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# **The Ghana Stock Exchange: Concentration, Diversification, Liquidity**

Mini-Dissertation

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Dedication:

I would like to dedicate this thesis to my mom Akosna Baaha, my dad Mr Kobina Kumi and finally to my girl friend, Sefalto Alipui for all their assistance, motivation and encouragement throughout this Mathematical Finance programme.

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## 0.1 Introduction

Analysts have found that concentration of portfolio weights affects portfolio risk. This is a unique feature in small markets where they tend to be concentrated in few stocks and the Ghana Stock Exchange (GSE) falls in that category. As a result portfolios based on the Ghana All Share index are highly concentrated.

The risk in a portfolio is mainly attributed to Covariance and weighting structure. Enough cannot be done about the covariance structure but the weighting structure can be controlled since it depends mainly on investment choices. The weighting structure determines the degree of concentration of a portfolio. The term concentration refers to the extent to which portfolio weights skew away from equally weighted distribution of portfolio weights. As at September 2009, the Ghana All Share index has about five (5) of the total of thirty-five (35) in the index accounting for about 82.25% of the index weight. Concentration can be measured using the Herfindahl-Hirschman index (HHI) or Richard Roll measure (RRC).

Diversification is concerned with generation of returns from different sources. The traditional method of measuring diversification has fallen short of what is usually expected hence the introduction of the new measure, portfolio diversification index or PDT.

Liquidity measures the effect the quantity of stocks traded has on the market price of stocks. Liquidity varies from time to time; hence its importance as a source of risk for investors.

The primary objective of this project is to determine the significance of concentration in portfolio risk, particularly from the Ghanaian perspective. Furthermore, we will be comparing diversification using the new measure and finally end with

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## 0.2 The Ghana Stock Exchange(GSE)

The Ghana Stock Exchange as a public company limited by guarantee has no owners or shareholders as such, but members are either corporate bodies or individuals.

There are three categories of members, namely Licensed Dealing Members (LDM), Associate Members and Government Securities Dealers (GSDs). An LDM is a corporate body licensed by the Exchange to deal in all securities. An Associate member is an individual or corporate body which has satisfied the Exchange's membership requirements but is not licensed to deal in securities. A GSD is a corporate body, which is approved by the Bank of Ghana and registered by the Exchange to deal only in government securities. GSE operates within a set of Rules, including membership, listing, trading, clearing settlement and depository. These are collectively referred to as the GSE Rule Book. Trading takes place every working day. The Exchange publishes the GSE All-Share Index, which comprises of all listed equities on GSE. It has its base as the average capitalization for the period covering November 12, 1990 to December 31, 1993. The base is 100. Currently (September 2009) the GSE has thirty-five (35) listed companies.

### 0.2.1 History

It all started on February, 1989 the then PFDC government set up a ten member committee chaired by the then governor of the Bank of Ghana Dr. G.K. Agama. The work of the committee was to consolidate all previous work connected to the Stock Exchange project and to fashion out modalities towards the actual establishment of the Exchange. As a result of the work of the committee, the Stock Exchange was born in July 1989 as a private company limited by guarantee under the Companies Code of 1963. It was given recognition as an authorized Stock Exchange under the Stock Exchange Act of 1971 (Act 384) in October 1990. The Council of the Exchange was inaugurated on November 12, 1990 and trading commenced on the same day. The Ghana Stock Exchange was officially launched on 11th January 1991. The Exchange, changed its status to a public company limited by guarantee in April 1994.

### 0.2.2 Regulations

The Exchange Control permission was given in 1993 to non-resident Ghanaians and non-resident foreigners to invest through the Exchange without any prior approval. However, each non-resident, foreign portfolio investor was not to hold more than 10% of a listed company's total issued shares while total holdings of non-resident foreigners in any one listed securities

was limited to 74% unless with prior exchange control approval from the Bank of Ghana.

The new Foreign Exchange Act of 2006 (Act 723) has done away with these limitations. Therefore non-resident investors can now invest in the Ghana Stock Exchange market with no limits or prior exchange control approval.

There is free and full foreign exchange remit ability for the original capital. Itics all capital gains and related earnings.

There is an 8% withholding tax (which is also the final tax on dividend income) for all investors, both resident and non-resident. Capital gains on listed securities are exempted from tax until November 2010.

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## 0.3 Concentration

Concentration refers to the extent to which portfolio weights skew away from an equally weighted distribution of portfolio weights. Thus, it is a reflection of how portfolio weights are concentrated in few stocks. The concept of concentration has received little attention in academic literatures, possibly because most financial research has been centered on the New York Stock Exchange, which has a far more even distribution of market capitalisation weights than the smaller markets.

Concentration can be measured using the Herfindahl-Hirschman Index (HHI) or the Richard Roll measure.

The Herfindahl-Hirschman index is a measure of concentration discovered by Hovenkamp in 1986. The computation was based on weights of the stocks in a portfolio. This is done by first squaring the weights followed by the sum of the squared weights. i.e.

$$HHI = \sum_{k=1}^N W_k^2 \quad (1)$$

Where  $W_k$  is the investment weight in the  $k$ th security and  $N$  represents the total number of assets or security in the portfolio. The HHI measure will be minimum when the investment weights are equal and will become larger the more the investment weights are skewed. A maximum value of one is obtained when a portfolio has only one security. The HHI measure is widely used than any other measure of concentration.

The Richard Roll measure of concentration (RRC) was originally used in measuring concentration in industries and has now been adapted in measuring concentration on a stock level. It is defined as

$$RRC = \frac{N}{N-1} \left( \sum_{k=1}^N W_k^2 - \frac{1}{N} \right) \quad (2)$$

A portfolio of equally weighted (i.e.  $1/N$ ) stocks have an RRC measure of zero. Clearly the greater the concentration of investment weights the larger the RRC measure.

### 0.3.1 Concentration in Ghana

The value obtained for concentration from using either the HHI measure or the RRC measure does not differ that much. In this section we will try to verify this and also go ahead to calculate the expected risk of the Ghana All Share Index. We will first use the equal weighting scheme and move on using the current (September, 2009) market capitalisation for all the stocks on the

exchange. The expected risk is calculated using the historical covariance matrix (from December, 2004 to September, 2009). In calculating the risk only 34 stocks are used. AngloGold Ashanti Limited is the excluded listed company because of the zero returns produced between December, 2004 to September, 2009.

