University of Cape Town
School of Education
Faculty of Humanities

What is constituted as mathematics when presented in contextually embedded forms? A study based on two activities from the 2003 Grade 9 Common Task for Assessment for Mathematics.

SUPERVISOR: Zain Davis

MINOR DISSERTATION SUBMITTED IN
PART-FULFILMENT OF THE MASTERS OF EDUCATION DEGREE
(MATHEMATICS EDUCATION)

EDN6057W

by

Rushdien Ebrahim
EBRRUS001

February 2011

I hereby declare that this minor dissertation is my own unaided work and that all sources of reference have been acknowledged. The dissertation is submitted for the degree of Master of Education at the University of Cape Town. It has not been submitted before for any
degree or examination at another university.

Signed: ___________________________

on this the ______ day of ____________________ 2011

ACKNOWLEDGEMENTS

I would like to thank my supervisor, Zain Davis, for his support, time and patience. His profound insight and careful guidance are enormously appreciated.

Thank you to Shaheeda Jaffer for sharing some of her research and ideas with me.

Many thanks to Yunus Omar for proofreading, editing and commenting on this manuscript.

This work is dedicated to the loving memory of my father, Achmat Essop Ebrahim, who instilled in me a love for reading and a thirst for knowledge.

ABBREVIATIONS

C2005	Curriculum 2005
CTA	Common Task for Assessment
DoE	Department of Education
GET	General Education and Training
MLMMS	Mathematical Literacy, Mathematics and Mathematical Sciences
OBE	Outcomes Based Education
RME	Realistic Mathematics Education
RNCS	Revised National Curriculum Statement
RNCSM	Revised National Curriculum Statement for Mathematics
TIMMS	Third International Mathematics and Science Study
SAM	Social Activity Method
SBA	School Based Assessment

LIST OF FIGURES

LIST OF TABLES

LIST OF ANNEXURES

TABLE OF CONTENTS

A B S T R A C T

What is constituted as Mathematics when it is presented in contextually embedded forms: A study based on the Grade 9 Common Tasks for Assessment (CTA) 2003

This study focuses on what comes to be constituted as mathematics when is presented in contextually embedded forms, how students constitute mathematical meaning from such texts, and what is constituted as mathematics when this particular form of mathematics is pedagogised.

Findings are based on data obtained by administering two tasks extracted from the 2003 Mathematics CTA to two groups of students in the form of two observed lessons and two tests. The study examines what is constituted as mathematics at the textual level of the CTA activities, the form of pedagogic modality, and the students' textual productions. The theoretical foundations of the study are based on the work of Bernstein and Dowling and these are supplemented by the recent work of Davis which concerns constitutions of mathematics in pedagogic situations. The analytical framework is concerned with the generation of data which describes how mathematics is constituted by each of the components of the study, as well as considering how the students' social class is implicated in this process.

The study is qualitative and focuses on a textual analysis of each of the components of the study. The analysis will show that largely procedural forms of mathematics are privileged and that the attempts to ground the mathematics within real life contexts creates further barriers to accessing principled forms of mathematics by this group of students. Furthermore, the nature of the activities as assessment tools also privileges procedural texts which attempt to mimic the forms of realisation perceived to be in the privileged solutions of the teacher's guide.

The study does not claim to make definitive or generalisable findings, but acknowledges its limitations. As such, it aims merely to contribute to an understanding of how mathematics is constituted by a group of students from a specific social class background.

Chapter 1

INTRODUCTION

1.1 Background

With the installation of South Africa's democratic government in 1994, a new education system became imperative. A key issue for curriculum developers at the time was to design a curriculum that would address the educational disparities perpetuated by having nineteen different racially defined and resourced education departments in existence at that time. One of the main goals of the new government was, therefore, to ensure that all children, irrespective of race, class, gender and religion had equal access to a quality education. To realise that ideal, a number of policies were put into place for the specific purpose of redress. It was thus that South Africa adopted a system of Outcomes Based Education (OBE) ostensibly aimed at ensuring that *learners* (my own italics – 'learner' replaced 'student': part of the many nomenclature changes that became part and parcel of the sweeping changes in education) have access to a quality lifelong education. The OBE approach is predicated on learner-centred approaches which were directed at attaining predetermined outcomes. Curriculum 2005 (hereinafter, C2005), the reviewed OBE curriculum, identified twelve critical outcomes which spelled out a broad range of knowledge, skills and values envisaged for each of these lifelong learners.

1.2 Implications for assessment

As part of a major attempt at national systemic change, Curriculum 2005 (C2005) encompassed many progressive education principles, such as child-centred learning, development of critical thinking and continuous assessment (Rogan & Grayson, 2003). The principles of assessment in this new curriculum are defined in the Revised National Curriculum Statement (RNCS) as:

> An outcomes-based framework uses assessment methods that are able to accommodate divergent contextual factors. Assessment should provide indications of learner achievement in the most effective and efficient manner, and ensure that learners integrate and apply skills. Assessment should also help students to make judgments about their own performance, set goals for progress and provoke further learning. (DoE, 2002)

Accompanying C2005, a new assessment program was introduced in 2002 at the exit level of Grade 9. A centre-piece of this program was an externally mandated assessment tool known as Common Tasks for Assessment (CTA), involving a series of tasks to be completed by all Grade 9 learners in all learning areas. According to the Revised National Curriculum Statement (R to 9) for Mathematics (RNCSM), the purpose of the CTA was:

- ensure consistency in teacher judgments;
- promote common standard setting;
- strengthen the capacity for school-based continuous assessment;
- increase the accuracy of the assessment process and tools;
- ensure that the school-based assessment tasks properly assess competencies and achievements, and
- ensure expanded opportunities for learners. (DoE, 2002)

In 2001 the Common Tasks for Assessment (CTAs) were introduced for each of the eight learning areas (formerly called subjects) for Grade 9 students as a pilot project. Grade 9, being the end of the General Education and Training (GET) band, is considered an exit year which enabled students to enter the market-place with a set of marketable skills which would enable them to find or create meaningful employment for themselves. As such, a rigid system of assessment was needed which would be centrally developed and administered to all Grade 9 students on the completion of the pilot project. The aim of the CTAs was to collect information about students' abilities and achievements through a range of activities which included practical work, projects, class work, presentations, pen-and-paper tests, etc. Theoretically the CTAs were intended to strengthen school-based-assessment by consolidating the process of continuous assessment (linked to the idea of lifelong learning) and were supposedly designed in such a way as to test a range of knowledge, skills and competencies.

A crucial concern that arises from the form the CTAs in mathematics assumed as they evolved since its first implementation, lies in the nature in which the subject of mathematics began to be defined. Several developments in the research of mathematics education began to influence how the designers of the CTAs not only viewed the subject, but also how they began to make assumptions about best practices for learning mathematics and as a consequence, how they regarded the best form of assessing development of skills, knowledge and competencies in the subject. In particular, the Realistic Mathematics Education (RME) project of the Netherlands and the work of Freudenthal in generating the principles behind RME had a major impact. Realistic Mathematics Education has its roots in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994). To this end, Freudenthal accentuated the actual activity of doing mathematics; he envisaged that an activityshould predominantly consist of organising or mathematising subject matter, taken from reality. Learners should therefore learn mathematics by mathematising subject matter from real contexts and their own mathematical activity rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability (Gravemeijer, 1994). These real situations can include contextual problems or mathematically authentic contexts for learners where they experience the problem presented as relevant and real. Therefore, mathematics came to be defined in terms of the ideal learner whose greater destiny is dependent on successfully integrating his/her mathematical ability for the greater good of society.

> Mathematics is a product of investigation by different cultures – a purposeful activity in the context of social, political and economic goals and constraints.
>
> Being Mathematically literate enables persons to contribute and participate with confidence in society. (DoE, 2002)

As an external assessment tool, the CTA is intended to make an objective evaluation of a child's mathematical knowledge and skills. Matson and Harley (2003, 287) point out that policy decisions around the CTA attempted to "seamlessly integrate conflicting demands and eloquently pacify the tensions to which they give rise" but the extent of the integration as it is meant to happen in practice is unclear.

1.3 The problem

This study will attempt to investigate *how mathematics is constituted when it is contextually embedded*. Therefore, this study will attempt to understand how the mathematics is constituted at three levels in the CTA assessment process:

1. at the textual level i.e. How does the text reveal mathematical objects and operations? What sort of mathematics is constituted in the text?
2. at the pedagogic level i.e. How is the embedded mathematical content revealed in lessons presented by a specific teacher to a specific group of students?
3. at individual students' level i.e. How does a specific group of students make sense of the embedded mathematics presented in the CTA tasks? Can any judgments be made about the impact or lack impact that this group's background may have on how they are able/unable to cope with the assessment tasks presented to them?

The main concern of this study is to investigate how embedding mathematics in real life contexts affect the transmission and acquisition of mathematical principles, particularly when these are administered to children of a social class similar to that of the Cooper & Dunne (2000) studies. In the development of their class categories they worked with various background factors such as parents' work, education and origins. Drawing on the work of Bourdieu (1987) who argued that social class differentiation can best be operationalised when clusters of occupations are considered, Cooper & Dunne devised three categories of social class:

1. Service class: this category is subdivided into higher and lower grades, each distinguished by employment status

2. Intermediate class: this class includes small business owners, foremen, technicians, and so forth
3. Working class: this class includes both skilled and unskilled manual labourers

Elements of each of these classes can be found among the group of students in this study. Therefore, drawing on the work of Cooper and Dunne, similar conclusions about social class categorisation are made for the purposes of this study. Like Cooper and Dunne, this study will also examine whether the mathematics CTA with its embedded mathematics, differentially constitutes mathematics for this group of children. Furthermore, this study will also consider the extent to which this group of students operationalises mathematical objects appropriately to draw conclusions that take into account the constraints of contexts in which the mathematics is embedded..

The CTAs present mathematics in contextualised formats, generally organised around predeterimed themes e.g. World Cup 2010, tourism, and so forth. The themes supposedly embed the mathematics in familiar contexts to the students and as such, it is proposed in RNCSM (DoE, 2002), will enable students to become familiar with the mathematics contained in real life. Contextualising the mathematics is also assumed to provide easier access to mathematics and where the mathematics is embedded, students would be able to grapple with it empirically and thereby reach some realisation of mathematical principles. Therefore, students do not necessarily need to know or have learned any specific mathematical content as they would be able to construct their own mathematics (Treffers, 1987; Gravemeijer, 2000). Furthermore, the CTA emphasised that real life mathematics be designed in such a way as to be an assessment tool. This, the designers of the CTAs believed, would permit learners to show what they can do in a real life situation.

> . . .this is a type of assessment that emphasises the learners' ability to use or transfer their knowledge, understanding and skills into action. [...] Performance-based assessment provides a systemic way of evaluating those reasoning skills and outcomes ... (DoE, 2002)

The move towards "real life" mathematics is a global trend and places much emphasis on the idea of mathematical literacy at the expense of a study of mathematics. According to Moschkovich (2002: 100,), real world mathematical activities are designed to mimic the kind of activities that students might engage in outside of school or in their future working lives. Cooper and Dunne (1998) note that in recent years, in the UK, there has been an increasing emphasis on understanding, investigation and the application of mathematics in 'realistic' settings, as opposed to an emphasis on 'abstract' algebraic approaches. Similarly, curriculum and teaching standards in the US incorporate calls to engage students in real world mathematics rather than mathematics in isolation (Moschkovich, 2002: 103,).

1.4 Design of the research project

The research question focuses on three different aspects.

The first focuses on the textual, symbolic and iconic representations of the tasks chosen from the 2003 CTA. This part of the study attempts to read the activities in conjunction with the solutions in the teacher's guide to identify the criteria privileged and how these criteria constitute mathematics.

The second focus is on two pedagogic instances in which the two CTA activities are presented in loosely structured mathematics lessons to a group of nine students. The aim of the lessons was to allow some form of mediation of meaning in order to facilitate the transmission of the mathematics inherent in the CTA activities, and in so doing identify what comes to be constituted as mathematics when the activities are pedagogised.

The third focus is on the textual productions and interviews conducted with eight students who did the CTA activities under test conditions. As such, they did not have anyone to assist them and had to draw solely on whatever mathematical resources they had at their disposal.

The study broadly follows in the tradition of the research work Luria (1939) conducted amongst Uzbekh peasants. Luria's approach was later replicated by Holland (1981). The purpose of these forms of research is ostensibly aimed at examining the kinds of orientation to knowledge different classes of people possess.

1.5 Guide to reading this dissertation

Chapter 1 describes the background and context of this study and outlines some of the key concerns that arose. A brief discussion on the design of the research project is outlined and sketches a rough outline of how the research questions are to be explicated and analysed.

Chapter 2 presents a review of the literature relating to the key issues of the research project. The literature under review is grouped according to the different concerns that come to bear on the questions raised by this study. These concerns are notions of what constitutes school mathematics, using real life or contextually embedded mathematics to facilitate access, the role of assessment, and what is constituted as mathematics in pedagogic practices.

Chapter 3 sets out the theoretical framework and methodology that underpins the analysis undertaken in this study. Bernstein's (1996) theory of the pedagogic device is considered with particular reference to his rules of recognition and realisation as explicated in his evaluative rule. Furthermore, his theory on framing is considered within the scope of this study. Dowling's (1998) domain's of practice is also outlined. Furthermore, various studies, both local and international, are recruited in support of the theoretical underpinnings of this study.

Chapter 4 sets out the method of the study. A description of the data collecting tools is provided. These include:

- a contextualization of the study
- a description and rationale of the CTA activities used to collect data
- a description of how students were selected for the study
- a description of and rationale for the interview method
- a description of the teacher recorded in the videod lessons
- a description of the videographed lessons
- a method of analyzing the data

Chapter 5 reports the findings of the study of the students' backgrounds and analyses the data collected using the analytical tools set out in chapter 4. This chapter analyses the data on the social background of the students and problematises the notion of class.

Chapter 6 presents an analysis of the CTA activities and the accompanying solutions in the teacher's guide presented to the students as assessment instruments. By performing a textual analysis on these activities the research attempts to show that a particular constitution of mathematics is being privileged. A central concern of this analysis revolves around what is constituted particularly when mathematical content is contextually embedded.

Chapter 7 presents an analysis of the two lessons presented to a selected group of grade 9 students and considers what is constituted as mathematics when the activities are pedagogised. Analysis is centred on how mathematical objects and operations are treated in the pedagogic process.

Chapter 8 presents an analysis of the students' textual productions and their verbal elaborations in the interviews to investigate how they constituted mathematical meaning from the activities and how they were able/unable to produce legitimate texts. The analysis also considers the nature of the mathematical texts produced by the students who completed the activities under test conditions.

Chapter 9 presents the concluding remarks, a summary of the main findings, and considers some of the limitations and constraints of this study.

Chapter 2

LITERATURE REVIEW

Various issues arise out of a consideration of the form that the assessment instruments, the CTAs, have assumed and how they are aligned with the Revised National Curriculum Statement for Mathematics (RNCSM). While the main purpose of this project is to consider how the CTAs construct students' mathematical identities and how these, in turn, position students towards the acquisition of the principles of mathematics, some consideration must also be given to how assessment instruments are conceptualised.

The early CTAs differ radically from conventional forms of assessment in terms of what knowledge is being assessed, the competencies being assessed and the mode of assessment. Changes such as the reconceptualisation of mathematical knowledge as a key to action and power, and not separate from everyday reality as well as embedding mathematics in a social sphere, are clearly evident in the CTAs.

The literature review explores some of the debates around issues related to contextually embedded mathematics as a means of providing access to the principles of mathematics (these include debates around so-called "real life" mathematics), debates around what constitutes valid assessment, as well as the different forms of mathematical constitution.

2.1. Developing mathematical meaning

How mathematical knowledge is constituted in an assessment tool and the types of mathematical meanings that are made available to students can be understood by examining ways in which students generally enlist from a repertoire of knowledge to produce legitimate texts. Schoenfeld's (1985, 1992, 1994) research on problem-solving explores how well people use the knowledge potentially at their disposal, and how the degree of efficient application of their knowledge affects their success or failure during problem solving. Schoenfeld claims that mathematical content alone is not enough to ensure success at problem-solving. Schoenfeld also revealed that the context in which mathematical competence was tested impacted significantly on their ability to solve problems. This was illustrated when a maximum-minimum problem was included in the context of a calculus course and was phrased differently: "What is the largest . . . ?" as opposed to "Find the maximum value . . ." Thus the research showed that the form of communication impacted on the development of the ways in which students understood things and how they made mathematical connections.

Schoenfeld's research speaks directly to a textual analysis of the CTAs by drawing the issue of mode of communication and the possible meanings those modes enable in students at grade nine level.

Similarly, and by extension of the contextualisation of mathematical activity, the work of Cooper & Dunne (2000) offers further elucidation on the types of complexities that arise when mathematical tasks/problems are embedded within a real-life context. However, before discussing their findings, contributing research which led to their study needs to be explored.

The work done by Verschaffel et al (2000) indicates that children seldom pay much heed to the realistic considerations when responding to contextually embedded problems. In an attempt to advance the work of Verschaffel they carried out further research to encourage children to respond more realistically to these types of contextually embedded problems and found that there was very little variation in the way in which students responded. Similarly, by drawing on the work of Säljö and Wyndham (1993) another variation had to be considered in that Säljö and Wyndham focused on the relationship between the concrete conditions for problem solving and the nature of the students' responses. They set a task which required students to calculate postage rates in the context of a mathematics lesson while others had to perform the same task in the context of a social studies lesson. They found that 57% of the students who did the task in the mathematics lesson context were able to compute an answer while only 29% could do so during the social studies context. It would seem then that the Säljö and Wyndham research suggests that the context in which the task is performed has a significant effect. In Bernstein's (1996) terms, students seem to "recognize" the task differently given the different contexts and that the strength of the "classification" of the task impacts on how well the rules of recognition are activated. The results of the study, they argue, shows that the "meaning of a task cannot be defined independently of the context in which the individual's assumptions of what are the relevant premises for his or her actions" (p. 334)

Cooper & Dunne (2000) operationalised their research by considering a different form of categorisation. Where the problems involved everyday objects and activities like shopping, banking, etc., these were labelled "realistic". Furthermore, their study also considered social class and sex differences insofar as student responses to these problems were concerned. In this regard they showed how students tended to bring extra-school realistic considerations into their solution to real-life problems and modified their research to take into consideration how too much or too little realistic compensation is drawn into their solutions.

It is Dowling's (1998) work that creates a structural framework for a sociological analysis of school mathematics texts. Dowling analysed secondary school textbooks and highlighted the manner in which differential forms of mathematics was being distributed to low and high ability students. He considers the Y series and G series of two texts within the SMP 11-16 scheme used in Mathematics in English schools and shows how the former apprentices students into the esoteric domain (the abstract principles of mathematics) and the latter restricts students to the public domain (the everyday, "real life" experiences). Dowling's analysis includes a focus on the relationship between mathematics and other practices and considers the way in which mathematics recontextualises these practices in constituting the public domain. Muller (2000) characterises the proponents of recontextualising forms of learning as progressivists and argues that "the naturalism of progressivism, from which stems the idea of the creative, active learner and the facilitating teacher, is rooted less in Dewey's pragmatism than it is in eighteenth century romanticism". Drawing on the ideas of Gramsci Muller proposes that the job of the school is to "accustom (the student) to reason, to think abstractly and schematically while remaining able to plunge back from abstraction into the real and immediate life, to see in each fact or datum what is general and what is particular, to distinguish the concept from the particular instance" (Gramsci quoted in Muller, 2000). Thus, Muller, argues that progressives who strive for greater integration of knowledge by means of the recontextualising process, devalue the position of the teacher as well as knowledge itself. This leads to an understipulated curriculum, one that emphasises processes, skills and values, instead of content. This he calls "connective coherence", a form of curriculum that promotes links with the real world, with other subjects and with topics within a subject. The design of the RNCSM displays the principles of "contiguity, worldly relevance, and interest" (ibid). This form of curriculum design then leads to the development of specific projections of teacher and student identity at a broad textual level.

2.2 Real-life mathematics

Drawing on the work of Bernstein, Ensor and Galant (2005) three key questions are raised when considering the use of real-life maths in curricula:

1. How is school mathematics different from everyday practices?
2. What is the relationship, if any, between the everyday life and school mathematics?
3. What conditions are necessary for students to gain mastery in school mathematics?

It is clear that school mathematics and everyday mathematics rest upon very different social relations, are differently structured, differently acquired and operate under different contextual constraints. Ensor and Galant (Ibid) further raise the following serious consequences when such integration is attempted:

1. Proponents of these constructivist approaches assume that all learners come to school with the same level of cultural and intellectual capital.
2. Vertical progression is often stunted, particularly for students with working class backgrounds.

The rhetoric of empowerment, thus, results in the opposite effect by disadvantaging the disadvantaged even more.

2.3 Realistic Mathematics Education (RME): a case for contextualised mathematics

Proponents of the methodology of Leen Streefland in the Realistic Mathematics Education (RME) tradition argue that real life mathematics “can function as anchoring points for the reinvention of mathematics by the students themselves” (Gravemeijer & Doorman, 1999). For them, contextually based mathematics can serve as a useful bridge to abstract mathematics and as such invert the traditional forms of pedagogy which focuses on simplifying complex abstract mathematical concepts. Invariably the process of simplification leads to a fragmentation of mathematical concepts in order to scaffold successive conceptual development. As a result, students are often left with an unsynthesised picture of the mathematical principles that the pedagogic process tries to unfold. In keeping with this constructivist approach, Gravemeijer & Doorman quote the work of Tall (1991) who argued for “more emphasis on visualizing mathematical concepts and more enactive experiences in mathematics education”. The processes of visualization and experiential discovery would lead to a greater appreciation of the underlying mathematical concepts. Freudenthal’s ideas (quoted in Gravemeijer & Doorman, 1999) are also enlisted to validate mathematising the everyday as well as mathematising mathematics itself. This quasi-inductive approach aims at reinventing the wheel in order to promote the notion of facilitated access to mathematics and to make mathematics a more interesting and relevant subject for young people.

While the RME paradigm is a noble one in terms of promoting access to mathematics, it fails to answer some key questions about the complexities that contextual mathematics raises. Gravemeijer and Doorman do not attempt to explain how the metaphor of the signifiers in this mathematical approach creates its metonymic links, how long the string of metonymic links should be, and how the signifieds are established. In addition the complex of social orientation to meaning and the class dynamics are not discussed at all. The assumptions that can be drawn from their description is that the RME classes are run by the ideal teachers who work with a homogenized group of idealized students.

2.4 Assessment

"Assessment and qualifications determine the level of inclusivity or exclusivity of the system, and the degree of 'fit' with the labour market" (Young, 2002). It is mainly through the process of assessment that systemic efficiency and accountability can be obtained. Muller identifies two principal axes of contestation in assessment and qualifications thinking. The first is between those who distinguish between different modes of knowledge, learning and qualification, and those who don't. He calls the former "dualists" and the latter "monists".

The second is between those for whom assessment in the classroom for pedagogic purposes is primary (decentralisers), and those for whom assessment as a signalling system for systemic performance is primary (centralisers). Each of these approaches to assessment serves different purposes. For the decentralisers, assessment is about providing feedback on individual performaces and for the centralisers, assessment is a gauging instrument to test the reliability and validity of a system.

In February 2002 the South African Department of Education (DoE) proposed that external assessment would take the form of CTAs. Muller argues that this is a form of performance assessment favoured by pedagogical progressives and very difficult to use for comparative and systemic purposes. Problems that arose out of the development of the CTAs were directly related to the underspecification of content and their obsession with connective coherence and relevance. The test development teams went to enormous lengths to ensure that all learners had equal access to the content of the tasks and this resulted in the tasks being over-elaborate and extremely long.

Cooper (2000) argues that the use of such realistic test items can cause a variety of problems for children. In addition to the amount of reading required, mathematics assessment based on realistic items often result in students failing to apply realistic considerations or they tend to get lost in the context of the problem. This results in students who are less knowledgeable about the peculiar ways boundaries are drawn between school and everyday knowledge, fail to demonstrate what they know and understand about mathematics as a result of drawing 'inappropriately', from the perspective of the test designers, on their 'everyday' knowledge of the world outside of the classroom (Cooper 1992).

Cooper (2000) argues that researchers who have presented their work using a psychological framework, tended to idealise the typical child and apply findings in far too generalised a manner. In the case of mathematics, these conclusions are often driven by tacit assumptions about the nature of mathematics itself. These include the belief in the universality of a discipline that many consider to be above and beyond 'everyday' cultural differences. School mathematics is not unchanging and changes are often driven by perceptions of economic and political imperatives.

There has been neglect, especially within research on mathematics education, of the ways in which cultural differences between children from differing social classes might influence their success and failure in mathematics (Apple, 1992, 1995a, 1995b). This is partly a result of a reaction against what were seen as 'deficit' theories and partly because of a relative falling away of concern with social class differences in educational achievement in comparison with the post-war period. Both Bernstein (e.g. 1996) in England and Bourdieu (e.g. 1986) in France have produced evidence which suggests, respectively, that both children and adults from the dominant and subordinate social classes of these societies differ considerably in their orientation to the boundary between 'everyday' and 'esoteric' knowledge. Cooper (ibid) argues that a shift to a curriculum and to assessment items that embed mathematical operations in 'everyday' contexts might not be neutral in its effects with respect to children from different social class backgrounds. Through a textual analysis of the RNCSM and the CTAs, this project will attempt to show that it is possible that assessment via 'realistic' items might serve to construct student identities of children from different social classes differentially i.e. middle class students would have greater cultural capital to draw from in order to construct meaningful mathematical knowledge. Changes in school subjects, and in assessment regimes in particular, should be examined not only from the point of view of their technical adequacy, that is their validity and reliability, but also in terms of the ways their demands intersect with the cultural resources of the various social groups confronted with them. Thus, Cooper (ibid) argues that an acceptance of realistic mathematics in the context of assessment needs to be examined more closely in order to understand how it might mislead researchers' perspectives about "which children know exactly what mathematics".

2.5 What is constituted as mathematics?

Different forms of mathematics are distinguishable in terms differentiating between mathematical competency and mathematical understanding. A vast body of literature appears to have consensus on what constitutes mathematical understanding. Mathematics understanding is based on a principled understanding of the rules and axioms that lend stability to mathematical procedures while mathematical competency emphasises problem solving techniques and procedures. This dichotomous relationship is well explicated by Skemp (1989) in which he distinguishes between "instrumental" and "relational" understanding. Relational understanding refers to cognition at high levels of abstraction so that knowledge can be related or transferred across any context. For this to occur, the principles underpinning the knowledge have to be known. Instrumental understanding is largely algorithmic and often independent of the underlying principles. Competency is emphasised and enhanced by means of exemplars. Similarly, Tall (1991) addresses the absence of integration between symbols and their contextual meaning and calls the accumulation of rote-learned rules “disjunctive realisation”. Research of this nature highlights an impoverished view of mathematics which cannot establish relations between various mathematical segments.

Jaffer (2010) argues that knowledge of the properties of the mathematical objects in operation provides students with combinatorial resources that enable the

> “production of a range of computational procedures that are distinct at the level of expression but equal at the level of value. An individual is free to choose different properties for a computational procedure which may be as efficient or elaborate depending on the choice of properties. Without the knowledge of the properties of the object, an indvidual is bound to a particular procedure. The freedom of choice resides in the knowledge of the properties of the object which regulates the production of mathematics.” (p. 301)

Therefore, students have a measure of flexibility in the selection of the forms of convergent legitimate texts that knowledge of the properties enables. Eco (1979) argues that the model reader of a text is always considered during the process of constructing the text so that the textual construction assumes an innate ability on the part of the reader to produce valid interpretations. Therefore, Jaffer argues that when students have access to the properties of the mathematical objects in question, the texts they produce correspond to “Eco’s open text since it delineates a ‘closed’ project for its readers/learners in that it aims to produce convergence of interpretation of the text” (Ibid).

2.6 Summary

The literature review highlights the contestations in several research projects about the nature of mathematical textual productions and how these contribute to the debate about what constitutes mathematics. Furthermore, the waters of this debate have been muddied by the assertion amongst some researchers that contextualised or real-life mathematics provides reasonable access to mathematics. Central to the debate is the dichotomous approaches of competency and principled understanding.

Therefore, the issues raised in the literature have significantly informed the research question of what is constituted as mathematics when it is presented in contextually embedded forms.

Chapter 3

THEORETICAL FRAMEWORK

The research question posed by this dissertation is concerned with how mathematics is constituted when it is contextually embedded. In attempting to answer this question the inter-relatedness of the following components of the research project have to be considered:

1. the presentation of the embedded mathematical texts (the CTA activities) and the mathematical meanings they constitute for the group of students participating in the study

2. the two instances of pedagogic practice

3. the texts the students produced.

All of these are considered against the social background of the students. The literature review outlined some of the underlying theoretical tensions that similar studies have highlighted as well as the areas of contestation within and between several of these studies. The general theoretical framework of this research project is underpinned by Bernstein's (2000) pedagogic device and supported by concepts of classification and framing as well as rules of recognition and realisation.

3.1 Bernstein's Pedagogic Device

Drawing on Chomsky's (1965) universal linguistic model, Bernstein developed the pedagogic device which outlines a universal set of principles that govern the production of school knowledge. Bernstein defines education as "a relay of power relations external to it." (1990: 168). He argues that any system of education is a system of technologies for the control of the pedagogic device and as such is a site for political contestation which attempts to maintain and to shift power. The official education system in any society exhibits realisations of the pedagogic device that reproduce the power relations of that society. Bernstein's conceptualisation of education as symbolic control elaborates links between state power and the specialisation of individual consciousness. For Bernstein the natural outcome of considering education as a form of symbolic control is a definition of the relationship between the "'carrier' (or relay) and the 'carried' (what is relayed)" (Bernstein, 1996: 41). The carrier is the pedagogic device and the carried is the pedagogic discourse. The pedagogic device provides an analytical description at the level of the classroom of a more general process, the recontextualisation of knowledge.

The structure of the pedagogic device determines how knowledge (intellectual, local, etc.) gets transformed into pedagogic communication. In other words, this structure and its rules attempt to regulate the 'pedagogic communication which the device makes possible', as well as the distinct classes of knowledge (thinkable or unthinkable), which incorporate the potential meaning, 'that is available to be pedagogised' (Bernstein, 2000). According to Bernstein (1996; 2000), the pedagogic device is a necessity for the reproduction and transformation of knowledge and culture. Additionally, its internal rules regulate the pedagogic discourse. The official field of education and its pedagogic device select and distribute the appropriate values of the fundamental principles of classification and framing in order to construct the suitable pedagogic modalities (Bernstein, 1971, 1986, 1990, 1996, 2000). Pedagogic modalities, in turn, select and legitimatise specific forms of school knowledge (abstract or concrete), as well as the processes of their realisation. Thus, they shape forms of consciousness and pedagogic identities.

3.2 The rules of the pedagogic device

The hierarchically dominant distributive rules express the existing power relations and constitute the basic pole of any contemporary pedagogic device. In Bernstein's words (1996: 42), the distributive rules '… regulate the relationships between power, social groups, forms of consciousness and practice'. They attempt to manage symbolic control, although ineffectively, within the wider social field and therefore within the official field of education. In this frame, distributive rules intervene and distribute who may transmit what to whom and under what conditions. Thus, they set the limits of a legitimate discourse. They intend to distribute the potential meaning of the available classes of knowledge, in order to construct forms of practices and, therefore, forms of consciousness (Bernstein, 2000).

The recontextualising rules regulate the delocation, relocation and refocusing of a discourse from the primary field of production into the field of its reproduction. In particular, they regulate the available potential meaning and provide for the construction of specific pedagogic discourses. Within the pedagogic discourse, which represents a body of school knowledge, two particular discourses are embedded: instructional discourse and regulative discourse.

The regulative discourse, the discourse that gives rise to the form of evaluation of the CTAs, constitutes a discourse of social/moral order, which provides the how of the pedagogic discourse. Bernstein implies that the dominant agents of the educational field select the appropriate theory of instruction, in order to support their functional logic (in the RNCSM, constructivism). Regulative discourse was 'a discourse of social order' (Bernstein, 1996: 46). Instructional discourse was 'a discourse of skill of various kinds and their relations to each other', (ibid). The important point is that Bernstein did not draw this analytical distinction to translate pedagogic practice simplistically into the transmission of skill though discourse on the one hand and the transmission of values on the other.

