Quantifying the Impact of Adding an Unlisted Credit Asset to a Portfolio of Listed Credit Assets

Vuyo Takalani Makhuvha

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

June 29, 2017

Abstract

This dissertation derives an objective method to quantify the impact of adding a unlisted credit asset to a portfolio of listed credit assets. It derives a two step approach for making these assessments. The first, modelling, draws from the liquidity risk methods employed in Ericsson and Renault (2006) to derive a suitable risk metric for unlisted assets. By modifying the ratio statistic introduced in Altman and Saunders (1997), a new mean-variance risk measure is proposed utilising the volatility measure derived from the proposed model. While the method leads to portfolio management decisions, certain components need to be reconsidered to allow for better decision-making.

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Chapter 1

Introduction

It is fairly easy to assess the impact of adding a liquid, listed credit asset into a portfolio of such assets as the required parameters are easily observable in the market. There are a host of portfolio credit models used to assess the risk and value of credit portfolios from a default perspective.

The impact of including unlisted credit assets, however, is not as easy to judge. Practitioners often make subjective decisions when deliberating the inclusion of the unlisted assets since there is little agreement on how to make adjustments to account for the fact that these assets are unlisted. Thus, the impact of adding these assets is unknown and remains speculation. While one would assume the addition of an unlisted asset results in some advantage because of diversification, this is not formally weighed up against the impact it has on the riskiness of the portfolio. Additionally, estimating some parameters required for mathematical models used when making portfolio management decisions is difficult because of the lack of available information. This issue can be largely attributed to the inability to observe the volatility of these assets since this parameter is usually determined using the market price of options on the assets or using historical time series.

For this reason, firms tend to make decisions regarding the management of portfolios with both listed and unlisted assets from an economic capital perspective or based on qualitative reasoning and the fundamentals of the firms issuing the assets.

This dissertation devises a quantitative method for making portfolio management decisions regarding the addition of unlisted credit assets to a portfolio of listed credit assets, with a strong focus on proposing a practical solution which can be used mainly by institutional investors to make portfolio management decisions within South Africa. This method rests on the premise that the major difference between an unlisted credit asset and a listed credit asset written by the same entity is the additional liquidity risk in holding the unlisted asset.

In order to model the credit assets individually, a structural model is used to take into account both the systematic and idiosyncratic credit risk and in the case of unlisted assets, liquidity risk. A risk-return approach is then applied to make portfolio management decisions with regards to credit portfolios.

The dissertation is structured as follows: Chapter 2 reviews credit risk models with a focus on relevant extensions of the Merton model. Chapter 3 details the model employed. Chapter 4 details the portfolio management decision process and in Chapter 5 analysis of the results are presented and discussed. Chapter 6 concludes.

Chapter 2

Credit Risk Modelling

There are two well-known approaches employed when dealing with default risk, namely structural models and reduced-form models. The structural approach was developed in the works of Black and Scholes (1973) and Merton (1974) and is commonly referred to as the Merton model. It involves using the fundamentals of a firm to model the default event. The firm assets, V_t, are modelled by geometric Brownian motion with the following dynamics:

$$\frac{dV_t}{V_t} = r\,dt + \sigma_V\,dW_t,$$

where r is the risk-neutral rate, σ_V is the volatility of the firm assets and W_t is standard Brownian motion. Using the Merton model, the value of the bond B_t, is modelled as a derivative with V_t being the underlying asset. Default occurs when the value of the firm assets at maturity T falls below a threshold barrier, D. The equity of the firm is valued as

$$B_T = \max(V_T - D, 0).$$

By the Black-Scholes equation, the initial value of the equity can be expressed as

$$B_t = V_t\Phi(d_+) - De^{-r(T-t)}\Phi(d_-),$$

where

$$d_\pm = \frac{\ln(\frac{V_t}{D}) + (r \pm \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}}.$$

The probability of default can thus be computed as

$$\text{PD}_T = \text{P}[V_T < D] = \Phi(-d_-).$$

The reduced-form approach was originally introduced in Heath *et al.* (1992) and is also referred to as the hazard rate model. It models the default process as a Poisson process. In the simplest case, a homogeneous hazard rate λ is used to determine the probability of default and the expected default time. The probability of default under this model is thus

$$\mathrm{PD}_T = \mathbb{E}[\mathbb{I}_{\tau \leq T}] = \mathbb{P}(\tau \leq T) = 1 - e^{-\lambda T}.$$

The relationship between the Poisson process and the exponential distribution, allows for the expected default time to be computed as

$$\mathbb{E}[\tau] = \int_0^\infty t\lambda e^{-\lambda t}\, dt = \frac{1}{\lambda}.$$

Both models have their merits and different extensions have been made in order to make various aspects more realistic.

2.1 Adaptations of the Merton Model

There have been many criticisms of the original Merton model. In an attempt to address these, extensions have been made to the model. One such extension is the first passage time (FPT) model which allows default to occur at any time t before maturity. Black and Cox (1976) modelled the firm assets as geometric Brownian motion and allowed default to occur the first time the asset value crossed a constant threshold D. The time of default, τ, is thus defined as

$$\tau = \inf[s \geq t | V_s \leq D].$$

The firm asset value is then set to D at time of default. These FPT models have been extended in various ways, some notable extensions are those by Leland (1994); Leland and Toft (1996).

Another major criticism of the Merton model is that the credit spreads generated tend to underestimate those observed in the market. Many adaptations to the original model have been made in an attempt to rectify this, including allowing for stochastic interest rates or volatility, adjusting for corporate taxes and incorporating liquidity risks. One adaptation is the inclusion of a jump component to the FPT model. Merton (1976) introduces a Poisson distributed jump component which can be interpreted as an 'abnormal' change in the firm asset value. This is attributed to the arrival of important information. This model is commonly referred to as the Merton jump-diffusion model. The diffusion of the firm assets is written as

$$\frac{dV_t}{V_t} = (\mu - \lambda k)\, dt + \sigma\, dW_t + dP_t,$$

where μ is the return on assets, λ is the mean number of arrivals per unit of time, $k \equiv \mathbb{E}(Y_t - 1)$, where $(Y_t - 1)$ is the random variable representing the relative asset jump size if the Poisson event occurs, W_t is standard Brownian motion and P_t is the Poisson process. W_t and P_t are assumed independent. The random variable, k, can follow any appropriate distribution, some common choices are the Poisson, uniform and log-normal distribution. There are many merits to the jump-diffusion model, one is that the total change in the firm asset value is now composed of two different types of changes. The first is what Merton (1976) terms as ”normal” changes due to the slow and steady decline of the firm asset value. The second is ”abnormal” change captured by the jumps. This has many implications; firstly it allows the credit spread term structure to have varying shapes. It also allows for credit spreads of short term good quality bonds that are significantly greater than 0 and lastly, the value of the firm at default is now a random variable (Zhou, 1997).

2.1.1 Structural Models Incorporating Liquidity Risk

There has been some interest in modelling liquidity risk in an attempt to better explain the spreads in the market. There are two noteworthy papers which explore liquidity risk within the structural model. The first by Ericsson and Renault (2006) views liquidity as the “ability to sell a security promptly at a price close to its value in frictionless markets”. They capture liquidity risk by modelling the impact of illiquidity on the renegotiation of distressed debt as well as the impact of liquidity shocks experienced by the bondholder. The firm enters distress when its asset value is below some threshold. During this period, the bondholder is still able to trade in the bond. The period of distress is resolved either by liquidation or a restructuring where the bondholder receives equity in exchange for their existing bonds in order to avoid liquidation. They also model random liquidity shocks where the bondholder needs to sell the bond quickly due to some liquidity crisis and thus sells it at a discount which is some fraction of the price in a perfectly liquid market. Using this model, they are able to generate downward sloping liquidity spreads implying a decreasing term structure of liquidity spreads and credit spreads that better match those observed in the market.

