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University of Cape Town

**OPTIMUM CURRENCY AREAS IN AFRICA: A Genetic
Optimisation Approach**

University of Cape Town

**OPTIMUM CURRENCY AREAS IN AFRICA: A Genetic
Optimisation Approach**

by

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Mini-Dissertation

Presented to the School of Economics of
The University of Cape Town
in Partial Fulfilment
of the Requirements
for the Degree of

Masters in Economics

The University of Cape Town

February 2011

Dedication

Regression, regression...

Acknowledgements

I am eternally grateful to my parents and brother without whose support I could not have completed the degree. I am also grateful to my supervisor Dr. Mark Ellyne for his patience and accommodating nature. His insights and guidance were invaluable. Special mention must go to Simon Winberg for poring through and correcting my MATLAB code as well as discussing the ideas for no other reason except to help. My heartfelt thanks to the administrators at the School of Economics and all others whose names I did not mention, but who contributed in any form towards the successful completion of the dissertation.

OPTIMUM CURRENCY AREAS IN AFRICA: A Genetic Optimisation Approach

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The University of Cape Town, 2011

Supervisor: Mark Ellyne

Abstract: We seek to establish whether a single currency for the entire African continent is the best way to achieve the desired level of economic integration. We put forward multiple regional currency groups as an alternative scenario to one single currency group for the whole continent. Using a genetic optimisation algorithm adapted from Ghosh and Wolf (1994) we identify and group African countries with the most synchronised business cycles based on a simple macroeconomic model. The more synchronised business cycles are between currency group members, the lower the combined output loss. We seek to minimise the total output loss for the continent by grouping countries with highly synchronised business cycles in the same currency union. We compare the five optimal currency area arrangement suggested by the algorithm with the Abuja Treaty recommendation of five regional economic communities. Further, the paper asks how many currency groups are appropriate for Africa at this time. Based on a qualitative analysis we suggest three currency unions for Africa.

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1. Introduction

The first goal of this study is to establish whether a single currency for the entire African continent is the best way to achieve the desired level of economic integration. We propose multiple regional currency groups as an alternative scenario to a single currency group for the whole continent.

We use a genetic optimisation algorithm adapted from Ghosh and Wolf (1994) to identify and group African countries into optimal currency areas. The choice of groupings is determined by maximising the synchronicity of business cycles based on a simple macroeconomic model. The more synchronised business cycles are among currency group members, the lower the combined output loss for any given monetary policy action. Thus, we seek to minimise the total output loss for the continent by grouping countries with highly synchronised business cycles into the same currency union. We find that adopting a single currency costs Africa approximately 0.9% of GDP each year. This cost is reduced to 0.6% of annual GDP by adding a second currency group. Having five currency groups decreases the calculated output cost to 0.4% of annual continental GDP. We uncover the country composition of the currency areas for different numbers of groups during the exercise.

We compare the five optimal currency area arrangement suggested by the algorithm with the Abuja Treaty recommendation of five regional economic communities. The calculated output cost of the Abuja Treaty five currency arrangements is 0.84% of annual GDP. This result means that using the optimal arrangement would result in a 50% reduction in output losses in comparison.

Further, the paper asks how many currency groups are appropriate for Africa at this time. We observe diminishing returns to additional currency groups. The

composition of the currency groups remains fairly consistent as we increase the number of currency groups. The impact of the additional groups is that they allow the identification of a number of outlier countries that experienced uniquely asymmetric shocks that are fundamentally non-economic. Based on qualitative reasons we suggested three currency unions for Africa as the optimal number at this time. Then we compare this combination with the 5 regional groupings proposed by the AU.

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2. Literature Review

THE AFRICAN UNION AND INTER-GOVERNMENTAL ORGANISATIONS

The African Union, the would-be architect of the African Economic Community (AEC), considers RECs in Africa to be the building blocks for continental integration (Economic Commission for Africa; African Union, 2006). RECs are regional groupings or inter-governmental organizations in Africa. Eight¹ are officially recognized by the African Union.

Table 1. Composition of Regional Economic Communities

REC	Number of Members	Number in Other RECs
UMA	5	3
IGAD	8	8
EAC	3	3
CEN-SAD	21	21
COMESA	25	25
ECCAS	14	13
CEMAC	6	6
CEPGL	3	3
SADC	16	16
SACU	5	5
IOC	5	5
UEMOA	8	8
ECOWAS	15	13
MRU	3	3

Source: *Assessing Regional Integration in Africa II, 2006*

The African Union has chosen a linear model of integration (McCarthy, 2010) as its framework for the development of the AEC (Economic Commission for Africa; African

¹ Arab Maghreb Union (UMA), Community of Sahel-Saharan States (CEN-SAD), Common Market for Eastern and Southern Africa (COMESA), East African Community (EAC), Economic Community of West African States (ECOWAS), Economic Community of Central African States (ECCAS), Inter-Governmental Authority on Development (IGAD), and Southern African Development Community (SADC).

Union, 2008) . It consists of six phases, where each successive phase requires progressively more synchronicity in the functioning of the economies of the member countries. The sixth and final stage of the process involves establishing a continent-wide economic and monetary union and thus a currency union to be completed in 2028.

Some authors have contemplated the creation of currency unions based on the existing regional economic communities, for example (Khamfula & Mensteab, 2004) and (Masson & Pattillo, 2001) for SADC and ECOWAS respectively to name a few. The rationale for using RECs in this manner is in an attempt to build on their accumulated knowledge and expertise as well as the successes at integration. Some of these include the customs unions already established for groups of countries in West, Central and Southern Africa, a common passport in each of the ECOWAS and EAC regions to bolster factor mobility, substantial communications infrastructure development in UMA, COMESA, ECOWAS and SADC and energy sharing arrangements in SADC (Economic Commission for Africa; African Union, 2006). Another important reason is that since political and economic considerations are virtually inseparable, any efforts to integrate involve political manoeuvring. Using RECs for the initial stages of economic integrations takes advantage of the existing political links which would otherwise require more time and effort to establish.

Taken as a whole, Africa has performed poorly economically relative to the rest of the world. The benefits of globalisation such as expansion of trade have not been realised significantly as can be seen by considering Africa's declining share of world exports that dropped from 4 per cent in 1980 to 1.6 per cent in 2000. The poor

performance can be partially attributed to inappropriate inward-looking development strategies that capture rents rather than foster growth, obstacles to trades, undemocratic politics and tribal and ethnic rooted civil unrest. The formation of the African Union and its implementation plan, New Partnership for Africa's Development were a result of a stronger consensus for integration on the continent coinciding with the creation of the euro zone. Previous regional monetary integration initiatives in Africa include, in West Africa, the West African Monetary Zone (WAMZ) which was to eventually merge with WAEMU, the very recently reinvigorated efforts of the EAC in East Africa, and the efforts of SADC following SACU in southern Africa (Masson & Pattillo, 2004).

Using the existing RECs in this manner comes saddled with several problems. Most of the countries are members of more than one REC. Some of the challenges that arise as a result of this arrangement involve increased cost of membership and the accompanying dispersed resources, duplicated efforts, inconsistent objectives and conflicting operational mandates at a national level (Economic Commission for Africa; African Union, 2006). Awareness of these and other challenges necessitated the on-going rationalization efforts, that is, the downsizing of RECs to become more efficient. To meet the rationalisation concerns the Abuja Treaty proposed dividing the continent into five new RECs, the North African Economic Community (NAEC), West African Economic Community (WAEC), East African Economic Community (EAEC), Central African Economic Community (CAEC) and the Southern African Economic Community (SAEC). This arrangement constitutes one pole of the rationalization continuum, the strong form of rationalization. At the other end of the continuum lies the weak form of

rationalization where the RECs remain in their original form and their programmes are harmonized (Economic Commission for Africa; African Union, 2006).

Table 2. *Abuja Treaty RECs*

NAEC	WAEC	EAEC	CAEC	SAEC
Algeria	Benin	Burundi	Angola	Botswana
Egypt	Burkina Faso	Comoros	Cameroon	Lesotho
Libya	Cape Verde	Djibouti	Central African Rep.	Mozambique
Mauritania	Cote d'Ivoire	Eritrea	Chad	Namibia
Morocco	The Gambia	Ethiopia	Republic of Congo	South Africa
Tunisia	Gambia	Kenya	DRC	Swaziland
	Ghana	Madagascar	Equatorial Guinea	Zambia
	Guinea	Mauritius	Gabon	Zimbabwe
	Guinea Bissau	Malawi	Sao Tome and Principe	
	Liberia	Rwanda		
	Mali	Seychelles		
	Niger	Somalia		
	Nigeria	Sudan		
	Senegal	Tanzania		
	Sierra Leone	Uganda		
	Togo			

Source: *Assessing Regional Integration in Africa II, 2006*

THE THEORY OF OPTIMUM CURRENCY AREAS

When a country joins a currency union it is no longer able to change the price of its currency or to determine the quantity of it in circulation. Confronted by aggregate demand shifts, a currency union is optimal if there is either sufficient wage flexibility or labour mobility or a sufficiently centralized budgetary process for smooth transfers between countries to allow adjustment to the system (De Grauwe, 1997). The shift in aggregate demand from one region to another result in an output decline, additional unemployment, a current account deficit and in all likelihood, a budget deficit to finance unemployment benefits in the primary region. On the other hand, the latter region faces

increased output, a current account surplus and inflationary pressure. If wages are flexible, the fall in aggregate demand would result in wage decline, a corresponding downward shift of the supply curve and a new equilibrium.

Another way that adjustment can come about is through migration of workers. Mobile labour alleviates the need for wages to fall because domestic workers simply relocate to the thriving foreign region thereby avoiding domestic unemployment and dampening inflationary pressure in the foreign country (Mundell, 1961).