Bernstein described two models of how knowledge was recontextualised that can be seen as two opposing forms of power (classification) and control (framing). Broadly, “competence” (Ibid) models were characterised by weak classification and framing and resulted in few explicit (instructional) structures. When knowledge was recontextualised according to competence models, surveillance was centred more directly on the intentions, attributes and individuality of the student, and less on the criteria attributable to specialist discourses. “Performance” (Ibid) models were characterised by explicit (instructional) structures and strong classification. Although fewer of the student's personal attributes, intentions and style were used as criteria for control, students became easily classified as successful and less successful, precisely because the subject (discipline) criteria were publicly available. Within any classroom we would expect to find a patterning of practice which might tend more towards one of these modalities than the other. The models for recontextualising knowledge outlined above refer to a distinction between two forms of organic solidarity within the middle class, individualised and personalised.

Throughout his work, Bernstein's main concern was with the principles of social control. His secondary focus was on the distinct forms of experience. The theory elaborates the movement and congruence of ideas between family, students and the school and shows how schools reproduce social class structures. The theory outlines a complex system of mediation between the three sites that takes into account factions within social class, dispositions towards learning and the culture of the school.

Briefly, working class children, rather than middle class children, tend to have acquired codes that are less compatible with those required to be successful in schools. This is partly because teachers tend to be middle class. Underlying teachers’ classroom discourse is a representation of the ideal pupil or the ideal citizen. When students’ ways of talking, acting and producing texts conform to their teacher’s representation of the ideal student, they are less likely to experience reprimands. However, when their ways of behaving conflict with the underlying representation of the ideal student, they are more likely to experience reprimands. However, the danger of this happening depends on the model for recontextualising knowledge that is prevalent within classroom practice. Representations of the citizen underlying teachers’ classroom discourse are culturally specific and change according to the social anxieties present at any one time.

According to Bernstein’s description of the pedagogic device, instructional discourse is embedded in regulative discourse and as such, the instructions that teachers give as they induct students into subject discourse cannot be dislocated from an underlying notion of the student implicit within them. The underlying representation of the student can be viewed as a social representation of the category of the person within a particular socio-cultural and socio-political context. At the core of the pedagogic endeavour is the drive not just to educate but to educate someone – notably the ideal citizen. Therefore, a teacher's instructional discourse is underpinned by a social and political necessity to educate students as if they will become the teacher's representation of the 'ideal pupil/citizen'.

Bernstein assumes that the theories of instruction are not socio-ideologically neutral. Theories of instruction belong to the regulative discourse and they are also dominated by the distributive rules. The regulative discourse is more related to distributive rules. Therefore, concepts such as selection, sequencing rules, pacing, transmission and acquisition are not socially-ideologically neutral. The instructional discourse is dominated by the regulative discourse in the process of the construction of school knowledge, which imposes the how of the pedagogic discourse. The regulative discourse incorporates social order, which regulates school knowledge and school practices. Bernstein implies that the dominant agents of the educational field select the appropriate theory of instruction in order to support their functional logic. He assumes that theories of instruction which belong to the regulative discourse, are not socially-ideologically neutral. Therefore, they are also dominated by the distributive rules. However, theories of instruction have the ability to legitimise or challenge the decisions of the dominant agents of the educational field, and could restrict or boost the ideological contradictions in the arena of the pedagogic discourse.

The evaluative rule is considered by Bernstein (1996) to be the key to pedagogic practice. It spells out what constitutes legitimate forms of knowledge within the distributive rule. It is at the level of the evaluative rule that pedagogic practice in the classroom is manifested. In essence the pedagogic device is condensed in the evaluative rule. The evaluative rule acts selectively on contents, the form of transmission, and their distribution to different groups of pupils in different contexts and in so doing helps to shape identities and consciousnesses through regulating the way in which students demonstrate that they can produce the 'required' text. The evaluative rules may be strongly or weakly framed. Weakly framed evaluative rules do not always favour students from non-middle class backgrounds, because they may not be able to recognise the implicit rules buried within them. Singh (2001) argues that weakly framed pacing and sequencing of knowledge and strongly framed assessment favour working class students by giving them as much control over their learning as possible, while making the goalposts absolutely clear. Students can respond to the evaluative rules to the extent that they can recognise the kind of knowledge required and realise the required outcomes. That is, recognition and realisation rules are at the level of the acquirer, while the way in which knowledge is classified and framed, and the distributive and recontextualising rules are mediated through struggles between the official recontextualising field and the pedagogic recontextualising field.

Within the grade 9 learning area of Mathematical literacy, Mathematics and Mathematical Sciences (MLMMS), the Department of Education defines mathematics as:

> . . . the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction (DOE, 2002:. 2).

This definition places an emphasis on more social constructivist, student-centered, and integrated approaches to mathematics teaching and learning. MLMMS focuses its attention on constructing mathematical meaning in order to understand the world and make use of that understanding. Mathematical learning is to be relational, flexible, transferable, and integrated with everyday life and other learning areas. In this way the classification of MLMMS signals weakened boundaries which necessitate a shift in the form of evaluation which remains coherent with the distributive recontextualisation rules.

Students vary in the extent to which they have internalised the 'rules' they need to be 'successful' students. This variation is, Bernstein argues, associated with social class background. Students need to have both the 'recognition' rules and the 'realisation' rules to be 'successful'. Students need to recognise the type of knowledge they are dealing with. For example, in the mathematics of the CTA activities, students need to be able to recognise when appropriate real life considerations need to be made and when purely mathematical considerations are called for. This is not just a question of identifying and understanding specialised knowledge, but also of understanding knowledge in its context and decoding it.

3.3 Social class background

This study relies on the theory of symbolic control for defining social class. This results in a definition of social class that comprises two autonomous aspects of social class, namely power and social control. The parallel concepts of power and social control, and by definition social class, pervade the theory of symbolic control. Bernstein constructs these concepts through his concepts, namely boundary, frame, classification, code, and modes of pedagogic practice. Power is inscribed in voice, that is, forms of organization of educational knowledge whereas social control is inscribed in the materialization of voice in the teaching relationship which is the message (Atkinson, 1985). The embodiment of power and social control therefore constructs forms of communication. Being linguistic forms and by extension

vehicles of thought, these forms of communication constitute and construct consciousness. Through their variation, forms of communication as constitutive of consciousness in turn position individuals and groups in inter-group situations. They therefore, not only regulate speakers in relation to each other but also in relation to the forms of communication in which each is positioned (Atkinson, 1985). For Bernstein, then, power is conceived as the working of boundaries and positions and as the difference between ‘thinkable’ and ‘unthinkable.’ Therefore, power constructs voice or knowledge just as control shapes the message. Power is therefore a relationship, but not a simple one: it is multifaceted, diffuse, and everywhere.
Power is not separate from other social relationships, for example of knowledge or production but flows within them (Atkinson, 1985).

The implication of this view is that power is tied to ideology or consciousness. School and society both have a divisions of labor which are constructed from boundaries of insulation between categories (curricular, social) and the boundaries are more or less continuous across both spheres. Persons occupy places in the social division of labor as a result of power’s positioning. Power, however, is encoded in language. This definition makes possible the integration of social class, race, and gender (Atkinson, 1985). Social class therefore refers to modes of power as inscribed in voice and modes of social control as inscribed in messages that realize voice. The conditions of one’s occupation that are classically seen as constitutive of social class are thus resources for the construction of social class membership.

The new middle class exercises the function of capital (control and surveillance) without being part of that capital-owning class. Independently, power and control define class relations through which groups try to monopolize resources and opportunities for their own benefit and deny the same to others, non-members (Jary & Jary, 1991). Class factions that control specialized principles of communication that are applied directly to the means of physical resources and those that control principles that are applied to discursive resources are distinguished. A concept of the social division of labor of symbolic control was derived from this distinction. Agents of symbolic control or production could function in the field of symbolic control, the cultural field, or economic field. Ideological orientation, interests, and modes of cultural reproduction would be related to economic functions of the agents (symbolic control or production), field location (economic, cultural, symbolic) and hierarchical position (political)(Bernstein, 1990).In that perspective, there appears to be no individuals, only processes by which subjects are selectively created and constrained in and by the process of their creation. The subject never seems to act to create meanings, purposes, struggle with beliefs, to negotiate or change the given order. It privileges transmissions, their social costs, and the basis for change but the individual is not the basic unit of analysis, rather the social relation of teacher and student is, yet it focuses on controls. The perspective focuses on the construction of rules that generate discourse (Bernstein, 1990). However, whereas the principles of description were not meant to study the "full choreography of the interaction" in classrooms or portray the "full repertoire of arabesques of interaction" (Bernstein, 1990: 6) they are assumed to be capable of describing such aspects that relate to classroom interaction, their organizational context, and their relation to external agencies, for example family and work. They show how the social class background of pupils acts selectively on the form and content of pedagogy.

Middle class students may derive a relative educational advantage from their social status in comparison with their poorer peers. In concert with teachers' responses to the calls by policymakers to change their practice in order to better serve the poor, acknowledgement of the subtle cultural nuances of social class are assumed to differentiate local knowledge or spur new knowledge production by teachers in classrooms. Pedagogic practice is constructed not only from policy resources and teacher knowledge but also from students' activities.

Several definitions of social class exist (Anyon, 1981; Lareau, 2000; McNeil, 2000). For example, for Anyon (1981), social class is a series of relationships to several aspects of the processes by which goods, services, and culture are produced. Social class comprises relationships to the system of ownership of physical and cultural capital, the structure of authority at work and in society, and the content and process of one's work activity. Occupational status and income do contribute to this definition. However, as Anyon describes it, each relationship is necessary but not sufficient for determining a relation to the process of production in society. That perspective on social class as a multifaceted way of life animates this study. As such, social class is understood to shape the policymaking process, to shape organizational characteristics of schools and school community relations, and to influence micro-social processes in the teacher-student relationship inside classrooms. In short, this view of social class makes it possible to relate the pedagogic device to a specific social class (lower middle class) and how the dialectic between the three rules insofar as power (distributive), control (recontextualising) and evaluation (evaluative) stabilizes the device in the new curriculum of the RNCSM.

3.4 Dowling's Domains of action

Dowling's (1998) initial formulations around the notions of activity in his social activity method (SAM) argues that school mathematics is an activity that establishes a set of practices which affect divisions and distributions within mathematics and between mathematics and other practices. He considers what constitutes mathematical utterances and makes clear distinction between mathematics practices and non-mathematics practices in terms of language, grammar, expression and content. Mathematics is able to constitute principles of recognition and realisation which allows it to cast a gaze on non-mathematical practices and redescribe it in its own terms. Mathematics activity regulates who can say, do, or, mean what and these are reproduced in texts. Therefore, an activity is (re)produced by texts and because student texts are not viewed as pedagogic texts, these texts are seen as a product of the social within the practice of school mathematics.

In order to realize legitimate mathematical texts, students have to display a high degree of mathematical grammar. This raises the question of whether students need to have access to the grammar before they are able to produce legitimate texts, or whether this can occur regardless. The production of a mathematical algorithm which in itself is principled with strong grammar would not necessarily mean that it can be sustained by a principled verbal explanation.

Bernstein (1990) speaks of “restricted” and “elaborated codes” which stand in dichotomous opposition:

> The simpler the social division of labour, and the more specific and local the relation between an agent and its material base, the more direct the relation between meanings and a specific material base, and the greater the probability of a restricted coding orientation. The more complex the social division of labour, the less specific and local the relation between an agent and its material base, and the greater the probability of an elaborated coding orientation. (pg 20)

At this point Bernstein argues that the orientation to meaning depends on the degree of dependence on the material base and that the restricted code is characterised by dependence on a specific context. This suggests that meaning and understanding are only realised in a very specific “enactment of the practice within which it occurs” (Dowling, 1993: 65) and are not transferable. Thus restricted coding remains localised while elaborated coding enables generalization. Using these dichotomous ideas Dowling developed his SAM to describe mathematical texts in terms of variations in form of expression and content. He identifies four domains of action: the “esoteric”, the “expressive”, the “descriptive” and the “public domain”. The principles of recognition and realisation of the mathematics gaze are manifested in varying degrees in the domains so that strong expression and content leads to strong classification which characterizes the esoteric domain. The public domain with its forms of weak expression and content “has the appearance of a non-specialised practice” and is “subject to the regulative principles of the esoteric domain”. The regulative principles cannot be adequately expressed in this domain and as a result this domain serves as the entry point into mathematics for the novice.

	Content (signifieds)	
Expression (signifiers)	strong institutionalisation	weak institutionalisation
strong institutionalisation	*esoteric domain*	*descriptive domain*
weak institutionalisation	*expressive domain*	*public domain*

Figure 3.1 Dowling's Domains of action (2007)

For the purpose of analysing the recontextualisation of domestic practices in school mathematics texts, Dowling (2007) introduces a "relational space" of domains of action that differentiates between content and expression of a text, both being weakly or strongly institutionalised (see Figure 3.1). Esoteric domain text refers to the conventional institutionalised mathematical language and its strongly classified specific meanings. In descriptive domain text, the expression is conventional mathematical language though its object of reference is not institutionalised mathematics. In expressive domain text, a mathematical concept or procedure etc. is expressed via signifiers that are weakly institutionalised (in an extreme case via non-mathematical signifiers). Public domain text is text with both weakly institutionalised forms of expressions and content.

Dowling's concept of institutionalisation rebounds on itself and creates a dilemma by questioning the construction which legitimises the institution in the form of the distributive rule. In the RNCSM the use of real-life mathematics, drawing the everyday into the classroom, integration across the curriculum and implementing democratic pedagogic practices are the cornerstones of the mathematics curriculum. Therefore, recontextualisations of mathematics by means of so-called real-life examples and embedded contexts are totally legitimate and as such, strongly institutionalised. Dowling's domains of action and his 2007 theoretical formulations are, therefore, not sufficiently grounded to provide an analytical framework for this project. It is for this reason that there is a greater reliance on his earlier formulations to develop an analytical framework.

3.5 Emerging propositions

Several propositions pertinent to this research project arise from the theoretical framework outlined thus far.

3.5.1 Class and orientation to meaning

Luria's (1976) research suggests that differences exist in the degree of orientation to meaning between educated and uneducated classes of people and that personal experiential knowledge does not assist learning. It also demonstrated that individuals with no formal schooling background do not use the same conceptual grid to classify information as formally schooled persons (Luria, 1976). Luria's well-known investigations, undertaken 1930-1931, into the intellectual capacities of peasants in Soviet Uzbekistan, influenced Bernstein's research. The findings seem to suggest that cultural contexts, especially as mediated by linguistic forms, are deeply implicated in cognitive functioning, and that intelligence tests are appropriate only for those who have acquired the forms of thought characteristic of the dominant classes. Luria presented his subjects with classification problems such as 'find the odd one out in the collection hammer, saw, log, hatchet'; and the peasants might answer, typically, 'if we're getting firewood for the stove, we could get rid of the hammer, but if it's planks we're fixing, we can do it without the hatchet' (1976: 62). Luria expects the answer 'log', but the peasants' order of relevance is tied to their immediate, day to day, interests. In Bernstein's terms, their responses reveal a particularistic rather than a universalistic orientation to meaning. Holland's (1981) study, often cited by Bernstein and his students, is basically a replication of Luria's work. Holland asked young primary school children to classify common foods, and found a greater tendency among those from working-class families to group them by personal, idiosyncratic criteria such as 'it's what we have for breakfast', 'it's what Mum makes', and so on, rather than by general, material, criteria such as 'these come from the sea', these all have butter in them', and so on. The implications drawn from these experiments are often treated as if they were self-evident: the results are predicted by Bernstein's theory and it is thus implied that the account of linguistically mediated cognitive performances is to that extent sound. Cognitive thinking is mediated by language, proves to be highly sensitive to context and, it seems, is more or less readily open to change by pedagogic action.

Bernstein concludes that middle-class children enter school with the rudiments of formal conceptual knowledge, which working-class children lack and, therefore, middle class children are better oriented to the knowledge structures that occur in formal schooling. Thus, heavy reliance on previous knowledge is not necessarily of great educational value to all students, and abstract conceptual thinking must be actively and deliberately fostered in children who are deprived of such cognitive modeling in their home environments. The first proposition that emerges then, suggests that if we consider a group of students from a particular social background, they would possess a particular orientation to meaning which will have direct implications for their orientation to mathematics.

3.5.2 Orientation to mathematics

Children's failure to make appropriate realistic considerations when confronted with contextually embedded mathematical problems have often been blamed on poor teaching methods and have tended to ignore their socio-cultural backgrounds (eg. Verschaffel et al, 1994). Cooper & Dunne (2000) analysed the working and middle class students' responses to so-called realistic mathematics items like the 'lift problem'. Students have to calculate the number of trips needed by a lift with a capacity 14 when there are 296 people. This example, like numerous others used in their research, employs pseudo-realistic mathematics which requires students to employ just the right dose of realism. They argue that working class students tended to get lost in the context, or apply mathematical solutions simplistically. They also argue that children may "lack the relevant 'realistic' knowledge" when confronted with these embedded problems. Furthermore, their research highlights the tendency of children to become "conformist" by responding in a manner they perceive to be within the bounds of the mathematical conventions privileged by the teacher in the classroom.

The proposition that emerges from the Cooper & Dunne studies seem to suggest that in learning the "rules of the game" students suppress realistic considerations in favour of some algorithm that generates something resembling mathematics. Here the rules of realisation are not about generating a contextually appropriate solution, but about a response that will be acceptable to the teacher. Therefore, mathematical meaning is oriented towards mirroring the forms of mathematics that the teacher privileges.

3.5.3 What constitutes legitimate pedagogic modalities?

The distributive rule of the RNCSM privileges a pedagogic modality that is weakly framed. Its OBE imperative places the student at the centre of the pedagogic process and as such redefines the teacher's role as that of a facilitator. Evaluation, therefore, resides at the level of the textual production of the students for whom mathematics have to be realistic and relevant. When this form of evaluation is confronted with a pedagogic modality that is strongly framed, the mediation that occurs could be compromised. What type of mathematics is constituted and distributed when such pedagogy arises?

In addition working class children, rather than middle class children, tend to have acquired codes that are less compatible with those required to be successful in schools (Bernstein, 1977). This is partly because teachers tend to be middle class. Underlying teachers' classroom discourse is a representation of the ideal pupil or the ideal citizen. When students' ways of talking, acting and producing texts conform to their teacher's representation of the ideal student, they are less likely to experience reprimands. However, when their ways of behaving conflict with the underlying representation of the ideal student, they are more likely to experience reprimands. However the danger of this happening depends on the model for recontextualising knowledge that is prevalent within classroom practice. Representations of the citizen underlying teachers' classroom discourse are culturally specific and change according to the social anxieties present at any one time. At the core of the pedagogic endeavour is the drive not just to educate but to educate someone – notably the ideal citizen. Therefore, a teacher's instructional discourse is underpinned by a social and political necessity to educate students as if they will become the teacher's representation of the 'ideal pupil/citizen'.

3.6 Eco's open and closed texts and the construction of the model reader

Eco (1979) uses the notions of "open" and "closed" texts to talk about text in relation to the reader. These concepts were devised primarily for the analysis of literary texts which differ starkly from pedagogic texts. Jaffer (2010) argues that pedagogic texts consist of strings of "verbal, written or gestural significations" which may occur in different forms by both the teacher and the student. Pedagogic texts are also distinguished from literary texts insofar as the former is fundamentally evaluative. Eco defines open and closed texts as:

> Those texts that obsessively aim at arousing a precise response on the part of more or less precise empirical readers (be they children, soap opera addicts, doctors, law-abiding citizens, swingers, Presbyterians, farmers, middle-class women, scuba divers, effete snobs, or any other imaginable sociopsychological category) are in fact open to any possible 'aberrant' decoding. A text so immoderately 'open' to every possible interpretation will be called a *closed* one. (Eco, 1979: 8).)

> [Open texts] work at their peak revolutions per minute only when each interpretation is reechoed by the others, and vice versa. [...] You cannot use the text as you want, but only as the text wants you to use it. An open text, however 'open' it be, cannot afford whatever interpretation. An open text outlines a 'closed' project of its Model Reader as a component of its structural strategy. (Eco, 1979: 9)

The use of these concepts in pedagogic analysis is still fairly undeveloped and has, therefore, been recontextualised for the purposes of this study. Open texts refer to texts in which the underlying principles/propositions of the mathematical activity are made explicit in all the aspects of the study. This means that all interpretations or recontextualisations are governed by principles and rules so that texts produced are limited to an orientation that reflects them. Closed texts do not exhibit the underlying principles and result varied interpretations.

The use of Eco's concepts are used in tandem with resources made available by the general theoretical framework in order to examine the forms of mathematical activity that are constituted in the different aspects of this study. In particular, these concepts will be tested in the analytical framework against the four propositions listed above to answer the question: “How is mathematics constituted when presented in embedded contexts?”

Chapter 4

METHODOLOGY

In this chapter I shall set out the analytical framework that will be employed to analyse the textual presentations and the pedagogic encounters in order to answer the research question *How is mathematics constituted when it is presented as embedded in extra-mathematical contexts?* The analytical framework will enable me to examine the empirical texts produced by a specific pedagogic modality and the students' productions under the conditions of mediated classroom lessons and controlled tests. This project draws on the work of Luria (1976), which was extended in the studies of Holland (1981), both of which sought to understand how social class impacted on orientation to meaning. In a similar approach, the analytical framework of this project will attempt to produce data which may describe how the students in this research group made mathematical sense of contextually embedded mathematical objects. The general Bernsteinian theoretical framework provides the shape of the analytical framework and determines how data will be generated. In particular, Bernstein's concepts of code in context, and rules of recognition and realization will be used to generate data about processes of evaluation in both the pedagogical and assessment contexts. Dowling's (1998) ideas about the nature of mathematical texts provide a useful springboard for investigating the criteria privileged mathematical objects and operations are enacted. Eco's concepts of open and closed texts will be used to elaborate the privileged criteria and investigate how these objects and operations are rendered by the resources brought to bear on them.

4.1 Context of the Research

The research was conducted in a girls' secondary school that is situated in Cape Town. We shall refer to the school by the pseudonym Hopeville High School for the purpose of this research. For much of its long history it has served as an educational institution for white girls and only in the 1990s, with the end of legalized Apartheid, did the school open up to all people while remaining a single sex school. It is a well resourced school, offering a wide range of subjects and has had a very stable, well-qualified staff. The school draws its students from a wide range of geographical locations which may be characterized as working class since the majority of these areas are populated mainly by semi-skilled and unskilled workers according to the City of Cape Town's census data of 2008. Table 4.1 overleaf lists the research group's parents' occupations and gives indications of the skill levels of their work. Further exposition about the nature of these skills will be dealt with in a later chapter.

PARENT OCCUPATIONS
Saleslady
Primary school teacher
Retired
Tour Consultant & Unemployed Welder
Shop Assistant and Plumber
Hairdresser and Business Owner
Saleslady
Truck Driver
Shop Assistant
Data Capturer
Data Typist and Foreman
Nurse and Network Engineer
Property Finance and Security Manager
Nurse
Accountant
Finance Officer
Property Evaluator
Shop Assistant and Pastor
Finance Officer

Table 4.1 Occupations of parents of the 18 students in the research group. Where only one occupation is listed indicates single parent families. Mothers' occupations are listed first.

4.2 Sampling

From a population of 90 Grade 9 students, 18 students were selected based on the following criterion:

- the top 6 students
- the middle 6 students
- bottom 6 students

The selection of these students was based on their achievement in mathematics at the end of the second school term. These results, however, comprised the marks for both the continuous assessment tasks and the June examination. School based assessment (SBA)[1] comprised 65% of the end of term mark and the examination 35%. Using these combined marks is useful since the SBA items tend to be similar in nature to the types of items characterising the CTAs.

1. School Based Assessment (SBA) was previously called Continuous Assessment (CASS) and it is comprised of class tests, assignments, projects, investigations, etc. These activities are intended to cover all the outcomes stated in the RNCSM and should provide indications of students' progress and achievements of these outcomes. Previously CASS comprised 75% of grade 9 students' final mark out of 100 so that students were (and still are) able to meet the pass requirement of 40 % for Mathematics before they wrote a summative final examination.

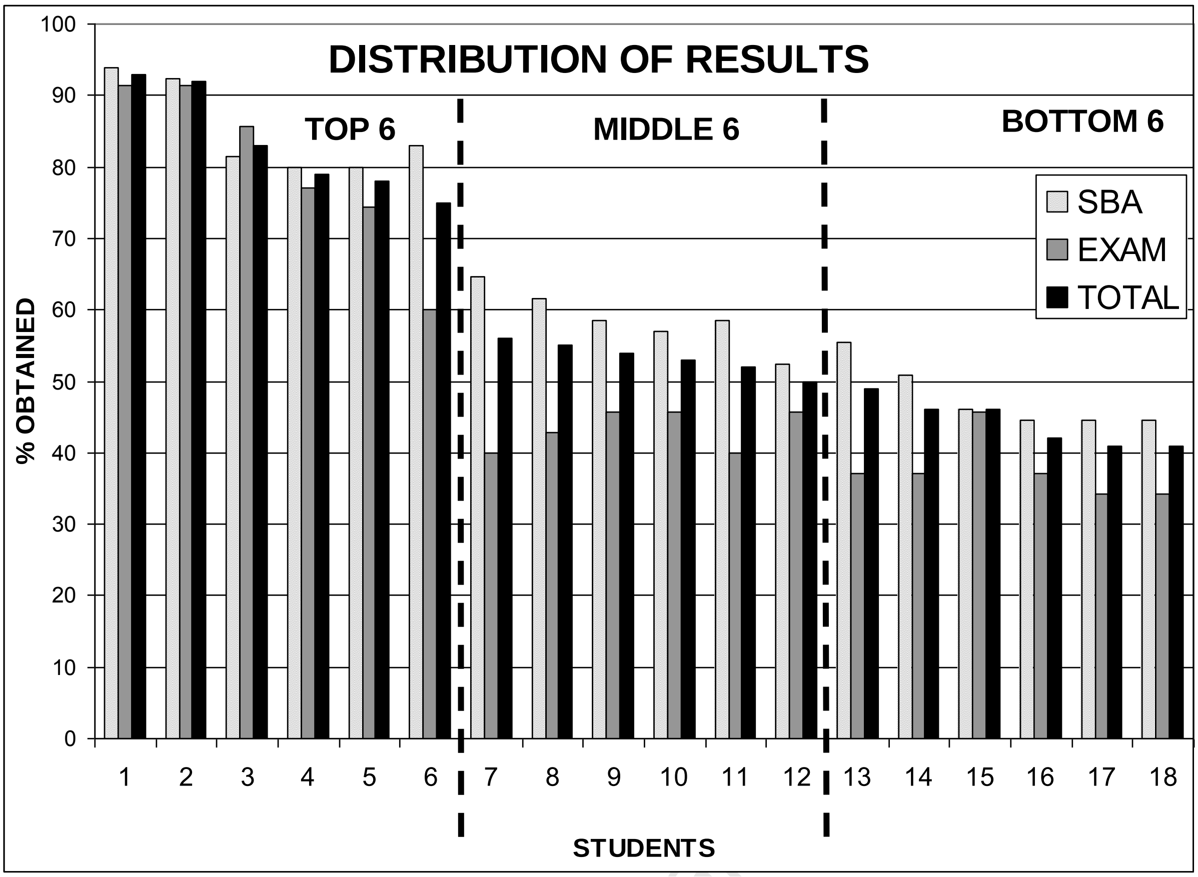


Figure 4.1 The distribution of the 2 components of marks used for evaluation of students at the end of term 2.

The graph above illustrates the distribution of the three sets of marks, the SBA marks, the June examination marks and the combined SBA and exam marks, for the 18 students in the research sample. All the marks were standardized to a percentage to allow for easier comparison. The SBA marks are derived from a range of different forms of assessment such as projects, investigations and tutorials. In all these forms of assessment time constraints were relaxed so that the students had several days/weeks to work on their own or in pairs and regular teacher guidance was provided to ensure that students remained on track. Therefore, this could possibly account for the better SBA marks they obtained.

The 18 students were split into 2 matched groups of 9, each consisting of 3 students from each performance grouping. The allocation of students into the two groups was determined alphabetically to ensure some form of randomness. The first group is labeled Group A and the second group Group B. Group A worked on the two CTA activities under controlled test conditions and had to rely on their own resource availability to generate solutions. Group B was presented with the same activities as classroom lessons by the teacher described above. (Refer to Appendix Y).

The sample of students used in this study derives from a fairly homogenous social background and yet their mathematical performances, on which the sampling was based, illustrate a large variance in their mathematical performance. Table 4.2 below summarises the results of the students in their respective groups and the average mark scored by each group for each activity is also indicated. Further discussion on these results will be undertaken in the analysis of the students' textual presentations. Ostensibly, discussion of these results will attempt to answer the question: Why, despite the social background homogeneity, is there such a variation in these students' mathematical ability, and is mathematics differentially constituted within the three ability groups?

GROUP A: TEST GROUP

	STUDENT	TERM 2 MARK	ACTIVITY 1	ACTIVITY 2
			21	26
1	KA	90.00	5	9
2	AJ	83.00	1	14
3	ZM	81.00	3	8
4	SZ	45.00	1	0
5	LA	45.00	1	0
6	LV	43.00	0	3
7	JA	30.00	1	0
8	JM	32.00	0	7
9	AB	27.00	1	0
		AVERAGE MARK	1.444	4.556

GROUP B: LESSONS GROUP

	First Name	TERM 2 MARK	ACTIVITY 1	ACTIVITY 2
			21	26
1	JR	90.00	20	12
2	LB	83.00	20	13
3	TS	79.00	14	11
4	TC	45.00	14	8
5	SV	44.00	1	0
6	IV	42.00	3	5
7	NF	28.00	0	6
8	AM	27.00	2	2
9	SJ	27.00	14	7
		AVERAGE MARK	9.778	7.111

Table 4.2 A summary of the results of the students in each group and in terms of their matched groupings

Investigating how mathematics is constituted when presented in contextually embedded forms is also considered in terms of the social background from which the students hail. In keeping with the arguments presented in the work of Cooper & Dunne (2000), the study aims to examine the responses to contextually embedded mathematics by the chosen sample of students and to draw broader theoretical conclusions about how mathematics is constituted by students from this particular social class.

4.3 Collecting information on social background

The work of Bernstein (1996), Dowling (1998) and Cooper & Dunne (2000) amongst others, indicate that the social class membership of learners is a variable that should be taken into account in educational research. The 2003 Trends in International Mathematics and Science Study (TIMMS) student background survey questionnaire was adapted to obtain information about the social class membership of the students who were used to gather research material for this project. The adapted questionnaire was circulated to all 90 Grade 9 students and 69 responses were obtained. The TIMSS (2003) data indicate that in almost every country students from homes with extensive educational resources have higher achievements in mathematics and other subjects than those from less educationally advantaged backgrounds. In line with the 2003 TIMSS study, the adapted questionnaire focused on a few central variables: level of parental education, students' educational aspirations, speaking the language of the test at home, having a range of study aids in the home, computer use at home and at school, and how often various mathematical operations were practiced. The TIMMS (Ibid) report argues that for most children, parents are their first and probably most important educators, therefore the level of education of the parents may be the most important educational resource in the home and it is this resource that this study focuses on in order to draw a broad classification of the sample group's background. Whether the data predict stable outcomes for this group of students and whether their parents' mode of labour can be argued to influence those outcomes, is partly what this study endeavours to reveal.

4.4 The research instruments

The activities used in the study were drawn from the 2003 grade 9 CTA and as they functioned as assessment instruments, they are evaluative and not pedagogic in nature. The activities differ in the form of their constitution. Activity one (overleaf) can be described as a sequenced exercise with references to explicit mathematical objects (eg. area, ratio, etc.) at the disposal of the students accompanied by a visual illustration to aid students. The nature of the questions is such that solutions to preceding questions are linked to consequent ones. In essence, the entire activity can be encapsulated in one instruction, namely, "calculate the area of the runways on the island".

ACTIVITY 1 (1 period : ±40min)

INSTRUCTIONS:

- **ALL ANSWERS MUST BE WRITTEN IN THE SPACES BELOW THE QUESTIONS**
- **ALL ADDITIONAL NOTES & CALCULATIONS MUST BE DONE ON THIS PAPER**
- **CALCULATORS MAY BE USED**
- **WRITE CLEARLY & LEGIBLY**

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is x metres and the length is y metres.