GSE All Share Index	Equal Weight	Market Capitalisation
Concentration HHI (all 35-stocks)	2.86%	46.33%
Concentration RRC (All 35-stocks)	0	44.76%
Expected Risk (only 34-stocks)	13.47%	32.97%

### 0.3.2 Impact of Concentration on Portfolio Risk

The process of reducing risk through portfolio formation is necessary in portfolio design. This concept was first described by Markowitz (1952) on how to combine assets into efficiently diversified portfolio. The Markowitz approach used two (2) important statistics that is mean and standard deviation. The mean and standard deviation is used to quantify the return and risk of a portfolio respectively. The total risk of a portfolio in this approach is represented as

$$\sigma_p^2 = \sum_{i=1}^N W_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, i \neq j}^N W_i W_j \sigma_{ij} \quad (3)$$

Where  $\sigma_p$  is total variance of the portfolio,  $W_i$  is the weight of the  $i$ th security,  $\sigma_i^2$  is the variance of the  $i$ th security and  $N$  is the total number of assets. Now we will illustrate the role concentration plays in portfolio risk. We kick off by making some unrealistic assumptions. Firstly we assume that the assets are uncorrelated, this reduce equation (3) to

$$\sigma_p^2 = \sum_{i=1}^N W_i^2 \sigma_i^2 \quad (4)$$

since the covariance between the securities are zero. We further proceed by assuming each asset has the same average variance,  $\sigma^2$  this leaves us with

$$\sigma_p^2 = \sigma^2 \sum_{i=1}^N W_i^2 \quad (5)$$

From the definition of the HHI measure we arrive at

$$\sigma_p^2 = \sigma^2 HHI \quad (6)$$

The expression above makes it evidently clear that under these oversimplified assumptions the only factor that affects the portfolio risk is the degree of

concentration of the portfolio. The same assumptions lead to

$$\sigma_p^2 = \sigma^2 \left( \frac{N-1}{N} RRC + \frac{1}{N} \right) \quad (7)$$

But this time around we are using the Richard Roll measure. This confirms the direct impact of concentration on risk.

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## 0.4 Diversification

Diversification is fundamental to portfolio construction and it is one of the core objectives for combining assets. Companies merge to ensure returns are generated from different sources. The importance attached to diversification has improved in recent years as managers are interested in the value of diversification as they add more assets to their portfolios. A portfolio is said to be well diversified when the return as well as the risk, arises from many unrelated sources.

Diversification was first measured using market index as a benchmark. This method has however been found to be inexact, because the market index of some small stock markets has been found to be poorly diversified due to the level of concentration in those markets. The disability of the traditional measure of diversification led to the discovery of a new measure called the portfolio diversification index or PDI. The PDI is consistent with the definition of diversification, The PDI is based on the number of independent factors observed in a portfolio. These factors are quantified using the principal component analysis or PCA.

Diversification can be measured by two approaches known as the traditional method and the Portfolio Diversification Index (PDI),

### 0.4.1 Traditional Approach

Diversification was first measured using this method. This method is dependent on the market index. The measurement is a simple regression of the portfolio returns against the market index returns. The variance of the residuals determines the amount of diversifiable risk remaining in a portfolio thus smaller residual variance indicates a better amount of diversification in the portfolio.

The inability to free the traditional approach from the overall market index makes it fall short, in measuring diversification effectively. This is because the market index of some markets particular developing markets might not be diversified across industries. This can mainly be attributed to one factor, and that is concentration. Concentration in few industries can swamp the diversification effect of other industries.

### 0.4.2 Portfolio Diversification Index (PDI)

The traditional approach of measuring diversification makes it clear that, to free the measure from the influences of the overall market index, hence the introduction of the PDI.

The PDI measure was proposed by researchers Alexander Rudin and Jonathan

Morgan in 2006. This premise of diversification was based on the number of independent factors observed in a portfolio. These independent factors are quantified using the principal components analysis (PCA), termed as the portfolio diversification index (PDI). A portfolio is completely undiversified if the PDI measure is equal to one (1) i.e. dominated by a single factor. The portfolio will be completely diversified if the PDI measure is equal to the number of assets in the portfolio. Thus, the smaller the PDI measure the less diversified the portfolio. Conversely, the larger the PDI measure the more diversified the portfolio.

The PDI can be carried out in two ways that is by forming a portfolio of assets or by using a collection of assets. The difference between the two approaches is that for the portfolio approach the analysis is performed on the covariance matrix while in the portfolio-free approach the analysis is done on the correlation matrix. The reason is that when calculating the PDI of a portfolio we want to decompose the portfolio variance and so we need to take account of the fact that different assets have different volatilities. In the case of the portfolio-free method we are only interested in the potential diversification that is available. Due to this reason the variance of each asset is less important than the correlation between assets. This is because it is always possible to down-weight highly volatile assets when constructing a portfolio. In this project we will stick to the portfolio approach.

PDI, which measures the number of unique investments in a portfolio, is useful to assess marginal and cumulative diversification benefits across asset classes and across time. Its implementation in hedge fund strategies reveals that various hedge funds offer less diversification than may have been thought, and that there has been reduced diversification in the past several years. Graphically the EDI is the centre of gravity (or balancing point) of the independent factors.

#### 0.4.3 Effect of Concentration on Diversification

We make a more realistic assumption Hillier diversification. Assuming all assets forming our portfolio have equal weights i.e. we invest  $1/N$  in each asset,  $N$  being the number of assets.

$$\sigma_p^2 = \frac{\sigma^2}{N} + \frac{N-1}{N} \sigma_{ij} \quad (8)$$

A rearrange of this expression can be found in the appendix. The first and second term of equation (8) convey the key concepts of diversification. The second term conveys the lesson that the more assets with low covariance we can include in the portfolio the lower the portfolio's risk. The first term however conveys the idea that as the number of assets in the portfolio is increased, the portfolio's risk reduces by the factor  $1/N$ , until for very large  $N$

where the first term becomes insignificant, Hence for large N the portfolio's risk converges to the average covariance across the assets. The question we would like to answer is what happens if the assets are not equally weighted but highly concentrated? To answer this question we return to equation (3) to assess the impact of weights skewing away from equal weights.

$$\sigma_p^2 = \sum_{i=1}^N W_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, i \neq j}^N W_i W_j \sigma_{ij}$$

It is clear from

higher than average variances, then the risk of the portfolio will be higher which rises geometrically with the increase in weight. The second term is probably more important as it, suggests that if assets with larger weights also have larger covariance with each other, the portfolio risk will be higher. This is not the case on the Ghana Stock Exchange(GSE) as stocks with large Market capitalisation tends to be less correlated with each other. An Example is the second and third large stocks in the GSE All Share Index, Ecobank Transnational Inc. and Golden Star Resource Limited has a correlation of -0.0282 which is less than the average correlation of 0.0405 amongst all the GSE All Share Index( i.e. only 34 stocks excluding AngloGold Ashanti Limited) .This is not the case in the South African stock market. Stocks with high market capitalisation tend to be highly correlated according to Bradfield and bigomari research on concentration in the pmth African stock market. Clearly the impact of eoncentration on risk boils down to distribution of weights among volatilities and correlations.

## 0.5 Liquidity

In business, economics or investment, market liquidity is an asset's ability to be sold without causing a significant movement in the price and with minimum loss of value. Liquidity in financial markets is dynamic and is an important source of risk due to the uncertainty attached to it. Money or cash on hand is the most liquid asset. Exchange of a less liquid asset with a more liquid asset is called liquidation.

