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UNIVERSITY OF CAPE TOWN

FACULTY OF EDUCATION

**AN ANALYSIS OF THE IMPACT OF A
GEOMETRY COURSE ON PRE-SERVICE TEACHERS
UNDERSTANDING OF GEOMETRY**

A minor dissertation presented in partial fulfilment
of the requirements for the Degree of

MASTER OF EDUCATION

by

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MARCH 1999

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DECLARATION

I hereby declare that the whole of this thesis, unless specifically indicated to the contrary in the text, is my own original work and that it has not been submitted for any degree in any other university.

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ABSTRACT

This dissertation examines the impact of a geometry course on pre-service teachers levels of understanding of geometry. It is located within the Van Hiele model of geometric development which provides a conceptual framework to assess and analyse the progress of students.

The study was conducted at a College of Education which prepares teachers for primary school teaching. It involved 26 second year, pre-service teachers over a 9 week period in a geometry course. The students were assessed for their levels of understanding before and after the course using a diagnostic instrument developed by Mayberry (1981) to assess the Van Hiele levels of pre-service teachers. An in-depth investigation of 8 students provided further insight into students' levels of understanding through course work and assignments.

The overall findings of the study revealed that the majority of students had low levels of understanding of geometry before and after the course. However, those who had taken high school mathematics performed better than those without, although few managed to reach the higher levels.

The results highlight the need for teachers to develop higher levels of understanding before being able to teach and design activities that are appropriate for learners. Little improvement in performance of learners on national and international competitions will occur while teachers continue to register low levels of understanding. It is crucial that time, resources and training are provided to all teachers if effective change is to occur in the mathematics classroom.

ACKNOWLEDGEMENTS

I would like to thank all those who made this research possible. In particular, I would like to thank the following:

My supervisor, Professor Chris Breen for his calm, reassuring and guided support throughout this past year.

The Rector, vice-rector and students at the Cape Town College of Education without whom the study would not have been possible.

Friends and colleagues who gave valuable input in terms of suggested improvements, proof-reading, editing and printing: Cheryl Reeves, Hanlie Murray, Pippa Campbell, Chris Dumas, Wendy Peck and the computer and printing department at Cape Town College of Education.

The Centre of Science Development for financial assistance. Opinions and conclusions expressed in the study are those of the author and are not necessarily to be attributed to the Centre for Science Development.

Finally and most importantly to Shawn, family and close friends who saw me through the good and the bad, and never failed with their generous support. Thank you very much for everything, without you, it would never have been possible.

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CHAPTER ONE: INTRODUCTION TO THE STUDY

1.1 BACKGROUND AND AIM OF THE STUDY

The changing face of education in South Africa has generated avid debate over the past year. The needs of a country pledged and committed to reform and economic growth has meant that the government has had to embark on a radical transformation of the education system to meet the demands of competing internationally. The legacy of the past and the policies associated with it have resulted in many young people leaving school without the knowledge and skills required to function effectively in the work place. As education tries to adapt to the wants of the economy and the demands of the workplace, on-going poor performance at school, especially in mathematics and science, does little to improve the potential of our country competing in the global market. If we want to succeed as a global player in a rapidly changing competitive world, then we have to prioritise the place of science and technology in education. Programme changes, initiatives and reforms must be implemented in science and mathematics education as soon as possible (Howie, 1997).

The poor quality of mathematics teaching and learning in South Africa is clearly reflected in the Grade 12 results in mathematics:

In 1997, less than half of all full-time Standard 10 (Grade 12) candidates were mathematics candidates. Of the 252 617 students, who wrote mathematics examinations, only 22 798 passed on the higher grade (Garson, 1998: 4).

Similarly, the poor results of our students in the Third International Mathematics and Science Study (TIMSS), in which 15 000 South African primary and high school pupils participated, did little to allay fears that our current system of mathematics and science education is failing our students. Although numerous questions have been raised about the TIMSS study, it does provide 'the country with valuable information that will enable policy makers to ascertain quantitatively and relatively objectively, for the first time the status quo in science and mathematics education in South Africa' (Howie, 1997: 60). The findings act as a warning mechanism of the effectiveness of our current teaching and learning program in mathematics education in South Africa. They highlight a lack of the

high level skills called for in commerce, science and engineering and 'crucial for any country's success' (Garson, 1998: 4).

The poor level of mathematics teaching practised in our schools is reflected in our weak performance in all aspects of the TIMMS items. It points to problems in the current system and a need to address these issues. Wedepohl (in Howie, 1997: 52-61) summarises the findings of a number of studies which have looked at education, including mathematics and science education in South African schools, more especially in previously disadvantaged communities. He identifies the following as possible roots of the problems:

1. Home environment: the majority of students come from the poorer socio-economic backgrounds. There are poor literacy rates amongst parents, which means that they often cannot help their children with their homework. Malnutrition which affects a large percentage of school going children, and contributes to poor levels of concentration in class.
2. General school environment: many schools have inadequate facilities ranging from a lack of running water to a shortage of reading and writing material. Classrooms are often over-crowded and, there is often weak leadership and poor attendance at schools. The legacy of the past as Essop (in Howie, 1997: 53) points out has left schools with disruption and 'malaise', which prevent people from moving forward, 'now that political liberation has been achieved, the culture of learning still has to be normalised'.
3. Peer environment: students who do well at school are not given enough support to study further. Equality of opportunity was denied to many of our students for far too long with the result that students are often not comfortable if they do better than their peers.
4. Gender: there is little significant encouragement for girls to enter traditional male domains, moreover the burden of housework often falls heavily on young women.

5. Homework: learners appear to spend less time on homework than their international counter-parts. Teachers need to motivate students by highlighting the positive improvements that can result from doing homework.
6. Language of instruction: most children continue to be instructed in a second or third language, which makes communication and conceptual understanding difficult, especially in mathematics and science. Teachers need to take extra care by making sure that students understand essential concepts.
7. Curricula: many of our courses continue to have a heavy content focus. Curriculum 2005 is trying to address this, but our teaching and assessment methods need to change to meet the demands of such a curriculum.
8. Student motivation: students perceive mathematics and science to be subjects which are difficult to pass. They are often not expected to achieve in these subjects and this has a direct influence on their motivation. Finally :
9. The quality of teachers and teacher training: it is a well-established fact that the current retrenchment package offered by the Department of Education has meant that many skilled mathematics and science teachers have left teaching. The result is that poorly or unqualified teachers have taken over teaching these teachers' classes. Poor content knowledge and lack of motivation of teachers can have a detrimental effect on the learning process.

The aim of this research is to focus on the quality of teachers and teacher training, the final point mentioned above. The need for a thorough investigation of the quality of program offered in teacher education has been an on-going concern of many of the academic institutions offering courses. Colleges of Education have a crucial role to play in training and equipping primary and secondary teachers to function effectively in the classroom. Consequently, it is critical that we examine the practices of colleges and monitor the progress of students. The National Teacher Education Audit (Hofmeyr & Hall, 1995) of the overall effectiveness of the Colleges of Education in teacher training did not paint a good picture of the current state of affairs:

In many colleges, students acquire only a superficial knowledge of their teaching subjects, so much so, that INSET agencies find that they have to spend considerable time improving teachers' subject knowledge before they can introduce innovative approaches. Syllabi are dated and concentrate on rote learning (Hofmeyr & Hall, 1995: 60).

The variability of quality of teacher education is great and though there are pockets of excellence, the quality is generally weak (ibid.). If Colleges of Education hope to make the transition into higher education, they will have to examine the quality of teacher education being offered to students. This could involve the assessment of programs offered to students at colleges, the up-grading of lecturer qualifications and the internal and external appraisal of course content and lecturers.

This study is an attempt to deal with some of these issues, it examines the quality of mathematics education, offered at a College of Education and tries to evaluate the effectiveness of a geometry course on students' understanding.

1.2 RATIONALE FOR THE STUDY

1.2.1 Geometry and perceptions of teachers and learners

Geometry covers approximately 25% of the primary school syllabus and is a component of the exit examination for Matriculants. As with all areas of mathematics, it is crucial that learners have a worthwhile experience in the early stages of geometry development. Successful progress through this section of mathematics is largely dependent on the initial experience of the learner in geometry. Too often, there is too much emphasis on skills and content development and too little time spent on conceptual development in the early stages of learning (Fuys & Liebov, 1997). The approach used in many schools over the years in South Africa and elsewhere has been to emphasise and instruct according to the axiomatic approach to geometry (Van Hiele, 1986: 40). In other words, students are taught the rules and how to apply them with little real understanding of the underlying relationships that exist. It is widely accepted that this view of geometry instruction is 'unpedagogical' and results in poor student performance (Wirszup, 1976: 96). This

approach requires students to operate at a level of thinking, which is far beyond their experience and does not provide the necessary instruction for the level of the learner.

Studies carried out by researchers in South Africa (De Villiers & Njisane, 1987; Govender, 1995) indicate that Grade 12 students and high school students in general are still functioning at more concrete and visual levels in spite of the fact that the National school exit examination requires a clear understanding of the underlying processes of defining. This, coupled with the fact that transition from concrete to abstract levels of thinking poses 'specific problems to second language speakers, since it also involves the acquisition of the technical terminology', it is little wonder that our students are performing so poorly (De Villiers, 1996: 10).

The geometry instruction which our primary schools offer is inadequate in terms of providing learners with the necessary thinking skills needed to operate at the level of axiomatic thinking required for most high school geometry courses. Too little informal geometry is covered in the primary school and a loaded high school formal curriculum which lacks creativity presents a static approach to the teaching of geometry (Breen, 1997: 21). Although mathematics is compulsory for the first two years of high schools, learners who are unable to understand and simply parrot the teacher's methods, pass through these two years of formal geometry with very little relational knowledge and understanding. This reinforces the earlier misconceptions that learners may have learnt and manifest in the form of under-generalisations (for example, learners include irrelevant characteristics), over-generalisations (for example, learners omit key properties) and incorrect understanding of the language and concepts employed in geometry (for example, describe 'diagonal' as 'slanty') (Fuys & Liebov, 1997). These misconceptions may remain with them for the rest of their lives.

Teachers frequently view geometry as a topic unrelated to other areas of mathematics and teach accordingly. They over-emphasise the importance of algebra and trigonometry at the expense of geometry and see little connection between the students' experience of the

world and the development of problem solving skills (Balomenos et al, 1987). It is taught as a repetitive sequence consisting of homework - discussion - new homework, which may not be the best way to develop logical thinking ability. The teacher reinforces the cycle by over-prescribing the textbook (Suydam, 1985). There is little interaction with concrete apparatus and an over-emphasis on developing formal understandings in geometry. The sequence of the textbook is followed very closely and no mention is made of the historical development of geometry or the contribution of different cultures towards its development. The teacher's objective is to complete the exercise at the end of each section with little or no reference to other aspects in geometry beyond the textbook. As a result of such instruction, high school learners have been exposed to a formal, axiomatic experience of geometry far too early. They are unable to understand the level of geometry and as a result, it is disliked by most. According to Hoffer (1981: 81) 'although several subjects were 'favourites', the subject that was almost universally disliked was geometry in high school'. When learners are asked why they disliked geometry, they answered: 'Had to prove theorems all year long'; 'Didn't understand what it was all about'; 'Got through the course by memorising proofs' and even 'We did more theorems than geometry' (ibid.).

Teachers are unaware of the difficulties learners have in learning geometry as a consequence of their instruction. The approach used in geometry is different to the approach used in other mathematics courses; because it requires the learner to comprehend the 'workings of a mathematical system at the same time' as 'learning the content of the course' (Usiskin, 1982: 1). Developing a conceptual language with learners is seldom addressed which could make the content become abstract and inaccessible for learners who are unfamiliar with the terminology.

Teachers' use of the visual aspects of geometry as a tool for teaching proof favours only one hemisphere of the brain (Ornstein, 1975 and Wheatley, 1977 in Hoffer, 1981: 18). Recent research concerning the two hemispheres of the brain indicates the left hemisphere has to do with 'logical and analytic functions' while the right hemisphere is concerned with

'spatial and holistic functions' (ibid.). Quite clearly for learners to have a worthwhile learning experience, both sides of the brain need to be stimulated.

Piaget (1955 in Wirszup, 1976: 76) maintains that geometry instruction begins too late and when it is eventually taught moves from a measurement (quantitative) position to the recognition of shape. As a result teachers ignore the 'qualitative phase of transforming spatial operations into logical ones' because they approach geometry instruction through the historical development of the subject. They move from the 'geometry of measurement' to the 'geometry of shape' and from the 'geometry of position' to the 'theoretical geometry' which is contrary to how children understand and learn geometry i.e. from the 'qualitative to the quantitative' (ibid.). This means that the 'child's development of geometric operations proceeds in the opposite direction' which is in opposition to the cognitive development of the child (ibid.).

1.2.2 Geometry and Teacher training

Research with pre-service teachers in America indicates that most teachers have low levels of understanding in geometry (Mayberry, 1983; Burger, 1992). A more recent research report done in South Africa (Mpofana in Ndaba, 1998) investigated 'aspects of pre-service and in-service training of mathematics teachers in Kwa-Zulu' and found that most first year student teachers had a poor perception of geometry. The study revealed that only 28.2% of students liked geometry while 75% found it difficult and problematic unlike algebra which recorded a 76.9% enjoyment aspect and 33.6% poor or negative attitudes. These results indicate that students have a negative attitude towards the subject, which may be a result of bad experiences at school. Unfortunately, this attitude tends to be carried over into the classroom situation perpetuating similar attitudes in learners. Very little will change in terms of geometry teaching within our schools unless we address the factors that contribute towards this attitude.

While attitude towards a subject is clearly an influential factor in instruction, the teacher's content knowledge of the subject is fundamental for effective teaching. It is very difficult for teachers to design appropriate learning experiences and to get the most from learners if they themselves are not competent in the knowledge, skills and processes necessary to teach the subject effectively. Teachers need to know more geometry than they are expected to teach and this can only happen if tertiary institutions include more geometry in their under-graduate courses. This is something, which is not being done in most institutions in this country at the moment (De Villiers, 1996; Breen, 1997).

Colleges of education who are primarily responsible for primary school teacher training have to reform and redesign their curriculum to include more geometry courses based on sound pedagogical perspectives. Laridon (1993: 44) suggests:

... any new curriculum is naturally doomed to failure unless the teachers who are going to implement it are well informed about the new directions to be taken both in content and approach

Teachers need to have a 'repertoire of skills and strategies to use in different situations and with different students' (Hofmeyr & Hall, 1995: 3). This may include an exploratory and discovery approach to teaching which acknowledges children's informal and intuitive methods to help them (re)construct geometry knowledge. It could involve the teaching of geometry from a dynamic approach greatly assisted by the use of concrete apparatus and technology in the classroom. There needs to be greater exploration of the potential and appropriateness of different teaching methodologies but the prescription of teaching methods should be avoided (Breen, 1997). In other words, we do not need to prescribe to teachers how things should be done as if there was only one true way to act but rather encourage teachers to apply the knowledge they have, in meeting the needs of their learners.

To make geometry relevant, truly contextualised and meaningful for a child means that the teacher needs to know her learners and be aware of how geometry development happens.

If she ignores any of these, she runs the risk of hampering the development of her learners by not supplying the necessary mathematical knowledge, skills and processes needed for further study in geometry.

1.3 PURPOSE OF THE STUDY

The aim of this research was to examine the impact of a geometry course on pre-service teachers' levels of understanding of geometry, at a College of Education for primary school teachers. It involved the assessment of students' levels of understanding prior to taking the course.

Students were then taken through a course, which included interactive tasks and relating geometry to everyday life and to the South African context. It introduced students to the language of geometry, to visualisation tasks, to a hands-on experience of geometry and to current research in the field. The course tried to emphasise the need for reform in mathematics teaching and learning:

...in the classroom, where students and mathematics come together, the mathematics must be worthwhile, the instructional strategies must be effective in promoting the students' learning (Lappan, 1998: 3)

The study concluded with the re-evaluation of students' understanding to determine if there had been changes in their levels of understanding.

1.4 OUTLINE OF CONTENTS

The report has been structured in the following way:

Chapter One contextualises the study by providing a background and brief discussion of the current state of mathematics and science education in South Africa. The study is located in the field of mathematics education, more especially in the teaching of geometry. It outlines the position of geometry in school mathematics, teacher and learner perceptions of geometry and the role of Colleges of Education in preparing competent and knowledgeable teachers of mathematics. The chapter concludes with the purpose of the study, which is to examine the impact of a geometry course taken at a College of Education on pre-service teachers' levels of understanding.

Chapter Two describes a brief history of teachers' perceptions of geometry instruction and how geometry instruction had evolved in a number of different countries. In particular, the chapter describes the Van Hiele model of development of geometric thought, widely accepted as a leading theory in describing how learners come to understand geometry.

Chapter Three describes the methodology of the study and defines more specifically the focus of the research. It explains how the research questions were addressed for the purpose of the study and explains the data collection process used to gather the necessary information.

Chapter Four provides an analysis of the impact of the course on students' levels of understanding. It examines the appropriateness of the Van Hiele model as a descriptor of student progress and assesses the potential of students to integrate the theoretical components of the course to select suitable learning activities.

Chapter Five restates the aim of the study and describes the limitations of the study in terms of methods. The chapter draws conclusions by relating the research findings to the research questions and tries to highlight the principle implications of the findings. The implications are used to make tentative recommendations for teacher education and future research based on these findings and the experiences of the researcher.

CHAPTER TWO: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews the literature relevant to the impact of a geometry course on pre-service teachers' levels of understanding. It provides an overall perspective of the relevance of geometry in mathematics education and considers the role of instruction. It links geometry and teaching to Van Hiele's theory which proposes that geometric thought develops in a series of five levels which are 'distinguished by characteristics of the thinking process and not merely by the acquisition of geometric knowledge' (Mayberry, 1981: 1).

2.2 GEOMETRY

There is a perception that primary school mathematics concentrates only on arithmetic and excludes other areas of mathematics. However, it is often forgotten that geometry is the child's first experience of mathematics that is the familiarisation with the physical environment. As the child orients her/himself in the real world, s/he develops a consciousness of space, which is only verbalised much later. This is in contrast to number development which starts as a 'language of counting and eventually leads to the counting of things' (Van Niekerk, 1995: 7). Although geometry and arithmetic understanding may not evolve in the same way, it does not mean that one should be privileged over the other.

Geometry should not be viewed as geometry as such but rather as the way in which we look at the world around us (ibid.). Geometry helps us to make sense of everyday situations and see the connectedness between other areas of mathematics and other school subjects (Lindquist & Shulte, 1987). As we recognise that spatial development often develops faster for younger children than numerical skills, this can be capitalised on by the teacher to cultivate a life-long interest in mathematics (Ortiz: 1994). It begins with the physical environment of the child, which helps form the mental images that can be verbalised and later represented in two dimensions. Hence a primary school curriculum should start with the real world of the child in their first year of schooling. It should

capitalise on the 'intuitive notions that children reveal when exposed to spatial situations' (van Niekerk, 1995: 7). These intuitive notions need to be reflected upon only when the experience and the context make sense for the child. It is crucial that children have a basic knowledge of geometry to 'interact effectively with their environment as well as for them to enter formal study of geometry' (Hershkowitz et al, 1987). Research (Usiskin, 1982 & Hoffer, 1983) indicates that if the child does not develop the necessary fundamental knowledge and skills in geometry, it can lead to misconceptions being formed and may lead to difficulties and poor performances in geometry later.

Learning and teaching geometry help us to recognise the importance of geometry as a deductive system which leads to a better understanding of 'logical reasoning and its relation to axioms, theorems and proofs as illustrated by Euclidean geometry' (Balomenos et al, 1987: 195). It helps develop reasoning abilities which can help the child to analyse the 'form of an argument and to recognise valid and invalid arguments in the context of geometric figures' (Ortiz, 1994: 231).

Geometry is useful in the development of other topics in mathematics and is 'especially suited for providing calculus readiness' and 'spatial visualisation' (Hershkowitz et al, 1987: 209). Many aspects of calculus depend on a well developed knowledge of geometry, students are frequently required to make pictorial representations, apply geometry knowledge such as Pythagoras's theorem, similar triangles, applying area and volume concepts, and solving problems in calculus. There is evidence to suggest that students perform poorly in calculus because they have weak understandings of geometry. It is very difficult for students to translate complicated verbal descriptions of a problem into a visual form if they do not have the necessary background. Solving problems of optimisation and related-rate requires students to apply their visual knowledge as well as their knowledge of calculus. If the fundamentals of either are not intact then the student is prone to make incorrect or blind application with disastrous results.

2.3 GEOMETRY TEACHING

There is consensus amongst mathematics educators that students need to develop more visualisation skills at an early stage in their development if they are to be successful in geometry in the future. Students are frequently introduced to formal geometry too soon without allowing sufficient time for exploration, investigation of properties and the gradual introduction of appropriate vocabulary (De Villiers, 1996). Teachers avoid informal exploration because of the time involved. The static nature of pencil and paper exploration does not always help students to visualise how the shape might change (ibid.). Fortunately the development of technology has helped to transform geometry teaching from a static domain to a more dynamic and interactive learning environment. Computer packages are designed with 'the specific intention of putting at the disposal of the student a micro-world type environment for the experimental exploration of elementary plane geometry' (ibid.). Through exploration and investigation, students are introduced early on to the art of problem posing which requires them to conjecture, refute, reformulate and explain their actions and decisions. The major drawback to this new environment is the lack of finance and resources needed to equip schools to teach, through using technology. Although alternative and more appropriate methods for teaching geometry are available, they are not accessible to the majority of the population. This forces us to try to find innovative ways to use available resources. A closer look at the Soviet Union curriculum, which incorporates the use of concrete materials, could contribute to the content and process reform needed to bring about change in geometry thinking in our schools.