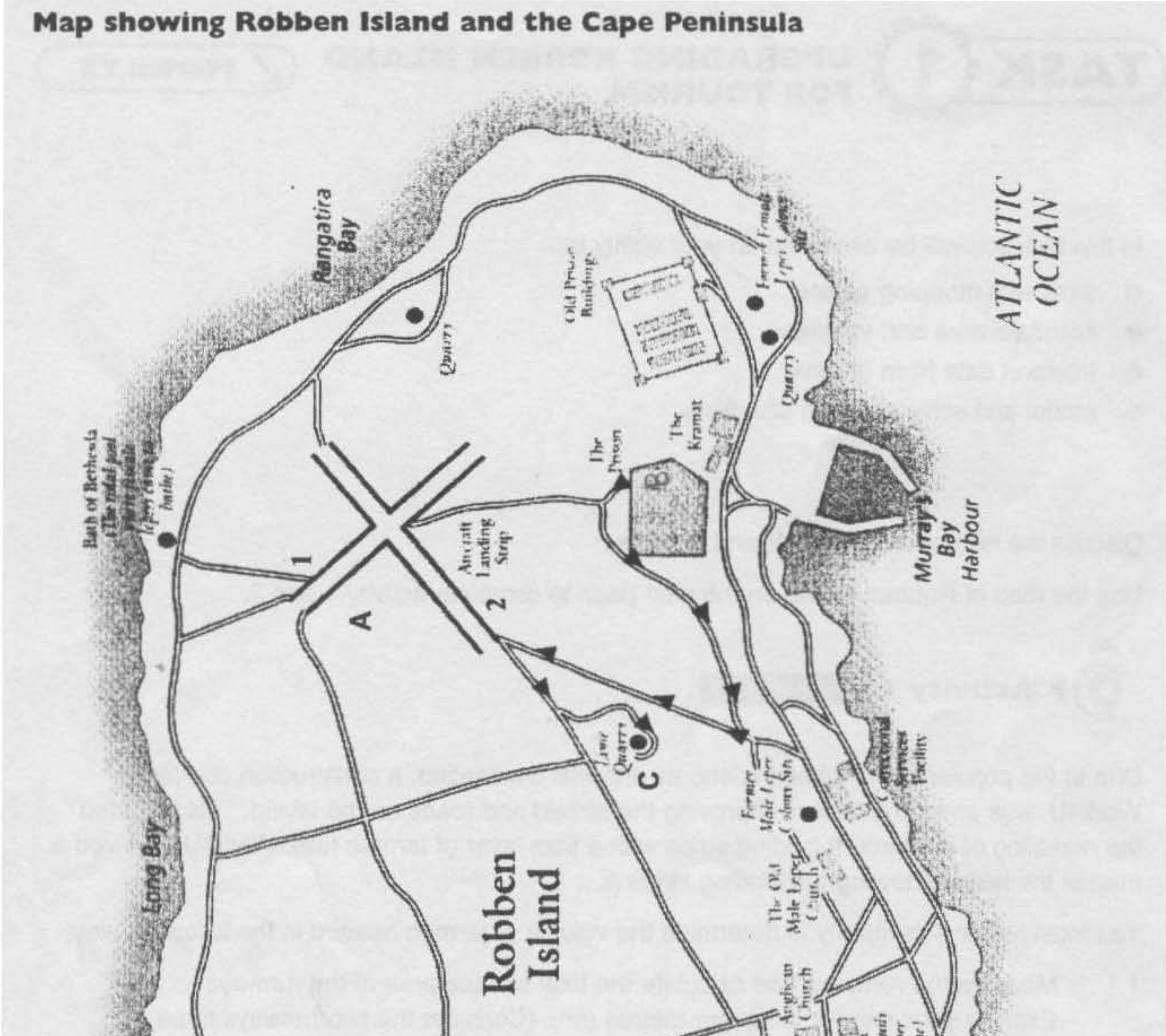
2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of x.

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of x. Simplify your answer.

4. If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer.

5. Why do you think the two runways intersect each other and are not parallel?

Figure 4.2 Activity One of the research instruments – an activity based on sequenced mathematical objects and operations. The accompanying map of Robben Island indicating the position and shape of the runways on the airfield.



- Learners measure distance and discuss differences in their measurements.
- Discuss the meaning of ratio and how ratios can be written as fractions.

	ACTIVITY ONE	MARK ALLOCATION/RUBRIC		SO
1	Area of runway: xym^2	Writing the expression	1 mark	
			[1]	SO9
2	$\frac{x}{y} = \frac{1}{20}$ or $x{:}y=1{:}20$	Interpreting the ratio	2 marks	
	Area of runway: $x(20x) = 20x^2$	Writing an algebraic equation	1 mark	
		Expressing area in terms of x.	1 mark	
		Substitution and calculation	2 marks	
			[6]	SO9
3	$20x^2 + 20x^2 - x^2 = 40x^2 - x^2$ $= 39x^2$	Showing that both runways are equal	2 marks	
		Adding areas of runways	1 mark	
		Subtracting intersecting area	1 mark	
		Subtracting correct intersecting area	1 mark	
		Simplifying the expression	1 mark	
			[6]	SO9
4	$39x^2 = 97500$ m^2 $\frac{39x^2}{39} = \frac{97500 \text{ m}^2}{39}$	Writing the equation (including unit)	2 marks	
	$\therefore x^2 = 2500$ m^2	Simplifying the equation	2 marks	
	$\therefore x = \pm\sqrt{2500 \text{ m}^2}$	Expressing the value of x as $\pm$	1 mark	
	$\therefore x = \pm 50$m	Calculation	1 mark	
	[x = +50m or x = -50m] x = 50m Accept true 50m(+50m). x represents length of runway		[6]	SO9
5	To cater for the maximum possible main wind directions	Similar answer	2 marks	
			[2]	SO10

Figure 4.3 The solutions to Activity One as supplied in the teacher's guide. Note that amendments have been made to the title and the numbering so that they would conform to the format of the activity presented to the students.

Activity two has a less structured approach with no sequentially arranged questions. A large amount of information is supplied in tabular and fragmented form accompanied by a broad overarching brief which requires a solution in which real life considerations have to be made. No hierarchical leading questions are supplied, but in essence, the activity is an income/expenditure exercise which engages basic forms of arithmetic.

ACTIVITY 2 (1 period : ±40min)

INSTRUCTIONS:

- **ALL ANSWERS MUST BE WRITTEN IN THE SPACE PROVIDED ON THE ANSWER SHEET**
- **ALL ADDITIONAL NOTES & CALCULATIONS MUST BE DONE ON THIS PAPER**
- **CALCULATORS MAY BE USED**
- **WRITE CLEARLY & LEGIBLY**

The tour operator plans to replace his old boat by the end of 2003. The value of the old boat will be between R300 000 and R700 000. The new boat, as a capital investment, will cost R3,5million. This capital investment may be spread over a number of years. He gathered the following information for the year 2003. Use the information below to make an informed decision about whether or not he will be able to buy the boat.

A boat carries a maximum of 60 passengers to Robben Island. Adults pay R100,00 and children 4 – 17 years pay R50,00 per return trip. His operating costs per return trip are:

EXPENSES	COST IN RAND
Fuel	200
Boat operator (hired)	100
Landing Fees	50
Maintenance cost	250
Owner's salary	350
TOTAL OPERATING COST	950

INFORMATION:

1. He can ferry only on days when it does not rain.
2. In the peak season (1 September to 30 April):
   - he operated his boat at 70% capacity
   - he managed an average of 6 trips per day
   - the ratio of adults to children was 1:1
3. In the off-peak season (1 May to 31 August)
   - he operated his boat at 30% capacity
   - he managed an average of 3 trips per day
   - the ratio of adults to children was 2:3
4. Rainy days per month for 2002/2003:

SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG
10	7	5	4	5	2	6	10	15	20	25	26

Figure 4.4 Activity Two of the research instruments – an activity based on integrated mathematical objects and operations .

5.6 Analysis of data

ACTIVITY TWO	CRITERIA	SO1	SO10
PEAK SEASON			
Persons per trip 70% of 60 = 42	Decide to calculate 70% of 60		1
∴ Adults = 21			
∴ Children = 21			
	Calculate 42 correctly	1	
	Calculate adults correctly	1	
	Calculate children correctly	1	
Cost per trip: R 950	To calculate the income/trip		1
Income per trip: 100(21) + 50(21) = R3150	To calculate the profit/trip		1
Profit per trip : R2200	To calculate profit for season		1
Profit for peak season = 6 x 193 x 2200			
= R 2 547 600,00			
	Correctly calculating profit/peak	1	
OFF-PEAK SEASON			
Persons per trip 30% of 60 = 18			
∴ Adults = 7	Decide to calculate 30% of 60		1
∴ Children = 11	Calculate 18 correctly	1	
	Calculate adults correctly	1	
	Calculate children correctly	1	
Cost per trip: R 950			
Income per trip: 100(7) + 50(11) = R1250	To calculate the income/trip		1
Profit per trip : R300	To calculate the profit /trip		1
Profit for off-peak season = 3 x 37 x 300	To calculate profit for season		1
= R 33 300,00			
	Correctly calculating profit/peak	1	
Total profit for year = R 2 580 900			
= R 2, 58m	How to find the total profit		1
	Correctly calculate the profit/year	1	
Cost of new boat = R 3,5m			
Profit = R 2,5m	How to determine the shortfall		1
Needed = R 1,0 m	Correctly calculate the shortfall	1	
Possible answers for decision:			
Sell old boat for R 0,5m and pay R 0,5m over next year	Reasoning and decision-making		2
	Making a decision		1
OR	Some calculations to back up decision-making	1	
Wait till next year to accumulate enough cash – price of boat might increase, etc.			
	TOTAL:	**11**	**13**

Figure 4.5 The solutions to Activity Two as presented in the teacher's guide. The title has been amended to conform the manner in which the activity was presented to the students in the research group.

4.5.1 Analysis of the lessons

Two lessons were presented to the 9 students in Group B and were video recorded. Full transcriptions of the lessons were produced from the video records. The video records and transcriptions constitute the archive of information from which data are generated.Each lesson transcript was segmented into units of analysis called evaluative events. For the purposes of this research project an evaluative event is identified in terms of the sequenced questions in activity one. Furthermore, within each evaluative event, sub-events can occur when specific mathematical objects or operations are identified. These events can be described in terms of the criteria they privilege and the type of orientation they favour towards the mathematical objects in question.

The notion of the evaluative event derives from Davis (2001, 2003) and is taken as a unit of mathematical activity in which criteria for the recognition and realisation of specific content is elaborated, in this case the specific criteria are directly related to those solutions that closely resembles the privileged solutions of the teacher's guide. In the context of teaching and learning Mathematics, pedagogy is understood as fundamentally evaluative. Evaluation distinguishes legitimate from non-legitimate knowledge statements for students and reveals criteria for the recognition and realisation of mathematical objects or procedures in pedagogic contexts. Therefore, what comes to be constituted as Mathematics and how Mathematics is constituted in a pedagogic context is rendered visible through the criteria that circulate in that context (Davis & Johnson, 2007). The generation of criteria for the recognition and realisation of specific mathematical objects or processes or procedures can be considered as constituting specific evaluative events and these are embedded in the contextualized forms presented in the CTA activities.

In Activity one an evaluative event starts with an introduction of each of the sequenced questions and elaborated either through a definition, a worked example or by directly referencing a solution to the problems. Sub-events occur when the teacher refers to mathematical objects or operations within each question and elaborates on them. The criteria used to elaborate those objects and operations will be primary to the pedagogic analysis. An evaluative event is terminated when the teacher decides to move onto the next question.

In activity two the absence of sequenced questions necessitates a different formulation of evaluative events. Therefore, evaluative events are directly linked to the mathematical objects and operations the teacher privileges as criteria for recontextualisation. The nature of the generalized approach in this activity as well as its basic constitution at the level of arithmetic calculations on income and expenditure renders the pedagogic criteria arbitrarily at the discretion of the teacher. In both activities the analysis seeks to draw out the criteria for the teacher's operational activity and, in particular, attempts to establish the nature of that activity using Davis' (2009) grounds model to establish what is constituted in each of the evaluative events indexed.

Ground	Domain of objects operated over
Iconic	Expression forms treated as images
Empirical	Graphical and/or symbolic expressions treated as in some way "measurable"
Propositional	Axioms, definitions and propositions that pertain to the mathematical object(s) signified by expressions
Procedural	Operations commonly used within a procedure

Figure 4.6 Davis' (2009) grounds categories used to analyse teachers' pedagogic modalities.

Thus, student may or may not have acquired the intended legitimate criteria by the conclusion of an evaluative event. Given that Mathematics necessarily uses previously established results to explore whatever content is engaged with in the present, evaluative events have to be thought of as elaborating new content and rehearsing and putting to use established, familiar contents. The new content elaborated as the principal topic of an evaluative event will be referred to as its primary content; the established contents that serve as resources will be referred to as its secondary content.

4.5.2 CTA activities

The two activities differ fundamentally in the form in which they are presented. The first activity is sequentially structured so that the activity is expanded over five questions. The questions are sequenced in such a manner that recognition of solutions to preceding questions is crucial to the realisation of solutions to consequent questions. The privileged criteria necessary for the production of legitimate realisation are contained in the teacher's guide. In the first CTA activity the mathematical objects are clearly available, but students are required to apply appropriate rules of recognition to produce legitimate texts. For example,

> Write an expression for the area of one runway if the width is x metres and the length is y metres. (Question1, Activity 1)

The structure of the activity suggests the construction of a particular competence required to solve the general problem presented to the student. The hierarchical nature of this activity raises the question about where competence resides. Does it reside purely in the organization of the text, or in the student who has to work through the series of questions?

The second activity has a different format in that students are presented with a set of information which has to be processed without any explicit leads being given. Recognition is not embedded in the form of questions with specific instructions, but resides at the level of the resources available to the individual student.

An explication of the underlying principles of the mathematical objects used in the activities is necessary to understand where competence will ultimately reside and how realisation of legitimate text may not necessarily be an indication of understanding of propositions active in the texts. It is, therefore, necessary to return to Bernstein's concepts of recognition and realization rules which will be used as analytical tools.

Rules of recognition are means by which students "recognize the speciality of the context they are in" (Bernstein, 1996: 31) while realisation rules are the means by which students produce legitimate texts. Given the context of the activity, students are able to recognise what is needed and how to respond appropriately. If realisation rules are appropriately applied, students may produce utterances considered legitimate in the given context. If students possess recognition rules, but are devoid of realisation rules, they may recognise the kind of response needed, but may not be able to produce legitimate texts. The sequential nature of Activity one constructs mathematical competence in such a manner that failure at the initial question as well resource paucity may result in a collapse of the entire activity.

By elucidating the mathematical objects and operations used in each activity and linking them to the underlying rules which regulate them, I hope to establish the resources that inhere to the problems. Furthermore, I shall attempt to construct networks of objects brought into operation in each task to analyse the textual possibilities made available to the students.

4.5.2.1 Properties of the mathematical objects

The mathematical objects in activity one are represented by the nouns and adjectives listed below:

Question 1: expression, area, width (specified at x metres) and length (specified at y metres).

Questions 2: ratio (specified at 1:20)

Question 3: perpendicular

Question 4: total area (specified at 97 500m^2), square metres (m^2)

Question 5: parallel

The school mathematical rules/properties that govern these objects include the following:

- Area is defined as part of a two-dimensional surface enclosed within a specified boundary or geometric figure. This implies that specific mathematical operators are required to derive the area of a specified geometric shape.
- multiplication rule for exponents, namely $a^m \times a^n = a^{m+n}$.

In activity two the explicit objects used are: maximum; ratio, percentage, average (although no computations of the sort are required). Ratio and percentage are different forms of proportional representation and “in the most concrete way fractions present themselves if a whole has been or is being split, cut, sliced, broken, coloured in equal parts” Freudenthal (1999: 140). Freudenthal (Ibid: 158) defines the rule for proportional representation as:

$(\frac{m}{n}\ of) = (m\ times) \circ (nth\ \ part\ \ of),$ which can be written as $(nth\ \ part) \circ (m\ times)$ where $\frac{m}{n}$ represents any rational number. Thus, the key to understanding proportional representation is to understand the rules governing rational numbers which constitute a subset of the real numbers.

The mathematical objects needed to realise legitimate texts in activity two reside at the basic level of addition, subtraction and multiplication. However, these are muted by the form in which the activity presents itself.

The rules that govern the real number ($\Re$) operations that are required in both activities are outlined by their following properties:

PROPERTY	FOR ADDITION	FOR MULTIPLICATION
Closure	$a+b\ \in \Re$	$a \times b\ \in \Re$
Associative	$(a+b)+c = a+(b+c)$	$(a \times b) \times c = a \times (b \times c)$
Commutative	$a+b = b+a$	$a \times b = b \times a$
Distributive	$a(b+c) = ab + ac$	
Identity	$0+a = a+0 = a$	$1 \times a = a \times 1 = a$
Inverse	$a+(-a) = 0$	$a \times \frac{1}{a} = 1\ ;\ a \neq 0$

Table 4.3 The additive and multiplicative properties of real numbers.

Analysis of the teacher’s methodology must be referenced against the mathematical properties outlined above and examined in terms of legitimate mathematical practices or pseudo-mathematical practices and how these may attempt to construct the competence of the students. The principles (outlined above) governing the mathematical objects to be acted upon in the activities makes the following network of possibilities available for selection for the computational contextualisations of area to the teacher:

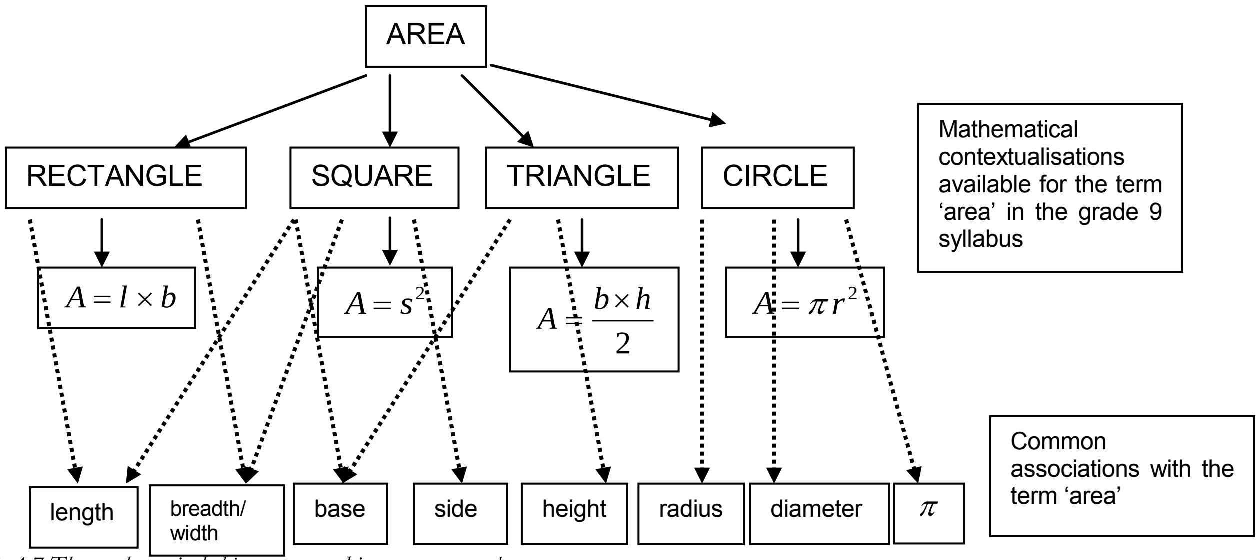


Fig 4.7 The mathematical object, area, and its component subsets.

Analysis of the lessons will index the objects and operations the teacher uses in her methodology and to distinguish between those practices that employ mathematical operations and those that are pseudo-mathematical. Pseudo-mathematical operations occur when the mathematical properties of an object or sets of objects are redescribed in metaphorical terms that index something other than the object's inherent properties. Mathematical operations can be divided into sub-categories, propositonal/principled, procedural, pseudo-mathematical and logical deductive. What distinguishes propositional/principled from procedural is the extent to which students are given access to the axiomatic properties that could avail a range of combinatorial resources to realise legitimate solutions. I refer to a modality that distributes a dependency on procedures, algorithms and formulae while denying access to the the knowledge of the properties of the object, as procedural. Logical deductive modality favours a use of chains of inference drawn from the given information.

4.5.3 Analysis of the tests and interviews

The students' texts were produced under two distinctly different conditions. Group A's responses to the two activities were produced under test conditions while group B produced their responses during and after the same activities had been taught to them. Members of group A were interviewed on completion of the tests to elicit how they derived the answers they produced and to gauge what were prioritised as criteria for their productions. Analysis of the students' texts will seek to answer the following questions:

- What procedures were used?
- What operations were involved?
- What objects were involved?

- What is the main operational activity?

In addition, a triangulation between the criteria privileged by the teacher, the criteria revealed in the interviews with the students who wrote the test and the solutions privileged in the teacher's guide should reveal attunement or differences in how mathematics is constituted.

4.6 Summary

The core of the analysis of this research project is essentially concerned with the operational activity of the agents involved in the different modes, namely, the tutored students' texts, the teacher, and the mode of the students involved in the tests. In particular, the analysis aims to describe the process of selection of operations on objects by these agents and interrogates the operational criteria favoured.

Chapter 5

BACKGROUND ANALYSIS

5.1 Introduction

The analysis to be presented will attempt to unravel the complexities of understanding how mathematics is constituted when it is presented in embedded formats. The analysis will examine the students' class backgrounds, problematise social class as well as examine the degree to which socialisation within specific classes, similar to those presented in the Cooper and Dunne (1998) studies, impact on mathematical performance. Furthermore, the analysis will focus exclusively on the three main aspects of the research:

- a textual analysis of the research activities will be undertaken using Dowlings' (1998) domains of practice as its basis
- an analysis of the lessons on the research activities will focus on what is constituted as mathematics using Davis' (Davis & Johnson, 2007) ground model
- an analysis of what criteria are privileged by the students in their constitution of mathematics

All of the above will be examined in terms of the students' background in order to draw some conclusions about how the rules of recognition and realization are accessed/acquired by this group of students.

This study attempts to unravel some of the complexities that come into operation when external assessment in the form of the Common Task for Assessment (CTA) is used in conjunction with the broad principles of real life or relevant mathematics. Furthermore, the study attempts to examine the difficulties presented to students when mathematics is embedded in a contextualized framework and also shows that assessment, particularly in this form, presents more challenges to working class children than anticipated by the designers of the CTA. The study also examines these issues in relation to the class backgrounds of the students and attempts to show that a complex interaction between students' attributes and abilities, task characteristics, and pedagogic intervention methodology result in the students' specific orientation to mathematical objects.

5.2 Students' background: Theoretical connections

Analysis of the issues raised by the student background data can usefully be accomplished by turning to Bernstein's theory (1971). Bernstein states that power relations are responsible for the creation of boundaries between groups of people, between different categories of discourse, such as school subjects, and between different agents. The power relation is defined by the term classification which determines the power attributed to one object over another. At the level of the curriculum it would exist, for example (though not exclusively), between the differences between subjects and the resulting power status accorded by one subject over another. This status of power can be seen, for instance, in the importance placed by society in subjects like mathematics and science. Bernstein argues that classification creates the space between the discourse of subjects like mathematics and other subjects. In fact, the meanings of subjects are only understandable in the relationship the subjects have with one another. It is the insulation of the subject that allows it to retain its identity and if that insulation is threatened then the subject is in danger of losing its identity. This has implications, particularly, for students whose orientation to knowledge is already of a weakened nature.

Bernstein argues that in recent times there has been a change in the classification of knowledge and, as such, it has become regionalised which has resulted in the weakening of classification and a shift in power structures. At the level of the classroom, the teacher has traditionally been the focus of power relations. Although the National Curriculum Statement (NCS) and its principles of outcomes based education (OBE) explicitly realigns the traditional power relations, the pedagogic style evidenced in this research project points towards a resistance to a redistribution of these power relations. Thus, the lessons observed and videotaped display a very strong teacher focus with a predominantly passive group of students. Furthermore, the social arrangement of the classroom is traditional with a clear pecking order in terms of teacher-student relations and student-student relations. The relationship between the physical arrangement of the classroom and the pedagogic methodology all come to bear on how the assessment of the CTA transpires.

Bernstein also argues that in order for an individual to operate effectively within a particular society, the individual needs to possess the appropriate recognition and realization rules of that society. It is the contention of this study that the code of the student forms the necessary foundation which will enable the student either to fit in seamlessly or not with the code of the schooling process of the particular school in which the study is conducted and the rules of recognition and realization necessary to understand mathematical texts and reproduce legitimate mathematical operations during the evaluation process. However, within the context of the school, the development of rules of recognition and realisation in an individual is strongly related to the concept of classification and therefore, the power relationships within the school culture.

Classification can be used to indicate the strength of the relationships between categories. In particular, it indicates the degree of insulation of a subject like mathematics and the degree of its distinction from other subjects. If its insulation is reduced, the strength of its classification also changes. Blurring the boundaries between subjects and within a subject is an indicator of the de-classification of categories according to Bernstein (2000). Therefore, there are two extremes of classification, one called the collection code, which represents strong classification and framing, and the other, the integrated code, representing weak classification and framing. These extreme classification codes have implications for how the evaluative process will be influenced by the differential distribution of the rules of recognition and realization for students from the socio-economic class of the group of students with whom this study was conducted.

While classification refers to the degree of insulation between categories, framing refers to how meanings are put together, the forms by which they are made public, and the nature of the social relationships they enable. In terms of the pedagogic context educational knowledge is defined in terms of a transmitter and an acquirer. The acquirer is identified in terms of how successfully he/she is able to access the rules of recognition and realization. For the acquirer the rules of recognition are the means by which "individuals are able to recognize the specialty of the context they are in" (Bernstein, 1996). In order to produce legitimate texts appropriate to the social context, the acquirer must produce legitimate texts and thereby, the acquirer gains the rules of realization. The rules of recognition are inextricably bound by the rules of recognition in that the rules of realization can be successfully acquired only if the rules of recognition have been mastered. Realisation rules therefore, lead to the production of legitimate texts and determine how these texts are made public. Cooper (1998) argues that these rules have the potential to operate independently of one another especially when applied to children from the working classes. These children often recognize the appropriate social context but lack the rules of realisation and as a result, are unable to produce legitimate texts as well as make them public appropriately. Holland's (1981) study revealed the differential acquisition abilities of children from the working and middle classes. She concluded that the middle-class children possessed more than one way of classification based on different principles and that these operated in a hierarchical manner. The order of the hierarchy was determined by a socially appropriate context i.e. the middle class children recognised the dominant code of the knowledge environment and as such, suppressed their non-specialised recognition rules. Working class children tended to select non-specialised recognition rules which resulted in the generation of non-specialised contexts. This is mainly the result of a discordance between the specialised code of schooling as opposed to the non-specialised home context. These two codes are at odds with each other and the use of the domestic code in the highly specialised code of the school context results in continued subjugation of the working class children within the pedagogic power relationship.

If Bernstein's theories of coding are to believed, one has to conclude that the working class children cannot adequately be educated especially since schooling is essentially a middle class conceit practiced by teachers who by the very nature of their higher education and practice, belong to the middle class. The teachers themselves do not possess the recognition rules of the students and therefore, have a completely different perception of what is happening within the school culture.

5.3 Students' background: Empirical connections

The education levels of the parents of the group of students who were directly involved in the data collection for this research project indicate that of the 18 students only one child's mother (a single parent) did not complete her high school education. Ironically this student was also the top performer in mathematics in the June examinations. Table 4.1 summarises the occupations of the parents of the 18 students in the research project. Where only one occupation is listed, the student hails from a single parent family or has no knowledge of the other parent (normally a father) or if the other parent is known, nothing is known about his occupation. Mothers' occupations are listed first.

PARENT OCCUPATIONS
Saleslady
Primary school teacher
Retired
Tour Consultant & Unemployed Welder
Shop Assistant and Plumber
Hairdresser and Business Owner
Saleslady
Truck Driver
Shop Assistant
Data Capturer
Data Typist and Foreman
Nurse and Network Engineer
Property Finance and Security Manager
Nurse
Accountant
Finance Officer
Property Evaluator
Shop Assistant and Pastor
Finance Officer

Pinning down an exact class orientation for this group of students is problematic for various reasons. While many of the parents in this group may be considered to occupy middle class occupations as well as having an education commensurate with a middle class orientation, one has to consider both these factors historically. These parents received the bulk of their education under the disparate Apartheid education departments and as such received unequal levels of education. With the demise of Apartheid and the demands for redress in the job market, many of these parents would have experienced the push towards employment equity as demanded by the new government's policies around social justice. Both of these factors give a skewed indication of middle class orientation and are misleading in terms of indicating the students' socialisation processes. It is not the scope of this study to delve into the historical roots of the research group's class origins, but we can make assumptions about the resources these students have access to from the data collected in the student questionnairres.

The nature of working class work (the historical roots of all the parents) requires a high level of obedience to authority and conformity to rigid routines. These jobs also lack complexity and seldom call for individual initiative and enterprise. Middle class jobs, in contrast, allow for greater autonomy and intellectual work (Kohn, 1983). Furthermore, Kohn argues that the different nature of work of these two classes impact on the manner in which each of these classes socialize their children. As such, working class children are subject to more authoritative rearing, while middle class children are exposed to more playful and intellectual instructional activities (Duberman, 1976). Bernstein's (1975) linguistic codes emphasised differences between social classes with the privileged classes having access to "elaborated codes". Elaborated codes allow children access to a language with explicit meanings independent of context. This code is also the code by which institutions like schools operate and as such allows middle class children a smooth transition form home into school. Working class children, on the other hand, have access to a "restricted code" in which the language is implicit and meanings context dependent. This code is not commensurate with the code of schools and as a consequence, working class children always experience a disjuncture in their movement from the home environment to the school. This situation was further elaborated by Holland's (1981) study of how children categorised pictures of food items. The significance of these studies point to the fact that children differed from class to class with respect to their orientation to knowledge and their access to cultural capital.

Cooper and Dunne employed the neo-Weberian social class typology developed by Goldthorpe (1987) in which he identified three distinct classes namely, the service class, the intermediate class and the working class. The service class is defined in terms of its level of property ownership and its concomitant prestige. The intermediate class, the so-called white- and blue collar workers, is a class with limited or no property. The intermediate class contains elements of the classical middle class but the emergence of a new middle class splits this class into two distinct groups: a deskilled routine white collar faction and an enskilled specialist faction (Abercrombie and Urry, 1983). Essentially, the deskilled white collar faction has undergone a process of proletarianisation accompanied by a feminisation of a large section of this fraction. The nature of their work is characterised by predetermined routines and high levels of mechanisation. It is mainly from this class that the students in the research group are drawn from. Goldthorpe's working class retains it's classical Marxist/Weberian definition of being propertyless manual workers alienated from the means of production. They have only their labour at their disposal in the market.

It is apparent from the parents' occupations that their jobs fit into Cooper and Dunne's intermediate and working class categories with some exceptions i.e. the teacher, nurse and accountant. From the routinised nature of the work done by most of these parents, most of the students in this research group can be linked to Bernstein's restricted code. In essence, neo-Weberian definitions of the lower middle class (intermediate class) distribute particular orientations to knowledge and limit the possibilities of orientation to meaning. Furthermore, if the training teachers and nurses (the enskilled exceptions in the group) are considered in historical terms then it reveals that teacher and nurse training under the Apartheid regime did not necessarily guarantee induction into a universal middle class, but the nature of the training provided by these colleges served merely to create racialised class differentiation with restricted access to the principles of these professions. The anomalous accountant, on further probing, is nothing more than a bookkeeper with some in-house training on operating specific accounting software.

Bernstein's code theory runs into an empirical anomaly with the student whose single parent dropped out of high school and now works as a saleslady. The nature of the work of a saleslady cannot by any stretch of the imagination be classified as a complex intellectual occupation. However, the fact that her child has been such a high achiever, not only in mathematics, but in all her other subjects, raise questions about the impact of socialisation opposed to the impact of schooling. If the logic of Bernstein's code theory is to be extended to its logical conclusion, then it should follow that students of higher social status should be greater achievers at all scholastic levels. The results of the two activities tell a more complex story. For the test group the range of marks for Activity one is 5, while that that of Activity two is 14. KA, the daughter of the saleslady, with her distinctly working class background, displays comparable results with those of the AJ, the daughter of a teacher, as well as ZM, whose father is retired and whose working background is unspecified.