In the absence of sufficient wage flexibility, labour mobility or a system of transfers, the adjustment problem remains. In this case it can be argued that individual currencies would be better, since each country can depreciate its currency accordingly to cut real wages and avoid further unemployment.

It follows that if asymmetric shocks are less likely among a group of countries then an exchange rate between them is unnecessary, and the group of countries can reap the benefits of currency union, such as growth through increased trade and economies of scale due to a larger market (Krugman P. R., 1979). To the extent that the countries chosen to be in a currency union still experience asymmetric shocks, however, sufficient wage flexibility, labour mobility and a system of transfers would bring about adjustment toward equilibrium.

A section of the literature based on empirical evidence questions the relevance of asymmetric shocks, and suggests that the frequency of asymmetric shocks is itself endogenous and lower upon formation of the monetary union (EC Commission, 1990). An opposing view is argued by Krugman (1991) that regional concentration of industrial

activities can be a result of trade integration via economies of scale. In the African context, for ECOWAS in particular, Masson and Pattillo (2005) find that terms of trade are a particularly important source of shocks for ECOWAS countries whose exports are primary commodities. Similarly, Wang et al (2007) note the same asymmetry for the Common Monetary Area in Southern Africa based on the differences in commodity exports. The establishment of Free Trade Agreements in regional blocs such as SADC reduces the incidence of asymmetric shocks from terms of trade differences increasing the likelihood of success of monetary union for them.

Additionally, asymmetric shocks can arise from the fiscal discretion afforded member countries when it comes to spending and taxation as well as public nature of economic institutions such as the wage bargaining process making this analysis relevant. At this time there is no conclusive empirical evidence or theoretical foundation to dismiss the importance of asymmetric shocks in the formation of currency areas.

For a comprehensive discussion of the costs and benefits of monetary unions in the African context see ECA and AU (2008) based on De Grauwe (1997). The advantages of a common currency depend on the savings of transaction costs which in turn depend on the extent of trade among countries. The low extent of trade among African countries suggests that the savings of this type will not be substantial (Masson & Pattillo, 2004). Studies that suggest that growth of trade can also be brought about monetary union imply that the preceding observation is insufficient grounds to abandon monetary union initiatives. The benefit will largely accrue from the envisioned stability of the new common currency stemming from the associated fiscal discipline.

3. Methodology

We follow a framework for estimating the cost of relinquishing the exchange rate as a tool for economic adjustment adapted from Ghosh and Wolf (G&W) (1994).

MACROECONOMIC MODEL

G&W begin by assuming the Cobb-Douglas form for the production function of each country where the only factor of production is labour. The output of a given country is subject to productivity shocks. We assume nominal wage rigidity, a perfectly competitive product market with profit maximizing behaviour and full employment during wage setting.

Assume the occurrence of a negative shock, such as a decline in the aggregate demand for a country's output. The rigidity of the nominal wages means that wages cannot fall to bring about adjustment and unemployment must increase. One way of avoiding higher unemployment in this scenario is for the monetary authorities to increase the price level in the economy above the expected level and reduce real wages using the exchange rate. If the central bank depreciates the currency just enough to offset the shock and maintain full employment then the output of the economy remains at full employment level.

The central bank of a country with its own currency can behave in the manner described above to enable adjustment in the economy. On the other hand, a common central bank for a currency union comprising countries that experience asymmetric shocks, by nature or magnitude, cannot be as effective for each of its member countries because the degree of depreciation of the currency required for full adjustment in each of

the economies is different. The common central bank must strike a balance in the level of depreciation to serve its entire membership. One way of doing this is to calculate a GDP-weighted average shock for its members and depreciate the common currency to offset the average shock level. All the countries will then be steered toward full employment but individually there will be a gap between the output that they could have achieved at full employment with an independent central bank and the output they are actually attaining as part of the currency union.

Therefore, for any allocation of countries into currency groups we can calculate each country's output loss emanating from its unique grouping as:

$$L_t^i = Q_t^i - \hat{Q}_t^i = Q_t^i - Q_t^i e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} = Q_t^i \left(e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} \right)$$

Where, Q_t^i is the output of country i at time t , β is the labour share, ε_t^i is the unique shock suffered by country i at time t and $\bar{\varepsilon}_t$ is the GDP-weighted depreciation level of the currency union. The total output loss of all countries in the pool represents the loss value of a given currency group configuration and it is given by:

$$L = \sum_{i=1}^n L_t^i$$

A lower total loss suggests a more optimal configuration. Intuitively, countries experiencing shocks in a similar fashion will require similar exchange rate policy responses. Thus the common central bank actions will be more closely tailored to each member's needs and output losses are minimized. A detailed technical treatment of the Ghosh-Wolf framework is presented in Appendix B.

Model Caveats

The mechanism that allows negative productivity shocks to impinge on the output of a country relies on the inflexibility of nominal wages. In the CFA zone countries, Rama (2000) finds evidence of real wage rigidity in the period from 1985 to 1993. Krishnan *et al.* (1998) observe real wage rigidity in Ethiopia. However, individual country studies such as one by Milne and Neizert (1994) on Kenya following negative shocks resulting from oil price fluctuation and severe droughts find that the real wage is actually quite flexible. Thus there appears to be no overwhelming evidence against wage rigidity in Africa.

An implicit assumption of the G&W framework is that labour is not mobile between the countries in a monetary union. In reality, major moves are in progress to encourage labour mobility in SADC and ECOWAS to name a few. This presents an alternative mechanism for restoring macroeconomic balance. Migration of labour reduces the actual cost of monetary union.

Another criticism that is often levelled at optimum currency area theory is that the exchange rate is an ineffective monetary policy tool in diverse economies because it is a blunt tool that cannot be applied selectively to different industries or sectors. This means that it is not a huge loss upon joining a monetary union. African countries are hardly diverse with most relying heavily on one or two sector more-often-than-not natural resources and/or agriculture. Therefore joining a currency union certainly sacrifices a valuable macroeconomic adjustment tool for most African countries. As such, optimum currency area theory insights are relevant if the countries pursue currency union.

GENETIC ALGORITHM

This paper adopts a similar approach to that of Ghosh and Wolf (1994) of solving the problem of finding the best currency groupings for African countries based on optimal currency area theory using a genetic algorithm. Genetic algorithms are adaptive algorithms for finding the global optimum solution for an optimization problem (Cao & Wu, 1999). The use of a genetic algorithm is necessary because testing every possible combination for all possible numbers of groups would require evaluation of $1.996133794 \times 10^{16}$ arrangements. Using a genetic algorithm enables us to zero in on the optimal arrangements with about 378 000 evaluations. Time, computation power restrictions and efficiency considerations led us to take the genetic optimisation route. Implementation of the algorithm is adapted from MATLAB code available in Cao and Wu (1999). An example is given in what follows to illustrate the functioning of the algorithm.

Example

This descriptive example explains what the MATLAB program provided in the appendix does on a larger scale. Suppose that our aim is to find the optimal allocation of five countries into three currency unions. The optimal allocation is the allocation that minimizes the loss function described in section 3.1 and given by equation 15 in the appendix. A candidate solution to this problem x assigns each of the five countries into one of the three monetary unions. For example,

$$x_1 = [2 \quad 2 \quad 3 \quad 1 \quad 1]$$

where the five positions in the vector represent countries A, B, C, D and E respectively. This means that for this particular candidate solution countries A and B are in a union together, as are D and E and finally country C has its own currency.

The first task is to create a number of vectors like x where the countries A, B, C, D and E are each randomly allocated to one of three currency unions. An example would be the first equation below,

$$P_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 & 1 & 1 \\ 3 & 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 1 & 3 & 2 & 3 \\ 3 & 3 & 3 & 2 & 1 \end{bmatrix}, \begin{bmatrix} v(x_1) \\ v(x_2) \\ v(x_3) \\ v(x_4) \\ v(x_5) \\ v(x_6) \end{bmatrix} = \begin{bmatrix} -30 \\ -40 \\ -50 \\ -55 \\ -45 \\ -35 \end{bmatrix}$$

P_1 is the first population of candidate vectors. Each row of the population matrix represents a candidate solution with a randomly generated allocation of the countries into three groups. Note that a population does not exhaust all the possible combinations. A loss value $v(x_i)$, $i = 1:6$ is calculated for each candidate solution using the loss function. The best currency grouping for the generation corresponds to the least severe loss value, i.e. x_1 , for which $v(x_1) = -30$, and it is recorded. Subsequent populations of candidate vectors are generated by applying the genetic operators of selection, cross-over and mutation which are described below.

Selection Operator

The selection stage involves choosing the better candidates of the preceding population to be part of the next population probabilistically. The calculated probability that a particular candidate solution is selected is inversely proportional to its loss value i.e. the

better candidates are more likely to be selected to be part of the new population. A proportion of the less desirable candidate solutions, based on their relatively more severe loss values are excluded from the subsequent population. They are replaced with an equal number of new randomly generated candidate solutions.

Cross-over Operator

The cross-over stage creates new candidates using two adjacent candidates from the previous population by swapping segments beyond a randomly chosen cross-over point. Suppose the two candidates to be *mated* are x_1 and x_2 given above. Also suppose the randomly chosen cross-over point is position 2.

$$x_1 = [2 \quad 2 \quad 3 \quad 1 \quad 1]$$

$$x_2 = [3 \quad 1 \quad 3 \quad 2 \quad 2]$$

The new candidates x_1^{new} and x_2^{new} after *mating* are:

$$x_1^{new} = [2 \quad 1 \quad 3 \quad 2 \quad 2]$$

$$x_2^{new} = [3 \quad 2 \quad 3 \quad 1 \quad 1]$$

The newly created candidates replace the original candidates in the subsequent population. The procedure is repeated for pairs of adjacent candidates (rows) in the population matrix.