A liquid asset can be rapidly sold without losing its value any time within trading hours. In a liquid market, there is the existence of competition to buy and sell always. Another elegant definition of liquidity is the probability that the next trade is executed at a price equal to the last (in a liquid market there are large scale of buyers and sellers ready to trade. This can be related to the concept of market depth, which can be measured as the units that can be sold or bought for a given impact. The opposite concept is that of market breadth measured as the price impact per unit of liquidity and it consists of assets.

In illiquid stock markets, investors are unable to sell large amounts of shares without a sharp decline in the price of the shares. An illiquid asset is an asset which is not readily saleable due to uncertainty about its value or lacking a market in which it is regularly traded. The mortgage related assets which resulted in subprime mortgage crisis are examples of illiquid assets as their value is not readily determinable despite being secured by real property. Another example is asset such as a large block of stock, the sale of which affects the market value.

A number of factors can lead to a decline in asset market liquidity. Firstly, if a country's financial intermediaries experience a substantial deterioration in their balance sheets. If market makers and other investors' faces credit constraints, they may reduce their ability to take advantage of high returns by providing liquidity to an illiquid market. Also when the country is operating in a deflationary environment in which savers are able to earn real returns simply by holding money. This will again reduce their incentives to take speculative risks by liquidity to the market.

Speculators and market makers are key contributors to the liquidity of a market, or asset. Speculators and market makers are individuals or institutions that seek to profit from anticipated increases or decreases in a particular market price. Doing this helps provide the capital needed to facilitate the liquidity.

In 2003 Pastor and Stambaugh did a liquidity study on the United States equity market. Our review will be different from their study. Their method was based on the extent to which the volume of stock traded affects the market price of stocks. The impact the quantity of stocks traded has on market price of stocks is experienced more in illiquid market but in liquid markets the effect is not so obvious. Amihud and Mendelson (1986) is an

early study of the relationship between market liquidity and stock returns. Campbell, Grossman and Wang (1993) constructed a model in which risk-averse market makers require a premium to buy large quantities of stock.

### 0.5.1 Measuring Stock Market Liquidity

In measuring liquidity in stock markets (e.g. the Ghana stock) we will use Pastor and Stambaugh's (2003) method as a guide. In doing so, we will use the Ghana all share index, indexed by  $k$ , we estimate the excess returns for each firm within the specified period. The ordinary least squares (OLS) estimation method is used we estimate the following equation:

$$r_{k,d,t}^{xs} = a_{k,t}^0 + a_{k,t}^1 \cdot r_{k,d-1,t} + a_{k,t}^2 \cdot \text{Sign}(r_{k,d-1,t}^{xs}) \cdot \text{Vol}_{k,d-1,t} + \varepsilon_{k,d,t} \quad (9)$$

Where  $r_{k,d,t}$  is the return on the stock of company  $k$  on day  $d$  of month  $t$ . We define  $r_{d,t}^{MKT}$  as equally-weighted return on Ghanaian stocks. The excess return  $r_{k,d,t}^{xs} = r_{k,d,t} - r_{d,t}^{MKT}$  is a measure of the difference between return on stock  $k$  and the market return. The  $\text{Sign}(r_{k,d-1,t}^{xs})$  variable is equal to 1 when lagged excess returns are positive and -1 when lagged excess returns are negative. We define  $\text{Vol}_{k,d,t}$  as the value of shares traded, measured in billions of Ghana Cedis. The signing of the trading volume is meant to distinguish whether trades are driven by selling pressure from investors or by buying pressure. When investors are selling shares in a company to market makers or other short-term providers such as speculators, excess return on that company should be negative. When investors are buying from market makers, excess returns, should be positive. The parameter  $a_{k,t}^2$  measures the degree to which sales affect returns and thus might be thought of as a measure of liquidity in that particular market.

## 0.6 Methodology

In this section we will discuss the procedures to be followed in concentration and diversification.

### 0.6.1 Concentration

In measuring concentration in the Ghana stock market, we will closely follow a concentration research carried out by Cadiz Financial Strategists on the South African stock market in 2004. They used a prior 3-years data. A monthly historical data between the periods, December 2004 and September 2009 will be used in measuring concentration on the Ghana Stock Exchange. About 13 stocks have to be backfilled using the average monthly sector prices depending on the sector that particular stock falls under. There are 35 companies listed on the exchange but we will use only 34 stocks because the monthly returns obtained by AngloGold Ashanti Limited (AGA) (the stock with the largest market capitalisation) between the periods of December 2004 and September 2009 was zero throughout this period. Also risk computed for each portfolio be converted to annual risk. The returns for each stock is calculated using the formula,

$$r_i = \ln \left( \frac{P_i}{P_{i-1}} \right) \quad (10)$$

$r_i$  is the current return of the stock.  $P_i$  is the current monthly price.  $P_{i-1}$  is the previous monthly price.

The objective is to assess empirically how various levels of concentration affect the resulting risk of a portfolio. This is done using simulation methodology for different portfolio sizes.

Throughout our analysis on concentration we will base our variance and covariance estimates of risk on the 34 stocks.

Three different scenarios would be considered in relation to the portfolio weighting scheme and the correlation structure of the stocks. 121

- Scenario 1 considers equally-weighted stocks, assuming zero correlation between the 34 stocks.
- Scenario 2 considers equally-weighted stocks, assuming, their historical correlation structure.
- Scenario 3 considers market capitalisation-weighted stocks, assuming their historical correlation structure.

All estimate of risk are between the periods, 31 December 2004 to 30 September 2009, historical data and also the methodology is based on simulations run of 500 times for each portfolio size (i.e. for each number of stocks).

We find the returns for all the 34 stocks in Microsoft excel using the historical data. This matrix of returns is imported to matlab and the covariance for these returns is computed using the matlab in-built function cov. The covariance matrix for the returns obtained is copied and pasted in a new excel sheet. On the same sheet the market capitalisation weight is placed at a column next to the covariance matrix. This column vector of market capitalisation is rescaled to one and should correspond to the various assets representing each row for the covariance matrix.

With the covariance matrix and the market capitalisation in place we can now use our Visual Basic (VBA) program, found at the appendix. The program forms various portfolios from 1 to the total 34 stocks. For each portfolio formed the program rebases the portfolio weights to sum to one, being it scenario 1, 2 or 3. Not only that, the risk (standard deviation) is computed for each of the portfolios. This computation process is repeated 500 times to obtain 500 estimates of risk for each portfolio. The average of the 500 estimates of risk is calculated and displayed on the excel sheet already holding the covariance matrix and the market capitalisation weights. The result displayed on the spreadsheet is the risk for the portfolio of 1 stock through to the portfolio of 34 stocks. Note that the program only terminates after the formation of the portfolio of all the 34 stocks and also the market capitalisation is used only in scenario 3. Before I end, it is worth knowing that for scenario 1 both the upper and lower triangular of the covariance matrix will be filled with zeros, only the leading diagonal will keep its values (i.e. the standard deviation for each stock). The actual covariance matrix is used for the other two scenarios (i.e. 2 and 3).

Below is a vivid outline of the procedure to be followed.