5.4 Conclusion

The aim of this analysis is to attempt to assign a precise social class delineation to this group of students in order to compare this with the mathematical orientation(s) privileged by them. The overall nature of this classification seems to point quite firmly towards the lower middle class or intermediate class. The nature of the work of parents in this class is described by neo-Marxists and neo-Weberians as routinised tasks with a strong bent towards proletarianisation. In essence, the skills needed to perform these jobs are highly specified and localised to such a degree that no form of higher learning is required to perform the jobs' tasks. Bernstein (2001) refers to these forms of work as that which requires "trainability". Therefore, in much the same way that Bernstein characterises the working class code as restricted, the same code can be extended to backgrounds of the students in the research group by virtue of the nature of the work their parents perform.

The purpose of analysing the students' background is to establish whether specific orientations to knowledge and consequently, orientation to a particular form of mathematical constitution, place this group of students at an advantage or disadvantage. The range of possible meanings established can be described in terms of the specific social class from which these students hail. The orientations to knowledge and meaning of this social class have to be measured in terms of the textual meanings that the assessment activities privilege and the degree to which the rules of recognition and realisation are revealed by the text to the students. Furthermore, social class must also be linked to how the pedagogic process explicates the rules of recognition and realisation.

Chapter 6

TEXTUAL ANALYSIS

6.1 The mathematical activities: Content Analysis

In this chapter I shall consider how mathematics is constituted at the textual level of the CTA activities presented to the two groups of students. Using Dowling's (1998) domains of practice, I shall explore the mathematical objects and operations and how they function in their contextually embedded forms. I shall argue that the different formats of the two activities construct students' competence differentially.

6.2 Activity 1

6.2.1 Concerns: The map

The signifying mode in Dowling's (Ibid) terms describes the relation between the form of expressions and the content of the text that is implicated in sign production. "Signifying mode" refers specifically to the repertoire of resources associated with generalizing and localizing strategies. Dowling identifies three signifying modes viz. iconic (pictures, photographs, maps), the indexical (diagrams, graphs) and the symbolic (mathematical signs, variables, symbols, etc). The map would falls under the category iconic even though the runway is intended to index a particular geometric shape, namely the rectangle.

The use of the map has great potential to mislead students insofar as the orientation of the image is concerned. The orientation of the enlarged Robben Island differs from that of the small insert. Furthermore, no directional orientation (i.e. there is no north indicator as on the smaller insert) is indicated for the larger Robben Island and as such the students is required to treat this as insignificant and extraneous to the mathematical tasks presented. The larger map has to be regarded only insofar as it indexes the mathematical content of the rectangular area contained within the shapes of the two runways represented on the Robben Island map. However, the particular directional orientation of the larger map ignores the effects that the directional error and the compass omission could have on student responses to the question about the prevailing winds in and around Cape Town. Therefore, the representation of the large Robben Island could lead to highly localised forms of responses as well as inappropriate application of so-called real-life considerations. Inaccurate symbolic representations could also have a negative effect on students from poorly resourced backgrounds. The specific representation of Robben Island in relation to the insert requires of students a high level of manipulation of the larger map in relation to the smaller insert in order to obtain the correct directional orientation.

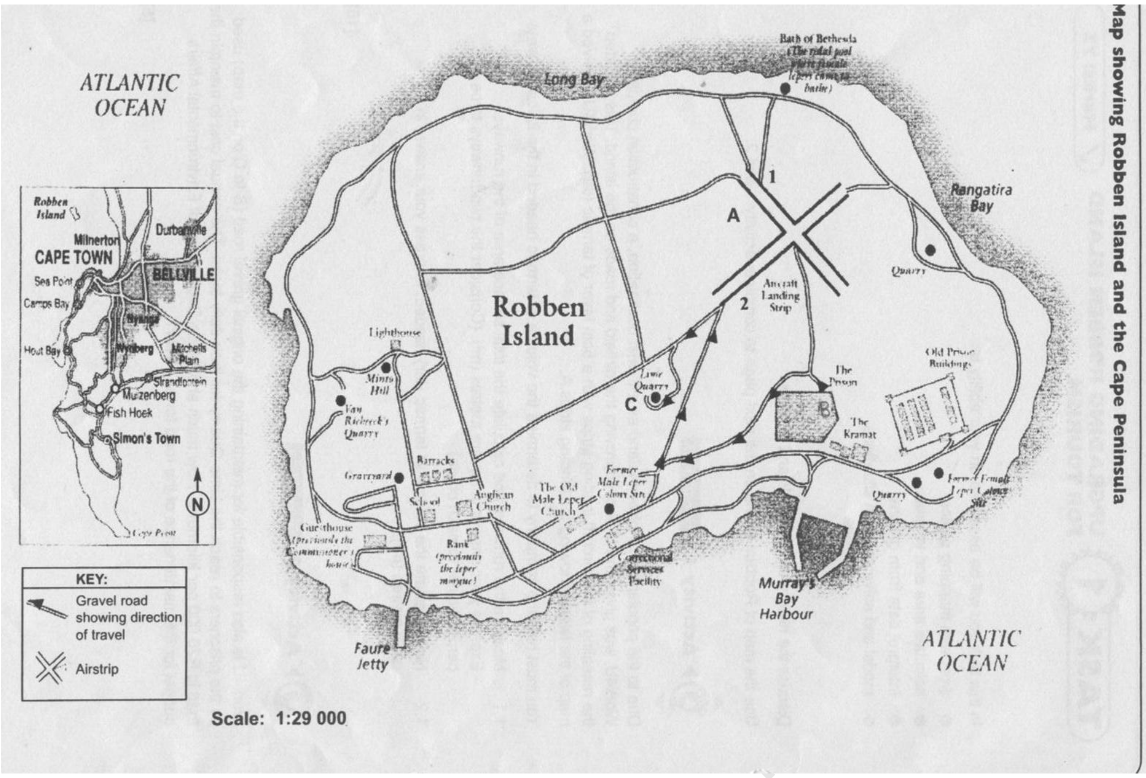


Fig 6.1 The map of Robben Island accompanying Activity 1 and a satellite image of the same island with its correct orienation. Note the difference in the actual north-south orientation of the island in the insert as opposed to the east-west orientation of the enlarged map.

6.2.2 Concerns: The text

Collapsing the distinction between "runway" and "airfield" has implications for answers to question 4. The boundary of the airfield is not demarcated and thus makes it impossible to make any conclusions about the derivation of the area. The equation/formula will have 2 unknowns (width & length) and this could lead to speculative answers, namely, finding factors of 97 500 or simply concluding that no answer exists.

6.2.3 The questions

The general context of the CTA revolves around Robben Island as a tourist attraction and as such the CTA consists of five tasks each dealing with a specific context related to Robben Island. Activity One forms part of Task One and deals with the mythical company "Work4U" which is enlisted to upgrade various facilities on Robben Island. All of these activities have been mythologised in terms of some mathematical operations. The general problem presented by this activity is concerned with finding the total surface area of two perpendicular runways. The activity is set in the form of a series of sequenced questions so that solutions to consequent questions are necessarily dependent on the questions preceding it. This format, thus, constructs the competence of the student through the text by making explicit the mathematical objects and operations required to generate legitimate texts.

6.2.3.1 Activity 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is x metres and the length is y metres.

In question one a mathematical gaze has been cast over a non-specialised setting viz. the area of the runway. Mathematical forms of expression – in this case, algebra – have been imposed on the setting in such a manner as to constitute Dowling's (1998) descriptive domain. In this way the specialised, esoteric terms of algebra in the context of area calculations have been superimposed on the non-specialised content. The mathematical objects of length and width operate independently of the indexical symbol of the runway on Robben Island. These mathematical objects also enable students to recognise a mathematical procedure and to realise the production of legitimate texts. The specific objects are:

- expressions: an indication that some form of algebraic symbolization is required
- area: a principled understanding of the concept as a bounded two dimensional shape is possible or the principle could simply be applied at the very stable level of a formula
- width (x metres): one of the dimensions that encloses the area of a shape
- length (y metres): one of the dimensions that encloses the area of a shape

The concept of the two dimensions, the differential values given to these dimensions, the map and the image of the runway combine to construct the recognition of a particular geometric shape, namely, the rectangle, and, consequently, the realisation of a solution commensurate with that in the teacher's guide.

1. *If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of x.*

The privileged solution for question two in the Teacher's Guide indicates that marks are allocated for very specific steps in rendering a solution. These indicate the specific objects and operations which count for legitimate mathematical texts. The objects needed to generate this solution are independent of the solution in questions 1 and therefore, the first two questions do not follow sequentially. The second criterion allocates a mark for writing an algebraic equation, but the solution offered is not consistent with the equation required, namely, $y = x.20 \ or \ y = 20x$.

$\frac{x}{y} = \frac{1}{20}$ or $x{:}y = 1{:}20$	Interpreting the ratio	2 marks
Area of runway: $x(20x) = 20x^2$	Writing an algebraic equation	1 mark
	Expressing area in terms of x.	1 mark
	Substitution and calculation	2 marks

Fig 6.2 Extract from the teacher's guide: The solution to Activity 2 question 2 as well as the criteria for allocating marks.

The rule of realisation in this question resides in the recognition that a correct solution to question one should be employed in operationalising legitimate text in question two. The form that any procedure should encompass is contained in the mathematical object suggested "ratio" and at a more complex level, the use of an equation in order to find an expression in terms of x.

3. *If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of x. Simplify your answer.*

4. *If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer.*

Both these questions are linked to the solution to question 2 and as such if an incorrect solution was generated there, it is bound to have an effect on solutions to question 3 and 4.

3	$20x^2 + 20x^2 - x^2 = 40x^2 - x^2$ $= 39x^2$	Showing that both runways are equal Adding areas of runways Subtracting intersecting area Subtracting correct intersecting area Simplifying the expression	2 marks 1 mark 1 mark 1 mark 1 mark **[6]**	**SO9**
4	$39x^2 = 97500\ m^2$ $\frac{39x^2}{39} = \frac{97500\ m^2}{39}$ $\therefore x^2 = 2500\ m^2$ $\therefore x = \pm\sqrt{2500\ m^2}$ $\therefore x = \pm 50m$ [x = +50m or x = -50m] x = 50m Accept true 50m(+50m). x represents length of runway	Writing the equation (including unit) Simplifying the equation Expressing the value of x as ± Calculation	2 marks 2 marks 1 mark 1 mark **[6]**	**SO9**

Figure 6.3 The teacher's guide's solutions to question three and four of activity one.

Several mathematical objects are invoked as recognition rules in the question. These include:

- a recognition that the runways have exactly the same dimensions ; the teacher's guide privileges the notion of equality of the runways and thus, adding the two areas
- being perpendicular, they intersect and have an overlapping area which forms a square ; thus, the need for subtraction
- an expression which does not require a solution as in an equation
- answers should be expressed in its simplest form
- the value of x, representing a distance, is absolute

5. Why do you think the two runways intersect each other and are not parallel?

This question removes the student from the specialised practice of algebraic calculations in the previous four and places the student exclusively in the public domain. There are no mathematical tasks involved in the question as it merely requires the student to apply some form of general knowledge about weather conditions in Cape Town. Alternatively, an element of cross-curricular appropriation is involved in the sense that the student is required to apply knowledge gained in Geography. While this has nothing do with mathematical content, it could be argued that a degree of compliance with the integration principles of the RNCS is achieved.

6.3 Activity 2

In keeping with the general theme of Robben Island as a tourist destination, activity two is embedded in the context of a tourist boat owner's operational costs. The aims relevant to this activity are spelt out in the original CTA as:

- *use given information to make an informed decision*
- *critically analyse investment scenarios*

The nature of this activity is significantly different from that of activity one. Activity one can be described as belonging to Dowling's (1998) "descriptive domain" since a mathematical gaze is cast over a non-mathematical context. The purpose of this exercise is, arguably, to create a familiar context which would encourage students to engage meaningfully with the mathematical content. The mathematical objects and operations, although weakly masked by the contextualisation, reside entirely at the basic level of the additive properties of real numbers. However, the difficulty for the students resided in the lack of formalised mathematical processes which they can draw from to generate meaningful solutions. The privileged solutions given in the teacher's guide overleaf, outline the criteria for legitimate realisation.

	CRITERIA
PEAK SEASON	
Persons per trip 70% of 60 = 42 ∴ Adults = 21 ∴ Children = 21	Decide to calculate 70% of 60 Calculate 42 correctly Calculate adults correctly Calculate children correctly
Cost per trip: R 950 Income per trip: 100(21) + 50(21) = R3150 Profit per trip : R2200 Profit for peak season = 6 x 193 x 2200 = R 2 547 600,00	To calculate the income/trip To calculate the profit/trip To calculate profit for season Correctly calculating profit/peak
OFF-PEAK SEASON	
Persons per trip 30% of 60 = 18 ∴ Adults = 7 ∴ Children = 11	Decide to calculate 30% of 60 Calculate 18 correctly Calculate adults correctly Calculate children correctly
Cost per trip: R 950 Income per trip: 100(7) + 50(11) = R1250 Profit per trip : R300 Profit for off-peak season = 3 x 37 x 300 = R 33 300,00	To calculate the income/trip To calculate the profit /trip To calculate profit for season Correctly calculating profit/peak
Total profit for year = R 2 580 900 = R 2, 58m	How to find the total profit Correctly calculate the profit/year
Cost of new boat = R 3,5m Profit = R 2,5m Needed = R 1,0 m	How to determine the shortfall Correctly calculate the shortfall
Possible answers for decision: Sell old boat for R 0,5m and pay R 0,5m over next year OR Wait till next year to accumulate enough cash – price of boat might increase, etc.	Reasoning and decision-making Making a decision Some calculations to back up decision-making

Figure 6.4 The teacher's guide's solutions to Activity two

The nature of this activity places it in Dowling's (1998) "public domain". The activity has very low levels of discursive saturation which indicates that the realisation of the regulating principles of the activity is hidden in the discourse.

For Dowling, activities displaying low level of discursive saturation (DS^-) are regulated by implicit principles and as such, lead to highly particularised or, as Bernstein would put it, context dependent utterances. Because of this context dependency, the forms of realisation produced by students under these type of activities, can only be interpreted in the context of their immediate production i.e. it does not lead to ever higher levels of abstraction and no generalised mathematical statements can be generated from these utterances. The two types of discursive saturation, high (DS^+) and low (DS^-), construct different positions in mathematical activities. Those activities displaying DS^+ tend to be of an esoteric nature and lead to the creation of subject positions which ultimately "apprentices" students into the principles of the mathematical discourse. Activity two, being of a DS^- nature, objectifies the student's position and leads to a dependent position which denies access to the regulative principles of mathematics. Therefore, Dowling (1998) argues that an activity, like activity two, with its public domain positioning, makes possible only an objectified position.

It is clear from the privileged solutions in the teacher's guide that solutions generated from the context are not totally dependent on the context i.e. real life considerations are muted and only operate at the decision making stage of the activity. Furthermore, the solutions do not require a consideration of real life contingencies in the same manner as those in the Cooper (1998) studies. The activity itself hinges on the ability of students to separate the given data in a meaningful way in order to generate solutions on a piece meal basis. Recognition rules reside in the students' ability to identify the basic arithmetical operations required to perform the necessary transformations to realize legitimate texts.

6.4 Conclusion

The two activities display different levels of discursive saturation, and, according to Dowling, should position students differently towards the mathematical principles at the textual level. This, along with the social background of the students, indicates that the embedded contexts of these activities constitute a particular form of mathematics that attempts to engage students' interest at a rather contrived level. The mathematics itself is forced onto the context in activity one, and, in activity two, the paucity of mathematical objects has the capacity to render the generation of solutions either hopeless or meaningless depending on the students' mathematical ability. The constitution of the mathematics in these activities, if read against the privileged solutions of the teacher's guide, do not provide much scope for solutions generated by means of individual methods. Furthermore, the activities do not require iteration back to the context from which they were produced in order to take their contexts into account. The only place where contextual considerations was possible arose through a poor semantic use i.e. interchanging runway and airfield. This may have implications for the legitimacy of the concerns raised by Cooper and Dunne (1998) with regard to the social class of the students and their orientation to the mathematical objects embedded in the contexts. The overall constitution of the mathematics is at a basic level as well as at a level that requires a firm curricular foundation in appropriate mathematical objects at grade 9 level.

Chapter 7

ANALYSIS OF PEDAGOGY

In this chapter I shall consider what is constituted as mathematics when it is contextually embedded and recontextualised by a pedagogic modality. In particular, I shall concentrate on how the pedagogic methodology constitutes mathematical meaning insofar as the CTA activities are concerned and what type of student texts it enables. The analysis will draw broadly on Bernstein's concepts of framing and rules of recognition and realisation. More specifically, analysis of the lessons will be done using Davis' (2007) ground model by focusing in each lesson on how evaluative events constitute mathematics. Arising from the textual analysis of chapter 6, the pedagogic analysis will attempt to unravel the complexities that occur when the type of mathematics presented in the CTA activities (ostensibly set as assessment tasks) are presented as formal mathematics lessons to the selected group of students with different levels of mathematical ability, but similar class backgrounds.

7.1 Overview

Mrs WA started teaching in 1969 and is beginning to reach the end of her teaching career after more than four decades as a mathematics teacher. It is fair to say that she has seen many changes in her life-time, but she considers none as dramatic as those introduced in 1994 by one of the former Ministers of Education, Sibusiso Bhengu. Mrs WA started her teaching career at Hope Park and for most of her career taught in a privileged white only education department. She characterises her own style of teaching as traditional and continues to subscribe to the methodology that she employed during her formative years in education. This is evident in the classroom arrangement we see in the video-taped lessons. The traditional school desks, most of which have been around since 1910 and have been kept in excellent condition, are arranged in rows with the students facing a chalkboard at the front of the class. However, the desks are coupled so that there are three rows of two desks pushed together so that a pair of students is able to sit quite close to each other and this facilitates pair work (the closest thing to group work in Mrs WA's class). Throughout the observed lessons Mrs WA spends most of the lessons in front of the class from which she directs proceedings and presents her lessons. There is an absence of modern electronic technology – the closest being an overhead projector which she does not use during either of the observed lessons. All her written work is presented on the chalkboard. The students selected to participate in these lessons sit in alphabetical order as directed by the teacher. They too have minimal technology available – a calculator and basic stationery are all that is required.

7.2 Framing

Bernstein (1975) refers to framing as the “pedagogical relationship” between the teacher and the students being taught.

> *Thus frame refers to the degree of control teacher and pupil possess over the selection, organization, pacing and timing of the knowledge transmitted and received in the pedagogical relationship.* (Ibid, p.89)

Therefore, the framing of the pedagogic encounter refers to the degree of control the teacher exercises in how the material in the CTA activities are to be presented, how the content of these activities are to be organized and at what pace the appropriate knowledge is to be transmitted. Framing, in Bernsteinian terms includes the “selection” of the topics to be taught. However, as the CTA activities have a predetermined content, they possess an internally motivated selection of which the mediation of the meaning of the mathematical objects and operations contained in the activities are at the discretion of the teacher. In as much as the students have had no say over the selection of the mathematical content to be learnt, Mrs WA’s modality seems to be prescribed by the mathematical objects and operations made available by the text. She was also not privy to the rules of realization as spelt out in the privileged solutions in the teacher’s guide.

The tables below gives a break-down of the time spent on non-teaching, teaching and students working on their own in each of the activities:

ACTIVITY ONE	TIME SPENT	% TIME
Instructions/ reading	6 minutes	13%
Teaching/explanations	23 minutes	50
Students working on problems	17 minutes	37%
TOTAL TIME FOR LESSON	46 minutes	

ACTIVITY TWO	TIME SPENT	% TIME
Instructions/ reading	5 minutes	12%
Teaching/explanations	17 minutes	40%
Students working on problems	20 minutes	48%
TOTAL TIME FOR LESSON	42 minutes	

Table 7.1 A breakdown of time spent on teaching and non-teaching activities in the two lessons

The data indicate that in both lessons activity was strongly centred on the teacher and as such, strong framing is evident. It is not quite clear from the data or the lesson observations whether the strong framing is a result of the general modality of the teacher or of a form of control that emanates from the evaluative nature of the CTA activities, i.e. that solutions had to be generated under controlled conditions.

7.3 The lessons

To reiterate, Mrs WA was not privy to the privileged solutions contained in the teacher's guide and, therefore, her pedagogic interpretation of both the activities are entirely her own. The lesson for each activity is closely aligned to the form of the activity. Mrs WA's lessons are structured to follow the internal logic of the activities and she leads students by means of generic recontextualisations as well as using the activity's questions as a springboard for generating students' solutions.

In lesson one she commences with a literal interpretation of "activity". For her the notion of activity implies a physical and mental action:

> *I've got an activity for you and it means you must be active. You mustn't all sit motionless.*

Her statement signals her expectation of how the ideal student would be required to participate in her classroom and in generating the solutions to the activity.. The seating arrangement of the classroom enables students to work in pairs, but this social arrangement does not imply that the lesson will be student centred. Student involvement is mainly at the level of teacher prompting. At the beginning of the lessons a student is asked to read the instructions and later a choral response is asked for when all of the students respond to various prompts from the teacher. At the level of the classroom activity, these forms of participation mythologogise mathematical activity and creates the illusion of egalitarianism insofar as mathematical ability is concerned i.e. it claims that all of these students are equally capable of doing mathematics.

7.3.1 Privileged criteria

The evaluative events for activity one are directly linked to the sequenced questions in the activity and the criteria focus on how mathematical objects and operations contained in each question are treated by the pedagogy. In activity two the lack of sequenced questioning and minimally distinguishable mathematical objects mean that evaluative events can only be identified in terms of the arbitrary pedagogic criteria chosen by the teacher. These criteria give an indication of how mathematical objects and operations are to be regarded and operated upon.

7.3.1.1 Criteria that focus on associations

The evaluative nature of the CTA activities as assessment tools has consequences for the pedagogic approach that the teacher adopts. She is particularly concerned with generating explanations that would enable students to produce legitimate texts. As such, the alignment of her pedagogic modality with the outline of activities is ostensibly linked to the mathematical objects listed in the activities. Association is, therefore, defined in terms of how explicitly and implicitly available mathematical objects are associated with a procedure or algorithm to effect a solution.

In activity one the mathematical objects appear more explicitly than in activity two and as a consequence, the teacher draws on associated objects in order to facilitate recognition and draw out realisation that would lead to the production of legitimate texts. Consider the following exchange between her and the group of students:

WA OK, and they are talking about . . . in the beginning they talk about . . . What are they asking us to find with regard to the runway? So, we want to find an expression for the . . . what?

S4 Area

WA Area, right so we need to find how we are going to get the area? Now what exactly is area? How do we work out area?

ALL Length times breadth.

WA (Writes on board “Area = . . .) Why do you say straight away length times breadth? What sort of area is that?

S1 A rectangle.

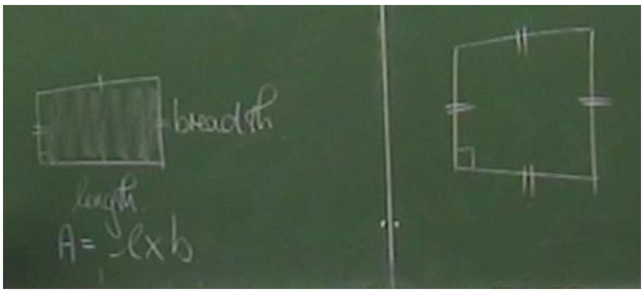
WA A rectangle. Why do we choose a rectangle.

S2 Because the width and length are different.

WA OK, and that would be a rectangle. A rectangle usually looks something like that (draws a rectangle on the board) And a rectangle . . . Where do we get that name from? (Draws a ninety degree symbol inside rectangle) . . . because there is a right angle. And the lines are equally far apart which we refer to as . . .

ALL Parallel

WA This we refer to as the length and this as the breadth. (Write words next to rectangle) And then we say area is length times breadth. What if we had all the sides the same? With a right angle and . . . (Draws a square marking all sides equal) What shape is that?



(Extract from Activity One, lines 23 - 33)

Mrs WA draws a comparison between two similar rectangles, namely the rectangle and the square, but specifies the shape by the different dimensions given in the text. She also specifies a rectangle by referring to one of its properties, namely, the right angle. Her associations exclude the possibility that the shape may be referenced as a parallelogram and as such, she does not delve into a discussion on the principles governing the area of geometric figures. The idea of area is mainly conveyed by shading the enclosed area of the rectangle.

In another example she discusses question 2 which deals with the ratio of the width of the runway to its length and uses a few examples associated with the concept of ratio.

WA	If I work with area it is going to be metres squared or kilometre squared . . . because I am working with the same thing. Right, now let's go to no. 2 We'll come back to no. 1 later when you answer the question. If the width and the length of the airfield is in the ratio of 1:20 . . . What is a ratio?
S3	When you compare . . .
WA	Compare! Yes, that's nice. So they say the length . . . Which one comes first?
S3	Width
WA	Width . . . we must be careful . . . compared to the length. Now, be very careful to read the question. Width compared to length. And it is in the ratio 1:20 (writes on board width: length and underneath 1:20) So, we started off with the runway and said this width and this is length. What have I forgotten of?

(Extract from Lesson One, lines 47 -51)

The concept of ratio is also likened to the notion of a comparison (although this idea came from one of the students, but validated by the teacher). Therefore, the teacher privileges the associative ideas contained in the text to derive at very specific conclusions that converge directly with the solutions given in the teacher's guide.

7.3.1.2 Criteria that focus on procedures and conventions

Mrs WA's lessons draw extensively on procedures without much explanation of the underlying principles that give rise to them or that stabilises a particular criterion. Her discussion on area adheres almost entirely to the activity's text while minimally referencing other geometric figures and in so doing avoids the principles driving the concept of area or the properties of real numbers that inhere in those forms of operations.

WA	This we refer to as the length and this as the breadth. (Write words next to rectangle) And then we say area is length times breadth. What if we had all the sides the same? with a right angle and . . . (Draws a square marking all sides equal) what shape is that?
ALL	Square.
WA	A square and we still use l x b but we know that l and b are exactly the . . .
ALL	Same
WA	But a circle would be a completely different formula, but we chose that because of the shape of the . . .
ALL	The runway.
WA	Right, because that is what we are asked to find. Now remember with area . . . area is all of that . . (Shades interior of rectangle) That is why we take l x b. So, we are quite sure what area means. So we want an expression for the area of one runway if the width is x metres and the length y metres. So, let's say the length is 10m and the width is 2 metres and we want for find the area. It would be . . . 10 x 2 . . . what will that give you?
ALL	20
WA	But, now what about this (points to m)
ALL	Metres squared.
WA	Right, you are all sure about that now? If I worked with centimeters and said cm x cm, what would that give?
S3	Centimetres squared
WA	And km x km?
S3	Kilometres squared

(Extract from Lesson 1, lines 33-46)

Mrs WA's explanation of area is strongly procedural as it concentrates mainly on the position the mathematical objects occupy in the text and the map. No attempt is made to explicate these objects beyond the context of the activity and pedagogy is centred on producing a convergence of solutions that would lead to realisation of legitimate texts that are commensurate with the demands of the activity. Similarly, Mrs WA's reference to the "metres squared" masks the rule of exponents $a^m \times a^n = a^{m+n}$ which is transferable beyond any context. In Eco's (1979) terms this constitutes the construction of a closed text as students are taught in a manner that is wholly dependent on the text (and consequently context dependent) and no generalisable principles are conveyed to them.

7.3.1.3 Criteria that focus on disaggregation

Disaggregation refers to the process whereby the component part of an operation or sets of objects are separated to be dealt with piece-meal. In much the same way that it could be argued that the entire Activity One is a disaggregation of the central problem, namely, to calculate the area of the Robben Island runways by sequencing the problem, the focus here is on how both activities are broken up into smaller calculations/procedures to effect solutions. This process is to some degree a result of the teacher's construction of the model student and a judgment of what she is capable of. In the picture below Mrs WA uses disaggregation by separating the two runways in order to assist students to calculate the total surface area of the runways in question 3 of Activity One.

WA In other words they are exactly crossing each other at ninety degrees like this . . . (draws runways crossing on board). Find an expression for the total area of the 2 runways. I want you to look at these 2 runways and tell me what you notice about them. Are they 2 separate runways? Do they overlap? What do you notice?

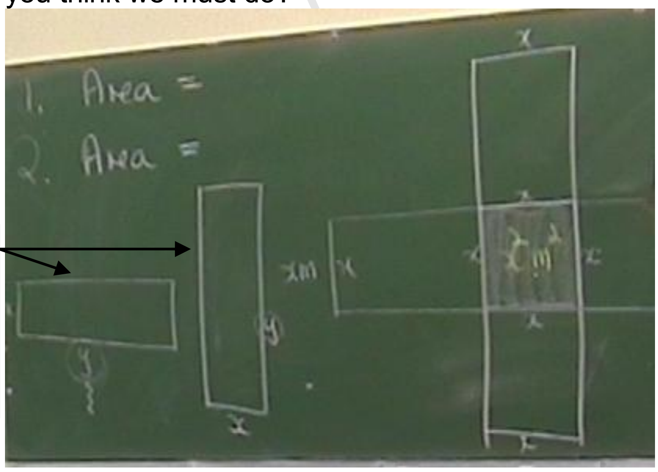
S1 They overlap.

WA Where do they overlap?

S1 In the middle.

WA In the middle. So this part here . . . (Shades overlapping runways on board) . . . belongs to both runways. So what is going to happen if you work out area. We work out the area of the one and we work out the area of the other. (Draws 2 separate rectangles representing runways on the board) . . . so that's 2 separate runways. This one is x metres and this y metres. This y we've spoken about is slightly different. Because . . . this x here. . . remember what would it be. For instance, p and 20p. If it is a it is 20a. So if we have the x and the y there it is going to be different. We also have an x and y here and the y is also going to be different. Then we've got this area and this area and we must put them together. (Points to overlap on drawing). What do you notice about these (emphasises overlap) What do you think we must do?

The runways have been separated to facilitate the calculation of the surface area



S1 We must subtract that area.

WA OK, we must do subtraction because it is happening in both runways. Now how do you think we are going to do that?

S2 Use x because the width is the same on both sides.

WA OK, use x

S2 x times x is x squared

WA This side here is x and this side is the same. What sort of figure is the overlap?

ALL Square

WA What is the area of that?

ALL L x l

(Extract from Lesson One, lines 89 -102)

The criteria involving disaggregation is an attempt to regulate the students' textual productions so that solutions generated have a close fit with what might count for legitimate texts in the teacher's guide. In the above evaluative event (question 3, Activity one), the sub-events reside at the level of the disaggregated objects, but are iterative of the general problem, namely calculation of area.

7.4 Findings

Table 8.1 below summarises the number of times the three criteria evident in the evaluative events and sub-events are evident in the two lessons observed. Notice that in the explanation of a sub-event the teacher moved between the three focus criteria several times as she attempts to elaborate on an evaluative event and its component sub-events.

ACTIVITY			CRITERIA		
	Evaluative events	Sub-events	Focus on associations	Focus on procedures	Focus on disaggregation
One	4	8	9	9	7
Two	1	6	1	6	6

Table 7.2 A breakdown of the criteria privileged in the two lessons.

In keeping with Davis and Johnson (2007: 121-136) this study affirms that a large proportion of teaching time was spent on the exposition of the assessment activities. However, the overtly evaluative nature of these activities means that they are not intended to serve a pedagogic function and as such, are internally motivated to stimulate a very particular response, namely, to generate legitimate solutions that are commensurate with the solutions contained in the teacher's guide. As a direct consequence, the lessons observed had very little evidence of exposure to the underlying principles and axioms of the mathematical objects and operations contained in the activities.

The lessons are strongly focused on the use of associated ideas that emanate from the text, map and data and these are used as stimuli to regulate how students' texts would be produced. This is not to say that any pedagogic recontextualisation should be exhaustive, but rather that the types of associations drawn are itself regulated by the image of the model student that the teacher may have preconceived.

The lessons are also strongly focused on procedure and disaggregation. Discussion of these is clustered together because disaggregation occurs in the lessons for the express purpose of generating simplified procedures. In the discussion of the textual analysis of the activities I argued that in Activity One the sequenced format of the activity constructs the competence of the student at the level of the text. In activity two the paucity of mathematical objects constructs the competence of the student at the level of the resources individual students bring to the task. By disaggregating and proceduralising, the teacher constructs the competence of the students by the resources she makes available through her recontextualisation of the mathematical objects and operations and as such, she may be placing a limit on access to more generalisable principles and higher levels of abstraction. This highly context-dependent pedagogic modality has implications when considered against Eco's open and closed texts and the construction of the model reader. It modalises an orientation to the specific objects contained in the activities and prevents descriptions of principles beyond that which the text and the modality makes available. The result is that students, irrespective of resource availability, are confined to the realisation of closed texts.