The second paper by Tychon *et al.* (2005) explores liquidity through the bargaining process between the bondholders and other investors in the market. It seeks to capture the effect of a lack of marketability which they describe as the inability to sell an asset at any price at any point in time and the lack of liquidity which they define as the need for the asset holder to sell at a greater discount in order to sell immediately. In the model, the bondholder is randomly matched with an investor in the secondary market who has a belief about the cost of bankruptcy which informs their belief on the value of the firm at bankruptcy. These investors all carry their own beliefs about this cost. Depending on both parties’ (bondholder and investor) beliefs about this cost, a sale may occur. This price is dependent on each party's view of bankruptcy costs as well as the bargaining power of the participants.

Chapter 3

Modelling the Credit Assets

This chapter describes the model used to evaluate each credit asset in the portfolio individually. The model is an adapted Merton model which is used to find an appropriate risk metric which can be used to make portfolio management decisions. It utilizes Monte Carlo techniques to evaluate the present value of cashflows associated with zero coupon or coupon-bearing bonds. This is done by simulating paths of the firm asset. For each path, the firm either defaults before or at time T, where T is the time to maturity of the asset, when its value drops below a threshold D, in which case the bondholder receives a discounted value of the firm asset at default or the firm does not default before time T and receives the redemption on the bond. For unlisted assets, the bondholder may also sell the credit asset given some liquidity event. The cashflows received through the life of the asset are then discounted to calculate the present value for a certain path. These are then averaged to evaluate the present value of the asset. This present value is then used to find a risk measure denoted as $\hat{\sigma}$.

In order to make portfolio management decisions on the portfolio, a risk metric needs to be generated for each asset in the portfolio. This is done by modelling the assets individually as described above and solving for $\hat{\sigma}$ in the following equation:

$$y_{t,T} - (r - \delta) = -\frac{1}{T-t}\ln\big(\Phi(d_-(\hat{\sigma})) + \frac{1}{L}\Phi(-d_+(\hat{\sigma}))\big),$$

where $y_{t,T}$ denotes the yield implied by the present value of the corporate bond, δ denotes the continuous coupon rate, Φ denotes the cumulative standard normal distribution and L denotes the leverage of the firm, i.e. $L = \frac{V_t}{D\exp^{-r(T-t)}}$, and

$$d_\pm(\hat{\sigma}) = \frac{\ln(\frac{V_t}{D}) + ((r-\delta) \pm \frac{1}{2}\hat{\sigma}^2)(T-t)}{\hat{\sigma}\sqrt{T-t}}.$$

This chapter will firstly describe the firm asset, then the default event and its related cashflows will be discussed. This is followed by the motivation behind the liquidity event and its application in other structural models as well as the cashflows related to it. Lastly, the parameter estimation method used to calculate values of parameters related both to the firm assets and the liquidity event is described.

3.1 The Firm Assets

The valuation framework is used to evaluate the cashflows of coupon-bearing or zero coupon bonds in order to obtain the expected present value of the bond. This present value is then used to calculate $\hat{\sigma}$. Each bond pays some $\frac{c}{n} \geq 0$, n times a year given that the firm is still solvent, until the time T where the principal P is paid out. The firm defaults when its asset value V_t crosses a lower boundary D. This represents the point when the assets of the firm can no longer cover its obligations. The firm asset value is modelled using a Merton jump-diffusion model. It can be characterized by the following stochastic differential equation:

$$dv_t = d\text{ln}V_t = (\mu - \lambda k - \frac{1}{2}\sigma^2)\,dt + \sigma\,dW_t + d\Big(\sum_{i=1}^{n} J_i\Big),$$

where all parameters are as described in Section 1.1, the $J_i \sim \mathcal{N}(\mu_j, \sigma_j)$ and the last term is 0 if $n = 0$. The model discretises the period $[0, T]$ into periods of length Δt. At each point t, given that the firm was solvent at $t-1$, the firm is either still solvent and so the bond is still active or has defaulted. If the firm has not yet defaulted and t is a coupon paying point, the coupon $\frac{c}{n}$ is paid out. If the firm defaults in the period $[t-1, t]$ the bondholder receives the asset value of the firm at default less the loss given default (LGD). The LGD is made up of the costs of bankruptcy or the discount at which the assets of the firm can be sold off quickly. Figure 3.1. is a representation of the cashflows associated with a simple listed coupon-paying bond. The first scenario is a depiction of the cashflows associated with a bond where the firm does not default and so the cashflows are made up of the coupon payments and the principal received at maturity. The second is a depiction of a firm that defaults at some $t < T$, thus the cashflows are the coupons until default and the amount received after the firm is liquidated, $(1 - \text{LGD})V_\tau$.

3.2 Liquidity Risk

While listed and unlisted credit assets face similar risks, for some of these risks the magnitude differs. One such risk is the liquidity or marketability risk. Since the unlisted assets are not sold on an exchange, there is a much greater risk that the bondholder is unable to sell the asset if they so wish or will have to accept

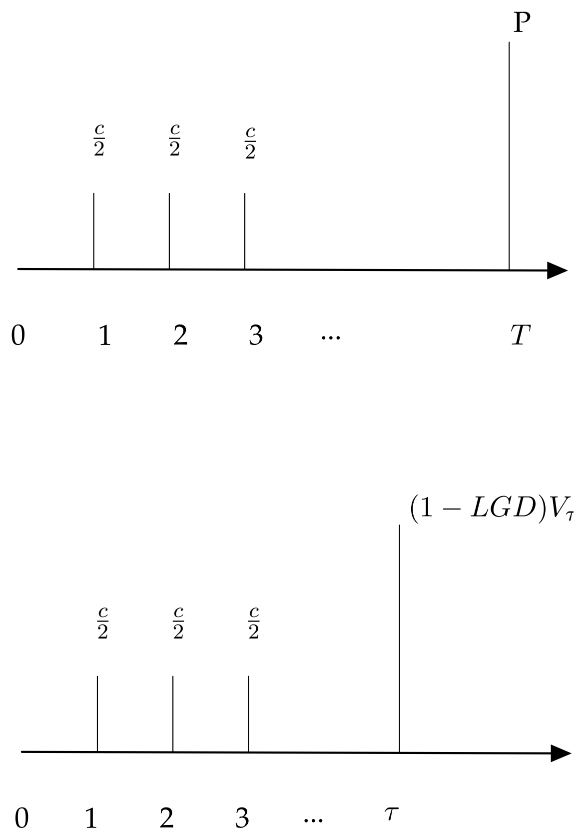

Fig. 3.1: The cashflows of a listed asset if default does not occur (top) and if default occurs at $\tau \leq T$ (bottom).

substantial discounts in order to do so. It is for this reason that this liquidity risk is included in the model for unlisted assets. This is done on the premise that for two bonds issued by the same issuer, which differ only in the fact that one is sold on the market while the other is not, the unlisted asset will face additional liquidity risk.