Some thirty years ago, Soviet dissatisfaction with the poor performance of students in geometry initiated major research into the area of geometric development and instruction. Though much of the Soviet research took place in the 1960's to 1970's, much of what emerged then is still relevant today. As a result of this work the following set of guidelines were drawn up for the teaching and content of grade 1-3 geometry:

- there must be emphasis on qualitative geometric operations (study of shapes, mutual positions, relations, et.) and measurement should be left until later;

- the study of geometry should be included from the beginning and should be continuous, in other words there should be no gaps in learning or lack of exposure to geometry over long periods of time;
- the exposure to the geometry content should be uniform and not restricted to particular work at a set time. This also depends on the level at which the learner is operating. In the early stages of development, the work is integrated with other mathematical concepts but as the child develops geometrically, specific time must be allocated to geometry teaching to nurture higher level thinking;
- there should be a diversity of exposure to 2-D and 3-D geometry as the two are interrelated and not seen as separate by learners. The fault with much of current practice is that we present the topic as distinct separate parts when learners see them as interrelated;
- there must be an 'organic connection' between the development of geometric and arithmetic thinking;
- learners need to be familiar with all the concepts to be studied in geometry over the 9 years of schooling, from the beginning. It does not mean that they have to be able to deduce the properties at an early stage but that their intuitive understandings have to be capitalised on;
- there needs to be many opportunities for interaction with concrete material which can help learners with spatial perception and development;
- the learners' understanding of the real world needs to be linked to their geometry learning and should include other disciplines where possible;
- although early geometry development concentrates on the holistic view of shape and space, the study of relations between figures and logical development needs to be highlighted so that all learners can operate successfully on other levels;
- the exposure to materials that help develop geometrical thinking must also lead to the use of the appropriate vocabulary and the correct terminology needed to operate at a particular level.

Interestingly, the Japanese have also begun to introduce much more geometry at an early age with the result that they are showing better performance rates in international competitions (De Villiers, 1996). They see the intention of geometry teaching to help learners to:

develop their abilities and attitudes to mathematically consider their daily-life problems, to think logically and to solve them, acquiring the fundamental knowledge and skills regarding geometrical figures (Nohda, 1992).

As a result of research with students, Shaughnessy & Burger (1985) also support the need to introduce informal geometry much earlier on in school and recommends that it be seen as a basic skill, as is arithmetic. It should be developed parallel to development of number. If geometry is approached in this way, it will have as high a pay-off for problem solving development as does arithmetic computation.

Another aspect of geometry teaching, which needs reform, is the nature of the course. The content of geometry courses needs to be expanded beyond the common understanding that we have of it now. It needs to include other approaches to Euclidean geometry such as vectors and transformation geometry as well as non-Euclidean geometries such as fractals and spherical geometry. The extended use of transformation geometry can also provide the 'conceptual structures' needed for formal activities at the next level (ibid.). It is inappropriate to have students measure the base angles of an isosceles triangle to 'discover' that base angles of isosceles triangles are equal. It is better to allow the student to fold the base angles on top of one another to help build the necessary 'conceptual structure' which can be treated later in a more formal deductive context.

It is also important, as Van Niekerk (1995) suggests, that spatial orientation which can lead to spatial insight, be nurtured and developed in the early stages. Students in the primary school need to solve complicated real world problems. This can involve some link with other disciplines and an emphasis on the development of intuitive understandings of

the processes involved in geometry. Geometry can no longer be presented as a ready-made product that needs to be memorised and regurgitated:

Rather than giving the child the opportunity to organise spatial experiences, the subject matter is offered as a pre-organised structure. All concepts, definitions, and deductions are preconceived by the teacher, who knows what is its use in every detail - or rather by the textbook author who has carefully built all his secrets into the structure (Freudenthal, 1973 in De Villiers, 1996: 16)

We also need to remember that there is little point in changing teaching and learning if we are not prepared to also look at our assessment methods. We cannot expect to use the same paper and pencil tests if we want to examine students thought processes in more depth.

Language can also be a stumbling block when learning geometry. As De Villiers (1996: 10) points out, it is a particular problem for second language speakers, as they have to acquire the technical terminology necessary to develop a particular level of reasoning. This problem is most significant in the transition between levels and time needs to be allocated for students to make that leap. It means that part of the curriculum content may need to be sacrificed to provide time for the necessary language acquisition.

2.4 THE VAN HIELE MODEL

The role of geometry and how it links to education needed to be guided by some theoretical framework. It was decided to use the Van Hiele model of development in geometry because its 'emphasis on developing successively higher thought levels, appears to signal direction and potential for improving the teaching of mathematics' (Fuys et al, 1988: 191). The model has been used in numerous studies e.g. Mayberry (1983); Fuys et al (1988); Senk (1985); and Shaughnessy & Burger (1985) to name but a few, to analysis student's reasoning processes on geometry tasks and is seen as a useful framework of reference (Burger, 1992). The results of research indicate that most primary school teachers are operating at low levels of conceptual development and reason geometry

holistically and inductively. Although few primary school teachers are required to teach deductive geometry, it seems appropriate for teachers in training to be able to operate at this level so that they can best inform their own teaching practice.

The Van Hiele model is useful in that it may help us to determine just how much or how little our learners understand geometry and perhaps inform us on how to teach more effectively (Pegg & Davey, 1991).

2.4.1 History of Van Hiele's work

During the 1950's while Piaget wrote extensively on the stages of development and how they relate to learning, the Van Hiele's, a Dutch husband and wife team, were completing companion doctoral dissertations at the University of Utrecht in 1957. Their work was a reflection of their teaching experiences with high school geometry students which looked at ways of helping students develop insight into geometry (Hoffer, 1983). They were concerned about the poor performances of high school students in geometry and decided to investigate further. While Dina concentrated on the teaching of geometry and how students' levels of thinking could be shifted through instruction, her husband, Pierre-Marie explicated the theory (Usiskin, 1982). He tried to explain and describe students' levels of understanding while she prescribed the content and learning activities needed to change students' insight (De Villiers, 1996). She died shortly after completing her PhD and he continued to develop the theory. He wrote three papers during 1958-59: 2 in English and one in Dutch, which was translated into French. During the 1960's, the Van Hiele work was developed in the Soviet Union and was referred to extensively in a 1963 report by A. M. Pyshkalo and used to inform their curriculum for geometry. The Soviets used the work of Van Hieles, especially Pierre's 1959 paper, to analyse student materials used in the Soviet Union in 1960 and the levels were also applied in numerous other research studies (Hoffer, 1983: 209). The Soviet studies found that most students were still at the level of recognising shapes but they could not identify the properties or see relationships at the end of grade 5 and that understanding of solids took even longer to develop. It was

also possible to shift students' levels by putting more structure to the learning experience and it was further proposed that most students have the capacity for higher-order thinking much earlier than they were being taught at school (Pyshkalo, in Hoffer, 1983).

Professor Hans Freudenthal (1973) mentor to the Van Hieles, promoted their theory in his book *Mathematics as an Educational Task* in which 'he asserts that mathematical induction actually developed along levels' (Hoffer, 1983: 210). As a result of the work in the Netherlands and the Soviet Union on the Van Hiele model, it came to the attention of Izaak Wirszup who wrote a paper in 1976 promoting the theory in the United States.

The theory was then taken and applied in a number of contexts, which revealed that many of the textbooks written for instructional purposes contain problems that are far beyond the level of the students. Informal surveys indicated that students were often on levels far lower than the teacher's expectation which meant that the teacher's instruction was not appropriate to the student's level of understanding. Many of the chapters in textbooks relating to geometry were covered superficially by the teacher or completely overlooked as teachers did not see geometry as an essential component of mathematics. As a result of numerous studies on the Van Hiele model which were carried out in the USA and elsewhere, it is regarded by some as the 'most persuasive model for research into geometry learning, teaching and curriculum' (Volmink, 1988: 82).

2.4.2 Aim of the Van Hiele model

The Van Hiele's formulated the levels in response to analysis of their own teaching of geometry and subsequently developed the model to impact on instructional practice (Fuys et al, 1988: 188). They applied the levels and phases to a teaching experiment within a classroom setting over an extended period of time (i.e., one year) to try to understand what best develops insight in students in the study of geometry (Fuys et al, 1988: 13; Schoenfeld, 1986: 230; Hoffer, 1983: 205). Their research focused on levels of thinking in geometry and the role of instruction in helping learners move from one level to the next (Fuys et al, 1988: 4)

They found that secondary school geometry involves thinking at a relatively high level before learners had had sufficient experiences in thinking at prerequisite lower levels. Their work tried to establish why many learners have difficulty with the higher order cognitive processes, particularly proof, required for success in high school geometry (Usiskin, 1982; Senk, 1985; De Villiers, 1987)

Their description of each of the levels can be used to describe characteristics of the thinking process and to guide instruction as well as to assess learners' abilities. It also provides a set of empirical guidelines for teacher - learner communication (Crowley, 1987: 1, Mayberry, 1983; Schoenfeld, 1987). It can be used as a template for textbook evaluation, which also means that we can design textbooks that can build on learners' learned knowledge that they have acquired during instruction.

Ultimately the Van Hiele model provides us with a peephole through which we can use our mathematical eye to view children's interaction with mathematics (Hoffer, 1983: 215). There are obviously questions as to whether the levels as proposed by Van Hiele actually exist but the fact that they are used as descriptive frames by reference, guarantees their existence. On the other hand, the levels should not be used to label people because the value of the Van Hiele model resides not so much in 'stratification of learner thought as in a prescription for instruction, not only in geometry but in most structured disciplines' (Hoffer, 1983: 224; Schoenfeld, 1986: 230).

2.4.3 Components of the model

There are a number of components, which are part of the Van Hiele theory (Usiskin, 1982; Volmink, 1988). They consist of the five levels of understanding, the behavioural characteristics of each level, the properties of the theory and the phases of learning. Each will be discussed in more detail below.

2.4.3.1 The five levels of understanding

Both the number and the numbering of the levels have been variable and there is empirical evidence to support the existence of a level prior to level 1. There have been several different numbering systems used for the levels. This study has adopted one system which differs from the traditional Van Hiele numbering system i.e. levels 0 - 4 and will for the sake of consistency be called levels 1 -5. This numbering has been transported to the work of other researchers in the field. The levels are described in Hoffer (1981: 20-21); Burger & Shaughnessy (1986: 31); Crowley (1987: 3), and Mayberry (1981: 47) as:

Level 1: Recognition/Visualisation. The learner learns some basic geometrical vocabulary, recognises a shape as a whole, and if given a picture can produce it. S/he reasons by means of visual considerations without explicit regard to the properties of its components. The learner can discriminate a given figure from others that look similar.

Level 2: Analysis. The learner can analyse properties of figures and necessary properties are established. Reasoning occurs by means of an informal analysis of component parts and attributes. Figures are recognised as having parts and are recognised by their parts. Arguments can be resolved by referring to the definition and accepting the implications of the definition as binding. Interrelationships between figures are not seen and definitions are not fully yet understood.

Level 3: Ordering/Informal deduction. The learner logically orders figures and understands interrelationships between figures and the importance of accurate definitions. The learner can distinguish between necessity and sufficiency of a set of properties in determining a concept. Learners can deduce properties of a figure and recognise classes of figures. The significance of deduction as a whole or the role of axioms is not fully comprehended and empirically obtained results are often used in conjunction with deduction techniques.

Level 4: Deduction. The learner understands the significance of deduction and the role of postulates, theorems, and proof. S/he can reason formally within the context of a mathematical system. The distinctions between a statement and its converse can be made. The learner can construct and not just memorise proofs and the possibility of developing a proof in more than one way can be seen.

Level 5: Rigor. The learner understands the importance of precision in dealing with foundations and interrelationships between structures. S/he can compare systems based on different axioms and can study various geometries in the absence of concrete models

2.4.3.2 Properties/characteristics of the Van Hiele levels

The theory has a number of defining characteristics (Holmes, 1995; Fuys et al, 1988: 5-6; Van Hiele, 1986; Clements & Battista, 1992: 426-427) which will be discussed further and rest on the premise that learning is a discontinuous process. Discontinuity in the learning process is construed by Van Hiele in Wirszup (1976: 79) to mean that there are jumps in the learning curve which are said to reveal the presence of levels. While you are operating at a particular level of reasoning, you use a language and sets of relations that apply to that level. You are unable to fully comprehend those on another level but once you have reached the new level, the process of thinking is more continuous. The passage from one level of thinking to another is not spontaneous; Van Hiele (1986: 63) sees the process as complicated and acknowledges that it is difficult to help a learner with this learning process. The development to higher levels of thinking proceeds under the influence of learning, which means that the content and method of instruction are the driving force behind the movement. There is no skipping of levels and it is very difficult for a learner to develop complete understanding at a higher level unless s/he has fully mastered the previous level. If the 'network of relations' is not founded on previous experience then the new networks will be forgotten in a

short time. A learner can learn to apply a network in particular contexts but if the context is changed to an unfamiliar domain and the learner cannot construct a network himself then s/he has not achieved 'optimal mathematical training' (ibid.). The development of appropriate networks requires a certain amount of time, of which a large part, in the early stages should be spent at developing level 1 and 2 understanding.

While the levels are discrete in nature, there are a number of characteristics/properties, which contribute to a more precise understanding of the levels of thought: Crowley (1987: 4) identifies the properties as sequential, advancement, intrinsic and extrinsic, linguistics, and mismatch and Usiskin (1982: 5)

1. fixed sequence property : levels are sequential and hierarchical (Hoffer, 1981). As with most developmental theories, a person must proceed through each of the levels in turn. This means that for a learner to function successfully at a particular level, s/he must have acquired the strategies of the preceding levels (Crowley, 1987: 4). When the learner does not develop these strategies, there is a discontinuity in the understanding and lack of progress ensues.

2. adjacency : intrinsic and extrinsic thought (what is implicit at one level becomes explicit at the next level, (Fuys et al: 1988: 8)) for example, at level 1, a learner identifies a shape by its appearance as a whole. The learner may realise that it has properties but is unable to analyse them until the figure is explored and its properties discovered (Crowley, 1987: 4). Concepts understood at one level become explicitly understood at the next level (Van Hiele, 1985: 246)

3. distinction : each level has its own thought processes, content and vocabulary or linguistic symbols and network of relations (Volmink, 1988; Van Hiele, 1986). Van Hiele acknowledges that it is possible to teach learners content that is above

their current level of understanding (e.g. area of a triangle). However he believes that the learner is simply memorising the information by reducing the new subject matter to a lower level and may often not understand the concept fully (Crowley, 1987). The material that is taught to learners above their operating level is subject to 'reduction of level' (Fuys et al, 1988:7), for example a lot of learners know the formula for the area of a rectangle but cannot explain how the formula was derived, hence the concept has been reduced to the level of rote and memorisation. The movement from one level to another is closely linked to the broadening of language:

A relation that is 'exact' on one level can be revealed to be 'inexact' on another.... Numerous linguistic symbols appear on two successive levels; they establish, moreover, the connection between the different levels and assume the continuity of thought in this discontinuous domain. But their meaning is different: it is shown by other relations among these symbols (Van Hiele in Wirszup, 1976: 82).

4. separation : different levels of understanding. If the learner is operating on a different level of understanding to the teacher then they will not understand one another and the learner will not develop further levels of understanding. Moreover, two people who reason at different levels cannot understand each other, since ideas are assimilated as one moves through a level which involves using differing linguistic symbols and relationships.

5. attainment : movement from one level to the next depends largely on the instruction process rather than on maturation and 'one goes through various phases of learning in proceeding from one level to the next' (Fuys et al, 1988: 7) and (Wirszup, 1976; 82). There is no 'developmental timetable' which determines when a learner should move through the levels (Schoenfeld, 1986), it depends entirely on the teaching instruction. Research shows that it is possible to remain on a particular level for a long time (Usiskin, 1982).

2.4.3.3 Phases of learning

There are five phases of learning which were developed by Dina van Hiele-Geldof in her dissertation work. They are the result of observations made during a yearlong teaching experiment, which she conducted, with 12 year olds in a school setting. They provide guidelines to the teacher on how to sequence and deliver geometry activities within a level and, if followed accordingly, will 'promote the acquisition of a level (Van Hiele-Geldof in Crowley, 1987).

- **Phase 1: Inquiry/Information.** This is understood as the conversation between the teacher and the learner in which questions are asked and observations are made by the teacher to determine prior learning. According to Van Hiele (1986: 177) the learner is also becoming familiar with the 'context of the field of study involved'. During this phase the teacher introduces the activity with the appropriate vocabulary and objects of study. This phase is known as inquiry both on behalf of the teacher to help her assess the level of the knowledge of the learner, and by the learner in becoming associated with the new context. It is alternatively known as 'information' for much of the same reasons.
- **Phase 2: Directed Orientation.** This phase is also known as 'bound orientation' as there is a careful sequencing of tasks by the teacher. It is the time in which the learner becomes aware of the 'principal connection of the network of relations to be formed' (Van Hiele, 1986: 177). The tasks are one-stepped and designed to elicit specific responses. They allow learner exploration through materials which helps the learner to make connections and come to some realisation about the new concept being taught.
- **Phase 3: Explication.** This is often referred to incorrectly as 'explanation', which seems to infer that the teacher is exposing information. In fact, it means quite the opposite, this is the time in which the learner is provided the opportunity to express his/her opinions about the tasks and the new relations observed while operating on the materials. The learner makes his/her observation explicit and begins to use more accurate and appropriate vocabulary with the help of the teacher. Van Hiele (1986: 177) sees it as the time when the learner is provided the

opportunity to discuss the relations found and in 'this way he learns to speak a technical language'.

- Phase 4: Free Orientation. During this phase, the learner is engaged in multi-step tasks, which are open-ended and can be solved in a variety of ways. Once again, the tasks are organised in such a way as to encourage the learner to see connections and relationships more explicitly. It is a further development of the second phase in which the learner 'learns to find his way in the network of relations with the help of the connections he has at his disposal' (Van Hiele, 1986: 177).
- Phase 5: Integration. This is the final stage of the teaching process, which indicates that the learner has reached a new level of thought. It requires the learner to summarise and reflect on the field of study that he has been engaged, with the purpose of developing an overview. At this stage, the learner is expected to have unified and internalised the object and the relations under study (Hoffer, 1983). The teacher assists the process 'by furnishing global surveys' of what the learner has learned (Van Hiele in Crowley, 1987: 6). Having completed the process in which the old domain of thinking has been replaced by the new, the learner is ready to begin the phases of learning at the next level.

Although the phases of learning provide a useful guide to a teacher, s/he needs to remember that 'explanation by the teacher is only possible after the learner has already formed an ordered field of thinking' (Van Hiele, 1986: 178). The teacher creates the situation to accelerate development, s/he is the guide in the learning process and her help is principally indirect. S/he is not restricted by the form of instruction as different phases necessitate different approaches to teaching:

...it may happen that in classroom teaching the fifth phase is stressed, in Montessori teaching the second phase, in Socratic teaching the third phase. But all these forms of teaching are good, as well as many others that are not mentioned here, if they use all phases to full advantage- and they fail if they place too much stress on one single phase (ibid.)

2.4.4 Early Research on the Van Hiele model in the USA

As a result of the work of Wirszup (1976) and his account of the Soviets' developments in the teaching and learning of geometry, as well as the results of the second international mathematics studies (1981-82), there was an urgency to examine the Van Hiele work in more detail. For many, the theory of Van Hiele did not differ from what had already been established in number theory.

During the early 1980's, three major American research studies were undertaken, the Oregon project directed by William Burger; the Brooklyn project directed by Dorothy Geddes; and the Chicago project directed by Zalman Usiskin. Each of the projects had a different purpose as well as different subjects, however the findings of each helped to inform, elaborate and re-define aspects of the Van Hiele theory.

The Oregon project looked at 'the extent to which the Van Hiele levels serve as a model to assess learner understanding of geometry' (Hoffer, 1983: 212). It asked the following questions (Burger & Shaughnessy, 1986: 32):

1. Are the Van Hiele levels useful in describing learners' thinking processes on geometry tasks?
2. Can the levels be characterised operationally by learner behaviour?
3. Can an interview procedure be developed to reveal predominant levels of reasoning on specific geometry tasks?

The Brooklyn study tried to determine if the model was useful in describing how learners learn geometry and how the model could be interpreted in the context of an American curriculum and environment (ibid.). It also included the investigation of the effects of instruction on a learner's predominant Van Hiele level.

The third project's aim was to 'determine the effects of the learner's stage of cognitive development and performance on a test of mathematics prerequisites on learner achievement in standard geometry concepts and proof' (Hoffer, 1983: 213). It tried to

measure the geometric abilities of learners as a function of Van Hiele levels (Burger & Shaughnessy, 1986: 32).

While these studies were taking place, Mayberry (1981), also in America, was completing her doctoral degree on the Van Hiele levels of geometric thought in pre-service teachers. She developed a test, which tried to determine what levels the student teachers were on, and whether the results of such a test would support the hierarchical nature of the Van Hieles' work. She also looked for evidence which supported the notion that student teachers could have the same level across different concept strands.