Chapter 8

ANALYSIS OF STUDENT TEXTS AND INTERVIEWS

In this chapter I shall consider what is constituted as mathematics by the untutored group of students who completed the assessment activities under test conditions. The texts will be analysed in conjunction with the interviews using Dowling's (1998) domains of practice. These two sets of data are grouped as the interviews provide elaboration on the texts the students produced. As such, one is able to form some conclusions about the resources students were able to draw on to generate solutions as well as make a judgment about the nature of the criteria they used to solve the problems.

8.1 Textual responses

Table 8.1 below summarises how the nine students in the test group responded to each questions in Activity One and to Activity Two as a whole. The response categories are: no response offered, incorrect response, partially correct but incomplete (in other words some of the teacher's guide criteria have been fulfilled), and correct response. As this study is mainly concerned with what is constituted as mathematics when presented in contextually embedded forms, question 5 of activity one is omitted from the summary as the question is non-mathematical in nature.

	NO RESPONSES	INCORRECT RESPONSES	PARTIALLY CORRECT RESPONSES	CORRECT RESPONSES
ACTIVITY 1				
QUESTION 1	0	1	0	8
QUESTION 2	2	7	0	0
QUESTION 3	3	6	0	0
QUESTION 4	3	6	0	0
ACTIVITY 2	0	5	4	0

Table 8.1 How students in the test group responded to the two activities.

All the students' textual productions for Activity One can be classified as per Dowling's esoteric domain while those for Activity Two show evidence of both the esoteric and public domains independently or in combination. Students favoured some form of transformation of the embedded content into the specialized signs and symbols of the mathematical topic perceived.

There seems to be an instinctive understanding that school mathematics privileges a specific textual format which may be produced or reproduced without necessarily understanding the mathematical principles of the topic referenced. This is generally achieved through procedural techniques such as algorithms or formulae. While the topics presented in both the activities are covered in the grade 9 curriculum, two of the students felt ill-equipped to deal with the tasks and expresses the need to have prepared for the test.

LA I did not know what was happening.
RE What did you not know? You did not understand the language or the maths?
LA I did not understand the language . . . the way they put the questions.
JA I agree. I had to study for this and I did not expect this would come in.
(Interview with LA & JA, lines 3 – 6)

The interview extract with LA and JA is indicative of students who were ill-equipped with mathematical resources to deal with Activity One and, in particular, the contextual embedded content made it difficult for these students to identify the mathematical objects and operations required to navigate through the task. JA expresses an expectation of what the test should have looked like and while the issue of her expectations is not probed, there is the sense that what she did expect was more discursive representation of the mathematical topic dealt with. Furthermore, the sequencing of the problem fails to assist poorly resourced students. If they fail to produce legitimate solutions at the start of the activity, the rest the activity also fails.

High and low discursive saturation (DS^+ and DS^-) indicates the degree of the discursive in the texts and verbal elaborations. For Dowling (1998: 30) texts displaying generalising strategies are DS^+ while fragmenting strategies are DS^-. Both of these are useful indicators of performances and are reflected in the relationship between novices and adepts. However, this part of Dowling's work is limited when applied as analytical tools for student texts since students tend to economise their performances in line with their perceived demands of the test. The belief that all texts exhibiting esoteric competence can be independent of their material product does not reflect school mathematics practices as it does not make allowance for rehearsed procedures that produce stable solutions during the evaluative process. Dowling's suggestion that pedagogising action presents students with simplified practices which at best can be described as DS^- makes the assumption that students are able to produce texts that are generalizing by virtue of the esoteric competence inherent in the procedures. This view of texts in the esoteric domain fails to recognize the way in which standardised assessment instruments like the CTA activities, encourage a hybridization of the principled, regulating rules that school mathematics produces. When the assessment of mathematics is recontextualised and embedded in contexts, students privilege the perceived expected outcomes in order to "play by the rules" (my emphasis). To return to JA's comment above "I had to study for this and I did not expect this would come in" says something about the way she regards mathematics as well as her expectation of how the rules of school mathematics would be adhered to.

8.2 Analysis of student texts and interviews

Eight out of the nine students in the test group successfully answered Question One of Activity One. Slight variations occur in some of the texts but, none which violate the principles of the objects. For example, two answers below are regarded as correct because the question does not stipulate that answers had to be simplified:

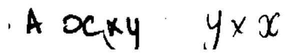


Figure 8.1 Two examples of correct but unsimplified responses to Question One of Activity One. Note how these students construct esoteric texts following a procedure based on calculating the area of a rectangle.

After Question One none of the students were able to produce correct solutions even though some of them attempted to bring their limited resources to the tasks to produce texts that conform to the esoteric nature expected in school mathematics. In Activity One and two specific criteria can be detected in the student texts as well as their verbal elaborations.

8.2.1 Criteria that focuses on associations

In much the same way that the teacher used associations from the activities' texts and from recontextualisations, students used cues from the activities' texts to constitute mathematical solutions. At the heart of the associations students make are the linguistic cues which indicate mathematical objects or operations. When asked how they knew that the area of the runway was a rectangle, students in three of the four interviews responded by pointing to the words "length" and "breadth" or that these dimensions had differential values, x and y.

R.E	How did you work out how to do no.1?
A.J.	I said the area is x times y.
RE	How did you know to use x times y?
AJ	They give it in the first question.
RE	But how did you know to multiply x and y?
AJ & JM	That's how you work out the area.
RE	Did you look at the picture at the back? (Refers to map of Robben Island)
AJ & JM	Yes
RE	What did the picture tell you? Where's the runway? Point it out to me.
BOTH	There it is. (pointing to the runway on the map)
RE	What did you conclude about the runway?
AJ	I did not really conclude anything.
JM	It looks like a cross.
RE	You saw it as a cross? Did you see anything else?
JM	Umm …. Not actually.
AJ	There's 4 quadrants.

(Interview with AJ & JM, lines 1 -16)

Even though the map indicating the runways on Robben Island is meant to reinforce the notion of a rectangle, AJ and JM, like the other students failed to take its significance into account. In the interview they misread the shape as a "cross" and as "quadrants". Therefore, the pictorial association proved to be superfluous as the students attempted to operate at the discursive level of the text by associating the mathematical objects with a familiar procedure.

A further example of application of associative criteria can be found in the textual production of JA in response to activity one, question 2.

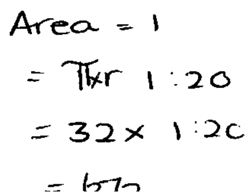


Figure 8.2 JA's response to activity two, question two. An example of how mathematically unrelated objects are associated with objects recognized in the activity text.

JA invokes the use of π and *r* by associating the "ratio" with the letter r and as well as the rational form of π which is often taught to be $\frac{22}{7}$. Some distant stirring of a formula for involving π and *r* is also introduced so that what comes to be constituted as mathematics at least adheres to its esoteric constructions which seems familiar in the context of school mathematics and at the level of an assessment task. Thus, what is produced by the student is partly a product of the perception of what the teacher expects of the model mathematics student and less what the student actually knows.

8.2.2 Criteria that focuses on disaggregation and reduction

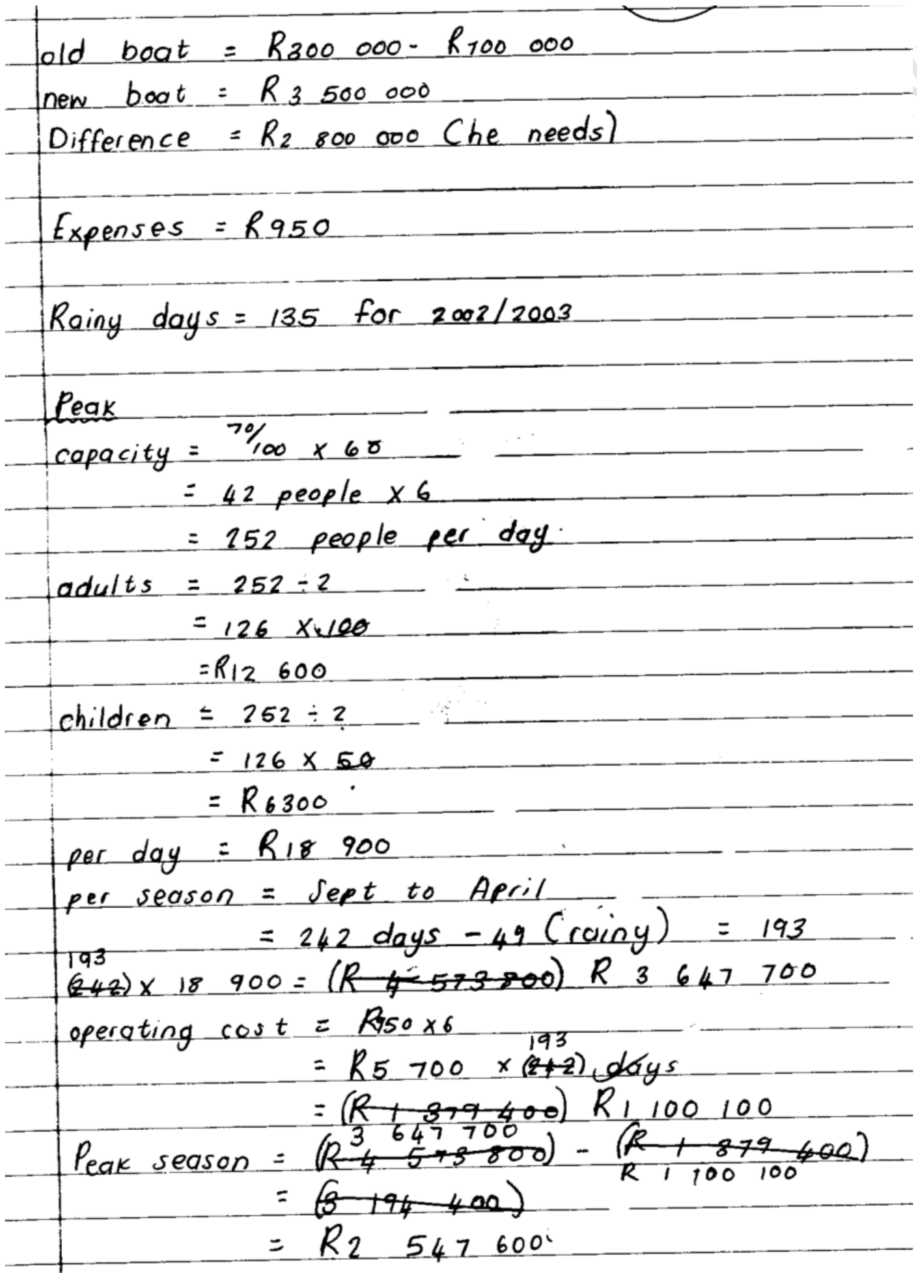
These criteria are observable mainly in Activity Two in which the mathematical objects were more obscure and had to be separated from the context in order to reduce the problem to manageable chunks. It is partly as a result of this obscurity that half the students in the group were unable to produce any coherent responses. Not surprisingly this group of students corresponds with the middle to low performing students. Students who managed partial solutions did so by disaggregating the mathematical objects from the context following some form of intuitive logic and reducing these to mathematical operations which essentially consist of the additive properties of the set of natural numbers. AJ explains how she got started on the problem and progressed using disaggregation and reduction.

RE	So you assumed for the rest of the problem that he would get R700 000?
AJ	Yes . . . And then I said the new boat would cost R3,5 million, so I got the difference which is R2,8 million.
RE	Where will he get the rest of the money from?
AJ	From trips that he would do. Selling would add to it and the trips to Robben Island.
RE	Let's look at these trips. Were the calculations on these a simple

		exercise?
	AJ	No.
	RE	Why not?
	AJ	You first have to work out the days it rained and the peak season . . . How many days there was in peak season. Then you had to work his expenses per trip and . . .

(Interview with AJ & JM, lines 103 – 110)

AJ's initial assumption revolves around a decision about the value of the old boat and in doing so she has decided on the vital point of entry into the problem. In addition, by settling on this course AJ discards contextual or real-life considerations and reduces the problem to a series of arithmetic algorithms. This is consistent with the solutions in the teacher's guide which also privileges the reduction of the problem to a series of disaggregated procedures. Part of AJ's solution is reproduced below:


old boat = R300 000 - R100 000
new boat = R 3 500 000
Difference = R2 800 000 (he needs)

Expenses = R950

Rainy days = 135 for 2002/2003

Peak
capacity = 70/100 x 60
= 42 people x 6
= 252 people per day
adults = 252 ÷ 2
= 126 x 100
= R12 600
children = 252 ÷ 2
= 126 x 50
= R6300
per day = R18 900
per season = Sept to April
= 242 days - 49 (rainy) = 193
193 (242) x 18 900 = (R 4 573 800) R 3 647 700
operating cost = R950 x 6
= R5 700 x 193 (242) days
= (R 1 379 400) R1 100 100
Peak season = (R 3 647 700 4 573 800) - (R 1 379 400) R 1 100 100
= (3 194 400)
= R2 547 600

Figure 8.3 AJ's part solution to activity two illustrates how contextual as well as integrative concerns are discarded.

In contrast, students with less mathematical resources at their disposal had difficulty with finding an entry point into the activity and as such, disaggregation and reduction occurred at an inappropriate point. This invariably led to the collapse at the textual level. In part the collapse can be attributed to possible misreading of the text. AB's solution (partly reproduced) is an illustration of a possible misreading. She has taken the minimum resale value of the old boat mentioned in the activity and added it to its maximum value.

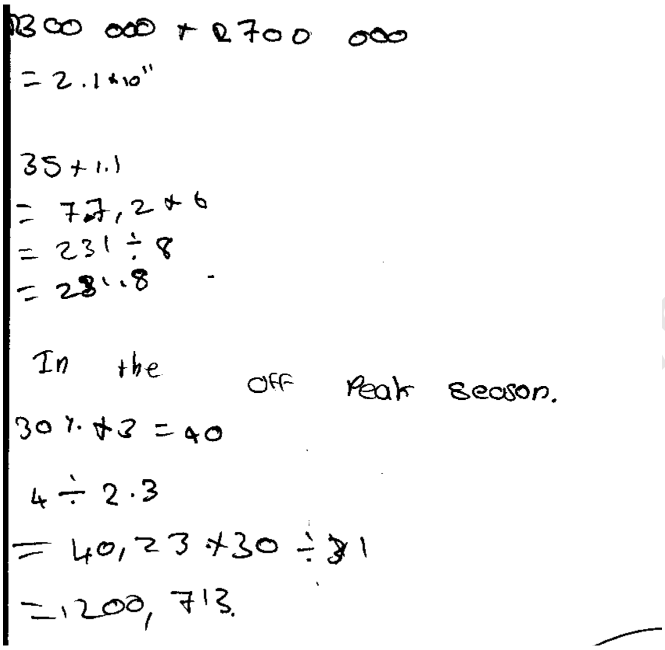


Figure 8.4 AB's solution attempts to produce an esoteric response, but fails to recognize any legitimate operations to do so.

8.2.3 Criteria that focuses on procedures

At the heart of all the various forms of mathematical solutions produced by this group of students is their reliance on some sort of procedure, algorithm or formula. When these are not readily available or obvious, students produce aberrant texts that are wildly divergent from any principled mathematical meaning. Procedures, algorithms and formulae are stabilised forms of the principles or axioms that underscore them. However, their stability is reliant on the extent to which they are coherently and appropriately applied. Furthermore, they begin to lose their stability once adjustments (major or minor) are imposed on them or when misreading is applied to them. KA's solution to Question 2 of Activity One illustrates such a misreading. She interchanges the measurements for width and length to produce the following:

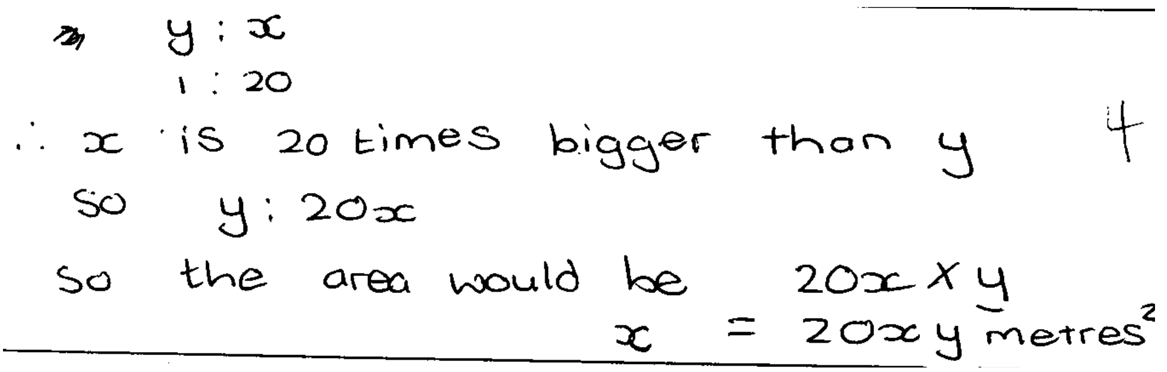


Figure 8.5 KA's solution to question 2 of activity two illustrates a recognition of mathematical operators, but fails to realize legitimate text.

In the interview with KA she is able to elaborate a coherent procedure that is absent from her text with the aid of my prompts which also privileges a procedural approach to the problem.

RE	Now, ratios. The width and the length of the runway is in the ratio 1:20. Express your answer in terms of x. What's wrong with your answer KA?
KA	I did not write it in terms of x.
RE	What have you got?
KA	There wasn't supposed to be a y.
RE	That's right. You've got a y in there so you haven't simplified right to the end. What should you have tried to get all your numbers as? Which letter should you have tried to eliminate?
KA	The y.
RE	How would you do that?
KA	x =20y . . . x over y equal 1 over 20.
RE	And if you solve for y?
ZM	y = 20x

(Interview with KA & ZM, lines 29 – 38)

KA's algorithm reveals a disjuncture between the written text and her verbal description. At the level of the discursive, KA's written text lacks the substance that she is able to provide in the verbal description. While her verbal description lacks an understanding of the properties of rational numbers, her procedure is stable.

8.3 Conclusion

In the analysis of the student texts and interviews I have attempted to unveil the strategies and criteria students used in order to construct meaningful mathematical statements. The students' inability to produce legitimate texts largely points to their lack of resources to cope with the two activities. The contextual embedding of the mathematical objects created further complications for most of the students in the group, irrespective of the ability. This points to two possible interpretations:

- The students have been schooled in a form of mathematics that privileges procedures drawn from associations and reduced to stable forms of algorithms that converge meaning at the level of the mathematical topic

- The activities, being assessment tools, condition, by means of associative criteria, responses that privilege the reduction of the content to mathematical procedures that are perceived to be required (in other words, students attempt to mimic the solutions contained in the teacher's guide.

In Dowling's terms, the algorithmic mode privileges formula and procedures and is positioned as an authorial mode for the production of legitimate texts. The production of legitimate texts not only means that an understanding of the underlying principles of the mathematical objects is not necessarily in place, but the inability of this group to produce coherent procedures in most cases, points to limitations in contextual/real-life mathematics. It also points to limitations in the particular orientation of the school mathematics to which this group have been exposed. Therefore, the limited access to mathematics discourse seemed to encourage the recruitment of fragments of mathematics knowledge on an ad hoc basis in order to produce esoteric texts that conform to those recognised as possessing the form of school mathematics.

Chapter 9

CONCLUSION AND RECOMMENDATIONS

In this chapter I present a summary of the findings of this study as well as point out some its limitations. This dissertation set out to explore what is constituted as mathematics when it is contextually embedded. The study sought, firstly, to explore this question by textually analyzing two activities in which mathematical content is presented in contextualized forms. Secondly, it explored what comes to be constituted as mathematics when these activities are presented to two groups of students where one group is tutored and the other group is not. Thirdly, these questions are explored taking into consideration the specific social class of the students.

9.1 Connecting theory and methodology

The analysis of the components of this study along with Bernstein's pedagogic device provides a framework for engaging in a systematic analysis of the school mathematics curriculum. However, the development of such an analytic tool is not the goal of this study. The pedagogic recontextualisations of mathematics and the texts that students produce are reflections of what is privileged as mathematics by RNCSM. As such students' productions are reflections of pedagogic practices and the way in which they are regulated. The constraints on the agents of pedagogic practices like the teacher, the students, and the curriculum practices dictate the forms of assessment and ultimately, what will count as legitimate texts.

Several issues concerning mathematics pedagogy, assessment and contextually embedded mathematics have been highlighted in the literature review. These issues point to the continuing debate around what constitutes school mathematics and, in particular, the debate around the usefulness of using real-life mathematics as a means of recruiting the familiar interests of the students and thereby easing their entry into the study of mathematical concepts. The literature review can be organised into two groups. The first group focuses on analyses of students' mathematical knowledge and emphasises what students ought to know. The second group considers the way in which the social organisation of mathematics impacts on the construction of mathematical knowledge. Research studies tended to ignore the way in which textual analysis could be used to show how mathematics knowledge was constructed and how textual analysis reflected the social organisation of school mathematics practices in general instead of conceptual mathematical knowledge. The production of school knowledge is a contested arena and socio-political interests are served by legitimating that which ultimately gets counted as school mathematics. It is for this reason that the theories of Basil Bernstein are invoked to provide the general theoretical basis for this study. The research around the social organisation of mathematics provides the background to the distinction drawn between the constitution of propositional mathematical knowledge and other forms mathematical knowledge, some of which include strong orientations to procedural mathematics as well as context dependent mathematical knowledge which has its roots in the applicability of the subject in the real world.

The literature review contributes to the adoption of the theoretical orientation which underpins this study. Bernstein's pedagogic device forms the theoretical foundation and provides the ideas needed to understand how the CTA activities are functions of a particular form of curriculum design. In essence, the strongly constructivist principles of the RNCSM and the OBE approach necessitate a student-centred approach and, therefore, legitimates the contextually embedded approach to mathematics. The CTA activities are assessment tools which stabilise Bernstein's pedagogic device at the level of evaluation.

Furthermore, theoretical concepts are also used to explore the dichotomous relationship between mathematics as a set of principles and laws and mathematics as a set of rehearsed procedures. Dowling's (1998) work considers the sociology of mathematics education through his focus on a semiotic analysis of mathematics textbooks. He explores the sociology of interpersonal relations and class relations and how different forms of mathematics are distributed to different ability and class groups. Central to his work is the notion that a mathematical gaze appropriates all that it surveys and all descriptions are judged in terms of the extent to which the gaze sanctions discursive practices instead of everyday real-life practices. Mathematics brings principles of recognition and realisation which enables it to project a mathematics gaze on non-mathematical practices and redescribe them in terms of mathematics. Therefore, Dowling's four domains of practice enable a description of mathematical texts by considering the strength of its signifiers and signifieds in terms of content and expression. These domains provide a way of reading mathematical textual productions as material instances by considering how they are structured and, therefore, what form of mathematical practice they privilege. Therefore, the study considered the structure and presentation of the CTA activities as well as the written and verbal productions of the students to examine what is constituted as mathematics in each of these instances. In addition, the work of Cooper and Dunne (1998) elaborates Dowling's theoretical propositions about the type of mathematics that is distributed to students from different social class backgrounds. Their work, like the work of Holland (1981) and Bernstein (1996), draws on the antecedent work of Luria (1976) to explore the contextual considerations and resources that students from different social class come to rely on when confronted with contextually embedded mathematics.

In order to analyse what is constituted as mathematics in pedagogy, different theoretical antecedents are used. The work of Davis (2007), although still formative in its formulations, uses sophisticated semiotic principles to construct the notion of ground. In essence, it considers the Hegelian concept to establish what is grounded in pedagogic treatment of any mathematical content. In particular, Davis' work explores the types of criteria that are privileged in the pedagogy of mathematics in 5 working class schools in the Western Cape.

All of the analytical components of the study are underpinned by Eco's theory of open and closed texts. The use of the idea of open and closed texts is only formative in their application and limited to considering the extent to which textual productions and utterances have access to or make available the underlying principles of the mathematical objects contained in the CTA activities.

9.2 Discussion of analysis: constraints and limitations

The analysis of students' texts is specific to the small sample group of students described in the study and applicable to their specific social class background. The most striking feature of the students' texts is their poor ability to engage with mathematical knowledge and discourse and yet, this does not prevent them from producing discursive texts. The criteria privileged in their textual productions indicate that the students in the test group were quite capable of putting together a string of discursive esoteric texts in spite of the fact that generally they were at a loss in terms of what the activities required of them. The esoteric domain productions appear to have high discursive saturation since they have appropriated the mathematising gaze from the esoteric domain. However, the location of the students' texts in the esoteric domain cannot make assumptions about the expertise of the students as authors of the texts. The accompanying verbal elaborations indicate that where esoteric descriptions are rendered, they prove to reside exclusively at the level of the procedural. Furthermore, the text signals that the interaction required something beyond mere (re)production of algorithms and that the context of the situation has to be considered.

The limitation of the criteria identified is inherent in this specific study and need to be explored further across a wider sample of students to establish validity and greater reliability. However, whether further criteria are identified is not the central concern. The central concern of any set of criteria is to identify what is privileged in mathematical operations and, ultimately, what is constituted as mathematics by these criteria.

The pedagogic actions focus almost exclusively on procedures, algorithms and formulae by drawing out the associated objects from the text and separating them segmentally. The absence of principled mathematics seems to suggest that this is a trend in school mathematics. That which is privileged as mathematical knowledge seems to be something other than the properties of the objects and operations privileged in the pedagogy. The tendency of school mathematics practices to produce texts that are insular and topic-dependent points to a segmental rather than integrated orientation in mathematical knowledge. This study makes no claims about the pervasiveness of the forms of pedagogic practice described above. Instead, further empirical evidence needs to be drawn from a larger sample of schools of different social class groupings in order to make more reliable generalised claims.

A major limitation of this study is in the nature of the CTA activities used to gather textual data. These activities are not meant to function as pedagogic instruments and as such they signal the need for a specific form of response. Ultimately, successful performance in these tasks is reliant on the students' ability to (re)produce texts that closely resemble the privileged solutions in the teacher's guide. Recognition and realisation, therefore, reside at the level of mimicking the types of texts that students think is expected of them.

Even though this study draws on the type of work Cooper and Dunne (1998) conducted, there is a difficulty in drawing parallels with their findings. The CTA activities make claims to recruiting real life mathematics, but do so in a contrived manner. Mathematical topics are superimposed on the contexts in such a manner that the mathematical objects and operations are given precedence over the context. The contexts become almost superfluous. This is further evidenced in the teacher's guide which itself privileges a series of mathematical procedures and algorithms as legitimate texts. Therefore, Cooper and Dunne's (1998) claims that working class children, with their limited resources, privilege contextual objects in the problem and bring personal experiences to bear on their solution processes, cannot be upheld by this study. Dowling's domains of practice also prove to be limited to superficial descriptions of the texts produced only in terms of its discursivity. It is able to tell us very little about the extent of students' access to the underlying principles of the mathematical objects.

9.3 Curriculum and practice

The components of the analysis seem to suggest that the curriculum shapes mathematical activity in such a way as to impoverish the construction of mathematics knowledge. This is in contrast to one of the aims of mathematics outlined in the RNCSM which is to:

> develop deep conceptual understandings in order to make sense of Mathematics
>
> (DoE, 2002: 5)

The lessons observed as well the mathematical productions of the students in the research group indicate that this goal is not being realized. Furthermore, the OBE principle of student centred learning is nowhere evident in the lessons observed. Students as authors of texts emerge as a consequence of the interaction situations of classroom practice.

The main concerns of this study were the way in which students made mathematical sense of texts which contained contextually embedded mathematical content. Part of this concern was the way in which students acquired mathematics in what appears to be isolated segments and yet, some of them are able to construct a form of mathematics that appears to be highly proficient in terms of its dicursivity. Out of this emerges the consideration of the way in which the cumulative organisation of school mathematics (re)produces students with an underdeveloped view of mathematical discourse.

A further concern arises from the RNCSM's claims about the "kind of learner that is envisaged" (pg 3) and its contradiction to the research findings. The RNCSM claims to develop mathematical knowledge, skills and values in students that will enable them to:

> display critical and insightful reasoning and interpretive and communicative skills when dealing with mathematical and contextualized problems (DoE, 2002: 5)

At the level of the assessment tasks very little substance exists that requires "critical and insightful reasoning". Indeed, the teacher's guide gives testimony to this by reducing the activities to two sets of solutions in which algorithmic reductions are privileged. As such, students themselves, instead of being pedagogised into the RNCSM's lofty goals, are objectified by the mathematics curriculum by the way in which the evaluative rule is applied. Pedagogic action and evaluation seem to develop a focus on competence and performance rather than mathematics subjects. Furthermore, relations in pedagogic practices are also influenced by the discursive regulations of the teacher as well as the instruments of evaluation. Lastly, the RNCSM's claim to enabling students to "transfer mathematical knowledge and skills between Learning Areas and within Mathematics" (DoE, 2002: 5) appears to be partly true. There is no evidence in the CTA activities of anything outside of mathematics being referenced except in a purely token manner which in any case references no mathematical knowledge at all (refer to Activity One, question 5). The principle of integration appears to operate largely at the level of the mathematical outcomes and assessment standards outlined in the RNCSM. This form of integration in the assessment process is questionable in the light of the segmental, disaggregated forms of pedagogy and textual productions that emerge from this study.

In conclusion, student texts are indicators of pedagogic practices and, in particular, they highlight the type of criteria favoured in the constitution of mathematics. As such, they also give an indication of how it is that students recruit and are recruited by mathematics knowledge. Assessment plays a significant role in skewing the way in which this occurs. The way in which students position themselves as authors of texts is related to the textual strategies and criteria which underpin their production. The segmental strategies employed are cause for concern because they prevent the development of abstracting skills necessary for higher levels of mathematics. The pervasiveness of these strategies in teaching and learning appear to be linked not only to external assessment processes, but also to perceptions of the model learner of mathematics along social class delineations.

BIBLIOGRAPHY

Abercrombie,N. & Urry,J. (1983) *Capital. Labour and the Middle Class*. London: Allen & Unwin.

Anyon, J. (1981) Social class and school knowledge, *Curriculum Inquiry*, 11: 3–42.

Apple, M. (1992). The text and cultural politics. *Educational Researcher*, 21, 7.

Apple, M. (1995a) Education, culture and class power: Basil Bernstein and the neo-Marxist sociology of education. In Sadovnik, A (ed) *Knowledge and Pedagogy: the Sociology of Basil Bernstein*. Norwood, NJ: Ablex.

Apple, M. (1995b) Taking power seriously: new directions in equity in mathematics education and beyond. In Sadovnik, W.G., Fennema, E., and Adajian, L.B. (eds) *New Directions for Equity in Mathematics Education*. Cambridge: Cambridge University Press.

Atkinson, P. (1985) *Language, Structure and Reproduction*. London, Methuen.

Bernstein, B. (1971) *Class, codes and control 1: theoretical studies towards a sociology of language.* London, Routledge & Kegan Paul.

Bernstein, B. (1975) *Class, codes and control, Volume 3: Towards a theory of educational transmissions*. London: Routledge & Kegan Paul.

Bernstein, B. (1977) *Class, Codes and Control, Vol. 3*. London: Routledge and Kegan Paul.

Bernstein, B. (1986) On pedagogic discourse. In J. G. Richardson (Ed.), *Handbook of theory andresearch for sociology of education*. New York: Greenwood Press.

Bernstein, B. (1990) *The structuring of pedagogic discourse: class, codes and control 4*. London, Routledge.

Bernstein, B. (1996) *Pedagogy, symbolic control and identity: theory, research, critique* London, Taylor & Francis.

Bernstein, B. (2000) *Pedagogy, symbolic control and identity: Theory, research, critique*. (revised edition).London: Rowman & Littlefield. (1st edition 1996, London: Taylor and Francis).

Bernstein, B. (2001) From pedagogies to knowledges. In A. Morais, I. Neves, B. Davies & H. Daniels, (Eds.), *Towards a sociology of pedagogy: The contribution of Basil Bernstein to research*. New York: Peter Lang.