- Randomly select a stock(k-1) out of the total 34 stocks.
- Use the number of stocks(k) selected to form a portfolio based on the scenario under consideration.
- Ensure the total portfolio weight sum to one.
- Compute the risk (standard deviation) of the portfolio.
- The risk computation should be repeated 500 times to obtain 500 estimates of risk
- Record the mean of the 500 estimates of risk.
- Proceed by increasing the number of stocks (k) by one and follow the same procedure above.
- Stop when the portfolio consists of all the 34 stocks (i.e. k=34)

### 0.6.2 Portfolio Diversification Index(PDI)

We will once again follow a Cadiz Financial Strategists research on the new Measure of non fnlio diversification in 2006.

Firstly, we select six equities from the Ghana All Share Index and form portfolio of six assets each. Six different portfolios are formed. Monthly market price for the stocks selected between the periods of 31st January, 2005 to 30th September, 2009 is used to obtain the returns for each of the asset within this duration using equation 10 above.

[6]

We now have to assign weights to each of the six stocks in the portfolios. The current (September, 2009) market capitalisation of the stocks are used as their portfolio weights. The constraint on the weights is that the weight of all the assets in a portfolio should sum up to one. For this reason we add up the market capitalisation of all the assets and for each asset we divide the market capitalisation of that asset by this sum (i.e. the total market capitalisation for that portfolio). After getting these weights for the assets, the portfolio return is obtained by multiplying the weight of each asset by its column vector of returns. A 50-by-6 matrix is obtained as the return for the portfolio. All the above task was done using Microsoft Excel. This 56-by-6 matrix is imported to matlab.

After the importation of the matrix to matlab an in-built function cov is used to find the covariance matrix of the 56-by-6 matrix. Since we are using portfolios, all the analysis will be performed on the covariance matrix. A principal component analysis is performed on the covariance matrix by using matlab in-built function Pcov. From this analysis, we obtain eigenvalues ranked from the largest to the smallest. Not only that, we also obtain percentage contribution of each factor (eigen-value) towards the total volatility and the coefficient of the principal components.

Finally, with all these results obtained, we can now go ahead and calculate the PDI measure for the portfolio using

$$PDI = 2 \sum_{k=1}^N k \lambda_k - 1 \quad (11)$$

Where N is the number of assets and M is the fraction contribution of factor k to total volatility. The fraction contribution is obtained by dividing the percentage contribution by 100.

Risk is very important in portfolio construction, so we will use the covariance matrix and the weight of each of the asset in the portfolio to calculate the portfolio's expected risk. The Markowitz (1952) formula for calculating risk in a portfolio is used, i.e.  $\sigma_p^2$

$$\sigma_p^2 = W^T (CovarianceMatrix) W \quad (12)$$

$NV$  is the  $6 \times 1$  column vector of weights of all the assets.  $1111-1$  is the transpose of the  $cA_{11111111}$  vector of weights. The portfolio risk is the square root of the result °Manic( from equation (12).

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## 0.7 Analysis and Results

In this chapter there will be intensive analysis on the results obtained from the methodology section.

### 0.7.1 Concentration

#### Scenario 1: Equal Weights with Zero Correlation

The assumption that assets are equally weighted with zero correlation with each other does not really occur in practice. Equation (8) reduces to:

$$\sigma_p^2 = \frac{\sigma^2}{N}$$

because the covariance term will be zero under the assumption of independence. This can be written as:

$$\sigma_p = \sqrt{\frac{\sigma^2}{N}} = \frac{\sqrt{\sigma^2}}{\sqrt{N}} \quad (13)$$

The numerator is the average standard deviation of the stocks in the portfolio.

More importantly the portfolio standard deviation decreases by a factor of  $\frac{1}{\sqrt{N}}$ .

Below is a graph of equally-weighted portfolio size (i.e. increasing in size from 1 to 34 stocks) with its associated standard deviation (risk). Another assumption is that the stocks are not correlated. Thus we can use equation (13) to compute the total risk of the portfolios. One interesting observation is that the portfolio consisting of only one stock has a standard deviation (63.19%) approximately equal to the average standard deviation of the 34 stocks (i.e. 63.39%). Also it can be seen from the graph that as the number of stocks in the portfolio increases the risk decreases. [8]

1.jpg

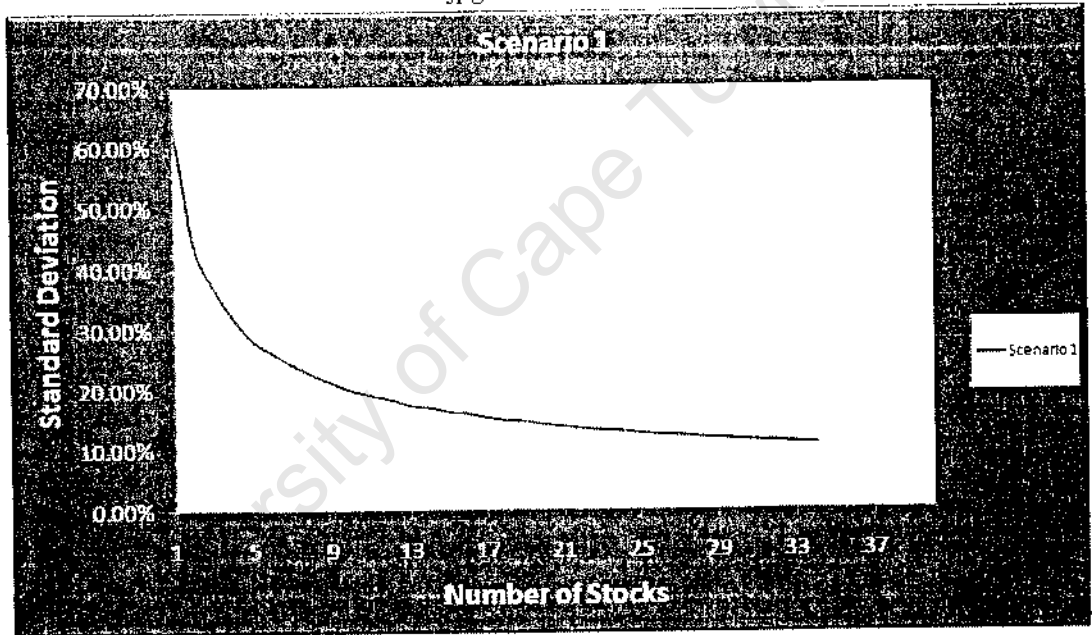


Figure 1: Equal Weight and Zero Correlation

From Figure 1 we can see that the risk of single stocks in the GSE All Share Index- under scenario 1 is 63.19 Vr per annum(p.a.)(the true average standard deviation of the 31 stocks is actually 63.39%) Figure 1 also shows that the portfolio risk for all the 34 stocks is 10.87% p.a. compared to the current (September 2009) GSE All Share Index risk of 32.97% p.a. excluding AngloGold Ashanti Limited(AGA). significant risk reduction of 22.10%(32.97-10.87%) is due to the assumption of equal weights and no correlation. Thus, confirms the impact of concentration and correlation has on portfolios risk in the Ghanaian setting.