The Ericsson and Renault (2006) model is one of the few structural models incorporating illiquidity. This is done in many ways, one being the inclusion of a liquidity shock where the bondholder is forced to sell the asset. Ericsson and Renault (2006) attribute this shock to shortages in cash, changes in capital requirements or a need to rebalance the portfolio. It is not a result of the lack of liquidity expected when a firm defaults. The shock is Poisson distributed with a time-inhomogeneous hazard rate that is correlated with the firm asset value. When the bondholder is forced to sell the asset, they receive offers for $\tilde{\delta}_t$, the discount factor applied to the present value of a similar liquid bond, from the N participants in the market at that point. The bondholder then sells the bond to the participant who offers the highest $\tilde{\delta}_t$.

In this model, there is the inclusion of a liquidity premium for unlisted assets modelled using a simplified version of the liquidity shock in Ericsson and Renault (2006). This event can occur at any point in time when the firm is solvent. When there is a shock, the bondholder is forced to sell the asset at a discounted value. As in Ericsson and Renault (2006), the liquidity event can represent a shortage of holdings in cash, capital constraints or the need to immediately rebalance the portfolio for whatever reason. It should not be confused with a liquidity crisis due to distress caused by the default of a firm, instead it represents any point in time while the firm is solvent where the bondholder attempts to sell the asset quickly for whatever reason. The arrival time of the liquidity event, $\bar{\tau}$, is exponentially distributed with constant λ_L and defined as

$$\bar{\tau} = \inf\{s \geq t | \mathbb{I}_L = 1, V_s > D\},$$

where $\mathbb{I}_L$ is the indicator function indicating a liquidity event has occurred. At this point in time, the credit asset is sold for $\beta B_{\bar{\tau}}$ where $B_{\bar{\tau}}$ is the present value of the hypothetical listed equivalent of the bond. The value β denotes the discount factor. As in Ericsson and Renault (2006), this can be randomly sampled from an appropriate distribution, ideally one derived from the appetite of the market of investors available to trade in the bond. In this model it is simply set to $\beta = 0.8$. Hence, for an unlisted asset given that the firm was solvent at $t-1$, there are three possible scenarios at time t. The firm may be solvent , may have defaulted due to the firm asset level having crossed the boundary or may have been sold off due to a liquidity shock at t. Figure 3.2 depicts the three scenarios possible for an unlisted asset in the model and the cashflows associated.

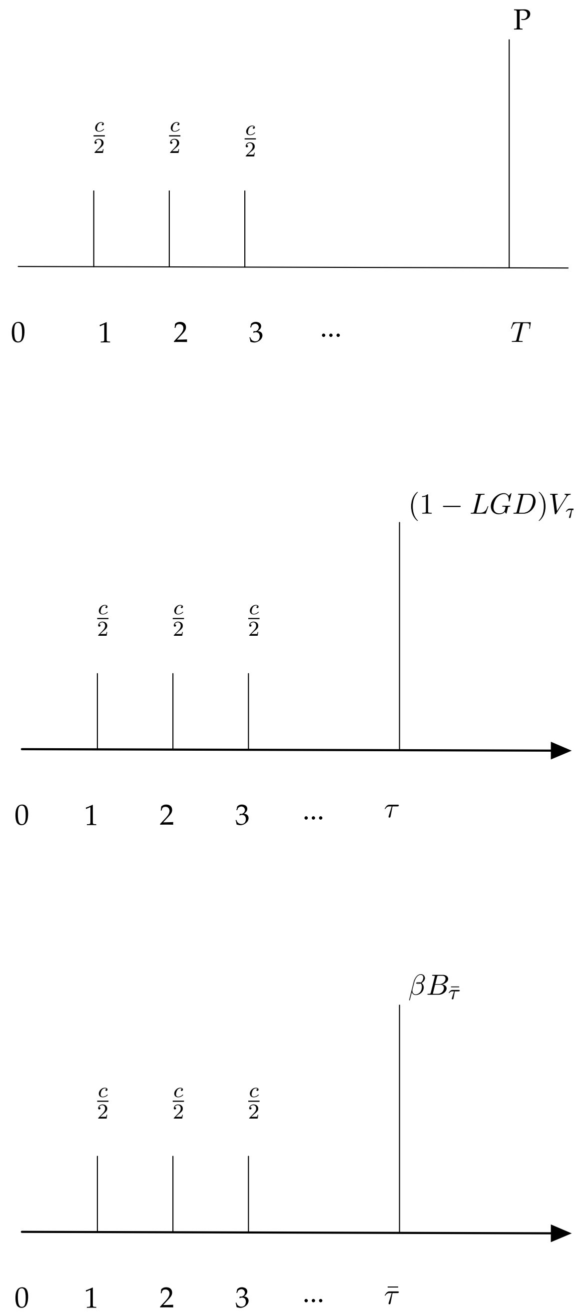

Fig. 3.2: The cashflows of an unlisted asset if default does not occur (top), if default occurs at $\tau \leq T$ (centre) and if a liquidity shock occurs at $\bar{\tau} \leq T$ (bottom).

3.3 Parameter Estimation

There are two sets of parameters that need to be estimated. The first is the set of parameters which pertain to the firm asset level. As mentioned above, the firm asset is modelled as a Poisson-mixing-normal Merton jump-diffusion satisfying the SDE:

$$dV_t = V_t[\mu_d\, dt + \sigma_d\, dW_t + J\, dP_t],$$

where $\mu_d = (\mu - \lambda k)$ is the mean return rate, σ_d is the diffusive volatility, J is a random jump amplitude with log-return mean and variance, μ_j and σ_j, respectively. P_t is a standard Poisson process with jump rate λ and independent of W_t. $J\, dP_t$ can be defined by a stochastic integral of Poisson random measure $\mathcal{P}(dt, dq)$ or as a sum of $dP(t)$ jumps,

$$J\, dP(t) = \int_{\mathcal{Q}} J(q)\mathcal{P}(dt, dq) = \sum_{i=1}^{dP_t} J(Q_i).$$

The parameters $\{\mu_d, \sigma_d, \mu_j, \sigma_j, \lambda\}$ need to be estimated. These are estimated in two steps. Firstly, the method described in Tauchen and Zhou (2011) is implemented to estimate the parameters related to jumps $\{\mu_j, \sigma_j, \lambda\}$. The relationships derived by Hanson and Westman (2002) are then used to calculate the remaining firm asset parameters, $\{\mu_d, \sigma_d\}$. The method described by Tauchen and Zhou (2011) extends the works of Barndorff-Nielsen *et al.* (2003); Barndorff-Nielsen and Shephard (2004, 2006) on jump detection using bipower variation. There are a number of jump detection methods each based on different assumptions and with its own challenges. Detecting jumps is challenging especially when using low frequency data. More recently there has been a focus on using high-frequency data to estimate the parameters. One such method requiring the use of high frequency data is pioneered by Barndorff-Nielsen *et al.* (2003). Commonly referred to as the bipower variation method, the jump detection process relies on various measures proposed by Barndorff-Nielsen and Shephard (2004), the first two being the realised variance and the realised bipower variation. They define the intra-daily return (the return on the t^{th} day at the j^{th} point in the day) as

$$r_{t,j} = v_{t,j\cdot\Delta} - v_{t,(j-1)\cdot\Delta}.$$

where Δ is the time interval between observation points. The realised variance is defined as

$$RV_t = \sum_{j=1}^{m} r_{t,j} \to \int_{t-1}^{t} \sigma_s^2 ds + \int_{t-1}^{t} J_s^2 ds,$$

and the realised bipower variation as

$$BV_t = \frac{\pi}{2}\frac{m}{m-1}\sum_{j=2}^{m}|r_{t,j}||r_{t,j-1}| \to \int_{t-1}^{t}\sigma_s^2 ds.$$

