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Vibration Serviceability of Long Span Slender Floors

By

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Abstract

The modern trend towards long slender floors has led to major vibration serviceability issues in building floors under rhythmic loading. This happens as a result of reduced natural frequencies and damping ratios due to reduced structural rigidity (Devin et al, 2015). Even though these vibrations do not lead to structural failure in most cases, they can often create such an excessive discomfort to the occupants that they render the structure unusable (Orvin et al, 2016). A serviceable long-span floor is a floor that does not only carry the permanent and imposed loads applied onto it, but one that is stiff enough to prevent excessive deflections as well as vibrations (Cement and Concrete Association of Australia, 2003).

Common structural systems used for long span floors are plane trusses, space frames and space grids. These types of structures are generally lighter and stronger because individual members carry externally applied loads mainly through tension and compression which makes them more structurally efficient than solid-web girders which are subjected to flexure (Porwal et al, 2017). This research focusses on the vibration serviceability of long span floors subjected to rhythmic loads such as aerobics, dance-type loads or similar audience participation activities.

The most important parameter in the design for vibration serviceability is the natural frequency and simplified methods in which it can be estimated for concrete floors supported on a steel frame or truss exist. According to the National Building Code of Canada (NBC) resonance for structures exposed to human activity can occur if the natural frequency of the structure is below 10 Hz which takes into consideration the believe by the NBC that resonance is attainable if the forcing frequency of a repetitive motion such as dancing is around half the natural frequency of the floor (Murray et al, 2003). Another important parameter is acceleration. Critical floor accelerations occur at resonance but their effect on vibration serviceability of the floor depend on human perception. This is dependent on the activity type the occupants are involved in. Acceptable acceleration limits are recommended in the NBC and the International Standards Organization (ISO 2631-2, 1989) for various occupancies such as office or residential, dining or weightlifting and rhythmic activity (Murray et al, 2003). Because the criterion by the NBC is based on the beam theory, models the floor as a single degree-of-freedom and only considers the fundamental mode of vibration, Ji et al (1994) and Ellis et al (2004) proposed an alternative criterion which entails the characterization of the load which pertains to the determination of the load model in a form of a Fourier series, evaluation of the characteristics of the floor vibration and calculation of the response of the floor to the dancing loads. Of all the design codes reviewed, only the British Standard provided the load model which can be used in the analysis of structures subjected to rhythmic loads. The UK National Annex to Eurocode 1: Part 1-1 does, however, recommend that a designer consult literature that provide rhythmic load models such as that by Ellis et al (2004).

Measures that can be applied to improve the vibration serviceability of planned or existing structures include increasing the stiffness of the structure, increasing the damping, installing tuned vibration absorbers and restricting the usage of the structure (Erlina et al, 2017).

Three different floor systems were studied in this research, namely, the plane truss, space frame and space grid floors and different measures were applied to evaluate their effect on the vibration

behaviour of these floor systems under rhythmic loading. The results showed that increasing the number of column supports, adding extra edge supports around the floor perimeter and increasing the floor depth all improve the vibration serviceability of the floor. The results also showed that the space frame floor performed better under rhythmic loading than the plane truss and space grid floors. The space frame, however, required large members (resulting in a relatively heavy structure) to work and is complicated and costly to construct. It was therefore determined that when taking into consideration both the vibration serviceability of the structure and the ease and cost of construction that the space grid was the best structural system to employ. Physical tests need to be carried out on existing long span floors under rhythmic loads to verify the FE analysis results obtained in this study. Guidelines in design codes should also be updated to make special provision for long-span floors subjected to rhythmic loading.

Acknowledgements

I firstly would like to thank the Lord God almighty for giving me the courage to undertake and complete this study and also my family and friends for their support and prayers.

I would also like to thank my supervisor Prof. Pilate Moyo for agreeing to supervise me and for his guidance and patience throughout the duration of this thesis.

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g – gravitational acceleration

E_s – modulus of elasticity for steel

I – moment of inertia

I_t – transformed moment of inertia

w – uniformly distributed weight per unit length

L – span of member

Δ – midspan deflection of the member relative to its supports due to the weight supported

f_j – beam or joist panel mode frequency

f_g – girder panel mode frequency

Δ_j – beam or joist deflection due to the weight supported

Δ_g – girder deflection due to the weight supported

Δ_c – axial shortening of the column due to the weight supported

I_{eff} – the "effective" transformed moment of inertia which accounts for shear deformation

I_{comp} – the fully composite moment of inertia

I_{chords} – the moment of inertia of the joist chords alone

δ – deflection

F_i – axial force due to working loads

L_i – length of the web member

A_i – cross-sectional area of the web member

γ_i – angle of the web member to the vertical

a_p – peak acceleration

α_i – dynamic coefficient

w_p – effective weight per unit area of participants distributed over floor panel

w_t – effective distributed weight per unit area of floor panel, including occupants

f_n – natural frequency

f – forcing frequency

β – damping ratio

a_m – peak acceleration for the i^th^ harmonic

a_0 – peak acceleration limit

K_p – impact factor

F_{max} – peak dynamic load per unit area

G – weight of dancer per unit area

t_p – contact duration

T_p – dancing load period

r_n – Fourier coefficient (or dynamic load factor) of the n^{th} term

n – number of Fourier terms

φ_n – phase lag of the n^{th} term

$\Phi(x, t)$ – the dimensionless fundamental mode with a unit peak value

$A(t)$ – the amplitude of vibration

A – displacement

$\ddot{A}$ – acceleration

B – structural factor

m – structural mass

D – dynamic magnification factor for displacement

D^a – dynamic magnification factor for acceleration

ξ – damping ratio, Ji et al (1994)

β – frequency ratio, w_p/w_s, Ji et al (1994)

1. Introduction

1.1 Background

Apart from designing safe buildings, one of the main goals of a structural engineer is to design structures that are cost efficient and easy to construct. One of the ways the designers achieved this a few decades ago was by keeping floor spans in the range of 6 to 9 metres. In modern times, however, clients and tenants have developed an affinity towards large column-free spaces which has resulted into the design of much larger spans ranging from 9 to 16 metres (Cement and Concrete Association of Australia, 2003). This is achieved by the use of modern construction techniques and lightweight materials. This development has, however, come with a downside in a form of vibration serviceability (Orvin et al, 2016).

Floor vibrations has long been an issue in buildings. It is the annoying vibration a person sitting in a chair feels when another person walks into the room or when items are accidentally dropped due to the excessive vibrations caused by children playing (Kalkert et al, 1992). The modern trend towards longer spans and slender floors has amplified this problem due to the reduced natural frequencies and damping ratios of the floors (Devin et al, 2015). Though these vibrations do not necessarily cause structural failure, they can render the structure unusable if they create excessive discomfort to the occupants of the structure. This can occur when the floor has a natural frequency below 10 Hz, in which case it can experience resonance under human motions (Orvin et al, 2016).

Aerobics, dancing and other rhythmic human activity have been reported to be the cause of some of the annoying vibrations experienced in buildings in recent years. Sometimes this is due to resonance taking place during the rhythmic activity and sometimes it is due to other occupancies such offices and restaurants, where people are more sensitive to vibrations, being present in the building. A serviceable long-span floor is a floor that does not only carry the permanent and imposed loads applied onto it, but one that is stiff enough to prevent excessive deflections as well as vibrations (Cement and Concrete Association of Australia, 2003).

1.2 Objective of Study

The primary objective of this thesis is to study the vibration serviceability of different long-span floor systems supported by steel truss and space grid systems and subjected to rhythmic loading, as well as to develop an optimised structural system that will best meet the serviceability limit state conditions for vibrations.

This study will also look at the provisions given by current design codes and guidelines for vibration serviceability of long-span floors whilst also reviewing the available load models for rhythmic human loads, especially those that would occur in spaces such as concert halls where long clear spans are often required.

1.3 Scope and Limitations of Study

This study focuses on:

i) Investigating the design guidelines for vibration serviceability of long-span floors
ii) Studying the rhythmic load models available in literature
iii) Investigating typical structural systems used for long-span floors
iv) Creating finite element (FE) models for various long-span floor systems
v) Drawing comparisons between the vibration serviceability of different floor systems
vi) Determining the best floor system for the purposes of vibration serviceability of long-span floors subjected to rhythmic loading

Limitations:

i) Although the development of rhythmic load models is covered in the literature review, and is significant in the design of floors subjected to rhythmic loading, this research only makes use of the dynamic properties of various long span floor structures susceptible to rhythmic loading to determine their vibration serviceability. It therefore does not focus on the application of rhythmic load models.
ii) This study does not entail conducting of physical tests on existing structures to determine their vibration serviceability but instead relies solely on the results obtained from FE analyses

1.4 Thesis Outline

This report consists of the following chapters:

- Chapter 1 is an introduction to the report.
- Chapter 2 is a review of long-span floor systems.
- Chapter 3 looks at the methods developed for estimating the natural frequency of steel framed floor systems.
- Chapter 4 provides a literature review of the early studies, vibration serviceability of floors and the design criteria for rhythmic excitation.
- Chapter 5 describes the methodology for modelling and analysing the various floor systems
- Chapter 6 provides the results obtained from the FE analysis of the floor models.
- Chapter 7 discusses the findings of this research
- Chapter 8 draws conclusions based on the findings of the study
- Chapter 9 gives recommendations for future study
- List of references follows the recommendations.
- The appendices follow the list of references.

2. Background to Long-Span Floor Systems

Modern technology has made the design of long clear spans possible. However, along with these advancements came floor vibration issues. Where waffle slabs and concrete joist slabs were enough to meet vibration serviceability requirements, new techniques now need to be developed to address floor vibration challenges (Tang et al, 2009). Before delving into vibration serviceability of long-span floors, however, it is first important to study the different techniques that have been employed to design and construct long-span floors.

The key to achieving long spans is using lighter and stronger material. Early floor systems such as waffle slabs and concrete joist slabs would require very deep floors to achieve long spans. This does not only increase the weight of the floor, and hence larger columns and foundations, but also makes the overall structure less cost efficient. One structural system that has proven to be an efficient way of reducing structural weights whilst retaining the floor strength is the truss design. This type of structural systems is well known to be able to span for over 30m (Tang et al, 2009).

Trusses are triangular frameworks in which members carry externally applied loads mainly through tension and compression. This gives them an advantage over solid-web girders in long-span structures because axially loaded steel is generally more structurally efficient than steel subjected to flexure. This subsequently allows for the utilisation of less material and in turn an overall lighter structure compared to solid web girders. Another characteristic of trusses that gives them an edge over other options as flooring support systems is that they are deeper and are an open-web system that allows for ducts and ventilation pipes to be accommodated within the truss without further increasing the depth of the ceiling (Porwal et al, 2017). This study will focus on three types of floors, namely, the plane truss floor, the space frame floor and space grid floor.

2.1 Plane Trusses

Plane trusses are the type of trusses whereby the external load and the members lie in the same plane. The most common of these are pitched roof trusses which consist of sloping top chords to facilitate natural drainage of rainwater and clearance of dust/snow accumulation. For long span pitched roofs the trapezoidal trusses are preferred.

Another type of truss is the parallel chord truss. This flat truss configuration is also used on roofs but unlike the previous configurations, can also be used for floor support as prefabricated floor joists, beams and girders as shown in Figure 2-1(a). Other common parallel chord truss configurations are the Warren truss, the lattice girder, the K type web and the diamond type web as shown in Figures 2-1(b), 2-1(c), 2-1(d) and 2-1(e), respectively. The K and diamond type web configurations are considered more suitable for very deep and very shallow trusses for the purposes of maintaining an inclination of around 45 degrees for web members (Porwal et al, 2017).

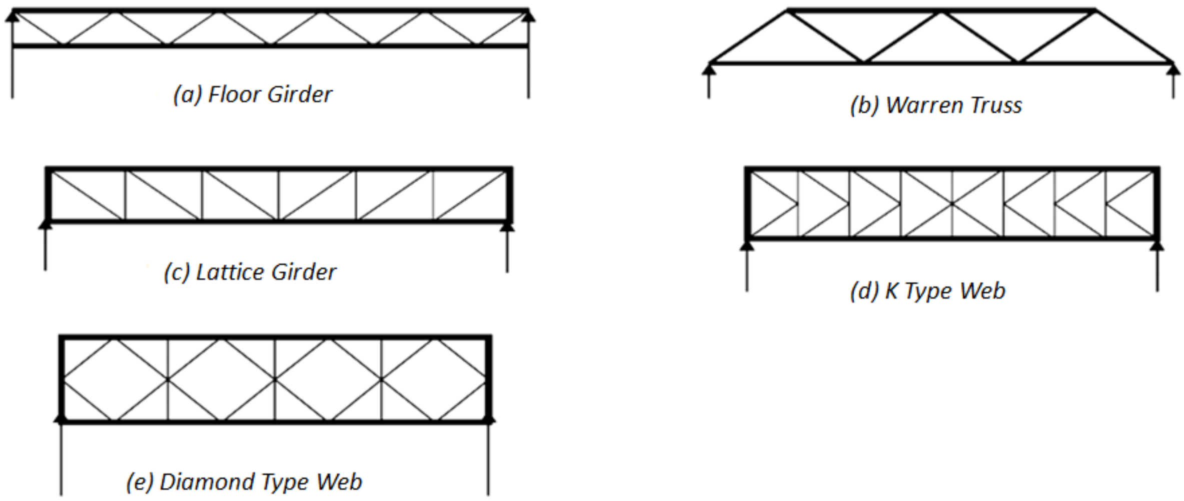


Figure 2-1: *Parallel chord trusses (Porwal et al, 2017)*

2.2 Space Frames

The two most popular definitions of space frames in literature are given by the International Association for Shell and Spatial Structures (IASS) Working Group on Spatial Steel Structures (1984) and the ASCE Task Committee on Latticed Structures of the Committee on Special Structures of the Committee on Metals of the Structural Division (1976) and they state as follows:

International Association for Shell and Spatial Structures (IASS)-

"A space frame is a structure system assembled of linear elements so arranged that forces are transferred in a three-dimensional manner. In some cases, the constituent element may be two-dimensional. Macroscopically a space frame often takes the form of a flat or curved surface."

American Society for Civil Engineers (ASCE)-

"A latticed structure is a structure system in the form of a network of elements (as opposed to a continuous surface). Rolled, extruded or fabricated sections comprise the member elements. Another characteristic of latticed structural system is that their load-carrying mechanism is three dimensional in nature."

The difference between a space frame (also known as a three dimensional truss) and a space grid is that a space frame has rigid joints (see Figure 2-2) which result in internal torsional and bending moments in members, whereas a space grid has hinged joints and its members are assumed to experience only pure axial loads. In practice, however, there are no absolutely rigid or hinged connections (Lan, 1999).

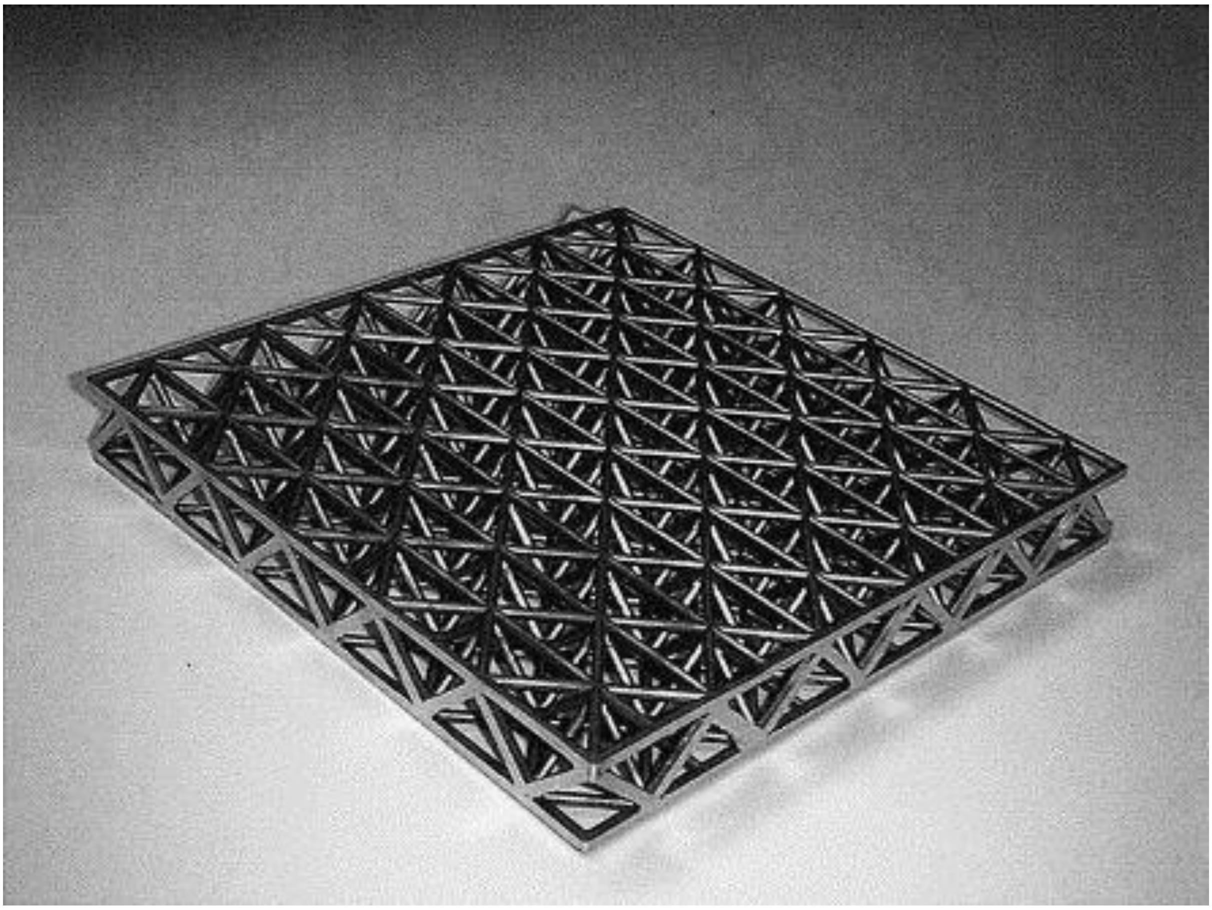


Figure 2-2: *Example of a space frame structure (Wallach et al, 2001)*

2.3 Space Grids

Grid structures are categorized as either flat surface grids or latticed shells. The most common form of flat surface grid is a double-layered grid but triple-layered and other multi-layered grids are also used where the spans are long, and a large structural depth is required. Latticed shells on the other hand are curved single layer grids which can be of many different forms and shapes (Lan, 1999). Space grids can also be characterized as two- and three-way space grids depending on whether the elements intersecting at a node run in two or three directions (Ramaswamy et al, 2002).

Double-layered grids consist of two parallel planar grid networks forming top and bottom layers which are interconnected by vertical and diagonal web members. Because they are jointed together by pin connections (see Figure 2-3), members in a double-layered grid have no resistance to moments or torsion and therefore resist loads in a purely axial manner, i.e. through tension and compression (Lan, 1999).

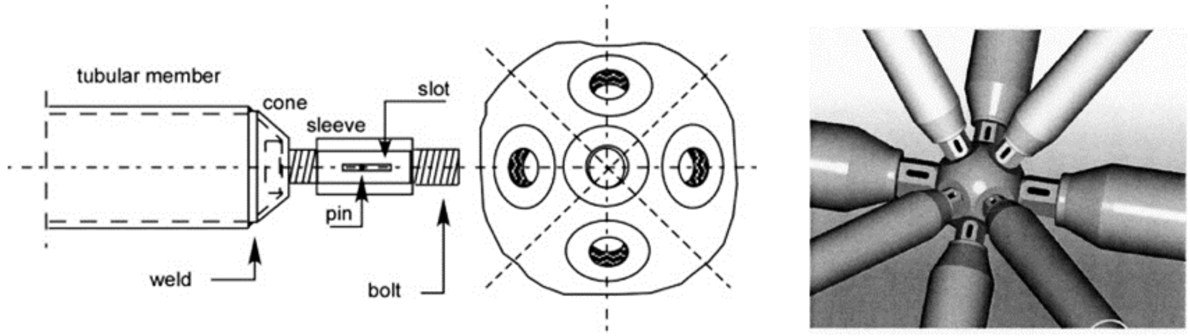


Figure 2-3: *Typical member connection for a double-layered grid structure (Adams, 2012)*

Double-layered grids are usually composed of basic elements such as (Lan, 1999):

1) a planar latticed truss as shown in Figure 2-4(a)
2) a pyramid with a square base (octahedron) as shown in Figure 2-4(b)
3) a pyramid with a triangular base (tetrahedron) as shown in Figure 2-4(c)

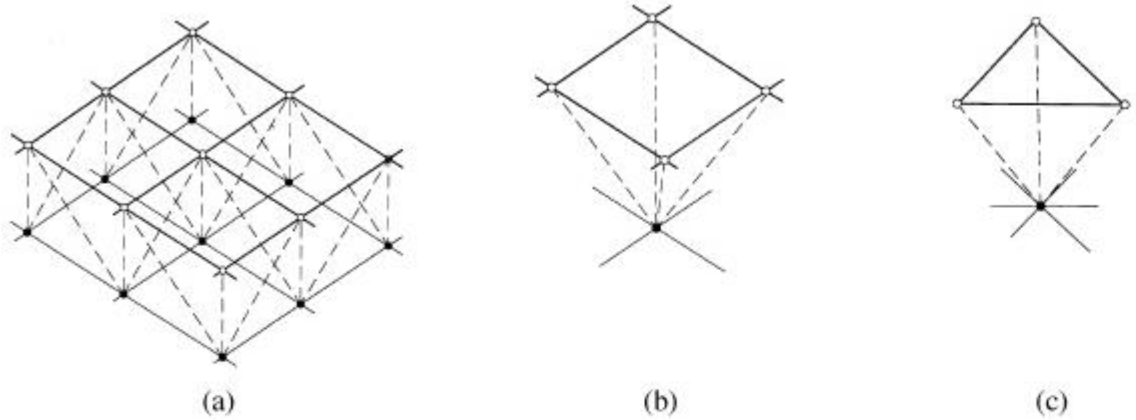


Figure 2-4: *Basic elements of double-layered grids (Lan, 1999)*

Many other types of double-layered grids can be formed by varying the directions of the of the top and bottom layers with respect to each other and also by varying the positions of the top and bottom nodes with respect to each other. Even more variations can be made by changing the sizes of the openings in one of the layers. Double-layer grids with aligned layers (i.e. no difference in directions) and no offset of nodes with respect to each other are known as latticed grids and ones with variations between top and bottom layers are called space grids. Their different characteristics have been listed below (Lan, 1999).

Latticed grids:

1) consist of intersecting vertical latticed trusses and form a regular grid,
2) two parallel grids are similar in design, with one layer directly over the top of another, and
3) top and bottom grids are directionally the same (see Figure 2-5).

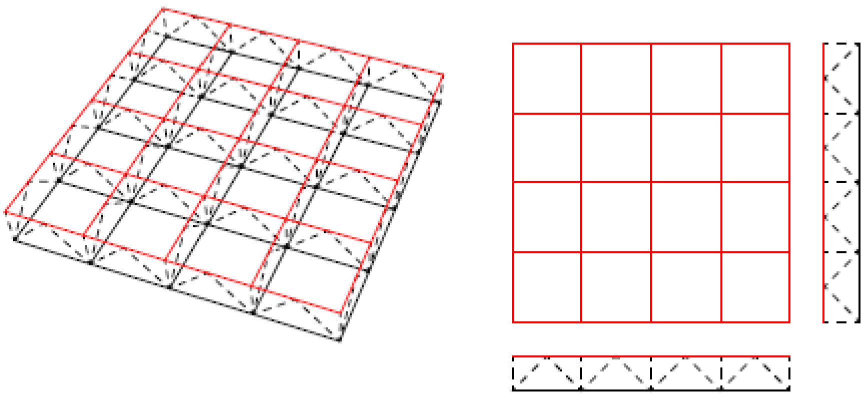


Figure 2-5: *Example of a latticed grid structure (Buijsen, 2011)*

Space grids:

1) consist of a combination of square or triangular pyramids, and
2) covers both the offset grids and differential grids.

Offset grids consist of parallel grids with an identical layout, one grid being offset from the other in plane but remaining directionally the same (see Figure 2-6). Differential grids on the other hand consist of two parallel top and bottom grids with different layout but are chosen to coordinate and form a regular pattern (Makowski, 1992).

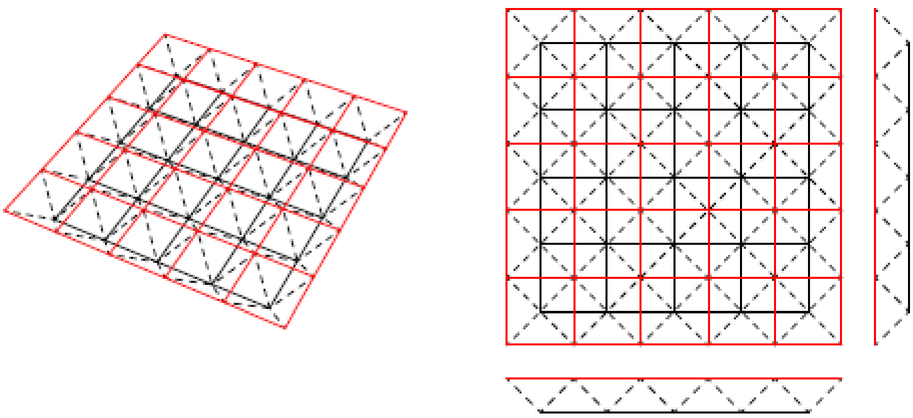


Figure 2-6: *Example of a space grid structure (Buijsen, 2011)*

Advantages of space grids:

1) Space grids consist of a very low self-weight which makes them more economical in covering large column-free spaces. This does not only make them easy to transport and assemble, but also reduces the number of columns required to carry the dead load.
2) Load transfer mechanism is primarily axial, i.e. compression and tension, hence allowing all material to operate at maximum capacity.
3) They are considerably rigid due to a three-dimensional distribution of loads along the elements. This allows flexibility in the layout and positioning of columns.
4) Space grids are built from factory manufactured units which come in standard sizes. They are therefore easy to assemble even by unskilled labour therefore reducing construction time and labour costs.
5) Services such as cabling and air conditioning can be integrated within the space grids due to their open nature.
6) Construction is safer since the structure can be assembled on the ground and therefore reducing the risk of working at heights.
7) They are generally more visually appealing and can be formed into many different kinds of shapes including free-form shapes. (Lan, 1999)(Ramaswamy et al, 2002)

Disadvantages of space grids:

1) Costs can be high compared to other options, e.g. portal frames.
2) At some viewing angles the structure loses its regularity and looks too dense/busy.
3) Depending on the number of joints in the grids, erection times can be very long, even longer than for other structural systems.
4) Since space grid components are made of steel or aluminium, some coating needs to be applied and this can be expensive due to a large number of bars in the structure. (Chilton, 2000)

3. Natural Frequency of Steel Framed Floor Systems

Natural frequency is the most important parameter in the design for vibration serviceability and simplified methods in which it can be estimated for concrete floors supported on a steel frame or truss are provided in this section.

3.1 Fundamental Relationships

In the following estimations for the vertical fundamental natural frequency, it is assumed that the floor consists of a concrete slab supported on steel beams or joists which are in turn supported on walls or steel girders sitting on columns. Assuming that the system is linear, the procedure into these estimations involves separating the vibration modes into “beam or joist panel” modes and “girder panel” modes and then combining them to obtain the final mode of vibration. Both these modes can be estimated individually from Equation (3-1), which is the fundamental natural frequency equation of a uniformly loaded, simply supported, beam

$$f_n = \frac{\pi}{2}\left[\frac{gE_sI_t}{wL^4}\right]^{1/2} \qquad (3-1)$$

where

f_n – fundamental natural frequency, Hz

g – gravitational acceleration

E_s – modulus of elasticity for steel

I_t – transformed moment of inertia (effective transformed moment of inertia, if shear deformations are included)

w – uniformly distributed weight per unit length (working, not design, live and dead loads)

L – span of the member

If $\Delta = 5wL^4/(384E_sI_t)$, where Δ is the midspan deflection of the member relative to its supports due to the weight supported, Equation (3-1) can be rewritten as shown in Equation (3-2).

$$f_n = 0.18\sqrt{\frac{g}{\Delta}} \qquad (3-2)$$

The combined mode or system frequency can be estimated from Equation (3-3), which is known as the Dunkerley relationship

$$\frac{1}{f_n^2} = \frac{1}{f_j^2} + \frac{1}{f_g^2} \qquad (3-3)$$

where

f_j – beam or joist panel mode frequency

f_g – girder panel mode frequency

or alternatively as shown in Equation (3-4) which is derived from Equation (3-2)

$$f_n = 0.18\sqrt{\frac{g}{(\Delta_j + \Delta_g)}} \qquad (3-4)$$

where

Δ_j – beam or joist deflection due to the weight supported

Δ_g – girder deflection due to the weight supported

If vertical column frequencies are also critical, as can be the case in tall buildings, Equation (3-4) can be modified to include the column deflection to obtain Equation (3-5) below (Murray et al, 2003)

$$f_n = 0.18\sqrt{\frac{g}{(\Delta_j + \Delta_g + \Delta_c)}} \qquad (3-5)$$

where

Δ_c – axial shortening of the column due to the weight supported

3.2 Composite Action

Composite action should be taken into consideration where the floor slab (or deck) is attached to the supporting members either through shear connectors, deck-to-member spot welds or by friction between the concrete and metal surfaces. The increased stiffness due to composite action should be accounted for in the calculations by incorporating the supported slab into the calculation of the transformed moment of inertia of the beams or joists and girders. In this calculation, it is recommended that the effective width of the concrete slab be taken as the lesser of the member spacing and 0.4 times the member span. It is the lesser of half the member spacing and 0.2 times the member span, plus the projection of the free edge of the slab beyond the member centreline. For slabs consisting of concrete on a metal deck, it is recommended that the modulus of elasticity of the concrete be taken as 1.35 times that specified in the design standards (Murray et al, 2003). A typical detail of a concrete on metal deck slab with a shear connector is shown in Figure 3-1 below.

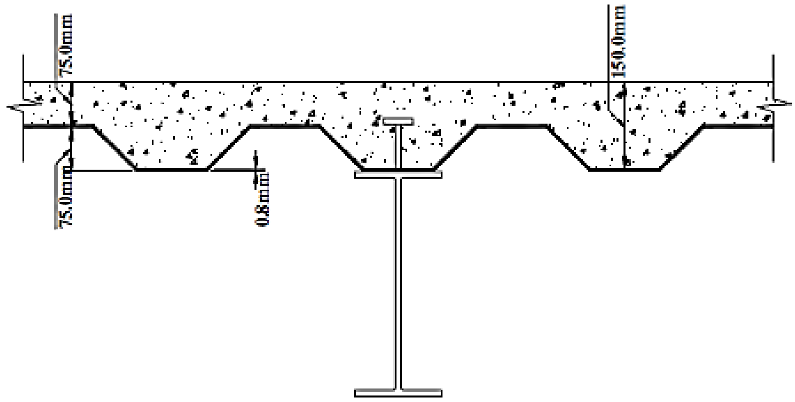


Figure 3-1: *Typical cross-section of a composite slab with a shear connector (da Silva et al, 2001)*

3.3 Distributed Weight

As explained in Section 3.1, the uniformly distributed load, *w*, used in the aforementioned frequency equations is the working load (not design load) supported by the floor. For typical office areas with desks, bookshelves and file cabinets, it is recommended that a live load of 0.5 kN/m^2 be used whereas a live load of 0.25 kN/m^2 is recommended for residential floors. For structures such as footbridges, gymnasiums and shopping centres, it is suggested that the live load be ignored. In cases where a member is not uniformly loaded but carries a midspan point load, it is suggested that the calculated deflection be factored by *4/π* to take into consideration the difference in frequency between a uniformly loaded simply-supported member and a simply-supported member carrying a point load midspan (Murray et al, 2003).

3.4 Flexural Deflection of Continuous Members

Equations in Section 3.1 for simply supported beams, joist or girders also apply for members continuous over the supports given that the member has equal spans. This is because adjacent spans of a continuous member deflect in opposite directions. Where spans are not equal, it is recommended that the deflections be determined as shown in Equations (3-6) and (3-7), where Δ_{ss} is a flexural deflection of a simply supported member of equivalent dimensions to the main (larger) span, L_m, due to the applied uniformly distributed load. For two continuous spans

$$\Delta = \left[\frac{0.4 + \frac{k_m}{k_s}\left(1 + 0.6\frac{L_S^2}{L_M^2}\right)}{1 + \frac{k_m}{k_s}}\right]\Delta_{SS} \qquad (3-6)$$

For three continuous spans

$$\Delta = \left[\frac{0.6 + 2\frac{k_m}{k_s}\left(1 + 1.2\frac{L_S^2}{L_M^2}\right)}{3 + 2\frac{k_m}{k_s}}\right]\Delta_{SS} \qquad (3-7)$$

where

$k_m = I_M/L_M$

$k_m = I_S/L_S$

I – moment of inertia

L_M and L_S are as shown in Figure 3-2.

Further procedures on how to estimate deflections for members fixed to columns and cantilevers are described by Murray et al (2003).

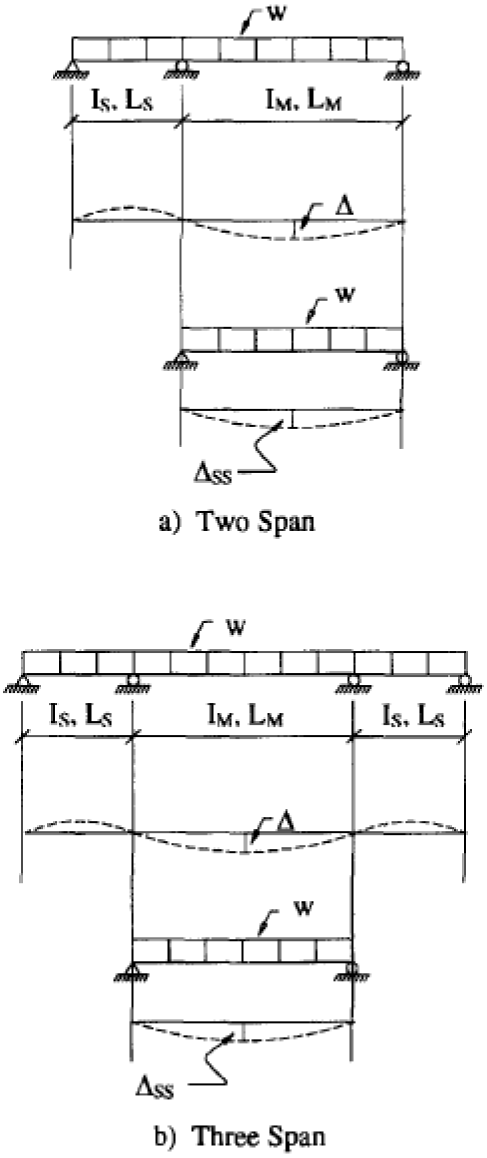


Figure 3-2: *Modal flexural deflections, Δ, for beams continuous over supports (Murray et al, 2003)*

3.5 Deflection due to Shear in Beams and Trusses

Apart from flexure, another cause of deflection in beams and trusses is shear. Shear may come in two forms:

1) Direct shear due to shear strain in the web of a beam or girder, or due to the change in length of the web members of a truss;
2) Indirect shear in trusses due to the eccentricity of member forces through joints.

These shear deflections are usually small relative to flexural deflections and may be ignored for rolled steel sections. For simply supported trusses, shear deflections can be accounted for by transforming the moment of inertia as shown in Equation (3-8) below:

$$I_{eff} = \frac{I_{comp}}{1 + \frac{0.15 I_{comp}}{I_{chords}}} \tag{3-8}$$

where

I_{eff} – the "effective" transformed moment of inertia which accounts for shear deformation

I_{comp} – the fully composite moment of inertia

I_{chords} – the moment of inertia of the joist chords alone

This equation applies to simply supported trusses with span to depth ratios equal or greater than 12. For deeper trusses, shear deflections may be estimated as follows, assuming that there is no eccentricity at the joints:

1) Determining web member forces due to the weight supported.
2) Determining the change in length of the *i*th member using Equation (3-9)

$$\delta_i = \frac{F_i L_i}{A_i E_s} \tag{3-9}$$

where

F_i – axial force due to working loads

L_i – length of the web member

A_i – cross-sectional area of the web member

3) Determining shear increments from Equation (3-10)

$$v_i = \frac{\delta_i}{\cos \gamma_i} \tag{3-10}$$

where

γ_i – angle of the web member to the vertical

4) Summing up all the shear increments for each member from the support to midspan

The shear deflection obtained from this method should be added to the flexural deflection in order to obtain an estimate total deflection to be used in the frequency equations. Alternatively, deflections and fundamental natural frequencies can be estimated by finite element analyses (Murray et al, 2003).

4. Literature Review

4.1 Review of Early Studies

Records on vibration serviceability go as far back as the 19th century where Thomas Tredgold, a famous carpenter and one of the fifty founders of the Institute of Civil Engineers (ICE), was recorded to have stated in 1828 that:

"Girders should always, for long bearings, be made as deep as they can be got; an inch or two taken from the height of a room is of little consequence compared with a ceiling disfigured with cracks, besides the inconvenience of not being able to move on the floor without shaking everything in the room".

Studies by Hyde et al (1929) also show that in 1901, H.R.A. Mallock conducted an experimental investigation on the vibrations caused by the Central London higher. His study followed numerous complaints about vibrations caused on houses near Hyde Park by passing trains. This is reported to be one of the earliest pieces of research on human sensitivity to vibrations in buildings. Mallock concluded in this study that peak floor accelerations of 5% g caused human discomfort while he considered accelerations five times less to be "noticeable". He also stated that human perception to floor vibrations varies from person to person (Pavic, 1999).

Hyde and Lintern conducted a similar study to Mallock and as part of their investigation designed a vibration measuring equipment as they deemed Mallock's one to be unsuitable and obsolete. They designed their instrumentation under the following requirements:

1) Portability
2) A low natural frequency of the main transducer
3) An easily reproducible graphical recording
4) Adequate mechanical scaling
5) Long data recording times, and
6) High sensitivity

These are the same requirements that modern digital data acquisition systems suitable for field measurements and the assessment of vibration serviceability in buildings still need to satisfy. Using the mechanical vibration measuring device they designed, Hyde and Lintern measured the maximum floor vibrations of 2% g, which they concluded only caused discomfort if they occurred over extended periods (Pavic, 1999).

4.2 Evaluation of Vibration Serviceability of Floors

Vibrations occur everywhere, all the time, and cannot be avoided. They only become a problem when they become excessive and thus causing annoyance, malfunction of sensitive equipment, damage or structural failure. Most often, however, the most important factor when dealing with vibration serviceability is human perception. This is, reportedly, a difficult issue to deal with, however, measures must be taken in design to address it (Pavic, 1999).

According to the ISO 10137 procedures, the first step towards the assessment of vibration serviceability of floors is to identify and characterise the following three key factors (Pavic, 1999):

1) The vibration source,
2) The transmission path, and
3) The receiver

4.2.1 Vibration Source

The general classification of floor vibration sources is either as internal vibration sources or as external vibration sources. Internal floor vibrations can be due to:

- Human excitation – walking, running, jumping and stomping.
- Machinery – elevators, lift trucks, punches and presses.
- Construction activities within the building.

ISO 10137 classifies vibration sources into two classes:

- Class A – vibration sources which vary both in time and in space (e.g. walking).
- Class B – vibration sources which vary only in time (e.g. mounted machinery)

ISO 10137 says of Class A vibration serviceability problems:

"The complexity of these problems is one reason why many of them have been treated by empirical methods, or by extensive use of measurements on similar existing structures" (Pavic, 1999).

4.2.2 Transmission Path

The transmission path is a medium through which excitations travel from the vibration source to the receiver. This may be in a form of columns, beams, walls, floors, etc. After leaving the source, vibration excitations are modified by the transmission path's physical properties – mass, stiffness and damping – before they can reach and be felt by the receiver (Pavic, 1999).

