and P-ISMC during the acceleration and deceleration phases. For the velocity steady state error, P-PI gave a slightly better performance than P-ISMC for the case without disturbances, but the opposite results was obtained for the case with disturbances. From these results, P-ISMC appears to be robust against the disturbances.

In terms of final position steady state error, in simulation both controllers had the same performance. However, in practice P-PI performed slightly better than P-ISMC for the cases with and without disturbances. The better performance of the P-PI arises from the lower tracking error achieved during the acceleration and deceleration phases (see Figures 4.7 and 4.8). Conversely, chattering is observed during the acceleration and deceleration which had a large influence on the tracking performance of P-ISMC during these acceleration and deceleration phases. The main reason is because the design of the discrete P-ISMC that was used in the experiment was based on continuous time. In other words, the continuous time P-ISMC controller has been approximated to a discrete form by means of the data acquisition card (DAC/ADC) and the sampling time. Although a sampling time of 0.109 seconds is reasonable in the experimental setup, the approximated discrete controller may induce some chattering if the sampling time is chosen to be too small.

Regarding the speed overshoot, in simulations both controllers had zero overshoot. Experimentally, by adjusting the parameter values of the P-ISMC, a zero overshoot can be obtained in both cases of disturbances. However, the experimental P-PI controller's velocity overshoot performance varied for the cases with disturbances and without

disturbances. In summary, although there were slight differences between the P-PI and P-ISMC in simulation and experimental results, both controllers' performance was still within the controllers' performance measures that are presented in section 2.5.

6.2. Analysis and Discussion of ANOVA Results

In the previous chapter (i.e. Chapter 5), four dependent variables (*Final Position Error*, *Settling Time*, *Overshoot* and *Image Blurring Index*) were tested by four separate three-way ANOVAs (with Factors or Independent Variables: Controller, Speed, Position). In this section the discussion only the statistically significant ($p < 0.05$) main effects and interactions that involve the *Controller* factor are discussed here, since they are relevant to the focus of this dissertation and the following are the discussion for the results obtained in Chapter 5.

6.2.1. Final Position Error

The tracking steady state error is an important criterion for determining the controller's performance with respect to position. A good controller will achieve minimum *Final Position Error*, and cause the system to go as precisely as possible to the required reference position [79].

The three-way ANOVA test in section 5.2.1 revealed that there was no significant difference at the *Controller* main effect ($p > 0.05$). However, *Controller*Speed*, *Controller*Position* and *Controller*Speed*Position* interactions were found to be significantly different for the *Controller* factor ($p < 0.05$), see Table 5.4. So therefore the Controllers were different but only at certain positions, and *Speed*Position* combinations.

Referring to Summary 5.2.1. it is not clear which Controller is the best performer with respect to FPE, although it seems that the P-ISMC+BD Controller performs better at higher speeds. However if you consider within-Controller (note that the within-Controller significant differences have not been indicated in Figure 5.7 but are listed in Table A.3) results in Figure 5.7, then it is apparent that there is no systematic increase or decrease of FPE in this Controller with respect to Speed since FPE decreases from 3mm/s to 6mm/s, but then increases to its maximum value at 9mm/s. However the P-ISMC and P-PI controllers show a systematic increase in FPE as the Speed increases. The non-systematic performance of the P-ISMC+BD must be due to the effect of the boundary layer, so even though it is statistically better at 9 mm/s in these experiments, this controller must be considered unsuitable for good FPE performance

6.2.2. Settling Time

In Control Engineering, Settling Time is an important performance measure. It tells us how fast a system responds to a given command – within an acceptable measure of precision – to become settled [80].

The three-way ANOVA test in section 5.2.2 revealed that there is a significant difference for all main effects (*Controller*, *Speed*, *Position*) as well as the *Controller*Speed* and the *Speed*Position* interaction but were no significant differences for the *Controller*Position* and the *Controller*Speed*Position* interactions(See Table 5.9).

Referring to Summary 5.2.2 even though the direct effect (Figure 5.19a) shows the P-ISMC controller to be the best, when this is studied in more detail in the speed interaction (Figure 5.17) the results indicate that both P-ISMC and P-PI controllers are equivalent at lower (3 and 6 mm/s) speeds (Figure 5.17) but that there is no difference between the controllers at high (9mm/s) speeds. Therefore despite the main effect results, the conclusion is that the P-ISMC and P-PI controllers are equivalent. All effects indicate that P-ISMC+BD is the worst controller here (except at high speed 9mm/s).

6.2.3. Overshoot Size

Overshoot is an important measure that indicates the controller's performance with respect to the system's transient response. Minimizing the size of overshoot can improve the transient response by making the Settling Time shorter [81].

The three-way ANOVA test in section 5.2.1 revealed that there was significant different at the *Controller* main effects ($p > 0.05$). in addition, the *Controller*Speed*, *Controller*Position* and *Controller*speed*Position* interactions were found to be significant different for the *Controller* factor ($p < 0.05$) See Table 5.15).

Referring to Summary 5.2.3 Therefore, in terms of *Overshoot*, P-ISMC is generally the best controller and P-ISMC+BD is generally the worst, however at different *Speed* and *Position* interactions the relative performances may vary.

6.2.4. Image Blurring Index

The image blurring index value is an important criterion in determining the image quality. An image that is well focused has a higher image blurring index value [47], [48], [49], [50]. The three-way ANOVA and Tukey's *post-hoc* test in section 5.2.4 revealed that there are no significant differences between the P-PI, P-ISMC and P-ISMC+BD controllers in terms of mean level of *Image Blurring Index* and hence the image quality, as summarized in Tables 5.23 and 5.24, however the main control effects were

marginal significantly different at the 10% level, see Table 5.23. In addition , the multiple comparison results tests (Tukey's HSD *post-hoc* test, Table 5.24) was close to the marginal significant level of 0.10 for the P-ISMC, P-PI and P-ISMC+BD controllers. This result suggests that in order to increase the statistical power and hence obtain better statistically significant result the addition of more images may be needed. Because, a larger number of images may clarify if this result was real or simply due to low statistical power. Furthermore, expert Radiologist opinion on the differences between the images would also provide more insight, and should be consulted before a conclusion can be reached. Therefore the interim conclusion is that the Controller does not appear to make a difference in image quality, however this requires further investigation.

7. Summary Conclusions and Recommendations

7.1. Summary Conclusions

It was concluded that compared to each other on control perspective (FPE, *Settling Time*, and *Overshoot*) the P-ISMC was found to be best controller in terms of *Overshoot* and both P-ISMC and P-PI were best controllers in terms of FPE and Settling Time. Generally P-ISMC+BD controller appears to be the worst controller except in certain interaction instances. However when it comes to image quality improvement, the controllers have very similar performances as measured by the *image blurring index*~~~~~. This may be because a few numbers of images have been selected for the Image Blurring index test. Therefore, additional images should be added for future analyses and simulations.

Both controllers' robustness and disturbance rejection comparison were also carried out using control input and system output disturbances. This was done by simulation and experiment. Both controllers provide good system output disturbance rejection. However, in terms of control input disturbance rejection, P-ISMC was better than P-PI. Using the boundary layer width could greatly improve the control system response by the P-PI but the robustness against parameter variations and disturbance rejection of the controller depends on the chosen boundary layer width.

Finally, the accurate tracking of the trapezoidal motion profile was not investigated in this dissertation. It could be achieved, for example, with type 2 controllers or the pole-zero cancelation technique if necessary [79]. Also, other motion profiles were also not tested for better tracking improvement in this dissertation. This could be done, for example, by using the S-curve motion profile [6].

7.2. Future Work and Recommendations

This section recommends some future work. The recommendations are based on the findings of this dissertation, but were not within its the scope.

- 7.2.1 A DC motor was used to test both controllers for point-to-point motion control applications. For a more accurate comparison, a model of the Statscan™ X-ray machine is needed, as long as the velocity in radian/seconds and the position in radians are linearized to mm/sec and mm, respectively, to match linear positioning specifications. The model will provide more realistic dynamic information about the Statscan™ X-ray system without the need to modify the code and use the time delay integrator simulator code.
- 7.2.2 The P-ISMC controller design in the dissertation did not improve the image quality as expected. Alternatively, we recommend the model reference discrete-time sliding mode control (MRDSMC) and a novel sliding mode control for positioning application to be investigated, finding out whether or not they will improve the vibration and position error as indicated in the literature (see Chapter2) or more images should be added for testing *Image Blurring Index*.
- 7.2.3 Data for ANOVA tests (see Appendix A and B) were generated using a root locus tuning process, sliding mode control tuning guidelines and TDI simulator code. However, for a proper comparison, an optimization tuning method should be used instead. Further, the optimization method can be extended to design the optimal controller instead of what has been designed. Moreover, the images for image quality tests can be taken directly from the physical system instead of from the TDI simulator. This will enable the designer to consider the amount of vibration produced at various speeds.
- 7.2.4 Vibration can cause the detector to move relative to the X-ray source, although both units are mounted on a rigid C-arm. This will cause horizontal banding in the image (see Fig. 1.2(a)). In order to overcome this problem, the current P-PI controller configuration implemented on the Statscan™ X-ray machine can possibly be modified or designed by using fuzzy logic algorithms or by using iterative learning control algorithms.

- 7.2.5 In addition, designed optimal controllers, the model reference discrete-time sliding mode control MRDSMC and a novel aforementioned design in points 7.2.2 and 7.2.4 can be tested on an S-curve in comparison with the P-PI controller since the trapezoidal motion profile can suddenly change during the acceleration phases. Alternatively, a smooth trajectory can be investigated as a second direction of high precision research (see Chapter 2). Hence, it would be interesting to achieve high precision and fast dynamics for systems with large inertia and friction.
- 7.2.6 Nowadays, since the control algorithms are widely implemented on digital computers or digital microcontrollers, we recommend the chosen sliding mode control methodology to be developed to discrete time control systems for avoiding the limited sampling frequency, sample/hold effects and discretization error, as well as chattering, as seen in Figures 4.6, 4.9 and 4.10 [82].

8. List of References

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9. Appendices

Appendix A: Three-Way ANOVA

This section provides a brief description of the statistical hypothesis testing method called Analysis of Variance or ANOVA for comparing multiple means. More details are available in the literature [53], [54], [55], [78].

ANOVA is often used when comparing the means of several factors. This is in contrast to the independent t-test that compares the means of just two normally distributed groups, or the Wilcoxon or Mann-Whitney tests that compare the medians of just two groups (i.e., not normally distributed in this case).

Given a set of FACTORS (e.g., *Controller Type*, *Speed*, *Position*) that produce a set of DEPENDENT VARIABLES (e.g., *Settling Time*, *Image Blurring Index*, etc.), ANOVA produces an F-RATIO which is the ratio of 'explained' to 'unexplained' (i.e., random) variance. Explained variance is the variance due to the factor. There is an F-ratio for each factor (direct effect), as well as for interactions between factors (interaction effect). For example, if the controllers are only different in performance (as measured, for example, by *Settling Time*) at a specific speed but not at other speeds, then there would also be a significant ($p \leq 0.05$) *Controller*Speed* interaction.

Therefore an F-ratio greater than 1 indicates that the factor has a statistical effect greater than random or chance variation. The F-ratio may be converted in a 'p-VALUE' (the probability of a type 1 error, where the null hypothesis is rejected despite being true i.e. the probability of a 'false positive' result) ranging from 0 to 1, if the data are normally distributed. The typical significance threshold is $p \leq 0.05$ or a less than or equal to 5% chance of a false positive.

Where there are three factors (also referred to as independent variables) such as *Controller*, *Speed* and *Position*, and more than one dependent variable (e.g., *Settling Time*, Damping Factor (*Overshoot*), and *Image Blurring Index*) then Multiple ANOVA or MANOVA may be used. Alternatively, three different ANOVAs, each relating to a specific dependent variable may be used. Note that each factor may have multiple 'levels', e.g., if there are three different controllers then the Controller factor has three levels.

Therefore, a 3-way ANOVA of, for example, dependent variable *Settling Time* would result in seven different F-values (and hence also p-values). In the previous example, this would be the three DIRECT EFFECTS or 1-way interaction (*Controller*, *Speed*, *Position*), three TWO-WAY INTERACTIONS (*Controller*Speed*, *Controller*Position*, *Speed*Position*), and one THREE WAY INTERACTION (*Controller*Speed*Position*).

However, ANOVA makes a number of assumptions that have to be checked before the results may be accepted. In particular, the variance of the groups is assumed to be equal, and the residuals are assumed to be normal.

LEVENE'S TEST may be used to assess the equality of variance, and hence may be used to test the first assumption of ANOVA. If the variances are not equal, then the BROWNE-FORSYTHE test, which produces an F-value corrected for unequal variance and compares the medians (preferred when the data are not normally distributed, i.e., the second assumption of ANOVA), may be used instead.