These quantities inform the ratio statistics method employed in detecting these jumps. Both converge uniformly as $\Delta \to 0$ or $m = 1/\Delta \to \infty$. The difference between these two quantities is zero if there is no jump and is strictly positive if there is one. Hence, the realised jumps are

$$RJ_t = \frac{RV_t - BV_t}{RV_t},$$

with appropriate scaling this converges to the standard normal distribution,

$$ZJ_t = \frac{RJ_t}{\sqrt{[(\frac{\pi}{2})^2 + \pi - 5]\frac{1}{m}\text{max}(1, \frac{TP_t}{BV_t^2})}} \xrightarrow{d} \mathcal{N}(0,1),$$

where TP_t denotes the tripower quadricity,

$$\text{TP}_t = m\mu_{4/3}^{-3}\frac{m}{m-2}\sum_{j=3}^{m}|r_{t,j-2}|^{4/3}|r_{t,j-1}|^{4/3}|r_{t,j}|^{4/3} \to \int_{t-1}^{t}\sigma_s^4 ds,$$

where for $k > 0$,

$$\mu_k = 2^k\Gamma((k+1)/2)/\Gamma(1/2).$$

Thus the estimated jumps, $\hat{J}_t$, are

$$\hat{J}_t = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times \mathbb{I}_{\text{ZJ}_t \geq \Phi_\alpha^{-1}}}.$$

More detail can be found in Barndorff-Nielsen and Shephard (2004, 2006).

In order to estimate the parameters $\{\mu_d, \sigma_d, \lambda_j, \mu_j, \sigma_j\}$ for the Merton jump-diffusion, Tauchen and Zhou (2011) propose a method informed by Barndorff-Nielsen and Shephard (2004, 2006). This method will be followed in the dissertation in order to estimate parameters relating to the jumps. The major assumptions underlying this methodology is that the jumps on the financial markets are rare and large. Tauchen and Zhou (2011) define λ as the number of estimated jumps ($\hat{J}_t$) divided by the number of days observed and μ_j and σ_j^2 as the mean and variance of these realized jumps.

Hanson and Westman (2002) describe a maximum likelihood (MLE) method employed to estimate parameters for a Merton jump-diffusion model with Poisson-mixing-normal distributed jumps. In their analysis, they derive the relationship between the parameters related to the jumps and μ_d and σ_d.These are

$$\sigma_d^2 = (M_2 - \lambda\Delta t(\sigma_j^2 + \mu_j))/\Delta t,$$

$$\mu_d = (M_1 - \lambda \Delta t \mu_j)/\Delta t,$$

where M_1 and M_2 are the first two central moments of the observed distribution. These relationships are employed to calculate the remaining parameters we wish to estimate μ_d and σ_d.

The last parameter required is the liquidity parameter λ_L. This is estimated by by decomposing the total spread of a bond into the credit spread attributable to the default risk and a residual spread which is attributed to the liquidity premium. This is graphically represented in Figure 3.3.

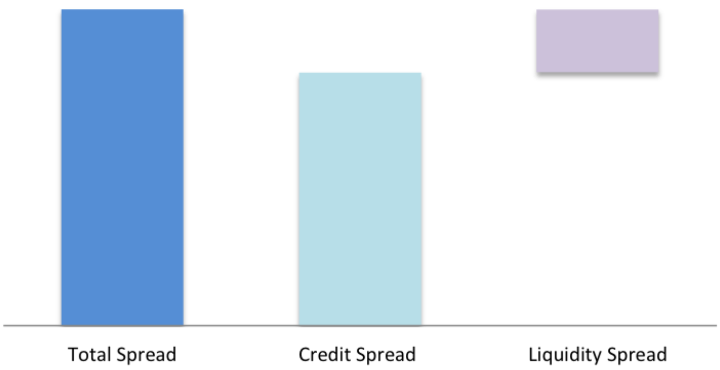

Fig. 3.3: The decomposition of the spread into credit and liquidity spread

Under the no-arbitrage condition:

$$e^{r_{0,T}T} = (1 - q_T + q_T(1 - \text{LGD}))e^{(r_{0,T}+s_{0,T})T},$$

hence

$$s_{0,T} = -\frac{\ln(1 - q_T + q_T(1 - \text{LGD}))}{T},$$

where $r_{0,T}$ is the spot risk-free rate, $s_{0,T}$ represents the spot credit spread on a T-bond, q_T is the default probability and LGD is the loss given default. Using the EDF for the bond as the default probability and a LGD of 45%, the spread attributable to default risk can be calculated. This is then subtracted from the yield spread of the bond so as to calculate the residual spread which is attributed fully to liquidity.

The parameter λ_L is derived as follows:

$$s^*_{0,T} = -\frac{\ln(1 - q^*_T + q^*_T(1 - z))}{T},$$

where $s^*_{0,T}$ represents the difference between the yield spread and the spot credit spread on a T-bond, q^*_T is the probability of a liquidity shock and z is the discount at which the bond is sold given a liquidity event. Using the residual spread and the discount value, the value for q_T^* is hence,

$$\lambda_L = -\frac{\ln(1 - q_T^*)}{T}.$$

Chapter 4

Portfolio management

In the past, financial institutions have relied heavily on subjective analysis or banker "expert" systems in order to make credit risk management decisions. However, these institutions are trying to move away from these approaches towards more objective methods. Hence, there is an increasing need for more robust credit management approaches. Altman and Saunders (1997) describe how practitioners and academics are developing more sophisticated early warning systems, moving away from assessing credit instruments individually to a concentrated credit risk approach, developing models to better price this risk and attempting to model the credit risk associated with off-balance sheet instruments.

There are a number of ways to assess a credit instrument on an individual basis. Accounting based credit scoring systems compare key ratios of borrowers with industry or group norms — these are often combined or weighted to produce a credit risk score or derive a probability of default that can be compared to those of other borrowers. These are often criticized as they rely on book value accounting data and their linear nature in a space that is inherently non-linear (Altman and Saunders, 1997). Other models include 'risk to ruin' models, which model a path the firm takes to bankruptcy. Closely related to the 'risk to ruin' model is the option pricing model which is largely discussed in Chapter 2. Some concerns related to these models are their use of the volatility of the stock price as a proxy for the variability in the firm asset value and whether the use of proxies for unlisted companies is effective. There are also models which make use of the term structure of the yield spread between risk-free and corporate bonds in order to derive an implied probability of default.

4.1 Modern Portfolio Theory and Bond Portfolios

Modern portfolio theory, pioneered by Markowitz (1959) is well understood in an equity portfolio context. However, there is very little published on its application to bond portfolios and while effective risk reduction methods are of great importance to financial institututions, robust diversification techniques similar to those achieved with modern portfolio theory have been elusive.

Modern portfolio theory can simply be summarised as a method of holding a combination of assets in order to receive the maximum return for a given level of risk or the least risk for a given level of return. It relies on two main assumptions:

1. Investors are risk averse;

2. Log-returns are jointly normally distributed.

One can easily believe that the first assumption still holds for bond portfolios. The second assumption is more difficult to deal with. First of all, empirical analysis tells us that the log-returns of equity assets are not normal but leptokurtic. But this non-normality can be ignored because for equity portfolio management, practitioners are not particularly interested in the entire distribution, in fact the focus is the area surrounding the mean. Traditionally, practitioners in the credit environment are focussed on the distribution of losses which are far from normally distributed. Practitioners also focus on the tail of this distribution and for this reason the non-normality cannot be ignored in the case of the distribution of losses.