4.2.3 Receiver

According to ISO 10137, the receiver is the object or person for which the vibration effects are to be assessed. The person in this case is an occupant of the building whereas the object can be the windows, walls, beams, slabs, instruments, machinery, etc. Several criteria have been developed to determine the amount of vibration experienced by the receiver (Pavic, 1999).

There are two types of vibration serviceability assessment:

1) The assessment by calculations during the floor design stage
2) The assessment by measuring vibrations on an existing floor structure

Both of these cases require that the vibration source, transmission path and receiver be characterised but differ when it comes to how the responses at the receiver are determined. For the calculation-type assessment a mathematical model of the structure has to be developed and then analysed under a specified vibration excitation, whereas for the measurement-type assessment responses are measured on a real, existing physical structure, also under a predetermined vibration excitation (Pavic, 1999).

4.3 Rhythmic Excitation Design Criteria

4.3.1 Criterion in the National Building Code of Canada

An increase in the occurrence of vibration problems in buildings due to rhythmic activities has produced a need for a rhythmic design criterion. Reports of accelerations as high as 50 percent of the gravitational acceleration have been recorded in literature and have been reported to have caused structural fatigue problems. In the early studies conducted by the National Building Code of Canada (NBC), it was reported that resonance for structures exposed to human activity could occur if the natural frequency of the structure is below 5 Hz. That value was since revised to 10 Hz in 1975 as the NBC believed that resonance is also attainable if the forcing frequency of a repetitive motion such as dancing is around half the floor’s natural frequency, i.e. the beat being danced to is on every second cycle of the floor vibration (Murray et al, 2003).

In 1985 a criterion was developed by the NBC for the design of floor structures under rhythmic loading. This criterion was based on the dynamic response of structural systems to rhythmic loading applied over the entire floor, or a portion of it, and can be used to evaluate structural systems supporting known rhythmic loads. Rhythmic loads may entail aerobics, dancing, audience participation or similar events (Murray et al, 2003). Included in this criterion was a new clause requiring that dynamic analyses should be performed for floors subjected to rhythmic loading, with fundamental natural frequencies below 6 Hz. This provided an alternative to avoiding resonance as was required in the earlier additions (Ji et al, 1994). In 1990 this criterion was expanded in the NBC commentary to take into consideration sensitive occupancies. It uses the acceleration limits shown in Table 4-1 (Murray et al, 2003).

Table 4-1: *Recommended acceleration limits for vibrations due to rhythmic activities (Murray et al, 2003)*

Occupancies Affected by the Vibration	**Acceleration Limit, % gravity**
Office or residential	0.4-0.7
Dining or weightlifting	1.5-2.5
Rhythmic activity only	4-7

4.3.2 Criterion by the American Institute of Steel Construction

The *Steel Design Guide Series 11* published by the American Institute of Steel Construction in collaboration with the Canadian Institute of Steel Construction provides a comprehensive guideline on the design for floor vibrations due to human activity. The guideline is based on the 1990 NBC design criterion.

This criterion can be used to evaluate structural systems subjected to activities such as aerobics and dancing provided the loading function is known. Common forcing frequencies and dynamic coefficients for rhythmic activities are given in Table 4-2. A typical time record of the dynamic loading function for eight people jumping at 2.1 Hz, together with its associated spectrum, is shown in Figure 4-1 (Murray et al, 2003).

Table 4-2: *Common forcing frequencies (f) and dynamic coefficients*(α_i) (Murray et al, 2003)*

Harmonic	**Person Walking**		**Aerobics Class**		**Group Dancing**	
i	**f, Hz**	**α_i**	**f, Hz**	**α_i**	**f, Hz**	**α_i**
1	1.6-2.2	0.5	2-2.75	1.5	1.5-3	0.5
2	3.2-4.4	0.2	4-5.5	0.6	-	-
3	4.8-6.6	0.1	6-8.25	0.1	-	-
4	6.4-8.8	0.05	-	-	-	-
*dynamic coefficient = peak sinusoidal force/weight of person(s)						

The peak acceleration of the floor due to a harmonic rhythmic force can be determined from Equation (4-1), which assumes that only one mode of vibration exists for the floor structure.

$$\frac{a_p}{g} = \frac{\frac{1.3\alpha_i w_p}{w_t}}{\sqrt{\left[\left(\frac{f_n}{f}\right)^2 - 1\right]^2 + \left[\frac{2\beta f_n}{f}\right]^2}} \qquad (4-1)$$

where

a_p/g – peak acceleration as a fraction of the acceleration due to gravity

α_i – dynamic coefficient (see Table 4-1)

w_p – effective weight per unit area of participants distributed over floor panel

w_t – effective distributed weight per unit area of floor panel, including occupants

f_n – natural frequency of the floor structure

f – forcing frequency (= $i \cdot f_{step}$ where f_{step} is the step frequency)

β – damping ratio

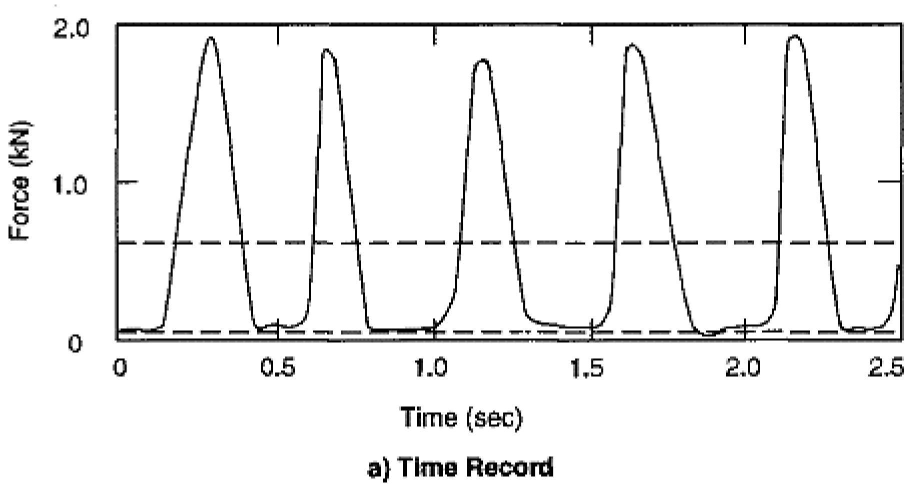
Since $f_n = f$ at resonance, Equation (4-1) can be simplified as follows:

$$\frac{a_p}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha_i w_p}{w_t} \tag{4-2}$$

Above resonance ($f_n > 1.2f$), it can be simplified as follows:

$$\frac{a_p}{g} = \frac{1.3}{(f_n/f)^2 - 1} \cdot \frac{\alpha_i w_p}{w_t} \tag{4-3}$$

The most critical accelerations occur at resonance and can be determined from Equation (4-2). Accelerations for the first two harmonics are sometimes also high enough to cause discomfort and may be determined from Equation (4-3). The expression for determining the effective maximum acceleration, which takes into account all the harmonics, is given in Equation (4-4) (Murray et al, 2003).



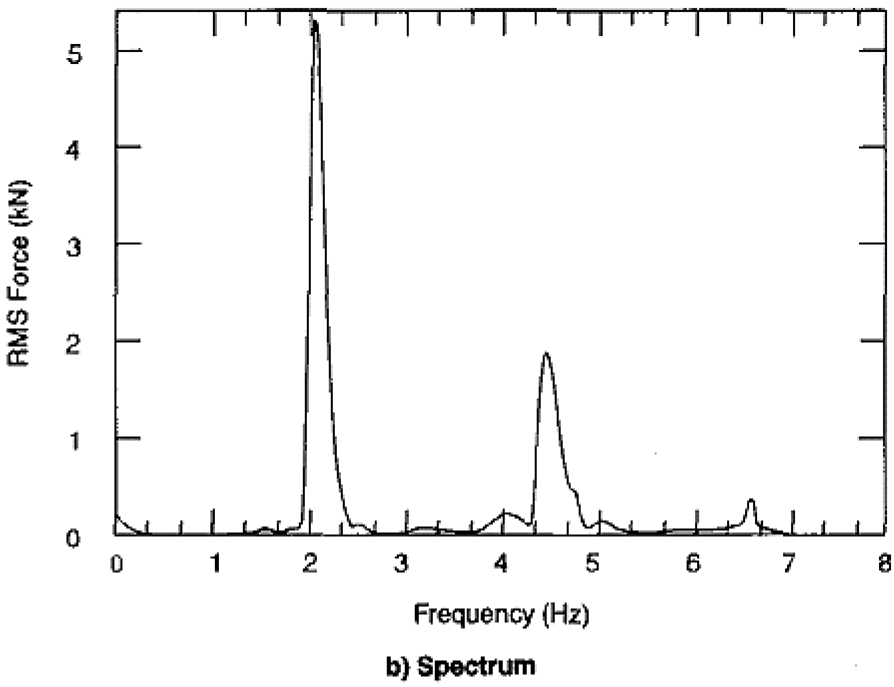


Figure 4-1: *Example loading function and spectrum from rhythm activity (Murray et al, 2003)*

$$a_m = \left[\sum a_i^{1.5}\right]^{1/1.5} \qquad (4-4)$$

where

a_m – peak acceleration for the i[th] harmonic

The result obtained from this expression can then be compared to the peak accelerations recommended by the International Standards Organization (ISO 2631-2, 1989) for human comfort shown in Figure 4-2 below. These values are not far from the values recommended by the NBC in Table 4-1.

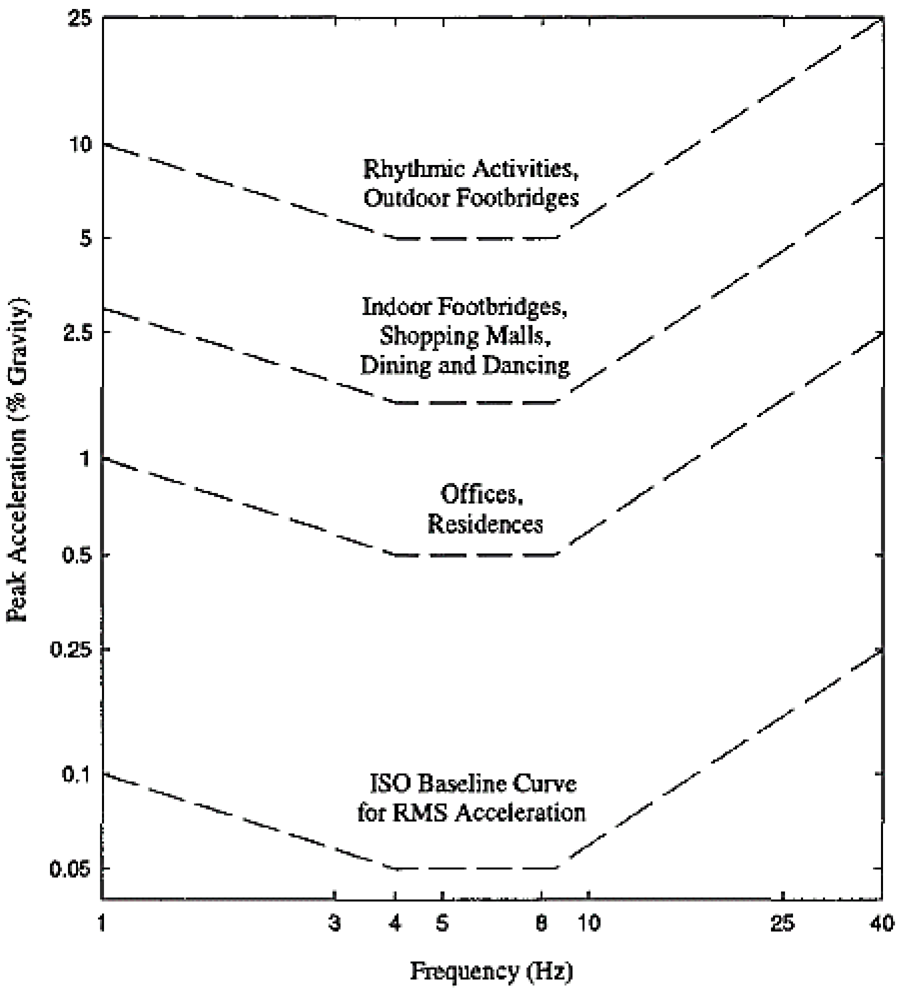


Figure 4-2: *Recommended peak acceleration for human comfort for vibrations due to human activities (Murray et al, 2003)*

To evaluate the effect of the rhythm activity on the neighbouring occupancies, the result obtained from Equation (4-3) should be reduced in accordance with the vibration mode shape of the structural system before it can be compared to the values given in Figure 4-2. For design purposes, it is important that the natural frequency of the structural system be made greater than the forcing frequency of the highest harmonic that can cause a large resonant vibration. An expression to represent this condition is obtained by inverting Equation (4-3) to give Equation (4-5):

$$f_n \geq f\sqrt{1 + \frac{k}{(a_0/g)}\frac{\alpha_i w_p}{w_t}} \qquad (4-5)$$

where

a_p/g – peak acceleration as a fraction of the acceleration due to gravity

α_i – dynamic coefficient (see Table 4-2)

w_p – effective weight per unit area of participants distributed over floor panel

w_t – effective distributed weight per unit area of floor panel, including occupants

f_n – natural frequency of the floor structure

$(f_n)_{req'd}$ – minimum natural frequency required to prevent unacceptable vibrations at each forcing frequency, f

f – forcing frequency (= $i \cdot f_{step}$, where f_{step} is the step frequency) (see Table 4-3)

i – number of harmonic = 1, 2, or 3 (see Table 4-3)

k – constant (1.3 for dancing, 1.7 for lively concert or sports event, 2.0 for aerobics)

a_0/g – ratio of peak acceleration limit (from Figure 4-2 in the frequency range of 4-8 Hz) to the acceleration due to gravity

Equations (4-2) to (4-4) are considered to give more accurate results, however (Murray et al, 2003). Typical minimum required natural frequencies based on Equation (4-5) are given in Table 4-4. Minimum natural frequencies recommended by Bachmann et al (1987) for different types of construction are shown in Table 4-5.

Table 4-3: *Estimated loading during rhythmic events (Murray et al, 2003)*

Activity	Forcing Frequency f, Hz	Weight of Participants* w_p		Dynamic Coefficient α_i	Dynamic Load $\alpha_i w_P$	
		kPa	psf		kPa	psf
Dancing and Dining • First Harmonic	1.5-3	0.6	12.5	0.5	0.3	6.2
Lively Concert or Sports Event • First Harmonic • Second Harmonic	1.5-3 3-5	1.5 1.5	31 31	0.25 0.05	0.4 0.075	7.8 1.6
Jumping Exercises • First harmonic • Second Harmonic • Third Harmonic	2-2.75 4-5.5 6-8.25	0.2 0.2 0.2	4.2 4.2 4.2	1.5 0.6 0.1	0.3 0.12 0.02	6.3 2.5 0.42
*Based on maximum density of participants on the occupied area of the floor for commonly encountered conditions. For special events the density of participants can be greater.						

Table 4-4: *Application of design criterion, Equation (4-5), for rhythmic events (Murray et al, 2003)*

Activity Acceleration Limit Construction	Forcing Frequency[1] f, Hz	Effective Weight of Participants w_p		Total Weight w_t		Minimum Required Fundamental Natural Frequency[1] f_n, Hz
		kPa	psf	kPa	psf	
Dancing and Dining $a_0/g = 0.02$						
• Heavy floor 5 kPa (100 psf)	3	0.6	12.5	5.6	112.5	6.4
• Light floor 2.5 kPa (50 psf)	3	0.6	12.5	3.1	62.5	8.1
Lively Concert or Sports Event $a_0/g = 0.05$						
• Heavy floor 5 kPa (100 psf)	5	1.5	31	6.5	131	5.9(2)
• Light floor 2.5 kPa (50 psf)	5	1.5	31	4	81	6.4(2)
Aerobics only $a_0/g = 0.06$						
• Heavy floor 5 kPa (100 psf)	8.25	0.2	4.2	5.2	104.2	8.8(2)
• Light floor 2.5 kPa (50 psf)	8.25	0.2	4.2	2.7	54.2	9.2(2)
Jumping Exercises Shared with Weight Lifting $a_0/g = 0.02$						
• Heavy floor 5 kPa (100 psf)	8.25	0.12	2.5	5.12	102.5	9.2(2)
• Light floor 2.5 kPa (50 psf)	5.5	0.12	2.5	2.62	52.5	10.6(2)

Notes to Table 4-4:
[1] Equation (4-5) is supplied to all harmonics listed in Table 4-3 and the governing forcing frequency is shown.
[2] May be reduced if, according to Equation (4-2), damping mass is sufficient to reduce 2nd and 3rd harmonic resonance to an acceptable level.
[3] From Equation (4-5)

Table 4-5: *Recommended natural frequencies of structures with human-induced vibrations (Bachmann et al, 1987)*

Structure Type	Construction Type			
	Reinforced Concrete	Pre-stressed Concrete	Composite Steel-concrete	Steel
Gymnasiums and sport halls	>7.5	>8.0	>8.5	>9.0
Dance halls and concert halls without fixed seating	>6.5	>7.0	>7.5	>8.0
Concert halls, theatres and spectator galleries with fixed seating				
• With classical concerts or "soft" pop music concert	>3.4	>3.4	>3.4	>3.4
• With "hard" pop music concerts	>6.5	>6.5	>6.5	>6.5
• In horizontal directions	>2.5	>2.5	>2.5	>2.5
Note for footbridges: Avoidance of 1.6 – 2.4 Hz (with low damping also 3.5 - 4.5 Hz)				

4.3.2.1 Estimation of Floor and Load Parameters

Fundamental Natural Frequency, f_n

The fundamental natural frequency of the structure is the most important parameter in preventing vibration problems due to rhythmic excitation. It is especially important for rhythmic excitation than it is for other types of loading such as walking excitation. It can be estimated from Equation (4-6) and should take the entire structure into consideration, including beams, girders and columns, all the way down into the foundations.

$$f_n = 0.18\sqrt{\frac{g}{(\Delta_j + \Delta_g + \Delta_c)}} \qquad (4-6)$$

where

Δ_j – the elastic deflection of the floor joist or beam at mid-span due to bending and shear

Δ_g – the elastic deflection of the girder supporting the beams due to bending and shear

Δ_c – the elastic shortening of the column or wall (and the ground if it is soft) due to axial strain

Details on how each of these deflections can be estimated are discussed in the previous chapter, however, it should be noted that in this study the deflections and the fundamental natural frequencies will be obtained from a finite element model. As a summary, Murray et al (2003) suggests that the flexural stiffness of floor components; namely, slab, beams and girders; should be based on composite action and that the column deflections will be very small relative to the other deflections for buildings with few stories (i.e. one to five).

Acceleration Limit, a_0/g

The ratio of peak acceleration limit to the acceleration due to gravity, a_0/g, can be obtained from Equations (4-2) to (4-4). Accelerations in vibration sensitive occupancies are estimated by using the obtained acceleration limit and the fundamental mode shape of the structure. These accelerations are then compared to the limits given in Table 4-2. A simpler and more conservative, but less accurate, approach requires that the a_0/g values provided in Table 4-4 be used in Equation (4-5) (Murray et al, 2003).

Rhythmic Loading Parameters: w_p, α_i and f

The values of w_p, weight of participants, can be obtained from Table 4-3 for cases where the rhythmic activity occupies the entire floor area. Where the rhythmic activity occupies only a portion of the floor area, the values of w_p provided in Table 4-3 should be reduced such that the resulting moments and deflections are equivalent to those of a partially loaded floor. Values of α_i *and f* can also be obtained from Table 4-3 (Murray et al, 2003).

Effective Weight, w_t

The effective weight of the floor is the distributed self-weight of the floor plus the weight of participants. This weight is increased if the floor supports additional weight, e.g. that of the floor above, and if the columns experience excessive vibrations due additional loads they are supporting. The effective weight of the floor can also increase due continuity of members over the supports into adjacent floor panels. Equation (4-7) is used to estimate an increase in w_t due to an additional point load, W_c, exerted onto the floor.

$$\Delta w_t = \frac{2W_c y^2}{LB} \qquad (4-7)$$

where

y – ratio of modal displacement at the location of the weight to maximum modal displacement

L – span

B – effective width of the panel (can be estimated as the width occupied by the participants) (Murray et al, 2003).

Damping Ratio, β

A value of approximately 0.06 is recommended as damping ratio. This value takes into consideration the damping of the floor and the additional damping due to participants and is only applied to Equation (4-2) when resonance occurs (Murray et al, 2003).

4.3.2.2 Design Procedure

The design criterion adopted by Murray et al (2003) consists of the following three stages:

1) Approximation of the natural frequency of the structure using Equation (3-5) and an approximation of the minimum natural frequency from Table 4-4.
2) Application of Equation (4-5) or Equations (4-2) to (4-4) to obtain a more accurate minimum frequency and recalculating the structure's natural frequency using Equation (3-5), taking into consideration the shear deformation and the continuity of beams and girders.

3) Determination of natural frequencies, mode shapes and vibration accelerations throughout the building using computer analyses and comparing the obtained accelerations in critical areas of the building to the acceleration limits provided in Table 4-2.

4.3.3 Design Criteria Found in Literature

The most in-depth study into rhythmic load models was done by Ji et al (1994) and in their study they highlight two possible ways of dealing with rhythmic loading on a floor, namely:

1) Avoiding resonance by designing a floor with a sufficiently high fundamental natural frequency. According to Ellis et al (2004), this is achievable if the vertical natural frequencies are kept above 8.4 Hz and horizontal natural frequencies above 4.0 Hz.
2) Calculating how the floor responds to given loads and checking whether it is satisfactory

In their study of floor vibration induced by dance-type loads where jumping is involved, Ji et al (1994) focus on the latter. This study was aimed at providing an alternative criterion to one recommended by BS 6399: Part 1 which recommends an application of an equivalent static design load of 5 kN/m^2 on dance floors, and the one recommended by the NBC which is based on the beam theory, models the floor as a single degree-of-freedom and only considers the fundamental mode of vibration. They noted that both these methods are not adequate.

They split the problem as follows:

1) Characterisation of the load.
2) Evaluation of the characteristics of the floor vibration.
3) Calculation of the response of the floor to the dancing loads.

4.3.3.1 Characterisation of Dance-Type Loads

Characterisation of dance-type loads requires an estimation of the number and weight, or density, of the people who will be dancing on an area of the floor. The dancing frequency depends on the beat frequency of the music being danced to and the type of dance, but it is generally estimated to range from 1.5-3.5 Hz.

Figure 4-3 shows the normalised load-time history for jump dancing including the first six Fourier terms. It is characterised by a high contact force over the contact duration, t_p, followed by a zero force when the feet leave the floor. This can be represented by a sequence of semi-sinusoidal pulses which are expressed in Equation (4-8) which is a semi-sinusoidal function for a single period as follows

$$F(t) = \begin{cases} K_p G \sin(\pi t/t_p) & 0 \leq t \leq t_p \\ 0 & t_p \leq t \leq T_p \end{cases} \qquad (4-8)$$

where

K_p – impact factor (F_{max}/G)

F_{max} – peak dynamic load per unit area

G – weight of dancer per unit area

t_p – contact duration

T_p – dancing load period

The contact duration, t_p, can vary from from 0 to T_p depending on the type of movements, with it being equal to T_p where the dancer is always in contact with the floor (Ji et al, 1994).

The contact ratio, α, is expressed as shown in Equation (4-9) with typical values for various activities given in Table 4-6.

$$\alpha = \frac{t_p}{T_p} \leq 1 \qquad (4-9)$$

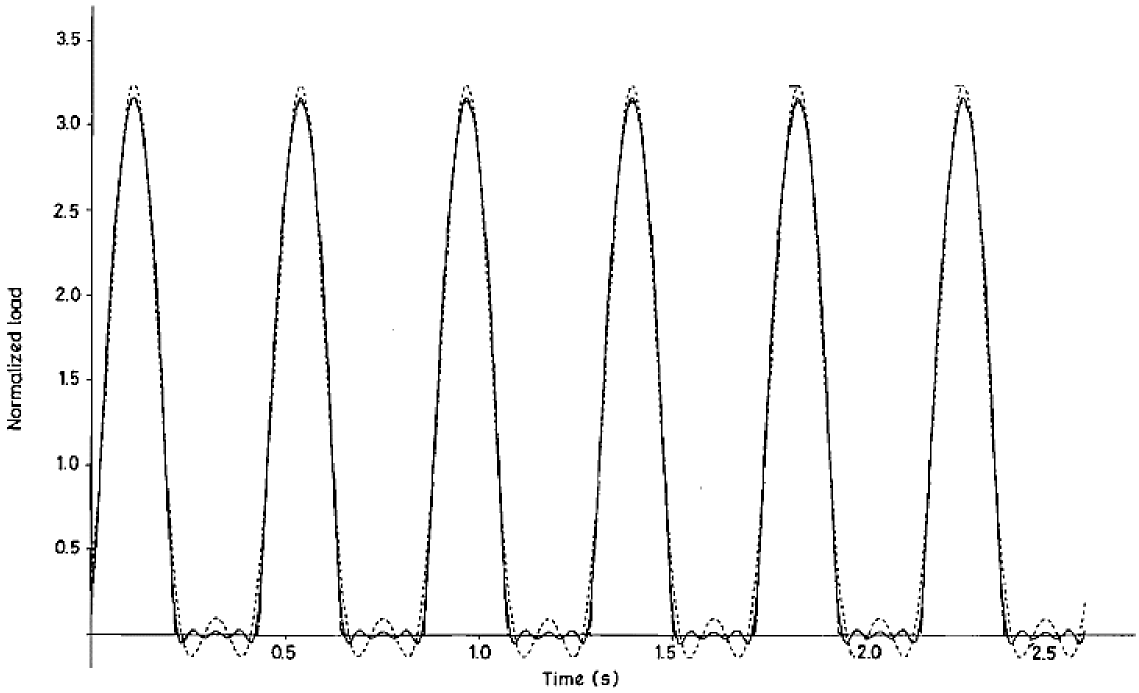


Figure 4-3: *Load-time history for jumping (Ji et al, 1994)*

Table 4-6: *Typical values of the contact ratio, α, for various activities (Ellis et al, 2004)*

Activity	$\boldsymbol{\alpha}$
Low impact aerobics	2/3
Rhythmic exercises, high impact exercises	1/2
Normal jumping	1/3

Expressing Equation (4-8) in terms of Fourier series, and taking into consideration the observation made by Tuan et al (1985) that the average vertical load according to the time history corresponding to jump dancing is equal to the weight of the dancer, gives Equation (4-10):

$$F(t) = G\left(1 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2n\pi}{T_p}t + \varphi_n\right)\right) \qquad (4-10)$$

where

r_n – Fourier coefficient (or dynamic load factor) of the n[th] term

n – number of Fourier terms

φ_n – phase lag of the n[th] term

$$r_n = \begin{cases} \pi/2 & if\ 2n\alpha = 1 \\ \left|\dfrac{2\cos(n\pi\alpha)}{1-(2n\alpha)^2}\right| & if\ 2n\alpha \neq 1 \end{cases}$$

$$\varphi_n = \begin{cases} 0 & & if\ 2n\alpha = 1 \\ \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) - \pi & if\ \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} < 0 & \\ -\dfrac{\pi}{2} & if\ \sin(2n\pi\alpha) = 0 & if\ 2n\alpha \neq 1 \\ \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) & if\ \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} > 0 & \end{cases}$$

The first six Fourier coefficients and phase lags for various contact ratios are shown in Table 4-7 below (Ellis et al, 2004).

Table 4-7: *Fourier coefficients and phase lags for different contact ratios (Ellis et al, 2004)*

		n=1	n=2	n=3	n=4	n=5	n=6
α=2/3	r_n	9/7	9/55	2/15	9/247	9/391	2/63
	ϕ_n	-π/6	- 5π/6	-π/2	-π/6	-5π/6	-π/2
α=1/2	r_n	π/2	2/3	0	2/15	0	2/35
	ϕ_n	0	-π/2	0	-π/2	0	-π/2
α=1/3	r_n	9/5	9/7	2/3	9/55	9/91	2/15
	ϕ_n	π/6	-π/6	-π/2	-5π/6	-π/6	-π/2

The above load model applies to an individual dancing/jumping. Where a crowd is involved, the load equation becomes as shown in Equation (4-11) below

$$F(x,y,t) = G(x,y)\left(1 + \sum_{n=1}^{\infty} r_{n,v} \sin\left(\frac{2n\pi}{T_p}t + \varphi_n\right)\right) \qquad (4-11)$$

where

$F(x,y,t)$ – distributed force which varies with time

$G(x,y)$ – density and distribution of human loads

$r_{n,v}$ – nth Fourier (load) coefficient induced by *v* persons

The load varies due to the difference in the spatial distribution of people and also due to the different jumping styles and coordination of the jumpers. Ellis et al (2004) conducted experiments for crowd loading on a 9 x 6m floor for crowds between 2 and 64 people and in their experiment determined that the Fourier coefficient reduces with an increase in crowd size, *v*. They came up with the following equations for the first three Fourier coefficients, denoted as FC, below:

- First FC, $r_{1,v}$ = 1.61 x $v^{-0.082}$
- Second FC, $r_{2,v}$ = 0.94 x $v^{-0.24}$
- Third FC, $r_{3,v}$ = 0.44 x $v^{-0.31}$

They suggested that these values should be used in conjunction with the phase angles provided in Table 4-7 for calculation purposes. Ellis et al (2004) further developed a numerical model that would be able to cover crowds of more than 64 people and from their models determined that the values of FC provided by the equations above may only apply to small crowds as they discovered that the FC values levelled out for large crowds. This is represented graphically in Figure 4-4. The values they obtained for the first three FCs for a crowd of 8192 people were as follows:

- First FC, $r_{1,v}$ = 1.40
- Second FC, $r_{2,v}$ = 0.47
- Third FC, $r_{3,v}$ = 0.072

They also noted that these values may increase with more synchronised crowds such as professional dancers and therefore resulting in higher peak loads.

There are other scenarios where rhythmic excitation does not involve everyone jumping, such as with pop concerts. In such cases some people will be dancing and jumping, others just standing while others will be sitting. As a result of those who are not dancing or jumping, the overall damping of the floor will increase due to human-structure interaction. Damping values reaching as high as 16 % have been measured. Due to less synchronisation than when everyone is jumping, the peak loads are also less, hence leading to a smaller structural response. FC values of 0.42, 0.087 and 0.017, assuming that the FCs varied with the crowd size, were determined for a crowd where 90 % of the people were standing, dancing and moving in response to music as opposed to 1.61, 0.94 and 0.44 where everyone is jumping (Ellis et al, 2004).

4.3.3.2 Structural Response

In the analysis of a structure under rhythmic loading, the two elements of the structure's response that are of utmost importance are the displacement and acceleration of the structure. Acceleration is necessary for serviceability assessments while displacements are necessary safety assessments (Ellis et al, 2004). This section looks at how these two components of the structure's response are determined.

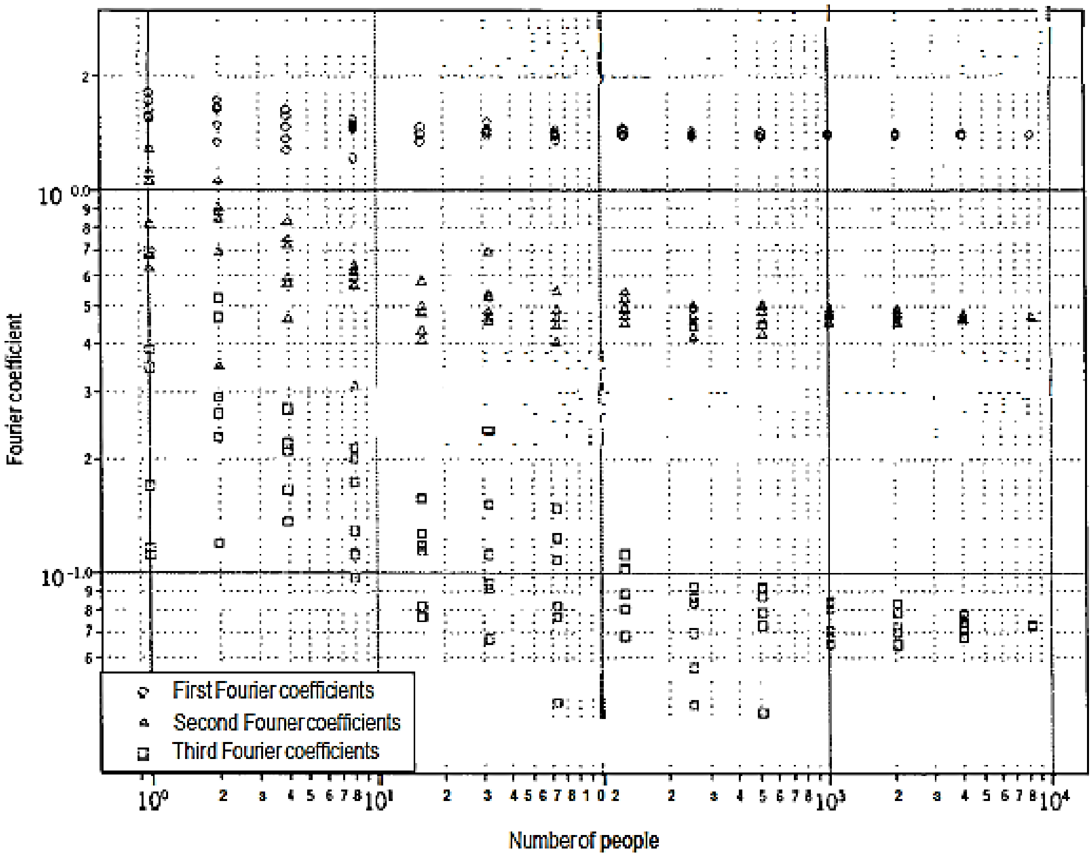


Figure 4-4: *Fourier coefficients against group size (Ellis et al, 2004)*

Vibration modes to consider

For the simplest types of floors, that is, symmetric single bay floors under a symmetric load, the fundamental mode of vibration is of utmost importance. In more complex structures another mode may be more critical (e.g. a cantilevered tier of a grandstand whereby the vertical mode of vibration of the cantilever or part of the cantilever, instead of the fundamental mode of the entire grandstand, is most critical). In such cases the most critical mode is referred to as a principal mode of vibration. For simplicity, derivations for displacement and acceleration are done using a symmetric single bay floor under a symmetric load, in which case the fundamental mode is most critical. Due to the dominance of the fundamental mode in such a scenario, other modes of vibration need not be considered. However, it is important to note that this does not apply for more complex structures where it may be necessary to include several low frequency modes as they may contribute significantly to the displacement (Ellis et al, 2004).

Number of Fourier components to consider

The number of Fourier components to be considered depends on their contribution to the overall response. It is essential that the number of Fourier terms considered includes resonant excitation (i.e. whereby the natural frequency of the structure, f_s, is equal to the dance frequency, f, or to its integer multiples). To ensure this, the first I Fourier terms are considered for serviceability assessments (i.e. acceleration calculations) where I is defined as the first integer greater than f_s/f. For safety assessments (i.e. displacement calculations), it is recommended that the first three Fourier terms be considered (Ellis et al, 2004).

Expressions for structural displacement and acceleration

In order to determine the expressions for displacement and acceleration, it is first important that we know the mode shape. Assuming a single bay floor with symmetric boundary conditions and under a uniformly distributed load, the expression of the fundamental mode is as shown in Equation (4-12):

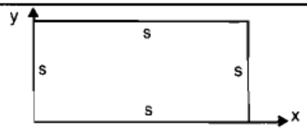
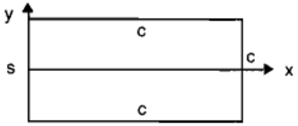
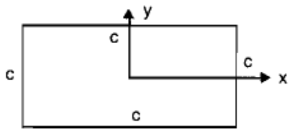
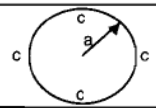
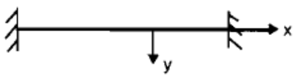
$$w(x,y,t) = A(t)\Phi(x,t) \qquad (4-12)$$

where

$\Phi(x,t)$ – the dimensionless fundamental mode with a unit peak value

$A(t)$ – the amplitude of vibration corresponding to that mode and is a function of time

Table 4-8: *Approximate structural factors for several common cases (Ji et al, 1994)*

Structures and boundaries	Assumed fundamental mode	Approximate fundamental frequency (ω)	Structural factors
Plate 	$\sin(\pi x/L_x)\ \sin(\pi y/L_y)$	$\pi^2\lambda(1+\eta^2)/L_x^2$	$(4/\pi)^2 \doteq 1.62$
	$\sin(\pi x/L_x)(1-4y^2/L_y^2)^2$	$22.45\lambda\left(1+\frac{\pi^2}{21}\eta^2+\frac{\pi^4}{504}\eta^4\right)^{1/2}/L_x^2$	$\frac{4}{\pi}\frac{21}{16} \doteq 1.67$
	$(1-4x^2/L_x^2)^2\ (1-4y^2/L_y^2)^2$	$22.45\lambda\left(1+\frac{4}{7}\eta^2+\eta^4\right)^{1/2}/L_x^2$	$\left(\frac{21}{16}\right)^2 \doteq 1.72$
	$(1-r^2/a^2)^2$	$10.22\lambda/a^2$	$5/3 \doteq 1.67$
Beams 	$\sin(\pi x/L_x)$	$\pi^2\lambda/L_x^2$	$4/\pi \doteq 1.27$
	$(1-4x^2/L_x^2)^2$	$22.37\lambda/L_x^2$	$21/16 \doteq 1.31$

S – simply supported boundary C – clamped boundary $\lambda = (D/m)^{1/2}$ D – flexural rigidity m – floor mass $\eta = L_x/L_y$

From the fundamental mode we can determine the structural factor, *B*, as shown in Equation (4-13):

$$B = \frac{\iint_s \Phi(x, y)\,dx\,dy}{\iint \Phi^2(x, y)\,dx\,dy} \qquad (4-13)$$

Typical fundamental mode shapes and their corresponding structural factors for different types of structures and boundary conditions are given in Table 4-8 where L_x and L_y are the length and width of the plate.

For structures with a uniformly distributed mass (i.e. *m(x,y) = m*) and under a uniformly distributed load, the structural displacement and acceleration can be determined from Equations (4-14) to (4-18).

$$A = B\frac{G}{m\omega_p^2}(1 + D) \qquad (4-14)$$

$$\ddot{A} = B\frac{G}{m}D^a \qquad (4-15)$$

$$D = \frac{r_n \sin\left(n\omega_p t - \theta_n + \phi_n\right)}{\sqrt{(1 - n^2\beta^2)^2 + (2n\xi\beta)^2}} \qquad (4-16)$$

$$D^a = \frac{r_n n^2\beta^2 \sin\left(n\omega_p t - \theta_n + \phi_n\right)}{\sqrt{(1 - n^2\beta^2)^2 + (2n\xi\beta)^2}} \qquad (4-17)$$

$$\phi_n = \left\{\begin{matrix} \tan^{-1}\left(\frac{2n\xi\beta}{1 - n^2\beta^2}\right) & if\ 1 - n^2\beta^2 > 0 \\ \frac{\pi}{2} & if\ 1 - n^2\beta^2 = 0 \\ \tan^{-1}\left(\frac{2n\xi\beta}{1 - n^2\beta^2} + \pi\right) & if\ 1 - n^2\beta^2 < 0 \end{matrix}\right\} \qquad (4-18)$$

where

A – displacement

$\ddot{A}$ – acceleration

B – structural factor

G – human load

m – structural mass

D – dynamic magnification factor for displacement

D^a – dynamic magnification factor for acceleration

ξ – damping ratio

β – frequency ratio, w_p/w_s

n – number of Fourier terms

Detailed derivations of these equations are provided by (Ji et al, 1994).