Because ANOVA compares many means, when significant effects are detected, it is often followed by pair-wise tests, for example, the TUKEY'S TEST, applied to the significant direct and interaction effects. Where there is no significant effect, then applying TUKEY'S TEST following ANOVA is unnecessary. A more formal mathematical explanation follows.

A.1. ANOVA Experimental model

Given a three-way experimental model with levels of a factor A, B and C with a total of ijk factorial combinations of n measured values, this model can be defined in parametric form:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} \quad (A.1)$$

where, $i, j, k = 1, 2, \dots$ and μ is a constant. Parameters α_i , β_j and γ_k are the main effect of the i^{th} , j^{th} and k^{th} level of factor *Controller* (A), *Speed* (B) and *Position* (C), respectively. Then $(\alpha\beta)_{ij}$, $(\alpha\gamma)_{ik}$ and $(\beta\gamma)_{jk}$ are the interactions between A and B, A and C and B and C, respectively. Finally, $(\alpha\beta\gamma)_{ijk}$ is the interaction between A, B and C. The term y_{ijk} is the fitness value of experiment with combination factor level at i, j and k .

The null hypotheses (no difference in the population means) of the model can be defined as:

$$\begin{aligned} H_{0A}: \alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_a = 0 \\ H_{0B}: \beta_1 = \beta_2 = \beta_3 = \dots = \beta_b = 0 \\ H_{0C}: \gamma_1 = \gamma_2 = \gamma_3 = \dots = \gamma_c = 0 \\ H_{0ABC}: \delta_{111} = \dots = \delta_{11c} = \dots = \delta_{11b} = \dots = \delta_{11a} = \dots = \delta_{abc} = 0 \end{aligned} \quad (A.2)$$

where α, β, γ and δ are the effects of the A factor, B factor and C factor and their interactions.

The test of the differences in means for statistical differences is accomplished by analysing the variances. This is done by partitioning the total variances into orthogonal components, namely the component due to the random error (i.e., within-group measured by sums of squared (SS)) and the components due to the differences between

means [54], [55]. Therefore, for statistical significance, the hypotheses of ANOVA can be rejected or accepted based on the latter variance components. The test procedure of a three-way ANOVA is summarized in Table A.1. From Table A.1, the variance estimates can be formed by dividing the sum of squares by its degree-of-freedom as shown in Table A.2, where each mean square in Table A.2 estimates the error variance (σ_e^2). In each case, that additional parameter contains exactly one of the four hypotheses given in Equation (A.2). In each case, when a null hypothesis is true, the term for that hypothesis equals zero. Finally, an F-ratio can be constructed by dividing the mean squares by mean squares error (MS_{Error}).

Table A.1: ANOVA for three factors and their corresponding degree-of-freedom

Source of Variation	Sum Squares	degree-of-freedom
SS of main effect in A	$SS_A = \frac{1}{a^2} \sum_{i=1}^a \left(\sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2$	
SS of main effect in B	$SS_B = \frac{1}{a^2} \sum_{j=1}^a \left(\sum_{i=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2$	$df_B = a-1$
SS of main effect in C	$SS_C = \frac{1}{a^2} \sum_{k=1}^a \left(\sum_{i=1}^a \sum_{j=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2$	$df_C = a-1$
SS of interaction between A and B	$SS_{AB} = \frac{1}{a} \sum_{i=1}^a \sum_{j=1}^a \left(\sum_{k=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - SS_A - SS_B$	$df_{AB} = (a-1)(a-1)$
SS of interaction between A and C	$SS_{AC} = \frac{1}{a} \sum_{i=1}^a \sum_{k=1}^a \left(\sum_{j=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - SS_A - SS_C$	$df_{AC} = (a-1)(a-1)$
SS of interaction between B and C	$SS_{BC} = \frac{1}{a} \sum_{j=1}^a \sum_{k=1}^a \left(\sum_{i=1}^a y_{ijk} \right)^2 - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - SS_{AB} - SS_C$	$df_{CA} = (a-1)(a-1)$
SS of interaction between A,B and C	$SS_{ABC} = \sum_{j=1}^a \sum_{k=1}^a \sum_{i=1}^a y_{ijk} - \frac{1}{a^3} \left(\sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a y_{ijk} \right)^2 - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC}$	$df_{ABC} = (a-1)(a-1)(a-1)$

Table A.2: The calculation of mean squares, parameter estimated and F-ratio

Mean Square (MS)	Parameter estimated	F-ratio
$MS_A = \frac{SS_A}{df_A}$	$\sigma_e^2 + \frac{3.a^2 \sum \alpha_i^2}{a-1}$	$F_A = \frac{MS_A}{MS_{Error}}$
$MS_B = \frac{SS_B}{df_B}$	$\sigma_e^2 + \frac{3.a^2 \sum \beta_j^2}{a-1}$	$F_B = \frac{MS_B}{MS_{Error}}$
$MS_C = \frac{SS_C}{df_C}$	$\sigma_e^2 + \frac{3.a^2 \sum \gamma_k^2}{a-1}$	$F_C = \frac{MS_C}{MS_{Error}}$
$MS_{AB} = \frac{SS_{AB}}{df_{AB}}$	$\sigma_e^2 + \frac{3 \sum \delta_{ij}^2}{(a-1)(a-1)}$	$F_{AB} = \frac{MS_{AB}}{MS_{Error}}$
$MS_{BC} = \frac{SS_{BC}}{df_{BC}}$	$\sigma_e^2 + \frac{3 \sum \delta_{jk}^2}{(a-1)(a-1)}$	$F_{BC} = \frac{MS_{BC}}{MS_{Error}}$
$MS_{CA} = \frac{SS_{CA}}{df_{CA}}$	$\sigma_e^2 + \frac{3 \sum \delta_{ki}^2}{(a-1)(a-1)}$	$F_{CA} = \frac{MS_{CA}}{MS_{Error}}$
$MS_{ABC} = \frac{SS_{ABC}}{df_{ABC}}$	$\sigma_e^2 + \frac{3 \sum \delta_{ijk}^2}{(a-1)(a-1)(a-1)}$	$\frac{MS_{ABC}}{MS_{Error}}$
$MS_{Error} = \frac{SS_{Error}}{df_{Error}}$	σ_e^2	

The general F-ratio formula can be defined as

$$F\text{-ratio} = \frac{\text{Error variation} + \text{Factor variation}}{\text{Error variation}} \quad (A.3)$$

This equation shows that if the null hypothesis is true, then the second term in the numerator equals zero and the F-ratio will equal 1. To test the F-ratio, a sampling F-distribution of the F-ratio is used. In each case, the additional parameter must equal less than 0.05 for a significant difference.

A.2. Brown-Forsythe Test

When the assumption of equal variances does not hold (Levene's homogeneity test violated) the Browne-Forsythe test is conducted to test for the equality of means. The Brown-Forsythe test [83] is defined as:

$$B = \frac{\sum_{i=1}^k n_i (x - x_0)^2}{\sum_{i=1}^k n_i \left(1 - \frac{n_i}{n}\right) s_i^2} \quad (A.4)$$

where:

$$x = \sum_{i=1}^{n_i} \frac{x_0}{n}, x_0 = \sum_{i=1}^{n_i} n_i \frac{x}{n} \text{ and } s_i^2 = \sum_{i=0}^n \frac{(x_0 - x)^2}{n_i - 1}$$

A.3. Tukey *Post-Hoc* Test

When differences between groups exist, a *post-hoc* test is conducted to identify which group differs from the others. In this section, the Tukey *post-hoc* test is defined. This test establishes a value for the smallest possible significant difference between two condition means; any mean difference greater than that amount is significant [54]. The Tukey critical mean difference is defined as:

$$D = q(\alpha, df, a) \sqrt{\frac{MS_{Error} \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}{2}} \quad (A.5)$$

where, $q(\alpha, df, a)$ is a standard range of values in statistic which is obtained by using the number of degrees-of-freedom for the error and number of groups being compared, where n_i and n_j are the two group sizes.

A.4. ANOVA's factors Interactions Pairwise Comparisons Results

This section presents the interactions results of the various factors (Controller, Speed and Position) for the dependent variables *Final Position Error*, *Settling time* and the *Overshoot* respectively.

A.4.1 Interaction Results for the *Final Position Error*

**Table A.3: Controller*Speed Interaction Pairwise Comparison for the *Final Position Error*.
See Figure 5.7.**

Speed	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
3	P-ISMIC	P-ISMIC+BD	-0.409*	0.090	< 0.001	-0.587	-0.231
		P-PI	0.032	0.090	0.726	-0.147	0.210
	P-ISMIC+BD	P-ISMIC	0.409*	0.090	< 0.001	0.231	0.587
		P-PI	0.441*	0.090	< 0.001	0.263	0.619
	P-PI	P-ISMIC	-0.032	0.090	0.726	-0.210	0.147
		P-ISMIC+BD	-0.441*	0.090	< 0.001	-0.619	-0.263
6	P-ISMIC	P-ISMIC+BD	0.184*	0.090	0.043	0.006	0.362
		P-PI	0.046	0.090	0.610	-0.132	0.224
	P-ISMIC+BD	P-ISMIC	-0.184*	0.090	0.043	-0.362	-0.006
		P-PI	-0.138	0.090	0.126	-0.317	0.040
	P-PI	P-ISMIC	-0.046	0.090	0.610	-0.224	0.132
		P-ISMIC+BD	0.138	0.090	0.126	-0.040	0.317
9	P-ISMIC	P-ISMIC+BD	0.135	0.090	0.135	-0.043	0.313
		P-PI	-0.048	0.090	0.598	-0.226	0.131
	P-ISMIC+BD	P-ISMIC	-0.135	0.090	0.135	-0.313	0.043
		P-PI	-0.183*	0.090	0.044	-0.361	-0.005
	P-PI	P-ISMIC	0.048	0.090	0.598	-0.131	0.226
		P-ISMIC+BD	0.183*	0.090	0.044	0.005	0.361
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.4: Controller* Position Interaction Pairwise Comparison for the *Final Position Error*. See Figure 5.8.

Position	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
25	P-ISMIC	P-ISMIC+BD	0.096	0.090	0.288	-0.082	0.274
		P-PI	-0.017	0.090	0.849	-0.195	0.161
	P-ISMIC+BD	P-ISMIC	-0.096	0.090	0.288	-0.274	0.082
		P-PI	-0.113	0.090	0.211	-0.291	0.065
	P-PI	P-ISMIC	0.017	0.090	0.849	-0.161	0.195
		P-ISMIC+BD	0.113	0.090	0.211	-0.065	0.291
50	P-ISMIC	P-ISMIC+BD	0.168	0.090	0.065	-0.010	0.346
		P-PI	0.095	0.090	0.293	-0.083	0.273
	P-ISMIC+BD	P-ISMIC	-0.168	0.090	0.065	-0.346	0.010
		P-PI	-0.073	0.090	0.420	-0.251	0.105
	P-PI	P-ISMIC	-0.095	0.090	0.293	-0.273	0.083
		P-ISMIC+BD	0.073	0.090	0.420	-0.105	0.251
100	P-ISMIC	P-ISMIC+BD	-0.353*	0.090	0.000	-0.532	-0.175
		P-PI	-0.048	0.090	0.595	-0.226	0.130
	P-ISMIC+BD	P-ISMIC	0.353*	0.090	< 0.001	0.175	0.532
		P-PI	0.306*	0.090	0.001	0.127	0.484
	P-PI	P-ISMIC	0.048	0.090	0.595	-0.130	0.226
		P-ISMIC+BD	-0.306*	0.090	0.001	-0.484	-0.127
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

**Table A.5: *Speed* Position* Interaction Pairwise Comparison for the *Final Position Error*.
See Figure 5.9.**

Position	(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
25	3	6	-0.264*	0.090	0.004	-0.442	-0.086
		9	-0.874*	0.090	< 0.001	-1.052	-0.696
	6	3	0.264*	0.090	0.004	0.086	0.442
		9	-0.610*	0.090	< 0.001	-0.788	-0.432
	9	3	0.874*	0.090	< 0.001	0.696	1.052
		6	0.610*	0.090	< 0.001	0.432	0.788
50	3	6	-0.011	0.090	.903	-0.189	0.167
		9	-0.418*	0.090	< 0.001	-0.596	-0.240
	6	3	0.011	0.090	0.903	-0.167	0.189
		9	-0.407*	0.090	< 0.001	-0.586	-0.229
	9	3	0.418*	0.090	< 0.001	0.240	0.596
		6	0.407*	0.090	< 0.001	0.229	0.586
100	3	6	0.094	0.090	0.299	-0.084	0.272
		9	-0.202*	0.090	0.027	-0.380	-0.023
	6	3	-0.094	0.090	0.299	-0.272	0.084
		9	-0.295*	0.090	0.001	-0.473	-0.117
	9	3	0.202*	0.090	0.027	0.023	0.380
		6	0.295*	0.090	0.001	0.117	0.473
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.6: Controller*Speed* Position Interaction Pairwise Comparison for the Final Position Error. See Figure 5.10.