The above result for the 34 stocks portfolio could have been estimated directly using equation (13).i.e. 631

$$\sigma_p = \sqrt{\frac{\sigma^2}{34}} = \sqrt{\frac{(0.6339)^2}{34}} = 10.87\%p.a.$$

We now contrast our result scenario 1 to Bradfield and Kgomari research on concentration in the South African stock market in 2004. They found that under these two assumptions(i.e. equally-weighted and no correlation) a risk reduction of 19%,(22%-3%) was achieved, because under scenario 1 the portfolio risk for all the 165 stocks in the South African stock market was 3% while the risk for the South African All Share Index(ALSE) was 22% (September 2003 market capitalisation was used); but under the Ghana stock exchange a risk reduction of 22.10% was obtained.

#### Scenario 2:Equal Weights - Historical Correlation Structure Assumed

In scenario 2 we simulate portfolios based on the assumptions of historical correlation structure estimated from December 2004 to September 2009 but the assumption of equal weight is maintained. The result obtained is represented graphically in Figure 2 below;

2.jpg

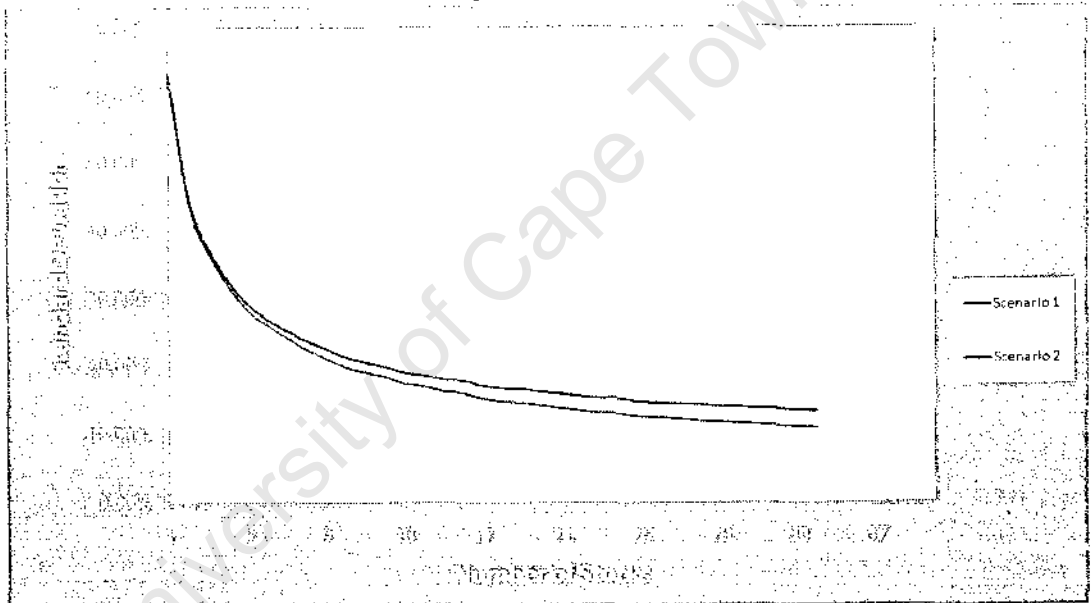


Figure 2: Equal weighted portfolio- historical correlation structure

The introduction of correlation between stocks in scenario 2 causes the portfolio risk of scenario 2 to be higher than the portfolio risk under the assumption of equally-weighted and no correlation between stocks. An example is, in scenario 2, the risk for the portfolio of 34 stocks is 10.877p.a. compared to the 13.4782p.a. obtained under the assumption of equal weights and historical correlation (scenario 2). A 2.5970p.a. (13-17% increase in risk) increase in risk testifies to the fact that the stocks are fairly correlated. We can conclude that to reduce the risk of our portfolio we should add more uncorrelated assets to the portfolio, which might be difficult to come across in the real world.

The simulated result obtained above could be generated using equation (8) and the derivation which assumes normal correlation structure but equal weighting scheme.

If we include all the 34 stocks in an equally weighted portfolio then the portfolio risk can be computed analytically as:

$$\sigma_p = \sqrt{\frac{(0.6339)^2}{34} + \frac{(34-1)(0.0065)}{34}} = 0.1347$$

Which conforms to the simulated portfolio risk of 34 stocks.

Once again, comparing our result to Bradfield and Agomari result, we realise that the elimination of the assumption of no correlation and introducing correlation between the stocks increases the risk from 37p.a. to 167p.a. (for the 165-stock portfolio). This substantial increase in risk of 137% (167-3%) was as a result of the relationship between stocks in the South African market. This increase in risk makes it clear that stocks in the South African market are highly correlated than stocks in the Ghana stock exchange. The risk increase was also due to the high correlation existing between stocks with high market capitalisation in the South African market compared to that of the Ghanaian Market where stocks with high market capitalisation tend to be less correlated.

### Scenario 3: Market capitalisation weights - historical correlation structure assumed

In scenarios 1 and 2 we constructed portfolios of equal weights which gave us an insight about the impact correlation has on portfolio risk. The third scenario will help us to assess the impact concentration has on portfolio risk. Under this scenario we designed a weighting scheme based on the market capitalisation-weights (taken as at the end of September 2009) of stocks together with the relationship between the stocks. The process followed does not differ that much from scenario 2 above. The only difference is that the GSE All Share market capitalisation weights are used.

The portfolio's risk can once again be computed directly using either equation (3) or (12). Below is a graphical representation of all the three scenarios discussed above.

3.jpg

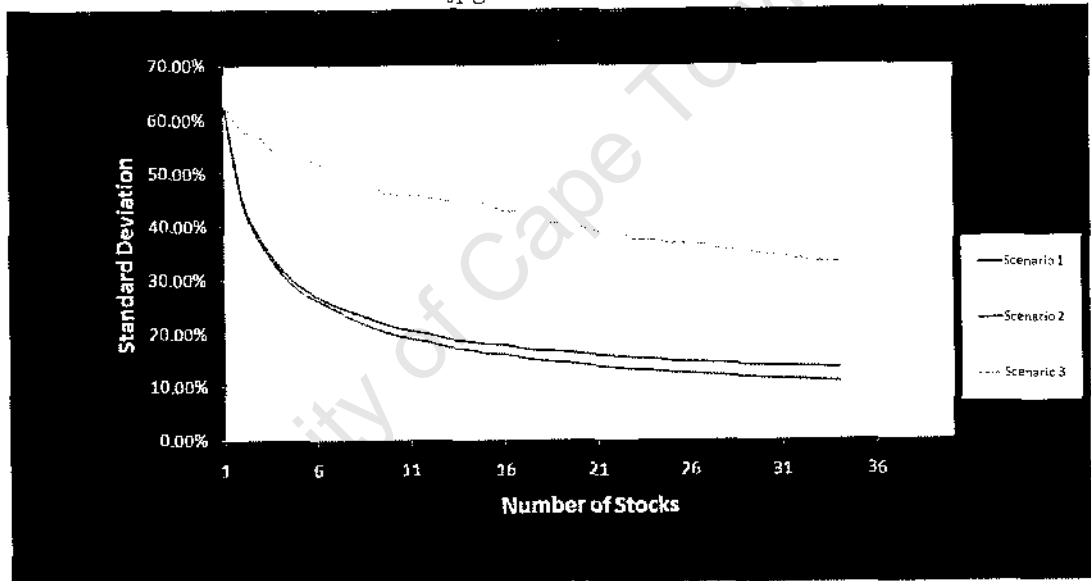


Figure 3: Market capitalisation - weighted portfolios - historical correlation structure