4.3.4 Design Criteria Found in Design Codes

4.3.4.1 The British Standard

BS 6399: Part 1: 1996 - *Code of practice for dead and imposed loads* makes provision for rhythmic loading in clause 9.2 and Annex A and it adopts a similar load model to the one derived by Ji et al (1994) as described in Section 4.3.2. BS 6399: Part 1: 1996 makes two additions to this model:

1) For large crowds it adopts the approach by Ellis et al (2000) which suggests that Equation (4-11) be modified to Equation (4-19) below, where C_e is the dynamic load factor with a recommended value of 0.67.

$$F(x,y,t) = G(x,y)\left(1 + C_e \sum_{n=1}^{\infty} r_{n,v} \sin\left(\frac{2n\pi}{T_p} t + \varphi_n\right)\right) \qquad (4-19)$$

2) In addition to the vertical loads, it further makes provision for the horizontal load which it recommends should be estimated as 10 % of the vertical load.

BS 6399: Part 1: 1996 has, however, been superseded by BS EN 1991-1-1:2002 which is the UK National Annex to Eurocode 1: Part 1-1 – *General actions – densities, self-weight, imposed loads for buildings*. Clause NA.2.1.2 of this annex which makes provision for synchronized rhythmical movements makes the following recommendations:

1) In order to avoid resonance, the vertical natural frequency of the structure should be kept above 8.4 Hz and the horizontal natural frequency above 4.0 Hz. This is in line with values recommended by Ellis et al (2004) as indicated in Section 4.3.2 of this document.
2) Dynamic analyses should be carried out in the design of structural elements subject to jumping and dance loads. Specialist advice and guidance documents such as one by Ellis et al (2004) are recommended.
3) The designer should follow the specific guidelines outlined by the certifying authority of the type of structure being designed.

Clause NA.2.1.4 further makes provision for lightweight and long-span structures and it stipulates that the intended use of the structure and the potential number and behaviour of occupants should be taken into consideration when designing such structures as such structures are likely to be subjected to synchronized movements by occupants, which may in turn exacerbate the dynamic loads on the structure. Specialist advice and specialist guidance documents are also recommended.

4.3.4.2 Eurocode

Provisions for dynamic loads can be found in clause 5.1.3 of EN 1990: 2002+A1: 2005 (E). It states that a serviceability limit state verification should be carried out where the dynamic actions may cause vibrations and frequencies above the serviceability limits. Amongst the aspects that should be considered, Annex A, clause A1.4.4, of the same code highlights the importance of user comfort and the functionality of the structure (e.g. sensitivity of the building contents, cracks in partitions, etc.). Clause A1.4.4 further provides the following recommendations in dealing with the vibration serviceability limit state:

1) The natural frequency of the structure should be kept above the forcing frequency
2) In a case where the natural frequency is lower than the forcing frequency, in-depth analysis of the structural response should be carried out.

EN 1990: 2002+A1: 2005 (E) does not provide further detail on how the above analysis should be carried out but rather refers to ISO 10137 for further guidance.

4.3.4.3 South African National Standard

Considerations for rhythmic loading, and dynamic loading in general, are mentioned in clauses 4.2.3, 4.2.5 and 8.3.1.2 of SANS 10160-2:2009, however, this code does not provide any guidance on how the vibration serviceability limit state analysis should be conducted. Table 1 of SANS 10160-2:2010 recommends an application of a category C5 imposed load equal to 5,0 kN/m^2 for areas susceptible to large crowds such as concert halls, sports halls, etc.

4.3.5 Measures to Deal with Vibration Serviceability

Ji et al (1994) proposed two methods of dealing with vibration serviceability, namely, i) avoiding resonance by designing a floor with a sufficiently high fundamental natural frequency, and ii) calculating how the floor responds to given loads and checking whether it is satisfactory by comparing it to a certain acceptance criteria. In their study on the vibration serviceability of the Grha Sabha Pramana Auditorium, Erlina et al (2017) discussed a third method, that is, adopting certain measures to moderate the vibration serviceability of planned or existing structures. These measures include:

1) Increasing the stiffness of the structure
2) Increasing the damping
3) Installing tuned vibration absorbers
4) Restricting the usage of the structure to limit rhythmic activity to certain areas or using a structure for a totally different purpose

From their research, Erlina et al (2017) found out that the use of a fluid viscous damper (i.e. tuned vibration absorber) was very effective at improving the vibration serviceability of the floor structure with peak accelerations decreasing from 0.0159 g to 0.00157 g and damping ratios increasing from 4.58 % to 14.51 %.

4.4 Summary of Literature Review

Modern technology has made the design of long slender floors possible. However, this has brought with it vibration serviceability issues in many buildings. Where traditional waffle slabs and concrete joist slabs were adequate to meet vibration serviceability requirements, new techniques now need to be developed to address floor vibration challenges. A popular solution to long clear spans is the truss design system. In this system members carry externally applied loads mainly through tension and compression and this allows them to span longer because axially loaded steel is generally more structurally efficient than steel subjected to flexure and concrete. This study focuses on two types of trusses, namely, plane trusses and space trusses, and also on space grids.

Studies into vibration serviceability go as far back as the 19th century. Excessive floor vibrations may cause annoyance, malfunctioning of sensitive equipment, structural damage or failure. Most often, however, the most important factor when dealing with vibration serviceability is human perception of floor accelerations. The most critical accelerations occur at resonance, that is, when the applied vibrations match the natural frequency of the floor. The peak accelerations recommended by the International Standards Organization (ISO 2631-2, 1989) for human comfort are shown in Figure 4-2. Estimated loading during rhythmic events is given in Table 4-3 below and typical minimum required floor natural frequencies are given in Table 4-4.

There are two possible ways of dealing with rhythmic loading on a floor:

1) Avoiding resonance by designing a floor with a sufficiently high fundamental natural frequency
2) Calculating how the floor responds to given loads and checking whether it is satisfactory

The latter involves characterisation of the load, evaluation of the characteristics of the floor vibration and calculation of the response of the floor to rhythmic loads.

Of the studied design codes, only the old British Standard provides a rhythmic load model to be applied on the floor for design purposes. The UK annex to Eurocode 1: Part 1-1, which supersedes the old British Standard, does not provide a specific load model but instead recommends that the designer should make use of literature where such load models can be found. The Eurocode and the South African National Standard recognise the need to perform extra analyses to account for rhythmic loading, however, they do not provide in-depth guidance on how such analyses should be carried out.

Apart from the load model in the British Standard, none of the studied design codes gives guidance on how to determine the structural response of different floor systems to rhythmic loading and how long slender floors may behave differently from floors with shorter spans. The UK annex to Eurocode 1: Part 1-1 does, however, note the importance of taking into consideration the intended use of lightweight, long-span structures and the potential number and behaviour of their occupants as such structures are likely to be subjected to synchronized movements by occupants. The response of the floor (i.e. acceleration) is crucial because it affects the comfort of the occupants of the structure and lack of coverage in the area may be the reason for continued vibration issues in long slender floors exposed to rhythmic loading. This report therefore seeks to cover this area with the aid of finite element modelling.

5. Methodology

5.1 Developing the Finite Element Model

Structural Configuration and Properties

Three floor systems were designed to cover an approximate area of 90000 x 54400 mm with beam and column supports located as shown in Figure 5-1. The first batch of floor systems consisted of a 3600 mm deep steel structure supporting a 170mm deep concrete-steel composite slab. The three floor systems considered were the plane truss, space frame and space grid.

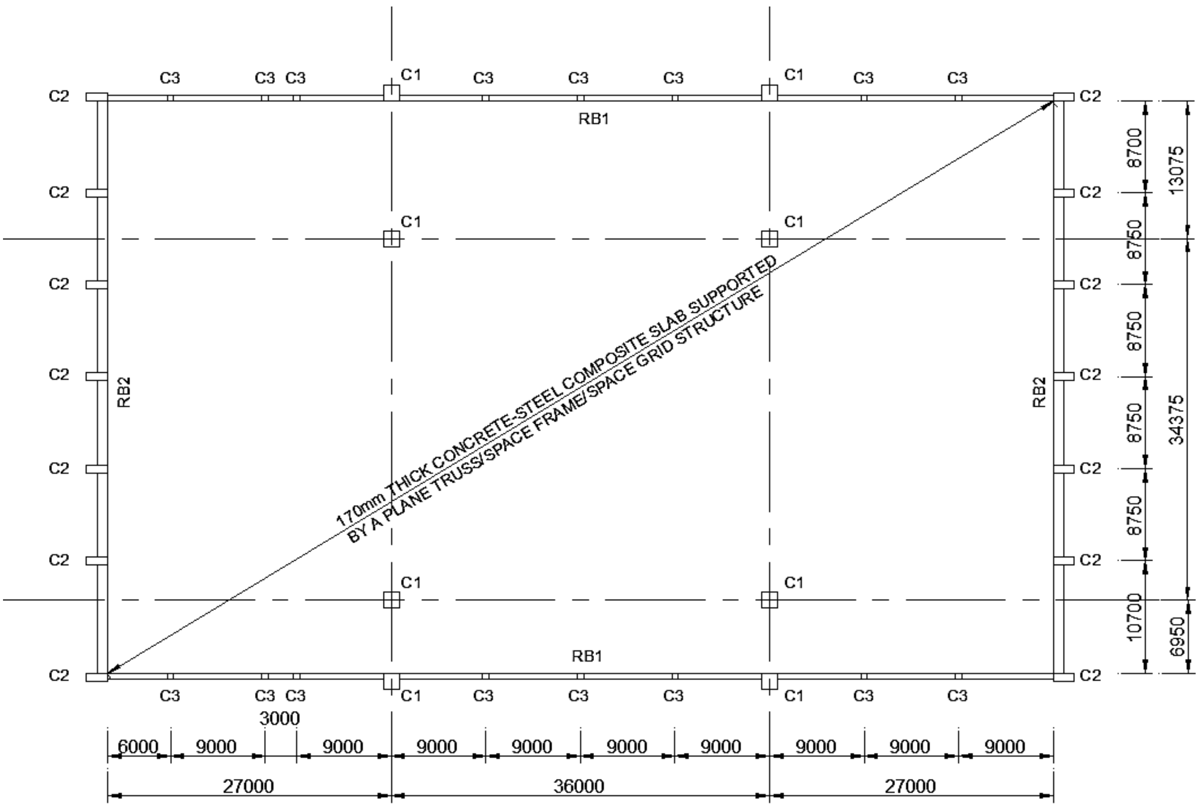


Figure 5-1: *Floor plan layout*

The four columns C1 acted as interior supports while the ring beams RB1 and RB2, columns C2 and C3 and the rest of columns C1 acted as supports along the perimeter of the building. Ring beams support the bottom edges and top edges of the floors, as shown in Figure 5-2. An assumption was made that all beams and columns were designed to adequately carry all the structural loads imposed on them and that the deflections of the supports were negligible. The columns and beams were therefore not designed in this study, but instead all supports were modelled as either pin supports or fixed supports.

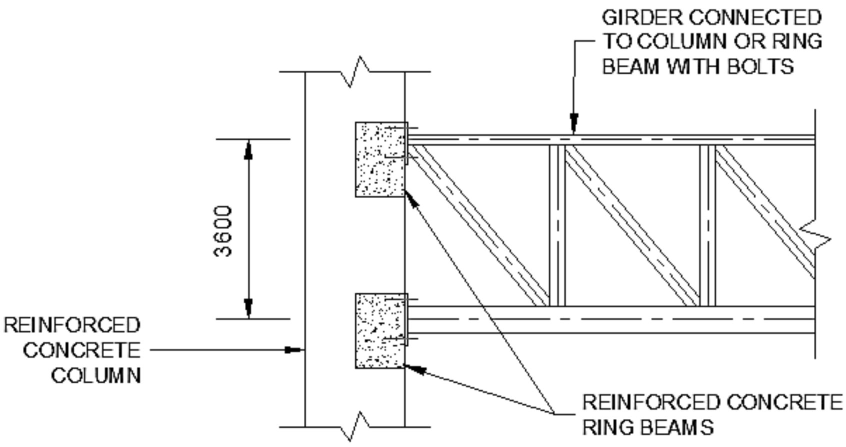


Figure 5-2:*Typical floor support*

To take into consideration the effects of concrete-steel composite behaviour, the modulus of elasticity of the concrete slab supported by the steel structure was factored by 1.35 as recommended by Murray et al (2003).

Loads

The static analyses were performed at ultimate limit state (ULS) and serviceability limit state (SLS) for each structure on PROKON Frame Analysis (with the aid of Design Links) to determine the minimum member sizes and mechanical properties. The following load combinations were used:

ULS = 1.6 LL + 1.2 DL

SLS = 1.0 LL + 1.0 DL

where

LL – live load

DL – dead load

The dead load consisted of self-weight while the live load of 5.0 kN/m^2 was obtained from Table 1 of *SANS 10160-2:2010 Edition 1*, under load category C, subcategory C5 for "*Areas susceptible to large crowds, for example, in buildings for public events like concert halls, exhibition halls, sports halls including stands, terraces, access areas, escape routes and railway platforms*".

Deflections

Each floor system was designed to satisfy the maximum permissible deflection under SLS loads which is recommended in clause 3.2.3.2.1.1 of *SANS 10100-1 Edition 2.2*. The recommended value is equal to span/250. To check this, the largest span (36000 mm) was considered and the maximum permissible deflection was determined to be 36000 mm/250 = 144 mm.

The sections below go into more detail on how each floor system was designed. Deflection results from the static analyses of the 3.6 m deep floors of each floor type can be found in Appendix A. The floor layout drawings are provided in Appendix C. It is to be noted that the dimensions, and the interior column positions, of the space grid and space frame floors were adjusted slightly to accommodate their geometry.

5.1.1 Plane Trusses

The plane truss floor model consisted of the following:

- Two main girders spanning over two columns and across the shorter span (54400 mm), each consisting of three parallel trusses welded together at certain intervals. The welds we modelled as steel members fixed to adjacent girders
- Secondary girders running perpendicular to the main girders, spaced between 3750 mm and 8750 mm, and connected with bolts (i.e. fixed supports) to the main girders
- All girders were supported with fixed supports all around the floor perimeter and pinned at the interior column supports
- All steel members were made of grade 350W steel

The element sections used are shown in Table 5-1.

Table 5-1: *Plane truss sections*

Main Girders:	Top chords	- 305x165x54 I-Section
	Bottom chords	- 1030x400 (8W,25F) Plate Girder
	Vertical web members	- 305x305x118 H-Section
	Diagonal web members	- 305x305x137 H-Section
Secondary Girders:	Top chords	- 203x133x25 I-Section
	Bottom chords	- 533x210x122 I-Section
	Vertical members	- 203x203x52 H-Section
	Diagonal members	- 203x203x60 H-Section
	Bracing	- 203x133x25 I-Section

The 3-dimensional layout of the plane truss floor is shown in Figure 5-3.

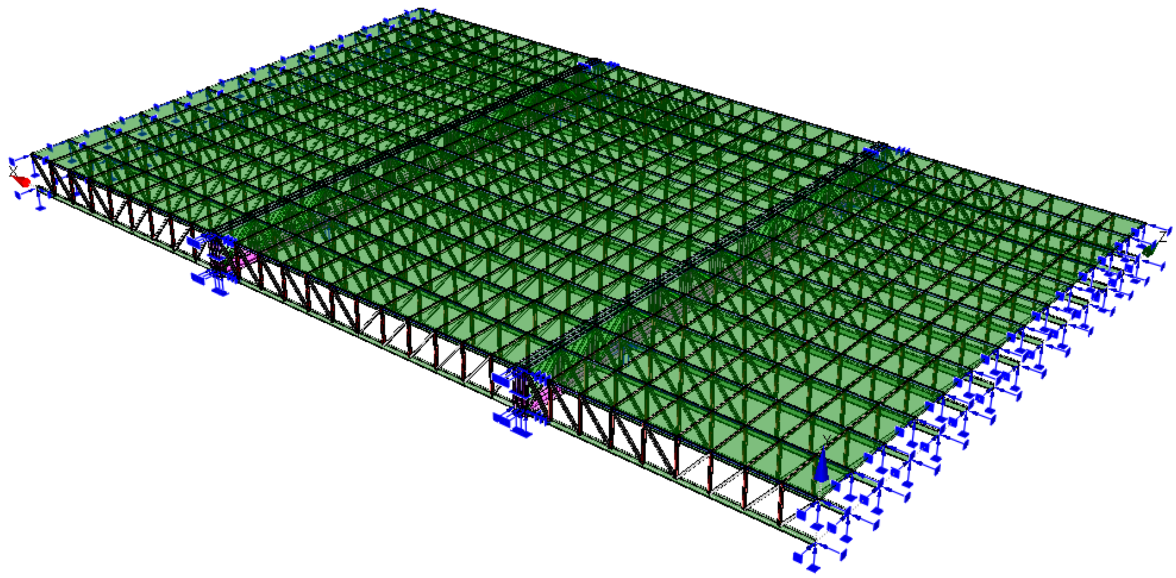


Figure 5-3: *Typical 3-dimensional layout of a plane truss floor FE model on PROKON*

5.1.2 Space Grid

The space grid floor model consisted of the following:

- Steel circular hollow sections connected with ball joints (i.e. pin joints) at all nodes. Torsional restraint was added to all nodes to establish structural stability. It was also necessary to fix one member to each node while the rest of the members were free to rotate about it. This was as per the PROKON Frame Analysis guidelines
- All steel members were made of grade 350W steel
- The floor was simply supported all around its perimeter and pinned at the interior column supports.

The element sections used are shown in Table 5-2 and the 3-dimensional PROKON layout in Figure 5-4.

Table 5-2: *Space grid sections*

Top chords	- 152.4x6.0 Circular Hollow Section
Bottom chords	- 406.4x12.0 Circular Hollow Section
Vertical and Diagonal members	- 406.4x12.0 Circular Hollow Section

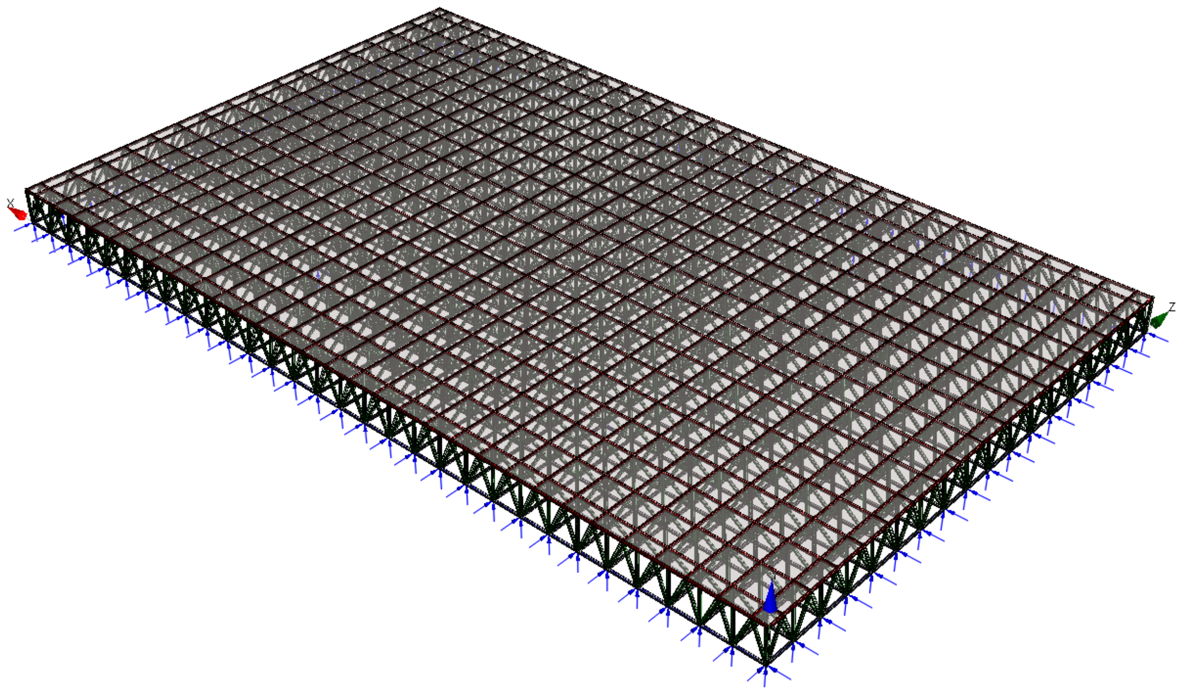


Figure 5-4: *Typical 3-dimensional layout of a space grid floor FE model on PROKON*

5.1.3 Space Frame

The space frame floor model consisted of the following:

- I-sections for top chords, H-sections for vertical and diagonal members and plate girders for bottom chords. All members were interconnected with fixed supports
- All steel members were made of grade 350W steel
- The floor was fixed all around its perimeter and pinned at the interior column supports.

The element sections used are shown in Table 5-3 and the 3-dimensional PROKON layout in Figure 5-5.

Table 5-3: *Space frame sections*

Top chords	-	203x133x25 I-Section
Bottom chords	-	1030x400 (8W,25F) Plate Girder
Vertical and Diagonal members	-	305x305x137 H-Section

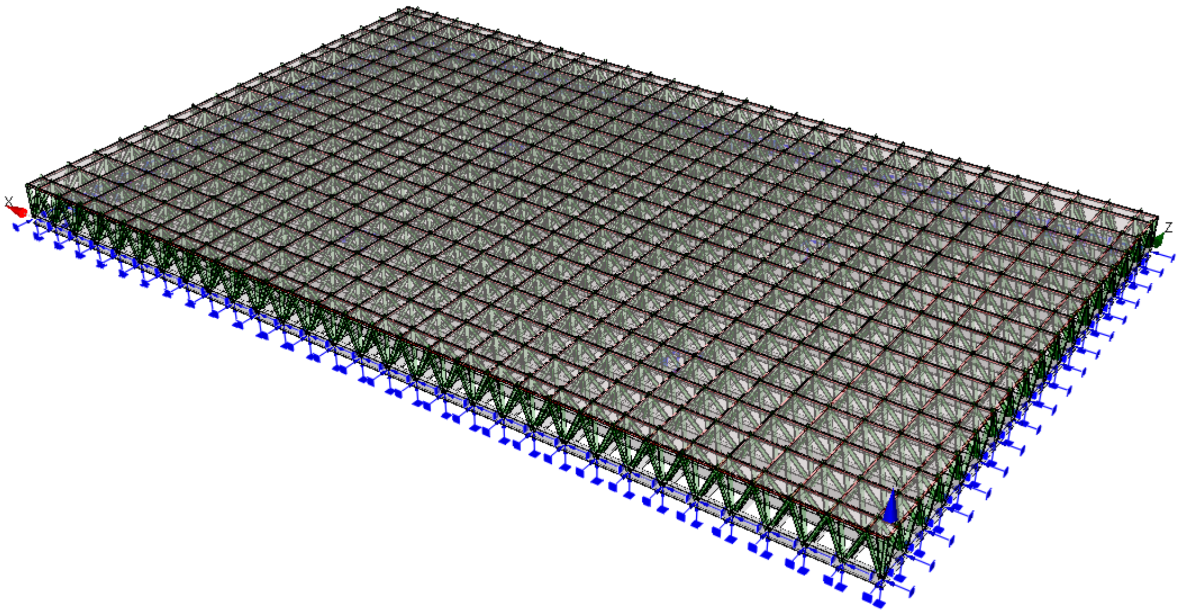


Figure 5-5: *Typical 3-dimensional layout of a space frame floor FE model on PROKON*

5.1.4 Variations applied to each floor type

For each floor type the floor depth, column supports and edge supports were varied in order to assess their effect on the dynamic properties of the different floors. The floor depths were varied from 3.6m to 3m, 4m and 5m. When the depths were changed the horizontal spacings between the elements of the truss, frame and grid were left unaltered. The space grid and space frame FE models only had bottom edge supports initially, so an extra variation was added by adding top edge supports to them.

Figures 5-6 to 5-8 show how the column support variations were applied. Figure 5-6 shows the floor plan with the column spacing reduced from 34375mm (see Figure 5-1) in the direction of the short span to 21875mm. Figure 5-7 shows the floor plan with additional columns added in-between the initial columns in the direction of the short span while Figure 5-8 shows the floor plan with additional columns added in-between the initial columns in the direction of the long span.

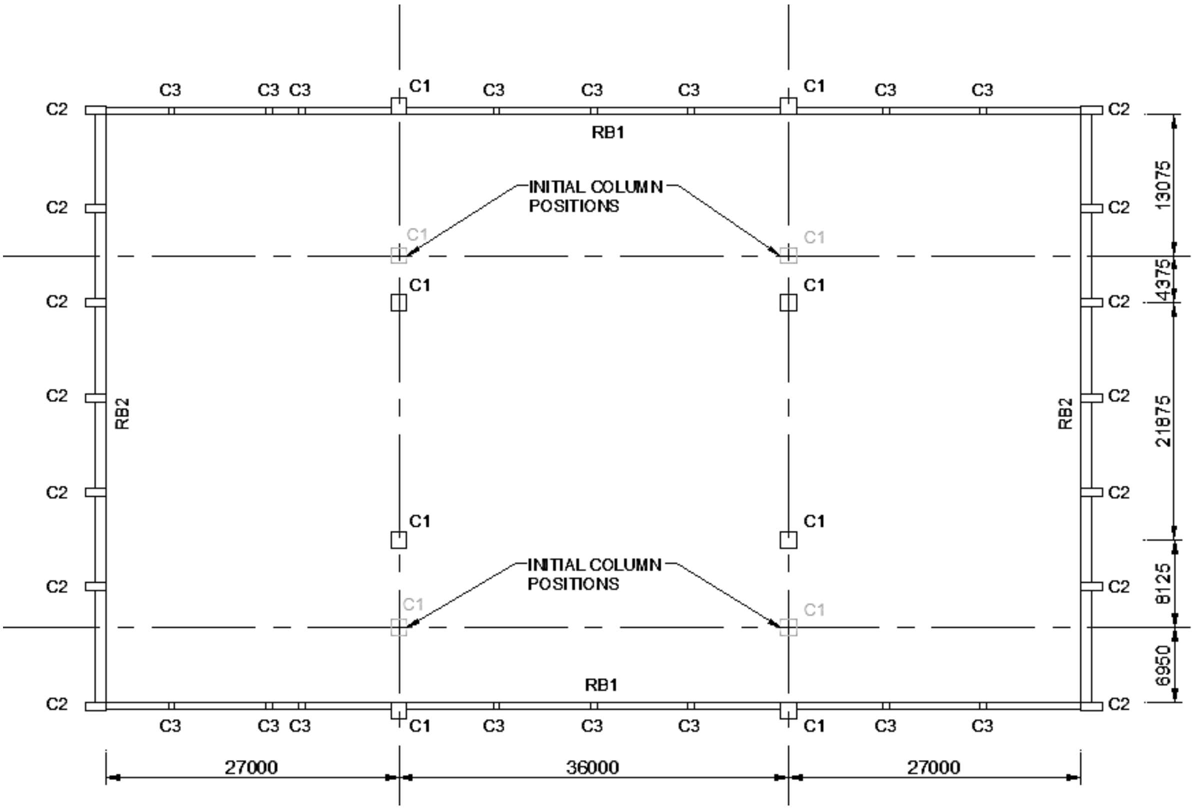


Figure 5-6: *Floor plan layout with reduced column spacing*

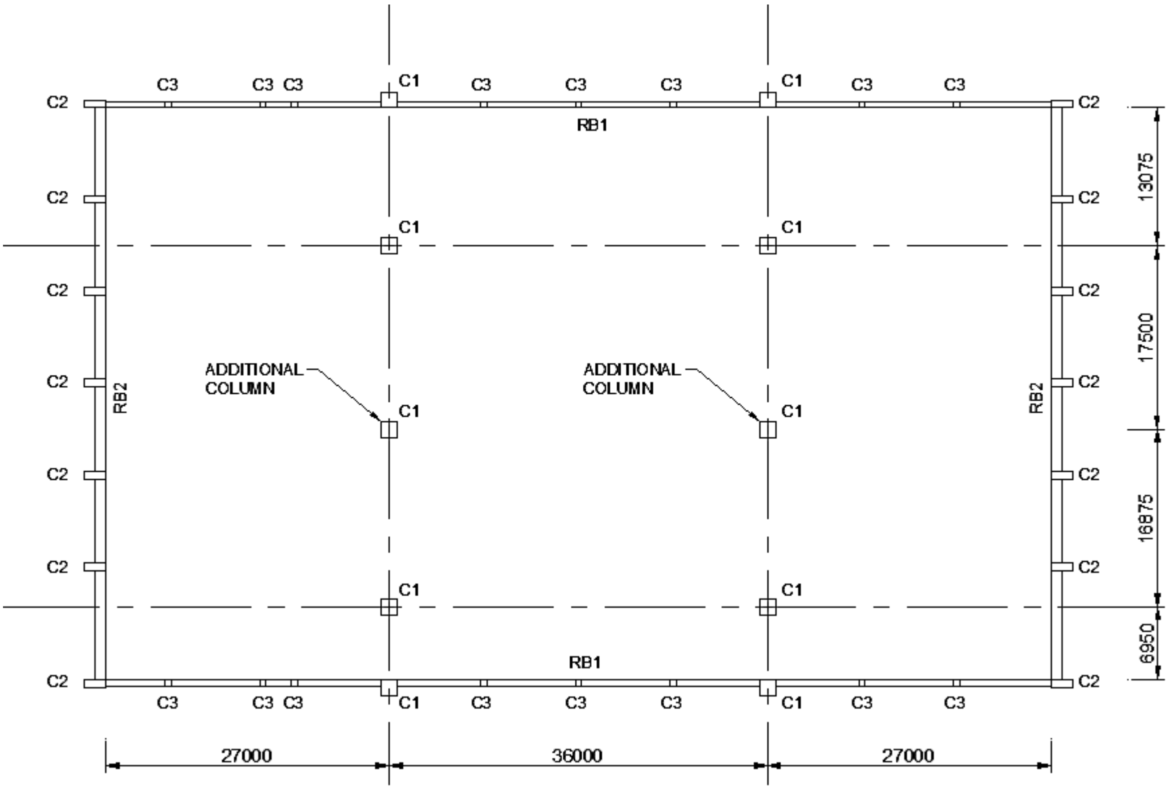


Figure 5-7: *Floor plan layout with additional columns along the short span*

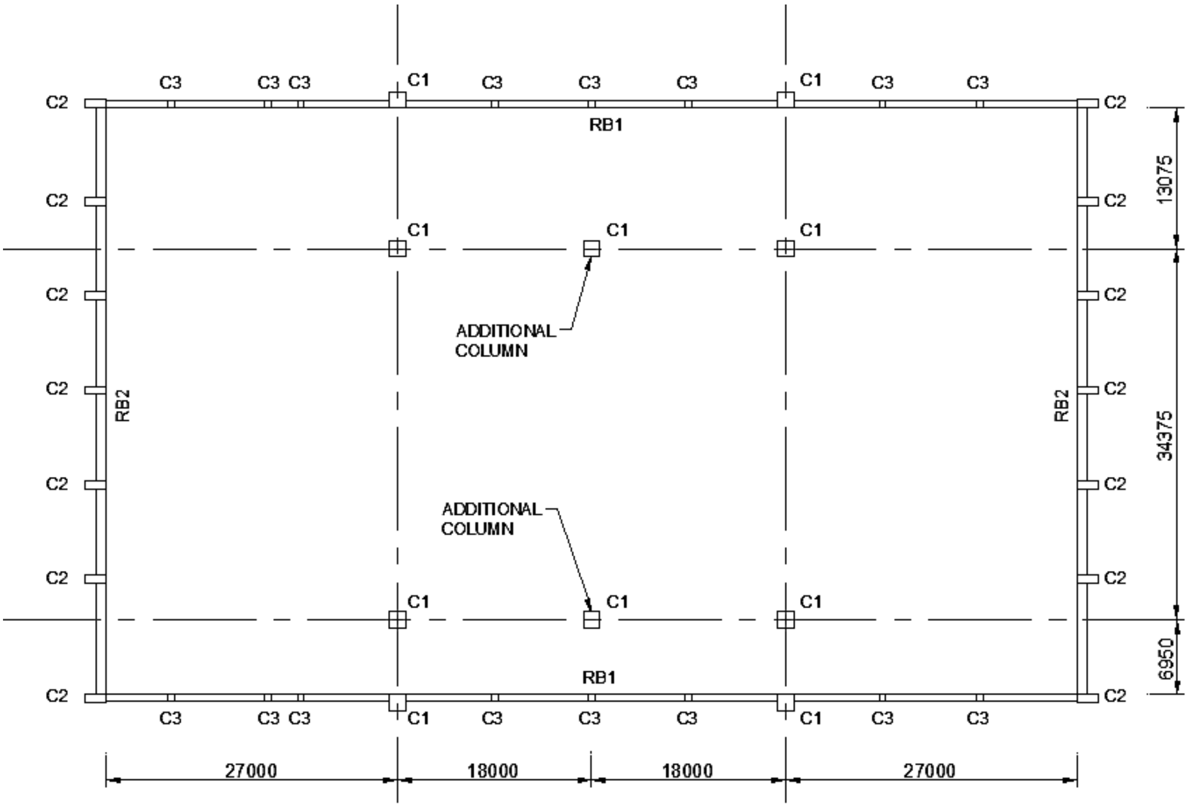


Figure 5-8: *Floor plan layout with additional columns along the long span*

Once the static analyses were completed and adequate designs were determined, the dynamic analyses were performed.

5.2 Dynamic Analyses

After the static analyses were performed and the structural elements were designed, structural modal analyses were performed to determine the dynamic properties (i.e. natural frequencies and mode shapes) of the different floor systems up to the tenth mode of vibration. The natural frequencies were then used as an indicator of susceptibility to rhythmic excitation, with higher frequencies reflecting lower risk. The obtained results are provided in the next section.

6. FE Analysis Results

6.1 Natural Frequencies

FE analysis results for the dynamic analyses performed for the three floor systems are shown in Tables 6-1 to 6-3 where the natural frequencies for the first ten modes of vibration for each floor are provided. Floors of various depths and support conditions were analysed to evaluate the effects of floor depth and support conditions of the vibration behaviour of the floor. Support conditions were varied by adding an extra row of supports around the top edge of the floor and by adding extra column supports.

Table 6-1: *Natural frequencies for plane truss floor systems*

Floor Property	Modes of Vibration									
	1	2	3	4	5	6	7	8	9	10
3.6m deep floor	3.6	4.6	4.8	5.1	5.5	5.6	5.8	6.1	6.1	6.3
3m deep floor	3.5	4.4	4.7	5.1	5.5	5.7	6.0	6.2	6.9	7.1
4m deep floor	3.7	4.7	4.8	5.1	5.3	5.5	5.7	5.7	5.8	5.9
5m deep floor	3.8	4.3	4.3	4.5	4.6	4.7	4.7	4.7	4.7	4.8
*top edge supports										
*reduced column spacing	4.2	4.5	4.8	5.0	5.5	5.9	6.5	6.6	6.6	6.6
*extra columns in short span	4.5	4.6	4.8	5.3	5.5	5.8	6.1	6.3	6.4	6.5
*extra columns in long span	5.2	5.8	6.7	6.8	6.9	7.0	7.1	7.1	7.1	7.2

*modification applied to a 3.6m deep floor

Table 6-2: *Natural frequencies for the space frame floor systems*

Floor Property	Modes of Vibration									
	1	2	3	4	5	6	7	8	9	10
3.6m deep floor	5.7	7.6	9.5	11.2	13.0	13.9	14.0	17.3	17.8	17.9
3m deep floor	5.5	7.3	9.1	11.0	12.7	13.8	13.8	17.3	17.5	18.8
4m deep floor	6.0	7.8	9.9	11.5	13.3	14.2	14.3	16.7	18.2	18.4
5m deep floor	6.0	7.7	9.9	10.8	12.4	13.4	13.8	14.8	17.0	17.7
*top edge supports	6.2	8.3	10.3	12.8	14.0	15.3	15.5	18.3	18.7	20.3
*reduced column spacing	7.3	10.1	10.6	11.7	14.5	14.5	14.5	17.6	18.3	18.8
*extra columns in short span	7.7	10.8	11.0	11.5	13.9	14.2	15.5	17.8	18.3	19.3
*extra columns in long span	6.6	7.7	10.2	13.0	13.6	14.4	15.5	16.9	18.1	18.3

*modification applied to a 3.6m deep floor

Table 6-3: *natural frequencies for the space grid floor systems*

Floor Property	Modes of Vibration									
	1	2	3	4	5	6	7	8	9	10
3.6m deep floor	4.9	6.5	8.2	9.4	11.1	12.1	15.4	15.5	15.5	15.6
3m deep floor	4.6	6.1	7.6	9.0	10.5	11.5	11.6	14.7	14.7	15.9
4m deep floor	5.2	6.8	8.6	9.7	11.5	12.3	12.5	15.0	15.9	16.1
5m deep floor	5.3	6.8	8.8	9.3	10.9	11.8	12.2	13.1	15.0	15.5
*top edge supports	5.3	7.1	8.8	11.1	12.1	13.2	13.4	15.9	16.3	17.8
*reduced column spacing	6.3	8.7	9.2	9.8	12.5	12.5	12.5	15.6	16.1	16.3
*extra columns in short span	6.7	9.4	9.5	9.8	12.1	12.3	13.4	15.6	16.1	16.8
*extra columns in long span	5.7	6.6	8.8	11.1	11.3	12.4	13.5	14.8	15.8	16.2

*modification applied to a 3.6m deep floor

6.2 Mode Shapes

This section provides mode shapes for the 3.6m deep floor of each system. The first three mode shapes for the plane truss are shown in Figures 6-1 to 6-3 whilst the first three mode shapes for the space frame and space grid are shown in Figures 6-4 to 6-6 and Figures 6-7 to 6-9, respectively. The rest of the mode shape are provided in the Appendix B.