Speed	Position	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
							Lower Bound	Upper Bound
3	25	P-ISMIC	P-ISMIC+BD	0.275	0.156	0.081	-0.034	0.583
			P-PI	0.029	0.156	0.852	-0.279	0.338
		P-ISMIC+BD	P-ISMIC	-0.275	0.156	0.081	-0.583	0.034
			P-PI	-0.245	0.156	0.118	-0.554	0.063
		P-PI	P-ISMIC	-0.029	0.156	0.852	-0.338	0.279
			P-ISMIC+BD	0.245	0.156	0.118	-0.063	0.554
	50	P-ISMIC	P-ISMIC+BD	-0.445*	0.156	0.005	-0.753	-0.136
			P-PI	0.033	0.156	0.834	-0.276	0.341
		P-ISMIC+BD	P-ISMIC	0.445*	0.156	0.005	0.136	0.753
			P-PI	0.478*	0.156	0.003	0.169	0.786
		P-PI	P-ISMIC	-0.033	0.156	0.834	-0.341	0.276
			P-ISMIC+BD	-0.478*	0.156	0.003	-0.786	-0.169
	100	P-ISMIC	P-ISMIC+BD	-1.058*	0.156	0.000	-1.366	-0.749
			P-PI	0.033	0.156	0.833	-0.276	0.341
		P-ISMIC+BD	P-ISMIC	1.058*	0.156	0.000	0.749	1.366
			P-PI	1.091*	0.156	<0.001	0.782	1.399
		P-PI	P-ISMIC	-0.033	0.156	0.833	-0.341	0.276
			P-ISMIC+BD	-1.091*	0.156	<0.001	-1.399	-0.782
6	25	P-ISMIC	P-ISMIC+BD	0.331*	0.156	0.036	0.022	0.639
			P-PI	-0.027	0.156	0.861	-0.336	0.281
		P-ISMIC+BD	P-ISMIC	-0.331*	0.156	0.036	-0.639	-0.022
			P-PI	-0.358*	0.156	0.023	-0.667	-0.050
		P-PI	P-ISMIC	0.027	0.156	0.861	-0.281	0.336
			P-ISMIC+BD	0.358*	0.156	0.023	0.050	0.667
	50	P-ISMIC	P-ISMIC+BD	0.489*	0.156	0.002	0.181	0.798
			P-PI	0.175	0.156	0.264	-0.134	0.483
		P-ISMIC+BD	P-ISMIC	-0.489*	0.156	0.002	-0.798	-0.181

			P-PI	-0.314*	0.156	0.046	-0.623	-0.006
		P-PI	P-ISMC	-0.175	0.156	0.264	-0.483	0.134
			P-ISMC+BD	0.314*	0.156	0.046	0.006	0.623
	100	P-ISMC	P-ISMC+BD	-0.267	0.156	0.089	-0.575	0.042
			P-PI	-0.010	0.156	0.951	-0.318	0.299
		P-ISMC+BD	P-ISMC	0.267	0.156	0.089	-0.042	0.575
			P-PI	0.257	0.156	0.101	-0.051	0.566
		P-PI	P-ISMC	0.010	0.156	0.951	-0.299	0.318
			P-ISMC+BD	-0.257	0.156	0.101	-0.566	0.051
		9	25	P-ISMC	P-ISMC+BD	-0.318*	0.156	0.044
P-PI	-0.053				0.156	0.733	-0.362	0.255
P-ISMC+BD	P-ISMC			0.318*	0.156	0.044	0.009	0.626
	P-PI			0.264	0.156	0.092	-0.044	0.573
P-PI	P-ISMC			0.053	0.156	0.733	-0.255	0.362
	P-ISMC+BD			-0.264	0.156	0.092	-0.573	0.044
50	P-ISMC		P-ISMC+BD	0.459*	0.156	0.004	0.151	0.768
			P-PI	0.078	0.156	0.619	-0.231	0.386
	P-ISMC+BD		P-ISMC	-0.459*	0.156	0.004	-0.768	-0.151
			P-PI	-0.381*	0.156	0.016	-0.690	-0.073
	P-PI		P-ISMC	-0.078	0.156	0.619	-0.386	0.231
			P-ISMC+BD	0.381*	0.156	0.016	0.073	0.690
100	P-ISMC		P-ISMC+BD	0.264	0.156	0.092	-0.044	0.573
			P-PI	-0.167	0.156	0.285	-0.476	0.141
	P-ISMC+BD		P-ISMC	-0.264	0.156	0.092	-0.573	0.044
			P-PI	-0.431*	0.156	0.007	-0.740	-0.123
	P-PI		P-ISMC	0.167	0.156	0.285	-0.141	0.476
			P-ISMC+BD	0.431*	0.156	0.007	0.123	0.740
Based on estimated marginal means								
*. The mean difference is significant at the .050 level.								

4.4.2. Pairwise Interaction Results for *Settling Time*

Table A.7: Controller*Speed Interaction Pairwise Comparison for the *Settling Tim*. See Figure 5.17.

Speed	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
3	P-ISMIC	P-ISMIC+BD	-0.397*	0.032	< 0.001	-0.461	-0.333
		P-PI	0.029	0.032	0.363	-0.034	0.093
	P-ISMIC+BD	P-ISMIC	0.397*	0.032	< 0.001	0.333	0.461
		P-PI	0.426*	0.032	< 0.001	0.363	0.490
	P-PI	P-ISMIC	-0.029	0.032	0.363	-0.093	0.034
		P-ISMIC+BD	-0.426*	0.032	< 0.001	-0.490	-0.363
6	P-ISMIC	P-ISMIC+BD	-0.155*	0.032	< 0.001	-0.219	-0.091
		P-PI	-0.018	0.032	0.571	-0.082	0.045
	P-ISMIC+BD	P-ISMIC	0.155*	0.032	< 0.001	0.091	0.219
		P-PI	0.137*	0.032	< 0.001	0.073	0.200
	P-PI	P-ISMIC	0.018	0.032	0.571	-0.045	0.082
		P-ISMIC+BD	-0.137*	0.032	< 0.001	-0.200	-0.073
9	P-ISMIC	P-ISMIC+BD	-0.014	0.033	0.669	-0.081	0.052
		P-PI	-0.024	0.034	0.489	-0.091	0.044
	P-ISMIC+BD	P-ISMIC	0.014	0.033	0.669	-0.052	0.081
		P-PI	-0.009	0.033	0.777	-0.074	0.056
	P-PI	P-ISMIC	0.024	0.034	0.489	-0.044	0.091
		P-ISMIC+BD	0.009	0.033	0.777	-0.056	0.074
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.8: *Speed*Position* Interaction Pairwise Comparison for the *Settling Time*. See Figure 5.18.

Position	(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
25	3	6	-1.610*	0.032	< 0.001	-1.674	-1.547
		9	-3.905*	0.033	< 0.001	-3.971	-3.838
	6	3	1.610*	0.032	< 0.001	1.547	1.674
		9	-2.295*	0.033	< 0.001	-2.361	-2.228
	9	3	3.905*	0.033	< 0.001	3.838	3.971
		6	2.295*	0.033	< 0.001	2.228	2.361
50	3	6	-1.787*	0.032	< 0.001	-1.851	-1.723
		9	-3.846*	0.033	< 0.001	-3.911	-3.781
	6	3	1.787*	0.032	< 0.001	1.723	1.851
		9	-2.059*	0.033	< 0.001	-2.124	-1.994
	9	3	3.846*	0.033	< 0.001	3.781	3.911
		6	2.059*	0.033	< 0.001	1.994	2.124
100	3	6	-1.818*	0.032	< 0.001	-1.881	-1.754
		9	-3.919*	0.032	< 0.001	-3.983	-3.855
	6	3	1.818*	0.032	< 0.001	1.754	1.881
		9	-2.101*	0.032	< 0.001	-2.165	-2.038
	9	3	3.919*	0.032	< 0.001	3.855	3.983
		6	2.101*	0.032	< 0.001	2.038	2.165
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

4.4.3. Pairwise Interaction Results for the *Overshoot*

Table A.9: *Controller*Speed* Interaction Pairwise Comparison for *Overshoot*. See Figure 5.25.

Speed	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
3	P-ISMIC	P-ISMIC+BD	-3.147*	0.401	< 0.001	-3.941	-2.353
		P-PI	-1.949*	0.401	< 0.001	-2.743	-1.154
	P-ISMIC+BD	P-ISMIC	3.147*	0.401	< 0.001	2.353	3.941
		P-PI	1.199*	0.401	0.003	0.405	1.993
	P-PI	P-ISMIC	1.949*	0.401	< 0.001	1.154	2.743
		P-ISMIC+BD	-1.199*	0.401	0.003	-1.993	-0.405
6	P-ISMIC	P-ISMIC+BD	-1.431*	0.401	0.001	-2.225	-0.636
		P-PI	0.005	0.401	0.990	-0.789	0.799
	P-ISMIC+BD	P-ISMIC	1.431*	0.401	0.001	0.636	2.225
		P-PI	1.435*	0.401	0.001	0.641	2.230
	P-PI	P-ISMIC	-0.005	0.401	0.990	-0.799	0.789
		P-ISMIC+BD	-1.435*	0.401	0.001	-2.230	-0.641
9	P-ISMIC	P-ISMIC+BD	-1.855*	0.401	< 0.001	-2.649	-1.061
		P-PI	0.211	0.401	0.600	-0.583	1.005
	P-ISMIC+BD	P-ISMIC	1.855*	0.401	< 0.001	1.061	2.649
		P-PI	2.066*	0.401	< 0.001	1.272	2.860
	P-PI	P-ISMIC	-0.211	0.401	0600	-1.005	0.583
		P-ISMIC+BD	-2.066*	0.401	< 0.001	-2.860	-1.272
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.10: Controller*Position Interaction Pairwise Comparisons for Overshoot. See Figure 5.26.

Position	(I) Controller	(J) Controller	Mean Difference (I- J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
25	P-ISM C	P-ISM C+BD	-1.128*	0.401	0.006	-1.923	-0.334
		P-PI	-0.871*	0.401	0.032	-1.665	-0.077
	P-ISM C+BD	P-ISM C	1.128*	0.401	0.006	0.334	1.923
		P-PI	0.257	0.401	0.522	-0.537	1.051
	P-PI	P-ISM C	0.871*	0.401	0.032	0.077	1.665
		P-ISM C+BD	-0.257	0.401	0.522	-1.051	0.537
50	P-ISM C	P-ISM C+BD	-2.860*	0.401	< 0.001	-3.654	-2.066
		P-PI	-0.703	0.401	0.082	-1.497	0.091
	P-ISM C+BD	P-ISM C	2.860*	0.401	< 0.001	2.066	3.654
		P-PI	2.157*	0.401	< 0.001	1.363	2.952
	P-PI	P-ISM C	0.703	0.401	0.082	-0.091	1.497
		P-ISM C+BD	-2.157*	0.401	< 0.001	-2.952	-1.363
100	P-ISM C	P-ISM C+BD	-2.444*	0.401	0.000	-3.239	-1.650
		P-PI	-0.159	0.401	0.693	-0.953	0.635
	P-ISM C+BD	P-ISM C	2.444*	0.401	< 0.001	1.650	3.239
		P-PI	2.286*	0.401	< 0.001	1.492	3.080
	P-PI	P-ISM C	0.159	0.401	0.693	-0.635	0.953
		P-ISM C+BD	-2.286*	0.401	< 0.001	-3.080	-1.492
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.11: *Speed*Position* Interaction Pairwise Comparison for *Overshoot*. See Figure 5.27.

Position	(I) Speed	(J) Speed	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
25	3	6	-0.952*	0.401	0.019	-1.746	-0.158
		9	-3.906*	0.401	< 0.001	-4.700	-3.112
	6	3	0.952*	0.401	0.019	0.158	1.746
		9	-2.954*	0.401	< 0.001	-3.748	-2.160
	9	3	3.906*	0.401	< 0.001	3.112	4.700
		6	2.954*	0.401	< 0.001	2.160	3.748
50	3	6	0.107	0.401	0.791	-0.688	0.901
		9	0.537	0.401	0.183	-0.257	1.331
	6	3	-0.107	0.401	0.791	-0.901	0.688
		9	0.431	0.401	0.285	-0.363	1.225
	9	3	-0.537	0.401	0.183	-1.331	0.257
		6	-0.431	0.401	0.285	-1.225	0.363
100	3	6	0.590	0.401	0.144	-0.204	1.384
		9	0.413	0.401	0.305	-0.381	1.207
	6	3	-0.590	0.401	0.144	-1.384	0.204
		9	-0.177	0.401	0.659	-0.972	0.617
	9	3	-0.413	0.401	0.305	-1.207	.0381
		6	0.177	.0401	0.659	-0.617	0.972
Based on estimated marginal means							
*. The mean difference is significant at the .050 level.							

Table A.12: Controller*Speed*Position Interaction Pairwise Comparisons for *Overshoot*.
See Figure 5.28.