A key feature of modern portfolio theory is the diversification effect. The variance of the portfolio is defined as:

$$\sigma_p^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} \omega_j \omega_k \sigma_j \sigma_k \rho_{j,k}$$

and so unless all the assets in the portfolio are perfectly correlated (i.e. $\rho_{j,k} = 1 \forall j,k,j \neq k$), the variance of the portfolio will be less than the weighted sum of variances of the assets in the portfolio. This is known as the diversification effect. Smithson (2003) raises that there is greater potential for a diversification effect for bond portfolios since the typical correlation of equity returns is 20% to 70% while the correlation of defaults for bonds is 5% to 15%. It is however, also pointed out that "full diversification" of credit portfolios will require a portfolio of considerably more assets than the number of assets required for an equity portfolio.

Essentially modern portfolio theory relies on the expected return of respective assets and a covariance matrix. Historically this has been challenging for credit portfolios as there is often a focus on the tail of loss distributions and difficulty deriving a relevant covariance matrix.

4.2 The Altman and Saunders (1997) Approach

Altman and Saunders (1997) detail a mean-variance approach to assessing credit portfolios. By doing this they derive a variation on the Sharpe ratio which they term the high yield portfolio ratio (HYPR).

In order to measure the expected return of the portfolio, they calculate the expected annual return for each fixed income instrument. It is the yield promised to the financial institution (yield-to-maturity or yield-to-failure) from which the expected annual loss (EAL) for that contract is subtracted.

The expected annual return (EAR_i) for a firm i is

$$\text{EAR}_i := \text{YTM}_i - \text{EAL}_i.$$

Altman and Saunders (1997) derive their EAL from tables of failure rates and losses produced in prior work. The expected portfolio return (R_p) is then the sum of the EAR for each firm, weighted by the proportion of the bond relative to the portfolio (X_i):

$$R_p := \sum_i X_i \text{EAR}_i.$$

The variance of the portfolio (V_p) is

$$V_p := \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_j \sigma_i \rho_{ij}.$$

The HYPR is then defined as

$$\text{HYPR} := \frac{R_p - R_B}{\sqrt{V_p}},$$

where R_B is a benchmark return. The HYPR is thus analogous to the Sharpe Ratio and provides a single number which allows one to compare different portfolios to one another.

It must be noted, however, that this approach is not well suited for long-term credit portfolios, because of the difference in the distribution of returns. For equity assets, the upside is theoretically unlimited and the investor can lose all their investment. For fixed income assets the return is limited and the investor can also lose all their investment in the event of default (Altman and Saunders, 1997). This analysis is therefore only valid for a credit portfolio in the short-term.

4.3 A Risk-return Metric

Using the $\hat{\sigma}$ taken from the model and a correlation, we can thus create a risk return measure. The proposed measure is defined as

$$\hat{\lambda} = \frac{\sum_{i=1}^{N} \omega_i \mathrm{YTM}_i}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \hat{\sigma}_i \hat{\sigma}_j \rho_{ij}}},$$

where ω_i are the weights of each asset i in the portfolio, $\hat{\sigma}_i$ are the variances derived from the model described in Chapter 3 and YTM_i and ρ_{ij} are as described as above.

The numerator of this statistic is the weighted average of the yield promised by the borrower. This makes up the return component of the metric. The denominator is the 'risk-adjusted' variance derived from the model. The measure is thus similar to the Sharpe ratio in that the return of the portfolio is compared to its risk. It can then be used to make portfolio management decisions. If the measure of the portfolio without the asset is greater than it is with the asset, then it is not beneficial to include the credit asset to the portfolio. Conversely, if the measure is higher for the portfolio including the asset then the portfolio with the unlisted asset included is preferred. This statistic differs from the Altman and Saunders (1997) high yield portfolio ratio as the variance of the each asset used is a 'risk-adjusted' or 'model-derived' variance. Because the variance is already somewhat adjusted for credit risk, the numerator is simply the promised yield. However, this ratio is similar to the one proposed by Altman and Saunders (1997) as it is also a short-term view of the portfolio given the distribution of the return.

Chapter 5

Application

The model proposed in Chapter 3 in conjunction with the result given in the previous section can be applied by practitioners to make portfolio management decisions. The method is proposed as follows:

1. Estimate parameters for the Merton jump-diffusion and the liquidation premium using share price information and the necessary credit information pertaining to the proxy firm for the unlisted asset.

2. Model each asset individually using the valuation model described in Chapter 3.

3. Calculate $\hat{\sigma}$ for each asset using its respective present value derived from the model.

4. Build a correlation matrix based on share price information.

5. Calculate the risk metric for the listed portfolio without the unlisted asset and the portfolio including the unlisted asset.

6. Compare the two metrics in order to make a portfolio management decision.

A portfolio of Rand denominated coupon bonds has been compiled from ten listed issuers to study the portfolio management decisions made using the risk measure proposed in the previous section. Each asset is modelled individually using the methodology outlined in Chapter 3. Four unlisted assets have been chosen to analyse the impact of the addition of an unlisted asset to the portfolio. The four unlisted assets are identical in all aspects apart from the firm offering them. The firms offering the bond were chosen in order to have all combinations of two characteristics; namely size and whether the industry in which it operates is represented in the listed asset portfolio. Hence, there is an asset which is small and whose industry is already represented in the portfolio; as well as large that is represented and a small and large asset which do not belong to any of the industries

Small and represented (Texton)	Small and unrepresented (Imbalie)
Large and represented (Mondi)	Large and unrepresented (Tiger Brands)

Tab. 5.1: Choosing the unlisted firms

already covered by the portfolio. For example, Mondi is chosen as a firm as it is large and the industry it operates in is already represented in the listed portfolio by Sappi. This is illustrated in table above. The value of the risk metric for the listed portfolio is compared to the value after the addition of each unlisted asset. When the value of this metric with the additional unlisted asset is greater than it was before, it suggests that the asset is a valuable addition. Market information is required for two reasons. Firstly, information is required from the balance sheet in order to establish the initial asset level and the default barrier. This has been taken from the Thompson Reuters terminal. Market information is also required to estimate the asset parameters and the liquidity parameter for the unlisted asset. In Chapter 3, the parameter estimation method described relies on the use of high frequency data to estimate the jump-related parameters using the bipower variation method. The high frequency data taken from the Thompson Reuters terminal is the data for the last 3 months. This means there are few points available from which one can estimate the parameters. This is not ideal. Furthermore, there are many missing points, resulting in an even smaller set of information. This constraint must be taken into consideration when analysing the results of the investigation. The remaining asset parameters are then estimated using the relationships stated in the same chapter and daily share price information which is also retrieved from the terminal. The EDFs required for liquidity parameter estimation are extracted from Moody's CreditEdge and credit spread information is sourced from Bloomberg. A correlation matrix required to calculate the variance of the portfolio is derived using the correlation matrix tool on the Reuters Eikon teminal.