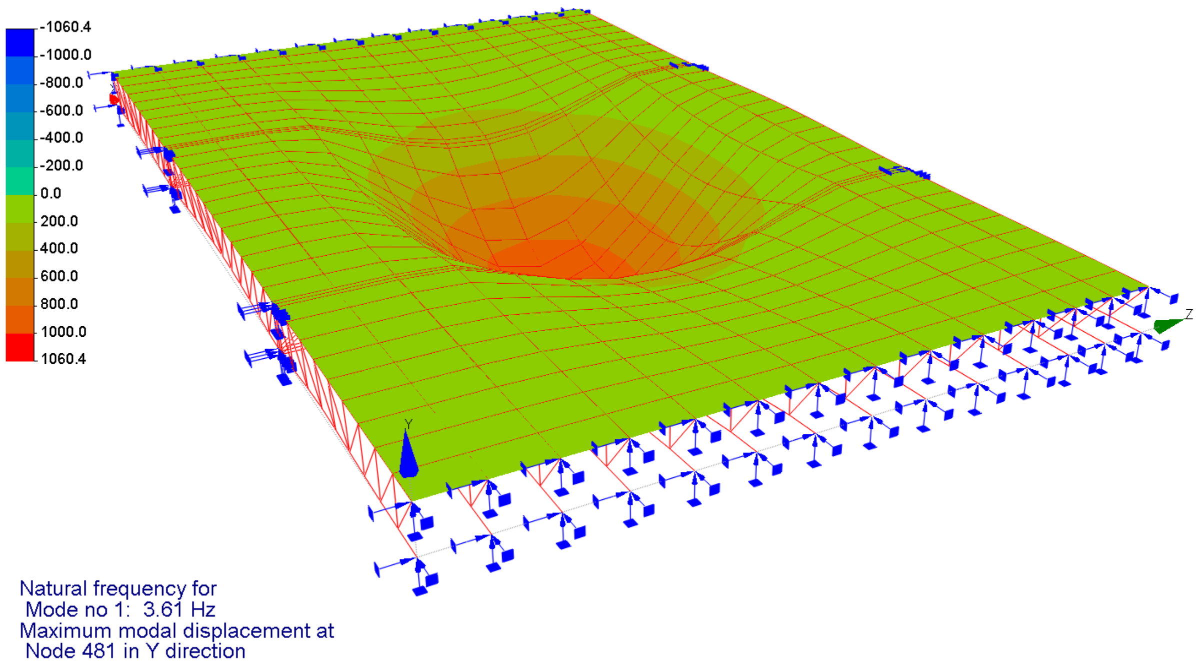


Figure 6-1: *First mode of vibration for the plane truss*

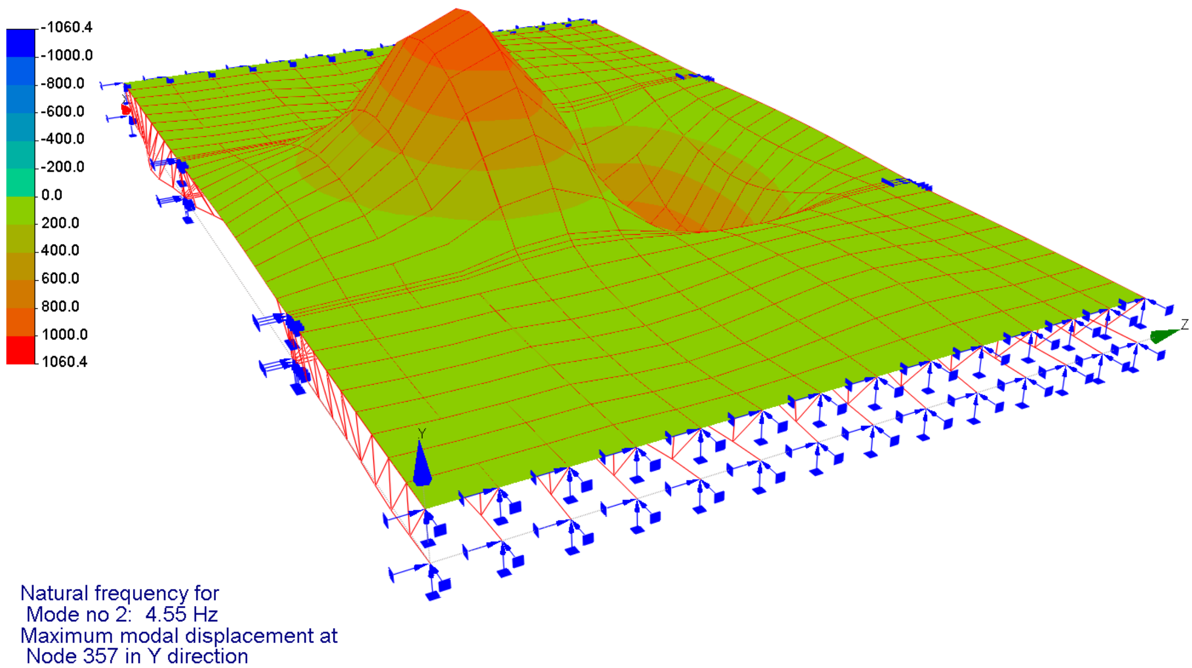


Figure 6-2: *Second mode of vibration for the plane truss*

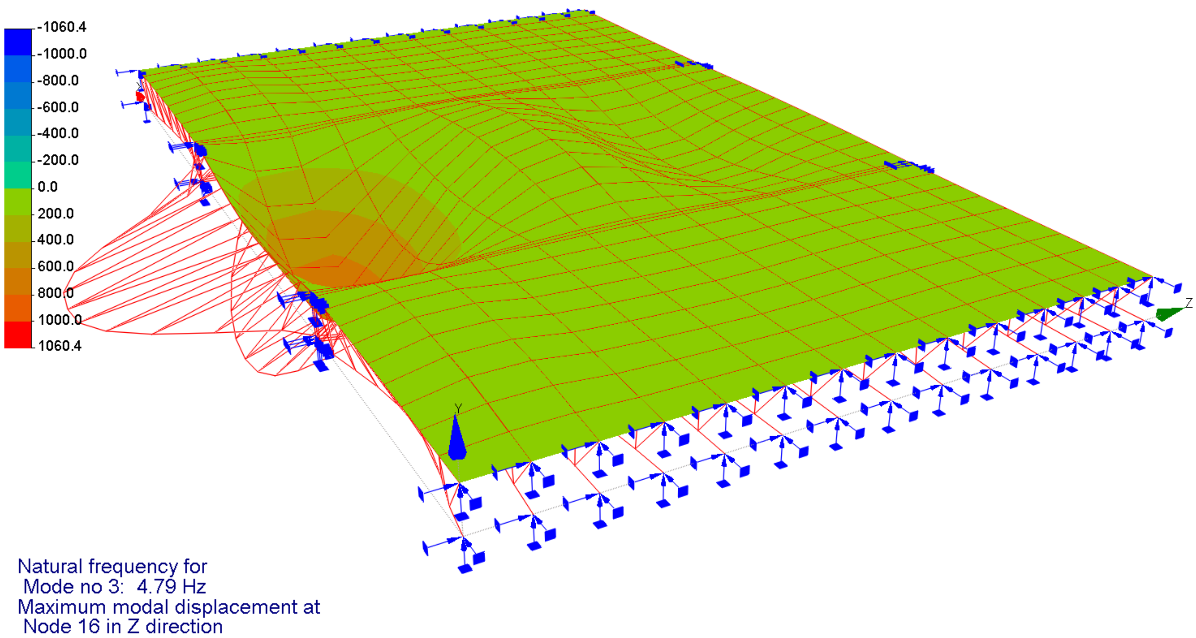


Figure 6-3: *Third mode of vibration for the plane truss*

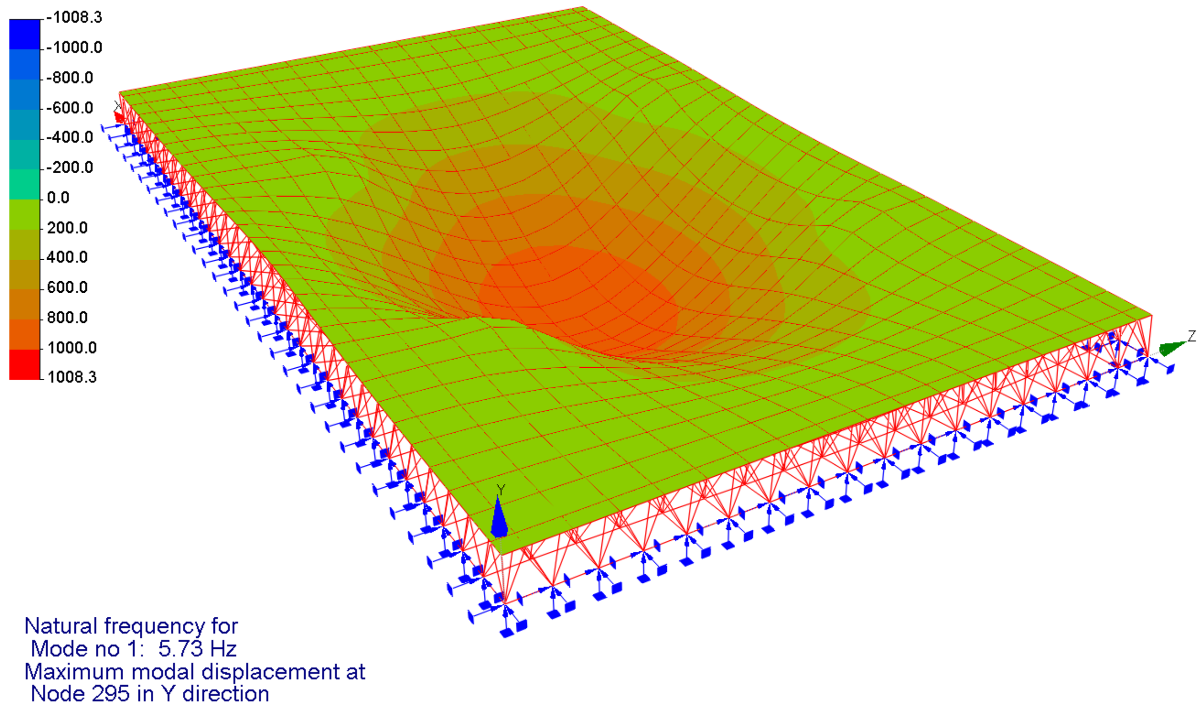


Figure 6-4: *First mode of vibration for the space frame*

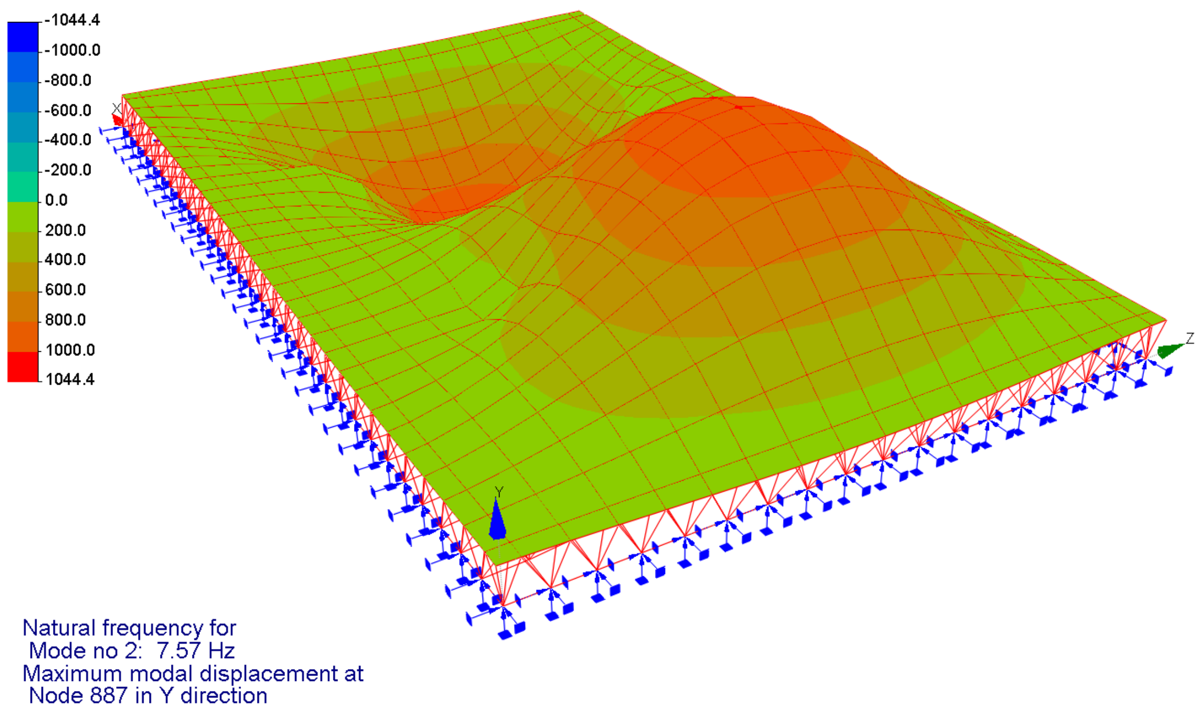


Figure 6-5: *Second mode of vibration for the space frame*

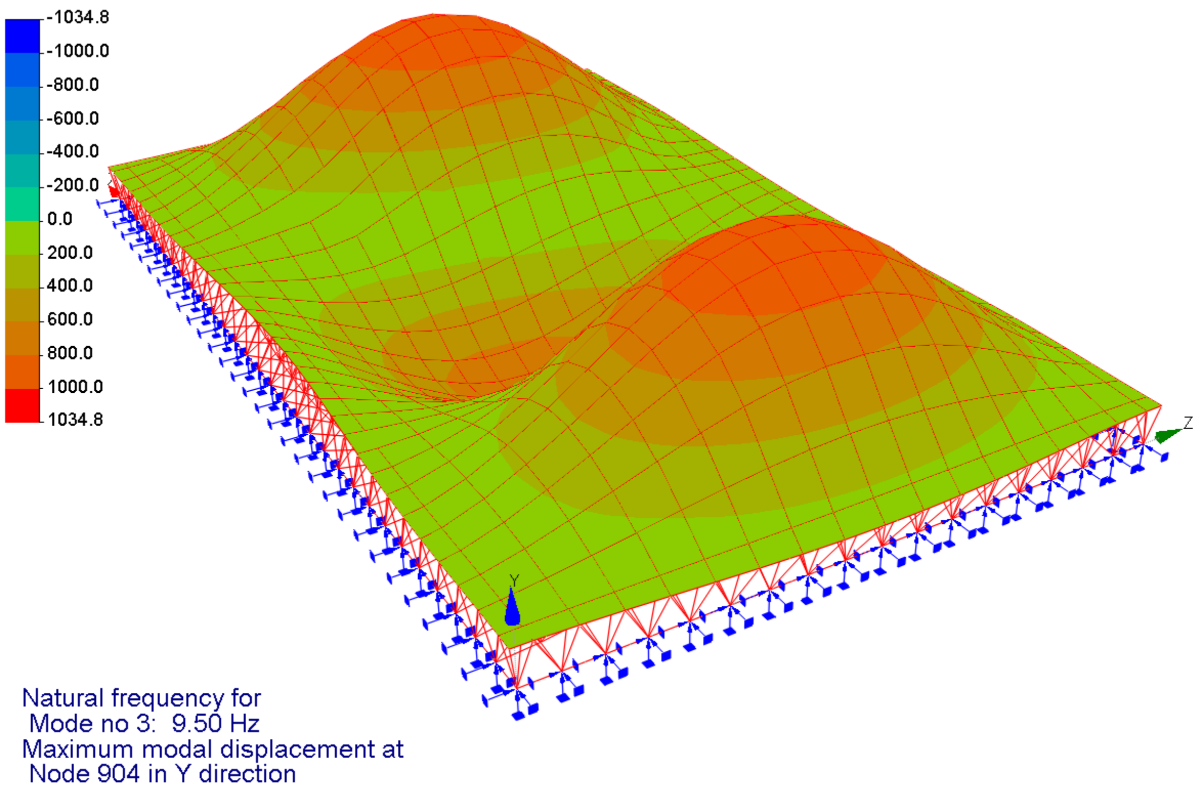


Figure 6-6: *Third mode of vibration for the space frame*

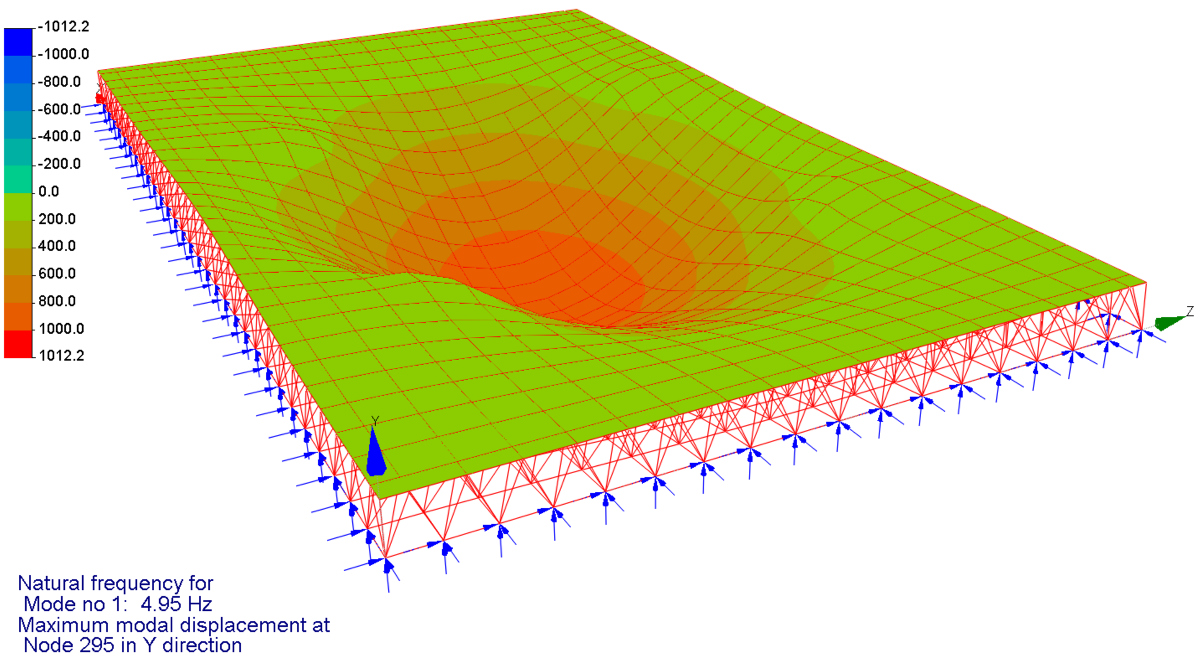


Figure 6-7: *First mode of vibration for the space grid*

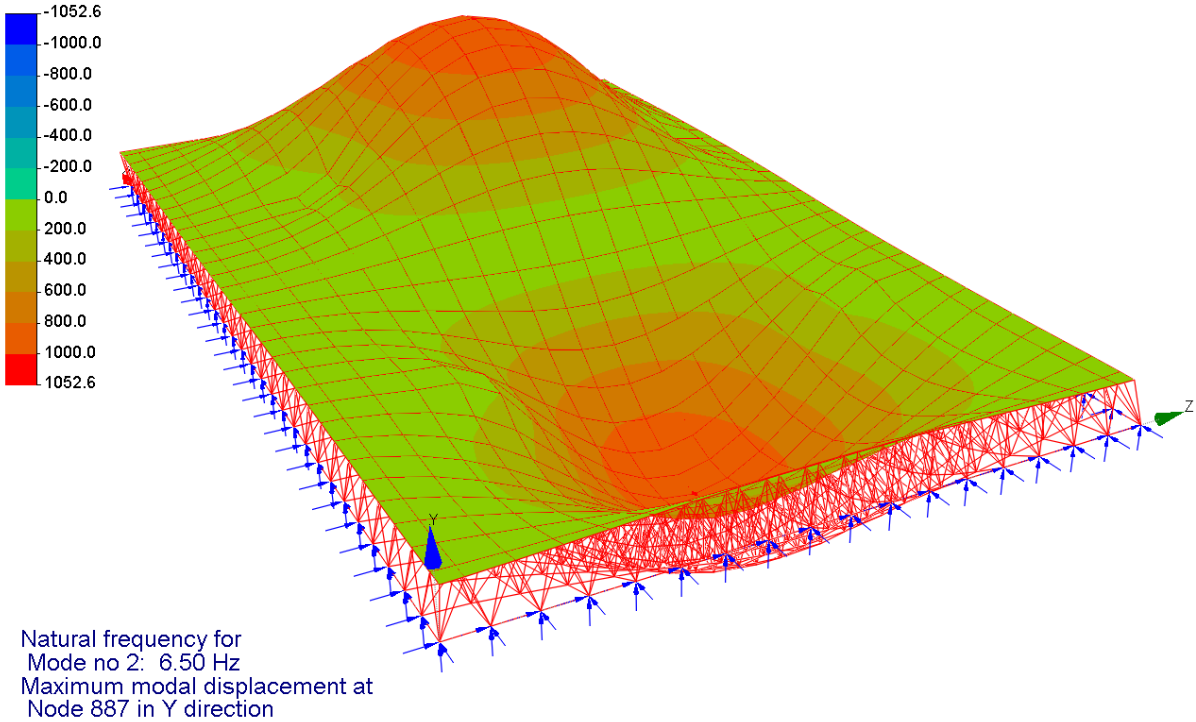


Figure 6-8: *Second mode of vibration for the space grid*

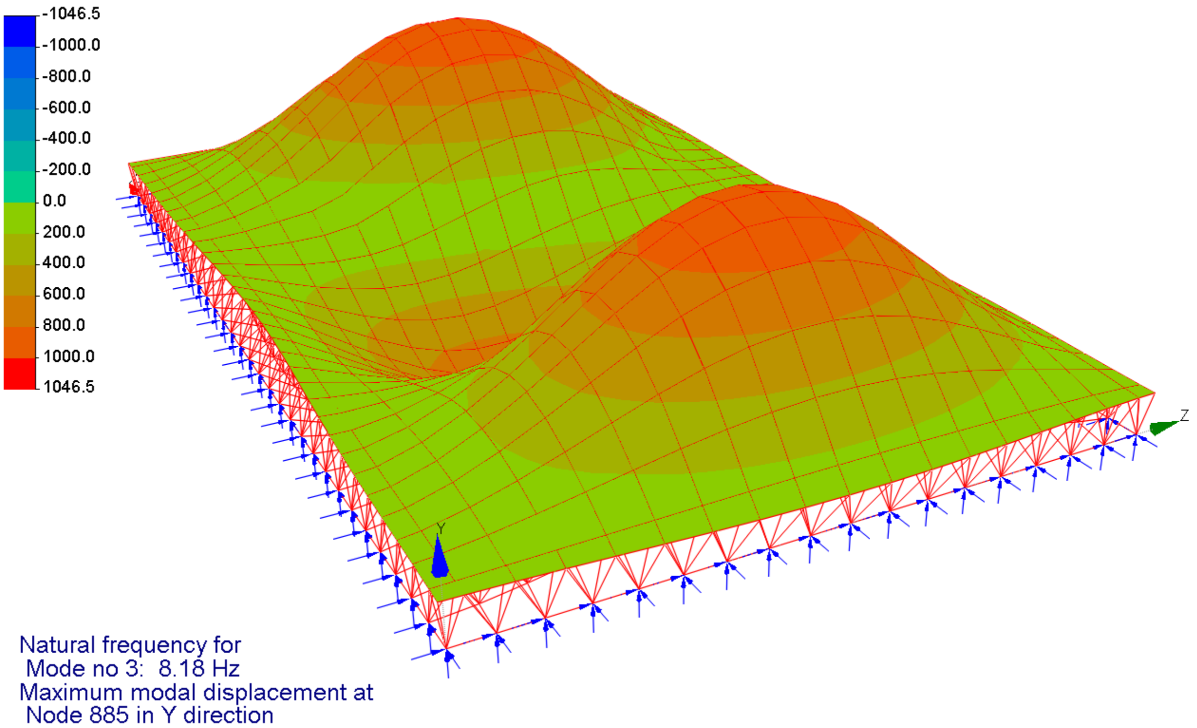


Figure 6-9: *Third mode of vibration for the space grid*

7. Discussions

In this section the results provided in Tables 6-1 to 6-3 are discussed and the effects of floor depth, support conditions and floor type are evaluated to determine their effect on the natural frequency of long span floor systems and subsequently their effect on the vibration serviceability of long span floors.

7.1 Effect of Floor Depth

Figures 7-1, 7-2 and 7-3 show the relationship between floor depth and the natural frequency of the plane truss, space frame and space grid floors, respectively. For each floor type the initial depth of 3.6m was reduced to 3m and then increased to 4m and 5m. The spacing of the top and bottom layer members was kept unchanged so the change in depth was achieved by changing the inclination and length of the diagonal elements and the length of vertical elements. The first ten modes of vibration corresponding to each floor depth are compared in the figures below.

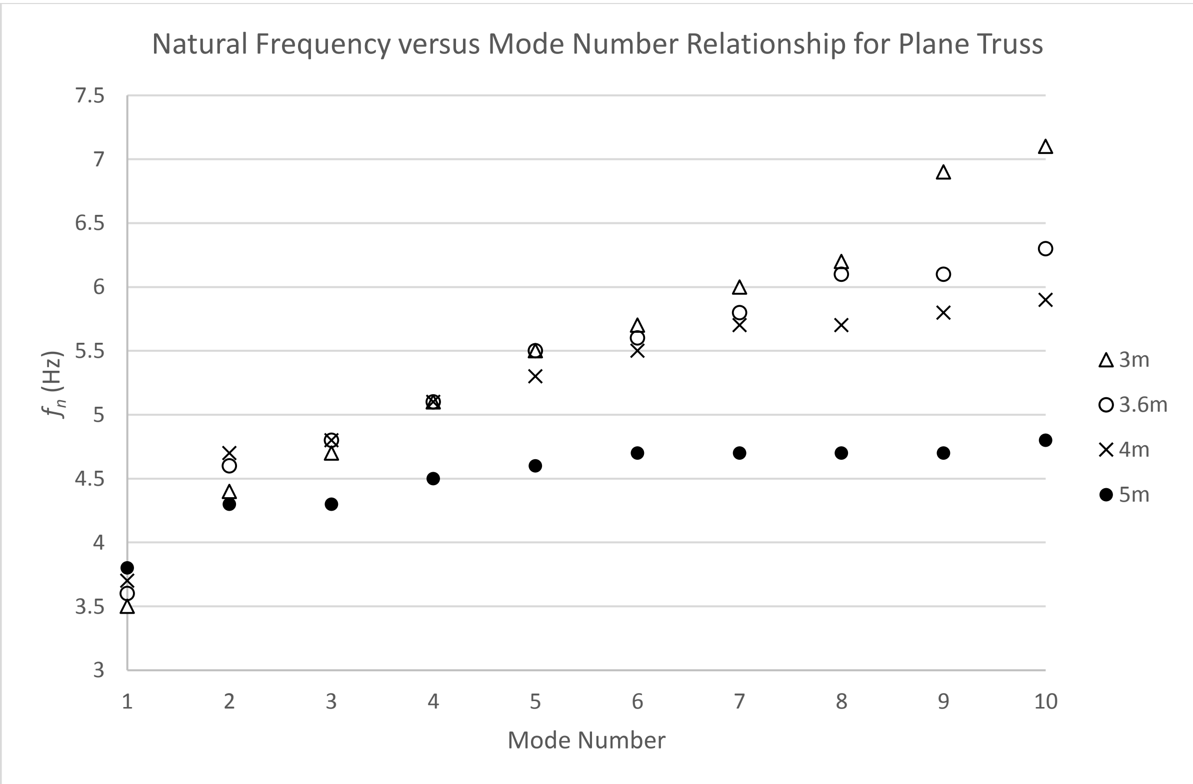


Figure 7-1: *Relationship between natural frequency and mode number for plane truss floors of various depths*

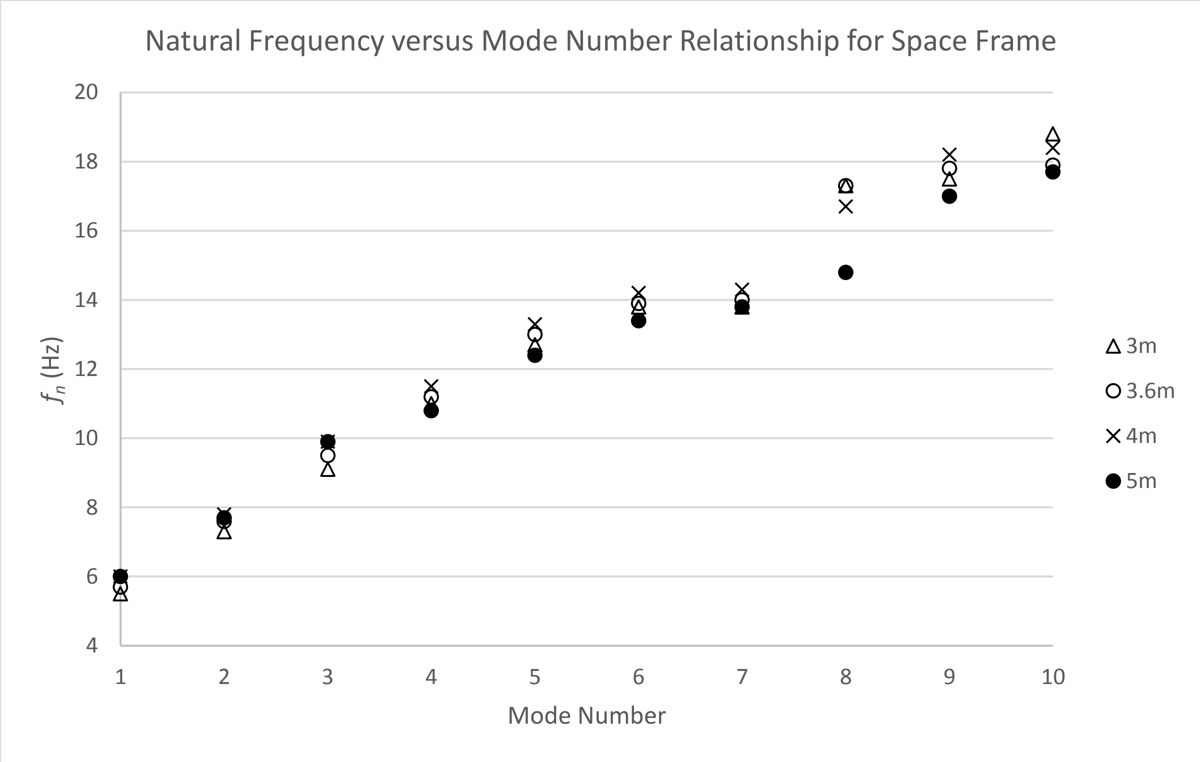


Figure 7-2: *Relationship between natural frequency and mode number for space frame floors of various depths*

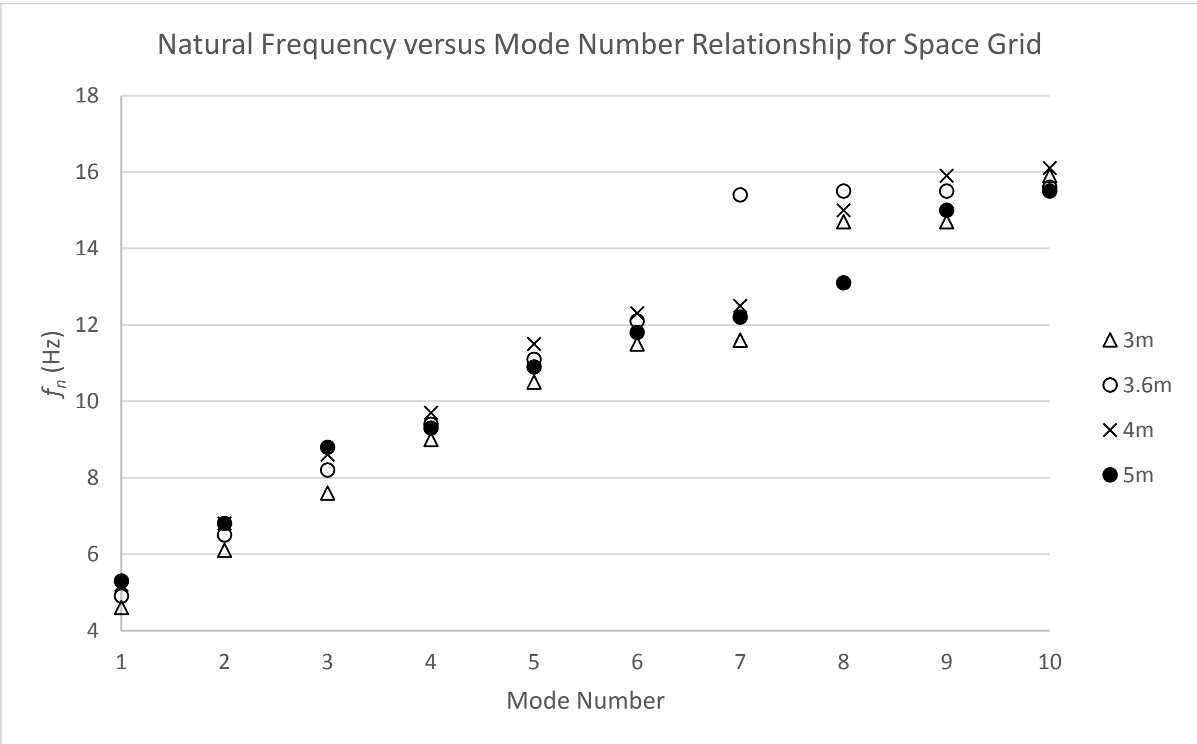


Figure 7-3: *Relationship between natural frequency and mode number for space grid floors of various depths*

Looking at the natural frequency versus mode number relationships for the plane truss system in Figure 7-1, it can be seen that the fundamental natural frequency is lowest for the 3m deep plane truss and increases with depth, with the highest fundamental natural frequency observed for a 5m deep floor. It can be seen, however, that the natural frequencies for deeper floors are closely spaced and therefore have flatter distributions. As a result of this, deeper floors have lower natural frequencies than shallower floors at higher modes of vibration.

The natural frequency versus mode number relationships for the space frame (see Figure 7-2) are similar to those of the plane truss system for the fundamental natural frequency, which increases with increasing floor depth. Unlike the plane truss, however, this relationship between natural frequency and floor depth continues up to the seventh mode of vibration with the exception of the 5m deep floor whose natural frequencies start to flatten out after the third mode of vibration. The relationship is inconclusive beyond the seventh mode.

The natural frequency versus mode number relationships for the space grid system (see Figure 7-3) follow the same pattern as those of the space frame system up to the sixth mode of vibration, with inconclusive results obtained beyond the sixth mode.

7.2 Effect of Column Position

The discussions in this section are based on the results obtained for 3.6m deep floors and the 3.6m deep floors with reduced column spacing for each floor type as provided in Tables 6-1 to 6-3. By reducing the column spacing and essentially reducing the middle span (which is also the largest span) on all three floor systems, the natural frequency is increased. This holds true for the first ten modes of vibration except for modes 2 to 5 for the plane truss system, where the natural frequencies either remained unchanged or the plane truss floor with reduced column spacings had slightly lower natural frequencies.

7.3 Effect of an Increase in the Number of Columns

Adding extra columns effectively reduces the clear spans and therefore also has a similar effect as reducing the column spacings. Figures 7-4 and 7-5 show how extra columns were added. Comparing the results of 3.6m deep floors of each system to those of floors with additional columns, it is evident that, in general, the natural frequency increases when additional columns are added. This is consistent with the findings for reduced column spacings.

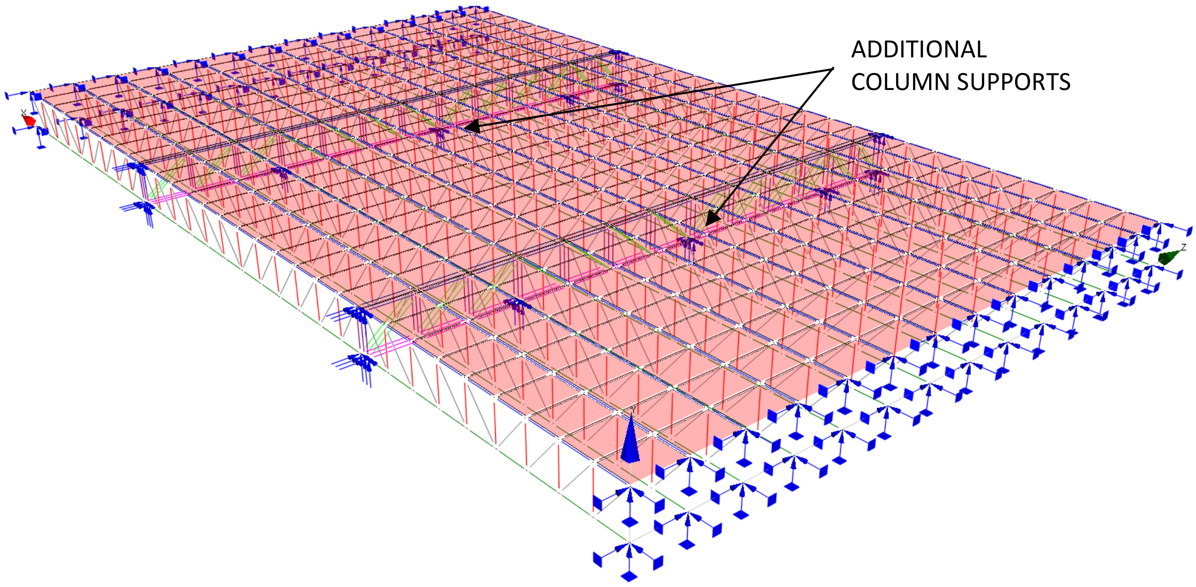


Figure 7-4: *Typical FE model of a floor with extra columns on the short span*

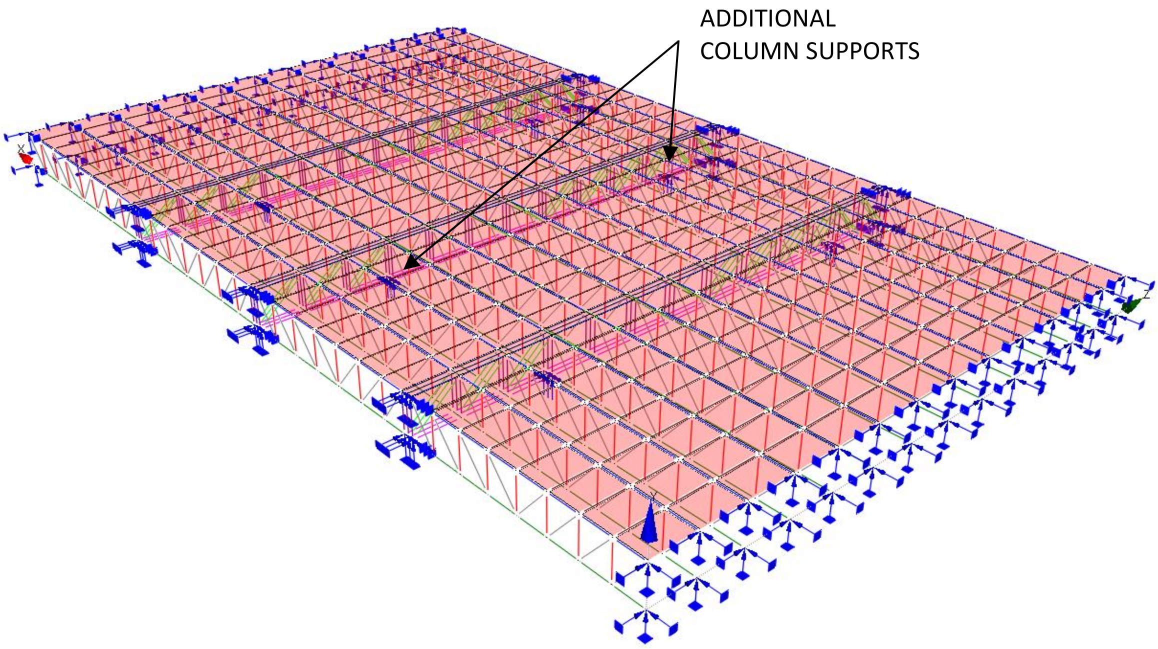


Figure 7-5: *Typical FE model of a floor with extra columns on the long span*

7.4 Effect of an Addition of Top Edge Supports

Looking at the results for **top edge supports* and comparing them to the results for the 3.6m deep floor for the space frame and space grid floor systems, it can be seen that the natural frequencies for the first ten modes of vibration increase when the top edge supports are added. This is an expected result since the addition of top edge supports makes the floor more rigid and from basic dynamic analysis, it is known that an increase in rigidity in a structure increases its natural frequency. A typical detail and a model snapshot of a floor structure supported on both the bottom and top edges are shown in Figure 7-6.

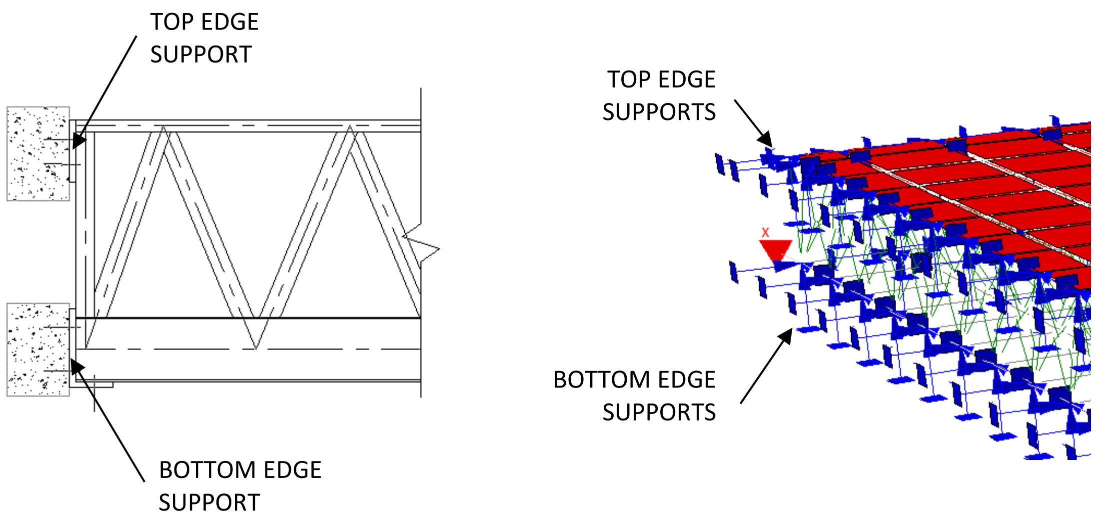


Figure 7-6: *Typical floor structure supported on both the bottom and top edges*

7.5 Comparison of the Floor Systems

A graphical comparison is provided in Figure 7-7 for the natural frequencies of a 3.6m deep plane truss, space frame and space grid floors. According to these results, the space frame had the highest natural frequencies of the three systems whereas the plane truss had the lowest. This can be regarded as an indication of the relative stiffness of the three floor systems and is justified as the plane truss system consists of members only in two planes (a single plane for an individual girder). The space frames and space grids, however, are three-dimensional in nature and as a result are more rigid. The rigid connections between the space frame elements also makes it stiffer than the space grid. It is also important to note, however, that a space frame requires a heavier structure for it to work as compared to the other systems and may as a result be a less economical option.