Speed	Position	(I) Controller	(J) Controller	Mean Difference (I-J)	Std. Error	p	95% Confidence Interval for Difference	
							Lower Bound	Upper Bound
3	25	P-ISMIC	P-ISMIC+BD	-3.185*	0.694	< 0.001	-4.561	-1.810
			P-PI	-1.543*	0.694	0.028	-2.918	-0.167
		P-ISMIC+BD	P-ISMIC	3.185*	0.694	< 0.001	1.810	4.561
			P-PI	1.643*	0.694	0.020	0.267	3.018
		P-PI	P-ISMIC	1.543*	0.694	0.028	0.167	2.918
			P-ISMIC+BD	-1.643*	0.694	0.020	-3.018	-0.267
	50	P-ISMIC	P-ISMIC+BD	-3.701*	0.694	< 0.001	-5.077	-2.326
			P-PI	-2.611*	0.694	< 0.001	-3.986	-1.235
		P-ISMIC+BD	P-ISMIC	3.701*	0.694	< 0.001	2.326	5.077
			P-PI	1.091	0.694	0.119	-0.285	2.466
		P-PI	P-ISMIC	2.611*	0.694	< 0.001	1.235	3.986
			P-ISMIC+BD	-1.091	0.694	0.119	-2.466	0.285
	100	P-ISMIC	P-ISMIC+BD	-2.555*	0.694	< 0.001	-3.931	-1.180
			P-PI	-1.692*	0.694	0.016	-3.068	-0.317
		P-ISMIC+BD	P-ISMIC	2.555*	0.694	< 0.001	1.180	3.931
			P-PI	0.863	0.694	0.216	-0.513	2.238
		P-PI	P-ISMIC	1.692*	0.694	0.016	0.317	3.068
			P-ISMIC+BD	-0.863	0.694	0.216	-2.238	0.513
6	25	P-ISMIC	P-ISMIC+BD	0.324	0.694	0.642	-1.052	1.699
			P-PI	-1.055	0.694	0.132	-2.430	0.321
		P-ISMIC+BD	P-ISMIC	-0.324	0.694	0.642	-1.699	1.052
			P-PI	-1.378*	0.694	0.050	-2.754	-0.003
		P-PI	P-ISMIC	1.055	0.694	0.132	-0.321	2.430
			P-ISMIC+BD	1.378*	0.694	0.050	0.003	2.754
	50	P-ISMIC	P-ISMIC+BD	-2.126*	0.694	0.003	-3.502	-0.751
			P-PI	0.858	0.694	0.219	-0.517	2.234
		P-ISMIC+BD	P-ISMIC	2.126*	0.694	0.003	0.751	3.502
			P-PI	2.985*	0.694	< 0.001	1.609	4.360
		P-PI	P-ISMIC	-0.858	0.694	.219	-2.234	0.517
			P-ISMIC+BD	0.858	0.694	0.219	-0.517	2.234

			P-ISM+BD	-2.985*	0.694	< 0.001	-4.360	-1.609
	100	P-ISM	P-ISM+BD	-2.489*	0.694	0.001	-3.864	-1.113
			P-PI	0.211	0.694	.762	-1.165	1.586
		P-ISM+BD	P-ISM	2.489*	0.694	0.001	1.113	3.864
			P-PI	2.699*	0.694	< 0.001	1.324	4.075
		P-PI	P-ISM	-0.211	0.694	0.762	-1.586	1.165
			P-ISM+BD	-2.699*	0.694	< 0.001	-4.075	-1.324
9	25	P-ISM	P-ISM+BD	-0.524	0.694	0.452	-1.899	0.852
			P-PI	-0.017	0.694	0.981	-1.392	1.359
		P-ISM+BD	P-ISM	0.524	0.694	0.452	-0.852	1.899
			P-PI	0.507	0.694	0.467	-0.868	1.883
		P-PI	P-ISM	0.017	0.694	0.981	-1.359	1.392
			P-ISM+BD	-0.507	0.694	0.467	-1.883	0.868
	50	P-ISM	P-ISM+BD	-2.753*	0.694	< 0.001	-4.128	-1.377
			P-PI	-0.356	0.694	0.609	-1.731	1.020
		P-ISM+BD	P-ISM	2.753*	0.694	< 0.001	1.377	4.128
			P-PI	2.397*	0.694	0.001	1.021	3.772
		P-PI	P-ISM	0.356	0.694	0.609	-1.020	1.731
			P-ISM+BD	-2.397*	0.694	0.001	-3.772	-1.021
	100	P-ISM	P-ISM+BD	-2.290*	0.694	0.001	-3.665	-0.914
			P-PI	1.005	0.694	.150	-0.370	2.381
		P-ISM+BD	P-ISM	2.290*	0.694	0.001	0.914	3.665
			P-PI	3.295*	0.694	< 0.001	1.919	4.670
		P-PI	P-ISM	-1.005	0.694	0.150	-2.381	0.370
			P-ISM+BD	-3.295*	0.694	< 0.001	-4.670	-1.919
Based on estimated marginal means								
*. The mean difference is significant at the .050 level.								

Appendix B: Time Delay Integration (TDI)

Time Delay Integration, or TDI, is a method of multiple scanning and averaging of a row of pixels, in order to increase the signal- to-noise ratio (SNR) and thereby improve image quality. An illustrative example of a simple (either TDI or non-TDI mode) detector using a CCD array of 3 rows and 14 columns is provided in figure B1.

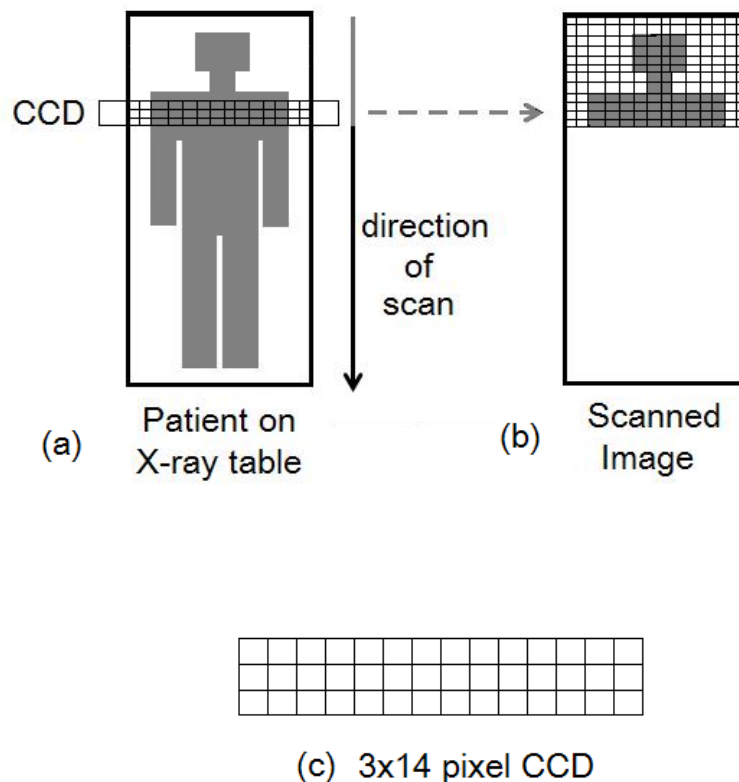


Figure B1: A 3x14 pixel CCD detector. The patient with a superimposed detector is illustrated in (a) and the corresponding partially scanned image is in (b), while (c) illustrates the CCD. Note that this CCD could be moved forward 3 pixels at a time in 'non-TDI' mode, or 1 pixel at a time in 'TDI' mode.

The detector in Figure B1 may be improved using Time Delay Integration, or TDI. This involves moving the scanner forward 1 pixel at a time. Therefore each pixel row is scanned 3 times in the example. The pixel rows may be integrated (i.e., added) at each step, and averaged at the end (i.e. divided by 3 in the example). This process has the advantage of cancelling out random noise, since the noise would not be identical each time the same pixel row is scanned. The larger the number of pixel rows, the greater the improvement in SNR. In addition, TDI also reduces the blurring effect of vibration of the C-arm, since there are multiple integrated line scans.

If there were no noise or vibration, then TDI and non-TDI images would be identical. However the absence of noise or vibration is difficult to achieve in a real setting.

Described more generally and formally, the detector is organized into an array of CDD camera elements consisting of M columns and N rows. The process of TDI is analogous to taking multiple exposures of the same object and summing them [11]. In this mode, the object is scanned along the linear direction and because of the relative motion between the detector and object; it is possible to produce two dimensional images. They are obtained by integrating the charge in the detector elements by the exposure to an optical image and subsequently reading the resultant charge package. During the read-out the whole image is transferred along the column. As each row of charge reaches the end of CCD, it is transferred to a serial register which then shifts it in the opposite direction. However, the shifting of the charge package must be done in synchrony with the movement of the image. In this way, the read-out and integration take place at the same time [72]. For synchrony, the charge transferring speed along columns should equal the detector translating speed [11].

According to the literature [73], obtaining good quality image with TDI mode depends on the modulation transfer function (MTF) which can be determined from the inherent aperture response of individual CCD elements. The $MTF_{y,ap}$ of an individual CCD is given as product of two *sinc* functions:

$$MTF_{y,ap} = (\text{sinc}(f\Delta y_1) \cdot \text{sinc}(f\Delta y_2)) \quad (B1)$$

where, $\text{sinc}(x) = \sin(\pi x)/\pi x$, f = spatial frequency, Δy_1 represents the physical aperture, and Δy_2 is the aperture due to charge diffusion.

In TDI mode the image degradation can however occur due to asynchrony between the detector speed (v_d) and the TDI clocking mechanism (v_c), i.e., if the scanning speed is not matched to the clocking speed ($v_d \neq v_c$). The $MTF_{y,\Delta v}$ due to asynchrony is then defined as follows:

$$MTF_{y,\Delta v} = (\text{sinc}(f\Delta y m \Delta v/v_c)) \quad (B2)$$

where, $\Delta v \equiv |v_d - v_c|$, and Δy is the detector spacing in the y-direction.

B.1. Operation of TDI Simulator for Distorted Images

To allow for offline simulation of the TDI process, a simulator was developed and coded in MATLAB by Dr LR John (UCT). Given a distance profile and an image, this simulator is able to generate both a non-TDI version and a TDI version of the corresponding 'distorted' copy of that image. This allows the effect of the distance profile (as generated by the different controllers, at different speeds and positions) on the scanned image to be investigated. It is useful because the impact of the different controllers on actual X-ray images may be simulated and the outcome is in a form (i.e. image) that is meaningful to the radiologist end-user.

The TDI simulator is therefore used to scan and TDI-'distort' Statscan™'s images based on the simulated distance profiles of the different controllers.

The flow chart of the TDI simulator is illustrated in Figure B2, namely, file load, image conversion type, TDI-CCD matrix formation, distance profile loading and scanning mode. As illustrated in that figure, the first step is the loading of the image (e.g., Figure B4 (a)) from the file into the code. This is followed by conversion of the Statscan™ image matrix into a double variable since it needs to be large enough to integrate or sum the pixels without a data overflow. Next, the empty TDI CDD matrix is formed. Then the double size image is called and the distance profile (e.g., Figure B4(b)) is loaded into the TDI simulator to start scanning. Finally, the TDI scanning iteration process is started at the top of the image to the final destination of the distance profile.

B.1.1. File load

This step concerns loading the image and distance profile. The first step of this portion of code is loading the image, followed by the distance profile as explained in step 4. The four Statscan™'s images and 27 distance profiles from three controllers were individually loaded for each (i.e., 4 x 27) simulation from a file saved in the PC.

B.1.2. Image Conversion Type

This step converts the loaded image to a double size type image. The image is converted to double precision in order to allow for row addition or integration without data overflow. In this step, the pre-allocated memory and the details such as size, height and width of the image are also prepared.

B.1.3.TDI CCD Matrix Formation

This portion of the code creates the detector array partially based on Statscan™ documentation. In the Lodox system, the detector consists of 12 CCD cameras. Each CCD camera is a CCD30-11 (see section B.4) with an actual CDD pixel size of 26 μm , but following optical magnification by a fiber optic taper (see schematic of detector in Figure 3.13), the recorded image pixel size is 60 μm . The twelve CDDs form one unit detector with a width of 690 mm and a length of 15 mm[74].

However, the TDI CCD matrix was created as a single large CCD (i.e., instead of the 12 CCDs used in the actual Statscan™ implementation) in the code as follows:

- The same TDI pixel size is used, i.e., 0.06mm (60 μm).
- The length of the simulated detector in pixels is set based on a 15mm length (i.e., identical to the Statscan™ specifications).
- The width of the simulated detector is set to match the width of the image to be scanned.

Using the above calculation an empty TDI CCD matrix with known height and width length is created using TDI simulator.

B.1.4. Loading the Distance Profile

This section of the code loads the distance profile (in mm) from a separate PC file. The distance profile would correspond to the output of the controller simulation programs. The distance profile may be used as is (i.e., without noise) or with noise added according to the user's selection option. The distance profile is then converted from mm to pixels by dividing the loaded distance profile by the CCD's pixel size (i.e., 0.06 mm).