2.4.4.1 Results of studies

Results of this research indicated that learners were familiar with geometrical terms but had incomplete understandings of what they were, i.e. learners drew all triangles as equilateral only and did not include other types of triangles. They appeared to parrot lists of properties they had learned but were unable to apply the list to a shape (Shaughnessy & Burger, 1985). Learners used the physical properties of a figure to describe a shape, which meant that a convex quadrilateral was often identified as a triangle, or an open curve as a circle. They were drawn to the visual representation of a diagram and would often change the location of the figure to decide on its properties. This also meant that if it did not look like the picture in the textbook then somehow the figure was perceived as incorrect and was discarded.

There is substantial evidence to show that learners have little understanding of the distinction between definitions, postulates and axioms. They also do not have a full understanding of the property "necessary" and "sufficient", and allow "necessary" to become "sufficient", for example, when told a figure has four sides, they immediately assume that it is a square. Usiskin (1982) found that 60% of all high school learners do not study proof and of the 40% that do, 11% study proof

but are unable to do anything with it, 9% can do only trivial proofs, 7% have moderate success and 13% are successful with proof. Senk (in Suydam, 1985) cites comparable results. She found that there was no difference between the results of boys and girls in geometry learning or in proof writing. This seems contrary to the popular belief that boys are better than girls in higher order thinking tasks. There is substantial evidence to support the hypothesis that small percentages of learners have any understanding of proof and slightly higher numbers see no reason to prove something that is obvious. At least 70% of learners cannot distinguish between deductive and inductive reasoning and do not understand that induction is inadequate to support mathematical generalisations (Williams, in Suydam, 1985). Many learners entering high school geometry or alternatively having done geometry at high school did not have the necessary level to cope with the work at that level (Mayberry, 1981; Usiskin, 1982). In fact, Shaughnessy & Burger (1985: 425) and Usiskin (1982) found that no high school learners were operating at level 4, which could suggest that that such reasoning is rare at this age. Usiskin (1982) found most Grade 10 learners were not on the first level while the course required at least level 3 to deal with proof writing. There is also evidence to indicate that those who have at least level 2 or are developing level 3 understanding, perform better in the long term on proof writing (Usiskin, 1982). A study of 2000 high school learners showed that those with level 3 understanding were most likely to understand and produce proofs by the end of the course.

Mayberry (1983) also found that prospective elementary school teachers think predominately in non-deductive, holistic ways (level 1) and found subjects do not think at the same level across content strands.

Burger & Shaughnessy (1986), in their report based on the interviews with learners, questioned the discreteness of some of the levels. In particular, level 2 and 3, analysis and ordering, would seem to be less discrete than thus far believed.

Usiskin (1982) and Burger & Shaughnessy (1986) found that though some learners could be assigned to a level, there were those who were difficult to classify because they appeared to be in transition from one level to the next. Evidence points to the dynamic rather than static nature of the levels, which seem to be more continuous than their discrete descriptions would have us believe (Burger & Shaughnessy, 1986: 31-48).

They also found that many post-geometry learners regressed a level on some activities and tended to use level 2 reasoning, “inductive” when forced to answer a question. It was proposed that post-geometry learners who may have more appropriate use of the geometric terms are at much the same level of reasoning as pre-geometry learners. This was also apparent in this study as many of the high school geometry learners performed marginally better if not the same as the non high geometry learners. Learners appear to exhibit different Van Hiele levels on different tasks but tend to resort to the comfort zone of level 2 reasoning though they know that level 3 was available. Shaughnessy & Burger (1985: 423) noted that if conflict occurred between visual and analytical reasoning, the visual usually won.

The analysis of some primary schools texts indicates a ‘deficiency in level 2 thinking, especially in the exercises and tests provided’ (Fuys et al, 1988: 177). This means that learners are not getting enough experience to develop level 2 thinking and are not being encouraged to use appropriate language for the level. More tasks need to be set asking learners to describe rather than to identify the properties of figures. The problems were almost entirely visual or entirely proof orientated with no questions that required the learner to apply analytical thinking. The words and objects used by the teacher seem to differ from the way the teacher and textbook intended, and misconceptions are often reinforced by inappropriate examples laid out in the textbook (Hoffer, 1983). This could be as a result of

limited examples and pictures, poor explanations and improper sequencing of material (ibid.).

Each of the studies found evidence to indicate that instructional experiences can shift learners' levels of understanding (Fuys et al, 1988). As the Van Hiele model highlighted the distinct lack of harmony in the teaching and learning of mathematics, so these studies corroborate this finding. Learners and teachers think and speak on different levels to one another (Usiskin, 1982; Burger, 1992). Teachers tend to use inappropriate activities for the levels of the learners, which in turn is reinforced by the examples in the textbook. As the learner becomes alienated by the language and examples used, so the development of geometric thinking is retarded.

2.4.5 Van Hiele and more recent research in South Africa, Australia and the USA, post 1990's.

Much of earlier research has focused on determining learners' levels of thinking, verifying and exploring the levels, evaluating the geometric content of textbooks and offering guidance to curriculum designers.

The assessment of the Van Hiele levels has taken two principal forms:

1. A learner's Van Hiele level of thinking on a topic is determined through the performance of the learner on a written test (Guiterriz et al, 1991; Mayberry, 1983 & Usiskin, 1982)
2. A learner is interviewed during while completing a series of activities and his/her response is then matched to a particular level (Burger & Shaughnessy, 1986; Fuys et al, 1988; Mayberry, 1983).

The results of such assessment procedures mean that a learner can be assigned to a particular level on the assumption that there is a dominant level of thinking. However learners may think on different levels for different concepts and be in transition between levels (Mayberry, 1981; Burger 1992; Fuys et al, 1988). Burger & Shaughnessy (1986) highlighted the problems that reviewers had in allocating levels and explained how

researchers eventually agreed on a specific level. Fuys et al (1988) were not as successful and allocated learners to level 1-2 to signify the transition.

Gutierrez et al (1991) developed an alternative method of evaluation which took account of learners in transition. Their work differed from the Van Hiele theory in so far as they believed that the levels were not discrete but rather continuous. The assessment concentrated on a learner's capacity to use each of the levels rather than assign a specific level to a learner. The notion that levels could be continuous means that acquisition does not happen immediately but may take months, even years. Gutierrez et al (1991) based their theory on observations made when learners answered questions. Although learners indicated a dominant level of thinking, they could also display some reflection typical of another level

... they maintain that initially learners are not aware of the new, higher level of thinking. They have no acquisition of that level. As they become aware of the new level, an attempt to work at the level is made and a low degree of acquisition is acquired. Continual growth in awareness is shown in an increasing degree of thinking by the learners at this level, through an intermediate degree of acquisition, a high degree, until they have a complete acquisition of the thinking at that level (Gutierrez in Lawrie, 1998: 176).

The Gutierrez method of assessment takes into account all responses, complete, correct or otherwise and proposes that it offers a more realistic evaluation or measure of the learner's level of reasoning in geometry. It provides:

... results in a qualitative assessment of a learner's degree of reasoning in each of the four levels (Gutierrez et al, 1991: 249-250).

It can give vital insight into the quality of the questions asked. It allows for more flexible interpretation of learner responses because it 'measures a learner's capacity to use each one of the Van Hiele levels in every statement made' (Lawrie, 1998: 180).

The assessment instrument took the form of a spatial geometry test, which evaluated learners' levels on 3-D geometry tasks. The test was given to 50 learners from a variety of backgrounds and ages, the results indicate the following:

- the higher the level, the lower the degree of acquisition which is in line with the hierarchical nature of the Van Hiele levels;
- the alternative method of evaluation offered a more flexible interpretation of the theory;
- learners can develop two consecutive levels of reasoning at the same time but the lower level was likely to be more complete than the higher level;
- learners showed better acquisition of level 3 than of level 2;
- not all learners showed single levels of reasoning but often reflected several levels of thinking at the same time depending on the difficulty of the problem;
- people do not think in the 'simple, linear manner which the assignment of one level would have us believe'. This does not mean that Gutierrez et al (1991: 249-250) are rejecting the Van Hiele theory and its hierarchy, but are suggesting that the theory be expanded to account for the 'complexity of the human reasoning processes'.

Lawrie (1998) took the Gutierrez alternative assessment instrument and applied it to the results of the written version of the Mayberry test that she had given to Australian pre-service teachers. Mayberry had tried to design a reliable diagnostic instrument that could measure student teacher levels of reasoning in geometry. She developed the test on the understanding that the Van Hiele levels were discontinuous (1981: 22). Each item tested for understanding of a particular concept on a particular level and analysed the responses to reflect the level of thinking. There was no grading of items according to levels of difficulty and no grading of responses for depth of understanding. As the Gutierrez assessment recognised the levels as dynamic and continuous, rather than discrete and static, Lawrie hoped that this instrument would help provide a more realistic picture of the student teachers' levels of understanding. She found that the following was minimised in the results:

- incorrect assignment of a level to a question;
- the effect of unequal distribution of questions across levels;
- results from lucky guesses such as found in the true or false questions ; and

- inequalities associated with the success criteria used in the Mayberry assessment method.

The results of Lawrie's analysis, using the Gutierrez instrument, reflected a fairer and more concise picture of students' Van Hiele levels of reasoning and helped eliminate some of the problems she had experienced with the Mayberry marking proforma. Lawrie (1998: 182) found that this research provides further empirical evidence of the robust nature of the Van Hiele levels and about what it means to understand at different levels. She found in using the assessment tool that some of the better students had problems in interpreting the thrust of level 1 questions. She suggested that it could have been a fault of the Mayberry test items which may not have been clear at that level. She (Lawrie, 1998: 182) supports the use of the Gutierrez assessment tool but also points to some unusual behaviour patterns which need further investigation and study:

- the fact that the better students were not able to perform adequately at level 1 yet sufficiently at level 2 and 3 could point to a limit in the ability of the coding system. It may be better suited to analysing level 2, 3 and 4 responses rather than to responses based on visual recognition;
- the system of coding reflects a more accurate picture of a students' reasoning skills but it is time consuming and not very practical for large scale use in mathematics classrooms;
- the allocation of a credit of 100 tends for level (n-1) for an attempt at level n, does not always seem justified and possibly presents a contradiction to other aspects of their coding system.

The controversy over the discontinuous nature or continuous nature of the levels was best highlighted by the Gutierrez research but there are other aspects of the model that have also caused concern for mathematics educators. These are:

1. the one dimensional nature of the levels and the belief that the theory fails to describe the complex nature of the learner behaviour (Pegg, 1997: 391). The limited description of the level 2 and level 3 which Pegg (1997) has broadened to account for the reasoning that happens at that level ; and
2. the contradiction in the Van Hiele literature regarding the level placement of hierarchical class inclusion (De Villiers, 1997: 25).

Pegg (1997: 391) found the level descriptors outlined by the Van Hiele theory did not help to explain textbook or examination questions that lie outside these descriptors. In order to address this problem and to make the theory more inclusive he expanded the descriptors for level 2 to level 2A and 2B. Level 2A: “figures” are identified in terms of a single property and level 2B “figures” are identified in terms of properties which are seen as independent of one another. He found that the Van Hiele level descriptors were too narrow and offered little guidance for analysing typical geometry questions found in school geometry. As most learners who enter junior high school tend to respond to questions at level 2, level 2A for Pegg represents the ‘culmination in the thinking process of the development of a single property’. Hence it ‘represents an important interface between the visual/intuitive thinking at level 1 and the identification of several isolated concepts/properties at level 2B’ (ibid.). The broadening of the descriptors for level 2 and level 3 meant that the theory was expanded to become more inclusive but remained true to the original descriptions. The Van Hiele descriptors give little guidance to support coding the nature of thinking associated with geometry questions. They are an indicator of a learner’s level of thinking developed through a teaching/learning process. If this is combined with the SOLO taxonomy (an instrument which analyses the response of a learner at a particular time in a particular circumstance), it becomes an effective tool to categorise the type of questions found in most school geometry textbooks. Level 3 reasoning is then broadened to include the acceptance and use of relationships between properties and figures (Pegg, 1997: 395). Pegg (1997) then applies the reviewed descriptors to provide empirical evidence to support his changes to the model. His work

highlights 'the potentially undesirable consequences of focusing on learners' thinking rather than on learners' responses (Pegg, 1996).

The second area of controversy in current research on the Van Hiele theory has been highlighted in the work De Villiers (1987, 1994, and 1996). He has done much research in the area of proof, more specifically in the role and function of hierarchical classification, which is part of Van Hiele's level 3 "reasoning". This level deals with learners' understanding of formal geometry definitions, which cannot happen if learners do not understand the underlying processes. Learners operating at level 1 and 2 develop spontaneous definitions, which tend to be partitional. In other words, they tend to list properties and avoid inclusion. According to Van Hiele, by level 3 learners definitions are typically hierarchical which means that they allow for inclusion and would not be understood by those operating at lower levels. There is a belief amongst many teachers and textbooks that 'only the conventional hierarchical classification is mathematically acceptable, whereas a partition classification is mathematically illogical and therefore unacceptable' (De Villiers, 1994: 17). De Villiers (1994) argues that the only reason there is a preference for hierarchical classification is because it is more functional. He found through his research that many learners were able to classify hierarchically but did not see the need to do so. They were happy to formulate their own definitions, which often included redundant information and had to be led, through instruction, to make the definitions more economical. De Villiers argues strongly that the problem that learners have with definition is located in the fact that they cannot see the reason for economic definitions. This does not mean that they cannot classify hierarchically but rather that the school experience has to highlight the reasons why it may be more appropriate:

It seems clear that unless the role and function of a hierarchical classification is meaningfully discussed in class as described in De Villiers (1994), many learners will have difficulty in understanding why their intuitive, partitional definitions are not used (De Villiers, 1996).

If we want hierarchical classification to be more meaningful for learners then (De Villiers, 1994: 17):

1. It is essential that an appropriate negotiation of linguistic meaning should have taken place.
2. It is absolutely vital that a negotiation of functional meaning also takes place.

This means the language that is used in level 3 must be understood by all. For example, when a learner makes a statement such as 'a square is a rectangle', the 'is' needs to be discussed so that the geometrical meaning of the statement is clear. At the same time, it is also crucial to discuss the definitions proposed by the learners and to highlight the important functions of hierarchical classification. They need to realise that hierarchical classification can lead to more economical definitions, to suggest alternative definitions, and to provide a useful global perspective, etc.

De Villiers (1994) highlights the difficulty that learners have with formal definitions at level 3 but he does not dispute the location of such behaviour at this level. Pegg (1992 in Lawrie & Pegg, 1997) goes somewhat further to question the validity of such a behavioural characteristic at this level. He disputes the assumption that a learner is not operating at level 3 if s/he cannot make the statement that a square is a rectangle i.e. class inclusion. As suggested by De Villiers (1996) earlier, class inclusion has a lot to do with the instruction or the lack of instruction needed to develop this level of thinking. He proposes and is presently researching the fact that class inclusion does not appear to be part of a 'natural mathematical development' (ibid.). It needs to be part of instruction if learners are expected to develop this reasoning skill. However, if it is excluded, it does not necessarily imply that the learner is not operating at the level 3.

2.4.6 Van Hiele and Piaget

Though much of the work of Piaget and Van Hiele relates to teaching and learning, their theories differ in a number of ways. Most importantly they are interested in different aspects of teaching and learning. Piaget as a psychologist is interested in 'general laws

that govern human behaviour' while Van Hiele wants to 'find those practices that will improve instruction' (Orton, 1995). In other words while Piaget tries to explain how the child's mind develops and highlights the 'stages of development', Van Hiele is interested in ways of shifting the development, the instructional practices that are necessary to bring about a change in insight, that is 'levels of thinking'. Van Hiele (1986: 5) is opposed to the idea that movement from one level of thinking to another is purely based on biological development and cannot be stimulated by a learning process. Although he (Van Hiele, 1986: 5-6) acknowledges the influence of Piaget's work on his own work, he highlights a number of differences between their work.

1. As mentioned earlier, he distinguishes between the purpose of his work and that of Piaget's. He is interested in how to stimulate the movement between levels while Piaget is concerned with the psychology of development. Van Hiele's work reflects a psychological/pedagogical theory with particular focus on the structuring of learner experience as a way of moving between the levels (Schoenfeld, 1986).
2. Piaget identifies two levels in a child's geometric development, the first of which identifies the child's perception of space as topological and later develops to the understanding of projective and Euclidean space, the transition from concrete operations to formal operations (Orton, 1995). Van Hiele identifies five discrete levels, which a learner has to complete in order to master the necessary subject matter. Though there is some debate as to the number of levels that exist, the theory is not prescriptive and does not 'provide a determinist, structuralist view of a fixed progression of clearly defined stages' which a learner has to pass through to reach the next level (Schoenfeld, 1986: 250). The theory offers an 'empirically based description of what appears to be relatively stable, qualitatively different levels or states of understanding' (ibid.).
3. The role of language is crucial to the Van Hiele theory in that it is used to 'redirect the objects of thought, thereby enabling the child to think at the next higher level' (Orton, 1995). Piaget describes the role of language somewhat differently and identifies actions as more significant to the development of logico-mathematical structures than language (ibid.). For Van Hiele (1986: 97) language is used at each phase of the

learning process. In the first phase a conversation is held between the teacher and the learner to establish the context, the teacher then guides the learner to read the relations between symbols within the context (phase 2). Through class discussion the relationships are made explicit (phase 3), this develops into the formation of a series of networks (phase 4) and finally the network is understood and assimilated into the learner's current system of relations (phases), then the next level of thinking is attained.

4. While Piaget intimates that 'the human spirit develops in the direction of certain theoretical concepts', he does not take cognisance of the changing nature of the human spirit which reflects on theoretical concepts (Van Hiele, 1986: 6). Concepts develop as human constructs, which are influenced by the socio, political and economic nature of the people of that time.
5. Van Hiele sees progress and movement through the levels as a result of building on the processes and structures of the previous levels. His theory suggests the need for a solid 'empirical grounding' for the 'apprehension and manipulation of abstract geometric objects' which is meaningful for learners (Schoenfeld, 1986: 261-262). It is pointless to equip learners with the skills and knowledge to operate within a restricted context when they are to apply the tools more globally. For Piaget, structures of a higher level do not develop as a study of the lower levels. He believes that children are born with a higher structure and they need to become aware of this.
6. The final distinction outlined by Van Hiele is how each defines structure. For Piaget, mathematical structure always defines the whole structure while for Van Hiele, a structure is a given thing which obeys certain rules. If it is a strong structure, it will be possible to superpose a mathematical structure onto it.

2.5 RÉSUMÉ

This chapter outlines the development of geometry and teaching over the past 20-40 years. It is primarily concerned with the didactical approach teachers adopt to geometry instruction. The Van Hiele model of geometric development is used as the conceptual framework on which to consider current geometry teaching. It provides the most comprehensive theory related to the assessment and instruction of geometry available to date.

The model has been researched in many parts of the world in high schools as well as in pre-service teacher education. The observations made by Van Hiele and his wife of their teaching experience has influenced the geometry curriculum of the Netherlands, Russia, America and South Africa. Each of the countries mentioned has interpreted and implemented the theory to various degrees of success. It is for this reason that the model was chosen to help evaluate the impact of a course on students' levels of understanding.

CHAPTER THREE: METHODOLOGY

3.1 INTRODUCTION

This chapter reviews the procedures used in the study to investigate the effectiveness of a geometry course on student teachers' levels of understanding. It includes a description of the subjects and the selection of an appropriate assessment tool, as well as the course design and course work done by learners.

3.2 RESEARCH DESIGN

Given the evidence of poor performance in geometry amongst high school learners and the link between the levels of development and instruction, it is necessary to examine the link between the study of geometry and the education of primary school teachers. Research indicates that most students entering Colleges of Education are operating at level 1 and 2, regardless of the level of mathematics achieved at school (Mayberry, 1981; Burger, 1992). This means that the students are sometimes at a level no higher than they are expected to instruct. The poor performance of high school learners may be a result of inappropriate levels of instruction in the primary school. If teachers involved in primary school education are at levels no higher than 1 or 2 in geometry, it is difficult for such teachers to develop level 3 thinking in their learners. It is crucial that colleges of education evaluate the impact of courses on developing potential primary school mathematics teachers, especially in the field of geometry, for reasons outlined already. This study looks at a geometry course offered to second year pre-service teachers and investigates the impact the course has had on students' levels of understanding in geometry.

The main focus of the research was to examine the impact of the course on students' levels of thinking. However, it was also important to assess how students' levels of understanding developed during the course and to examine how this was reflected in their course work. The inquiry was restricted to four principal objectives.

The research design intended to:

1. Determine if a geometry course could shift pre-service teachers' levels of thinking;
2. Analyse the Mayberry test as an effective instrument in assigning students to a particular level;
3. Ascertain whether the Van Hiele model is useful to describe students' progress throughout the course ; and
4. Assess how pre-service teachers integrate the theoretical and content elements of the course to select suitable learning activities for learners.

3.3 RESEARCH QUESTIONS

The research questions focused on the following:

1. Can a geometry course shift pre-service teacher's levels of understanding?
2. Does the Van Hiele model appropriately describe students' progress through the course?
3. Do pre-service teachers integrate the theoretical and content elements of the course to select suitable activities for learners?