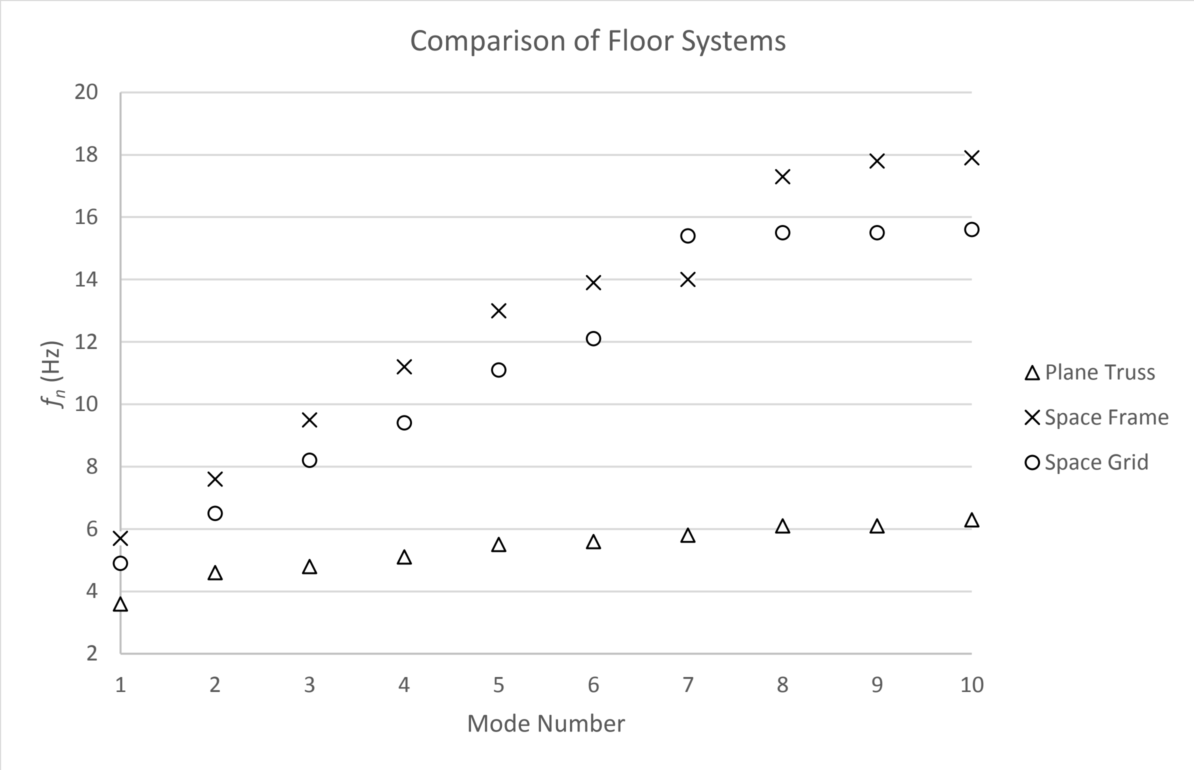


Figure 7-7: *Comparison of natural frequencies of the three floor systems*

7.6 Comparison of Results to the Minimum Recommended Values

In order to avoid vibration serviceability issues in floors subjected to rhythmic excitation, Murray et al (2003) recommend a minimum fundamental frequency of 5.9 Hz (see Table 4-4). The 1975 NBC Commentary states that resonance can occur for floor frequencies less than 10 Hz (see Section 4.3.1) while Ellis et al (2004) and the UK National Annex to Eurocode 1: Part 1-1 state that it can be avoided by keeping vertical natural frequencies above 8.4 Hz.

Looking at the results provided in Table 6-1, the plane truss floors are susceptible to resonance due to any of the first ten modes of vibration according to the above provisions. Up to fifty modes of vibration were checked and it was observed that they were all below 8.4 Hz. All the fundamental natural frequencies also do not satisfy the recommendation by Murray et al (2003) as they were all below 5.9 Hz.

Table 6-2 shows that even though all the fundamental natural frequencies of the space frame floors were below the 8.4 Hz, all the natural frequencies from the third mode upwards, and including some frequencies on the second mode, were above 8.4 Hz. All the natural frequencies on the fourth mode and upwards were above 10 Hz. All except the 3m and 3.6m deep floors had fundamental natural frequencies exceeding 5.9 Hz recommended by Murray et al (2003).

Only two of the space grid floors had fundamental natural frequencies exceeding 5.9 Hz (see Table 6-3). All floors were also susceptible to resonance due to the first four modes according to the NBC except for the floor with top edge support and additional columns on the long span, which only had the first three modes at less than 10 Hz. According to Ellis et al (2004) and the UK National Annex to Eurocode 1: Part 1-1, resonance cannot occur on the third mode of vibration and upwards except for the 3m deep floor which is also susceptible on the third mode.

7.7 Mode Shapes

The mode shapes of a structure are affected by both the geometry of the structure and its support conditions. The first mode of vibration takes on a shape that requires the least resistance to form with the higher mode shapes forming at higher resistances. Shorter spans provide higher resistance than longer spans and fixed, or continuous, supports provide higher resistance than simple supports.

Looking at Figures 6-1 to 6-9, the first mode shape for all three structures was a single wave forming in the middle span of the floor, which was the largest span for all three structures and therefore provided the least resistance. The second mode shapes for the space grid and the space frame consisted of two waves forming in the direction of the long span, which was the direction of the second least resistance for both structures. For the plane truss floor, however, the main girders provided extra resistance in the direction of the long span and hence the second wave formed in the direction on the short span, which then became the direction of the second least resistance. It can be seen in Figure 6-3 that the third wave for the plane truss floor also formed the direction of the short span, between the two main girders, indicating that this was the direction that provided the third least resistance. Without extra resistance in the direction of the long span, however, the third mode shapes for the space frame and space grid consisted of three waves in the direction of the long span.

It is evident from the mode shapes provided in Appendix A that the support conditions had little effect on the mode shapes for the lower modes of vibration of the space grid and space frame floors with apparent differences only observed from the eighth mode of vibration and upwards.

8. Conclusions

The following conclusions are made based on the discussions above:

1) The space frame is the best system to employ for a long-span floor subjected to rhythmic excitation if the cost of construction is not an important factor. Its larger element sizes make it more rigid than the space grid and its three-dimensional load carrying capacity makes it more rigid than a plane truss system. In the analyses performed on a 3.6m deep floor, the natural frequency of the space frame floor was 58 % higher than that of the plane truss floor and 16 % higher that of the space grid floor for the first mode of vibration. It was 184 % higher than that of the plane truss floor and 15 % higher than that of the space grid floor on the tenth mode of vibration. The space frame is, however, much heavier than the space grid and more difficult to construct.
2) The space grid is stiffer than the plane truss system due to its three-dimensional load carrying capacity and is also lighter and easier to construct than the space frame. As a result, it may be the most suitable system to employ where the cost and ease of construction are also important factors.
3) Any measure that improves the stiffness of the floor - whether it is increasing the floor depth, reducing the clear spans or additional supports – increases the natural frequency of the floor and therefore improves the vibration serviceability of the floor. Even though an improvement will occur, selecting the right floor system is important because the improvement may still not be sufficient for some floor systems.
4) The static imposed load of 5 kN/m^2 recommended by SANS 10160 -2 for buildings subjected to large crowds and rhythmic loading, such as concert halls and sports halls, is not sufficient. Dynamic analyses need to be performed to ensure dynamically serviceable structures.

9. Recommendations

Based on the conclusions above, the following recommendations are made:

1) Physical tests need to be carried out on existing long span floors under rhythmic loads to verify the FE analysis results obtained in this study.
2) Further research needs to be conducted to study the response of long-span slender floors to rhythmic loads modelled in accordance with the rhythmic load models recommended in literature.
3) An update to the South African National Standard is required to make provision for long-span slender floors subjected to rhythmic loads as the current provisions are inadequate.
4) Designers should increasingly be encouraged to opt for the space grid design over the plane truss design for long-span floor structures, especially those involving potential rhythmic activity.

10. References

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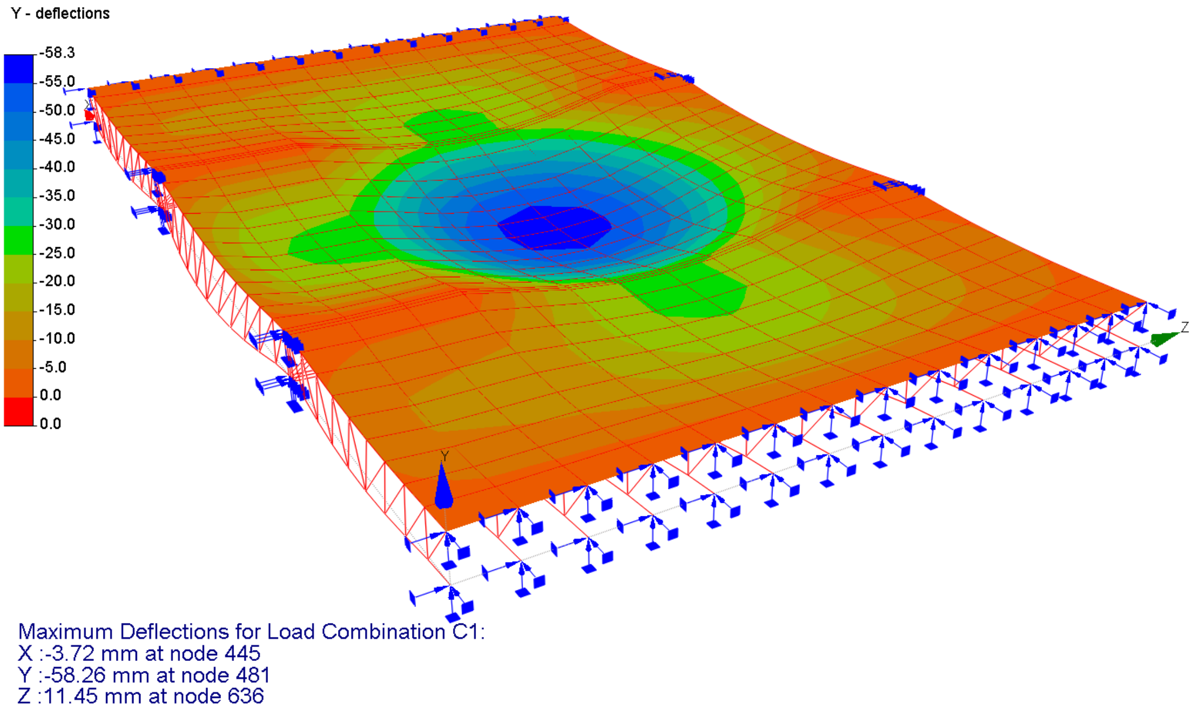
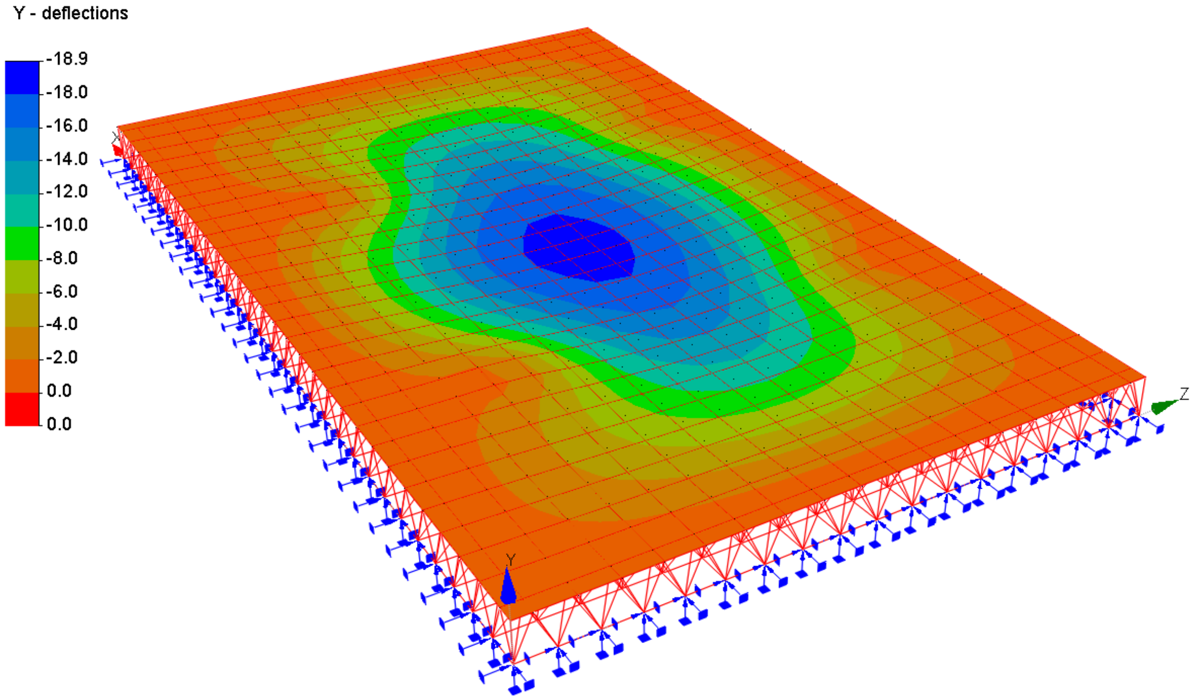
Appendix A. Deflections Under Static Loads

3.6m Plane Truss




```
=============================================== STATISTICAL DATA ===============================================

Weights of beam elements:        (Added to load case DL)

Section Designation                            Total   Mass
                                               (kN)      (kg)

 S_DIAG 203x203x60      H2              1141.843    116395.9
 S_VERT 203x203x52      H2               695.750     70922.5
 S_B.CH 533x210x122     I1              1479.878    150854.1
 S_T.CH 203x133x25      I1               303.565     30944.4
 M_B.CH 1030x400 (8     I1               698.692     71222.4
 M_T.CH 305x165x54      I1               171.406     17472.5
 M_DIAG 305x305x137     H1               577.381     58856.4
 M_VERT 305x305x118     H1               349.272     35603.7
 CONNEC 533x210x122     I1                67.267      6857.0
 BRACIN 203x133x25      I1               361.912     36892.2
                                        ---------------------
                                        5846.967    596021.1

 Total weight of structure = 26659.59 kN

 No. of real numbers in stiffness matrix = 0  (0 bytes)
 Time used to analyse = 0: 0:1.170 seconds
 Total number of : Nodes             = 980
                   Beam Elements     = 2419
                   Cable Elements    = 0
                   Shell Elements    = 442
                   Supports          = 92
                   Section Properties = 10
                   Load Cases        = 2
                   Load Combinations = 1

================================================ END OF OUTPUT ================================================
```

3.6m Deep Space Frame



Maximum Deflections for Load Combination C1:
X :-1.18 mm at node 284
Y :-18.88 mm at node 295
Z :-1.78 mm at node 1111

```
============================================== STATISTICAL DATA ==============================================

Weights of beam elements:        (Added to load case DL)

Section Designation                          Total   Mass
                                              (kN)       (kg)

 B. CHO 1030x400 (8      I1              7134.085    727225.8
 T. CHO 203x133x25       I1               856.979     87357.7
 DIAG.   305x305x137     H1             12553.904   1279704.8
                                        ----------------------
                                        20544.968   2094288.3

 Total weight of structure = 41678.09 kN

 No. of real numbers in Stiffness matrix = 0  (0 bytes)
 Time used to analyse = 0: 0:1.648 seconds
 Total number of : Nodes              = 1166
                   Beam Elements      = 4464
                   Cable Elements     = 0
                   Shell Elements     = 558
                   Supports           = 98
                   Section Properties = 3
                   Load Cases         = 2
                   Load Combinations  = 1

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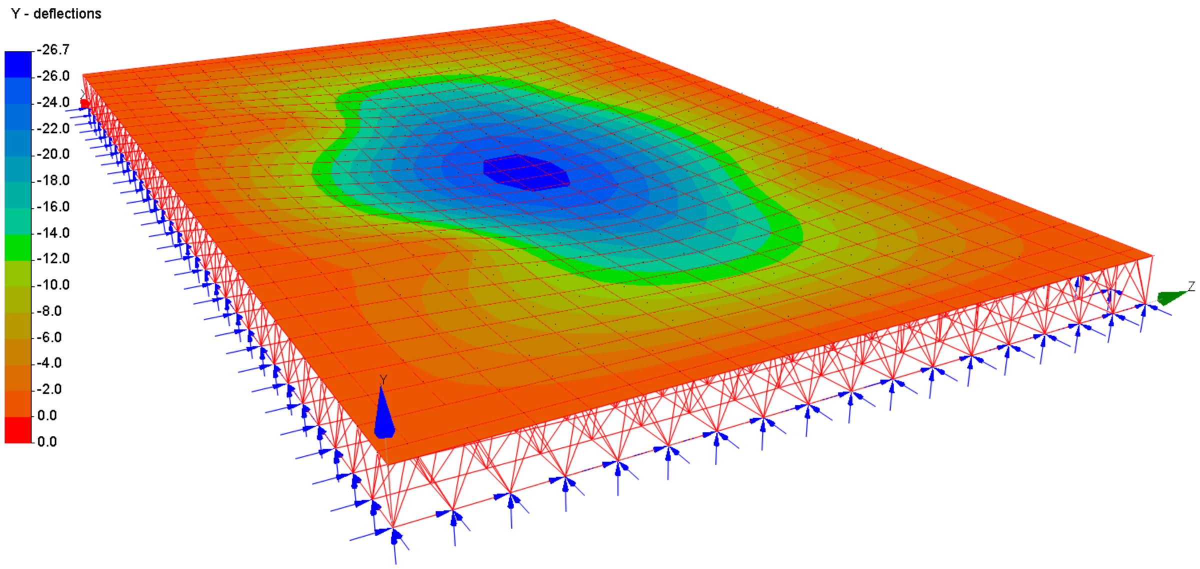
3.6m Deep Space Grid



Maximum Deflections for Load Combination C1:
X :1.99 mm at node 306
Y :-26.72 mm at node 295
Z :-3.06 mm at node 207

```
=============================================== STATISTICAL DATA ===============================================

Weights of beam elements:       (Added to load case DL)

Section Designation                         Total    Mass
                                             (kN)       (kg)

 B. CHO 406.4x12.0      01              3823.664    389772.1
 T. CHO 152.4x6.0       01               736.467     75073.0
 DIAG.  406.4x12.0      01             10750.182   1095839.2
                                        ---------------------
                                       15310.313   1560684.3

 Total weight of structure = 36443.44 kN

 No. of real numbers in stiffness matrix = 0  (0 bytes)
 Time used to analyse = 0: 0:1.709 seconds
 Total number of : Nodes              = 1166
                   Beam Elements      = 4464
                   Cable Elements     = 0
                   Shell Elements     = 558
                   Supports           = 98
                   Section Properties = 3
                   Load Cases         = 2
                   Load Combinations  = 1

================================================ END OF OUTPUT ================================================
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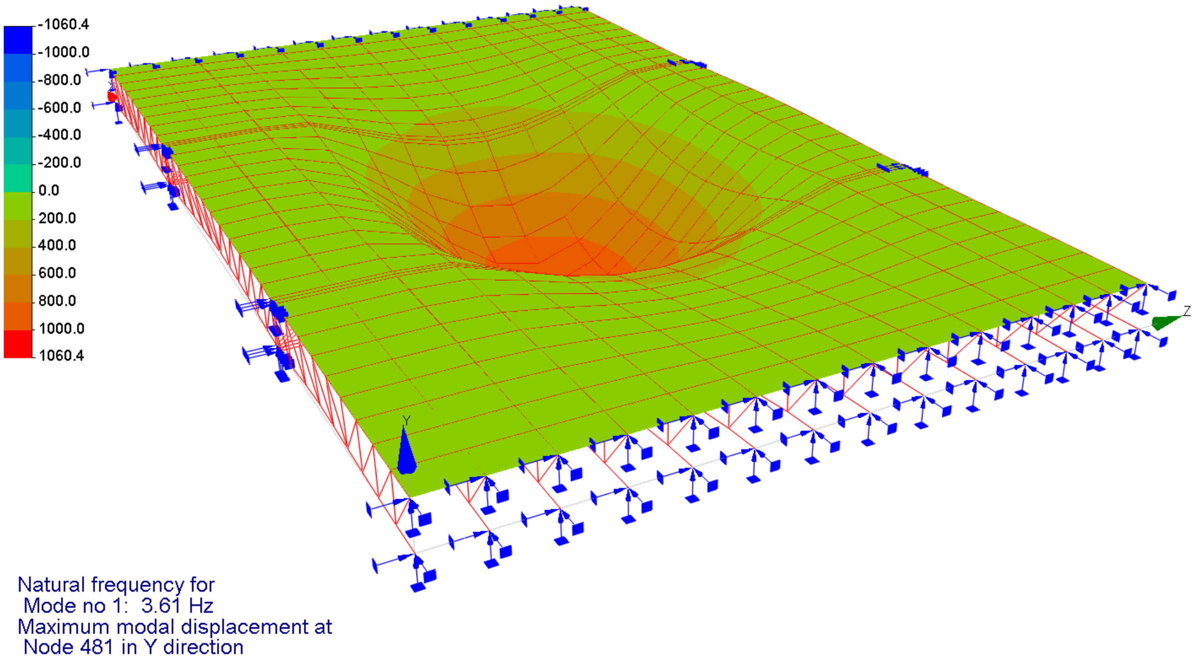
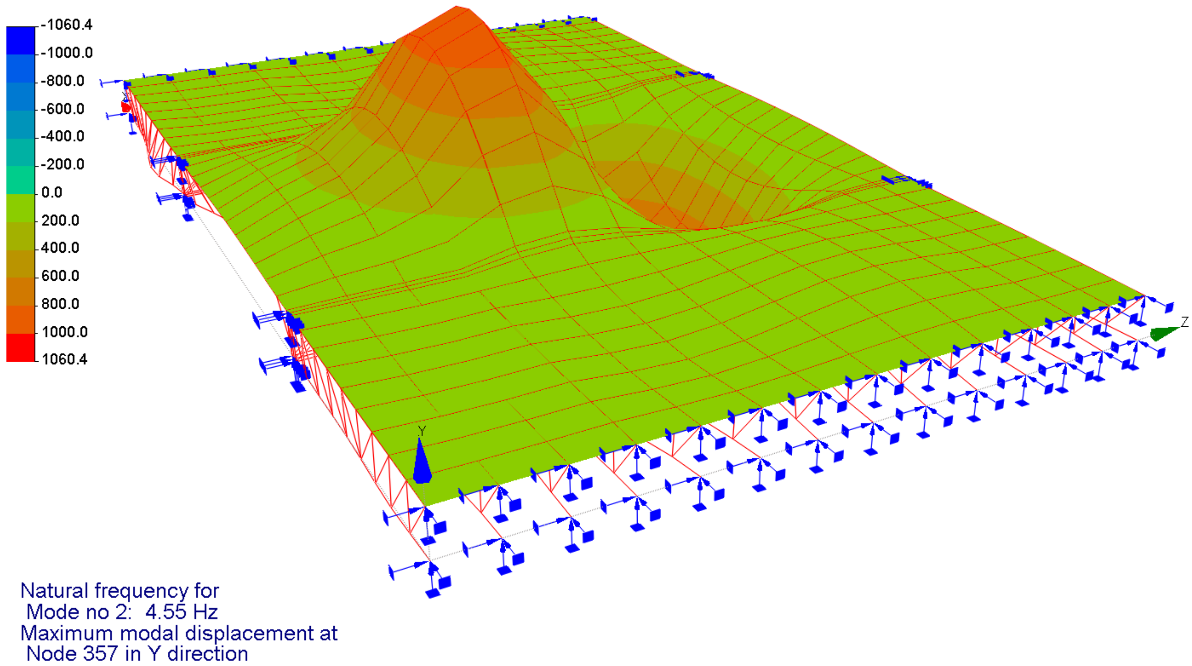
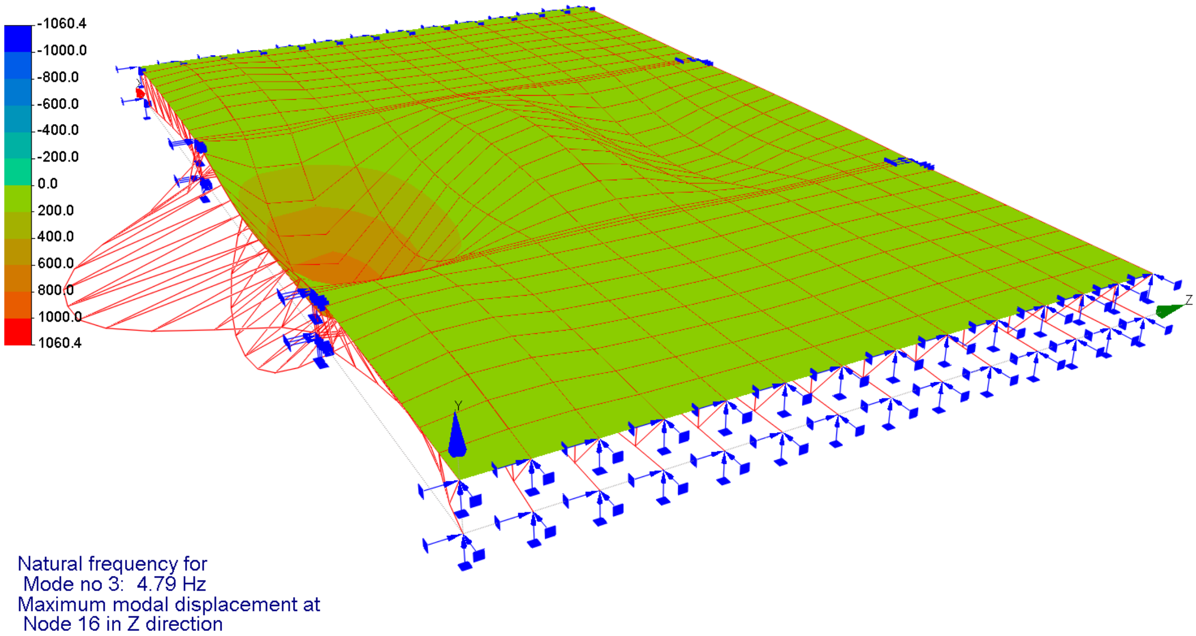
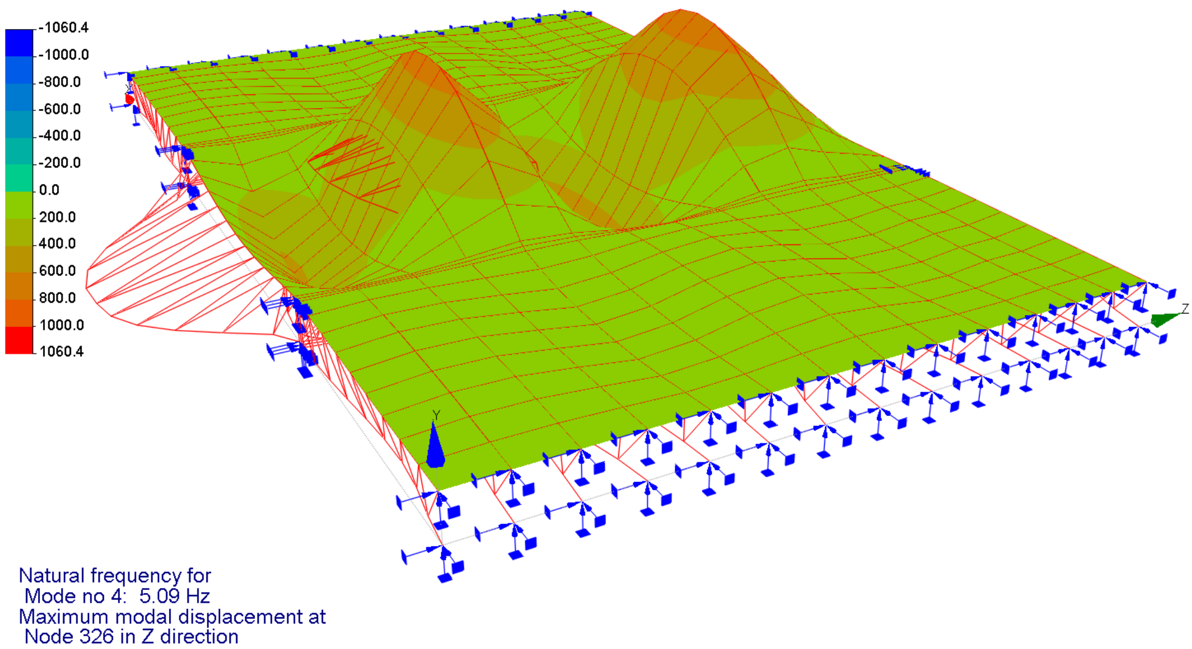
Appendix B. Mode Shapes

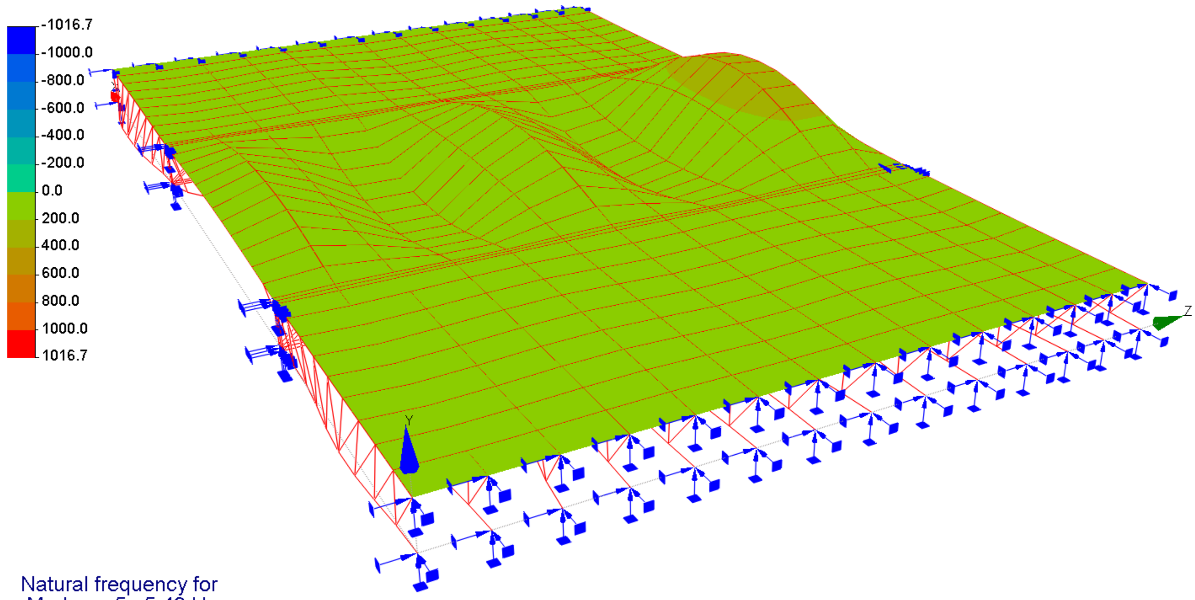
Mode Shapes for the 3.6m Deep Plane Truss Floor



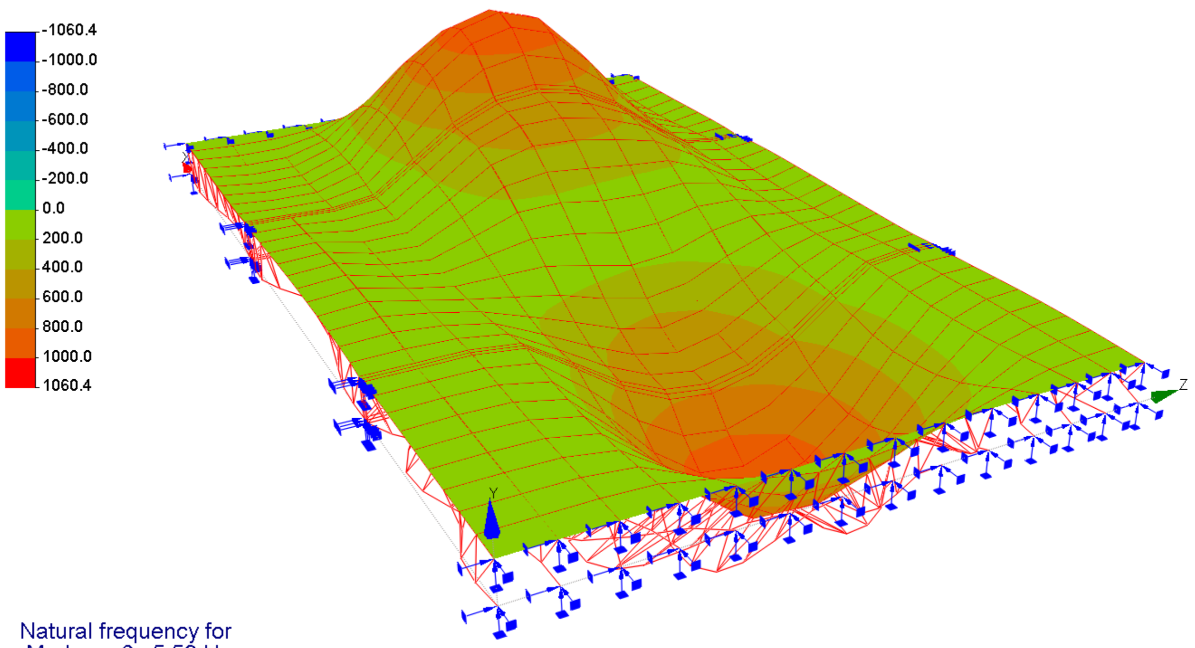



-1060.4
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1060.4
Y
Z
Natural frequency for
Mode no 3: 4.79 Hz
Maximum modal displacement at
Node 16 in Z direction

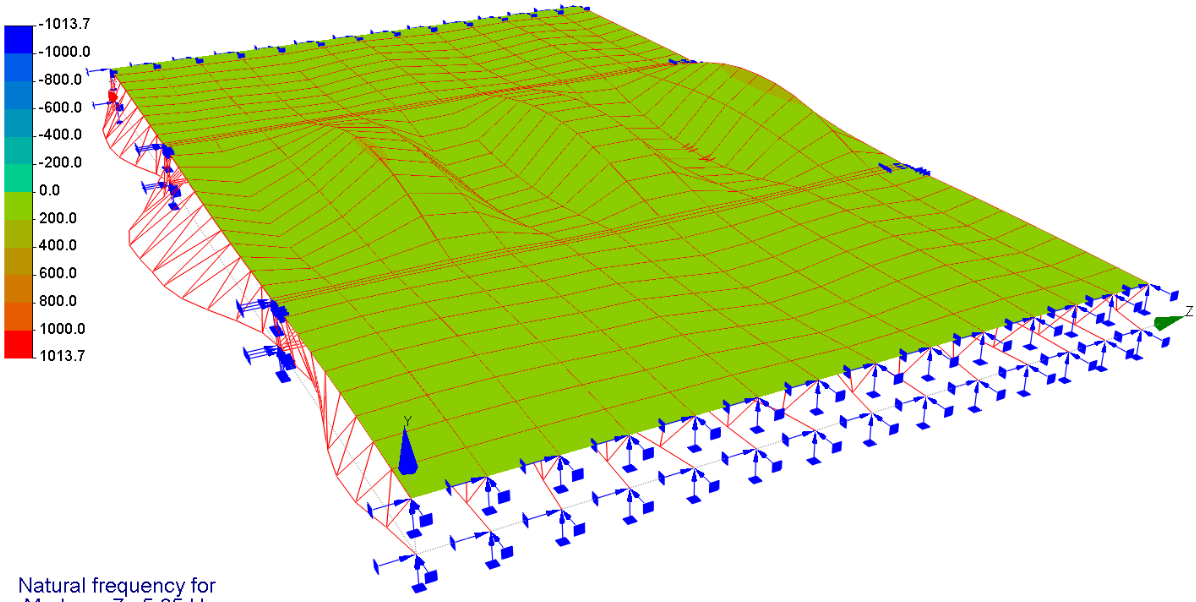

-1060.4
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1060.4
Y
Z
Natural frequency for
Mode no 4: 5.09 Hz
Maximum modal displacement at
Node 326 in Z direction



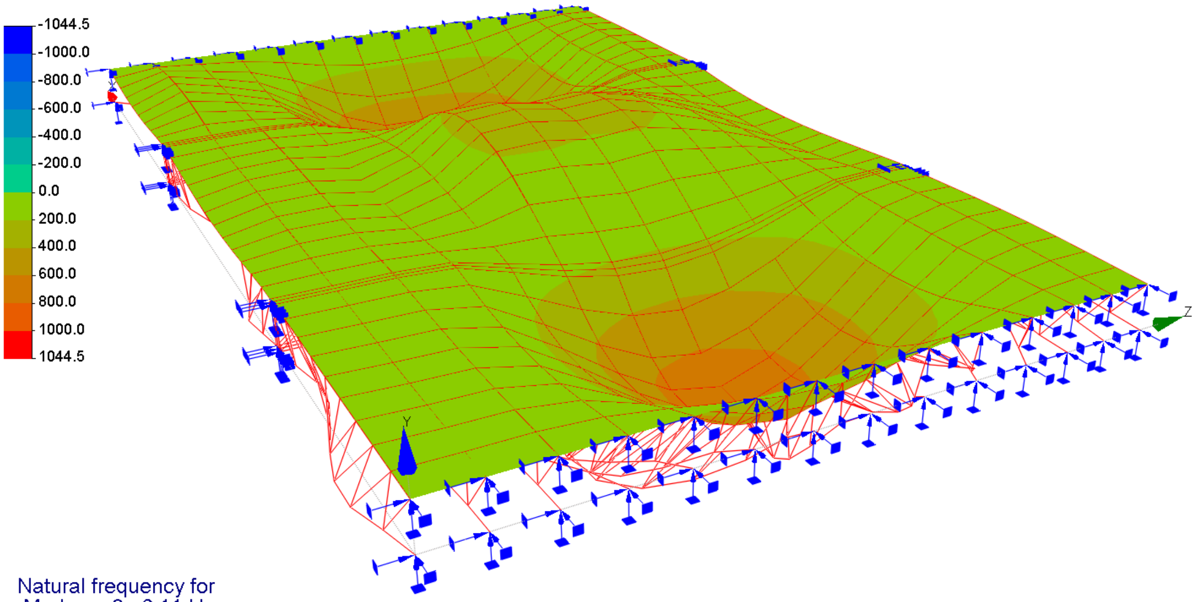
Natural frequency for
Mode no 5: 5.49 Hz
Maximum modal displacement at
Node 821 in Z direction