By using an iteration loop, scaled random noise may be added to the distance profile if the 'ADD_NOISE' option is set to 'true'. Note that the starting value of the distance profile is changed from zero to one for the MATLAB simulation

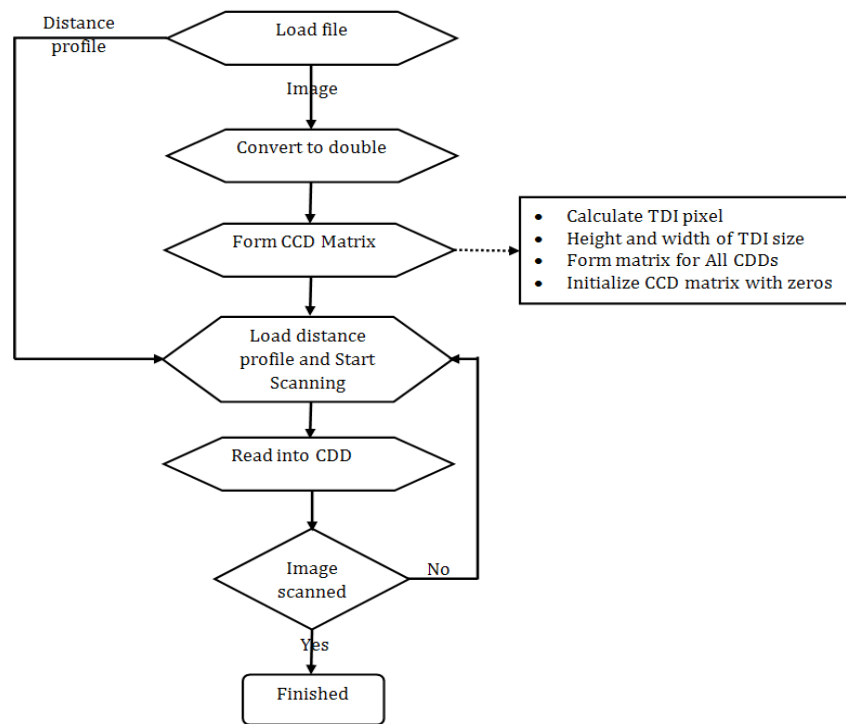


Figure B2: The flow chart of the TDI simulator operation steps.

B.1.5. TDI Scanning Mode

This section of the code scans the loaded image with the TDI CCD matrix. The scanning is limited to the length of the distance profile. Before scanning, the image size (X) must be modified (X_modified) to add a blank area at the beginning and at the end. This helps the TDI software scanning algorithm to scan all selected test images without failing.

Based on the TDI integration mode, the image scanning is established with the following sequence:

- Start scanning the image from the top by using the distance profile, segment by segment. Note that the distance profile determines the instantaneous position of the CCD. If there is noise, then there will be variation in the CCD position. However this position is recorded and known. The TDI embedded algorithm will always clock out the top row at each iteration. This results in distortion of the scanned image. Whilst this row-by-row clock-out process may be modified by low-level TDI algorithm changes in an attempt to reduce blurring, it will still not solve all the noise-induced problems since there will be banding in images due to variations in SNR (since SNR depends on the number of times that a row is scanned, and this would vary if there is noise that affects the smoothness of the distance profile). Such a banded image would not be acceptable in a radiological context.

- Read the current image segment and add or integrate it into the TDI CCD matrix.
- Clock out the top row of the CCD_matrix into the scanned image variable (X_scanned).
- Then shift the CCD_matrix up by 1 row.
- Then advance the CCD down by 1 row (i.e., 1 pixel) and repeat the process above.

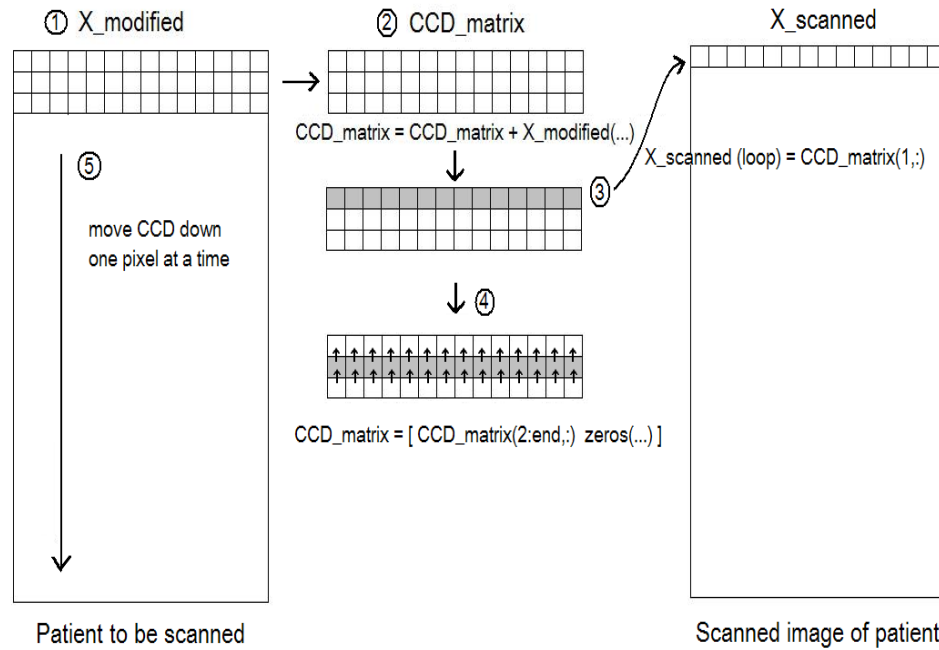


Figure B3: An illustration of the TDI process implemented in the simulator. The 'patient' is on the left, with a 3x14 pixel CCD located at the 'head' of the patient (1). Data from this CCD are then added or integrated into a CCD_matrix variable (2). The top row is then clocked out to the scanned image X_scanned (3). The CCD_matrix is then shifted one row up. The CCD is then moved one pixel down and the process is repeated.

B.2. Image Scanning for TDI Simulator Test

In order to test the effectiveness of the TDI simulator, it is used with sample Statscan™ chest images in three cases; (a) TDI without vibration, (b) TDI with vibration noise added and (c) non-TDI with the same vibration noise.

Figure B4 shows the TDI scan of the image when there is no vibration with B4(a) representing the original, B4(b) representing the distance profile, and B4(c) representing the corresponding TDI scanned image. Comparing the original image and the scanned image, it can be seen clearly that there is no visible difference between the original image and scanned image in terms of image degradation. A straight line without noise was used as an ideal distance profile. This also verifies the correct operation of the TDI simulator program.

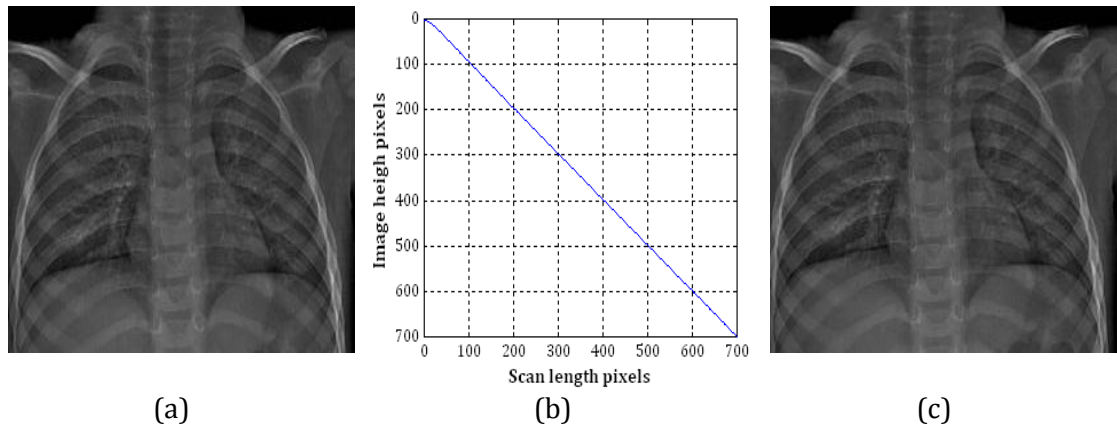


Figure B4: (a). Original image. (b) Active distance profile without vibration. (c) Scanned image.

Figure B5 shows the TDI scan of the same image when there is added noise or vibration. Comparing the original and scanned images, it can be seen clearly that there is a difference between the original and scanned images, with the scanned image showing degradation in the form of blurring. This is due to the combination of a noisy distance profile and the TDI scanning mode.

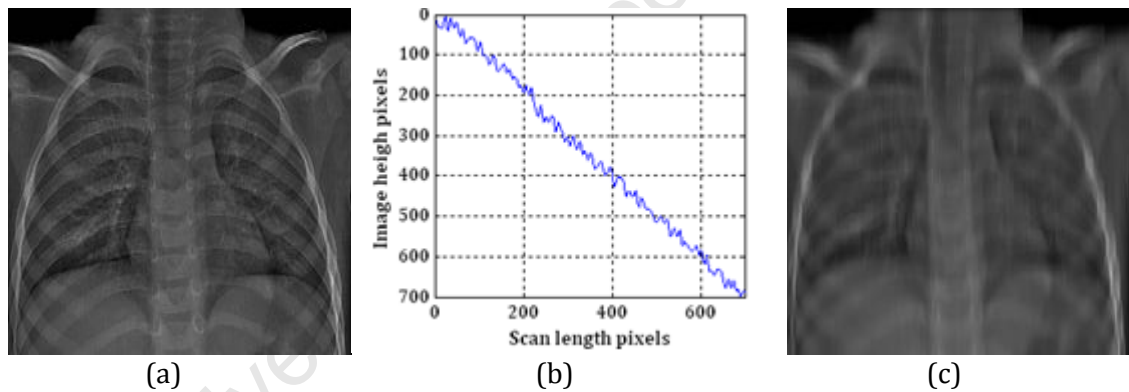


Figure B5: (a). Original image. (b). Active distance profile with vibration (c). TDI scanned image.

However, if the same distance profile from Figure B5.(b) is scanned in the non-TDI mode, the blurring is worsened, as illustrated in Figure B6. This demonstrates that the TDI process also reduces the effect of the distance profile noise, i.e., vibration of the C-arm.

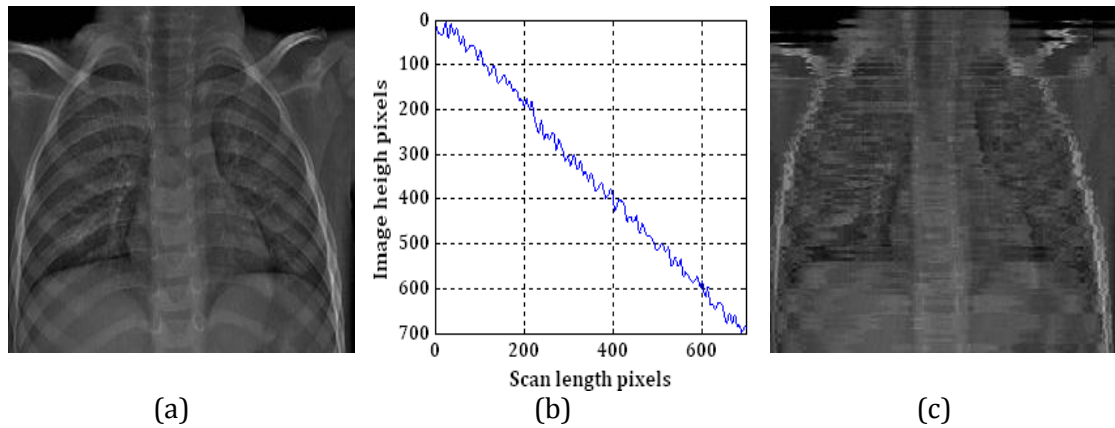


Figure B6: (a). Original image. (b).Distance profile with vibration (c). Non-TDI scanned image.