Bourdieu, P. (1974) The school as a conservative force: Scholastic and cultural inequalities. In Egglestone, J. (Ed). *Contemporary research in the sociology of education*. London: Methuen, pp. 33-46.

Bourdieu, P. (1982) The school as a conservative force: Scholastic and cultural inequalities. In E. Bredo and W. Feinberg (eds.), *Knowledge and Values in Social and Educational Research*. Temple University Press, Philadelphia, 391407.

Bourdieu, P. (1983) 'The forms of capital', in J. G. Richardson (ed.), *Handbook of Theory and Research for the Sociology of Education*, Greenwood press, New York.

Bourdieu, P. (1985) The social space and genesis of groups, *Social Science Information*, 24(2), 195-220.

Bourdieu, P. (1986) *Distinction: a social critique of the judgement of taste* (London, Routledge Kegan Paul).

Bourdieu, P. (1987) What makes a social class? On the theoretical and practical existence of groups. *Berkley Journal of Sociology*, 32

Bourdieu, P. (1992) 'Thinking about limits', *Theory, Culture and Society* 9, 37-49.

Chomsky, N. (1965) *Aspects of the theory of syntax*, The MIT Press, Cambridge, Ma.

Cooper, B. (1992) Testing national curriculum mathematics: some critical comments on the treatment of 'real' contexts for mathematics, *The Curriculum Journal*, 3(3), 231–243

Cooper, B. & Dunne, M. (2000) *Assessing children's Mathematical knowledge: Social Class, Sex and Problem Solving*. Oxford University Press

Davis, Z. (2001) Measure for measure: evaluative judgement in school mathematics pedagogic texts. *Pythagoras*. 56, 2 - 11.

Davis, Z. (2003) Bernstein avec Lacan: Jouissance and pedagogic discourse. In Trueit D, Doll JR, Wang H & Pinar W (eds) *The internationalization of curriculum studies: Selected proceedings from the LSU Conference 2000*. New York: Peter Lang Publishers.

Davis, Z., & Johnson, Y. (2007) Failing by example: initial remarks on the constitution of Mathematics in schools, with special reference to the teaching and learning of Mathematics in five secondary schools. In M. Setati, N. Chitera, & A. Essien (Ed.), *Proceedings of the 13th Annual Congress of the Association for Mathematics Education in South Africa: the Beauty, Utility and Applicability of Mathematics*. 1, pp. 121-136. Uplands College, Mpumalanga: AMESA.

Department of Education. (2002) *Revised National Curriculum Statement. Mathematics. Grade R – 9* . Pretoria: Department of Education.

Dowling, P.C., (1993) 'A Language for the Sociological Description of Pedagogic Texts with Particular Reference to the Secondary School Mathematics Scheme SMP 11-16, PhD thesis, Institute of Education, University of London.

Dowling, P. C. (1998) *The Sociology of Mathematics Education: Mathematical Myths/Pedagogic Texts*. London: Falmer.

Dowling, P.C. (2009) *Sociology as Method: Departures from the forensics of culture, text and knowledge*. Rotterdam: Sense.

Duberman, L. (1976) *Social Inequality: Class and Caste in America*, Philadelphia, PA: J.B. Lippincott.

Eco, U. (1979) *The Role of the Reader. Explorations in the Semiotics of Texts*. Bloomington: Indiana University Press.

Ensor, P. and Galant, J. (2005) Knowledge and pedagogy: Sociological research in mathematics education in South Africa from the early 1990s. In R. Vithal, J. Adler and C. Keitel (eds), *Researching mathematics education in South Africa: Perspectives, practices and possibilities*: 281-306. Pretoria: Human Sciences Research Council.

Freudenthal, H. (1973) *Mathematics as an Educational Task*. Dordrecht: Reidel.

Gravemeijer, K.P.E. (1994) *Developing realistic mathematics education*. Utrecht: Cdß Press

Gravemeijer, K., & Doorman, M. (1999) Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39,* 111-129.

Goldthorpe,J.H. (1987) *Social Mobility and Class Structure in Modern Britain*,. Oxford: Clarendon Press (revised edition, first published 1980)

Holland, J. (1981) Social class and changes in orientations to meaning. *Sociology*, 15: pp.1–18.

Jaffer, S. (2010) Investigating the use of procedural and iconic resources in the pedagogising of mathematics in five secondary schools. In M.D. de Villiers (ed.) *Proceedings of the 16th Annual National Congress of the Association for Mathematics Education of South Africa, March 2010*, UKZN, pp. 299-310.

Jary, D., & Jary, J. (1991) *The HarperCollins dictionary of sociology*. New York: HarperCollins.

Kohn, M.L. (1983) 'On the transmission of values in the family: a preliminary formulation', *Research in Sociology of Education and Socialization*, 4: 1–12.

Lareau, A. (2000) *Home advantage: Social class and parental intervention in elementary school*. Lanham, MD: Rowman & Littlefield Publishers, Inc.

Luria, A.R. (1976) *Cognitive development*. Cambridge, MA: Harvard University Press. (First edition 1974).

Mattson, E & Harley, K. (2003) Teacher identities and strategic mimicry in the policy/practice gap. In K.Lewin, M. Samuel & Y. Sayed (Eds.), *Changing patterns of teacher education in South Africa* (pp. 284-305). Cape Town, South Africa: Heinemann.

McNeil, L. M. (2000) Creating new inequalities. *Phi Delta Kappan* 81(10), 728-735.

Moschkovich, J.N. (2002) Bringing together workplace and academic mathematical practices during classroom assessments. In M.E. Brenner & J.N. Moschovich (Eds.), *Everyday and academic mathematics in the classroom.* Reston, VA: National Council of Teachers of Mathematics.

Muller, J. (2000) Reclaiming Knowledge: Social Theory, Curriculum and Education Policy. New York, London: Falmer.

Muller, J. (2000) *Reclaiming Knowledge: Social Theory, Curriculum and Education Policy*. London: RoutledgeFalmer

Moore, R., & Muller, J. (1999) The discourse of "voice" and the problem of knowledge and identity in the sociology of education. *British Journal of Sociology of Education*, 20 (2), 189–206.

Muller, J & Taylot, N (1995) "School and everyday life: Knowledge sacred and profane" in *Social Epistimology* 9, 3: 257 - 275

Rogan, J. M. & Grayson, D. (2003) Towards a Theory of Curriculum Implementation with Particular Reference to Science Education in Developing Countries. *International Journal of Science Education* 25, 10

Saljo R., & Wyndhamn. J. (1993) Solving everyday problems in the formal setting. An emiprical study of the school as context for thought. In S. Chaiklin. & J. Lave (Eds.), *Understanding pracfice. Perspectives on acfivity and contevt*. Cambridge: Cambridge University Press.

Schoenfeld, A. (1985) *Mathematical problem Solving*. Orlando: Academic Press.

Schoenfeld, A. (1994) What do we know about curricula? *Journal of Mathematical Behaviour*, 13, 1, 55 – 80.

Singh, Parlo (2001) "Pedagogic Discourses and Student Resistance", Morais, Ana, Neves, Isabel, Davies, Brian and Daniels, Harry, *Towards a Sociology of Pedagogy. The Contribution of Basil Bernstein to Research,* New York: Peter Lang,

Tall, D. (1991) *Advanced Mathematical Thinking*, (D. Tall ed.), Kluwer, Dordrecht.

Taylor, N., Muller, J. and Vinjevold, P. (2003) *Getting schools working*. Cape Town: Pearson Education South Africa.

TIMSS (2003) Released Items: Eighth Grade Mathematics. Retrieved 2 November 2006, from http://timss.bc.edu/PDF/T03_released_M8.pdf

Treffers, A. (1987) Three *dimensions: A model of goal and theory description in mathematics instruction—The Wiskobas Project*. Dordrecht, The Netherlands: Reidel.

Verschaffel, L., De Corte, E., & Lasure, S. (1994) Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction*, 4.

Verschaffel, L., Greer, B. & De Corte, E. (2000) *Making sense of word problems* (Lisse, Swets & Zeitlinger).

Walkerdine, V (1988) *The mastery of reason: Cognitive development and the production of rationality*. London, Routledge

Walkerdine, V (1990) Diference, cognition and mathematics education. *For the Learning of Mathematics* 10, 3: 51 - 56

Weber, M. (1964) *The Theory of Social and Economic Organization*. New York: The Free Press.

Young, M. (2002) 'Durkheim, Vygotsky and the curriculum', paper prepared for the 5th Congress of the International Society for Cultural Research and Activity Theory, Amsterdam, Vrije Universiteit, 18–22 June.

ANNEXURE 1

STUDENT QUESTIONNAIRE:

GRADE 9

STUDENT QUESTIONNAIRE: Grade 9

General Directions

In this questionnaire, you will find questions about yourself. Some questions ask for facts while other questions ask for your opinions.

Read each question carefully and respond as accurately as possible. You may ask for help if you do not understand something or are not sure how to respond.

Some of the questions will be followed by a few possible choices indicated with a circle
with a number in it. For these questions, shade in the circle with the response of your
choice as shown in Examples 1, 2, and 3.

Example 1

Do you go to school?

*Fill in **one** circle only*

Yes -- ●

No -- ②

Example 2

How often do you do these things?

*Fill in **one** circle for each line*

	Every day	At least once a week	Once or twice a month	A few times a year	Never
a) I listen to music	①	②	●	④	⑤
b) I talk with my friends	●	②	③	④	⑤
c) I play sports	①	●	③	④	⑤

Read each question carefully, and pick the answer you think is best. Fill in the circle

next to or below your answer. If you decide to change your answer, erase your
first
answer and then fill in the circle next to or under your new answer. Ask for help
if you
do not understand something or are not sure how to answer.

Thank you for your time, effort, and thought in completing this questionnaire.

About You

1 When were you born?

A. Fill in the circle next to the year you were born

B. Fill in the circle next to the month you were born

Year	Month
① 1988	① January
② 1989	② February
③ 1990	③ March
④ 1991	④ April
⑤ 1992	⑤ May
⑥ 1993	⑥ June
⑦ 1994	⑦ July
⑧ 1995	⑧ August
⑨ Other	⑨ September
	⑩ October
	⑪ November
	⑫ December

2 How often do you speak English at home?

*Fill in **one** circle only*

Always --- ①

Almost always ------------------------------------ ②

Sometimes --------------------------------------- ③

Never -- ④

3 **About how many books are there in your home? (Do not count magazines, newspapers, or your school books.)**

*Fill in **one** circle only*

None or very few
(0-10 books) --------------------------------------- ①

Enough to fill one shelf
(11-25 books) -------------------------------------- ②

Enough to fill one bookcase
(26-100 books) ------------------------------------ ③

Enough to fill two bookcases
(101-200 books) ---------------------------------- ④

Enough to fill three or more bookcases
(more than 200 books) -------------------------- ⑤

4 **Do you have any of these items at your home?**

*Fill in **one** circle for each line*

Yes **No**

a) Calculator ---------------------------------- ① ------ ②

b) Computer (do not include PlayStation®, GameCube®, XBox®, or other TV/video game computers) --- ① ------ ②

c) Study desk/table for your use ----------- ① ------ ②

d) Dictionary ---------------------------------- ① ------ ②

5 A. What is the highest level of education completed by your mother (or stepmother or female guardian)?

*Fill in **one** circle only*

Did not finish school or did not go to school --- ①

Finished school at Grade 8 level ------------------------- ②

Finished school at Grade 9 level ------------------------- ③

Finished school at Grade 10 level ----------------------- ④

Finished school at Grade 11 level ----------------------- ⑤

Matriculated --- ⑥

Has a university degree ------------------------------------ ⑦

B. What is the highest level of education completed by your father (or stepfather or male guardian)?

*Fill in **one** circle only*

Did not finish school or did not go to school --- ①

Finished school at Grade 8 level ------------------------- ②

Finished school at Grade 9 level ------------------------- ③

Finished school at Grade 10 level ----------------------- ④

Finished school at Grade 11 level ----------------------- ⑤

Matriculated --- ⑥

Has a university degree ------------------------------------ ⑦

6 How far in school do you expect to go?

*Fill in **one** circle only*

Finish school at Grade 9 -- ①

Finish school at Grade 10 -- ②

Finish school at Grade 11 -- ③

Matriculate -- ④

Go to university --- ⑤

Go to a technical institute -- ⑥

MATHEMATICS IN SCHOOL

7 How much do you agree with these statements about learning mathematics?

*Fill in **one** circle for each line*

Agree a lot	Agree a little	Disagree a little	Disagree a lot

		↓	↓	↓	↓
a)	I usually do well in mathematics	①	②	③	④
b)	I would like to take more mathematics in school	①	②	③	④
c)	Mathematics is more difficult for me than for many of my classmates	①	②	③	④
d)	I enjoy learning mathematics	①	②	③	④
e)	Sometimes, when I do not initially understand a new topic in mathematics, I know that I will never really understand it	①	②	③	④
f)	Mathematics is not one of my strengths	①	②	③	④
g)	I learn things quickly in mathematics	①	②	③	④

8 How much do you agree with these statements about mathematics?

*Fill in **one** circle for each line*

		Agree a lot	Agree a little	Disagree a little	Disagree a lot
		↓	↓	↓	↓
a)	I think learning mathematics will help me in my daily life	①	②	③	④
b)	I need mathematics to learn other school subjects	①	②	③	④
c)	I need to do well in mathematics to get into the <university> of my choice	①	②	③	④
d)	I would like a job that involved using mathematics	①	②	③	④
e)	I need to do well in mathematics to get the job I want	①	②	③	④

9 How often do you do these things in your mathematics lessons?

*Fill in **one** circle for each line*

Every or

		almost every lesson Never	About half the lessons	Some	lessons
a)	We practice adding, subtracting, multiplying, and dividing without using a calculator	①	②	③	④
b)	We work on fractions and decimals	①	②	③	④
c)	We interpret data in tables, charts, or graphs	①	②	③	④
d)	We write equations and functions to represent relationships	①	②	③	④
e)	We work together in small groups	①	②	③	④
f)	We relate what we are learning in mathematics to our daily lives	①	②	③	④
g)	We explain our answers	①	②	③	④
h)	We decide on our own procedures for solving complex problems	①	②	③	④
i)	We review our homework	①	②	③	④
j)	We listen to the teacher give a lecture-style presentation	①	②	③	④
k)	We work problems on our own	①	②	③	④
l)	We begin our homework in class	①	②	③	④
m)	We have a quiz or test	①	②	③	④
n)	We use calculators	①	②	③	④

SCIENCE IN SCHOOL

10 How much do you agree with these statements about learning science?

*Fill in **one** circle for each line*

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) I usually do well in science	①	②	③	④
b) I would like to take more science in school	①	②	③	④
c) Science is more difficult for me than for many of my classmates	①	②	③	④
d) I enjoy learning science	①	②	③	④
e) Sometimes, when I do not initially understand a new topic in science, I know that I will never really understand it	①	②	③	④
f) Science is not one of my strengths	①	②	③	④
g) I learn things quickly in science	①	②	③	④

11 How much do you agree with these statements about science?

*Fill in **one** circle for each line*

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) I think learning science will help me in my daily life	①	②	③	④
b) I need science to learn other school subjects	①	②	③	④
c) I need to do well in science to get into the <university> of my choice	①	②	③	④
d) I would like a job that involved using science	①	②	③	④
e) I need to do well in science to get the job I want	①	②	③	④

12 How often do you do these things in your science lessons?

*Fill in **one** circle for each line*

	Every or almost every lesson	About half the lessons	Some lessons	Never
a) We watch the teacher demonstrate an experiment or investigation	①	②	③	④
b) We formulate hypotheses or predictions to be tested	①	②	③	④
c) We design or plan an experiment or investigation	①	②	③	④
d) We conduct an experiment or investigation	①	②	③	④
e) We work in small groups on an experiment or investigation	①	②	③	④
f) We write explanations about what was observed and why it happened	①	②	③	④
g) We study the impact of technology on society	①	②	③	④
h) We relate what we are learning in science to our daily lives	①	②	③	④
i) We present our work to the class	①	②	③	④
j) We review our homework	①	②	③	④
k) We listen to the teacher give a lecture-style presentation	①	②	③	④
l) We work problems on our own	①	②	③	④
m) We begin our homework in class	①	②	③	④
n) We have a quiz or test	①	②	③	④

COMPUTERS

13 A. Do you ever use a computer? (Do not include PlayStation®, GameCube®, XBox®, or other TV/video game computers).

	Yes	No
*Fill in **one** circle only*	①	②

*If **No**, please go to question **14***

B. Where do you use a computer?

*Fill in **one** circle for each line*

		Yes	No
a)	At home	①	②
b)	At school	①	②
c)	At a library	①	②
d)	At a friend's home	①	②
e)	At an Internet café	①	②
f)	Elsewhere	①	②

C. How often do you do these things with a computer?

*Fill in **one** circle for each line*

At least Once or A
few
Every once a twice a
times
day week month a
year Never

a) I look up ideas and information for mathematics	①	②	③	④	⑤
b) I look up ideas and information for science	①	②	③	④	⑤
c) I write reports for school	①	②	③	④	⑤
d) I process and analyze data	①	②	③	④	⑤

YOUR SCHOOL

14 How much do you agree with these statements about your school?

*Fill in **one** circle for each line*

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) I like being in school	①	②	③	④
b) I think that students in my school try to do their best	①	②	③	④
c) I think that teachers in my school care about the students	①	②	③	④
d) I think that teachers in my school want students to do their best	①	②	③	④

15 <u>In school</u>, did any of these things happen during the last month?

*Fill in **one** circle for each line*

Yes **No**

a) Something of mine was stolen	①	②
b) I was hit or hurt by other student(s) (e.g., shoving, hitting, kicking)	①	②
c) I was made to do things I didn't want to do by other students	①	②
d) I was made fun of or called names	①	②
e) I was left out of activities by other students	①	②

THINGS YOU DO OUTSIDE OF SCHOOL

16 On a normal school day, how much time do you spend before or after school doing each of these things?

*Fill in **one** circle for each line*

More than 2

Less

but less 4 or

No than

1-2 than more

time 1 hour hours 4

hours hours

		↓	↓	↓	↓	↓
a)	I watch television and videos	①	②	③	④	⑤
b)	I play computer games	①	②	③	④	⑤
c)	I play or talk with friends	①	②	③	④	⑤
d)	I do jobs at home	①	②	③	④	⑤
e)	I work at a paid job	①	②	③	④	⑤
f)	I play sports	①	②	③	④	⑤
g)	I read a book for enjoyment	①	②	③	④	⑤
h)	I use the internet	①	②	③	④	⑤
i)	I do homework	①	②	③	④	⑤

17 A. During this school year, how often have you had extra lessons or tutoring in mathematics that is not part of your regular class?

*Fill in **one** circle only*

Every or almost every day ---------------------- ①

Once or twice a week ---------------------------- ②

Sometimes -- ③

Never or almost never --------------------------- ④

B. During this school year, how often have you had extra lessons or tutoring in science that is not part of your regular class?

*Fill in **one** circle only*

Every or almost every day ---------------------- ①

Once or twice a week ---------------------------- ②

Sometimes -- ③

Never or almost never --------------------------- ④

18 A. How often does your teacher give you homework in mathematics?

*Fill in **one** circle only*

Every day -- ①

3 or 4 times a week ------------------------------ ②

1 or 2 times a week ------------------------------ ③

Less than once a week -------------------------- ④

Never -- ⑤

*If **Never**, please go to question* **19**

B. When your teacher gives you mathematics homework, about how many minutes are you usually given?

*Fill in **one** circle only*

Fewer than 15 minutes ------------------------ ①

15–30 minutes ------------------------------------ ②

31–60 minutes ------------------------------------ ③

61–90 minutes ------------------------------------ ④

More than 90 minutes --------------------------- ⑤

---

19 A. How often does your teacher give you homework in science?

*Fill in **one** circle only*

Every day -- ①

3 or 4 times a week ------------------------------ ②

1 or 2 times a week ------------------------------ ③

Less than once a week -------------------------- ④

Never -- ⑤

*If **Never**, please go to question* **20**

B. When your teacher gives you science homework, about how many minutes are you usually given?

*Fill in **one** circle only*

Fewer than 15 minutes ------------------------ ①

15–30 minutes ----------------------------------- ②

31–60 minutes ----------------------------------- ③

61–90 minutes ----------------------------------- ④

More than 90 minutes -------------------------- ⑤

MORE ABOUT YOU

20 Including yourself, how many people live in your home?

*Fill in **one** circle only*

2 --- ②

3 --- ③

4 --- ④

5 --- ⑤

6 --- ⑥

7 --- ⑦

8 or more -- ⑧

21 A. Was your mother (or stepmother or female guardian) born in this country?

Yes

No

*Fill in **one** circle only* ------------------- ① ------ ②

B. Was your father (or stepfather or male guardian) born in this country?

Yes

No

*Fill in **one** circle only* ------------------- ① ------ ②

22 A. Were you born in this country?

Yes **No**

*Fill in **one** circle only* ------------------- ① ------ ②

*If **Yes**, you have completed the questionnaire*

B. If you were not born in this country, how old were you when you came to South Africa?

*Fill in **one** circle only*

Older than 10 years old ------------------------- ①

5 to 10 years old ---------------------------------- ②

Younger than 5 years old ----------------------- ③

Thank You
for completing
this questionnaire

ANNEXURE 2

SUMMARY OF DATA GATHERED FROM THE STUDENT QUESTIONNAIRES

PART 1: PERSONAL INFORMATION

1	AGE			
	14	**15**	**16**	**16+**
	1	55	11	2

2	ENGLISH AT HOME			
	ALWAYS	**ALMOST ALWAYS**	**SOMETIMES**	**NEVER**
	29	27	13	0

3	NO BOOKS AT HOME				
	0-10	**11-25**	**26-100**	**101-200**	**200+**
	17	26	21	3	2

4	RESOURCE ITEMS			
	CALCULATOR	**COMPUTER**	**DESK**	**DICTIONARY**
	68	52	56	67

5A	MOTHER EDUCATION						
	OUT	**Gr 8**	**Gr 9**	**Gr 10**	**Gr 11**	**Gr 12**	**UNIV**
	1	2	5	9	9	30	11

5B	FATHER EDUCATION						
	OUT	**8**	**9**	**10**	**11**	**12**	**UNIV**
	5	6	4	5	4	30	9

6	OWN EXPECTATIONS					
	9	**10**	**11**	**12**	**UNIV**	**TECH**
	0	1	0	2	62	4

20	NUMBER PEOPLE IN HOME					
2	**3**	**4**	**5**	**6**	**7**	**8+**
6	13	21	17	5	2	4

PART 2: SCHOOL INFORMATION

7	MATHS IN SCHOOL				
		AGREE		DISAGREE	
		LOT	LITTLE	LITTLE	LOT
A	DO WELL	13	27	18	10
B	LIKE TO DO MORE	37	19	8	4
C	IS DIFFICULT	7	22	15	22
D	ENJOY	27	23	10	8
E	NEVER UNDERSTAND	7	12	22	27
F	NOT A STRENGTH	26	16	12	15
G	LEARN QUICKLY	13	32	15	9

8	VIEWS ON MATHS				
		AGREE		DISAGREE	
		LOT	LITTLE	LITTLE	LOT
A	HELP DAILY LIFE	47	19	2	1
B	NEED FOR OTHER SUBJ	30	27	9	2
C	GET INTO UNIV	60	5	3	1
D	JOB INVOLVING MATHS	17	25	14	13
E	TO GET A JOB I WANT	37	18	14	0

9	MATHS PRACTICE				
		EVERY LESSON	HALF THE LESSONS	SOME LESSONS	NEVER
A	PRACTICE BASIC OPERATIONS	16	14	31	6
B	FRACTIONS & DECIMALS	17	12	38	2
C	INTERPRET GRAPHS, TABLES,ETC	22	19	29	0
D	WRITE EQUATIONS, FUNCTIONS	24	15	29	0
E	SMALL GROUPS	3	5	37	23
F	RELATE TO DAILY LIVES	18	12	27	7
G	EXPLAIN ANSWERS	33	14	17	3
H	DECIDE OWN PROCEDURES	5	21	31	6
I	REVIEW HOMEWORK	35	11	19	2
J	LECTURE STYLE TEACHING	23	20	21	3
K	WORK PROBLEMS ON OWN	12	31	20	4
L	BEGIN HOMEWORK IN CLASS	16	13	35	3
M	HAVE TESTS	2	13	44	7
N	USE CALCULATORS	50	8	9	1

13A	COMPUTER USE	YES	NO
		66	3

13B	COMPUTERS WHERE?	HOME	SCHOOL	LIBRARY	FRIEND	INTERNET CAFÉ	OTHER
		48	57	17	24	25	25

13C	DOING THINGS ON COMPUTER					
		EVERY	WEEKLY	2 MONTH	FEW YEAR	0
A	SEARCH INFO MATHS	1	16	18	9	21
B	SEARCH INFO SCIENCE	2	16	26	9	13
C	WRITE REPORTS	2	12	15	11	25
D	PROCESS & ANALYSE	4	15	12	15	19

14	YOUR SCHOOL	AGREE		DISAGREE	
		LOT	LITTLE	LOT	LITTLE
A	LIKE SCHOOL	21	30	8	10
B	STUDENTS TRY BEST	22	34	11	2
C	TEACHERS CARE	33	22	11	1
D	TEACHERS WANT BEST	56	10	1	

15	HAPPENINGS AT SCHOOL		
		YES	NO
A	SOMETHING STOLEN	29	40
B	HIT OR HURT	5	64
C	DO THINGS DIDN'T WANT TO	1	67
D	MADE FUN OF	9	60
E	LEFT OUT OF ACTIVITIES	6	63

16	OUTSIDE SCHOOL	0	<1	1-2	2 & 4	4+
A	TV & VIDEOS	6	18	25	11	9
B	COMPUTER GAMES	48	12	8		
C	PLAY/TALK FRIENDS	12	19	16	14	6
D	JOBS AT HOME	13	34	12	6	3
E	PAID JOB	61	3	2	1	2
F	PLAY SPORTS	34	10	12	8	4
G	READ BOOKS	9	28	22	3	5
H	USE INTERNET	12	28	19	7	3
I	DO HOMEWORK		12	29	16	11

17A	EXTRA MATHS	EVERY	1/2 WEEK	SOMETIMES	NEVER
		1	28	22	17

18A	HOMEWORK GIVEN	EVERY	3/4WEEK	1/2 WEEK	<WEEK	NEVER
		28	33	7	1	

ANNEXURE 3

TRANSCRIPTS OF TWO OBSERVED LESSONS

LESSON 1: ACTIVITY 1

WA	Afternoon girls. Pretend that you know me and that we are not going to be like sheep and enjoy today. Nobody is going to be nervous about anything. I've got an activity for you and it means you must be active. You mustn't all sit motionless. Take these papers and write your names on them. We have 40 minutes. Now look at this. It says activity 1 which means there will be another. You don't need any paper. First of all, look at the instructions. . . Who's going to read for us?	1
S1	(Reads instructions)	2
WA	All answers must be done! MUST BE DONE. Alright?	3
S1	(Continues to read)	4
WA	Have you got that? Let's see. All calculators out . . .Ready to go?	5
S1	(Continues to read)	6
WA	Now what do they start off saying? We have Bongani and he is a maths enthusiast working for who?	7
ALL	Work for U.	8
WA	And he challenged the rest of the staff and you to solve the following problem. What's the problem? What does it say? Everybody.	9
ALL	Write an expression for the area of one runway with a width of x meters and a length of y metres.	10
WA	OK, what is it about this runway? Do we know anything about runways? Have you got a map?	11
ALL	Yes	12
WA	What is it of?	13
ALL	Robben Island	14
WA	Where's Robben Island?	15
S2	Off the coast. It's close to us . . .	16
WA	You all know where it is? And you can see it from here. It's a big island and there is a map of it. It's got roads on it and down at the bottom I can see a harbour. If I look straight up from the harbour, . . . Can you see, not very clear, but it says " Landing strip". Can you see that? At point . . . ?	17
S3	2	18
WA	And . . . ?	19
S3	1	20
WA	There is an A there also. OK, so we have a landing strip there . . . and that must be the runway we are talking about? What sort of shape is that runway and landing strip?	21
S2	A cross.	22
WA	OK, and they are talking about . . . in the beginning they talk about . . . What are they asking us to find with regard to the runway? So, we want to find an expression for the . . . what?	23
S4	Area	24
WA	Area, right so we need to find how we are going to get the area? Now what exactly is area? How do we work out area?	25
ALL	Length times breadth.	26
WA	(Writes on board "Area = . . .) Why do you say straight away length times breadth? What sort of area is that?	27
S1	A rectangle.	28
WA	A rectangle. Why do we choose a rectangle.	29
S2	Because the width and length are different.	30

WA	OK, and that would be a rectangle. A rectangle usually looks something like that (draws a rectangle on the board) And a rectangle . . . Where do we get that name from? (Draws a ninety degree symbol inside rectangle) . . . because there is a right angle. And the lines are equally far apart which we refer to as . . .	31
ALL	Parallel	32
WA	This we refer to as the length and this as the breadth. (Write words next to rectangle) And then we say area is length times breadth. What if we had all the sides the same? with a right angle and . . . (Draws a square marking all sides equal) what shape is that?	33
ALL	Square.	34
WA	A square and we still use l x b but we know that l and b are exactly the . . .	35
ALL	Same	36
WA	But a circle would be a completely different formula, but we chose that because of the shape of the . . .	37
ALL	The runway.	38
WA	Right, because that is what we are asked to find. Now remember with area . . . area is all of that . . (Shades interior of rectangle) That is why we take l x b. So, we are quite sure what area means. So we want an expression for the area of one runway if the width is x metres and the length y metres. So, let's say the length is 10m and the width is 2 metres and we want for find the area. It would be . . . 10 x 2 . . . what will that give you?	39
ALL	20	40
WA	But, now what about this (points to m)	41
ALL	Metres squared.	42
WA	Right, you are all sure about that now? If I worked with centimeters and said cm x cm, what would that give?	43
S3	Centimetres squared	44
WA	And km x km?	45
S3	Kilometres squared	46
WA	If I work with area it is going to be metres squared or kilometer squared . . . because I am working with the same thing. Right, now let's go to no. 2 We'll come back to no. 1 later when you answer the question. If the width and the length of the airfield is in the ratio of 1:20 . . . What is a ratio?	47
S3	When you compare . . .	48
WA	Compare! Yes, that's nice. So they say the length . . . Which one comes first?	49
S3	Width	50
WA	Width . . . we must be careful . . . compared to the length. Now, be very careful to read the question. Width compared to length. And it is in the ratio 1:20 (writes on board width: length and underneath 1:20) So, we started off with the runway and said this width and this is length. What have I forgotten of?	51
S1	Metres	52
WA	Can't be a runway in cm . . . it is too small for a plane. Now we say that you width and your length are in this ratio. So the length would be . . . tell me . . .	53
S3	20 . .	54
WA	Would be 20 times the width. So if you width is 1m how long would the	55

	length be?	
S1	20	56
WA	And if it was 2m?	57
S2	40m	58
WA	And if it was 6m?	59
S1	240m	60
WA	How did you get that?	61
S1	6 x 20	62
WA	Each time you are multiplying by 20 . . . Are you sure you are correct?	63
S1	No, it is 120.	64
WA	Good. Now what if I said to you that the width was a metres . . . Now?	65
S1	20a metres	66
WA	Width was p metres?	67
S1	20p metres	68
WA	And x metres?	69
S1	20x	70
WA	Because it is always 20 times whatever the width is. Now, can you answer no. 1 and 2 on your paper? Write clearly, legibly and all calculations and everything must be shown.	71
S2	Must we work out the whole area?	72
WA	The whole runway. It says, there's the runway . . . the width is x metres (draws rectangle on board) and the length is y metres. There are 2 of them, but you must work out one. So find the area of one runway and then answer no. 2. (Students are given time to work on the 2 questions while WA checks quietly what they do and offer advice) (3 minutes) Now, let's have a look at no. 1 and think about how you did it. We started off and spoke about area and what did we say the area of a rectangle would be? There is one runway and there is another. But we must deal with one at a time. So, what is the area of a rectangle everyone?	73
ALL	L x b	74
WA	Length times breadth. And if we are measuring this runway, the width would be x metres and length y metres. Then we say l x b. (Writes l x b on board) and we said be careful because if this is 2 metres and this is 5 metres and multiply them together, the answer must be . . .	75
S4	Metres	76
WA	But what else?	77
S4	Metres squared	78
WA	OK, don't forget that. It is part of it. Now, we haven't got a number here and there. So we must work out the area using the x and y. We are doing a maths activity so the x and y can stand for anything. They are variables. If we work out the area then we will have length times breadth. Don't forget if we multiply them together, what is going to happen here? (Points to m)	79
ALL	Metres squared	80
WA	Now do all have that area worked out? Now finish it off. Let's go to no. 2. We said the width compared to the length was as 1:20 so that if your width was 1m then your length would be 20m. And if your width was a metres, your length would be 20a metres. And if your width was p metres then your length would be 20p metres. And if	81

	your width was x metres, then your length would be 20x metres. Let's see what you've found? (Students work on no.2) Let's look at no. 3. Read no. 3 for us.	
S4	Reads Q3	82
WA	Again we have the runway and what do they say about them? They are . . .	83
ALL	Identical	84
WA	Perpendicular? What does that mean?	85
S2	They cross . . .	86
WA	At . . . ?	87
S2	Ninety degrees.	88
WA	In other words they are exactly crossing each other at ninety degrees like this . . . (draws runways crossing on board). Find an expression for the total area of the 2 runways. I want you to look at these 2 runways and tell me what you notice about them. Are they 2 separate runways? Do they overlap? What do you notice?	89
S1	They overlap.	90
WA	Where do they overlap?	91
S1	In the middle.	92
WA	In the middle. So this part here . . . (Shades overlapping runways on board) . . . belongs to both runways. So what is going to happen if you work out area. We work out the area of the one and we work out the area of the other. (Draws 2 separate rectangles representing runways on the board) . . . so that's 2 separate runways. This one is x metres and this y metres. This y we've spoken about is slightly different. Because . . . this x here. . . remember what would it be. For instance, p and 20p. If it is a it is 20a. So if we have the x and the y there it is going to be different. We also have an x and y here and the y is also going to be different. Then we've got this area and this area and we must put them together. (Points to overlap on drawing). What do you notice about these (emphasizes overlap) What do you think we must do?	93
S1	We must subtract that area.	94
WA	OK, we must do subtraction because it is happening in both runways. Now how do you think we are going to do that?	95
S2	Use x because the width is the same on both sides.	96
WA	OK, use x	97
S2	x times x is x squared	98
WA	This side here is x and this side is the same. What sort of figure is the overlap?	99
ALL	Square	100
WA	What is the area of that?	101
ALL	L x l	102
WA	Or l x b or side times side. So what is this going to be?	103
S5	X squared	104
WA	What's been left out?	105
S2	Metres squared	106
WA	That's right. We want to find the area of these whole 2 runways. How?	107
S1	Multiply our answer by 2.	108
WA	Which answer?	109
S1	Answer in no. 2	110