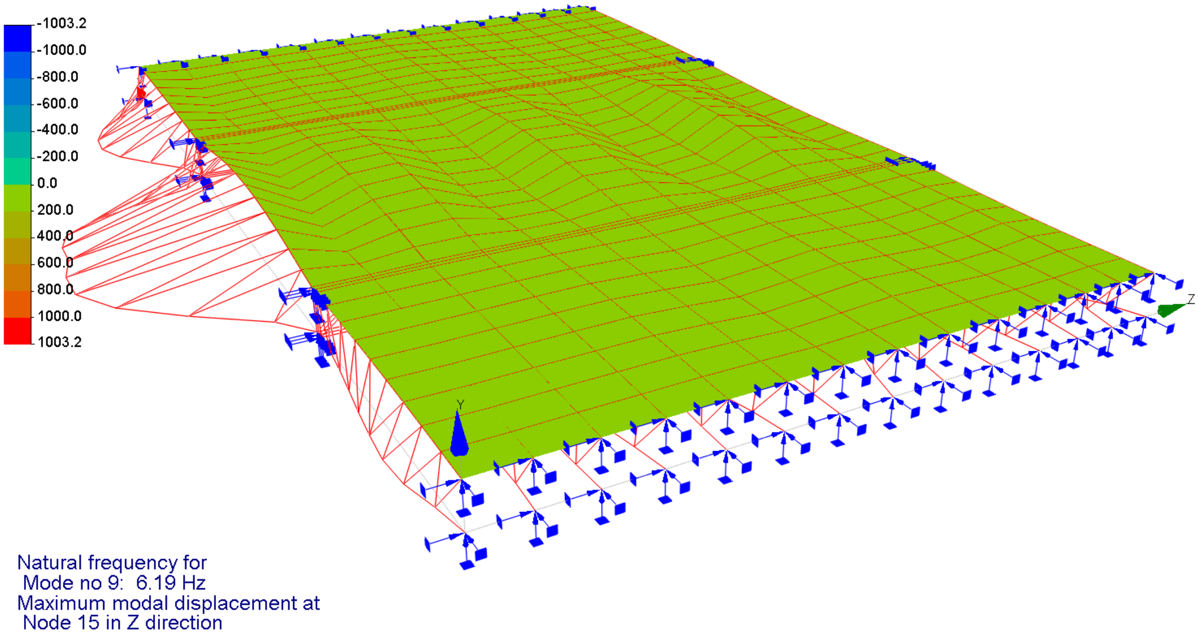
Natural frequency for
Mode no 6: 5.59 Hz
Maximum modal displacement at
Node 490 in Y direction



Natural frequency for
Mode no 7: 5.85 Hz
Maximum modal displacement at
Node 821 in Z direction



Natural frequency for
Mode no 8: 6.11 Hz
Maximum modal displacement at
Node 26 in Z direction


-1003.2
-1000.0
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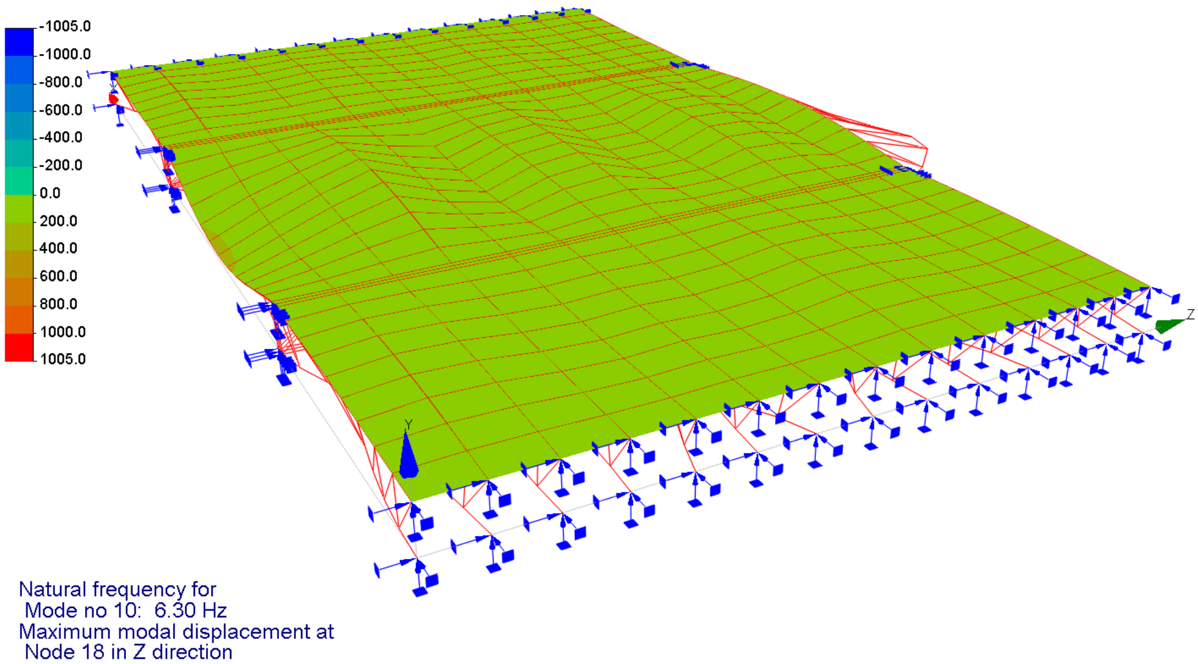
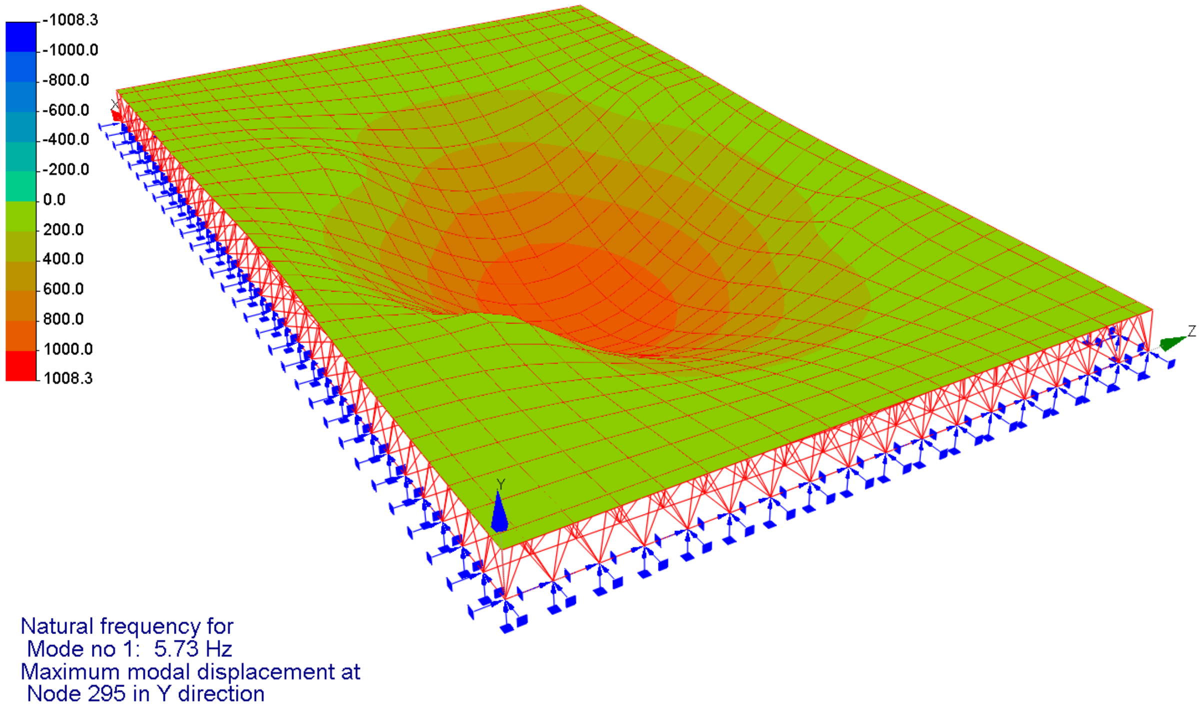
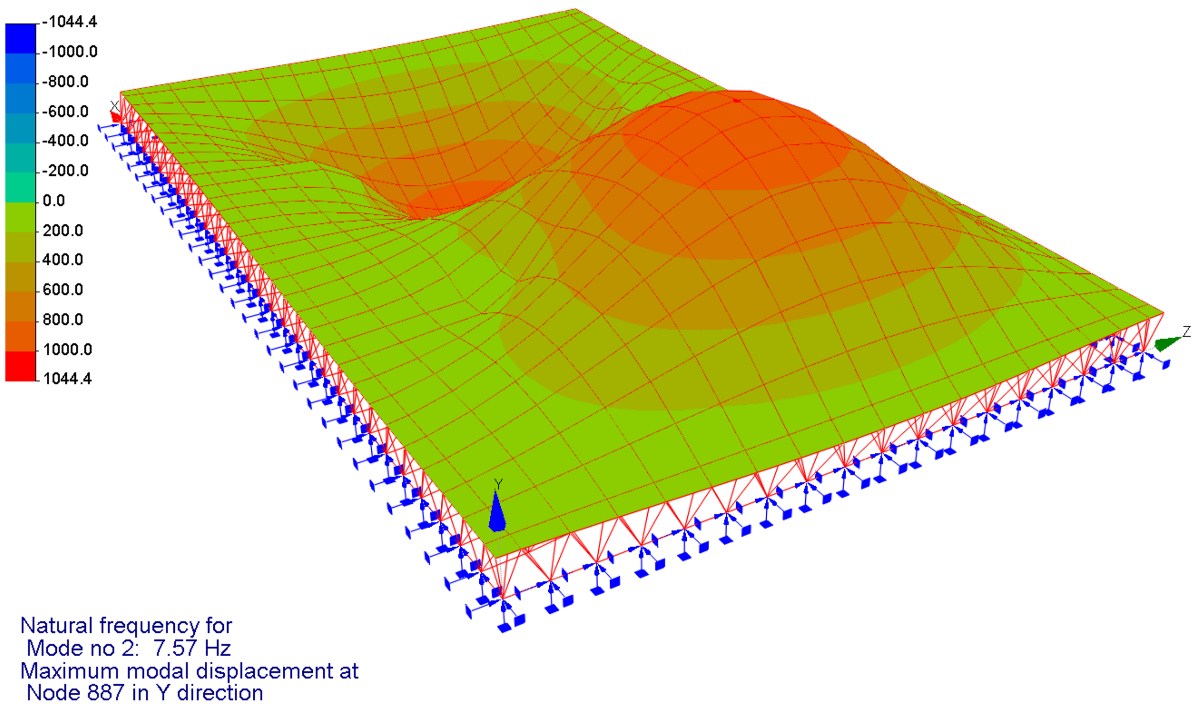
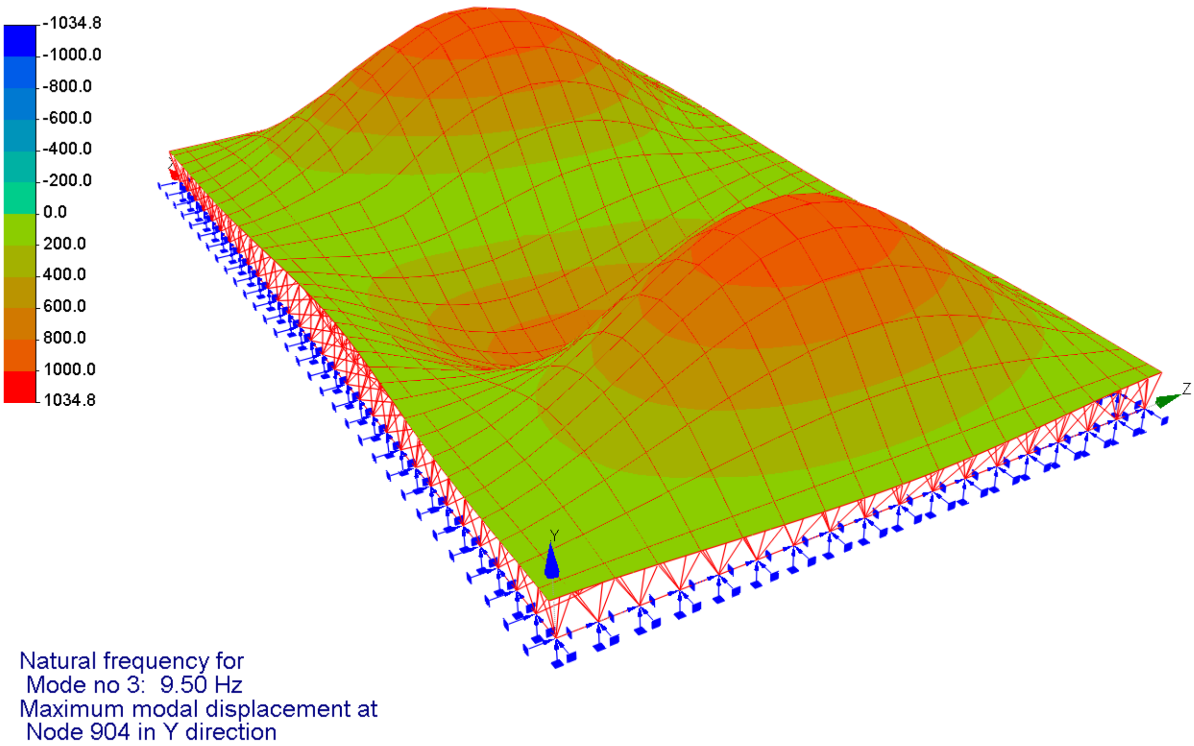
0.0
200.0
400.0
600.0
800.0
1000.0
1003.2
Y
Z
Natural frequency for
Mode no 9: 6.19 Hz
Maximum modal displacement at
Node 15 in Z direction

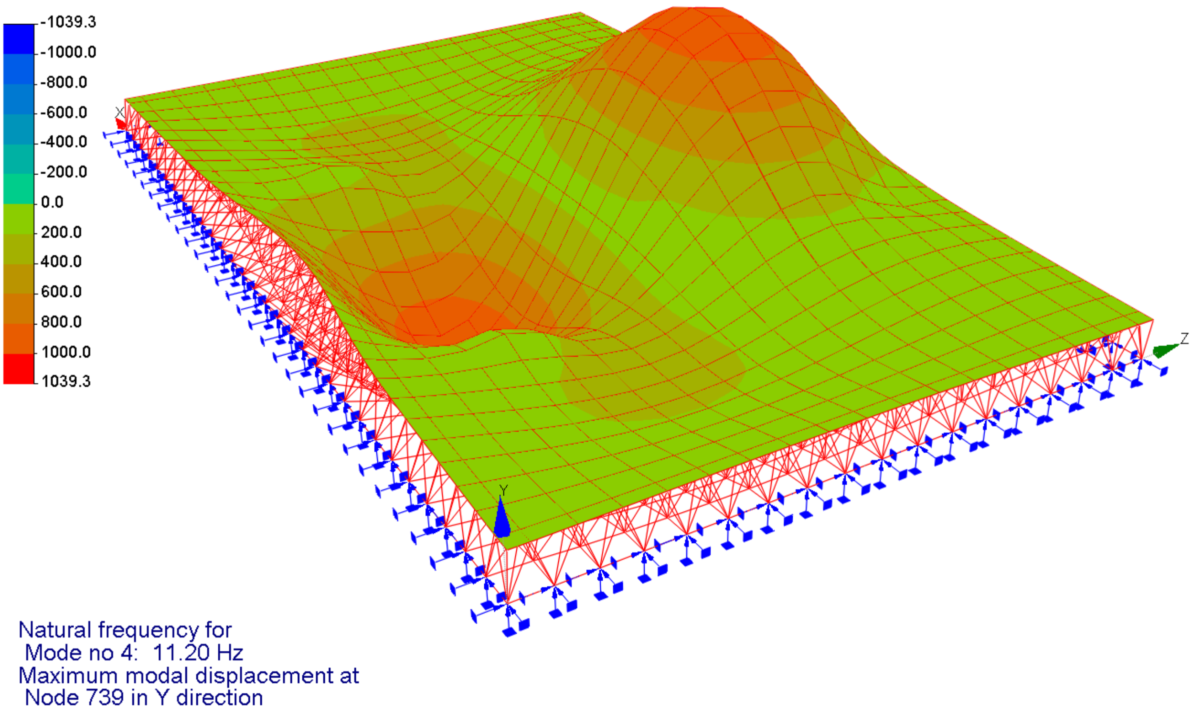

-1005.0
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1005.0
Y
Z
Natural frequency for
Mode no 10: 6.30 Hz
Maximum modal displacement at
Node 18 in Z direction

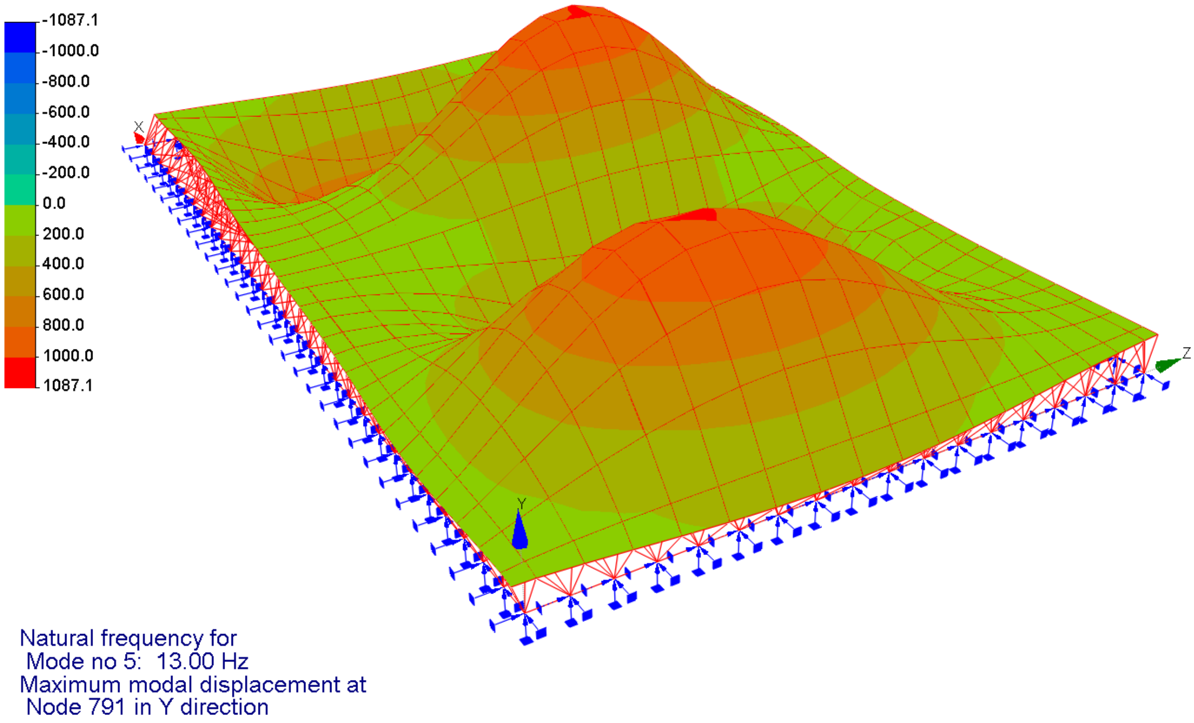
Mode Shapes for the 3.6m Deep Space Frame Floor

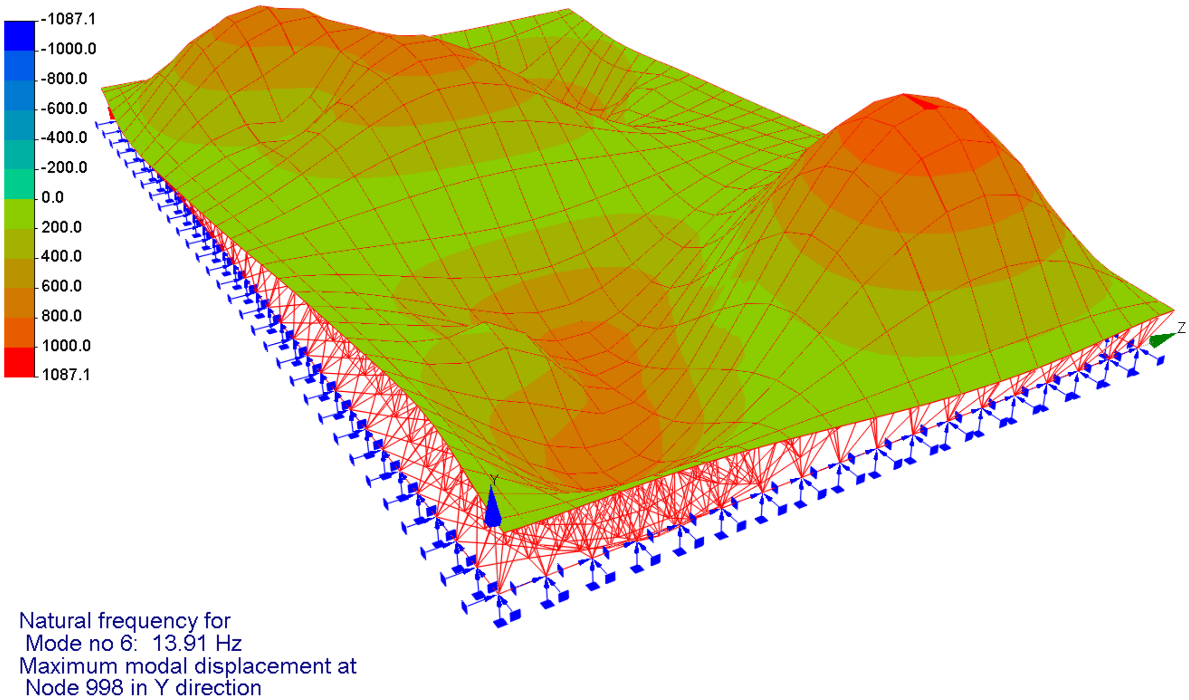


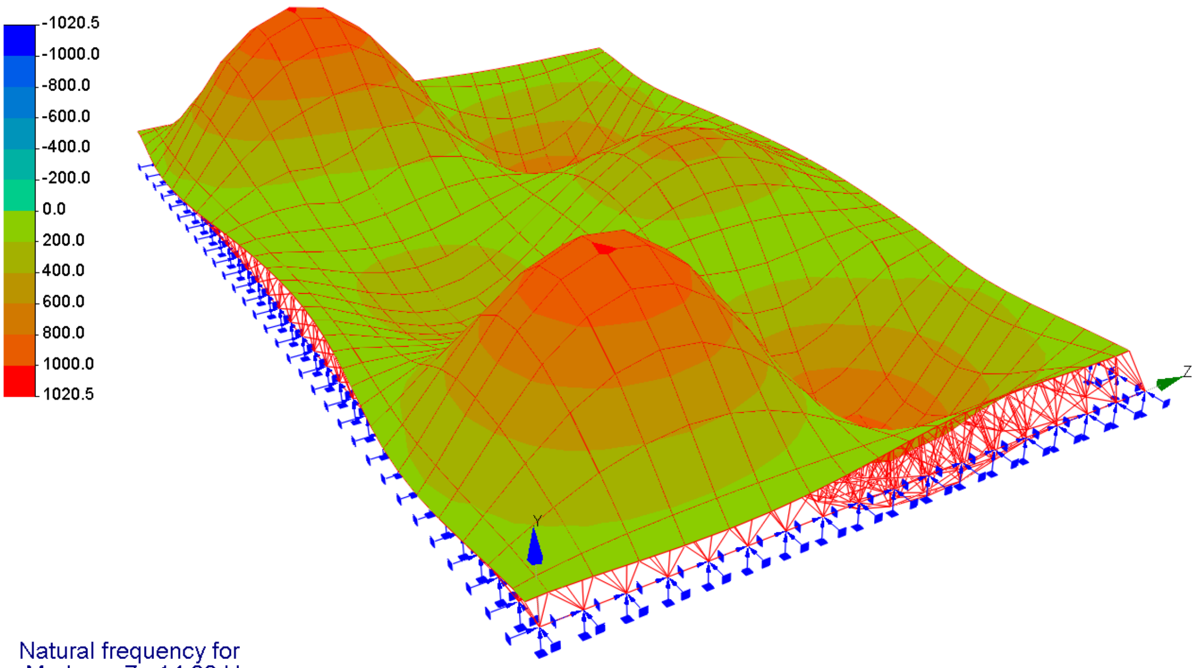



-1034.8
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1034.8
X
Y
Z
Natural frequency for
Mode no 3: 9.50 Hz
Maximum modal displacement at
Node 904 in Y direction

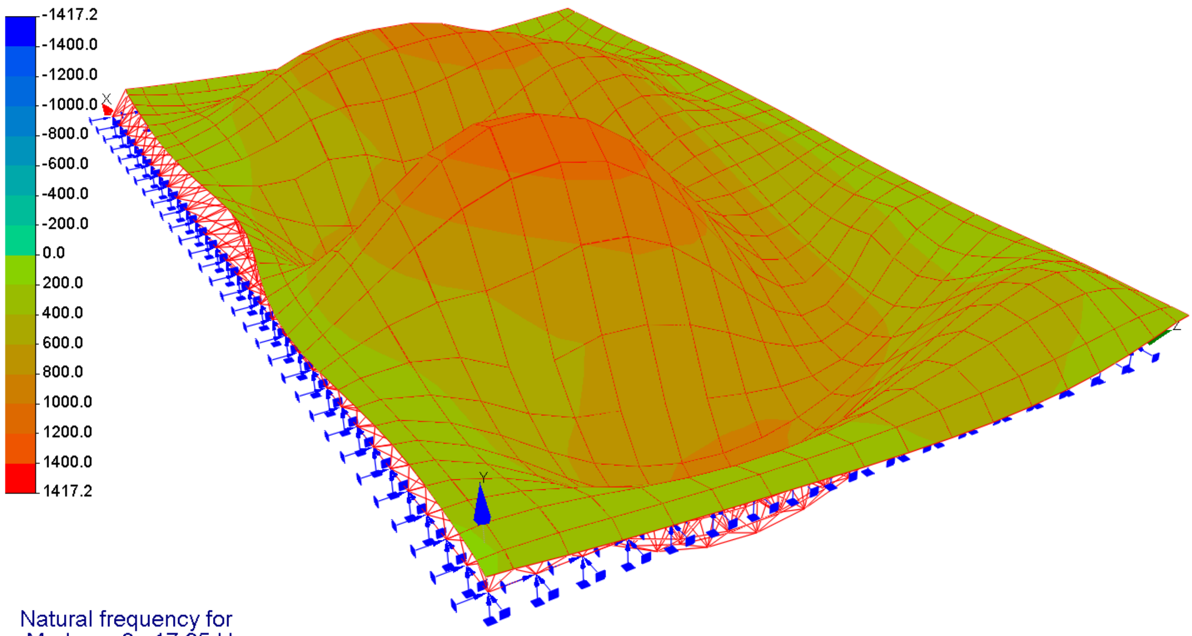

-1039.3
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1039.3
X
Y
Z
Natural frequency for
Mode no 4: 11.20 Hz
Maximum modal displacement at
Node 739 in Y direction


-1087.1
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1087.1
X
Y
Z
Natural frequency for
Mode no 5: 13.00 Hz
Maximum modal displacement at
Node 791 in Y direction

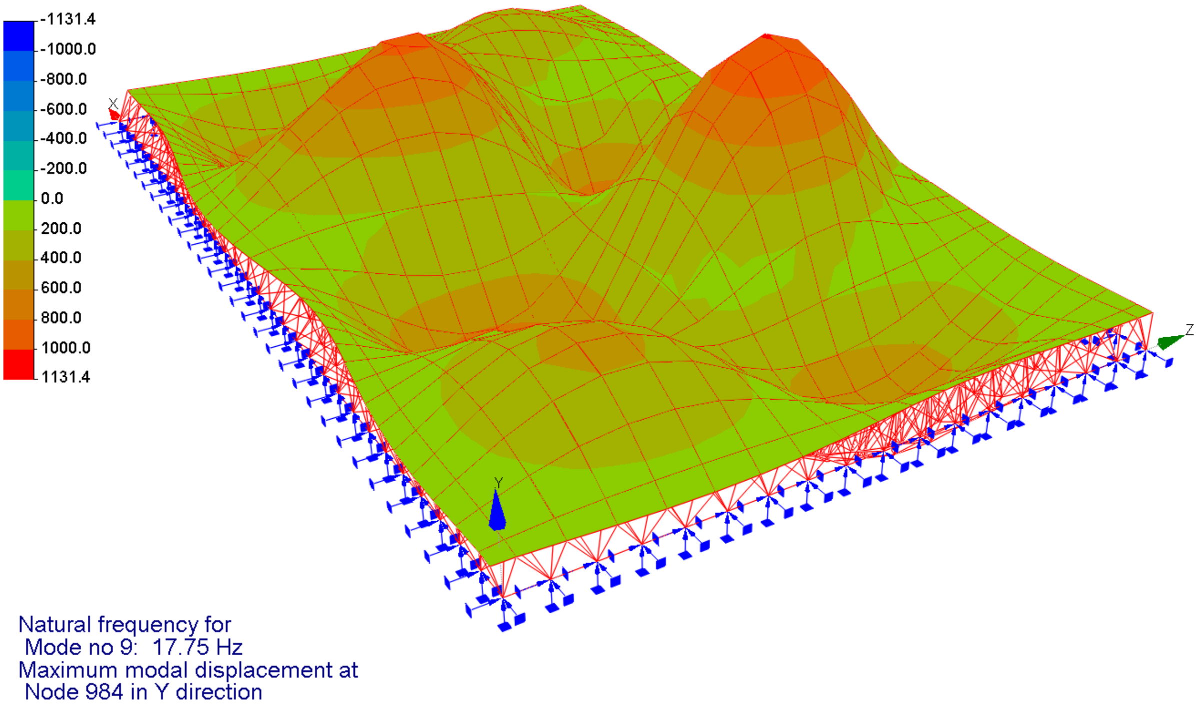

-1087.1
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1087.1
Y
Z
Natural frequency for
Mode no 6: 13.91 Hz
Maximum modal displacement at
Node 998 in Y direction

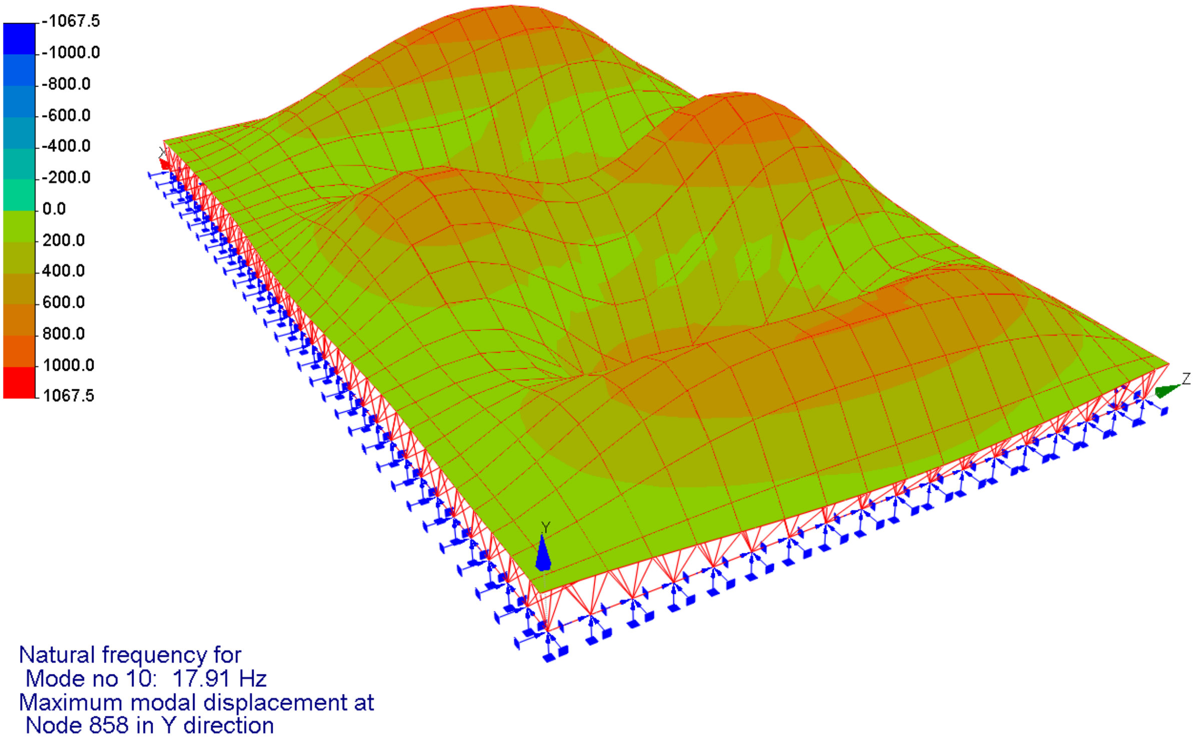


Natural frequency for
Mode no 7: 14.00 Hz
Maximum modal displacement at
Node 729 in Y direction

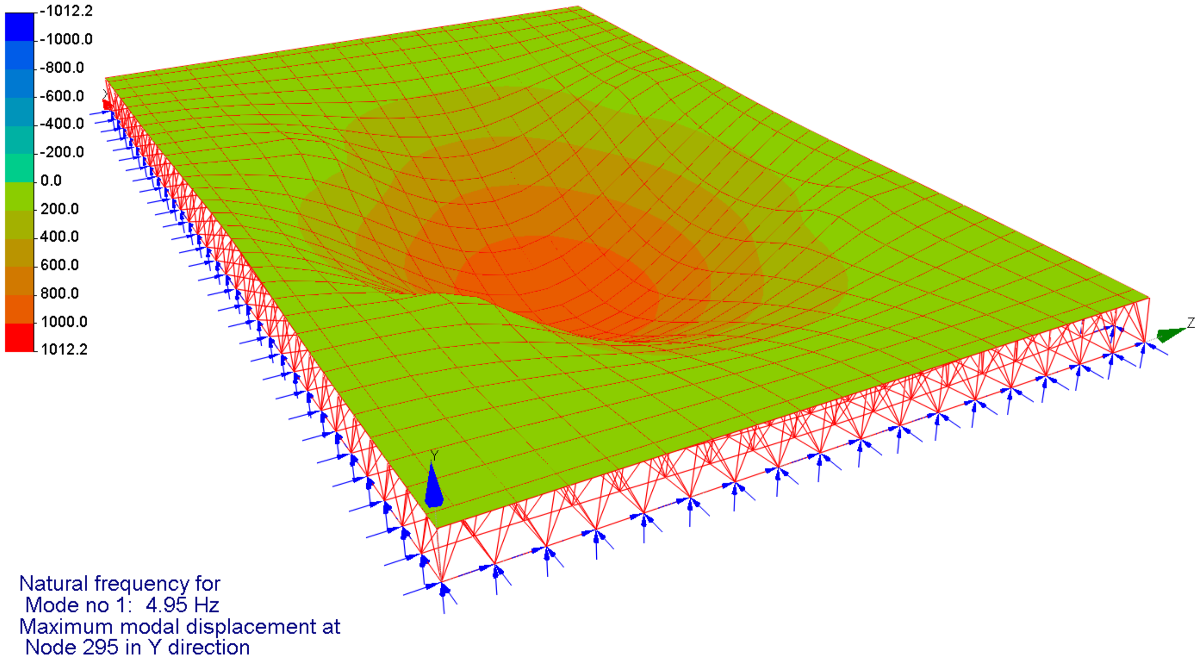


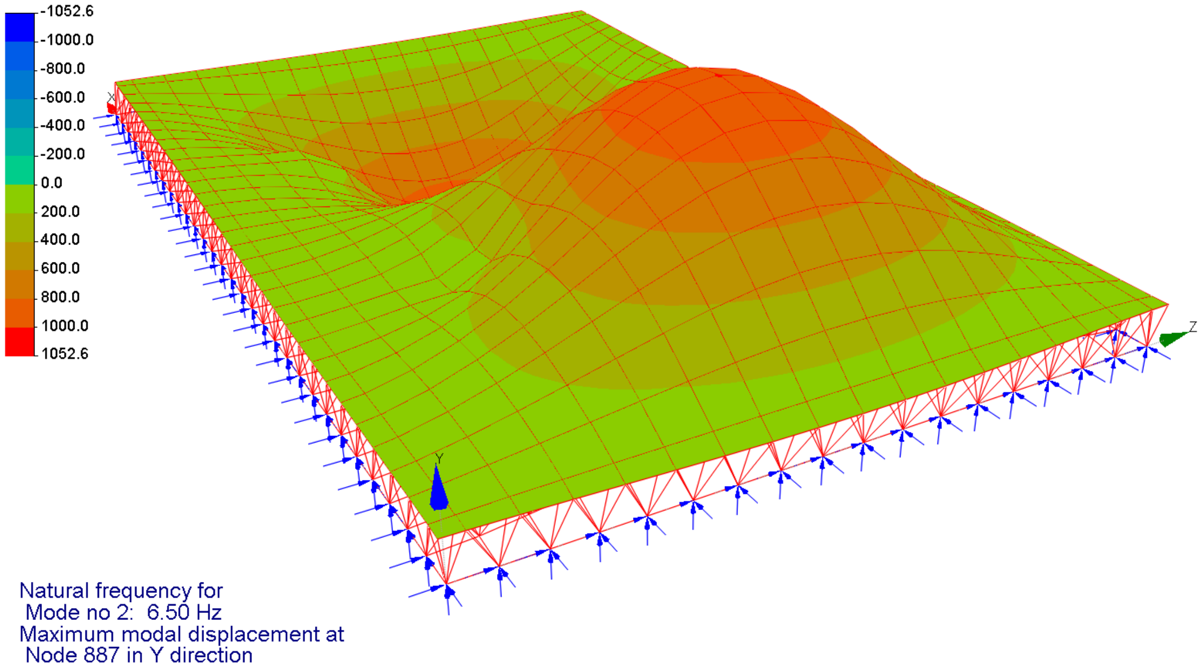
Natural frequency for
Mode no 8: 17.35 Hz
Maximum modal displacement at
Node 767 in Y direction

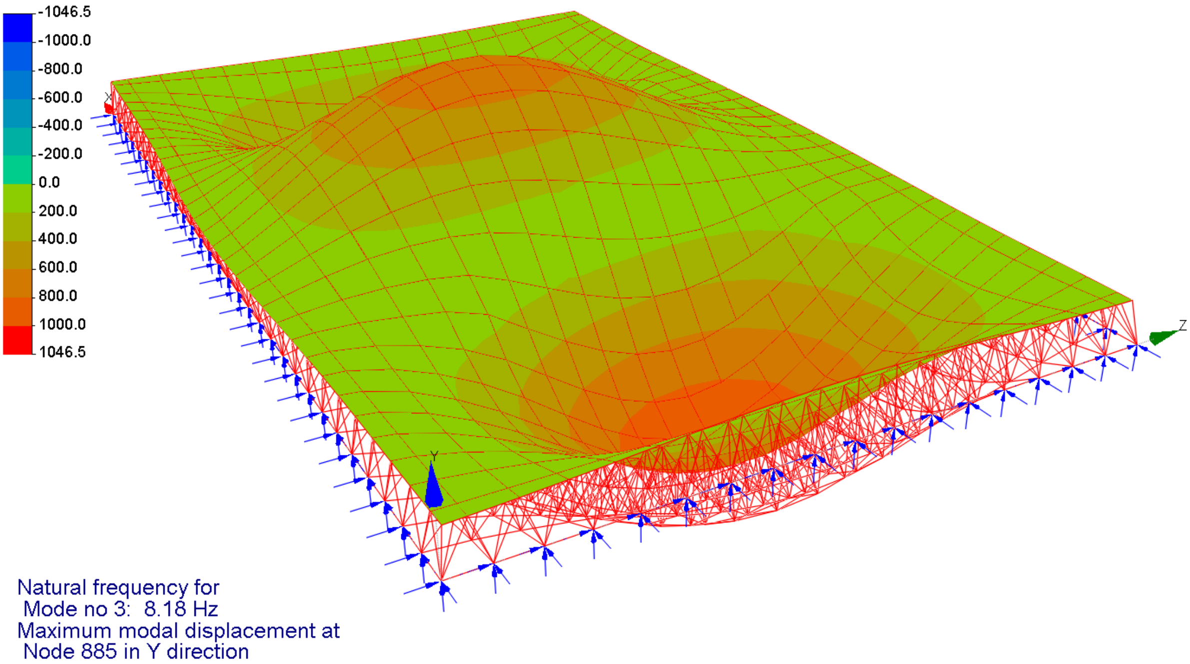

-1131.4
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1131.4
X
Y
Z
Natural frequency for
Mode no 9: 17.75 Hz
Maximum modal displacement at
Node 984 in Y direction