Figure 3 confirms that the risk estimate for the 34-stock portfolio is identical to the GSE All Share index risk of 32.97% p.a. This is because the

The important thing about Figure 3 is that we are able to assess the impact concentration has on portfolio risk in the Ghanaian market. The Figure reveals that the difference in risk between equally weighted portfolios and market capitalisation-weighted portfolios on the GSE for the total 34 stocks is 19.51% p.a. (i.e. 32.97% - 13.47%). This significant difference is as a result of concentration. We can conclude that three-fifths (3/5) of the GSE All Share index risk is due to the effect of concentration.

Another interesting revelation is that a randomly selected 4 stock portfolio with equal weights has a risk of 32.58% which is approximately equal to the risk (32.97%) of the GSE All Share Index consisting of market capitalisation weights of all the 34 stocks.

Finally, we will compare scenario 3 results to the results obtained by Bradford and Kgornari from their study of concentration in the South African market. From their research on the South African market in 2004, they found that a 165-stock portfolio under the same assumptions of scenario 3 has a risk of 22% p.a. compared to the 16% p.a. they obtained under equally-weighted and historical correlation assumptions. Thus, due to concentration, there was an increase in risk of 6% p.a. (22% - 16%). Contrasting this to the 19.51% p.a. increase in risk obtained from the Ghana stock exchange, it can be said that the Ghanaian stock market tends to be more concentrated than the South African stock market.

### 0.7.2 Portfolio Diversification Index

Portfolio	PDI	Concentration	Expected Risk
Portfolio 1	2.057 (3)	0.243 (1)	0.248(4)
Portfolio 2	1.791(5)	0.489 (6)	0.208 (3)
Portfolio 3	2.664 (2)	0.305 (3)	0.191(1)
Portfolio 4	1.159 (6)	0.389(5)	0.285 (6)
Portfolio 5	3.193 (1)	0.261 (2)	0.202 (2)
Portfolio 6	1.978 (4)	0.307 (4)	0.269(5)

The table above shows the results for PDI, concentration and expected risk for each of the six portfolios. Rankings are shown in parenthesis.

From Column two (PDI), portfolio 5 is the most diversified portfolio ranked one, simply because it has the highest PDI measure (3.193) compared to the other five portfolios. Portfolio 4 is the least diversified portfolio that is why it is ranked sixth (6th) because it has the smallest PDI value (1.159). In general, the higher the PDI measure the more diversified the portfolio. Portfolio 1 is the less concentrated portfolio among the rest. This is because it has the smallest concentration measure, 0.243 and portfolio 2 is

the most concentrated portfolio dumm to its high concentration value: (1189). The smaller the concentration measure the lesser concentrated the portfolio becomes.

Finally we take a look at the last column (expected risk): Portfolio 3 is the portfolio with the minimum risk, 0.191p.a. The most risky portfolio is portfolio 4 having an expected risk of 0.285p.a. Risk is quantified by its size that is bigger value represent higher risk for that portfolio,

We noticed that the risk and the concentration are closely related to the PDT measures across the portfolios. The more diversified a portfolio (i.e. higher PDI value), typically the lower its risk and concentration measure, thus highlighting the linkage between these concepts. An example is portfolio 1, it is ranked sixth as the least diversified portfolio and ranked again sixth as the portfolio with the highest risk among the other portfolios and finally ranked number five in terms of concentration, justifying the relationship between these three measures. The same concept holds for the rest of the portfolios as it can be seen from the rankings.

Below is a pictorial representation showing the relationships between risk, concentration and PDT.

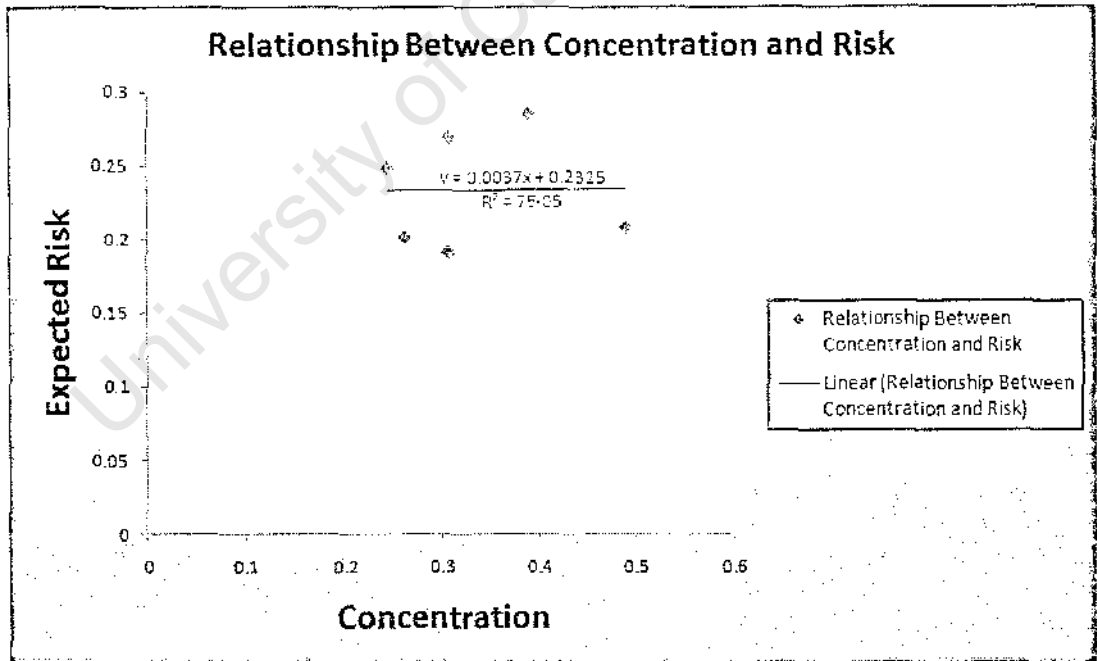


Figure 1: Relationship between concentration and risk

Figure 4 above shows that there is a positive relationship between concentration and risk that is the higher the concentration measure the higher the