B.3.TDI Simulator Implemented Code

```
% Time domain integration - for the Lodox scanner
%
% Copyright 2010, Lester John, Biomedical Engineering, UCT, South Africa
% Lester.John@uct.ac.za
%
% USAGE: tdi_image =tdi_lodox();

ADD_NOISE =true;
AUTO_DISTANCE_PROFILE = false; % if 'false' then load distance profile from a file

X = double(X); % convert image to double
X_scanned =zeros(size(X)); % pre-allocate memory
X_scanned_nonTDI=zeros(size(X));
[height,width] = size(X); % image details in pixels

TDI_pixel_size = 0.06; % TDI_pixel dimensions from Statscan
documentation, 60 um
TDI_height = 15/TDI_pixel_size; % note this simulator CCD height as per Statscan

if (height < (TDI_height*10))
    TDI_height = round(height/10); % reduce the sensor height to 1/10 of image
    % height for small images (e.g. 30 pixels) in case
    % the
    % image file has been clipped
end

TDI_width = width; % i.e. set this to the same as the image width.
CCD_matrix = zeros(TDI_height,TDI_width);

% distance profile for C arm scan pixels
if(AUTO_DISTANCE_PROFILE)
    distance_profile = [0:height-1]; % ideal noiseless distance profile

    if(ADD_NOISE)
        for loop = 1:length(distance_profile);
            distance_profile(loop)= round(distance_profile(loop)+ 45*rand/1.2); %
        end
    end
end
add some noise
```

```

        end
    end
    else
        % load distance profile from a file
        if ~exist('distance_profile_filename'); % UI read distance profile file if none has
        been provided
            [filename_dist, pathname_dist, filterindex_dist] = uigetfile('*.txt', 'Pick a distance
            profile data file');
            distance_profile = eval(['load('',pathname_dist, filename_dist,'')']); % in mm;
            output from control simulation program
            distance_profile = round(distance_profile/TDI_pixel_size); % convert
            to pixels
        else
            distance_profile = eval(['load('',distance_profile_filename,'')']); % in mm;
            output from control simulation program
            distance_profile = round(distance_profile/TDI_pixel_size); % convert
            to pixels
        end
        if(ADD_NOISE)
            for loop = 1:length(distance_profile);
                distance_profile(loop)= round(distance_profile(loop)+ 45*rand/1.2); % more
            noise
            end
        end
        end
        distance_profile = distance_profile+1; % ensure
        that the first or lowest value is 1 and not 0.
        original_distance_profile = distance_profile;

        X_modified = [zeros(TDI_height-1,TDI_width);X;zeros(1.5*TDI_height-1,TDI_width)]; %
        increased blank area at end for jumps during % noise testing

        % plot 3 images showing TDI scan lines

        if(distance_profile(end)< height);
            abc=1;
        elseif (distance_profile(end)> height); %
        truncate distance profile

            truncate_distance_profile_index = find(distance_profile < height);
            truncate_distance_profile_index = truncate_distance_profile_index(end); % store
            the last index where next value is > height
            distance_profile = distance_profile(1:truncate_distance_profile_index);
        end

        % evenly resample distance_profile data using a spline, see
        <http://www.mathworks.com/help/techdoc/ref/spline.html>
        x = 1:length(distance_profile);
        y = distance_profile;
        xx = 1:length(distance_profile)/height:length(distance_profile);
        yy = spline(x,y,xx);
        figure; plot(x,y,'o',xx,yy);
        distance_profile=round(yy);

        % Scan image with TDI CCD array
        for loop = 1:length(distance_profile); % note: image scan limited by
        distance profile

```

```

    current_image_row = distance_profile(loop);
    % read image segment into CCD matrix
    CCD_matrix(:, :) = CCD_matrix +
X_modified(current_image_row:current_image_row+TDI_height-1,:);
    X_scanned(loop,:) = CCD_matrix(1,:); % clock / read out
the last CCD row = (first row in matrix)
    X_scanned_nonTDI(loop,:) = X_modified(current_image_row,:); % this is the non-
TDI equivalent scan
    CCD_matrix(:, :) = [CCD_matrix(2:end,:); zeros(1,TDI_width)]; % shift the data
up and clear first CCD row
end
X_scanned = X_scanned/TDI_height; % rescale the scanned image so
that the intensity matches
X_scanned = uint8(X_scanned); % convert the image back to
uint8
X_scanned_nonTDI = uint8(X_scanned_nonTDI);

% Remove the zero-padded areas
ind = find(X_scanned, 1, 'first'); % find the first non-zero row.
X_scanned_zero_padded = X_scanned(1:ind-1,:); % temporary store
X_scanned = X_scanned(ind:end,:); % cut out those rows
X_scanned = [X_scanned;X_scanned_zero_padded]; % paste it at the end

% repeat for the nonTDI scan
ind2 = find(X_scanned_nonTDI, 1, 'first'); % find the
first non-zero row.
X_scanned_zero_padded_nonTDI = X_scanned_nonTDI(1:ind2-1,:); %
temporary store
X_scanned_nonTDI = X_scanned_nonTDI(ind2:end,:); % cut
those rows
X_scanned_nonTDI = [X_scanned_nonTDI;X_scanned_zero_padded_nonTDI]; % paste it
at the end

```

B.4. STATSCAN Detector CCD Sensor CCD30-11

e2v

e2v technologies

FEATURES

- 1024 by 256 Pixel Format
- 26 μm Square Pixels
- Image Area 26.6 x 6.7 mm
- Wide Dynamic Range
- Symmetrical Anti-static Gate Protection
- Advanced Inverted Mode Operation (AIMO)
- Anti-blooming Readout Register
- Zero Light Emitting Output Amplifier

APPLICATIONS

- Spectroscopy
- Scientific Imaging
- TDI Operation

INTRODUCTION

The CCD30-11 is a high performance CCD sensor designed as an upgrade for the CCD15-11, for use in the scientific spectroscopy instrument market. With an array of 1024 x 256 26 μm square pixels it has an imaging area to suit most spectrometer outputs of 26.6 x 6.7 mm (1.05 x 0.26 inch).

The readout register is organised along the long (1024 pixel) edge of the sensor and contains an anti-blooming drain to allow high speed binning operations of low level signals which may be adjacent to much stronger signals. The novel output amplifier design has no light emission.

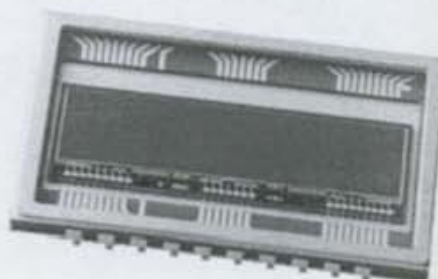
Standard three phase clocking and isolated buried channel charge transfer are employed and Advanced Inverted Mode Operation (AIMO) is included as standard.

The CCD30-11 is packaged in a 20-pin DIL ceramic package and is pin compatible (but not completely clock compatible) with the CCD15-11, with the exception that temperature sensing diodes are not provided.

In common with all other e2v technologies CCD sensors, the CCD30-11 is available with either a fibre-optic window, a UV sensitive coating or a CsI coating for hard X-ray detection. In addition a high performance electronics drive unit is available to enable the CCD30-11 to be evaluated easily.

Designers are advised to contact e2v technologies should they be considering using CCD sensors in abnormal environments or if they require customised packaging or performance features.

CCD30-11 Inverted Mode Sensor High Performance CCD Sensor



TYPICAL PERFORMANCE

Pixel readout frequency	20 - 5000	kHz
Output amplifier sensitivity	1.5	$\mu\text{V}/\text{e}^-$
Peak signal	500	ke^-/pixel
Dynamic range	125 000:1	
Spectral range	420 - 1060	nm
Readout noise (at 253 K, 20 kHz)	4	$\text{e}^- \text{ rms}$
QE at 700 nm	47	%
Peak output voltage	750	mV

GENERAL DATA

Format

Image area	26.6 x 6.7	mm
Active pixels (H)	1024	
(V)	256	(usable)
Pixel size	26 x 26	μm

Package

Package size	32.89 x 20.07 mm	
Number of pins	20	
Inter-pin spacing	2.54 mm	
Inter-row spacing	15.24 mm	
Window material	quartz or removable glass	

e2v technologies (uk) limited, Waterhouse Lane, Chelmsford, Essex CM1 2QU, UK. Telephone: +44 (0)1245 483493 Facsimile: +44 (0)1245 483482
e-mail: enquiries@e2v.com Internet: www.e2v.com Holding Company: e2v technologies plc
e2v technologies inc, 4 Westchester Plaza, PO Box 1462, Elmsford, NY10523-1462 USA Telephone: (914) 582-4050 Facsimile: (914) 582-0146
e-mail: enquiries@e2vtechnologies-usa.com

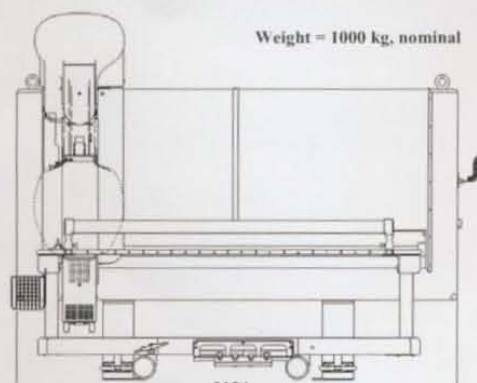
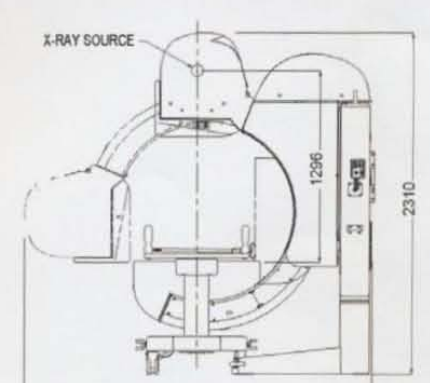
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A1A-100007 Issue 8, March 2006

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B.5. STATSCAN Technical Specifications

Statscan Forensic Imaging System – Abbreviated Product Specifications

Image Quality	
Contrast Resolution	>16000 grey levels (14 bits)
Fundamental pixel size.	60 μm (1x1 binning)
Maximum Size	1800 mm x 680 mm (full body size, measured at lowest table height)
Spatial Resolution	Ranges from 1.04 lp/mm to 5.0 lp/mm. The correct spatial resolution is automatically chosen according to the procedure selected for each x-ray. It can also be set by the user
Scanner throughput	
Time to complete a full field scan	<13 seconds (nominally 12.88s at normal speed)
Best case time between two successive x-rays on the same patient.	28 seconds
Image flexibility	
Radial angles	0 to 90 degrees. The supine or prone patient can be x-rayed in any radial angle from AP (or PA) to the supine lateral positions.
Output	
Radiation Type	This equipment emits ionized x-radiation through an adjustable width narrow fan-beam scanning across the patient. Each part of the body is exposed for only 11msecs during a whole body scan.
X-ray Tube	Rotating Anode "CT" type. CE / FDA approved 2MHU or 3MHU Options
X-ray Generator	Peak Power 64kW
Flexible-Format Image Information	
Image size range	680mm x 1800mm maximum for full body. Adjustable from 100mm x 100mm to maximum in any area on the patient without moving the body
Detector System	
Detector	Proprietary ultra low noise TDI CCD detector (similar to spiral MDCT).
Image Viewing and Manipulation	
Compatibility	PACS, networking and DICOM 3.0 compatible. DICOM send, print and worklist input from an HIS or RIS system is standard. Confirmation statement available on request
Image recall	Images viewed by Previous, Next, Latest, Patient name, Referring doctor, Hospital number etc.
Database	Images in the database belonging to a study or patient can be easily retrieved.
Zoom / Pan / Rotate / Aspect	Real time "Mouse wheel" zoom, pan, rotate, and aspect adjustment all standard.
Image Manipulation / lucid™ image enhancement	Windowing and leveling of grey scales automatically adjusted for optimal viewing. Initial "best view" with lucid™ enhanced image processing simultaneously adjusts for soft tissue and bone.
Compliance	
US FDA Pre- Market Clearance (510k) No. K013999 Accession # 0310920 EU CE-Mark, ISO 13485:2003, ISO 9001:2000 Japan Marketing Certificate #220AIB2X00001000 QMS Conformity Certificate #S080212	
     	
Physical Characteristics - Front View	
	
Physical Characteristics - Side View	
	

These specifications are current and subject to change, Lodox Systems reserves the right to revise these specifications to ensure a well-engineered product.