Mutation Operator

At this stage each candidate is probabilistically perturbed by randomly changing the currency group of a randomly chosen country in the original candidate solution. For instance, with a probability of, say, 0.05 we perform the following. We generate a

random number between 1 and the total number of countries e.g. 4. What this means is that we will change the currency group of the country in position 4. We also generate a random number between 1 and the number of currency groups that we want e.g. 2. Supposing the country in position 4 was initially in currency group 1, we would then move it to group 2.

$$x_1^{before} = [2 \quad 2 \quad 3 \quad 1 \quad 1]$$

$$x_1^{after} = [2 \quad 2 \quad 3 \quad 2 \quad 1]$$

Taken together, the genetic operators are employed to introduce systematic, controlled variation into the population of candidates. The algorithm mimics the actual process of evolution in nature.

After the application of the mutation operator we obtain the new population of candidate vectors. As before, loss values are calculated for each candidate and the best candidate of the generation is discovered and recorded. For continuity, we include the previous best candidate as a candidate in each subsequent population so that a different generational best candidate necessarily implies an improved currency grouping. The composition of the running optimal candidates is only tweaked gradually to preserve the desirable links and at the same time explore the possibilities for adjustment cautiously.

In a global sense, the running best candidate is noted and updated only if the current generational best candidate is better i.e. has a less severe loss value. The cycle of selection, cross-over and mutation is repeated as the running best candidate converges to the optimal currency groupings. The optimization criteria employed are two-fold. We terminate the cycle if the running best candidate is stable i.e. unchanged for a

predetermined number of cycles even after increasing the probability of cross-over and mutation.

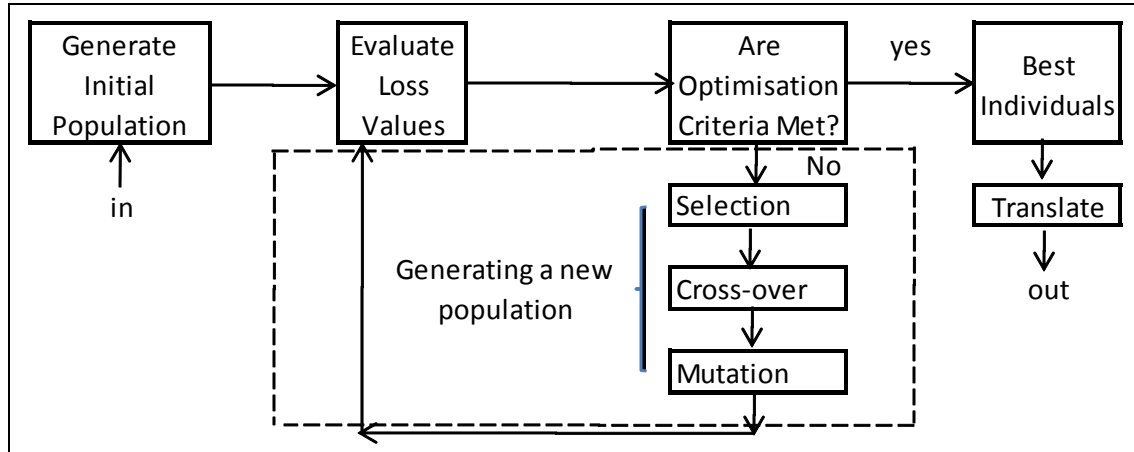


Figure 1: Illustration of Genetic Optimisation Algorithm

The interested reader can find a similar yet more detailed explanation of the genetic operators and their implementation in MATLAB in Cao & Wu (1999).

In addition to employing the same algorithm as in the current study on various regions of the world, Ghosh and Wolf (1994) also investigate impact of imposing the condition of contiguity for each currency area. For a currency union to be contiguous, each member country must share a geographical border with at least one other country also in the union. This feature is very relevant because shared borders can be logically expected to increase the incidence and extent of trade between countries and will likely be greatly influential in real-world policy-making.

The requirement of contiguity means that economic agents in any one of the member countries have access to any the other member countries geographically by navigating through other member countries. That is, they can be guaranteed, theoretically, hassle-free passage to anywhere in the union.

While the concept of restricting the possible monetary union allocations is quite straightforward conceptually, its translation into the algorithm is much more intricate. In brief, the algorithm requires crucial adjustment at four key points to incorporate contiguity. Firstly, the random generation of candidate solutions at the outset of the algorithm will take considerably more time and finesse to execute because we have to ensure that each candidate solution generated satisfies the contiguity condition. The same checks have to be implemented after each implementation of the genetic operators of selection, mutation and crossover.

Considerable progress has been made at the time of writing regarding the MATLAB code to improve the algorithm to incorporate the contiguity dimension. It has been provided in the appendix and it illustrates the convolution of the exercise. A follow-up study that incorporates this feature among the many other recommendations can be expected are in the works. One might say that shared borders are one, albeit very important, factor representing existing ties between subsets of countries. The impact of shared colonial history or REC membership can be incorporated into the study and capture a highly similar, possibly more relevant, yet less permanent effect.

DATA

The data used in this study are annual constant 2005 US dollar Gross Domestic Product at market prices for the fifty-four African countries. The data are expressed in logarithmic form to dampen their variability. The sample period is from 1970 to 2009. The choice of sample period is based on availability of data. The data were provided by the United Nations Statistics Division.

We use the Hodrick-Prescott filter to decompose the log (GDP) series into the sum of a slow moving secular trend and a transitory deviation from it which we refer to as the business cycle. We use the deviation at a given point in time as the shock that the economy suffers. The decomposition of the logged output series into a growth and a cyclical component assumes that the series does not contain any seasonality. Additionally, the cyclical component is not separated from any irregular movements resulting in error in the measurement of the cycle. Also, by assuming a constant value for lambda in the detrending procedure the HP filter implicitly assumes that the relative variances of demand and supply disturbances to output are time-invariant making them indistinguishable to the HP filter (Razzak & Dennis, 1995).

An alternative to the Hodrick-Prescott filter would be to extract the shocks to real output the study would use a three-step procedure employed by (Wang, Masha, Shirono, & Harris, 2007) for comparison with the current use of deviations of the actual series from the HP filtered version. First, a unit root (Augmented Dickey Fuller) test would be carried out on the time series data of the natural log of per capita real GDP of the countries in a candidate currency area for the sample period. If the data are found to be integrated of order 1 (I(1)), the next step would use data of first differences selected based on lag tests. The next step would be to extract the underlying disturbances or shocks from the data by regressing on the lagged first differences of the per capita GDP data. However, the automated design of the algorithm is not easily adaptable at this stage to incorporate the full procedure.

The Blanchard-Quah decomposition methodology that would be appropriate because it uses two series per given country to identify two structural shocks, a permanent supply shock and a temporary demand shock in the framework of Vector Autoregression (VAR) (Blanchard & Quah, 1989). However, this is not feasible in our study because the only complete data for all African countries that we could find was GDP data.

We assume the labour share in the economy of each country to be constant at two-thirds.

4. Empirical Results

GENETIC ALGORITHM OUTPUT

Figure 1 below plots the percentage of GDP loss values of the optimal currency configurations for various numbers of currency groupings obtained from the genetic algorithm. Note that the marginal benefit of an additional currency group appears to decrease progressively as the total number of currency groups increases. Each additional currency yields smaller and smaller improvement in terms of output losses. Thus, there are diminishing returns to currency groups. The cause of diminishing marginal returns stems from the limited asymmetry of shocks that the member countries experience. The addition of a currency group reduces the overall asymmetry of shocks. Subsequent addition of currency groups have less asymmetry to mitigate.

With fifty-four separate currencies the loss from forgone macroeconomic stabilization is zero by design. Adopting a single currency, on the other hand costs Africa approximately 0.9% of GDP each year. From Figure 1 we observe that the largest marginal benefit of a single currency is observed when we add a second currency. A

saving of approximately 0.3% of the continental GDP is made by this action. Having five currency groups instead decreases output losses by 0.5% of continental GDP. To achieve a further 0.5% decrease in output losses would require an additional forty-nine currency groups.

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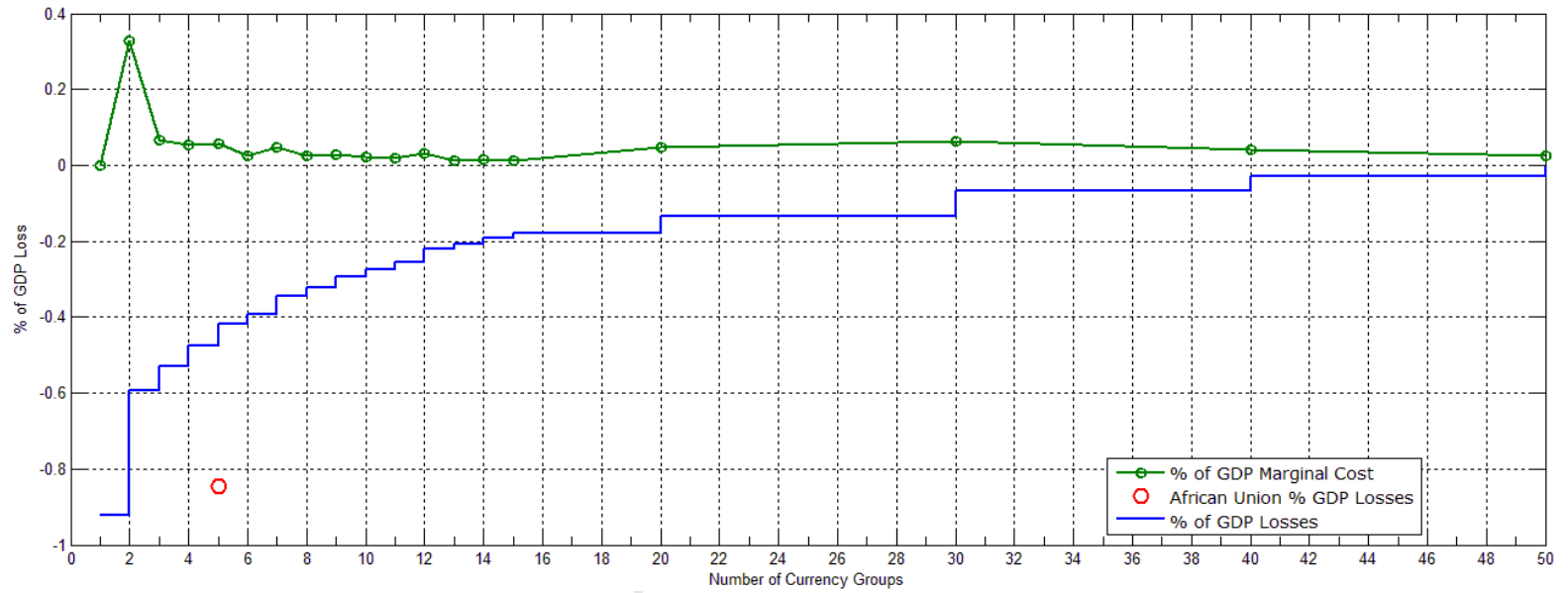