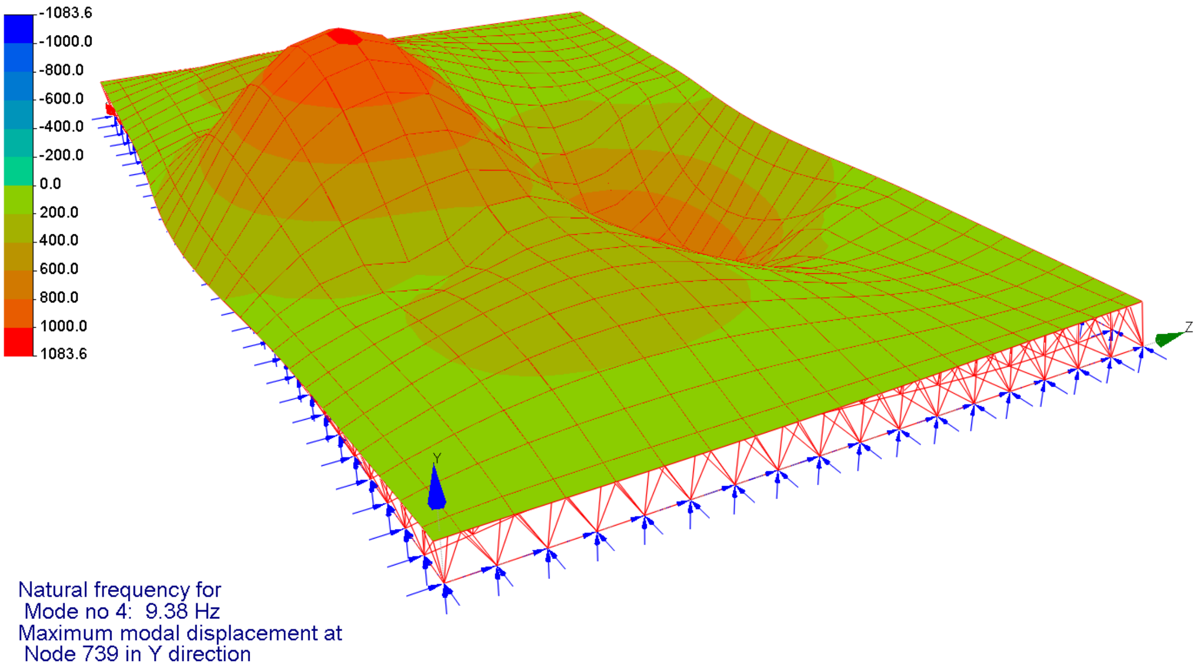

-1067.5
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1067.5
Y
Z
Natural frequency for
Mode no 10: 17.91 Hz
Maximum modal displacement at
Node 858 in Y direction

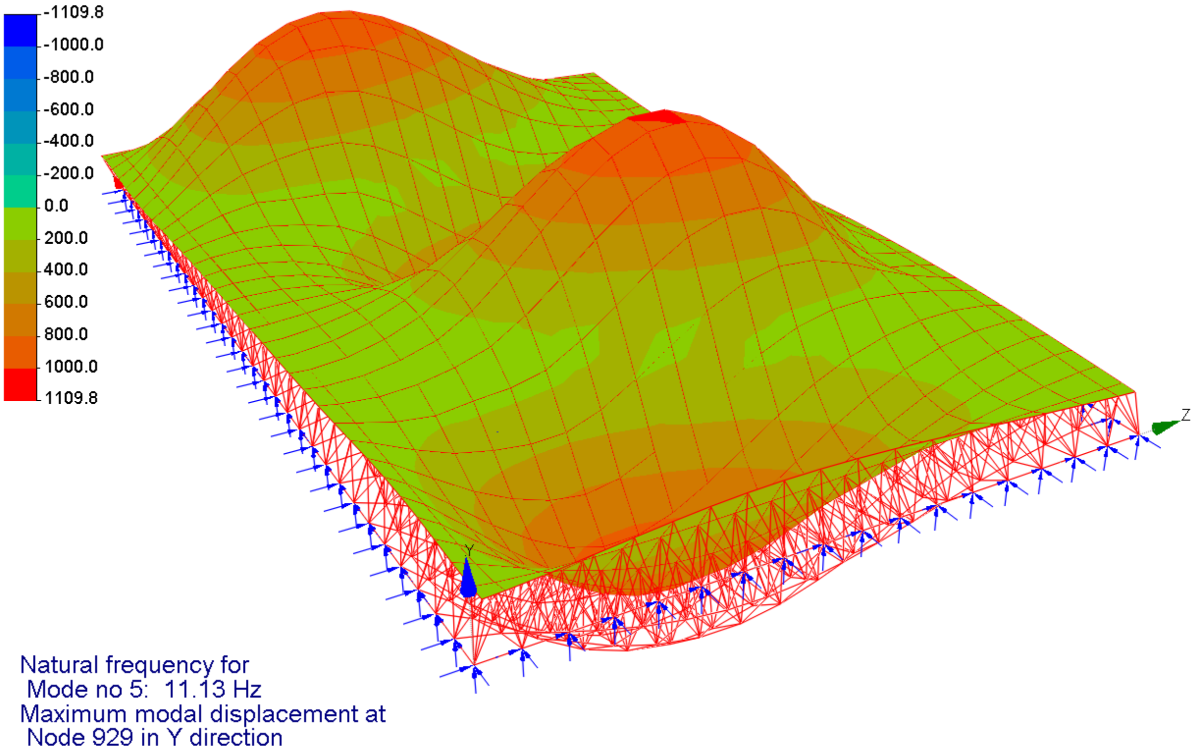
Mode Shapes for the 3.6m Deep Space Grid Floor

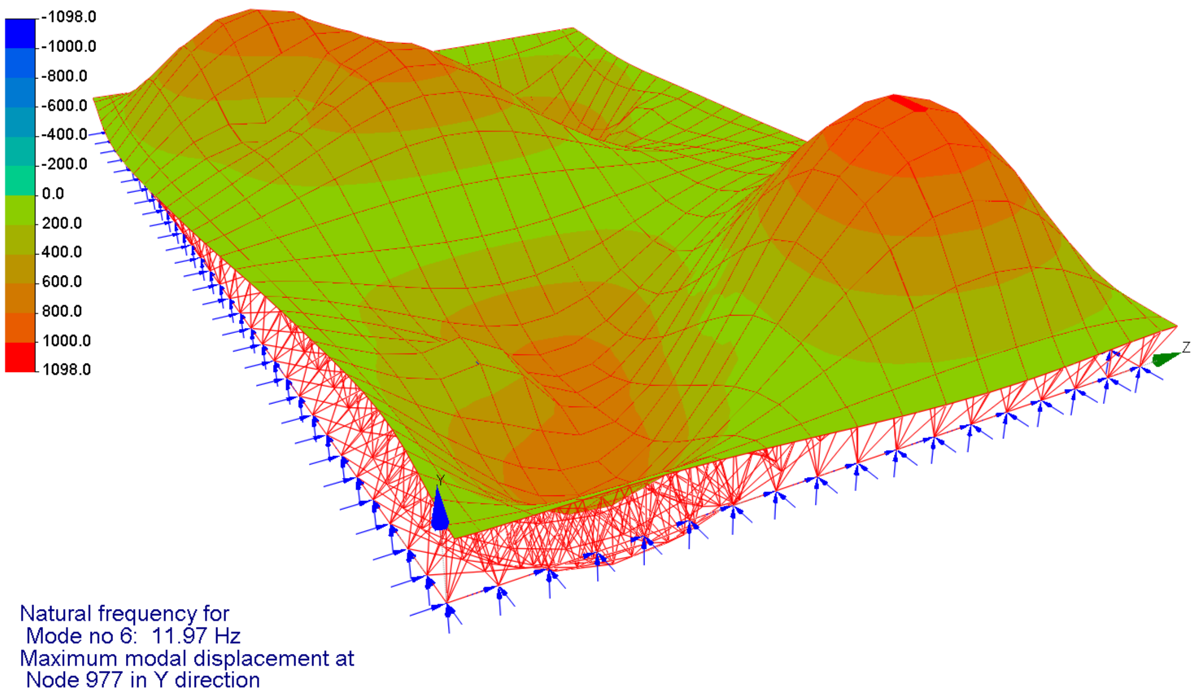


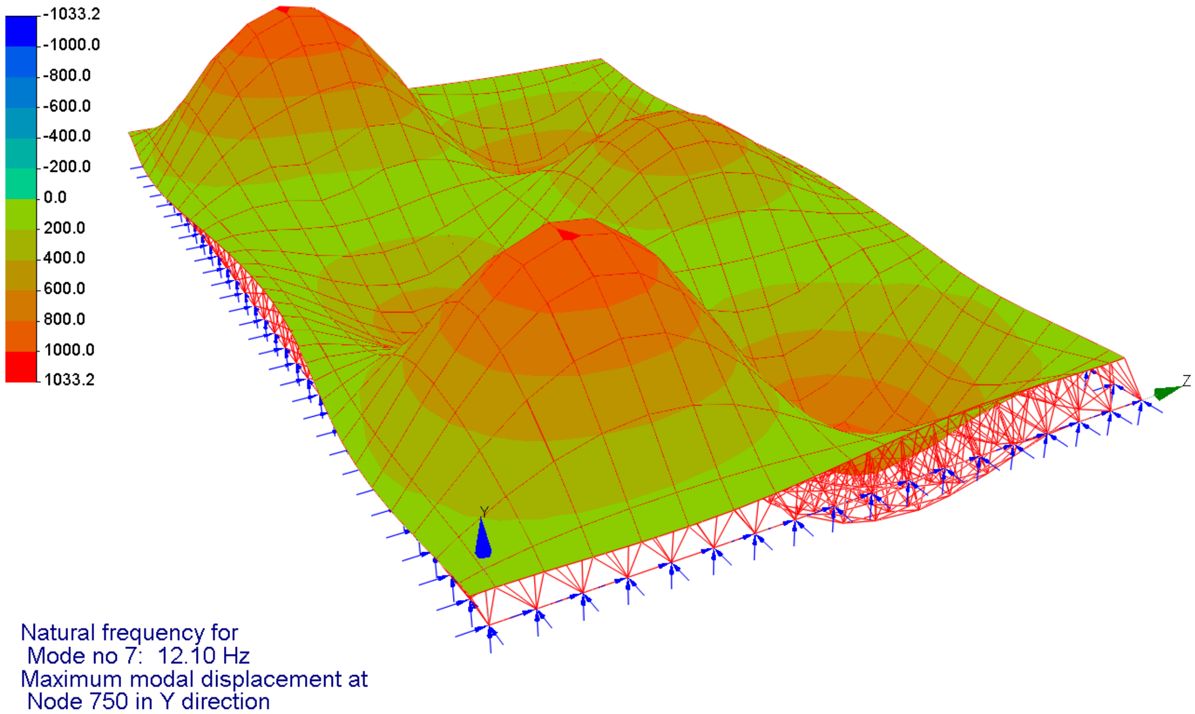


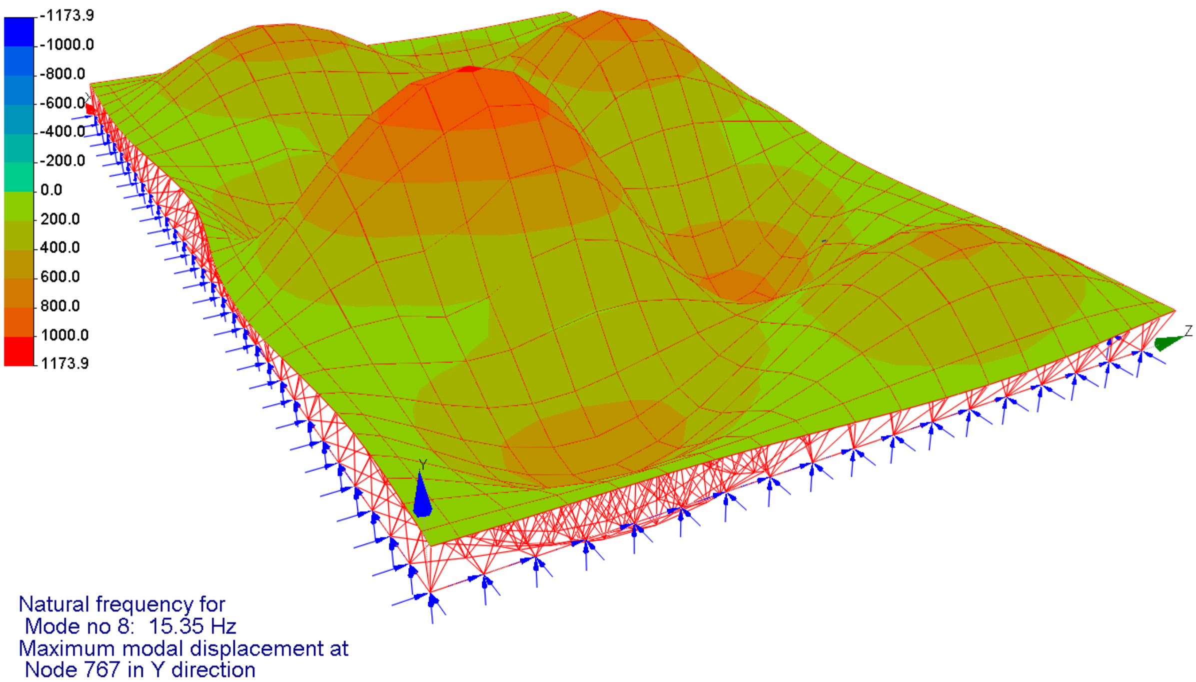

-1046.5
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1046.5
Y
Z
Natural frequency for
Mode no 3: 8.18 Hz
Maximum modal displacement at
Node 885 in Y direction

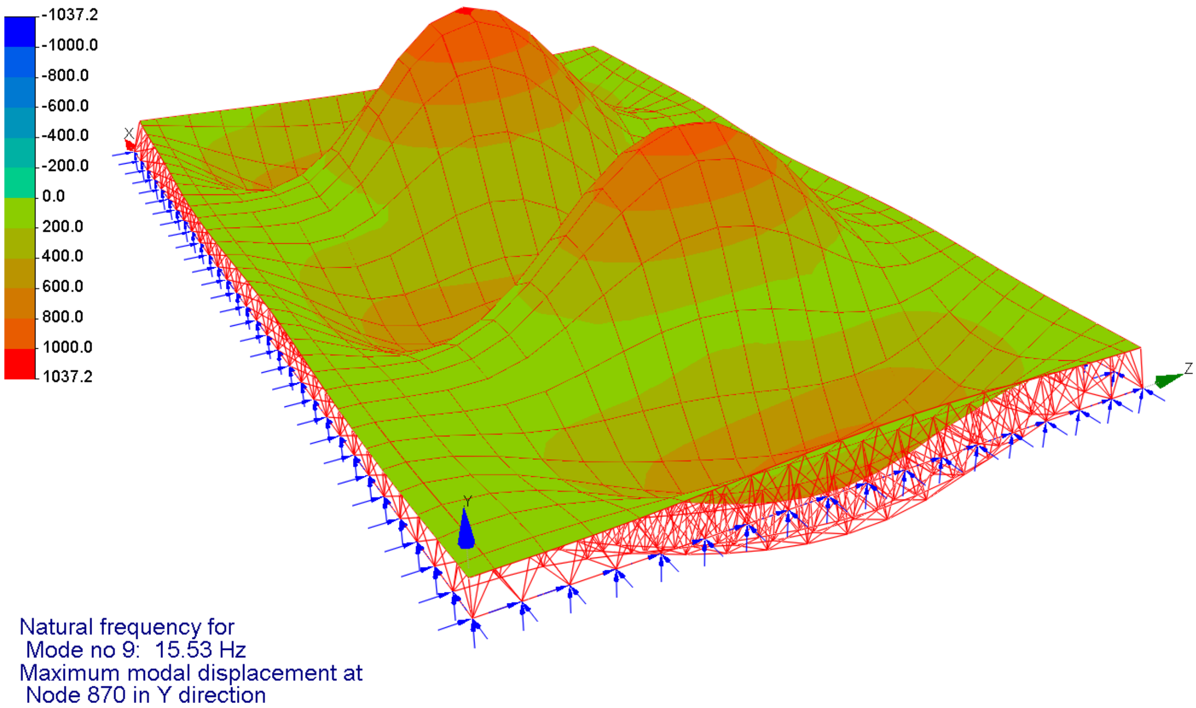

-1083.6
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1083.6
Y
Z
Natural frequency for
Mode no 4: 9.38 Hz
Maximum modal displacement at
Node 739 in Y direction

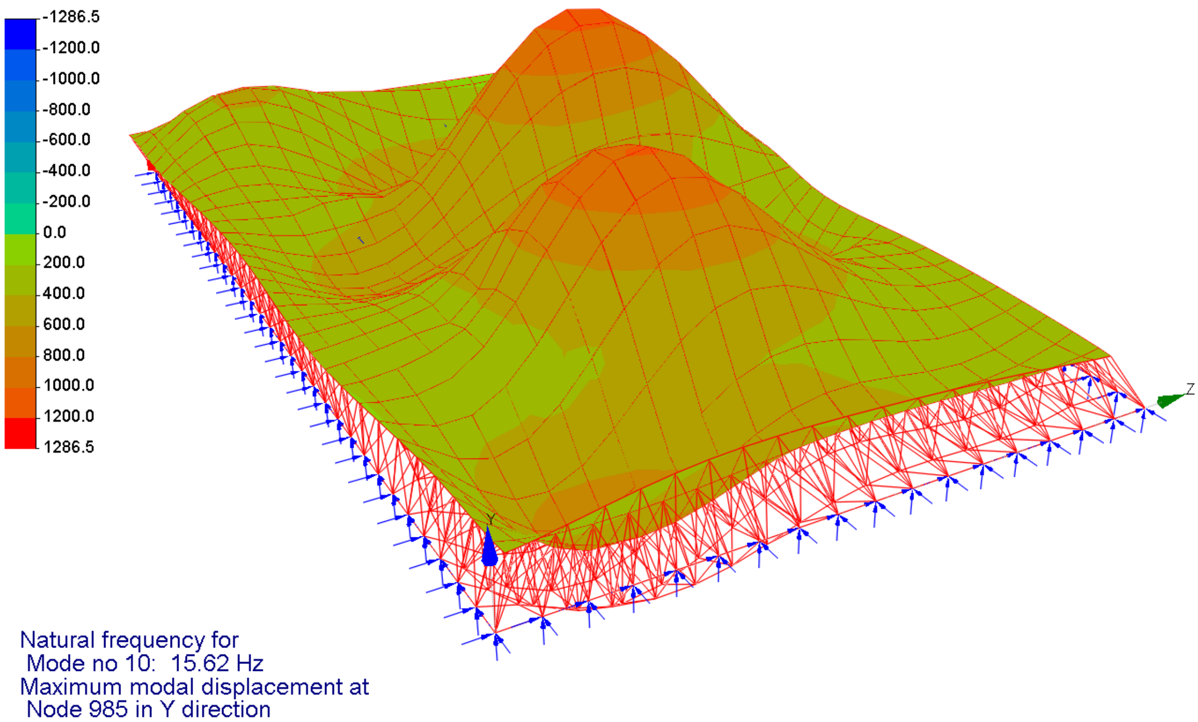

-1109.8
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1109.8
Y
Z
Natural frequency for
Mode no 5: 11.13 Hz
Maximum modal displacement at
Node 929 in Y direction


-1098.0
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1098.0
Y
Z
Natural frequency for
Mode no 6: 11.97 Hz
Maximum modal displacement at
Node 977 in Y direction

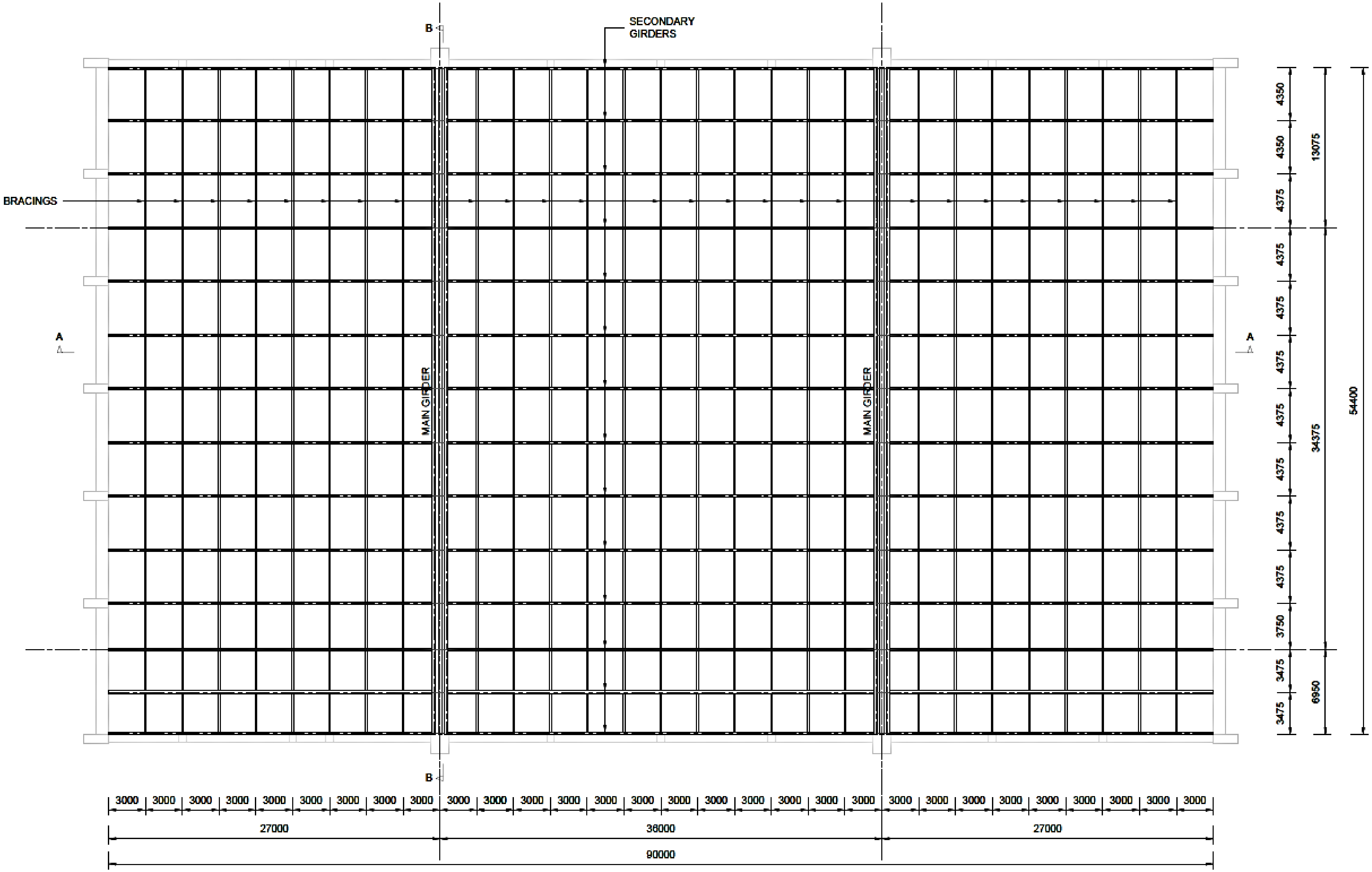

-1033.2
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1033.2
Y
Z
Natural frequency for
Mode no 7: 12.10 Hz
Maximum modal displacement at
Node 750 in Y direction


-1173.9
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1173.9
Y
Z
Natural frequency for
Mode no 8: 15.35 Hz
Maximum modal displacement at
Node 767 in Y direction

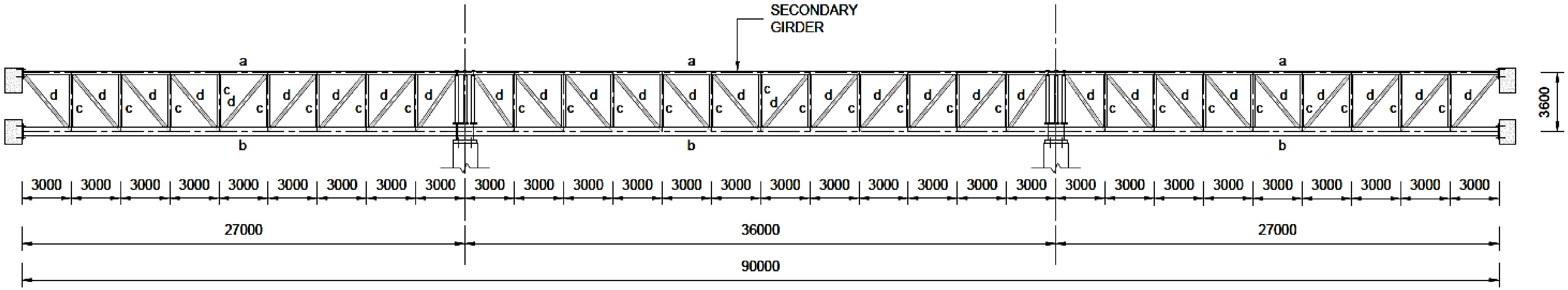

-1037.2
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1037.2
X
Y
Z
Natural frequency for
Mode no 9: 15.53 Hz
Maximum modal displacement at
Node 870 in Y direction


-1286.5
-1200.0
-1000.0
-800.0
-600.0
-400.0
-200.0
0.0
200.0
400.0
600.0
800.0
1000.0
1200.0
1286.5
Y
Z
Natural frequency for
Mode no 10: 15.62 Hz
Maximum modal displacement at
Node 985 in Y direction

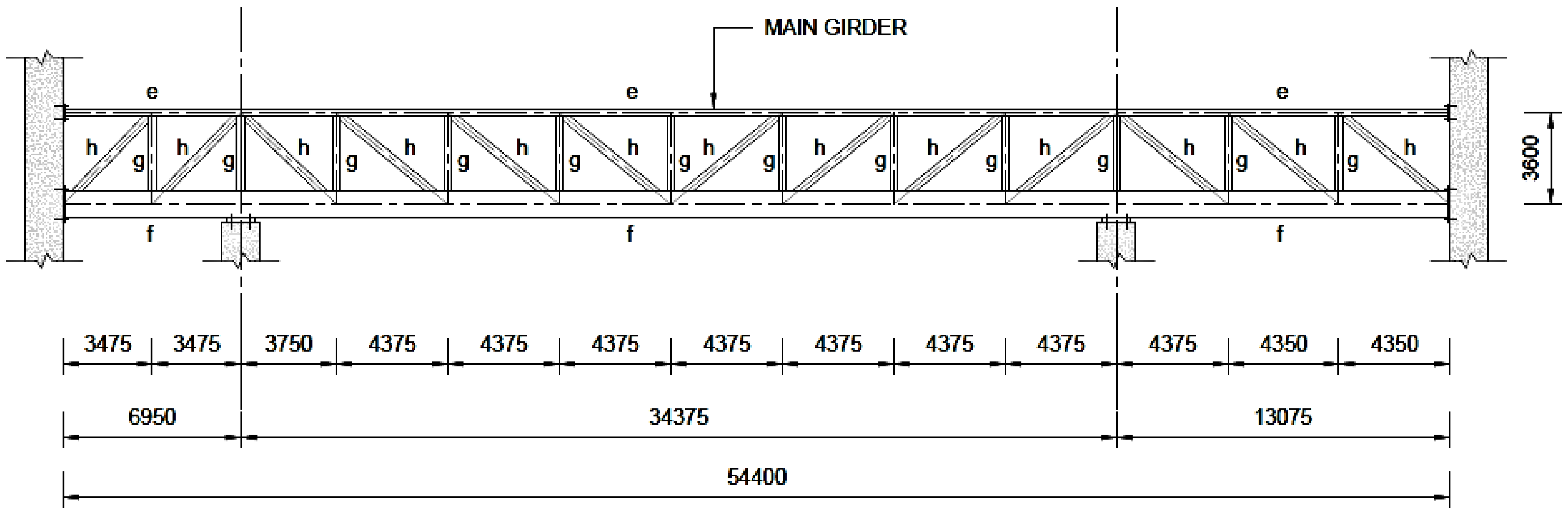
Appendix C. Floor Layouts



PLANE TRUSS PLAN VIEW



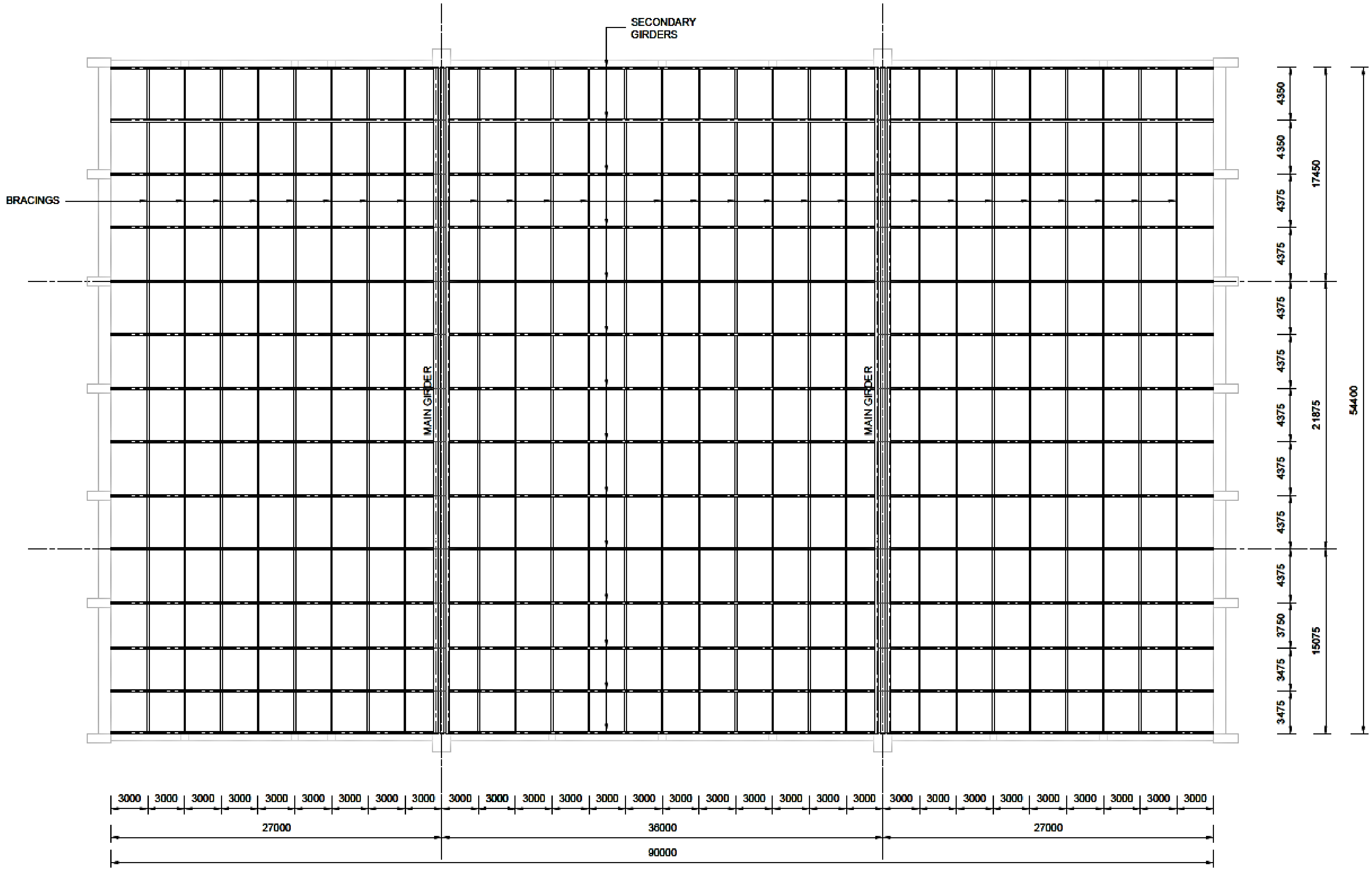
SECTION A-A



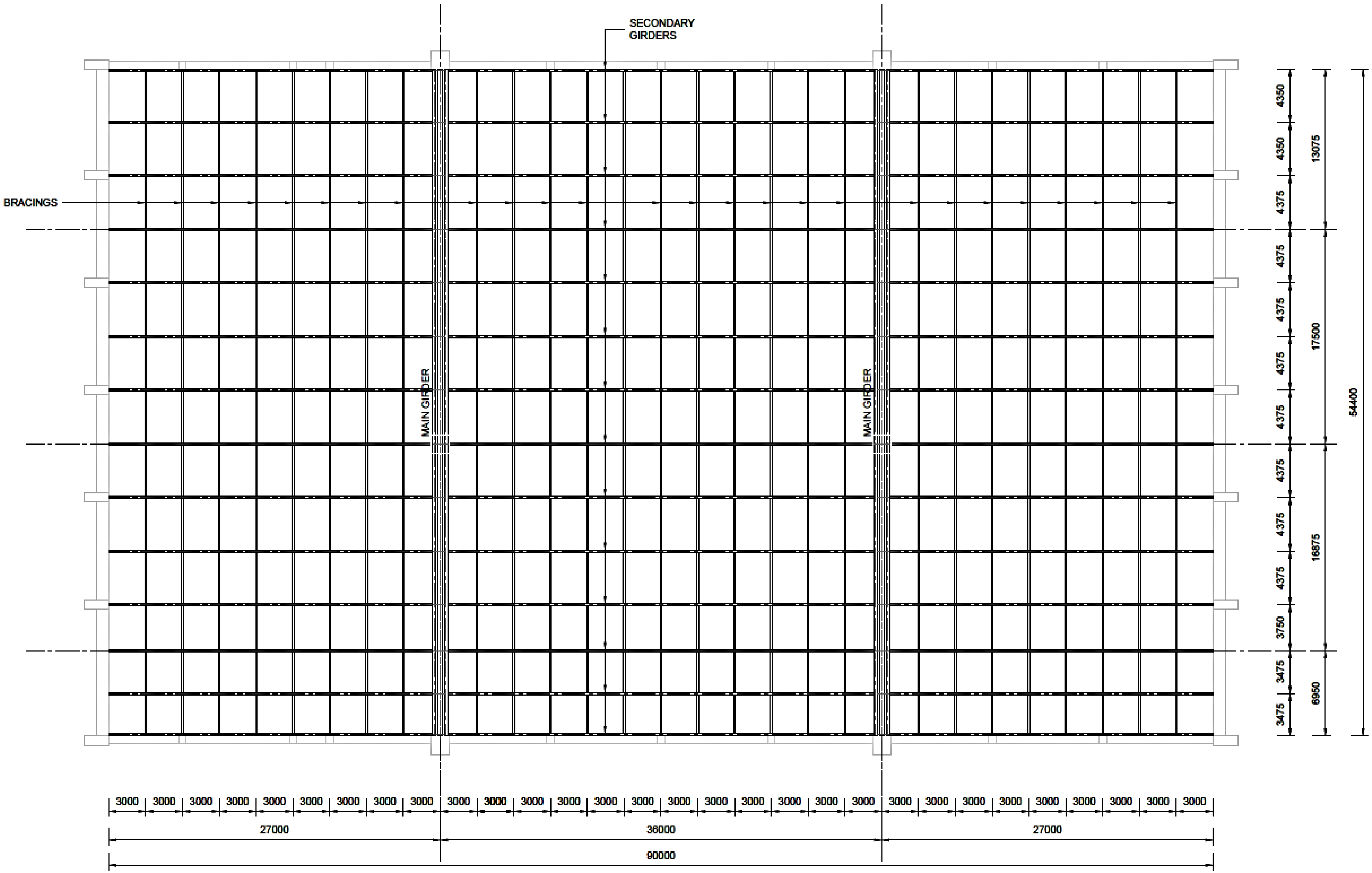
SECTION B-B

MEMBER SIZES

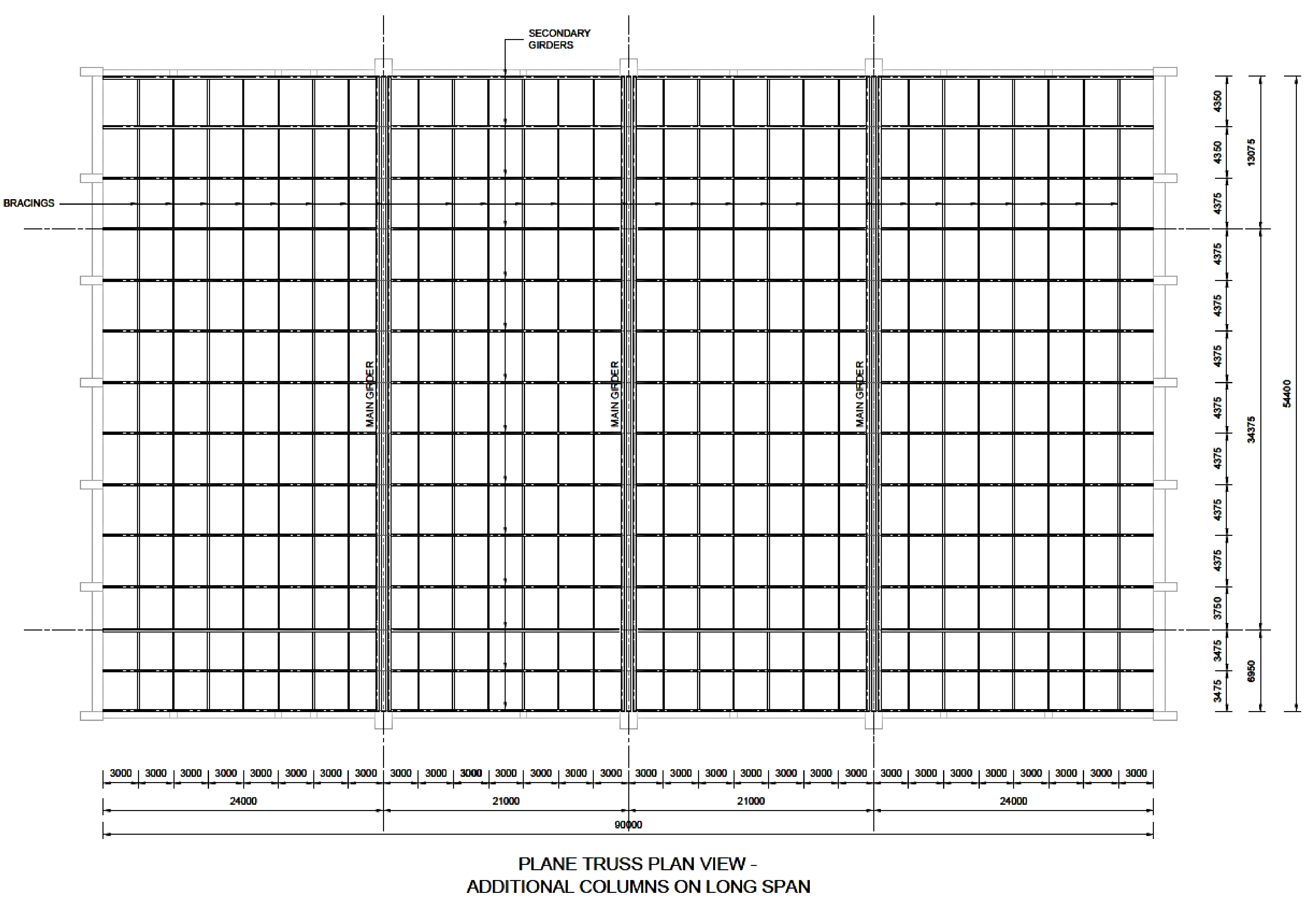
a	-	203x133x25
b	-	533x210x122
c	-	203x203x52
d	-	203x203x60
e	-	305x165x54
f	-	1030x400 (8W,25F)
g	-	305x305x118
h	-	305x305x137