Contact: Lodox Systems (PTY) LTD. Phone +27 11 444 9118. WEB: sales@lodox.com herman.potgieter@lodox.com www.lodox.com or:
 North America: Lodox NA LLC. Phone +1 248 446 8480 web: marty.kulis@lodox.com

B.6. STATSCAN Full Body Scanning Time

13 seconds in the golden hour, COULD MEAN A LIFETIME

Statscan

Emergency Trauma & Disaster Casualty Assessment

- Designed to meet the needs of emergency centers at any level worldwide
- Emergency and trauma patients of all types can be quickly and flexibly assessed
- Smaller format detailed studies can be accomplished with extreme ease
- Full body X-ray images can be completed in 13 seconds or less
- Ease of operation allows non-specialized personnel to perform procedures
- Reduced risk of radiation exposure to staff and patients
- Exams typically take as little as 4 - 5 minutes from beginning to end



Paediatric Patient Radiography

- Unique linear slot-scanning X-ray technology
- Highly focused fan beam that effectively eliminates large X-ray doses
- The user can effectively control the actual dose administered to a child
- Adjustable: scan speed, digital resolution and X-ray technical parameters
- Paediatric imaging can be fully customised to fit the diagnostic requirements
- Paediatric imaging specialists can individually control and administer only the lowest possible dose required to achieve the highest quality results needed
- The ability to create images using a very short effective exposure time

Bariatric Patient Radiography

- The system is designed to meet the needs of bariatric patients via a multi-faceted approach
- Statscan's highly focused primary beam remains true to form
- Cleanest (scatter-free) X-ray beam is used to produce the resultant images
- Creates image quality that is innately more diagnostic
- 14 bit image processing that provides over 16,000 levels of grey scale contrast
- Resolution is optimised to maximize image quality
- Optional higher weight limit table design



High Throughput General Radiography

- Flexible format DR capability for large or small field-of-view X-ray images
- Fewer views to be taken to complete an exam
- Excellent spatial and contrast resolution
- Linear slot scan DR technology increases workflow, reduces costs and increases productivity
- PACS compatibility for the "all digital imaging department"
- General Radiography is made even easier with our Rad-MP optional System Upgrade



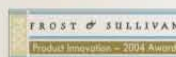
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Lodox Systems (Pty) Ltd, P.O. Box 651807, Benmore, 2010, South Africa. Phone: + 27 11 444 9118 Fax: +27 11 262 0085

Lodox NA, LLC, 143 Burton Street, Painesville, Ohio 44077. Phone: (440) 209-1246 Fax: (440) 257-3984, U.S. Toll Free: (866) 61-lodox (615-6369)

E-mail your inquiries to: info@lodox.com Website: www.lodox.com



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Appendix C: Potentiometer and Tachometer Calibrations

The modelling setup (see Figure 3.2) has two types of sensors: the potentiometer to measure angular position and the tachometer for angular velocity. In order to determine the relationship between the angular position (radians), the angular velocity (radians/sec) and the sensors outputs in volts, the voltage output of the both sensors needs to be calibrated corresponding to actual angular position and angular velocity measurements. The calibration details are given in the following sections.

C.1. Potentiometer Calibration

Table C.1 shows the calibration data between the angular position and the potentiometer output voltage. The calibration data is plotted and a linear least square method in Microsoft excels is applied to find the best fit line (see Figure C.1). From the gradient of the best fit line, the conversion between the angular position and the potentiometer is as follows:

$$V_p = k_\theta \theta = 0.05 * \theta \quad (C.1)$$

where k_θ represents the proportionality constant in volts·rad⁻¹, V_p is the potentiometer voltage output (in volts) and θ is the angular position (radians). From the equation we observe k_θ is 0.05 volts·rad⁻¹.

Table C.1: Angular displacement and corresponding output voltage

Voltage	0	0.95	1.96	2.97	4	5	6	7.03	8	9	9.9	10.9	11.9	12.9
Angle	0	20	40	60	80	100	120	140	160	180	200	220	240	260



Figure C1: The relationship between angular displacement and output voltage.

C.2. Tachometer Calibration

Table C.2 shows the calibration data between the angular velocity and the tachometer output. The calibration data are plotted and a linear least-square method in Microsoft Excel is applied to find the best fit line (see Figure C.2). From the gradient of the best fit line, the conversion between the angular velocity and the tachometer is obtained by using the potentiometer calibration done earlier, the time for each sample and the output voltage of the tachometer. Angular velocity is calculated corresponding to the particular output voltage of the tachometer as follows:

$$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} \quad (\text{C.2})$$

where ω is an angular velocity, θ_1 and θ_2 are the angular positions at the previous sampling (t_1) and the current sampling time (t_2). The relationship of the output voltage and the angular velocity is obtained by calculating the gradient of the fit line as:

$$Vt = k_t * \omega = 0.0087 * \omega \quad (\text{C.3})$$

where k_t is the proportionality constant in volts·sec·rad⁻¹, Vt is the tachometer voltage output (in volts) and θ is the angular position (radians). From the equation we observe k_t is 0.0087 volts·sec·rad⁻¹.

Table C.2: Angular velocity and corresponding output voltage

Voltage	1.4	2.2	2.9	3.7	4.4	5.2	5.8	6.7
Angular Velocity	136	238	336	422	497	603	669	729

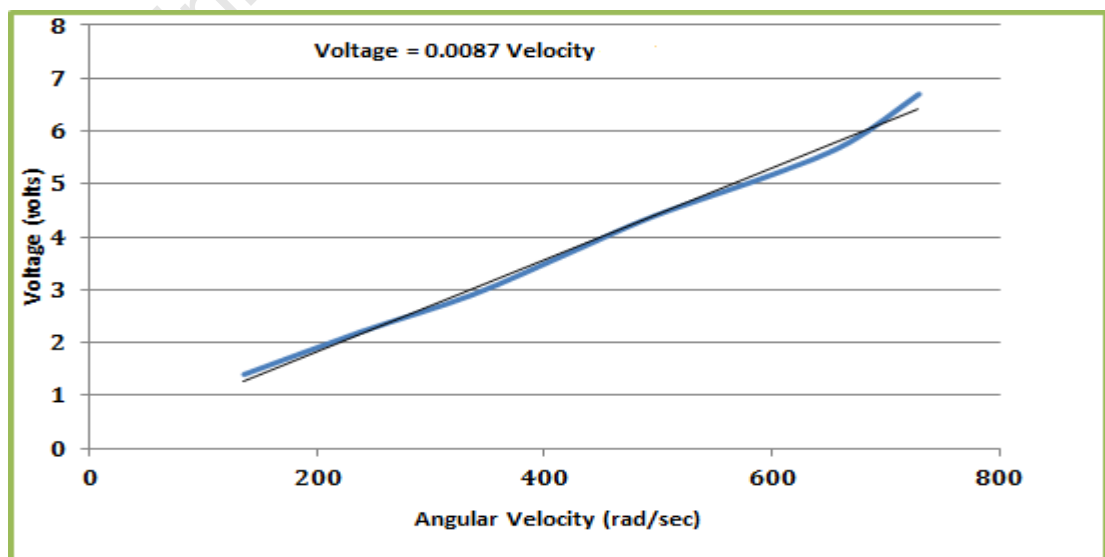


Figure C.2: The relationship between angular velocity and output voltage

C.3. Motion Conversion

This section deals with the conversion from the rotational motion to linear motion. Consider a rack-and-pinion mechanism shown in Figure C.3. This mechanism usually consists of a rotary motor directly coupled to the pinion, producing linear motion along the length of the rack [84]. The rotary input at the pinion shaft is presented in terms of angular velocity ω and the applied torque T . On the other hand, the linear power related with the rack is presented in terms of its linear velocity v and force F [85]

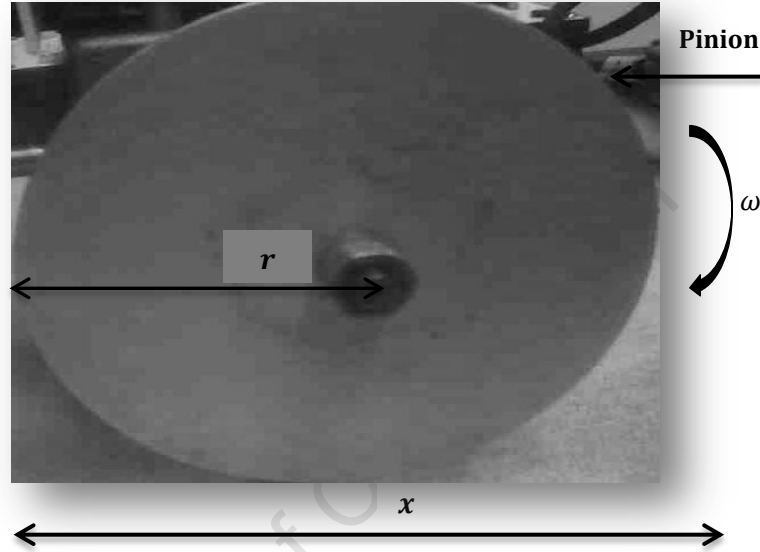


Figure C.3: Disk with a 50 mm radius representing dynamics of a Rack and Pinion

For the mechanism shown in Figure C.3, let v be the linear velocity related to angular velocity ω of the pinion on the rack, r be the radius of the pinion, T_m be the applied torque, F be the linear force and x be the linear position of the pinion on the rack. Therefore, the linear velocity of the rack in mm/sec is correlated to the angular velocity of the pinion in radians/sec directly by the pinion radius r through the following equations:

$$v = r \omega \quad (C.4)$$

and the linear force related to the torque on the pinion shaft T_m by the relationship. It should be noted that the negative force is required to balance the positive torque.

$$F = -\left(\frac{1}{r}\right) T_m \quad (C.5)$$

Equations (C.4) and (C.5) define the conversion relationship between the rotational and linear motions, which may be written in the matrix form as:

$$\begin{bmatrix} v \\ F \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & -\frac{1}{r} \end{bmatrix} \begin{bmatrix} \omega \\ T_m \end{bmatrix} \quad (\text{C.6})$$

Since the position output is in radians we need to transform it to millimeter so that both velocity and position are consistent in terms of units. This easily can be done by integrating the linear velocity from Equation (C.4),

$$x = \int_0^t v \, d\tau \quad (\text{C.7})$$

where v is the linear velocity, t is time of conversion length and x is the linear position in millimetres.

An example:

Consider the mechanism shown in Figure C.3, where the pinion or the disc turns with an angular velocity of 0.1 rad/sec and its radius (r) is 50 mm/rad.

- What is the linear velocity that corresponds to this angular velocity?

Solutions:

- $v = r \omega = 50 \text{ mm/rad} * 0.1 \text{ rad/sec} = 5 \text{ mm/sec}.$
- Owing to this, then through integration the linear position can be obtained.

Appendix D: Chattering Simulations to Step Input

Figures D.1 shows the chattering simulation responses without disturbances (D.1.a) and with disturbances (b), where the graphs are (a) the time responses for velocity output and observer output, (b) the controller signal output and (C) the switching surface, respectively. Looking to the corresponding responses, they are oscillatory (chattering). The system stability is guaranteed, since the system trajectories converged to the switching function and then to the original (see switching surface simulations (c)). The responses were found to prove that the ISMC controller design is guaranteed to track the step input.

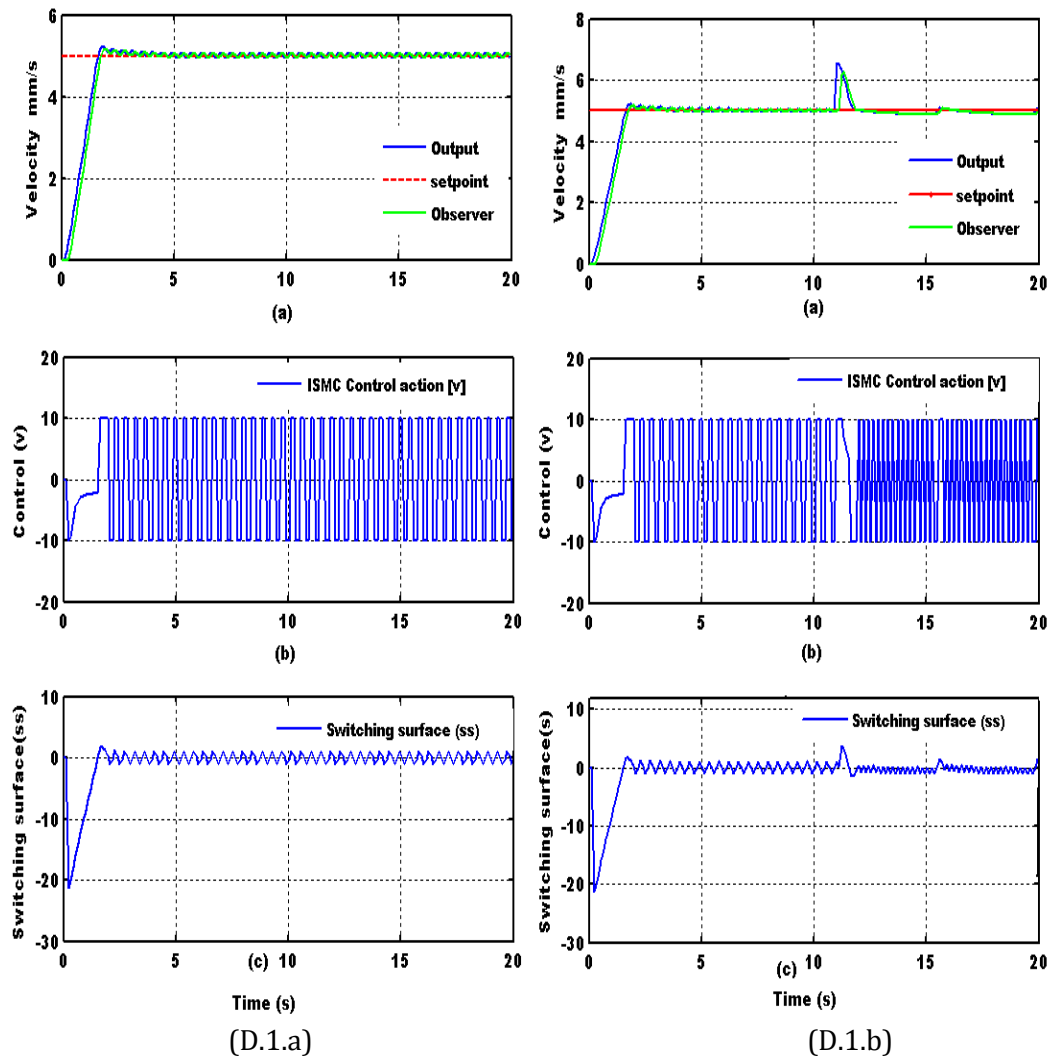


Figure D.1: Chattering Simulations showing time responses of ISMC controller without disturbances (a) and simulation results with disturbances (b).