Figure 2: *Plot of Output Losses vs. Number of Currency Groups for 1970 to 2009*

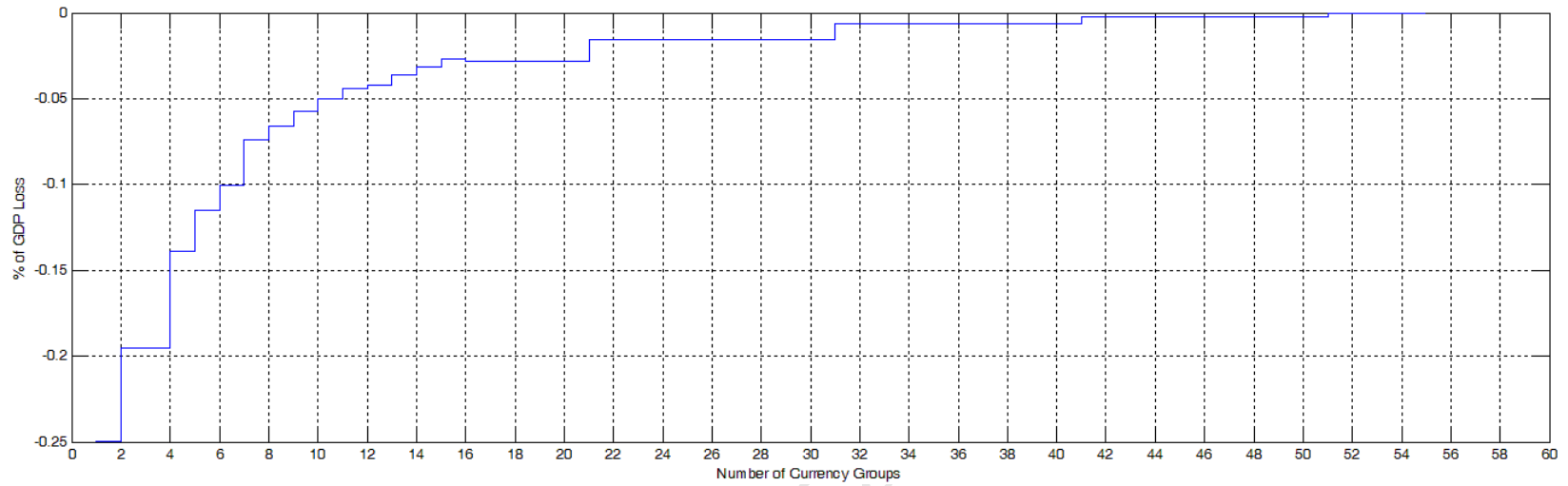


Figure 3: *Plot of Output Losses vs. Number of Currency Groups for 2000 to 2009*

The sample period spans over forty years from 1970 to 2009. While this might be considered to be quite long for such a study because the nature of the shocks faced by the economies of the countries could have changed dramatically over time as well as the structure of the economies themselves, using the entire period maintains a level of objectivity because the selection of a sub-period that meets certain conditions such as relative peace can easily degenerate into a subjective exercise. However, in this study we have also run the algorithm on a data set of the most recent ten year of data available, that is, from 2000 to 2009. The results are plotted in figure 4 below and discussed thereafter. Several insights can be gleaned from an analysis of the optimal country groupings provided in Appendix A. We take a closer look at the optimal country groups indicated by the genetic algorithms for different total numbers of currency groups.

We analyse the optimum currency groups by considering the countries that progressively get excluded from the larger blocs as we increase the number of currency groups (see Table 2 below). What the genetic algorithm essentially does is to provide a business cycle dissonance ordering i.e. countries with the most uniquely uncorrelated business cycles over the entire period are singled out earliest as we increase the number of groups. For larger numbers of groups, where the marginal benefits of an additional currency is negligible, a prohibitively large number of runs of the algorithm is required to pinpoint the absolute optimal arrangement. As a result we observe shifting of countries between the major groups for an additional currency group.

From the union of all countries where we have only one currency for the whole continent to two currency groups, Liberia is left out on its own in the optimal solution.

Prematurely looking forward to scenarios of more currencies we also observe that the recommendation that Liberia maintain its own currency is a recurring feature. We consider the reason for this below.

Adding an additional currency for a total of three currencies breaks up the major bloc from the two currency arrangement into two roughly equal sized portions. Another increment to four currency groups gives rise to another splinter of countries consisting of Chad, Gabon and Swaziland.

Proceeding in the manner described above, a pattern emerges as we increase the number of currency groups that are possible on the continent. While a few of the countries move between the major three or four currency blocs, presumably adjusting to harness the increased flexibility afforded by the additional currency and minimize output loss, a subset of countries are systematically gradually excluded from these blocs individually or in small groups of two. These countries are deemed to have business cycles so uncorrelated to any of the major blocs that the continent is better off when they manage their own individual or sub-group exchange rates. We conclude that these countries fall outside any blocs owing to domestic political problems and civil war, so economic factors are not governing business cycles or the generation of shocks.

<i>No. of Groups</i>		<i>Sub-groups and Individual Countries</i>				
2	Liberia					
3	Liberia					
4	Liberia	Chad				
		Gabon				
		Swaziland				
5	Liberia	Chad	Eq. Guinea			
		Gabon	Eritrea			
		Swaziland	Ethiopia			
6	Liberia	Chad	Eritrea			
		Gabon	Ethiopia			
7	Liberia	Chad	Eritrea	Eq. Guinea		
		Gabon	Ethiopia	Rwanda		
		Swaziland				
8	Liberia	Chad	Eritrea	Eq. Guinea	Sierra Leone	
		Gabon	Ethiopia	Rwanda		
9	Liberia	Chad	Eritrea	Eq. Guinea	Sierra Leone	Angola
		Gabon	Ethiopia	Rwanda		DRC
10	Liberia	Chad	Eritrea	Eq. Guinea	Sierra Leone	Angola
		Gabon	Ethiopia	Rwanda		DRC

Table 3: Outliers

<i>No. of Groups</i>	<i>Sub-groups and Individual Countries</i>			
2	-			
3	Chad			
	Eq. Guinea			
	Sierra Leone			
4	Chad	Liberia		
	Eq. Guinea			
5	Chad	Liberia		
	Eq. Guinea			
6	Eq. Guinea	Liberia	Chad	
			Nigeria	
7	Eq. Guinea	Liberia	Chad	
			Nigeria	
8	Eq. Guinea	Liberia	Chad	Nigeria
9	Eq. Guinea	Liberia	Chad	
10	Eq. Guinea	Liberia	Chad	Nigeria

Table 4: Outliers for 2000 to 2009

The genetic algorithm is applied in the study primarily to identify the optimal groupings of countries for each number of currency groups from one to fifty-four (trivial). The interpretation of the table of outliers given above is used merely as a way of making sense of the model output. It does not imply that the status quo or starting point for the analysis is a single currency for the entire continent.

The reasoning from either perspective also results in the same relationship between the number of currency groups and the loss value in the context of this investigation. Consider a policy maker looking to reduce the total number of currencies in

Africa from fifty-four, the status quo, to fifty-three with the lowest possible output loss. This is achieved by taking the two countries with the greatest symmetry in shocks and forming a currency union between them. To reduce the number of currencies on the continent even further, countries or country groups are progressively united to share a common currency if they have the greatest symmetry of shocks. Therefore the curve starts out from the left with a low gradient because countries with highly similar shock structures are joined but progressively, subsets of countries with less and less symmetry of shocks have to be united to reduce the total number of currencies resulting in larger and larger output losses. As a result we observe larger output losses when we have very few currencies and reduce the total number by one. The graph of output loss versus number of currency unions that would emerge is identical to the one obtained in the current study empirically regardless of whether the starting point is one or the maximum number of currencies.

We casually observe that the countries that are isolated by the algorithm using 40 years' worth of annual log (GDP) data seem to be country that have experienced strife of one form or the other for very large periods in the sample. We further speculate that the underlying source of dissonance of the business cycles of the countries given in Table 2 above is, in fact, the prevalence of strife for sustained periods in the sample. We infer that the currency groups that emerge containing these countries do not reflect their underlying fundamental economic structures but are the result of social and political shocks, thus they should all be allowed to keep their own currency until they are stable enough to join of the other optimum currency areas. Admittedly, the analysis given above is hardly

rigorous. To mention a few, the Democratic Republic of Congo, Angola, Rwanda, Ethiopia, Eritrea, Equatorial Guinea, Chad, Liberia, Sierra Leone have all experienced at least one of coup d'états, civil war, alleged dictatorship or genocide. Political strife is fairly commonplace in the history of most African countries. The isolated countries seem to have experienced political trouble for larger periods in the dataset.