3.4 SITE AND SAMPLE SELECTED FOR THE STUDY

3.4.1 Site

As mentioned previously, the study was conducted at a College of Education, which specialises in the training of primary school teachers. Students have to complete a three year course in order to obtain a Diploma in Education. There are a number of compulsory components to the course, one of which is mathematics. Students are obliged to pass all three years of mathematics at the college to be awarded a diploma. The mathematics course is a combination of content and pedagogy and concentrates primarily on the current South African primary school syllabus. This includes number (whole, rational and real), geometry, measurement, data handling and pre-algebra. The students need to have a matriculation exemption to gain access to the college, which means that they have to have 3 subjects on the higher grade. More often than not, students have not taken mathematics beyond Grade 9.

3.4.2 Sample

The class used for this particular study was comprised of 26 students. 5 of the students had taken mathematics to Grade 12, 3 students dropped mathematics between grade 10 - 12 and 18 did not have mathematics further than Grade 9. This particular college of education requires first year student teachers of mathematics to take courses on number, measurement, data handling and didactics. Once students have become proficient in the content and the pedagogy of these topics, they proceed to the study of geometry in their second year. There are three categories of students at the second year at the college, Pre-primary (Pre-school), Junior primary (Grade 1-3) and Senior Primary (Grade 4-7). As most geometry teaching happens in Senior Primary, the need for teachers teaching in this phase to be adequately prepared in geometry is crucial. Given the needs of the schools, it was decided to conduct the research with prospective Senior Primary teachers, the bulk of whom did not have mathematics beyond Grade 9. The fact that most students (69%) did not have high school mathematics suggests that they would probably be operating at below Van Hiele level 2.

As the majority of students in this class had not had an in-depth exposure to mathematics, they arrived at college with a fear of the subject and spent the first year at college coming to terms with this. This meant that a secure, helpful working relationship between students was paramount for a co-operative learning environment. Students worked in mixed ability groups of their own choice, which helped act as a support mechanism to provide them with a friendly, informal context in which to learn mathematics.

Once the year group was selected for the study, the class was assessed as a whole by means of a pre-test (see 3.5.1.2) to establish the levels of understanding of individuals. The pre-test was analysed, after which it was decided to select two focus groups to help monitor student development more closely. The work of the focus groups was to be analysed in terms of the Van Hiele theory to determine the appropriateness of the model in describing students' progress. The first group comprised of 3 female students with grade

12 mathematics and 1 older male student with no mathematics beyond Grade 10. They were one of the stronger groups in the class based on the previous year's results and were all first language English speakers. The second group was an all-female group with poor content knowledge of mathematics. No one in the group had mathematics beyond Grade 9 and they were all second language English speakers who had to take the course in English. Despite the difficulties, they appeared to work well as a group and attempted to make sense of the mathematics being taught.

Table 3.1
Student Profile

Students	Gender	Age	First Language	Highest level of mathematics
Focus Group A				
S1	Female	22	English	Grade 12 - Standard grade
S2	Female	22	English	Grade 12 - Lower grade
S7	Female	20	English	Grade 12 - Standard grade
S8	Male	32	English	Grade 10/11
Focus Group B				
S12	Female	40	Xhosa	Grade 9
S15	Female	21	Xhosa	Grade 9
S19	Female	26	Xhosa	Grade 9
S20	Female	24	Xhosa	Grade 9

3.4.3 The Geometry Course

The course was held over nine weeks. Lectures took place three times a week and each lecture was 50 minutes long.

3.4.3.1 Aims of the course

The teaching objectives of the course are to make students better able to:

- identify and solve problems in which responses display that responsible decisions using critical and creative thinking have been made (Critical Outcome - CO1);
- work effectively and co-operatively with others on geometrical problems (CO1);
- communicate effectively using the appropriate notation, symbols, conventions and expressions (Specific Outcome - SO9);
- question, examine, conjecture and experiment (SO9);
- recognise and represent mathematical forms in the natural world and in cultural representations (SO8);
- describe the position, change and orientation of an object (SO7);
- explore patterns in abstract and natural contexts using mathematical processes (SO2);
- experience mathematics as a human activity (SO3) and:
- show that all peoples of the world have contributed to the development of mathematics (SO3).

The NCTM Standards Document (1989: 112) suggests that a mathematics curriculum should include the study of the geometry of one, two and three dimensions in a variety of situations. In more specific terms the students must be able to:

- identify, describe, compare, and classify geometric figures;
- visualise and represent geometric figures with special attention to developing spatial sense;

- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- classify figures in terms of congruency and similarity;
- deduce properties of and relationships between figures ; and
- develop an appreciation of geometry as a means of describing the physical world.

3.4.3.2 Content of the course

The course was designed for a 9-week period that allowed students to engage with some primary school geometry content as well as occupy students interactively with:

a) A panoramic view of geometry by

- identifying geometry in nature and the world around us;
- identifying and/or copying linear patterns in everyday life using 2-D and 3-D shapes (floor tiles, walls, paving, etc.) as well as identifying artistic patterns in South African cultures (beading, paintings, baskets, Ndebele houses, fabrics, etc.);
- arranging geometric shapes in a logical sequence and identifying missing terms of geometric patterns;
- extending or creating linear patterns using 2-D shapes ; and
- using concrete objects to extend, create and depict tiling or grid patterns (leaves, petals, cobweb, etc.)

b) Tangrams by

- sorting, grouping, matching, comparing, classifying and recognising the following shapes: triangle, circle, square, rectangle, quadrilaterals and other polygons;
- identifying the different types of triangles;

- listing all possible 2-D regular and irregular shapes;
- developing an intuitive understanding of the underlying properties of shapes;
- discussing and experimenting with the concept of congruency by placing triangles on top of one another;
- using tangram pieces, trying to use visualisation skills to remake the picture provided;
- discussing ways of extending the activities for primary classroom use; and
- linking tangrams to other geometry concepts.

c) Properties of regular polygons by

- identifying the properties of regular 2-D shapes;
- classifying shapes according to properties;
- identifying the properties of quadrilaterals;
- drawing a development map of quadrilaterals and justifying the diagram;
- exploring the notion of class inclusion and minimum properties in quadrilaterals;
- identifying lines of symmetry found in 2-D regular shapes;
- estimating and measuring angles, naming the different types; and
- calculating angles from given information in diagrams.

d) Perimeter and Area of shapes by

- calculating area and perimeter of irregular and regular shapes;
- measuring area and perimeter using appropriate units;
- demonstrating an understanding of the relationship between area and perimeter through concrete activities; and
- recognising and using appropriate units of measure.

e) Van Hiele theory and reading by

- outlining the main points of the Van Hiele theory;

- giving examples of activities related to the different levels;
- discussing and debating the relevance of Davey & Holliday (1992) expansion of the Van Hiele theory;
- relate the Van Hiele theory to different concepts in geometry; and
- relate Van Hiele theory to the classroom tasks and skills.

f) Pentominoes by

- introducing the concept of monimoes, dominoes, trominoes, etc. through practical experience;
- combining each category in as many ways as possible;
- finding the maximum numbers of ways five squares can be combined so that only one full side is touching each time;
- calculating the perimeter and area of each combination;
- developing an understanding of the relationship between area and perimeter; and
- demonstrating an understanding of the relationship between area and perimeter.

g) Tessellations

- introducing the vocabulary of transformation geometry;
- identifying and describing informally 2-D shapes that can cover a surface with gaps (regular pentagons, circles, pentominoes, etc.);
- tessellating regular and irregular polygons (tessellations);
- calculating interior and exterior angles of polygons; and
- recognising and labelling regular and irregular tessellations.

3.4.3.3 Instructional methods

The teaching of geometry throughout the course highlighted the need to develop verbal, visual, drawing, logical and applied skills in learners. Students were

required to utilise the skills in tasks and presentations in class. There was an attempt through class based discussion to link the skills and the Van Hiele levels of development. The skills were linked to geometry content through:

- manipulation of real life objects, for example: tangrams, 3-D shapes, tessellation's activity;
- use of pencils, cardboard, crayons, glue to construct polygons, polyhedra, pentominoes, tessellations;
- providing verbal, visual and drawing experience through the pentominoes activity, and the triangle and quads activity on the geo-board, etc.;
- discussion of the findings of the groups, done after most activities;
- the role of writing in the mathematics classroom through investigation write ups;
- establishing links between real life experiences and the world of geometry, for example: the Panoramic View of geometry activity, finding shapes in nature, identifying real-life examples; and
- investigation and problem-solving tasks such as tessellations and transformation activities, area and perimeter applied problems.

3.5 DATA SOURCES FOR THE STUDY

3.5.1 Focus One: Can a geometry course shift pre-service teachers' levels of understanding of geometry?

Having determined the need to evaluate the students' current level of thinking, it was crucial to find an appropriate assessment tool. It was accepted that a student's response to questions about an unknown topic might not give an accurate assessment of the potential of the student to think at certain levels (Fuys et al, 1988: 13). However, assessing the students before and after an intervention program would help to indicate the progress (or lack of progress) a student can make within a level.

There were a number of tests to choose from but there were also certain constraints that

limited the choice for this study. The test had to be completed within 50 minutes as that is the standard time for each lecture. The language needed to encourage students to display what they knew and not restrict them because of lack of understanding or difficulty with the terminology. It needed to cover concepts that students would have met in their primary schooling, as many had not taken high school geometry courses. It had to be user-friendly for students, so as not to discourage their efforts.

As the Van Hiele framework is currently the best known developed theory in terms of describing students' levels of thinking in geometry, the test needed to work within the parameters of this theory. This meant finding an instrument that would assess students' understandings, based on the Van Hiele theory. Identifying an effective assessment tool required a search of relevant literature to find the various instruments available and then to make an appropriate selection based on the needs mentioned.

3.5.1.1 Identifying an effective assessment tool to determine students' levels

The search produced a variety of instruments, some of which will be described, by highlighting the strengths and weaknesses of each. The first of these was developed by Usiskin & Senk (1990: 242) working through the Cognitive Development and Achievement in Secondary School Geometry project (CDASSG). They developed a test, which has been used to determine Van Hiele levels for a set of individuals and to test the Van Hiele theory. The items were written to correspond directly to statements from the Van Hiele's about characteristic behaviour students' exhibit at each level. They were concerned that the theory had not been sufficiently tested to make it scientific. They believed that there are two questions that need to be answered to make a theory scientific:

- a) Is the theory descriptive, in the sense that a unique level can be assigned to each student; and if so
- b) Is the theory predictive, in the sense that the student's Van Hiele level can be utilised to predict his or her performance in the traditional tenth-graded geometry course (Usiskin & Senk, 1990: 242).

The CDASSG Van Hiele Level Test was a 25-item multiple choice test with 5 foils per item and 5 items per level and had to be completed in 35 minutes. The test was administered to all students enrolled in the 10th grade geometry course in 13 schools selected as representative of schools in the United States. Based on the results of this test, Usiskin (1982) was able to draw conclusions on the testability of the theory; the ability to classify students according to levels; changes in levels; relationships between concurrent and future geometry achievement and the levels; and student performance in geometry.

Although the Usiskin test has been used repeatedly to determine students' Van Hiele levels, the authors point out, it was not intended for that specific use (Usiskin & Senk, 1990: 242). Usiskin believes that the test his project developed has been misinterpreted. It was not initially designed to help classify students but rather to test the theory of Van Hiele 'by writing items that correspond to the Van Hiele descriptions of the levels and measuring the extent to which students' responses formed a hierarchy' (ibid.). However what Usiskin does acknowledge is that, though the test was intended for other purposes, the fact that it has been used so prolifically 'indicates a perceived need for instruments that assess students in geometry and there are few, if any, instruments to choose from (Usiskin & Senk, 1990: 244).

The second test considered was designed by the RUMEUS group at the University of Stellenbosch, South Africa. It was a longer open-ended test which consisted of 29 questions (60 items) and covered the following concepts: lines, angles, triangles, polygons, congruency and constructions (Smith, 1989). It was also used to measure Van Hiele levels in secondary school students. Results indicate that it 'outperformed the CDASSG test in virtually all aspects, most notably in regard to the placement of pupils on a Van Hiele level, its reliability and hierarchical structure' (Smith in Usiskin & Senk, 1990: 245).

Mayberry (1983: 58-61) designed a third test to examine the following hypotheses with undergraduate pre-service teachers:

- H1: for each geometric concept, a students at level N will answer all questions at a level below N but will not meet the criterion on questions above level N;
- H2: a student will meet the criterion at the same level on all geometric concepts tested.

The study defined and studied the five learning levels hypothesised by the Van Hiele. Tasks were designed for the five levels using seven common geometric concepts, namely 'squares', 'right triangles', 'isosceles triangles', 'circles', 'parallel lines', 'similarity' and 'congruence'. There were 128 questions comprising of 62 items (14 at level 1, 25 at level 2, 70 at level 3, 15 at level 4 and 4 at level 5) given to 19 pre-service elementary school teachers, 13 of whom had studied high school geometry. These were reviewed by 11 mathematics educators who had a 'special interest and expertise in geometry' for validation, including Van Hiele himself (ibid.). The test was designed to be administered using two one-hour interviews and candidates were given 'paper, a straight-edge, a pencil and instructions to draw diagrams and handle cards as necessary' (ibid.).

The results of the study indicated that answers supported the hypothesis that the Van Hiele levels form a hierarchy. However students did not appear to 'think at the same level across different concepts' (Mayberry, 1981: 101-102). It was also apparent from the results that pre-service teachers have low levels of understanding even though 68% had taken high school geometry.

The fourth assessment tool considered was a revised version of the Mayberry test (Lawrie & Pegg, 1997). Her work was used in an Australian context with 60 first-year primary-teacher trainees at the University of New England. The aim of the Lawrie & Pegg (1997) study was to replicate Mayberry's work in some alternate

format and to analyse the validity of the test questions. The original interview schedule was amended to produce a written version of the Mayberry test, which allows for the assessment of larger numbers of students.

The test included most of the original questions, but with adjustments so that 'the intention of each was clear' (Lawrie, 1998: 178). Not all weaknesses could be corrected without a major reconstruction and losing the essence of the original test. The full test, which examined the seven original concepts selected by Mayberry, was recommended to take 2 hours to complete. Lawrie (1998) divided the test into two one-hour papers (random distribution to students), each testing four concepts (Paper I - 'square', 'right triangle', 'circle', 'congruency'; Paper II, 'square', 'isosceles triangle', 'parallel lines', 'similarity'). Students were interviewed on eight selected questions, with probing to check for reliability of level assessment in the written test. Interview questions included two on 'squares' and one for each other concept. Basically, the questions selected were those that could be answered at several levels. Interview time was 30-40 minutes for each student. Selection was stratified, across gender, concept and level of result (high, middle and lower groupings).

The findings of the written test found similar levels of understanding (no greater than level 2) amongst Australian students, as the Mayberry test. Students who had taken high school geometry (level 3 or 4) could not display 'overall level 3 understanding in their responses' (Lawrie & Pegg, 1997: 187). There were problems with the assignment of levels for some students, which meant that a number of students did not validate the level hierarchy. It was concluded that aspects of the Mayberry items had the 'potential to lead to an incorrect assessment of a student's level of understanding' (ibid.).

The final assessment tool considered for use in this study was developed by the Fuys et al (1988) study. They required a research tool that would allow them to:

characterise the thinking in geometry of sixth and ninth graders in terms of levels... in particular at what levels are students? Do they show potential for progress within a level or to a higher level and what difficulties do they encounter (Fuys et al, 1998: 1).

The program developed and validated a set of three modules based on properties of quadrilaterals, angle relationships for polygons and area of quadrilaterals. The modules were informed by the work of the Van Hiele and contained instructional activities as well as key assessment tasks, which correlated with specific level descriptors. The modules tried to include assessment tasks that allowed the researcher to examine the students' current levels of thinking about a topic and their potential levels assessed through the students' performance during the learning situation. The assessment was conducted over 6-8 sessions of 45 minutes each with each student. The subjects worked through the modules with the interviewer and all sessions were videotaped. A detailed and elaborate protocol form was developed to analyse students' responses.

Findings indicated that most students were operating at level 1 and 2 with some at level 3. For both sixth and ninth graders who completed the second or third module, it was clear that the highest level achieved in one concept remained consistent across other concepts (Fuys et al, 1988: 40). This meant that though students may lapse to a lower level in learning a new concept, they were able to move more quickly to higher levels than students who originally operated on lower levels.

Results also indicated that language was a highly influential factor in geometry development. Students confused common usage of words with the mathematical definition. Therefore assessment of students understandings of terminology has to be part of instruction. It appeared from this work that the more able the student is to talk about geometry in an appropriate context, the easier the progress through the levels. Good visual materials and manipulatives which students can select from to explain their thoughts can help bridge the language gap, hence the importance of including these in classroom activities.

Visual perception difficulties were also apparent in students' understandings. They may have had the correct definition of a concept while at the same time had a 'special visual image associated tightly with the concept' which leads to application difficulties (Fuys et al, 1988: 137). This difficulty often arose as a result of students' experiences, which had been limited to a specific orientation; for example shapes were often displayed in the upright position only.

3.5.1.2 Selecting the most appropriate instrument

Each test was considered for its application to this context and a choice was then made. The tests are outlined below, both in terms of their merits and demerits.

The Usiskin test has proved itself in that it has been re-used many times and the results are not significantly different. It has the advantage of being shorter than most of the other tests and its multiple choice format makes it more convenient and quicker to apply (Smith, in Usiskin, 1990: 245). It is a useful instrument for assigning students to levels because of its easy marking system and level allocation. However, the specialised terminology which students have to negotiate could have been a problem for many of the subjects used in this study, as English was not their first language. The test does not allow for verbal or written descriptions, which means that students are not provided with an opportunity to show what they know. The choice of answers is sometimes confusing because it appears as though there is more than one correct answer. Moreover, the multiple-choice questions allow students to guess.

The RUMEUS test is comprised of open-ended questions, which give students a better opportunity to show what they know. However, having 60 test items could make the marking an administrative nightmare especially as the variety of responses makes it difficult to mark and assign students to levels. In addition, some of the concept categories were not all appropriate to the student's prior

learning experience. The time needed to administer this test fell outside the time available for this particular study.

Given the nature of this study, the assessment tool used for the Fuy's study was far too detailed and did not allow for the evaluation of large numbers of students in a short period of time. It required a large amount of time for individual interviews and because of the limited manpower available, this was a major stumbling block. It is an interesting and dynamic tool for more detailed study of students' levels of understanding and is the first instrument to look at a student's potential level. However, the assessment tool required for this research aimed to evaluate the student's level at the beginning and the end of the course and was not concerned with the potential of the student at this point.

The final test available, for use was the written version of the Mayberry test as the interview version of the same test would have required too many manpower hours from both the researcher and the students. The written test could be administered more easily and because it was a tried and tested method of assessment, it helped place students on the correct level of understanding. The test could be broken up into one-hour segments and concepts chosen based on the prior knowledge and experience of the students. The questions allowed students to make selections and justify their answers. The language was simpler which would hopefully help avoid confusion about what was required from the questions. Furthermore, marking seemed relatively straightforward.

Though the written version of the Mayberry test seemed the most appropriate, there were some disadvantages to the instrument. Some of the questions were a little ambiguous, especially level 4 which required students to deal with "never", "sometimes" and "always" options. The limited number of questions at each level meant that students could be allocated to an incorrect level based on some error

responses. Some questions allowed students to make lucky guesses thereby giving a false impression of the student's level of thinking. The marking proforma was vague in places and meant that marking was not as easy as initially envisaged.

Given the background knowledge of the subjects and the fact that not all of the students were first language English speakers, it was decided to eliminate the Usiskin and the RUMEUS tests. The Fuys test required too much time in assessment. Therefore the written version of the Mayberry test was chosen as it allowed for the assessment of large numbers of students in a relatively short period of time. It combined multiple choice and open-ended questions, which provided more opportunity for students to display what they know instead of what they do not know. The written version of the Mayberry test was chosen as the assessment instrument as it was more advantageous in terms of time needed to administer the test, variation in concepts, assessment at all four levels and a relatively straightforward marking proforma than the other tests.

Although the test examined students' levels in seven geometric concepts: 'square', 'right triangle', 'congruency', 'similarity', 'parallel lines', 'circle' and 'isosceles triangles', this study chose to select only four concepts, namely 'squares', 'right triangles', 'congruency' and 'circles' (see Appendix A). The choice was based on students' experience in primary school as well as content covered in that phase of schooling. All four of the concepts chosen are dealt with at various stages in the primary school, and early high school thus the terminology should have been familiar to the students. The Mayberry test takes each concept and assesses it on five different levels. As most of the class did not have mathematics beyond grade 9, it was decided to exclude level 5 questions from the study.

Each level contained a number of questions each with an accompanying success criterion. Level 1 questions required the student to name a particular figure and to choose (discriminate) the same figure from a variety of images. The student was

successful if he/she was able to obtain 50% for each of the two questions, i.e. the criteria for success was 50%.

Level 2 required the student to describe the properties of a given figure although the student was not required to relate the properties to one another. In transforming the original test into a written test, the success criterion was changed from 80% to 75% for allocation to this level. The conversion of the original Mayberry test required some modification to 'ensure that the intention of each question was clear' (Lawrie & Pegg, 1997: 185)

Level 3 required the student to:

- a) make an assumption about a figure based on a given list of properties;
- b) display understanding of class inclusion (For example, is a square also a rectangle?);
- c) recognise relationships between properties; and
- d) make logical implications from properties provided.

The student was deemed to be functioning successfully at this level if two-thirds of the work was correct, again a deviation from Mayberry that required 65% correct responses to meet the success criterion.