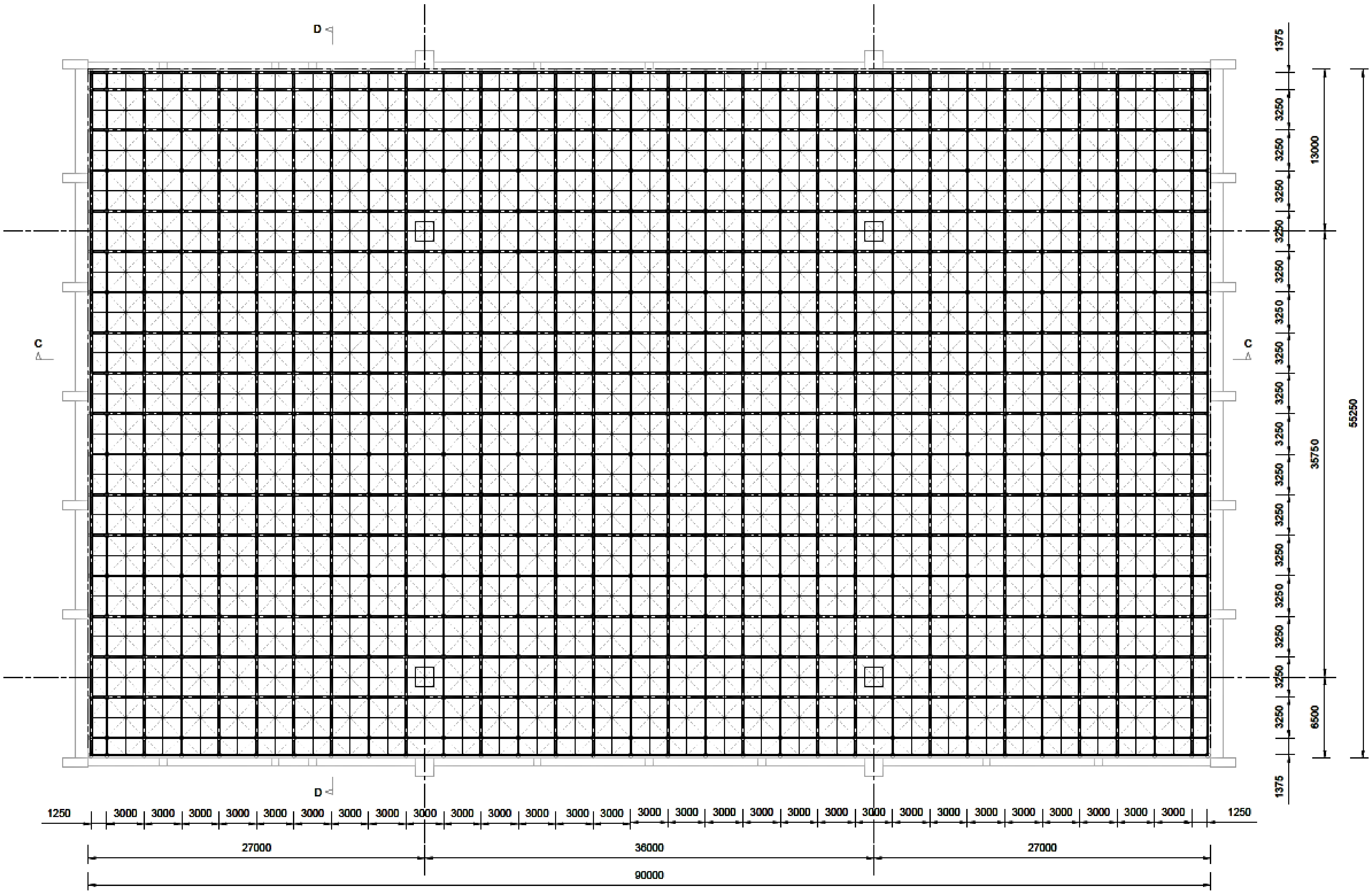
PLANE TRUSS PLAN VIEW -
COLUMNS MOVED CLOSER



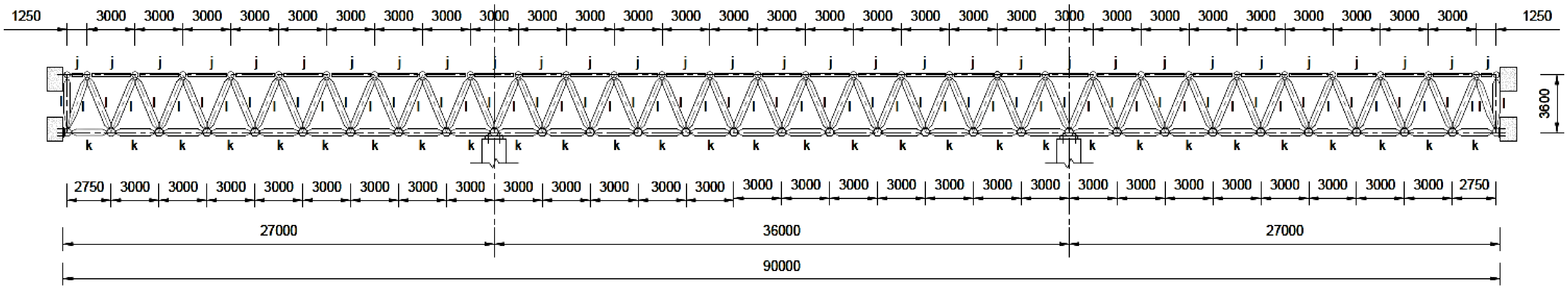
PLANE TRUSS PLAN VIEW -
ADDITIONAL COLUMNS ON SHORT SPAN



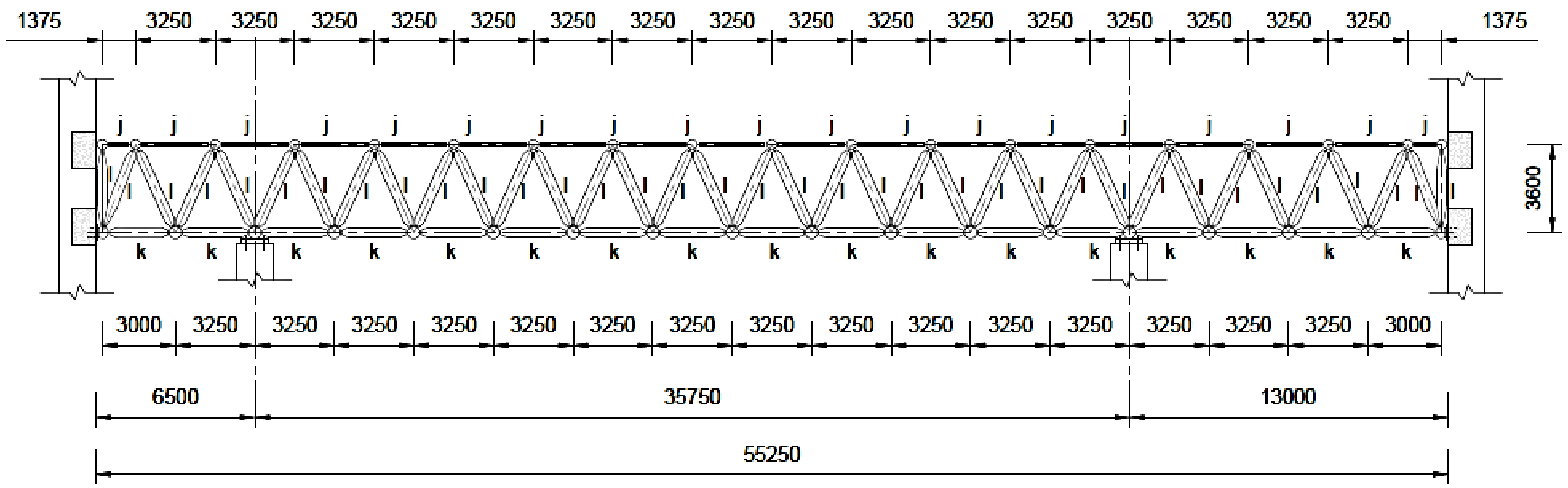
PLANE TRUSS PLAN VIEW -
ADDITIONAL COLUMNS ON LONG SPAN



SPACE GRID PLAN VIEW



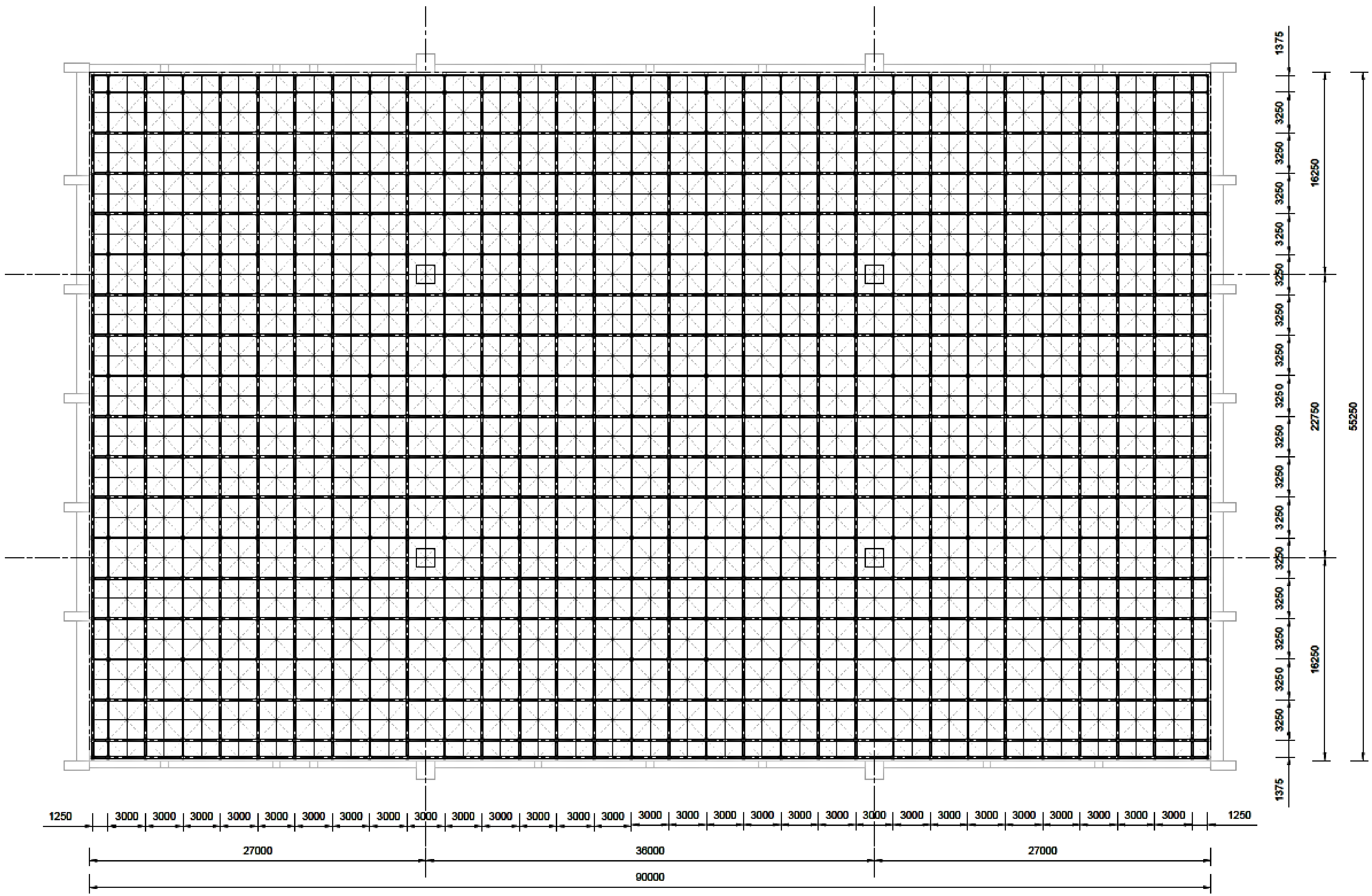
SECTION C-C



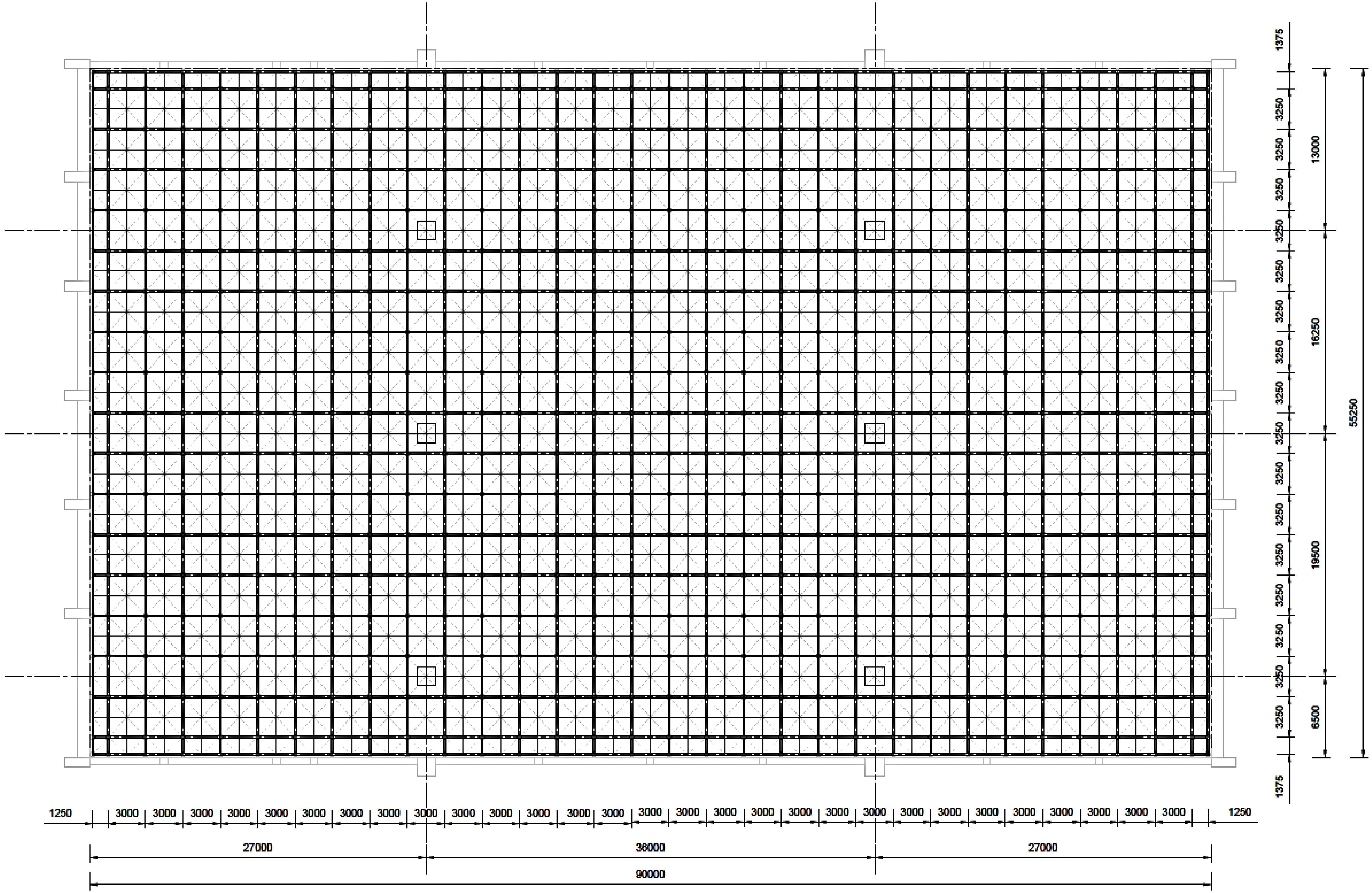
SECTION D-D

MEMBER SIZES

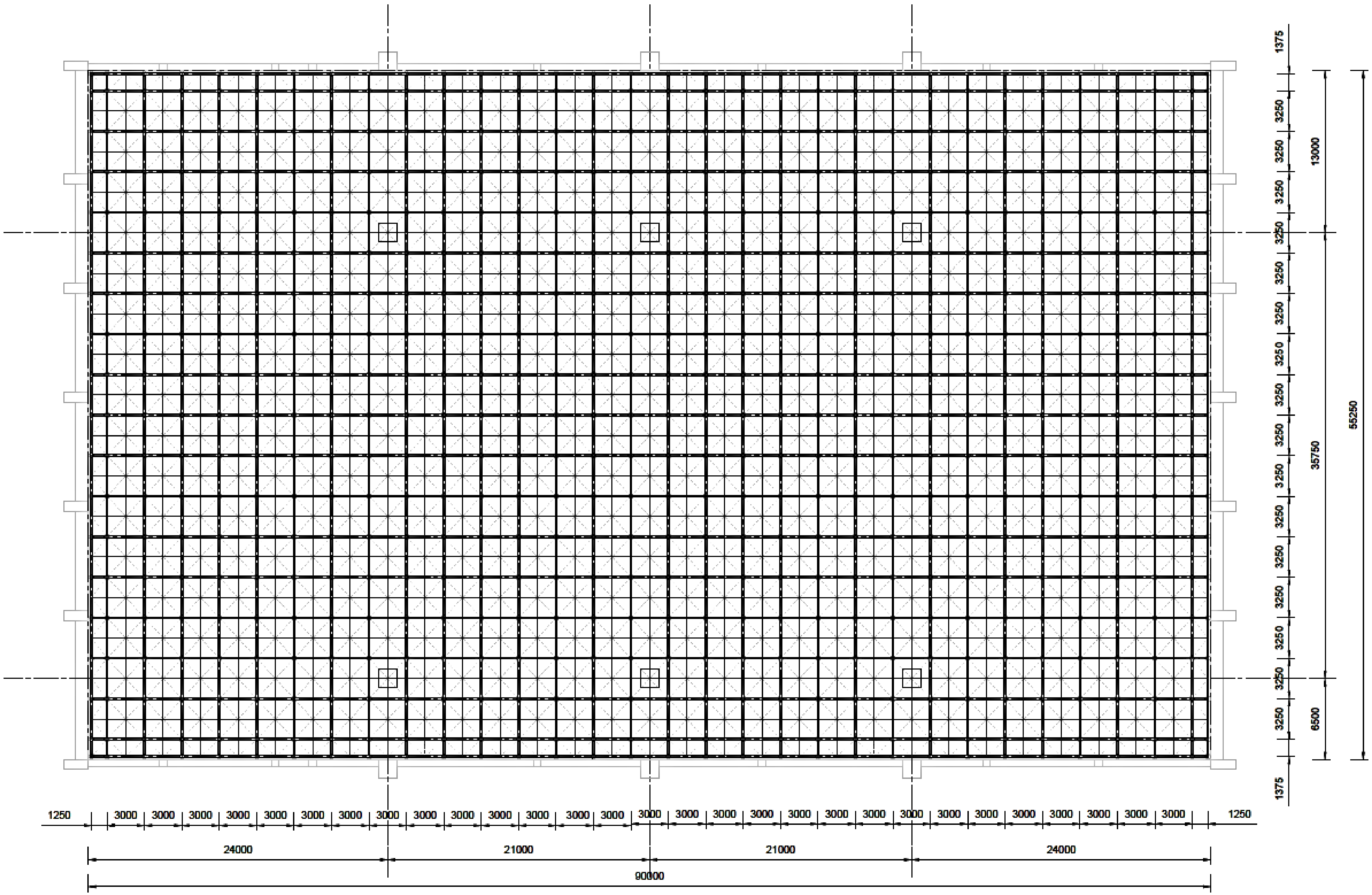
j - ○ 152.4x6.0 CHS
k - ○ 406.4x12.0 CHS
l - ○ 406.4x12.0 CHS



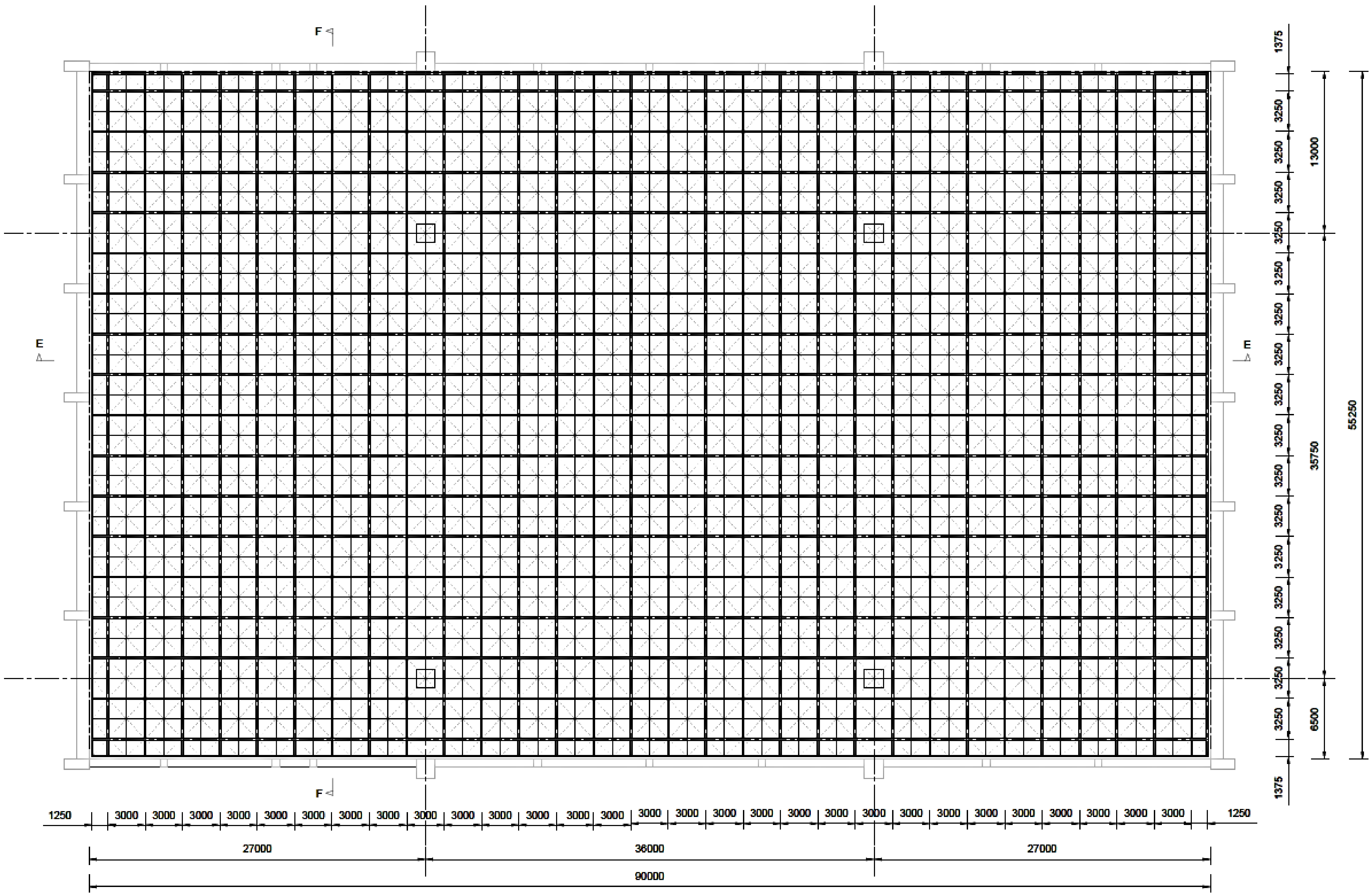
SPACE GRID PLAN VIEW -
COLUMNS MOVED CLOSER



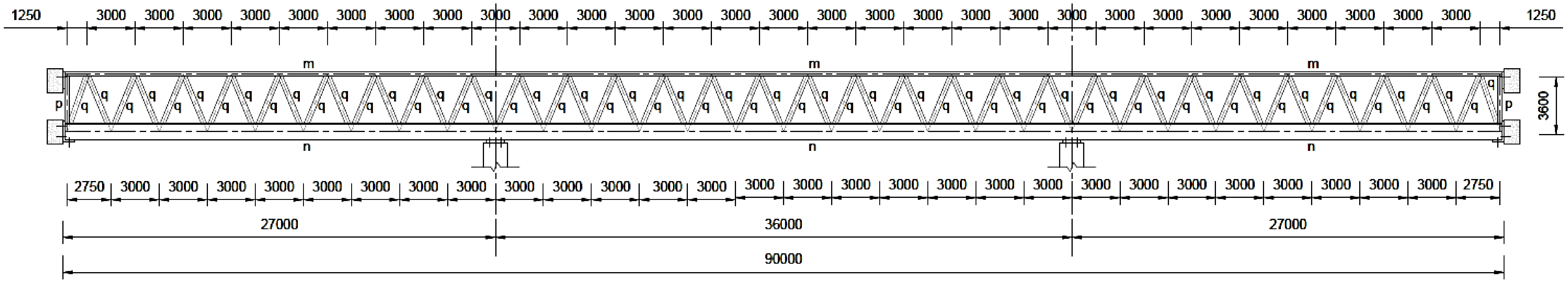
SPACE GRID PLAN VIEW -
ADDITIONAL COLUMNS ON SHORT SPAN



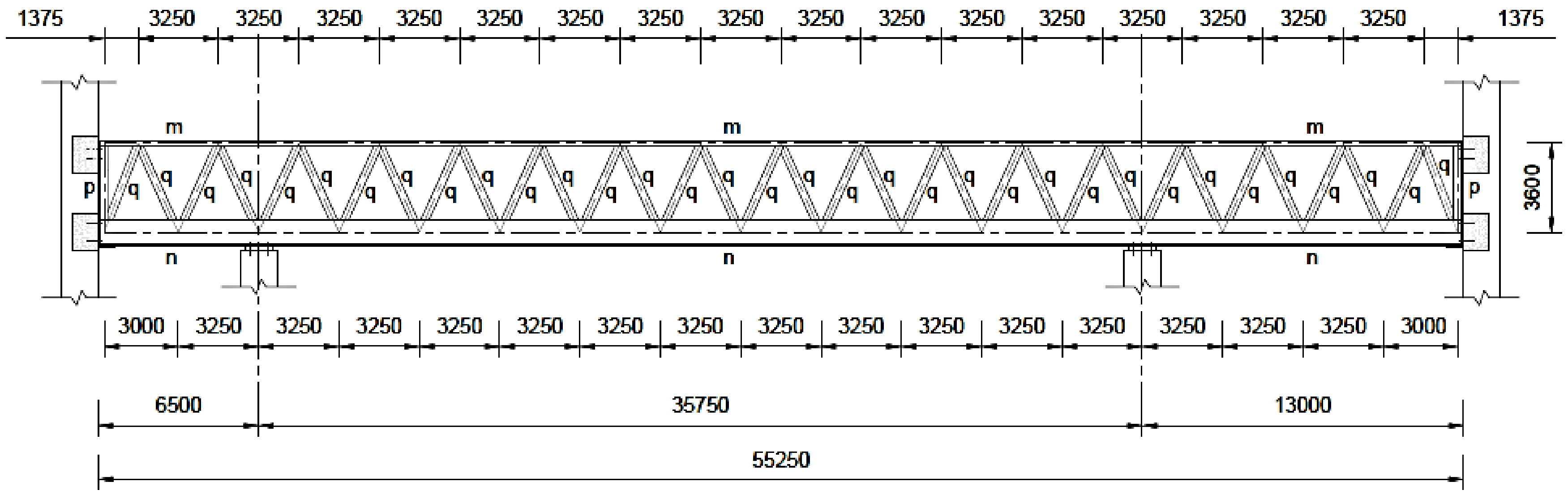
SPACE GRID PLAN VIEW -
ADDITIONAL COLUMNS ON LONG SPAN



SPACE FRAME PLAN VIEW



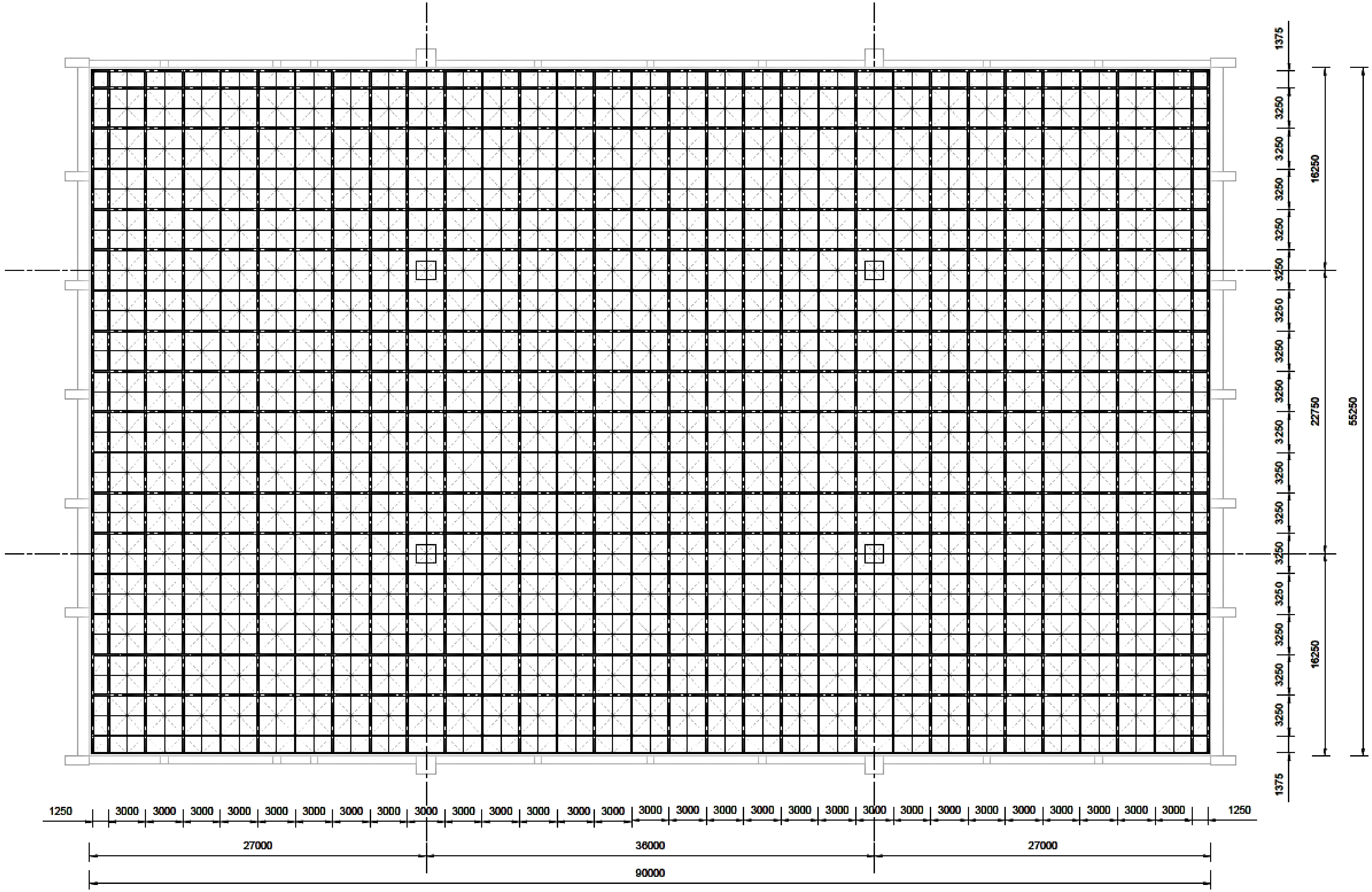
SECTION E-E



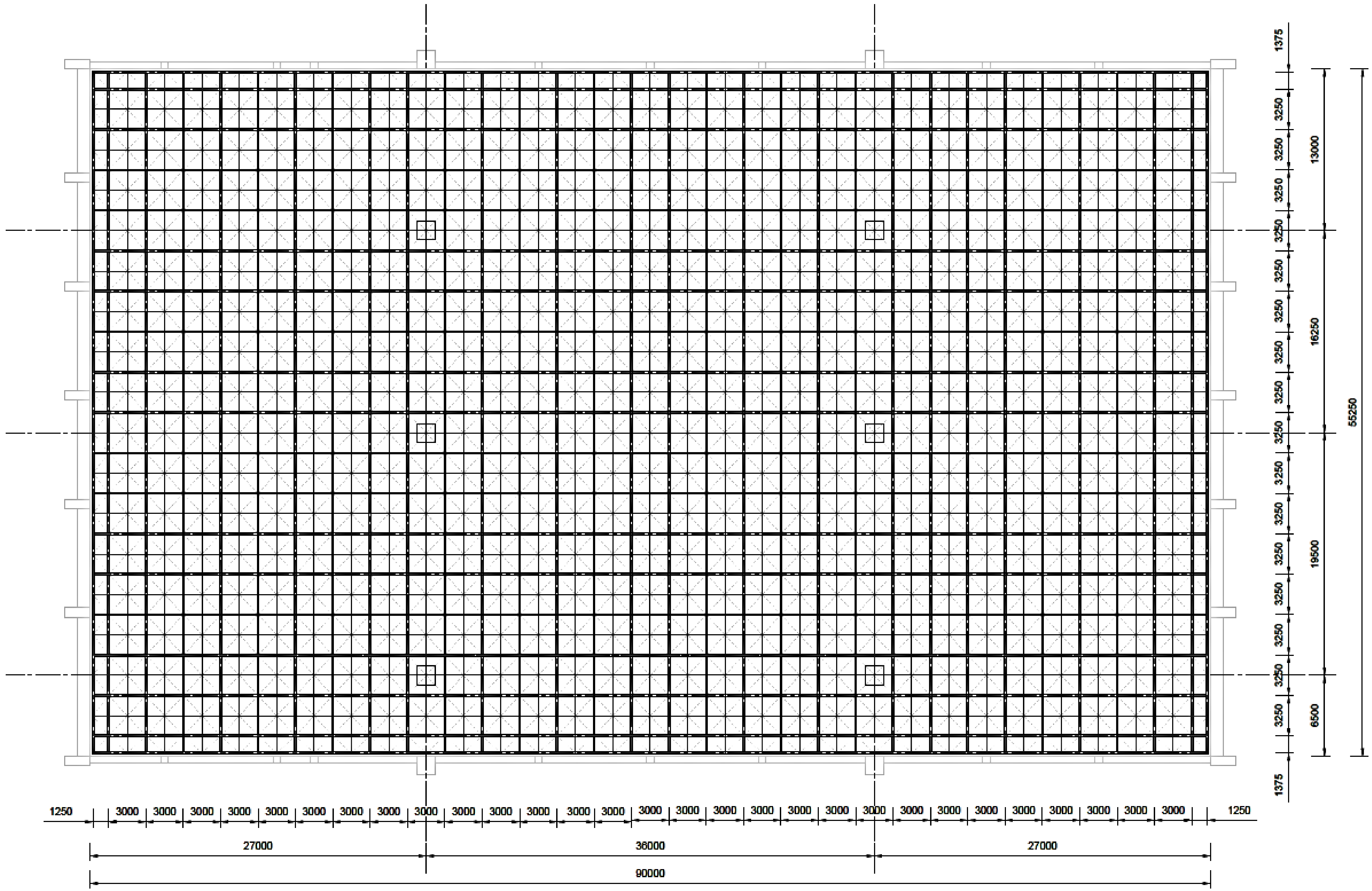
SECTION F-F

MEMBER SIZES

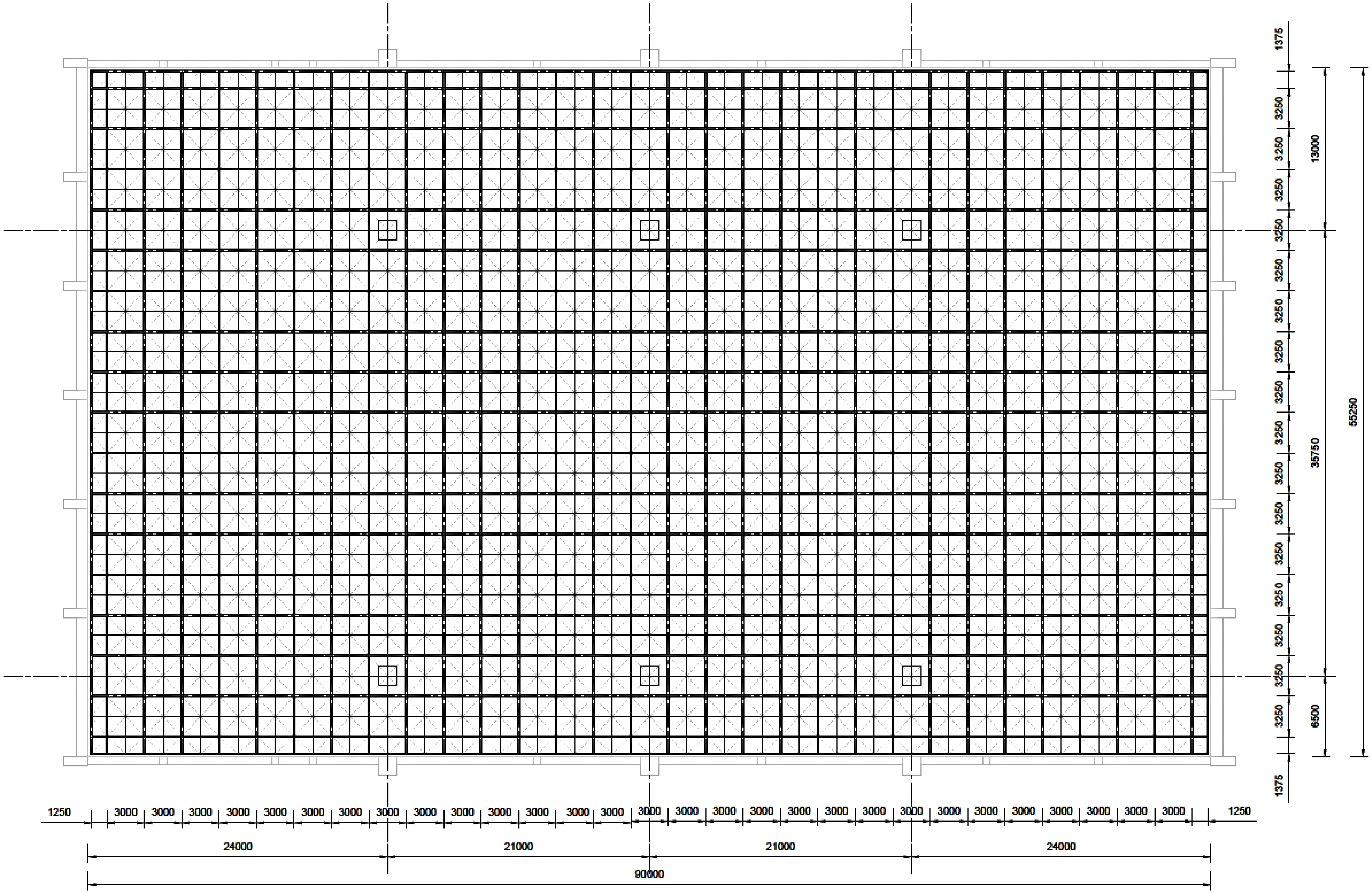
m -	I	203x133x25
n -	I	1030x400 (8W,25F)
p -	I	305x305x137
q -	I	305x305x137



SPACE FRAME PLAN VIEW -
COLUMNS MOVED CLOSER



SPACE FRAME PLAN VIEW -
ADDITIONAL COLUMNS ON SHORT SPAN



SPACE FRAME PLAN VIEW -
ADDITIONAL COLUMNS ON LONG SPAN