Model Recommended Currency Groups

The main limitation of the G&W framework is that it only focuses on minimising losses and ignores gains. Reason being that an additional currency will always give rise to an improvement. If minimizing losses were the only criterion then the optimal number of currencies would be fifty-four, a separate currency for each country. Currency unions provide gains through increased trade and economies of scale due to a larger market among other things. The optimal number of currencies can be rigorously determined through marginal analysis. The number of currency groups should only be increased as long as the marginal benefit of doing so exceeds the marginal cost. The marginal benefits of currency union are not explicitly modelled or assessed in the analysis in the current study. This deficiency reduces the capacity of the model and the algorithm to make definitive recommendations regarding the number of currency unions that are optimal for Africa. However, to the extent that asymmetrical shocks are detrimental to the well-functioning of currency unions, the methodology alerts us to the pattern of country groupings based on the correlation of their business cycles.

However, based on the optimal group patterns that emerge as we increase the number of currency groups we can make a few tentative recommendations. We begin by noting on

figure 1 that the marginal cost in terms of forgone macroeconomic stabilization in moving from sixteen to fifteen currencies is negligible relative to a corresponding increase from, say, one to two currency groups for Africa. We mark this as the ceiling of the number of currency groups in consideration. We also notice that three or four groupings emerge after excluding the politically fragile countries. One or two countries, Tunisia and The Gambia, move back and forth between two of the groups a phenomenon which can be attributed to sampling error. We present the groups below as the recommended groups for currency unions in Africa.

1	Namibia	Zanzibar	Egypt	Ghana	Sao Tome and Principe
Burundi	Nigeria	2	The Gambia	Kenya	Togo
Djibouti	Senegal	Algeria	South Africa	Lesotho	
Guinea Bissau	Somalia	Benin	Tunisia	Libya	
Guinea	Sudan	Botswana	Zimbabwe	Madagascar	
Mali	Swaziland	Cameroon	3	Malawi	
Mauritania	Tanzania	Cape Verde	Burkina Faso	Mozambique	
Mauritius	Uganda	Comoros	Central African Republic	Niger	
Morocco	Zambia	Republic of Congo	Cote d'Ivoire	Seychelles	

Table 4: *Model Recommended Currency Groups*

The map below illustrates the recommended groups geographically. The countries shaded grey are the ones that exhibit particularly uniquely uncorrelated business cycles likely due to their extreme political instability during the period and are not included in any of the currency groups.

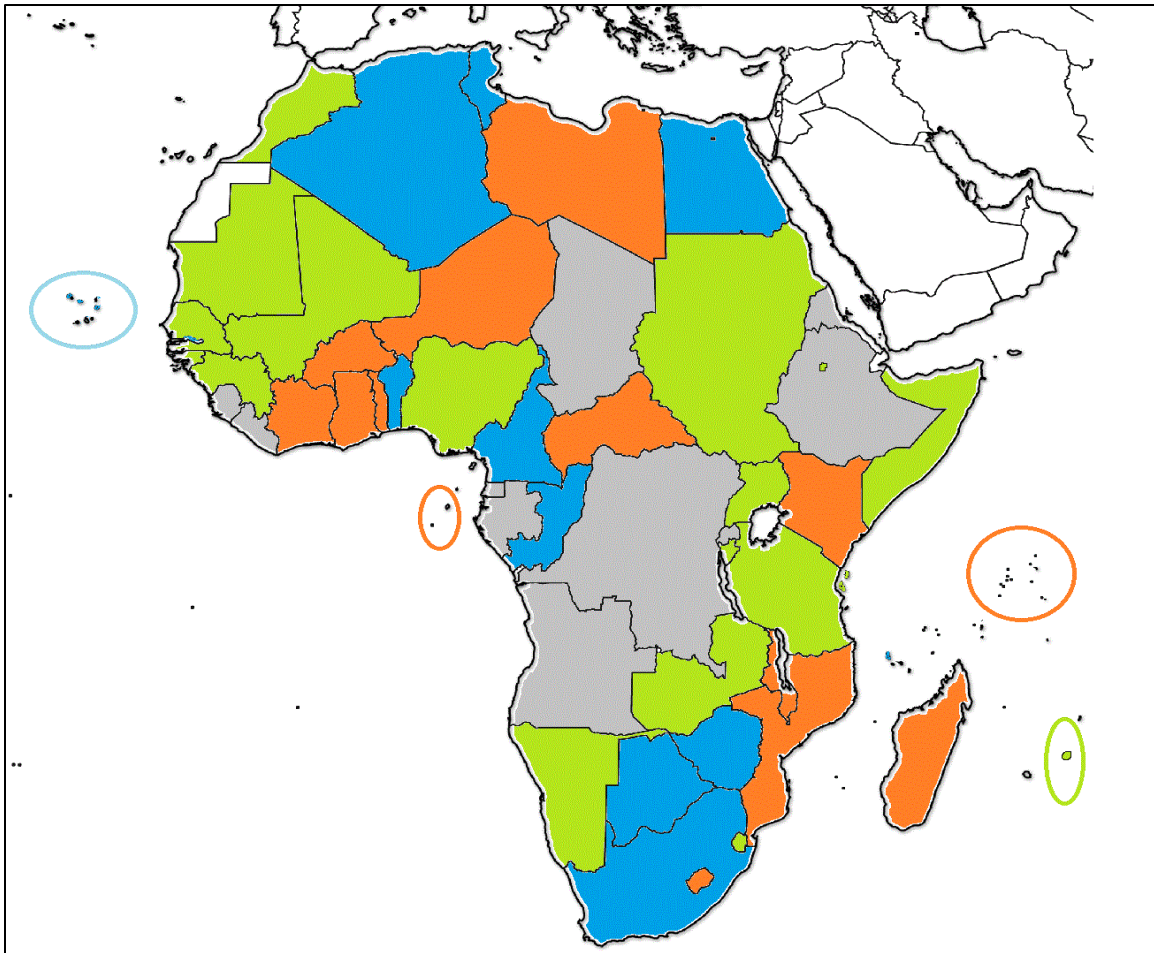


Figure 4: Illustration of Model Recommended Groups

Model Recommended Currency Groups vs. Merger and Absorption Currency unions

The Merger and Absorption rationalization scenario has five RECs namely, SAEC, EAEC, WAEC, CAEC and NAEC mainly based on geographic location.

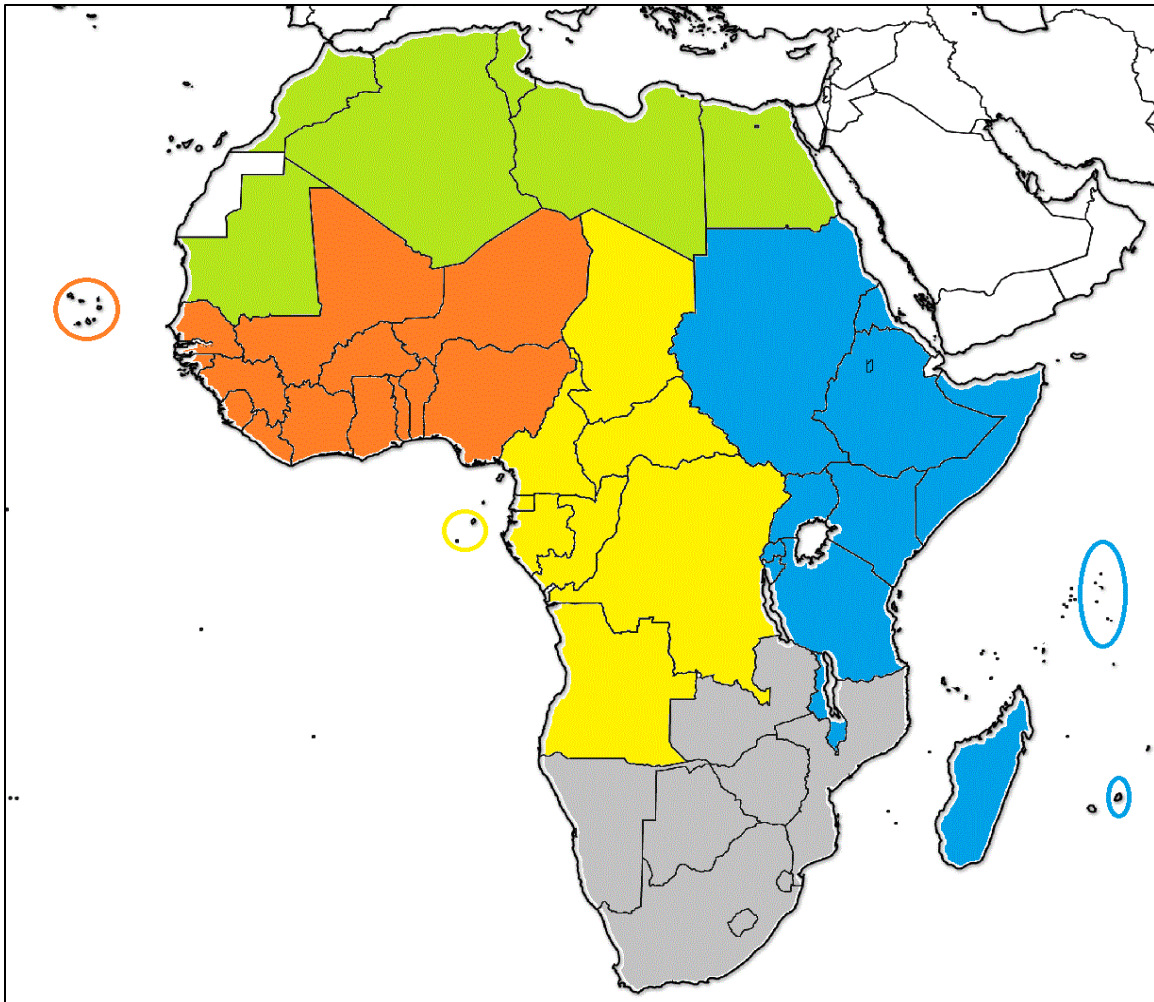


Figure 5: Illustration of African Union recommended Groups

Evaluating the potential of this arrangement using the framework of the current study reveals output losses of -0.8447% of continental GDP. The optimal grouping derived from the genetic algorithm incurs output losses of -0.4186% continental GDP. There is a more than 50% reduction in output losses from employing the recommended arrangement from genetic optimization.