A very important consideration is how to choose the proxies in situations when information concerning a specific firm is unavailable. While the listed portfolio was made up of listed firms that have at least one listed credit asset, the unlisted assets were chosen from firms without any listed credit assets and so proxies were required when estimating the liquidity parameter. For the two assets in industries already represented in the listed portfolio, the firms in the portfolio were chosen as proxy firms and their yield spread information was used to derive a liquidity parameter. However, for unlisted assets whose industries were not represented in the listed portfolio, a proxy firm is chosen that has a similar size, industry and similar age. It must be noted that the act of choosing proxies does bring some

Listed Asset	$\hat{\sigma}$	EDF (%)
ABSA_1	0,5366	0,5784
ABSA_2	0,5932	0,5784
BARCLAYS	0,5457	0,5784
GROWTHPOINT	0,6694	0,4574
INVESTEC	0,567	1,1332
LIBERTY_1	0,6047	0,6644
LIBERTY_2	0,6212	0,6644
NORTHAM	0,6893	0,3879
SAPPI	0,5902	0,3763
STANDARD	0,4451	0,4643
TELKOM	1,1622E-16	0,8663
TOYOTA_1	0,7029	-
TOYOTA_2	0,6854	-

Tab. 5.2: Derived volatility ($\hat{\sigma}$) and Structural PDs

subjectivity into the portfolio analysis. It is also difficult to argue how best to choose these proxies as it may be argued that using counterparts with listed assets in the same industry is in fact not a criterion for choosing a proxy for a firm that has no listed bond offerings. Furthermore, it is debatable whether proxies provide a fair picture of the riskiness of a firm.

5.1 Model Results

Table 5.2 details the $\hat{\sigma}$ derived for each of the assets in the original listed portfolio as well as the expected default frequencies (EDFs) from Moody's CreditEdge. These EDFs are the probability that the issuer will default within one year. For both measures, a larger value would signify greater riskiness. This is not a perfect comparison, since the metric derived for each individual asset is being compared to that derived for each issuer. One anomaly is the $\hat{\sigma}$ derived for the Telkom SA SOC corporate bond. The $\hat{\sigma}$ for this bond is almost zero and so suggests that there is no risk in holding it. This can be explained by the fact that the coupon on the bond is 6% which is lower than the 7,825% yield on the 2-year SA Government T-bond which was used as the risk-free rate. This means that the issuer can easily earn more than the coupon rate in the market making it very easy to maintain the loan. This along with the fact that Telkom holds very little debt (making the threshold considerably low), results in the $\hat{\sigma}$ close to zero.

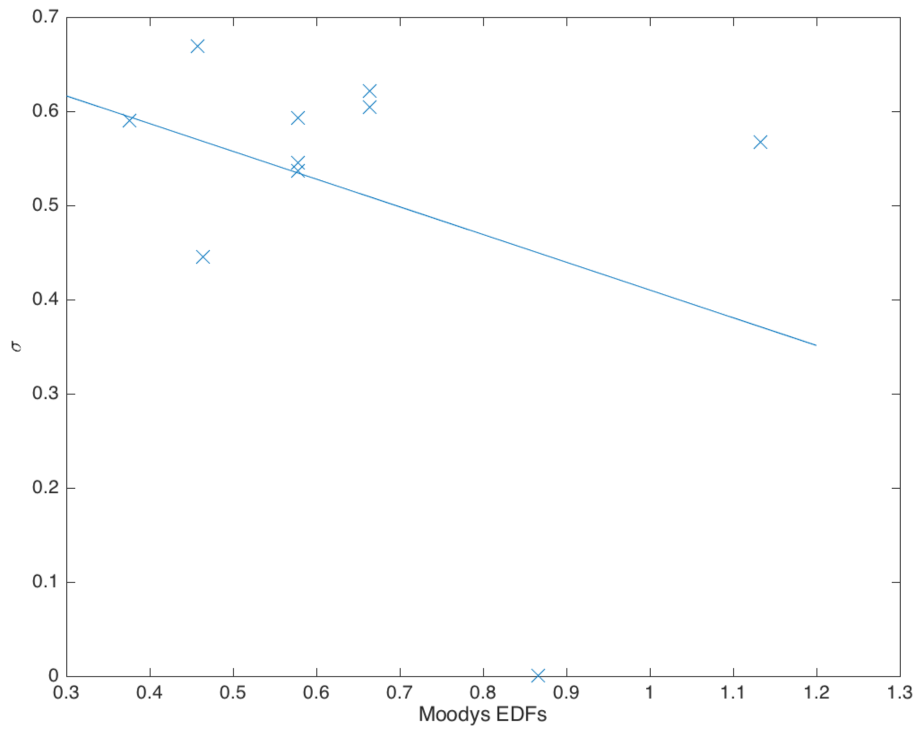

Fig. 5.1: Comparison of the risk metric derived $\hat{\sigma}$ and Reuters Structural PDs

Figure 5.1 is a graphical representation of the results displayed in Table 5.2. It suggests that the model's view of the riskiness of the asset is not aligned to those held by the Moody's model. When assessing both the table and the graph it is clear that the model and Moody's do not view the riskiness of these issues the same way, in fact there may even be an inverse relationship, i.e., an asset the model views as risky is not very risky according to Moody's. This suggests that the model currently does not capture the market sentiment. Because the $\hat{\sigma}$ for Telkom SA SOC is significantly lower than any of the others, removing it from this analysis may provide a better picture. Figure 5.2 makes the same comparison as before without Telkom and the line of best fit suggests that the two metrics are not aligned. It is evident that there is no real relationship between the two sets of metrics.

Another way to assess whether the model carries the same sentiment as other models is by comparing a ranking of the listed assets by $\hat{\sigma}$ to their respective credit ratings. Table 5.3 compares the ranked $\hat{\sigma}$ derived from the model to the Moody's Analytics Market Implied Ratings pertaining to the issuer. The Market Implied Ratings use CDS, equity and bond market information to derive a rating in line with Moody's standard rating system. It must be noted once again, that the riskiness of

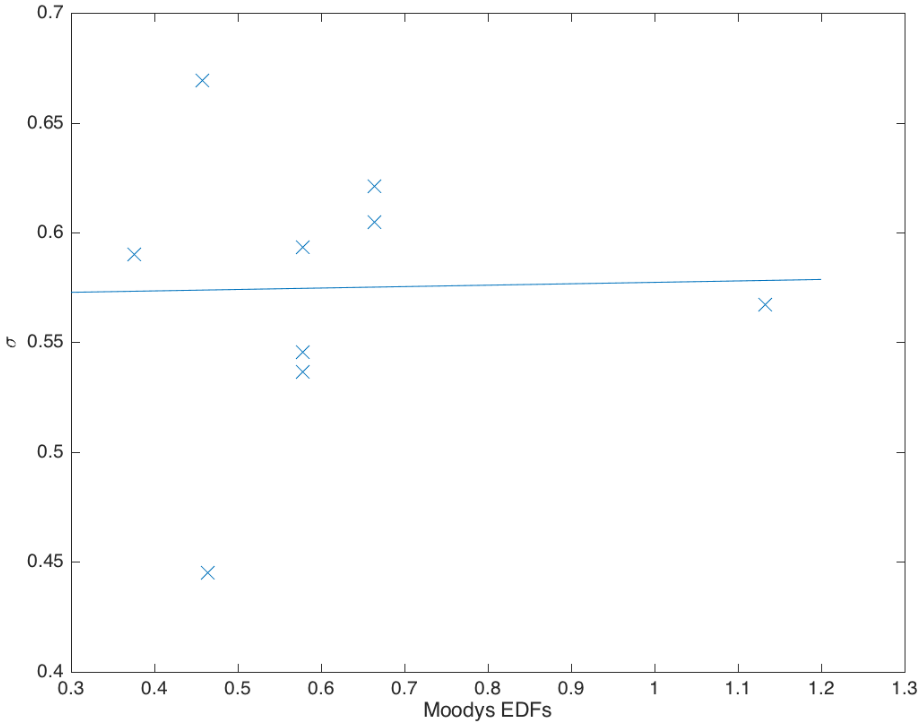

Fig. 5.2: Comparison of the risk metric derived $\hat{\sigma}$ and Reuters Structural PDs (excluding Telkom)

the asset is being compared to the riskiness of the issuer and so these comparisons are not entirely like-for-like. In Table 5.3, Telkom which is the least risky asset according to our model has the lowest credit rating. However, since we acknowledge that Telkom is a peculiar asset, we still see the same type of discrepancies with the other assets. The issuer with the higher credit rating, Growthpoint, is near the bottom of the table making it one of the riskiest assets in the portfolio. These results affirm the fact that there is no relationship between traditional credit ratings (made by credit rating firms) and the riskiness of the asset according to the $\hat{\sigma}$. One might even suggest that there is an inverse relationship and that a risky asset according to the model, implies a trusted asset according to rating agencies. The discrepancy could be because the $\hat{\sigma}$ is derived using only equity market information and the debt and equity information of the firm while credit ratings are made on a lot more information in the market and about the specific issuer. It may be useful then to include more information in the valuation model or to make adjustments to the $\hat{\sigma}$ given certain CDS, bond market or fundamental information.