WA	We will take that area (shades individual rectangles) and that area. It is l x b.	111
S2	Add it together.	112
WA	And then?	113
S2	Subtract the square	114
WA	Because these 2 overlap and occur in both of them. Now remember instead of y, what have we got? Go back. One is a and the other is 20a. One is p and the other is 20p. One is x and the other is . . .	115
ALL	20x	116
WA	So what will we put in place of this and this. We know now what to use. So we will find this area and this area, add them together and subtract that area. We will have (*writes* $A_1 + A_2 - A_3$ *on the board*) because they are the same and subtract the middle area. (3 minutes students work on finding solutions) OK, so let's just look here. Say the one runway is a metres wide and the length would be . . .	117
S3	20a metres	118
WA	Good, if I want to work out the area of that runway, it will be l x b. Who will give me the answer?	119
S3	20a squared	120
WA	Right, so a x a = a^2, 1 x 20 = 20 and m x m = m^2. That is the one. How much will the other be?	121
S6	$20a^2m^2$	122
WA	Why? Because they are identical . . . So the area of one is (writes $20a^2m^2 + 20a^2m^2$ on board) . . . and how will I find out the area of the actual runway? What must I do?	123
S7	Subtract	124
WA	Subtract the one in the middle because it happens twice, because they overlap. So what will this be?	125
S4	X squared	126
WA	Not x. We are working with this one here.	127
S4	$20a^2m^2$	128
WA	$20a^2m^2 + 20a^2m^2 - a^2m^2$. Now I've got 20 apples + 20 apples minus 1 apple. Do that. What am I going to get here? . . . 40 . . . So it will give us . . . $39a^2m^2$. Now check your no. 3 Let's look at Q4.	129
S5	(Reads Q4)	130
WA	Right, now we find the total area. We found that the area of one and the area of the other, added them and subtracted the middle area. What were you using?	131
S6	X	132
WA	Now we have our answer coming to 97 $500m^2$. So say your answer came to $5a^2 = 125$. How will I solve it?	133
S1	Divide both sides by 5	134
WA	(Writes what students said on board) So then the 5s go away and 5 goes into 125 25 times Now we have $a^2 = 25$.	135
S1	Square root both sides	136
WA	Find the square root. But now with the square root I don't know if it is a plus or a minus (writes $a = \pm\sqrt{25}$) Now what am I going to times a minus?	137
S8	A minus	138

WA	Which is . . .?	139
S8	A plus	140
WA	What is a plus times a plus?	141
ALL	A plus	142
WA	So therefore it will be a $\pm\sqrt{25}$. So a is plus or minus 5. Usually we can tell whether one of these answers won't work. Now we are dealing with a runway and we can't say this runway will have minus 5 metres. So therefore, which will we use?	143
S5	Plus 5m	144
WA	The positive one. Now see if you can sort out the value of x now. Remember if your length was a and width 20a, what would area be? (Students work on no. 4) Remember when you square root you have more than one answer and you have to choose the correct one. The question says motivate your answer. Right, last question.	145
S7	(Reads Q5)	146
WA	What is intersect?	147
ALL	Crossing	148
WA	Right, why do they intersect rather than run parallel? Imagine if you had 2 runways like that . . . (draws parallel runways on board). Is the island quite big? You need quite a lot of space to get airborne. What else happens in Cape Town? We had it yesterday.	149
S2	The wind blew.	150
WA	Wind. Now, when planes take off, how are they going to get airborne? Is the wind important?	151
S2	The wind will help	152
WA	But, it might be quite dangerous here if (points to parallel runways) I've got one plane going here and another going there. They can't get too close or otherwise . . . There are lots of things you can think of there . . . Consider those. (Students write up answers) Time's up. You have to hand in now girls.	153

LESSON 2: ACTIVITY 2

WA	You have activity 2 and you have the instructions which are the same as yesterday's. Who's going to read?	1
S8	Reads instructions	2
WA	It looks like it deals with boats and it carries on from yesterday. Where were we yesterday?	3
ALL	Robben Island	4
WA	So we need a boat to go to Robben Island. We are going to replace the old boat with a new one. He's going to buy a boat and it is going to cost him R3,5 million. The old boat is only worth between R300 000 and R700 000.How much will the new boat cost?	5
ALL	R3,5 million	6
WA	How much will the old one be worth?	7

ALL	R700 000	8
WA	Looks like there's a lot already in this activity. Let's put down some things. We've got the old boat and it will cost R300 000 to R700 000. New boat R3,5 million (writes this info on the board). Carry on reading.	9
S8	Reads rest of activity	10
WA	Let's write down 60 passengers. Adults . . . how much?	11
S1	R100 return	12
WA	What does that mean?	13
S1	There and back	14
WA	What about children?	15
S2	R50	16
WA	He has lots of expenses. Does it really matter what they are? What is the total expenses?	17
S4	R950	18
WA	So expenses . . . R950 per trip. He can only carry on days when it does not rain. In peak season . . . Peak season is from the beginning . . .	19
S1	September	20
WA	Until . . .	21
S1	End of April	22
WA	And he operated at 70% capacity. Now, 70% capacity . . . we are talking about 70% of 60 passengers. How ill we work that out? What does percentage mean?	23
S6	A fraction out of a 100.	24
WA	Good, so we have 70% of 60 and how many people will that be?	25
S3	42	26
WA	Sure, . . . 42 people. Right 6 trips per day. And what's the third thing?	27
S2	The ratio is 1:1	28
WA	What is ratio?	29
S3	A comparison	30
WA	We had that yesterday. Now we are saying the ratio of adults to children is 1 is to 1. What does that mean?	31
S2	There is the same number of adults as children.	32
WA	We have 42 people . . .	33
S2	21 adults and 21 children.	34
WA	OK, anymore information? Let's go to off peak season. Someone read.	35
S9	Reads	36
WA	What is that again . . . 30%. Now, we've got to work out what is 30% of 60 passengers. Come on. . .	37
S1	18	38
WA	18 people. . . Carry on.	39
S9	Reads	40
WA	3 trips . . . and ratio 2:3. Now let's go back to 1:1. We had 42 people and 1:1 that's 2 parts, right? How did I get that? 1 and 1 is 2. Now, we've got 2:3. How many parts?	41
S5	5	42
WA	5 parts, 18 people. So divide 18 by 5. Does it go?	43
S1	No. Mustn't you say 2 over 5 times 18?	44
WA	OK, 2 over 5 times 18. Then what? What does that come to?	45
S1	7,2	46
WA	7,2 people. What do you think 0,2 persons mean?	47
S2	We must round off.	48

WA	OK, let's see 3 over 5 times 18. What is that?	49
S1	10,8	50
WA	OK, we have a problem here. These are people. So we cannot say I am going to take 0,8 of you and the rest must stay on shore. So what are we going to do?	51
S1	Round it off.	52
WA	How.?	53
S2	Less than 0,5, round down.	54
WA	So, what was the 2 part of 2:3?	55
S3	Adults	56
WA	How many adults will there be?	57
S3	7	58
WA	Not 7,2. And how many children?	59
S2	11	60
WA	11 children and that comes to 18. So, now point no. 4. What does it say?	61
S5	The rainy days.	62
WA	And what happens on these rainy days?	63
S2	The boat cannot go out.	64
WA	The boat stays on shore. So now we've got September, October, November, December, January, February, March, April, May, June, July, August. (Writes all these on the board) Call to me.	65
S4	10, 7, 5, 4, 5, 2, 6, 10, 15, 20, 25, 26	66
WA	Which is the peak season? September to April. How many trips?	67
S6	6	68
WA	Right, now we are going to find out how many days were available.	69
S2	Subtract the rainy days from the days of each month.	70
WA	What must we fill in now? Subtract the days in the month. How do we remember the days in the months? How does it go? 30 days . . . Don't you know that story? 30 days have September, April, June, . . . Didn't you learn that? No, you children are deprived, my goodness. What do you do? Count your knuckles? I'll say 30 days have September, April, June and November, all the rest have 31 excepting February alone which have 28 days here and 29 in each leap year. So, is this a leap year?	71
ALL	No	72
WA	So what will we put . . .	73
S1	28 days	74
WA	Now, let's check the numbers. These are the rainy days and these are the nice days. So we will say 30 minus 10.	75
S2	20	76
WA	Next	77
S1	24, 25, 27, 26, 26, 25, 20, 16, 10, 6, 5	78
WA	OK, now we have to divide the table into the seasons. (Underlines peak) How many days in peak?	79
S2	Add the days.	80
S1	193 days	81
WA	Check or else we will have all these figures wrong. Everybody happy? Now off peak.	82
S1	37	83
WA	Look at this and this (*Points to 193 and 37 days*) Now, we have all the	84

	info we need. How will we divide this info up? You've got expenses and income. What we need to look at is how much we are spending and how much we are getting in. How do we work out our expenses?	
S3	950	85
WA	Per trip. Now remember in the season he does 6 trips per day. That means 950 for every trip. So 950 x 6 and in off season 950 x 3 for every day. Now, we've got 193, we've how many trips . . . ?	86
S3	6 trips . . .	87
WA	(Writes 193 x 6 x 950) And in the off peak 37 days and 3 trips costing 950 (writes 950 x 37 x 3) Does he have any other expenses?	88
ALL	No	89
WA	Now what about income? How will we work out income? Where will we get money from? How much is he earning each trips?	90
S1	R100 per adult and R50 per child.	91
WA	Right, how many trips?	92
ALL	6	93
WA	And so 6 trips . . . how much rand per adult and 6 trips . . . and how much rand per child? 21 children and R50 and how often will that happen?	94
S4	193 days	95
WA	So, we have 193 days, 6 trips, 21 adults at R100 and 193 days, 6 trips and 21 children at R50. Now that is in the season and that is a lot of money. What will happen in off season? 37 days times 3 trips times 7 adults at R100 and 37 days x 3 trips x 11 children at R50. And then we will add it all together and get our income. What is the whole purpose of doing this?	96
S1	To see if he will have enough money to buy the new boat.	97
WA	So, we have 193 days and 37 days. You should be able to work it all out. Off you go. You can write anywhere. Write clearly. If you want to ask anything, ask me, not you neighbour. (Students work on the activity – teacher walks around checking – generally left on their own to solve the problem) (20 minutes later) Time's up. Put your last figures down and hand in your work.	98

ANNEXURE 3

TRANSCRIPTS OF FOUR STUDENT INTERVIEWS

INTERVIEW 1: A.J & J.M

R.E	How did you work out how to do no.1?	1
A.J.	I said the area is x times y.	2
RE	How did you know to use x times y?	3
AJ	They give it in the first question.	4
RE	But how did you know to multiply x and y?	5
AJ & JM	That's how you work out the area.	6
RE	Did you look at the picture at the back? (Refers to map of Robben Island)	7
AJ & JM	Yes	8
RE	What did the picture tell you? Where's the runway? Point it out to me.	9
BOTH	There it is. (pointing to the runway on the map)	10
RE	What did you conclude about the runway?	11
AJ	I did not really conclude anything.	12
JM	It looks like a cross.	13
RE	You saw it as a cross? Did you see anything else L?	14
JM	Umm …. Not actually.	15
AJ	There's 4 quadrants.	16
JM	Yes.	17
RE	Now why are you switching to quadrants? Did you think about quadrants when you worked on this problem?	18
BOTH	No.	19
JM	Sir, it is 2 rectangles crossing each other.	20
RE	So, how else would you have known they are rectangles if you hadn't seen the picture? What in this sentence told you were dealing with a rectangle?	21
AJ	Runway. The word runway.	22
RE	Runway? Only that? Nothing else?	23
AJ	The width and the length.	24
RE	What about those?	25
AJ	It is just stated that	26
JM	That could be a square also.	27
RE	But why couldn't it be a square in this case?	28
AJ	Runways aren't square.	29
RE	OK, runways aren't square. Any other reasons why its is not a square?	30
JM	Oh, a square is all sides are equal.	31
RE	How do you know that	32
JM	x & y are used indicating 2 different sizes.	33
RE	Ok, so they are denoting 2 different lengths. Now question 2. The ratio story. If the width and length is in the ratio 1:20, state your answer in terms of x. So, why couldn't you do that one AJ?	34
AJ	Umm . . . I didn't know what to do. I didn't understand it properly.	35
RE	How would you express x and y as a ratio?	36
AJ	X is to y.	37
RE	Now why did you tell me you couldn't understand it?	38
AJ	It seemed complicated with that one in brackets.	39
RE	If you had x:y as ratio, what will you do with that 1:20?	40

AJ	Replace it.	41
RE	Just replace it?	42
AJ	No I would . . . . I don't know.	43
RE	Won't you equate the 2? Say x:y is equal to . . . . ?	44
AJ	1:20	45
RE	Yes, What do you do with that afterwards to find a solution? To find the unknown?	46
AJ	Divide.	47
RE	By what?	48
AJ	I said x multiplied by 20x. Ok, I first said 1:20 and then 1x is to 20x and then I had x multiplied by 20x will give you $20x^2m^2$.	49
RE	So what were you supposed to do with the $20x^2m^2$? What did you do with it afterwards?	50
AJ	Umm . . . . I don't know. I just left it $20x^2$.	51
RE	What did $20x^2m^2$ mean to you?	52
JM	Oh, . . . that was the length . . . umm . . . no, that was the runway.	53
RE	What about the runway? The length or the width?	54
JM	Umm . . . I think it was the width of the runway.	55
RE	It says express your answer in terms of x. So what was x really?	56
JM	X is the unknown. x is metres.	57
RE	So what is it? The width or the length?	58
JM	The width.	59
RE	AJ, if the airfield has 2 identical perpendicular runways, write down . . .	60
	What does identical mean?	61
AJ	The same	62
RE	And perpendicular?	63
AJ	Parallel, . . . no!	64
JM	When they cross at 90^0.	65
RE	Ok, if they intersect at 90^0.	66
AJ	Oh, yes I forgot that . . .	67
RE	AJ, why couldn't you do no. 3?	68
AJ	I did try.	69
RE	But your answer was never going to be correct. Why?	70
AJ	Ok, what did I write. . . . Was this supposed to be length and breadth? I actually can't answer because I don't know what I wrote.	71
RE	But it doesn't matter what you wrote or how you did no.3. You were just not going to get it right. Why not?	72
AJ	Cos, I didn't understand when I read the question.	73
JM	No, your answer in no.2 was wrong!	74
RE	You needed to work out no. 2 to be able to do no. 3 and do no. 3 to go onto no.4.	75
AJ	Oh yes …. All the questions follow on each other.	76
RE	So, why did you break down? Is it because you didn't know enough mathematics?	77
AJ	Yes, I think so.	78
RE	So, you haven't been taught any of these like, ratio, expressions, . . .	79
AJ	We have. I just forgot it	80
RE	What if I gave you a question which simply said x:y equal 1:20? Let me write it down. If this was given, what would you have done? Would you be able to solve it?	81

AJ	No.	82
JM	I don’t think so either.	83
RE	What if I said x over y is equal to 1 over 20? Would you be able to do that?	84
AJ	Oh, yes.	85
RE	How?	86
AJ	I will cross multiply.	87
RE	Move on to Q5. Why do you think the 2 runways intersect each other and are not parallel? Any maths involved in that?	88
BOTH	Not really.	89
RE	So, no calculations involved? What did you need to know about to answer that question?	90
JM	You need to know like the size of the island. That maybe counts. They are asking why did they intersect and also the wind direction and that.	91
RE	AJ what did the word runway make you think about? What is it?	92
AJ	The airfield is the runway.	93
RE	Isn’t it something on which an aeroplane	94
AJ	. . . lands or take off!	95
RE	Now, they wanted to know why were these runways built in these particular directions and JM said something about the direction of the wind. So, what do you know?	96
AJ	Not much . . . I had no clue about answering that question.	97
		98
RE	Now Activity 2. Our tour operator with his visits to Robben Island. AJ, tell us, where did you start?	99
AJ	I first . . . checked how much . . . if he sold his old boat . . . how much he would get for it.	100
RE	How much?	101
AJ	At maximum R700 000.	102
RE	So you assumed for the rest of the problem that he would get R700 000?	103
AJ	Yes . . . And then I said the new boat would cost R3,5 million, so I got the difference which is R2,8 million.	104
RE	Where will he get the rest of the money from?	105
AJ	From trips that he would do. Selling would add to it and the trips to Robben Island.	106
RE	Let’s look at these trips. Were the calculations on these a simple exercise?	107
AJ	No.	108
RE	Why not?	109
AJ	You first have to work out the days it rained and the peak season . . . How many days there was in peak season. Then you had to work his expenses per trip and . . .	110
RE	Before we talk about expenses, what’s the difference between peak and off peak seasons?	111
JM	The amount of people . . .	112
AJ	More people umm . . .	113
RE	So?	114
AJ	He had more trips in peak season.	115
RE	And therefore . .?	116

BOTH	He made more money.	117
RE	So peak season was the time he would make the most money. Now, all the trips that he made . . . the money he made . . . was it pure profit?	118
AJ	No, he had expenses to cover.	119
RE	Which was R950 per trip. During peak season . . . how many people could he get into his boat, by the way?	120
JM	40	121
RE	No, no, in total I mean. How many people could the boat hold?	122
JM	Oh, 42 people.	123
RE	Are you sure? The capacity of the boat?	124
JM	Oh, the boat could hold up to 70 people.	125
AJ	The capacity was 70%.	126
JM	The boat could hold 60 people and he operated at 70% which gave 42 people.	127
RE	Now, he had to consider the ratio. What does 1:1 mean?	128
AJ	It means that there was the same amount of adults as children. There was umm . . . 21 adults and 21 children	129
RE	Did the children pay the same fee as the adults?	130
BOTH	No.	131
AJ	Children paid R50 and adults R100.	132
RE	How did you work out the number of days he would be operating?	134
AJ	I took the amount of days in each month . . .	135
JM	Minus the amount of rainy days.	136
AJ	That is only off peak season.	137
RE	What did you do with all these days?	138
AJ	Add it.	139
JM	You work out the days he was operating and you have to say the amount of days in the month minus the rainy days would give you the days he could operate.	140
RE	How many days in total could he operate in the peak season?	141
JM	193 days.	142
RE	193, right. How many trips per day?	143
AJ	Umm . . 6 trips per day . . . Divide it . . .	144
RE	Why?	145
JM	I said the days multiplied by the amount of trips.	146
RE	193 x 6?	147
JM	Yes, but multiplied the expenses also.	148
RE	Did you work that out separately?	149
JM	No.	150
AJ	Oh, it is the amount of passengers he carries . . . so you have to say the amount of people per day.	151
JM	I worked out the expenses . . . I said 193 x 950 x 6 for the amount of days for expenses per trip also to get my answer.	152
AL	Mustn't . . . you mustn't times by 950.	153
RE	Why not?	154
JM	Yes, that is to work out expenses because it is 950 per trip.	155
RE	When you come to the profit, how did you go about finding that?	156
JM	Oh, the profit?	157
RE	Still peak season.	158
JM	Umm . . I said 193, the amount of days, times 6, the amount of trips,	159

	times 21 adults times R100. Adults paid R100. And I did the same for children but I multiplied by R50.	
RE	Then you got the total earnings for the peak season and subtracted the expenses	160
	AJ, it seems you did not factor those expenses into your calculations.	161
AJ	I only minused it afterwards. I would say here 950 x 6 trips and I would get R5700 and that times 193 days.	162
RE	So, you got a total of R2,5 million. Now, off peak. Was the ratio the same? The percentages the same?	163
BOTH	No.	164
RE	How many passengers could he take during the off peak season?	165
JM	60 people but at 30% capacity is 18 people.	166
RE	Same number of adults as children?	167
BOTH	No.	168
RE	So?	169
JM	7 adults . . .	170
RE	Sure ?	171
JM	The ratio is 2:3 but . . . for every 2 adults there were 3 children.	172
RE	How many of each were there?	173
JM	7 adults and 11 children.	174
RE	Now. Overall was this a very complex problem?	175
AJ	Complex as in . . .?	176
RE	Were there many difficult calculations and tasks?	178
BOTH	No, not really.	179
RE	What were the main mathematical operations involved?	180
JM	Multiplying, dividing, subtracting . . .	181
RE	Did you do any division?	182
JM	I didn't divide anything . . . except when I figured out the amount of people to work out the percentages. There was also plus and minus.	183
RE	So, AJ, why was it so difficult?	184
AJ	I actually did not find it difficult, but then my answers were . . . I actually thought they were all right. But I went wrong with this peak and off peak earnings and when I checked how much money he made. It went wrong there.	185
RE	From all the text you had to read, did you have a problem figuring out the mathematics?	186
AJ	I did get stuck, but once I got going I thought it was right so I kept going. It felt right because it was actually not difficult.	187
RE	So, was there a trick to doing this?	188
AJ	It was a lot of things to do . . . You had to understand it. The trick was knowing your mathematics . . . I don't know. Reading and understanding it.	189
RE	Thanks girls, that' it.	190

INTERVIEW 2: L.V & S.Z

R.E	We've done the first activity. What were the general problems you had with it?	1
SV	Sir, I didn't really understand the whole thing. The questions weren't like . . . you know what I mean?	2
RE	How did you expect them to be?	3
SZ	Like a normal test.	4
LV	I didn't understand what was going on.	5
RE	If I gave you some sums and said for instance, find the value of x.	6
LV	That would be easier.	7
RE	Look at Q1. It says write an expression. What is an expression?	8
SZ	Like what that means. I don't know.	9
RE	You didn't know what an expressions is?	10
LV	No.	11
RE	Then it says: "Find an expression for the area." So. Presumably, you did not have a problem with area? When you have to find an expression for the runway . . . Do you know what a runway is?	12
BOTH	Yes.	13
RE	Did you ever go back to the picture at the back? To see what it is you have to work with? Did you look at the picture?	14
LV	I did . . . but I didn't understand it. I looked at it a few times but it did not make any sense to me.	15
RE	So how did you get that answer correct?	16
SZ	Because that just made sense because area is equal to x times y because they give it to you in the question.	17
RE	So, you knew that if you get a length and a width . . . to find the area you must multiply the 2. Why didn't you say, for instance, half base times height? Or Pi time radius squared?	18
SZ	Because it is the area of a runway . . .	19
RE	So, you expected it to be what shape?	20
LV	A rectangle?	21
RE	If you looked you looked at the picture again then we can see that shape. Ok, from then onwards both of you could not manage the rest. Let's look at Q2. Why couldn't you go beyond that point? Do you know how to work with ratios?	22
BOTH	Not really.	23
LV	Isn't it for every one centimeter there is 20 metres. So if you have one width, the length is 20 times that width.	24
RE	If the width is 2m, what would the length be?	25
LV	40	26
RE	You have to express area in terms of x.	27
SZ	I got 40.	28
RE	But you did not go further to solve the problem. That problem was linked to Q1. You had to set up some sort of an equation using ratios. OK, if the airfield has 2 identical and perpendicular runways . . . So what do you understand by identical?	29
BOTH	The same.	30
RE	And perpendicular?	31
SZ	Same length . . .	32
RE	Look at the picture again . . . What do you think . . . ?	33

LV	Crossing?	34
RE	At what angle?	35
SZ	Ninety degrees.	36
RE	All the rest depend on how you managed No. 2 . Then they give you the actual area and you have to say what the width, x, is equal to. But, again, you could not do that . .	37
SZ	We weren't able to do the questions before . . .	38
RE	The last question did not have any mathematics. Why not?	39
SZ	Because it was like a simple question.	40
RE	Is there anything in the question that told you to some sort of maths?	41
LV	No.	42
RE	How do you know that?	43
SZ	Because they ask "why".	44
RE	Why couldn't you answer the question?	45
SZ	I couldn't really understand all the words that was in there.	46
RE	Why do the 2 runways cross and not run side by side?	47
LV	Because they are in different places.	48
RE	Anything else?	49
SZ	Maybe it is the direction in which the plane wants to fly.	50
LV	The one will fly out and another will land in different directions.	51
RE	What is important in taking off and landing for aeroplanes?	52
LV	They must do it at the right times	53
RE	When is that?	54
LV	They must take off when the aeroplane . . .	55
RE	What does that have to do with the direction of the runway?	56
LV	They can go faster and they can go straight.	57
RE	But what if I changed direction? I'm still going straight.	58
SZ	That is when other planes must land.	59
LV	The plane must get up . . .	60
RE	Let's leave that there and move on to activity 2. Give me your reflections. This was long one and I see you scored no marks on this activity, SZ Why did you struggle?	61
SZ	They asked too much questions. You had to work out too many things.	62
LV	You don't know what is going on and you must work out one thing at a time.	63
RE	Isn't it possible to have done it step by step? If you had to redo it, what would be your first step?	64
SZ	You have calculations, like ratios and things and percentages . . .	65
RE	Were they difficult calculations?	66
BOTH	No, they were confusing.	67
LV	Umm . . . I would start by the amount of people that can get on the baot and the prices and that and the months . . .	68
RE	OK, you are starting with too many things.	69
SZ	I would start in the peak season and how many people he could take.	70
RE	How do you get that?	71
SZ	I will say the number of days and he had 70% capacity . . . like 70 people if there was like a 100. And count the days and times that by 70.	72
RE	But he isn't taking 70 people . . .	73
SZ	It's just a percentage.	74
RE	How many people can actually fit on the boat? What's the capacity?	75
LV	60	76

RE	But at no time is his boat full. In peak it is only 70%. So you . . .	77
LV	. . . had to get 70% of 60.	78
RE	That's 42 . . . Of these he takes children and adults in the ratio of 1:1. What does that mean?	79
LV	1 child means . . . there's one adult. Therefore, 21 children and 21 adults.	80
RE	How much did they pay?	81
LV	R50 a child and R100 for adults.	82
RE	So that's 50 x 21 and 100 x 21 and you add the answers. OK?	83
LV	That gives the amount for one trip.	84
RE	How many trips per day?	85
SZ	6	86
RE	So multiply by 6 and multiply by another number and that is the number of days. He can't operate in September for the whole month because 10 days it rained. So . . .	87
LV	He could only operate for 20 days.	88
RE	And October? And so on. So now you begin to see steps. What was the whole purpose of this?	89
LV	To see if he will have enough money to buy a new boat.	90
RE	How much will he get for his old boat?	91
LV	R700 000 . . . the rest he must get from his trips.	92
RE	Does he make a clear profit on each trip?	93
LV	No . . . what do you mean by clear profit?	94
RE	All the money he makes goes into his own pocket. He pays for nothing else.	95
LV	No, because he has to pay for fuel.	96
SZ	He has to pay for expenses.	97
LV	You take the money he earned and subtract his expenses.	98
RE	So there you have some steps and you were able to make sense of the problem in this time together. Why is that?	99
SZ	You explained it to us and it made more sense.	100
RE	Thank you girls, our time's up.	101

INTERVIEW 3: K.A & Z.M

R.E	Let's look at Q1 on Activity 1 and tell me something about your reasoning. The first question says "write an expression for the area of the runway if the width is x metres and the length is y metres". You both understand an expression?	1
BOTH	Yes.	2
RE	What is it?	3
ZM	It is when you take that 2 and put it in the place of numbers.	4
KA	Variables replace actual numbers. Like x and y.	5
RE	And the area story. It says write an expression for the area. So, when you looked at this picture (map of Robben Island) . . . Where's the runway?	6
BOTH	(Point to runway on map)	7
RE	So, what did you have to do?	8
ZM	Length times breadth.	9
RE	Why not half base time height?	10
ZM	That's the area of a triangle.	11
KA	It forms a rectangle.	12
RE	What else tells you it is a rectangle?	13
KA	It tells you the length and the width.	14
RE	What particular things tell you its is a rectangle? Why not a square?	15
KA	The runway is rectangular . . . . it can be a square but . . .	16
RE	Why not in this problem?	17
KA	Because it must be long for the plane to land on.	18
RE	Leave the plane for now. You said it is a rectangle. Why in this case is it so?	19
ZM	Because they didn't say x and y are equal to each other	20
RE	If x and y were equal to each other, what would they have said?	21
ZM	x squared, y squared.	22
RE	If it was a square . . .	23
ZM	All the sides would have been equal, but the sides are x and y.	24
RE	You were able to see a rectangle in the oicture?	25
BOTH	Yes.	26
RE	In your answer KA, you said x times y equals xy. ZM why didn't you simplify your answer? Why did you leave it as x times y?	27
ZM	When I wrote x times y I didn't think it would matter because there were no numbers so it was the same thing for me. It was the final answer.	28
RE	Now, ratios. The width and the length of the runway is in the ratio 1:20. Express your answer in terms of x. What's wrong with your answer KA?	29
KA	I did not write it in terms of x.	30
RE	What have you got?	31
KA	There wasn't supposed to be a y.	32
RE	That's right. You've got a y in there so you haven't simplified right to the end. What should have tried to get all your numbers as? Which letter should you have tried to eliminate?	33
KA	The y.	34
RE	How would you do that?	35
KA	x =20y . . . x over y equal 1 over 20.	36
RE	And if you solve for y?	37