This study implicitly tests for a correlation between shocks and geographical proximity between countries. By comparing the optimal groups obtained with the geographically inclined Abuja Treaty regional economic communities we see the degree to which neighbouring countries are allocated to the same currency groups. There is limited evidence for geographic symmetry of shocks. In SAEC, South Africa, Botswana and Zimbabwe are grouped together whilst in WAEC, Mali, Senegal, Guinea-Bissau, Guinea and Nigeria maintain common membership. The fact that South Africa is not grouped with the other Common Monetary Area members, Lesotho, Namibia and Swaziland in the optimal grouping based on correlation of business cycles confirms the findings of Wang et al (2007).

Model Recommended Currency Groups vs. Rooted Communities

We focus our analysis on rooted communities as defined by the Economic Commission for Africa (Economic Commission for Africa; African Union, 2006). These are ECCAS, ECOWAS, SADC, COMESA and CENSAD. COMESA and SADC members are most represented in group one and least represented in group two. CENSAD members are comparatively most represented in group three closely followed by group one. ECOWAS members feature most prominently in group three.

5. Conclusion and Recommendations

The first lesson learned from employing the Ghosh and Wolf (G&W) methodology to examine optimal currency areas in Africa is that the basic methodology is somewhat limited by focusing only on minimizing an output loss function and ignoring the maximization of benefits from alternative country combination, including the contiguity of currency union members. Incorporating the benefits of currency union would augment the findings of this study greatly. Nonetheless, the approach identifies the decreasing marginal losses as more currency areas are added and allows the analyst to strategically identify an optimal number of currency areas.

We find that adopting a single currency costs Africa approximately 0.9% of GDP each year. This cost is reduced to 0.6% of annual GDP by adding a second currency group. Having five currency groups decreases the calculated output cost to 0.4% of annual continental GDP. We uncover the country composition of the currency areas for different numbers of groups during the exercise.

Our methodology also allowed us to identify a number of countries that do not appear ready to join any OCA. These outlier countries (noted in Table 3) did not synchronize with any larger blocks of countries, mainly because their economies were still driven more by noneconomic shocks.

Based on the G&W output loss minimising model and some qualitative analysis, we suggest that Africa would currently be better off with three regional currency areas rather than one continental currency. We agree that beginning with multiple regional OCAs would be superior to trying to establish a single currency area for the continent at

once. Using the G&W approach, we also found that 3 currency areas would be appropriate and appear superior to the 5 currency areas proposed by the AU. The calculated output cost of the Abuja Treaty's five currency arrangements is 0.84% of annual GDP. This result means that using the optimal arrangement would result in a 50% reduction in output losses in comparison.

We believe that this methodology provides a good starting point for identification of OCAs, and can further be enhanced by using the permanent and temporary shocks obtained using the Blanchard-Quah decomposition to separate out supply and demand shocks respectively.² Additionally, other variables, such as real money balances, might be used to identify the level of synchronicity among economies. Running the algorithm on such a variable would confirm if the identified currency groups in this study remain appropriate.

²It was not possible to use this method because of lack of employment data for many of the countries for the sample period.

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6. Appendix

APPENDIX A: OPTIMAL CURRENCY GROUPS

1	Cape Verde	Ethiopia	Madagascar	Rwanda	Togo
Liberia	CAR	Gabon	Malawi	South Africa	Tunisia
2	Cote d'Ivoire	Chad	Mali	Senegal	Uganda
Algeria	Comoros	The Gambia	Mauritania	Seychelles	Zambia
Angola	Congo	Ghana	Mauritius	Sierra Leone	Zanzibar
Benin	Djibouti	Guinea Bissau	Morocco	Somalia	Zimbabwe
Botswana	DRC	Guinea	Mozambique	Sao Tome and Principe	
Burkina Faso	Egypt	Kenya	Namibia	Sudan	
Burundi	Equatorial Guinea	Lesotho	Niger	Swaziland	
Cameroon	Eritrea	Libya	Nigeria	Tanzania	

Table 1: Two Currency Groups

1	Eritrea	Burundi	Guinea	Mozambique	Swaziland
Liberia	Ethiopia	CAR	Kenya	Namibia	Tanzania
2	Niger	Chad	Lesotho	Nigeria	Togo
Benin	Rwanda	Cote d'Ivoire	Libya	South Africa	Tunisia
Cameroon	Zimbabwe	Djibouti	Madagascar	Senegal	Uganda
Cape Verde	3	DRC	Malawi	Seychelles	Zambia
Comoros	Algeria	Gabon	Mali	Sierra Leone	Zanzibar
Congo	Angola	The Gambia	Mauritania	Somalia	
Egypt	Botswana	Ghana	Mauritius	Sao Tome and Principe	
Equatorial Guinea	Burkina Faso	Guinea Bissau	Morocco	Sudan	

Table 2: Three Currency Groups

1	Congo	Zimbabwe	Kenya	4	Togo
Liberia	DRC	3	Lesotho	Namibia	Tunisia
2	Egypt	Burkina Faso	Libya	Niger	Uganda
Algeria	Equatorial Guinea	Burundi	Madagascar	Nigeria	Zambia
Angola	Eritrea	CAR	Malawi	Seychelles	Zanzibar
Benin	Ethiopia	Cote d'Ivoire	Mali	Sierra Leone	5
Botswana	The Gambia	Djibouti	Mauritania	Somalia	Chad
Cameroon	Rwanda	Ghana	Mauritius	Sao Tome and Principe	Gabon
Cape Verde	South Africa	Guinea Bissau	Morocco	Sudan	Swaziland
Comoros	Senegal	Guinea	Mozambique	Tanzania	

Table 3: Four Currency Groups

1	Congo	Zambia	Kenya	Namibia	Zanzibar
Liberia	Djibouti	Zimbabwe	Lesotho	Niger	4
2	DRC	3	Libya	Nigeria	Chad
Algeria	Egypt	Burkina Faso	Madagascar	Seychelles	Gabon
Angola	The Gambia	Burundi	Malawi	Sierra Leone	Swaziland
Benin	Rwanda	CAR	Mali	Somalia	5
Botswana	South Africa	Cote d'Ivoire	Mauritania	Sao Tome and Principe	Equatorial Guinea
Cameroon	Senegal	Ghana	Mauritius	Sudan	Eritrea
Cape Verde	Tunisia	Guinea Bissau	Morocco	Tanzania	Ethiopia
Comoros	Uganda	Guinea	Mozambique	Togo	

Table 4: Five Currency Groups

1	Egypt	Mozambique	Togo	Guinea Bissau	Angola
Liberia	The Gambia	Namibia	Tunisia	Lesotho	Benin
2	Ghana	Niger	Uganda	Libya	Botswana
Algeria	Guinea	South Africa	Zambia	4	Cameroon
Burkina Faso	Kenya	Senegal	Zanzibar	Eritrea	Cape Verde
Burundi	Madagascar	Seychelles	3	Ethiopia	Congo
CAR	Malawi	Somalia	Mauritius	5	DRC
Cote d'Ivoire	Mali	Sao Tome and Principe	Nigeria	Chad	Equatorial Guinea
Comoros	Mauritania	Sudan	Sierra Leone	Gabon	Rwanda
Djibouti	Morocco	Tanzania	Swaziland	6	Zimbabwe

Table 5: Six Currency Groups

1	Gabon	Benin	South Africa	Cote d'Ivoire	Morocco	Zanzibar
Liberia	Swaziland	Botswana	Senegal	Ghana	Mozambique	
2	5	Cameroon	Tunisia	Guinea	Namibia	
Equatorial Guinea	Guinea Bissau	Cape Verde	Uganda	Kenya	Niger	
Rwanda	Libya	Comoros	Zambia	Lesotho	Seychelles	
3	Nigeria	Congo	Zimbabwe	Madagascar	Somalia	
Eritrea	Sierra Leone	Djibouti	7	Malawi	Sao Tome and Principe	
Ethiopia	6	DRC	Burkina Faso	Mali	Sudan	
4	Algeria	Egypt	Burundi	Mauritania	Tanzania	
Chad	Angola	The Gambia	CAR	Mauritius	Togo	

Table 6: Seven Currency Groups

1	5	DRC	Libya	The Gambia	Sudan
Liberia	Sierra Leone	Egypt	Madagascar	Guinea Bissau	Swaziland
2	6	South Africa	Malawi	Guinea	Tanzania
Eritrea	Algeria	Zimbabwe	Mozambique	Mali	Tunisia
Ethiopia	Angola	7	Niger	Mauritania	Uganda
3	Benin	Burkina Faso	Seychelles	Mauritius	Zambia
Equatorial Guinea	Botswana	CAR	Sao Tome and Principe	Morocco	Zanzibar
Rwanda	Cameroon	Cote d'Ivoire	Togo	Namibia	
4	Cape Verde	Ghana	8	Nigeria	
Chad	Comoros	Kenya	Burundi	Senegal	
Gabon	Congo	Lesotho	Djibouti	Somalia	