risk. Hence the slope (0.003) of the line is positive

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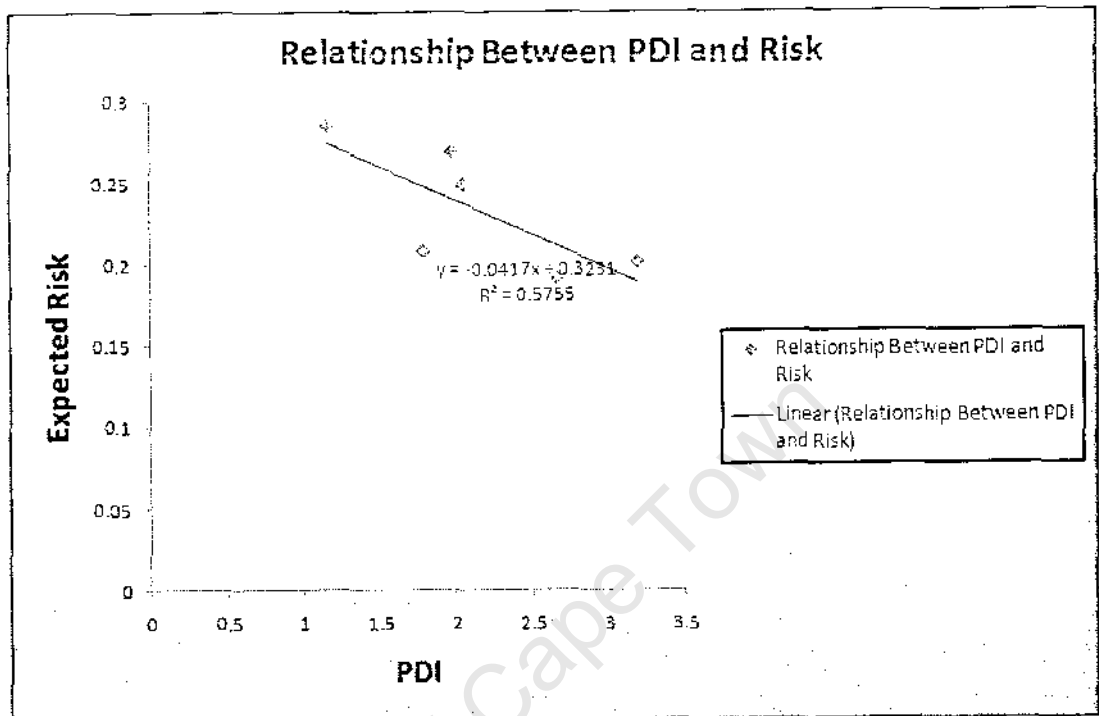


Figure 5: Relationship between PDI and risk

The graph showing the relationship between PH and risk has a negative slope (-0.041). [his once again tomtit-1ns the fact that PDI and risk moves in opposite direction that is the higher the PDT ineasure the lesser the risk. [5], t7j

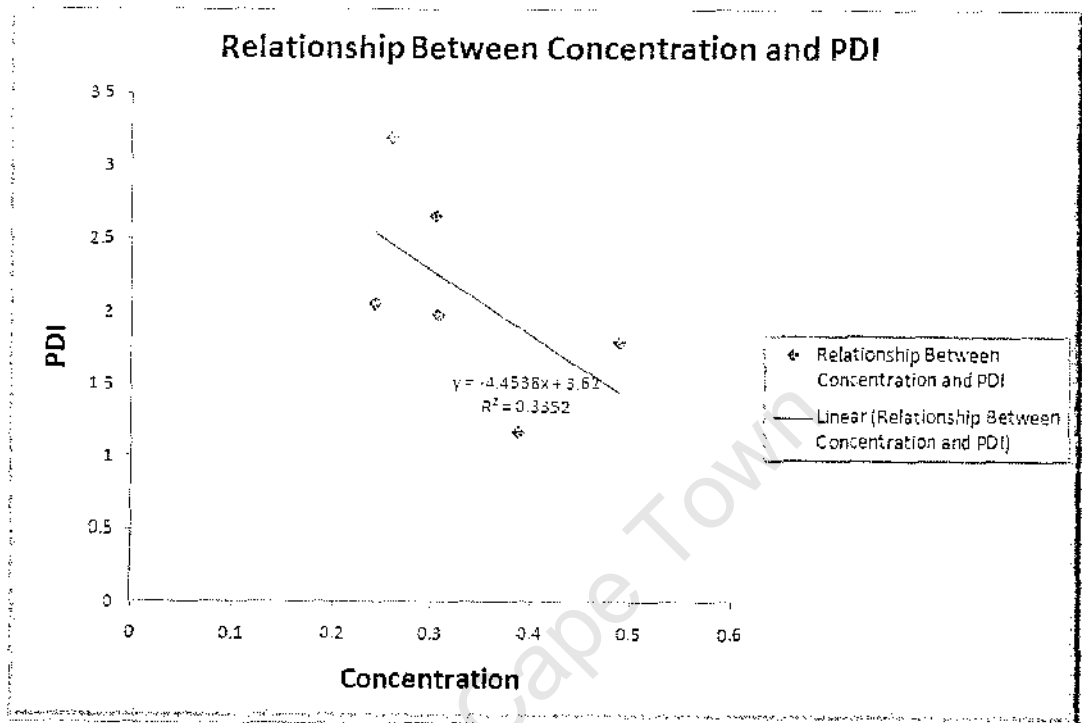


Figure 6: Relationship between concentration and PDI

Figure 6 shows the negative relationship between RJR and concentration that is, higher Pill in is ahDrnpanied by lower corontration measure,

## 0.8 Conclusion

Risk is very important when it comes to investment. The amount of risk an investor is willing to take differs from one investor to the other. Currently most investors are averse to risk. Concentration plays a role in portfolio risk and it is evidently clear that concentration does have an impact on portfolio risk i.e. higher level of concentration leads to higher level of risk even though correlation between stocks has a huge impact on portfolio risk as well. Risk and concentration are positively related.

Diversification cannot be ignored when investing because every investor will be excited knowing his/her portfolio of wealth hails from different sources. We did measure the extent of diversification achieved from an in-coming portfolio of stocks using the GSE All Share Index. The PDI measure was used instead of the traditional approach because of the flaw associated with the traditional method. Risk and diversification were found to move in opposite directions. The analysis above speaks boldly about the effect of concentration and correlation on portfolio risk. These two factors (i.e. concentration, correlation) account for the difference in risk for the three scenarios. For scenario 1 and 2 the difference in risk was solely due to correlation between the stocks. Risk difference between scenario 2 and 3 was due to both concentration and correlation between the stocks. Finally, only concentration led to the difference in risk between scenario 2 and 3.

Lastly, on the Ghanaian market stocks with high market capitalisation have low correlation with each other. The results obtained from the three scenarios make it evidently clear that the Ghanaian market is highly concentrated.