Level 4 questions required students to:

- a) give reasons for the steps involved in proof-writing;
- b) answer questions that were based on steps given in a proof; and
- c) do a simple proof.

The success criterion for the written test for this level is 50% although the Mayberry version requires a 60% success.

3.5.1.3 Data collection

Prior to the commencement of the course students were given 50 minutes to complete 33 questions on the following four concepts: 'square', 'right triangles',

'circles' and 'congruency'. After which they took a 9-week (24 hours) space and shape course geared for the teaching of geometry in primary school. On completion of the course, the students wrote a post-test on the same 33 items to determine whether there had been a shift in levels of thinking in the above concepts. The course was not designed to develop these particular concepts but rather to familiarise students with the content of the primary school syllabus and to expose them to current teaching practice in geometry instruction.

3.5.2 Focus Two: Does the van Hiele model appropriately describe students' progress through the course?

During the course, the focus groups' written work was collected, to be used in the assessment of the appropriateness of the Van Hiele model in describing students progress. This was then compared with students performance on the pre and post-test, to try to validate the initial findings of the test results.

The classroom based tasks included:

3.5.2.1 Prior Learning Questionnaire (see Appendix B)

It was crucial to establish the prior knowledge of the groups so that the planning for the course content could be finalised. Students were required to complete a series of questions relating to their prior experience in geometry and to highlight possible areas of concern. They were given a list of syllabus items that are currently part of the primary school curriculum and asked to indicate terms or concepts that were unfamiliar to them. The questions had to be completed overnight and handed in at the next lecture.

3.5.2.2 Poster

The nature of the class work varied and started with a group activity, which required the students to work in groups of 4 and make a poster of 'what is geometry?' They were given paper, glue, cutting equipment and newsprint and

told to fill the paper with their impressions of geometry. They could choose to write words or to make pictorial representations of their impressions of geometry. The presentation was left to the discretion of each group.

3.5.2.3 Examination (see Appendix C)

There was an examination at the end of the module to assess students' understandings of the geometry they were expected to have learned on the course. It was a two-hour examination that included questions on fractions, decimals and geometry. The examination consisted of 8 questions, 5 of which covered geometry and focused on the content and didactics of the course given to the students.

Question One required students to identify quadrilaterals, sort them into groups and to state the minimum properties necessary to define two of the shapes. The second question concentrated on rectangles, the similarities and differences of shapes and required the student to draw sketches of two different kinds of trapeziums. Thirdly, the students were asked about the Van Hiele levels of thinking and how they relate to geometry. They were asked to classify the previously asked questions according to the Van Hiele levels as well as to justify their conclusions. Question Four related to perimeter and area and required some problem solving tasks, which were situated in real life contexts. The last question focused on calculating angles in diagrams that contained some of the information needed to solve the problem. The questions attempted to determine the students' level of understanding of the course content and each of the questions assessed different levels of understanding of polygons. An authority in the research of Van Hiele's theory verified the levels of the questions.

3.5.2.4 Data collection

The second focus question, the appropriateness of the Van Hiele model in describing students' work produced during the course, required the introduction of focus research groups. The class had traditionally been working in groups of 4 during their mathematics lectures, which allowed the researcher to monitor the progress of the students more closely. Two groups were chosen as focus groups and samples of their work throughout the course were collected for further analysis. Their performance in the end of term examination was also used to verify their progress and together with the sample course work, helped trace the path of student development. It allowed the researcher to observe the students' development through the course, using the Van Hiele framework and determine whether the course work results correlated with students' levels on the pre- and post-test.

3.5.3 Focus Three: Do pre-service teachers integrate the theoretical and content elements of the course to select suitable activities for learners?

3.5.3.1 Assignment of Focus Groups

An assignment was given to students to assess the extent to which they could link the theory and content in the course to design learning activities.

Instructions were given as follows:

1. Outline your understanding of how children learn geometry. Refer to the Van Hiele course notes. You can also use the method books in the library on 510.7 and the material behind the desk in the library.
2. Plan a sequence of activities (3 or more) to teach any one of the following topics to a particular grade (refer to the WCED syllabus handout for the appropriate content):
 - Shapes - polygons
 - Shapes - polyhedra
 - Angles
 - Symmetry

- Tessellations
 - Area and perimeter
3. Explain in detail why each activity was chosen. Try to make links with the literature you have read on how children learn geometry (including the Van Hiele levels). You need to explain why you chose particular activities and why you sequenced them in a particular way.
 4. Include objectives for each activity and say how the activities are to be used in a mathematics classroom to achieve these objectives. Include a test question with which you could determine whether the objectives had been met.

The activities you choose can be taken from textbooks, journals, internet, etc. Make sure that all work is referenced properly.

3.5.3.2 Data collection

Finally, the assessment of students ability to select appropriate learning experiences for learners, which reflects the integration of the theoretical and practical nature of the course, was again done through the focus groups. The assignment work of the eight students involved in the focus groups was assessed to determine how successfully they had managed to integrate the mathematical content and theoretical elements of the course. Students' work was examined with the help of a template designed from work on the application of the Van Hiele model. This work makes recommendations about the kind of activities learners should be engaged in at the appropriate level (Holmes, 1995).

3.6 RÉSUMÉ

This chapter is specifically concerned with the research design of this study. It outlines the aims and objectives of the research and highlights three focus questions.

There is a description of the site, sample and content of the course as well as a detailed description of the data collection methods used. Data sources for the study have been related to each of the research questions and an outline of the content of each of the items is included. There is a detailed discussion of assessment instruments, as the study required the measurement of students' levels of understanding and a justification is provided for the eventual selection of the Mayberry instrument.

CHAPTER FOUR: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter presents the analysis of data obtained from the pre- and post-tests administered to pre-service primary school teachers before and after taking a course in geometry. The tests were used to 'elicit information about the geometric thought processes' of students and to determine whether a geometry course could shift students levels of thinking (Mayberry, 1981: 69). In addition, data was collected from two focus groups consisting of 8 students. The questions under investigation in this study were:

1. Can a geometry course shift pre-service teachers' levels of understanding of geometry?
2. Does the Van Hiele model appropriately describe students' progress throughout the course?
3. Do pre-service teachers integrate the theoretical and content elements of the course to select suitable learning activities for learners?

Each of the above questions is analysed and interpreted separately and common understandings/conclusions, which emerge, are summarised in Chapter 5.

4.2 FOCUS ONE: Can a geometry course shift pre-service teachers' levels of understanding of geometry?

4.2.1 Marking of pre- and post-test

The responses of each student were analysed using the amended Mayberry marking proforma developed by Lawrie & Pegg (1997). There was opportunity to discuss marking of some of the test items through personal correspondence with Lawrie (see Appendix D). Each question for a level in a particular concept was given a '1' if answered sufficiently at a particular level and '0' if not (see Appendix E). If a student achieved a pattern (1,1,0,0) result in a concept, it was taken to mean that s/he had a level 2 understanding of that concept and no understanding of the concept at level 3 and 4. This particular student can recognise the concept and identify its properties but is unable to see relationships between these properties.

The table below reflects the acceptable response patterns: a (0,0,0,0) result means a student has 0 level understanding, in other words, no recognition of the concept. (1,0,0,0) equals level 1 understanding for the given concept; (1,1,0,0) means the student has reached level 2; (1,1,1,0) equals level 3 understanding and (1,1,1,1) is level 4. Students who did not fall into any of these categories were recorded as an error response (¹). For example, if a student obtained a (1,0,1,0) result in the marking proforma, it meant that s/he had a level 1 understanding, no level 2 and a successful level 3 understanding of a particular concept. According to Van Hiele the levels are sequential in that movement can only occur from level 0 to level 4 in that order. This means that a response (1,0,1,0) would not be an acceptable progression in terms of understanding, according to Van Hiele, therefore it was recorded as an error response (¹).

The test questions and success criteria can be found in Appendices (A & E). There were 26 students, four concepts ('square', 'right triangles', 'circles' and 'congruency') and four possible levels of understanding (level 1 - 4). The results of the pre- and post-tests are reflected on Table 4.1, in which S1-S8 signifies students who studied geometry beyond grade 9, although not all of these students took geometry to grade 12, and S9-S26 denotes students who had geometry up to grade 9. There were two students (*) who were repeating the course having failed the previous year. Students who made unusual shifts up and down in levels of understanding of concepts have also been highlighted for further comment (²).

4.2.2 Pre-and post-test results

The following table reflects the class pre- and post-test performance of students in the written version of the Mayberry test.

Table 4.1
Pre and post-test response pattern results

Student No.	Squares		Right Triangles		Circles		Congruency	
	Pre-	Post	Pre-	Post	Pre-	Post	Pre-	Post
S1	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,1,0,0	1,1,0,0
S2	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,1,0,0	1,0,0,0 ²	1,0,0,0	1,1,0,0
S3	1,1,0,0	1,1,0,0	1,0,0,0	1,1,1,0	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0
S4	1,1,0,0	1,1,0,0	1,1,0,0	1,1,1,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0
S5	1,1,1,1	1,1,1,1	1,1,1,0	1,1,1,1	1,1,1,0	1,1,1,1	1,0,1,0 ¹	1,1,1,0
S6	1,1,1,0	1,1,1,0	1,1,1,0	1,1,1,1	1,1,0,0	1,1,1,0	1,1,1,0	1,1,1,1
S7	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0
S8	1,1,0,0	1,1,1,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,1,0
S9	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0
S10	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S11	1,0,0,0	1,0,0,0	1,0,0,0	0,0,0,0 ²	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0
S12	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S13	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0
S14	1,0,0,0	1,0,0,0	0,0,0,0	0,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,1,0,0
S15	1,0,0,0	1,0,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0
S16	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0
S17	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S18	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S19	1,1,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,1,0,0	1,1,0,0	1,1,0,0	1,1,0,0
S20	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
*S21	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0
*S22	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S23	1,0,0,0	1,0,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	1,0,0,0
S24	1,0,0,0	1,1,1,0 ²	1,0,0,0	1,0,0,0	1,1,0,0	1,0,0,0 ²	1,0,0,0	1,0,0,0
S25	1,1,0,0	1,1,0,0	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	0,0,0,0	0,0,0,0
S26	1,0,0,0	1,1,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0	1,0,0,0

*Repeat students

¹Error response pattern

²Unusual response patterns

The results were summarised into percentage of students at each level for the given concepts in the pre- and post-test. This allowed for examination of the data in more detail and provided an overall view of the performance of 26 pre-service teachers in geometry.

Table 4.2
Summary of results (% of sample)

Pre-test

Concept	Level 0	Level 1	Level 2	Level 3	Level 4
Squares	0%	34%	58%	4%	4%
Right Triangles	4%	73%	15%	8%	0%
Circles	0%	65%	31%	4%	0%
Congruency	31%	46%	15% *	8%	0%

*includes one (1,0,1,0) response

Post-test

Concept	Level 0	Level 1	Level 2	Level 3	Level 4
Squares	0%	15%	69%	12%	4%
Right Triangles	8%	31%	46%	8%	8%
Circles	0%	58%	34%	4%	4%
Congruency	4%	58%	27%	8%	4%

4.2.2.1 Shifts in levels across concepts

Table 4.2 revealed the majority of students in this particular class had either levels 1 or 2 understanding of the 'square', 'right triangle', 'circle' and 'congruency' concepts both in pre- and post-test results. Though post-test results indicated some shifts in levels of understanding, the majority of students remained at level 1 or 2. There was an 8% no change in levels of understanding in pre- and post-tests, 31% of students made at least one change in level of understanding, 46% made two changes and 15% made three changes in level of understanding. No student from this sample made a shift in all four concepts.

Students who had taken mathematics in grade 12 performed marginally better than those who had not taken mathematics to this level. However, 50% of the students who had not taken mathematics beyond grade 9, performed as well as two of the grade 12 students (see S2 and S7 results). Four students from the same group (S9, S13, S19, S21) achieved higher levels of understanding than a lower grade (LG) mathematics grade 12 student (S7). There was very little movement in levels of understanding between levels 3 and 4 and this was only amongst students that had taken mathematics beyond grade 9.

Overall, the results of the post-test confirms the findings of Mayberry (1983) and Lawrie & Pegg (1997) which indicates that the majority of pre-service teachers in this study are operating at levels 1 and 2. This means that students can recognise concepts and list their properties but have difficulty understanding the relationships between the properties. The Lawrie and Mayberry results revealed higher levels of understanding than those found in this sample, however both of those studies comprised of 60 - 70% of students who had taken high school mathematics compared to +/-30% in this study.

Table 4.3

**Highest level reached by the Australian students for each concept
(% of sample: 60 first-year primary-teacher trainees) - Lawrie & Pegg (1997)**

Concept	No Level	Level 1	Level 2	Level 3	Level 4
Squares	0%	3%	84%	7%	7%
Right Triangles	3%	19%	55%	19%	3%
Circles	0%	13%	19%	52%	16%
Congruency	0%	32%	35%	3%	29%

Table 4.4

**Highest level reached by the Mayberry students for each concept
(% of sample: 19 pre-service teachers) - Mayberry (1983)**

Concept	No Level	Level 1	Level 2	Level 3	Level 4
Squares	0%	11%	32%	26%	32%
Right Triangles	26%	21%	21%	16%	16%
Circles	5%	11%	16%	21%	47%
Congruency	0%	21%	32%	21%	26%

4.2.2.2 Shifts in levels within concepts

There was an overall 54% increase in levels of understanding in 'right triangles', 50% in 'congruency', 38% in 'circles' and 23% in 'squares'. The largest shift in understanding occurred in 'right triangles'. This was most noticeable in the shift from level 1 to level 2 (35%). The second biggest shift in levels of understanding was in 'congruency' from no level of understanding to level 1 (27%). The third biggest shift was in 'circles' from level 1 to level 2 (19%) and the final biggest shift was in 'squares' from level 1 to level 2 (15%). The results of level 0 understanding which meant students could not recognise or visualise the concept was highest in 'congruency' (31%). This shifted substantially in the post-test to reveal that only 4% of the class were unable to recognise 'congruency' upon completion of the geometry course.

Pre-test results revealed that students had highest level 1 understanding of 'right triangles' (73%) and 'circles' (65%). This meant that almost three-quarters of the class could recognise and name the properties of these concepts. However, post-test findings indicated that well over half the class continued to have only level 1 understanding of 'circles' having completed the course.

The concept with the highest level 2 understanding was 'squares' followed by 'circles' and post-test results indicated that this continued to be the pattern.

There were few students with level 3 understanding of concepts in the pre-test, the highest percentage being in right triangles and congruency. This changed in the post-test in which 12% of students showed level 3 understanding of 'squares' with no shift in the 'right triangle' and 'congruency' concepts. All students that reached level 3 understanding in the respective concepts had taken mathematics beyond grade 9.

There was very little evidence of level four understanding in the pre-test except for the 'square' concept. Post-test results indicated that there was a shift in all other concepts amongst students that had taken grade 12 mathematics (see S5 and S6). It is significant that students who had taken mathematics in grade 12 and are assumed to be at Van Hiele level 3 and 4 could not display an overall level 3 or 4 understanding in their responses (Lawrie & Pegg, 1997). It was also apparent that those who had achieved well in the grade 12 examination had higher levels of understanding although the standard grade student (S5) did better on the pre- and post-tests than the student who had taken higher grade mathematics in grade 12 (S6). This would support claims that students who have taken high school mathematics can regress in levels (Burger & Shaughnessy, 1986).

4.2.3 Problems with data collection

The analysis of the instrument examines the role of language, the subjects and problems associated with the implementation of the test. Mayberry (1981) and Lawrie & Pegg (1997) both deal with the shortcomings of the instrument, and it was interesting to note the similarity between their comments and the findings that emerged from this study. The analysis therefore includes some of their thoughts as well as patterns observed from this research.

Students firstly had problems with understanding the requirements of the questions. Questions which asked the students to choose between "certain", "possible" and "never" level 4 options confused students which meant that they often ignored the requirement to

justify their choice. This was only realised in the post-test, by which time it was too late to determine how much effect it would have had on the pre-test results.

Secondly, the sample used for this particular group were heterogeneous and included students from a variety of different schools, diverse cultural backgrounds and with different home/primary languages. The medium of instruction at the college is English but it is not the home language for many of the students. The results of the pre- and post-tests highlight the general understandings of students in geometry regardless of the backgrounds and suggests that cultural and language differences did not play a big part in the overall performance of students in this study. The fact that those students who had taken high school geometry had slightly higher results on some levels corresponded in part with the Mayberry study.

Finally, one of the most difficult problems with the Mayberry test is the fact that no memorandum/solution was included with the test. This assumes that the evaluator and test designer share the same understanding of what the Van Hiele levels mean. Difficulties occurred in the written test in which students did not convey all they meant or included information that was irrelevant. The assessor was then forced to place students on a particular level given the nature of the student's response to other questions on the same concept. Consequently, the allocation of levels then became subject to the knowledge and experience of the researcher, which raises questions of subjectivity.

Lawrie & Pegg (1997: 188-190) identified a number of problems that they had experienced with the implementation of the written version of Mayberry's test. They recorded a high level of response pattern errors (inconsistencies in responses) which caused major problems in validating the test. Upon investigation they found that items of the Mayberry test had the potential to assign students to levels that were not always appropriate.

There were four main elements of the test, which accounted for this:

- Incorrect assignment of a level to certain items (see Appendix A for test items)

Certain items in the Mayberry test did not appear to match the levels they were assigned to, an example of which is item, 31 and 32 which relate to the concept of 'congruency'. It appears as though item 31 requires more sophisticated levels of reasoning than item 32, yet both are rated as level 4 questions in the test. Students' responses in this study to the items, supported the hypothesis as many of the students could answer item 32 correctly but were unable to do item 31. This raises the question of levels. The Van Hiele (in Lawrie & Pegg, 1997) theory indicates that the ability to give a proof of congruency is in fact level 3 understanding, it however makes no distinction between establishing proof when given visual prompts or proving congruency in an 'unprompted situation'. Similar patterns were also observed in this study in the concept of 'squares'. Students with little or no evidence of level 3 understanding of 'squares' made reasonable attempts to answer item 25 which was categorised a level 4 question. Again, the influence of teaching minimum properties or the memorisation of facts cannot be discounted, as few students were able to successfully answer item 26. Therefore item 25 should have been re-allocated to level 3 to reflect a more appropriate analysis of the student's level of thinking.

- Unequal treatment of concepts across levels

It appears that the 'circle' concept did not make clear distinctions between level 2, 3 and 4 questions. As a result, students in this study performed unduly well on a concept for which they have had little exposure. There seems to be an over emphasis on the properties of 'circles' i.e. radius and diameter properties (items 11 and 21), and very little clear distinction between the level 3 (item 30) and level 4 (item 21) questions. The Van Hieles do not write about circles so it is difficult to verify the choices made by Mayberry, however Lawrie & Pegg (1997) found that Australian students did extremely well in this concept and not so well on the more familiar concepts. This study recorded similar findings. In the pre-test results, there were +/- 96% of the students on level 1 or 2 in the 'circles' concept compared to 'squares' at 92%, 'right

triangles' at 88% and 'congruency' at 62%. The post-test results continued to register a higher percentage of students that had either level 1 or 2 understanding, 'circles' (92%), 'squares' (85%), 'congruency' (85%) and 'right triangles' (77%). The fact that students did better overall on circles is difficult to understand when the concept is hardly dealt with in South African primary schools and supports Lawrie's claim that the concepts have been treated unequally across levels. In fact, circles had the biggest downward shift in the post-test, three students (S2, S11 and S24) who had originally been allocated to level 2 understanding of circles, achieved only level 1 in the post-test, which highlights the guessing element of these particular questions.

One of the same students (S11) also showed a downward shift in the 'right triangles' concept. In the pre-test she had successfully chosen the right triangles from the figures provided but was unable to repeat the same action when re-tested on the same item.

- Uneven distribution of questions across levels

It was found in this study and in Lawrie's that test items were not distributed evenly across the concepts. Item 12 is a case in point as revealed in the response pattern of Student 5 (S5) in the 'congruency' concept, level 2. Although s/he indicated that the sides and angles were equal and chose the correct corresponding side, s/he did not choose the correct angle, which meant that s/he did not meet the success criteria for level 2. However the same student successfully managed to find the relationships, implications and class inclusion in the level 3 questions (items 22, 23, 24) of this concept, indicating that s/he did have the necessary understanding for this level.

This error may not have occurred if more opportunities to display level 2 understanding existed in other test items. It indicates to us that the test is simply a guide to a student's thinking and is not a foolproof method of evaluation.

A detailed examination of the distribution of the test items across levels revealed that the Mayberry test appeared to allow for equal opportunity for a concept to be assessed at level 1. However, in level 2, there were a number of questions related to 'squares' and 'circles' but only one question each on 'right triangles' and 'congruency'. This meant that students had only one opportunity to display their level of understanding in

these particular concepts, which was not a fair distribution of questions across the levels. Level 3 questions were more evenly distributed across the four concepts though the concept of definition was ignored in 'congruency' and the 'squares' concept had more level 3 questions than the other concepts. It was also clear from the marking proforma that a student had more than one opportunity to display level 4 understanding in the 'square' and 'right triangle' concepts compared to a single assessment of 'circles' and 'congruency'.