Table 7: Eight Currency Groups

1	5	Guinea	Swaziland	Cape Verde	CAR	Seychelles
Liberia	Chad	Mali	Tanzania	Comoros	Cote d'Ivoire	Sao Tome
2	Gabon	Mauritania	Uganda	Congo	Ghana	Togo
Sierra Leone	6	Mauritius	Zambia	Egypt	Kenya	
3	Angola	Morocco	Zanzibar	The Gambia	Lesotho	
Equatorial Guinea	DRC	Namibia	8	South Africa	Libya	
Rwanda	7	Nigeria	Algeria	Tunisia	Madagascar	
4	Burundi	Senegal	Benin	Zimbabwe	Malawi	
Eritrea	Djibouti	Somalia	Botswana	9	Mozambique	
Ethiopia	Guinea Bissau	Sudan	Cameroon	Burkina Faso	Niger	

Table 8: Nine Currency Groups

APPENDIX B: GHOSH-WOLF (1994) MODEL

Output for country i at time t :

$$Q_t^i = (L_t^i)^\beta e^{\theta_t^i} \quad (1)$$

Marginal Revenue Product for Labour:

$$MRP_L = p_t^i \frac{\partial Q_t^i}{\partial L_t^i} = p_t^i \beta (L_t^i)^{\beta-1} e^{\theta_t^i} \quad (2)$$

Under Perfection Competition:

$$w_t^i = MRP_L = p_t^i \beta (L_t^i)^{\beta-1} e^{\theta_t^i}$$

(3)

Taking Logarithms and rearranging:

$$\log w_t^i - \log p_t^i = \log \beta + (\beta - 1) \log L_t^i + \theta_t^i$$

(4)

Assumed wage-setting equation at time $t-1$ (assuming labour market equilibrium) is:

$$\log w_t^i - E_{t-1} \log p_t^i = \log \beta + (\beta - 1) \log \bar{L} + E_{t-1} \theta_t^i$$

(5)

Assuming wage rigidity (i.e. not flexible downwards) a negative shock (i.e. $\theta_t^i > E_{t-1} \theta_t^i$)

causes the quantity of labour demanded at time t to deviate from full employment level

and thus satisfy the following equation:

$$\log w_t^i - \log p_t^i = \log \beta + (\beta - 1) \log L_t^i + \theta_t^i$$

(6)

Making $\log w_t^i$ the subject of the formula in (5), substituting into (6) and rearranging

gives:

$$E_{t-1} \log p_t^i - \log p_t^i = \theta_t^i - E_{t-1} \theta_t^i + (\beta - 1) \log(L_t^i / \bar{L})$$

(7)

To maintain full employment [i.e. $L_t^i = \bar{L} \Rightarrow \log(L_t^i / \bar{L}) = \log(1) = 0$] we have that:

$$E_{t-1} \log p_t^i - \log p_t^i = \theta_t^i - E_{t-1} \theta_t^i = -\varepsilon_t^i \quad (8)$$

To maintain full employment in the country the currency must depreciate by a margin that is commensurate with the size of the shock experienced to cut real wages. When a shock occurs with an asymmetric impact on two countries, i and j , in a monetary union the depreciation required for i is different from that required for j to ensure full adjustment and maintenance of full employment in each individual country i.e. $\varepsilon_t^i \neq \varepsilon_t^j$. The common monetary authority strikes a balance by depreciating the common currency by a margin that lies somewhere between the magnitudes of the two individual shock impacts ε_t^i and ε_t^j . This can be done by taking a GDP-weighted average of the two shock impacts $\bar{\varepsilon}_t$. The cost of monetary unions in this regard lies in the fact that the measure taken to depreciate the common currency will be insufficient for the country with the high shock impact and excessive for the country with the low shock impact.

The depreciation of the common currency $-\bar{\varepsilon}_t$ is given by:

$$E_{t-1} \log p_t^i - \log p_t^i = -\bar{\varepsilon}_t \quad (9)$$

The individual shock impact is still given by:

$$\theta_t^i - E_{t-1} \theta_t^i = -\varepsilon_t^i \quad (10)$$

Substituting (9) and (10) into (7) and rearranging gives the labour demand after the depreciation of the common currency by the regional central bank:

$$L_t^i = \bar{L} e^{(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} \quad (11)$$

Outside of the monetary union country i would attain reach full-employment level because it can depreciate to the full extent of the country-specific shock impact so its output would be given by:

$$Q_t^i = e^{\theta_t^i} (\bar{L})^\beta \quad (12)$$

We can obtain the output for country i in a monetary union using result (11) as follows:

$$\hat{Q}_t^i = e^{\theta_t^i} (L_t^i)^\beta = e^{\theta_t^i} \left(\bar{L} e^{(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} \right)^\beta = e^{\theta_t^i} (\bar{L})^\beta \cdot e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} = Q_t^i e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} \quad (13)$$

Thus the output loss suffered as a result of being in the monetary union is calculated as the difference:

$$L_t^i = Q_t^i - \hat{Q}_t^i = Q_t^i - Q_t^i e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} = Q_t^i \left(e^{\beta(\varepsilon_t^i - \bar{\varepsilon}_t)/(1-\beta)} \right) \quad (14)$$

If $\varepsilon_t^i > \bar{\varepsilon}_t$

The total cost of forming the currency unions is given by:

$$L = \sum_{i=1}^n L_t^i \quad (15)$$

APPENDIX C: MATLAB CODE

whileloopfn

```
function [xopt] = whileloopfn(num,k,lim,data,carry)
%
%WHILELOOPFN runs the genetic algorithm a specified number of times for
a
%specified number of countries,currency groups, dataset and initial
vector
%
% Syntax:
%
% [xopt] = whileloopfn(num,k,lim,data,carry)
%
% Description:
%
% runs the genetic algorithm a specified number of times for a
% specified number of countries,currency groups, dataset and initial
vector
%
% Input Arguments:
%
% num - the total number of countries in the group under investigation
%
% k - number of currency groups
%
% lim - the number of times that an optimal vector of allocations must
% remain unchanged before the algorithm terminates
%
% data - the data. Annual time series logGDP data.
%
% carry = the starting vector for the algorithm. It can be used for
% continuity so that a previously discovered optimal vector is included
in
% the new initial population. at the beginning it is specified as a one
% currency union arrangement i.e. all ones
%
% Output Arguments:
%
% xopt = the best vector for the parameters set.
%
%
% we begin by initializing the counting index that will halt the loop
when
% it exceed lim
index=0;
%
% we execute the beginfn function that creates a population of 26
candidate
% vector solutions of size (1x26)
[pop]=beginfn(num,k,carry);
%
```

```

while index<lim
% the for loop below calculates the loss value of each candidate vector
and
% assigns that value to the last column
    for t=1:26
        x=pop(t,1:num+1);
        [pop(t,num+1)]=fitnessfn(x,k,data);
    end
%
% the candidate vectors are arranged by their loss values with the
best
% vector (i.e. the one with the lowest loss) at the bottom
    popranked=sortrows(pop,num+1);
%
% we assign the best vector for the current generation to xopt to save
it.
    xopt=popranked(26,:);
% we generate a vector of the distinct groups in the optimal vector
    check = unique(popranked(26,1:num));
% we check the size of this vector because we want to ensure that the
% number of unions is actually equal to k instead of some smaller
number
    dim = size(check);
% the if loop below ensures that the current best vector only replaces
the
% running best vector if it has a better fitness values and has the
right
% number of groups
    if (popranked(26,num+1)>xopt(num+1)) && dim(2)==k
        xopt=popranked(26,:); index=0;
% if the running best vector is unchanged, the index increases to
signify
% that the running best survived another population
    else index=index+1;
    end
% the for loop below rearranges the rows of the population matrix to
put
% the best vectors on top.
    for t=1:26
        descendranked(t,:)=popranked(27-t,:);
    end
% the for loop below recalibrates the fitness values as positive
numbers
% so that we can implement probabilistic selection of each vector into
the
% next generation depending on the loss value
    for t=1:26
        descendranked(t,num+1)=1000+descendranked(t,num+1);
    end
% we run the selection operator
    pop=roulettefn(descendranked,k,num);
% we run the crossover operator
    for t=1:2:25

```

```

        x1=pop(t,1:num+1);
        x2=pop(t+1,1:num+1);
        [pop(t,1:num+1),pop(t+1,1:num+1)]=crossfn(x1,x2,0.99,num);
    end
% we run the mutation operator
    for t=1:26
        x3=pop(t,1:num+1);
        pop(t,1:num+1)=mutatefn(x3,0.99,k,num);
    end
% we include the running best vector as a member of the next generation
of
% candidate solution vectors.
    pop(26,:)=xopt;
end

```

beginfn

```

function [pop]=beginfn(num,k,carryvect)
%BEGINFN Creates the initial population of candidate row vectors
randomly
%
% Syntax:
%
% [pop]=beginfn(num,k,carryvect)
%
% Description:
%
% Creates the initial population of candidate row vectors randomly
%
% Input Arguments:
%
% carryvect - the starting vector for the algorithm. It can be used for
% continuity so that a previously discovered optimal vector is included
in
% the new initial population. at the beginning it is specified as a one
% currency union arrangement i.e. all ones
%
% k - number of currency groups
%
% num - the total number of countries in the group under investigation
%
% Output Arguments:
%
% pop - initial population of candidate row vectors randomly generated
pop = round(rand(25,num+1)*(k-1))+ones(25,num+1);
pop = [pop; carryvect];
end