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## 0.10 Appendix

Proof that equally weighted portfolios converge to the average covariance of stocks in the market

We assume that assets are correlated as they usually are, but are equally-weighted, then portfolio risk will converge to the average covariance of the assets. We begin with the usual definition of portfolio variance and follow the proof as shown in Grubar(2003).

$$\sigma_p^2 = \sum_{i=1}^N W_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N W_i W_j \sigma_{ij}$$

For equally-weighted portfolios the investment weight for each stock becomes  $1/N$ , hence

$$\begin{aligned} \sigma_p^2 &= \sum_{i=1}^N \left(\frac{1}{N}\right)^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left(\frac{1}{N}\right)^2 \sigma_{ij} \\ &= \frac{1}{N} \sum_{i=1}^N \left(\frac{\sigma_i^2}{N}\right) + \frac{N-1}{N} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left(\frac{\sigma_{ij}}{N(N-1)}\right) \\ \sigma_p^2 &= \frac{\sigma_i^2}{N} + \frac{(N-1)\sigma_{ij}}{N} \end{aligned}$$

For large values of  $N$ , the above expression reduces to the average covariance.

### Simulation Results

Portfolio Size	Portfolio Risk		
	Scenario 1	Scenario 2	Scenario 3
1	0.6319	0.6319	0.6319
2	0.4401	0.4452	0.5820
3	0.3699	0.3758	0.5668
4	0.3184	0.3258	0.5300
5	0.2812	0.2906	0.5270
6	0.2607	0.2691	0.5200
7	0.2417	0.2522	0.5015
8	0.2254	0.2388	0.4800
9	0.2112	0.2263	0.4708
10	0.1985	0.2138	0.4592
11	0.1903	0.2058	0.4575
12	0.1835	0.1994	0.4540
13	0.1740	0.1904	0.4472
14	0.1699	0.1862	0.4460
15	0.1628	0.1807	0.4414
16	0.1603	0.1794	0.4280
17	0.1533	0.1713	0.4266
18	0.1486	0.1680	0.4077
19	0.1462	0.1662	0.4062
20	0.1421	0.1626	0.3978
21	0.1382	0.1600	0.3902
22	0.1347	0.1566	0.3875
23	0.1319	0.1543	0.3745
24	0.1305	0.1535	0.3738
25	0.1266	0.1488	0.3679
26	0.1244	0.1470	0.3669
27	0.1226	0.1460	0.3627
28	0.1199	0.1446	0.3544
29	0.1180	0.1421	0.3527
30	0.1159	0.1408	0.3459
31	0.1140	0.1389	0.3440
32	0.1124	0.1378	0.3368
33	0.1105	0.1361	0.3332
34	0.1087	0.1347	0.3297

## VBA CODE

Option Base 1

'Scenario1: Using equal weight and no correlation structure

'Scenario2: Using equal weight and historical correlation structure

Sub tst()

Dim numPool() As Integer, numDrawn() As Integer, Weights As Variant

Dim maxNum As Integer, Draw As Integer, numSamples As Integer, numStocks As Integer

Dim var As Variant

Dim i As Integer, j As Integer, k As Integer

Dim wsf As WorksheetFunction

Set wsf = Excel.WorksheetFunction

numStocks = 34 'number of stock on the Ghana stock exchange

numSamples = 500 'number of times to run for each k selected

ReDim numPool(numStocks), numDrawn(numStocks), Weights(numStocks, 1), sample(numStocks)

' For k stocks

For k = 1 To numStocks

sample(k) = 0

' l number of times

For l = 1 To numSamples

For i = 1 To numStocks

numPool(i) = i

Next i

maxNum = numStocks

' select k stocks from the universe of 34 stocks

```

For i = 1 To k
    Draw = Round(Rnd() * (maxNum - 1), 0) + 1
    numDrawn(i) = numPool(Draw)
    If Draw <> maxNum Then
        numPool(Draw) = numPool(maxNum)
    End If
    maxNum = maxNum - 1
Next i

```

' Reset weights of all the 34 stocks to zero

```

For i = 1 To numStocks
    Weights(i, 1) = 0
Next i

```

' Set equal weights for selected stocks with the rest still having zero weight

```

For i = 1 To k
    Weights(numDrawn(i), 1) = 1 / k
Next i

```

```

Range("a1:a34") = Weights

```

' Calculate portfolio variance

```

var = wsf.MMult(wsf.Transpose(Range("a1:a34")), wsf.MMult(Range("b1:ai34"), Range("a1:a34")))

```

' Add to running total

```

sample(k) = sample(k) + var(1)

```

```

Next I

```

' Calculate average standard deviation

```

sample(k) = Sqr(sample(k) / numSamples) * Sqr(12)

```

```

    Next k

'display the standard deviation for each portfolio of k stocks formed in the spreadsheet
    Range("a38:ah38") = sample

End Sub

'Scenario3: Using market captalization weights and historical correlation structure
Sub tst()

    Dim numPool() As Integer, numDrawn() As Integer, Weights As Variant, MktCap As Variant

    Dim maxNum As Integer, Draw As Integer, numSamples As Integer, numStocks As Integer

    Dim var As Variant

    Dim i As Integer, j As Integer, k As Integer

    Dim wsf As WorksheetFunction

        Set wsf = Excel.WorksheetFunction

        numStocks = 34 'number of stock on the Ghana stock exchange

        numSamples = 500 'number of times to run for each k stocks selected

        ReDim numPool(numStocks), numDrawn(numStocks), Weights(numStocks, 1), sample(numStocks)

MktCap = Range("ak1:ak34") 'calls the weight of the market cap from excel

    ' For k stocks

    For k = 1 To numStocks

        sample(k) = 0

    ' i number of times

        For I = 1 To numSamples

            For i = 1 To numStocks

                numPool(i) = i

            Next i

            maxNum = numStocks

        ' select k stocks from the universe of 34 stocks

```

```

For i = 1 To k
    Draw = Round(Rnd() * (maxNum - 1), 0) + 1
    numDrawn(i) = numPool(Draw)
    If Draw <> maxNum Then
        numPool(Draw) = numPool(maxNum)
    End If
    maxNum = maxNum - 1
Next i

```

```
' Reset weights for all the 34 stocks to be zero
```

```

For i = 1 To numStocks
    Weights(i, 1) = 0
Next i

```

```
' Use the market capitalization weights for selected stocks
```

```
Sum = 0
```

```

For i = 1 To k
    Sum = Sum + MktCap(numDrawn(i), 1)
Next i

```

```
For i = 1 To k
```

```
    Weights(numDrawn(i), 1) = MktCap(numDrawn(i), 1) / Sum
```

```
Next i
```

```
Range("a1:a34") = Weights
```

```
' Calculate portfolio variance
```

```
var = wsf.MMult(wsf.Transpose(Range("a1:a34")), wsf.MMult(Range("b1:ai34"), Range("a1:a34")))
```

```
' Add to running total
```

```
sample(k) = sample(k) + var(1)
```

```
Next i
```

```
' Calculate average standard deviation
```

```
sample(k) = Sqr(sample(k) / numSamples) * Sqr(12)
```

```
Next k
```

```
' Displays the results in the spreadsheet
```

```
Range("a38:h38") = sample
```

```
End Sub
```

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