One merit of the model is that the $\hat{\sigma}$ appear to converge as the number of sample

Asset	$\hat{\sigma}$	Moody's Rating
TELKOM	1,16E-16	Ba3
STANDARD	0,4451	Baa3
ABSA 1	0,5366	Baa3
BARCLAYS	0,5457	Baa3
INVESTEC	0,567	Ba1
SAPPI	0,5902	Baa3
ABSA 2	0,5932	Baa3
LIBERTY 1	0,6047	Ba1
LIBERTY 2	0,6212	Ba1
GROWTHPOINT	0,6694	Baa1
NORTHAM	0,6893	Baa2
TOYOTA 1	0,7029	-
TOYOTA 2	0,6854	-

Tab. 5.3: Listed assets ranked according to derived volatilities

paths increases. In Figure 5.3, the derived volatility for three different assets are pictured. For all three, there is a clear convergence towards a certain value.

5.2 Portfolio Management

Each unlisted asset is chosen for a particular reason. Mondi and Tiger Brands are established firms with high market capitalisation, while Imbalie and Texton are much smaller. Mondi belongs to an industry already represented in the portfolio by Sappi, and Texton is a constituent of the mining industry which is represented by Northam in the listed portfolio. The other two are chosen to be from industries not already listed in the portfolio. Apart from the issuer, all four unlisted assets are the same. Table 5.4 contains the $\hat{\sigma}$ values for the four unlisted assets which pay 3 different coupons as well as the value of the risk metric for the listed portfolio without any of the unlisted assets and the portfolios with the additional asset in all three cases. Firstly, the $\hat{\sigma}$ are considerably higher and almost double those of the listed assets. It may be that these are inflated representations of the riskiness of these unlisted assets and this is attributable to the fact that the whole residual spread is assigned as a liquidity risk in the event that the asset needs to be sold. It may be more reasonable to assign only a portion of this residual spread to this risk as the spread could be due to other factors including taxation and other risks. This should result in more reasonable figures although deciding how much of the

	9%		10,5%		12%	
	$\hat{\sigma}$	Metric value	$\hat{\sigma}$	Metric value	$\hat{\sigma}$	Metric value
Listed Portfolio	-	0,1136	-	0,1136	-	0,1136
MONDI	1,0329	0,1077	1,0473	0,1092	1,0662	0,1106
IMBALIE	1,4649	0,1245	1,4714	0,1265	1,4918	0,1285
TEXTON	1,6493	0,1082	1,6649	0,1098	1,6463	0,1117
TIGER	1,2818	0,0995	1,2962	0,1009	1,2441	0,1054

Tab. 5.4: $\hat{\sigma}$ and risk metric value for unlisted assets

residual spread is attributable to the liquidation premium would be difficult and would bring more subjectivity to the analysis. Although these $\hat{\sigma}$ values appear to be significantly high, according to the risk metric, some unlisted assets would still be included into the portfolio. The Imbalie unlisted asset would be included in the portfolio for all three coupon rates. This is an interesting result as it has the highest $\hat{\sigma}$ and so one may expect it to be too risky. If one takes a look at the correlation matrix however, Imbalie is the least correlated with the rest of the portfolio. For Imbalie, the diversification effect outweighs the high risk it adds to the portfolio and makes it a suitable inclusion to the listed portfolio for all three coupon rates. This is useful to practitioners as, although they value the diversification that these unlisted assets add to their portfolio, they often have no way to quantify the trade-off between the diversification and the additional risk. It is possible that these inflated $\hat{\sigma}$ resulted in too few assets being chosen and has resulted in the risk metric being too 'strict'. Once again, only allocating a portion of the yield spread as a result of liquidity premium may resolve this.

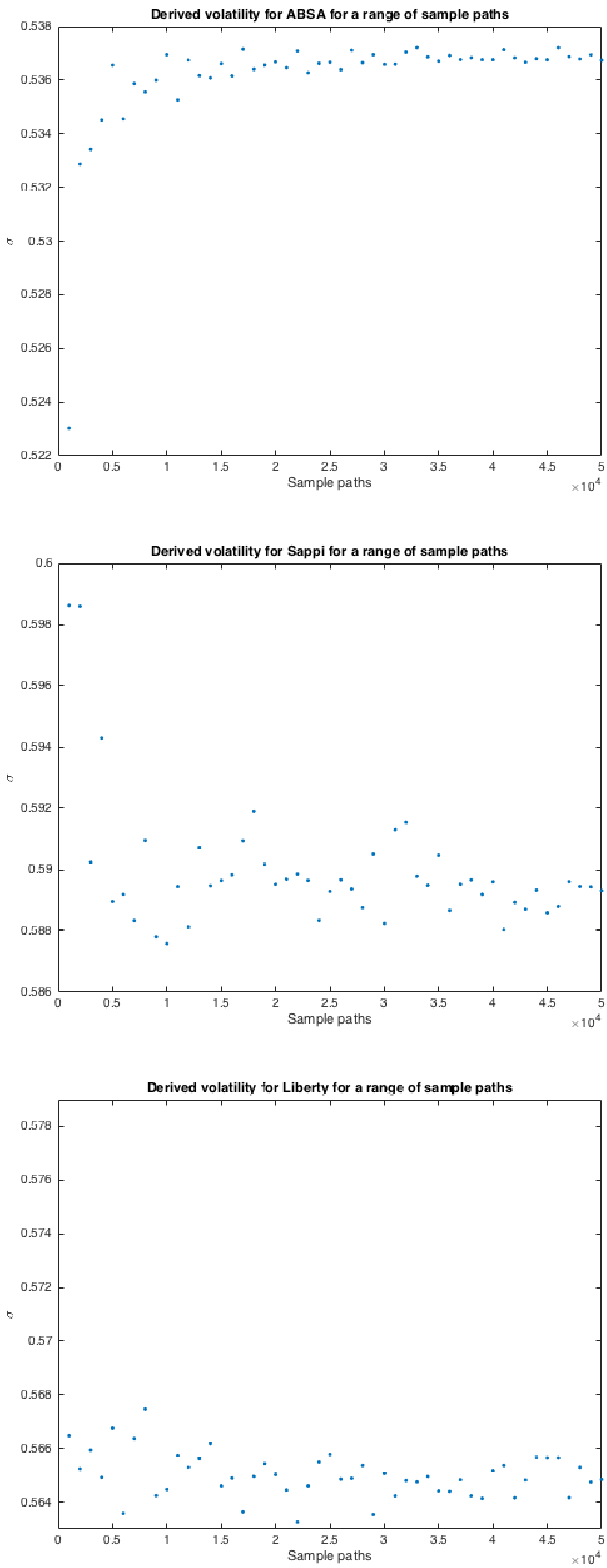

Fig. 5.3: Derived volatility of three assets for a range of sample paths

Chapter 6

Discussion

The proposed risk metric provides an objective manner for portfolio management decisions to be made. However, in order for the metric to allow for the best decision to be made many components need to be accurate and well chosen.