```

fitnessfn

```
function [loss, count] = fitnessfn(x,r,data)
%FITNESSFN Calculates the fitness of a (1x54) vector of currency groups
%allocations
%
% Syntax:
%
% [loss, count] = fitnessfn(x,r,data)
%
% Description:
%
% Calculates the fitness of a (1x54) vector of currency groups
% allocation
%
% Input Arguments:
%
% x - a (1x54) vector of currency groups allocations
%
% r - number of currency groups
%
% data - the data. Annual time series logGDP data.
%
% Output Arguments:
%
% loss - output loss of the currency group allocation vector
%
% count - this is the actual number of currency groups implied by the
% vector. It can happen that a randomly generated vector has fewer
% currency
% groups than specified in beginfn. We use this index to ensure that
% the
% vector has the required number of groups.
loss = 0;
count = 0;
for m = 1:r
    Mat_m = [];
    for n = 1:54
        if x(n) == m
            Mat_m = [Mat_m data(:,n)];
            map(n) = 1;
        else
            Mat_m = Mat_m;
            map(n) = 0;
        end
    end
end
check = isempty(Mat_m);
if check == 0
    count = count + 1;
    [T_m C_m] = hpfilter(Mat_m, 100);
    Err_m = C_m;
    Mrow_tots = sum(Mat_m, 2);
    Pr_m = Mat_m.*Err_m;
```

```

Prrow_tots = sum(Pr_m,2);
w_Err_m = Prrow_tots./Mrow_tots;
row_col = size(Mat_m);
col = row_col(2);
err_diff_m = Err_m - repmat(w_Err_m,1,col);
Ones_m = ones(40,col);
Exp_m = exp(err_diff_m*2);
Temp_m = Ones_m - Exp_m;
loss_m = sum(sum(Mat_m.*Temp_m));
loss = loss + loss_m;
check = 1;
end
end
end

```

roulettefn

```

function [newpop] = roulettefn(oldpop,u,num)
%ROULETTEFN For proportional selection of candidate solutions based on
%fitness value
%
% Syntax:
%
% [newpop] = roulettefn(oldpop,u,num)
%
% Description:
%
% Selects each member of the old population into the new population
with a
% probability directly proportional to its fitness value. It adds new
% randomly created solutions to the selected ones from the old
populations
% to maintain a constant population size
%
% Input Arguments:
%
% oldpop - a matrix made up of 26 candidate solutions as row vectors
%
% u - number of currency groups
%
% num - the total number of countries in the group under investigation
%
% Output Arguments:
%
% newpop - a new matrix made up of 26 candidate row vectors. Some of
the
% vectors are the fitter ones from oldpop and the rest are randomly
% generated ones.

```

```

%we begin by calculating the total fitness of all the candidates in
oldpop

```

```

%after crossover and mutation have been implemented.
totalfit = sum(oldpop(:,num+1));

%calculate each candidate's fitness as a fraction of the total fitness
prob = oldpop(:,num+1)/totalfit;

%obtain the cumulative probabilities of the candidates in the
population
prob = cumsum(prob);

%create a vector with randomly generated elements and sort them. we
will
%use this vector to filter in candidates that make it into the new
%population
rns = sort(rand(26,1));

%now for the loop that implements the roulette wheel selection
fitin = 1; %index for the oldpop
newin = 1; %index for the newpop
newpop = []; % create empty newpop
for t = 1:26
    if (rns(newin) < prob(fitin))
        newpop(newin,:) = oldpop(fitin,:);
        newin = newin + 1;
    else
        fitin = fitin + 1;
    end
end
%we need to generate more candidate solution vectors to fill out the
extra
%rows where those from oldpop did not get selected

%count the number of rows in newpop
a = size(newpop);
rows = a(1);

%create a matrix of size
popsiz = size(oldpop,1);
append = round(ones(popsiz-rows,num+1)+ rand(popsiz-rows, num+1)*(u-
1));

%form the new population by combining the selected rows of oldpop with
the
%newly generated matrix
newpop = [newpop; append];
end

crossfn
function [chi1, chi2] = crossfn(par1, par2, pc,num)
%CROSSFN takes in two adjacent candidate vectors as parent1 and

```

```

%parent2 from the matrix pop and produces two new candidate vectors,
child1
%and child2 that are either different from the parents with probability
pc
%or identical to them with probability (1-pc)
%
% Syntax:
%
% [chi1, chi2] = crossfn(par1, par2, pc,num)
%
% Description:
%
% this function takes in two adjacent candidate vectors as parent1 and
% parent2 from the matrix pop and produces two new candidate vectors,
child1
% and child2 that are either different from the parents with
probability pc
% or identical to them with probability (1-pc)
%
% Input Arguments:
%
% par1 - a (1x55) candidate row vector
%
% par2 - a (1x55) candidate row vector adjacent to par1
%
% num - the total number of countries in the group under investigation
%
% pc - probability of cross-over
%
% Output Arguments:
%
% chi1 - a (1x55) candidate row vector created from par1 and par2 by
% exchanging portions or it is the same as par1
%
% chi2 - a (1x55) candidate row vector created from par1 and par2 by
% exchanging portions or it is the same as par2
%

if (rand < pc)

%randomly generating a cutoff point
cpoint = round(rand*num)+1;

%swapping portions of parent1 and parent2 to make child1 and child2
chi1 = [par1(:,1:cpoint) par2(:,cpoint+1:num+1)];
chi2 = [par2(:,1:cpoint) par1(:,cpoint+1:num+1)];

else
chi1 = par1;
chi2 = par2;
end

```

end

mutatefn

```
function [child] = mutatefn(parent, pm, w,num)
%MUTATEFN mutatefn switches the currency union of one country in the
%vector of allocations to another randomly chosen one with probability
pm
%
% Syntax:
%
% [child] = mutatefn(parent, pm, w,num)
%
% Description:
%
% switches the currency union of one country in the vector parent
% to another randomly chosen one with probability pm
%
% Input Arguments:
%
% parent - a (1x55) row vector of currency group allocations for 54
% countries and the corresponding fitness value
%
% pm - probability of mutation
%
% w - number of currency groups
%
% num - the total number of countries in the group under investigation
%
% Output Arguments:
%
% child - a new (1x55) row vector after mutation (possibly unchanged)

if (rand < pm)
    mpoint = round(rand*num)+1;
    child = parent;
    child(mpoint) = round(rand*(w-1))+1;
else
    child = parent;
end
end
```

decipherfn

```
function [africa] = decipherfn(optimal,j)
%DECIPHERFN displays the currency group members for a given (1x55)
%candidate row vector
%
% Syntax:
%
```

```

% [africa] = decipherfn(optimal,j)
%
% Description:
%
% displays the currency group members for a given (1x55)
% candidate row vector
%
% Input Arguments:
%
% optimal - a (1x55) candidate row vector
%
% j - number of currency groups. j is the number of currency unions as
% given by count (not by k because the one specified may differ from
% the
% one that actually results from the random generation of the pop
% matrix
%
% Output Arguments:
%
% africa - the names of countries grouped in their respective currency
% groups

countries = ['dza'; 'ago'; 'ben'; 'bwa'; 'bfa'; 'bdi'; 'cmr';
'cpv'; 'caf'; 'tcd'; 'civ'; 'com'; 'cog'; 'dji'; 'zar'; 'egy';
'gnq'; 'eri'; 'eth'; 'gab'; 'gmb'; 'gha'; 'gnb'; 'gin'; 'ken';
'lso'; 'lbr'; 'lby'; 'mdg'; 'mwi'; 'mli'; 'mrt'; 'mus'; 'mar';
'moz'; 'nam'; 'ner'; 'nga'; 'rwa'; 'zaf'; 'sen'; 'syc'; 'sle';
'som'; 'stp'; 'sdn'; 'swz'; 'tza'; 'tgo'; 'tun'; 'uga'; 'zmb';
'znz'; 'zwe'];
africa = cell(1,j);%create a cell to save the country groups of
different
                %sizes

for m = 1:j
union = []; %initialise vector containing names of countries in union m
count = 0;

c1 = [' '];

disp(c1);
    for n = 1:54
        if optimal(n) == m
            count = count + 1;
            union = [union; countries(n,:)];%column vector of union
members
        else
            end
        end
        disp(union);

        africa{1,m} = union;

end

```



```

while colsunas >= 1

    % count the number of elements in assigned
    dimas = size(assigned);
    colsas = dimas(2);

    % choose a random number in the range (1:no. of rows of
assigned)
    initial = round(rand*(colsas-1)) + 1;

    % the randomly chosen, already assigned country is in position
    % given by
    seedcountry = assigned(initial);
    % display(seedcountry);

    % create a vector to identify the countries that are neighbours
    % with the seedcountry by using the border matrix

    % initialise the vector of neighbours
    neighbours = [];

    % go through the vector of unassigned countries to identify the
    % neighbours of the seedcountry that are as yet unassigned
    for l = 1:colsunas
        tempvar = unassigned(l);
        if (border(l,seedcountry)==1) && (candidate(tempvar)==0)
            neighbours = [neighbours l];
        else
            end
        end
    end
    % display(neighbours);

    % choose one of the countries in the neighbours vector randomly
and
    % put it in the same group as the seedcountry. Do this by
    % generating a number in the range (1:no of cols of neighbours
    % vector)

    % count the number of elements in neighbours
    dimne = size(neighbours);
    colsne = dimne(2);

    % choosing a random position in the neighbour vector
    newmemberposition = round(rand*(colsne-1)) + 1;

    % assign the group number to the new member of the group in the
    % candidate vector
    index = neighbours(newmemberposition);
    candidate(index) = candidate(seedcountry);

```

```

        % identify the position in unassigned that contains the new
member
        % initialise the holding variable
        eliminatecol = 0;
        for m = 1:colsunas
            if unassigned(m)==index
                eliminatecol = m;
            else
                end
            end
        display(m)
        display(eliminatecol);
        % reduce the size of the unassigned vector by removing the
country
        % that was given a group
        assigned = [assigned index];
        unassigned(eliminatecol) = [];

        % count the number of elements in unassigned
        dimunas = size(unassigned);
        colsunas = dimunas(2);

    end

    % append the candidate vector to pop
    pop = [pop; candidate];

end

end

```