Appendix E: The Overshoot, Final Position Error and Settling Time Data

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-PI	3	25	0.0171	0.3466	2.4240
P-PI	3	25	0.0025	0.4445	2.5700
P-PI	3	25	0.0172	0.2391	2.6120
P-PI	3	25	0.0152	0.4276	2.7500
P-PI	3	25	0.0030	0.3265	2.7800
P-PI	3	50	0.0198	0.3211	2.4300
P-PI	3	50	0.0127	0.4724	2.5720
P-PI	3	50	0.0077	0.377	2.6120
P-PI	3	50	0.0164	0.4228	2.7500
P-PI	3	50	0.0147	0.5615	2.8000
P-PI	3	100	0.0199	0.2673	2.4600
P-PI	3	100	0.0178	0.5286	2.5500
P-PI	3	100	0.0164	0.346	2.5800
P-PI	3	100	0.0164	0.7515	2.6000
P-PI	3	100	0.0153	0.3343	2.6500
P-PI	6	25	0.0366	0.441	4.3500
P-PI	6	25	0.0124	0.6503	4.3300
P-PI	6	25	0.1891	0.9077	4.3400

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-PI	6	25	0.1372	0.5863	4.3000
P-PI	6	25	0.0226	0.8026	4.3000
P-PI	6	50	0.0016	0.7848	4.5400
P-PI	6	50	0.0012	0.568	4.5700
P-PI	6	50	0.0016	0.0553	4.6000
P-PI	6	50	0.0090	0.8846	4.5300
P-PI	6	50	0.0013	1.0000	4.5000
P-PI	6	100	0.0020	1.0000	4.5400
P-PI	6	100	0.0080	0.4072	4.5500
P-PI	6	100	0.0060	0.6973	4.4700
P-PI	6	100	0.0013	0.3522	4.4500
P-PI	6	100	0.0012	0.7615	4.5300
P-PI	9	25	0.1964	1.2066	6.5300
P-PI	9	25	0.2325	0.9632	5.5500
P-PI	9	25	0.4200	0.6899	6.5900
P-PI	9	25	0.4571	1.4606	6.6600
P-PI	9	25	0.6037	1.1225	6.8600
P-PI	9	50	0.0170	1.23	6.6000
P-PI	9	50	0.0017	1.6778	6.6400
P-PI	9	50	0.0016	1.0902	6.6500
P-PI	9	50	0.0014	0.563	6.6700

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-PI	9	50	0.0013	1.0423	6.7300
P-PI	9	100	0.0018	1	6.6400
P-PI	9	100	0.0073	1.2324	6.6800
P-PI	9	100	0.0015	1	6.6400
P-PI	9	100	0.0014	0.9678	6.6800
P-PI	9	100	0.0013	1.905	6.7000
P-ISMC	3	25	0.0002	0.3649	2.5000
P-ISMC	3	25	0.0000	0.4751	2.6000
P-ISMC	3	25	0.0010	0.2551	2.6400
P-ISMC	3	25	0.0078	0.4791	2.7800
P-ISMC	3	25	0.0096	0.3561	2.7900
P-ISMC	3	50	0.0010	0.3459	2.5000
P-ISMC	3	50	0.0010	0.5036	2.6200
P-ISMC	3	50	0.0010	0.403	2.6500
P-ISMC	3	50	0.0010	0.4746	2.7800
P-ISMC	3	50	0.0010	0.5909	2.8000
P-ISMC	3	100	0.0010	0.2932	2.5000
P-ISMC	3	100	0.0071	0.5598	2.6200
P-ISMC	3	100	0.0087	0.3718	2.5500
P-ISMC	3	100	0.0078	0.8036	2.6000
P-ISMC	3	100	0.00064	0.3636	2.6500
P-ISMC	6	25	0.0042	0.3564	4.3300
P-ISMC	6	25	0.0010	0.6439	4.3500
P-ISMC	6	25	0.1205	0.9169	4.4000
P-ISMC	6	25	0.3065	0.4948	4.3200
P-ISMC	6	25	0.0088	0.8393	4.3000

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-ISMC	6	50	0.0082	0.8026	4.5000
P-ISMC	6	50	0.0079	0.573	4.5300
P-ISMC	6	50	0.0069	1.0783	4.4700
P-ISMC	6	50	0.0084	0.9017	4.4360
P-ISMC	6	50	0.0007	0.8104	4.4800
P-ISMC	6	100	0.0010	0.8593	4.5200
P-ISMC	6	100	0.0078	0.411	4.5500
P-ISMC	6	100	0.0007	0.7296	4.4800
P-ISMC	6	100	0.0084	0.3694	4.4600
P-ISMC	6	100	0.0095	0.8010	4.5000
P-ISMC	9	25	0.162	0.8919	6.5500
P-ISMC	9	25	0.3199	0.6976	5.5800
P-ISMC	9	25	0.4574	1.5374	6.6000
P-ISMC	9	25	0.4801	1.25	6.7000
P-ISMC	9	25	0.428	0.7997	6.9000
P-ISMC	9	50	0.0010	1.3046	6.5500
P-ISMC	9	50	0.0010	1.7616	5.5800
P-ISMC	9	50	0.0010	1.1434	6.6100
P-ISMC	9	50	0.0010	0.6406	6.6400
P-ISMC	9	50	0.0142	1.1415	6.6700
P-ISMC	9	100	0.0074	1.3394	6.5500
P-ISMC	9	100	0.0090	1.1622	6.5800
P-ISMC	9	100	0.0100	1.0077	6.6100
P-ISMC	9	100	0.0010	0.9358	6.6400
P-ISMC	9	100	0.0082	0.8248	6.6700
P-ISMC+BD	3	25	0.0418	0.0626	2.8500

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-ISMV+BD	3	25	0.0418	0.1824	2.9700
P-ISMV+BD	3	25	0.0418	0.0422	3.0100
P-ISMV+BD	3	25	0.0418	0.1987	3.1400
P-ISMV+BD	3	25	0.0405	0.0715	3.1700
P-ISMV+BD	3	50	0.0405	1.0340	2.8500
P-ISMV+BD	3	50	0.0405	0.8467	3.0000
P-ISMV+BD	3	50	0.0405	0.9368	3.0200
P-ISMV+BD	3	50	0.0405	0.8221	3.1500
P-ISMV+BD	3	50	0.0405	0.9029	3.1800
P-ISMV+BD	3	100	0.0405	1.8106	2.8500
P-ISMV+BD	3	100	0.0405	1.4875	3.0000
P-ISMV+BD	3	100	0.0405	1.6441	3.0200
P-ISMV+BD	3	100	0.0405	1.1608	3.1450
P-ISMV+BD	3	100	0.0405	1.5776	3.1800
P-ISMV+BD	6	25	0.0005	0.1422	4.4200
P-ISMV+BD	6	25	0.0010	0.3273	4.4500
P-ISMV+BD	6	25	0.1463	0.3816	4.4600
P-ISMV+BD	6	25	0.1978	0.1811	4.5200
PISMV+BD	6	25	0.0187	0.5642	4.5700
P-ISMV+BD	6	50	0.0405	0.3159	4.6000
P-ISMV+BD	6	50	0.0405	0.0766	4.6200
P-ISMV+BD	6	50	0.0405	0.5948	4.6700
P-ISMV+BD	6	50	0.0405	0.4091	4.6700
P-ISMV+BD	6	50	0.0405	0.3244	4.8000
P-ISMV+BD	6	100	0.0405	0.6865	4.6000
P-ISMV+BD	6	100	0.0405	1.14	4.6200

<i>Controller</i>	<i>Speed</i>	<i>Position</i>	<i>Overshoot</i>	<i>p_error</i>	<i>Settling</i>
P-ISMC+BD	6	100	0.0405	0.8121	4.6300
P-ISMC+BD	6	100	0.0405	1.1618	4.6500
P-ISMC+BD	6	100	0.0405	0.7044	4.6700
P-ISMC+BD	9	25	0.5406	1.2355	6.6200
P-ISMC+BD	9	25	0.5021	1.0734	6.6500
P-ISMC+BD	9	25	0.5729	1.7446	6.6800
P-ISMC+BD	9	25	0.5339	1.5673	6.7200
P-ISMC+BD	9	25	0.8045	1.1438	6.7500
P-ISMC+BD	9	50	0.0614	0.6623	6.5200
P-ISMC+BD	9	50	0.0502	1.1244	6.5800
P-ISMC+BD	9	50	0.0645	0.5092	6.6100
P-ISMC+BD	9	50	0.0010	1.0027	6.6400
P-ISMC+BD	9	50	0.0678	0.3974	6.6700
P-ISMC+BD	9	100	0.0405	0.4916	6.5800
P-ISMC+BD	9	100	0.0405	0.6795	6.6300
P-ISMC+BD	9	100	0.0405	0.8441	6.6700
P-ISMC+BD	9	100	0.0518	0.9058	6.7000
P-ISMC+BD	9	100	0.14869	1.0273	6.7700

Appendix F: Energy Laplacian *Image Blurring Index* Outcomes

<i>Speed</i>	<i>Position</i>	<i>Controller</i>	<i>Hand</i>	<i>Chest</i>	<i>Skull</i>	<i>Knee</i>
3	25	P-PI	1.61E-05	2.40E-06	1.92E-05	1.92E-05
3	50	P-PI	1.45E-05	3.18E-05	6.48E-05	6.48E-05
3	100	P-PI	1.50E-05	3.15E-05	3.03E-05	3.03E-05
6	25	P-PI	1.03E-04	1.62E-05	8.72E-05	8.72E-05
6	50	P-PI	8.73E-05	1.87E-05	3.80E-05	3.80E-05
6	100	P-PI	1.55E-05	1.13E-05	6.17E-05	6.17E-05
9	25	P-PI	7.80E-05	1.10E-05	3.54E-05	3.54E-05
9	50	P-PI	7.97E-05	3.66E-05	8.04E-05	8.04E-05
9	100	P-PI	8.89E-05	4.76E-05	8.55E-05	8.55E-05
3	25	P-ISMC	1.62E-04	1.98E-05	3.91E-05	3.91E-05
3	50	P-ISMC	7.17E-05	2.51E-06	7.53E-05	7.53E-05
3	100	P-ISMC	7.33E-05	4.20E-05	5.47E-05	5.47E-05
6	25	P-ISMC	4.36E-05	3.64E-05	3.57E-05	3.57E-05
6	50	P-ISMC	8.36E-05	3.51E-05	8.71E-05	8.71E-05
6	100	P-ISMC	7.79E-05	2.76E-06	9.69E-05	9.69E-05
9	25	P-ISMC	8.76E-05	5.54E-05	4.88E-05	4.88E-05
9	50	P-ISMC	4.69E-05	3.08E-05	7.79E-05	7.79E-05
9	100	P-ISMC	8.06E-05	4.24E-05	7.70E-05	7.70E-05
3	25	P-ISMC+BD	7.64E-05	1.59E-05	1.95E-05	1.95E-05
3	50	P-ISMC+BD	6.62E-05	1.67E-05	3.89E-05	3.89E-05
3	100	P-ISMC+BD	1.38E-05	1.67E-05	3.20E-05	3.20E-05
6	25	P-ISMC+BD	8.78E-05	3.55E-05	8.48E-05	8.48E-05
6	50	P-ISMC+BD	1.09E-04	1.99E-05	8.49E-05	8.49E-05

<i>Speed</i>	<i>Position</i>	<i>Controller</i>	<i>Hand</i>	Chest	Skull	Skull
6	100	P-ISMC+BD	4.83E-05	1.68E-05	7.50E-05	7.50E-05
9	25	P-ISMC+BD	1.47E-05	2.46E-05	1.44E-05	1.44E-05
9	50	P-ISMC+BD	1.59E-05	6.81E-05	5.50E-05	5.50E-05
9	100	P-ISMC+BD	1.48E-05	4.76E-05	4.78E-04	4.78E-04

Appendix G: Resources Used

These are included on a compact disk (CD) that is submitted together with this dissertation. The main files are the Time Delay Integration Simulation Code, both controllers' simulation codes and the Image Blurring Index code which generated the distorted images, the controllers' simulation performance, distance profiles generation and *Image Blurring Index*, respectively. The other files are C/C++ files that were used for experimental testing. Text files of the data extracted from step tests of the DC motor process are also included.