- **Unbalanced distribution of question focus within levels**

Mayberry's scoring seems to 'adversely affect' those students who may not have been exposed to a 'particular aspect of a form of reasoning'. This can be seen most clearly in the 'square' concept, which includes class inclusion for level 3 thinking which, is problematic if a student has not been exposed to this aspect of reasoning. Lawrie & Pegg (1997) believe that this inclusion creates a false impression of a student's level of understanding as class inclusion (a square is also a rectangle) is closely linked to the student's instructional experience and may not mean that a student is not operating at level 3. Many students are able to identify minimum properties and see relationships and implications without being able to look for inclusions. Pegg suggests a re-definition of the descriptions of level 3 to include the 'willingness, ability and the perceived need to discuss the issue'. Shaughnessy & Burger (1985: 423) also found that class inclusions were seldom recognised without a lot of probing even by students who had taken a year course in geometry.

4.2.4 Summary of findings of pre- and post-test results

The results indicate that the majority of pre-service students in this study are operating at level 1 or/and 2. Low levels of understanding amongst pre-service teachers are consistent with the Mayberry and Lawrie & Pegg findings. Given the clustering of results in this area, it makes it difficult to confirm the hierarchical nature of the levels, which is similar to the findings of the Mayberry study (1981). Although the results were clustered in the lower levels, the data did support the notion that the 'higher the level, the lower the

degree of acquisition' (Lawrie, 1998: 180). Students in the study also do not appear to 'think at the same level across different concepts' as evident in the pre- and post-test results.

Although prior exposure to a concept clearly has had an influence on test ratings, the extent of this influence is difficult to determine given the lack of information on student backgrounds. Various studies indicate that ex-high school mathematics students may regress if they have not been exposed to geometry for some time, the poor performances of the S1-S8 group, in the pre- and post-tests supported this. However, there is evidence to indicate that students who had taken mathematics to grade 12 (S5 and S6) and did well in the final examination tended to record higher levels of understanding than those that had not taken mathematics to grade 12 or who had done poorly at that level. The tests revealed that non-high school mathematics students do not appear to be any worse off than those who had taken high school geometry and not done well. It was also apparent not all students who had taken mathematics to grade 12 had an overall level 3 and 4 understanding of geometry concepts.

The allocation to a level depends very much on prior exposure to that concept and to how much was understood of the concept. Very little work in this particular course reflected the development of the 'circle' concept and it was obvious from the lack of shifts in levels of responses that students had little or no opportunity to improve their levels of understanding.

Students were overly familiar with the 'square' concept, which made it difficult to determine whether they had simply regurgitated the properties or whether they really understood them. The post-test results indicate that 69% of the students were on level 2 for this concept and there was a recorded 23% shift upwards in levels within the concept, compared to 53% for right triangles, 38% in circles and 50% in congruency. This reflected some movement within the levels for this concept but considering the fact that well over

50% of the class had some level 2 understanding beforehand, perhaps more activities should have been geared towards developing level 3 thinking. The low level 3 ratings could indicate the course was focusing on the development of level 1 and 2 understanding and not providing students with enough opportunity to develop informal deductive reasoning. Alternatively, such low level 1 and 2 understandings mean that students need to develop these levels more consistently in all concepts before they can begin to develop level 3 understanding.

Poor levels of understanding in the pre-test on the 'congruency' concept could be attributed to the fact that this concept is only developed in school mathematics at Grade 9 when students may have already developed an anxiety towards mathematics. Students also had problems in understanding what the term meant and therefore could not readily answer the question. It was interesting that the pre-test and post-test results indicate that the move from level 0 to level 1 was most significant in 'congruency'. The concept was introduced in the course as a result of responses in the pre-test. This supports the Van Hiele theory that if students are instructed at the appropriate level, they will develop accordingly, if there is a mismatch between the teacher and the learner then concept development cannot occur (Usiskin, 1982; Burger, 1992).

4.3 FOCUS TWO: Does the Van Hiele model appropriately describe students' progress through the course?

It would have been difficult to examine the appropriateness of the Van Hiele model using the whole sample in the time available, therefore focus groups were introduced. As explained in chapter 3, the groups were chosen for the particular purpose of tracking the students progress from the pre-test through the course to the post-test. The purpose was to determine whether their performance in course work corresponded with their Van Hiele levels on the pre- and post-tests and to examine the appropriateness of the model in interpreting student performance. The focus groups comprised of the following students: group A (S1, S2, S7, and S8) and group B (S12, S15, S19, and S20). This analysis looked at students' Van Hiele levels and how these relate student performance in the end

group A (S1, S2, S7, and S8) and group B (S12, S15, S19, and S20). This analysis looked at students' Van Hiele levels and how these relate student performance in the end of module examination. An overview of the knowledge, skills and attitudes students bring to the learning environment is included in the analysis. An attempt has been made to use the Van Hiele model to reflect on students' progress throughout the course.

4.3.1 Prior knowledge and poster

Students in group A appeared to have had a good experience of school mathematics and most had taken mathematics beyond grade 9. They recalled instruction as traditional, which generally meant that it had followed a set procedure: explanation of an example and practice of similar examples

‘they (teachers) read over the introduction of a chapter and then did a few examples on the board’ (S7) and ‘they showed us by using examples, then expected us to do it’ (S2).

They had covered content related to both regular and irregular shapes as well as angles. When asked about how they would teach geometry, given their experience of teaching so far, the consensus was they would use a mixture of tasks involving problem-solving, experimentation, drawing, construction and discussion linked to real-life experiences and use opportunities to take children outside:

‘first of all I would start teaching geometry by taking students outdoors and ask them to draw the shapes they see’ (S1) and ‘basically it would consist of letting the child experience doing problems’ (S8).

When asked to illustrate what geometry meant for the group, they included a variety of images that have been categorised into 5 topics: direction, application, school content (vocabulary and theorems), measurement and construction. Direction included the image of a compass with North, South, East and West bearings. The application category referred to the use of geometry in real-life contexts and included images of a tree, star, flower, building, bridge, river, leaf, and a mountain. The school content category had regular and irregular polygons: rectangle, square, parallelogram, right triangle, an equilateral triangle, circle and different types of angles, etc. Measurement included

markings on a railway track and a syringe with volume with calibrations on the outside. The final category denoted as “construction” for clarity purposes contained the model of a box, which was drawn as a net. There appeared to be a diversity of thinking on what geometry was and how it was linked to everyday life.

Group B was comprised of students who had not taken mathematics beyond grade 9. They had dropped mathematics as soon as was legally possible as they believed that they were not good at it. They remembered lots about rules and vocabulary related to geometry. This was later confirmed through their graphical representation (poster) of what geometry is for them. In terms of teaching, they highlighted the need for the use of pictures, the application of geometry through inclusion of body parts and nature, and for children to be actively involved through group work:

‘I can teach it by letting them cut out all shapes’ (S15), ‘I can also asked them to do the mobiles for geometry’ (S19) and “When I teach geometry I would like to use parts of the bodies and nature because is too easy to remember’ (S20).

Group B’s poster did not contain as much variety in terms of imagery as group A. The content of the poster could be divided into 2 main topics: school content and application. The school content category contained similar images to the previous group, namely, equilateral triangle, rectangle, square, rectangle, circle, octagon, parallel lines, right angles and vertically opposite angles with much of the emphasis on the rules that are learned in geometry. The applied geometry included only one image, which was that of a flower. It was obvious from the poster that these students saw geometry as partially linked to real-life but emphasised theorems that they had been taught at school. They appeared to have a good knowledge of Euclidean geometry but could not see its application beyond the syllabus.

4.3.2 Test and examination results

The levels indicated in Table 4.5 and 4.6 provide a summary of the focus groups’ pre-, post tests and examination results. Some students changed level between the tests which is indicated on the table, a > indicates an increase and < indicates a decrease in level of

understanding which is followed by the new level obtained. If a student did not change levels then the pre-test result was not altered. The students took the post-test before they wrote the end of module examination so we can assume that they were operating on the post-test level in these concepts for the examination.

Table 4.5
Pre- and post-test and examination results (Group A)

Group A	Squares	Triangles	Circles	Congruency	Examination Results
S1	L2	L1 > L2	L1 > L2	L2	55%
S2	L2	L1	L2 < L1	L1 > L2	53%
S7	L2	L2	L2	L1	67%
S8	L2>L3	L2	L2	L2>L3	83%

Table 4.6
Pre- and post-test and examination results (Group B)

Group B	Squares	Triangles	Circles	Congruency	Examination Results
S12	L2	L1	L1	L0 > L1	45%
S15	L1	L1 > L2	L1	L1	22%
S19	L2	L1	L2	L2	57%
S20	L1 > L2	L1	L1	L0>L1	25%

Most of the students in group A appear to be operating at level 2 for each of the given concepts. The content of the examination was related to the course work and did not always correspond with the concepts tested in Mayberry. The examination contained a variety of questions, which required level 1,2 and 3 understanding (see Appendix C). It was apparent from these results that students' performance in examinations is very much related to their level of understanding of the concept at that time. It also supports the Van Hiele theory that if work is assessed at a particular level then students need to be operating

at that level to be successful. Hence, the performance of S15 and S20, who are most obviously operating at level 1, is not surprising given the level of the examination.

The questions in the examination were fairly typical of questions found in most grade 8 and 9 textbooks. Thus, the findings imply that students need to be operating at least at level 2 or 3 in most concepts in order to pass a Grade 8 test.

It is apparent that students can move up a level and it is equally apparent that they can also drop a level as can be seen in S3's results. The drop in level in the 'circle' concept is a result of the poor nature of the level 2 circle questions as highlighted in the analysis on the assessment tool. It allowed students to make uninformed guesses, which could give a false impression of a student's level of understanding.

4.3.3 Examination questions and group performances

The overall examination results helped provide some comparative value to the pre- and post-test results. However, consideration of student performance on individual questions was necessary to give substance to the patterns observed above. Each groups' response to questions was analysed to find evidence to support some of the observations mentioned already. The following table is an average of each group's performance in particular questions:

Table 4.7

Summary of examination results (% average per question per group)

Question	Level of response possible	Group A	Group B
1.1	level 1		
1.2	level 1	54%	43%
1.3	level 1		
1.4	level 1-4		
2.1	level 1	90%	56%
2.2	level 1 or 3		
3	not applicable	67%	48%
4	level 2	60%	18%
5.1	level 2	56%	10%
5.2	level 2, early level 3		

The first question which involved the identification of quadrilaterals was judged to be level 1 (see Appendix F) as no properties were included in the sketches and the decision had to be made on appearance alone (Mayberry, 1981: 47). When students were asked to minimise properties (level 4) in question 1.4, neither group could provide a suitable answer.

It is obvious from the results of question 2 that most students have a firm level 1 understanding of quadrilaterals. Question 3 required students to discuss the Van Hiele levels of understanding and to link these to questions 1 and 2. The results of the examination and students assignment work indicated that students in both groups had not fully comprehended the theory nor were they able to recognise the levels within examination questions.

Question 4 linked perimeter and area to a real context. The question required students to problem solve the context and to use the appropriate measurement concept. It was categorised as requiring level 2 understanding. In group A, all except one student had a good understanding of the question while 2 students in group B showed similar levels.

The final question required students to calculate angles from a given sketch using real numbers and entailed providing reasons. In group A, both students who were strong level 2 and early level 3 in the post test results did well on this question. Students who operated at level 1 for other concepts, from group A and B, did not do well on this question. The implication is that students operating on a lower level to that required in the assessment task tend to perform poorly, compared to those on the same level of thinking as required by the question.

4.3.4 Summary of findings

It is apparent from the patterns mentioned earlier that the Van Hiele model is useful in allowing us to examine in detail the understandings students displayed through the use of

The test results indicate students' current levels of understanding in a given concept often equates with levels in other concepts (refer examination questions). The model highlights the fact that students do not do well in questions that are beyond their level of understanding, the proof of which can be seen in S15 and S20's examination results. It emphasises the need for learners to develop a good level 1 and 2 understanding in primary school so that they are able to cope with the geometry requirements of high school. At the same time we need to recognise that most primary school teachers are themselves operating at the same levels as their learners regardless of the amount of mathematics that they may have done at school.

The poster and the written accounts of prior learning seen through the lens of the Van Hiele model allow us to analyse the experiences of students. The over-emphasis at school level of the acquisition of geometry rules in an attempt to develop informal deduction (level 3) necessary for deductive reasoning (level 4), means that students often resort to memorisation. Van Hiele (Crowley, 1987: 4) points out that it is possible to teach

... a skilful pupil abilities above his actual level, like one can train young children in the arithmetic of fractions without telling them what fractions mean....

In situations such as these, the learner reduces the subject matter to a lower level of understanding so that some understanding can occur. The implications for instruction is that teachers need to construct learning experiences that are best suited to the learner's level of understanding. It also means that Colleges of Education will need to make sure that they are able to produce teachers that have a firm grounding in geometry principles to at least level 3.

The data collected in this study reveals a strong consonance between time spent engaged in the learning of a subject and levels of understanding. It would appear that the longer the time spent in the study of mathematics, the more the likelihood that students will develop a positive attitude towards the subject, which means that those with higher levels of understanding tend to be those who have taken mathematics beyond grade 9. On the other hand, this does not mean that those with less than grade 7 mathematics cannot

progress and do well (see for example S12 and S19). There is no guarantee that students taking grade 12 mathematics will have developed level 4 understanding but the data presented here implies that it is more likely that such students will have a higher level of understanding than students who drop mathematics in grade 7.

The usefulness of the model in interpreting students' progress means that it 'can be used to guide instruction as well as assess student abilities' (Crowley, 1987: 1). It also helps the teacher to understand her learners better and to design learning experiences accordingly.

4.4 FOCUS THREE: Do pre-service teachers integrate the theoretical and content elements of the course to select suitable activities for learners?

4.4.1 Course assignment

The extent to which students could take the mathematical content and methodology they had been given in class and design appropriate learning activities for their learners was another focus of this research. Students were given the task of designing suitable learning activities in terms of the Van Hiele theory and syllabus requirements. As the subjects were second year students and still had to complete another year of study for certification, they did not have to design original tasks and could take ideas from books available in the library, as long as they justified their choice.

It was crucial for future course design to try to establish the extent to which students were able to assimilate theory and content and to try to uncover possible problem areas. Much of the course time had been spent on developing students' knowledge of the primary school geometry curriculum and discussing methods of instruction. Students had been given practical examples of the Van Hiele theory and its application in geometry instruction. Hence, it was necessary to determine if the course was able to shift students in terms of their content knowledge, and to observe how this was translated into the designing of learning activities (see Appendix G for assessment rubric).

in terms of their content knowledge, and to observe how this was translated into the designing of learning activities (see Appendix G for assessment rubric).

Table 4.8
Focus Groups: Student progress in the course

Students	Prior mathematics	Post-test results	Examination result	Examination result	Assignment result
Group A		Square/Triangle/ Circle/Congruency	Q3 - theory	Q1,2,4,5- content	
S1	Grade 12 (SG-15%)	2 2 2 2	60%	54%	35%
S2	Grade 12 (LG -)	2 1 1 2	60%	52%	53%
S7	Grade 12 (SG-E)	2 2 2 1	40%	72%	45%
S8	Grade 10	2 3 2 3	80%	84%	70%
Group B					
S12	Grade 9	2 1 1 1	30%	48%	45%
S15	Grade 9	1 2 1 1	0%	26%	35%
S19	Grade 9	2 1 2 2	40%	60%	60%
S20	Grade 9	2 1 1 1	0%	30%	45%

The content and theory knowledge reflected by the examination mark appeared to have a significant influence on the student's ability to select appropriate activities for the teaching of geometry (in all except one student - S7). Those who had done poorly in the examination both in theory and content tended to select inappropriate geometry activities for classroom instruction (see S15, S20 and S12). For example, S12 revealed some understanding of the concept of perimeter and area but in designing an appropriate learning experience resorted to telling the learner the rule:

... when they finish they must count the many blocks are found in the figure in the length and in the base. When they finish they must multiply the base by the length to get the area.

It appears that students with poor content and theoretical knowledge of a subject have great difficulty in selecting and designing appropriate learning activities. Poor content knowledge is also linked to low levels of understanding on the Van Hiele scale (see results

of S12, S15 and S20). Low levels of understanding in geometry makes it more likely for students to perform poorly in new concept development of other aspects of geometry (see examination results of S15 and S20). Therefore, until levels of understanding improve, which are linked to how content is presented and acquired, pre-service teachers will continue to find it difficult to fully exploit a learning situation.

Students used material available in the library for the selection of teaching activities. These resources appeared to develop concepts in geometry from recognition to analysis leading to informal deduction. S20 referred to a text, which indicated:

I think geometry is to provide the types of experiences necessary to enable a child to make the transition to deductive geometry in a natural and meaningful way.

Therefore, if correctly designed and structured material is available for teachers with poor content knowledge, it may remedy some of the poor instruction currently taking place in our schools.

Both groups of students emphasised the need for fun, interesting and relevant activities as a crucial element of the classroom activity. There had to be opportunity for visual, verbal, drawing, logical and applied skill development using suitable apparatus

‘these activities were chosen to allow the children to experiment and play around with symmetry. They will be able to use their visual skills to identify the shapes they are experimenting with...the objective of this activity is to see if the children understand the terminology..’(S2)

Although both focus groups recognised the need to develop skills within geometry. There was very little reference or linkage to the Van Hiele theory with the exception of one student

‘The learners should at the end of the activity be able to see the differences between, acute, right, obtuse and reflex angles. They should also be able to describe the angles in their own words. This activity is aimed at Van Hiele level of visualisation. It is aimed for the beginning of teaching the above-mentioned angles. This also ties in to Hoffa’s (Hoffer) visual and verbal skills’ (S8)

4.4.2 Summary of findings

The theoretical aspects of the course which students (Group A & B) appeared to understand and utilise were based almost entirely on the need for skill development in geometry. Geometry teaching for these students was influenced by their lack of interesting and relevant experiences during their own school experience. An examination of data from their prior learning experience supports the view reflected here. These pre-service teachers recognise the need for a change in instructional methods but do not appear to have related this to the levels of understanding of their learners. There seems to be little cognisance of the fact that although activities may be more interesting and skills related, it is very difficult for a learner to develop appropriate understanding if the activity is at an inappropriate level for the learner. Students need to realise that a positive, “fun” learning experience does not automatically lead to a better understanding of geometry concepts. In all of the activities suggested, by students in both groups, there was insufficient time given for concept development in the early stages of learning. Research indicates that teachers move too quickly from visualisation (Level 1) to analysis (Level 2) without giving learners sufficient time and experience to grasp the fundamentals of concepts.

It is obvious from students’ work that although some integration of the theoretical aspects of the course were successfully married with geometry content, a lot more time needs to be given to identifying the Van Hiele levels in activities. Students need to have a strong base of content knowledge of the topic combined with a thorough understanding of how learners come to make sense of geometry before they can design learning activities that meet the needs of the learner. They need to have lots of opportunities to design activities that relate to learners’ levels which means learning to practically apply the Van Hiele theory.

4.5 RÉSUMÉ

The purpose of this chapter was to look at the data collected from students involved in a geometry course. It meant assessing students' levels of understanding and monitoring their progress as they developed further understanding of geometry, both its processes and content, and its theoretical underpinnings.

There was an opportunity to focus on the development of 8 particular students and to establish a profile of their thinking. Having determined the extent of their understanding and its application to new concept development, it was then possible to examine their progress in detail and to draw some conclusions.

It appears that students' levels of understanding directly impacts on how they accommodate exposure to new concepts in geometry and how they design and choose learning activities. Students who have weak content knowledge and poor levels of understanding of some of the fundamental concepts in geometry find it difficult to fully comprehend the need to accommodate levels of understanding of learners. They tend to over emphasize the need for interaction and 'real-life' experiences to the detriment of structured well-planned learning experiences. It is clear pre-service teachers need to be apprenticed in this task if any real change is to occur in geometry instruction in our schools.

CHAPTER FIVE: CONCLUSION AND RECOMMENDATIONS

The final chapter of this dissertation draws together elements of the research completed so far. It links the aims of the study to the analysis of the data and highlights some of the obvious conclusions. The implications of such conclusions are discussed in light of the limitations and size of the sample of the study. Recommendations are made in terms of future research and implications for effective teacher training are discussed so that the teaching of geometry can be more worthwhile for teachers and learners.

5.1 AIM OF THE STUDY

The aim of the study was to examine the impact of a geometry course on student teachers' levels of understanding of geometry. It was an attempt to address the following questions:

1. Can a geometry course shift pre-service teachers' levels of understanding of geometry?
2. Does the Van Hiele model appropriately describe students' progress throughout the course?
3. Do pre-service teachers integrate the theoretical and content elements of the course to select suitable learning activities for learners?

The Van Hiele model of development provided the theoretical base on which the impact of the course could be monitored. It provided a conceptual framework to assess and analyse the progress of students while at the same time allowing for a deeper understanding of student reasoning. An instrument was chosen to assess students' levels of understanding prior to the course and some of the subsequent written work of students was used to verify the initial allocation of levels. Students were re-tested on completion of the course and an attempt was made to assess students' ability to link the theory and content of the course to design suitable learning activities.

5.2 LIMITATIONS OF THE STUDY IN TERMS OF RESEARCH METHODS

There are a number of possible limitations in the study, in terms of research methods. Firstly, choosing an assessment instrument, which provides opportunities for learners to show what they are capable of, is not easy. Considering the varied language and mathematical background of the students, meant that it was almost impossible to find an instrument that was all inclusive of student diversity. There was no opportunity or time to follow up discrepancies in test results through individual interviews, therefore inconsistencies in results were left to the interpretation of the researcher. This ultimately means that it is difficult to account for the interference of bias and subjectivity (Cohen and Manion, 1984).