One component is the estimation of the relevant parameters. The Merton jump-diffusion is considered a better choice than geometric Brownian Motion for the progression of the firm asset value through time. However, the method of estimating its parameters is a lot less defined. It is therefore important to consider whether the bipower variation method employed is the correct way to estimate these parameters as well as whether it is suitable for use by credit practitioners. One might question its suitability because of the need to use high frequency data which may not be available for all the firms required or may be too onerous to obtain and use. One might even go further and question whether it is necessary to model the firm asset value as a Merton jump-diffusion and whether it is not sufficient to use geometric Brownian motion.

Secondly, it is clear that the liquidity premium results in $\hat{\sigma}$ values for unlisted assets that are incredibly high. This may lead to highly inaccurate estimation of the riskiness of the unlisted asset and result in acceptable assets being rejected because the risk parameter is an overestimate. One must thus consider whether only a portion of the residual credit spread should be used to derive the liquidity parameter. While this should lead to reduced $\hat{\sigma}$, it adds more subjectivity to the analysis as the practitioner would have to decide on how much of the residual spread is attributable to the liquidity premium.

Another decision that needs to be made subjectively is the choice of proxy firm for firms where information cannot be obtained. While organisations of similar size and industries have been used as proxies this may not entirely be the best choice. It may be important to consider if there is an objective and accurate way to choose proxy assets for firms that either have no listed bonds or are in fact not listed.

Finally, the $\hat{\sigma}$ is derived from only share price information and so ranks assets differently to credit ratings. For that reason, it is possible that in order to make more accurate decisions the $\hat{\sigma}$ should be adjusted given certain CDS, bond market or fundamental information.

This dissertation aimed to produce a Sharpe-like ratio which can be used to make objective portfolio management decisions regarding the inclusion of unlisted credit assets to a portfolio of listed credit assets. The dissertation fulfills its objectives by modelling the listed and unlisted assets and proposing a risk metric for bond portfolios that is developed from the mean-variance approach used for evaluating equity portfolios. However, there are a number of considerations as discussed above that need to be made in order to improve on this method.

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Appendix A

Appendix

A.1 Portfolio Information

Tab. A.1: Listed Portfolio

Label	Asset	Coupon	% of original portfolio	% of adapted portfolio
ABSA_1	ASASJ__17/03/2026	9,308	2,8%	2,6%
ABSA_2	ASASJ__21/12/2026	10,5	0,8%	0,8%
BARCLAYS	BACR__19/11/2024	10,667	7,8%	7,1%
GROWTHPOINT	GRTSJ__17/10/2021	9,098	10,9%	9,9%
INVESTEC	IPFSJ__22/12/2022	9,158	2,6%	2,4%
LIBERTY_1	LGLSJ__14/08/2020	9,165	14,7%	13,5%
LIBERTY_2	LGLSJ__04/10/2022	9,638	12,6%	11,5%
NORTHAM	NPLAZA__12/05/2021	13,5	3,7%	3,4%
SAPPI	SAPSJ__16/04/2020	8,06	14,7%	13,5%
STANDARD	STABAN__22/03/2021	9,358	0,2%	0,2%
TELKOM	TKGSJ__24/02/2020	6	14,7%	13,5%
TOYOTA_1	TOYOTA__28/10/2021	9,008	5,9%	5,4%
TOYOTA_2	TOYOTA__06/07/2020	8,858	8,4%	7,7%

Tab. A.2: Correlation Matrix

	ABSA_1	ABSA_2	BARCLAYS	GROWTHPOINT	INVESTEC	LIBERTY_1	LIBERTY_2	NORTHAM
ABSA_1	1	1	0,0002	0,0596	0,0477	0,0444	0,0444	0,0567
ABSA_2	1	1	0,0002	0,0596	0,0477	0,0444	0,0444	0,0567
BARCLAYS	0,0002	0,0002	1	0,4744	0,177	0,4343	0,4343	0,1114
GROWTHPOINT	0,0596	0,0596	0,4744	1	0,1175	0,4139	0,4139	0,0294
INVESTEC	0,0477	0,0477	0,177	0,1175	1	0,0263	0,0263	-0,004
LIBERTY_1	0,0444	0,0444	0,4343	0,4139	0,0263	1	1	0,1268
LIBERTY_2	0,0444	0,0444	0,4343	0,4139	0,0263	1	1	0,1268
NORTHAM	0,0567	0,0567	0,1114	0,0294	-0,004	0,1268	0,1268	1
SAPPI	0,0069	0,0069	0,0617	0,0726	-0,0541	0,1185	0,1185	0,0287
STANDARD	0,0025	0,0025	0,7557	0,497	0,0932	0,5242	0,5242	0,1844
TOYOTA_1	0,042	0,042	0,1045	0,008	0,014	0,1317	0,1317	0,0045
TOYOTA_2	0,042	0,042	0,1045	0,008	0,014	0,1317	0,1317	0,0045
MONDI	0,0077	0,0077	0,0996	0,0925	0,0271	0,221	0,221	-0,032
IMBALIE	0,0485	0,0485	0,0609	0,028	0,1037	0,0188	0,0188	0,039
TEXTON	0,022	0,022	0,1085	0,0844	0,0873	0,0969	0,0969	0,0733
TIGER	0,0273	0,0273	0,4318	0,3841	0,1336	0,369	0,369	0,2033

SAPPI	STANDARD	TOYOTA_1	TOYOTA_2	MONDI	IMBALIE	TEXTON	TIGER
0,0069	0,0025	0,042	0,0077	0,0077	0,0485	0,022	0,0273
0,0069	0,0025	0,042	0,0077	0,0077	0,0485	0,022	0,0273
0,0617	0,7557	0,1045	0,0996	0,0996	0,0609	0,1085	0,4318
0,0726	0,497	0,008	0,0925	0,0925	0,028	0,0844	0,3841
-0,0541	0,0932	0,014	0,0271	0,0271	0,1037	0,0873	0,1336
0,1185	0,5242	0,1317	0,221	0,221	0,0188	0,0969	0,369
0,1185	0,5242	0,1317	0,221	0,221	0,0188	0,0969	0,369
0,0287	0,1844	0,0045	-0,032	-0,032	0,039	0,0733	0,2033
1	0,0678	0,1584	0,3953	0,3953	-0,0176	0,2115	0,1483
0,0678	1	0,1689	0,1647	0,1647	0,072	0,0314	0,4849
0,1584	0,1689	1	0,1694	0,1694	-0,0603	0,1908	0,0914
0,1584	0,1689	1	0,1694	0,1694	-0,0603	0,1908	0,0914
0,3953	0,1647	0,1694	1	1	-0,0322	0,1478	0,1796
-0,0176	0,072	-0,0603	-0,0322	-0,0322	1	0,0527	0,1247
0,2115	0,0314	0,1908	0,1478	0,1478	0,0527	1	0,0746
0,1483	0,4849	0,0914	0,1796	0,1796	0,1247	0,0746	1