ZM	y = 20x	38
RE	So what is another name for y.	39
KA	It's 20x . . .	40
RE	What should you have done with the 20x?	41
KA	Divide.	42
ZM	To give you x. So 20x multiplied by x will give you the area, $20x^2$.	43
RE	Y is the same as 20x . . . we have a value for y which we can work with. So our answer will be in terms of x. Next question was a bit of a mess. What do you understand by identical?	44
KA	They are exactly the same.	45
RE	Perpendicular?	46
ZM	Umm . . . they cut at a certain point . .	47
RE	Be more specific.	48
KA	Ninety degrees.	49
ZM	No, when you multiply the gradients it must give you -1.	50
RE	Leave that. What does perpendicular mean? Look at the picture.	51
ZM	Directly opposite.	52
KA	It refers to a ninety degree angle . . .	53
RE	Yes, they intersect at ninety degrees. Now write down an expression for the total area in terms of x.	54
ZM	If you got number 2 wrong then no3 will also be wrong.	55
RE	But, let's see whether you can work it out.	56
ZM	4 times twenty x squared.	57
RE	Why?	58
ZM	2 times twenty x squared.	59
RE	It will work. Why?	60
KA	The 2 runways are the same.	61
RE	But something happens in the middle.	62
ZM	They intersect . . .	63
RE	If you work out the areas of the 2 rectangles, what will you end up with?	64
ZM	Minus the square in the middle or else you will get the same area twice in the 2 rectangles . . . . Therefore, take away one of the areas	65
RE	Now what is the answer?	66
ZM	2 times twenty x squared is 400 . . . no, it's 40x squared.	67
RE	So the total is 40x squared but the middle block must betaken away. What's the area of the middle block?	68
KA	x times x is x squared.	69
ZM	39x squared.	70
RE	There you have it. The total area is 39x squared. Next, . . . if the total area is 97 500 metres squared, determine the width of the runway using the expression in no.3.	71
ZM	Divide by 4 . . .	72
KA	No, . . . you must use that one we did now . . the 39x squared . . . then divide it by 20x.	73
RE	Why?	74
KA	We need to find the width so . . . This is the total using the width and the length. . . so, . . .	75
RE	But the total is given . . .	76
ZM	No, that divided by 39x squared . . .	77
RE	If I asked you to work out the area of a rectangle . . .	78

ZM	Area = length times breadth.	79
RE	What in this formula (A = l x b) is given?	80
ZM	You say 97 500 = length times 39x squared . . .	81
KA	97 500 = x times y because that is the length and the breadth.	82
RE	But you must use the expression in there . . . What can you come up with?	83
ZM	How about if you say 97 500 = 39xsquared . . . divide the same by a w.	84
RE	Why introduced a w?	85
ZM	I don't know . . . it is width.	86
RE	You already did l x b = xy. Then we did the ratio and we got y = 20x and therefore, the area is 40x squared.	87
KA	You must divide by 20 xsquared. . .	88
RE	Why?	89
KA	Because that is the width.	90
RE	What did you multiply to get 39x squared?	91
ZM	2 times 39x squared minus that other area . . . no, . . .	92
RE	Let's take the 20x, how did we get it?	93
ZM	Cross multiplied . . .	94
RE	We determined that the total area of the runway is given by 39x squared. The x stands for the width, Now, solve for x.	95
KA	Divide by 39 . . . both sides. So, x squared = the answer and square root both sides.	96
RE	Good, last question. Why do they intersect at ninety degrees and not run parallel to each other?	97
ZM	So they can get more . . .	98
KA	They ride, they go up and fly . . . Stuff inside them make them go up.	99
ZM	The speed makes them go up.	100
KA	The winds come in different directions and the plane goes in another direction . . no, wait, . . . it must go with the wind.	101
RE	Sure?	102
KA	They must go in the direction of the wind . . . it must blow from behind . . . No, it must come from the front.	103
RE	OK, Let's leave that there and go to Activity 2. Our tour operator . . . how did you start this problem?	104
KA	First you have the cost of the boat and he already has R700 000 and still . . .I worked out how much he still needs. So it is the R3,5 million minus R700 000 and he still needed R2,8 million which he must raise from riding boat trips to Rbben Island . . . the old boat . . .	105
RE	Talk to me about the peak and off peak seasons. What did you make of those?	106
KA	In peak he got more money than off peak.	107
RE	Why?	108
KA	Because, there's a lot of people . . . it's the holidays . . .	109
RE	But did a lot of people determine how much he could operate . . . What determined how many days he could operate?	110
ZM	The weather.	111
RE	Good, the amount of rainy days. How did you work with these? By the way which were the peak months?	112
ZM	September till April . . . 7 months.	113
KA	8 months . . . When foreigners come to Cape Town . . . Peak season	114

	is summer which is 3 months . . . no, six months of summer and six months of winter.	
RE	In the question they told you peak season is from 1st Sept to 30th April. How many months is that?	115
KA	8 months.	116
ZM	You take the number of days in each month and minus the rainy days . . .	117
RE	How many trips per day?	118
KA	6	119
RE	What is the maximum number of passengers he could carry?	120
ZM	1 is to 1.	121
RE	What is the capacity of the boat?	122
ZM	60	123
KA	In peak it was 42 because it is 70% of 60.	124
RE	What does the ratio 1:1 mean?	125
ZM	It means the number of adults were the same as the children	126
RE	And in off peak?	127
KA	18	128
RE	The ratio is 2:3 . . . so how many adults and children could he take?	129
KA	7 adults and 11 children. The ratio must be 2:3 so 2 over 5 . . . 2 over 5 times 18 for adults and 3 over 5 times 18 for children and you get 7 and 11.	130
RE	Now when he makes a trip or these 6 trips a day . . . . is all the money profit?	131
ZM	No, he has operating costs of R950 per trip.	132
RE	Where did you go wrong with these calculations?	134
ZM	I went wrong by when they ask if it is going to be enough . . . I multiplied the . . .	135
RE	How did you work out his profit during peak?	136
ZM	The number he got from taking children and whatever which gave you 18 900 per day and you take that and multiply it by 6 . . .	137
RE	In peak season, how did you work out how much he would make from a trip?	138
ZM	You added that . . .	139
KA	The children paid R50 and adults R100. Therefore, R50 times 21 and R100 times 21 and add the answers and you got how much he made . . . That must be multiplied by 6 because it is 6 trip a day.	140
RE	That must be multiplied by how much?	141
ZM	193	142
RE	That gives a total . . .	143
ZM	. . . during peak season	144
RE	And you had to do the same thing for off peak. So how much did he make? I got a total of R 2 554 600. What did you get?	145
KA	Umm . . . I got R3 647 700.	146
RE	What did you not subtract?	147
ZM	We should have said 950 x 193 x 6 and minus it from that total.	148
RE	How much did he need for the boat?	149
KA	R2,8 million.	150
RE	But he only made R2,5million. So, he was short by about R300 000. Ok, that’s it. Our times up.	151

INTERVIEW 4: L.A & J.A

RE	Let's look at activity 1. Did you know what to do?	1
JA	No, sir. I did not understand the questions so clearly.	2
LA	I did not know what was happening.	3
RE	What did you not know? You did not understand the language or the maths?	4
LA	I did not understand the language . . . the way they put the questions.	5
JA	I agree. I had to study for this and I did not expect this would come in.	6
RE	Let's look at Q1. Write an expression for the area of one runway if the width is x metres and the length is y metres. What's the problem?	7
LA	No, that was easy.	8
RE	What is an expression?	9
LA	A sum without words and letters . . . I mean without numbers.	10
RE	So you said area = x times y. Area = length times breadth, I take it. Did you ever look at the picture?	11
JA	No, I didn't know there was a picture. Not till afterwards.	12
RE	If you look at the picture, what did they ask you to work out? Point to me on the picture? (No response) No idea? Do you know what a runway is?	13
JA	It is that (points to picture) The road where planes land on.	14
RE	So isn't that what you had to work out?	15
LA	Yes, but I didn't understand the way they put the question.	16
RE	What was the difficult part to understand? Which words?	17
LA	When they say like width and length is okay, but they say ratio and things like that . . .	18
RE	Let's just deal with Q1 for now. I take it you had no problems with it? No Q2 refers to Q1. You did not know what to do. Why?	19
JA	I just wrote my own thing.	20
RE	What part didn't you understand?	21
JA	The whole thing.	22
LA	From ratio onwards.	23
RE	If you had to express x and y as a ratio, how would you do it?	24
LA	Say now x and y is a number you must times that with 20 to get the actual thing.	25
RE	But you don't know how to turn x and y into a ratio and how to relate it to 1:20. So from there onwards you could not get Q4. Now, the total area for the runway is given and again you needed to be able to do the previous question for this one. The last question asked why you think the runways crossed each othet and did not run parallel. What did you understand by intersect?	26
LA	They cross each other.	27
RE	And parallel?	28
JA	That er . . . down.	29
LA	Parallel is like when they don't touch each other. They never meet. They intersect and parallel I didn't understand. It intersect and now they are parallel.	30
RE	If you look at the picture, what are they asking you about the runway? Is intersect important or something else?	31
JA	Something further. When they talk about being parallel.	32
RE	So what did you have to try and work out?	33
JA	No idea, sir.	34
RE	What are runways used for?	35

LA	Planes. They land and take off on runways. The length of the runway is important I think . . .	36
RE	Alright let's go to Activity 2. The boat operator. What is he trying to do?	37
LA	He wants to buy a new boat.	38
RE	What is he guaranteed of having when he buys his new boat?	39
LA	The money he will get if he sells his old boat.	40
RE	How much will he get? The smallest amount?	41
JA	R300 000	42
RE	And biggest?	43
JA	R700 000	44
RE	So we assume he will get R700 000. How much will the new boat cost?	45
JA	R3,5 million	46
RE	How will he get the rest?	47
JA	He must work more. He must have regular trips to Robben Island.	48
RE	What's the catch for him?	49
LA	There's a peak and off peak season.	50
RE	What are those?	51
LA	Peak season is when he goes to Robben Island a lot and off peak a little . . .	52
RE	Why little in off peak?	53
JA	Because of the weather.	54
RE	It rains a lot which prevents him from going out. The table gives the number of rainy days in the year. The peak is from 1st Sept to 30 April. How many peak months?	55
JA	8	56
RE	How many days in total was that?	57
LA	I didn't do it like that. I took every month and I took the total number of days in the month and subtracted the rainy days. Then I took the number of children and adults and the capacity of the boat . . .	58
RE	What were the main mathematical processes in this problem.	59
LA	I can't remember.	60
RE	Were there many difficult calculations?	61
LA	No, not really, but there were many steps you had to cover.	62
RE	What did these steps involve?	63
LA	Adding, subtracting . . . stuff like that.	64
RE	Each trip he took, did he make clear profit?	65
JA	It doesn't look like it . . . I know it but . . .	66
LA	Expenses . . . something he must subtract from his profits. Like petrol, fees, and so on . . .	67
RE	What happens in peak season? How many people does he take?	68
JA	60	69
RE	Does he really take 60 people?	70
JA	No . . .	71
LA	He was only 70% full.	72
RE	If the boat is 70% full, what is 70% of 60?	73
LA	42 passengers.	74
RE	What is the ratio of adults to children in peak season?	75
LA	1:1	76
RE	What does that mean?	77
LA	1 adult for 1 child.	78
RE	Out of 42 people, how many were adults and . . .	79

LA	42, . . . no, half of that. 21 adults and 21 children.	80
RE	Did they pay the same amounts?	81
JA	No, children paid R50 and adults R100.	82
LA	You have to say R50 x21 and R100 x 21	83
RE	In off peak, what percentage of the boat was full?	84
LA	30%	85
RE	30% of 60?	86
LA	18 passengers.	87
RE	And the ratio?	88
LA	2 adults to 3 children.	89
RE	What will there be more of on the boat?	90
JA	Children	91
RE	Out of the 18 passengers, how many were children and adults?	92
LA	You get half of 18 and then divide that one by 2 . . .no . . .	93
RE	What must you do with the ratio? Can't you turn it into a fraction?	94
LA	But, I don't know how to do it? I was lost.	95
RE	During which period did he make most of his money?	96
JA	Off peak season.	97
RE	Now why is that answer wrong? Did he have more passengers during off peak season?	98
JA	During peak he had 42 and off peak 18. Oh, I see . . .	99
RE	OK, our times up. Thanks girls.	100

ANNEXURE 4

TEXTUAL PRODUCTIONS OF NINE STUDENTS WHO WROTE THE TWO TESTS

AB: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is x metres and the length is y metres. (1)

$x = 0,6$
$y = 0,5$
$L \times b$
$\therefore A\ x \times y$

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of x. (6)

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of x. Simplify your answer. (6)

4. If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

AB: ACTIVITY 2

march 31 september 30
Jan 31 Feb 28 April 30 may 30 november 30 august 31
December 31 october 31 June 30 July 31

R300 000 + R2700 000
= 2.1 × 10"

0/26

35 × 1.1
= 77,2 × 6
= 231 ÷ 8
= 281.8

In the off peak season.
30% × 3 = 40

4 ÷ 2.3
= 40,23 × 30 ÷ 31
= 1200, 713.

If I add all the expenses It all equal ~~me~~ plus
is total ~~to R1400~~ It will add up to R1900

If I add all the amount up that
I got then yes he would be able to buy
his boot....

AJ: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

$A = y_m \times x_m$
$A = xym^2$

1

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

$A = y \times x$
$A = xym^2$
$xym^2 = 20xym^2$

$y = 20x$

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

$x = 1 \times b + 1 \times b$
$x = (1 \times b)^2$

$x = 1 \times b$
$x = b1 \times 4$
$x = 4b1$

r

4. If the total area of the airfield is 97 500m², determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

Area = 97 500m²

W = 97500m² / ~~2~~ 4

~~(W = 48 750m²)~~

W = 24 375m²

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

They both have to reach the same destination point.

Didnt understand at all

Feel really dumb right now

Wrote my own nonsense sorry!

AJ: ACTIVITY 2

old boat = R300 000 - R100 000
new boat = R 3 500 000
Difference = R2 800 000 (he needs)

Expenses = R950

Rainy days = 135 for 2002/2003

Peak
capacity = 70/100 x 60
= 42 people x 6
= 252 people per day
adults = 252 ÷ 2
= 126 x 100
= R12 600
children = 252 ÷ 2
= 126 x 50
= R6300
per day = R18 900
per season = Sept to April
= 242 days - 49 (rainy) = 193
193
~~(242)~~ x 18 900 = (~~R 4 573 800~~) R 3 647 700
operating cost = R950 x 6
193
= R5 700 x ~~(242)~~ days
= (~~R 1 379 400~~) R1 100 100
Peak season = (~~R 4 573 800~~) 3 647 700 - (~~R 1 379 400~~) R 1 100 100
= (~~3 194 400~~)
= R2 547 600

off peak

capacity = $\frac{30}{100} \times 60$

= 18 people × ~~6~~ 3

= ~~108~~ 54 people per day

adults = $\frac{2}{5}$ × ~~18~~ 54

= (~~7 × 100~~) 22 × 100

= (~~R 700~~) R 2 200

Children = $\frac{3}{5}$ × (~~18~~) 54

= (~~11~~) 32 × 50

= (~~R 550~~) R1600

per day = R 2200 + R 1600

= (~~R 600~~) R 3800

per season = May - August

= 123 days - 86 (rainy)

= 37 days

37 × 3800 = R 1 40 600 per season

operating cost = 950 × 37

= R 35 150

R 1 40 600 - R 35 150 = R 105 450

Peak + off peak

= R 2 547 600 + R 105 450

= R 2 653 050

~~old~~ new old

R 3 500 000 - R 700 000

= R 2 800 000 (he needs)

He will not be able to buy the boat. Sadly.

JA: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

Area = 1
∴ = $x \times y$

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

Area = 1
= πr 1:20
= 32x 1:20
= 672.

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

Area = 1
∴ = $x \times y$
= xy.

4. If the total area of the airfield is 97 500m², determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

Area = 97 500 m²

= 97500 m × 97500

∴ = 950625

∴ = 29530

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

They are intersecting because they are not parrel lines. It would also be a better connection if it do intersect.

JA: ACTIVITY 2

Old boat R300.000 and between R700.000
new boat R3.5 million.$

60 passengers

30 adults - R100.00
30 children R50.00

1 September to 30 April
35 adults
35 Children
6 trips per day
(ratio of adults to children 1:1)

$\frac{70}{100} \times 200 = 140$ → fuel

$\frac{70}{100} \times 6 = 4,2$ → trips

1 May to 31 August
30% capacity
15 adults
15 children
3 trips per day
ratio of adults to children 2:3

$\frac{30}{100} \times 3 = 0.9$ → trips.

ratio 2:5.

Expenses

Fuel - 200
Boat operator - 100
Landing fees - 50
Maintenance cost - 250
owner's salary - 350

TOTAL 950

He will not be able to buy there boat because he has to look at the amount of his expences befor buying the boat.

He wont be able to make any profit out of it.

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

A = 2l × 2b
A = x × y = a

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

A = 2l × 2b
A = x × y = a
A = 40x × y

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

A = 2l × 2b
= 2x × 2y
= 40x × y
= $\frac{40x}{y}$

9 50625 ÷ 40

4. **If the total area of the airfield is 97 500m², determine the width of the runway using the expression in (3). Motivate your choice of answer.** **(6)**

$x = 40$

$A = 975000 m^2$

$A = 9,50625''$

$x = 9,50625'' \times 40$

$x = 2,445377552^{12}$

$x = 2,45 m^2$

$x = 6,0025 m$

5. **Why do you think the two runways intersect each other and are not parallel?** **(2)**

[21]

If the two run ways were parallel the airoplain whouldn't have any means of breaking proporly which could cause accidents.

ACTIVITY 2 (1 period : ±40min)

INSTRUCTIONS:

- **ALL ANSWERS MUST BE WRITTEN IN THE SPACE PROVIDED ON THE ANSWER SHEET**
- **ALL ADDITIONAL NOTES & CALCULATIONS MUST BE DONE ON THIS PAPER**
- **CALCULATORS MAY BE USED**
- **WRITE CLEARLY & LEGIBLY**

The tour operator plans to replace his old boat by the end of 2003. The value of the old boat will be between R300 000 and R700 000. The new boat, as a capital investment, will cost R3,5million. This capital investment may be spread over a number of years. He gathered the following information for the year 2003. Use the information below to make an informed decision about whether or not he will be able to buy the boat.

A boat carries a maximum of 60 passengers to Robben Island. Adults pay R100,00 and children 4 – 17 years pay R50,00 per return trip. His operating costs per return trip are:

EXPENSES	COST IN RAND
Fuel	200
Boat operator (hired)	100
Landing Fees	50
Maintenance cost	250
Owner's salary	350
TOTAL OPERATING COST	950

INFORMATION:

1. He can ferry only on days when it does not rain.
2. In the peak season (1 September to 30 April):
   - he operated his boat at 70% capacity
   - he managed an average of 6 trips per day
   - the ratio of adults to children was 1:1
3. In the off-peak season (1 May to 31 August)
   - he operated his boat at 30% capacity
   - he managed an average of 3 trips per day
   - the ratio of adults to children was 2:3
4. Rainy days per month for 2002/2003:

SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG
10	7	5	4	5	2	6	10	15	20	25	26

Through out the year (Sep 02 – Aug 03) there was only 230 days which he could do business. 193 of those days were peak days/during peak season, 37 of those (230) days were off peak days.

Peak Season.

70 = capacity
60 = total number of passangers.
~~60 : 70 = 86% of people~~ 70 − 60 = 10.
If 10 people were on the boat per trip and 5 of them children and 5 adults;

5 x 50 = R250
5 x 100 = R500 ∴ R750 per trip will be made.
R750 x 6 = R4500.
R4500 x 193 = R868500.

Off-peak season.

30 = capacity
60 = total number of passangers.

60 − 30 = 30

If 30 people were on bord 15 of which are adults and 15 of which children;

15 x 50 = R750

15 x 100 = R1500 ∴ R2 250 per trip will be made.

R2250 x 3 = R6750.

R6750 x 37 = R249 750.

A total of R1118 250 will be made with in the year.
R1118250 − R1000 000 = R118 250 will be left after buying the boat.

Yes, the tour operator will be able to buy the boat.

KA: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

Area = ~~(LxB)~~ B x L (width x length)
~~(width)~~ = ~~(xy)~~ $x \times y$
= xy metres^2

1

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

$y : x$
$1 : 20$
∴ x is 20 times bigger than y
so $y : 20x$
so the area would be $20x \times y$
x = $20xy$ metres^2

4

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

One runway is $20xy$ metres
two identical runways = $20xy \times 20xy$
= $400x^2y^2$
e.g. A perpendicular runway would look like this: ⊥ so 2
∴ $20xy \times 20xy$
x = $400x^2y^2$ metres^2

/

4. **If the total area of the airfield is 97 500m², determine the width of the runway using the expression in (3). Motivate your choice of answer.** **(6)**

$97\,500m^2 = 20xy \times 20xy$

$= 400x^2y^2$

$=$

97 500

$39x^2$

$97x = A$

$\frac{97\,500}{400}$

$= 243{,}75$ $\frac{97\,500}{} = \frac{39x^2 \times w}{}$

5. **Why do you think the two runways intersect each other and are not parallel?** **(2)**

[2]

Because ~~(to)~~ people come from different directions to the ~~(airfield)~~ airfield so it makes it easier for them and also because they are travelling to ~~(a)~~ different places which are in different directions. Which makes it more convenient for them

KA: ACTIVITY 2

Cost of boat = R3 500 000
He already has about R700 000 from his old boat so he still needs about R2 800 000 (R3 500 000 – R700 000)

Peak season → ~~(6 trips)~~ 1 Sep – 30 Apr ~~(From (2) 1)~~ So it is 30 + 31 + 30 + 31 + 31 + 28 + 31 + 30 = 242 days

~~(6 trips per day at)~~ operated at 70% capacity → ~~(...)~~ boat carries maximum of 60 passangers

∴ $\frac{70}{100} \times \frac{60}{1} = 42$ passengers on the boat on each trip

242 days and 6 trips per day
∴ 242 X 6 = 1 452 total trips

42 passengers with a ratio of adults to children = 1 : 1
So there were 21 adults and 21 children (42 ÷ 2) on each trip

Adults pay R100 each
∴ R100 X 21
= R2 100 ← adults pay in total for each trip

Children pay R50 each
∴ R 50 X 21
= 1 050 ← children pay in total for each trip

1 452 X 2 100
=R3 049 200 ← Amount of money all adults pay in total for peak season

1 452 X 1050
=R1 524 600 ← Amount ~~of~~ ~~money~~ all adults pay in total for peak season

R 3 049 200 + R1 524 600
=R4 573 800 ← so he should ~~get~~ make a total of R4 573 800 in peak season
but....
It (~~rained~~) rained on 10+7+5+4+5+2+6+10 = 49 days

∴ 49 X 6 = 294 ← he missed out on 294 trips

∴ 294 X 2100
=R6 1 7 400 ← he misses out on R617 400 on adults
and...
294 X 1050
= (~~617 400~~) R308 700 ← he misses out on R308 700 from (~~adults~~) children

∴ R617 400 + R 308 700 =R926 100

R 4 573 800 − R 926 100
= R3 647 700 ← Amount of money he made in total for peak season

Off-peak season → 1 May − 31 August
So it is 31 + 30 + 31 + 31 = 123 days

but it rained for ~~(10)~~ 15 + 20 + 25 + 26
= 86 days

∴ He only ~~of~~ operated for
123 − 86 = 37 days

Boat operated at 30% capacity with maximum ~~(60)~~ of 60 passengers
∴ $\frac{30}{100} \times \frac{60}{1} = 18$ passengers on each boat trip.

86 days and 3 trips per day
= 86 × 3
= 258 trips in total

18 passengers with a ratio of adults to children = 2:3
2 : 3 = 2 + 3 = 5
$\frac{2}{5} \times 18 = 7$ adults
$\frac{3}{5} \times 18 = 11$ children
18

Children pay R50 each
∴ 11 X 50 = R550 ∠ children pay in total for each trip

Adults pay R100
∴ 7 X 100 = ~~(R520)~~ R700 ∠ Adults pay in total for each trip

258 X R550 = 141 900 ∠ Total amount of money children ~~(payed)~~ paid in off-peak season

258 X 700 = 180 600 ∠ Total amount of money adults pay in off-peak season

R 141 900 + R180 600
= R322 500 ∠ Total amount of money he makes in off-peak season.

peak season + off-peak season
3 ~~647~~ 700 + ~~3222~~ 500
= 3 970 200 ∠ Amount of money he makes for all his trips

1452 + 258
= 1710 ← Total return trips

Operating cost of each return trip is R950
∴ 1710 × 950
=R1 624 500 ← Total operating cost for ~~ea~~ all trips made

R 3970 200 − R 1624 500
= R 2345 700 ← Total money he has made

R700 000 + R 2345 700
= R3 045 700 ← Total money he has ~~for the new boat~~

~~R 3 500 000 − R 3 045 700~~
~~= R 454 300~~

~~So he his able to buy the new boat and will have about R 454 300 left after buying the boat~~

He needed R 2800 000 and made ~~=~~ R2 345 700

∴ R 2800 000 − R 2345 700
= R454 300 ← amount He has a shortfall of

Which means he has not made enough money. He still needs R454 300

~~R 3m~~ R 3045 700 - R3 500 000
= -R454 300

Proves that he ~~is~~ still needs R 454 300

LA: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is x metres and the length is y metres. (1)

$y \times x$ 1

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of x. (6)

[redacted]

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of x. Simplify your answer. (6)

4. If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

LA: ACTIVITY 2

3.5000000 - 700 000 = 343 000 00

PEAK SEASON

September

30 - 10 = 20 days

84 × 150 = 12600

12600 - 950 = 11650

11 650 × 6 = 69900

699 00 × 20 = 139 8000

October

31 - 7 = 24

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

699 00 × 24 = 1677600

November

30 - 5 = 25

84 × 150 = 12600

12600 - 950 = 11650

11 650 × 6 = 69900

699 00 × 25 = 1747500

December

31 - 4 = 27

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

69900 × 27 = 1 887 300

January

31 - 5 = 26

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

69900 × 26 = 18 17 400

February

28 - 2 = 26

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

69900 × 26 = 18 17 400

March

31 - 6 = 25

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

69900 × 25 = 1 74 7500

Aprill

30 - 10 = 20

84 × 150 = 12600

12600 - 950 = 11650

11650 × 6 = 69900

69900 × 20 = 139 8000

OFF PEAK SEASON

May

31 − 15 = 16
18 ÷ 2 = 9
9 × 3 = 27
27 × 150 = 4050
4050 × 3 = 12150
12150 × 16 = 194400

JUNE

30 − 20 = 10
18 ÷ 2 = 9
9 × 3 = 27
27 × 150 = 4050
4050 × 3 = 12150
12150 × 10 = 121500

July

31 − 25 = 6
18 ÷ 2 = 9
9 × 3 = 27
27 × 150 = 4050
4050 × 3 = 12150
12150 × 6 = 72900

August

31 − 26 = 5
18 ÷ 2 = 9
9 × 3 = 27
27 × 150 = 4050
4050 × 3 = 12150
12150 × 5 = 60750

Total PEAK : 13490700
OFF PEAK : 449550
grand Total : 13940250

He will not be able to buy the boat in a years time, because he earns to little.

LV: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is x metres and the length is y metres. (1)

runway = x m + y m

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of x. (6)

For every 1 metre there is 20 metres, so if x is equal to 2 then it will be 40 in real life.

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of x. Simplify your answer. (6)

runway = x × 2
= 40 metres.

4. If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

• 97 500 ÷ 2 will give us the width.
• = 48750 is the width of the airfield

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

They are to land to go straight thats why they ~~too~~ intrersept each other. And each runway ~~are go~~ is going to different placer.

LV: ACTIVITY 2

= R700000

= Adult= R 100 x 21= 2100 }
= Children=R50 x 21 = 1050 } on the boat

= The amount of days he goes 142 peak season

R203700 in peak season

off-peak season

Adults = 100 x 6 = R600 }
Children = 50 x 3 = 150 } on the boat

The amount of days he goes in off-peak season
= 37 days

R600 + R150 x 37 = R6150 in off peak season

R 35100 + R203700 = R209850 − 200 − 100 − 50 − 250 − 350 −
= 207950
+ 700000
R 907950

No, he wont beable to aford to new boat

SZ: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

X = 2 metres.
y = 4metres.
$x = y$
A = l x b
= 4 x 2
A = x x y

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

A = x x y
A = 2 x 4 ->
x = 8 ∴ 1 = 2
20 = 4 x 20
= 80
The real ratio would be 2:80 in reality.
x = 2
y = 80

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

x

4. If the total area of the airfield is 97 500m², determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

A = 97 500m²

W = 97500 / 2

W = 48 750 ->

The width of the runway would be 48,750 m² ->

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

~~Because a runway cant be parallel to one another, when you modelling you the people all around needs to see what you wearing. So you going to have to walk all around inorder for them to see you so it can't be parallel.~~

Because when the aeroplanes go and land they have to go in there parking places.

SZ: ACTIVITY 2

1 Expences = R1900

peak season = ~~30 April~~ 1 September = 30 day

November 30 → October = 30 days

December = 31 days × 6

January = 31 days

Febuary - 28 days

March = 31 days

April = 30 days

2 Days of ~~peek~~ season x trips per day

= 1458 trips during peek season.

* 35 Children and 35 ~~Children~~ adults

* 35 children × R 50,00
= R1750 →

* 35 adults × R100,00
= R 3500 →

Rainy days

213 days - 49 days that it rained

* 182 days in which he could visit Robben Island on peak season.

Off peek season May = 31, June = 30, July = 31, August 31 × 3 trips per day

= 123 trips during off peek season.

* R trips off peek - rainy days

* ∴ 123 - ~~56~~ 86

= 37 days in which he could go on the boat to Robben island.

2 So the amount he made at on peek is : 182 days X 6
= 1092 trips on all days

35 children X R50,00
= R1750 X 182 days
= R318 500 made by children alone

35 adults X R100 00
= R 3500 X R182 days
= R637000

* R637000 + R 318 500
= R955500 amount of peak season alone.

off peek

10 adults X R100,00
= R1000

20 children X R50,00
= R1000

R 2000 X 37 days
= R74000 ->

On peek + off peak
R955500 + R 74000
= R 1029500 ->

= R 1029500 + R 700000 (in which he could get for his boat "Max"
= R 1729500

- My decision is that he will not be able to buy the boat it takes him long to get that amount of money and by the time he has the right amount it won't be that price anymore.

ZA: ACTIVITY 1

Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.

1. Write an expression for the area of one runway if the width is *x* metres and the length is *y* metres. (1)

Area = l x b

∴ A = y x x

2. If the width and the length of the runway of an airfield in (1) above, is in the ratio 1:20, express your answer in (1) in terms of *x*. (6)

y = 1:20
x = 1:20

A = x(1 : 20) x y(1 : 20)
= x : 20x x y : 20y
xy : 400xy

$\frac{x}{1} = \frac{1}{x}$

3. If the airfield has two identical, and perpendicular runways, write down an expression for the total area in terms of *x*. Simplify your answer. (6)

/

4. If the total area of the airfield is 97 500m^2, determine the width of the runway using the expression in (3). Motivate your choice of answer. (6)

5. Why do you think the two runways intersect each other and are not parallel? (2)

[21]

They are perpendicular, meaning that if you multiply both gradients you get a total of -1.

1. Operating ~~Income~~ Expenses (per trip) = R950

2. Ratio = 1:1 $\frac{70}{100} \times \frac{60}{1} = 42$ people

∴ Adults = 21 Children = 21

~~42~~ × 6 = 252 (252 ÷ 2 = 126)

Children = 126 × 50 = R6 300

Adults = 126 × 100 = R12 600

Peak season = (30 - 10 = (20); 31 - 7 = (24); 30 - 5 = (25); 31 - 4 = (27);
31 - 5 = (26); 28 - 2 = (26); 31 - 6 = (25); 30 - 10 = (20))

= 20 + 24 + 25 + ~~27~~ + 26 + 26 + 25 + 20

= 193 days

6 300 + 12 600 = R18 900 per day

R18 900 × 193 = R3 647 700

3. Ratio = 2 : 3 $\frac{30}{100} \times \frac{60}{1}$ = 5 people 2 : 3 = 5

∴ Adults = $\frac{2}{5} \times \frac{5}{1} = 2$ Children : $\frac{3}{5} \times \frac{5}{1} = 3$

Children = 3 × 50 = R1 500

Adults = 2 × 100 = R200

Off peak season = (31 - 15 = (16); 30 - 20 = (10); 31 - 25 = (6); 31 - 26 = (5))

= 16 + 10 + 6 + 5 = 37 days

1 500 + 200 = 1 700 per day

1700 × 37 = R62 900

R3 647 700 + R62 900 = R3 710 600

∴ Money available = R3 710 600 + R700 000 = R4 410 600

Total operating cost per day = 950

Peak season = 950 x 193 = R183 350

Off peak season = 950 x 37 = R35 150

Total operating costs = R35 150 + R183 350

= R218 500

∴ Money available in total = R4 410 600 − R218 500

= R4 192 100

∴ He will be able to buy the boat and still have a profit of (R4 192 100 − R3 500 000) R692 100 in the business.