Secondly, the Mayberry test evaluated students' understandings of only four particular concepts: 'squares', 'right triangles', 'circles' and 'congruency'. Although the content of the course did expose students to other concepts, it was not possible given the limited time frame of this study to determine the shift in levels of understanding in the other concept areas covered.

Thirdly, there is contention that the pedagogical sequence suggested by the Van Hiele's is viewed by some as 'more common sense than justified theory' (Schoenfeld, 1986: 262). The foundation of their theory is based solely on the empirical evidence observed during their teaching experiment and the justifications given for their findings are 'loose by any rigorous standards' (ibid.). This implies that the theoretical framework may not be as rigid as research would have us believe.

Fourthly, the lack of detailed information on students prior knowledge of geometry and the length of time between dropping mathematics and taking the geometry course meant that students could have regressed in levels of thinking. In addition, the sample size of 26 students, which was later concentrated on two focus groups of eight students, does not lend itself to generalisations of student thinking. A larger sample would be necessary with the introduction of more comparative groups.

lend itself to generalisations of student thinking. A larger sample would be necessary with the introduction of more comparative groups.

Finally, the students' written work, as well as the assessment of the final assignment, was subject to the researcher's interpretation of the Van Hiele theory. Although much of the interpretation was verified through correspondence with Lawrie, the issue of bias and subjectivity cannot be ignored.

In conclusion, though limitations do exist and need to be accounted for, this study on the impact of a course on students' levels of understanding, allows us to take a closer look at the sense students make of geometry. It is an opportunity to evaluate a course given to pre-service teachers at a college of education and to determine whether the course meets the needs and levels of the students. It provides the opportunity for the course to be re-interpreted so that it can become a more effective instrument in developing pre-service teachers' understanding of geometry.

5.3 CONCLUSIONS AND IMPLICATIONS

5.3.1 Focus One: Can a geometry course shift pre-service teachers' levels of understanding of geometry?

Yes, as evidence indicates in this study. However, given the low levels of improvement in students' understanding, it is difficult to establish significant change in a nine-week period. Teaching was not specifically geared to the levels of the students, as the purpose of the research was to examine the impact of the current course on students' understanding. Perhaps if instruction was matched more appropriately with the students' levels of thinking, there may have been a bigger improvement.

There is ample evidence to support the theory that geometric thought does develop in a sequence of levels and that failure to develop on one level prevents achievement on higher levels (Mayberry 1981: 96). The general Van Hiele level of the pre-service elementary

teachers in the study was low and pre-college students are operating largely at level 1 and level 2 understanding. Those with some high school geometry appear to be in a slightly stronger position although this is not guaranteed.

Few students leave high school with level 4 understanding although this is difficult to determine from this data, as there were too few successful high school mathematics students.

5.3.2 Focus Two: Does the Van Hiele model appropriately describe students' progress throughout the course?

Although the model is useful for evaluation purposes and instruction, it can also assist with the tracking of student development. It was apparent from the research that students with limited perspectives of what geometry is tended to be the same students with low levels of understanding. This was further translated into the acquisition of new concepts in which the same students continued to have low levels of understanding. It is apparent that the higher the level of understanding, the more likely the student is to reach higher levels more rapidly in learning new concepts.

The model highlighted that a prolonged period of exposure to mathematics learning can act as a positive factor in developing understanding. Students who had taken mathematics to grade 12 tended to perform better than those that had dropped mathematics in grade 9. The performance of students in the examination depended very much on their level of understanding and the level required by the examination question. Students who had a mismatch between their levels of thinking and that required by the assessment task did not do as well as those on the same level.

5.3.3 Focus Three: Do pre-service teachers integrate the theoretical and content elements of the course to select suitable learning activities for learners?

The results of the focus groups (eight students) indicate that students struggled to make connections with the theory and geometric content of the course. The notion of meeting or confronting a level as Van Hiele described in 1980 may indeed be a real phenomenon in

mathematics teaching. This means that teachers and students may have different meanings about different concepts. For example: if a teacher talks about triangles, she has a particular picture in her mind which may include all types of triangles, the learner on the other hand may be thinking of the equilateral triangle and nothing more. If the concept is taken further by the teacher and used to develop other concepts, the student may be completely confused and once again be forced to memorise because of lack of understanding.

The Van Hiele model is useful in describing how students think and is concerned with the teaching needed to match the level of the child. However, pre-service teachers who realise that the teaching they experienced at school may have deficiencies in that it did not match the level of the learner, often embrace alternative teaching strategies with open, naïve abandon. They are determined to change their classroom practice and include all sorts of interesting activities and materials. Yet somehow the need to extend intuitive understandings is lost. They see elementary geometry as fun, exciting and relevant to the children's lives which indeed it should be, but they neglect to take the geometry further, once again perpetuating poor learning and teaching experiences.

It is more likely that teachers are unaware of the thinking, which is necessary in geometry development. They do not see that the importance of the empirical experience that helps build inductive and deductive skills. They do not see the 'foundations on which geometric performance are based include both inductive and deductive competencies' (Schoenfeld, 1986: 226). The students are a product of 'learned behaviour' patterns, they teach geometry as they were taught, they select activities and instruct accordingly.

If we want to 'develop a thoughtful view of the kinds of things we expect students to learn - and the kind of connections we expect them to make', then Van Hiele (1986: 96) believes that students must 'have been busy in some way with mathematics in order to develop by careful analysis the logical structure of the internal connections of mathematics' (Schoenfeld, 1986: 263). We need to make sure that our geometry courses

5.4 RECOMMENDATIONS

There are a number of issues that need to be addressed if we hope to improve teachers' understanding of geometry. Firstly, it is necessary to design appropriate experiences for pre-service and in-service elementary and high school teachers to help them understand the levels and achieve those levels appropriate for their teaching (Mayberry 1981: 99). The design of materials for instruction at each of the levels needs to be developed by those who understand the principles of the theory.

Secondly, elementary textbooks need to be restructured with an emphasis on activities, which would allow students to develop level 3 thinking in formal geometry courses. In conjunction with this, there is a need for extensive in-service and pre-service training of elementary teachers as many of the new innovations find their way into the textbook but not into the classroom (Mayberry 1981: 96). This means that high school teachers will also need to be aware of the kind of activities they will need to include in their classrooms, as some primary school learners may not be at level 3 when they reach high school.

Thirdly, the Van Hiele theory is useful in assisting teachers to design learning experiences that build on the informal geometry experience and knowledge of the learners to develop the formal structures needed later. A geometry course for primary school children which takes cognisance of the necessary theory also needs to include activities that utilises children's spatial skills and leads to insight (van Niekerk, 1995). This means that certain key aspects of geometry need to be included in the curriculum. The Dutch have proposed four key areas of spatial development, namely; visual geometry; geometry of shapes and figures; geometry of location; and calculations in geometry. While the school curriculum has tended to emphasis the geometry of shape and calculations, we have at the same time ignored the visual and location aspects, as well as the South African context. Curriculum 2005 makes provision for this. However, until teachers are instructed in the practice, it will be difficult for them to change instruction.

Van Niekerk (1995: 11) suggests that the design and implementation of a new geometry curriculum must consider the following:

2005 makes provision for this. However, until teachers are instructed in the practice, it will be difficult for them to change instruction.

Van Niekerk (1995: 11) suggests that the design and implementation of a new geometry curriculum must consider the following:

1. educators should realise the extreme importance of geometry as part of the existing school curriculum
2. an experimental curriculum must be designed by people who specialise in this field
3. the curriculum must be properly tested in the actual classrooms of all the different groups in South Africa, before implementation
4. the success of a geometry curriculum is solely in the hands of the teachers of South Africa, if they are not convinced that their children need geometry for a better life, then very little will change in the classroom.

In addition, more time needs to be given to geometry instruction in the primary school and in junior high school. Much of the research indicates that if students are exposed to the spatial and visual aspects of geometry early on, it helps them to develop geometrical competence more readily. In the Soviet Union, the geometry curriculum was reformed to include the study of shapes, their properties and measurement in the first three years of primary school. This meant that students had completed level 2 tasks by age 10. They then spent the next 7 years studying semi-deductive geometry i.e. level 3 before they began level 4 geometry. The findings of this work indicates that the learner is better prepared for level 4 reasoning than most American learners. Many of the secondary learners interviewed in the Shaughnessy & Burger (1985) study were not sufficiently grounded in basic geometry and hence would be unable to re-invent Euclidean geometry and would most likely be forced to resort to memorisation.

Finally, those responsible for education in this country need to realise that policies must be developed which make teachers and schools accountable for change. If teachers do not have understanding of the conceptual development that needs to be nurtured in learners through instruction, then levels of understanding will continue to remain low. Until the

knowledge, skills and attitudes of teachers are advanced through further INSET work, we will continue to perform poorly on global assessment instruments. Further resources, in terms of time, money and research need to be employed in the up-grading of our teachers in the current education system. If teachers are themselves unable to function at higher levels of understanding, it is little wonder our learners achieve so poorly.

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APPENDIX A
AMENDED SET OF MAYBERRY TEST ITEMS

LEVELS OF THINKING IN GEOMETRY

AMENDED SET OF MAYBERRY TEST ITEMS

MARCH 1998

Name of student: _____

Year Group: _____

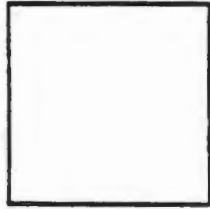
The purpose of this test is to find out how students think in geometry.

Please do your utmost best. The information will be used to make geometry easier.

INSTRUCTIONS

1. This test contains 33 questions. It is not expected that you know everything on this test but try to answer as much as possible.
2. Read questions carefully.
3. You have 50 minutes to complete this test.
4. Work on the test page where possible if you need to work on a separate page, please indicate where question you are referring to.
5. Good luck and thank you for your co-operation.

1. This figure is which of the following?



- a) triangle
- b) quadrilateral
- c) square
- d) parallelogram
- e) rectangle

2.



Are all of these triangles?

YES

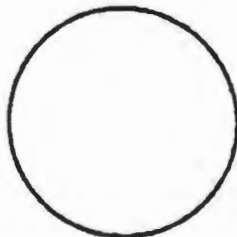
NO

Explain: _____

Do they appear to be a special kind of triangle?

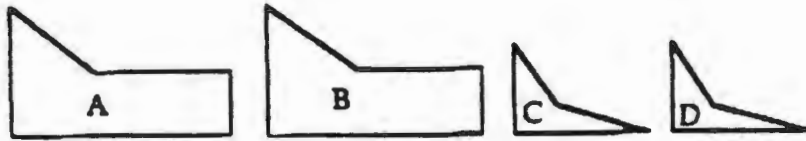
If so what kind? _____

3.



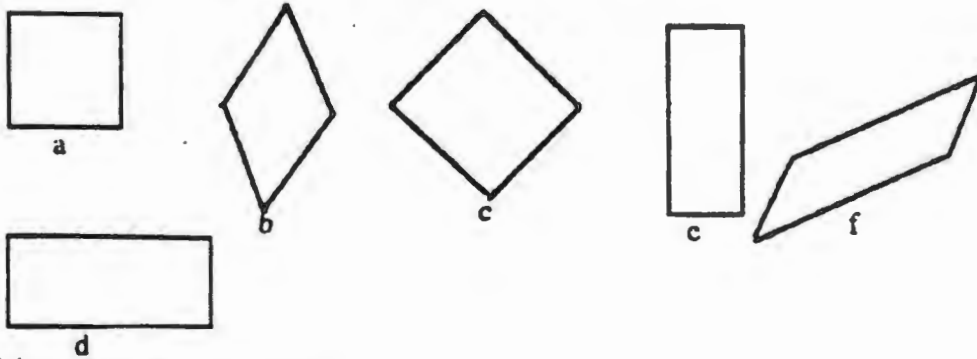
Name this figure: _____

4.



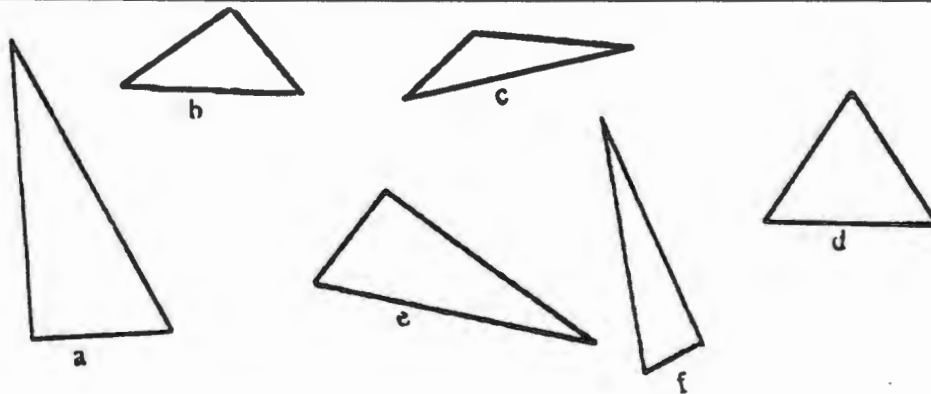
What is true of A and B? What is true of C and D? _____
What word describes this? _____

5.



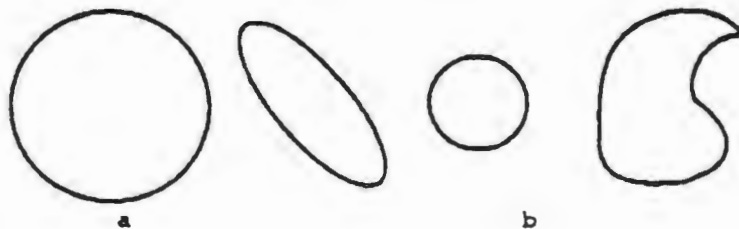
Which of these figures are squares? _____

6.



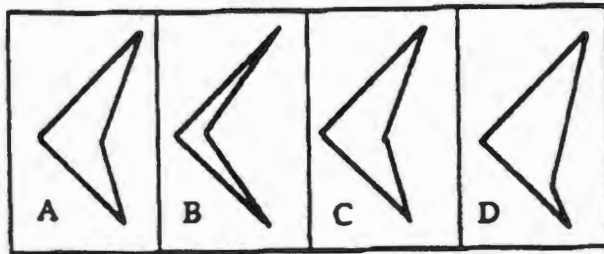
Which of these appear to be right triangles? _____

7.



Which of these are circles? _____

8.



Which figure appears to be congruent? _____

9.

Draw a square

What must be true about the sides? _____

What must be true about the angles? _____

10.

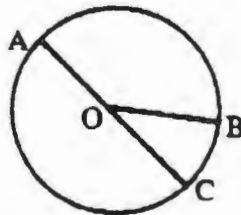
Does a right triangle always have a longest side? _____

If so, which one? _____

Does a right triangle always have a largest angle? _____

If so, which one? _____

11.



This figure is a circle. O is the centre.

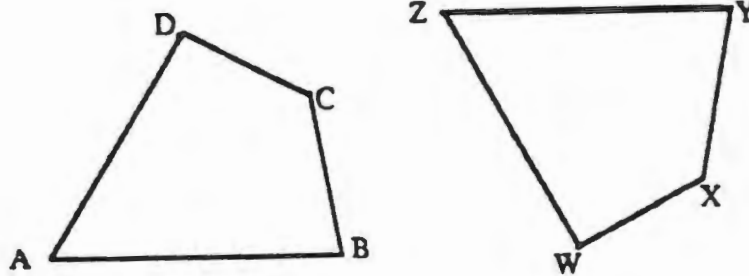
Name the following line segments.

OB is a _____ of the circle.

OC is a _____ of the circle.

AC is a _____ of the circle.

12.

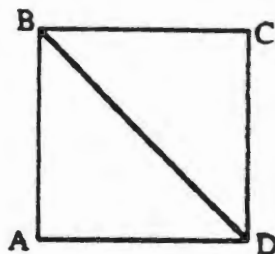


These are congruent figures.

What is true about their sides? _____ $AD =$ _____

What is true about their angles? _____ $\angle B =$ _____

13.



ABCD is a square, BD is a diagonal.

(a) Name an angle congruent to $\angle ABD$ _____

(b) How do you know? _____

14. Circle the smallest combination of the following which guarantees a figure to be a square.

- a. It is a parallelogram.
 - b. It is a rectangle.
 - c. It has right angles.
 - d. Opposite sides are parallel.
 - e. Adjacent sides are equal in length.
 - f. Opposite sides are equal in length.
-

15.

(a) Name some ways in which squares and rectangles are alike? _____

(b) Are all squares also rectangles? Why? _____

16. Circle the smallest combination of the following which guarantees a triangle to be a right triangle?

- a. It has two acute angles.
- b. The measure of the angles add up to 180° .
- c. An altitude is also a side.
- d. The measure of two angles add up to 90° .

17. QAB is a triangle.

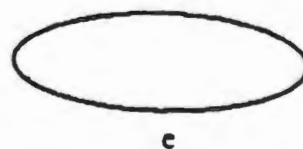
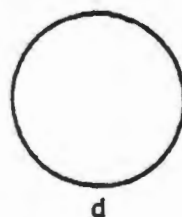
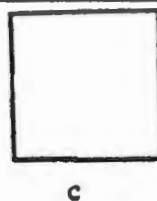
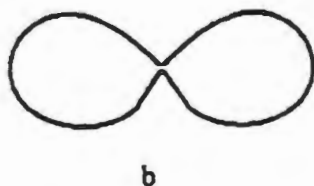
- a) Suppose angle Q is a right angle. Does that tell you anything about angles A and B? If so, what? _____
- b) Suppose angle Q is less than 90° . Could the triangle be a right triangle? Why? _____
- c) Suppose angle Q is more than 90° . Could the triangle be a right triangle? Why? _____

18.

Which are true? Give reasons.

- a) All isosceles triangles are right triangles. _____
- b) Some right triangles are isosceles triangles. _____

19.



Tell why each of these figures is or is not a circle.

- a) _____
 - b) _____
 - c) _____
 - d) _____
 - e) _____
 - f) Can you give a general rule to fit all the above answers?
-

20.

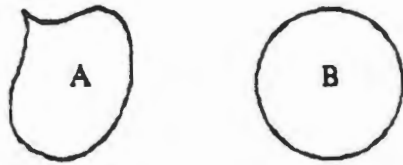


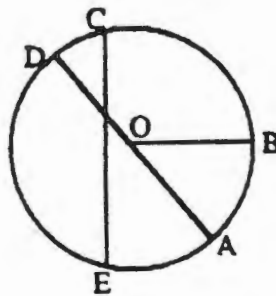
Figure A is a simple closed curve. Figure B is a circle.

Is figure B a simple closed curve? _____

How are these figures alike? How are they different? _____

(T - F) All simple closed curves are circles. _____

21.



This figure is a circle with centre O.

Would the following be

- a) certain b) possible c) impossible

Give reasons for your answer.

- 1) $OB = OA$ _____
- 2) $OD = OA$ _____
- 3) $2OB = AD$ _____
- 4) $AD = EC$ _____

22. Will figures A and B be similar

I - always

II - sometimes or

III - never?

Give reasons for your answers.

A

- a) a square
- b) an isosceles triangle
- c) a triangle congruent to B
- d) a rectangle
- e) a rectangle

B

- a) a square _____
- b) an isosceles triangle _____
- c) a triangle congruent to A _____
- d) a square _____
- e) a triangle _____

23. $\triangle ABC$ is congruent to $\triangle DEF$ (in that order).

Are the following

- a) certain b) possible c) impossible

Give reasons for your answers.

- a) $AB = DE$
b) $\angle A = \angle E$
c) $\angle A < \angle D$
d) $AB = EF$

24. Will figures A and B be congruent

I - always

II - sometimes or

III - never?

Give reasons for your answers.

A

- a) a square
b) a square with a 10cm side
c) a right triangle with a 10cm hypotenuse
d) a circle with 10cm chord
e) a triangle similar to B

B

- a) a triangle _____
b) a square with a 10cm side _____
c) a right triangle with a 10cm hypotenuse _____
d) a circle with 10cm chord _____
e) a triangle similar to A _____

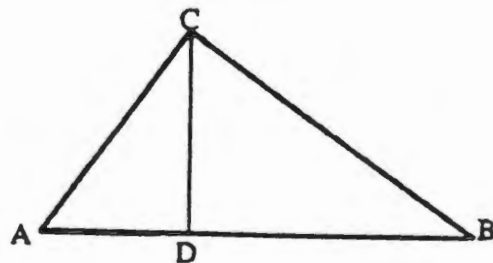
25. ABCD is a four sided figure. Suppose that opposite sides are parallel.

What are the fewest facts necessary to prove that ABCD is a square?

26. Figure ABCD is a parallelogram. $AB = BC$ and $\angle ABC$ is a right angle.

Is ABCD a square? Prove your answer.

27.

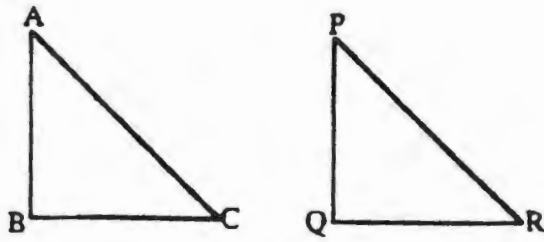


CD is perpendicular to AB. $\angle C$ is a right angle.

If you measure $\angle ACD$ and $\angle B$, you find that they are the same measure.

Would this equality be true for all right triangles? Why or why not? _____

28.



Figures ABC and PQR are right isosceles triangles with angles B and Q being right angles. Prove that $\angle A = \angle P$ and $\angle C = \angle R$

29.

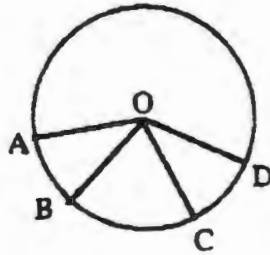


Figure O is a circle. O is the centre.
 $\angle AOB = \angle COD$, so $AB = CD$.

What have we proved? _____

30.

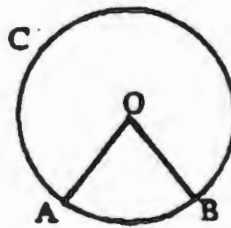
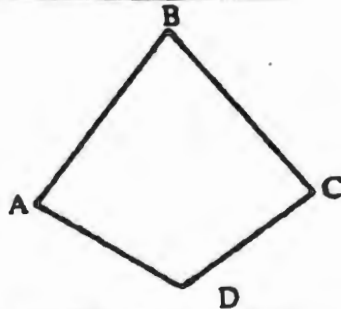


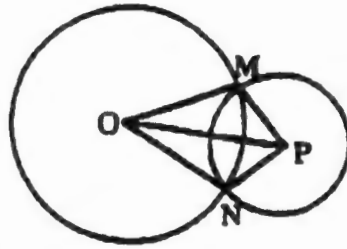
Figure C is a circle. O is the centre.
Prove that triangle AOB is isosceles.

31.



In this figure AB and CB are the same length. AD and CD are the same length.
Will $\angle A$ and $\angle C$ be the same size? Why or why not? _____

32.



These circles with centres O and P intersect at M and N.

Prove: $\triangle OMP \cong \triangle ONP$.

33. Prove that the perpendicular from a point not on the line is the shortest line segment that can be drawn from the point to the line.

APPENDIX B
PRIOR LEARNING QUESTIONNAIRE

HOMEWORK ASSIGNMENT FOR FRIDAY, 24 APRIL 1998

Geometry and Me???

It is difficult for a teacher to design activities if s/he is not aware of the prior knowledge and experience of the learner, therefore I would like you to please answer the following questions about your experience with mathematics/geometry to date??

1. How far did you do mathematics at school? (Standard 5, 6, 7, 8, 9, 10)?
2. Did you pass mathematics at that level? If not, what level did you achieve at?
3. When was the last time you did mathematics before you came to the college?
4. If you did not come straight to the college from matric what did you do in between, e.g. worked, had babies, looked after the home, or tried to find a job?
5. If you did not do mathematics for matric, explain what influenced your decision not to take it?
6. What things do you remember of the geometry you did at school?
7. How did they teach geometry when you were at school?
8. How would you teach geometry (refer to syllabus overleaf) and be honest?
9. What things do you need to know more about in geometry? Please refer to the list at the back of this page and highlight the words/concepts that are not familiar or that you may not have a thorough understanding of.

GEOMETRY

STANDARD 2

1. Shape in 3 dimensions : sphere, cylinder, cone, cube, pyramid, rectangular block and triangular block

- Recognising and naming the above shapes involving physical objects in the classroom, the home, photographs and other places and the ability to talk about them.
- Sorting shapes in different ways and giving reasons for each method of sorting (vocab.: includes solid, hollow, flat, curved, face, edge, corner or vertex, symmetry)
- Investigating the number of faces, edges and corners of the solids
- Fitting shapes together to form new 3-D shapes

2. Shape in 2 dimensions : circle , rectangle, square, triangle

- Tracing the faces of the solids on paper and recognising and naming the 2-D shapes
- Identifying shapes in a variety of orientations from drawings, photographs, patterns, cut-outs and physical objects in the class-room, the home and other places
- Sorting the shapes in different ways and giving reasons for each method of sorting (vocab.: includes flat, round, side, corner, square-corner, axis of symmetry)
- Creating shapes on dotted paper, square paper, geo-boards
- Fitting shapes together to form new 2-D shapes; tangrams

STANDARD 3

1. Quadrilaterals

- Creating new 4-sided shapes on geo-boards
- Sorting 4-sided shapes in different ways and giving reasons for each method of sorting
- Identifying and naming a parallelogram, trapezium, rhombus, kite

2. Line symmetry

- Concept formation through practical experience
- Line symmetry of non-geometric objects from the environment and nature
- Line symmetry of 2-D shapes, plane symmetry for 3-D shapes, classifying shapes by means of the number of lines or planes of symmetry
- Creating symmetrical shapes by giving line(s) of symmetry e.g. complete mirror image
- Using shapes to create patterns and testing the patterns for line and plane symmetry

STANDARD 4

1. Angles

- The idea of an angle
- Use of the protractor to measure and draw angles for $[0^\circ; 180^\circ]$
- Types of angles: acute, right, obtuse and straight
- Estimation of angle sizes
- Naming of angles e.g. ABC or B

2. Circle

The concept of a circle should be formed through practical experience, by drawing circles and introducing terminology such as centre point, radius, diameter and circumference.

3. Rotational symmetry

- a) Concept formation through practical experience
- b) Investigation of rotational symmetry in geometric shapes

4. The square and rectangle

- a) Investigation of their properties: length of sides, size of angles, line and rotational symmetry
- b) Polyominoes: investigation of the different types of shapes formed by combining squares and the perimeters and areas of these shapes
- c) Polycubes: investigating the different types of shapes formed by combining cubes and the volumes and surface areas of these shapes

5. Tessellations

Learners should learn about tessellations by creating their own with squares, rectangles, right-angled triangles and equilateral triangles.

STANDARD 5 : GEOMETRY AND MENSURATION

1. Points, lines and angles

- a) Use of a protractor to measure and draw angles for $[0^\circ; 360^\circ]$
- b) Acute, right, obtuse and reflex angles
- c) Estimation of angle sizes
- d) Naming of angles
- e) Solution of simple and relevant problems
- f) Practical acquaintance with the concepts: plane, point, line, line segment, horizontal, vertical, perpendicular and parallel lines and planes

2. Polygons

- a) Experimental investigation of the properties of polygons with at most 8 sides, including: concave and convex polygons, polygons with axes of symmetry and regular polygons. Associated vocab: triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, vertex, side and diagonal
- b) Determining of size of angles of regular polygons through experimentation
- c) Determining the perimeter of polygons by calculation or measurement
- d) Classification of triangles and quadrilaterals (vocab: isosceles, equilateral, scalene, right, acute and obtuse- angled triangles and square, rectangle, trapezium, kite, rhombus and parallelogram for the quadrilaterals)
- e) Exploring possible tessellations of the plane with triangles, quadrilaterals and hexagons as well as polygons which do not tessellate
- f) Exploration of the rigidity of polygons
- g) Area of rectangles: derive the formula and calculation of the area of rectangles in relevant units, as well as applications in problem-solving

3. Solids

- a) Classification of solids: emphasis on the distinction between rectangular and non-rectangular solids
- b) Nets of cubes and rectangular prisms
- c) Volume: develop concept of volume through packing activities, volume by displacement, formula for volume of a rectangular prism (vocab: edge, face, vertex) and estimation of volumes.

**APPENDIX C
EXAMINATION**

Cape Town College of Education

EXAMINATION JUNE 1998

SUBJECT
MATHEMATICS

COURSE
DE II (SP)

TIME
3 HRS

EXAMINER
S. McAULIFFE

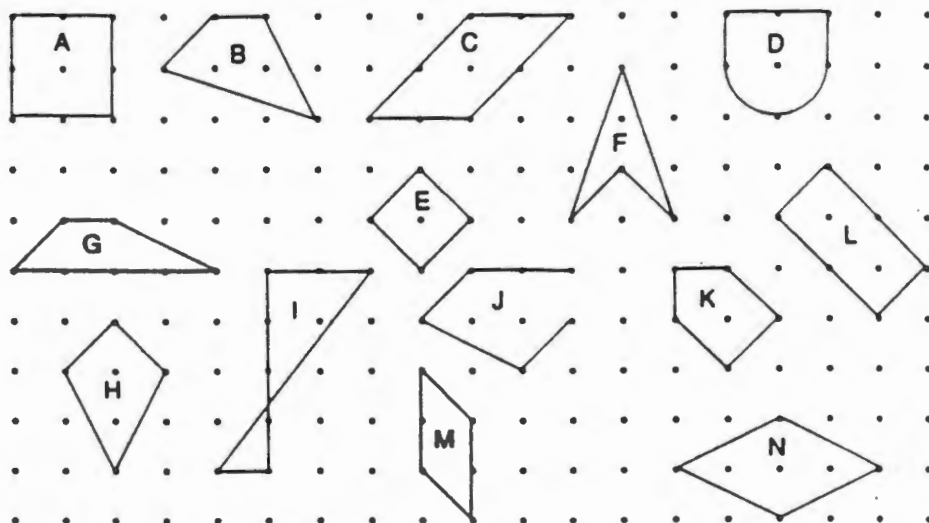
MARKS
120

INSTRUCTIONS:

1. ANSWER ALL QUESTIONS
 2. A CALCULATOR MAY BE USED IN THE EXAM
 3. SHOW ALL YOUR CALCULATIONS IN YOUR ANSWER BOOK (NO WORK ON THE QUESTION PAPER WILL BE MARKED)
-

QUESTION 1

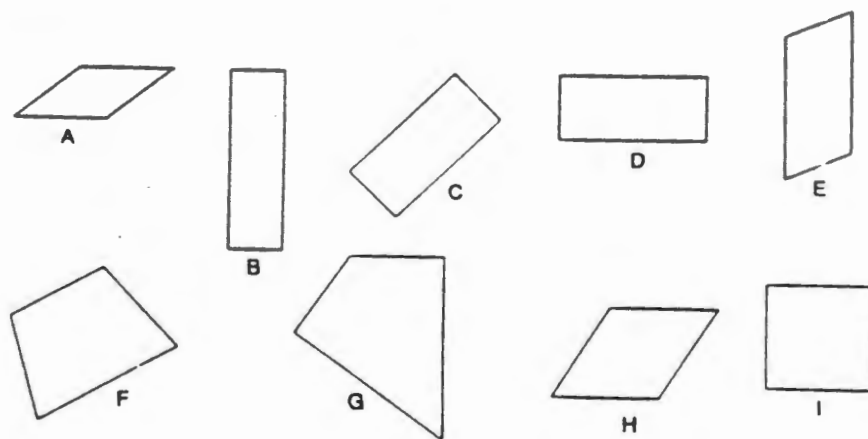
Below are a number of figures drawn on dotted paper.



- 1.1. Which of the above figure are quadrilaterals ?
- 1.2. Explain why each of the figures which you did not write down in 1.1. is not a quadrilateral.
- 1.3. Sort all the quadrilaterals into a number of different groups and explain how you choose the groups for sorting the shapes.
- 1.4. What type of quadrilateral is figure A and N and state the minimum properties necessary to define this figure?

[20]

QUESTION 2



- 2.1
 - a) In which respects are the figures above the same ?
 - b) How do they differ ?
 - c) Figure F is called a trapezium. Draw two different trapeziums in you answer book
- 2.2 Which of the figure shown are not rectangles and if a figure is not a rectangle, explain why not.

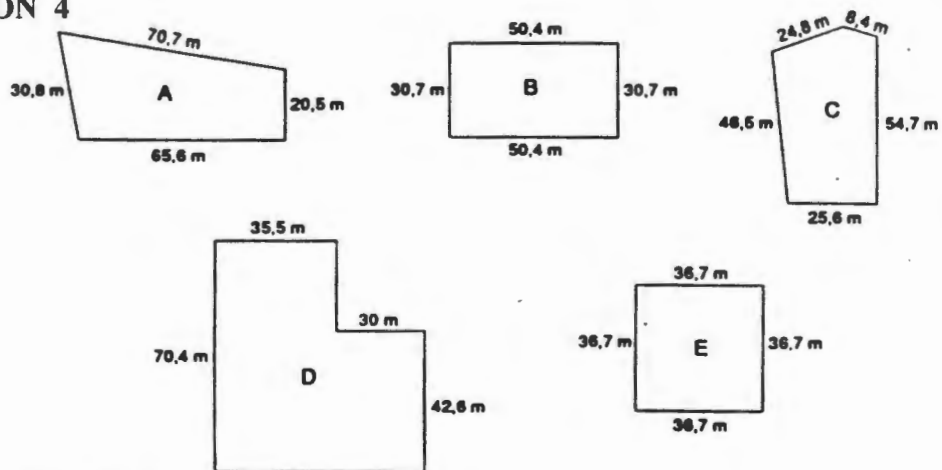
[8]

QUESTION 3

- 3.1 What do the van Hiele's say about levels of thinking and geometry ? (4)
- 3.2 How would you classify each of the above question (1 and 2) according to the van H. Levels ?
Justify your answer. (6)

[10]

QUESTION 4



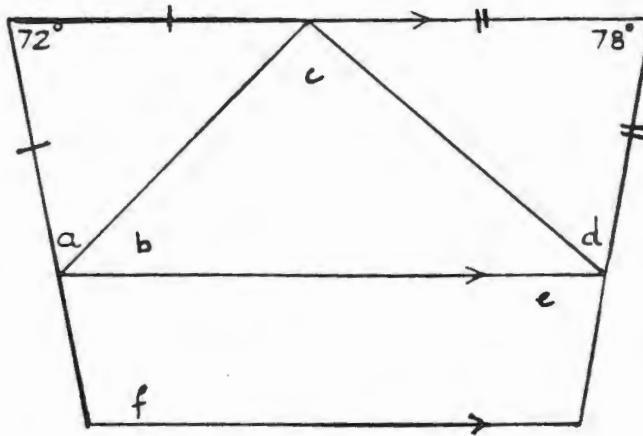
The town council of Newtown decided that all the plots should be fenced to prevent dogs from straying in the town. The sketches above show the shapes and dimensions of a few plots in Newton.

- 4.1 How many meters of fencing wire are needed to fence each plot ?
(ignore the entrance to each plot)
- 4.2 Which plot is the largest and justify your answer.
- 4.3 To fence the plots, there must be a pole at each corner of the plot to support the wire as well as two poles at each entrance. How many poles will the council need to buy ? (explain your reasoning)

[12]

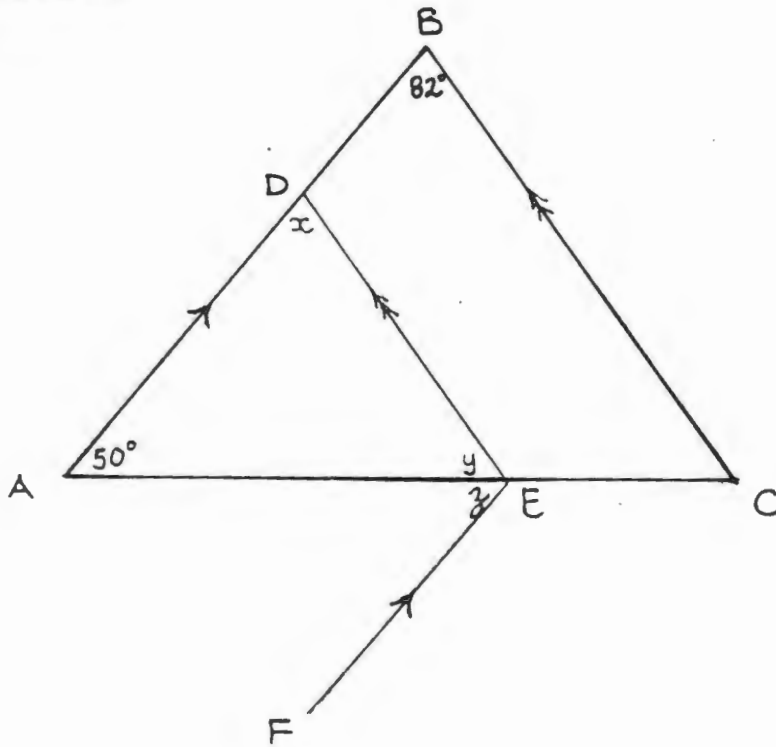
QUESTION 5

- 5.1 Calculate the size of angles a - f in the sketch below (do not measure them or redraw the sketch).
No reasons required.



(6)

- 5.2 Using the given sketch, calculate the sizes of angles x, y and z. Give reasons for each answer.



(6)

[12]

APPENDIX D
PERSONAL CORRESPONDENCE WITH LAWRIE (1998)

From: Ms Christine Lawrie <clawri2@metz.une.edu.au>

To: sharonmc <sharonmc@mweb.co.za>

Subject: comments on questions

Date: Wednesday, October 21, 1998 5: 08 AM

Dear Sharon,

Enclosed comments on your questions in the June examination. I have discussed these with John.

Q1

1.1 Level 1 - OK

1.2 Level 1 only; as there are no properties marked on diagrams, decisions will be made solely on shape.

1.3 Mostly Level 1 answers would be expected for the above reason. Level 2 students generally require markings, hence, you may get comments that there are no markings.

You could well get students giving a Level 3 response, but you would need other indications that they were actually applying L 3 thinking.

1.4 I would expect answers from all four levels, the level determined through the student's responses elsewhere. The question could be rephrased. Its intent is not really clear when put as one sentence. How about

a) In shape N, list all properties. (L 2)

b) How does shape A compare? (L 3)

c) What are the minimum number of properties necessary to define shape N? (L 4)

Q2

2.1 Level 1 agreed

2.2 Level 1, for the reasons given in Q1. You could also get some Level 3 responses (class inclusion)

Q3

3 Agreed, all Level 2

Q4

4.1 Only Level 2, since no reasons are required.

4.2 Probably still only Level 2, since using real numbers allows students to close at each calculation. You may wish to argue that there could be some reasoning given which indicated L 3, but it could only be early L 3.

I hope these are useful comments.

Regards,

Chris.

APPENDIX E
AMENDED MARKING PROFORMAS

AMENDED MARKING PROFORMAS

PAPER 1

(amended assessment)

Concept	Level	Question type	Question number	Score	Question criteria	Mark	Level criteria	Total score	
Square	1	Name	1	1			1 of 2		
		Discriminate	5	2	2/2=1				
	2	Properties		9	2			*3 of 4	
				13a	1				
				15a	1				
	3	Definition		14	1			*4 of 6	
				15b	1				
				22a	1				
				22d	1				
				24b	1				
	4	Proof		13b	1			*4 of 6	
			25	1					
			26	1					
Right Triangle	1	Name	2	1			1 of 2		
		Discriminate	6	4	3/4=1				
	2	Properties		10	4			3 of 4	
				16	1				
	3	Definition		17	3			5 of 7	
				18	2				
				24	1				
	4	Proof		27				2 of 3	
				28	1				
				33	1				
Circle	1	Name	3	1			1 of 2		
		Discriminate	7	2	2/2=1				
	2	Properties		11	3	2/3=1		*7 of 10	
				19a-e	5				
				21	4				
	3	Definition		19f	1			*4 of 6	
				24d	1				
				30	1				
				20	3				
	4	Proof		29	1		*1		
Congruency	1	Name	4	1			1 of 2		
		Discriminate	8	1					
	2	Properties		12	4			3 of 4	
				23	4				
	3	Relationships			4	Deduct 1 point if miss a and c		*7 of 11	
				24	5				
				32	1				
4	Proof		22c	1			*1		
			31	1					

APPENDIX F
PERSONAL CORRESPONDENCE WITH LAWRIE (1998)

From: Ms Christine Lawrie <clawri2@metz.une.edu.au>
To: Sharon Mc Auliffe <sharonmc@ilink.nis.za>
Subject: Re: South Africa Contact
Date: Monday, April 06, 1998 9:44 AM

Dear Sharon,

I entered the total for the level in the last column, e.g., 3, then marked a zero or one in the margin as success for the level. This gave an instant check of how well each student met the criterion. You don't have to use my system. My supervisor acted as an independent assessor to validate my assessment of the responses. Most responses for Levels 1 and 2 have only one possible correct answer. However, with Levels 3 and 4, many responses were difficult to evaluate, and these were all checked and discussed between us. This is not necessary for answers which are obviously correct or incorrect. I found, with the higher levels, that I often needed to look at a student's other responses to determine whether, for example, a statement of a few properties was a list of all they knew (Level 2), or whether it was an attempt to give necessary and sufficient details (Levels 3 to 4). Also, many proofs are incomplete. For these, you need to decide between you, how much is necessary to indicate Level 4 understanding of the essence of proof.

Regards, Chris.

APPENDIX G
ASSESSMENT RUBRIX

DE 2 Senior Primary Mathematics Assignment Feedback

Assignment Questions	1	2	3	4	5
• Children's understanding of Geometry and source of content.					
• Sequencing of activities and links to WCED syllabus					
• Explanation of why activities were chosen with reference to van Hiele and why they were put in a particular sequence.					
• Objectives/Outcomes of activities. Detail of how they are to be used and test question.					
Grade/Mark and comment:					
<p>Key:</p> <p>1 - Little or no comment on the important issues</p> <p>2 - Weak content and poor understanding of assignment</p> <p>3 - Good attempt at answering the questions</p> <p>4 - Very good, clear and concise explanations</p> <p>5 - Excellent reporting of key issues in the assignment</p>					